Instructor's Solutions Manual

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Algebra and Trigonometry

Fourth Edition

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Publishing as Prentice Hall, Upper Saddle River, NJ 07458.

ISBN-13: 978-0-321-57543-2
ISBN-10: 0-321-57543-1

1 2 3 4 5 6 BRR 12 11 10 09 08

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Algebra and Trigonometry 4e

Chapter P
Fundamental Concepts of Algebra

Section P.1

Check Point Exercises

1. $8+6(x-3)^2 = 8+6(13-3)^2$
$= 8+6(10)^2$
$= 8+6(100)$
$= 8+600$
$= 608$

2. Since 2010 is 10 years after 2000, substitute 10 for x.
$T = 17x^2 + 261x + 3257$
$= 17(10)^2 + 261(10) + 3257$
$= 7567$
If trends continue, the tuition and fees will be $7567

3. The elements common to {3, 4, 5, 6, 7} and {3, 7, 8, 9} are 3 and 7.
$\{3,4,5,6,7\} \cap \{3,7,8,9\} = \{3,7\}$

4. The union is the set containing all the elements of either set.
$\{3,4,5,6,7\} \cup \{3,7,8,9\} = \{3,4,5,6,7,8,9\}$

5. $\left\{-9,\ -1.3,\ 0,\ 0.\bar{3},\ \frac{\pi}{2},\ \sqrt{9},\ \sqrt{10}\right\}$

a. Natural numbers: $\sqrt{9}$ because $\sqrt{9} = 3$

b. Whole numbers: $0,\ \sqrt{9}$

c. Integers: $-9,\ 0,\ \sqrt{9}$

d. Rational numbers: $-9,\ -1.3,\ 0,\ 0.\bar{3},\ \sqrt{9}$

e. Irrational numbers: $\frac{\pi}{2},\ \sqrt{10}$

f. Real numbers:
$\left\{-9,\ -1.3,\ 0,\ 0.\bar{3},\ \frac{\pi}{2},\ \sqrt{9},\ \sqrt{10}\right\}$

6. a. $\left|1-\sqrt{2}\right|$
Because $\sqrt{2} \approx 1.4$, the number inside the absolute value bars is negative. The absolute value of x when $x < 0$ is $-x$. Thus,
$\left|1-\sqrt{2}\right| = -\left(1-\sqrt{2}\right) = \sqrt{2}-1$

b. $|\pi - 3|$
Because $\pi \approx 3.14$, the number inside the absolute value bars is positive. The absolute value of a positive number is the number itself. Thus,
$|\pi - 3| = \pi - 3.$

c. $\frac{|x|}{x}$
Because $x > 0$, $|x| = x$.
Thus, $\frac{|x|}{x} = \frac{x}{x} = 1$

7. $|-4-(5)| = |-9| = 9$
The distance between –4 and 5 is 9.

8. $7(4x^2+3x)+2(5x^2+x)$
$= 7(4x^2+3x)+2(5x^2+x)$
$= 28x^2+21x+10x^2+2x$
$= 38x^2+23x$

9. $6+4[7-(x-2)]$
$= 6+4[7-x+2)]$
$= 6+4[9-x]$
$= 6+36-4x$
$= 42-4x$

Exercise Set P.1

1. $7+5(10) = 7+50 = 57$

2. $8+6(5) = 8+30 = 38$

3. $6(3)-8 = 18-8 = 10$

4. $8(3)-4 = 24-4 = 20$

5. $8^2 + 3(8) = 64 + 24 = 88$

6. $6^2 + 5(6) = 36 + 30 = 66$

7. $7^2 - 6(7) + 3 = 49 - 42 + 3 = 7 + 3 = 10$

8. $8^2 - 7(8) + 4 = 64 - 56 + 4 = 8 + 4 = 12$

9. $4 + 5(9 - 7)^3 = 4 + 5(2)^3$
$= 4 + 5(8) = 4 + 40 = 44$

10. $6 + 5(8 - 6)^3 = 6 + 5(2)^3$
$= 6 + 5(8)$
$= 6 + 40 = 46$

11. $8^2 - 3(8 - 2) = 64 - 3(6)$
$= 64 - 18 = 46$

12. $8^2 - 4(8 - 3) = 64 - 4(5) = 64 - 20 = 44$

13. $\frac{5(x+2)}{2x-14} = \frac{5(10+2)}{2(10)-14}$
$= \frac{5(12)}{6}$
$= 5 \cdot 2$
$= 10$

14. $\frac{7(x-3)}{2x-16} = \frac{7(9-3)}{2(9)-16} = \frac{7(6)}{2} = 7 \cdot 3 = 21$

15. $\frac{2x+3y}{x+1}; x = -2, y = 4$
$= \frac{2(-2)+3(4)}{-2+1} = \frac{-4+12}{-1} = \frac{8}{-1} = -8$

16. $\frac{2x+y}{xy-2x}$; $x = -2$ and $y = 4$
$\frac{2(-2)+4}{(-2)(4)-2(-2)} = \frac{-4+4}{-8+4} = \frac{0}{4} = 0$

17. $C = \frac{5}{9}(50-32) = \frac{5}{9}(18) = 10$
10°C is equivalent to 50°F.

18. $C = \frac{5}{9}(F-32) = \frac{5}{9}(86-32) = \frac{5}{9}(54) = 30$
30°C is equivalent to 86°F.

19. $h = 4 + 60t - 16t^2 = 4 + 60(2) - 16(2)^2$
$= 4 + 120 - 16(4) = 4 + 120 - 64$
$= 124 - 64 = 60$
Two seconds after it is kicked, the ball's height is 60 feet.

20. $h = 4 + 60t - 16t^2$
$= 4 + 60(3) - 16(3)^2$
$= 4 + 180 - 16(9)$
$= 4 + 180 - 144$
$= 184 - 144 = 40$
Three seconds after it is kicked, the ball's height is 40 feet.

21. $\{1, 2, 3, 4\} \cap \{2, 4, 5\} = \{2, 4\}$

22. $\{1, 3, 7\} \cap \{2, 3, 8\} = \{3\}$

23. $\{s, e, t\} \cap \{t, e, s\} = \{s, e, t\}$

24. $\{r, e, a, l\} \cap \{l, e, a, r\} = \{r, e, a, l\}$

25. $\{1, 3, 5, 7\} \cap \{2, 4, 6, 8, 10\} = \{\ \ \}$
The empty set is also denoted by $\varnothing$.

26. $\{1, 3, 5, 7\} \cap \{-5, -3, -1\} = \{\ \ \}$ or $\varnothing$

27. $\{a, b, c, d\} \cap \varnothing = \varnothing$

28. $\{w, y, z\} \cap \varnothing = \varnothing$

29. $\{1, 2, 3, 4\} \cup \{2, 4, 5\} = \{1, 2, 3, 4, 5\}$

30. $\{1, 3, 7, 8\} \cup \{2, 3, 8\} = \{1, 2, 3, 7, 8\}$

31. $\{1, 3, 5, 7\} \cup \{2, 4, 6, 8, 10\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 10\}$

32. $\{0, 1, 3, 5\} \cup \{2, 4, 6\} = \{0, 1, 2, 3, 4, 5, 6\}$

33. $\{a, e, i, o, u\} \cup \varnothing = \{a, e, i, o, u\}$

34. $\{e, m, p, t, y\} \cup \varnothing = \{e, m, p, t, y\}$

35. **a.** $\sqrt{100}$

b. $0, \sqrt{100}$

c. $-9, 0, \sqrt{100}$

d. $-9, -\frac{4}{5}, 0, 0.25, 9.2, \sqrt{100}$

e. $\sqrt{3}$

f. $-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}$

36. a. $\sqrt{49}$

b. $0, \sqrt{49}$

c. $-7, 0, \sqrt{49}$

d. $-7, -0.\overline{6}, 0, \sqrt{49}$

e. $\sqrt{50}$

f. $-7, -0.\overline{6}, 0, \sqrt{49}, \sqrt{50}$

37. a. $\sqrt{64}$

b. $0, \sqrt{64}$

c. $-11, 0, \sqrt{64}$

d. $-11, -\frac{5}{6}, 0, 0.75, \sqrt{64}$

e. $\sqrt{5}, \pi$

f. $-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}$

38. a. $\sqrt{4}$

b. $0, \sqrt{4}$

c. $-5, 0, \sqrt{4}$

d. $-5, -0.\overline{3}, 0, \sqrt{4}$

e. $\sqrt{2}$

f. $-5, -0.\overline{3}, 0, \sqrt{2}, \sqrt{4}$

39. 0

40. Answers may vary. An example is $\frac{1}{2}$.

41. Answers may vary. An example is 2.

42. Answers may vary. An example is -2.

43. true; –13 is to the left of –2 on the number line.

44. false; –6 is to the left of 2 on the number line.

45. true; 4 is to the right of –7 on the number line.

46. true; –13 is to the left of –5 on the number line.

47. true; $-\pi = -\pi$

48. true; –3 is to the right of –13 on the number line.

49. true; 0 is to the right of –6 on the number line.

50. true; 0 is to the right of –13 on the number line.

51. $|300| = 300$

52. $|-203| = 203$

53. $|12-\pi| = 12-\pi$

54. $|7-\pi| = 7-\pi$

55. $|\sqrt{2}-5| = 5-\sqrt{2}$

56. $|\sqrt{5}-13| = 13-\sqrt{5}$

57. $\frac{-3}{|-3|} = \frac{-3}{3} = -1$

58. $\frac{-7}{|-7|} = \frac{-7}{7} = -1$

59. $||-3|-|-7|| = |3-7| = |-4| = 4$

60. $||-5|-|-13|| = |5-13| = |-8| = 8$

61. $|x+y| = |2+(-5)| = |-3| = 3$

62. $|x-y| = |2-(-5)| = |7| = 7$

63. $|x|+|y| = |2|+|-5| = 2+5 = 7$

64. $|x|-|y| = |2|-|-5| = 2-5 = -3$

65. $\frac{y}{|y|} = \frac{-5}{|-5|} = \frac{-5}{5} = -1$

66. $\frac{|x|}{x} + \frac{|y|}{y} = \frac{|2|}{2} + \frac{|-5|}{-5} = \frac{2}{2} + \frac{5}{-5} = 1 + (-1) = 0$

67. The distance is $|2-17| = |-15| = 15$.

68. The distance is $|4-15| = |-11| = 11$.

69. The distance is $|-2-5| = |-7| = 7$.

70. The distance is $|-6-8| = |-14| = 14$.

71. The distance is $|-19-(-4)| = |-19+4| = |-15| = 15$.

72. The distance is $|-26-(-3)| = |-26+3| = |-23| = 23$.

73. The distance is
$|-3.6-(-1.4)| = |-3.6+1.4| = |-2.2| = 2.2$.

74. The distance is
$|-5.4-(-1.2)| = |-5.4+1.2| = |-4.2| = 4.2$.

75. 6 + (–4) = (–4) + 6;
commutative property of addition

76. $11\cdot(7+4) = 11\cdot 7 + 11\cdot 4$;
distributive property of multiplication over addition

77. 6 + (2 + 7) = (6 + 2) + 7;
associative property of addition

78. $6\cdot(2\cdot 3) = 6\cdot(3\cdot 2)$;
commutative property of multiplication

79. (2 + 3) + (4 + 5) = (4 + 5) + (2 + 3);
commutative property of addition

80. $7\cdot(11\cdot 8) = (11\cdot 8)\cdot 7$;
commutative property of multiplication

81. 2 (–8 + 6) = –16 + 12;
distributive property of multiplication over addition

82. $-8(3+11) = -24 + (-88)$;
distributive property of multiplication over addition

83. $\frac{1}{x+3}(x+3) = 1; x \neq -3$,
inverse property of multiplication

84. $(x+4) + [-(x+4)] = 0$;
inverse property of addition

85.
$$\begin{aligned} 5(3x+4)-4 &= 5\cdot 3x + 5\cdot 4 - 4 \\ &= 15x + 20 - 4 \\ &= 15x + 16 \end{aligned}$$

86.
$$\begin{aligned} 2(5x+4)-3 &= 2\cdot 5x + 2\cdot 4 - 3 \\ &= 10x + 8 - 3 \\ &= 10x + 5 \end{aligned}$$

87.
$$\begin{aligned} 5(3x-2)+12x &= 5\cdot 3x - 5\cdot 2 + 12x \\ &= 15x - 10 + 12x \\ &= 27x - 10 \end{aligned}$$

88.
$$\begin{aligned} 2(5x-1)+14x &= 2\cdot 5x - 2\cdot 1 + 14x \\ &= 10x - 2 + 14x \\ &= 24x - 2 \end{aligned}$$

89.
$$\begin{aligned} &7(3y-5)+2(4y+3) \\ &= 7\cdot 3y - 7\cdot 5 + 2\cdot 4y + 2\cdot 3 \\ &= 21y - 35 + 8y + 6 \\ &= 29y - 29 \end{aligned}$$

90.
$$\begin{aligned} &4(2y-6)+3(5y+10) \\ &= 4\cdot 2y - 4\cdot 6 + 3\cdot 5y + 3\cdot 10 \\ &= 8y - 24 + 15y + 30 \\ &= 23y + 6 \end{aligned}$$

91.
$$\begin{aligned} 5(3y-2)-(7y+2) &= 15y - 10 - 7y - 2 \\ &= 8y - 12 \end{aligned}$$

92.
$$\begin{aligned} 4(5y-3)-(6y+3) &= 20y - 12 - 6y - 3 \\ &= 14y - 15 \end{aligned}$$

93.
$$\begin{aligned} 7-4[3-(4y-5)] &= 7-4[3-4y+5] \\ &= 7-4[8-4y] \\ &= 7-32+16y \\ &= 16y-25 \end{aligned}$$

94.
$$\begin{aligned} 6-5[8-(2y-4)] &= 6-5[8-2y+4] \\ &= 6-5[12-2y] \\ &= 6-60+10y \\ &= 10y-54 \end{aligned}$$

95. $18x^2 + 4 - \left[6\left(x^2 - 2\right) + 5\right]$
$= 18x^2 + 4 - \left[6x^2 - 12 + 5\right]$
$= 18x^2 + 4 - \left[6x^2 - 7\right]$
$= 18x^2 + 4 - 6x^2 + 7$
$= 18x^2 - 6x^2 + 4 + 7$
$= \left(18 - 6\right)x^2 + 11 = 12x^2 + 11$

96. $14x^2 + 5 - \left[7\left(x^2 - 2\right) + 4\right]$
$= 14x^2 + 5 - \left[7x^2 - 14 + 4\right]$
$= 14x^2 + 5 - \left[7x^2 - 10\right]$
$= 14x^2 + 5 - 7x^2 + 10$
$= 14x^2 - 7x^2 + 5 + 10$
$= \left(14 - 7\right)x^2 + 15$
$= 7x^2 + 15$

97. $-(-14x) = 14x$

98. $-\left(-17y\right) = 17y$

99. $-(2x - 3y - 6) = -2x + 3y + 6$

100. $-\left(5x - 13y - 1\right) = -5x + 13y + 1$

101. $\frac{1}{3}(3x) + \left[(4y) + (-4y)\right] = x + 0$
$= x$

102. $\frac{1}{2}\left(2y\right) + \left[\left(-7x\right) + 7x\right] = y + 0 = y$

103. $|-6| \; \square \; |-3|$
$6 \; \square \; 3$
$6 > 3$
Since $6 > 3$, $|-6| > |-3|$.

104. $|-20| \; \square \; |-50|$
$20 \; \square \; 50$
$20 < 50$
Since $20 < 50$, $|-20| < |-50|$.

105. $\left|\frac{3}{5}\right| \; \square \; |-0.6|$
$|0.6| \; \square \; |-0.6|$
$0.6 \; \square \; 0.6$
$0.6 = 0.6$
Since $0.6 = 0.6$, $\left|\frac{3}{5}\right| = |-0.6|$.

106. $\left|\frac{5}{2}\right| \; \square \; |-2.5|$
$|2.5| \; \square \; |-2.5|$
$2.5 \; \square \; 2.5$
$2.5 = 2.5$
Since $2.5 = 2.5$, $\left|\frac{5}{2}\right| = |-2.5|$.

107. $\frac{30}{40} - \frac{3}{4} \; \square \; \frac{14}{15} \cdot \frac{15}{14}$
$\frac{30}{40} - \frac{30}{40} \; \square \; \frac{\cancel{14}}{\cancel{15}} \cdot \frac{\cancel{15}}{\cancel{14}}$
$0 \; \square \; 1$
$0 < 1$
Since $0 < 1$, $\frac{30}{40} - \frac{3}{4} < \frac{14}{15} \cdot \frac{15}{14}$.

108. $\frac{17}{18} \cdot \frac{18}{17} \; \square \; \frac{50}{60} - \frac{5}{6}$
$\frac{\cancel{17}}{\cancel{18}} \cdot \frac{\cancel{18}}{\cancel{17}} \; \square \; \frac{50}{60} - \frac{50}{60}$
$1 \; \square \; 0$
$1 > 0$
Since $1 > 0$, $\frac{17}{18} \cdot \frac{18}{17} > \frac{50}{60} - \frac{5}{6}$.

109. $\frac{8}{13} \div \frac{8}{13} \, \square \, |-1|$

$\frac{8}{13} \cdot \frac{13}{8} \, \square \, 1$

$1 \, \square \, 1$

$1 = 1$

Since $1 = 1$, $\frac{8}{13} \div \frac{8}{13} = |-1|$.

110. $|-2| \, \square \, \frac{4}{17} \div \frac{4}{17}$

$2 \, \square \, \frac{4}{17} \cdot \frac{17}{4}$

$2 \, \square \, 1$

$2 > 1$

Since $2 > 1$, $|-2| > \frac{4}{17} \div \frac{4}{17}$.

111.
$$\begin{aligned} 8^2 - 16 \div 2^2 \cdot 4 - 3 &= 64 - 16 \div 4 \cdot 4 - 3 \\ &= 64 - 4 \cdot 4 - 3 \\ &= 64 - 16 - 3 \\ &= 48 - 3 \\ &= 45 \end{aligned}$$

112.
$$\begin{aligned} 10^2 - 100 \div 5^2 \cdot 2 - 3 &= 100 - 100 \div 25 \cdot 2 - 3 \\ &= 100 - 4 \cdot 2 - 3 \\ &= 100 - 8 - 3 \\ &= 92 - 3 \\ &= 89 \end{aligned}$$

113.
$$\begin{aligned} \frac{5 \cdot 2 - 3^2}{[3^2 - (-2)]^2} &= \frac{5 \cdot 2 - 9}{[9 - (-2)]^2} \\ &= \frac{10 - 9}{[9 + 2]^2} \\ &= \frac{10 - 9}{11^2} \\ &= \frac{1}{121} \end{aligned}$$

114.
$$\begin{aligned} \frac{10 \div 2 + 3 \cdot 4}{(12 - 3 \cdot 2)^2} &= \frac{5 + 12}{(12 - 6)^2} \\ &= \frac{17}{6^2} \\ &= \frac{17}{36} \end{aligned}$$

115.
$$\begin{aligned} 8 - 3[-2(2 - 5) - 4(8 - 6)] &= 8 - 3[-2(-3) - 4(2)] \\ &= 8 - 3[6 - 8] \\ &= 8 - 3[-2] \\ &= 8 + 6 \\ &= 14 \end{aligned}$$

116.
$$\begin{aligned} 8 - 3[-2(5 - 7) - 5(4 - 2)] &= 8 - 3[-2(-2) - 5(2)] \\ &= 8 - 3[4 - 10] \\ &= 8 - 3[-6] \\ &= 8 + 18 \\ &= 26 \end{aligned}$$

117.
$$\begin{aligned} \frac{2(-2) - 4(-3)}{5 - 8} &= \frac{-4 + 12}{-3} \\ &= \frac{8}{-3} \\ &= -\frac{8}{3} \end{aligned}$$

118.
$$\begin{aligned} \frac{6(-4) - 5(-3)}{9 - 10} &= \frac{-24 + 15}{-1} \\ &= \frac{-9}{-1} \\ &= 9 \end{aligned}$$

119.
$$\begin{aligned} \frac{(5 - 6)^2 - 2|3 - 7|}{89 - 3 \cdot 5^2} &= \frac{(-1)^2 - 2|-4|}{89 - 3 \cdot 25} \\ &= \frac{1 - 2(4)}{89 - 75} \\ &= \frac{1 - 8}{14} \\ &= \frac{-7}{14} \\ &= -\frac{1}{2} \end{aligned}$$

120.
$$\begin{aligned} \frac{12 \div 3 \cdot 5|2^2 + 3^2|}{7 + 3 - 6^2} &= \frac{12 \div 3 \cdot 5|4 + 9|}{7 + 3 - 36} \\ &= \frac{4 \cdot 5|13|}{10 - 36} \\ &= \frac{20(13)}{-26} \\ &= \frac{260}{-26} \\ &= -10 \end{aligned}$$

121. $x - (x + 4) = x - x - 4 = -4$

122. $x-(8-x)=x-8+x=2x-8$

123. $6(-5x)=-30x$

124. $10(-4x)=-40x$

125. $5x-2x=3x$

126. $6x-(-2x)=6x+2x=8x$

127. $8x-(3x+6)=8x-3x-6=5x-6$

128. $8-3(x+6)=8-3x-18=-3x-10$

129. **a.** $H=\frac{7}{10}(220-a)$

$H=\frac{7}{10}(220-20)$

$=\frac{7}{10}(200)$

$=140$

The lower limit of the heart rate for a 20-year-old with this exercise goal is 140 beats per minute.

b. $H=\frac{4}{5}(220-a)$

$H=\frac{4}{5}(220-20)$

$=\frac{4}{5}(200)$

$=160$

The upper limit of the heart rate for a 20-year-old with this exercise goal is 160 beats per minute.

130. **a.** $H=\frac{1}{2}(220-a)$

$H=\frac{1}{2}(220-30)$

$=\frac{1}{2}(190)$

$=95$

The lower limit of the heart rate for a 30-year-old with this exercise goal is 95 beats per minute.

b. $H=\frac{3}{5}(220-a)$

$H=\frac{3}{5}(220-30)$

$=\frac{3}{5}(190)$

$=114$

The upper limit of the heart rate for a 30-year-old with this exercise goal is 114 beats per minute.

131. **a.** $T=15,395+988x-2x^2$

$=15,395+988(7)-2(7)^2$

$=22,213$

The formula estimates the cost to have been $22,213 in 2007.

b. This underestimates the value in the graph by $5.

c. $T=15,395+988x-2x^2$

$=15,395+988(10)-2(10)^2$

$=25,075$

The formula projects the cost to be $25,075 in 2010.

132. **a.** $T=15,395+988x-2x^2$

$=15,395+988(6)-2(6)^2$

$=21,251$

The formula estimates the cost to have been $21,251 in 2006.

b. This underestimates the value in the graph by $16.

c. $T=15,395+988x-2x^2$

$=15,395+988(12)-2(12)^2$

$=26,963$

The formula projects the cost to be $26,963 in 2012.

133. **a.** $0.05x+0.12(10,000-x)$

$=0.05x+1200-0.12x$

$=1200-0.07x$

b. $1200-0.07x=1200-0.07(6000)$

$=\$780$

134. **a.** $0.06t + 0.5(50 - t) = 0.06t + 25 - 0.5t$
$= 25 - 0.44t$

b. $0.06(20) + 0.5(50 - 20)$
$= 1.2 + 0.5(30)$
$= 1.2 + 15$
$= 16.2$ miles

135. – 144. Answers may vary.

145. does not make sense; Explanations will vary. Sample explanation: Models do not always accurately predict future values.

146. does not make sense; Explanations will vary. Sample explanation: To use the model, substitute 0 for *x*.

147. makes sense

148. does not make sense; Explanations will vary. Sample explanation: The commutative property changes order and the associative property changes groupings.

149. false; Changes to make the statement true will vary. A sample change is: Some rational numbers are not integers.

150. false; Changes to make the statement true will vary. A sample change is: All whole numbers are integers

151. true

152. false; Changes to make the statement true will vary. A sample change is: Some irrational numbers are negative.

153. false; Changes to make the statement true will vary. A sample change is: The term *x* has a coefficient of 1.

154. false; Changes to make the statement true will vary. A sample change is:
$5 + 3(x - 4) = 5 + 3x - 12 = 3x - 7.$

155. false; Changes to make the statement true will vary. A sample change is: $-x - x = -2x.$

156. true

157. $\sqrt{2} \approx 1.4$
$1.4 < 1.5$
$\sqrt{2} < 1.5$

158. $-\pi > -3.5$

159. $-\dfrac{3.14}{2} = -1.57$
$-\dfrac{\pi}{2} \approx -1.571$
$-1.57 > -1.571$
$-\dfrac{3.14}{2} > -\dfrac{\pi}{2}$

160. **a.** $b^4 \cdot b^3 = (b \cdot b \cdot b \cdot b)(b \cdot b \cdot b) = b^7$

b. $b^5 \cdot b^5 = (b \cdot b \cdot b \cdot b \cdot b)(b \cdot b \cdot b \cdot b \cdot b) = b^{10}$

c. add the exponents

161. **a.** $\dfrac{b^7}{b^3} = \dfrac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b \cdot b} = b^4$

b. $\dfrac{b^8}{b^2} = \dfrac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b} = b^6$

c. subtract the exponents

162. $6.2 \times 10^3 = 6.2 \times 10 \times 10 \times 10 = 6200$
It moves the decimal point 3 places to the right.

Section P.2

Check Point Exercises

1. **a.** $3^3 3^2 = 3^{3+2} = 3^5$ or 243

b. $\left(4x^3y^4\right)\left(10x^2y^6\right) = 4 \cdot 10 \cdot x^3 \cdot x^2 \cdot y^4 \cdot y^6$
$= 40x^{3+2} \cdot y^{4+6}$
$= 40x^5 \cdot y^{10}$

2. **a.** $\dfrac{(-3)^6}{(-3)^3} = (-3)^3 = -27$

b. $\dfrac{27x^{14}y^8}{3x^3y^5} = \dfrac{27}{3} \cdot \dfrac{x^{14}}{x^3} \cdot \dfrac{y^8}{y^5} = 9x^{14-3}y^{8-5} = 9x^{11}y^3$

3. **a.** $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

b. $(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{-27} = -\frac{1}{27}$

c. $\frac{1}{4^{-2}} = \frac{1}{\frac{1}{4^2}} = 1 \cdot \frac{4^2}{1} = 4^2 = 16$

d. $3x^{-6}y^4 = 3 \cdot \frac{1}{x^6} \cdot y^4 = \frac{3y^4}{x^6}$

4. **a.** $\left(3^3\right)^2 = 3^{3\cdot 2} = 3^6$ or 729

b. $\left(y^7\right)^{-2} = y^{7(-2)} = y^{-14} = \frac{1}{y^{14}}$

c. $\left(b^{-3}\right)^{-4} = b^{-3(-4)} = b^{12}$

5. $(-4x)^3 = (-4)^3(x)^3 = -64x^3$

6. **a.** $\left(-\frac{2}{y}\right)^5 = \frac{(-2)^5}{y^5} = \frac{-32}{y^5}$

b. $\left(\frac{x^5}{3}\right)^3 = \frac{\left(x^5\right)^3}{3^3} = \frac{x^{15}}{27}$

7. **a.** $\left(2x^3y^6\right)^4 = (2)^4\left(x^3\right)^4\left(y^6\right)^4 = 16x^{12}y^{24}$

b.
$$\left(-6x^2y^5\right)\left(3xy^3\right) = (-6)\cdot 3\cdot x^2\cdot x\cdot y^5\cdot y^3$$
$$= -18x^3y^8$$

c.
$$\frac{100x^{12}y^2}{20x^{16}y^{-4}} = \left(\frac{100}{20}\right)\left(\frac{x^{12}}{x^{16}}\right)\left(\frac{y^2}{y^{-4}}\right)$$
$$= 5x^{12-16}y^{2-(-4)}$$
$$= 5x^{-4}y^6$$
$$= \frac{5y^6}{x^4}$$

d.
$$\left(\frac{5x}{y^4}\right)^{-2} = \frac{(5)^{-2}(x)^{-2}}{\left(y^4\right)^{-2}}$$
$$= \frac{(5)^{-2}(x)^{-2}}{\left(y^4\right)^{-2}}$$
$$= \frac{5^{-2}x^{-2}}{y^{-8}}$$
$$= \frac{y^8}{5^2x^2}$$
$$= \frac{y^8}{25x^2}$$

8. **a.** $-2.6\times 10^9 = -2,600,000,000$

b. $3.017\times 10^{-6} = 0.000003017$

9. **a.** $5,210,000,000 = 5.21\times 10^9$

b. $-0.00000006893 = -6.893\times 10^{-8}$

10.
$$410\times 10^7 = \left(4.1\times 10^2\right)\times 10^7$$
$$= 4.1\times\left(10^2\times 10^7\right)$$
$$= 4.1\times 10^9$$

11 **a.**
$$\left(7.1\times 10^5\right)\left(5\times 10^{-7}\right)$$
$$= 7.1\cdot 5\times 10^5\cdot 10^{-7}$$
$$= 35.5\times 10^{-2}$$
$$= \left(3.55\times 10^1\right)\times 10^{-2}$$
$$= 3.55\times\left(10^1\times 10^{-2}\right)$$
$$= 3.55\times 10^{-1}$$

b.
$$\frac{1.2\times 10^6}{3\times 10^{-3}} = \frac{1.2}{3}\cdot\frac{10^6}{10^{-3}}$$
$$= 0.4\times 10^{6-(-3)}$$
$$= 0.4\times 10^9$$
$$= 4\times 10^8$$

12.
$$\frac{13\times 10^9}{5.1\times 10^6} = \frac{13}{5.1}\cdot\frac{10^9}{10^6}$$
$$\approx 2.5\cdot 10^3$$
$$\approx 2500$$

The average Pell grant was $2500 in 2006.

Exercise Set P.2

1. $5^2 \cdot 2 = (5 \cdot 5) \cdot 2 = 25 \cdot 2 = 50$

2. $6^2 \cdot 2 = (6 \cdot 6) \cdot 2 = 36 \cdot 2 = 72$

3. $(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = 64$

4. $(-2)^4 = (-2)(-2)(-2)(-2) = 16$

5. $-2^6 = -2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = -64$

6. $-2^4 = -2 \cdot 2 \cdot 2 \cdot 2 = -16$

7. $(-3)^0 = 1$

8. $(-9)^0 = 1$

9. $-3^0 = -1$

10. $-9^0 = -1$

11. $4^{-3} = \frac{1}{4^3} = \frac{1}{4 \cdot 4 \cdot 4} = \frac{1}{64}$

12. $2^{-6} = \frac{1}{2^6} = \frac{1}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \frac{1}{64}$

13. $2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

14. $3^3 \cdot 3^2 = 3^{3+2} = 3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$

15. $(2^2)^3 = 2^{2 \cdot 3} = 2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$

16. $(3^3)^2 = 3^{3 \cdot 2} = 3^6 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 729$

17. $\frac{2^8}{2^4} = 2^{8-4} = 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

18. $\frac{3^8}{3^4} = 3^{8-4} = 3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

19. $3^{-3} \cdot 3 = 3^{-3+1} = 3^{-2} = \frac{1}{3^2} = \frac{1}{3 \cdot 3} = \frac{1}{9}$

20. $2^{-3} \cdot 2 = 2^{-3+1} = 2^{-2} = \frac{1}{2^2} = \frac{1}{2 \cdot 2} = \frac{1}{4}$

21. $\frac{2^3}{2^7} = 2^{3-7} = 2^{-4} = \frac{1}{2^4} = \frac{1}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{1}{16}$

22. $\frac{3^4}{3^7} = 3^{4-7} = 3^{-3} = \frac{1}{3^3} = \frac{1}{3 \cdot 3 \cdot 3} = \frac{1}{27}$

23. $x^{-2}y = \frac{1}{x^2} \cdot y = \frac{y}{x^2}$

24. $xy^{-3} = x \cdot \frac{1}{y^3} = \frac{x}{y^3}$

25. $x^0 y^5 = 1 \cdot y^5 = y^5$

26. $x^7 \cdot y^0 = x^7 \cdot 1 = x^7$

27. $x^3 \cdot x^7 = x^{3+7} = x^{10}$

28. $x^{11} \cdot x^5 = x^{11+5} = x^{16}$

29. $x^{-5} \cdot x^{10} = x^{-5+10} = x^5$

30. $x^{-6} \cdot x^{12} = x^{-6+12} = x^6$

31. $(x^3)^7 = x^{3 \cdot 7} = x^{21}$

32. $(x^{11})^5 = x^{11 \cdot 5} = x^{55}$

33. $(x^{-5})^3 = x^{-5 \cdot 3} = x^{-15} = \frac{1}{x^{15}}$

34. $(x^{-6})^4 = x^{-6 \cdot 4} = x^{-24} = \frac{1}{x^{24}}$

35. $\frac{x^{14}}{x^7} = x^{14-7} = x^7$

36. $\frac{x^{30}}{x^{10}} = x^{30-10} = x^{20}$

37. $\frac{x^{14}}{x^{-7}} = x^{14-(-7)} = x^{14+7} = x^{21}$

38. $\frac{x^{30}}{x^{-10}} = x^{30-(-10)} = x^{30+10} = x^{40}$

39. $(8x^3)^2 = 8^2(x^3)^2 = 8^2 x^{3 \cdot 2} = 64x^6$

40. $(6x^4)^2 = (6)^2(x^4)^2 = 6^2 x^{4 \cdot 2} = 36x^8$

41. $\left(-\frac{4}{x}\right)^3 = \frac{(-4)^3}{x^3} = -\frac{64}{x^3}$

42. $\left(-\frac{6}{y}\right)^3 = \frac{(-6)^3}{y^3} = -\frac{216}{y^3}$

43. $(-3x^2y^5)^2 = (-3)^2(x^2)^2 \cdot (y^5)^2$
$= 9x^{2\cdot 2}y^{5\cdot 2}$
$= 9x^4y^{10}$

44. $(-3x^4y^6)^3 = (-3)^3(x^4)^3(y^6)^3$
$= -27x^{4\cdot 3}y^{6\cdot 3}$
$= -27x^{12}y^{18}$

45. $(3x^4)(2x^7) = 3\cdot 2x^4\cdot x^7 = 6x^{4+7} = 6x^{11}$

46. $(11x^5)(9x^{12}) = 11\cdot 9x^5x^{12} = 99x^{5+12} = 99x^{17}$

47. $(-9x^3y)(-2x^6y^4) = (-9)(-2)x^3x^6yy^4$
$= 18x^{3+6}y^{1+4}$
$= 18x^9y^5$

48. $(-5x^4y)(-6x^7y^{11}) = (-5)(-6)x^4x^7yy^{11}$
$= 30x^{4+7}y^{1+11}$
$= 30x^{11}y^{12}$

49. $\frac{8x^{20}}{2x^4} = \left(\frac{8}{2}\right)\left(\frac{x^{20}}{x^4}\right) = 4x^{20-4} = 4x^{16}$

50. $\frac{20x^{24}}{10x^6} = \left(\frac{20}{10}\right)\left(\frac{x^{24}}{x^6}\right) = 2x^{24-6} = 2x^{18}$

51. $\frac{25a^{13}\cdot b^4}{-5a^2\cdot b^3} = \left(\frac{25}{-5}\right)\left(\frac{a^{13}}{a^2}\right)\left(\frac{b^4}{b^3}\right)$
$= -5a^{13-2}b^{4-3}$
$= -5a^{11}b$

52. $\frac{35a^{14}b^6}{-7a^7b^3} = \left(\frac{35}{-7}\right)\left(\frac{a^{14}}{a^7}\right)\left(\frac{b^6}{b^3}\right)$
$= -5a^{14-7}b^{6-3}$
$= -5a^7b^3$

53. $\frac{14b^7}{7b^{14}} = \left(\frac{14}{7}\right)\left(\frac{b^7}{b^{14}}\right) = 2\cdot b^{7-14} = 2b^{-7} = \frac{2}{b^7}$

54. $\frac{20b^{10}}{10b^{20}} = \left(\frac{20}{10}\right)\left(\frac{b^{10}}{b^{20}}\right)$
$= 2b^{10-20}$
$= 2b^{-10}$
$= \frac{2}{b^{10}}$

55. $(4x^3)^{-2} = (4^{-2})(x^3)^{-2}$
$= 4^{-2}x^{-6}$
$= \frac{1}{4^2x^6}$
$= \frac{1}{16x^6}$

56. $(10x^2)^{-3} = 10^{-3}x^{2\cdot(-3)}$
$= 10^{-3}x^{-6}$
$= \frac{1}{10^3x^6}$
$= \frac{1}{1000x^6}$

57. $\frac{24x^3\cdot y^5}{32x^7y^{-9}} = \frac{3}{4}x^{3-7}y^{5-(-9)}$
$= \frac{3}{4}x^{-4}y^{14}$
$= \frac{3y^{14}}{4x^4}$

58. $\frac{10x^4y^9}{30x^{12}y^{-3}} = \frac{1}{3}x^{4-12}y^{9-(-3)}$
$= \frac{1}{3}x^{-8}y^{12}$
$= \frac{y^{12}}{3x^8}$

59. $\left(\frac{5x^3}{y}\right)^{-2} = \frac{5^{-2}x^{-6}}{y^{-2}} = \frac{y^2}{25x^6}$

60. $\left(\frac{3x^4}{y}\right)^{-3} = \left(\frac{y}{3x^4}\right)^3$
$= \frac{y^3}{3^3x^{4\cdot 3}}$
$= \frac{y^3}{27x^{12}}$

61. $\left(\dfrac{-15a^4b^2}{5a^{10}b^{-3}}\right)^3 = \left(\dfrac{-3b^{2-(-3)}}{a^{10-4}}\right)^3$

$= \left(\dfrac{-3b^5}{a^6}\right)^3$

$= \dfrac{-27b^{15}}{a^{18}}$

62. $\left(\dfrac{-30a^{14}b^8}{10a^{17}b^{-2}}\right)^3 = \left(\dfrac{-3b^{8-(-2)}}{a^{17-14}}\right)^3$

$= \left(\dfrac{-3b^{10}}{a^3}\right)^3$

$= \dfrac{-27b^{30}}{a^9}$

63. $\left(\dfrac{3a^{-5}b^2}{12a^3b^{-4}}\right)^0 = 1$

64. $\left(\dfrac{4a^{-5}b^3}{12a^3b^{-5}}\right)^0 = 1$

65. $3.8\times10^2 = 380$

66. $9.2\times10^2 = 920$

67. $6\times10^{-4} = 0.0006$

68. $7\times10^{-5} = 0.00007$

69. $-7.16\times10^6 = -7,160,000$

70. $-8.17\times10^6 = -8,170,000$

71. $7.9\times10^{-1} = 0.79$

72. $6.8\times10^{-1} = 0.68$

73. $-4.15\times10^{-3} = -0.00415$

74. $-3.14\times10^{-3} = -0.00314$

75. $-6.00001\times10^{10} = -60,000,100,000$

76. $-7.00001\times10^{10} = -70,000,100,000$

77. $32,000 = 3.2\times10^4$

78. $64,000 = 6.4\times10^4$

79. $638,000,000,000,000,000$

$= 6.38\times10^{17}$

80. $579,000,000,000,000,000 = 5.79\times10^{17}$

81. $-5716 = -5.716\times10^3$

82. $-3829 = -3.829\times10^3$

83. $0.0027 = 2.7\times10^{-3}$

84. $0.0083 = 8.3\times10^{-3}$

85. $-0.00000000504 = -5.04\times10^{-9}$

86. $-0.00000000405 = -4.05\times10^{-9}$

87. $(3\times10^4)(2.1\times10^3) = (3\times2.1)(10^4\times10^3)$

$= 6.3\times10^{4+3} = 6.3\times10^7$

88. $(2\times10^4)(4.1\times10^3) = 8.2\times10^7$

89. $(1.6\times10^{15})(4\times10^{-11}) = (1.6\times4)(10^{15}\times10^{-11})$

$= 6.4\times10^{15+(-11)}$

$= 6.4\times10^4$

90. $(1.4\times10^{15})(3\times10^{-11}) = (1.4\times3)(10^{15}\times10^{-11})$

$= 4.2\times10^{15+(-11)}$

$= 4.2\times10^4$

91. $(6.1\times10^{-8})(2\times10^{-4}) = (6.1\times2)(10^{-8}\times10^{-4})$

$= 12.2\times10^{-8+(-4)}$

$= 12.2\times10^{-12}$

$= 1.22\times10^{-11}$

92. $(5.1\times10^{-8})(3\times10^{-4}) = 15.3\times10^{-12}$

$= 1.53\times10^{-11}$

93. $(4.3\times10^8)(6.2\times10^4)$

$= (4.3\times6.2)(10^8\times10^4)$

$= 26.66\times10^{8+4}$

$= 26.66\times10^{12}$

$= 2.666\times10^{13} \approx 2.67\times10^{13}$

94. $\left(8.2\times10^{8}\right)\left(4.6\times10^{4}\right)$
$=37.72\times10^{8+4}=37.72\times10^{12}$
$=3.772\times10^{13}\approx3.77\times10^{13}$

95. $\dfrac{8.4\times10^{8}}{4\times10^{5}}=\dfrac{8.4}{4}\times\dfrac{10^{8}}{10^{5}}$
$=2.1\times10^{8-5}=2.1\times10^{3}$

96. $\dfrac{6.9\times10^{8}}{3\times10^{5}}=2.3\times10^{8-5}=2.3\times10^{3}$

97. $\dfrac{3.6\times10^{4}}{9\times10^{-2}}=\dfrac{3.6}{9}\times\dfrac{10^{4}}{10^{-2}}$
$=0.4\times10^{4-(-2)}$
$=0.4\times10^{6}=4\times10^{5}$

98. $\dfrac{1.2\times10^{4}}{2\times10^{-2}}=0.6\times10^{4-(-2)}=0.6\times10^{6}$
$=\left(6\times10^{-1}\right)\times10^{6}=6\times10^{5}$

99. $\dfrac{4.8\times10^{-2}}{2.4\times10^{6}}=\dfrac{4.8}{2.4}\times\dfrac{10^{-2}}{10^{6}}$
$=2\times10^{-2-6}=2\times10^{-8}$

100. $\dfrac{7.5\times10^{-2}}{2.5\times10^{6}}=3\times10^{-2-6}=3\times10^{-8}$

101. $\dfrac{2.4\times10^{-2}}{4.8\times10^{-6}}=\dfrac{2.4}{4.8}\times\dfrac{10^{-2}}{10^{-6}}$
$=0.5\times10^{-2-(-6)}$
$=0.5\times10^{4}=5\times10^{3}$

102. $\dfrac{1.5\times10^{-2}}{5\times10^{-6}}=0.5\times10^{-2-(-6)}$
$=0.5\times10^{4}=5\times10^{3}$

103. $\dfrac{480{,}000{,}000{,}000}{0.00012}=\dfrac{4.8\times10^{11}}{1.2\times10^{-4}}$
$=\dfrac{4.8}{1.2}\times\dfrac{10^{11}}{10^{-4}}$
$=4\times10^{11-(-4)}$
$=4\times10^{15}$

104. $\dfrac{282{,}000{,}000{,}000}{0.00141}=\dfrac{2.82\times10^{11}}{1.41\times10^{-3}}$
$=2\times10^{11-(-3)}$
$=2\times10^{14}$

105. $\dfrac{0.00072\times0.003}{0.00024}$
$=\dfrac{\left(7.2\times10^{-4}\right)\left(3\times10^{-3}\right)}{2.4\times10^{-4}}$
$=\dfrac{7.2\times3}{2.4}\times\dfrac{10^{-4}\cdot10^{-3}}{10^{-4}}=9\times10^{-3}$

106. $\dfrac{66000\times0.001}{0.003\times0.002}=\dfrac{\left(6.6\times10^{4}\right)\left(1\times10^{-3}\right)}{\left(3\times10^{-3}\right)\left(2\times10^{-3}\right)}$
$=\dfrac{6.6\times10^{1}}{6\times10^{-6}}=1.1\times10^{1-(-6)}$
$=1.1\times10^{7}$

107. $\dfrac{\left(x^{-2}y\right)^{-3}}{\left(x^{2}y^{-1}\right)^{3}}=\dfrac{x^{6}y^{-3}}{x^{6}y^{-3}}$
$=x^{6-6}y^{-3-(-3)}=x^{0}y^{0}=1$

108. $\dfrac{\left(xy^{-2}\right)^{-2}}{\left(x^{-2}y\right)^{-3}}=\dfrac{x^{-2}y^{4}}{x^{6}y^{-3}}$
$=x^{-2-6}y^{4-(-3)}=x^{-8}y^{7}=\dfrac{y^{7}}{x^{8}}$

109. $\left(2x^{-3}yz^{-6}\right)\left(2x\right)^{-5}=2x^{-3}yz^{-6}\cdot2^{-5}x^{-5}$
$=2^{-4}x^{-8}yz^{-6}=\dfrac{y}{2^{4}x^{8}z^{6}}=\dfrac{y}{16x^{8}z^{6}}$

110. $\left(3x^{-4}yz^{-7}\right)\left(3x\right)^{-3}=3x^{-4}yz^{-7}\cdot3^{-3}x^{-3}$
$=3^{-2}x^{-7}yz^{-7}=\dfrac{y}{3^{2}x^{7}z^{7}}=\dfrac{y}{9x^{7}z^{7}}$

111. $\left(\dfrac{x^{3}y^{4}z^{5}}{x^{-3}y^{-4}z^{-5}}\right)^{-2}=\left(x^{6}y^{8}z^{10}\right)^{-2}$
$=x^{-12}y^{-16}z^{-20}=\dfrac{1}{x^{12}y^{16}z^{20}}$

112. $\left(\dfrac{x^{4}y^{5}z^{6}}{x^{-4}y^{-5}z^{-6}}\right)^{-4}=\left(x^{8}y^{10}z^{12}\right)^{-4}$
$=x^{-32}y^{-40}z^{-48}=\dfrac{1}{x^{32}y^{40}z^{48}}$

113. $\dfrac{(2^{-1}x^{-2}y^{-1})^{-2}(2x^{-4}y^{3})^{-2}(16x^{-3}y^{3})^{0}}{(2x^{-3}y^{-5})^{2}}$

$=\dfrac{(2^{2}x^{2}y^{2})(2^{-2}x^{8}y^{-6})(1)}{(2^{2}x^{-6}y^{-10})}$

$=\dfrac{x^{18}y^{6}}{4}$

114. $\dfrac{(2^{-1}x^{-3}y^{-1})^{-2}(2x^{-6}y^{4})^{-2}(9x^{3}y^{-3})^{0}}{(2x^{-4}y^{-6})^{2}}$

$=\dfrac{(2^{2}x^{6}y^{2})(2^{-2}x^{12}y^{-8})(1)}{(2^{2}x^{-8}y^{-12})}$

$=\dfrac{x^{26}y^{6}}{4}$

115. **a.** 2.52×10^{12}

b. 3×10^{8}

c. $\dfrac{2.52\times10^{12}}{3\times10^{8}}=\dfrac{2.52}{3}\times\dfrac{10^{12}}{10^{8}}$

$=0.84\times10^{4}$

$=8400$

$8400 per American

116. **a.** 2.27×10^{12}

b. 2.98×10^{8}

c. $\dfrac{2.27\times10^{12}}{2.98\times10^{8}}=\dfrac{2.27}{2.98}\times\dfrac{10^{12}}{10^{8}}$

$=0.7617\times10^{4}$

$=7617$

$7617 per American

117. $1450\times10^{9}\cdot6.60=1.45\times10^{12}\cdot6.6$

$=1.45\cdot6.6\times10^{12}$

$=9.57\times10^{12}$

Box-office receipts were $\$9.57\times10^{12}$ in 2006.

118. $1400\times10^{9}\cdot6.40=1.4\times10^{12}\cdot6.4$

$=1.4\cdot6.4\times10^{12}$

$=8.96\times10^{12}$

Box-office receipts were $\$8.96\times10^{12}$ in 2005.

119. $5.3\times10^{-23}\cdot20,000=5.3\times10^{-23}\cdot2\times10^{4}$

$=5.3\cdot2\times10^{-23}\cdot10^{4}$

$=10.6\times10^{-19}$

$=1.06\times10^{1}\cdot10^{-19}$

$=1.06\times10^{-18}$

The mass is 1.06×10^{-18} gram.

120. $1.67\times10^{-24}\cdot80,000=1.67\times10^{-24}\cdot8\times10^{4}$

$=1.67\cdot8\times10^{-24}\cdot10^{4}$

$=13.36\times10^{-20}$

$=1.336\times10^{1}\cdot10^{-20}$

$=1.336\times10^{-19}$

The mass is 1.336×10^{-19} gram.

121. $3.2\times10^{7}\cdot127=3.2\times10^{7}\cdot1.27\times10^{2}$

$=3.2\cdot1.27\times10^{7}\cdot10^{2}$

$=4.064\times10^{9}$

Americans eat 4.064×10^{9} chickens per year.

122. 365 days equals $365\cdot24$ or 8760 hours.
8760 hours equals $8760\cdot60$ or 525,600 minutes.
525,600 min. equals
$525,600\cdot60$ or 31,536,000 seconds.
There are 3.1536×10^{7} seconds in a year.

123. – 130. Answers may vary.

131. does not make sense; Explanations will vary.
Sample explanation: $36(x^{3})^{9}=36x^{27}$ not $36x^{12}$.

132. makes sense

133. does not make sense; Explanations will vary.
Sample explanation: 4.6×10^{12} represents over 4 trillion. The entire world population is measured in billions (10^{9}).

134. makes sense

135. false; Changes to make the statement true will vary.
A sample change is: $4^{-2}>4^{-3}$.

136. true

137. false; Changes to make the statement true will vary. A sample change is: $(-2)^4 \neq 2^{-4}$ because $16 \neq \frac{1}{16}$.

138. false; Changes to make the statement true will vary. A sample change is: $5^2 \cdot 5^{-2} = 2^5 \cdot 2^{-5}$.

139. false; Changes to make the statement true will vary. A sample change is: $534.7 \neq 5347$.

140. false; Changes to make the statement true will vary. A sample change is:
$$\frac{8\times 10^{30}}{2\times 10^{-5}} = 4\times 10^{30-(-5)} = 4\times 10^{35}.$$

141. false; Changes to make the statement true will vary. A sample change is:
$$(7\times 10^5) + (2\times 10^{-3}) = 700{,}000.002.$$

142. true

143. The doctor has gathered:
$$2^{-1} + 2^{-2} = \frac{1}{2} + \frac{1}{2^2} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$
So, $1 - \frac{3}{4} = \frac{1}{4}$ is remaining.

144. $b^A = MN, b^C = M, b^D = N$
$b^A = b^C b^D$
$A = C + D$

145. $\frac{70 \text{ bts}}{\text{min}} \cdot \frac{60 \text{ min}}{\text{hr}} \cdot \frac{24 \text{ hrs}}{\text{day}} \cdot \frac{365 \text{ days}}{\text{yr}} \cdot 80 \text{ yrs}$
$= 70 \cdot 60 \cdot 24 \cdot 365 \cdot 80$ beats
$= 2943360000$ beats
$= 2.94336\times 10^9$ beats
$\approx 2.94\times 10^9$ beats
The heartbeats approximately 2.94×10^9 times over a lifetime of 80 years.

146. Answers may vary.

147. **a.** $\sqrt{16}\cdot\sqrt{4} = 4\cdot 2 = 8$

b. $\sqrt{16\cdot 4} = \sqrt{64} = 8$

c. $\sqrt{16}\cdot\sqrt{4} = \sqrt{16\cdot 4}$

148. **a.** $\sqrt{300} \approx 17.32$

b. $10\sqrt{3} \approx 17.32$

c. $\sqrt{300} = 10\sqrt{3}$

149. **a.** $21x + 10x = 31x$

b. $21\sqrt{2} + 10\sqrt{2} = 31\sqrt{2}$

Section P.3

Check Point Exercises

1. **a.** $\sqrt{81} = 9$

b. $-\sqrt{9} = -3$

c. $\sqrt{\frac{1}{25}} = \frac{1}{5}$

d. $\sqrt{36+64} = \sqrt{100} = 10$

e. $\sqrt{36} + \sqrt{64} = 6 + 8 = 14$

2. **a.** $\sqrt{75} = \sqrt{25\cdot 3} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

b. $\sqrt{5x}\cdot\sqrt{10x} = \sqrt{5x\cdot 10x}$
$= \sqrt{50x^2}$
$= \sqrt{25\cdot 2x^2}$
$= \sqrt{25x^2}\cdot\sqrt{2}$
$= 5x\sqrt{2}$

3. **a.** $\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$

b. $\frac{\sqrt{150x^3}}{\sqrt{2x}} = \sqrt{\frac{150x^3}{2x}}$
$= \sqrt{75x^2}$
$= \sqrt{25x^2}\cdot\sqrt{3}$
$= 5x\sqrt{3}$

4. **a.** $8\sqrt{13}+9\sqrt{13}=(8+9)\sqrt{3}$
$=17\sqrt{13}$

b. $\sqrt{17x}-20\sqrt{17x}$
$=1\sqrt{17x}-20\sqrt{17x}$
$=(1-20)\sqrt{17x}$
$=-19\sqrt{17x}$

5. **a.** $5\sqrt{27}+\sqrt{12}$
$=5\sqrt{9\cdot 3}+\sqrt{4\cdot 3}$
$=5\cdot 3\sqrt{3}+2\sqrt{3}$
$=15\sqrt{3}+2\sqrt{3}$
$=(15+2)\sqrt{3}$
$=17\sqrt{3}$

b. $6\sqrt{18x}-4\sqrt{8x}$
$=6\sqrt{9\cdot 2x}-4\sqrt{4\cdot 2x}$
$=6\cdot 3\sqrt{2x}-4\cdot 2\sqrt{2x}$
$=18\sqrt{2x}-8\sqrt{2x}$
$=(18-8)\sqrt{2x}$
$=10\sqrt{2x}$

6. **a.** If we multiply numerator and denominator by $\sqrt{3}$, the denominator becomes $\sqrt{3}\cdot\sqrt{3}=\sqrt{9}=3.$ Therefore, multiply by 1, choosing $\frac{\sqrt{3}}{\sqrt{3}}$ for 1.

$$\frac{5}{\sqrt{3}}=\frac{5}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{5\sqrt{3}}{\sqrt{9}}=\frac{5\sqrt{3}}{3}$$

b. The *smallest* number that will produce a perfect square in the denominator of $\frac{6}{\sqrt{12}}$ is $\sqrt{3}$ because $\sqrt{12}\cdot\sqrt{3}=\sqrt{36}=6.$ So multiply by 1, choosing $\frac{\sqrt{3}}{\sqrt{3}}$ for 1.

$$\frac{6}{\sqrt{12}}=\frac{6}{\sqrt{12}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{6\sqrt{3}}{\sqrt{36}}=\frac{6\sqrt{3}}{6}=\sqrt{3}$$

7. Multiply by $\frac{4-\sqrt{5}}{4-\sqrt{5}}$.

$$\frac{8}{4+\sqrt{5}}=\frac{8}{4+\sqrt{5}}\cdot\frac{4-\sqrt{5}}{4-\sqrt{5}}$$
$$=\frac{8(4-\sqrt{5})}{4^2-(\sqrt{5})^2}$$
$$=\frac{8(4-\sqrt{5})}{16-5}$$
$$=\frac{8(4-\sqrt{5})}{11}\text{ or }\frac{32-8\sqrt{5}}{11}$$

8. **a.** $\sqrt[3]{40}=\sqrt[3]{8\cdot 5}=\sqrt[3]{8}\cdot\sqrt[3]{5}=2\sqrt[3]{5}$

b. $\sqrt[5]{8}\cdot\sqrt[5]{8}=\sqrt[5]{64}=\sqrt[5]{32}\cdot\sqrt[5]{2}=2\sqrt[5]{2}$

c. $\sqrt[3]{\frac{125}{27}}=\frac{\sqrt[3]{125}}{\sqrt[3]{27}}=\frac{5}{3}$

9. $3\sqrt[3]{81}-4\sqrt[3]{3}$
$=3\sqrt[3]{27\cdot 3}-4\sqrt[3]{3}$
$=3\cdot 3\sqrt[3]{3}-4\sqrt[3]{3}$
$=9\sqrt[3]{3}-4\sqrt[3]{3}$
$=(9-4)\sqrt[3]{3}$
$=5\sqrt[3]{3}$

10. **a.** $25^{\frac{1}{2}}=\sqrt{25}=5$

b. $8^{\frac{1}{3}}=\sqrt[3]{8}=2$

c. $-81^{\frac{1}{4}}=-\sqrt[4]{81}=-3$

d. $(-8)^{\frac{1}{3}}=\sqrt[3]{-8}=-2$

e. $27^{-\frac{1}{3}}=\frac{1}{27^{\frac{1}{3}}}=\frac{1}{\sqrt[3]{27}}=\frac{1}{3}$

11. **a.** $27^{\frac{4}{3}}=\left(\sqrt[3]{27}\right)^4=(3)^4=81$

b. $4^{\frac{3}{2}}=\left(\sqrt[2]{4}\right)^3=(2)^3=8$

c. $32^{-\frac{2}{5}}=\frac{1}{32^{\frac{2}{5}}}=\frac{1}{\left(\sqrt[5]{32}\right)^2}=\frac{1}{2^2}=\frac{1}{4}$

12. **a.**
$$\left(2x^{4/3}\right)\left(5x^{8/3}\right)$$
$$= 2\cdot 5x^{4/3}\cdot x^{8/3}$$
$$= 10x^{(4/3)+(8/3)}$$
$$= 10x^{12/3}$$
$$= 10x^4$$

b.
$$\frac{20x^4}{5x^{3/2}} = \left(\frac{20}{5}\right)\left(\frac{x^4}{x^{3/2}}\right)$$
$$= 4x^{4-(3/2)}$$
$$= 4x^{(8/2)-(3/2)}$$
$$= 4x^{5/2}$$

13. $\sqrt[6]{x^3} = x^{3/6} = x^{1/2} = \sqrt{x}$

Exercise Set P.3

1. $\sqrt{36} = \sqrt{6^2} = 6$

2. $\sqrt{25} = \sqrt{5^2} = 5$

3. $-\sqrt{36} = -\sqrt{6^2} = -6$

4. $-\sqrt{25} = -\sqrt{5^2} = -5$

5. $\sqrt{-36}$, The square root of a negative number is not real.

6. $\sqrt{-25}$, The square root of a negative number is not real.

7. $\sqrt{25-16} = \sqrt{9} = 3$

8. $\sqrt{144+25} = \sqrt{169} = 13$

9. $\sqrt{25} - \sqrt{16} = 5-4 = 1$

10. $\sqrt{144} + \sqrt{25} = 12+5 = 17$

11. $\sqrt{(-13)^2} = \sqrt{169} = 13$

12. $\sqrt{(-17)^2} = \sqrt{289} = 17$

13. $\sqrt{50} = \sqrt{25\cdot 2} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$

14. $\sqrt{27} = \sqrt{9\cdot 3} = \sqrt{9}\sqrt{3} = 3\sqrt{3}$

15.
$$\sqrt{45x^2} = \sqrt{9x^2\cdot 5}$$
$$= \sqrt{9x^2}\sqrt{5}$$
$$= \sqrt{9}\sqrt{x^2}\sqrt{5}$$
$$= 3|x|\sqrt{5}$$

16.
$$\sqrt{125x^2} = \sqrt{25x^2\cdot 5}$$
$$= \sqrt{25x^2}\sqrt{5}$$
$$= \sqrt{25}\sqrt{x^2}\sqrt{5}$$
$$= 5|x|\sqrt{5}$$

17.
$$\sqrt{2x}\cdot\sqrt{6x} = \sqrt{2x\cdot 6x}$$
$$= \sqrt{12x^2}$$
$$= \sqrt{4x^2}\cdot\sqrt{3}$$
$$= 2x\sqrt{3}$$

18.
$$\sqrt{10x}\cdot\sqrt{8x} = \sqrt{10x\cdot 8x}$$
$$= \sqrt{80x^2}$$
$$= \sqrt{16x^2}\cdot\sqrt{5}$$
$$= 4x\sqrt{5}$$

19. $\sqrt{x^3} = \sqrt{x^2}\cdot\sqrt{x} = x\sqrt{x}$

20. $\sqrt{y^3} = \sqrt{y^2}\cdot\sqrt{y} = y\sqrt{y}$

21.
$$\sqrt{2x^2}\cdot\sqrt{6x} = \sqrt{2x^2\cdot 6x}$$
$$= \sqrt{12x^3}$$
$$= \sqrt{4x^2}\cdot\sqrt{3x}$$
$$= 2x\sqrt{3x}$$

22.
$$\sqrt{6x}\cdot\sqrt{3x^2} = \sqrt{6x\cdot 3x^2}$$
$$= \sqrt{18x^3}$$
$$= \sqrt{9x^2}\cdot\sqrt{2x}$$
$$= 3x\sqrt{2x}$$

23. $\sqrt{\dfrac{1}{81}} = \dfrac{\sqrt{1}}{\sqrt{81}} = \dfrac{1}{9}$

24. $\sqrt{\dfrac{1}{49}} = \dfrac{\sqrt{1}}{\sqrt{49}} = \dfrac{1}{7}$

25. $\sqrt{\dfrac{49}{16}} = \dfrac{\sqrt{49}}{\sqrt{16}} = \dfrac{7}{4}$

26. $\sqrt{\dfrac{121}{9}} = \dfrac{\sqrt{121}}{\sqrt{9}} = \dfrac{11}{3}$

27. $\dfrac{\sqrt{48x^3}}{\sqrt{3x}} = \sqrt{\dfrac{48x^3}{3x}} = \sqrt{16x^2} = 4x$

28. $\dfrac{\sqrt{72x^3}}{\sqrt{8x}} = \sqrt{\dfrac{72x^3}{8x}} = \sqrt{9x^2} = 3x$

29. $\dfrac{\sqrt{150x^4}}{\sqrt{3x}} = \sqrt{\dfrac{150x^4}{3x}}$
$= \sqrt{50x^3}$
$= \sqrt{25x^2} \cdot \sqrt{2x}$
$= 5x\sqrt{2x}$

30. $\dfrac{\sqrt{24x^4}}{\sqrt{3x}} = \sqrt{\dfrac{24x^4}{3x}}$
$= \sqrt{8x^3}$
$= \sqrt{4x^2} \cdot \sqrt{2x}$
$= 2x\sqrt{2x}$

31. $\dfrac{\sqrt{200x^3}}{\sqrt{10x^{-1}}} = \sqrt{\dfrac{200x^3}{10x^{-1}}}$
$= \sqrt{20x^{3-(-1)}}$
$= \sqrt{20x^4}$
$= \sqrt{4 \cdot 5x^4}$
$= 2x^2\sqrt{5}$

32. $\dfrac{\sqrt{500x^3}}{\sqrt{10x^{-1}}} = \sqrt{\dfrac{500x^3}{10x^{-1}}} = \sqrt{50x^{3-(-1)}}$
$= \sqrt{50x^4} = \sqrt{25 \cdot 2x^4} = 5x^2\sqrt{2}$

33. $7\sqrt{3} + 6\sqrt{3} = (7+6)\sqrt{3} = 13\sqrt{3}$

34. $8\sqrt{5} + 11\sqrt{5} = (8+11)\sqrt{5} = 19\sqrt{5}$

35. $6\sqrt{17x} - 8\sqrt{17x} = (6-8)\sqrt{17x} = -2\sqrt{17x}$

36. $4\sqrt{13x} - 6\sqrt{13x} = (4-6)\sqrt{13x} = -2\sqrt{13x}$

37. $\sqrt{8} + 3\sqrt{2} = \sqrt{4 \cdot 2} + 3\sqrt{2}$
$= 2\sqrt{2} + 3\sqrt{2}$
$= (2+3)\sqrt{2}$
$= 5\sqrt{2}$

38. $\sqrt{20} + 6\sqrt{5} = \sqrt{4 \cdot 5} + 6\sqrt{5}$
$= 2\sqrt{5} + 6\sqrt{5}$
$= (2+6)\sqrt{5}$
$= 8\sqrt{5}$

39. $\sqrt{50x} - \sqrt{8x} = \sqrt{25 \cdot 2x} - \sqrt{4 \cdot 2x}$
$= 5\sqrt{2x} - 2\sqrt{2x}$
$= (5-2)\sqrt{2x}$
$= 3\sqrt{2x}$

40. $\sqrt{63x} - \sqrt{28x} = \sqrt{9 \cdot 7x} - \sqrt{4 \cdot 7x}$
$= 3\sqrt{7x} - 2\sqrt{7x}$
$= (3-2)\sqrt{7x}$
$= \sqrt{7x}$

41. $3\sqrt{18} + 5\sqrt{50} = 3\sqrt{9 \cdot 2} + 5\sqrt{25 \cdot 2}$
$= 3 \cdot 3\sqrt{2} + 5 \cdot 5\sqrt{2}$
$= 9\sqrt{2} + 25\sqrt{2}$
$= (9+25)\sqrt{2}$
$= 34\sqrt{2}$

42. $4\sqrt{12} - 2\sqrt{75} = 4\sqrt{4 \cdot 3} - 2\sqrt{25 \cdot 3}$
$= 4 \cdot 2\sqrt{3} - 2 \cdot 5\sqrt{3}$
$= 8\sqrt{3} - 10\sqrt{3}$
$= (8-10)\sqrt{3}$
$= -2\sqrt{3}$

43. $3\sqrt{8} - \sqrt{32} + 3\sqrt{72} - \sqrt{75}$
$= 3\sqrt{4 \cdot 2} - \sqrt{16 \cdot 2} + 3\sqrt{36 \cdot 2} - \sqrt{25 \cdot 3}$
$= 3 \cdot 2\sqrt{2} - 4\sqrt{2} + 3 \cdot 6\sqrt{2} - 5\sqrt{3}$
$= 6\sqrt{2} - 4\sqrt{2} + 18\sqrt{2} - 5\sqrt{3}$
$= 20\sqrt{2} - 5\sqrt{3}$

44. $3\sqrt{54}-2\sqrt{24}-\sqrt{96}+4\sqrt{63}$
$=3\sqrt{9\cdot 6}-2\sqrt{4\cdot 6}-\sqrt{16\cdot 6}+4\sqrt{9\cdot 7}$
$=3\cdot 3\sqrt{6}-2\cdot 2\sqrt{6}-4\sqrt{6}+4\cdot 3\sqrt{7}$
$=9\sqrt{6}-4\sqrt{6}-4\sqrt{6}+12\sqrt{7}$
$=\sqrt{6}+12\sqrt{7}$

45. $\frac{1}{\sqrt{7}}=\frac{1}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}=\frac{\sqrt{7}}{7}$

46. $\frac{2}{\sqrt{10}}=\frac{2}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}}=\frac{2\sqrt{10}}{10}=\frac{\sqrt{10}}{5}$

47. $\frac{\sqrt{2}}{\sqrt{5}}=\frac{\sqrt{2}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{10}}{5}$

48. $\frac{\sqrt{7}}{\sqrt{3}}=\frac{\sqrt{7}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{21}}{3}$

49. $\frac{13}{3+\sqrt{11}}=\frac{13}{3+\sqrt{11}}\cdot\frac{3-\sqrt{11}}{3-\sqrt{11}}$
$=\frac{13(3-\sqrt{11})}{3^2-(\sqrt{11})^2}$
$=\frac{13(3-\sqrt{11})}{9-11}$
$=\frac{13(3-\sqrt{11})}{-2}$

50. $\frac{3}{3+\sqrt{7}}=\frac{3}{3+\sqrt{7}}\cdot\frac{3-\sqrt{7}}{3-\sqrt{7}}$
$=\frac{3(3-\sqrt{7})}{3^2-(\sqrt{7})^2}$
$=\frac{3(3-\sqrt{7})}{9-7}$
$=\frac{3(3-\sqrt{7})}{2}$

51. $\frac{7}{\sqrt{5}-2}=\frac{7}{\sqrt{5}-2}\cdot\frac{\sqrt{5}+2}{\sqrt{5}+2}$
$=\frac{7(\sqrt{5}+2)}{(\sqrt{5})^2-2^2}$
$=\frac{7(\sqrt{5}+2)}{5-4}$
$=7(\sqrt{5}+2)$

52. $\frac{5}{\sqrt{3}-1}=\frac{5}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1}$
$=\frac{5(\sqrt{3}+1)}{(\sqrt{3})^2-1^2}$
$=\frac{5(\sqrt{3}+1)}{3-1}$
$=\frac{5(\sqrt{3}+1)}{2}$

53. $\frac{6}{\sqrt{5}+\sqrt{3}}=\frac{6}{\sqrt{5}+\sqrt{3}}\cdot\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$
$=\frac{6(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$
$=\frac{6(\sqrt{5}-\sqrt{3})}{5-3}$
$=\frac{6(\sqrt{5}-\sqrt{3})}{2}$
$=3(\sqrt{5}-\sqrt{3})$

54. $\frac{11}{\sqrt{7}-\sqrt{3}}=\frac{11}{\sqrt{7}-\sqrt{3}}\cdot\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$
$=\frac{11(\sqrt{7}+\sqrt{3})}{(\sqrt{7})^2-(\sqrt{3})^2}$
$=\frac{11(\sqrt{7}+\sqrt{3})}{7-3}$
$=\frac{11(\sqrt{7}+\sqrt{3})}{4}$

55. $\sqrt[3]{125}=\sqrt[3]{5^3}=5$

56. $\sqrt[3]{8}=\sqrt[3]{2^3}=2$

57. $\sqrt[3]{-8}=\sqrt[3]{(-2)^3}=-2$

58. $\sqrt[3]{-125}=\sqrt[3]{(-5)^3}=-5$

59. $\sqrt[4]{-16}$ is not a real number.

60. $\sqrt[4]{-81}$ is not a real number.

61. $\sqrt[4]{(-3)^4}=|-3|=3$

62. $\sqrt[4]{(-2)^4}=|-2|=2$

63. $\sqrt[5]{(-3)^5} = -3$

64. $\sqrt[5]{(-2)^5} = -2$

65. $\sqrt[5]{-\frac{1}{32}} = \sqrt[5]{-\frac{1}{2^5}} = -\frac{1}{2}$

66. $\sqrt[6]{\frac{1}{64}} = \frac{\sqrt[6]{1}}{\sqrt[6]{2^6}} = \frac{1}{2}$

67. $\sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = \sqrt[3]{8}\,\sqrt[3]{4} = 2 \cdot \sqrt[3]{4}$

68. $\sqrt[3]{150}$ cannot be simplified further.

69. $\sqrt[3]{x^4} = \sqrt[3]{x^3 \cdot x} = x \cdot \sqrt[3]{x}$

70. $\sqrt[3]{x^5} = \sqrt[3]{x^3 x^2} = x\sqrt[3]{x^2}$

71. $\sqrt[3]{9} \cdot \sqrt[3]{6} = \sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27}\sqrt[3]{2} = 3\sqrt[3]{2}$

72. $\sqrt[3]{12} \cdot \sqrt[3]{4} = \sqrt[3]{48} = \sqrt[3]{8 \cdot 6} = 2\sqrt[3]{6}$

73. $\frac{\sqrt[5]{64x^6}}{\sqrt[5]{2x}} = \sqrt[5]{\frac{64x^6}{2x}} = \sqrt[5]{32x^5} = 2x$

74. $\frac{\sqrt[4]{162x^5}}{\sqrt[4]{2x}} = \sqrt[4]{\frac{162x^5}{2x}} = \sqrt[4]{81x^4} = 3x$

75. $4\sqrt[5]{2} + 3\sqrt[5]{2} = 7\sqrt[5]{2}$

76. $6\sqrt[5]{3} + 2\sqrt[5]{3} = 8\sqrt[5]{3}$

77. $5\sqrt[3]{16} + \sqrt[3]{54} = 5\sqrt[3]{8 \cdot 2} + \sqrt[3]{27 \cdot 2}$
$= 5 \cdot 2\sqrt[3]{2} + 3\sqrt[3]{2}$
$= 10\sqrt[3]{2} + 3\sqrt[3]{2}$
$= 13\sqrt[3]{2}$

78. $3\sqrt[3]{24} + \sqrt[3]{81} = \sqrt[3]{8 \cdot 3} + \sqrt[3]{27 \cdot 3}$
$= 3 \cdot 2\sqrt[3]{3} + 3\sqrt[3]{3}$
$= 6\sqrt[3]{3} + 3\sqrt[3]{3}$
$= 9\sqrt[3]{3}$

79. $\sqrt[3]{54xy^3} - y\sqrt[3]{128x}$
$= \sqrt[3]{27 \cdot 2xy^3} - y\sqrt[3]{64 \cdot 2x}$
$= 3y\sqrt[3]{2x} - 4y\sqrt[3]{2x}$
$= -y\sqrt[3]{2x}$

80. $\sqrt[3]{24xy^3} - y\sqrt[3]{81x}$
$= \sqrt[3]{8 \cdot 3xy^3} - y\sqrt[3]{27 \cdot 3x}$
$= 2y\sqrt[3]{3x} - 3y\sqrt[3]{3x}$
$= -y\sqrt[3]{3x}$

81. $\sqrt{2} + \sqrt[3]{8} = \sqrt{2} + 2$

82. $\sqrt{3} + \sqrt[3]{15}$ will not simplify

83. $36^{1/2} = \sqrt{36} = 6$

84. $121^{1/2} = \sqrt{121} = 11$

85. $8^{1/3} = \sqrt[3]{8} = 2$

86. $27^{1/3} = \sqrt[3]{27} = 3$

87. $125^{2/3} = \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

88. $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 4$

89. $32^{-4/5} = \frac{1}{32^{4/5}} = \frac{1}{2^4} = \frac{1}{16}$

90. $16^{-5/2} = \frac{1}{16^{5/2}} = \frac{1}{(\sqrt{16})^5} = \frac{1}{4^5} = \frac{1}{1024}$

91. $\left(7x^{1/3}\right)\left(2x^{1/4}\right) = 7 \cdot 2x^{1/3} \cdot x^{1/4}$
$= 14 \cdot x^{1/3+1/4}$
$= 14x^{7/12}$

92. $(3x^{2/3})(4x^{3/4}) = 3 \cdot 4x^{2/3} \cdot x^{3/4}$
$= 12 \cdot x^{2/3+3/4}$
$= 12x^{17/12}$

93. $\frac{20x^{1/2}}{5x^{1/4}} = \left(\frac{20}{5}\right)\left(\frac{x^{1/2}}{x^{1/4}}\right)$
$= 4 \cdot x^{1/2-1/4}$
$= 4x^{1/4}$

94. $\frac{72x^{3/4}}{9x^{1/3}}=\left(\frac{72}{9}\right)\left(\frac{x^{3/4}}{x^{1/3}}\right)=8\cdot x^{3/4-1/3}=8x^{5/12}$

95. $\left(x^{2/3}\right)^3=x^{2/3\cdot 3}=x^2$

96. $(x^{4/5})^5=x^{4/5\cdot 5}=x^4$

97. $(25x^4y^6)^{1/2}=25^{1/2}x^{4\cdot 1/2}y^{6\cdot 1/2}=5x^2|y|^3$

98. $(125x^9y^6)^{1/3}=125^{1/3}x^{9/3}y^{6/3}=5x^3y^2$

99. $\frac{\left(3y^{\frac{1}{4}}\right)^3}{y^{\frac{1}{12}}}=\frac{27y^{\frac{3}{4}}}{y^{\frac{1}{12}}}=27y^{\frac{3}{4}-\frac{1}{12}}$

$=27y^{\frac{8}{12}}=27y^{\frac{2}{3}}$

100. $\frac{\left(2y^{1/5}\right)^4}{y^{3/10}}=\frac{2^4\left(y^{1/5}\right)^4}{y^{3/10}}$

$=\frac{16y^{4/5}}{y^{3/10}}=16y^{4/5-3/10}=16y^{1/2}$

101. $\sqrt[4]{5^2}=5^{2/4}=5^{1/2}=\sqrt{5}$

102. $\sqrt[4]{7^2}=7^{2/4}=7^{1/2}=\sqrt{7}$

103. $\sqrt[3]{x^6}=x^{6/3}=x^2$

104. $\sqrt[4]{x^{12}}=x^{12/4}=|x|^3$

105. $\sqrt[6]{x^4}={}^{6/2}\sqrt{x^{4/2}}=\sqrt[3]{x^2}$

106. $\sqrt[9]{x^6}={}^{9/3}\sqrt{x^{6/3}}=\sqrt[3]{x^2}$

107. $\sqrt[9]{x^6y^3}=x^{\frac{6}{9}}y^{\frac{3}{9}}=x^{\frac{2}{3}}y^{\frac{1}{3}}=\sqrt[3]{x^2y}$

108. $\sqrt[12]{x^4y^8}=|x|^{\frac{4}{12}}|y|^{\frac{8}{12}}=|x|^{\frac{1}{3}}|y|^{\frac{2}{3}}=\sqrt[3]{|x|y^2}$

109. $\sqrt[3]{\sqrt[4]{16}+\sqrt{625}}=\sqrt[3]{2+25}=\sqrt[3]{27}=3$

110. $\sqrt[3]{\sqrt{\sqrt{169}+\sqrt{9}}+\sqrt{\sqrt[3]{1000}+\sqrt[3]{216}}}$

$=\sqrt[3]{\sqrt{13+3}+\sqrt{10+6}}$

$=\sqrt[3]{\sqrt{16}+\sqrt{16}}$

$=\sqrt[3]{4+4}=\sqrt[3]{8}$

$=2$

111. $\left(49x^{-2}y^4\right)^{-1/2}\left(xy^{1/2}\right)$

$=(49)^{-1/2}\left(x^{-2}\right)^{-1/2}\left(y^4\right)^{-1/2}\left(xy^{1/2}\right)$

$=\frac{1}{49^{1/2}}x^{(-2)(-1/2)}y^{(4)(-1/2)}\left(xy^{1/2}\right)$

$=\frac{1}{7}x^1y^{-2}\cdot xy^{1/2}=\frac{1}{7}x^{1+1}y^{-2+(1/2)}$

$=\frac{1}{7}x^2y^{-3/2}=\frac{x^2}{7y^{3/2}}$

112. $\left(8x^{-6}y^3\right)^{1/3}\left(x^{5/6}y^{-1/3}\right)^6$

$=8^{1/3}x^{(-6)(1/3)}y^{(3)(1/3)}x^{(5/6)(6)}y^{(-1/3)(6)}$

$=2x^{-2}y^1x^5y^{-2}=2x^{-2+5}y^{1+(-2)}$

$=2x^3y^{-1}=\frac{2x^3}{y}$

113. $\left(\frac{x^{-5/4}y^{1/3}}{x^{-3/4}}\right)^{-6}=\left(x^{(-5/4)-(-3/4)}y^{1/3}\right)^{-6}$

$=\left(x^{-2/4}y^{1/3}\right)^{-6}=x^{(-2/4)(-6)}y^{(1/3)(-6)}$

$=x^3y^{-2}=\frac{x^3}{y^2}$

114. $\left(\frac{x^{1/2}y^{-7/4}}{y^{-5/4}}\right)^{-4}=\left(x^{1/2}y^{(-7/4)-(-5/4)}\right)^{-4}$

$=\left(x^{1/2}y^{-2/4}\right)^{-4}=x^{(1/2)(-4)}y^{(-2/4)(-4)}$

$=x^{-2}y^2=\frac{y^2}{x^2}$

115. **a.** In 2004, we have $x = 5$.

$y = 20.8\sqrt{5} + 21 \approx 67.5$

According to the model, 67.5% of email was spam in 2004.
This underestimates the actual value shown in the bar graph by 0.5%.

b. In 2011, we have $x = 12$.

$y = 20.8\sqrt{12} + 21 \approx 93.1$

According to the model, 93.1% of email will be spam in 2011.
This overestimates the value given in the bar graph by 21.1%.

116. **a.** For 2020: $E = 5\sqrt{x} + 34.1$

$= 5\sqrt{10} + 34.1$

For 2050: $E = 5\sqrt{x} + 34.1$

$= 5\sqrt{40} + 34.1$

$= 5 \cdot 2\sqrt{10} + 34.1$

$= 10\sqrt{10} + 34.1$

Difference:

$(10\sqrt{10} + 34.1) - (5\sqrt{10} + 34.1)$

$= 10\sqrt{10} + 34.1 - 5\sqrt{10} - 34.1$

$= 10\sqrt{10} - 5\sqrt{10} + 34.1 - 34.1$

$= 5\sqrt{10}$

The difference is $5\sqrt{10}$.

b. $5\sqrt{10} \approx 15.8$

This underestimates the difference projected by the graph of $65.8 - 47.3 = 18.5$ by 2.7. This represents a difference of 2.7 million people.

117. $\dfrac{2}{\sqrt{5} - 1} \cdot \dfrac{\sqrt{5} + 1}{\sqrt{5} + 1} = \dfrac{2(\sqrt{5} + 1)}{5 - 1}$

$= \dfrac{2(\sqrt{5} + 1)}{4}$

$= \dfrac{\sqrt{5} + 1}{2}$

≈ 1.62

About 1.62 to 1.

118. $R_a = R_f\sqrt{1 - \left(\dfrac{v}{c}\right)^2}$

$= R_f\sqrt{1 - \left(\dfrac{0.9c}{c}\right)^2}$

$= R_f\sqrt{1 - (0.9)^2}$

$= R_f\sqrt{0.19}$

$\approx 0.44R_f$

$R_a = 0.44R_f$

$44 = 0.44R_f$

$\dfrac{44}{0.44} = \dfrac{0.44R_f}{0.44}$

$100 = R_f$

If you are gone for 44 weeks, then 100 weeks will have passed for your friend.

119. Perimeter:

$P = 2l + 2w$

$= 2 \cdot \sqrt{125} + 2 \cdot 2\sqrt{20}$

$= 2 \cdot \sqrt{25 \cdot 5} + 4\sqrt{4 \cdot 5}$

$= 2 \cdot 5\sqrt{5} + 4 \cdot 2\sqrt{5}$

$= 10\sqrt{5} + 8\sqrt{5}$

$= 18\sqrt{5}$ feet

Area:

$A = lw$

$= \sqrt{125} \cdot 2\sqrt{20}$

$= 2\sqrt{125 \cdot 20}$

$= 2\sqrt{2500}$

$= 2 \cdot 50$

$= 100$ square feet

120. Perimeter:

$$\begin{aligned} P &= 2l + 2w \\ &= 2\cdot 4\sqrt{20} + 2\cdot\sqrt{80} \\ &= 8\sqrt{4\cdot 5} + 2\sqrt{16\cdot 5} \\ &= 8\cdot 2\sqrt{5} + 2\cdot 4\sqrt{5} \\ &= 16\sqrt{5} + 8\sqrt{5} \\ &= 24\sqrt{5} \text{ feet} \end{aligned}$$

Area:

$$\begin{aligned} A &= lw \\ &= 4\sqrt{20}\cdot\sqrt{80} \\ &= 4\sqrt{20\cdot 80} \\ &= 4\sqrt{1600} \\ &= 4\cdot 40 \\ &= 160 \text{ square feet} \end{aligned}$$

121. – 128. Answers may vary.

129. does not make sense; Explanations will vary. Sample explanation: The denominator is rationalized correctly.

130. makes sense

131. does not make sense; Explanations will vary. Sample explanation: $2\sqrt{20} + 4\sqrt{75}$ simplifies to $4\sqrt{5} + 20\sqrt{3}$ and thus the radical terms are not common.

132. does not make sense; Explanations will vary. Sample explanation: Finding the *n*th root first often gives smaller numbers on the middle step.

133. false; Changes to make the statement true will vary. A sample change is: $7^{1/2}\cdot 7^{1/2} = 7^1 = 7$.

134. false; Changes to make the statement true will vary. A sample change is: $(8)^{-\frac{1}{3}} = \dfrac{1}{(8)^{\frac{1}{3}}} = \dfrac{1}{\sqrt[3]{8}} = \dfrac{1}{2}$.

135. false; Changes to make the statement true will vary. The cube root of –8 is the real number –2.

136. false; Changes to make the statement true will vary. A sample change is: $\dfrac{\sqrt{20}}{8} = \dfrac{\sqrt{5}}{4}$.

137.

$$\begin{aligned} \left(5+\sqrt{\boxed{3}}\right)\left(5-\sqrt{\boxed{3}}\right) &= 22 \\ 25-\boxed{3} &= 22 \\ \boxed{3} &= 3 \end{aligned}$$

138. $\sqrt{\boxed{25}x^{\boxed{14}}} = 5x^7$

139.

$$\begin{aligned} &\sqrt{13+\sqrt{2}+\frac{7}{3+\sqrt{2}}} \\ &= \sqrt{13+\sqrt{2}+\frac{7}{3+\sqrt{2}}\cdot\frac{3-\sqrt{2}}{3-\sqrt{2}}} \\ &= \sqrt{13+\sqrt{2}+\frac{21-7\sqrt{2}}{9-2}} \\ &= \sqrt{13+\sqrt{2}+\frac{21-7\sqrt{2}}{7}} \\ &= \sqrt{13+\sqrt{2}+3-\sqrt{2}} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

140. a. $3^{\frac{1}{2}} \boxed{>} 3^{\frac{1}{3}}$

Calculator Check: $1.7321 > 1.4422$

b. $\sqrt{7}+\sqrt{18} \boxed{>} \sqrt{7+18}$

Calculator Check: $6.8884 > 5$

141. a. $2^{\frac{5}{2}} \cdot 2^{\frac{3}{4}} \div 2^{\frac{1}{4}} = \dfrac{2^{\frac{5}{2}} \cdot 2^{\frac{3}{4}}}{2^{\frac{1}{4}}} = 2^{\frac{5}{2}+\frac{3}{4}-\frac{1}{4}} = 2^3 = 8$

Her son is 8 years old.

b. Son's portion:

$$\frac{8^{-\frac{4}{3}} + 2^{-2}}{16^{-\frac{3}{4}} + 2^{-1}} = \frac{\dfrac{1}{\left(\sqrt[3]{8}\right)^4} + \dfrac{1}{2^2}}{\dfrac{1}{\left(\sqrt[4]{16}\right)^3} + \dfrac{1}{2}}$$

$$= \frac{\dfrac{1}{2^4} + \dfrac{1}{4}}{\dfrac{1}{2^3} + \dfrac{1}{2}}$$

$$= \frac{\dfrac{1}{16} + \dfrac{1}{4}}{\dfrac{1}{8} + \dfrac{1}{2}}$$

$$= \frac{\dfrac{5}{16}}{\dfrac{5}{8}}$$

$$= \frac{8}{16}$$

$$= \frac{1}{2}$$

Mom's portion:

$$\frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$$

142. $(2x^3y^2)(5x^4y^7) = 10x^7y^9$

143. $2x^4(8x^4 + 3x) = 2x^4(8x^4) + 2x^4(3x)$

$= 16x^8 + 6x^5$

144. $2x(x^2 + 4x + 5) + 3(x^2 + 4x + 5)$

$= 2x^3 + 8x^2 + 10x + 3x^2 + 12x + 15$

$= 2x^3 + 8x^2 + 3x^2 + 10x + 12x + 15$

$= 2x^3 + 11x^2 + 22x + 15$

Section P.4

Check Point Exercises

1. **a.** $(-17x^3 + 4x^2 - 11x - 5) + (16x^3 - 3x^2 + 3x - 15)$
$= (-17x^3 + 16x^3) + (4x^2 - 3x^2) + (-11x + 3x) + (-5 - 15)$
$= -x^3 + x^2 - 8x - 20$

b. $(13x^2 - 9x^2 - 7x + 1) - (-7x^3 + 2x^2 - 5x + 9)$
$= (13x^3 - 9x^2 - 7x + 1) + (7x^3 - 2x^2 + 5x - 9)$
$= (13x^3 + 7x^3) + (-9x^2 - 2x^2) + (-7x + 5x) + (1 - 9)$
$= 20x^3 - 11x^2 - 2x - 8$

2. $(5x - 2)(3x^2 - 5x + 4)$
$= 5x(3x^2 - 5x + 4) - 2(3x^2 - 5x + 4)$
$= 5x \cdot 3x^2 - 5x \cdot 5x + 5x \cdot 4 - 2 \cdot 3x^2 + 2 \cdot 5x - 2 \cdot 4$
$= 15x^3 - 25x^2 + 20x - 6x^2 + 10x - 8$
$= 15x^3 - 31x^2 + 30x - 8$

3. $(7x - 5)(4x - 3) = 7x \cdot 4x + 7x(-3) + (-5)4x + (-5)(-3)$
$= 28x^2 - 21x - 20x + 15$
$= 28x^2 - 41x + 15$

4. **a.** Use the special-product formula shown.
$(A + B)(A - B) = A^2 - B^2$
$(7x + 8)(7x - 8) = (7x)^2 - (8)^2$
$= 49x^2 - 64$

b. Use the special-product formula shown.
$(A + B)(A - B) = A^2 - B^2$
$(2y^3 - 5)(2y^3 + 5) = (2y^3 + 5)(2y^3 - 5)$
$= (2y^3)^2 - (5)^2$
$= 4y^6 - 25$

5. **a.** Use the special-product formula shown.
$(A + B)^2 = A^2 + 2AB + B^2$
$(x + 10)^2 = x^2 + 2(x)(10) + 10^2$
$= x^2 + 20x + 100$

b. Use the special-product formula shown.
$(A + B)^2 = A^2 + 2AB + B^2$
$(5x + 4)^2 = (5x)^2 + 2(5x)(4) + 4^2$
$= 25x^2 + 40x + 16$

6. **a.** Use the special-product formula shown.

$$(A-B)^2 = A^2 - 2AB + B^2$$
$$(x-9)^2 = x^2 - 2(x)(9) + 9^2$$
$$= x^2 - 18x + 81$$

b. Use the special-product formula shown.

$$(A-B)^2 = A^2 - 2AB + B^2$$
$$(7x-3)^2 = (7x)^2 - 2(7x)(3) + 3^2$$
$$= 49x^2 - 42x + 9$$

7. $(x^3 - 4x^2y + 5xy^2 - y^3) - (x^3 - 6x^2y + y^3)$
$= (x^3 - 4x^2y + 5xy^2 - y^3) + (-x^3 + 6x^2y - y^3)$
$= (x^3 - x^3) + (-4x^2y + 6x^2y) + (5xy^2) + (-y^3 - y^3)$
$= 2x^2y + 5xy^2 - 2y^3$

8. **a.** $(7x-6y)(3x-y) = (7x)(3x) + (7x)(-y) + (-6y)(3x) + (-6y)(-y)$
$= 21x^2 - 7xy - 18xy + 6y^2$
$= 21x^2 - 25xy + 6y^2$

b. $(2x+4y)^2 = (2x)^2 + 2(2x)(4y) + (4y)^2$
$= 4x^2 + 16xy + 16y^2$

Exercise Set P.4

1. yes; $2x + 3x^2 - 5 = 3x^2 + 2x - 5$

2. no; The term $3x^{-1}$ does not have a whole number exponent.

3. no; The form of a polynomial involves addition and subtraction, not division.

4. yes; $x^2 - x^3 + x^4 - 5 = x^4 - x^3 + x^2 - 5$

5. $3x^2$ has degree 2
$-5x$ has degree 1
4 has degree 0
$3x^2 - 5x + 4$ has degree 2.

6. $-4x^3$ has degree 3
$7x^2$ has degree 2
-11 has degree 0
$-4x^3 + 7x^2 - 11$ has degree 3.

7. x^2 has degree 2
$-4x^3$ has degree 3
$9x$ has degree 1
$-12x^4$ has degree 4
63 has degree 0
$x^2-4x^3+9x-12x^4+63$ has degree 4.

8. x^2 has degree 2
$-8x^3$ has degree 3
$15x^4$ has degree 4
91 has degree 0
$x^2-8x^3+15x^4+91$ has degree 4.

9. $$\begin{aligned}(-6x^3+5x^2-8x+9)+(17x^3+2x^2-4x-13) &= (-6x^3+17x^3)+(5x^2+2x^2)+(-8x-4x)+(9-13)\\ &= 11x^3+7x^2-12x-4\end{aligned}$$
The degree is 3.

10. $$\begin{aligned}(-7x^3+6x^2-11x+13)+(19x^3-11x^2+7x-17) &= (-7x^3+19x^3)+(6x^2-11x^2)+(-11x+7x)+(13-17)\\ &= 12x^3-5x^2-4x-4\end{aligned}$$
The degree is 3.

11. $$\begin{aligned}(17x^3-5x^2+4x-3)-(5x^3-9x^2-8x+11) &= (17x^3-5x^2+4x-3)+(-5x^3+9x^2+8x-11)\\ &= (17x^3-5x^3)+(-5x^2+9x^2)+(4x+8x)+(-3-11)\\ &= 12x^3+4x^2+12x-14\end{aligned}$$
The degree is 3.

12. $$\begin{aligned}(18x^4-2x^3-7x+8)-(9x^4-6x^3-5x+7) &= (18x^4-2x^3-7x+8)+(-9x^4+6x^3+5x-7)\\ &= (18x^4-9x^4)+(-2x^3+6x^3)+(-7x+5x)+(8-7)\\ &= 9x^4+4x^3-2x+1\end{aligned}$$
The degree is 4.

13. $$\begin{aligned}(5x^2-7x-8)+(2x^2-3x+7)-(x^2-4x-3) &= (5x^2-7x-8)+(2x^2-3x+7)+(-x^2+4x+3)\\ &= (5x^2+2x^2-x^2)+(-7x-3x+4x)+(-8+7+3)\\ &= 6x^2-6x+2\end{aligned}$$
The degree is 2.

14. $$\begin{aligned}(8x^2+7x-5)-(3x^2-4x)-(-6x^3-5x^2+3) &= (8x^2+7x-5)+(-3x^2+4x)+(6x^3+5x^2-3)\\ &= 6x^3+(8x^2-3x^2+5x^2)+(7x+4x)+(-5-3)\\ &= 6x^3+10x^2+11x-8\end{aligned}$$
The degree is 3.

15. $$\begin{aligned}(x+1)(x^2-x+1) &= x(x^2)-x\cdot x+x\cdot 1+1(x^2)-1\cdot x+1\cdot 1\\ &= x^3-x^2+x+x^2-x+1\\ &= x^3+1\end{aligned}$$

16. $$\begin{aligned}\left(x+5\right)\left(x^2-5x+25\right) &= x(x^2)-x(5x)+x(25)+5(x^2)-5(5x)+5(25)\\ &= x^3-5x^2+25x+5x^2-25x+125)\\ &= x^3+125\end{aligned}$$

17. $(2x-3)(x^2-3x+5) = (2x)(x^2)+(2x)(-3x)+(2x)(5)+(-3)(x^2)+(-3)(-3x)+(-3)(5)$
$= 2x^3-6x^2+10x-3x^2+9x-15$
$= 2x^3-9x^2+19x-15$

18. $(2x-1)(x^2-4x+3) = (2x)(x^2)+(2x)(-4x)+(2x)(3)+(-1)(x^2)+(-1)(-4x)+(-1)(3)$
$= 2x^3-8x^2+6x-x^2+4x-3$
$= 2x^3-9x^2+10x-3$

19. $(x+7)(x+3) = x^2+3x+7x+21 = x^2+10x+21$

20. $(x+8)(x+5) = x^2+5x+8x+40 = x^2+13x+40$

21. $(x-5)(x+3) = x^2+3x-5x-15 = x^2-2x-15$

22. $(x-1)(x+2) = x^2+2x-x-2 = x^2+x-2$

23. $(3x+5)(2x+1) = (3x)(2x)+3x(1)+5(2x)+5 = 6x^2+3x+10x+5 = 6x^2+13x+5$

24. $(7x+4)(3x+1) = (7x)(3x)+7x(1)+4(3x)+4(1) = 21x^2+7x+12x+4 = 21x^2+19x+4$

25. $(2x-3)(5x+3) = (2x)(5x)+(2x)(3)+(-3)(5x)+(-3)(3) = 10x^2+6x-15x-9 = 10x^2-9x-9$

26. $(2x-5)(7x+2) = (2x)(7x)+(2x)(2)+(-5)(7x)+(-5)(2) = 14x^2+4x-35x-10 = 14x^2-31x-10$

27. $(5x^2-4)(3x^2-7) = (5x^2)(3x^2)+(5x^2)(-7)+(-4)(3x^2)+(-4)(-7) = 15x^4-35x^2-12x^2+28 = 15x^4-47x^2+28$

28. $(7x^2-2)(3x^2-5) = (7x^2)(3x^2)+(7x^2)(-5)+(-2)(3x^2)+(-2)(-5) = 21x^4-35x^2-6x^2+10 = 21x^4-41x^2+10$

29. $\left(8x^3+3\right)\left(x^2-5\right) = \left(8x^3\right)\left(x^2\right)+\left(8x^3\right)\left(-5\right)+\left(3\right)\left(x^2\right)+\left(3\right)\left(-5\right) = 8x^5-40x^3+3x^2-15$

30. $\left(7x^3+5\right)\left(x^2-2\right) = \left(7x^3\right)\left(x^2\right)+\left(7x^3\right)\left(-2\right)+\left(5\right)\left(x^2\right)+\left(5\right)\left(-2\right) = 7x^5-14x^3+5x^2-10$

31. $(x+3)(x-3) = x^2-3^2 = x^2-9$

32. $(x+5)(x-5) = x^2-5^2 = x^2-25$

33. $(3x+2)(3x-2) = (3x)^2-2^2 = 9x^2-4$

34. $(2x+5)(2x-5) = (2x)^2-5^2 = 4x^2-25$

35. $(5-7x)(5+7x) = 5^2-(7x)^2 = 25-49x^2$

36. $(4-3x)(4+3x) = 4^2-(3x)^2 = 16-9x^2$

37. $(4x^2+5x)(4x^2-5x) = (4x^2)^2-(5x)^2 = 16x^4-25x^2$

38. $(3x^2+4x)(3x^2-4x) = (3x^2)^2-(4x)^2 = 9x^4-16x^2$

39. $\left(1-y^5\right)\left(1+y^5\right)=(1)^2-\left(y^5\right)^2=1-y^{10}$

40. $\left(2-y^5\right)\left(2+y^5\right)=(2)^2-\left(y^5\right)^2=4-y^{10}$

41. $(x+2)^2=x^2+2\cdot x\cdot 2+2^2=x^2+4x+4$

42. $(x+5)^2=x^2+2\cdot x\cdot 5+5^2=x^2+10x+25$

43. $(2x+3)^2=(2x)^2+2(2x)(3)+3^2=4x^2+12x+9$

44. $(3x+2)^2=(3x)^2+2(3x)(2)+2^2=9x^2+12x+4$

45. $(x-3)^2=x^2-2\cdot x\cdot 3+3^2=x^2-6x+9$

46. $(x-4)^2=x^2-2\cdot x\cdot 4+4^2=x^2-8x+16$

47. $(4x^2-1)^2=(4x^2)^2-2(4x^2)(1)+1^2=16x^4-8x^2+1$

48. $(5x^2-3)^2=(5x^2)^2-2(5x^2)(3)+3^2=25x^4-30x^2+9$

49. $(7-2x)^2=7^2-2(7)(2x)+(2x)^2=49-28x+4x^2=4x^2-28x+49$

50. $(9-5x)^2=9^2-2(9)(5x)+(5x)^2=81-90x+25x^2$ or $25x^2-90x+81$

51. $(x+1)^3=x^3+3\cdot x^2\cdot 1+3x\cdot 1^2+1^3=x^3+3x^2+3x+1$

52. $(x+2)^3=x^3+3\cdot x^2\cdot 2+3\cdot x\cdot 2^2+2^3=x^3+6x^2+12x+8$

53. $(2x+3)^3=(2x)^3+3\cdot(2x)^2\cdot 3+3(2x)\cdot 3^2+3^3=8x^3+36x^2+54x+27$

54. $(3x+4)^3=(3x)^3+3(3x)^2\cdot 4+3(3x)\cdot 4^2+4^3=27x^3+108x^2+144x+64$

55. $(x-3)^3=x^3-3\cdot x^3\cdot 3+3\cdot x\cdot 3^2-3^3=x^3-9x^2+27x-27$

56. $(x-1)^3=x^3-3x^2\cdot 1+3x\cdot 1^2-1^3=x^3-3x^2+3x-1$

57. $(3x-4)^3=(3x)^3-3(3x)^2\cdot 4+3(3x)\cdot 4^2-4^3=27x^3-108x^2+144x-64$

58. $(2x-3)^3=(2x)^3-3(2x)^2\cdot 3+3(2x)\cdot 3^2-3^3=8x^3-36x^2+54x-27$

59.
$$\begin{aligned}(5x^2y-3xy)+(2x^2y-xy)&=(5x^2y+2x^2y)+(-3xy-xy)\\&=(5+2)x^2y+(-3-1)xy\\&=7x^2y-4xy \text{ is of degree 3.}\end{aligned}$$

60.
$$\begin{aligned}(-2x^2y+xy)+(4x^2y+7xy)&=(-2x^2y+4x^2y)+(xy+7xy)\\&=(-2+4)x^2y+(1+7)xy\\&=2x^2y+8xy \text{ is of degree 3.}\end{aligned}$$

61. $(4x^2y + 8xy + 11) + (-2x^2y + 5xy + 2) = (4x^2y - 2x^2y) + (8xy + 5xy) + (11 + 2)$
$= (4 - 2)x^2y + (8 + 5)xy + 13$
$= 2x^2y + 13xy + 13$ is of degree 3.

62. $(7x^4y^2 - 5x^2y^2 + 3xy) + (-18x^4y^2 - 6x^2y^2 - xy) = (7x^4y^2 - 18x^4y^2) + (-5x^2y^2 - 6x^2y^2) + (3xy - xy)$
$= (7 - 18)x^4y^2 + (-5 - 6)x^2y^2 + (3 - 1)xy$
$= -11x^4y^2 - 11x^2y^2 + 2xy$ is of degree 6.

63. $(x^3 + 7xy - 5y^2) - (6x^3 - xy + 4y^2) = (x^3 + 7xy - 5y^2)$
$= (x^3 - 6x^3) + (7xy + xy) + (-5y^2 - 4y^2)$
$= (1 - 6)x^3 + (7 + 1)xy + (-5 - 4)y^2$
$= -5x^3 + 8xy - 9y^2$ is of degree 3.

64. $(x^4 - 7xy - 5y^3) - (6x^4 - 3xy + 4y^3) = (x^4 - 7xy - 5y^3) + (-6x^4 + 3xy - 4y^3)$
$= (x^4 - 6x^4) + (-7xy + 3xy) + (-5y^3 - 4y^3)$
$= (1 - 6)x^4 + (-7 + 3)xy + (-5 - 4)y^3$
$= -5x^4 - 4xy - 9y^3$ is of degree 4.

65. $(3x^4y^2 + 5x^3y - 3y) - (2x^4y^2 - 3x^3y - 4y + 6x) = (3x^4y^2 + 5x^3y - 3y) + (-2x^4y^2 + 3x^3y + 4y - 6x)$
$= (3x^4y^2 - 2x^4y^2) + (5x^3y + 3x^3y) + (-3y + 4y) - 6x$
$= (3 - 2)x^4y^2 + (5 + 3)x^3y + (-3 + 4)y - 6x$
$= x^4y^2 + 8x^3y + y - 6x$ is of degree 6.

66. $(5x^4y^2 + 6x^3y - 7y) - (3x^4y^2 - 5x^3y - 6y + 8x) = (5x^4y^2 + 6x^3y - 7y) + (-3x^4y^2 + 5x^3y + 6y - 8x)$
$= (5x^4y^2 - 3x^4y^2) + (6x^3y + 5x^3y) + (-7y + 6y) - 8x$
$= (5 - 3)x^4y^2 + (6 + 5)x^3y + (-7 + 6)y - 8x$
$= 2x^4y^2 + 11x^3y - y - 8x$ is of degree 6.

67. $(x + 5y)(7x + 3y) = x(7x) + x(3y) + (5y)(7x) + (5y)(3y)$
$= 7x^2 + 3xy + 35xy + 15y^2$
$= 7x^2 + 38xy + 15y^2$

68. $(x + 9y)(6x + 7y) = x(6x) + x(7y) + (9y)(6x) + (9y)(7y)$
$= 6x^2 + 7xy + 54xy + 63y^2$
$= 6x^2 + 61xy + 63y^2$

69. $(x - 3y)(2x + 7y) = x(2x) + x(7y) + (-3y)(2x) + (-3y)(7y)$
$= 2x^2 + 7xy - 6xy - 21y^2$
$= 2x^2 + xy - 21y^2$

70. $(3x - y)(2x + 5y) = (3x)(2x) + (3x)(5y) + (-y)(2x) + (-y)(5y)$
$= 6x^2 + 15xy - 2xy - 5y^2$
$= 6x^2 + 13xy - 5y^2$

71. $$\begin{aligned}(3xy-1)(5xy+2) &= (3xy)(5xy)+(3xy)(2)+(-1)(5xy)+(-1)(2)\\ &= 15x^2y^2+6xy-5xy-2\\ &= 15x^2y^2+xy-2\end{aligned}$$

72. $$\begin{aligned}(7x^2y+1)(2x^2y-3) &= (7x^2y)(2x^2y)+(7x^2y)(-3)+(1)2x^2y+(1)(-3)\\ &= 14x^4y^2-21x^2y+2x^2y-3\\ &= 14x^4y^2-19x^2y-3\end{aligned}$$

73. $(7x+5y)^2=(7x)^2+2(7x)(5y)+(5y)^2=49x^2+70xy+25y^2$

74. $(9x+7y)^2=(9x)^2+2(9x)(7y)+(7y)^2=81x^2+126xy+49y^2$

75. $(x^2y^2-3)^2=(x^2y^2)^2-2(x^2y^2)(3)+3^2=x^4y^4-6x^2y^2+9$

76. $(x^2y^2-5)^2=(x^2y^2)^2-2(x^2y^2)(5)+5^2=x^4y^4-10x^2y^2+25$

77. $$\begin{aligned}(x-y)(x^2+xy+y^2) &= x(x^2)+x(xy)+x(y^2)+(-y)(x^2)+(-y)(xy)+(-y)(y^2)\\ &= x^3+x^2y+xy^2-x^2y-xy^2-y^3\\ &= x^3-y^3\end{aligned}$$

78. $$\begin{aligned}(x+y)(x^2-xy+y^2) &= x(x^2)+x(-xy)+x(y^2)+y(x^2)+y(-xy)+y(y^2)\\ &= x^3-x^2y+xy^2+x^2y-xy^2+y^3\\ &= x^3+y^3\end{aligned}$$

79. $(3x+5y)(3x-5y)=(3x)^2-(5y)^2=9x^2-25y^2$

80. $(7x+3y)(7x-3y)=(7x)^2-(3y)^2=49x^2-9y^2$

81. $\left(7xy^2-10y\right)\left(7xy^2+10y\right)=\left(7xy^2\right)^2-\left(10y\right)^2=49x^2y^4-100y^2$

82. $\left(3xy^2-4y\right)\left(3xy^2+4y\right)=\left(3xy^2\right)^2-\left(4y\right)^2=9x^2y^4-16y^2$

83. $$\begin{aligned}(3x+4y)^2-(3x-4y)^2 &= \left[(3x)^2+2(3x)(4y)+(4y)^2\right]-\left[(3x)^2-2(3x)(4y)+(4y)^2\right]\\ &= \left(9x^2+24xy+16y^2\right)-\left(9x^2-24xy+16y^2\right)\\ &= 9x^2+24xy+16y^2-9x^2+24xy-16y^2\\ &= 48xy\end{aligned}$$

84. $$\begin{aligned}(5x+2y)^2-(5x-2y)^2 &= \left[(5x)^2+2(5x)(2y)+(2y)^2\right]-\left[(5x)^2-2(5x)(2y)+(2y)^2\right]\\ &= \left(25x^2+20xy+4y^2\right)-\left(25x^2-20xy+4y^2\right)\\ &= 25x^2+20xy+4y^2-25x^2+20xy-4y^2\\ &= 40xy\end{aligned}$$

85. $(5x-7)(3x-2)-(4x-5)(6x-1)$

$=\left[15x^2-10x-21x+14\right]-\left[24x^2-4x-30x+5\right]$

$=\left(15x^2-31x+14\right)-\left(24x^2-34x+5\right)$

$=15x^2-31x+14-24x^2+34x-5$

$=-9x^2+3x+9$

86. $(3x+5)(2x-9)-(7x-2)(x-1)$

$=\left(6x^2-27x+10x-45\right)-\left(7x^2-7x-2x+2\right)$

$=\left(6x^2-17x-45\right)-\left(7x^2-9x+2\right)$

$=6x^2-17x-45-7x^2+9x-2$

$=-x^2-8x-47$

87. $(2x+5)(2x-5)\left(4x^2+25\right)$

$=\left[(2x)^2-5^2\right]\left(4x^2+25\right)$

$=\left(4x^2-25\right)\left(4x^2+25\right)$

$=\left(4x^2\right)^2-(25)^2$

$=16x^4-625$

88. $(3x+4)(3x-4)\left(9x^2+16\right)$

$=\left[(3x)^2-4^2\right]\left(9x^2+16\right)$

$=\left(9x^2-16\right)\left(9x^2+16\right)$

$=\left(9x^2\right)^2-(16)^2$

$=81x^4-256$

89. $\dfrac{(2x-7)^5}{(2x-7)^3}=(2x-7)^{5-3}$

$=(2x-7)^2$

$=(2x)^2-2(2x)(7)+(7)^2$

$=4x^2-28x+49$

90. $\dfrac{(5x-3)^6}{(5x-3)^4}=(5x-3)^{6-4}$

$=(5x-3)^2$

$=(5x)^2-2(5x)(3)+(3)^2$

$=25x^2-30x+9$

91. **a.** $M=177x^2+288x+7075$

$M=177(16)^2+288(16)+7075=56,995$

The model estimates the median annual income for a man with 16 years of education to be \$56,995.
The model underestimates the actual value of \$57,220 shown in the bar graph by \$225.

b. $M-W=(-18x^3+923x^2-9603x+48,446)-(17x^3-450x^2+6392x-14,764)$

$M-W=-18x^3+923x^2-9603x+48,446-17x^3+450x^2-6392x+14,764$

$M-W=-18x^3-17x^3+923x^2+450x^2-9603x-6392x+48,446+14,764$

$M-W=-35x^3+1373x^2-15,995x+63,210$

c. $M-W=-35x^3+1373x^2-15,995x+63,210$

$M-W=-35(14)^3+1373(14)^2-15,995(14)+63,210=12,348$

The difference in the median income between men and women with 14 years experience is \$12,348.

d. $44,404-33,481=10,923$

The actual difference displayed in the graph in the median income between men and women with 14 years experience is \$10,923.
The model overestimates this difference by $\$12,348-\$10,923=\$1425$.

92. **a.** $W=255x^2-2956x+24,336$

$W=255(18)^2-2956(18)+24,336=53,748$

The model estimates the median annual income for a woman with 18 years of education to be \$53,748.
The model overestimates the actual value of \$51,316 shown in the bar graph by \$2432.

b. $M-W=(-18x^3+923x^2-9603x+48,446)-(17x^3-450x^2+6392x-14,764)$

$M-W=-18x^3+923x^2-9603x+48,446-17x^3+450x^2-6392x+14,764$

$M-W=-18x^3-17x^3+923x^2+450x^2-9603x-6392x+48,446+14,764$

$M-W=-35x^3+1373x^2-15,995x+63,210$

c. $M-W=-35x^3+1373x^2-15,995x+63,210$

$M-W=-35(16)^3+1373(16)^2-15,995(16)+63,210=15,418$

The difference in the median income between men and women with 16 years experience is \$15,418.

d. $57,220-41,681=15,539$

The actual difference displayed in the graph in the median income between men and women with 16 years experience is \$15,539.
The model underestimates this difference by $\$15,539-\$15,418=\$121$.

93. $x(8-2x)(10-2x) = x(80-36x+4x^2)$

$= 80x - 36x^2 + 4x^3$

$= 4x^3 - 36x^2 + 80x$

94. $x(8-2x)(5-2x) = x(40-26x+4x^2)$

$= 40x - 26x^2 + 4x^3$

$= 4x^3 - 26x^2 + 40x$

95. $(x+9)(x+3)-(x+5)(x+1)$

$= x^2+12x+27-(x^2+6x+5)$

$= x^2+12x+27-x^2-6x-5$

$= 6x+22$

96. $(x+4)(x+3)-(x+2)(x+1)$

$= x^2+7x+12-(x^2+3x+2)$

$= x^2+7x+12-x^2-3x-2$

$= 4x+10$

97. – 102. Answers may vary.

103. makes sense

104. does not make sense; Explanations will vary. Sample explanation: FOIL is used to multiply two binomials.

105. makes sense

106. makes sense, although answers may vary

107. false; Changes to make the statement true will vary. A sample change is: $(3x^3+2)(3x^3-2)=9x^6-4$

108. false; Changes to make the statement true will vary. A sample change is: $(x-5)^2 = x^2-10x+25$

109. false; Changes to make the statement true will vary. A sample change is: $(x+1)^2 = x^2+2x+1$

110. true

111. $[(7x+5)+4y][(7x+5)-4y] = (7x+5)^2-4y^2$

$= (7x)^2+2(7x)(5)+5^2-16y^2$

$= 49x^2+70x+25-16y^2$

112. $[(3x+y)+1]^2$

$= (3x+y)^2+2(3x+y)(1)+1^2$

$= (3x)^2+2(3x)y+y^2+6x+2y+1$

$= 9x^2+6xy+y^2+6x+2y+1$

113. $(x^n+2)(x^n-2)-(x^n-3)^2$

$(x^n+2)(x^n-2)-(x^n-3)^2$
$=(x^{2n}-4)-(x^{2n}-6x^n+9)$
$=x^{2n}-4-x^{2n}+6x^n-9$
$=6x^n-13$

114. $(x+3)(x-1)+((x+3)-x)(x-(x-1))$
$=(x+3)(x-1)+3(x-x+1)$
$=x^2-x+3x-3+3$
$=x^2+2x$

115. $(x+3)(x+\boxed{4})=x^2+7x+12$

116. $(x-\boxed{2})(x-12)=x^2-14x+24$

117. $(4x+1)(2x-\boxed{3})=8x^2-10x-3$

Mid-Chapter P Check Point

1. $(3x+5)(4x-7)=(3x)(4x)+(3x)(-7)+(5)(4x)+(5)(-7)$
$=12x^2-21x+20x-35$
$=12x^2-x-35$

2. $(3x+5)-(4x-7)=3x+5-4x+7$
$=3x-4x+5+7$
$=-x+12$

3. $\sqrt{6}+9\sqrt{6}=10\sqrt{6}$

4. $3\sqrt{12}-\sqrt{27}=3\cdot 2\sqrt{3}-3\sqrt{3}=6\sqrt{3}-3\sqrt{3}=3\sqrt{3}$

5. $7x+3[9-(2x-6)]=7x+3[9-2x+6]=7x+3[15-2x]=7x+45-6x=x+45$

6. $(8x-3)^2=(8x)^2-2(8x)(3)+(3)^2=64x^2-48x+9$

7. $\left(x^{\frac{1}{3}}y^{-\frac{1}{2}}\right)^6=x^{\frac{1}{3}\cdot 6}y^{-\frac{1}{2}\cdot 6}=x^2y^{-3}=\dfrac{x^2}{y^3}$

8. $\left(\dfrac{2}{7}\right)^0-32^{-\frac{2}{5}}=1-\dfrac{1}{\left(\sqrt[5]{32}\right)^2}=1-\dfrac{1}{(2)^2}=1-\dfrac{1}{4}=\dfrac{3}{4}$

9. $(2x-5)-(x^2-3x+1)=2x-5-x^2+3x-1=-x^2+5x-6$

10. $$\begin{aligned}(2x-5)(x^2-3x+1) &= 2x(x^2-3x+1)-5(x^2-3x+1)\\ &= 2x(x^2-3x+1)-5(x^2-3x+1)\\ &= 2x^3-6x^2+2x-5x^2+15x-5\\ &= 2x^3-6x^2-5x^2+2x+15x-5\\ &= 2x^3-11x^2+17x-5\end{aligned}$$

11. $x^3+x^3-x^3\cdot x^3=2x^3-x^6=-x^6+2x^3$

12. $$\begin{aligned}(9a-10b)(2a+b) &= (9a)(2a)+(9a)(b)+(-10b)(2a)+(-10b)(b)\\ &= (9a)(2a)+(9a)(b)+(-10b)(2a)+(-10b)(b)\\ &= 18a^2+9ab-20ab-10b^2\\ &= 18a^2-11ab-10b^2\end{aligned}$$

13. $\{a,c,d,e\}\cup\{c,d,f,h\}=\{a,c,d,e,f,h\}$

14. $\{a,c,d,e\}\cap\{c,d,f,h\}=\{c,d\}$

15. $$\begin{aligned}\left(3x^2y^3-xy+4y^2\right)-\left(-2x^2y^3-3xy+5y^2\right) &= 3x^2y^3-xy+4y^2+2x^2y^3+3xy-5y^2\\ &= 3x^2y^3-xy+4y^2+2x^2y^3+3xy-5y^2\\ &= 3x^2y^3+2x^2y^3-xy+3xy+4y^2-5y^2\\ &= 5x^2y^3+2xy-y^2\end{aligned}$$

16. $\dfrac{24x^2y^{13}}{-2x^5y^{-2}}=-12x^{2-5}y^{13-(-2)}=-12x^{-3}y^{15}=-\dfrac{12y^{15}}{x^3}$

17. $\left(\dfrac{1}{3}x^{-5}y^4\right)\left(18x^{-2}y^{-1}\right)=6x^{-5-2}y^{4-1}=\dfrac{6y^3}{x^7}$

18. $\sqrt[12]{x^4}=x^{\frac{4}{12}}=\left|x^{\frac{1}{3}}\right|=\left|\sqrt[3]{x}\right|$

19. $\dfrac{24\times10^3}{2\times10^6}=\dfrac{24}{2}\cdot\dfrac{10^3}{10^6}=12\times10^{-3}=\left(1.2\times10^1\right)\times10^{-3}=1.2\times\left(10^1\times10^{-3}\right)=1.2\times10^{-2}$

20. $\dfrac{\sqrt[3]{32}}{\sqrt[3]{2}}=\sqrt[3]{\dfrac{32}{2}}=\sqrt[3]{16}=\sqrt[3]{2^4}=2\sqrt[3]{2}$

21. $(x^3+2)(x^3-2)=x^6-4$

22. $(x^2+2)^2=(x^2)^2+2(x^2)(2)+(2)^2=x^4+4x^2+4$

23. $\sqrt{50}\cdot\sqrt{6}=5\sqrt{2}\cdot\sqrt{6}=5\sqrt{2\cdot6}=5\sqrt{12}=5\cdot2\sqrt{3}=10\sqrt{3}$

24. $\dfrac{11}{7-\sqrt{3}}=\dfrac{11}{7-\sqrt{3}}\cdot\dfrac{7+\sqrt{3}}{7+\sqrt{3}}=\dfrac{77+11\sqrt{3}}{49-3}=\dfrac{77+11\sqrt{3}}{46}$

25. $\frac{11}{\sqrt{3}} = \frac{11}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{11\sqrt{3}}{3}$

26. $\left\{-11,\ -\frac{3}{7},\ 0,\ 0.45,\ \sqrt{25}\right\}$

27. Since $2-\sqrt{13}<0$ then $\left|2-\sqrt{13}\right| = \sqrt{13}-2$

28. Since $x<0$ then $|x| = -x$. Thus $x^2|x| = -x^2x = -x^3$

29. $140 \cdot 3.0\times10^8 = 420\times10^8 = 4.2\times10^2\times10^8 = 4.2\times10^{10}$
The total annual spending on ice cream is $\$4.2\times10^{10}$

30. $\frac{3\times10^{10}}{7.5\times10^9} = \frac{3}{7.5}\cdot\frac{10^{10}}{10^9} = 0.4\times10 = 4$
A human brain has 4 times as many neurons as a gorilla brain.

31. **a.** Model 1:
$N = 6.8x + 64$
$N = 6.8(0) + 64$
$N = 64$
Model 2:
$N = -0.5x^2 + 9.5x + 62$
$N = -0.5(0)^2 + 9.5(0) + 62$
$N = 62$

Model 2 best describes the data in 2000.

b. $N = -0.5x^2 + 9.5x + 62$
$N = -0.5(6)^2 + 9.5(6) + 62$
$N = 101$
Model 2 underestimates the number of channels in 2006 by 3.

c. $N = 6.8x + 64$
$N = 6.8(10) + 64$
$N = 132$
Model 1 predicts there will be 132 channels in 2010.

Section P.5

Check Point Exercises

1. **a.** $10x^3 - 4x^2$
$= 2x^2(5x) - 2x^2(2)$
$= 2x^2(5x - 2)$

b. $2x(x-7) + 3(x-7)$
$= (x-7)(2x+3)$

2. $x^3 + 5x^2 - 2x - 10$
$= (x^3 + 5x^2) - (2x + 10)$
$= x^2(x+5) - 2(x+5)$
$= (x+5)(x^2 - 2)$

3. Find two numbers whose product is 40 and whose sum is 13. The required integers are 8 and 5. Thus,
$x^2 + 13x + 40 = (x+5)(x+8)$ or $(x+8)(x+5)$

4. Find two numbers whose product is –14 and whose sum is –5. The required integers are –7 and 2. Thus,
$x^2 - 5x - 14 = (x-7)(x+2)$ or $(x+2)(x-7)$.

5. Find two First terms whose product is $6x^2$.
$6x^2 + 19x - 7 = (6x \quad)(x \quad)$
$6x^2 + 19x - 7 = (3x \quad)(2x \quad)$

Find two Last terms whose product is –7.
The possible factors are 1(–7) and –1(7).

Try various combinations of these factors to find the factorization in which the sum of the Outside and Inside products is 19*x*.

Possible Factors of $6x^2 + 19x - 7$	Sum of Outside and Inside Products (Should Equal 19*x*)
$(6x+1)(x-7)$	$-42x + x = -41x$
$(6x-7)(x+1)$	$6x - 7x = -x$
$(6x-1)(x+7)$	$42x - x = 41x$
$(6x+7)(x-1)$	$-6x + 7x = x$
$(3x+1)(2x-7)$	$-21x + 2x = -19x$
$(3x-7)(2x+1)$	$3x - 14x = -11x$
$(3x-1)(2x+7)$	$21x - 2x = 19x$
$(3x+7)(2x-1)$	$-3x + 14x = 11x$

Thus, $6x^2 + 19x - 7 = (3x-1)(2x+7)$ or $(2x+7)(3x-1)$.

6. Find two First terms whose product is $3x^2$.
$3x^2 - 13xy + 4y^2 = (3x \quad)(x \quad)$

Find two Last terms whose product is $4y^2$.
The possible factors are $(2y)(2y)$, $(-2y)(-2y)$, $(4y)(y)$, and $(-4y)(-y)$.

Try various combinations of these factors to find the factorization in which the sum of the Outside and Inside products is $-13xy$.

$3x^2 - 13xy + y^2 = (3x - y)(x - 4y)$ or $(x - 4y)(3x - y)$.

7. Express each term as the square of some monomial. Then use the formula for factoring $A^2 - B^2$.

a. $x^2 - 81 = x^2 - 9^2 = (x+9)(x-9)$

b. $36x^2 - 25 = (6x)^2 - 5^2 = (6x+5)(6x-5)$

8. Express $81x^4 - 16$ as the difference of two squares and use the formula for factoring $A^2 - B^2$.
$81x^4 - 16 = (9x^2)^2 - 4^2 = (9x^2+4)(9x^2-4)$

The factor $9x^2 - 4$ is the difference of two squares and can be factored. Express $9x^2 - 4$ as the difference of two squares and again use the formula for factoring $A^2 - B^2$.
$(9x^2+4)(9x^2-4) = (9x^2+4)\left[(3x)^2 - 2^2\right] = (9x^2+4)(3x+2)(3x-2)$

Thus, factored completely,
$81x^4 - 16 = (9x^2+4)(3x+2)(3x-2).$

9. **a.** $x^2 + 14x + 49 = x^2 + 2 \cdot x \cdot 7 + 7^2 = (x+7)^2$

b. Since $16x^2 = (4x)^2$ and $49 = 7^2$, check to see if the middle term can be expressed as twice the product of $4x$ and 7.

Since $2 \cdot 4x \cdot 7 = 56x$, $16x^2 - 56x + 49$ is a perfect square trinomial. Thus,
$$\begin{aligned}16x^2 - 56x + 49 &= (4x)^2 - 2 \cdot 4x \cdot 7 + 7^2 \\ &= (4x-7)^2\end{aligned}$$

10. **a.**
$$\begin{aligned}x^3 + 1 &= x^3 + 1^3 \\ &= (x+1)(x^2 - x \cdot 1 + 1^2) \\ &= (x+1)(x^2 - x + 1)\end{aligned}$$

b.
$$\begin{aligned}125x^3 - 8 &= (5x)^3 - 2^3 \\ &= (5x-2)\left[(5x)^2 + (5x)(2) + 2^2\right] \\ &= (5x-2)(25x^2 + 10x + 4)\end{aligned}$$

11. Factor out the greatest common factor.
$3x^3 - 30x^2 + 75x = 3x\left(x^2 - 10x + 25\right)$
Factor the perfect square trinomial.
$3x\left(x^2 - 10x + 25\right) = 3x\left(x-5\right)^2$

12. Reorder to write as a difference of squares.

$x^2 - 36a^2 + 20x + 100$
$= x^2 + 20x + 100 - 36a^2$
$= \left(x^2 + 20x + 100\right) - 36a^2$
$= \left(x + 10\right)^2 - 36a^2$
$= \left(x + 10 + 6a\right)\left(x + 10 - 6a\right)$

13. $x\left(x-1\right)^{-\frac{1}{2}} + \left(x-1\right)^{\frac{1}{2}}$
$= \left(x-1\right)^{-\frac{1}{2}}\left[x + \left(x-1\right)^{\frac{1}{2}-\left(-\frac{1}{2}\right)}\right]$
$= \left(x-1\right)^{-\frac{1}{2}}\left[x + \left(x-1\right)\right]$
$= \left(x-1\right)^{-\frac{1}{2}}\left(2x-1\right)$
$= \dfrac{\left(2x-1\right)}{\left(x-1\right)^{\frac{1}{2}}}$

Exercise Set P.5

1. $18x + 27 = 9 \cdot 2x + 9 \cdot 3 = 9(2x + 3)$

2. $16x - 24 = 8(2x) + 8(-3) = 8(2x - 3)$

3. $3x^2 + 6x = 3x \cdot x + 3x \cdot 2 = 3x(x + 2)$

4. $4x^2 - 8x = 4x(x) + 4x(-2) = 4x(x - 2)$

5. $9x^4 - 18x^3 + 27x^2$
$= 9x^2(x^2) + 9x^2(-2x) + 9x^2(3)$
$= 9x^2(x^2 - 2x + 3)$

6. $6x^4 - 18x^3 + 12x^2$
$= 6x^2(x^2) + 6x^2(-3x) + 6x^2(2)$
$= 6x^2\left(x^2 - 3x + 2\right)$

7. $x(x + 5) + 3(x + 5) = (x + 5)(x + 3)$

8. $x(2x + 1) + 4(2x + 1) = (2x + 1)(x + 4)$

9. $x^2(x - 3) + 12(x - 3) = (x - 3)(x^2 + 12)$

10. $x^2\left(2x + 5\right) + 17\left(2x + 5\right) = \left(2x + 5\right)\left(x^2 + 17\right)$

11. $x^3 - 2x^2 + 5x - 10 = x^2(x - 2) + 5(x - 2)$
$= (x^2 + 5)(x - 2)$

12. $x^3 - 3x^2 + 4x - 12 = x^2(x-3) + 4(x-3)$
$= (x-3)(x^2+4)$

13. $x^3 - x^2 + 2x - 2 = x^2(x-1) + 2(x-1)$
$= (x-1)(x^2+2)$

14. $x^3 + 6x^2 - 2x - 12 = x^2(x+6) - 2(x+6)$
$= (x+6)(x^2-2)$

15. $3x^3 - 2x^2 - 6x + 4 = x^2(3x-2) - 2(3x-2)$
$= (3x-2)(x^2-2)$

16. $x^3 - x^2 - 5x + 5 = x^2(x-1) - 5(x-1)$
$= (x-1)(x^2-5)$

17. $x^2 + 5x + 6 = (x+2)(x+3)$

18. $x^2 + 8x + 15 = (x+3)(x+5)$

19. $x^2 - 2x - 15 = (x-5)(x+3)$

20. $x^2 - 4x - 5 = (x-5)(x+1)$

21. $x^2 - 8x + 15 = (x-5)(x-3)$

22. $x^2 - 14x + 45 = (x-5)(x-9)$

23. $3x^2 - x - 2 = (3x+2)(x-1)$

24. $2x^2 + 5x - 3 = (2x-1)(x+3)$

25. $3x^2 - 25x - 28 = (3x-28)(x+1)$

26. $3x^2 - 2x - 5 = (3x-5)(x+1)$

27. $6x^2 - 11x + 4 = (2x-1)(3x-4)$

28. $6x^2 - 17x + 12 = (2x-3)(3x-4)$

29. $4x^2 + 16x + 15 = (2x+3)(2x+5)$

30. $8x^2 + 33x + 4 = (8x+1)(x+4)$

31. $9x^2 - 9x + 2 = (3x-1)(3x-2)$

32. $9x^2 + 5x - 4 = (9x-4)(x+1)$

33. $20x^2 + 27x - 8 = (5x+8)(4x-1)$

34. $15x^2 - 19x + 6 = (3x - 2)(5x - 3)$

35. $2x^2 + 3xy + y^2 = (2x + y)(x + y)$

36. $3x^2 + 4xy + y^2 = (3x + y)(x + y)$

37. $6x^2 - 5xy - 6y^2 = (3x + 2y)(2x - 3y)$

38. $6x^2 - 7xy - 5y^2 = (3x - 5y)(2x + y)$

39. $x^2 - 100 = x^2 - 10^2 = (x + 10)(x - 10)$

40. $x^2 - 144 = x^2 - 12^2 = (x + 12)(x - 12)$

41. $36x^2 - 49 = (6x)^2 - 7^2 = (6x + 7)(6x - 7)$

42. $64x^2 - 81 = (8x)^2 - 9^2 = (8x + 9)(8x - 9)$

43.
$$\begin{aligned} 9x^2 - 25y^2 &= (3x)^2 - (5y)^2 \\ &= (3x + 5y)(3x - 5y) \end{aligned}$$

44.
$$\begin{aligned} 36x^2 - 49y^2 &= (6x)^2 - (7y)^2 \\ &= (6x + 7y)(6x - 7y) \end{aligned}$$

45.
$$\begin{aligned} x^4 - 16 &= (x^2)^2 - 4^2 \\ &= (x^2 + 4)(x^2 - 4) \\ &= (x^2 + 4)(x + 2)(x - 2) \end{aligned}$$

46.
$$\begin{aligned} x^4 - 1 &= (x^2)^2 - 1^2 = (x^2 + 1)(x^2 - 1) \\ &= (x^2 + 1)(x + 1)(x - 1) \end{aligned}$$

47.
$$\begin{aligned} 16x^4 - 81 &= (4x^2)^2 - 9^2 \\ &= (4x^2 + 9)(4x^2 - 9) \\ &= (4x^2 + 9)[(2x)^2 - 3^2] \\ &= (4x^2 + 9)(2x + 3)(2x - 3) \end{aligned}$$

48.
$$\begin{aligned} 81x^4 - 1 &= (9x^2)^2 - 1^2 \\ &= (9x^2 + 1)(9x^2 - 1) \\ &= (9x^2 + 1)[(3x)^2 - 1^2] \\ &= (9x^2 + 1)(3x + 1)(3x - 1) \end{aligned}$$

49. $x^2 + 2x + 1 = x^2 + 2 \cdot x \cdot 1 + 1^2 = (x + 1)^2$

50. $x^2 + 4x + 4 = x^2 + 2 \cdot x \cdot 2 + 2^2 = (x + 2)^2$

51.
$$\begin{aligned} x^2 - 14x + 49 &= x^2 - 2 \cdot x \cdot 7 + 7^2 \\ &= (x - 7)^2 \end{aligned}$$

52. $x^2 - 10x + 25 = x^2 - 2 \cdot x \cdot 5 + 5^2 = (x-5)^2$

53. $4x^2 + 4x + 1 = (2x)^2 + 2 \cdot 2x \cdot 1 + 1^2$
$= (2x+1)^2$

54. $25x^2 + 10x + 1 = (5x)^2 + 2 \cdot 5x \cdot 1 + 1^2 = (5x+1)^2$

55. $9x^2 - 6x + 1 = (3x)^2 - 2 \cdot 3x \cdot 1 + 1^2$
$= (3x-1)^2$

56. $64x^2 - 16x + 1 = (8x)^2 - 2 \cdot 8x \cdot 1 + 1^2 = (8x-1)^2$

57. $x^3 + 27 = x^3 + 3^3$
$= (x+3)(x^2 - x \cdot 3 + 3^2)$
$= (x+3)(x^2 - 3x + 9)$

58. $x^3 + 64 = x^3 + 4^3$
$= (x+4)(x^2 - x \cdot 4 + 4^2)$
$= (x+4)(x^2 - 4x + 16)$

59. $x^3 - 64 = x^3 - 4^3$
$= (x-4)(x^2 + x \cdot 4 + 4^2)$
$= (x-4)(x^2 + 4x + 16)$

60. $x^3 - 27 = x^3 - 3^3$
$= (x-3)(x^2 + x \cdot 3 + 3^2)$
$= (x-3)(x^2 + 3x + 9)$

61. $8x^3 - 1 = (2x)^3 - 1^3$
$= (2x-1)[(2x)^2 + (2x)(1) + 1^2]$
$= (2x-1)(4x^2 + 2x + 1)$

62. $27x^3 - 1 = (3x)^3 - 1^3$
$= (3x-1)[(3x)^2 + (3x)(1) + 1^2]$
$= (3x-1)(9x^2 + 3x + 1)$

63. $64x^3 + 27 = (4x)^3 + 3^3$
$= (4x+3)[(4x)^2 - (4x)(3) + 3^2]$
$= (4x+3)(16x^2 - 12x + 9)$

64. $8x^3 + 125 = (2x)^3 + 5^3$
$= (2x+5)[(2x)^2 - (2x)(5) + 5^2]$
$= (2x+5)(4x^2 - 10x + 25)$

65. $3x^3 - 3x = 3x(x^2 - 1) = 3x(x+1)(x-1)$

66. $5x^3 - 45x = 5x(x^2 - 9) = 5x(x+3)(x-3)$

67. $4x^2 - 4x - 24 = 4(x^2 - x - 6)$
$= 4(x+2)(x-3)$

68. $6x^2 - 18x - 60 = 6(x^2 - 3x - 10)$
$= 6(x+2)(x-5)$

69. $2x^4 - 162 = 2(x^4 - 81)$
$= 2[(x^2)^2 - 9^2]$
$= 2(x^2 + 9)(x^2 - 9)$
$= 2(x^2 + 9)(x^2 - 3^2)$
$= 2(x^2 + 9)(x+3)(x-3)$

70. $7x^4 - 7 = 7(x^4 - 1)$
$= 7[(x^2)^2 - 1^2]$
$= 7(x^2 + 1)(x^2 - 1)$
$= 7(x^2 + 1)(x+1)(x-1)$

71. $x^3 + 2x^2 - 9x - 18 = (x^3 + 2x^2) - (9x + 18)$
$= x^2(x+2) - 9(x+2)$
$= (x^2 - 9)(x+2)$
$= (x^2 - 3^2)(x+2)$
$= (x-3)(x+3)(x+2)$

72. $x^3 + 3x^2 - 25x - 75 = (x^3 + 3x^2) - (25x + 75)$
$= x^2(x+3) - 25(x+3)$
$= (x^2 - 25)(x+3)$
$= (x^2 - 5^2)(x+3)$
$= (x-5)(x+5)(x+3)$

73. $2x^2 - 2x - 112 = 2(x^2 - x - 56) = 2(x-8)(x+7)$

74. $6x^2 - 6x - 12 = 6(x^2 - x - 2)$
$= 6(x-2)(x+1)$

75. $x^3 - 4x = x(x^2 - 4)$
$= x(x^2 - 2^2)$
$= x(x-2)(x+2)$

76. $9x^3 - 9x = 9x(x^2 - 1) = 9x(x-1)(x+1)$

77. $x^2 + 64$ is prime.

78. $x^2 + 36$ is prime.

79. $x^3+2x^2-4x-8=(x^3+2x^2)+(-4x-8)$
$=x^2(x+2)-4(x+2)=(x^2-4)(x+2)=(x^2-2^2)(x+2)=(x-2)(x+2)(x+2)=(x-2)(x+2)^2$

80. x^3+2x^2-x-2
$=(x^3+2x^2)+(-x-2)=x^2(x+2)-1(x+2)=(x^2-1)(x+2)=(x^2-1^2)(x+2)=(x-1)(x+1)(x+2)$

81. y^5-81y
$=y(y^4-81)=y[(y^2)^2-9^2]=y(y^2+9)(y^2-9)=y(y^2+9)(y^2-3^2)=y(y^2+9)(y+3)(y-3)$

82. y^5-16y
$=y(y^4-16)=y[(y^2)^2-4^2]=y(y^2+4)(y^2-4)=y(y^2+4)(y^2-2^2)=y(y^2+4)(y+2)(y-2)$

83. $20y^4-45y^2=5y^2(4y^2-9)=5y^2[(2y)^2-3^2]=5y^2(2y+3)(2y-3)$

84. $48y^4-3y^2=3y^2(16y^2-1)=3y^2[(4y)^2-1^2]=3y^2(4y+1)(4y-1)$

85. $x^2-12x+36-49y^2$
$=\left(x^2-12x+36\right)-49y^2=\left(x-6\right)^2-49y^2=(x-6+7y)\left(x-6-7y\right)$

86. $x^2-10x+25-36y^2=(x^2-10x+25)-36y^2=(x-5)^2-36y^2=\left(x-5+6y\right)\left(x-5-6y\right)$

87. $9b^2x-16y-16x+9b^2y$
$=\left(9b^2x+9b^2y\right)+\left(-16x-16y\right)=9b^2\left(x+y\right)-16\left(x+y\right)=\left(x+y\right)\left(9b^2-16\right)=\left(x+y\right)\left(3b+4\right)\left(3b-4\right)$

88. $16a^2x-25y-25x+16a^2y$
$=\left(16a^2x+16a^2y\right)+\left(-25y-25x\right)=16a^2\left(x+y\right)-25\left(x+y\right)=\left(x+y\right)\left(16a^2-25\right)=\left(x+y\right)\left(4a+5\right)\left(4a-5\right)$

89. $x^2y-16y+32-2x^2$
$=\left(x^2y-16y\right)+\left(-2x^2+32\right)=y\left(x^2-16\right)-2\left(x^2-16\right)=\left(x^2-16\right)\left(y-2\right)=\left(x+4\right)\left(x-4\right)\left(y-2\right)$

90. $12x^2y-27y-4x^2+9$
$=\left(12x^2y-27y\right)+\left(-4x^2+9\right)=3y\left(4x^2-9\right)-1\left(4x^2-9\right)=\left(4x^2-9\right)\left(3y-1\right)=\left(2x+3\right)\left(2x-3\right)\left(3y-1\right)$

91. $2x^3-8a^2x+24x^2+72x$
$=2x\left(x^2-4a^2+12x+36\right)=2x\left[\left(x^2+12x+36\right)-4a^2\right]=2x\left[\left(x+6\right)^2-4a^2\right]=2x\left(x+6-2a\right)\left(x+6+2a\right)$

92. $2x^3-98a^2x+28x^2+98x$
$=2x\left(x^2-49a^2+14x+49\right)=2x\left[\left(x^2+14x+49\right)-49a^2\right]=2x\left[\left(x+7\right)^2-49a^2\right]=2x\left(x+7-7a\right)\left(x+7+7a\right)$

93. $x^{\frac{3}{2}}-x^{\frac{1}{2}}=x^{\frac{1}{2}}\left(x^{\frac{3}{2}-\frac{1}{2}}\right)-1=x^{\frac{1}{2}}\left(x-1\right)$

94. $x^{\frac{3}{4}}-x^{\frac{1}{4}}=x^{\frac{1}{4}}\left(x^{\frac{3}{4}-\frac{1}{4}}-1\right)=x^{\frac{1}{4}}\left(x^{\frac{1}{2}}-1\right)$

95. $4x^{-\frac{2}{3}}+8x^{\frac{1}{3}}=4x^{-\frac{2}{3}}\left(1+2x^{\frac{1}{3}-\left(-\frac{2}{3}\right)}\right)=4x^{-\frac{2}{3}}(1+2x)=\dfrac{4(1+2x)}{x^{\frac{2}{3}}}$

96. $12x^{-\frac{3}{4}}+6x^{\frac{1}{4}}=6x^{-\frac{3}{4}}\left(2+x^{\frac{1}{4}-\left(-\frac{3}{4}\right)}\right)=6x^{-\frac{3}{4}}(2+x)=\dfrac{6(x+2)}{x^{\frac{3}{4}}}$

97. $(x+3)^{\frac{1}{2}}-(x+3)^{\frac{3}{2}}=(x+3)^{\frac{1}{2}}\left[1-(x+3)^{\frac{3}{2}-\frac{1}{2}}\right]=(x+3)^{\frac{1}{2}}\left[1-(x+3)\right]=(x+3)^{\frac{1}{2}}(-x-2)=-(x+3)^{\frac{1}{2}}(x+2)$

98. $\left(x^2+4\right)^{\frac{3}{2}}+\left(x^2+4\right)^{\frac{7}{2}}=\left(x^2+4\right)^{\frac{3}{2}}\left[1+\left(x^2+4\right)^{\frac{7}{2}-\frac{3}{2}}\right]=\left(x^2+4\right)^{\frac{3}{2}}\left[1+\left(x^2+4\right)^{2}\right]=\left(x^2+4\right)^{\frac{3}{2}}\left(x^4+8x^2+17\right)$

99. $(x+5)^{-\frac{1}{2}}-(x+5)^{-\frac{3}{2}}=(x+5)^{-\frac{3}{2}}\left[(x+5)^{-\frac{1}{2}-\left(-\frac{3}{2}\right)}-1\right]=(x+5)^{-\frac{3}{2}}\left[(x+5)-1\right]=(x+5)^{-\frac{3}{2}}(x+4)=\dfrac{x+4}{(x+5)^{\frac{3}{2}}}$

100. $\left(x^2+3\right)^{-\frac{2}{3}}+\left(x^2+3\right)^{-\frac{5}{3}}=\left(x^2+3\right)^{-\frac{5}{3}}\left[\left(x^2+3\right)^{-\frac{2}{3}-\left(-\frac{5}{3}\right)}+1\right]=\left(x^2+3\right)^{-\frac{5}{3}}\left[\left(x^2+3\right)+1\right]=\dfrac{x^2+4}{(x^2+3)^{5/3}}$

101. $(4x-1)^{\frac{1}{2}}-\frac{1}{3}(4x-1)^{\frac{3}{2}}$

$=(4x-1)^{\frac{1}{2}}\left[1-\frac{1}{3}(4x-1)^{\frac{3}{2}-\frac{1}{2}}\right]=(4x-1)^{\frac{1}{2}}\left[1-\frac{1}{3}(4x-1)\right]=(4x-1)^{\frac{1}{2}}\left[1-\frac{4}{3}x+\frac{1}{3}\right]$

$=(4x-1)^{\frac{1}{2}}\left(\frac{4}{3}-\frac{4}{3}x\right)=(4x-1)^{\frac{1}{2}}\frac{4}{3}(1-x)=\dfrac{-4(4x-1)^{\frac{1}{2}}(x-1)}{3}$

102. $-8(4x+3)^{-2}+10(5x+1)(4x+3)^{-1}=2(4x+3)^{-2}\left[-4+5(5x+1)(4x+3)\right]=\dfrac{2(100x^2+95x+11)}{(4x+3)^2}$

103. $10x^2(x+1)-7x(x+1)-6(x+1)=(x+1)\left(10x^2-7x-6\right)=(x+1)(5x-6)(2x+1)$

104. $12x^2(x-1)-4x(x-1)-5(x-1)=(x-1)\left(12x^2-4x-5\right)=(x-1)(6x-5)(2x+1)$

105. $6x^4+35x^2-6=\left(x^2+6\right)\left(6x^2-1\right)$

106. $7x^4+34x^2-5=\left(7x^2-1\right)\left(x^2+5\right)$

107. $y^7+y=y\left(y^6+1\right)=y\left[\left(y^2\right)^3+1^3\right]=y\left(y^2+1\right)\left(y^4-y^2+1\right)$

108. $(y+1)^3+1=(y+1)^3+1^3=\left[(y+1)+1\right]\left[(y+1)^2-(y+1)+1\right]=(y+2)\left[\left(y^2+2y+1\right)-y-1+1\right]$
$=(y+2)\left(y^2+2y+1-y-1+1\right)=(y+2)\left(y^2+y+1\right)$

109. $x^4-5x^2y^2+4y^4=\left(x^2-4y^2\right)\left(x^2-y^2\right)=(x+2y)(x-2y)(x+y)(x-y)$

110. $x^4-10x^2y^2+9y^4=\left(x^2-9y^2\right)\left(x^2-y^2\right)=(x+3y)(x-3y)(x+y)(x-y)$

111. $(x-y)^4-4(x-y)^2$
$=(x-y)^2\left((x-y)^2-4\right)=(x-y)^2\left((x-y)+2\right)\left((x-y)-2\right)=(x-y)^2\left(x-y+2\right)\left(x-y-2\right)$

112. $(x+y)^4-100(x+y)^2=(x+y)^2\left((x+y)^2-100\right)=(x+y)^2\left(x+y-10\right)\left(x+y+10\right)$

113. $2x^2-7xy^2+3y^4=\left(2x-y^2\right)\left(x-3y^2\right)$

114. $3x^2+5xy^2+2y^4=\left(3x+2y^2\right)\left(x+y^2\right)$

115. a. $(x-0.4x)-0.4(x-0.4x)=(x-0.4x)(1-0.4)=(0.6x)(0.6)=0.36x$

b. No, the computer is selling at 36% of its original price.

116. a. $(x-0.3x)-0.3(x-0.3x)=(x-0.3x)(1-0.3)=(0.7x)(0.7)=0.49x$

b. No, the computer is selling at 49% of its original price.

117. a. $(3x)^2-4\cdot 2^2=9x^2-16$

b. $9x^2-16=(3x+4)(3x-4)$

118. a. $(7x)^2-4\cdot 3^2=49x^2-36$

b. $49x^2-36=(7x+6)(7x-6)$

119. a. $x(x+y)-y(x+y)$

b. $x(x+y)-y(x+y)=(x+y)(x-y)$

120. a. $x^2+xy+xy+y^2=x^2+2xy+y^2$

b. $x^2+2xy+y^2=(x+y)^2$

121. $V_{\text{shaded}} = V_{\text{outside}} - V_{\text{inside}}$
$= a \cdot a \cdot 4a - b \cdot b \cdot 4a$
$= 4a^3 - 4ab^2$
$= 4a\left(a^2 - b^2\right)$
$= 4a\left(a+b\right)\left(a-b\right)$

122. $V_{shaded} = V_{outside} - V_{inside}$
$= a \cdot a \cdot 3a - b \cdot b \cdot 3a$
$= 3a^3 - 3ab^2$
$= 3a\left(a^2 - b^2\right)$
$= 3a\left(a+b\right)\left(a-b\right)$

123. – 129. Answers may vary.

130. makes sense

131. makes sense

132. does not make sense; Explanations will vary. Sample explanation: $4x^2 - 100 = 4(x^2 - 25) = 4(x+5)(x-5)$

133. makes sense

134. false; Changes to make the statement true will vary. A sample change is:
$x^4 - 16 = (x^2 + 4)(x^2 - 4) = (x^2 + 4)(x+2)(x-2)$

135. true

136. false; Changes to make the statement true will vary. A sample change is: The binomial $x^2 + 36$ is prime.

137. false; Changes to make the statement true will vary. A sample change is: $x^3 - 64 = \left(x-4\right)\left(x+4x+16\right)$

138. $x^{2n} + 6x^n + 8 = \left(x^n + 4\right)\left(x^n + 2\right)$

139. $-x^2 - 4x + 5 = -1\left(x^2 + 4x - 5\right) = -1\left(x+5\right)\left(x-1\right) = -\left(x+5\right)(x-1)$

140. $x^4 - y^4 - 2x^3y + 2xy^3$
$= \left(x^4 - y^4\right) + \left(-2x^3y + 2xy^3\right)$
$= \left(x^2 - y^2\right)\left(x^2 + y^2\right) - 2xy\left(x^2 - y^2\right)$
$= \left(x^2 - y^2\right)\left(x^2 + y^2 - 2xy\right)$
$= \left(x-y\right)\left(x+y\right)\left(x^2 - 2xy + y^2\right)$
$= \left(x-y\right)\left(x+y\right)\left(x-y\right)^2$
$= \left(x-y\right)^3\left(x+y\right)$

141. $(x-5)^{-\frac{1}{2}}(x+5)^{-\frac{1}{2}}-(x+5)^{\frac{1}{2}}(x-5)^{-\frac{3}{2}}=(x-5)^{-\frac{3}{2}}(x+5)^{-\frac{1}{2}}\left[(x-5)^{-\frac{1}{2}-\left(-\frac{3}{2}\right)}-(x+5)^{\frac{1}{2}-\left(-\frac{1}{2}\right)}\right]$

$$=(x-5)^{-\frac{3}{2}}(x+5)^{-\frac{1}{2}}\left[(x-5)-(x+5)\right]$$

$$=(x-5)^{-\frac{3}{2}}(x+5)^{-\frac{1}{2}}(-10)=\frac{-10}{(x-5)^{3/2}(x+5)^{1/2}}$$

142. $x^2+bx+15$, b =16, – 16, 8 or –8

143. b = 0, 3, 4, or $-c(c+4)$, where $c > 0$ is an integer.

144. $\frac{x^2+6x+5}{x^2-25}=\frac{(x+5)(x+1)}{(x+5)(x-5)}=\frac{x+1}{x-5}$

145. $\frac{5}{4}\cdot\frac{8}{15}=\frac{5}{4}\cdot\frac{4\cdot 2}{5\cdot 3}=\frac{1}{1}\cdot\frac{2}{3}=\frac{2}{3}$

146. $\frac{1}{2}+\frac{2}{3}=\frac{3}{6}+\frac{4}{6}=\frac{7}{6}$

Section P.6

Check Point Exercises

1. **a.** The denominator would equal zero if $x=-5$, so –5 must be excluded from the domain.

b. $x^2-36=(x+6)(x-6)$

The denominator would equal zero if $x=-6$ or $x = 6$, so –6 and 6 must both must be excluded from the domain.

2. **a.**

$$\frac{x^3+3x^2}{x+3}=\frac{x^2(x+3)}{x+3}=\frac{x^2(x+3)}{x+3}=x^2,\ x\neq -3$$

Because the denominator is $x + 3$, $x\neq -3$

b.

$$\frac{x^2-1}{x^2+2x+1}=\frac{(x-1)(x+1)}{(x+1)(x+1)}=\frac{x-1}{x+1},\ x\neq -1$$

Because the denominator is $(x+1)(x+1), x\neq -1$

3.

$$\frac{x+3}{x^2-4}\cdot\frac{x^2-x-6}{x^2+6x+9}$$
$$=\frac{x+3}{(x+2)(x-2)}\cdot\frac{(x-3)(x+2)}{(x+3)(x+3)}$$
$$=\frac{x+3}{(x+2)(x-2)}\cdot\frac{(x-3)(x+2)}{(x+3)(x+3)}$$
$$=\frac{x-3}{(x-2)(x+3)},\ x\neq -2,\ x\neq 2,\ x\neq -3$$

Because the denominator has factors of $x+2$, $x-2$, and $x+3$, $x\neq -2$, $x\neq 2$, and $x\neq -3$.

4. $\dfrac{x^2-2x+1}{x^3+x} \div \dfrac{x^2+x-2}{3x^2+3}$

$= \dfrac{x^2-2x+1}{x^3+x} \cdot \dfrac{3x^2+3}{x^2+x-2}$

$= \dfrac{(x-1)(x-1)}{x(x^2+1)} \cdot \dfrac{3(x^2+1)}{(x+2)(x-1)}$

$= \dfrac{3(x-1)}{x(x+2)},\ x \neq 0,\ x \neq -2,\ x \neq 1$

5. $\dfrac{x}{x+1} - \dfrac{3x+2}{x+1} = \dfrac{x-3x-2}{x+1}$

$= \dfrac{-2x-2}{x+1}$

$= \dfrac{-2(x+1)}{x+1}$

$= -2, x \neq -1$

6. $\dfrac{3}{x+1} + \dfrac{5}{x-1}$

$= \dfrac{3x(x-1)+5(x+1)}{(x+1)(x-1)}$

$= \dfrac{3x-3+5x+5}{(x+1)(x-1)}$

$= \dfrac{8x+2}{(x+1)(x-1)}$

$= \dfrac{2(4x+1)}{(x+1)(x-1)}$

$= \dfrac{2(4x+1)}{(x+1)(x-1)},\ x \neq -1$ and $x \neq 1$.

7. Factor each denominator completely.

$x^2-6x+9 = (x-3)^2$

$x^2-9 = (x+3)(x-3)$

List the factors of the first denominator.

$x-3,\ x-3$

Add any unlisted factors from the second denominator.

$x-3,\ x-3,\ x+3$

The least common denominator is the product of all factors in the final list.

$(x-3)(x-3)(x+3)$ or $(x-3)^2(x+3)$ is the least common denominator.

8. Find the least common denominator.

$x^2 - 10x + 25 = (x-5)^2$

$2x - 10 = 2(x-5)$

The least common denominator is $2(x - 5)(x - 5)$. Write all rational expressions in terms of the least common denominator.

$$\frac{x}{x^2-10x+25} - \frac{x-4}{2x-10}$$
$$= \frac{x}{(x-5)(x-5)} - \frac{x-4}{2(x-5)}$$
$$= \frac{2x}{2(x-5)(x-5)} - \frac{(x-4)(x-5)}{2(x-5)(x-5)}$$

Add numerators, putting this sum over the least common denominator.

$$= \frac{2x-(x-4)(x-5)}{2(x-5)(x-5)}$$
$$= \frac{2x-(x^2-5x-4x+20)}{2(x-5)(x-5)}$$
$$= \frac{2x-x^2+5x+4x-20}{2(x-5)(x-5)}$$
$$= \frac{2x-x^2+5x+4x-20}{2(x-5)(x-5)}$$
$$= \frac{-x^2+11x-20}{2(x-5)(x-5)}$$
$$= \frac{-x^2+11x-20}{2(x-5)^2},\ x \neq 5$$

9. $$\frac{\frac{1}{x}-\frac{3}{2}}{\frac{1}{x}+\frac{3}{4}} = \frac{\frac{2}{2x}-\frac{3x}{2x}}{\frac{4}{4x}+\frac{3x}{4x}},\ x \neq 0$$
$$= \frac{\frac{2-3x}{2x}}{\frac{4+3x}{4x}},\ x \neq \frac{-4}{3}$$
$$= \frac{2-3x}{2x} \div \frac{4+3x}{4x}$$
$$= \frac{2-3x}{2x} \cdot \frac{4x}{4+3x}$$
$$= \frac{2-3x}{4+3x} \cdot \frac{4}{2}$$
$$= \frac{2-3x}{4+3x} \cdot \frac{2}{1}$$
$$= \frac{2(2-3x)}{4+3x},\ x \neq 0 \textit{ and } x \neq \frac{-4}{3}$$

10. Multiply each of the three terms, $\frac{1}{x+7}$, $\frac{1}{x}$, and 7 by the least common denominator of $x(x+7)$.

$$\frac{\frac{1}{x+7}-\frac{1}{x}}{7} = \frac{x(x+7)\left(\frac{1}{x+7}\right)-x(x+7)\left(\frac{1}{x}\right)}{7x(x+7)}$$
$$= \frac{x-(x+7)}{7x(x+7)}$$
$$= \frac{-7}{7x(x+7)}$$
$$= -\frac{1}{x(x+7)},\ x \neq 0,\ x \neq -7$$

Exercise Set P.6

1. $\frac{7}{x-3},\ x \neq 3$

2. $\frac{13}{x+9},\ x \neq -9$

3. $\frac{x+5}{x^2-25} = \frac{x+5}{(x+5)(x-5)},\ x \neq 5, -5$

4. $\frac{x+7}{x^2-49} = \frac{x+7}{(x+7)(x-7)},\ x \neq 7, -7$

5. $\frac{x-1}{x^2+11x+10} = \frac{x-1}{(x+1)(x+10)},\ x \neq -1, -10$

6. $\frac{x-3}{x^2+4x-45} = \frac{x-3}{(x+9)(x-5)},\ x \neq -9, 5$

7. $$\frac{3x-9}{x^2-6x+9} = \frac{3(x-3)}{(x-3)(x-3)}$$
$$= \frac{3}{x-3},\ x \neq 3$$

8. $\frac{4x-8}{x^2-4x+4} = \frac{4(x-2)}{(x-2)(x-2)} = \frac{4}{x-2},\ x \neq 2$

9. $\frac{x^2-12x+36}{4x-24} = \frac{(x-6)(x-6)}{4(x-6)} = \frac{x-6}{4}.$

$x \neq 6$

10. $\dfrac{x^2-8x+16}{3x-12}=\dfrac{(x-4)(x-4)}{3(x-4)}=\dfrac{x-4}{3},\ x\neq 4$

11. $\dfrac{y^2+7y-18}{y^2-3y+2}=\dfrac{(y+9)(y-2)}{(y-2)(y-1)}=\dfrac{y+9}{y-1},$

$y\neq 1, 2$

12. $\dfrac{y^2-4y-5}{y^2+5y+4}=\dfrac{(y-5)(y+1)}{(y+4)(y+1)}=\dfrac{y-5}{y+4},\ y\neq -4, -1$

13. $\dfrac{x^2+12x+36}{x^2-36}=\dfrac{(x+6)^2}{(x+6)(x-6)}=\dfrac{x+6}{x-6},$

$x\neq 6, -6$

14. $\dfrac{x^2-14x+49}{x^2-49}=\dfrac{(x-7)^2}{(x-7)(x+7)}$

$=\dfrac{x-7}{x+7},$

$x\neq 7, -7$

15. $\dfrac{x-2}{3x+9}\cdot\dfrac{2x+6}{2x-4}=\dfrac{x-2}{3(x+3)}\cdot\dfrac{2(x+3)}{2(x-2)}$

$=\dfrac{2}{6}=\dfrac{1}{3},\ x\neq 2,-3$

16. $\dfrac{6x+9}{3x-15}\cdot\dfrac{x-5}{4x+6}=\dfrac{3(2x+3)}{3(x-5)}\cdot\dfrac{x-5}{2(2x+3)}$

$=\dfrac{3}{6}$

$=\dfrac{1}{2},$

$x\neq 5,-\dfrac{3}{2}$

17. $\dfrac{x^2-9}{x^2}\cdot\dfrac{x^2-3x}{x^2+x-12}$

$=\dfrac{(x-3)(x+3)}{x^2}\cdot\dfrac{x(x-3)}{(x+4)(x-3)}$

$=\dfrac{(x-3)(x+3)}{x(x+4)},\ x\neq 0,-4,3$

18. $\dfrac{x^2-4}{x^2-4x+4}\cdot\dfrac{2x-4}{x+2}=\dfrac{(x+2)(x-2)}{(x-2)^2}\cdot\dfrac{2(x-2)}{x+2}$

$=2,$

$x\neq 2, -2$

19. $\dfrac{x^2-5x+6}{x^2-2x-3}\cdot\dfrac{x^2-1}{x^2-4}$

$=\dfrac{(x-3)(x-2)}{(x-3)(x+1)}\cdot\dfrac{(x+1)(x-1)}{(x-2)(x+2)}$

$=\dfrac{x-1}{x+2},\ x\neq -2, -1, 2, 3$

20. $\dfrac{x^2+5x+6}{x^2+x-6}\cdot\dfrac{x^2-9}{x^2-x-6}$

$=\dfrac{(x+3)(x+2)}{(x+3)(x-2)}\cdot\dfrac{(x-3)(x+3)}{(x-3)(x+2)}=\dfrac{x+3}{x-2},$

$x\neq -3, -2, 2, 3$

21. $\dfrac{x^3-8}{x^2-4}\cdot\dfrac{x+2}{3x}=\dfrac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}\cdot\dfrac{x+2}{3x}$

$=\dfrac{x^2+2x+4}{3x},\ x\neq -2, 0, 2$

22. $\dfrac{x^2+6x+9}{x^3+27}\cdot\dfrac{1}{x+3}$

$=\dfrac{(x+3)(x+3)}{(x+3)(x^2-3x+9)}\cdot\dfrac{1}{x+3}=\dfrac{1}{x^2-3x+9},$

$x\neq -3$

23. $\dfrac{x+1}{3}\div\dfrac{3x+3}{7}=\dfrac{x+1}{3}\div\dfrac{3(x+1)}{7}$

$=\dfrac{x+1}{3}\cdot\dfrac{7}{3(x+1)}$

$=\dfrac{7}{9},\ x\neq -1$

24. $\dfrac{x+5}{7}\div\dfrac{4x+20}{9}=\dfrac{x+5}{7}\div\dfrac{4(x+5)}{9}$

$=\dfrac{x+5}{7}\cdot\dfrac{9}{4(x+5)}$

$=\dfrac{9}{28},$

$x\neq -5$

25. $\dfrac{x^2-4}{x}\div\dfrac{x+2}{x-2}=\dfrac{(x-2)(x+2)}{x}\cdot\dfrac{x-2}{x+2}$

$=\dfrac{(x-2)^2}{x};\ x\neq 0,-2,2$

26. $\frac{x^2-4}{x-2}\div\frac{x+2}{4x-8}=\frac{(x-2)(x+2)}{x-2}\div\frac{x+2}{4(x-2)}$
$=\frac{(x-2)(x+2)}{x-2}\cdot\frac{4(x-2)}{x+2}$
$=4(x-2),$
$x\neq 2,-2$

27. $\frac{4x^2+10}{x-3}\div\frac{6x^2+15}{x^2-9}$
$=\frac{2(2x^2+5)}{x-3}\div\frac{3(2x^2+5)}{(x-3)(x+3)}$
$=\frac{2(2x^2+5)}{x-3}\cdot\frac{(x-3)(x+3)}{3(2x^2+5)}$
$=\frac{2(x+3)}{3},\ x\neq 3,-3$

28. $\frac{x^2+x}{x^2-4}\div\frac{x^2-1}{x^2+5x+6}$
$=\frac{x(x+1)}{(x-2)(x+2)}\div\frac{(x-1)(x+1)}{(x+2)(x+3)}$
$=\frac{x(x+1)}{(x-2)(x+2)}\cdot\frac{(x+2)(x+3)}{(x-1)(x+1)}$
$=\frac{x(x+3)}{(x-2)(x-1)},$
$x\neq 2,1,-1,-2,-3$

29. $\frac{x^2-25}{2x-2}\div\frac{x^2+10x+25}{x^2+4x-5}$
$=\frac{(x-5)(x+5)}{2(x-1)}\div\frac{(x+5)^2}{(x+5)(x-1)}$
$=\frac{(x-5)(x+5)}{2(x-1)}\cdot\frac{(x+5)(x-1)}{(x+5)^2}$
$=\frac{x-5}{2},\ x\neq 1,-5$

30. $\frac{x^2-4}{x^2+3x-10}\div\frac{x^2+5x+6}{x^2+8x+15}$
$=\frac{(x+2)(x-2)}{(x+5)(x-2)}\div\frac{(x+2)(x+3)}{(x+3)(x+5)}$
$=\frac{(x+2)(x-2)}{(x+5)(x-2)}\cdot\frac{(x+3)(x+5)}{(x+2)(x+3)}$
$=1$
$x\neq 2,-2,-3,-5$

31. $\frac{x^2+x-12}{x^2+x-30}\cdot\frac{x^2+5x+6}{x^2-2x-3}\div\frac{x+3}{x^2+7x+6}$
$=\frac{(x+4)(x-3)}{(x+6)(x-5)}\cdot\frac{(x+2)(x+3)}{(x+1)(x-3)}\cdot\frac{(x+6)(x+1)}{x+3}$
$=\frac{(x+4)(x+2)}{x-5}$
$x\neq -6,-3,-1,3,5$

32. $\frac{x^3-25x}{4x^2}\cdot\frac{2x^2-2}{x^2-6x+5}\div\frac{x^2+5x}{7x+7}$
$=\frac{x(x-5)(x+5)}{4x^2}\cdot\frac{2(x-1)(x+1)}{(x-1)(x-5)}\cdot\frac{7(x+1)}{x(x+5)}$
$=\frac{7(x+1)^2}{2x^2}$
$x\neq 0,1,-1,5,-5$

33. $\frac{4x+1}{6x+5}+\frac{8x+9}{6x+5}=\frac{4x+1+8x+9}{6x+5}$
$=\frac{12x+10}{6x+5}$
$=\frac{2(6x+5)}{6x+5}=2,\ x\neq -\frac{5}{6}$

34. $\frac{3x+2}{3x+4}+\frac{3x+6}{3x+4}=\frac{3x+2+3x+6}{3x+4}$
$=\frac{6x+8}{3x+4}$
$=\frac{2(3x+4)}{3x+4}$
$=2$
$x\neq -\frac{4}{3}$

35. $\frac{x^2-2x}{x^2+3x}+\frac{x^2+x}{x^2+3x}=\frac{x^2-2x+x^2+x}{x^2+3x}$

$=\frac{2x^2-x}{x^2+3x}$

$=\frac{x(2x-1)}{x(x+3)}$

$=\frac{2x-1}{x+3},\ x\neq 0,-3$

36. $\frac{x^2-4x}{x^2-x-6}+\frac{4x-4}{x^2-x-6}=\frac{x^2-4x+4x-4}{x^2-x-6}$

$=\frac{x^2-4}{(x-3)(x+2)}$

$=\frac{(x-2)(x+2)}{(x-3)(x+2)}$

$=\frac{x-2}{x-3},$

$x\neq -2,\ 3$

37. $\frac{4x-10}{x-2}-\frac{x-4}{x-2}=\frac{4x-10-(x-4)}{x-2}$

$=\frac{4x-10-x+4}{x-2}$

$=\frac{3x-6}{x-2}$

$=\frac{3(x-2)}{x-2}$

$=3,\ x\neq 2$

38. $\frac{2x+3}{3x-6}-\frac{3-x}{3x-6}=\frac{2x+3-(3-x)}{3x-6}$

$=\frac{2x+3-3+x}{3x-6}$

$=\frac{3x}{3(x-2)}$

$=\frac{x}{x-2},$

$x\neq 2$

39. $\frac{x^2+3x}{x^2+x-12}-\frac{x^2-12}{x^2+x-12}$

$=\frac{x^2+3x-(x^2-12)}{x^2+x-12}$

$=\frac{x^2+3x-x^2+12}{x^2+x-12}$

$=\frac{3x+12}{x^2+x-12}$

$=\frac{3(x+4)}{(x+4)(x-3)}$

$=\frac{3}{x-3},\ x\neq 3,-4$

40. $\frac{x^2-4x}{x^2-x-6}-\frac{x-6}{x^2-x-6}$

$=\frac{x^2-4x-(x-6)}{x^2-x-6}$

$=\frac{x^2-4x-x+6}{x^2-x-6}$

$=\frac{x^2-5x+6}{x^2-x-6}$

$=\frac{(x-2)(x-3)}{(x-3)(x+2)}$

$=\frac{x-2}{x+2},\ x\neq -2,\ 3$

41. $\frac{3}{x+4}+\frac{6}{x+5}=\frac{3(x+5)+6(x+4)}{(x+4)(x+5)}$

$=\frac{3x+15+6x+24}{(x+4)(x+5)}$

$=\frac{9x+39}{(x+4)(x+5)},\ x\neq -4,\ -5$

42. $\frac{8}{x-2}+\frac{2}{x-3}=\frac{8(x-3)+2(x-2)}{(x-2)(x-3)}$

$=\frac{8x-24+2x-4}{(x-2)(x-3)}$

$=\frac{10x-28}{(x-2)(x-3)},$

$x\neq 2,\ 3$

43. $\frac{3}{x+1}-\frac{3}{x}=\frac{3x-3(x+1)}{x(x+1)}$

$=\frac{3x-3x-3}{x(x+1)}=-\frac{3}{x(x+1)}$, $x\neq -1, 0$

44. $\frac{4}{x}-\frac{3}{x+3}=\frac{4(x+3)-3x}{x(x+3)}$

$=\frac{4x+12-3x}{x(x+3)}$

$=\frac{x+12}{x(x+3)}$

$x\neq -3, 0$

45. $\frac{2x}{x+2}+\frac{x+2}{x-2}=\frac{2x(x-2)+(x+2)(x+2)}{(x+2)(x-2)}$

$=\frac{2x^2-4x+x^2+4x+4}{(x+2)(x-2)}$

$=\frac{3x^2+4}{(x+2)(x-2)}$, $x\neq -2, 2$

46. $\frac{3x}{x-3}-\frac{x+4}{x+2}=\frac{3x(x+2)-(x+4)(x-3)}{(x-3)(x+2)}$

$=\frac{3x^2+6x-(x^2+x-12)}{(x-3)(x+2)}$

$=\frac{2x^2+5x+12}{(x-3)(x+2)}$,

$x\neq 3, -2$

47. $\frac{x+5}{x-5}+\frac{x-5}{x+5}$

$=\frac{(x+5)(x+5)+(x-5)(x-5)}{(x-5)(x+5)}$

$=\frac{x^2+10x+25+x^2-10x+25}{(x-5)(x+5)}$

$=\frac{2x^2+50}{(x-5)(x+5)}$, $x\neq -5, 5$

48. $\frac{x+3}{x-3}+\frac{x-3}{x+3}=\frac{(x+3)(x+3)+(x-3)(x-3)}{(x-3)(x+3)}$

$=\frac{x^2+6x+9+x^2-6x+9}{(x-3)(x+3)}$

$=\frac{2x^2+18}{(x-3)(x+3)}$,

$x\neq -3, 3$

49. $\frac{3}{2x+4}+\frac{2}{3x+6}=\frac{3}{2(x+2)}+\frac{2}{3(x+2)}$

$=\frac{9}{6(x+2)}+\frac{4}{6(x+2)}$

$=\frac{9+4}{6(x+2)}$

$=\frac{13}{6(x+2)}$

$x\neq -2$

50. $\frac{5}{2x+8}+\frac{7}{3x+12}=\frac{5}{2(x+4)}+\frac{7}{3(x+4)}$

$=\frac{15}{6(x+4)}+\frac{14}{6(x+4)}$

$=\frac{15+14}{6(x+4)}$

$=\frac{29}{6(x+4)}$

$x\neq -4$

51. $\frac{4}{x^2+6x+9}+\frac{4}{x+3}=\frac{4}{(x+3)^2}+\frac{4}{x+3}$

$=\frac{4+4(x+3)}{(x+3)^2}=\frac{4+4x+12}{(x+3)^2}=\frac{4x+16}{(x+3)^2}$,

$x\neq -3$

52. $\frac{3}{5x+2}+\frac{5x}{25x^2-4}=\frac{3}{5x+2}+\frac{5x}{(5x-2)(5x+2)}$

$=\frac{3(5x-2)+5x}{(5x-2)(5x+2)}$

$=\frac{15x-6+5x}{(5x-2)(5x+2)}$

$=\frac{20x-6}{(5x-2)(5x+2)}$,

$x\neq -\frac{2}{5}, \frac{2}{5}$

53. $\dfrac{3x}{x^2+3x-10}-\dfrac{2x}{x^2+x-6}$

$=\dfrac{3x}{(x+5)(x-2)}-\dfrac{2x}{(x+3)(x-2)}$

$=\dfrac{3x(x+3)-2x(x+5)}{(x+5)(x-2)(x+3)}$

$=\dfrac{3x^2+9x-2x^2-10x}{(x+5)(x-2)(x+3)}$

$=\dfrac{x^2-x}{(x+5)(x-2)(x+3)}$, $x \neq -5, 2, -3$

54. $\dfrac{x}{x^2-2x-24}-\dfrac{x}{x^2-7x+6}$

$=\dfrac{x}{(x-6)(x+4)}-\dfrac{x}{(x-6)(x-1)}$

$=\dfrac{x(x-1)-x(x+4)}{(x-6)(x+4)(x-1)}$

$=\dfrac{x^2-x-x^2-4x}{(x-6)(x+4)(x-1)}$

$=-\dfrac{5x}{(x-6)(x-1)(x+4)}$,

$x \neq 6, 1, -4$

55. $\dfrac{x+3}{x^2-1}-\dfrac{x+2}{x-1}$

$=\dfrac{x+3}{(x+1)(x-1)}-\dfrac{x+2}{x-1}$

$=\dfrac{x+3}{(x+1)(x-1)}-\dfrac{(x+1)(x+2)}{(x+1)(x-1)}$

$=\dfrac{x+3}{(x+1)(x-1)}-\dfrac{x^2+3x+2}{(x+1)(x-1)}$

$=\dfrac{x+3-x^2-3x-2}{(x+1)(x-1)}$

$=\dfrac{-x^2-2x+1}{(x+1)(x-1)}$

$x \neq 1, -1$

56. $\dfrac{x+5}{x^2-4}-\dfrac{x+1}{x-2}$

$=\dfrac{x+5}{(x+2)(x-2)}-\dfrac{x+1}{x-2}$

$=\dfrac{x+5}{(x+2)(x-2)}-\dfrac{(x+2)(x+1)}{(x+2)(x-2)}$

$=\dfrac{x+5}{(x+2)(x-2)}-\dfrac{x^2+3x+2}{(x+2)(x-2)}$

$=\dfrac{x+5-x^2-3x-2}{(x+2)(x-2)}$

$=\dfrac{-x^2-2x+3}{(x+2)(x-2)}$

$x \neq 2, -2$

57. $\dfrac{4x^2+x-6}{x^2+3x+2}-\dfrac{3x}{x+1}+\dfrac{5}{x+2}$

$=\dfrac{4x^2+x-6}{(x+1)(x+2)}+\dfrac{-3x}{x+1}+\dfrac{5}{x+2}$

$=\dfrac{4x^2+x-5}{(x+1)(x+2)}+\dfrac{-3x(x+2)}{(x+1)(x+2)}+\dfrac{5(x+1)}{(x+1)(x+2)}$

$=\dfrac{4x^2+x-6-3x^2-6x+5x+5}{(x+1)(x+2)}$

$=\dfrac{x^2-1}{(x+1)(x+2)}$

$=\dfrac{(x-1)(x+1)}{(x+1)(x+2)}$

$=\dfrac{x-1}{x+2}; x \neq -2, -1$

58. $\dfrac{6x^2+17x-40}{x^2+x-20}+\dfrac{3}{x-4}-\dfrac{5x}{x+5}$

$=\dfrac{6x^2+17x-40}{(x+5)(x-4)}+\dfrac{3}{x-4}-\dfrac{5x}{x+5}$

$=\dfrac{6x^2+17x-40+3(x+5)-5x(x-4)}{(x+5)(x-4)}$

$=\dfrac{6x^2+17x-40+3x+15-5x^2+20x}{(x+5)(x-4)}$

$=\dfrac{x^2+40x-25}{(x+5)(x-4)}; x \neq -5, 4$

59. $\dfrac{\frac{x}{3}-1}{x-3}=\dfrac{3\left[\frac{x}{3}-1\right]}{3[x-3]}=\dfrac{x-3}{3(x-3)}=\dfrac{1}{3}$, $x \neq 3$

60. $\dfrac{\frac{x}{4}-1}{x-4}=\dfrac{4\left[\frac{x}{4}-1\right]}{4(x-4)}=\dfrac{x-4}{4(x-4)}=\dfrac{1}{4},\ x\neq 4$

61. $\dfrac{1+\frac{1}{x}}{3-\frac{1}{x}}=\dfrac{x\left[1+\frac{1}{x}\right]}{x\left[3-\frac{1}{x}\right]}=\dfrac{x+1}{3x-1},\ x\neq 0,\dfrac{1}{3}$

62. $\dfrac{8+\frac{1}{x}}{4-\frac{1}{x}}=\dfrac{x\left[8+\frac{1}{x}\right]}{x\left[4-\frac{1}{x}\right]}=\dfrac{8x+1}{4x-1},\ x\neq 0,\dfrac{1}{4}$

63. $\dfrac{\frac{1}{x}+\frac{1}{y}}{x+y}=\dfrac{xy\left[\frac{1}{x}+\frac{1}{y}\right]}{xy[x+y]}=\dfrac{y+x}{xy(x+y)}=\dfrac{1}{xy},$

$x\neq 0,\ y\neq 0,\ x\neq -y$

64. $\dfrac{1-\frac{1}{x}}{xy}=\dfrac{x\left[1-\frac{1}{x}\right]}{x(xy)}=\dfrac{x-1}{x^2y},\ x\neq 0,\ y\neq 0$

65. $\dfrac{x-\frac{x}{x+3}}{x+2}=\dfrac{(x+3)\left[x-\frac{x}{x+3}\right]}{(x+3)(x+2)}=\dfrac{x(x+3)-x}{(x+3)(x+2)}$

$=\dfrac{x^2+3x-x}{(x+3)(x+2)}=\dfrac{x^2+2x}{(x+3)(x+2)}$

$=\dfrac{x(x+2)}{(x+3)(x+2)}=\dfrac{x}{x+3},\ x\neq -2,-3$

66. $\dfrac{x-3}{x-\frac{3}{x-2}}=\dfrac{(x-2)[x-3]}{(x-2)\left[x-\frac{3}{x-2}\right]}=\dfrac{(x-2)(x-3)}{x(x-2)-3}$

$=\dfrac{(x-2)(x-3)}{x^2-2x-3}$

$=\dfrac{(x-2)(x-3)}{(x-3)(x+1)}=\dfrac{x-2}{x+1},\ x\neq 2,3,-1$

67. $\dfrac{\frac{3}{x-2}-\frac{4}{x+2}}{\frac{7}{x^2-4}}=\dfrac{\frac{3}{x-2}-\frac{4}{x+2}}{\frac{7}{(x-2)(x+2)}}$

$=\dfrac{\left[\frac{3}{x-2}-\frac{4}{x+2}\right](x-2)(x+2)}{\left[\frac{7}{(x-2)(x+2)}\right](x-2)(x+2)}$

$=\dfrac{3(x+2)-4(x-2)}{7}$

$=\dfrac{3x+6-4x+8}{7}=\dfrac{-x+14}{7}$

$=-\dfrac{x-14}{7}\quad x\neq -2,2$

68. $\dfrac{\frac{x}{x-2}+1}{\frac{3}{x^2-4}+1}=\dfrac{\frac{x}{x-2}+1}{\frac{3}{(x-2)(x+2)}+1}$

$=\dfrac{\left[\frac{x}{x-2}+1\right](x-2)(x+2)}{\left[\frac{3}{(x-2)(x+2)}+1\right](x-2)(x+2}$

$=\dfrac{x(x+2)+(x-2)(x+2)}{3+(x-2)(x+2)}$

$=\dfrac{x^2+2x+x^2-4}{3+x^2-4}=\dfrac{2x^2+2x-4}{x^2-1}$

$=\dfrac{2(x^2+x-2)}{(x-1)(x+1)}$

$=\dfrac{2(x+2)(x-1)}{(x-1)(x+1)}=\dfrac{2(x+2)}{x+1},$

$x\neq 1,-1,\ 2,\ -2$

69. $\dfrac{\frac{1}{x+1}}{\frac{1}{x^2-2x-3}+\frac{1}{x-3}}=\dfrac{\frac{1}{x+1}}{\frac{1}{(x+1)(x-3)}+\frac{1}{x-3}}$

$=\dfrac{\frac{(x+1)(x-3)}{x+1}}{\frac{(x+1)(x-3)}{(x+1)(x-3)}+\frac{(x+1)(x-3)}{x-3}}$

$=\dfrac{x-3}{1+x+1}$

$=\dfrac{x-3}{x+2}\quad x\neq -2,-1,3$

70. $$\frac{\frac{6}{x^2+2x-15}-\frac{1}{x-3}}{\frac{1}{x+5}+1}=\frac{\frac{6}{(x+5)(x-3)}-\frac{1}{x-3}}{\frac{1}{x+5}+1}$$

$$=\frac{\frac{6(x+5)(x-3)}{(x+5)(x-3)}-\frac{(x+5)(x-3)}{x-3}}{\frac{(x+5)(x-3)}{x+5}+(x+5)(x-3)}$$

$$=\frac{6-(x+5)}{(x-3)+(x+5)(x-3)}$$

$$=\frac{6-x-5}{x-3+x^2+2x-15}$$

$$=\frac{1-x}{x^2+3x-18}$$

$$=\frac{1-x}{(x+6)(x-3)}\quad x\neq -6,-5,3$$

71. $$\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}=\frac{\frac{x^2(x+h)^2}{(x+h)^2}-\frac{x^2(x+h)^2}{x^2}}{hx^2(x+h)^2}$$

$$=\frac{x^2-(x+h)^2}{hx^2(x+h)^2}$$

$$=\frac{x^2-(x^2+2hx+h^2)}{hx^2(x+h)^2}$$

$$=\frac{x^2-x^2-2hx-h^2}{hx^2(x+h)^2}$$

$$=\frac{-2hx-h^2}{hx^2(x+h)^2}$$

$$=\frac{-h(2x+h)}{hx^2(x+h)^2}$$

$$=-\frac{(2x+h)}{x^2(x+h)^2}$$

72. $$\frac{\frac{x+h}{x+h+1}-\frac{x}{x+1}}{h}=\frac{\frac{(x+h)(x+h+1)(x+1)}{x+h+1}-\frac{x(x+h+1)(x+1)}{x+1}}{h(x+h+1)(x+1)}$$

$$=\frac{(x+h)(x+1)-x(x+h+1)}{h(x+h+1)(x+1)}$$

$$=\frac{x^2+x+hx+h-x^2-hx-x}{h(x+h+1)(x+1)}$$

$$=\frac{h}{h(x+h+1)(x+1)}$$

$$=\frac{1}{(x+h+1)(x+1)}$$

73. $\left(\frac{2x+3}{x+1}\cdot\frac{x^2+4x-5}{2x^2+x-3}\right)-\frac{2}{x+2}=\left(\frac{\cancel{(2x+3)}}{x+1}\cdot\frac{(x+5)\cancel{(x-1)}}{\cancel{(2x+3)}\cancel{(x-1)}}\right)-\frac{2}{x+2}=\frac{x+5}{x+1}-\frac{2}{x+2}$

$=\frac{(x+5)(x+2)}{(x+1)(x+2)}-\frac{2(x+1)}{(x+1)(x+2)}=\frac{(x+5)(x+2)-2(x+1)}{(x+1)(x+2)}=\frac{x^2+2x+5x+10-2x-2}{(x+1)(x+2)}=\frac{x^2+5x+8}{(x+1)(x+2)}$

74. $\frac{1}{x^2-2x-8}\cdot\left(\frac{1}{x-4}-\frac{1}{x+2}\right)=\frac{1}{(x-4)(x+2)}\div\left(\frac{(x+2)}{(x-4)(x+2)}-\frac{(x-4)}{(x-4)(x+2)}\right)$

$=\frac{1}{(x-4)(x+2)}\div\left(\frac{x+2-x+4}{(x-4)(x+2)}\right)=\frac{1}{(x-4)(x+2)}\div\left(\frac{6}{(x-4)(x+2)}\right)=\frac{1}{(x-4)(x+2)}\cdot\frac{(x-4)(x+2)}{6}=\frac{1}{6}$

75. $\left(2-\frac{6}{x+1}\right)\left(1+\frac{3}{x-2}\right)=\left(\frac{2(x+1)}{(x+1)}-\frac{6}{(x+1)}\right)\left(\frac{(x-2)}{(x-2)}+\frac{3}{(x-2)}\right)$

$=\left(\frac{2x+2-6}{x+1}\right)\left(\frac{x-2+3}{x-2}\right)=\left(\frac{2x-4}{x+1}\right)\left(\frac{x+1}{x-2}\right)=\frac{2\cancel{(x-2)}\cancel{(x+1)}}{\cancel{(x+1)}\cancel{(x-2)}}=2$

76. $\left(4-\frac{3}{x+2}\right)\left(1+\frac{5}{x-1}\right)=\left(\frac{4(x+2)}{x+2}-\frac{3}{x+2}\right)\left(\frac{(x-1)}{x-1}+\frac{5}{x-1}\right)$

$=\left(\frac{4x+8-3}{x+2}\right)\left(\frac{x-1+5}{x-1}\right)=\frac{4x+5}{x+2}\cdot\frac{x+4}{x-1}=\frac{(4x+5)(x+4)}{(x+2)(x-1)}$

77. $\frac{y^{-1}-(y+5)^{-1}}{5}=\frac{\frac{1}{y}-\frac{1}{y+5}}{5}$

LCD $=y(y+5)$

$\frac{\frac{1}{y}-\frac{1}{y+5}}{5}=\frac{y(y+5)\left(\frac{1}{y}-\frac{1}{y+5}\right)}{y(y+5)(5)}=\frac{y+5-y}{5y(y+5)}=\frac{5}{5y(y+5)}=\frac{1}{y(y+5)}$

78. $\frac{y^{-1}-(y+2)^{-1}}{2}=\frac{\frac{1}{y}-\frac{1}{y+2}}{2}$

LCD $=y(y+2)$

$\frac{\frac{1}{y}-\frac{1}{y+2}}{2}=\frac{y(y+2)\left(\frac{1}{y}-\frac{1}{y+2}\right)}{y(y+2)(2)}=\frac{y+2-y}{2y(y+2)}=\frac{2}{2y(y+2)}=\frac{1}{y(y+2)}$

79. $\left(\frac{1}{a^3-b^3}\cdot\frac{ac+ad-bc-bd}{1}\right)-\frac{c-d}{a^2+ab+b^2}=\left(\frac{1}{(a-b)\left(a^2+ab+b^2\right)}\cdot\frac{a(c+d)-b(c+d)}{1}\right)-\frac{c-d}{a^2+ab+b^2}$

$=\left(\frac{1}{\cancel{(a-b)}\left(a^2+ab+b^2\right)}\cdot\frac{(c+d)\cancel{(a-b)}}{1}\right)-\frac{c-d}{a^2+ab+b^2}=\frac{c+d}{a^2+ab+b^2}-\frac{c-d}{a^2+bd+b^2}$

$=\frac{c+d-c+d}{a^2+ab+b^2}=\frac{2d}{a^2+ab+b^2}$

80. $\frac{ab}{a^2+ab+b^2}+\left(\frac{ac-ad-bc+bd}{ac-ad+bc-bd}\div\frac{a^3-b^3}{a^3+b^3}\right)=\frac{ab}{a^2+ab+b^2}+\left(\frac{a(c-d)-b(c-d)}{a(c-d)+b(c-d)}\cdot\frac{a^3+b^3}{a^3-b^3}\right)$

$=\frac{ab}{a^2+ab+b^2}+\left(\frac{\cancel{(c-d)}\cancel{(a-b)}}{\cancel{(c-d)}\cancel{(a+b)}}\cdot\frac{\cancel{(a+b)}\left(a^2-ab+b^2\right)}{\cancel{(a-b)}\left(a^2+ab+b^2\right)}\right)=\frac{ab}{a^2+ab+b^2}+\frac{a^2-ab+b^2}{a^2+ab+b^2}$

$=\frac{ab+a^2-ab+b^2}{a^2+ab+b^2}=\frac{a^2+b^2}{a^2+ab+b^2}$

81. a. $\frac{130x}{100-x}$ is equal to

1. $\frac{130\cdot 40}{100-40}=\frac{130\cdot 40}{60}=86.67$,
   when $x = 40$
2. $\frac{130\cdot 80}{100-80}=\frac{130\cdot 80}{20}=520$,
   when $x = 80$
3. $\frac{130\cdot 90}{100-90}=\frac{130\cdot 90}{10}=1170$,
   when $x = 90$

It costs \$86,670,000 to inoculate 40% of the population against this strain of flu, and \$520,000,000 to inoculate 80% of the population, and \$1,170,000,000 to inoculate 90% of the population.

b. For $x = 100$, the function is not defined.

c. As x approaches 100, the value of the function increases rapidly. So it costs an astronomical amount of money to inoculate almost all of the people, and it is impossible to inoculate 100% of the population.

82. $\dfrac{2d}{\frac{d}{r_1}+\frac{d}{r_2}}$

LCD = r_1r_2

$$\frac{2d}{\frac{d}{r_1}+\frac{d}{r_2}} = \frac{r_1r_2(2d)}{r_1r_2\left(\frac{d}{r_1}+\frac{d}{r_2}\right)}$$
$$= \frac{2r_1r_2d}{r_2d+r_1d}$$
$$= \frac{2r_1r_2d}{d(r_2+r_1)} = \frac{2r_1r_2}{r_2+r_1}$$

If r_1 = 40 and r_2 = 30, the value of this expression will be

$$\frac{2\cdot 40\cdot 30}{30+40} = \frac{2400}{70}$$
$$= 34\frac{2}{7}.$$

Your average speed will be $34\frac{2}{7}$ miles per hour.

83. **a.** Substitute 4 for *x* in the model.

$W = -66x^2 + 526x + 1030$

$W = -66(4)^2 + 526(4) + 1030$

$W = 2078$

According to the model, women between the ages of 19 and 30 with this lifestyle need 2078 calories per day. This underestimates the actual value shown in the bar graph by 22 calories.

b. Substitute 4 for *x* in the model.

$M = -120x^2 + 998x + 590$

$M = -120(4)^2 + 998(4) + 590$

$M = 2662$

According to the model, men between the ages of 19 and 30 with this lifestyle need 2662 calories per day. This underestimates the actual value shown in the bar graph by 38 calories.

c.
$$\frac{W}{M} = \frac{-66x^2+526x+1030}{-120x^2+998x+590}$$
$$= \frac{2(-33x^2+263x+515)}{2(-60x^2+499x+295)}$$
$$= \frac{-33x^2+263x+515}{-60x^2+499x+295}$$

84.
$$P = 2L + 2W$$
$$= 2\left(\frac{x}{x+3}\right)+2\left(\frac{x}{x-4}\right)$$
$$= \frac{2x}{x+3}+\frac{2x}{x+4}$$
$$= \frac{2x(x+4)}{(x+3)(x+4)}+\frac{2x(x+3)}{(x+3)(x+4)}$$
$$= \frac{2x^2+8x+2x^2+6x}{(x+3)(x+4)}$$
$$= \frac{4x^2+14x}{(x+3)(x+4)}$$

85.
$$P = 2L + 2W$$
$$= 2\left(\frac{x}{x+5}\right)+2\left(\frac{x}{x+6}\right)$$
$$= \frac{2x}{x+5}+\frac{2x}{x+6}$$
$$= \frac{2x(x+6)}{(x+5)(x+6)}+\frac{2x(x+5)}{(x+5)(x+6)}$$
$$= \frac{2x^2+12x+2x^2+10x}{(x+5)(x+6)}$$
$$= \frac{4x^2+22x}{(x+5)(x+6)}$$

86. – 97. Answers may vary.

98. does not make sense; Explanations will vary. Sample explanation: $\dfrac{3x-3}{4x(x-1)} = \dfrac{3(1)-3}{4(1)(1-1)} = \dfrac{0}{0}$ which is undefined.

99. does not make sense; Explanations will vary. Sample explanation: The numerator and denominator of $\dfrac{7}{14+x}$ do not share a common factor.

100. does not make sense; Explanations will vary. Sample explanation: The first step is to invert the second fraction.

101. makes sense

102. false; Changes to make the statement true will vary. A sample change is: $\frac{x^2-25}{x-5}=\frac{(x+5)(x-5)}{x-5}=x+5$

103. true

104. true

105. false; Changes to make the statement true will vary. A sample change is: $6+\frac{1}{x}=\frac{6x}{x}+\frac{1}{x}=\frac{6x+1}{x}$

106.
$$\frac{1}{x^n-1}-\frac{1}{x^n+1}-\frac{1}{x^{2n}-1}$$
$$=\frac{x^n+1}{x^{2n}-1}-\frac{x^n-1}{x^{2n}-1}-\frac{1}{x^{2n}-1}$$
$$=\frac{x^n+1-x^n+1-1}{x^{2n}-1}$$
$$=\frac{1}{x^{2n}-1}$$

107.
$$\left(1-\frac{1}{x}\right)\left(1-\frac{1}{x+1}\right)\left(1-\frac{1}{x+2}\right)\left(1-\frac{1}{x+3}\right)=\left(\frac{x}{x}-\frac{1}{x}\right)\left(\frac{x+1}{x+1}-\frac{1}{x+1}\right)\left(\frac{x+2}{x+2}-\frac{1}{x+2}\right)\left(\frac{x+3}{x+3}-\frac{1}{x+3}\right)$$
$$=\left(\frac{x-1}{x}\right)\left(\frac{(x+1)-1}{x+1}\right)\left(\frac{(x+2)-1}{x+2}\right)\left(\frac{(x+3)-1}{x+3}\right)$$
$$=\frac{x-1}{\cancel{x}}\cdot\frac{\cancel{x}}{\cancel{x+1}}\cdot\frac{\cancel{x+1}}{\cancel{x+2}}\cdot\frac{\cancel{x+2}}{x+3}=\frac{x-1}{x+3}$$

108.
$$(x-y)^{-1}+(x-y)^{-2}=\frac{1}{(x-y)}+\frac{1}{(x-y)^2}=\frac{(x-y)}{(x-y)(x-y)}+\frac{1}{(x-y)^2}=\frac{x-y+1}{(x-y)^2}$$

109. It cubes x.
$$\frac{\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}}{\frac{1}{x^4}+\frac{1}{x^5}+\frac{1}{x^6}}=\frac{\frac{x^6}{x}+\frac{x^6}{x^2}+\frac{x^6}{x^3}}{\frac{x^6}{x^4}+\frac{x^6}{x^5}+\frac{x^6}{x^6}}=\frac{x^5+x^4+x^3}{x^2+x+1}=\frac{x^3\left(x^2+x+1\right)}{x^2+x+1}=x^3$$

110. $y = 4 - x^2$

x	$y = 4 - x^2$
-3	$4-(-3)^2 = -5$
-2	$4-(-2)^2 = 0$
-1	$4-(-1)^2 = 3$
0	$4-(0)^2 = 4$
1	$4-(1)^2 = 3$
2	$4-(2)^2 = 0$
3	$4-(3)^2 = -5$

111. $y = 1 - x^2$

x	$y = 1 - x^2$
-3	$1-(-3)^2 = -8$
-2	$1-(-2)^2 = -3$
-1	$1-(-1)^2 = 0$
0	$1-(0)^2 = 1$
1	$1-(1)^2 = 0$
2	$1-(2)^2 = -3$
3	$1-(3)^2 = -8$

112. $y = |x+1|$

x	$y = \|x+1\|$
-4	$\|-4+1\| = 3$
-3	$\|-3+1\| = 2$
-2	$\|-2+1\| = 1$
-1	$\|-1+1\| = 0$
0	$\|0+1\| = 1$
1	$\|1+1\| = 2$
2	$\|2+1\| = 3$

Chapter P Review Exercises

1. $$\begin{aligned} 3+6(x-2)^3 &= 3+6(4-2)^3 \\ &= 3+6(2)^3 \\ &= 3+6(8) \\ &= 3+48 \\ &= 51 \end{aligned}$$

2. $$\begin{aligned} x^2-5(x-y) &= 6^2-5(6-2) \\ &= 36-5(4) \\ &= 36-20 \\ &= 16 \end{aligned}$$

3. $$\begin{aligned} S &= 0.015x^2+x+10 \\ S &= 0.015(60)^2+(60)+10 \\ &= 0.015(3600)+60+10 \\ &= 54+60+10 \\ &= 124 \end{aligned}$$

4. $A=\{a,b,c\}\quad B=\{a,c,d,e\}$
$\{a,b,c\}\cap\{a,c,d,e\}=\{a,c\}$

5. $A=\{a,b,c\}\quad B=\{a,c,d,e\}$
$\{a,b,c\}\cup\{a,c,d,e\}=\{a,b,c,d,e\}$

6. $A=\{a,b,c\}\quad C=\{a,d,f,g\}$
$\{a,b,c\}\cup\{a,d,f,g\}=\{a,b,c,d,f,g\}$

7. $A=\{a,b,c\}\quad C=\{a,d,f,g\}$
$\{a,d,f,g\}\cap\{a,b,c\}=\{a\}$

8. **a.** $\sqrt{81}$

b. $0, \sqrt{81}$

c. $-17, 0, \sqrt{81}$

d. $-17, -\frac{9}{13}, 0, 0.75, \sqrt{81}$

e. $\sqrt{2}, \pi$

f. $-17, -\frac{9}{13}, 0, 0.75, \sqrt{2}, \pi, \sqrt{81}$

9. $|-103| = 103$

10. $\left|\sqrt{2}-1\right| = \sqrt{2}-1$

11. $\left|3-\sqrt{17}\right| = \sqrt{17}-3$ since $\sqrt{17}$ is greater than 3.

12. $|4-(-17)| = |4+17| = |21| = 21$

13. 3 + 17 = 17 + 3;
commutative property of addition.

14. $(6\cdot 3)\cdot 9 = 6\cdot(3\cdot 9)$;
associative property of multiplication.

15. $\sqrt{3}(\sqrt{5}+\sqrt{3}) = \sqrt{15}+3$;
distributive property of multiplication over addition.

16. $(6\cdot 9)\cdot 2 = 2\cdot(6\cdot 9)$;
commutative property of multiplication.

17. $\sqrt{3}(\sqrt{5}+\sqrt{3}) = (\sqrt{5}+\sqrt{3})\sqrt{3}$;
commutative property of multiplication.

18. $(3\cdot 7)+(4\cdot 7) = (4\cdot 7)+(3\cdot 7)$;
commutative property of addition.

19. $5(2x-3)+7x = 10x-15+7x = 17x-15$

20. $\frac{1}{5}(5x)+\left[(3y)+(-3y)\right]-(-x) = x+\left[0\right]+x = 2x$

21. $3(4y-5)-(7y+2) = 12y-15-7y-2 = 5y-17$

22.
$$\begin{aligned} 8-2[3-(5x-1)] &= 8-2[3-5x+1] \\ &= 8-2[4-5x] \\ &= 8-8+10x \\ &= 10x \end{aligned}$$

23. $P = -0.05x^2+3.6x-15$
$P = -0.05(21)^2+3.6(21)-15$
$= 38.55$
38.55% of 21 year olds have been tested.
This overestimates the percent displayed by the bar graph by 3.55%.

24. $(-3)^3(-2)^2 = (-27)\cdot(4) = -108$

25.
$$\begin{aligned} 2^{-4}+4^{-1} &= \frac{1}{2^4}+\frac{1}{4} \\ &= \frac{1}{16}+\frac{1}{4} \\ &= \frac{1}{16}+\frac{4}{16} \\ &= \frac{5}{16} \end{aligned}$$

26. $5^{-3}\cdot 5 = 5^{-3}5^1 = 5^{-3+1} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

27. $\frac{3^3}{3^6} = 3^{3-6} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

28.
$$\begin{aligned} (-2x^4y^3)^3 &= (-2)^3(x^4)^3(y^3)^3 \\ &= (-2)^3x^{4\cdot 3}y^{3\cdot 3} \\ &= -8x^{12}y^9 \end{aligned}$$

29.
$$\begin{aligned} &(-5x^3y^2)(-2x^{-11}y^{-2}) \\ &= (-5)(-2)x^3x^{-11}y^2y^{-2} \\ &= 10\cdot x^{3-11}y^{2-2} \\ &= 10x^{-8}y^0 \\ &= \frac{10}{x^8} \end{aligned}$$

30.
$$\begin{aligned} (2x^3)^{-4} &= (2)^{-4}(x^3)^{-4} \\ &= 2^{-4}x^{-12} \\ &= \frac{1}{2^4x^{12}} \\ &= \frac{1}{16x^{12}} \end{aligned}$$

31.
$$\begin{aligned} \frac{7x^5y^6}{28x^{15}y^{-2}} &= \left(\frac{7}{28}\right)(x^{5-15})(y^{6-(-2)}) \\ &= \frac{1}{4}x^{-10}y^8 \\ &= \frac{y^8}{4x^{10}} \end{aligned}$$

32. $3.74\times 10^4 = 37,400$

33. $7.45\times 10^{-5} = 0.0000745$

34. $3,590,000 = 3.59\times 10^6$

35. $0.00725 = 7.25\times 10^{-3}$

36.
$$\begin{aligned} (3\times 10^3)(1.3\times 10^2) &= (3\times 1.3)\times(10^3\times 10^2) \\ &= 3.9\times 10^5 \\ &= 390,000 \end{aligned}$$

37.
$$\begin{aligned} \frac{6.9\times 10^3}{3\times 10^5} &= \left(\frac{6.9}{3}\right)\times 10^{3-5} \\ &= 2.3\times 10^{-2} \\ &= 0.023 \end{aligned}$$

38. $257\times 10^9 = 2.57\times 10^2\cdot 10^9 = 2.57\times 10^{11}$

39. $175\times 10^6 = 1.75\times 10^2\cdot 10^6 = 1.75\times 10^8$

40. $\frac{2.57\times10^{11}}{1.75\times10^{8}}=\frac{2.57}{1.75}\cdot\frac{10^{11}}{10^{8}}\approx1.469\times10^{3}=1469$
The average tax return cost \$1469.

41. $\sqrt{300}=\sqrt{100\cdot3}=\sqrt{100}\cdot\sqrt{3}=10\sqrt{3}$

42. $\sqrt{12x^2}=\sqrt{4x^2\cdot3}=\sqrt{4x^2}\cdot\sqrt{3}=2|x|\sqrt{3}$

43. $\sqrt{10x}\cdot\sqrt{2x}=\sqrt{20x^2}$
$=\sqrt{4x^2}\cdot\sqrt{5}$
$=2x\sqrt{5}$

44. $\sqrt{r^3}=\sqrt{r^2}\cdot\sqrt{r}=r\sqrt{r}$

45. $\sqrt{\frac{121}{4}}=\frac{\sqrt{121}}{\sqrt{4}}=\frac{11}{2}$

46. $\frac{\sqrt{96x^3}}{\sqrt{2x}}=\sqrt{\frac{96x^3}{2x}}$
$=\sqrt{48x^2}$
$=\sqrt{16x^2}\cdot\sqrt{3}$
$=4x\sqrt{3}$

47. $7\sqrt{5}+13\sqrt{5}=(7+13)\sqrt{5}=20\sqrt{5}$

48. $2\sqrt{50}+3\sqrt{8}=2\sqrt{25\cdot2}+3\sqrt{4\cdot2}$
$=2\cdot5\sqrt{2}+3\cdot2\sqrt{2}$
$=10\sqrt{2}+6\sqrt{2}$
$=16\sqrt{2}$

49. $4\sqrt{72}-2\sqrt{48}=4\sqrt{36\cdot2}-2\sqrt{16\cdot3}$
$=4\cdot6\sqrt{2}-2\cdot4\sqrt{3}$
$=24\sqrt{2}-8\sqrt{3}$

50. $\frac{30}{\sqrt{5}}=\frac{30}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{30\sqrt{5}}{5}=6\sqrt{5}$

51. $\frac{\sqrt{2}}{\sqrt{3}}=\frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$

52. $\frac{5}{6+\sqrt{3}}=\frac{5}{6+\sqrt{3}}\cdot\frac{6-\sqrt{3}}{6-\sqrt{3}}$
$=\frac{5(6-\sqrt{3})}{36-3}$
$=\frac{5(6-\sqrt{3})}{33}$

53.
$\frac{14}{\sqrt{7}-\sqrt{5}}=\frac{14}{\sqrt{7}-\sqrt{5}}\cdot\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}$
$=\frac{14(\sqrt{7}+\sqrt{5})}{7-5}$
$=\frac{14(\sqrt{7}+\sqrt{5})}{2}$
$=7(\sqrt{7}+\sqrt{5})$

54. $\sqrt[3]{125}=5$

55. $\sqrt[5]{-32}=-2$

56. $\sqrt[4]{-125}$ is not a real number.

57. $\sqrt[4]{(-5)^4}=\sqrt[4]{625}=\sqrt[4]{5^4}=5$

58. $\sqrt[3]{81}=\sqrt[3]{27\cdot3}=\sqrt[3]{27}\cdot\sqrt[3]{3}=3\sqrt[3]{3}$

59. $\sqrt[3]{y^5}=\sqrt[3]{y^3y^2}=y\sqrt[3]{y^2}$

60. $\sqrt[4]{8}\cdot\sqrt[4]{10}=\sqrt[4]{80}=\sqrt[4]{16\cdot5}=\sqrt[4]{16}\cdot\sqrt[4]{5}=2\sqrt[4]{5}$

61. $4\sqrt[3]{16}+5\sqrt[3]{2}=4\sqrt[3]{8\cdot2}+5\sqrt[3]{2}$
$=4\cdot2\sqrt[3]{2}+5\sqrt[3]{2}$
$=8\sqrt[3]{2}+5\sqrt[3]{2}$
$=13\sqrt[3]{2}$

62. $\frac{\sqrt[4]{32x^5}}{\sqrt[4]{16x}}=\sqrt[4]{\frac{32x^5}{16x}}=\sqrt[4]{2x^4}=x\sqrt[4]{2}$

63. $16^{1/2}=\sqrt{16}=4$

64. $25^{-1/2}=\frac{1}{25^{1/2}}=\frac{1}{\sqrt{25}}=\frac{1}{5}$

65. $125^{1/3}=\sqrt[3]{125}=5$

66. $27^{-1/3}=\frac{1}{27^{1/3}}=\frac{1}{\sqrt[3]{27}}=\frac{1}{3}$

67. $64^{2/3} = (\sqrt[3]{64})^2 = 4^2 = 16$

68. $27^{-4/3} = \dfrac{1}{27^{4/3}} = \dfrac{1}{(\sqrt[3]{27})^4} = \dfrac{1}{3^4} = \dfrac{1}{81}$

69. $(5x^{2/3})(4x^{1/4}) = 5 \cdot 4x^{2/3+1/4} = 20x^{11/12}$

70. $\dfrac{15x^{3/4}}{5x^{1/2}} = \left(\dfrac{15}{5}\right)x^{3/4-1/2} = 3x^{1/4}$

71. $(125 \cdot x^6)^{2/3} = (\sqrt[3]{125x^6})^2$
$= (5x^2)^2$
$= 25x^4$

72. $\sqrt[6]{y^3} = (y^3)^{1/6} = y^{3 \cdot 1/6} = y^{1/2} = \sqrt{y}$

73. $(-6x^3 + 7x^2 - 9x + 3) + (14x^3 + 3x^2 - 11x - 7) = (-6x^3 + 14x^3) + (7x^2 + 3x^2) + (-9x - 11x) + (3 - 7)$
$= 8x^3 + 10x^2 - 20x - 4$
The degree is 3.

74. $(13x^4 - 8x^3 + 2x^2) - (5x^4 - 3x^3 + 2x^2 - 6) = (13x^4 - 8x^3 + 2x^2) + (-5x^4 + 3x^3 - 2x^2 + 6)$
$= (13x^4 - 5x^4) + (-8x^3 + 3x^3) + (2x^2 - 2x^2) + 6$
$= 8x^4 - 5x^3 + 6$
The degree is 4.

75. $(3x - 2)(4x^2 + 3x - 5) = (3x)(4x^2) + (3x)(3x) + (3x)(-5) + (-2)(4x^2) + (-2)(3x) + (-2)(-5)$
$= 12x^3 + 9x^2 - 15x - 8x^2 - 6x + 10$
$= 12x^3 + x^2 - 21x + 10$

76. $(3x - 5)(2x + 1) = (3x)(2x) + (3x)(1) + (-5)(2x) + (-5)(1)$
$= 6x^2 + 3x - 10x - 5$
$= 6x^2 - 7x - 5$

77. $(4x + 5)(4x - 5) = (4x^2) - 5^2 = 16x^2 - 25$

78. $(2x + 5)^2 = (2x)^2 + 2(2x) \cdot 5 + 5^2 = 4x^2 + 20x + 25$

79. $(3x - 4)^2 = (3x)^2 - 2(3x) \cdot 4 + (-4)^2 = 9x^2 - 24x + 16$

80. $(2x + 1)^3 = (2x)^3 + 3(2x)^2(1) + 3(2x)(1)^2 + 1^3 = 8x^3 + 12x^2 + 6x + 1$

81. $(5x - 2)^3 = (5x)^3 - 3(5x)^2(2) + 3(5x)(2)^2 - 2^3 = 125x^3 - 150x^2 + 60x - 8$

82. $(7x^2 - 8xy + y^2) + (-8x^2 - 9xy - 4y^2) = (7x^2 - 8x^2) + (-8xy - 9xy) + (y^2 - 4y^2)$
$= -x^2 - 17xy - 3y^2$
The degree is 2.

83. $(13x^3y^2 - 5x^2y - 9x^2) - (-11x^3y^2 - 6x^2y + 3x^2 - 4)$
$= (13x^3y^2 - 5x^2y - 9x^2) + (11x^3y^2 + 6x^2y - 3x^2 + 4)$
$= (13x^3y^2 + 11x^3y^2) + (-5x^2y + 6x^2y) + (-9x^2 - 3x^2) + 4$
$= 24x^3y^2 + x^2y - 12x^2 + 4$
The degree is 5.

84. $(x+7y)(3x-5y) = x(3x) + (x)(-5y) + (7y)(3x) + (7y)(-5y)$
$= 3x^2 - 5xy + 21xy - 35y^2$
$= 3x^2 + 16xy - 35y^2$

85. $(3x-5y)^2 = (3x)^2 - 2(3x)(5y) + (-5y)^2$
$= 9x^2 - 30xy + 25y^2$

86. $(3x^2+2y)^2 = (3x^2)^2 + 2(3x^2)(2y) + (2y)^2$
$= 9x^4 + 12x^2y + 4y^2$

87. $(7x+4y)(7x-4y) = (7x)^2 - (4y)^2$
$= 49x^2 - 16y^2$

88. $(a-b)(a^2+ab+b^2)$
$= a(a^2) + a(ab) + a(b^2) + (-b)(a^2)$
$+(-b)(ab) + (-b)(b^2)$
$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$
$= a^3 - b^3$

89. $15x^3 + 3x^2 = 3x^2 \cdot 5x + 3x^2 \cdot 1$
$= 3x^2(5x+1)$

90. $x^2 - 11x + 28 = (x-4)(x-7)$

91. $15x^2 - x - 2 = (3x+1)(5x-2)$

92. $64 - x^2 = 8^2 - x^2 = (8-x)(8+x)$

93. $x^2 + 16$ is prime.

94. $3x^4 - 9x^3 - 30x^2 = 3x^2(x^2 - 3x - 10)$
$= 3x^2(x-5)(x+2)$

95. $20x^7 - 36x^3 = 4x^3(5x^4 - 9)$

96. $x^3 - 3x^2 - 9x + 27 = x^2(x-3) - 9(x-3)$
$= (x^2-9)(x-3)$
$= (x+3)(x-3)(x-3)$
$= (x+3)(x-3)^2$

97. $16x^2 - 40x + 25 = (4x-5)(4x-5)$
$= (4x-5)^2$

98. $x^4 - 16 = (x^2)^2 - 4^2$
$= (x^2+4)(x^2-4)$
$= (x^2+4)(x+2)(x-2)$

99. $y^3 - 8 = y^3 - 2^3 = (y-2)(y^2+2y+4)$

100. $x^3 + 64 = x^3 + 4^3 = (x+4)(x^2-4x+16)$

101. $3x^4 - 12x^2 = 3x^2(x^2-4)$
$= 3x^2(x-2)(x+2)$

102. $27x^3 - 125 = (3x)^3 - 5^3$
$= (3x-5)[(3x)^2 + (3x)(5) + 5^2]$
$= (3x-5)(9x^2+15x+25)$

103. $x^5 - x = x(x^4-1)$
$= x(x^2-1)(x^2+1)$
$= x(x-1)(x+1)(x^2+1)$

104. $x^3 + 5x^2 - 2x - 10 = x^2(x+5) - 2(x+5)$
$= (x^2-2)(x+5)$

105. $x^2 + 18x + 81 - y^2 = \left(x^2+18x+81\right) - y^2$
$= \left(x+9\right)^2 - y^2$
$= \left(x+9-y\right)\left(x+9+y\right)$

106. $16x^{-3/4} + 32x^{1/4} = 16x^{-3/4}\left(1 + 2x^{1/4-(-3/4)}\right)$
$= 16x^{-3/4}\left(1+2x\right)$
$= \dfrac{\left(1+2x\right)}{16x^{3/4}}$

107. $\left(x^2-4\right)\left(x^2+3\right)^{\frac{1}{2}} - \left(x^2-4\right)^2\left(x^2+3\right)^{\frac{3}{2}}$
$= \left(x^2-4\right)\left(x^2+3\right)^{\frac{1}{2}}\left[1 - \left(x^2-4\right)\left(x^2+3\right)\right]$
$= \left(x-2\right)\left(x+2\right)\left(x^2+3\right)^{\frac{1}{2}}\left[1 - \left(x-2\right)\left(x+2\right)\left(x^2+3\right)\right]$
$= (x-2)(x+2)(x^2+3)^{\frac{1}{2}}(-x^4+x^2+13)$

108. $12x^{-\frac{1}{2}} + 6x^{-\frac{3}{2}} = 6x^{-\frac{3}{2}}\left(2x+1\right) = \dfrac{6(2x+1)}{x^{\frac{3}{2}}}$

109. $\dfrac{x^3+2x^2}{x+2} = \dfrac{x^2(x+2)}{x+2} = x^2,\ x \neq -2$

110. $\dfrac{x^2+3x-18}{x^2-36} = \dfrac{(x+6)(x-3)}{(x+6)(x-6)} = \dfrac{x-3}{x-6}$,
$x \neq -6, 6$

111. $\dfrac{x^2+2x}{x^2+4x+4} = \dfrac{x(x+2)}{(x+2)^2} = \dfrac{x}{x+2}$,
$x \neq -2$

112. $\dfrac{x^2+6x+9}{x^2-4} \cdot \dfrac{x+3}{x-2} = \dfrac{(x+3)^2}{(x-2)(x+2)} \cdot \dfrac{x+3}{x-2}$
$= \dfrac{(x+3)^3}{(x-2)^2(x+2)}$,
$x \neq 2, -2$

113. $\dfrac{6x+2}{x^2-1} \div \dfrac{3x^2+x}{x-1}$
$= \dfrac{2(3x+1)}{(x-1)(x+1)} \div \dfrac{x(3x+1)}{x-1}$
$= \dfrac{2(3x+1)}{(x-1)(x+1)} \cdot \dfrac{x-1}{x(3x+1)}$
$= \dfrac{2}{x(x+1)}$,
$x \neq 0, 1, -1, -\dfrac{1}{3}$

114. $\dfrac{x^2-5x-24}{x^2-x-12} \div \dfrac{x^2-10x+16}{x^2+x-6}$
$= \dfrac{(x-8)(x+3)}{(x-4)(x+3)} \div \dfrac{(x-2)(x-8)}{(x+3)(x-2)}$
$= \dfrac{x-8}{x-4} \cdot \dfrac{x+3}{x-8}$
$= \dfrac{x+3}{x-4}$,
$x \neq -3, 4, 2, 8$

115. $\dfrac{2x-7}{x^2-9}-\dfrac{x-10}{x^2-9}=\dfrac{2x-7-(x-10)}{x^2-9}$

$=\dfrac{x+3}{(x+3)(x-3)}$

$=\dfrac{1}{x-3},$

$x \neq 3, -3$

116. $\dfrac{3x}{x+2}+\dfrac{x}{x-2}=\dfrac{3x}{x+2}\cdot\dfrac{x-2}{x-2}+\dfrac{x}{x-2}\cdot\dfrac{x+2}{x+2}$

$=\dfrac{3x^2-6x+x^2+2x}{(x+2)(x-2)}$

$=\dfrac{4x^2-4x}{(x+2)(x-2)}$

$=\dfrac{4x(x-1)}{(x+2)(x-2)},$

$x \neq 2, -2$

117. $\dfrac{x}{x^2-9}+\dfrac{x-1}{x^2-5x+6}$

$=\dfrac{x}{(x-3)(x+3)}+\dfrac{x-1}{(x-2)(x-3)}$

$=\dfrac{x}{(x-3)(x+3)}\cdot\dfrac{x-2}{x-2}+\dfrac{x-1}{(x-2)(x-3)}\cdot\dfrac{x+3}{x+3}$

$=\dfrac{x(x-2)+(x-1)(x+3)}{(x-3)(x+3)(x-2)}$

$=\dfrac{x^2-2x+x^2+2x-3}{(x-3)(x+3)(x-2)}$

$=\dfrac{2x^2-3}{(x-3)(x+3)(x-2)}$

$x \neq 3, -3, 2$

118. $\dfrac{4x-1}{2x^2+5x-3}-\dfrac{x+3}{6x^2+x-2}$

$=\dfrac{4x-1}{(2x-1)(x+3)}-\dfrac{x+3}{(2x-1)(3x+2)}$

$=\dfrac{4x-1}{(2x-1)(x+3)}\cdot\dfrac{3x+2}{3x+2}$

$-\dfrac{x+3}{(2x-1)(3x+2)}\cdot\dfrac{x+3}{x+3}$

$=\dfrac{12x^2+8x-3x-2-x^2-6x-9}{(2x-1)(x+3)(3x+2)}$

$=\dfrac{11x^2-x-11}{(2x-1)(x+3)(3x+2)},$

$x \neq \dfrac{1}{2}, -3, -\dfrac{2}{3}$

119. $\dfrac{\frac{1}{x}-\frac{1}{2}}{\frac{1}{3}-\frac{x}{6}}=\dfrac{\frac{1}{x}-\frac{1}{2}}{\frac{1}{3}-\frac{x}{6}}\cdot\dfrac{6x}{6x}$

$=\dfrac{6-3x}{2x-x^2}$

$=\dfrac{-3(x-2)}{-x(x-2)}$

$=\dfrac{3}{x},$

$x \neq 0, 2$

120. $\dfrac{3+\frac{12}{x}}{1-\frac{16}{x^2}}=\dfrac{3+\frac{12}{x}}{1-\frac{16}{x^2}}\cdot\dfrac{x^2}{x^2}$

$=\dfrac{3x^2+12x}{x^2-16}$

$=\dfrac{3x(x+4)}{(x+4)(x-4)}$

$=\dfrac{3x}{x-4},$

$x \neq 0, 4, -4$

121. $\dfrac{3-\frac{1}{x+3}}{3+\frac{1}{x+3}} = \dfrac{3-\frac{1}{x+3}}{3+\frac{1}{x+3}} \cdot \dfrac{x+3}{x+3}$

$= \dfrac{3(x+3)-1}{3(x+3)+1}$

$= \dfrac{3x+9-1}{3x+9+1}$

$= \dfrac{3x+8}{3x+10},$

$x \neq -3, -\dfrac{10}{3}$

Chapter P Test

1. $5(2x^2-6x)-(4x^2-3x) = 10x^2-30x-4x^2+3x$
$= 6x^2-27x$

2. $7+2[3(x+1)-2(3x-1)]$
$= 7+2[3x+3-6x+2]$
$= 7+2[-3x+5]$
$= 7-6x+10$
$= -6x+17$

3. $\{1,2,5\} \cap \{5,a\} = \{5\}$

4. $\{1,2,5\} \cup \{5,a\} = \{1,2,5,a\}$

5. $(2x^2y^3-xy+y^2)-(-4x^2y^3-5xy-y^2)$
$= 2x^2y^3-xy+y^2+4x^2y^3+5xy+y^2$
$= 2x^2y^3+4x^2y^3-xy+5xy+y^2+y^2$
$= 6x^2y^3+4xy+2y^2$

6. $\dfrac{30x^3y^4}{6x^9y^{-4}} = 5x^{3-9}y^{4-(-4)} = 5x^{-6}y^8 = \dfrac{5y^8}{x^6}$

7. $\sqrt{6r}\cdot\sqrt{3r} = \sqrt{18r^2} = \sqrt{9r^2}\cdot\sqrt{2} = 3r\sqrt{2}$

8. $4\sqrt{50}-3\sqrt{18} = 4\sqrt{25\cdot 2}-3\sqrt{9\cdot 2}$
$= 4\cdot 5\sqrt{2}-3\cdot 3\sqrt{2}$
$= 20\sqrt{2}-9\sqrt{2}$
$= 11\sqrt{2}$

9. $\dfrac{3}{5+\sqrt{2}} = \dfrac{3}{5+\sqrt{2}}\cdot\dfrac{5-\sqrt{2}}{5-\sqrt{2}}$

$= \dfrac{3(5-\sqrt{2})}{25-2}$

$= \dfrac{3(5-\sqrt{2})}{23}$

10. $\sqrt[3]{16x^4} = \sqrt[3]{8x^3\cdot 2x}$
$= \sqrt[3]{8x^3}\cdot\sqrt[3]{2x}$
$= 2x\sqrt[3]{2x}$

11. $\dfrac{x^2+2x-3}{x^2-3x+2} = \dfrac{(x+3)(x-1)}{(x-2)(x-1)} = \dfrac{x+3}{x-2},$
$x \neq 2, 1$

12. $\dfrac{5\times 10^{-6}}{20\times 10^{-8}} = \dfrac{5}{20}\cdot\dfrac{10^{-6}}{10^{-8}} = 0.25\times 10^2 = 2.5\times 10^1$

13. $(2x-5)(x^2-4x+3)$
$= 2x^3-8x^2+6x-5x^2+20x-15$
$= 2x^3-13x^2+26x-15$

14. $(5x+3y)^2 = (5x)^2+2(5x)(3y)+(3y)^2$
$= 25x^2+30xy+9y^2$

15. $\dfrac{2x+8}{x-3} \div \dfrac{x^2+5x+4}{x^2-9}$

$= \dfrac{2(x+4)}{x-3} \div \dfrac{(x+1)(x+4)}{(x-3)(x+3)}$

$= \dfrac{2(x+4)}{x-3}\cdot\dfrac{(x-3)(x+3)}{(x+1)(x+4)}$

$= \dfrac{2(x+3)}{x+1},$

$x \neq 3, -1, -4, -3$

16. $\dfrac{x}{x+3}+\dfrac{5}{x-3}$

$= \dfrac{x}{x+3}\cdot\dfrac{x-3}{x-3}+\dfrac{5}{x-3}\cdot\dfrac{x+3}{x+3}$

$= \dfrac{x(x-3)+5(x+3)}{(x+3)(x-3)}$

$= \dfrac{x^2-3x+5x+15}{(x+3)(x-3)}$

$= \dfrac{x^2+2x+15}{(x+3)(x-3)}, x \neq 3, -3$

17. $\dfrac{2x+3}{x^2-7x+12}-\dfrac{2}{x-3}$

$=\dfrac{2x+3}{(x-3)(x-4)}-\dfrac{2}{x-3}$

$=\dfrac{2x+3}{(x-3)(x-4)}-\dfrac{2}{x-3}\cdot\dfrac{x-4}{x-4}$

$=\dfrac{2x+3-2(x-4)}{(x-3)(x-4)}$

$=\dfrac{2x+3-2(x-4)}{(x-3)(x-4)}$

$=\dfrac{2x+3-2x+8}{(x-3)(x-4)}$

$=\dfrac{11}{(x-3)(x-4)},$

$x\neq 3,\ 4$

18. $\dfrac{\frac{1}{x}-\frac{1}{3}}{\frac{1}{x}}=\dfrac{\frac{1}{x}-\frac{1}{3}}{\frac{1}{x}}\cdot\dfrac{3x}{3x}=\dfrac{3-x}{3},$

$x\neq 0$

19. $x^2-9x+18=(x-3)(x-6)$

20. $x^3+2x^2+3x+6=x^2(x+2)+3(x+2)$

$=(x^2+3)(x+2)$

21. $25x^2-9=(5x)^2-3^2=(5x-3)(5x+3)$

22. $36x^2-84x+49=(6x)^2-2(6x)\cdot 7+7^2$

$=(6x-7)^2$

23. $y^3-125=y^3-5^3=(y-5)(y^2+5y+25)$

24. $(x^2+10x+25)-9y^2$

$=(x+5)^2-9y^2$

$=(x+5-3y)(x+5+3y)$

25. $x(x+3)^{-\frac{3}{5}}+(x+3)^{\frac{2}{5}}$

$=(x+3)^{-\frac{3}{5}}\left[x+(x+3)\right]$

$=(x+3)^{-\frac{3}{5}}(2x+3)=\dfrac{2x+3}{(x+3)^{\frac{3}{5}}}$

26. $-7,-\dfrac{4}{5},0,0.25,\sqrt{4},\dfrac{22}{7}$ are rational numbers.

27. 3(2 + 5) = 3(5 + 2);
commutative property of addition

28. $6(7+4)=6\cdot 7+6\cdot 4$
distributive property of multiplication over addition

29. $0.00076=7.6\times 10^{-4}$

30. $27^{-\frac{5}{3}}=\dfrac{1}{27^{\frac{5}{3}}}=\dfrac{1}{\left(\sqrt[3]{27}\right)^5}=\dfrac{1}{(3)^5}=\dfrac{1}{243}$

31. $2\left(6.6\times 10^9\right)=13.2\times 10^9=1.32\times 10^{10}$

32. a. 2003 is 14 years after 1989.

$M=-0.28n+47$

$M=-0.28(14)+47$

$=43.08$

In 2003, 43.08% of bachelor's degrees were awarded to men. This overestimates the actual percent shown by the bar graph by 0.08%.

b. $R=\dfrac{M}{W}=\dfrac{-0.28n+47}{0.28n+53}$

c. $R=\dfrac{-0.28n+47}{0.28n+53}$

$R=\dfrac{-0.28(25)+47}{0.28(25)+53}$

$=\dfrac{2}{3}$

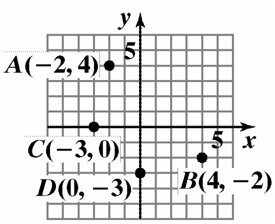
Three women will receive bachelor's degrees for every two men. This describes the projections exactly.

Chapter 1
Equations and Inequalities

Section 1.1

Check Point Exercises

1.



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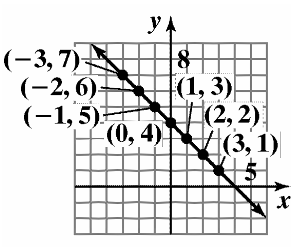


$y = 4 - x$

$x = -3,\ y = 7$

$x = -2,\ y = 6$

$x = -1,\ y = 5$

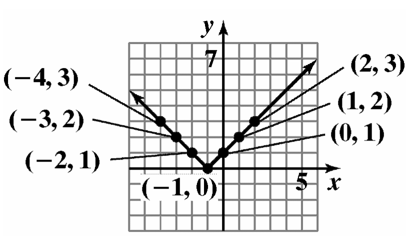
$x = 0,\ y = 4$

$x = 1,\ y = 3$

$x = 2,\ y = 2$

$x = 3,\ y = 1$

3.



$y = |x + 1|$

$x = -4,\ y = 3$

$x = -3,\ y = 2$

$x = -2,\ y = 1$

$x = -1,\ y = 0$

$x = 0,\ y = 1$

$x = 1,\ y = 2$

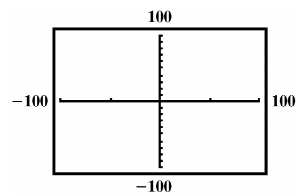
$x = 2,\ y = 3$

4. The meaning of a $[-100,100,50]$ by $[-100,100,10]$ viewing rectangle is as follows:

$$[\underbrace{-100}_{\text{minimum } x\text{-value}},\ \underbrace{100}_{\text{maximum } x\text{-value}},\ \underbrace{50}_{\text{distance between } x\text{-axis tick marks}}]$$

by

$$[\underbrace{-100}_{\text{minimum } y\text{-value}},\ \underbrace{100}_{\text{maximum } y\text{-value}},\ \underbrace{10}_{\text{distance between } y\text{-axis tick marks}}]$$



5. **a.** The graph crosses the x-axis at (–3, 0). Thus, the x-intercept is –3. The graph crosses the y-axis at (0, 5). Thus, the y-intercept is 5.

b. The graph does not cross the x-axis. Thus, there is no x-intercept. The graph crosses the y-axis at (0, 4). Thus, the y-intercept is 4.

c. The graph crosses the x- and y-axes at the origin (0, 0). Thus, the x-intercept is 0 and the y-intercept is 0.

6. **a.** $d = 4n + 5$

$d = 4(15) + 5 = 65$

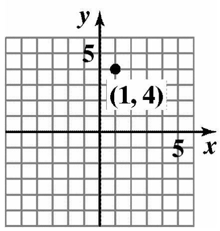
65% of marriages end in divorce after 15 years when the wife is under 18 at the time of marriage.

b. According to the line graph, 60% of marriages end in divorce after 15 years when the wife is under 18 at the time of marriage.

c. The mathematical model overestimates the actual percentage shown in the graph by 5%.

Exercise Set 1.1

1.



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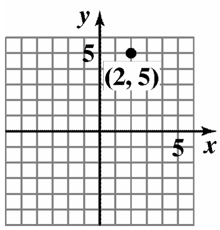

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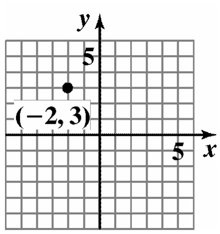

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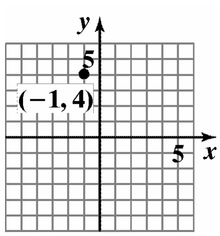

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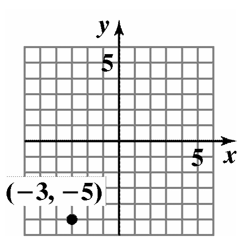

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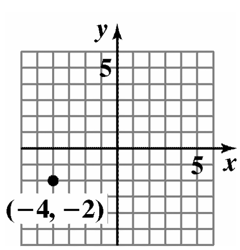

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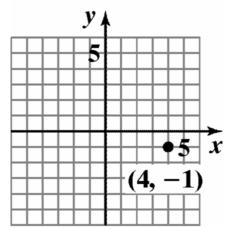

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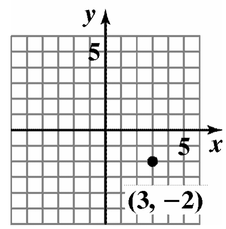

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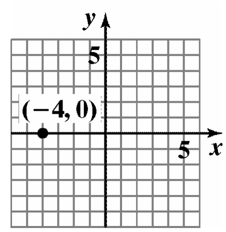

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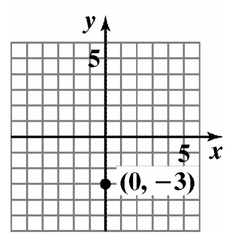

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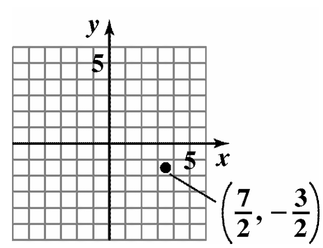
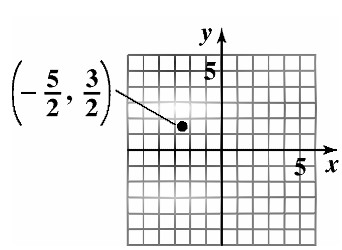


12.



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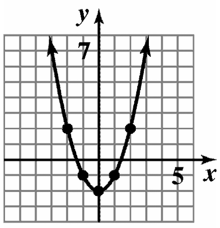


$y = x^2 - 2$

$x = -3, y = 7$

$x = -2, y = 2$

$x = -1, y = -1$

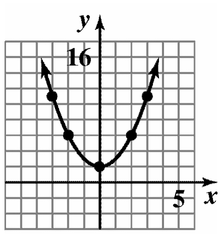
$x = 0, y = -2$

$x = 1, y = -1$

$x = 2, y = 2$

$x = 3, y = 7$

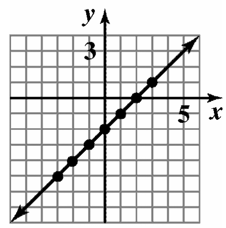
14.



$y = x^2 + 2$

$x = -3, y = 11$
$x = -2, y = 6$
$x = -1, y = 3$
$x = 0, y = 2$
$x = 1, y = 3$
$x = 2, y = 6$
$x = 3, y = 11$

15.



$y = x - 2$

$x = -3, y = -5$

$x = -2, y = -4$

$x = -1, y = -3$

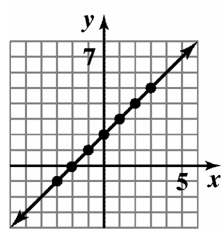
$x = 0, y = -2$

$x = 1, y = -1$

$x = 2, y = 0$

$x = 3, y = 1$

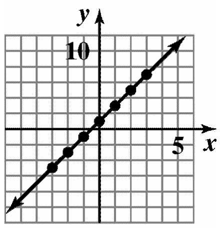
16.



$y = x + 2$

$x = -3, y = -1$
$x = -2, y = 0$
$x = -1, y = 1$
$x = 0, y = 2$
$x = 1, y = 3$
$x = 2, y = 4$
$x = 3, y = 5$

17.



$y = 2x + 1$

$x = -3,\ y = -5$

$x = -2,\ y = -3$

$x = -1,\ y = -1$

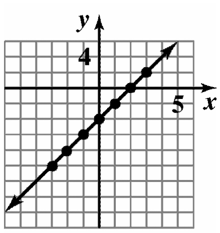
$x = 0,\ y = 1$

$x = 1,\ y = 3$

$x = 2,\ y = 5$

$x = 3,\ y = 7$

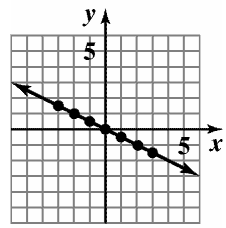
18.



$y = 2x - 4$

$x = -3,\ y = -10$
$x = -2,\ y = -8$
$x = -1,\ y = -6$
$x = 0,\ y = -4$
$x = 1,\ y = -2$
$x = 2,\ y = 0$
$x = 3,\ y = 2$

19.



$y = -\frac{1}{2}x$

$x = -3,\ y = \frac{3}{2}$

$x = -2,\ y = 1$

$x = -1,\ y = \frac{1}{2}$

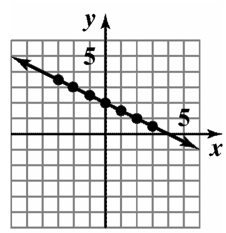
$x = 0,\ y = 0$

$x = 1,\ y = -\frac{1}{2}$

$x = 2,\ y = -1$

$x = 3,\ y = -\frac{3}{2}$

20.



$y = -\frac{1}{2}x + 2$

$x = -3,\ y = \frac{7}{2}$

$x = -2,\ y = 3$

$x = -1,\ y = \frac{5}{2}$

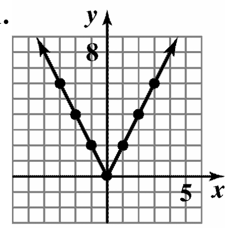
$x = 0,\ y = 2$

$x = 1,\ y = \frac{3}{2}$

$x = 2,\ y = 1$

$x = 3,\ y = \frac{1}{2}$

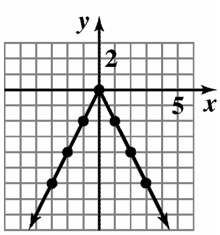
21.



$y = 2|x|$

$x = -3, y = 6$
$x = -2, y = 4$
$x = -1, y = 2$
$x = 0, y = 0$
$x = 1, y = 2$
$x = 2, y = 4$
$x = 3, y = 6$

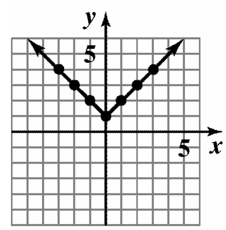
22.



$y = -2|x|$

$x = -3, y = -6$
$x = -2, y = -4$
$x = -1, y = -2$
$x = 0, y = 0$
$x = 1, y = -2$
$x = 2, y = -4$
$x = 3, y = -6$

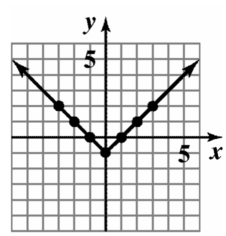
23.



$y = |x| + 1$

$x = -3, y = 4$
$x = -2, y = 3$
$x = -1, y = 2$
$x = 0, y = 1$
$x = 1, y = 2$
$x = 2, y = 3$
$x = 3, y = 4$

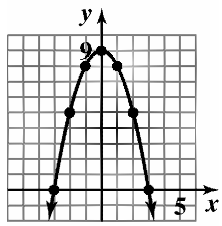
24.



$y = |x| - 1$

$x = -3, y = 2$
$x = -2, y = 1$
$x = -1, y = 0$
$x = 0, y = -1$
$x = 1, y = 0$
$x = 2, y = 1$
$x = 3, y = 2$

25.



$y = 9 - x^2$

$x = -3, y = 0$

$x = -2, y = 5$

$x = -1, y = 8$

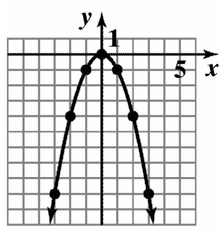
$x = 0, y = 9$

$x = 1, y = 8$

$x = 2, y = 5$

$x = 3, y = 0$

26.



$y = -x^2$

$x = -3, y = -9$

$x = -2, y = -4$

$x = -1, y = -1$

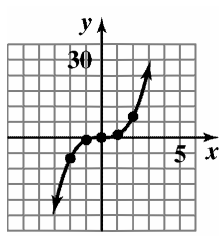
$x = 0, y = 0$

$x = 1, y = -1$

$x = 2, y = -4$

$x = 3, y = -9$

27.



$y = x^3$

$x = -3, y = -27$

$x = -2, y = -8$

$x = -1, y = 1$

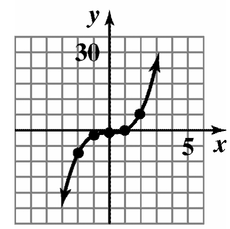
$x = 0, y = 0$

$x = 1, y = 1$

$x = 2, y = 8$

$x = 3, y = 27$

28.



$y = x^3 - 1$

$x = -3, y = -28$

$x = -2, y = -9$

$x = -1, y = -2$

$x = 0, y = -1$

$x = 1, y = 0$

$x = 2, y = 7$

$x = 3, y = 26$

29. (c) x-axis tick marks –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5; y-axis tick marks are the same.

30. (d) x-axis tick marks –10, –8, –6, –4, –2, 0, 2, 4, 6, 8, 10; y-axis tick marks –4, –2, 0, 2, 4

31. (b); x-axis tick marks –20, –10, 0, 10, 20, 30, 40, 50, 60, 70, 80; y-axis tick marks –30, –20, –10, 0, 10, 20, 30, 40, 50, 60, 70

32. (a) x-axis tick marks –40, –20, 0, 20, 40; y-axis tick marks –1000, –900, –800, –700, . . . , 700, 800, 900, 1000

33. The equation that corresponds to Y_2 in the table is (c), $y_2 = 2 - x$. We can tell because all of the points $(-3, 5)$, $(-2, 4)$, $(-1, 3)$, $(0, 2)$, $(1, 1)$, $(2, 0)$, and $(3, -1)$ are on the line $y = 2 - x$, but all are not on any of the others.

34. The equation that corresponds to Y_1 in the table is (b), $y_1 = x^2$. We can tell because all of the points $(-3, 9)$, $(-2, 4)$, $(-1, 1)$, $(0, 0)$, $(1, 1)$, $(2, 4)$, and $(3, 9)$ are on the graph $y = x^2$, but all are not on any of the others.

35. No. It passes through the point $(0, 2)$.

36. Yes. It passes through the point $(0, 0)$.

37. $(2, 0)$

38. $(0, 2)$

39. The graphs of Y_1 and Y_2 intersect at the points $(-2, 4)$ and $(1, 1)$.

40. The values of Y_1 and Y_2 are the same when $x = -2$ and $x = 1$.

41. a. 2; The graph intersects the x-axis at (2, 0).

b. –4; The graph intersects the y-axis at (0,–4).

42. a. 1; The graph intersects the x-axis at (1, 0).

b. 2; The graph intersects the y-axis at (0, 2).

43. a. 1, –2; The graph intersects the x-axis at (1, 0) and (–2, 0).

b. 2; The graph intersects the y-axis at (0, 2).

44. a. 1, –1; The graph intersects the x-axis at (1, 0) and (–1, 0).

b. 1; The graph intersect the y-axis at (0, 1).

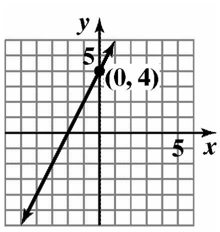
45. a. –1; The graph intersects the x-axis at (–1, 0).

b. none; The graph does not intersect the y-axis.

46. a. none; The graph does not intersect the x-axis.

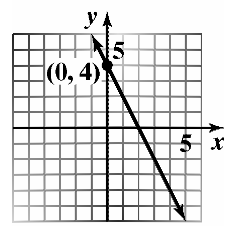
b. 2; The graph intersects the y-axis at (0, 2).

47.



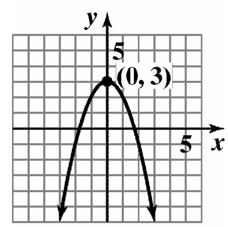
$y = 2x + 4$

48.



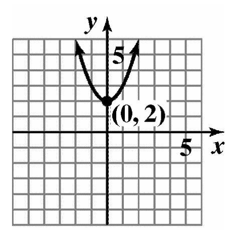
$y = 4 - 2x$

49.



$y = 3 - x^2$

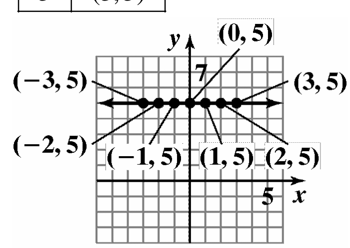
50.



$y = x^2 + 2$

51.

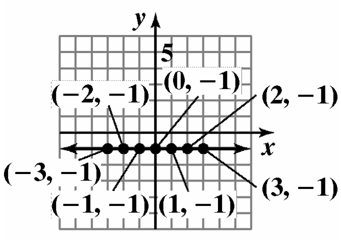
x	(x, y)
–3	$(-3, 5)$
–2	$(-2, 5)$
–1	$(-1, 5)$
0	$(0, 5)$
1	$(1, 5)$
2	$(2, 5)$
3	$(3, 5)$



$y = 5$

52.

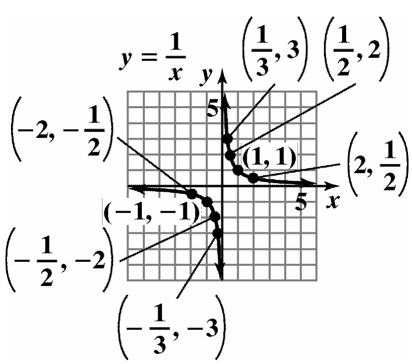
x	(x, y)
-3	$(-3,-1)$
-2	$(-2,-1)$
-1	$(-1,-1)$
0	$(0,-1)$
1	$(1,-1)$
2	$(2,-1)$
3	$(3,-1)$



$y = -1$

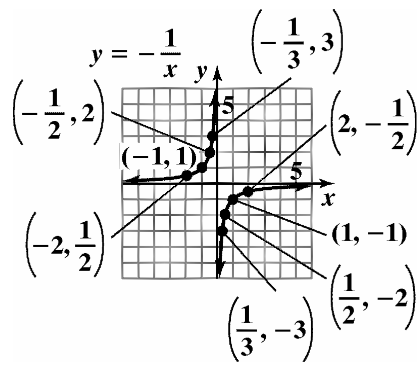
53.

x	(x, y)
-2	$\left(-2,-\frac{1}{2}\right)$
-1	$(-1,-1)$
$-\frac{1}{2}$	$\left(-\frac{1}{2},-2\right)$
$-\frac{1}{3}$	$\left(-\frac{1}{3},-3\right)$
$\frac{1}{3}$	$\left(\frac{1}{3},3\right)$
$\frac{1}{2}$	$\left(\frac{1}{2},2\right)$
1	$(1,1)$
2	$\left(2,\frac{1}{2}\right)$



54.

x	(x, y)
-2	$\left(-2,\frac{1}{2}\right)$
-1	$(-1,1)$
$-\frac{1}{2}$	$\left(-\frac{1}{2},2\right)$
$-\frac{1}{3}$	$\left(-\frac{1}{3},3\right)$
$\frac{1}{3}$	$\left(\frac{1}{3},-3\right)$
$\frac{1}{2}$	$\left(\frac{1}{2},-2\right)$
1	$(1,-1)$
2	$\left(2,-\frac{1}{2}\right)$



55. **a.** According to the line graph, 20% of seniors used marijuana in 2005.

b. 2005 is 25 years after 1980.

$M = -0.4n + 28$

$M = -0.4(25) + 28 = 18$

According to formula, 18% of seniors used marijuana in 2005. This underestimates the value in the graph by 2%.

c. According to the line graph, about 45% of seniors used alcohol in 2006.

d. 2006 is 26 years after 1980.

$A = -n + 70$

$A = -(26) + 70 = 44$

According to formula, 44% of seniors used alcohol in 2006. This underestimates the value in the graph.

e. The minimum for marijuana was reached in 1990.
According to the line graph, about 14% of seniors used marijuana in 1990.

56. **a.** According to the line graph, 50% of seniors used alcohol in 2000.

b. 2000 is 20 years after 1980.
$A = -n + 70$
$A = -(20) + 70 = 50$
According to formula, 50% of seniors used alcohol in 2000. This matches the value in the graph.

c. According to the line graph, about 22% of seniors used marijuana in 2000.

d. 2000 is 20 years after 1980.
$M = -0.4n + 28$
$M = -0.4(20) + 28 = 20$
According to formula, 20% of seniors used marijuana in 2000. This underestimates the value in the graph.

e. The maximum for alcohol was reached in 1980. According to the line graph, about 72% of seniors used alcohol in 1980.

57. At age 8, women have the least number of awakenings, averaging about 1 awakening per night.

58. At age 65, men have the greatest number of awakenings, averaging about 8 awakenings per night.

59. The difference between the number of awakenings for 25-year-old men and women is about 1.9.

60. The difference between the number of awakenings for 18-year-old men and women is about 1.1.

61. – 66. Answers may vary.

67. makes sense

68. does not make sense; Explanations will vary. Sample explanation: Most graphing utilities do not display numbers an the axes.

69. does not make sense; Explanations will vary. Sample explanation: These three points are not collinear.

70. does not make sense; Explanations will vary. Sample explanation: As the time of day goes up, the total calories burned will also go up.

71. false; Changes to make the statement true will vary. A sample change is: The product of the coordinates of a point in quadrant III is also positive.

72. false; Changes to make the statement true will vary. A sample change is: A point on the *x*-axis will have $y = 0$.

73. true

74. false; Changes to make the statement true will vary. A sample change is: $3(5) - 2(2) \neq -4$.

75. (a)

76. (d)

77. (b)

78. (c)

79. (b)

80. (a)

81. (c)

82. (b)

83.
$$\begin{aligned} 2(x-3)-17 &= 13-3(x+2) \\ 2(6-3)-17 &= 13-3(6+2) \\ 2(3)-17 &= 13-3(8) \\ 6-17 &= 13-24 \\ -11 &= -11, \text{ true} \end{aligned}$$

84.
$$\begin{aligned} 12\left(\frac{x+2}{4}-\frac{x-1}{3}\right) &= 12\left(\frac{x+2}{4}\right)-12\left(\frac{x-1}{3}\right) \\ &= 3(x+2)-4(x-1) \\ &= 3x+6-4x+4 \\ &= -x+10 \end{aligned}$$

85.
$$\begin{aligned} (x-3)\left(\frac{3}{x-3}+9\right) &= (x-3)\left(\frac{3}{x-3}\right)+(x-3)(9) = \\ &= 3+9x-27 \\ &= 9x-24 \end{aligned}$$

Section 1.2

Check Point Exercises

1.
$$4x+5=29$$
$$4x+5-5=29-5$$
$$4x=24$$
$$\frac{4x}{4}=\frac{24}{4}$$
$$x=6$$
Check:
$$4x+5=29$$
$$4(6)+5=29$$
$$24+5=29$$
$$29=29 \text{ true}$$
The solution set is {6}.

2.
$$4(2x+1)-29=3(2x-5)$$
$$8x+4-29=6x-15$$
$$8x-25=6x-15$$
$$8x-25-6x=6x-15-6x$$
$$2x-25=-15$$
$$2x-25+25=-15+25$$
$$2x=10$$
$$\frac{2x}{2}=\frac{10}{2}$$
$$x=5$$
Check:
$$4(2x+1)-29=3(2x-5)$$
$$4[2(5)+1]-29=3[2(5)-5]$$
$$4[10+1]-29=3[10-5]$$
$$4[11]-29=3[5]$$
$$44-29=15$$
$$15=15 \text{ true}$$
The solution set is {5}.

3.
$$\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}$$
$$28\cdot\frac{x-3}{4}=28\left(\frac{5}{14}-\frac{x+5}{7}\right)$$
$$7(x-3)=2(5)-4(x+5)$$
$$7x-21=10-4x-20$$
$$7x-21=-4x-10$$
$$7x+4x=-10+21$$
$$11x=11$$
$$\frac{11x}{11}=\frac{11}{11}$$
$$x=1$$
Check:
$$\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}$$
$$\frac{1-3}{4}=\frac{5}{14}-\frac{1+5}{7}$$
$$\frac{-2}{4}=\frac{5}{14}-\frac{6}{7}$$
$$-\frac{1}{2}=-\frac{1}{2}$$
The solution set is {1}.

4.
$$\frac{5}{2x}=\frac{17}{18}-\frac{1}{3x},\ x\neq 0$$
$$18x\cdot\frac{5}{2x}=18x\left(\frac{17}{18}-\frac{1}{3x}\right)$$
$$18\cdot\frac{5}{2x}=18x\cdot\frac{17}{18}-18x\cdot\frac{1}{3x}$$
$$45=17x-6$$
$$45+6=17x-6+6$$
$$51=17x$$
$$\frac{51}{17}=\frac{17x}{17}$$
$$3=x$$
The solution set is {3}.

5. $\frac{x}{x-2}=\frac{2}{x-2}-\frac{2}{3}, \quad x\neq 2$

$3(x-2)\cdot\frac{x}{x-2}=3(x-2)\left[\frac{2}{x-2}-\frac{2}{3}\right]$

$3(x-2)\cdot\frac{x}{x-2}=(3x-2)\cdot\frac{2}{x-2}-3(x-2)\cdot\frac{2}{3}$

$3x=6-(x-2)\cdot 2$

$3x=6-2(x-2)$

$3x=6-2x+4$

$3x=10-2x$

$3x+2x=10-2x+2x$

$5x=10$

$\frac{5x}{5}=\frac{10}{5}$

$x=2$

The solution set is the empty set, $\varnothing$.

6. Set $y_1=y_2$.

$\frac{1}{x+4}+\frac{1}{x-4}=\frac{22}{x^2-16}$

$\frac{1}{x+4}+\frac{1}{x-4}=\frac{22}{(x+4)(x-4)}$

$\frac{(x+4)(x-4)}{x+4}+\frac{(x+4)(x-4)}{x-4}=\frac{22(x+4)(x-4)}{(x+4)(x-4)}$

$(x-4)+(x+4)=22$

$x-4+x+4=22$

$2x=22$

$x=11$

Check:

$\frac{1}{x+4}+\frac{1}{x-4}=\frac{22}{x^2-16}$

$\frac{1}{11+4}+\frac{1}{11-4}=\frac{22}{11^2-16}$

$\frac{1}{15}+\frac{1}{7}=\frac{22}{105}$

$\frac{22}{105}=\frac{22}{105}$ true

7. $4x-7=4(x-1)+3$

$4x-7=4(x-1)+3$

$4x-7=4x-4+3$

$4x-7=4x-1$

$-7=-1$

The original equation is equivalent to the statement $-7=-1$, which is false for every value of x.
The solution set is the empty set, $\varnothing$.
The equation is an inconsistent equation.

8. $D=\frac{10}{9}x+\frac{53}{9}$

$10=\frac{10}{9}x+\frac{53}{9}$

$9\cdot 10=9\left(\frac{10}{9}x+\frac{53}{9}\right)$

$90=10x+53$

$90-53=10x+53-53$

$37=10x$

$\frac{37}{10}=\frac{10x}{10}$

$3.7=x$

$x=3.7$

The formula indicates that if the low-humor group averages a level of depression of 10 in response to a negative life event, the intensity of that event is 3.7. This is shown as the point whose corresponding value on the vertical axis is 10 and whose value on the horizontal axis is 3.7.

Exercise Set 1.2

1. $7x-5=72$

$7x=77$

$x=11$

Check:

$7x-5=72$

$7(11)-5=72$

$77-5=72$

$72=72$

The solution set is $\{11\}$.

2. $6x-3=63$

$6x=66$

$x=11$

The solution set is $\{11\}$.

Check:

$6x-3=63$

$6(11)-3=63$

$66-3=63$

$63=63$

3. $11x-(6x-5)=40$
$11x-6x+5=40$
$5x+5=40$
$5x=35$
$x=7$
The solution set is $\{7\}$.

Check:
$11x-(6x-5)=40$
$11(7)-[6(7)-5]=40$
$77-(42-5)=40$
$77-(37)=40$
$40=40$

4. $5x-(2x-10)=35$
$5x-2x+10=35$
$3x+10=35$
$3x=25$
$x=\frac{25}{3}$
The solution set is $\left\{\frac{25}{3}\right\}$.

Check:
$5x-(2x-10)=35$
$5\left(\frac{25}{3}\right)-\left[2\left(\frac{25}{3}\right)-10\right]=35$
$\frac{125}{3}-\left[\frac{50}{3}-10\right]=35$
$\frac{125}{3}-\frac{20}{3}=35$
$\frac{105}{3}=35$
$35=35$

5. $2x-7=6+x$
$x-7=6$
$x=13$
The solution set is $\{13\}$.

Check:
$2(13)-7=6+13$
$26-7=19$
$19=19$

6. $3x+5=2x+13$
$x+5=13$
$x=8$
The solution set is $\{8\}$.

Check:
$3x+5=2x+13$
$3(8)+5=2(8)+13$
$24+5=16+13$
$29=29$

7. $7x+4=x+16$
$6x+4=16$
$6x=12$
$x=2$
The solution set is $\{2\}$.

Check:
$7(2)+4=2+16$
$14+4=18$
$18=18$

8. $13x+14=12x-5$
$x+14=-5$
$x=-19$
The solution set is $\{-19\}$.

Check:
$13x+14=12x-5$
$13(-19)+14=12(-19)-5$
$-247+14=-228-5$
$-233=-233$

9. $3(x-2)+7=2(x+5)$
$3x-6+7=2x+10$
$3x+1=2x+10$
$x+1=10$
$x=9$
The solution set is {9}.

Check:

$$3(9-2)+7=2(9+5)$$
$$3(7)+7=2(14)$$
$$21+7=28$$
$$28=28$$

10. $2(x-1)+3=x-3(x+1)$
$2x-2+3=x-3x-3$
$2x+1=-2x-3$
$4x+1=-3$
$4x=-4$
$x=-1$
The solution set is {–1}.

Check:
$$2(x-1)+3=x-3(x+1)$$
$$2(-1-1)+3=-1-3(-1+1)$$
$$2(-2)+3=-1-3(0)$$
$$-4+3=-1+0$$
$$-1=-1$$

11. $3(x-4)-4(x-3)=x+3-(x-2)$
$3x-12-4x+12=x+3-x+2$
$-x=5$
$x=-5$
The solution set is {–5}.

Check:
$$3(-5-4)-4(-5-3)=-5+3-(-5-2)$$
$$3(-9)-4(-8)=-2-(-7)$$
$$-27+32=-2+7$$
$$5=5$$

12. $2-(7x+5)=13-3x$
$2-7x-5=13-3x$
$-7x-3=13-3x$
$-4x=16$
$x=-4$
The solution set is {–4}.

Check:
$$2-(7x+5)=13-3x$$
$$2-[7(-4)+5]=13-3(-4)$$
$$2-[-28+5]=13+12$$
$$2-[-23]=15$$
$$2+23=25$$
$$25=25$$

13. $16=3(x-1)-(x-7)$
$16=3x-3-x+7$
$16=2x+4$
$12=2x$
$6=x$
The solution set is {6}.

Check:
$$16=3(6-1)-(6-7)$$
$$16=3(5)-(-1)$$
$$16=15+1$$
$$16=16$$

14. $5x-(2x+2)=x+(3x-5)$
$5x-2x-2=x+3x-5$
$3x-2=4x-5$
$-x=-3$
$x=3$
The solution set is {3}.

Check:
$$5x-(2x+2)=x+(3x-5)$$
$$5(3)-[2(3)+2]=3+[3(3)-5]$$
$$15-[6+2]=3+[9-5]$$
$$15-8=3+4$$
$$7=7$$

15.
$$\begin{aligned}25-[2+5y-3(y+2)] &= -3(2y-5)-[5(y-1)-3y+3]\\25-[2+5y-3y-6] &= -6y+15-[5y-5-3y+3]\\25-[2y-4] &= -6y+15-[2y-2]\\25-2y+4 &= -6y+15-2y+2\\-2y+29 &= -8y+17\\6y &= -12\\y &= -2\end{aligned}$$

The solution set is {–2}.

Check:

$$\begin{aligned}25-[2+5y-3(y+2) &= -3(2y-5)-[5(y-1)-3y+3]\\25-[2+5(-2)-3(-2+2) &= -3[2(-2)-5]-[5(-2-1)-3(-2)+3]\\25-[2-10-3(0)] &= -3[-4-5]-[5(-3)+6+3]\\25-[-8] &= -3(-9)-[-15+9]\\25+8 &= 27-(-6)\\33 &= 27+6\\33 &= 33\end{aligned}$$

16.
$$\begin{aligned}45-[4-2y-4(y+7)] &= -4(1+3y)-[4-3(y+2)-2(2y-5)]\\45-[4-2y-4y-28] &= -4-12y-[4-3y-6-4y+10]\\45-[-6y-24] &= -4-12y-[-7y+8]\\45+6y+24 &= -4-12y+7y-8\\6y+69 &= -5y-12\\11y &= -81\\y &= -\frac{81}{11}\end{aligned}$$

The solution set is $\left\{-\frac{81}{11}\right\}$.

17.
$$\begin{aligned}\frac{x}{3} &= \frac{x}{2}-2\\6\left[\frac{x}{3} = \frac{x}{2}-2\right]&\\2x &= 3x-12\\12 &= 3x-2x\\x &= 12\end{aligned}$$

The solution set is {12}.

18.
$$\begin{aligned}\frac{x}{5} &= \frac{x}{6}+1\\30\left[\frac{x}{5} = \frac{x}{6}+1\right]&\\6x &= 5x+30\\6x-5x &= 30\\x &= 30\end{aligned}$$

The solution set is {30}.

19. $20-\frac{x}{3}=\frac{x}{2}$

$$6\left[20-\frac{x}{3}=\frac{x}{2}\right]$$

$$120-2x=3x$$

$$120=3x+2x$$

$$120=5x$$

$$x=\frac{120}{5}$$

$$x=24$$

The solution set is $\{24\}$.

20. $\frac{x}{5}-\frac{1}{2}=\frac{x}{6}$

$$30\left[\frac{x}{5}-\frac{1}{2}=\frac{x}{6}\right]$$

$$6x-15=5x$$

$$6x-5x=15$$

$$x=15$$

The solution set is $\{15\}$.

21. $\frac{3x}{5}=\frac{2x}{3}+1$

$$15\left[\frac{3x}{5}=\frac{2x}{3}+1\right]$$

$$9x=10x+15$$

$$9x-10x=15$$

$$-x=15$$

$$x=-15$$

The solution set is $\{-15\}$.

22. $\frac{x}{2}=\frac{3x}{4}+5$

$$4\left[\frac{x}{2}=\frac{3x}{4}+5\right]$$

$$2x=3x+20$$

$$2x-3x=20$$

$$-x=20$$

$$x=-20$$

The solution set is $\{-20\}$.

23. $\frac{3x}{5}-x=\frac{x}{10}-\frac{5}{2}$

$$10\left[\frac{3x}{5}-x=\frac{x}{10}-\frac{5}{2}\right]$$

$$6x-10x=x-25$$

$$-4x-x=-25$$

$$-5x=-25$$

$$x=5$$

The solution set is $\{5\}$.

24. $2x-\frac{2x}{7}=\frac{x}{2}+\frac{17}{2}$

$$14\left[2x-\frac{2x}{7}=\frac{x}{2}+\frac{17}{2}\right]$$

$$28x-4x=7x+119$$

$$24x-7x=119$$

$$17x=119$$

$$x=7$$

The solution set is $\{7\}$.

25. $\frac{x+3}{6}=\frac{3}{8}+\frac{x-5}{4}$

$$24\left[\frac{x+3}{6}=\frac{3}{8}+\frac{x-5}{4}\right]$$

$$4x+12=9+6x-30$$

$$4x-6x=-21-12$$

$$-2x=-33$$

$$x=\frac{33}{2}$$

The solution set is $\left\{\frac{33}{2}\right\}$.

26. $\frac{x+1}{4}=\frac{1}{6}+\frac{2-x}{3}$

$$12\left[\frac{x+1}{4}=\frac{1}{6}+\frac{2-x}{3}\right]$$

$$3x+3=2+8-4x$$

$$3x+4x=10-3$$

$$7x=7$$

$$x=1$$

The solution set is $\{1\}$.

27. $\frac{x}{4}=2+\frac{x-3}{3}$

$12\left[\frac{x}{4}=2+\frac{x-3}{3}\right]$

$3x = 24 + 4x - 12$

$3x - 4x = 12$

$-x = 12$

$x = -12$

The solution set is $\{-12\}$.

28. $5+\frac{x-2}{3}=\frac{x+3}{8}$

$24\left[5+\frac{x-2}{3}=\frac{x+3}{8}\right]$

$120+8x-16=3x+9$

$8x-3x=9-104$

$5x=-95$

$x=-19$

The solution set is $\{-19\}$.

29. $\frac{x+1}{3}=5-\frac{x+2}{7}$

$21\left[\frac{x+1}{3}=5-\frac{x+2}{7}\right]$

$7x + 7 = 105 - 3x - 6$

$7x + 3x = 99 - 7$

$10x = 92$

$x=\frac{92}{10}$

$x=\frac{46}{5}$

The solution set is $\left\{\frac{46}{5}\right\}$.

30. $\frac{3x}{5}-\frac{x-3}{2}=\frac{x+2}{3}$

$30\left[\frac{3x}{5}-\frac{x-3}{2}=\frac{x+2}{3}\right]$

$18x-15x+45=10x+20$

$3x-10x=20-45$

$-7x=-25$

$x=\frac{25}{7}$

The solution set is $\left\{\frac{25}{7}\right\}$.

31. **a.** $\frac{4}{x}=\frac{5}{2x}+3\ (x\neq 0)$

b. $\frac{4}{x}=\frac{5}{2x}+3$

$8=5+6x$

$3=6x$

$\frac{1}{2}=x$

The solution set is $\left\{\frac{1}{2}\right\}$.

32. **a.** $\frac{5}{x}=\frac{10}{3x}+4\ (x\neq 0)$

b. $\frac{5}{x}=\frac{10}{3x}+4$

$15=10+12x$

$5=12x$

$x=\frac{5}{12}$

The solution set is $\left\{\frac{5}{12}\right\}$.

33. **a.** $\frac{2}{x}+3=\frac{5}{2x}+\frac{13}{4}\ (x\neq 0)$

b. $\frac{2}{x}+3=\frac{5}{2x}+\frac{13}{4}$

$8+12x=10+13x$

$-x=2$

$x=-2$

The solution set is $\{-2\}$.

34. **a.** $\frac{7}{2x}-\frac{5}{3x}=\frac{22}{3}\ (x\neq 0)$

b. $\frac{7}{2x}-\frac{5}{3x}=\frac{22}{3}$

$21-10=44x$

$11=44x$

$x=\frac{1}{4}$

The solution set is $\left\{\frac{1}{4}\right\}$.

35. **a.** $\frac{2}{3x}+\frac{1}{4}=\frac{11}{6x}-\frac{1}{3}\ (x \neq 0)$

b. $\frac{2}{3x}+\frac{1}{4}=\frac{11}{6x}-\frac{1}{3}$

$8+3x=22-4x$

$7x=14$

$x=2$

The solution set is $\{2\}$.

36. **a.** $\frac{5}{2x}-\frac{8}{9}=\frac{1}{18}-\frac{1}{3x}\ (x \neq 0)$

b. $\frac{5}{2x}-\frac{8}{9}=\frac{1}{18}-\frac{1}{3x}$

$45-16x=x-6$

$-17x=-51$

$x=3$

The solution set is $\{3\}$.

37. **a.** $\frac{x-2}{2x}+1=\frac{x+1}{x}\quad (x \neq 0)$

b. $\frac{x-2}{2x}+1=\frac{x+1}{x}$

$x-2+2x=2x+2$

$x-2=2$

$x=4$

The solution set is $\{4\}$.

38. **a.** $\frac{4}{x}=\frac{9}{5}-\frac{7x-4}{5x}\quad (x \neq 0)$

b. $\frac{4}{x}=\frac{9}{5}-\frac{7x-4}{5x}$

$20=9x-7x+4$

$16=2x$

$8=x$

The solution set is $\{8\}$.

39. **a.** $\frac{1}{x-1}+5=\frac{11}{x-1}\ (x \neq 1)$

b. $\frac{1}{x-1}+5=\frac{11}{x-1}$

$1+5(x-1)=11$

$1+5x-5=11$

$5x-4=11$

$5x=15$

$x=3$

The solution set is $\{3\}$.

40. **a.** $\frac{3}{x+4}-7=\frac{-4}{x+4}\ (x \neq -4)$

b. $\frac{3}{x+4}-7=\frac{-4}{x+4}$

$3-7(x+4)=-4$

$3-7x-28=-4$

$-7x=21$

$x=-3$

The solution set is $\{-3\}$.

41. **a.** $\frac{8x}{x+1}=4-\frac{8}{x+1}\ (x \neq -1)$

b. $\frac{8x}{x+1}=4-\frac{8}{x+1}$

$8x=4(x+1)-8$

$8x=4x+4-8$

$4x=-4$

$x=-1 \Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

42. **a.** $\frac{2}{x-2}=\frac{x}{x-2}-2\ (x \neq 2)$

b. $\frac{2}{x-2}=\frac{x}{x-2}-2$

$2=x-2(x-2)$

$2=x-2x+4$

$x=2 \Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

43. **a.** $\frac{3}{2x-2}+\frac{1}{2}=\frac{2}{x-1}\ (x \neq 1)$

b. $\frac{3}{2x-2}+\frac{1}{2}=\frac{2}{x-1}$

$\frac{3}{2(x-1)}+\frac{1}{2}=\frac{2}{x-1}$

$3+1(x-1)=4$

$3+x-1=4$

$x=2$

The solution set is $\{2\}$.

44. **a.** $\frac{3}{x+3}=\frac{5}{2x+6}+\frac{1}{x-2}\ (x\neq -3, x\neq 2)$

b. $\frac{3}{x+3}=\frac{5}{2(x+3)}+\frac{1}{x-2}$

$6(x-2)=5(x-2)+2(x+3)$

$6x-12=5x-10+2x+6$

$-x=8$

$x=-8$

The solution set is {–8}.

45. **a.** $\frac{3}{x+2}+\frac{2}{x-2}=\frac{8}{(x+2)(x-2)};(x\neq -2,2)$

b. $\frac{3}{x+2}+\frac{2}{x-2}=\frac{8}{(x+2)(x-2)}$

$(x\neq 2, x\neq -2)$

$3(x-2)+2(x+2)=8$

$3x-6+2x+4=8$

$5x=10$

$x=2\Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

46. **a.** $\frac{5}{x+2}+\frac{3}{x-2}=\frac{12}{(x+2)(x-2)}$

$(x\neq 2, x\neq -2)$

b. $\frac{5}{x+2}+\frac{3}{x-2}=\frac{12}{(x+2)(x-2)}$

$5(x-2)+3(x+2)=12$

$5x-10+3x+6=12$

$8x=16$

$x=2\Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

47. **a.** $\frac{2}{x+1}-\frac{1}{x-1}=\frac{2x}{x^2-1}\ (x\neq 1, x\neq -1)$

b.

$\frac{2}{x+1}-\frac{1}{x-1}=\frac{2x}{x^2-1}$

$\frac{2}{x+1}-\frac{1}{x-1}=\frac{2x}{(x+1)(x-1)}$

$2(x-1)-1(x+1)=2x$

$2x-2-x-1=2x$

$-x=3$

$x=-3$

The solution set is {–3}.

48. **a.** $\frac{4}{x+5}+\frac{2}{x-5}=\frac{32}{x^2-25};\ x\neq 5,-5$

b. $\frac{4}{x+5}+\frac{2}{x-5}=\frac{32}{(x+5)(x-5)}$

$(x\neq 5, x\neq -5)$

$4(x-5)+2(x+5)=32$

$4x-20+2x+10=32$

$6x=42$

$x=7$

The solution set is {7}.

49. **a.** $\frac{1}{x-4}-\frac{5}{x+2}=\frac{6}{(x-4)(x+2)};(x\neq -2,4)$

b. $\frac{1}{x-4}-\frac{5}{x+2}=\frac{6}{x^2-2x-8}$

$\frac{1}{x-4}-\frac{5}{x+2}=\frac{6}{(x-4)(x+2)}$

$(x\neq 4, x\neq -2)$

$1(x+2)-5(x-4)=6$

$x+2-5x+20=6$

$-4x=-16$

$x=4\Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

50. **a.** $\frac{6}{x+3}-\frac{5}{x-2}=\frac{-20}{x^2+x-6};\ x\neq -3,2$

b. $\frac{6}{x+3}-\frac{5}{x-2}=\frac{-20}{(x-2)(x+3)}$

$(x\neq -3, x\neq 2)$

$6(x-2)-5(x+3)=-20$

$6x-12-5x-15=-20$

$x=7$

The solution set is {7}.

51. Set $y_1=y_2$.

$5(2x-8)-2=5(x-3)+3$

$10x-40-2=5x-15+3$

$10x-42=5x-12$

$10x-5x=-12+42$

$5x=30$

$x=6$

The solution set is {6}.

52. Set $y_1 = y_2$.

$$\begin{aligned} 7(3x-2)+5 &= 6(2x-1)+24 \\ 21x-14+5 &= 12x-6+24 \\ 21x-9 &= 12x+18 \\ 21x-12x &= 18+9 \\ 9x &= 27 \\ x &= 3 \end{aligned}$$

The solution set is $\{3\}$.

53. Set $y_1 - y_2 = 1$.

$$\begin{aligned} \frac{x-3}{5}-\frac{x-5}{4} &= 1 \\ 20\cdot\frac{x-3}{5}-20\cdot\frac{x-5}{4} &= 20\cdot 1 \\ 4(x-3)-5(x-5) &= 20 \\ 4x-12-5x+25 &= 20 \\ -x+13 &= 20 \\ -x &= 7 \\ x &= -7 \end{aligned}$$

The solution set is $\{-7\}$.

54. Set $y_1 - y_2 = -4$.

$$\begin{aligned} \frac{x+1}{4}-\frac{x-2}{3} &= -4 \\ 12\cdot\frac{x+1}{4}-12\cdot\frac{x-2}{3} &= 12(-4) \\ 3(x+1)-4(x-2) &= -48 \\ 3x+3-4x+8 &= -48 \\ -x+11 &= -48 \\ -x &= -59 \\ x &= 59 \end{aligned}$$

The solution set is $\{59\}$.

55. Set $y_1 + y_2 = y_3$.

$$\begin{aligned} \frac{5}{x+4}+\frac{3}{x+3} &= \frac{12x+19}{x^2+7x+12} \\ \frac{5}{x+4}+\frac{3}{x+3} &= \frac{12x+19}{(x+4)(x+3)} \\ (x+4)(x+3)\left(\frac{5}{x+4}+\frac{3}{x+3}\right) &= (x+4)(x+3)\frac{12x+19}{(x+4)(x+3)} \\ 5(x+3)+3(x+4) &= 12x+19 \\ 5x+15+3x+12 &= 12x+19 \\ 8x+27 &= 12x+19 \\ -4x &= -8 \\ x &= 2 \end{aligned}$$

The solution set is $\{2\}$.

56. Set $y_1 + y_2 = y_3$.

$$\frac{2x-1}{x^2+2x-8}+\frac{2}{x+4}=\frac{1}{x-2}$$

$$\frac{2x-1}{(x+4)(x-2)}+\frac{2}{x+4}=\frac{1}{x-2}$$

$$(x+4)(x-2)\left(\frac{2x-1}{(x+4)(x-2)}+\frac{2}{x+4}\right)=(x+4)(x-2)\frac{1}{x-2}$$

$$2x-1+2(x-2)=x+4$$

$$2x-1+2x-4=x+4$$

$$4x-5=x+4$$

$$3x=9$$

$$x=3$$

The solution set is {3}.

57.

$$0=4[x-(3-x)]-7(x+1)$$

$$0=4[x-3+x]-7x-7$$

$$0=4[2x-3]-7x-7$$

$$0=8x-12-7x-7$$

$$0=x-19$$

$$-x=-19$$

$$x=19$$

The solution set is {19}.

58.

$$0=2[3x-(4x-6)]-5(x-6)$$

$$0=2[3x-4x+6]-5x+30$$

$$0=2[-x+6]-5x+30$$

$$0=-2x+12-5x+30$$

$$0=-7x+42$$

$$7x=42$$

$$x=6$$

The solution set is {6}.

59.

$$0=\frac{x+6}{3x-12}-\frac{5}{x-4}-\frac{2}{3}$$

$$0=\frac{x+6}{3(x-4)}-\frac{5}{x-4}-\frac{2}{3}$$

$$3(x-4)\cdot 0=3(x-4)\left(\frac{x+6}{3(x-4)}-\frac{5}{x-4}-\frac{2}{3}\right)$$

$$0=\frac{3(x-4)(x+6)}{3(x-4)}-\frac{5\cdot 3(x-4)}{x-4}-\frac{2\cdot 3(x-4)}{3}$$

$$0=(x+6)-15-2(x-4)$$

$$0=x+6-15-2x+8$$

$$0=-x-1$$

$$x=-1$$

The solution set is {–1}.

60. $0=\frac{1}{5x+5}-\frac{3}{x+1}+\frac{7}{5}$

$$0=\frac{1}{5(x+1)}-\frac{3}{x+1}+\frac{7}{5}$$
$$5(x+1)\cdot 0=5(x+1)\left(\frac{1}{5(x+1)}-\frac{3}{x+1}+\frac{7}{5}\right)$$
$$0=\frac{1\cdot 5(x+1)}{5(x+1)}-\frac{3\cdot 5(x+1)}{x+1}+\frac{7\cdot 5(x+1)}{5}$$
$$0=1-15+7(x+1)$$
$$0=1-15+7x+7$$
$$0=-7+7x$$
$$-7x=-7$$
$$x=1$$

The solution set is {1}.

61. $4(x-7) = 4x-28$
$4x-28 = 4x-28$
The given equation is an identity.

62. $4(x-7) = 4x+28$
$4x-28 = 4x+28$
The given equation is an inconsistent equation.

63. $2x+3 = 2x-3$
$3 = -3$
The given equation is an inconsistent equation.

64. $\frac{7x}{x}=7$
$7x=7x$
The given equation is an identity.

65. $4x+5x = 8x$
$9x = 8x$
$x = 0$
The given equation is a conditional equation.

66. $8x+2x = 9x$
$10x = 9x$
$x = 0$
The given equation is a conditional equation.

67. $\frac{2x}{x-3}=\frac{6}{x-3}+4$

$$2x=6+4(x-3)$$
$$2x=6+4x-12$$
$$-2x=-6$$
$$x=3 \Rightarrow \text{ no solution}$$

The given equation is an inconsistent equation.

68. $\frac{3}{x-3} = \frac{x}{x-3} + 3$

$3 = x + 3(x-3)$

$3 = x + 3x - 9$

$-4x = -12$

$x = 3 \Rightarrow$ no solution

The given equation is an inconsistent equation.

69. $\frac{x+5}{2} - 4 = \frac{2x-1}{3}$

$3(x+5) - 24 = 2(2x-1)$

$3x + 15 - 24 = 4x - 2$

$-x = 7$

$x = -7$

The solution set is $\{-7\}$.

The given equation is a conditional equation.

70. $\frac{x+2}{7} = 5 - \frac{x+1}{3}$

$3(x+2) = 105 - 7(x+1)$

$3x + 6 = 105 - 7x - 7$

$10x = 92$

$x = \frac{92}{10}$

$x = \frac{46}{5}$

The solution set is $\left\{\frac{46}{5}\right\}$.

The given equation is a conditional equation.

71. $\frac{2}{x-2} = 3 + \frac{x}{x-2}$

$2 = 3(x-2) + x$

$2 = 3x - 6 + x$

$-4x = -8$

$x = 2 \Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

The given equation is an inconsistent equation.

72. $\frac{6}{x+3} + 2 = \frac{-2x}{x+3}$

$6 + 2(x+3) = -2x$

$6 + 2x + 6 = -2x$

$4x = -12$

$x = -3 \Rightarrow$ no solution

This equation is not true for any real numbers.

The given equation is an inconsistent equation.

73. $8x - (3x + 2) + 10 = 3x$
$8x - 3x - 2 + 10 = 3x$
$2x = -8$
$x = -4$
The solution set is {–4}.
The given equation is a conditional equation.

74. $2(x + 2) + 2x = 4(x + 1)$
$2x + 4 + 2x = 4x + 4$
$0 = 0$
This equation is true for all real numbers.
The given equation is an identity.

75. $\frac{2}{x}+\frac{1}{2}=\frac{3}{4}$
$8+2x=3x$
$-x=-8$
$x=8$
The solution set is {8}.
The given equation is a conditional equation.

76. $\frac{3}{x}-\frac{1}{6}=\frac{1}{3}$
$18-x=2x$
$-3x=-18$
$x=6$
The solution set is {6}.
The given equation is a conditional equation.

77. $\frac{4}{x-2}+\frac{3}{x+5}=\frac{7}{(x+5)(x-2)}$
$4(x+5)+3(x-2)=7$
$4x+20+3x-6=7$
$7x=-7$
$x=-1$
The solution set is {–1}.
The given equation is a conditional equation.

78. $\frac{1}{x-1}=\frac{1}{(2x+3)(x-1)}+\frac{4}{2x+3}$
$1(2x+3)=1+4(x-1)$
$2x+3=1+4x-4$
$-2x=-6$
$x=3$
The solution set is {3}.
The given equation is a conditional equation.

79.
$$\frac{4x}{x+3}-\frac{12}{x-3}=\frac{4x^2+36}{x^2-9};x\neq 3,-3$$
$$4x(x-3)-12(x+3)=4x^2+36$$
$$4x^2-12x-12x-36=4x^2+36$$
$$4x^2-24x-36=4x^2+36$$
$$-24x-36=36$$
$$-24x=72$$
$$x=-3 \quad \text{No solution}$$
The solution set is { }.
The given equation is an inconsistent equation.

80.
$$\frac{4}{x^2+3x-10}-\frac{1}{x^2+x-6}=\frac{3}{x^2-x-12}$$
$$\frac{4}{(x+5)(x-2)}-\frac{1}{(x+3)(x-2)}=\frac{3}{(x+3)(x-4)},x\neq -5,2,-3,4$$
$$4(x+3)(x-4)-1(x+5)(x-4)=3(x+5)(x-2)$$
$$4x^2-4x-48-x^2-x+20=3x^2+9x-30$$
$$3x^2-5x-28=3x^2+9x-30$$
$$2=14x$$
$$\frac{1}{7}=x$$
The solution set is $\left\{\frac{1}{7}\right\}$.
The given equation is a conditional equation.

81. The equation is $3(x-4)=3(2-2x)$, and the solution is $x=2$.

82. The equation is $3(2x-5)=5x+2$, and the solution is $x=17$.

83. The equation is $-3(x-3)=5(2-x)$, and the solution is $x=0.5$.

84. The equation is $2x-5=4(3x+1)-2$, and the solution is $x=-0.7$.

85. Solve: $4(x-2)+2=4x-2(2-x)$
$$4x-8+2=4x-4+2x$$
$$4x-6=6x-4$$
$$-2x-6=-4$$
$$-2x=2$$
$$x=-1$$
Now, evaluate x^2-x for $x=-1$:
$$x^2-x=(-1)^2-(-1)$$
$$=1-(-1)=1+1=2$$

86. Solve: $2(x-6)=3x+2(2x-1)$

$$\begin{aligned}2x-12&=3x+4x-2\\2x-12&=7x-2\\-5x-12&=-2\\-5x&=10\\x&=-2\end{aligned}$$

Now, evaluate x^2-x for $x=-2$:

$$\begin{aligned}x^2-x&=(-2)^2-(-2)\\&=4-(-2)=4+2=6\end{aligned}$$

87. Solve for x: $\dfrac{3(x+3)}{5}=2x+6$

$$\begin{aligned}3(x+3)&=5(2x+6)\\3x+9&=10x+30\\-7x+9&=30\\-7x&=21\\x&=-3\end{aligned}$$

Solve for y: $-2y-10=5y+18$

$$\begin{aligned}-7y-10&=18\\-7y&=28\\y&=-4\end{aligned}$$

Now, evaluate $x^2-(xy-y)$ for $x=-3$ and $y=-4$:

$$\begin{aligned}&x^2-(xy-y)\\&=(-3)^2-[-3(-4)-(-4)]\\&=(-3)^2-[12-(-4)]\\&=9-(12+4)=9-16=-7\end{aligned}$$

88. Solve for x: $\dfrac{13x-6}{4}=5x+2$

$$\begin{aligned}13x-6&=4(5x+2)\\13x-6&=20x+8\\-7x-6&=8\\-7x&=14\\x&=-2\end{aligned}$$

Solve for y: $5-y=7(y+4)+1$

$$\begin{aligned}5-y&=7y+28+1\\5-y&=7y+29\\5-8y&=29\\-8y&=24\\y&=-3\end{aligned}$$

Now, evaluate $x^2-(xy-y)$ for $x=-2$ and $y=-3$:

$$\begin{aligned}&x^2-(xy-y)\\&=(-2)^2-[-2(-3)-(-3)]\\&=(-2)^2-[6-(-3)]\\&=4-(6+3)=4-9=-5\end{aligned}$$

89. $\left[(3+6)^2\div 3\right]\cdot 4=-54x$

$$\begin{aligned}\left(9^2\div 3\right)\cdot 4&=-54x\\(81\div 3)\cdot 4&=-54x\\27\cdot 4&=-54x\\108&=-54x\\-2&=x\end{aligned}$$

The solution set is $\{-2\}$.

90. $2^3-\left[4(5-3)^3\right]=-8x$

$$\begin{aligned}8-\left[4(2)^3\right]&=-8x\\8-4\cdot 8&=-8x\\8-32&=-8x\\-24&=-8x\\3&=x\end{aligned}$$

The solution set is $\{3\}$.

91. $5-12x=8-7x-\left[6\div 3\left(2+5^3\right)+5x\right]$

$$\begin{aligned}5-12x&=8-7x-\left[6\div 3(2+125)+5x\right]\\5-12x&=8-7x-[6\div 3\cdot 127+5x]\\5-12x&=8-7x-[2\cdot 127+5x]\\5-12x&=8-7x-[254+5x]\\5-12x&=8-7x-254-5x\\5-12x&=-12x-246\\5&=-246\end{aligned}$$

The final statement is a contradiction, so the equation has no solution. The solution set is $\varnothing$.

92. $2(5x+58)=10x+4(21\div 3.5-11)$

$$\begin{aligned}10x+116&=10x+4(6-11)\\10x+116&=10x+4(-5)\\10x+116&=10x-20\\116&=-20\end{aligned}$$

The final statement is a contradiction, so the equation has no solution. The solution set is $\varnothing$.

93. $$\begin{aligned}0.7x+0.4(20)&=0.5(x+20)\\0.7x+8&=0.5x+10\\0.2x+8&=10\\0.2x&=2\\x&=10\end{aligned}$$
The solution set is $\{10\}$.

94. $$\begin{aligned}0.5(x+2)&=0.1+3(0.1x+0.3)\\0.5x+1&=0.1+0.3x+0.9\\0.5x+1&=0.3x+1\\0.2x+1&=1\\0.2x&=0\\x&=0\end{aligned}$$
The solution set is $\{0\}$.

95. $$\begin{aligned}4x+13-\{2x-[4(x-3)-5]\}&=2(x-6)\\4x+13-\{2x-[4x-12-5]\}&=2x-12\\4x+13-\{2x-[4x-17]\}&=2x-12\\4x+13-\{2x-4x+17\}&=2x-12\\4x+13-\{-2x+17\}&=2x-12\\4x+13+2x-17&=2x-12\\6x-4&=2x-12\\4x-4&=-12\\4x&=-8\\x&=-2\end{aligned}$$
The solution set is $\{-2\}$.

96. $$\begin{aligned}-2\{7-[4-2(1-x)+3]\}&=10-[4x-2(x-3)]\\-2\{7-[4-2+2x+3]\}&=10-[4x-2x+6]\\-2\{7-[2x+5]\}&=10-[2x+6]\\-2\{7-2x-5\}&=10-2x-6\\-2\{-2x+2\}&=-2x+4\\4x-4&=-2x+4\\6x-4&=4\\6x&=8\\x&=\frac{8}{6}=\frac{4}{3}\end{aligned}$$
The solution set is $\left\{\frac{4}{3}\right\}$.

97. $$\begin{aligned}\frac{W}{2}-3H&=53\\\frac{W}{2}-3(6)&=53\\\frac{W}{2}-18&=53\\\frac{W}{2}-18+18&=53+18\\\frac{W}{2}&=71\\2\cdot\frac{W}{2}&=2\cdot 71\\W&=142\end{aligned}$$
According to the formula, the healthy weight of a person of height 5’6” is 142 pounds. This is 13 pounds below the upper end of the range shown in the bar graph.

98. $$\begin{aligned}\frac{W}{2}-3H&=53\\\frac{W}{2}-3(12)&=53\\\frac{W}{2}-36&=53\\\frac{W}{2}-36+36&=53+36\\\frac{W}{2}&=89\\2\cdot\frac{W}{2}&=2\cdot 89\\W&=178\end{aligned}$$
According to the formula, the healthy weight of a person of height 6’ is 178 pounds. This is 6 pounds below the upper end of the range shown in the bar graph.

99. **a.** What cost \$10,000 in 1975 would cost about \$32,000 in 2000.

b. $C = 865x + 15{,}316$
$= 865(20) + 15{,}316$
$= 32{,}616$
According to Model 1, what cost \$10,000 in 1975 would cost about \$32,616 in 2000. This estimate is \$616 higher than the estimate read from the graph.

c. $C = -2x^2 + 900x + 15{,}397$
$= -2(20)^2 + 900(20) + 15{,}397$
$= 32{,}597$
According to Model 2, what cost \$10,000 in 1975 would cost about \$32,597 in 2000. This estimate is \$597 higher than the estimate read from the graph.

100. **a.** What cost \$10,000 in 1975 would have cost about \$24,000 in 1990.

b. $C = 865x + 15{,}316$
$= 865(10) + 15{,}316$
$= 23{,}966$
According to Model 1, what cost \$10,000 in 1975 would have cost about \$23,966 in 1990. This estimate is \$34 lower than the estimate read from the graph.

c. $C = -2x^2 + 900x + 15{,}397$
$= -2(10)^2 + 900(10) + 15{,}397$
$= 24{,}197$
According to Model 2, what cost \$10,000 in 1975 would have cost about \$24,197 in 1990. This estimate is \$197 higher than the estimate read from the graph.

101.
$$C = 865x + 15{,}316$$
$$43{,}861 = 865x + 15{,}316$$
$$28{,}545 = 865x$$
$$\frac{28{,}545}{865} = \frac{865x}{865}$$
$$33 = x$$
Model 1 predicts the cost will be \$43,861 33 years after 1980, or 2013.

102.
$$C = 865x + 15{,}316$$
$$54{,}241 = 865x + 15{,}316$$
$$38{,}925 = 865x$$
$$\frac{38{,}925}{865} = \frac{865x}{865}$$
$$45 = x$$
Model 1 predicts the cost will be \$54,241 33 years after 1980, or 2025.

103. 11 learning trials; represented by the point $(11, 0.95)$ on the graph.

104. 1 learning trial; represented by the point $(1, 0.5)$ on the graph.

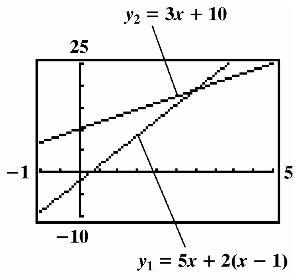
105.
$$C = \frac{x + 0.1(500)}{x + 500}$$
$$0.28 = \frac{x + 0.1(500)}{x + 500}$$
$$0.28(x + 500) = x + 0.1(500)$$
$$0.28x + 140 = x + 50$$
$$-0.72x = -90$$
$$\frac{-0.72x}{-0.72} = \frac{-90}{-0.72}$$
$$x = 125$$
125 liters of pure peroxide must be added.

106. **a.** $C = \dfrac{x + 0.35(200)}{x + 200}$

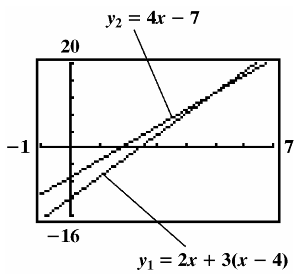
b.
$$0.74 = \frac{x + 0.35(200)}{x + 200}$$
$$0.74(x + 200) = x + 0.35(200)$$
$$0.74x + 148 = x + 70$$
$$-0.26x = -78$$
$$\frac{-0.26x}{-0.26} = \frac{-78}{-0.26}$$
$$x = 300$$
300 liters of pure acid must be added.

107. – 115. Answers may vary.

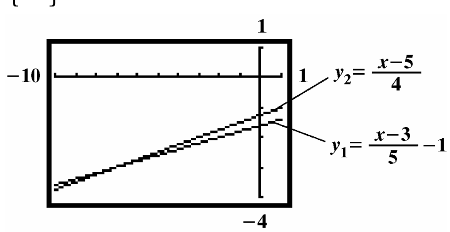
116. $\{3\}$



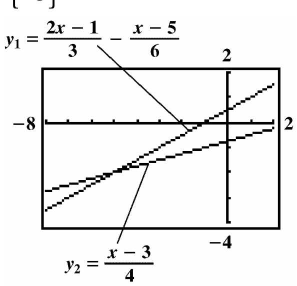
117. $\{5\}$



118. $\{-7\}$



119. $\{-5\}$



120. does not make sense; Explanations will vary. Sample explanation: Substitute $n = 6$ into the equation to find P.

121. makes sense

122. makes sense

123. makes sense

124. false; Changes to make the statement true will vary. A sample change is: $x = 0$ is a solution.

125. false; Changes to make the statement true will vary. A sample change is: In the first equation, $x \neq 4$.

126. true

127. false; Changes to make the statement true will vary. A sample change is: If $a = 0$, then $ax + b = 0$ is equivalent to $b = 0$, which either has no solution ($b \neq 0$) or infinitely many solutions ($b = 0$).

128. Answers may vary.

129.
$$\frac{7x+4}{b} + 13 = x$$
$$\frac{7(-6)+4}{b} + 13 = -6$$
$$\frac{-42+4}{b} + 13 = -6$$
$$\frac{-38}{b} + 13 = -6$$
$$\frac{-38}{b} = -19$$
$$-38 = -19b$$
$$b = 2$$

130.
$$\frac{4x-b}{x-5} = 3$$
$$4x - b = 3(x-5)$$
The solution set will be $\varnothing$ if $x = 5$.
$$4(5) - b = 3(5-5)$$
$$20 - b = 0$$
$$20 = b$$
$$b = 20$$

131. $x + 150$

132. $20 + 0.05x$

133. $4x + 400$

Section 1.3

Check Point Exercises

1. Let x = the average salary for women
Let x + 14,037 = the average salary for men

$$x+(x+14,037)=130,015$$
$$x+x+14,037=130,015$$
$$2x+14,037=130,015$$
$$2x=115,978$$
$$x=57,989$$
$$x+14,037=72,026$$

In 2007 the average teaching salary for women was \$57,989 and the average salary for men was \$72,026.

2. Let x = the number of years since 1969.

$$88-1.1x=33$$
$$-1.1x=33-88$$
$$-1.1x=-55$$
$$x=\frac{-55}{-1.1}$$
$$x=50$$

33% of female freshmen will respond this way 50 years after 1969, or 2019.

3. Let x = the number of minutes at which the costs of the two plans are the same.

$$\overbrace{15+0.08x}^{\text{Plan A}}=\overbrace{3+0.12x}^{\text{Plan B}}$$
$$15+0.08x-15=3+0.12x-15$$
$$0.08x=0.12x-12$$
$$0.08x-0.12x=0.12x-12-0.12x$$
$$-0.04x=-12$$
$$\frac{-0.04x}{-0.04}=\frac{-12}{-0.04}$$
$$x=300$$

The two plans are the same at 300 minutes.

4. Let x = the computer's price before the reduction.

$$x-0.30x=840$$
$$0.70x=840$$
$$x=\frac{840}{0.70}$$
$$x=1200$$

Before the reduction the computer's price was \$1200.

5. Let x = the amount invested at 9%.
Let 5000 – x = the amount invested at 11%.

$$0.09x+0.11(5000-x)=487$$
$$0.09x+550-0.11x=487$$
$$-0.02x+550=487$$
$$-0.02x=-63$$
$$x=\frac{-63}{-0.02}$$
$$x=3150$$
$$5000-x=1850$$

\$3150 was invested at 9% and \$1850 was invested at 11%.

6. Let x = the width of the court.
Let x + 44 = the length of the court.

$$2l+2w=P$$
$$2(x+44)+2x=288$$
$$2x+88+2x=288$$
$$4x+88=288$$
$$4x=200$$
$$x=\frac{200}{4}$$
$$x=50$$
$$x+44=94$$

The dimensions of the court are 50 by 94.

7.

$$2l+2w=P$$
$$2l+2w-2l=P-2l$$
$$2w=P-2l$$
$$\frac{2w}{2}=\frac{P-2l}{2}$$
$$w=\frac{P-2l}{2}$$

8.

$$P=C+MC$$
$$P=C(1+M)$$
$$\frac{P}{1+M}=\frac{C(1+M)}{1+M}$$
$$\frac{P}{1+M}=C$$
$$C=\frac{P}{1+M}$$

Exercise Set 1.3

1. Let x = the number
$$5x-4=26$$
$$5x=30$$
$$x=6$$
The number is 6.

2. Let x = the number
$$2x-3=11$$
$$2x=14$$
$$x=7$$
The number is 7.

3. Let x = the number
$$x-0.20x=20$$
$$0.80x=20$$
$$x=25$$
The number is 25.

4. Let x = the number
$$x-0.30x=28$$
$$0.70x=28$$
$$x=40$$
The number is 40.

5. Let x = the number
$$0.60x+x=192$$
$$1.6x=192$$
$$x=120$$
The number is 120.

6. Let x = the number
$$0.80x+x=252$$
$$1.8x=252$$
$$x=140$$
The number is 140.

7. Let x = the number
$$0.70x=224$$
$$x=320$$
The number is 320.

8. Let x = the number
$$0.70x=252$$
$$x=360$$
The number is 360.

9. Let x = the number
$x+26$ = the other number
$$x+(x+26)=64$$
$$x+x+26=64$$
$$2x+26=64$$
$$2x=38$$
$$x=19$$
If x = 19, then $x+26=45$.
The numbers are 19 and 45.

10. Let x = the number,
Let x +24 = the other number
$$x+(x+24)=58$$
$$x+x+24=58$$
$$2x+24=58$$
$$2x=34$$
$$x=17$$
If x = 17, then x +24 = 41.
The numbers are 17 and 41.

11.
$$y_1-y_2=2$$
$$(13x-4)-(5x+10)=2$$
$$13x-4-5x-10=2$$
$$8x-14=2$$
$$8x=16$$
$$\frac{8x}{8}=\frac{16}{8}$$
$$x=2$$

12.
$$y_1-y_2=3$$
$$(10x+6)-(12x-7)=3$$
$$10x+6-12x+7=3$$
$$-2x+13=3$$
$$-2x=-10$$
$$\frac{-2x}{-2}=\frac{-10}{-2}$$
$$x=5$$

13.
$$y_1=8y_2+14$$
$$10(2x-1)=8(2x+1)+14$$
$$20x-10=16x+8+14$$
$$20x-10=16x+22$$
$$4x=32$$
$$\frac{4x}{4}=\frac{32}{4}$$
$$x=8$$

14.
$$\begin{aligned} y_1 &= 12y_2 - 51 \\ 9(3x-5) &= 12(3x-1) - 51 \\ 27x - 45 &= 36x - 12 - 51 \\ 27x - 45 &= 36x - 63 \\ -9x &= -18 \\ \frac{-9x}{-9} &= \frac{-18}{-9} \\ x &= 2 \end{aligned}$$

15.
$$\begin{aligned} 3y_1 - 5y_2 &= y_3 - 22 \\ 3(2x+6) - 5(x+8) &= (x) - 22 \\ 6x + 18 - 5x - 40 &= x - 22 \\ x - 22 &= x - 22 \\ x - x &= -22 + 22 \\ 0 &= 0 \end{aligned}$$
The solution set is the set of all real numbers.

16.
$$\begin{aligned} 2y_1 - 3y_2 &= 4y_3 - 8 \\ 2(2.5) - 3(2x+1) &= 4(x) - 8 \\ 5 - 6x - 3 &= 4x - 8 \\ -6x + 2 &= 4x - 8 \\ -10x &= -10 \\ \frac{-10x}{-10} &= \frac{-10}{-10} \\ x &= 1 \end{aligned}$$

17.
$$\begin{aligned} 3y_1 + 4y_2 &= 4y_3 \\ 3\left(\frac{1}{x}\right) + 4\left(\frac{1}{2x}\right) &= 4\left(\frac{1}{x-1}\right) \\ \frac{3}{x} + \frac{2}{x} &= \frac{4}{x-1} \\ \frac{5}{x} &= \frac{4}{x-1} \\ \frac{5x(x-1)}{x} &= \frac{4x(x-1)}{x-1} \\ 5(x-1) &= 4x \\ 5x - 5 &= 4x \\ x &= 5 \end{aligned}$$

18.
$$\begin{aligned} 6y_1 - 3y_2 &= 7y_3 \\ 6\left(\frac{1}{x}\right) - 3\left(\frac{1}{x^2-x}\right) &= 7\left(\frac{1}{x-1}\right) \\ \frac{6}{x} - \frac{3}{x^2-x} &= \frac{7}{x-1} \\ \frac{6}{x} - \frac{3}{x(x-1)} &= \frac{7}{x-1} \\ x(x-1)\left(\frac{6}{x} - \frac{3}{x(x-1)}\right) &= x(x-1)\frac{7}{x-1} \\ \frac{6x(x-1)}{x} - \frac{3x(x-1)}{x(x-1)} &= \frac{7x(x-1)}{x-1} \\ 6(x-1) - 3 &= 7x \\ 6x - 6 - 3 &= 7x \\ 6x - 9 &= 7x \\ 6x - 7x &= 9 \\ -x &= 9 \\ \frac{-x}{-1} &= \frac{9}{-1} \\ x &= -9 \end{aligned}$$

19. Let $x =$ the time spent listening to radio.
Let $x + 581 =$ the time spent watching TV.
$$\begin{aligned} x + (x + 581) &= 2529 \\ x + x + 581 &= 2529 \\ 2x + 581 - 581 &= 2529 - 581 \\ 2x &= 1948 \\ x &= 974 \\ x + 581 &= 1555 \end{aligned}$$
Americans spent 974 hours listening to radio and 1555 hours watching TV.

20. Let $x =$ number of weeks Americans spend on vacation.
Let $x + 4 =$ number of weeks Italians spend on vacation.
$$\begin{aligned} x + (x + 4) &= 11.8 \\ x + x + 4 &= 11.8 \\ 2x + 4 - 4 &= 11.8 - 4 \\ 2x &= 7.8 \\ x &= 3.9 \\ x + 4 &= 7.9 \end{aligned}$$
Americans spend an average of 3.9 weeks on vacation and Italians spend an average of 7.9 weeks.

21. Let $x =$ the average salary for carpenters.
Let $2x - 7740 =$ the average salary for computer programmers.

$$\begin{aligned} x + (2x - 7740) &= 99,000 \\ x + 2x - 7740 &= 99,000 \\ 3x - 7740 &= 99,000 \\ 3x - 7740 + 7740 &= 99,000 + 7740 \\ 3x &= 106,740 \\ x &= 35,580 \\ 2x - 7740 &= 63,420 \end{aligned}$$

The average salary for carpenters is $35,580 and the average salary for computer programmers is $63,420.

22. Let $x =$ the average salary for janitors.
Let $3x - 3500 =$ the average salary for registered nurses.

$$\begin{aligned} x + (3x - 3500) &= 74,060 \\ x + 3x - 3500 &= 74,060 \\ 4x - 3500 &= 74,060 \\ 4x - 3500 + 3500 &= 74,060 + 3500 \\ 4x &= 77,560 \\ x &= 19,390 \\ 3x - 3500 &= 54,760 \end{aligned}$$

The average salary for janitors is $19,390 and the average salary for registered nurses is $54,760.

23. Let x = the number of years since 1983.

$$\begin{aligned} 43 + 1.5x &= 100 \\ 1.5x &= 100 - 43 \\ 1.5x &= 57 \\ x &= \frac{57}{1.5} \\ x &= 38 \end{aligned}$$

All American adults will approve 38 years after 1983, or 2021.

24. Let x = the number of years since 1986.

$$\begin{aligned} 43 + 0.6x &= 61 \\ 0.6x &= 61 - 43 \\ 0.6x &= 18 \\ x &= \frac{18}{0.6} \\ x &= 30 \end{aligned}$$

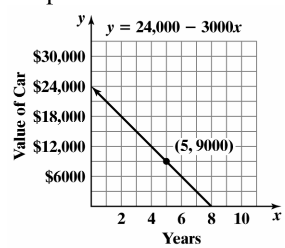
61% of American adults will approve 30 years after 1986, or 2016.

25. **a.** $y = 24,000 - 3000x$

b.
$$\begin{aligned} y &= 24,000 - 3000x \\ 9000 &= 24,000 - 3000x \\ 9000 - 24,000 &= -3000x \\ -15,000 &= -3000x \\ x &= \frac{-15,000}{-3000} \\ x &= 5 \end{aligned}$$

The car's value will drop to $9000 after 5 years.

c. Graph:

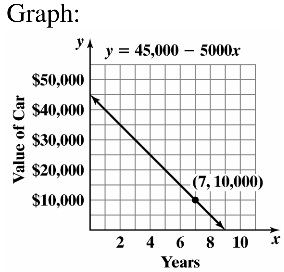


26. **a.** $y = 45,000 - 5000x$

b.
$$\begin{aligned} y &= 45,000 - 5000x \\ 10,000 &= 45,000 - 5000x \\ 10,000 - 45,000 &= -5000x \\ -35,000 &= -5000x \\ x &= \frac{-35,000}{-5000} \\ x &= 7 \end{aligned}$$

The car's value will drop to $10,000 after 7 years.

c. Graph:



27. Let x = the number of months.
The cost for Club A: $25x + 40$
The cost for Club B: $30x + 15$

$$\begin{aligned} 25x + 40 &= 30x + 15 \\ -5x + 40 &= 15 \\ -5x &= -25 \\ x &= 5 \end{aligned}$$

The total cost for the clubs will be the same at 5 months. The cost will be
$25(5) + 40 = 30(5) + 15 = \$165$

28. Let g = the number of video games rented

$9g = 4g + 50$

$5g = 50$

$g = 10$

The total amount spent at each store will be the same after 10 rentals.

$9g = 9(10) = 90$

The total amount spent will be \$90.

29. Let x = the number of uses.

Cost without discount pass: $1.25x$

Cost with discount pass: $15 + 0.75x$

$1.25x = 15 + 0.75x$

$0.50x = 15$

$x = 30$

The bus must be used 30 times in a month for the costs to be equal.

30. Cost per crossing: $\$5x$

Cost with discount pass: $\$30 + \$3.50x$

$5x = 30 + 3.50x$

$1.50x = 30$

$x = 20$

The bridge must be used 20 times in a month for the costs to be equal.

31. a. Let x = the number of years (after 2005).

College A's enrollment: $13{,}300 + 1000x$

College B's enrollment: $26{,}800 - 500x$

$13{,}300 + 1000x = 26{,}800 - 500x$

$13{,}300 + 1500x = 26{,}800$

$1500x = 13{,}500$

$x = 9$

The two colleges will have the same enrollment in the year $2005 + 9 = 2014$.

That year the enrollments will be

$13{,}300 + 1000(9)$

$= 26{,}800 - 500(9)$

$= 22{,}300$ students

b. Check points to determine that

$y_1 = 13{,}300 + 1000x$ and

$y_2 = 26{,}800 - 500x$.

32. Let x = the number of years after 2000

$10{,}600{,}000 - 28{,}000x = 10{,}200{,}000 - 12{,}000x$

$-16{,}000x = -400{,}000$

$x = 25$

The countries will have the same population 25 years after the year 2000, or the year 2025.

$10{,}200{,}000 - 12{,}000x = 10{,}200{,}000 - 12{,}000(25)$

$= 10{,}200{,}000 - 300{,}000$

$= 9{,}900{,}000$

The population in the year 2025 will be 9,900,000.

33. Let x = the cost of the television set.

$x - 0.20x = 336$

$0.80x = 336$

$x = 420$

The television set's price is \$420.

34. Let x = the cost of the dictionary

$x - 0.30x = 30.80$

$0.70x = 30.80$

$x = 44$

The dictionary's price before the reduction was \$44.

35. Let x = the nightly cost

$x + 0.08x = 162$

$1.08x = 162$

$x = 150$

The nightly cost is \$150.

36. Let x = the nightly cost

$x + 0.05x = 252$

$1.05x = 252$

$x = 240$

The nightly cost is \$240.

37. Let c = the dealer's cost

$584 = c + 0.25c$

$584 = 1.25c$

$467.20 = c$

The dealer's cost is \$467.20.

38. Let c = the dealer's cost

$15 = c + 0.25c$

$15 = 1.25c$

$12 = c$

The dealer's cost is \$12.

39. Let x = the amount invested at 6%.
Let 7000 – x = the amount invested at 8%.

$$0.06x + 0.08(7000 - x) = 520$$
$$0.06x + 560 - 0.08x = 520$$
$$-0.02x + 560 = 520$$
$$-0.02x = -40$$
$$x = \frac{-40}{-0.02}$$
$$x = 2000$$
$$7000 - x = 5000$$

\$2000 was invested at 6% and \$5000 was invested at 8%.

40. Let x = the amount invested at 5%.
Let 11,000 – x = the amount invested at 8%.

$$0.05x + 0.08(11,000 - x) = 730$$
$$0.05x + 880 - 0.08x = 730$$
$$-0.03x + 880 = 730$$
$$-0.03x = -150$$
$$x = \frac{-150}{-0.03}$$
$$x = 5000$$
$$11,000 - x = 6000$$

\$5000 was invested at 5% and \$6000 was invested at 8%.

41. Let x = amount invested at 12%
8000 – x = amount invested at 5% loss

$$.12x - .05(8000 - x) = 620$$
$$.12x - 400 + .05x = 620$$
$$.17x = 1020$$
$$x = 6000$$
$$8000\text{-}x = 2000$$

\$6000 at 12%, \$2000 at 5% loss

42. Let x = amount at 14%
12000 – x = amount at 6%

$$.14x - 0.6(12000 - x) = 680$$
$$.14x - 720 + .06x = 680$$
$$.2x = 1400$$
$$x = 7000$$
$$12000 - 7000 = 5000$$

\$7000 at 14%, \$5000 at 6% loss

43. Let w = the width of the field
Let $2w$ = the length of the field

$$P = 2(\text{length}) + 2(\text{width})$$
$$300 = 2(2w) + 2(w)$$
$$300 = 4w + 2w$$
$$300 = 6w$$
$$50 = w$$

If $w = 50$, then $2w = 100$. Thus, the dimensions are 50 yards by 100 yards.

44. Let w = the width of the swimming pool,
Let $3w$ = the length of the swimming pool

$$P = 2(\text{length}) + 2(\text{width})$$
$$320 = 2(3w) + 2(w)$$
$$320 = 6w + 2w$$
$$320 = 8w$$
$$40 = w$$

If w = 40, $3w$ = 3(40) = 120.
The dimensions are 40 feet by 120 feet.

45. Let w = the width of the field
Let $2w + 6$ = the length of the field

$$228 = 6w + 12$$
$$216 = 6w$$
$$36 = w$$

If $w = 36$, then $2w + 6 = 2(36) + 6 = 78$. Thus, the dimensions are 36 feet by 78 feet.

46. Let w = the width of the pool,
Let $2w - 6$ = the length of the pool

$$P = 2(\text{length}) + 2(\text{width})$$
$$126 = 2(2w - 6) + 2(w)$$
$$126 = 4w - 12 + 2w$$
$$126 = 6w - 12$$
$$138 = 6w$$
$$23 = w$$

Find the length.

$$2w - 6 = 2(23) - 6 = 46 - 6 = 40$$

The dimensions are 23 meters by 40 meters.

47. Let x = the width of the frame.
Total length: $16+2x$
Total width: $12+2x$

$P=2(\text{length})+2(\text{width})$
$72=2(16+2x)+2(12+2x)$
$72=32+4x+24+4x$
$72=8x+56$
$16=8x$
$2=x$

The width of the frame is 2 inches.

48. Let w = the width of the path
Let 40 + 2w = the width of the pool and path
Let 60 + 2w = the length of the pool and path

$2(40+2w)+2(60+2w)=248$
$80+4w+120+4w=248$
$200+8w=248$
$8w=48$
$w=6$

The width of the path is 6 feet.

49. Let x = number of hours
$35x$ = labor cost
$35x + 63 = 448$
$35x = 385$
$x = 11$
It took 11 hours.

50. Let x = number of hours
$63x$ = labor cost
$63x + 532 = 1603$
$63x = 1071$
$x = 17$
17 hours were required to repair the yacht.

51. Let x = inches over 5 feet
$100 + 5x = 135$
$5x = 35$
$x = 7$
A height of 5 feet 7 inches corresponds to 135 pounds.

52. Let g = the gross amount of the paycheck

$\text{Yearly Salary}=2(12)g+750$
$33150=24g+750$
$32400=24g$
$1350=g$

The gross amount of each paycheck is $1350.

53. Let x = the weight of unpeeled bananas.
$\frac{7}{8}x =$ weight of peeled bananas

$x=\frac{7}{8}x+\frac{7}{8}$
$\frac{1}{8}x=\frac{7}{8}$
$x=7$

The banana with peel weighs 7 ounces.

54. Let x = the length of the call.

$0.43+0.32(x-1)+2.10=5.73$
$0.43+0.32x-0.32+2.10=5.73$
$0.32x+2.21=5.73$
$0.32x=3.52$
$x=11$

The person talked for 11 minutes.

55. $A = lw$
$w=\frac{A}{l}$
area of rectangle

56. $D = RT$
$R=\frac{D}{T}$
distance, rate, time equation

57. $A=\frac{1}{2}bh$
$2A=bh$
$b=\frac{2A}{h}$;
area of triangle

58. $V=\frac{1}{3}Bh$
$3V=Bh$
$B=\frac{3V}{h}$
volume of a cone

59. $I = Prt$
$P=\frac{I}{rt}$;
interest

60. $C = 2\pi r$

$r = \frac{C}{2\pi}$;

circumference of a circle

61. $E = mc^2$

$m = \frac{E}{c^2}$;

Einstein's equation

62. $V = \pi r^2 h$

$h = \frac{V}{\pi r^2}$;

volume of a cylinder

63. $T = D + pm$

$T - D = pm$

$\frac{T-D}{m} = \frac{pm}{m}$

$\frac{T-D}{m} = p$

total of payment

64. $P = C + MC$

$P - C = MC$

$\frac{P-C}{C} = M$

markup based on cost

65. $A = \frac{1}{2}h(a+b)$

$2A = h(a+b)$

$\frac{2A}{h} = a + b$

$\frac{2A}{h} - b = a$

area of trapezoid

66. $A = \frac{1}{2}h(a+b)$

$2A = h(a+b)$

$\frac{2A}{h} = a + b$

$\frac{2A}{h} - a = b$

area of trapezoid

67. $S = P + Prt$

$S - P = Prt$

$\frac{S-P}{Pt} = r$;

interest

68. $S = P + Prt$

$S - P = Prt$

$\frac{S-P}{Pr} = t$;

interest

69. $B = \frac{F}{S-V}$

$B(S-V) = F$

$S - V = \frac{F}{B}$

$S = \frac{F}{B} + V$

70. $S = \frac{C}{1-r}$

$S(1-r) = C$

$1 - r = \frac{C}{S}$

$-r = \frac{C}{S} - 1$

$r = -\frac{C}{S} + 1$

markup based on selling price

71. $IR + Ir = E$

$I(R+r) = E$

$I = \frac{E}{R+r}$

electric current

72. $A = 2lw + 2lh + 2wh$

$A - 2lw = h(2l + 2w)$

$\frac{A-2lw}{2l+2w} = h$

surface area

73. $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

$qf + pf = pq$

$f(q+p) = pq$

$f = \frac{pq}{p+q}$

thin lens equation

74. $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$

$R_1R_2 = RR_2 + RR_1$

$R_1R_2 - RR_1 = RR_2$

$R_1(R_2 - R) = RR_2$

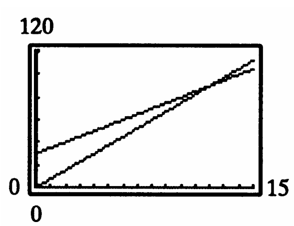
$R_1 = \frac{RR_2}{R_2 - R}$

resistance

75. – 79. Answers may vary.

80. **a.** $F = 30 + 5x$
$F = 7.5x$

b.



c. Calculator shows the graphs to intersect at (12, 90); the two options both cost $90 when 12 hours court time is used per month.

d. $30 + 5x = 7.5x$

$30 = 2.5x$

$x = 12$

Rent the court 12 hours per month.

81. does not make sense; Explanations will vary. Sample explanation: Though mathematical models can often provide excellent estimates about future attitudes, they cannot guaranty perfect precision.

82. makes sense

83. does not make sense; Explanations will vary. Sample explanation: Solving a formula for one of its variables does not produce a numerical value for the variable.

84. does not make sense; Explanations will vary. Sample explanation: The correct equation is $x - 0.35x = 780$.

85. $0.1x + .9(1000 - x) = 420$

$0.1 + 900 - 0.9x = 420$

$-0.8x = -480$

$x = 600$

600 students at the north campus, 400 students at south campus.

86. Let x = original price
$x - 0.4x = 0.6x$ = price after first reduction
$0.6x - 0.4(0.6x)$ = price after second reduction

$0.6x - 0.24x = 72$

$0.36x = 72$

$x = 200$

The original price was $200.

87. Let x = woman's age
$3x$ = Coburn's age
$3x + 20 = 2(x + 20)$
$3x + 20 = 2x + 40$
$x + 20 = 40$
$x = 20$
Coburn is 60 years old the woman is 20 years old.

88. Let x = correct answers
$26 - x$ = incorrect answers

$8x - 5(26 - x) = 0$

$8x - 130 + 5x = 0$

$13x - 130 = 0$

$13x = 130$

$x = 10$

10 problems were solved correctly.

89. Let x = mother's amount
$2x$ = boy's amount
$\frac{x}{2}$ = girl's amount

$x + 2x + \frac{x}{2} = 14,000$

$\frac{7}{2}x = 14,000$

$x = \$4,000$

The mother received $4000, the boy received $8000, and the girl received $2000.

90. Let x = the number of plants originally stolen
After passing the first security guard, the thief has:
$$x-\left(\frac{1}{2}x+2\right)=x-\frac{1}{2}x-2=\frac{1}{2}x-2$$
After passing the second security guard, the thief has:
$$\frac{1}{2}x-2-\left(\frac{\frac{1}{2}x-2}{2}+2\right)=\frac{1}{4}x-3$$
After passing the third security guard, the thief has:
$$\frac{1}{4}x-3-\left(\frac{\frac{1}{4}x-3}{2}+2\right)=\frac{1}{8}x-\frac{7}{2}$$
Thus, $\frac{1}{8}x-\frac{7}{2}=1$
$$x-28=8$$
$$x=36$$
The thief stole 36 plants.

91. $V=C-\frac{C-S}{L}N$
$$VL=CL-CN+SN$$
$$VL-SN=CL-CN$$
$$VL-SN=C(L-N)$$
$$\frac{VL-SN}{L-N}=C$$

92. Answers may vary

93. $(7-3x)(-2-5x)=-14-35x+6x+15x^2$
$$=-14-29x+15x^2$$
or
$$=15x^2-29x-14$$

94. $\sqrt{18}-\sqrt{8}=\sqrt{9\cdot 2}-\sqrt{4\cdot 2}$
$$=3\sqrt{2}-2\sqrt{2}$$
$$=\sqrt{2}$$

95. $\frac{7+4\sqrt{2}}{2-5\sqrt{2}}\cdot\frac{2+5\sqrt{2}}{2+5\sqrt{2}}=\frac{14+35\sqrt{2}+8\sqrt{2}+40}{4+10\sqrt{2}-10\sqrt{2}-50}$
$$=\frac{54+43\sqrt{2}}{-46}$$
$$=-\frac{54+43\sqrt{2}}{46}$$

Section 1.4

Check Point Exercises

1. **a.** $(5-2i)+(3+3i)$
$$=5-2i+3+3i$$
$$=(5+3)+(-2+3)i$$
$$=8+i$$

b. $(2+6i)-(12-i)$
$$=2+6i-12+i$$
$$=(2-12)+(6+1)i$$
$$=-10+7i$$

2. **a.** $7i(2-9i)=7i(2)-7i(9i)$
$$=14i-63i^2$$
$$=14i-63(-1)$$
$$=63+14i$$

b. $(5+4i)(6-7i)=30-35i+24i-28i^2$
$$=30-35i+24i-28(-1)$$
$$=30+28-35i+24i$$
$$=58-11i$$

3. $\frac{5+4i}{4-i}=\frac{5+4i}{4-i}\cdot\frac{4+i}{4+i}$
$$=\frac{20+5i+16i+4i^2}{16+4i-4i-i^2}$$
$$=\frac{20+21i-4}{16+1}$$
$$=\frac{16+21i}{17}$$
$$=\frac{16}{17}+\frac{21}{17}i$$

4. **a.** $\sqrt{-27}+\sqrt{-48}=i\sqrt{27}+i\sqrt{48}$
$$=i\sqrt{9\cdot 3}+i\sqrt{16\cdot 3}$$
$$=3i\sqrt{3}+4i\sqrt{3}$$
$$=7i\sqrt{3}$$

b. $(-2+\sqrt{-3})^2=(-2+i\sqrt{3})^2$
$$=(-2)^2+2(-2)(i\sqrt{3})+(i\sqrt{3})^2$$
$$=4-4i\sqrt{3}+3i^2$$
$$=4-4i\sqrt{3}+3(-1)$$
$$=1-4i\sqrt{3}$$

c. $\frac{-14+\sqrt{-12}}{2}=\frac{-14+i\sqrt{12}}{2}$
$=\frac{-14+2i\sqrt{3}}{2}$
$=\frac{-14}{2}+\frac{2i\sqrt{3}}{2}$
$=-7+i\sqrt{3}$

Exercise Set 1.4

1. $(7+2i)+(1-4i)=7+2i+1-4i$
$=7+1+2i-4i$
$=8-2i$

2. $(-2+6i)+(4-i)$
$=-2+6i+4-i$
$=-2+4+6i-i$
$=2+5i$

3. $(3+2i)-(5-7i)=3-5+2i+7i$
$=3+2i-5+7i$
$=-2+9i$

4. $(-7+5i)-(-9-11i)=-7+5i+9+11i$
$=-7+9+5i+11i$
$=2+16i$

5. $6-(-5+4i)-(-13-i)=6+5-4i+13+i$
$=24-3i$

6. $7-(-9+2i)-(-17-i)=7+9-2i+17+i$
$=33-i$

7. $8i-(14-9i)=8i-14+9i$
$=-14+8i+9i$
$=-14+17i$

8. $15i-(12-11i)=15i-12+11i$
$=-12+15i+11i$
$=-12+26i$

9. $-3i(7i-5)=-21i^2+15i$
$=-21(-1)+15i$
$=21+15i$

10. $-8i(2i-7)=-16i^2+56i=-16(-1)+56i$
$=9-25i^2=9+25=34=16+56i$

11. $(-5+4i)(3+i)=-15-5i+12i+4i^2$
$=-15+7i-4$
$=-19+7i$

12. $(-4-8i)(3+i)=-12-4i-24i-8i^2$
$=-12-28i+8$
$=-4-28i$

13. $(7-5i)(-2-3i)=-14-21i+10i+15i^2$
$=-14-15-11i$
$=-29-11i$

14. $(8-4i)(-3+9i)=-24+72i+12i-36i^2$
$=-24+36+84i$
$=12+84i$

15. $(3+5i)(3-5i)=9-15i+15i-25i^2$
$=9+25$
$=34$

16. $(2+7i)(2-7i)=4-49i^2=4+49=53$

17. $(-5+i)(-5-i)=25+5i-5i-i^2$
$=25+1$
$=26$

18. $(-7+i)(-7-i)=49+7i-7i-i^2$
$=49+1$
$=50$

19. $(2+3i)^2=4+12i+9i^2$
$=4+12i-9$
$=-5+12i$

20. $(5-2i)^2=25-20i+4i^2$
$=25-20i-4$
$=21-20i$

21. $\frac{2}{3-i}=\frac{2}{3-i}\cdot\frac{3+i}{3+i}$
$=\frac{2(3+i)}{9+1}$
$=\frac{2(3+i)}{10}$
$=\frac{3+i}{5}$
$=\frac{3}{5}+\frac{1}{5}i$

22. $\dfrac{3}{4+i}=\dfrac{3}{4+i}\cdot\dfrac{4-i}{4-i}$
$=\dfrac{3(4-i)}{16-i^2}$
$=\dfrac{3(4-i)}{17}$
$=\dfrac{12}{17}-\dfrac{3}{17}i$

23. $\dfrac{2i}{1+i}=\dfrac{2i}{1+i}\cdot\dfrac{1-i}{1-i}=\dfrac{2i-2i^2}{1+1}=\dfrac{2+2i}{2}=1+i$

24. $\dfrac{5i}{2-i}=\dfrac{5i}{2-i}\cdot\dfrac{2+i}{2+i}$
$=\dfrac{10i+5i^2}{4+1}$
$=\dfrac{-5+10i}{5}$
$=-1+2i$

25. $\dfrac{8i}{4-3i}=\dfrac{8i}{4-3i}\cdot\dfrac{4+3i}{4+3i}$
$=\dfrac{32i+24i^2}{16+9}$
$=\dfrac{-24+32i}{25}$
$=-\dfrac{24}{25}+\dfrac{32}{25}i$

26. $\dfrac{-6i}{3+2i}=\dfrac{-6i}{3+2i}\cdot\dfrac{3-2i}{3-2i}=\dfrac{-18i+12i^2}{9+4}$
$=\dfrac{-12-18i}{13}=-\dfrac{12}{13}-\dfrac{18}{13}i$

27. $\dfrac{2+3i}{2+i}=\dfrac{2+3i}{2+i}\cdot\dfrac{2-i}{2-i}$
$=\dfrac{4+4i-3i^2}{4+1}$
$=\dfrac{7+4i}{5}$
$=\dfrac{7}{5}+\dfrac{4}{5}i$

28. $\dfrac{3-4i}{4+3i}=\dfrac{3-4i}{4+3i}\cdot\dfrac{4-3i}{4-3i}$
$=\dfrac{12-25i+12i^2}{16+9}$
$=\dfrac{-25i}{25}$
$=-i$

29. $\sqrt{-64}-\sqrt{-25}=i\sqrt{64}-i\sqrt{25}$
$=8i-5i=3i$

30. $\sqrt{-81}-\sqrt{-144}=i\sqrt{81}-i\sqrt{144}=9i-12i$
$=-3i$

31. $5\sqrt{-16}+3\sqrt{-81}=5(4i)+3(9i)$
$=20i+27i=47i$

32. $5\sqrt{-8}+3\sqrt{-18}$
$=5i\sqrt{8}+3i\sqrt{18}=5i\sqrt{4\cdot 2}+3i\sqrt{9\cdot 2}$
$=10i\sqrt{2}+9i\sqrt{2}$
$=19i\sqrt{2}$

33. $\left(-2+\sqrt{-4}\right)^2=(-2+2i)^2$
$=4-8i+4i^2$
$=4-8i-4$
$=-8i$

34. $\left(-5-\sqrt{-9}\right)^2=(-5-i\sqrt{9})^2=(-5-3i)^2$
$=25+30i+9i^2$
$=25+30i-9$
$=16+30i$

35. $\left(-3-\sqrt{-7}\right)^2=\left(-3-i\sqrt{7}\right)^2$
$=9+6i\sqrt{7}+i^2(7)$
$=9-7+6i\sqrt{7}$
$=2+6i\sqrt{7}$

36. $\left(-2+\sqrt{-11}\right)^2=\left(-2+i\sqrt{11}\right)^2$
$=4-4i\sqrt{11}+i^2(11)$
$=4-11-4i\sqrt{11}$
$=-7-4i\sqrt{11}$

37. $\dfrac{-8+\sqrt{-32}}{24}=\dfrac{-8+i\sqrt{32}}{24}$
$=\dfrac{-8+i\sqrt{16\cdot 2}}{24}$
$=\dfrac{-8+4i\sqrt{2}}{24}$
$=-\dfrac{1}{3}+\dfrac{\sqrt{2}}{6}i$

38. $\dfrac{-12+\sqrt{-28}}{32}=\dfrac{-12+i\sqrt{28}}{32}=\dfrac{-12+i\sqrt{4\cdot 7}}{32}$
$=\dfrac{-12+2i\sqrt{7}}{32}=-\dfrac{3}{8}+\dfrac{\sqrt{7}}{16}i$

39. $\dfrac{-6-\sqrt{-12}}{48}=\dfrac{-6-i\sqrt{12}}{48}$
$=\dfrac{-6-i\sqrt{4\cdot 3}}{48}$
$=\dfrac{-6-2i\sqrt{3}}{48}$
$=-\dfrac{1}{8}-\dfrac{\sqrt{3}}{24}i$

40. $\dfrac{-15-\sqrt{-18}}{33}=\dfrac{-15-i\sqrt{18}}{33}=\dfrac{-15-i\sqrt{9\cdot 2}}{33}$
$=\dfrac{-15-3i\sqrt{2}}{33}=-\dfrac{5}{11}-\dfrac{\sqrt{2}}{11}i$

41. $\sqrt{-8}\left(\sqrt{-3}-\sqrt{5}\right)=i\sqrt{8}(i\sqrt{3}-\sqrt{5})$
$=2i\sqrt{2}\left(i\sqrt{3}-\sqrt{5}\right)$
$=-2\sqrt{6}-2i\sqrt{10}$

42. $\sqrt{-12}\left(\sqrt{-4}-\sqrt{2}\right)=i\sqrt{12}(i\sqrt{4}-\sqrt{2})$
$=2i\sqrt{3}\left(2i-\sqrt{2}\right)$
$=4i^2\sqrt{3}-2i\sqrt{6}$
$=-4\sqrt{3}-2i\sqrt{6}$

43. $\left(3\sqrt{-5}\right)\left(-4\sqrt{-12}\right)=\left(3i\sqrt{5}\right)\left(-8i\sqrt{3}\right)$
$=-24i^2\sqrt{15}$
$=24\sqrt{15}$

44. $\left(3\sqrt{-7}\right)\left(2\sqrt{-8}\right)$
$=(3i\sqrt{7})(2i\sqrt{8})=(3i\sqrt{7})(2i\sqrt{4\cdot 2})$
$=\left(3i\sqrt{7}\right)\left(4i\sqrt{2}\right)=12i^2\sqrt{14}=-12\sqrt{14}$

45. $(2-3i)(1-i)-(3-i)(3+i)$
$=\left(2-2i-3i+3i^2\right)-\left(3^2-i^2\right)$
$=2-5i+3i^2-9+i^2$
$=-7-5i+4i^2$
$=-7-5i+4(-1)$
$=-11-5i$

46. $(8+9i)(2-i)-(1-i)(1+i)$
$=\left(16-8i+18i-9i^2\right)-\left(1^2-i^2\right)$
$=16+10i-9i^2-1+i^2$
$=15+10i-8i^2$
$=15+10i-8(-1)$
$=23+10i$

47. $(2+i)^2-(3-i)^2$
$=\left(4+4i+i^2\right)-\left(9-6i+i^2\right)$
$=4+4i+i^2-9+6i-i^2$
$=-5+10i$

48. $(4-i)^2-(1+2i)^2$
$=\left(16-8i+i^2\right)-\left(1+4i+4i^2\right)$
$=16-8i+i^2-1-4i-4i^2$
$=15-12i-3i^2$
$=15-12i-3(-1)$
$=18-12i$

49. $5\sqrt{-16}+3\sqrt{-81}$
$=5\sqrt{16}\sqrt{-1}+3\sqrt{81}\sqrt{-1}$
$=5\cdot 4i+3\cdot 9i$
$=20i+27i$
$=47i$ or $0+47i$

50. $5\sqrt{-8}+3\sqrt{-18}$
$=5\sqrt{4}\sqrt{2}\sqrt{-1}+3\sqrt{9}\sqrt{2}\sqrt{-1}$
$=5\cdot 2\sqrt{2}\,i+3\cdot 3\sqrt{2}\,i$
$=10i\sqrt{2}+9i\sqrt{2}$
$=(10+9)i\sqrt{2}$
$=19i\sqrt{2}$ or $0+19i\sqrt{2}$

51. $f(x)=x^2-2x+2$
$f(1+i)=(1+i)^2-2(1+i)+2$
$=1+2i+i^2-2-2i+2$
$=1+i^2$
$=1-1$
$=0$

52. $f(x)=x^2-2x+5$
$f(1-2i)=(1-2i)^2-2(1-2i)+5$
$=1-4i+4i^2-2+4i+5$
$=4+4i^2$
$=4-4$
$=0$

53. $f(x)=\dfrac{x^2+19}{2-x}$
$f(3i)=\dfrac{(3i)^2+19}{2-3i}$
$=\dfrac{9i^2+19}{2-3i}$
$=\dfrac{-9+19}{2-3i}$
$=\dfrac{10}{2-3i}$
$=\dfrac{10}{2-3i}\cdot\dfrac{2+3i}{2+3i}$
$=\dfrac{20+30i}{4-9i^2}$
$=\dfrac{20+30i}{4+9}$
$=\dfrac{20+30i}{13}$
$=\dfrac{20}{13}+\dfrac{30}{13}i$

54. $f(x)=\dfrac{x^2+11}{3-x}$
$f(4i)=\dfrac{(4i)^2+11}{3-4i}=\dfrac{16i^2+11}{3-4i}$
$=\dfrac{-16+11}{3-4i}$
$=\dfrac{-5}{3-4i}$
$=\dfrac{-5}{3-4i}\cdot\dfrac{3+4i}{3+4i}$
$=\dfrac{-15-20i}{9-16i^2}$
$=\dfrac{-15-20i}{9+16}$
$=\dfrac{-15-20i}{25}$
$=\dfrac{-15}{25}-\dfrac{20}{25}i$
$=-\dfrac{3}{5}-\dfrac{4}{5}i$

55. $E=IR=(4-5i)(3+7i)$
$=12+28i-15i-35i^2$
$=12+13i-35(-1)$
$=12+35+13i=47+13i$
The voltage of the circuit is
(47 + 13*i*) volts.

56. $E=IR=(2-3i)(3+5i)$
$=6+10i-9i-15i^2=6+i-15(-1)$
$=6+i+15=21+i$
The voltage of the circuit is $(21+i)$ volts.

57. Sum:
$(5+i\sqrt{15})+(5-i\sqrt{15})$
$=5+i\sqrt{15}+5-i\sqrt{15}$
$=5+5$
$=10$
Product:
$(5+i\sqrt{15})(5-i\sqrt{15})$
$=25-5i\sqrt{15}+5i\sqrt{15}-15i^2$
$=25+15$
$=40$

58. – 66. Answers may vary.

67. makes sense

68. does not make sense; Explanations will vary. Sample explanation: Imaginary numbers are not undefined.

69. does not make sense; Explanations will vary. Sample explanation: $i = \sqrt{-1}$; It is not a variable in this context.

70. makes sense

71. false; Changes to make the statement true will vary. A sample change is: All irrational numbers are complex numbers.

72. false; Changes to make the statement true will vary. A sample change is: $(3 + 7i)(3 - 7i) = 9 + 49 = 58$ which is a real number.

73. false; Changes to make the statement true will vary. A sample change is:
$$\frac{7+3i}{5+3i} = \frac{7+3i}{5+3i} \cdot \frac{5-3i}{5-3i} = \frac{44-6i}{34} = \frac{22}{17} - \frac{3}{17}i$$

74. true

75.
$$\begin{aligned}\frac{4}{(2+i)(3-i)} &= \frac{4}{6-2i+3i-i^2} \\ &= \frac{4}{6+i+1} \\ &= \frac{4}{7+i} \\ &= \frac{4}{7+i} \cdot \frac{7-i}{7-i} \\ &= \frac{28-4i}{49-i^2} \\ &= \frac{28-4i}{49+1} \\ &= \frac{28-4i}{50} \\ &= \frac{28}{50} - \frac{4}{50}i \\ &= \frac{14}{25} - \frac{2}{25}i\end{aligned}$$

76.
$$\begin{aligned}&\frac{1+i}{1+2i} + \frac{1-i}{1-2i} \\ &= \frac{(1+i)(1-2i)}{(1+2i)(1-2i)} + \frac{(1-i)(1+2i)}{(1+2i)(1-2i)} \\ &= \frac{(1+i)(1-2i)+(1-i)(1+2i)}{(1+2i)(1-2i)} \\ &= \frac{1-2i+i-2i^2+1+2i-i-2i^2}{1-4i^2} \\ &= \frac{1-2i+i+2+1+2i-i+2}{1+4} \\ &= \frac{6}{5} \\ &= \frac{6}{5} + 0i\end{aligned}$$

77.
$$\begin{aligned}\frac{8}{1+\frac{2}{i}} &= \frac{8}{\frac{i}{i}+\frac{2}{i}} \\ &= \frac{8}{\frac{2+i}{i}} \\ &= \frac{8i}{2+i} \\ &= \frac{8i}{2+i} \cdot \frac{2-i}{2-i} \\ &= \frac{16i-8i^2}{4-i^2} \\ &= \frac{16i+8}{4+1} \\ &= \frac{8+16i}{5} \\ &= \frac{8}{5} + \frac{16}{5}i\end{aligned}$$

78. $2x^2 + 7x - 4 = (2x-1)(x+4)$

79. $x^2 - 6x + 9 = (x-3)(x-3) = (x-3)^2$

80.
$$\begin{aligned}\frac{-b-\sqrt{b^2-4ac}}{2a} &= \frac{-(9)-\sqrt{(9)^2-4(2)(-5)}}{2(2)} \\ &= \frac{-9-\sqrt{81+40}}{4} \\ &= \frac{-9-\sqrt{121}}{4} \\ &= \frac{-9-11}{4} \\ &= -5\end{aligned}$$

Section 1.5

Check Point Exercises

1. **a.**
$$3x^2-9x=0$$
$$3x(x-3)=0$$
$$3x=0 \quad \text{or} \quad x-3=0$$
$$x=0 \qquad x=3$$
The solution set is $\{0, 3\}$.

b.
$$2x^2+x=1$$
$$2x^2+x-1=0$$
$$(2x-1)(x+1)=0$$
$$2x-1=0 \quad \text{or} \quad x+1=0$$
$$2x=1 \qquad x=-1$$
$$x=\frac{1}{2}$$
The solution set is $\left\{-1, \frac{1}{2}\right\}$.

2. **a.**
$$3x^2=21$$
$$\frac{3x^2}{3}=\frac{21}{3}$$
$$x^2=7$$
$$x=\pm\sqrt{7}$$
The solution set is $\left\{-\sqrt{7}, \sqrt{7}\right\}$.

b.
$$5x^2+45=0$$
$$5x^2=-45$$
$$x^2=-9$$
$$x=\pm\sqrt{-9}$$
$$x=\pm 3i$$

c.
$$(x+5)^2=11$$
$$x+5=\pm\sqrt{11}$$
$$x=-5\pm\sqrt{11}$$
The solution set is $\left\{-5+\sqrt{11}, -5-\sqrt{11}\right\}$.

3. **a.** The coefficient of the x-term is 6. Half of 6 is 3, and 3^2 is 9.
9 should be added to the binomial.
$$x^2+6x+9=(x+3)^2$$

b. The coefficient of the x-term is –5.
Half of –5 is $-\frac{5}{2}$, and $\left(-\frac{5}{2}\right)^2$ is $\frac{25}{4}$.
$\frac{25}{4}$ should be added to the binomial.
$$x^2-5x+\frac{25}{4}=\left(x-\frac{5}{2}\right)^2$$

c. The coefficient of the x-term is $\frac{2}{3}$.
Half of $\frac{2}{3}$ is $\frac{1}{3}$, and $\left(\frac{1}{3}\right)^2$ is $\frac{1}{9}$.
$\frac{1}{9}$ should be added to the binomial.
$$x^2+\frac{2}{3}x+\frac{1}{9}=\left(x+\frac{1}{3}\right)^2$$

4.
$$x^2+4x-1=0$$
$$x^2+4x=1$$
$$x^2+4x+4=1+4$$
$$(x+2)^2=5$$
$$x+2=\pm\sqrt{5}$$
$$x=-2\pm\sqrt{5}$$

5.
$$2x^2+3x-4=0$$
$$x^2+\frac{3}{2}x-2=0$$
$$x^2+\frac{3}{2}x=2$$
$$x^2+\frac{3}{2}x+\frac{9}{16}=2+\frac{9}{16}$$
$$\left(x+\frac{3}{4}\right)^2=\frac{41}{16}$$
$$x+\frac{3}{4}=\pm\sqrt{\frac{41}{16}}$$
$$x+\frac{3}{4}=\pm\frac{\sqrt{41}}{4}$$
$$x=-\frac{3}{4}\pm\frac{\sqrt{41}}{4}$$
$$x=\frac{-3\pm\sqrt{41}}{4}$$

6. $2x^2 + 2x - 1 = 0$

$a = 2, b = 2, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{-2 \pm \sqrt{4+8}}{4}$$
$$= \frac{-2 \pm \sqrt{12}}{4}$$
$$= \frac{-2 \pm 2\sqrt{3}}{4}$$
$$= \frac{2(-1 \pm \sqrt{3})}{4}$$
$$= \frac{-1 \pm \sqrt{3}}{2}$$

The solution set is $\left\{\frac{-1+\sqrt{3}}{2}, \frac{-1-\sqrt{3}}{2}\right\}$.

7. $x^2 - 2x + 2 = 0$

$a = 1, b = -2, c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$
$$x = \frac{2 \pm \sqrt{4-8}}{2}$$
$$x = \frac{2 \pm \sqrt{-4}}{2}$$
$$x = \frac{2 \pm 2i}{2}$$
$$x = 1 \pm i$$

The solution set is $\{1+i, 1-i\}$.

8. **a.** $a = 1, \quad b = 6, \quad c = 9$

$$b^2 - 4ac = (6)^2 - 4(1)(9)$$
$$= 36 - 36$$
$$= 0$$

Since $b^2 - 4ac = 0$, the equation has one real solution.

b. $a = 2, \quad b = -7, \quad c = -4$

$$b^2 - 4ac = (-7)^2 - 4(2)(-4)$$
$$= 49 + 32$$
$$= 81$$

Since $b^2 - 4ac > 0$, the equation has two real solutions. Since 81 is a perfect square, the two solutions are rational.

c. $a = 3, \quad b = -2, \quad c = 4$

$$b^2 - 4ac = (-2)^2 - 4(3)(4)$$
$$= 4 - 48$$
$$= -44$$

Since $b^2 - 4ac < 0$, the equation has two imaginary solutions that are complex conjugates.

9. $P = 0.01A^2 + 0.05A + 107$

$115 = 0.01A^2 + 0.05A + 107$

$0 = 0.01A^2 + 0.05A - 8$

$a = 0.01, \quad b = 0.05, \quad c = -8$

$$A = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$A = \frac{-(0.05) \pm \sqrt{(0.05)^2 - 4(0.01)(-8)}}{2(0.01)}$$
$$A = \frac{-0.05 \pm \sqrt{0.3225}}{0.02}$$
$$A \approx \frac{-0.05 + \sqrt{0.3225}}{0.02} \qquad A \approx \frac{-0.05 - \sqrt{0.3225}}{0.02}$$
$$A \approx 26 \qquad A \approx -31$$

Age cannot be negative, reject the negative answer. Thus, a woman whose normal systolic blood pressure is 115 mm Hg is 26 years old.

10. Let w = the screen's width.

$$w^2 + l^2 = d^2$$
$$w^2 + 15^2 = 25^2$$
$$w^2 + 225 = 625$$
$$w^2 = 400$$
$$w = \pm\sqrt{400}$$
$$w = \pm 20$$

Reject the negative value.
The width of the television is 20 inches.

Exercise Set 1.5

1. $x^2 - 3x - 10 = 0$
$(x + 2)(x - 5) = 0$
$x + 2 = 0$ or $x - 5 = 0$
$x = -2$ or $x = 5$
The solution set is $\{-2, 5\}$.

2. $x^2 - 13x + 36 = 0$
$(x - 4)(x - 9) = 0$
$x - 4 = 0$ or $x - 9 = 0$
$x = 4$ or $x = 9$
The solution set is $\{4, 9\}$.

3. $x^2 = 8x - 15$
$x^2 - 8x + 15 = 0$
$(x - 3)(x - 5) = 0$
$x - 3 = 0$ or $x - 5 = 0$
$x = 3$ or $x = 5$
The solution set is $\{3, 5\}$.

4. $x^2 = -11x - 10$
$x^2 + 11x + 10 = 0$
$(x + 10)(x + 1) = 0$
$x + 10 = 0$ or $x + 1 = 0$
$x = -10$ or $x = -1$
The solution set is $\{-10, -1\}$.

5. $6x^2 + 11x - 10 = 0$
$(2x + 5)(3x - 2) = 0$
$2x + 5 = 0$ or $3x - 2 = 0$
$2x = -5$ $\quad 3x = 2$
$x = -\frac{5}{2}$ or $x = \frac{2}{3}$
The solution set is $\left\{-\frac{5}{2}, \frac{2}{3}\right\}$.

6. $9x^2 + 9x + 2 = 0$
$(3x + 2)(3x + 1) = 0$
$3x + 2 = 0$ or $3x + 1 = 0$
$x = -\frac{2}{3}$ or $x = -\frac{1}{3}$
The solution set is $\left\{-\frac{2}{3}, -\frac{1}{3}\right\}$.

7. $3x^2 - 2x = 8$
$3x^2 - 2x - 8 = 0$
$(3x + 4)(x - 2) = 0$
$3x + 4 = 0$ or $x - 2 = 0$
$3x = -4$
$x = -\frac{4}{3}$ or $x = 2$
The solution set is $\left\{-\frac{4}{3}, 2\right\}$.

8. $4x^2 - 13x = -3$
$4x^2 - 13x + 3 = 0$
$(4x - 1)(x - 3) = 0$
$4x - 1 = 0$ or $x - 3 = 0$
$4x = 1$
$x = \frac{1}{4}$ or $x = 3$
The solution set is $\left\{\frac{1}{4}, 3\right\}$.

9. $3x^2 + 12x = 0$
$3x(x + 4) = 0$
$3x = 0$ or $x + 4 = 0$
$x = 0$ or $x = -4$
The solution set is $\{-4, 0\}$.

10. $5x^2 - 20x = 0$
$5x(x - 4) = 0$
$5x = 0$ or $x - 4 = 0$
$x = 0$ or $x = 4$
The solution set is $\{0, 4\}$.

11. $2x(x - 3) = 5x^2 - 7x$
$2x^2 - 6x - 5x^2 + 7x = 0$
$-3x^2 + x = 0$
$x(-3x + 1) = 0$
$x = 0$ or $-3x + 1 = 0$
$-3x = -1$
$x = \frac{1}{3}$
The solution set is $\left\{0, \frac{1}{3}\right\}$.

12. $16x(x-2) = 8x - 25$
$16x^2 - 32x - 8x + 25 = 0$
$16x^2 - 40x + 25 = 0$
$(4x-5)(4x-5) = 0$
$4x - 5 = 0$
$4x = 5$
$x = \frac{5}{4}$
The solution set is $\left\{\frac{5}{4}\right\}$.

13. $7 - 7x = (3x+2)(x-1)$
$7 - 7x = 3x^2 - x - 2$
$7 - 7x - 3x^2 + x + 2 = 0$
$-3x^2 - 6x + 9 = 0$
$-3(x+3)(x-1) = 0$
$x + 3 = 0$ or $x - 1 = 0$
$x = -3$ or $x = 1$
The solution set is $\{-3, 1\}$.

14. $10x - 1 = (2x+1)^2$
$10x - 1 = 4x^2 + 4x + 1$
$10x - 1 - 4x^2 - 4x - 1 = 0$
$-4x^2 + 6x - 2 = 0$
$-2(2x-1)(x-1) = 0$
$2x - 1 = 0$ or $x - 1 = 0$
$2x = 1$
$x = \frac{1}{2}$ or $x = 1$
The solution set is $\left\{\frac{1}{2}, 1\right\}$.

15. $3x^2 = 27$
$x^2 = 9$
$x = \pm\sqrt{9} = \pm 3$
The solution set is $\{-3, 3\}$.

16. $5x^2 = 45$
$x^2 = 9$
$x = \pm\sqrt{9} = \pm 3$
The solution set is $\{-3, 3\}$.

17. $5x^2 + 1 = 51$
$5x^2 = 50$
$x^2 = 10$
$x = \pm\sqrt{10}$
The solution set is $\left\{-\sqrt{10}, \sqrt{10}\right\}$.

18. $3x^2 - 1 = 47$
$3x^2 = 48$
$x^2 = 16$
$x = \pm\sqrt{16} = \pm 4$
The solution set is $\{-4, 4\}$.

19. $2x^2 - 5 = -55$
$2x^2 = -50$
$x^2 = -25$
$x = \pm\sqrt{-25} = \pm 5i$
The solution set is $\{5i, -5i\}$.

20. $2x^2 - 7 = -15$
$2x^2 = -8$
$x^2 = -4$
$x = \pm\sqrt{-4} = \pm 2i$
The solution set is $\{2i, -2i\}$.

21. $(x+2)^2 = 25$
$x + 2 = \pm\sqrt{25}$
$x + 2 = \pm 5$
$x = -2 \pm 5$
$x = -2 + 5$ or $x = -2 - 5$
$x = 3$ $\quad$ $x = -7$
The solution set is $\{-7, 3\}$.

22. $(x-3)^2 = 36$
$x - 3 = \pm\sqrt{36}$
$x - 3 = \pm 6$
$x = 3 \pm 6$
$x = 3 + 6$ or $x = 3 - 6$
$x = 9$ $\quad$ $x = -3$
The solution set is $\{-3, 9\}$.

23. $3(x-4)^2 = 15$
$(x-4)^2 = 5$
$x - 4 = \pm\sqrt{5}$
$x = 4 \pm \sqrt{5}$
The solution set is $\left\{4 + \sqrt{5}, 4 - \sqrt{5}\right\}$.

24. $3(x+4)^2 = 21$

$(x+4)^2 = 7$

$x+4 = \pm\sqrt{7}$

$x = -4 \pm \sqrt{7}$

The solution set is $\{-4+\sqrt{7}, -4-\sqrt{7}\}$.

25. $(x+3)^2 = -16$

$x+3 = \pm\sqrt{-16}$

$x+3 = \pm 4i$

$x = -3 \pm 4i$

The solution set is $\{-3+4i, -3-4i\}$.

26. $(x-1)^2 = -9$

$x-1 = \pm\sqrt{-9}$

$x-1 = \pm 3i$

$x = 1 \pm 3i$

The solution set is $\{1+3i, 1-3i\}$.

27. $(x-3)^2 = -5$

$x-3 = \pm\sqrt{-5}$

$x-3 = \pm i\sqrt{5}$

$x = 3 \pm i\sqrt{5}$

The solution set is $\{3+i\sqrt{5}, 3-i\sqrt{5}\}$.

28. $(x+2)^2 = -7$

$x+2 = \pm\sqrt{-7}$

$x+2 = \pm i\sqrt{7}$

$x = -2 \pm i\sqrt{7}$

The solution set is $\{-2+i\sqrt{7}, -2-i\sqrt{7}\}$.

29. $(3x+2)^2 = 9$

$3x+2 = \pm\sqrt{9} = \pm 3$

$3x+2 = -3$ or $3x+2 = 3$

$3x = -5$ $\quad$ $3x = 1$

$x = -\frac{5}{3}$ or $x = \frac{1}{3}$

The solution set is $\left\{-\frac{5}{3}, \frac{1}{3}\right\}$.

30. $(4x-1)^2 = 16$

$4x-1 = \pm\sqrt{16} = \pm 4$

$4x-1 = -4$ or $4x-1 = 4$

$4x = -3$ $\quad$ $4x = 5$

$x = \frac{-3}{4}$ or $x = \frac{5}{4}$

The solution set is $\left\{-\frac{3}{4}, \frac{5}{4}\right\}$.

31. $(5x-1)^2 = 7$

$5x-1 = \pm\sqrt{7}$

$5x = 1 \pm \sqrt{7}$

$x = \frac{1 \pm \sqrt{7}}{5}$

The solution set is $\left\{\frac{1-\sqrt{7}}{5}, \frac{1+\sqrt{7}}{5}\right\}$.

32. $(8x-3)^2 = 5$

$8x-3 = \pm\sqrt{5}$

$8x = 3 \pm \sqrt{5}$

$x = \frac{3 \pm \sqrt{5}}{8}$

The solution set is $\left\{\frac{3-\sqrt{5}}{8}, \frac{3+\sqrt{5}}{8}\right\}$.

33. $(3x-4)^2 = 8$

$3x-4 = \pm\sqrt{8} = \pm 2\sqrt{2}$

$3x = 4 \pm 2\sqrt{2}$

$x = \frac{4 \pm 2\sqrt{2}}{3}$

The solution set is $\left\{\frac{4-2\sqrt{2}}{3}, \frac{4+2\sqrt{2}}{3}\right\}$.

34. $(2x+8)^2 = 27$

$2x+8 = \pm\sqrt{27} = \pm 3\sqrt{3}$

$2x = -8 + 3\sqrt{3}$

$x = \frac{-8 \pm 3\sqrt{3}}{2}$

The solution set is $\left\{\frac{-8-3\sqrt{3}}{2}, \frac{-8+3\sqrt{3}}{2}\right\}$.

35. x^2+12x

$\left(\frac{12}{2}\right)^2=6^2=36$

$x^2+12x+36=(x+6)^2$

36. x^2+16x

$\left(\frac{16}{2}\right)^2=8^2=64;$

$x^2+16x+64=(x+8)^2$

37. x^2-10x

$\left(\frac{10}{2}\right)^2=5^2=25$

$x^2-10x+25=(x-5)^2$

38. x^2-14x

$\left(\frac{-14}{2}\right)^2=(-7)^2=49;$

$x^2-14x+49=(x-7)^2$

39. x^2+3x

$\left(\frac{3}{2}\right)^2=\frac{9}{4}$

$x^2+3x+\frac{9}{4}=\left(x+\frac{3}{2}\right)^2$

40. x^2+5x

$\left(\frac{5}{2}\right)^2=\frac{25}{4};$

$x^2+5x+\frac{25}{4}=\left(x+\frac{5}{2}\right)^2$

41. x^2-7x

$\left(\frac{7}{2}\right)^2=\frac{49}{4}$

$x^2-7x+\frac{49}{4}=\left(x-\frac{7}{2}\right)^2$

42. x^2-9x

$\left(\frac{-9}{2}\right)^2=\frac{81}{4};$

$x^2-9x+\frac{81}{4}=\left(x-\frac{9}{2}\right)^2$

43. $x^2-\frac{2}{3}x$

$\left(\frac{\frac{2}{3}}{2}\right)^2=\left(\frac{1}{3}\right)^2=\frac{1}{9}$

$x^2-\frac{2}{3}x+\frac{1}{9}=\left(x-\frac{1}{3}\right)^2$

44. $x^2+\frac{4}{5}x$

$\left(\frac{\frac{4}{5}}{2}\right)^2=\left(\frac{2}{5}\right)^2=\frac{4}{25};$

$x^2+\frac{4}{5}x+\frac{4}{25}=\left(x+\frac{2}{5}\right)^2$

45. $x^2-\frac{1}{3}x$

$\left(\frac{\frac{1}{3}}{2}\right)^2=\left(\frac{1}{6}\right)^2=\frac{1}{36}$

$x^2-\frac{1}{3}x+\frac{1}{36}=\left(x-\frac{1}{6}\right)^2$

46. $x^2-\frac{1}{4}x$

$\left(\frac{\frac{-1}{4}}{2}\right)^2=\left(\frac{-1}{8}\right)^2=\frac{1}{64};$

$x^2-\frac{1}{4}x+\frac{1}{64}=\left(x-\frac{1}{8}\right)^2$

47.
$$\begin{aligned} x^2+6x&=7\\ x^2+6x+9&=7+9\\ (x+3)^2&=16\\ x+3&=\pm4\\ x&=-3\pm4 \end{aligned}$$
The solution set is {–7, 1}.

48.
$$\begin{aligned} x^2+6x&=-8\\ x^2+6x+9&=-8+9\\ (x+3)^2&=1\\ x+3&=\pm1\\ x&=-3\pm1 \end{aligned}$$
The solution set is {–4, –2}.

49.
$$x^2-2x=2$$
$$x^2-2x+1=2+1$$
$$(x-1)^2=3$$
$$x-1=\pm\sqrt{3}$$
$$x=1\pm\sqrt{3}$$
The solution set is $\left\{1+\sqrt{3},1-\sqrt{3}\right\}$.

50.
$$x^2+4x=12$$
$$x^2+4x+4=12+4$$
$$(x+2)^2=16$$
$$x+2=\pm 4$$
$$x=-2\pm 4$$
The solution set is {–6, 2}.

51.
$$x^2-6x-11=0$$
$$x^2-6x=11$$
$$x^2-6x+9=11+9$$
$$(x-3)^2=20$$
$$x-3=\pm\sqrt{20}$$
$$x=3\pm 2\sqrt{5}$$
The solution set is $\left\{3+2\sqrt{5},3-2\sqrt{5}\right\}$.

52.
$$x^2-2x-5=0$$
$$x^2-2x=5$$
$$x^2-2x+1=5+1$$
$$(x-1)^2=6$$
$$x-1=\pm\sqrt{6}$$
$$x=1\pm\sqrt{6}$$
The solution set is $\left\{1+\sqrt{6},1-\sqrt{6}\right\}$.

53.
$$x^2+4x+1=0$$
$$x^2+4x=-1$$
$$x^2+4x+4=-1+4$$
$$(x+2)^2=3$$
$$x+2=\pm\sqrt{3}$$
$$x=-2\pm\sqrt{3}$$
The solution set is $\left\{-2+\sqrt{3},-2-\sqrt{3}\right\}$.

54.
$$x^2+6x-5=0$$
$$x^2+6x=5$$
$$x^2+6x+9=5+9$$
$$(x+3)^2=14$$
$$x+3=\pm\sqrt{14}$$
$$x=-3\pm\sqrt{14}$$
The solution set is $\left\{-3+\sqrt{14},-3-\sqrt{14}\right\}$.

55.
$$x^2-5x+6=0$$
$$x^2-5x=-6$$
$$x^2-5x+\frac{25}{4}=-6+\frac{25}{4}$$
$$\left(x-\frac{5}{2}\right)^2=\frac{1}{4}$$
$$x-\frac{5}{2}=\pm\sqrt{\frac{1}{4}}$$
$$x-\frac{5}{2}=\pm\frac{1}{2}$$
$$x=\frac{5}{2}\pm\frac{1}{2}$$
$$x=\frac{5}{2}+\frac{1}{2} \quad \text{or} \quad x=\frac{5}{2}-\frac{1}{2}$$
$$x=3 \qquad\qquad x=2$$
The solution set is {2, 3}.

56.
$$x^2+7x-8=0$$
$$x^2+7x=8$$
$$x^2+7x+\frac{49}{4}=8+\frac{49}{4}$$
$$\left(x+\frac{7}{2}\right)^2=\frac{81}{4}$$
$$x+\frac{7}{2}=\pm\sqrt{\frac{81}{4}}$$
$$x+\frac{7}{2}=\pm\frac{9}{2}$$
$$x=-\frac{7}{2}\pm\frac{9}{2}$$
$$x=-\frac{7}{2}+\frac{9}{2} \quad \text{or} \quad x=-\frac{7}{2}-\frac{9}{2}$$
$$x=1 \qquad\qquad x=-8$$
The solution set is {–8, 1}.

57. $x^2+3x-1=0$

$x^2+3x=1$

$x^2+3x+\frac{9}{4}=1+\frac{9}{4}$

$\left(x+\frac{3}{2}\right)^2=\frac{13}{4}$

$x+\frac{3}{2}=\pm\frac{\sqrt{13}}{2}$

$x=\frac{-3\pm\sqrt{13}}{2}$

The solution set is $\left\{\frac{-3+\sqrt{13}}{2},\frac{-3-\sqrt{13}}{2}\right\}$.

58. $x^2-3x-5=0$

$x^2-3x=5$

$x^2-3x+\frac{9}{4}=5+\frac{9}{4}$

$\left(x-\frac{3}{2}\right)^2=\frac{29}{4}$

$x-\frac{3}{2}=\frac{\pm\sqrt{29}}{2}$

$x=\frac{3\pm\sqrt{29}}{2}$

The solution set is $\left\{\frac{3+\sqrt{29}}{2},\frac{3-\sqrt{29}}{2}\right\}$.

59. $2x^2-7x+3=0$

$x^2-\frac{7}{2}x+\frac{3}{2}=0$

$x^2-\frac{7}{2}x=\frac{-3}{2}$

$x^2-\frac{7}{2}x+\frac{49}{16}=-\frac{3}{2}+\frac{49}{16}$

$\left(x-\frac{7}{4}\right)^2=\frac{25}{16}$

$x-\frac{7}{4}=\pm\frac{5}{4}$

$x=\frac{7}{4}\pm\frac{5}{4}$

The solution set is $\left\{\frac{1}{2},3\right\}$.

60. $2x^2+5x-3=0$

$x^2+\frac{5}{2}x-\frac{3}{2}=0$

$x^2+\frac{5}{2}x=\frac{3}{2}$

$x^2+\frac{5}{2}x+\frac{25}{16}=\frac{3}{2}+\frac{25}{16}$

$\left(x+\frac{5}{4}\right)^2=\frac{49}{16}$

$x+\frac{5}{4}=\pm\frac{7}{4}$

$x=-\frac{5}{4}\pm\frac{7}{4}$

$x=\frac{1}{2};-3$

The solution set is $\left\{-3,\frac{1}{2}\right\}$.

61. $4x^2-4x-1=0$

$4x^2-4x-1=0$

$x^2-x-\frac{1}{4}=0$

$x^2-x=\frac{1}{4}$

$x^2-x+\frac{1}{4}=\frac{1}{4}+\frac{1}{4}$

$\left(x-\frac{1}{2}\right)^2=\frac{2}{4}$

$x-\frac{1}{2}=\frac{\pm\sqrt{2}}{2}$

$x=\frac{1\pm\sqrt{2}}{2}$

The solution set is $\left\{\frac{1+\sqrt{2}}{2},\frac{1-\sqrt{2}}{2}\right\}$.

62. $2x^2 - 4x - 1 = 0$

$$x^2 - 2x - \frac{1}{2} = 0$$
$$x^2 - 2x + 1 = \frac{1}{2} + 1$$
$$x^2 - 2x = \frac{1}{2}$$
$$(x-1)^2 = \frac{3}{2}$$
$$x - 1 = \pm\sqrt{\frac{3}{2}}$$
$$x = 1 \pm \frac{\sqrt{6}}{2}$$
$$x = \frac{2 \pm \sqrt{6}}{2}$$

The solution set is $\left\{\frac{2+\sqrt{6}}{2}, \frac{2-\sqrt{6}}{2}\right\}$.

63. $3x^2 - 2x - 2 = 0$

$$x^2 - \frac{2}{3}x - \frac{2}{3} = 0$$
$$x^2 - \frac{2}{3}x = \frac{2}{3}$$
$$x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{2}{3} + \frac{1}{9}$$
$$\left(x - \frac{1}{3}\right)^2 = \frac{7}{9}$$
$$x - \frac{1}{3} = \frac{\pm\sqrt{7}}{3}$$
$$x = \frac{1 \pm \sqrt{7}}{3}$$

The solution set is $\left\{\frac{1+\sqrt{7}}{3}, \frac{1-\sqrt{7}}{3}\right\}$.

64. $3x^2 - 5x - 10 = 0$

$$x^2 - \frac{5}{3}x - \frac{10}{3} = 0$$
$$x^2 - \frac{5}{3}x = \frac{10}{3}$$
$$x^2 - \frac{5}{3}x + \frac{25}{36} = \frac{10}{3} + \frac{25}{36}$$
$$\left(x - \frac{5}{6}\right)^2 = \frac{145}{36}$$
$$x - \frac{5}{6} = \frac{\pm\sqrt{145}}{6}$$
$$x = \frac{5 \pm \sqrt{145}}{6}$$

The solution set is $\left\{\frac{5\pm\sqrt{145}}{6}, \frac{5-\sqrt{145}}{6}\right\}$.

65. $x^2 + 8x + 15 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(15)}}{2(1)}$$
$$x = \frac{-8 \pm \sqrt{64 - 60}}{2}$$
$$x = \frac{-8 \pm \sqrt{4}}{2}$$
$$x = \frac{-8 \pm 2}{2}$$

The solution set is $\{-5, -3\}$.

66. $x^2 + 8x + 12 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(12)}}{2(1)}$$
$$x = \frac{-8 \pm \sqrt{64 - 48}}{2}$$
$$x = \frac{-8 \pm \sqrt{16}}{2}$$
$$x = \frac{-8 \pm 4}{2}$$

The solution set is $\{-6, -2\}$.

67. $x^2+5x+3=0$

$$x=\frac{-5\pm\sqrt{5^2-4(1)(3)}}{2(1)}$$
$$x=\frac{-5\pm\sqrt{25-12}}{2}$$
$$x=\frac{-5\pm\sqrt{13}}{2}$$

The solution set is $\left\{\frac{-5+\sqrt{13}}{2},\frac{-5-\sqrt{13}}{2}\right\}$.

68. $x^2+5x+2=0$

$$x=\frac{-5\pm\sqrt{5^2-4(1)(2)}}{2(1)}$$
$$x=\frac{-5\pm\sqrt{25-8}}{2}$$
$$x=\frac{-5\pm\sqrt{17}}{2}$$

The solution set is $\left\{\frac{-5+\sqrt{17}}{2},\frac{-5-\sqrt{17}}{2}\right\}$.

69. $3x^2-3x-4=0$

$$x=\frac{3\pm\sqrt{(-3)^2-4(3)(-4)}}{2(3)}$$
$$x=\frac{3\pm\sqrt{9+48}}{6}$$
$$x=\frac{3\pm\sqrt{57}}{6}$$

The solution set is $\left\{\frac{3+\sqrt{57}}{6},\frac{3-\sqrt{57}}{6}\right\}$

70. $5x^2+x-2=0$

$$x=\frac{-1\pm\sqrt{1^2-4(5)(-2)}}{2(5)}$$
$$x=\frac{-1\pm\sqrt{1+40}}{10}$$
$$x=\frac{-1\pm\sqrt{41}}{10}$$

The solution set is $\left\{\frac{-1+\sqrt{41}}{10},\frac{-1-\sqrt{41}}{10}\right\}$.

71. $4x^2=2x+7$

$$4x^2-2x-7=0$$
$$x=\frac{2\pm\sqrt{(-2)^2-4(4)(-7)}}{2(4)}$$
$$x=\frac{2\pm\sqrt{4+112}}{8}$$
$$x=\frac{2\pm\sqrt{116}}{8}$$
$$x=\frac{2\pm2\sqrt{29}}{8}$$
$$x=\frac{1\pm\sqrt{29}}{4}$$

The solution set is $\left\{\frac{1+\sqrt{29}}{4},\frac{1-\sqrt{29}}{4}\right\}$.

72. $3x^2=6x-1$

$$3x^2-6x+1=0$$
$$x=\frac{6\pm\sqrt{(-6)^2-4(3)(1)}}{2(3)}$$
$$x=\frac{6\pm\sqrt{36-12}}{6}$$
$$x=\frac{6\pm\sqrt{24}}{6}$$
$$x=\frac{6\pm2\sqrt{6}}{6}$$
$$x=\frac{3\pm\sqrt{6}}{3}$$

The solution set is $\left\{\frac{3+\sqrt{6}}{3},\frac{3-\sqrt{6}}{3}\right\}$.

73. $x^2-6x+10=0$

$$x=\frac{6\pm\sqrt{(-6)^2-4(1)(10)}}{2(1)}$$
$$x=\frac{6\pm\sqrt{36-40}}{2}$$
$$x=\frac{6\pm\sqrt{-4}}{2}$$
$$x=\frac{6\pm2i}{2}$$
$$x=3\pm i$$

The solution set is $\{3+i,3-i\}$.

74. $x^2-2x+17=0$

$x=\dfrac{2\pm\sqrt{(-2)^2-4(1)(17)}}{2(1)}$

$x=\dfrac{2\pm\sqrt{4-68}}{2}$

$x=\dfrac{2\pm\sqrt{-64}}{2}$

$x=\dfrac{2\pm 8i}{2}$

$x=1\pm 4i$

The solution set is $\{1+4i, 1-4i\}$.

75. $x^2-4x-5=0$

$(-4)^2-4(1)(-5)$

= 16 + 20

= 36; 2 unequal real solutions

76. $4x^2-2x+3=0$

$(-2)^2-4(4)(3)$

= 4 – 48

= –44; 2 complex imaginary solutions

77. $2x^2-11x+3=0$

$(-11)^2-4(2)(3)$

= 121 – 24

= 97; 2 unequal real solutions

78. $2x^2+11x-6=0$

$11^2-4(2)(-6)$

= 121 + 48

= 169; 2 unequal real solutions

79. $x^2-2x+1=0$

$(-2)^2-4(1)(1)$

= 4 – 4

= 0; 1 real solution

80. $3x^2=2x-1$

$3x^2-2x+1=0$

$(-2)^2-4(3)(1)$

= 4 – 12

= –8; 2 complex imaginary solutions

81. $x^2-3x-7=0$

$(-3)^2-4(1)(-7)$

= 9 + 28

= 37; 2 unequal real solutions

82. $3x^2+4x-2=0$

$4^2-4(3)(-2)$

= 16 + 24

= 40; 2 unequal real solutions

83. $2x^2-x=1$

$2x^2-x-1=0$

$(2x+1)(x-1)=0$

$2x+1=0$ or $x-1=0$

$2x=-1$

$x=-\dfrac{1}{2}$ or $x=1$

The solution set is $\left\{-\dfrac{1}{2}, 1\right\}$.

84. $3x^2-4x=4$

$3x^2-4x-4=0$

$(3x+2)(x-2)=0$

$3x+2$ or $x-2=0$

$3x=-2$

$x=-\dfrac{2}{3}$ or $x=-3$

The solution set is $\left\{-\dfrac{2}{3}, 2\right\}$.

85. $5x^2+2=11x$

$5x^2-11x+2=0$

$(5x-1)(x-2)=0$

$5x-1=0$ or $x-2=0$

$5x=1$

$x=\dfrac{1}{5}$ or $x=2$

The solution set is $\left\{\dfrac{1}{5}, 2\right\}$.

86. $5x^2=6-13x$

$5x^2+13x-6=0$

$(5x-2)(x+3)=0$

$5x-2=0$ or $x+3$

$5x=2$

$x=\dfrac{2}{5}$ or $x=-3$

The solution set is $\left\{-3, \dfrac{2}{5}\right\}$.

87. $3x^2 = 60$

$x^2 = 20$

$x = \pm\sqrt{20}$

$x = \pm 2\sqrt{5}$

The solution set is $\left\{-2\sqrt{5}, 2\sqrt{5}\right\}$.

88. $2x^2 = 250$

$x^2 = 125$

$x = \pm\sqrt{125}$

$x = \pm 5\sqrt{5}$

The solution set is $\left\{-5\sqrt{5}, 5\sqrt{5}\right\}$.

89. $x^2 - 2x = 1$

$x^2 - 2x + 1 = 1 + 1$

$(x-1)^2 = 2$

$x - 1 = \pm\sqrt{2}$

$x = 1 \pm \sqrt{2}$

The solution set is $\left\{1+\sqrt{2}, 1-\sqrt{2}\right\}$.

90. $2x^2 + 3x = 1$

$2x^2 + 3x - 1 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{9+8}}{4}$$

$$x = \frac{-3 \pm \sqrt{17}}{4}$$

The solution set is $\left\{\frac{-3+\sqrt{17}}{4}, \frac{-3-\sqrt{17}}{4}\right\}$.

91. $(2x+3)(x+4) = 1$

$2x^2 + 8x + 3x + 12 = 1$

$2x^2 + 11x + 11 = 0$

$$x = \frac{-11 \pm \sqrt{11^2 - 4(2)(11)}}{2(2)}$$

$$x = \frac{-11 \pm \sqrt{121-88}}{4}$$

$$x = \frac{-11 \pm \sqrt{33}}{4}$$

The solution set is $\left\{\frac{-11+\sqrt{33}}{4}, \frac{-11-\sqrt{33}}{4}\right\}$.

92. $(2x-5)(x+1) = 2$

$2x^2 + 2x - 5x - 5 = 2$

$2x^2 - 3x - 7 = 0$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9+56}}{4}$$

$$x = \frac{3 \pm \sqrt{65}}{4}$$

The solution set is $\left\{\frac{3+\sqrt{65}}{4}, \frac{3-\sqrt{65}}{4}\right\}$.

93. $(3x-4)^2 = 16$

$3x - 4 = \pm\sqrt{16}$

$3x - 4 = \pm 4$

$3x = 4 \pm 4$

$3x = 8$ or $3x = 0$

$x = \frac{8}{3}$ or $x = 0$

The solution set is $\left\{0, \frac{8}{3}\right\}$.

94. $(2x+7)^2 = 25$

$2x + 7 = \pm 5$

$2x = -7 \pm 5$

$2x = -12$ or $2x = -2$

$x = 6$ or $x = -1$

The solution set is $\{-6, -1\}$.

95. $3x^2 - 12x + 12 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x - 2 = 0$

$x = 2$

The solution set is $\{2\}$.

96. $9 - 6x + x^2 = 0$

$x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$x - 3 = 0$

$x = 3$

The solution set is $\{3\}$.

97. $4x^2 - 16 = 0$
$4x^2 = 16$
$x^2 = 4$
$x = \pm 2$
The solution set is $\{-2, 2\}$.

98. $3x^2 - 27 = 0$
$3x^2 = 27$
$x^2 = 9$
$x = \pm 3$
The solution set is $\{-3, 3\}$.

99. $x^2 - 6x + 13 = 0$
$x^2 - 6x = -13$
$x^2 - 6x + 9 = -13 + 9$
$(x-3)^2 = -4$
$x - 3 = \pm 2i$
$x = 3 \pm 2i$
The solution set is $\{3 + 2i, 3 - 2i\}$.

100. $x^2 - 4x + 29 = 0$
$x^2 - 4x = -29$
$x^2 - 4x + 4 = -29 + 4$
$(x-2)^2 = -25$
$x - 2 = \pm 5i$
$x = 2 \pm 5i$
The solution set is $\{2 + 5i, 2 - 5i\}$.

101. $x^2 = 4x - 7$
$x^2 - 4x = -7$
$x^2 - 4x + 4 = -7 + 4$
$(x-2)^2 = -3$
$x - 2 = \pm i\sqrt{3}$
$x = 2 \pm i\sqrt{3}$
The solution set is $\{2 + i\sqrt{3}, 2 - i\sqrt{3}\}$.

102. $5x^2 = 2x - 3$
$5x^2 - 2x + 3 = 0$
$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(5)(3)}}{2(5)}$$
$$x = \frac{2 \pm \sqrt{4 - 60}}{10}$$
$$x = \frac{2 \pm \sqrt{-56}}{10}$$
$$x = \frac{2 \pm 2i\sqrt{14}}{10}$$
$$x = \frac{1 \pm i\sqrt{14}}{5}$$
The solution set is $\left\{\frac{1 + i\sqrt{14}}{5}, \frac{1 - i\sqrt{14}}{5}\right\}$.

103. $2x^2 - 7x = 0$
$x(2x - 7) = 0$
$x = 0$ or $2x - 7 = 0$
$2x = 7$
$x = 0$ or $x = \frac{7}{2}$
The solution set is $\left\{0, \frac{7}{2}\right\}$.

104. $2x^2 + 5x = 3$
$2x^2 + 5x - 3 = 0$
$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-3)}}{2(2)}$$
$$x = \frac{-5 \pm \sqrt{25 + 24}}{4}$$
$$x = \frac{-5 \pm \sqrt{49}}{4}$$
$$x = \frac{-5 \pm 7}{4}$$
$$x = -3, \frac{1}{2}$$
The solution set is $\left\{-3, \frac{1}{2}\right\}$.

105. $\frac{1}{x}+\frac{1}{x+2}=\frac{1}{3}; x\neq 0,-2$

$3x+6+3x=x^2+2x$

$0=x^2-4x-6$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-6)}}{2(1)}$

$x=\frac{4\pm\sqrt{16+24}}{2}$

$x=\frac{4\pm\sqrt{40}}{2}$

$x=\frac{4\pm 2\sqrt{10}}{2}$

$x=2\pm\sqrt{10}$

The solution set is $\{2+\sqrt{10},\ 2-\sqrt{10}\}$.

106. $\frac{1}{x}+\frac{1}{x+3}=\frac{1}{4}; x\neq 0,-3$

$4x+12+4x=x^2+3x$

$0=x^2-5x-12$

$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-12)}}{2(1)}$

$x=\frac{5\pm\sqrt{25+48}}{2}$

$x=\frac{5\pm\sqrt{73}}{2}$

The solution set is $\left\{\frac{5+\sqrt{73}}{2},\frac{5-\sqrt{73}}{2}\right\}$.

107. $\frac{2x}{x-3}+\frac{6}{x+3}=\frac{-28}{x^2-9}; x\neq 3,-3$

$2x(x+3)+6(x-3)=-28$

$2x^2+6x+6x-18=-28$

$2x^2+12x+10=0$

$x^2+6x+5=0$

$(x+1)(x+5)=0$

The solution set is $\{-5,\ -1\}$.

108. $\frac{3}{x-3}+\frac{5}{x-4}=\frac{x^2-20}{x^2-7x+12}; x\neq 3,4$

$3x-12+5x-15=x^2-20$

$0=x^2-8x+7$

$0=(x-7)(x-1)$

$x=7 \qquad x=1$

The solution set is $\{1, 7\}$.

109. $x^2-4x-5=0$

$(x+1)(x-5)=0$

$x+1=0$ or $x-5=0$

$x=-1 \qquad x=5$

This equation matches graph (d).

110. $x^2-6x+7=0$

$a=1,\quad b=-6,\quad c=7$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(7)}}{2(1)}$

$x=\frac{6\pm\sqrt{8}}{2}$

$x=3\pm\sqrt{2}$

$x\approx 1.6,\quad x\approx 4.4$

This equation matches graph (a).

111. $0=-(x+1)^2+4$

$(x+1)^2=4$

$x+1=\pm 2$

$x=-1\pm 2$

$x=-3,\quad x=1$

This equation matches graph (f).

112. $0=-(x+3)^2+1$

$(x+3)^2=1$

$x+3=\pm 1$

$x=-3\pm 1$

$x=-4,\quad x=-2$

This equation matches graph (e).

113. $x^2 - 2x + 2 = 0$

$a = 1, \quad b = -2, \quad c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{-4}}{2}$$

$$x = \frac{2 \pm 2i}{2}$$

$$x = 1 \pm i$$

This equation has no real roots. Thus, its equation has no x-intercepts. This equation matches graph (b).

114. $x^2 + 6x + 9 = 0$

$(x+3)(x+3) = 0$

$x + 3 = 0$

$x = -3$

This equation matches graph (c).

115. $y = 2x^2 - 3x$

$2 = 2x^2 - 3x$

$0 = 2x^2 - 3x - 2$

$0 = (2x+1)(x-2)$

$x = -\frac{1}{2}, \quad x = 2$

116. $y = 5x^2 + 3x$

$2 = 5x^2 + 3x$

$0 = 5x^2 + 3x - 2$

$0 = (x+1)(5x-2)$

$x = -1, \quad x = \frac{2}{5}$

117. $y_1 y_2 = 14$

$(x-1)(x+4) = 14$

$x^2 + 3x - 4 = 14$

$x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$

$x = -6, \quad x = 3$

118. $y_1 y_2 = -30$

$(x-3)(x+8) = -30$

$x^2 + 5x - 24 = -30$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x = -3, \quad x = -2$

119. $y_1 + y_2 = 1$

$$\frac{2x}{x+2} + \frac{3}{x+4} = 1$$

$$(x+2)(x+4)\left(\frac{2x}{x+2} + \frac{3}{x+4}\right) = 1(x+2)(x+4)$$

$$\frac{2x(x+2)(x+4)}{x+2} + \frac{3(x+2)(x+4)}{x+4} = (x+2)(x+4)$$

$$2x(x+4) + 3(x+2) = (x+2)(x+4)$$

$$2x^2 + 8x + 3x + 6 = x^2 + 6x + 8$$

$$x^2 + 5x - 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{33}}{2}$$

The solution set is $\left\{\frac{-5+\sqrt{33}}{2}, \frac{-5-\sqrt{33}}{2}\right\}$.

120. $y_1 + y_2 = 3$

$$\frac{3}{x-1} + \frac{8}{x} = 3$$

$$x(x-1)\left(\frac{3}{x-1} + \frac{8}{x}\right) = 3(x)(x-1)$$

$$\frac{3x(x-1)}{x-1} + \frac{8x(x-1)}{x} = 3x(x-1)$$

$$3x + 8(x-1) = 3x^2 - 3x$$

$$3x + 8x - 8 = 3x^2 - 3x$$

$$11x - 8 = 3x^2 - 3x$$

$$0 = 3x^2 - 14x + 8$$

$$0 = (3x-2)(x-4)$$

$$x = \frac{2}{3}, \quad x = 4$$

The solution set is $\left\{\frac{2}{3}, 4\right\}$.

121.
$$y_1 - y_2 = 0$$
$$\left(2x^2 + 5x - 4\right) - \left(-x^2 + 15x - 10\right) = 0$$
$$2x^2 + 5x - 4 + x^2 - 15x + 10 = 0$$
$$3x^2 - 10x + 6 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(6)}}{2(3)}$$
$$x = \frac{10 \pm \sqrt{28}}{6}$$
$$x = \frac{10 \pm 2\sqrt{7}}{6}$$
$$x = \frac{5 \pm \sqrt{7}}{3}$$

The solution set is $\left\{\frac{5+\sqrt{7}}{3}, \frac{5-\sqrt{7}}{3}\right\}$.

122.
$$y_1 - y_2 = 0$$
$$\left(-x^2 + 4x - 2\right) - \left(-3x^2 + x - 1\right) = 0$$
$$-x^2 + 4x - 2 + 3x^2 - x + 1 = 0$$
$$2x^2 + 3x - 1 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-1)}}{2(2)}$$
$$x = \frac{-3 \pm \sqrt{17}}{4}$$

The solution set is $\left\{\frac{-3+\sqrt{17}}{4}, \frac{-3-\sqrt{17}}{4}\right\}$.

123. Values that make the denominator zero must be excluded**.**
$$2x^2 + 4x - 9 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(2)(-9)}}{2(2)}$$
$$x = \frac{-4 \pm \sqrt{88}}{4}$$
$$x = \frac{-4 \pm 2\sqrt{22}}{4}$$
$$x = \frac{-2 \pm \sqrt{22}}{2}$$

124. Values that make the denominator zero must be excluded**.**
$$2x^2 - 8x + 5 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(5)}}{2(2)}$$
$$x = \frac{8 \pm \sqrt{24}}{4}$$
$$x = \frac{8 \pm 2\sqrt{6}}{4}$$
$$x = \frac{4 \pm \sqrt{6}}{2}$$

125. $x^2 - \left(6 + 2x\right) = 0$
$$x^2 - 2x - 6 = 0$$

Apply the quadratic formula.

$a = 1 \quad b = -2 \quad c = -6$
$$x = \frac{-\left(-2\right) \pm \sqrt{\left(-2\right)^2 - 4\left(1\right)\left(-6\right)}}{2\left(1\right)}$$
$$= \frac{2 \pm \sqrt{4 - \left(-24\right)}}{2}$$
$$= \frac{2 \pm \sqrt{28}}{2}$$
$$= \frac{2 \pm \sqrt{4 \cdot 7}}{2} = \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$$

We disregard $1 - \sqrt{7}$ because it is negative, and we are looking for a positive number.

Thus, the number is $1 + \sqrt{7}$.

126. Let x = the number.

$2x^2-(1+2x)=0$

$2x^2-2x-1=0$

Apply the quadratic formula.

$a=2 \quad b=-2 \quad c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(2)(-1)}}{2(2)}$$

$$=\frac{2\pm\sqrt{4-(-8)}}{4}$$

$$=\frac{2\pm\sqrt{12}}{4}=\frac{2\pm\sqrt{4\cdot 3}}{4}=\frac{2\pm 2\sqrt{3}}{4}=\frac{1\pm\sqrt{3}}{2}$$

We disregard $\frac{1+\sqrt{3}}{2}$ because it is positive, and we are looking for a negative number. The number is $\frac{1-\sqrt{3}}{2}$.

127. $$\frac{1}{x^2-3x+2}=\frac{1}{x+2}+\frac{5}{x^2-4}$$

$$\frac{1}{(x-1)(x-2)}=\frac{1}{x+2}+\frac{5}{(x+2)(x-2)}$$

Multiply both sides of the equation by the least common denominator, $(x-1)(x-2)(x+2)$. This results in the following:

$x+2=(x-1)(x-2)+5(x-1)$

$x+2=x^2-2x-x+2+5x-5$

$x+2=x^2+2x-3$

$0=x^2+x-5$

Apply the quadratic formula:

$a=1 \quad b=1 \quad c=-5$.

$$x=\frac{-1\pm\sqrt{1^2-4(1)(-5)}}{2(1)}=\frac{-1\pm\sqrt{1-(-20)}}{2}$$

$$=\frac{-1\pm\sqrt{21}}{2}$$

The solutions are $\frac{-1\pm\sqrt{21}}{2}$, and the solution set is $\left\{\frac{-1\pm\sqrt{21}}{2}\right\}$.

128. $$\frac{x-1}{x-2}+\frac{x}{x-3}=\frac{1}{x^2-5x+6}$$

$$\frac{x-1}{x-2}+\frac{x}{x-3}=\frac{1}{(x-2)(x-3)}$$

Multiply both sides of the equation by the least common denominator, $(x-2)(x-3)$. This results in the following:

$(x-3)(x-1)+x(x-2)=1$

$x^2-x-3x+3+x^2-2x=1$

$2x^2-6x+3=1$

$2x^2-6x+2=0$

Apply the quadratic formula:

$a=2 \quad b=-6 \quad c=2$.

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(2)}}{2(2)}$$

$$=\frac{6\pm\sqrt{36-16}}{4}=\frac{6\pm\sqrt{20}}{4}$$

$$=\frac{6\pm\sqrt{4\cdot 5}}{4}=\frac{6\pm 2\sqrt{5}}{4}$$

$$=\frac{3\pm\sqrt{5}}{2}$$

The solutions are $\frac{3\pm\sqrt{5}}{2}$, and the solution set is $\left\{\frac{3\pm\sqrt{5}}{2}\right\}$.

129. $\sqrt{2}x^2+3x-2\sqrt{2}=0$

Apply the quadratic formula:

$a=\sqrt{2} \quad b=3 \quad c=-2\sqrt{2}$

$$x=\frac{-3\pm\sqrt{3^2-4\left(\sqrt{2}\right)\left(-2\sqrt{2}\right)}}{2\left(\sqrt{2}\right)}$$

$$=\frac{-3\pm\sqrt{9-(-16)}}{2\sqrt{2}}$$

$$=\frac{-3\pm\sqrt{25}}{2\sqrt{2}}=\frac{-3\pm 5}{2\sqrt{2}}$$

Evaluate the expression to obtain two solutions.

$$x=\frac{-3-5}{2\sqrt{2}} \quad \text{or} \quad x=\frac{-3+5}{2\sqrt{2}}$$

$$=\frac{-8}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \qquad =\frac{2}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-8\sqrt{2}}{4} \qquad =\frac{2\sqrt{2}}{4}$$

$$=-2\sqrt{2} \qquad =\frac{\sqrt{2}}{2}$$

The solutions are $-2\sqrt{2}$ and $\frac{\sqrt{2}}{2}$, and the solution set is $\left\{-2\sqrt{2},\ \frac{\sqrt{2}}{2}\right\}$.

130. $\sqrt{3}x^2+6x+7\sqrt{3}=0$

Apply the quadratic formula:

$a=\sqrt{3} \quad b=6 \quad c=7\sqrt{3}$

$$x=\frac{-6\pm\sqrt{6^2-4\left(\sqrt{3}\right)\left(7\sqrt{3}\right)}}{2\left(\sqrt{3}\right)}$$

$$=\frac{-6\pm\sqrt{36-84}}{2\sqrt{3}}$$

$$=\frac{-6\pm\sqrt{-48}}{2\sqrt{3}}$$

$$=\frac{-6\pm\sqrt{16\cdot 3\cdot(-1)}}{2\sqrt{3}}$$

$$=\frac{-6\pm 4\sqrt{3}i}{2\sqrt{3}}$$

$$=\frac{-6}{2\sqrt{3}}\pm\frac{4\sqrt{3}i}{2\sqrt{3}}=-\sqrt{3}\pm 2i$$

The solutions are $-\sqrt{3}\pm 2i$, and the solution set is $\left\{-\sqrt{3}\pm 2i\right\}$.

131. $N=\frac{x^2-x}{2}$

$$21=\frac{x^2-x}{2}$$

$$42=x^2-x$$

$$0=x^2-x-42$$

$$0=(x+6)(x-7)$$

$$x+6=0 \quad \text{or} \quad x-7=0$$

$$x=-6 \qquad x=7$$

Reject the negative value.
There were 7 players.

132. $N=\frac{x^2-x}{2}$

$$36=\frac{x^2-x}{2}$$

$$72=x^2-x$$

$$0=x^2-x-72$$

$$0=(x+8)(x-9)$$

$$x+8=0 \quad \text{or} \quad x-9=0$$

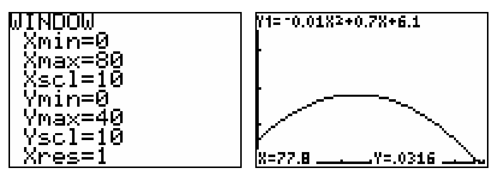
$$x=-8 \qquad x=9$$

Reject the negative value.
There were 9 players.

133. This is represented on the graph as point (7, 21).

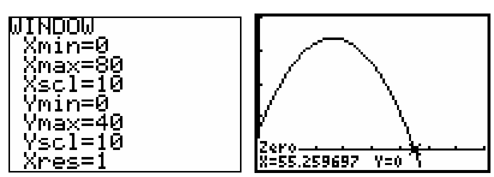
134. This is represented on the graph as point (9, 36).

135. Let $y_1=-0.01x^2+0.7x+6.1$



Using the TRACE feature, we find that the height of the shot put is approximately 0 feet when the distance is 77.8 feet. Graph (b) shows the shot' path.

136. Let $y_1=-0.04x^2+2.1x+6.1$



Using the ZERO feature, we find that the height of the shot put is approximately 0 feet when the distance is 55.3 feet. Graph (a) shows the shot's path.

137. **a.** $\dfrac{1}{\Phi-1}$

b.
$$\frac{\Phi}{1}=\frac{1}{\Phi-1}$$
$$(\Phi-1)\frac{\Phi}{1}=(\Phi-1)\frac{1}{\Phi-1}$$
$$\Phi^2-\Phi=1$$
$$\Phi^2-\Phi-1=0$$
$$\Phi=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$\Phi=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-1)}}{2(1)}$$
$$\Phi=\frac{1\pm\sqrt{1+4}}{2}$$
$$\Phi=\frac{1\pm\sqrt{5}}{2},\text{ reject negative}$$
$$\Phi=\frac{1+\sqrt{5}}{2}$$

c. The golden ratio is $\dfrac{1+\sqrt{5}}{2}$ to 1

138. $x^2=6^2+3^2$
$x^2=36+9$
$x^2=45$
$x=\pm\sqrt{45}$
$x=\pm3\sqrt{5}$
We disregard $-3\sqrt{5}$ because we can't have a negative measurement. The path is $3\sqrt{5}$ miles, or approximately 6.7 miles.

139. $x^2=4^2+2^2$
$x^2=16+4$
$x^2=20$
$x=\pm\sqrt{20}$
$x=\pm2\sqrt{5}$
We disregard $-2\sqrt{5}$ because we can't have a negative measurement. The path is $2\sqrt{5}$ miles, or approximately 4.5 miles.

140. $x^2+10^2=30^2$
$x^2+100=900$
$x^2=800$
Apply the square root property.
$x=\pm\sqrt{800}=\pm\sqrt{400\cdot2}=\pm20\sqrt{2}$
We disregard $-20\sqrt{2}$ because we can't have a negative length measurement. The solution is $20\sqrt{2}$. We conclude that the ladder reaches $20\sqrt{2}$ feet, or approximately 28.3 feet, up the house.

141. $90^2+90^2=x^2$
$8100+8100=x^2$
$16200=x^2$
$x\approx\pm127.28$
The distance is 127.28 feet.

142. **a.** $h^2=a^2+a^2$
$h^2=2a^2$
$h=\sqrt{2a^2}$
$h=a\sqrt{2}$

b. The length of the hypotenuse of an isosceles right triangle is the length of the leg times $\sqrt{2}$.

143. Let w = the width
Let $w+3$ = the length
$\text{Area}=lw$
$54=(w+3)w$
$54=w^2+3w$
$0=w^2+3w-54$
$0=(w+9)(w-6)$
Apply the zero product principle.
$w+9=0 \qquad w-6=0$
$w=-9 \qquad w=6$
The solution set is $\{-9,6\}$. Disregard –9 because we can't have a negative length measurement. The width is 6 feet and the length is $6+3=9$ feet.

144. Let w = the width
Let $w + 3$ = the width
Area = lw

$$180 = (w+3)w$$
$$180 = w^2 + 3w$$
$$0 = w^2 + 3w - 180$$
$$0 = (w+15)(w-12)$$
$$w+15 = 0 \qquad w-12 = 0$$
$$\cancel{w = -15} \qquad w = 12$$

The width is 12 yards and the length is 12 yards + 3 yards = 15 yards.

145. Let x = the length of the side of the original square
Let $x + 3$ = the length of the side of the new, larger square

$$(x+3)^2 = 64$$
$$x^2 + 6x + 9 = 64$$
$$x^2 + 6x - 55 = 0$$
$$(x+11)(x-5) = 0$$

Apply the zero product principle.

$$x+11 = 0 \qquad x-5 = 0$$
$$x = -11 \qquad x = 5$$

The solution set is $\{-11, 5\}$. Disregard –11 because we can't have a negative length measurement. This means that x, the length of the side of the original square, is 5 inches.

146. Let x = the side of the original square,
Let $x + 2$ = the side of the new, larger square

$$(x+2)^2 = 36$$
$$x^2 + 4x + 4 = 36$$
$$x^2 + 4x - 32 = 0$$
$$(x+8)(x-4) = 0$$
$$x+8 = 0 \qquad x-4 = 0$$
$$\cancel{x = -8} \qquad x = 4$$

The length of the side of the original square, is 4 inches.

147. Let x = the width of the path

$$(20+2x)(10+2x) = 600$$
$$200 + 40x + 20x + 4x^2 = 600$$
$$200 + 60x + 4x^2 = 600$$
$$4x^2 + 60x + 200 = 600$$
$$4x^2 + 60x - 400 = 0$$
$$4(x^2 + 15x - 100) = 0$$
$$4(x+20)(x-5) = 0$$

Apply the zero product principle.

$$4(x+20) = 0 \qquad x-5 = 0$$
$$x+20 = 0 \qquad x = 5$$
$$x = -20$$

The solution set is $\{-20, 5\}$. Disregard –20 because we can't have a negative width measurement. The width of the path is 5 meters.

148. Let x = the width of the path

$$(12+2x)(15+2x) = 378$$
$$180 + 24x + 30x + 4x^2 = 378$$
$$4x^2 + 54x + 180 = 378$$
$$4x^2 + 54x - 198 = 0$$
$$2(2x^2 + 27x - 99) = 0$$
$$2(2x+33)(x-3) = 0$$
$$2(2x+33) = 0 \qquad x-3 = 0$$
$$2x+33 = 0 \qquad x = 3$$
$$2x = -33$$
$$\cancel{x = -\frac{33}{2}}$$

The width of the path is 3 meters.

149. $x(x)(2) = 200$

$$2x^2 = 200$$
$$x^2 = 100$$
$$x = \pm 10$$

The length and width are 10 inches.

150. $x(x)(3) = 75$

$$3x^2 = 75$$
$$x^2 = 25$$
$$x = \pm 5$$

The length and width is 5 inches.

151. $x(20-2x)=13$

$20x-2x^2=13$

$0=2x^2-20x+13$

$x=\dfrac{-(-20)\pm\sqrt{(-20)^2-4(2)(13)}}{2(2)}$

$x=\dfrac{20\pm\sqrt{296}}{4}$

$x=\dfrac{10\pm17.2}{4}$

$x=9.3, 0.7$

9.3 in and 0.7 in

152. $\left(\dfrac{x}{4}\right)^2+\left(\dfrac{8-x}{4}\right)^2=2$

$\dfrac{x^2}{16}+\dfrac{64-16x+x^2}{16}=2$

$x^2+64-16x+x^2=32$

$2x^2-16x+32=0$

$x^2-8x+16=0$

$(x-4)(x-4)=0$

$x=4$ in

Both are 4 inches.

153. – 163. Answers may vary.

164. does not make sense; Explanations will vary. Sample explanation: The factoring method would be quicker.

165. does not make sense; Explanations will vary. Sample explanation: Higher degree polynomial equations can have only one x-intercept.

166. does not make sense; Explanations will vary. Sample explanation: The solutions are not irrational.

167. makes sense

168. false; Changes to make the statement true will vary. A sample change is: $(2x-3)^2=25$

$2x-3=\pm5$

169. true

170. false; Changes to make the statement true will vary. A sample change is: The quadratic formula is developed by completing the square.

171. false; Changes to make the statement true will vary. A sample change is: The first step is to collect all the terms on one side and have 0 on the other.

172. $(x+3)(x-5)=0$

$x^2-5x+3x-15=0$

$x^2-2x-15=0$

173. $s=-16t^2+v_0t$

$0=-16t^2+v_0t-s$

$a=-16,\ b=v_0,\ c=-s$

$t=\dfrac{-v_0\pm\sqrt{(v_0)^2-4(-16)(-s)}}{2(-16)}$

$t=\dfrac{-v_0\pm\sqrt{(v_0)^2-64s}}{-32}$

$t=\dfrac{v_0\pm\sqrt{v_0^{\,2}-64s}}{32}$

174. The dimensions of the pool are 12 meters by 8 meters. With the tile, the dimensions will be 12 + 2x meters by 8 + 2x meters. If we take the area of the pool with the tile and subtract the area of the pool without the tile, we are left with the area of the tile only.

$(12+2x)(8+2x)-12(8)=120$

$\cancel{96}+24x+16x+4x^2-\cancel{96}=120$

$4x^2+40x-120=0$

$x^2+10x-30=0$

$a=1\qquad b=10\qquad c=-30$

$x=\dfrac{-10\pm\sqrt{10^2-4(1)(-30)}}{2(1)}$

$=\dfrac{-10\pm\sqrt{100+120}}{2}$

$=\dfrac{-10\pm\sqrt{220}}{2}\approx\dfrac{-10\pm14.8}{2}$

Evaluate the expression to obtain two solutions.

$x=\dfrac{-10+14.8}{2}$ or $x=\dfrac{-10-14.8}{2}$

$x=\dfrac{4.8}{2}$ $\qquad x=\dfrac{-24.8}{2}$

$x=2.4$ $\qquad x=-12.4$

We disregard –12.4 because we can't have a negative width measurement. The solution is 2.4 and we conclude that the width of the uniform tile border is 2.4 meters. This is more than the 2-meter requirement, so the tile meets the zoning laws.

175. $x^3 + x^2 - 4x - 4 = x^2(x+1) - 4(x+1)$
$= (x+1)(x^2 - 4)$
$= (x+1)(x+2)(x-2)$

176. $(\sqrt{x+4}+1)^2 = \sqrt{x+4}^2 + 2(\sqrt{x+4})(1) + 1^2$
$= x + 4 + 2\sqrt{x+4} + 1$
$= x + 5 + 2\sqrt{x+4}$

177. $5x^{2/3} + 11x^{1/3} + 2 = 0$
$5(-8)^{2/3} + 11(-8)^{1/3} + 2 = 0$
$5(-2)^2 + 11(-2)^1 + 2 = 0$
$5(4) + 11(-2) + 2 = 0$
$20 - 22 + 2 = 0$
$0 = 0$, true
The statement is true.

Mid-Chapter 1 Check Point

1. $-5 + 3(x+5) = 2(3x-4)$
$-5 + 3x + 15 = 6x - 8$
$3x + 10 = 6x - 8$
$-3x = -18$
$\frac{-3x}{-3} = \frac{-18}{-3}$
$x = 6$
The solution set is $\{6\}$.

2. $5x^2 - 2x = 7$
$5x^2 - 2x - 7 = 0$
$(5x-7)(x+1) = 0$
$5x - 7 = 0$ or $x + 1 = 0$
$5x = 7$ $\quad x = -1$
$x = \frac{7}{5}$
The solution set is $\left\{-1, \frac{7}{5}\right\}$.

3. $\frac{x-3}{5} - 1 = \frac{x-5}{4}$
$20\left(\frac{x-3}{5} - 1\right) = 20\left(\frac{x-5}{4}\right)$
$\frac{20(x-3)}{5} - 20(1) = \frac{20(x-5)}{4}$
$4(x-3) - 20 = 5(x-5)$
$4x - 12 - 20 = 5x - 25$
$4x - 32 = 5x - 25$
$-x = 7$
$x = -7$
The solution set is $\{-7\}$.

4. $3x^2 - 6x - 2 = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-2)}}{2(3)}$
$x = \frac{6 \pm \sqrt{60}}{6}$
$x = \frac{6 \pm 2\sqrt{15}}{6}$
$x = \frac{3 \pm \sqrt{15}}{3}$
The solution set is $\left\{\frac{3+\sqrt{15}}{3}, \frac{3-\sqrt{15}}{3}\right\}$.

5. $4x - 2(1-x) = 3(2x+1) - 5$
$4x - 2(1-x) = 3(2x+1) - 5$
$4x - 2 + 2x = 6x + 3 - 5$
$6x - 2 = 6x - 2$
$0 = 0$
The solution set is all real numbers.

6.
$$5x^2+1=37$$
$$5x^2=36$$
$$\frac{5x^2}{5}=\frac{36}{5}$$
$$x^2=\frac{36}{5}$$
$$x=\pm\sqrt{\frac{36}{5}}$$
$$x=\pm\frac{6}{\sqrt{5}}$$
$$x=\pm\frac{6}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}$$
$$x=\pm\frac{6\sqrt{5}}{5}$$
The solution set is $\left\{-\frac{6\sqrt{5}}{5},\frac{6\sqrt{5}}{5}\right\}$.

7.
$$x(2x-3)=-4$$
$$2x^2-3x=-4$$
$$2x^2-3x+4=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(4)}}{2(2)}$$
$$x=\frac{3\pm\sqrt{-23}}{4}$$
$$x=\frac{3\pm i\sqrt{23}}{4}$$
The solution set is $\left\{\frac{3+i\sqrt{23}}{4},\frac{3-i\sqrt{23}}{4}\right\}$.

8.
$$\frac{3x}{4}-\frac{x}{3}+1=\frac{4x}{5}-\frac{3}{20}$$
$$\frac{3x}{4}-\frac{x}{3}+1=\frac{4x}{5}-\frac{3}{20}$$
$$60\left(\frac{3x}{4}-\frac{x}{3}+1\right)=60\left(\frac{4x}{5}-\frac{3}{20}\right)$$
$$\frac{60(3x)}{4}-\frac{60x}{3}+60(1)=\frac{60(4x)}{5}-\frac{60(3)}{20}$$
$$45x-20x+60=48x-9$$
$$25x+60=48x-9$$
$$-23x=-69$$
$$\frac{-23x}{-23}=\frac{-69}{-23}$$
$$x=3$$
The solution set is $\{3\}$.

9.
$$(x+3)^2=24$$
$$x+3=\pm\sqrt{24}$$
$$x=-3\pm2\sqrt{6}$$
The solution set is $\{-3+2\sqrt{6},-3-2\sqrt{6}\}$.

10.
$$\frac{1}{x^2}-\frac{4}{x}+1=0$$
$$x^2\left(\frac{1}{x^2}-\frac{4}{x}+1\right)=x^2(0)$$
$$\frac{x^2}{x^2}-\frac{4x^2}{x}+x^2=0$$
$$1-4x+x^2=0$$
$$x^2-4x+1=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$$
$$x=\frac{4\pm\sqrt{12}}{2}$$
$$x=\frac{4\pm2\sqrt{3}}{2}$$
$$x=2\pm\sqrt{3}$$
The solution set is $\{2+\sqrt{3},2-\sqrt{3}\}$.

11.
$$3x+1-(x-5)=2x-4$$
$$2x+6=2x-4$$
$$6=-4$$
The solution set is $\varnothing$.

12. $\frac{2x}{x^2+6x+8}=\frac{x}{x+4}-\frac{2}{x+2}, \quad x\neq -2, x\neq -4$

$\frac{2x}{(x+4)(x+2)}=\frac{x}{x+4}-\frac{2}{x+2}$

$\frac{2x(x+4)(x+2)}{(x+4)(x+2)}=(x+4)(x+2)\left(\frac{x}{x+4}-\frac{2}{x+2}\right)$

$2x=\frac{x(x+4)(x+2)}{x+4}-\frac{2(x+4)(x+2)}{x+2}$

$2x=x(x+2)-2(x+4)$

$2x=x^2+2x-2x-8$

$0=x^2-2x-8$

$0=(x+2)(x-4)$

$x+2=0$ or $x-4=0$

$x=-2$ $\quad x=4$

–2 must be rejected.

The solution set is $\{4\}$.

13. Let $y=0$.

$0=x^2+6x+2$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(6)\pm\sqrt{(6)^2-4(1)(2)}}{2(1)}$

$x=\frac{-6\pm\sqrt{28}}{2}$

$x=\frac{-6\pm 2\sqrt{7}}{2}$

$x=-3\pm\sqrt{7}$

x-intercepts: $-3+\sqrt{7}$ and $-3-\sqrt{7}$.

14. Let $y=0$.

$0=4(x+1)-3x-(6-x)$

$0=4x+4-3x-6+x$

$0=2x-2$

$-2x=-2$

$x=1$

x-intercept: 1.

15. Let $y=0$.

$0=2x^2+26$

$-2x^2=26$

$x^2=-13$

$x=\pm\sqrt{-13}$

$x=\pm i\sqrt{13}$

There are no x-intercepts.

16. Let $y=0$.

$0=\frac{x^2}{3}+\frac{x}{2}-\frac{2}{3}$

$6(0)=6\left(\frac{x^2}{3}+\frac{x}{2}-\frac{2}{3}\right)$

$0=\frac{6\cdot x^2}{3}+\frac{6\cdot x}{2}-\frac{6\cdot 2}{3}$

$0=2x^2+3x-4$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(3)\pm\sqrt{(3)^2-4(2)(-4)}}{2(2)}$

$x=\frac{-3\pm\sqrt{41}}{4}$

x-intercepts: $\frac{-3+\sqrt{41}}{4}$ and $\frac{-3-\sqrt{41}}{4}$.

17. Let $y=0$.

$0=x^2-5x+8$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(8)}}{2(1)}$

$x=\frac{5\pm\sqrt{-7}}{2}$

$x=\frac{5\pm i\sqrt{7}}{2}$

There are no x-intercepts.

18. $y_1=y_2$

$3(2x-5)-2(4x+1)=-5(x+3)-2$

$6x-15-8x-2=-5x-15-2$

$-2x-17=-5x-17$

$3x=0$

$x=0$

The solution set is $\{0\}$.

19.
$$y_1 y_2 = 10$$
$$(2x+3)(x+2) = 10$$
$$2x^2 + 7x + 6 = 10$$
$$2x^2 + 7x - 4 = 0$$
$$(2x-1)(x+4) = 0$$
$$2x - 1 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = \frac{1}{2} \qquad x = -4$$
The solution set is $\left\{-4, \frac{1}{2}\right\}$.

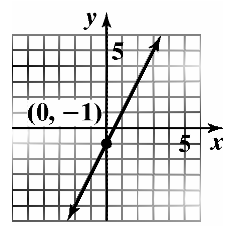
20.
$$x^2 + 10x - 3 = 0$$
$$x^2 + 10x \quad = 3$$
Since $b = 10$, we add $\left(\frac{10}{2}\right)^2 = 5^2 = 25$.
$$x^2 + 10x + 25 = 3 + 25$$
$$(x+5)^2 = 28$$
Apply the square root principle:
$$x + 5 = \pm\sqrt{28}$$
$$x + 5 = \pm\sqrt{4 \cdot 7} = \pm 2\sqrt{7}$$
$$x = -5 \pm 2\sqrt{7}$$
The solutions are $-5 \pm 2\sqrt{7}$, and the solution set is $\left\{-5 \pm 2\sqrt{7}\right\}$.

21.
$$2x^2 + 5x + 4 = 0$$
$$a = 2 \quad b = 5 \quad c = 4$$
$$b^2 - 4ac = 5^2 - 4(2)(4)$$
$$= 25 - 32 = -7$$
Since the discriminant is negative, there are no real solutions. There are two imaginary solutions that are complex conjugates.

22.
$$10x(x+4) = 15x - 15$$
$$10x^2 + 40x = 15x - 15$$
$$10x^2 - 25x + 15 = 0$$
$$a = 10 \quad b = -25 \quad c = 15$$
$$b^2 - 4ac = (-25)^2 - 4(10)(15)$$
$$= 625 - 600 = 25$$
Since the discriminant is positive and a perfect square, there are two rational solutions.

23.

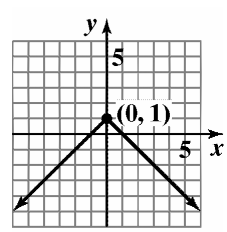
x	(x, y)
−2	−5
−1	−3
0	−1
1	1
2	3



$y = 2x - 1$

24.

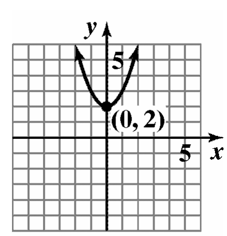
x	(x, y)
−3	−2
−2	−1
−1	0
0	1
1	0
2	−1
3	−2



$y = 1 - |x|$

25.

x	(x, y)
−2	6
−1	3
0	2
1	3
2	6



$y = x^2 + 2$

26.
$$L = a + (n-1)d$$
$$L = a + dn - d$$
$$-dn = a - d - L$$
$$\frac{-dn}{-d} = \frac{a}{-d} - \frac{d}{-d} - \frac{L}{-d}$$
$$n = -\frac{a}{d} + 1 + \frac{L}{d}$$
$$n = \frac{L}{d} - \frac{a}{d} + 1$$
$$n = \frac{L-a}{d} + 1$$

27.
$$A = 2lw + 2lh + 2wh$$
$$-2lw - 2lh = 2wh - A$$
$$l(-2w - 2h) = 2wh - A$$
$$l = \frac{2wh - A}{-2w - 2h}$$
$$l = \frac{A - 2wh}{2w + 2h}$$

28.
$$f = \frac{f_1 f_2}{f_1 + f_2}$$
$$(f_1 + f_2)(f) = (f_1 + f_2)\frac{f_1 f_2}{f_1 + f_2}$$
$$f_1 f + f_2 f = f_1 f_2$$
$$f_1 f - f_1 f_2 = -f_2 f$$
$$f_1(f - f_2) = -f_2 f$$
$$f_1 = \frac{-f_2 f}{f - f_2}$$
$$f_1 = \frac{ff_2}{f - f_2}$$

29. Let x = the average credit card debt for college sophomores.
Let $x + 421 =$ the average credit card debt for college juniors.
Let $x + 1265 =$ the average credit card debt for college seniors.
$$x + (x + 421) + (x + 1265) = 6429$$
$$x + x + 421 + x + 1265 = 6429$$
$$3x + 1686 = 6429$$
$$3x = 4743$$
$$x = 1581$$
$$x + 421 = 2002$$
$$x + 1265 = 2846$$
The average credit card debt for college sophomores, juniors, and seniors, is \$1581, \$2002, and \$2846, respectfully.

30. Let x = the number of years after 2000.
$$2.1 - 0.18x = 0$$
$$-0.18x = -2.1$$
$$\frac{-0.18x}{-0.18} = \frac{-2.1}{-0.18}$$
$$x \approx 12$$
If the trend continues, there will be no payphones 12 years after 2000, or 2012.

31. Let x = the amount invested at 8%.
Let $25{,}000 - x$ = the amount invested at 9%.
$$0.08x + 0.09(25{,}000 - x) = 2135$$
$$0.08x + 2250 - 0.09x = 2135$$
$$-0.01x + 2250 = 2135$$
$$-0.01x = -115$$
$$x = \frac{-115}{-0.01}$$
$$x = 11{,}500$$
$$25{,}000 - x = 13{,}500$$
\$11,500 was invested at 8% and \$13,500 was invested at 9%.

32. Let x = the number of prints.
Photo Shop A: $0.11x + 1.60$
Photo Shop B: $0.13x + 1.20$
$$0.13x + 1.20 = 0.11x + 1.60$$
$$0.02x + 1.20 = 1.60$$
$$0.02x = 0.40$$
$$x = 20$$
The cost will be the same for 20 prints.
That common price is
$$0.11(20) + 1.60 = 0.13(20) + 1.20$$
$$= \$3.80$$

33. Let x = the price before the reduction.
$$x - 0.40x = 468$$
$$0.60x = 468$$
$$\frac{0.60x}{0.60} = \frac{468}{0.60}$$
$$x = 780$$
The price before the reduction was \$780.

34. Let x = the amount invested at 4%.
Let $4000 - x$ = the amount invested that lost 3%.

$$0.04x - 0.03(4000 - x) = 55$$
$$0.04x - 120 + 0.03x = 55$$
$$0.07x - 120 = 55$$
$$0.07x = 175$$
$$x = \frac{175}{0.07}$$
$$x = 2500$$
$$4000 - x = 1500$$

\$2500 was invested at 4% and \$1500 lost 3%.

35. Let x = the width of the rectangle
Let $2x + 5$ = the length of the rectangle

$$2l + 2w = P$$
$$2(2x+5) + 2x = 46$$
$$4x + 10 + 2x = 46$$
$$6x + 10 = 46$$
$$6x = 36$$
$$\frac{6x}{6} = \frac{36}{6}$$
$$x = 6$$
$$2x + 5 = 17$$

The dimensions of the rectangle are 6 by 17.

36. Let x = the width of the rectangle
Let $2x - 1$ = the length of the rectangle

$$lw = A$$
$$(2x-1)x = 28$$
$$2x^2 - x = 28$$
$$2x^2 - x - 28 = 0$$
$$(2x+7)(x-4) = 0$$
$$2x + 7 = 0 \quad \text{or} \quad x - 4 = 0$$
$$2x = -7 \qquad x = 4$$
$$x = -\frac{7}{2}$$

$-\frac{7}{2}$ must be rejected.

If $x = 4$, then $2x - 1 = 7$

The dimensions of the rectangle are 4 by 7.

37. Let x = the height up the pole at which the wires are attached.

$$x^2 + 5^2 = 13^2$$
$$x^2 + 25 = 169$$
$$x^2 = 144$$
$$x = \pm 12$$

–12 must be rejected.
The wires are attached 12 feet up the pole.

38. **a.**

$$S = 2x^2 - 12x + 82$$
$$72 = 2x^2 - 12x + 82$$
$$0 = 2x^2 - 12x + 10$$
$$0 = x^2 - 6x + 5$$
$$0 = (x-1)(x-5)$$
$$x - 1 = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = 1 \qquad x = 5$$

Spending will be \$72 billion 1 year and 5 years after 2000, or in 2001 and in 2005.

b. The points (1, 72) and (5, 72).

39.

$$P = 0.004x^2 - 0.37x + 14.1$$
$$25 = 0.004x^2 - 0.37x + 14.1$$
$$0 = 0.004x^2 - 0.37x - 10.9$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-0.37) \pm \sqrt{(-0.37)^2 - 4(0.004)(-10.9)}}{2(0.004)}$$
$$x = \frac{0.37 \pm \sqrt{0.1369 + 0.1744}}{0.008}$$
$$x \approx 116, \quad x \approx -23 \text{ (rejected)}$$

The percentage of foreign born Americans will be 25% about 116 years after 1920, or 2036.

40. $(6 - 2i) - (7 - i) = 6 - 2i - 7 + i = -1 - i$

41. $3i(2 + i) = 6i + 3i^2 = -3 + 6i$

42.

$$(1+i)(4-3i) = 4 - 3i + 4i - 3i^2$$
$$= 4 + i + 3 = 7 + i$$

43.

$$\frac{1+i}{1-i} = \frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{1+i+i+i^2}{1-i^2}$$
$$= \frac{1+2i-1}{1+1}$$
$$= \frac{2i}{2}$$
$$= i$$

44. $\sqrt{-75} - \sqrt{-12} = 5i\sqrt{3} - 2i\sqrt{3} = 3i\sqrt{3}$

45.

$$\left(2 - \sqrt{-3}\right)^2 = \left(2 - i\sqrt{3}\right)^2$$
$$= 4 - 4i\sqrt{3} + 3i^2$$
$$= 4 - 4i\sqrt{3} - 3$$
$$= 1 - 4i\sqrt{3}$$

Section 1.6

Check Point Exercises

1. $4x^4 = 12x^2$

$4x^4 - 12x^2 = 0$

$4x^2\left(x^2 - 3\right) = 0$

$4x^2 = 0$ or $x^2 - 3 = 0$

$x^2 = 0$ $\quad x^2 = 3$

$x = \pm\sqrt{0}$ $\quad x = \pm\sqrt{3}$

$x = 0$ $\quad x = \pm\sqrt{3}$

The solution set is $\left\{-\sqrt{3}, 0, \sqrt{3}\right\}$.

2. $2x^3 + 3x^2 = 8x + 12$

$x^2(2x+3) - 4(2x+3) = 10$

$(2x+3)(x^2-4) = 0$

$2x + 3 = 0$ or $x^2 - 4 = 0$

$2x = -3$ $\quad x^2 = 4$

$x = -\frac{3}{2}$ $\quad x = \pm 2$

The solution set is $\left\{-2, -\frac{3}{2}, 2\right\}$.

3. $\sqrt{x+3} + 3 = x$

$\sqrt{x+3} = x - 3$

$\left(\sqrt{x+3}\right)^2 = (x-3)^2$

$x + 3 = x^2 - 6x + 9$

$0 = x^2 - 7x + 6$

$0 = (x-6)(x-1)$

$x - 6 = 0$ or $x - 1 = 0$

$x = 6$ $\quad x = 1$

1 does not check and must be rejected.

The solution set is $\{6\}$.

4. $\sqrt{x+5} - \sqrt{x-3} = 2$

$\sqrt{x+5} = 2 + \sqrt{x-3}$

$\left(\sqrt{x+5}\right)^2 = \left(2 + \sqrt{x-3}\right)^2$

$x + 5 = (2)^2 + 2(2)\left(\sqrt{x-3}\right) + \left(\sqrt{x-3}\right)^2$

$x + 5 = 4 + 4\sqrt{x-3} + x - 3$

$4 = 4\sqrt{x-3}$

$\frac{4}{4} = \frac{4\sqrt{x-3}}{4}$

$1 = \sqrt{x-3}$

$(1)^2 = \left(\sqrt{x-3}\right)^2$

$1 = x - 3$

$4 = x$

The check indicates that 4 is a solution.

The solution set is $\{4\}$.

5. **a.** $5x^{3/2} - 25 = 0$

$5x^{3/2} = 25$

$x^{3/2} = 5$

$\left(x^{3/2}\right)^{2/3} = (5)^{2/3}$

$x = 5^{2/3}$ or $\sqrt[3]{25}$

Check:

$5\left(5^{2/3}\right)^{3/2} - 25 = 0$

$5(5) - 25 = 0$

$25 - 25 = 0$

$0 = 0$

The solution set is $\left\{5^{2/3}\right\}$ or $\left\{\sqrt[3]{25}\right\}$.

b. $x^{\frac{2}{3}} - 8 = -4$

$x^{2/3} = 4$

$\left(x^{2/3}\right)^{3/2} = 4^{3/2}$ $\quad$ or

$x = \left(2^2\right)^{3/2}$

$x = 2^3$ $\quad x = (-2)^3$

$x = 8$ $\quad x = -8$

The solution set is $\{-8, 8\}$.

6. $x^4 - 5x^2 + 6 = 0$

$\left(x^2\right)^2 - 5x^2 + 6 = 0$

Let $t = x^2$.

$t^2 - 5t + 6 = 0$

$(t-3)(t-2) = 0$

$t - 3 = 0$ or $t - 2 = 0$

$t = 3$ or $t = 2$

$x^2 = 3$ or $x^2 = 2$

$x = \pm\sqrt{3}$ or $x = \pm\sqrt{2}$

The solution set is $\left\{-\sqrt{3}, \sqrt{3}, -\sqrt{2}, \sqrt{2}\right\}$.

7. $3x^{2/3} - 11x^{1/3} - 4 = 0$

Let $t = x^{1/3}$.

$3t^2 - 11t - 4 = 0$

$(3t+1)(t-4) = 0$

$3t + 1 = 0$ or $t - 4 = 0$

$3t = -1$

$t = -\frac{1}{3}$ $t = 4$

$x^{1/3} = -\frac{1}{3}$ $x^{1/3} = 4$

$x = \left(-\frac{1}{3}\right)^3$ $x = 4^3$

$x = -\frac{1}{27}$ $x = 64$

The solution set is $\left\{-\frac{1}{27}, 64\right\}$.

8. $|2x - 1| = 5$

$2x - 1 = 5$ or $2x - 1 = -5$

$2x = 6$ $2x = -4$

$x = 3$ $x = -2$

The solution set is {–2, 3}.

9. $4|1 - 2x| - 20 = 0$

$4|1 - 2x| = 20$

$|1 - 2x| = 5$

$1 - 2x = 5$ or $1 - 2x = -5$

$-2x = 4$ $-2x = -6$

$x = -2$ $x = 3$

The solution set is {–2, 3}.

10. $M = 0.7\sqrt{x} + 12.5$

$16 = 0.7\sqrt{x} + 12.5$

$3.5 = 0.7\sqrt{x}$

$\frac{3.5}{0.7} = \sqrt{x}$

$\left(\frac{3.5}{0.7}\right)^2 = \left(\sqrt{x}\right)^2$

$25 = x$

There will be 16 cluttered minutes 25 years after 1996, or 2021.

Exercise Set 1.6

1. $3x^4 - 48x^2 = 0$

$3x^2(x^2 - 16) = 0$

$3x^2(x+4)(x-4) = 0$

$3x^2 = 0$ $x + 4 = 0$ $x - 4 = 0$

$x^2 = 0$ $x = -4$ $x = 4$

$x = 0$

The solution set is {–4, 0, 4}.

2. $5x^4 - 20x^2 = 0$

$5x^2(x^2 - 4) = 0$

$5x^2(x+2)(x-2) = 0$

$5x^2 = 0$ $x + 2 = 0$ $x - 2 = 0$

$x^2 = 0$

$x = 0$ $x = -2$ $x = 2$

The solution set is {–2, 0, 2}.

3. $3x^3 + 2x^2 = 12x + 8$

$3x^3 + 2x^2 - 12x - 8 = 0$

$x^2(3x+2) - 4(3x+2) = 0$

$(3x+2)(x^2 - 4) = 0$

$3x + 2 = 0$ $x^2 - 4 = 0$

$3x = -2$ $x^2 = 4$

$x = -\frac{2}{3}$ $x = \pm 2$

The solution set is $\left\{-2, -\frac{2}{3}, 2\right\}$.

4. $4x^3 - 12x^2 = 9x - 27$

$4x^3 - 12x^2 - 9x + 27 = 0$

$4x^2(x-3) - 9(x-3) = 0$

$(x-3)(4x^2 - 9) = 0$

$x - 3 = 0 \quad 4x^2 - 9 = 0$

$x = 3 \quad 4x^2 = 9$

$x^2 = \frac{9}{4}$

$x = \pm\frac{3}{2}$

The solution set is $\left\{-\frac{3}{2}, \frac{3}{2}, 3\right\}$.

5. $2x - 3 = 8x^3 - 12x^2$

$8x^3 - 12x^2 - 2x + 3 = 0$

$4x^2(2x-3) - (2x-3) = 0$

$(2x-3)(4x^2 - 1) = 0$

$2x - 3 = 0 \quad 4x^2 - 1 = 0$

$2x = 3 \quad 4x^2 = 1$

$x^2 = \frac{1}{4}$

$x = \frac{3}{2} \quad x = \pm\frac{1}{2}$

The solution set is $\left\{\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}\right\}$.

6. $x + 1 = 9x^3 + 9x^2$

$9x^3 + 9x^2 - x - 1 = 0$

$9x^2(x+1) - (x+1) = 0$

$(x+1)(9x^2 - 1) = 0$

$x + 1 = 0 \quad 9x^2 - 1 = 1$

$x = -1 \quad 9x^2 = 1$

$x^2 = \frac{1}{9}$

$x = \pm\frac{1}{3}$

The solution set is $\left\{-1, -\frac{1}{3}, \frac{1}{3}\right\}$.

7. $4y^3 - 2 = y - 8y^2$

$4y^3 + 8y^2 - y - 2 = 0$

$4y^2(y+2) - (y+2) = 0$

$(y+2)(4y^2 - 1) = 0$

$y + 2 = 0 \quad 4y^2 - 1 = 0$

$4y^2 = 1$

$y^2 = \frac{1}{4}$

$y = -2 \quad y = \pm\frac{1}{2}$

The solution set is $\left\{-2, \frac{1}{2}, -\frac{1}{2}\right\}$.

8. $9y^3 + 8 = 4y + 18y^2$

$9y^3 - 18y^2 - 4y + 8 = 0$

$9y^2(y-2) - 4(y-2) = 0$

$(y-2)(9y^2 - 4) = 0$

$y - 2 = 0 \quad 9y^2 - 4 = 0$

$y = 2 \quad 9y^2 = 4$

$y^2 = \frac{4}{9}$

$y = \pm\frac{2}{3}$

The solution set is $\left\{-\frac{2}{3}, \frac{2}{3}, 2\right\}$.

9. $2x^4 = 16x$

$2x^4 - 16x = 0$

$2x\left(x^3 - 8\right) = 0$

$2x = 0 \quad x^3 - 8 = 0$

$x = 0 \quad (x-2)(x^2 + 2x + 2) = 0$

$x - 2 = 0 \quad x^2 + 2x + 4 = 0$

$x = 2 \quad x = \frac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{-12}}{2}$

$x = \frac{-2 \pm 2i\sqrt{3}}{2}$

$x = -1 \pm i\sqrt{3}$

The solution set is $\left\{0, 2, -1 \pm i\sqrt{3}\right\}$.

10.
$$3x^4 = 81x$$
$$3x^4 - 81x = 0$$
$$3x(x^3 - 27) = 0$$
$$3x = 0 \qquad x^3 - 27 = 0$$
$$x = 0;$$
$$(x-3)(x^2+3x+9) = 0$$
$$x - 3 = 0 \qquad x^2 + 3x + 9 = 0$$
$$x = 3 \qquad x = \frac{-3 \pm \sqrt{3^2 - 4(1)(9)}}{2(1)}$$
$$x = \frac{-3 \pm \sqrt{9-36}}{2}$$
$$x = \frac{-3 \pm \sqrt{-27}}{2}$$
$$x = \frac{-3 \pm 3i\sqrt{3}}{2}$$
The solution set is $\left\{0, 3, \frac{-3 \pm 3i\sqrt{3}}{2}\right\}$.

11.
$$\sqrt{3x+18} = x$$
$$3x + 18 = x^2$$
$$x^2 - 3x - 18 = 0$$
$$(x+3)(x-6) = 0$$
$$x + 3 = 0 \qquad x - 6 = 0$$
$$x = -3 \qquad x = 6$$
$$\sqrt{3(-3)+18} = -3 \qquad \sqrt{3(6)+18} = 6$$
$$\sqrt{-9+18} = -3 \qquad \sqrt{18+18} = 6$$
$$\sqrt{9} = -3 \text{ False} \qquad \sqrt{36} = 6$$
The solution set is $\{6\}$.

12.
$$\sqrt{20-8x} = x$$
$$20 - 8x = x^2$$
$$x^2 + 8x - 20 = 0$$
$$(x+10)(x-2) = 0$$
$$x + 10 = 0 \qquad x - 2 = 0$$
$$x = -10 \qquad x = 2$$
$$\sqrt{20-8(-10)} = -10 \qquad \sqrt{20-8(2)} = 2$$
$$\sqrt{20+80} = -10 \qquad \sqrt{20-16} = 2$$
$$\sqrt{100} = -10 \text{ False} \qquad \sqrt{4} = 2$$
The solution set is $\{2\}$.

13.
$$\sqrt{x+3} = x - 3$$
$$x + 3 = x^2 - 6x + 9$$
$$x^2 - 7x + 6 = 0$$
$$(x-1)(x-6) = 0$$
$$x - 1 = 0 \quad x - 6 = 0$$
$$x = 1 \qquad x = 6$$
$$\sqrt{1+3} = 1 - 3 \qquad \sqrt{6+3} = 6 - 3$$
$$\sqrt{4} = -2 \quad \text{False} \quad \sqrt{9} = 3$$
The solution set is $\{6\}$.

14.
$$\sqrt{x+10} = x - 2$$
$$x + 10 = (x-2)^2$$
$$x + 10 = x^2 - 4x + 4$$
$$x^2 - 5x - 6 = 0$$
$$(x+1)(x-6) = 0$$
$$x + 1 = 0 \qquad x - 6 = 0$$
$$x = -1 \qquad x = 6$$
$$\sqrt{-1+10} = -1 - 2 \qquad \sqrt{6+10} = 6 - 2$$
$$\sqrt{9} = -3 \text{ False} \qquad \sqrt{16} = 4$$
The solution set is $\{6\}$.

15.
$$\sqrt{2x+13} = x + 7$$
$$2x + 13 = (x+7)^2$$
$$2x + 13 = x^2 + 14x + 49$$
$$x^2 + 12x + 36 = 0$$
$$(x+6)^2 = 0$$
$$x + 6 = 0$$
$$x = -6$$
$$\sqrt{2(-6)+13} = -6 + 7$$
$$\sqrt{-12+13} = 1$$
$$\sqrt{1} = 1$$
The solution set is $\{-6\}$.

16. $\sqrt{6x+1}=x-1$

$6x+1=(x-1)^2$

$6x+1=x^2-2x+1$

$x^2-8x=0$

$x(x-8)=0$

$x-8=0 \qquad x=0$

$x=8$

$\sqrt{6(0)+1}=0-1 \qquad \sqrt{6(8)+1}=8-1$

$\sqrt{0+1}=-1 \qquad \sqrt{48+1}=7$

$\sqrt{1}=-1$ False $\qquad \sqrt{49}=7$

The solution set is {8}.

17. $x-\sqrt{2x+5}=5$

$x-5=\sqrt{2x+5}$

$(x-5)^2=2x+5$

$x^2-10x+25=2x+5$

$x^2-12x+20=0$

$(x-2)(x-10)=0$

$x-2=0 \quad x-10=0$

$x=2 \qquad x=10$

$2-\sqrt{2(2)+5}=5 \qquad 10-\sqrt{2(10)+5}=5$

$2-\sqrt{9}=5 \qquad 10-\sqrt{25}=5$

$2-3=5$ False $\qquad 10-5=5$

The solution set is {10}.

18. $x-\sqrt{x+11}=1$

$x-1=\sqrt{x+11}$

$(x-1)^2=x+11$

$x^2-2x+1=x+11$

$x^2-3x-10=0$

$(x+2)(x-5)=0$

$x+2=0 \qquad x-5=0$

$x=-2 \qquad x=5$

$-2-\sqrt{-2+11}=1 \qquad 5-\sqrt{5+11}=1$

$-2-\sqrt{9}=1 \qquad 5-\sqrt{16}=1$

$-2-3=1$ False $\qquad 5-4=1$

The solution set is {5}.

19. $\sqrt{2x+19}-8=x$

$\sqrt{2x+19}=x+8$

$\left(\sqrt{2x+19}\right)^2=\left(x+8\right)^2$

$2x+19=x^2+16x+64$

$0=x^2+14x+45$

$0=(x+9)(x+5)$

$x+9=0$ or $x+5=0$

$x=-9 \qquad x=-5$

–9 does not check and must be rejected.

The solution set is {–5}.

20. $\sqrt{2x+15}-6=x$

$\sqrt{2x+15}=x+6$

$\left(\sqrt{2x+15}\right)^2=\left(x+6\right)^2$

$2x+15=x^2+12x+36$

$0=x^2+10x+21$

$0=(x+3)(x+7)$

$x+3=0$ or $x+7=0$

$x=-3 \qquad x=-7$

–7 does not check and must be rejected.

The solution set is {–3}.

21. $\sqrt{3x}+10=x+4$

$\sqrt{3x}=x-6$

$3x=(x-6)^2$

$3x=x^2-12x+36$

$x^2-15x+36=0$

$(x-12)(x-3)=0$

$x-12=0 \qquad x-3=0$

$x=12 \qquad x=3$

$\sqrt{3(12)}+10=12+4 \qquad \sqrt{3(3)}+10=3+4$

$\sqrt{36}+10=16 \qquad \sqrt{9}+10=7$

$6+10=16 \qquad 3+10=7$ False

The solution set is {12}.

22.
$$\sqrt{x}-3=x-9$$
$$\sqrt{x}=x-6$$
$$x=(x-6)^2$$
$$x=x^2-12x+36$$
$$x^2-13x+36=0$$
$$(x-9)(x-4)=0$$
$$x-9=0 \quad x-4=0$$
$$x=9 \quad x=4$$
$$\sqrt{9}-3=9-9 \qquad \sqrt{4}-3=4-9$$
$$3-3=9-9 \qquad 2-3=4-9 \text{ False}$$
The solution set is $\{9\}$.

23.
$$\sqrt{x+8}-\sqrt{x-4}=2$$
$$\sqrt{x+8}=\sqrt{x-4}+2$$
$$x+8=(\sqrt{x-4}+2)^2$$
$$x+8=x-4+4\sqrt{x-4}+4$$
$$x+8=x+4\sqrt{x-4}$$
$$8=4\sqrt{x-4}$$
$$2=\sqrt{x-4}$$
$$4=x-4$$
$$x=8$$
$$\sqrt{8+8}-\sqrt{8-4}=2$$
$$\sqrt{16}-\sqrt{4}=2$$
$$4-2=2$$
The solution set is $\{8\}$.

24.
$$\sqrt{x+5}-\sqrt{x-3}=2$$
$$\sqrt{x+5}=\sqrt{x-3}+2$$
$$x+5=(\sqrt{x-3}+2)^2$$
$$x+5=x-3+4\sqrt{x-3}+4$$
$$x+5=x+1+4\sqrt{x-3}$$
$$5=1+4\sqrt{x-3}$$
$$4=4\sqrt{x-3}$$
$$1=\sqrt{x-3}$$
$$1=x-3$$
$$x=4$$
$$\sqrt{4+5}-\sqrt{4-3}=2$$
$$\sqrt{9}-\sqrt{1}=2$$
$$3-1=2$$
The solution set is $\{4\}$.

25.
$$\sqrt{x-5}-\sqrt{x-8}=3$$
$$\sqrt{x-5}=\sqrt{x-8}+3$$
$$x-5=(\sqrt{x-8}+3)^2$$
$$x-5=x-8+6\sqrt{x-8}+9$$
$$x-5=x+1+6\sqrt{x-8}$$
$$-6=6\sqrt{x-8}$$
$$-1=\sqrt{x-8}$$
$$1=x-8$$
$$x=9$$
$$\sqrt{9-5}-\sqrt{9-8}=3$$
$$\sqrt{4}-\sqrt{1}=3$$
$$2-1=3 \text{ False}$$
The solution set is the empty set, $\varnothing$.

26.
$$\sqrt{2x-3}-\sqrt{x-2}=1$$
$$\sqrt{2x-3}=\sqrt{x-2}+1$$
$$2x-3=(\sqrt{x-2}+1)^2$$
$$2x-3=x-2+2\sqrt{x-2}+1$$
$$2x-3=x-1+2\sqrt{x-2}$$
$$x-2=2\sqrt{x-2}$$
$$\frac{x}{2}-1=\sqrt{x-2}$$
$$\left(\frac{x}{2}-1\right)^2=x-2$$
$$\frac{x^2}{4}-x+1=x-2$$
$$x^2-4x+4=4x-8$$
$$x^2-8x+12=0$$
$$(x-6)(x-2)=0$$
$$x-6=0 \quad x-2=0$$
$$x=6 \quad x=2$$
$$\sqrt{2(6)-3}-\sqrt{6-2}=1 \qquad \sqrt{2(2)-3}-\sqrt{2-2}=1$$
$$\sqrt{12-3}-\sqrt{4}=1 \qquad \sqrt{4-3}-\sqrt{0}=1$$
$$\sqrt{9}-\sqrt{4}=1 \qquad \sqrt{1}-0=1$$
$$3-2=1 \qquad 1-0=1$$
The solution set is $\{2, 6\}$.

27. $\sqrt{2x+3}+\sqrt{x-2}=2$

$\sqrt{2x+3}=2-\sqrt{x-2}$

$2x+3=(2-\sqrt{x-2})^2$

$2x+3=4-4\sqrt{x-2}+x-2$

$x+1=-4\sqrt{x-2}$

$(x+1)^2=16(x-2)$

$x^2+2x+1=16x-32$

$x^2-14x+33=0$

$(x-11)(x-3)=0$

$x-11=0 \quad x-3=0$

$x=11 \qquad x=3$

$\sqrt{2(11)+3}+\sqrt{11-2}=2$

$\sqrt{22+3}+\sqrt{9}=2$

$5+3=2$ False

$\sqrt{2(3)+3}+\sqrt{3-2}=2$

$\sqrt{6+3}+\sqrt{1}=2$

$3+1=2$ False

The solution set is the empty set, $\varnothing$.

28. $\sqrt{x+2}+\sqrt{3x+7}=1$

$\sqrt{x+2}=1-\sqrt{3x+7}$

$x+2=(1-\sqrt{3x+7})^2$

$x+2=1-2\sqrt{3x+7}+3x+7$

$-2x-6=-2\sqrt{3x+7}$

$x+3=\sqrt{3x+7}$

$(x+3)^2=3x+7$

$x^2+6x+9=3x+7$

$x^2+3x+2=0$

$(x+1)(x+2)=0$

$x+1=0 \quad x+2=0$

$x=-1 \qquad x=-2$

$\sqrt{-1+2}+\sqrt{3(-1)+7}=1$

$\sqrt{1}+\sqrt{4}=1$

$1+2=1$ False

$\sqrt{-2+2}+\sqrt{3(-2)+7}=1$

$\sqrt{0}+\sqrt{1}=1$

$0+1=1$

The solution set is $\{-2\}$.

29. $\sqrt{3\sqrt{x+1}}=\sqrt{3x-5}$

$3\sqrt{x+1}=3x-5$

$9(x+1)=9x^2-30x+25$

$9x^2-39x+16=0$

$x=\frac{39\pm\sqrt{945}}{18}=\frac{13\pm\sqrt{105}}{6}$

Check proposed solutions.

The solution set is $\left\{\frac{13+\sqrt{105}}{6}\right\}$.

30. $\sqrt{1+4\sqrt{x}}=1+\sqrt{x}$

$1+4\sqrt{x}=1+2\sqrt{x}+x$

$2\sqrt{x}=x$

$4x=x^2$

$x^2-4x=0$

$x(x-4)=0$

$x=0$ or $x=4$

The solution set is $\{0, 4\}$.

31. $x^{3/2}=8$

$(x^{3/2})^{2/3}=8^{2/3}$

$x=\sqrt[3]{8}^2$

$x=2^2$

$x=4$

$4^{3/2}=8$

$\sqrt{4}^3=8$

$2^3=8$

The solution set is $\{4\}$.

32. $x^{3/2}=27$

$(x^{3/2})^{2/3}=27^{2/3}$

$x=\sqrt[3]{27}^2$

$x=3^2$

$x=9$

$9^{3/2}=27$

$\sqrt{9}^3=27$

$3^3=27$

The solution set is $\{9\}$.

33. $(x-4)^{3/2}=27$

$((x-4)^{3/2})^{2/3}=27^{2/3}$

$x-4=\sqrt[3]{27}^{2}$

$x-4=3^2$

$x-4=9$

$x=13$

$(13-4)^{3/2}=27$

$9^{3/2}=27$

$\sqrt{9}^{3}=27$

$3^3=27$

The solution set is {13}.

34. $(x+5)^{3/2}=8$

$((x+5)^{3/2})^{2/3}=8^{2/3}$

$x+5=\sqrt[3]{8}^{2}$

$x+5=2^2$

$x+5=4$

$x=-1$

$(-1+5)^{3/2}=8$

$4^{3/2}=8$

$\sqrt{4}^{3}=8$

$2^3=8$

The solution set is {–1}.

35. $6x^{5/2}-12=0$

$6x^{5/2}=12$

$x^{5/2}=2$

$(x^{5/2})^{2/5}=2^{2/5}$

$x=\sqrt[5]{2^2}$

$x=\sqrt[5]{4}$

$6(\sqrt[5]{4})^{5/2}-12=0$

$6(4^{1/5})^{5/2}-12=0$

$6(4^{1/2})-12=0$

$6(2)-12=0$

The solution set is $\left\{\sqrt[5]{4}\right\}$.

36. $8x^{5/3}-24=0$

$8x^{5/3}=24$

$x^{5/3}=3$

$(x^{5/3})^{3/5}=3^{3/5}$

$x=\sqrt[5]{3^3}$

$x=\sqrt[5]{27}$

$8(\sqrt[5]{27})^{5/3}-24=0$

$8(27^{1/5})^{5/3}-24=0$

$8(27^{1/3})-24=0$

$8(3)-24=0$

The solution set is $\left\{\sqrt[5]{27}\right\}$.

37. $(x-4)^{2/3}=16$

$\left[(x-4)^{2/3}\right]^{3/2}=(16)^{3/2}$

$x-4=\left(2^4\right)^{3/2}$

$x-4=4^3 \qquad x-4=(-4)^3$

$x-4=64 \qquad x-4=-64$

$x=68 \qquad x=-60$

The solution set is {– 60, 68}.

38. $(x+5)^{\frac{2}{3}}=4$

$\left[(x+5)\frac{2}{3}\right]^{\frac{3}{2}}=(4)^{\frac{3}{2}}$

$x+5=\left(2^2\right)^{\frac{3}{2}}$

$x+5=2^3$ or $x+5=(-2)^3$

$x+5=8 \qquad x+5=-8$

$x=3 \qquad x=-13$

The solution set is {–13, 3}.

39. $(x^2 - x - 4)^{3/4} - 2 = 6$

$(x^2 - x - 4)^{3/4} = 8$

$((x^2 - x - 4)^{3/4})^{4/3} = 8^{4/3}$

$x^2 - x - 4 = \sqrt[3]{8}^4$

$x^2 - x - 4 = 2^4$

$x^2 - x - 4 = 16$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

$x - 5 = 0 \quad x + 4 = 0$

$x = 5 \qquad x = -4$

$(5^2 - 5 - 4)^{3/4} - 2 = 6$

$(25 - 9)^{3/4} - 2 = 6$

$16^{3/4} - 2 = 6$

$\sqrt[4]{16}^3 - 2 = 6$

$2^3 - 2 = 6$

$8 - 2 = 6$

$((-4)^2 - (-4) - 4)^{3/4} - 2 = 6$

$(16 + 4 - 4)^{3/4} - 2 = 6$

$16^{3/4} - 2 = 6$

$\sqrt[4]{16}^3 - 2 = 6$

$2^3 - 2 = 6$

$8 - 2 = 6$

The solution set is {5, –4}.

40. $(x^2 - 3x + 3)^{3/2} - 1 = 0$

$(x^2 - 3x + 3)^{3/2} = 1$

$x^2 - 3x + 3 = 1^{2/3}$

$x^2 - 3x + 3 = 1$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x - 1 = 0 \quad x - 2 = 0$

$x = 1 \qquad x = 2$

$(1^2 - 3(1) + 3)^{3/2} - 1 = 0$

$(1 - 3 + 3)^{3/2} - 1 = 0$

$1^{3/2} - 1 = 0$

$1 - 1 = 0$

$(2^2 - 3(2) + 3)^{3/2} - 1 = 0$

$(4 - 6 + 3)^{3/2} - 1 = 0$

$1^{3/2} - 1 = 0$

$1 - 1 = 0$

The solution set is {1, 2}.

41. $x^4 - 5x^2 + 4 = 0$ let $t = x^2$

$t^2 - 5t + 4 = 0$

$(t-1)(t-4) = 0$

$t - 1 = 0 \quad t - 4 = 0$

$t = 1 \qquad t = 4$

$x^2 = 1 \qquad x^2 = 4$

$x = \pm 1 \qquad x = \pm 2$

The solution set is {1, –1, 2, –2}

42. $x^4 - 13x^2 + 36 = 0$ let $t = x^2$

$t^2 - 13t + 36 = 0$

$(t-4)(t-9) = 0$

$t - 4 = 0 \quad t - 9 = 0$

$t = 4 \quad t = 9$

$x^2 = 4 \quad x^2 = 9$

$x = \pm 2 \quad x = \pm 3$

The solution set is {–3, –2, 2, 3}.

43. $9x^4 = 25x^2 - 16$

$9x^4 - 25x^2 + 16 = 0$ let $t = x^2$

$9t^2 - 25t + 16 = 0$

$(9t-16)(t-1) = 0$

$9t - 16 = 0 \quad t - 1 = 0$

$9t = 16 \quad t = 1$

$t = \frac{16}{9} \quad x^2 = 1$

$x = \pm 1$

$x^2 = \frac{16}{9}$

$x = \pm\frac{4}{3}$

The solution set is $\left\{1, -1, \frac{4}{3}, -\frac{4}{3}\right\}$.

44. $4x^4 = 13x^2 - 9$

$4x^4 - 13x^2 + 9 = 0$ let $t = x^2$

$4t^2 - 13t + 9 = 0$

$(4t-9)(t-1) = 0$

$4t - 9 = 0 \quad t - 1 = 0$

$4t = 9 \quad t = 1$

$t = \frac{9}{4} \quad x^2 = 1$

$x^2 = \frac{9}{4} \quad x = \pm 1$

$x = \pm\frac{3}{2}$

The solution set is $\left\{-\frac{3}{2}, -1, 1, \frac{3}{2}\right\}$.

45. $x - 13\sqrt{x} + 40 = 0$ Let $t = \sqrt{x}$.

$t^2 - 13t + 40 = 0$

$(t-8)(t-5) = 0$

$t - 8 = 0 \quad t - 5 = 0$

$t = 8 \quad t = 5$

$\sqrt{x} = 8 \quad \sqrt{x} = 5$

$x = 64 \quad x = 25$

The solution set is {25, 64}.

46. $2x - 7\sqrt{x} - 30 = 0$ Let $t = \sqrt{x}$.

$2t^2 - 7t - 30 = 0$

$(2t+5)(t-6) = 0$

$2t + 5 = 0$

$t = \frac{5}{2} \quad t - 6 = 0$

$t = 6$

$\sqrt{x} = \frac{5}{2} \quad \sqrt{x} = 6$

$x = 36$

$x = \frac{25}{4}$

The solution set is {36} since 25/4 does not check in the original equation.

47. $x^{-2} - x^{-1} - 20 = 0$ Let $t = x^{-1}$

$t^2 - t - 20 = 0$

$(t-5)(t+4) = 0$

$t - 5 = 0 \quad t + 4 = 0$

$t = 5 \quad t = -4$

$x^{-1} = 5 \quad x^{-1} = -4$

$\frac{1}{x} = 5 \quad \frac{1}{x} = -4$

$1 = 5x \quad 1 = -4x$

$\frac{1}{5} = x \quad -\frac{1}{4} = x$

The solution set is $\left\{-\frac{1}{4}, \frac{1}{5}\right\}$.

48. $x^{-2} - x^{-1} - 6 = 0$ Let t = x^{-1}.

$t^2 - t - 6 = 0$

$(t-3)(t+2) = 0$

$t - 3 = 0 \quad t + 2 = 0$

$t = 3 \quad t = -2$

$x^{-1} = 3 \quad x^{-1} = -2$

$\frac{1}{x} = 3 \quad \frac{1}{x} = -2$

$1 = 3x \quad 1 = -2x$

$\frac{1}{3} = x \quad -\frac{1}{2} = x$

The solution set is $\left\{-\frac{1}{2}, \frac{1}{3}\right\}$.

49. $x^{2/3} - x^{1/3} - 6 = 0$ let $t = x^{1/3}$

$t^2 - t - 6 = 0$

$(t-3)(t+2) = 0$

$t - 3 = 0 \quad t + 2 = 0$

$t = 3 \quad t = -2$

$x^{1/3} = 3 \quad x^{1/3} = -2$

$x = 3^3 \quad x = (-2)^3$

$x = 27 \quad x = -8$

The solution set is {27, –8}.

50. $2x^{2/3} + 7x^{1/3} - 15 = 0$ let $t = x^{1/3}$

$2t^2 + 7t - 15 = 0$

$(2t-3)(t+5) = 0$

$2t - 3 = 0 \quad t + 5 = 0$

$2t = 3 \quad t = -5$

$t = \frac{3}{2} \quad x^{1/3} = -5$

$x^{1/3} = \frac{3}{2} \quad x = (-5)^2$

$x = \left(\frac{3}{2}\right)^3 \quad x = -125$

$x = \frac{27}{8}$

The solution set is $\left\{-125, \frac{27}{8}\right\}$.

51. $x^{3/2} - 2x^{3/4} + 1 = 0$ let $t = x^{3/4}$

$t^2 - 2t + 1 = 0$

$(t-1)(t-1) = 0$

$t - 1 = 0$

$t = 1$

$x^{3/4} = 1$

$x = 1^{4/3}$

$x = 1$

The solution set is {1}.

52. $x^{2/5} + x^{1/5} - 6 = 0$ let $t = x^{1/5}$

$t^2 + t - 6 = 0$

$(t+3)(t-2) = 0$

$t + 3 = 0 \quad t - 2 = 0$

$t = -3 \quad t = 2$

$x^{1/5} = -3 \quad x^{1/5} = 2$

$x = (-3)^5 \quad x = 2^5$

$x = -243 \quad x = 32$

The solution set is {–243, 32}.

53. $2x-3x^{1/2}+1=0$ let $t=x^{1/2}$

$2t^2-3t+1=0$

$(2t-1)(t-1)=0$

$2t-1=0 \qquad t-1=0$

$2t=1$

$t=\dfrac{1}{2} \qquad t=1$

$x^{1/2}=\dfrac{1}{2} \qquad x^{1/2}=1$

$x=\left(\dfrac{1}{2}\right)^2 \qquad x=1^2$

$x=\dfrac{1}{4} \qquad x=1$

The solution set is $\left\{\dfrac{1}{4},1\right\}$.

54. $x+3x^{1/2}-4=0$ let $t=x^{1/2}$

$t^2+3t-4=0$

$(t-1)(t+4)=0$

$t-1=0 \qquad t+4=0$

$t=1 \qquad t=-4$

$x^{1/2}=1 \qquad x^{1/2}=-4$

$x=1^2 \qquad x=(-4)^2$

$x=1 \qquad x=16$

The solution set is {1}.

55. $(x-5)^2-4(x-5)-21=0$ let $t=x-5$

$t^2-4t-21=0$

$(t+3)(t-7)=0$

$t+3=0 \qquad t-7=0$

$t=-3 \qquad t=7$

$x-5=-3 \qquad x-5=7$

$x=2 \qquad x=12$

The solution set is {2, 12}.

56. $(x+3)^2+7(x+3)-18=0$ let $t=x+3$

$t^2+7t-18=0$

$(t+9)(t-2)=0$

$t+9=0 \qquad t-2=0$

$t=-9 \qquad t=2$

$x+3=-9 \qquad x+3=2$

$x=-12 \qquad x=-1$

The solution set is {–12, –1}.

57. $\left(x^2-x\right)^2-14\left(x^2-x\right)+24=0$

Let $t=x^2-x$.

$t^2-14t+24=0$

$(t-2)(t-12)=0$

$t=2$ or $t=12$

$x^2-x=2 \qquad$ or $\qquad x^2-x=12$

$x^2-x-2=0 \qquad x^2-x-12=0$

$(x-2)(x+1)=0 \qquad (x-4)(x+3)=0$

The solution set is {–3, –1, 2, 4}.

58. $\left(x^2-2x\right)^2-11\left(x^2-2x\right)+24=0$

Let $t=x^2-2x$

$t^2-11t+24=0$

$(t-3)(t-8)=0$

$t=3$ or $t=8$

$x^2-2x=3 \qquad$ or $\qquad x^2-2x=8$

$x^2-2x-3=0 \qquad x^2-2x-8=0$

$(x-3)(x+1)=0 \qquad (x-4)(x+2)=0$

The solution set is {–2, –1, 3, 4}.

59. $\left(y-\dfrac{8}{y}\right)^2+5\left(y-\dfrac{8}{y}\right)-14=0$

Let $t=y-\dfrac{8}{y}$.

$t^2+5t-14=0$

$(t+7)(t-2)=0$

$t=-7$ or $t=2$

$y-\dfrac{8}{y}=-7 \qquad$ or $\qquad y-\dfrac{8}{y}=2$

$y^2+7y-8=0 \qquad y^2-2y-8=0$

$(y+8)(y-1)=0 \qquad (y-4)(y+2)=0$

The solution set is {–8, –2, 1, 4}.

60. $\left(y-\frac{10}{y}\right)^2+6\left(y-\frac{10}{y}\right)-27=0$

Let $t=y-\frac{10}{y}$.

$t^2+6t-27=0$

$(t+9)(t-3)=0$

$t=-9$ or $t=3$

$y-\frac{10}{y}=-9$ or $y-\frac{10}{y}=3$

$y^2+9y-10=0$ $\quad$ $y^2-3y-10=0$

$(y+10)(y-1)=0$ $\quad$ $(y-5)(y+2)=0$

The solution set is {–10, –2, 1, 5}

61. $|x|=8$

$x=8, x=-8$

The solution set is {8, –8}.

62. $|x|=6$

$x=6, x=-6$

The solution set is {–6, 6}.

63. $|x-2|=7$

$x-2=7$ $\quad$ $x-2=-7$

$x=9$ $\quad$ $x=-5$

The solution set is {9, –5}.

64. $|x+1|=5$

$x+1=5$ $\quad$ $x+1=-5$

$x=4$ $\quad$ $x=-6$

The solution set is {–6, 4}.

65. $|2x-1|=5$

$2x-1=5$ $\quad$ $2x-1=-5$

$2x=6$ $\quad$ $2x=-4$

$x=3$ $\quad$ $x=-2$

The solution set is {3, –2}.

66. $|2x-3|=11$

$2x-3=11$ $\quad$ $2x-3=-11$

$2x=14$ $\quad$ $2x=-8$

$x=7$ $\quad$ $x=-4$

The solution set is {–4, 7}.

67. $2|3x-2|=14$

$|3x-2|=7$

$3x-2=7$ $\quad$ $3x-2=-7$

$3x=9$ $\quad$ $3x=-5$

$x=3$ $\quad$ $x=-5/3$

The solution set is {3, –5/3}

68. $3|2x-1|=21$

$|2x-1|=7$

$2x-1=7$ or $2x-1=-7$

$2x=8$ $\quad$ $2x=-6$

$x=4$ $\quad$ $x=-3$

The solution set is {4, –3}

69. $7|5x|+2=16$

$7|5x|=14$

$|5x|=2$

$5x=2$ $\quad$ $5x=-2$

$x=2/5$ $\quad$ $x=-2/5$

The solution set is $\left\{\frac{2}{5},-\frac{2}{5}\right\}$.

70. $7|3x|+2=16$

$7|3x|=14$

$|3x|=2$

$3x=2$ or $3x=-2$

$x=2/3$ $\quad$ $x=-2/3$

The solution set is {–2/3, 2/3}

71. $2\left|4-\frac{5}{2}x\right|+6=18$

$2\left|4-\frac{5}{2}x\right|=12$

$\left|4-\frac{5}{2}x\right|=6$

$4-\frac{5}{2}x=6$ or $4-\frac{5}{2}x=-6$

$-\frac{5}{2}x=2$ $\quad$ $-\frac{5}{2}x=-10$

$-\frac{2}{5}\left(-\frac{5}{2}\right)x=-\frac{2}{5}(2)$ $\quad$ $-\frac{2}{5}\left(-\frac{5}{2}\right)x=-\frac{2}{5}(-10)$

$x=-\frac{4}{5}$ $\quad$ $x=4$

The solution set is $\left\{-\frac{4}{5},4\right\}$.

72. $4\left|1-\frac{3}{4}x\right|+7=10$

$$4\left|1-\frac{3}{4}x\right|=3$$

$$\left|1-\frac{3}{4}x\right|=\frac{3}{4}$$

$$1-\frac{3}{4}x=\frac{3}{4} \quad \text{or} \quad 1-\frac{3}{4}x=-\frac{3}{4}$$

$$-\frac{3}{4}x=-\frac{1}{4} \qquad -\frac{3}{4}x=-\frac{7}{4}$$

$$-\frac{4}{3}\left(-\frac{3}{4}\right)x=-\frac{4}{3}\left(-\frac{1}{4}\right) \qquad -\frac{4}{3}\left(-\frac{3}{4}\right)x=-\frac{4}{3}\left(-\frac{7}{4}\right)$$

$$x=\frac{1}{3} \qquad x=\frac{7}{3}$$

The solution set is $\left\{\frac{1}{3},\frac{7}{3}\right\}$.

73. $|x+1|+5=3$

$|x+1|=-2$

No solution

The solution set is { }.

74. $|x+1|+6=2$

$|x+1|=-4$

No solution

The solution set is { }.

75. $|2x-1|+3=3$

$|2x-1|=0$

$2x-1=0$

$2x=1$

$x=1/2$

The solution set is {1/2}.

76. $|3x-2|+4=4$

$$|3x-2|=0$$

$$3x-2=0$$

$$3x=2$$

$$x=\frac{2}{3}$$

The solution set is {2/3}.

77. $|3x-1|=|x+5|$

$3x-1=x+5 \qquad 3x-1=-x-5$

$2x-1=5 \qquad 4x-1=-5$

$2x=6 \qquad 4x=-4$

$x=3 \qquad x=-1$

The solution set is {3, −1}.

78. $|2x-7|=|x+3|$

$$2x-7=x+3 \quad \text{or} \quad 2x-7=-(x+3)$$

$$x=10 \qquad 2x-7=-x-3$$

$$3x=4$$

$$x=\frac{4}{3}$$

The solution set is $\left\{10,\frac{4}{3}\right\}$

79. Set $y = 0$ to find the x–intercept(s).

$$0=\sqrt{x+2}+\sqrt{x-1}-3$$

$$-\sqrt{x+2}=\sqrt{x-1}-3$$

$$\left(-\sqrt{x+2}\right)^2=\left(\sqrt{x-1}-3\right)^2$$

$$x+2=\left(\sqrt{x-1}\right)^2-2\left(\sqrt{x-1}\right)(3)+(3)^2$$

$$x+2=x-1-6\sqrt{x-1}+9$$

$$x+2=x-1-6\sqrt{x-1}+9$$

$$2=8-6\sqrt{x-1}$$

$$-6=-6\sqrt{x-1}$$

$$\frac{-6}{-6}=\frac{-6\sqrt{x-1}}{-6}$$

$$1=\sqrt{x-1}$$

$$(1)^2=\left(\sqrt{x-1}\right)^2$$

$$1=x-1$$

$$2=x$$

The x-intercept is 2.

The corresponding graph is graph (c).

80. Set $y = 0$ to find the x–intercept(s).

$$0 = \sqrt{x-4} + \sqrt{x+4} - 4$$
$$-\sqrt{x-4} = \sqrt{x+4} - 4$$
$$\left(-\sqrt{x-4}\right)^2 = \left(\sqrt{x+4} - 4\right)^2$$
$$x - 4 = \left(\sqrt{x+4}\right)^2 - 2\left(\sqrt{x+4}\right)(4) + (4)^2$$
$$x - 4 = x + 4 - 8\sqrt{x+4} + 16$$
$$-4 = 20 - 8\sqrt{x+4}$$
$$-24 = -8\sqrt{x+4}$$
$$\frac{-24}{-8} = \frac{-8\sqrt{x+4}}{-8}$$
$$3 = \sqrt{x+4}$$
$$(3)^2 = \left(\sqrt{x+4}\right)^2$$
$$9 = x + 4$$
$$5 = x$$

The x-intercept is 5.
The corresponding graph is graph (a).

81. Set $y = 0$ to find the x–intercept(s).

$$0 = x^{\frac{1}{3}} + 2x^{\frac{1}{6}} - 3$$

Let $t = x^{\frac{1}{6}}$.

$$x^{\frac{1}{3}} + 2x^{\frac{1}{6}} - 3 = 0$$
$$\left(x^{\frac{1}{6}}\right)^2 + 2x^{\frac{1}{6}} - 3 = 0$$
$$t^2 + 2t - 3 = 0$$
$$(t+3)(t-1) = 0$$
$$t + 3 = 0 \quad \text{or} \quad t - 1 = 0$$
$$t = -3 \qquad\qquad t = 1$$

Substitute $x^{\frac{1}{6}}$ for t.

$$x^{\frac{1}{6}} = -3 \quad \text{or} \quad x^{\frac{1}{6}} = 1$$
$$\left(x^{\frac{1}{6}}\right)^6 = (-3)^6 \qquad \left(x^{\frac{1}{6}}\right)^6 = (1)^6$$
$$x = 729 \qquad\qquad x = 1$$

729 does not check and must be rejected.
The x-intercept is 1.
The corresponding graph is graph (e).

82. Set $y = 0$ to find the x–intercept(s).

$$0 = x^{-2} - x^{-1} - 6$$

Let $t = x^{-1}$.

$$x^{-2} - x^{-1} - 6 = 0$$
$$\left(x^{-1}\right)^2 - x^{-1} - 6 = 0$$
$$t^2 - t - 6 = 0$$
$$(t+2)(t-3) = 0$$
$$t + 2 = 0 \quad \text{or} \quad t - 3 = 0$$
$$t = -2 \qquad\qquad t = 3$$

Substitute x^{-1} for t.

$$x^{-1} = -2 \quad \text{or} \quad x^{-1} = 3$$
$$x = -\frac{1}{2} \qquad\qquad x = \frac{1}{3}$$

The x-intercepts are $-\frac{1}{2}$ and $\frac{1}{3}$.
The corresponding graph is graph (b).

83. Set $y = 0$ to find the x–intercept(s).

$$(x+2)^2 - 9(x+2) + 20 = 0$$

Let $t = x + 2$.

$$(x+2)^2 - 9(x+2) + 20 = 0$$
$$t^2 - 9t + 20 = 0$$
$$(t-5)(t-4) = 0$$
$$t - 5 = 0 \quad \text{or} \quad t - 4 = 0$$
$$t = 5 \qquad\qquad t = 4$$

Substitute $x + 2$ for t.

$$x + 2 = 5 \quad \text{or} \quad x + 2 = 4$$
$$x = 3 \qquad\qquad x = 2$$

The x-intercepts are 2 and 3.
The corresponding graph is graph (f).

84. Set $y = 0$ to find the x–intercept(s).

$0 = 2(x+2)^2 + 5(x+2) - 3$

Let $t = x+2$.

$2(x+2)^2 + 5(x+2) - 3 = 0$

$2t^2 + 5t - 3 = 0$

$(2t-1)(t+3) = 0$

$2t - 1 = 0$ or $t + 3 = 0$

$2t = 1$ $\quad t = -3$

$t = \frac{1}{2}$

Substitute $x+2$ for t.

$x + 2 = \frac{1}{2}$ or $x + 2 = -3$

$x = -5$

$x = \frac{1}{2} - 2$

$x = -\frac{3}{2}$

The x-intercepts are -5 and $-\frac{3}{2}$.

The corresponding graph is graph (d).

85. $|5-4x| = 11$

$5 - 4x = 11$ $\quad 5 - 4x = -11$

$-4x = 6$ or $-4x = -16$

$x = -\frac{3}{2}$ $\quad x = 4$

The solution set is $\left\{-\frac{3}{2}, 4\right\}$.

86. $|2-3x| = 13$

$2 - 3x = 13$ $\quad 2 - 3x = -13$

$-3x = 11$ or $-3x = -15$

$x = -\frac{11}{3}$ $\quad x = 5$

The solution set is $\left\{-\frac{11}{3}, 5\right\}$.

87. $x + \sqrt{x+5} = 7$

$\sqrt{x+5} = 7 - x$

$\left(\sqrt{x+5}\right)^2 = (7-x)^2$

$x + 5 = 49 - 14x + x^2$

$0 = x^2 - 15x + 44$

$0 = (x-4)(x-11)$

$x - 4 = 0$ or $x - 11 = 0$

$x = 4$ $\quad x = 11$

11 does not check and must be rejected.

The solution set is $\{4\}$.

88. $x - \sqrt{x-2} = 4$

$-\sqrt{x-2} = 4 - x$

$\left(-\sqrt{x-2}\right)^2 = (4-x)^2$

$x - 2 = 16 - 8x + x^2$

$0 = x^2 - 9x + 18$

$0 = (x-6)(x-3)$

$x - 6 = 0$ or $x - 3 = 0$

$x = 6$ $\quad x = 3$

3 does not check and must be rejected.

The solution set is $\{6\}$.

89. $2x^3 + x^2 - 8x + 2 = 6$

$2x^3 + x^2 - 8x - 4 = 0$

$x^2(2x+1) - 4(2x+1) = 0$

$(2x+1)(x^2-4) = 0$

$(2x+1)(x+2)(x-2) = 0$

$2x + 1 = 0$ or $x + 2 = 0$ or $x - 2 = 0$

$x = -\frac{1}{2}$ $\quad x = -2$ $\quad x = 2$

The solution set is $\left\{-\frac{1}{2}, -2, 2\right\}$.

90. $x^3 + 4x^2 - x + 6 = 10$

$x^3 + 4x^2 - x - 4 = 0$

$x^2(x+4) - 1(x+4) = 0$

$(x+4)(x^2-1) = 0$

$(x+4)(x+1)(x-1) = 0$

$x + 4 = 0$ or $x + 1 = 0$ or $x - 1 = 0$

$x = -4$ $\quad x = -1$ $\quad x = 1$

The solution set is $\{-4, -1, 1\}$.

91. $(x+4)^{\frac{3}{2}}=8$

$$\left((x+4)^{\frac{3}{2}}\right)^{\frac{2}{3}}=(8)^{\frac{2}{3}}$$
$$x+4=\left(\sqrt[3]{8}\right)^2$$
$$x+4=(2)^2$$
$$x+4=4$$
$$x=0$$

The solution set is $\{0\}$.

92. $(x-5)^{\frac{3}{2}}=125$

$$\left((x-5)^{\frac{3}{2}}\right)^{\frac{2}{3}}=(125)^{\frac{2}{3}}$$
$$x-5=\left(\sqrt[3]{125}\right)^2$$
$$x-5=(5)^2$$
$$x-5=25$$
$$x=30$$

The solution set is $\{30\}$.

93. $y_1 = y_2+3$

$$\left(x^2-1\right)^2=2\left(x^2-1\right)+3$$
$$\left(x^2-1\right)^2-2\left(x^2-1\right)-3=0$$

Let $t=x^2-1$ and substitute.

$$t^2-2t-3=0$$
$$(t+1)(t-3)=0$$
$t+1=0$ or $t-3=0$

$t=-1$ $\quad t=3$

Substitute x^2-1 for t.

$x^2-1=-1$ or $x^2-1=3$

$x^2=0$ $\quad x^2=4$

$x=0$ $\quad x=\pm 2$

The solution set is $\{-2,0,2\}$.

94. $y_1 = y_2+6$

$$6\left(\frac{2x}{x-3}\right)^2=5\left(\frac{2x}{x-3}\right)+6$$
$$6\left(\frac{2x}{x-3}\right)^2-5\left(\frac{2x}{x-3}\right)-6=0$$

Let $t=\frac{2x}{x-3}$ and substitute.

$$6t^2-5t-6=0$$
$$(3t+2)(2t-3)=0$$
$3t+2=0$ or $2t-3=0$

$t=-\frac{2}{3}$ $\quad t=\frac{3}{2}$

Substitute $\frac{2x}{x-3}$ for t.

$\frac{2x}{x-3}=-\frac{2}{3}$ or $\frac{2x}{x-3}=\frac{3}{2}$

First solve $\frac{2x}{x-3}=-\frac{2}{3}$

$$\frac{2x(3)(x-3)}{x-3}=-\frac{2(3)(x-3)}{3}$$
$$2x(3)=-2(x-3)$$
$$6x=-2x+6$$
$$8x=6$$
$$x=\frac{3}{4}$$

Next solve $\frac{2x}{x-3}=\frac{3}{2}$

$$\frac{2x(2)(x-3)}{x-3}=\frac{3(2)(x-3)}{2}$$
$$2x(2)=3(x-3)$$
$$4x=3x-9$$
$$x=-9$$

The solution set is $\left\{-9,\frac{3}{4}\right\}$.

95. $\left|x^2+2x-36\right|=12$

$x^2+2x-36=12$ $\quad x^2+2x-36=-12$

$x^2+2x-48=0$ or $x^2+2x-24=0$

$(x+8)(x-6)=0$ $\quad (x+6)(x-4)=0$

Setting each of the factors above equal to zero gives $x=-8,\ \ x=6,\ \ x=-6,$ and $x=4$.

The solution set is $\{-8,-6,4,6\}$.

96. $|x^2+6x+1|=8$

$x^2+6x+1=8$ or $x^2+6x+1=-8$

$x^2+6x-7=0$ $\quad$ $x^2+6x+9=0$

$(x+7)(x-1)=0$ $\quad$ $(x+3)(x+3)=0$

Setting each of the factors above equal to zero gives $x=-7,\quad x=-3,\quad$ and $x=1$.

The solution set is $\{-7,-3,1\}$.

97. $x(x+1)^3-42(x+1)^2=0$

$(x+1)^2(x(x+1)-42)=0$

$(x+1)^2(x^2+x-42)=0$

$(x+1)^2(x+7)(x-6)=0$

Setting each of the factors above equal to zero gives $x=-7,\quad x=-1,\quad$ and $x=6$.

The solution set is $\{-7,-1,6\}$.

98. $x(x-2)^3-35(x-2)^2=0$

$x(x-2)^3-35(x-2)^2=0$

$(x-2)^2(x(x-2)-35)=0$

$(x-2)^2(x^2-2x-35)=0$

$(x-2)^2(x+5)(x-7)=0$

Setting each of the factors above equal to zero gives $x=-5,\quad x=2,\quad$ and $x=7$.

The solution set is $\{-5,2,7\}$.

99. Let x = the number.

$\sqrt{5x-4}=x-2$

$\left(\sqrt{5x-4}\right)^2=(x-2)^2$

$5x-4=x^2-4x+4$

$0=x^2-9x+8$

$0=(x-8)(x-1)$

$x-8=0$ or $x-1=0$

$x=8$ $\quad$ $x=1$

Check $x=8$: $\sqrt{5(8)-4}=8-2$

$\sqrt{40-4}=6$

$\sqrt{36}=6$

$6=6$

Check $x=1$: $\sqrt{5(1)-4}=1-2$

$\sqrt{5-4}=-1$

$\sqrt{-1}\neq-1$

Discard $x=1$. The number is 8.

100. Let x = the number.

$\sqrt{x-3}=x-5$

$\left(\sqrt{x-3}\right)^2=(x-5)^2$

$x-3=x^2-10x+25$

$0=x^2-11x+28$

$0=(x-7)(x-4)$

$x-7=0$ or $x-4=0$

$x=7$ $\quad$ $x=4$

Check $x=7$: $\sqrt{7-3}=7-5$

$\sqrt{4}=2$

$2=2$

Check $x=4$: $\sqrt{4-3}=4-5$

$\sqrt{1}=-1$

$1\neq-1$

Discard 4. The number is 7.

101.

$r=\sqrt{\dfrac{3V}{\pi h}}$

$r^2=\left(\sqrt{\dfrac{3V}{\pi h}}\right)^2$

$r^2=\dfrac{3V}{\pi h}$

$\pi r^2h=3V$

$\dfrac{\pi r^2h}{3}=V$

$V=\dfrac{\pi r^2h}{3}$ or $V=\dfrac{1}{3}\pi r^2h$

102.
$$r = \sqrt{\frac{A}{4\pi}}$$
$$r^2 = \left(\sqrt{\frac{A}{4\pi}}\right)^2$$
$$r^2 = \frac{A}{4\pi}$$
$$4\pi r^2 = A \quad \text{or} \quad A = 4\pi r^2$$

103. Exclude any value that causes the denominator to equal zero.
$$|x+2| - 14 = 0$$
$$|x+2| = 14$$
$$x+2 = 14 \quad \text{or} \quad x+2 = -14$$
$$x = 12 \qquad\qquad x = -16$$
–16 and 12 must be excluded from the domain.

104. Exclude any value that causes the denominator to equal zero.
$$x^3 + 3x^2 - x - 3 = 0$$
$$x^2(x+3) - 1(x+3) = 0$$
$$(x+3)\left(x^2 - 1\right) = 0$$
$$(x+3)(x+1)(x-1) = 0$$
Setting each of the factors above equal to zero gives $x = -3,\quad x = -1,\quad \text{and } x = 1.$
–3, –1, and 1 must be excluded from the domain.

105.
$$t = \frac{\sqrt{d}}{2}$$
$$1.16 = \frac{\sqrt{d}}{2}$$
$$2.32 = \sqrt{d}$$
$$2.32^2 = \left(\sqrt{d}\right)^2$$
$$d \approx 5.4$$
The vertical distance was about 5.4 feet.

106.
$$t = \frac{\sqrt{d}}{2}$$
$$0.85 = \frac{\sqrt{d}}{2}$$
$$1.7 = \sqrt{d}$$
$$1.7^2 = \left(\sqrt{d}\right)^2$$
$$d \approx 2.9$$
The vertical distance was about 2.9 feet.

107. It is represented by the point $(5.4, 1.16)$.

108. It is represented by the point $(2.9, 0.85)$.

109.
$$J = 2.6\sqrt{x} + 35$$
$$50.6 = 2.6\sqrt{x} + 35$$
$$15.6 = 2.6\sqrt{x}$$
$$\frac{15.6}{2.6} = \sqrt{x}$$
$$\left(\frac{15.6}{2.6}\right)^2 = \left(\sqrt{x}\right)^2$$
$$36 = x$$
50.6% of full-time college students will have jobs 36 years after 1975, or 2011.

110.
$$J = 2.6\sqrt{x} + 35$$
$$53.2 = 2.6\sqrt{x} + 35$$
$$18.2 = 2.6\sqrt{x}$$
$$\frac{18.2}{2.6} = \sqrt{x}$$
$$\left(\frac{18.2}{2.6}\right)^2 = \left(\sqrt{x}\right)^2$$
$$49 = x$$
53.2% of full-time college students will have jobs 49 years after 1975, or 2024.

111. **a.** The solution is represented by the point (36, 50.6).

b. 51% and 52% are reasonable estimates.

112. **a.** The solution is represented by the point (49, 53.2).

b. 53% and 54% are reasonable estimates.

113.
$$365 = 0.2x^{3/2}$$
$$\frac{365}{0.2} = \frac{0.2x^{3/2}}{0.2}$$
$$1825 = x^{3/2}$$
$$1825^2 = \left(x^{3/2}\right)^2$$
$$3{,}330{,}625 = x^3$$
$$\sqrt[3]{3{,}330{,}625} = \sqrt[3]{x^3}$$
$$149.34 \approx x$$
The average distance of the Earth from the sun is approximately 149 million kilometers.

114.
$$f(x) = 0.2x^{3/2}$$
$$88 = 0.2x^{3/2}$$
$$\frac{88}{0.2} = \frac{0.2x^{3/2}}{0.2}$$
$$440 = x^{3/2}$$
$$440^2 = \left(x^{3/2}\right)^2$$
$$193{,}600 = x^3$$
$$\sqrt[3]{193{,}600} = \sqrt[3]{x^3}$$
$$58 \approx x$$
The average distance of Mercury from the sun is approximately 58 million kilometers.

115.
$$\sqrt{6^2 + x^2} + \sqrt{8^2 + (10-x)^2} = 18$$
$$\sqrt{36 + x^2} = 18 - \sqrt{64 + 100 - 20x + x^2}$$
$$36 + x^2 = 324 - 36\sqrt{x^2 - 20x + 164} + x^2 - 20x + 164$$
$$36\sqrt{x^2 - 20x + 164} = -20x + 452$$
$$9\sqrt{x^2 - 20x + 164} = -5x + 113$$
$$81(x^2 - 20x + 164) = 25x^2 - 1130x + 12769$$
$$81x^2 - 1620x + 13284 = 25x^2 - 1130x + 12769$$
$$56x^2 - 490x + 515 = 0$$
$$x = \frac{490 \pm \sqrt{(-490)^2 - 4(56)(515)}}{2(56)}$$
$$x = \frac{490 \pm 353.19}{112}$$
$$x \approx 1.2 \qquad x \approx 7.5$$
The point should be located approximately either 1.2 feet or 7.5 feet from the base of the 6-foot pole.

116. a. Distance from point $A = \sqrt{6^2 + x^2} + \sqrt{3^2 + (12-x)^2}$ or $A = \sqrt{x^2 + 36} + \sqrt{(12-x)^2 + 9}$.

b. Let the distance = 15.
$$\sqrt{6^2 + x^2} + \sqrt{3^2 + (12-x)^2} = 15$$
$$\sqrt{36 + x^2} = 15 - \sqrt{9 + 144 - 24x + x^2}$$
$$36 + x^2 = 225 - 30\sqrt{153 - 24x + x^2} + x^2 - 24x + 153$$
$$30\sqrt{x^2 - 24x + 153} = -24x + 342$$
$$5\sqrt{x^2 - 24x + 153} = -4x + 157$$
$$25(x^2 - 24x + 153) = 16x^2 - 456x + 3249$$
$$25x^2 - 600x + 3825 = 16x^2 - 456x + 3249$$
$$9x^2 - 144x + 576 = 0$$
$$x^2 - 16x + 64 = 0$$
$$(x-8)(x-8) = 0$$
$$x = 8$$
The distance is 8 miles.

117. – 123. Answers may vary.

124. $x^3+3x^2-x-3=0$
The solution set is {–3, –1, 1}.
$(-3)^3+3(-3)^2-(-3)-3=0$
$-27+27+3-3=0$
$(-1)^3+3(-1)^2-(-1)-3=0$
$-1+3+1-3=0$
$1^3+3(1)^2-(1)-3=0$
$1+3-1-3=0$

125. $-x^4+4x^3-4x^2=0$
The solution set is {0, 2}.
$-(0)^4+4(0)^3-4(0)^2=0$
$0=0$
$-(2)^4+4(2)^3-4(2)^2=0$
$-16+32-16=0$
$0=0$

126. $\sqrt{2x+13}-x-5=0$
The solution set is {–2}.
$\sqrt{2(-2)+13}-(-2)-5=0$
$\sqrt{-4+13}+2-5=0$
$\sqrt{9}-3=0$
$3-3=0$

127. does not make sense; Explanations will vary. Sample explanation: You should substitute into the original equation.

128. makes sense

129. does not make sense; Explanations will vary. Sample explanation: Changing the order of the terms does not change the fact that this equation is quadratic in form.

130. makes sense

131. false; Changes to make the statement true will vary. A sample change is: Squaring $x+2$ results in x^2+4x+4.

132. false; Changes to make the statement true will vary. A sample change is: 21 satisfies the linear equation but not the radical equation.

133. false; Changes to make the statement true will vary. A sample change is: To solve the equation, let $u^2=x$.

134. false; Changes to make the statement true will vary. A sample change is: Neither 6 nor –6 satisfies the absolute value equation.

135. $\sqrt{6x-2}=\sqrt{2x+3}-\sqrt{4x-1}$
$6x-2=2x+3-2\sqrt{(2x+3)(4x-1)}+4x-1$
$-4=-2\sqrt{(2x+3)(4x-1)}$
$2=\sqrt{8x^2+10x-3}$
$4=8x^2+10x-3$
$8x^2+10x-7=0$
$x=\dfrac{-10\pm\sqrt{10^2-4(8)(-7)}}{2(8)}$
$x=\dfrac{-10\pm\sqrt{100+224}}{16}$
$x=\dfrac{-10\pm\sqrt{324}}{16}$
$x=\dfrac{-10\pm18}{16}$
$x=\dfrac{-28}{26},\dfrac{8}{16}$
$x=\dfrac{1}{2}$
The solution set is $\left\{\dfrac{1}{2}\right\}$.

136. $5-\dfrac{2}{x}=\sqrt{5-\dfrac{2}{x}}$
or
$5-\dfrac{2}{x}=0$ $\qquad 5-\dfrac{2}{x}=1$
$5=\dfrac{2}{x}$ $\qquad -\dfrac{2}{x}=-4$
$5x=2$ $\qquad -4x=-2$
$x=\dfrac{2}{5}$ $\qquad x=\dfrac{1}{2}$
The solution set is $\left\{\dfrac{2}{5},\dfrac{1}{2}\right\}$.

137. $\sqrt[3]{x\sqrt{x}} = 9$

$\sqrt[3]{x\sqrt{x}} = 9$

$\sqrt[3]{x^1 x^{\frac{1}{2}}} = 9$

$\left(x^1 x^{\frac{1}{2}}\right)^{\frac{1}{3}} = 9$

$\left(x^{\frac{3}{2}}\right)^{\frac{1}{3}} = 9$

$x^{\frac{1}{2}} = 9$

$\left(x^{\frac{1}{2}}\right)^2 = (9)^2$

$x = 81$

The solution set is {81}.

138. $x^{5/6} + x^{2/3} - 2x^{1/2} = 0$

$x^{1/2}(x^{2/6} + x^{1/6} - 2) = 0$ let $t = x^{1/6}$

$x^{1/2}(t^2 + t - 2) = 0$

$x^{1/2} = 0 \quad t^2 + t - 2 = 0$

$(t-1)(t+2) = 0$

$t - 1 = 0 \qquad t + 2 = 0$

$t = 1 \qquad t = -2$

$x^{1/6} = 1 \qquad x^{1/6} = -2$

$x = 1^6 \qquad x = (-2)^6$

$x = 0 \qquad x = 1 \qquad x = 64$

64 does not check and must be rejected.
The solution set is {0, 1}.

139. $3 - 2x \le 11$

$3 - 2(-1) \le 11$

$3 + 2 \le 11$

$5 \le 11$, true

–1 is a solution.

140. $-2x - 4 = x + 5$

$-2x - x = 5 + 4$

$-3x = 9$

$x = \frac{9}{-3}$

$x = -3$

The solution set is {–3}.

141. $\frac{x+3}{4} = \frac{x-2}{3} + \frac{1}{4}$

$12\left(\frac{x+3}{4}\right) = 12\left(\frac{x-2}{3} + \frac{1}{4}\right)$

$3(x+3) = 4(x-2) + 3$

$3x + 9 = 4x - 8 + 3$

$3x + 9 = 4x - 5$

$3x - 4x = -5 - 9$

$-x = -14$

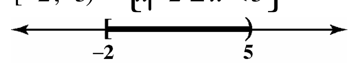
$x = 14$

The solution set is {14}.

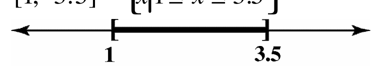
Section 1.7

Check Point Exercises

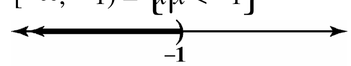
1. **a.** $[-2, 5) = \{x \mid -2 \le x < 5\}$

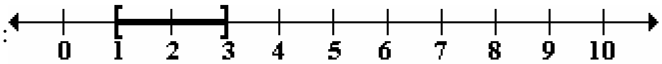


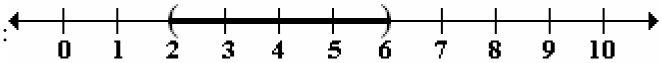
b. $[1, 3.5] = \{x \mid 1 \le x \le 3.5\}$



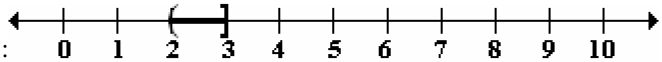
c. $[-\infty, -1) = \{x \mid x < -1\}$



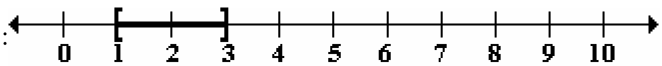
2. **a.** Graph $[1,3]$: 

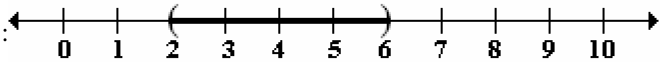
Graph $(2,6)$: 

To find the intersection, take the portion of the number line that the two graphs have in common.

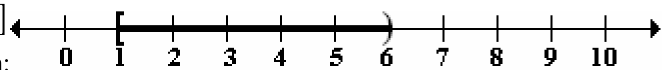
Numbers in both $[1,3]$ and $(2,6)$: 

Thus, $[1,3] \cap (2,6) = (2,3]$.

b. Graph $[1,3]$: 

Graph $(2,6)$: 

To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

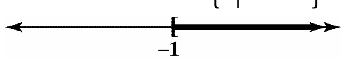
Numbers in either $[1,3]$ or $(2,6)$ or both: 

Thus, $[1,3] \cup (2,6) = [1,6)$.

3.

$$2 - 3x \le 5$$
$$-3x \le 3$$
$$x \ge -1$$

The solution set is $\{x | x \ge -1\}$ or $[-1, \infty)$.



4.

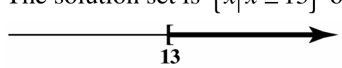
$$3x + 1 > 7x - 15$$
$$-4x > -16$$
$$\frac{-4x}{-4} < \frac{-16}{-4}$$
$$x < 4$$

The solution set is $\{x | x < 4\}$ or $(-\infty, 4]$.

5.

$$\frac{x-4}{2} \ge \frac{x-2}{3} + \frac{5}{6}$$
$$6\left(\frac{x-4}{2}\right) \ge 6\left(\frac{x-2}{3} + \frac{5}{6}\right)$$
$$3(x-4) \ge 2(x-2) + 5$$
$$3x - 12 \ge 2x - 4 + 5$$
$$3x - 12 \ge 2x + 1$$
$$3x - 2x \ge 1 + 12$$
$$x \ge 13$$

The solution set is $\{x | x \ge 13\}$ or $[13, \infty)$.



6. **a.** $3(x+1) > 3x+2$

$3x+3 > 3x+2$

$3 > 2$

$3 > 2$ is true for all values of *x*.

The solution set is $\{x \mid x \text{ is a real number}\}$.

b. $x+1 \le x-1$

$1 \le -1$

$1 \le -1$ is false for all values of *x*.

The solution set is $\varnothing$.

7. $1 \le 2x+3 < 11$

$-2 \le 2x < 8$

$-1 \le x < 4$

The solution set is $\{x \mid -1 \le x < 4\}$ or $[-1, 4)$.

8. $|x-2| < 5$

$-5 < x-2 < 5$

$-3 < x < 7$

The solution set is $\{x \mid -3 < x < 7\}$ or $(-3, 7)$.

9. $-3|5x-2| + 20 \ge -19$

$-3|5x-2| \ge -39$

$\frac{-3|5x-2|}{-3} \le \frac{-39}{-3}$

$|5x-2| \le 13$

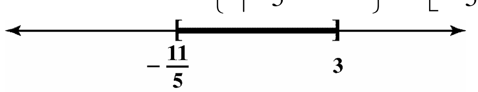
$-13 \le 5x-2 \le 13$

$-11 \le 5x \le 15$

$\frac{-11}{5} \le \frac{5x}{5} \le \frac{15}{5}$

$-\frac{11}{5} \le x \le 3$

The solution set is $\left\{x \,\middle|\, -\frac{11}{5} \le x \le 3\right\}$ or $\left[-\frac{11}{5}, 3\right]$.



10. $18 < |6 - 3x|$

$$\begin{array}{lcl} 6 - 3x < -18 & \text{or} & 6 - 3x > 18 \\ -3x < -24 & & -3x > 12 \\ \dfrac{-3x}{-3} > \dfrac{-24}{-3} & & \dfrac{-3x}{-3} < \dfrac{12}{-3} \\ x > 8 & & x < -4 \end{array}$$

The solution set is $\{x \mid x < -4 \text{ or } x > 8\}$

or $(-\infty, -4) \cup (8, \infty)$.

11. Let x = the number of miles driven in a week.

$260 < 80 + 0.25x$

$180 < 0.25x$

$720 < x$

Driving more than 720 miles in a week makes Basic the better deal.

Exercise Set 1.7

1. $1 < x \le 6$

2. $-2 < x \le 4$

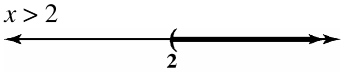
3. $-5 \le x < 2$

4. $-4 \le x < 3$

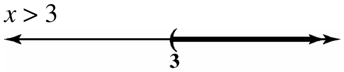
5. $-3 \le x \le 1$

6. $-2 \le x \le 5$

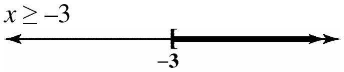
7. $x > 2$



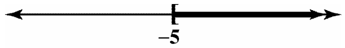
8. $x > 3$



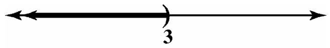
9. $x \ge -3$



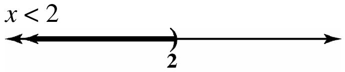
10. $x \geq -5$



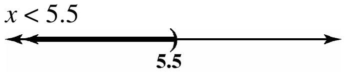
11. $x < 3$



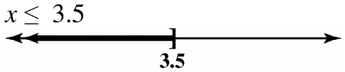
12. $x < 2$



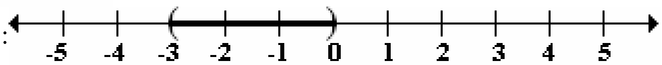
13. $x < 5.5$



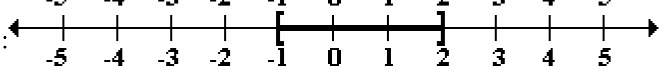
14. $x \leq 3.5$



15. Graph $(-3,0)$:

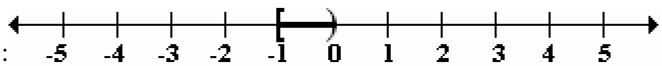


Graph $[-1,2]$:



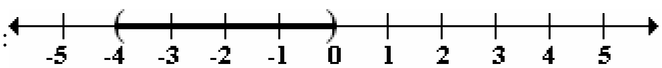
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $(-3,0)$ and $[-1,2]$:

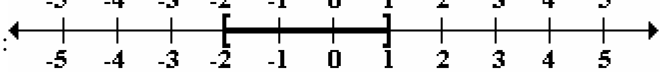


Thus, $(-3,0) \cap [-1,2] = [-1,0)$.

16. Graph $(-4,0)$:

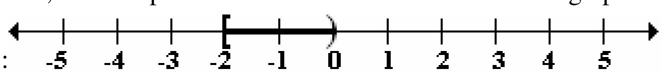


Graph $[-2,1]$:



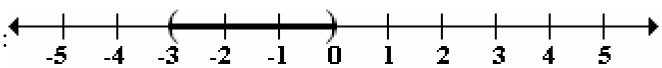
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $(-4,0)$ and $[-2,1]$:

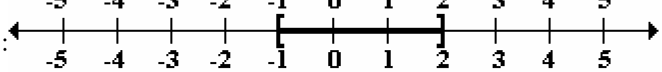


Thus, $(-4,0) \cap [-2,1] = [-2,0)$.

17. Graph $(-3,0)$:

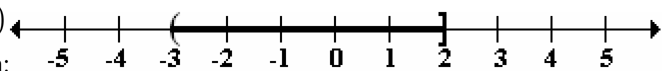


Graph $[-1,2]$:



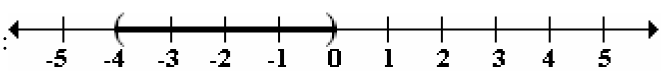
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $(-3,0)$ or $[-1,2]$ or both:

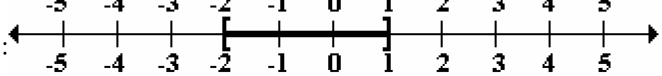


Thus, $(-3,0) \cup [-1,2] = (-3,2]$.

18. Graph $(-4,0)$:

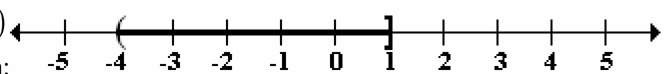


Graph $[-2,1]$:



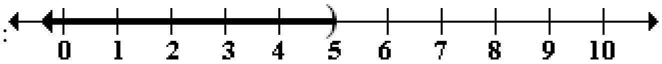
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $(-4,0)$ or $[-2,1]$ or both:

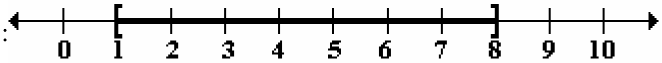


Thus, $(-4,0) \cup [-2,1] = (-4,1]$.

19. Graph $(-\infty,5)$:

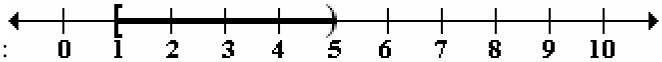


Graph $[1,8]$:



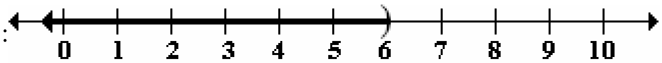
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $(-\infty,5)$ and $[1,8]$:

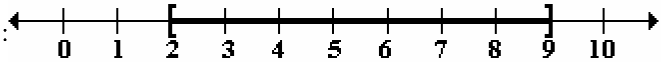


Thus, $(-\infty,5) \cap [1,8] = [1,5)$.

20. Graph $(-\infty,6)$:

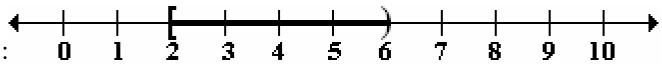


Graph $[2,9]$:



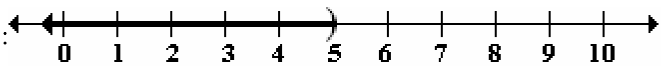
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $(-\infty,6)$ and $[2,9]$:

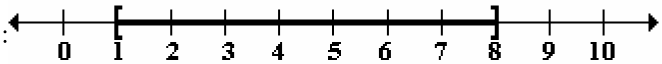


Thus, $(-\infty,6) \cap [2,9] = [2,6)$.

21. Graph $(-\infty,5)$:

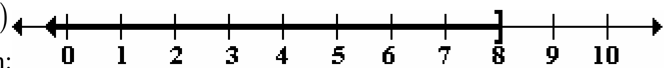


Graph $[1,8]$:



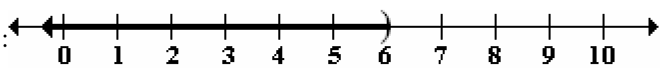
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $(-\infty,5)$ or $[1,8]$ or both:

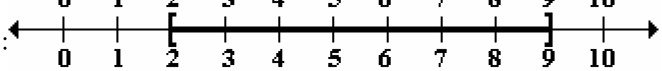


Thus, $(-\infty,5) \cup [1,8] = (-\infty,8]$.

22. Graph $(-\infty,6)$:

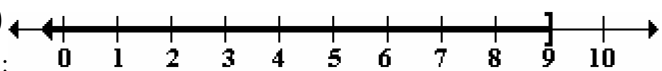


Graph $[2,9]$:



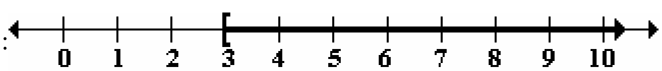
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $(-\infty,6)$ or $[2,9]$ or both:

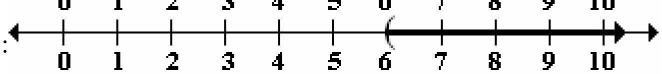


Thus, $(-\infty,6) \cup [2,9] = (-\infty,9]$.

23. Graph $[3,\infty)$:

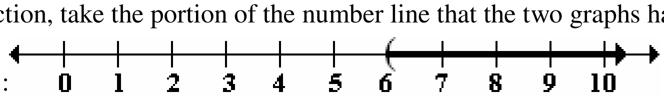


Graph $(6,\infty)$:



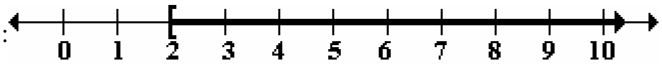
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $[3,\infty)$ and $(6,\infty)$:

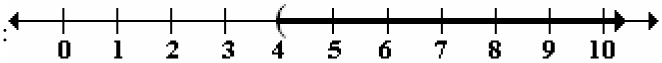


Thus, $[3,\infty) \cap (6,\infty) = (6,\infty)$.

24. Graph $[2,\infty)$:

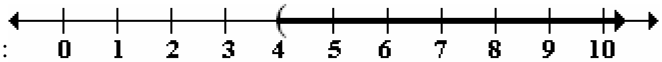


Graph $(4,\infty)$:



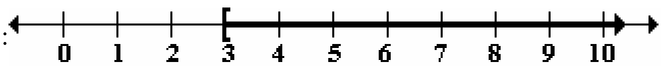
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $[2,\infty)$ and $(4,\infty)$:

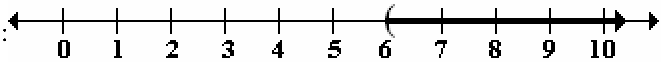


Thus, $[2,\infty) \cap (4,\infty) = (4,\infty)$.

25. Graph $[3,\infty)$:

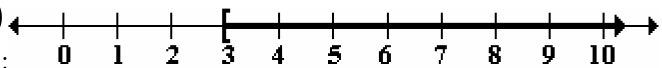


Graph $(6,\infty)$:



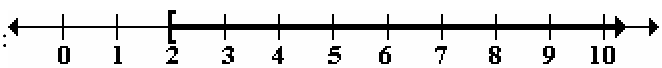
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $[3,\infty)$ or $(6,\infty)$ or both:

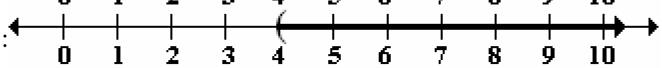


Thus, $[3,\infty) \cup (6,\infty) = [3,\infty)$.

26. Graph $[2,\infty)$:

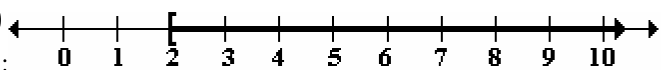


Graph $(4,\infty)$:



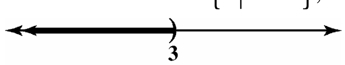
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $[2,\infty)$ or $(4,\infty)$ or both:

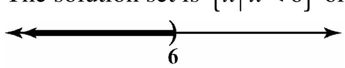


Thus, $[2,\infty) \cup (4,\infty) = [2,\infty)$.

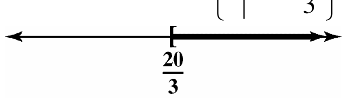
27. $5x + 11 < 26$
$5x < 15$
$x < 3$
The solution set is $\{x \mid x < 3\}$, or $(-\infty, 3)$.



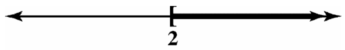
28. $2x + 5 < 17$
$2x < 12$
$x < 6$
The solution set is $\{x \mid x < 6\}$ or $(-\infty, 6)$.



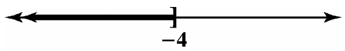
29. $3x - 7 \geq 13$
$3x \geq 20$
$x \geq \frac{20}{3}$
The solution set is $\left\{x \,\middle|\, x > \frac{20}{3}\right\}$, or $\left[\frac{20}{3}, \infty\right)$.



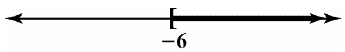
30. $8x - 2 \geq 14$
$8x \geq 16$
$x \geq 2$
The solution set is $\{x \mid x > 2\}$ or $[2, \infty)$.



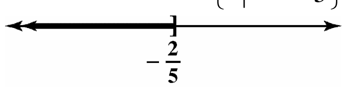
31. $-9x \geq 36$
$x \leq -4$
The solution set is $\{x \mid x \leq -4\}$, or $(-\infty, -4]$.



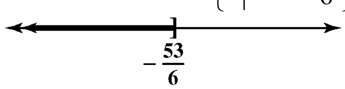
32. $-5x \leq 30$
$x \geq -6$
The solution set is $\{x \mid x \geq -6\}$ or $[-6, \infty)$.



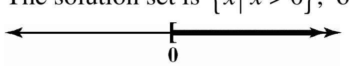
33. $8x - 11 \leq 3x - 13$
$8x - 3x \leq -13 + 11$
$5x \leq -2$
$x \leq -\dfrac{2}{5}$
The solution set is $\left\{x \,\middle|\, x \leq -\dfrac{2}{5}\right\}$, or $\left(-\infty, -\dfrac{2}{5}\right]$.



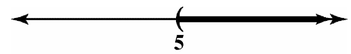
34. $18x + 45 \leq 12x - 8$
$18x - 12x \leq -8 - 45$
$6x \leq -53$
$x \leq -\dfrac{53}{6}$
The solution set is $\left\{x \,\middle|\, x \leq -\dfrac{53}{6}\right\}$ or $\left(-\infty, -\dfrac{53}{6}\right]$.



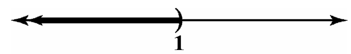
35. $4(x + 1) + 2 \geq 3x + 6$
$4x + 4 + 2 \geq 3x + 6$
$4x + 6 \geq 3x + 6$
$4x - 3x \geq 6 - 6$
$x \geq 0$
The solution set is $\{x \mid x > 0\}$, or $[0, \infty)$.



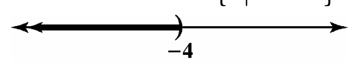
36. $8x + 3 > 3(2x + 1) + x + 5$
$8x + 3 > 6x + 3 + x + 5$
$8x + 3 > 7x + 8$
$8x - 7x > 8 - 3$
$x > 5$
The solution set is $\{x \mid x > 5\}$ or $(5, \infty)$.



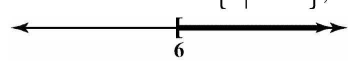
37. $2x - 11 < -3(x + 2)$
$2x - 11 < -3x - 6$
$5x < 5$
$x < 1$
The solution set is $\{x \mid x < 1\}$, or $(-\infty, 1)$.



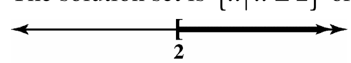
38. $-4(x + 2) > 3x + 20$
$-4x - 8 > 3x + 20$
$-7x > 28$
$x < -4$
The solution set is $\{x \mid x < -4\}$ or $(-\infty, -4)$.



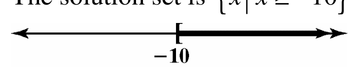
39. $1 - (x + 3) \geq 4 - 2x$
$1 - x - 3 \geq 4 - 2x$
$-x - 2 \geq 4 - 2x$
$x \geq 6$
The solution set is $\{x \mid x \geq 6\}$, or $[6, \infty)$.



40. $5(3 - x) \leq 3x - 1$
$15 - 5x \leq 3x - 1$
$-8x \leq -16$
$x \geq 2$
The solution set is $\{x \mid x \geq 2\}$ or $[2, \infty)$.



41. $\dfrac{x}{4} - \dfrac{3}{2} \leq \dfrac{x}{2} + 1$
$\dfrac{4x}{4} - \dfrac{4 \cdot 3}{2} \leq \dfrac{4 \cdot x}{2} + 4 \cdot 1$
$x - 6 \leq 2x + 4$
$-x \leq 10$
$x \geq -10$
The solution set is $\{x \mid x \geq -10\}$, or $[-10, \infty)$.



42. $\frac{3x}{10}+1\geq\frac{1}{5}-\frac{x}{10}$

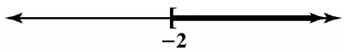
$10\left(\frac{3x}{10}+1\right)\geq10\left(\frac{1}{5}-\frac{x}{10}\right)$

$3x+10\geq2-x$

$4x\geq-8$

$x\geq-2$

The solution set is $\{x|\,x\geq-2\}$ or $[-2,\infty)$.

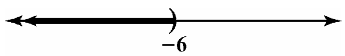


43. $1-\frac{x}{2}>4$

$-\frac{x}{2}>3$

$x<-6$

The solution set is $\{x|\,x,-6\}$, or $(-\infty,-6)$.

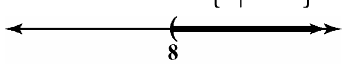


44. $7-\frac{4}{5}x<\frac{3}{5}$

$-\frac{4}{5}x<-\frac{32}{5}$

$x>8$

The solution set is $\{x|\,x>8\}$ or $(8,\infty)$.



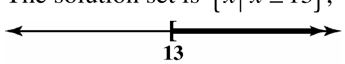
45. $\frac{x-4}{6}\geq\frac{x-2}{9}+\frac{5}{18}$

$3(x-4)\geq2(x-2)+5$

$3x-12\geq2x-4+5$

$x\geq13$

The solution set is $\{x|\,x\geq13\}$, or $[13,\infty)$.



46.

$\frac{4x-3}{6}+2\geq\frac{2x-1}{12}$

$2(4x-3)+24\geq2x-1$

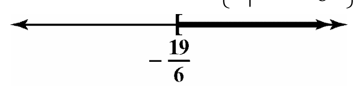
$8x-6+24\geq2x-1$

$6x+18\geq-1$

$6x\geq-19$

$x\geq-\frac{19}{6}$

The solution set is $\left\{x\,\middle|\,x\geq\frac{-19}{6}\right\}$ or $\left[\frac{-19}{6},\infty\right)$.



47. $4(3x-2)-3x<3(1+3x)-7$

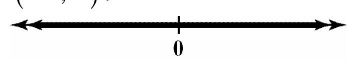
$12x-8-3x<3+9x-7$

$9x-8<-4+9x$

$-8<-4$

True for all x

The solution set is $\{x|\,x \text{ is any real number}\}$, or $(-\infty,\infty)$.



48. $3(x-8)-2(10-x)>5(x-1)$

$3x-24-20+2x>5x-5$

$5x-44>5x-5$

$-44>-5$

Not true for any x.

The solution set is the empty set, $\varnothing$.

49. $5(x-2)-3(x+4)\geq2x-20$

$5x-10-3x-12\geq2x-20$

$2x-22\geq2x-20$

$-22\geq-20$

Not true for any x.

The solution set is the empty set, $\varnothing$.

50. $6(x-1)-(4-x)\geq7x-8$

$6x-6-4+x\geq7x-8$

$7x-10\geq7x-8$

$-10\geq-8$

Not true for any x.

The solution set is the empty set, $\varnothing$.

51. $6<x+3<8$

$6-3<x+3-3<8-3$

$3<x<5$

The solution set is $\{x|\,3<x<5\}$, or $(3, 5)$.

52. $7 < x + 5 < 11$
$7 - 5 < x + 5 - 5 < 11 - 5$
$2 < x < 6$
The solution set is $\{x \mid 2 < x < 6\}$ or (2, 6).

53. $-3 \le x - 2 < 1$
$-1 \le x < 3$
The solution set is $\{x \mid -1 \le x < 3\}$, or [–1, 3).

54. $-6 < x - 4 \le 1$
$-2 < x \le 5$
The solution set is $\{x \mid -2 < x \le 5\}$ or (–2, 5].

55. $-11 < 2x - 1 \le -5$
$-10 < 2x \le -4$
$-5 < x \le -2$
The solution set is $\{x \mid -5 < x \le -2\}$, or (–5, –2].

56. $3 \le 4x - 3 < 19$
$6 \le 4x < 22$
$\frac{6}{4} \le x < \frac{22}{4}$
$\frac{3}{2} \le x < \frac{11}{2}$
The solution set is $\left\{x \,\middle|\, \frac{3}{2} \le x < \frac{11}{2}\right\}$ or $\left[\frac{3}{2}, \frac{11}{2}\right)$.

57. $-3 \le \frac{2}{3}x - 5 < -1$
$2 \le \frac{2}{3}x < 4$
$3 \le x < 6$
The solution set is $\{x \mid 3 \le x < 6\}$, or [3, 6).

58. $-6 \le \frac{1}{2}x - 4 < -3$
$-2 \le \frac{1}{2}x < 1$
$-4 \le x < 2$
The solution set is $\{x \mid -4 \ge x < 2\}$ or $[-4, 2)$.

59. $|x| < 3$
$-3 < x < 3$
The solution set is $\{x \mid -3 < x < 3\}$, or (–3, 3).

60. $|x| < 5$
$-5 < x < 5$
The solution set is $\{x \mid -5 < x < 5\}$ or (–5, 5).

61. $|x - 1| \le 2$
$-2 \le x - 1 \le 2$
$-1 \le x \le 3$
The solution set is $\{x \mid -1 \le x \le 3\}$, or [–1, 3].

62. $|x + 3| \le 4$
$-4 \le x + 3 \le 4$
$-7 \le x \le 1$
The solution set is $\{x \mid -7 \le x \le 1\}$ or [–7, 1].

63. $|2x - 6| < 8$
$-8 < 2x - 6 < 8$
$-2 < 2x < 14$
$-1 < x < 7$
The solution set is $\{x \mid -1 < x < 7\}$, or (–1, 7).

64. $|3x + 5| < 17$
$-17 < 3x + 5 < 17$
$-22 < 3x < 12$
The solution set is $\left\{x \,\middle|\, -\frac{22}{3} < x < 4\right\}$ or $\left(-\frac{22}{3}, 4\right)$.

65. $|2(x - 1) + 4| \le 8$
$-8 \le 2(x - 1) + 4 \le 8$
$-8 \le 2x - 2 + 4 \le 8$
$-8 \le 2x + 2 \le 8$
$-10 \le 2x \le 6$
$-5 \le x \le 3$
The solution set is $\{x \mid -5 \le x \le 3\}$, or [–5, 3].

66. $|3(x - 1) + 2| \le 20$
$-20 \le 3(x - 1) + 2 \le 20$
$-20 \le 3x - 1 \le 20$
$-19 \le 3x \le 21$
$-\frac{19}{3} \le x \le 7$
The solution set is $\left\{x \,\middle|\, -\frac{19}{3} \le x \le 7\right\}$ or $\left[-\frac{19}{3}, 7\right]$.

67. $\left|\frac{2y + 6}{3}\right| < 2$
$-2 < \frac{2y + 6}{3} < 2$
$-6 < 2y + 6 < 6$
$-12 < 2y < 0$
$-6 < y < 0$
The solution set is $\{x \mid -6 < y < 0\}$, or (–6, 0).

68. $\left|\frac{3(x-1)}{4}\right| < 6$

$-6 < \frac{3(x-1)}{4} < 6$

$-24 < 3x - 3 < 24$

$-21 < 3x < 27$

$-7 < x < 9$

The solution set is $\{x \mid -7 < x < 9\}$ or (–7, 9).

69. $|x| > 3$

$x > 3$ or $x < -3$

The solution set is $\{x \mid x > 3 \text{ or } x < -3\}$, that is, $(-\infty, -3)$ or $(3, \infty)$.

70. $|x| > 5$

$x > 5$ or $x < -5$

The solution set is $\{x \mid x < -5 \text{ or } x > 5\}$, that is, all x in $(-\infty, -5)$ or $(5, \infty)$.

71. $|x - 1| \ge 2$

$x - 1 \ge 2$ or $x - 1 \le -2$

$x \ge 3$ $\quad$ $x \le -1$

The solution set is $\{x \mid x \le -1 \text{ or } x \ge 3\}$, that is, $(-\infty, -1]$ or $[3, \infty)$.

72. $|x + 3| \ge 4$

$x + 3 \ge 4$ or $x + 3 \le -4$

$x \ge 1$ $\quad$ $x \le -7$

The solution set is $\{x \mid x \le -7 \text{ or } x \ge 1\}$ that is, $(-\infty, -7)$ or $(1, \infty)$.

73. $|3x - 8| > 7$

$3x - 8 > 7$ or $3x - 8 < -7$

$3x > 15$ $\quad$ $3x < 1$

$x > 5$ $\quad$ $x < \frac{1}{3}$

The solution set is $\left\{x \,\middle|\, x < \frac{1}{3} \text{ or } x > 5\right\}$, that is, $\left(-\infty, \frac{1}{3}\right)$ or $(5, \infty)$.

74. $|5x - 2| > 13$

$5x - 2 > 13$ or $5x - 2 < -13$

$5x > 15$ $\quad$ $5x < -11$

$x > 3$ $\quad$ $x < -\frac{11}{5}$

The solution set is $\left\{x \,\middle|\, x < \frac{-11}{5} \text{ or } x > 3\right\}$, that is, all x in $\left(-\infty, \frac{-11}{5}\right)$ or $(3, \infty)$

75. $\left|\frac{2x+2}{4}\right| \ge 2$

$\frac{2x+2}{4} \ge 2$ or $\frac{2x+2}{4} \le -2$

$2x + 2 \ge 8$ $\quad$ $2x + 2 \le -8$

$2x \ge 6$ $\quad$ $2x \le -10$

$x \ge 3$ $\quad$ $x \le -5$

The solution set is $\{x \mid x \le -5 \text{ or } x \ge 3\}$, that is, $(-\infty, -5]$ or $[3, \infty)$.

76. $\left|\frac{3x-3}{9}\right| \ge 1$

$\frac{3x-3}{9} \ge 1$ or $\frac{3x-3}{9} \le -1$

$3x - 3 \ge 9$ $\quad$ $3x - 3 \le -9$

$3x \ge 12$ $\quad$ $3x \le -6$

$x \ge 4$ $\quad$ $x \le -2$

The solution set is $\{x \mid x \le -2 \text{ or } x \ge 4\}$, or $(-\infty, -2]$ or $[4, \infty)$.

77. $\left|3 - \frac{2}{3}x\right| > 5$

$3 - \frac{2}{3}x > 5$ or $3 - \frac{2}{3}x < -5$

$-\frac{2}{3}x > 2$ $\quad$ $-\frac{2}{3}x < -8$

$x < -3$ $\quad$ $x > 12$

The solution set is $\{x \mid x < -3 \text{ or } x > 12\}$, that is, $(-\infty, -3)$ or $(12, \infty)$.

78. $\left|3-\frac{3}{4}x\right|>9$

$3-\frac{3}{4}x>9$ or $3-\frac{3}{4}x<-9$

$-\frac{3}{4}x>6$ $\quad -\frac{3}{4}x<-12$

$x<-8$ $\quad x>16$

$\{x \mid x<-8 \text{ or } x>16\}$, that is all x in

$(-\infty,-8)$ or $(16,\infty)$.

79. $3|x-1|+2 \ge 8$

$3|x-1| \ge 6$

$|x-1| \ge 2$

$x-1 \ge 2$ or $x-1 \le -2$

$x \ge 3$ $\quad x \le -1$

The solution set is $\{x \mid x \le 1 \text{ or } x \ge 3\}$, that is,

$(-\infty,-1]$ or $[3,\infty)$.

80. $5|2x+1|-3 \ge 9$

$5|2x+1| \ge 12$

$|2x+1| \ge \frac{12}{5}$

$2x+1 \ge \frac{12}{5}$ $\quad 2x+1 \le -\frac{12}{5}$

$2x \ge \frac{7}{5}$ or $2x \le -\frac{17}{5}$

$x \ge \frac{7}{10}$ $\quad x \le -\frac{17}{10}$

The solution set is $\left\{x \,\middle|\, x \le -\frac{17}{10} \text{ or } x \ge \frac{7}{10}\right\}$.

81. $-2|x-4| \ge -4$

$\frac{-2|x-4|}{-2} \le \frac{-4}{-2}$

$|x-4| \le 2$

$-2 \le x-4 \le 2$

$2 \le x \le 6$

The solution set is $\{x \mid 2 \le x \le 6\}$.

82. $-3|x+7| \ge -27$

$\frac{-3|x+7|}{-3} \le \frac{-27}{-3}$

$|x+7| \le 9$

$-9 \le x+7 \le 9$

$-16 \le x \le 2$

The solution set is $\{x \mid -16 \le x \le 2\}$.

83. $-4|1-x| < -16$

$\frac{-4|1-x|}{-4} > \frac{-16}{-4}$

$|1-x| > 4$

$1-x>4$ $\quad 1-x<-4$

$-x>3$ or $-x<-5$

$x<-3$ $\quad x>5$

The solution set is $\{x \mid x<-3 \text{ or } x>5\}$.

84. $-2|5-x| < -6$

$-2|5-x| < -6$

$\frac{-2|5-x|}{-2} > \frac{-6}{-2}$

$|5-x| > 3$

$5-x>3$ $\quad 5-x<-3$

$-x>-2$ or $-x<-8$

$x<2$ $\quad x>8$

The solution set is $\{x \mid x<2 \text{ or } x>8\}$.

85. $3 \le |2x-1|$

$2x-1 \ge 3$ $\quad 2x-1 \le -3$

$2x \ge 4$ or $2x \le -2$

$x \ge 2$ $\quad x \le -1$

The solution set is $\{x \mid x \le -1 \text{ or } x \ge 2\}$.

86. $9 \le |4x+7|$

$4x+7 \ge 9$ or $4x+7 \le -9$

$4x \ge 2$ $\quad 4x \le -16$

$x \ge \frac{2}{4}$ $\quad x \le -4$

$x \ge \frac{1}{2}$

The solution set is $\left\{x \,\middle|\, x \le -4 \text{ or } x \ge \frac{1}{2}\right\}$.

87. $5 > |4-x|$ is equivalent to $|4-x| < 5$.

$-5 < 4 - x < 5$

$-9 < -x < 1$

$\frac{-9}{-1} > \frac{-x}{-1} > \frac{1}{-1}$

$9 > x > -1$

$-1 < x < 9$

The solution set is $\{x | -1 < x < 9\}$.

88. $2 > |11-x|$ is equivalent to $|11-x| < 2$.

$-2 < 11 - x < 2$

$-13 < -x < -9$

$\frac{-13}{-1} > \frac{-x}{-1} > \frac{-9}{-1}$

$13 > x > 9$

$9 < x < 13$

The solution set is $\{x | 9 < x < 13\}$.

89. $1 < |2-3x|$ is equivalent to $|2-3x| > 1$.

$2 - 3x > 1$ or $2 - 3x < -1$

$-3x > -1$ $-3x < -3$

$\frac{-3x}{-3} < \frac{-1}{-3}$ $\frac{-3x}{-3} > \frac{-3}{-3}$

$x < \frac{1}{3}$ $x > 1$

The solution set is $\left\{x \middle| x < \frac{1}{3} \text{ or } x > 1\right\}$.

90. $4 < |2-x|$ is equivalent to $|2-x| > 4$.

$2 - x > 4$ or $2 - x < -4$

$-x > 2$ $-x < -6$

$\frac{-x}{-1} < \frac{2}{-1}$ $\frac{-x}{-1} > \frac{-6}{-1}$

$x < -2$ $x > 6$

The solution set is $\{x | x < -2 \text{ or } x > 6\}$.

91. $12 < \left|-2x + \frac{6}{7}\right| + \frac{3}{7}$

$\frac{81}{7} < \left|-2x + \frac{6}{7}\right|$

$-2x + \frac{6}{7} > \frac{81}{7}$ or $-2x + \frac{6}{7} < -\frac{81}{7}$

$-2x > \frac{75}{7}$ $-2x < -\frac{87}{7}$

$x < -\frac{75}{14}$ $x > \frac{87}{14}$

The solution set is $\left\{x \middle| x < -\frac{75}{14} \text{ or } x > \frac{87}{14}\right\}$, that is,

$\left(-\infty, -\frac{75}{14}\right)$ or $\left(\frac{87}{14}, \infty\right)$.

92. $1 < \left|x - \frac{11}{3}\right| + \frac{7}{3}$

$-\frac{4}{3} < \left|x - \frac{11}{3}\right|$

Since $\left|x - \frac{11}{3}\right| > -\frac{4}{3}$ is true for all x,

the solution set is $\{x | x \text{ is any real number}\}$

or $(-\infty, \infty)$.

93. $4 + \left|3 - \frac{x}{3}\right| \ge 9$

$\left|3 - \frac{x}{3}\right| \ge 5$

$3 - \frac{x}{3} \ge 5$ or $3 - \frac{x}{3} \le -5$

$-\frac{x}{3} \ge 2$ $-\frac{x}{3} \le -8$

$x \le -6$ $x \ge 24$

The solution set is $\{x | x \le -6 \text{ or } x \ge 24\}$, that is,

$(-\infty, -6]$ or $[24, \infty)$.

94. $\left|2-\frac{x}{2}\right|-1\le 1$

$\left|2-\frac{x}{2}\right|\le 2$

$-2\le 2-\frac{x}{2}\le 2$

$-4\le -\frac{x}{2}\le 0$

$8\ge x\ge 0$

The solution set is $\{x \mid 0\le x\le 8\}$ or $[0,8]$.

95. $y_1 \le y_2$

$\frac{x}{2}+3\le \frac{x}{3}+\frac{5}{2}$

$6\left(\frac{x}{2}+3\right)\le 6\left(\frac{x}{3}+\frac{5}{2}\right)$

$\frac{6x}{2}+6(3)\le \frac{6x}{3}+\frac{6(5)}{2}$

$3x+18\le 2x+15$

$x\le -3$

The solution set is $(-\infty,-3]$.

96. $y_1 > y_2$

$\frac{2}{3}(6x-9)+4>5x+1$

$3\left(\frac{2}{3}(6x-9)+4\right)>3(5x+1)$

$2(6x-9)+12>15x+3$

$12x-18+12>15x+3$

$12x-6>15x+3$

$-3x>9$

$\frac{-3x}{-3}<\frac{9}{-3}$

$x<-3$

The solution set is $(-\infty,-3)$.

97. $y\ge 4$

$1-(x+3)+2x\ge 4$

$1-x-3+2x\ge 4$

$x-2\ge 4$

$x\ge 6$

The solution set is $[6,\infty)$.

98. $y\le 0$

$2x-11+3(x+2)\le 0$

$2x-11+3x+6\le 0$

$5x-5\le 0$

$5x\le 5$

$x\le 1$

The solution set is $(-\infty,1]$.

99. $y<8$

$|3x-4|+2<8$

$|3x-4|<6$

$-6<3x-4<6$

$-2<3x<10$

$\frac{-2}{3}<\frac{3x}{3}<\frac{10}{3}$

$\frac{-2}{3}<x<\frac{10}{3}$

The solution set is $\left(\frac{-2}{3},\frac{10}{3}\right)$.

100. $y>9$

$|2x-5|+1>9$

$|2x-5|>8$

$2x-5<-8$ or $2x-5>8$

$2x<-3$ $\quad$ $2x>13$

$x<-\frac{3}{2}$ $\quad$ $x>\frac{13}{2}$

The solution set is $\left(-\infty,-\frac{3}{2}\right)\cup\left(\frac{13}{2},\infty\right)$.

101. $y\le 4$

$7-\left|\frac{x}{2}+2\right|\le 4$

$-\left|\frac{x}{2}+2\right|\le -3$

$\left|\frac{x}{2}+2\right|\ge 3$

$\frac{x}{2}+2\ge 3$ or $\frac{x}{2}+2\le -3$

$x+4\ge 6$ $\quad$ $x+4\le -6$

$x\ge 2$ $\quad$ $x\le -10$

The solution set is $(-\infty,-10]\cup[2,\infty)$.

102.
$$y \ge 6$$
$$8-|5x+3| \ge 6$$
$$-|5x+3| \ge -2$$
$$-(-|5x+3|) \le -(-2)$$
$$|5x+3| \le 2$$
$$-2 \le 5x+3 \le 2$$
$$-5 \le 5x \le -1$$
$$\frac{-5}{5} \le \frac{5x}{5} \le \frac{-1}{5}$$
$$-1 \le x \le -\frac{1}{5}$$
The solution set is $\left[-1,-\frac{1}{5}\right]$.

103. The graph's height is below 5 on the interval $(-1,9)$.

104. The graph's height is at or above 5 on the interval $(-\infty,-1]\cup[9,\infty)$.

105. The solution set is $\{x\,|\,-1 \le x < 2\}$ or $[-1,2)$.

106. The solution set is $\{x\,|\,1 < x \le 4\}$ or $(1,4]$.

107. Let x be the number.
$|4-3x| \ge 5$ or $|3x-4| \ge 5$

$3x-4 \ge 5$ or $3x-4 \le -5$
$3x \ge 9$ or $3x \le -1$
$x \ge 3$ or $x \le -\frac{1}{3}$

The solution set is $\left\{x\,|\,x \le -\frac{1}{3} \text{ or } x \ge 3\right\}$ or $\left(-\infty,-\frac{1}{3}\right]\cup[3,\infty)$.

108. Let x be the number.
$|5-4x| \le 13$ or $|4x-5| \le 13$

$$-13 \le 4x-5 \le 13$$
$$-8 \le 4x \le 18$$
$$-2 \le x \le \frac{9}{2}$$
The solution set is $\left\{x\,|\,-2 \le x \le \frac{9}{2}\right\}$ or $\left[-2,-\frac{2}{9}\right]$.

109. $(0,4)$

110. $[0,5]$

111. passion $\le$ intimacy or intimacy $\ge$ passion

112. commitment $\ge$ intimacy or intimacy $\le$ commitment

113. passion<commitment or commitment > passion

114. commitment > passion or passion < commitment

115. 9, after 3 years

116. after approximately $5\frac{1}{2}$ years

117.
$$3.1x+25.8 > 63$$
$$3.1x > 37.2$$
$$x > 12$$
Since x is the number of years after 1994, we calculate 1994+12=2006. 63% of voters will use electronic systems after 2006.

118.
$$-2.5x+63.1 < 38.1$$
$$-2.5x < 25$$
$$x > 10$$
1994 + 10 = 2004
In years after 2004, fewer than 38.1% of U.S. voters will use punch cards or lever machines.

119.
$$28 \le 20+0.40(x-60) \le 40$$
$$28 \le 20+0.40x-24 \le 40$$
$$28 \le 0.40x-4 \le 40$$
$$32 \le 0.40x \le 44$$
$$80 \le x \le 110$$
Between 80 and 110 ten minutes, inclusive.

120.
$$15 \le \frac{5}{9}(F-32) \le 35$$
$$\frac{9}{5}(15) \le \frac{9}{5}\left(\frac{5}{9}(F-32)\right) \le \frac{9}{5}(35)$$
$$9(3) \le F-32 \le 9(7)$$
$$27 \le F-32 \le 63$$
$$59 \le F \le 95$$
The range for Fahrenheit temperatures is 59°F to 95°F, inclusive or [59°F, 95°F].

121. $\left|\frac{h-50}{5}\right| \geq 1.645$

$\frac{h-50}{5} \geq 1.645$ or $\frac{h-50}{5} \leq -1.645$

$h-50 \geq 8.225$ $\quad$ $h-50 \leq -8.225$

$h \geq 58.225$ $\quad$ $h \leq 41.775$

The number of outcomes would be 59 or more, or 41 or less.

122. $50 + 0.20x < 20 + 0.50x$

$30 < 0.3x$

$100 < x$

Basic Rental is a better deal when driving more than 100 miles per day.

123. $15 + 0.08x < 3 + .12x$

$12 < 0.04x$

$300 < x$

Plan A is a better deal when driving more than 300 miles a month.

124. $1800 + 0.03x < 200 + 0.08x$

$1600 < 0.05x$

$32000 < x$

A home assessment of greater than $32,000 would make the first bill a better deal.

125. $2 + 0.08x < 8 + 0.05x$

$0.03x < 6$

$x < 200$

The credit union is a better deal when writing less than 200 checks.

126. $2x > 10{,}000 + 0.40x$

$1.6x > 10{,}000$

$\frac{1.6x}{1.6} > \frac{10{,}000}{1.6}$

$x > 6250$

More than 6250 tapes need to be sold a week to make a profit.

127. $3000 + 3x < 5.5x$

$3000 < 2.5x$

$1200 < x$

More then 1200 packets of stationary need to be sold each week to make a profit.

128. $265 + 65x \leq 2800$

$65x \leq 2535$

$x \leq 39$

39 bags or fewer can be lifted safely.

129. $245 + 95x \leq 3000$

$95x \leq 2755$

$x \leq 29$

29 bags or less can be lifted safely.

130. Let x = the grade on the final exam.

$\frac{86+88+92+84+x+x}{6} \geq 90$

$86+88+92+84+x+x \geq 540$

$2x + 350 \geq 540$

$2x \geq 190$

$x \geq 95$

You must receive at least a 95% to earn an A.

131. a. $\frac{86+88+x}{3} \geq 90$

$\frac{174+x}{3} \geq 90$

$174 + x \geq 270$

$x \geq 96$

You must get at least a 96.

b. $\frac{86+88+x}{3} < 80$

$\frac{174+x}{3} < 80$

$174 + x < 240$

$x < 66$

This will happen if you get a grade less than 66.

132. Let x = the number of hours the mechanic works on the car.

$226 \leq 175 + 34x \leq 294$

$51 \leq 34x \leq 119$

$1.5 \leq x \leq 3.5$

The man will be working on the job at least 1.5 and at most 3.5 hours.

133. Let x = the number of times the bridge is crossed per three month period
The cost with the 3-month pass is
$C_3 = 7.50 + 0.50x.$
The cost with the 6-month pass is $C_6 = 30.$

Because we need to buy two 3-month passes per 6-month pass, we multiply the cost with the 3-month pass by 2.

$$2(7.50 + 0.50x) < 30$$
$$15 + x < 30$$
$$x < 15$$

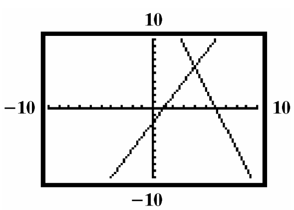
We also must consider the cost without purchasing a pass. We need this cost to be less than the cost with a 3-month pass.

$$3x > 7.50 + 0.50x$$
$$2.50x > 7.50$$
$$x > 3$$

The 3-month pass is the best deal when making more than 3 but less than 15 crossings per 3-month period.

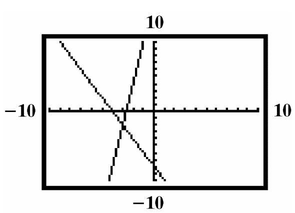
134. – 141. Answers may vary.

142.



$x < 4$

143.



$x < -3$

144. Verify exercise 142.

X	Y1	Y2
2	12	2
3	9	4
4	6	6
5	3	8
6	0	10
7	-3	12
8	-6	14

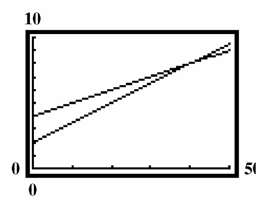
X=4

Verify exercise 143.

X	Y1	Y2
-5	2	-14
-4	0	-8
-3	-2	-2
-2	-4	4
-1	-6	10
0	-8	16
1	-10	22

X= -3

145 a. The cost of Plan A is $4 + 0.10x$;
The cost of Plan B is $2 + 0.15x$.



c. 41 or more checks make Plan A better.

d.
$$4 + 0.10x < 2 + 0.15x$$
$$2 < 0.05x$$
$$x > 40$$

The solution set is $\{x \mid x > 40\}$ or $(40, \infty)$.

146. makes sense

147. makes sense

148. makes sense

149. makes sense

150. true

151. false; Changes to make the statement true will vary.
A sample change is: $(-\infty, 3) \cup (-\infty, -2) = (-\infty, 3)$

152. false; Changes to make the statement true will vary.
A sample change is: $3x > 6$ is equivalent to $x > 2$.

153. true

154. Because $x > y$, $y - x$ represents a negative number. When both sides are multiplied by $(y - x)$ the inequality must be reversed.

155. a. $|x-4|<3$

b. $|x-4|\geq 3$

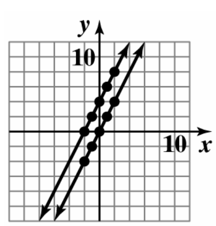
156. Answers may vary.

157. Set 1 has each x-coordinate paired with only one y-coordinate.

158.

x	$y = 2x$	(x, y)
–2	$y = 2(-2) = -4$	$(-2,-4)$
–1	$y = 2(-1) + 4 = 2$	$(-1,-2)$
0	$y = 2(0) = 0$	$(0,0)$
1	$y = 2(1) = 2$	$(1,2)$
2	$y = 2(2) = 4$	$(2,4)$

x	$y = 2x + 4$	(x, y)
–2	$y = 2(-2) + 4 = 0$	$(-2,0)$
–1	$y = 2(-1) + 4 = 2$	$(-1,2)$
0	$y = 2(0) + 4 = 4$	$(0,4)$
1	$y = 2(1) + 4 = 6$	$(1,6)$
2	$y = 2(2) + 4 = 8$	$(2,8)$



159. a. When the x-coordinate is 2, the y-coordinate is 3.

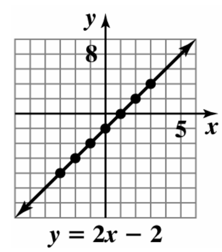
b. When the y-coordinate is 4, the x-coordinates are –3 and 3.

c. The x-coordinates are all real numbers.

d. The y-coordinates are all real numbers greater than or equal to 1.

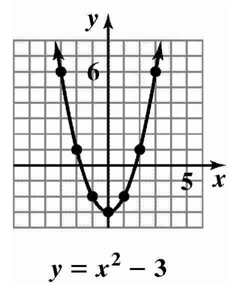
Chapter 1 Review Exercises

1.



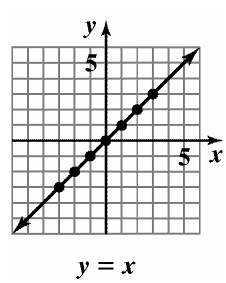
$x = -3, y = -8$
$x = -2, y = -6$
$x = -1, y = -4$
$x = 0, y = -2$
$x = 1, y = 0$
$x = 2, y = 2$
$x = 3, y = 4$

2.

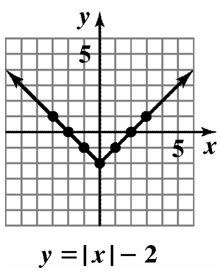


$x = -3, y = 6$
$x = -2, y = 1$
$x = -1, y = -2$
$x = 0, y = -3$
$x = 1, y = -2$
$x = 2, y = 1$
$x = 3, y = 6$

3.



$x = -3, y = -3$
$x = -2, y = -2$
$x = -1, y = -1$
$x = 0, y = 0$
$x = 1, y = 1$
$x = 2, y = 2$
$x = 3, y = 3$

4. 

$x = -3, y = 1$

$x = -2, y = 0$

$x = -1, y = -1$

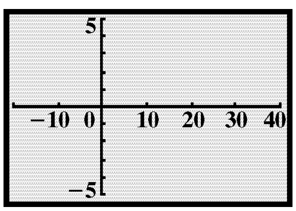
$x = 0, y = -2$

$x = 1, y = -1$

$x = 2, y = 0$

$x = 3, y = 1$

5. A portion of Cartesian coordinate plane with minimum x-value equal to –20, maximum x-value equal to 40, x-scale equal to 10 and with minimum y-value equal to –5, maximum y-value equal to 5, and y-scale equal to 1.



6. x-intercept: –2; The graph intersects the x-axis at (–2, 0).
y-intercept: 2; The graph intersects the y-axis at (0, 2).

7. x-intercepts: 2, –2; The graph intersects the x-axis at (–2, 0) and (2, 0).
y-intercept: –4; The graph intercepts the y-axis at (0, –4).

8. x-intercept: 5; The graph intersects the x-axis at (5, 0).
y-intercept: None; The graph does not intersect the y-axis.

9. The coordinates are (1985, 50%).

10. The top marginal tax rate in 2005 was 35%.

11. The highest marginal tax rate occurred in 1945 and was about 94%.

12. The lowest marginal tax rate occurred in 1990 and was about 28%.

13. During the ten-year period from 1950 to 1960, the top marginal tax rate remained constant at about 91%.

14. During the five-year period from 1930 to 1935, the top marginal tax rate increased about 38%.

15. $2x - 5 = 7$
$2x = 12$
$x = 6$
The solution set is {6}.
This is a conditional equation.

16. $5x + 20 = 3x$
$2x = -20$
$x = -10$
The solution set is {–10}.
This is a conditional equation.

17. $7(x - 4) = x + 2$
$7x - 28 = x + 2$
$6x = 30$
$x = 5$
The solution set is {5}.
This is a conditional equation.

18. $1 - 2(6 - x) = 3x + 2$
$1 - 12 + 2x = 3x + 2$
$-11 - x = 2$
$-x = 13$
$x = -13$
The solution set is {–13}.
This is a conditional equation.

19. $2(x-4)+3(x+5) = 2x-2$
$2x-8+3x+15 = 2x-2$
$5x+7 = 2x-2$
$3x = -9$
$x = -3$
The solution set is {–3}.
This is a conditional equation.

20. $2x - 4(5x + 1) = 3x + 17$
$2x - 20x - 4 = 3x + 17$
$-18x - 4 = 3x + 17$
$-21x = 21$
$x = -1$
The solution set is {–1}.
This is a conditional equation.

21. $7x+5=5(x+3)+2x$
$7x+5=5x+15+2x$
$7x+5=7x+15$
$5=15$
The solution set is $\varnothing$.
This is an inconsistent equation.

22. $7x+13=2(2x-5)+3x+23$
$7x+13=2(2x-5)+3x+23$
$7x+13=4x-10+3x+23$
$7x+13=7x+13$
$13=13$
The solution set is all real numbers.
This is an identity.

23. $\frac{2x}{3}=\frac{x}{6}+1$
$2(2x)=x+6$
$4x=x+6$
$3x=6$
$x=2$
The solution set is $\{2\}$.
This is a conditional equation.

24. $\frac{x}{2}-\frac{1}{10}=\frac{x}{5}+\frac{1}{2}$
$5x-1=2x+5$
$3x=6$
$x=2$
The solution set is $\{2\}$.
This is a conditional equation.

25. $\frac{2x}{3}=6-\frac{x}{4}$
$4(2x)=12(6)-3x$
$8x=72-3x$
$11x=72$
$x=\frac{72}{11}$
The solution set is $\left\{\frac{72}{11}\right\}$.
This is a conditional equation.

26. $\frac{x}{4}=2-\frac{x-3}{3}$
$\frac{12\cdot x}{4}=12(2)-\frac{12(x-3)}{3}$
$3x=24-4x+12$
$7x=36$
$x=\frac{36}{7}$
The solution set is $\left\{\frac{36}{7}\right\}$.
This is a conditional equation.

27. $\frac{3x+1}{3}-\frac{13}{2}=\frac{1-x}{4}$
$4(3x+1)-6(13)=3(1-x)$
$12x+4-78=3-3x$
$12x-74=3-3x$
$15x=77$
$x=\frac{77}{15}$
The solution set is $\left\{\frac{77}{15}\right\}$.
This is a conditional equation.

28. $\frac{9}{4}-\frac{1}{2x}=\frac{4}{x}$
$9x-2=16$
$9x=18$
$x=2$
The solution set is $\{2\}$.
This is a conditional equation.

29. $\frac{7}{x-5}+2=\frac{x+2}{x-5}$
$7+2(x-5)=x+2$
$7+2x-10=x+2$
$2x-3=x+2$
$x=5$
5 does not check and must be rejected.
The solution set is the empty set, $\varnothing$.
This is an inconsistent equation.

30. $\frac{1}{x-1}-\frac{1}{x+1}=\frac{2}{x^2-1}$

$\frac{1}{x-1}-\frac{1}{x+1}=\frac{2}{(x+1)(x-1)}$

$x+1-(x-1)=2$

$x+1-x+1=2$

$2=2$

The solution set is all real numbers except –1 and 1.
This is a conditional equation.

31. $\frac{5}{x+3}+\frac{1}{x-2}=\frac{8}{x^2+x-6}$

$\frac{5}{x+3}+\frac{1}{x-2}=\frac{8}{(x+3)(x-2)}$

$\frac{5(x+3)(x-2)}{x+3}+\frac{(x+3)(x-2)}{x-2}=\frac{8(x+3)(x-2)}{(x+3)(x-2)}$

$5(x-2)+1(x+3)=8$

$5x-10+x+3=8$

$6x-7=8$

$6x=15$

$x=\frac{15}{6}$

$x=\frac{5}{2}$

The solution set is $\left\{\frac{5}{2}\right\}$.

This is a conditional equation.

32. $\frac{1}{x+5}=0$

$(x+5)\frac{1}{x+5}=(x+5)(0)$

$1=0$

The solution set is the empty set, $\varnothing$.
This is an inconsistent equation.

33. $\frac{4}{x+2}+\frac{3}{x}=\frac{10}{x^2+2x}$

$\frac{4}{x+2}+\frac{3}{x}=\frac{10}{x(x+2)}$

$\frac{4\cdot x(x+2)}{x+2}+\frac{3\cdot x(x+2)}{x}=\frac{10\cdot x(x+2)}{x(x+2)}$

$4x+3(x+2)=10$

$4x+3x+6=10$

$7x+6=10$

$7x=4$

$x=\frac{4}{7}$

The solution set is $\left\{\frac{4}{7}\right\}$.

This is a conditional equation.

34. $3-5(2x+1)-2(x-4)=0$

$3-5(2x+1)-2(x-4)=0$

$3-10x-5-2x+8=0$

$-12x+6=0$

$-12x=-6$

$x=\frac{-6}{-12}$

$x=\frac{1}{2}$

The solution set is $\left\{\frac{1}{2}\right\}$.

This is a conditional equation.

35. $\frac{x+2}{x+3}+\frac{1}{x^2+2x-3}-1=0$

$\frac{x+2}{x+3}+\frac{1}{(x+3)(x-1)}-1=0$

$\frac{x+2}{x+3}+\frac{1}{(x+3)(x-1)}=1$

$\frac{(x+2)(x+3)(x-1)}{x+3}+1=(x+3)(x-1)$

$(x+2)(x-1)+1=(x+3)(x-1)$

$x^2+x-2+1=x^2+2x-3$

$x-1=2x-3$

$-x=-2$

$x=2$

The solution set is $\{2\}$.

This is a conditional equation.

36. Let x = millions of barrels of oil consumed each day by Japan.
Let $x+0.8=$ millions of barrels of oil consumed each day by China.
Let $x+15=$ millions of barrels of oil consumed each day by the United States.

$$x+(x+0.8)+(x+15)=32.3$$
$$x+x+0.8+x+15=32.3$$
$$3x+15.8=32.3$$
$$3x=16.5$$
$$x=5.5$$
$$x+0.8=6.3$$
$$x+15=20.5$$

The daily oil consumption of the United States, China, and Japan is 20.5 million barrels, 6.3 million barrels, and 5.5 million barrels, respectfully.

37. Let x = the number of years after 2000.

$$17.5+0.4x=25.1$$
$$0.4x=7.6$$
$$x=19$$

The percentage of people in the U.S. that will speak a language other than English at home will reach 25.1% 19 years after 2000, or 2019.

38.
$$15+.05x=5+.07x$$
$$10=.02x$$
$$500=x$$

Both plans cost the same at 500 minutes.

39. Let x = the original price of the phone

$$48=x-0.20x$$
$$48=0.80x$$
$$60=x$$

The original price is $60.

40. Let x = the amount sold to earn $800 in one week

$$800=300+0.05x$$
$$500=0.05x$$
$$10,000=x$$

Sales must be $10,000 in one week to earn $800.

41. Let x = the amount invested at 4%
Let y = the amount invested at 7%

$$x+y=9000$$
$$0.04x+0.07y=555$$

Multiply the first equation by –0.04 and add.

$$-0.04x-0.04y=-360$$
$$0.04x+0.07y=555$$
$$0.03y=195$$
$$y=6500$$

Back-substitute 6500 for y in one of the original equations to find x.

$$x+y=9000$$
$$x+6500=9000$$
$$x=2500$$

There was $2500 invested at 4% and $6500 invested at 7%.

42. Let $x=$ the amount invested at 2%
Let $8000-x=$ the amount invested at 5%.

$$0.05(8000-x)=0.02x+85$$
$$400-0.05x=0.02x+85$$
$$-0.05x-0.02x=85-400$$
$$-0.07x=-315$$
$$\frac{-0.07x}{-0.07}=\frac{-315}{-0.07}$$
$$x=4500$$
$$8000-x=3500$$

$4500 was invested at 2% and $3500 was invested at 5%.

43. Let w = the width of the playing field,
Let $3w – 6$ = the length of the playing field

$$P=2(\text{length})+2(\text{width})$$
$$340=2(3w-6)+2w$$
$$340=6w-12+2w$$
$$340=8w-12$$
$$352=8w$$
$$44=w$$

The dimensions are 44 yards by 126 yards.

44. **a.** Let x = the number of years (after 2007).
College A's enrollment: $14,100+1500x$
College B's enrollment: $41,700-800x$
$14,100+1500x=41,700-800x$

b. Check some points to determine that $y_1=14,100+1500x$ and $y_2=41,700-800x$. Since $y_1=y_2=32,100$ when $x=12$, the two colleges will have the same enrollment in the year $2007+12=2019$. That year the enrollments will be 32,100 students.

45. $vt + gt^2 = s$
$$gt^2 = s - vt$$
$$\frac{gt^2}{t^2} = \frac{s - vt}{t^2}$$
$$g = \frac{s - vt}{t^2}$$

46. $T = gr + gvt$
$$T = g(r + vt)$$
$$\frac{T}{r + vt} = \frac{g(r + vt)}{r + vt}$$
$$\frac{T}{r + vt} = g$$
$$g = \frac{T}{r + vt}$$

47. $T = \frac{A - P}{\Pr}$
$$Pr(T) = Pr\frac{A - P}{\Pr}$$
$$PrT = A - P$$
$$PrT + P = A$$
$$P(rT + 1) = A$$
$$P = \frac{A}{1 + rT}$$

48. $(8 - 3i) - (17 - 7i) = 8 - 3i - 17 + 7i$
$$= -9 + 4i$$

49. $4i(3i - 2) = (4i)(3i) + (4i)(-2)$
$$= 12i^2 - 8i$$
$$= -12 - 8i$$

50. $(7 - i)(2 + 3i)$
$$= 7 \cdot 2 + 7(3i) + (-i)(2) + (-i)(3i)$$
$$= 14 + 21i - 2i + 3$$
$$= 17 + 19i$$

51. $(3 - 4i)^2 = 3^2 + 2 \cdot 3(-4i) + (-4i)^2$
$$= 9 - 24i - 16$$
$$= -7 - 24i$$

52. $(7 + 8i)(7 - 8i) = 7^2 + 8^2 = 49 + 64 = 113$

53. $\frac{6}{5 + i} = \frac{6}{5 + i} \cdot \frac{5 - i}{5 - i}$
$$= \frac{30 - 6i}{25 + 1}$$
$$= \frac{30 - 6i}{26}$$
$$= \frac{15 - 3i}{13}$$
$$= \frac{15}{13} - \frac{3}{13}i$$

54. $\frac{3 + 4i}{4 - 2i} = \frac{3 + 4i}{4 - 2i} \cdot \frac{4 + 2i}{4 + 2i}$
$$= \frac{12 + 6i + 16i + 8i^2}{16 - 4i^2}$$
$$= \frac{12 + 22i - 8}{16 + 4}$$
$$= \frac{4 + 22i}{20}$$
$$= \frac{1}{5} + \frac{11}{10}i$$

55. $\sqrt{-32} - \sqrt{-18} = i\sqrt{32} - i\sqrt{18}$
$$= i\sqrt{16 \cdot 2} - i\sqrt{9 \cdot 2}$$
$$= 4i\sqrt{2} - 3i\sqrt{2}$$
$$= (4i - 3i)\sqrt{2}$$
$$= i\sqrt{2}$$

56. $(-2 + \sqrt{-100})^2 = (-2 + i\sqrt{100})^2$
$$= (-2 + 10i)^2$$
$$= 4 - 40i + (10i)^2$$
$$= 4 - 40i - 100$$
$$= -96 - 40i$$

57. $\frac{4 + \sqrt{-8}}{2} = \frac{4 + i\sqrt{8}}{2} = \frac{4 + 2i\sqrt{2}}{2} = 2 + i\sqrt{2}$

58. $2x^2 + 15x = 8$
$$2x^2 + 15x - 8 = 0$$
$$(2x - 1)(x + 8) = 0$$
$$2x - 1 = 0 \quad x + 8 = 0$$
$$x = \frac{1}{2} \text{ or } x = -8$$
The solution set is $\left\{\frac{1}{2}, -8\right\}$.

59. $5x^2 + 20x = 0$

$5x(x+4) = 0$

$5x = 0 \quad x + 4 = 0$

$x = 0$ or $x = -4$

The solution set is $\{0, -4\}$.

60. $2x^2 - 3 = 125$

$2x^2 = 128$

$x^2 = 64$

$x = \pm 8$

The solution set is $\{8, -8\}$.

61. $\frac{x^2}{2} + 5 = -3$

$\frac{x^2}{2} = -8$

$x^2 = -16$

$\sqrt{x^2} = \pm\sqrt{-16}$

$x = \pm 4i$

62. $(x+3)^2 = -10$

$\sqrt{(x+3)^2} = \pm\sqrt{-10}$

$x + 3 = \pm i\sqrt{10}$

$x = -3 \pm i\sqrt{10}$

63. $(3x-4)^2 = 18$

$\sqrt{(3x-4)^2} = \pm\sqrt{18}$

$3x - 4 = \pm 3\sqrt{2}$

$3x = 4 \pm 3\sqrt{2}$

$\frac{3x}{3} = \frac{4 \pm 3\sqrt{2}}{3}$

$x = \frac{4 \pm 3\sqrt{2}}{3}$

64. $x^2 + 20x$

$\left(\frac{20}{2}\right)^2 = 10^2 = 100$

$x^2 + 20x + 100 = (x+10)^2$

65. $x^2 - 3x$

$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$x^2 - 3x + \frac{9}{4} = \left(x - \frac{3}{2}\right)^2$

66. $x^2 - 12x = -27$

$x^2 - 12x + 36 = -27 + 36$

$(x-6)^2 = 9$

$x - 6 = \pm 3$

$x = 6 \pm 3$

$x = 9, 3$

The solution set is $\{9, 3\}$.

67. $3x^2 - 12x + 11 = 0$

$x^2 - 4x = -\frac{11}{3}$

$x^2 - 4x + 4 = -\frac{11}{3} + 4$

$(x-2)^2 = \frac{1}{3}$

$x - 2 = \pm\sqrt{\frac{1}{3}}$

$x = 2 \pm \frac{\sqrt{3}}{3}$

The solution set is $\left\{2 + \frac{\sqrt{3}}{3}, 2 - \frac{\sqrt{3}}{3}\right\}$.

68. $x^2 = 2x + 4$

$x^2 - 2x - 4 = 0$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4 + 16}}{2}$

$x = \frac{2 \pm \sqrt{20}}{2}$

$x = \frac{2 \pm 2\sqrt{5}}{2}$

$x = 1 \pm \sqrt{5}$

The solution set is $\{1 + \sqrt{5}, 1 - \sqrt{5}\}$.

69. $x^2-2x+19=0$

$x=\dfrac{2\pm\sqrt{(-2)^2-4(1)(19)}}{2(1)}$

$x=\dfrac{2\pm\sqrt{4-76}}{2}$

$x=\dfrac{2\pm\sqrt{-72}}{2}$

$x=\dfrac{2\pm 6i\sqrt{2}}{2}$

$x=1\pm 3i\sqrt{2}$

The solution set is $\{1+3i\sqrt{2}, 1-3i\sqrt{2}\}$.

70. $2x^2=3-4x$

$2x^2+4x-3=0$

$x=\dfrac{-4\pm\sqrt{4^2-4(2)(-3)}}{2(2)}$

$x=\dfrac{-4\pm\sqrt{16+24}}{4}$

$x=\dfrac{-4\pm\sqrt{40}}{4}$

$x=\dfrac{-4\pm 2\sqrt{10}}{4}$

$x=\dfrac{-2\pm\sqrt{10}}{2}$

The solution set is $\left\{\dfrac{-2+\sqrt{10}}{2}, \dfrac{-2-\sqrt{10}}{2}\right\}$.

71. $x^2-4x+13=0$

$(-4)^2-4(1)(13)$

$= 16 - 52$

$= -36$; 2 complex imaginary solutions

72. $9x^2=2-3x$

$9x^2+3x-2=0$

$3^2-4(9)(-2)$

$= 9 + 72$

$= 81$; 2 unequal real solutions

73. $2x^2-11x+5=0$

$(2x-1)(x-5)=0$

$2x-1=0 \quad x-5=0$

$x=\dfrac{1}{2}$ or $x=5$

The solution set is $\left\{5, \dfrac{1}{2}\right\}$.

74. $(3x+5)(x-3)=5$

$3x^2+5x-9x-15=5$

$3x^2-4x-20=0$

$x=\dfrac{4\pm\sqrt{(-4)^2-4(3)(-20)}}{2(3)}$

$x=\dfrac{4\pm\sqrt{16+240}}{6}$

$x=\dfrac{4\pm\sqrt{256}}{6}$

$x=\dfrac{4\pm 16}{6}$

$x=\dfrac{20}{6}, \dfrac{-12}{6}$

$x=\dfrac{10}{3}, -2$

The solution set is $\left\{-2, \dfrac{10}{3}\right\}$.

75. $3x^2-7x+1=0$

$x=\dfrac{7\pm\sqrt{(-7)^2-4(3)(1)}}{2(3)}$

$x=\dfrac{7\pm\sqrt{49-12}}{6}$

$x=\dfrac{7\pm\sqrt{37}}{6}$

The solution set is $\left\{\dfrac{7+\sqrt{37}}{6}, \dfrac{7-\sqrt{37}}{6}\right\}$.

76. $x^2-9=0$

$x^2=9$

$x=\pm 3$

The solution set is $\{-3, 3\}$.

77. $(x-3)^2-25=0$

$(x-3)^2=25$

$x-3=\pm 5$

$x=3\pm 5$

$x=8,-2$

The solution set is {8, –2}.

78. $3x^2-x+2=0$

$x=\dfrac{1\pm\sqrt{(-1)^2-4(3)(2)}}{2(3)}$

$x=\dfrac{1\pm\sqrt{1-24}}{6}$

$x=\dfrac{1\pm\sqrt{-23}}{6}$

$x=\dfrac{1\pm i\sqrt{23}}{6}$

The solution set is $\left\{\dfrac{1+i\sqrt{23}}{6},\dfrac{1-i\sqrt{23}}{6}\right\}$.

79. $3x^2-10x=8$

$3x^2-10x-8=0$

$(3x+2)(x-4)=0$

$3x+2=0$ or $x-4=0$

$3x=-2$ $\quad x=4$

$x=-\dfrac{2}{3}$

The solution set is $\left\{-\dfrac{2}{3},4\right\}$.

80. $(x+2)^2+4=0$

$(x+2)^2=-4$

$\sqrt{(x+2)^2}=\pm\sqrt{-4}$

$x+2=\pm 2i$

$x=-2\pm 2i$

The solution set is $\{-2+2i,-2-2i\}$.

81. $\dfrac{5}{x+1}+\dfrac{x-1}{4}=2$

$\dfrac{5\cdot 4(x+1)}{x+1}+\dfrac{(x-1)\cdot 4(x+1)}{4}=2\cdot 4(x+1)$

$20+(x-1)(x+1)=8(x+1)$

$20+x^2-1=8x+8$

$x^2-8x-11=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(-8)\pm\sqrt{(-8)^2-4(1)(11)}}{2(1)}$

$x=\dfrac{8\pm\sqrt{20}}{2}$

$x=\dfrac{8\pm 2\sqrt{5}}{2}$

$x=4\pm\sqrt{5}$

The solution set is $\{4+\sqrt{5},4-\sqrt{5}\}$.

82. $W(t)=3t^2$

$588=3t^2$

$196=t^2$

Apply the square root property.

$t^2=196$

$t=\pm\sqrt{196}$

$t=\pm 14$

The solutions are –14 and 14. We disregard –14, because we cannot have a negative time measurement. The fetus will weigh 588 grams after 14 weeks.

83. **a.** $M=-1.8x^2+21x+15$

$M=-1.8(4)^2+21(4)+15$

$=70.2$

According to formula, 70.2% of new cellphones will play music in 2009. This overestimates the value in the graph by 0.2%.

b. $M = -1.8x^2 + 21x + 15$

$75 = -1.8x^2 + 21x + 15$

$0 = -1.8x^2 + 21x - 60$

$0 = 1.8x^2 - 21x + 60$

$0 = 1.8x^2 - 21x + 60$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-21) \pm \sqrt{(-21)^2 - 4(1.8)(60)}}{2(1.8)}$

$x = 5 \quad x \approx 7$

75% of new cellphones will play music 5 years after 2005, or 2010.

84. $A = lw$

$15 = l(2l - 7)$

$15 = 2l^2 - 7l$

$0 = 2l^2 - 7l - 15$

$0 = (2l + 3)(l - 5)$

$l = 5$

$2l - 7 = 3$

The length is 5 yards, the width is 3 yards.

85. Let x = height of building

$2x$ = shadow height

$x^2 + (2x)^2 = 300^2$

$x^2 + 4x^2 = 90,000$

$5x^2 = 90,000$

$x^2 = 18,000$

$x \approx \pm 134.164$

Discard negative height.
The building is approximately 134 meters high.

86. $2x^4 = 50x^2$

$2x^4 - 50x^2 = 0$

$2x^2(x^2 - 25) = 0$

$x = 0$

$x = \pm 5$

The solution set is $\{-5, 0, 5\}$.

87. $2x^3 - x^2 - 18x + 9 = 0$

$x^2(2x-1) - 9(2x-1) = 0$

$(x^2 - 9)(2x - 1) = 0$

$x = \pm 3,\ x = \dfrac{1}{2}$

The solution set is $\left\{-3, \dfrac{1}{2}, 3\right\}$.

88. $\sqrt{2x-3} + x = 3$

$\sqrt{2x-3} = 3 - x$

$2x - 3 = 9 - 6x + x^2$

$x^2 - 8x + 12 = 0$

$x^2 - 8x = -12$

$x^2 - 8x + 16 = -12 + 16$

$(x-4)^2 = 4$

$x - 4 = \pm 2$

$x = 4 + 2$

$x = 6, 2$

The solution set is $\{2\}$.

89. $\sqrt{x-4} + \sqrt{x+1} = 5$

$\sqrt{x-4} = 5 - \sqrt{x+1}$

$x - 4 = 25 - 10\sqrt{x+1} + (x+1)$

$x - 4 = 26 + x - 10\sqrt{x+1}$

$-30 = -10\sqrt{x+1}$

$3 = \sqrt{x+1}$

$9 = x + 1$

$x = 8$

The solution set is $\{8\}$.

90. $3x^{\frac{3}{4}} - 24 = 0$

$3x^{\frac{3}{4}} = 24$

$x^{\frac{3}{4}} = 8$

$\left(x^{\frac{3}{4}}\right)^{\frac{4}{3}} = (8)^{\frac{4}{3}}$

$x = 16$

The solution set is $\{16\}$.

91. $(x-7)^{\frac{2}{3}} = 25$

$\left[(x-7)^{\frac{2}{3}}\right]^{\frac{3}{2}} = 25^{\frac{3}{2}}$

$x-7 = \left(5^2\right)^{\frac{3}{2}}$

$x-7 = 5^3$

$x-7 = 125$

$x = 132$

The solution set is {132}.

92. $x^4 - 5x^2 + 4 = 0$

Let $t = x^2$

$t^2 - 5t + 4 = 0$

$t = 4$ or $t = 1$

$x^2 = 4$ $\quad x^2 = 1$

$x = \pm 2$ $\quad x = \pm 1$

The solution set is {–2, –1, 1, 2}.

93. $x^{1/2} + 3x^{1/4} - 10 = 0$

Let $t = x^{1/4}$

$t^2 + 3t - 10 = 0$

$(t+5)(t-2) = 0$

$t = -5$ or $t = 2$

$x^{\frac{1}{4}} = -5$ $\quad x^{\frac{1}{4}} = 2$

$\left(x^{\frac{1}{4}}\right)^4 = (-5)^4$ $\quad \left(x^{\frac{1}{4}}\right)^4 = (2)^4$

$x = 625$ $\quad x = 16$

625 does not check and must be rejected.
The solution set is {16}.

94. $|2x+1| = 7$

$2x+1 = 7$ or $2x+1 = -7$

$2x = 6$ $\quad 2x = -8$

$x = 3$ $\quad x = -8$

The solution set is {–4, 3}.

95. $2|x-3| - 6 = 10$

$2|x-3| = 16$

$|x-3| = 8$

$x-3 = 8$ or $x-3 = -8$

$x = 11$ $\quad x = -5$

The solution set is {–5, 11}.

96. $3x^{4/3} - 5x^{2/3} + 2 = 0$

Let $t = x^{\frac{2}{3}}$.

$3t^2 - 5t + 2 = 0$

$(3t-2)(t-1) = 0$

$3t - 2 = 0$ or $t - 1 = 0$

$3t = 2$ $\quad t = 1$

$t = \frac{2}{3}$ $\quad x^{\frac{2}{3}} = 1$

$x^{\frac{2}{3}} = \frac{2}{3}$ $\quad \left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \pm(1)^{\frac{3}{2}}$

$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \pm\left(\frac{2}{3}\right)^{\frac{3}{2}}$ $\quad x = \pm 1$

$x = \pm\sqrt[2]{\left(\frac{2}{3}\right)^3}$

$x = \pm\frac{2}{3}\sqrt{\frac{2}{3}}$

$x = \pm\frac{2}{3}\cdot\frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$x = \pm\frac{2\sqrt{6}}{9}$

The solution set is $\left\{-\frac{2\sqrt{6}}{9}, \frac{2\sqrt{6}}{9}, -1, 1\right\}$.

97. $2\sqrt{x-1} = x$

$4(x-1) = x^2$

$4x - 4 = x^2$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

The solution set is {2}.

98. $|2x-5| - 3 = 0$

$2x-5 = 3$ or $2x-5 = -3$

$2x = 8$ $\quad 2x = 2$

$x = 4$ $\quad x = 1$

The solution set is {4, 1}.

99. $x^3 + 2x^2 - 9x - 18 = 0$

$x^2(x+2) - 9(x+2) = 0$

$(x+2)(x^2-9) = 0$

$(x+2)(x+3)(x-3) = 0$

The solution set is {–3, –2, 3}.

100. $\sqrt{8-2x}-x=0$

$\sqrt{8-2x}=x$

$\left(\sqrt{8-2x}\right)^2=(x)^2$

$8-2x=x^2$

$0=x^2+2x-8$

$0=(x+4)(x-2)$

$x+4=0$ or $x-2=0$

$x=-4$ $\quad x=2$

–4 does not check.

The solution set is $\{2\}$.

101. $x^3+3x^2-2x-6=0$

$x^2(x+3)-2(x+3)=0$

$(x+3)(x^2-2)=0$

$x+3=0$ or $x^2-2=0$

$x=-3$ $\quad x^2=2$

$x=\pm\sqrt{2}$

The solution set is $\{-3,-\sqrt{2},\sqrt{2}\}$.

102. $-4|x+1|+12=0$

$-4|x+1|=-12$

$|x+1|=3$

$x+1=3$ or $x+1=-3$

$x=2$ $\quad x=-4$

The solution set is $\{-4, 2\}$.

103. $p=-2.5\sqrt{t}+17$

$7=-2.5\sqrt{t}+17$

$-10=-2.5\sqrt{t}$

$4=\sqrt{t}$

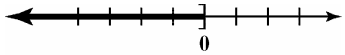
$16=t$

The percentage will drop to 7% 16 years after 1993, or 2009.

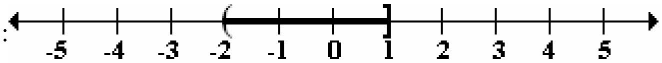
104. $\{x|-3\le x<5\}$

105. $\{x|x>-2\}$

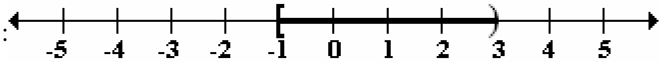
106. $\{x|x\le 0\}$



107. Graph $(-2,1]$:

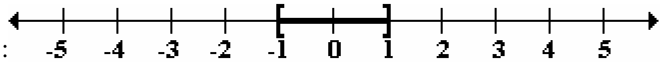


Graph $[-1,3)$:



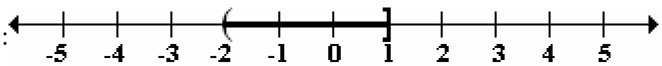
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $(-2,1]$ and $[-1,3)$:

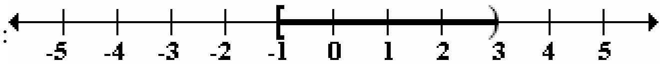


Thus, $(-2,1] \cap [-1,3) = [-1,1]$.

108. Graph $(-2,1]$:

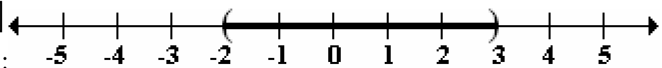


Graph $[-1,3)$:



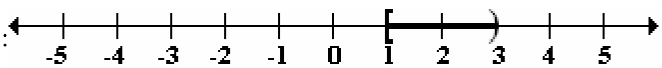
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $(-2,1]$ or $[-1,3)$ or both:

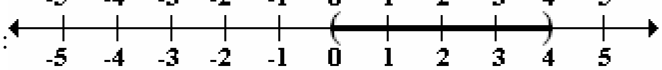


Thus, $(-2,1] \cup [-1,3) = (-2,3)$.

109. Graph $[1,3)$:

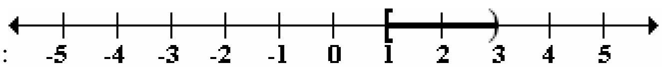


Graph $(0,4)$:



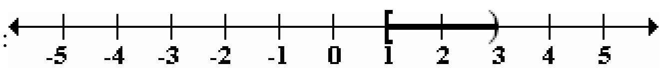
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $[1,3)$ and $(0,4)$:

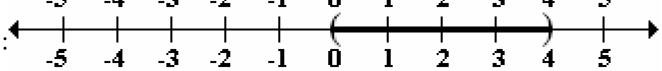


Thus, $[1,3) \cap (0,4) = [1,3)$.

110. Graph $[1,3)$:

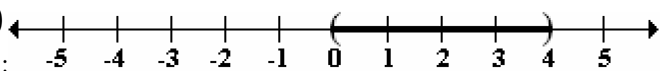


Graph $(0,4)$:



To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $[1,3)$ or $(0,4)$ or both:

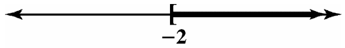


Thus, $[1,3) \cup (0,4) = (0,4)$.

111. $-6x + 3 \le 15$

$-6x \le 12$

$x \ge 2$

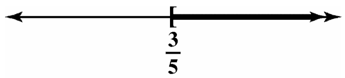


The solution set is $[-2,\infty)$.

112. $6x-9 \ge -4x-3$

$10x \ge 6$

$x \ge \dfrac{3}{5}$



The solution set is $\left[\dfrac{3}{5},\infty\right)$.

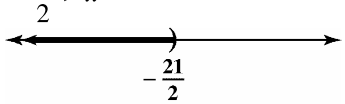
113. $\dfrac{x}{3}-\dfrac{3}{4}-1 > \dfrac{x}{2}$

$12\left(\dfrac{x}{3}-\dfrac{3}{4}-1\right) > 12\left(\dfrac{x}{2}\right)$

$4x - 9 - 12 > 6x$

$-21 > 2x$

$-\dfrac{21}{2} > x$



The solution set is $\left(-\infty,-\dfrac{21}{2}\right)$.

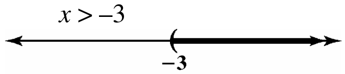
114. $6x + 5 > -2(x - 3) - 25$

$6x + 5 > -2x + 6 - 25$

$8x + 5 > -19$

$8x > -24$

$x > -3$



The solution set is $(-3,\infty)$.

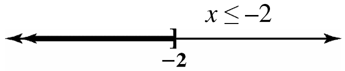
115. $3(2x - 1) - 2(x - 4) \ge 7 + 2(3 + 4x)$

$6x - 3 - 2x + 8 \ge 7 + 6 + 8x$

$4x + 5 \ge 8x + 13$

$-4x \ge 8$

$x \le -2$



The solution set is $[-\infty,-2)$.

116. $5(x-2)-3(x+4) \ge 2x-20$

$5x-10-3x-12 \ge 2x-20$

$2x-22 \ge 2x-20$

$-22 \ge -20$

The solution set is $\varnothing$.

117. $7 < 2x + 3 \le 9$

$4 < 2x \le 6$

$2 < x \le 3$

$(2, 3]$

The solution set is $[2,3)$.

118. $|2x+3| \le 15$

$-15 \le 2x + 3 \le 15$

$-18 \le 2x \le 12$

$-9 \le x \le 6$

The solution set is $[-9,6]$.

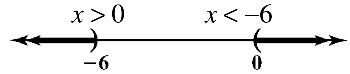
119. $\left|\dfrac{2x+6}{3}\right| > 2$

$\dfrac{2x+6}{3} > 2 \quad \dfrac{2x+6}{3} < -2$

$2x+6 > 6 \quad 2x+6 < -6$

$2x > 0 \quad 2x < -12$

$x > 0 \quad x < -6$



The solution set is $(-\infty,-6)$ or $(0,\infty)$.

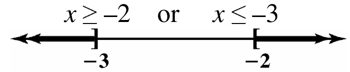
120. $|2x+5|-7 \ge -6$

$|2x+5| \ge 1$

$2x + 5 \ge 1$ or $2x + 5 \le -1$

$2x \ge -4 \quad 2x \le -6$

$x \ge -2$ or $x \le -3$



The solution set is $(-\infty,-3]$ or $[-2,\infty)$.

121. $-4|x+2|+5 \le -7$

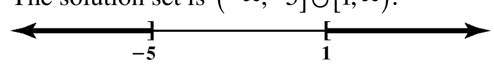
$-4|x+2| \le -12$

$|x+2| \ge 3$

$x+2 \ge 3$ or $x+2 \le -3$

$x \ge 1 \quad x \le -5$

The solution set is $(-\infty,-5]\cup[1,\infty)$.



122. $y_1 > y_2$

$-10-3(2x+1)>8x+1$
$-10-6x-3>8x+1$
$-6x-13>8x+1$
$-14x>14$
$\frac{-14x}{-14}<\frac{14}{-14}$
$x<-1$

The solution set is $(-\infty,-1)$.

123. $3-|2x-5|\ge -6$
$-|2x-5|\ge -9$
$\frac{-|2x-5|}{-1}\le\frac{-9}{-1}$
$|2x-5|\le 9$
$-9\le 2x-5\le 9$
$-4\le 2x\le 14$
$-2\le x\le 7$

The solution set is $[-2,7]$.

124. $0.20x+24\le 40$
$0.20x\le 16$
$\frac{0.20x}{0.20}\le\frac{16}{0.20}$
$x\le 80$

A customer can drive no more than 80 miles.

125. $80\le\frac{95+79+91+86+x}{5}<90$
$400\le 95+79+91+86+x<450$
$400\le 351+x<450$
$49\le x<99$

A grade of at least 49% but less than 99% will result in a B.

126. $0.075x\ge 9000$
$\frac{0.075x}{0.075}\ge\frac{9000}{0.075}$
$x\ge 120{,}000$

The investment must be at least $120,000.

Chapter 1 Test

1. $7(x-2)=4(x+1)-21$
$7x-14=4x+4-21$
$7x-14=4x-17$
$3x=-3$
$x=-1$

The solution set is $\{-1\}$.

2. $-10-3(2x+1)-8x-1=0$
$-10-6x-3-8x-1=0$
$-14x-14=0$
$-14x=14$
$x=-1$

The solution set is $\{-1\}$.

3. $\frac{2x-3}{4}=\frac{x-4}{2}-\frac{x+1}{4}$
$2x-3=2(x-4)-(x+1)$
$2x-3=2x-8-x-1$
$2x-3=x-9$
$x=-6$

The solution set is $\{-6\}$.

4. $\frac{2}{x-3}-\frac{4}{x+3}=\frac{8}{(x-3)(x+3)}$
$2(x+3)-4(x-3)=8$
$2x+6-4x+12=8$
$-2x+18=8$
$-2x=-10$
$x=5$

The solution set is $\{5\}$.

5. $2x^2-3x-2=0$
$(2x+1)(x-2)=0$
$2x+1=0$ or $x-2=0$
$x=-\frac{1}{2}$ or $x=2$

The solution set is $\left\{-\frac{1}{2}, 2\right\}$.

6. $(3x-1)^2 = 75$

$3x-1 = \pm\sqrt{75}$

$3x = 1 \pm 5\sqrt{3}$

$x = \frac{1 \pm 5\sqrt{3}}{3}$

The solution set is $\left\{\frac{1-5\sqrt{3}}{3}, \frac{1+5\sqrt{3}}{3}\right\}$.

7. $(x+3)^2 + 25 = 0$

$(x+3)^2 = -25$

$x+3 = \pm\sqrt{-25}$

$x = -3 \pm 5i$

The solution set is $\{-3+5i, -3-5i\}$.

8. $x(x-2) = 4$

$x^2 - 2x - 4 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2}$

$x = \frac{2 \pm 2\sqrt{5}}{2}$

$x = 1 \pm \sqrt{5}$

The solution set is $\{1-\sqrt{5}, 1+\sqrt{5}\}$.

9. $4x^2 = 8x - 5$

$4x^2 - 8x + 5 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{8 \pm \sqrt{(-8)^2 - 4(4)(5)}}{2(4)}$

$x = \frac{8 \pm \sqrt{-16}}{8}$

$x = \frac{8 \pm 4i}{8}$

$x = 1 \pm \frac{1}{2}i$

The solution set is $\left\{1+\frac{1}{2}i, 1-\frac{1}{2}i\right\}$.

10. $x^3 - 4x^2 - x + 4 = 0$

$x^2(x-4) - 1(x-4) = 0$

$(x^2-1)(x-4) = 0$

$(x-1)(x+1)(x-4) = 0$

$x = 1$ or $x = -1$ or $x = 4$

The solution set is $\{-1, 1, 4\}$.

11. $\sqrt{x-3} + 5 = x$

$\sqrt{x-3} = x - 5$

$x - 3 = x^2 - 10x + 25$

$x^2 - 11x + 28 = 0$

$x = \frac{11 \pm \sqrt{11^2 - 4(1)(28)}}{2(1)}$

$x = \frac{11 \pm \sqrt{121-112}}{2}$

$x = \frac{11 \pm \sqrt{9}}{2}$

$x = \frac{11 \pm 3}{2}$

$x = 7$ or $x = 4$

4 does not check and must be rejected.

The solution set is $\{7\}$.

12. $\sqrt{8-2x} - x = 0$

$\sqrt{8-2x} = x$

$\left(\sqrt{8-2x}\right)^2 = (x)^2$

$8 - 2x = x^2$

$0 = x^2 + 2x - 8$

$0 = (x+4)(x-2)$

$x+4 = 0$ or $x-2 = 0$

$x = -4$ $\quad x = 2$

–4 does not check and must be rejected.

The solution set is $\{2\}$.

13. $\sqrt{x+4} + \sqrt{x-1} = 5$

$\sqrt{x+4} = 5 - \sqrt{x-1}$

$x + 4 = 25 - 10\sqrt{x-1} + (x-1)$

$x + 4 = 25 - 10\sqrt{x-1} + x - 1$

$-20 = -10\sqrt{x-1}$

$2 = \sqrt{x-1}$

$4 = x - 1$

$x = 5$

The solution set is $\{5\}$.

14. $5x^{3/2} - 10 = 0$

$5x^{3/2} = 10$

$x^{3/2} = 2$

$x = 2^{2/3}$

$x = \sqrt[3]{4}$

The solution set is $\left\{\sqrt[3]{4}\right\}$.

15. $x^{2/3} - 9x^{1/3} + 8 = 0$ let $t = x^{1/3}$

$t^2 - 9t + 8 = 0$

$(t-1)(t-8) = 0$

$t = 1 \qquad t = 8$

$x^{1/3} = 1 \qquad x^{1/3} = 8$

$x = 1 \qquad x = 512$

The solution set is {1, 512}.

16. $\left|\frac{2}{3}x - 6\right| = 2$

$\frac{2}{3}x - 6 = 2 \qquad \frac{2}{3}x - 6 = -2$

$\frac{2}{3}x = 8 \qquad \frac{2}{3}x = 4$

$x = 12 \qquad x = 6$

The solution set is {6, 12}.

17. $-3|4x-7| + 15 = 0$

$-3|4x-7| = -15$

$|4x-7| = 5$

$4x - 7 = 5$ or $4x - 7 = -5$

$4x = 12 \qquad 4x = 2$

$x = 3 \qquad x = \frac{1}{2}$

The solution set is $\left\{\frac{1}{2}, 3\right\}$

18. $\frac{1}{x^2} - \frac{4}{x} + 1 = 0$

$\frac{x^2}{x^2} - \frac{4x^2}{x} + x^2 = 0$

$1 - 4x + x^2 = 0$

$x^2 - 4x + 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{4 \pm \sqrt{12}}{2}$

$x = \frac{4 \pm 2\sqrt{3}}{2}$

$x = 2 \pm \sqrt{3}$

The solution set is $\left\{2+\sqrt{3}, 2-\sqrt{3}\right\}$.

19. $\frac{2x}{x^2 + 6x + 8} + \frac{2}{x+2} = \frac{x}{x+4}$

$\frac{2x}{(x+4)(x+2)} + \frac{2}{x+2} = \frac{x}{x+4}$

$\frac{2x(x+4)(x+2)}{(x+4)(x+2)} + \frac{2(x+4)(x+2)}{x+2} = \frac{x(x+4)(x+2)}{x+4}$

$2x + 2(x+4) = x(x+2)$

$2x + 2x + 8 = x^2 + 2x$

$2x + 8 = x^2$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$x - 4 = 0$ or $x + 2 = 0$

$x = 4 \qquad x = -2$ (rejected)

The solution set is $\{4\}$.

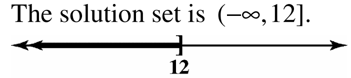
20. $3(x + 4) \geq 5x - 12$

$3x + 12 \geq 5x - 12$

$-2x \geq -24$

$x \leq 12$

The solution set is $(-\infty, 12]$.



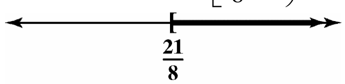
21. $\frac{x}{6}+\frac{1}{8}\le\frac{x}{2}-\frac{3}{4}$

$4x+3\le 12x-18$

$-8x\le -21$

$x\ge\frac{21}{8}$

The solution set is $\left[\frac{21}{8},\infty\right)$.



22. $-3\le\frac{2x+5}{3}<6$

$-9\le 2x+5<18$

$-14\le 2x<13$

$-7\le x<\frac{13}{2}$

The solution set is $\left[-7,\frac{13}{2}\right)$.

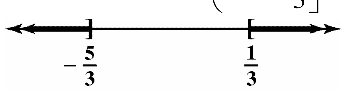
23. $|3x+2|\ge 3$

$3x+2\ge 3$ or $3x+2\le -3$

$3x\ge 1$ $\quad$ $3x\le -5$

$x\ge\frac{1}{3}$ $\quad$ $x\le -\frac{5}{3}$

The solution set is $\left(-\infty,-\frac{5}{3}\right]\cup\left[\frac{1}{3},\infty\right)$.



24. $-3\le y\le 7$

$-3\le 2x-5\le 7$

$2\le 2x\le 12$

$1\le x\le 6$

The solution set is $[1,6]$.

25. $y\ge 1$

$\left|\frac{2-x}{4}\right|\ge 1$

$\frac{2-x}{4}\ge 1$ or $\frac{2-x}{4}\le -1$

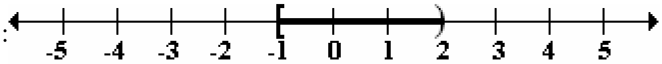
$2-x\ge 4$ $\quad$ $2-x\le -4$

$-x\ge 2$ $\quad$ $-x\le -6$

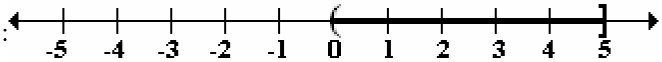
$x\le -2$ $\quad$ $x\ge 6$

The solution set is $(-\infty,-2]\cup[6,\infty)$.

26. Graph $[-1, 2)$:

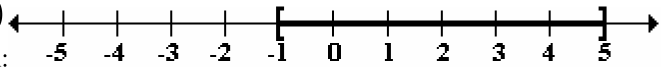


Graph $(0, 5]$:



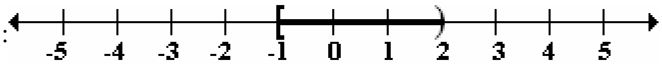
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $[-1, 2)$ or $(0, 5]$ or both:

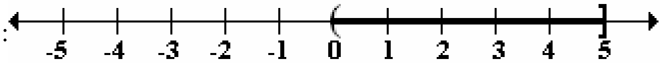


Thus,
$[-1, 2) \cup (0, 5] = [-1, 5]$.

27. Graph $[-1, 2)$:

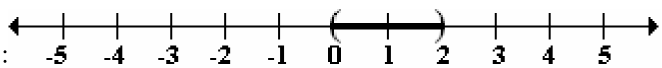


Graph $(0, 5]$:



To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $[-1, 2)$ and $(0, 5]$:

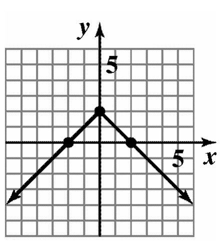


Thus, $[-1, 2) \cap (0, 5] = (0, 2)$.

28.
$$V = \frac{1}{3}lwh$$
$$3V = lwh$$
$$\frac{3V}{lw} = \frac{lwh}{lw}$$
$$\frac{3V}{lw} = h$$
$$h = \frac{3V}{lw}$$

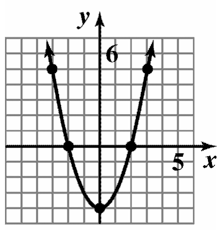
29.
$$y - y_1 = m(x - x_1)$$
$$y - y_1 = mx - mx_1$$
$$-mx = y_1 - mx_1 - y$$
$$\frac{-mx}{-m} = \frac{y_1 - mx_1 - y}{-m}$$
$$x = \frac{y - y_1}{m} + x_1$$

30.



$y = 2 - |x|$

31.



$y = x^2 - 4$

32. $(6-7i)(2+5i) = 12+30i-14i-35i^2$
$= 12+16i+35$
$= 47+16i$

33. $\frac{5}{2-i} = \frac{5}{2-i}\cdot\frac{2+i}{2+i}$
$= \frac{5(2+i)}{4+1}$
$= \frac{5(2+i)}{5}$
$= 2+i$

34. $2\sqrt{-49}+3\sqrt{-64} = 2(7i)+3(8i)$
$= 14i+24i$
$= 38i$

35. $43x+575 = 1177$
$43x = 602$
$x = 14$
The system's income will be \$1177 billion 14 years after 2004, or 2018.

36. $B = 0.07x^2+47.4x+500$
$1177 = 0.07x^2+47.4x+500$
$0 = 0.07x^2+47.4x-677$
$0 = 0.07x^2+47.4x-677$
$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x = \frac{-(47.4)\pm\sqrt{(47.4)^2-4(0.07)(-677)}}{2(0.07)}$
$x \approx 14, \quad x \approx -691$ (rejected)
The system's income will be \$1177 billion 14 years after 2004, or 2018.

37. The formulas model the data quite well.

38. Let $x =$ the number drive-in theaters.
Let $x+16 =$ the number movie theaters.
Let $x+64 =$ the number video rental stores.
$(x)+(x+16)+(x+64) = 83$
$x+x+16+x+64 = 83$
$3x+80 = 83$
$3x = 3$
$x = 1$
$x+16 = 17$
$x+64 = 65$
For every one million U.S. residents, there is 1 drive-in theater, 17 movie theaters, and 65 video rental stores..

39. $29700+150x = 5000+1100x$
$24700 = 950x$
$26 = x$
In 26 years, the cost will be \$33,600.

40. Let x = amount invested at 8%
$10000 - x$ = amount invested at 10%
$0.08x+0.1(10000-x) = 940$
$0.08x+1000-0.1x = 940$
$-0.02x = -60$
$x = 3000$
$10000-x = 7000$
\$3000 at 8%, \$7000 at 10%

41. $l = 2w+4$
$A = lw$
$48 = (2w+4)w$
$48 = 2w^2+4w$
$0 = 2w^2+4w-48$
$0 = w^2+2w-24$
$0 = (w+6)(w-4)$

$w+6 = 0 \qquad w-4 = 0$
$w = -6 \qquad w = 4$
$2w+4 = 2(4)+4 = 12$
width is 4 feet, length is 12 feet

42. $24^2+x^2 = 26^2$
$576+x^2 = 676$
$x^2 = 100$
$x = \pm 10$
The wire should be attached 10 feet up the pole.

43. Let x = the original selling price

$20 = x - 0.60x$

$20 = 0.40x$

$50 = x$

The original price is \$50.

44. Let x = the number of local calls

The monthly cost using Plan *A* is $C_A = 25$.

The monthly cost using Plan *B* is $C_B = 13 + 0.06x$.

For Plan *A* to be better deal, it must cost less than Plan *B*.

$C_A < C_B$

$25 < 13 + 0.06x$

$12 < 0.06x$

$200 < x$

$x > 200$

Plan *A* is a better deal when more than 200 local calls are made per month.

Chapter 2
Functions and Graphs

Section 2.1

Check Point Exercises

1. The domain is the set of all first components: {0, 10, 20, 30, 36}. The range is the set of all second components: {9.1, 6.7, 10.7, 13.2, 17.4}.

2. a. The relation is not a function since the two ordered pairs (5, 6) and (5, 8) have the same first component but different second components.

b. The relation is a function since no two ordered pairs have the same first component and different second components.

3. a. $2x + y = 6$

$y = -2x + 6$

For each value of *x*, there is one and only one value for *y*, so the equation defines *y* as a function of *x*.

b. $x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$y = \pm\sqrt{1 - x^2}$

Since there are values of *x* (all values between –1 and 1 exclusive) that give more than one value for *y* (for example, if *x* = 0, then $y = \pm\sqrt{1 - 0^2} = \pm 1$), the equation does not define *y* as a function of *x*.

4. a. $f(-5) = (-5)^2 - 2(-5) + 7$

$= 25 - (-10) + 7$

$= 42$

b. $f(x + 4) = (x + 4)^2 - 2(x + 4) + 7$

$= x^2 + 8x + 16 - 2x - 8 + 7$

$= x^2 + 6x + 15$

c. $f(-x) = (-x)^2 - 2(-x) + 7$

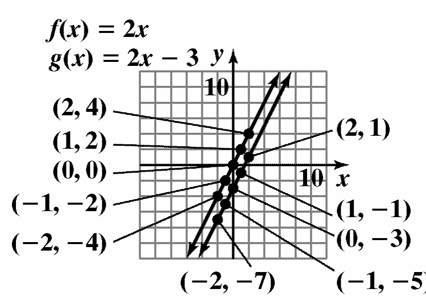
$= x^2 - (-2x) + 7$

$= x^2 + 2x + 7$

5.

x	$f(x) = 2x$	(x, y)
-2	–4	$(-2, -4)$
-1	–2	$(-1, -2)$
0	0	$(0, 0)$
1	2	$(1, 2)$
2	4	$(2, 4)$

x	$g(x) = 2x - 3$	(x, y)
-2	$g(-2) = 2(-2) - 3 = -7$	$(-2, -7)$
-1	$g(-1) = 2(-1) - 3 = -5$	$(-1, -5)$
0	$g(0) = 2(0) - 3 = -3$	$(0, -3)$
1	$g(1) = 2(1) - 3 = -1$	$(1, -1)$
2	$g(2) = 2(2) - 3 = 1$	$(2, 1)$



The graph of *g* is the graph of *f* shifted down 3 units.

6. The graph (c) fails the vertical line test and is therefore not a function.
y is a function of *x* for the graphs in (a) and (b).

7. a. $f(5) = 400$

b. $x = 9$, $f(9) = 100$

c. The minimum T cell count in the asymptomatic stage is approximately 425.

8. **a.** domain: $\{x|-2 \le x \le 1\}$ or $[-2,1]$.
range: $\{y|0 \le y \le 3\}$ or $[0,3]$.

b. domain: $\{x|-2 < x \le 1\}$ or $(-2,1]$.
range: $\{y|-1 \le y < 2\}$ or $[-1,2)$.

c. domain: $\{x|-3 \le x < 0\}$ or $[-3,0)$.
range: $\{y|y = -3,-2,-1\}$.

Exercise Set 2.1

1. The relation is a function since no two ordered pairs have the same first component and different second components. The domain is {1, 3, 5} and the range is {2, 4, 5}.

2. The relation is a function because no two ordered pairs have the same first component and different second components The domain is {4, 6, 8} and the range is {5, 7, 8}.

3. The relation is not a function since the two ordered pairs (3, 4) and (3, 5) have the same first component but different second components (the same could be said for the ordered pairs (4, 4) and (4, 5)). The domain is {3, 4} and the range is {4, 5}.

4. The relation is not a function since the two ordered pairs (5, 6) and (5, 7) have the same first component but different second components (the same could be said for the ordered pairs (6, 6) and (6, 7)). The domain is {5, 6} and the range is {6, 7}.

5. The relation is a function because no two ordered pairs have the same first component and different second components The domain is {3, 4, 5, 7} and the range is {–2, 1, 9}.

6. The relation is a function because no two ordered pairs have the same first component and different second components The domain is {–2, –1, 5, 10} and the range is {1, 4, 6}.

7. The relation is a function since there are no same first components with different second components. The domain is {–3, –2, –1, 0} and the range is {–3, –2, –1, 0}.

8. The relation is a function since there are no ordered pairs that have the same first component but different second components. The domain is {–7, –5, –3, 0} and the range is {–7, –5, –3, 0}.

9. The relation is not a function since there are ordered pairs with the same first component and different second components. The domain is {1} and the range is {4, 5, 6}.

10. The relation is a function since there are no two ordered pairs that have the same first component and different second components. The domain is {4, 5, 6} and the range is {1}.

11. $x + y = 16$
$y = 16 - x$
Since only one value of *y* can be obtained for each value of *x*, *y* is a function of *x*.

12. $x + y = 25$
$y = 25 - x$
Since only one value of *y* can be obtained for each value of *x*, *y* is a function of *x*.

13. $x^2 + y = 16$
$y = 16 - x^2$
Since only one value of *y* can be obtained for each value of *x*, *y* is a function of *x*.

14. $x^2 + y = 25$
$y = 25 - x^2$
Since only one value of *y* can be obtained for each value of *x*, *y* is a function of *x*.

15. $x^2 + y^2 = 16$
$y^2 = 16 - x^2$
$y = \pm\sqrt{16 - x^2}$
If $x = 0$, $y = \pm 4$.
Since two values, $y = 4$ and $y = -4$, can be obtained for one value of *x*, *y* is not a function of *x*.

16. $x^2 + y^2 = 25$
$y^2 = 25 - x^2$
$y = \pm\sqrt{25 - x^2}$
If $x = 0$, $y = \pm 5$.
Since two values, $y = 5$ and $y = -5$, can be obtained for one value of *x*, *y* is not a function of *x*.

17. $x = y^2$
$y = \pm\sqrt{x}$
If $x = 1$, $y = \pm 1$.
Since two values, $y = 1$ and $y = -1$, can be obtained for $x = 1$, *y* is not a function of *x*.

18. $4x = y2$

$y = \pm\sqrt{4x} = \pm 2\sqrt{x}$

If $x = 1$, then $y = \pm 2$.

Since two values, $y = 2$ and $y = -2$, can be obtained for $x = 1$, y is not a function of x.

19. $y = \sqrt{x+4}$

Since only one value of y can be obtained for each value of x, y is a function of x.

20. $y = -\sqrt{x+4}$

Since only one value of y can be obtained for each value of x, y is a function of x.

21. $x + y^3 = 8$

$y^3 = 8 - x$

$y = \sqrt[3]{8-x}$

Since only one value of y can be obtained for each value of x, y is a function of x.

22. $x + y^3 = 27$

$y^3 = 27 - x$

$y = \sqrt[3]{27-x}$

Since only one value of y can be obtained for each value of x, y is a function of x.

23. $xy + 2y = 1$

$y(x+2) = 1$

$y = \dfrac{1}{x+2}$

Since only one value of y can be obtained for each value of x, y is a function of x.

24. $xy - 5y = 1$

$y(x-5) = 1$

$y = \dfrac{1}{x-5}$

Since only one value of y can be obtained for each value of x, y is a function of x.

25. $|x| - y = 2$

$-y = -|x| + 2$

$y = |x| - 2$

Since only one value of y can be obtained for each value of x, y is a function of x.

26. $|x| - y = 5$

$-y = -|x| + 5$

$y = |x| - 5$

Since only one value of y can be obtained for each value of x, y is a function of x.

27. **a.** $f(6) = 4(6) + 5 = 29$

b. $f(x + 1) = 4(x + 1) + 5 = 4x + 9$

c. $f(-x) = 4(-x) + 5 = -4x + 5$

28. **a.** $f(4) = 3(4) + 7 = 19$

b. $f(x + 1) = 3(x + 1) + 7 = 3x + 10$

c. $f(-x) = 3(-x) + 7 = -3x + 7$

29. **a.** $g(-1) = (-1)^2 + 2(-1) + 3$

$= 1 - 2 + 3$

$= 2$

b. $g(x+5) = (x+5)^2 + 2(x+5) + 3$

$= x^2 + 10x + 25 + 2x + 10 + 3$

$= x^2 + 12x + 38$

c. $g(-x) = (-x)^2 + 2(-x) + 3$

$= x^2 - 2x + 3$

30. **a.** $g(-1) = (-1)^2 - 10(-1) - 3$

$= 1 + 10 - 3$

$= 8$

b. $g(x+2) = (x+2)^2 - 10(8+2) - 3$

$= x^2 + 4x + 4 - 10x - 20 - 3$

$= x^2 - 6x - 19$

c. $g(-x) = (-x)^2 - 10(-x) - 3$

$= x^2 + 10x - 3$

31. **a.** $h(2) = 2^4 - 2^2 + 1$

$= 16 - 4 + 1$

$= 13$

b. $h(-1) = (-1)^4 - (-1)^2 + 1$

$= 1 - 1 + 1$

$= 1$

c. $h(-x) = (-x)^4 - (-x)^2 + 1 = x^4 - x^2 + 1$

d. $h(3a) = (3a)^4 - (3a)^2 + 1$

$= 81a^4 - 9a^2 + 1$

32. a. $h(3) = 3^3 - 3 + 1 = 25$

b. $h(-2) = (-2)^3 - (-2) + 1$
$= -8 + 2 + 1$
$= -5$

c. $h(-x) = (-x)^3 - (-x) + 1 = -x^3 + x + 1$

d. $h(3a) = (3a)^3 - (3a) + 1$
$= 27a^3 - 3a + 1$

33. a. $f(-6) = \sqrt{-6+6} + 3 = \sqrt{0} + 3 = 3$

b. $f(10) = \sqrt{10+6} + 3$
$= \sqrt{16} + 3$
$= 4 + 3$
$= 7$

c. $f(x-6) = \sqrt{x-6+6} + 3 = \sqrt{x} + 3$

34. a. $f(16) = \sqrt{25-16} - 6 = \sqrt{9} - 6 = 3 - 6 = -3$

b. $f(-24) = \sqrt{25-(-24)} - 6$
$= \sqrt{49} - 6$
$= 7 - 6 = 1$

c. $f(25-2x) = \sqrt{25-(25-2x)} - 6$
$= \sqrt{2x} - 6$

35. a. $f(2) = \dfrac{4(2)^2 - 1}{2^2} = \dfrac{15}{4}$

b. $f(-2) = \dfrac{4(-2)^2 - 1}{(-2)^2} = \dfrac{15}{4}$

c. $f(-x) = \dfrac{4(-x)^2 - 1}{(-x)^2} = \dfrac{4x^2 - 1}{x^2}$

36. a. $f(2) = \dfrac{4(2)^3 + 1}{2^3} = \dfrac{33}{8}$

b. $f(-2) = \dfrac{4(-2)^3 + 1}{(-2)^3} = \dfrac{-31}{-8} = \dfrac{31}{8}$

c. $f(-x) = \dfrac{4(-x)^3 + 1}{(-x)^3} = \dfrac{-4x^3 + 1}{-x^3}$
or $\dfrac{4x^3 - 1}{x^3}$

37. a. $f(6) = \dfrac{6}{|6|} = 1$

b. $f(-6) = \dfrac{-6}{|-6|} = \dfrac{-6}{6} = -1$

c. $f(r^2) = \dfrac{r^2}{|r^2|} = \dfrac{r^2}{r^2} = 1$

38. a. $f(5) = \dfrac{|5+3|}{5+3} = \dfrac{|8|}{8} = 1$

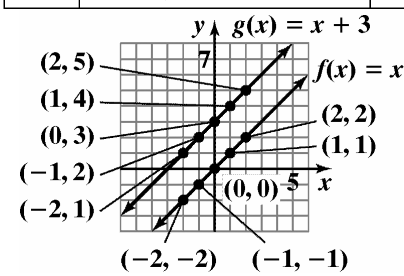
b. $f(-5) = \dfrac{|-5+3|}{-5+3} = \dfrac{|-2|}{-2} = \dfrac{2}{-2} = -1$

c. $f(-9-x) = \dfrac{|-9-x+3|}{-9-x+3}$
$= \dfrac{|-x-6|}{-x-6} = \begin{cases} 1, & \text{if } x < -6 \\ -1, & \text{if } x > -6 \end{cases}$

39.

x	$f(x) = x$	(x, y)
–2	$f(-2) = -2$	$(-2,-2)$
–1	$f(-1) = -1$	$(-1,-1)$
0	$f(0) = 0$	$(0,0)$
1	$f(1) = 1$	$(1,1)$
2	$f(2) = 2$	$(2,2)$

x	$g(x) = x+3$	(x, y)
–2	$g(-2) = -2+3 = 1$	$(-2,1)$
–1	$g(-1) = -1+3 = 2$	$(-1,2)$
0	$g(0) = 0+3 = 3$	$(0,3)$
1	$g(1) = 1+3 = 4$	$(1,4)$
2	$g(2) = 2+3 = 5$	$(2,5)$

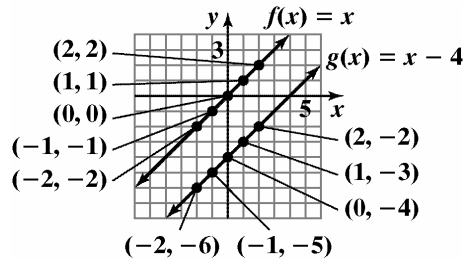


The graph of g is the graph of f shifted up 3 units.

40.

x	$f(x) = x$	(x, y)
–2	$f(-2) = -2$	$(-2,-2)$
–1	$f(-1) = -1$	$(-1,-1)$
0	$f(0) = 0$	$(0,0)$
1	$f(1) = 1$	$(1,1)$
2	$f(2) = 2$	$(2,2)$

x	$g(x) = x-4$	(x, y)
–2	$g(-2) = -2-4 = -6$	$(-2,-6)$
–1	$g(-1) = -1-4 = -5$	$(-1,-5)$
0	$g(0) = 0-4 = -4$	$(0,-4)$
1	$g(1) = 1-4 = -3$	$(1,-3)$
2	$g(2) = 2-4 = -2$	$(2,-2)$

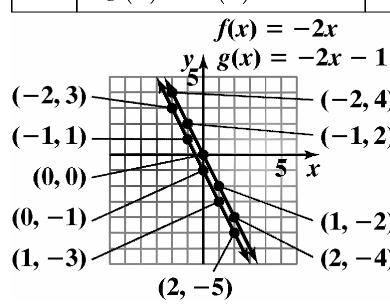


The graph of g is the graph of f shifted down 4 units.

41.

x	$f(x) = -2x$	(x, y)
–2	$f(-2) = -2(-2) = 4$	$(-2,4)$
–1	$f(-1) = -2(-1) = 2$	$(-1,2)$
0	$f(0) = -2(0) = 0$	$(0,0)$
1	$f(1) = -2(1) = -2$	$(1,-2)$
2	$f(2) = -2(2) = -4$	$(2,-4)$

x	$g(x) = -2x-1$	(x, y)
–2	$g(-2) = -2(-2)-1 = 3$	$(-2,3)$
–1	$g(-1) = -2(-1)-1 = 1$	$(-1,1)$
0	$g(0) = -2(0)-1 = -1$	$(0,-1)$
1	$g(1) = -2(1)-1 = -3$	$(1,-3)$
2	$g(2) = -2(2)-1 = -5$	$(2,-5)$

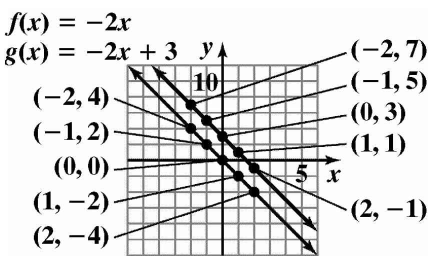


The graph of g is the graph of f shifted down 1 unit.

42.

x	$f(x) = -2x$	(x, y)
–2	$f(-2) = -2(-2) = 4$	$(-2, 4)$
–1	$f(-1) = -2(-1) = 2$	$(-1, 2)$
0	$f(0) = -2(0) = 0$	$(0, 0)$
1	$f(1) = -2(1) = -2$	$(1, -2)$
2	$f(2) = -2(2) = -4$	$(2, -4)$

x	$g(x) = -2x + 3$	(x, y)
–2	$g(-2) = -2(-2) + 3 = 7$	$(-2, 7)$
–1	$g(-1) = -2(-1) + 3 = 5$	$(-1, 5)$
0	$g(0) = -2(0) + 3 = 3$	$(0, 3)$
1	$g(1) = -2(1) + 3 = 1$	$(1, 1)$
2	$g(2) = -2(2) + 3 = -1$	$(2, -1)$

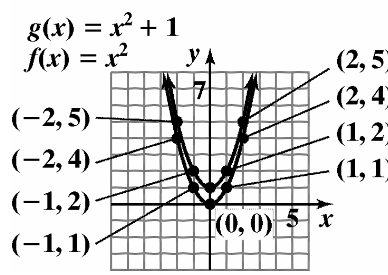


The graph of g is the graph of f shifted up 3 units.

43.

x	$f(x) = x^2$	(x, y)
–2	$f(-2) = (-2)^2 = 4$	$(-2, 4)$
–1	$f(-1) = (-1)^2 = 1$	$(-1, 1)$
0	$f(0) = (0)^2 = 0$	$(0, 0)$
1	$f(1) = (1)^2 = 1$	$(1, 1)$
2	$f(2) = (2)^2 = 4$	$(2, 4)$

x	$g(x) = x^2 + 1$	(x, y)
–2	$g(-2) = (-2)^2 + 1 = 5$	$(-2, 5)$
–1	$g(-1) = (-1)^2 + 1 = 2$	$(-1, 2)$
0	$g(0) = (0)^2 + 1 = 1$	$(0, 1)$
1	$g(1) = (1)^2 + 1 = 2$	$(1, 2)$
2	$g(2) = (2)^2 + 1 = 5$	$(2, 5)$

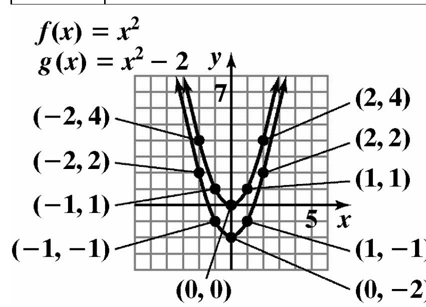


The graph of g is the graph of f shifted up 1 unit.

44.

x	$f(x) = x^2$	(x, y)
–2	$f(-2) = (-2)^2 = 4$	$(-2, 4)$
–1	$f(-1) = (-1)^2 = 1$	$(-1, 1)$
0	$f(0) = (0)^2 = 0$	$(0, 0)$
1	$f(1) = (1)^2 = 1$	$(1, 1)$
2	$f(2) = (2)^2 = 4$	$(2, 4)$

x	$g(x) = x^2 - 2$	(x, y)
–2	$g(-2) = (-2)^2 - 2 = 2$	$(-2, 2)$
–1	$g(-1) = (-1)^2 - 2 = -1$	$(-1, -1)$
0	$g(0) = (0)^2 - 2 = -2$	$(0, -2)$
1	$g(1) = (1)^2 - 2 = -1$	$(1, -1)$
2	$g(2) = (2)^2 - 2 = 2$	$(2, 2)$

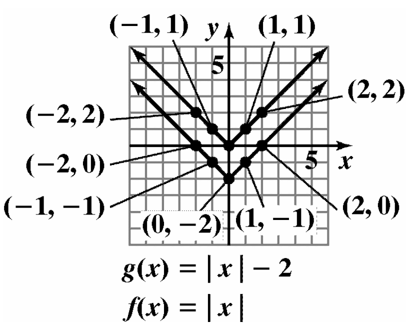


The graph of g is the graph of f shifted down 2 units.

45.

x	$f(x)=\lvert x\rvert$	(x, y)
-2	$f(-2)=\lvert -2\rvert=2$	$(-2,2)$
-1	$f(-1)=\lvert -1\rvert=1$	$(-1,1)$
0	$f(0)=\lvert 0\rvert=0$	$(0,0)$
1	$f(1)=\lvert 1\rvert=1$	$(1,1)$
2	$f(2)=\lvert 2\rvert=2$	$(2,2)$

x	$g(x)=\lvert x\rvert-2$	(x, y)
-2	$g(-2)=\lvert -2\rvert-2=0$	$(-2,0)$
-1	$g(-1)=\lvert -1\rvert-2=-1$	$(-1,-1)$
0	$g(0)=\lvert 0\rvert-2=-2$	$(0,-2)$
1	$g(1)=\lvert 1\rvert-2=-1$	$(1,-1)$
2	$g(2)=\lvert 2\rvert-2=0$	$(2,0)$

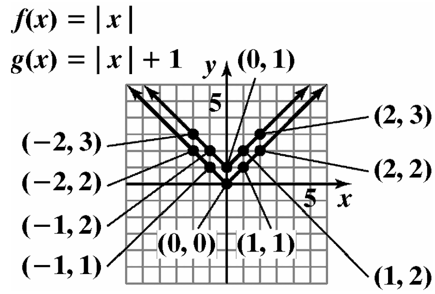


The graph of g is the graph of f shifted down 2 units.

46.

x	$f(x)=\lvert x\rvert$	(x, y)
-2	$f(-2)=\lvert -2\rvert=2$	$(-2,2)$
-1	$f(-1)=\lvert -1\rvert=1$	$(-1,1)$
0	$f(0)=\lvert 0\rvert=0$	$(0,0)$
1	$f(1)=\lvert 1\rvert=1$	$(1,1)$
2	$f(2)=\lvert 2\rvert=2$	$(2,2)$

x	$g(x)=\lvert x\rvert+1$	(x, y)
-2	$g(-2)=\lvert -2\rvert+1=3$	$(-2,3)$
-1	$g(-1)=\lvert -1\rvert+1=2$	$(-1,2)$
0	$g(0)=\lvert 0\rvert+1=1$	$(0,1)$
1	$g(1)=\lvert 1\rvert+1=2$	$(1,2)$
2	$g(2)=\lvert 2\rvert+1=3$	$(2,3)$

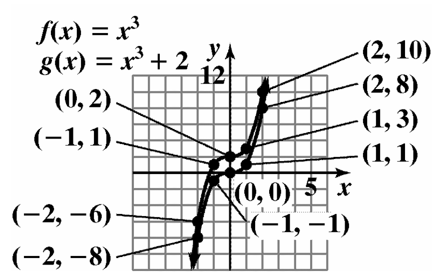


The graph of g is the graph of f shifted up 1 unit.

47.

x	$f(x)=x^3$	(x, y)
-2	$f(-2)=(-2)^3=-8$	$(-2,-8)$
-1	$f(-1)=(-1)^3=-1$	$(-1,-1)$
0	$f(0)=(0)^3=0$	$(0,0)$
1	$f(1)=(1)^3=1$	$(1,1)$
2	$f(2)=(2)^3=8$	$(2,8)$

x	$g(x)=x^3+2$	(x, y)
-2	$g(-2)=(-2)^3+2=-6$	$(-2,-6)$
-1	$g(-1)=(-1)^3+2=1$	$(-1,1)$
0	$g(0)=(0)^3+2=2$	$(0,2)$
1	$g(1)=(1)^3+2=3$	$(1,3)$
2	$g(2)=(2)^3+2=10$	$(2,10)$

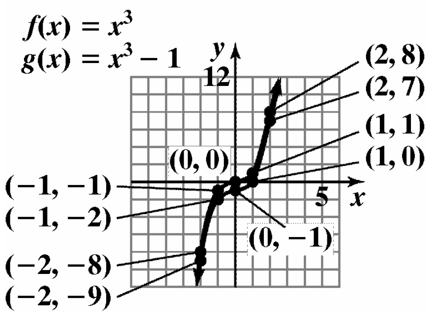


The graph of g is the graph of f shifted up 2 units.

48.

x	$f(x)=x^3$	(x, y)
–2	$f(-2)=(-2)^3=-8$	$(-2,-8)$
–1	$f(-1)=(-1)^3=-1$	$(-1,-1)$
0	$f(0)=(0)^3=0$	$(0,0)$
1	$f(1)=(1)^3=1$	$(1,1)$
2	$f(2)=(2)^3=8$	$(2,8)$

x	$g(x)=x^3-1$	(x, y)
–2	$g(-2)=(-2)^3-1=-9$	$(-2,-9)$
–1	$g(-1)=(-1)^3-1=-2$	$(-1,-2)$
0	$g(0)=(0)^3-1=-1$	$(0,-1)$
1	$g(1)=(1)^3-1=0$	$(1,0)$
2	$g(2)=(2)^3-1=7$	$(2,7)$

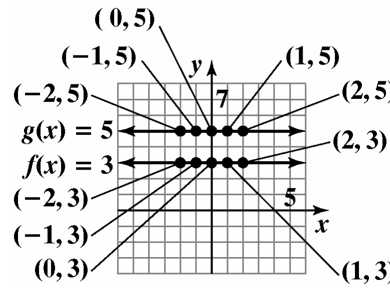


The graph of g is the graph of f shifted down 1 unit.

49.

x	$f(x)=3$	(x, y)
–2	$f(-2)=3$	$(-2,3)$
–1	$f(-1)=3$	$(-1,3)$
0	$f(0)=3$	$(0,3)$
1	$f(1)=3$	$(1,3)$
2	$f(2)=3$	$(2,3)$

x	$g(x)=5$	(x, y)
–2	$g(-2)=5$	$(-2,5)$
–1	$g(-1)=5$	$(-1,5)$
0	$g(0)=5$	$(0,5)$
1	$g(1)=5$	$(1,5)$
2	$g(2)=5$	$(2,5)$

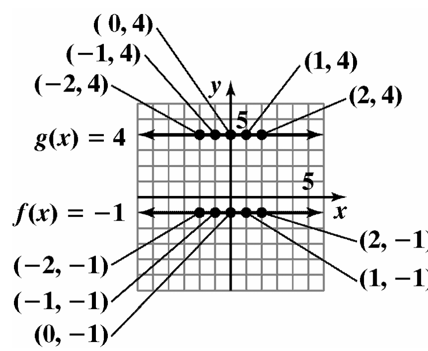


The graph of g is the graph of f shifted up 2 units.

50.

x	$f(x)=-1$	(x, y)
–2	$f(-2)=-1$	$(-2,-1)$
–1	$f(-1)=-1$	$(-1,-1)$
0	$f(0)=-1$	$(0,-1)$
1	$f(1)=-1$	$(1,-1)$
2	$f(2)=-1$	$(2,-1)$

x	$g(x)=4$	(x, y)
–2	$g(-2)=4$	$(-2,4)$
–1	$g(-1)=4$	$(-1,4)$
0	$g(0)=4$	$(0,4)$
1	$g(1)=4$	$(1,4)$
2	$g(2)=4$	$(2,4)$

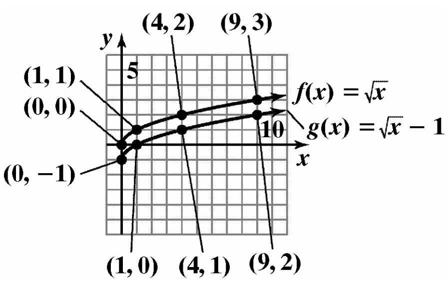


The graph of g is the graph of f shifted up 5 units.

51.

x	$f(x)=\sqrt{x}$	(x, y)
0	$f(0)=\sqrt{0}=0$	$(0,0)$
1	$f(1)=\sqrt{1}=1$	$(1,1)$
4	$f(4)=\sqrt{4}=2$	$(4,2)$
9	$f(9)=\sqrt{9}=3$	$(9,3)$

x	$g(x)=\sqrt{x}-1$	(x, y)
0	$g(0)=\sqrt{0}-1=-1$	$(0,-1)$
1	$g(1)=\sqrt{1}-1=0$	$(1,0)$
4	$g(4)=\sqrt{4}-1=1$	$(4,1)$
9	$g(9)=\sqrt{9}-1=2$	$(9,2)$

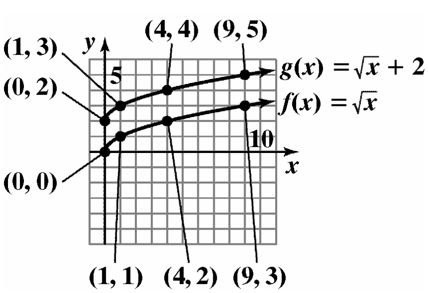


The graph of g is the graph of *f* shifted down 1 unit.

52.

x	$f(x)=\sqrt{x}$	(x, y)
0	$f(0)=\sqrt{0}=0$	$(0,0)$
1	$f(1)=\sqrt{1}=1$	$(1,1)$
4	$f(4)=\sqrt{4}=2$	$(4,2)$
9	$f(9)=\sqrt{9}=3$	$(9,3)$

x	$g(x)=\sqrt{x}+2$	(x, y)
0	$g(0)=\sqrt{0}+2=2$	$(0,2)$
1	$g(1)=\sqrt{1}+2=3$	$(1,3)$
4	$g(4)=\sqrt{4}+2=4$	$(4,4)$
9	$g(9)=\sqrt{9}+2=5$	$(9,5)$

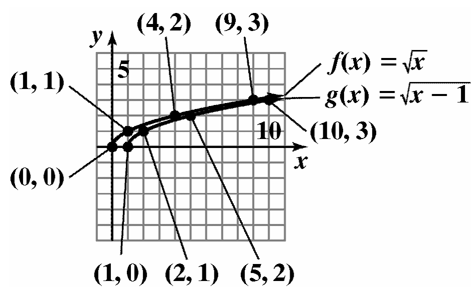


The graph of g is the graph of *f* shifted up 2 units.

53.

x	$f(x)=\sqrt{x}$	(x, y)
0	$f(0)=\sqrt{0}=0$	$(0,0)$
1	$f(1)=\sqrt{1}=1$	$(1,1)$
4	$f(4)=\sqrt{4}=2$	$(4,2)$
9	$f(9)=\sqrt{9}=3$	$(9,3)$

x	$g(x)=\sqrt{x-1}$	(x, y)
1	$g(1)=\sqrt{1-1}=0$	$(1,0)$
2	$g(2)=\sqrt{2-1}=1$	$(2,1)$
5	$g(5)=\sqrt{5-1}=2$	$(5,2)$
10	$g(10)=\sqrt{10-1}=3$	$(10,3)$

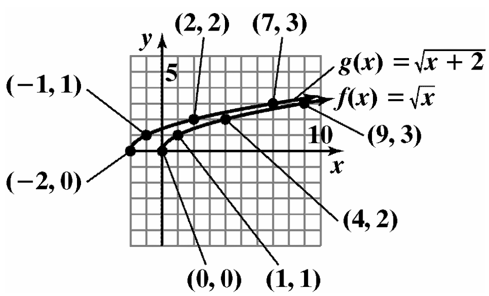


The graph of g is the graph of *f* shifted right 1 unit.

54.

x	$f(x)=\sqrt{x}$	(x, y)
0	$f(0)=\sqrt{0}=0$	$(0,0)$
1	$f(1)=\sqrt{1}=1$	$(1,1)$
4	$f(4)=\sqrt{4}=2$	$(4,2)$
9	$f(9)=\sqrt{9}=3$	$(9,3)$

x	$g(x)=\sqrt{x+2}$	(x, y)
–2	$g(-2)=\sqrt{-2+2}=0$	$(-2,0)$
–1	$g(-1)=\sqrt{-1+2}=1$	$(-1,1)$
2	$g(2)=\sqrt{2+2}=2$	$(2,2)$
7	$g(7)=\sqrt{7+2}=3$	$(7,3)$



The graph of g is the graph of *f* shifted left 2 units.

55. function

56. function

57. function

58. not a function

59. not a function

60. not a function

61. function

62. not a function

63. function

64. function

65. $f(-2)=-4$

66. $f(2)=-4$

67. $f(4)=4$

68. $f(-4)=4$

69. $f(-3)=0$

70. $f(-1)=0$

71. $g(-4)=2$

72. $g(2)=-2$

73. $g(-10)=2$

74. $g(10)=-2$

75. When $x=-2,\ g(x)=1.$

76. When $x=1,\ g(x)=-1.$

77. **a.** domain: $(-\infty, \infty)$

b. range: $[-4, \infty)$

c. x-intercepts: –3 and 1

d. y-intercept: –3

e. $f(-2)=-3$ and $f(2)=5$

78. **a.** domain: $(-\infty, \infty)$

b. range: $(-\infty, 4]$

c. x-intercepts: –3 and 1

d. y-intercept: 3

e. $f(-2)=3$ and $f(2)=-5$

79. **a.** domain: $(-\infty, \infty)$

b. range: $[1, \infty)$

c. x-intercept: none

d. y-intercept: 1

e. $f(-1)=2$ and $f(3)=4$

80. **a.** domain: $(-\infty, \infty)$
b. range: $[0, \infty)$
c. x-intercept: -1
d. y-intercept: 1
e. $f(-4) = 3$ and $f(3) = 4$

81. **a.** domain: $[0, 5)$
b. range: $[-1, 5)$
c. x-intercept: 2
d. y-intercept: -1
e. $f(3) = 1$

82. **a.** domain: $(-6, 0]$
b. range: $[-3, 4)$
c. x-intercept: -3.75
d. y-intercept: -3
e. $f(-5) = 2$

83. **a.** domain: $[0, \infty)$
b. range: $[1, \infty)$
c. x-intercept: none
d. y-intercept: 1
e. $f(4) = 3$

84. **a.** domain: $[-1, \infty)$
b. range: $[0, \infty)$
c. x-intercept: -1
d. y-intercept: 1
e. $f(3) = 2$

85. **a.** domain: $[-2, 6]$
b. range: $[-2, 6]$
c. x-intercept: 4
d. y-intercept: 4
e. $f(-1) = 5$

86. **a.** domain: $[-3, 2]$
b. range: $[-5, 5]$
c. x-intercept: $-\frac{1}{2}$
d. y-intercept: 1
e. $f(-2) = -3$

87. **a.** domain: $(-\infty, \infty)$
b. range: $(-\infty, -2]$
c. x-intercept: none
d. y-intercept: -2
e. $f(-4) = -5$ and $f(4) = -2$

88. **a.** domain: $(-\infty, \infty)$
b. range: $[0, \infty)$
c. x-intercept: $\{x \mid x \le 0\}$
d. y-intercept: 0
e. $f(-2) = 0$ and $f(2) = 4$

89. **a.** domain: $(-\infty, \infty)$
b. range: $(0, \infty)$
c. x-intercept: none
d. y-intercept: 1.5
e. $f(4) = 6$

90. **a.** domain: $(-\infty, 1) \cup (1, \infty)$
b. range: $(-\infty, 0) \cup (0, \infty)$
c. x-intercept: none
d. y-intercept: -1
e. $f(2) = 1$

91. **a.** domain: $\{-5, -2, 0, 1, 3\}$
b. range: $\{2\}$
c. x-intercept: none
d. y-intercept: 2
e. $f(-5) + f(3) = 2 + 2 = 4$

92. **a.** domain: $\{-5, -2, 0, 1, 4\}$

b. range: $\{-2\}$

c. x-intercept: none

d. y-intercept: -2

e. $f(-5)+f(4)=-2+(-2)=-4$

93.
$$g(1)=3(1)-5=3-5=-2$$
$$f(g(1))=f(-2)=(-2)^2-(-2)+4$$
$$=4+2+4=10$$

94.
$$g(-1)=3(-1)-5=-3-5=-8$$
$$f(g(-1))=f(-8)=(-8)^2-(-8)+4$$
$$=64+8+4=76$$

95. $\sqrt{3-(-1)}-(-6)^2+6\div(-6)\cdot 4$
$$=\sqrt{3+1}-36+6\div(-6)\cdot 4$$
$$=\sqrt{4}-36+-1\cdot 4$$
$$=2-36+-4$$
$$=-34+-4$$
$$=-38$$

96. $|-4-(-1)|-(-3)^2+-3\div 3\cdot -6$
$$=|-4+1|-9+-3\div 3\cdot -6$$
$$=|-3|-9+-1\cdot -6$$
$$=3-9+6=-6+6=0$$

97. $f(-x)-f(x)$
$$=(-x)^3+(-x)-5-(x^3+x-5)$$
$$=-x^3-x-5-x^3-x+5=-2x^3-2x$$

98. $f(-x)-f(x)$
$$=(-x)^2-3(-x)+7-(x^2-3x+7)$$
$$=x^2+3x+7-x^2+3x-7$$
$$=6x$$

99. **a.** {(Iceland, 9.7), (Finland, 9.6), (New Zealand, 9.6), (Denmark, 9.5)}

b. Yes, the relation is a function. Each element in the domain corresponds to only one element in the range.

c. {(9.7, Iceland), (9.6, Finland), (9.6, New Zealand), (9.5, Denmark)}

d. No, the relation is not a function. 9.6 in the domain corresponds to both Finland and New Zealand in the range.

100. a. $\{(\text{Bangladesh, 1.7}), (\text{Chad, 1.7}), (\text{Haiti, 1.8}), (\text{Myanmar, 1.8})\}$

b. Yes, the relation is a function. Each element in the domain corresponds to only one element in the range.

c. $\{(\text{1.7, Bangladesh}), (\text{1.7, Chad}), (\text{1.8, Haiti}), (\text{1.8, Myanmar})\}$

d. No, the relation is not a function. 1.7 in the domain corresponds to both Bangladesh and Chad in the range.

101. a. $f(70) = 83$ which means the chance that a 60-year old will survive to age 70 is 83%.

b. $g(70) = 76$ which means the chance that a 60-year old will survive to age 70 is 76%.

c. Function *f* is the better model.

102. a. $f(90) = 25$ which means the chance that a 60-year old will survive to age 90 is 25%.

b. $g(90) = 10$ which means the chance that a 60-year old will survive to age 90 is 10%.

c. Function *f* is the better model.

103. a. $T(x) = -0.125x^2 + 5.25x + 72$

$T(20) = -0.125(20)^2 + 5.25(20) + 72 = 127$

Americans ordered an average of 127 takeout meals per person 20 years after 1984, or 2004.
This is represented on the graph by the point (20,127).

b. $R(x) = -0.6x + 94$

$R(0) = -0.6(0) + 94 = 94$

Americans ordered an average of 94 meals in restaurants per person 0 years after 1984, or 1984.
This is represented on the graph by the point (0,94).

c. According to the graphs, the average number of takeout orders approximately equaled the average number of in-restaurant meals 4 years after 1984, or 1988.

$T(x) = -0.125x^2 + 5.25x + 72$

$T(4) = -0.125(4)^2 + 5.25(4) + 72 = 91$

In 1988 Americans ordered an average of 91 takeout meals per person.

$R(x) = -0.6x + 94$

$R(4) = -0.6(4) + 94 = 91.6$

In 1988 Americans ordered an average of 91.6 meals in restaurants per person.

104. a. $T(x) = -0.125x^2 + 5.25x + 72$

$T(18) = -0.125(18)^2 + 5.25(18) + 72 = 126$

Americans ordered an average of 126 takeout meals per person 18 years after 1984, or 2002.
This is represented on the graph by the point (18,126).

b. $R(x) = -0.6x + 94$

$R(20) = -0.6(20) + 94 = 82$

Americans ordered an average of 82 meals in restaurants per person 20 years after 1984, or 2004.
This is represented on the graph by the point (20,82).

105. $C(x) = 100,000 + 100x$

$C(90) = 100,000 + 100(90) = \$109,000$

It will cost $109,000 to produce 90 bicycles.

106. $V(x) = 22,500 - 3200x$

$V(3) = 22,500 - 3200(3) = \$12,900$

After 3 years, the car will be worth $12,900.

107.

$$T(x) = \frac{40}{x} + \frac{40}{x+30}$$

$$T(30) = \frac{40}{30} + \frac{40}{30+30} = \frac{80}{60} + \frac{40}{60} = \frac{120}{60} = 2$$

If you travel 30 mph going and 60 mph returning, your total trip will take 2 hours.

108. $S(x) = 0.10x + 0.60(50 - x)$

$S(30) = 0.10(30) + 0.60(50 - 30) = 15$

When 30 mL of the 10% mixture is mixed with 20 mL of the 60% mixture, there will be 15 mL of sodium-iodine in the vaccine.

109. – 117. Answers may vary.

118. makes sense

119. does not make sense; Explanations will vary. Sample explanation: The parentheses used in function notation, such as $f(x)$, do not imply multiplication.

120. does not make sense; Explanations will vary. Sample explanation: The domain is the number of years worked for the company.

121. does not make sense; Explanations will vary. Sample explanation: This would not be a function because some elements in the domain would correspond to more than one age in the range.

122. false; Changes to make the statement true will vary. A sample change is: The domain is $[-4, 4]$.

123. false; Changes to make the statement true will vary. A sample change is: The range is $[-2, 2)$.

124. true

125. false; Changes to make the statement true will vary. A sample change is: $f(0) = 0.8$

126. $f(a+h) = 3(a+h) + 7 = 3a + 3h + 7$

$f(a) = 3a + 7$

$$\frac{f(a+h) - f(a)}{h} = \frac{(3a+3h+7) - (3a+7)}{h} = \frac{3a+3h+7-3a-7}{h} = \frac{3h}{h} = 3$$

127. Answers may vary.
An example is $\{(1,1),(2,1)\}$

128. It is given that $f(x+y) = f(x) + f(y)$ and $f(1) = 3$.

To find $f(2)$, rewrite 2 as 1 + 1.

$f(2) = f(1+1) = f(1) + f(1)$
$= 3 + 3 = 6$

Similarly:

$f(3) = f(2+1) = f(2) + f(1)$
$= 6 + 3 = 9$

$f(4) = f(3+1) = f(3) + f(1)$
$= 9 + 3 = 12$

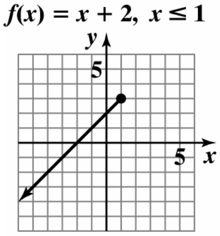
While $f(x+y) = f(x) + f(y)$ is true for this function, it is not true for all functions. It is not true for $f(x) = x^2$, for example.

129. $C(t) = 20 + 0.40(t - 60)$

$C(100) = 20 + 0.40(100 - 60)$
$= 20 + 0.40(40)$
$= 20 + 16$
$= 36$

For 100 calling minutes, the monthly cost is $36.

130. $\boldsymbol{f(x) = x + 2,\ x \le 1}$



131. $2(x+h)^2 + 3(x+h) + 5 - (2x^2 + 3x + 5)$

$= 2(x^2 + 2xh + h^2) + 3x + 3h + 5 - 2x^2 - 3x - 5$

$= 2x^2 + 4xh + 2h^2 + 3x + 3h + 5 - 2x^2 - 3x - 5$

$= 2x^2 - 2x^2 + 4xh + 2h^2 + 3x - 3x + 3h + 5 - 5$

$= 4xh + 2h^2 + 3h$

Section 2.2

Check Point Exercises

1. The function is increasing on the interval (–∞, –1), decreasing on the interval (–1, 1), and increasing on the interval (1, ∞).

2. **a.** $f(-x) = (-x)^2 + 6 = x^2 + 6 = f(x)$
The function is even.

b. $g(-x) = 7(-x)^3 - (-x) = -7x^3 + x = -f(x)$
The function is odd.

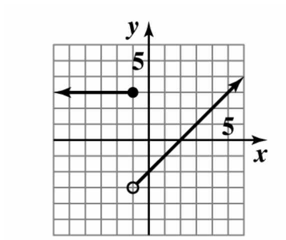
c. $h(-x) = (-x)^5 + 1 = -x^5 + 1$
The function is neither even nor odd.

3. $C(t) = \begin{cases} 20 & \text{if } 0 \le t \le 60 \\ 20 + 0.40(t - 60) & \text{if } t > 60 \end{cases}$

b. Since $0 \le 40 \le 60$, $C(40) = 20$
With 40 calling minutes, the cost is \$20.
This is represented by $(40, 20)$.

c. Since $80 > 60$,
$C(80) = 20 + 0.40(80 - 60) = 28$
With 80 calling minutes, the cost is \$28.
This is represented by $(80, 28)$.

4.



$f(x) = \begin{cases} 3 & \text{if } x \le -1 \\ x - 2 & \text{if } x > -1 \end{cases}$

5. **a.**

$$f(x) = -2x^2 + x + 5$$
$$f(x+h) = -2(x+h)^2 + (x+h) + 5$$
$$= -2(x^2 + 2xh + h^2) + x + h + 5$$
$$= -2x^2 - 4xh - 2h^2 + x + h + 5$$

b.

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{-2x^2 - 4xh - 2h^2 + x + h + 5 - \left(-2x^2 + x + 5\right)}{h}$$
$$= \frac{-2x^2 - 4xh - 2h^2 + x + h + 5 + 2x^2 - x - 5}{h}$$
$$= \frac{-4xh - 2h^2 + h}{h}$$
$$= \frac{h\left(-4x - 2h + 1\right)}{h}$$
$$= -4x - 2h + 1$$

Exercise Set 2.2

1. **a.** increasing: $(-1, \infty)$

b. decreasing: $(-\infty, -1)$

c. constant: none

2. **a.** increasing: (–∞, –1)

b. decreasing: (–1, ∞)

c. constant: none

3. **a.** increasing: $(0, \infty)$

b. decreasing: none

c. constant: none

4. **a.** increasing: (–1, ∞)

b. decreasing: none

c. constant: none

5. **a.** increasing: none

b. decreasing: (–2, 6)

c. constant: none

6. **a.** increasing: (–3, 2)

b. decreasing: none

c. constant: none

7. **a.** increasing: $(-\infty, -1)$

b. decreasing: none

c. constant: $(-1, \infty)$

8. **a.** increasing: $(0, \infty)$

b. decreasing: none

c. constant: $(-\infty, 0)$

9. **a.** increasing: $(-\infty, 0)$ or $(1.5, 3)$

b. decreasing: $(0, 1.5)$ or $(3, \infty)$

c. constant: none

10. **a.** increasing: $(-5, -4)$ or $(-2, 0)$ or $(2, 4)$

b. decreasing: $(-4, -2)$ or $(0, 2)$ or $(4, 5)$

c. constant: none

11. **a.** increasing: $(-2, 4)$

b. decreasing: none

c. constant: $(-\infty, -2)$ or $(4, \infty)$

12. **a.** increasing: none

b. decreasing: $(-4, 2)$

c. constant: $(-\infty, -4)$ or $(2, \infty)$

13. **a.** $x = 0$, relative maximum = 4

b. $x = -3, 3$, relative minimum = 0

14. **a.** $x = 0$, relative maximum = 2

b. $x = -3, 3$, relative minimum = -1

15. **a.** $x = -2$, relative maximum = 21

b. $x = 1$, relative minimum = -6

16. **a.** $x = 1$, relative maximum = 30

b. $x = 4$, relative minimum = 3

17. $f(x) = x^3 + x$
$f(-x) = (-x)^3 + (-x)$
$f(-x) = -x^3 - x = -(x^3 + x)$
$f(-x) = -f(x)$, odd function

18. $f(x) = x^3 - x$
$f(-x) = (-x)^3 - (-x)$
$f(-x) = -x^3 + x = -(x^3 - x)$
$f(-x) = -f(x)$, odd function

19. $g(x) = x^2 + x$
$g(-x) = (-x)^2 + (-x)$
$g(-x) = x^2 - x$, neither

20. $g(x) = x^2 - x$
$g(-x) = (-x)^2 - (-x)$
$g(-x) = x^2 + x$, neither

21. $h(x) = x^2 - x^4$
$h(-x) = (-x)^2 - (-x)^4$
$h(-x) = x^2 - x^4$
$h(-x) = h(x)$, even function

22. $h(x) = 2x^2 + x^4$
$h(-x) = 2(-x)^2 + (-x)^4$
$h(-x) = 2x^2 + x^4$
$h(-x) = h(x)$, even function

23. $f(x) = x^2 - x^4 + 1$
$f(-x) = (-x)^2 - (-x)^4 + 1$
$f(-x) = x^2 - x^4 + 1$
$f(-x) = f(x)$, even function

24. $f(x) = 2x^2 + x^4 + 1$
$f(-x) = 2(-x)^2 + (-x)^4 + 1$
$f(-x) = 2x^2 + x^4 + 1$
$f(-x) = f(x)$, even function

25. $f(x)=\frac{1}{5}x^6-3x^2$

$f(-x)=\frac{1}{5}(-x)^6-3(-x)^2$

$f(-x)=\frac{1}{5}x^6-3x^2$

$f(-x)=f(x)$, even function

26. $f(x)=2x^3-6x^5$

$f(-x)=2(-x)^3-6(-x)^5$

$f(-x)=-2x^3+6x^5$

$f(-x)=-(2x^3-6x^5)$

$f(-x)=-f(x)$, odd function

27. $f(x)=x\sqrt{1-x^2}$

$f(-x)=-x\sqrt{1-(-x)^2}$

$f(-x)=-x\sqrt{1-x^2}$

$=-\left(x\sqrt{1-x^2}\right)$

$f(-x)=-f(x)$, odd function

28. $f(x)=x^2\sqrt{1-x^2}$

$f(-x)=(-x)^2\sqrt{1-(-x)^2}$

$f(-x)=x^2\sqrt{1-x^2}$

$f(-x)=f(x)$, even function

29. The graph is symmetric with respect to the *y*-axis. The function is even.

30. The graph is symmetric with respect to the origin. The function is odd.

31. The graph is symmetric with respect to the origin. The function is odd.

32. The graph is not symmetric with respect to the *y*-axis or the origin. The function is neither even nor odd.

33. **a.** domain: $(-\infty,\infty)$

b. range: $[-4,\infty)$

c. *x*-intercepts: 1, 7

d. *y*-intercept: 4

e. $(4,\infty)$

f. $(0,4)$

g. $(-\infty,0)$

h. $x=4$

i. $y=-4$

j. $f(-3)=4$

k. $f(2)=-2$ and $f(6)=-2$

l. neither ; $f(-x)\neq x$, $f(-x)\neq -x$

34. **a.** domain: $(-\infty,\infty)$

b. range: $(-\infty,4]$

c. *x*-intercepts: –4, 4

d. *y*-intercept: 1

e. $(-\infty,-2)$ or $(0,3)$

f. $(-2,0)$ or $(3,\infty)$

g. $(-\infty,-4]$ or $[4,\infty)$

h. $x=-2$ and $x=3$

i. $f(-2)=4$ and $f(3)=2$

j. $f(-2)=4$

k. $x=-4$ and $x=4$

l. neither ; $f(-x)\neq x$, $f(-x)\neq -x$

35. **a.** domain: $(-\infty,3]$

b. range: $(-\infty,4]$

c. *x*-intercepts: –3, 3

d. $f(0)=3$

e. $(-\infty,1)$

f. $(1,3)$

g. $(-\infty, -3]$

h. $f(1) = 4$

i. $x = 1$

j. positive; $f(-1) = +2$

36. a. domain: $(-\infty, 6]$

b. range: $(-\infty, 1]$

c. zeros of f: –3, 3

d. $f(0) = 1$

e. $(-\infty, -2)$

f. $(2, 6)$

g. $(-2, 2)$

h. $(-3, 3)$

i. $x = -5$ and $x = 5$

j. negative; $f(4) = -1$

k. neither

l. no; f(2) is not greater than the function values to the immediate left.

37. a. $f(-2) = 3(-2) + 5 = -1$

b. $f(0) = 4(0) + 7 = 7$

c. $f(3) = 4(3) + 7 = 19$

38. a. $f(-3) = 6(-3) - 1 = -19$

b. $f(0) = 7(0) + 3 = 3$

c. $f(4) = 7(4) + 3 = 31$

39. a. $g(0) = 0 + 3 = 3$

b. $g(-6) = -(-6 + 3) = -(-3) = 3$

c. $g(-3) = -3 + 3 = 0$

40. a. $g(0) = 0 + 5 = 5$

b. $g(-6) = -(-6 + 5) = -(-1) = 1$

c. $g(-5) = -5 + 5 = 0$

41. a. $h(5) = \dfrac{5^2 - 9}{5 - 3} = \dfrac{25 - 9}{2} = \dfrac{16}{2} = 8$

b. $h(0) = \dfrac{0^2 - 9}{0 - 3} = \dfrac{-9}{-3} = 3$

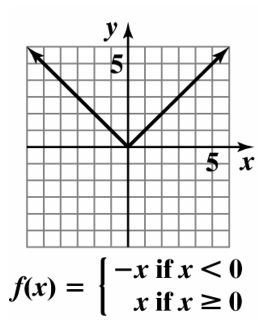
c. $h(3) = 6$

42. a. $h(7) = \dfrac{7^2 - 25}{7 - 5} = \dfrac{49 - 25}{2} = \dfrac{24}{2} = 12$

b. $h(0) = \dfrac{0^2 - 25}{0 - 5} = \dfrac{-25}{-5} = 5$

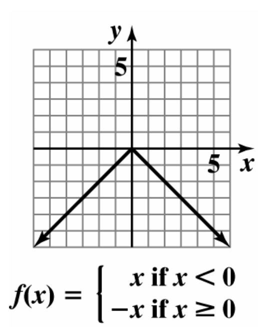
c. $h(5) = 10$

43. a.



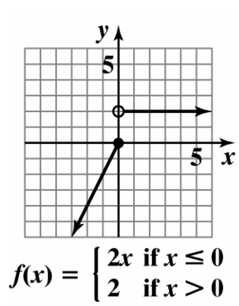
b. range: $[0, \infty)$

44. a.



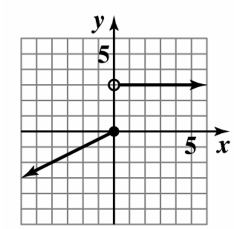
b. range: $(-\infty, 0]$

45. a.



b. range: $(-\infty, 0] \cup \{2\}$

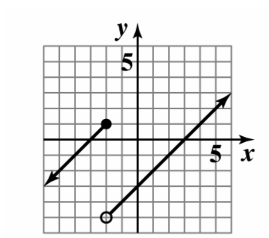
46. **a.**



$$f(x) = \begin{cases} \frac{1}{2}x & \text{if } x \le 0 \\ 3 & \text{if } x > 0 \end{cases}$$

b. range: $(-\infty, 0] \cup \{3\}$

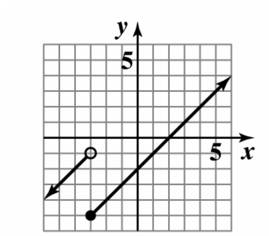
47. **a.**



$$f(x) = \begin{cases} x + 3 & \text{if } x < -2 \\ x - 3 & \text{if } x \ge -2 \end{cases}$$

b. range: $(-\infty, \infty)$

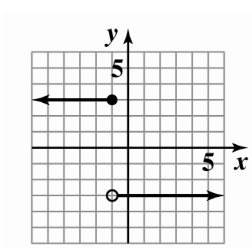
48. **a.**



$$f(x) = \begin{cases} x + 2 & \text{if } x < -3 \\ x - 2 & \text{if } x \ge -3 \end{cases}$$

b. range: $(-\infty, \infty)$

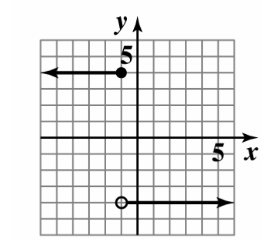
49. **a.**



$$f(x) = \begin{cases} 3 & \text{if } x \le -1 \\ -3 & \text{if } x > -1 \end{cases}$$

b. range: $\{-3, 3\}$

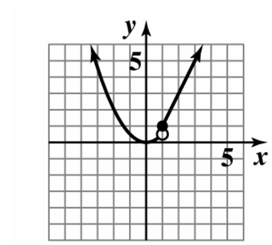
50. **a.**



$$f(x) = \begin{cases} 4 & \text{if } x \le -1 \\ -4 & \text{if } x > -1 \end{cases}$$

b. range: $\{-4, 4\}$

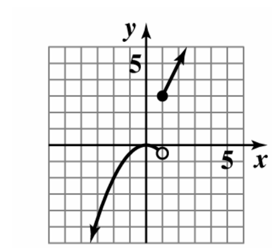
51. **a.**



$$f(x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x < 1 \\ 2x - 1 & \text{if } x \ge 1 \end{cases}$$

b. range: $[0, \infty)$

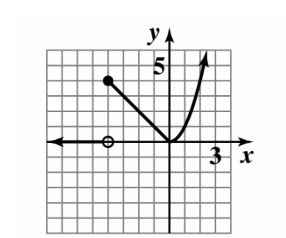
52. **a.**



$$f(x) = \begin{cases} -\frac{1}{2}x^2 & \text{if } x < 1 \\ 2x + 1 & \text{if } x \ge 1 \end{cases}$$

b. range: $(-\infty, 0] \cup [3, \infty)$

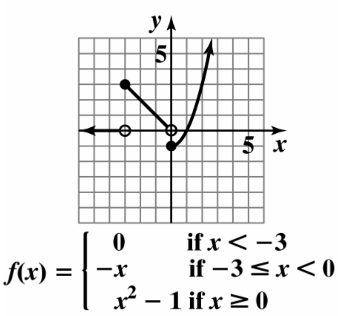
53. **a.**



$$f(x) = \begin{cases} 0 & \text{if } x < -4 \\ -x & \text{if } -4 \le x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$$

b. range: $[0, \infty)$

54. **a.**



b. range: $[-1, \infty)$

55. $\frac{f(x+h)-f(x)}{h}$

$= \frac{4(x+h)-4x}{h}$

$= \frac{4x+4h-4x}{h}$

$= \frac{4h}{h}$

$= 4$

56. $\frac{f(x+h)-f(x)}{h}$

$= \frac{7(x+h)-7x}{h}$

$= \frac{7x+7h-7x}{h}$

$= \frac{7h}{h}$

$= 7$

57. $\frac{f(x+h)-f(x)}{h}$

$= \frac{3(x+h)+7-(3x+7)}{h}$

$= \frac{3x+3h+7-3x-7}{h}$

$= \frac{3h}{h}$

$= 3$

58. $\frac{f(x+h)-f(x)}{h}$

$= \frac{6(x+h)+1-(6x+1)}{h}$

$= \frac{6x+6h+1-6x-1}{h}$

$= \frac{6h}{h}$

$= 6$

59. $\frac{f(x+h)-f(x)}{h}$

$= \frac{(x+h)^2-x^2}{h}$

$= \frac{x^2+2xh+h^2-x^2}{h}$

$= \frac{2xh+h^2}{h}$

$= \frac{h(2x+h)}{h}$

$= 2x+h$

60. $\frac{f(x+h)-f(x)}{h}$

$= \frac{2(x+h)^2-2x^2}{h}$

$= \frac{2(x^2+2xh+h^2)-2x^2}{h}$

$= \frac{2x^2+4xh+2h^2-2x^2}{h}$

$= \frac{4xh+2h^2}{h}$

$= \frac{h(4x+2h)}{h}$

$= 4x+2h$

61. $\frac{f(x+h)-f(x)}{h}$

$= \frac{(x+h)^2-4(x+h)+3-(x^2-4x+3)}{h}$

$= \frac{x^2+2xh+h^2-4x-4h+3-x^2+4x-3}{h}$

$= \frac{2xh+h^2-4h}{h}$

$= \frac{h(2x+h-4)}{h}$

$= 2x+h-4$

62. $\frac{f(x+h)-f(x)}{h}$

$=\frac{(x+h)^2-5(x+h)+8-(x^2-5x+8)}{h}$

$=\frac{x^2+2xh+h^2-5x-5h+8-x^2+5x-8}{h}$

$=\frac{2xh+h^2-5h}{h}$

$=\frac{h(2x+h-5)}{h}$

$=2x+h-5$

63. $\frac{f(x+h)-f(x)}{h}$

$=\frac{2(x+h)^2+(x+h)-1-(2x^2+x-1)}{h}$

$=\frac{2x^2+4xh+2h^2+x+h-1-2x^2-x+1}{h}$

$=\frac{4xh+2h^2+h}{h}$

$=\frac{h(4x+2h+1)}{h}$

$=4x+2h+1$

64. $\frac{f(x+h)-f(x)}{h}$

$=\frac{3(x+h)^2+(x+h)+5-(3x^2+x+5)}{h}$

$=\frac{3x^2+6xh+3h^2+x+h+5-3x^2-x-5}{h}$

$=\frac{6xh+3h^2+h}{h}$

$=\frac{h(6x+3h+1)}{h}$

$=6x+3h+1$

65. $\frac{f(x+h)-f(x)}{h}$

$=\frac{-(x+h)^2+2(x+h)+4-(-x^2+2x+4)}{h}$

$=\frac{-x^2-2xh-h^2+2x+2h+4+x^2-2x-4}{h}$

$=\frac{-2xh-h^2+2h}{h}$

$=\frac{h(-2x-h+2)}{h}$

$=-2x-h+2$

66. $\frac{f(x+h)-f(x)}{h}$

$=\frac{-(x+h)^2-3(x+h)+1-(-x^2-3x+1)}{h}$

$=\frac{-x^2-2xh-h^2-3x-3h+1+x^2+3x-1}{h}$

$=\frac{-2xh-h^2-3h}{h}$

$=\frac{h(-2x-h-3)}{h}$

$=-2x-h-3$

67. $\frac{f(x+h)-f(x)}{h}$

$=\frac{-2(x+h)^2+5(x+h)+7-(-2x^2+5x+7)}{h}$

$=\frac{-2x^2-4xh-2h^2+5x+5h+7+2x^2-5x-7}{h}$

$=\frac{-4xh-2h^2+5h}{h}$

$=\frac{h(-4x-2h+5)}{h}$

$=-4x-2h+5$

68. $\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{-3(x+h)^2+2(x+h)-1-(-3x^2+2x-1)}{h}$

$= \dfrac{-3x^2-6xh-3h^2+2x+2h-1+3x^2-2x+1}{h}$

$= \dfrac{-6xh-3h^2+2h}{h}$

$= \dfrac{h(-6x-3h+2)}{h}$

$= -6x-3h+2$

69. $\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{-2(x+h)^2-(x+h)+3-(-2x^2-x+3)}{h}$

$= \dfrac{-2x^2-4xh-2h^2-x-h+3+2x^2+x-3}{h}$

$= \dfrac{-4xh-2h^2-h}{h}$

$= \dfrac{h(-4x-2h-1)}{h}$

$= -4x-2h-1$

70. $\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{-3(x+h)^2+(x+h)-1-(-3x^2+x-1)}{h}$

$= \dfrac{-3x^2-6xh-3h^2+x+h-1+3x^2-x+1}{h}$

$= \dfrac{-6xh-3h^2+h}{h}$

$= \dfrac{h(-6x-3h+1)}{h}$

$= -6x-3h+1$

71. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{6-6}{h} = \dfrac{0}{h} = 0$

72. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{7-7}{h} = \dfrac{0}{h} = 0$

73. $\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

$= \dfrac{\frac{x}{x(x+h)}+\frac{-(x+h)}{x(x+h)}}{h}$

$= \dfrac{\frac{x-x-h}{x(x+h)}}{h}$

$= \dfrac{\frac{-h}{x(x+h)}}{h}$

$= \dfrac{-h}{x(x+h)}\cdot\dfrac{1}{h}$

$= \dfrac{-1}{x(x+h)}$

74. $\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{\frac{1}{2(x+h)}-\frac{1}{2x}}{h}$

$= \dfrac{\frac{x}{2x(x+h)}-\frac{x+h}{2x(x+h)}}{h}$

$= \dfrac{\frac{-h}{2x(x+h)}}{h}$

$= \dfrac{-h}{2x(x+h)}\cdot\dfrac{1}{h}$

$= \dfrac{-1}{2x(x+h)}$

75. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

$=\dfrac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$=\dfrac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$

$=\dfrac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$

$=\dfrac{1}{\sqrt{x+h}+\sqrt{x}}$

76. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{\sqrt{x+h-1}-\sqrt{x-1}}{h}$

$=\dfrac{\sqrt{x+h-1}-\sqrt{x-1}}{h}\cdot\dfrac{\sqrt{x+h-1}+\sqrt{x-1}}{\sqrt{x+h-1}+\sqrt{x-1}}$

$=\dfrac{x+h-1-(x-1)}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}$

$=\dfrac{x+h-1-x+1}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}$

$=\dfrac{h}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}$

$=\dfrac{1}{\sqrt{x+h-1}+\sqrt{x-1}}$

77. $\sqrt{f(-1.5)+f(-0.9)}-\left[f(\pi)\right]^2+f(-3)\div f(1)\cdot f(-\pi)$

$=\sqrt{1+0}-[-4]^2+2\div(-2)\cdot 3$

$=\sqrt{1}-16+(-1)\cdot 3$

$=1-16-3$

$=-18$

78. $\sqrt{f(-2.5)-f(1.9)}-\left[f(-\pi)\right]^2+f(-3)\div f(1)\cdot f(\pi)$

$\sqrt{f(-2.5)-f(1.9)}-\left[f(-\pi)\right]^2+f(-3)\div f(1)\cdot f(\pi)$

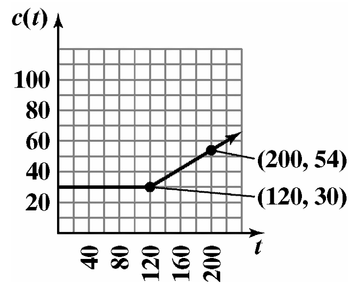
$=\sqrt{2-(-2)}-[3]^2+2\div(-2)\cdot(-4)$

$=\sqrt{4}-9+(-1)(-4)$

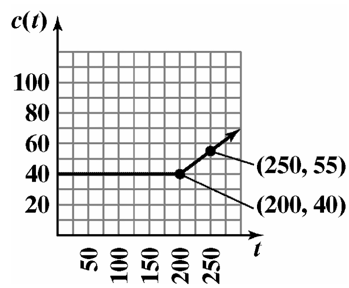
$=2-9+4$

$=-3$

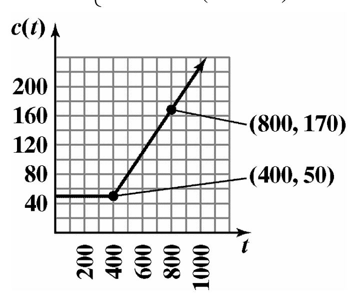
79. $30+0.30(t-120)=30+0.3t-36=0.3t-6$



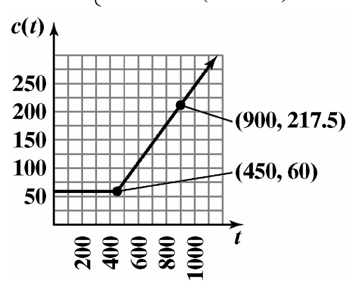
80. $40+0.30(t-200)=40+0.3t-60=0.3t-20$



81. $C(t)=\begin{cases}50 & \text{if } 0\le t\le 400\\ 50+0.30(t-400) & \text{if } t>400\end{cases}$



82. $C(t)=\begin{cases}60 & \text{if } 0\le t\le 450\\ 60+0.35(t-450) & \text{if } t>450\end{cases}$



83. increasing: (25, 55); decreasing: (55, 75)

84. increasing: (25, 65); decreasing: (65, 75)

85. The percent body fat in women reaches a maximum at age 55. This maximum is 38%.

86. The percent body fat in men reaches a maximum at age 65. This maximum is 26%.

87. domain: [25, 75]; range: [34, 38]

88. domain: [25, 75]; range: [23, 26]

89. This model describes percent body fat in men.

90. This model describes percent body fat in women.

91.

$$T(20,000) = 782.50 + 0.15(20,000 - 7825) = 2608.75$$

A single taxpayer with taxable income of \$20,000 owes \$2608.75.

92.

$$T(50,000) = 4386.25 + 0.25(50,000 - 31,850) = 8923.75$$

A single taxpayer with taxable income of \$50,000 owes \$8923.75.

93. $39,148.75 + 0.33(x - 160,850)$

94. $101,469.25 + 0.35(x - 349,700)$

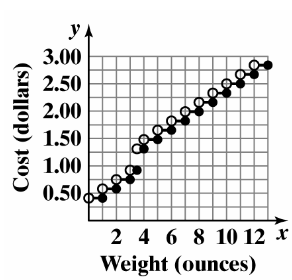
95. $f(3) = 0.76$
The cost of mailing a first-class letter weighing 3 ounces is \$0.76.

96. $f(3.5) = 0.93$
The cost of mailing a first-class letter weighing 3.5 ounces is \$0.93.

97. The cost to mail a letter weighing 1.5 ounces is \$0.59.

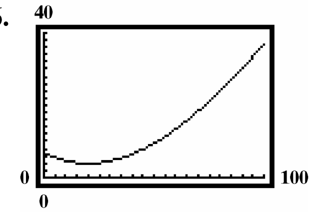
98. The cost to mail a letter weighing 1.8 ounces is \$0.59.

99.



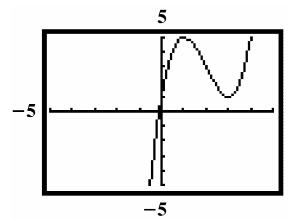
100. – 105. Answers may vary.

106.



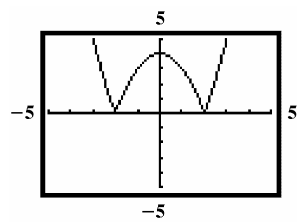
The number of doctor visits decreases during childhood and then increases as you get older.
The minimum is (20.29, 3.99), which means that the minimum number of doctor visits, about 4, occurs at around age 20.

107.



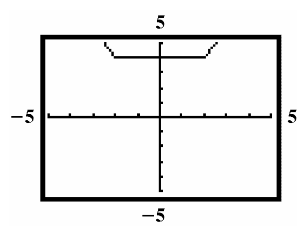
Increasing: $(-\infty, 1)$ or $(3, \infty)$
Decreasing: (1, 3)

108.



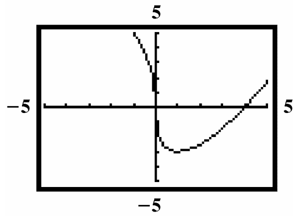
Increasing: (–2, 0) or (2, ∞)
Decreasing: (–∞, –2) or (0, 2)

109.



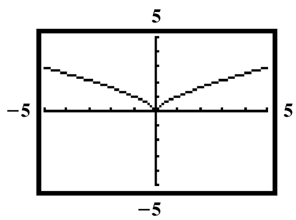
Increasing: $(2, \infty)$
Decreasing: $(-\infty, -2)$
Constant: (–2, 2)

110.



Increasing: $(1, \infty)$
Decreasing: $(-\infty, 1)$

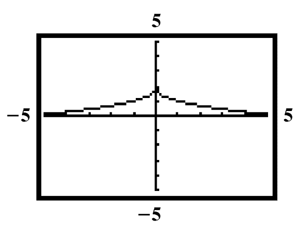
111.



Increasing: $(0, \infty)$

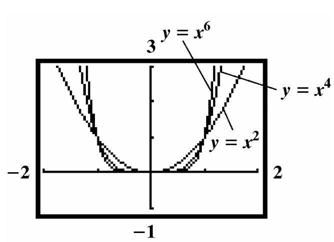
Decreasing: $(-\infty, 0)$

112.

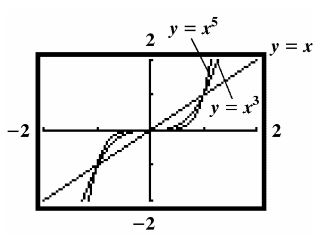


Increasing: $(-\infty, 0)$
Decreasing: $(0, \infty)$

113. a.



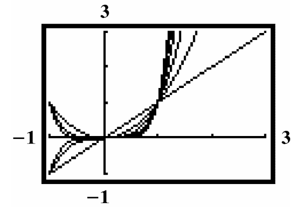
b.



c. Increasing: $(0, \infty)$
Decreasing: $(-\infty, 0)$

d. $f(x) = x^n$ is increasing from $(-\infty, \infty)$ when n is odd.

e.



114. does not make sense; Explanations will vary. Sample explanation: It's possible the graph is not defined at a.

115. makes sense

116. makes sense

117. makes sense

118. answers may vary

119. answers may vary

120. a. h is even if both f and g are even or if both f and g are odd.
f and g are both even:

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{g(x)} = h(x)$$

f and g are both odd:

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{-f(x)}{-g(x)} = \frac{f(x)}{g(x)} = h(x)$$

b. h is odd if f is odd and g is even or if f is even and g is odd.
f is odd and g is even:

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{-f(x)}{g(x)} = -\frac{f(x)}{g(x)} = -h(x)$$

f is even and g is odd:

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f(x)}{g(x)} = -h(x)$$

121. answers may vary

122. $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-1}{-2-(-3)} = \dfrac{3}{1} = 3$

123. When $y = 0$:

$$4x - 3y - 6 = 0$$
$$4x - 3(0) - 6 = 0$$
$$4x - 6 = 0$$
$$4x = 6$$
$$x = \frac{3}{2}$$

The point is $\left(\frac{3}{2}, 0\right)$.

When $x = 0$:

$$4x - 3y - 6 = 0$$
$$4(0) - 3y - 6 = 0$$
$$-3y - 6 = 0$$
$$-3y = 6$$
$$x = -2$$

The point is $(0, -2)$.

124. $3x + 2y - 4 = 0$

$$2y = -3x + 4$$
$$y = \frac{-3x + 4}{2}$$

or

$$y = -\frac{3}{2}x + 2$$

Section 2.3

Check Point Exercises

1. **a.** $m = \frac{-2-4}{-4-(-3)} = \frac{-6}{-1} = 6$

b. $m = \frac{5-(-2)}{-1-4} = \frac{7}{-5} = -\frac{7}{5}$

2.

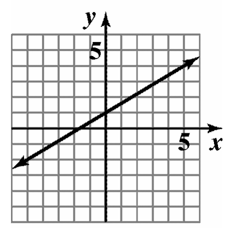
$$y - y_1 = m(x - x_1)$$
$$y - (-5) = 6(x - 2)$$
$$y + 5 = 6x - 12$$
$$y = 6x - 17$$

3. $m = \frac{-6-(-1)}{-1-(-2)} = \frac{-5}{1} = -5$,

so the slope is –5. Using the point (–2, –1), we get the point slope equation:

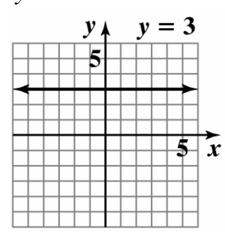
$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -5[x - (-2)]$$
$$y + 1 = -5(x + 2). \text{Solve the equation for } y:$$
$$y + 1 = -5x - 10$$
$$y = -5x - 11.$$

4. The slope m is $\frac{3}{5}$ and the y-intercept is 1, so one point on the line is (1, 0). We can find a second point on the line by using the slope $m = \frac{3}{5} = \frac{\text{Rise}}{\text{Run}}$: starting at the point (0, 1), move 3 units up and 5 units to the right, to obtain the point (5, 4).

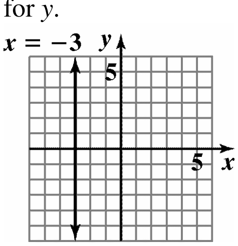


$f(x) = \frac{3}{5}x + 1$

5. $y = 3$ is a horizontal line.



6. All ordered pairs that are solutions of $x = -3$ have a value of x that is always –3. Any value can be used for y.

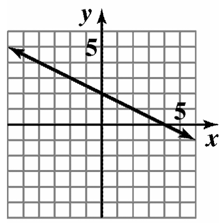


7. $3x+6y-12=0$

$$6y=-3x+12$$

$$y=\frac{-3}{6}x+\frac{12}{6}$$

$$y=-\frac{1}{2}x+2$$



3x + 6y − 12 = 0

The slope is $-\frac{1}{2}$ and the y-intercept is 2.

8. Find the x-intercept:

$$3x-2y-6=0$$

$$3x-2(0)-6=0$$

$$3x-6=0$$

$$3x=6$$

$$x=2$$

Find the y-intercept:

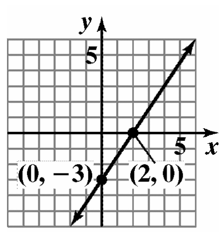
$$3x-2y-6=0$$

$$3(0)-2y-6=0$$

$$-2y-6=0$$

$$-2y=6$$

$$y=-3$$



3x − 2y = 6

9. First find the slope.

$$m=\frac{\text{Change in } y}{\text{Change in } x}=\frac{57.64-57.04}{354-317}=\frac{0.6}{37}\approx 0.016$$

Use the point-slope form and then find slope-intercept form.

$$y-y_1=m(x-x_1)$$

$$y-57.04=0.016(x-317)$$

$$y-57.04=0.016x-5.072$$

$$y=0.016x+51.968$$

$$f(x)=0.016x+52.0$$

Find the temperature at a concentration of 600 parts per million.

$$f(x)=0.016x+52.0$$

$$f(600)=0.016(600)+52.0$$

$$=61.6$$

The temperature at a concentration of 600 parts per million would be 61.6°F.

Exercise Set 2.3

1. $m=\frac{10-7}{8-4}=\frac{3}{4}$; rises

2. $m=\frac{4-1}{3-2}=\frac{3}{1}=3$; rises

3. $m=\frac{2-1}{2-(-2)}=\frac{1}{4}$; rises

4. $m=\frac{4-3}{2-(-1)}=\frac{1}{3}$; rises

5. $m=\frac{2-(-2)}{3-4}=\frac{0}{-1}=0$; horizontal

6. $m=\frac{-1-(-1)}{3-4}=\frac{0}{-1}=0$; horizontal

7. $m=\frac{-1-4}{-1-(-2)}=\frac{-5}{1}=-5$; falls

8. $m=\frac{-2-(-4)}{4-6}=\frac{2}{-2}=-1$; falls

9. $m=\frac{-2-3}{5-5}=\frac{-5}{0}$ undefined; vertical

10. $m = \frac{5-(-4)}{3-3} = \frac{9}{0}$ undefined; vertical

11. $m = 2,\ x_1 = 3,\ y_1 = 5;$
point-slope form: $y - 5 = 2(x - 3);$
slope-intercept form: $y - 5 = 2x - 6$
$y = 2x - 1$

12. point-slope form: $y - 3 = 4(x - 1);$
$m = 4, x_1 = 1, y_1 = 3;$
slope-intercept form: $y = 4x - 1$

13. $m = 6,\ x_1 = -2,\ y_1 = 5;$
point-slope form: $y - 5 = 6(x + 2);$
slope-intercept form: $y - 5 = 6x + 12$
$y = 6x + 17$

14. point-slope form: $y + 1 = 8(x - 4);$
$m = 8, x_1 = 4, y_1 = -1;$
slope-intercept form: $y = 8x - 33$

15. $m = -3,\ x_1 = -2,\ y_1 = -3;$
point-slope form: $y + 3 = -3(x + 2);$
slope-intercept form: $y + 3 = -3x - 6$
$y = -3x - 9$

16. point-slope form: $y + 2 = -5(x + 4);$
$m = -5, x_1 = -4, y_1 = -2;$
slope-intercept form: $y = -5x - 22$

17. $m = -4,\ x_1 = -4,\ y_1 = 0;$
point-slope form: $y - 0 = -4(x + 4);$
slope-intercept form: $y = -4(x + 4)$
$y = -4x - 16$

18. point-slope form: $y + 3 = -2(x - 0)$
$m = -2, x_1 = 0, y_1 = -3;$
slope-intercept form: $y = -2x - 3$

19. $m = -1,\ x_1 = \frac{-1}{2},\ y_1 = -2;$
point-slope form: $y + 2 = -1\left(x + \frac{1}{2}\right);$
slope-intercept form: $y + 2 = -x - \frac{1}{2}$
$y = -x - \frac{5}{2}$

20. point-slope form: $y + \frac{1}{4} = -1(x + 4);$
$m = -1, x_1 = -4, y_1 = -\frac{1}{4};$
slope-intercept form: $y = -x - \frac{17}{4}$

21. $m = \frac{1}{2},\ x_1 = 0,\ y_1 = 0;$
point-slope form: $y - 0 = \frac{1}{2}(x - 0);$
slope-intercept form: $y = \frac{1}{2}x$

22. point-slope form: $y - 0 = \frac{1}{3}(x - 0);$
$m = \frac{1}{3}, x_1 = 0, y_1 = 0;$
slope-intercept form: $y = \frac{1}{3}x$

23. $m = -\frac{2}{3},\ x_1 = 6,\ y_1 = -2;$
point-slope form: $y + 2 = -\frac{2}{3}(x - 6);$
slope-intercept form: $y + 2 = -\frac{2}{3}x + 4$
$y = -\frac{2}{3}x + 2$

24. point-slope form: $y + 4 = -\frac{3}{5}(x - 10);$
$m = -\frac{3}{5}, x_1 = 10, y_1 = -4;$
slope-intercept form: $y = -\frac{3}{5}x + 2$

25. $m = \frac{10-2}{5-1} = \frac{8}{4} = 2;$
point-slope form: $y - 2 = 2(x - 1)$ using $(x_1,\ y_1) = (1,\ 2)$, or $y - 10 = 2(x - 5)$ using $(x_1,\ y_1) = (5,\ 10);$
slope-intercept form: $y - 2 = 2x - 2$ or
$y - 10 = 2x - 10,$
$y = 2x$

26. $m = \frac{15-5}{8-3} = \frac{10}{5} = 2$;
point-slope form: $y - 5 = 2(x - 3)$ using $(x_1, y_1) = (3,5)$, or $y - 15 = 2(x - 8)$ using $(x_1, y_1) = (8,15)$;
slope-intercept form: $y = 2x - 1$

27. $m = \frac{3-0}{0-(-3)} = \frac{3}{3} = 1$;
point-slope form: $y - 0 = 1(x + 3)$ using $(x_1, y_1) = (-3, 0)$, or $y - 3 = 1(x - 0)$ using $(x_1, y_1) = (0, 3)$; slope-intercept form: $y = x + 3$

28. $m = \frac{2-0}{0-(-2)} = \frac{2}{2} = 1$;
point-slope form: $y - 0 = 1(x + 2)$ using $(x_1, y_1) = (-2,0)$, or $y - 2 = 1(x - 0)$ using $(x_1, y_1) = (0,2)$;
slope-intercept form: $y = x + 2$

29. $m = \frac{4-(-1)}{2-(-3)} = \frac{5}{5} = 1$;
point-slope form: $y + 1 = 1(x + 3)$ using $(x_1, y_1) = (-3, -1)$, or $y - 4 = 1(x - 2)$ using $(x_1, y_1) = (2, 4)$; slope-intercept form:
$y + 1 = x + 3$ or
$y - 4 = x - 2$
$y = x + 2$

30. $m = \frac{-1-(-4)}{1-(-2)} = \frac{3}{3} = 1$;
point-slope form: $y + 4 = 1(x + 2)$ using $(x_1, y_1) = (-2,-4)$, or $y + 1 = 1(x - 1)$ using $(x_1, y_1) = (1,-1)$
slope-intercept form: $y = x - 2$

31. $m = \frac{6-(-2)}{3-(-3)} = \frac{8}{6} = \frac{4}{3}$;
point-slope form: $y + 2 = \frac{4}{3}(x + 3)$ using $(x_1, y_1) = (-3, -2)$, or $y - 6 = \frac{4}{3}(x - 3)$ using $(x_1, y_1) = (3, 6)$;
slope-intercept form: $y + 2 = \frac{4}{3x} + 4$ or
$y - 6 = \frac{4}{3}x - 4,$
$y = \frac{4}{3}x + 2$

32. $m = \frac{-2-6}{3-(-3)} = \frac{-8}{6} = -\frac{4}{3}$;
point-slope form: $y - 6 = -\frac{4}{3}(x + 3)$ using $(x_1, y_1) = (-3,6)$, or $y + 2 = -\frac{4}{3}(x - 3)$ using $(x_1, y_1) = (3,-2)$;
slope-intercept form: $y = -\frac{4}{3}x + 2$

33. $m = \frac{-1-(-1)}{4-(-3)} = \frac{0}{7} = 0$;
point-slope form: $y + 1 = 0(x + 3)$ using $(x_1, y_1) = (-3, -1)$, or $y + 1 = 0(x - 4)$ using $(x_1, y_1) = (4, -1)$;
slope-intercept form: $y + 1 = 0$, so
$y = -1$

34. $m = \frac{-5-(-5)}{6-(-2)} = \frac{0}{8} = 0$;
point-slope form: $y + 5 = 0(x + 2)$ using $(x_1, y_1) = (-2,-5)$, or $y + 5 = 0(x - 6)$ using $(x_1, y_1) = (6,-5)$;
slope-intercept form: $y + 5 = 0$, so
$y = -5$

35. $m = \frac{0-4}{-2-2} = \frac{-4}{-4} = 1$;
point-slope form: $y - 4 = 1(x - 2)$ using $(x_1, y_1) = (2, 4)$, or $y - 0 = 1(x + 2)$ using $(x_1, y_1) = (-2, 0)$;
slope-intercept form: $y - 9 = x - 2$, or
$y = x + 2$

36. $m = \dfrac{0-(-3)}{-1-1} = \dfrac{3}{-2} = -\dfrac{3}{2}$

point-slope form: $y+3 = -\dfrac{3}{2}(x-1)$ using $(x_1, y_1) = (1,-3)$, or $y-0 = -\dfrac{3}{2}(x+1)$ using $(x_1, y_1) = (-1,0)$;

slope-intercept form: $y+3 = -\dfrac{3}{2}x + \dfrac{3}{2}$, or

$$y = -\frac{3}{2}x - \frac{3}{2}$$

37. $m = \dfrac{4-0}{0-\left(-\frac{1}{2}\right)} = \dfrac{4}{\frac{1}{2}} = 8$;

point-slope form: $y - 4 = 8(x - 0)$ using $(x_1,\ y_1) = (0,\ 4)$, or $y - 0 = 8\left(x+\frac{1}{2}\right)$ using $(x_1,\ y_1) = \left(-\frac{1}{2},\ 0\right)$; or $y-0 = 8\left(x+\frac{1}{2}\right)$

slope-intercept form: $y = 8x+4$

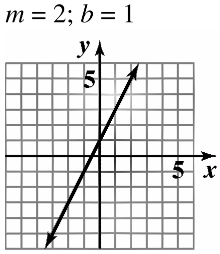
38. $m = \dfrac{-2-0}{0-4} = \dfrac{-2}{-4} = \dfrac{1}{2}$;

point-slope form: $y-0 = \dfrac{1}{2}(x-4)$ using $(x_1, y_1) = (4,0)$,

or $y+2 = \dfrac{1}{2}(x-0)$ using $(x_1, y_1) = (0,-2)$;

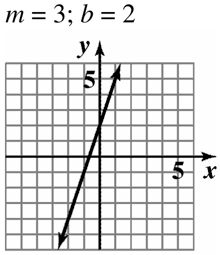
slope-intercept form: $y = \dfrac{1}{2}x - 2$

39. $m = 2$; $b = 1$



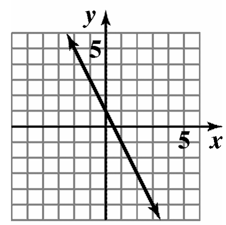
$y = 2x + 1$

40. $m = 3$; $b = 2$



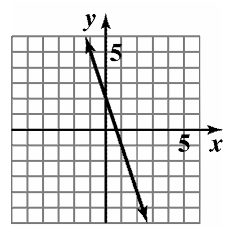
$y = 3x + 2$

41. $m = -2$; $b = 1$



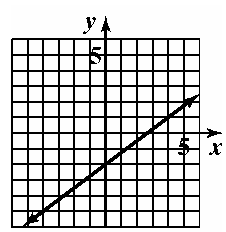
$f(x) = -2x + 1$

42. $m = -3$; $b = 2$



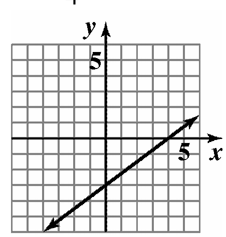
$f(x) = -3x + 2$

43. $m = \dfrac{3}{4}$; $b = -2$



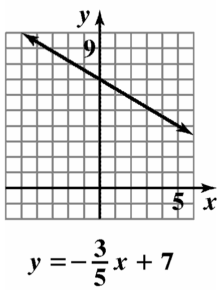
$f(x) = \dfrac{3}{4}x - 2$

44. $m = \dfrac{3}{4}$; $b = -3$

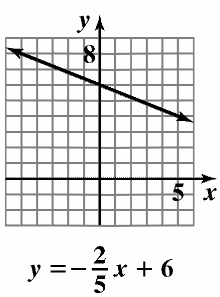


$f(x) = \dfrac{3}{4}x - 3$

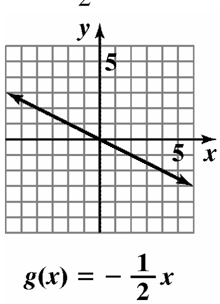
45. $m = -\frac{3}{5};\ b = 7$



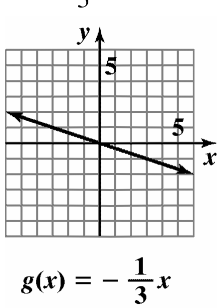
46. $m = -\frac{2}{5};\ b = 6$



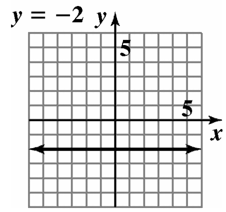
47. $m = -\frac{1}{2};\ b = 0$



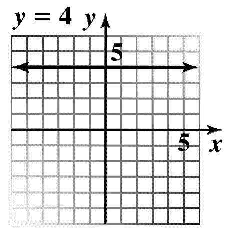
48. $m = -\frac{1}{3};\ b = 0$



49.



50.



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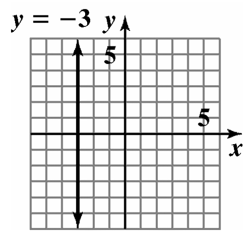
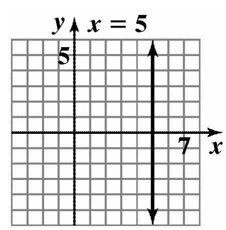


52.



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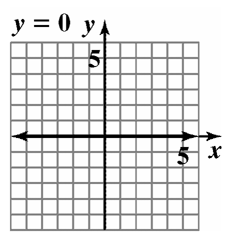

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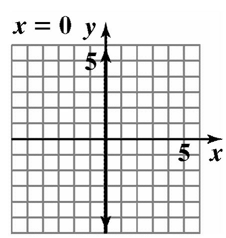
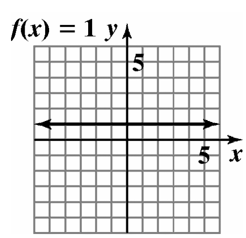
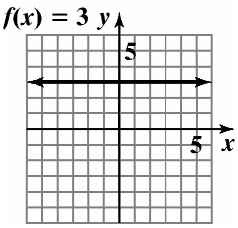


55.



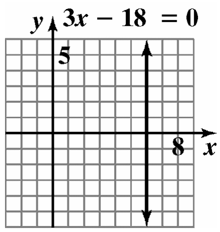
56. $f(x) = 3$



57. $3x - 18 = 0$

$3x = 18$

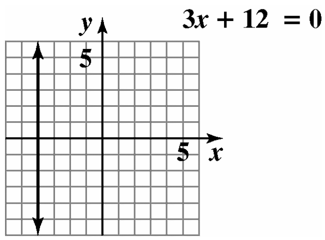
$x = 6$



58. $3x + 12 = 0$

$3x = -12$

$x = -4$



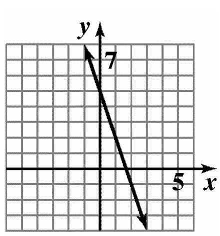
59. **a.** $3x + y - 5 = 0$

$y - 5 = -3x$

$y = -3x + 5$

b. $m = -3;\ b = 5$

c.



3x + y − 5 = 0

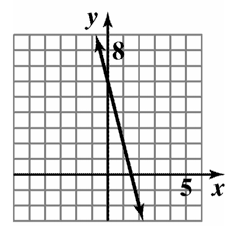
60. **a.** $4x + y - 6 = 0$

$y - 6 = -4x$

$y = -4x + 6$

b. $m = -4; b = 6$

c.



4x + y − 6 = 0

61. **a.** $2x + 3y - 18 = 0$

$2x - 18 = -3y$

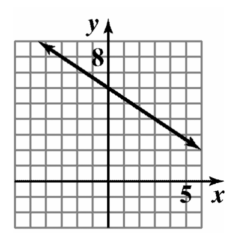
$-3y = 2x - 18$

$y = \frac{2}{-3}x - \frac{18}{-3}$

$y = -\frac{2}{3}x + 6$

b. $m = -\frac{2}{3};\ b = 6$

c.



2x + 3y − 18 = 0

62. **a.** $4x + 6y + 12 = 0$

$4x + 12 = -6y$

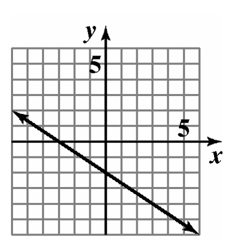
$-6y = 4x + 12$

$y = \frac{4}{-6}x + \frac{12}{-6}$

$y = -\frac{2}{3}x - 2$

b. $m = -\frac{2}{3};\ b = -2$

c.

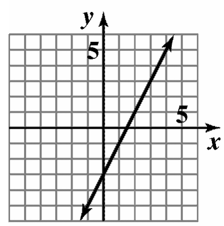


4x + 6y + 12 = 0

63. **a.**
$$8x-4y-12=0$$
$$8x-12=4y$$
$$4y=8x-12$$
$$y=\frac{8}{4}x-\frac{12}{4}$$
$$y=2x-3$$

b. $m=2;\ b=-3$

c.

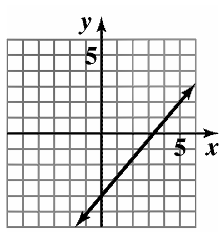


8x − 4y − 12 = 0

64. **a.**
$$6x-5y-20=0$$
$$6x-20=5y$$
$$5y=6x-20$$
$$y=\frac{6}{5}x-\frac{20}{5}$$
$$y=\frac{6}{5}x-4$$

b. $m=\frac{6}{5};b=-4$

c.

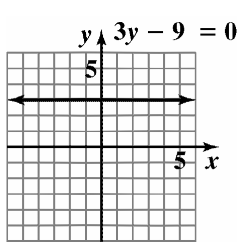


6x − 5y − 20 = 0

65. **a.**
$$3y-9=0$$
$$3y=9$$
$$y=3$$

b. $m=0;\ b=3$

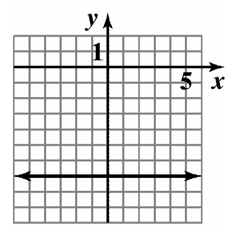
c.



66. **a.**
$$4y+28=0$$
$$4y=-28$$
$$y=-7$$

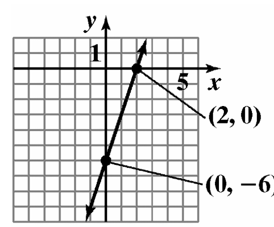
b. $m=0;\ b=-7$

c.



4y + 28 = 0

67. Find the x-intercept:
$$6x-2y-12=0$$
$$6x-2(0)-12=0$$
$$6x-12=0$$
$$6x=12$$
$$x=2$$
Find the y-intercept:
$$6x-2y-12=0$$
$$6(0)-2y-12=0$$
$$-2y-12=0$$
$$-2y=12$$
$$y=-6$$



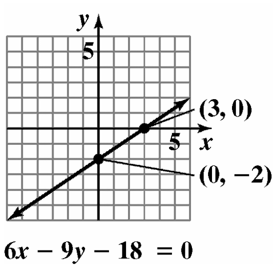
6x − 2y − 12 = 0

68. Find the x-intercept:

$$6x-9y-18=0$$
$$6x-9(0)-18=0$$
$$6x-18=0$$
$$6x=18$$
$$x=3$$

Find the y-intercept:

$$6x-9y-18=0$$
$$6(0)-9y-18=0$$
$$-9y-18=0$$
$$-9y=18$$
$$y=-2$$

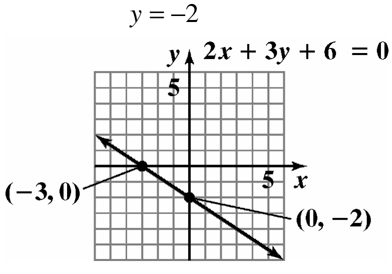


69. Find the x-intercept:

$$2x+3y+6=0$$
$$2x+3(0)+6=0$$
$$2x+6=0$$
$$2x=-6$$
$$x=-3$$

Find the y-intercept:

$$2x+3y+6=0$$
$$2(0)+3y+6=0$$
$$3y+6=0$$
$$3y=-6$$
$$y=-2$$

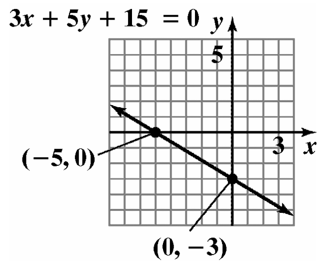


70. Find the x-intercept:

$$3x+5y+15=0$$
$$3x+5(0)+15=0$$
$$3x+15=0$$
$$3x=-15$$
$$x=-5$$

Find the y-intercept:

$$3x+5y+15=0$$
$$3(0)+5y+15=0$$
$$5y+15=0$$
$$5y=-15$$
$$y=-3$$

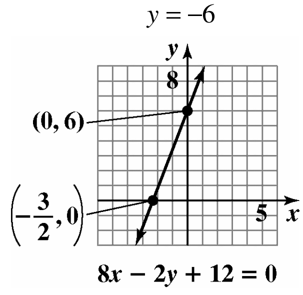


71. Find the x-intercept:

$$8x-2y+12=0$$
$$8x-2(0)+12=0$$
$$8x+12=0$$
$$8x=-12$$
$$\frac{8x}{8}=\frac{-12}{8}$$
$$x=\frac{-3}{2}$$

Find the y-intercept:

$$8x-2y+12=0$$
$$8(0)-2y+12=0$$
$$-2y+12=0$$
$$-2y=-12$$
$$y=-6$$

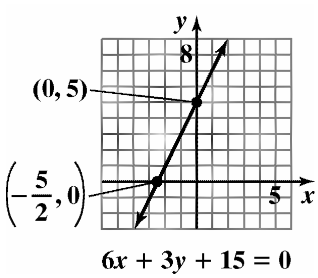


72. Find the x-intercept:

$$\begin{aligned} 6x-3y+15&=0\\ 6x-3(0)+15&=0\\ 6x+15&=0\\ 6x&=-15\\ \frac{6x}{6}&=\frac{-15}{6}\\ x&=-\frac{5}{2} \end{aligned}$$

Find the y-intercept:

$$\begin{aligned} 6x-3y+15&=0\\ 6(0)-3y+15&=0\\ -3y+15&=0\\ -3y&=-15\\ y&=5 \end{aligned}$$



73. $m=\frac{0-a}{b-0}=\frac{-a}{b}=-\frac{a}{b}$

Since a and b are both positive, $-\frac{a}{b}$ is negative. Therefore, the line falls.

74. $m=\frac{-b-0}{0-(-a)}=\frac{-b}{a}=-\frac{b}{a}$

Since a and b are both positive, $-\frac{b}{a}$ is negative. Therefore, the line falls.

75. $m=\frac{(b+c)-b}{a-a}=\frac{c}{0}$

The slope is undefined.
The line is vertical.

76. $m=\frac{(a+c)-c}{a-(a-b)}=\frac{a}{b}$

Since a and b are both positive, $\frac{a}{b}$ is positive. Therefore, the line rises.

77.

$$\begin{aligned} Ax+By&=C\\ By&=-Ax+C\\ y&=-\frac{A}{B}x+\frac{C}{B} \end{aligned}$$

The slope is $-\frac{A}{B}$ and the $y-$intercept is $\frac{C}{B}$.

78.

$$\begin{aligned} Ax&=By-C\\ Ax+C&=By\\ \frac{A}{B}x+\frac{C}{B}&=y \end{aligned}$$

The slope is $\frac{A}{B}$ and the $y-$intercept is $\frac{C}{B}$.

79.

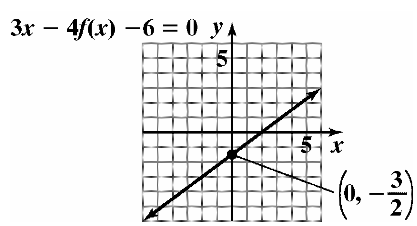
$$\begin{aligned} -3&=\frac{4-y}{1-3}\\ -3&=\frac{4-y}{-2}\\ 6&=4-y\\ 2&=-y\\ -2&=y \end{aligned}$$

80.

$$\begin{aligned} \frac{1}{3}&=\frac{-4-y}{4-(-2)}\\ \frac{1}{3}&=\frac{-4-y}{4+2}\\ \frac{1}{3}&=\frac{-4-y}{6}\\ 6&=3(-4-y)\\ 6&=-12-3y\\ 18&=-3y\\ -6&=y \end{aligned}$$

81.

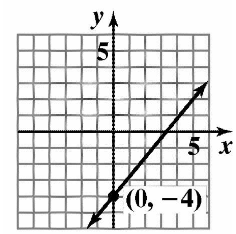
$$\begin{aligned} 3x-4f(x)&=6\\ -4f(x)&=-3x+6\\ f(x)&=\frac{3}{4}x-\frac{3}{2} \end{aligned}$$



82. $6x - 5f(x) = 20$

$-5f(x) = -6x + 20$

$f(x) = \frac{6}{5}x - 4$



6x − 5f(x) −20 = 0

83. Using the slope-intercept form for the equation of a line:

$-1 = -2(3) + b$

$-1 = -6 + b$

$5 = b$

84. $-6 = -\frac{3}{2}(2) + b$

$-6 = -3 + b$

$-3 = b$

85. m_1, m_3, m_2, m_4

86. b_2, b_1, b_4, b_3

87. **a.** First, find the slope using $(20, 38.9)$ and $(10, 31.1)$.

$m = \frac{38.9 - 31.1}{20 - 10} = \frac{7.8}{10} = 0.78$

Then use the slope and one of the points to write the equation in point-slope form.

$y - y_1 = m(x - x_1)$

$y - 31.1 = 0.78(x - 10)$

or

$y - 38.9 = 0.78(x - 20)$

b. $y - 31.1 = 0.78(x - 10)$

$y - 31.1 = 0.78x - 7.8$

$y = 0.78x + 23.3$

$f(x) = 0.78x + 23.3$

c. $f(40) = 0.78(40) + 23.3 = 54.5$

The linear function predicts the percentage of never married American females, ages 25 – 29, to be 54.5% in 2020.

88. **a.** First, find the slope using $(20, 51.7)$ and $(10, 45.2)$.

$m = \frac{51.7 - 45.2}{20 - 10} = \frac{6.5}{10} = 0.65$

Then use the slope and one of the points to write the equation in point-slope form.

$y - y_1 = m(x - x_1)$

$y - 45.2 = 0.65(x - 10)$

or

$y - 51.7 = 0.65(x - 20)$

b. $y - 45.2 = 0.65(x - 10)$

$y - 45.2 = 0.65x - 6.5$

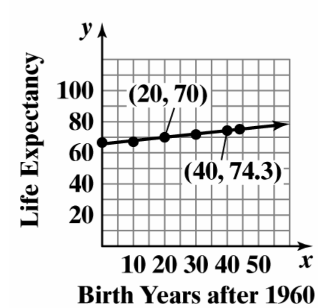
$y = 0.65x + 38.7$

$f(x) = 0.65x + 38.7$

c. $f(35) = 0.65(35) + 38.7 = 61.45$

The linear function predicts the percentage of never married American males, ages 25 – 29, to be 61.45% in 2015.

89. **a.** **Life Expectancy for United States Males, by Year of Birth**



b. $m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{74.3 - 70.0}{40 - 20} = 0.215$

$y - y_1 = m(x - x_1)$

$y - 70.0 = 0.215(x - 20)$

$y - 70.0 = 0.215x - 4.3$

$y = 0.215x + 65.7$

$E(x) = 0.215x + 65.7$

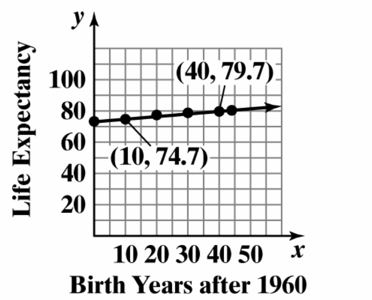
c. $E(x) = 0.215x + 65.7$

$E(60) = 0.215(60) + 65.7$

$= 78.6$

The life expectancy of American men born in 2020 is expected to be 78.6.

90. a. **Life Expectancy for United States Females, by Year of Birth**



b. $m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{79.7-74.7}{40-10} \approx 0.17$

$$y - y_1 = m(x - x_1)$$
$$y - 74.7 = 0.17(x-10)$$
$$y - 74.7 = 0.17x - 1.7$$
$$y = 0.17x + 73$$
$$E(x) = 0.17x + 73$$

c. $E(x) = 0.17x + 73$

$$E(60) = 0.17(60) + 73$$
$$= 83.2$$

The life expectancy of American women born in 2020 is expected to be 83.2.

91. (10, 230) (60, 110) Points may vary.

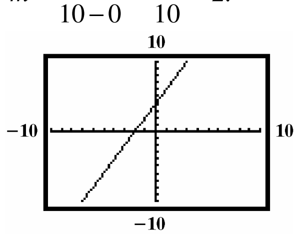
$$m = \frac{110-230}{60-10} = -\frac{120}{50} = -2.4$$
$$y - 230 = -2.4(x-10)$$
$$y - 230 = -2.4x + 24$$
$$y = -2.4x + 254$$

Answers may vary for predictions.

92. – 99. Answers may vary.

100. Two points are (0,4) and (10,24).

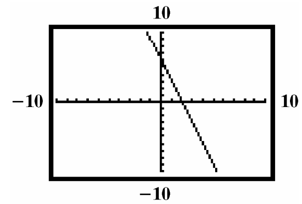
$$m = \frac{24-4}{10-0} = \frac{20}{10} = 2.$$



101. Two points are (0, 6) and (10, –24).

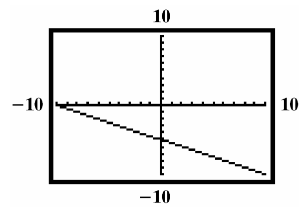
$$m = \frac{-24-6}{10-0} = \frac{-30}{10} = -3.$$

Check: $y = mx + b$: $y = -3x + 6$.



102. Two points are (0,–5) and (10,–10).

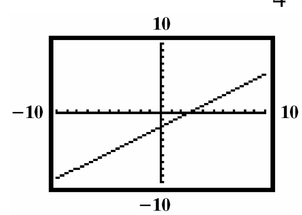
$$m = \frac{-10-(-5)}{10-0} = \frac{-5}{10} = -\frac{1}{2}.$$



103. Two points are (0, –2) and (10, 5.5).

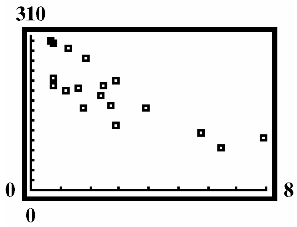
$$m = \frac{5.5-(-2)}{10-0} = \frac{7.5}{10} = 0.75 \text{ or } \frac{3}{4}.$$

Check: $y = mx + b$: $y = \frac{3}{4}x - 2$.



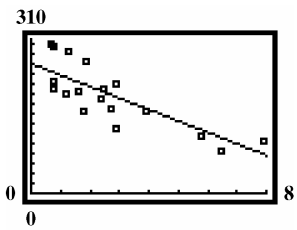
104. **a.** Enter data from table.

b.



c. $a = -22.96876741$
$b = 260.5633751$
$r = -0.8428126855$

d.



105. does not make sense; Explanations will vary. Sample explanation: Linear functions never change from increasing to decreasing.

106. does not make sense; Explanations will vary. Sample explanation: Since college cost are going up, this function has a positive slope.

107. does not make sense; Explanations will vary. Sample explanation: The slope of line's whose equations are in this form can be determined in several ways. One such way is to rewrite the equation in slope-intercept form.

108. makes sense

109. false; Changes to make the statement true will vary. A sample change is: It is possible for *m* to equal *b*.

110. false; Changes to make the statement true will vary. A sample change is: Slope-intercept form is $y = mx + b$. Vertical lines have equations of the form $x = a$. Equations of this form have undefined slope and cannot be written in slope-intercept form.

111. true

112. false; Changes to make the statement true will vary. A sample change is: The graph of $x = 7$ is a vertical line through the point (7, 0).

113. We are given that the $x-$intercept is -2 and the $y-$intercept is 4. We can use the points $(-2,0)$ and $(0,4)$ to find the slope.

$$m = \frac{4-0}{0-(-2)} = \frac{4}{0+2} = \frac{4}{2} = 2$$

Using the slope and one of the intercepts, we can write the line in point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 2(x-(-2))$$
$$y = 2(x+2)$$
$$y = 2x + 4$$
$$-2x + y = 4$$

Find the *x*– and *y*–coefficients for the equation of the line with right-hand-side equal to 12. Multiply both sides of $-2x + y = 4$ by 3 to obtain 12 on the right-hand-side.

$$-2x + y = 4$$
$$3(-2x + y) = 3(4)$$
$$-6x + 3y = 12$$

Therefore, the coefficient of x is –6 and the coefficient of y is 3.

114. We are given that the $y-$intercept is -6 and the slope is $\frac{1}{2}$.

So the equation of the line is $y = \frac{1}{2}x - 6$.

We can put this equation in the form $ax + by = c$ to find the missing coefficients.

$$y = \frac{1}{2}x - 6$$
$$y - \frac{1}{2}x = -6$$
$$2\left(y - \frac{1}{2}x\right) = 2(-6)$$
$$2y - x = -12$$
$$x - 2y = 12$$

Therefore, the coefficient of x is 1 and the coefficient of y is –2.

115. Answers may vary.

116. Let (25, 40) and (125, 280) be ordered pairs (M, E) where M is degrees Madonna and E is degrees Elvis. Then
$m = \frac{280-40}{125-25} = \frac{240}{100} = 2.4$. Using $(x_1, y_1) = (25, 40)$, point-slope form tells us that
$E - 40 = 2.4\ (M - 25)$ or
$E = 2.4\ M - 20$.

117. Answers may vary.

118. Since the slope is the same as the slope of $y = 2x+1$, then $m = 2$.

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 2(x-(-3))$$
$$y - 1 = 2(x+3)$$
$$y - 1 = 2x+6$$
$$y = 2x+7$$

119. Since the slope is the negative reciprocal of $-\frac{1}{4}$, then $m = 4$.

$$y - y_1 = m(x - x_1)$$
$$y-(-5) = 4(x-3)$$
$$y+5 = 4x-12$$
$$-4x + y + 17 = 0$$
$$4x - y - 17 = 0$$

120. $\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{f(4)-f(1)}{4-1}$
$= \frac{4^2-1^2}{4-1}$
$= \frac{15}{3}$
$= 5$

Section 2.4

Check Point Exercises

1. The slope of the line $y = 3x+1$ is 3.

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 3(x-(-2))$$
$$y - 5 = 3(x+2) \text{ point-slope}$$
$$y - 5 = 3x+6$$
$$y = 3x+11 \text{ slope-intercept}$$

2. **a.** Write the equation in slope-intercept form:

$$x+3y-12 = 0$$
$$3y = -x+12$$
$$y = -\frac{1}{3}x+4$$

The slope of this line is $-\frac{1}{3}$ thus the slope of any line perpendicular to this line is 3.

b. Use $m = 3$ and the point (–2, –6) to write the equation.

$$y - y_1 = m(x - x_1)$$
$$y-(-6) = 3(x-(-2))$$
$$y+6 = 3(x+2)$$
$$y+6 = 3x+6$$
$$-3x + y = 0$$
$$3x - y = 0 \text{ general form}$$

3. $m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{12.7-9.0}{2005-1990} = \frac{3.7}{15} \approx 0.25$

The slope indicates that the number of U.S. men living alone is projected to increase by 0.25 million each year.

4. **a.** $\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{1^3-0^3}{1-0} = 1$

b. $\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{2^3-1^3}{2-1} = \frac{8-1}{1} = 7$

c. $\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{0^3-(-2)^3}{0-(-2)} = \frac{8}{2} = 4$

5. $\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{f(3)-f(1)}{3-1} = \frac{0.05-0.03}{3-1} = 0.01$

Exercise Set 2.4

1. Since L is parallel to $y = 2x$, we know it will have slope $m = 2$. We are given that it passes through $(4, 2)$. We use the slope and point to write the equation in point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 2(x - 4)$$

Solve for y to obtain slope-intercept form.

$$y - 2 = 2(x - 4)$$
$$y - 2 = 2x - 8$$
$$y = 2x - 6$$

In function notation, the equation of the line is $f(x) = 2x - 6.$

2. L will have slope $m = -2$. Using the point and the slope, we have $y - 4 = -2(x - 3)$. Solve for y to obtain slope-intercept form.

$$y - 4 = -2x + 6$$
$$y = -2x + 10$$
$$f(x) = -2x + 10$$

3. Since L is perpendicular to $y = 2x$, we know it will have slope $m = -\frac{1}{2}$. We are given that it passes through $(2, 4)$. We use the slope and point to write the equation in point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -\frac{1}{2}(x - 2)$$

Solve for y to obtain slope-intercept form.

$$y - 4 = -\frac{1}{2}(x - 2)$$
$$y - 4 = -\frac{1}{2}x + 1$$
$$y = -\frac{1}{2}x + 5$$

In function notation, the equation of the line is $f(x) = -\frac{1}{2}x + 5.$

4. L will have slope $m = \frac{1}{2}$. The line passes through (–1, 2). Use the slope and point to write the equation in point-slope form.

$$y - 2 = \frac{1}{2}(x - (-1))$$
$$y - 2 = \frac{1}{2}(x + 1)$$

Solve for y to obtain slope-intercept form.

$$y - 2 = \frac{1}{2}x + \frac{1}{2}$$
$$y = \frac{1}{2}x + \frac{1}{2} + 2$$
$$y = \frac{1}{2}x + \frac{5}{2}$$
$$f(x) = \frac{1}{2}x + \frac{5}{2}$$

5. $m = -4$ since the line is parallel to $y = -4x + 3$; $x_1 = -8$, $y_1 = -10$;

point-slope form: $y + 10 = -4(x + 8)$

slope-intercept form: $y + 10 = -4x - 32$

$y = -4x - 42$

6. $m = -5$ since the line is parallel to $y = -5x + 4$;

$x_1 = -2, y_1 = -7$;

point-slope form: $y + 7 = -5(x + 2)$

slope-intercept form: $y + 7 = -5x - 10$

$y = -5x - 17$

7. $m = -5$ since the line is perpendicular to $y = \frac{1}{5}x + 6$; $x_1 = 2$, $y_1 = -3$;

point-slope form: $y + 3 = -5(x - 2)$

slope-intercept form: $y + 3 = -5x + 10$

$y = -5x + 7$

8. $m = -3$ since the line is perpendicular to $y = \frac{1}{3}x + 7$;

$x_1 = -4, y_1 = 2$;

point-slope form: $y - 2 = -3(x + 4)$

slope-intercept form: $y - 2 = -3x - 12$

$y = -3x - 10$

9. $2x-3y-7=0$

$$-3y=-2x+7$$
$$y=\frac{2}{3}x-\frac{7}{3}$$

The slope of the given line is $\frac{2}{3}$, so $m=\frac{2}{3}$ since the lines are parallel.

point-slope form: $y-2=\frac{2}{3}(x+2)$

general form: $2x-3y+10=0$

10. $3x-2y-=0$

$$-2y=-3x+5$$
$$y=\frac{3}{2}x-\frac{5}{2}$$

The slope of the given line is $\frac{3}{2}$, so $m=\frac{3}{2}$ since the lines are parallel.

point-slope form: $y-3=\frac{3}{2}(x+1)$

general form: $3x-2y+9=0$

11. $x-2y-3=0$

$$-2y=-x+3$$
$$y=\frac{1}{2}x-\frac{3}{2}$$

The slope of the given line is $\frac{1}{2}$, so $m=-2$ since the lines are perpendicular.

point-slope form: $y+7=-2(x-4)$

general form: $2x+y-1=0$

12. $x+7y-12=0$

$$7y=-x+12$$
$$y=\frac{-1}{7}x+\frac{12}{7}$$

The slope of the given line is $-\frac{1}{7}$, so $m=7$ since the lines are perpendicular.

point-slope form: $y+9=7(x-5)$

general form: $7x-y-44=0$

13. $\frac{15-0}{5-0}=\frac{15}{5}=3$

14. $\frac{24-0}{4-0}=\frac{24}{4}=6$

15. $\frac{5^2+2\cdot5-(3^2+2\cdot3)}{5-3}=\frac{25+10-(9+6)}{2}$

$$=\frac{20}{2}$$
$$=10$$

16. $\frac{6^2-2(6)-(3^2-2\cdot3)}{6-3}=\frac{36-12-(9-6)}{3}=\frac{21}{3}=7$

17. $\frac{\sqrt{9}-\sqrt{4}}{9-4}=\frac{3-2}{5}=\frac{1}{5}$

18. $\frac{\sqrt{16}-\sqrt{9}}{16-9}=\frac{4-3}{7}=\frac{1}{7}$

19. Since the line is perpendicular to $x=6$ which is a vertical line, we know the graph of f is a horizontal line with 0 slope. The graph of f passes through $(-1,5)$, so the equation of f is $f(x)=5$.

20. Since the line is perpendicular to $x=-4$ which is a vertical line, we know the graph of f is a horizontal line with 0 slope. The graph of f passes through $(-2,6)$, so the equation of f is $f(x)=6$.

21. First we need to find the equation of the line with $x-$intercept of 2 and $y-$intercept of -4. This line will pass through $(2,0)$ and $(0,-4)$. We use these points to find the slope.

$$m=\frac{-4-0}{0-2}=\frac{-4}{-2}=2$$

Since the graph of f is perpendicular to this line, it will have slope $m=-\frac{1}{2}$.

Use the point $(-6,4)$ and the slope $-\frac{1}{2}$ to find the equation of the line.

$$y-y_1=m(x-x_1)$$
$$y-4=-\frac{1}{2}(x-(-6))$$
$$y-4=-\frac{1}{2}(x+6)$$
$$y-4=-\frac{1}{2}x-3$$
$$y=-\frac{1}{2}x+1$$
$$f(x)=-\frac{1}{2}x+1$$

22. First we need to find the equation of the line with $x-$intercept of 3 and $y-$intercept of -9. This line will pass through $(3,0)$ and $(0,-9)$. We use these points to find the slope.

$$m=\frac{-9-0}{0-3}=\frac{-9}{-3}=3$$

Since the graph of f is perpendicular to this line, it will have slope $m=-\frac{1}{3}$.

Use the point $(-5,6)$ and the slope $-\frac{1}{3}$ to find the equation of the line.

$$y-y_1=m(x-x_1)$$
$$y-6=-\frac{1}{3}(x-(-5))$$
$$y-6=-\frac{1}{3}(x+5)$$
$$y-6=-\frac{1}{3}x-\frac{5}{3}$$
$$y=-\frac{1}{3}x+\frac{13}{3}$$
$$f(x)=-\frac{1}{3}x+\frac{13}{3}$$

23. First put the equation $3x-2y-4=0$ in slope-intercept form.

$$3x-2y-4=0$$
$$-2y=-3x+4$$
$$y=\frac{3}{2}x-2$$

The equation of f will have slope $-\frac{2}{3}$ since it is perpendicular to the line above and the same $y-$intercept -2.

So the equation of f is $f(x)=-\frac{2}{3}x-2.$

24. First put the equation $4x-y-6=0$ in slope-intercept form.

$$4x-y-6=0$$
$$-y=-4x+6$$
$$y=4x-6$$

The equation of f will have slope $-\frac{1}{4}$ since it is perpendicular to the line above and the same $y-$intercept -6.

So the equation of f is $f(x)=-\frac{1}{4}x-6.$

25. $P(x)=-1.2x+47$

26. $P(x)=1.3x+23$

27. $m=\frac{1163-617}{1998-1994}=\frac{546}{4}\approx 137$

There was an average increase of approximately 137 discharges per year.

28. $m=\frac{612-1273}{2006-2001}=\frac{-661}{5}\approx -132$

There was an average decrease of approximately 132 discharges per year.

29. **a.** $f(x)=1.1x^3-35x^2+264x+557$

$f(0)=1.1(0)^3-35(0)^2+264(0)+557=557$

$f(4)=1.1(4)^3-35(4)^2+264(4)+557=1123.4$

$m=\frac{1123.4-557}{4-0}\approx 142$

b. This overestimates by 5 discharges per year.

30. **a.** $f(x)=1.1x^3-35x^2+264x+557$

$f(0)=1.1(7)^3-35(7)^2+264(7)+557=1067.3$

$f(12)=1.1(12)^3-35(12)^2+264(12)+557=585.8$

$m=\frac{585.8-1067.3}{12-7}\approx -96$

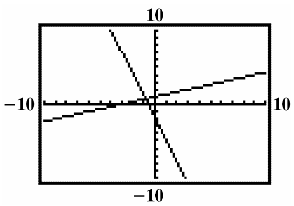
b. This underestimates the decrease by 36 discharges per year.

31. – 36. Answers may vary.

37.
$$y = \frac{1}{3}x + 1$$
$$y = -3x - 2$$

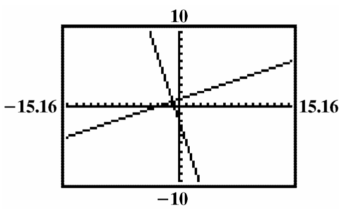
a. The lines are perpendicular because their slopes are negative reciprocals of each other. This is verified because product of their slopes is –1.

b.



The lines do not appear to be perpendicular.

c.



The lines appear to be perpendicular. The calculator screen is rectangular and does not have the same width and height. This causes the scale of the *x*–axis to differ from the scale on the *y*–axis despite using the same scale in the window settings. In part (b), this causes the lines not to appear perpendicular when indeed they are. The zoom square feature compensates for this and in part (c), the lines appear to be perpendicular.

38. makes sense

39. makes sense

40. does not make sense; Explanations will vary. Sample explanation: Slopes can be used for segments of the graph.

41. makes sense

42. Write $Ax + By + C = 0$ in slope-intercept form.

$$Ax + By + C = 0$$
$$By = -Ax - C$$
$$\frac{By}{B} = \frac{-Ax}{B} - \frac{C}{B}$$
$$y = -\frac{A}{B}x - \frac{C}{B}$$

The slope of the given line is $-\frac{A}{B}$.

The slope of any line perpendicular to $Ax + By + C = 0$ is $\frac{B}{A}$.

43. The slope of the line containing $(1,-3)$ and $(-2,4)$ has slope $m=\frac{4-(-3)}{-2-1}=\frac{4+3}{-3}=\frac{7}{-3}=-\frac{7}{3}$

Solve $Ax+y-2=0$ for y to obtain slope-intercept form.

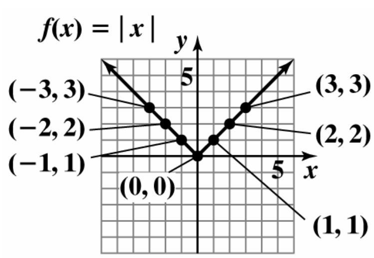
$Ax+y-2=0$

$y=-Ax+2$

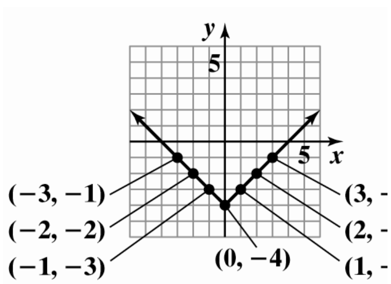
So the slope of this line is $-A$.

This line is perpendicular to the line above so its slope is $\frac{3}{7}$. Therefore, $-A=\frac{3}{7}$ so $A=-\frac{3}{7}$.

44. a.

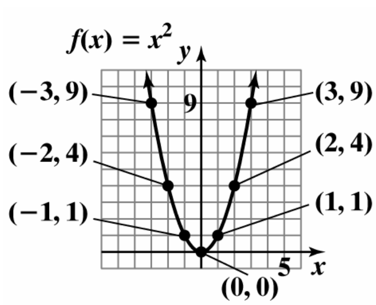


b.

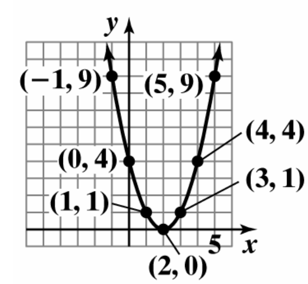


c. The graph in part (b) is the graph in part (a) shifted down 4 units.

45. a.

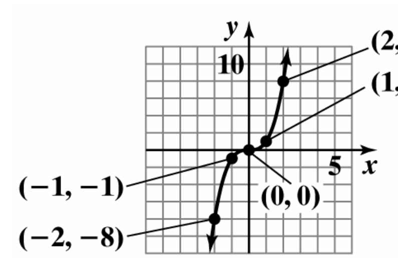


b.

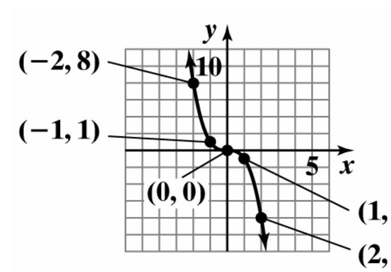


c. The graph in part (b) is the graph in part (a) shifted to the right 2 units.

46. a.



b.



c. The graph in part (b) is the graph in part (a) reflected across the *y*-axis.

Mid-Chapter 2 Check Point

1. The relation is not a function.
The domain is $\{1,2\}$.
The range is $\{-6,4,6\}$.

2. The relation is a function.
The domain is $\{0,2,3\}$.
The range is $\{1,4\}$.

3. The relation is a function.
The domain is $\{x\,|\,-2\le x<2\}$.
The range is $\{y\,|\,0\le y\le 3\}$.

4. The relation is not a function.
The domain is $\{x\,|\,-3<x\le 4\}$.
The range is $\{y\,|\,-1\le y\le 2\}$.

5. The relation is not a function.
The domain is $\{-2,-1,0,1,2\}$.
The range is $\{-2,-1,1,3\}$.

6. The relation is a function.
The domain is $\{x \mid x \le 1\}$.
The range is $\{y \mid y \ge -1\}$.

7. $x^2 + y = 5$

$y = -x^2 + 5$

For each value of x, there is one and only one value for y, so the equation defines y as a function of x.

8. $x + y^2 = 5$

$y^2 = 5 - x$

$y = \pm\sqrt{5-x}$

Since there are values of x that give more than one value for y (for example, if $x = 4$, then $y = \pm\sqrt{5-4} = \pm 1$), the equation does not define y as a function of x.

9. Each value of x corresponds to exactly one value of y.

10. Domain: $(-\infty, \infty)$

11. Range: $(-\infty, 4]$

12. x-intercepts: –6 and 2

13. y-intercept: 3

14. increasing: $(-\infty, -2)$

15. decreasing: $(-2, \infty)$

16. $x = -2$

17. $f(-2) = 4$

18. $f(-4) = 3$

19. $f(-7) = -2$ and $f(3) = -2$

20. $f(-6) = 0$ and $f(2) = 0$

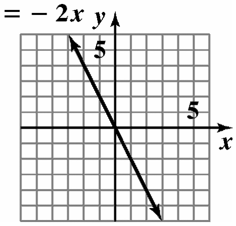
21. $(-6, 2)$

22. $f(100)$ is negative.

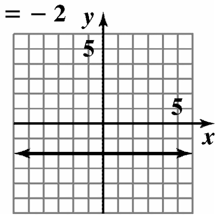
23. neither; $f(-x) \ne x$ and $f(-x) \ne -x$

24. $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(4) - f(-4)}{4 - (-4)} = \dfrac{-5-3}{4+4} = -1$

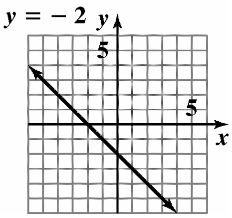
25. $y = -2x$



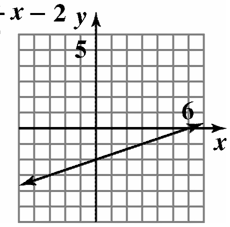
26. $y = -2$



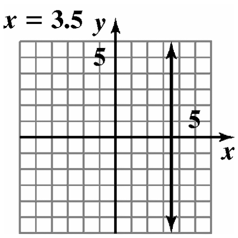
27. $x + y = -2$



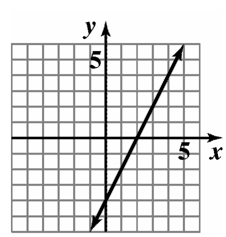
28. $y = \frac{1}{3}x - 2$



29. $x = 3.5$

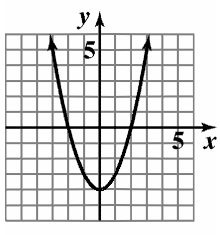


30.



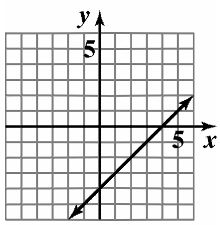
$4x - 2y = 8$

31.



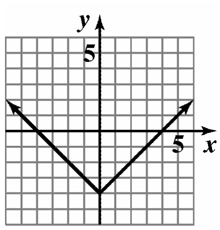
$f(x) = x^2 - 4$

32.



$f(x) = x - 4$

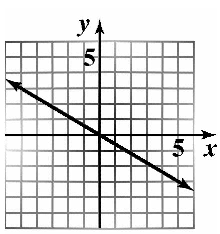
33.



$f(x) = |x| - 4$

34. $5y = -3x$

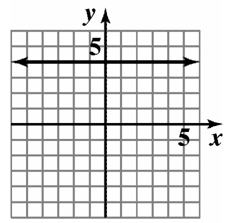
$y = -\frac{3}{5}x$



$5y = -3x$

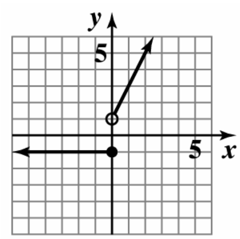
35. $5y = 20$

$y = 4$



$5y = 20$

36.



$f(x) = \begin{cases} -1 \text{ if } x \le 0 \\ 2x + 1 \text{ if } x > 0 \end{cases}$

37. **a.** $f(-x) = -2(-x)^2 - x - 5 = -2x^2 - x - 5$

neither; $f(-x) \neq x$ and $f(-x) \neq -x$

b. $\frac{f(x+h) - f(x)}{h}$

$= \frac{-2(x+h)^2 + (x+h) - 5 - (-2x^2 + x - 5)}{h}$

$= \frac{-2x^2 - 4xh - 2h^2 + x + h - 5 + 2x^2 - x + 5}{h}$

$= \frac{-4xh - 2h^2 + h}{h}$

$= \frac{h(-4x - 2h + 1)}{h}$

$= -4x - 2h + 1$

38. $C(x) = \begin{cases} 30 & \text{if} \quad 0 \le t \le 200 \\ 30 + 0.40(t - 200) & \text{if} \quad t > 200 \end{cases}$

a. $C(150) = 30$

b. $C(250) = 30 + 0.40(250 - 200) = 50$

39. $y - y_1 = m(x - x_1)$
$y - 3 = -2\left(x - (-4)\right)$
$y - 3 = -2(x + 4)$
$y - 3 = -2x - 8$
$y = -2x - 5$
$f(x) = -2x - 5$

40. $m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{1 - (-5)}{2 - (-1)} = \frac{6}{3} = 2$
$y - y_1 = m(x - x_1)$
$y - 1 = 2\left(x - 2\right)$
$y - 1 = 2x - 4$
$y = 2x - 3$
$f(x) = 2x - 3$

41. $3x - y - 5 = 0$
$-y = -3x + 5$
$y = 3x - 5$
The slope of the given line is 3, and the lines are parallel, so $m = 3$.
$y - y_1 = m(x - x_1)$
$y - (-4) = 3(x - 3)$
$y + 4 = 3x - 9$
$y = 3x - 13$
$f(x) = 3x - 13$

42. $2x - 5y - 10 = 0$
$-5y = -2x + 10$
$\frac{-5y}{-5} = \frac{-2x}{-5} + \frac{10}{-5}$
$y = \frac{2}{5}x - 2$

The slope of the given line is $\frac{2}{5}$, and the lines are perpendicular, so $m = -\frac{5}{2}$.
$y - y_1 = m(x - x_1)$
$y - (-3) = -\frac{5}{2}\left(x - (-4)\right)$
$y + 3 = -\frac{5}{2}x - 10$
$y = -\frac{5}{2}x - 13$
$f(x) = -\frac{5}{2}x - 13$

43. $m_1 = \frac{\text{Change in } y}{\text{Change in } x} = \frac{0 - (-4)}{7 - 2} = \frac{4}{5}$
$m_2 = \frac{\text{Change in } y}{\text{Change in } x} = \frac{6 - 2}{1 - (-4)} = \frac{4}{5}$
The slope of the lines are equal thus the lines are parallel.

44. **a.** $m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{42 - 26}{180 - 80} = \frac{16}{100} = 0.16$

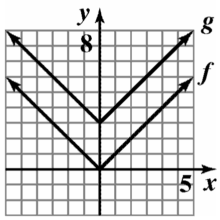
b. For each minute of brisk walking, the percentage of patients with depression in remission increased by <u>0.16</u>%. The rate of change is <u>0.16</u>% per <u>minute of brisk walking</u>.

45. $\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(2) - f(-1)}{2 - (-1)}$
$= \frac{\left(3(2)^2 - 2\right) - \left(3(-1)^2 - (-1)\right)}{2 + 1}$
$= 2$

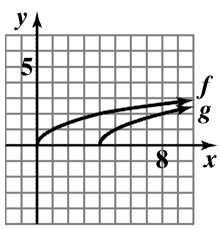
Section 2.5

Check Point Exercises

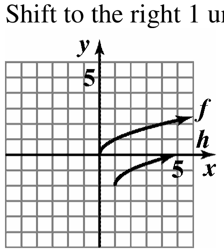
1. Shift up vertically 3 units.



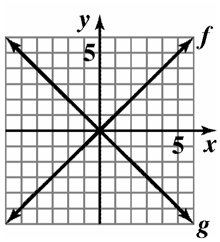
2. Shift to the right 4 units.



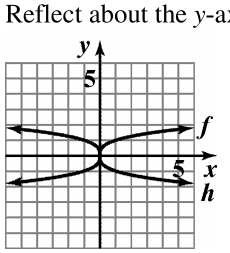
3. Shift to the right 1 unit and down 2 units.



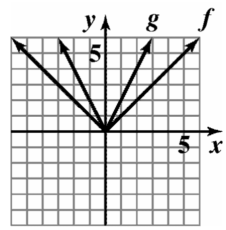
4. Reflect about the *x*-axis.



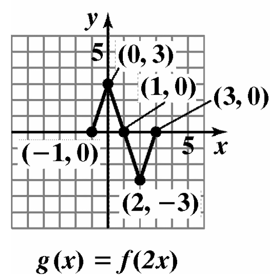
5. Reflect about the *y*-axis.



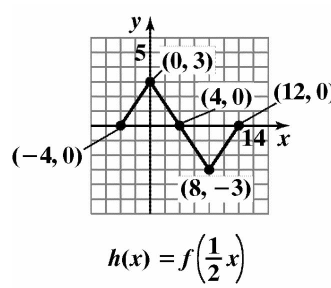
6. Vertically stretch the graph of $f(x) = |x|$.



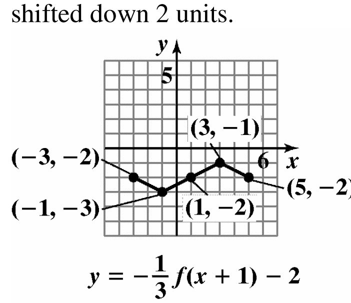
7. **a.** Horizontally shrink the graph of $y = f(x)$.



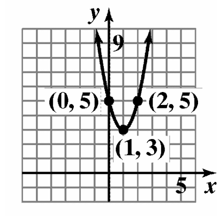
b. Horizontally stretch the graph of $y = f(x)$.



8. The graph of $y = f(x)$ is shifted 1 unit left, shrunk by a factor of $\frac{1}{3}$, reflected about the x-axis, then shifted down 2 units.



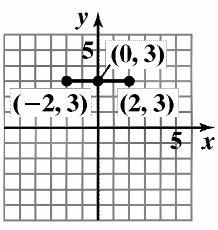
9. The graph of $f(x) = x^2$ is shifted 1 unit right, stretched by a factor of 2, then shifted up 3 units.



$g(x) = 2(x-1)^2 + 3$

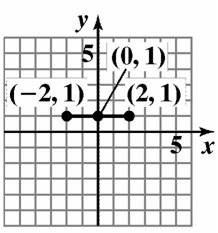
Exercise Set 2.5

1.



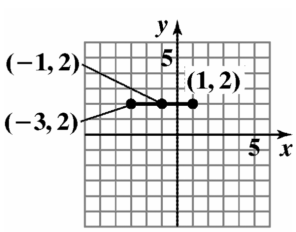
$g(x) = f(x) + 1$

2.



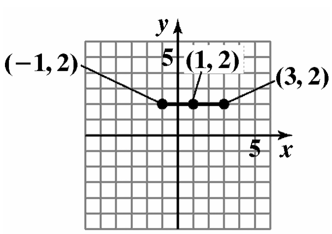
$g(x) = f(x) - 1$

3.



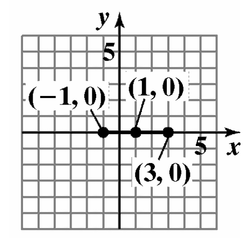
$g(x) = f(x+1)$

4.



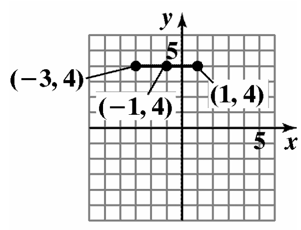
$g(x) = f(x-1)$

5.



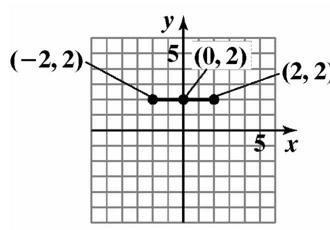
$g(x) = f(x-1) - 2$

6.



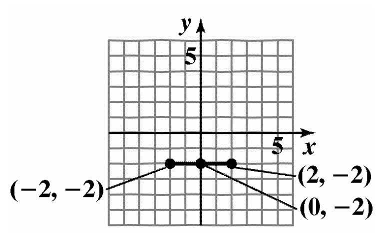
$g(x) = f(x+1) + 2$

7.



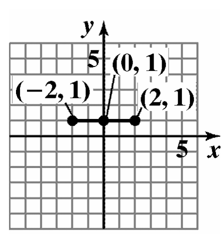
$g(x) = -f(x)$

8.



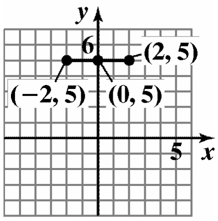
$g(x) = -f(x)$

9.



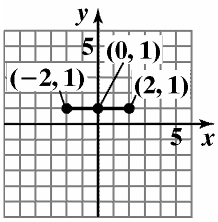
$g(x) = -f(x) + 3$

10.



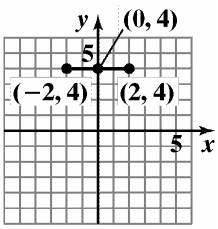
$g(x) = f(-x) + 3$

11.



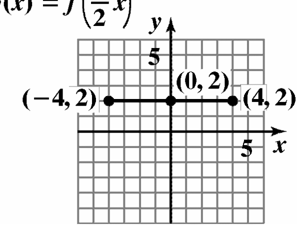
$g(x) = \frac{1}{2}f(x)$

12.

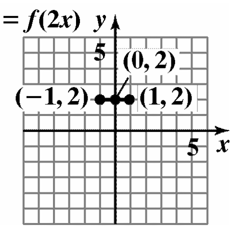


$g(x) = 2f(x)$

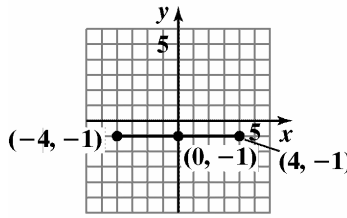
13. $g(x) = f\left(\frac{1}{2}x\right)$



14. $g(x) = f(2x)$

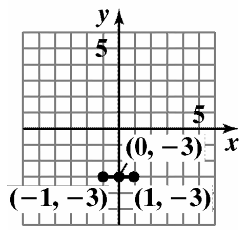


15.



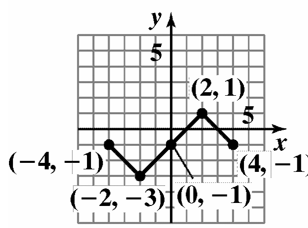
$g(x) = -f\left(\frac{1}{2}x\right) + 1$

16.



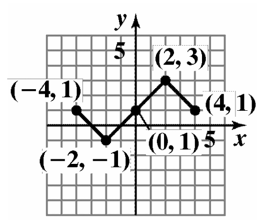
$g(x) = -f(2x) - 1$

17.



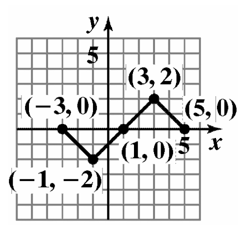
$g(x) = f(x) - 1$

18.



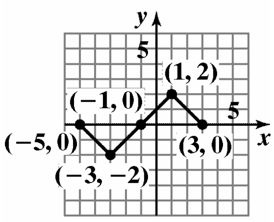
$g(x) = f(x) + 1$

19.



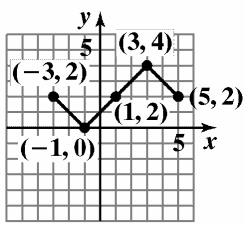
$g(x) = f(x - 1)$

20.



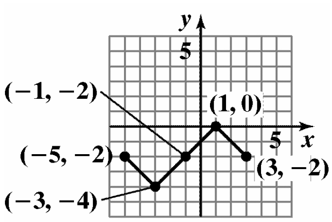
$g(x) = f(x + 1)$

21.



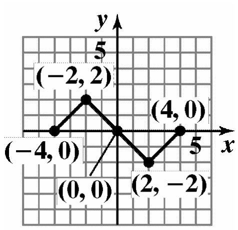
$g(x) = f(x - 1) + 2$

22.



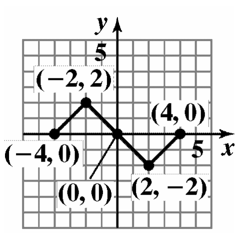
$g(x) = f(x + 1) - 2$

23.



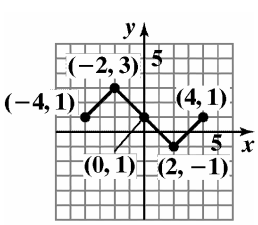
$g(x) = -f(x)$

24.



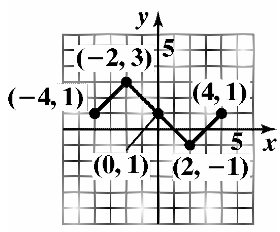
$g(x) = f(-x)$

25.



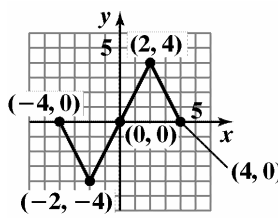
$g(x) = f(-x) + 1$

26.



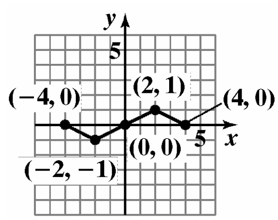
$g(x) = -f(x) + 1$

27.



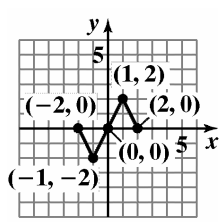
$g(x) = 2f(x)$

28.



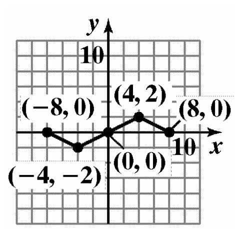
$g(x) = \frac{1}{2}f(x)$

29.



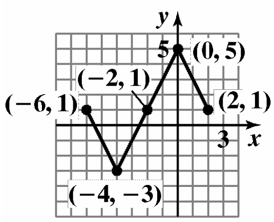
$g(x) = f(2x)$

30.



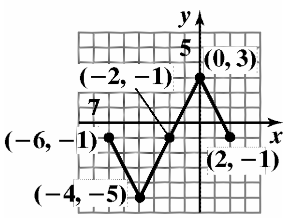
$g(x) = f\left(\frac{1}{2}x\right)$

31.



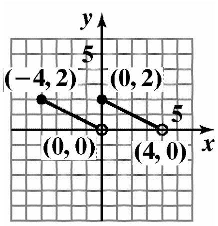
$g(x) = 2f(x+2) + 1$

32.



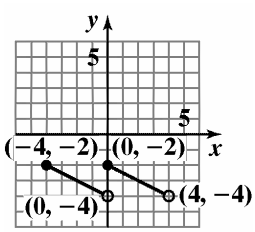
$g(x) = 2f(x+2) - 1$

33.



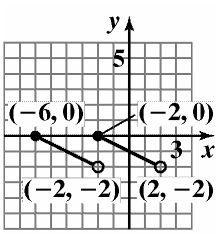
$g(x) = f(x) + 2$

34.



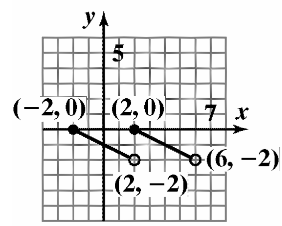
$g(x) = f(x) - 2$

35.



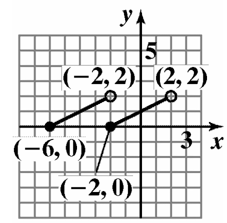
$g(x) = f(x+2)$

36.



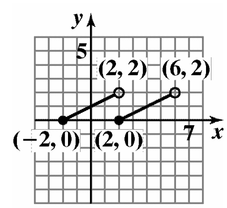
$g(x) = f(x-2)$

37.



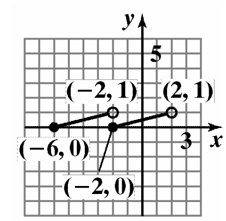
$g(x) = -f(x+2)$

38.



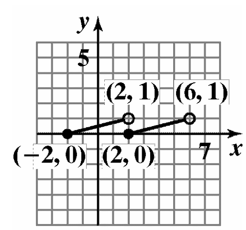
$g(x) = -f(x-2)$

39.



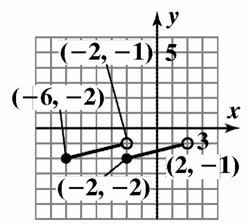
$g(x) = -\frac{1}{2}f(x+2)$

40.



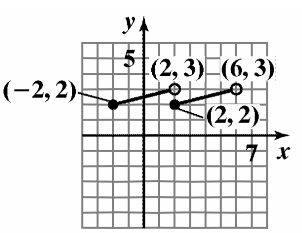
$g(x) = -\frac{1}{2}f(x-2)$

41.



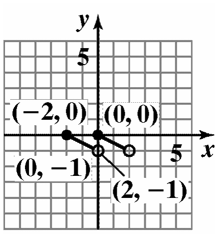
$g(x) = -\frac{1}{2}f(x+2) - 2$

42.



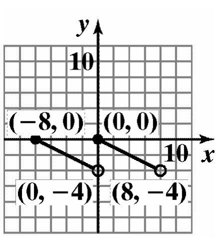
$g(x) = -\frac{1}{2}f(x-2) + 2$

43.



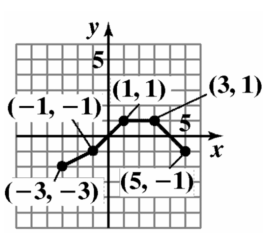
$g(x) = \frac{1}{2}f(2x)$

44.



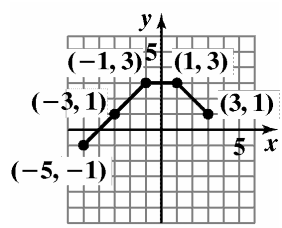
$g(x) = 2f\left(\frac{1}{2}x\right)$

45.



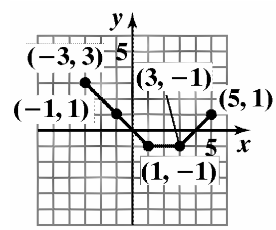
$g(x) = f(x-1) - 1$

46.



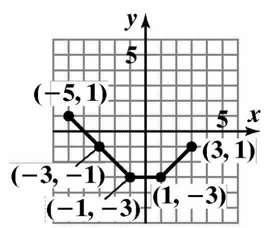
$g(x) = f(x+1) + 1$

47.



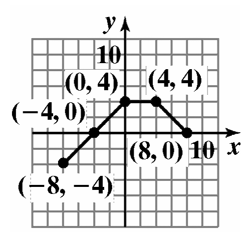
$g(x) = -f(x-1) + 1$

48.



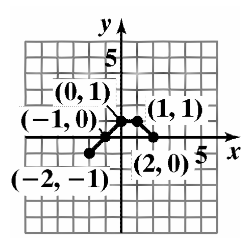
$g(x) = -f(x+1) - 1$

49.



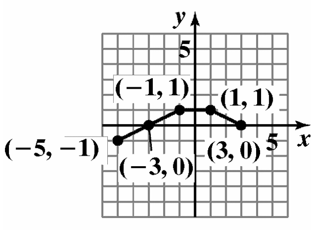
$g(x) = 2f\left(\frac{1}{2}x\right)$

50.



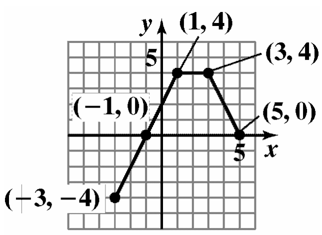
$g(x) = \frac{1}{2}f(2x)$

51.



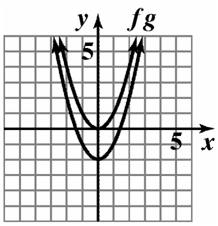
$g(x) = \frac{1}{2}f(x+1)$

52.



$g(x) = 2f(x-1)$

53.



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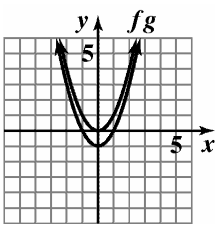

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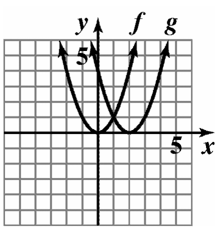

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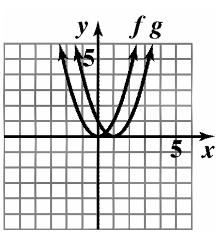

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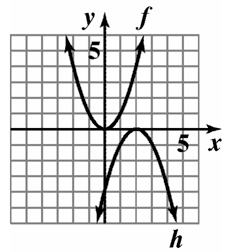

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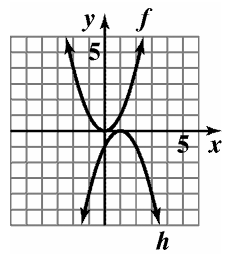

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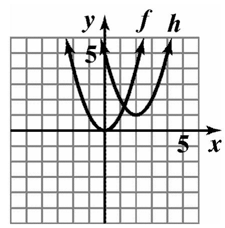

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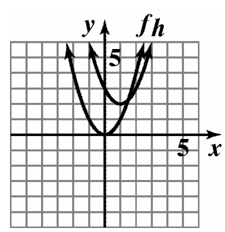

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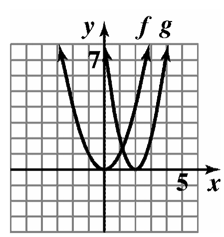

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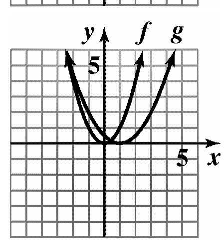
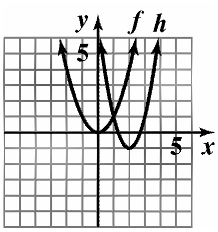
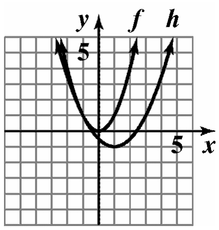
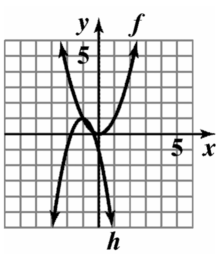
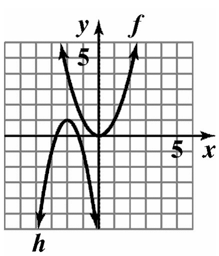
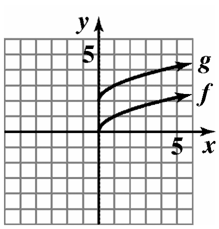


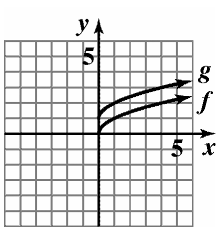
63. 

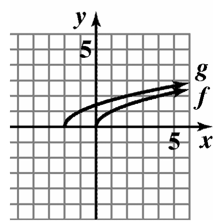
64. 

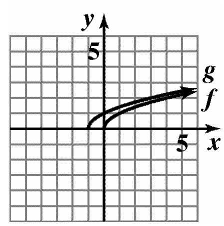
65. 

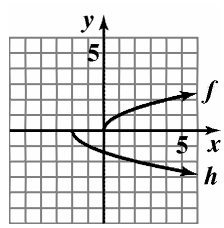
66. 

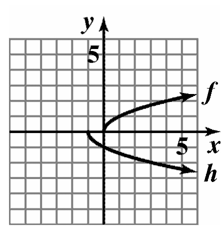
67. 

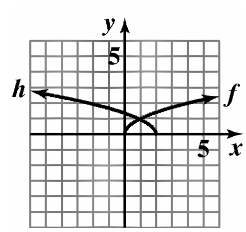
68. 

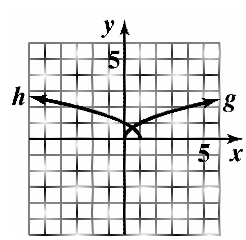
69. 

70. 

71. 

72. 

73. 

74. 

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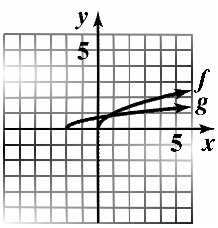

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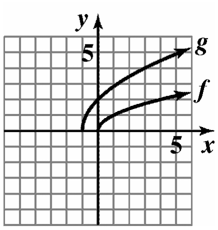

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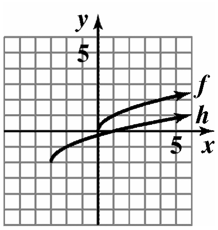

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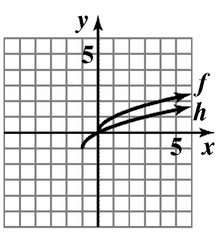

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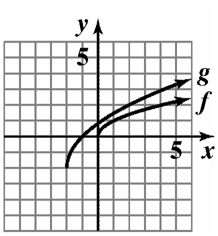

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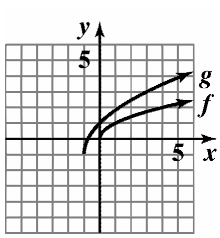

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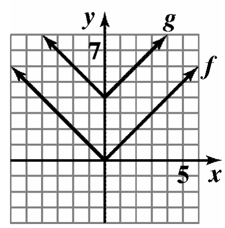

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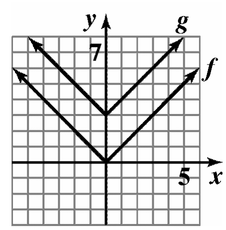

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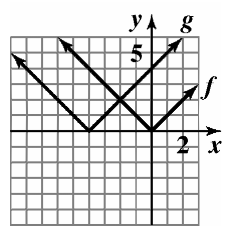

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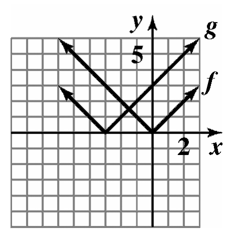

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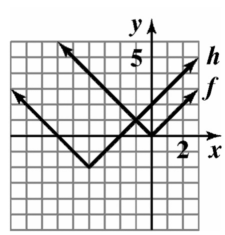

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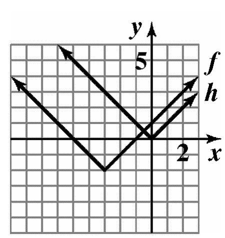

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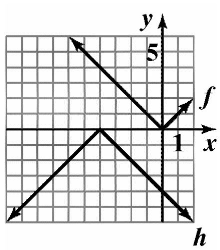

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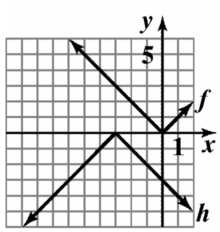

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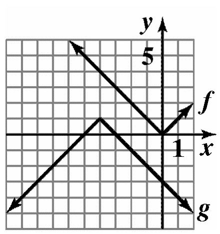

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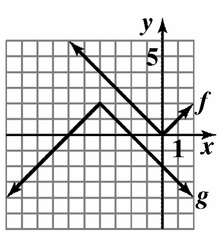

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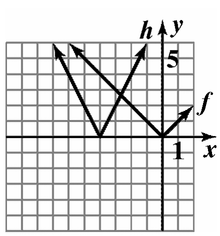

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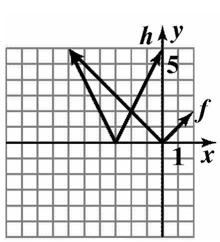

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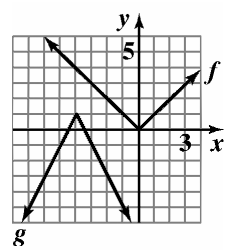

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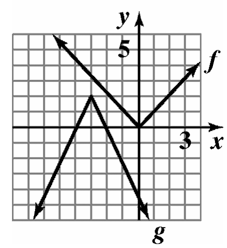

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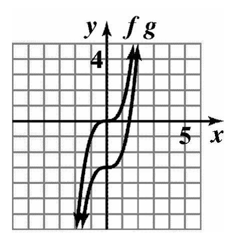

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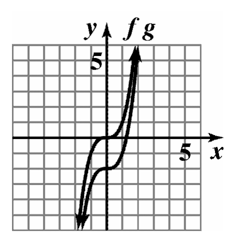

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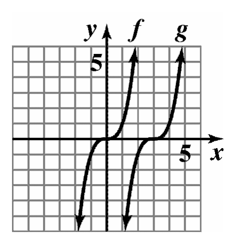

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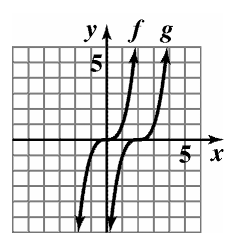

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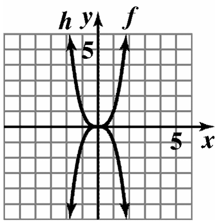

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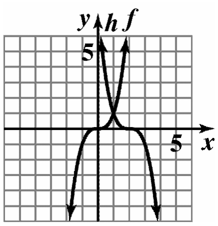

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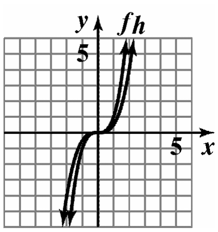

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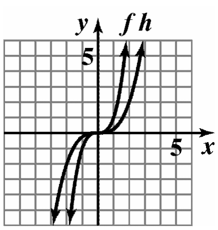

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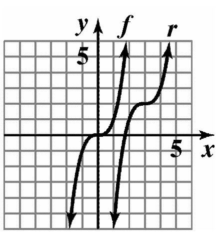

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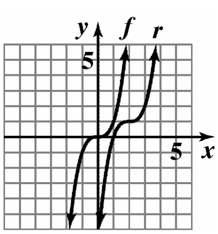

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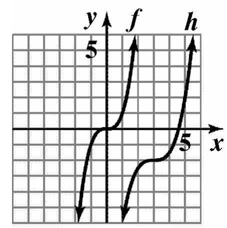

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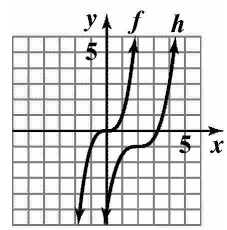

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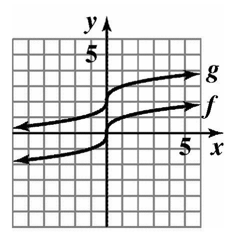

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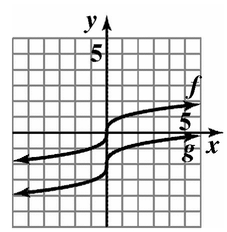

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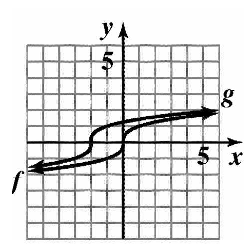

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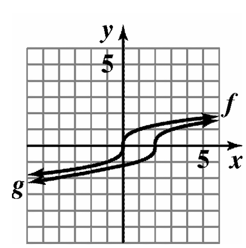
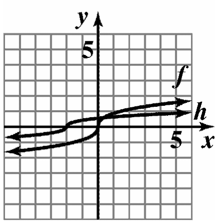
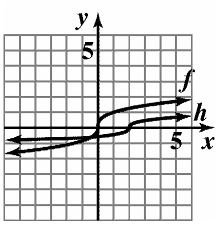
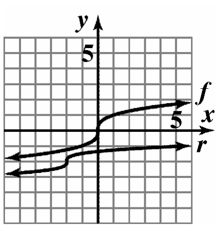
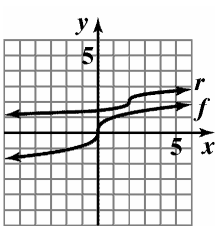
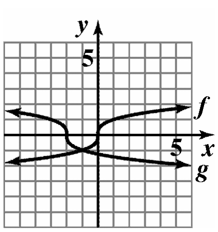


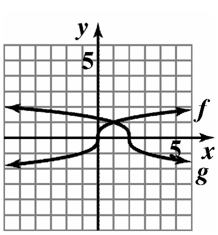
111. 

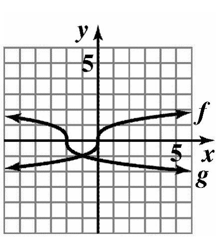
112. 

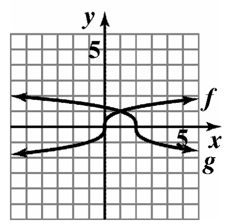
113. 

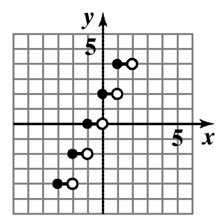
114. 

115. 

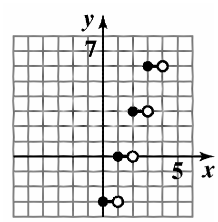
116. 

117. 

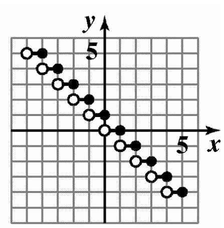
118. 

119. 

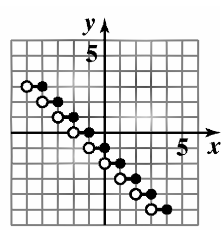
$g(x) = 2\ \text{int}\ (x + 1)$

120. 

$g(x) = 3\ \text{int}\ (x - 1)$

121. 

$h(x) = \text{int}\ (-x) + 1$

122. 

$h(x) = \text{int}\ (-x) - 1$

123. $y = \sqrt{x-2}$

124. $y = -x^3 + 2$

125. $y = (x+1)^2 - 4$

126. $y = \sqrt{x-2} + 1$

127. a. First, vertically stretch the graph of $f(x)=\sqrt{x}$ by the factor 2.9; then shift the result up 20.1 units.

b. $f(x)=2.9\sqrt{x}+20.1$

$f(48)=2.9\sqrt{48}+20.1\approx 40.2$

The model describes the actual data very well.

c. $\frac{f(x_2)-f(x_1)}{x_2-x_1}$

$=\frac{f(10)-f(0)}{10-0}$

$=\frac{\left(2.9\sqrt{10}+20.1\right)-\left(2.9\sqrt{0}+20.1\right)}{10-0}$

$=\frac{29.27-20.1}{10}$

≈ 0.9

0.9 inches per month

d. $\frac{f(x_2)-f(x_1)}{x_2-x_1}$

$=\frac{f(60)-f(50)}{60-50}$

$=\frac{\left(2.9\sqrt{60}+20.1\right)-\left(2.9\sqrt{50}+20.1\right)}{60-50}$

$=\frac{42.5633-40.6061}{10}$

≈ 0.2

This rate of change is lower than the rate of change in part (c). The relative leveling off of the curve shows this difference.

128. a. First, vertically stretch the graph of $f(x)=\sqrt{x}$ by the factor 3.1; then shift the result up 19 units.

b. $f(x)=3.1\sqrt{x}+19$

$f(48)=3.1\sqrt{48}+19\approx 40.5$

The model describes the actual data very well.

c. $\frac{f(x_2)-f(x_1)}{x_2-x_1}$

$=\frac{f(10)-f(0)}{10-0}$

$=\frac{\left(3.1\sqrt{10}+19\right)-\left(3.1\sqrt{0}+19\right)}{10-0}$

$=\frac{28.8031-19}{10}$

≈ 1.0

1.0 inches per month

d. $\frac{f(x_2)-f(x_1)}{x_2-x_1}$

$=\frac{f(60)-f(50)}{60-50}$

$=\frac{\left(3.1\sqrt{60}+19\right)-\left(3.1\sqrt{50}+19\right)}{60-50}$

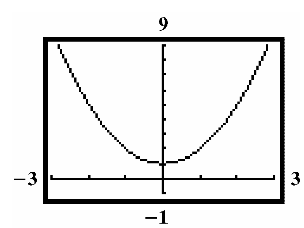
$=\frac{43.0125-40.9203}{10}$

≈ 0.2

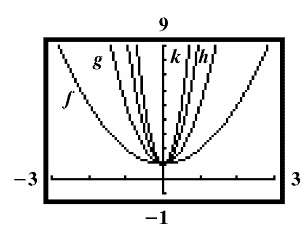
This rate of change is lower than the rate of change in part (c). The relative leveling off of the curve shows this difference.

129. – 134. Answers may vary.

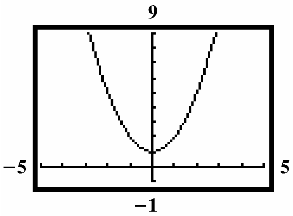
135. a.



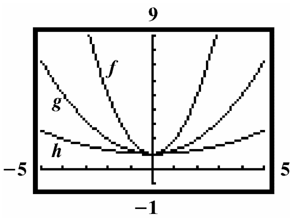
b.



136. a.



b.



137. makes sense

138. makes sense

139. does not make sense; Explanations will vary. Sample explanation: The reprogram should be $y = f(t+1)$.

140. does not make sense; Explanations will vary. Sample explanation: The reprogram should be $y = f(t-1)$.

141. false; Changes to make the statement true will vary. A sample change is: The graph of g is a translation of f three units to the left and three units upward.

142. false; Changes to make the statement true will vary. A sample change is: The graph of f is a reflection of the graph of $y = \sqrt{x}$ in the x-axis, while the graph of g is a reflection of the graph of $y = \sqrt{x}$ in the y-axis.

143. false; Changes to make the statement true will vary. A sample change is: The stretch will be 5 units and the downward shift will be 10 units.

144. true

145. $g(x) = -(x+4)^2$

146. $g(x) = -|x-5|+1$

147. $g(x) = -\sqrt{x-2}+2$

148. $g(x) = -\frac{1}{4}\sqrt{16-x^2}-1$

149. $(-a, b)$

150. $(a, 2b)$

151. $(a+3, b)$

152. $(a, b-3)$

153.
$$\begin{aligned}(2x-1)(x^2+x-2) &= 2x(x^2+x-2)-1(x^2+x-2)\\ &= 2x^3+2x^2-4x-x^2-x+2\\ &= 2x^3+2x^2-x^2-4x-x+2\\ &= 2x^3+x^2-5x+2\end{aligned}$$

154.
$$\begin{aligned}\left(f(x)\right)^2-2f(x)+6 &= (3x-4)^2-2(3x-4)+6\\ &= 9x^2-24x+16-6x+8+6\\ &= 9x^2-24x-6x+16+8+6\\ &= 9x^2-30x+30\end{aligned}$$

155. $\dfrac{2}{\frac{3}{x}-1} = \dfrac{2x}{\frac{3x}{x}-x} = \dfrac{2x}{3-x}$

Section 2.6

Check Point Exercises

1. a. The function $f(x) = x^2+3x-17$ contains neither division nor an even root. The domain of f is the set of all real numbers or $(-\infty, \infty)$.

b. The denominator equals zero when $x = 7$ or $x = -7$. These values must be excluded from the domain.
domain of $g = (-\infty,-7)\cup(-7,7)\cup(7,\infty)$.

c. Since $h(x) = \sqrt{9x-27}$ contains an even root; the quantity under the radical must be greater than or equal to 0.
$$\begin{aligned}9x-27 &\ge 0\\ 9x &\ge 27\\ x &\ge 3\end{aligned}$$
Thus, the domain of h is $\{x | x \ge 3\}$, or the interval $[3, \infty)$.

2. **a.** $(f+g)(x) = f(x)+g(x)$
$= x-5+\left(x^2-1\right)$
$= x-5+x^2-1$
$= -x^2+x-6$

b. $(f-g)(x) = f(x)-g(x)$
$= x-5-\left(x^2-1\right)$
$= x-5-x^2+1$
$= -x^2+x-4$

c. $(fg)(x) = (x-5)\left(x^2-1\right)$
$= x\left(x^2-1\right)-5\left(x^2-1\right)$
$= x^3-x-5x^2+5$
$= x^3-5x^2-x+5$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
$= \frac{x-5}{x^2-1},\ x \neq \pm 1$

3. **a.** $(f+g)(x) = f(x)+g(x)$
$= \sqrt{x-3}+\sqrt{x+1}$

b. domain of *f*: $x-3 \geq 0$
$x \geq 3$
$[3, \infty)$
domain of g: $x+1 \geq 0$
$x \geq -1$
$[-1, \infty)$
The domain of *f* + *g* is the set of all real numbers that are common to the domain of *f* and the domain of *g*. Thus, the domain of *f* + *g* is [3, ∞).

4. **a.** $(f \circ g)(x) = f(g(x))$
$= 5\left(2x^2-x-1\right)+6$
$= 10x^2-5x-5+6$
$= 10x^2-5x+1$

b. $(g \circ f)(x) = g(f(x))$
$= 2(5x+6)^2-(5x+6)-1$
$= 2(25x^2+60x+36)-5x-6-1$
$= 50x^2+120x+72-5x-6-1$
$= 50x^2+115x+65$

c. $(f \circ g)(x) = 10x^2-5x+1$
$(f \circ g)(-1) = 10(-1)^2-5(-1)+1$
$= 10+5+1$
$= 16$

5. **a.** $(f \circ g)(x) = \dfrac{4}{\frac{1}{x}+2} = \dfrac{4x}{1+2x}$

b. domain: $\left\{x \middle| x \neq 0,\ x \neq -\frac{1}{2}\right\}$

6. $h(x) = f \circ g$ where $f(x) = \sqrt{x}$; $g(x) = x^2+5$

Exercise Set 2.6

1. The function contains neither division nor an even root. The domain $= (-\infty, \infty)$

2. The function contains neither division nor an even root. The domain $= (-\infty, \infty)$

3. The denominator equals zero when $x = 4$. This value must be excluded from the domain.
domain: $(-\infty, 4) \cup (4, \infty)$.

4. The denominator equals zero when $x = -5$. This value must be excluded from the domain.
domain: $(-\infty, -5) \cup (-5, \infty)$.

5. The function contains neither division nor an even root. The domain $= (-\infty, \infty)$

6. The function contains neither division nor an even root. The domain $= (-\infty, \infty)$

7. The values that make the denominator equal zero must be excluded from the domain.
domain: $(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$

8. The values that make the denominator equal zero must be excluded from the domain.
domain: $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$

9. The values that make the denominators equal zero must be excluded from the domain.
domain: $(-\infty, -7) \cup (-7, 9) \cup (9, \infty)$

10. The values that make the denominators equal zero must be excluded from the domain.
domain: $(-\infty,-8)\cup(-8,10)\cup(10,\infty)$

11. The first denominator cannot equal zero. The values that make the second denominator equal zero must be excluded from the domain.
domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

12. The first denominator cannot equal zero. The values that make the second denominator equal zero must be excluded from the domain.
domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

13. Exclude x for $x=0$.
Exclude x for $\frac{3}{x}-1=0$.

$$\frac{3}{x}-1=0$$
$$x\left(\frac{3}{x}-1\right)=x(0)$$
$$3-x=0$$
$$-x=-3$$
$$x=3$$

domain: $(-\infty,0)\cup(0,3)\cup(3,\infty)$

14. Exclude x for $x=0$.
Exclude x for $\frac{4}{x}-1=0$.

$$\frac{4}{x}-1=0$$
$$x\left(\frac{4}{x}-1\right)=x(0)$$
$$4-x=0$$
$$-x=-4$$
$$x=4$$

domain: $(-\infty,0)\cup(0,4)\cup(4,\infty)$

15. Exclude x for $x-1=0$.

$$x-1=0$$
$$x=1$$

Exclude x for $\frac{4}{x-1}-2=0$.

$$\frac{4}{x-1}-2=0$$
$$(x-1)\left(\frac{4}{x-1}-2\right)=(x-1)(0)$$
$$4-2(x-1)=0$$
$$4-2x+2=0$$
$$-2x+6=0$$
$$-2x=-6$$
$$x=3$$

domain: $(-\infty,1)\cup(1,3)\cup(3,\infty)$

16. Exclude x for $x-2=0$.

$$x-2=0$$
$$x=2$$

Exclude x for $\frac{4}{x-2}-3=0$.

$$\frac{4}{x-2}-3=0$$
$$(x-2)\left(\frac{4}{x-2}-3\right)=(x-2)(0)$$
$$4-3(x-2)=0$$
$$4-3x+6=0$$
$$-3x+10=0$$
$$-3x=-10$$
$$x=\frac{10}{3}$$

domain: $(-\infty,2)\cup\left(2,\frac{10}{3}\right)\cup\left(\frac{10}{3},\infty\right)$

17. The expression under the radical must not be negative.

$$x-3\ge 0$$
$$x\ge 3$$

domain: $[3,\infty)$

18. The expression under the radical must not be negative.

$$x+2\ge 0$$
$$x\ge -2$$

domain: $[-2,\infty)$

19. The expression under the radical must be positive.
$x-3>0$
$x>3$
domain: $(3,\infty)$

20. The expression under the radical must be positive.
$x+2>0$
$x>-2$
domain: $(-2,\infty)$

21. The expression under the radical must not be negative.
$5x+35\geq 0$
$5x\geq -35$
$x\geq -7$
domain: $[-7,\infty)$

22. The expression under the radical must not be negative.
$7x-70\geq 0$
$7x\geq 70$
$x\geq 10$
domain: $[10,\infty)$

23. The expression under the radical must not be negative.
$24-2x\geq 0$
$-2x\geq -24$
$\frac{-2x}{-2}\leq\frac{-24}{-2}$
$x\leq 12$
domain: $(-\infty,12]$

24. The expression under the radical must not be negative.
$84-6x\geq 0$
$-6x\geq -84$
$\frac{-6x}{-6}\leq\frac{-84}{-6}$
$x\leq 14$
domain: $(-\infty,14]$

25. The expressions under the radicals must not be negative.
$x-2\geq 0$ and $x+3\geq 0$
$x\geq 2$ $\quad$ $x\geq -3$
To make both inequalities true, $x\geq 2$.
domain: $[2,\infty)$

26. The expressions under the radicals must not be negative.
$x-3\geq 0$ and $x+4\geq 0$
$x\geq 3$ $\quad$ $x\geq -4$
To make both inequalities true, $x\geq 3$.
domain: $[3,\infty)$

27. The expression under the radical must not be negative.
$x-2\geq 0$
$x\geq 2$
The denominator equals zero when $x=5$.
domain: $[2,5)\cup(5,\infty)$.

28. The expression under the radical must not be negative.
$x-3\geq 0$
$x\geq 3$
The denominator equals zero when $x=6$.
domain: $[3,6)\cup(6,\infty)$.

29. Find the values that make the denominator equal zero and must be excluded from the domain.
$x^3-5x^2-4x+20$
$=x^2(x-5)-4(x-5)$
$=(x-5)(x^2-4)$
$=(x-5)(x+2)(x-2)$
–2, 2, and 5 must be excluded.
domain: $(-\infty,-2)\cup(-2,2)\cup(2,5)\cup(5,\infty)$

30. Find the values that make the denominator equal zero and must be excluded from the domain.
$x^3-2x^2-9x+18$
$=x^2(x-2)-9(x-2)$
$=(x-2)(x^2-9)$
$=(x-2)(x+3)(x-3)$
–3, 2, and 3 must be excluded.
domain: $(-\infty,-3)\cup(-3,2)\cup(2,3)\cup(3,\infty)$

31. $(f+g)(x) = 3x + 2$
domain: $(-\infty, \infty)$
$$(f-g)(x) = f(x) - g(x) = (2x+3) - (x-1) = x + 4$$
domain: $(-\infty, \infty)$
$$(fg)(x) = f(x) \cdot g(x) = (2x+3)\cdot(x-1) = 2x^2 + x - 3$$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x+3}{x-1}$$
domain: $(-\infty,1)\cup(1,\infty)$

32. $(f+g)(x) = 4x - 2$
domain: $(-\infty, \infty)$
$(f-g)(x) = (3x-4) - (x+2) = 2x - 6$
domain: $(-\infty, \infty)$
$(fg)(x) = (3x-4)(x+2) = 3x^2 + 2x - 8$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{3x-4}{x+2}$$
domain: $(-\infty,-2)\cup(-2,\infty)$

33. $(f+g)(x) = 3x^2 + x - 5$
domain: $(-\infty, \infty)$
$(f-g)(x) = -3x^2 + x - 5$
domain: $(-\infty, \infty)$
$(fg)(x) = (x-5)(3x^2) = 3x^3 - 15x^2$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{x-5}{3x^2}$$
domain: $(-\infty,0)\cup(0,\infty)$

34. $(f+g)(x) = 5x^2 + x - 6$
domain: $(-\infty, \infty)$
$(f-g)(x) = -5x^2 + x - 6$
domain: $(-\infty, \infty)$
$(fg)(x) = (x-6)(5x^2) = 5x^3 - 30x^2$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{x-6}{5x^2}$$
domain: $(-\infty,0)\cup(0,\infty)$

35. $(f+g)(x) = 2x^2 - 2$
domain: $(-\infty, \infty)$
$(f-g)(x) = 2x^2 - 2x - 4$
domain: $(-\infty, \infty)$
$$(fg)(x) = (2x^2 - x - 3)(x+1) = 2x^3 + x^2 - 4x - 3$$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{2x^2 - x - 3}{x+1} = \frac{(2x-3)(x+1)}{(x+1)} = 2x - 3$$
domain: $(-\infty,-1)\cup(-1,\infty)$

36. $(f+g)(x) = 6x^2 - 2$
domain: $(-\infty, \infty)$
$(f-g)(x) = 6x^2 - 2x$
domain: $(-\infty, \infty)$
$(fg)(x) = (6x^2 - x - 1)(x-1) = 6x^3 - 7x^2 + 1$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{6x^2 - x - 1}{x-1}$$
domain: $(-\infty,1)\cup(1,\infty)$

37. $(f+g)(x) = (3 - x^2) + (x^2 + 2x - 15) = 2x - 12$
domain: $(-\infty, \infty)$
$$(f-g)(x) = (3 - x^2) - (x^2 + 2x - 15) = -2x^2 - 2x + 18$$
domain: $(-\infty, \infty)$
$$(fg)(x) = (3 - x^2)(x^2 + 2x - 15) = -x^4 - 2x^3 + 18x^2 + 6x - 45$$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{3 - x^2}{x^2 + 2x - 15}$$
domain: $(-\infty,-5)\cup(-5,3)\cup(3,\infty)$

38. $(f+g)(x)=(5-x^2)+(x^2+4x-12)$
$=4x-7$
domain: $(-\infty, \infty)$
$(f-g)(x)=(5-x^2)-(x^2+4x-12)$
$=-2x^2-4x+17$
domain: $(-\infty, \infty)$
$(fg)(x)=(5-x^2)(x^2+4x-12)$
$=-x^4-4x^3+17x^2+20x-60$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x)=\frac{5-x^2}{x^2+4x-12}$
domain: $(-\infty,-6)\cup(-6,2)\cup(2,\infty)$

39. $(f+g)(x)=\sqrt{x}+x-4$
domain: $[0,\infty)$
$(f-g)(x)=\sqrt{x}-x+4$
domain: $[0,\infty)$
$(fg)(x)=\sqrt{x}(x-4)$
domain: $[0,\infty)$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x}}{x-4}$
domain: $[0,4)\cup(4,\infty)$

40. $(f+g)(x)=\sqrt{x}+x-5$
domain: $[0,\infty)$
$(f-g)(x)=\sqrt{x}-x+5$
domain: $[0,\infty)$
$(fg)(x)=\sqrt{x}(x-5)$
domain: $[0,\infty)$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x}}{x-5}$
domain: $[0,5)\cup(5,\infty)$

41. $(f+g)(x)=2+\frac{1}{x}+\frac{1}{x}=2+\frac{2}{x}=\frac{2x+2}{x}$
domain: $(-\infty,0)\cup(0,\infty)$
$(f-g)(x)=2+\frac{1}{x}-\frac{1}{x}=2$
domain: $(-\infty,0)\cup(0,\infty)$
$(fg)(x)=\left(2+\frac{1}{x}\right)\cdot\frac{1}{x}=\frac{2}{x}+\frac{1}{x^2}=\frac{2x+1}{x^2}$
domain: $(-\infty,0)\cup(0,\infty)$
$\left(\frac{f}{g}\right)(x)=\frac{2+\frac{1}{x}}{\frac{1}{x}}=\left(2+\frac{1}{x}\right)\cdot x=2x+1$
domain: $(-\infty,0)\cup(0,\infty)$

42. $(f+g)(x)=6-\frac{1}{x}+\frac{1}{x}=6$
domain: $(-\infty,0)\cup(0,\infty)$
$(f-g)(x)=6-\frac{1}{x}-\frac{1}{x}=6-\frac{2}{x}=\frac{6x-2}{x}$
domain: $(-\infty,0)\cup(0,\infty)$
$(fg)(x)=\left(6-\frac{1}{x}\right)\cdot\frac{1}{x}=\frac{6}{x}-\frac{1}{x^2}=\frac{6x-1}{x^2}$
domain: $(-\infty,0)\cup(0,\infty)$
$\left(\frac{f}{g}\right)(x)=\frac{6-\frac{1}{x}}{\frac{1}{x}}=\left(6-\frac{1}{x}\right)\cdot x=6x-1$
domain: $(-\infty,0)\cup(0,\infty)$

43. $(f+g)(x) = f(x) + g(x)$

$$= \frac{5x+1}{x^2-9} + \frac{4x-2}{x^2-9}$$
$$= \frac{9x-1}{x^2-9}$$

domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$(f-g)(x) = f(x) - g(x)$

$$= \frac{5x+1}{x^2-9} - \frac{4x-2}{x^2-9}$$
$$= \frac{x+3}{x^2-9}$$
$$= \frac{1}{x-3}$$

domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{5x+1}{x^2-9}\cdot\frac{4x-2}{x^2-9}$$
$$= \frac{(5x+1)(4x-2)}{\left(x^2-9\right)^2}$$

domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{5x+1}{x^2-9}}{\frac{4x-2}{x^2-9}}$$
$$= \frac{5x+1}{x^2-9}\cdot\frac{x^2-9}{4x-2}$$
$$= \frac{5x+1}{4x-2}$$

The domain must exclude –3, 3, and any values that make $4x-2=0$.

$$4x-2=0$$
$$4x=2$$
$$x=\frac{1}{2}$$

domain: $(-\infty,-3)\cup(-3,\tfrac{1}{2})\cup(\tfrac{1}{2},3)\cup(3,\infty)$

44. $(f+g)(x) = f(x) + g(x)$

$$= \frac{3x+1}{x^2-25} + \frac{2x-4}{x^2-25}$$
$$= \frac{5x-3}{x^2-25}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$(f-g)(x) = f(x) - g(x)$

$$= \frac{3x+1}{x^2-25} - \frac{2x-4}{x^2-25}$$
$$= \frac{x+5}{x^2-25}$$
$$= \frac{1}{x-5}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{3x+1}{x^2-25}\cdot\frac{2x-4}{x^2-25}$$
$$= \frac{(3x+1)(2x-4)}{\left(x^2-25\right)^2}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{3x+1}{x^2-25}}{\frac{2x-4}{x^2-25}}$$
$$= \frac{3x+1}{x^2-25}\cdot\frac{x^2-25}{2x-4}$$
$$= \frac{3x+1}{2x-4}$$

The domain must exclude –5, 5, and any values that make $2x-4=0$.

$$2x-4=0$$
$$2x=4$$
$$x=2$$

domain: $(-\infty,-5)\cup(-5,2)\cup(2,5)\cup(5,\infty)$

45. $(f+g)(x) = \sqrt{x+4} + \sqrt{x-1}$

domain: $[1,\infty)$

$(f-g)(x) = \sqrt{x+4} - \sqrt{x-1}$

domain: $[1,\infty)$

$(fg)(x) = \sqrt{x+4}\cdot\sqrt{x-1} = \sqrt{x^2+3x-4}$

domain: $[1,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+4}}{\sqrt{x-1}}$$

domain: $(1,\infty)$

46. $(f+g)(x)=\sqrt{x+6}+\sqrt{x-3}$
domain: $[3, \infty)$
$(f-g)(x)=\sqrt{x+6}-\sqrt{x-3}$
domain: $[3, \infty)$
$(fg)(x)=\sqrt{x+6}\cdot\sqrt{x-3}=\sqrt{x^2+3x-18}$
domain: $[3, \infty)$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x+6}}{\sqrt{x-3}}$
domain: $(3, \infty)$

47. $(f+g)(x)=\sqrt{x-2}+\sqrt{2-x}$
domain: $\{2\}$
$(f-g)(x)=\sqrt{x-2}-\sqrt{2-x}$
domain: $\{2\}$
$(fg)(x)=\sqrt{x-2}\cdot\sqrt{2-x}=\sqrt{-x^2+4x-4}$
domain: $\{2\}$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x-2}}{\sqrt{2-x}}$
domain: $\varnothing$

48. $(f+g)(x)=\sqrt{x-5}+\sqrt{5-x}$
domain: $\{5\}$
$(f-g)(x)=\sqrt{x-5}-\sqrt{5-x}$
domain: $\{5\}$
$(fg)(x)=\sqrt{x-5}\cdot\sqrt{5-x}=\sqrt{-x^2+10x-25}$
domain: $\{5\}$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x-5}}{\sqrt{5-x}}$
domain: $\varnothing$

49. $f(x) = 2x;\ g(x) = x + 7$

a. $(f\circ g)(x)=2(x+7)=2x+14$

b. $(g\circ f)(x)=2x+7$

c. $(f\circ g)(2)=2(2)+14=18$

50. $f(x) = 3x;\ g(x) = x - 5$

a. $(f\circ g)(x)=3(x-5)=3x-15$

b. $(g\circ f)(x)=3x-5$

c. $(f\circ g)(2)=3(2)-15=-9$

51. $f(x) = x + 4;\ g(x) = 2x + 1$

a. $(f\circ g)(x)=(2x+1)+4=2x+5$

b. $(g\circ f)(x)=2(x+4)+1=2x+9$

c. $(f\circ g)(2)=2(2)+5=9$

52. $f(x) = 5x + 2;\ g(x) = 3x - 4$

a. $(f\circ g)(x)=5(3x-4)+2=15x-18$

b. $(g\circ f)(x)=3(5x+2)-4=15x+2$

c. $(f\circ g)(2)=15(2)-18=12$

53. $f(x) = 4x - 3;\ g(x)=5x^2-2$

a. $(f\circ g)(x)=4(5x^2-2)-3$
$=20x^2-11$

b. $(g\circ f)(x)=5(4x-3)^2-2$
$=5(16x^2-24x+9)-2$
$=80x^2-120x+43$

c. $(f\circ g)(2)=20(2)^2-11=69$

54. $f(x)=7x+1;\ g(x)=2x^2-9$

a. $(f\circ g)(x)=7(2x^2-9)+1=14x^2-62$

b. $(g\circ f)(x)=2(7x+1)^2-9$
$=2(49x^2+14x+1)-9$
$=98x^2+28x-7$

c. $(f\circ g)(2)=14(2)^2-62=-6$

55. $f(x)=x^2+2;\ g(x)=x^2-2$

a. $(f\circ g)(x)=(x^2-2)^2+2$
$=x^4-4x^2+4+2$
$=x^4-4x^2+6$

b. $(g\circ f)(x)=(x^2+2)^2-2$
$=x^4+4x^2+4-2$
$=x^4+4x^2+2$

c. $(f\circ g)(2)=2^4-4(2)^2+6=6$

56. $f(x) = x^2 + 1; g(x) = x^2 - 3$

a. $(f \circ g)(x) = (x^2 - 3)^2 + 1$
$= x^4 - 6x^2 + 9 + 1$
$= x^4 - 6x^2 + 10$

b. $(g \circ f)(x) = (x^2 + 1)^2 - 3$
$= x^4 + 2x^2 + 1 - 3$
$= x^4 + 2x^2 - 2$

c. $(f \circ g)(2) = 2^4 - 6(2)^2 + 10 = 2$

57. $f(x) = 4 - x; \quad g(x) = 2x^2 + x + 5$

a. $(f \circ g)(x) = 4 - \left(2x^2 + x + 5\right)$
$= 4 - 2x^2 - x - 5$
$= -2x^2 - x - 1$

b. $(g \circ f)(x) = 2\left(4 - x\right)^2 + \left(4 - x\right) + 5$
$= 2(16 - 8x + x^2) + 4 - x + 5$
$= 32 - 16x + 2x^2 + 4 - x + 5$
$= 2x^2 - 17x + 41$

c. $(f \circ g)(2) = -2(2)^2 - 2 - 1 = -11$

58. $f(x) = 5x - 2; \quad g(x) = -x^2 + 4x - 1$

a. $(f \circ g)(x) = 5\left(-x^2 + 4x - 1\right) - 2$
$= -5x^2 + 20x - 5 - 2$
$= -5x^2 + 20x - 7$

b. $(g \circ f)(x) = -\left(5x - 2\right)^2 + 4\left(5x - 2\right) - 1$
$= -(25x^2 - 20x + 4) + 20x - 8 - 1$
$= -25x^2 + 20x - 4 + 20x - 8 - 1$
$= -25x^2 + 40x - 13$

c. $(f \circ g)(2) = -5(2)^2 + 20(2) - 7 = 13$

59. $f(x) = \sqrt{x}; \; g(x) = x - 1$

a. $(f \circ g)(x) = \sqrt{x - 1}$

b. $(g \circ f)(x) = \sqrt{x} - 1$

c. $(f \circ g)(2) = \sqrt{2 - 1} = \sqrt{1} = 1$

60. $f(x) = \sqrt{x}; g(x) = x + 2$

a. $(f \circ g)(x) = \sqrt{x + 2}$

b. $(g \circ f)(x) = \sqrt{x} + 2$

c. $(f \circ g)(2) = \sqrt{2 + 2} = \sqrt{4} = 2$

61. $f(x) = 2x - 3; \; g(x) = \dfrac{x + 3}{2}$

a. $(f \circ g)(x) = 2\left(\dfrac{x + 3}{2}\right) - 3$
$= x + 3 - 3$
$= x$

b. $(g \circ f)(x) = \dfrac{(2x - 3) + 3}{2} = \dfrac{2x}{2} = x$

c. $(f \circ g)(2) = 2$

62. $f(x) = 6x - 3; g(x) = \dfrac{x + 3}{6}$

a. $(f \circ g)(x) = 6\left(\dfrac{x + 3}{6}\right) - 3 = x + 3 - 3 = x$

b. $(g \circ f)(x) = \dfrac{6x - 3 + 3}{6} = \dfrac{6x}{6} = x$

c. $(f \circ g)(2) = 2$

63. $f(x) = \dfrac{1}{x}; \quad g(x) = \dfrac{1}{x}$

a. $(f \circ g)(x) = \dfrac{1}{\frac{1}{x}} = x$

b. $(g \circ f)(x) = \dfrac{1}{\frac{1}{x}} = x$

c. $(f \circ g)(2) = 2$

64. $f(x)=\frac{2}{x};\quad g(x)=\frac{2}{x}$

a. $(f\circ g)(x)=\frac{2}{\frac{2}{x}}=x$

b. $(g\circ f)(x)=\frac{2}{\frac{2}{x}}=x$

c. $(f\circ g)(2)=2$

65. a. $(f\circ g)(x)=f\left(\frac{1}{x}\right)=\frac{2}{\frac{1}{x}+3},x\neq 0$

$=\frac{2(x)}{\left(\frac{1}{x}+3\right)(x)}$

$=\frac{2x}{1+3x}$

b. We must exclude 0 because it is excluded from g.
We must exclude $-\frac{1}{3}$ because it causes the denominator of $f\circ g$ to be 0.
domain: $\left(-\infty,-\frac{1}{3}\right)\cup\left(-\frac{1}{3},0\right)\cup(0,\infty)$.

66. a. $f\circ g(x)=f\left(\frac{1}{x}\right)=\frac{5}{\frac{1}{x}+4}=\frac{5x}{1+4x}$

b. We must exclude 0 because it is excluded from g.
We must exclude $-\frac{1}{4}$ because it causes the denominator of $f\circ g$ to be 0.
domain: $\left(-\infty,-\frac{1}{4}\right)\cup\left(-\frac{1}{4},0\right)\cup(0,\infty)$.

67. a. $(f\circ g)(x)=f\left(\frac{4}{x}\right)=\frac{\frac{4}{x}}{\frac{4}{x}+1}$

$=\frac{\left(\frac{4}{x}\right)(x)}{\left(\frac{4}{x}+1\right)(x)}$

$=\frac{4}{4+x},x\neq -4$

b. We must exclude 0 because it is excluded from g.
We must exclude -4 because it causes the denominator of $f\circ g$ to be 0.
domain: $(-\infty,-4)\cup(-4,0)\cup(0,\infty)$.

68. a. $f\circ g(x)=f\left(\frac{6}{x}\right)=\frac{\frac{6}{x}}{\frac{6}{x}+5}=\frac{6}{6+5x}$

b. We must exclude 0 because it is excluded from g.
We must exclude $-\frac{6}{5}$ because it causes the denominator of $f\circ g$ to be 0.
domain: $\left(-\infty,-\frac{6}{5}\right)\cup\left(-\frac{6}{5},0\right)\cup(0,\infty)$.

69. a. $f\circ g(x)=f(x-2)=\sqrt{x-2}$

b. The expression under the radical in $f\circ g$ must not be negative.
$x-2\geq 0$
$x\geq 2$
domain: $[2,\infty)$.

70. a. $f\circ g(x)=f(x-3)=\sqrt{x-3}$

b. The expression under the radical in $f\circ g$ must not be negative.
$x-3\geq 0$
$x\geq 3$
domain: $[3,\infty)$.

71. **a.** $(f \circ g)(x) = f(\sqrt{1-x})$

$= \left(\sqrt{1-x}\right)^2 + 4$

$= 1 - x + 4$

$= 5 - x$

b. The domain of $f \circ g$ must exclude any values that are excluded from g.

$1 - x \geq 0$

$-x \geq -1$

$x \leq 1$

domain: $(-\infty, 1]$.

72. **a.** $(f \circ g)(x) = f(\sqrt{2-x})$

$= \left(\sqrt{2-x}\right)^2 + 1$

$= 2 - x + 1$

$= 3 - x$

b. The domain of $f \circ g$ must exclude any values that are excluded from g.

$2 - x \geq 0$

$-x \geq -2$

$x \leq 2$

domain: $(-\infty, 2]$.

73. $f(x) = x^4 \quad g(x) = 3x - 1$

74. $f(x) = x^3; g(x) = 2x - 5$

75. $f(x) = \sqrt[3]{x} \quad g(x) = x^2 - 9$

76. $f(x) = \sqrt{x}; g(x) = 5x^2 + 3$

77. $f(x) = |x| \quad g(x) = 2x - 5$

78. $f(x) = |x|; \ g(x) = 3x - 4$

79. $f(x) = \dfrac{1}{x} \quad g(x) = 2x - 3$

80. $f(x) = \dfrac{1}{x}; g(x) = 4x + 5$

81. $(f + g)(-3) = f(-3) + g(-3) = 4 + 1 = 5$

82. $(g - f)(-2) = g(-2) - f(-2) = 2 - 3 = -1$

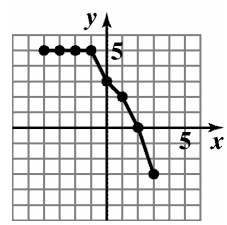
83. $(fg)(2) = f(2)g(2) = (-1)(1) = -1$

84. $\left(\dfrac{g}{f}\right)(3) = \dfrac{g(3)}{f(3)} = \dfrac{0}{-3} = 0$

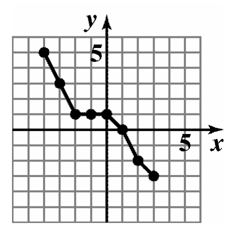
85. The domain of $f + g$ is $[-4, 3]$.

86. The domain of $\dfrac{f}{g}$ is $(-4, 3)$

87. The graph of $f + g$



88. The graph of $f - g$



89. $(f \circ g)(-1) = f(g(-1)) = f(-3) = 1$

90. $(f \circ g)(1) = f(g(1)) = f(-5) = 3$

91. $(g \circ f)(0) = g(f(0)) = g(2) = -6$

92. $(g \circ f)(-1) = g(f(-1)) = g(1) = -5$

93.

$$(f \circ g)(x) = 7$$
$$2\left(x^2 - 3x + 8\right) - 5 = 7$$
$$2x^2 - 6x + 16 - 5 = 7$$
$$2x^2 - 6x + 11 = 7$$
$$2x^2 - 6x + 4 = 0$$
$$x^2 - 3x + 2 = 0$$
$$(x-1)(x-2) = 0$$
$$x - 1 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = 1 \qquad\qquad x = 2$$

94.
$$(f \circ g)(x) = -5$$
$$1-2\left(3x^2+x-1\right) = -5$$
$$1-6x^2-2x+2 = -5$$
$$-6x^2-2x+3 = -5$$
$$-6x^2-2x+8 = 0$$
$$3x^2+x-4 = 0$$
$$(3x+4)(x-1) = 0$$
$$3x+4=0 \quad \text{or} \quad x-1=0$$
$$3x=-4 \qquad x=1$$
$$x=-\frac{4}{3}$$

95. **a.** $(B-D)(x) = B(x)-D(x)$
$$= (7.4x^2-15x+4046)-(-3.5x^2+20x+2405)$$
$$= 7.4x^2-15x+4046+3.5x^2-20x-2405$$
$$= 10.9x^2-35x+1641$$

b. $(B-D)(x) = 10.9x^2-35x+1641$
$$(B-D)(3) = 10.9(3)^2-35(3)+1641$$
$$= 1634.1$$
The change in population in the U.S. in 2003 was 1634.1 thousand.

c. $(B-D)(x)$ overestimates the actual change in population in the U.S. in 2003 by 0.1 thousand.

96. **a.** $(B+D)(x) = B(x)+D(x)$
$$= (7.4x^2-15x+4046)+(-3.5x^2+20x+2405)$$
$$= 7.4x^2-15x+4046-3.5x^2+20x+2405$$
$$= 3.9x^2+5x+6451$$

b. $(B+D)(x) = 3.9x^2+5x+6451$
$$(B+D)(5) = 3.9(5)^2+5(5)+6451$$
$$= 6573.5$$
The number of births and deaths in the U.S. in 2005 is 6573.5 thousand.

c. $(B+D)(x)$ underestimates the actual number of births and deaths in 2005 by 1.5 thousand.

97. $(R-C)(20{,}000)$
$= 65(20{,}000)-(600{,}000+45(20{,}000))$
$= -200{,}000$
The company lost $200,000 since costs exceeded revenues.
$(R - C)(30{,}000)$
$= 65(30{,}000) - (600{,}000 + 45(30{,}000))$
$= 0$
The company broke even.

98. **a.** The slope for f is -0.44 This is the decrease in profits for the first store for each year after 2004.

b. The slope of g is 0.51 This is the increase in profits for the second store for each year after 2004.

c. $f + g = -.044x + 13.62 + 0.51x + 11.14$
$= 0.07x + 24.76$
The slope for $f + g$ is 0.07 This is the profit for the two stores combined for each year after 2004.

99. **a.** f gives the price of the computer after a \$400 discount. g gives the price of the computer after a 25% discount.

b. $(f \circ g)(x) = 0.75x - 400$
This models the price of a computer after first a 25% discount and then a \$400 discount.

c. $(g \circ f)(x) = 0.75(x - 400)$
This models the price of a computer after first a \$400 discount and then a 25% discount.

d. The function $f \circ g$ models the greater discount, since the 25% discount is taken on the regular price first.

100. **a.** f gives the cost of a pair of jeans for which a \$5 rebate is offered.
g gives the cost of a pair of jeans that has been discounted 40%.

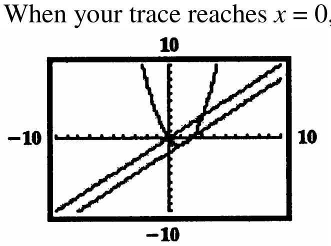
b. $(f \circ g)(x) = 0.6x - 5$
The cost of a pair of jeans is 60% of the regular price minus a \$5 rebate.

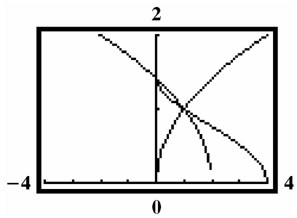
c. $(g \circ f)(x) = 0.6(x - 5)$
$= 0.6x - 3$
The cost of a pair of jeans is 60% of the regular price minus a \$3 rebate.

d. $f \circ g$ because of a \$5 rebate.

101. – 105. Answers may vary.

106. When your trace reaches $x = 0$, the y value disappears because the function is not defined at $x = 0$.



107. 

$(f \circ g)(x) = \sqrt{2-\sqrt{x}}$

The domain of g is $[0,\infty)$.

The expression under the radical in $f \circ g$ must not be negative.

$2-\sqrt{x} \ge 0$

$-\sqrt{x} \ge -2$

$\sqrt{x} \le 2$

$x \le 4$

domain: $[0,4]$

108. makes sense

109. makes sense

110. does not make sense; Explanations will vary. Sample explanation: It is common that $f \circ g$ and $g \circ f$ are not the same.

111. does not make sense; Explanations will vary. Sample explanation: The diagram illustrates $g\left(f(x)\right) = x^2 + 4.$

112. false; Changes to make the statement true will vary. A sample change is: $(f \circ g)(x) = f\left(\sqrt{x^2-4}\right)$

$$= \left(\sqrt{x^2-4}\right)^2 - 4$$
$$= x^2 - 4 - 4$$
$$= x^2 - 8$$

113. false; Changes to make the statement true will vary. A sample change is:

$f(x) = 2x; g(x) = 3x$

$(f \circ g)(x) = f(g(x)) = f(3x) = 2(3x) = 6x$

$(g \circ f)(x) = g(f(x)) = g(f(x)) = 3(2x) = 6x$

114. false; Changes to make the statement true will vary. A sample change is: $(f \circ g)(4) = f(g(4)) = f(7) = 5$

115. true

116. $(f \circ g)(x) = (f \circ g)(-x)$

$f(g(x)) = f(g(-x))$ since g is even

$f(g(x)) = f(g(x))$ so $f \circ g$ is even

117. Answers may vary.

118. $\{(4,-2),(1,-1),(1,1),(4,2)\}$
The element 1 in the domain corresponds to two elements in the range.
Thus, the relation is not a function.

119.
$$x = \frac{5}{y} + 4$$
$$y(x) = y\left(\frac{5}{y} + 4\right)$$
$$xy = 5 + 4y$$
$$xy - 4y = 5$$
$$y(x-4) = 5$$
$$y = \frac{5}{x-4}$$

$$x = y^2 - 1$$
$$x + 1 = y^2$$

120. $\sqrt{x+1} = \sqrt{y^2}$
$$\sqrt{x+1} = y$$
$$y = \sqrt{x+1}$$

Section 2.7

Check Point Exercises

1. $f\left(g(x)\right) = 4\left(\frac{x+7}{4}\right) - 7 = x$
$$g\left(f(x)\right) = \frac{(4x-7)+7}{4} = x$$
$$f\left(g(x)\right) = g\left(f(x)\right) = x$$

2. $f(x) = 2x + 7$
Replace $f(x)$ with *y*:
$y = 2x + 7$
Interchange *x* and *y*:
$x = 2y + 7$
Solve for *y*:
$$x = 2y + 7$$
$$x - 7 = 2y$$
$$\frac{x-7}{2} = y$$
Replace y with $f^{-1}(x)$:
$$f^{-1}(x) = \frac{x-7}{2}$$

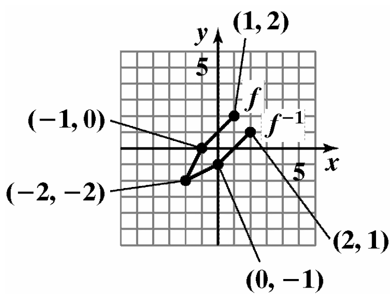
3. $f(x) = 4x^3 - 1$
Replace $f(x)$ with *y*:
$y = 4x^3 - 1$
Interchange *x* and *y*:
$x = 4y^3 - 1$
Solve for *y*:
$$x = 4y^3 - 1$$
$$x + 1 = 4y^3$$
$$\frac{x+1}{4} = y^3$$
$$\sqrt[3]{\frac{x+1}{4}} = y$$
Replace *y* with $f^{-1}(x)$:
$$f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}}$$
Alternative form for answer:
$$f(x)^{-1} = \sqrt[3]{\frac{x+1}{4}} = \frac{\sqrt[3]{x+1}}{\sqrt[3]{4}}$$
$$= \frac{\sqrt[3]{x+1}}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{2x+2}}{\sqrt[3]{8}}$$
$$= \frac{\sqrt[3]{2x+2}}{2}$$

4. $f(x) = \frac{3}{x} - 1$
Replace $f(x)$ with *y*:
$y = \frac{3}{x} - 1$
Interchange *x* and *y*:
$x = \frac{3}{y} - 1$
Solve for *y*:
$$x = \frac{3}{y} - 1$$
$$xy = 3 - y$$
$$xy + y = 3$$
$$y(x+1) = 3$$
$$y = \frac{3}{x+1}$$
Replace y with $f^{-1}(x)$:
$$f^{-1}(x) = \frac{3}{x+1}$$

5. The graphs of (b) and (c) pass the horizontal line test and thus have an inverse.

6. Find points of f^{-1}.

$f(x)$	$f^{-1}(x)$
$(-2,-2)$	$(-2,-2)$
$(-1,0)$	$(0,-1)$
$(1,2)$	$(2,1)$



7. $f(x) = x^2 + 1$

Replace $f(x)$ with *y*:

$y = x^2 + 1$

Interchange *x* and *y*:

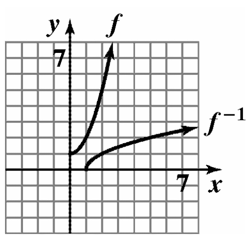
$x = y^2 + 1$

Solve for *y*:

$$x = y^2 + 1$$
$$x - 1 = y^2$$
$$\sqrt{x-1} = y$$

Replace *y* with $f^{-1}(x)$:

$f^{-1}(x) = \sqrt{x-1}$



Exercise Set 2.7

1. $f(x) = 4x;\ g(x) = \dfrac{x}{4}$

$$f(g(x)) = 4\left(\frac{x}{4}\right) = x$$

$$g(f(x)) = \frac{4x}{4} = x$$

f and *g* are inverses.

2. $f(x) = 6x;\ g(x) = \dfrac{x}{6}$

$$f(g(x)) = 6\left(\frac{x}{6}\right) = x$$

$$g(f(x)) = \frac{6x}{6} = x$$

f and *g* are inverses.

3. $f(x) = 3x + 8;\ g(x) = \dfrac{x-8}{3}$

$$f(g(x)) = 3\left(\frac{x-8}{3}\right) + 8 = x - 8 + 8 = x$$

$$g(f(x)) = \frac{(3x+8)-8}{3} = \frac{3x}{3} = x$$

f and *g* are inverses.

4. $f(x) = 4x + 9;\ g(x) = \dfrac{x-9}{4}$

$$f(g(x)) = 4\left(\frac{x-9}{4}\right) + 9 = x - 9 + 9 = x$$

$$g(f(x)) = \frac{(4x+9)-9}{4} = \frac{4x}{4} = x$$

f and *g* are inverses.

5. $f(x) = 5x - 9;\ g(x) = \dfrac{x+5}{9}$

$$f(g(x)) = 5\left(\frac{x+5}{9}\right) - 9$$
$$= \frac{5x+25}{9} - 9$$
$$= \frac{5x-56}{9}$$

$$g(f(x)) = \frac{5x-9+5}{9} = \frac{5x-4}{9}$$

f and *g* are not inverses.

6. $f(x) = 3x - 7;\ g(x) = \dfrac{x+3}{7}$

$$f(g(x)) = 3\left(\frac{x+3}{7}\right) - 7 = \frac{3x+9}{7} - 7 = \frac{3x-40}{7}$$

$$g(f(x)) = \frac{3x-7+3}{7} = \frac{3x-4}{7}$$

f and *g* are not inverses.

7. $f(x)=\frac{3}{x-4}$; $g(x)=\frac{3}{x}+4$

$f(g(x))=\frac{3}{\frac{3}{x}+4-4}=\frac{3}{\frac{3}{x}}=x$

$$\begin{aligned} g(f(x)) &= \frac{3}{\frac{3}{x-4}}+4 \\ &= 3\cdot\left(\frac{x-4}{3}\right)+4 \\ &= x-4+4 \\ &= x \end{aligned}$$

f and *g* are inverses.

8. $f(x)=\frac{2}{x-5}$; $g(x)=\frac{2}{x}+5$

$f(g(x))=\frac{2}{\left(\frac{2}{x}+5\right)-5}=\frac{2x}{2}=x$

$g\left(f(x)\right)=\frac{2}{\frac{2}{x-5}}+5=2\left(\frac{x-5}{2}\right)+5=x-5+5=x$

f and *g* are inverses.

9. $f(x)=-x$; $g(x)=-x$

$f(g(x))=-(-x)=x$

$g(f(x))=-(-x)=x$

f and *g* are inverses.

10. $f(x)=\sqrt[3]{x-4}$; $g(x)=x^3+4$

$f\left(g(x)\right)=\sqrt[3]{x^3+4-4}=\sqrt[3]{x^3}=x$

$g\left(f(x)\right)=\left(\sqrt[3]{x-4}\right)^3+4=x-4+4=x$

f and *g* are inverses.

11. a.

$$\begin{aligned} f(x) &= x+3 \\ y &= x+3 \\ x &= y+3 \\ y &= x-3 \\ f^{-1}(x) &= x-3 \end{aligned}$$

b. $f(f^{-1}(x))=x-3+3=x$

$f^{-1}(f(x))=x+3-3=x$

12. a.

$$\begin{aligned} f(x) &= x+5 \\ y &= x+5 \\ x &= y+5 \\ y &= x-5 \\ f^{-1}(x) &= x-5 \end{aligned}$$

b. $f\left(f^{-1}(x)\right)=x-5+5=x$

$f^{-1}\left(f(x)\right)=x+5-5=x$

13. a.

$$\begin{aligned} f(x) &= 2x \\ y &= 2x \\ x &= 2y \\ y &= \frac{x}{2} \\ f^{-1}(x) &= \frac{x}{2} \end{aligned}$$

b. $f(f^{-1}(x))=2\left(\frac{x}{2}\right)=x$

$f^{-1}(f(x))=\frac{2x}{2}=x$

14. a.

$$\begin{aligned} f(x) &= 4x \\ y &= 4x \\ x &= 4y \\ y &= \frac{x}{4} \\ f^{-1}(x) &= \frac{x}{4} \end{aligned}$$

b. $f\left(f^{-1}(x)\right)=4\left(\frac{x}{4}\right)=x$

$f^{-1}\left(f(x)\right)=\frac{4x}{4}=x$

15. a.

$$\begin{aligned} f(x) &= 2x+3 \\ y &= 2x+3 \\ x &= 2y+3 \\ x-3 &= 2y \\ y &= \frac{x-3}{2} \\ f^{-1}(x) &= \frac{x-3}{2} \end{aligned}$$

b.

$$\begin{aligned} f(f^{-1}(x)) &= 2\left(\frac{x-3}{2}\right)+3 \\ &= x-3+3 \\ &= x \end{aligned}$$

$f^{-1}(f(x))=\frac{2x+3-3}{2}=\frac{2x}{2}=x$

16. a.

$$f(x) = 3x - 1$$
$$y = 3x - 1$$
$$x = 3y - 1$$
$$x + 1 = 3y$$
$$y = \frac{x+1}{3}$$
$$f^{-1}(x) = \frac{x+1}{3}$$

b.

$$f\left(f^{-1}(x)\right) = 3\left(\frac{x+1}{3}\right) - 1 = x + 1 - 1 = x$$
$$f^{-1}\left(f(x)\right) = \frac{3x - 1 + 1}{3} = \frac{3x}{3} = x$$

17. a.

$$f(x) = x^3 + 2$$
$$y = x^3 + 2$$
$$x = y^3 + 2$$
$$x - 2 = y^3$$
$$y = \sqrt[3]{x-2}$$
$$f^{-1}(x) = \sqrt[3]{x-2}$$

b.

$$f(f^{-1}(x)) = \left(\sqrt[3]{x-2}\right)^3 + 2$$
$$= x - 2 + 2$$
$$= x$$
$$f^{-1}(f(x)) = \sqrt[3]{x^3 + 2 - 2} = \sqrt[3]{x^3} = x$$

18. a.

$$f(x) = x^3 - 1$$
$$y = x^3 - 1$$
$$x = y^3 - 1$$
$$x + 1 = y^3$$
$$y = \sqrt[3]{x+1}$$
$$f^{-1}(x) = \sqrt[3]{x+1}$$

b.

$$f(f^{-1}(x)) = \left(\sqrt[3]{x+1}\right)^3 - 1$$
$$= x + 1 - 1$$
$$= x$$
$$f^{-1}(f(x)) = \sqrt[3]{x^3 - 1 + 1} = \sqrt[3]{x^3} = x$$

19. a.

$$f(x) = (x+2)^3$$
$$y = (x+2)^3$$
$$x = (y+2)^3$$
$$\sqrt[3]{x} = y + 2$$
$$y = \sqrt[3]{x} - 2$$
$$f^{-1}(x) = \sqrt[3]{x} - 2$$

b.

$$f(f^{-1}(x)) = \left(\sqrt[3]{x} - 2 + 2\right)^3 = \left(\sqrt[3]{x}\right)^3 = x$$
$$f^{-1}(f(x)) = \sqrt[3]{(x+2)^3} - 2$$
$$= x + 2 - 2$$
$$= x$$

20. a.

$$f(x) = (x-1)^3$$
$$y = (x-1)^3$$
$$x = (y-1)^3$$
$$\sqrt[3]{x} = y - 1$$
$$y = \sqrt[3]{x} + 1$$

b.

$$f\left(f^{-1}(x)\right) = \left(\sqrt[3]{x} + 1 - 1\right)^3 = \left(\sqrt[3]{x}\right)^3 = x$$
$$f^{-1}\left(f(x)\right) = \sqrt[3]{(x-1^3} + 1 = x - 1 + 1 = x$$

21. a.

$$f(x) = \frac{1}{x}$$
$$y = \frac{1}{x}$$
$$x = \frac{1}{y}$$
$$xy = 1$$
$$y = \frac{1}{x}$$
$$f^{-1}(x) = \frac{1}{x}$$

b.

$$f(f^{-1}(x)) = \frac{1}{\frac{1}{x}} = x$$
$$f^{-1}(f(x)) = \frac{1}{\frac{1}{x}} = x$$

22. a.

$$f(x) = \frac{2}{x}$$
$$y = \frac{2}{x}$$
$$x = \frac{2}{y}$$
$$xy = 2$$
$$y = \frac{2}{x}$$
$$f^{-1}(x) = \frac{2}{x}$$

b. $$f(f^{-1}(x)) = \frac{2}{\frac{2}{x}} = 2 \cdot \frac{x}{2} = x$$

$$f^{-1}(f(x)) = \frac{2}{\frac{2}{x}} = 2 \cdot \frac{x}{2} = x$$

23. a.

$$f(x) = \sqrt{x}$$
$$y = \sqrt{x}$$
$$x = \sqrt{y}$$
$$y = x^2$$
$$f^{-1}(x) = x^2, x \geq 0$$

b. $f(f^{-1}(x)) = \sqrt{x^2} = |x| = x$ for $x \geq 0$.

$$f^{-1}(f(x)) = (\sqrt{x})^2 = x$$

24. a.

$$f(x) = \sqrt[3]{x}$$
$$y = \sqrt[3]{x}$$
$$x = \sqrt[3]{y}$$
$$y = x^3$$
$$f^{-1}(x) = x^3$$

b. $$f\left(f^{-1}(x)\right) = \sqrt[3]{x^3} = x$$

$$f^{-1}\left(f(x)\right) = \left(\sqrt[3]{x}\right)^3 = x$$

25. a.

$$f(x) = \frac{7}{x} - 3$$
$$y = \frac{7}{x} - 3$$
$$x = \frac{7}{y} - 3$$
$$xy = 7 - 3y$$
$$xy + 3y = 7$$
$$y(x+3) = 7$$
$$y = \frac{7}{x+3}$$
$$f^{-1}(x) = \frac{7}{x+3}$$

b. $$f\left(f^{-1}(x)\right) = \frac{7}{\frac{7}{x+3}} - 3 = x$$

$$f^{-1}\left(f(x)\right) = \frac{7}{\frac{7}{x} - 3 + 3} = x$$

26. a.

$$f(x) = \frac{4}{x} + 9$$
$$y = \frac{4}{x} + 9$$
$$x = \frac{4}{y} + 9$$
$$xy = 4 + 9y$$
$$xy - 9y = 4$$
$$y(x-9) = 4$$
$$y = \frac{4}{x-9}$$
$$f^{-1}(x) = \frac{4}{x-9}$$

b. $$f\left(f^{-1}(x)\right) = \frac{4}{\frac{4}{x-9}} + 9 = x$$

$$f^{-1}\left(f(x)\right) = \frac{4}{\frac{4}{x} + 9 - 9} = x$$

27. a. $f(x)=\dfrac{2x+1}{x-3}$

$y=\dfrac{2x+1}{x-3}$

$x=\dfrac{2y+1}{y-3}$

$x(y-3)=2y+1$

$xy-3x=2y+1$

$xy-2y=3x+1$

$y(x-2)=3x+1$

$y=\dfrac{3x+1}{x-2}$

$f^{-1}(x)=\dfrac{3x+1}{x-2}$

b. $f(f^{-1}(x))=\dfrac{2\left(\frac{3x+1}{x-2}\right)+1}{\frac{3x+1}{x-2}-3}$

$=\dfrac{2(3x+1)+x-2}{3x+1-3(x-2)}=\dfrac{6x+2+x-2}{3x+1-3x+6}$

$=\dfrac{7x}{7}=x$

$f^{-1}(f(x))=\dfrac{3\left(\frac{2x+1}{x-3}\right)+1}{\frac{2x+1}{x-3}-2}$

$=\dfrac{3(2x+1)+x-3}{2x+1-2(x-3)}$

$=\dfrac{6x+3+x-3}{2x+1-2x+6}=\dfrac{7x}{7}=x$

28. a. $f(x)=\dfrac{2x-3}{x+1}$

$y=\dfrac{2x-3}{x+1}$

$x=\dfrac{2y-3}{y+1}$

$xy+x=2y-3$

$y(x-2)=-x-3$

$y=\dfrac{-x-3}{x-2}$

$f^{-1}(x)=\dfrac{-x-3}{x-2},\quad x\neq 2$

b. $f(f^{-1}(x))=\dfrac{2\left(\frac{-x-3}{x-2}\right)-3}{\frac{-x-3}{x-2}+1}$

$=\dfrac{-2x-6-3x+6}{-x-3+x-2}=\dfrac{-5x}{-5}=x$

$f^{-1}(f(x))=\dfrac{-\left(\frac{2x-3}{x+1}\right)-3}{\frac{2x-3}{x+1}-2}$

$=\dfrac{-2x+3-3x-3}{2x-3-2x-2}=\dfrac{-5x}{-5}=x$

29. The function fails the horizontal line test, so it does not have an inverse function.

30. The function passes the horizontal line test, so it does have an inverse function.

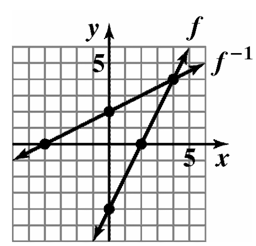
31. The function fails the horizontal line test, so it does not have an inverse function.

32. The function fails the horizontal line test, so it does not have an inverse function.

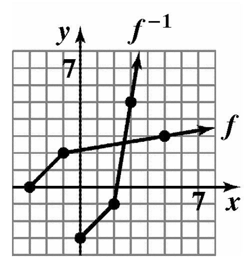
33. The function passes the horizontal line test, so it does have an inverse function.

34. The function passes the horizontal line test, so it does have an inverse function.

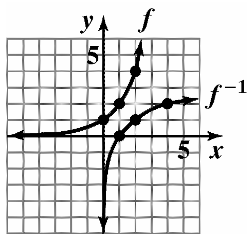
35.



36.



37.



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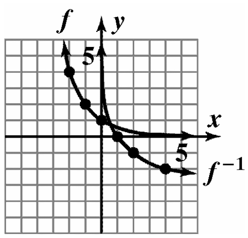
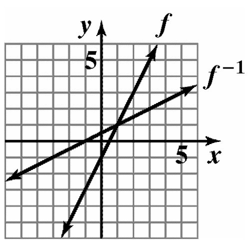


39. **a.**

$$f(x) = 2x - 1$$
$$y = 2x - 1$$
$$x = 2y - 1$$
$$x + 1 = 2y$$
$$\frac{x+1}{2} = y$$
$$f^{-1}(x) = \frac{x+1}{2}$$

b.

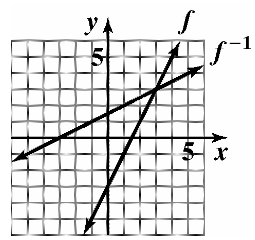


c. domain of f : $(-\infty, \infty)$
range of f : $(-\infty, \infty)$
domain of f^{-1}: $(-\infty, \infty)$
range of f^{-1}: $(-\infty, \infty)$

40. **a.**

$$f(x) = 2x - 3$$
$$y = 2x - 3$$
$$x = 2y - 3$$
$$x + 3 = 2y$$
$$\frac{x+3}{2} = y$$
$$f^{-1}(x) = \frac{x+3}{2}$$

b.

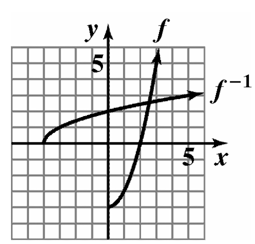


c. domain of f : $(-\infty, \infty)$
range of f : $(-\infty, \infty)$
domain of f^{-1}: $(-\infty, \infty)$
range of f^{-1}: $(-\infty, \infty)$

41. **a.**

$$f(x) = x^2 - 4$$
$$y = x^2 - 4$$
$$x = y^2 - 4$$
$$x + 4 = y^2$$
$$\sqrt{x+4} = y$$
$$f^{-1}(x) = \sqrt{x+4}$$

b.

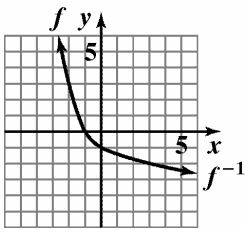


c. domain of f : $[0, \infty)$
range of f : $[-4, \infty)$
domain of f^{-1}: $[-4, \infty)$
range of f^{-1}: $[0, \infty)$

42. a.

$$\begin{aligned} f(x) &= x^2 - 1 \\ y &= x^2 - 1 \\ x &= y^2 - 1 \\ x + 1 &= y^2 \\ -\sqrt{x+1} &= y \\ f^{-1}(x) &= -\sqrt{x+1} \end{aligned}$$

b.

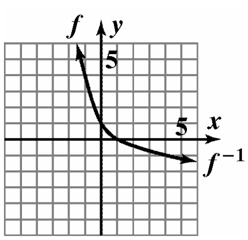


c. domain of f: $(-\infty, 0]$
range of f: $[-1, \infty)$
domain of f^{-1}: $[-1, \infty)$
range of f^{-1}: $(-\infty, 0]$

43. a.

$$\begin{aligned} f(x) &= (x-1)^2 \\ y &= (x-1)^2 \\ x &= (y-1)^2 \\ -\sqrt{x} &= y - 1 \\ -\sqrt{x} + 1 &= y \\ f^{-1}(x) &= 1 - \sqrt{x} \end{aligned}$$

b.

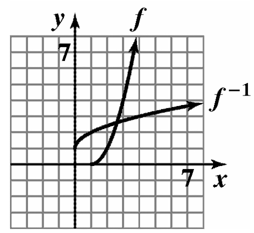


c. domain of f: $(-\infty, 1]$
range of f: $[0, \infty)$
domain of f^{-1}: $[0, \infty)$
range of f^{-1}: $(-\infty, 1]$

44. a.

$$\begin{aligned} f(x) &= (x-1)^2 \\ y &= (x-1)^2 \\ x &= (y-1)^2 \\ \sqrt{x} &= y - 1 \\ \sqrt{x} + 1 &= y \\ f^{-1}(x) &= 1 + \sqrt{x} \end{aligned}$$

b.

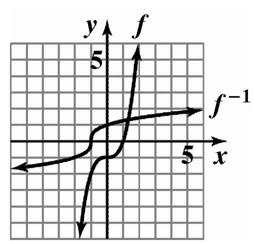


c. domain of f: $[1, \infty)$
range of f: $[0, \infty)$
domain of f^{-1}: $[0, \infty)$
range of f^{-1}: $[1, \infty)$

45. a.

$$\begin{aligned} f(x) &= x^3 - 1 \\ y &= x^3 - 1 \\ x &= y^3 - 1 \\ x + 1 &= y^3 \\ \sqrt[3]{x+1} &= y \\ f^{-1}(x) &= \sqrt[3]{x+1} \end{aligned}$$

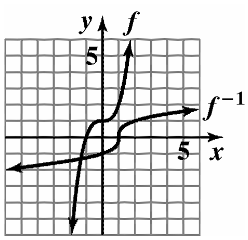
b.



c. domain of f: $(-\infty, \infty)$
range of f: $(-\infty, \infty)$
domain of f^{-1}: $(-\infty, \infty)$
range of f^{-1}: $(-\infty, \infty)$

46. a.
$$f(x) = x^3 + 1$$
$$y = x^3 + 1$$
$$x = y^3 + 1$$
$$x - 1 = y^3$$
$$\sqrt[3]{x-1} = y$$
$$f^{-1}(x) = \sqrt[3]{x-1}$$

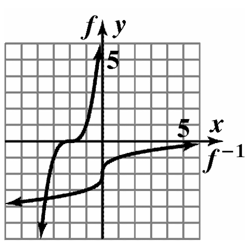
b.



c. domain of f: $(-\infty, \infty)$
range of f: $(-\infty, \infty)$
domain of f^{-1}: $(-\infty, \infty)$
range of f^{-1}: $(-\infty, \infty)$

47. a.
$$f(x) = (x+2)^3$$
$$y = (x+2)^3$$
$$x = (y+2)^3$$
$$\sqrt[3]{x} = y + 2$$
$$\sqrt[3]{x} - 2 = y$$
$$f^{-1}(x) = \sqrt[3]{x} - 2$$

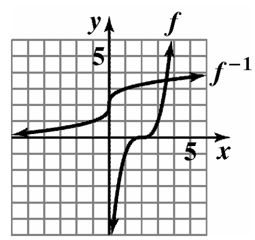
b.



c. domain of f: $(-\infty, \infty)$
range of f: $(-\infty, \infty)$
domain of f^{-1}: $(-\infty, \infty)$
range of f^{-1}: $(-\infty, \infty)$

48. a.
$$f(x) = (x-2)^3$$
$$y = (x-2)^3$$
$$x = (y-2)^3$$
$$\sqrt[3]{x} = y - 2$$
$$\sqrt[3]{x} + 2 = y$$
$$f^{-1}(x) = \sqrt[3]{x} + 2$$

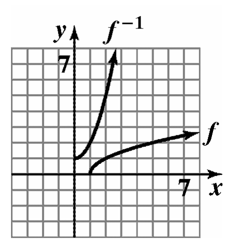
b.



c. domain of f: $(-\infty, \infty)$
range of f: $(-\infty, \infty)$
domain of f^{-1}: $(-\infty, \infty)$
range of f^{-1}: $(-\infty, \infty)$

49. a.
$$f(x) = \sqrt{x-1}$$
$$y = \sqrt{x-1}$$
$$x = \sqrt{y-1}$$
$$x^2 = y - 1$$
$$x^2 + 1 = y$$
$$f^{-1}(x) = x^2 + 1$$

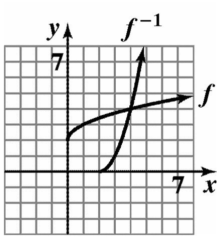
b.



c. domain of f: $[1, \infty)$
range of f: $[0, \infty)$
domain of f^{-1}: $[0, \infty)$
range of f^{-1}: $[1, \infty)$

50. **a.**
$$f(x)=\sqrt{x}+2$$
$$y=\sqrt{x}+2$$
$$x=\sqrt{y}+2$$
$$x-2=\sqrt{y}$$
$$(x-2)^2=y$$
$$f^{-1}(x)=(x-2)^2$$

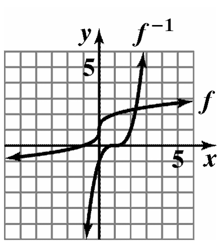
b.



c. domain of f: $[0,\infty)$
range of f: $[2,\infty)$
domain of f^{-1}: $[2,\infty)$
range of f^{-1}: $[0,\infty)$

51. **a.**
$$f(x)=\sqrt[3]{x}+1$$
$$y=\sqrt[3]{x}+1$$
$$x=\sqrt[3]{y}+1$$
$$x-1=\sqrt[3]{y}$$
$$(x-1)^3=y$$
$$f^{-1}(x)=(x-1)^3$$

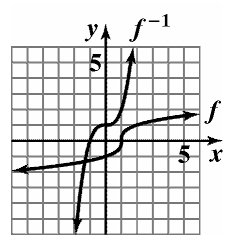
b.



c. domain of f: $(-\infty,\infty)$
range of f: $(-\infty,\infty)$
domain of f^{-1}: $(-\infty,\infty)$
range of f^{-1}: $(-\infty,\infty)$

52. **a.**
$$f(x)=\sqrt[3]{x-1}$$
$$y=\sqrt[3]{x-1}$$
$$x=\sqrt[3]{y-1}$$
$$x^3=y-1$$
$$x^3+1=y$$
$$f^{-1}(x)=x^3+1$$

b.



c. domain of f: $(-\infty,\infty)$
range of f: $(-\infty,\infty)$
domain of f^{-1}: $(-\infty,\infty)$
range of f^{-1}: $(-\infty,\infty)$

53. $f\left(g(1)\right)=f\left(1\right)=5$

54. $f\left(g(4)\right)=f\left(2\right)=-1$

55. $\left(g\circ f\right)\left(-1\right)=g\left(f(-1)\right)=g\left(1\right)=1$

56. $\left(g\circ f\right)\left(0\right)=g\left(f(0)\right)=g\left(4\right)=2$

57. $f^{-1}\left(g(10)\right)=f^{-1}\left(-1\right)=2$, since $f\left(2\right)=-1$.

58. $f^{-1}\left(g(1)\right)=f^{-1}\left(1\right)=-1$, since $f\left(-1\right)=1$.

59. $\left(f\circ g\right)\left(0\right)=f\left(g(0)\right)$
$$=f\left(4\cdot 0-1\right)$$
$$=f\left(-1\right)=2\left(-1\right)-5=-7$$

60. $\left(g\circ f\right)\left(0\right)=g\left(f(0)\right)$
$$=g\left(2\cdot 0-5\right)$$
$$=g\left(-5\right)=4\left(-5\right)-1=-21$$

61. Let $f^{-1}(1)=x$. Then

$$\begin{aligned} f(x)&=1\\ 2x-5&=1\\ 2x&=6\\ x&=3 \end{aligned}$$

Thus, $f^{-1}(1)=3$

62. Let $g^{-1}(7)=x$. Then

$$\begin{aligned} g(x)&=7\\ 4x-1&=7\\ 4x&=8\\ x&=2 \end{aligned}$$

Thus, $g^{-1}(7)=2$

63.
$$\begin{aligned} g\left(f\left[h(1)\right]\right)&=g\left(f\left[1^2+1+2\right]\right)\\ &=g\left(f(4)\right)\\ &=g\left(2\cdot 4-5\right)\\ &=g(3)\\ &=4\cdot 3-1=11 \end{aligned}$$

64.
$$\begin{aligned} f\left(g\left[h(1)\right]\right)&=f\left(g\left[1^2+1+2\right]\right)\\ &=f\left(g(4)\right)\\ &=f\left(4\cdot 4-1\right)\\ &=f(15)\\ &=2\cdot 15-5=25 \end{aligned}$$

65. **a.** $\{(17,9.7),(22,8.7),(30,8.4),(40,8.3),(50,8.2),(60,8.3)\}$

b. $\{(9.7,17),(8.7,22),(8.4,30),(8.3,40),(8.2,50),(8.3,60)\}$
f is not a one-to-one function because the inverse of *f* is not a function.

66. **a.** $\{(17,9.3),(22,9.1),(30,8.8),(40,8.5),(50,8.4),(60,8.5)\}$

b. $\{(9.3,17),(9.1,22),(8.8,30),(8.5,40),(8.4,50),(8.5,60)\}$
g is not a one-to-one function because the inverse of *g* is not a function.

67. **a.** It passes the horizontal line test and is one-to-one.

b. $f^{-1}(0.25) = 15$ If there are 15 people in the room, the probability that 2 of them have the same birthday is 0.25.
$f^{-1}(0.5) = 21$ If there are 21 people in the room, the probability that 2 of them have the same birthday is 0.5.
$f^{-1}(0.7) = 30$ If there are 30 people in the room, the probability that 2 of them have the same birthday is 0.7.

68. **a.** This function fails the horizontal line test. Thus, this function does not have an inverse.

b. The average happiness level is 3 at 12 noon and at 7 p.m. These values can be represented as $(12,3)$ and $(19,3)$.

c. The graph does not represent a one-to-one function. $(12,3)$ and $(19,3)$ are an example of two *x*-values that correspond to the same *y*-value.

69. $f(g(x))=\frac{9}{5}\left[\frac{5}{9}(x-32)\right]+32$

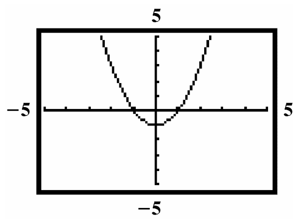
$=x-32+32$

$=x$

f and *g* are inverses.

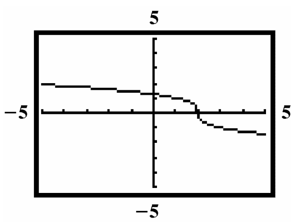
70. – 75. Answers may vary.

76.



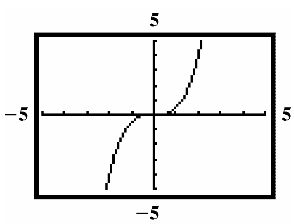
not one-to-one

77.



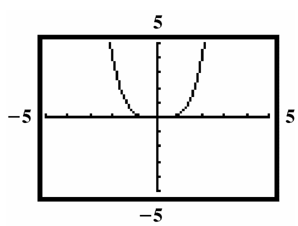
one-to-one

78.



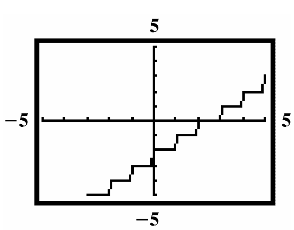
one-to-one

79.



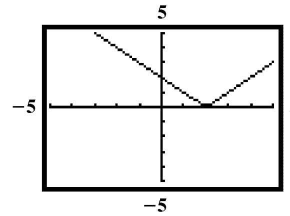
not one-to-one

80.



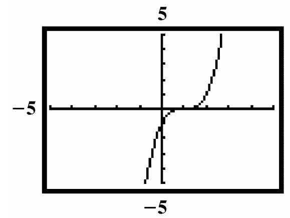
not one-to-one

81.



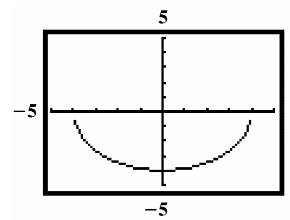
not one-to-one

82.



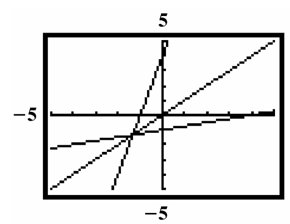
one-to-one

83.



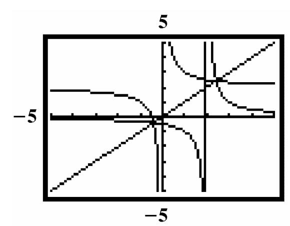
not one-to-one

84.



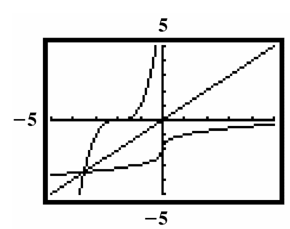
f and *g* are inverses

85.



f and *g* are inverses

86.



f and *g* are inverses

87. makes sense

88. makes sense

89. makes sense

90. makes sense

91. false; Changes to make the statement true will vary. A sample change is: The inverse is $\{(4,1), (7,2)\}$.

92. false; Changes to make the statement true will vary. A sample change is: $f(x) = 5$ is a horizontal line, so it does not pass the horizontal line test.

93. false; Changes to make the statement true will vary. A sample change is: $f^{-1}(x) = \frac{x}{3}$.

94. true

95. $(f \circ g)(x) = 3(x+5) = 3x+15.$

$$\begin{aligned} y &= 3x+15 \\ x &= 3y+15 \\ y &= \frac{x-15}{3} \\ (f \circ g)^{-1}(x) &= \frac{x-15}{3} \end{aligned}$$

$$\begin{aligned} g(x) &= x+5 \\ y &= x+5 \\ x &= y+5 \\ y &= x-5 \\ g^{-1}(x) &= x-5 \end{aligned}$$

$$\begin{aligned} f(x) &= 3x \\ y &= 3x \\ x &= 3y \\ y &= \frac{x}{3} \\ f^{-1}(x) &= \frac{x}{3} \end{aligned}$$

$$\left(g^{-1} \circ f^{-1}\right)(x) = \frac{x}{3} - 5 = \frac{x-15}{3}$$

96.

$$\begin{aligned} f(x) &= \frac{3x-2}{5x-3} \\ y &= \frac{3x-2}{5x-3} \\ x &= \frac{3y-2}{5y-3} \\ x(5y-3) &= 3y-2 \\ 5xy-3x &= 3y-2 \\ 5xy-3y &= 3x-2 \\ y(5x-3) &= 3x-2 \\ y &= \frac{3x-2}{5x-3} \\ f^{-1}(x) &= \frac{3x-2}{5x-3} \end{aligned}$$

Note: An alternative approach is to show that $(f \circ f)(x) = x.$

97. No, there will be 2 times when the spacecraft is at the same height, when it is going up and when it is coming down.

98.

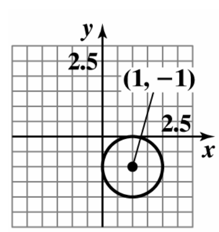
$$\begin{aligned} 8 + f^{-1}(x-1) &= 10 \\ f^{-1}(x-1) &= 2 \\ f(2) &= x-1 \\ 6 &= x-1 \\ 7 &= x \\ x &= 7 \end{aligned}$$

99. Answers may vary.

100.

$$\begin{aligned} \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} &= \sqrt{(1-7)^2+(-1-2)^2} \\ &= \sqrt{(-6)^2+(-3)^2} \\ &= \sqrt{36+9} \\ &= \sqrt{45} \\ &= 3\sqrt{5} \end{aligned}$$

101.



102. $y^2-6y-4=0$

$y^2-6y \quad =4$

$y^2-6y+9=4+9$

$(y-3)^2=13$

$y-3=\pm\sqrt{13}$

$y=3\pm\sqrt{13}$

Section 2.8

Check Point Exercises

1. $d=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$

$d=\sqrt{\left(1-(-4)\right)^2+\left(-3-9\right)^2}$

$=\sqrt{(5)^2+(-12)^2}$

$=\sqrt{25+144}$

$=\sqrt{169}$

$=13$

2. $\left(\frac{1+7}{2},\frac{2+(-3)}{2}\right)=\left(\frac{8}{2},\frac{-1}{2}\right)=\left(4,-\frac{1}{2}\right)$

3. $h=0,\ k=0,\ r=4;$

$(x-0)^2+(y-0)^2=4^2$

$x^2+y^2=16$

4. $h=0,\ k=-6,\ r=10;$

$(x-0)^2+[y-(-6)]^2=10^2$

$(x-0)^2+(y+6)^2=100$

$x^2+(y+6)^2=100$

5. **a.** $(x+3)^2+(y-1)^2=4$

$[x-(-3)]^2+(y-1)^2=2^2$

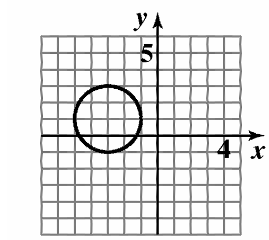
So in the standard form of the circle's equation $(x-h)^2+(y-k)^2=r^2$,

we have $h=-3,\ k=1,\ r=2.$

center: $(h,\ k)=(-3,\ 1)$

radius: $r=2$

b.



$(x+3)^2+(y-1)^2=4$

c. domain: $[-5,-1]$

range: $[-1,3]$

6. $x^2+y^2+4x-4y-1=0$

$x^2+y^2+4x-4y-1=0$

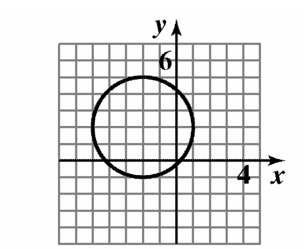
$\left(x^2+4x\quad\right)+\left(y^2-4y\quad\right)=0$

$\left(x^2+4x+4\right)+\left(y^2+4y+4\right)=1+4+4$

$(x+2)^2+(y-2)^2=9$

$[x-(-x)]^2+(y-2)^2=3^2$

So in the standard form of the circle's equation $(x-h)^2+(y-k)^2=r^2$, we have $h=-2,\ k=2,\ r=3$.



$x^2+y^2+4x-4y-1=0$

Exercise Set 2.8

1. $d=\sqrt{(14-2)^2+(8-3)^2}$

$=\sqrt{12^2+5^2}$

$=\sqrt{144+25}$

$=\sqrt{169}$

$=13$

2. $$\begin{aligned} d &= \sqrt{(8-5)^2+(5-1)^2} \\ &= \sqrt{3^2+4^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

3. $$\begin{aligned} d &= \sqrt{\left(-6-4\right)^2+\left(3-(-1)\right)^2} \\ &= \sqrt{\left(-10\right)^2+\left(4\right)^2} \\ &= \sqrt{100+16} \\ &= \sqrt{116} \\ &= 2\sqrt{29} \\ &\approx 10.77 \end{aligned}$$

4. $$\begin{aligned} d &= \sqrt{\left(-1-2\right)^2+\left(5-(-3)\right)^2} \\ &= \sqrt{\left(-3\right)^2+\left(8\right)^2} \\ &= \sqrt{9+64} \\ &= \sqrt{73} \\ &\approx 8.54 \end{aligned}$$

5. $$\begin{aligned} d &= \sqrt{(-3-0)^2+(4-0)^2} \\ &= \sqrt{3^2+4^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

6. $$\begin{aligned} d &= \sqrt{(3-0)^2+\left(-4-0\right)^2} \\ &= \sqrt{3^2+\left(-4\right)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

7. $$\begin{aligned} d &= \sqrt{[3-(-2)]^2+[-4-(-6)]^2} \\ &= \sqrt{5^2+2^2} \\ &= \sqrt{25+4} \\ &= \sqrt{29} \\ &\approx 5.39 \end{aligned}$$

8. $$\begin{aligned} d &= \sqrt{[2-(-4)]^2+[-3-(-1)]^2} \\ &= \sqrt{6^2+\left(-2\right)^2} \\ &= \sqrt{36+4} \\ &= \sqrt{40} \\ &= 2\sqrt{10} \\ &\approx 6.32 \end{aligned}$$

9. $$\begin{aligned} d &= \sqrt{(4-0)^2+[1-(-3)]^2} \\ &= \sqrt{4^2+4^2} \\ &= \sqrt{16+16} \\ &= \sqrt{32} \\ &= 4\sqrt{2} \\ &\approx 5.66 \end{aligned}$$

10. $$\begin{aligned} d &= \sqrt{\left(4-0\right)^2+[3-\left(-2\right)]^2} \\ &= \sqrt{4^2+[3+2]^2} \\ &= \sqrt{16+5^2} \\ &= \sqrt{16+25} \\ &= \sqrt{41} \\ &\approx 6.40 \end{aligned}$$

11. $$\begin{aligned} d &= \sqrt{(-.5-3.5)^2+(6.2-8.2)^2} \\ &= \sqrt{(-4)^2+(-2)^2} \\ &= \sqrt{16+4} \\ &= \sqrt{20} \\ &= 2\sqrt{5} \\ &\approx 4.47 \end{aligned}$$

12. $$\begin{aligned} d &= \sqrt{(1.6-2.6)^2+\left(-5.7-1.3\right)^2} \\ &= \sqrt{\left(-1\right)^2+\left(-7\right)^2} \\ &= \sqrt{1+49} \\ &= \sqrt{50} \\ &= 5\sqrt{2} \\ &\approx 7.07 \end{aligned}$$

13. $d=\sqrt{(\sqrt{5}-0)^2+[0-(-\sqrt{3})]^2}$
$=\sqrt{(\sqrt{5})^2+(\sqrt{3})^2}$
$=\sqrt{5+3}$
$=\sqrt{8}$
$=2\sqrt{2}$
≈ 2.83

14. $d=\sqrt{\left(\sqrt{7}-0\right)^2+\left[0-\left(-\sqrt{2}\right)\right]^2}$
$=\sqrt{\left(\sqrt{7}\right)^2+\left[-\sqrt{2}\right]^2}$
$=\sqrt{7+2}$
$=\sqrt{9}$
$=3$

15. $d=\sqrt{(-\sqrt{3}-3\sqrt{3})^2+(4\sqrt{5}-\sqrt{5})^2}$
$=\sqrt{(-4\sqrt{3})^2+(3\sqrt{5})^2}$
$=\sqrt{16(3)+9(5)}$
$=\sqrt{48+45}$
$=\sqrt{93}$
≈ 9.64

16. $d=\sqrt{\left(-\sqrt{3}-2\sqrt{3}\right)^2+\left(5\sqrt{6}-\sqrt{6}\right)^2}$
$=\sqrt{\left(-3\sqrt{3}\right)^2+\left(4\sqrt{6}\right)^2}$
$=\sqrt{9\cdot 3+16\cdot 6}$
$=\sqrt{27+96}$
$=\sqrt{123}$
≈ 11.09

17. $d=\sqrt{\left(\frac{1}{3}-\frac{7}{3}\right)^2+\left(\frac{6}{5}-\frac{1}{5}\right)^2}$
$=\sqrt{(-2)^2+1^2}$
$=\sqrt{4+1}$
$=\sqrt{5}$
≈ 2.24

18. $d=\sqrt{\left[\frac{3}{4}-\left(-\frac{1}{4}\right)\right]^2+\left[\frac{6}{7}-\left(-\frac{1}{7}\right)\right]^2}$
$=\sqrt{\left(\frac{3}{4}+\frac{1}{4}\right)^2+\left[\frac{6}{7}+\frac{1}{7}\right]^2}$
$=\sqrt{1^2+1^2}$
$=\sqrt{2}$
≈ 1.41

19. $\left(\frac{6+2}{2},\frac{8+4}{2}\right)=\left(\frac{8}{2},\frac{12}{2}\right)=(4,6)$

20. $\left(\frac{10+2}{2},\frac{4+6}{2}\right)=\left(\frac{12}{2},\frac{10}{2}\right)=(6,5)$

21. $\left(\frac{-2+(-6)}{2},\frac{-8+(-2)}{2}\right)$
$=\left(\frac{-8}{2},\frac{-10}{2}\right)=(-4,-5)$

22. $\left(\frac{-4+(-1)}{2},\frac{-7+(-3)}{2}\right)=\left(\frac{-5}{2},\frac{-10}{2}\right)$
$=\left(\frac{-5}{2},-5\right)$

23. $\left(\frac{-3+6}{2},\frac{-4+(-8)}{2}\right)$
$=\left(\frac{3}{2},\frac{-12}{2}\right)=\left(\frac{3}{2},-6\right)$

24. $\left(\frac{-2+(-8)}{2}\frac{-1+6}{2}\right)=\left(\frac{-10}{2},\frac{5}{2}\right)=\left(-5,\frac{5}{2}\right)$

25. $\left(\frac{\frac{-7}{2}+\left(-\frac{5}{2}\right)}{2},\frac{\frac{3}{2}+\left(-\frac{11}{2}\right)}{2}\right)$
$=\left(\frac{\frac{-12}{2}}{2},\frac{\frac{-8}{2}}{2}\right)=\left(-\frac{6}{2},\frac{-4}{2}\right)=\left(-3,-2\right)$

26. $\left(\dfrac{-\frac{2}{5}+\left(-\frac{2}{5}\right)}{2},\dfrac{\frac{7}{15}+\left(-\frac{4}{15}\right)}{2}\right)=\left(\dfrac{-\frac{4}{5}}{2},\dfrac{\frac{3}{15}}{2}\right)$

$=\left(-\frac{4}{5}\cdot\frac{1}{2},\frac{3}{15}\cdot\frac{1}{2}\right)=\left(-\frac{2}{5},\frac{1}{10}\right)$

27. $\left(\dfrac{8+(-6)}{2},\dfrac{3\sqrt{5}+7\sqrt{5}}{2}\right)$

$=\left(\frac{2}{2},\frac{10\sqrt{5}}{2}\right)=\left(1,5\sqrt{5}\right)$

28. $\left(\dfrac{7\sqrt{3}+3\sqrt{3}}{2},\dfrac{-6+(-2)}{2}\right)=\left(\dfrac{10\sqrt{3}}{2},\dfrac{-8}{2}\right)$

$=\left(5\sqrt{3},-4\right)$

29. $\left(\dfrac{\sqrt{18}+\sqrt{2}}{2},\dfrac{-4+4}{2}\right)$

$=\left(\dfrac{3\sqrt{2}+\sqrt{2}}{2},\dfrac{0}{2}\right)=\left(\dfrac{4\sqrt{2}}{2},0\right)=(2\sqrt{2},0)$

30. $\left(\dfrac{\sqrt{50}+\sqrt{2}}{2},\dfrac{-6+6}{2}\right)=\left(\dfrac{5\sqrt{2}+\sqrt{2}}{2},\dfrac{0}{2}\right)$

$=\left(\dfrac{6\sqrt{2}}{2},0\right)=\left(3\sqrt{2},0\right)$

31. $(x-0)^2+(y-0)^2=7^2$

$x^2+y^2=49$

32. $(x-0)^2+(y-0)^2=8^2$

$x^2+y^2=64$

33. $(x-3)^2+(y-2)^2=5^2$

$(x-3)^2+(y-2)^2=25$

34. $(x-2)^2+[y-(-1)]^2=4^2$

$(x-2)^2+(y+1)^2=16$

35. $[x-(-1)]^2+(y-4)^2=2^2$

$(x+1)^2+(y-4)^2=4$

36. $[x-(-3)]^2+(y-5)^2=3^2$

$(x+3)^2+(y-5)^2=9$

37. $[x-(-3)]^2+[y-(-1)]^2=\left(\sqrt{3}\right)^2$

$(x+3)^2+(y+1)^2=3$

38. $[x-(-5)]^2+[y-(-3)]^2=\left(\sqrt{5}\right)^2$

$(x+5)^2+(y+3)^2=5$

39. $[x-(-4)]^2+(y-0)^2=10^2$

$(x+4)^2+(y-0)^2=100$

40. $[x-(-2)]^2+(y-0)^2=6^2$

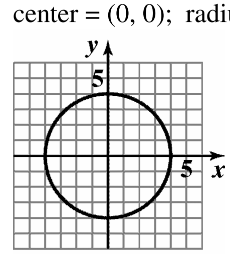
$(x+2)^2+y^2=36$

41. $x^2+y^2=16$

$(x-0)^2+(y-0)^2=y^2$

$h=0,\ k=0,\ r=4;$

center = (0, 0); radius = 4



$\mathbf{x^2+y^2=16}$

domain: $[-4,4]$

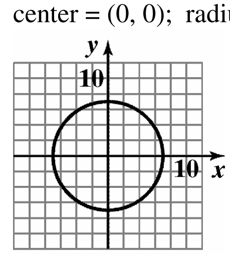
range: $[-4,4]$

42. $x^2+y^2=49$

$(x-0)^2+(y-0)^2=7^2$

$h=0,k=0,r=7;$

center = (0, 0); radius = 7



$\mathbf{x^2+y^2=49}$

domain: $[-7,7]$

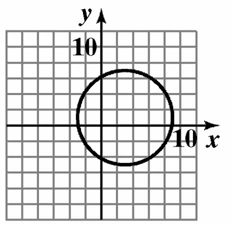
range: $[-7,7]$

43. $(x-3)^2+(y-1)^2=36$

$(x-3)^2+(y-1)^2=6^2$

$h=3,\ k=1,\ r=6;$

center = (3, 1); radius = 6



$(x-3)^2+(y-1)^2=36$

domain: $[-3,9]$

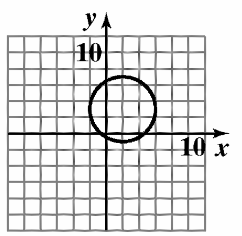
range: $[-5,7]$

44. $(x-2)^2+(y-3)^2=16$

$(x-2)^2+(y-3)^2=4^2$

$h=2, k=3, r=4;$

center = (2, 3); radius = 4



$(x-2)^2+(y-3)^2=16$

domain: $[-2,6]$

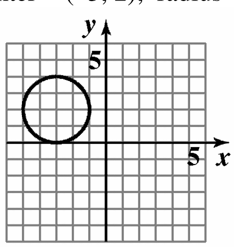
range: $[-1,7]$

45. $(x+3)^2+(y-2)^2=4$

$[x-(-3)]^2+(y-2)^2=2^2$

$h=-3,\ k=2,\ r=2$

center = (–3, 2); radius = 2



$(x+3)^2+(y-2)^2=4$

domain: $[-5,-1]$

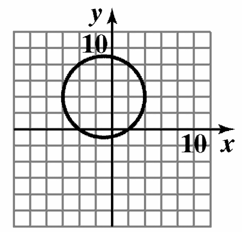
range: $[0,4]$

46. $(x+1)^2+(y-4)^2=25$

$[x-(-1)]^2+(y-4)^2=5^2$

$h=-1, k=4, r=5;$

center = (–1, 4); radius = 5



$(x+1)^2+(y-4)^2=25$

domain: $[-6,4]$

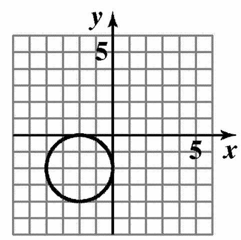
range: $[-1,9]$

47. $(x+2)^2+(y+2)^2=4$

$[x-(-2)]^2+[y-(-2)]^2=2^2$

$h=-2,\ k=-2,\ r=2$

center = (–2, –2); radius = 2



$(x+2)^2+(y+2)^2=4$

domain: $[-4,0]$

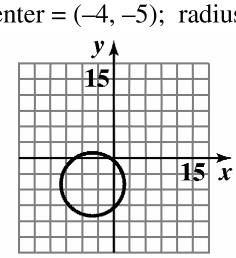
range: $[-4,0]$

48. $(x+4)^2+(y+5)^2=36$

$[x-(-4)]^2+[y-(-5)]^2=6^2$

$h=-4, k=-5, r=6;$

center = (–4, –5); radius = 6



$(x+4)^2+(y+5)^2=36$

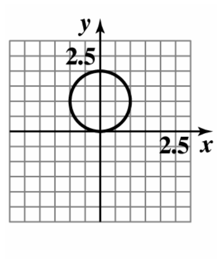
domain: $[-10,2]$

range: $[-11,1]$

49. $x^2 + (y-1)^2 = 1$

$h = 0, k = 1, r = 1;$

center = (0, 1); radius = 1



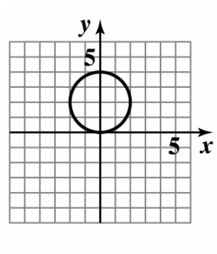
domain: $[-1,1]$

range: $[0,2]$

50. $x^2 + (y-2)^2 = 4$

$h = 0, k = 2, r = 2;$

center = (0,2); radius = 2



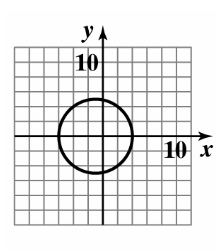
domain: $[-2,2]$

range: $[0,4]$

51. $(x+1)^2 + y^2 = 25$

$h = -1, k = 0, r = 5;$

center = (–1,0); radius = 5



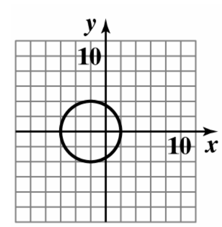
domain: $[-6,4]$

range: $[-5,5]$

52. $(x+2)^2 + y^2 = 16$

$h = -2, k = 0, r = 4;$

center = (–2,0); radius = 4



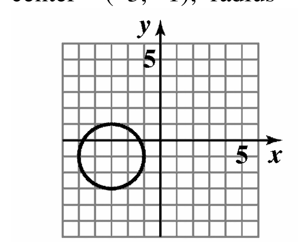
domain: $[-6,2]$

range: $[-4,4]$

53.

$$x^2 + y^2 + 6x + 2y + 6 = 0$$
$$(x^2 + 6x) + (y^2 + 2y) = -6$$
$$(x^2 + 6x + 9) + (y^2 + 2y + 1) = 9 + 1 - 6$$
$$(x+3)^2 + (y+1)^2 = 4$$
$$[x-(-3)]^2 + [9-(-1)]^2 = 2^2$$

center = (–3, –1); radius = 2

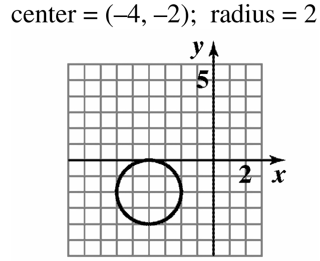


$\mathbf{x^2 + y^2 + 6x + 2y + 6 = 0}$

54. $x^2 + y^2 + 8x + 4y + 16 = 0$

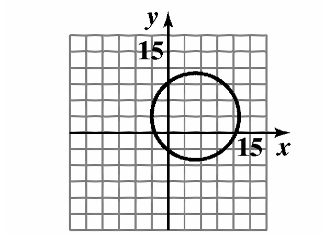
$$(x^2 + 8x) + (y^2 + 4y) = -16$$
$$(x^2 + 8x + 16) + (y^2 + 4y + 4) = 20 - 16$$
$$(x+4)^2 + (y+2)^2 = 4$$
$$[x-(-4)]^2 + [y-(-2)]^2 = 2^2$$

center = (–4, –2); radius = 2



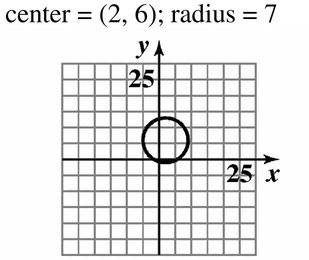
$\mathbf{x^2 + y^2 + 8x + 4y + 16 = 0}$

55.
$$x^2 + y^2 - 10x - 6y - 30 = 0$$
$$\left(x^2 - 10x\right) + \left(y^2 - 6y\right) = 30$$
$$\left(x^2 - 10x + 25\right) + \left(y^2 - 6y + 9\right) = 25 + 9 + 30$$
$$\left(x - 5\right)^2 + \left(y - 3\right)^2 = 64$$
$$(x - 5)^2 + (y - 3)^2 = 8^2$$
center = (5, 3); radius = 8



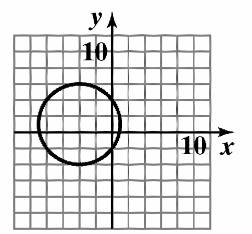
$x^2 + y^2 - 10x - 6y - 30 = 0$

56.
$$x^2 + y^2 - 4x - 12y - 9 = 0$$
$$\left(x^2 - 4x\right) + \left(y^2 - 12y\right) = 9$$
$$\left(x^2 - 4x + 4\right) + \left(y^2 - 12y + 36\right) = 4 + 36 + 9$$
$$\left(x - 2\right)^2 + \left(y - 6\right)^2 = 49$$
$$(x - 2)^2 + (y - 6)^2 = 7^2$$
center = (2, 6); radius = 7



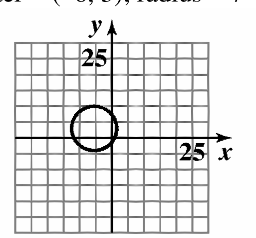
$x^2 + y^2 - 4x - 12y - 9 = 0$

57.
$$x^2 + y^2 + 8x - 2y - 8 = 0$$
$$\left(x^2 + 8x\right) + \left(y^2 - 2y\right) = 8$$
$$\left(x^2 + 8x + 16\right) + \left(y^2 - 2y + 1\right) = 16 + 1 + 8$$
$$\left(x + 4\right)^2 + \left(y - 1\right)^2 = 25$$
$$\left[x - (-4)\right]^2 + (y - 1)^2 = 5^2$$
center = (–4, 1); radius = 5



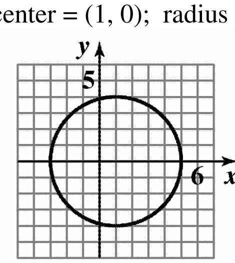
$x^2 + y^2 + 8x - 2y - 8 = 0$

58.
$$x^2 + y^2 + 12x - 6y - 4 = 0$$
$$\left(x^2 + 12x\right) + \left(y^2 - 6y\right) = 4$$
$$\left(x^2 + 12x + 36\right) + \left(y^2 - 6y + 9\right) = 36 + 9 + 4$$
$$\left[x - (-6)\right]^2 + \left(y - 3\right)^2 = 7^2$$
center = (–6, 3); radius = 7



$x^2 + y^2 + 12x - 6y - 4 = 0$

59.
$$x^2 - 2x + y^2 - 15 = 0$$
$$\left(x^2 - 2x\right) + y^2 = 15$$
$$\left(x^2 - 2x + 1\right) + \left(y - 0\right)^2 = 1 + 0 + 15$$
$$\left(x - 1\right)^2 + \left(y - 0\right)^2 = 16$$
$$\left(x - 1\right)^2 + \left(y - 0\right)^2 = 4^2$$
center = (1, 0); radius = 4



$x^2 - 2x + y^2 - 15 = 0$

60. $x^2 + y^2 - 6y - 7 = 0$

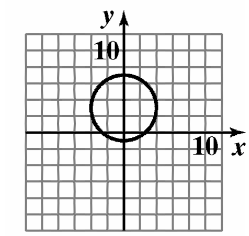
$x^2 + \left(y^2 - 6y\right) = 7$

$\left(x-0\right)^2 = \left(y^2 - 6y + 9\right) = 0 + 9 + 7$

$\left(x-0\right)^2 + \left(y-3\right)^2 = 16$

$(x-0)^2 + (y-3)^2 = 4^2$

center = (0, 3); radius = 4



$\boldsymbol{x^2 + y^2 - 6y - 7 = 0}$

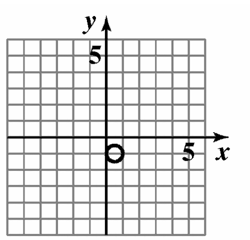
61. $x^2 + y^2 - x + 2y + 1 = 0$

$x^2 - x \quad + y^2 + 2y \quad = -1$

$x^2 - x + \frac{1}{4} + y^2 + 2y + 1 = -1 + \frac{1}{4} + 1$

$\left(x - \frac{1}{2}\right)^2 + \left(y+1\right)^2 = \frac{1}{4}$

center = $\left(\frac{1}{2}, -1\right)$; radius = $\frac{1}{2}$



$\boldsymbol{x^2 + y^2 - x + 2y + 1 = 0}$

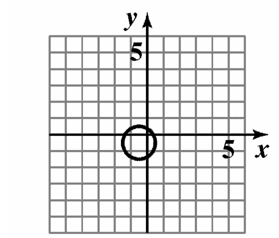
62. $x^2 + y^2 + x + y - \frac{1}{2} = 0$

$x^2 + x \quad + y^2 + y \quad = \frac{1}{2}$

$x^2 + x + \frac{1}{4} + y^2 + y + \frac{1}{4} = \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = 1$

center = $\left(\frac{1}{2}, \frac{1}{2}\right)$; radius = 1



$\boldsymbol{x^2 + y^2 + x + y - \frac{1}{2} = 0}$

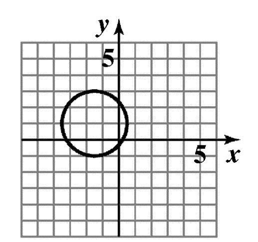
63. $x^2 + y^2 + 3x - 2y - 1 = 0$

$x^2 + 3x \quad + y^2 - 2y \quad = 1$

$x^2 + 3x + \frac{9}{4} + y^2 - 2y + 1 = 1 + \frac{9}{4} + 1$

$\left(x + \frac{3}{2}\right)^2 + \left(y-1\right)^2 = \frac{17}{4}$

center = $\left(-\frac{3}{2}, 1\right)$; radius = $\frac{\sqrt{17}}{2}$



$\boldsymbol{x^2 + y^2 + 3x - 2y - 1 = 0}$

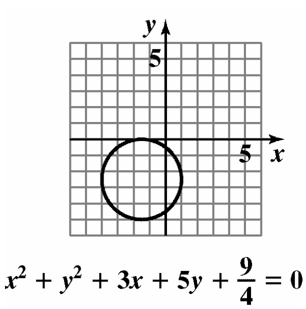
64. $x^2+y^2+3x+5y+\frac{9}{4}=0$

$$x^2+3x \quad +y^2+5y \quad =-\frac{9}{4}$$

$$x^2+3x+\frac{9}{4}+y^2+5y+\frac{25}{4}=-\frac{9}{4}+\frac{9}{4}+\frac{25}{4}$$

$$\left(x+\frac{3}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{25}{4}$$

center = $\left(-\frac{3}{2},-\frac{5}{2}\right)$; radius = $\frac{5}{2}$



65. a. Since the line segment passes through the center, the center is the midpoint of the segment.

$$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$=\left(\frac{3+7}{2},\frac{9+11}{2}\right)=\left(\frac{10}{2},\frac{20}{2}\right)$$

$$=(5,10)$$

The center is $(5,10)$.

b. The radius is the distance from the center to one of the points on the circle. Using the point $(3,9)$, we get:

$$d=\sqrt{(5-3)^2+(10-9)^2}$$

$$=\sqrt{2^2+1^2}=\sqrt{4+1}$$

$$=\sqrt{5}$$

The radius is $\sqrt{5}$ units.

c. $(x-5)^2+(y-10)^2=\left(\sqrt{5}\right)^2$

$$(x-5)^2+(y-10)^2=5$$

66. a. Since the line segment passes through the center, the center is the midpoint of the segment.

$$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$=\left(\frac{3+5}{2},\frac{6+4}{2}\right)=\left(\frac{8}{2},\frac{10}{2}\right)$$

$$=(4,5)$$

The center is $(4,5)$.

b. The radius is the distance from the center to one of the points on the circle. Using the point $(3,6)$, we get:

$$d=\sqrt{(4-3)^2+(5-6)^2}$$

$$=\sqrt{1^2+(-1)^2}=\sqrt{1+1}$$

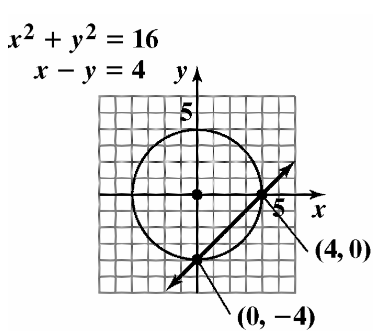
$$=\sqrt{2}$$

The radius is $\sqrt{2}$ units.

c. $(x-4)^2+(y-5)^2=\left(\sqrt{2}\right)^2$

$$(x-4)^2+(y-5)^2=2$$

67.



Intersection points: $(0,-4)$ and $(4,0)$

Check $(0,-4)$:

$0^2+(-4)^2=16$ $\quad$ $0-(-4)=4$

$16=16$ true $\quad$ $4=4$ true

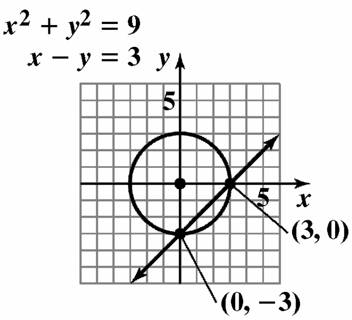
Check $(4,0)$:

$4^2+0^2=16$ $\quad$ $4-0=4$

$16=16$ true $\quad$ $4=4$ true

The solution set is $\{(0,-4),(4,0)\}$.

68.



Intersection points: $(0,-3)$ and $(3,0)$

Check $(0,-3)$:

$0^2+(-3)^2=9 \qquad 0-(-3)=3$

$9=9$ true $\qquad 3=3$ true

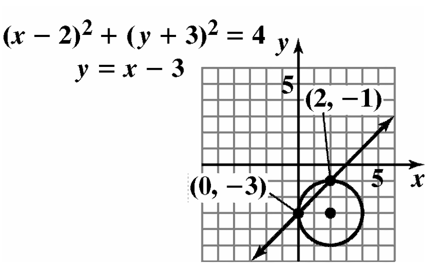
Check $(3,0)$:

$3^2+0^2=9 \qquad 3-0=3$

$9=9$ true $\qquad 3=3$ true

The solution set is $\{(0,-3),(3,0)\}$.

69.



Intersection points: $(0,-3)$ and $(2,-1)$

Check $(0,-3)$:

$(0-2)^2+(-3+3)^2=9 \qquad -3=0-3$

$(-2)^2+0^2=4 \qquad -3=-3$ true

$4=4$

true

Check $(2,-1)$:

$(2-2)^2+(-1+3)^2=4 \qquad -1=2-3$

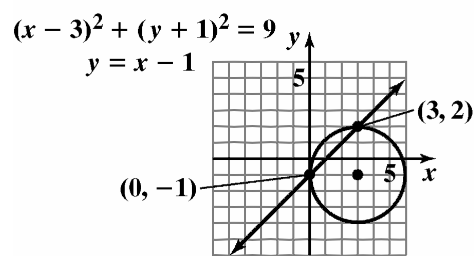
$0^2+2^2=4 \qquad -1=-1$ true

$4=4$

true

The solution set is $\{(0,-3),(2,-1)\}$.

70.



Intersection points: $(0,-1)$ and $(3,2)$

Check $(0,-1)$:

$(0-3)^2+(-1+1)^2=9 \qquad -1=0-1$

$(-3)^2+0^2=9 \qquad -1=-1$ true

$9=9$

true

Check $(3,2)$:

$(3-3)^2+(2+1)^2=9 \qquad 2=3-1$

$0^2+3^2=9 \qquad 2=2$ true

$9=9$

true

The solution set is $\{(0,-1),(3,2)\}$.

71. $d=\sqrt{(8495-4422)^2+(8720-1241)^2}\cdot\sqrt{0.1}$

$d=\sqrt{72,524,770}\cdot\sqrt{0.1}$

$d\approx 2693$

The distance between Boston and San Francisco is about 2693 miles.

72. $d=\sqrt{(8936-8448)^2+(3542-2625)^2}\cdot\sqrt{0.1}$

$d=\sqrt{1,079,033}\cdot\sqrt{0.1}$

$d\approx 328$

The distance between New Orleans and Houston is about 328 miles.

73. If we place L.A. at the origin, then we want the equation of a circle with center at $(-2.4,-2.7)$ and radius 30.

$(x-(-2.4))^2+(y-(-2.7))^2=30^2$

$(x+2.4)^2+(y+2.7)^2=900$

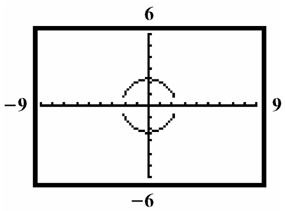
74. $C(0, 68+14)=(0, 82)$

$(x-0)^2+(y-82)^2=68^2$

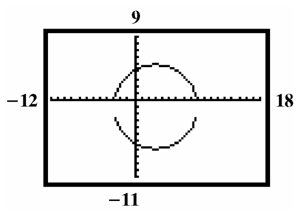
$x^2+(y-82)^2=4624$

75. – 82. Answers may vary.

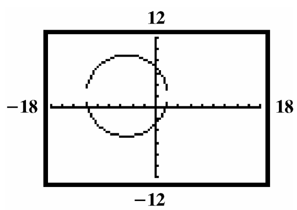
83.



84.



85.



86. makes sense

87. makes sense

88. does not make sense; Explanations will vary. Sample explanation: Since $r^2 = -4$ this is not the equation of a circle.

89. makes sense

90. false; Changes to make the statement true will vary. A sample change is: The equation would be $x^2 + y^2 = 256$.

91. false; Changes to make the statement true will vary. A sample change is: The center is at (3, –5).

92. false; Changes to make the statement true will vary. A sample change is: This is not an equation for a circle.

93. false; Changes to make the statement true will vary. A sample change is: Since $r^2 = -36$ this is not the equation of a circle.

94. The distance for A to B:

$$\begin{aligned}\overline{AB} &= \sqrt{(3-1)^2+[3+d-(1+d)]^2}\\ &= \sqrt{2^2+2^2}\\ &= \sqrt{4+4}\\ &= \sqrt{8}\\ &= 2\sqrt{2}\end{aligned}$$

The distance from B to C:

$$\begin{aligned}\overline{BC} &= \sqrt{(6-3)^2+[3+d-(6+d)]^2}\\ &= \sqrt{3^2+(-3)^2}\\ &= \sqrt{9+9}\\ &= \sqrt{18}\\ &= 3\sqrt{2}\end{aligned}$$

The distance for A to C:

$$\begin{aligned}\overline{AC} &= \sqrt{(6-1)^2+[6+d-(1+d)]^2}\\ &= \sqrt{5^2+5^2}\\ &= \sqrt{25+25}\\ &= \sqrt{50}\\ &= 5\sqrt{2}\end{aligned}$$

$$\begin{aligned}\overline{AB}+\overline{BC} &= \overline{AC}\\ 2\sqrt{2}+3\sqrt{2} &= 5\sqrt{2}\\ 5\sqrt{2} &= 5\sqrt{2}\end{aligned}$$

95. **a.** d_1 is distance from (x_1, x_2) to midpoint

$$d_1 = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$

$$d_1 = \sqrt{\left(\frac{x_1 + x_2 - 2x_1}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_1}{2}\right)^2}$$

$$d_1 = \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$$

$$d_1 = \sqrt{\frac{x_2 - 2x_1x_2 + x_1^2}{4} + \frac{y_2^2 - 2y_2y_1 + y_1^2}{4}}$$

$$d_1 = \sqrt{\frac{1}{4}\left(x_2 - 2x_1x_2 + x_1 + y_2^2 - 2y_2y_1 + y_1^2\right)}$$

$$d_1 = \frac{1}{2}\sqrt{x_2 - 2x_1x_2 + x_1 + y_2^2 - 2y_2y_1 + y_1^2}$$

d_2 is distance from midpoint to (x_2, y_2)

$$d_2 = \sqrt{\left(\frac{x_1 + x_2}{2} - x_2\right)^2 + \left(\frac{y_1 + y_2}{2} - y_2\right)^2}$$

$$d_2 = \sqrt{\left(\frac{x_1 + x_2 - 2x_2}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_2}{2}\right)^2}$$

$$d_2 = \sqrt{\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2}$$

$$d_2 = \sqrt{\frac{x_1^2 - 2x_1x_2 + x_2^2}{4} + \frac{y_1^2 - 2y_2y_1 + y_2^2}{4}}$$

$$d_2 = \sqrt{\frac{1}{4}\left(x_1^2 - 2x_1x_2 + x_2^2 + y_1^2 - 2y_2y_1 + y_2^2\right)}$$

$$d_2 = \frac{1}{2}\sqrt{x_1^2 - 2x_1x_2 + x_2^2 + y_1^2 - 2y_2y_1 + y_2^2}$$

$$d_1 = d_2$$

b. d_3 is the distance from (x_1, y_1) to $(x_2 y_2)$

$$d_3 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_3 = \sqrt{x_2^2 - 2x_1x_2 + x_1^2 + y_2^2 - 2y_2y_1 + y_1^2}$$

$d_1 + d_2 = d_3$ because $\frac{1}{2}\sqrt{a} + \frac{1}{2}\sqrt{a} = \sqrt{a}$

96. Both circles have center (2, –3). The smaller circle has radius 5 and the larger circle has radius 6. The smaller circle is inside of the larger circle. The area between them is given by

$$\pi(6)^2 - \pi(5)^2 = 36\pi - 25\pi$$
$$= 11\pi$$
$$\approx 34.56 \text{ square units.}$$

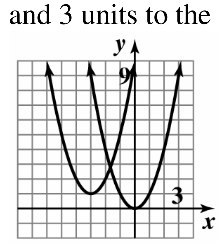
97. The circle is centered at (0,0). The slope of the radius with endpoints (0,0) and (3,–4) is $m = -\frac{-4-0}{3-0} = -\frac{4}{3}$. The line perpendicular to the radius has slope $\frac{3}{4}$. The tangent line has slope $\frac{3}{4}$ and passes through (3,–4), so its equation is:
$y + 4 = \frac{3}{4}(x - 3).$

98.
$$0 = -2(x-3)^2 + 8$$
$$2(x-3)^2 = 8$$
$$(x-3)^2 = 4$$
$$x - 3 = \pm\sqrt{4}$$
$$x = 3 \pm 2$$
$$x = 1, 5$$

99.
$$-x^2 - 2x + 1 = 0$$
$$x^2 + 2x - 1 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{8}}{2}$$
$$= \frac{2 \pm 2\sqrt{2}}{2}$$
$$= 1 \pm \sqrt{2}$$

The solution set is $\{1 \pm \sqrt{2}\}$.

100. The graph of g is the graph of f shifted 1 unit up and 3 units to the left.



$f(x) = x^2$
$g(x) = (x + 3)^2 + 1$

Chapter 2 Review Exercises

1. function
domain: {2, 3, 5}
range: {7}

2. function
domain: {1, 2, 13}
range: {10, 500, π}

3. not a function
domain: {12, 14}
range: {13, 15, 19}

4. $2x + y = 8$
$y = -2x + 8$
Since only one value of *y* can be obtained for each value of *x*, *y* is a function of *x*.

5. $3x^2 + y = 14$
$y = -3x^2 + 14$
Since only one value of *y* can be obtained for each value of *x*, *y* is a function of *x*.

6. $2x + y^2 = 6$
$y^2 = -2x + 6$
$y = \pm\sqrt{-2x+6}$
Since more than one value of *y* can be obtained from some values of *x*, *y* is not a function of *x*.

7. $f(x) = 5 - 7x$

a. $f(4) = 5 - 7(4) = -23$

b. $f(x+3) = 5 - 7(x+3)$
$= 5 - 7x - 21$
$= -7x - 16$

c. $f(-x) = 5 - 7(-x) = 5 + 7x$

8. $g(x) = 3x^2 - 5x + 2$

a. $g(0) = 3(0)^2 - 5(0) + 2 = 2$

b. $g(-2) = 3(-2)^2 - 5(-2) + 2$
$= 12 + 10 + 2$
$= 24$

c. $g(x-1) = 3(x-1)^2 - 5(x-1) + 2$
$= 3(x^2 - 2x + 1) - 5x + 5 + 2$
$= 3x^2 - 11x + 10$

d. $g(-x) = 3(-x)^2 - 5(-x) + 2$
$= 3x^2 + 5x + 2$

9. **a.** $g(13) = \sqrt{13-4} = \sqrt{9} = 3$

b. $g(0) = 4 - 0 = 4$

c. $g(-3) = 4 - (-3) = 7$

10. **a.** $f(-2) = \dfrac{(-2)^2 - 1}{-2-1} = \dfrac{3}{-3} = -1$

b. $f(1) = 12$

c. $f(2) = \dfrac{2^2 - 1}{2-1} = \dfrac{3}{1} = 3$

11. The vertical line test shows that this is not the graph of a function.

12. The vertical line test shows that this is the graph of a function.

13. The vertical line test shows that this is the graph of a function.

14. The vertical line test shows that this is not the graph of a function.

15. The vertical line test shows that this is not the graph of a function.

16. The vertical line test shows that this is the graph of a function.

17. **a.** domain: $[-3, 5)$

b. range: $[-5, 0]$

c. *x*-intercept: –3

d. *y*-intercept: –2

e. increasing: $(-2, 0)$ or $(3, 5)$
decreasing: $(-3, -2)$ or $(0, 3)$

f. $f(-2) = -3$ and $f(3) = -5$

18. **a.** domain: $(-\infty, \infty)$

b. range: $(-\infty, \infty)$

c. x-intercepts: –2 and 3

d. y-intercept: 3

e. increasing: (–5, 0)
decreasing: $(-\infty, -5)$ or $(0, \infty)$

f. $f(-2) = 0$ and $f(6) = -3$

19. **a.** domain: $(-\infty, \infty)$

b. range: [–2, 2]

c. x-intercept: 0

d. y-intercept: 0

e. increasing: (–2, 2)
constant: $(-\infty, -2)$ or $(2, \infty)$

f. $f(-9) = -2$ and $f(14) = 2$

20. **a.** 0, relative maximum –2

b. –2, 3, relative minimum –3, –5

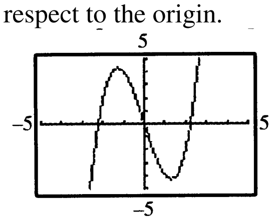
21. **a.** 0, relative maximum 3

b. –5, relative minimum –6

22. $f(x) = x^3 - 5x$

$$\begin{aligned} f(-x) &= (-x)^3 - 5(-x) \\ &= -x^3 + 5x \\ &= -f(x) \end{aligned}$$

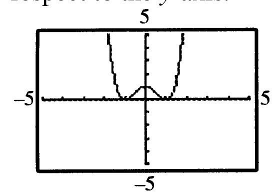
The function is odd. The function is symmetric with respect to the origin.



23. $f(x) = x^4 - 2x^2 + 1$

$$\begin{aligned} f(-x) &= (-x)^4 - 2(-x)^2 + 1 \\ &= x^4 - 2x^2 + 1 \\ &= f(x) \end{aligned}$$

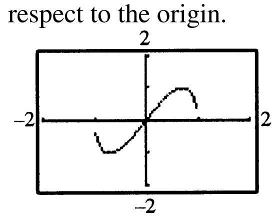
The function is even. The function is symmetric with respect to the y-axis.



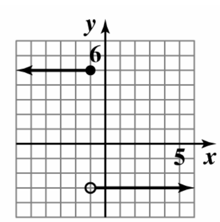
24. $f(x) = 2x\sqrt{1-x^2}$

$$\begin{aligned} f(-x) &= 2(-x)\sqrt{1-(-x)^2} \\ &= -2x\sqrt{1-x^2} \\ &= -f(x) \end{aligned}$$

The function is odd. The function is symmetric with respect to the origin.



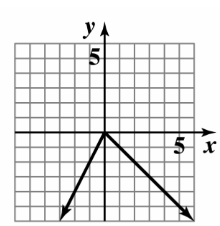
25. **a.**



$$f(x) = \begin{cases} 5 & \text{if } x \le -1 \\ -3 & \text{if } x > -1 \end{cases}$$

b. range: {–3, 5}

26. **a.**



$$f(x) = \begin{cases} 2x & \text{if } x < 0 \\ -x & \text{if } x \ge 0 \end{cases}$$

b. range: $\{y \mid y \le 0\}$

27. $\dfrac{8(x+h)-11-(8x-11)}{h}$
$=\dfrac{8x+8h-11-8x+11}{h}$
$=\dfrac{8h}{8}$
$=8$

28. $\dfrac{-2(x+h)^2+(x+h)+10-\left(-2x^2+x+10\right)}{h}$
$=\dfrac{-2\left(x^2+2xh+h^2\right)+x+h+10+2x^2-x-10}{h}$
$=\dfrac{-2x^2-4xh-2h^2+x+h+10+2x^2-x-10}{h}$
$=\dfrac{-4xh-2h^2+h}{h}$
$=\dfrac{h\left(-4x-2h+1\right)}{h}$
$-4x-2h+1$

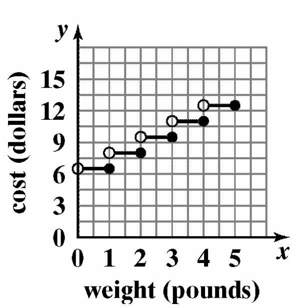
29. **a.** Yes, the eagle's height is a function of time since the graph passes the vertical line test.

b. Decreasing: (3, 12)
The eagle descended.

c. Constant: (0, 3) or (12, 17)
The eagle's height held steady during the first 3 seconds and the eagle was on the ground for 5 seconds.

d. Increasing: (17, 30)
The eagle was ascending.

30.



31. $m=\dfrac{1-2}{5-3}=\dfrac{-1}{2}=-\dfrac{1}{2}$; falls

32. $m=\dfrac{-4-(-2)}{-3-(-1)}=\dfrac{-2}{-2}=1$; rises

33. $m=\dfrac{\frac{1}{4}-\frac{1}{4}}{6-(-3)}=\dfrac{0}{9}=0$; horizontal

34. $m=\dfrac{10-5}{-2-(-2)}=\dfrac{5}{0}$ undefined; vertical

35. point-slope form: $y-2=-6(x+3)$
slope-intercept form: $y=-6x-16$

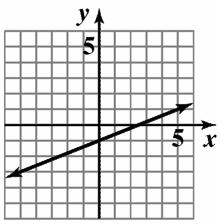
36. $m=\dfrac{2-6}{-1-1}=\dfrac{-4}{-2}=2$
point-slope form: $y-6=2(x-1)$
or $y-2=2(x+1)$
slope-intercept form: $y=2x+4$

37. $3x+y-9=0$
$y=-3x+9$
$m=-3$
point-slope form:
$y+7=-3(x-4)$
slope-intercept form:
$y=-3x+12-7$
$y=-3x+5$

38. perpendicular to $y=\dfrac{1}{3}x+4$
$m=-3$
point-slope form:
$y-6=-3(x+3)$
slope-intercept form:
$y=-3x-9+6$
$y=-3x-3$

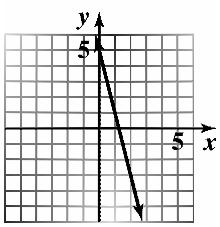
39. Write $6x-y-4=0$ in slope intercept form.
$6x-y-4=0$
$-y=-6x+4$
$y=6x-4$
The slope of the perpendicular line is 6, thus the slope of the desired line is $m=-\dfrac{1}{6}$.
$y-y_1=m(x-x_1)$
$y-(-1)=-\frac{1}{6}\left(x-(-12)\right)$
$y+1=-\frac{1}{6}(x+12)$
$y+1=-\frac{1}{6}x-2$
$6y+6=-x-12$
$x+6y+18=0$

40. slope: $\frac{2}{5}$; y-intercept: -1



$y = \frac{2}{5}x - 1$

41. slope: -4; y-intercept: 5

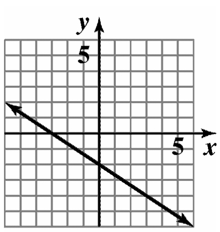


$f(x) = -4x + 5$

42. $2x + 3y + 6 = 0$

$$3y = -2x - 6$$
$$y = -\frac{2}{3}x - 2$$

slope: $-\frac{2}{3}$; y-intercept: -2

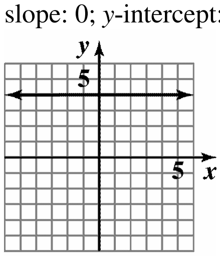


$2x + 3y + 6 = 0$

43. $2y - 8 = 0$

$$2y = 8$$
$$y = 4$$

slope: 0; y-intercept: 4



$2y - 8 = 0$

44. $2x - 5y - 10 = 0$

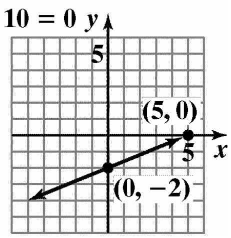
Find x-intercept:

$$2x - 5(0) - 10 = 0$$
$$2x - 10 = 0$$
$$2x = 10$$
$$x = 5$$

Find y-intercept:

$$2(0) - 5y - 10 = 0$$
$$-5y - 10 = 0$$
$$-5y = 10$$
$$y = -2$$

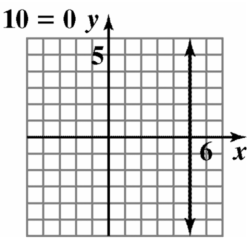
$2x - 5y - 10 = 0$



45. $2x - 10 = 0$

$$2x = 10$$
$$x = 5$$

$2x - 10 = 0$



46. **a.** $m = \frac{11 - 2.3}{90 - 15} = \frac{8.7}{75} = 0.116$

$$y - y_1 = m(x - x_1)$$
$$y - 11 = 0.116(x - 90)$$

or

$$y - 2.3 = 0.116(x - 15)$$

b. $y - 11 = 0.116(x - 90)$

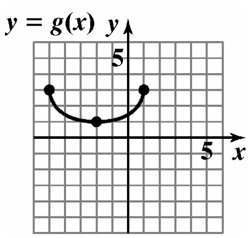
$$y - 11 = 0.116x - 10.44$$
$$y = 0.116x + 0.56$$
$$f(x) = 0.116x + 0.56$$

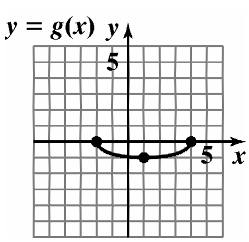
c. According to the graph, France has about 5 deaths per 100,000 persons.

d. $f(x) = 0.116x + 0.56$
$f(32) = 0.116(32) + 0.56$
$= 4.272$
≈ 4.3
According to the function, France has about 4.3 deaths per 100,000 persons.
This underestimates the value in the graph by 0.7 deaths per 100,000 persons.
The line passes below the point for France.

47. $m = \dfrac{1616 - 886}{2006 - 2002} = \dfrac{730}{4} = 182.5$
Corporate profits increased at a rate of \$182.5 billion per year. The rate of change is \$182.5 billion per year.

48. $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{[9^2 - 4(9)] - [4^2 - 4 \cdot 5]}{9 - 5} = 10$

49.


50.


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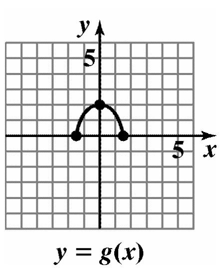
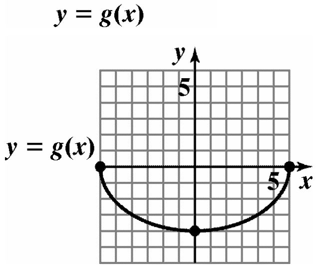


52.


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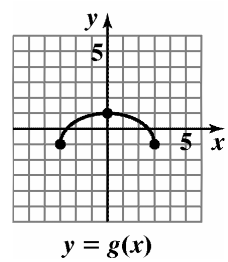

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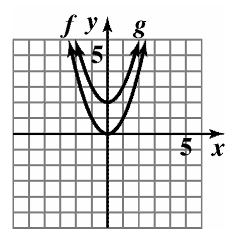

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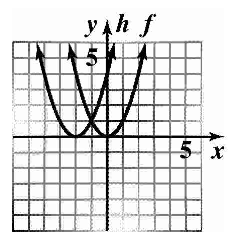

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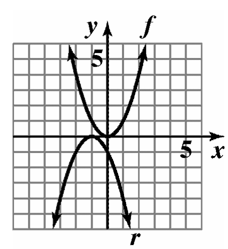

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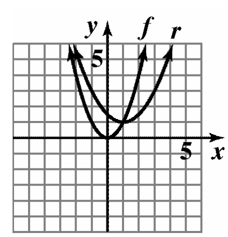

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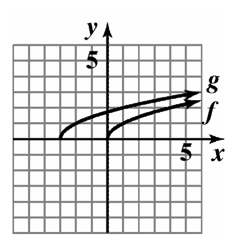
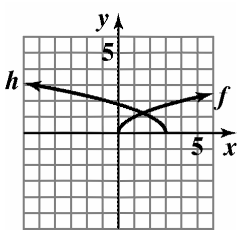
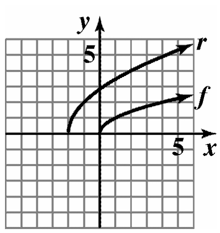


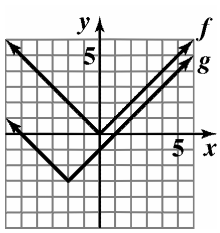
59.



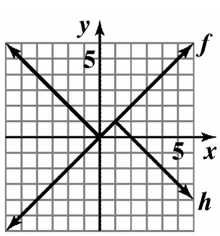
60.



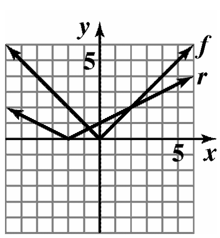
61.



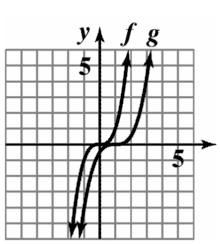
62.



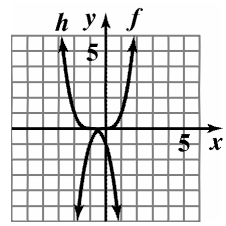
63.



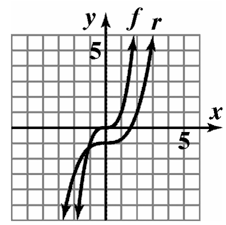
64.



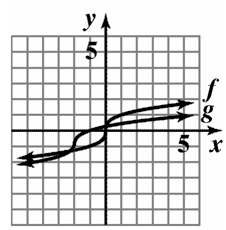
65.



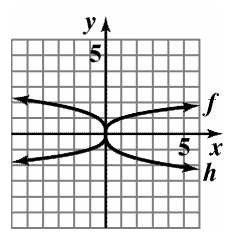
66.



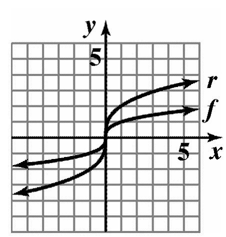
67.



68.



69.



70. domain: $(-\infty, \infty)$

71. The denominator is zero when $x = 7$. The domain is $(-\infty, 7) \cup (7, \infty)$.

72. The expressions under each radical must not be negative.
$8 - 2x \geq 0$
$-2x \geq -8$
$x \leq 4$
domain: $(-\infty, 4]$.

73. The denominator is zero when $x = -7$ or $x = 3$.
domain: $(-\infty,-7)\cup(-7,3)\cup(3,\infty)$

74. The expressions under each radical must not be negative. The denominator is zero when $x = 5$.
$x-2\ge 0$
$x\ge 2$
domain: $[2,5)\cup(5,\infty)$

75. The expressions under each radical must not be negative.
$x-1\ge 0$ and $x+5\ge 0$
$x\ge 1$ $\quad x\ge -5$
domain: $[1,\infty)$

76. $f(x) = 3x - 1$; $g(x) = x - 5$
$(f+g)(x) = 4x - 6$
domain: $(-\infty,\infty)$
$(f-g)(x) = (3x-1)-(x-5) = 2x+4$
domain: $(-\infty,\infty)$
$(fg)(x) = (3x-1)(x-5) = 3x^2-16x+5$
domain: $(-\infty,\infty)$
$\left(\dfrac{f}{g}\right)(x) = \dfrac{3x-1}{x-5}$
domain: $(-\infty,5)\cup(5,\infty)$

77. $f(x) = x^2+x+1$; $g(x) = x^2-1$
$(f+g)(x) = 2x^2+x$
domain: $(-\infty,\infty)$
$(f-g)(x) = (x^2+x+1)-(x^2-1) = x+2$
domain: $(-\infty,\infty)$
$(fg)(x) = (x^2+x+1)(x^2-1)$
$= x^4+x^3-x-1$
$\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2+x+1}{x^2-1}$
domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

78. $f(x)=\sqrt{x+7}$; $g(x)=\sqrt{x-2}$
$(f+g)(x)=\sqrt{x+7}+\sqrt{x-2}$
domain: $[2,\infty)$
$(f-g)(x)=\sqrt{x+7}-\sqrt{x-2}$
domain: $[2,\infty)$
$(fg)(x)=\sqrt{x+7}\cdot\sqrt{x-2}$
$=\sqrt{x^2+5x-14}$
domain: $[2,\infty)$
$\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{x+7}}{\sqrt{x-2}}$
domain: $(2,\infty)$

79. $f(x)=x^2+3$; $g(x)=4x-1$

a. $(f\circ g)(x)=(4x-1)^2+3$
$=16x^2-8x+4$

b. $(g\circ f)(x)=4(x^2+3)-1$
$=4x^2+11$

c. $(f\circ g)(3)=16(3)^2-8(3)+4=124$

80. $f(x)=\sqrt{x}$; $g(x)=x+1$

a. $(f\circ g)(x)=\sqrt{x+1}$

b. $(g\circ f)(x)=\sqrt{x}+1$

c. $(f\circ g)(3)=\sqrt{3+1}=\sqrt{4}=2$

81. a. $(f\circ g)(x)=f\left(\dfrac{1}{x}\right)$

$$=\frac{\frac{1}{x}+1}{\frac{1}{x}-2}=\frac{\left(\frac{1}{x}+1\right)x}{\left(\frac{1}{x}-2\right)x}=\frac{1+x}{1-2x}$$

b. $x\ne 0$ $\quad 1-2x\ne 0$
$x\ne\dfrac{1}{2}$
$(-\infty,0)\cup\left(0,\dfrac{1}{2}\right)\cup\left(\dfrac{1}{2},\infty\right)$

82. **a.** $(f\circ g)(x)=f(x+3)=\sqrt{x+3-1}=\sqrt{x+2}$

b. $x+2\ge 0$
$x\ge -2$ $\quad [-2,\infty)$

83. $f(x)=x^4 \qquad g(x)=x^2+2x-1$

84. $f(x)=\sqrt[3]{x} \qquad g(x)=7x+4$

85. $f(x)=\frac{3}{5}x+\frac{1}{2};\ g(x)=\frac{5}{3}x-2$

$$f(g(x))=\frac{3}{5}\left(\frac{5}{3}x-2\right)+\frac{1}{2}$$
$$=x-\frac{6}{5}+\frac{1}{2}$$
$$=x-\frac{7}{10}$$
$$g(f(x))=\frac{5}{3}\left(\frac{3}{5}x+\frac{1}{2}\right)-2$$
$$=x+\frac{5}{6}-2$$
$$=x-\frac{7}{6}$$

f and *g* are not inverses of each other.

86. $f(x)=2-5x;\ g(x)=\frac{2-x}{5}$

$$f(g(x))=2-5\left(\frac{2-x}{5}\right)$$
$$=2-(2-x)$$
$$=x$$
$$g(f(x))=\frac{2-(2-5x)}{5}=\frac{5x}{5}=x$$

f and *g* are inverses of each other.

87. **a.**
$$f(x)=4x-3$$
$$y=4x-3$$
$$x=4y-3$$
$$y=\frac{x+3}{4}$$
$$f^{-1}(x)=\frac{x+3}{4}$$

b.
$$f(f^{-1}(x))=4\left(\frac{x+3}{4}\right)-3$$
$$=x+3-3$$
$$=x$$
$$f^{-1}(f(x))=\frac{(4x-3)+3}{4}=\frac{4x}{4}=x$$

88. **a.**
$$f(x)=8x^3+1$$
$$y=8x^3+1$$
$$x=8y^3+1$$
$$x-1=8y^3$$
$$\frac{x-1}{8}=y^3$$
$$\sqrt[3]{\frac{x-1}{8}}=y$$
$$\frac{\sqrt[3]{x-1}}{2}=y$$
$$f^{-1}(x)=\frac{\sqrt[3]{x-1}}{2}$$

b.
$$f\left(f^{-1}(x)\right)=8\left(\frac{\sqrt[3]{x-1}}{2}\right)^3+1$$
$$=8\left(\frac{x-1}{8}\right)+1$$
$$=x-1+1$$
$$=x$$
$$f^{-1}\left(f(x)\right)=\frac{\sqrt[3]{\left(8x^3+1\right)-1}}{2}$$
$$=\frac{\sqrt[3]{8x^3}}{2}$$
$$=\frac{2x}{2}$$
$$=x$$

89. **a.**
$$f(x)=\frac{2}{x}+5$$
$$y=\frac{2}{x}+5$$
$$x=\frac{2}{y}+5$$
$$xy=2+5y$$
$$xy-5y=2$$
$$y(x-5)=2$$
$$y=\frac{2}{x-5}$$
$$f^{-1}(x)=\frac{2}{x-5}$$

b.
$$f\left(f^{-1}(x)\right)=\frac{2}{\frac{2}{x-5}}+5$$
$$=\frac{2(x-5)}{2}+5$$
$$=x-5+5$$
$$=x$$
$$f^{-1}\left(f(x)\right)=\frac{2}{\frac{2}{x}+5-5}$$
$$=\frac{2}{\frac{2}{x}}$$
$$=\frac{2x}{2}$$
$$=x$$

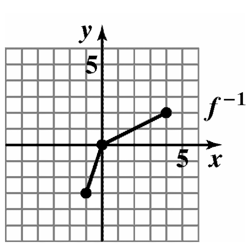
90. The inverse function exists.

91. The inverse function does not exist since it does not pass the horizontal line test.

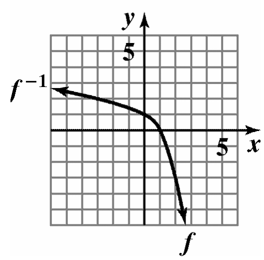
92. The inverse function exists.

93. The inverse function does not exist since it does not pass the horizontal line test.

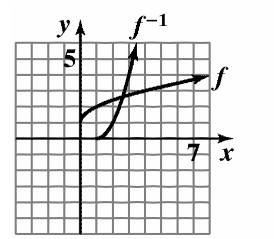
94.



95.
$$f(x)=1-x^2$$
$$y=1-x^2$$
$$x=1-y^2$$
$$y^2=1-x$$
$$y=\sqrt{1-x}$$
$$f^{-1}(x)=\sqrt{1-x}$$



96.
$$f(x)=\sqrt{x}+1$$
$$y=\sqrt{x}+1$$
$$x=\sqrt{y}+1$$
$$x-1=\sqrt{y}$$
$$(x-1)^2=y$$
$$f^{-1}(x)=(x-1)^2,\quad x\ge 1$$



$\boldsymbol{f(x)=\sqrt{x}+1}$
$\boldsymbol{g(x)=(x-1)^2, x\ge 1}$

97.
$$d=\sqrt{[3-(-2)]^2+[9-(-3)]^2}$$
$$=\sqrt{5^2+12^2}$$
$$=\sqrt{25+144}$$
$$=\sqrt{169}$$
$$=13$$

98. $d = \sqrt{[-2-(-4)]^2 + (5-3)^2}$
$= \sqrt{2^2 + 2^2}$
$= \sqrt{4+4}$
$= \sqrt{8}$
$= 2\sqrt{2}$
≈ 2.83

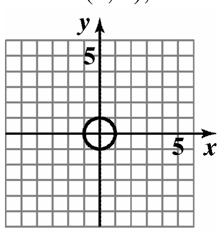
99. $\left(\frac{2+(-12)}{2}, \frac{6+4}{2}\right) = \left(\frac{-10}{2}, \frac{10}{2}\right) = (-5, 5)$

100. $\left(\frac{4+(-15)}{2}, \frac{-6+2}{2}\right) = \left(\frac{-11}{2}, \frac{-4}{2}\right) = \left(\frac{-11}{2}, -2\right)$

101. $x^2 + y^2 = 3^2$
$x^2 + y^2 = 9$

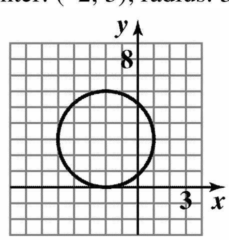
102. $(x-(-2))^2 + (y-4)^2 = 6^2$
$(x+2)^2 + (y-4)^2 = 36$

103. center: (0, 0); radius: 1



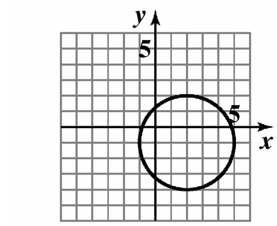
$\mathbf{x^2 + y^2 = 1}$
domain: $[-1, 1]$
range: $[-1, 1]$

104. center: (–2, 3); radius: 3



$\mathbf{(x+2)^2 + (y-3)^2 = 9}$
domain: $[-5, 1]$
range: $[0, 6]$

105. $x^2 + y^2 - 4x + 2y - 4 = 0$
$x^2 - 4x \quad + y^2 + 2y \quad = 4$
$x^2 - 4x + 4 + y^2 + 2y + 1 = 4 + 4 + 1$
$(x-2)^2 + (y+1)^2 = 9$
center: (2, –1); radius: 3



$\mathbf{x^2 + y^2 - 4x + 2y - 4 = 0}$
domain: $[-1, 5]$
range: $[-4, 2]$

Chapter 2 Test

1. (b), (c), and (d) are not functions.

2. **a.** $f(4) - f(-3) = 3 - (-2) = 5$

b. domain: (–5, 6]

c. range: [–4, 5]

d. increasing: (–1, 2)

e. decreasing: $(-5, -1)$ or $(2, 6)$

f. $2, f(2) = 5$

g. (–1, –4)

h. x-intercepts: –4, 1, and 5.

i. y-intercept: –3

3. **a.** –2, 2

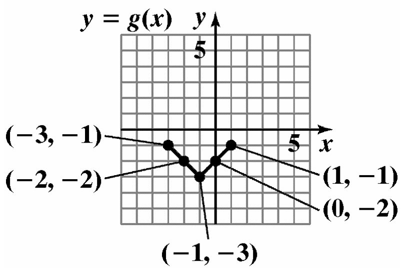
b. –1, 1

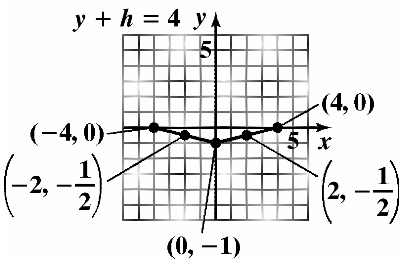
c. 0

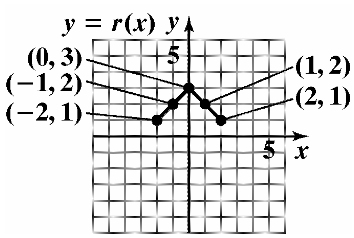
d. even; $f(-x) = f(x)$

e. no; f fails the horizontal line test

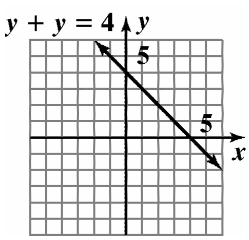
f. $f(0)$ is a relative minimum.

g. 

h. 

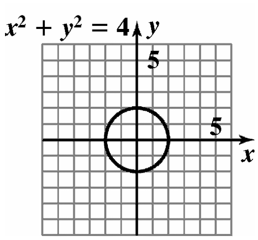
i. 

j. $\frac{f(x_2)-f(x_1)}{x_2-x_1}=\frac{-1-0}{1-(-2)}=-\frac{1}{3}$

4. 

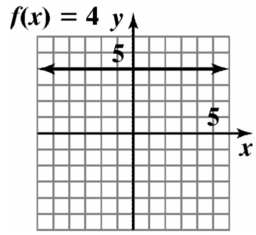
domain: $(-\infty,\infty)$

range: $(-\infty,\infty)$

5. 

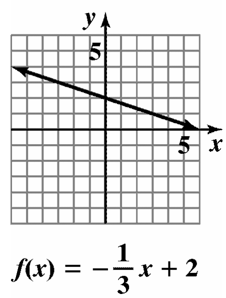
domain: $[-2,2]$

range: $[-2,2]$

6. 

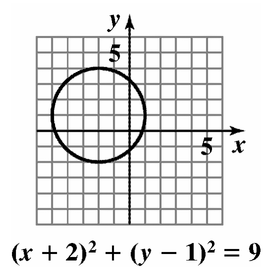
domain: $(-\infty,\infty)$

range: $\{4\}$

7. 

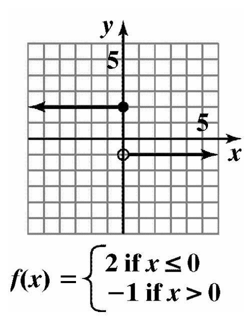
domain: $(-\infty,\infty)$

range: $(-\infty,\infty)$

8. 

domain: $[-5,1]$

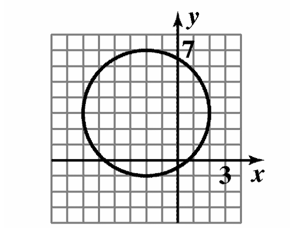
range: $[-2,4]$

9. 

domain: $(-\infty,\infty)$

range: $\{-1,2\}$

10.

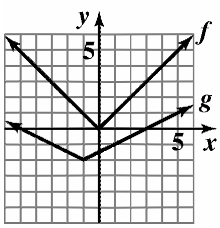


$x^2 + y^2 + 4x + 6y - 3 = 0$

domain: $[-6, 2]$

range: $[-1, 7]$

11.



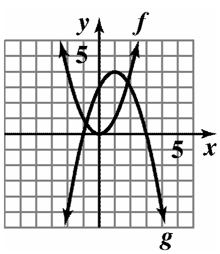
domain of *f*: $(-\infty, \infty)$

range of *f*: $[0, \infty)$

domain of *g*: $(-\infty, \infty)$

range of *g*: $[-2, \infty)$

12.



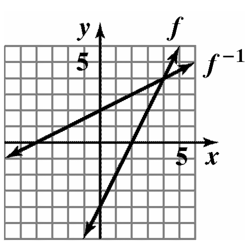
domain of *f*: $(-\infty, \infty)$

range of *f*: $[0, \infty)$

domain of *g*: $(-\infty, \infty)$

range of *g*: $(-\infty, 4]$

13.



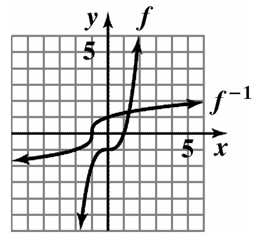
domain of *f*: $(-\infty, \infty)$

range of *f*: $(-\infty, \infty)$

domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

14.



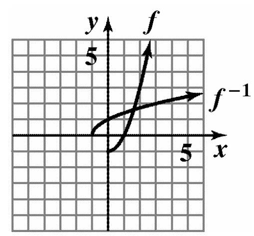
domain of *f*: $(-\infty, \infty)$

range of *f*: $(-\infty, \infty)$

domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

15.



domain of *f*: $[0, \infty)$

range of *f*: $[-1, \infty)$

domain of f^{-1}: $[-1, \infty)$

range of f^{-1}: $[0, \infty)$

16.
$$\begin{aligned} f(x) &= x^2 - x - 4 \\ f(x-1) &= (x-1)^2 - (x-1) - 4 \\ &= x^2 - 2x + 1 - x + 1 - 4 \\ &= x^2 - 3x - 2 \end{aligned}$$

17.
$$\begin{aligned} &\frac{f(x+h) - f(x)}{h} \\ &= \frac{(x+h)^2 - (x+h) - 4 - \left(x^2 - x - 4\right)}{h} \\ &= \frac{x^2 + 2xh + h^2 - x - h - 4 - x^2 + x + 4}{h} \\ &= \frac{2xh + h^2 - h}{h} \\ &= \frac{h\left(2x + h - 1\right)}{h} \\ &= 2x + h - 1 \end{aligned}$$

18.
$$\begin{aligned} (g - f)(x) &= 2x - 6 - \left(x^2 - x - 4\right) \\ &= 2x - 6 - x^2 + x + 4 \\ &= -x^2 + 3x - 2 \end{aligned}$$

19. $\left(\frac{f}{g}\right)(x) = \frac{x^2 - x - 4}{2x - 6}$

domain: $(-\infty, 3) \cup (3, \infty)$

20. $(f \circ g)(x) = f(g(x))$
$= (2x-6)^2 - (2x-6) - 4$
$= 4x^2 - 24x + 36 - 2x + 6 - 4$
$= 4x^2 - 26x + 38$

21. $(g \circ f)(x) = g(f(x))$
$= 2(x^2 - x - 4) - 6$
$= 2x^2 - 2x - 8 - 6$
$= 2x^2 - 2x - 14$

22. $g(f(-1)) = 2((-1)^2 - (-1) - 4) - 6$
$= 2(1 + 1 - 4) - 6$
$= 2(-2) - 6$
$= -4 - 6$
$= -10$

23. $f(x) = x^2 - x - 4$
$f(-x) = (-x)^2 - (-x) - 4$
$= x^2 + x - 4$
f is neither even nor odd.

24. $m = \frac{-8-1}{-1-2} = \frac{-9}{-3} = 3$

point-slope form: $y - 1 = 3(x - 2)$
or $y + 8 = 3(x + 1)$
slope-intercept form: $y = 3x - 5$

25. $y = -\frac{1}{4}x + 5$ so $m = 4$

point-slope form: $y - 6 = 4(x + 4)$
slope-intercept form: $y = 4x + 22$

26. Write $4x + 2y - 5 = 0$ in slope intercept form.

$4x + 2y - 5 = 0$
$2y = -4x + 5$
$y = -2x + \frac{5}{2}$

The slope of the parallel line is –2, thus the slope of the desired line is $m = -2$.

$y - y_1 = m(x - x_1)$
$y - (-10) = -2(x - (-7))$
$y + 10 = -2(x + 7)$
$y + 10 = -2x - 14$
$2x + y + 24 = 0$

27. **a.** First, find the slope using the points $(2, 476)$ and $(4, 486)$.

$m = \frac{486 - 476}{4 - 2} = \frac{10}{2} = 5$

Then use the slope and a point to write the equation in point-slope form.

$y - y_1 = m(x - x_1)$
$y - 486 = 5(x - 4)$
or
$y - 476 = 5(x - 2)$

b. $y - 486 = 5(x - 4)$
$y - 486 = 5x - 20$
$y = 5x + 466$
$f(x) = 5x + 466$

c. $f(10) = 5(10) + 466 = 516$

The function predicts that in 2010 the number of sentenced inmates in the U.S. will be 516 per 100,000 residents.

28. $\frac{3(10)^2 - 5 - [3(6)^2 - 5]}{10 - 6}$
$= \frac{205 - 103}{4}$
$= \frac{192}{4}$
$= 48$

29. $g(-1) = 3 - (-1) = 4$
$g(7) = \sqrt{7-3} = \sqrt{4} = 2$

30. The denominator is zero when $x = 1$ or $x = -5$.
domain: $(-\infty,-5)\cup(-5,1)\cup(1,\infty)$

31. The expressions under each radical must not be negative.
$x+5\ge 0$ and $x-1\ge 0$
$x\ge -5$ $\quad$ $x\ge 1$
domain: $[1,\infty)$

32. $(f\circ g)(x)=\dfrac{7}{\frac{2}{x}-4}=\dfrac{7x}{2-4x}$

$x\ne 0,\quad 2-4x\ne 0$
$x\ne \dfrac{1}{2}$
domain: $(-\infty,0)\cup\left(0,\dfrac{1}{2}\right)\cup\left(\dfrac{1}{2},\infty\right)$

33. $f(x)=x^7 \qquad g(x)=2x+3$

34. $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$\begin{aligned}d&=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\\&=\sqrt{(5-2)^2+(2-(-2))^2}\\&=\sqrt{3^2+4^2}\\&=\sqrt{9+16}\\&=\sqrt{25}\\&=5\end{aligned}$$

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{2+5}{2},\frac{-2+2}{2}\right)=\left(\frac{7}{2},0\right)$$

The length is 5 and the midpoint is $\left(\dfrac{7}{2},0\right)$.

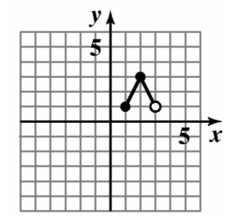
Cumulative Review Exercises (Chapters 1–2)

1. domain: $[0,2)$
range: $[0,2]$

2. $f(x)=1$ at $\frac{1}{2}$ and $\frac{3}{2}$.

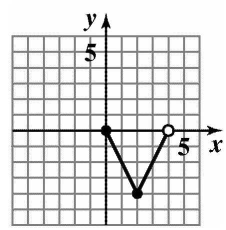
3. relative maximum: 2

4.



$g(x) = f(x - 1) + 1$

5.



6.
$$\begin{aligned}(x+3)(x-4)&=8\\x^2-x-12&=8\\x^2-x-20&=0\\(x+4)(x-5)&=0\end{aligned}$$
$x+4=0$ or $x-5=0$
$x=-4$ or $x=5$

7.
$$\begin{aligned}3(4x-1)&=4-6(x-3)\\12x-3&=4-6x+18\\18x&=25\\x&=\frac{25}{18}\end{aligned}$$

8.
$$\begin{aligned}\sqrt{x}+2&=x\\\sqrt{x}&=x-2\\(\sqrt{x})^2&=(x-2)^2\\x&=x^2-4x+4\\0&=x^2-5x+4\\0&=(x-1)(x-4)\end{aligned}$$
$x-1=0$ or $x-4=0$
$x=1$ or $x=4$

A check of the solutions shows that $x = 1$ is an extraneous solution. The only solution is $x = 4$.

9. $x^{2/3} - x^{1/3} - 6 = 0$

Let $u = x^{1/3}$. Then $u^2 = x^{2/3}$.

$u^2 - u - 6 = 0$

$(u+2)(u-3) = 0$

$u = -2$ or $u = 3$

$x^{1/3} = -2$ or $x^{1/3} = 3$

$x = (-2)^3$ or $x = 3^3$

$x = -8$ or $x = 27$

10. $\frac{x}{2} - 3 \le \frac{x}{4} + 2$

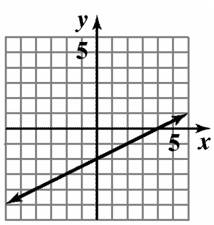
$4\left(\frac{x}{2} - 3\right) \le 4\left(\frac{x}{4} + 2\right)$

$2x - 12 \le x + 8$

$x \le 20$

The solution set is $(-\infty, 20]$.

11.

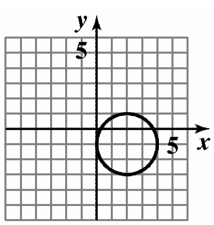


$3(x) - 6y - 12 = 0$

domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

12.

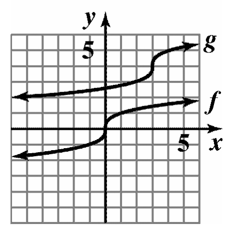


$(x-2)^2 + (y+1)^2 = 4$

domain: $[0, 4]$

range: $[-3, 1]$

13.



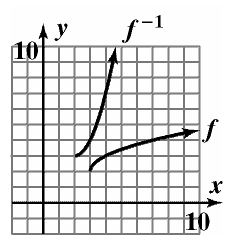
domain of *f*: $(-\infty, \infty)$

range of *f*: $(-\infty, \infty)$

domain of *g*: $(-\infty, \infty)$

range of *g*: $(-\infty, \infty)$

14.



domain of *f*: $[3, \infty)$

range of *f*: $[2, \infty)$

domain of f^{-1}: $[2, \infty)$

range of f^{-1}: $[3, \infty)$

15. $\frac{f(x+h) - f(x)}{h}$

$= \frac{\left(4 - (x+h)^2\right) - \left(4 - x^2\right)}{h}$

$= \frac{4 - (x^2 + 2xh + h^2) - \left(4 - x^2\right)}{h}$

$= \frac{4 - x^2 - 2xh - h^2 - 4 + x^2}{h}$

$= \frac{-2xh - h^2}{h}$

$= \frac{h(-2x - h)}{h}$

$= -2x - h$

16. $(f \circ g)(x) = f\left(g(x)\right)$

$(f \circ g)(x) = f\left(x+5\right)$

$$0 = 4 - \left(x+5\right)^2$$
$$0 = 4 - (x^2 + 10x + 25)$$
$$0 = 4 - x^2 - 10x - 25$$
$$0 = -x^2 - 10x - 21$$
$$0 = x^2 + 10x + 21$$
$$0 = (x+7)(x+3)$$

The value of $(f \circ g)(x)$ will be 0 when $x = -3$ or $x = -7$.

17. $y = -\frac{1}{4}x + \frac{1}{3}$, so $m = 4$.

point-slope form: $y - 5 = 4(x + 2)$
slope-intercept form: $y = 4x + 13$
general form: $4x - y + 13 = 0$

18. $0.07x + 0.09(6000 - x) = 510$

$$0.07x + 540 - 0.09x = 510$$
$$-0.02x = -30$$
$$x = 1500$$
$$6000 - x = 4500$$

\$1500 was invested at 7% and \$4500 was invested at 9%.

19. $200 + 0.05x = .15x$

$$200 = 0.10x$$
$$2000 = x$$

For \$2000 in sales, the earnings will be the same.

20. width = w
length = $2w + 2$
$2(2w + 2) + 2w = 22$
$4w + 4 + 2w = 22$
$6w = 18$
$w = 3$
$2w + 2 = 8$
The garden is 3 feet by 8 feet.

Chapter 3
Polynomial and Rational Functions

Section 3.1

Check Point Exercises

1. $f(x) = -(x-1)^2 + 4$

$f(x) = \overbrace{-}^{a=-1}\left(x - \overbrace{1}^{h=1}\right)^2 + \overbrace{4}^{k=4}$

Step 1: The parabola opens down because $a < 0$.
Step 2: find the vertex: (1, 4)
Step 3: find the x-intercepts:

$0 = -(x-1)^2 + 4$

$(x-1)^2 = 4$

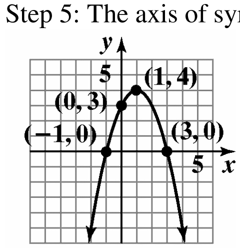
$x - 1 = \pm 2$

$x = 1 \pm 2$

$x = 3$ or $x = -1$
Step 4: find the y-intercept:

$f(0) = -(0-1)^2 + 4 = 3$

Step 5: The axis of symmetry is $x = 1$.



$f(x) = -(x-1)^2 + 4$

2. $f(x) = (x-2)^2 + 1$

Step 1: The parabola opens up because $a > 0$.
Step 2: find the vertex: (2, 1)
Step 3: find the x-intercepts:

$0 = (x-2)^2 + 1$

$(x-2)^2 = -1$

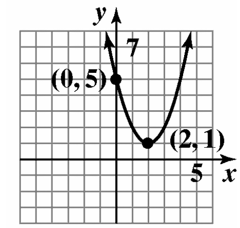
$x - 2 = \sqrt{-1}$

$x = 2 \pm i$

The equation has no real roots, thus the parabola has no x-intercepts.
Step 4: find the y-intercept:

$f(0) = (0-2)^2 + 1 = 5$

Step 5: The axis of symmetry is $x = 2$.



$f(x) = (x-2)^2 + 1$

3. $f(x) = -x^2 + 4x + 1$

Step 1: The parabola opens down because $a < 0$.
Step 2: find the vertex:

$x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$

$f(2) = -2^2 + 4(2) + 1 = 5$

The vertex is (2, 5).
Step 3: find the x-intercepts:

$0 = -x^2 + 4x + 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(1)}}{2(-1)}$

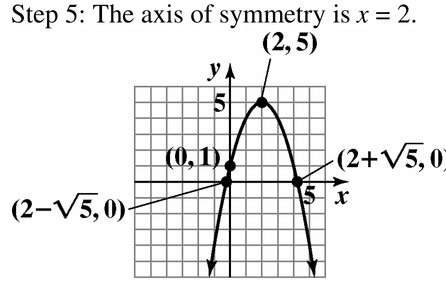
$x = \frac{-4 \pm \sqrt{20}}{-2}$

$x = 2 \pm \sqrt{5}$

The x-intercepts are $x \approx -0.2$ and $x \approx -4.2$.
Step 4: find the y-intercept:

$f(0) = -0^2 + 4(0) + 1 = 1$

Step 5: The axis of symmetry is $x = 2$.



$f(x) = -x^2 + 4x + 1$

4. $f(x)=4x^2-16x+1000$

a. a = 4. The parabola opens upward and has a minimum value.

b. $x=\frac{-b}{2a}=\frac{16}{8}=2$

$f(2)=4(2)^2-16(2)+1000=984$

The minimum point is 984 at $x=2$.

c. domain: $(-\infty,\infty)$ range: $[984,\infty)$

5. $y=-0.005x^2+2x+5$

a. The information needed is found at the vertex.
x-coordinate of vertex

$x=\frac{-b}{2a}=\frac{-2}{2(-0.005)}=200$

y-coordinate of vertex

$y=-0.005(200)^2+2(200)+5=205$

The vertex is (200,205).
The maximum height of the arrow is 205 feet.
This occurs 200 feet from its release.

b. The arrow will hit the ground when the height reaches 0.

$y=-0.005x^2+2x+5$

$0=-0.005x^2+2x+5$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-2\pm\sqrt{2^2-4(-0.005)(5)}}{2(-0.005)}$

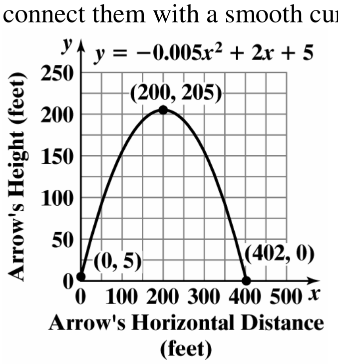
$x\approx-2$ or $x\approx 402$

The arrow travels 402 feet before hitting the ground.

c. The starting point occurs when $x=0$. Find the corresponding y-coordinate.

$y=-0.005(0)^2+2(0)+5=5$

Plot $(0,5)$, $(402,0)$, and $(200,205)$, and connect them with a smooth curve.



6. Let x = one of the numbers;
$x-8$ = the other number.

The product is $f(x)=x(x-8)=x^2-8x$

The x-coordinate of the minimum is

$x=-\frac{b}{2a}=-\frac{-8}{2(1)}=-\frac{-8}{2}=4.$

$f(4)=(4)^2-8(4)$
$=16-32=-16$

The vertex is $(4,-16)$.

The minimum product is -16. This occurs when the two number are 4 and $4-8=-4$.

7. Maximize the area of a rectangle constructed with 120 feet of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$2x+2y=120$

$2y=120-2x$

$y=\frac{120-2x}{2}=60-x$

We need to maximize $A=xy=x(60-x)$. Rewrite A as a function of x.

$A(x)=x(60-x)=-x^2+60x$

Since $a=-1$ is negative, we know the function opens downward and has a maximum at

$x=-\frac{b}{2a}=-\frac{60}{2(-1)}=-\frac{60}{-2}=30.$

When the length x is 30, the width y is
$y=60-x=60-30=30.$

The dimensions of the rectangular region with maximum area are 30 feet by 30 feet. This gives an area of $30\cdot30=900$ square feet.

Exercise Set 3.1

1. vertex: (1, 1)

$h(x)=(x-1)^2+1$

2. vertex: (–1, 1)

$g(x)=(x+1)^2+1$

3. vertex: (1, –1)

$j(x)=(x-1)^2-1$

4. vertex: (–1, –1)
$f(x) = (x+1)^2 - 1$

5. The graph is $f(x) = x^2$ translated down one.
$h(x) = x^2 - 1$

6. The point (–1, 0) is on the graph and $f(-1) = 0$. $f(x) = x^2 + 2x + 1$

7. The point (1, 0) is on the graph and $g(1) = 0$. $g(x) = x^2 - 2x + 1$

8. The graph is $f(x) = -x^2$ translated down one.
$j(x) = -x^2 - 1$

9. $f(x) = 2(x - 3)^2 + 1$
$h = 3, k = 1$
The vertex is at (3, 1).

10. $f(x) = -3(x - 2)^2 + 12$
$h = 2, k = 12$
The vertex is at (2, 12).

11. $f(x) = -2(x + 1)^2 + 5$
$h = -1, k = 5$
The vertex is at (–1, 5).

12. $f(x) = -2(x + 4)^2 - 8$
$h = -4, k = -8$
The vertex is at (–4, –8).

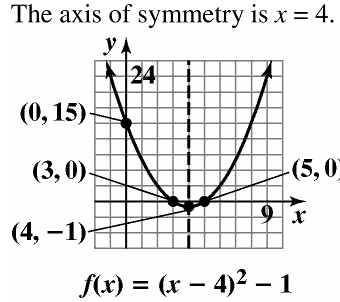
13. $f(x) = 2x^2 - 8x + 3$
$$x = \frac{-b}{2a} = \frac{8}{4} = 2$$
$f(2) = 2(2)^2 - 8(2) + 3$
$= 8 - 16 + 3 = -5$
The vertex is at (2, –5).

14. $f(x) = 3x^2 - 12x + 1$
$$x = \frac{-b}{2a} = \frac{12}{6} = 2$$
$f(2) = 3(2)^2 - 12(2) + 1$
$= 12 - 24 + 1 = -11$
The vertex is at (2, –11).

15. $f(x) = -x^2 - 2x + 8$
$$x = \frac{-b}{2a} = \frac{2}{-2} = -1$$
$f(-1) = -(-1)^2 - 2(-1) + 8$
$= -1 + 2 + 8 = 9$
The vertex is at (–1, 9).

16. $f(x) = -2x^2 + 8x - 1$
$$x = \frac{-b}{2a} = \frac{-8}{-4} = 2$$
$f(2) = -2(2)^2 + 8(2) - 1$
$= -8 + 16 - 1 = 7$
The vertex is at (2, 7).

17. $f(x) = (x-4)^2 - 1$
vertex: (4, –1)
x-intercepts:
$0 = (x-4)^2 - 1$
$1 = (x-4)^2$
$\pm 1 = x - 4$
$x = 3$ or $x = 5$
y-intercept:
$f(0) = (0-4)^2 - 1 = 15$
The axis of symmetry is $x = 4$.



domain: $(-\infty, \infty)$
range: $[-1, \infty)$

18. $f(x)=(x-1)^2-2$

vertex: (1, –2)
x-intercepts:

$0=(x-1)^2-2$

$(x-1)^2=2$

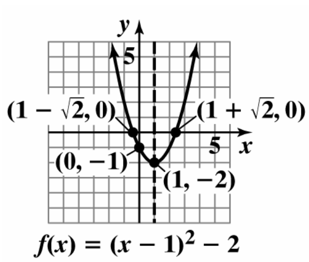
$x-1=\pm\sqrt{2}$

$x=1\pm\sqrt{2}$

y-intercept:

$f(0)=(0-1)^2-2=-1$

The axis of symmetry is $x = 1$.



domain: $(-\infty,\infty)$

range: $[-2,\infty)$

19. $f(x)=(x-1)^2+2$

vertex: (1, 2)
x-intercepts:

$0=(x-1)^2+2$

$(x-1)^2=-2$

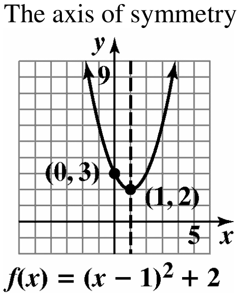
$x-1=\pm\sqrt{-2}$

$x=1\pm i\sqrt{2}$

No x-intercepts.
y-intercept:

$f(0)=(0-1)^2+2=3$

The axis of symmetry is $x = 1$.



domain: $(-\infty,\infty)$

range: $[2,\infty)$

20. $f(x)=(x-3)^2+2$

vertex: (3, 2)
x-intercepts:

$0=(x-3)^2+2$

$(x-3)^2=-2$

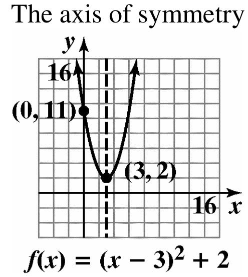
$x-3=\pm i\sqrt{2}$

$x=3\pm i\sqrt{2}$

No x-intercepts.
y-intercept:

$f(0)=(0-3)^2+2=11$

The axis of symmetry is $x = 3$.



domain: $(-\infty,\infty)$

range: $[2,\infty)$

21. $y-1=(x-3)^2$

$y=(x-3)^2+1$

vertex: (3, 1)
x-intercepts:

$0=(x-3)^2+1$

$(x-3)^2=-1$

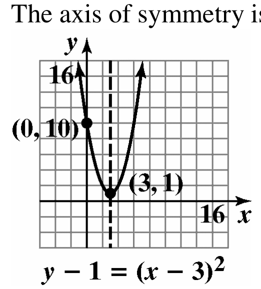
$x-3=\pm i$

$x=3\pm i$

No x-intercepts.
y-intercept: 10

$y=(0-3)^2+1=10$

The axis of symmetry is $x = 3$.



domain: $(-\infty,\infty)$

range: $[1,\infty)$

22. $y-3=(x-1)^2$

$y=(x-1)^2+3$

vertex: (1, 3)

x-intercepts:

$0=(x-1)^2+3$

$(x-1)^2=-3$

$x-1=\pm i\sqrt{3}$

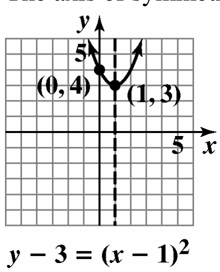
$x=1\pm i\sqrt{3}$

No x-intercepts

y-intercept:

$y=(0-1)^2+3=4$

The axis of symmetry is $x = 1$.



domain: $(-\infty,\infty)$

range: $[3,\infty)$

23. $f(x)=2(x+2)^2-1$

vertex: (–2, –1)

x-intercepts:

$0=2(x+2)^2-1$

$2(x+2)^2=1$

$(x+2)^2=\frac{1}{2}$

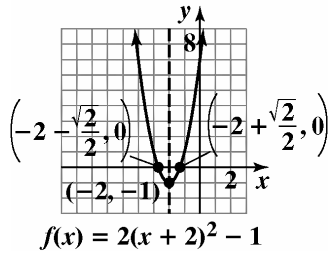
$x+2=\pm\frac{1}{\sqrt{2}}$

$x=-2\pm\frac{1}{\sqrt{2}}=-2\pm\frac{\sqrt{2}}{2}$

y-intercept:

$f(0)=2(0+2)^2-1=7$

The axis of symmetry is $x = -2$.



domain: $(-\infty,\infty)$

range: $[-1,\infty)$

24. $f(x)=\frac{5}{4}-\left(x-\frac{1}{2}\right)^2$

$f(x)=-\left(x-\frac{1}{2}\right)^2+\frac{5}{4}$

vertex: $\left(\frac{1}{2},\frac{5}{4}\right)$

x-intercepts:

$0=-\left(x-\frac{1}{2}\right)^2+\frac{5}{4}$

$\left(x-\frac{1}{2}\right)^2=\frac{5}{4}$

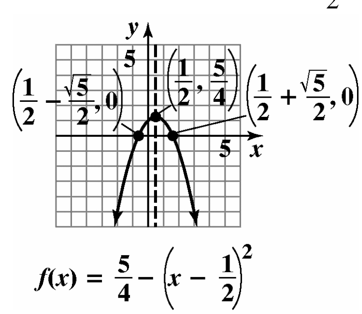
$x-\frac{1}{2}=\pm\frac{\sqrt{5}}{2}$

$x=\frac{1\pm\sqrt{5}}{2}$

y-intercept:

$f(0)=-\left(0-\frac{1}{2}\right)^2+\frac{5}{4}=1$

The axis of symmetry is $x=\frac{1}{2}$.



domain: $(-\infty,\infty)$

range: $\left(-\infty,\frac{5}{4}\right]$

25. $f(x) = 4 - (x-1)^2$

$f(x) = -(x-1)^2 + 4$

vertex: (1, 4)

x-intercepts:

$0 = -(x-1)^2 + 4$

$(x-1)^2 = 4$

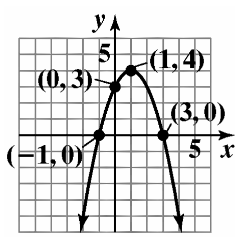
$x - 1 = \pm 2$

$x = -1$ or $x = 3$

y-intercept:

$f(x) = -(0-1)^2 + 4 = 3$

The axis of symmetry is $x = 1$.



$f(x) = 4 - (x-1)^2$

domain: $(-\infty, \infty)$

range: $(-\infty, 4]$

26. $f(x) = 1 - (x-3)^2$

$f(x) = -(x-3)^2 + 1$

vertex: (3, 1)

x-intercepts:

$0 = -(x-3)^2 + 1$

$(x-3)^2 = 1$

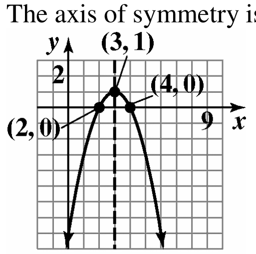
$x - 3 = \pm 1$

$x = 2$ or $x = 4$

y-intercept:

$f(0) = -(0-3)^2 + 1 = -8$

The axis of symmetry is $x = 3$.



$f(x) = 1 - (x-3)^2$

domain: $(-\infty, \infty)$

range: $(-\infty, 1]$

27. $f(x) = x^2 - 2x - 3$

$f(x) = (x^2 - 2x + 1) - 3 - 1$

$f(x) = (x-1)^2 - 4$

vertex: (1, –4)

x-intercepts:

$0 = (x-1)^2 - 4$

$(x-1)^2 = 4$

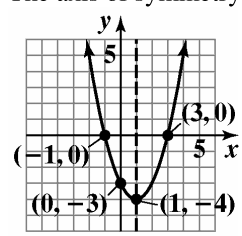
$x - 1 = \pm 2$

$x = -1$ or $x = 3$

y-intercept: –3

$f(0) = 0^2 - 2(0) - 3 = -3$

The axis of symmetry is $x = 1$.



$f(x) = x^2 + 3x - 10$

domain: $(-\infty, \infty)$

range: $[-4, \infty)$

28. $f(x) = x^2 - 2x - 15$

$f(x) = (x^2 - 2x + 1) - 15 - 1$

$f(x) = (x-1)^2 - 16$

vertex: (1, –16)

x-intercepts:

$0 = (x-1)^2 - 16$

$(x-1)^2 = 16$

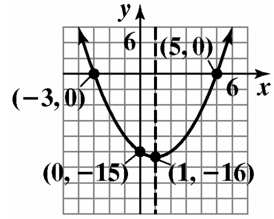
$x - 1 = \pm 4$

$x = -3$ or $x = 5$

y-intercept:

$f(0) = 0^2 - 2(0) - 15 = -15$

The axis of symmetry is $x = 1$.



$f(x) = x^2 - 2x - 15$

domain: $(-\infty, \infty)$

range: $[-16, \infty)$

29. $f(x) = x^2 + 3x - 10$

$f(x) = \left(x^2 + 3x + \frac{9}{4}\right) - 10 - \frac{9}{4}$

$f(x) = \left(x + \frac{3}{2}\right)^2 - \frac{49}{4}$

vertex: $\left(-\frac{3}{2}, -\frac{49}{4}\right)$

x-intercepts:

$0 = \left(x + \frac{3}{2}\right)^2 - \frac{49}{4}$

$\left(x + \frac{3}{2}\right)^2 = \frac{49}{4}$

$x + \frac{3}{2} = \pm\frac{7}{2}$

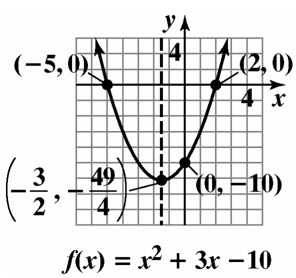
$x = -\frac{3}{2} \pm \frac{7}{2}$

$x = 2$ or $x = -5$

y-intercept:

$f(x) = 0^2 + 3(0) - 10 = -10$

The axis of symmetry is $x = -\frac{3}{2}$.



domain: $(-\infty, \infty)$

range: $\left[-\frac{49}{4}, \infty\right)$

30. $f(x) = 2x^2 - 7x - 4$

$f(x) = 2\left(x^2 - \frac{7}{2}x + \frac{49}{16}\right) - 4 - \frac{49}{8}$

$f(x) = 2\left(x - \frac{7}{4}\right)^2 - \frac{81}{8}$

vertex: $\left(\frac{7}{4}, -\frac{81}{8}\right)$

x-intercepts:

$0 = 2\left(x - \frac{7}{4}\right)^2 - \frac{81}{8}$

$2\left(x - \frac{7}{4}\right)^2 = \frac{81}{8}$

$\left(x - \frac{7}{4}\right)^2 = \frac{81}{16}$

$x - \frac{7}{4} = \pm\frac{9}{4}$

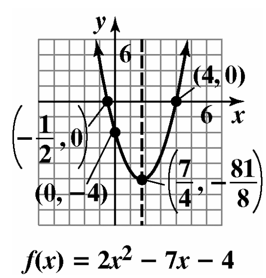
$x = \frac{7}{4} \pm \frac{9}{4}$

$x = -\frac{1}{2}$ or $x = 4$

y-intercept:

$f(0) = 2(0)^2 - 7(0) - 4 = -4$

The axis of symmetry is $x = \frac{7}{4}$.



domain: $(-\infty, \infty)$

range: $\left[-\frac{81}{8}, \infty\right)$

31. $f(x) = 2x - x^2 + 3$

$f(x) = -x^2 + 2x + 3$

$f(x) = -\left(x^2 - 2x + 1\right) + 3 + 1$

$f(x) = -\left(x-1\right)^2 + 4$

vertex: (1, 4)
x-intercepts:

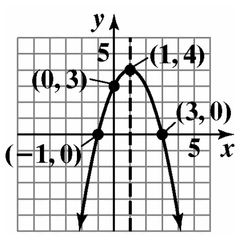
$0 = -\left(x-1\right)^2 + 4$

$\left(x-1\right)^2 = 4$

$x - 1 = \pm 2$
$x = -1$ or $x = 3$
y-intercept:

$f\left(0\right) = 2\left(0\right) - \left(0\right)^2 + 3 = 3$

The axis of symmetry is $x = 1$.



$f(x) = 2x - x^2 + 3$

domain: $\left(-\infty, \infty\right)$

range: $\left(-\infty, 4\right]$

32. $f(x) = 5 - 4x - x^2$

$f(x) = -x^2 - 4x + 5$

$f(x) = -\left(x^2 + 4x + 4\right) + 5 + 4$

$f(x) = -\left(x+2\right)^2 + 9$

vertex: (–2, 9)
x-intercepts:

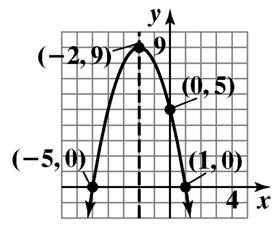
$0 = -\left(x+2\right)^2 + 9$

$\left(x+2\right)^2 = 9$

$x + 2 = \pm 3$
$x = -5, 1$
y-intercept:

$f(0) = 5 - 4\left(0\right) - \left(0\right)^2 = 5$

The axis of symmetry is $x = -2$.



$f(x) = 5 - 4x - x^2$

domain: $\left(-\infty, \infty\right)$

range: $\left(-\infty, 9\right]$

33. $f(x) = x^2 + 6x + 3$

$f(x) = (x^2 + 6x + 9) + 3 - 9$

$f(x) = \left(x+3\right)^2 - 6$

vertex: $\left(-3, -6\right)$
x-intercepts:

$0 = \left(x+3\right)^2 - 6$

$\left(x+3\right)^2 = 6$

$x + 3 = \pm\sqrt{6}$

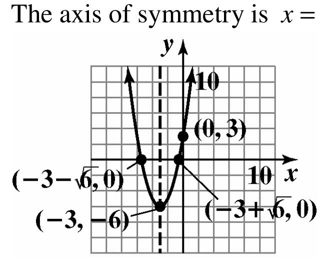
$x = -3 \pm \sqrt{6}$

y-intercept:

$f(0) = (0)^2 + 6(0) + 3$

$f(0) = 3$

The axis of symmetry is $x = -3$.



$f(x) = x^2 + 6x + 3$

domain: $\left(-\infty, \infty\right)$

range: $\left[-6, \infty\right)$

34. $f(x) = x^2 + 4x - 1$

$f(x) = (x^2 + 4x + 4) - 1 - 4$

$f(x) = (x+2)^2 - 5$

vertex: $(-2, -5)$

x-intercepts:

$0 = (x+2)^2 - 5$

$(x+2)^2 = 5$

$x + 2 = \pm\sqrt{5}$

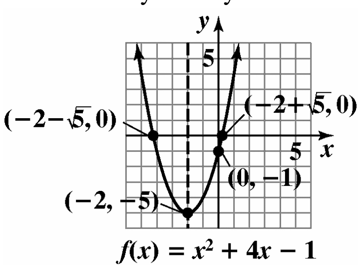
$x = -2 \pm \sqrt{5}$

y-intercept:

$f(0) = (0)^2 + 4(0) - 1$

$f(0) = -1$

The axis of symmetry is $x = -2$.



domain: $(-\infty, \infty)$

range: $[-5, \infty)$

35. $f(x) = 2x^2 + 4x - 3$

$f(x) = 2(x^2 + 2x \quad) - 3$

$f(x) = 2(x^2 + 2x + 1) - 3 - 2$

$f(x) = 2(x+1)^2 - 5$

vertex: $(-1, -5)$

x-intercepts:

$0 = 2(x+1)^2 - 5$

$2(x+1)^2 = 5$

$(x+1)^2 = \frac{5}{2}$

$x + 1 = \pm\sqrt{\frac{5}{2}}$

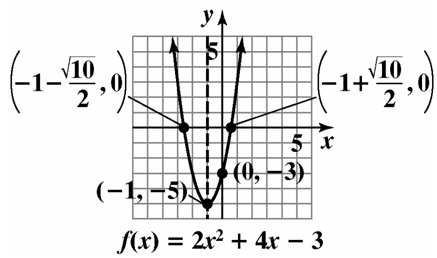
$x = -1 \pm \frac{\sqrt{10}}{2}$

y-intercept:

$f(0) = 2(0)^2 + 4(0) - 3$

$f(0) = -3$

The axis of symmetry is $x = -1$.



domain: $(-\infty, \infty)$

range: $[-5, \infty)$

36. $f(x) = 3x^2 - 2x - 4$

$f(x) = 3\left(x^2 - \frac{2}{3}x \quad\right) - 4$

$f(x) = 3\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) - 4 - \frac{1}{3}$

$f(x) = 3\left(x - \frac{1}{3}\right)^2 - \frac{13}{3}$

vertex: $\left(\frac{1}{3}, -\frac{13}{3}\right)$

x-intercepts:

$0 = 3\left(x - \frac{1}{3}\right)^2 - \frac{13}{3}$

$3\left(x - \frac{1}{3}\right)^2 = \frac{13}{3}$

$\left(x - \frac{1}{3}\right)^2 = \frac{13}{9}$

$x - \frac{1}{3} = \pm\sqrt{\frac{13}{9}}$

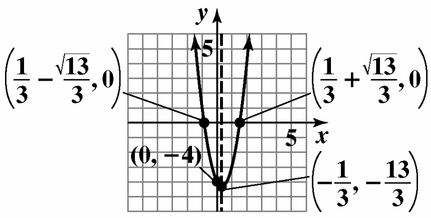
$x = \frac{1}{3} \pm \frac{\sqrt{13}}{3}$

y-intercept:

$f(0) = 3(0)^2 - 2(0) - 4$

$f(0) = -4$

The axis of symmetry is $x = \frac{1}{3}$.



$f(x) = 3x^2 - 2x - 4$

domain: $(-\infty, \infty)$

range: $\left[-\frac{13}{3}, \infty\right)$

37. $f(x) = 2x - x^2 - 2$

$f(x) = -x^2 + 2x - 2$

$f(x) = -\left(x^2 - 2x + 1\right) - 2 + 1$

$f(x) = -\left(x - 1\right)^2 - 1$

vertex: (1, –1)

x-intercepts:

$0 = -\left(x - 1\right)^2 - 1$

$\left(x - 1\right)^2 = -1$

$x - 1 = \pm i$

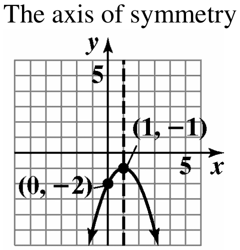
$x = 1 \pm i$

No x-intercepts.

y-intercept:

$f(0) = 2\left(0\right) - \left(0\right)^2 - 2 = -2$

The axis of symmetry is $x = 1$.



$f(x) = 2x - x^2 - 2$

domain: $(-\infty, \infty)$

range: $(-\infty, -1]$

38. $f(x) = 6 - 4x + x^2$

$f(x) = x^2 - 4x + 6$

$f(x) = \left(x^2 - 4x + 4\right) + 6 - 4$

$f(x) = \left(x - 2\right)^2 + 2$

vertex: (2, 2)

x-intercepts:

$0 = \left(x - 2\right)^2 + 2$

$\left(x - 2\right)^2 = -2$

$x - 2 = \pm i\sqrt{2}$

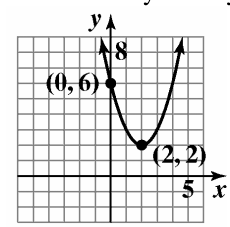
$x = 2 \pm i\sqrt{2}$

No x-intercepts

y-intercept:

$f(0) = 6 - 4\left(0\right) + \left(0\right)^2 = 6$

The axis of symmetry is $x = 2$.



$f(x) = 6 - 4x + x^2$

domain: $(-\infty, \infty)$

range: $[2, \infty)$

39. $f(x) = 3x^2 - 12x - 1$

a. $a = 3$. The parabola opens upward and has a minimum value.

b. $x = \frac{-b}{2a} = \frac{12}{6} = 2$

$f(2) = 3(2)^2 - 12(2) - 1$
$= 12 - 24 - 1 = -13$

The minimum is –13 at $x = 2$.

c. domain: $(-\infty, \infty)$ range: $[-13, \infty)$

40. $f(x) = 2x^2 - 8x - 3$

a. $a = 2$. The parabola opens upward and has a minimum value.

b. $x = \frac{-b}{2a} = \frac{8}{4} = 2$

$f(2) = 2(2)^2 - 8(2) - 3$
$= 8 - 16 - 3 = -11$

The minimum is –11 at $x = 2$.

c. domain: $(-\infty, \infty)$ range: $[-11, \infty)$

41. $f(x) = -4x^2 + 8x - 3$

a. a = -4. The parabola opens downward and has a maximum value.

b. $x = \frac{-b}{2a} = \frac{-8}{-8} = 1$

$f(1) = -4(1)^2 + 8(1) - 3$

$= -4 + 8 - 3 = 1$

The maximum is 1 at $x = 1$.

c. domain: $(-\infty, \infty)$ range: $(-\infty, 1]$

42. $f(x) = -2x^2 - 12x + 3$

a. $a = -2$. The parabola opens downward and has a maximum value.

b. $x = \frac{-b}{2a} = \frac{12}{-4} = -3$

$f(-3) = -2(-3)^2 - 12(-3) + 3$

$= -18 + 36 + 3 = 21$

The maximum is 21 at $x = -3$.

c. domain: $(-\infty, \infty)$ range: $(-\infty, 21]$

43. $f(x) = 5x^2 - 5x$

a. a = 5. The parabola opens upward and has a minimum value.

b. $x = \frac{-b}{2a} = \frac{5}{10} = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = 5\left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right)$

$= \frac{5}{4} - \frac{5}{2} = \frac{5}{4} - \frac{10}{4} = \frac{-5}{4}$

The minimum is $\frac{-5}{4}$ at $x = \frac{1}{2}$.

c. domain: $(-\infty, \infty)$ range: $\left[\frac{-5}{4}, \infty\right)$

44. $f(x) = 6x^2 - 6x$

a. a = 6. The parabola opens upward and has minimum value.

b. $x = \frac{-b}{2a} = \frac{6}{12} = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right)$

$= \frac{6}{4} - 3 = \frac{3}{2} - \frac{6}{2} = \frac{-3}{2}$

The minimum is $\frac{-3}{2}$ at $x = \frac{1}{2}$.

c. domain: $(-\infty, \infty)$ range: $\left[\frac{-3}{2}, \infty\right)$

45. Since the parabola opens up, the vertex $(-1, -2)$ is a minimum point.
domain: $(-\infty, \infty)$. range: $[-2, \infty)$

46. Since the parabola opens down, the vertex $(-3, -4)$ is a maximum point.
domain: $(-\infty, \infty)$. range: $(-\infty, -4]$

47. Since the parabola has a maximum, it opens down from the vertex $(10, -6)$.
domain: $(-\infty, \infty)$. range: $(-\infty, -6]$

48. Since the parabola has a minimum, it opens up from the vertex $(-6, 18)$.
domain: $(-\infty, \infty)$. range: $[18, \infty)$

49. $(h, k) = (5, 3)$

$f(x) = 2(x - h)^2 + k = 2(x - 5)^2 + 3$

50. $(h, k) = (7, 4)$

$f(x) = 2(x - h)^2 + k = 2(x - 7)^2 + 4$

51. $(h, k) = (-10, -5)$

$f(x) = 2(x - h)^2 + k$

$= 2[x - (-10)]^2 + (-5)$

$= 2(x + 10)^2 - 5$

52. $(h,k)=(-8,-6)$

$$f(x)=2(x-h)^2+k$$
$$=2[x-(-8)]^2+(-6)$$
$$=2(x+8)^2-6$$

53. Since the vertex is a maximum, the parabola opens down and $a=-3$.

$(h,k)=(-2,4)$

$$f(x)=-3(x-h)^2+k$$
$$=-3[x-(-2)]^2+4$$
$$=-3(x+2)^2+4$$

54. Since the vertex is a maximum, the parabola opens down and $a=-3$.

$(h,k)=(5,-7)$

$$f(x)=-3(x-h)^2+k$$
$$=-3(x-5)^2+(-7)$$
$$=-3(x-5)^2-7$$

55. Since the vertex is a minimum, the parabola opens up and $a=3$.

$(h,k)=(11,0)$

$$f(x)=3(x-h)^2+k$$
$$=3(x-11)^2+0$$
$$=3(x-11)^2$$

56. Since the vertex is a minimum, the parabola opens up and $a=3$.

$(h,k)=(9,0)$

$$f(x)=3(x-h)^2+k$$
$$=3(x-9)^2+0$$
$$=3(x-9)^2$$

57. **a.** $y=-0.01x^2+0.7x+6.1$

$a=-0.01,\ b=0.7,\ c=6.1$

x-coordinate of vertex

$$=\frac{-b}{2a}=\frac{-0.7}{2(-0.01)}=35$$

y-coordinate of vertex

$$y=-0.01x^2+0.7x+6.1$$
$$y=-0.01(35)^2+0.7(35)+6.1=18.35$$

The maximum height of the shot is about 18.35 feet. This occurs 35 feet from its point of release.

b. The ball will reach the maximum horizontal distance when its height returns to 0.

$$y=-0.01x^2+0.7x+6.1$$
$$0=-0.01x^2+0.7x+6.1$$
$$a=-0.01,\ b=0.7,\ c=6.1$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-0.7\pm\sqrt{0.7^2-4(-0.01)(6.1)}}{2(-0.01)}$$
$$x\approx 77.8 \text{ or } x\approx -7.8$$

The maximum horizontal distance is 77.8 feet.

c. The initial height can be found at $x=0$.

$$y=-0.01x^2+0.7x+6.1$$
$$y=-0.01(0)^2+0.7(0)+6.1=6.1$$

The shot was released at a height of 6.1 feet.

58. **a.** $y=-0.04x^2+2.1x+6.1$

$a=-0.04,\ b=2.1,\ c=6.1$

x-coordinate of vertex

$$=\frac{-b}{2a}=\frac{-2.1}{2(-0.04)}=26.25$$

y-coordinate of vertex

$$y=-0.04x^2+2.1x+6.1$$
$$y=-0.04(26.25)^2+2.1(26.25)+6.1\approx 33.7$$

The maximum height of the shot is about 33.7 feet. This occurs 26.25 feet from its point of release.

b. The ball will reach the maximum horizontal distance when its height returns to 0.

$$y=-0.04x^2+2.1x+6.1$$
$$0=-0.04x^2+2.1x+6.1$$
$$a=-0.04,\ b=2.1,\ c=6.1$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-2.1\pm\sqrt{2.1^2-4(-0.04)(6.1)}}{2(-0.04)}$$
$$x\approx 55.3 \text{ or } x\approx -2.8$$

The maximum horizontal distance is 55.3 feet.

c. The initial height can be found at $x = 0$.

$y = -0.04x^2 + 2.1x + 6.1$

$y = -0.04(0)^2 + 2.1(0) + 6.1 = 6.1$

The shot was released at a height of 6.1 feet.

59. $f(x) = 0.004x^2 - 0.094x + 2.6$

a. $f(25) = 0.004(25)^2 - 0.094(25) + 2.6$

$= 2.75$

According to the function, U.S. adult wine consumption in 2005 was 2.75 gallons per person. This underestimates the graph's value by 0.05 gallon.

b. $year = -\frac{b}{2a} = -\frac{-0.094}{2(0.004)} \approx 12$

Wine consumption was at a minimum about 12 years after 1980, or 1992.

$f(12) = 0.004(12)^2 - 0.094(12) + 2.6 \approx 2.048$

Wine consumption was about 2.048 gallons per U.S. adult in 1992.

This seems reasonable as compared to the values in the graph.

60. $f(x) = -0.03x^2 + 0.14x + 1.43$

a. $f(5) = -0.03(5)^2 + 0.14(5) + 1.43$

$= 1.38$

According to the function, 1.38 billion movie tickets were sold in 2005. This underestimates the graph's value by 0.03 billion.

b. $year = -\frac{b}{2a} = -\frac{0.14}{2(-0.03)} \approx 2$

Movie attendance was at a minimum about 2 years after 2000, or 2002.

$f(2) = -0.03(2)^2 + 0.14(2) + 1.43$

$= 1.59$

Movie attendance was about 1.59 billion in 2002.

This differs from the value in the graph by 0.04 billion.

61. Let x = one of the numbers;

$16 - x$ = the other number.

The product is $f(x) = x(16 - x)$

$= 16x - x^2 = -x^2 + 16x$

The x-coordinate of the maximum is

$x = -\frac{b}{2a} = -\frac{16}{2(-1)} = -\frac{16}{-2} = 8.$

$f(8) = -8^2 + 16(8) = -64 + 128 = 64$

The vertex is (8, 64). The maximum product is 64. This occurs when the two number are 8 and $16 - 8 = 8$.

62. Let x = one of the numbers

Let $20 - x$ = the other number

$P(x) = x(20 - x) = 20x - x^2 = -x^2 + 20x$

$x = -\frac{b}{2a} = -\frac{20}{2(-1)} = -\frac{20}{-2} = 10$

The other number is $20 - x = 20 - 10 = 10$.

The numbers which maximize the product are 10 and 10. The maximum product is $10 \cdot 10 = 100$.

63. Let x = one of the numbers;

$x - 16$ = the other number.

The product is $f(x) = x(x - 16) = x^2 - 16x$

The x-coordinate of the minimum is

$x = -\frac{b}{2a} = -\frac{-16}{2(1)} = -\frac{-16}{2} = 8.$

$f(8) = (8)^2 - 16(8)$

$= 64 - 128 = -64$

The vertex is $(8, -64)$. The minimum product is -64. This occurs when the two number are 8 and $8 - 16 = -8$.

64. Let x = the larger number. Then $x - 24$ is the smaller number. The product of these two numbers is given by

$P(x) = x(x - 24) = x^2 - 24x$

The product is minimized when

$x = -\frac{b}{2a} = -\frac{(-24)}{2(1)} = 12$

Since $12 - (-12) = 24$, the two numbers whose difference is 24 and whose product is minimized are 12 and -12.

The minimum product is

$P(12) = 12(12 - 24) = -144$.

65. Maximize the area of a rectangle constructed along a river with 600 feet of fencing.
Let x = the width of the rectangle;
$600-2x$ = the length of the rectangle
We need to maximize.

$$A(x)=x(600-2x)$$
$$=600x-2x^2=-2x^2+600x$$

Since $a=-2$ is negative, we know the function opens downward and has a maximum at

$$x=-\frac{b}{2a}=-\frac{600}{2(-2)}=-\frac{600}{-4}=150.$$

When the width is $x=150$ feet, the length is $600-2(150)=600-300=300$ feet.
The dimensions of the rectangular plot with maximum area are 150 feet by 300 feet. This gives an area of $150\cdot 300=45,000$ square feet.

66. From the diagram, we have that x is the width of the rectangular plot and $200-2x$ is the length. Thus, the area of the plot is given by

$$A=l\cdot w=(200-2x)(x)=-2x^2+200x$$

Since the graph of this equation is a parabola that opens down, the area is maximized at the vertex.

$$x=-\frac{b}{2a}=-\frac{200}{2(-2)}=50$$
$$A=-2(50)^2+200(50)=-5000+10,000$$
$$=5000$$

The maximum area is 5000 square feet when the length is 100 feet and the width is 50 feet.

67. Maximize the area of a rectangle constructed with 50 yards of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x+2y=50$$
$$2y=50-2x$$
$$y=\frac{50-2x}{2}=25-x$$

We need to maximize $A=xy=x(25-x)$. Rewrite A as a function of x.

$$A(x)=x(25-x)=-x^2+25x$$

Since $a=-1$ is negative, we know the function opens downward and has a maximum at

$$x=-\frac{b}{2a}=-\frac{25}{2(-1)}=-\frac{25}{-2}=12.5.$$

When the length x is 12.5, the width y is $y=25-x=25-12.5=12.5.$
The dimensions of the rectangular region with maximum area are 12.5 yards by 12.5 yards. This gives an area of $12.5\cdot 12.5=156.25$ square yards.

68. Let x = the length of the rectangle
Let y = the width of the rectangle

$$2x+2y=80$$
$$2y=80-2x$$
$$y=\frac{80-2x}{2}$$
$$y=40-x$$
$$A(x)=x(40-x)=-x^2+40x$$
$$x=-\frac{b}{2a}=-\frac{40}{2(-1)}=-\frac{40}{-2}=20.$$

When the length x is 20, the width y is $y=40-x=40-20=20.$
The dimensions of the rectangular region with maximum area are 20 yards by 20 yards. This gives an area of $20\cdot 20=400$ square yards.

69. Maximize the area of the playground with 600 feet of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x + 3y = 600$$
$$3y = 600 - 2x$$
$$y = \frac{600 - 2x}{3}$$
$$y = 200 - \frac{2}{3}x$$

We need to maximize $A = xy = x\left(200 - \frac{2}{3}x\right)$.

Rewrite A as a function of x.

$$A(x) = x\left(200 - \frac{2}{3}x\right) = -\frac{2}{3}x^2 + 200x$$

Since $a = -\frac{2}{3}$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{200}{2\left(-\frac{2}{3}\right)} = -\frac{200}{-\frac{4}{3}} = 150.$$

When the length x is 150, the width y is

$$y = 200 - \frac{2}{3}x = 200 - \frac{2}{3}(150) = 100.$$

The dimensions of the rectangular playground with maximum area are 150 feet by 100 feet. This gives an area of $150 \cdot 100 = 15{,}000$ square feet.

70. Maximize the area of the playground with 400 feet of fencing.
Let x = the length of the rectangle. Let y = the width of the rectangle.
Since we need an equation in one variable, use the perimeter to express y in terms of x.

$$2x + 3y = 400$$
$$3y = 400 - 2x$$
$$y = \frac{400 - 2x}{3}$$
$$y = \frac{400}{3} - \frac{2}{3}x$$

We need to maximize $A = xy = x\left(\frac{400}{3} - \frac{2}{3}x\right)$.

Rewrite A as a function of x.

$$A(x) = x\left(\frac{400}{3} - \frac{2}{3}x\right) = -\frac{2}{3}x^2 + \frac{400}{3}x$$

Since $a = -\frac{2}{3}$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{\frac{400}{3}}{2\left(-\frac{2}{3}\right)} = -\frac{\frac{400}{3}}{-\frac{4}{3}} = 100.$$

When the length x is 100, the width y is

$$y = \frac{400}{3} - \frac{2}{3}x = \frac{400}{3} - \frac{2}{3}(100) = \frac{200}{3} = 66\frac{2}{3}.$$

The dimensions of the rectangular playground with maximum area are 100 feet by $66\frac{2}{3}$ feet. This gives an area of $100 \cdot 66\frac{2}{3} = 6666\frac{2}{3}$ square feet.

71. Maximize the cross-sectional area of the gutter:

$$A(x) = x(20 - 2x)$$
$$= 20x - 2x^2 = -2x^2 + 20x.$$

Since $a = -2$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{20}{2(-2)} = -\frac{20}{-4} = 5.$$

When the height x is 5, the width is

$$20 - 2x = 20 - 2(5) = 20 - 10 = 10.$$
$$A(5) = -2(5)^2 + 20(5)$$
$$= -2(25) + 100 = -50 + 100 = 50$$

The maximum cross-sectional area is 50 square inches. This occurs when the gutter is 5 inches deep and 10 inches wide.

72.
$$A(x) = x(12 - 2x) = 12x - 2x^2$$
$$= -2x^2 + 12x$$
$$x = -\frac{b}{2a} = -\frac{12}{2(-2)} = -\frac{12}{-4} = 3$$

When the height x is 3, the width is

$$12 - 2x = 12 - 2(3) = 12 - 6 = 6.$$
$$A(3) = -2(3)^2 + 12(3) = -2(9) + 36$$
$$= -18 + 36 = 18$$

The maximum cross-sectional area is 18 square inches. This occurs when the gutter is 3 inches deep and 6 inches wide.

73. a. $C(x) = 0.55x + 525$

b. $P(x) = R - C$

$P(x) = -0.001x^2 + 3x - 0.55x - 525$

$P(x) = -0.001x^2 + 2.45x - 525$

c. $x = \dfrac{-2.45}{2(-0.001)} = 1225$

$P(x) = -0.001(1225)^2 + 2.45(1225) - 525$

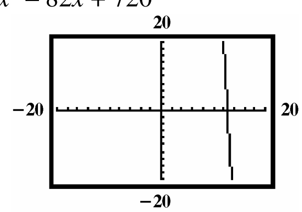
$= 975.63$

The maximum profit will be \$975.63 per week obtained by selling 1225 sandwiches.

74. – 80. Answers may vary.

81. $y = 2x^2 - 82x + 720$

a.



You can only see a little of the parabola.

b. $a = 2;\ b = -82$

$x = -\dfrac{b}{2a} = -\dfrac{-82}{4} = 20.5$

$y = 2(20.5)^2 - 82(20.5) + 720$

$= 840.5 - 1681 + 720$

$= -120.5$

vertex: (20.5, –120.5)

c. Ymax = 750

d. You can choose Xmin and Xmax so the x-value of the vertex is in the center of the graph. Choose Ymin to include the y-value of the vertex.

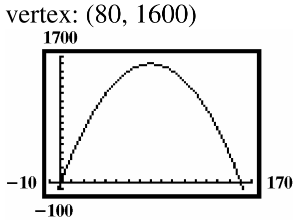
82. $y = -0.25x^2 + 40x$

$x = \dfrac{-b}{2a} = \dfrac{-40}{-0.5} = 80$

$y = -0.25(80)^2 + 40(80)$

$= 1600$

vertex: (80, 1600)



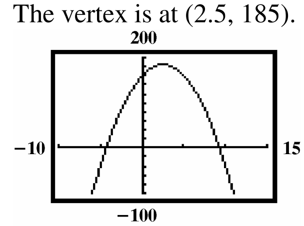
83. $y = -4x^2 + 20x + 160$

$x = \dfrac{-b}{2a} = \dfrac{-20}{-8} = 2.5$

$y = -4(2.5)^2 + 20(2.5) + 160$

$= -2.5 + 50 + 160 = 185$

The vertex is at (2.5, 185).



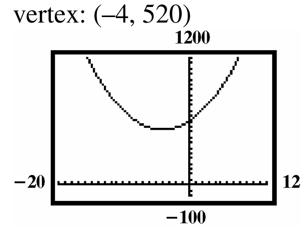
84. $y = 5x^2 + 40x + 600$

$x = \dfrac{-b}{2a} = \dfrac{-40}{10} = -4$

$y = 5(-4)^2 + 40(-4) + 600$

$= 80 - 160 + 600 = 520$

vertex: (–4, 520)



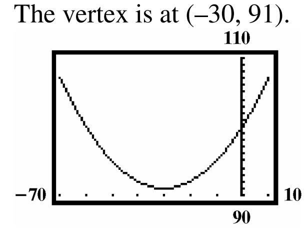
85. $y = 0.01x^2 + 0.6x + 100$

$x = \dfrac{-b}{2a} = \dfrac{-0.6}{0.02} = -30$

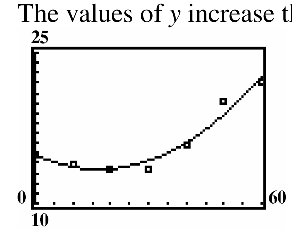
$y = 0.01(-30)^2 + 0.6(-30) + 100$

$= 9 - 18 + 100 = 91$

The vertex is at (–30, 91).



86. a. The values of y increase then decrease.



b. $y = 0.005x^2 - 0.170x + 14.817$

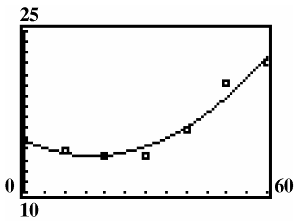
c. $x = \frac{-(-0.170)}{2(.005)} = 17; \quad 1940 + 17 = 1957$

$y = 0.005(17)^2 - 0.170(17) + 14.817$

≈ 13.372

The worst gas mileage was 13.372 mpg in 1957.

d.



87. does not make sense; Explanations will vary. Sample explanation: Some parabolas have the *y*-axis as the axis of symmetry.

88. makes sense

89. does not make sense; Explanations will vary. Sample explanation: If it is thrown vertically, its path will be a line segment.

90. does not make sense; Explanations will vary. Sample explanation: The football's path is better described by a quadratic model.

91. true

92. false; Changes to make the statement true will vary. A sample change is: The vertex is $(5, -1)$.

93. false; Changes to make the statement true will vary. A sample change is: The graph has no *x*–intercepts. To find *x*–intercepts, set *y* = 0 and solve for *x*.

$0 = -2(x+4)^2 - 8$

$2(x+4)^2 = -8$

$(x+4)^2 = -4$

Because the solutions to the equation are imaginary, we know that there are no *x*–intercepts.

94. false; Changes to make the statement true will vary. A sample change is: The *x*-coordinate of the maximum is $-\frac{b}{2a} = -\frac{1}{2(-1)} = -\frac{1}{-2} = \frac{1}{2}$ and the *y*–coordinate of the vertex of the parabola is $f\left(-\frac{b}{2a}\right) = f\left(\frac{1}{2}\right) = \frac{5}{4}$.

The maximum *y*–value is $\frac{5}{4}$.

95. $f(x) = 3(x+2)^2 - 5$; (–1, –2)
axis: $x = -2$
(–1, –2) is one unit right of (–2, –2). One unit left of (–2, –2) is (–3, –2).
point: (–3, –2)

96. Vertex (3, 2) Axis: $x = 3$
second point (0, 11)

97. We start with the form $f(x) = a(x-h)^2 + k$. Since we know the vertex is $(h,k) = (-3,-4)$, we have $f(x) = a(x+3)^2 - 4$. We also know that the graph passes through the point $(1,4)$, which allows us to solve for *a*.

$4 = a(1+3)^2 - 4$

$8 = a(4)^2$

$8 = 16a$

$\frac{1}{2} = a$

Therefore, the function is $f(x) = \frac{1}{2}(x+3)^2 - 4$.

98. We know $(h,k) = (-3,-4)$, so the equation is of the form $f(x) = a(x-h)^2 + k$

$= a[x-(-3)]^2 + (-1)$

$= a(x+3)^2 - 1$

We use the point $(-2,-3)$ on the graph to determine the value of *a*: $f(x) = a(x+3)^2 - 1$

$-3 = a(-2+3)^2 - 1$

$-3 = a(1)^2 - 1$

$-3 = a - 1$

$-2 = a$

Thus, the equation of the parabola is $f(x) = -2(x+3)^2 - 1$.

99. $3x + 4y = 1000$

$$4y = 1000 - 3x$$
$$y = 250 - 0.75x$$
$$A(x) = x(250 - 0.75x)$$
$$= 250x - 0.75x^2$$
$$x = \frac{-b}{2a} = \frac{-250}{-1.5} = 166\frac{2}{3}$$
$$y = 250 - 0.75\left(166\frac{2}{3}\right) = 125$$

The dimensions are $x = 166\frac{2}{3}$ ft, y = 125 ft.

The maximum area is about 20,833 ft^2.

100. Let x = the number of trees over 50 that will be planted.
The function describing the annual yield per lemon tree when $x+50$ trees are planted per acre is

$$f(x) = (x+50)(320-4x)$$
$$= 320x - 4x^2 + 16000 - 200x$$
$$= -4x^2 + 120x + 16000.$$

This represents the number of lemon trees planted per acre multiplied by yield per tree.
The x–coordinate of the maximum is
$-\frac{b}{2a} = -\frac{120}{2(-4)} = -\frac{120}{-8} = 15$ and the y–coordinate of the vertex of the parabola is

$$f\left(-\frac{b}{2a}\right) = f(15)$$
$$= -4(15)^2 + 120(15) + 16000$$
$$= -4(225) + 1800 + 16000$$
$$= -900 + 1800 + 16000$$
$$= 16900$$

The maximum lemon yield is 16,900 pounds when 50 + 15 = 65 lemon trees are planted per acre.

101. Answers may vary.

102. $x^3 + 3x^2 - x - 3 = x^2(x+3) - 1(x+3)$

$$= (x+3)(x^2-1)$$
$$= (x+3)(x+1)(x-1)$$

103. $f(x) = x^3 - 2x - 5$

$$f(2) = (2)^3 - 2(2) - 5 = -1$$
$$f(3) = (3)^3 - 2(3) - 5 = 16$$

The graph passes through (2, –1), which is below the x-axis, and (3, 16), which is above the x-axis. Since the graph of f is continuous, it must cross the x-axis somewhere between 2 and 3 to get from one of these points to the other.

104. $f(x) = x^4 - 2x^2 + 1$

$$f(-x) = (-x)^4 - 2(-x)^2 + 1$$
$$= x^4 - 2x^2 + 1$$

Since $f(-x) = f(x)$, the function is even.
Thus, the graph is symmetric with respect to the y-axis.

Section 3.2

Check Point Exercises

1. Since n is even and $a_n > 0$, the graph rises to the left and to the right.

2. It is not necessary to multiply out the polynomial to determine its degree. We can find the degree of the polynomial by adding the degrees of each of its factors. $f(x) = 2\underbrace{x^3}_{\text{degree 3}}\underbrace{(x-1)}_{\text{degree 1}}\underbrace{(x+5)}_{\text{degree 1}}$ has degree $3+1+1=5$.
$f(x) = 2x^3(x-1)(x+5)$ is of odd degree with a positive leading coefficient. Thus its graph falls to the left and rises to the right.

3. Since n is odd and the leading coefficient is negative, the function falls to the right. Since the ratio cannot be negative, the model won't be appropriate.

4. The graph does not show the function's end behavior. Since $a_n > 0$ and n is odd, the graph should fall to the left.

5. $f(x) = x^3 + 2x^2 - 4x - 8$

$$0 = x^2(x+2) - 4(x+2)$$
$$0 = (x+2)(x^2-4)$$
$$0 = (x+2)^2(x-2)$$

x = 2 or x = –2
The zeros are 2 and –2.

6. $f(x) = x^4 - 4x^2$

$x^4 - 4x^2 = 0$

$x^2(x^2 - 4) = 0$

$x^2(x+2)(x-2) = 0$

$x = 0$ or $x = -2$ or $x = 2$

The zeros are 0, –2, and 2.

7. $f(x) = -4\left(x + \frac{1}{2}\right)^2 (x-5)^3$

$-4\left(x + \frac{1}{2}\right)^2 (x-5)^3 = 0$

$x = -\frac{1}{2}$ or $x = 5$

The zeros are $-\frac{1}{2}$, with multiplicity 2, and 5, with multiplicity 3.

Because the multiplicity of $-\frac{1}{2}$ is even, the graph touches the x-axis and turns around at this zero. Because the multiplicity of 5 is odd, the graph crosses the x-axis at this zero.

8. $f(-3) = 3(-3)^3 - 10(-3) + 9 = -42$

$f(-2) = 3(-2)^3 - 10(-2) + 9 = 5$

The sign change shows there is a zero between –3 and –2.

9. $f(x) = x^3 - 3x^2$

Since $a_n > 0$ and n is odd, the graph falls to the left and rises to the right.

$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$x = 0$ or $x = 3$

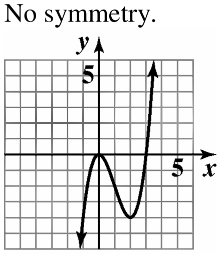
The x-intercepts are 0 and 3.

$f(0) = 0^3 - 3(0)^2 = 0$

The y-intercept is 0.

$f(-x) = (-x)^3 - 3(-x)^2 = -x^3 - 3x^2$

No symmetry.



$f(x) = x^3 - 3x^2$

Exercise Set 3.2

1. polynomial function;
degree: 3

2. polynomial function;
degree: 4

3. polynomial function;
degree: 5

4. polynomial function;
degree: 7

5. not a polynomial function

6. not a polynomial function

7. not a polynomial function

8. not a polynomial function

9. not a polynomial function

10. polynomial function;
degree: 2

11. polynomial function

12. Not a polynomial function because graph is not smooth.

13. Not a polynomial function because graph is not continuous.

14. polynomial function

15. (b)

16. (c)

17. (a)

18. (d)

19. $f(x) = 5x^3 + 7x^2 - x + 9$

Since $a_n > 0$ and n is odd, the graph of $f(x)$ falls to the left and rises to the right.

20. $f(x) = 11x^3 - 6x^2 + x + 3$

Since $a_n > 0$ and n is odd, the graph of $f(x)$ falls to the left and rises to the right.

21. $f(x) = 5x^4 + 7x^2 - x + 9$

Since $a_n > 0$ and n is even, the graph of $f(x)$ rises to the left and to the right.

22. $f(x) = 11x^4 - 6x^2 + x + 3$
Since $a_n > 0$ and n is even, the graph of $f(x)$ rises to the left and to the right.

23. $f(x) = -5x^4 + 7x^2 - x + 9$
Since $a_n < 0$ and n is even, the graph of $f(x)$ falls to the left and to the right.

24. $f(x) = -11x^4 - 6x^2 + x + 3$
Since $a_n < 0$ and n is even, the graph of $f(x)$ falls to the left and to the right.

25. $f(x) = 2(x-5)(x+4)^2$
x = 5 has multiplicity 1;
The graph crosses the x-axis.
$x = -4$ has multiplicity 2;
The graph touches the x-axis and turns around.

26. $f(x) = 3(x+5)(x+2)^2$
x = –5 has multiplicity 1;
The graph crosses the x-axis.
$x = -2$ has multiplicity 2;
The graph touches the x-axis and turns around.

27. $f(x) = 4(x-3)(x+6)^3$
x = 3 has multiplicity 1;
The graph crosses the x-axis.
$x = -6$ has multiplicity 3;
The graph crosses the x-axis.

28. $f(x) = -3\left(x + \frac{1}{2}\right)(x-4)^3$

$x = -\frac{1}{2}$ has multiplicity 1;
The graph crosses the x-axis.
$x = 4$ has multiplicity 3;
The graph crosses the x-axis.

29. $f(x) = x^3 - 2x^2 + x$
$= x\left(x^2 - 2x + 1\right)$
$= x(x-1)^2$
$x = 0$ has multiplicity 1;
The graph crosses the x-axis.
$x = 1$ has multiplicity 2;
The graph touches the x-axis and turns around.

30. $f(x) = x^3 + 4x^2 + 4x$
$= x\left(x^2 + 4x + 4\right)$
$= x(x+2)^2$
$x = 0$ has multiplicity 1;
The graph crosses the x-axis.
$x = -2$ has multiplicity 2;
The graph touches the x-axis and turns around.

31. $f(x) = x^3 + 7x^2 - 4x - 28$
$= x^2(x+7) - 4(x+7)$
$= \left(x^2 - 4\right)(x+7)$
$= (x-2)(x+2)(x+7)$
$x = 2$, $x = -2$ and $x = -7$ have multiplicity 1;
The graph crosses the x-axis.

32. $f(x) = x^3 + 5x^2 - 9x - 45$
$= x^2(x+5) - 9(x+5)$
$= \left(x^2 - 9\right)(x+5)$
$= (x-3)(x+3)(x+5)$
$x = 3$, $x = -3$ and $x = -5$ have multiplicity 1;
The graph crosses the x-axis.

33. $f(x) = x^3 - x - 1$
$f(1) = -1$
$f(2) = 5$
The sign change shows there is a zero between the given values.

34. $f(x) = x^3 - 4x^2 + 2$
$f(0) = 2$
$f(1) = -1$
The sign change shows there is a zero between the given values.

35. $f(x) = 2x^4 - 4x^2 + 1$
$f(-1) = -1$
$f(0) = 1$
The sign change shows there is a zero between the given values.

36. $f(x) = x^4 + 6x^3 - 18x^2$
$f(2) = -8$
$f(3) = 81$
The sign change shows there is a zero between the given values.

37. $f(x) = x^3 + x^2 - 2x + 1$
$f(-3) = -11$
$f(-2) = 1$
The sign change shows there is a zero between the given values.

38. $f(x) = x^5 - x^3 - 1$
$f(1) = -1$
$f(2) = 23$
The sign change shows there is a zero between the given values.

39. $f(x) = 3x^3 - 10x + 9$
$f(-3) = -42$
$f(-2) = 5$
The sign change shows there is a zero between the given values.

40. $f(x) = 3x^3 - 8x^2 + x + 2$
$f(2) = -4$
$f(3) = 14$
The sign change shows there is a zero between the given values.

41. $f(x) = x^3 + 2x^2 - x - 2$

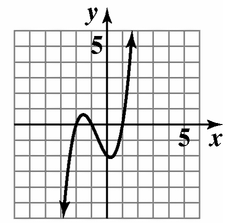
a. Since $a_n > 0$ and n is odd, $f(x)$ rises to the right and falls to the left.

b.
$$x^3 + 2x^2 - x - 2 = 0$$
$$x^2(x+2) - (x+2) = 0$$
$$(x+2)(x^2 - 1) = 0$$
$$(x+2)(x-1)(x+1) = 0$$
$$x = -2,\ x = 1,\ x = -1$$
The zeros at –2, –1, and 1 have odd multiplicity so $f(x)$ crosses the x-axis at these points.

c.
$$f(0) = (0)^3 + 2(0)^2 - 0 - 2$$
$$= -2$$
The y-intercept is –2.

d.
$$f(-x) = (-x) + 2(-x)^2 - (-x) - 2$$
$$= -x^3 + 2x^2 + x - 2$$
$$-f(x) = -x^3 - 2x^2 + x + 2$$
The graph has neither origin symmetry nor y-axis symmetry.

e. The graph has 2 turning points and $2 \le 3 - 1$.



$y = x^3 + 2x^2 - x - 2$

42. $f(x) = x^3 + x^2 - 4x - 4$

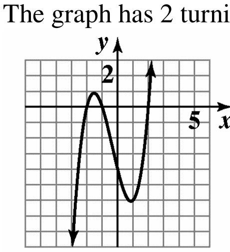
a. Since $a_n > 0$ and n is odd, $f(x)$ rises to the right and falls to the left.

b.
$$x^3 + x^2 - 4x - 4 = 0$$
$$x^2(x+1) - 4(x+1) = 0$$
$$(x+1)(x^2 - 4) = 0$$
$$(x+1)(x-2)(x+2) = 0$$
$x = -1$, or $x = 2$, or $x = -2$
The zeros at –2, –1 and 2 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The x-intercepts are –2, –1, and 2.

c. $f(0) = 0^3 + (0)^2 - 4(0) - 4 = -4$
The y-intercept is –4.

d.
$$f(-x) = -x^3 + x^2 + 4x - 4$$
$$-f(x) = -x^3 - x^2 + 4x + 4$$
neither symmetry

e. The graph has 2 turning points and $2 \le 3 - 1$.



$y = x^3 + x^2 - 4x - 4$

43. $f(x) = x^4 - 9x^2$

a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

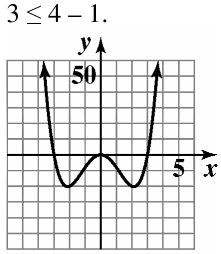
b.
$$x^4 - 9x^2 = 0$$
$$x^2\left(x^2 - 9\right) = 0$$
$$x^2\left(x-3\right)\left(x+3\right) = 0$$
$x = 0, x = 3, x = -3$

The zeros at –3 and 3 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The root at 0 has even multiplicity, so $f(x)$ touches the x-axis at 0.

c. $f(0) = (0)^4 - 9(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = x^4 - 9x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = x^4 - 9x^2$

44. $f(x) = x^4 - x^2$

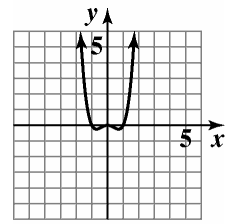
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b.
$$x^4 - x^2 = 0$$
$$x^2\left(x^2 - 1\right) = 0$$
$$x^2\left(x-1\right)\left(x+1\right) = 0$$
$x = 0, x = 1, x = -1$
f touches but does not cross the x-axis at 0.

c. $f(0) = (0)^4 - (0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = x^4 - x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = x^4 - x^2$

45. $f(x) = -x^4 + 16x^2$

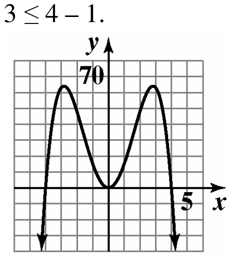
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b.
$$-x^4 + 16x^2 = 0$$
$$x^2\left(-x^2 + 16\right) = 0$$
$$x^2\left(4-x\right)\left(4+x\right) = 0$$
$x = 0, x = 4, x = -4$
The zeros at –4 and 4 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The root at 0 has even multiplicity, so $f(x)$ touches the x-axis at 0.

c. $f(0) = (0)^4 - 9(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = -x^4 + 16x^2$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = -x^4 + 16x^2$

46. $f(x) = -x^4 + 4x^2$

a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-x^4 + 4x^2 = 0$

$x^2(4 - x^2) = 0$

$x^2(2 - x)(2 + x) = 0$

$x = 0, x = 2, x = -2$

The x-intercepts are –2,0, and 2. Since f has a double root at 0, it touches but does not cross the x-axis at 0.

c. $f(0) = -(0)^4 + 4(0)^2 = 0$

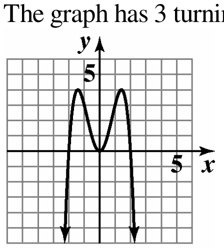
The y-intercept is 0.

d. $f(-x) = -x^4 + 4x^2$

$f(-x) = f(x)$

The graph has y-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = -x^4 + 4x^2$

47. $f(x) = x^4 - 2x^3 + x^2$

a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x^4 - 2x^3 + x^2 = 0$

$x^2(x^2 - 2x + 1) = 0$

$x^2(x - 1)(x - 1) = 0$

$x = 0, x = 1$

The zeros at 1 and 0 have even multiplicity, so $f(x)$ touches the x-axis at 0 and 1.

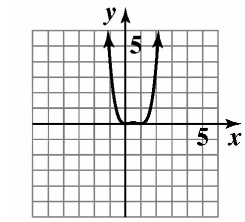
c. $f(0) = (0)^4 - 2(0)^3 + (0)^2 = 0$

The y-intercept is 0.

d. $f(-x) = x^4 + 2x^3 + x^2$

The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = x^4 - 2x^3 + x^2$

48. $f(x) = x^4 - 6x^3 + 9x^2$

a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x^4 - 6x^3 + 9x^2 = 0$

$x^2(x^2 - 6x + 9) = 0$

$x^2(x - 3)^2 = 0$

$x = 0, x = 3$

The zeros at 3 and 0 have even multiplicity, so $f(x)$ touches the x-axis at 3 and 0.

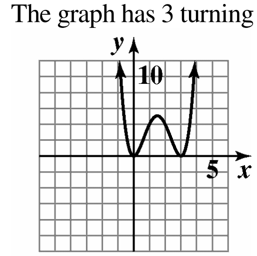
c. $f(0) = (0)^4 - 6(0)^3 + 9(0)^2 = 0$

The y-intercept is 0.

d. $f(-x) = x^4 + 6x^3 + 9x^2$

The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = x^4 - 6x^3 + 9x^2$

49. $f(x) = -2x^4 + 4x^3$

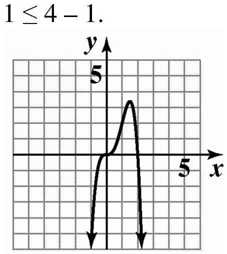
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-2x^4 + 4x^3 = 0$
$x^3(-2x+4) = 0$
$x = 0, x = 2$
The zeros at 0 and 2 have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. $f(0) = -2(0)^4 + 4(0)^3 = 0$
The y-intercept is 0.

d. $f(-x) = -2x^4 - 4x^3$
The graph has neither y-axis nor origin symmetry.

e. The graph has 1 turning point and $1 \le 4 - 1$.



$f(x) = -2x^4 + 4x^3$

50. $f(x) = -2x^4 + 2x^3$

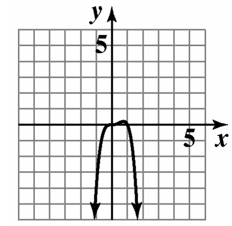
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-2x^4 + 2x^3 = 0$
$x^3(-2x+2) = 0$
$x = 0, x = 1$
The zeros at 0 and 1 have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. The y-intercept is 0.

d. $f(-x) = -2x^4 - 2x^3$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 4 - 1$.



$f(x) = -2x^4 + 2x^3$

51. $f(x) = 6x^3 - 9x - x^5$

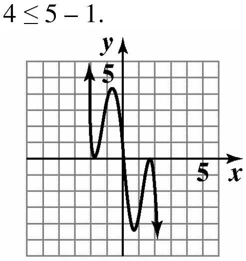
a. Since $a_n < 0$ and n is odd, $f(x)$ rises to the left and falls to the right.

b. $-x^5 + 6x^3 - 9x = 0$
$-x(x^4 - 6x^2 + 9) = 0$
$-x(x^2 - 3)(x^2 - 3) = 0$
$x = 0, \ x = \pm\sqrt{3}$
The root at 0 has odd multiplicity so $f(x)$ crosses the x-axis at (0, 0). The zeros at $-\sqrt{3}$ and $\sqrt{3}$ have even multiplicity so $f(x)$ touches the x-axis at $\sqrt{3}$ and $-\sqrt{3}$.

c. $f(0) = -(0)^5 + 6(0)^3 - 9(0) = 0$
The y-intercept is 0.

d. $f(-x) = x^5 - 6x^3 + 9x$
$f(-x) = -f(x)$
The graph has origin symmetry.

e. The graph has 4 turning point and $4 \le 5 - 1$.



$f(x) = 6x^3 - 9x - x^5$

52. $f(x) = 6x - x^3 - x^5$

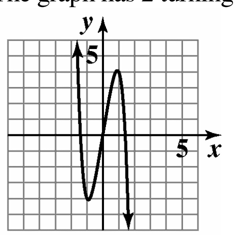
a. Since $a_n < 0$ and n is odd, $f(x)$ rises to the left and falls to the right.

b. $-x^5 - x^3 + 6x = 0$
$-x(x^4 + x^2 - 6) = 0$
$-x(x^2 + 3)(x^2 - 2) = 0$
$x = 0, x = \pm\sqrt{2}$
The zeros at $-\sqrt{2}$, 0, and $\sqrt{2}$ have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. $f(0) = -(0)^5 - (0)^3 + 6(0) = 0$
The y-intercept is 0.

d. $f(-x) = x^5 + x^3 - 6x$
$f(-x) = -f(x)$
The graph has origin symmetry.

e. The graph has 2 turning points and $2 \le 5 - 1$.



$f(x) = 6x - x^3 - x^5$

53. $f(x) = 3x^2 - x^3$

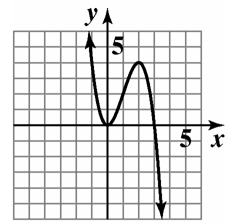
a. Since $a_n < 0$ and n is odd, $f(x)$ rises to the left and falls to the right.

b. $-x^3 + 3x^2 = 0$
$-x^2(x - 3) = 0$
$x = 0, x = 3$
The zero at 3 has odd multiplicity so $f(x)$ crosses the x-axis at that point. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

c. $f(0) = -(0)^3 + 3(0)^2 = 0$
The y-intercept is 0.

d. $f(-x) = x^3 + 3x^2$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning point and $2 \le 3 - 1$.



$f(x) = 3x^2 - x^3$

54. $f(x) = \frac{1}{2} - \frac{1}{2}x^4$

a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

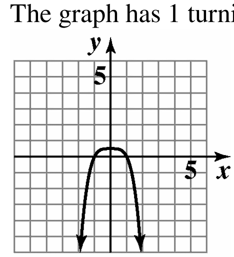
b. $-\frac{1}{2}x^4 + \frac{1}{2} = 0$
$-\frac{1}{2}(x^4 - 1) = 0$
$-\frac{1}{2}(x^2 + 1)(x^2 - 1) = 0$
$-\frac{1}{2}(x^2 + 1)(x - 1)(x + 1) = 0$
$x = \pm 1$
The zeros at –1 and 1 have odd multiplicity, so $f(x)$ crosses the x-axis at these points.

c. $f(0) = -\frac{1}{2}(0)^4 + \frac{1}{2} = \frac{1}{2}$

The y-intercept is $\frac{1}{2}$.

d. $f(-x) = \frac{1}{2} - \frac{1}{2}x^4$
$f(-x) = f(x)$
The graph has y-axis symmetry.

e. The graph has 1 turning point and $1 \le 4 - 1$.



$f(x) = \frac{1}{2} - \frac{1}{2}x^4$

55. $f(x) = -3(x-1)^2(x^2-4)$

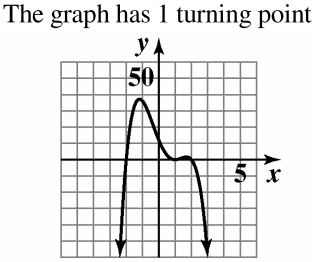
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-3(x-1)^2(x^2-4) = 0$
$x = 1, x = -2, x = 2$
The zeros at –2 and 2 have odd multiplicity, so $f(x)$ crosses the x-axis at these points. The root at 1 has even multiplicity, so $f(x)$ touches the x-axis at (1, 0).

c. $f(0) = -3(0-1)^2(0^2-4)^3$
$= -3(1)(-4) = 12$
The y-intercept is 12.

d. $f(-x) = -3(-x-1)^2(x^2-4)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 1 turning point and $1 \le 4 - 1$.



$f(x) = -3(x-1)^2(x^2-4)$

56. $f(x) = -2(x-4)^2(x^2-25)$

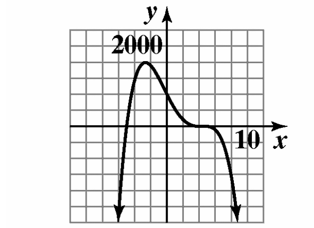
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $-2(x-4)^2(x^2-25) = 0$
$x = 4, x = -5, x = 5$
The zeros at –5 and 5 have odd multiplicity so $f(x)$ crosses the x-axis at these points. The root at 4 has even multiplicity so $f(x)$ touches the x-axis at (4, 0).

c. $f(0) = -2(0-4)^2(0^2-25)$
$= -2(16)(-25)$
$= 800$
The y-intercept is 800.

d. $f(-x) = -2(-x-4)^2(x^2-2)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 1 turning point and $1 \le 4 - 1$.



$f(x) = -2(x-4)^2(x^2-25)$

57. $f(x) = x^2(x-1)^3(x+2)$

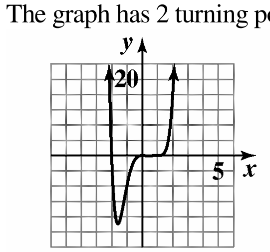
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x = 0, x = 1, x = -2$
The zeros at 1 and –2 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0).

c. $f(0) = 0^2(0-1)^3(0+2) = 0$
The y-intercept is 0.

d. $f(-x) = x^2(-x-1)^3(-x+2)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 6 - 1$.



$f(x) = x^2(x-1)^3(x+2)$

58. $f(x) = x^3(x+2)^2(x+1)$

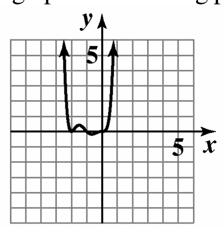
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.

b. $x = 0, x = -2, x = -1$
The roots at 0 and –1 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at –2 has even multiplicity so $f(x)$ touches the axis at (–2, 0).

c. $f(0) = 0^3(0+2)^2(0+1) = 0$
The y-intercept is 0.

d. $f(-x) = -x^3(-x+2)^2(-x+1)$
The graph has neither *y*-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 6 - 1$.



$f(x) = x^3(x+2)^2(x+1)$

59. $f(x) = -x^2(x-1)(x+3)$

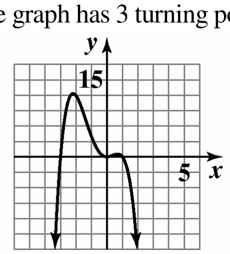
a. Since $a_n < 0$ and *n* is even, *f*(*x*) falls to the left and the right.

b. $x = 0, x = 1, x = -3$
The zeros at 1 and –3 have odd multiplicity so *f*(*x*) crosses the *x*-axis at those points. The root at 0 has even multiplicity so *f*(*x*) touches the axis at (0, 0).

c. $f(0) = -0^2(0-1)(0+3) = 0$
The *y*-intercept is 0.

d. $f(-x) = -x^2(-x-1)(-x+3)$
The graph has neither *y*-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = -x^2(x-1)(x+3)$

60. $f(x) = -x^2(x+2)(x-2)$

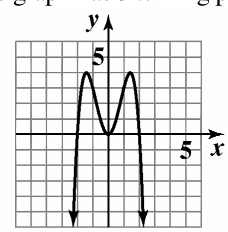
a. Since $a_n < 0$ and *n* is even, *f*(*x*) falls to the left and the right.

b. $x = 0, x = 2, x = -2$
The zeros at 2 and –2 have odd multiplicity so *f*(*x*) crosses the *x*-axis at those points. The root at 0 has even multiplicity so *f*(*x*) touches the axis at (0, 0).

c. $f(0) = -0^2(0+2)(0-2) = 0$
The *y*-intercept is 0.

d. $f(-x) = -x^2(-x+2)(-x-2)$
$f(-x) = -x^2(-1)(x-2)(-1)(x+2)$
$f(-x) = -x^2(x+2)(x-2)$
$f(-x) = f(x)$
The graph has *y*-axis symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = -x^2(x+2)(x-2)$

61. $f(x) = -2x^3(x-1)^2(x+5)$

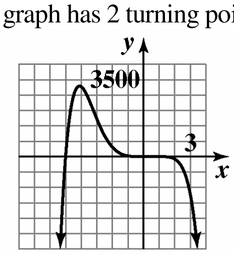
a. Since $a_n < 0$ and *n* is even, *f*(*x*) falls to the left and the right.

b. $x = 0, x = 1, x = -5$
The roots at 0 and –5 have odd multiplicity so *f*(*x*) crosses the *x*-axis at those points. The root at 1 has even multiplicity so *f*(*x*) touches the axis at (1, 0).

c. $f(0) = -2(0)^3(0-1)^2(0+5) = 0$
The *y*-intercept is 0.

d. $f(-x) = 2x^3(-x-1)^2(-x+5)$
The graph has neither *y*-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 6 - 1$.



$f(x) = -2x^3(x-1)^2(x+5)$

62. $f(x) = -3x^3(x-1)^2(x+3)$

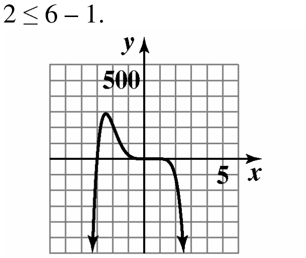
a. Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

b. $x = 0$, $x = 1$, $x = -3$
The roots at 0 and –3 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 1 has even multiplicity so $f(x)$ touches the axis at (1, 0).

c. $f(0) = -3(0)^3(0-1)^2(0+3) = 0$
The y-intercept is 0.

d. $f(-x) = 3x^3(-x-1)^2(-x+3)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points and $2 \le 6 - 1$.



$f(x) = -3x^3(x-1)^2(x+3)$

63. $f(x) = (x-2)^2(x+4)(x-1)$

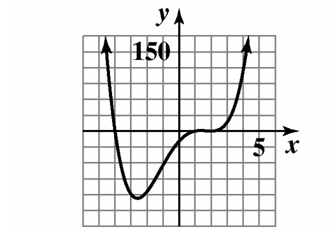
a. Since $a_n > 0$ and n is even, $f(x)$ rises to the left and rises the right.

b. $x = 2$, $x = -4$, $x = 1$
The zeros at –4 and 1 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 2 has even multiplicity so $f(x)$ touches the axis at (2, 0).

c. $f(0) = (0-2)^2(0+4)(0-1) = -16$
The y-intercept is –16.

d. $f(-x) = (-x-2)^2(-x+4)(-x-1)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 3 turning points and $3 \le 4 - 1$.



$f(x) = (x-2)^2(x+4)(x-1)$

64. $f(x) = (x+3)(x+1)^3(x+4)$

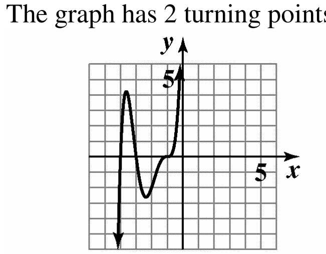
a. Since $a_n > 0$ and n is odd, $f(x)$ falls to the left and rises to the right.

b. $x = -3$, $x = -1$, $x = -4$
The zeros at all have odd multiplicity so $f(x)$ crosses the x-axis at these points.

c. $f(0) = (0+3)(0+1)^3(0+4) = 12$
The y-intercept is 12.

d. $f(-x) = (-x+3)(-x+1)^3(-x+4)$
The graph has neither y-axis nor origin symmetry.

e. The graph has 2 turning points



$f(x) = (x+3)(x+1)^3(x+4)$

65. a. The x-intercepts of the graph are -2, 1, and 4, so they are the zeros. Since the graph actually crosses the x-axis at all three places, all three have odd multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -2, 1, and 4 are the zeros, $x+2$, $x-1$, and $x-4$ are factors of the function. The lowest odd multiplicity is 1. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x) = (x+2)(x-1)(x-4)$.

c. $f(0) = (0+2)(0-1)(0-4) = 8$

66. a. The x-intercepts of the graph are -3, 2, and 5, so they are the zeros. Since the graph actually crosses the x-axis at all three places, all three have odd multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -3, 2, and 5 are the zeros, $x+3$, $x-2$, and $x-5$ are factors of the function. The lowest odd multiplicity is 1. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x)=(x+3)(x-2)(x-5)$.

c. $f(0)=(0+3)(0-2)(0-5)=30$

67. a. The x-intercepts of the graph are -1 and 3, so they are the zeros. Since the graph crosses the x-axis at -1, it has odd multiplicity. Since the graph touches the x-axis and turns around at 3, it has even multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -1 and 3 are the zeros, $x+1$ and $x-3$ are factors of the function. The lowest odd multiplicity is 1, and the lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x)=(x+1)(x-3)^2$.

c. $f(0)=(0+1)(0-3)^2=9$

68. a. The x-intercepts of the graph are -2 and 1, so they are the zeros. Since the graph crosses the x-axis at -2, it has odd multiplicity. Since the graph touches the x-axis and turns around at 1, it has even multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -2 and 1 are the zeros, $x+2$ and $x-1$ are factors of the function. The lowest odd multiplicity is 1, and the lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be positive. Thus, the function is $f(x)=(x+2)(x-1)^2$.

c. $f(0)=(0+2)(0-1)^2=2$

69. a. The x-intercepts of the graph are -3 and 2, so they are the zeros. Since the graph touches the x-axis and turns around at both -3 and 2, both have even multiplicity.

b. Since the graph has three turning points, the function must be at least of degree 4. Since -3 and 2 are the zeros, $x+3$ and $x-2$ are factors of the function. The lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be negative. Thus, the function is $f(x)=-(x+3)^2(x-2)^2$.

c. $f(0)=-(0+3)^2(0-2)^2=-36$

70. a. The x-intercepts of the graph are -1 and 4, so they are the zeros. Since the graph touches the x-axis and turns around at both -1 and 4, both have even multiplicity.

b. Since the graph has two turning points, the function must be at least of degree 3. Since -1 and 4 are the zeros, $x+1$ and $x-4$ are factors of the function. The lowest even multiplicity is 2. From the end behavior, we can tell that the leading coefficient must be negative. Thus, the function is $f(x)=-(x+1)^2(x-4)^2$.

c. $f(0)=-(0+1)^2(0-4)^2=-16$

71. a. The x-intercepts of the graph are -2, -1, and 1, so they are the zeros. Since the graph crosses the x-axis at -1 and 1, they both have odd multiplicity. Since the graph touches the x-axis and turns around at -2, it has even multiplicity.

b. Since the graph has five turning points, the function must be at least of degree 6. Since -2, -1, and 1 are the zeros, $x+2$, $x+1$, and $x-1$ are factors of the function. The lowest even multiplicity is 2, and the lowest odd multiplicity is 1. However, to reach degree 6, one of the odd multiplicities must be 3. From the end behavior, we can tell that the leading coefficient must be positive. The function is $f(x)=(x+2)^2(x+1)(x-1)^3$.

c. $f(0)=(0+2)^2(0+1)(0-1)^3=-4$

72. a. The x-intercepts of the graph are -2, -1, and 1, so they are the zeros. Since the graph crosses the x-axis at -2 and 1, they both have odd multiplicity. Since the graph touches the x-axis and turns around at -1, it has even multiplicity.

b. Since the graph has five turning points, the function must be at least of degree 6. Since -2, -1, and 1 are the zeros, $x+2$, $x+1$, and $x-1$ are factors of the function. The lowest even multiplicity is 2, and the lowest odd multiplicity is 1. However, to reach degree 6, one of the odd multiplicities must be 3. From the end behavior, we can tell that the leading coefficient must be positive. The function is $f(x)=(x+2)(x+1)^2(x-1)^3$.

c. $f(0)=(0+2)(0+1)^2(0-1)^3=-2$

73. a.

$$f(x)=-3402x^2+42,203x+308,453$$
$$f(3)=-3402(3)^2+42,203(3)+308,453$$
$$=404,444$$
$$g(x)=2769x^3-28,324x^2+107,555x+261,931$$
$$g(3)=2769(3)^3-28,324(3)^2+107,555(3)+261,931$$
$$=404,443$$

Function f provides a better description of the actual number.

b. Since the degree of f is even and the leading coefficient is negative, the graph falls to the right. The function will not be a useful model over an extended period of time because it will eventually give negative values.

74. a.

$$f(x)=-3402x^2+42,203x+308,453$$
$$f(5)=-3402(5)^2+42,203(5)+308,453$$
$$=434,418$$
$$g(x)=2769x^3-28,324x^2+107,555x+261,931$$
$$g(5)=2769(5)^3-28,324(5)^2+107,555(5)+261,931$$
$$=437,731$$

Function g provides a better description of the actual number.

b. Since the degree of g is odd and the leading coefficient is negative, the graph rises to the right. Based on the end behavior, the function will be a useful model over an extended period of time.

75. a. The woman's heart rate was increasing from 1 through 4 minutes and from 8 through 10 minutes.

b. The woman's heart rate was decreasing from 4 through 8 minutes and from 10 through 12 minutes.

c. There were 3 turning points during the 12 minutes.

d. Since there were 3 turning points, a polynomial of degree 4 would provide the best fit.

e. The leading coefficient should be negative. The graph falls to the left and to the right.

f. The woman's heart rate reached a maximum of about 116 ± 1 beats per minute. This occurred after 10 minutes.

g. The woman's heart rate reached a minimum of about 64 ± 1 beats per minute. This occurred after 8 minutes.

76. a. The percentage of students with B+ averages or better was increasing from 1960 through 1975 and from 1985 through 2000.

b. The percentage of students with B+ averages or better was decreasing from 1975 through 1985 and from 2000 through 2005.

c. There were 3 turning points during the period shown.

d. Since there were 3 turning points, a polynomial of degree 4 would provide the best fit.

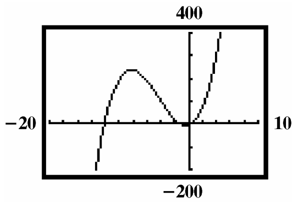
e. The leading coefficient should be negative. The graph falls to the left and to the right.

f. The percentage reached a maximum of about $69\pm1\%$ in 2000.

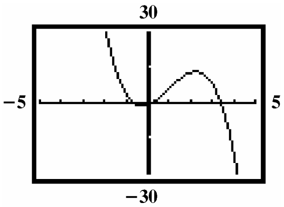
g. The percentage reached a minimum of about $18\pm1\%$ in 1960.

77. – 93. Answers may vary.

94.



95.



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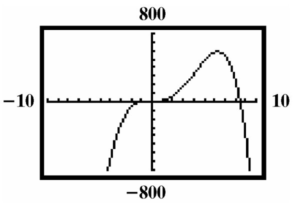
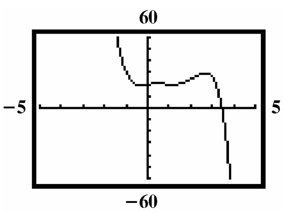
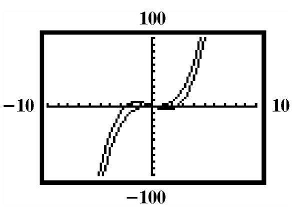


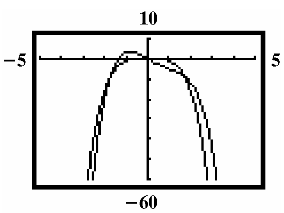
97.



98.



99.



100. makes sense

101. does not make sense; Explanations will vary. Sample explanation: Since $(x+2)$ is raised to an odd power, the graph crosses the x-axis at –2.

102. does not make sense; Explanations will vary. Sample explanation: A forth degree function has at most 3 turning points.

103. makes sense

104. false; Changes to make the statement true will vary. A sample change is: $f(x)$ falls to the left and rises to the right.

105. false; Changes to make the statement true will vary. A sample change is: Such a function falls to the right and will eventually have negative values.

106. true

107. false; Changes to make the statement true will vary. A sample change is: A function with origin symmetry either falls to the left and rises to the right, or rises to the left and falls to the right.

108. $f(x) = x^3 + x^2 - 12x$

109. $f(x) = x^3 - 2x^2$

110. $\frac{737}{21} = 35 + \frac{2}{21}$

111. $6x^3 - x^2 - 5x + 4$

112. $2x^3 - x^2 - 11x + 6 = (x-3)(2x^2 + 3x - 2)$
$= (x-3)(2x-1)(x+2)$

Section 3.3

Check Point Exercises

1.

$$\begin{array}{r} x+5 \\ x+9\overline{)\,x^2+14x+45} \\ \underline{x^2+9x} \\ 5x+45 \\ \underline{5x+45} \\ 0 \end{array}$$

The answer is $x + 5$.

2.

$$\begin{array}{r} 2x^2+3x-2 \\ x-3\overline{)\,2x^3-3x^2-11x+7} \\ \underline{2x^3-6x^2} \\ 3x^2-11x \\ \underline{3x^2-9x} \\ -2x+7 \\ \underline{-2x+6} \\ 1 \end{array}$$

The answer is $2x^2 + 3x - 2 + \frac{1}{x-3}$.

3.
$$\begin{array}{r} 2x^2+7x+14 \\ x^2-2x\overline{)2x^4+3x^3+0x^2-7x-10} \\ \underline{2x^4-4x^3} \\ 7x^3+0x^2 \\ \underline{7x^3-14x^2} \\ 14x^2-7x \\ \underline{14x^2-28x} \\ 21x-10 \end{array}$$

The answer is $2x^2+7x+14+\dfrac{21x-10}{x^2-2x}$.

4.

–2	1	0	–7	–6
		–2	4	6
	1	–2	–3	0

The answer is x^2-2x-3.

5.

–4	3	4	–5	3
		–12	32	–108
	3	–8	27	–105

$f(-4)=-105$

6.

–1	15	14	–3	–2
		–15	1	2
	15	–1	–2	0

$$15x^2-x-2=0$$
$$(3x+1)(5x-2)=0$$
$$x=-\frac{1}{3} \quad \text{or} \quad x=\frac{2}{5}$$

The solution set is $\left\{-1,-\dfrac{1}{3},\dfrac{2}{5}\right\}$.

Exercise Set 3.3

1.
$$\begin{array}{r} x+3 \\ x+5\overline{)x^2+8x+15} \\ \underline{x^2+5x} \\ 3x+15 \\ \underline{3x+15} \\ 0 \end{array}$$

The answer is $x+3$.

2.
$$\begin{array}{r} x+5 \\ x-2\overline{)x^2+3x-10} \\ \underline{x^2-2x} \\ 5x-10 \\ \underline{5x-10} \\ 0 \end{array}$$

The answer is $x+5$.

3.
$$\begin{array}{r} x^2+3x+1 \\ x+2\overline{)x^3+5x^2+7x+2} \\ \underline{x^3+2x^2} \\ 3x^2+7x \\ \underline{3x^2+6x} \\ x+2 \\ \underline{x+2} \\ 0 \end{array}$$

The answer is x^2+3x+1.

4.
$$\begin{array}{r} x^2+x-2 \\ x-3\overline{)x^3-2x^2-5x+6} \\ \underline{x^3-3x^2} \\ x^2-5x \\ \underline{x^2-3x} \\ -2x+6 \\ \underline{-2x+6} \\ 0 \end{array}$$

The answer is x^2+x-2.

5.
$$\begin{array}{r} 2x^2+3x+5 \\ 3x-1\overline{)6x^3+7x^2+12x-5} \\ \underline{6x^3-2x^2} \\ 9x^2+12x \\ \underline{9x^2-3x} \\ 15x-5 \\ \underline{15x-5} \\ 0 \end{array}$$

The answer is $2x^2+3x+5$.

6.

$$\begin{array}{r} 2x^2+3x+5 \\ 3x+4\overline{)6x^3+17x^2+27x+20} \\ \underline{6x^3+8x^2} \\ 9x^2+27x \\ \underline{9x^2+12x} \\ 15x+20 \\ \underline{15x+20} \\ 0 \end{array}$$

The answer is $2x^2+3x+5$.

7.

$$\begin{array}{r} 4x+3+\dfrac{2}{3x-2} \\ 3x-2\overline{)12x^2+x-4} \\ \underline{12x^2-8x} \\ 9x-4 \\ \underline{9x-6} \\ 2 \end{array}$$

The answer is $4x+3+\dfrac{2}{3x-2}$.

8.

$$\begin{array}{r} 2x-3+\dfrac{3}{2x-1} \\ 2x-1\overline{)4x^2-8x+6} \\ \underline{4x^2-2x} \\ -6x+6 \\ \underline{-6x+6} \\ 3 \end{array}$$

The answer is $2x-3+\dfrac{3}{2x-1}$.

9.

$$\begin{array}{r} 2x^2+x+6-\dfrac{38}{x+3} \\ x+3\overline{)2x^3+7x^2+9x-20} \\ \underline{2x^3+6x^2} \\ x^2+9x \\ \underline{x^2+3x} \\ 6x-20 \\ \underline{6x+18} \\ -38 \end{array}$$

The answer is $2x^2+x+6-\dfrac{38}{x+3}$.

10.

$$\begin{array}{r} 3x+7+\dfrac{26}{x-3} \\ x-3\overline{)3x^2-2x+5} \\ \underline{3x^2-9x} \\ 7x+5 \\ \underline{7x-21} \\ 26 \end{array}$$

The answer is $3x+7+\dfrac{26}{x-3}$.

11.

$$\begin{array}{r} 4x^3+16x^2+60x+246+\dfrac{984}{x-4} \\ x-4\overline{)4x^4-4x^2+6x} \\ \underline{4x^4-16x^3} \\ 16x^3-4x^2 \\ \underline{16x^3-64x^2} \\ 60x^2+6x \\ \underline{60x^2-240x} \\ 246x \\ \underline{246x-984} \\ 984 \end{array}$$

The answer is

$$4x^3+16x^2+60x+246+\frac{984}{x-4}.$$

12.

$$\begin{array}{r} x^3+3x^2+9x+27 \\ x-3\overline{)x^4 \qquad\qquad\qquad -81} \\ \underline{x^4-3x^3} \\ 3x^3 \\ \underline{3x^2-9x^2} \\ 9x^2 \\ \underline{9x^2-27x} \\ 27x-81 \\ \underline{27x-81} \\ 0 \end{array}$$

The answer is $x^3+3x^2+9x+27$.

13.
$$\begin{array}{r} 2x+5 \\ 3x^2-x-3\overline{)6x^3+13x^2-11x-15} \\ \underline{6x^3-2x^2-6x} \\ 15x^2-5x-15 \\ \underline{15x^2-5x-15} \\ 0 \end{array}$$

The answer is $2x+5$.

14.
$$\begin{array}{r} x^2+x-3 \\ x^2+x-2\overline{)x^4+2x^3-4x^2-5x-6} \\ \underline{x^4+x^3-2x^2} \\ x^3-2x^2-5x \\ \underline{x^3+x^2-2x} \\ -3x^2-3x-6 \\ \underline{-3x^2-3x+6} \\ -12 \end{array}$$

The answer is $x^2+x-3-\frac{12}{x^2+x-2}$.

15.
$$\begin{array}{r} 6x^2+3x-1 \\ 3x^2+1\overline{)18x^4+9x^3+3x^2} \\ \underline{18x^4+6x^2} \\ 9x^3-3x^2 \\ \underline{9x^3+3x} \\ -3x^2-3x \\ \underline{-3x^2-1} \\ -3x+1 \end{array}$$

The answer is $6x^2+3x-1-\frac{3x-1}{3x^2+1}$.

16.
$$\begin{array}{r} x^2-4x+1 \\ 2x^3+1\overline{)2x^5-8x^4+2x^3+x^2} \\ \underline{2x^5+x^2} \\ -8x^4+2x^3 \\ \underline{-8x^4-4x} \\ 2x^3+4x \\ \underline{2x^3+1} \\ 4x-1 \end{array}$$

The answer is $x^2-4x+1+\frac{4x-1}{2x^3+1}$.

17. $(2x^2+x-10)\div(x-2)$

$$\begin{array}{r|rrr} 2 & 2 & 1 & -10 \\ & & 4 & 10 \\ \hline & 2 & 5 & 0 \end{array}$$

The answer is $2x+5$.

18. $(x^2+x-2)\div(x-1)$

$$\begin{array}{r|rrr} 1 & 1 & 1 & -2 \\ & & 1 & 2 \\ \hline & 1 & 2 & 0 \end{array}$$

The answer is $x+2$.

19. $(3x^2+7x-20)\div(x+5)$

$$\begin{array}{r|rrr} -5 & 3 & 7 & -20 \\ & & -15 & 40 \\ \hline & 3 & -8 & 20 \end{array}$$

The answer is $3x-8+\frac{20}{x+5}$.

20. $(5x^2-12x-8)\div(x+3)$

$$\begin{array}{r|rrr} -3 & 5 & 12 & -8 \\ & & -15 & 81 \\ \hline & 5 & -27 & 73 \end{array}$$

The answer is $5x-27+\frac{73}{x+3}$.

21. $(4x^3-3x^2+3x-1)\div(x-1)$

$$\begin{array}{r|rrrr} 1 & 4 & -3 & 3 & -1 \\ & & 4 & 1 & 4 \\ \hline & 4 & 1 & 4 & 3 \end{array}$$

The answer is $4x^2+x+4+\frac{3}{x-1}$.

22. $\left(5x^3-6x^2+3x+11\right)\div(x-2)$

2	5	–6	3	11
		10	8	22
	5	4	11	33

The answer is $5x^2+4x+11+\frac{33}{x-2}$.

23. $\left(6x^5-2x^3+4x^2-3x+1\right)\div(x-2)$

2	6	0	–2	4	–3	1
		12	24	44	96	186
	6	12	22	48	93	187

The answer is

$6x^4+12x^3+22x^2+48x+93+\frac{187}{x-2}$.

24. $\left(x^5+4x^4-3x^2+2x+3\right)\div(x-3)$

3	1	4	0	–3	2	3
		3	21	63	180	546
	1	7	21	60	182	549

The answer is

$x^4+7x^3+21x^2+60x+182+\frac{549}{x-3}$.

25. $\left(x^2-5x-5x^3+x^4\right)\div(5+x)\Rightarrow$

$\left(x^4-5x^3+x^2-5x\right)\div(x+5)$

–5	1	–5	1	–5	0
		–5	50	–255	1300
	1	–10	51	–260	1300

The answer is

$x^3-10x^2+51x-260+\frac{1300}{x+5}$.

26. $\left(x^2-6x-6x^3+x^4\right)\div(6+x)\Rightarrow$

$\left(x^4-6x^3+x^2-6x\right)\div(x+6)$

–6	1	–6	1	–6	0
		–6	72	–438	2664
	1	–12	73	–444	2664

The answer is $x^3-12x^2+73x-444+\frac{2664}{x+6}$.

27. $\frac{x^5+x^3-2}{x-1}$

1	1	0	1	0	0	–2
		1	1	2	2	2
	1	1	2	2	2	0

The answer is $x^4+x^3+2x^2+2x+2$.

28. $\frac{x^7+x^5-10x^3+12}{x+2}$

–2	1	0	1	0	–10	0	0	12
		–2	4	–10	20	–20	40	–80
	1	–2	5	–10	10	–20	40	–68

The answer is $x^6-2x^5+5x^4-10x^3+10x^2$

$-20x+40-\frac{68}{x+2}$.

29. $\frac{x^4-256}{x-4}$

4	1	0	0	0	–256
		4	16	64	256
	1	4	16	64	0

The answer is $x^3+4x^2+16x+64$.

30. $\frac{x^7-128}{x-2}$

2	1	0	0	0	0	0	0	–128
		2	4	8	16	32	64	128
	1	2	4	8	16	32	64	0

The answer is

$x^6+2x^5+4x^4+8x^3+16x^2+32x+64$.

31. $\dfrac{2x^5-3x^4+x^3-x^2+2x-1}{x+2}$

−2	2	−3	1	−1	2	−1
		−4	14	−30	62	−128
	2	−7	15	−31	64	−129

The answer is

$2x^4-7x^3+15x^2-31x+64-\dfrac{129}{x+2}$.

32. $\dfrac{x^5-2x^4-x^3+3x^2-x+1}{x-2}$

2	1	−2	−1	3	−1	1
		2	0	−2	2	2
	1	0	−1	1	1	3

The answer is $x^4-x^2+x+1+\dfrac{3}{x-2}$.

33. $f(x)=2x^3-11x^2+7x-5$

4	2	−11	7	−5
		8	−12	−20
	2	−3	−5	−25

$f(4)=-25$

34.

3	1	−7	5	−6
		3	−12	−21
	1	−4	−7	−27

$f(3)=-27$

35. $f(x)=3x^3-7x^2-2x+5$

−3	3	−7	−2	5
		−9	48	−138
	3	−16	46	−133

$f(-3)=-133$

36.

−2	4	5	−6	−4
		−8	6	0
	4	−3	0	−4

$f(-2)=-4$

37. $f(x)=x^4+5x^3+5x^2-5x-6$

3	1	5	5	−5	−6
		3	24	87	246
	1	8	29	82	240

$f(3)=240$

38.

2	1	−5	5	5	−6
		2	−6	−2	6
	1	−3	−1	3	0

$f(2)=0$

39. $f(x)=2x^4-5x^3-x^2+3x+2$

$-\frac{1}{2}$	2	−5	−1	3	2
		−1	3	−1	−1
	2	−6	2	2	1

$f\left(-\frac{1}{2}\right)=1$

40.

$-\frac{2}{3}$	6	10	5	1	1
		−4	−4	$-\frac{2}{3}$	$-\frac{2}{9}$
	6	6	1	$\frac{1}{3}$	$\frac{7}{9}$

$f\left(-\frac{2}{3}\right)=\frac{7}{9}$

41. Dividend: x^3-4x^2+x+6
Divisor: $x+1$

−1	1	−4	1	6
		−1	5	−6
	1	−5	6	0

The quotient is x^2-5x+6.

$(x+1)(x^2-5x+6)=0$

$(x+1)(x-2)(x-3)=0$

$x=-1,\ x=2,\ x=3$

The solution set is $\{-1, 2, 3\}$.

42. Dividend: x^3-2x^2-x+2
Divisor: $x+1$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -1 & 2 \\ & & -1 & 3 & -2 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

The quotient is x^2-3x+2.
$(x+1)(x^2-3x+2)=0$
$(x+1)(x-2)(x-1)=0$
$x=-1,\ x=2,\ x=1$
The solution set is $\{-1, 2, 1\}$.

43. $2x^3-5x^2+x+2=0$

$$\begin{array}{r|rrrr} 2 & 2 & -5 & 1 & 2 \\ & & 4 & -2 & -2 \\ \hline & 2 & -1 & -1 & 0 \end{array}$$

$(x-2)(2x^2-x-1)=0$
$(x-2)(2x+1)(x-1)=0$
$x=2,\ x=-\frac{1}{2},\ x=1$
The solution set is $\left\{-\frac{1}{2}, 1, 2\right\}$.

44. $2x^3-3x^2-11x+6=0$

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -11 & 6 \\ & & -4 & 14 & -6 \\ \hline & 2 & -7 & 3 & 0 \end{array}$$

$(x+2)(2x^2-7x+3)=0$
$(x+2)(2x-1)(x-3)=0$
$x=-2,\ x=\frac{1}{2},\ x=3$
The solution set is $\left\{-2, \frac{1}{2}, 3\right\}$.

45. $12x^3+16x^2-5x-3=0$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 12 & 16 & -5 & -3 \\ & & -18 & 3 & 3 \\ \hline & 12 & -2 & -2 & 0 \end{array}$$

$\left(x+\frac{3}{2}\right)(12x^2-2x-2)=0$
$\left(x+\frac{3}{2}\right)2\left(6x^2-x-1\right)=0$
$\left(x+\frac{3}{2}\right)2(3x+1)(2x-1)=0$
$x=-\frac{3}{2},\ x=-\frac{1}{3},\ x=\frac{1}{2}$
The solution set is $\left\{-\frac{3}{2}, -\frac{1}{3}, \frac{1}{2}\right\}$.

46. $3x^3+7x^2-22x-8=0$

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & 7 & -22 & -8 \\ & & -1 & -2 & 8 \\ \hline & 3 & 6 & -24 & 0 \end{array}$$

$\left(x+\frac{1}{3}\right)3x^2+6x-24=0$
$\left(x+\frac{1}{3}\right)3(x+4)(x-2)=0$
$x=-4,\ x=2,\ x=-\frac{1}{3}$
The solution set is $\left\{-4, -\frac{1}{3}, 2\right\}$.

47. The graph indicates that 2 is a solution to the equation.

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -5 & -6 \\ & & 2 & 8 & 6 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

The remainder is 0, so 2 is a solution.
$x^3+2x^2-5x-6=0$
$(x-2)(x^2+4x+3)=0$
$(x-2)(x+3)(x+1)=0$
The solutions are 2, -3, and -1, or $\{-3,-1,2\}$.

48. The graph indicates that -3 is a solution to the equation.

$$\begin{array}{r|rrrr} -3 & 2 & 1 & -13 & 6 \\ & & -6 & 15 & -6 \\ \hline & 2 & -5 & 2 & 0 \end{array}$$

The remainder is 0, so -3 is a solution.

$$2x^3 + x^2 - 13x + 6 = 0$$
$$(x+3)(2x^2 - 5x + 2) = 0$$
$$(x+3)(2x-1)(x-2) = 0$$

The solutions are -3, $\frac{1}{2}$, and 2, or $\left\{-3, \frac{1}{2}, 2\right\}$.

49. The table indicates that 1 is a solution to the equation.

$$\begin{array}{r|rrrr} 1 & 6 & -11 & 6 & -1 \\ & & 6 & -5 & 1 \\ \hline & 6 & -5 & 1 & 0 \end{array}$$

The remainder is 0, so 1 is a solution.

$$6x^3 - 11x^2 + 6x - 1 = 0$$
$$(x-1)(6x^2 - 5x + 1) = 0$$
$$(x-1)(3x-1)(2x-1) = 0$$

The solutions are 1, $\frac{1}{3}$, and $\frac{1}{2}$, or $\left\{\frac{1}{3}, \frac{1}{2}, 1\right\}$.

50. The table indicates that 1 is a solution to the equation.

$$\begin{array}{r|rrrr} 1 & 2 & 11 & -7 & -6 \\ & & 2 & 13 & -6 \\ \hline & 2 & 13 & 6 & 0 \end{array}$$

The remainder is 0, so 1 is a solution.

$$2x^3 + 11x^2 - 7x - 6 = 0$$
$$(x-1)(2x^2 + 13x + 6) = 0$$
$$(x-1)(2x+1)(x+6) = 0$$

;olutions are 1, $-\frac{1}{2}$, and -6, or $\left\{-6, -\frac{1}{2}, 1\right\}$.

51. **a.** $14x^3 - 17x^2 - 16x - 177 = 0$

$$\begin{array}{r|rrrr} 3 & 14 & -17 & -16 & -177 \\ & & 42 & 75 & 177 \\ \hline & 14 & 25 & 59 & 0 \end{array}$$

The remainder is 0 so 3 is a solution.

$$14x^3 - 17x^2 - 16x - 177$$
$$= (x-3)(14x^2 + 25x + 59)$$

b. $f(x) = 14x^3 - 17x^2 - 16x + 34$

We need to find x when $f(x) = 211$.

$$f(x) = 14x^3 - 17x^2 - 16x + 34$$
$$211 = 14x^3 - 17x^2 - 16x + 34$$
$$0 = 14x^3 - 17x^2 - 16x - 177$$

This is the equation obtained in part **a.** One solution is 3. It can be used to find other solutions (if they exist).

$$14x^3 - 17x^2 - 16x - 177 = 0$$
$$(x-3)(14x^2 + 25x + 59) = 0$$

The polynomial $14x^2 + 25x + 59$ cannot be factored, so the only solution is $x = 3$. The female moth's abdominal width is 3 millimeters.

52. **a.**

$$\begin{array}{r|rrrr} 2 & 2 & 14 & 0 & -72 \\ & & 4 & 36 & 72 \\ \hline & 2 & 18 & 36 & 0 \end{array}$$

$$2h^3 + 14h^2 - 72 = (h-2)(2h^2 + 18h + 36)$$

b. $V = lwh$

$$72 = (h+7)(2h)(h)$$
$$72 = 2h^3 + 14h^2$$
$$0 = 2h^3 + 14h^2 - 72$$
$$0 = (h-2)(2h^2 + 18h + 36)$$
$$0 = (h-2)(2(h^2 + 9h + 18))$$
$$0 = (h-2)(2(h+6)(h+3))$$
$$0 = 2(h-2)(h+6)(h+3)$$

$2(h-2) = 0 \quad h + 6 = 0 \quad h + 3 = 0$

$h - 2 = 0 \quad$ ~~$h = -6$~~ $\quad$ ~~$h = -3$~~

$h = 2$

The height is 2 inches, the width is $2 \cdot 2 = 4$ inches and the length is $2 + 7 = 9$ inches. The dimensions are 2 inches by 4 inches by 9 inches.

53. $A = l \cdot w$ so

$$l = \frac{A}{w} = \frac{0.5x^3 - 0.3x^2 + 0.22x + 0.06}{x + 0.2}$$

–0.2	0.5	–0.3	0.22	0.06
		–0.1	0.08	–0.06
	0.5	–0.4	0.3	0

Therefore, the length of the rectangle is $0.5x^2 - 0.4x + 0.3$ units.

54. $A = l \cdot w$ so,

$$l = \frac{A}{w} = \frac{8x^3 - 6x^2 - 5x + 3}{x + \frac{3}{4}}$$

$-\frac{3}{4}$	8	–6	–5	3
		–6	9	–3
	8	–12	4	0

Therefore, the length of the rectangle is $8x^2 - 12x + 4$ units.

55. a.

$$f(30) = \frac{80(30) - 8000}{30 - 110} = 70$$

(30, 70) At a 30% tax rate, the government tax revenue will be $70 ten billion.

b.

110	80	-8000
		8800
	80	800

$$f(x) = 80 + \frac{800}{x - 110}$$

$$f(30) = 80 + \frac{800}{80 - 110} = 70$$

(30, 70) same answer as in **a.**

c. $f(x)$ is not a polynomial function. It is a rational function because it is the quotient of two linear polynomials.

56. a. $$f(40) = \frac{80(40) - 8000}{40 - 110} = 68.57$$

(40, 68.57) At a 40% tax rate, the government's revenue is $68.57 ten billion.

b.

110	80	-8000
		8800
	80	800

$$f(x) = 80 + \frac{800}{x - 110}$$

$$f(40) = 80 + \frac{800}{40 - 110} = 68.57$$

c. $f(x)$ is not a polynomial function. It is a rational function because it is the quotient of two linear polynomials.

57. – 65. Answers may vary.

66. does not make sense; Explanations will vary. Sample explanation: The division must account for the zero coefficients on the x^4, x^3, x^2 and x terms.

67. makes sense

68. does not make sense; Explanations will vary. Sample explanation: The remainder theorem provides an alternative method for evaluating a function at a given value.

69. does not make sense; Explanations will vary. Sample explanation: The zeros of *f* are the same as the solutions of $f(x) = 0$.

70. false; Changes to make the statement true will vary. A sample change is: The degree of the quotient is 3, since $\frac{x^6}{x^3} = x^3$.

71. true

72. true

73. false; Changes to make the statement true will vary. A sample change is: The divisor is a factor of the divided only if the remainder is the whole number 0.

74.

$$
\begin{array}{r}
5x^2+2x-4 \\
4x+3\overline{)20x^3+23x^2-10x+k} \\
\underline{20x^3+15x^2} \\
8x^2-10 \\
\underline{8x^2+6x} \\
-16x+k \\
\underline{-16x-12}
\end{array}
$$

To get a remainder of zero, k must equal –12.
$k=-12$

75. $f(x)=d(x)\cdot q(x)+r(x)$

$2x^2-7x+9=d(x)(2x-3)+3$

$2x^2-7x+6=d(x)(2x-3)$

$\dfrac{2x^2-7x+6}{2x-3}=d(x)$

$$
\begin{array}{r}
x-2 \\
2x-3\overline{)2x^2-7x+6} \\
\underline{2x^2-3x} \\
-4x+6 \\
\underline{-4x+6}
\end{array}
$$

The polynomial is $x-2$.

76.

$$
\begin{array}{r}
x^{2n}-x^n+1 \\
x^n+1\overline{)x^{3n}\qquad\qquad+1} \\
\underline{x^{3n}+x^{2n}} \\
-x^{2n} \\
\underline{-x^{2n}-x^n} \\
x^n+1 \\
\underline{x^n+1} \\
0
\end{array}
$$

77. $2x-4=2(x-2)$

Use synthetic division to divide by $x-2$. Then divide the quotient by 2.

78. $x^4-4x^3-9x^2+16x+20=0$

$$
\begin{array}{r|rrrrr}
5 & 1 & -4 & -9 & 16 & 20 \\
 & & 5 & 5 & -20 & -20 \\
\hline
 & 1 & 1 & -4 & -4 & 0
\end{array}
$$

The remainder is zero and 5 is a solution to the equation.

$x^4-4x^3-9x^2+16x+20$

$=(x-5)\left(x^3+x^2-4x-4\right)$

To solve the equation, we set it equal to zero and factor.

$(x-5)\left(x^3+x^2-4x-4\right)=0$

$(x-5)\left(x^2(x+1)-4(x+1)\right)=0$

$(x-5)(x+1)\left(x^2-4\right)=0$

$(x-5)(x+1)(x+2)(x-2)=0$

Apply the zero product principle.

$x-5=0 \qquad x+1=0$

$x=5 \qquad x=-1$

$x+2=0 \qquad x-2=0$

$x=-2 \qquad x=2$

The solutions are –2, –1, 2 and 5 and the solution set is $\{-2,-1,2,5\}$.

79. $x^2+4x-1=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(4)\pm\sqrt{(4)^2-4(1)(-1)}}{2(1)}$

$x=\dfrac{-4\pm\sqrt{20}}{2}$

$x=\dfrac{-4\pm2\sqrt{5}}{2}$

$x=-2\pm\sqrt{5}$

The solution set is $\left\{-2\pm\sqrt{5}\right\}$.

80. $x^2+4x+6=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(4)\pm\sqrt{(4)^2-4(1)(6)}}{2(1)}$$

$$x=\frac{-4\pm\sqrt{-8}}{2}$$

$$x=\frac{-4\pm 2i\sqrt{2}}{2}$$

$$x=-2\pm i\sqrt{2}$$

The solution set is $\left\{-2\pm i\sqrt{2}\right\}$.

81. $f(x)=a_n(x^4-3x^2-4)$

$$f(3)=-150$$

$$a_n\left((3)^4-3(3)^2-4\right)=-150$$

$$a_n\left(81-27-4\right)=-150$$

$$a_n\left(50\right)=-150$$

$$a_n=-3$$

Section 3.4

Check Point Exercises

1. $p:\pm1,\ \pm2,\ \pm3,\ \pm6$

$q:\pm1$

$\frac{p}{q}:\pm1,\ \pm2,\ \pm3,\ \pm6$

are the possible rational zeros.

2. $p:\pm1,\ \pm3$

$q:\pm1,\ \pm2,\ \pm4$

$\frac{p}{q}:\pm1,\ \pm3,\ \pm\frac{1}{2},\ \pm\frac{1}{4},\ \pm\frac{3}{2},\ \pm\frac{3}{4}$

are the possible rational zeros.

3. $\pm1,\ \pm2,\ \pm4,\ \pm5,\ \pm10,\ \pm20$ are possible rational zeros

1	1	8	11	–20
		1	9	20
	1	9	20	0

1 is a zero.

$x^2+9x+20=0$

$(x+4)(x+5)=0$

$x=-4$ or $x=-5$

The solution set is $\{1,-4,-5\}$.

4. $\pm1,\ \pm2$ are possible rational zeros

2	1	1	-5	–2
		2	6	2
	1	3	1	0

2 is a zero.

$x^2+3x+1=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-3\pm\sqrt{3^2-4(1)(1)}}{2(1)}$$

$$=\frac{-3\pm\sqrt{5}}{2}$$

The solution set is $\left\{2,\frac{-3+\sqrt{5}}{2},\frac{-3-\sqrt{5}}{2}\right\}$.

5. $\pm1,\ \pm13$ are possible rational zeros.

1	1	–6	22	–30	13
		1	–5	17	–13
	1	–5	17	–13	0

1 is a zero.

1	1	5	17	–13
		1	–4	13
	1	–4	13	0

1 is a double root.

$x^2-4x+13=0$

$$x=\frac{4\pm\sqrt{16-52}}{2}=\frac{4\pm\sqrt{-36}}{2}=2+3i$$

The solution set is $\{1,\ 2+3i,\ 2-3i\}$.

6. $(x+3)(x-i)(x+i) = (x+3)(x^2+1)$

$f(x) = a_n(x+3)(x^2+1)$

$f(1) = a_n(1+3)(1^2+1) = 8a_n = 8$

$a_n = 1$

$f(x) = (x+3)(x^2+1)$ or x^3+3x^2+x+3

7. $f(x) = x^4 - 14x^3 + 71x^2 - 154x + 120$

$f(-x) = x^4 + 14x^3 + 71x^2 + 154x + 120$

Since $f(x)$ has 4 changes of sign, there are 4, 2, or 0 positive real zeros.
Since $f(-x)$ has no changes of sign, there are no negative real zeros.

Exercise Set 3.4

1. $f(x) = x^3 + x^2 - 4x - 4$

$p: \pm 1, \pm 2, \pm 4$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

2. $f(x) = x^3 + 3x^2 - 6x - 8$

$p: \pm 1, \pm 2, \pm 4, \pm 8$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8$

3. $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$

p: ±1, ±2, ±3, ±6

q: ±1, ±3

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{3}, \pm\frac{2}{3}$

4. $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$

p: ±1, ±3, ±5, ±15

q: ±1, ±2

$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

5. $f(x) = 4x^4 - x^3 + 5x^2 - 2x - 6$

p: ±1, ±2, ±3, ±6

q: ±1, ±2, ±4

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{3}{2}, \pm\frac{3}{4}$

6. $f(x) = 3x^4 - 11x^3 - 3x^2 - 6x + 8$

p: ±1, ±2, ±4, ±8

q: ±1, ±3

$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

7. $f(x) = x^5 - x^4 - 7x^3 + 7x^2 - 12x - 12$

$p: \pm 1, \pm 2, \pm 3 \pm 4 \pm 6 \pm 12$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 2, \pm 3 \pm 4 \pm 6 \pm 12$

8. $f(x) = 4x^5 - 8x^4 - x + 2$

$p: \pm 1, \pm 2$

$q: \pm 1, \pm 2, \pm 4$

$\frac{p}{q}: \pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{4}$

9. $f(x) = x^3 + x^2 - 4x - 4$

a. $p: \pm 1, \pm 2, \pm 4$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

b.

2	1	1	–4	–4
		2	6	4
	1	3	2	0

2 is a zero.
2, –2, –1 are rational zeros.

c. $x^3 + x^2 - 4x - 4 = 0$

$(x-2)(x^2+3x+2) = 0$

$(x-2)(x+2)(x+1) = 0$

$x-2=0 \quad x+2=0 \quad x+1=0$

$x = 2, \quad x = -2, \quad x = -1$

The solution set is {2, –2, –1}.

10. **a.** $f(x) = x^3 - 2x - 11x + 12$

p: ±1, ±2, ±3, ±4, ±6, ±12
q: ±1

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b.

4	1	–2	–11	12
		4	8	–12
	1	2	–3	0

4 is a zero.
4, –3, 1 are rational zeros.

c. $x^3 - 2x^2 - 11x + 12 = 0$

$(x-4)\ \left(x^2 + 2x - 3\right) = 0$

$(x-4)\ (x+3)(x-1) = 0$

$x = 4,\ \ x = -3,\ \ x = 1$

The solution set is{4, –3, 1}.

11. $f(x) = 2x^3 - 3x^2 - 11x + 6$

a. $p: \pm 1, \pm 2,\ \pm 3, \pm 6$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

b.

3	2	–3	–11	6
		6	9	–6
	2	3	–2	0

3 is a zero.

$3, \frac{1}{2}, -2$ are rational zeros.

c. $2x^3 - 3x^2 - 11x + 6 = 0$

$(x-3)(2x^2 + 3x - 2) = 0$

$(x-3)(2x-1)(x+2) = 0$

$x = 3,\ x = \frac{1}{2},\ x = -2$

The solution set is $\left\{3, \frac{1}{2}, -2\right\}$.

12. **a.** $f(x) = 2x^3 - 5x^2 + x + 2$

p: ±1, ±2
q: ±1, ±2

$\frac{p}{q}: \pm 1, \pm 2, \pm \frac{1}{2}$

b.

2	2	–5	1	2
		4	–2	–2
	2	–1	–1	0

2 is a zero.

$2, -\frac{1}{2}, 1$ are rational zeros.

c. $2x^3 - 5x^2 + x + 2 = 0$

$(x-2)\ \left(2x^2 - x - 1\right) = 0$

$(x-2)\ \ (2x+1)(x-1) = 0$

$x = 2,\ \ x = -\frac{1}{2},\ \ x = 1$

The solution set is $\left\{2, -\frac{1}{2}, 1\right\}$.

13. **a.** $f(x) = x^3 + 4x^2 - 3x - 6$

p: ±1, ±2, ±3, ±6
q: ±1

$\frac{p}{q}$: ±1, ±2, ±3, ±6

b.

–1	1	4	–3	–6
		–1	–3	6
	1	3	–6	0

–1 is a rational zero.

c. $x^2 + 3x - 6 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-6)}}{2(1)}$

$= \frac{-3 \pm \sqrt{33}}{2}$

The solution set is $\left\{-1, \frac{-3+\sqrt{33}}{2}, \frac{-3-\sqrt{33}}{2}\right\}$.

14. a. $f(x) = 2x^3 + x^2 - 3x + 1$

p: ±1
q: ±1, ±2

$\frac{p}{q}: \pm 1, \pm \frac{1}{2}$

b.

$\frac{1}{2}$	2	1	–3	1
		1	1	–1
	2	2	–2	0

$\frac{1}{2}$ is a rational zero.

c. $2x^2 + 2x - 2 = 0$

$x^2 + x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

The solution set is $\left\{ \frac{1}{2}, \frac{-1+\sqrt{5}}{2}, \frac{-1-\sqrt{5}}{2} \right\}$.

15. a. $f(x) = 2x^3 + 6x^2 + 5x + 2$

p: ±1, ±2
q: ±1, ±2

$\frac{p}{q}: \pm 1, \pm 2, \pm \frac{1}{2}$

b.

–2	2	6	5	2
		–4	–4	–2
	2	2	1	0

–2 is a rational zero.

c. $2x^2 + 2x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{-2 \pm \sqrt{-4}}{4}$$

$$= \frac{-2 \pm 2i}{4}$$

$$= \frac{-1 \pm i}{2}$$

The solution set is $\left\{ -2, \frac{-1+i}{2}, \frac{-1-i}{2} \right\}$.

16. a. $f(x) = x^3 - 4x^2 + 8x - 5$

p: ±1, ±5
q: ±1

$\frac{p}{q}: \pm 1, \pm 5$

b.

1	1	–4	8	–5
		1	–3	5
	1	–3	5	0

1 is a rational zero.

c. $x^2 - 3x + 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{-11}}{2}$$

$$= \frac{3 \pm i\sqrt{11}}{2}$$

The solution set is $\left\{ 1, \frac{3+i\sqrt{11}}{2}, \frac{3-i\sqrt{11}}{2} \right\}$.

17. $x^3 - 2x^2 - 11x + 12 = 0$

a. $p: \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$
$q: \pm1$
$\frac{p}{q}: \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

b.

4	1	–2	–11	12
		4	8	–12
	1	2	–3	0

4 is a root.
–3, 1, 4 are rational roots.

c. $x^3 - 2x^2 - 11x + 12$
$(x-4)(x^2+2x-3) = 0$
$(x-4)(x+3)(x-1) = 0$
$x-4=0 \quad x+3=0 \quad x-1=0$
$x=4 \quad x=-3 \quad x=1$
The solution set is $\{-3, 1, 4\}$.

18. a. $x^3 - 2x^2 - 7x - 4 = 0$
p: ±1, ±2, ±4
q: ±1
$\frac{p}{q}: \pm1, \pm2, \pm4$

b.

4	1	–2	–7	–4
		4	8	4
	1	2	1	0

4 is a root.
–1, 4 are rational roots.

c. $x^3 + 2x^2 - 7x - 4 = 0$
$(x-4)\ (x^2+2x+1) = 0$
$(x-4) \quad (x+1)^2$
$x = 4, \quad x = -1$
The solution set is $\{4, -1\}$.

19. $x^3 - 10x - 12 = 0$

a. $p: \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$
$q: \pm1$
$\frac{p}{q}: \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

b.

–2	1	0	–10	–12
		–2	4	12
	1	–2	–6	0

–2 is a rational root.

c. $x^3 - 10x - 12 = 0$
$(x+2)(x^2-2x-6) = 0$
$x = \frac{2 \pm \sqrt{4+24}}{2} = \frac{2 \pm \sqrt{28}}{2}$
$= \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$
The solution set is $\left\{-2, 1+\sqrt{7}, 1-\sqrt{7}\right\}$.

20. a. $x^3 - 5x^2 + 17x - 13 = 0$
p: ±1, ±13
q: ±1
$\frac{p}{q}: \pm1, \pm13$

b.

1	1	–5	17	–13
		1	–4	13
	1	–4	13	0

1 is a rational root.

c. $x^3 - 5x^2 + 17x - 13 = 0$
$(x-1)\ (x^2 - 4x + 13) = 0$
$x = \frac{4 + \sqrt{16-52}}{2} = \frac{4 \pm \sqrt{-36}}{2}$
$= \frac{4 \pm 6i}{2} = 2 \pm 3i$
The solution set is $\{1, 2+3i, 2, -3i\}$.

21. $6x^3 + 25x^2 - 24x + 5 = 0$

a. $p: \pm1, \pm5$
$q: \pm1, \pm2, \pm3, \pm6$
$\frac{p}{q}: \pm1, \pm5, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$

b.

–5	6	25	–24	5
		–30	25	–5
	6	–5	1	0

–5 is a root.
$-5, \frac{1}{2}, \frac{1}{3}$ are rational roots.

c. $6x^3 + 25x^2 - 24x + 5 = 0$
$(x+5)(6x^2 - 5x + 1) = 0$
$(x+5)(2x-1)(3x-1) = 0$
$x+5=0 \quad 2x-1=0 \quad 3x-1=0$
$x=-5, \quad x=\frac{1}{2}, \quad x=\frac{1}{3}$
The solution set is $\left\{-5, \frac{1}{2}, \frac{1}{3}\right\}$.

22. a. $2x^3 - 5x^2 - 6x + 4 = 0$
p: ±1, ±2, ±4
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm 2 \pm 4 \pm \frac{1}{2}$

b.

$\frac{1}{2}$	2	–5	–6	4
		1	–2	–4
	2	–4	–8	0

$\frac{1}{2}$ is a rational root.

c. $2x^3 - 5x^2 - 6x + 4 = 0$
$(x - \frac{1}{2})\left(2x^2 - 4x - 8\right) = 0$
$2\left(x - \frac{1}{2}\right)\left(x^2 - 2x - 4\right) = 0$
$x = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$
The solution set is $\left\{\frac{1}{2}, 1+\sqrt{5}, 1-\sqrt{5}\right\}$.

23. $x^4 - 2x^3 - 5x^2 + 8x + 4 = 0$

a. $p: \pm 1, \pm 2, \pm 4$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

b.

2	1	–2	–5	8	4
		2	0	–10	–4
	1	0	–5	–2	0

2 is a root.
$-2, 2$ are rational roots.

c. $x^4 - 2x^3 - 5x^2 + 8x + 4 = 0$
$(x-2)(x^3 - 5x - 2) = 0$

–2	1	0	–5	–2
		–2	4	2
	1	–2	–1	0

–2 is a zero of $x^3 - 5x - 2 = 0$.

$(x-2)(x+2)\left(x^2 - 2x - 1\right) = 0$
$x = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$
$= 1 \pm \sqrt{2}$
The solution set is
$\left\{-2, 2, 1+\sqrt{2}, 1-\sqrt{2}\right\}$.

24. a. $x^4 - 2x^2 - 16x - 15 = 0$
p: ±1, ±3, ±5, ±15
q: ±1
$\frac{p}{q}: \pm 1, \pm 3 \pm 5 \pm 15$

b.

3	1	0	–2	–16	–15
		3	9	21	15
	1	3	7	5	0

3 is a root.
$-1, 3$ are rational roots.

c. $x^4 - 2x^2 - 16x - 15 = 0$
$(x-3)\left(x^3 + 3x^2 + 7x + 5\right) = 0$

–1	1	3	7	5
		–1	–2	–5
	1	2	5	0

-1 is a root of $x^3 + 3x^2 + 7x + 5$

$(x-3)\ (x+1)\ \left(x^2 + 2x + 5\right)$
$x = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm \sqrt{-16}}{2}$
$= \frac{-2 \pm 4i}{2} = -1 \pm 2i$
The solution set is $\{3, -1, -1+2i, -1-2i\}$.

25. $(x-1)\ (x+5i)\ (x-5i)$

$= (x-1)\left(x^2+25\right)$

$= x^3+25x-x^2-25$

$= x^3-x^2+25x-25$

$f(x)=a_n\left(x^3-x^2+25x-25\right)$

$f(-1)=a_n(-1-1-25-25)$

$-104=a_n(-52)$

$a_n=2$

$f(x)=2\left(x^3-x^2+25x-25\right)$

$f(x)=2x^3-2x^2+50x-50$

26. $(x-4)\ (x+2i)\ (x-2i)$

$= (x-4)\left(x^2+4\right)$

$= x^3-4x^2+4x-16$

$f(x)=a_n\left(x^3-4x^2+4x-16\right)$

$f(-1)=a_n(-1-4-4-16)$

$-50=a_n(-25)$

$a_n=2$

$f(x)=2\left(x^3-4x^2+4x-16\right)$

$f(x)=2x^3-8x^2+8x-32$

27. $(x+5)(x-4-3i)(x-4+3i)$

$=(x+5)\left(x^2-4x+3ix-4x+16-12i\right.$

$\left.-3ix+12i-9i^2\right)$

$=(x+5)\left(x^2-8x+25\right)$

$=\left(x^3-8x^2+25x+5x^2-40x+125\right)$

$=x^3-3x^2-15x+125$

$f(x)=a_n(x^3-3x^2-15x+125)$

$f(2)=a_n\left(2^3-3(2)^2-15(2)+125\right)$

$91=a_n(91)$

$a_n=1$

$f(x)=1\left(x^3-3x^2-15x+125\right)$

$f(x)=x^3-3x^2-15x+125$

28. $(x-6)\ (x+5+2i)\ (x+5-2i)$

$=(x-6)\left(x^2+5x-2ix+5x+25-10i+2ix+10i-4i^2\right)$

$=(x-6)\left(x^2+10x+29\right)$

$=x^3+10x^2+29x-6x^2-60x-174$

$=x^3+4x^2-31x-174$

$f(x)=a_n\left(x^3+4x^2-31x-174\right)$

$f(2)=a_n(8+16-62-174)$

$-636=a_n(-212)$

$a_n=3$

$f(x)=3\left(x^3+4x^2-31x-174\right)$

$f(x)=3x^3+12x^2-93x-522$

29. $(x-i)(x+i)(x-3i)(x+3i)$

$=\left(x^2-i^2\right)\left(x^2-9i^2\right)$

$=\left(x^2+1\right)\left(x^2+9\right)$

$=x^4+10x^2+9$

$f(x)=a_n(x^4+10x^2+9)$

$f(-1)=a_n((-1)^4+10(-1)^2+9)$

$20=a_n(20)$

$a_n=1$

$f(x)=x^4+10x^2+9$

30. $(x+2)\left(x+\frac{1}{2}\right)(x-i)(x+i)$

$=\left(x^2+\frac{5}{2}x+1\right)\left(x^2+1\right)$

$=x^4+x^2+\frac{5}{2}x^3+\frac{5}{2}x+x^2+1$

$=x^4+\frac{5}{2}x^3+2x^2+\frac{5}{2}x+1$

$f(x)=a_n\left(x^4+\frac{5}{2}x^3+2x^2+\frac{5}{2}x+1\right)$

$f(1)=a_n\left[(1)^4+\frac{5}{2}(1)^3+2(1)^2+\frac{5}{2}(1)+1\right]$

$18=a_n(9)$

$a_n=2$

$f(x)=2\left(x^4+\frac{5}{2}x^3+2x^2+\frac{5}{2}x+1\right)$

$f(x)=2x^4+5x^3+4x^2+5x+2$

31. $(x+2)\ (x-5)\ (x-3+2i)\ (x-3-2i)$

$=\left(x^2-3x-10\right)\left(x^2-3x-2ix-3x+9+6i+2ix-6i-4i^2\right)$

$=\left(x^2-3x-10\right)\left(x^2-6x+13\right)$

$=x^4-6x+13x^2-3x^3+18x^2-39x-10x^2+60x-130$

$=x^4-9x^3+21x^2+21x-130$

$f(x)=a_n\left(x^4-9x^3+21x^2+21x-130\right)$

$f(1)=a_n(1-9+21+21-130)$

$-96=a_n(-96)$

$a_n=1$

$f(x)=x^4-9x^3+21x^2+21x-130$

32. $(x+4)\ (3x-1)\ (x-2+3i)\ (x-2-3i)$

$=\left(3x^2+11x-4\right)\left(x^2-2x-3ix-2x+4+6i+3ix-6i-9i^2\right)$

$=\left(3x^2+11x-4\right)\left(x^2-4x+13\right)$

$=3x^4-12x^3+39x^2+11x^3-44x^2+143x-4x^2+16x-52$

$=3x^4-x^3-9x^2+159x-52$

$f(x)=a_n\left(3x^4-x^3-9x^2+159x-52\right)$

$f(1)=a_n(3-1-9+159-52)$

$100=a_n(100)$

$a_n=1$

$f(x)=3x^4-x^3-9x^2+159x-52$

33. $f(x)=x^3+2x^2+5x+4$

Since $f(x)$ has no sign variations,
no positive real roots exist.

$f(-x)=-x^3+2x^2-5x+4$

Since $f(-x)$ has 3 sign variations,
3 or 1 negative real roots exist.

34. $f(x)=x^3+7x^2+x+7$

Since $f(x)$ has no sign variations no positive real roots exist.

$f(-x)=-x^3+7x^2-x+7$

Since $f(-x)$ has 3 sign variations, 3 or 1 negative real roots exist.

35. $f(x)=5x^3-3x^2+3x-1$

Since $f(x)$ has 3 sign variations, 3 or 1 positive real roots exist.

$f(-x)=-5x^3-3x^2-3x-1$

Since $f(-x)$ has no sign variations, no negative real roots exist.

36. $f(x) = -2x^3 + x^2 - x + 7$
Since $f(x)$ has 3 sign variations,
3 or 1 positive real roots exist.
$f(-x) = 2x^3 + x^2 + x + 7$
Since $f(-x)$ has no sign variations,
no negative real roots exist.

37. $f(x) = 2x^4 - 5x^3 - x^2 - 6x + 4$
Since $f(x)$ has 2 sign variations, 2 or 0 positive real roots exist.
$f(-x) = 2x^4 + 5x^3 - x^2 + 6x + 4$
Since $f(-x)$ has 2 sign variations, 2 or 0 negative real roots exist.

38. $f(x) = 4x^4 - x^3 + 5x^2 - 2x - 6$
Since $f(x)$ has 3 sign variations, 3 or 1 positive real roots exist.
$f(-x) = 4x^4 + x^3 + 5x^2 + 2x - 6$
Since $f(x)$ has 1 sign variations, 1 negative real roots exist.

39. $f(x) = x^3 - 4x^2 - 7x + 10$
$p: \pm 1, \pm 2, \pm 5, \pm 10$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 5, \pm 10$
Since $f(x)$ has 2 sign variations, 0 or 2 positive real zeros exist.
$f(-x) = -x^3 - 4x^2 + 7x + 10$
Since $f(-x)$ has 1 sign variation, exactly one negative real zeros exists.

–2	1	–4	–7	10
		–2	12	–10
	1	–6	5	0

–2 is a zero.

$$f(x) = (x+2)(x^2 - 6x + 5)$$
$$= (x+2)(x-5)(x-1)$$

$x = -2, x = 5, x = 1$
The solution set is $\{-2, 5, 1\}$.

40. $f(x) = x^3 + 12x^2 + 2x + 10$
p: ±1, ±2, ±5, ±10
q: ±1,
$\frac{p}{q}: \pm 1, \pm 2 \pm 5 \pm 10$

Since $f(x)$ has no sign variations, no positive zeros exist.
$f(-x) = -x^3 + 12x^2 - 21x + 10$
Since $f(-x)$ has 3 sign variations, 3 or 1 negative zeros exist.

–1	1	12	21	10
		–1	–11	–10
	1	11	10	0

–1 is a zero.

$$f(x) = (x+1)(x^2 + 11x + 10)$$
$$= (x+1)(x+10)(x+1)$$
$$x = -1, x = -10$$

The solution set is $\{-1, -10\}$.

41. $2x^3 - x^2 - 9x - 4 = 0$
$p: \pm 1, \pm 2, \pm 4$
$q: \pm 1, \pm 2$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4 \pm \frac{1}{2}$
1 positive real root exists.
$f(-x) = -2x^3 - x^2 + 9x - 4$ 2 or no negative real roots exist.

$-\frac{1}{2}$	2	–1	–9	–4
		–1	1	4
	2	–2	–8	0

$-\frac{1}{2}$ is a root.

$$\left(x + \frac{1}{2}\right)(2x^2 - 2x - 8) = 0$$
$$2\left(x + \frac{1}{2}\right)(x^2 - x - 4) = 0$$
$$x = \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$$

The solution set is $\left\{-\frac{1}{2}, \frac{1+\sqrt{17}}{2}, \frac{1-\sqrt{17}}{2}\right\}$.

42. $3x^3-8x^2-8x+8=0$

p: ±1, ±2, ±4, ±8

q: ±1, ±3

$\frac{p}{q}: \pm1, \pm2 \pm4 \pm8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

Since $f(x)$ has 2 sign variations, 2 or no positive real roots exist.

$$f(-x)=-3x^3-8x^2+8x+8$$

Since $f(-x)$ has 1 sign changes, exactly 1 negative real zero exists.

$\frac{2}{3}$	3	–8	–8	8
		2	–4	–8
	3	–6	–12	0

$\frac{2}{3}$ is a zero.

$$f(x)=\left(x-\frac{2}{3}\right)\left(3x^2-6x-12\right)$$

$$x=\frac{6\pm\sqrt{36+144}}{6}=\frac{6\pm6\sqrt{5}}{6}$$

$$=1\pm\sqrt{5}$$

The solution set is $\left\{\frac{2}{3}, 1+\sqrt{5}, 1-\sqrt{5}\right\}$.

43. $f(x)=x^4-2x^3+x^2+12x+8$

$p: \pm1,\ \pm2,\ \pm4,\ \pm8$

$q: \pm1$

$\frac{p}{q}: \pm1,\ \pm2,\ \pm4,\ \pm8$

Since $f(x)$ has 2 sign changes, 0 or 2 positive roots exist.

$f(-x)=(-x)^4-2(-x)^3+(-x)^2-12x+8$

$\quad =x^4+2x^3+x^2-12x+8$

Since $f(-x)$ has 2 sign changes, 0 or 2 negative roots exist.

–1	1	–2	1	12	8
		–1	4	–4	–8
	1	–3	4	8	0

–1	1	–3	4	8
		–1	4	–8
	1	–4	8	0

$0=x^2-4x+8$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(8)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{16-32}}{2}$$

$$x=\frac{4\pm\sqrt{-16}}{2}$$

$$x=\frac{4\pm4i}{2}$$

$$x=2\pm2i$$

The solution set is { –1, –1, 2 + 2i, 2 – 2i}.

44. $f(x)=x^4-4x^3-x^2+14x+10$

p: ±1, ±2, ±5, ±10

q: ±1

$\frac{p}{q}$: ±1, ±2, ±5, ±10

–1	1	-4	-1	14	10
		–1	5	-4	-10
	1	-5	4	10	0

–1	1	-5	4	10
		–1	6	–10
	1	-6	10	0

$$f(x)=(x-1)(x-1)(x^2-6x+10)$$

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(10)}}{2(1)} \qquad x=1$$

$$x=\frac{6\pm\sqrt{36-40}}{2}$$

$$x=\frac{6\pm\sqrt{-4}}{2}$$

$$x=\frac{6\pm2i}{2}$$

$$x=3\pm i$$

The solution set is {–1, 3 – i, 3 + i}

45. $x^4-3x^3-20x^2-24x-8=0$
$p: \pm 1, \pm 2, \pm 4, \pm 8$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4 \pm 8$
1 positive real root exists.
3 or 1 negative real roots exist.

–1	1	–3	–20	–24	–8
		–1	4	16	8
	1	–4	–16	–8	0

$(x+1)\left(x^3-4x^2-16x-8\right)=0$

–2	1	–4	–16	–8
		–2	12	8
	1	–6	–4	0

$(x+1)(x+2)\left(x^2-6x-4\right)=0$

$x=\frac{6\pm\sqrt{36+16}}{2}=\frac{6\pm\sqrt{52}}{2}$

$=\frac{6\pm 2\sqrt{13}}{2}=\frac{3\pm\sqrt{13}}{2}$

The solution set is
$\left\{-1, -2, 3\pm\sqrt{13}, 3-\sqrt{13}\right\}$.

46. $x^4-x^3+2x^2-4x-8=0$
p: ±1, ±2, ±4, ±8
q: ±1
$\frac{p}{q}: \pm 1, \pm 2 \pm 4 \pm 8$
1 negative real root exists.

–1	1	–1	2	–4	–8
		–1	2	–4	8
	1	–2	4	–8	0

$(x+1)\left(x^3-2x^2+4x-8\right)$

2	1	–2	4	–8
		2	0	8
	1	0	4	0

$(x+1)\quad (x-2)\quad \left(x^2+4\right)$

$x+1=0\quad x-2=0\quad x^2+4=0$

$x=-1\quad x=2\quad x^2=-4$

$x=\pm 2i$

The solution set is $\{-1, 2, 2i, -2i\}$.

47. $f(x)=3x^4-11x^3-x^2+19x+6$
$p: \pm 1, \pm 2, \pm 3, \pm 6$
$q: \pm 1, \pm 3$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{3}, \pm\frac{2}{3}$
2 or no positive real zeros exists.
$f(-x)=3x^4+11x^3-x^2-19x+6$
2 or no negative real zeros exist.

–1	3	–11	–1	19	6
		–3	14	–13	–6
	3	–14	13	6	0

$f(x)=(x+1)\left(3x^3-14x^2+13x+6\right)$

2	3	–14	13	6
		6	–16	–6
	3	–8	–3	0

$f(x)=(x+1)(x-2)\left(3x^2-8x-3\right)$
$=(x+1)(x-2)(3x+1)(x-3)$

$x=-1,\ x=2\ x=-\frac{1}{3},\ x=3$

The solution set is $\left\{-1, 2, -\frac{1}{3}, 3\right\}$.

48. $f(x)=2x^4+3x^3-11x^2-9x+15$
p: ±1, ±3, ±5, ±15
q: ±1, ±2
$\frac{p}{q}: \pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$
2 or no positive real zeros exist.
$f(-x)=2x^4-3x^3-11x^2+9x+15$
2 or no negative real zeros exist.

1	2	3	–11	–9	15
		2	5	–6	–15
	2	5	–6	–15	0

$f(x)=(x-1)\left(2x^3+5x^2-6x-15\right)$

$-\frac{5}{2}$	2	5	–6	–15
		–5	0	15
	2	0	–6	0

$f(x)=(x-1)\left(x+\frac{5}{2}\right)\left(2x^2-6\right)$
$=2(x-1)\left(x+\frac{5}{2}\right)\left(x^2-3\right)$
$x^2-3=0$
$x^2=3$
$x=\pm\sqrt{3}$
$x=1,\ x=-\frac{5}{2},\ x=\sqrt{3},\ x=-\sqrt{3}$
The solution set is $\left\{1,-\frac{5}{2},\sqrt{3},-\sqrt{3}\right\}$.

49. $4x^4-x^3+5x^2-2x-6=0$
$p: \pm1, \pm2, \pm3, \pm6$
$q: \pm1, \pm2, \pm4$
$\frac{p}{q}: \pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$
3 or 1 positive real roots exists.
1 negative real root exists.

1	4	–1	5	–2	–6
		4	3	8	6
	4	3	8	6	0

$(x-1)(4x^3+3x^2+8x+6)=0$
$4x^3+3x^2+8x+6=0$ has no positive real roots.

$-\frac{3}{4}$	4	3	8	6
		–3	0	–6
	4	0	8	0

$(x-1)\left(x+\frac{3}{4}\right)\left(4x^2+8\right)=0$
$4(x-1)\left(x+\frac{3}{4}\right)\left(x^2+2\right)=0$
$x^2+2=0$
$x^2=-2$
$x=\pm i\sqrt{2}$
The solution set is $\left\{1, -\frac{3}{4}, i\sqrt{2}, -i\sqrt{2}\right\}$.

50. $3x^4-11x^3-3x^2-6x+8=0$
p: ±1, ±2, ±4, ±8
q: ±1, ±3
$\frac{p}{q}: \pm1, \pm2 \pm4 \pm8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$
2 or no positive real roots exist.
$f(-x)=3x^4+11x^3-3x^2+6x+8$ 2 or no negative real roots exist.

4	3	–11	–3	–6	8
		12	4	4	–8
	3	1	1	–2	0

$(x-4)\left(3x^3+x^2+x-2\right)=0$
Another positive real root must exist.

$\frac{2}{3}$	3	1	1	–2
		2	2	2
	3	3	3	0

$(x-4)\left(x-\frac{2}{3}\right)\left(3x^2+3x+3\right)=0$
$3(x-4)\left(x-\frac{2}{3}\right)\left(x^2+x+1\right)=0$
$x=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm i\sqrt{3}}{2}$
The solution set is $\left\{4, \frac{2}{3}, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right\}$.

51. $2x^5 + 7x^4 - 18x^2 - 8x + 8 = 0$

$p: \pm 1, \pm 2, \pm 4, \pm 8$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$

2 or no positive real roots exists.
3 or 1 negative real root exist.

–2	2	7	0	–18	–8	8
		–4	–6	12	12	–8
	2	3	–6	–6	4	0

$(x+2)(2x^4 + 3x^3 - 6x^2 - 6x + 4) = 0$

$4x^3 + 3x^2 + 8x + 6 = 0$ has no positive real roots.

–2	2	3	–6	–6	4
		–4	2	8	–4
	2	–1	–4	2	0

$(x+2)^2(2x^3 - x^2 - 4x + 2)$

$\frac{1}{2}$	2	–1	–4	2
		1	0	2
	2	0	–4	0

$(x+2)^2\left(x - \frac{1}{2}\right)\left(2x^2 - 4\right) = 0$

$2(x+2)^2\left(x - \frac{1}{2}\right)\left(x^2 - 2\right) = 0$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

The solution set is $\left\{-2, \frac{1}{2}, \sqrt{2}, -\sqrt{2}\right\}$.

52. $4x^5 + 12x^4 - 41x^3 - 99x^2 + 10x + 24 = 0$

p: ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24

q: ±1, ±2, ±4

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}$

2 or no positive real roots exist.

$f(-x) = -4x^5 + 12x^4 + 41x^3 - 99x^2 - 10x + 24$

3 or 1 negative real roots exist.

3	4	12	–41	–99	10	24
		12	72	93	–18	–24
	4	24	31	–6	–8	0

$(x-3)\left(4x^4 + 24x^3 + 31x^2 - 6x - 8\right) = 0$

–2	4	24	31	–6	–8
		–8	–32	2	8
	4	16	–1	–4	0

$(x-3)(x+2)\left(4x^3 + 16x^2 - x - 4\right) = 0$

–4	4	16	–1	4
		–16	0	4
	4	0	–1	0

$(x-3)(x+2)(x+4)\left(4x^2 - 1\right) = 0$

$4x^2 - 1 = 0$

$4x^2 = 1$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$

The solution set is $\left\{3, -2, -4, \frac{1}{2}, -\frac{1}{2}\right\}$.

53. $f(x) = -x^3 + x^2 + 16x - 16$

a. From the graph provided, we can see that -4 is an x-intercept and is thus a zero of the function. We verify this below:

–4	–1	1	16	–16
		4	–20	16
	–1	5	–4	0

Thus, $-x^3 + x^2 + 16x - 16 = 0$

$(x+4)\left(-x^2 + 5x - 4\right) = 0$

$-(x+4)\left(x^2 - 5x + 4\right) = 0$

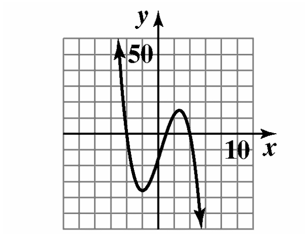
$-(x+4)(x-1)(x-4) = 0$

$x + 4 = 0$ or $x - 1 = 0$ or $x - 4 = 0$

$x = -4$ $\quad x = 1$ $\quad x = 4$

The zeros are -4, 1, and 4.

b.



$f(x) = -x^3 + x^2 + 16x - 16$

54. $f(x) = -x^3 + 3x^2 - 4$

a. From the graph provided, we can see that -1 is an x-intercept and is thus a zero of the function. We verify this below:

$\underline{-1}\rfloor$	-1	3	0	-4
		1	-4	4
	-1	4	-4	0

Thus,
$$-x^3 + 3x^2 - 4 = 0$$
$$(x+1)(-x^2 + 4x - 4) = 0$$
$$-(x+1)(x^2 - 4x + 4) = 0$$
$$-(x+1)(x-2)^2 = 0$$

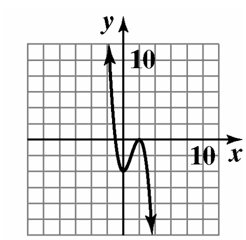
$x + 1 = 0$ or $(x-2)^2 = 0$

$x = -1$ $\qquad x - 2 = 0$

$x = 2$

The zeros are -1 and 2.

b.



$f(x) = -x^3 + 3x^2 - 4$

55. $f(x) = 4x^3 - 8x^2 - 3x + 9$

a. From the graph provided, we can see that -1 is an x-intercept and is thus a zero of the function. We verify this below:

$\underline{-1}\rfloor$	4	-8	-3	9
		-4	12	-9
	4	-12	9	0

Thus,
$$4x^3 - 8x^2 - 3x + 9 = 0$$
$$(x+1)(4x^2 - 12x + 9) = 0$$
$$(x+1)(2x-3)^2 = 0$$

$x + 1 = 0$ or $(2x-3)^2 = 0$

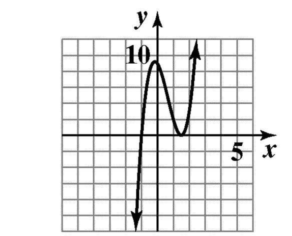
$x = -1$ $\qquad 2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

The zeros are -1 and $\frac{3}{2}$.

b.



$f(x) = 4x^3 - 8x^2 - 3x + 9$

56. $f(x) = 3x^3 + 2x^2 + 2x - 1$

a. From the graph provided, we can see that $\frac{1}{3}$ is an x-intercept and is thus a zero of the function. We verify this below:

$\underline{\frac{1}{3}}\rfloor$	3	2	2	-1
		1	1	1
	3	3	3	0

Thus,
$$3x^3 + 2x^2 + 2x - 1 = 0$$
$$\left(x - \frac{1}{3}\right)(3x^2 + 3x + 3) = 0$$
$$3\left(x - \frac{1}{3}\right)(x^2 + x + 1) = 0$$

Note that $x^2 + x + 1$ will not factor, so we use the quadratic formula:

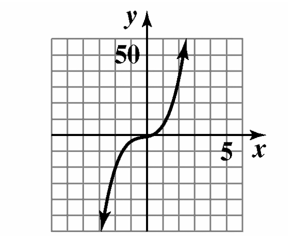
$x - \frac{1}{3} = 0$ or $x^2 + x + 1 = 0$

$x = \frac{1}{3}$ $\qquad a = 1 \quad b = 1 \quad c = 1$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{-3}}{2}$$
$$= \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

The zeros are $\frac{1}{3}$ and $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.

b.



$f(x) = 3x^3 + 2x^2 + 2x - 1$

57. $f(x) = 2x^4 - 3x^3 - 7x^2 - 8x + 6$

a. From the graph provided, we can see that $\frac{1}{2}$ is an x-intercept and is thus a zero of the function. We verify this below:

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & -3 & -7 & -8 & 6 \\ & & 1 & -1 & -4 & -6 \\ \hline & 2 & -2 & -8 & -12 & 0 \end{array}$$

Thus,
$$2x^4 - 3x^3 - 7x^2 - 8x + 6 = 0$$
$$\left(x - \frac{1}{2}\right)\left(2x^3 - 2x^2 - 8x - 12\right) = 0$$
$$2\left(x - \frac{1}{2}\right)\left(x^3 - x^2 - 4x - 6\right) = 0$$

To factor $x^3 - x^2 - 4x - 6$, we use the Rational Zero Theorem to determine possible rational zeros.

Factors of the constant term -6:
$\pm 1, \ \pm 2, \ \pm 3, \ \pm 6$
Factors of the leading coefficient 1: ± 1

The possible rational zeros are:

$$\frac{\text{Factors of } -6}{\text{Factors of } 1} = \frac{\pm 1, \ \pm 2, \ \pm 3, \ \pm 6}{\pm 1}$$
$$= \pm 1, \ \pm 2, \ \pm 3, \ \pm 6$$

We test values from above until we find a zero. One possibility is shown next:

Test 3:

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -4 & -6 \\ & & 3 & 6 & 6 \\ \hline & 1 & 2 & 2 & 0 \end{array}$$

The remainder is 0, so 3 is a zero of *f*.

$$2x^4 - 3x^3 - 7x^2 - 8x + 6 = 0$$
$$\left(x - \frac{1}{2}\right)\left(2x^3 - 2x^2 - 8x - 12\right) = 0$$
$$2\left(x - \frac{1}{2}\right)\left(x^3 - x^2 - 4x - 6\right) = 0$$
$$2\left(x - \frac{1}{2}\right)(x - 3)\left(x^2 + 2x + 2\right) = 0$$

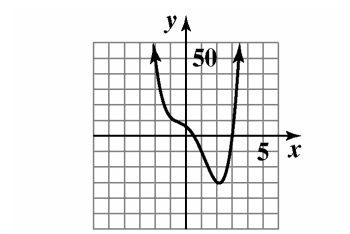
Note that $x^2 + x + 1$ will not factor, so we use the quadratic formula:

$a = 1 \quad b = 2 \quad c = 2$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)}$$
$$= \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

The zeros are $\frac{1}{2}$, 3, and $-1 \pm i$.

b.



$f(x) = 2x^4 - 3x^3 - 7x^2 - 8x + 6$

58. $f(x) = 2x^4 + 2x^3 - 22x^2 - 18x + 36$

a. From the graph provided, we can see that 1 and 3 are x-intercepts and are thus zeros of the function. We verify this below:

$$\begin{array}{r|rrrrr} 1 & 2 & 2 & -22 & -18 & 36 \\ & & 2 & 4 & -18 & -36 \\ \hline & 2 & 4 & -18 & -36 & 0 \end{array}$$

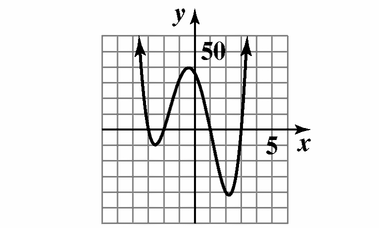
Thus, $2x^4 + 2x^3 - 22x^2 - 18x + 36$
$$= (x - 1)\left(2x^3 + 4x^2 - 18x - 36\right)$$

$$\begin{array}{r|rrrr} 3 & 2 & 4 & -18 & -36 \\ & & 6 & 30 & 36 \\ \hline & 2 & 10 & 12 & 0 \end{array}$$

Thus,
$$2x^4 + 2x^3 - 22x^2 - 18x + 36 = 0$$
$$(x - 1)(x - 3)\left(2x^2 + 10x + 12\right) = 0$$
$$2(x - 1)(x - 3)\left(x^2 + 5x + 6\right) = 0$$
$$2(x - 1)(x - 3)(x + 3)(x + 2) = 0$$
$$x = 1, \ \ x = 3, \ \ x = -3, \ \ x = -2$$

The zeros are -3, -2, 1, and 3.

b.



$f(x) = 2x^4 + 2x^3 - 22x^2 - 18x + 36$

59. $f(x) = 3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$

a. From the graph provided, we can see that 1 and 2 are x-intercepts and are thus zeros of the function. We verify this below:

$$\begin{array}{r|rrrrrr} 1 & 3 & 2 & -15 & -10 & 12 & 8 \\ & & 3 & 5 & -10 & -20 & -8 \\ \hline & 3 & 5 & -10 & -20 & -8 & 0 \end{array}$$

Thus, $3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$
$= (x-1)(3x^4 + 5x^3 - 10x^2 - 20x - 8)$

$$\begin{array}{r|rrrrr} 2 & 3 & 5 & -10 & -20 & -8 \\ & & 6 & 22 & 24 & 8 \\ \hline & 3 & 11 & 12 & 4 & 0 \end{array}$$

Thus, $3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$
$= (x-1)(3x^4 + 5x^3 - 10x^2 - 20x - 8)$
$= (x-1)(x-2)(3x^3 + 11x^2 + 12x + 4)$

To factor $3x^3 + 11x^2 + 12x + 4$, we use the Rational Zero Theorem to determine possible rational zeros.

Factors of the constant term 4: $\pm1, \pm2, \pm4$
Factors of the leading coefficient 3: $\pm1, \pm3$

The possible rational zeros are:

$$\frac{\text{Factors of 4}}{\text{Factors of 3}} = \frac{\pm1, \pm2, \pm4}{\pm1, \pm3}$$

$$= \pm1, \pm2, \pm4, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$$

We test values from above until we find a zero. One possibility is shown next:

Test -1:

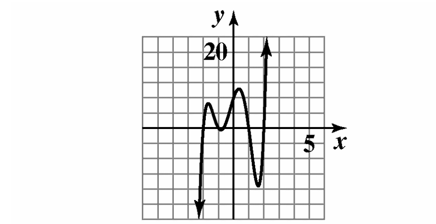
$$\begin{array}{r|rrrr} -1 & 3 & 11 & 12 & 4 \\ & & -3 & -8 & -4 \\ \hline & 3 & 8 & 4 & 0 \end{array}$$

The remainder is 0, so -1 is a zero of f. We can now finish the factoring:

$$3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8 = 0$$
$$(x-1)(3x^4 + 5x^3 - 10x^2 - 20x - 8) = 0$$
$$(x-1)(x-2)(3x^3 + 11x^2 + 12x + 4) = 0$$
$$(x-1)(x-2)(x+1)(3x^2 + 8x + 4) = 0$$
$$(x-1)(x-2)(x+1)(3x+2)(x+2) = 0$$
$$x = 1,\ x = 2,\ x = -1,\ x = -\frac{2}{3},\ x = -2$$

The zeros are -2, -1, $-\frac{2}{3}$, 1 and 2.

b.



$f(x) = 3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$

60. $f(x) = -5x^4 + 4x^3 - 19x^2 + 16x + 4$

a. From the graph provided, we can see that 1 is an x-intercept and is thus a zero of the function. We verify this below:

$$\begin{array}{r|rrrrr} 1 & -5 & 4 & -19 & 16 & 4 \\ & & -5 & -1 & -20 & -4 \\ \hline & -5 & -1 & -20 & -4 & 0 \end{array}$$

Thus, $-5x^4 + 4x^3 - 19x^2 + 16x + 4 = 0$
$$(x-1)(-5x^3 - x^2 - 20x - 4) = 0$$
$$-(x-1)(5x^3 + x^2 + 20x + 4) = 0$$

To factor $5x^3 + x^2 + 20x + 4$, we use the Rational Zero Theorem to determine possible rational zeros.

Factors of the constant term 4: $\pm1, \pm2, \pm4$
Factors of the leading coefficient 5: $\pm1, \pm5$

The possible rational zeros are:

$$\frac{\text{Factors of 4}}{\text{Factors of 5}} = \frac{\pm1, \pm2, \pm4}{\pm1, \pm5}$$

$$= \pm1, \pm2, \pm4, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{4}{5}$$

We test values from above until we find a zero. One possibility is shown next:

Test $-\frac{1}{5}$:

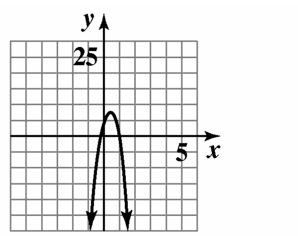
$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & 1 & 20 & 4 \\ & & -1 & 0 & -4 \\ \hline & 5 & 0 & 20 & 0 \end{array}$$

The remainder is 0, so $-\frac{1}{5}$ is a zero of *f*.

$$-5x^4+4x^3-19x^2+16x+4=0$$
$$(x-1)(-5x^3-x^2-20x-4)=0$$
$$-(x-1)(5x^3+x^2+20x+4)=0$$
$$-(x-1)\left(x+\frac{1}{5}\right)(5x^2+20)=0$$
$$-5(x-1)\left(x+\frac{1}{5}\right)(x^2+4)=0$$
$$-5(x-1)\left(x+\frac{1}{5}\right)(x+2i)(x-2i)=0$$
$$x=1,\ x=-\frac{1}{5},\ x=-2i,\ x=2i$$

The zeros are $-\frac{1}{5}$, 1, and $\pm 2i$.

b.



$f(x) = -5x^4 + 4x^3 - 19x^2 + 16x + 4$

61. $V(x)=x(x+10)(30-2x)$

$$2000=x(x+10)(30-2x)$$
$$2000=-2x^3+10x^2+300x$$
$$2x^3-10x^2-300x+2000=0$$
$$x^3-5x^2-150x+1000=0$$

Find the roots.

$$\begin{array}{r|rrrr} 10 & 1 & -5 & -150 & 1000 \\ & & 10 & 50 & -1000 \\ \hline & 1 & 5 & -100 & 0 \end{array}$$

Use the remaining quadratic to find the other 2 roots.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(5)\pm\sqrt{(5)^2-4(1)(-100)}}{2(1)}$$
$$x\approx -12.8,\ 7.8$$

Since the depth must be positive, reject the negative value.
The depth can be 10 inches or 7.8 inches to obtain a volume of 2000 cubic inches.

62. $V(x)=x(x+10)(30-2x)$

$$1500=x(x+10)(30-2x)$$
$$1500=-2x^3+10x^2+300x$$
$$2x^3-10x^2-300x+1500=0$$
$$x^3-5x^2-150x+750=0$$

Find the roots.

$$\begin{array}{r|rrrr} 5 & 1 & -5 & -150 & 750 \\ & & 5 & 0 & -750 \\ \hline & 1 & 0 & -150 & 0 \end{array}$$

Use the remaining quadratic to find the other 2 roots.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(0)\pm\sqrt{(0)^2-4(1)(-150)}}{2(1)}$$
$$x\approx -12.2,\ 12.2$$

Since the depth must be positive, reject the negative value.
The depth can be 5 inches or 12.2 inches to obtain a volume of 1500 cubic inches.

63. **a.** The answers correspond to the points (7.8, 2000) and (10, 2000).

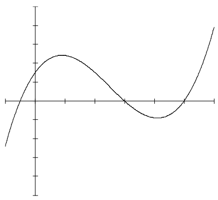
b. The range is (0, 15).

64. **a.** The answers correspond to the points (5, 1500) and (12.2, 1500).

b. The range is (0, 15).

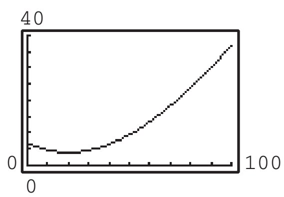
65. – 71. Answers may vary.

72. $2x^3 - 15x^2 + 22x + 15 = 0$
p: ±1, ±3, ±5, ±15
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$



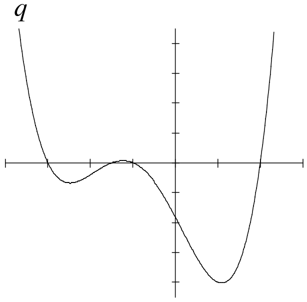
From the graph we see that the solutions are $-\frac{1}{2}$, 3 and 5.

73. $6x^3 - 19x^2 + 16x - 4 = 0$
p: ±1, ±2, ±4
q: ±1, ±2, ±3, ±6
$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6}$



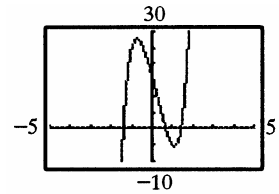
From the graph, we see that the solutions are $\frac{1}{2}, \frac{2}{3}$ and 2.

74. $2x^4 + 7x^3 - 4x^2 - 27x - 18 = 0$
$p: \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$
$q: \pm 1, \pm 2$
$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$



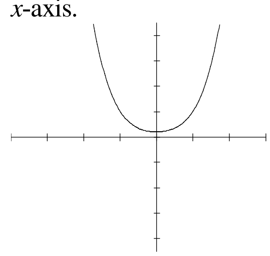
From the graph we see the solutions are $-3, -\frac{3}{2}, -1, 2$.

75. $4x^4 + 4x^3 + 7x^2 - x - 2 = 0$
p: ±1, ±2
q: ±1, ±2, ±4
$\frac{p}{q}: \pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$



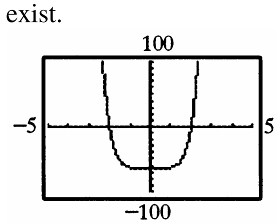
From the graph, we see that the solutions are $-\frac{1}{2}$ and $\frac{1}{2}$.

76. $f(x) = 3x^4 + 5x^2 + 2$
Since $f(x)$ has no sign variations, it has no positive real roots.
$f(-x) = 3x^4 + 5x^2 + 2$
Since $f(-x)$ has no sign variations, no negative roots exist.
The polynomial's graph doesn't intersect the x-axis.



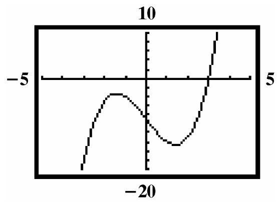
From the graph, we see that there are no real solutions.

77. $f(x) = x^5 - x^4 + x^3 - x^2 + x - 8$
$f(x)$ has 5 sign variations, so either 5, 3, or 1 positive real roots exist.
$f(-x) = -x^5 - x^4 - x^3 - x^2 - x - 8$
$f(-x)$ has no sign variations, so no negative real roots exist.



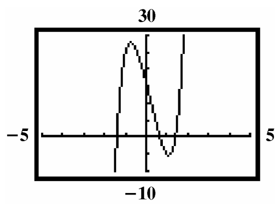
78. Odd functions must have at least one real zero. Even functions do not.

79. $f(x) = x^3 - 6x - 9$



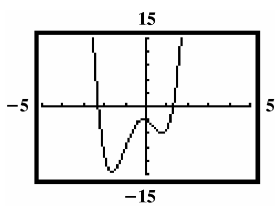
1 real zero
2 nonreal complex zeros

80. $f(x) = 3x^5 - 2x^4 + 6x^3 - 4x^2 - 24x + 16$

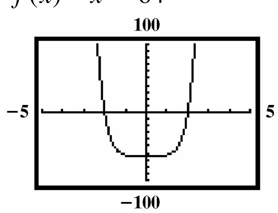


3 real zeros
2 nonreal complex zeros

81. $f(x) = 3x^4 + 4x^3 - 7x^2 - 2x - 3$



82. $f(x) = x^6 - 64$



2 real zeros
4 nonreal complex zeros

83. makes sense

84. does not make sense; Explanations will vary. Sample explanation: The quadratic formula is can be applied only of equations of degree 2.

85. makes sense

86. makes sense

87. false; Changes to make the statement true will vary. A sample change is: The equation has 0 sign variations, so no positive roots exist.

88. false; Changes to make the statement true will vary. A sample change is: Descartes' Rule gives the maximum possible number of real roots.

89. true

90. false; Changes to make the statement true will vary. A sample change is: Polynonials of degree *n* have at most *n* distinct solutions.

91. $(2x+1)(x+5)(x+2) - 3x(x+5) = 208$

$(2x^2+11x+5)(x+2) - 3x^2 - 15x = 208$

$2x^3 + 4x^2 + 11x^2 + 22x + 5x$

$+10 - 3x^2 - 15x = 208$

$2x^3 + 15x^2 + 27x - 3x^2 - 15x - 198 = 0$

$2x^3 + 12x^2 + 12x - 198 = 0$

$2(x^3 + 6x^2 + 6x - 99) = 0$

3	1	6	6	–99
		3	27	99
	1	9	33	0

$x^2 + 9x + 33 = 0$

$b^2 - 4ac = -51$

$x = 3$ in.

92. Answers will vary

93. Because the polynomial has two obvious changes of direction; the smallest degree is 3.

94. Because the polynomial has no obvious changes of direction but the graph is obviously not linear, the smallest degree is 3.

95. Because the polynomial has two obvious changes of direction and two roots have multiplicity 2, the smallest degree is 5.

96. Two roots appear twice, the smallest degree is 5.

97. Answers may vary.

98. The function is undefined at $x = 1$ and $x = 2$.

99. The equation of the vertical asymptote is $x = 1$.

100. The equation of the horizontal asymptote is $y = 0$.

Mid-Chapter 3 Check Point

1. $f(x)=(x-3)^2-4$

The parabola opens up because $a>0$.
The vertex is (3, –4).
x-intercepts:

$0=(x-3)^2-4$

$(x-3)^2=4$

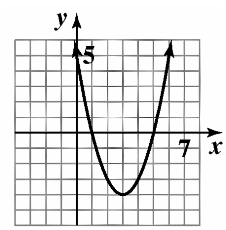
$x-3=\pm\sqrt{4}$

$x=3\pm 2$

The equation has x-intercepts at $x=1$ and $x=5$.
y-intercept:

$f(0)=(0-3)^2-4=5$

domain: $(-\infty,\infty)$ range: $[-4,\infty)$



$f(x)=(x-3)^2-4$

2. $f(x)=5-(x+2)^2$

The parabola opens down because $a<0$.
The vertex is (–2, 5).
x-intercepts:

$0=5-(x+2)^2$

$(x+2)^2=5$

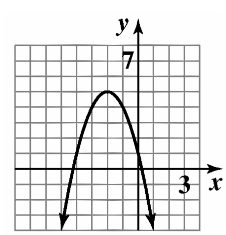
$x+2=\pm\sqrt{5}$

$x=-2\pm\sqrt{5}$

y-intercept:

$f(0)=5-(0+2)^2=1$

domain: $(-\infty,\infty)$ range: $(-\infty,5]$



$f(x)=5-(x+2)^2$

3. $f(x)=-x^2-4x+5$

The parabola opens down because $a<0$.

vertex: $x=-\dfrac{b}{2a}=-\dfrac{-4}{2(-1)}=-2$

$f(-2)=-(-2)^2-4(-2)+5=9$

The vertex is (–2, 9).
x-intercepts:

$0=-x^2-4x+5$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(-4)\pm\sqrt{(-4)^2-4(-1)(5)}}{2(-1)}$

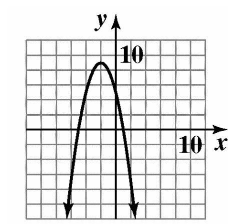
$x=\dfrac{4\pm\sqrt{36}}{-2}$

$x=-2\pm 3$

The x-intercepts are $x=1$ and $x=-5$.
y-intercept:

$f(0)=-0^2-4(0)+5=5$

domain: $(-\infty,\infty)$ range: $(-\infty,9]$



$f(x)=-x^2-4x+5$

4. $f(x) = 3x^2 - 6x + 1$

The parabola opens up because $a > 0$.

vertex: $x = -\frac{b}{2a} = -\frac{-6}{2(3)} = 1$

$f(1) = 3(1)^2 - 6(1) + 1 = -2$

The vertex is (1, –2).

x-intercepts:

$0 = 3x^2 - 6x + 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)}$

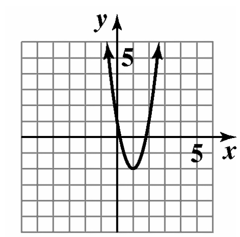
$x = \frac{6 \pm \sqrt{24}}{6}$

$x = \frac{3 \pm \sqrt{6}}{3}$

y-intercept:

$f(0) = 3(0)^2 - 6(0) + 1 = 1$

domain: $(-\infty, \infty)$ range: $[-2, \infty)$



$f(x) = 3x^2 - 6x + 1$

5. $f(x) = (x-2)^2(x+1)^3$

$(x-2)^2(x+1)^3 = 0$

Apply the zero-product principle:

$(x-2)^2 = 0$ or $(x+1)^3 = 0$

$x - 2 = 0$ $\quad$ $x + 1 = 0$

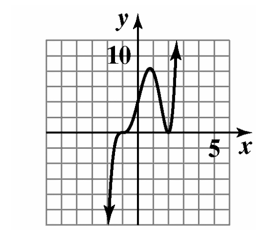
$x = 2$ $\quad$ $x = -1$

The zeros are -1 and 2.

The graph of *f* crosses the x-axis at -1, since the zero has multiplicity 3. The graph touches the x-axis and turns around at 2 since the zero has multiplicity 2.

Since *f* is an odd-degree polynomial, degree 5, and since the leading coefficient, 1, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = (x-2)^2(x+1)^3$

6. $f(x) = -(x-2)^2(x+1)^2$

$-(x-2)^2(x+1)^2 = 0$

Apply the zero-product principle:

$(x-2)^2 = 0$ or $(x+1)^2 = 0$

$x - 2 = 0$ $\quad$ $x + 1 = 0$

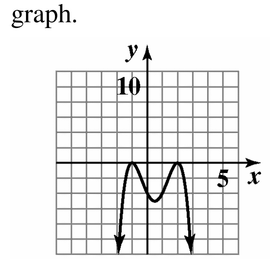
$x = 2$ $\quad$ $x = -1$

The zeros are -1 and 2.

The graph touches the x-axis and turns around both at -1 and 2 since both zeros have multiplicity 2.

Since *f* is an even-degree polynomial, degree 4, and since the leading coefficient, -1, is negative, the graph falls to the left and falls to the right.

Plot additional points as necessary and construct the graph.



$f(x) = -(x-2)^2(x+1)^2$

7. $f(x) = x^3 - x^2 - 4x + 4$

$$x^3 - x^2 - 4x + 4 = 0$$
$$x^2(x-1) - 4(x-1) = 0$$
$$(x^2 - 4)(x-1) = 0$$
$$(x+2)(x-2)(x-1) = 0$$

Apply the zero-product principle:

$x + 2 = 0$ or $x - 2 = 0$ or $x - 1 = 0$

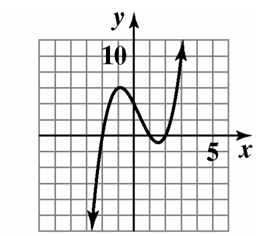
$x = -2$ $\quad x = 2$ $\quad x = 1$

The zeros are -2, 1, and 2.

The graph of *f* crosses the *x*-axis at all three zeros, -2, 1, and 2, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, 1, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = x^3 - x^2 - 4x + 4$

8. $f(x) = x^4 - 5x^2 + 4$

$$x^4 - 5x^2 + 4 = 0$$
$$(x^2 - 4)(x^2 - 1) = 0$$
$$(x+2)(x-2)(x+1)(x-1) = 0$$

Apply the zero-product principle,

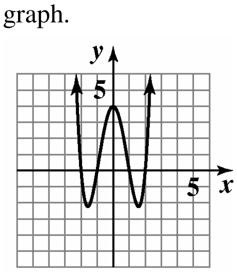
$x = -2,\ x = 2,\ x = -1,\ x = 1$

The zeros are -2, -1, 1, and 2.

The graph crosses the *x*-axis at all four zeros, -2, -1, 1, and 2., since all have multiplicity 1.

Since *f* is an even-degree polynomial, degree 4, and since the leading coefficient, 1, is positive, the graph rises to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = x^4 - 5x^2 + 4$

9. $f(x) = -(x+1)^6$

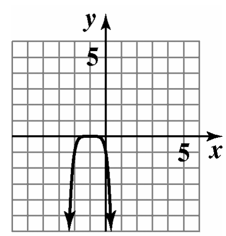
$$-(x+1)^6 = 0$$
$$(x+1)^6 = 0$$
$$x + 1 = 0$$
$$x = -1$$

The zero is are -1.

The graph touches the *x*-axis and turns around at -1 since the zero has multiplicity 6.

Since *f* is an even-degree polynomial, degree 6, and since the leading coefficient, -1, is negative, the graph falls to the left and falls to the right.

Plot additional points as necessary and construct the graph.



$f(x) = -(x + 1)^6$

10. $f(x) = -6x^3 + 7x^2 - 1$

To find the zeros, we use the Rational Zero Theorem:
List all factors of the constant term -1: ± 1
List all factors of the leading coefficient -6:
$\pm 1,\ \pm 2,\ \pm 3,\ \pm 6$

The possible rational zeros are:

$$\frac{\text{Factors of } -1}{\text{Factors of } -6} = \frac{\pm 1}{\pm 1,\ \pm 2,\ \pm 3,\ \pm 6}$$
$$= \pm 1,\ \pm\frac{1}{2},\ \pm\frac{1}{3},\ \pm\frac{1}{6}$$

We test values from the above list until we find a zero. One is shown next:

Test 1:

$$\begin{array}{r|rrrr} 1 & -6 & 7 & 0 & -1 \\ & & -6 & 1 & 1 \\ \hline & -6 & 1 & 1 & 0 \end{array}$$

The remainder is 0, so 1 is a zero. Thus,

$$-6x^3 + 7x^2 - 1 = 0$$
$$(x-1)(-6x^2 + x + 1) = 0$$
$$-(x-1)(6x^2 - x - 1) = 0$$
$$-(x-1)(3x+1)(2x-1) = 0$$

Apply the zero-product property:

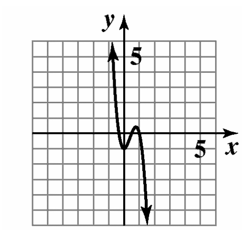
$x = 1, \quad x = -\frac{1}{3}, \quad x = \frac{1}{2}$

The zeros are $-\frac{1}{3}, \ \frac{1}{2}$, and 1.

The graph of *f* crosses the *x*-axis at all three zeros, $-\frac{1}{3}, \ \frac{1}{2}$, and 1, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, -6, is negative, the graph rises to the left and falls to the right.

Plot additional points as necessary and construct the graph.



$f(x) = -6x^3 + 7x^2 - 1$

11. $f(x) = 2x^3 - 2x$

$$2x^3 - 2x = 0$$
$$2x(x^2 - 1) = 0$$
$$2x(x+1)(x-1) = 0$$

Apply the zero-product principle:

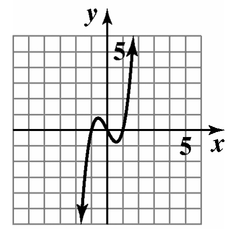
$x = 0, \qquad x = -1, \qquad x = 1$

The zeros are -1, 0, and 1.

The graph of *f* crosses the *x*-axis at all three zeros, -1, 0, and 1, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, 2, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = 2x^3 - 2x$

12. $f(x) = x^3 - 2x^2 + 26x$

$$x^3 - 2x^2 + 26x = 0$$
$$x(x^2 - 2x + 26) = 0$$

Note that $x^2 - 2x + 26$ does not factor, so we use the quadratic formula:

$x = 0 \quad$ or $\quad x^2 - 2x + 26 = 0$

$$a = 1, \ b = -2, \ c = 26$$

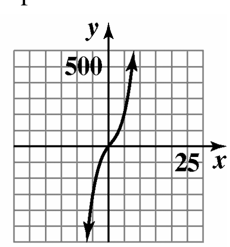
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(26)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{-100}}{2} = \frac{2 \pm 10i}{2} = 1 \pm 5i$$

The zeros are 0 and $1 \pm 5i$.

The graph of *f* crosses the *x*-axis at 0 (the only real zero), since it has multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, 1, is positive, the graph falls to the left and rises to the right.

Plot additional points as necessary and construct the graph.



$f(x) = x^3 - 2x^2 + 26x$

13. $f(x) = -x^3 + 5x^2 - 5x - 3$

To find the zeros, we use the Rational Zero Theorem:
List all factors of the constant term -3: $\pm1, \ \pm3$
List all factors of the leading coefficient -1: ±1
The possible rational zeros are:

$$\frac{\text{Factors of } -3}{\text{Factors of } -1} = \frac{\pm1, \ \pm3}{\pm1} = \pm1, \ \pm3$$

We test values from the previous list until we find a zero. One is shown next:

Test 3:

$$\begin{array}{r|rrrr} 3 & -1 & 5 & -5 & -3 \\ & & -3 & 6 & 3 \\ \hline & -1 & 2 & 1 & 0 \end{array}$$

The remainder is 0, so 3 is a zero. Thus,

$$-x^3 + 5x^2 - 5x - 3 = 0$$
$$(x-3)\left(-x^2 + 2x + 1\right) = 0$$
$$-(x-3)\left(x^2 - 2x - 1\right) = 0$$

Note that $x^2 - 2x - 1$ does not factor, so we use the quadratic formula:

$x - 3 = 0$ or $x^2 - 2x - 1 = 0$
$x = 3$ $\quad a = 1, \ b = -2, \ c = -1$

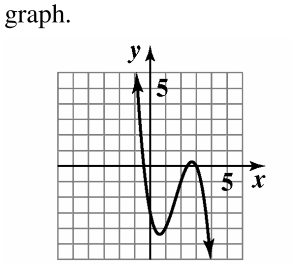
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

The zeros are 3 and $1 \pm \sqrt{2}$.

The graph of *f* crosses the *x*-axis at all three zeros, 3 and $1 \pm \sqrt{2}$, since all have multiplicity 1.

Since *f* is an odd-degree polynomial, degree 3, and since the leading coefficient, -1, is negative, the graph rises to the left and falls to the right.

Plot additional points as necessary and construct the graph.



$f(x) = -x^3 + 5x^2 - 5x - 3$

14. $x^3 - 3x + 2 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term 2: $\pm1, \ \pm2$
Factors of the leading coefficient 1: ±1

The possible rational zeros are:

$$\frac{\text{Factors of } 2}{\text{Factors of } 1} = \frac{\pm1, \ \pm2}{\pm1} = \pm1, \ \pm2$$

We test values from above until we find a root. One is shown next:

Test 1:

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

The remainder is 0, so 1 is a root of the equation. Thus,

$$x^3 - 3x + 2 = 0$$
$$(x-1)\left(x^2 + x - 2\right) = 0$$
$$(x-1)(x+2)(x-1) = 0$$
$$(x-1)^2(x+2) = 0$$

Apply the zero-product property:

$(x-1)^2 = 0$ or $x + 2 = 0$
$x - 1 = 0$ $\quad x = -2$
$x = 1$

The solutions are -2 and 1, and the solution set is $\{-2, 1\}$.

15. $6x^3-11x^2+6x-1=0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term -1: ±1
Factors of the leading coefficient 6:
$\pm1,\ \pm2,\ \pm3,\ \pm6$

The possible rational zeros are:

$$\frac{\text{Factors of } -1}{\text{Factors of } 6}=\frac{\pm1}{\pm1,\ \pm2,\ \pm3,\ \pm6}$$
$$=\pm1,\ \pm\frac{1}{2},\ \pm\frac{1}{3},\ \pm\frac{1}{6}$$

We test values from above until we find a root. One is shown next:

Test 1:

1	6	−11	6	−1
		6	−5	1
	6	−5	1	0

The remainder is 0, so 1 is a root of the equation. Thus,

$$6x^3-11x^2+6x-1=0$$
$$(x-1)(6x^2-5x+1)=0$$
$$(x-1)(3x-1)(2x-1)=0$$

Apply the zero-product property:

$x-1=0$ or $3x-1=0$ or $2x-1=0$

$x=1$ $\quad x=\frac{1}{3}$ $\quad x=\frac{1}{2}$

The solutions are $\frac{1}{3}$, $\frac{1}{2}$ and 1, and the solution set is $\left\{\frac{1}{3},\ \frac{1}{2},\ 1\right\}$.

16. $(2x+1)(3x-2)^3(2x-7)=0$

Apply the zero-product property:

$2x+1=0$ or $(3x-2)^3=0$ or $2x-7=0$

$x=-\frac{1}{2}$ $\quad 3x-2=0$ $\quad x=\frac{7}{2}$ T

$x=\frac{2}{3}$

he solutions are $-\frac{1}{2}$, $\frac{2}{3}$ and $\frac{7}{2}$, and the solution set is $\left\{-\frac{1}{2},\ \frac{2}{3},\ \frac{7}{2}\right\}$.

17. $2x^3+5x^2-200x-500=0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term -500:
$\pm1,\ \pm2,\ \pm4,\ \pm5,\ \pm10,\ \pm20,\ \pm25,$
$\pm50,\ \pm100,\ \pm125,\ \pm250,\ \pm500$
Factors of the leading coefficient 2: $\pm1,\ \pm2$

The possible rational zeros are:

$$\frac{\text{Factors of } 500}{\text{Factors of } 2}=\pm1,\ \pm2,\ \pm4,\ \pm5,$$
$$\pm10,\ \pm20,\ \pm25,\ \pm50,\ \pm100,\ \pm125,$$
$$\pm250,\ \pm500,\ \pm\frac{1}{2},\ \pm\frac{5}{2},\ \pm\frac{25}{2},\ \pm\frac{125}{2}$$

We test values from above until we find a root. One is shown next:

Test 10:

10	2	5	−200	−500
		20	250	500
	2	25	50	0

The remainder is 0, so 10 is a root of the equation. Thus,

$$2x^3+5x^2-200x-500=0$$
$$(x-10)(2x^2+25x+50)=0$$
$$(x-10)(2x+5)(x+10)=0$$

Apply the zero-product property:

$x-10=0$ or $2x+5=0$ or $x+10=0$

$x=10$ $\quad x=-\frac{5}{2}$ $\quad x=-10$

The solutions are $-10,\ -\frac{5}{2},$ and 10, and the solution set is $\left\{-10,\ -\frac{5}{2},\ 10\right\}$.

18. $x^4 - x^3 - 11x^2 = x + 12$

$x^4 - x^3 - 11x^2 - x - 12 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term -12:
$\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 12$

Factors of the leading coefficient 1: ± 1

The possible rational zeros are:

$$\frac{\text{Factors of } -12}{\text{Factors of } 1} = \frac{\pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 12}{\pm 1}$$
$$= \pm 1,\ \pm 2,\ \pm 3,\ \pm 4,\ \pm 6,\ \pm 12$$

We test values from this list we find a root. One possibility is shown next:

Test -3:

-3	1	−1	−11	−1	−12
		−3	12	−3	12
	1	−4	1	−4	0

The remainder is 0, so -3 is a root of the equation. Using the Factor Theorem, we know that $x-1$ is a factor. Thus,

$$x^4 - x^3 - 11x^2 - x - 12 = 0$$
$$(x+3)(x^3 - 4x^2 + x - 4) = 0$$
$$(x+3)\left[x^2(x-4) + 1(x-4)\right] = 0$$
$$(x+3)(x-4)(x^2+1) = 0$$

As this point we know that -3 and 4 are roots of the equation. Note that x^2+1 does not factor, so we use the square-root principle: $x^2 + 1 = 0$

$$x^2 = -1$$
$$x = \pm\sqrt{-1} = \pm i$$

The roots are -3, 4, and $\pm i$, and the solution set is $\{-3,\ 4,\ \pm i\}$.

19. $2x^4 + x^3 - 17x^2 - 4x + 6 = 0$

We begin by using the Rational Zero Theorem to determine possible rational roots.

Factors of the constant term 6: $\pm 1,\ \pm 2,\ \pm 3,\ \pm 6$

Factors of the leading coefficient 4: $\pm 1,\ \pm 2$

The possible rational roots are:

$$\frac{\text{Factors of } 6}{\text{Factors of } 2} = \frac{\pm 1,\ \pm 2,\ \pm 3,\ \pm 6}{\pm 1,\ \pm 2}$$
$$= \pm 1,\ \pm 2,\ \pm 3,\ \pm 6,\ \pm\frac{1}{2},\ \pm\frac{3}{2}$$

We test values from above until we find a root. One possibility is shown next:

Test -3:

-3	2	1	−17	−4	6
		−6	15	6	−6
	2	−5	−2	2	0

The remainder is 0, so -3 is a root. Using the Factor Theorem, we know that $x+3$ is a factor of the polynomial. Thus,

$$2x^4 + x^3 - 17x^2 - 4x + 6 = 0$$
$$(x+3)(2x^3 - 5x^2 - 2x + 2) = 0$$

To solve the equation above, we need to factor $2x^3 - 5x^2 - 2x + 2$. We continue testing potential roots:

Test $\frac{1}{2}$:

$\frac{1}{2}$	2	−5	−2	2
		1	−2	−2
	2	−4	−4	0

The remainder is 0, so $\frac{1}{2}$ is a zero and $x - \frac{1}{2}$ is a factor.

Summarizing our findings so far, we have

$$2x^4 + x^3 - 17x^2 - 4x + 6 = 0$$
$$(x+3)(2x^3 - 5x^2 - 2x + 2) = 0$$
$$(x+3)\left(x - \frac{1}{2}\right)(2x^2 - 4x - 4) = 0$$
$$2(x+3)\left(x - \frac{1}{2}\right)(x^2 - 2x - 2) = 0$$

At this point, we know that -3 and $\frac{1}{2}$ are roots of the equation. Note that $x^2 - 2x - 2$ does not factor, so we use the quadratic formula:

$x^2 - 2x - 2 = 0$

$a = 1, \;\; b = -2, \;\; c = -2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

The solutions are -3, $\frac{1}{2}$, and $1 \pm \sqrt{3}$, and the solution set is $\left\{-3, \; \frac{1}{2}, \; 1 \pm \sqrt{3}\right\}$.

20. $P(x) = -x^2 + 150x - 4425$

Since $a = -1$ is negative, we know the function opens down and has a maximum at

$$x = -\frac{b}{2a} = -\frac{150}{2(-1)} = -\frac{150}{-2} = 75.$$

$$P(75) = -75^2 + 150(75) - 4425$$
$$= -5625 + 11{,}250 - 4425 = 1200$$

The company will maximize its profit by manufacturing and selling 75 cabinets per day. The maximum daily profit is \$1200.

21. Let x = one of the numbers;
$-18 - x$ = the other number

The product is $f(x) = x(-18 - x) = -x^2 - 18x$

The x-coordinate of the maximum is

$$x = -\frac{b}{2a} = -\frac{-18}{2(-1)} = -\frac{-18}{-2} = -9.$$

$$f(-9) = -9[-18 - (-9)]$$
$$= -9(-18 + 9) = -9(-9) = 81$$

The vertex is $(-9, 81)$. The maximum product is 81. This occurs when the two number are -9 and $-18 - (-9) = -9$.

22. Let x = height of triangle;
$40 - 2x$ = base of triangle

$$A = \frac{1}{2}bh = \frac{1}{2}x(40 - 2x)$$
$$A(x) = 20x - x^2$$

The height at which the triangle will have maximum area is $x = -\frac{b}{2a} = -\frac{20}{2(-1)} = 10.$

$$A(10) = 20(10) - (10)^2 = 100$$

The maximum area is 100 squares inches.

23.

$$\begin{array}{r} 2x^2 - x - 3 \\ 3x^2 - 1 \overline{)\, 6x^4 - 3x^3 - 11x^2 + 2x + 4} \\ \underline{6x^4 \qquad - 2x^2} \qquad\qquad \\ -3x^3 - 9x^2 + 2x \qquad \\ \underline{-3x^3 \qquad + x} \qquad \\ -9x^2 + x + 4 \\ \underline{-9x^2 \qquad + 3} \\ x + 1 \end{array}$$

$$2x^2 - x - 3 + \frac{x+1}{3x^2 - 1}$$

24. $(2x^4 - 13x^3 + 17x^2 + 18x - 24) \div (x - 4)$

4	2	–13	17	18	–24
		8	-20	–12	24
	2	–5	–3	6	0

The quotient is $2x^3 - 5x^2 - 3x + 6$.

25. $(x-1)(x-i)(x+i)=(x-1)(x^2+1)$

$f(x)=a_n(x-1)(x^2+1)$

$f(-1)=a_n(-1-1)\left((-1)^2+1\right)=-4a_n=8$

$a_n=-2$

$f(x)=-2(x-1)(x^2+1)$ or $-2x^3+2x^2-2x+2$

26. $(x-2)(x-2)(x-3i)(x+3i)$

$=(x-2)(x-2)(x^2+9)$

$f(x)=a_n(x-2)(x-2)(x^2+9)$

$f(0)=a_n(0-2)(0-2)(0^2+9)$

$36=36a_n$

$a_n=1$

$f(x)=1(x-2)(x-2)(x^2+9)$

$f(x)=x^4-4x^3+13x^2-36x+36$

27. $f(x)=x^3-x-5$

$f(1)=1^3-1-5=-5$

$f(2)=2^3-2-5=1$

Yes, the function must have a real zero between 1 and 2 because $f(1)$ and $f(2)$ have opposite signs.

Section 3.5

Check Point Exercises

1. **a.** $x-5=0$

$x=5$

$\{x|x\neq 5\}$

b. $x^2-25=0$

$x^2=25$

$x=\pm 5$

$\{x\,|\,x\neq 5, x\neq -5\}$

c. The denominator cannot equal zero. All real numbers.

2. **a.** $x^2-1=0$

$x^2=1$

$x=1, x=-1$

b. $g(x)=\dfrac{x-1}{x^2-1}=\dfrac{x-1}{(x-1)(x+1)}=\dfrac{1}{x+1}$

$x=-1$

c. The denominator cannot equal zero. No vertical asymptotes.

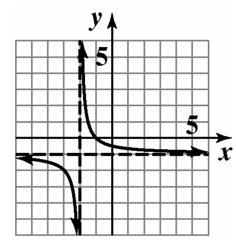
3. **a.** Since $n=m$, $y=\dfrac{9}{3}=3$

$y=3$ is a horizontal asymptote.

b. Since $n<m$, $y=0$ is a horizontal asymptote.

c. Since $n>m$, there is no horizontal asymptote.

4. Begin with the graph of $f(x)=\dfrac{1}{x}$.



$g(x)=\dfrac{1}{x+2}-1$

Shift the graph 2 units to the left by subtracting 2 from each x-coordinate. Shift the graph 1 unit down by subtracting 1 from each y-coordinate.

5. $f(x)=\dfrac{3x-3}{x-2}$

$f(-x)=\dfrac{3(-x)-3}{-x-2}=\dfrac{-3x-3}{-x-2}=\dfrac{3x+3}{x+2}$

no symmetry

$f(0)=\dfrac{3(0)-3}{0-2}=\dfrac{3}{2}$

The y-intercept is $\dfrac{3}{2}$.

$3x-3=0$

$3x=3$

$x=1$

The x-intercept is 1.

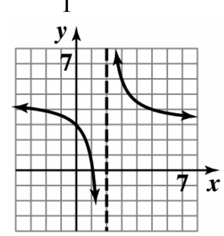
Vertical asymptote:

$x-2=0$

$x=2$

Horizontal asymptote:

$y=\dfrac{3}{1}=3$



$f(x)=\dfrac{3x-3}{x-2}$

6. $f(x) = \dfrac{2x^2}{x^2 - 9}$

$f(-x) = \dfrac{2(-x)^2}{(-x)^2 - 9} = \dfrac{2x^2}{x^2 - 9} = f(x)$

The y-axis symmetry.

$f(0) = \dfrac{2(0)^2}{0^2 - 9} = 0$

The y-intercept is 0.

$2x^2 = 0$ k

$x = 0$

The x-intercept is 0.

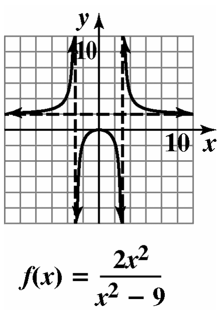
vertical asymptotes:

$x^2 - 9 = 0$

$x = 3, x = -3$

horizontal asymptote:

$y = \dfrac{2}{1} = 2$



7. $f(x) = \dfrac{x^4}{x^2 + 2}$

$f(-x) = \dfrac{(-x)^4}{(-x)^2 + 2} = \dfrac{x^4}{x^2 + 2} = f(x)$

y-axis symmetry

$f(0) = \dfrac{0^4}{0^2 + 2} = 0$

The y-intercept is 0.

$x^4 = 0$

$x = 0$

The x-intercept is 0.

vertical asymptotes:

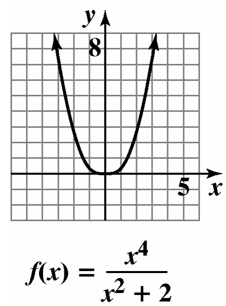
$x^2 + 2 = 0$

$x^2 = -2$

no vertical asymptotes

horizontal asymptote:

Since $n > m$, there is no horizontal asymptote.



8.

2	2	–5	7
		4	–2
	2	–1	5

the equation of the slant asymptote is $y = 2x - 1$.

9. **a.** $C(x) = 500{,}000 + 400x$

b. $\bar{C}(x) = \dfrac{500{,}000 + 400x}{x}$

c. $\bar{C}(1000) = \dfrac{500{,}000 + 400(1000)}{1000} = 900$

$\bar{C}(10{,}000) = \dfrac{500{,}000 + 400(10{,}000)}{10{,}000} = 450$

$\bar{C}(100{,}000) = \dfrac{500{,}000 + 400(100{,}000)}{100{,}000} = 405$

The average cost per wheelchair of producing 1000, 10,000, and 100,000 wheelchairs is \$900, \$450, and \$405, respectively.

d. $y = \dfrac{400}{1} = 400$

The cost per wheelchair approaches \$400 as more wheelchairs are produced.

Exercise Set 3.5

1. $f(x)=\dfrac{5x}{x-4}$
$\{x|x\neq 4\}$

2. $f(x)=\dfrac{7x}{x-8}$
$\{x|x\neq 8\}$

3. $g(x)=\dfrac{3x^2}{(x-5)(x+4)}$
$\{x|x\neq 5, x\neq -4\}$

4. $g(x)=\dfrac{2x^2}{(x-2)(x+6)}$
$\{x|x\neq 2, x\neq -6\}$

5. $h(x)=\dfrac{x+7}{x^2-49}$
$x^2-49=(x-7)(x+7)$
$\{x|x\neq 7, x\neq -7\}$

6. $h(x)=\dfrac{x+8}{x^2-64}$
$x^2-64=(x-8)(x+8)$
$\{x|x\neq 8, x\neq -8\}$

7. $f(x)=\dfrac{x+7}{x^2+49}$
all real numbers

8. $f(x)=\dfrac{x+8}{x^2+64}$
all real numbers

9. $-\infty$

10. $+\infty$

11. $-\infty$

12. $+\infty$

13. 0

14. 0

15. $+\infty$

16. $-\infty$

17. $-\infty$

18. $+\infty$

19. 1

20. 1

21. $f(x)=\dfrac{x}{x+4}$
$x+4=0$
$x=-4$
vertical asymptote: $x=-4$

22. $f(x)=\dfrac{x}{x-3}$
$x-3=0$
$x=3$
vertical asymptote: $x=3$

23. $g(x)=\dfrac{x+3}{x(x+4)}$
$x(x+4)=0$
$x=0, x=-4$
vertical asymptotes: $x=0,\ x=-4$

24. $g(x)=\dfrac{x+3}{x(x-3)}$
$x(x-3)=0$
$x=0, x=3$
vertical asymptotes: $x=0, x=3$

25. $h(x)=\dfrac{x}{x(x+4)}=\dfrac{1}{x+4}$
$x+4=0$
$x=-4$
vertical asymptote: $x=-4$

26. $h(x)=\dfrac{x}{x(x-3)}=\dfrac{1}{x-3}$
$x-3=0$
$x=3$
vertical asymptote: $x=3$

27. $r(x) = \frac{x}{x^2 + 4}$

$x^2 + 4$ has no real zeros

There are no vertical asymptotes.

28. $r(x) = \frac{x}{x^2 + 3}$

$x^2 + 3$ has no real zeros

There is no vertical asymptotes.

29. $f(x) = \frac{12x}{3x^2 + 1}$

$n < m$

horizontal asymptote: $y = 0$

30. $f(x) = \frac{15x}{3x^2 + 1}$

$n < m$

horizontal asymptote: $y = 0$

31. $g(x) = \frac{12x^2}{3x^2 + 1}$

$n = m$,

horizontal asymptote: $y = \frac{12}{3} = 4$

32. $g(x) = \frac{15x^2}{3x^2 + 1}$

$n = m$

horizontal asymptote: $y = \frac{15}{3} = 5$

33. $h(x) = \frac{12x^3}{3x^2 + 1}$

$n > m$

no horizontal asymptote

34. $h(x) = \frac{15x^3}{3x^2 + 1}$

$n > m$

no horizontal asymptote

35. $f(x) = \frac{-2x + 1}{3x + 5}$

$n = m$

horizontal asymptote: $y = -\frac{2}{3}$

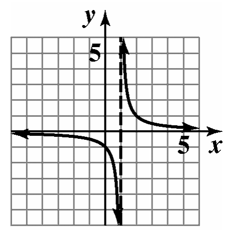
36. $f(x) = \frac{-3x + 7}{5x - 2}$

$n = m$ s

horizontal asymptote: $y = -\frac{3}{5}$

37. $g(x) = \frac{1}{x - 1}$

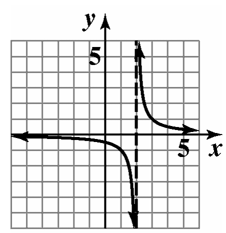
Shift the graph of $f(x) = \frac{1}{x}$ 1 unit to the right.



$$\mathbf{g(x) = \frac{1}{x - 1}}$$

38. $g(x) = \frac{1}{x - 2}$

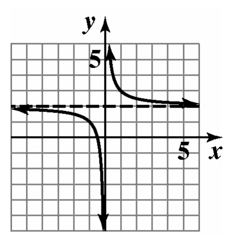
Shift the graph of $f(x) = \frac{1}{x}$ 2 units to the right.



$$\mathbf{g(x) = \frac{1}{x - 2}}$$

39. $h(x) = \frac{1}{x} + 2$

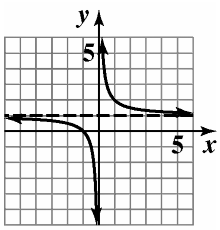
Shift the graph of $f(x) = \frac{1}{x}$ 2 units up.



$$\mathbf{h(x) = \frac{1}{x} + 2}$$

40. $h(x)=\frac{1}{x}+1$

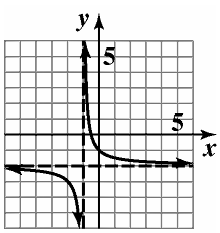
Shift the graph of $f(x)=\frac{1}{x}$ 1 unit up.



$h(x) = \frac{1}{x} + 1$

41. $g(x)=\frac{1}{x+1}-2$

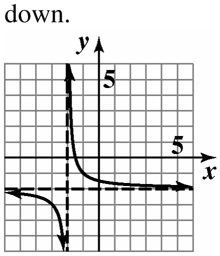
Shift the graph of $f(x)=\frac{1}{x}$ 1 unit left and 2 units down.



$g(x) = \frac{1}{x+1} - 2$

42. $g(x)=\frac{1}{x+2}-2$

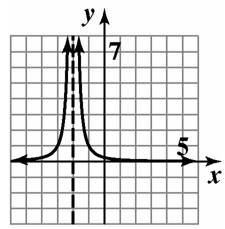
Shift the graph of $f(x)=\frac{1}{x}$ 2 units left and 2 units down.



$g(x) = \frac{1}{x+2} - 2$

43. $g(x)=\frac{1}{(x+2)^2}$

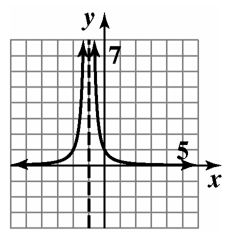
Shift the graph of $f(x)=\frac{1}{x^2}$ 2 units left.



$g(x) = \frac{1}{(x+2)^2}$

44. $g(x)=\frac{1}{(x+1)^2}$

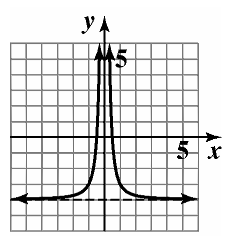
Shift the graph of $f(x)=\frac{1}{x^2}$ 1 unit left.



$g(x) = \frac{1}{(x+1)^2}$

45. $h(x)=\frac{1}{x^2}-4$

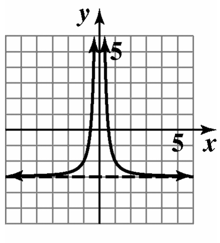
Shift the graph of $f(x)=\frac{1}{x^2}$ 4 units down.



$h(x) = \frac{1}{x^2} - 4$

46. $h(x) = \dfrac{1}{x^2} - 3$

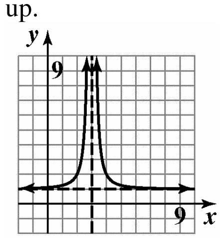
Shift the graph of $f(x) = \dfrac{1}{x^2}$ 3 units down.



$h(x) = \dfrac{1}{x^2} - 3$

47. $h(x) = \dfrac{1}{(x-3)^2} + 1$

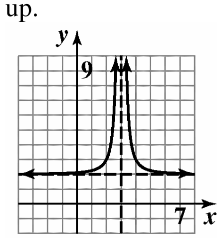
Shift the graph of $f(x) = \dfrac{1}{x^2}$ 3 units right and 1 unit up.



$h(x) = \dfrac{1}{(x-3)^2} + 1$

48. $h(x) = \dfrac{1}{(x-3)^2} + 2$

Shift the graph of $f(x) = \dfrac{1}{x^2}$ 3 units right and 2 units up.



$h(x) = \dfrac{1}{(x-3)^2} + 2$

49. $f(x) = \dfrac{4x}{x-2}$

$f(-x) = \dfrac{4(-x)}{(-x)-2} = \dfrac{4x}{x+2}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{4(0)}{0-2} = 0$

x-intercept: $4x = 0$

$x = 0$

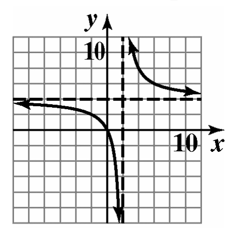
vertical asymptote:

$x - 2 = 0$

$x = 2$

horizontal asymptote:

$n = m$, so $y = \dfrac{4}{1} = 4$



$f(x) = \dfrac{4x}{x-2}$

50. $f(x) = \dfrac{3x}{x-1}$

$f(-x) = \dfrac{3(-x)}{(-x)-1} = \dfrac{3x}{x+1}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{3(0)}{0-1} = 0$

x-intercept: $3x = 0$

$x = 0$

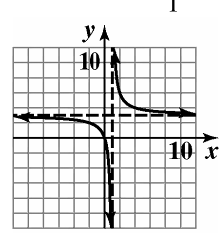
vertical asymptote:

$x - 1 = 0$

$x = 1$

horizontal asymptote:

$n = m$, so $y = \dfrac{3}{1} = 3$



$f(x) = \dfrac{3x}{x-1}$

51. $f(x) = \dfrac{2x}{x^2 - 4}$

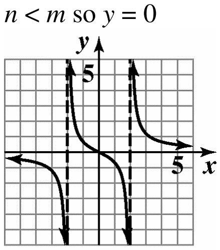
$f(-x) = \dfrac{2(-x)}{(-x)^2 - 4} = -\dfrac{2x}{x^2 - 4} = -f(x)$

Origin symmetry

y-intercept: $\dfrac{2(0)}{0^2 - 4} = \dfrac{0}{-4} = 0$

x-intercept:
$2x = 0$
$x = 0$

vertical asymptotes:
$x^2 - 4 = 0$
$x = \pm 2$
horizontal asymptote:
$n < m$ so $y = 0$



$f(x) = \dfrac{2x}{x^2 - 4}$

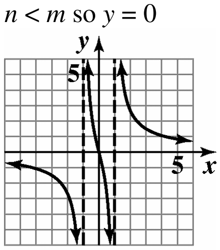
52. $f(x) = \dfrac{4x}{x^2 - 1}$

$f(-x) = \dfrac{4(-x)}{(-x)^2 - 1} = -\dfrac{4x}{x^2 - 1} = -f(x)$

Origin symmetry

y-intercept: $\dfrac{4(0)}{0^2 - 1} = 0$

x-intercept: $4x = 0$
$x = 0$
vertical asymptotes:
$x^2 - 1 = 0$
$(x-1)(x+1) = 0$
$x = \pm 1$
horizontal asymptote:
$n < m$ so $y = 0$



$f(x) = \dfrac{4x}{x^2 - 1}$

53. $f(x) = \dfrac{2x^2}{x^2 - 1}$

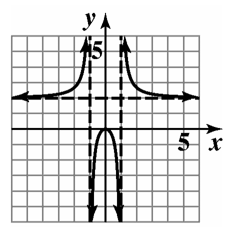
$f(-x) = \dfrac{2(-x)^2}{(-x)^2 - 1} = \dfrac{2x^2}{x^2 - 1} = f(x)$

y-axis symmetry

y-intercept: $y = \dfrac{2(0)^2}{0^2 - 1} = \dfrac{0}{1} = 0$

x-intercept:
$2x^2 = 0$
$x = 0$
vertical asymptote:
$x^2 - 1 = 0$
$x^2 = 1$
$x = \pm 1$
horizontal asymptote:

$n = m$, so $y = \dfrac{2}{1} = 2$



$f(x) = \dfrac{2x^2}{x^2 - 1}$

54. $f(x)=\dfrac{4x^2}{x^2-9}$

$f(-x)=\dfrac{4(-x)^2}{(-x)^2-9}=\dfrac{4x^2}{x^2-9}=f(x)$

y-axis symmetry

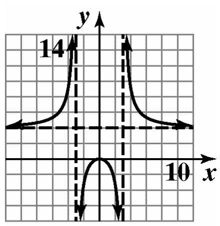
y-intercept: $y=\dfrac{4(0)^2}{0^2-9}=0$

x-intercept:
$4x^2=0$
$x=0$

vertical asymptotes:
$x^2-9=0$
$(x-3)(x+3)=0$
$x=\pm3$

horizontal asymptote:

$n=m$, so $y=\dfrac{4}{1}=4$



$f(x)=\dfrac{4x^2}{x^2-9}$

55. $f(x)=\dfrac{-x}{x+1}$

$f(-x)=\dfrac{-(-x)}{(-x)+1}=\dfrac{x}{-x+1}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$
no symmetry

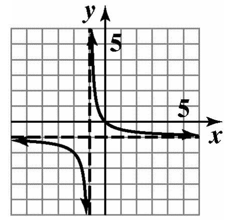
y-intercept: $y=\dfrac{-(0)}{0+1}=\dfrac{0}{1}=0$

x-intercept:
$-x=0$
$x=0$

vertical asymptote:
$x+1=0$
$x=-1$

horizontal asymptote:

$n=m$, so $y=\dfrac{-1}{1}=-1$



$f(x)=\dfrac{-x}{x+1}$

56. $f(x)=\dfrac{-3x}{x+2}$

$f(-x)=\dfrac{-3(-x)}{(-x)+2}=\dfrac{3x}{-x+2}$

$f–x)\neq f(x), f(-x)\neq -f(x)$
no symmetry

y-intercept:

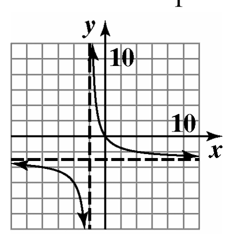
$y=\dfrac{-3(0)}{0+2}=0$

x-intercept:
$-3x=0$
$x=0$

vertical asymptote:
$x+2=0$
$x=-2$

horizontal asymptote:

$n=m$, so $y=\dfrac{-3}{1}=-3$



$f(x)=\dfrac{-3x}{x+2}$

57. $f(x) = -\dfrac{1}{x^2 - 4}$

$f(-x) = -\dfrac{1}{(-x)^2 - 4} = -\dfrac{1}{x^2 - 4} = f(x)$

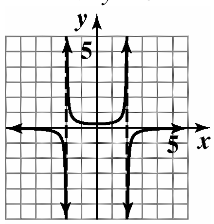
y-axis symmetry

y-intercept: $y = -\dfrac{1}{0^2 - 4} = \dfrac{1}{4}$

x-intercept: $-1 \neq 0$
no x-intercept
vertical asymptotes:

$x^2 - 4 = 0$
$x^2 = 4$
$x = \pm 2$

horizontal asymptote:
$n < m$ or $y = 0$



$f(x) = -\dfrac{1}{x^2 - 4}$

58. $f(x) = -\dfrac{2}{x^2 - 1}$

$f(-x) = -\dfrac{2}{(-x)^2 - 1} = -\dfrac{2}{x^2 - 1} = f(x)$

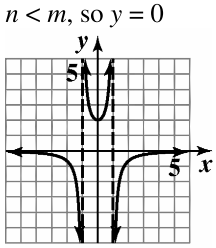
y-axis symmetry
y-intercept:

$y = -\dfrac{2}{0^2 - 1} = -\dfrac{2}{-1} = 2$

x-intercept:
$-2 = 0$
no x-intercept
vertical asymptotes:

$x^2 - 1 = 0$
$(x - 1)(x + 1)$
$x = \pm 1$

horizontal asymptote:
$n < m$, so $y = 0$



$f(x) = -\dfrac{2}{x^2 - 1}$

59. $f(x) = \dfrac{2}{x^2 + x - 2}$

$f(-x) = -\dfrac{2}{(-x)^2 - x - 2} = \dfrac{2}{x^2 - x - 2}$

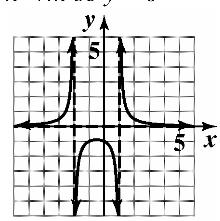
$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

y-intercept: $y = \dfrac{2}{0^2 + 0 - 2} = \dfrac{2}{-2} = -1$

x-intercept: none
vertical asymptotes:

$x^2 + x - 2 = 0$
$(x + 2)(x - 1) = 0$
$x = -2, x = 1$

horizontal asymptote:
$n < m$ so $y = 0$



$f(x) = \dfrac{2}{x^2 + x - 2}$

60. $f(x) = \dfrac{-2}{x^2 - x - 2}$

$f(-x) = \dfrac{-2}{(-x)^2 - (-x) - 2} = \dfrac{-2}{x^2 + x - 2}$

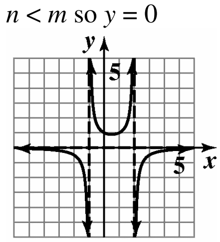
$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

y-intercept: $y = \dfrac{-2}{0^2 - 0 - 2} = 1$

x-intercept: none
vertical asymptotes:

$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$
$x = 2, x = -1$

horizontal asymptote:
$n < m$ so $y = 0$



$f(x) = \dfrac{-2}{x^2 - x - 2}$

61. $f(x) = \frac{2x^2}{x^2+4}$

$f(-x) = \frac{2(-x)^2}{(-x)^2+4} = \frac{2x^2}{x^2+4} = f(x)$

y axis symmetry

y-intercept: $y = \frac{2(0)^2}{0^2+4} = 0$

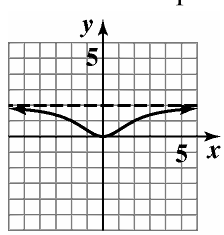
x-intercept: $2x^2 = 0$

$x = 0$

vertical asymptote: none

horizontal asymptote:

$n = m$, so $y = \frac{2}{1} = 2$



$f(x) = \frac{2x^2}{x^2 + 4}$

62. $f(x) = \frac{4x^2}{x^2+1}$

$f(-x) = \frac{4(-x)^2}{(-x)^2+1} = \frac{4x^2}{x^2+1} = f(x)$

y axis symmetry

y-intercept: $y = \frac{4(0)^2}{0^2+1} = 0$

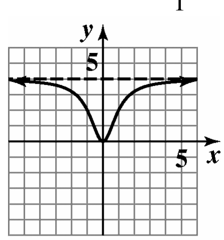
x-intercept: $4x^2 = 0$

$x = 0$

vertical asymptote: none

horizontal asymptote:

$n = m$, so $y = \frac{4}{1} = 4$



$f(x) = \frac{4x^2}{x^2 + 1}$

63. $f(x) = \frac{x+2}{x^2+x-6}$

$f(-x) = \frac{-x+2}{(-x)^2-(-x)-6} = \frac{-x+2}{x^2+x-6}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \frac{0+2}{0^2+0-6} = -\frac{2}{6} = -\frac{1}{3}$

x-intercept:

$x + 2 = 0$

$x = -2$

vertical asymptotes:

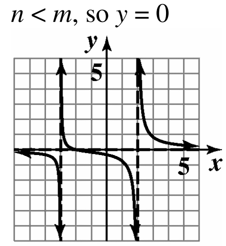
$x^2 + x - 6 = 0$

$(x+3)(x-2)$

$x = -3, x = 2$

horizontal asymptote:

$n < m$, so $y = 0$



$f(x) = \frac{x + 2}{x^2 + x - 6}$

64. $f(x)=\dfrac{x-4}{x^2-x-6}$

$f(-x)=\dfrac{-x-4}{(-x)^2-(-x)-6}=-\dfrac{x+4}{x^2+x-6}$

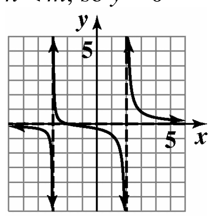
$f(-x)\neq f(x), f(-x)\neq -f(x)$
no symmetry

y-intercept: $y=\dfrac{0-4}{0^2-0-6}=\dfrac{2}{3}$

x-intercept:
$x-4=0, x=4$

vertical asymptotes:
$x^2-x-6=0$
$(x-3)(x+2)$
$x=3, x=-2$

horizontal asymptote:
$n<m$, so $y=0$



$f(x)=\dfrac{x+2}{x^2-x-6}$

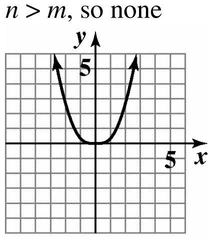
65. $f(x)=\dfrac{x^4}{x^2+2}$

$f(-x)=\dfrac{(-x)^4}{(-x)^2+2}=\dfrac{x^4}{x^2+2}=f(x)$

y-axis symmetry

y-intercept: $y=\dfrac{0^4}{0^2+2}=0$

x-intercept: $x^4=0$
$x=0$
vertical asymptote: none
horizontal asymptote:
$n>m$, so none



$f(x)=\dfrac{x^4}{x^2+2}$

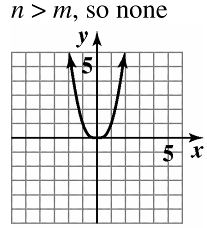
66. $f(x)=\dfrac{2x^4}{x^2+1}$

$f(-x)=\dfrac{2(-x)^4}{(-x)^2+1}=\dfrac{2x^4}{x^2+1}=f(x)$

y-axis symmetry

y-intercept: $y=\dfrac{2(0^4)}{0^2+2}=0$

x-intercept: $2x^4=0$
$x=0$
vertical asymptote: none
horizontal asymptote:
$n>m$, so none



$f(x)=\dfrac{2x^4}{x^2+1}$

67. $f(x)=\dfrac{x^2+x-12}{x^2-4}$

$f(-x)=\dfrac{(-x)^2-x-12}{(-x)^2-4}=\dfrac{x^2-x-12}{x^2-4}$

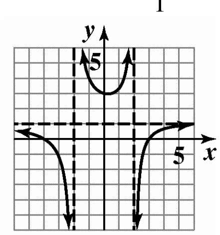
$f(-x)\neq f(x), f(-x)\neq -f(x)$
no symmetry

y-intercept: $y=\dfrac{0^2+0-12}{0^2-4}=3$

x-intercept: $x^2+x-12=0$
$(x-3)(x+4)=0$
$x=3, x=-4$

vertical asymptotes:
$x^2-4=0$
$(x-2)(x+2)=0$
$x=2, x=-2$

horizontal asymptote:
$n=m$, so $y=\dfrac{1}{1}=1$



$f(x)=\dfrac{x^2+x-12}{x^2-4}$

68. $f(x) = \dfrac{x^2}{x^2 + x - 6}$

$f(-x) = \dfrac{(-x)^2}{(-x)^2 - x - 6} = \dfrac{x^2}{x^2 - x - 6}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{0^2}{0^2 + 0 - 6} = 0$

x-intercept: $x^2 = 0, x = 0$

vertical asymptotes:

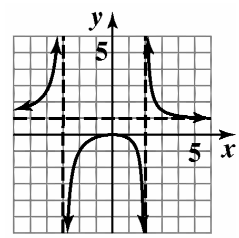
$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3, x = 2$

horizontal asymptote:

$n = m$, so $y = \dfrac{1}{1} = 1$



$f(x) = \dfrac{x^2}{x^2 + x - 6}$

69. $f(x) = \dfrac{3x^2 + x - 4}{2x^2 - 5x}$

$f(-x) = \dfrac{3(-x)^2 - x - 4}{2(-x)^2 + 5x} = \dfrac{3x^2 - x - 4}{2x^2 + 5x}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{3(0)^2 + 0 - 4}{2(0)^2 - 5(0)} = \dfrac{-4}{0}$

no y-intercept

x-intercepts:

$3x^2 + x - 4 = 0$

$(3x+4)(x-1) = 0$

$3x + 4 = 0 \quad x - 1 = 0$

$3x = -4$

$x = -\dfrac{4}{3}, x = 1$

vertical asymptotes:

$2x^2 - 5x = 0$

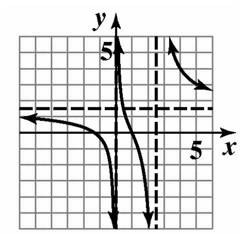
$x(2x - 5) = 0$

$x = 0, 2x = 5$

$x = \dfrac{5}{2}$

horizontal asymptote:

$n = m$, so $y = \dfrac{3}{2}$



$f(x) = \dfrac{3x^2 + x - 4}{2x^2 - 5x}$

70. $f(x) = \dfrac{x^2 - 4x + 3}{(x+1)^2}$

$f(-x) = \dfrac{(-x)^2 - 4(-x) + 3}{(-x+1)^2} = \dfrac{x^2 + 4x + 3}{(-x+1)^2}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{0^2 - 4(0) + 3}{(0+1)^2} = \dfrac{3}{1} = 3$

x-intercept:

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 3$ and $x = 1$

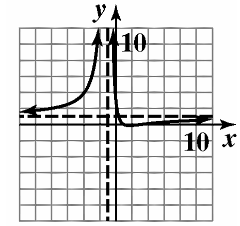
vertical asymptote:

$(x+1)^2 = 0$

$x = -1$

horizontal asymptote:

$n = m$, so $y = \dfrac{1}{1} = 1$



$f(x) = \dfrac{x^2 - 4x + 3}{(x+1)^2}$

71. **a.** Slant asymptote:

$f(x) = x - \frac{1}{x}$

$y = x$

b. $f(x) = \frac{x^2 - 1}{x}$

$f(-x) = \frac{(-x)^2 - 1}{(-x)} = \frac{x^2 - 1}{-x} = -f(x)$

Origin symmetry

y-intercept: $y = \frac{0^2 - 1}{0} = \frac{-1}{0}$

no y-intercept

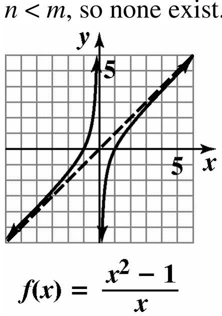
x-intercepts: $x^2 - 1 = 0$

$x = \pm 1$

vertical asymptote: $x = 0$

horizontal asymptote:

$n < m$, so none exist.



72. $f(x) = \frac{x^2 - 4}{x}$

a. slant asymptote:

$f(x) = x - \frac{4}{x}$

$y = x$

b. $f(x) = \frac{x^2 - 4}{x}$

$f(-x) = \frac{(-x)^2 - 4}{-x} = \frac{x^2 - 4}{-x} = -f(x)$

origin symmetry

y-intercept: $y = \frac{0^2 - 4}{0} = -\frac{4}{0}$

no y-intercept

x-intercept:

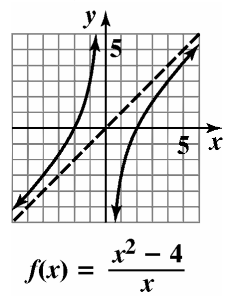
$x^2 - 4 = 0$

$x = \pm 2$

vertical asymptote: $x = 0$

horizontal asymptote:

$n > m$, so none exist.



73. **a.** Slant asymptote:

$f(x) = x + \frac{1}{x}$

$y = x$

b. $f(x) = \frac{x^2 + 1}{x}$

$f(-x) = \frac{(-x)^2 + 1}{-x} = \frac{x^2 + 1}{-x} = -f(x)$

Origin symmetry

y-intercept: $y = \frac{0^2 + 1}{0} = \frac{1}{0}$

no y-intercept

x-intercept:

$x^2 + 1 = 0$

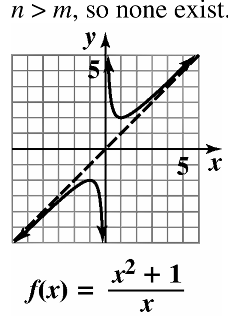
$x^2 = -1$

no x-intercept

vertical asymptote: $x = 0$

horizontal asymptote:

$n > m$, so none exist.



74. $f(x)=\dfrac{x^2+4}{x}$

a. slant asymptote:

$g(x)=x+\dfrac{4}{x}$

$y=x$

b. $f(x)=\dfrac{x^2+4}{x}$

$f(-x)=\dfrac{(-x)^2+4}{-x}=\dfrac{x^2+4}{-x}=-f(x)$

origin symmetry

y-intercept: $y=\dfrac{0^2+4}{0}=\dfrac{4}{0}$

no y-intercept

$x^2+4=0$

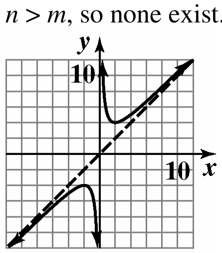
$x^2=-4$

no x-intercept

vertical asymptote: $x=0$

horizontal asymptote:

$n>m$, so none exist.



$f(x)=\dfrac{x^2+4}{x}$

75. a. Slant asymptote:

$f(x)=x+4+\dfrac{6}{x-3}$

$y=x+4$

b. $f(x)=\dfrac{x^2+x-6}{x-3}$

$f(-x)=\dfrac{(-x)^2+(-x)-6}{-x-3}=\dfrac{x^2-x-6}{-x-3}$

$f(-x)\neq g(x)$, $g(-x)\neq -g(x)$

No symmetry

y-intercept: $y=\dfrac{0^2+0-6}{0-3}=\dfrac{-6}{-3}=2$

x-intercept:

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3$ and $x=2$

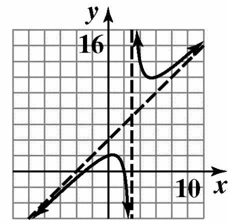
vertical asymptote:

$x-3=0$

$x=3$

horizontal asymptote:

$n>m$, so none exist.



$f(x)=\dfrac{x^2+x-6}{x-3}$

76. $f(x)=\dfrac{x^2-x+1}{x-1}$

a. slant asymptote:

$g(x)=x+\dfrac{1}{x-1}$

$y=x$

b. $f(x)=\dfrac{x^2-x-1}{x-1}$

$f(-x)=\dfrac{(-x)^2-(-x)+1}{-x-1}=\dfrac{x^2+x+1}{-x-1}$

no symmetry

$f(-x)\neq f(x), f(-x)\neq -g(x)$

y-intercept: $y=\dfrac{0^2-0+1}{0-1}=\dfrac{1}{-1}=-1$

x-intercept:

$x^2-x+1=0$

no x-intercept

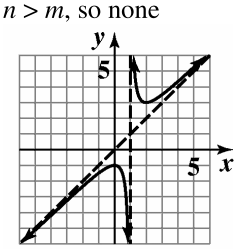
vertical asymptote:

$x-1=0$

$x=1$

horizontal asymptote:

$n>m$, so none



$f(x)=\dfrac{x^2-x+1}{x-1}$

77. $f(x)=\dfrac{x^3+1}{x^2+2x}$

a. slant asymptote:

$$\begin{array}{r} x-2 \\ x^2+2x\overline{)x^3 \qquad +1} \\ \underline{x^3+2x^2} \\ -2x^2 \\ \underline{-2x^2+4x} \\ -4x+1 \end{array}$$

$y=x-2$

b. $f(-x)=\dfrac{(-x)^3+1}{(-x)^2+2(-x)}=\dfrac{-x^3+1}{x^2-2x}$

$f(-x)\neq f(x),\ f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0^3+1}{0^2+2(0)}=\dfrac{1}{0}$

no *y*-intercept

x-intercept: $x^3+1=0$

$x^3=-1$

$x=-1$

vertical asymptotes:

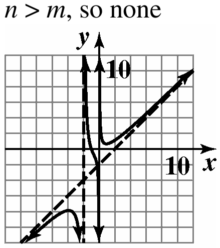
$x^2+2x=0$

$x(x+2)=0$

$x=0,\ x=-2$

horizontal asymptote:

$n>m$, so none



$f(x)=\dfrac{x^3+1}{x^2+2x}$

78. $f(x)=\dfrac{x^3-1}{x^2-9}$

a. slant asymptote:

$$\begin{array}{r} x+\dfrac{9x-1}{x^2-9} \\ x^2-9\overline{)x^3 \qquad -1} \\ \underline{x^3-9x} \\ 9x-1 \end{array}$$

$y=x$

b. $f(-x)=\dfrac{(-x)^3-1}{(-x)^2-9}=\dfrac{-x^3-1}{x^2-9}$

$f(-x)\neq f(x),\ f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0^3-1}{0^2-9}=\dfrac{1}{9}$

x-intercept: $x^3-1=0$

$x^3=1$

$x=1$

vertical asymptotes:

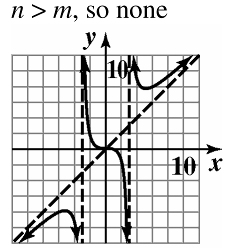
$x^2-9=0$

$(x-3)(x+3)=0$

$x=3,\ x=-3$

horizontal asymptote:

$n>m$, so none



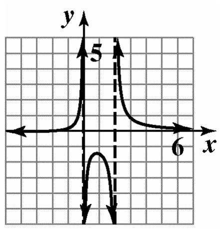
$f(x)=\dfrac{x^3-1}{x^2-9}$

79. $\frac{5x^2}{x^2-4}\cdot\frac{x^2+4x+4}{10x^3}$

$=\frac{\cancel{5}\,\cancel{x^2}}{\cancel{(x+2)}(x-2)}\cdot\frac{(x+2)^{\cancel{2}}}{\underset{2}{\cancel{10}}\,x^{\cancel{3}1}}$

$=\frac{x+2}{2x(x-2)}$

So, $f(x)=\frac{x+2}{2x(x-2)}$



$f(x)=\frac{x+2}{2x(x-2)}$

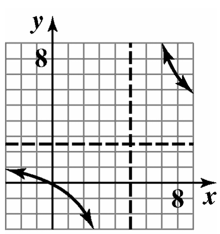
80. $\frac{x-5}{10x-2}\div\frac{x^2-10x+25}{25x^2-1}$

$=\frac{x-5}{10x-2}\cdot\frac{25x^2-1}{x^2-10x+25}$

$=\frac{\cancel{x-5}}{2\cancel{(5x-1)}}\cdot\frac{(5x+1)\cancel{(5x-1)}}{(x-5)^{\cancel{2}}}$

$=\frac{5x+1}{2(x-5)}$

So, $f(x)=\frac{5x+1}{2(x-5)}$



$f(x)=\frac{5x+1}{2(x-5)}$

81. $\frac{x}{2x+6}-\frac{9}{x^2-9}$

$\frac{x}{2x+6}-\frac{9}{x^2-9}$

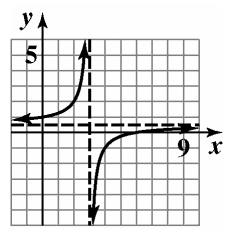
$=\frac{x}{2(x+3)}-\frac{9}{(x+3)(x-3)}$

$=\frac{x(x-3)-9(2)}{2(x+3)(x-3)}$

$=\frac{x^2-3x-18}{2(x+3)(x-3)}$

$=\frac{(x-6)\cancel{(x+3)}}{2\cancel{(x+3)}(x-3)}=\frac{x-6}{2(x-3)}$

So, $f(x)=\frac{x-6}{2(x-3)}$



$f(x)=\frac{x-6}{2(x-3)}$

82. $\dfrac{2}{x^2+3x+2}-\dfrac{4}{x^2+4x+3}$

$=\dfrac{2}{(x+2)(x+1)}-\dfrac{4}{(x+3)(x+1)}$

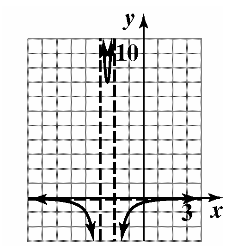
$=\dfrac{2(x+3)-4(x+2)}{(x+2)(x+1)(x+3)}$

$=\dfrac{2x+6-4x-8}{(x+2)(x+1)(x+3)}$

$=\dfrac{-2x-2}{(x+2)(x+1)(x+3)}$

$=\dfrac{-2\cancel{(x+1)}}{(x+2)\cancel{(x+1)}(x+3)}=\dfrac{-2}{(x+2)(x+3)}$

So, $f(x)=\dfrac{-2}{(x+2)(x+3)}$



$f(x)=\dfrac{-2}{(x+2)(x+3)}$

83. $\dfrac{1-\dfrac{3}{x+2}}{1+\dfrac{1}{x-2}}=\dfrac{1-\dfrac{3}{x+2}}{1+\dfrac{1}{x-2}}\cdot\dfrac{(x+2)(x-2)}{(x+2)(x-2)}$

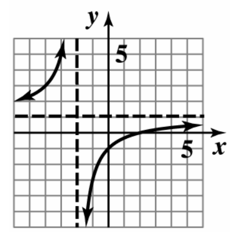
$=\dfrac{(x+2)(x-2)-3(x-2)}{(x+2)(x-2)+(x+2)}$

$=\dfrac{x^2-4-3x+6}{x^2-4+x+2}$

$=\dfrac{x^2-3x+2}{x^2+x-2}$

$=\dfrac{(x-2)\cancel{(x-1)}}{(x+2)\cancel{(x-1)}}=\dfrac{x-2}{x+2}$

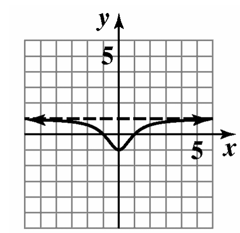
So, $f(x)=\dfrac{x-2}{x+2}$



$f(x)=\dfrac{x-2}{x+2}$

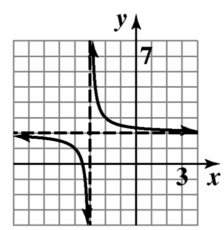
84. $\dfrac{x-\dfrac{1}{x}}{x+\dfrac{1}{x}}\cdot\dfrac{x}{x}=\dfrac{x^2-1}{x^2+1}=\dfrac{(x-1)(x+1)}{x^2+1}$

So, $f(x)=\dfrac{(x-1)(x+1)}{x^2+1}$



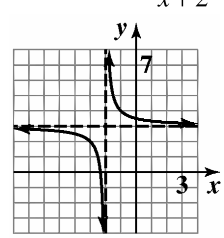
$f(x)=\dfrac{(x-1)(x+1)}{x^2+1}$

85. $g(x)=\dfrac{2x+7}{x+3}=\dfrac{1}{x+3}+2$



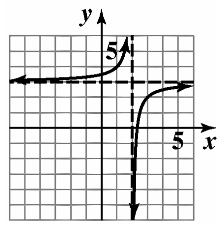
$f(x)=\dfrac{1}{x+3}+2$

86. $g(x)=\dfrac{3x+7}{x+2}=\dfrac{1}{x+2}+3$



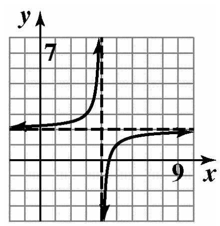
$f(x)=\dfrac{1}{x+2}+3$

87. $g(x) = \dfrac{3x-7}{x-2} = \dfrac{-1}{x-2} + 3$



$f(x) = \dfrac{-1}{x-2} + 3$

88. $g(x) = \dfrac{2x-9}{x-4} = \dfrac{-1}{x-4} + 2$



$f(x) = \dfrac{-1}{x-4} + 2$

89. a. $C(x) = 100x + 100{,}000$

b. $\overline{C}(x) = \dfrac{100x + 100{,}000}{x}$

c. $\overline{C}(500) = \dfrac{100(500) + 100{,}000}{500} = \$300$

When 500 bicycles are manufactured, it costs \$300 to manufacture each.

$\overline{C}(1000) = \dfrac{100(1000) + 100{,}000}{1000} = \$200$

When 1000 bicycles are manufactured, it costs \$200 to manufacture each.

$\overline{C}(2000) = \dfrac{100(2000) + 100{,}000}{2000} = \$150$

When 2000 bicycles are manufactured, it costs \$150 to manufacture each.

$\overline{C}(4000) = \dfrac{100(4000) + 100{,}000}{4000} = \$125$

When 4000 bicycles are manufactured, it costs \$125 to manufacture each.
The average cost decreases as the number of bicycles manufactured increases.

d. $n = m$, so $y = \dfrac{100}{1} = 100$.

As greater numbers of bicycles are manufactured, the average cost approaches \$100.

90. a. $C(x) = 30x + 300{,}000$

b. $\overline{C} = \dfrac{300{,}000 + 30x}{x}$

c. $\overline{C}(1000) = \dfrac{300000 + 30(1000)}{1000} = 330$

When 1000 shoes are manufactured, it costs \$330 to manufacture each.

$\overline{C}(10000) = \dfrac{300000 + 30(10000)}{10000} = 60$

When 10,000 shoes are manufactured, it costs \$60 to manufacture each.

$\overline{C}(100{,}00) = \dfrac{300{,}000 + 30(100{,}000)}{100{,}000} = 33$

When 100,000 shoes are manufactured, it costs \$33 to manufacture each.
The average cost decreases as the number of shoes manufactured increases.

d. $n = m$, so $y = \dfrac{30}{1} = 30$.

As greater numbers of shoes are manufactured, the average cost approaches \$30.

91. a. From the graph the pH level of the human mouth 42 minutes after a person eats food containing sugar will be about 6.0.

b. From the graph, the pH level is lowest after about 6 minutes.

$$f(6) = \frac{6.5(6)^2 - 20.4(6) + 234}{6^2 + 36}$$
$$= 4.8$$

The pH level after 6 minutes (i.e. the lowest pH level) is 4.8.

c. From the graph, the pH level appears to approach 6.5 as time goes by. Therefore, the normal pH level must be 6.5.

d. $y = 6.5$

Over time, the pH level rises back to the normal level.

e. During the first hour, the pH level drops quickly below normal, and then slowly begins to approach the normal level.

92. **a.** From the graph, the drug's concentration after three hours appears to be about 1.5 milligrams per liter.

$$C(3)=\frac{5(3)}{3^2+1}=\frac{15}{10}=1.5$$

This verifies that the drug's concentration after 3 hours will be 1.5 milligrams per liter.

b. The degree of the numerator, 1, is less than the degree of the denominator, 2, so the the horizontal asymptote is $y=0$.

Over time, the drug's concentration will approach 0 milligrams per liter.

93. $P(10)=\dfrac{100(10-1)}{10}=90 \quad (10, 90)$

For a disease that smokers are 10 times more likely to contact than non-smokers, 90% of the deaths are smoking related.

94. $P(9)=\dfrac{100(9-1)}{9}=89 \quad (9, 89)$

For a disease that smokers are 9 times more likely to have than non-smokers, 89% of the deaths are smoking related.

95. $y = 100$ As incidence of the diseases increases, the percent of death approaches, but never gets to be, 100%.

96. No, the percentage approaches 100%, but never reaches 100%.

97. **a.** $f(x)=\dfrac{11x^2+40x+1040}{12x^2+230x+2190}$

b. According to the graph, $\dfrac{1707.2}{2708.7}$ or about 63% of federal expenditures were spent on human resources in 2006.

c. According to the function,

$$f(36)=\frac{11(36)^2+40(36)+1040}{12(36)^2+230(36)+2190}=\frac{16736}{26022} \text{ or}$$

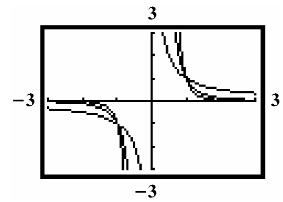
about 64% of federal expenditures were spent on human resources in 2006. This overestimates the actual percent found in the graph by 1%.

d. The horizontal asymptote is $y=\dfrac{11}{12}$.

If trends continue, $\dfrac{11}{12}$ or about 92% of federal expenditures will spent on human resources over time.

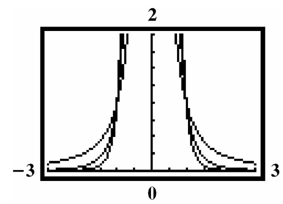
98. – 107. Answers may vary.

108.



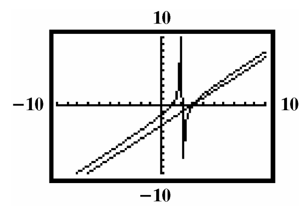
The graph approaches the horizontal asymptote faster and the vertical asymptote slower as n increases.

109.



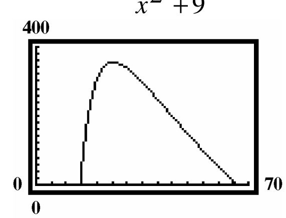
The graph approaches the horizontal asymptote faster and the vertical asymptote slower as n increases.

110.



$g(x)$ is the graph of a line where $f(x)$ is the graph of a rational function with a slant asymptote.
In $g(x)$, $x-2$ is a factor of x^2-5x+6.

111. **a.** $f(x)=\dfrac{27725(x-14)}{x^2+9}-5x$



b. The graph increases from late teens until about the age of 25, and then the number of arrests decreases.

c. At age 25 the highest number arrests occurs. There are about 356 arrests for every 100,000 drivers.

112. does not make sense; Explanations will vary. Sample explanation: A rational function can have at most one horizontal asymptote.

113. does not make sense; Explanations will vary. Sample explanation: The function has one vertical asymptote, $x = 2$.

114. makes sense

115. does not make sense; Explanations will vary. Sample explanation: As production level increases, the average cost for a company to produce each unit of its product decreases.

116. false; Changes to make the statement true will vary. A sample change is: The graph of a rational function may have both a vertical asymptote and a horizontal asymptote.

117. true

118. true

119. true

120. – 123. Answers may vary.

124.
$$\begin{aligned} 2x^2 + x &= 15 \\ 2x^2 + x - 15 &= 0 \\ (2x-5)(x+3) &= 0 \end{aligned}$$
$$2x - 5 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = \frac{5}{2} \qquad\qquad x = -3$$
The solution set is $\left\{-3, \frac{5}{2}\right\}$.

125.
$$\begin{aligned} x^3 + x^2 &= 4x + 4 \\ x^3 + x^2 - 4x - 4 &= 0 \\ x^2(x+1) - 4(x+1) &= 0 \\ (x+1)(x^2-4) &= 0 \\ (x+1)(x+2)(x-2) &= 0 \end{aligned}$$
The solution set is $\{-2, -1, 2\}$.

126.
$$\begin{aligned} \frac{x+1}{x+3} - 2 &= \frac{x+1}{x+3} - \frac{2(x+3)}{x+3} \\ &= \frac{x+1}{x+3} - \frac{2x+6}{x+3} \\ &= \frac{x+1-2x-6}{x+3} \\ &= \frac{-x-5}{x+3} \text{ or } -\frac{x+5}{x+3} \end{aligned}$$

Section 3.6

Check Point Exercises

1. $x^2 - x > 20$

$x^2 - x - 20 > 0$

$(x+4)(x-5) > 0$

Solve the related quadratic equation.

$(x+4)(x-5) = 0$

Apply the zero product principle.

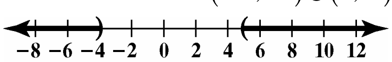
$x+4=0$ or $x-5=0$

$x=-4$ $\quad x=5$

The boundary points are –2 and 4.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-4)$	–5	$(-5)^2-(-5)>20$ $30>20$, true	$(-\infty,-4)$ belongs to the solution set.
$(-4,5)$	0	$(0)^2-(0)>20$ $0>20$, false	$(-4,5)$ does not belong to the solution set.
$(5,\infty)$	10	$(10)^2-(10)>20$ $90>20$, true	$(5,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-4)\cup(5,\infty)$ or $\{x \mid x<-4 \text{ or } x>5\}$.



2. $x^3+3x^2 \le x+3$

$x^3+3x^2-x-3 \le 0$

$(x+1)(x-1)(x+3) \le 0$

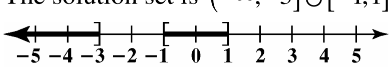
$(x+1)(x-1)(x+3) = 0$

$x+1=0$ or $x-1=0$ or $x+3=0$

$x=-1$ $\quad x=1$ $\quad x=-3$

Test Interval	Test Number	Test	Conclusion
$(-\infty,-3)$	-4	$(-4)^3+3(-4)^2 \le (-4)+3$ $-16 \le -1$ true	$(-\infty,-3)$ belongs to the solution set.
$(-3,-1]$	–2	$(-2)^3+3(-2)^2 \le (-2)+3$ $4 \le 1$ false	$(-3,-1]$ does not belong to the solution set.
$[-1,1]$	0	$(0)^3+3(0)^2 \le (0)+3$ $0 \le 3$ true	$[-1,1]$ belongs to the solution set.
$[1,\infty)$	2	$(6+3)(6-5)>0$ true	$[1,\infty)$ does not belong to the solution set.

The solution set is $(-\infty,-3]\cup[-1,1]$ or $\{x \mid x \le -3 \text{ or } -1 \le x \le 1\}$.

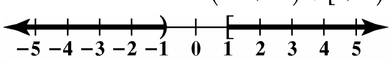


3.

$$\frac{2x}{x+1} \ge 1$$
$$\frac{2x}{x+1} - 1 \ge 0$$
$$\frac{x-1}{x+1} \ge 0$$
$$x-1=0 \quad \text{or} \quad x+1=0$$
$$x=1 \qquad\qquad x=-1$$

Test Interval	Test Number	Test	Conclusion
$(-\infty,-1)$	–2	$\frac{2(-2)}{-2+1} \ge 1$ $4 \ge 1$, true	$(-\infty,-1)$ belongs to the solution set.
$(-1,1]$	0	$\frac{2(0)}{0+1} \ge 1$ $0 \ge 1$, false	$(-1,1]$ does not belong to the solution set.
$[1,\infty)$	2	$\frac{2(2)}{2+1} \ge 1$ $\frac{4}{3} \ge 1$, true	$[1,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-1) \cup [1,\infty)$ or $\{x \mid x < -1 \text{ or } x \ge 1\}$.



4.

$$-16t^2 + 80t > 64$$
$$-16t^2 + 80t - 64 > 0$$
$$-16(t-1)(t-4) > 0$$
$$t-1=0 \quad \text{or} \quad t-4=0$$
$$t=1 \qquad\qquad t=4$$

Test Interval	Test Number	Test	Conclusion
$(-\infty,1)$	0	$-16(0)^2 + 80(0) > 64$ $0 > 64$, false	$(-\infty,1)$ does not belong to the solution set.
$(1,4)$	2	$-16(2)^2 + 80(2) > 64$ $96 > 64$, true	$(1,4)$ belongs to the solution set.
$(4,\infty)$	5	$-16(5)^2 + 80(5) > 64$ $0 > 64$, false	$(4,\infty)$ does not belong to the solution set.

The object will be more than 64 feet above the ground between 1 and 4 seconds.

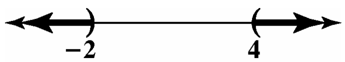
Exercise Set 3.6

1. $(x-4)(x+2) > 0$
$x = 4$ or $x = -2$

T	F	T

$-2 \quad 4$

Test –3: $(-3-4)(-3+2) > 0$
$7 > 0$ True
Test 0: $(0-4)(0+2) > 0$
$-8 > 0$ False
Test 5: $(5-4)(5+2) > 0$
$7 > 0$ True
$(-\infty, -2)$ or $(4, \infty)$

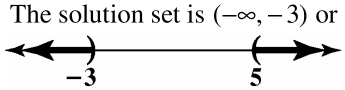


2. $(x+3)(x-5) > 0$
$x = -3$ or $x = 5$

T	F	T

$-3 \quad 5$

Test –4: $(-4+3)(-4-5) > 0$
$9 > 0$ True
Test 0: $(0+3)(0-5) > 0$
$-15 > 0$ False
Test 6: $(6+3)(6-5) > 0$
$18 > 0$ True
The solution set is $(-\infty, -3)$ or $(5, \infty)$.

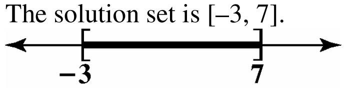


3. $(x-7)(x+3) \le 0$
$x = 7$ or $x = -3$

F	T	F

$-3 \quad 7$

Test –4: $(-4-7)(-4+3) \le 0$
$11 \le 0$ False
Test 0: $(0-7)(0+3) \le 0$
$-21 \le 0$ True
Test 8: $(8-7)(8+3) \le 0$
$11 \le 0$ False
The solution set is $[-3, 7]$.

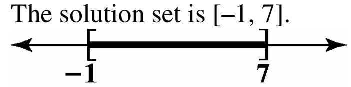


4. $(x+1)(x-7) \le 0$
$x = -1$ or $x = 7$

F	T	F

$-1 \quad 7$

Test –2: $(-2+1)(-2-7) \le 0$
$9 \le 0$ False
Test 0: $(0+1)(0-7) \le 0$
$-7 \le 0$ True
Test 8: $(8+1)(8-7) \le 0$
$9 \le 0$ False
The solution set is $[-1, 7]$.

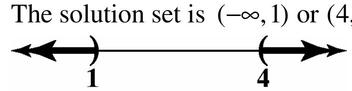


5. $x^2 - 5x + 4 > 0$
$(x-4)(x-1) > 0$
$x = 4$ or $x = 1$

T	F	T

$1 \quad 4$

Test 0: $0^2 - 5(0) + 4 > 0$
$4 > 0$ True
Test 2: $2^2 - 5(2) + 4 > 0$
$-2 > 0$ False
Test 5: $5^2 - 5(5) + 4 > 0$
$4 > 0$ True
The solution set is $(-\infty, 1)$ or $(4, \infty)$.

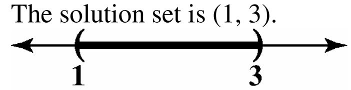


6. $x^2 - 4x + 3 < 0$
$(x-1)(x-3) < 0$
$x = 1$ or $x = 3$

F	T	F

$1 \quad 3$

Test 0: $0^2 - 4(0) + 3 < 0$
$3 < 0$ False
Test 2: $2^2 - 4(2) + 3 < 0$
$-1 < 0$ True
Test 4: $4^2 - 4(4) + 3 < 0$
$3 < 0$ False
The solution set is $(1, 3)$.



7. $x^2 + 5x + 4 > 0$

$(x+1)(x+4) > 0$

$x = -1$ or $x = -4$

T	F	T
	–4	–1

Test –5: $(-5)^2 + 5(-5) + 4 > 0$

$4 > 0$ True

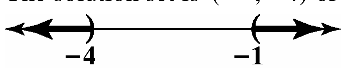
Test –3: $(-3)^2 + 5(-3) + 4 > 0$

$-2 > 0$ False

Test 0: $0^2 + 5(0) + 4 > 0$

$4 > 0$ True

The solution set is $(-\infty, -4)$ or $(-1, \infty)$.



8. $x^2 + x - 6 > 0$

$(x+3)(x-2) > 0$

$x = -3$ or $x = 2$

T	F	T
	–3	2

Test –4: $(-4)^2 - 4 - 6 > 0$

$6 > 0$ True

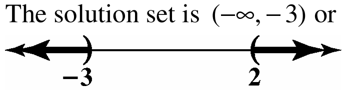
Test 0: $(0)^2 + 0 - 6 > 0$

$-6 > 0$ False

Test 3: $3^2 + 3 - 6 > 0$

$6 > 0$ True

The solution set is $(-\infty, -3)$ or $(2, \infty)$.



9. $x^2 - 6x + 9 < 0$

$(x-3)(x-3) < 0$

$x = 3$

F	F
	3

Test 0: $0^2 - 6(0) + 9 < 0$

$9 < 0$ False

Test 4: $4^2 - 6(4) + 9 < 0$

$1 < 0$ False

The solution set is the empty set, $\varnothing$.

10. $x^2 - 2x + 1 > 0$

$(x-1)(x-1) > 0$

$x = 1$

T	T
	1

Test 0: $0^2 - 2(0) + 1 > 0$

$1 > 0$ True

Test 2: $2^2 - 2(2) + 1 > 0$

$1 > 0$ True

The solution set is $(-\infty, 1)$ or $(1, \infty)$.

11. $3x^2 + 10x - 8 \le 0$

$(3x-2)(x+4) \le 0$

$x = \frac{2}{3}$ or $x = -4$

F	T	F
	–4	$\frac{2}{3}$

Test –5: $3(-5)^2 + 10(-5) - 8 \le 0$

$17 \le 0$ False

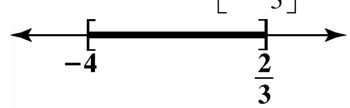
Test 0: $3(0)^2 + 10(0) - 8 \le 0$

$8 \le 0$ True

Test 1: $3(1)^2 + 10(1) - 8 \le 0$

$5 \le 0$ False

The solution set is $\left[-4, \frac{2}{3}\right]$.



12. $9x^2+3x-2 \ge 0$

$(3x-1)(3x+2) \ge 0$

$3x=1 \quad 3x=-2$

$x=\frac{1}{3} \quad x=\frac{-2}{3}$

T	F	T

$\frac{-2}{3}$ $\quad$ $\frac{1}{3}$

Test –1: $9(-1)^2+3(-1)-2 \ge 0$

$4 \ge 0$ True

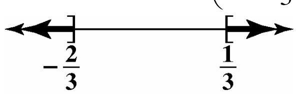
Test 0: $9(0)^2+3(0)-2 \ge 0$

$-2 \ge 0$ False

Test 1: $9(1)^2+3(1)-2 \le 0$

$10 \ge 0$ True

The solution set is $\left(-\infty, \frac{-2}{3}\right]$ or $\left[\frac{1}{3}, \infty\right)$.



13. $2x^2+x<15$

$2x^2+x-15<0$

$(2x-5)(x+3)<0$

$2x-5=0$ or x+3=0

$2x=5$

$x=\frac{5}{2}$ or $x=-3$

F	T	F

-3 $\quad$ $\frac{5}{2}$

Test –4: $2(-4)^2+(-4)<15$

$28 < 15$ False

Test 0: $2(0)^2+0<15$

$0 < 15$ True

Test 3: $2(3)^2+3<15$

$21 < 15$ False

The solution set is $\left(-3, \frac{5}{2}\right)$.

14. $6x^2+x>1$

$6x^2+x-1>0$

$(2x+1)(3x-1)>0$

$2x+1=0$ or $3x-1=0$

$2x=-1 \quad 3x=1$

$x=-\frac{1}{2} \quad x=\frac{1}{3}$

T	F	T

$-\frac{1}{2}$ $\quad$ $\frac{1}{3}$

Test –1: $6(-1)^2+(-1)>1$

$5 > 1$ True

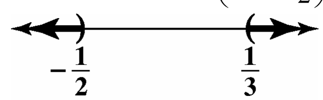
Test 0: $6(0)^2+0>1$

$0 > 1$ False

Test 1: $6(1)^2+1>1$

$7 > 1$ True

The solution set is $\left(-\infty, -\frac{1}{2}\right)$ or $\left(\frac{1}{3}, \infty\right)$.



15. $4x^2 + 7x < -3$

$4x^2 + 7x + 3 < 0$

$(4x+3)(x+1) < 0$

$4x + 3 = 0$ or $x+1=0$

$4x\text{-}3=0$

$x = -\frac{3}{4}$ or $x = -1$

F	T	F

-1 $\quad -\frac{3}{4}$

Test –2: $4(-2)^2 + 7(-2) < -3$

$2 < -3$ False

Test $-\frac{7}{8}$: $4\left(-\frac{7}{8}\right)^2 + 7\left(-\frac{7}{8}\right) < -3$

$\frac{49}{16} - \frac{49}{8} < -3$

$-\frac{49}{16} < -3$ True

Test 0: $4(0)^2 + 7(0) < -3$

$0 < -3$ False

The solution set is $\left(-1, -\frac{3}{4}\right)$.

16. $3x^2 + 16x < -5$

$3x^2 + 16x + 5 < 0$

$(3x+1)(x+5) < 0$

$3x + 1 = 0$ or $x + 5 = 0$

$3x = -1$

$x = -\frac{1}{3}$ $\qquad x = -5$

F	T	F

-5 $\quad -\frac{1}{3}$

Test –6: $3(-6)^2 + 16(-6) < -5$

$12 < -5$ False

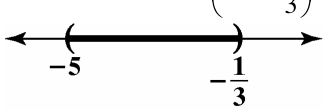
Test –2: $3(-2)^2 + 16(-2) < -5$

$-20 < -5$ True

Test 0: $3(0)^2 + 16(0) < -5$

$0 < -5$ False

The solution set is $\left(-5, -\frac{1}{3}\right)$.



17. $5x \le 2 - 3x^2$

$3x^2 + 5x - 2 \le 0$

$(3x-1)(x+2) \le 0$

$3x - 1 = 0$ or $x + 2 = 0$

$3x = 1$

$3x - 1=0$ or $x + 2 = 0$

$3x = 1$

$x = \frac{1}{3}$ or $x = -2$

F	T	F

-2 $\quad \frac{1}{3}$

Test –3: $5(-3) \le 2 - 3(-3)^2$

$-15 \le -25$ False

Test 0: $5(0) \le 2 - 3(0)^2$

$0 \le 2$ True

Test 1: $5(1) \le 2 - 3(1)^2$

$5 \le -1$ False

The solution set is $\left[-2, \frac{1}{3}\right]$.

18. $4x^2 + 1 \ge 4x$

$4x^2 - 4x + 1 \ge 0$

$(2x-1)(2x-1) \ge 0$

$2x - 1 = 0$

$x = \frac{1}{2}$

T	T

$\frac{1}{2}$

Test 0: $4(0)^2 + 1 \ge 4(0)$

$1 \ge 0$ True

Test 1: $4(1)^2 + 1 \ge 4(1)$

$5 \ge 4$ True

The solution set is $(-\infty, \infty)$.

19. $x^2 - 4x \geq 0$

$x(x-4) \geq 0$

$x = 0$ or $x - 4 = 0$

$x = 4$

T	F	T
	0	4

Test –1: $(-1)^2 - 4(-1) \geq 0$

$5 \geq 0$ True

Test 1: $(1)^2 - 4(1) \geq 0$

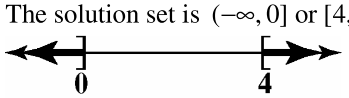
$-3 \geq 0$ False

$0 \leq 2$ True

Test 5: $5^2 - 4(5) \geq 0$

$5 \geq 0$ True

The solution set is $(-\infty, 0]$ or $[4, \infty)$.



20. $x^2 + 2x < 0$

$x(x+2) < 0$

$x = 0$ or $x = -2$

F	T	F
	–2	0

Test –3: $(-3)^2 + 2(-3) < 0$

$3 < 0$ False

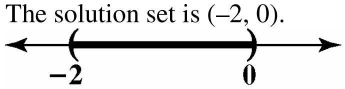
Test –1: $(-1)^2 + 2(-1) < 0$

$-1 < 0$ True

Test 1: $(1)^2 + 2(1) < 0$

$3 < 0$ False

The solution set is (–2, 0).



21. $2x^2 + 3x > 0$

$x(2x+3) > 0$

$x = 0$ or $x = -\frac{3}{2}$

T	F	T
	$-\frac{3}{2}$	0

Test –2: $2(-2)^2 + 3(-2) > 0$

$2 > 0$ True

Test –1: $2(-1)^2 + 3(-1) > 0$

$-1 > 0$ False

Test 1: $2(1)^2 + 3(1) > 0$

$5 > 0$ True

The solution set is $\left(-\infty, -\frac{3}{2}\right)$ or $(0, \infty)$.

22. $3x^2 - 5x \leq 0$

$x(3x-5) \leq 0$

$x = 0$ or $x = \frac{5}{3}$

F	T	F
	0	$\frac{5}{3}$

Test –1: $3(-1)^2 - 5(-1) \leq 0$

$8 \leq 0$ False

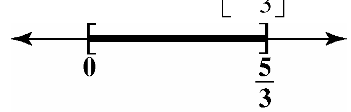
Test 1: $3(1)^2 - 5(1) \leq 0$

$-2 \leq 0$ True

Test 2: $3(2)^2 - 5(2) \leq 0$

$2 \leq 0$ False

The solution set is $\left[0, \frac{5}{3}\right]$.



23. $-x^2 + x \geq 0$

$x^2 - x \leq 0$

$x(x-1) \leq 0$

$x = 0 \quad \text{or} \quad x = 1$

F	T	F
	0	1

Test –1: $-(-1)^2 + (-1) \geq 0$

$-2 \geq 0$ False

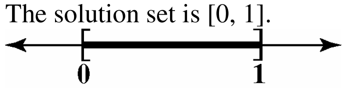
Test $\frac{1}{2}$: $-\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) \geq 0$

$\frac{1}{4} \geq 0$ True

Test 2: $-(2)^2 + 2 \geq 0$

$-2 \geq 0$ False

The solution set is [0, 1].



24. $-x^2 + 2x \geq 0$

$x(-x+2) \geq 0$

$x = 0$ or $x = 2$

F	T	F
	0	2

Test –1: $-(-1)^2 + 2(-1) \geq 0$

$-3 \geq 0$ False

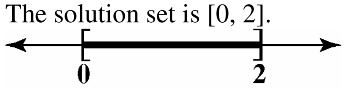
Test 1: $-(1)^2 + 2(1) \geq 0$

$1 \geq 0$ True

Test 3: $-(3)^2 + 2(3) \geq 0$

$-3 \geq 0$ False

The solution set is [0, 2].



25. $x^2 \leq 4x - 2$

$x^2 - 4x + 2 \leq 0$

Solve $x^2 - 4x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

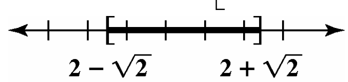
$$= \frac{4 \pm \sqrt{8}}{2}$$

$$= 2 \pm \sqrt{2}$$

$x \approx 0.59$ or $x \approx 3.41$

F	T	F
	0.59	3.41

The solution set is $\left[2-\sqrt{2}, 2+\sqrt{2}\right]$ or $[0.59, 3.41]$.



26. $x^2 \leq 2x + 2$

$x^2 - 2x - 2 \leq 0$

Solve $x^2 - 2x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$$

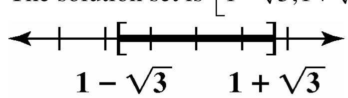
$$= \frac{2 \pm \sqrt{12}}{2}$$

$$= 1 \pm \sqrt{3}$$

$x \approx -0.73$ or $x \approx 2.73$

F	T	F
	–0.73	2.73

The solution set is $\left[1-\sqrt{3}, 1+\sqrt{3}\right]$ or $[-0.73, 2.73]$.



27. $x^2 - 6x + 9 < 0$

Solve $x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$(x-3)^2 = 0$

$x = 3$

F	F

3

The solution set is the empty set, $\varnothing$.

28. $4x^2 - 4x + 1 \ge 0$

Solve $4x^2 - 4x + 1 = 0$

$(2x-1)(2x-1) = 0$

$(2x-1)^2 = 0$

$x = \frac{1}{2}$

T	T

$\frac{1}{2}$

The solution set is $(-\infty, \infty)$.

29. $(x-1)(x-2)(x-3) \ge 0$

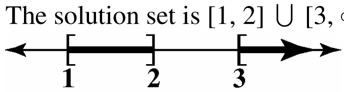
Boundary points: 1, 2, and 3

Test one value in each interval.

F	T	F	T

1 2 3

The solution set is [1, 2] ∪ [3, ∞).



30. $(x+1)(x+2)(x+3) \ge 0$

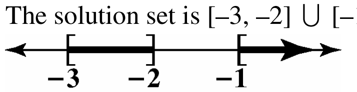
Boundary points: –1, –2, and –3

Test one value in each interval.

F	T	F	T

–3 –2 –1

The solution set is [–3, –2] ∪ [–1, ∞).



31. $x(3-x)(x-5) \le 0$

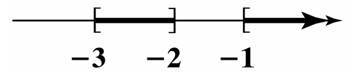
Boundary points: 0, 3, and 5

Test one value in each interval.

F	T	F	T

0 3 5

The solution set is [0, 3] ∪ [5, ∞).



32. $x(4-x)(x-6) \le 0$

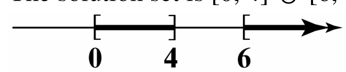
Boundary points: 0, 3, and 5

Test one value in each interval.

F	T	F	T

0 4 6

The solution set is [0, 4] ∪ [6, ∞).



33. $(2-x)^2\left(x-\frac{7}{2}\right) < 0$

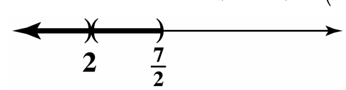
Boundary points: 2, and $\frac{7}{2}$

Test one value in each interval.

T	T	F

2 $\frac{7}{2}$

The solution set is $(-\infty, 2) \cup \left(2, \frac{7}{2}\right)$.



34. $(5-x)^2\left(x-\frac{13}{2}\right) < 0$

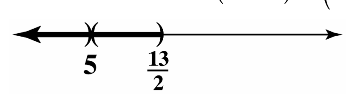
Boundary points: 5, and $\frac{13}{2}$

Test one value in each interval.

T	T	F

5 $\frac{13}{2}$

The solution set is $(-\infty, 5) \cup \left(5, \frac{13}{2}\right)$.



35. $x^3 + 2x^2 - x - 2 \ge 0$

$x^2(x+2) - 1(x+2) \ge 0$

$(x+2)(x^2 - 1) \ge 0$

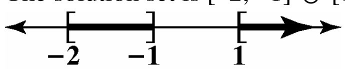
$(x+2)(x-1)(x+1) \ge 0$

Boundary points: –2, –1, and 2

Test one value in each interval.

F	T	F	T
	–2	–1	2

The solution set is [–2, –1] ∪ [1, ∞).



36. $x^3 + 2x^2 - 4x - 8 \ge 0$

$x^2(x+2) - 4(x+1) \ge 0$

$(x+2)(x^2 - 4) \ge 0$

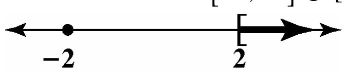
$(x+2)(x+2)(x-2) \ge 0$

Boundary points: –2, and 2

Test one value in each interval.

F	F	T
	–2	2

The solution set is $[-2,-2]$ ∪ [2, ∞).



37. $x^3 + 2x^2 - x - 2 \ge 0$

$x^2(x-3) - 9(x-3) \ge 0$

$(x-3)(x^2 - 9) \ge 0$

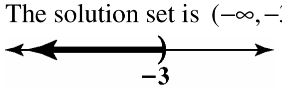
$(x-3)(x+3)(x-3) \ge 0$

Boundary points: –3 and 3

Test one value in each interval.

T	F	F
	–3	3

The solution set is $(-\infty, -3]$.



38. $x^3 + 7x^2 - x - 7 < 0$

$x^2(x+7) - (x+7) < 0$

$(x+7)(x^2 - 1) < 0$

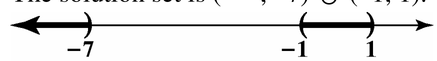
$(x+7)(x+1)(x-1) < 0$

Boundary points: –7, –1 and 1

Test one value in each interval.

T	F	T	F
	–7	–1	1

The solution set is (–∞ , –7) ∪ (–1, 1).



39. $x^3 + x^2 + 4x + 4 > 0$

$x^2(x+1) + 4(x+1) \ge 0$

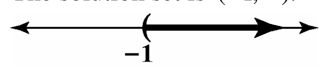
$(x+1)(x^2 + 4) \ge 0$

Boundary point: –1

Test one value in each interval.

F	T
	–1

The solution set is $(-1, \infty)$.



40. $x^3 - x^2 + 9x - 9 > 0$

$x^2(x-1) + 9(x-1) \ge 0$

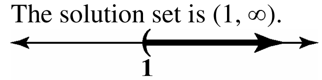
$(x-1)(x^2 + 9) \ge 0$

Boundary point: 1.

Test one value in each interval.

F	T
	1

The solution set is (1, ∞).



41. $x^3 - 9x^2 \ge 0$

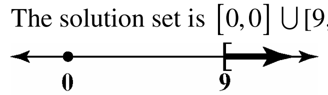
$x^2(x-9) \ge 0$

Boundary points: 0 and 9

Test one value in each interval.

F	F	T
	0	9

The solution set is $[0,0]$ ∪ [9, ∞).



42. $x^3 - 4x^2 \le 0$

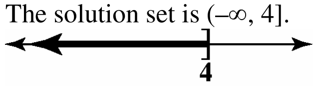
$x^2(x-4) \le 0$

Boundary points: 0 and 4.
Test one value in each interval.

T	T	F

0 4

The solution set is $(-\infty, 4]$.



43. $\dfrac{x-4}{x+3} > 0$

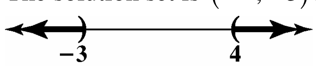
$x - 4 = 0 \quad x + 3 = 0$

$x = 4 \quad x = -3$

T	F	T

–3 4

The solution set is $(-\infty, -3) \cup (4, \infty)$.



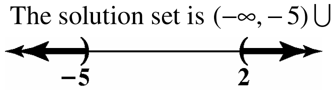
44. $\dfrac{x+5}{x-2} > 0$

$x = -5$ or $x = 2$

T	F	T

–5 2

The solution set is $(-\infty, -5) \cup (2, \infty)$.



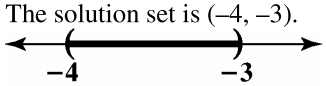
45. $\dfrac{x+3}{x+4} < 0$

$x = -3$ or $x = -4$

F	T	F

–4 –3

The solution set is (–4, –3).



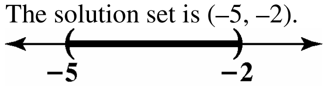
46. $\dfrac{x+5}{x+2} < 0$

$x = -5$ or $x = -2$

F	T	F

–5 –2

The solution set is (–5, –2).



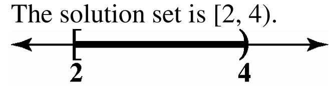
47. $\dfrac{-x+2}{x-4} \ge 0$

$x = 2$ or $x = 4$

F	T	F

2 4

The solution set is [2, 4).



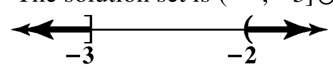
48. $\dfrac{-x-3}{x+2} \le 0$

$x = -3$ or $x = -2$

T	F	T

–3 –2

The solution set is $(-\infty, -3] \cup (-2, \infty)$.



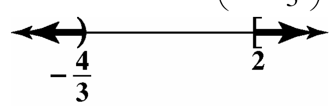
49. $\dfrac{4-2x}{3x+4} \le 0$

$x = 2$ or $x = -\dfrac{4}{3}$

T	F	T

$-\frac{4}{3}$ 2

The solution set is $\left(-\infty, \dfrac{-4}{3}\right) \cup [2, \infty)$.



50. $\dfrac{3x+5}{6-2x} \ge 0$

$x = -\dfrac{5}{3}$ or $x = 3$

F	T	F

$-\frac{5}{3}$ 3

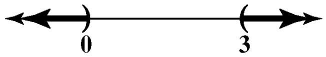
The solution set is $\left[-\dfrac{5}{3}, 3\right)$.

51. $\dfrac{x}{x-3} > 0$

$x = 0$ or $x = 3$

T	F	T
0	3	

The solution set is $(-\infty, 0) \cup (3, \infty)$.

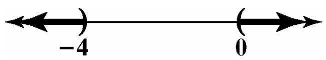


52. $\dfrac{x+4}{x} > 0$

$x = -4$ or $x = 0$

T	F	T
–4	0	

The solution set is $(-\infty, -4) \cup (0, \infty)$.



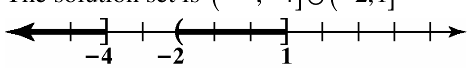
53. $\dfrac{(x+4)(x-1)}{x+2} \le 0$

$x = -4$ or $x = -2$ or $x = 1$.

T	F	T	F
–4	–2	1	

Values of $x = -4$ or $x = 1$ result in $f(x) = 0$ and, therefore must be included in the solution set.

The solution set is $(-\infty, -4] \cup (-2, 1]$



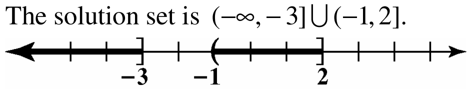
54. $\dfrac{(x+3)(x-2)}{x+1} \le 0$

$x = -3$ or $x = -1$ or $x = 2$.

T	F	T	F
–3	–1	2	

Values of $x = -3$ or $x = 2$ result in $f(x) = 0$ and, therefore must be included in the solution set.

The solution set is $(-\infty, -3] \cup (-1, 2]$.

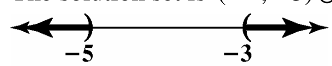


55.
$$\frac{x+1}{x+3} < 2$$
$$\frac{x+1}{x+3} - 2 < 0$$
$$\frac{x+1-2(x+3)}{x+3} < 0$$
$$\frac{x+1-2x-6}{x+3} < 0$$
$$\frac{-x-5}{x+3} < 0$$

$x =$ or $x = -3$

T	F	T
–5	–3	

The solution set is $(-\infty, -5) \cup (-3, \infty)$.

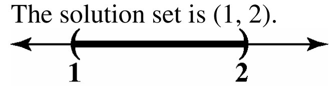


56.
$$\frac{x}{x-1} > 2$$
$$\frac{x}{x-1} - 2 > 0$$
$$\frac{x}{x-1} - \frac{2(x-1)}{x-1} > 0$$
$$\frac{x-2x+2}{x-1} > 0$$
$$\frac{-x+2}{x-1} > 0$$

$x = 2$ or $x = 1$

F	T	F
1	2	

The solution set is (1, 2).

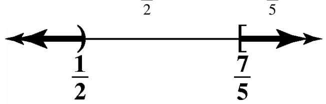


57.

$$\frac{x+4}{2x-1} \le 3$$
$$\frac{x+4}{2x-1} - 3 \le 0$$
$$\frac{x+4-3(2x-1)}{2x-1} \le 0$$
$$\frac{x+4-6x+3}{2x-1} \le 0$$
$$\frac{-5x+7}{2x-1} \le 0$$

$x = \frac{7}{5}$ or $x = \frac{1}{2}$

T	F	T
	$\frac{1}{2}$	$\frac{7}{5}$



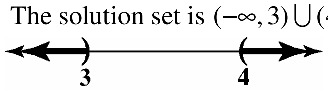
58.

$$\frac{1}{x-3} < 1$$
$$\frac{1}{x-3} - 1 < 0$$
$$\frac{1}{x-3} - \frac{x-3}{x-3} < 0$$
$$\frac{1-x+3}{x-3} < 0$$
$$\frac{-x+4}{x-3} < 0$$

$x = 4$ or $x = 3$

T	F	T
	3	4

The solution set is $(-\infty, 3) \cup (4, \infty)$.



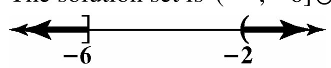
59.

$$\frac{x-2}{x+2} \le 2$$
$$\frac{x-2}{x+2} - 2 \le 0$$
$$\frac{x-2-2(x+2)}{x+2} \le 0$$
$$\frac{x-2-2x-4}{x+2} \le 0$$
$$\frac{-x-6}{x+2} \le 0$$

$x = -6$ or $x = -2$

T	F	T
	-6	-2

The solution set is $(-\infty, -6] \cup (-2, \infty)$.



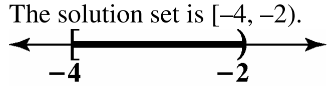
60.

$$\frac{x}{x+2} \ge 2$$
$$\frac{x}{x+2} - 2 \ge 0$$
$$\frac{x}{x+2} - \frac{2(x+2)}{x+2} \ge 0$$
$$\frac{x-2x-4}{x+2} \ge 0$$
$$\frac{-x-4}{x+2} \ge 0$$

$x = -4$ or $x = -2$

F	T	F
	–4	–2

The solution set is $[-4, -2)$.



61. $f(x) = \sqrt{2x^2 - 5x + 2}$

The domain of this function requires that

$2x^2 - 5x + 2 \ge 0$

Solve $2x^2 - 5x + 2 = 0$

$(x-2)(2x-1) = 0$

$x = \frac{1}{2}$ or $x = 2$

T	F	T
	$\frac{1}{2}$	2

The domain is $\left(-\infty, \frac{1}{2}\right] \cup [2, \infty)$.

62. $f(x) = \dfrac{1}{\sqrt{4x^2 - 9x + 2}}$

The domain of this function requires that $4x^2 - 9x + 2 > 0$

Solve $4x^2 - 9x + 2 = 0$

$(x-2)(4x-1) = 0$

$x = \dfrac{1}{4}$ or $x = 2$

T	F	T
	$\frac{1}{4}$	2

The domain is $\left(-\infty, \dfrac{1}{4}\right) \cup (2, \infty)$.

63. $f(x) = \sqrt{\dfrac{2x}{x+1} - 1}$

The domain of this function requires that $\dfrac{2x}{x+1} - 1 \ge 0$ or $\dfrac{x-1}{x+1} \ge 0$

$x = -1$ or $x = 1$

T	F	T
	-1	1

The value $x = 1$ results in 0 and, thus, it must be included in the domain.

The domain is $(-\infty, -1) \cup [1, \infty)$.

64. $f(x) = \sqrt{\dfrac{x}{2x-1} - 1}$

The domain of this function requires that $\dfrac{x}{2x-1} - 1 \ge 0$ or $\dfrac{-x+1}{2x-1} \ge 0$

$x = \dfrac{1}{2}$ or $x = 1$

F	T	F
	$\frac{1}{2}$	1

The value $x = 1$ results in 0 and, thus, it must be included in the domain.

The domain is $\left(\dfrac{1}{2}, 1\right]$.

65. $\left|x^2+2x-36\right|>12$

Express the inequality without the absolute value symbol:

$x^2+2x-36<-12 \quad \text{or} \quad x^2+2x-36>12$

$x^2+2x-24<0 \qquad\qquad x^2+2x-48>0$

Solve the related quadratic equations.

$x^2+2x-24=0 \quad \text{or} \quad x^2+2x-48=0$

$(x+6)(x-4)=0 \qquad (x+8)(x-6)=0$

Apply the zero product principle.

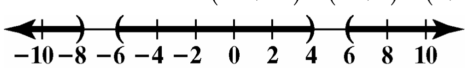
$x+6=0 \quad \text{or} \quad x-4=0 \quad \text{or} \quad x+8=0 \quad \text{or} \quad x-6=0$

$x=-6 \qquad x=4 \qquad x=-8 \qquad x=6$

The boundary points are $-8,\ -6,\ 4$ and 6.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-8)$	-9	$\left\|(-9)^2+2(-9)-36\right\|>12$ $27>12$, True	$(-\infty,-8)$ belongs to the solution set.
$(-8,-6)$	-7	$\left\|(-7)^2+2(-7)-36\right\|>12$ $1>12$, False	$(-8,-6)$ does not belong to the solution set.
$(-6,4)$	0	$\left\|0^2+2(0)-36\right\|>12$ $36>12$, True	$(-6,4)$ belongs to the solution set.
$(4,6)$	5	$\left\|5^2+2(5)-36\right\|>12$ $1>12$, False	$(4,6)$ does not belong to the solution set.
$(6,\infty)$	7	$\left\|7^2+2(7)-36\right\|>12$ $27>12$, True	$(6,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-8)\cup(-6,4)\cup(6,\infty)$ or $\{x \mid x<-8 \text{ or } -6<x<4 \text{ or } x>6\}$.



66. $\left|x^2+6x+1\right|>8$

Express the inequality without the absolute value symbol:

$x^2+6x+1<-8 \quad \text{or} \quad x^2+6x+1>8$

$x^2+6x+9<0 \qquad\qquad x^2+6x-7>0$

Solve the related quadratic equations.

$x^2+6x+9=0 \quad \text{or} \quad x^2+6x-7=0$

$(x+3)^2=0 \qquad (x+7)(x-1)=0$

$x+3=\pm\sqrt{0} \quad \text{or} \quad x+7=0 \quad \text{or} \quad x-1=0$

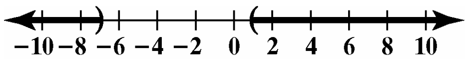
$x+3=0 \qquad x=-7 \qquad x=1$

$x=-3$

The boundary points are $-7,-3,$ and 1.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-7)$	-8	$\lvert(-8)^2+6(-8)+1\rvert>8$ $17\ge 8$, True	$(-\infty,-7)$ belongs to the solution set.
$(-7,-3)$	-5	$\lvert(-5)^2+6(-5)+1\rvert>8$ $4\ge 8$, False	$(-7,-3)$ does not belong to the solution set.
$(-3,1)$	0	$\lvert 0^2+6(0)+1\rvert>8$ $1\ge 8$, False	$(-3,1)$ does not belong to the solution set.
$(1,\infty)$	2	$\lvert 2^2+6(2)+1\rvert>8$ $17\ge 8$, True	$(1,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-7)\cup(1,\infty)$ or $\{x\,|\,x<-7 \text{ or } x>1\}$.



67. $\dfrac{3}{x+3}>\dfrac{3}{x-2}$

Express the inequality so that one side is zero.

$$\frac{3}{x+3}-\frac{3}{x-2}>0$$

$$\frac{3(x-2)}{(x+3)(x-2)}-\frac{3(x+3)}{(x+3)(x-2)}>0$$

$$\frac{3x-6-3x-9}{(x+3)(x-2)}<0$$

$$\frac{-15}{(x+3)(x-2)}<0$$

Find the values of x that make the denominator zero.

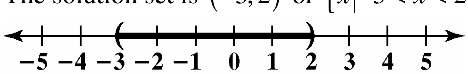
$x+3=0 \qquad x-2=0$

$x=-3 \qquad x=2$

The boundary points are -3 and 2.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-3)$	-4	$\dfrac{3}{-4+3}>\dfrac{3}{-4-2}$ $-3>\dfrac{1}{2}$, False	$(-\infty,-3)$ does not belong to the solution set.
$(-3,2)$	0	$\dfrac{3}{0+3}>\dfrac{3}{0-2}$ $1>-\dfrac{3}{2}$, True	$(-3,2)$ belongs to the solution set.
$(2,\infty)$	3	$\dfrac{3}{3+3}>\dfrac{3}{3-2}$ $\dfrac{1}{2}>3$, False	$(2,\infty)$ does not belong to the solution set.

The solution set is $(-3,2)$ or $\{x\,|\,-3<x<2\}$.



68. $\frac{1}{x+1} > \frac{2}{x-1}$

Express the inequality so that one side is zero.

$$\frac{1}{x+1} - \frac{2}{x-1} > 0$$

$$\frac{x-1}{(x+1)(x-1)} - \frac{2(x+1)}{(x+1)(x-1)} > 0$$

$$\frac{x-1-2x-2}{(x+1)(x-1)} < 0$$

$$\frac{-x-3}{(x+1)(x-1)} < 0$$

Find the values of *x* that make the numerator and denominator zero.

$-x-3=0 \qquad x+1=0 \qquad x-1=0$

$-3=x \qquad x=-1 \qquad x=1$

The boundary points are -3, -1, and 1.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-3)$	-4	$\frac{1}{-4+1} > \frac{2}{-3-1}$ $-\frac{1}{3} > -\frac{1}{2}$, True	$(-\infty,-3)$ belongs to the solution set.
$(-3,-1)$	-2	$\frac{1}{-2+1} > \frac{2}{-2-1}$ $-1 > -\frac{2}{3}$, False	$(-3,-1)$ does not belong to the solution set.
$(-1,1)$	0	$\frac{1}{0+1} > \frac{2}{0-1}$ $1 > -2$, True	$(-3,1)$ belongs to the solution set.
$(1,\infty)$	2	$\frac{1}{2+1} > \frac{2}{2-1}$ $\frac{1}{3} > 1$, False	$(1,\infty)$ does not belong to the solution set.

The solution set is $(-\infty,-3)\cup(-1,1)$ or $\{x \mid x<-3 \text{ or } -1<x<1\}$.

69. $\dfrac{x^2-x-2}{x^2-4x+3}>0$

Find the values of x that make the numerator and denominator zero.

$x^2-x-2=0 \qquad x^2-4x+3=0$

$(x-2)(x+1)=0 \qquad (x-3)(x-1)=0$

Apply the zero product principle.

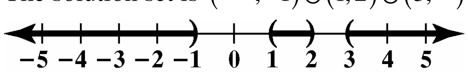
$x-2=0$ or $x+1=0 \qquad x-3=0$ or $x-1=0$

$x=2 \qquad x=-1 \qquad x=3 \qquad x=1$

The boundary points are -1, 1, 2 and 3.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-1)$	-2	$\dfrac{(-2)^2-(-2)-2}{(-2)^2-4(-2)+3}>0$ $\dfrac{4}{15}>0$, True	$(-\infty,-1)$ belongs to the solution set.
$(-1,1)$	0	$\dfrac{0^2-0-2}{0^2-4(0)+3}>0$ $-\dfrac{2}{3}>0$, False	$(-1,1)$ does not belong to the solution set.
$(1,2)$	1.5	$\dfrac{1.5^2-1.5-2}{1.5^2-4(1.5)+3}>0$ $\dfrac{5}{3}>0$, True	$(1,2)$ belongs to the solution set.
$(2,3)$	2.5	$\dfrac{2.5^2-2.5-2}{2.5^2-4(2.5)+3}>0$ $-\dfrac{7}{3}>0$, False	$(2,3)$ does not belong to the solution set.
$(3,\infty)$	4	$\dfrac{4^2-4-2}{4^2-4(4)+3}>0$ $\dfrac{10}{3}>0$, True	$(3,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-1)\cup(1,2)\cup(3,\infty)$ or $\{x \mid x<-1 \text{ or } 1<x<2 \text{ or } x>3\}$.



70. $\frac{x^2-3x+2}{x^2-2x-3}>0$

Find the values of x that make the numerator and denominator zero.

$x^2-3x+2=0$ $\qquad$ $x^2-2x-3=0$

$(x-2)(x-1)=0$ $\qquad$ $(x-3)(x+1)=0$

Apply the zero product principle.

$x-2=0$ or $x-1=0$ $\qquad$ $x-3=0$ or $x+1=0$

$x=2$ $\qquad$ $x=1$ $\qquad$ $x=3$ $\qquad$ $x=-1$

The boundary points are -1, 1, 2 and 3.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-1)$	-2 $\frac{x^2-3x+2}{x^2-2x-3}>0$	$\frac{(-2)^2-3(-2)+2}{(-2)^2-2(-2)-3}>0$ $\frac{12}{5}>0$, True	$(-\infty,-1)$ belongs to the solution set.
$(-1,1)$	0	$\frac{0^2-3(0)+2}{0^2-2(0)-3}>0$ $-\frac{2}{3}>0$, False	$(-1,1)$ does not belong to the solution set.
$(1,2)$	1.5	$\frac{1.5^2-3(1.5)+2}{1.5^2-2(1.5)-3}>0$ $\frac{1}{15}>0$, True	$(1,2)$ belongs to the solution set.
$(2,3)$	2.5	$\frac{2.5^2-3(2.5)+2}{2.5^2-2(2.5)-3}>0$ $-\frac{3}{7}>0$, False	$(2,3)$ does not belong to the solution set.
$(3,\infty)$	4	$\frac{4^2-3(4)+2}{4^2-2(4)-3}>0$ $\frac{6}{5}>0$, True	$(3,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-1)\cup(1,2)\cup(3,\infty)$ or $\{x \mid x<-1 \text{ or } 1<x<2 \text{ or } x>3\}$.

71. $2x^3+11x^2 \geq 7x+6$

$2x^3+11x^2-7x-6 \geq 0$

The graph of $f(x)=2x^3+11x^2-7x-6$ appears to cross the *x*-axis at -6, $-\frac{1}{2}$, and 1. We verify this numerically by substituting these values into the function:

$f(-6)=2(-6)^3+11(-6)^2-7(-6)-6=2(-216)+11(36)-(-42)-6=-432+396+42-6=0$

$f\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)^3+11\left(-\frac{1}{2}\right)^2-7\left(-\frac{1}{2}\right)-6=2\left(-\frac{1}{8}\right)+11\left(\frac{1}{4}\right)-\left(-\frac{7}{2}\right)-6=-\frac{1}{4}+\frac{11}{4}+\frac{7}{2}-6=0$

$f(1)=2(1)^3+11(1)^2-7(1)-6=2(1)+11(1)-7-6=2+11-7-6=0$

Thus, the boundaries are -6, $-\frac{1}{2}$, and 1. We need to find the intervals on which $f(x) \geq 0$. These intervals are indicated on the graph where the curve is above the *x*-axis. Now, the curve is above the *x*-axis when $-6<x<-\frac{1}{2}$ and when $x>1$. Thus, the solution set is $\left\{x \,\middle|\, -6 \leq x \leq -\frac{1}{2} \text{ or } x \geq 1\right\}$ or $\left[-6,-\frac{1}{2}\right] \cup [1,\infty)$.

72. $2x^3+11x^2<7x+6$

$2x^3+11x^2-7x-6<0$

In Problem 63, we verified that the boundaries are -6, $-\frac{1}{2}$, and 1. We need to find the intervals on which $f(x)<0$. These intervals are indicated on the graph where the curve is below the *x*-axis. Now, the curve is below the *x*-axis when $x<-6$ and when $-\frac{1}{2}<x<1$. Thus, the solution set is $\left\{x \,\middle|\, x<-6 \text{ or } -\frac{1}{2}<x<1\right\}$ or $(-\infty,-6) \cup \left(-\frac{1}{2},1\right)$.

73. $\frac{1}{4(x+2)} \leq -\frac{3}{4(x-2)}$

$\frac{1}{4(x+2)}+\frac{3}{4(x-2)} \leq 0$

Simplify the left side of the inequality:

$\frac{x-2}{4(x+2)}+\frac{3(x+2)}{4(x-2)}=\frac{x-2+3x+6}{4(x+2)(x-2)}=\frac{4x+4}{4(x+2)(x-2)}=\frac{4(x+1)}{4(x+2)(x-2)}=\frac{x+1}{x^2-4}$.

The graph of $f(x)=\frac{x+1}{x^2-4}$ crosses the *x*-axis at -1, and has vertical asymptotes at $x=-2$ and $x=2$. Thus, the boundaries are -2, -1, and 1. We need to find the intervals on which $f(x) \leq 0$. These intervals are indicated on the graph where the curve is below the *x*-axis. Now, the curve is below the *x*-axis when $x<-2$ and when $-1<x<2$. Thus, the solution set is $\{x \mid x<-2 \text{ or } -1 \leq x<2\}$ or $(-\infty,-2) \cup [-1,2)$.

74. $$\frac{1}{4(x+2)} > -\frac{3}{4(x-2)}$$
$$\frac{1}{4(x+2)} + \frac{3}{4(x-2)} > 0$$
$$\frac{x+1}{(x+2)(x-2)} > 0$$

The boundaries are -2, -1, and 2. We need to find the intervals on which $f(x) > 0$. These intervals are indicated on the graph where the curve is above the *x*-axis. The curve is above the *x*-axis when $-2 < x < -1$ and when $x > 2$. Thus, the solution set is $\{x \mid -2 < x < -1 \text{ or } x > 2\}$ or $(-2,-1)\cup(2,\infty)$.

75. $s(t) = -16t^2 + 8t + 87$

The diver's height will exceed that of the cliff when $s(t) > 87$

$$-16t^2 + 8t + 87 > 87$$
$$-16t^2 + 8t > 0$$
$$-8t(2t-1) > 0$$

The boundaries are 0 and $\frac{1}{2}$. Testing each interval shows that the diver will be higher than the cliff for the first half second after beginning the jump. The interval is $\left(0, \frac{1}{2}\right)$.

76. $s(t) = -16t^2 + 48t + 160$

The ball's height will exceed that of the rooftop when $s(t) > 160$

$$-16t^2 + 48t + 160 > 160$$
$$-16t^2 + 48t > 0$$
$$-16t(t-3) > 0$$

The boundaries are 0 and 3. Testing each interval shows that the ball will be higher than the rooftop for the first three seconds after the throw. The interval is $(0,3)$.

77. $f(x) = 0.0875x^2 - 0.4x + 66.6$

$g(x) = 0.0875x^2 + 1.9x + 11.6$

a. $f(35) = 0.0875(35)^2 - 0.4(35) + 66.6 \approx 160$ feet

$g(35) = 0.0875(35)^2 + 1.9(35) + 11.6 \approx 185$ feet

b. Dry pavement: graph (b)
Wet pavement: graph (a)

c. The answers to part (a) model the actual stopping distances shown in the figure extremely well. The function values and the data are identical.

d. $0.0875x^2 - 0.4x + 66.6 > 540$

$0.0875x^2 - 0.4x + 473.4 > 0$

Solve the related quadratic equation.

$0.0875x^2 - 0.4x + 473.4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-0.4) \pm \sqrt{(-0.4)^2 - 4(0.0875)(473.4)}}{2(0.0875)}$$

$x \approx -71$ or 76

Since the function's domain is $x \geq 30$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(30, 76)$	50	$0.0875(50)^2 - 0.4(50) + 66.6 > 540$ $265.35 > 540$, False	$(30, 76)$ does not belong to the solution set.
$(76, \infty)$	100	$0.0875(100)^2 - 0.4(100) + 66.6 > 540$ $901.6 > 540$, True	$(76, \infty)$ belongs to the solution set.

On dry pavement, stopping distances will exceed 540 feet for speeds exceeding 76 miles per hour. This is represented on graph (b) to the right of point (76, 540).

78. $f(x) = 0.0875x^2 - 0.4x + 66.6$

$g(x) = 0.0875x^2 + 1.9x + 11.6$

a. $f(55) = 0.0875(55)^2 - 0.4(55) + 66.6 \approx 309$ feet

$g(55) = 0.0875(55)^2 + 1.9(55) + 11.6 \approx 381$ feet

b. Dry pavement: graph (b)
Wet pavement: graph (a)

c. The answers to part (a) model the actual stopping distances shown in the figure extremely well.

d. $0.0875x^2 + 1.9x + 11.6 > 540$

$0.0875x^2 + 1.9x + 528.4 > 0$

Solve the related quadratic equation.

$0.0875x^2 + 1.9x + 528.4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1.9) \pm \sqrt{(1.9)^2 - 4(0.0875)(528.4)}}{2(0.0875)}$$

$x \approx -89$ or 68

Since the function's domain is $x \geq 30$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(30,68)$	50	$0.0875(50)^2 + 1.9(50) + 11.6 > 540$ $325.35 > 540$, False	$(30,68)$ does not belong to the solution set.
$(68,\infty)$	100	$0.0875(100)^2 + 1.9(100) + 11.6 > 540$ $1076.6 > 540$, True	$(68,\infty)$ belongs to the solution set.

On wet pavement, stopping distances will exceed 540 feet for speeds exceeding 68 miles per hour. This is represented on graph (a) to the right of point (68, 540).

79. Let x = the length of the rectangle.

Since $\text{Perimeter} = 2(\text{length}) + 2(\text{width})$, we know

$$50 = 2x + 2(\text{width})$$
$$50 - 2x = 2(\text{width})$$
$$\text{width} = \frac{50-2x}{2} = 25 - x$$

Now, $A = (\text{length})(\text{width})$, so we have that

$$A(x) \leq 114$$
$$x(25-x) \leq 114$$
$$25x - x^2 \leq 114$$

Solve the related equation

$$25x - x^2 = 114$$
$$0 = x^2 - 25x + 114$$
$$0 = (x-19)(x-6)$$

Apply the zero product principle:

$x - 19 = 0$ or $x - 6 = 0$

$x = 19$ $\quad x = 6$

The boundary points are 6 and 19.

Test Interval	Test Number	Test	Conclusion
$(-\infty,6)$	0	$25(0) - 0^2 \leq 114$ $0 \leq 114$, True	$(-\infty,6)$ belongs to the solution set.
$(6,19)$	10	$25(10) - 10^2 \leq 114$ $150 \leq 114$, False	$(6,19)$ does not belong to the solution set.
$(19,\infty)$	20	$25(20) - 20^2 \leq 114$ $100 \leq 114$, True	$(19,\infty)$ belongs to the solution set.

If the length is 6 feet, then the width is 19 feet. If the length is less than 6 feet, then the width is greater than 19 feet. Thus, if the area of the rectangle is not to exceed 114 square feet, the length of the shorter side must be 6 feet or less.

80. $2l + 2w = P$

$2l + 2w = 180$

$2l = 180 - 2w$

$l = 90 - w$

We want to restrict the area to 800 square feet. That is,

$$A \le 800$$
$$l \cdot w \le 800$$
$$(90 - w)w \le 800$$
$$90w - w^2 \le 800$$
$$-w^2 + 90w - 800 \le 0$$
$$w^2 - 90w + 800 \ge 0$$
$$w^2 - 90w + 800 = 0$$
$$(w-80)(w-10) = 0$$
$$w - 80 = 0 \quad \text{or} \quad w - 10 = 0$$
$$w = 80 \qquad\qquad w = 10$$

Assuming the width is the shorter side, we ignore the larger solution.

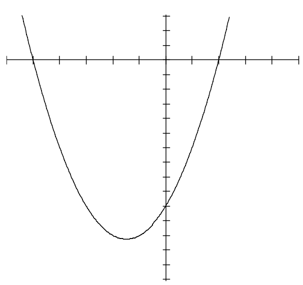
Test Interval	Test Number	Test	Conclusion
$(0,10)$	5	$90(5)-(5)^2 \le 800$ true	$(0,10)$ is part of the solution set
$(10,45)$	20	$90(20)-(20)^2 \le 800$ false	$(10,45)$ is not part of the solution set

The solution set is $\{w \mid 0 < w \le 10\}$ or $(0,10]$.

The length of the shorter side cannot exceed 10 feet.

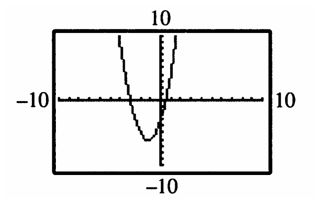
81. – 85. Answers may vary.

86.



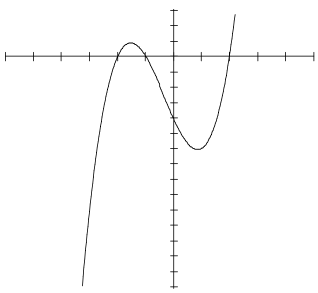
The solution set is $(-\infty, -5) \cup (2, \infty)$.

87. Graph $y_1 = 2x^2 + 5x - 3$ in a standard window. The graph is below or equal to the *x*-axis for $-3 \le x \le \frac{1}{2}$.



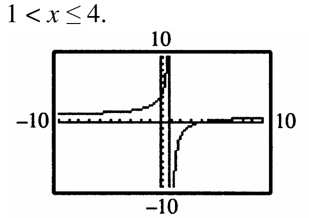
The solution set is $\left\{x \middle| -3 \le x \le \frac{1}{2}\right\}$ or $\left[-3, \frac{1}{2}\right]$.

88.



The solution set is (–2, –1) or (2, ∞).

89. Graph $y_1 = \frac{x-4}{x-1}$ in a standard viewing window. The graph is below the *x*-axis for $1 < x \le 4$.



The solution set is (1, 4].

90. Graph $y_1 = \frac{x+2}{x-3}$ and $y_2 = 2$

y_1 less than or equal to y_2 for $x < 3$ or $x \ge 8$.

The solution set is $(-\infty, 3) \cup [8, \infty)$

91. Graph $y_1 = \frac{1}{x+1}$ and $y_2 = \frac{2}{x+4}$

y_1 less than or equal to y_2 for $-4 < x < -1$ or $x \ge 2$.

The solution set is $(-4, -1) \cup [2, \infty)$

92. **a.** $f(x) = 0.1125x^2 - 0.1x + 55.9$

b. $0.1125x^2 - 0.1x + 55.9 > 455$

$0.1125x^2 - 0.1x + 399.1 > 0$

Solve the related quadratic equation.

$0.1125x^2 - 0.1x + 399.1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-0.1) \pm \sqrt{(-0.1)^2 - 4(0.1125)(399.1)}}{2(0.1125)}$$

$x \approx -59$ or 60

Since the function's domain must be $x \ge 0$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(0, 60)$	50	$0.1125(50)^2 - 0.1(50) + 55.9 > 455$ $332.15 > 455$, False	$(0, 60)$ does not belong to the solution set.
$(60, \infty)$	100	$0.1125(100)^2 - 0.1(100) + 55.9 > 455$ $1170.9 > 455$, True	$(60, \infty)$ belongs to the solution set.

On dry pavement, stopping distances will exceed 455 feet for speeds exceeding 60 miles per hour.

93. **a.** $f(x) = 0.1375x^2 + 0.7x + 37.8$

b. $0.1375x^2 + 0.7x + 37.8 > 446$

$0.1375x^2 + 0.7x + 408.2 > 0$

Solve the related quadratic equation.

$0.1375x^2 + 0.7x + 408.2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(0.7) \pm \sqrt{(0.7)^2 - 4(0.1375)(408.2)}}{2(0.1375)}$$

$x \approx -57$ or 52

Since the function's domain must be $x \ge 0$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(0, 52)$	10	$0.1375(10)^2 + 0.7(10) + 37.8 > 446$ $58.55 > 446$, False	$(0, 52)$ does not belong to the solution set.
$(52, \infty)$	100	$0.1375(100)^2 + 0.7(100) + 37.8 > 446$ $1482.8 > 446$, True	$(52, \infty)$ belongs to the solution set.

On wet pavement, stopping distances will exceed 446 feet for speeds exceeding 52 miles per hour.

94. makes sense

95. does not make sense; Explanations will vary. Sample explanation: Polynomials are defined for all values.

96. makes sense

97. does not make sense; Explanations will vary. Sample explanation: To solve this inequality you must first subtract 2 from both sides.

98. false; Changes to make the statement true will vary. A sample change is: The solution set is $\{x | x < -5 \text{ or } x > 5\}$ or $(-\infty, -5) \cup (5, \infty)$.

99. false; Changes to make the statement true will vary. A sample change is: The inequality cannot be solved by multiplying both sides by $x + 3$. We do not know if $x + 3$ is positive or negative. Thus, we would not know whether or not to reverse the order of the inequality.

100. false; Changes to make the statement true will vary. A sample change is: The inequalities have different solution sets. The value, 1, is included in the domain of the first inequality, but not included in the domain of the second inequality.

101. true

102. One possible solution: $x^2 - 2x - 15 \le 0$

103. One possible solution: $\dfrac{x-3}{x+4} \ge 0$

104. Because any non-zero number squared is positive, the solution is all real numbers except 2.

105. Because any number squared other than zero is positive, the solution includes only 2.

106. Because any number squared is positive, the solution is the empty set, $\varnothing$.

107. Because any number squared other than zero is positive, and the reciprocal of zero is undefined, the solution is all real numbers except 2.

108. a. The solution set is all real numbers.

b. The solution set is the empty set, $\varnothing$.

c. $4x^2 - 8x + 7 > 0$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(4)(7)}}{2(4)}$$

$$x = \frac{8 \pm \sqrt{64 - 112}}{8}$$

$$x = \frac{8 \pm \sqrt{-48}}{8} \Rightarrow \text{imaginary}$$

no critical values

Test 0: $4(0)^2 - 8(0) + 7 > 0$

$7 > 0$ True

The inequality is true for all numbers.

$4x^2 - 8x + 7 < 0$

no critical values

Test 0: $4(0)^2 - 8(0) + 7 = 7 < 0$ False

The solution set is the empty set.

109. $\sqrt{27 - 3x^2} \ge 0$

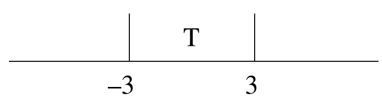
$27 - 3x^2 \ge 0$

$9 - x^2 \ge 0$

$(3 - x)(3 + x) \ge 0$

$3 - x = 0 \qquad 3 + x = 0$

$x = 3$ or $\qquad x = -3$



Test –4: $\sqrt{27 - 3(-4)^2} \ge 0$

$\sqrt{27 - 48} \ge 0$

$\sqrt{-21} \ge 0$

no graph- imaginary

Test 0: $\sqrt{27 - 3(0)^2} \ge 0$

$\sqrt{27} \ge 0$ True

Test 4: $\sqrt{27 - 3(4)^2} \ge 0$

$\sqrt{27 - 48} \ge 0$

$\sqrt{-21} \ge 0$

no graph -imaginary

The solution set is [–3, 3].

110. **a.**
$$y = kx^2$$
$$64 = k \cdot 2^2$$
$$64 = 4k$$
$$16 = k$$

b.
$$y = kx^2$$
$$y = 16x^2$$

c.
$$y = kx^2$$
$$y = 16x^2$$
$$y = 16 \cdot 5^2$$
$$y = 400$$

111. **a.**
$$y = \frac{k}{x}$$
$$12 = \frac{k}{8}$$
$$96 = k$$

b.
$$y = \frac{k}{x}$$
$$y = \frac{96}{x}$$

c.
$$y = \frac{96}{x}$$
$$y = \frac{96}{3}$$
$$y = 32$$

112.
$$S = \frac{kA}{P}$$
$$12,000 = \frac{k \cdot 60,000}{40}$$
$$\frac{12,000 \cdot 40}{60,000} = k$$
$$8 = k$$

Section 3.7

Check Point Exercises

1. y varies directly as x is expressed as $y = kx.$
The volume of water, W, varies directly as the time, t can be expressed as $W = kt$.
Use the given values to find k.
$$W = kt$$
$$30 = k(5)$$
$$6 = k$$
Substitute the value of k into the equation.
$$W = kt$$
$$W = 6t$$
Use the equation to find W when $t = 11$.
$$W = 6t$$
$$= 6(11)$$
$$= 66$$
A shower lasting 11 minutes will use 66 gallons of water.

2. y varies directly as the cube of x is expressed as $y = kx^3$.
The weight, w, varies directly as the cube of the length, l can be expressed as $w = kl^2$.
Use the given values to find k.
$$w = kl^3$$
$$2025 = k(15)^3$$
$$0.6 = k$$
Substitute the value of k into the equation.
$$w = kl^3$$
$$w = 0.6l^3$$
Use the equation to find w when $l = 25$.
$$w = 0.6l^3$$
$$= 0.6(25)^3$$
$$= 9375$$
The 25-foot long shark was 9375 pounds.

3. y varies inversely as x is expressed as $y = \frac{k}{x}$.
The length, L, varies inversely as the frequency, f can be expressed as $L = \frac{k}{f}$.
Use the given values to find k.

$$L = \frac{k}{f}$$
$$8 = \frac{k}{640}$$
$$5120 = k$$

Substitute the value of k into the equation.

$$L = \frac{k}{f}$$
$$L = \frac{5120}{f}$$

Use the equation to find f when $L = 10$.

$$L = \frac{5120}{f}$$
$$10 = \frac{5120}{f}$$
$$10f = 5120$$
$$f = 512$$

A 10 inch violin string will have a frequency of 512 cycles per second.

4. let M represent the number of minutes
let Q represent the number of problems
let P represent the number of people
M varies directly as Q and inversely as P is expressed as $M = \frac{kQ}{P}$.
Use the given values to find k.

$$M = \frac{kQ}{P}$$
$$32 = \frac{k(16)}{4}$$
$$8 = k$$

Substitute the value of k into the equation.

$$M = \frac{kQ}{P}$$
$$M = \frac{8Q}{P}$$

Use the equation to find M when $P = 8$ and $Q = 24$.

$$M = \frac{8Q}{P}$$
$$M = \frac{8(24)}{8}$$
$$M = 24$$

It will take 24 minutes for 8 people to solve 24 problems.

5. V varies jointly with h and r^2 and can be modeled as $V = khr^2$.
Use the given values to find k.

$$V = khr^2$$
$$120\pi = k(10)(6)^2$$
$$\frac{\pi}{3} = k$$

Therefore, the volume equation is $V = \frac{1}{3}hr^2$.

$$V = \frac{\pi}{3}(2)(12)^2 = 96\pi \text{ cubic feet}$$

Exercise Set 3.7

1. Use the given values to find k.

$$y = kx$$
$$65 = k \cdot 5$$
$$\frac{65}{5} = \frac{k \cdot 5}{5}$$
$$13 = k$$

The equation becomes $y = 13x$.
When $x = 12$, $y = 13x = 13 \cdot 12 = 156$.

2.
$$y = kx$$
$$45 = k \cdot 5$$
$$9 = k$$

$$y = 9x = 9 \cdot 13 = 117$$

3. Since y varies inversely with x, we have $y = \frac{k}{x}$.

Use the given values to find k.

$$y = \frac{k}{x}$$
$$12 = \frac{k}{5}$$
$$5 \cdot 12 = 5 \cdot \frac{k}{5}$$
$$60 = k$$

The equation becomes $y = \frac{60}{x}$.

When $x = 2$, $y = \frac{60}{2} = 30$.

4.
$$y = \frac{k}{x}$$
$$6 = \frac{k}{3}$$
$$18 = k$$

$$y = \frac{18}{9} = 2$$

5. Since y varies inversely as x and inversely as the square of z, we have $y = \frac{kx}{z^2}$.

Use the given values to find k.

$$y = \frac{kx}{z^2}$$
$$20 = \frac{k(50)}{5^2}$$
$$20 = \frac{k(50)}{25}$$
$$20 = 2k$$
$$10 = k$$

The equation becomes $y = \frac{10x}{z^2}$.

When $x = 3$ and $z = 6$,

$$y = \frac{10x}{z^2} = \frac{10(3)}{6^2} = \frac{10(3)}{36} = \frac{30}{36} = \frac{5}{6}.$$

6.
$$a = \frac{kb}{c^2}$$
$$7 = \frac{k(9)}{(6)^2}$$
$$7 = \frac{k(9)}{36}$$
$$7 = \frac{k}{4}$$
$$28 = k$$

$$a = \frac{28(4)}{(8)^2} = \frac{28(4)}{64} = \frac{7}{4}$$

7. Since y varies jointly as x and z, we have $y = kxz$.

Use the given values to find k.

$$y = kxz$$
$$25 = k(2)(5)$$
$$25 = k(10)$$
$$\frac{25}{10} = \frac{k(10)}{10}$$
$$\frac{5}{2} = k$$

The equation becomes $y = \frac{5}{2}xz$.

When $x = 8$ and $z = 12$, $y = \frac{5}{2}(8)(12) = 240$.

8.
$$C = kAT$$
$$175 = k(2100)(4)$$
$$175 = k(8400)$$
$$\frac{1}{48} = k$$

$$C = \frac{1}{48}(2400)(6) = \frac{14400}{48} = 300$$

9. Since y varies jointly as a and b and inversely as the square root of c, we have $y=\dfrac{kab}{\sqrt{c}}$.

Use the given values to find k.

$$y=\frac{kab}{\sqrt{c}}$$
$$12=\frac{k(3)(2)}{\sqrt{25}}$$
$$12=\frac{k(6)}{5}$$
$$12(5)=\frac{k(6)}{5}(5)$$
$$60=6k$$
$$\frac{60}{6}=\frac{6k}{6}$$
$$10=k$$

The equation becomes $y=\dfrac{10ab}{\sqrt{c}}$.

When $a = 5$, $b = 3$, $c = 9$,

$$y=\frac{10ab}{\sqrt{c}}=\frac{10(5)(3)}{\sqrt{9}}=\frac{150}{3}=50.$$

10.

$$y=\frac{kmn^2}{p}$$
$$15=\frac{k(2)(1)^2}{6}$$
$$15=\frac{2k}{6}$$
$$15(6)=\frac{2k}{6}(6)$$
$$90=2k$$
$$k=45$$

$$y=\frac{45mn^2}{p}=\frac{45(3)(4)^2}{10}=\frac{2160}{10}=216$$

11. $x=kyz$;

Solving for y:

$$x=kyz$$
$$\frac{x}{kz}=\frac{kyz}{yz}.$$
$$y=\frac{x}{kz}$$

12. $x=kyz^2$;

Solving for y:

$$x=kyz^2$$
$$\frac{x}{kz^2}=\frac{kyz^2}{kz^2}$$
$$y=\frac{x}{kz^2}$$

13. $x=\dfrac{kz^3}{y}$;

Solving for y

$$x=\frac{kz^3}{y}$$
$$xy=y\cdot\frac{kz^3}{y}$$
$$xy=kz^3$$
$$\frac{xy}{x}=\frac{kz^3}{x}$$
$$y=\frac{kz^3}{x}$$

14.

$$x=\frac{k\sqrt[3]{z}}{y}$$
$$yx=y\cdot\frac{k\sqrt[3]{z}}{y}$$
$$yx=k\sqrt[3]{z}$$
$$\frac{yx}{x}=\frac{k\sqrt[3]{z}}{x}$$
$$y=\frac{k\sqrt[3]{z}}{x}$$

15. $x=\dfrac{kyz}{\sqrt{w}}$;

Solving for y:

$$x=\frac{kyz}{\sqrt{w}}$$
$$x\left(\sqrt{w}\right)=\left(\sqrt{w}\right)\frac{kyz}{\sqrt{w}}$$
$$x\sqrt{w}=kyz$$
$$\frac{x\sqrt{w}}{kz}=\frac{kyz}{kz}$$
$$y=\frac{x\sqrt{w}}{kz}$$

16.
$$x = \frac{kyz}{w^2}$$
$$\left(\frac{w^2}{kz}\right)x = \frac{w^2}{kz}\frac{kyz}{w^2}$$
$$y = \frac{xw^2}{kz}$$

17. $x = kz(y+w)$;
Solving for y:
$$x = kz(y+w)$$
$$x = kzy + kzw$$
$$x - kzw = kzy$$
$$\frac{x-kzw}{kz} = \frac{kzy}{kz}$$
$$y = \frac{x-kzw}{kz}$$

18.
$$x = kz(y-w)$$
$$x = kzy - kzw$$
$$x + kzw = kzy$$
$$\frac{x+kzw}{kz} = \frac{kzy}{kz}$$
$$y = \frac{x+kzw}{kz}$$

19. $x = \frac{kz}{y-w}$;
Solving for y:
$$x = \frac{kz}{y-w}$$
$$(y-w)x = (y-w)\frac{kz}{y-w}$$
$$xy - wx = kz$$
$$xy = kz + wx$$
$$\frac{xy}{x} = \frac{kz+wx}{x}$$
$$y = \frac{xw+kz}{x}$$

20.
$$x = \frac{kz}{y+w}$$
$$(y+w)x = (y+w)\frac{kz}{y+w}$$
$$yx + xw = kz$$
$$yx = kz - xw$$
$$\frac{yx}{x} = \frac{kz-xw}{x}$$
$$y = \frac{kz-xw}{x}$$

21. Since T varies directly as B, we have $T = kB$.
Use the given values to find k.
$$T = kB$$
$$3.6 = k(4)$$
$$\frac{3.6}{4} = \frac{k(4)}{4}$$
$$0.9 = k$$
The equation becomes $T = 0.9B$.
When $B = 6$, $T = 0.9(6) = 5.4$.
The tail length is 5.4 feet.

22.
$$M = kE$$
$$60 = k(360)$$
$$\frac{60}{360} = \frac{k(360)}{360}$$
$$\frac{1}{6} = k$$

$$M = \frac{1}{6}(186) = 31$$

A person who weighs 186 pounds on Earth will weigh 31 pounds on the moon.

23. Since B varies directly as D, we have $B = kD$.
Use the given values to find k.

$$B = kD$$
$$8.4 = k(12)$$
$$\frac{8.4}{12} = \frac{k(12)}{12}$$
$$k = \frac{8.4}{12} = 0.7$$

The equation becomes $B = 0.7D$.
When $B = 56$,

$$56 = 0.7D$$
$$\frac{56}{0.7} = \frac{0.7D}{0.7}$$
$$D = \frac{56}{0.7} = 80$$

It was dropped from 80 inches.

24.
$$d = kf$$
$$9 = k(12)$$
$$\frac{9}{12} = \frac{k(12)}{12}$$
$$0.75 = k$$
$$d = 0.75f$$
$$15 = 0.75f$$
$$\frac{15}{0.75} = \frac{0.75f}{0.75}$$
$$20 = f$$

A force of 20 pounds is needed.

25. Since a man's weight varies directly as the cube of his height, we have $w = kh^3$.
Use the given values to find k.

$$w = kh^3$$
$$170 = k(70)^3$$
$$170 = k(343,000)$$
$$\frac{170}{343,000} = \frac{k(343,000)}{343,000}$$
$$0.000496 = k$$

The equation becomes $w = 0.000496h^3$.
When h = 107,

$$w = 0.000496(107)^3$$
$$= 0.000496(1,225,043) \approx 607.$$

Robert Wadlow's weight was approximately 607 pounds.

26.
$$h = kd^2$$
$$50 = k \cdot 10^2$$
$$0.5 = k$$
$$h = 0.5d^2$$

a.
$$h = 0.5d^2$$
$$h = 0.5(30)^2$$
$$h = 450$$

A water pipe with a 30 centimeter diameter can serve 450 houses.

b.
$$h = 0.5d^2$$
$$1250 = 0.5d^2$$
$$d^2 = 625$$
$$d = \sqrt{625}$$
$$d = 25$$

A water pipe with a 25 centimeter diameter can serve 1250 houses.

27. Since the banking angle varies inversely as the turning radius, we have $B = \frac{k}{r}$.
Use the given values to find k.

$$B = \frac{k}{r}$$
$$28 = \frac{k}{4}$$
$$28(4) = 28\left(\frac{k}{4}\right)$$
$$112 = k$$

The equation becomes $B = \frac{112}{r}$.

When $r = 3.5$, $B = \frac{112}{r} = \frac{112}{3.5} = 32$.

The banking angle is 32° when the turning radius is 3.5 feet.

28.
$$t = \frac{k}{d}$$
$$4.4 = \frac{k}{1000}$$
$$(1000)4.4 = (1000)\frac{k}{1000}$$
$$4400 = k$$
$$t = \frac{4400}{d} = \frac{4400}{5000} = 0.88$$

The water temperature is 0.88° Celsius at a depth of 5000 meters.

29. Since intensity varies inversely as the square of the distance, we have pressure, we have

$I = \frac{k}{d}.$

Use the given values to find *k*.

$$I = \frac{k}{d^2}.$$
$$62.5 = \frac{k}{3^2}$$
$$62.5 = \frac{k}{9}$$
$$9(62.5) = 9\left(\frac{k}{9}\right)$$
$$562.5 = k$$

The equation becomes $I = \frac{562.5}{d^2}.$

When $d = 2.5$, $I = \frac{562.5}{2.5^2} = \frac{562.5}{6.25} = 90$

The intensity is 90 milliroentgens per hour.

30.

$$i = \frac{k}{d^2}$$
$$3.75 = \frac{k}{40^2}$$
$$3.75 = \frac{k}{1600}$$
$$(1600)3.75 = (1600)\frac{k}{1600}$$
$$6000 = k$$

$$i = \frac{6000}{d^2} = \frac{6000}{50^2} = \frac{6000}{2500} = 2.4$$

The illumination is 2.4 foot-candles at a distance of 50 feet.

31. Since index varies directly as weight and inversely as the square of one's height, we have $I = \frac{kw}{h^2}.$

Use the given values to find *k*.

$$I = \frac{kw}{h^2}$$
$$35.15 = \frac{k(180)}{60^2}$$
$$35.15 = \frac{k(180)}{3600}$$
$$(3600)35.15 = \frac{k(180)}{3600}$$
$$126540 = k(180)$$
$$k = \frac{126540}{180} = 703$$

The equation becomes $I = \frac{703w}{h^2}.$

When $w = 170$ and $h = 70$,

$I = \frac{703(170)}{(70)^2} \approx 24.4.$

This person has a BMI of 24.4 and is not overweight.

32.

$$i = \frac{km}{c}$$
$$125 = \frac{k(25)}{20}$$
$$20(125) = (20)\frac{k(25)}{20}$$
$$2500 = 25k$$
$$\frac{2500}{25} = \frac{25k}{25}$$
$$100 = k$$

$$i = \frac{100m}{c}$$
$$80 = \frac{100(40)}{c}$$
$$80 = \frac{4000}{c}$$
$$80c = c \cdot \frac{4000}{c}$$
$$80c = 4000$$
$$\frac{80c}{80} = \frac{4000}{80}$$
$$c = 50$$

The chronological age is 50.

33. Since heat loss varies jointly as the area and temperature difference, we have $L = kAD$. Use the given values to find *k*.

$$L = kAD$$
$$1200 = k(3\cdot 6)(20)$$
$$1200 = 360k$$
$$\frac{1200}{360} = \frac{360k}{360}$$
$$k = \frac{10}{3}$$

The equation becomes $L = \frac{10}{3}AD$

When $A = 6\cdot 9 = 54$, $D = 10$,

$L = \frac{10}{3}(9\cdot 6)(10) = 1800$.

The heat loss is 1800 Btu .

34.
$$e = kmv^2$$
$$36 = k(8)(3)^2$$
$$36 = k(8)(9)$$
$$36 = 72k$$
$$\frac{36}{72} = \frac{72k}{72}$$
$$k = 0.5$$

$$e = 0.5mv^2 = 0.5(4)(6)^2 = 0.5(4)(36) = 72$$

A mass of 4 grams and velocity of 6 centimeters per second has a kinetic energy of 72 ergs.

35. Since intensity varies inversely as the square of the distance from the sound source, we have $I = \frac{k}{d^2}$. If you move to a seat twice as far, then $d = 2d$. So we have $I = \frac{k}{(2d)^2} = \frac{k}{4d^2} = \frac{1}{4}\cdot\frac{k}{d^2}$. The intensity will be multiplied by a factor of $\frac{1}{4}$. So the sound intensity is $\frac{1}{4}$ of what it was originally.

36.
$$t = \frac{k}{a}$$
$$t = \frac{k}{3a} = \frac{1}{3}\cdot\frac{k}{a}$$

A year will seem to be $\frac{1}{3}$ of a year.

37. a. Since the average number of phone calls varies jointly as the product of the populations and inversely as the square of the distance, we have $C = \frac{kP_1P_2}{d^2}$.

b. Use the given values to find *k*.

$$C = \frac{kP_1P_2}{d^2}$$
$$326,000 = \frac{k(777,000)(3,695,000)}{(420)^2}$$
$$326,000 = \frac{k(2.87\times 10^{12})}{176,400}$$
$$326,000 = 16269841.27k$$
$$0.02 \approx k$$

The equation becomes $C = \frac{0.02P_1P_2}{d^2}$.

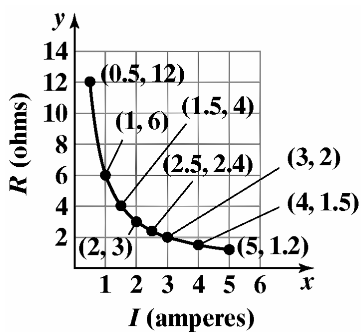
c.
$$C = \frac{0.02(650,000)(220,000)}{(400)^2}$$
$$= 17,875$$

There are approximately 17,875 daily phone calls.

38.
$$f = kas^2$$
$$150 = k(4\cdot 5)(30)^2$$
$$150 = k(20)(900)$$
$$150 = 18000k$$
$$\frac{150}{18000} = \frac{18000k}{150}$$
$$\frac{1}{120} = k$$
$$f = \frac{1}{120}as^2 = \frac{1}{120}(3\cdot 4)(60)^2$$
$$= \frac{1}{120}(12)(3600)$$
$$= 360$$

Yes, the wind will exert a force of 360 pounds on the window.

39. a.



b. Current varies inversely as resistance. Answers will vary.

c. Since the current varies inversely as resistance we have $R = \frac{k}{I}$. Using one of the given ordered pairs to find k.

$$12 = \frac{k}{0.5}$$
$$12(0.5) = \frac{k}{0.5}(0.5)$$
$$k = 6$$

The equation becomes $R = \frac{6}{I}$.

40. – 48. Answers may vary.

49. does not make sense; Explanations will vary. Sample explanation: For an inverse variation, the independent variable can not be zero.

50. does not make sense; Explanations will vary. Sample explanation: A direct variation with a positive constant of variation will have both variables increase simultaneously.

51. makes sense

52. makes sense

53. Pressure, P, varies directly as the square of wind velocity, v, can be modeled as $P = kv^2$.
If $v = x$ then $P = k(x)^2 = kx^2$
If $v = 2x$ then $P = k(2x)^2 = 4kx^2$
If the wind speed doubles the pressure is 4 times more destructive.

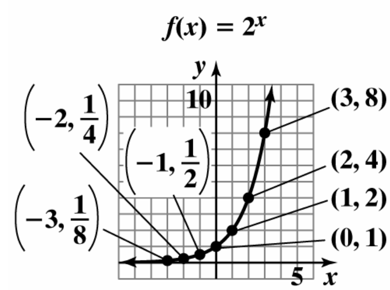
54. Illumination, I, varies inversely as the square of the distance, d, can be modeled as $I = \frac{k}{d^2}$.
If $d = 15$ then $I = \frac{k}{15^2} = \frac{k}{225}$
If $d = 30$ then $I = \frac{k}{30^2} = \frac{k}{900}$
Note that $\frac{900}{225} = 4$
If the distance doubles the illumination is 4 times less intense.

55. The Heat, H, varies directly as the square of the voltage, v, and inversely as the resistance, r.
$$H = \frac{kv^2}{r}$$
If the voltage remains constant, to triple the heat the resistant must be reduced by a multiple of 3.

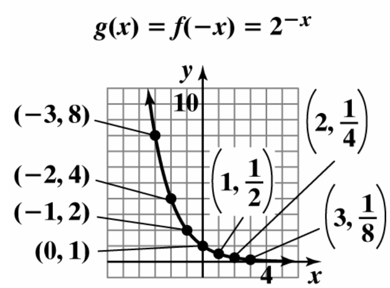
56. Illumination, I, varies inversely as the square of the distance, d, can be modeled as $I = \frac{k}{d^2}$.
If $I = x$ then $x = \frac{k}{d^2} \Rightarrow d = \sqrt{\frac{k}{x}}$.
If $I = \frac{1}{50}x$ then $\frac{1}{50}x = \frac{k}{d^2} \Rightarrow d = \sqrt{\frac{50k}{x}} = \sqrt{50}\sqrt{\frac{k}{x}}$.
Since $\sqrt{50} \approx 7$, the Hubble telescope is able to see about 7 times farther than a ground-based telescope.

57. Answers may vary.

58.

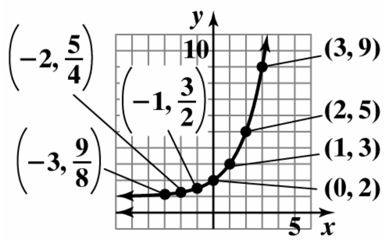


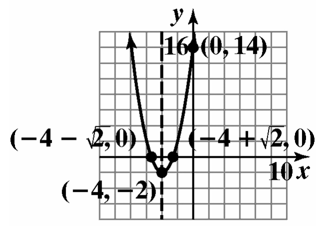
59.



60.

$h(x) = f(x) + 1 = 2^x + 1$





$f(x) = (x + 4)^2 - 2$

The axis of symmetry is $x = -4$.
domain: $(-\infty, \infty)$ range: $[-2, \infty)$

Chapter 3 Review Exercises

1. $f(x) = -(x+1)^2 + 4$

vertex: (–1, 4)
x-intercepts:

$0 = -(x+1)^2 + 4$

$(x+1)^2 = 4$

$x + 1 = \pm 2$

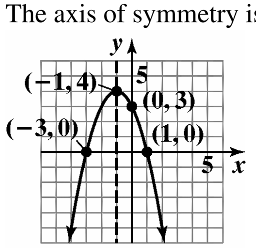
$x = -1 \pm 2$

$x = -3$ or $x = 1$

y-intercept:

$f(0) = -(0+1)^2 + 4 = 3$

The axis of symmetry is $x = -1$.



$f(x) = -(x + 1)^2 + 4$

domain: $(-\infty, \infty)$ range: $(-\infty, 4]$

2. $f(x) = (x+4)^2 - 2$

vertex: (–4, –2)
x-intercepts:

$0 = (x+4)^2 - 2$

$(x+4)^2 = 2$

$x + 4 = \pm\sqrt{2}$

$x = -4 \pm \sqrt{2}$

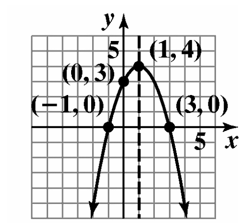
y-intercept:

$f(0) = (0+4)^2 - 2 = 14 = -1$

3. $f(x) = -x^2 + 2x + 3$

$= -(x^2 - 2x + 1) + 3 + 1$

$f(x) = -(x-1)^2 + 4$



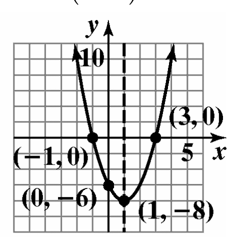
$f(x) = -x^2 + 2x + 3$

domain: $(-\infty, \infty)$ range: $(-\infty, 4]$

4. $f(x) = 2x^2 - 4x - 6$

$f(x) = 2(x^2 - 2x + 1) - 6 - 2$

$2(x-1)^2 - 8$



$f(x) = 2x^2 - 4x - 6$

axis of symmetry: $x = 1$
domain: $(-\infty, \infty)$ range: $[-8, \infty)$

5. $f(x) = -x^2 + 14x - 106$

a. Since $a < 0$ the parabola opens down with the maximum value occurring at

$x = -\frac{b}{2a} = -\frac{14}{2(-1)} = 7$.

The maximum value is $f(7)$.

$f(7) = -(7)^2 + 14(7) - 106 = -57$

b. domain: $(-\infty, \infty)$ range: $(-\infty, -57]$

6. $f(x)=2x^2+12x+703$

a. Since $a>0$ the parabola opens up with the minimum value occurring at

$$x=-\frac{b}{2a}=-\frac{12}{2(2)}=-3.$$

The minimum value is $f(-3)$.

$$f(-3)=2(-3)^2+12(-3)+703=685$$

b. domain: $(-\infty,\infty)$ range: $[685,\infty)$

7. a. The maximum height will occur at the vertex.

$$f(x)=-0.025x^2+x+6$$

$$x=-\frac{b}{2a}=-\frac{1}{2(-0.025)}=20$$

$$f(20)=-0.025(20)^2+(20)+6=16$$

The maximum height of 16 feet occurs when the ball is 20 yards downfield.

b. $f(x)=-0.025x^2+x+6$

$$f(0)=-0.025(0)^2+(0)+6=6$$

The ball was tossed at a height of 6 feet.

c. The ball is at a height of 0 when it hits the ground.

$$f(x)=-0.025x^2+x+6$$

$$0=-0.025x^2+x+6$$

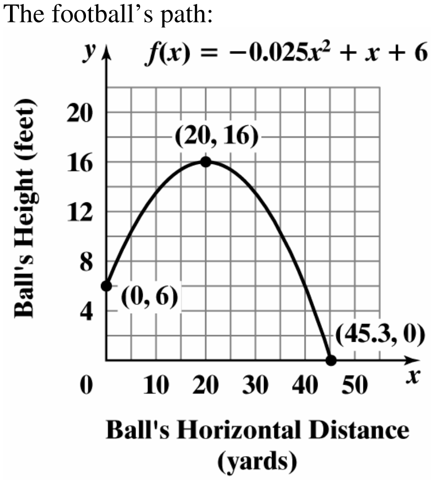
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(1)\pm\sqrt{(1)^2-4(-0.025)(6)}}{2(-0.025)}$$

$x\approx 45.3,\ -5.3$ (reject)

The ball will hit the ground 45.3 yards downfield.

d. The football's path:



8. Maximize the area using $A=lw$.

$$A(x)=x(1000-2x)$$

$$A(x)=-2x^2+1000x$$

Since $a=-2$ is negative, we know the function opens downward and has a maximum at

$$x=-\frac{b}{2a}=-\frac{1000}{2(-2)}=-\frac{1000}{-4}=250.$$

The maximum area is achieved when the width is 250 yards. The maximum area is

$$A(250)=250(1000-2(250))$$
$$=250(1000-500)$$
$$=250(500)=125,000.$$

The area is maximized at 125,000 square yards when the width is 250 yards and the length is $1000-2\cdot 250=500$ yards.

9. Let x = one of the numbers
Let $14+x$ = the other number

We need to minimize the function

$$P(x)=x(14+x)$$
$$=14x+x^2$$
$$=x^2+14x.$$

The minimum is at

$$x=-\frac{b}{2a}=-\frac{14}{2(1)}=-\frac{14}{2}=-7.$$

The other number is $14+x=14+(-7)=7$.

The numbers which minimize the product are 7 and -7. The minimum product is $-7\cdot 7=-49$.

10. $f(x)=-x^3+12x^2-x$

The graph rises to the left and falls to the right and goes through the origin, so graph (c) is the best match.

11. $g(x)=x^6-6x^4+9x^2$

The graph rises to the left and rises to the right, so graph (b) is the best match.

12. $h(x)=x^5-5x^3+4x$

The graph falls to the left and rises to the right and crosses the y-axis at zero, so graph (a) is the best match.

13. $f(x)=-x^4+1$

$f(x)$ falls to the left and to the right so graph (d) is the best match.

14. The leading coefficient is –0.87 and the degree is 3. This means that the graph will fall to the right. This function is not useful in modeling the number of thefts over an extended period of time. The model predicts that eventually, the number of thefts would be negative. This is impossible.

15. In the polynomial, $f(x) = -x^4 + 21x^2 + 100$, the leading coefficient is –1 and the degree is 4. Applying the Leading Coefficient Test, we know that even-degree polynomials with negative leading coefficient will fall to the left and to the right. Since the graph falls to the right, we know that the elk population will die out over time.

16. $f(x) = -2(x-1)(x+2)^2(x+5)^3$

$x = 1$, multiplicity 1, the graph crosses the x-axis
$x = -2$, multiplicity 2, the graph touches the x-axis
$x = -5$, multiplicity 5, the graph crosses the x-axis

17. $f(x) = x^3 - 5x^2 - 25x + 125$

$= x^2(x-5) - 25(x-5)$

$= (x^2 - 25)(x-5)$

$= (x+5)(x-5)^2$

$x = -5$, multiplicity 1, the graph crosses the x-axis
$x = 5$, multiplicity 2, the graph touches the x-axis

18. $f(x) = x^3 - 2x - 1$

$f(1) = (1)^3 - 2(1) - 1 = -2$

$f(2) = (2)^3 - 2(2) - 1 = 3$

The sign change shows there is a zero between the given values.

19. $f(x) = x^3 - x^2 - 9x + 9$

a. Since n is odd and $a_n > 0$, the graph falls to the left and rises to the right.

b. $f(-x) = (-x)^3 - (-x)^2 - 9(-x) + 9$

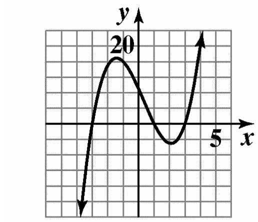
$= -x^3 - x^2 + 9x + 9$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

c. $f(x) = (x-3)(x+3)(x-1)$

zeros: 3, –3, 1



$f(x) = x^3 - x^2 - 9x + 9$

20. $f(x) = 4x - x^3$

a. Since n is odd and $a_n < 0$, the graph rises to the left and falls to the right.

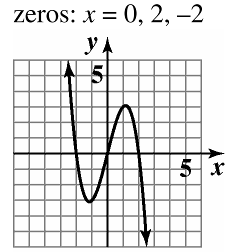
b. $f(-x) = -4x + x^3$

$f(-x) = -f(x)$

origin symmetry

c. $f(x) = x(x^2 - 4) = x(x-2)(x+2)$

zeros: $x = 0, 2, -2$



$f(x) = 4x - x^3$

21. $f(x) = 2x^3 + 3x^2 - 8x - 12$

a. Since h is odd and $a_n > 0$, the graph falls to the left and rises to the right.

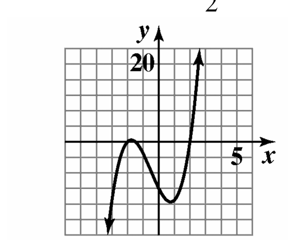
b. $f(-x) = -2x^3 + 3x^2 + 8x - 12$

$f(-x) \neq f(x),\ f(-x) = -f(x)$

no symmetry

c. $f(x) = (x - 2)(x + 2)(2x + 3)$

zeros: $x = 2, -2, -\frac{3}{2}$



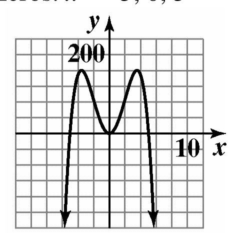
$f(x) = 2x^3 + 3x^2 - 8x - 12$

22. $g(x) = -x^4 + 25x^2$

a. The graph falls to the left and to the right.

b. $f(-x) = -(-x)^4 + 25(-x)^2$
$= -x^4 + 25x^2 = f(x)$
y-axis symmetry

c. $-x^4 + 25x^2 = 0$
$-x^2(x^2 - 25) = 0$
$-x^2(x-5)(x+5) = 0$
zeros: $x = -5, 0, 5$



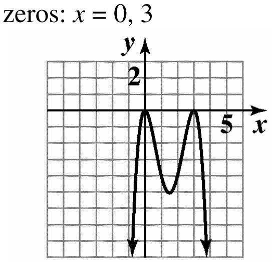
$f(x) = -x^4 + 25x^2$

23. $f(x) = -x^4 + 6x^3 - 9x^2$

a. The graph falls to the left and to the right.

b. $f(-x) = -(-x)^4 + 6(-x)^3 - 9(-x)$
$= -x^4 - 6x^3 - 9x^2 f(-x) \neq f(x)$
$f(-x) \neq -f(x)$
no symmetry

c. $= -x^2(x^2 - 6x + 9) = 0$
$-x^2(x-3)(x-3) = 0$
zeros: $x = 0, 3$



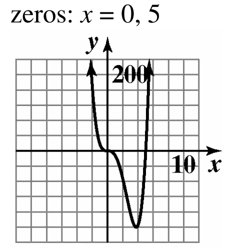
$f(x) = -x^4 + 6x^3 - 9x^2$

24. $f(x) = 3x^4 - 15x^3$

a. The graph rises to the left and to the right.

b. $f(-x) = 3(-x)^4 - 15(-x)^2 = 3x^4 + 15x^3$
$f(-x) \neq f(x),\ f(-x) \neq -f(x)$
no symmetry

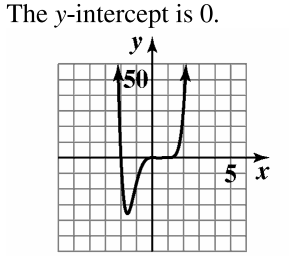
c $3x^4 - 15x^3 = 0$
$3x^3(x-5) = 0$
zeros: $x = 0, 5$



$f(x) = 3x^4 - 15x^3$

25. $f(x) = 2x^2(x-1)^3(x+2)$

Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.
$x = 0, x = 1, x = -2$
The zeros at 1 and –2 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0)
$f(0) = 2(0)^2(0-1)^3(0+2) = 0$
The y-intercept is 0.



$f(x) = 2x^2(x-1)^3(x+2)$

26. $f(x) = -x^3(x+4)^2(x-1)$

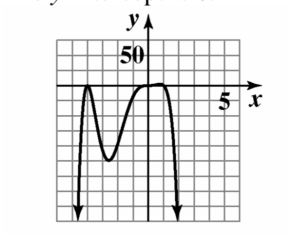
Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.

$x = 0, x = -4, x = 1$

The roots at 0 and 1 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at –4 has even multiplicity so $f(x)$ touches the axis at (–4, 0)

$f(0) = -(0)^3(0+4)^2(0-1) = 0$

The y-intercept is 0.



$f(x) = -x^3(x+4)^2(x-1)$

27.

$$
\begin{array}{r}
4x^2 - 7x + 5 \\
x+1 \overline{)\,4x^3 - 3x^2 - 2x + 1} \\
\underline{4x^3 + 4x^2} \qquad\qquad \\
-7x^2 - 2x \qquad \\
\underline{-7x^2 - 7x} \qquad \\
5x + 1 \\
\underline{5x + 5} \\
-4
\end{array}
$$

Quotient: $4x^2 - 7x + 5 - \dfrac{4}{x+1}$

28.

$$
\begin{array}{r}
2x^2 - 4x + 1 \\
5x-3 \overline{)\,10x^3 - 26x^2 + 17x - 13} \\
\underline{10x^3 + 6x^2} \qquad\qquad\quad \\
-20x^2 + 17x \qquad \\
\underline{-20x^2 + 12x} \qquad \\
5x - 13 \\
\underline{5x - \ 3} \\
-10
\end{array}
$$

Quotient: $2x^2 - 4x + 1 - \dfrac{10}{5x-3}$

29.

$$
\begin{array}{r}
2x^2 + 3x - 1 \\
2x^2+1 \overline{)\,4x^4 + 6x^3 + 3x - 1} \\
\underline{4x^2 + 2x^2} \qquad\qquad\quad \\
6x^3 - 2x^2 + 3x \\
\underline{6x^2 + 3x} \\
-2x^2 - 1 \\
\underline{-2x^2 - 1} \\
0
\end{array}
$$

30. $(3x^4 + 11x^3 - 20x^3 + 7x + 35) \div (x+5)$

–5	3	11	–20	7	35
		–15	20	0	–35
	3	–4	0	7	0

Quotient: $3x^3 - 4x^2 + 7$

31. $(3x^4 - 2x^2 - 10x) \div (x-2)$

2	3	0	–2	–10	0
		6	12	20	20
	3	6	10	10	20

Quotient: $3x^3 + 6x^2 + 10x + 10 + \dfrac{20}{x-2}$

32. $f(x) = 2x^3 - 7x^2 + 9x - 3$

–13	2	–7	9	–3
		–26	429	–5694
	2	–33	438	–5697

Quotient: $f(-13) = -5697$

33. $f(x) = 2x^3 + x^2 - 13x + 6$

2	2	1	–13	6
		4	10	–6
	2	5	–3	0

$f(x) = (x-2)(2x^2 + 5x - 3)$
$= (x-2)(2x-1)(x+3)$

Zeros: $x = 2, \dfrac{1}{2}, -3$

34. $x^3 - 17x + 4 = 0$

4	1	0	–17	4
		4	16	–4
	1	4	–1	0

$(x-4)(x^2+4x-1)=0$

$x = \frac{-4 \pm \sqrt{16+4}}{2} = \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$

The solution set is $\{4, -2+\sqrt{5}, -2-\sqrt{5}\}$.

35. $f(x) = x^4 - 6x^3 + 14x^2 - 14x + 5$

$p: \pm 1, \pm 5$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 5$

36. $f(x) = 3x^5 - 2x^4 - 15x^3 + 10x^2 + 12x - 8$

$p: \pm 1, \pm 2, \pm 4, \pm 8$

$q: \pm 1, \pm 3$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{8}{3}, \pm\frac{4}{3}, \pm\frac{2}{3}, \pm\frac{1}{3}$

37. $f(x) = 3x^4 - 2x^3 - 8x + 5$

$f(x)$ has 2 sign variations, so $f(x) = 0$ has 2 or 0 positive solutions.

$f(-x) = 3x^4 + 2x^3 + x + 5$

$f(-x)$ has no sign variations, so $f(x) = 0$ has no negative solutions.

38. $f(x) = 2x^5 - 3x^3 - 5x^2 + 3x - 1$

$f(x)$ has 3 sign variations, so $f(x) = 0$ has 3 or 1 positive real roots.

$f(-x) = -2x^5 + 3x^3 - 5x^2 - 3x - 1$

$f(-x)$ has 2 sign variations, so $f(x) = 0$ has 2 or 0 negative solutions.

39. $f(x) = f(-x) = 2x^4 + 6x^2 + 8$

No sign variations exist for either $f(x)$ or $f(-x)$, so no real roots exist.

40. $f(x) = x^3 + 3x^2 - 4$

a. $p: \pm 1, \pm 2, \pm 4$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

b. 1 sign variation $\Rightarrow$ 1 positive real zero

$f(-x) = -x^3 + 3x^2 - 4$

2 sign variations $\Rightarrow$ 2 or no negative real zeros

c.

1	1	3	0	–4
		1	4	–4
	1	4	4	0

1 is a zero.

1, –2 are rational zeros.

d. $(x-1)(x^2+4x+4)=0$

$(x-1)(x+2)^2 = 0$

$x = 1$ or $x = -2$

The solution set is $\{1, -2\}$.

41. $f(x) = 6x^3 + x^2 - 4x + 1$

a. $p: \pm 1$

$q: \pm 1, \pm 2, \pm 3, \pm 6$

$\frac{p}{q}: \pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}$

b. $f(x) = 6x^3 + x^2 - 4x + 1$

2 sign variations; 2 or 0 positive real zeros.

$f(-x) = -6x^3 + x^2 + 4x + 1$

1 sign variation; 1 negative real zero.

c.

–1	6	1	–4	1
		–6	5	–1
	6	–5	1	0

–1 is a zero.

$-1, \frac{1}{3}, \frac{1}{2}$ are rational zeros.

d. $6x^3 + x^2 - 4x + 1 = 0$

$(x+1)(6x^2 - 5x + 1) = 0$

$(x+1)(3x-1)(2x-1) = 0$

$x = -1$ or $x = \frac{1}{3}$ or $x = \frac{1}{2}$

The solution set is $\left\{-1, \frac{1}{3}, \frac{1}{2}\right\}$.

42. $f(x) = 8x^3 - 36x^2 + 46x - 15$

a. p: ±1, ±3, ±5, ±15
q: ±1, ±2, ±4, ±8
$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8},$
$\pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{3}{8}, \pm \frac{5}{2}, \pm \frac{5}{4},$
$\pm \frac{5}{8}, \pm \frac{15}{2}, \pm \frac{15}{4}, \pm \frac{15}{8}$

b. $f(x) = 8x^3 - 36x^2 + 46x - 15$
3 sign variations; 3 or 1 positive real solutions.
$f(-x) = -8x^3 - 36x^2 - 46x - 15$
0 sign variations; no negative real solutions.

c.

$\frac{1}{2}$	8	–36	46	–15
		4	–16	15
	8	–32	30	0

$\frac{1}{2}$ is a zero.
$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ are rational zeros.

d.

$$8x^3 - 36x^2 + 46x - 15 = 0$$
$$\left(x - \frac{1}{2}\right)(8x^2 - 32x + 30) = 0$$
$$2\left(x - \frac{1}{2}\right)(4x - 16x + 15) = 0$$
$$2\left(x - \frac{1}{2}\right)(2x - 5)(2x - 3) = 0$$

$x = \frac{1}{2}$ or $x = \frac{5}{2}$ or $x = \frac{3}{2}$

The solution set is $\left\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right\}$.

43. $2x^3 + 9x^2 - 7x + 1 = 0$

a. p: ±1
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm \frac{1}{2}$

b. $f(x) = 2x^3 + 9x^2 - 7x + 1$
2 sign variations; 2 or 0 positive real zeros.
$f(-x) = -2x^3 + 9x^2 + 7x + 1$
1 sign variation; 1 negative real zero.

c.

$\frac{1}{2}$	2	9	–7	1
		1	5	–1
	2	10	–2	0

$\frac{1}{2}$ is a rational zero.

d.

$$2x^3 + 9x^2 - 7x + 1 = 0$$
$$\left(x - \frac{1}{2}\right)(2x^2 + 10x - 2) = 0$$
$$2\left(x - \frac{1}{2}\right)(x^2 + 5x - 1) = 0$$

Solving $x^2 + 5x - 1 = 0$ using the quadratic formula gives $x = \frac{-5 \pm \sqrt{29}}{2}$

The solution set is $\left\{\frac{1}{2}, \frac{-5 + \sqrt{29}}{2}, \frac{-5 - \sqrt{29}}{2}\right\}$.

44. $x^4 - x^3 - 7x^2 + x + 6 = 0$

a. $p = \frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6$

b. $f(x) = x^4 - x^3 - 7x^2 + x + 6$
2 sign variations; 2 or 0 positive real zeros.
$f(-x) = x^4 + x^3 - 7x^2 - x + 6$
2 sign variations; 2 or 0 negative real zeros.

c.

1	1	–1	–7	1	6
		1	0	–7	–6
	1	0	–7	–6	0

–1	1	0	–7	–6
		–1	1	6
	1	–1	–6	0

$-2, -1, 1, 3$ are rational zeros.

d. $x^4 - x^3 - 7x^2 + x + 6 = 0$

$(x-1)(x+1)(x^2 - x + 6) = 0$

$(x-1)(x+1)(x-3)(x+2) = 0$

The solution set is $\{-2, -1, 1, 3\}$.

45. $4x^4 + 7x^2 - 2 = 0$

a. p: ±1, ±2
q: ±1, ±2, ±4

$\frac{p}{q}: \pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$

b. $f(x) = 4x^4 + 7x^2 - 2$
1 sign variation; 1 positive real zero.
$f(-x) = 4x^4 + 7x^2 - 2$
1 sign variation; 1 negative real zero.

c.

$\frac{1}{2}$	4	0	7	0	–2
		2	1	4	2
	4	2	8	4	0

$-\frac{1}{2}$	4	2	8	4
		–2	0	–4
	4	0	8	0

$-\frac{1}{2}, \frac{1}{2}$ are rational zeros.

d. $4x^4 + 7x^2 - 2 = 0$

$\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)(4x^2 + 8) = 0$

$4\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)(x^2 + 2) = 0$

Solving $x^2 + 2 = 0$ using the quadratic formula gives $x = \pm 2i$

The solution set is $\left\{-\frac{1}{2}, \frac{1}{2}, 2i, -2i\right\}$.

46. $f(x) = 2x^4 + x^3 - 9x^2 - 4x + 4$

a. p: ±1, ±2, ±4
q: ±1, ±2

$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$

b. $f(x) = 2x^4 + x^3 - 9x^2 - 4x + 4$
2 sign variations; 2 or 0 positive real zeros.
$f(-x) = 2x^4 - x^3 - 9x^2 + 4x + 4$
2 sign variations; 2 or 0 negative real zeros.

c.

2	2	1	–9	–4	4
		4	10	2	–4
	2	5	1	–2	0

–1	2	5	1	–2
		–2	–3	2
	2	3	–2	0

$-2, -1, \frac{1}{2}, 2$ are rational zeros.

d. $2x^2 + 3x - 2 = 0$

$(2x-1)(x+2) = 0$

$x = -2$ or $x = \frac{1}{2}$

The solution set is $\left\{-2, -1, \frac{1}{2}, 2\right\}$.

47. $f(x) = a_n(x-2)(x-2+3i)(x-2-3i)$

$f(x) = a_n(x-2)(x^2 - 4x + 13)$

$f(1) = a_n(1-2)\left[1^2 - 4(1) + 13\right]$

$-10 = -10a_n$

$a_n = 1$

$f(x) = 1(x-2)(x^2 - 4x + 13)$

$f(x) = x^3 - 4x^2 + 13x - 2x^2 + 8x - 26$

$f(x) = x^3 - 6x^2 + 21x - 26$

48. $f(x) = a_n(x-i)(x+i)(x+3)^2$

$f(x) = a_n(x^2+1)(x^2+6x+9)$

$f(-1) = a_n\left[(-1)^2 + 1\right]\left[(-1)^2 + 6(-1) + 9\right]$

$16 = 8a_n$

$a_n = 2$

$f(x) = 2(x^2+1)(x^2+6x+9)$

$f(x) = 2(x^4 + 6x^3 + 9x^2 + x^2 + 6x + 9)$

$f(x) = 2x^4 + 12x^3 + 20x^2 + 12x + 18$

49. $f(x) = 2x^4 + 3x^3 + 3x - 2$

p: ±1, ±2
q: ±1, ±2

$\frac{p}{q}: \pm 1, \pm 2, \pm \frac{1}{2}$

–2	2	3	0	3	–2
		–4	2	–4	2
	2	–1	2	–1	0

$$2x^4 + 3x^3 + 3x - 2 = 0$$
$$(x+2)(2x^3 - x^2 + 2x - 1) = 0$$
$$(x+2)[x^2(2x-1) + (2x-1)] = 0$$
$$(x+2)(2x-1)(x^2+1) = 0$$

$x = -2,\ x = \frac{1}{2}$ or $x = \pm i$

The zeros are $-2,\ \frac{1}{2},\ \pm i$.

$f(x) = (x-i)(x+i)(x+2)(2x-1)$

50. $g(x) = x^4 - 6x^3 + x^2 + 24x + 16$

p: ±1, ±2, ±4, ±8, ±16
q: ±1

$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

–1	1	–6	1	24	16
		–1	7	–8	–16
	1	–7	8	16	0

$$x^4 - 6x^3 + x^2 + 24x + 16 = 0$$
$$(x+1)(x^3 - 7x^2 + 8x + 16) = 0$$

–1	1	–7	8	16
		–1	8	–16
	1	–8	16	0

$$(x+1)^2(x^2 - 8x + 16) = 0$$
$$(x+1)^2(x-4)^2 = 0$$

$x = -1$ or $x = 4$

$g(x) = (x+1)^2(x-4)^2$

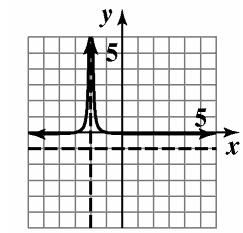
51. 4 real zeros, one with multiplicity two

52. 3 real zeros; 2 nonreal complex zeros

53. 2 real zeros, one with multiplicity two; 2 nonreal complex zeros

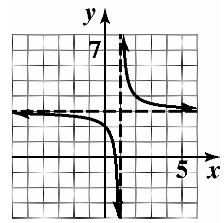
54. 1 real zero; 4 nonreal complex zeros

55. $g(x) = \frac{1}{(x+2)^2} - 1$



$g(x) = \frac{1}{(x+2)^2} - 1$

56. $h(x) = \frac{1}{x-1} + 3$



$h(x) = \frac{1}{x-1} + 3$

57. $f(x) = \frac{2x}{x^2 - 9}$

Symmetry: $f(-x) = -\frac{2x}{x^2 - 9} = -f(x)$

origin symmetry

x-intercept:

$$0 = \frac{2x}{x^2 - 9}$$
$$2x = 0$$
$$x = 0$$

y-intercept: $y = \frac{2(0)}{0^2 - 9} = 0$

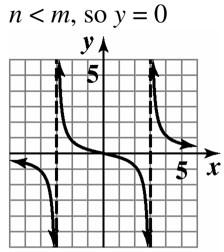
Vertical asymptote:

$$x^2 - 9 = 0$$
$$(x-3)(x+3) = 0$$

$x = 3$ and $x = -3$

Horizontal asymptote:

$n < m$, so $y = 0$



$f(x) = \frac{2x}{x^2 - 9}$

58. $g(x) = \dfrac{2x-4}{x+3}$

Symmetry: $g(-x) = \dfrac{-2x-4}{x+3}$

$g(-x) \neq g(x)$, $g(-x) \neq -g(x)$

No symmetry

x-intercept:

$2x - 4 = 0$

$x = 2$

y-intercept: $y = \dfrac{2(0)-4}{(0)+3} = -\dfrac{4}{3}$

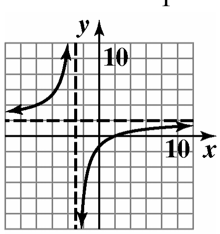
Vertical asymptote:

$x + 3 = 0$

$x = -3$

Horizontal asymptote:

$n = m$, so $y = \dfrac{2}{1} = 2$



$f(x) = \dfrac{2x-4}{x+3}$

59. $h(x) = \dfrac{x^2-3x-4}{x^2-x-6}$

Symmetry: $h(-x) = \dfrac{x^2+3x-4}{x^2+x-6}$

$h(-x) \neq h(x)$, $h(-x) \neq -h(x)$

No symmetry

x-intercepts:

$x^2 - 3x - 4 = 0$

$(x-4)(x+1)$

$x = 4 \quad x = -1$

y-intercept: $y = \dfrac{0^2-3(0)-4}{0^2-0-6} = \dfrac{2}{3}$

Vertical asymptotes:

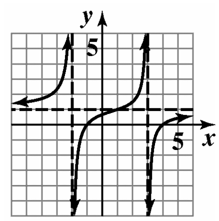
$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, -2$

Horizontal asymptote:

$n = m$, so $y = \dfrac{1}{1} = 1$



$h(x) = \dfrac{x^2-3x-4}{x^2-x-6}$

60. $r(x) = \dfrac{x^2+4x+3}{(x+2)^2}$

Symmetry: $r(-x) = \dfrac{x^2-4x+3}{(-x+2)^2}$

$r(-x) \neq r(x)$, $r(-x) \neq -r(x)$

No symmetry

x-intercepts:

$x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$x = -3, -1$

y-intercept: $y = \dfrac{0^2+4(0)+3}{(0+2)^2} = \dfrac{3}{4}$

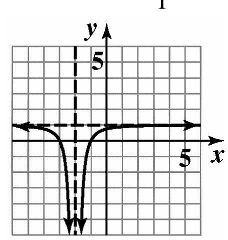
Vertical asymptote:

$x + 2 = 0$

$x = -2$

Horizontal asymptote:

$n = m$, so $y = \dfrac{1}{1} = 1$

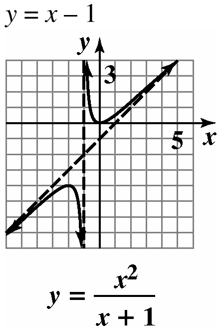


$r(x) = \dfrac{x^2+4x+3}{(x+4)^2}$

61. $y = \dfrac{x^2}{x+1}$

Symmetry: $f(-x) = \dfrac{x^2}{-x+1}$
$f(-x) \neq f(x), f(-x) \neq -f(x)$
No symmetry
x-intercept:
$x^2 = 0$
$x = 0$

y-intercept: $y = \dfrac{0^2}{0+1} = 0$
Vertical asymptote:
$x + 1 = 0$
$x = -1$
$n > m$, no horizontal asymptote.
Slant asymptote:
$y = x - 1 + \dfrac{1}{x+1}$
$y = x - 1$

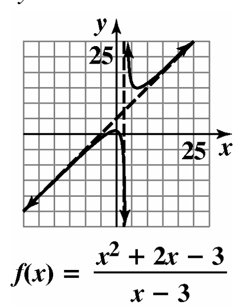


62. $y = \dfrac{x^2 + 2x - 3}{x - 3}$

Symmetry: $f(-x) = \dfrac{x^2 - 2x - 3}{-x - 3}$
$f(-x) \neq f(x), f(-x) \neq -f(x)$
No symmetry
x-intercepts:
$x^2 + 2x - 3 = 0$
$(x + 3)(x - 1) = 0$
$x = -3, 1$

y-intercept: $y = \dfrac{0^2 + 2(0) - 3}{0 - 3} = \dfrac{-3}{-3} = 1$
Vertical asymptote:
$x - 3 = 0$
$x = 3$
Horizontal asymptote:
$n > m$, so no horizontal asymptote.

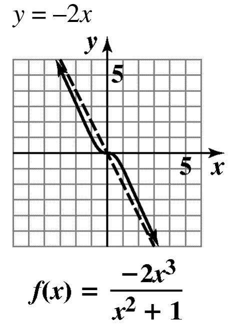
Slant asymptote:
$y = x + 5 + \dfrac{12}{x - 3}$
$y = x + 5$



63. $f(x) = \dfrac{-2x^3}{x^2 + 1}$

Symmetry: $f(-x) = \dfrac{2}{x^2 + 1} = -f(x)$
Origin symmetry
x-intercept:
$-2x^3 = 0$
$x = 0$

y-intercept: $y = \dfrac{-2(0)^3}{0^2 + 1} = \dfrac{0}{1} = 0$
Vertical asymptote:
$x^2 + 1 = 0$
$x^2 = -1$
No vertical asymptote.
Horizontal asymptote:
$n > m$, so no horizontal asymptote.
Slant asymptote:
$f(x) = -2x + \dfrac{2x}{x^2 + 1}$
$y = -2x$



64. $g(x)=\dfrac{4x^2-16x+16}{2x-3}$

Symmetry: $g(-x)=\dfrac{4x^2+16x+16}{-2x-3}$

$g(-x)\neq g(x)$, $g(-x)\neq -g(x)$

No symmetry

x-intercept:

$4x^2-16x+16=0$

$4(x-2)^2=0$

$x=2$

y-intercept:

$y=\dfrac{4(0)^2-16(0)+16}{2(0)-3}=-\dfrac{16}{3}$

Vertical asymptote:

$2x-3=0$

$x=\dfrac{3}{2}$

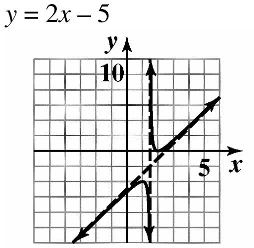
Horizontal asymptote:

$n>m$, so no horizontal asymptote.

Slant asymptote:

$g(x)=2x-5+\dfrac{1}{2x-3}$

$y=2x-5$



$g(x)=\dfrac{4x^2-16x+16}{2x-3}$

65. a. $C(x)=50{,}000+25x$

b. $\overline{C}(x)=\dfrac{25x+50{,}000}{x}$

c. $\overline{C}(50)=\dfrac{25(50)+50{,}000}{50}=1025$

When 50 calculators are manufactured, it costs $1025 to manufacture each.

$\overline{C}(100)=\dfrac{25(100)+50{,}000}{100}=525$

When 100 calculators are manufactured, it costs $525 to manufacture each.

$\overline{C}(1000)=\dfrac{25(1000)+50{,}000}{1000}=75$

When 1,000 calculators are manufactured, it costs $75 to manufacture each.

$\overline{C}(100{,}000)=\dfrac{25(100{,}000)+50{,}000}{100{,}000}=25.5$ Wh

en 100,000 calculators are manufactured, it costs $25.50 to manufacture each.

d. $n=m$, so $y=\dfrac{25}{1}=25$ is the horizontal asymptote. Minimum costs will approach $25.

66. $f(x)=\dfrac{150x+120}{0.05x+1}$

$n=m$, so $y=\dfrac{150}{0.05}=3000$

The number of fish available in the pond approaches 3000.

67. $P(x)=\dfrac{72{,}900}{100x^2+729}$

$n<m$ so $y=0$

As the number of years of education increases the percentage rate of unemployment approaches zero.

68. a. $P(x)=M(x)+F(x)$

$=1.58x+114.4+1.48x+120.6$

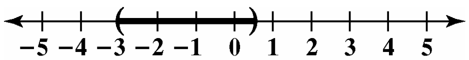
$=3.06x+235$

b. $R(x)=\dfrac{M(x)}{P(x)}=\dfrac{1.58x+114.4}{3.06x+235}$

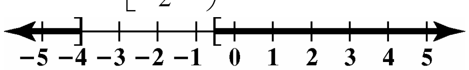
c. $y=\dfrac{1.58}{3.06}\approx 0.52$

Over time, the percentage of men in the U.S. population will approach 52%.

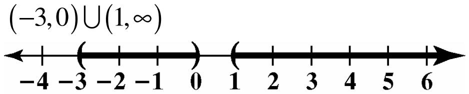
69. $2x^2 + 5x - 3 < 0$
Solve the related quadratic equation.
$2x^2 + 5x - 3 = 0$
$(2x-1)(x+3) = 0$
The boundary points are -3 and $\frac{1}{2}$.
Testing each interval gives a solution set of $\left(-3, \frac{1}{2}\right)$



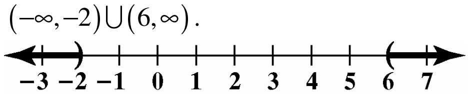
70. $2x^2 + 9x + 4 \geq 0$
Solve the related quadratic equation.
$2x^2 + 9x + 4 = 0$
$(2x+1)(x+4) = 0$
The boundary points are -4 and $-\frac{1}{2}$.
Testing each interval gives a solution set of
$(-\infty, -4] \cup \left[-\frac{1}{2}, \infty\right)$



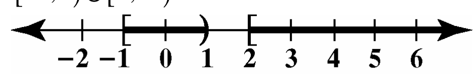
71. $x^3 + 2x^2 > 3x$
Solve the related equation.
$x^3 + 2x^2 = 3x$
$x^3 + 2x^2 - 3x = 0$
$x(x^2 + 2x - 3) = 0$
$x(x+3)(x-1) = 0$
The boundary points are -3, 0, and 1.
Testing each interval gives a solution set of
$(-3, 0) \cup (1, \infty)$



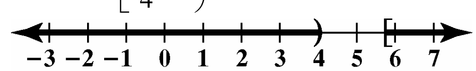
72. $\frac{x-6}{x+2} > 0$
Find the values of x that make the numerator and denominator zero.
The boundary points are –2 and 6.
Testing each interval gives a solution set of
$(-\infty, -2) \cup (6, \infty)$.



73. $\frac{(x+1)(x-2)}{x-1} \geq 0$
Find the values of x that make the numerator and denominator zero.
The boundary points are –1, 1 and 2. We exclude 1 from the solution set, since this would make the denominator zero.
Testing each interval gives a solution set of
$[-1, 1) \cup [2, \infty)$.



74. $\frac{x+3}{x-4} \leq 5$
Express the inequality so that one side is zero.
$\frac{x+3}{x-4} - 5 \leq 0$
$\frac{x+3}{x-4} - \frac{5(x-4)}{x-4} \leq 0$
$\frac{-4x+23}{x-4} \leq 0$
Find the values of x that make the numerator and denominator zero.
The boundary points are 4 and $\frac{23}{4}$. We exclude 4 from the solution set, since this would make the denominator zero.
Testing each interval gives a solution set of
$(-\infty, 4) \cup \left[\frac{23}{4}, \infty\right)$.



75. **a.** $g(x) = 0.125x^2 + 2.3x + 27$

$g(35) = 0.125(35)^2 + 2.3(35) + 27 \approx 261$

The stopping distance on wet pavement for a motorcycle traveling 35 miles per hour is about 261 feet. This overestimates the distance shown in the graph by 1 foot.

b. $f(x) = 0.125x^2 - 0.8x + 99$

$0.125x^2 - 0.8x + 99 > 267$

$0.125x^2 - 0.8x - 168 > 0$

Solve the related quadratic equation.

$0.125x^2 - 0.8x - 168 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4(0.125)(-168)}}{2(0.125)}$$

$x = -33.6,\ 40$

Testing each interval gives a solution set of $(-\infty, -33.6) \cup (40, \infty)$.

Thus, speeds exceeding 40 miles per hour on dry pavement will require over 267 feet of stopping distance.

76. $s = -16t^2 + v_0 t + s_0$

$32 < -16t^2 + 48t + 0$

$0 < -16t^2 + 48t - 32$

$0 < -16\left(t^2 - 3t + 2\right)$

$0 < -16(t-2)(t-1)$

F	T	F
	1	2

The projectile's height exceeds 32 feet during the time period from 1 to 2 seconds.

77. $w = ks$

$28 = k \cdot 250$

0.112=k

Thus, $w = 0.112s$.

$w = 0.112(1200) = 134.4$

1200 cubic centimeters of melting snow will produce 134.4 cubic centimeters of water.

78. $d = kt^2$

$144 = k(3)^2$

$k = 16$

$d = 16t^2$

$d = 16(10)^2 = 1{,}600$ ft

79. $p = \dfrac{k}{w}$

$660 = \dfrac{k}{1.6}$

$1056 = k$

Thus, $p = \dfrac{1056}{w}$.

$p = \dfrac{1056}{2.4} = 440$

The pitch is 440 vibrations per second.

80. $l = \dfrac{k}{d^2}$

$28 = \dfrac{k}{8^2}$

$k = 1792$

$l = \dfrac{1792}{d^2}$

$l = \dfrac{1792}{4^2} = 112$ decibels

81. $t = \dfrac{kc}{w}$

$10 = \dfrac{k \cdot 30}{6}$

$10 = 5h$

$h = 2$

$t = \dfrac{2c}{w}$

$t = \dfrac{2(40)}{5} = 16$ hours

82. $V = khB$

$175 = k \cdot 15 \cdot 35$

$k = \dfrac{1}{3}$

$V = \dfrac{1}{3}hB$

$V = \dfrac{1}{3} \cdot 20 \cdot 120 = 800\ \text{ft}^3$

83. **a.** Use $L = \frac{k}{R}$ to find k.

$$L = \frac{k}{R}$$
$$30 = \frac{k}{63}$$
$$63 \cdot 30 = 63 \cdot \frac{k}{63}$$
$$1890 = k$$

Thus, $L = \frac{1890}{R}$.

b. This is an approximate model.

c. $L = \frac{1890}{R}$

$L = \frac{1890}{27} = 70$

The average life span of an elephant is 70 years.

Chapter 3 Test

1. $f(x) = (x+1)^2 + 4$

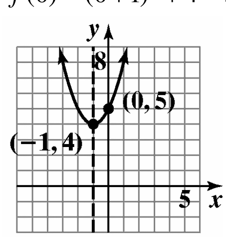
vertex: (–1, 4)
axis of symmetry: $x = -1$
x-intercepts:

$(x+1)^2 + 4 = 0$

$x^2 + 2x + 5 = 0$

$x = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$

no x-intercepts
y-intercept:
$f(0) = (0+1)^2 + 4 = 5$



$\boldsymbol{f(x) = (x+1)^2 + 4}$

domain: $(-\infty, \infty)$; range: $[4, \infty)$

2. $f(x) = x^2 - 2x - 3$

$x = \frac{-b}{2a} = \frac{2}{2} = 1$

$f(1) = 1^2 - 2(1) - 3 = -4$

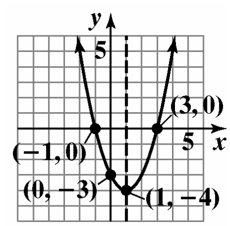
vertex: (1, –4)
axis of symmetry $x = 1$
x-intercepts:

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

y-intercept:
$f(0) = 0^2 - 2(0) - 3 = -3$



$\boldsymbol{f(x) = x^2 - 2x - 3}$

domain: $(-\infty, \infty)$; range: $[-4, \infty)$

3. $f(x) = -2x^2 + 12x - 16$

Since the coefficient of x^2 is negative, the graph of $f(x)$ opens down and $f(x)$ has a maximum point.

$x = \frac{-12}{2(-2)} = 3$

$f(3) = -2(3)^2 + 12(3) - 16$
$= -18 + 36 - 16$
$= 2$

Maximum point: (3, 2)
domain: $(-\infty, \infty)$; range: $(-\infty, 2]$

4. $f(x) = -x^2 + 46x - 360$

$x = -\frac{b}{2a} = \frac{-46}{-2} = 23$

23 computers will maximize profit.

$f(23) = -(23)^2 + 46(23) - 360 = 169$

Maximum daily profit = \$16,900.

5. Let x = one of the numbers;
$14-x$ = the other number.
The product is $f(x)=x(14-x)$

$f(x)=x(14-x)=-x^2+14x$

The x-coordinate of the maximum is

$x=-\frac{b}{2a}=-\frac{14}{2(-1)}=-\frac{14}{-2}=7.$

$f(7)=-7^2+14(7)=49$

The vertex is (7, 49). The maximum product is 49. This occurs when the two number are 7 and $14-7=7$.

6. **a.** $f(x)=x^3-5x^2-4x+20$

$x^3-5x^2-4x+20=0$

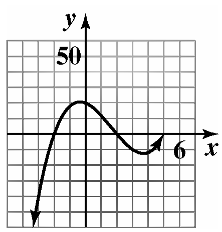
$x^2(x-5)-4(x-5)=0$

$(x-5)(x-2)(x+2)=0$

$x=5, 2, -2$

The solution set is {5, 2, –2}.

b. The degree of the polynomial is odd and the leading coefficient is positive. Thus the graph falls to the left and rises to the right.



7. $f(x)=x^5-x$

Since the degree of the polynomial is odd and the leading coefficient is positive, the graph of f should fall to the left and rise to the right. The x-intercepts should be –1 and 1.

8. **a.** The integral root is 2.

b.

2	6	–19	16	–4
		12	–14	4
	6	–7	2	0

$6x^2-7x+2=0$

$(3x-2)(2x-1)=0$

$x=\frac{2}{3}$ or $x=\frac{1}{2}$

The other two roots are $\frac{1}{2}$ and $\frac{2}{3}$.

9. $2x^3+11x^2-7x-6=0$

p: ±1, ±2, ±3, ±6
q: ±1, ±2

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$

10. $f(x)=3x^5-2x^4-2x^2+x-1$

$f(x)$ has 3 sign variations.

$f(-x)=-3x^5-2x^4-2x^2-x-1$

$f(-x)$ has no sign variations.
There are 3 or 1 positive real solutions and no negative real solutions.

11. $x^3+9x^2+16x-6=0$

Since the leading coefficient is 1, the possible rational zeros are the factors of 6

$p=\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6$

–3	1	9	16	–6
		–3	–18	6
	1	6	–2	0

Thus $x=3$ is a root.

Solve the quotient $x^2+6x-2=0$ using the quadratic formula to find the remaining roots.

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(6)\pm\sqrt{(6)^2-4(1)(-2)}}{2(1)}$

$=\frac{-6\pm\sqrt{44}}{2}$

$=-3\pm\sqrt{11}$

The zeros are -3, $-3+\sqrt{11}$, and $-3-\sqrt{11}$.

12. $f(x) = 2x^4 - x^3 - 13x^2 + 5x + 15$

a. Possible rational zeros are:

$p: \pm 1, \ \pm 3, \ \pm 5, \ \pm 15$

$q: \pm 1, \ \pm 2$

$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

b. Verify that -1 and $\frac{3}{2}$ are zeros as it appears in the graph:

$$\begin{array}{r|rrrrr} -1 & 2 & -1 & -13 & 5 & 15 \\ & & -2 & 3 & 10 & -15 \\ \hline & 2 & -3 & -10 & 15 & 0 \end{array}$$

$$\begin{array}{r|rrrr} \frac{3}{2} & 2 & -3 & -10 & 15 \\ & & 3 & 0 & -15 \\ \hline & 2 & 0 & -10 & 0 \end{array}$$

Thus, -1 and $\frac{3}{2}$ are zeros, and the polynomial factors as follows:

$$2x^4 - x^3 - 13x^2 + 5x + 15 = 0$$

$$(x+1)(2x^3 - 3x^2 - 10x + 15) = 0$$

$$(x+1)\left(x - \frac{3}{2}\right)(2x^2 - 10) = 0$$

Find the remaining zeros by solving:

$$2x^2 - 10 = 0$$

$$2x^2 = 10$$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

The zeros are -1, $\frac{3}{2}$, and $\pm\sqrt{5}$.

13. $f(x)$ has zeros at –2 and 1. The zero at –2 has multiplicity of 2.

$x^3 + 3x^2 - 4 = (x-1)(x+2)^2$

14. $f(x) = a_0(x+1)(x-1)(x+i)(x-i)$

$= a_0(x^2 - 1)(x^2 + 1)$

$= a_0(x^4 - 1)$

Since $f(3) = 160$, then

$$a_0(3^4 - 1) = 160$$

$$a_0(80) = 160$$

$$a_0 = \frac{160}{80}$$

$$a_0 = 2$$

$f(x) = 2(x^4 - 1) = 2x^4 - 2$

15. $f(x) = -3x^3 - 4x^2 + x + 2$

The graph shows a root at $x = -1$.
Use synthetic division to verify this root.

$$\begin{array}{r|rrrr} -1 & -3 & -4 & 1 & 2 \\ & & 3 & 1 & 4 \\ \hline & -3 & -1 & 2 & 0 \end{array}$$

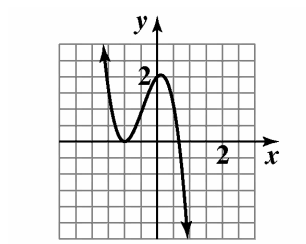
Factor the quotient to find the remaining zeros.

$$-3x^2 - x + 2 = 0$$

$$-(3x-2)(x+1) = 0$$

The zeros (x-intercepts) are –1 and $\frac{2}{3}$.

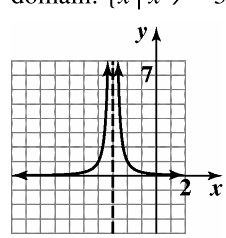
The y-intercept is $f(0) = 2$



$f(x) = -3x^3 - 4x^2 + x + 2$

16. $f(x) = \frac{1}{(x+3)^2}$

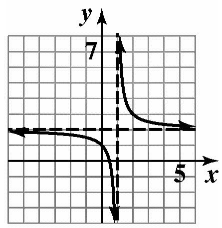
domain: $\{x \mid x \neq -3\}$ or $(-\infty, -3) \cup (-3, \infty)$



$f(x) = \frac{1}{(x+3)^2}$

17. $f(x)=\dfrac{1}{x-1}+2$

domain: $\{x \mid x \neq 1\}$ or $(-\infty,1)\cup(1,\infty)$



$f(x) = \dfrac{1}{x-1} + 2$

18. $f(x)=\dfrac{x}{x^2-16}$

domain: $\{x \mid x \neq 4, x \neq -4\}$

Symmetry: $f(-x)=\dfrac{-x}{x^2-16}=-f(x)$

y-axis symmetry

x-intercept: $x = 0$

y-intercept: $y=\dfrac{0}{0^2-16}=0$

Vertical asymptotes:

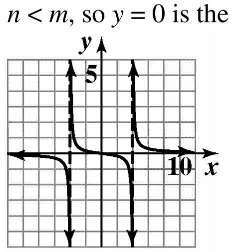
$x^2-16=0$

$(x-4)(x+4)=0$

$x = 4, -4$

Horizontal asymptote:

$n < m$, so $y = 0$ is the horizontal asymptote.



$f(x) = \dfrac{x}{x^2-16}$

19. $f(x)=\dfrac{x^2-9}{x-2}$

domain: $\{x \mid x \neq 2\}$

Symmetry: $f(-x)=\dfrac{x^2-9}{-x-2}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

No symmetry

x-intercepts:

$x^2-9=0$

$(x-3)(x+3)=0$

$x = 3, -3$

y-intercept: $y=\dfrac{0^2-9}{0-2}=\dfrac{9}{2}$

Vertical asymptote:

$x - 2 = 0$

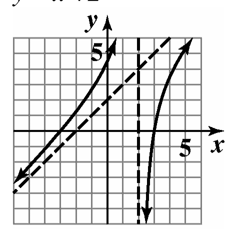
$x = 2$

Horizontal asymptote:

$n > m$, so no horizontal asymptote exists.

Slant asymptote: $f(x)=x+2-\dfrac{5}{x-2}$

$y = x + 2$



$f(x) = \dfrac{x^2-9}{x-2}$

20. $f(x)=\dfrac{x+1}{x^2+2x-3}$

$x^2+2x-3=(x+3)(x-1)$

domain: $\{x \mid x \neq -3, x \neq 1\}$

Symmetry: $f(-x)=\dfrac{-x+1}{x^2-2x-3}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

No symmetry

x-intercept:

$x + 1 = 0$

$x = -1$

y-intercept: $y=\dfrac{0+1}{0^2+2(0)-3}=-\dfrac{1}{3}$

Vertical asymptotes:

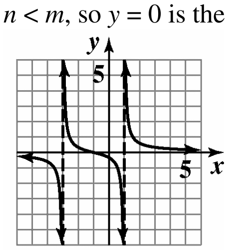
$x^2+2x-3=0$

$(x+3)(x-1)=0$

x –3, 1

Horizontal asymptote:

$n < m$, so $y = 0$ is the horizontal asymptote.



$f(x) = \dfrac{x+1}{x^2+2x-3}$

21. $f(x) = \dfrac{4x^2}{x^2 + 3}$

domain: all real numbers

Symmetry: $f(-x) = \dfrac{4x^2}{x^2 + 3} = f(x)$

y-axis symmetry

x-intercept:

$4x^2 = 0$

$x = 0$

y-intercept: $y = \dfrac{4(0)^2}{0^2 + 3} = 0$

Vertical asymptote:

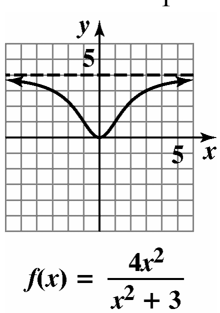
$x^2 + 3 = 0$

$x^2 = -3$

No vertical asymptote.

Horizontal asymptote:

$n = m$, so $y = \dfrac{4}{1} = 4$ is the horizontal asymptote.



22. **a.** $\overline{C}(x) = \dfrac{300{,}000 + 10x}{x}$

b. Since the degree of the numerator equals the degree of the denominator, the horizontal asymptote is $x = \dfrac{10}{1} = 10$.

This represents the fact that as the number of satellite radio players produced increases, the production cost approaches \$10 per radio.

23.

$$x^2 < x + 12$$
$$x^2 - x - 12 < 0$$
$$(x+3)(x-4) < 0$$

Boundary values: –3 and 4

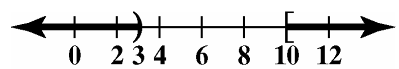
Solution set: $(-3, 4)$

24.

$$\frac{2x+1}{x-3} \le 3$$
$$\frac{2x+1}{x-3} - 3 \le 0$$
$$\frac{10-x}{x-3} \le 0$$

Boundary values: 3 and 10

Solution set: $(-\infty, 3) \cup [10, \infty)$



25.

$$i = \frac{k}{d^2}$$
$$20 = \frac{k}{15^2}$$
$$4500 = k$$

$$i = \frac{4500}{d^2} = \frac{4500}{10^2} = 45 \text{ foot-candles}$$

Cumulative Review Exercises (Chapters P–3)

1. domain: $(-2, 2)$ range: $[0, \infty)$

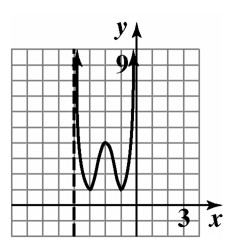
2. The zero at –1 touches the x-axis at turns around so it must have a minimum multiplicity of 2.
The zero at 1 touches the x-axis at turns around so it must have a minimum multiplicity of 2.

3. There is a relative maximum at the point (0, 3).

4. $(f \circ f)(-1) = f(f(-1)) = f(0) = 3$

5. $f(x) \to \infty$ as $\underline{x \to -2^+}$ or as $\underline{x \to 2^-}$

6.



7. $|2x-1|=3$

$2x-1=3$

$2x=4$

$x=2$

$2x-1=-3$

$2x=-2$

$x=-1$

The solution set is $\{2, -1\}$.

8. $3x^2-5x+1=0$

$x=\dfrac{5\pm\sqrt{25-12}}{6}=\dfrac{5\pm\sqrt{13}}{6}$

The solution set is $\left\{\dfrac{5+\sqrt{13}}{6}, \dfrac{5-\sqrt{13}}{6}\right\}$.

9. $9+\dfrac{3}{x}=\dfrac{2}{x^2}$

$9x^2+3x=2$

$9x^2+3x-2=0$

$(3x-1)(3x+2)=0$

$3x-1=0 \qquad 3x+2=0$

$x=\dfrac{1}{3} \quad$ or $\quad x=-\dfrac{2}{3}$

The solution set is $\left\{\dfrac{1}{3}, -\dfrac{2}{3}\right\}$.

10. $x^3+2x^2-5x-6=0$

p: ±1, ±2, ±3, ±6

q: ±1

$\dfrac{p}{q}: \pm1, \pm2, \pm3, \pm6$

–3	1	2	–5	–6
		–3	3	6
	1	–1	–2	0

$x^3+2x^2-5x-6=0$

$(x+3)(x^2-x-2)=0$

$(x+3)(x+1)(x-2)=0$

$x=-3$ or $x=-1$ or $x=2$

The solution set is $\{-3, -1, 2\}$.

11. $|2x-5|>3$

$2x-5>3$

$2x>8$

$x>4$

$2x-5<-3$

$2x<2$

$x<1$

$(-\infty, 1)$ or $(4, \infty)$

12. $3x^2>2x+5$

$3x^2-2x-5>0$

$3x^2-2x-5=0$

$(3x-5)(x+1)=0$

$x=\dfrac{5}{3}$ or $x=-1$

Test intervals are $(-\infty, -1)$, $\left(-1, \dfrac{5}{3}\right)$, $\left(\dfrac{5}{3}, \infty\right)$.

Testing points, the solution is $(-\infty, -1)$ or $\left(\dfrac{5}{3}, \infty\right)$.

13. $f(x)=x^3-4x^2-x+4$

x-intercepts:

$x^3-4x^2-x+4=0$

$x^2(x-4)-1(x-4)=0$

$(x-4)(x^2-1)=0$

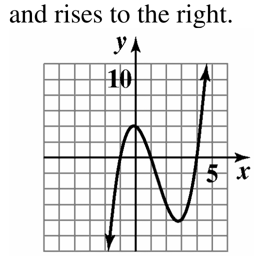
$(x-4)(x+1)(x-1)=0$

$x=-1, 1, 4$

x-intercepts:

$f(0)=0^3-4(0)^2-0+4=4$

The degree of the polynomial is odd and the leading coefficient is positive. Thus the graph falls to the left and rises to the right.



$f(x) = x^3 - 4x^2 - x + 4$

14. $f(x) = x^2 + 2x - 8$

$$x = \frac{-b}{2a} = \frac{-2}{2} = -1$$

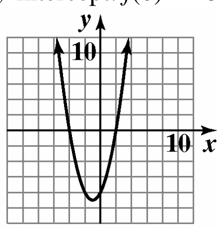
$$f(-1) = (-1)^2 + 2(-1) - 8$$
$$= 1 - 2 - 8 = -9$$

vertex: (–1, –9)

x-intercepts:

$$x^2 + 2x - 8 = 0$$
$$(x+4)(x-2) = 0$$
$$x = -4 \text{ or } x = 2$$

y-intercept: $f(0) = -8$



$f(x) = x^2 + 2x - 8$

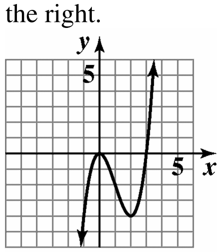
15. $f(x) = x^2(x-3)$

zeros: $x = 0$ (multiplicity 2) and $x = 3$

y-intercept: $y = 0$

$f(x) = x^3 - 3x^2$

$n = 3, a_n = 0$ so the graph falls to the left and rises to the right.



$f(x) = x^2(x - 3)$

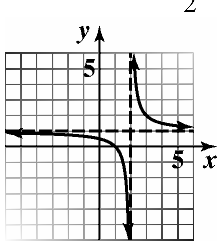
16. $f(x) = \dfrac{x-1}{x-2}$

vertical asymptote: $x = 2$

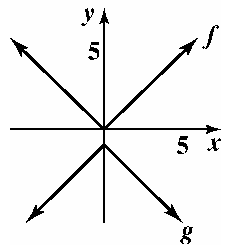
horizontal asymptote: $y = 1$

x-intercept: $x = 1$

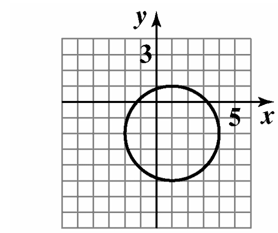
y-intercept: $y = \dfrac{1}{2}$



17.



18.



$x^2 + y^2 - 2x + 4y - 4 = 0$

19. $(f \circ g)(x) = f\left(g(x)\right)$

$$= 2(4x-1)^2 - (4x-1) - 1$$
$$= 32x^2 - 20x + 2$$

20. $\dfrac{f(x+h) - f(x)}{h}$

$$= \frac{\left[2(x+h)^2 - (x+h) - 1\right] - \left[2x^2 - x - 1\right]}{h}$$

$$= \frac{2x^2 + 4hx - x + 2h^2 - h - 1 - 2x^2 + x + 1}{h}$$

$$= \frac{4hx + 2h^2 - h}{h}$$

$$= 4x + 2h - 1$$

Chapter 4
Exponential and Logarithmic Functions

Section 4.1

Check Point Exercises

1. $f(x) = 42.2(1.56)^x$

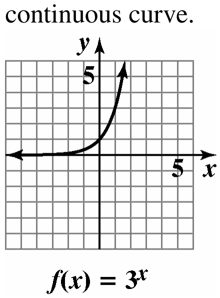
$f(3) = 42.2(1.56)^3 \approx 160.20876 \approx 160$

According to the function, the average amount spent after three hours of shopping at the mall is \$160. This overestimates the actual amount shown by \$11.

2. Begin by setting up a table of coordinates.

x	$f(x) = 3^x$
–3	$f(-3) = 3^{-3} = \frac{1}{27}$
–2	$f(-2) = 3^{-2} = \frac{1}{9}$
–1	$f(-1) = 3^{-1} = \frac{1}{3}$
0	$f(0) = 3^0 = 1$
1	$f(1) = 3^1 = 3$
2	$f(2) = 3^2 = 9$
3	$f(3) = 3^3 = 27$

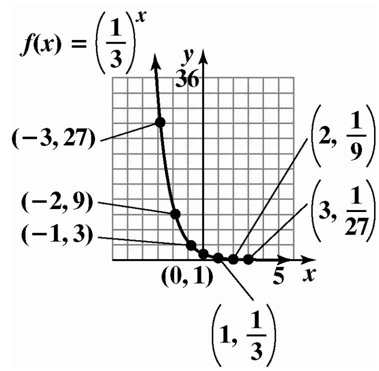
Plot these points, connecting them with a continuous curve.



3. Begin by setting up a table of coordinates.

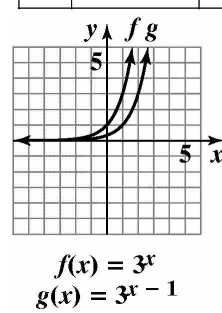
x	$f(x) = \left(\frac{1}{3}\right)^x$
–2	$\left(\frac{1}{3}\right)^{-2} = 9$
–1	$\left(\frac{1}{3}\right)^{-1} = 3$
0	$\left(\frac{1}{3}\right)^{0} = 1$
1	$\left(\frac{1}{3}\right)^{1} = \frac{1}{3}$
2	$\left(\frac{1}{3}\right)^{2} = \frac{1}{9}$

Plot these points, connecting them with a continuous curve.



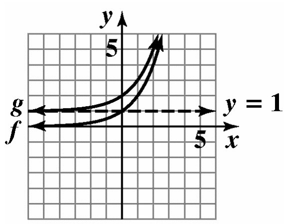
4. Note that the function $g(x) = 3^{x-1}$ has the general form $g(x) = b^{x+c}$ where $c = -1$. Because $c < 0$, we graph $g(x) = 3^{x-1}$ by shifting the graph of $f(x) = 3^x$ one unit to the right. Construct a table showing some of the coordinates for f and g.

x	$f(x) = 3^x$	$g(x) = 3^{x-1}$
–2	$3^{-2} = \frac{1}{9}$	$3^{-2-1} = 3^{-3} = \frac{1}{27}$
–1	$3^{-1} = \frac{1}{3}$	$3^{-1-1} = 3^{-2} = \frac{1}{9}$
0	$3^0 = 1$	$3^{0-1} = 3^{-1} = \frac{1}{3}$
1	$3^1 = 3$	$3^{1-1} = 3^0 = 1$
2	$3^2 = 9$	$3^{2-1} = 3^1 = 3$



5. Note that the function $g(x) = 2^x + 1$ has the general form $g(x) = b^x + c$ where c = 1. Because $c > 0$, we graph $g(x) = 2^x + 1$ by shifting the graph of $f(x) = 2^x$ up one unit. Construct a table showing some of the coordinates for *f* and *g*.

x	$f(x) - 2^x$	$g(x) = 2^x + 1$
-2	$2^{-2} = \frac{1}{4}$	$2^{-2} + 1 = \frac{1}{4} + 1 = \frac{5}{4}$
-1	$2^{-1} = \frac{1}{2}$	$2^{-1} + 1 = \frac{1}{2} + 1 = \frac{3}{2}$
0	$2^0 = 1$	$2^0 + 1 = 1 + 1 = 2$
1	$2^1 = 2$	$2^1 + 1 = 2 + 1 = 3$
2	$2^2 = 4$	$2^2 + 1 = 4 + 1 = 5$



$f(x) = 2^x$
$g(x) = 2^x + 1$

6. 2012 is 34 years after 1978.

$f(x) = 1066e^{0.042x}$

$f(34) = 1066e^{0.042(34)} \approx 4446$

In 2012 the gray wolf population of the Western Great Lakes is projected to be about 4446.

7. **a.** $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 10{,}000\left(1 + \frac{0.08}{4}\right)^{4(5)}$

$= \$14{,}859.47$

b. $A = Pe^{rt}$

$A = 10{,}000e^{0.08(5)}$

$= \$14{,}918.25$

Exercise Set 4.1

1. $2^{3.4} \approx 10.556$

2. $3^{2.4} \approx 13.967$

3. $3^{\sqrt{5}} \approx 11.665$

4. $5^{\sqrt{3}} \approx 16.242$

5. $4^{-1.5} = 0.125$

6. $6^{-1.2} \approx 0.116$

7. $e^{2.3} \approx 9.974$

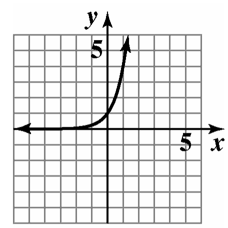
8. $e^{3.4} \approx 29.964$

9. $e^{-0.95} \approx 0.387$

10. $e^{-0.75} \approx 0.472$

11.

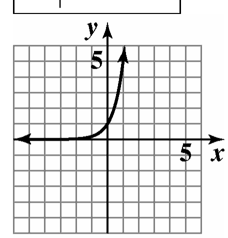
x	$f(x) = 4^x$
-2	$4^{-2} = \frac{1}{16}$
-1	$4^{-1} = \frac{1}{4}$
0	$4^0 = 1$
1	$4^1 = 4$
2	$4^2 = 16$



$f(x) = 4^x$

12.

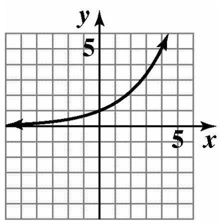
x	$g(x) = 5^x$
-2	$5^{-2} = \frac{1}{25}$
-1	$5^{-1} = \frac{1}{5}$
0	$5^0 = 1$
1	$5^1 = 5$
2	$5^2 = 25$



$f(x) = 5^x$

13.

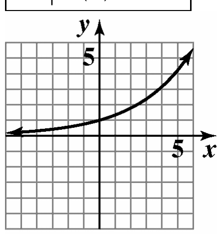
x	$g(x)=\left(\frac{3}{2}\right)^x$
-2	$\left(\frac{3}{2}\right)^{-2}=\frac{4}{9}$
-1	$\left(\frac{3}{2}\right)^{-1}=\frac{2}{3}$
0	$\left(\frac{3}{2}\right)^{0}=1$
1	$\left(\frac{3}{2}\right)^{1}=\frac{3}{2}$
2	$\left(\frac{3}{2}\right)^{2}=\frac{9}{4}$



$g(x) = \left(\frac{3}{2}\right)^x$

14.

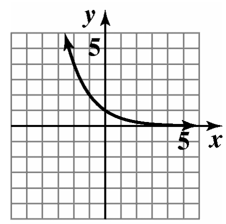
x	$g(x)=\left(\frac{4}{3}\right)^x$
-2	$\left(\frac{4}{3}\right)^{-2}=\frac{9}{16}$
-1	$\left(\frac{4}{3}\right)^{-1}=\frac{3}{4}$
0	$\left(\frac{4}{3}\right)^{0}=1$
1	$\left(\frac{4}{3}\right)^{1}=\frac{4}{3}$
2	$\left(\frac{4}{3}\right)^{2}=\frac{16}{9}$



$g(x) = \left(\frac{4}{3}\right)^x$

15.

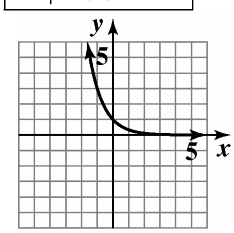
x	$h(x)=\left(\frac{1}{2}\right)^x$
-2	$\left(\frac{1}{2}\right)^{-2}=4$
-1	$\left(\frac{1}{2}\right)^{-1}=2$
0	$\left(\frac{1}{2}\right)^{0}=1$
1	$\left(\frac{1}{2}\right)^{1}=\frac{1}{2}$
2	$\left(\frac{1}{2}\right)^{2}=\frac{1}{4}$



$h(x) = \left(\frac{1}{2}\right)^x$

16.

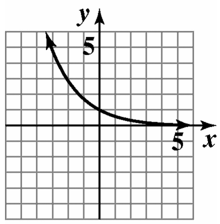
x	$h(x)=\left(\frac{1}{3}\right)^x$
-2	$\left(\frac{1}{3}\right)^{-2}=9$
-1	$\left(\frac{1}{3}\right)^{-1}=3$
0	$\left(\frac{1}{3}\right)^{0}=1$
1	$\left(\frac{1}{3}\right)^{1}=\frac{1}{3}$
2	$\left(\frac{1}{3}\right)^{2}=\frac{1}{9}$



$h(x) = \left(\frac{1}{3}\right)^x$

17.

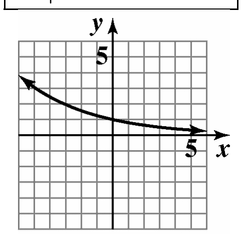
x	$f(x)=(0.6)^x$
−2	$(0.6)^{-2}=2.\overline{7}$
−1	$(0.6)^{-1}=1.\overline{6}$
0	$(0.6)^0=1$
1	$(0.6)^1=0.6$
2	$(0.6)^2=0.36$



f(x) = (0.6)^x

18.

x	$f(x)=(0.8)^x$
−2	$(0.8)^{-2}=1.5625$
−1	$(0.8)^{-1}=1.25$
0	$(0.8)^0=1$
1	$(0.8)^1=0.8$
2	$(0.8)^2=0.64$



f(x) = (0.8)^x

19. This is the graph of $f(x)=3^x$ reflected about the *x*-axis and about the *y*-axis, so the function is $H(x)=-3^{-x}$.

20. This is the graph of $f(x)=3^x$ shifted one unit to the right, so the function is $g(x)=3^{x-1}$.

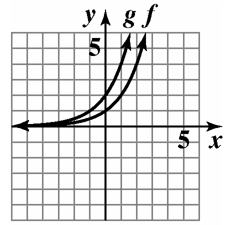
21. This is the graph of $f(x)=3^x$ reflected about the *x*-axis, so the function is $F(x)=-3^x$.

22. This is the graph of *f(x)* = 3^x.

23. This is the graph of $f(x)=3^x$ shifted one unit downward, so the function is $h(x)=3^x-1$.

24. This is the graph of $f(x)=3^x$ reflected about the y-axis, so the function is $G(x)=3^{-x}$.

25. The graph of $g(x)=2^{x+!}$ can be obtained by shifting the graph of $f(x)=2^x$ one unit to the left.

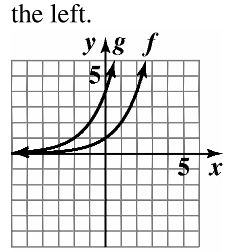


f(x) = 2^x
g(x) = 2^{x+1}

asymptote: $y=0$

domain: $(-\infty,\infty)$; range: $(0,\infty)$

26. The graph of $g(x)=2^{x+2}$ can be obtained by shifting the graph of $f(x)=2^x$ two units to the left.

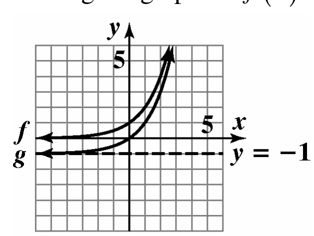


f(x) = 2^x
g(x) = 2^{x+2}

asymptote: $y=0$

domain: $(-\infty,\infty)$; range: $(0,\infty)$

27. The graph of $g(x)=2^x-1$ can be obtained by shifting the graph of $f(x)=2^x$ downward one unit.

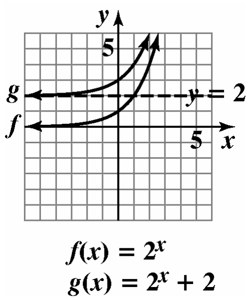


f(x) = 2^x
g(x) = 2^x − 1

asymptote: $y=-1$

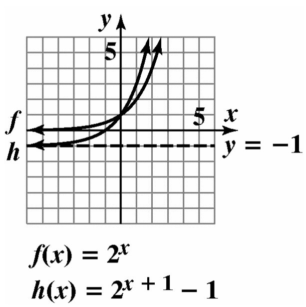
domain: $(-\infty,\infty)$; range: $(-1,\infty)$

28. The graph of $g(x) = 2^x + 2$ can be obtained by shifting the graph of $f(x) = 2^x$ two units upward.



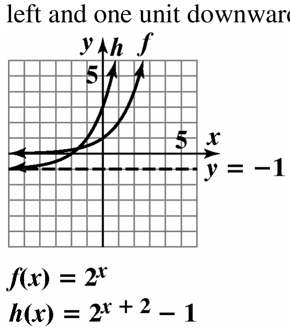
asymptote: $y = 2$
domain: $(-\infty, \infty)$; range: $(2, \infty)$

29. The graph of $h(x) = 2^{x+1} - 1$ can be obtained by shifting the graph of $f(x) = 2^x$ one unit to the left and one unit downward.



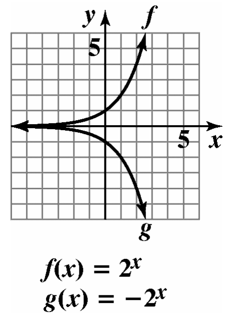
asymptote: $y = -1$
domain: $(-\infty, \infty)$; range: $(-1, \infty)$

30. The graph of $h(x) = 2^{x+2} - 1$ can be obtained by shifting the graph of $f(x) = 2^x$ two units to the left and one unit downward.



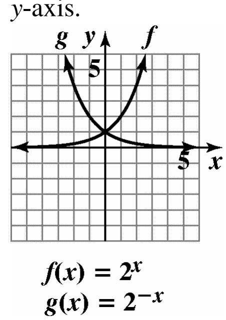
asymptote: $y = -1$
domain: $(-\infty, \infty)$; range: $(-1, \infty)$

31. The graph of $g(x) = -2^x$ can be obtained by reflecting the graph of $f(x) = 2^x$ about the *x*-axis.



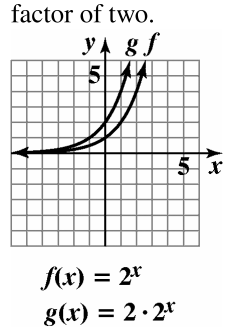
asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(-\infty, 0)$

32. The graph of $g(x) = 2^{-x}$ can be obtained by reflecting the graph of $f(x) = 2^x$ about the *y*-axis.



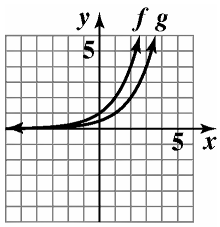
asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(0, \infty)$

33. The graph of $g(x) = 2 \cdot 2^x$ can be obtained by vertically stretching the graph of $f(x) = 2^x$ by a factor of two.



asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(0, \infty)$

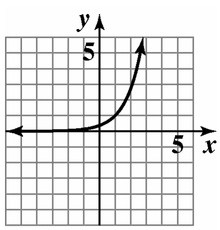
34. The graph of $g(x) = \frac{1}{2} \cdot 2^x$ can be obtained by vertically shrinking the graph of $f(x) = 2^x$ by a factor of one-half.



$f(x) = 2^x$
$g(x) = \frac{1}{2} \cdot 2^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(0, \infty)$

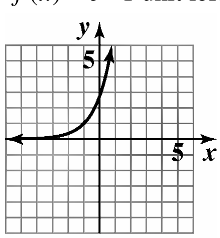
35. The graph of $g(x) = e^{x-1}$ can be obtained by moving $f(x) = e^x$ 1 unit right.



$g(x) = e^{x-1}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(0, \infty)$

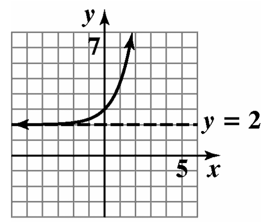
36. The graph of $g(x) = e^{x+1}$ can be obtained by moving $f(x) = e^x$ 1 unit left.



$g(x) = e^{x+1}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(0, \infty)$

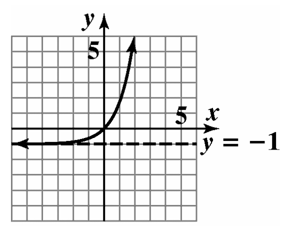
37. The graph of $g(x) = e^x + 2$ can be obtained by moving $f(x) = e^x$ 2 units up.



$g(x) = e^x + 2$

asymptote: $y = 2$
domain: $(-\infty, \infty)$; range: $(2, \infty)$

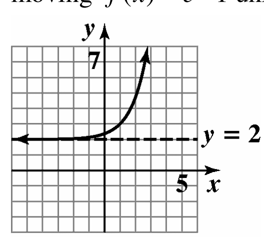
38. The graph of $g(x) = e^x - 1$ can be obtained by moving $f(x) = e^x$ 1 unit down.



$g(x) = e^x - 1$

asymptote: $y = -1$
domain: $(-\infty, \infty)$; range: $(-1, \infty)$

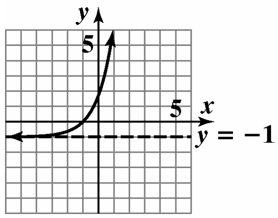
39. The graph of $h(x) = e^{x-1} + 2$ can be obtained by moving $f(x) = e^x$ 1 unit right and 2 units up.



$h(x) = e^{x-1} + 2$

asymptote: $y = 2$
domain: $(-\infty, \infty)$; range: $(2, \infty)$

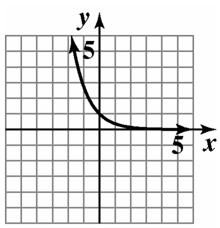
40. The graph of $h(x) = e^{x+1} - 1$ can be obtained by moving $f(x) = e^x$ 1 unit left and 1 unit down.



$h(x) = e^{x+1} - 1$

asymptote: $y = -1$
domain: $(-\infty, \infty)$; range: $(-1, \infty)$

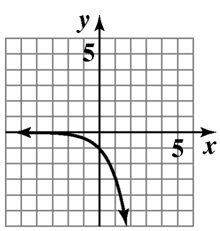
41. The graph of $h(x) = e^{-x}$ can be obtained by reflecting $f(x) = e^x$ about the y-axis.



$h(x) = e^{-x}$

asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(0, \infty)$

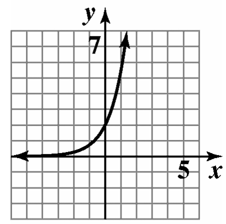
42. The graph of $h(x) = -e^x$ can be obtained by reflecting $f(x) = e^x$ about the x-axis.



$h(x) = -e^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(-\infty, 0)$

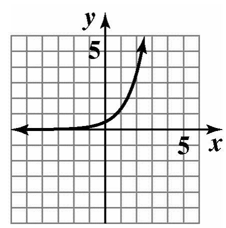
43. The graph of $g(x) = 2e^x$ can be obtained by stretching $f(x) = e^x$ vertically by a factor of 2.



$g(x) = 2e^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(0, \infty)$

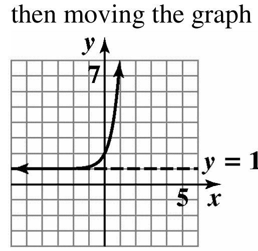
44. The graph of $g(x) = \frac{1}{2}e^x$ can be obtained by shrinking $f(x) = e^x$ vertically by a factor of $\frac{1}{2}$.



$g(x) = \frac{1}{2}e^x$

asymptote: $y = 0$
domain: $(-\infty, \infty)$; range: $(0, \infty)$

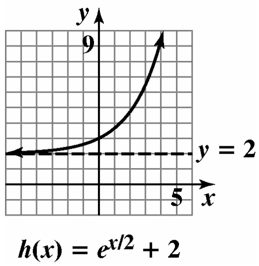
45. The graph of $h(x) = e^{2x} + 1$ can be obtained by stretching $f(x) = e^x$ horizontally by a factor of 2 and then moving the graph up 1 unit.



$h(x) = e^{2x} + 1$

asymptote: $y = 1$
domain: $(-\infty, \infty)$; range: $(1, \infty)$

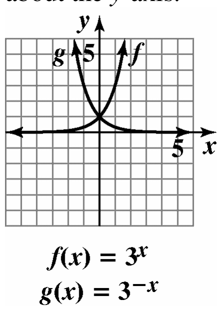
46. The graph of $h(x)=e^{\frac{1}{2}x}+2$ can be obtained by shrinking $f(x)=e^x$ horizontally by a factor of $\frac{1}{2}$ and then moving the graph up 2 units.



asymptote: $y=2$

domain: $(-\infty,\infty)$; range: $(2,\infty)$

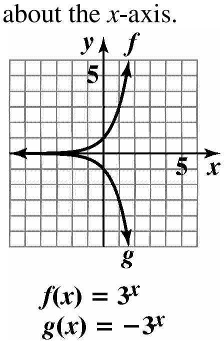
47. The graph of $g(x)$ can be obtained by reflecting $f(x)$ about the y-axis.



asymptote of $f(x)$: $y=0$

asymptote of $g(x)$: $y=0$

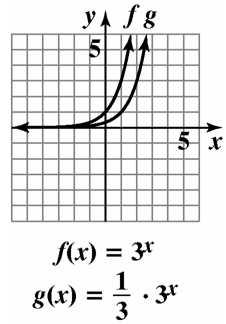
48. The graph of $g(x)$ can be obtained by reflecting $f(x)$ about the x-axis.



asymptote of $f(x)$: $y=0$

asymptote of $g(x)$: $y=0$

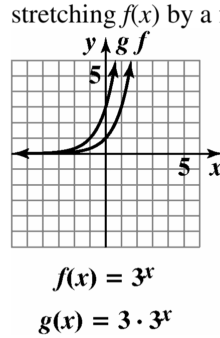
49. The graph of $g(x)$ can be obtained by vertically shrinking $f(x)$ by a factor of $\frac{1}{3}$.



asymptote of $f(x)$: $y=0$

asymptote of $g(x)$: $y=0$

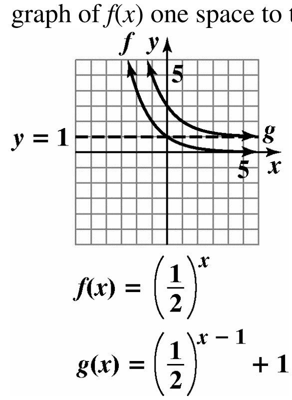
50. The graph of $g(x)$ can be obtained by horizontally stretching $f(x)$ by a factor of 3..



asymptote of $f(x)$: $y=0$

asymptote of $g(x)$: $y=0$

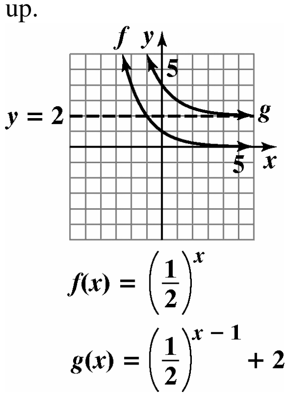
51. The graph of $g(x)$ can be obtained by moving the graph of $f(x)$ one space to the right and one space up.



asymptote of $f(x)$: $y=0$

asymptote of $g(x)$: $y=1$

52. The graph of $g(x)$ can be obtained by moving the graph of $f(x)$ one space to the right and two spaces up.



asymptote of $f(x)$: $y = 0$
asymptote of $g(x)$: $y = 2$

53. a. $A = 10,000\left(1+\frac{0.055}{2}\right)^{2(5)}$
$\approx \$13,116.51$

b. $A = 10,000\left(1+\frac{0.055}{4}\right)^{4(5)}$
$\approx \$13,140.67$

c. $A = 10,000\left(1+\frac{0.055}{12}\right)^{12(5)}$
$\approx \$13,157.04$

d. $A = 10,000e^{0.055(5)}$
$\approx \$13,165.31$

54. a. $A = 5000\left(1+\frac{0.065}{2}\right)^{2(10)} \approx \$9479.19$

b. $A = 5000\left(1+\frac{0.065}{4}\right)^{4\cdot 10} \approx \$9527.79$

c. $A = 5000\left(1+\frac{0.065}{12}\right)^{12(10)} \approx = \$9560.92$

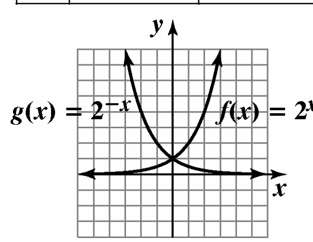
d. $A = 5000(e)^{0.065(10)} \approx 9577.70$

55. $A = 12,000\left(1+\frac{0.07}{12}\right)^{12(3)}$
$\approx 14,795.11$ (7% yield)
$A = 12,000e^{0.0685(3)}$
$\approx 14,737.67$ (6.85% yield)
Investing \$12,000 for 3 years at 7% compounded monthly yields the greater return.

56. $A = 6000\left(1+\frac{0.0825}{4}\right)^{4(4)}$
$\approx$ \$8317.84 (8.25% yield)
$A = 6000\left(1+\frac{0.083}{2}\right)^{2(4)}$
$\approx$ \$ 8306.64 (8.3% yield)
Investing \$6000 for 4 years at 8.25% compounded quarterly yields the greater return.

57.

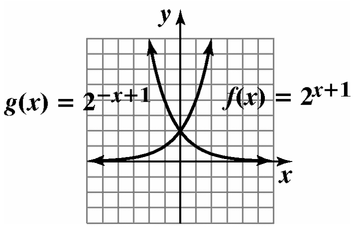
x	$f(x) = 2^x$	$g(x) = 2^{-x}$
-2	$\frac{1}{4}$	4
-1	$\frac{1}{2}$	2
0	1	1
1	2	$\frac{1}{2}$
2	4	$\frac{1}{4}$



The point of intersection is $(0,1)$.

58.

x	$f(x)=2^{x+1}$	$g(x)=2^{-x+1}$
−2	$\frac{1}{2}$	8
−1	1	4
0	2	2
1	4	1
2	8	$\frac{1}{2}$

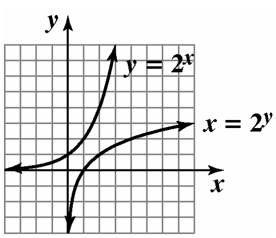


The point of intersection is $(0,2)$.

59.

x	$y=2^x$
−2	$\frac{1}{4}$
−1	$\frac{1}{2}$
0	1
1	2
2	4

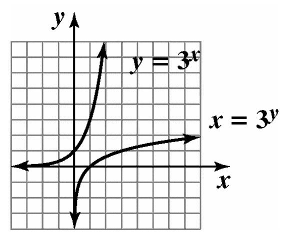
y	$x=2^y$
−2	$\frac{1}{4}$
−1	$\frac{1}{2}$
0	1
1	2
2	4



60.

x	$y=3^x$
−2	$\frac{1}{9}$
−1	$\frac{1}{3}$
0	1
1	3
2	9

y	$x=3^y$
−2	$\frac{1}{9}$
−1	$\frac{1}{3}$
0	1
1	3
2	9



61. The graph is of the form $y=b^x$.
Substitute values from the point (1, 4) to find b.

$y=b^x$

$4=b^1$

$4=b$

The equation of the graph is $y=4^x$

62. The graph is of the form $y=b^x$.
Substitute values from the point (1, 6) to find b.

$y=b^x$

$6=b^1$

$6=b$

The equation of the graph is $y=6^x$

63. The graph is of the form $y=-b^x$.
Substitute values from the point (1, –*e*) to find b.

$y=-b^x$

$-e=-b^1$

$e=b$

The equation of the graph is $y=-e^x$

64. The graph is of the form $y = b^x$.
Substitute values from the point (–1, *e*) to find b.

$$y = b^x$$
$$e = b^{-1}$$
$$e = \frac{1}{b}$$
$$eb = 1$$
$$b = \frac{1}{e}$$

The equation of the graph is $y = \left(\frac{1}{e}\right)^x = e^{-x}$

65. **a.** $f(0) = 574(1.026)^0$
$= 574(1) = 574$
India's population in 1974 was 574 million.

b. $f(27) = 574(1.026)^{27} \approx 1148$
India's population in 2001 will be 1148 million.

c. Since $2028 - 1974 = 54$, find
$f(54) = 574(1.026)^{54} \approx 2295$.
India's population in 2028 will be 2295 million.

d. $2055 - 1974 = 81$, find
$f(54) = 574(1.026)^{81} \approx 4590$.
India's population in 2055 will be 4590 million.

e. India's population appears to be doubling every 27 years.

66. $f(80) = 1000(0.5)^{\frac{80}{30}} = 157.49$
Chernobyl will not be safe for human habitation by 2066. There will still be 157.5 kilograms of cesium-137 in Chernobyl's atmosphere.

67. $S = 465,000(1 + 0.06)^{10}$
$= 465,000(1.06)^{10} \approx \$832,744$

68. $S = 510,000(1 + 0.03)^5$
$= 510,000(1.03)^5$
$\approx \$591,230$

69. $2^{1.7} \approx 3.249009585$
$2^{1.73} \approx 3.317278183$
$2^{1.732} \approx 3.321880096$
$2^{1.73205} \approx 3.321995226$
$2^{1.7320508} \approx 3.321997068$
$2^{\sqrt{3}} \approx 3.321997085$
The closer the exponent is to $\sqrt{3}$, the closer the value is to $2^{\sqrt{3}}$.

70. $2^3 \approx 8$
$2^{3.1} \approx 8.5741877$
$2^{3.14} \approx 8.815240927$
$2^{3.141} \approx 8.821353305$
$2^{3.1415} \approx 8.824411082$
$2^{3.14159} \approx 8.824961595$
$2^{3.141593} \approx 8.824979946$
$2^{\pi} \approx 8.824977827$
The closer the exponent gets to π, the closer the value is to 2^{π}.

71. **a.** 2005 is 50 years after 1955.
$f(x) = 0.15x + 1.44$
$f(50) = 0.15(50) + 1.44 \approx 8.9$
According to the linear model, there were about 8.9 million words in the federal tax code in 2005.

b. 2005 is 50 years after 1955.
$g(x) = 1.87e^{0.0344x}$
$g(50) = 1.87e^{0.0344(50)} \approx 10.4$
According to the exponential model, there were about 10.4 million words in the federal tax code in 2005.

c. The linear model is the better model for the data in 2005.

72. **a.** 1975 is 20 years after 1955.

$f(x) = 0.15x + 1.44$

$f(20) = 0.15(20) + 1.44 \approx 4.4$

According to the linear model, there were about 4.4 million words in the federal tax code in 1975.

b. 1975 is 20 years after 1955.

$g(x) = 1.87e^{0.0344x}$

$g(20) = 1.87e^{0.0344(20)} \approx 3.7$

According to the exponential model, there were about 3.7 million words in the federal tax code in 1975.

c. The exponential model is the better model for the data in 1975.

73. **a.** $f(0) = 80e^{-0.5(0)} + 20$

$= 80e^{0} + 20$

$= 80(1) + 20$

$= 100$

100% of the material is remembered at the moment it is first learned.

b. $f(1) = 80e^{-0.5(1)} + 20 \approx 68.5$

68.5% of the material is remembered 1 week after it is first learned.

c. $f(4) = 80e^{-0.5(4)} + 20 \approx 30.8$

30.8% of the material is remembered 4 week after it is first learned.

d. $f(52) = 80e^{-0.5(52)} + 20 \approx 20$

20% of the material is remembered 1 year after it is first learned.

74. **a.** $24\left(1+\frac{0.05}{12}\right)^{12(379)} \approx \$3,917,360,753$

b. $24e^{0.05(379)} \approx \$4,074,662,794$

75. $f(x) = 6.19(1.029)^{x}$

$f(56) = 6.19(1.029)^{56} \approx 30.7$

$g(x) = \frac{37.3}{1+6.1e^{-0.052x}}$

$g(56) = \frac{37.3}{1+6.1e^{-0.052(56)}} \approx 27.9$

Function $g(x)$ is a better model for the graph's value of 28.0 in 2006.

76. $f(x) = 6.19(1.029)^{x}$

$f(40) = 6.19(1.029)^{40} \approx 19.4$

$g(x) = \frac{37.3}{1+6.1e^{-0.052x}}$

$g(40) = \frac{37.3}{1+6.1e^{-0.052(40)}} \approx 21.1$

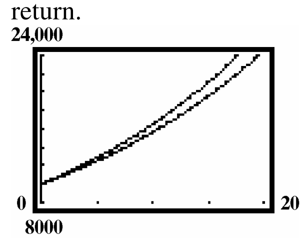
Function $g(x)$ is a better model for the graph's value of 21.3 in 1990.

77. – 80. Answers may vary.

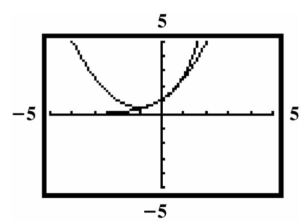
81. **a.** $A = 10,000\left(1+\frac{0.05}{4}\right)^{4t}$

$A = 10,000\left(1+\frac{0.045}{12}\right)^{12t}$

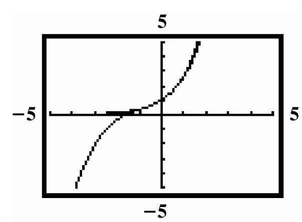
b. 5% compounded quarterly offers the better return.



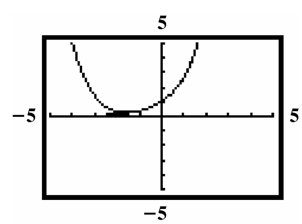
82. **a.**



b.



c.



d. Answers may vary.

83. does not make sense; Explanations will vary. Sample explanation: The horizontal asymptote is $y = 0$.

84. makes sense

85. does not make sense; Explanations will vary. Sample explanation: An exponential model is better than a linear model.

86. makes sense

87. false; Changes to make the statement true will vary. A sample change is: The amount of money will not increase without bound.

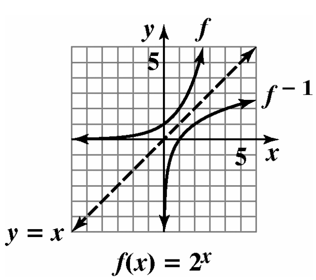
88. false; Changes to make the statement true will vary. A sample change is: The functions do not have the same graph. $f(x) = 3^{-x}$ reflects the graph of $y = 3^x$ about the y-axis while $f(x) = -3^x$ reflects the graph of $y = 3^x$ about the x-axis.

89. false; Changes to make the statement true will vary. A sample change is: If $f(x) = 2^x$ then $f(a+b) = f(a) \cdot f(b)$.

90. true

91. $y = 3^x$ is (d). y increases as x increases, but not as quickly as $y = 5^x$. $y = 5^x$ is (c). $y = \left(\frac{1}{3}\right)^x$ is (a). $y = \left(\frac{1}{3}\right)^x$ is the same as $y = 3^{-x}$, so it is (d) reflected about the y-axis. $y = \left(\frac{1}{5}\right)^x$ is (b). $y = \left(\frac{1}{5}\right)^x$ is the same as $y = 5^{-x}$, so it is (c) reflected about the y-axis.

92.



93. a.

$$\cosh(-x) = \frac{e^{-x} + e^{-(-x)}}{2} = \frac{e^{-x} + e^{x}}{2} = \frac{e^{x} + e^{-x}}{2} = \cosh x$$

b.

$$\sinh(-x) = \frac{e^{-x} - e^{-(-x)}}{2} = \frac{e^{-x} - e^{x}}{2} = \frac{-\left(-e^{-x} + e^{x}\right)}{2} = -\frac{e^{x} - e^{-x}}{2} = -\sinh x$$

c.

$$(\cosh x)^2 - (\sinh x)^2 \stackrel{?}{=} 1$$

$$\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 \stackrel{?}{=} 1$$

$$\frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4} \stackrel{?}{=} 1$$

$$\frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{4} \stackrel{?}{=} 1$$

$$\frac{4}{4} \stackrel{?}{=} 1$$

$$1 = 1$$

94. We do not know how to solve $x = 2^y$ for y.

95. $\frac{1}{2}$; i.e. $25^{1/2} = 5$

96. $(x-3)^2 > 0$

Solving the related equation, $(x-3)^2 > 0$, gives $x = 3$.

Note that the boundary value $x = 3$ does not satisfy the inequality.

Testing each interval gives a solution set of $(-\infty, 3) \cup (3, \infty)$.

Section 4.2

Check Point Exercises

1. a. $3 = \log_7 x$ means $7^3 = x$.

b. $2 = \log_b 25$ means $b^2 = 25$.

c. $\log_4 26 = y$ means $4^y = 26$.

2. a. $2^5 = x$ means $5 = \log_2 x$.

b. $b^3 = 27$ means $3 = \log_b 27$.

c. $e^y = 33$ means $y = \log_e 33$.

3. **a.** Question: 10 to what power gives 100?
$\log_{10} 100 = 2$ because $10^2 = 100$.

b. Question: 5 to what power gives $\frac{1}{125}$?
$\log_5 \frac{1}{125} = -3$ because $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$.

c. Question: 36 to what power gives 6?
$\log_{36} 6 = \frac{1}{2}$ because $36^{1/2} = \sqrt{36} = 6$

d. Question: 3 to what power gives $\sqrt[7]{3}$?
$\log_3 \sqrt[7]{3} = \frac{1}{7}$ because $3^{1/7} = \sqrt[7]{3}$.

4. **a.** Because $\log_b b = 1$, we conclude $\log_9 9 = 1$.

b. Because $\log_b 1 = 0$, we conclude $\log_8 1 = 0$.

5. **a.** Because $\log_b b^x = x$, we conclude $\log_7 7^8 = 8$.

b. Because $b^{\log_b x} = x$, we conclude $3^{\log_3 17} = 17$.

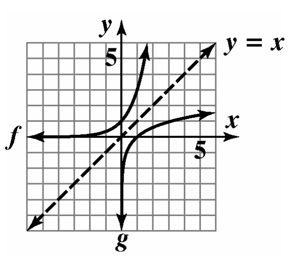
6. First, set up a table of coordinates for $f(x) = 3^x$.

x	-2	-1	0	1	2	3
$f(x) = 3^x$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

Reversing these coordinates gives the coordinates for the inverse function $g(x) = \log_3 x$.

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27
$g(x) = \log_3 x$	-2	-1	0	1	2	3

The graph of the inverse can also be drawn by reflecting the graph of $f(x) = 3^x$ about the line $y = x$.



f*(*x*) = 3*^x
g*(*x*) = log₃ *x

7. The domain of h consists of all x for which $x - 5 > 0$. Solving this inequality for x, we obtain $x > 5$. Thus, the domain of h is $(5, \infty)$.

8. Substitute the boy's age, 10, for x and evaluate the function at 10.
$f(10) = 29 + 48.8 \log(10 + 1)$
$= 29 + 48.8 \log(11)$
≈ 80
Thus, a 10-year-old boy is approximately 80% of his adult height.

9. Because $I = 10{,}000\, I_0$,
$R = \log \frac{10{,}000 I_0}{I_0}$
$= \log 10{,}000$
$= 4$
The earthquake registered 4.0 on the Richter scale.

10. **a.** The domain of f consists of all x for which $4 - x > 0$. Solving this inequality for x, we obtain $x < 4$. Thus, the domain of f is $(-\infty, 4)$

b. The domain of g consists of all x for which $x^2 > 0$. Solving this inequality for x, we obtain $x < 0$ or $x > 0$. Thus the domain of g is $(-\infty, 0) \cup (0, \infty)$.

11. Find the temperature increase after 30 minutes by substituting 30 for x and evaluating the function at 30.
$f(x) = 13.4 \ln x - 11.6$
$f(30) = 13.4 \ln 30 - 11.6$
≈ 34
The function models the actual increase shown in the graph quite well.

Exercise Set 4.2

1. $2^4 = 16$

2. $2^6 = 64$

3. $3^2 = x$

4. $9^2 = x$

5. $b^5 = 32$

6. $b^3 = 27$

7. $6^y = 216$

8. $5^y = 125$

9. $\log_2 8 = 3$

10. $\log_5 625 = 4$

11. $\log_2 \frac{1}{16} = -4$

12. $\log_5 \frac{1}{125} = -3$

13. $\log_8 2 = \frac{1}{3}$

14. $\log_{64} 4 = \frac{1}{3}$

15. $\log_{13} x = 2$

16. $\log_{15} x = 2$

17. $\log_b 1000 = 3$

18. $\log_b 343 = 3$

19. $\log_7 200 = y$

20. $\log_8 300 = y$

21. $\log_4 16 = 2$ because $4^2 = 16$.

22. $\log_7 49 = 2$ because $7^2 = 49$.

23. $\log_2 64 = 6$ because $2^6 = 64$.

24. $\log_3 27 = 3$ because $3^3 = 27$.

25. $\log_5 \frac{1}{5} = -1$ because $5^{-1} = \frac{1}{5}$.

26. $\log_6 \frac{1}{6} = -1$ because $6^{-1} = \frac{1}{6}$.

27. $\log_2 \frac{1}{8} = -3$ because $2^{-3} = \frac{1}{8}$.

28. $\log_3 \frac{1}{9} = -2$ because $3^{-2} = \frac{1}{9}$.

29. $\log_7 \sqrt{7} = \frac{1}{2}$ because $7^{\frac{1}{2}} = \sqrt{7}$.

30. $\log_6 \sqrt{6} = \frac{1}{2}$ because $6^{\frac{1}{2}} = \sqrt{6}$.

31. $\log_2 \frac{1}{\sqrt{2}} = -\frac{1}{2}$ because $2^{-\frac{1}{2}} = \frac{1}{\sqrt{2}}$.

32. $\log_3 \frac{1}{\sqrt{3}} = -\frac{1}{2}$ because $3^{-\frac{1}{2}} = \frac{1}{\sqrt{3}}$.

33. $\log_{64} 8 = \frac{1}{2}$ because $64^{1/2} = \sqrt{64} = 8$.

34. $\log_{81} 9 = \frac{1}{2}$ because $81^{1/2} = \sqrt{81} = 9$.

35. Because $\log_b b = 1$, we conclude $\log_5 5 = 1$.

36. Because $\log_b b = 1$, we conclude $\log_{11} 11 = 1$.

37. Because $\log_b 1 = 0$, we conclude $\log_4 1 = 0$.

38. Because $\log_b 1 = 0$, we conclude $\log_6 1 = 0$.

39. Because $\log_b b^x = x$, we conclude $\log_5 5^7 = 7$.

40. Because $\log_b b^x = x$, we conclude $\log_4 4^6 = 6$.

41. Because $b^{\log_b x} = x$, we conclude $8^{\log_8 19} = 19$.

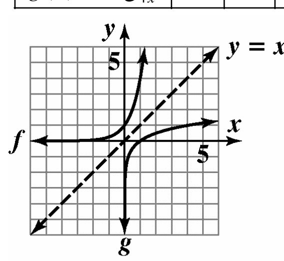
42. Because $b^{\log_b x} = x$, we conclude $7^{\log_7 23} = 23$.

43. First, set up a table of coordinates for $f(x) = 4^x$.

x	-2	-1	0	1	2	3
$f(x) = 4x$	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16	64

Reversing these coordinates gives the coordinates for the inverse function $g(x) = \log_4 x$.

x	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16	64
$g(x) = \log_{4x}$	-2	-1	0	1	2	3



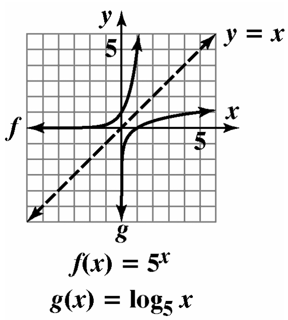
$f(x) = 4^x$
$g(x) = \log_4 x$

44. First, set up a table of coordinates for $f(x) = 5^x$.

x	-2	-1	0	1	2	3
$f(x) = 5^x$	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25	125

Reversing these coordinates gives the coordinates for the inverse function $g(x) = \log_5 x$.

x	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25	125
$g(x) = \log_5 x$	-2	-1	0	1	2	3

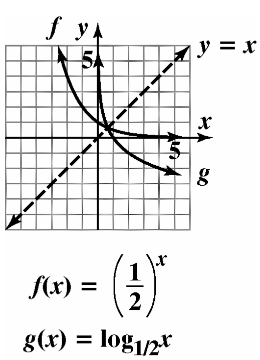


45. First, set up a table of coordinates for $f(x) = \left(\frac{1}{2}\right)^x$.

x	-2	-1	0	1	2	3
$f(x) = \left(\frac{1}{2}\right)^x$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

Reversing these coordinates gives the coordinates for the inverse function $g(x) = \log_{1/2} x$.

x	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
$g(x) = \log_{1/2} x$	-2	-1	0	1	2	3

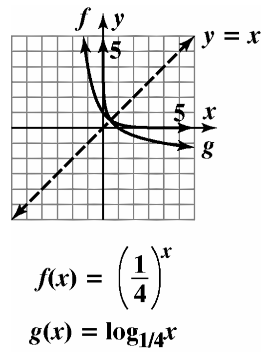


46. First, set up a table of coordinates for $f(x) = \left(\frac{1}{4}\right)^x$.

x	-2	-1	0	1	2	3
$f(x) = \left(\frac{1}{4}\right)^x$	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$

Reversing these coordinates gives the coordinates for the inverse function $g(x) = \log_{1/4} x$.

x	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
$g(x) = \log_{1/4} x$	-2	-1	0	1	2	3



47. This is the graph of $f(x) = \log_3 x$ reflected about the x-axis and shifted up one unit, so the function is $H(x) = 1 - \log_3 x$.

48. This is the graph of $f(x) = \log_3 x$ reflected about the y-axis, so the function is $G(x) = \log_3(-x)$.

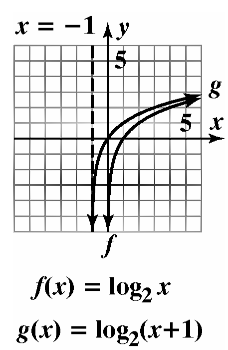
49. This is the graph of $f(x) = \log_3 x$ shifted down one unit, so the function is $h(x) = \log_3 x - 1$.

50. This is the graph of $f(x) = \log_3 x$ reflected about the x-axis, so the function is $F(x) = -\log_3 x$.

51. This is the graph of $f(x) = \log_3 x$ shifted right one unit, so the function is $g(x) = \log_3(x-1)$.

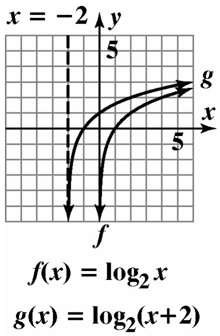
52. This is the graph of $f(x) = \log_3 x$.

53.



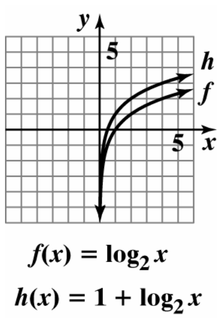
vertical asymptote: $x = -1$
domain: $(-1, \infty)$; range: $(-\infty, \infty)$

54.



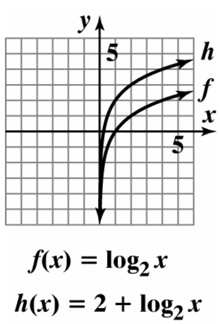
vertical asymptote: $x = -2$
domain: $(-2,\infty)$; range: $(-\infty,\infty)$

55.



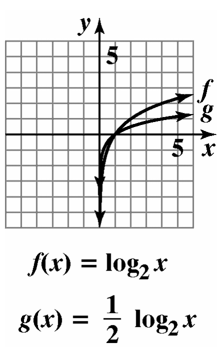
vertical asymptote: $x = 0$
domain: $(0,\infty)$; range: $(-\infty,\infty)$

56.



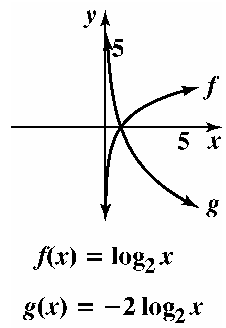
vertical asymptote: $x = 0$
domain: $(0,\infty)$; range: $(-\infty,\infty)$

57.



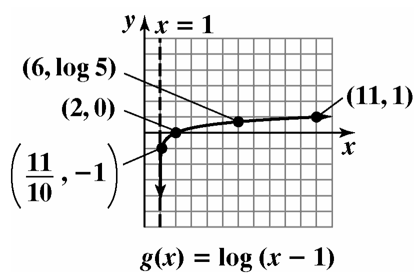
vertical asymptote: $x = 0$
domain: $(0,\infty)$; range: $(-\infty,\infty)$

58.



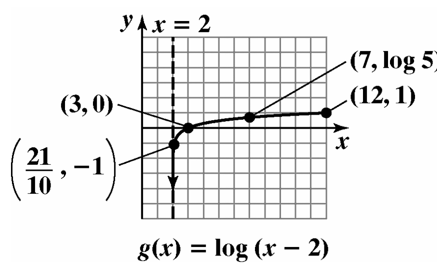
vertical asymptote: $x = 0$
domain: $(0,\infty)$; range: $(-\infty,\infty)$

59.



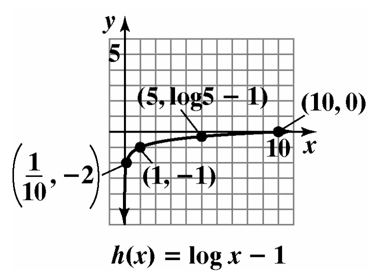
vertical asymptote: $x = 1$
domain: $(1,\infty)$; range: $(-\infty,\infty)$

60.



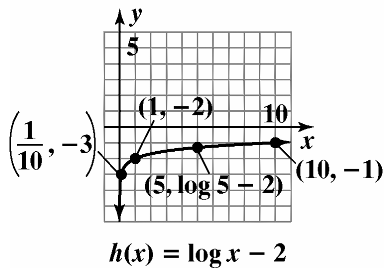
vertical asymptote: $x = 2$
domain: $(2,\infty)$; range: $(-\infty,\infty)$

61.



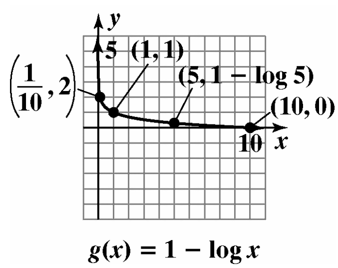
vertical asymptote: $x = 0$
domain: $(0,\infty)$; range: $(-\infty,\infty)$

62.



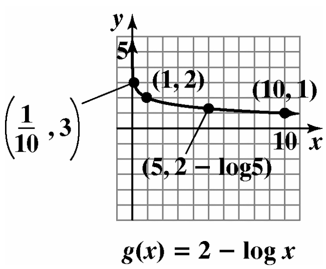
vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

63.



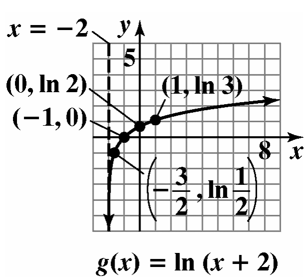
vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

64.



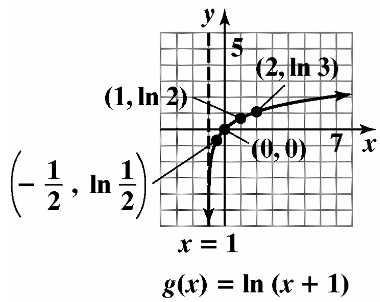
vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

65.



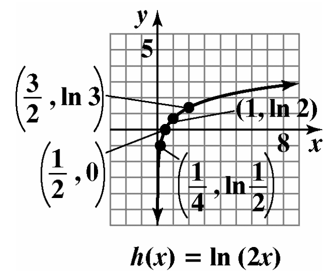
vertical asymptote: $x = -2$
domain: $(-2, \infty)$; range: $(-\infty, \infty)$

66.



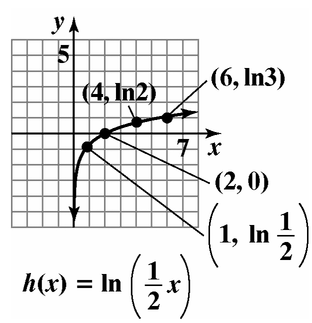
vertical asymptote: $x = -1$
domain: $(-1, \infty)$; range: $(-\infty, \infty)$

67.



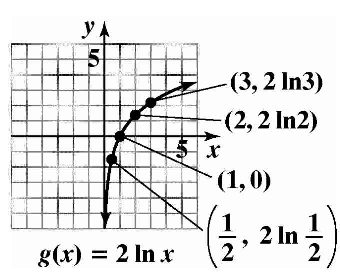
vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

68.



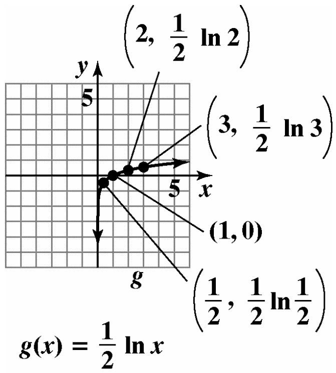
vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

69.



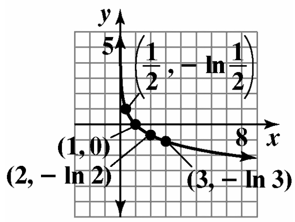
vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

70.



vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

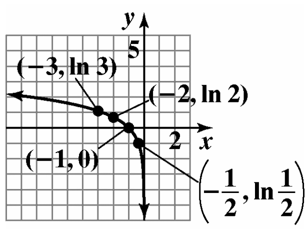
71.



$h(x) = -\ln x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

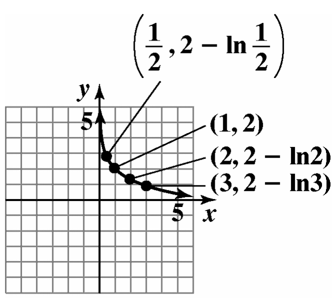
72.



$h(x) = \ln(-x)$

vertical asymptote: $x = 0$
domain: $(-\infty, 0)$; range: $(-\infty, \infty)$

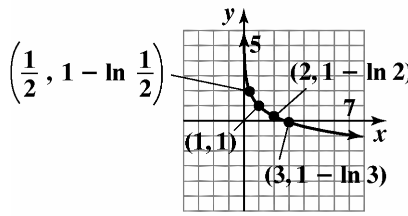
73.



$g(x) = 2 - \ln x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

74.



$g(x) = 1 - \ln x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

75. The domain of f consists of all x for which $x + 4 > 0$. Solving this inequality for x, we obtain $x > -4$. Thus, the domain of f is $(-4, \infty)$.

76. The domain of f consists of all x for which $x + 6 > 0$. Solving this inequality for x, we obtain $x > -6$. Thus, the domain of f is $(-6, \infty)$.

77. The domain of f consists of all x for which $2 - x > 0$. Solving this inequality for x, we obtain $x < 2$. Thus, the domain of f is $(-\infty, 2)$.

78. The domain of f consists of all x for which $7 - x > 0$. Solving this inequality for x, we obtain $x < 7$. Thus, the domain of f is $(-\infty, 7)$.

79. The domain of f consists of all x for which $(x - 2)^2 > 0$. Solving this inequality for x, we obtain $x < 2$ or $x > 2$. Thus, the domain of f is $(-\infty, 2)$ or $(2, \infty)$.

80. The domain of f consists of all x for which $(x - 7)^2 > 0$. Solving this inequality for x, we obtain $x < 7$ or $x > 7$. Thus, the domain of f is $(-\infty, 7)$ or $(7, \infty)$.

81. $\log 100 = \log_{10} 100 = 2$ because $10^2 = 100$.

82. $\log 1000 = \log_{10} 1000 = 3$ because $10^3 = 1000$.

83. Because $\log 10^x = x$, we conclude $\log 10^7 = 7$.

84. Because $\log 10^x = x$, we conclude $\log 10^8 = 8$.

85. Because $10^{\log x} = x$, we conclude $10^{\log 33} = 33$.

86. Because $10^{\log x} = x$, we conclude $10^{\log 53} = 53$.

87. $\ln 1 = 0$ because $e^0 = 1$.

88. $\ln\ e = \log_e e = 1$ because $e^1 = e$.

89. Because $\ln e^x = x$, we conclude $\ln e^6 = 6$.

90. Because $\ln\ e^x = x$, we conclude $\ln\ e^7 = 7$.

91. $\ln \frac{1}{e^6} = \ln e^{-6}$

Because $\ln e^x = x$ we conclude

$\ln e^{-6} = -6$, so $\ln \frac{1}{e^6} = -6$.

92. $\ln\ \frac{1}{e^7} = \ln\ e^{-7}$ Because $\ln\ e^x = x$, we conclude

$\ln\ e^{-7} = -7$, so $\ln\ \frac{1}{e^7} = -7$.

93. Because $e^{\ln x} = x$, we conclude $e^{\ln 125} = 125$.

94. Because $e^{\ln x} = x$, we conclude $e^{\ln 300} = 300$.

95. Because $\ln e^x = x$, we conclude $\ln e^{9x} = 9x$.

96. Because $\ln\ e^x = x$, we conclude $\ln\ e^{13x} = 13x$.

97. Because $e^{\ln x} = x$, we conclude $e^{\ln 5x^2} = 5x^2$.

98. Because $e^{\ln x} = x$, we conclude $e^{\ln 7x^2} = 7x^2$.

99. Because $10^{\log x} = x$, we conclude $10^{\log\sqrt{x}} = \sqrt{x}$.

100. Because $10^{\log x} = x$, we conclude $10^{\log\sqrt[3]{x}} = \sqrt[3]{x}$.

101. $\log_3(x-1) = 2$

$3^2 = x-1$

$9 = x-1$

$10 = x$

The solution is 10, and the solution set is {10}.

102. $\log_5(x+4) = 2$

$5^2 = x+4$

$25 = x+4$

$21 = x$

The solution is 21, and the solution set is {21}.

103. $\log_4 x = -3$

$4^{-3} = x$

$x = \frac{1}{4^3} = \frac{1}{64}$

The solution is $\frac{1}{64}$, and the solution set is $\left\{\frac{1}{64}\right\}$.

104. $\log_{64} x = \frac{2}{3}$

$64^{\frac{2}{3}} = x$

$x = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$

The solution is 16, and the solution set is {16}.

105. $\log_3(\log_7 7) = \log_3 1 = 0$

106. $\log_5(\log_2 32) = \log_5\left(\log_2 2^5\right) = \log_5 5 = 1$

107. $\log_2(\log_3 81) = \log_2\left(\log_3 3^4\right)$

$= \log_2 4 = \log_2 2^2 = 2$

108. $\log(\ln e) = \log 1 = 0$

109. For $f(x) = \ln(x^2 - x - 2)$ to be real, $x^2 - x - 2 > 0$.
Solve the related equation to find the boundary points:

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

The boundary points are –1 and 2. Testing each interval gives a domain of $(-\infty,-1)\cup(2,\infty)$.

110. For $f(x) = \ln(x^2 - 4x - 12)$ to be real,
$x^2 - 4x - 12 > 0$.
Solve the related equation to find the boundary points:

$x^2 - 4x - 12 = 0$

$(x+2)(x-6) = 0$

The boundary points are –2 and 6. Testing each interval gives a domain of $(-\infty,-2)\cup(6,\infty)$.

111. For $f(x) = \ln\left(\frac{x+1}{x-5}\right)$ to be real, $\frac{x+1}{x-5} > 0$.
The boundary points are –1 and 5. Testing each interval gives a domain of $(-\infty,-1)\cup(5,\infty)$.

112. For $f(x)=\ln\left(\frac{x-2}{x+5}\right)$ to be real, $\frac{x-2}{x+5}>0$.

The boundary points are –5 and 2. Testing each interval gives a domain of $(-\infty,-5)\cup(2,\infty)$.

113. $f(13) = 62 + 35\log(13–4) \approx 95.4$

She is approximately 95.4% of her adult height.

114. $f(10)=62+35\log(10-4)\approx 89.2$.

She is approximately 89.2% of her adult height.

115. a. 2004 is 35 years after 1969.

$f(x)=-7.49\ln x+53$

$f(35)=-7.49\ln 35+53\approx 26.4$

According to the function, 26.4% of first-year college men expressed antifeminist views in 2004. This underestimates the value in the graph by 1%.

b. 2010 is 41 years after 1969.

$f(x)=-7.49\ln x+53$

$f(41)=-7.49\ln 41+53\approx 25.2$

According to the function, 25.2% of first-year college men will express antifeminist views in 2010.

116. a. 2004 is 35 years after 1969.

$f(x)=-4.86\ln x+32.5$

$f(35)=-4.86\ln 35+32.5\approx 15.2$

According to the function, 15.2% of first-year college women expressed antifeminist views in 2004. This underestimates the value in the graph by 0.6%.

b. 2010 is 41 years after 1969.

$f(x)=-4.86\ln x+32.5$

$f(41)=-4.86\ln 41+32.5\approx 14.5$

According to the function, 14.5% of first-year college women will express antifeminist views in 2010.

117. $D = 10\log\left[10^{12}(6.3\times10^{6})\right] \approx 188$

Yes, the sound can rupture the human eardrum.

118. $D=10\log\left[10^{12}\left(3.2\times10^{-6}\right)\right]\approx 65.05$

A normal conversation is about 65 decibels.

119. a. $f(0) = 88–15\ln(0 + 1) = 88$

The average score on the original exam was 88.

b. $f(2) = 88–15\ln(2 + 1) = 71.5$

$f(4) = 88–15\ln(4 + 1) = 63.9$

$f(6) = 88–15\ln(6 + 1) = 58.8$

$f(8) = 88–15\ln(8 + 1) = 55$

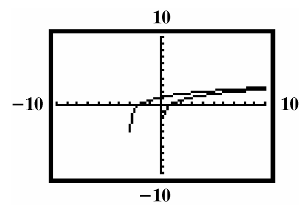
$f(10) = 88–15\ln(10 + 1) = 52$

$f(12) = 88–15\ln(12 + 1) = 49.5$

The average score after 2 months was about 71.5, after 4 months was about 63.9, after 6 months was about 58.8, after 8 months was about 55, after 10 months was about 52, and after one year was about 49.5.

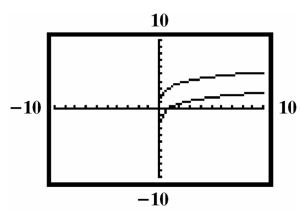
120. – 127. Answers may vary.

128.



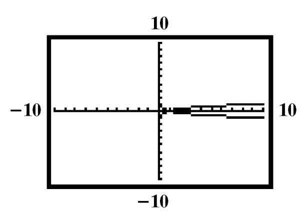
$g(x)$ is $f(x)$ shifted 3 units left.

129.



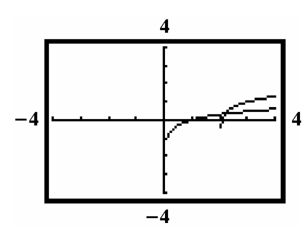
$g(x)$ is $f(x)$ shifted 3 units upward.

130.



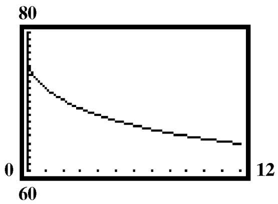
$g(x)$ is $f(x)$ reflected about the x-axis.

131.



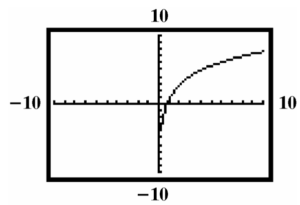
$g(x)$ is $f(x)$ shifted right 2 units and upward 1 unit.

132.

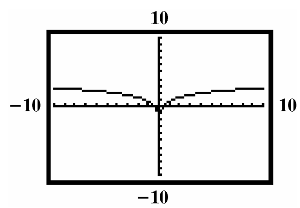


The score falls below 65 after 9 months.

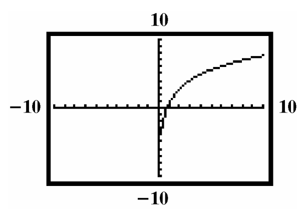
133. a.



b.



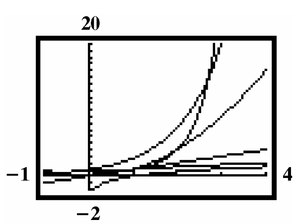
c.



d They are the same.
$\log_b MN = \log_b M + \log_b N$

e. The sum of the logarithms of its factors.

134.



$y = \ln x$, $y = \sqrt{x}$, $y = x$,
$y = x^2$, $y = e^x$, $y = x^x$

135. makes sense

136. makes sense

137. makes sense

138. does not make sense; Explanations will vary. Sample explanation: An earthquake of magnitude 8 on the Richter scale is $10^{8-4} = 10^4 = 10{,}000$ times as intense as an earthquake of magnitude 4.

139. false; Changes to make the statement true will vary. A sample change is: $\dfrac{\log_2 8}{\log_2 4} = \dfrac{3}{2}$

140. false; Changes to make the statement true will vary. A sample change is: We cannot take the log of a negative number.

141. false; Changes to make the statement true will vary. A sample change is: The domain of $f(x) = \log_2 x$ is $(0, \infty)$.

142. true

143. $\dfrac{\log_3 81 - \log_\pi 1}{\log_{2\sqrt{2}} 8 - \log 0.001} = \dfrac{4-0}{2-(-3)} = \dfrac{4}{5}$

144. $\log_4\left[\log_3\left(\log_2 8\right)\right]$
$= \log_4\left[\log_3\left(\log_2 2^3\right)\right]$
$= \log_4\left[\log_3 3\right] = \log_4 1 = 0$

145. $\log_4 60 < \log_4 64 = 3$ so $\log_4 60 < 3$.
$\log_3 40 > \log_3 27 = 3$ so $\log_3 40 > 3$.
$\log_4 60 < 3 < \log_3 40$
$\log_3 40 > \log_4 60$

146. Answers may vary.

147. a. $\log_2 32 = \log_2 2^5 = 5$

b. $\log_2 8 + \log_2 4 = \log_2 2^3 + \log_2 2^2 = 3 + 2 = 5$

c. $\log_2(8 \cdot 4) = \log_2 8 + \log_2 4$

148. a. $\log_2 16 = \log_2 2^4 = 4$

b. $\log_2 32 - \log_2 2 = \log_2 2^5 - \log_2 2 = 5 - 1 = 4$

c. $\log_2\left(\dfrac{32}{2}\right) = \log_2 32 - \log_2 2$

149. a. $\log_3 81 = \log_3 3^4 = 4$

b. $2\log_3 9 = 2\log_3 3^2 = 2 \cdot 2 = 4$

c. $\log_3 9^2 = 2\log_3 9$

Section 4.3

Check Point Exercises

1. a. $\log_6(7\cdot 11)=\log_6 7+\log_6 11$

b. $\log(100x)=\log 100+\log x$
$=2+\log x$

2. a. $\log_8\left(\frac{23}{x}\right)=\log_8 23-\log_8 x$

b. $\ln\left(\frac{e^5}{11}\right)=\ln e^5-\ln 11$
$=5-\ln 11$

3. a. $\log_6 3^9=9\log_6 3$

b. $\ln\sqrt[3]{x}=\ln x^{1/3}=\frac{1}{3}\ln x$

c. $\log(x+4)^2=2\log(x+4)$

4. a. $\log_b x^4\sqrt[3]{y}$
$=\log_b x^4 y^{1/3}$
$=\log_b x^4+\log_b y^{1/3}$
$=4\log_b x+\frac{1}{3}\log_b y$

b. $\log_5\frac{\sqrt{x}}{25y^3}$
$=\log_5\frac{x^{1/2}}{25y^3}$
$=\log_5 x^{1/2}-\log_5 25y^3$
$=\log_5 x^{1/2}-\left(\log_5 5^2+\log_5 y^3\right)$
$=\frac{1}{2}\log_5 x-\log_5 5^2-\log_5 y^3$
$=\frac{1}{2}\log_5 x-2\log_5 5-3\log_5 y$
$=\frac{1}{2}\log_5 x-2-3\log_5 y$

5. a. $\log 25+\log 4=\log(25\cdot 4)=\log 100=2$

b. $\log(7x+6)-\log x=\log\frac{7x+6}{x}$

6. a. $\ln x^2+\frac{1}{3}\ln(x+5)$
$=\ln x^2+\ln(x+5)^{1/3}$
$=\ln x^2(x+5)^{1/3}$
$=\ln x^2\sqrt[3]{x+5}$

b. $2\log(x-3)-\log x$
$=\log(x-3)^2-\log x$
$=\log\frac{(x-3)^2}{x}$

c. $\frac{1}{4}\log_b x-2\log_b 5-10\log_b y$
$=\log_b x^{1/4}-\log_b 5^2-\log_b y^{10}$
$=\log_b x^{1/4}-\left(\log_b 25-\log_b y^{10}\right)$
$=\log_b x^{1/4}-\log_b 25y^{10}$
$=\log_b\frac{x^{1/4}}{25y^{10}}$ or $\log_b\frac{\sqrt[4]{x}}{25y^{10}}$

7. $\log_7 2506=\frac{\log 2506}{\log 7}\approx 4.02$

8. $\log_7 2506=\frac{\ln 2506}{\ln 7}\approx 4.02$

Exercise Set 4.3

1. $\log_5(7\cdot 3)=\log_5 7+\log_5 3$

2. $\log_8(13\cdot 7)=\log_8 13+\log_8 7$

3. $\log_7(7x)=\log_7 7+\log_7 x=1+\log_7 x$

4. $\log_9 9x=\log_9 9+\log_9 x=1+\log_9 x$

5. $\log(1000x)=\log 1000+\log x=3+\log x$

6. $\log(10,000x)=\log 10,000+\log x=4+\log x$

7. $\log_7\left(\frac{7}{x}\right)=\log_7 7-\log_7 x=1-\log_7 x$

8. $\log_9\left(\frac{9}{x}\right)=\log_9 9-\log_9 x=1-\log_9 x$

9. $\log\left(\frac{x}{100}\right)=\log x-\log 100=\log_x-2$

10. $\log\left(\frac{x}{1000}\right) = \log x - \log 1000 = \log x - 3$

11. $\log_4\left(\frac{64}{y}\right) = \log_4 64 - \log_4 y$

$= 3 - \log_4 y$

12. $\log_5\left(\frac{125}{y}\right) = \log_5 125 - \log_5 y = 3 - \log_5 y$

13. $\ln\left(\frac{e^2}{5}\right) = \ln e^2 - \ln 5 = 2\ln e - \ln 5 = 2 - \ln 5$

14. $\ln\left(\frac{e^4}{8}\right) = \ln e^4 - \ln 8 = 4\ln e - \ln 8 = 4 - \ln 8$

15. $\log_b x^3 = 3\log_b x$

16. $\log_b x^7 = 7\log_b x$

17. $\log N^{-6} = -6\log N$

18. $\log M^{-8} = -8\log M$

19. $\ln \sqrt[5]{x} = \ln x^{(1/5)} = \frac{1}{5}\ln x$

20. $\ln \sqrt[7]{x} = \ln x^{\frac{1}{7}} = \frac{1}{7}\ln x$

21. $\log_b x^2 y = \log_b x^2 + \log_b y = 2\log_b x + \log_b y$

22. $\log_b xy^3 = \log_b x + \log_b y^3 = \log_b x + 3\log_b y$

23. $\log_4\left(\frac{\sqrt{x}}{64}\right) = \log_4 x^{1/2} - \log_4 64 = \frac{1}{2}\log_4 x - 3$

24. $\log_5\left(\frac{\sqrt{x}}{25}\right) = \log_5 x^{1/2} - \log_5 25 = \frac{1}{2}\log_5 x - 2$

25. $\log_6\left(\frac{36}{\sqrt{x+1}}\right) = \log_6 36 - \log_6 (x+1)^{1/2}$

$= 2 - \frac{1}{2}\log_6 (x+1)$

26. $\log_8\left(\frac{64}{\sqrt{x+1}}\right) = \log_8 64 - \log_8 (x+1)^{1/2}$

$= 2 - \frac{1}{2}\log_8 (x+1)$

27. $\log_b\left(\frac{x^2 y}{z^2}\right) = \log_b\left(x^2 y\right) - \log_b z^2$

$= \log_b x^2 + \log_b y - \log_b z^2$

$= 2\log_b x + \log_b y - 2\log_b z$

28. $\log_b\left(\frac{x^3 y}{z^2}\right) = \log_b\left(x^3 y\right) - \log_b z^2$

$\log_b\left(\frac{x^3 y}{z^2}\right) = \log_b x^3 + \log_b y - \log_b z^2$

$= 3\log_b x + \log_b y - 2\log_b z$

29. $\log\sqrt{100x} = \log(100x)^{1/2}$

$= \frac{1}{2}\log(100x)$

$= \frac{1}{2}(\log 100 + \log x)$

$= \frac{1}{2}(2 + \log x)$

$= 1 + \frac{1}{2}\log x$

30. $\ln\sqrt{ex} = \ln(ex)^{1/2}$

$= \frac{1}{2}\ln(ex)$

$= \frac{1}{2}(\ln e + \ln x)$

$= \frac{1}{2}(1 + \ln x)$

$= \frac{1}{2} + \frac{1}{2}\ln x$

31. $\log\sqrt[3]{\frac{x}{y}} = \log\left(\frac{x}{y}\right)^{1/3}$

$= \frac{1}{3}\left[\log\left(\frac{x}{y}\right)\right]$

$= \frac{1}{3}(\log x - \log y)$

$= \frac{1}{3}\log x - \frac{1}{3}\log y$

32. $\log \sqrt[5]{\frac{x}{y}} = \log\left(\frac{x}{y}\right)^{1/5}$

$= \frac{1}{5}\left[\log\left(\frac{x}{y}\right)\right]$

$= \frac{1}{5}(\log x - \log y)$

$= \frac{1}{5}\log x - \frac{1}{5}\log y$

33. $\log_b \frac{\sqrt{x}y^3}{z^3}$

$= \log_b x^{1/2} + \log_b y^3 - \log_b z^3$

$= \frac{1}{2}\log_b x + 3\log_b y - 3\log_b z$

34. $\log_b \frac{\sqrt[3]{x}y^4}{z^5}$

$= \log_b x^{1/3} + \log_b y^4 - \log_b z^5$

$= \frac{1}{3}\log_b x + 4\log_b y - 5\log_b z$

35. $\log_5 \sqrt[3]{\frac{x^2 y}{25}}$

$= \log_5 x^{2/3} + \log_5 y^{1/3} - \log_5 25^{1/3}$

$= \frac{2}{3}\log_5 x + \frac{1}{3}\log_5 y - \log_5 5^{2/3}$

$= \frac{2}{3}\log_5 x + \frac{1}{3}\log_5 y - \frac{2}{3}$

36. $\log_2 \sqrt[5]{\frac{xy^4}{16}}$

$= \log_2 x^{1/5} + \log_2 y^{4/5} - \log_2 16^{1/5}$

$= \frac{1}{5}\log_2 x + \frac{4}{5}\log_2 y - \frac{1}{5}\log_2 16$

$= \frac{1}{5}\log_2 x + \frac{4}{5}\log_2 y - \frac{4}{5}$

37. $\ln\left[\frac{x^3\sqrt{x^2+1}}{(x+1)^4}\right]$

$= \ln x^3 + \ln\sqrt{x^2+1} - \ln(x+1)^4$

$= 3\ln x + \frac{1}{2}\ln(x^2+1) - 4\ln(x+1)$

38. $\ln\left[\frac{x^4\sqrt{x^2+3}}{(x+3)^5}\right]$

$= \ln\left[\frac{x^4(x^2+3)^{1/2}}{(x+3)^5}\right]$

$= \ln x^4 + \ln\left(x^2+3\right)^{1/2} - \ln\left(x+3\right)^5$

$= 4\ln x + \frac{1}{2}\ln\left(x^2+3\right) - 5\ln\left(x+3\right)$

39. $\log\left[\frac{10x^2\sqrt[3]{1-x}}{7(x+1)^2}\right]$

$= \log 10 + \log x^2 + \log\sqrt[3]{1-x} - \log 7 - \log(x+1)^2$

$= 1 + 2\log x + \frac{1}{3}\log(1-x) - \log 7 - 2\log(x+1)$

40. $\log\left[\frac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right]$

$= \log 100 + \log x^3 + \log\left(5-x\right)^{\frac{1}{3}} - \log 3 - \log\left(x+7\right)^2$

$= 2 + 3\log x + \frac{1}{3}\log(5-x) - \log 3 - 2\log(x+7)$

41. $\log 5 + \log 2 = \log(5\cdot 2) = \log 10 = 1$

42. $\log 250 + \log 4 = \log 1000 = 3$

43. $\ln x + \ln 7 = \ln(7x)$

44. $\ln x + \ln 3 = \ln(3x)$

45. $\log_2 96 - \log_2 3 = \log_2\left(\frac{96}{3}\right) = \log_2 32 = 5$

46. $\log_3 405 - \log_3 5 = \log_3\left(\frac{405}{5}\right)$

$= \log_3 81$

$= 4$

47. $\log(2x+5) - \log x = \log\left(\frac{2x+5}{x}\right)$

48. $\log(3x+7) - \log x = \log\left(\frac{3x+7}{x}\right)$

49. $\log x + 3\log y = \log x + \log y^3 = \log(xy^3)$

50. $\log x + 7\log y = \log x + \log y^7 = \log(xy^7)$

51. $\frac{1}{2}\ln x + \ln y = \ln x^{1/2} + \ln y$

$= \ln\left(x^{\frac{1}{2}}y\right)$ or $\ln\left(y\sqrt{x}\right)$

52. $\frac{1}{3}\ln x + \ln y = \ln x^{1/3} + \ln y$

$= \ln\left(x^{1/3}y\right)$ or $\ln\left(y\sqrt[3]{x}\right)$

53. $2\log_b x + 3\log_b y = \log_b x^2 + \log_b y^3$

$= \log_b(x^2y^3)$

54. $5\log_b x + 6\log_b y = \log_b x^5 + \log_b y^6$

$= \log_b\left(x^5y^6\right)$

55. $5\ln x - 2\ln y = \ln x^5 - \ln y^2 = \ln\left(\frac{x^5}{y^2}\right)$

56. $7\ln x - 3\ln y = \ln x^7 - \ln y^3 = \ln\left(\frac{x^7}{y^3}\right)$

57. $3\ln x - \frac{1}{3}\ln y = \ln x^3 - \ln y^{1/3}$

$= \ln\left(\frac{x^3}{y^{1/3}}\right)$ or $\ln\left(\frac{x^3}{\sqrt[3]{y}}\right)$

58. $2\ln x - \frac{1}{2}\ln y = \ln x^2 - \ln y^{1/2}$

$= \ln\left(\frac{x^2}{y^{1/2}}\right)$ or $\ln\left(\frac{x^2}{\sqrt{y}}\right)$

59. $4\ln(x+6) - 3\ln x = \ln(x+6)^4 - \ln x^3$

$= \ln\frac{(x+6)^4}{x^3}$

60. $8\ln(x+9) - 4\ln x = \ln(x+9)^8 - \ln x^4$

$= \ln\frac{(x+9)^8}{x^4}$

61. $3\ln x + 5\ln y - 6\ln z$

$= \ln x^3 + \ln y^5 - \ln z^6$

$= \ln\frac{x^3y^5}{z^6}$

62. $4\ln x + 7\ln y - 3\ln z$

$= \ln x^4 + \ln y^7 - \ln z^3$

$= \ln\frac{x^4y^7}{z^3}$

63. $\frac{1}{2}\left(\log x + \log y\right)$

$= \frac{1}{2}(\log xy)$

$= \log(xy)^{1/2}$

$= \log\sqrt{xy}$

64. $\frac{1}{3}\left(\log_4 x - \log_4 y\right)$

$= \frac{1}{3}\log_4\frac{x}{y}$

$= \log_4\left(\frac{x}{y}\right)^{1/3}$

$= \log_4\sqrt[3]{\frac{x}{y}}$

65. $\frac{1}{2}(\log_5 x + \log_5 y) - 2\log_5(x+1)$

$= \frac{1}{2}\log_5 xy - \log_5(x+1)^2$

$= \log_5(xy)^{1/2} - \log_5(x+1)^2$

$= \log_5\frac{(xy)^{1/2}}{(x+1)^2}$

$= \log_5\frac{\sqrt{xy}}{(x+1)^2}$

66. $\frac{1}{3}\left(\log_4 x - \log_4 y\right) + 2\log_4(x+1)$

$= \frac{1}{3}\log_4\frac{x}{y} + \log_4(x+1)^2$

$= \log_4\left[\left(\frac{x}{y}\right)^{1/3}(x+1)^2\right]$

$= \log_4\left[(x+1)^2\sqrt[3]{\frac{x}{y}}\right]$

67. $\frac{1}{3}[2\ln(x+5)-\ln x-\ln(x^2-4)]$

$=\frac{1}{3}[\ln(x+5)^2-\ln x-\ln(x^2-4)]$

$=\frac{1}{3}\left[\ln\frac{(x+5)^2}{x(x^2-4)}\right]$

$=\ln\left[\frac{(x+5)^2}{x(x^2-4)}\right]^{1/3}$

$=\ln\sqrt[3]{\frac{(x+5)^2}{x(x^2-4)}}$

68. $\frac{1}{3}\left[5\ln(x+6)-\ln x-\ln(x^2-25)\right]$

$=\frac{1}{3}\ln\left[\frac{(x+6)^5}{x(x^2-25)}\right]$

$=\ln\left[\frac{(x+6)^5}{x(x^2-25)}\right]^{\frac{1}{3}}$

69. $\log x+\log\left(x^2-1\right)-\log 7-\log(x+1)$

$=\log x+\log\left(x^2-1\right)-\left(\log 7+\log(x+1)\right)$

$=\log\left(x\left(x^2-1\right)\right)-\log\left(7(x+1)\right)$

$=\log\frac{x\left(x^2-1\right)}{7(x+1)}$

$=\log\frac{x(x+1)(x-1)}{7(x+1)}$

$=\log\frac{x(x-1)}{7}$

70. $\log x+\log\left(x^2-4\right)-\log 15-\log(x+2)$

$=\log x+\log\left(x^2-4\right)-\left(\log 15+\log(x+2)\right)$

$=\log\left(x\left(x^2-4\right)\right)-\log\left(15(x+2)\right)$

$=\log\frac{x\left(x^2-4\right)}{15(x+2)}$

$=\log\frac{x(x+2)(x-2)}{15(x+2)}$

$=\log\frac{x(x-2)}{15}$

71. $\log_5 13=\frac{\log 13}{\log 5}\approx 1.5937$

72. $\log_6 17=\frac{\log 17}{\log 6}\approx 1.5812$

73. $\log_{14} 87.5=\frac{\ln 87.5}{\ln 14}\approx 1.6944$

74. $\log_{16} 57.2=\frac{\ln 57.2}{\ln 16}\approx 1.4595$

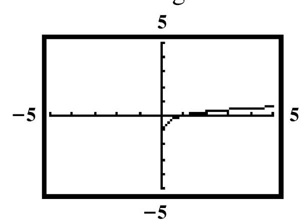
75. $\log_{0.1} 17=\frac{\log 17}{\log 0.1}\approx -1.2304$

76. $\log_{0.3} 19=\frac{\log 19}{\log 0.3}\approx -2.4456$

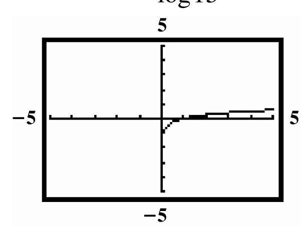
77. $\log_{\pi} 63=\frac{\ln 63}{\ln \pi}\approx 3.6193$

78. $\log_{\pi} 400=\frac{\ln 400}{\ln \pi}\approx 5.2340$

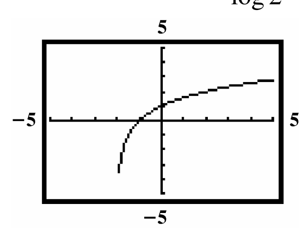
79. $y=\log_3 x=\frac{\log x}{\log 3}$



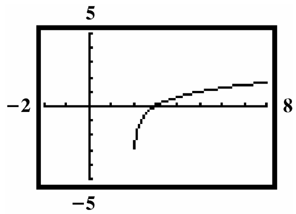
80. $y=\log_{15} x=\frac{\log x}{\log 15}$



81. $y=\log_2(x+2)=\frac{\log(x+2)}{\log 2}$



82. $y = \log_3(x-2) = \dfrac{\log(x-2)}{\log 3}$



83. $\log_b \dfrac{3}{2} = \log_b 3 - \log_b 2 = C - A$

84. $\log_b 6 = \log_b (2\cdot 3)$
$= \log_b 2 + \log_b 3 = A + C$

85. $\log_b 8 = \log_b 2^3 = 3\log_b 2 = 3A$

86. $\log_b 81 = \log_b 3^4 = 4\log_b 3 = 4C$

87.
$$\log_b \sqrt{\frac{2}{27}} = \log_b \left(\frac{2}{27}\right)^{\frac{1}{2}}$$
$$= \frac{1}{2}\log_b\left(\frac{2}{3^3}\right)$$
$$= \frac{1}{2}\left(\log_b 2 - \log_b 3^3\right)$$
$$= \frac{1}{2}\left(\log_b 2 - 3\log_b 3\right)$$
$$= \frac{1}{2}\log_b 2 - \frac{3}{2}\log_b 3$$
$$= \frac{1}{2}A - \frac{3}{2}C$$

88.
$$\log_b \sqrt{\frac{3}{16}} = \log_b\left(\frac{\sqrt{3}}{4}\right)$$
$$= \log_b \sqrt{3} - \log_b 4$$
$$= \log_b 3^{\frac{1}{2}} - \log 2^2$$
$$= \frac{1}{2}\log_b 3 - 2\log 2$$
$$= \frac{1}{2}C - 2A$$

89. false; $\ln e = 1$

90. false; $\ln e^e = 0$

91. false; $\log_4 (2x)^3 = 3\log_4(2x)$

92. true; $\ln(8x^3) = \ln(2^3x^3) = \ln(2x)^3 = 3\ln(2x)$

93. true; $x\log 10^x = x\cdot x = x^2$

94. false; $\ln(x\cdot 1) = \ln x + \ln 1$

95. true; $\ln(5x) + \ln 1 = \ln 5x + 0 = \ln 5x$

96. false; $\ln x + \ln(2x) = \ln(x\cdot 2x)) = \ln 2x^2$

97. false; $\log(x+3) - \log(2x) = \log\dfrac{x+3}{2x}$

98. false; $\log\dfrac{x+2}{x-1} = \log(x+2) - \log(x-1)$

99. true; quotient rule

100. true; product rule

101. true; $\log_3 7 = \dfrac{\log 7}{\log 3} = \dfrac{1}{\frac{\log 3}{\log 7}} = \dfrac{1}{\log_7 3}$

102. false; $e^x = \ln e^{e^x}$

103. a. $D = 10\log\left(\dfrac{I}{I_0}\right)$

b.
$$D_1 = 10\log\left(\frac{100I}{I_0}\right)$$
$$= 10\log(100I - I_0)$$
$$= 10\log 100 + 10\log I - 10\log I_0$$
$$= 10(2) + 10\log I - 10\log I_0$$
$$= 20 + 10\log\left(\frac{I}{I_0}\right)$$

This is 20 more than the loudness level of the softer sound. This means that the 100 times louder sound will be 20 decibels louder.

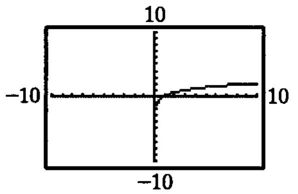
104. a. $t = \dfrac{1}{c}\ln\left(\dfrac{A}{A-N}\right)$

b.
$$t = \frac{1}{0.03}\left[\ln\frac{65}{65-30}\right]$$
$$t = \frac{1}{0.03}\ln\left(\frac{65}{35}\right)$$
$$t \approx 20.63$$

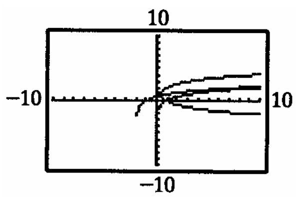
It will take the chimpanzee a little more than 20.5 weeks to master 30 signs.

105. – 112. Answers may vary.

113. **a.** $y = \log_3 x = \dfrac{\ln x}{\ln 3}$

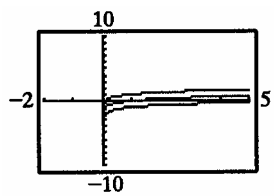


b.



To obtain the graph of $y = 2 + \log 3x$, shift the graph of $y = \log 3x$ two units upward. To obtain the graph of $y = \log 3(x + 2)$, shift the graph of $y = \log 3x$ two units left. To obtain the graph of $y = -\log 3x$, reflect the graph of $y = \log 3x$ about the x-axis.

114.

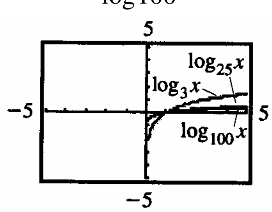


Using the product rule, $\log(10x) = \log x + 1$ and $1\log(0.1x) = \log x - 1$. Hence, these two graphs are just vertical shifts of $y = \log x$.

115. $\log_3 x = \dfrac{\log x}{\log 3}$;

$\log_{25} x = \dfrac{\log x}{\log 25}$;

$\log_{100} x = \dfrac{\log x}{\log 100}$



a. top graph: $y = \log_{100} x$
bottom graph: $y = \log_3 x$

b. top graph: $y = \log_3 x$
bottom graph: $y = \log_{100} x$

c. Comparing graphs of $\log_b x$ for $b > 1$, the graph of the equation with the largest b will be on the top in the interval (0, 1) and on the bottom in the interval $(1, \infty)$.

116. – 120. Answers may vary.

121. makes sense

122. makes sense

123. makes sense

124. does not make sense; Explanations will vary.

Sample explanation:
$$\log_4 \sqrt{\frac{x}{y}} = \log_4 \left(\frac{x}{y}\right)^{\frac{1}{2}} = \frac{1}{2}\log_4\left(\frac{x}{y}\right) = \frac{1}{2}\left(\log_4 x - \log_4 y\right) = \frac{1}{2}\log_4 x - \frac{1}{2}\log_4 y$$

125. true

126. false; Changes to make the statement true will vary. A sample change is:
$\dfrac{\log_7 49}{\log_7 7} = \dfrac{\log_7 49}{1} = \log_7 49 = 2$, but
$\log_7 49 - \log_7 7 = 2 - 1 = 1$.

127. false; Changes to make the statement true will vary. A sample change is: $\log_b\left(x^3 + y^3\right)$ cannot be simplified. If we were taking the logarithm of a product and not a sum, we would have been able to simplify as follows.
$$\log_b\left(x^3 y^3\right) = \log_b x^3 + \log_b y^3 = 3\log_b x + 3\log_b y$$

128. false; Changes to make the statement true will vary. A sample change is:
$$\log_b (xy)^5 = 5\log_b (xy) = 5\left(\log_b x + \log_b y\right) = 5\log_b x + 5\log_b y$$

129. $\log e = \log_{10} e = \dfrac{\ln e}{\ln 10} = \dfrac{1}{\ln 10}$

130. $\log_7 9 = \dfrac{\log 9}{\log 7} = \dfrac{\log 3^2}{\log 7} = \dfrac{2\log 3}{\log 7}$

$= \dfrac{2A}{B}$

131. $e^{\ln 8x^5 - \ln 2x^2} = e^{\ln\left(\frac{8x^5}{2x^2}\right)} = e^{\ln\left(4x^3\right)} = 4x^3$

132. $\dfrac{\log_b(x+h) - \log_b x}{h}$

$= \dfrac{\log_b \frac{x+h}{x}}{h}$

$= \dfrac{\log_b\left(1 + \frac{h}{x}\right)}{h}$

$= \dfrac{1}{h}\log_b\left(1 + \dfrac{h}{x}\right)$

$= \log_b\left(1 + \dfrac{x}{h}\right)^{1/h}$

133. $a(x-2) = b(2x+3)$

$ax - 2a = 2bx + 3b$

$ax - 2bx = 2a + 3b$

$x(a - 2b) = 2a + 3b$

$x = \dfrac{2a+3b}{a-2b}$

134. $x(x-7) = 3$

$x^2 - 7x = 3$

$x^2 - 7x - 3 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-3)}}{2(1)}$

$x = \dfrac{7 \pm \sqrt{61}}{2}$

The solution set is $\left\{\dfrac{7 \pm \sqrt{61}}{2}\right\}$.

135. $\dfrac{x+2}{4x+3} = \dfrac{1}{x}$

$x(4x+3)\left(\dfrac{x+2}{4x+3}\right) = x(4x+3)\left(\dfrac{1}{x}\right)$

$x(x+2) = 4x+3$

$x^2 + 2x = 4x + 3$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

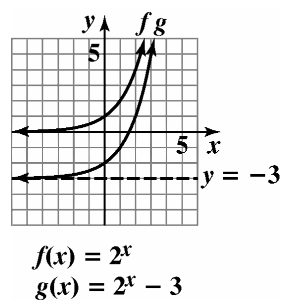
$x + 1 = 0$ or $x - 3 = 0$

$x = -1$ $\quad x = 3$

The solution set is $\{-1, 3\}$.

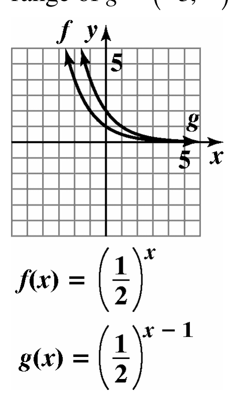
Mid-Chapter 4 Check Point

1.



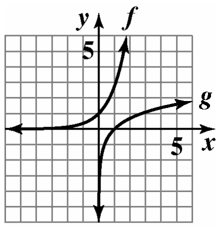
asymptote of f: $y = 0$
asymptote of g: $y = -3$
domain of f = domain of g = $(-\infty, \infty)$
range of f = $(0, \infty)$
range of g = $(-3, \infty)$

2.



asymptote of f: $y = 0$
asymptote of g: $y = 0$
domain of f = domain of g = $(-\infty, \infty)$
range of f = range of g = $(0, \infty)$

3.



$f(x) = e^x$
$g(x) = \ln x$

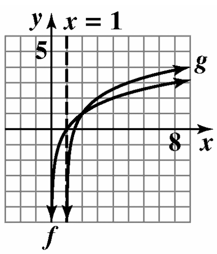
asymptote of f: $y = 0$

asymptote of g: $x = 0$

domain of f = range of g = $(-\infty, \infty)$

range of f = domain of g = $(0, \infty)$

4.



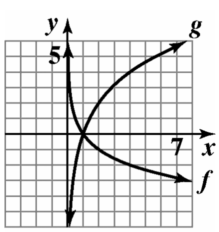
$f(x) = \log_2 x$
$g(x) = \log_2 (x - 1) + 1$

asymptote of f: $x = 0$
asymptote of g: $x = 1$
domain of f = $(0, \infty)$

domain of g = $(1, \infty)$

range of f = range of g = $(-\infty, \infty)$

5.



$f(x) = \log_{1/2} x$
$g(x) = -2\log_{1/2} x$

asymptote of f: $x = 0$
asymptote of g: $x = 0$
domain of f = domain of g = $(0, \infty)$

range of f = range of g = $(-\infty, \infty)$

6. $f(x) = \log_3(x+6)$
The argument of the logarithm must be positive:
$x + 6 > 0$
$x > -6$
domain: $\{x \mid x > -6\}$ or $(-6, \infty)$.

7. $f(x) = \log_3 x + 6$
The argument of the logarithm must be positive:
$x > 0$
domain: $\{x \mid x > 0\}$ or $(0, \infty)$.

8. $\log_3 (x+6)^2$
The argument of the logarithm must be positive.
Now $(x+6)^2$ is always positive, except when
$x = -6$
domain: $\{x \mid x \neq 0\}$ or $(-\infty, -6) \cup (-6, \infty)$.

9. $f(x) = 3^{x+6}$
domain: $\{x \mid x$ is a real number$\}$ or $(-\infty, \infty)$.

10. $\log_2 8 + \log_5 25 = \log_2 2^3 + \log_5 5^2$
$= 3 + 2 = 5$

11. $\log_3 \frac{1}{9} = \log_3 \frac{1}{3^2} = \log_3 3^{-2} = -2$

12. Let $\log_{100} 10 = y$
$100^y = 10$
$(10^2)^y = 10^1$
$10^{2y} = 10^1$
$2y = 1$
$y = \frac{1}{2}$

13. $\log \sqrt[3]{10} = \log 10^{\frac{1}{3}} = \frac{1}{3}$

14. $\log_2(\log_3 81) = \log_2(\log_3 3^4)$
$= \log_2 4 = \log_2 2^2 = 2$

15. $\log_3\left(\log_2 \frac{1}{8}\right) = \log_3\left(\log_2 \frac{1}{2^3}\right)$
$= \log_3\left(\log_2 2^{-3}\right)$
$= \log_3(-3)$
$=$ not possible
This expression is impossible to evaluate.

16. $6^{\log_6 5} = 5$

17. $\ln e^{\sqrt{7}} = \sqrt{7}$

18. $10^{\log 13} = 13$

19. $\log_{100} 0.1 = y$
$100^y = 0.1$
$\left(10^2\right)^y = \frac{1}{10}$
$10^{2y} = 10^{-1}$
$2y = -1$
$y = -\frac{1}{2}$

20. $\log_\pi \pi^{\sqrt{\pi}} = \sqrt{\pi}$

21. $\log\left(\frac{\sqrt{xy}}{1000}\right) = \log\left(\sqrt{xy}\right) - \log 1000$
$= \log(xy)^{\frac{1}{2}} - \log 10^3$
$= \frac{1}{2}\log(xy) - 3$
$= \frac{1}{2}(\log x + \log y) - 3$
$= \frac{1}{2}\log x + \frac{1}{2}\log y - 3$

22. $\ln\left(e^{19}x^{20}\right) = \ln e^{19} + \ln x^{20}$
$= 19 + 20\ln x$

23. $8\log_7 x - \frac{1}{3}\log_7 y = \log_7 x^8 - \log_7 y^{\frac{1}{3}}$
$= \log_7\left(\frac{x^8}{y^{\frac{1}{3}}}\right)$
$= \log_7\left(\frac{x^8}{\sqrt[3]{y}}\right)$

24. $7\log_5 x + 2\log_5 x = \log_5 x^7 + \log_5 x^2$
$= \log_5\left(x^7 \cdot x^2\right)$
$= \log_5 x^9$

25. $\frac{1}{2}\ln x - 3\ln y - \ln(z-2)$
$= \ln x^{\frac{1}{2}} - \ln y^3 - \ln(z-2)$
$= \ln\sqrt{x} - \left[\ln y^3 + \ln(z-2)\right]$
$= \ln\sqrt{x} - \ln\left[y^3(z-2)\right]$
$= \ln\left[\frac{\sqrt{x}}{y^3(z-2)}\right]$

26. Continuously: $A = 8000e^{0.08(3)}$
$\approx 10{,}170$
Monthly: $A = 8000\left(1 + \frac{0.08}{12}\right)^{12\cdot 3}$
$\approx 10{,}162$
$10{,}170 - 10{,}162 = 8$
Interest returned will be \$8 more if compounded continuously.

Section 4.4

Check Point Exercises

1. **a.**
$$5^{3x-6}=125$$
$$5^{3x-6}=5^3$$
$$3x-6=3$$
$$3x=9$$
$$x=3$$

b.
$$8^{x+2}=4^{x-3}$$
$$\left(2^3\right)^{x+2}=\left(2^2\right)^{x-3}$$
$$2^{3x+6}=2^{2x-6}$$
$$3x+6=2x-6$$
$$x=-12$$

2. **a.**
$$5^x=134$$
$$\ln 5^x=\ln 134$$
$$x\ln 5=\ln 134$$
$$x=\frac{\ln 134}{\ln 5}\approx 3.04$$
The solution set is $\left\{\frac{\ln 134}{\ln 5}\right\}$, approximately 3.04.

b.
$$10^x=8000$$
$$\log 10^x=\log 8000$$
$$x\log 10=\log 8000$$
$$x=\log 8000\approx 3.90$$
The solution set is $\{\log 8000\}$, approximately 3.90.

3.
$$7e^{2x}=63$$
$$e^{2x}=9$$
$$\ln e^{2x}=\ln 9$$
$$2x=\ln 9$$
$$x=\frac{\ln 9}{2}\approx 1.10$$
The solution set is $\left\{\frac{\ln 9}{2}\right\}$, approximately 1.10.

4.
$$3^{2x-1}=7^{x+1}$$
$$\ln 3^{2x-1}=\ln 7^{x+1}$$
$$(2x-1)\ln 3=(x+1)\ln 7$$
$$2x\ln 3-\ln 3=x\ln 7+\ln 7$$
$$2x\ln 3-x\ln 7=\ln 3+\ln 7$$
$$x(2\ln 3-\ln 7)=\ln 3+\ln 7$$
$$x=\frac{\ln 3+\ln 7}{2\ln 3-\ln 7}$$
$$x\approx 12.11$$

5.
$$e^{2x}-8e^x+7=0$$
$$\left(e^x-7\right)\left(e^x-1\right)=0$$
$e^x-7=0$ or $e^x-1=0$

$e^x=7$ $\quad$ $e^x=1$

$\ln e^x=\ln 7$ $\quad$ $\ln e^x=\ln 1$

$x=\ln 7$ $\quad$ $x=0$

The solution set is {0, ln7}. The solutions are 0 and (approximately) 1.95.

6. **a.**
$$\log_2(x-4)=3$$
$$2^3=x-4$$
$$8=x-4$$
$$12=x$$
Check:
$$\log_2(x-4)=3$$
$$\log_2(12-4)=3$$
$$\log_2 8=3$$
$$3=3$$
The solution set is {12}.

b. $4\ln 3x = 8$

$\ln 3x = 2$

$e^{\ln 3x} = e^2$

$3x = e^2$

$x = \frac{e^2}{3} \approx 2.46$

Check

$4\ln 3x = 8$

$4\ln 3\left(\frac{e^2}{3}\right) = 8$

$4\ln e^2 = 8$

$4(2) = 8$

$8 = 8$

The solution set is $\left\{\frac{e^2}{3}\right\}$, approximately 2.46.

7. $\log x + \log(x-3) = 1$

$\log x(x-3) = 1$

$10^1 = x(x-3)$

$10 = x^2 - 3x$

$0 = x^2 - 3x - 10$

$0 = (x-5)(x+2)$

$x-5=0$ or $x+2=0$

$x=5$ or $x=-2$

Check

Checking 5:

$\log 5 + \log(5-3) = 1$

$\log 5 + \log 2 = 1$

$\log(5\cdot 2) = 1$

$\log 10 = 1$

$1 = 1$

Checking –2:

$\log x + \log(x-3) = 1$

$\log(-2) + \log(-2-3)\ 0\ 1$

Negative numbers do not have logarithms so –2 does not check.

The solution set is {5}.

8. $\ln(x-3) = \ln(7x-23) - \ln(x+1)$

$\ln(x-3) = \ln\frac{7x-23}{x+1}$

$x-3 = \frac{7x-23}{x+1}$

$(x-3)(x+1) = 7x-23$

$x^2-2x-3 = 7x-23$

$x^2-9x+20 = 0$

$(x-4)(x-5) = 0$

$x=4$ or $x=5$

Both values produce true statements.

The solution set is {4, 5}

9. For a risk of 7%, let $R = 7$ in

$R = 6e^{12.77x}$

$6e^{12.77x} = 7$

$e^{12.77x} = \frac{7}{6}$

$\ln e^{12.77x} = \ln\left(\frac{7}{6}\right)$

$12.77x = \ln\left(\frac{7}{6}\right)$

$x = \frac{\ln\left(\frac{7}{6}\right)}{12.77} \approx 0.01$

For a blood alcohol concentration of 0.01, the risk of a car accident is 7%.

10. $A = P\left(1+\frac{r}{n}\right)^{nt}$

$3600 = 1000\left(1+\frac{0.08}{4}\right)^{4t}$

$1000\left(1\_\frac{0.08}{4}\right)^{4t} = 3600$

$1000(1+0.02)^{4t} = 3600$

$1000(1.02)^{4t} = 3600$

$(1.02)^{4t} = \ln 3.6$

$4t\ln(1.02) = \ln 3.6$

$t = \frac{\ln 3.6}{4\ln 1.02}$

≈ 16.2

After approximately 16.2 years, the $1000 will grow to an accumulated value of $3600.

11. $f(x) = 54.8 - 12.3\ln x$

Solve equation when $f(x) = 25$.

$$54.8 - 12.3\ln x = 25$$
$$-12.3\ln x = -29.8$$
$$\ln x = \frac{-29.8}{-12.3}$$
$$\log_e x = \frac{29.8}{12.3}$$
$$x = e^{\frac{29.8}{12.3}}$$
$$x \approx 11.277$$

An annual income of approximately $11,000 corresponds to 25% of Americans reporting fair or poor health.

Exercise Set 4.4

1. $2^x = 64$
$$2^x = 2^6$$
$$x = 6$$
The solution is 6, and the solution set is $\{6\}$.

2. $3^x = 81$
$$3^x = 3^4$$
$$x = 4$$
The solution set is $\{4\}$.

3. $5^x = 125$
$$5^x = 5^3$$
$$x = 3$$
The solution is 3, and the solution set is $\{3\}$.

4. $5^x = 625$
$$5^x = 5^4$$
$$x = 4$$
The solution set is $\{4\}$.

5. $2^{2x-1} = 32$
$$2^{2x-1} = 2^5$$
$$2x - 1 = 5$$
$$2x = 6$$
$$x = 3$$
The solution is 3, and the solution set is $\{3\}$.

6. $3^{2x+1} = 27$
$$3^{2x+1} = 3^3$$
$$2x + 1 = 3$$
$$2x = 2$$
$$x = 1$$
The solution set is $\{1\}$.

7. $4^{2x-1} = 64$
$$4^{2x-1} = 4^3$$
$$2x - 1 = 3$$
$$2x = 4$$
$$x = 2$$
The solution is 2, and the solution set is $\{2\}$.

8. $5^{3x-1} = 125$
$$5^{3x-1} = 5^3$$
$$3x - 1 = 3$$
$$3x = 4$$
$$x = \frac{4}{3}$$
The solution set is $\left\{\frac{4}{3}\right\}$.

9. $32^x = 8$
$$\left(2^5\right)^x = 2^3$$
$$2^{5x} = 2^3$$
$$5x = 3$$
$$x = \frac{3}{5}$$
The solution is $\frac{3}{5}$, and the solution set is $\left\{\frac{3}{5}\right\}$.

10. $4^x = 32$
$$\left(2^2\right)^x = 2^5$$
$$2^{2x} = 2^5$$
$$2x = 5$$
$$x = \frac{5}{2}$$
The solution set is $\left\{\frac{5}{2}\right\}$.

11.
$$9^x = 27$$
$$(3^2)^x = 3^3$$
$$3^{2x} = 3^3$$
$$2x = 3$$
$$x = \frac{3}{2}$$
The solution is $\frac{3}{2}$, and the solution set is $\left\{\frac{3}{2}\right\}$.

12.
$$125^x = 625$$
$$(5^3)^x = 5^4$$
$$5^{3x} = 5^4$$
$$3x = 4$$
$$x = \frac{4}{3}$$
The solution set is $\left\{\frac{4}{3}\right\}$.

13.
$$3^{1-x} = \frac{1}{27}$$
$$3^{1-x} = \frac{1}{3^3}$$
$$3^{1-x} = 3^{-3}$$
$$1 - x = -3$$
$$-x = -4$$
$$x = 4$$
The solution set is $\{4\}$.

14.
$$5^{2-x} = \frac{1}{125}$$
$$5^{2-x} = \frac{1}{5^3}$$
$$5^{2-x} = 5^{-3}$$
$$2 - x = -3$$
$$-x = -5$$
$$x = 5$$
The solution set is $\{5\}$.

15.
$$6^{\frac{x-3}{4}} = \sqrt{6}$$
$$6^{\frac{x-3}{4}} = 6^{\frac{1}{2}}$$
$$\frac{x-3}{4} = \frac{1}{2}$$
$$2(x-3) = 4(1)$$
$$2x - 6 = 4$$
$$2x = 10$$
$$x = 5$$
The solution is 5, and the solution set is $\{5\}$.

16.
$$7^{\frac{x-2}{6}} = \sqrt{7}$$
$$7^{\frac{x-2}{6}} = 7^{\frac{1}{2}}$$
$$\frac{x-2}{6} = \frac{1}{2}$$
$$2(x-2) = 6(1)$$
$$2x - 4 = 6$$
$$2x = 10$$
$$x = 5$$
The solution set is $\{5\}$.

17.
$$4^x = \frac{1}{\sqrt{2}}$$
$$(2^2)^x = \frac{1}{2^{\frac{1}{2}}}$$
$$2^{2x} = 2^{-\frac{1}{2}}$$
$$2x = -\frac{1}{2}$$
$$x = \frac{1}{2}\left(-\frac{1}{2}\right) = -\frac{1}{4}$$
The solution is $-\frac{1}{4}$, and the solution set is $\left\{-\frac{1}{4}\right\}$.

18.
$$9^x = \frac{1}{\sqrt[3]{3}}$$
$$(3^2)^x = \frac{1}{3^{\frac{1}{3}}}$$
$$3^{2x} = 3^{-\frac{1}{3}}$$
$$2x = -\frac{1}{3}$$
$$x = \frac{1}{2}\left(-\frac{1}{3}\right) = -\frac{1}{6}$$
The solution set is $\left\{-\frac{1}{6}\right\}$.

19.
$$8^{x+3}=16^{x-1}$$
$$\left(2^3\right)^{x+3}=\left(2^4\right)^{x-1}$$
$$2^{3x+9}=2^{4x-4}$$
$$3x+9=4x-4$$
$$13=x$$
The solution set is $\{13\}$.

20.
$$8^{1-x}=4^{x+2}$$
$$\left(2^3\right)^{1-x}=\left(2^2\right)^{x+2}$$
$$2^{3-3x}=2^{2x+4}$$
$$3-3x=2x+4$$
$$-5x=1$$
$$x=-\frac{1}{5}$$
The solution set is $\left\{-\frac{1}{5}\right\}$.

21.
$$e^{x+1}=\frac{1}{e}$$
$$e^{x+1}=e^{-1}$$
$$x+1=-1$$
$$x=-2$$
The solution set is $\{-2\}$.

22.
$$e^{x+4}=\frac{1}{e^{2x}}$$
$$e^{x+4}=e^{-2x}$$
$$x+4=-2x$$
$$3x=-4$$
$$x=-\frac{4}{3}$$
The solution set is $\left\{-\frac{4}{3}\right\}$.

23.
$$10^x=3.91$$
$$\ln 10^x=\ln 3.91$$
$$x\ln 10=\ln 3.91$$
$$x=\frac{\ln 3.91}{\ln 10}\approx 0.59$$

24.
$$10^x=8.07$$
$$\ln 10^x=\ln 8.07$$
$$x\ln 10=\ln 8.07$$
$$x=\frac{\ln 8.07}{\ln 10}\approx 0.91$$

25.
$$e^x=5.7$$
$$\ln e^x=5.7$$
$$x=\ln 5.7\approx 1.74$$

26.
$$e^x=0.83$$
$$\ln e^x=\ln 0.83$$
$$x=\ln 0.83\approx -0.19$$

27.
$$5^x=17$$
$$\ln 5^x=\ln 17$$
$$x\ln 5=\ln 17$$
$$x=\frac{\ln 17}{\ln 5}\approx 1.76$$

28.
$$19^x=143$$
$$x\ln 19=\ln 143$$
$$x=\frac{\ln 143}{\ln 19}\approx 1.69$$

29.
$$5e^x=23$$
$$e^x=\frac{23}{5}$$
$$\ln e^x=\ln\frac{23}{5}$$
$$x=\ln\frac{23}{5}\approx 1.53$$

30.
$$9e^x=107$$
$$e^x=\frac{107}{9}$$
$$\ln e^x=\ln\frac{107}{9}$$
$$x=\ln\frac{107}{9}\approx 2.48$$

31.
$$3e^{5x}=1977$$
$$e^{5x}=659$$
$$\ln e^{5x}=\ln 659$$
$$x=\frac{\ln 659}{5}\approx 1.30$$

32. $4e^{7x} = 10,273$

$e^{7x} = \frac{10,273}{4}$

$\ln e^{7x} = \ln\left(\frac{10,273}{4}\right)$

$7x = \ln\left(\frac{10,273}{4}\right)$

$x = \frac{1}{7}\ln\left(\frac{10,273}{4}\right) \approx 1.12$

33. $e^{1-5x} = 793$

$\ln e^{1-5x} = \ln 793$

$(1-5x)(\ln e) = \ln 793$

$1-5x = \ln 793$

$5x = 1 - \ln 793$

$x = \frac{1-\ln 793}{5} \approx -1.14$

34. $e^{1-8x} = 7957$

$\ln e^{1-8x} = \ln 7957$

$(1 - 8x) \ln e = \ln 7957$

$1 - 8x = \ln 7957$

$8x = 1 - \ln 7957$

$x = \frac{1-\ln 7957}{8} \approx -1.00$

35. $e^{5x-3} - 2 = 10,476$

$e^{5x-3} = 10,478$

$\ln e^{5x-3} = \ln 10,478$

$(5x-3)\ln e = \ln 10,478$

$5x-3 = \ln 10,478$

$5x = \ln 10,478 + 3$

$x = \frac{\ln 10,478+3}{5} \approx 2.45$

36. $e^{4x-5} - 7 = 11,243$

$e^{4x-5} = 11,250$

$\ln e^{4x-5} = \ln 11,250$ $(4x - 5)\ln e = \ln 11,250$

$4x - 5 = \ln 11,250$

$x = \frac{\ln 11,250+5}{4} \approx 3.58$

37. $7^{x+2} = 410$

$\ln 7^{x+2} = \ln 410$

$(x+2)\ln 7 = \ln 410$

$x+2 = \frac{\ln 410}{\ln 7}$

$x = \frac{\ln 410}{\ln 7} - 2 \approx 1.09$

38. $5^{x-3} = 137$

$\ln 5^{x-3} = \ln 137$

$(x - 3) \ln 5 = \ln 137$

$x-3 = \frac{\ln 137}{\ln 5}$

$x = 3 + \frac{\ln 137}{\ln 5} \approx 6.06$

39. $7^{0.3x} = 813$

$\ln 7^{0.3x} = \ln 813$

$0.3x \ln 7 = \ln 813$

$x = \frac{\ln 813}{0.3 \ln 7} \approx 11.48$

40. $3^{x/7} = 0.2$

$\ln 3^{x/7} = \ln 0.2$

$\frac{x}{7}\ln 3 = \ln 0.2$

$x \ln 3 = 7 \ln 0.2$

$x = \frac{7\ln 0.2}{\ln 3} \approx -10.25$

41. $5^{2x+3} = 3^{x-1}$

$\ln 5^{2x+3} = \ln 3^{x-1}$

$(2x+3)\ln 5 = (x-1)\ln 3$

$2x\ln 5 + 3\ln 5 = x\ln 3 - \ln 3$

$3\ln 5 + \ln 3 = x\ln 3 - 2x\ln 5$

$3\ln 5 + \ln 3 = x(\ln 3 - 2\ln 5)$

$\frac{3\ln 5+\ln 3}{\ln 3 - 2\ln 5} = x$

$-2.80 \approx x$

42. $7^{2x+1}=3^{x+2}$

$$\ln 7^{2x+1}=\ln 3^{x+2}$$
$$(2x+1)\ln 7=(x+2)\ln 3$$
$$2x+1=(x+2)\frac{\ln 3}{\ln 7}$$
$$2x+1=x\frac{\ln 3}{\ln 7}+\frac{2\ln 3}{\ln 7}$$
$$2x-x\frac{\ln 3}{\ln 7}=\frac{2\ln 3}{\ln 7}-1$$
$$x\left(2-\frac{\ln 3}{\ln 7}\right)=\frac{2\ln 3}{\ln 7}-1$$
$$x=\frac{\frac{2\ln 3}{\ln 7}-1}{2-\frac{\ln 3}{\ln 7}}\approx 0.09$$

43. $e^{2x}-3e^x+2=0$

$(e^x-2)(e^x-1)=0$

$e^x-2=0$ or $e^x-1=0$

$e^x=2$ $\quad e^x=1$

$\ln e^x=\ln 2$ $\quad \ln e^x=\ln 1$

$x=\ln 2$ $\quad x=0$

The solution set is {0, ln 2}. The solutions are 0 and approximately 0.69.

44. $e^{2x}-2e^x-3=0$

$(e^x-3)(e^x+1)=0$

$e^x-3=0$ or $e^x+1=0$

$e^x=3$ $\quad e^x=-1$

$\ln e^x=\ln 3$ $\quad \ln e^x=\ln(-1)$

$x=\ln 3$ $\quad$ no solution

The solution set is {ln 3}. The solutions is approximately 1.10.

45. $e^{4x}+5e^{2x}-24=0$

$(e^{2x}+8)(e^{2x}-3)=0$

$e^{2x}+8=0$ or $e^{2x}-3=0$

$e^{2x}=-8$ $\quad e^{2x}=3$

$\ln e^{2x}=\ln(-8)$ $\quad \ln e^{2x}=\ln 3$

$2x=\ln(-8)$ $\quad 2x=\ln 3$

$\ln(-8)$ does not exist $\quad x=\frac{\ln 3}{2}$

$x=\frac{\ln 3}{2}\approx 0.55$

46. $e^{4x}-3e^{2x}-18=0$

$(e^{2x}-6)(e^{2x}+3)=0$

$e^{2x}-6=0$ or $e^{2x}+3=0$

$e^{2x}=6$ $\quad e^{2x}=-3$

$\ln e^{2x}=\ln 6$ $\quad \ln e^{2x}=\ln(-3)$

$2x=\ln 6$ $\quad \ln(-3)$ does not exist.

$x=\frac{\ln 6}{2}\approx 0.90$

47. $3^{2x}+3^x-2=0$

$(3^x+2)(3^x-1)=0$

$3^x+2=0$ $\quad 3^x-1=0$

$3^x=-2$ $\quad 3^x=1$

$\log 3^x=\log(-2)$ $\quad \log 3^x=\log 1$

does not exist $\quad \log 3=0$

$x=\frac{0}{\log 3}$

$x=0$

The solution set is {0}.

48. $2^{2x}+2^x-12=0$

$(2^x+4)(2^x-3)=0$

$2^x+4=0$ $\quad 2^x-3=0$

$2^x=-4$ $\quad 2^x=3$

$\ln 2^x=\ln(-4)$ $\quad \ln 2^x=\ln 3$

does not exist $\quad x\ln 2=\ln 3$

$x=\frac{\ln 3}{\ln 2}$

$x\approx 1.58$

49. $\log_3 x=4$

$3^4=x$

$81=x$

50. $\log_5 x=3$

$5^3=x$

$125=x$

51. $\ln x=2$

$e^2=x$

$7.39\approx x$

52. $\ln x=3$

$e^3=x$

$20.09\approx x$

53. $\log_4(x+5)=3$
$4^3 = x+5$
$59 = x$

54. $\log_5(x-7)=2$
$5^2 = x-7$
$32 = x$

55. $\log_3(x-4)=-3$
$3^{-3} = x-4$
$\frac{1}{27} = x-4$
$4\frac{1}{27} = x$
$4.04 \approx x$

56. $\log_7(x+2)=-2$
$7^{-2} = x+2$
$\frac{1}{49} = x+2$
$-1\frac{48}{49} = x$
$-1.98 \approx x$

57. $\log_4(3x+2)=3$
$4^3 = 3x+2$
$64 = 3x+2$
$62 = 3x$
$\frac{62}{3} = x$
$20.67 \approx x$

58. $\log_2(4x+1)=5$
$2^5 = 4x+1$
$32 = 4x+1$
$31 = 4x$
$\frac{31}{4} = x$
$7.75 = x$

59. $5\ln 2x = 20$
$\ln 2x = 4$
$e^{\ln 2x} = e^4$
$2x = e^4$
$x = \frac{e^4}{2} \approx 27.30$

60. $6\ln 2x = 30$
$\ln 2x = 5$
$e^{\ln 2x} = e^5$
$2x = e^5$
$x = \frac{e^5}{2} \approx 74.21$

61. $6+2\ln x = 5$
$2\ln x = -1$
$\ln x = -\frac{1}{2}$
$e^{\ln x} = e^{-1/2}$
$x = e^{-1/2} \approx 0.61$

62. $7+3\ln x = 6$
$3\ln x = -1$
$\ln x = -\frac{1}{3}$
$e^{\ln x} = e^{-1/3}$
$x = e^{-1/3} \approx 0.72$

63. $\ln\sqrt{x+3} = 1$
$e^{\ln\sqrt{x+3}} = e^1$
$\sqrt{x+3} = e$
$x+3 = e^2$
$x = e^2 - 3 \approx 4.39$

64. $\ln\sqrt{x+4} = 1$
$e^{\ln\sqrt{x+4}} = e^1$
$\sqrt{x+4} = e$
$x+4 = e^2$
$x = e^2 - 4 \approx 3.39.$

65. $\log_5 x + \log_5(4x-1) = 1$
$\log_5(4x^2 - x) = 1$
$4x^2 - x = 5$
$4x^2 - x - 5 = 0$
$(4x-5)(x+1) = 0$
$x = \frac{5}{4}$ or $x = -1$

$x = -1$ does not check because $\log_5(-1)$ does not exist.

The solution set is $\left\{\frac{5}{4}\right\}$.

66. $\log_6(x+5)+\log_6 x=2$

$\log_6 x(x+5)=2$

$x(x+5)=6^2$

$x^2+5x=36$

$x^2+5x-36=0$

$(x+9)(x-4)=0$

$x=-9$ or $x=4$

$x=-9$ does not check because $\log_6(-9+5)$ does not exist.

The solution set is $\{4\}$.

67. $\log_3(x-5)+\log_3(x+3)=2$

$\log_3\left[(x-5)(x+3)\right]=2$

$(x-5)(x+3)=3^2$

$x^2-2x-15=9$

$x^2-2x-24=0$

$(x-6)(x+4)=0$

$x=6$ or $x=-4$

$x=-4$ does not check because $\log_3(-4-5)$ does not exist. The solution set is $\{6\}$.

68. $\log_2(x-1)+\log_2(x+1)=3$

$\log_2\left[(x-1)(x+1)\right]=3$

$(x-1)(x+1)=2^3$

$x^2-1=8$

$x^2=9$

$x=3$ or $x=-3$

$x=-3$ does not check because $\log_2(-3-1)$ does not exist.

The solution set is $\{3\}$.

69. $\log_2(x+2)-\log_2(x-5)=3$

$\log_2\left(\frac{x+2}{x-5}\right)=3$

$\frac{x+2}{x-5}=2^3$

$\frac{x+2}{x-5}=8$

$x+2=8(x-5)$

$x+2=8x-40$

$7x=42$

$x=6$

70. $\log_4(x+2)-\log_4(x-1)=1$

$\log_4\left(\frac{x+2}{x-1}\right)=1$

$\frac{x+2}{x-1}=4^1$

$\frac{x+2}{x-1}=4$

$x+2=4(x-1)$

$x+2=4x-4$

$3x=6$

$x=2$

71. $2\log_3(x+4)=\log_3 9+2$

$2\log_3(x+4)=2+2$

$2\log_3(x+4)=4$

$\log_3(x+4)=2$

$3^2=x+4$

$9=x+4$

$5=x$

72. $3\log_2(x-1)=5-\log_2 4$

$3\log_2(x-1)=5-2$

$3\log_2(x-1)=3$

$\log_2(x-1)=1$

$2^1=x-1$

$3=x$

73. $\log_2(x-6)+\log_2(x-4)-\log_2 x=2$

$\log_2\frac{(x-6)(x-4)}{x}=2$

$\frac{(x-6)(x-4)}{x}=2^2$

$x^2-10x+24=4x$

$x^2-14x+24=0$

$(x-12)(x-2)=0$

$x-12=0$ or $x-2=0$

$x=12$ $\quad x=2$

The solution set is $\{12\}$ since $\log_2(2-6)=\log_2(-4)$ is not possible.

74. $\log_2(x-3)+\log_2 x-\log_2(x+2)=2$

$$\log_2\frac{(x-3)x}{(x+2)}=2$$
$$2^2=\frac{x^2-3x}{x+2}$$
$$4(x+2)=x^2-3x$$
$$4x+8=x^2-3x$$
$$0=x^2-7x-8$$
$$0=(x+1)(x-8)$$

$x+1=0$ or $x-8=0$

$x=-1$ $\quad x=8$

$\log_2(-1-3)=\log_2(-4)$ does not exist, so the solution set is $\{8\}$

75. $\log(x+4)=\log x+\log 4$

$\log(x+4)=\log 4x$

$x+4=4x$

$4=3x$

$x=\frac{4}{3}$

This value is rejected. The solution set is $\left\{\frac{4}{3}\right\}$.

76. $\log(5x+1)=\log(2x+3)+\log 2$

$\log(5x+1)=\log(4x+6)$

$5x+1=4x+6$

$x=5$

77. $\log(3x-3)=\log(x+1)+\log 4$

$\log(3x-3)=\log(4x+4)$

$3x-3=4x+4$

$-7=x$

This value is rejected. The solution set is $\{\ \}$.

78. $\log(2x-1)=\log(x+3)+\log 3$

$\log(2x-1)=\log(3x+9)$

$2x-1=3x+9$

$-10=x$

This value is rejected. The solution set is $\{\ \}$.

79. $2\log x=\log 25$

$\log x^2=\log 25$

$x^2=25$

$x=\pm 5$

–5 is rejected. The solution set is $\{5\}$.

80. $3\log x=\log 125$

$\log x^3=\log 125$

$x^3=125$

$x=5$

81. $\log(x+4)-\log 2=\log(5x+1)$

$\log\frac{x+4}{2}=\log(5x+1)$

$\frac{x+4}{2}=5x+1$

$x+4=10x+2$

$-9x=-2$

$x=\frac{2}{9}$

$x\approx 0.22$

82. $\log(x+7)-\log 3=\log(7x+1)$

$\log\frac{x+7}{3}=\log(7x+1)$

$\frac{x+7}{3}=7x+1$

$x+7=21x+3$

$-20x=-4$

$x=\frac{1}{5}$

$x\approx 0.2$

83. $2\log x-\log 7=\log 112$

$\log x^2-\log 7=\log 112$

$\log\frac{x^2}{7}=\log 112$

$\frac{x^2}{7}=112$

$x^2=784$

$x=\pm 28$

–28 is rejected. The solution set is $\{28\}$.

84. $\log(x-2)+\log 5=\log 100$

$\log(5x-10)=\log 100$

$5x-10=100$

$5x=110$

$x=22$

85. $\log x + \log(x+3) = \log 10$

$\log(x^2 + 3x) = \log 10$

$x^2 + 3x = 10$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = -5$ or $x = 2$

–5 is rejected. The solution set is {2}.

86. $\log(x+3) + \log(x-2) = \log 14$

$\log(x^2 + x - 6) = \log 14$

$x^2 + x - 6 = 14$

$x^2 + x - 20 = 0$

$(x+5)(x-4) = 0$

$x = -5$ or $x = 4$

–5 is rejected. The solution set is {4}.

87. $\ln(x-4) + \ln(x+1) = \ln(x-8)$

$\ln(x^2 - 3x - 4) = \ln(x-8)$

$x^2 - 3x - 4 = x - 8$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x = 2$

2 is rejected. The solution set is { }.

88. $\log_2(x-1) - \log_2(x+3) = \log_2\left(\frac{1}{x}\right)$

$\log_2 \frac{x-1}{x+3} = \log_2\left(\frac{1}{x}\right)$

$\frac{x-1}{x+3} = \frac{1}{x}$

$x^2 - x = x + 3$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$

–1 is rejected. The solution set is {3}.

89. $\ln(x-2) - \ln(x+3) = \ln(x-1) - \ln(x+7)$

$\ln \frac{x-2}{x+3} = \ln \frac{x-1}{x+7}$

$\frac{x-2}{x+3} = \frac{x-1}{x+7}$

$(x-2)(x+7) = (x+3)(x-1)$

$x^2 + 5x - 14 = x^2 + 2x - 3$

$3x = 11$

$x = \frac{11}{3}$

$x \approx 3.67$

90. $\ln(x-5) - \ln(x+4) = \ln(x-1) - \ln(x+2)$

$\ln \frac{x-5}{x+4} = \ln \frac{x-1}{x+2}$

$\frac{x-5}{x+4} = \frac{x-1}{x+2}$

$(x-5)(x+2) = (x+4)(x-1)$

$x^2 - 3x - 10 = x^2 + 3x - 4$

$-6x = 6$

$x = -1$

–1 is rejected. The solution set is { }.

91. $5^{2x} \cdot 5^{4x} = 125$

$5^{2x+4x} = 5^3$

$5^{6x} = 5^3$

$6x = 3$

$x = \frac{1}{2}$

92. $3^{x+2} \cdot 3^x = 81$

$3^{(x+2)+x} = 3^4$

$3^{2x+2} = 3^4$

$2x + 2 = 4$

$2x = 2$

$x = 1$

93. $2|\ln x| - 6 = 0$

$2|\ln x| = 6$

$|\ln x| = 3$

$\ln x = 3$ or $\ln x = -3$

$x = e^3$ $\quad$ $x = e^{-3}$

$x \approx 20.09$ $\quad$ $x \approx 0.05$

94. $3|\log x| - 6 = 0$

$3|\log x| = 6$

$|\log x| = 2$

$\log x = 2$ or $\log x = -2$

$x = 10^2$ $\quad$ $x = 10^{-2}$

$x = 100$ $\quad$ $x = 0.01$

95. $3^{x^2} = 45$

$\ln 3^{x^2} = \ln 45$

$x^2 \ln 3 = \ln 45$

$x^2 = \dfrac{\ln 45}{\ln 3}$

$x = \pm\sqrt{\dfrac{\ln 45}{\ln 3}} \approx \pm 1.86$

96. $5^{x^2} = 50$

$\ln 5^{x^2} = \ln 50$

$x^2 \ln 5 = \ln 50$

$x^2 = \dfrac{\ln 50}{\ln 5}$

$x = \pm\sqrt{\dfrac{\ln 50}{\ln 5}} \approx \pm 1.56$

97. $\ln(2x+1) + \ln(x-3) - 2\ln x = 0$

$\ln(2x+1) + \ln(x-3) - \ln x^2 = 0$

$\ln \dfrac{(2x+1)(x-3)}{x^2} = 0$

$\dfrac{(2x+1)(x-3)}{x^2} = e^0$

$\dfrac{2x^2 - 5x - 3}{x^2} = 1$

$2x^2 - 5x - 3 = x^2$

$x^2 - 5x - 3 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-3)}}{2(1)}$

$x = \dfrac{5 \pm \sqrt{37}}{2}$

$x = \dfrac{5 + \sqrt{37}}{2} \approx 5.54$

$x = \dfrac{5 - \sqrt{37}}{2} \approx -0.54$ (rejected)

The solution set is $\left\{\dfrac{5 + \sqrt{37}}{2}\right\}$.

98. $\ln 3 - \ln(x+5) - \ln x = 0$

$\ln \dfrac{3}{x(x+5)} = 0$

$e^0 = \dfrac{3}{x(x+5)}$

$1 = \dfrac{3}{x(x+5)}$

$x(x+5) = 3$

$x^2 + 5x = 3$

$x^2 + 5x - 3 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}\ x^2 + 5x - 3 = 0$

$x = \dfrac{-(5) \pm \sqrt{(5)^2 - 4(1)(-3)}}{2(1)}$

$x = \dfrac{-5 \pm \sqrt{37}}{2}$

$x = \dfrac{-5 + \sqrt{37}}{2} \approx 0.54$

$x = \dfrac{-5 - \sqrt{37}}{2} \approx -5.54$ (rejected)

The solution set is $\left\{\dfrac{-5 + \sqrt{37}}{2}\right\}$.

99.
$$5^{x^2-12}=25^{2x}$$
$$5^{x^2-12}=\left(5^2\right)^{2x}$$
$$5^{x^2-12}=5^{4x}$$
$$x^2-12=4x$$
$$x^2-4x-12=0$$
$$(x-6)(x+2)=0$$
Apply the zero product property:
$x-6=0$ or $x+2=0$
$x=6$ $\quad$ $x=-2$
The solutions are -2 and 6, and the solution set is $\{-2,\ 6\}$.

100.
$$3^{x^2-12}=9^{2x}$$
$$3^{x^2-12}=\left(3^2\right)^{2x}$$
$$3^{x^2-12}=3^{4x}$$
$$x^2-12=4x$$
$$x^2-4x-12=0$$
$$(x-6)(x+2)=0$$
Apply the zero product property:
$x-6=0$ or $x+2=0$
$x=6$ $\quad$ $x=-2$
The solutions are -2 and 6, and the solution set is $\{-2,\ 6\}$.

101. a. 2005 is 0 years after 2005.
$$A=36.1e^{0.0126t}$$
$$A=36.1e^{0.0126(0)}=36.1$$
The population of California was 36.1 million in 2005.

b.
$$A=36.1e^{0.0126t}$$
$$40=36.1e^{0.0126t}$$
$$\frac{40}{36.1}=e^{0.0126t}$$
$$\ln\frac{40}{36.1}=\ln e^{0.0126t}$$
$$0.0126t=\ln\frac{40}{36.1}$$
$$t=\frac{\ln\frac{40}{36.1}}{0.0126}\approx 8$$
The population of California will reach 40 million about 8 years after 2005, or 2013

102. a. 2005 is 0 years after 2005.
$$A=22.9e^{0.0183t}$$
$$A=22.9e^{0.0183(0)}=22.9$$
The population of Texas was 22.9 million in 2005.

b.
$$A=22.9e^{0.0183t}$$
$$27=22.9e^{0.0183t}$$
$$\frac{27}{22.9}=e^{0.0183t}$$
$$\ln\frac{27}{22.9}=\ln e^{0.0183t}$$
$$0.0183t=\ln\frac{27}{22.9}$$
$$t=\frac{\ln\frac{27}{22.9}}{0.0183}\approx 9$$
The population of Texas will reach 27 million about 9 years after 2005, or 2014.

103.
$$f(x)=20(0.975)^x$$
$$1=20(0.975)^x$$
$$\frac{1}{20}=0.975^x$$
$$\ln\frac{1}{20}=\ln 0.975^x$$
$$\ln\frac{1}{20}=x\ln 0.975$$
$$x=\frac{\ln\frac{1}{20}}{\ln 0.975}$$
$$x\approx 118$$
There is 1% of surface sunlight at 118 feet. This is represented by the point (118,1).

104. $f(x) = 20(0.975)^x$

$$3 = 20(0.975)^x$$

$$\frac{3}{20} = 0.975^x$$

$$\ln\frac{3}{20} = \ln 0.975^x$$

$$\ln\frac{3}{20} = x\ln 0.975$$

$$x = \frac{\ln\frac{3}{20}}{\ln 0.975}$$

$$x \approx 75$$

There is 3% of surface sunlight at 75 feet. This is represented by the point (75,3).

105. $20,000 = 12,500\left(1+\frac{0.0575}{4}\right)^{4t}$

$$12,500(1.014375)^{4t} = 20,000$$

$$(1.014375)^{4t} = 1.6$$

$$\ln(1.014375)^{4t} = \ln 1.6$$

$$4t\ln(1.014375) = \ln 1.6$$

$$t = \frac{\ln 1.6}{4\ln 1.014375} \approx 8.2$$

8.2 years

106. $15,000 = 7250\left(1+\frac{0.065}{12}\right)^{12t}$

$$7250(1.005416667)^{12t} = 15,000$$

$$(1.005416667)^{12t} = \frac{60}{29}$$

$$\ln(1.005416667)^{12t} = \ln\left(\frac{60}{29}\right)$$

$$12t\ln(1.00541667) = \ln\left(\frac{60}{29}\right)$$

$$t = \frac{\ln\left(\frac{60}{29}\right)}{12\ln 1.00541667} \approx 11.2 \text{ years}$$

107. $1400 = 1000\left(1+\frac{r}{360}\right)^{360\cdot 2}$

$$\left(1+\frac{r}{360}\right)^{720} = 1.4$$

$$\ln\left(1+\frac{r}{360}\right)^{720} = \ln 1.4$$

$$720\ln\left(1+\frac{r}{360}\right) = \ln 1.4$$

$$\ln\left(1+\frac{r}{360}\right) = \frac{\ln 1.4}{720}$$

$$e^{\ln(1+r/360)} = e^{(\ln 1.4)/720}$$

$$1+\frac{r}{360} = e^{(\ln 1.4)/720} - 1$$

$$r = 360(e^{(\ln 1.4)/720)} - 1$$

$$\approx 0.168$$

16.8%

108. $9000 = 5000\left(1+\frac{r}{360}\right)^{(360\cdot 4)}$

$$\left(1+\frac{r}{360}\right)^{1440} = 1.8$$

$$\ln\left(1+\frac{r}{360}\right)^{1440} = \ln 1.8$$

$$1440\ln\left(1+\frac{r}{360}\right) = \ln 1.8$$

$$\ln\left(1+\frac{r}{360}\right) = \frac{\ln 1.8}{1440}$$

$$e^{\ln(1+r/360)} = e^{(\ln 1.8)/1440}$$

$$1+\frac{r}{360} = e^{(\ln 1.8)/1440} - 1$$

$$\frac{r}{360} = e^{(\ln 1.8)/1440} - 1$$

$$r = 360(e^{(\ln 1.8)/1440} - 1 \approx 0.147$$

14.7%

109. accumulated amount = 2(8000) = 16,000

$$16,000 = 8000e^{0.08t}$$

$$e^{0.08t} = 2$$

$$\ln e^{0.08t} = \ln 2$$

$$0.08t = \ln 2$$

$$t = \frac{\ln 2}{0.08}$$

$$t \approx 8.7$$

The amount would double in 8.7 years.

110. $12,000 = 8000e^{r \cdot 2}$

$$e^{2r} = 1.5$$
$$\ln e^{2r} = \ln 1.5$$
$$2r = \ln 1.5$$
$$r = \frac{\ln 1.5}{2} \approx 0.203$$

20.3%

111. accumulated amount = 3(2350) = 7050

$$7050 = 2350e^{r \cdot 7}$$
$$e^{7r} = 3$$
$$\ln e^{7r} = \ln 3$$
$$7r = \ln 3$$
$$r = \frac{\ln 3}{7} \approx 0.157$$

15.7%

112. $25,000 = 17,425e^{0.0425t}$

$$e^{0.0425t} = \frac{1000}{697}$$
$$\ln e^{0.0425t} = \ln\left(\frac{1000}{697}\right)$$
$$0.0425t = \ln\left(\frac{1000}{697}\right)$$
$$t = \frac{\ln\left(\frac{1000}{697}\right)}{0.0425} \approx 8.5 \text{ years}$$

113. a. 2007 is 5 years after 2002.

$$f(x) = 8 + 38\ln x$$
$$f(5) = 8 + 38\ln 5 \approx 69$$

According to the function, 69% of new cellphones will have cameras in 2007. This overestimates the value shown in the graph by 1%.

b.

$$f(x) = 8 + 38\ln x$$
$$87 = 8 + 38\ln x$$
$$79 = 38\ln x$$
$$\frac{79}{38} = \ln x$$
$$x = e^{\frac{79}{38}}$$
$$x \approx 8$$

If the trend continues, 87% of new cellphones will have cameras 8 years after 2002, or 2010.

114. a. 2006 is 4 years after 2002.

$$f(x) = 8 + 38\ln x$$
$$f(4) = 8 + 38\ln 4 \approx 61$$

According to the function, 61% of new cellphones will have cameras in 2006. This underestimates the value shown in the graph by 2%.

b.

$$f(x) = 8 + 38\ln x$$
$$100 = 8 + 38\ln x$$
$$92 = 38\ln x$$
$$\frac{92}{38} = \ln x$$
$$x = e^{\frac{92}{38}}$$
$$x \approx 11$$

If the trend continues, 100% of new cellphones will have cameras 11 years after 2002, or 2013.

115.

$$P(x) = 95 - 30\log_2 x$$
$$40 = 95 - 30\log_2 x$$
$$30\log_2 x = 45$$
$$\log_2 x = 1.5$$
$$x = 2^{1.5} \approx 2.8$$

Only half the students recall the important features of the lecture after 2.8 days.
(2.8, 50)

116.

$$P(x) = 95 - 30\log_2 x$$
$$0 = 95 - 30\log_2 x$$
$$30\log_2 x = 95$$
$$\log_2 x = \frac{95}{30}$$
$$2^{\frac{95}{30}} = x$$
$$9.0 \approx x$$

(9.0, 0)

117. a.

$$\text{pH} = -\log x$$
$$5.6 = -\log x$$
$$-5.6 = \log x$$
$$x = 10^{-5.6}$$

The hydrogen ion concentration is $10^{-5.6}$ mole per liter.

b.
$$\begin{aligned} pH &= -\log x \\ 2.4 &= -\log x \\ -2.4 &= \log x \\ x &= 10^{-2.4} \end{aligned}$$
The hydrogen ion concentration is $10^{-2.4}$ mole per liter.

c. $\dfrac{10^{-2.4}}{10^{-5.6}} = 10^{-2.4-(-5.6)} = 10^{3.2}$

The concentration of the acidic rainfall in part (b) is $10^{3.2}$ times greater than the normal rainfall in part (a).

118. a.
$$\begin{aligned} pH &= -\log x \\ 2.3 &= -\log x \\ -2.3 &= \log x \\ x &= 10^{-2.3} \end{aligned}$$
The hydrogen ion concentration is $10^{-2.3}$ mole per liter.

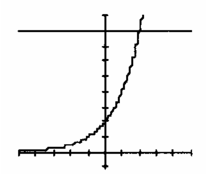
b.
$$\begin{aligned} pH &= -\log x \\ 1 &= -\log x \\ -1 &= \log x \\ x &= 10^{-1} \end{aligned}$$
The hydrogen ion concentration is 10^{-1} mole per liter.

c. $\dfrac{10^{-1}}{10^{-2.3}} = 10^{-1-(-2.3)} = 10^{1.3}$

The concentration of the acidic stomach in part (b) is $10^{1.3}$ times greater than the lemon juice in part (a).

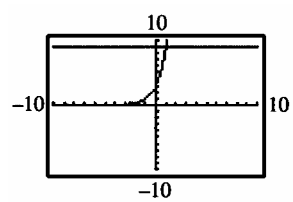
119. – 122. Answers may vary.

123.



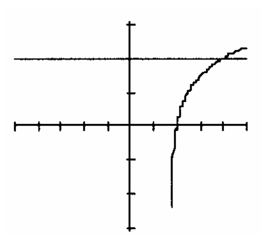
The intersection point is (2, 8).
Verify: $x = 2$
$$\begin{aligned} 2^{x+1} &= 8 \\ 2^{2+1} &= 2 \\ 2^3 &= 8 \\ 8 &= 8 \end{aligned}$$
The solution set is $\{2\}$.

124.



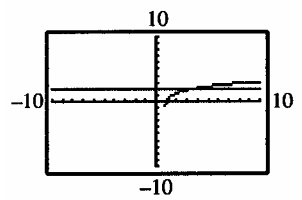
$\{1\}$
The intersection point is (1, 9).
Verify $x = 1$:
$$\begin{aligned} 3^{x+1} &= 9 \\ 3^{1+1} &= 9 \\ 3^2 &= 9 \\ 9 &= 9 \end{aligned}$$
The solution set is $\{1\}$.

125.



The intersection point is (4, 2).
Verify: $x = 4$
$$\begin{aligned} \log_3(4\cdot 4-7) &= 2 \\ \log_3 9 &= 2 \\ 2 &= 2 \end{aligned}$$
The solution set is $\{4\}$.

126.



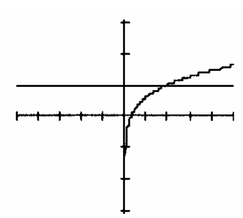
The intersection point is $\left(\frac{11}{3}, 2\right)$.

Verify: $x = \frac{11}{3}$

$$\log_3\left(3 \cdot \frac{11}{3} - 2\right) = 2$$
$$\log_3(11\text{-}2) = 2$$
$$\log_3 9 = 2$$
$$2 = 2$$

The solution set is $\left\{\frac{11}{3}\right\}$.

127.



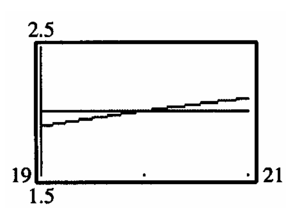
The intersection point is (2, 1).

Verify: $x = 2$

$$\log(2+3) + \log 2 = 1$$
$$\log 5 + \log 2 = 1$$
$$\log(5 \cdot 2) = 1$$
$$\log 10 = 1$$
$$1 = 1$$

The solution set is {2}.

128.



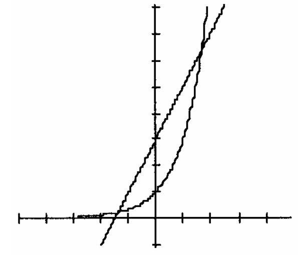
The intersection point is (20, 2).

Verify $x = 20$:

$$\log(x - 15) + \log x = 2$$
$$\log(20 - 15) + \log 20 = 2$$
$$\log 5 + \log 20 = 2$$
$$\log 100 = 2$$
$$100 = 10^2$$
$$100 = 100$$

The solution set is {20}.

129.



There are 2 points of intersection, approximately (–1.391606, 0.21678798) and (1.6855579, 6.3711158).

Verify $x \approx -1.391606$

$3^x = 2x + 3$

$3^{-1.391606} \approx 2(-1.391606) + 3$

$0.2167879803 \approx 0.216788$

Verify $x \approx 1.6855579$

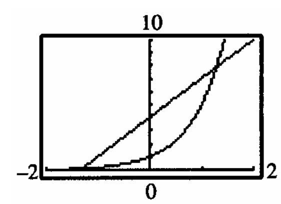
$3^x = 2x + 3$

$3^{1.6855579} \approx 2(1.6855579) + 3$

$6.37111582 \approx 6.371158$

The solution set is {–1.391606, 1.6855579}.

130.



There are 2 points of intersection, approximately (–1.291641, 0.12507831) and (1.2793139, 7.8379416).

Verify: $x \approx -1.291641$

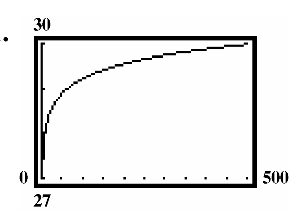
$$5^x = 3x + 4$$
$$5^{-1.291641} = 3(-1.291641) + 4$$
$$0.1250782178 \approx 0.125077$$

Verify: $x \approx 1.2793139$

$$5^{1.2793139} = 3(1.2793139) + 4$$
$$7.837941942 \approx 7.8379417$$

The solution set is {–1.291641, 1,2793139}.

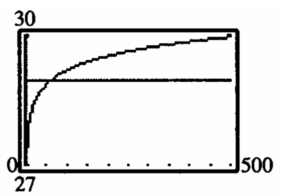
131.



As the distance from the eye increases, barometric air pressure increases, leveling off at about 30 inches of mercury.

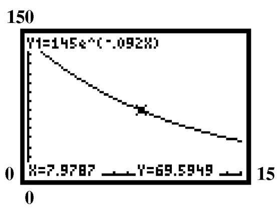
132.
$$29 = 0.48\ln(x+1) + 27$$
$$0.48\ln(x+1) = 2$$
$$\ln(x+1) = \frac{1}{0.24}$$
$$e^{\ln(x+1)} = e^{1/0.24}$$
$$x+1 = e^{1/0.24}$$
$$x = e^{1/0.24} - 1 \approx 63.5$$

The barometric air pressure is 29 inches of mercury at a distance of about 63.5 miles from the eye of a hurricane.



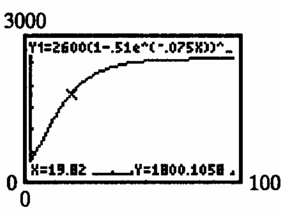
The point of intersection is approximately (63.5, 29).

133.



When $P = 70$, $t \approx 7.9$, so it will take about 7.9 minutes.
Verify:
$$70 = 45e^{-0.092(7.9)}$$
$$70 \approx 70.10076749$$
The runner's pulse will be 70 beats per minute after about 7.9 minutes.

134.



An adult female elephant weighing 1800 kilograms is about 20 years old.

135. does not make sense; Explanations will vary. Sample explanation: $2^x = 15$ requires logarithms. $2^x = 16$ can be solved by rewriting 16 as 2^4.
$$2^x = 15$$
$$\ln 2^x = \ln 15$$
$$x\ln 2 = \ln 15$$
$$x = \frac{\ln 15}{\ln 2}$$

$$2^x = 16$$
$$2^x = 2^4$$
$$x = 4$$

136. does not make sense; Explanations will vary. Sample explanation: The first equation is solved by rewriting it in exponential form. The second equation is solved by using the one-to-one property of logarithms.

137. makes sense

138. makes sense

139. false; Changes to make the statement true will vary. A sample change is: If $\log(x+3) = 2$, then $10^2 = x + 3$.

140. false; Changes to make the statement true will vary. A sample change is: If $\log(7x+3) - \log(2x+5) = 4$, then $\log\left(\frac{7x+3}{2x+5}\right) = 4$, and $10^4 = \frac{7x+3}{2x+5}$.

141. true

142. false; Changes to make the statement true will vary. A sample change is: $x^{10} = 5.71$ is not an exponential equation, because there is not a variable in an exponent.

143. Account paying 3% interest:

$$A = 4000\left(1+\frac{0.03}{1}\right)^{1\cdot t}$$

Account paying 5% interest:

$$A = 2000\left(1+\frac{0.05}{1}\right)^{1\cdot t}$$

The two accounts will have the same balance when

$$4000(1.03)^t = 2000(1.05)^t$$
$$(1.03)^t = 0.5(1.05)^t$$
$$\left(\frac{1.03}{1.05}\right)^t = 0.5$$
$$\ln\left(\frac{1.03}{1.05}\right)^t = \ln 0.5$$
$$t\ln\left(\frac{1.03}{1.05}\right) = \ln 0.5$$
$$t = \frac{\ln 0.5}{\ln\left(\frac{1.03}{1.05}\right)} \approx 36$$

The accounts will have the same balance in about 36 years.

144.

$$(\ln x)^2 = \ln x^2$$
$$(\ln x)^2 = 2\ln x$$
$$(\ln x)^2 - 2\ln x = 0$$
$$\ln x(\ln x - 2) = 0$$

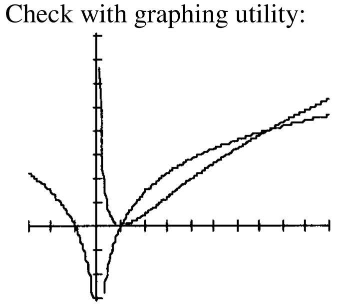
$\ln x = 2$ or $\ln x = 0$

$e^{\ln x} = e^2$ $\quad x = 1$

$x = e^2$

The solution set is $\{1, e^2\}$.

Check with graphing utility:



There are two points of intersection: (1, 0) and approximately (7.3890561, 4). Since $e^2 \approx 7.3890566099$, the graph verifies $x = 1$ and $x = e^2$, so the solution set is $\{1, e^2\}$ as determined algebraically.

145 $(\log x)(2\log x + 1) = 6$

$$2(\log x)^2 + \log x - 6 = 0$$
$$(2\log x - 3)(\log x + 2) = 0$$

$2\log x - 3 = 0$ or $\log x + 2 = 0$

$2\log x = 3$ $\quad \log x = -2$

$\log x = \frac{3}{2}$ $\quad x = 10^{-2}$

$x = 10^{3/2}$ $\quad x = \frac{1}{100}$

$x = 10\sqrt{10}$

The solution set is $\left\{\frac{1}{100}, 10\sqrt{10}\right\}$.

Check by direct substitution:

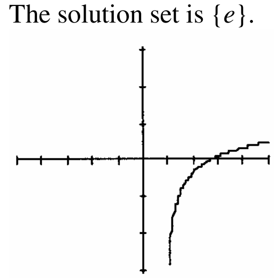
Check: $x = 10\sqrt{10} = 10^{3/2}$

$$(\log x)(2\log x + 1) = 6$$
$$\left(\log 10^{3/2}\right)(2\log 10^{3/2} + 1) = 6$$
$$\left(\frac{3}{2}\right)\left(2\cdot\frac{3}{2}+1\right) = 6$$
$$\left(\frac{3}{2}\right)(3+1) = 6$$
$$\left(\frac{3}{2}\right)(4) = 6$$
$$6 = 6$$

146. $\ln(\ln x) = 0$

$$e^{\ln(\ln x)} = e^0$$
$$\ln x = 1$$
$$e^{\ln x} = e^1$$
$$x = e$$

The solution set is $\{e\}$.



The graph of $\ln(\ln(x))$ crosses the graph $y = 0$ at approximately 2.718.

147. $A = 10e^{-0.003t}$

a. 2006: $A = 10e^{-0.003(0)} = 10$ million

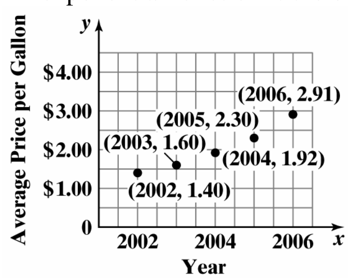
2007: $A = 10e^{-0.003(1)} \approx 9.97$ million

2008: $A = 10e^{-0.003(2)} \approx 9.94$ million

2009: $A = 10e^{-0.003(3)} \approx 9.91$ million

b. The population is decreasing.

148. An exponential function is the best choice.



149. a. $e^{\ln 3} = 3$

b. $$e^{\ln 3} = 3$$
$$\left(e^{\ln 3}\right)^x = 3^x$$
$$e^{(\ln 3)x} = 3^x$$

Section 4.5

Check Point Exercises

1. a. $A_0 = 643$. Since 2006 is 16 years after 1990, when $t = 16$, $A = 906$.

$$A = A_0 e^{kt}$$
$$906 = 643e^{k(16)}$$
$$\frac{906}{643} = e^{16k}$$
$$\ln\left(\frac{906}{643}\right) = \ln e^{16k}$$
$$\ln\left(\frac{906}{643}\right) = 16k$$
$$k = \frac{\ln\left(\frac{906}{643}\right)}{16} \approx 0.021$$

Thus, the growth function is $A = 643e^{0.021t}$.

b.
$$A = 643e^{0.021t}$$
$$2000 = 643e^{0.021t}$$
$$\frac{2000}{643} = e^{0.021t}$$
$$\ln\left(\frac{2000}{643}\right) = \ln e^{0.021t}$$
$$\ln\left(\frac{2000}{643}\right) = 0.021t$$
$$t = \frac{\ln\left(\frac{2000}{643}\right)}{0.021} \approx 54$$

Africa's population will reach 2000 million approximately 54 years after 1990, or 2044.

2. a. In the exponential decay model $A = A_0 e^{kt}$, substitute $\frac{A_0}{2}$ for A since the amount present after 28 years is half the original amount.

$$\frac{A_0}{2} = A_0 e^{k \cdot 28}$$
$$e^{28k} = \frac{1}{2}$$
$$\ln e^{28k} = \ln\frac{1}{2}$$
$$28k = \ln\frac{1}{2}$$
$$k = \frac{\ln^{1/2}}{28} \approx -0.0248$$

So the exponential decay model is $A = A_0 e^{-0.0248t}$

b. Substitute 60 for A_0 and 10 for A in the model from part (a) and solve for t.

$$10 = 60e^{-0.0248t}$$
$$e =^{-0.0248t} = \frac{1}{6}$$
$$\ln e^{-0.0248t} = \ln\frac{1}{6}$$
$$-0.0248t = \ln\frac{1}{6}$$
$$t = \frac{\ln\frac{1}{6}}{-0.0248} \approx 72$$

The strontium-90 will decay to a level of 10 grams about 72 years after the accident.

3. **a.** The time prior to learning trials corresponds to $t = 0$.

$$f(0) = \frac{0.8}{1+e^{-0.2(0)}} = 0.4$$

The proportion of correct responses prior to learning trials was 0.4.

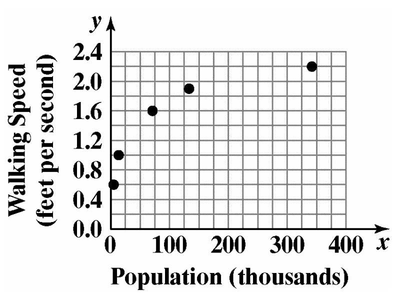
b. Substitute 10 for t in the model:

$$f(10) = \frac{0.8}{1+e^{-0.2(10)}} \approx 0.7$$

The proportion of correct responses after 10 learning trials was 0.7.

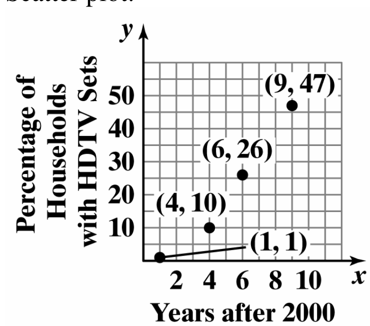
c. In the logistic growth model, $f(t) = \dfrac{c}{1+ae^{-bt}}$, the constant c represents the limiting size that $f(t)$ can attain. The limiting size of the proportion of correct responses as continued learning trials take place is 0.8.

4. Scatter plot:



Because the data in the scatter plot increase rapidly at first and then begin to level off, the shape suggests that a logarithmic function is a good choice for modeling the data.

5. Scatter plot:



Because the data in the scatter plot appear to increase more and more rapidly, the shape suggests that an exponential function is a good choice for modeling the data.

6. $y = ab^x$ is equivalent to $y = ae^{(\ln b)x}$.

For $y = 4(7.8)^x$, $a = 4$, $b = 7.8$.

Thus, $y = 4(7.8)^x$ is equivalent to $y = 4e^{(\ln 7.8)x}$ in terms of a natural logarithm. Rounded to three decimal places, the model is approximately equivalent to $y = 4e^{2.054x}$.

Exercise Set 4.5

1. Since 2006 is 0 years after 2006, find A when $t = 0$:

$$A = 127.5e^{0.001t}$$
$$A = 127.5e^{0.001(0)}$$
$$A = 127.5e^{0}$$
$$A = 127.5(1)$$
$$A = 127.5$$

In 2006, the population of Japan was 127.5 million.

2. Since 2006 is 0 years after 2006, find A when $t = 0$:

$$A = 26.8e^{0.027t}$$
$$A = 26.8e^{0.027(0)}$$
$$A = 26.8e^{0}$$
$$A = 26.8(1)$$
$$A = 26.8$$

In 2006, the population of Iraq was 26.8 million.

3. Iraq has the greatest growth rate at 2.7% per year.

4. Since $k = -0.004$, Russia has a decreasing population. The population is dropping at of 0.4% per year.

5. Substitute $A = 1238$ into the model for India and solve for *t*:

$$1238 = 1095.4e^{0.014t}$$
$$\frac{1238}{1095.4} = e^{0.014t}$$
$$\ln\frac{1238}{1095.4} = \ln e^{0.014t}$$
$$\ln\frac{1238}{1095.4} = 0.014t$$
$$t = \frac{\ln\frac{1238}{1095.4}}{0.014} \approx 9$$

The population of India will be 1238 million approximately 9 years after 2006, or 2015.

6. Substitute $A = 1416$ into the model for India and solve for *t*:

$$1416 = 1095.4e^{0.014t}$$
$$\frac{1416}{1095.4} = e^{0.014t}$$
$$\ln\frac{1416}{1095.4} = \ln e^{0.014t}$$
$$\ln\frac{1416}{1095.4} = 0.014t$$
$$t = \frac{\ln\frac{1416}{1095.4}}{0.014} \approx 18$$

The population of India will be 1416 million approximately 18 years after 2006, or 2024.

7. **a.** $A_0 = 6.04$. Since 2050 is 50 years after 2000, when $t = 50$, $A = 10$.

$$A = A_0e^{kt}$$
$$10 = 6.04e^{k(50)}$$
$$\frac{10}{6.04} = e^{50k}$$
$$\ln\left(\frac{10}{6.04}\right) = \ln e^{50k}$$
$$\ln\left(\frac{10}{6.04}\right) = 50k$$
$$k = \frac{\ln\left(\frac{10}{6.04}\right)}{50} \approx 0.01$$

Thus, the growth function is $A = 6.04e^{0.01t}$.

b.

$$9 = 6.04e^{0.01t}$$
$$\frac{9}{6.04} = e^{0.01t}$$
$$\ln\left(\frac{9}{6.04}\right) = \ln e^{0.01t}$$
$$\ln\left(\frac{9}{6.04}\right) = 0.01t$$
$$t = \frac{\ln\left(\frac{9}{6.04}\right)}{0.01} \approx 40$$

Now, 2000 + 40 = 2040, so the population will be 9 million is approximately the year 2040

8. **a.** $A_0 = 3.2$. Since 2050 is 50 years after 2000, when $t = 50$, $A = 12$.

$$A = A_0e^{kt}$$
$$12 = 3.2e^{k(50)}$$
$$\frac{12}{3.2} = e^{50k}$$
$$\ln\left(\frac{12}{3.2}\right) = \ln e^{50k}$$
$$\ln\left(\frac{12}{3.2}\right) = 50k$$
$$k = \frac{\ln\left(\frac{12}{3.2}\right)}{50} \approx 0.026$$

Thus, the growth function is $A = 3.2e^{0.026t}$.

b.

$$3.2e^{0.026t} = 9$$
$$e^{0.026t} = \frac{9}{3.2}$$
$$\ln e^{0.026t} = \ln\left(\frac{9}{3.2}\right)$$
$$0.026t = \left(\frac{9}{3.2}\right)$$
$$t = \frac{\ln\left(\frac{9}{3.2}\right)}{0.026} \approx 40$$

Now, 2000 + 40 = 2040, so the population will be 9 million is approximately the year 2040.

9.

$$P(x) = 91.1e^{0.0147t}$$
$$P(18) = 91.1e^{0.0147(18)}$$
$$P(18) = 91.1e^{0.0147(18)} \approx 118.7$$

The population is projected to be 118.7 million in 2025.

10. $P(x) = 164.7e^{0.0157t}$

$P(18) = 164.7e^{0.0157(18)}$

$P(18) = 164.7e^{0.0157(18)} \approx 218.5$

The population is projected to be 218.5 million in 2025.

11. $P(x) = 44.4e^{kt}$

$55.2 = 44.4e^{18k}$

$\frac{55.2}{44.4} = e^{18k}$

$\ln\left(\frac{55.2}{44.4}\right) = \ln e^{18k}$

$\ln\left(\frac{55.2}{44.4}\right) = 18k$

$\frac{\ln\left(\frac{55.2}{44.4}\right)}{18} = k$

$k \approx 0.0121$

The growth rate is 0.0121.

12. $P(x) = 19.4e^{kt}$

$32.4 = 19.4e^{18k}$

$\frac{32.4}{19.4} = e^{18k}$

$\ln\left(\frac{32.4}{19.4}\right) = \ln e^{18k}$

$\ln\left(\frac{32.4}{19.4}\right) = 18k$

$\frac{\ln\left(\frac{32.4}{19.4}\right)}{18} = k$

$k \approx 0.0285$

The growth rate is 0.0285.

13. $P(x) = 44.0e^{kt}$

$40.0 = 44.0e^{18k}$

$\frac{40.0}{44.0} = e^{18k}$

$\ln\left(\frac{40.0}{44.0}\right) = \ln e^{18k}$

$\ln\left(\frac{40.0}{44.0}\right) = 18k$

$\frac{\ln\left(\frac{40.0}{44.0}\right)}{18} = k$

$k \approx -0.0053$

The growth rate is –0.0053.

14. $P(x) = 7.3e^{kt}$

$6.3 = 7.3e^{18k}$

$\frac{6.3}{7.3} = e^{18k}$

$\ln\left(\frac{6.3}{7.3}\right) = \ln e^{18k}$

$\ln\left(\frac{6.3}{7.3}\right) = 18k$

$\frac{\ln\left(\frac{6.3}{7.3}\right)}{18} = k$

$k \approx -0.0082$

The growth rate is –0.0082.

15. $A = 16e^{-0.000121t}$

$A = 16e^{-0.000121(5715)}$

$A = 16e^{-0.691515}$

$A \approx 8.01$

Approximately 8 grams of carbon-14 will be present in 5715 years.

16. $A = 16e^{-0.000121t}$

$A = 16e^{-0.000121(11430)}$

$A = 16e^{-1.38303}$

$A \approx 4.01$

Approximately 4 grams of carbon-14 will be present in 11,430 years.

17. After 10 seconds, there will be $16 \cdot \frac{1}{2} = 8$ grams present. After 20 seconds, there will be $8 \cdot \frac{1}{2} = 4$ grams present. After 30 seconds, there will be $4 \cdot \frac{1}{2} = 2$ grams present. After 40 seconds, there will be $2 \cdot \frac{1}{2} = 1$ grams present. After 50 seconds, there will be $1 \cdot \frac{1}{2} = \frac{1}{2}$ gram present.

18. After 25,000 years, there will be $16 \cdot \frac{1}{2} = 8$ grams present. After 50,000 years, there will be $8 \cdot \frac{1}{2} = 4$ grams present. After 75,000 years, there will be $4 \cdot \frac{1}{2} = 2$ grams present. After 100,000 years, there will be $2 \cdot \frac{1}{2} = 1$ gram present. After 125,000 years, there will be $1 \cdot \frac{1}{2} = \frac{1}{2}$ gram present.

19.
$$A = A_0 e^{-0.000121t}$$
$$15 = 100e^{-0.000121t}$$
$$\frac{15}{100} = e^{-0.000121t}$$
$$\ln 0.15 = \ln e^{-0.000121t}$$
$$\ln 0.15 = -0.000121t$$
$$t = \frac{\ln 0.15}{-0.000121} \approx 15,679$$
The paintings are approximately 15,679 years old.

20.
$$A = A_0 e^{-0.000121t}$$
$$88 = 100e^{-0.000121t}$$
$$\frac{88}{100} = e^{-0.000121t}$$
$$\ln 0.88 = \ln e^{-0.000121t}$$
$$\ln 0.88 = -0.000121t$$
$$t = \frac{\ln 0.88}{-0.000121} \approx 1056$$
In 1989, the skeletons were approximately 1056 years old.

21.
$$0.5 = e^{kt}$$
$$0.5 = e^{-0.055t}$$
$$\ln 0.5 = \ln e^{-0.055t}$$
$$\ln 0.5 = -0.055t$$
$$\frac{\ln 0.5}{-0.055} = t$$
$$t \approx 12.6$$
The half-life is 12.6 years.

22.
$$0.5 = e^{kt}$$
$$0.5 = e^{-0.063t}$$
$$\ln 0.5 = \ln e^{-0.063t}$$
$$\ln 0.5 = -0.063t$$
$$\frac{\ln 0.5}{-0.063} = t$$
$$t \approx 11.0$$
The half-life is 11.0 years.

23.
$$0.5 = e^{kt}$$
$$0.5 = e^{1620k}$$
$$\ln 0.5 = \ln e^{1620k}$$
$$\ln 0.5 = 1620k$$
$$\frac{\ln 0.5}{1620} = k$$
$$k \approx -0.000428$$
The decay rate is 0.0428% per year.

24.
$$0.5 = e^{kt}$$
$$0.5 = e^{4560k}$$
$$\ln 0.5 = \ln e^{4560k}$$
$$\ln 0.5 = 4560k$$
$$\frac{\ln 0.5}{4560} = k$$
$$k \approx -0.000152$$
The decay rate is 0.0152% per year.

25.
$$0.5 = e^{kt}$$
$$0.5 = e^{17.5k}$$
$$\ln 0.5 = \ln e^{17.5k}$$
$$\ln 0.5 = 17.5k$$
$$\frac{\ln 0.5}{17.5} = k$$
$$k \approx -0.039608$$
The decay rate is 3.9608% per day.

26.
$$0.5 = e^{kt}$$
$$0.5 = e^{113k}$$
$$\ln 0.5 = \ln e^{113k}$$
$$\ln 0.5 = 113k$$
$$\frac{\ln 0.5}{113} = k$$
$$k \approx -0.006134$$
The decay rate is 0.6134% per hour.

27. **a.**
$$\frac{1}{2} = 1e^{k1.31}$$
$$\ln\frac{1}{2} = \ln e^{1.31k}$$
$$\ln\frac{1}{2} = 1.31k$$
$$k = \frac{\ln\frac{1}{2}}{1.31} \approx -0.52912$$
The exponential model is given by $A = A_0e^{-0.52912t}$.

b.
$$A = A_0e^{-0.52912t}$$
$$0.945A_0 = A_0e^{-0.52912t}$$
$$0.945 = e^{-0.52912t}$$
$$\ln 0.945 = \ln e^{-0.52912t}$$
$$\ln 0.945 = -0.52912t$$
$$t = \frac{\ln 0.945}{-0.52912} \approx 0.1069$$
The age of the dinosaur ones is approximately 0.1069 billion or 106,900,000 years old.

28. First find the decay equation.
$$0.5 = e^{kt}$$
$$0.5 = e^{7340k}$$
$$\ln 0.5 = \ln e^{7340k}$$
$$\ln 0.5 = 7340k$$
$$\frac{\ln 0.5}{7340} = k$$
$$k \approx -0.000094$$
$$A = e^{-0.000094t}$$
Next use the decay equation answer question.
$$A = e^{-0.000094t}$$
$$0.2 = e^{-0.000094t}$$
$$\ln 0.2 = \ln e^{-0.000094t}$$
$$\ln 0.2 = -0.000094t$$
$$\frac{\ln 0.2}{-0.000094} = t$$
$$t \approx 17121.7$$
It will take 17121.7 years.

29. First find the decay equation.
$$0.5 = e^{kt}$$
$$0.5 = e^{22k}$$
$$\ln 0.5 = \ln e^{22k}$$
$$\ln 0.5 = 22k$$
$$\frac{\ln 0.5}{22} = k$$
$$k \approx -0.031507$$
$$A = e^{-0.031507t}$$
Next use the decay equation answer question.
$$A = e^{-0.031507t}$$
$$0.8 = e^{-0.031507t}$$
$$\ln 0.8 = \ln e^{-0.031507t}$$
$$\ln 0.8 = -0.031507t$$
$$\frac{\ln 0.8}{-0.031507} = t$$
$$t \approx 7.1$$
It will take 7.1 years.

30. First find the decay equation.

$$0.5 = e^{kt}$$
$$0.5 = e^{12k}$$
$$\ln 0.5 = \ln e^{12k}$$
$$\ln 0.5 = 12k$$
$$\frac{\ln 0.5}{12} = k$$
$$k \approx -0.057762$$
$$A = e^{-0.057762t}$$

Next use the decay equation answer question.

$$A = e^{-0.057762t}$$
$$0.7 = e^{-0.057762t}$$
$$\ln 0.7 = \ln e^{-0.057762t}$$
$$\ln 0.7 = -0.057762t$$
$$\frac{\ln 0.7}{-0.057762} = t$$
$$t \approx 6.2$$

It will take 6.2 hours.

31. First find the decay equation.

$$0.5 = e^{kt}$$
$$0.5 = e^{36k}$$
$$\ln 0.5 = \ln e^{36k}$$
$$\ln 0.5 = 36k$$
$$\frac{\ln 0.5}{36} = k$$
$$k \approx -0.019254$$
$$A = e^{-0.019254t}$$

Next use the decay equation answer question.

$$A = e^{-0.019254t}$$
$$0.9 = e^{-0.019254t}$$
$$\ln 0.9 = \ln e^{-0.019254t}$$
$$\ln 0.9 = -0.019254t$$
$$\frac{\ln 0.9}{-0.019254} = t$$
$$t \approx 5.5$$

It will take 5.5 hours.

32.

$$A = A_0 e^{kt}$$
$$1000 = 1400 e^{k5}$$
$$\frac{1000}{1400} = e^{5k}$$
$$\ln \frac{5}{7} = 5k$$
$$k = \frac{\ln \frac{5}{7}}{5} \approx -0.0673$$

The exponential model is given by $A = A_0 e^{-0.0673t}$.

$$100 = 1000 e^{-0.0673t}$$
$$\frac{100}{1000} = e^{-0.0673t}$$
$$\ln \frac{1}{10} = \ln e^{-0.0673t}$$
$$\ln \frac{1}{10} = -0.0673t$$
$$t = \frac{\ln \frac{1}{10}}{-0.0673} \approx 34.2$$

The population will drop below 100 birds approximately 34 years from now. (This is 39 years from the time the population was 1400.)

33.

$$2A_0 = A_0 e^{kt}$$
$$2 = e^{kt}$$
$$\ln 2 = \ln e^{kt}$$
$$\ln 2 = kt$$
$$t = \frac{\ln 2}{k}$$

The population will double in $t = \frac{\ln 2}{k}$ years.

34.

$$A = A_0 e^{kt}$$
$$3A_0 = A_0 e^{kt}$$
$$3 = e^{kt}$$
$$\ln 3 = \ln e^{kt}$$
$$\ln 3 = kt$$
$$t = \frac{\ln 3}{k}$$

The population will triple in $t = \frac{\ln 3}{k}$ years.

35. $A = 4.1e^{0.01t}$

a. $k = 0.01$, so New Zealand's growth rate is 1%.

b.
$$\begin{aligned} A &= 4.1e^{0.01t} \\ 2\cdot 4.1 &= 4.1e^{0.01t} \\ 2 &= e^{0.01t} \\ \ln 2 &= \ln e^{0.01t} \\ \ln 2 &= 0.01t \\ t &= \frac{\ln 2}{0.01} \approx 69 \end{aligned}$$

New Zealand's population will double in approximately 69 years.

36. $A = 107.4e^{0.012t}$

a. $k = 0.012$, so Mexico's growth rate is 1.2%.

b.
$$\begin{aligned} A &= 107.4e^{0.012t} \\ 2\cdot 107.4 &= 107.4e^{0.012t} \\ 2 &= e^{0.012t} \\ \ln 2 &= \ln e^{0.012t} \\ \ln 2 &= 0.012t \\ t &= \frac{\ln 2}{0.012} \approx 58 \end{aligned}$$

Mexico's population will double in approximately 58 years.

37. a. When the epidemic began, $t = 0$.

$$f(0) = \frac{100{,}000}{1+5000e^{0}} \approx 20$$

Twenty people became ill when the epidemic began.

b. $$f(4) = \frac{100{,}000}{1+5{,}000e^{-4}} \approx 1080$$

About 1080 people were ill at the end of the fourth week.

c. In the logistic growth model,

$$f(t) = \frac{c}{1+ae^{-bt}},$$

the constant c represents the limiting size that $f(t)$ can attain. The limiting size of the population that becomes ill is 100,000 people.

38. $$f(x) = \frac{11.82}{1+3.81e^{-0.027(x)}}$$

$$f(51) = \frac{11.82}{1+3.81e^{-0.027(51)}} \approx 6.0$$

The function models the data very well.

39. $$f(x) = \frac{11.82}{1+3.81e^{-0.027(x)}}$$

$$f(54) = \frac{11.82}{1+3.81e^{-0.027(57)}} \approx 6.5$$

The function models the data very well.

40.
$$\begin{aligned} f(x) &= \frac{11.82}{1+3.81e^{-0.027(x)}} \\ 7 &= \frac{11.82}{1+3.81e^{-0.027(x)}} \\ 7\left(1+3.81e^{-0.027(x)}\right) &= 11.82 \\ 7+26.67e^{-0.027(x)} &= 11.82 \\ 26.67e^{-0.027(x)} &= 4.82 \\ e^{-0.027(x)} &= \frac{4.82}{26.67} \\ \ln e^{-0.027(x)} &= \ln\frac{4.82}{26.67} \\ -0.027x &= \ln\frac{4.82}{26.67} \\ x &= \frac{\ln\frac{4.82}{26.67}}{-0.027} \\ x &\approx 63 \end{aligned}$$

The world population will reach 7 billion 63 years after 1949, or 2012.

41.
$$\begin{aligned} f(x) &= \frac{11.82}{1+3.81e^{-0.027(x)}} \\ 8 &= \frac{11.82}{1+3.81e^{-0.027(x)}} \\ 8\left(1+3.81e^{-0.027(x)}\right) &= 11.82 \\ 8+30.48e^{-0.027(x)} &= 11.82 \\ 30.48e^{-0.027(x)} &= 3.82 \\ e^{-0.027(x)} &= \frac{3.82}{30.48} \\ \ln e^{-0.027(x)} &= \ln\frac{3.82}{30.48} \\ -0.027x &= \ln\frac{3.82}{30.48} \\ x &= \frac{\ln\frac{3.82}{30.48}}{-0.027} \\ x &\approx 77 \end{aligned}$$

The world population will reach 8 billion 77 years after 1949, or 2026.

42. $f(x) = \dfrac{11.82}{1 + 3.81e^{-0.027(x)}}$

As x increases, the exponent of e will decrease. This will make $e^{-0.027(x)}$ become very close to 0 and make the denominator become very close to 1. Thus, the limiting size of this function is 11.82 billion.

43. $P(20) = \dfrac{90}{1 + 271e^{-0.122(20)}} \approx 3.7$

The probability that a 20-year-old has some coronary heart disease is about 3.7%.

44. $P(80) = \dfrac{90}{1 + 271e^{-0.122(80)}} \approx 88.6$

The probability that an 80-year-old has some coronary heart disease is about 88.6%.

45.

$$0.5 = \frac{0.9}{1 + 271e^{-0.122t}}$$
$$0.5\left(1 + 271e^{-0.122t}\right) = 0.9$$
$$1 + 271e^{-0.122t} = 1.8$$
$$271e^{-0.122t} = 0.8$$
$$e^{-0.122t} = \frac{0.8}{271}$$
$$\ln e^{-0.122t} = \ln\frac{0.8}{271}$$
$$-0.122t = \ln\frac{0.8}{271}$$
$$t = \frac{\ln\frac{0.8}{271}}{-0.122} \approx 48$$

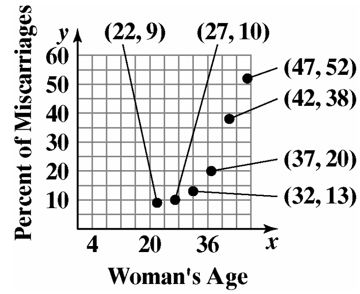
The probability of some coronary heart disease is 50% at about age 48.

46.

$$70 = \frac{90}{1 + 271e^{-0.122x}}$$
$$70(1 + 271e^{-0.122x}) = 90$$
$$1 + 271e^{-0.122x} = \frac{90}{70}$$
$$271e^{-0.122x} = \frac{2}{7}$$
$$e^{-0.122x} = \frac{2}{1897}$$
$$-0.122x = \ln\frac{2}{1897}$$
$$x = \frac{\ln\frac{2}{1897}}{-0.122}$$
$$x \approx 56$$

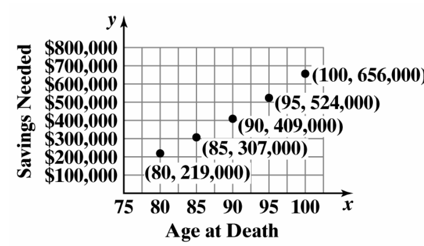
The probability of some coronary heart disease is 70% at about age 56.

47. **a.**



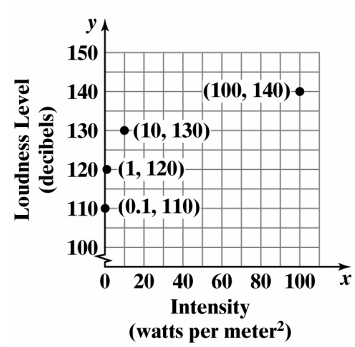
b. An exponential function appears to be the best choice for modeling the data.

48. **a.**



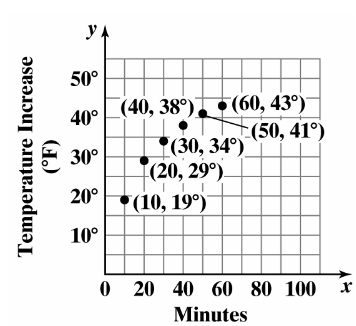
b. An exponential function appears to be the best choice for modeling the data.

49. **a.**



b. A logarithmic function appears to be the best choice for modeling the data.

50. **a.**



b. A logarithmic function appears to be the best choice for modeling the data.

51. $y = 100(4.6)^x$ is equivalent to
$y = 100e^{(\ln 4.6)x}$;
Using $\ln 4.6 \approx 1.526$,
$y = 100e^{1.526x}$.

52. $y = 1000(7.3)^x$ is equivalent to
$y = 1000e^{(\ln 7.3)x}$;
Using $\ln 7.3 \approx 1.988$,
$y = 1000e^{1.988x}$.

53. $y = 2.5(0.7)^x$ is equivalent to
$y = 2.5e^{(\ln 0.7)x}$;
Using $\ln 0.7 \approx -0.357$,
$y = 2.5e^{-0.357x}$.

54. $y = 4.5(0.6)^x$ is equivalent to
$y = 4.5e^{(\ln 0.6)x}$;
Using $\ln 0.6 \approx -0.511$,
$y = 4.5e^{-0.511x}$.

55. – 63. Answers may vary.

64. a. The exponential model is $y = 200.9(1.011)^x$.
Since $r \approx 0.999$ is very close to 1, the model fits the data well.

b. $y = 200.9(1.011)^x$
$y = 200.9e^{(\ln 1.011)x}$
$y = 200.9e^{0.0109x}$
Since $k = .0109$, the population of the United States is increasing by about 1% each year.

65. The logarithmic model is $y = 193.16 + 23.574 \ln x$. Since $r = 0.878$ is fairly close to 1, the model fits the data okay, but not great.

66. The linear model is $y = 2.654x + 198.015$. Since $r \approx 0.997$ is close to 1, the model fits the data very well.

67. The power regression model is $y = 195.871x^{0.097}$. Since $r = 0.901$, the model fits the data fairly well.

68. Using r, the model of best fit is the exponential model $y = 200.9(1.011)^x$.
The model of second best fit is the linear model $y = 2.654x + 198.015$.
Using the exponential model:

$$315 = 200.9(1.011)^x$$
$$\frac{315}{200.9} = (1.011)^x$$
$$\ln\left(\frac{315}{200.9}\right) = \ln(1.011)^x$$
$$\ln\left(\frac{315}{200.9}\right) = x\ln(1.011)$$
$$x = \frac{\ln\left(\frac{315}{200.9}\right)}{\ln(1.011)} \approx 41$$

$1969 + 41 = 2010$
Using the linear model:

$$y = 2.654x + 198.015$$
$$315 = 2.654x + 198.015$$
$$116.985 = 2.654x$$
$$x = \frac{116.985}{2.654} \approx 44$$

$1969 + 44 = 2013$
According to the exponential model, the U.S. population will reach 315 million around the year 2010. According to the linear model, the U.S. population will reach 315 million around the year 2013. Both results are reasonably close to the result found in Example 1 (2010).
Explanations will vary.

69. a. Exponential Regression:
$y = 3.46(1.02)^x$; $r \approx 0.994$
Logarithmic Regression:
$y = 14.752 \ln x - 26.512$; $r \approx 0.673$
Linear Regression:
$y = 0.557x - 10.972$; $r \approx 0.947$
The exponential model has an r value closer to 1. Thus, the better model is $y = 3.46(1.02)^x$.

b. $y = 3.46(1.02)^x$
$y = 3.46e^{(\ln 1.02)x}$
$y = 3.46e^{0.02x}$
The 65-and-over population is increasing by approximately 2% each year.

70. Models and predictions will vary. Sample models are provided

Exercise 47: $y = 1.402(1.078)^x$

Exercise 48: $y = 2896.7(1.056)^x$

Exercise 49: $y = 120 + 4.343\ln x$

Exercise 50: $y = -11.629 + 13.424\ln x$

71. does not make sense; Explanations will vary. Sample explanation: Since the car's value is decreasing (depreciating), the growth rate is negative.

72. does not make sense; Explanations will vary. Sample explanation: This is not necessarily so. Growth rate measures how fast a population is growing relative to that population. It does not indicate how the size of a population compares to the size of another population.

73. makes sense

74. makes sense

75. true

76. true

77. true

78. true

79. Use $T_0 = 210$, $C = 70$, $t = 30$, and $T = 140$ to determine the constant k:

$$T = C + (T_0 - C)e^{-kt}$$
$$140 = 70 + (210 - 70)e^{-k(30)}$$
$$140 = 70 + 140e^{-30k}$$
$$70 = 140e^{-30k}$$
$$\frac{70}{140} = \frac{140e^{-30k}}{140}$$
$$0.5 = e^{-30k}$$
$$\ln 0.5 = \ln e^{-30k}$$
$$\ln 0.5 = -30k$$
$$k = \frac{\ln 0.5}{-30} \approx 0.0231$$

Thus, the model for these conditions is $T = 70 + 140e^{-0.0231t}$.

Evaluate the model for $t = 40$:

$$T = 70 + 140e^{-0.0231(40)} \approx 126$$

Thus, the temperature of the cake after 40 minutes will be approximately $126°F$.

80. Answers may vary.

81.
$$\frac{5\pi}{4} = 2\pi x$$
$$\frac{5\pi}{4\cdot 2\pi} = \frac{2\pi x}{2\pi}$$
$$\frac{5}{8} = x$$

The solution set is $\left\{\frac{5}{8}\right\}$.

82.
$$\frac{17\pi}{6} - 2\pi = \frac{17\pi}{6} - \frac{12\pi}{6} = \frac{17\pi - 12\pi}{6} = \frac{5\pi}{6}$$

83.
$$-\frac{\pi}{12} + 2\pi = -\frac{\pi}{12} + \frac{24\pi}{12} = \frac{-\pi + 24\pi}{12} = \frac{23\pi}{12}$$

Chapter 4 Review Exercises

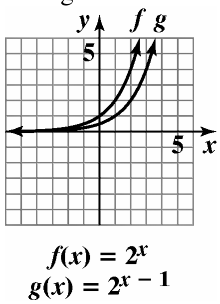
1. This is the graph of $f(x) = 4^x$ reflected about the y-axis, so the function is $g(x) = 4^{-x}$.

2. This is the graph of $f(x) = 4^x$ reflected about the x-axis and about the y-axis, so the function is $h(x) = -4^{-x}$.

3. This is the graph of $f(x) = 4^x$ reflected about the x-axis and about the y-axis then shifted upward 3 units, so the function is $r(x) = -4^{-x} + 3$.

4. This is the graph of $f(x) = 4^x$.

5. The graph of $g(x)$ shifts the graph of $f(x)$ one unit to the right.



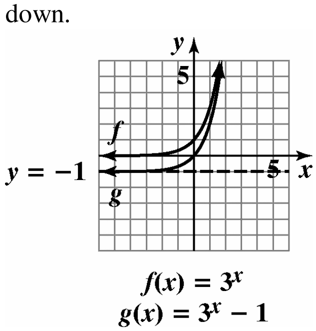
asymptote of f: $y = 0$

asymptote of g: $y = 0$

domain of f = domain of g = $(-\infty, \infty)$

range of f = range of g = $(0, \infty)$

6. The graph of $g(x)$ shifts the graph of $f(x)$ one unit down.



asymptote of f: $y = 0$

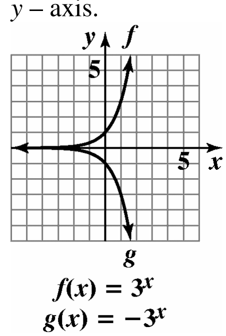
asymptote of g: $y = -1$

domain of f = domain of g = $(-\infty, \infty)$

range of f = $(0, \infty)$

range of g = $(-1, \infty)$

7. The graph of $g(x)$ reflects the graph of $f(x)$ about the y – axis.



asymptote of f: $y = 0$

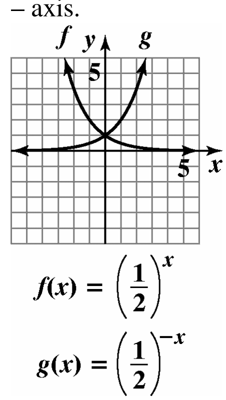
asymptote of g: $y = 0$

domain of f = domain of g = $(-\infty, \infty)$

range of f = $(0, \infty)$

range of g = $(-\infty, 0)$

8. The graph of $g(x)$ reflects the graph of $f(x)$ about the x – axis.



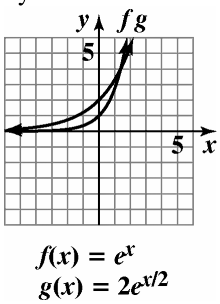
asymptote of f: $y = 0$

asymptote of g: $y = 0$

domain of f = domain of g = $(-\infty, \infty)$

range of f = range of g = $(0, \infty)$

9. The graph of $g(x)$ vertically stretches the graph of $f(x)$ by a factor of 2.



asymptote of f: $y = 0$
asymptote of g: $y = 0$
domain of f = domain of g = $(-\infty, \infty)$
range of f = range of g = $(0, \infty)$

10. 5.5% compounded semiannually:
$A = 5000\left(1+\frac{0.055}{2}\right)^{2\cdot 5} \approx 6558.26$
5.25% compounded monthly:
$A = 5000\left(1+\frac{0.0525}{12}\right)^{12\cdot 5} \approx 6497.16$
5.5% compounded semiannually yields the greater return.

11. 7% compounded monthly:
$A = 14,000\left(1+\frac{0.07}{12}\right)^{12\cdot 10} \approx 28,135.26$
6.85% compounded continuously:
$A = 14,000e^{0.0685(10)} \approx 27,772.81$
7% compounded monthly yields the greater return.

12. a. When first taken out of the microwave, the temperature of the coffee was 200°.

b. After 20 minutes, the temperature of the coffee was about 120°.
$T = 70 + 130e^{-0.04855(20)} \approx 119.23$
Using a calculator, the temperature is about 119°.

c. The coffee will cool to about 70°; The temperature of the room is 70°.

13. $49^{1/2} = 7$

14. $4^3 = x$

15. $3^y = 81$

16. $\log_6 216 = 3$

17. $\log_b 625 = 4$

18. $\log_{13} 874 = y$

19. $\log_4 64 = 3$ because $4^3 = 64$.

20. $\log_5 \frac{1}{25} = -2$ because $5^{-2} = \frac{1}{25}$.

21. $\log_3(-9)$ cannot be evaluated since $\log_b x$ is defined only for $x > 0$.

22. $\log_{16} 4 = \frac{1}{2}$ because $16^{1/2} = \sqrt{16} = 4$.

23. Because $\log_b b = 1$,
we conclude $\log_{17} 17 = 1$.

24. Because $\log_b b^x = x$,
we conclude $\log_3 3^8 = 8$.

25. Because $\ln e^x = x$,
we conclude $\ln e^5 = 5$.

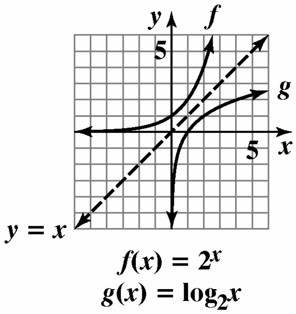
26. $\log_3 \frac{1}{\sqrt{3}} = \log_3 \frac{1}{3^{\frac{1}{2}}} = \log_3 3^{-\frac{1}{2}} = -\frac{1}{2}$

27. $\ln \frac{1}{e^2} = \ln e^{-2} = -2$

28. $\log \frac{1}{1000} = \log \frac{1}{10^3} = \log 10^{-3} = -3$

29. Because $\log_b = 1$,
we conclude $\log_8 8 = 1$.
So, $\log_3(\log_8 8) = \log_3 1$.
Because $\log_b 1 = 0$
we conclude $\log_3 1 = 0$.
Therefore, $\log_3(\log_8 8) = 0$.

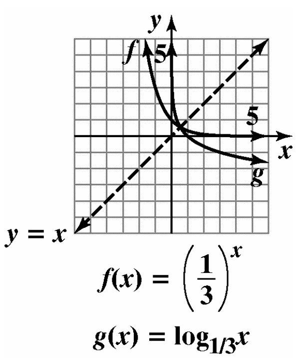
30.



domain of f = range of g = $(-\infty, \infty)$

range of f = domain of g = $(0, \infty)$

31.



domain of f = range of g = $(-\infty, \infty)$

range of f = domain of g = $(0, \infty)$

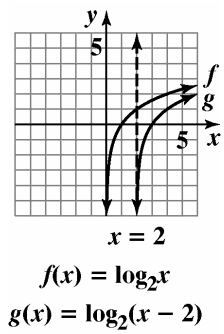
32. This is the graph of $f(x) = \log x$ reflected about the y-axis, so the function is $g(x) = \log(-x)$.

33. This is the graph of $f(x) = \log x$ shifted left 2 units, reflected about the y-axis, then shifted upward one unit, so the function is $r(x) = 1 + \log(2 - x)$.

34. This is the graph of $f(x) = \log x$ shifted left 2 units then reflected about the y-axis, so the function is $h(x) = \log(2 - x)$.

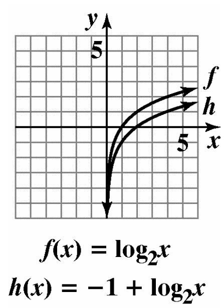
35. This is the graph of $f(x) = \log x$.

36.



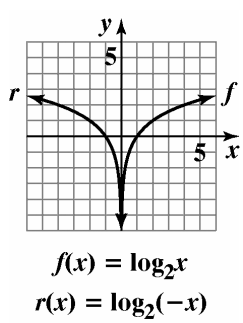
x-intercept: (3, 0)
vertical asymptote: $x = 2$
domain: $(2, \infty)$
range: $(-\infty, \infty)$

37.



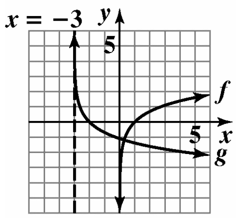
x-intercept: (2, 0)
vertical asymptote: $x = 0$
domain: $(0, \infty)$
range: $(-\infty, \infty)$

38.



x-intercept: (–1, 0)
vertical asymptote: $x = 0$
domain: $(-\infty, 0)$
range: $(-\infty, \infty)$

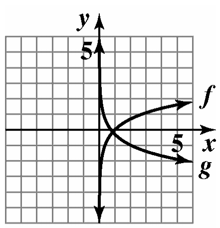
39.



$f(x) = \log x$
$g(x) = -\log(x+3)$

asymptote of *f*: $x = 0$
asymptote of *g*: $x = -3$
domain of *f* = $(0, \infty)$
domain of *g* = $(-3, \infty)$
range of *f* = range of *g* = $(-\infty, \infty)$

40.



$f(x) = \ln x$
$g(x) = -\ln(2x)$

asymptote of *f*: $x = 0$
asymptote of *g*: $x = 0$
domain of *f* = domain of *g* = $(0, \infty)$
range of *f* = range of *g* = $(-\infty, \infty)$

41. The domain of *f* consists of all *x* for which $x + 5 > 0$. Solving this inequality for *x*, we obtain $x > -5$. Thus the domain of *f* is $(-5, \infty)$

42. The domain of *f* consists of all *x* for which $3 - x > 0$. Solving this inequality for *x*, we obtain $x < 3$. Thus, the domain of *f* is $(-\infty, 3)$.

43. The domain of *f* consists of all *x* for which $(x-1)^2 > 0$.
Solving this inequality for *x*, we obtain $x < 1$ or $x > 1$.
Thus, the domain of *f* is $(-\infty, 1) \cup (1, \infty)$.

44. Because $\ln e^x = x$, we conclude $\ln e^{6x} = 6x$.

45. Because $e^{\ln x} = x$, we conclude $e^{\ln \sqrt{x}} = \sqrt{x}$.

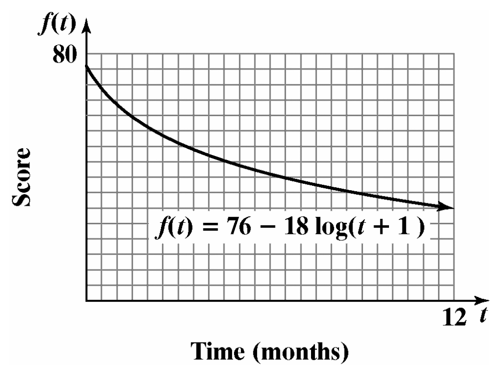
46. Because $10^{\log x} = x$, we conclude $10^{\log 4x^2} = 4x^2$.

47. $R = \log \dfrac{1000 I_0}{I_0} = \log 1000 = 3$
The Richter scale magnitude is 3.0.

48. **a.** $f(0) = 76 - 18\log(0+1) = 76$
When first given, the average score was 76.

b.
$f(2) = 76 - 18\log(2+1) \approx 67$
$f(4) = 76 - 18\log(4+1) \approx 63$
$f(6) = 76 - 18\log(6+1) \approx 61$
$f(8) = 76 - 18\log(8+1) \approx 59$
$f(12) = 76 - 18\log(12+1) \approx 56$
After 2, 4, 6, 8, and 12 months, the average scores are about 67, 63, 61, 59, and 56, respectively.

c.



Retention decreases as time passes.

49. $t = \dfrac{1}{0.06}\ln\left(\dfrac{12}{12-5}\right) \approx 8.98$
It will take about 9 weeks.

50. $\log_6(36x^3)$
$= \log_6 36 + \log_6 x^3$
$= \log_6 36 + 3\log_6 x$
$= 2 + 3\log_6 x$

51. $\log_4 \dfrac{\sqrt{x}}{64} = \log_4 x^{1/2} - \log_4 64$
$= \dfrac{1}{2}\log_4 x - 3$

52. $\log_2 \frac{xy^2}{64} = \log_2 xy^2 - \log_2 64$
$= \log_2 x + \log_2 y^2 - \log_2 64$
$= \log_2 x + 2\log_2{}^y - 6$

53. $\ln \sqrt[3]{\frac{x}{e}}$
$= \ln\left(\frac{x}{e}\right)^{1/3}$
$= \frac{1}{3}[\ln x - \ln e]$
$= \frac{1}{3}\ln x - \frac{1}{3}\ln e$
$= \frac{1}{3}\ln x - \frac{1}{3}$

54. $\log_b 7 + \log_b 3$
$= \log_b (7 \cdot 3)$
$= \log_b 21$

55. $\log 3 - 3\log x$
$= \log 3 - \log x^3$
$= \log \frac{3}{x^3}$

56. $3\ln x + 4\ln y$
$= \ln x^3 + \ln y^4$
$= \ln\left(x^3 y^4\right)$

57. $\frac{1}{2}\ln x - \ln y$
$= \ln x^{1/2} - \ln y$
$= \ln \frac{\sqrt{x}}{y}$

58. $\log_6 72,348 = \frac{\log 72,348}{\log 6} \approx 6.2448$

59. $\log_4 0.863 = \frac{\ln 0.863}{\ln 4} \approx -0.1063$

60. true; $(\ln x)(\ln 1) = (\ln x)(0) = 0$

61. false; $\log(x+9) - \log(x+1) = \log \frac{(x+9)}{(x+1)}$

62. false; $\log_2 x^4 = 4\log_2 x$

63. true; $\ln e^x = x \ln e$

64. $2^{4x-2} = 64$
$2^{4x-2} = 2^6$
$4x - 2 = 6$
$4x = 8$
$x = 2$

65. $125^x = 25$
$\left(5^3\right)^x = 5^2$
$5^{3x} = 5^2$
$3x = 2$
$x = \frac{2}{3}$

66. $10^x = 7000$
$\log 10^x = \log 7000$
$x \log 10 = \log 7000$
$x = \log 7000$
$x \approx 3.85$

67. $9^{x+2} = 27^{-x}$
$\left(3^2\right)^{x+2} = \left(3^3\right)^{-x}$
$3^{2x+4} = 3^{-3x}$
$2x + 4 = -3x$
$5x = -4$
$x = -\frac{4}{5}$

68. $8^x = 12,143$
$\ln 8^x = \ln 12,143$
$x \ln 8 = \ln 12,143$
$x = \frac{\ln 12,143}{\ln 8} \approx 4.52$

69. $9e^{5x} = 1269$
$e^{5x} = 141$
$\ln e^{5x} = \ln 141$
$5x = \ln 141$
$x = \frac{\ln 141}{5} \approx 0.99$

70. $e^{12-5x} - 7 = 123$

$e^{12-5x} = 130$

$\ln e^{12-5x} = \ln 130$

$12-5x = \ln 130$

$5x = 12 - \ln 130$

$x = \dfrac{12 - \ln 130}{5} \approx 1.43$

71. $5^{4x+2} = 37{,}500$

$\ln 5^{4x+2} = \ln 37{,}500$

$(4x+2)\ln 5 = \ln 37{,}500$

$4x \ln 5 + 2\ln 5 = \ln 37{,}500$

$4x \ln 5 = \ln 37{,}500 - 2\ln 5$

$x = \dfrac{\ln 37{,}500 - 2\ln 5}{4\ln 5} \approx 1.14$

72. $3^{x+4} = 7^{2x-1}$

$\ln 3^{x+4} = \ln 7^{2x-1}$

$(x+4)\ln 3 = (2x-1)\ln 7$

$x\ln 3 + 4\ln 3 = 2x\ln 7 - \ln 7$

$x\ln 3 - 2x\ln 7 = -4\ln 3 - \ln 7$

$x(\ln 3 - 2\ln 7) = -4\ln 3 - \ln 7$

$x = \dfrac{-4\ln 3 - \ln 7}{\ln 3 - 2\ln 7}$

$x = \dfrac{4\ln 3 + \ln 7}{2\ln 7 - \ln 3}$

$x \approx 2.27$

73. $e^{2x} - e^x - 6 = 0$

$(e^x - 3)(e^x + 2) = 0$

$e^x - 3 = 0$ or $e^x + 2 = 0$

$e^x = 3$ $\quad e^x = -2$

$\ln e^x = \ln 3$ $\quad \ln e^x - \ln(-2)$

$x = \ln 3$ $\quad x = \ln(-2)$

$x = \ln 3 \approx 1.099$ $\ln(-2)$ does not exist.

The solution set is $\{\ln 3\}$, approximately 1.10.

74. $\log_4(3x-5) = 3$

$3x - 5 = 4^3$

$3x - 5 = 64$

$3x = 69$

$x = 23$

The solutions set is {23}.

75. $3 + 4\ln(2x) = 15$

$4\ln(2x) = 12$

$\ln(2x) = 3$

$2x = e^3$

$x = \dfrac{e^3}{2}$

$x \approx 10.04$

The solutions set is $\left\{\dfrac{e^3}{2}\right\}$.

76. $\log_2(x+3) + \log_2(x-3) = 4$

$\log_2(x+3)(x-3) = 4$

$\log_2(x^2 - 9) = 4$

$x^2 - 9 = 2^4$

$x^2 - 9 = 16$

$x^2 = 25$

$x = \pm 5$

$x = -5$ does not check because $\log_2(-5+3)$ does not exist.

The solution set is {5}.

77. $\log_3(x-1) - \log_3(x+2) = 2$

$\log_3 \dfrac{x-1}{x+2} = 2$

$\dfrac{x-1}{x-2} = 3^2$

$\dfrac{x-1}{x+2} = 9$

$x - 1 = 9(x+2)$

$x - 1 = 9x + 18$

$8x = -19$

$x = -\dfrac{19}{8}$

$x = -\dfrac{19}{8}$ does not check because $\log_3\left(-\dfrac{19}{8} - 1\right)$ does not exist.

The solution set is $\varnothing$.

78. $\ln(x+4) - \ln(x+1) = \ln x$

$$\ln\frac{x+4}{x+1} = \ln x$$
$$\frac{x+4}{x+1} = x$$
$$x(x+1) = x+4$$
$$x^2 + x = x+4$$
$$x^2 = 4$$
$$x = \pm 2$$

$x = -2$ does not check and must be rejected.
The solution set is $\{2\}$.

79. $\log_4(2x+1) = \log_4(x-3) + \log_4(x+5)$

$$\log_4(2x+1) = \log_4(x-3) + \log_4(x+5)$$
$$\log_4(2x+1) = \log_4(x^2+2x-15)$$
$$2x+1 = x^2+2x-15$$
$$16 = x^2$$
$$x^2 = 16$$
$$x = \pm 4$$

$x = -4$ does not check and must be rejected.
The solution set is $\{4\}$.

80. $P(x) = 14.7e^{-0.21x}$

$$4.6 = 14.7e^{-0.21x}$$
$$\frac{4.6}{14.7} = e^{-0.21x}$$
$$\ln\frac{4.6}{14.7} = \ln e^{-0.21x}$$
$$\ln\frac{4.6}{14.7} = -0.21x$$
$$t = \frac{\ln\frac{4.6}{14.7}}{-0.21} \approx 5.5$$

The peak of Mt. Everest is about 5.5 miles above sea level.

81. $f(t) = 364(1.005)^t$

$$560 = 364(1.005)^t$$
$$\frac{560}{364} = (1.005)^t$$
$$\ln\frac{560}{364} = \ln(1.005)^t$$
$$\ln\frac{560}{364} = t\ln 1.005$$
$$t = \frac{\ln\frac{560}{364}}{\ln 1.005} \approx 86.4$$

The carbon dioxide concentration will be double the pre-industrial level approximately 86 years after the year 2000 in the year 2086.

82. $W(x) = 0.37\ln x + 0.05$

$$3.38 = 0.37\ln x + 0.05$$
$$3.33 = 0.37\ln x$$
$$\frac{3.33}{0.37} = \ln x$$
$$9 = \ln x$$
$$e^9 = e^{\ln x}$$
$$x = e^9 \approx 8103$$

The population of New York City is approximately 8103 thousand, or 8,103,000.

83. $20,000 = 12,500\left(1+\frac{0.065}{4}\right)^{4t}$

$$12,500(1.01625)^{4t} = 20,000$$
$$(1.01625)^{4t} = 1.6$$
$$\ln(1.01625)^{4t} = \ln 1.6$$
$$4t\ln 1.01625 = \ln 1.6$$
$$t = \frac{\ln 1.6}{4\ln 1.01625} \approx 7.3$$

It will take about 7.3 years.

84. $3 \cdot 50,000 = 50,000e^{0.075t}$

$$50,000e^{0.075t} = 150,000$$
$$e^{0.075} = 3$$
$$\ln e^{0.075t} = \ln 3$$
$$0.075t = \ln 3$$
$$t = \frac{\ln 3}{0.075} \approx 14.6$$

It will take about 14.6 years.

85. When an investment value triples, $A = 3P$.

$$3P = Pe^{5r}$$
$$e^{5r} = 3$$
$$\ln e^{5r} = \ln 3$$
$$5r = \ln 3$$
$$r = \frac{\ln 3}{5} \approx 0.2197$$

The interest rate would need to be about 22%

86. **a.**

$$35.3 = 22.4e^{k10}$$
$$\frac{35.3}{22.4} = e^{10k}$$
$$\ln\frac{35.3}{22.4} = \ln e^{10k}$$
$$\ln\frac{35.3}{22.4} = 10k$$
$$\frac{\ln\frac{35.3}{22.4}}{10} = k$$
$$0.045 \approx k$$
$$A = 22.4e^{0.045t}$$

b. $A = 22.4e^{0.045(20)} \approx 55.1$

In 2010, the population will be about 55.1 million.

c.

$$60 = 22.4e^{0.045t}$$
$$\frac{60}{22.4} = e^{0.045t}$$
$$\ln\frac{60}{22.4} = \ln e^{0.045t}$$
$$\ln\frac{60}{22.4} = 0.045t$$
$$\frac{\ln\frac{60}{22.4}}{0.045} = t$$
$$22 \approx t$$

The population will reach 60 million about 22 years after 1990, in 2012.

87. Use the half-life of 140 days to find k.

$$A = A_0e^{kt}$$
$$\tfrac{1}{2} = e^{k\cdot 140}$$
$$\tfrac{1}{2} = e^{140k}$$
$$\ln\tfrac{1}{2} = \ln e^{140k}$$
$$\ln\tfrac{1}{2} = 140k$$
$$\frac{\ln\frac{1}{2}}{140} = k$$
$$k \approx -0.004951$$

Use $A = A_0e^{kt}$ to find t.

$$A = A_0e^{-0.004951t}$$
$$0.2 = e^{-0.004951t}$$
$$\ln 0.2 = \ln e^{-0.004951t}$$
$$\ln 0.2 = -0.004951t$$
$$t = \frac{\ln 0.2}{-0.004951}$$
$$t \approx 325$$

It will take about 325 days for the substance to decay to 20% of its original amount.

88. **a.** $f(0) = \dfrac{500,000}{1 + 2499e^{-0.92(0)}} = 200$

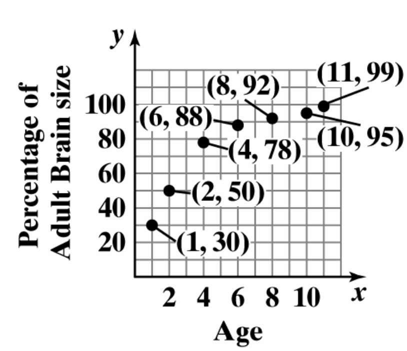
200 people became ill when the epidemic began.

b. $f(6) = \dfrac{500,000}{1 + 2499e^{-0.92(6)}} = 45,411$

45,410 were ill after 6 weeks.

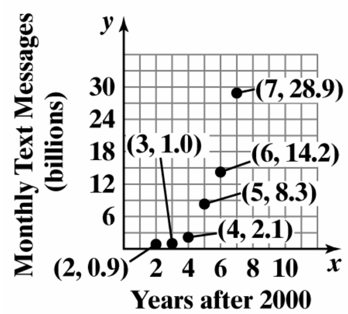
c. 500,000 people

89. **a.**



b. A logarithmic function appears to be the better choice for modeling the data.

90. **a.**



b. An exponential function appears to be the better choice for modeling the data.

91. $y = 73(2.6)^x$

$y = 73e^{(\ln 2.6)x}$

$y = 73e^{0.956x}$

92. $y = 6.5(0.43)^x$

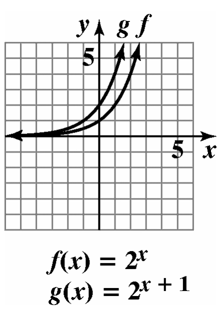
$y = 6.5e^{(\ln 0.43)x}$

$y = 6.5e^{-0.844x}$

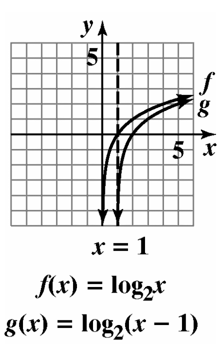
93. Answers may vary.

Chapter 4 Test

1.



2.



3. $5^3 = 125$

4. $\log_{36} 6 = \dfrac{1}{2}$

5. The domain of *f* consists of all *x* for which $3 - x > 0$. Solving this inequality for *x*, we obtain $x < 3$.
Thus, the domain of *f* is $(-\infty, 3)$.

6. $\log_4\left(64x^5\right) = \log_4 64 + \log_4 x^5$
$= 3 + 5\log_4 x$

7. $\log_3 \dfrac{\sqrt[3]{x}}{81} = \log_3 x^{\frac{1}{3}} - \log_3 81$
$= \dfrac{1}{3}\log_3 x - 4$

8. $6\log x + 2\log y = \log x^6 + \log y^2$
$= \log\left(x^6 y^2\right)$

9. $\ln 7 - 3\ln x = \ln 7 - \ln x^3$
$= \ln \dfrac{7}{x^3}$

10. $\log_{15} 71 = \dfrac{\log 71}{\log 15} \approx 1.5741$

11.
$3^{x-2} = 9^{x+4}$
$3^{x-2} = \left(3^2\right)^{x+4}$
$3^{x-2} = 3^{2x+8}$
$x - 2 = 2x + 8$
$-x = 10$
$x = -10$

12.
$5^x = 1.4$
$\ln 5^x = \ln 1.4$
$x\ln 5 = \ln 1.4$
$x = \dfrac{\ln 1.4}{\ln 5} \approx 0.2091$

13.
$400e^{0.005x} = 1600$
$e^{0.005x} = 4$
$\ln e^{0.005x} = \ln 4$
$0.005x = \ln 4$
$x = \dfrac{\ln 4}{0.005} \approx 277.2589$

14. $e^{2x} - 6e^{x} + 5 = 0$

$(e^x - 5)(e^x - 1) = 0$

$e^x - 5 = 0$ or $e^x - 1 = 0$

$e^x = 5 \qquad e^x = 1$

$\ln e^x = \ln 5 \qquad \ln e^x = \ln 1$

$x = \ln 5 \qquad x = \ln 1$

$x \approx 1.6094 \qquad x = 0$

The solution set is $\{0, \ln 5\}$; $\ln \approx 1.6094$.

15. $\log_6(4x-1) = 3$

$4x - 1 = 6^3$

$4x - 1 = 216$

$4x = 217$

$x = \frac{217}{4} = 54.25$

16. $2\ln 3x = 8$

$\ln 3x = 4$

$3x = e^4$

$x = \frac{e^4}{3} \approx 18.1994$

17. $\log x + \log(x+15) = 2$

$\log(x^2 + 15x) = 2$

$x^2 + 15x = 10^2$

$x^2 + 15x - 100 = 0$

$(x+20)(x-5) = 0$

$x + 20 = 0$ or $x - 5 = 0$

$x = -20 \qquad x = 5$

$x = -20$ does not check because $\log(-20)$ does not exist.

The solution set is {5}.

18. $\ln(x-4) - \ln(x+1) = \ln 6$

$\ln \frac{x-4}{x+1} = \ln 6$

$\frac{x-4}{x+1} = 6$

$6(x+1) = x - 4$

$6x + 6 = x - 4$

$5x = -10$

$x = -2$

$x = -2$ does not check and must be rejected.

The solution set is { }.

19. $D = 10\log \frac{10^{12} I_0}{I_0}$

$= 10\log 10^{12}$

$= 10 \cdot 12$

$= 120$

The loudness of the sound is 120 decibels.

20. Since $\ln e^x = x$, $\ln e^{5x} = 5x$.

21. $\log_b b = 1$ because $b^1 = b$.

22. $\log_6 1 = 0$ because $6^0 = 1$.

23. 6.5% compounded semiannually:

$A = 3{,}000\left(1 + \frac{0.065}{2}\right)^{2(10)} \approx \$5{,}687.51$

6% compounded continuously:

$A = 3{,}000e^{0.06(10)} \approx \$5{,}466.36$

6.5% compounded semiannually yields about $221 more than 6% compounded continuously.

24. $8000 = 4000\left(1 + \frac{0.05}{4}\right)^{4t}$

$\frac{8000}{4000} = (1 + 0.0125)^{4t}$

$2 = (1.0125)^{4t}$

$\ln 2 = \ln(1.0125)^{4t}$

$\ln 2 = 4t\ln(1.0125)$

$\frac{\ln 2}{4\ln(1.0125)} = \frac{4t\ln(1.0125)}{4\ln(1.0125)}$

$t = \frac{\ln 2}{4\ln(1.0125)} \approx 13.9$

It will take approximately 13.9 years for the money to grow to $8000.

25. $2 = 1e^{r10}$

$2 = e^{10r}$

$\ln 2 = \ln e^{10r}$

$\ln 2 = 10r$

$r = \frac{\ln 2}{10} \approx 0.069$

The money will double in 10 years with an interest rate of approximately 6.9%.

26. a.
$$A = 82.4e^{-0.002(x)}$$
$$A = 82.4e^{-0.002(0)} \approx 82.4$$
In 2006, the population of Germany was 82.4 million.

b. The population of Germany is decreasing. We can tell because the model has a negative $k = -0.002$.

c.
$$81.5 = 82.4e^{-0.002t}$$
$$\frac{81.5}{82.4} = e^{-0.002t}$$
$$\ln\frac{81.5}{82.4} = \ln e^{-0.002t}$$
$$\ln\frac{81.5}{82.4} = -0.002t$$
$$t = \frac{\ln\frac{81.5}{82.4}}{-0.002} \approx 5$$
The population of Germany will be 81.5 million approximately 5 years after 2006 in the year 2011.

27. In 1990, $t = 0$ and $A_0 = 509$
In 2000, $t = 2000 - 1990 = 10$ and $A = 729$.
$$729 = 509e^{k10}$$
$$\frac{729}{509} = e^{10k}$$
$$\ln\frac{729}{509} = \ln e^{10k}$$
$$\ln\frac{729}{509} = 10k$$
$$\frac{\ln\frac{729}{509}}{10} = k$$
$$0.036 \approx k$$
The exponential growth function is
$A = 509e^{0.036t}$.

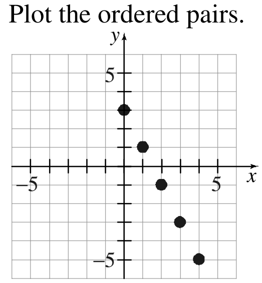
28. First find the decay equation.
$$0.5 = e^{kt}$$
$$0.5 = e^{7.2k}$$
$$\ln 0.5 = \ln e^{7.2k}$$
$$\ln 0.5 = 7.2k$$
$$\frac{\ln 0.5}{7.2} = k$$
$$k \approx -0.096270$$
$$A = e^{-0.096270t}$$
Next use the decay equation answer question.
$$A = e^{-0.096270t}$$
$$0.3 = e^{-0.096270t}$$
$$\ln 0.3 = \ln e^{-0.096270t}$$
$$\ln 0.3 = -0.096270t$$
$$\frac{\ln 0.3}{-0.096270} = t$$
$$t \approx 12.5$$
It will take 12.5 days.

29. a. $f(0) = \dfrac{140}{1+9e^{-0.165(0)}} = 14$
Fourteen elk were initially introduced to the habitat.

b. $f(10) = \dfrac{140}{1+9e^{-0.165(10)}} \approx 51$
After 10 years, about 51 elk are expected.

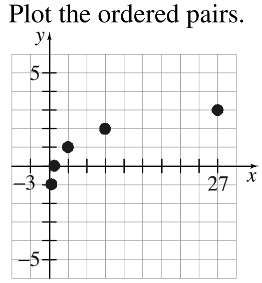
c. In the logistic growth model,
$$f(t) = \frac{c}{1+ae^{-bt}},$$
the constant c represents the limiting size that $f(t)$ can attain. The limiting size of the elk population is 140 elk.

30. Plot the ordered pairs.



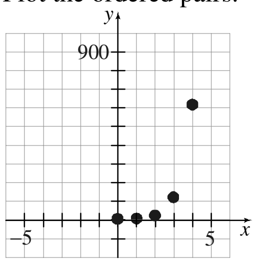
The values appear to belong to a linear function.

31. Plot the ordered pairs.



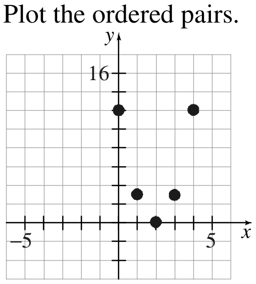
The values appear to belong to a logarithmic function.

32. Plot the ordered pairs.



The values appear to belong to an exponential function.

33. Plot the ordered pairs.



The values appear to belong to a quadratic function.

34.
$$y = 96\left(0.38\right)^x$$
$$y = 96e^{(\ln 0.38)x}$$
$$y = 96e^{-0.968x}$$

Cumulative Review Exercises (Chapters 1–4)

1. $|3x-4| = 2$

$3x - 4 = 2$ or $3x - 4 = -2$

$3x = 6$ $\quad 3x = 2$

$x = 2$ $\quad x = \dfrac{2}{3}$

The solution set is $\left\{\dfrac{2}{3},\ 2\right\}$.

2.
$$\sqrt{2x-5} - \sqrt{x-3} = 1$$
$$\sqrt{2x-5} = 1 + \sqrt{x-3}$$
$$\left(\sqrt{2x-5}\right)^2 = \left(1+\sqrt{x-3}\right)^2$$
$$2x - 5 = 1 + 2\sqrt{x-3} + x - 3$$
$$2x - 5 = x - 2 + 2\sqrt{x-3}$$
$$x - 3 = 2\sqrt{x-3}$$
$$\left(x-3\right)^2 = \left(2\sqrt{x-3}\right)^2$$
$$(x-3)^2 = 4(x-3)$$
$$x^2 - 6x + 9 = 4x - 12$$
$$x^2 - 10x + 21 = 0$$
$$(x-3)(x-7) = 0$$

$x = 3$ or $x = 7$

Both solutions satisfy the original equation when checked.

The solution set is {3, 7}.

3. $x^4 + x^3 - 3x^2 - x + 2 = 0$

p: ±1, ±2

q: ±1

$\dfrac{p}{q}$: ±1, ±2

−2	1	1	−3	−1	2
		−2	2	2	−2
	1	−1	−1	1	0

$$(x+2)(x^3 - x^2 - x + 1) = 0$$
$$(x+2)[x^2(x-1) - (x-1)] = 0$$
$$(x+2)(x^2-1)(x-1) = 0$$
$$(x+2)(x+1)(x-1)(x-1) = 0$$
$$(x+2)(x+1)(x-1)^2 = 0$$

$x + 2 = 0$ or $x + 1 = 0$ or $x - 1 = 0$

$x = -2$ $\quad x = -1$ $\quad x = 1$

The solution set is {–2, –1, 1}.

4.
$$e^{5x} - 32 = 96$$
$$e^{5x} = 128$$
$$\ln e^{5x} = \ln 128$$
$$5x = \ln 128$$
$$x = \frac{\ln 128}{5} \approx 0.9704$$

The solution set is $\left\{\dfrac{\ln 128}{5}\right\}$, approximately 0.9704.

5. $\log_2(x+5)+\log_2(x-1)=4$

$\log_2[(x+5)(x-1)]=4$

$(x+5)(x-1)=2^4$

$x^2+4x-5=16$

$x^2+4x-21=0$

$(x+7)(x-3)=0$

$x+7=0$ or $x-3=0$

$x=-7$ $\quad x=3$

$x=-7$ does not check because $\log_2(-7+5)$ does not exist.

The solution set is $\{3\}$.

6. $\ln(x+4)+\ln(x+1)=2\ln(x+3)$

$\ln((x+4)(x+1))=\ln(x+3)^2$

$(x+4)(x+1)=(x+3)^2$

$x^2+5x+4=x^2+6x+9$

$5x+4=6x+9$

$-x=5$

$x=-5$

$x=-5$ does not check and must be rejected.

The solution set is { }.

7. $14-5x\ge -6$

$-5x\ge -20$

$x\le 4$

The solution set is $(-\infty,4]$.

8. $|2x-4|\le 2$

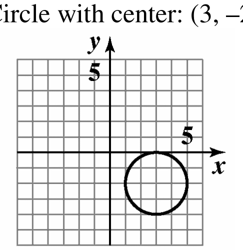
$2x-4\le 2$ and $2x-4\ge -2$

$2x\le 6$ $\quad 2x\ge 2$

$x\le 3$ and $x\ge 1$

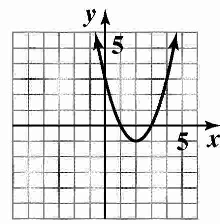
The solution set is $[1,3]$.

9. Circle with center: (3, –2) and radius of 2



$(x-3)^2+(y+2)^2=4$

10. Parabola with vertex: (2, –1)



$f(x)=(x-2)^2-1$

11. x-intercepts:

$x^2-1=0$

$x^2=1$

$x=\pm 1$

The x-intercepts are $(1,0)$ and $(-1,0)$.

vertical asymptotes:

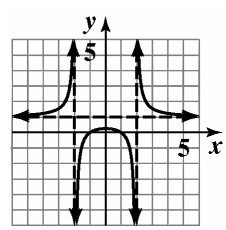
$x^2-4=0$

$x^2=4$

$x=\pm 2$

The vertical asymptotes are $x=2$ and $x=-2$.

Horizontal asymptote: y 5 1



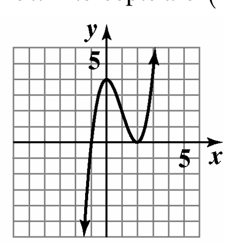
$f(x)=\dfrac{x^2-1}{x^2-4}$

12. x-intercepts:

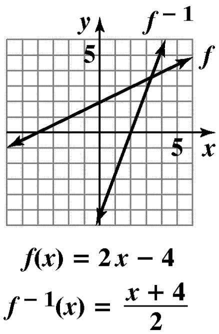
$x-2=0$ or $x+1=0$

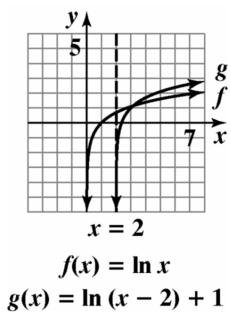
$x=2$ or $x=-1$

The x-intercepts are $(2,0)$ and $(-1,0)$.



$f(x)=(x-2)^2(x+1)$

13. 

14. 

15. $m = \dfrac{3-(-3)}{1-3} = \dfrac{6}{-2} = -3$

Using (1, 3) point-slope form:

$y - 3 = -3(x-1)$

slope-intercept form:

$$y - 3 = -3(x-1)$$
$$y - 3 = -3x + 3$$
$$y = -3x + 6$$

16. $(f \circ g)(x) = f(x+2)$

$$= (x+2)^2$$
$$= x^2 + 4x + 4$$

$(g \circ f)(x) = g(x^2)$

$$= x^2 + 2$$

17. y varies inversely as the square of x is expressed as $y = \dfrac{k}{x^2}$.

The hours, H, vary inversely as the square of the number of cups of coffee, C can be expressed as $H = \dfrac{k}{C^2}$.

Use the given values to find k.

$$H = \frac{k}{C^2}$$
$$8 = \frac{k}{2^2}$$
$$32 = k$$

Substitute the value of k into the equation.

$$H = \frac{k}{C^2}$$
$$H = \frac{32}{C^2}$$

Use the equation to find H when $C = 4$.

$$H = \frac{32}{C^2}$$
$$H = \frac{32}{4^2}$$
$$H = 2$$

If 4 cups of coffee are consumed you should expect to sleep 2 hours.

18. $s(t) = -16t^2 + 64t + 5$

The ball reaches its maximum height at

$t = \dfrac{-b}{2a} = \dfrac{-(64)}{2(-16)} = 2$ seconds.

The maximum height is $s(2)$.

$s(2) = -16(2)^2 + 64(2) + 5 = 69$ feet.

19. $s(t) = -16t^2 + 64t + 5$

Let $s(t) = 0$:

$0 = -16t^2 + 64t + 5$

Use the quadratic formula to solve.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{-(64) \pm \sqrt{(64)^2 - 4(-16)(5)}}{2(-16)}$$

$t \approx 4.1, \quad t \approx -0.1$

The negative value is rejected.

The ball hits the ground after about 4.1 seconds.

20. $40x + 10(1.5x) = 660$

$$40x + 15x = 660$$
$$55x = 660$$
$$x = 12$$

Your normal hourly salary is \$12 per hour.

Chapter 5
Trigonometric Functions

Section 5.1

Check Point Exercises

1. The radian measure of a central angle is the length of the intercepted arc, *s,* divided by the circle's radius, *r.* The length of the intercepted arc is 42 feet: $s = 42$ feet. The circle's radius is 12 feet: $r = 12$ feet. Now use the formula for radian measure to find the radian measure of θ.

$$\theta = \frac{s}{r} = \frac{42 \text{ feet}}{12 \text{ feet}} = 3.5$$

Thus, the radian measure of θ is 3.5

2. **a.** $60° = 60° \cdot \frac{\pi \text{ radians}}{180°} = \frac{60\pi}{180}$ radians $= \frac{\pi}{3}$ radians

b. $270° = 270° \cdot \frac{\pi \text{ radians}}{180°} = \frac{270\pi}{180}$ radians $= \frac{3\pi}{2}$ radians

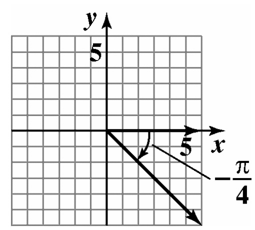
c. $-300° = -300° \cdot \frac{\pi \text{ radians}}{180°} = \frac{-300\pi}{180}$ radians $= -\frac{5\pi}{3}$ radians

3. **a.** $\frac{\pi}{4}$ radians $= \frac{\pi \text{ radians}}{4} \cdot \frac{180^{\circ}}{\pi \text{ radians}} = \frac{180^{\circ}}{4} = 45^{\circ}$

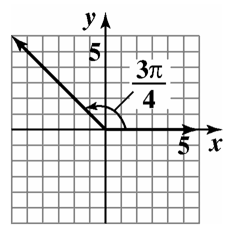
b. $-\frac{4\pi}{3}$ radians $= -\frac{4\pi \text{ radians}}{3} \cdot \frac{180^{\circ}}{\pi} = -\frac{4 \cdot 180^{\circ}}{3} = -240^{\circ}$

c. 6 radians $= 6 \text{ radians} \cdot \frac{180^{\circ}}{\pi \text{ radians}} = \frac{6 \cdot 180^{\circ}}{\pi} \approx 343.8^{\circ}$

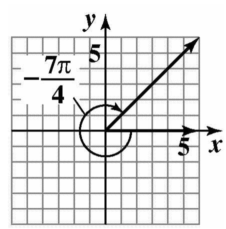
4. **a.**



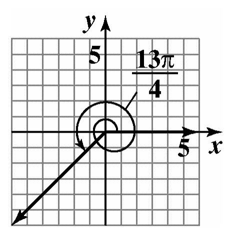
b.



c.



d.



5. **a.** For a 400º angle, subtract 360º to find a positive coterminal angle.
$400^{\circ} - 360^{\circ} = 40^{\circ}$

b. For a –135º angle, add 360º to find a positive coterminal angle.
$-135^{\circ} + 360^{\circ} = 225^{\circ}$

6. **a.** $\frac{13\pi}{5} - 2\pi = \frac{13\pi}{5} - \frac{10\pi}{5} = \frac{3\pi}{5}$

b. $-\frac{\pi}{15} + 2\pi = -\frac{\pi}{15} + \frac{30\pi}{15} = \frac{29\pi}{15}$

7. **a.** $855° - 360° \cdot 2 = 855° - 720° = 135°$

b. $\frac{17\pi}{3} - 2\pi \cdot 2 = \frac{17\pi}{3} - 4\pi$
$= \frac{17\pi}{3} - \frac{12\pi}{3} = \frac{5\pi}{3}$

c. $-\frac{25\pi}{6} + 2\pi \cdot 3 = -\frac{25\pi}{6} + 6\pi$
$= -\frac{25\pi}{6} + \frac{36\pi}{6} = \frac{11\pi}{6}$

8. The formula $s = r\theta$ can only be used when θ is expressed in radians. Thus, we begin by converting 45° to radians. Multiply by $\frac{\pi \text{ radians}}{180°}$.

$45° = 45° \cdot \frac{\pi \text{ radians}}{180°} = \frac{45}{180}\pi \text{ radians}$
$= \frac{\pi}{4} \text{ radians}$

Now we can use the formula $s = r\theta$ to find the length of the arc. The circle's radius is 6 inches : r = 6 inches. The measure of the central angle in radians is $\frac{\pi}{4} : \theta = \frac{\pi}{4}$. The length of the arc intercepted by this central angle is

$s = r\theta = (6 \text{ inches})\left(\frac{\pi}{4}\right) = \frac{6\pi}{4} \text{ inches} \approx 4.71 \text{ inches}.$

9. We are given ω, the angular speed.
$\omega = 45$ revolutions per minute
We use the formula $v = r\omega$ to find v, the linear speed. Before applying the formula, we must express ω in radians per minute.

$\omega = \frac{45 \text{ revolutions}}{1 \text{ minute}} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$
$= \frac{90\pi \text{ radians}}{1 \text{ minute}}$

The angular speed of the propeller is 90π radians per minute. The linear speed is

$v = r\omega = 1.5 \text{ inches} \cdot \frac{90\pi}{1 \text{ minute}} = \frac{135\pi \text{ inches}}{\text{minute}}$

The linear speed is 135π inches per minute, which is approximately 424 inches per minute.

Exercise Set 5.1

1. obtuse

2. obtuse

3. acute

4. acute

5. straight

6. right

7. $\theta = \frac{s}{r} = \frac{40 \text{ inches}}{10 \text{ inches}} = 4 \text{ radians}$

8. $\theta = \frac{s}{r} = \frac{30 \text{ feet}}{5 \text{ feet}} = 6 \text{ radians}$

9. $\theta = \frac{s}{r} = \frac{8 \text{ yards}}{6 \text{ yards}} = \frac{4}{3} \text{ radians}$

10. $\theta = \frac{s}{r} = \frac{18 \text{ yards}}{8 \text{ yards}} = 2.25 \text{ radians}$

11. $\theta = \frac{s}{r} = \frac{400 \text{ centimeters}}{100 \text{ centimeters}} = 4 \text{ radians}$

12. $\theta = \frac{s}{r} = \frac{600 \text{ centimeters}}{100 \text{ centimeters}} = 6 \text{ radians}$

13. $45° = 45° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{45\pi}{180} \text{ radians}$
$= \frac{\pi}{4} \text{ radians}$

14. $18° = 18° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{18\pi}{180} \text{ radians}$
$= \frac{\pi}{10} \text{ radians}$

15. $135° = 135° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{135\pi}{180} \text{ radians}$
$= \frac{3\pi}{4} \text{ radians}$

16. $150° = 150° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{150\pi}{180} \text{ radians}$
$= \frac{5\pi}{6} \text{ radians}$

17. $300° = 300° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{300\pi}{180} \text{ radians}$
$= \frac{5\pi}{3} \text{ radians}$

18. $330° = 330° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{330\pi}{180} \text{ radians}$
$= \frac{11\pi}{6} \text{ radians}$

19. $-225° = -225° \cdot \frac{\pi \text{ radians}}{180°}$
$= -\frac{225\pi}{180} \text{ radians}$
$= -\frac{5\pi}{4} \text{ radians}$

20. $-270° = -270° \cdot \frac{\pi \text{ radians}}{180°}$
$= -\frac{270\pi}{180} \text{ radians}$
$= -\frac{3\pi}{2} \text{ radians}$

21. $\frac{\pi}{2} \text{ radians} = \frac{\pi \text{ radians}}{2} \cdot \frac{180^{\circ}}{\pi \text{ radians}}$
$= \frac{180^{\circ}}{2}$
$= 90^{\circ}$

22. $\frac{\pi}{9} \text{ radians} = \frac{\pi \text{ radians}}{9} \cdot \frac{180^{\circ}}{\pi \text{ radians}}$
$= \frac{180^{\circ}}{9} = 20^{\circ}$

23. $\frac{2\pi}{3} \text{ radians} = \frac{2\pi \text{ radians}}{3} \cdot \frac{180^{\circ}}{\pi \text{ radians}}$
$= \frac{2 \cdot 180^{\circ}}{3}$
$= 120^{\circ}$

24. $\frac{3\pi \text{ radians}}{4} \cdot \frac{180^{0}}{\pi \text{ radians}} = \frac{3 \cdot 180^{\circ}}{4} = 135^{\circ}$

25. $\frac{7\pi}{6} \text{ radians} = \frac{7\pi \text{ radians}}{6} \cdot \frac{180^{\circ}}{\pi \text{ radians}}$
$= \frac{7 \cdot 180^{\circ}}{6}$
$= 210^{\circ}$

26. $\frac{11\pi \text{ radians}}{6} \cdot \frac{180^{\circ}}{\pi \text{ radians}} = \frac{11 \cdot 180^{\circ}}{6} = 330^{\circ}$

27. $-3\pi \text{ radians} = -3\pi \text{ radians} \cdot \frac{180^{\circ}}{\pi \text{ radians}}$
$= -3 \cdot 180^{\circ}$
$= -540^{\circ}$

28. $-4\pi \text{ radians} \cdot \frac{180^{\circ}}{\pi \text{ radians}} = -4 \cdot 180^{\circ} = -720^{\circ}$

29. $18° = 18° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{18\pi}{180} \text{ radians}$
$\approx 0.31 \text{ radians}$

30. $76° = 76° \cdot \frac{\pi \text{ radians}}{180°}$
$= \frac{76\pi}{180} \text{ radians}$
$\approx 1.33 \text{ radians}$

31. $-40° = -40° \cdot \frac{\pi \text{ radians}}{180°}$
$= -\frac{40\pi}{180} \text{ radians}$
$\approx -0.70 \text{ radians}$

32. $-50° = -50° \cdot \frac{\pi \text{ radians}}{180°}$

$= -\frac{50\pi}{180}$ radians

≈ -0.87 radians

33. $200° = 200° \cdot \frac{\pi \text{ radians}}{180°}$

$= \frac{200\pi}{180}$ radians

≈ 3.49 radians

34. $250° = 250° \cdot \frac{\pi \text{ radians}}{180°}$

$= \frac{250\pi}{180}$ radians

≈ 4.36 radians

35. $2 \text{ radians} = 2 \text{ radians} \cdot \frac{180^{\circ}}{\pi \text{ radians}}$

$= \frac{2 \cdot 180^{\circ}}{\pi}$

$\approx 114.59^{\circ}$

36. $3 \text{ radians} \cdot \frac{180^{\circ}}{\pi \text{ radians}} = \frac{3 \cdot 180^{\circ}}{\pi} \approx 171.89^{\circ}$

37. $\frac{\pi}{13} \text{ radians} = \frac{\pi \text{ radians}}{13} \cdot \frac{180^{\circ}}{\pi \text{ radians}}$

$= \frac{180^{\circ}}{13}$

$\approx 13.85^{\circ}$

38. $\frac{\pi}{17} \text{radians} \cdot \frac{180^{\circ}}{\pi \text{radians}} = \frac{180^{\circ}}{17} \approx 10.59^{\circ}$

39. $-4.8 \text{ radians} = -4.8 \text{ radians} \cdot \frac{180^{\circ}}{\pi \text{ radians}}$

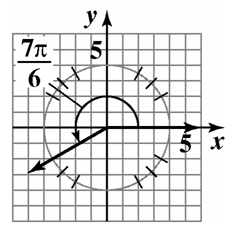
$= \frac{-4.8 \cdot 180^{\circ}}{\pi}$

$\approx -275.02^{\circ}$

40. $-5.2 \text{ radians} \cdot \frac{180^{\circ}}{\pi \text{radians}} = \frac{-5.2 \cdot 180^{\circ}}{\pi}$

$\approx -297.94^{\circ}$

41.



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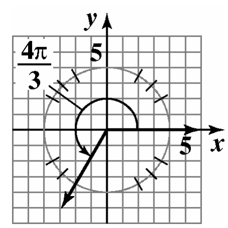

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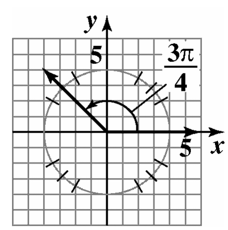

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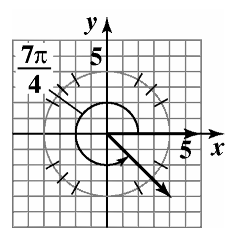

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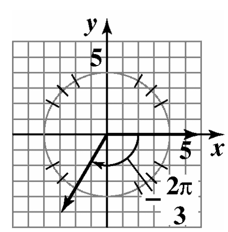

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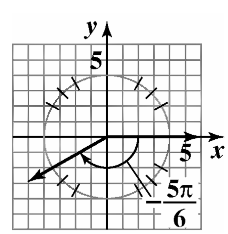

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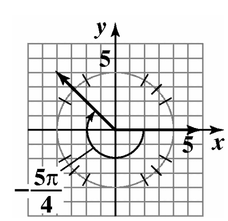

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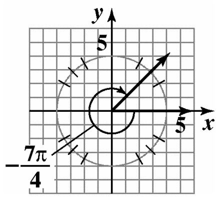

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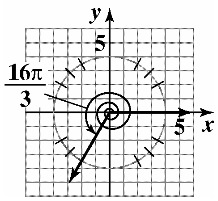

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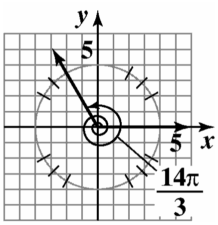

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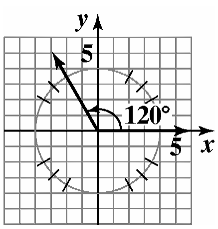

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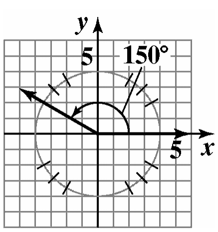

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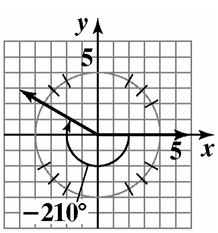

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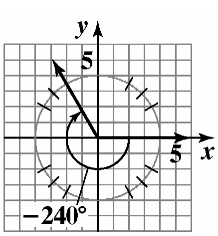

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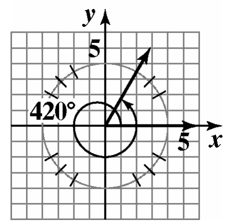

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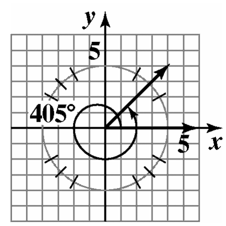


57. $395° - 360° = 35°$

58. $415° - 360° = 55°$

59. $-150° + 360° = 210°$

60. $-160° + 360° = 200°$

61. $-765° + 360° \cdot 3 = -765° + 1080° = 315°$

62. $-760° + 360° \cdot 3 = -760° + 1080° = 320°$

63. $\frac{19\pi}{6} - 2\pi = \frac{19\pi}{6} - \frac{12\pi}{6} = \frac{7\pi}{6}$

64. $\frac{17\pi}{5} - 2\pi = \frac{17\pi}{5} - \frac{10\pi}{5} = \frac{7\pi}{5}$

65. $\frac{23\pi}{5} - 2\pi \cdot 2 = \frac{23\pi}{5} - 4\pi = \frac{23\pi}{5} - \frac{20\pi}{5} = \frac{3\pi}{5}$

66. $\frac{25\pi}{6} - 2\pi \cdot 2 = \frac{25\pi}{6} - 4\pi = \frac{25\pi}{6} - \frac{24\pi}{6} = \frac{\pi}{6}$

67. $-\frac{\pi}{50} + 2\pi = -\frac{\pi}{50} + \frac{100\pi}{50} = \frac{99\pi}{50}$

68. $-\frac{\pi}{40} + 2\pi = -\frac{\pi}{40} + \frac{80\pi}{40} = \frac{79\pi}{40}$

69. $-\frac{31\pi}{7} + 2\pi \cdot 3 = -\frac{31\pi}{7} + 6\pi$

$= -\frac{31\pi}{7} + \frac{42\pi}{7} = \frac{11\pi}{7}$

70. $-\frac{38\pi}{9}+2\pi\cdot 3=-\frac{38\pi}{9}+6\pi$

$=-\frac{38\pi}{9}+\frac{54\pi}{9}=\frac{16\pi}{9}$

71. $r=12$ inches, $\theta=45°$

Begin by converting 45° to radians, in order to use the formula $s=r\theta$.

$45°=45°\cdot\frac{\pi \text{ radians}}{180°}=\frac{\pi}{4}$ radians

Now use the formula $s=r\theta$.

$s=r\theta=12\cdot\frac{\pi}{4}=3\pi$ inches ≈ 9.42 inches

72. $r=16$ inches, $\theta=60°$

Begin by converting 60° to radians, in order to use the formula $s=r\theta$.

$60°=60°\cdot\frac{\pi \text{ radians}}{180°}=\frac{\pi}{3}$ radians

Now use the formula $s=r\theta$.

$s=r\theta=16\cdot\frac{\pi}{3}=\frac{16\pi}{3}$ inches ≈ 16.76 inches

73. $r=8$ feet, $\theta=225°$

Begin by converting 225° to radians, in order to use the formula $s=r\theta$.

$225°=225°\cdot\frac{\pi \text{ radians}}{180°}=\frac{5\pi}{4}$ radians

Now use the formula $s=r\theta$.

$s=r\theta=8\cdot\frac{5\pi}{4}=10\pi$ feet ≈ 31.42 feet

74. $r=9$ yards, $\theta=315°$

Begin by converting 315° to radians, in order to use the formula $s=r\theta$.

$315°=315°\cdot\frac{\pi \text{ radians}}{180°}=\frac{7\pi}{4}$ radians

Now use the formula $s=r\theta$.

$s=r\theta=9\cdot\frac{7\pi}{4}=\frac{63\pi}{4}$ yards ≈ 49.48 yards

75. 6 revolutions per second

$=\frac{6 \text{ revolutions}}{1 \text{ second}}\cdot\frac{2\pi \text{ radians}}{1 \text{ revolutions}}=\frac{12\pi \text{ radians}}{1 \text{ seconds}}$

$=12\pi$ radians per second

76. 20 revolutions per second

$=\frac{20 \text{ revolutions}}{1 \text{ second}}\cdot\frac{2\pi \text{ radians}}{1 \text{ revolution}}=\frac{40\pi \text{ radians}}{1 \text{ second}}$

$=40\pi$ radians per second

77. $-\frac{4\pi}{3}$ and $\frac{2\pi}{3}$

78. $-\frac{7\pi}{6}$ and $\frac{5\pi}{6}$

79. $-\frac{3\pi}{4}$ and $\frac{5\pi}{4}$

80. $-\frac{\pi}{4}$ and $\frac{7\pi}{4}$

81. $-\frac{\pi}{2}$ and $\frac{3\pi}{2}$

82. $-\pi$ and π

83. $\frac{55}{60}\cdot 2\pi=\frac{11\pi}{6}$

84. $\frac{35}{60}\cdot 2\pi=\frac{7\pi}{6}$

85. 3 minutes and 40 seconds equals 220 seconds.

$\frac{220}{60}\cdot 2\pi=\frac{22\pi}{3}$

86. 4 minutes and 25 seconds equals 265 seconds.

$\frac{265}{60}\cdot 2\pi=\frac{53\pi}{6}$

87. First, convert to degrees.

$\frac{1}{6}$ revolution $=\frac{1}{6}$ revolution $\cdot\frac{360°}{1 \text{ revolution}}$

$=\frac{1}{6}\cdot 360°=60°$

Now, convert 60° to radians.

$60°=60°\cdot\frac{\pi \text{ radians}}{180°}=\frac{60\pi}{180}$ radians

$=\frac{\pi}{3}$ radians

Therefore, $\frac{1}{6}$ revolution is equivalent to 60° or $\frac{\pi}{3}$ radians.

88. First, convert to degrees.

$$\frac{1}{3}\text{revolutions} = \frac{1}{3}\text{ revolutions}\cdot\frac{360^{\circ}}{1\text{ revolution}}$$
$$= \frac{1}{3}\cdot 360^{\circ} = 120^{\circ}$$

Now, convert 120° to radians.

$$120^\circ = 120^\circ\cdot\frac{\pi\text{ radians}}{180^\circ} = \frac{120\pi}{180}\text{ radians}$$
$$= \frac{2\pi}{3}\text{ radians}$$

Therefore, $\frac{1}{3}$ revolution is equivalent to 120° or $\frac{2\pi}{3}$ radians.

89. The distance that the tip of the minute hand moves is given by its arc length, s. Since $s = r\theta$, we begin by finding r and θ. We are given that $r = 8$ inches. The minute hand moves from 12 to 2 o'clock, or $\frac{1}{6}$ of a complete revolution. The formula $s = r\theta$ can only be used when θ is expressed in radians. We must convert $\frac{1}{6}$ revolution to radians.

$$\frac{1}{6}\text{ revolution} = \frac{1}{6}\text{ revolution}\cdot\frac{2\pi\text{ radians}}{1\text{ revolution}}$$
$$= \frac{\pi}{3}\text{ radians}$$

The distance the tip of the minute hand moves is

$$s = r\theta = (8\text{ inches})\left(\frac{\pi}{3}\right) = \frac{8\pi}{3}\text{ inches}$$
$$\approx 8.38\text{ inches.}$$

90. The distance that the tip of the minute hand moves is given by its arc length, s. Since $s = r\theta$, we begin by finding r and θ. We are given that $r = 6$ inches. The minute hand moves from 12 to 4 o'clock, or $\frac{1}{3}$ of a complete revolution. The formula $s = r\theta$ can only be used when θ is expressed in radians. We must convert $\frac{1}{3}$ revolution to radians.

$$\frac{1}{3}\text{ revolution} = \frac{1}{3}\text{revolution}\cdot\frac{2\pi\text{ radians}}{1\text{ revolution}}$$
$$= \frac{2\pi}{3}\text{ radians}$$

The distance the tip of the minute hand moves is

$$s = r\theta = (6\text{ inches})\left(\frac{2\pi}{3}\right) = \frac{12\pi}{3}\text{ inches}$$
$$= 4\pi\text{ inches} \approx 12.57\text{ inches.}$$

91. The length of each arc is given by $s = r\theta$. We are given that $r = 24$ inches and $\theta = 90°$. The formula $s = r\theta$ can only be used when θ is expressed in radians.

$$90^\circ = 90^\circ\cdot\frac{\pi\text{ radians}}{180^\circ} = \frac{90\pi}{180}\text{ radians}$$
$$= \frac{\pi}{2}\text{ radians}$$

The length of each arc is

$$s = r\theta = (24\text{ inches})\left(\frac{\pi}{2}\right) = 12\pi\text{ inches}$$
$$\approx 37.70\text{ inches.}$$

92. The distance that the wheel moves is given by $s = r\theta$. We are given that $r = 80$ centimeters and $\theta = 60°$. The formula $s = r\theta$ can only be used when θ is expressed in radians.

$$60^\circ = 60^\circ\cdot\frac{\pi\text{ radians}}{180^\circ} = \frac{60\pi}{180}\text{ radians}$$
$$= \frac{\pi}{3}\text{ radians}$$

The length that the wheel moves is

$$s = r\theta = (80\text{ centimeters})\left(\frac{\pi}{3}\right) = \frac{80\pi}{3}\text{ centimeters}$$
$$\approx 83.78\text{ centimeters.}$$

93. Recall that $\theta = \frac{s}{r}$. We are given that $s = 8000$ miles and $r = 4000$ miles.

$$\theta = \frac{s}{r} = \frac{8000\text{ miles}}{4000\text{ miles}} = 2\text{ radians}$$

Now, convert 2 radians to degrees.

$$2\text{ radians} = 2\text{ radians}\cdot\frac{180^{\circ}}{\pi\text{ radians}} \approx 114.59^{\circ}$$

94. Recall that $\theta = \frac{s}{r}$. We are given that $s = 10{,}000$ miles and $r = 4000$ miles.

$$\theta = \frac{s}{r} = \frac{10{,}000\text{ miles}}{4000\text{ miles}} = 2.5\text{ radians}$$

Now, convert 2.5 radians to degrees.

$$2.5\text{ radians}\cdot\frac{180^{\circ}}{2\pi\text{ radians}} \approx 143.24^{\circ}$$

95. Recall that $s = r\theta$. We are given that $r = 4000$ miles and $\theta = 30°$. The formula $s = r\theta$ can only be used when θ is expressed in radians.

$$30° = 30° \cdot \frac{\pi \text{ radians}}{180°} = \frac{30\pi}{180} \text{ radians}$$

$$= \frac{\pi}{6} \text{ radians}$$

$$s = r\theta = (4000 \text{ miles})\left(\frac{\pi}{6}\right) \approx 2094 \text{ miles}$$

To the nearest mile, the distance from *A* to *B* is 2094 miles.

96. Recall that $s = r\theta$. We are given that $r = 4000$ miles and $\theta = 10°$. We can only use the formula $s = r\theta$ when θ is expressed in radians.

$$10° = 10° \cdot \frac{\pi \text{ radians}}{180°} = \frac{10\pi}{180} \text{ radians}$$

$$= \frac{\pi}{18} \text{ radians}$$

$$s = r\theta = (4000 \text{ miles})\left(\frac{\pi}{18}\right) \approx 698 \text{ miles}$$

To the nearest mile, the distance from *A* to *B* is 698 miles.

97. Linear speed is given by $v = r\omega$. We are given that $\omega = \frac{\pi}{12}$ radians per hour and $r = 4000$ miles. Therefore,

$$v = r\omega = (4000 \text{ miles})\left(\frac{\pi}{12}\right)$$

$$= \frac{4000\pi}{12} \text{ miles per hour}$$

$$\approx 1047 \text{ miles per hour}$$

The linear speed is about 1047 miles per hour.

98. Linear speed is given by $v = r\omega$. We are given that $r = 25$ feet and the wheel rotates at 3 revolutions per minute. We need to convert 3 revolutions per minute to radians per minute.

3 revolutions per minute

$$= 3 \text{ revolutions per minute} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$

$$= 6\pi \text{ radians per minute}$$

$$v = r\omega = (25 \text{ feet})(6\pi) \approx 471 \text{ feet per minute}$$

The linear speed of the Ferris wheel is about 471 feet per minute.

99. Linear speed is given by $v = r\omega$. We are given that $r = 12$ feet and the wheel rotates at 20 revolutions per minute.

20 revolutions per minute

$$= 20 \text{ revolutions per minute} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$

$$= 40\pi \text{ radians per minute}$$

$$v = r\omega = (12 \text{ feet})(40\pi)$$

$$\approx 1508 \text{ feet per minute}$$

The linear speed of the wheel is about 1508 feet per minute.

100. Begin by converting 2.5 revolutions per minute to radians per minute.

2.5 revolutions per minute

$$= 2.5 \text{ revolutions per minute} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$

$$= 5\pi \text{ radians per minute}$$

The linear speed of the animals in the outer rows is

$$v = r\omega = (20 \text{ feet})(5\pi) \approx 100 \text{ feet per minute}$$

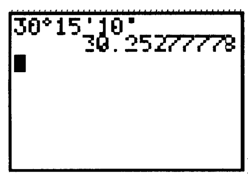
The linear speed of the animals in the inner rows is

$$v = r\omega = (10 \text{ feet})(5\pi) \approx 50 \text{ feet per minute}$$

The difference is $100\pi - 50\pi = 50\pi$ feet per minute or about 157.08 feet per minute.

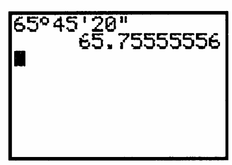
101. – 112. Answers may vary.

113.



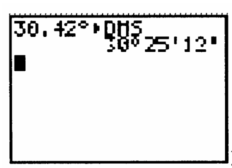
30.25°

114.



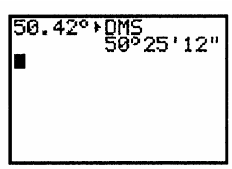
65.76°

115.



30° 25'12"

116.



50° 25' 12"

117. does not make sense; Explanations will vary. Sample explanation: Angles greater than π will exceed a straight angle.

118. does not make sense; Explanations will vary. Sample explanation: It is possible for π to be used in an angle measured using degrees.

119. makes sense

120. does not make sense; Explanations will vary. Sample explanation: That will not be possible if the angle is a multiple of 2π.

121. A right angle measures 90° and

$90° = \frac{\pi}{2}$ radians ≈ 1.57 radians.

If $\theta = \frac{3}{2}$ radians $= 1.5$ radians, θ is smaller than a right angle.

122. $s = r\theta$

Begin by changing $\theta = 20°$ to radians.

$$20° = 20° \cdot \frac{\pi}{180°} = \frac{\pi}{9} \text{ radians}$$

$$100 = \frac{\pi}{9} r$$

$$r = \frac{900}{\pi} \approx 286 \text{ miles}$$

To the nearest mile, a radius of 286 miles should be used.

123. $s = r\theta$

Begin by changing $\theta = 26°$ to radians.

$$26° = 26° \cdot \frac{\pi}{180°} = \frac{13\pi}{90} \text{ radians}$$

$$s = 4000 \cdot \frac{13\pi}{90}$$

$$\approx 1815 \text{ miles}$$

To the nearest mile, Miami, Florida is 1815 miles north of the equator.

124. First find the hypotenuse.

$$c^2 = a^2 + b^2$$
$$c^2 = 5^2 + 12^2$$
$$c^2 = 25 + 144$$
$$c^2 = 169$$
$$c = 13$$

Next write the ratio.

$$\frac{a}{c} = \frac{5}{13}$$

125. First find the hypotenuse.

$$c^2 = a^2 + b^2$$
$$c^2 = 1^2 + 1^2$$
$$c^2 = 1 + 1$$
$$c^2 = 2$$
$$c = \sqrt{2}$$

Next write the ratio and simplify.

$$\frac{a}{c} = \frac{1}{\sqrt{2}}$$
$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{2}}{2}$$

126. $$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2}{c^2} + \frac{b^2}{c^2}$$
$$= \frac{a^2 + b^2}{c^2}$$

Since $c^2 = a^2 + b^2$, continue simplifying by substituting c^2 for $a^2 + b^2$.

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2}{c^2} + \frac{b^2}{c^2}$$
$$= \frac{a^2 + b^2}{c^2}$$
$$= \frac{\overbrace{a^2 + b^2}^{c^2}}{c^2}$$
$$= \frac{c^2}{c^2}$$
$$= 1$$

Section 5.2

Checkpoint Exercises

1. Use the Pythagorean Theorem, $c^2 = a^2 + b^2$, to find c.

$a = 3, b = 4$

$c^2 = a^2 + b^2 = 3^2 + 4^2 = 9 + 16 = 25$

$c = \sqrt{25} = 5$

Referring to these lengths as opposite, adjacent, and hypotenuse, we have

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{3}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{4}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{4}{3}$$

2. Use the Pythagorean Theorem, $c^2 = a^2 + b^2$, to find b.

$a^2 + b^2 = c^2$

$1^2 + b^2 = 5^2$

$1 + b^2 = 25$

$b^2 = 24$

$b = \sqrt{24} = 2\sqrt{6}$

Note that side a is opposite θ and side b is adjacent to θ.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{5}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2\sqrt{6}}{5}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{1} = 5$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{2\sqrt{6}}{1} = 2\sqrt{6}$$

3. Apply the definitions of these three trigonometric functions.

$$\csc 45° = \frac{\text{length of hypotenuse}}{\text{length of side opposite } 45°} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\sec 45° = \frac{\text{length of hypotenuse}}{\text{length of side adjacent to } 45°} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\cot 45° = \frac{\text{length of side adjacent to } 45°}{\text{length of side opposite } 45°} = \frac{1}{1} = 1$$

4. $$\tan 60° = \frac{\text{length of side opposite } 60°}{\text{length of side adjacent to } 60°} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\tan 30° = \frac{\text{length of side opposite } 30°}{\text{length of side adjacent to } 30°} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{3}$$

5. $$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}} = \frac{2}{3} \cdot \frac{3}{\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{5}}{3}} = \frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{2}{\sqrt{5}}} = \frac{\sqrt{5}}{2}$$

6. We can find the value of $\cos\theta$ by using the Pythagorean identity.

$$\sin^2\theta+\cos^2\theta=1$$
$$\left(\frac{1}{2}\right)^2+\cos^2\theta=1$$
$$\frac{1}{4}+\cos^2\theta=1$$
$$\cos^2\theta=1-\frac{1}{4}$$
$$\cos^2\theta=\frac{3}{4}$$
$$\cos\theta=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$$

7. **a.** $\sin 46^{\circ}=\cos(90^{\circ}-46^{\circ})=\cos 44^{\circ}$

b. $\cot\frac{\pi}{12}=\tan\left(\frac{\pi}{2}-\frac{\pi}{12}\right)$
$=\tan\left(\frac{6\pi}{12}-\frac{\pi}{12}\right)$
$=\tan\frac{5\pi}{12}$

8.

	Many Scientific Calculators			
	Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
a.	sin 72.8°	Degree	72.8 [SIN]	0.9553
b.	csc 1.5	Radian	1.5 [SIN] [1/*x*]	1.0025

	Many Graphing Calculators			
	Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
a.	sin 72.8°	Degree	[SIN] 72.8 [ENTER]	0.9553
b.	csc 1.5	Radian	[(] [SIN] 1.5 [)] [x^{-1}] [ENTER]	1.0025

9. Because we have a known angle, an unknown opposite side, and a known adjacent side, we select the tangent function.

$$\tan 24^{0}=\frac{a}{750}$$
$$a=750\tan 24^{0}$$
$$a\approx 750(0.4452)\approx 334$$

The distance across the lake is approximately 334 yards.

10. $\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{14}{10}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
TAN⁻¹ (14 ÷ 10) ENTER	TAN (14 ÷ 10) ENTER

The display should show approximately 54. Thus, the angle of elevation of the sun is approximately 54°.

Exercise Set 5.2

1. $c^2 = 9^2 + 12^2 = 225$

$c = \sqrt{225} = 15$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{9}{15} = \frac{3}{5}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{15} = \frac{4}{5}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{9}{12} = \frac{3}{4}$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{15}{9} = \frac{5}{3}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{15}{12} = \frac{5}{4}$

$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{12}{9} = \frac{4}{3}$

2. $c^2 = 6^2 + 8^2 = 100$

$c = \sqrt{100} = 10$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{8}{10} = \frac{4}{5}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{6}{8} = \frac{3}{4}$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{10}{6} = \frac{5}{3}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{10}{8} = \frac{5}{4}$

$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{8}{6} = \frac{4}{3}$

3. $a^2 + 21^2 = 29^2$

$a^2 = 841 - 441 = 400$

$a = \sqrt{400} = 20$

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{20}{29}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{21}{29}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{20}{21}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{29}{20}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{29}{21}$

$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{21}{20}$

4. $a^2 + 15^2 = 17^2$

$a^2 = 289 - 225 = 64$

$a = \sqrt{64} = 8$

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{8}{17}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{15}{17}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{8}{15}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{17}{8}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{17}{15}$

$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{15}{8}$

5. $10^2 + b^2 = 26^2$

$b^2 = 676 - 100 = 576$

$b = \sqrt{576} = 24$

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{10}{26} = \dfrac{5}{13}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{24}{26} = \dfrac{12}{13}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{10}{24} = \dfrac{5}{12}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{26}{10} = \dfrac{13}{5}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{26}{24} = \dfrac{13}{12}$

$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{24}{10} = \dfrac{12}{5}$

6. $a^2 + 40^2 = 41^2$

$a^2 = 1681 - 1600 = 81$

$a = \sqrt{81} = 9$

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{9}{41}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{40}{41}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{9}{40}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{41}{9}$

$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{41}{40}$

$\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{40}{9}$

7. $21^2 + b^2 = 35^2$

$b^2 = 1225 - 441 = 784$

$b = \sqrt{784} = 28$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{28}{35} = \frac{4}{5}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{21}{35} = \frac{3}{5}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{28}{21} = \frac{4}{3}$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{35}{28} = \frac{5}{4}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{35}{21} = \frac{5}{3}$

$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{21}{28} = \frac{3}{4}$

8. $a^2 + 24^2 = 25^2$

$a^2 = 625 - 576 = 49$

$a = \sqrt{49} = 7$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{24}{25}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{7}{25}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{24}{7}$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{25}{24}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{25}{7}$

$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{7}{24}$

9. $\cos 30° = \frac{\text{length of side adjacent to } 30°}{\text{length of hypotenuse}}$

$= \frac{\sqrt{3}}{2}$

10. $\tan 30° = \frac{\text{length of side opposite } 30°}{\text{length of side adjacent to } 30°}$

$= \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

11. $\sec 45° = \frac{\text{length of hypotenuse}}{\text{length of side adjacent to } 45°}$

$= \frac{\sqrt{2}}{1} = \sqrt{2}$

12. $\csc 45° = \frac{\text{length of hypotenuse}}{\text{length of side opposite } 45°}$

$= \frac{\sqrt{2}}{1} = \sqrt{2}$

13. $\tan\frac{\pi}{3} = \tan 60°$

$= \frac{\text{length of side opposite } 60°}{\text{length of side adjacent to } 60°}$

$= \frac{\sqrt{3}}{1} = \sqrt{3}$

14. $\cot\frac{\pi}{3} = \cot 60° = \frac{\text{length of side adjacent to } 60°}{\text{length of side opposite } 60°}$

$= \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

15. $\sin\frac{\pi}{4} - \cos\frac{\pi}{4} = \sin 45° - \cos 45°$

$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

16. $\tan\frac{\pi}{4} + \csc\frac{\pi}{6} = \tan 45° + \csc 30°$

$= \frac{1}{1} + \frac{2}{1} = 1 + 2 = 3$

17. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{8}{17}}{\frac{15}{17}} = \frac{8}{15}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{8}{17}} = \frac{17}{8}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{15}{17}} = \frac{17}{15}$

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{15}{17}}{\frac{8}{17}} = \frac{15}{8}$

18. $\tan\theta = \dfrac{\frac{3}{5}}{\frac{4}{5}} = \dfrac{3}{4}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{3}{5}} = \dfrac{5}{3}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{4}{5}} = \dfrac{5}{4}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\frac{4}{5}}{\frac{3}{5}} = \dfrac{4}{3}$

19. $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \dfrac{1}{2\sqrt{2}}$

$= \dfrac{1}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{4}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{1}{3}} = \dfrac{3}{1} = 3$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{2\sqrt{2}}{3}} = \dfrac{3}{2\sqrt{2}}$

$= \dfrac{3}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{3\sqrt{2}}{4}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = \dfrac{2\sqrt{2}}{1} = 2\sqrt{2}$

20. $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{6}{7}}{\frac{\sqrt{13}}{7}} = \dfrac{6}{\sqrt{13}} = \dfrac{6}{\sqrt{13}} \cdot \dfrac{\sqrt{13}}{\sqrt{13}} = \dfrac{6\sqrt{13}}{13}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{6}{7}} = \dfrac{7}{6}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{\sqrt{13}}{7}} = \dfrac{7}{\sqrt{13}} \cdot \dfrac{\sqrt{13}}{\sqrt{13}} = \dfrac{7\sqrt{13}}{13}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\frac{\sqrt{13}}{7}}{\frac{6}{7}} = \dfrac{\sqrt{13}}{6}$

21. $\sin^2\theta + \cos^2\theta = 1$

$\left(\dfrac{6}{7}\right)^2 + \cos^2\theta = 1$

$\dfrac{36}{49} + \cos^2\theta = 1$

$\cos^2\theta = 1 - \dfrac{36}{49}$

$\cos^2\theta = \dfrac{13}{49}$

$\cos\theta = \sqrt{\dfrac{13}{49}} = \dfrac{\sqrt{13}}{7}$

22. $\sin^2\theta + \cos^2\theta = 1$

$\left(\dfrac{7}{8}\right)^2 + \cos^2\theta = 1$

$\dfrac{49}{64} + \cos^2\theta = 1$

$\cos^2\theta = 1 - \dfrac{49}{64}$

$\cos^2\theta = \dfrac{15}{64}$

$\cos\theta = \sqrt{\dfrac{15}{64}} = \dfrac{\sqrt{15}}{8}$

23. $\sin^2\theta+\cos^2\theta=1$

$$\left(\frac{\sqrt{39}}{8}\right)^2+\cos^2\theta=1$$

$$\frac{39}{64}+\cos^2\theta=1$$

$$\cos^2\theta=1-\frac{39}{64}$$

$$\cos^2\theta=\frac{25}{64}$$

$$\cos\theta=\sqrt{\frac{25}{64}}=\frac{5}{8}$$

24. $\sin^2\theta+\cos^2\theta=1$

$$\left(\frac{\sqrt{21}}{5}\right)^2+\cos^2\theta=1$$

$$\frac{21}{25}+\cos^2\theta=1$$

$$\cos^2\theta=1-\frac{21}{25}$$

$$\cos^2\theta=\frac{4}{25}$$

$$\cos\theta=\sqrt{\frac{4}{25}}=\frac{2}{5}$$

25. $\sin 37^0\csc 37^0=\sin 37^0\cdot\frac{1}{\sin 37^0}=1$

26. $\cos 53^0\sec 53^0=\cos 53^0\frac{1}{\cos 52^0}=1$

27. $\sin^2\theta+\cos^2\theta=1$

$$\sin^2\frac{\pi}{9}+\cos^2\frac{\pi}{9}=1$$

28. We can find the value of the expression by using the Pythagorean identity $\sin^2\theta+\cos^2\theta=1$

$$\sin^2\frac{\pi}{10}+\cos^2\frac{\pi}{10}=1$$

29. $1+\tan^2\theta=\sec^2\theta$

$$1+\tan^2 23^0=\sec^2 23^0$$

$$1=\sec^2 23^0-\tan^2 23^0$$

30. We can find the value of the expression by using the Pythagorean identity $1+\cot^2\theta=\csc^2\theta$

$\csc^2 63^0-\cot^2 63^0=1$

31. $\sin 7°=\cos(90°-7°)=\cos 83°$

32. $\sin 19°=\cos(90°-19°)=\cos 71°$

33. $\csc 25°=\sec(90°-25^o)=\sec 65°$

34. $\csc 35°=\sec(90°-35°)=\sec 55°$

35. $\tan\frac{\pi}{9}=\cot\left(\frac{\pi}{2}-\frac{\pi}{9}\right)$

$$=\cot\left(\frac{9\pi}{18}-\frac{2\pi}{18}\right)$$

$$=\cot\frac{7\pi}{18}$$

36. $\tan\frac{\pi}{7}=\cot\left(\frac{\pi}{2}-\frac{\pi}{7}\right)=\cot\left(\frac{7\pi}{14}-\frac{2\pi}{14}\right)=\cot\frac{5\pi}{14}$

37. $\cos\frac{2\pi}{5}=\sin\left(\frac{\pi}{2}-\frac{2\pi}{5}\right)$

$$=\sin\left(\frac{5\pi}{10}-\frac{4\pi}{10}\right)$$

$$=\sin\frac{\pi}{10}$$

38. $\cos\frac{3\pi}{8}=\sin\left(\frac{\pi}{2}-\frac{3\pi}{8}\right)=\sin\left(\frac{4\pi}{8}-\frac{3\pi}{8}\right)=\sin\frac{\pi}{8}$

39.

Many Scientific Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
sin 38°	Degree	38 [SIN]	.6157

Many Graphing Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
sin 38°	Degree	[SIN] 38 [ENTER]	.6157

40.

Many Scientific Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
cos 21°	Degree	21 [COS]	.9336

Many Graphing Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
cos 21°	Degree	[COS] 21 [ENTER]	.9336

41.

Many Scientific Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
tan 32.7°	Degree	32.7 [TAN]	.6420

Many Graphing Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
tan 32.7°	Degree	[TAN] 32.7 [ENTER]	.6420

42.

Many Scientific Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
tan 52.6°	Degree	52.6 [TAN]	1.3079

Many Graphing Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
tan 52.6°	Degree	[TAN] 52.6 [ENTER]	1.3079

43.

Many Scientific Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
csc 17°	Degree	17 [SIN] [1/x]	3.4203

Many Graphing Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
csc17°	Degree	[(] [SIN] 17 [)] [x^{-1}] [ENTER]	3.4203

44.

Many Scientific Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
sec 55°	Degree	55 [COS] [1/x]	1.7434

Many Graphing Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
sec 55°	Degree	[(] [COS] 55 [)] [x^{-1}] [ENTER]	1.7434

45.

Many Scientific Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
$\cos\frac{\pi}{10}$	Radian	[π] ÷ [10] [=] [COS]	.9511

Many Graphing Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
$\cos\frac{\pi}{10}$	Radian	[COS] [(] π ÷ 10 [)] [ENTER]	.9511

46.

Many Scientific Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
$\sin\frac{3\pi}{10}$	Radian	3 × π ÷ 10 [=] [SIN]	.8090

Many Graphing Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
$\sin\frac{3\pi}{10}$	Radian	[SIN] [(] 3π ÷ 10 [)] [ENTER]	.8090

47.

Many Scientific Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
$\cot\frac{\pi}{12}$	Radian	π ÷ 12 [=] [TAN] [1/x]	3.7321

Many Graphing Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
$\cot\frac{\pi}{12}$	Radian	[(] [TAN] [(] π ÷ 12 [)] [)] [x^{-1}] [ENTER]	3.7321

48.

Many Scientific Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
$\cot\frac{\pi}{18}$	Radian	π ÷ 18 [=] [TAN] [1/x]	5.6713

Many Graphing Calculators			
Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
$\cot\frac{\pi}{18}$	Radian	[(] [TAN] [(] π ÷ 18 [)] [)] [x^{-1}] [ENTER]	5.6713

49. $\tan 37° = \frac{a}{250}$

$a = 250\tan 37°$

$a \approx 250(0.7536) \approx 188$ cm

50. $\tan 61° = \dfrac{a}{10}$

$a = 10\tan 61°$

$a \approx 10(1.8040) \approx 18$ cm

51. $\cos 34° = \dfrac{b}{220}$

$b = 220\cos 34°$

$b \approx 220(0.8290) \approx 182$ in.

52. $\sin 34° = \dfrac{a}{13}$

$a = 13\sin 34°$

$a \approx 13(0.5592) \approx 7$ m

53. $\sin 23° = \dfrac{16}{c}$

$c = \dfrac{16}{\sin 23°} \approx \dfrac{16}{0.3907} \approx 41$ m

54. $\tan 44° = \dfrac{23}{b}$

$b = \dfrac{23}{\tan 44°} \approx \dfrac{23}{0.9657} \approx 24$ yd

55.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to the nearest degree)
.2974 [SIN^{-1}]	[SIN^{-1}] .2974 [ENTER]	17

If $\sin\theta = 0.2974$, then $\theta \approx 17°$.

56.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to the nearest degree)
.877 [COS^{-1}]	[COS^{-1}] .877 [ENTER]	29

If $\cos\theta = 0.877$, then $\theta \approx 29°$.

57.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to the nearest degree)
4.6252 [TAN^{-1}]	[TAN^{-1}] 4.6252 [ENTER]	78

If $\tan\theta = 4.6252$, then $\theta \approx 78°$.

58.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to the nearest degree)
26.0307 [TAN^{-1}]	[TAN^{-1}] 26.0307 [ENTER]	88

If $\tan\theta = 26.0307$, then $\theta \approx 88°$.

59.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to three places)
.4112 $\boxed{\text{COS}^{-1}}$	$\boxed{\text{COS}^{-1}}$ .4112 $\boxed{\text{ENTER}}$	1.147

If $\cos\theta = 0.4112$, then $\theta \approx 1.147$ radians.

60.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to three places)
.9499 $\boxed{\text{SIN}^{-1}}$	$\boxed{\text{SIN}^{-1}}$ .9499 $\boxed{\text{ENTER}}$	1.253

If $\sin\theta = 0.9499$, then $\theta = 1.253$ radians.

61.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to three places)
.4169 $\boxed{\text{TAN}^{-1}}$	$\boxed{\text{TAN}^{-1}}$ .4169 $\boxed{\text{ENTER}}$	.395

If $\tan\theta = 0.4169$, then $\theta \approx 0.395$ radians.

62.

Scientific Calculator	**Graphing Calculator**	**Display** (rounded to three places)
.5117 $\boxed{\text{TAN}^{-1}}$	$\boxed{\text{TAN}^{-1}}$ .5117 $\boxed{\text{ENTER}}$	.473

If $\tan\theta = 0.5117$, then $\theta = 0.473$

63. $$\begin{aligned}\frac{\tan\frac{\pi}{3}}{2} - \frac{1}{\sec\frac{\pi}{6}} &= \frac{\sqrt{3}}{2} - \frac{1}{\frac{1}{\cos\frac{\pi}{6}}}\\ &= \frac{\sqrt{3}}{2} - \frac{1}{\frac{1}{\frac{\sqrt{3}}{2}}}\\ &= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\\ &= 0\end{aligned}$$

64. $$\begin{aligned}\frac{1}{\cot\frac{\pi}{4}} - \frac{2}{\csc\frac{\pi}{6}} &= \frac{1}{\frac{1}{\tan\frac{\pi}{4}}} - \frac{2}{\frac{1}{\sin\frac{\pi}{6}}}\\ &= \frac{1}{\frac{1}{1}} - \frac{2}{\frac{1}{1/2}}\\ &= \frac{1}{1} - \frac{2}{2}\\ &= 1 - 1\\ &= 0\end{aligned}$$

65. $$\begin{aligned}&1 + \sin^2 40° + \sin^2 50°\\ &= 1 + \sin^2(90° - 50°) + \sin^2 50°\\ &= 1 + \cos^2 50° + \sin^2 50°\\ &= 1 + 1\\ &= 2\end{aligned}$$

66. $1-\tan^2 10°+\csc^2 80°$
$=1-\cot^2 80°+\csc^2 80°$
$=1+\csc^2 80°-\cot^2 80°$
$=1+1$
$=2$

67. $\csc 37°\sec 53°-\tan 53°\cot 37°$
$=\sec 53°\sec 53°-\tan 53°\tan 53°$
$=\sec^2 53°-\tan^2 53°$
$=1$

68. $\cos 12°\sin 78°+\cos 78°\sin 12°$
$=\sin 78°\sin 78°+\cos 78°\cos 78°$
$=\sin^2 78°+\cos^2 78°$
$=1$

69. $f(\theta)=2\cos\theta-\cos 2\theta$

$$f\left(\frac{\pi}{6}\right)=2\cos\frac{\pi}{6}-\cos\left(2\cdot\frac{\pi}{6}\right)$$
$$=2\left(\frac{\sqrt{3}}{2}\right)-\cos\left(\frac{\pi}{3}\right)$$
$$=\frac{2\sqrt{3}}{2}-\frac{1}{2}$$
$$=\frac{2\sqrt{3}-1}{2}$$

70. $f(\theta)=2\sin\theta-\sin\frac{\theta}{2}$

$$f\left(\frac{\pi}{3}\right)=2\sin\frac{\pi}{3}-\sin\frac{\frac{\pi}{3}}{2}$$
$$=2\left(\frac{\sqrt{3}}{2}\right)-\sin\left(\frac{\pi}{6}\right)$$
$$=\frac{2\sqrt{3}}{2}-\frac{1}{2}$$
$$=\frac{2\sqrt{3}-1}{2}$$

71. $\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta=\frac{1}{4}$

72. $\csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{1}{3}}=3$

73. $\tan 40° = \dfrac{a}{630}$

$a = 630\tan 40°$

$a \approx 630(0.8391) \approx 529$

The distance across the lake is approximately 529 yards.

74. $\tan 40° = \dfrac{h}{35}$

$h = 35\tan 40°$

$h \approx 35(0.8391) \approx 29$

The tree's height is approximately 29 feet.

75. $\tan\theta = \dfrac{125}{172}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
125 [÷] 172 [=] [TAN^{-1}]	[TAN^{-1}] [(] 125 [÷] 172 [)] [ENTER]

The display should show approximately 36. Thus, the angle of elevation of the sun is approximately 36°.

76. $\tan\dfrac{555}{1320}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
555 ÷ 1320 [=] [TAN^{-1}]	[TAN^{-1}] [(] 555 ÷ 1320 [)] [ENTER]

The display should show approximately 23. Thus, the angle of elevation is approximately 23°.

77. $\sin 10° = \dfrac{500}{c}$

$c = \dfrac{500}{\sin 10°} \approx \dfrac{500}{0.1736} \approx 2880$

The plane has flown approximately 2880 feet.

78. $\sin 5° = \dfrac{a}{5000}$

$a = 5000\sin 5° \approx 5000(0.0872) = 436$

The driver's increase in altitude was approximately 436 feet.

79. $\cos\theta = \dfrac{60}{75}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
60 [÷] 75 [=] [COS^{-1}]	[COS^{-1}] [(] 60 [÷] 75 [)] [ENTER]

The display should show approximately 37. Thus, the angle between the wire and the pole is approximately 37°.

80. $\cos\theta = \frac{55}{80}$

Use a calculator in degree mode to find θ.

Many Scientific Calculators	Many Graphing Calculators
55 ÷ 80 [=] [COS^{-1}]	[COS^{-1}] [(] 55 ÷ 80 [)] [ENTER]

The display should show approximately 47. Thus, the angle between the wire and the pole is approximately 47°.

81. – 91. Answers may vary.

92.

θ	0.4	0.3	0.2	0.1	0.01	0.001	0.0001	0.00001
sinθ	0.3894	0.2955	0.1987	0.0998	0.0099998	9.999998×10^{-4}	9.99999998×10^{-5}	1×10^{-5}
$\frac{\sin\theta}{\theta}$	0.9736	0.9851	0.9933	0.9983	0.99998	0.9999998	0.999999998	1

$\frac{\sin\theta}{\theta}$ approaches 1 as θ approaches 0.

93.

θ	0.4	0.3	0.2	0.1	0.01	0.001	0.0001	0.00001
$\cos\theta$	0.92106	0.95534	0.98007	0.99500	0.99995	0.9999995	0.999999995	1
$\frac{\cos\theta-1}{\theta}$	–0.19735	–0.148878	–0.099667	–0.04996	–0.005	–0.0005	–0.00005	0

$\frac{\cos\theta-1}{\theta}$ approaches 0 as θ approaches 0.

94. does not make sense; Explanations will vary. Sample explanation: An increase in the size of a triangle does not affect the ratios of the sides.

95. does not make sense; Explanations will vary. Sample explanation: This value is irrational. Irrational numbers are rounded on calculators.

96. makes sense

97. makes sense

98. false; Changes to make the statement true will vary. A sample change is: $\frac{\tan 45°}{\tan 15°} \neq \tan\left(\frac{45°}{15°}\right)$

99. true

100. false; Changes to make the statement true will vary. A sample change is: $\sin 45° + \cos 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} \neq 1$

101. true

102. In a right triangle, the hypotenuse is greater than either other side. Therefore both $\frac{\text{opposite}}{\text{hypotenuse}}$ and $\frac{\text{adjacent}}{\text{hypotenuse}}$ must be less than 1 for an acute angle in a right triangle.

103. Use a calculator in degree mode to generate the following table. Then use the table to describe what happens to the tangent of an acute angle as the angle gets close to 90°.

θ	60	70	80	89	89.9	89.99	89.999	89.9999
tanθ	1.7321	2.7475	5.6713	57	573	5730	57,296	572,958

As θ approaches 90°, tanθ increases without bound. At 90°, tanθ is undefined.

104. a. Let a = distance of the ship from the lighthouse.

$$\tan 35° = \frac{250}{a}$$

$$a = \frac{250}{\tan 35°} \approx \frac{250}{0.7002} \approx 357$$

The ship is approximately 357 feet from the lighthouse.

b. Let b = the plane's height above the lighthouse.

$$\tan 22° = \frac{b}{357}$$

$$b = 357\tan 22° \approx 357(0.4040) \approx 144$$

$$144 + 250 = 394$$

The plane is approximately 394 feet above the water.

105. a. $\frac{y}{r}$

b. First find r: $r = \sqrt{x^2 + y^2}$

$$r = \sqrt{(-3)^2 + 4^2}$$

$$r = 5$$

$\frac{y}{r} = \frac{4}{5}$, which is positive.

106. a. $\frac{x}{r}$

b. First find r: $r = \sqrt{x^2 + y^2}$

$$r = \sqrt{(-3)^2 + 5^2}$$

$$r = \sqrt{34}$$

$\frac{x}{r} = \frac{-3}{\sqrt{34}} = \frac{-3}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{-3\sqrt{34}}{34}$, which is negative.

107. a. $\theta' = 360^\circ - 345^\circ = 15^\circ$

b. $\theta' = \pi - \frac{5\pi}{6} = \frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6}$

Section 5.3

Checkpoint Exercises

1. $r=\sqrt{x^2+y^2}$

$r=\sqrt{1^2+(-3)^2}=\sqrt{1+9}=\sqrt{10}$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$\sin\theta=\frac{y}{r}=\frac{-3}{\sqrt{10}}=-\frac{3\sqrt{10}}{10}$

$\cos\theta=\frac{x}{r}=\frac{1}{\sqrt{10}}=\frac{\sqrt{10}}{10}$

$\tan\theta=\frac{y}{x}=\frac{-3}{1}=-3$

$\csc\theta=\frac{r}{y}=\frac{\sqrt{10}}{-3}=-\frac{\sqrt{10}}{3}$

$\sec\theta=\frac{r}{x}=\frac{\sqrt{10}}{1}=\sqrt{10}$

$\cot\theta=\frac{x}{y}=\frac{1}{-3}=-\frac{1}{3}$

2. a. $\theta=0°=0$ radians

The terminal side of the angle is on the positive *x*-axis. Select the point $P=(1,0)$: $x=1, y=0, r=1$

Apply the definitions of the cosine and cosecant functions.

$\cos 0°=\cos 0=\frac{x}{r}=\frac{1}{1}=1$

$\csc 0°=\csc 0=\frac{r}{y}=\frac{1}{0}$, undefined

b. $\theta=90°=\frac{\pi}{2}$ radians

The terminal side of the angle is on the positive *y*-axis. Select the point $P=(0,1)$: $x=0, y=1, r=1$

Apply the definitions of the cosine and cosecant functions.

$\cos 90°=\cos\frac{\pi}{2}=\frac{x}{r}=\frac{0}{1}=0$

$\csc 90°=\csc\frac{\pi}{2}=\frac{r}{y}=\frac{1}{1}=1$

c. $\theta=180°=\pi$ radians

The terminal side of the angle is on the negative *x*-axis. Select the point $P=(-1,0)$: $x=-1, y=0, r=1$

Apply the definitions of the cosine and cosecant functions.

$\cos 180°=\cos\pi=\frac{x}{r}=\frac{-1}{1}=-1$

$\csc 180°=\csc\pi=\frac{r}{y}=\frac{1}{0}$, undefined

d. $\theta=270°=\frac{3\pi}{2}$ radians

The terminal side of the angle is on the negative *y*-axis. Select the point $P=(0,-1)$: $x=0, y=-1, r=1$

Apply the definitions of the cosine and cosecant functions.

$\cos 270°=\cos\frac{3\pi}{2}=\frac{x}{r}=\frac{0}{1}=0$

$\csc 270°=\csc\frac{3\pi}{2}=\frac{r}{y}=\frac{1}{-1}=-1$

3. Because $\sin\theta<0$, θ cannot lie in quadrant I; all the functions are positive in quadrant I. Furthermore, θ cannot lie in quadrant II; $\sin\theta$ is positive in quadrant II. Thus, with $\sin\theta<0$, θ lies in quadrant III or quadrant IV. We are also given that $\cos\theta<0$. Because quadrant III is the only quadrant in which cosine is negative and the sine is negative, we conclude that θ lies in quadrant III.

4. Because the tangent is negative and the cosine is negative, θ lies in quadrant II. In quadrant II, *x* is negative and *y* is positive. Thus,

$\tan\theta=-\frac{1}{3}=\frac{y}{x}=\frac{1}{-3}$

$x=-3,\ y=1$

Furthermore,

$r=\sqrt{x^2+y^2}=\sqrt{(-3)^2+1^2}=\sqrt{9+1}=\sqrt{10}$

Now that we know *x, y,* and *r,* we can find $\sin\theta$ and $\sec\theta$.

$\sin\theta=\frac{y}{r}=\frac{1}{\sqrt{10}}=\frac{1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}}=\frac{\sqrt{10}}{10}$

$\sec\theta=\frac{r}{x}=\frac{\sqrt{10}}{-3}=-\frac{\sqrt{10}}{3}$

5. **a.** Because 210° lies between 180° and 270°, it is in quadrant III. The reference angle is $\theta' = 210° - 180° = 30°$.

b. Because $\frac{7\pi}{4}$ lies between $\frac{3\pi}{2} = \frac{6\pi}{4}$ and $2\pi = \frac{8\pi}{4}$, it is in quadrant IV. The reference angle is $\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$.

c. Because –240° lies between –180° and –270°, it is in quadrant II. The reference angle is $\theta = 240 - 180 = 60°$.

d. Because 3.6 lies between $\pi \approx 3.14$ and $\frac{3\pi}{2} \approx 4.71$, it is in quadrant III. The reference angle is $\theta' = 3.6 - \pi \approx 0.46$.

6. **a.** $665° - 360° = 305°$
This angle is in quadrant IV, thus the reference angle is $\theta' = 360° - 305° = 55°$.

b. $\frac{15\pi}{4} - 2\pi = \frac{15\pi}{4} - \frac{8\pi}{4} = \frac{7\pi}{4}$
This angle is in quadrant IV, thus the reference angle is $\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$.

c. $-\frac{11\pi}{3} + 2 \cdot 2\pi = -\frac{11\pi}{3} + \frac{12\pi}{3} = \frac{\pi}{3}$
This angle is in quadrant I, thus the reference angle is $\theta' = \frac{\pi}{3}$.

7. **a.** 300° lies in quadrant IV. The reference angle is $\theta' = 360° - 300° = 60°$.
$\sin 60° = \frac{\sqrt{3}}{2}$
Because the sine is negative in quadrant IV,
$\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}$.

b. $\frac{5\pi}{4}$ lies in quadrant III. The reference angle is
$\theta' = \frac{5\pi}{4} - \pi = \frac{5\pi}{4} - \frac{4\pi}{4} = \frac{\pi}{4}$.
$\tan\frac{\pi}{4} = 1$
Because the tangent is positive in quadrant III,
$\tan\frac{5\pi}{4} = +\tan\frac{\pi}{4} = 1$.

c. $-\frac{\pi}{6}$ lies in quadrant IV. The reference angle is
$\theta' = \frac{\pi}{6}$.
$\sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3}$
Because the secant is positive in quadrant IV,
$\sec\left(-\frac{\pi}{6}\right) = +\sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3}$.

8. **a.** $\frac{17\pi}{6} - 2\pi = \frac{17\pi}{6} - \frac{12\pi}{6} = \frac{5\pi}{6}$ lies in quadrant II. The reference angle is $\theta' = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$.
The function value for the reference angle is
$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.
Because the cosine is negative in quadrant II,
$\cos\frac{17\pi}{6} = \cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$.

b. $\frac{-22\pi}{3} + 8\pi = \frac{-22\pi}{3} + \frac{24\pi}{3} = \frac{2\pi}{3}$ lies in quadrant II. The reference angle is
$\theta' = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$.
The function value for the reference angle is
$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.
Because the sine is positive in quadrant II,
$\sin\frac{-22\pi}{3} = \sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

Exercise Set 5.3

1. We need values for *x, y,* and *r*. Because $P = (-4, 3)$ is a point on the terminal side of θ, $x = -4$ and $y = 3$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-4)^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$ Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{3}{5}$$

$$\cos\theta = \frac{x}{r} = \frac{-4}{5} = -\frac{4}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{3}{-4} = -\frac{3}{4}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{5}{-4} = -\frac{5}{4}$$

$$\cot\theta = \frac{x}{y} = \frac{-4}{3} = -\frac{4}{3}$$

2. We need values for *x, y,* and *r,* Because $P = (-12, 5)$ is a point on the terminal side of θ, $x = -12$ and $y = 5$. Furthermore,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-12)^2 + 5^2} = \sqrt{144+25}$$
$$= \sqrt{169} = 13$$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{5}{13}$$

$$\cos\theta = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}$$

$$\tan\theta = \frac{y}{x} = \frac{5}{-12} = -\frac{5}{12}$$

$$\csc\theta = \frac{r}{y} = \frac{13}{5}$$

$$\sec\theta = \frac{r}{x} = \frac{13}{-12} = -\frac{13}{12}$$

$$\cot\theta = \frac{x}{y} = \frac{-12}{5} - \frac{12}{5}$$

3. We need values for *x, y,* and *r*. Because $P = (2, 3)$ is a point on the terminal side of θ, $x = 2$ and $y = 3$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}} = \frac{3}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\cos\theta = \frac{x}{r} = \frac{2}{\sqrt{13}} = \frac{2}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\tan\theta = \frac{y}{x} = \frac{3}{2}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{13}}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{2}$$

$$\cot\theta = \frac{x}{y} = \frac{2}{3}$$

4. We need values for *x, y,* and *r,* Because $P = (3, 7)$ is a point on the terminal side of θ, $x = 3$ and $y = 7$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 7^2} = \sqrt{9+49} = \sqrt{58}$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{7}{\sqrt{58}} = \frac{7}{\sqrt{58}}\cdot\frac{\sqrt{58}}{\sqrt{58}} = \frac{7\sqrt{58}}{58}$$

$$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{58}} = \frac{3}{\sqrt{58}}\cdot\frac{\sqrt{58}}{\sqrt{58}} = \frac{3\sqrt{58}}{58}$$

$$\tan\theta = \frac{y}{x} = \frac{7}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{58}}{7}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{58}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{3}{7}$$

5. We need values for *x, y,* and *r*. Because $P = (3, -3)$ is a point on the terminal side of θ, $x = 3$ and $y = -3$.

Furthermore, $r = \sqrt{x^2 + y^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9+9}$
$= \sqrt{18} = 3\sqrt{2}$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-3}{3\sqrt{2}} = -\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-3}{3} = -1$$

$$\csc\theta = \frac{r}{y} = \frac{3\sqrt{2}}{-3} = -\sqrt{2}$$

$$\sec\theta = \frac{r}{x} = \frac{3\sqrt{2}}{3} = \sqrt{2}$$

$$\cot\theta = \frac{x}{y} = \frac{3}{-3} = -1$$

6. We need values for *x, y,* and *r,* Because $P = (5, -5)$ is a point on the terminal side of θ, $x = 5$ and $y = -5$.

Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{5 + (-5)^2} = \sqrt{25+25} = \sqrt{50}$
$= 5\sqrt{2}$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-5}{5\sqrt{2}} = \frac{-1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-5}{5} = -1$$

$$\csc\theta = \frac{r}{y} = \frac{5\sqrt{2}}{-5} = -\sqrt{2}$$

$$\sec\theta = \frac{r}{x} = \frac{5\sqrt{2}}{5} = \sqrt{2}$$

$$\cot\theta = \frac{x}{y} = \frac{5}{-5} = -1$$

7. We need values for *x, y,* and *r.* Because $P = (-2, -5)$ is a point on the terminal side of θ, $x = -2$ and $y = -5$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (-5)^2} = \sqrt{4+25} = \sqrt{29}$ Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{29}} = \frac{-5}{\sqrt{29}}\cdot\frac{\sqrt{29}}{\sqrt{29}} = -\frac{5\sqrt{29}}{29}$$

$$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{29}} = \frac{-2}{\sqrt{29}}\cdot\frac{\sqrt{29}}{\sqrt{29}} = -\frac{2\sqrt{29}}{29}$$

$$\tan\theta = \frac{y}{x} = \frac{-5}{-2} = \frac{5}{2}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{-5} = -\frac{\sqrt{29}}{5}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{29}}{-2} = -\frac{\sqrt{29}}{2}$$

$$\cot\theta = \frac{x}{y} = \frac{-2}{-5} = \frac{2}{5}$$

8. We need values for *x, y,* and *r,* Because $P = (-1, -3)$ is a point on the terminal side of θ, $x = -1$ and $y = -3$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10}$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-3}{\sqrt{10}} = \frac{-3}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$

$$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{10}} = \frac{-1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$

$$\tan\theta = \frac{y}{x} = \frac{-3}{-1} = 3$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{-1} = -\sqrt{10}$$

$$\cot\theta = \frac{x}{y} = \frac{-1}{-3} = \frac{1}{3}$$

9. $\theta = \pi$ radians
The terminal side of the angle is on the negative *x*-axis. Select the point $P = (-1, 0)$:
$x = -1,\ y = 0,\ r = 1$ Apply the definition of the cosine function.
$$\cos\pi = \frac{x}{r} = \frac{-1}{1} = -1$$

10. $\theta = \pi$ radians
The terminal side of the angle is on the negative *x*-axis. Select the point $P = (-1, 0)$: $x = -1, y = 0,\ r = 1$
Apply the definition of the tangent function.
$$\tan\pi = \frac{y}{x} = \frac{0}{-1} = 0$$

11. $\theta = \pi$ radians
The terminal side of the angle is on the negative *x*-axis. Select the point $P = (-1, 0)$:
$x = -1,\ y = 0,\ r = 1$ Apply the definition of the secant function.
$$\sec\pi = \frac{r}{x} = \frac{1}{-1} = -1$$

12. $\theta = \pi$ radians
The terminal side of the angle is on the negative *x*-axis. Select the point $P = (-1, 0)$: $x = -1, y = 0,\ r = 1$
Apply the definition of the cosecant function.
$$\csc\pi = \frac{r}{y} = \frac{1}{0}, \text{ undefined}$$

13. $\theta = \frac{3\pi}{2}$ radians
The terminal side of the angle is on the negative *y*-axis. Select the point $P = (0, -1)$:
$x = 0,\ y = -1,\ r = 1$ Apply the definition of the tangent function. $\tan\frac{3\pi}{2} = \frac{y}{x} = \frac{-1}{0}$, undefined

14. $\theta = \frac{3\pi}{2}$ radians
The terminal side of the angle is on the negative *y*-axis. Select the point $P = (0, -1)$: $x = 0, y = -1,\ r = 1$
Apply the definition of the cosine function.
$$\cos\frac{3\pi}{2} = \frac{x}{r} = \frac{0}{1} = 0$$

15. $\theta = \frac{\pi}{2}$ radians
The terminal side of the angle is on the positive *y*-axis. Select the point $P = (0, 1)$:
$x = 0,\ y = 1,\ r = 1$ Apply the definition of the cotangent function. $\cot\frac{\pi}{2} = \frac{x}{y} = \frac{0}{1} = 0$

16. $\theta = \frac{\pi}{2}$ radians
The terminal side of the angle is on the positive *y*-axis. Select the point $P = (0, 1)$: $x = 0, y = 1,\ r = 1$
Apply the definition of the tangent function.
$$\tan\frac{\pi}{2} = \frac{y}{x} = \frac{1}{0}, \text{ undefined}$$

17. Because $\sin\theta > 0,\ \theta$ cannot lie in quadrant III or quadrant IV; the sine function is negative in those quadrants. Thus, with $\sin\theta > 0,\ \theta$ lies in quadrant I or quadrant II. We are also given that $\cos\theta > 0$. Because quadrant I is the only quadrant in which the cosine is positive and sine is positive, we conclude that θ lies in quadrant I.

18. Because $\sin\theta < 0,\ \theta$ cannot lie in quadrant I or quadrant II; the sine function is positive in those two quadrants. Thus, with $\sin\theta < 0,\ \theta$ lies in quadrant III or quadrant IV. We are also given that $\cos\theta > 0$. Because quadrant IV is the only quadrant in which the cosine is positive and the sine is negative, we conclude that θ lies in quadrant IV.

19. Because $\sin\theta < 0,\ \theta$ cannot lie in quadrant I or quadrant II; the sine function is positive in those two quadrants. Thus, with $\sin\theta < 0,\ \theta$ lies in quadrant III or quadrant IV. We are also given that $\cos\theta < 0$. Because quadrant III is the only quadrant in which the cosine is positive and the sine is negative, we conclude that θ lies in quadrant III.

20. Because $\tan\theta < 0,\ \theta$ cannot lie in quadrant I or quadrant III; the tangent function is positive in those two quadrants. Thus, with $\tan\theta < 0,\ \theta$ lies in quadrant II or quadrant IV. We are also given that $\sin\theta < 0$. Because quadrant IV is the only quadrant in which the sine is negative and the tangent is negative, we conclude that θ lies in quadrant IV.

21. Because $\tan\theta < 0$, θ cannot lie in quadrant I or quadrant III; the tangent function is positive in those quadrants. Thus, with $\tan\theta < 0$, θ lies in quadrant II or quadrant IV. We are also given that $\cos\theta < 0$. Because quadrant II is the only quadrant in which the cosine is negative and the tangent is negative, we conclude that θ lies in quadrant II.

22. Because $\cot\theta > 0$, θ cannot lie in quadrant II or quadrant IV; the cotangent function is negative in those two quadrants. Thus, with $\cot\theta > 0$, θ lies in quadrant I or quadrant III. We are also given that $\sec\theta < 0$. Because quadrant III is the only quadrant in which the secant is negative and the cotangent is positive, we conclude that θ lies in quadrant III.

23. In quadrant III x is negative and y is negative. Thus,

$\cos\theta = -\frac{3}{5} = \frac{x}{r} = \frac{-3}{5}$, $x = -3$, $r = 5$. Furthermore,

$$r^2 = x^2 + y^2$$
$$5^2 = (-3)^2 + y^2$$
$$y^2 = 25 - 9 = 16$$
$$y = -\sqrt{16} = -4$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-4}{5} = -\frac{4}{5}$$
$$\tan\theta = \frac{y}{x} = \frac{-4}{-3} = \frac{4}{3}$$
$$\csc\theta = \frac{r}{y} = \frac{5}{-4} = -\frac{5}{4}$$
$$\sec\theta = \frac{r}{x} = \frac{5}{-3} = -\frac{5}{3}$$
$$\cot\theta = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$$

24. In quadrant III, x is negative and y is negative. Thus,

$\sin\theta = -\frac{12}{13} = \frac{y}{r} = \frac{-12}{13}$, $y = -12$, $r = 13$.

Furthermore,

$$x^2 + y^2 = r^2$$
$$x^2 + (-12)^2 = 13^2$$
$$x^2 = 169 - 144 = 25$$
$$x = -\sqrt{25} = -5$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\cos\theta = \frac{x}{r} = \frac{-5}{13} = -\frac{5}{13}$$
$$\tan\theta = \frac{y}{x} = \frac{-12}{-5} = \frac{12}{5}$$
$$\csc\theta = \frac{r}{y} = \frac{13}{-12} = -\frac{13}{12}$$
$$\sec\theta = \frac{r}{x} = \frac{13}{-5} = -\frac{13}{5}$$
$$\cot\theta = \frac{x}{y} = \frac{-5}{-12} = \frac{5}{12}$$

25. In quadrant II x is negative and y is positive. Thus,

$\sin\theta = \frac{5}{13} = \frac{y}{r}$, $y = 5$, $r = 13$. Furthermore,

$$x^2 + y^2 = r^2$$
$$x^2 + 5^2 = 13^2$$
$$x^2 = 169 - 25 = 144$$
$$x = -\sqrt{144} = -12$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\cos\theta = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}$$
$$\tan\theta = \frac{y}{x} = \frac{5}{-12} = -\frac{5}{12}$$
$$\csc\theta = \frac{r}{y} = \frac{13}{5}$$
$$\sec\theta = \frac{r}{x} = \frac{13}{-12} = -\frac{13}{12}$$
$$\cot\theta = \frac{x}{y} = \frac{-12}{5} = -\frac{12}{5}$$

26. In quadrant IV, x is positive and y is negative. Thus,

$\cos\theta = \frac{4}{5} = \frac{x}{r}$, $x = 4$, $r = 5$. Furthermore,

$x^2 + y^2 = r^2$

$4^2 + y^2 = 5^2$

$y^2 = 25 - 16 = 9$

$y = -\sqrt{9} = -3$

Now that we know *x, y,* and *r,* we can find the remaining trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{-3}{5} = -\frac{3}{5}$

$\tan\theta = \frac{y}{x} = \frac{-3}{4} = -\frac{3}{4}$

$\csc\theta = \frac{r}{y} = \frac{5}{-3} = -\frac{5}{3}$

$\sec\theta = \frac{r}{x} = \frac{5}{4}$

$\cot\theta = \frac{x}{y} = \frac{4}{-3} = -\frac{4}{3}$

27. Because $270° < \theta < 360°$, θ is in quadrant IV. In quadrant IV x is positive and y is negative. Thus,

$\cos\theta = \frac{8}{17} = \frac{x}{r}$, $x = 8$,

$r = 17$. Furthermore

$x^2 + y^2 = r^2$

$8^2 + y^2 = 17^2$

$y^2 = 289 - 64 = 225$

$y = -\sqrt{225} = -15$

Now that we know *x, y,* and *r,* we can find the remaining trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{-15}{17} = -\frac{15}{17}$

$\tan\theta = \frac{y}{x} = \frac{-15}{8} = -\frac{15}{8}$

$\csc\theta = \frac{r}{y} = \frac{17}{-15} = -\frac{17}{15}$

$\sec\theta = \frac{r}{x} = \frac{17}{8}$

$\cot\theta = \frac{x}{y} = \frac{8}{-15} = -\frac{8}{15}$

28. Because $270° < \theta < 360°$, θ is in quadrant IV. In quadrant IV, x is positive and y is negative. Thus,

$\cos\theta = \frac{1}{3} = \frac{x}{r}$, $x = 1$, $r = 3$. Furthermore,

$x^2 + y^2 = r^2$

$1^2 + y^2 = 3^2$

$y^2 = 9 - 1 = 8$

$y = -\sqrt{8} = -2\sqrt{2}$

Now that we know *x, y,* and *r,* we can find the remaining trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{2}}{3} = -\frac{2\sqrt{2}}{3}$

$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{2}}{1} = -2\sqrt{2}$

$\csc\theta = \frac{r}{y} = \frac{3}{-2\sqrt{2}} = \frac{3}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{3\sqrt{2}}{4}$

$\sec\theta = \frac{r}{x} = \frac{3}{1} = 3$

$\cot\theta = \frac{x}{y} = \frac{1}{-2\sqrt{2}} = \frac{1}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4}$

29. Because the tangent is negative and the sine is positive, θ lies in quadrant II. In quadrant II, x is negative and y is positive. Thus,

$\tan\theta = -\frac{2}{3} = \frac{y}{x} = \frac{2}{-3}$, $x = -3$, $y = 2$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$

Now that we know *x, y,* and *r,* we can find the remaining trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{13}} = \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$

$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{13}} = \frac{-3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{13}}{2}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{-3} = -\frac{\sqrt{13}}{3}$

$\cot\theta = \frac{x}{y} = \frac{-3}{2} = -\frac{3}{2}$

30. Because the tangent is negative and the sine is positive, θ lies in quadrant II. In quadrant II, x is negative and y is positive. Thus,

$\tan\theta = -\frac{1}{3} = \frac{y}{x} = \frac{1}{-3},\ y = 1,\ x = -3$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{10}} = \frac{-3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{1} = \sqrt{10}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$

$\cot\theta = \frac{x}{y} = \frac{-3}{1} = -3$

31. Because the tangent is positive and the cosine is negative, θ lies in quadrant III. In quadrant III, x is negative and y is negative. Thus, $\tan\theta = \frac{4}{3} = \frac{y}{x} = \frac{-4}{-3}$,

$x = -3$, $y = -4$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{-4}{5} = -\frac{4}{5}$

$\cos\theta = \frac{x}{r} = \frac{-3}{5} = -\frac{3}{5}$

$\csc\theta = \frac{r}{y} = \frac{5}{-4} = -\frac{5}{4}$

$\sec\theta = \frac{r}{x} = \frac{5}{-3} = -\frac{5}{3}$

$\cot\theta = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$

32. Because the tangent is positive and the cosine is negative, θ lies in quadrant III. In quadrant III, x is negative and y is negative. Thus,

$\tan\theta = \frac{5}{12} = \frac{y}{x} = \frac{-5}{-12},\ x = -12,\ y = -5$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-12)^2 + (-5)^2} = \sqrt{144+25}$

$= \sqrt{169} = 13$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{-5}{13} = -\frac{5}{13}$

$\cos\theta = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}$

$\csc\theta = \frac{r}{y} = \frac{13}{-5} = -\frac{13}{5}$

$\sec\theta = \frac{r}{x} = \frac{13}{-12} = -\frac{13}{12}$

$\cot\theta = \frac{x}{y} = \frac{-12}{-5} = \frac{12}{5}$

33. Because the secant is negative and the tangent is positive, θ lies in quadrant III. In quadrant III, x is negative and y is negative. Thus,

$\sec\theta = -3 = \frac{r}{x} = \frac{3}{-1},\ x = -1,\ r = 3$. Furthermore,

$x^2 + y^2 = r^2$

$(-1)^2 + y^2 = 3^2$

$y^2 = 9 - 1 = 8$

$y = -\sqrt{8} = -2\sqrt{2}$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{2}}{3} = -\frac{2\sqrt{2}}{3}$

$\cos\theta = \frac{x}{r} = \frac{-1}{3} = -\frac{1}{3}$

$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{2}}{-1} = 2\sqrt{2}$

$\csc\theta = \frac{r}{y} = \frac{3}{-2\sqrt{2}} = \frac{3}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{3\sqrt{2}}{4}$

$\cot\theta = \frac{x}{y} = \frac{-1}{-2\sqrt{2}} = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$

34. Because the cosecant is negative and the tangent is positive, θ lies in quadrant III. In quadrant III, x is negative and y is negative. Thus,

$\csc\theta = -4 = \frac{r}{y} = \frac{4}{-1}$, $y = -1$, $r = 4$. Furthermore,

$$x^2 + y^2 = r^2$$
$$x^2 + (-1)^2 = 4^2$$
$$x^2 = 16 - 1 = 15$$
$$x = -\sqrt{15}$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-1}{4} = -\frac{1}{4}$$
$$\cos\theta = \frac{x}{r} = \frac{-\sqrt{15}}{4} = -\frac{\sqrt{15}}{4}$$
$$\tan\theta = \frac{y}{x} = \frac{-1}{-\sqrt{15}} = \frac{1}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{15}}{15}$$
$$\sec\theta = \frac{r}{x} = \frac{4}{-\sqrt{15}} = -\frac{4}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = -\frac{4\sqrt{15}}{15}$$
$$\cot\theta = \frac{x}{y} = \frac{-\sqrt{15}}{-1} = \sqrt{15}$$

35. Because 160° lies between 90° and 180°, it is in quadrant II. The reference angle is $\theta' = 180° - 160° = 20°$.

36. Because 170° lies between 90° and 180°, it is in quadrant II. The reference angle is $\theta' = 180° - 170° = 10°$.

37. Because 205° lies between 180° and 270°, it is in quadrant III. The reference angle is $\theta' = 205° - 180° = 25°$.

38. Because 210° lies between 180° and 270°, it is in quadrant III. The reference angle is $\theta' = 210° - 180° = 30°$.

39. Because 355° lies between 270° and 360°, it is in quadrant IV. The reference angle is $\theta' = 360° - 355° = 5°$.

40. Because 351° lies between 270° and 360°, it is in quadrant IV. The reference angle is $\theta' = 360° - 351° = 9°$.

41. Because $\frac{7\pi}{4}$ lies between $\frac{3\pi}{2} = \frac{6\pi}{4}$ and $2\pi = \frac{8\pi}{4}$, it is in quadrant IV. The reference angle is $\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$.

42. Because $\frac{5\pi}{4}$ lies between $\pi = \frac{4\pi}{4}$ and $\frac{3\pi}{2} = \frac{6\pi}{4}$, it is in quadrant III. The reference angle is $\theta' = \frac{5\pi}{4} - \pi = \frac{5\pi}{4} - \frac{4\pi}{4} = \frac{\pi}{4}$.

43. Because $\frac{5\pi}{6}$ lies between $\frac{\pi}{2} = \frac{3\pi}{6}$ and $\pi = \frac{6\pi}{6}$, it is in quadrant II. The reference angle is $\theta' = \pi - \frac{5\pi}{6} = \frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6}$.

44. Because $\frac{5\pi}{7} = \frac{10\pi}{14}$ lies between $\frac{\pi}{2} = \frac{7\pi}{14}$ and $\pi = \frac{14\pi}{14}$, it is in quadrant II. The reference angle is $\theta' = \pi - \frac{5\pi}{7} = \frac{7\pi}{7} - \frac{5\pi}{7} = \frac{2\pi}{7}$.

45. $-150° + 360° = 210°$
Because the angle is in quadrant III, the reference angle is $\theta' = 210° - 180° = 30°$.

46. $-250° + 360° = 110°$
Because the angle is in quadrant II, the reference angle is $\theta' = 180° - 110° = 70°$.

47. $-335° + 360° = 25°$
Because the angle is in quadrant I, the reference angle is $\theta' = 25°$.

48. $-359° + 360° = 1°$
Because the angle is in quadrant I, the reference angle is $\theta' = 1°$.

49. Because 4.7 lies between $\pi \approx 3.14$ and $\frac{3\pi}{2} \approx 4.71$, it is in quadrant III. The reference angle is $\theta' = 4.7 - \pi \approx 1.56$.

50. Because 5.5 lies between $\frac{3\pi}{2} \approx 4.71$ and $2\pi \approx 6.28$, it is in quadrant IV. The reference angle is $\theta' = 2\pi - 5.5 \approx 0.78$.

51. $565° - 360° = 205°$
Because the angle is in quadrant III, the reference angle is $\theta' = 205° - 180° = 25°$.

52. $553° - 360° = 193°$
Because the angle is in quadrant III, the reference angle is $\theta' = 193° - 180° = 13°$.

53. $\frac{17\pi}{6} - 2\pi = \frac{17\pi}{6} - \frac{12\pi}{6} = \frac{5\pi}{6}$
Because the angle is in quadrant II, the reference angle is $\theta' = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$.

54. $\frac{11\pi}{4} - 2\pi = \frac{11\pi}{4} - \frac{8\pi}{4} = \frac{3\pi}{4}$
Because the angle is in quadrant II, the reference angle is $\theta' = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$.

55. $\frac{23\pi}{4} - 4\pi = \frac{23\pi}{4} - \frac{16\pi}{4} = \frac{7\pi}{4}$
Because the angle is in quadrant IV, the reference angle is $\theta' = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$.

56. $\frac{17\pi}{3} - 4\pi = \frac{17\pi}{3} - \frac{12\pi}{3} = \frac{5\pi}{3}$
Because the angle is in quadrant IV, the reference angle is $\theta' = 2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$.

57. $-\frac{11\pi}{4} + 4\pi = -\frac{11\pi}{4} + \frac{16\pi}{4} = \frac{5\pi}{4}$
Because the angle is in quadrant III, the reference angle is $\theta' = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$.

58. $-\frac{17\pi}{6} + 4\pi = -\frac{17\pi}{6} + \frac{24\pi}{6} = \frac{7\pi}{6}$
Because the angle is in quadrant III, the reference angle is $\theta' = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$.

59. $-\frac{25\pi}{6} + 6\pi = -\frac{25\pi}{6} + \frac{36\pi}{6} = \frac{11\pi}{6}$
Because the angle is in quadrant IV, the reference angle is $\theta' = 2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$.

60. $-\frac{13\pi}{3} + 6\pi = -\frac{13\pi}{3} + \frac{18\pi}{3} = \frac{5\pi}{3}$
Because the angle is in quadrant IV, the reference angle is $\theta' = 2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$.

61. 225° lies in quadrant III. The reference angle is $\theta' = 225° - 180° = 45°$.
$\cos 45° = \frac{\sqrt{2}}{2}$
Because the cosine is negative in quadrant III,
$\cos 225° = -\cos 45° = -\frac{\sqrt{2}}{2}$.

62. 300° lies in quadrant IV. The reference angle is $\theta' = 360° - 300° = 60°$.
$\sin 60° = \frac{\sqrt{3}}{2}$
Because the sine is negative in quadrant IV,
$\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}$.

63. 210° lies in quadrant III. The reference angle is $\theta' = 210° - 180° = 30°$.
$\tan 30° = \frac{\sqrt{3}}{3}$
Because the tangent is positive in quadrant III,
$\tan 210° = \tan 30° = \frac{\sqrt{3}}{3}$.

64. 240° lies in quadrant III. The reference angle is $\theta' = 240° - 180° = 60°$.
$\sec 60° = 2$
Because the secant is negative in quadrant III,
$\sec 240° = -\sec 60° - 2$.

65. 420° lies in quadrant I. The reference angle is $\theta' = 420° - 360° = 60°$.
$\tan 60° = \sqrt{3}$
Because the tangent is positive in quadrant I,
$\tan 420° = \tan 60° = \sqrt{3}$.

66. 405° lies in quadrant I. The reference angle is $\theta' = 405° - 360° = 45°$.
$\tan 45° = 1$
Because the tangent is positive in quadrant I,
$\tan 405° = \tan 45° = 1$.

67. $\frac{2\pi}{3}$ lies in quadrant II. The reference angle is

$\theta' = \pi - \frac{2\pi}{3} = \frac{3\pi}{3} - \frac{2\pi}{3} = \frac{\pi}{3}$.

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Because the sine is positive in quadrant II,

$\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

68. $\frac{3\pi}{4}$ lies in quadrant II. The reference angle is

$\theta' = \pi - \frac{3\pi}{4} = \frac{4\pi}{4} - \frac{3\pi}{4} = \frac{\pi}{4}$.

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

Because the cosine is negative in quadrant II,

$\cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$.

69. $\frac{7\pi}{6}$ lies in quadrant III. The reference angle is

$\theta' = \frac{7\pi}{6} - \pi = \frac{7\pi}{6} - \frac{6\pi}{6} = \frac{\pi}{6}$.

$\csc\frac{\pi}{6} = 2$

Because the cosecant is negative in quadrant III,

$\csc\frac{7\pi}{6} = -\csc\frac{\pi}{6} = -2$.

70. $\frac{7\pi}{4}$ lies in quadrant IV. The reference angle is

$\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$.

$\cot\frac{\pi}{4} = 1$

Because the cotangent is negative in quadrant IV,

$\cot\frac{7\pi}{4} = -\cot\frac{\pi}{4} = -1$.

71. $\frac{9\pi}{4}$ lies in quadrant I. The reference angle is

$\theta' = \frac{9\pi}{4} - 2\pi = \frac{9\pi}{4} - \frac{8\pi}{4} = \frac{\pi}{4}$.

$\tan\frac{\pi}{4} = 1$

Because the tangent is positive in quadrant I,

$\tan\frac{9\pi}{4} = \tan\frac{\pi}{4} = 1$

72. $\frac{9\pi}{2}$ lies on the positive *y*-axis. The reference angle is

$\theta' = \frac{9\pi}{2} - 4\pi = \frac{9\pi}{2} - \frac{8\pi}{2} = \frac{\pi}{2}$.

Because $\tan\frac{\pi}{2}$ is undefined, $\tan\frac{9\pi}{2}$ is also undefined.

73. –240° lies in quadrant II. The reference angle is $\theta' = 240° - 180° = 60°$.

$\sin 60° = \frac{\sqrt{3}}{2}$

Because the sine is positive in quadrant II,

$\sin(-240°) = \sin 60° = \frac{\sqrt{3}}{2}$.

74. –225° lies in quadrant II. The reference angle is $\theta' = 225° - 180° = 45°$.

$\sin 45° = \frac{\sqrt{2}}{2}$

Because the sine is positive in quadrant II,

$\sin(-225°) = \sin 45° = \frac{\sqrt{2}}{2}$.

75. $-\frac{\pi}{4}$ lies in quadrant IV. The reference angle is

$\theta' = \frac{\pi}{4}$.

$\tan\frac{\pi}{4} = 1$

Because the tangent is negative in quadrant IV,

$\tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$

76. $-\frac{\pi}{6}$ lies in quadrant IV. The reference angle is

$\theta = \frac{\pi}{6}$. $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

Because the tangent is negative in quadrant IV,

$\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$.

77. $\sec 495° = \sec 135° = -\sqrt{2}$

78. $\sec 510° = \sec 150° = -\frac{2\sqrt{3}}{3}$

79. $\cot\frac{19\pi}{6} = \cot\frac{7\pi}{6} = \sqrt{3}$

80. $\cot\frac{13\pi}{3} = \cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

81. $\cos\frac{23\pi}{4} = \cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$

82. $\cos\frac{35\pi}{6} = \cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$

83. $\tan\left(-\frac{17\pi}{6}\right) = \tan\frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

84. $\tan\left(-\frac{11\pi}{4}\right) = \tan\frac{\pi}{4} = 1$

85. $\sin\left(-\frac{17\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

86. $\sin\left(-\frac{35\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

87. $\sin\frac{\pi}{3}\cos\pi - \cos\frac{\pi}{3}\sin\frac{3\pi}{2}$

$= \left(\frac{\sqrt{3}}{2}\right)(-1) - \left(\frac{1}{2}\right)(-1)$

$= -\frac{\sqrt{3}}{2} + \frac{1}{2}$

$= \frac{1-\sqrt{3}}{2}$

88. $\sin\frac{\pi}{4}\cos 0 - \sin\frac{\pi}{6}\cos\pi$

$= \left(\frac{\sqrt{2}}{2}\right)(1) - \left(\frac{1}{2}\right)(-1)$

$= \frac{\sqrt{2}}{2} + \frac{1}{2}$

$= \frac{\sqrt{2}+1}{2}$

89. $\sin\frac{11\pi}{4}\cos\frac{5\pi}{6} + \cos\frac{11\pi}{4}\sin\frac{5\pi}{6}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= -\frac{\sqrt{6}+\sqrt{2}}{4}$

90. $\sin\frac{17\pi}{3}\cos\frac{5\pi}{4} + \cos\frac{17\pi}{3}\sin\frac{5\pi}{4}$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

91. $\sin\frac{3\pi}{2}\tan\left(-\frac{15\pi}{4}\right) - \cos\left(-\frac{5\pi}{3}\right)$

$= (-1)(1) - \left(\frac{1}{2}\right)$

$= -1 - \frac{1}{2}$

$= -\frac{2}{2} - \frac{1}{2}$

$= -\frac{3}{2}$

92. $\sin\frac{3\pi}{2}\tan\left(-\frac{8\pi}{3}\right)+\cos\left(-\frac{5\pi}{6}\right)$
$= (-1)(\sqrt{3})+\left(-\frac{\sqrt{3}}{2}\right)$
$= -\sqrt{3}-\frac{\sqrt{3}}{2}$
$= -\frac{2\sqrt{3}}{2}-\frac{\sqrt{3}}{2}$
$= -\frac{3\sqrt{3}}{2}$

93. $f\left(\frac{4\pi}{3}+\frac{\pi}{6}\right)+f\left(\frac{4\pi}{3}\right)+f\left(\frac{\pi}{6}\right)$
$= \sin\left(\frac{4\pi}{3}+\frac{\pi}{6}\right)+\sin\frac{4\pi}{3}+\sin\frac{\pi}{6}$
$= \sin\frac{3\pi}{2}+\sin\frac{4\pi}{3}+\sin\frac{\pi}{6}$
$= (-1)+\left(-\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)$
$= -\frac{\sqrt{3}+1}{2}$

94. $g\left(\frac{5\pi}{6}+\frac{\pi}{6}\right)+g\left(\frac{5\pi}{6}\right)+g\left(\frac{\pi}{6}\right)$
$= \cos\left(\frac{5\pi}{6}+\frac{\pi}{6}\right)+\cos\frac{5\pi}{6}+\cos\frac{\pi}{6}$
$= \cos\pi+\cos\frac{5\pi}{6}+\cos\frac{\pi}{6}$
$= (-1)+\left(-\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)$
$= -1$

95. $(h\circ g)\left(\frac{17\pi}{3}\right)=h\left(g\left(\frac{17\pi}{3}\right)\right)$
$= 2\left(\cos\left(\frac{17\pi}{3}\right)\right)$
$= 2\left(\frac{1}{2}\right)$
$= 1$

96. $(h\circ f)\left(\frac{11\pi}{4}\right)=h\left(f\left(\frac{11\pi}{4}\right)\right)$
$= 2\left(\sin\left(\frac{11\pi}{4}\right)\right)$
$= 2\left(\frac{\sqrt{2}}{2}\right)$
$= \sqrt{2}$

97. The average rate of change is the slope of the line through the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$

$$m=\frac{f(x_2)-f(x_1)}{x_2-x_1}$$
$$=\frac{\sin\left(\frac{3\pi}{2}\right)-\sin\left(\frac{5\pi}{4}\right)}{\frac{3\pi}{2}-\frac{5\pi}{4}}$$
$$=\frac{-1-\left(-\frac{\sqrt{2}}{2}\right)}{\frac{\pi}{4}}$$
$$=\frac{-1+\frac{\sqrt{2}}{2}}{\frac{\pi}{4}}$$
$$=\frac{4\left(-1+\frac{\sqrt{2}}{2}\right)}{4\left(\frac{\pi}{4}\right)}$$
$$=\frac{2\sqrt{2}-4}{\pi}$$

98. The average rate of change is the slope of the line through the points $(x_1, g(x_1))$ and $(x_2, g(x_2))$

$$m = \frac{g(x_2) - g(x_1)}{x_2 - x_1} = \frac{\cos(\pi) - \cos\left(\frac{3\pi}{4}\right)}{\pi - \frac{3\pi}{4}} = \frac{-1 - \left(-\frac{\sqrt{2}}{2}\right)}{\frac{\pi}{4}} = \frac{4\left(-1 + \frac{\sqrt{2}}{2}\right)}{4\left(\frac{\pi}{4}\right)} = \frac{2\sqrt{2} - 4}{\pi}$$

99. $\sin\theta = \frac{\sqrt{2}}{2}$ when the reference angle is $\frac{\pi}{4}$ and θ is in quadrants I or II.

QI: $\theta = \frac{\pi}{4}$

QII: $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

100. $\cos\theta = \frac{1}{2}$ when the reference angle is $\frac{\pi}{3}$ and θ is in quadrants I or IV.

QI: $\theta = \frac{\pi}{3}$

QIV: $\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

101. $\sin\theta = -\frac{\sqrt{2}}{2}$ when the reference angle is $\frac{\pi}{4}$ and θ is in quadrants III or IV.

QIII: $\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

QIV: $\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

$$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$$

102. $\cos\theta = -\frac{1}{2}$ when the reference angle is $\frac{\pi}{3}$ and θ is in quadrants II or III.

QII: $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

QIII: $\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

103. $\tan\theta = -\sqrt{3}$ when the reference angle is $\frac{\pi}{3}$ and θ is in quadrants II or IV.

QII: $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

QIV: $\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$$

104. $\tan\theta = -\frac{\sqrt{3}}{3}$ when the reference angle is $\frac{\pi}{6}$ and θ is in quadrants II or IV.

QII	QIV
$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$	$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

105. – 109. Answers may vary.

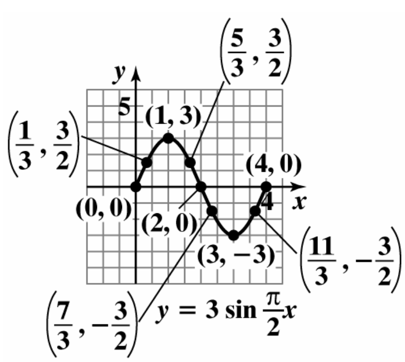
110. does not make sense; Explanations will vary. Sample explanation: Sine is defined for all values of the angle.

111. does not make sense; Explanations will vary. Sample explanation: Sine and cosecant have the same sign within any quadrant because they are reciprocals of each other.

112. does not make sense; Explanations will vary. Sample explanation: It is also possible that $y = -3$ and $x = -5$.

113. makes sense

114.



115. domain: $\{x \mid -1 \le x \le 1\}$ or $[-1,1]$
range: $\{y \mid -1 \le y \le 1\}$ or $[-1,1]$

116. **a.** $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

The sine function is not even because $\sin(-\theta) \ne \sin\theta$

b. $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

The cosine function is not odd because $\cos(-\theta) \ne -\cos\theta$

Section 5.4

Check Point Exercises

1. $P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\sin t = y = \frac{1}{2}$

$\cos t = x = \frac{\sqrt{3}}{2}$

$\tan t = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

$\csc t = \frac{1}{y} = 2$

$\sec t = \frac{1}{x} = \frac{2\sqrt{3}}{3}$

$\cot t = \frac{x}{y} = \sqrt{3}$

2. The point P on the unit circle that corresponds to $t = \pi$ has coordinates (–1, 0). Use $x = -1$ and $y = 0$ to find the values of the trigonometric functions.

$\sin \pi = y = 0$

$\cos \pi = x = -1$

$\tan \pi = \frac{y}{x} = \frac{0}{-1} = 0$

$\sec \pi = \frac{1}{x} = \frac{1}{-1} = -1$

$\cot \pi = \frac{x}{y} = \frac{-1}{0} = \text{undefined}$

$\csc \pi = \frac{1}{y} = \frac{1}{0} = \text{undefined}$

3. **a.** $\cos(-60^\circ) = \cos 60^\circ = \frac{1}{2}$

b. $\tan\left(-\frac{\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$

4. **a.** $\cos 405^\circ = \cos(360^\circ + 45^\circ)$

$= \cos 45^\circ = \frac{\sqrt{2}}{2}$

b. $\sin \frac{7\pi}{3} = \sin\left(\frac{\pi}{3} + 2\pi\right) = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Exercise Set 5.4

1. The point P on the unit circle has coordinates $\left(-\frac{15}{17}, \frac{8}{17}\right)$. Use $x = -\frac{15}{17}$ and $y = \frac{8}{17}$ to find the values of the trigonometric functions.

$\sin t = y = \frac{8}{17}$

$\cos t = x = -\frac{15}{17}$

$\tan t = \frac{y}{x} = \frac{\frac{8}{17}}{-\frac{15}{17}} = -\frac{8}{15}$

$\csc t = \frac{1}{y} = \frac{17}{8}$

$\sec t = \frac{1}{x} = -\frac{17}{15}$

$\cot t = \frac{x}{y} = -\frac{15}{8}$

2. The point P on the unit circle has coordinates $\left(-\frac{5}{13}, -\frac{12}{13}\right)$ Use $x = -\frac{5}{13}$ and $y = -\frac{12}{13}$ to find the values of the trigonometric functions.

$\sin t = y = -\frac{12}{13}$

$\cos t = x = -\frac{5}{13}$

$\tan t = \frac{y}{x} = \frac{-\frac{12}{13}}{-\frac{5}{13}} = \frac{12}{5}$

$\csc t = \frac{1}{y} = -\frac{13}{12}$

$\sec t = \frac{1}{x} = -\frac{13}{5}$

$\cot t = \frac{x}{y} = \frac{5}{12}$

3. The point P on the unit circle that corresponds to $t=-\frac{\pi}{4}$ has coordinates $\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$. Use $x=\frac{\sqrt{2}}{2}$ and $y=-\frac{\sqrt{2}}{2}$ to find the values of the trigonometric functions.

$\sin t=y=-\frac{\sqrt{2}}{2}$

$\cos t=x=\frac{\sqrt{2}}{2}$

$\tan t=\frac{y}{x}=\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}=-1$

$\csc t=\frac{1}{y}=-\sqrt{2}$

$\sec t=\frac{1}{x}=\sqrt{2}$

$\cot t=\frac{x}{y}=-1$

4. The point P on the unit circle that corresponds to $t=\frac{3\pi}{4}$ has coordinates $\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$. Use $x=-\frac{\sqrt{2}}{2}$ and $y=\frac{\sqrt{2}}{2}$ to find the values of the trigonometric functions.

$\sin t=y=\frac{\sqrt{2}}{2}$

$\cos t=x=-\frac{\sqrt{2}}{2}$

$\tan t=\frac{y}{x}=\frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}=-1$

$\csc t=\frac{1}{y}=\sqrt{2}$

$\sec t=\frac{1}{x}=-\sqrt{2}$

$\cot t=\frac{x}{y}=-1$

5. $\sin\frac{\pi}{6}=\frac{1}{2}$

6. $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

7. $\cos\frac{5\pi}{6}=-\frac{\sqrt{3}}{2}$

8. $\cos\frac{2\pi}{3}=-\frac{1}{2}$

9. $\tan\pi=\frac{0}{-1}=0$

10. $\tan 0=\frac{0}{1}=0$

11. $\csc\frac{7\pi}{6}=\frac{1}{-\frac{1}{2}}=-2$

12. $\csc\frac{4\pi}{3}=\frac{1}{-\frac{\sqrt{3}}{2}}=\frac{-2\sqrt{3}}{3}$

13. $\sec\frac{11\pi}{6}=\frac{1}{\frac{\sqrt{3}}{2}}=\frac{2\sqrt{3}}{3}$

14. $\sec\frac{5\pi}{3}=\frac{1}{\frac{1}{2}}=2$

15. $\sin\frac{3\pi}{2}=-1$

16. $\cos\frac{3\pi}{2}=0$

17. $\sec\frac{3\pi}{2}=\text{undefined}$

18. $\tan\frac{3\pi}{2}=\text{undefined}$

19. **a.** $\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

b. $\cos\left(-\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

20. **a.** $\cos\frac{\pi}{3}=\frac{1}{2}$

b. $\cos\left(-\frac{\pi}{3}\right)=\cos\frac{\pi}{3}=\frac{1}{2}$

21. **a.** $\sin\frac{5\pi}{6}=\frac{1}{2}$

b. $\sin\left(-\frac{5\pi}{6}\right)=-\sin\frac{5\pi}{6}=-\frac{1}{2}$

22. **a.** $\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$

b. $\sin\left(-\frac{2\pi}{3}\right)=-\sin\frac{2\pi}{3}=-\frac{\sqrt{3}}{2}$

23. **a.** $\tan\frac{5\pi}{3}=\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}=-\sqrt{3}$

b. $\tan\left(-\frac{5\pi}{3}\right)=-\tan\frac{5\pi}{3}=\sqrt{3}$

24. **a.** $\tan\frac{11\pi}{6}=\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}=-\frac{\sqrt{3}}{3}$

b. $\tan\left(-\frac{11\pi}{6}\right)=-\tan\frac{11\pi}{6}=\frac{\sqrt{3}}{3}$

25. **a.** $\sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2}$

b. $\sin\frac{11\pi}{4}=\sin\left(\frac{3\pi}{4}+2\pi\right)=\sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2}$

26. **a.** $\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$

b. $\cos\frac{11\pi}{4}=\cos\left(\frac{3\pi}{4}+2\pi\right)=\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$

27. **a.** $\cos\frac{\pi}{2}=0$

b.
$$\begin{aligned}\cos\frac{9\pi}{2}&=\cos\left(\frac{\pi}{2}+4\pi\right)\\&=\cos\left[\frac{\pi}{2}+2(2\pi)\right]\\&=\cos\frac{\pi}{2}\\&=0\end{aligned}$$

28. **a.** $\sin\frac{\pi}{2}=1$

b. $\sin\frac{9\pi}{2}=\sin\left(\frac{\pi}{2}+4\pi\right)=\sin\frac{\pi}{2}=1$

29. **a.** $\tan\pi=\frac{0}{-1}=0$

b.
$$\begin{aligned}\tan 17\pi&=\tan(\pi+16\pi)\\&=\tan[\pi+8(2\pi)]\\&=\tan\pi\\&=0\end{aligned}$$

30. **a.** $\cot\frac{\pi}{2}=\frac{0}{1}=0$

b. $\cot\frac{15\pi}{2}=\cot\left(\frac{\pi}{2}+7\pi\right)=\cot\frac{\pi}{2}=0$

31. **a.** $\sin\frac{7\pi}{4}=-\frac{\sqrt{2}}{2}$

b.
$$\begin{aligned}\sin\frac{47\pi}{4}&=\sin\left(\frac{7\pi}{4}+10\pi\right)\\&=\sin\left[\frac{7\pi}{4}+5(2\pi)\right]\\&=\sin\frac{7\pi}{4}\\&=-\frac{\sqrt{2}}{2}\end{aligned}$$

32. **a.** $\cos\frac{7\pi}{4}=\frac{\sqrt{2}}{2}$

b. $\cos\frac{47\pi}{4}=\cos\left(\frac{7\pi}{4}+10\pi\right)=\cos\frac{7\pi}{4}=\frac{\sqrt{2}}{2}$

33. $\sin(-t)-\sin t=-\sin t-\sin t=-2\sin t=-2a$

34. $\tan(-t)-\tan t=-\tan t-\tan t=-2\tan t=-2c$

35. $4\cos(-t)-\cos t=4\cos t-\cos t=3\cos t=3b$

36. $3\cos(-t)-\cos t=3\cos t-\cos t=2\cos t=2b$

37. $\sin(t+2\pi)-\cos(t+4\pi)+\tan(t+\pi)$
$=\sin(t)-\cos(t)+\tan(t)$
$=a-b+c$

38. $\sin(t+2\pi)+\cos(t+4\pi)-\tan(t+\pi)$
$=\sin(t)+\cos(t)-\tan(t)$
$=a+b-c$

39. $\sin(-t-2\pi)-\cos(-t-4\pi)-\tan(-t-\pi)$
$=-\sin(t+2\pi)-\cos(t+4\pi)+\tan(t+\pi)$
$=-\sin(t)-\cos(t)+\tan(t)$
$=-a-b+c$

40. $\sin(-t-2\pi)+\cos(-t-4\pi)-\tan(-t-\pi)$
$=-\sin(t+2\pi)+\cos(t+4\pi)+\tan(t+\pi)$
$=-\sin(t)+\cos(t)+\tan(t)$
$=-a+b+c$

41. $\cos t+\cos(t+1000\pi)-\tan t-\tan(t+999\pi)$
$-\sin t+4\sin(t-1000\pi)$
$=\cos t+\cos t-\tan t-\tan t-\sin t+4\sin t$
$=2\cos t-2\tan t+3\sin t$
$=3a+2b-2c$

42. $-\cos t+7\cos(t+1000\pi)+\tan t+\tan(t+999\pi)$
$+\sin t+\sin(t-1000\pi)$
$=-\cos t+7\cos t+\tan t+\tan t+\sin t+\sin t$
$=6\cos t+2\tan t+2\sin t$
$=2a+6b+2c$

43. **a.** $H=12+8.3\sin\left[\frac{2\pi}{365}(80-80)\right]$
$=12+8.3\sin 0=12+8.3(0)$
$=12$
There are 12 hours of daylight in Fairbanks on March 21.

b. $H=12+8.3\sin\left[\frac{2\pi}{365}(172-80)\right]$
$\approx 12+8.3\sin 1.5837$
≈ 20.3
There are about 20.3 hours of daylight in Fairbanks on June 21.

c. $H=12+8.3\sin\left[\frac{2\pi}{365}(355-80)\right]$
$\approx 12+8.3\sin 4.7339$
≈ 3.7
There are about 3.7 hours of daylight in Fairbanks on December 21.

44. **a.** $H=12+24\sin\left[\frac{2\pi}{365}(80-80)\right]$
$=12+24\sin 0=12+24(0)$
$=12$
There are 12 hours of daylight in San Diego on March 21.

b. $H=12+24\sin\left[\frac{2\pi}{365}(172-80)\right]$
$\approx 12+24\sin 1.5837$
≈ 14.3998
There are about 14.4 hours of daylight in San Diego on June 21.

c. $H=12+24\sin\left[\frac{2\pi}{365}(355-80)\right]$
$\approx 12+24\sin 4.7339$
≈ 9.6
There are about 9.6 hours of daylight in San Diego on December 21.

45. **a.** For $t = 7$,
$E=\sin\frac{\pi}{14}\cdot 7=\sin\frac{\pi}{2}=1$

For $t=14$,
$E=\sin\frac{\pi}{14}\cdot 14=\sin\pi=0$
For $t=21$,
$E=\sin\frac{\pi}{14}\cdot 21=\sin\frac{3\pi}{2}=-1$
For $t=28$,
$E=\sin\frac{\pi}{14}\cdot 28=\sin 2\pi=\sin 0=0$
For $t=35$,
$E=\sin\frac{\pi}{14}\cdot 35=\sin\frac{5\pi}{2}=\sin\frac{\pi}{2}=1$
Observations may vary.

b. Because $E(35) = E(7) = 1$, the period is $35 - 7 = 28$ or 28 days.

46. a. At 6 A.M., $t = 0$.

$H = 10 + 4\sin\frac{\pi}{6}\cdot 0$

$= 10 + 4\sin 0 = 10 + 4\cdot 0 = 10$

The height is 10 feet.

At 9 A.M., $t = 3$.

$H = 10 + 4\sin\frac{\pi}{6}\cdot 3$

$= 10 + 4\sin\frac{\pi}{2} = 10 + 4(1) = 14$

The height is 14 feet.

At noon, $t = 6$.

$H = 10 + 4\sin\frac{\pi}{6}\cdot 6$

$= 10 + 4\sin\pi = 10 + 4\cdot 0 = 10$

The height is 10 feet.

At 6 P.M., $t = 12$.

$H = 10 + 4\sin\frac{\pi}{6}\cdot 12$

$= 10 + 4\sin 2\pi = 10 + 4\cdot 0 = 10$

The height is 10 feet.

At midnight, $t = 18$.

$H = 10 + 4\sin\frac{\pi}{6}\cdot 18$

$= 10 + 4\sin 3\pi = 10 + 4\sin\pi$

$= 10 + 4\cdot 0 = 10$

The height is 10 feet.

At 3 A.M., $t = 21$.

$H = 10 + 4\sin\frac{\pi}{6}\cdot 21$

$= 10 + 4\sin\frac{7\pi}{2} = 10 + 4\sin\frac{3\pi}{2}$

$= 10 + 4(-1) = 6$

The height is 6 feet.

b. The sine function has a minimum at $\frac{3\pi}{2}$. Thus, we find a low tide at $\frac{\pi}{6}t = \frac{3\pi}{2}$ or $t = 9$. This value of t corresponds to 3 P.M. For $t = 9$,

$h = 10 + 4\sin\frac{\pi}{6}\cdot 9$

$= 10 + 4\sin\frac{3\pi}{2} = 10 + 4(-1) = 6$

The height is 6 feet. From part *a*, the height at 3 A.M. is also 6 feet. Thus, low tide is at 3 A.M. and 3 P.M.

The sine function has a maximum at $\frac{\pi}{2}$. Thus, we find a high tide at $\frac{\pi}{6}t = \frac{\pi}{2}$ or $t = 3$. This value of t corresponds to 9 a.m. From part *a*, the height at 9 A.M. is 14 feet. Because the sine has a period of 2π we also find a maximum at $\frac{5\pi}{2}$.

We find another high tide at $\frac{\pi}{6}t = \frac{5\pi}{2}$ or $t = 15$. This value of t corresponds to 9 P.M. Thus, high tide is at 9 A.M. and 9 P.M.

c. The period of the sine function is 2π or on the interval $[0, 2\pi]$. The cycle of the sine function starts at $\frac{\pi}{6}t = \frac{5\pi}{2}$ or $t = 0$, and ends at $\frac{\pi}{6}t = 2\pi$ or $t = 12$. Thus, the period is 12 hours, which means high and low tides occur every 12 hours.

47. – 52. Answers may vary.

53. makes sense

54. does not make sense; Explanations will vary. Sample explanation: $\sin t$ cannot be less than -1. Note that $-\frac{\sqrt{10}}{2} \approx -1.58 < -1$.

55. does not make sense; Explanations will vary. Sample explanation: Cosine is not an odd function.

56. makes sense

57. $\cos 0^\circ + \cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \cdots + \cos 179^\circ + \cos 180^\circ$

$= (\cos 0^\circ + \cos 180^\circ) + (\cos 1^\circ + \cos 179^\circ) + (\cos 2^\circ + \cos 178^\circ) + \cdots$

$= \overbrace{(\cos 0^\circ + \cos 180^\circ)}^{0} + \overbrace{(\cos 1^\circ + \cos 179^\circ)}^{0} + \overbrace{(\cos 2^\circ + \cos 178^\circ)}^{0} + \cdots$

$= 0 + 0 + 0 + \cdots + 0$

$= 0$

58. $f(a) + f(a + 2\pi) + f(a + 4\pi) + f(a + 6\pi)$

$= \sin(a) + \sin(a + 2\pi) + \sin(a + 4\pi) + \sin(a + 6\pi)$

$= \sin(a) + \sin(a) + \sin(a) + \sin(a)$

$= f(a) + f(a) + f(a) + f(a)$

$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

$= 1$

59. $f(a) + 2f(-a) = \sin(a) + 2\sin(-a)$

$= \sin(a) - 2\sin(a)$

$= f(a) - 2f(a)$

$= \frac{1}{4} - 2\left(\frac{1}{4}\right)$

$= -\frac{1}{4}$

60. $\cos 765^0 = \cos(360^0 + 360^0 + 45^0) = \cos 45^0$

Using a right triangle we have a known angle, an unknown opposite side, *h*, and a known hypotenuse.

Thus, $\cos 45^{\text{o}} = \frac{h}{40}$

$h = 40\cos 45^0 = 40(0.7071) \approx 28$

Because *h* represents your height below the center of the Ferris wheel, we subtract 28 from 40 and add 5 to determine our height above the ground. $40 - 28 + 5 = 17$

You are about 17 feet above the ground.

61. $y = \frac{1}{2}\cos(4x + \pi)$

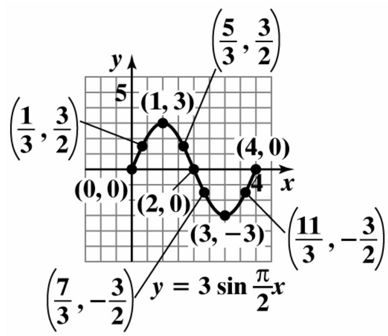
x	$-\frac{\pi}{4}$	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
y	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$

62. $y = 4\sin\left(2x - \frac{2\pi}{3}\right)$

x	$\frac{\pi}{3}$	$\frac{7\pi}{12}$	$\frac{5\pi}{6}$	$\frac{13\pi}{12}$	$\frac{4\pi}{3}$
y	0	4	0	–4	0

63. $y = 3\sin\frac{\pi}{2}x$

x	0	$\frac{1}{3}$	1	$\frac{5}{3}$	2	$\frac{7}{3}$	3	$\frac{11}{3}$	4
y	0	$\frac{3}{2}$	3	$\frac{3}{2}$	0	$-\frac{3}{2}$	−3	$-\frac{3}{2}$	0



Mid-Chapter 5 Check Point

1. $10° = 10° \cdot \frac{\pi \text{ radians}}{180°} = \frac{10\pi}{180}$ radians

$= \frac{\pi}{18}$ radians

2. $-105° = -105° \cdot \frac{\pi \text{ radians}}{180°} = -\frac{105\pi}{180}$ radians

$= -\frac{7\pi}{12}$ radians

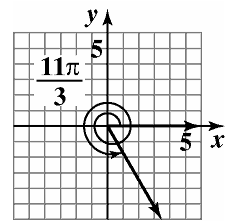
3. $\frac{5\pi}{12}$ radians $= \frac{5\pi \text{ radians}}{12} \cdot \frac{180^{\text{o}}}{\pi \text{ radians}} = 75^{\text{o}}$

4. $-\frac{13\pi}{20}$ radians $= -\frac{13\pi \text{ radians}}{20} \cdot \frac{180^{\text{o}}}{\pi \text{ radians}}$

$= -117^{\text{o}}$

5. **a.** $\frac{11\pi}{3} - 2\pi = \frac{11\pi}{3} - \frac{6\pi}{3} = \frac{5\pi}{3}$

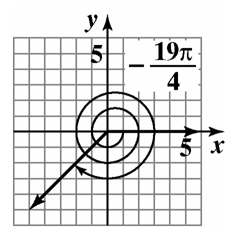
b.



c. Since $\frac{5\pi}{3}$ is in quadrant IV, the reference angle is $2\pi - \frac{5\pi}{3} = \frac{6\pi}{3} - \frac{5\pi}{3} = \frac{\pi}{3}$

6. **a.** $-\frac{19\pi}{4} + 6\pi = -\frac{19\pi}{4} + \frac{24\pi}{4} = \frac{5\pi}{4}$

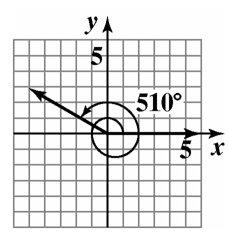
b.



c. Since $\frac{5\pi}{4}$ is in quadrant III, the reference angle is $\frac{5\pi}{4} - \pi = \frac{5\pi}{4} - \frac{4\pi}{4} = \frac{\pi}{4}$

7. **a.** $510° - 360° = 150°$

b.



c. Since $150°$ is in quadrant II, the reference angle is $180° - 150° = 30°$

8. Use the Pythagorean theorem to find *b*.

$a^2+b^2=c^2$

$5^2+b^2=6^2$

$25+b^2=36$

$b^2=11$

$b=\sqrt{11}$

$\sin\theta=\dfrac{\text{opposite}}{\text{hypotenuse}}=\dfrac{5}{6}$

$\cos\theta=\dfrac{\text{adjacent}}{\text{hypotenuse}}=\dfrac{\sqrt{11}}{6}$

$\tan\theta=\dfrac{\text{opposite}}{\text{adjacent}}=\dfrac{5\sqrt{11}}{11}$

$\csc\theta=\dfrac{\text{hypotenuse}}{\text{opposite}}=\dfrac{6}{5}$

$\sec\theta=\dfrac{\text{hypotenuse}}{\text{adjacent}}=\dfrac{6}{\sqrt{11}}=\dfrac{6\sqrt{11}}{11}$

$\cot\theta=\dfrac{\text{adjacent}}{\text{opposite}}=\dfrac{\sqrt{11}}{5}$

9. $r=\sqrt{x^2+y^2}$

$r=\sqrt{3^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$\sin\theta=\dfrac{y}{r}=\dfrac{-2}{\sqrt{13}}=-\dfrac{2\sqrt{13}}{13}$

$\cos\theta=\dfrac{x}{r}=\dfrac{3}{\sqrt{13}}=\dfrac{3\sqrt{13}}{13}$

$\tan\theta=\dfrac{y}{x}=\dfrac{-2}{3}=-\dfrac{2}{3}$

$\csc\theta=\dfrac{r}{y}=\dfrac{\sqrt{13}}{-2}=-\dfrac{\sqrt{13}}{2}$

$\sec\theta=\dfrac{r}{x}=\dfrac{\sqrt{13}}{3}$

$\cot\theta=\dfrac{x}{y}=\dfrac{3}{-2}=-\dfrac{3}{2}$

10. $r=\sqrt{x^2+y^2}$

$r=\sqrt{\left(-\dfrac{3}{5}\right)^2+\left(-\dfrac{4}{5}\right)^2}=\sqrt{\dfrac{9}{25}+\dfrac{16}{25}}=\sqrt{\dfrac{25}{25}}=1$

Now that we know *x, y,* and *r,* we can find the six trigonometric functions of θ.

$\sin\theta=\dfrac{y}{r}=\dfrac{-\frac{4}{5}}{1}=-\dfrac{4}{5}$

$\cos\theta=\dfrac{x}{r}=\dfrac{-\frac{3}{5}}{1}=-\dfrac{3}{5}$

$\tan\theta=\dfrac{y}{x}=\dfrac{-\frac{4}{5}}{-\frac{3}{5}}=\dfrac{4}{3}$

$\csc\theta=\dfrac{r}{y}=\dfrac{1}{-\frac{4}{5}}=-\dfrac{5}{4}$

$\sec\theta=\dfrac{r}{x}=\dfrac{1}{-\frac{3}{5}}=-\dfrac{5}{3}$

$\cot\theta=\dfrac{x}{y}=\dfrac{-\frac{3}{5}}{-\frac{4}{5}}=\dfrac{3}{4}$

11. Because the tangent is negative and the cosine is negative, θ is in quadrant II. In quadrant II, *x* is negative and *y* is positive. Thus,

$\tan\theta=-\dfrac{3}{4}=\dfrac{x}{y},\quad x=-4,\ y=3$. Furthermore,

$r^2=x^2+y^2$

$r^2=(-3)^2+4^2$

$r^2=9+16=25$

$r=5$

Now that we know *x, y,* and *r,* we can find the remaining trigonometric functions of θ.

$\sin\theta=\dfrac{y}{r}=\dfrac{3}{5}$

$\cos\theta=\dfrac{x}{r}=\dfrac{-4}{5}=-\dfrac{4}{5}$

$\csc\theta=\dfrac{r}{y}=\dfrac{5}{3}$

$\sec\theta=\dfrac{r}{x}=\dfrac{5}{-3}=-\dfrac{5}{4}$

$\cot\theta=\dfrac{x}{y}=\dfrac{-3}{4}=-\dfrac{4}{3}$

12. Since $\cos\theta = \frac{3}{7} = \frac{x}{r}$, $x = 3,\ r = 7$. Furthermore,

$$x^2 + y^2 = r^2$$
$$3^2 + y^2 = 7^2$$
$$9 + y^2 = 49$$
$$y^2 = 40$$
$$y = \pm\sqrt{40} = \pm 2\sqrt{10}$$

Because the cosine is positive and the sine is negative, θ is in quadrant IV. In quadrant IV, x is positive and y is negative.

Therefore $y = -2\sqrt{10}$

Use x, y, and r to find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{10}}{7} = -\frac{2\sqrt{10}}{7}$$
$$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{10}}{3} = -\frac{2\sqrt{10}}{3}$$
$$\csc\theta = \frac{r}{y} = \frac{7}{-2\sqrt{10}} = -\frac{7\sqrt{10}}{20}$$
$$\sec\theta = \frac{r}{x} = \frac{7}{3}$$
$$\cot\theta = \frac{x}{y} = \frac{3}{-2\sqrt{10}} = -\frac{3\sqrt{10}}{20}$$

13. $$\tan\theta = \frac{\text{side opposite } \theta}{\text{side adjacent } \theta}$$

$$\tan 41^\circ = \frac{a}{60}$$
$$a = 60\tan 41^\circ$$
$$a \approx 52 \text{ cm}$$

14. $$\cos\theta = \frac{\text{side adjacent } \theta}{\text{hypotenuse}}$$

$$\cos 72^\circ = \frac{250}{c}$$
$$c = \frac{250}{\cos 72^\circ}$$
$$c \approx 809 \text{ m}$$

15. Since $\cos\theta = \frac{1}{6} = \frac{x}{r}$, $x = 1,\ r = 6$. Furthermore,

$$x^2 + y^2 = r^2$$
$$1^2 + y^2 = 6^2$$
$$1 + y^2 = 36$$
$$y^2 = 35$$
$$y = \pm\sqrt{35}$$

Since θ is acute, $y = +\sqrt{35} = \sqrt{35}$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta = \frac{y}{x} = \frac{\sqrt{35}}{1} = \sqrt{35}$$

16. $\tan 30^\circ = \frac{\sqrt{3}}{3}$

17. $\cot 120^\circ = \frac{1}{\tan 120^\circ} = \frac{1}{-\tan 60^\circ} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$

18. $\cos 240^\circ = -\cos 60^\circ = -\frac{1}{2}$

19. $\sec\frac{11\pi}{6} = \frac{1}{\cos\frac{11\pi}{6}} = \frac{1}{\cos\frac{\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

20. $\sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7} = 1$

21.
$$\sin\left(-\frac{2\pi}{3}\right) = \sin\left(-\frac{2\pi}{3} + 2\pi\right)$$
$$= \sin\frac{4\pi}{3} = -\sin\frac{\pi}{3}$$
$$= -\frac{\sqrt{3}}{2}$$

22.
$$\csc\left(\frac{22\pi}{3}\right) = \csc\left(\frac{22\pi}{3} - 6\pi\right) = \csc\frac{4\pi}{3}$$
$$= \frac{1}{\sin\frac{4\pi}{3}} = \frac{1}{-\sin\frac{\pi}{3}} = \frac{1}{-\frac{\sqrt{3}}{2}}$$
$$= -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

23. $\cos 495° = \cos(495° - 360°) = \cos 135°$
$= -\cos 45° = -\dfrac{\sqrt{2}}{2}$

24. $\tan\left(-\dfrac{17\pi}{6}\right) = \tan\left(-\dfrac{17\pi}{6} + 4\pi\right) = \tan\dfrac{7\pi}{6}$
$= \tan\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{3}$

25. $\sin^2\dfrac{\pi}{2} - \cos\pi = (1)^2 - (-1) = 1 + 1 = 2$

26. $\cos\left(\dfrac{5\pi}{6} + 2\pi n\right) + \tan\left(\dfrac{5\pi}{6} + n\pi\right)$
$= \cos\dfrac{5\pi}{6} + \tan\dfrac{5\pi}{6} = -\cos\dfrac{\pi}{6} - \tan\dfrac{\pi}{6}$
$= -\dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{3}}{3} = -\dfrac{3\sqrt{3}}{6} - \dfrac{2\sqrt{3}}{6}$
$= -\dfrac{5\sqrt{3}}{6}$

27. Begin by converting from degrees to radians.
$36° = 36° \cdot \dfrac{\pi \text{ radians}}{180°} = \dfrac{\pi}{5} \text{ radians}$
$s = r\theta = 40 \cdot \dfrac{\pi}{5} = 8\pi \approx 25.13 \text{ cm}$

28. Linear speed is given by $v = r\omega$. It is given that r = 10 feet and the merry-go-round rotates at 8 revolutions per minute. Convert 8 revolutions per minute to radians per minute.
$8 \text{ revolutions per minute}$
$= 8 \text{ revolutions per minute} \cdot \dfrac{2\pi \text{ radians}}{1 \text{ revolution}}$
$= 16\pi \text{ radians per minute}$
$v = r\omega = (10)(16\pi) = 160\pi \approx 502.7 \text{ feet per minute}$ T
he linear speed of the horse is about 502.7 feet per minute.

29. $\sin\theta = \dfrac{\text{side opposite } \theta}{\text{hypotenuse}}$
$\sin 6° = \dfrac{h}{5280}$
$h = 5280 \sin 6°$
$h \approx 551.9 \text{ feet}$

30. $\tan\theta = \dfrac{\text{side opposite } \theta}{\text{side adjacent } \theta}$
$\tan\theta = \dfrac{50}{60}$
$\theta = \tan^{-1}\left(\dfrac{50}{60}\right)$
$\theta \approx 40°$

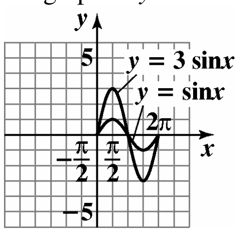
Section 5.5

Checkpoint Exercises

1. The equation $y = 3\sin x$ is of the form $y = A\sin x$ with A = 3. Thus, the amplitude is $|A| = |3| = 3$ The period for both $y = 3\sin x$ and $y = \sin x$ is 2π. We find the three x–intercepts, the maximum point, and the minimum point on the interval $[0, 2\pi]$ by dividing the period, 2π, by 4, $\dfrac{\text{period}}{4} = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$, then by adding quarter-periods to generate x-values for each of the key points. The five x-values are
$x = 0$
$x = 0 + \dfrac{\pi}{2} = \dfrac{\pi}{2}$
$x = \dfrac{\pi}{2} + \dfrac{\pi}{2} = \pi$
$x = \pi + \dfrac{\pi}{2} = \dfrac{3\pi}{2}$
$x = \dfrac{3\pi}{2} + \dfrac{\pi}{2} = 2\pi$
Evaluate the function at each value of x.

x	$y = 3\sin x$	coordinates
0	$y = 3\sin 0 = 3 \cdot 0 = 0$	(0, 0)
$\dfrac{\pi}{2}$	$y = 3\sin\dfrac{\pi}{2} = 3 \cdot 1 = 3$	$\dfrac{\pi}{2}$, 3
π	$y = 3\sin x = 3 \cdot 0 = 0$	$(\pi, 0)$
$\dfrac{3\pi}{2}$	$y = 3\sin\dfrac{3\pi}{2}$ $= 3(-1) = -3$	$\dfrac{3\pi}{2}, -3$
2π	$y = 3\sin 2\pi = 3 \cdot 0 = 0$	$(2\pi, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



2. The equation $y = -\frac{1}{2}\sin x$ is of the form $y = A\sin x$ with $A = -\frac{1}{2}$. Thus, the amplitude is $|A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$. The period for both $y = -\frac{1}{2}\sin x$ and $y = \sin x$ is 2π.

Find the x–values for the five key points by dividing the period, 2π, by 4, $\frac{\text{period}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$, then by adding quarter- periods. The five x-values are

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

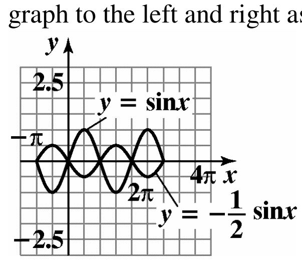
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -\frac{1}{2}\sin x$	coordinates
0	$y = -\frac{1}{2}\sin 0$ $= -\frac{1}{2}\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = -\frac{1}{2}\sin\frac{\pi}{2}$ $= -\frac{1}{2}\cdot 1 = -\frac{1}{2}$	$\left(\frac{\pi}{2}, -\frac{1}{2}\right)$
π	$y = -\frac{1}{2}\sin\pi$ $= -\frac{1}{2}\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = -\frac{1}{2}\sin\frac{3\pi}{2}$ $= -\frac{1}{2}(-1) = \frac{1}{2}$	$\left(\frac{3\pi}{2}, \frac{1}{2}\right)$
2π	$y = -\frac{1}{2}\sin 2\pi$ $= -\frac{1}{2}\cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$. Extend the pattern of each graph to the left and right as desired.



3. The equation $y = 2\sin\frac{1}{2}x$ is of the form

$y = A\sin Bx$ with $A = 2$ and $B = \frac{1}{2}$.

The amplitude is $|A| = |2| = 2$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 4\pi$.

Find the x–values for the five key points by dividing the period, 4π, by 4, $\frac{\text{period}}{4} = \frac{4\pi}{4} = \pi$, then by adding quarter-periods.

The five x-values are

$x = 0$

$x = 0 + \pi = \pi$

$x = \pi + \pi = 2\pi$

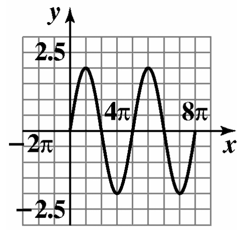
$x = 2\pi + \pi = 3\pi$

$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = 2\sin\frac{1}{2}x$	coordinates
0	$y = 2\sin\ \frac{1}{2}\cdot 0$ $= 2\sin 0$ $= 2\cdot 0 = 0$	(0, 0)
π	$y = 2\sin\ \frac{1}{2}\cdot\pi$ $= 2\sin\frac{\pi}{2} = 2\cdot 1 = 2$	$(\pi, 2)$
2π	$y = 2\sin\ \frac{1}{2}\cdot 2\pi$ $= 2\sin\pi = 2\cdot 0 = 0$	$(2\pi, 0)$
3π	$y = 2\sin\ \frac{1}{2}\cdot 3\pi$ $= 2\sin\frac{3\pi}{2}$ $= 2\cdot(-1) = -2$	$(3\pi, -2)$
4π	$y = 2\sin\ \frac{1}{2}\cdot 4\pi$ $= 2\sin 2\pi = 2\cdot 0 = 0$	$(4\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function. Extend the pattern of the graph another full period to the right.



$y = 2\sin\frac{1}{2}x$

4. The equation $y = 3\sin\ 2x - \frac{\pi}{3}$ is of the form

$y = A\sin(Bx - C)$ with $A = 3$, $B = 2$, and $C = \frac{\pi}{3}$. The amplitude is $|A| = |3| = 3$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$.

The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{3}}{2} = \frac{\pi}{3}\cdot\frac{1}{2} = \frac{\pi}{6}$.

Find the x-values for the five key points by dividing the period, π, by 4, $\frac{\text{period}}{4} = \frac{\pi}{4}$, then by adding quarter-periods to the value of x where the cycle begins, $x = \frac{\pi}{6}$.

The five x-values are

$x = \frac{\pi}{6}$

$x = \frac{\pi}{6} + \frac{\pi}{4} = \frac{2\pi}{12} + \frac{3\pi}{12} = \frac{5\pi}{12}$

$x = \frac{5\pi}{12} + \frac{\pi}{4} = \frac{5\pi}{12} + \frac{3\pi}{12} = \frac{8\pi}{12} = \frac{2\pi}{3}$

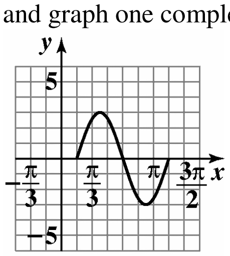
$x = \frac{2\pi}{3} + \frac{\pi}{4} = \frac{8\pi}{12} + \frac{3\pi}{12} = \frac{11\pi}{12}$

$x = \frac{11\pi}{12} + \frac{\pi}{4} = \frac{11\pi}{12} + \frac{3\pi}{12} = \frac{14\pi}{12} = \frac{7\pi}{6}$

Evaluate the function at each value of x.

x	$y=3\sin\ 2x-\frac{\pi}{3}$	coordinates
$\frac{\pi}{6}$	$y=3\sin\ 2\cdot\frac{\pi}{6}-\frac{\pi}{3}$ $=3\sin 0=3\cdot 0=0$	$\frac{\pi}{6}$, 0
$\frac{5\pi}{12}$	$y=3\sin\left(2\cdot\frac{5\pi}{12}-\frac{\pi}{3}\right)$ $=3\sin\frac{3\pi}{6}=3\sin\frac{\pi}{2}$ $=3\cdot 1=3$	$\frac{5\pi}{12}$, 3
$\frac{2\pi}{3}$	$y=3\sin\left(2\cdot\frac{2\pi}{3}-\frac{\pi}{3}\right)$ $=3\sin\frac{3\pi}{3}=3\sin\pi$ $=3\cdot 0=0$	$\frac{2\pi}{3}$, 0
$\frac{11\pi}{12}$	$y=3\sin\left(2\cdot\frac{11\pi}{12}-\frac{\pi}{3}\right)$ $=3\sin\frac{9\pi}{6}=3\sin\frac{3\pi}{2}$ $=3(-1)=-3$	$\frac{11\pi}{12}$, -3
$\frac{7\pi}{6}$	$y=3\sin\ 2\cdot\frac{7\pi}{6}-\frac{\pi}{3}$ $=3\sin\frac{6\pi}{3}=3\sin 2\pi$ $=3\cdot 0=0$	$\frac{7\pi}{6}$, 0

Connect the five key points with a smooth curve and graph one complete cycle of the given graph.



$y = 3\sin\left(2x - \frac{\pi}{3}\right)$

5. The equation $y=-4\cos\pi x$ is of the form $y=A\cos Bx$ with $A=-4$, and $B=\pi$.
Thus, the amplitude is $|A|=|-4|=4$.
The period is $\frac{2\pi}{B}=\frac{2\pi}{\pi}=2$.
Find the x-values for the five key points by dividing the period, 2, by 4, $\frac{\text{period}}{4}=\frac{2}{4}=\frac{1}{2}$, then by adding quarter periods to the value of x where the cycle begins. The five x-values are

$x=0$

$x=0+\frac{1}{2}=\frac{1}{2}$

$x=\frac{1}{2}+\frac{1}{2}=1$

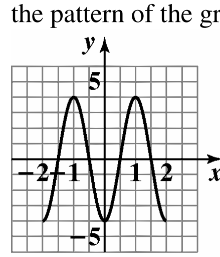
$x=1+\frac{1}{2}=\frac{3}{2}$

$x=\frac{3}{2}+\frac{1}{2}=2$

Evaluate the function at each value of x.

x	$y=-4\cos\pi x$	coordinates
0	$y=-4\cos(\pi\cdot 0)$ $=-4\cos 0=-4$	(0, –4)
$\frac{1}{2}$	$y=-4\cos\left(\pi\cdot\frac{1}{2}\right)$ $=-4\cos\frac{\pi}{2}=0$	$\frac{1}{2}$, 0
1	$y=-4\cos(\pi\cdot 1)$ $=-4\cos\pi=4$	(1, 4)
$\frac{3}{2}$	$y=-4\cos\left(\pi\cdot\frac{3}{2}\right)$ $=-4\cos\frac{3\pi}{2}=0$	$\frac{3}{2}$, 0
2	$y=-4\cos(\pi\cdot 2)$ $=-4\cos 2\pi=-4$	(2, –4)

Connect the five key points with a smooth curve and graph one complete cycle of the given function. Extend the pattern of the graph another full period to the left.



$y = -4\cos\pi x$

6. $y=\frac{3}{2}\cos(2x+\pi)=\frac{3}{2}\cos(2x-(-\pi))$

The equation is of the form $y=A\cos(Bx-C)$ with $A=\frac{3}{2}$, $B=2$, and $C=-\pi$.

Thus, the amplitude is $|A|=\left|\frac{3}{2}\right|=\frac{3}{2}$.

The period is $\frac{2\pi}{B}=\frac{2\pi}{2}=\pi$.

The phase shift is $\frac{C}{B}=\frac{-\pi}{2}=-\frac{\pi}{2}$.

Find the x-values for the five key points by dividing the period, π, by 4, $\frac{\text{period}}{4}=\frac{\pi}{4}$, then by adding quarter-periods to the value of x where the cycle begins, $x=-\frac{\pi}{2}$.

The five x-values are

$x=-\frac{\pi}{2}$

$x=-\frac{\pi}{2}+\frac{\pi}{4}=-\frac{\pi}{4}$

$x=-\frac{\pi}{4}+\frac{\pi}{4}=0$

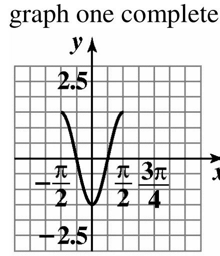
$x=0+\frac{\pi}{4}=\frac{\pi}{4}$

$x=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$

Evaluate the function at each value of x.

x	$y=\frac{3}{2}\cos(2x+\pi)$	coordinates
$-\frac{\pi}{2}$	$y=\frac{3}{2}\cos(-\pi+\pi)$ $=\frac{3}{2}\cdot 1=\frac{3}{2}$	$-\frac{\pi}{2}, \frac{3}{2}$
$-\frac{\pi}{4}$	$y=\frac{3}{2}\cos\ -\frac{\pi}{2}+\pi$ $=\frac{3}{2}\cdot 0=0$	$-\frac{\pi}{4}, 0$
0	$y=\frac{3}{2}\cos(0+\pi)$ $=\frac{3}{2}\cdot -1=-\frac{3}{2}$	$0, -\frac{3}{2}$
$\frac{\pi}{4}$	$y=\frac{3}{2}\cos\ \frac{\pi}{2}+\pi$ $=\frac{3}{2}\cdot 0=0$	$\frac{\pi}{4}, 0$
$\frac{\pi}{2}$	$y=\frac{3}{2}\cos(\pi+\pi)$ $=\frac{3}{2}\cdot 1=\frac{3}{2}$	$\frac{\pi}{2}, \frac{3}{2}$

Connect the five key points with a smooth curve and graph one complete cycle of the given graph.



$y=\frac{3}{2}\cos(2x+\pi)$

7. The graph of $y = 2\cos x + 1$ is the graph of $y = 2\cos x$ shifted one unit upwards. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

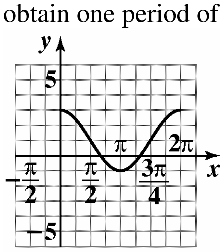
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 2\cos x + 1$	coordinates
0	$y = 2\cos 0 + 1$ $= 2\cdot 1 + 1 = 3$	(0, 3)
$\frac{\pi}{2}$	$y = 2\cos\frac{\pi}{2} + 1$ $= 2\cdot 0 + 1 = 1$	$\frac{\pi}{2}$, 1
π	$y = 2\cos\pi + 1$ $= 2\cdot(-1) + 1 = -1$	$(\pi, -1)$
$\frac{3\pi}{2}$	$y = 2\cos\frac{3\pi}{2} + 1$ $= 2\cdot 0 + 1 = 1$	$\frac{3\pi}{2}$, 1
2π	$y = 2\cos 2\pi + 1$ $= 2\cdot 1 + 1 = 3$	$(2\pi, 3)$

By connecting the points with a smooth curve, we obtain one period of the graph.



$y = 2\cos x + 1$

8. A, the amplitude, is the maximum value of y. The graph shows that this maximum value is 4, Thus, $A = 4$. The period is $\frac{\pi}{2}$, and period $= \frac{2\pi}{B}$. Thus,

$\frac{\pi}{2} = \frac{2\pi}{B}$

$\pi B = 4\pi$

$B = 4$

Substitute these values into $y = A\sin Bx$.
The graph is modeled by $y = 4\sin 4x$.

9. Because the hours of daylight ranges from a minimum of 10 hours to a maximum of 14 hours, the curve oscillates about the middle value, 12 hours. Thus, $D = 12$. The maximum number of hours is 2 hours above 12 hours. Thus, $A = 2$. The graph shows that one complete cycle occurs in 12–0, or 12 months. The period is 12. Thus,

$12 = \frac{2\pi}{B}$

$12B = 2\pi$

$B = \frac{2\pi}{12} = \frac{\pi}{6}$

The graph shows that the starting point of the cycle is shifted from 0 to 3. The phase shift, $\frac{C}{B}$, is 3.

$3 = \frac{C}{B}$

$3 = \frac{C}{\frac{\pi}{6}}$

$\frac{\pi}{2} = C$

Substitute these values into $y = A\sin(Bx - C) + D$.
The number of hours of daylight is modeled by

$y = 2\sin\left(\frac{\pi}{6}x - \frac{\pi}{2}\right) + 12$.

Exercise Set 5.5

1. The equation $y = 4\sin x$ is of the form $y = A\sin x$ with $A = 4$. Thus, the amplitude is $|A| = |4| = 4$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

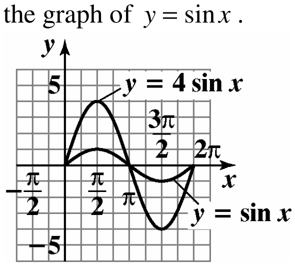
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 4\sin x$	coordinates
0	$y = 4\sin 0 = 4\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = 4\sin\frac{\pi}{2} = 4\cdot 1 = 4$	$\frac{\pi}{2}, 4$
π	$y = 4\sin\pi = 4\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = 4\sin\frac{3\pi}{2}$ $= 4(-1) = -4$	$\frac{3\pi}{2}, -4$
2π	$y = 4\sin 2\pi = 4\cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



2. The equation $y = 5\sin x$ is of the form $y = A\sin x$ with $A = 5$. Thus, the amplitude is $|A| = |5| = 5$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

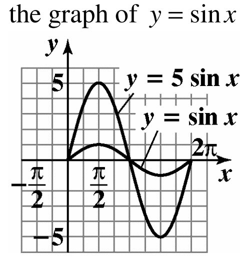
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 5\sin x$	coordinates
0	$y = 5\sin 0 = 5\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = 5\sin\frac{\pi}{2} = 5\cdot 1 = 5$	$\frac{\pi}{2}, 5$
π	$y = 5\sin\pi = 5\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = 5\sin\frac{3\pi}{2} = 5(-1) = -5$	$\frac{3\pi}{2}, -5$
2π	$y = 5\sin 2\pi = 5\cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



3. The equation $y = \frac{1}{3}\sin x$ is of the form $y = A\sin x$ with $A = \frac{1}{3}$. Thus, the amplitude is $|A| = \left|\frac{1}{3}\right| = \frac{1}{3}$.

The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

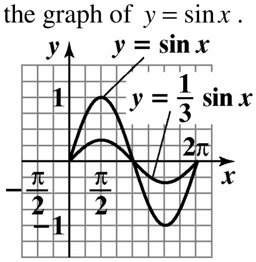
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \frac{1}{3}\sin x$	coordinates
0	$y = \frac{1}{3}\sin 0 = \frac{1}{3}\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = \frac{1}{3}\sin\frac{\pi}{2} = \frac{1}{3}\cdot 1 = \frac{1}{3}$	$\frac{\pi}{2}, \frac{1}{3}$
π	$y = \frac{1}{3}\sin\pi = \frac{1}{3}\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = \frac{1}{3}\sin\frac{3\pi}{2}$ $= \frac{1}{3}(-1) = -\frac{1}{3}$	$\frac{3\pi}{2}, -\frac{1}{3}$
2π	$y = \frac{1}{3}\sin 2\pi = \frac{1}{3}\cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



4. The equation $y = \frac{1}{4}\sin x$ is of the form $y = A\sin x$ with $A = \frac{1}{4}$. Thus, the amplitude is $|A| = \left|\frac{1}{4}\right| = \frac{1}{4}$.

The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

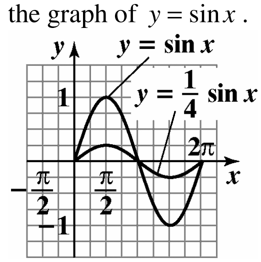
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \frac{1}{4}\sin x$	coordinates
0	$y = \frac{1}{4}\sin 0 = \frac{1}{4}\cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = \frac{1}{4}\sin\frac{\pi}{2} = \frac{1}{4}\cdot 1 = \frac{1}{4}$	$\frac{\pi}{2}, \frac{1}{4}$
π	$y = \frac{1}{4}\sin\pi = \frac{1}{4}\cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = \frac{1}{4}\sin\frac{3\pi}{2} = \frac{1}{4}(-1) = -\frac{1}{4}$	$\frac{3\pi}{2}, -\frac{1}{4}$
2π	$y = \frac{1}{4}\sin 2\pi = \frac{1}{4}\cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



5. The equation $y = -3\sin x$ is of the form $y = A\sin x$ with $A = -3$. Thus, the amplitude is $|A| = |-3| = 3$.
The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

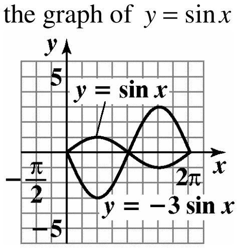
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -3\sin x$	coordinates
0	$y = -3\sin x$ $= -3 \cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = -3\sin\frac{\pi}{2}$ $= -3 \cdot 1 = -3$	$\left(\frac{\pi}{2}, -3\right)$
π	$y = -3\sin\pi$ $= -3 \cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = -3\sin\frac{3\pi}{2}$ $= -3(-1) = 3$	$\left(\frac{3\pi}{2}, 3\right)$
2π	$y = -3\sin 2\pi$ $= -3 \cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



6. The equation $y = -4\sin x$ is of the form $y = A\sin x$ with $A = -4$. Thus, the amplitude is $|A| = |-4| = 4$.
The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

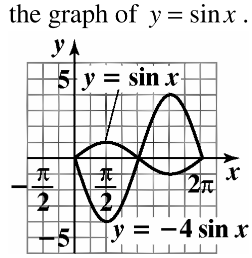
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -4\sin x$	coordinates
0	$y = -4\sin 0 = -4 \cdot 0 = 0$	(0, 0)
$\frac{\pi}{2}$	$y = -4\sin\frac{\pi}{2} = -4 \cdot 1 = -4$	$\left(\frac{\pi}{2}, -4\right)$
π	$y = -4\sin\pi = -4 \cdot 0 = 0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y = -4\sin\frac{3\pi}{2} = -4(-1) = 4$	$\left(\frac{3\pi}{2}, 4\right)$
2π	$y = -4\sin 2\pi = -4 \cdot 0 = 0$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \sin x$.



7. The equation $y = \sin 2x$ is of the form $y = A\sin Bx$ with $A = 1$ and $B = 2$. The amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

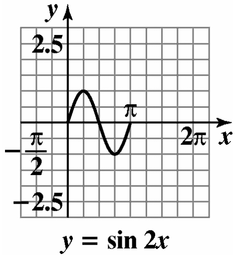
$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

Evaluate the function at each value of x.

x	$y = \sin 2x$	coordinates
0	$y = \sin 2 \cdot 0 = \sin 0 = 0$	(0, 0)
$\frac{\pi}{4}$	$y = \sin\left(2 \cdot \frac{\pi}{4}\right) = \sin\frac{\pi}{2} = 1$	$\left(\frac{\pi}{4}, 1\right)$
$\frac{\pi}{2}$	$y = \sin\left(2 \cdot \frac{\pi}{2}\right) = \sin \pi = 0$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$y = \sin\left(2 \cdot \frac{3\pi}{4}\right) = \sin\frac{3\pi}{2} = -1$	$\left(\frac{3\pi}{4}, -1\right)$
π	$y = \sin(2 \cdot \pi) = \sin 2\pi = 0$	$(\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



8. The equation $y = \sin 4x$ is of the form $y = A\sin Bx$ with $A = 1$ and $B = 4$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{4} = \frac{\pi}{2}$. The quarter-period is $\frac{\frac{\pi}{2}}{4} = \frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{8} = \frac{\pi}{8}$

$x = \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi}{4}$

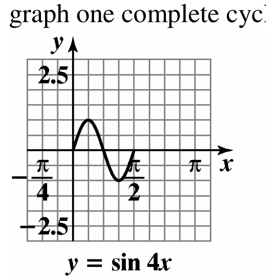
$x = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$

$x = \frac{3\pi}{8} + \frac{\pi}{8} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	$y = \sin 4x$	coordinates
0	$y = \sin(4 \cdot 0) = \sin 0 = 0$	(0, 0)
$\frac{\pi}{8}$	$y = \sin\left(4 \cdot \frac{\pi}{8}\right) = \sin\frac{\pi}{2} = 1$	$\left(\frac{\pi}{8}, 1\right)$
$\frac{\pi}{4}$	$y = \sin\left(4 \cdot \frac{\pi}{4}\right) = \sin \pi = 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{3\pi}{8}$	$y = \sin\left(4 \cdot \frac{3\pi}{8}\right) = \sin\frac{3\pi}{2} = -1$	$\left(\frac{3\pi}{8}, -1\right)$
$\frac{\pi}{2}$	$y = \sin 2\pi = 0$	$\left(\frac{\pi}{2}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



9. The equation $y = 3\sin\frac{1}{2}x$ is of the form $y = A\sin Bx$ with $A = 3$ and $B = \frac{1}{2}$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$. The quarter-period is $\frac{4\pi}{4} = \pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \pi = \pi$

$x = \pi + \pi = 2\pi$

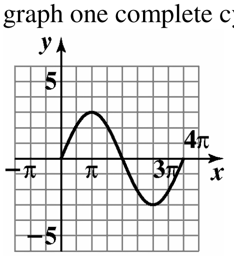
$x = 2\pi + \pi = 3\pi$

$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = 3\sin\frac{1}{2}x$	coordinates
0	$y = 3\sin\left(\frac{1}{2}\cdot 0\right)$ $= 3\sin 0 = 3\cdot 0 = 0$	(0, 0)
π	$y = 3\sin\left(\frac{1}{2}\cdot \pi\right)$ $= 3\sin\frac{\pi}{2} = 3\cdot 1 = 3$	$(\pi, 3)$
2π	$y = 3\sin\left(\frac{1}{2}\cdot 2\pi\right)$ $= 3\sin\pi = 3\cdot 0 = 0$	$(2\pi, 0)$
3π	$y = 3\sin\left(\frac{1}{2}\cdot 3\pi\right)$ $= 3\sin\frac{3\pi}{2}$ $= 3(-1) = -3$	$(3\pi, -3)$
4π	$y = 3\sin\left(\frac{1}{2}\cdot 4\pi\right)$ $= 3\sin 2\pi = 3\cdot 0 = 0$	$(4\pi, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = 3\sin\frac{1}{2}x$

10. The equation $y = 2\sin\frac{1}{4}x$ is of the form $y = A\sin Bx$ with $A = 2$ and $B = \frac{1}{4}$. Thus, the amplitude is $|A| = |2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{4}} = 2\pi \cdot 4 = 8\pi$. The quarter-period is $\frac{8\pi}{4} = 2\pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + 2\pi = 2\pi$

$x = 2\pi + 2\pi = 4\pi$

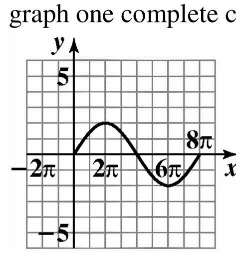
$x = 4\pi + 2\pi = 6\pi$

$x = 6\pi + 2\pi = 8\pi$

Evaluate the function at each value of x.

x	$y = 2\sin\frac{1}{4}x$	coordinates
0	$y = 2\sin\left(\frac{1}{4}\cdot 0\right)$ $= 2\sin 0 = 2\cdot 0 = 0$	(0, 0)
2π	$y = 2\sin\left(\frac{1}{4}\cdot 2\pi\right)$ $= 2\sin\frac{\pi}{2} = 2\cdot 1 = 2$	$(2\pi, 2)$
4π	$y = 2\sin\pi = 2\cdot 0 = 0$	$(4\pi, 0)$
6π	$y = 2\sin\frac{3\pi}{2} = 2(-1) = -2$	$(6\pi, -2)$
8π	$y = 2\sin 2\pi = 2\cdot 0 = 0$	$(8\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 2\sin\frac{1}{4}x$

11. The equation $y = 4\sin \pi x$ is of the form $y = A\sin Bx$ with $A = 4$ and $B = \pi$. The amplitude is $|A| = |4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$. The quarter-period is $\frac{2}{4} = \frac{1}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{2} = \frac{1}{2}$

$x = \frac{1}{2} + \frac{1}{2} = 1$

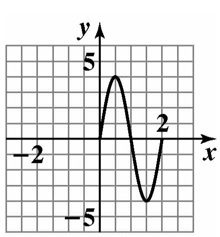
$x = 1 + \frac{1}{2} = \frac{3}{2}$

$x = \frac{3}{2} + \frac{1}{2} = 2$

Evaluate the function at each value of x.

x	$y = 4\sin \pi x$	coordinates
0	$y = 4\sin(\pi \cdot 0)$ $= 4\sin 0 = 4 \cdot 0 = 0$	(0, 0)
$\frac{1}{2}$	$y = 4\sin\ \pi \cdot \frac{1}{2}$ $= 4\sin \frac{\pi}{2} = 4(1) = 4$	$\frac{1}{2}$, 4
1	$y = 4\sin(\pi \cdot 1)$ $= 4\sin \pi = 4 \cdot 0 = 0$	(1, 0)
$\frac{3}{2}$	$y = 4\sin\ \pi \cdot \frac{3}{2}$ $= 4\sin \frac{3\pi}{2}$ $= 4(-1) = -4$	$\frac{3}{2}$, − 4
2	$y = 4\sin(\pi \cdot 2)$ $= 4\sin 2\pi = 4 \cdot 0 = 0$	(2, 0)

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = 4\sin \pi x$

12. The equation $y = 3\sin 2\pi x$ is of the form $y = A\sin Bx$ with $A = 3$ and $B = 2\pi$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{4} = \frac{1}{4}$

$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

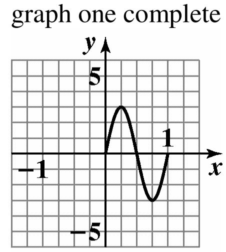
$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = 3\sin 2\pi x$	coordinates
0	$y = 3\sin(2\pi \cdot 0)$ $= 3\sin 0 = 3 \cdot 0 = 0$	(0, 0)
$\frac{1}{4}$	$y = 3\sin\ 2\pi \cdot \frac{1}{4}$ $= 3\sin \frac{\pi}{2} = 3 \cdot 1 = 3$	$\frac{1}{4}$, 3
$\frac{1}{2}$	$y = 3\sin\ 2\pi \cdot \frac{1}{2}$ $= 3\sin \pi = 3 \cdot 0 = 0$	$\frac{1}{2}$, 0
$\frac{3}{4}$	$y = 3\sin\ 2\pi \cdot \frac{3}{4}$ $= 3\sin \frac{3\pi}{2} = 3(-1) = -3$	$\frac{3}{4}$, −3
1	$y = 3\sin(2\pi \cdot 1)$ $= 3\sin 2\pi = 3 \cdot 0 = 0$	(1, 0)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 3\sin 2\pi x$

13. The equation $y = -3\sin 2\pi x$ is of the form $y = A\sin Bx$ with $A = -3$ and $B = 2\pi$. The amplitude is $|A| = |-3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{4} = \frac{1}{4}$

$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

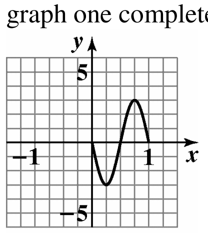
$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = -3\sin 2\pi x$	coordinates
0	$y = -3\sin(2\pi \cdot 0)$ $= -3\sin 0$ $= -3 \cdot 0 = 0$	(0, 0)
$\frac{1}{4}$	$y = -3\sin\ 2\pi \cdot \frac{1}{4}$ $= -3\sin\frac{\pi}{2}$ $= -3 \cdot 1 = -3$	$\frac{1}{4}, -3$
$\frac{1}{2}$	$y = -3\sin\ 2\pi \cdot \frac{1}{2}$ $= -3\sin\pi$ $= -3 \cdot 0 = 0$	$\frac{1}{2}, 0$
$\frac{3}{4}$	$y = -3\sin\ 2\pi \cdot \frac{3}{4}$ $= -3\sin\frac{3\pi}{2}$ $= -3(-1) = 3$	$\frac{3}{4}, 3$
1	$y = -3\sin(2\pi \cdot 1)$ $= -3\sin 2\pi$ $= -3 \cdot 0 = 0$	(1, 0)

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = -3\sin 2\pi x$

14. The equation $y = -2\sin \pi x$ is of the form $y = A\sin Bx$ with $A = -2$ and $B = \pi$. The amplitude is $|A| = |-2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$. The quarter-period is $\frac{2}{4} = \frac{1}{2}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{2} = \frac{1}{2}$

$x = \frac{1}{2} + \frac{1}{2} = 1$

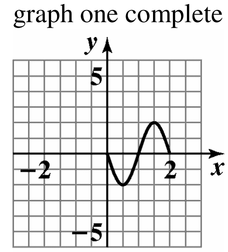
$x = 1 + \frac{1}{2} = \frac{3}{2}$

$x = \frac{3}{2} + \frac{1}{2} = 2$

Evaluate the function at each value of x.

x	$y = -2\sin \pi x$	coordinates
0	$y = -2\sin(\pi \cdot 0)$ $= -2\sin 0 = -2 \cdot 0 = 0$	(0, 0)
$\frac{1}{2}$	$y = -2\sin\ \pi \cdot \frac{1}{2}$ $= -2\sin\frac{\pi}{2} = -2 \cdot 1 = -2$	$\frac{1}{2}, -2$
1	$y = -2\sin(\pi \cdot 1)$ $= -2\sin\pi = -2 \cdot 0 = 0$	(1, 0)
$\frac{3}{2}$	$y = -2\sin\ \pi \cdot \frac{3}{2}$ $= -2\sin\frac{3\pi}{2} = -2(-1) = 2$	$\frac{3}{2}, 2$
2	$y = -2\sin(\pi \cdot 2)$ $= -2\sin 2\pi = -2 \cdot 0 = 0$	(2, 0)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -2\sin \pi x$

15. The equation $y = -\sin\frac{2}{3}x$ is of the form $y = A\sin Bx$ with $A = -1$ and $B = \frac{2}{3}$.

The amplitude is $|A| = |-1| = 1$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{2}{3}} = 2\pi \cdot \frac{3}{2} = 3\pi$.

The quarter-period is $\frac{3\pi}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{3\pi}{4} = \frac{3\pi}{2}$

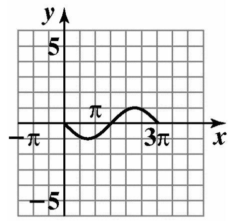
$x = \frac{3\pi}{2} + \frac{3\pi}{4} = \frac{9\pi}{4}$

$x = \frac{9\pi}{4} + \frac{3\pi}{4} = 3\pi$

Evaluate the function at each value of x.

x	$y = -\sin\frac{2}{3}x$	coordinates
0	$y = -\sin\frac{2}{3}\cdot 0$ $= -\sin 0 = 0$	(0, 0)
$\frac{3\pi}{4}$	$y = -\sin\frac{2}{3}\cdot\frac{3\pi}{4}$ $= -\sin\frac{\pi}{2} = -1$	$\left(\frac{3\pi}{4}, -1\right)$
$\frac{3\pi}{2}$	$y = -\sin\frac{2}{3}\cdot\frac{3\pi}{2}$ $= -\sin\pi = 0$	$\left(\frac{3\pi}{2}, 0\right)$
$\frac{9\pi}{4}$	$y = -\sin\frac{2}{3}\cdot\frac{9\pi}{4}$ $= -\sin\frac{3\pi}{2}$ $= -(-1) = 1$	$\left(\frac{9\pi}{4}, 1\right)$
3π	$y = -\sin\frac{2}{3}\cdot 3\pi$ $= -\sin 2\pi = 0$	$(3\pi, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = -\sin\frac{2}{3}x$

16. The equation $y = -\sin\frac{4}{3}x$ is of the form $y = A\sin Bx$ with $A = -1$ and $B = \frac{4}{3}$.

The amplitude is $|A| = |-1| = 1$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{4}{3}} = 2\pi \cdot \frac{3}{4} = \frac{3\pi}{2}$.

The quarter-period is $\frac{\frac{3\pi}{2}}{4} = \frac{3\pi}{2}\cdot\frac{1}{4} = \frac{3\pi}{8}$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3\pi}{8} = \frac{3\pi}{8}$

$x = \frac{3\pi}{8} + \frac{3\pi}{8} = \frac{3\pi}{4}$

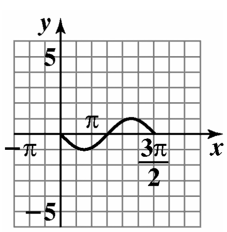
$x = \frac{3\pi}{4} + \frac{3\pi}{8} = \frac{9\pi}{8}$

$x = \frac{9\pi}{8} + \frac{3\pi}{8} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	$y=-\sin\frac{4}{3}x$	coordinates
0	$y=-\sin\ \frac{4}{3}\cdot 0$ $=-\sin 0=0$	(0, 0)
$\frac{3\pi}{8}$	$y=-\sin\ \frac{4}{3}\cdot\frac{3\pi}{8}$ $=-\sin\frac{\pi}{2}=-1$	$\frac{3\pi}{8}, -1$
$\frac{3\pi}{4}$	$y=-\sin\ \frac{4}{3}\cdot\frac{3\pi}{4}$ $=-\sin\pi=0$	$\frac{3\pi}{4}, 0$
$\frac{9\pi}{8}$	$y=-\sin\ \frac{4}{3}\cdot\frac{9\pi}{8}$ $=-\sin\frac{3\pi}{2}=-(-1)=1$	$\frac{9\pi}{8}, 1$
$\frac{3\pi}{2}$	$y=-\sin\ \frac{4}{3}\cdot\frac{3\pi}{2}$ $=-\sin 2\pi=0$	$\frac{3\pi}{2}, 0$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y=-\sin\frac{4}{3}x$

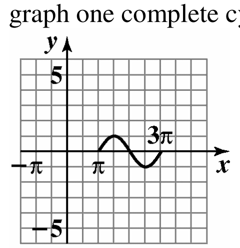
17. The equation $y=\sin(x-\pi)$ is of the form $y=A\sin(Bx-C)$ with $A = 1$, $B = 1$, and $C=\pi$. The amplitude is $|A|=|1|=1$. The period is $\frac{2\pi}{B}=\frac{2\pi}{1}=2\pi$. The phase shift is $\frac{C}{B}=\frac{\pi}{1}=\pi$. The quarter-period is $\frac{2\pi}{4}=\frac{\pi}{2}$. The cycle begins at $x=\pi$. Add quarter-periods to generate x-values for the key points.

$$x=\pi$$
$$x=\pi+\frac{\pi}{2}=\frac{3\pi}{2}$$
$$x=\frac{3\pi}{2}+\frac{\pi}{2}=2\pi$$
$$x=2\pi+\frac{\pi}{2}=\frac{5\pi}{2}$$
$$x=\frac{5\pi}{2}+\frac{\pi}{2}=3\pi$$

Evaluate the function at each value of x.

x	$y=\sin(x-\pi)$	coordinates
π	$y=\sin(\pi-\pi)$ $=\sin 0=0$	$(\pi, 0)$
$\frac{3\pi}{2}$	$y=\sin\ \frac{3\pi}{2}-\pi$ $=\sin\frac{\pi}{2}=1$	$\frac{3\pi}{2}, 1$
2π	$y=\sin(2\pi-\pi)$ $=\sin\pi=0$	$(2\pi, 0)$
$\frac{5\pi}{2}$	$y=\sin\ \frac{5\pi}{2}-\pi$ $=\sin\frac{3\pi}{2}=-1$	$\frac{5\pi}{2}, -1$
3π	$y=\sin(3\pi-\pi)$ $=\sin 2\pi=0$	$(3\pi, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y=\sin(x-\pi)$

18. The equation $y = \sin\ x - \frac{\pi}{2}$ is of the form

$y = A\sin(Bx - C)$ with $A = 1$, $B = 1$, and $C = \frac{\pi}{2}$. The amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

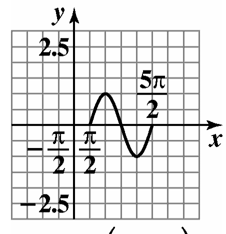
$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

$x = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$

Evaluate the function at each value of x.

x	$y = \sin\ x - \frac{\pi}{2}$	coordinates
$\frac{\pi}{2}$	$y = \sin\ \frac{\pi}{2} - \frac{\pi}{2} = \sin 0 = 0$	$\frac{\pi}{2}$, 0
π	$y = \sin\ \pi - \frac{\pi}{2} = \sin\frac{\pi}{2} = 1$	$(\pi, 1)$
$\frac{3\pi}{2}$	$y = \sin\ \frac{3\pi}{2} - \frac{\pi}{2}$ $= \sin\pi = 0$	$\frac{3\pi}{2}$, 0
2π	$y = \sin\ 2\pi - \frac{\pi}{2}$ $= \sin\frac{3\pi}{2} = -1$	$(2\pi, -1)$
$\frac{5\pi}{2}$	$y = \sin\ \frac{5\pi}{2} - \frac{\pi}{2}$ $= \sin 2\pi = 0$	$\frac{5\pi}{2}$, 0

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = \sin\left(x - \frac{\pi}{2}\right)$

19. The equation $y = \sin(2x - \pi)$ is of the form $y = A\sin(Bx - C)$ with $A = 1$, $B = 2$, and $C = \pi$. The amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

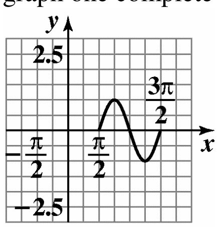
$x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	$y = \sin(2x - \pi)$	coordinates
$\frac{\pi}{2}$	$y = \sin\ 2\cdot\frac{\pi}{2} - \pi$ $= \sin(\pi - \pi)$ $= \sin 0 = 0$	$\frac{\pi}{2}$, 0
$\frac{3\pi}{4}$	$y = \sin\ 2\cdot\frac{3\pi}{4} - \pi$ $= \sin\ \frac{3\pi}{2} - \pi$ $= \sin\frac{\pi}{2} = 1$	$\frac{3\pi}{4}$, 1
π	$y = \sin(2\cdot\pi - \pi)$ $= \sin(2\pi - \pi)$ $= \sin\pi = 0$	$(\pi, 0)$

$\frac{5\pi}{4}$	$y=\sin\ 2\cdot\frac{5\pi}{4}-\pi$ $=\sin\ \frac{5\pi}{2}-\pi$ $=\sin\frac{3\pi}{2}=-1$	$\frac{5\pi}{4},\ -1$
$\frac{3\pi}{2}$	$y=\sin\ 2\cdot\frac{3\pi}{2}-\pi$ $=\sin(3\pi-\pi)$ $=\sin 2\pi=0$	$\frac{3\pi}{2},\ 0$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = \sin(2x - \pi)$

20. The equation $y=\sin\ 2x-\frac{\pi}{2}$ is of the form

$y=A\sin(Bx-C)$ with $A = 1$, $B = 2$, and $C=\frac{\pi}{2}$. The amplitude is $|A|=|1|=1$.

The period is $\frac{2\pi}{B}=\frac{2\pi}{2}=\pi$.

The phase shift is $\frac{C}{B}=\frac{\frac{\pi}{2}}{2}=\frac{\pi}{2}\cdot\frac{1}{2}=\frac{\pi}{4}$.

The quarter-period is $\frac{\pi}{4}$.

The cycle begins at $x=\frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x=\frac{\pi}{4}$

$x=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$

$x=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$

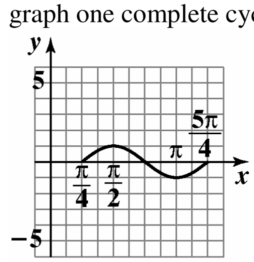
$x=\frac{3\pi}{4}+\frac{\pi}{4}=\pi$

$x=\pi+\frac{\pi}{4}=\frac{5\pi}{4}$

Evaluate the function at each value of x.

x	$y=\sin\ 2x-\frac{\pi}{2}$	coordinates
$\frac{\pi}{4}$	$y=\sin\ 2\cdot\frac{\pi}{4}-\frac{\pi}{2}$ $=\sin\ \frac{\pi}{2}-\frac{\pi}{2}\ =\sin 0=0$	$\frac{\pi}{4},\ 0$
$\frac{\pi}{2}$	$y=\sin\ 2\cdot\frac{\pi}{2}-\frac{\pi}{2}$ $=\sin\ \pi-\frac{\pi}{2}\ =\sin\frac{\pi}{2}=1$	$\frac{\pi}{2},\ 1$
$\frac{3\pi}{4}$	$y=\sin\ 2\cdot\frac{3\pi}{4}-\frac{\pi}{2}$ $=\sin\ \frac{3\pi}{2}-\frac{\pi}{2}$ $=\sin\pi=0$	$\frac{3\pi}{4},\ 0$
π	$y=\sin\ 2\cdot\pi-\frac{\pi}{2}$ $=\sin\ 2\pi-\frac{\pi}{2}$ $=\sin\frac{3\pi}{2}=-1$	$(\pi,\ -1)$
$\frac{5\pi}{4}$	$y=\sin\ 2\cdot\frac{5\pi}{4}-\frac{\pi}{2}$ $=\sin\ \frac{5\pi}{2}-\frac{\pi}{2}$ $=\sin 2\pi=0$	$\frac{5\pi}{4},\ 0$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = \sin\left(2x - \frac{\pi}{2}\right)$

21. The equation $y = 3\sin(2x - \pi)$ is of the form $y = A\sin(Bx - C)$ with $A = 3$, $B = 2$, and $C = \pi$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

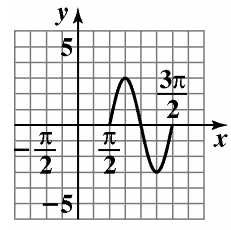
$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	$y = 3\sin(2x - \pi)$	coordinates
$\frac{\pi}{2}$	$y = 3\sin\ 2\cdot\frac{\pi}{2} - \pi$ $= 3\sin(\pi - \pi)$ $= 3\sin 0 = 3\cdot 0 = 0$	$\frac{\pi}{2}$, 0
$\frac{3\pi}{4}$	$y = 3\sin\ 2\cdot\frac{3\pi}{4} - \pi$ $= 3\sin\ \frac{3\pi}{2} - \pi$ $= 3\sin\frac{\pi}{2} = 3\cdot 1 = 3$	$\frac{3\pi}{4}$, 3
π	$y = 3\sin(2\cdot\pi - \pi)$ $= 3\sin(2\pi - \pi)$ $= 3\sin\pi = 3\cdot 0 = 0$	$(\pi, 0)$
$\frac{5\pi}{4}$	$y = 3\sin\ 2\cdot\frac{5\pi}{4} - \pi$ $= 3\sin\ \frac{5\pi}{2} - \pi$ $= 3\sin\frac{3\pi}{2}$ $= 3(-1) = -3$	$\frac{5\pi}{4}$, -3
$\frac{3\pi}{2}$	$y = 3\sin\ 2\cdot\frac{3\pi}{2} - \pi$ $= 3\sin(3\pi - \pi)$ $= 3\sin 2\pi = 3\cdot 0 = 0$	$\frac{3\pi}{2}$, 0

Connect the five points with a smooth curve and graph one complete cycle of the given function.



y = 3 sin (2x − π)

22. The equation $y = 3\sin\ 2x - \frac{\pi}{2}$ is of the form $y = A\sin(Bx - C)$ with $A = 3$, $B = 2$, and $C = \frac{\pi}{2}$.

The amplitude is $|A| = |3| = 3$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$.

The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{2}\cdot\frac{1}{2} = \frac{\pi}{4}$.

The quarter-period is $\frac{\pi}{4}$.

The cycle begins at $x = \frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

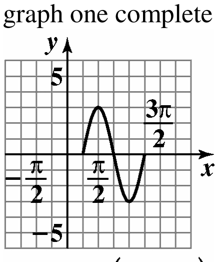
$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Evaluate the function at each value of x.

x	$y = 3\sin\ 2x - \frac{\pi}{2}$	coordinates
$\frac{\pi}{4}$	$y = 3\sin\ 2\cdot\frac{\pi}{4} - \frac{\pi}{2}$ $= \sin\ \frac{\pi}{2} - \frac{\pi}{2}$ $= 3\sin 0 = 3\cdot 0 = 0$	$\frac{\pi}{4}, 0$
$\frac{\pi}{2}$	$y = 3\sin\ 2\cdot\frac{\pi}{2} - \frac{\pi}{2}$ $= 3\sin\ \pi - \frac{\pi}{2}$ $= 3\sin\frac{\pi}{2} = 3\cdot 1 = 3$	$\frac{\pi}{2}, 3$
$\frac{3\pi}{4}$	$y = 3\sin\ 2\cdot\frac{3\pi}{4} - \frac{\pi}{2}$ $= 3\sin\ \frac{3\pi}{2} - \frac{\pi}{2}$ $= 3\sin\pi = 3\cdot 0 = 0$	$\frac{3\pi}{4}, 0$
π	$y = 3\sin\ 2\cdot\pi - \frac{\pi}{2}$ $= 3\sin\ 2\pi - \frac{\pi}{2}$ $= 3\sin\frac{3\pi}{2} = 3\cdot(-1) = -3$	$(\pi, -3)$
$\frac{5\pi}{4}$	$y = 3\sin\ 2\cdot\frac{5\pi}{4} - \frac{\pi}{2}$ $= 3\sin\ \frac{5\pi}{2} - \frac{\pi}{2}$ $= 3\sin 2\pi = 3\cdot 0 = 0$	$\frac{5\pi}{4}, 0$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 3\sin\left(2x - \frac{\pi}{2}\right)$

23. $y = \frac{1}{2}\sin\ x + \frac{\pi}{2}\ = \frac{1}{2}\sin\ x - \ -\frac{\pi}{2}$

The equation $y = \frac{1}{2}\sin\ x - \ -\frac{\pi}{2}$ is of the form

$y = A\sin(Bx - C)$ with $A = \frac{1}{2}$, $B = 1$, and $C = -\frac{\pi}{2}$.

The amplitude is $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$. The period is

$\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2}$.

The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at

$x = -\frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} + \frac{\pi}{2} = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

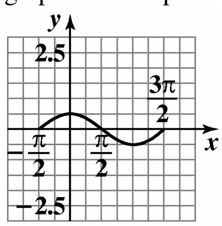
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	$y = \frac{1}{2}\sin\ x + \frac{\pi}{2}$	coordinates
$-\frac{\pi}{2}$	$y = \frac{1}{2}\sin\ -\frac{\pi}{2} + \frac{\pi}{2}$ $= \frac{1}{2}\sin 0 = \frac{1}{2}\cdot 0 = 0$	$-\frac{\pi}{2}, 0$
0	$y = \frac{1}{2}\sin\ 0 + \frac{\pi}{2}$ $= \frac{1}{2}\sin\frac{\pi}{2} = \frac{1}{2}\cdot 1 = \frac{1}{2}$	$0, \frac{1}{2}$
$\frac{\pi}{2}$	$y = \frac{1}{2}\sin\ \frac{\pi}{2} + \frac{\pi}{2}$ $= \frac{1}{2}\sin\pi = \frac{1}{2}\cdot 0 = 0$	$\frac{\pi}{2}, 0$

π	$y=\frac{1}{2}\sin\ \pi+\frac{\pi}{2}$ $=\frac{1}{2}\sin\frac{3\pi}{2}$ $=\frac{1}{2}\cdot(-1)=-\frac{1}{2}$	$\pi,\ -\frac{1}{2}$
$\frac{3\pi}{2}$	$y=\frac{1}{2}\sin\ \frac{3\pi}{2}+\frac{\pi}{2}$ $=\frac{1}{2}\sin 2\pi$ $=\frac{1}{2}\cdot 0=0$	$\frac{3\pi}{2},\ 0$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y=\frac{1}{2}\sin\left(x+\frac{\pi}{2}\right)$

24. $y=\frac{1}{2}\sin(x+\pi)=\frac{1}{2}\sin(x-(-\pi))$

The equation $y=\frac{1}{2}\sin(x-(-\pi))$ is of the form

$y=A\sin(Bx-C)$ with $A=\frac{1}{2}$, $B=1$, and $C=-\pi$.

The amplitude is $|A|=\left|\frac{1}{2}\right|=\frac{1}{2}$. The period is

$\frac{2\pi}{B}=\frac{2\pi}{1}=2\pi$. The phase shift is $\frac{C}{B}=\frac{-\pi}{1}=-\pi$.

The quarter-period is $\frac{2\pi}{4}=\frac{\pi}{2}$. The cycle begins at $x=-\pi$. Add quarter-periods to generate x-values for the key points.

$x=-\pi$

$x=-\pi+\frac{\pi}{2}=-\frac{\pi}{2}$

$x=-\frac{\pi}{2}+\frac{\pi}{2}=0$

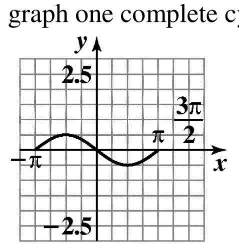
$x=0+\frac{\pi}{2}=\frac{\pi}{2}$

$x=\frac{\pi}{2}+\frac{\pi}{2}=\pi$

Evaluate the function at each value of x.

x	$y=\frac{1}{2}\sin(x+\pi)$	coordinates
$-\pi$	$y=\frac{1}{2}\sin(-\pi+\pi)$ $=\frac{1}{2}\sin 0=\frac{1}{2}\cdot 0=0$	$(-\pi,\ 0)$
$-\frac{\pi}{2}$	$y=\frac{1}{2}\sin\ -\frac{\pi}{2}+\pi$ $=\frac{1}{2}\sin\frac{\pi}{2}=\frac{1}{2}\cdot 1=\frac{1}{2}$	$-\frac{\pi}{2},\ \frac{1}{2}$
0	$y=\frac{1}{2}\sin(0+\pi)$ $=\frac{1}{2}\sin\pi=\frac{1}{2}\cdot 0=0$	$(0,\ 0)$
$\frac{\pi}{2}$	$y=\frac{1}{2}\sin\ \frac{\pi}{2}+\pi$ $=\frac{1}{2}\sin\frac{3\pi}{2}=\frac{1}{2}\cdot(-1)=-\frac{1}{2}$	$\frac{\pi}{2},\ -\frac{1}{2}$
π	$y=\frac{1}{2}\sin(\pi+\pi)$ $=\frac{1}{2}\sin 2\pi=\frac{1}{2}\cdot 0=0$	$(\pi,\ 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y=\frac{1}{2}\sin\left(x+\pi\right)$

25. $y = -2\sin\ 2x + \dfrac{\pi}{2}\ = -2\sin\ 2x - \ -\dfrac{\pi}{2}$

The equation $y = -2\sin\ 2x - \ -\dfrac{\pi}{2}$ is of the form $y = A\sin(Bx - C)$ with $A = -2$, $B = 2$, and $C = -\dfrac{\pi}{2}$. The amplitude is $|A| = |-2| = 2$. The period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{2} = \pi$. The phase shift is $\dfrac{C}{B} = \dfrac{-\frac{\pi}{2}}{2} = -\dfrac{\pi}{2}\cdot\dfrac{1}{2} = -\dfrac{\pi}{4}$. The quarter-period is $\dfrac{\pi}{4}$. The cycle begins at $x = -\dfrac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = -\dfrac{\pi}{4}$

$x = -\dfrac{\pi}{4} + \dfrac{\pi}{4} = 0$

$x = 0 + \dfrac{\pi}{4} = \dfrac{\pi}{4}$

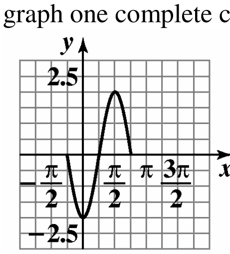
$x = \dfrac{\pi}{4} + \dfrac{\pi}{4} = \dfrac{\pi}{2}$

$x = \dfrac{\pi}{2} + \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

Evaluate the function at each value of x.

x	$y = -2\sin\ 2x + \dfrac{\pi}{2}$	coordinates
$-\dfrac{\pi}{4}$	$y = -2\sin\ 2\cdot\ -\dfrac{\pi}{4}\ + \dfrac{\pi}{2}$ $= -2\sin\ -\dfrac{\pi}{2} + \dfrac{\pi}{2}$ $= -2\sin 0 = -2\cdot 0 = 0$	$-\dfrac{\pi}{4}, 0$
0	$y = -2\sin\ 2\cdot 0 + \dfrac{\pi}{2}$ $= -2\sin\ 0 + \dfrac{\pi}{2}$ $= -2\sin\dfrac{\pi}{2}$ $= -2\cdot 1 = -2$	$(0, -2)$
$\dfrac{\pi}{4}$	$y = -2\sin\ 2\cdot\dfrac{\pi}{4} + \dfrac{\pi}{2}$ $= -2\sin\ \dfrac{\pi}{2} + \dfrac{\pi}{2}$ $= -2\sin\pi$ $= -2\cdot 0 = 0$	$\dfrac{\pi}{4}, 0$
$\dfrac{\pi}{2}$	$y = -2\sin\ 2\cdot\dfrac{\pi}{2} + \dfrac{\pi}{2}$ $= -2\sin\ \pi + \dfrac{\pi}{2}$ $= -2\sin\dfrac{3\pi}{2}$ $= -2(-1) = 2$	$\dfrac{\pi}{2}, 2$
$\dfrac{3\pi}{4}$	$y = -2\sin\ 2\cdot\dfrac{3\pi}{4} + \dfrac{\pi}{2}$ $= -2\sin\ \dfrac{3\pi}{2} + \dfrac{\pi}{2}$ $= -2\sin 2\pi$ $= -2\cdot 0 = 0$	$\dfrac{3\pi}{4}, 0$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = -2\sin\left(2x + \frac{\pi}{2}\right)$

26. $y = -3\sin\left(2x + \frac{\pi}{2}\right) = -3\sin\left(2x - \left(-\frac{\pi}{2}\right)\right)$

The equation $y = -3\sin\left(2x - \left(-\frac{\pi}{2}\right)\right)$ is of the form

$y = A\sin(Bx - C)$ with $A = -3$, $B = 2$, and $C = -\frac{\pi}{2}$.
The amplitude is $|A| = |-3| = 3$. The period is
$\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is
$\frac{C}{B} = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{2}\cdot\frac{1}{2} = -\frac{\pi}{4}$. The quarter-period is $\frac{\pi}{4}$.
The cycle begins at $x = -\frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{4}$

$x = -\frac{\pi}{4} + \frac{\pi}{4} = 0$

$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

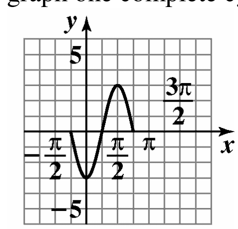
$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

Evaluate the function at each value of x.

x	$y = -3\sin\left(2x + \frac{\pi}{2}\right)$	coordinates
$-\frac{\pi}{4}$	$y = -3\sin\left(2\cdot\left(-\frac{\pi}{4}\right) + \frac{\pi}{2}\right)$ $= -3\sin\left(-\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= -3\sin 0 = -3\cdot 0 = 0$	$\left(-\frac{\pi}{4}, 0\right)$
0	$y = -3\sin\left(2\cdot 0 + \frac{\pi}{2}\right)$ $= -3\sin\left(0 + \frac{\pi}{2}\right)$ $= -3\sin\frac{\pi}{2} = -3\cdot 1 = -3$	$(0, -3)$
$\frac{\pi}{4}$	$y = -3\sin\left(2\cdot\frac{\pi}{4} + \frac{\pi}{2}\right)$ $= -3\sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= -3\sin\pi = -3\cdot 0 = 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$y = -3\sin\left(2\cdot\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= -3\sin\left(\pi + \frac{\pi}{2}\right)$ $= -3\sin\frac{3\pi}{2} = -3\cdot(-1) = 3$	$\left(\frac{\pi}{2}, 3\right)$
$\frac{3\pi}{4}$	$y = -3\sin\left(2\cdot\frac{3\pi}{4} + \frac{\pi}{2}\right)$ $= -3\sin\left(\frac{3\pi}{2} + \frac{\pi}{2}\right)$ $= -3\sin 2\pi = -3\cdot 0 = 0$	$\left(\frac{3\pi}{4}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -3\sin\left(2x + \frac{\pi}{2}\right)$

27. $y = 3\sin(\pi x + 2)$
The equation $y = 3\sin(\pi x - (-2))$ is of the form
$y = A\sin(Bx - C)$ with $A = 3$, $B = \pi$, and $C = -2$.
The amplitude is $|A| = |3| = 3$. The period is
$\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$. The phase shift is $\frac{C}{B} = \frac{-2}{\pi} = -\frac{2}{\pi}$. The quarter-period is $\frac{2}{4} = \frac{1}{2}$. The cycle begins at $x = -\frac{2}{\pi}$. Add quarter-periods to generate x-values for the key points.

$x=-\frac{2}{\pi}$

$x=-\frac{2}{\pi}+\frac{1}{2}=\frac{\pi-4}{2\pi}$

$x=\frac{\pi-4}{2\pi}+\frac{1}{2}=\frac{\pi-2}{\pi}$

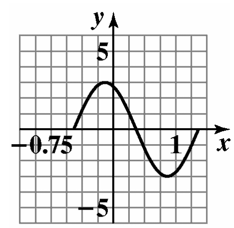
$x=\frac{\pi-2}{\pi}+\frac{1}{2}=\frac{3\pi-4}{2\pi}$

$x=\frac{3\pi-4}{2\pi}+\frac{1}{2}=\frac{2\pi-2}{\pi}$

Evaluate the function at each value of x.

x	$y=3\sin(\pi x+2)$	coordinates
$-\frac{2}{\pi}$	$y=3\sin\ \pi\ -\frac{2}{\pi}\ +2$ $=3\sin(-2+2)$ $=3\sin 0=3\cdot 0=0$	$-\frac{2}{\pi},\ 0$
$\frac{\pi-4}{2\pi}$	$y=3\sin\ \pi\ \frac{\pi-4}{2\pi}\ +2$ $=3\sin\ \frac{\pi-4}{2}+2$ $=3\sin\ \frac{\pi}{2}-2+2$ $=3\sin\frac{\pi}{2}$ $=3\cdot 1=3$	$\frac{\pi-4}{2\pi},\ 3$
$\frac{\pi-2}{\pi}$	$y=3\sin\ \pi\ \frac{\pi-2}{\pi}\ +2$ $=3\sin(\pi-2+2)$ $=3\sin\pi=3\cdot 0=0$	$\frac{\pi-2}{\pi},\ 0$
$\frac{3\pi-4}{2\pi}$	$y=3\sin\ \pi\ \frac{3\pi-4}{2\pi}\ +2$ $=3\sin\ \frac{3\pi-4}{2}+2$ $=3\sin\ \frac{3\pi}{2}-2+2$ $=3\sin\frac{3\pi}{2}$ $=3(-1)=-3$	$\frac{5\pi}{4},\ -3$
$\frac{2\pi-2}{\pi}$	$y=3\sin\ \pi\ \frac{2\pi-2}{\pi}\ +2$ $=3\sin(2\pi-2+2)$ $=3\sin 2\pi=3\cdot 0=0$	$\frac{2\pi-2}{\pi},\ 0$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = 3\sin(\pi x + 2)$

28. $y=3\sin(2\pi x+4)=3\sin(2\pi x-(-4))$

The equation $y=3\sin(2\pi x-(-4))$ is of the form $y=A\sin(Bx-C)$ with $A = 3$, $B=2\pi$, and $C = -4$. The amplitude is $|A|=|3|=3$. The period is $\frac{2\pi}{B}=\frac{2\pi}{2\pi}=1$. The phase shift is $\frac{C}{B}=\frac{-4}{2\pi}=-\frac{2}{\pi}$.

The quarter-period is $\frac{1}{4}$. The cycle begins at $x=-\frac{2}{\pi}$. Add quarter-periods to generate x-values for the key points.

$x=-\frac{2}{\pi}$

$x=-\frac{2}{\pi}+\frac{1}{4}=\frac{\pi-8}{4\pi}$

$x=\frac{\pi-8}{4\pi}+\frac{1}{4}=\frac{\pi-4}{2\pi}$

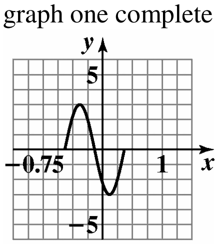
$x=\frac{\pi-4}{2\pi}+\frac{1}{4}=\frac{3\pi-8}{4\pi}$

$x=\frac{3\pi-8}{4\pi}+\frac{1}{4}=\frac{\pi-2}{\pi}$

Evaluate the function at each value of x.

x	$y = 3\sin(2\pi x + 4)$	coordinates
$-\frac{2}{\pi}$	$y = 3\sin\ 2\pi\ -\frac{2}{\pi}\ \ +4$ $= 3\sin(-4+4)$ $= 3\sin 0 = 3\cdot 0 = 0$	$-\frac{2}{\pi},\ 0$
$\frac{\pi-8}{4\pi}$	$y = 3\sin\ 2\pi\ \frac{\pi-8}{4\pi}\ \ +4$ $= 3\sin\ \frac{\pi-8}{2}+4$ $= 3\sin\ \frac{\pi}{2}-4+4$ $= 3\sin\frac{\pi}{2} = 3\cdot 1 = 3$	$\frac{\pi-8}{4\pi},\ 3$
$\frac{\pi-4}{2\pi}$	$y = 3\sin\ 2\pi\ \frac{\pi-4}{2\pi}\ \ +4$ $= 3\sin(\pi-4+4)$ $= 3\sin\pi = 3\cdot 0 = 0$	$\frac{\pi-4}{2\pi},\ 0$
$\frac{3\pi-8}{4\pi}$	$y = 3\sin\ 2\pi\ \frac{3\pi-8}{4\pi}\ \ +4$ $= 3\sin\ \frac{3\pi-8}{2}+4$ $= 3\sin\ \frac{3\pi}{2}-4+4$ $= 3\sin\frac{3\pi}{2} = 3(-1) = -3$	$\frac{3\pi-8}{4\pi},\ -3$
$\frac{\pi-2}{\pi}$	$y = 3\sin\ 2\pi\ \frac{\pi-2}{\pi}\ \ +4$ $= 3\sin(2\pi-4+4)$ $= 3\sin 2\pi = 3\cdot 0 = 0$	$\frac{\pi-2}{\pi},0$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



***y* = 3 sin (2π*x* + 4)**

29. $y = -2\sin(2\pi x + 4\pi) = -2\sin(2\pi x - (-4\pi))$
The equation $y = -2\sin(2\pi x - (-4\pi))$ is of the form $y = A\sin(Bx - C)$ with $A = -2,\ B = 2\pi$, and $C = -4\pi$. The amplitude is $|A| = |-2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The phase shift is $\frac{C}{B} = \frac{-4\pi}{2\pi} = -2$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = -2$. Add quarter-periods to generate *x*-values for the key points.

$x = -2$

$x = -2 + \frac{1}{4} = -\frac{7}{4}$

$x = -\frac{7}{4} + \frac{1}{4} = -\frac{3}{2}$

$x = -\frac{3}{2} + \frac{1}{4} = -\frac{5}{4}$

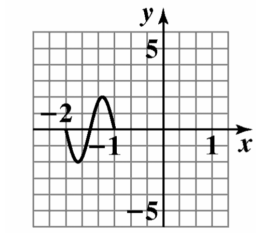
$x = -\frac{5}{4} + \frac{1}{4} = -1$

Evaluate the function at each value of *x*.

x	$y = -2\sin(2\pi x + 4\pi)$	coordinates
-2	$y = -2\sin(2\pi(-2) + 4\pi)$ $= -2\sin(-4\pi + 4\pi)$ $= -2\sin 0$ $= -2\cdot 0 = 0$	$(-2, 0)$
$-\frac{7}{4}$	$y = -2\sin\ 2\pi\ -\frac{7}{4}\ \ +4\pi$ $= -2\sin\ -\frac{7\pi}{2}+4\pi$ $= -2\sin\frac{\pi}{2} = -2\cdot 1 = -2$	$-\frac{7}{4},\ -2$
$-\frac{3}{2}$	$y = -2\sin\ 2\pi\ -\frac{3}{2}\ \ +4\pi$ $= -2\sin(-3\pi + 4\pi)$ $= -2\sin\pi = -2\cdot 0 = 0$	$-\frac{3}{2},\ 0$
$-\frac{5}{4}$	$y = -2\sin\ 2\pi\ -\frac{5}{4}\ \ +4\pi$ $= -2\sin\ -\frac{5\pi}{2}+4\pi$ $= -2\sin\frac{3\pi}{2}$ $= -2(-1) = 2$	$-\frac{5}{4},\ 2$

-1	$y=-2\sin(2\pi(-1)+4\pi)$ $=-2\sin(-2\pi+4\pi)$ $=-2\sin 2\pi$ $=-2\cdot 0=0$	$(-1, 0)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = -2\sin(2\pi x + 4\pi)$

30. $y=-3\sin(2\pi x+4\pi)=-3\sin(2\pi x-(-4\pi))$

The equation $y=-3\sin(2\pi x-(-4\pi))$ is of the form $y=A\sin(Bx-C)$ with $A=-3$, $B=2\pi$, and $C=-4\pi$. The amplitude is $|A|=|-3|=3$. The period is $\frac{2\pi}{B}=\frac{2\pi}{2\pi}=1$. The phase shift is $\frac{C}{B}=\frac{-4\pi}{2\pi}=-2$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x=-2$. Add quarter-periods to generate x-values for the key points.

$x=-2$

$x=-2+\frac{1}{4}=-\frac{7}{4}$

$x=-\frac{7}{4}+\frac{1}{4}=-\frac{3}{2}$

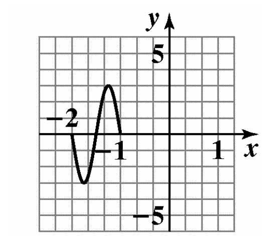
$x=-\frac{3}{2}+\frac{1}{4}=-\frac{5}{4}$

$x=-\frac{5}{4}+\frac{1}{4}=-1$

Evaluate the function at each value of x.

x	$y=-3\sin(2\pi x+4\pi)$	coordinates
-2	$y=-3\sin(2\pi(-2)+4\pi)$ $=-3\sin(-4\pi+4\pi)$ $=-3\sin 0=-3\cdot 0=0$	$(-2, 0)$
$-\frac{7}{4}$	$y=-3\sin\ 2\pi\ -\frac{7}{4}\ +4\pi$ $=-3\sin\ -\frac{7\pi}{2}+4\pi$ $=-3\sin\frac{\pi}{2}=-3\cdot 1=-3$	$-\frac{7}{4}, -3$
$-\frac{3}{2}$	$y=-3\sin\ 2\pi\ -\frac{3}{2}\ +4\pi$ $=-3\sin(-3\pi+4\pi)$ $=-3\sin\pi=-3\cdot 0=0$	$-\frac{3}{2}, 0$
$-\frac{5}{4}$	$y=-3\sin\ 2\pi\ -\frac{5}{4}\ +4\pi$ $=-3\sin\ -\frac{5\pi}{2}+4\pi$ $=-3\sin\frac{3\pi}{2}=-3(-1)=3$	$-\frac{5}{4}, 3$
-1	$y=-3\sin(2\pi(-1)+4\pi)$ $=-3\sin(-2\pi+4\pi)$ $=-3\sin 2\pi=-3\cdot 0=0$	$(-1, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -3\sin(2\pi x + 4\pi)$

31. The equation $y = 2\cos x$ is of the form $y = A\cos x$ with $A = 2$. Thus, the amplitude is $|A| = |2| = 2$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

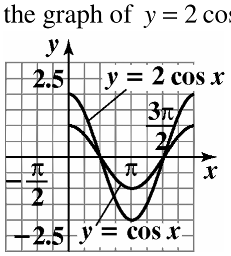
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 2\cos x$	coordinates
0	$y = 2\cos 0$ $= 2\cdot 1 = 2$	(0, 2)
$\frac{\pi}{2}$	$y = 2\cos\frac{\pi}{2}$ $= 2\cdot 0 = 0$	$\frac{\pi}{2}$, 0
π	$y = 2\cos\pi$ $= 2\cdot(-1) = -2$	$(\pi, -2)$
$\frac{3\pi}{2}$	$y = 2\cos\frac{3\pi}{2}$ $= 2\cdot 0 = 0$	$\frac{3\pi}{2}$, 0
2π	$y = 2\cos 2\pi$ $= 2\cdot 1 = 2$	$(2\pi, 2)$

Connect the five points with a smooth curve and graph one complete cycle of the given function with the graph of $y = 2\cos x$.



32. The equation $y = 3\cos x$ is of the form $y = A\cos x$ with $A = 3$. Thus, the amplitude is $|A| = |3| = 3$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

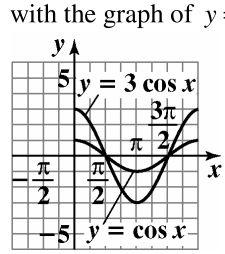
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = 3\cos x$	coordinates
0	$y = 3\cos 0 = 3\cdot 1 = 3$	(0, 3)
$\frac{\pi}{2}$	$y = 3\cos\frac{\pi}{2} = 3\cdot 0 = 0$	$\frac{\pi}{2}$, 0
π	$y = 3\cos\pi = 3\cdot(-1) = -3$	$(\pi, -3)$
$\frac{3\pi}{2}$	$y = 3\cos\frac{3\pi}{2} = 3\cdot 0 = 0$	$\frac{3\pi}{2}$, 0
2π	$y = 3\cos 2\pi = 3\cdot 1 = 3$	$(2\pi, 3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \cos x$.



33. The equation $y = -2\cos x$ is of the form $y = A\cos x$ with $A = -2$. Thus, the amplitude is $|A| = |-2| = 2$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

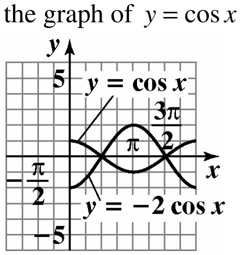
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -2\cos x$	coordinates
0	$y = -2\cos 0$ $= -2\cdot 1 = -2$	$(0, -2)$
$\frac{\pi}{2}$	$y = -2\cos\frac{\pi}{2}$ $= -2\cdot 0 = 0$	$\frac{\pi}{2}, 0$
π	$y = -2\cos\pi$ $= -2\cdot(-1) = 2$	$(\pi, 2)$
$\frac{3\pi}{2}$	$y = -2\cos\frac{3\pi}{2}$ $= -2\cdot 0 = 0$	$\frac{3\pi}{2}, 0$
2π	$y = -2\cos 2\pi$ $= -2\cdot 1 = -2$	$(2\pi, -2)$

Connect the five points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \cos x$.



34. The equation $y = -3\cos x$ is of the form $y = A\cos x$ with $A = -3$. Thus, the amplitude is $|A| = |-3| = 3$. The period is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

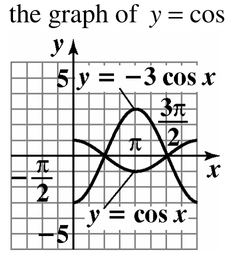
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = -3\cos x$	coordinates
0	$y = -3\cos 0 = -3\cdot 1 = -3$	$(0, -3)$
$\frac{\pi}{2}$	$y = -3\cos\frac{\pi}{2} = -3\cdot 0 = 0$	$\frac{\pi}{2}, 0$
π	$y = -3\cos\pi = -3\cdot(-1) = 3$	$(\pi, 3)$
$\frac{3\pi}{2}$	$y = -3\cos\frac{3\pi}{2} = -3\cdot 0 = 0$	$\frac{3\pi}{2}, 0$
2π	$y = -3\cos 2\pi = -3\cdot 1 = -3$	$(2\pi, -3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function with the graph of $y = \cos x$.



35. The equation $y = \cos 2x$ is of the form $y = A\cos Bx$ with $A = 1$ and $B = 2$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

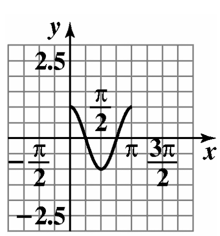
$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

Evaluate the function at each value of x.

x	$y = \cos 2x$	coordinates
0	$y = \cos(2 \cdot 0)$ $= \cos 0 = 1$	(0, 1)
$\frac{\pi}{4}$	$y = \cos\ 2 \cdot \frac{\pi}{4}$ $= \cos\frac{\pi}{2} = 0$	$\frac{\pi}{4}, 0$
$\frac{\pi}{2}$	$y = \cos\ 2 \cdot \frac{\pi}{2}$ $= \cos \pi = -1$	$\frac{\pi}{2}, -1$
$\frac{3\pi}{4}$	$y = \cos\ 2 \cdot \frac{3\pi}{4}$ $= \cos\frac{3\pi}{2} = 0$	$\frac{3\pi}{4}, 0$
π	$y = \cos(2 \cdot \pi)$ $= \cos 2\pi = 1$	$(\pi, 1)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



y = cos 2x

36. The equation $y = \cos 4x$ is of the form $y = A\cos Bx$ with $A = 1$ and $B = 4$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{4} = \frac{\pi}{2}$. The quarter-period is $\frac{\frac{\pi}{2}}{4} = \frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{8} = \frac{\pi}{8}$

$x = \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi}{4}$

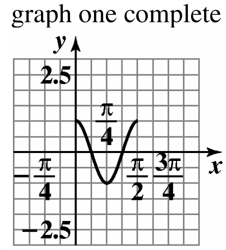
$x = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$

$x = \frac{3\pi}{8} + \frac{\pi}{8} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	$y = \cos 4x$	coordinates
0	$y = \cos(4 \cdot 0) = \cos 0 = 1$	(0, 1)
$\frac{\pi}{8}$	$y = \cos\ 4 \cdot \frac{\pi}{8}\ = \cos\frac{\pi}{2} = 0$	$\frac{\pi}{8}, 0$
$\frac{\pi}{4}$	$y = \cos\ 4 \cdot \frac{\pi}{4}\ = \cos \pi = -1$	$\frac{\pi}{4}, -1$
$\frac{3\pi}{8}$	$y = \cos\ 4 \cdot \frac{3\pi}{8}$ $= \cos\frac{3\pi}{2} = 0$	$\frac{3\pi}{8}, 0$
$\frac{\pi}{2}$	$y = \cos\ 4 \cdot \frac{\pi}{2}\ = \cos 2\pi = 1$	$\frac{\pi}{2}, 1$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



y = cos 4x

37. The equation $y = 4\cos 2\pi x$ is of the form $y = A\cos Bx$ with $A = 4$ and $B = 2\pi$. Thus, the amplitude is $|A| = |4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{4} = \frac{1}{4}$

$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

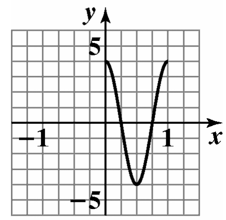
$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = 4\cos 2\pi x$	coordinates
0	$y = 4\cos(2\pi\cdot 0)$ $= 4\cos 0$ $= 4\cdot 1 = 4$	(0, 4)
$\frac{1}{4}$	$y = 4\cos\ 2\pi\cdot\frac{1}{4}$ $= 4\cos\frac{\pi}{2}$ $= 4\cdot 0 = 0$	$\frac{1}{4}$, 0
$\frac{1}{2}$	$y = 4\cos\ 2\pi\cdot\frac{1}{2}$ $= 4\cos\pi$ $= 4\cdot(-1) = -4$	$\frac{1}{2}$, −4
$\frac{3}{4}$	$y = 4\cos\ 2\pi\cdot\frac{3}{4}$ $= 4\cos\frac{3\pi}{2}$ $= 4\cdot 0 = 0$	$\frac{3}{4}$, 0
1	$y = 4\cos(2\pi\cdot 1)$ $= 4\cos 2\pi$ $= 4\cdot 1 = 4$	(1, 4)

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = 4\cos 2\pi x$

38. The equation $y = 5\cos 2\pi x$ is of the form $y = A\cos Bx$ with $A = 5$ and $B = 2\pi$. Thus, the amplitude is $|A| = |5| = 5$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{4} = \frac{1}{4}$

$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

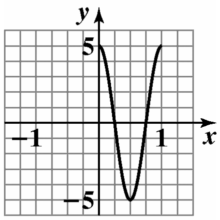
$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = 5\cos 2\pi x$	coordinates
0	$y = 5\cos(2\pi\cdot 0)$ $= 5\cos 0 = 5\cdot 1 = 5$	(0, 5)
$\frac{1}{4}$	$y = 5\cos\ 2\pi\cdot\frac{1}{4}$ $= 5\cos\frac{\pi}{2} = 5\cdot 0 = 0$	$\frac{1}{4}$, 0
$\frac{1}{2}$	$y = 5\cos\ 2\pi\cdot\frac{1}{2}$ $= 5\cos\pi = 5\cdot(-1) = -5$	$\frac{1}{2}$, −5
$\frac{3}{4}$	$y = 5\cos\ 2\pi\cdot\frac{3\pi}{4}$ $= 5\cos\frac{3\pi}{2} = 5\cdot 0 = 0$	$\frac{3}{4}$, 0
1	$y = 5\cos(2\pi\cdot 1)$ $= 5\cos 2\pi = 5\cdot 1 = 5$	(1, 5)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 5\cos 2\pi x$

39. The equation $y = -4\cos\frac{1}{2}x$ is of the form $y = A\cos Bx$ with $A = -4$ and $B = \frac{1}{2}$. Thus, the amplitude is $|A| = |-4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$. The quarter-period is $\frac{4\pi}{4} = \pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \pi = \pi$

$x = \pi + \pi = 2\pi$

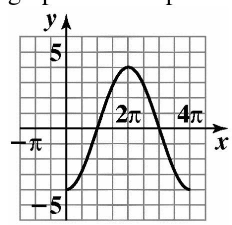
$x = 2\pi + \pi = 3\pi$

$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = -4\cos\frac{1}{2}x$	coordinates
0	$y = -4\cos\left(\frac{1}{2} \cdot 0\right)$ $= -4\cos 0$ $= -4 \cdot 1 = -4$	$(0, -4)$
π	$y = -4\cos\left(\frac{1}{2} \cdot \pi\right)$ $= -4\cos\frac{\pi}{2}$ $= -4 \cdot 0 = 0$	$(\pi, 0)$
2π	$y = -4\cos\left(\frac{1}{2} \cdot 2\pi\right)$ $= -4\cos\pi$ $= -4 \cdot (-1) = 4$	$(2\pi, 4)$
3π	$y = -4\cos\left(\frac{1}{2} \cdot 3\pi\right)$ $= -4\cos\frac{3\pi}{2}$ $= -4 \cdot 0 = 0$	$(3\pi, 0)$
4π	$y = -4\cos\left(\frac{1}{2} \cdot 4\pi\right)$ $= -4\cos 2\pi$ $= -4 \cdot 1 = -4$	$(4\pi, -4)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = -4\cos\frac{1}{2}x$

40. The equation $y = -3\cos\frac{1}{3}x$ is of the form $y = A\cos Bx$ with $A = -3$ and $B = \frac{1}{3}$. Thus, the amplitude is $|A| = |-3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{3}} = 2\pi \cdot 3 = 6\pi$. The quarter-period is $\frac{6\pi}{4} = \frac{3\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{3\pi}{2} = 3\pi$

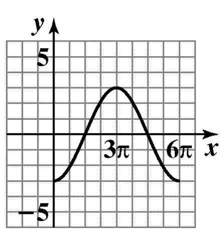
$x = 3\pi + \frac{3\pi}{2} = \frac{9\pi}{2}$

$x = \frac{9\pi}{2} + \frac{3\pi}{2} = 6\pi$

Evaluate the function at each value of x.

x	$y=-3\cos\frac{1}{3}x$	coordinates
0	$y=-3\cos\frac{1}{3}\cdot 0$ $=-3\cos 0=-3\cdot 1=-3$	(0, –3)
$\frac{3\pi}{2}$	$y=-3\cos\frac{1}{3}\cdot\frac{3\pi}{2}$ $=-3\cos\frac{\pi}{2}=-3\cdot 0=0$	$\frac{3\pi}{2}$, 0
3π	$y=-3\cos\frac{1}{3}\cdot 3\pi$ $=-3\cos\pi=-3\cdot(-1)=3$	$(3\pi, 3)$
$\frac{9\pi}{2}$	$y=-3\cos\frac{1}{3}\cdot\frac{9\pi}{2}$ $=-3\cos\frac{3\pi}{2}=-3\cdot 0=0$	$\frac{9\pi}{2}$, 0
6π	$y=-3\cos\frac{1}{3}\cdot 6\pi$ $=-3\cos 2\pi=-3\cdot 1=-3$	$(6\pi, -3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y=-3\cos\frac{1}{3}x$

41. The equation $y=-\frac{1}{2}\cos\frac{\pi}{3}x$ is of the form $y=A\cos Bx$ with $A=-\frac{1}{2}$ and $B=\frac{\pi}{3}$. Thus, the amplitude is $|A|=\left|-\frac{1}{2}\right|=\frac{1}{2}$. The period is $\frac{2\pi}{B}=\frac{2\pi}{\frac{\pi}{3}}=2\pi\cdot\frac{3}{\pi}=6$. The quarter-period is $\frac{6}{4}=\frac{3}{2}$. The cycle begins at $x=0$. Add quarter-periods to generate x-values for the key points.

$x=0$

$x=0+\frac{3}{2}=\frac{3}{2}$

$x=\frac{3}{2}+\frac{3}{2}=3$

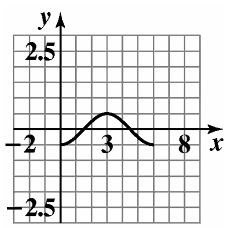
$x=3+\frac{3}{2}=\frac{9}{2}$

$x=\frac{9}{2}+\frac{3}{2}=6$

Evaluate the function at each value of x.

x	$y=-\frac{1}{2}\cos\frac{\pi}{3}x$	coordinates
0	$y=-\frac{1}{2}\cos\frac{\pi}{3}\cdot 0$ $=-\frac{1}{2}\cos 0$ $=-\frac{1}{2}\cdot 1=-\frac{1}{2}$	$0, -\frac{1}{2}$
$\frac{3}{2}$	$y=-\frac{1}{2}\cos\frac{\pi}{3}\cdot\frac{3}{2}$ $=-\frac{1}{2}\cos\frac{\pi}{2}$ $=-\frac{1}{2}\cdot 0=0$	$\frac{3}{2}$, 0
3	$y=-\frac{1}{2}\cos\frac{\pi}{3}\cdot 3$ $=-\frac{1}{2}\cos\pi$ $=-\frac{1}{2}\cdot(-1)=\frac{1}{2}$	$3, \frac{1}{2}$
$\frac{9}{2}$	$y=-\frac{1}{2}\cos\frac{\pi}{3}\cdot\frac{9}{2}$ $=-\frac{1}{2}\cos\frac{3\pi}{2}$ $=-\frac{1}{2}\cdot 0=0$	$\frac{9}{2}$, 0
6	$y=-\frac{1}{2}\cos\frac{\pi}{3}\cdot 6$ $=-\frac{1}{2}\cos 2\pi$ $=-\frac{1}{2}\cdot 1=-\frac{1}{2}$	$6, -\frac{1}{2}$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



$y = -\frac{1}{2}\cos\frac{\pi}{3}x$

42. The equation $y = -\frac{1}{2}\cos\frac{\pi}{4}x$ is of the form $y = A\cos Bx$ with $A = -\frac{1}{2}$ and $B = \frac{\pi}{4}$. Thus, the amplitude is $|A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{4}} = 2\pi \cdot \frac{4}{\pi} = 8$. The quarter-period is $\frac{8}{4} = 2$.

The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + 2 = 2$

$x = 2 + 2 = 4$

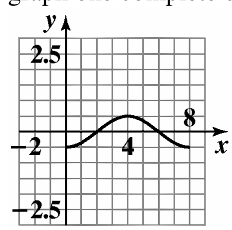
$x = 4 + 2 = 6$

$x = 6 + 2 = 8$

Evaluate the function at each value of x.

x	$y = -\frac{1}{2}\cos\frac{\pi}{4}x$	coordinates
0	$y = -\frac{1}{2}\cos\frac{\pi}{4}\cdot 0$ $= -\frac{1}{2}\cos 0 = -\frac{1}{2}\cdot 1 = -\frac{1}{2}$	$0, -\frac{1}{2}$
2	$y = -\frac{1}{2}\cos\frac{\pi}{4}\cdot 2$ $= -\frac{1}{2}\cos\frac{\pi}{2} = -\frac{1}{2}\cdot 0 = 0$	(2, 0)
4	$y = -\frac{1}{2}\cos\frac{\pi}{4}\cdot 4$ $= -\frac{1}{2}\cos\pi = -\frac{1}{2}\cdot(-1) = \frac{1}{2}$	$4, \frac{1}{2}$
6	$y = -\frac{1}{2}\cos\frac{\pi}{4}\cdot 6$ $= -\frac{1}{2}\cos\frac{3\pi}{2} = -\frac{1}{2}\cdot 0 = 0$	(6, 0)
8	$y = -\frac{1}{2}\cos\frac{\pi}{4}\cdot 8$ $= -\frac{1}{2}\cos 2\pi = -\frac{1}{2}\cdot 1 = -\frac{1}{2}$	$8, -\frac{1}{2}$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -\frac{1}{2}\cos\frac{\pi}{4}x$

43. The equation $y = \cos\left(x - \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx - C)$ with $A = 1$, and $B = 1$, and $C = \frac{\pi}{2}$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

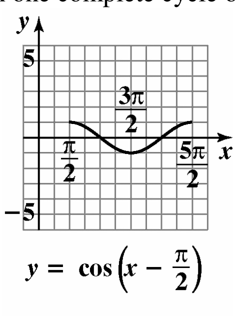
$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

$x = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 1\right)$
π	$(\pi, 0)$
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, -1\right)$
2π	$(2\pi, 0)$
$\frac{5\pi}{2}$	$\left(\frac{5\pi}{2}, 1\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



44. The equation $y = \cos\left(x + \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx - C)$ with $A = 1$, and $B = 1$, and $C = -\frac{\pi}{2}$. Thus, the amplitude is $|A| = |1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2}$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = -\frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} + \frac{\pi}{2} = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

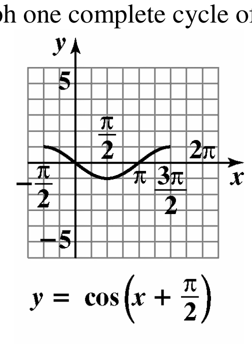
$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$-\frac{\pi}{2}$	$\left(-\frac{\pi}{2}, 1\right)$
0	$(0, 0)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, -1\right)$
π	$(\pi, 0)$
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, 1\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



45. The equation $y = 3\cos(2x - \pi)$ is of the form $y = A\cos\left(Bx - C\right)$ with $A = 3$, and $B = 2$, and $C = \pi$. Thus, the amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

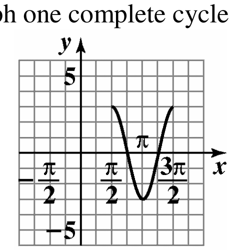
$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{2}$	$\frac{\pi}{2}, 3$
$\frac{3\pi}{4}$	$\frac{3\pi}{4}, 0$
π	$(\pi, -3)$
$\frac{5\pi}{4}$	$\frac{5\pi}{4}, 0$
$\frac{3\pi}{2}$	$\frac{3\pi}{2}, 3$

Connect the five points with a smooth curve and graph one complete cycle of the given function



$y = 3\cos(2x - \pi)$

46. The equation $y = 4\cos(2x - \pi)$ is of the form $y = A\cos(Bx - C)$ with $A = 4$, and $B = 2$, and $C = \pi$. Thus, the amplitude is $|A| = |4| = 4$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

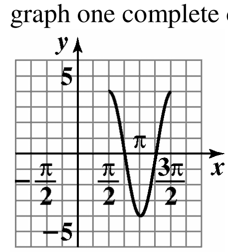
$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{2}$	$\frac{\pi}{2}, 4$
$\frac{3\pi}{4}$	$\frac{3\pi}{4}, 0$
π	$(\pi, -4)$
$\frac{5\pi}{4}$	$\frac{5\pi}{4}, 0$
$\frac{3\pi}{2}$	$\frac{3\pi}{2}, 4$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 4\cos(2x - \pi)$

47. $y=\frac{1}{2}\cos\left(3x+\frac{\pi}{2}\right)=\frac{1}{2}\cos\left(3x-\left(-\frac{\pi}{2}\right)\right)$

The equation $y=\frac{1}{2}\cos\left(3x-\left(-\frac{\pi}{2}\right)\right)$ is of the form

$y=A\cos(Bx-C)$ with $A=\frac{1}{2}$, and $B = 3$, and

$C=-\frac{\pi}{2}$. Thus, the amplitude is $|A|=\left|\frac{1}{2}\right|=\frac{1}{2}$. The

period is $\frac{2\pi}{B}=\frac{2\pi}{3}$. The phase shift is

$\frac{C}{B}=\frac{-\frac{\pi}{2}}{3}=-\frac{\pi}{2}\cdot\frac{1}{3}=-\frac{\pi}{6}$. The quarter-period is

$\frac{\frac{2\pi}{3}}{4}=\frac{2\pi}{3}\cdot\frac{1}{4}=\frac{\pi}{6}$. The cycle begins at $x=-\frac{\pi}{6}$. Add quarter-periods to generate x-values for the key points.

$x=-\frac{\pi}{6}$

$x=-\frac{\pi}{6}+\frac{\pi}{6}=0$

$x=0+\frac{\pi}{6}=\frac{\pi}{6}$

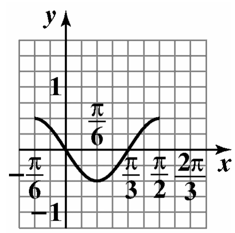
$x=\frac{\pi}{6}+\frac{\pi}{6}=\frac{\pi}{3}$

$x=\frac{\pi}{3}+\frac{\pi}{6}=\frac{\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$-\frac{\pi}{6}$	$\left(-\frac{\pi}{6}, \frac{1}{2}\right)$
0	(0, 0)
$\frac{\pi}{6}$	$\left(\frac{\pi}{6}, -\frac{1}{2}\right)$
$\frac{\pi}{3}$	$\left(\frac{\pi}{3}, 0\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, \frac{1}{2}\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



$y=\frac{1}{2}\cos\left(3x+\frac{\pi}{2}\right)$

48. $y=\frac{1}{2}\cos(2x+\pi)=\frac{1}{2}\cos(2x-(-\pi))$

The equation $y=\frac{1}{2}\cos(2x-(-\pi))$ is of the form

$y=A\cos(Bx-C)$ with $A=\frac{1}{2}$, and $B = 2$, and

$C=-\pi$. Thus, the amplitude is $|A|=\left|\frac{1}{2}\right|=\frac{1}{2}$. The

period is $\frac{2\pi}{B}=\frac{2\pi}{2}=\pi$. The phase shift is

$\frac{C}{B}=\frac{-\pi}{2}=-\frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle

begins at $x=-\frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x=-\frac{\pi}{2}$

$x=-\frac{\pi}{2}+\frac{\pi}{4}=-\frac{\pi}{4}$

$x=-\frac{\pi}{4}+\frac{\pi}{4}=0$

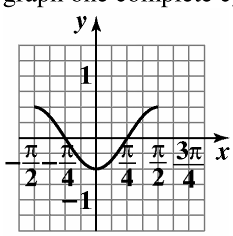
$x=0+\frac{\pi}{4}=\frac{\pi}{4}$

$x=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$-\frac{\pi}{2}$	$-\frac{\pi}{2}, \frac{1}{2}$
$-\frac{\pi}{4}$	$-\frac{\pi}{4}, 0$
0	$0, -\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{\pi}{4}, 0$
$\frac{\pi}{2}$	$\frac{\pi}{2}, \frac{1}{2}$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = \frac{1}{2}\cos(2x + \pi)$

49. The equation $y = -3\cos\ 2x - \frac{\pi}{2}$ is of the form $y = A\cos(Bx - C)$ with $A = -3$, and $B = 2$, and $C = \frac{\pi}{2}$. Thus, the amplitude is $|A| = |-3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

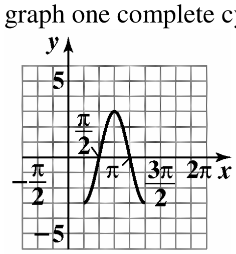
$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{4}$	$\frac{\pi}{4}, -3$
$\frac{\pi}{2}$	$\frac{\pi}{2}, 0$
$\frac{3\pi}{4}$	$\frac{3\pi}{4}, 3$
π	$(\pi, 0)$
$\frac{5\pi}{4}$	$\frac{5\pi}{4}, -3$

Connect the five points with a smooth curve and graph one complete cycle of the given function



$y = -3\cos\left(2x - \frac{\pi}{2}\right)$

50. The equation $y = -4\cos\ 2x - \frac{\pi}{2}$ is of the form $y = A\cos(Bx - C)$ with $A = -4$, and $B = 2$, and $C = \frac{\pi}{2}$. Thus, the amplitude is $|A| = |-4| = 4$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{2}\cdot\frac{1}{2} = \frac{\pi}{4}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

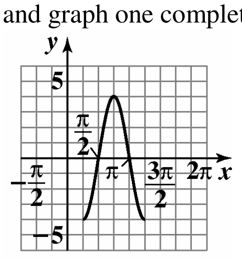
$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{4}$	$\frac{\pi}{4}, -4$
$\frac{\pi}{2}$	$\frac{\pi}{2}, 0$
$\frac{3\pi}{4}$	$\frac{3\pi}{4}, 4$
π	$(\pi, 0)$
$\frac{5\pi}{4}$	$\frac{5\pi}{4}, -4$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -4\cos\left(2x - \frac{\pi}{2}\right)$

51. $y = 2\cos(2\pi x + 8\pi) = 2\cos(2\pi x - (-8\pi))$

The equation $y = 2\cos(2\pi x - (-8\pi))$ is of the form $y = A\cos(Bx - C)$ with $A = 2$, $B = 2\pi$, and $C = -8\pi$. Thus, the amplitude is $|A| = |2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The phase shift is $\frac{C}{B} = \frac{-8\pi}{2\pi} = -4$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = -4$. Add quarter-periods to generate x-values for the key points.

$x = -4$

$x = -4 + \frac{1}{4} = -\frac{15}{4}$

$x = -\frac{15}{4} + \frac{1}{4} = -\frac{7}{2}$

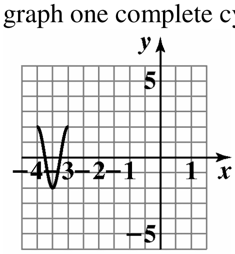
$x = -\frac{7}{2} + \frac{1}{4} = -\frac{13}{4}$

$x = -\frac{13}{4} + \frac{1}{4} = -3$

Evaluate the function at each value of x.

x	coordinates
-4	$(-4, 2)$
$-\frac{15}{4}$	$-\frac{15}{4}, 0$
$-\frac{7}{2}$	$-\frac{7}{2}, -2$
$-\frac{13}{4}$	$-\frac{13}{4}, 0$
-3	$(-3, 2)$

Connect the five points with a smooth curve and graph one complete cycle of the given function



$y = 2\cos(2\pi x + 8\pi)$

52. $y = 3\cos(2\pi x + 4\pi) = 3\cos(2\pi x - (-4\pi))$
The equation $y = 3\cos(2\pi x - (-4\pi))$ is of the form $y = A\cos(Bx - C)$ with $A = 3$, and $B = 2\pi$, and $C = -4\pi$. Thus, the amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The phase shift is $\frac{C}{B} = \frac{-4\pi}{2\pi} = -2$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = -2$. Add quarter-periods to generate x-values for the key points.

$x = -2$

$x = -2 + \frac{1}{4} = -\frac{7}{4}$

$x = -\frac{7}{4} + \frac{1}{4} = -\frac{3}{2}$

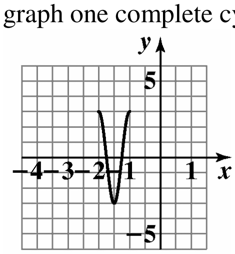
$x = -\frac{3}{2} + \frac{1}{4} = -\frac{5}{4}$

$x = -\frac{5}{4} + \frac{1}{4} = -1$

Evaluate the function at each value of x.

x	coordinates
-2	$(-2, 3)$
$-\frac{7}{4}$	$-\frac{7}{4}, 0$
$-\frac{3}{2}$	$-\frac{3}{2}, -3$
$-\frac{5}{4}$	$-\frac{5\pi}{4}, 0$
-1	$(-1, 3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 3\cos(2\pi x + 4\pi)$

53. The graph of $y = \sin x + 2$ is the graph of $y = \sin x$ shifted up 2 units upward. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

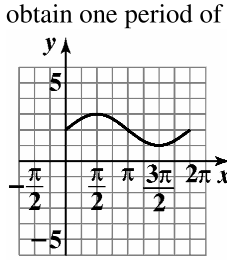
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \sin x + 2$	coordinates
0	$y = \sin 0 + 2$ $= 0 + 2 = 2$	$(0, 2)$
$\frac{\pi}{2}$	$y = \sin\frac{\pi}{2} + 2$ $= 1 + 2 = 3$	$\frac{\pi}{2}, 3$
π	$y = \sin\pi + 2$ $= 0 + 2 = 2$	$(\pi, 2)$
$\frac{3\pi}{2}$	$y = \sin\frac{3\pi}{2} + 2$ $= -1 + 2 = 1$	$\frac{3\pi}{2}, 1$
2π	$y = \sin 2\pi + 2$ $= 0 + 2 = 2$	$(2\pi, 2)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = \sin x + 2$

54. The graph of $y = \sin x - 2$ is the graph of $y = \sin x$ shifted 2 units downward. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

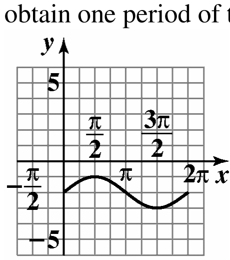
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \sin x - 2$	coordinates
0	$y = \sin 0 - 2 = 0 - 2 = -2$	$(0, -2)$
$\frac{\pi}{2}$	$y = \sin\frac{\pi}{2} - 2 = 1 - 2 = -1$	$\frac{\pi}{4}, -1$
π	$y = \sin \pi - 2 = 0 - 2 = -2$	$(\pi, -2)$
$\frac{3\pi}{2}$	$y = \sin\frac{3\pi}{2} - 2 = -1 - 2 = -3$	$\frac{3\pi}{2}, -3$
2π	$y = \sin 2\pi - 2 = 0 - 2 = -2$	$(2\pi, -2)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = \sin x - 2$

55. The graph of $y = \cos x - 3$ is the graph of $y = \cos x$ shifted 3 units downward. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

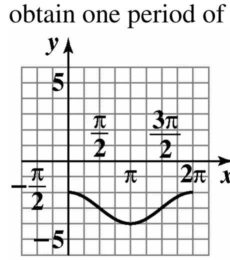
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \cos x - 3$	coordinates
0	$y = \cos 0 - 3$ $= 1 - 3 = -2$	$(0, -2)$
$\frac{\pi}{2}$	$y = \cos\frac{\pi}{2} - 3$ $= 0 - 3 = -3$	$\frac{\pi}{2}, -3$
π	$y = \cos \pi - 3$ $= -1 - 3 = -4$	$(\pi, -4)$
$\frac{3\pi}{2}$	$y = \cos\frac{3\pi}{2} - 3$ $= 0 - 3 = -3$	$\frac{3\pi}{2}, -3$
2π	$y = \cos 2\pi - 3$ $= 1 - 3 = -2$	$(2\pi, -2)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = \cos x - 3$

56. The graph of $y = \cos x + 3$ is the graph of $y = \cos x$ shifted 3 units upward. The period for both functions is 2π. The quarter-period is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

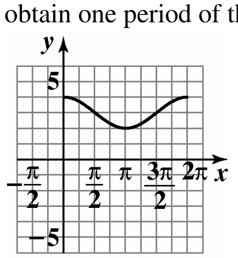
$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

Evaluate the function at each value of x.

x	$y = \cos x + 3$	coordinates
0	$y = \cos 0 + 3 = 1 + 3 = 4$	(0, 4)
$\frac{\pi}{2}$	$y = \cos\frac{\pi}{2} + 3 = 0 + 3 = 3$	$\left(\frac{\pi}{2}, 3\right)$
π	$y = \cos\pi + 3 = -1 + 3 = 2$	$(\pi, 2)$
$\frac{3\pi}{2}$	$y = \cos\frac{3\pi}{2} + 3 = 0 + 3 = 3$	$\left(\frac{3\pi}{2}, 3\right)$
2π	$y = \cos 2\pi + 3 = 1 + 3 = 4$	$(2\pi, 4)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = \cos x + 3$

57. The graph of $y = 2\sin\frac{1}{2}x + 1$ is the graph of $y = 2\sin\frac{1}{2}x$ shifted one unit upward. The amplitude for both functions is $|2| = 2$. The period for both functions is $\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$. The quarter-period is $\frac{4\pi}{4} = \pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \pi = \pi$

$x = \pi + \pi = 2\pi$

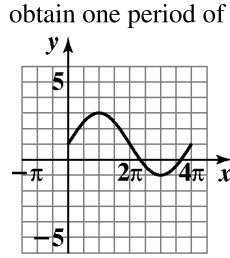
$x = 2\pi + \pi = 3\pi$

$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = 2\sin\frac{1}{2}x + 1$	coordinates
0	$y = 2\sin\left(\frac{1}{2}\cdot 0\right) + 1$ $= 2\sin 0 + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	(0, 1)
π	$y = 2\sin\left(\frac{1}{2}\cdot\pi\right) + 1$ $= 2\sin\frac{\pi}{2} + 1$ $= 2\cdot 1 + 1 = 2 + 1 = 3$	$(\pi, 3)$
2π	$y = 2\sin\left(\frac{1}{2}\cdot 2\pi\right) + 1$ $= 2\sin\pi + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	$(2\pi, 1)$
3π	$y = 2\sin\left(\frac{1}{2}\cdot 3\pi\right) + 1$ $= 2\sin\frac{3\pi}{2} + 1$ $= 2\cdot(-1) + 1$ $= -2 + 1 = -1$	$(3\pi, -1)$
4π	$y = 2\sin\left(\frac{1}{2}\cdot 4\pi\right) + 1$ $= 2\sin 2\pi + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	$(4\pi, 1)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = 2\sin\frac{1}{2}x + 1$

58. The graph of $y = 2\cos\frac{1}{2}x + 1$ is the graph of $y = 2\cos\frac{1}{2}x$ shifted one unit upward. The amplitude for both functions is $|2| = 2$. The period for both functions is $\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$. The quarter-period is $\frac{4\pi}{4} = \pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \pi = \pi$

$x = \pi + \pi = 2\pi$

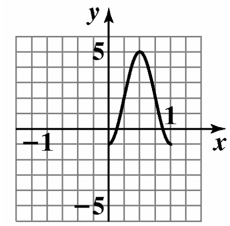
$x = 2\pi + \pi = 3\pi$

$x = 3\pi + \pi = 4\pi$

Evaluate the function at each value of x.

x	$y = 2\cos\frac{1}{2}x + 1$	coordinates
0	$y = 2\cos\ \frac{1}{2}\cdot 0\ \ +1$ $= 2\cos 0 + 1$ $= 2\cdot 1 + 1 = 2 + 1 = 3$	(0, 3)
π	$y = 2\cos\ \frac{1}{2}\cdot \pi\ \ +1$ $= 2\cos\frac{\pi}{2} + 1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	$(\pi, 1)$
2π	$y = 2\cos\ \frac{1}{2}\cdot 2\pi\ \ +1$ $= 2\cos\pi + 1$ $= 2\cdot(-1) + 1 = -2 + 1 = -1$	$(2\pi, -1)$
3π	$y = 2\cos\ \frac{1}{2}\cdot 3\pi\ \ +1$ $= 2\cdot 0 + 1 = 0 + 1 = 1$	$(3\pi, 1)$
4π	$y = 2\cos\ \frac{1}{2}\cdot 4\pi\ \ +1$ $= 2\cos 2\pi + 1$ $= 2\cdot 1 + 1 = 2 + 1 = 3$	$(4\pi, 3)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = -3\cos 2\pi x + 2$

59. The graph of $y = -3\cos 2\pi x + 2$ is the graph of $y = -3\cos 2\pi x$ shifted 2 units upward. The amplitude for both functions is $|-3| = 3$. The period for both functions is $\frac{2\pi}{2\pi} = 1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{4} = \frac{1}{4}$

$x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$x = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

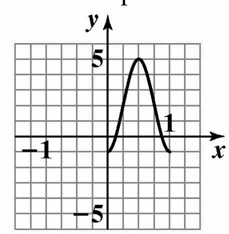
$x = \frac{3}{4} + \frac{1}{4} = 1$

Evaluate the function at each value of x.

x	$y = -3\cos 2\pi x + 2$	coordinates
0	$y = -3\cos(2\pi\cdot 0) + 2$ $= -3\cos 0 + 2$ $= -3\cdot 1 + 2$ $= -3 + 2 = -1$	(0, –1)
$\frac{1}{4}$	$y = -3\cos\ 2\pi\cdot\frac{1}{4}\ \ +2$ $= -3\cos\frac{\pi}{2} + 2$ $= -3\cdot 0 + 2$ $= 0 + 2 = 2$	$\frac{1}{4}$, 2
$\frac{1}{2}$	$y = -3\cos\ 2\pi\cdot\frac{1}{2}\ \ +2$ $= -3\cos\pi + 2$ $= -3\cdot(-1) + 2$ $= 3 + 2 = 5$	$\frac{1}{2}$, 5

$\frac{3}{4}$	$y=-3\cos\ 2\pi\cdot\frac{3}{4}\ \ +2$ $=-3\cos\frac{3\pi}{2}+2$ $=-3\cdot 0+2$ $=0+2=2$	$\frac{3}{4}, 2$
1	$y=-3\cos(2\pi\cdot 1)+2$ $=-3\cos 2\pi+2$ $=-3\cdot 1+2$ $=-3+2=-1$	$(1,-1)$

By connecting the points with a smooth curve we obtain one period of the graph.



***y* = −3 cos 2π*x* + 2**

60. The graph of $y=-3\sin 2\pi x+2$ is the graph of $y=-3\sin 2\pi x$ shifted two units upward. The amplitude for both functions is $|A|=|-3|=3$. The period for both functions is $\frac{2\pi}{2\pi}=1$. The quarter-period is $\frac{1}{4}$. The cycle begins at $x = 0$. Add quarter–periods to generate *x*-values for the key points.

$x=0$

$x=0+\frac{1}{4}=\frac{1}{4}$

$x=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

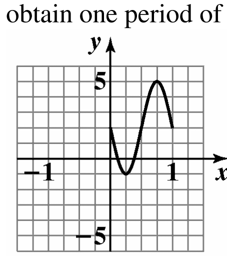
$x=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$

$x=\frac{3}{4}+\frac{1}{4}=1$

Evaluate the function at each value of *x*.

x	$y=-3\sin 2\pi x+2$	coordinates
0	$y=-3\sin(2\pi\cdot 0)+2$ $=-3\sin 0+2$ $=-3\cdot 0+2=0+2=2$	(0, 2)
$\frac{1}{4}$	$y=-3\sin\ 2\pi\cdot\frac{1}{4}\ \ +2$ $=-3\sin\frac{\pi}{2}+2$ $=-3\cdot 1+2=-3+2=-1$	$\frac{1}{4}, -1$
$\frac{1}{2}$	$y=-3\sin\ 2\pi\cdot\frac{1}{2}\ \ +2$ $=-3\sin\pi+2$ $=-3\cdot 0+2=0+2=2$	$\frac{1}{2}, 2$
$\frac{3}{4}$	$y=-3\sin\ 2\pi\cdot\frac{3}{4}\ \ +2$ $=-3\sin\frac{3\pi}{2}+2$ $=-3\cdot(-1)+2=3+2=5$	$\frac{3}{4}, 5$
1	$y=-3\sin(2\pi\cdot 1)+2$ $=-3\sin 2\pi+2$ $=-3\cdot 0+2=0+2=2$	(1, 2)

By connecting the points with a smooth curve we obtain one period of the graph.



***y* = −3 sin 2π*x* + 2**

61. Using $y=A\cos Bx$ the amplitude is 3 and $A=3$, The period is 4π and thus

$B=\frac{2\pi}{\text{period}}=\frac{2\pi}{4\pi}=\frac{1}{2}$

$y=A\cos Bx$

$y=3\cos\frac{1}{2}x$

62. Using $y = A\sin Bx$ the amplitude is 3 and $A = 3$, The period is 4π and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{4\pi} = \frac{1}{2}$$

$$y = A\sin Bx$$

$$y = 3\sin\frac{1}{2}x$$

63. Using $y = A\sin Bx$ the amplitude is 2 and $A = -2$, The period is π and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi} = 2$$

$$y = A\sin Bx$$

$$y = -2\sin 2x$$

64. Using $y = A\cos Bx$ the amplitude is 2 and $A = -2$, The period is 4π and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi} = 2$$

$$y = A\cos Bx$$

$$y = -2\cos 2x$$

65. Using $y = A\sin Bx$ the amplitude is 2 and $A = 2$, The period is 4 and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$y = A\sin Bx$$

$$y = 2\sin\left(\frac{\pi}{2}x\right)$$

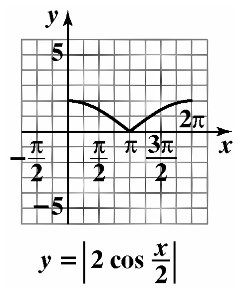
66. Using $y = A\cos Bx$ the amplitude is 2 and $A = 2$, The period is 4 and thus

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{4} = \frac{\pi}{2}$$

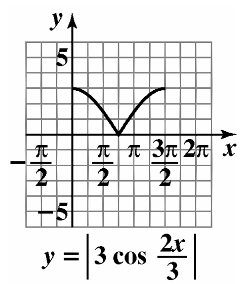
$$y = A\cos Bx$$

$$y = 2\cos\left(\frac{\pi}{2}x\right)$$

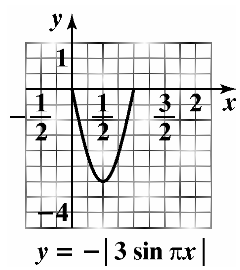
67.



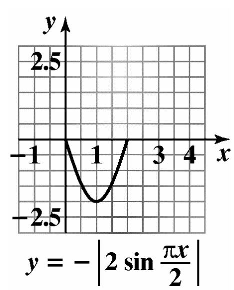
68.



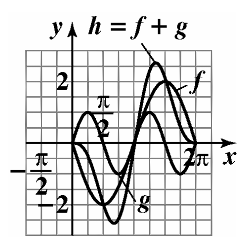
69.



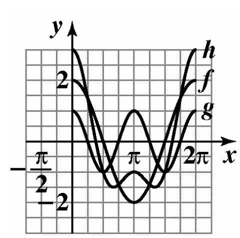
70.



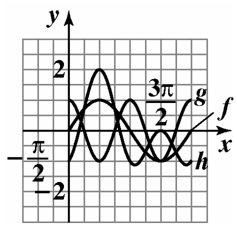
71.



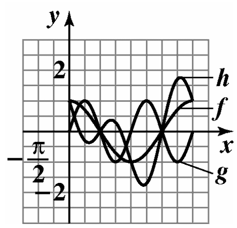
72.



73.



74.



75. The period of the physical cycle is 33 days.

76. The period of the emotional cycle is 28 days.

77. The period of the intellectual cycle is 23 days.

78. In the month of February, the physical cycle is at a minimum on February 18. Thus, the author should not run in a marathon on February 18.

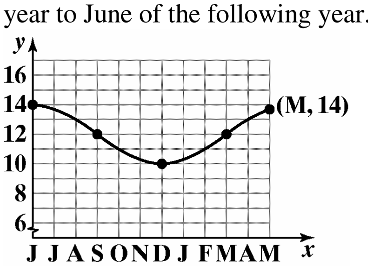
79. In the month of March, March 21 would be the best day to meet an on-line friend for the first time, because the emotional cycle is at a maximum.

80. In the month of February, the intellectual cycle is at a maximum on February 11. Thus, the author should begin writing the on February 11.

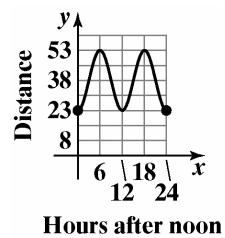
81. Answers may vary.

82. Answers may vary.

83. The information gives the five key point of the graph.
(0, 14) corresponds to June,
(3, 12) corresponds to September,
(6, 10) corresponds to December,
(9, 12) corresponds to March,
(12, 14) corresponds to June
By connecting the five key points with a smooth curve we graph the information from June of one year to June of the following year.



84. The information gives the five key points of the graph.
(0, 23) corresponds to Noon,
(3, 38) corresponds to 3 P.M.,
(6, 53) corresponds to 6 P.M.,
(9, 38) corresponds to 9 P.M.,
(12, 23) corresponds to Midnight.
By connecting the five key points with a smooth curve we graph information from noon to midnight. Extend the graph one cycle to the right to graph the information for $0 \le x \le 24$.



85. The function $y = 3\sin\frac{2\pi}{365}(x-79)+12$ is of the form

$$y = A\sin B\left(x-\frac{C}{B}\right)+D \text{ with}$$

$$A = 3 \text{ and } B = \frac{2\pi}{365}.$$

a. The amplitude is $|\,A\,| = |\,3\,| = 3$.

b. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{2\pi}{365}} = 2\pi \cdot \frac{365}{2\pi} = 365$.

c. The longest day of the year will have the most hours of daylight. This occurs when the sine function equals 1.

$$y = 3\sin\frac{2\pi}{365}(x-79)+12$$
$$y = 3(1)+12$$
$$y = 15$$

There will be 15 hours of daylight.

d. The shortest day of the year will have the least hours of daylight. This occurs when the sine function equals –1.

$$y = 3\sin\frac{2\pi}{365}(x-79)+12$$
$$y = 3(-1)+12$$
$$y = 9$$

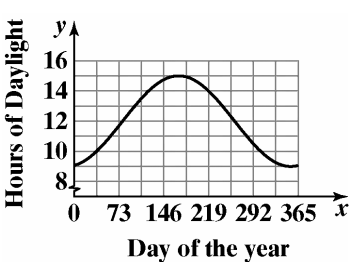
There will be 9 hours of daylight.

e. The amplitude is 3. The period is 365. The phase shift is $\frac{C}{B} = 79$. The quarter-period is $\frac{365}{4} = 91.25$. The cycle begins at x = 79. Add quarter-periods to find the x-values of the key points.

$$x = 79$$
$$x = 79 + 91.25 = 170.25$$
$$x = 170.25 + 91.25 = 261.5$$
$$x = 261.5 + 91.25 = 352.75$$
$$x = 352.75 + 91.25 = 444$$

Because we are graphing for $0 \le x \le 365$, we will evaluate the function for the first four x-values along with x = 0 and x = 365. Using a calculator we have the following points.
(0, 9.07) (79, 12) (170.25, 15)
(261.5, 12) (352.75, 9) (365, 9.07)

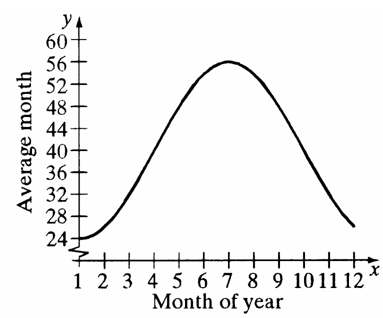
By connecting the points with a smooth curve we obtain one period of the graph, starting on January 1.



86. The function $y = 16\sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right)+40$ is in the form $y = A\sin(Bx - C) + D$ with A = 16, $B = \frac{\pi}{6}$, and $C = \frac{2\pi}{3}$. The amplitude is $|A| = |16| = 16$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{6}} = 2\pi \cdot \frac{6}{\pi} = 12$. The phase shift is $\frac{C}{B} = \frac{\frac{2\pi}{3}}{\frac{\pi}{6}} = \frac{2\pi}{3}\cdot\frac{6}{\pi} = 4$. The quarter-period is $\frac{12}{4} = 3$. The cycle begins at x = 4. Add quarter-periods to find the x-values for the key points.

$$x = 4$$
$$x = 4 + 3 = 7$$
$$x = 7 + 3 = 10$$
$$x = 10 + 3 = 13$$
$$x = 13 + 3 = 16$$

Because we are graphing for $1 \le x \le 12$, we will evaluate the function for the three x-values between 1 and 12, along with x = 1 and x = 12. Using a calculator we have the following points.
(1, 24) (4, 40) (7, 56) (10, 40) (12, 26.1)
By connecting the points with a smooth curve we obtain the graph for $1 \le x \le 12$.



The highest average monthly temperature is 56° in July.

87. Because the depth of the water ranges from a minimum of 6 feet to a maximum of 12 feet, the curve oscillates about the middle value, 9 feet. Thus, $D = 9$. The maximum depth of the water is 3 feet above 9 feet. Thus, $A = 3$. The graph shows that one complete cycle occurs in 12-0, or 12 hours. The period is 12.
Thus,

$$12 = \frac{2\pi}{B}$$
$$12B = 2\pi$$
$$B = \frac{2\pi}{12} = \frac{\pi}{6}$$

Substitute these values into $y = A\cos Bx + D$. The depth of the water is modeled by $y = 3\cos\frac{\pi x}{6} + 9$.

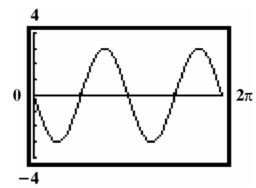
88. Because the depth of the water ranges from a minimum of 3 feet to a maximum of 5 feet, the curve oscillates about the middle value, 4 feet. Thus, $D = 4$. The maximum depth of the water is 1 foot above 4 feet. Thus, $A = 1$. The graph shows that one complete cycle occurs in 12–0, or 12 hours. The period is 12.
Thus,

$$12 = \frac{2\pi}{B}$$
$$12B = 2\pi$$
$$B = \frac{2\pi}{12} = \frac{\pi}{6}$$

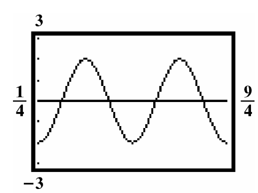
Substitute these values into $y = A\cos Bx + D$. The depth of the water is modeled by $y = \cos\frac{\pi x}{6} + 4$.

89. – 100. Answers may vary.

101. The function $y = 3\sin(2x + \pi) = 3\sin(2x - (-\pi))$ is of the form $y = A\sin(Bx - C)$ with $A = 3$, $B = 2$, and $C = -\pi$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The cycle begins at $x = \frac{C}{B} = \frac{-\pi}{2} = -\frac{\pi}{2}$. We choose $-\frac{\pi}{2} \le x \le \frac{3\pi}{2}$, and $-4 \le y \le 4$ for our graph.

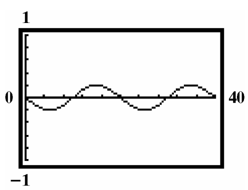


102. The function $y = -2\cos\left(2\pi x - \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx - C)$ with $A = -2$, $B = 2\pi$, and $C = \frac{\pi}{2}$. The amplitude is $|A| = |-2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$. The cycle begins at $x = \frac{C}{B} = \frac{\frac{\pi}{2}}{2\pi} = \frac{\pi}{2}\cdot\frac{1}{2\pi} = \frac{1}{4}$. We choose $\frac{1}{4} \le x \le \frac{9}{4}$, and $-3 \le y \le 3$ for our graph.

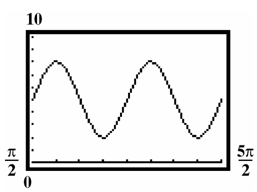


103. The function

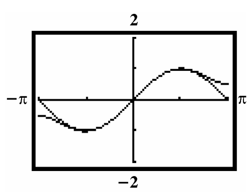
$y = 0.2\sin\left(\frac{\pi}{10}x + \pi\right) = 0.2\sin\left(\frac{\pi}{10}x - (-\pi)\right)$ is of the form $y = A\sin(Bx - C)$ with $A = 0.2$, $B = \frac{\pi}{10}$, and $C = -\pi$. The amplitude is $|A| = |0.2| = 0.2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{10}} = 2\pi \cdot \frac{10}{\pi} = 20$. The cycle begins at $x = \frac{C}{B} = \frac{-\pi}{\frac{\pi}{10}} = -\pi \cdot \frac{10}{\pi} = -10$. We choose $-10 \le x \le 30$, and $-1 \le y \le 1$ for our graph.



104. The function $y = 3\sin(2x - \pi) + 5$ is of the form $y = A\cos(Bx - C) + D$ with $A = 3$, $B = 2$, $C = \pi$, and $D = 5$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The cycle begins at $x = \frac{C}{B} = \frac{\pi}{2}$. Because $D = 5$, the graph has a vertical shift 5 units upward. We choose $\frac{\pi}{2} \le x \le \frac{5\pi}{2}$, and $0 \le y \le 10$ for our graph.

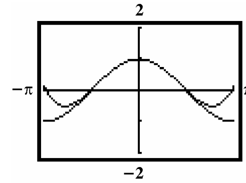


105.



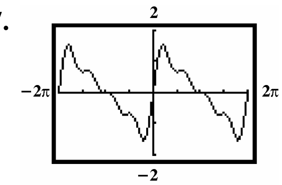
The graphs appear to be the same from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

106.



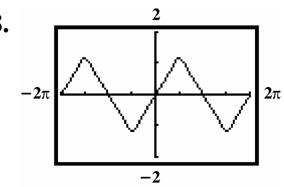
The graphs appear to be the same from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

107.



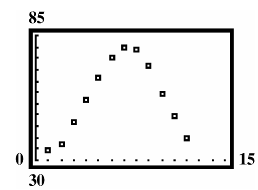
The graph is similar to $y = \sin x$, except the amplitude is greater and the curve is less smooth.

108.

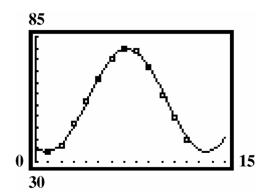


The graph is very similar to $y = \sin x$, except not smooth.

109. a.



b. $y = 22.61\sin(0.50x - 2.04) + 57.17$



110. Answers may vary.

111. makes sense

112. does not make sense; Explanations will vary. Sample explanation: It may be easier to start at the highest point.

113. makes sense

114. makes sense

115. a. Since $A=3$ and $D=-2$, the maximum will occur at $3-2=1$ and the minimum will occur at $-3-2=-5$. Thus the range is $[-5,1]$

Viewing rectangle: $\left[-\frac{\pi}{6},\frac{23\pi}{6},\frac{\pi}{6}\right]$ by $[-5,1,1]$

b. Since $A=1$ and $D=-2$, the maximum will occur at $1-2=-1$ and the minimum will occur at $-1-2=-3$. Thus the range is $[-3,-1]$

Viewing rectangle: $\left[-\frac{\pi}{6},\frac{7\pi}{6},\frac{\pi}{6}\right]$ by $[-3,-1,1]$

116. $A=\pi$

$$B=\frac{2\pi}{\text{period}}=\frac{2\pi}{1}=2\pi$$

$$\frac{C}{B}=\frac{C}{2\pi}=-2$$

$C=-4\pi$

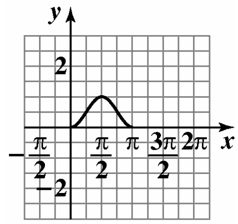
$y=A\cos(Bx-C)$

$y=\pi\cos(2\pi x+4\pi)$

or

$y=\pi\cos\left[2\pi(x+2)\right]$

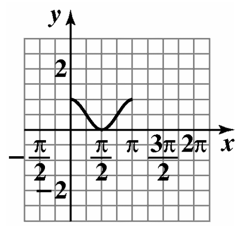
117. $y=\sin^2 x=\frac{1}{2}-\frac{1}{2}\cos 2x$



$y=\sin^2 x$

or $y=\frac{1}{2}-\frac{1}{2}\cos 2x$

118. $y=\cos^2 x=\frac{1}{2}+\frac{1}{2}\cos 2x$



$y=\cos^2 x$

or $y=\frac{1}{2}+\frac{1}{2}\cos 2x$

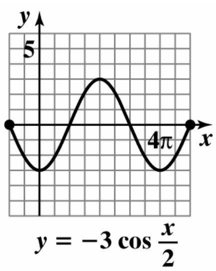
119. Answers may vary.

120. $-\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2}$

$$-\frac{\pi}{2} - \frac{\pi}{4} < x + \frac{\pi}{4} - \frac{\pi}{4} < \frac{\pi}{2} - \frac{\pi}{4}$$
$$-\frac{2\pi}{4} - \frac{\pi}{4} < x < \frac{2\pi}{4} - \frac{\pi}{4}$$
$$-\frac{3\pi}{4} < x < \frac{\pi}{4}$$
$$\left\{x \middle| -\frac{3\pi}{4} < x < \frac{\pi}{4}\right\} \text{ or } \left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$$

121. $\dfrac{-\frac{3\pi}{4} + \frac{\pi}{4}}{2} = \dfrac{-\frac{2\pi}{4}}{2} = \dfrac{-\frac{\pi}{2}}{2} = -\frac{\pi}{4}$

122. a.



b. The reciprocal function is undefined.

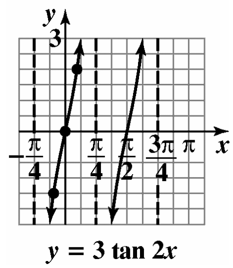
Section 5.6

Check Point Exercises

1. Solve the equations $2x = -\frac{\pi}{2}$ and $2x = \frac{\pi}{2}$

$x = -\frac{\pi}{4}$ $\qquad$ $x = \frac{\pi}{4}$

Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$. Midway between these asymptotes is x = 0. An *x*-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 3, the points on the graph midway between an *x*-intercept and the asymptotes have *y*-coordinates of –3 and 3. Use the two asymptotes, the *x*-intercept, and the points midway between to graph one period of $y = 3\tan 2x$ from $-\frac{\pi}{4}$ to $\frac{\pi}{4}$. In order to graph for $-\frac{\pi}{4} < x < \frac{3\pi}{4}$, Continue the pattern and extend the graph another full period to the right.



2. Solve the equations

$x - \frac{\pi}{2} = -\frac{\pi}{2}$ and $x - \frac{\pi}{2} = \frac{\pi}{2}$

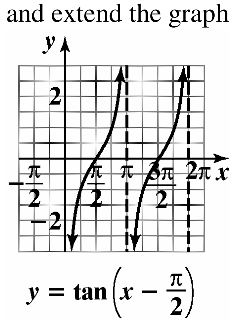
$x = \frac{\pi}{2} - \frac{\pi}{2}$ $\qquad$ $x = \frac{\pi}{2} + \frac{\pi}{2}$

$x = 0$ $\qquad$ $x = \pi$

Thus, two consecutive asymptotes occur at $x = 0$ and $x = \pi$.

x-intercept $= \frac{0 + \pi}{2} = \frac{\pi}{2}$

An *x*-intercept is $\frac{\pi}{2}$ and the graph passes through $\left(\frac{\pi}{2}, 0\right)$. Because the coefficient of the tangent is 1, the points on the graph midway between an *x*-intercept and the asymptotes have *y*-coordinates of –1 and 1. Use the two consecutive asymptotes, $x = 0$ and $x = \pi$, to graph one full period of $y = \tan\left(x - \frac{\pi}{2}\right)$ from 0 to π. Continue the pattern and extend the graph another full period to the right.



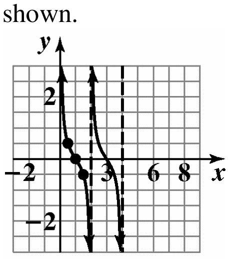
3. Solve the equations

$\frac{\pi}{2}x = 0$ and $\frac{\pi}{2}x = \pi$

$x = 0$ $\quad$ $x = \frac{\pi}{\frac{\pi}{2}}$

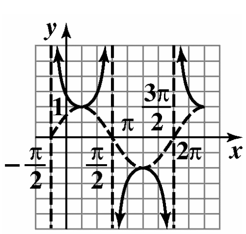
$x = 2$

Two consecutive asymptotes occur at $x = 0$ and $x = 2$. Midway between $x = 0$ and $x = 2$ is $x = 1$. An x-intercept is 1 and the graph passes through (1, 0). Because the coefficient of the cotangent is $\frac{1}{2}$, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of $-\frac{1}{2}$ and $\frac{1}{2}$. Use the two consecutive asymptotes, $x = 0$ and $x = 2$, to graph one full period of $y = \frac{1}{2}\cot\frac{\pi}{2}x$. The curve is repeated along the x-axis one full period as shown.



$y = \frac{1}{2}\cot\frac{\pi}{2}x$

4. The x-intercepts of $y = \sin\left(x + \frac{\pi}{4}\right)$ correspond to vertical asymptotes of $y = \csc\left(x + \frac{\pi}{4}\right)$.



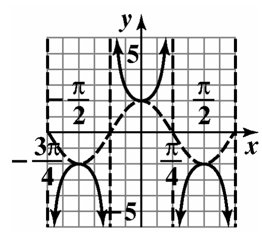
$y = \csc\left(x + \frac{\pi}{4}\right)$

5. Graph the reciprocal cosine function, $y = 2\cos 2x$. The equation is of the form $y = A\cos Bx$ with $A = 2$ and $B = 2$.

amplitude: $|A| = |2| = 2$

period: $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$

Use quarter-periods, $\frac{\pi}{4}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$, and π. Evaluating the function at each value of x, the key points are $(0, 2)$, $\left(\frac{\pi}{4}, 0\right)$, $\left(\frac{\pi}{2}, -2\right)$, $\left(\frac{3\pi}{4}, 0\right)$, $(\pi, 2)$. In order to graph for $-\frac{3\pi}{4} \le x \le \frac{3\pi}{4}$, Use the first four points and extend the graph $-\frac{3\pi}{4}$ units to the left. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 2\sec 2x$.



$y = 2\sec 2x$

Exercise Set 5.6

1. The graph has an asymptote at $x = -\frac{\pi}{2}$.

The phase shift, $\frac{C}{B}$, from $\frac{\pi}{2}$ to $-\frac{\pi}{2}$ is $-\pi$ units.

Thus, $\frac{C}{B} = \frac{C}{1} = -\pi$

$C = -\pi$

The function with $C = -\pi$ is $y = \tan(x + \pi)$.

2. The graph has an asymptote at $x = 0$.

The phase shift, $\frac{C}{B}$, from $\frac{\pi}{2}$ to 0 is $-\frac{\pi}{2}$ units. Thus,

$\frac{C}{B} = \frac{C}{1} = -\frac{\pi}{2}$

$C = -\frac{\pi}{2}$

The function with $C = -\frac{\pi}{2}$ is $y = \tan\left(x + \frac{\pi}{2}\right)$.

3. The graph has an asymptote at $x = \pi$.

$\pi = \frac{\pi}{2} + C$

$C = \frac{\pi}{2}$

The function is $y = -\tan\left(x - \frac{\pi}{2}\right)$.

4. The graph has an asymptote at $\frac{\pi}{2}$.

There is no phase shift. Thus, $\frac{C}{B} = \frac{C}{1} = 0$

$C = 0$

The function with $C = 0$ is $y = -\tan x$.

5. Solve the equations

$\frac{x}{4} = -\frac{\pi}{2}$ and $\frac{x}{4} = \frac{\pi}{2}$

$x = \left(-\frac{\pi}{2}\right)4$ $\quad x = \left(\frac{\pi}{2}\right)4$

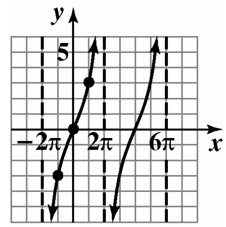
$x = -2\pi$ $\quad x = 2\pi$

Thus, two consecutive asymptotes occur at $x = -2\pi$ and $x = 2\pi$.

x-intercept $= \frac{-2\pi + 2\pi}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 3, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –3 and 3. Use the two consecutive asymptotes, $x = -2\pi$ and $x = 2\pi$, to graph one full period of $y = 3\tan\frac{x}{4}$ from -2π to 2π.

Continue the pattern and extend the graph another full period to the right.



$y = 3 \tan \frac{x}{4}$

6. Solve the equations

$\frac{x}{4} = -\frac{\pi}{2}$ and $\frac{x}{4} = \frac{\pi}{2}$

$x = \left(-\frac{\pi}{2}\right)4$ $\quad x = \left(\frac{\pi}{2}\right)4$

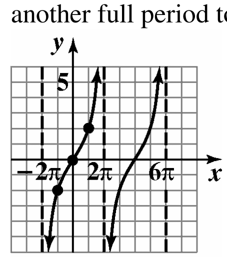
$x = -2\pi$ $\quad x = 2\pi$

Thus, two consecutive asymptotes occur at $x = -2\pi$ and $x = 2\pi$.

x-intercept $= \frac{-2\pi + 2\pi}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –2 and 2. Use the two consecutive asymptotes, $x = -2\pi$ and $x = 2\pi$,

to graph one full period of $y = 2\tan\frac{x}{4}$ from -2π to 2π. Continue the pattern and extend the graph another full period to the right.



$y = 2 \tan \frac{x}{4}$

7. Solve the equations $2x = -\frac{\pi}{2}$ and $2x = \frac{\pi}{2}$

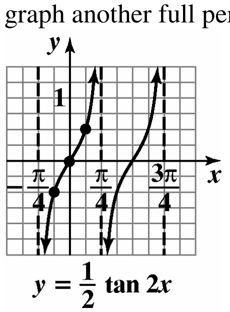
$$x = \frac{-\frac{\pi}{2}}{2} \qquad x = \frac{\frac{\pi}{2}}{2}$$

$$x = -\frac{\pi}{4} \qquad x = \frac{\pi}{4}$$

Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$.

x-intercept = $\frac{-\frac{\pi}{4}+\frac{\pi}{4}}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is $\frac{1}{2}$, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of $-\frac{1}{2}$ and $\frac{1}{2}$. Use the two consecutive asymptotes, $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$, to graph one full period of $y = \frac{1}{2}\tan 2x$ from $-\frac{\pi}{4}$ to $\frac{\pi}{4}$. Continue the pattern and extend the graph another full period to the right.



8. Solve the equations

$$2x = -\frac{\pi}{2} \quad \text{and} \quad 2x = \frac{\pi}{2}$$

$$x = \frac{-\frac{\pi}{2}}{2} \qquad x = \frac{\frac{\pi}{2}}{2}$$

$$x = -\frac{\pi}{4} \qquad x = \frac{\pi}{4}$$

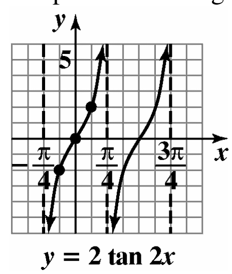
Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$.

x-intercept = $\frac{-\frac{\pi}{4}+\frac{\pi}{4}}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –2 and 2.

Use the two consecutive asymptotes, $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$, to graph one full period of $y = 2\tan 2x$ from $-\frac{\pi}{4}$ to $\frac{\pi}{4}$.

Continue the pattern and extend the graph another full period to the right.



9. Solve the equations

$$\frac{1}{2}x = -\frac{\pi}{2} \quad \text{and} \quad \frac{1}{2}x = \frac{\pi}{2}$$

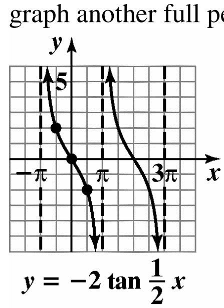
$$x = \left(-\frac{\pi}{2}\right)2 \qquad x = \left(\frac{\pi}{2}\right)2$$

$$x = -\pi \qquad x = \pi$$

Thus, two consecutive asymptotes occur at $x = -\pi$ and $x = \pi$.

x-intercept = $\frac{-\pi+\pi}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is –2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 2 and –2. Use the two consecutive asymptotes, $x = -\pi$ and $x = \pi$, to graph one full period of $y = -2\tan\frac{1}{2}x$ from $-\pi$ to π. Continue the pattern and extend the graph another full period to the right.



10. Solve the equations

$$\frac{1}{2}x = -\frac{\pi}{2} \qquad \text{and} \qquad \frac{1}{2}x = \frac{\pi}{2}$$

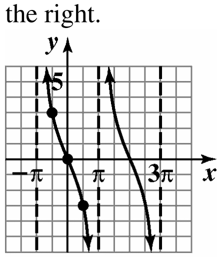
$$x = \left(-\frac{\pi}{2}\right)2 \qquad x = \left(\frac{\pi}{2}\right)2$$

$$x = -\pi \qquad x = \pi$$

Thus, two consecutive asymptotes occur at $x = -\pi$ and $x = \pi$.

$$x\text{-intercept} = \frac{-\pi + \pi}{2} = \frac{0}{2} = 0$$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is –3, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 3 and –3. Use the two consecutive asymptotes, $x = -\pi$ and $x = \pi$, to graph one full period of $y = -3\tan\frac{1}{2}x$ from $-\pi$ to π. Continue the pattern and extend the graph another full period to the right.



$y = -3\tan\frac{1}{2}x$

11. Solve the equations

$$x - \pi = -\frac{\pi}{2} \qquad \text{and} \qquad x - \pi = \frac{\pi}{2}$$

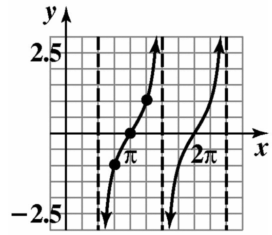
$$x = -\frac{\pi}{2} + \pi \qquad x = \frac{\pi}{2} + \pi$$

$$x = \frac{\pi}{2} \qquad x = \frac{3\pi}{2}$$

Thus, two consecutive asymptotes occur at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

$$x\text{-intercept} = \frac{\frac{\pi}{2} + \frac{3\pi}{2}}{2} = \frac{\frac{4\pi}{2}}{2} = \frac{4\pi}{4} = \pi$$

An x-intercept is π and the graph passes through $(\pi, 0)$. Because the coefficient of the tangent is 1, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –1 and 1. Use the two consecutive asymptotes, $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$, to graph one full period of $y = \tan(x - \pi)$ from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$. Continue the pattern and extend the graph another full period to the right.



$y = \tan(x - \pi)$

12. Solve the equations

$$x - \frac{\pi}{4} = -\frac{\pi}{2} \qquad \text{and} \qquad x - \frac{\pi}{4} = \frac{\pi}{2}$$

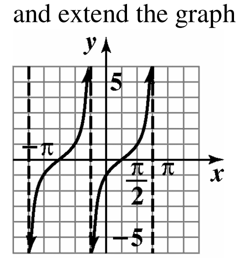
$$x = -\frac{2\pi}{4} + \frac{\pi}{4} \qquad x = \frac{2\pi}{4} + \frac{\pi}{4}$$

$$x = -\frac{\pi}{4} \qquad x = \frac{3\pi}{4}$$

Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{4}$ and $x = \frac{3\pi}{4}$.

$$x\text{-intercept} = \frac{-\frac{\pi}{4} + \frac{3\pi}{4}}{2} = \frac{\frac{2\pi}{4}}{2} = \frac{\pi}{4}$$

An x-intercept is $\frac{\pi}{4}$ and the graph passes through $\left(\frac{\pi}{4}, 0\right)$. Because the coefficient of the tangent is 1, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –1 and 1. Use the two consecutive asymptotes, $x = -\frac{\pi}{4}$ and $x = \frac{3\pi}{4}$, to graph one full period of $y = \tan\left(x - \frac{\pi}{4}\right)$ from 0 to π. Continue the pattern and extend the graph another full period to the right.



$y = \tan\left(x - \frac{\pi}{4}\right)$

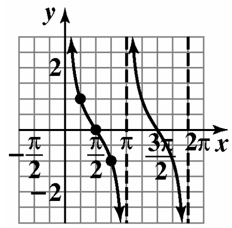
13. There is no phase shift. Thus,
$\frac{C}{B}=\frac{C}{1}=0$
$C=0$
Because the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of -1 and 1, $A=-1$. The function with $C=0$ and $A=-1$ is $y=-\cot x$.

14. The graph has an asymptote at $\frac{\pi}{2}$. The phase shift, $\frac{C}{B}$, from 0 to $\frac{\pi}{2}$ is $\frac{\pi}{2}$ units.
Thus, $\frac{C}{B}=\frac{C}{1}=\frac{\pi}{2}$
$C=\frac{\pi}{2}$
The function with $C=\frac{\pi}{2}$ is $y=-\cot\left(x-\frac{\pi}{2}\right)$.

15. The graph has an asymptote at $-\frac{\pi}{2}$. The phase shift, $\frac{C}{B}$, from 0 to $-\frac{\pi}{2}$ is $-\frac{\pi}{2}$ units. Thus, $\frac{C}{B}=\frac{C}{1}=-\frac{\pi}{2}$
$C=-\frac{\pi}{2}$
The function with $C=-\frac{\pi}{2}$ is $y=\cot\left(x+\frac{\pi}{2}\right)$.

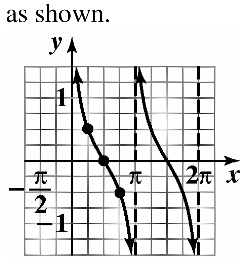
16. The graph has an asymptote at $-\pi$. The phase shift, $\frac{C}{B}$, from 0 to $-\pi$ is $-\pi$ units.
Thus, $\frac{C}{B}=\frac{C}{1}=-\pi$
$C=-\pi$
The function with $C=-\pi$ is $y=\cot(x+\pi)$.

17. Solve the equations $x=0$ and $x=\pi$. Two consecutive asymptotes occur at $x=0$ and $x=\pi$.
x-intercept $=\frac{0+\pi}{2}=\frac{\pi}{2}$
An x-intercept is $\frac{\pi}{2}$ and the graph passes through $\left(\frac{\pi}{2}, 0\right)$. Because the coefficient of the cotangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 2 and –2. Use the two consecutive asymptotes, $x=0$ and $x=\pi$, to graph one full period of $y=2\cot x$.
The curve is repeated along the x-axis one full period as shown.



$y = 2\cot x$

18. Solve the equations
$x=0$ and $x=\pi$
Two consecutive asymptotes occur at $x=0$ and $x=\pi$.
x-intercept = $\frac{0+\pi}{2}=\frac{\pi}{2}$
An x-intercept is $\frac{\pi}{2}$ and the graph passes through $\left(\frac{\pi}{2}, 0\right)$. Because the coefficient of the cotangent is $\frac{1}{2}$, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of $\frac{1}{2}$ and $-\frac{1}{2}$. Use the two consecutive asymptotes, $x=0$ and $x=\pi$, to graph one full period of $y=\frac{1}{2}\cot x$.
The curve is repeated along the x-axis one full period as shown.



$y = \frac{1}{2}\cot x$

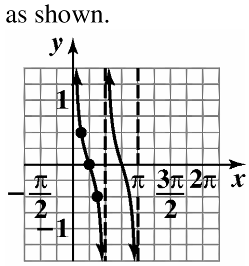
19. Solve the equations $2x = 0$ and $2x = \pi$

$x = 0$ $\quad x = \dfrac{\pi}{2}$

Two consecutive asymptotes occur at $x = 0$ and $x = \dfrac{\pi}{2}$.

$x\text{-intercept} = \dfrac{0 + \frac{\pi}{2}}{2} = \dfrac{\frac{\pi}{2}}{2} = \dfrac{\pi}{4}$

An x-intercept is $\dfrac{\pi}{4}$ and the graph passes through $\left(\dfrac{\pi}{4}, 0\right)$. Because the coefficient of the cotangent is $\dfrac{1}{2}$, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of $\dfrac{1}{2}$ and $-\dfrac{1}{2}$. Use the two consecutive asymptotes, $x = 0$ and $x = \dfrac{\pi}{2}$, to graph one full period of $y = \dfrac{1}{2}\cot 2x$. The curve is repeated along the x-axis one full period as shown.



$y = \dfrac{1}{2}\cot 2x$

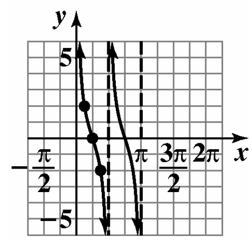
20. Solve the equations $2x = 0$ and $2x = \pi$

$x = 0$ $\quad x = \dfrac{\pi}{2}$

Two consecutive asymptotes occur at $x = 0$ and $x = \dfrac{\pi}{2}$.

$x\text{-intercept} = \dfrac{0 + \frac{\pi}{2}}{2} = \dfrac{\frac{\pi}{2}}{2} = \dfrac{\pi}{4}$

An x-intercept is $\dfrac{\pi}{4}$ and the graph passes through $\left(\dfrac{\pi}{4}, 0\right)$. Because the coefficient of the cotangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 2 and –2. Use the two consecutive asymptotes, $x = 0$ and $x = \dfrac{\pi}{2}$, to graph one full period of $y = 2\cot 2x$. The curve is repeated along the x-axis one full period as shown.



$y = 2\cot 2x$

21. Solve the equations $\dfrac{\pi}{2}x = 0$ and $\dfrac{\pi}{2}x = \pi$

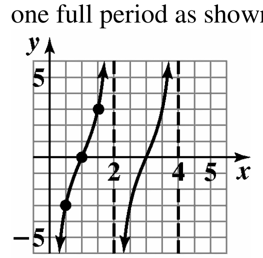
$x = 0$ $\quad x = \dfrac{\pi}{\frac{\pi}{2}}$

$x = 2$

Two consecutive asymptotes occur at $x = 0$ and $x = 2$.

$x\text{-intercept} = \dfrac{0+2}{2} = \dfrac{2}{2} = 1$

An x-intercept is 1 and the graph passes through (1, 0). Because the coefficient of the cotangent is –3, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –3 and 3. Use the two consecutive asymptotes, $x = 0$ and $x = 2$, to graph one full period of $y = -3\cot\dfrac{\pi}{2}x$. The curve is repeated along the x-axis one full period as shown.



$y = -3\cot\dfrac{\pi}{2}x$

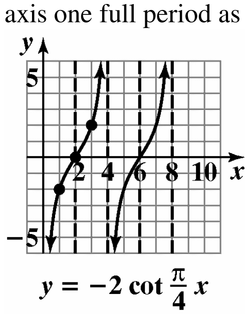
22. Solve the equations $\frac{\pi}{4}x = 0$ and $\frac{\pi}{4}x = \pi$

$$x = 0 \qquad x = \frac{\pi}{\frac{\pi}{4}}$$
$$x = 4$$

Two consecutive asymptotes occur at $x = 0$ and $x = 4$.

x-intercept $= \frac{0+4}{2} = \frac{4}{2} = 2$

An x-intercept is 2 and the graph passes through $(2, 0)$. Because the coefficient of the cotangent is –2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –2 and 2. Use the two consecutive asymptotes, $x = 0$ and $x = 4$, to graph one full period of $y = -2\cot\frac{\pi}{4}x$. The curve is repeated along the x-axis one full period as shown.



23. Solve the equations

$$x + \frac{\pi}{2} = 0 \qquad \text{and} \qquad x + \frac{\pi}{2} = \pi$$
$$x = 0 - \frac{\pi}{2} \qquad x = \pi - \frac{\pi}{2}$$
$$x = -\frac{\pi}{2} \qquad x = \frac{\pi}{2}$$

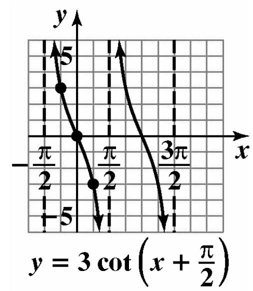
Two consecutive asymptotes occur at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$.

x-intercept $= \frac{-\frac{\pi}{2} + \frac{\pi}{2}}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the cotangent is 3, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 3 and –3.

Use the two consecutive asymptotes, $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$, to graph one full period of $y = 3\cot\left(x + \frac{\pi}{2}\right)$.

The curve is repeated along the x-axis one full period as shown.



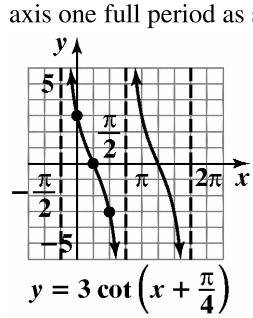
24. Solve the equations

$$x + \frac{\pi}{4} = 0 \qquad \text{and} \qquad x + \frac{\pi}{4} = \pi$$
$$x = 0 - \frac{\pi}{4} \qquad x = \pi - \frac{\pi}{4}$$
$$x = -\frac{\pi}{4} \qquad x = \frac{3\pi}{4}$$

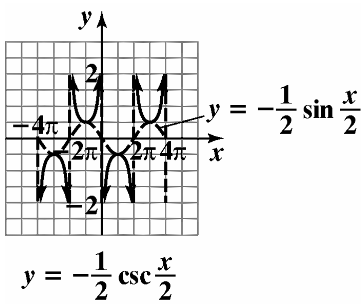
Two consecutive asymptotes occur at $x = -\frac{\pi}{4}$ and $x = \frac{3\pi}{4}$.

x-intercept $= \frac{-\frac{\pi}{4} + \frac{3\pi}{4}}{2} = \frac{\frac{2\pi}{4}}{2} = \frac{\pi}{4}$

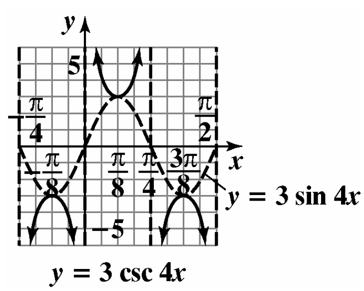
An x-intercept is $\frac{\pi}{4}$ and the graph passes through $\left(\frac{\pi}{4}, 0\right)$. Because the coefficient of the cotangent is 3, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 3 and –3. Use the two consecutive asymptotes, $x = -\frac{\pi}{4}$ and $x = \frac{3\pi}{4}$, to graph one full period of $y = 3\cot\left(x + \frac{\pi}{4}\right)$. The curve is repeated along the x-axis one full period as shown.



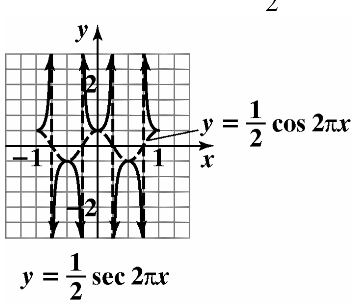
25. The x-intercepts of $y=-\frac{1}{2}\sin\frac{x}{2}$ corresponds to vertical asymptotes of $y=-\frac{1}{2}\csc\frac{x}{2}$. Draw the vertical asymptotes, and use them as a guide to sketch the graph of $y=-\frac{1}{2}\csc\frac{x}{2}$.



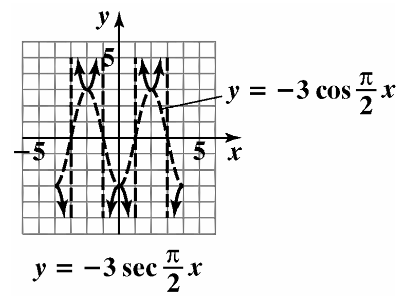
26. The x-intercepts of $y=3\sin 4x$ correspond to vertical asymptotes of $y=3\csc 4x$. Draw the vertical asymptotes, and use them as a guide to sketch the graph of $y=3\csc 4x$.



27. The x-intercepts of $y=\frac{1}{2}\cos 2\pi x$ corresponds to vertical asymptotes of $y=\frac{1}{2}\sec 2\pi x$. Draw the vertical asymptotes, and use them as a guide to sketch the graph of $y=\frac{1}{2}\sec 2\pi x$.



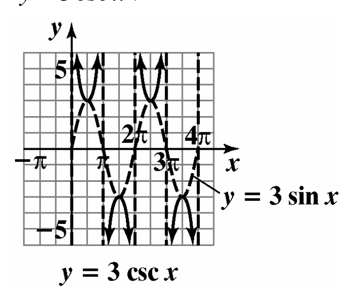
28. The x-intercepts of $y=-3\cos\frac{\pi}{2}x$ correspond to vertical asymptotes of $y=-3\sec\frac{\pi}{2}x$. Draw the vertical asymptotes, and use them as a guide to sketch the graph of $y=-3\sec\frac{\pi}{2}x$.



29. Graph the reciprocal sine function, $y=3\sin x$. The equation is of the form $y=A\sin Bx$ with $A=3$ and $B=1$.
amplitude: $|A|=|3|=3$

period: $\frac{2\pi}{B}=\frac{2\pi}{1}=2\pi$

Use quarter-periods, $\frac{\pi}{2}$, to find x-values for the five key points. Starting with $x=0$, the x-values are $0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}$, and 2π. Evaluating the function at each value of x, the key points are $(0, 0)$, $\left(\frac{\pi}{2}, 3\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, -3\right)$, and $(2\pi, 0)$. Use these key points to graph $y=3\sin x$ from 0 to 2π. Extend the graph one cycle to the right.
Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y=3\csc x$.



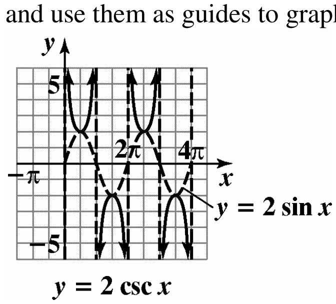
30. Graph the reciprocal sine function, $y = 2\sin x$. The equation is of the form $y = A\sin Bx$ with $A = 2$ and $B = 1$.

amplitude: $|A| = |2| = 2$

period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

Use quarter-periods, $\frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, and 2π. Evaluating the function at each value of x, the key points are $(0, 0), \left(\frac{\pi}{2}, 2\right), (\pi, 0), \left(\frac{3\pi}{2}, -2\right)$, and $(2\pi, 0)$.

Use these key points to graph $y = 2\sin x$ from 0 to 2π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 2\csc x$.

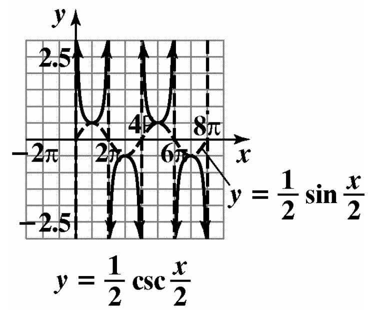


31. Graph the reciprocal sine function, $y = \frac{1}{2}\sin\frac{x}{2}$. The equation is of the form $y = A\sin Bx$ with $A = \frac{1}{2}$ and $B = \frac{1}{2}$.

amplitude: $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$

period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$

Use quarter-periods, π, to find x-values for the five key points. Starting with $x = 0$, the x-values are 0, $\pi, 2\pi, 3\pi$, and 4π. Evaluating the function at each value of x, the key points are (0, 0), $\left(\pi, \frac{1}{2}\right)$, $(2\pi, 0)$, $\left(3\pi, -\frac{1}{2}\right)$, and $(4\pi, 0)$. Use these key points to graph $y = \frac{1}{2}\sin\frac{x}{2}$ from 0 to 4π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = \frac{1}{2}\csc\frac{x}{2}$.



32. Graph the reciprocal sine function, $y = \frac{3}{2}\sin\frac{x}{4}$. The equation is of the form $y = A\sin Bx$ with $A = \frac{3}{2}$ and $B = \frac{1}{4}$.

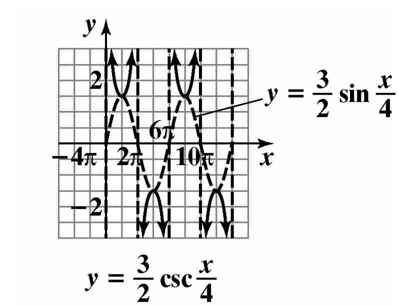
amplitude: $|A| = \left|\frac{3}{2}\right| = \frac{3}{2}$

period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{4}} = 2\pi \cdot 4 = 8\pi$

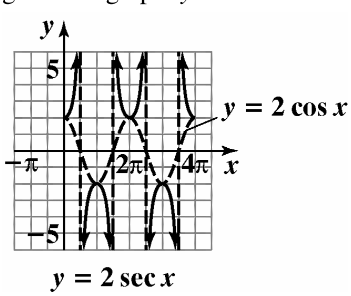
Use quarter-periods, 2π, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, 2\pi, 4\pi, 6\pi$, and 8π. Evaluating the function at each value of x, the key points are $(0, 0), \left(2\pi, \frac{3}{2}\right), (4\pi, 0), \left(6\pi, -\frac{3}{2}\right)$, and $(8\pi, 0)$.

Use these key points to graph $y = \frac{3}{2}\sin\frac{x}{4}$ from 0 to 8π. Extend the graph one cycle to the right.

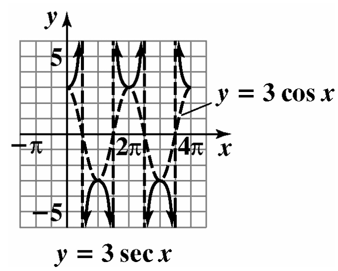
Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = \frac{3}{2}\csc\frac{x}{4}$.



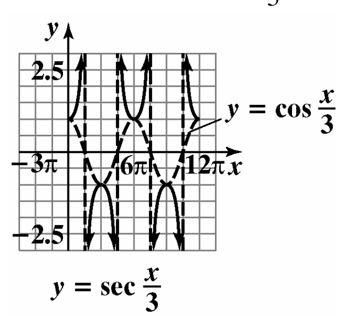
33. Graph the reciprocal cosine function, $y = 2\cos x$. The equation is of the form $y = A\cos Bx$ with $A = 2$ and $B = 1$.
amplitude: $|A| = |2| = 2$
period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$
Use quarter-periods, $\frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, 2π. Evaluating the function at each value of x, the key points are $(0, 2)$, $\left(\frac{\pi}{2}, 0\right)$, $(\pi, -2)$, $\left(\frac{3\pi}{2}, 0\right)$, and $(2\pi, 2)$. Use these key points to graph $y = 2\cos x$ from 0 to 2π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 2\sec x$.



34. Graph the reciprocal cosine function, $y = 3\cos x$. The equation is of the form $y = A\cos Bx$ with $A = 3$ and $B = 1$.
amplitude: $|A| = |3| = 3$
period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$
Use quarter-periods, $\frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, and 2π. Evaluating the function at each value of x, the key points are $(0, 3), \left(\frac{\pi}{2}, 0\right), (\pi, -3), \left(\frac{3\pi}{2}, 0\right), (2\pi, 3)$.
Use these key points to graph $y = 3\cos x$ from 0 to 2π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 3\sec x$.



35. Graph the reciprocal cosine function, $y = \cos\frac{x}{3}$. The equation is of the form $y = A\cos Bx$ with $A = 1$ and $B = \frac{1}{3}$.
amplitude: $|A| = |1| = 1$
period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{3}} = 2\pi \cdot 3 = 6\pi$
Use quarter-periods, $\frac{6\pi}{4} = \frac{3\pi}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are 0, $\frac{3\pi}{2}$, 3π, $\frac{9\pi}{2}$, and 6π. Evaluating the function at each value of x, the key points are $(0, 1)$, $\left(\frac{3\pi}{2}, 0\right)$, $(3\pi, -1)$, $\left(\frac{9\pi}{2}, 0\right)$, and $(6\pi, 1)$. Use these key points to graph $y = \cos\frac{x}{3}$ from 0 to 6π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = \sec\frac{x}{3}$.



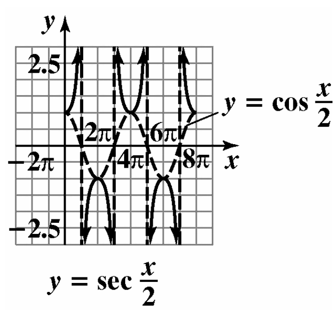
36. Graph the reciprocal cosine function, $y = \cos\frac{x}{2}$. The equation is of the form $y = A\cos Bx$ with $A = 1$ and $B = \frac{1}{2}$.

amplitude: $|A| = |1| = 1$

period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$

Use quarter-periods, π, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \pi, 2\pi, 3\pi,$ and 4π. Evaluating the function at each value of x, the key points are $(0, 1), (\pi, 0), (2\pi, -1), (3\pi, 0),$ and $(4\pi, 1)$.

Use these key points to graph $y = \cos\frac{x}{2}$ from 0 to 4π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = \sec\frac{x}{2}$.

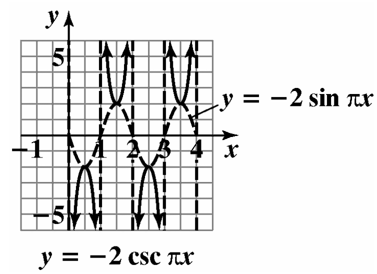


37. Graph the reciprocal sine function, $y = -2\sin\pi x$. The equation is of the form $y = A\sin Bx$ with $A = -2$ and $B = \pi$.

amplitude: $|A| = |-2| = 2$

period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$

Use quarter-periods, $\frac{2}{4} = \frac{1}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{1}{2}, 1, \frac{3}{2},$ and 2. Evaluating the function at each value of x, the key points are (0, 0), $\left(\frac{1}{2}, -2\right)$, (1, 0), $\left(\frac{3}{2}, 2\right)$, and (2, 0). Use these key points to graph $y = -2\sin\pi x$ from 0 to 2. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = -2\csc\pi x$.



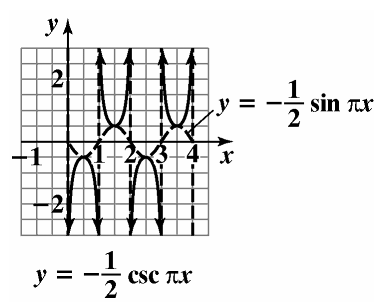
38. Graph the reciprocal sine function, $y = -\frac{1}{2}\sin\pi x$. The equation is of the form $y = A\sin Bx$ with $A = -\frac{1}{2}$ and $B = \pi$.

amplitude: $|A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$

period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$

Use quarter-periods, $\frac{2}{4} = \frac{1}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{1}{2}, 1, \frac{3}{2},$ and 2. Evaluating the function at each value of x, the key points are $(0, 0), \left(\frac{1}{2}, -\frac{1}{2}\right), (1, 0), \left(\frac{3}{2}, \frac{1}{2}\right),$ and $(2, 0)$.

Use these key points to graph $y = -\frac{1}{2}\sin\pi x$ from 0 to 2. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = -\frac{1}{2}\csc\pi x$.



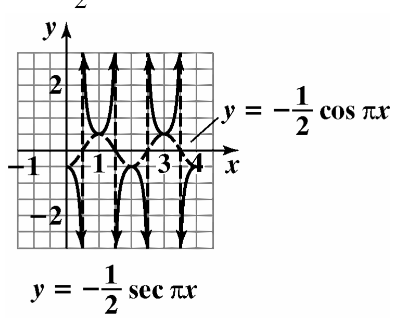
39. Graph the reciprocal cosine function, $y = -\frac{1}{2}\cos \pi x$.

The equation is of the form $y = A\cos Bx$ with $A = -\frac{1}{2}$ and $B = \pi$.

amplitude: $|A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$

period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$

Use quarter-periods, $\frac{2}{4} = \frac{1}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{1}{2}, 1, \frac{3}{2},$ and 2. Evaluating the function at each value of x, the key points are $\left(0, -\frac{1}{2}\right)$, $\left(\frac{1}{2}, 0\right), \left(1, \frac{1}{2}\right), \left(\frac{3}{2}, 0\right), \left(2, -\frac{1}{2}\right)$. Use these key points to graph $y = -\frac{1}{2}\cos \pi x$ from 0 to 2. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = -\frac{1}{2}\sec \pi x$.



40. Graph the reciprocal cosine function, $y = -\frac{3}{2}\cos \pi x$.

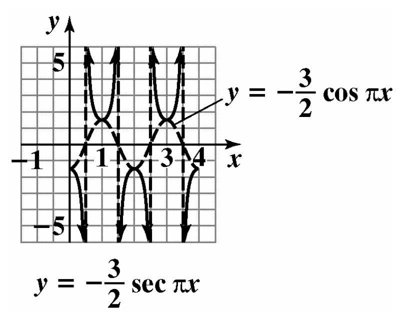
The equation is of the form $y = A\cos Bx$ with $A = -\frac{3}{2}$ and $B = \pi$.

amplitude: $|A| = \left|-\frac{3}{2}\right| = \frac{3}{2}$

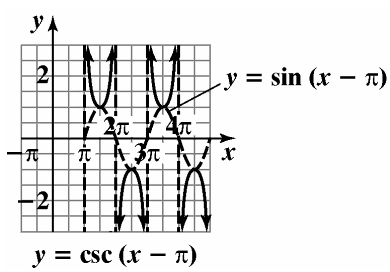
period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$

Use quarter-periods, $\frac{2}{4} = \frac{1}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{1}{2}, 1, \frac{3}{2},$ and 2. Evaluating the function at each value of x, the key points are $\left(0, -\frac{3}{2}\right), \left(\frac{1}{2}, 0\right), \left(1, \frac{3}{2}\right), \left(\frac{3}{2}, 0\right), \left(2, -\frac{3}{2}\right)$.

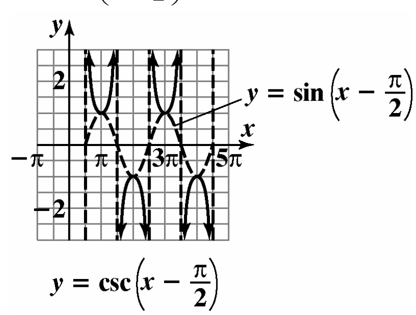
Use these key points to graph $y = -\frac{3}{2}\cos \pi x$ from 0 to 2. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = -\frac{3}{2}\sec \pi x$.



41. Graph the reciprocal sine function, $y = \sin(x - \pi)$. The equation is of the form $y = A\sin(Bx - C)$ with $A = 1$, and $B = 1$, and $C = \pi$.
amplitude: $|A| = |1| = 1$
period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$
phase shift: $\frac{C}{B} = \frac{\pi}{1} = \pi$
Use quarter-periods, $\frac{2\pi}{4} = \frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = \pi$, the x-values are $\pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}$, and 3π.
Evaluating the function at each value of x, the key points are $(\pi, 0)$, $\left(\frac{3\pi}{2}, 1\right)$, $(2\pi, 0)$, $\left(\frac{5\pi}{2}, -1\right)$, $(3\pi, 0)$. Use these key points to graph $y = \sin(x - \pi)$ from π to 3π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = \csc(x - \pi)$.



42. Graph the reciprocal sine function, $y = \sin\left(x - \frac{\pi}{2}\right)$.
The equation is of the form $y = A\sin(Bx - C)$ with $A = 1$, $B = 1$, and $C = \frac{\pi}{2}$.
amplitude: $|A| = |1| = 1$
period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$
phase shift: $\frac{C}{B} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$
Use quarter-periods, $\frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = \frac{\pi}{2}$, the x-values are $\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$, and $\frac{5\pi}{2}$. Evaluating the function at each value of x, the key points are $\left(\frac{\pi}{2}, 0\right)$, $(\pi, 1)$, $\left(\frac{3\pi}{2}, 0\right)$, $(2\pi, -1)$, and $\left(\frac{5\pi}{2}, 0\right)$.
Use these key points to graph $y = \sin\left(x - \frac{\pi}{2}\right)$ from $\frac{\pi}{2}$ to $\frac{5\pi}{2}$. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = \csc\left(x - \frac{\pi}{2}\right)$.



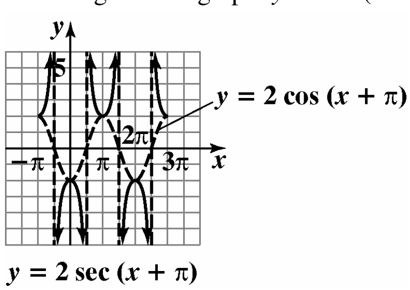
43. Graph the reciprocal cosine function, $y = 2\cos(x+\pi)$. The equation is of the form $y = A\cos(Bx+C)$ with $A = 2$, $B = 1$, and $C = -\pi$.

amplitude: $|A| = |2| = 2$

period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

phase shift: $\frac{C}{B} = \frac{-\pi}{1} = -\pi$

Use quarter-periods, $\frac{2\pi}{4} = \frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = -\pi$, the x-values are $-\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}$, and π. Evaluating the function at each value of x, the key points are $(-\pi, 2)$, $\left(-\frac{\pi}{2}, 0\right), (0, -2), \left(\frac{\pi}{2}, 0\right)$, and $(\pi, 2)$. Use these key points to graph $y = 2\cos(x+\pi)$ from $-\pi$ to π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 2\sec(x+\pi)$.



44. Graph the reciprocal cosine function, $y = 2\cos\left(x+\frac{\pi}{2}\right)$. The equation is of the form $y = A\cos(Bx+C)$ with $A = 2$ and $B = 1$, and $C = -\frac{\pi}{2}$.

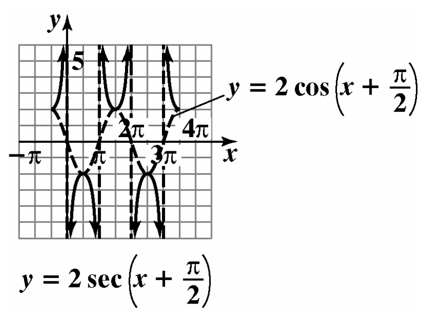
amplitude: $|A| = |2| = 2$

period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

phase shift: $\frac{C}{B} = \frac{-\frac{\pi}{2}}{1} = -\frac{\pi}{2}$

Use quarter-periods, $\frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = -\frac{\pi}{2}$, the x-values are $-\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi$, and $\frac{3\pi}{2}$. Evaluating the function at each value of x, the key points are $\left(-\frac{\pi}{2}, 2\right), (0, 0), \left(\frac{\pi}{2}, -2\right), (\pi, 0), \left(\frac{3\pi}{2}, 2\right)$.

Use these key points to graph $y = 2\cos\left(x+\frac{\pi}{2}\right)$ from $-\frac{\pi}{2}$ to $\frac{3\pi}{2}$. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 2\sec\left(x+\frac{\pi}{2}\right)$.



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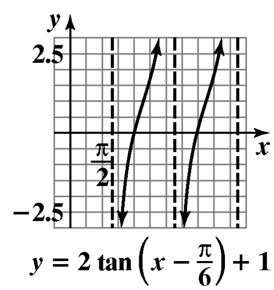

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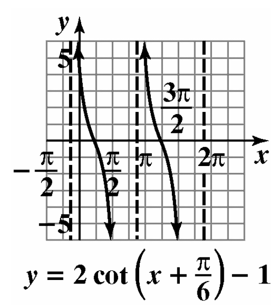

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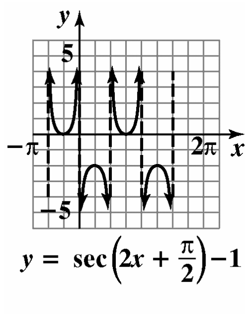

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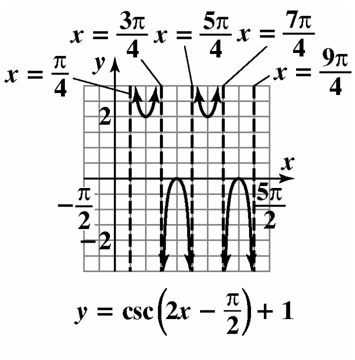

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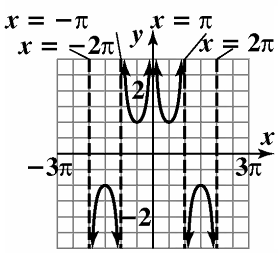

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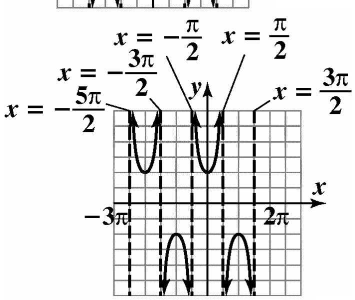

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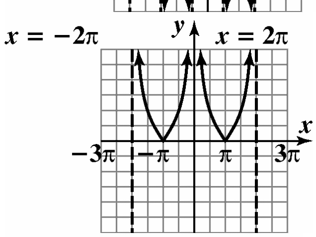
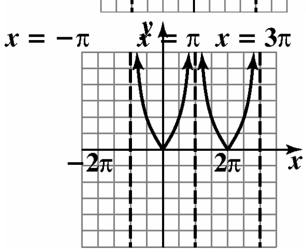
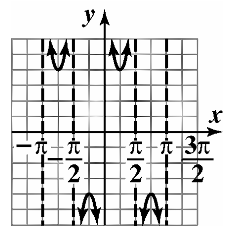


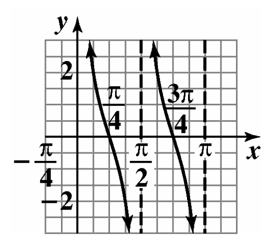
52.



53. $y = (f \circ h)(x) = f(h(x)) = 2\sec\left(2x - \dfrac{\pi}{2}\right)$



54. $y = (g \circ h)(x) = g(h(x)) = -2\tan\left(2x - \dfrac{\pi}{2}\right)$



55. Use a graphing utility with $y_1 = \tan x$ and $y_2 = -1$.
For the window use Xmin $= -2\pi$, Xmax $= 2\pi$, Ymin $= -2$, and Ymax $= 2$.
$x = -\dfrac{5\pi}{4},\ -\dfrac{\pi}{4},\ \dfrac{3\pi}{4},\ \dfrac{7\pi}{4}$
$x \approx -3.93,\ -0.79,\ 2.36,\ 5.50$

56. Use a graphing utility with $y_1 = 1/\tan x$ and $y_2 = -1$.
For the window use Xmin $= -2\pi$, Xmax $= 2\pi$, Ymin $= -2$, and Ymax $= 2$.
$x = -\dfrac{5\pi}{4},\ -\dfrac{\pi}{4},\ \dfrac{3\pi}{4},\ \dfrac{7\pi}{4}$
$x \approx -3.93,\ -0.79,\ 2.36,\ 5.50$

57. Use a graphing utility with $y_1 = 1/\sin x$ and $y_2 = 1$.
For the window use Xmin $= -2\pi$, Xmax $= 2\pi$, Ymin $= -2$, and Ymax $= 2$.
$x = -\dfrac{3\pi}{2},\ \dfrac{\pi}{2}$
$x \approx -4.71,\ 1.57$

58. Use a graphing utility with $y_1 = 1/\cos x$ and $y_2 = 1$.
For the window use Xmin $= -2\pi$, Xmax $= 2\pi$, Ymin $= -2$, and Ymax $= 2$.
$x = -2\pi,\ 0,\ 2\pi$
$x \approx -6.28,\ 0,\ 6.28$

59. $d = 12\tan 2\pi t$

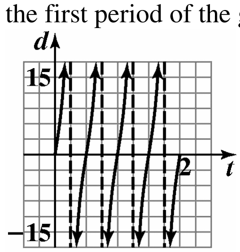
a. Solve the equations

$$2\pi t = -\frac{\pi}{2} \quad \text{and} \quad 2\pi t = \frac{\pi}{2}$$
$$t = \frac{-\frac{\pi}{2}}{2\pi} \qquad t = \frac{\frac{\pi}{2}}{2\pi}$$
$$t = -\frac{1}{4} \qquad t = \frac{1}{4}$$

Thus, two consecutive asymptotes occur at $x = -\frac{1}{4}$ and $x = \frac{1}{4}$.

x-intercept $= \frac{-\frac{1}{4} + \frac{1}{4}}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 12, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –12 and 12. Use the two consecutive asymptotes, $x = -\frac{1}{4}$ and $x = \frac{1}{4}$, to graph one full period of $d = 12\tan 2\pi t$. To graph on [0, 2], continue the pattern and extend the graph to 2. (Do not use the left hand side of the first period of the graph on [0, 2].)



$d = 12\tan 2\pi t$

b. The function is undefined for t = 0.25, 0.75, 1.25, and 1.75.
The beam is shining parallel to the wall at these times.

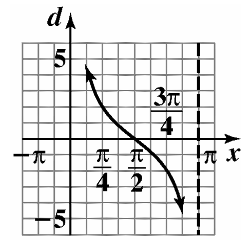
60. In a right triangle the angle of elevation is one of the acute angles, the adjacent leg is the distance d, and the opposite leg is 2 mi. Use the cotangent function.

$$\cot x = \frac{d}{2}$$
$$d = 2\cot x$$

Use the equations $x = 0$ and $x = \pi$.

Two consecutive asymptotes occur at $x = 0$ and $x = \pi$. Midway between $x = 0$ and $x = \pi$ is $x = \frac{\pi}{2}$.

An x-intercept is $\frac{\pi}{2}$ and the graph passes through $\left(\frac{\pi}{2}, 0\right)$. Because the coefficient of the cotangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –2 and 2. Use the two consecutive asymptotes, $x = 0$ and $x = \pi$, to graph $y = 2\cot x$ for $0 < x < \pi$.



$d = 2\cot x$

61. Use the function that relates the acute angle with the hypotenuse and the adjacent leg, the secant function.

$$\sec x = \frac{d}{10}$$
$$d = 10\sec x$$

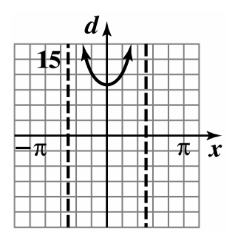
Graph the reciprocal cosine function, $y = 10\cos x$.
The equation is of the form $y = A\cos Bx$ with $A = 10$ and $B = 1$.

amplitude: $|A| = |10| = 10$

period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$

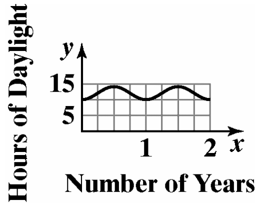
For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, use the x-values $-\frac{\pi}{2}$, 0, and $\frac{\pi}{2}$ to find the key points $\left(-\frac{\pi}{2}, 0\right)$, (0, 10), and $\left(\frac{\pi}{2}, 0\right)$.

Connect these points with a smooth curve, then draw vertical asymptotes through the x-intercepts, and use them as guides to graph $d = 10\sec x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

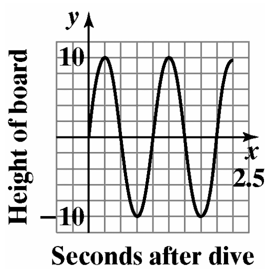


$d = 10\sec x$

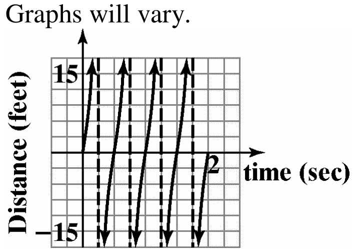
62. Graphs will vary.



63.



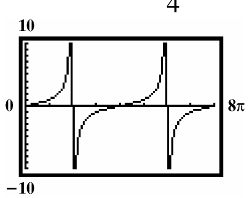
64. Graphs will vary.



65. – 76. Answers may vary.

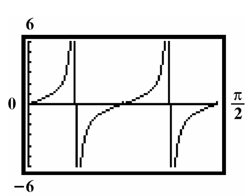
77. period: $\frac{\pi}{B} = \frac{\pi}{\frac{1}{4}} = \pi \cdot 4 = 4\pi$

Graph $y = \tan \frac{x}{4}$ for $0 \le x \le 8\pi$.



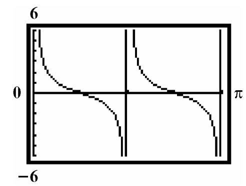
78. period: $\frac{\pi}{B} = \frac{\pi}{4}$

Graph $y = \tan 4x$ for $0 \le x \le \frac{\pi}{2}$.



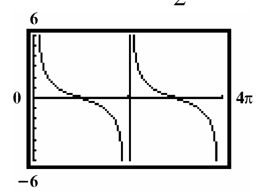
79. period: $\frac{\pi}{B} = \frac{\pi}{2}$

Graph $y = \cot 2x$ for $0 \le x \le \pi$.



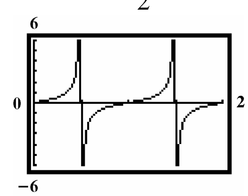
80. period: $\frac{\pi}{B} = \frac{\pi}{\frac{1}{2}} = \pi \cdot 2 = 2\pi$

Graph $y = \cot \frac{x}{2}$ for $0 \le x \le 4\pi$.



81. period: $\frac{\pi}{B} = \frac{\pi}{\pi} = 1$

Graph $y = \frac{1}{2} \tan \pi x$ for $0 \le x \le 2$.



82. Solve the equations

$\pi x + 1 = -\frac{\pi}{2}$ and $\pi x + 1 = \frac{\pi}{2}$

$\pi x = -\frac{\pi}{2} - 1$ $\qquad$ $\pi x = \frac{\pi}{2} - 1$

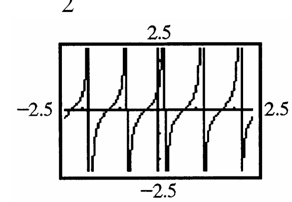
$x = \frac{\frac{-\pi}{2} - 1}{\pi}$ $\qquad$ $x = \frac{\frac{\pi}{2} - 1}{\pi}$

$x = \frac{-\pi - 2}{2\pi}$ $\qquad$ $x = \frac{\pi - 2}{2\pi}$

$x \approx -0.82$ $\qquad$ $x \approx 0.18$

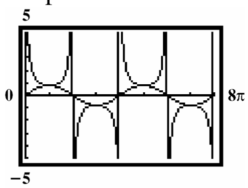
period: $\frac{\pi}{B} = \frac{\pi}{\pi} = 1$

Thus, we include $-0.82 \le x \le 1.18$ in our graph of $y = \frac{1}{2}\tan(\pi x + 1)$, and graph for $-0.85 \le x \le 1.2$.



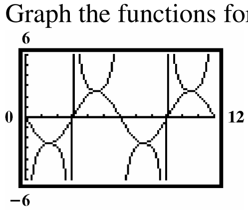
83. period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2 = 4\pi$

Graph the functions for $0 \le x \le 8\pi$.



84. period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{3}} = 2\pi \cdot \frac{3}{\pi} = 6$

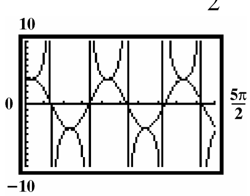
Graph the functions for $0 \le x \le 12$.



85. period: $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$

phase shift: $\frac{C}{B} = \frac{\frac{\pi}{6}}{2} = \frac{\pi}{12}$

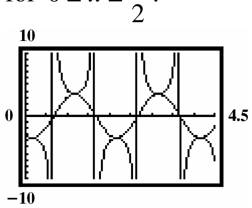
Thus, we include $\frac{\pi}{12} \le x \le \frac{25\pi}{12}$ in our graph, and graph for $0 \le x \le \frac{5\pi}{2}$.



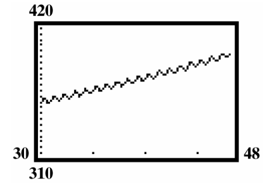
86. period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$

phase shift: $\frac{C}{B} = \frac{\frac{\pi}{6}}{\pi} = \frac{\pi}{6} \cdot \frac{1}{\pi} = \frac{1}{6}$

Thus, we include $\frac{1}{6} \le x \le \frac{25}{6}$ in our graph, and graph for $0 \le x \le \frac{9}{2}$.

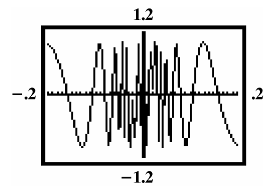


87.



The graph shows that carbon dioxide concentration rises and falls each year, but over all the concentration increased from 1990 to 2008.

88. $y = \sin\frac{1}{x}$



The graph is oscillating between 1 and –1. The oscillation is faster as x gets closer to 0. Explanations may vary.

89. makes sense

90. makes sense

91. does not make sense; Explanations will vary. Sample explanation: To obtain a cosecant graph, you can use a sine graph.

92. does not make sense; Explanations will vary. Sample explanation: To model a cyclical temperature, use sine or cosine.

93. The graph has the shape of a cotangent function with consecutive asymptotes at

$x = 0$ and $x = \frac{2\pi}{3}$. The period is $\frac{2\pi}{3} - 0 = \frac{2\pi}{3}$. Thus,

$$\frac{\pi}{B} = \frac{2\pi}{3}$$
$$2\pi B = 3\pi$$
$$B = \frac{3\pi}{2\pi} = \frac{3}{2}$$

The points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 1 and –1. Thus, $A = 1$. There is no phase shift. Thus, $C = 0$. An equation for this graph is $y = \cot\frac{3}{2}x$.

94. The graph has the shape of a secant function.
The reciprocal function has amplitude $|A| = 1$. The period is $\frac{8\pi}{3}$. Thus, $\frac{2\pi}{B} = \frac{8\pi}{3}$

$$8\pi B = 6\pi$$

$$B = \frac{6\pi}{8\pi} = \frac{3}{4}$$

There is no phase shift. Thus, $C = 0$. An equation for the reciprocal function is $y = \cos\frac{3}{4}x$. Thus, an equation for this graph is $y = \sec\frac{3}{4}x$.

95. The range shows that $A = 2.$
Since the period is $3\pi,$ the coefficient of x is given by B where $\frac{2\pi}{B} = 3\pi$

$$\frac{2\pi}{B} = 3\pi$$

$$3B\pi = 2\pi$$

$$B = \frac{2}{3}$$

Thus, $y = 2\csc\frac{2x}{3}$

96. The range shows that $A = \pi.$
Since the period is $2,$ the coefficient of x is given by B where $\frac{2\pi}{B} = 2$

$$\frac{2\pi}{B} = 2$$

$$2B = 2\pi$$

$$B = \pi$$

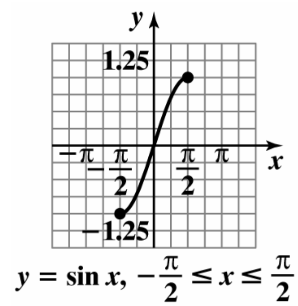
Thus, $y = \pi\csc\pi x$

97. **a.** Since A=1, the range is $(-\infty,-1]\cup[1,\infty)$
Viewing rectangle: $\left[-\frac{\pi}{6},\pi,\frac{7\pi}{6}\right]$ by $[-3,3,1]$

b. Since A=3, the range is $(-\infty,-3]\cup[3,\infty)$
Viewing rectangle: $\left[-\frac{1}{2},\frac{7}{2},1\right]$ by $[-6,6,1]$

98. $y = 2^{-x}\sin x$
2^{-x} decreases the amplitude as x gets larger.
Examples may vary.

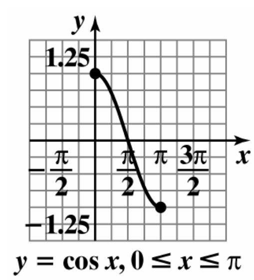
99. **a.**



b. yes; Explanations will vary.

c. The angle is $-\frac{\pi}{6}.$
This is represented by the point $\left(-\frac{\pi}{6},-\frac{1}{2}\right).$

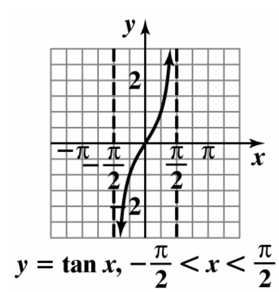
100. **a.**



b. yes; Explanations will vary.

c. The angle is $\frac{5\pi}{6}.$
This is represented by the point $\left(\frac{5\pi}{6},-\frac{\sqrt{3}}{2}\right).$

101. **a.**



b. yes; Explanations will vary.

c. The angle is $-\frac{\pi}{3}.$
This is represented by the point $\left(-\frac{\pi}{3},-\sqrt{3}\right).$

Section 5.7

Check Point Exercises

1. Let $\theta = \sin^{-1}\frac{\sqrt{3}}{2}$, then $\sin\theta = \frac{\sqrt{3}}{2}$.

The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = \frac{\sqrt{3}}{2}$ is $\frac{\pi}{3}$. Thus, $\theta = \frac{\pi}{3}$, or $\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$.

2. Let $\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$, then $\sin\theta = -\frac{\sqrt{2}}{2}$.

The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\cos\theta = -\frac{\sqrt{2}}{2}$ is $-\frac{\pi}{4}$. Thus $\theta = -\frac{\pi}{4}$, or $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$.

3. Let $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$, then $\cos\theta = -\frac{1}{2}$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = -\frac{1}{2}$ is $\frac{2\pi}{3}$. Thus, $\theta = \frac{2\pi}{3}$, or $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$.

4. Let $\theta = \tan^{-1}(-1)$, then $\tan\theta = -1$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = -1$ is $-\frac{\pi}{4}$. Thus $\theta = -\frac{\pi}{4}$ or $\tan^{-1}\theta = -\frac{\pi}{4}$.

5.

	Scientific Calculator Solution			
	Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
a.	$\cos^{-1}\left(\frac{1}{3}\right)$	Radian	1 [÷] 3 [=] [COS^{-1}]	1.2310
b.	$\tan^{-1}(-35.85)$	Radian	35.85 [+/−] [TAN^{-1}]	−1.5429

	Graphing Calculator Solution			
	Function	**Mode**	**Keystrokes**	**Display** (rounded to four places)
a.	$\cos^{-1}\left(\frac{1}{3}\right)$	Radian	[COS^{-1}] [(] 1 [÷] 3 [)] [ENTER]	1.2310
b.	$\tan^{-1}(-35.85)$	Radian	[TAN^{-1}] [−] 35.85 [ENTER]	−1.5429

6. **a.** $\cos\left(\cos^{-1} 0.7\right)$

$x = 0.7$, x is in $[-1,1]$ so $\cos(\cos^{-1} 0.7) = 0.7$

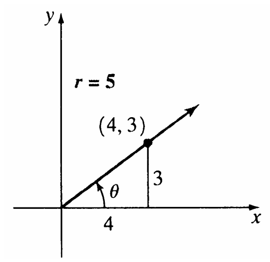
b. $\sin^{-1}(\sin \pi)$

$x = \pi$, x is not in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. x is in the domain of $\sin x$, so $\sin^{-1}(\sin \pi) = \sin^{-1}(0) = 0$

c. $\cos\left(\cos^{-1} \pi\right)$

$x = \pi$, x is not in $[-1,1]$ so $\cos\left(\cos^{-1} \pi\right)$ is not defined.

7. Let $\theta = \tan^{-1}\left(\frac{3}{4}\right)$, then $\tan\theta = \frac{3}{4}$. Because $\tan\theta$ is positive, θ is in the first quadrant.



Use the Pythagorean Theorem to find r.

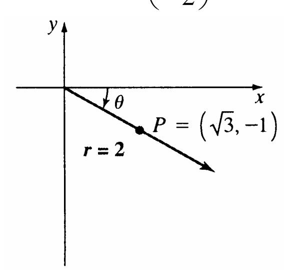
$r^2 = 3^2 + 4^2 = 9 + 16 = 25$

$r = \sqrt{25} = 5$

Use the right triangle to find the exact value.

$$\sin\left(\tan^{-1}\frac{3}{4}\right) = \sin\theta = \frac{\text{side opposite } \theta}{\text{hypotenuse}} = \frac{3}{5}$$

8. Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$, then $\sin\theta = -\frac{1}{2}$. Because $\sin\theta$ is negative, θ is in quadrant IV.



Use the Pythagorean Theorem to find x.

$x^2 + (-1)^2 = 2^2$

$x^2 + 1 = 4$

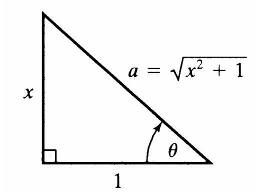
$x^2 = 3$

$x = \sqrt{3}$

Use values for x and r to find the exact value.

$$\cos\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = \cos\theta = \frac{x}{r} = \frac{\sqrt{3}}{2}$$

9. Let $\theta = \tan^{-1} x$, then $\tan\theta = x = \frac{x}{1}$.



Use the Pythagorean Theorem to find the third side, *a*.

$a^2 = x^2 + 1^2$

$a = \sqrt{x^2 + 1}$

Use the right triangle to write the algebraic expression.

$\sec\left(\tan^{-1} x\right) = \sec\theta = \frac{\sqrt{x^2+1}}{1} = \sqrt{x^2+1}$

Exercise Set 5.7

1. Let $\theta = \sin^{-1}\frac{1}{2}$, then $\sin\theta = \frac{1}{2}$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = \frac{1}{2}$ is $\frac{\pi}{6}$. Thus, $\theta = \frac{\pi}{6}$, or $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$.

2. Let $\theta = \sin^{-1} 0$, then $\sin\theta = 0$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = 0$ is 0. Thus $\theta = 0$, or $\sin^{-1} 0 = 0$.

3. Let $\theta = \sin^{-1}\frac{\sqrt{2}}{2}$, then $\sin\theta = \frac{\sqrt{2}}{2}$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = \frac{\sqrt{2}}{2}$ is $\frac{\pi}{4}$. Thus $\theta = \frac{\pi}{4}$, or $\sin^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$.

4. Let $\theta = \sin^{-1}\frac{\sqrt{3}}{2}$, then $\sin\theta = \frac{\sqrt{3}}{2}$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = \frac{\sqrt{3}}{2}$ is $\frac{\pi}{3}$. Thus $\theta = \frac{\pi}{3}$, or $\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$.

5. Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$, then $\sin\theta = -\frac{1}{2}$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = -\frac{1}{2}$ is $-\frac{\pi}{6}$. Thus $\theta = -\frac{\pi}{6}$, or $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$.

6. Let $\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, then $\sin\theta = -\frac{\sqrt{3}}{2}$.

The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = -\frac{\sqrt{3}}{2}$ is $-\frac{\pi}{3}$. Thus $\theta = -\frac{\pi}{3}$,

or $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$.

7. Let $\theta = \cos^{-1}\frac{\sqrt{3}}{2}$, then $\cos\theta = \frac{\sqrt{3}}{2}$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = \frac{\sqrt{3}}{2}$ is $\frac{\pi}{6}$. Thus $\theta = \frac{\pi}{6}$, or $\cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$.

8. Let $\theta = \cos^{-1}\frac{\sqrt{2}}{2}$, then $\cos\theta = \frac{\sqrt{2}}{2}$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = \frac{\sqrt{2}}{2}$ is $\frac{\pi}{4}$. Thus $\theta = \frac{\pi}{4}$, or $\cos^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$.

9. Let $\theta = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$, then $\cos\theta = -\frac{\sqrt{2}}{2}$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = -\frac{\sqrt{2}}{2}$ is $\frac{3\pi}{4}$. Thus $\theta = \frac{3\pi}{4}$, or $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$.

10. Let $\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, then $\cos\theta = -\frac{\sqrt{3}}{2}$.

The only angle in the interval $[0, \pi]$ that

satisfies $\cos\theta = -\frac{\sqrt{3}}{2}$ is $\frac{5\pi}{6}$. Thus $\theta = \frac{5\pi}{6}$, or $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$.

11. Let $\theta = \cos^{-1}0$, then $\cos\theta = 0$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = 0$ is $\frac{\pi}{2}$.

Thus $\theta = \frac{\pi}{2}$, or $\cos^{-1}0 = \frac{\pi}{2}$.

12. Let $\theta = \cos^{-1}1$, then $\cos\theta = 1$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = 1$ is 0.
Thus $\theta = 0$, or $\cos^{-1}1 = 0$.

13. Let $\theta = \tan^{-1}\frac{\sqrt{3}}{3}$, then $\tan\theta = \frac{\sqrt{3}}{3}$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = \frac{\sqrt{3}}{3}$ is $\frac{\pi}{6}$. Thus $\theta = \frac{\pi}{6}$, or $\tan^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{6}$.

14. Let $\theta = \tan^{-1}1$, then $\tan\theta = 1$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = 1$ is $\frac{\pi}{4}$.

Thus $\theta = \frac{\pi}{4}$, or $\tan^{-1}1 = \frac{\pi}{4}$.

15. Let $\theta = \tan^{-1} 0$, then $\tan\theta = 0$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = 0$ is 0. Thus $\theta = 0$, or $\tan^{-1} 0 = 0$.

16. Let $\theta = \tan^{-1}(-1)$, then $\tan\theta = -1$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = -1$ is $-\frac{\pi}{4}$. Thus $\theta = -\frac{\pi}{4}$, or $\tan^{-1}(-1) = -\frac{\pi}{4}$.

17. Let $\theta = \tan^{-1}\left(-\sqrt{3}\right)$, then $\tan\theta = -\sqrt{3}$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = -\sqrt{3}$ is $-\frac{\pi}{3}$. Thus $\theta = -\frac{\pi}{3}$, or $\tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$.

18. Let $\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$, then $\tan\theta = -\frac{\sqrt{3}}{3}$. The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = -\frac{\sqrt{3}}{3}$ is $-\frac{\pi}{6}$. Thus $\theta = -\frac{\pi}{6}$, or $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$.

19.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1} 0.3$	Radian	0.3 [SIN^{-1}]	0.30

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1} 0.3$	Radian	[SIN^{-1}] 0.3 [ENTER]	0.30

20.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1} 0.47$	Radian	0.47 [SIN^{-1}]	0.49

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1} 0.47$	Radian	[SIN^{-1}] 0.47 [ENTER]	0.49

21.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1}(-0.32)$	Radian	0.32 [+/–] [SIN⁻¹]	–0.33

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1}(-0.32)$	Radian	[SIN⁻¹] [–] 0.32 [ENTER]	–0.33

22.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1}(-0.625)$	Radian	0.625 [+/–] [SIN⁻¹]	–0.68

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\sin^{-1}(-0.625)$	Radian	[SIN⁻¹] [–] 0.625 [ENTER]	–0.68

23.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\left(\frac{3}{8}\right)$	Radian	3 [÷] 8 [=] [COS⁻¹]	1.19

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\left(\frac{3}{8}\right)$	Radian	[COS⁻¹] [(] 3 [÷] 8 [)] [ENTER]	1.19

24.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\left(\frac{4}{9}\right)$	Radian	4 [÷] 9 [=] [COS⁻¹]	1.11

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\left(\frac{4}{9}\right)$	Radian	[COS⁻¹] [(] 4 [÷] 9 [)] [ENTER]	1.11

25.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\frac{\sqrt{5}}{7}$	Radian	5 [√] [÷] 7 [=] [COS⁻¹]	1.25

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\frac{\sqrt{5}}{7}$	Radian	[COS⁻¹] [(] [√] 5 [÷] 7 [)] [ENTER]	1.25

26.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\frac{\sqrt{7}}{10}$	Radian	7 [√] [÷] 10 [=] [COS⁻¹]	1.30

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\cos^{-1}\frac{\sqrt{7}}{10}$	Radian	[COS⁻¹] [(] [√] 7 [÷] 10 [)] [ENTER]	1.30

27.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}(-20)$	Radian	20 [+/−] [TAN⁻¹]	–1.52

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}(-20)$	Radian	[TAN⁻¹] [−] 20 [ENTER]	–1.52

28.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}(-30)$	Radian	30 [+/−] [TAN⁻¹]	–1.54

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}(-30)$	Radian	[TAN⁻¹] [−] 30 [ENTER]	–1.54

29.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}\left(-\sqrt{473}\right)$	Radian	473 [√] [+/−] [TAN⁻¹]	−1.52

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}\left(-\sqrt{473}\right)$	Radian	[TAN⁻¹] [(] [−] [√] 473 [)] [ENTER]	−1.52

30.

Scientific Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}\left(-\sqrt{5061}\right)$	Radian	5061 [√] [+/−] [TAN⁻¹]	−1.56

Graphing Calculator Solution			
Function	**Mode**	**Keystrokes**	**Display** (rounded to two places)
$\tan^{-1}\left(-\sqrt{5061}\right)$	Radian	[TAN⁻¹] [(] [−] [√] 5061 [)] [ENTER]	−1.56

31. $\sin\left(\sin^{-1} 0.9\right)$

$x = 0.9$, x is in $[-1, 1]$, so $\sin(\sin^{-1} 0.9) = 0.9$

32. $\cos(\cos^{-1} 0.57)$

$x = 0.57$, x is in $[-1, 1]$,

so $\cos(\cos^{-1} 0.57) = 0.57$

33. $\sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$x = \frac{\pi}{3}$, x is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so $\sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

34. $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)$

$x = \frac{2\pi}{3}$, x is in $[0, \pi]$,

so $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) = \frac{2\pi}{3}$

35. $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$

$x = \frac{5\pi}{6}$, x is not in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, x is in the domain of $\sin x$, so $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

36. $\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$

$x = \frac{4\pi}{3}$, x is not in $[0, \pi]$,

x is in the domain of $\cos x$,

so $\cos^{-1}\left(\cos\frac{4\pi}{3}\right) = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

37. $\tan\left(\tan^{-1}125\right)$

$x = 125$, x is a real number, so $\tan\left(\tan^{-1}125\right) = 125$

38. $\tan(\tan^{-1}380)$

$x = 380$, x is a real number,

so $\tan(\tan^{-1}380) = 380$

39. $\tan^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right]$

$x = -\frac{\pi}{6}$, x is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so

$\tan^{-1}\left[\tan\left(-\frac{\pi}{6}\right)\right] = -\frac{\pi}{6}$

40. $\tan^{-1}\left[\tan\left(-\frac{\pi}{3}\right)\right]$

$x = -\frac{\pi}{3}$, x is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

so $\tan^{-1}\left[\tan\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$

41. $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

$x = \frac{2\pi}{3}$, x is not in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, x is in the domain of $\tan x$, so $\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$

42. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$x = \frac{3\pi}{4}$, x is not in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

x is in the domain of $\tan x$

so $\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$

43. $\sin^{-1}(\sin\pi)$

$x = \pi$, x is not in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$,

x is in the domain of $\sin x$, so

$\sin^{-1}(\sin\pi) = \sin^{-1}0 = 0$

44. $\cos^{-1}(\cos 2\pi)$

$x = 2\pi$, x is not in $[0, \pi]$,

x is in the domain of $\cos x$,

so $\cos^{-1}(\cos 2\pi) = \cos^{-1}1 = 0$

45. $\sin\left(\sin^{-1}\pi\right)$

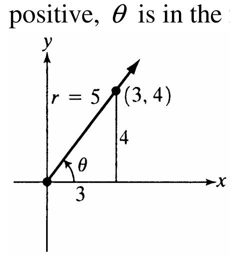
$x = \pi$, x is not in $[-1, 1]$, so $\sin\left(\sin^{-1}\pi\right)$ is not defined.

46. $\cos(\cos^{-1}3\pi)$

$x = 3\pi$, x is not in $[-1, 1]$

so $\cos(\cos^{-1}3\pi)$ is not defined.

47. Let $\theta = \sin^{-1}\frac{4}{5}$, then $\sin\theta = \frac{4}{5}$. Because $\sin\theta$ is positive, θ is in the first quadrant.



$x^2 + y^2 = r^2$

$x^2 + 4^2 = 5^2$

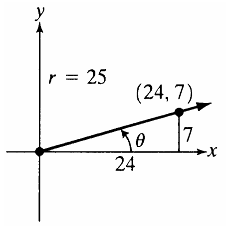
$x^2 = 25 - 16 = 9$

$x = 3$

$\cos\left(\sin^{-1}\frac{4}{5}\right) = \cos\theta = \frac{x}{r} = \frac{3}{5}$

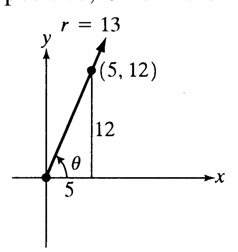
48. Let $\theta = \tan^{-1}\frac{7}{24}$, then $\tan\theta = \frac{7}{24}$.

Because $\tan\theta$ is positive, θ is in the first quadrant.



$$r^2 = x^2 + y^2$$
$$r^2 = 7^2 + 24^2$$
$$r^2 = 625$$
$$r = 25$$
$$\sin\left(\tan^{-1}\frac{7}{24}\right) = \sin\theta = \frac{y}{r} = \frac{7}{25}$$

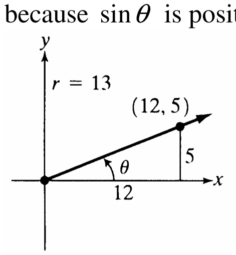
49. Let $\theta = \cos^{-1}\frac{5}{13}$, then $\cos\theta = \frac{5}{13}$. Because $\cos\theta$ is positive, θ is in the first quadrant.



$$x^2 + y^2 = r^2$$
$$5^2 + y^2 = 13^2$$
$$y^2 = 169 - 25$$
$$y^2 = 144$$
$$y = 12$$
$$\tan\left(\cos^{-1}\frac{5}{13}\right) = \tan\theta = \frac{y}{x} = \frac{12}{5}$$

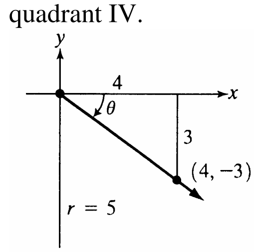
50. Let $\theta = \sin^{-1}\frac{5}{13}$ then $\sin\theta = \frac{5}{13}$.

because $\sin\theta$ is positive, θ is in the first quadrant.



$$x^2 + y^2 = r^2$$
$$x^2 + 5^2 = 13^2$$
$$x^2 = 144$$
$$x = 12$$
$$\cot\left(\sin^{-1}\frac{5}{13}\right) = \cot\theta = \frac{x}{y} = \frac{12}{5}$$

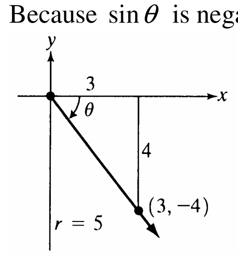
51. Let $\theta = \sin^{-1}\left(-\frac{3}{5}\right)$, then $\sin\theta = -\frac{3}{5}$. Because $\sin\theta$ is negative, θ is in quadrant IV.



$$x^2 + y^2 = r^2$$
$$x^2 + (-3)^2 = 5^2$$
$$x^2 = 16$$
$$x = 4$$
$$\tan\left[\sin^{-1}\left(-\frac{3}{5}\right)\right] = \tan\theta = \frac{y}{x} = -\frac{3}{4}$$

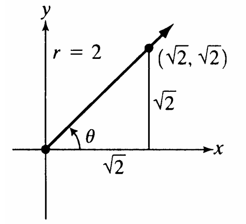
52. Let $\theta = \sin^{-1}\left(-\frac{4}{5}\right)$, then $\sin\theta = -\frac{4}{5}$.

Because $\sin\theta$ is negative, θ is in quadrant IV.



$$x^2 + y^2 = r^2$$
$$x^2 + (-4)^2 = 5^2$$
$$x^2 = 9$$
$$x = 3$$
$$\cos\left[\sin^{-1}\left(-\frac{4}{5}\right)\right] = \cos\theta = \frac{x}{r} = \frac{3}{5}$$

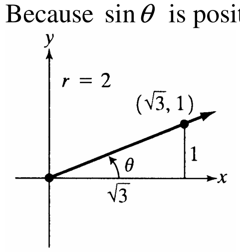
53. Let, $\theta = \cos^{-1}\frac{\sqrt{2}}{2}$, then $\cos\theta = \frac{\sqrt{2}}{2}$. Because $\cos\theta$ is positive, θ is in the first quadrant.



$$x^2 + y^2 = r^2$$
$$\left(\sqrt{2}\right)^2 + y^2 = 2^2$$
$$y^2 = 2$$
$$y = \sqrt{2}$$
$$\sin\left(\cos^{-1}\frac{\sqrt{2}}{2}\right) = \sin\theta = \frac{y}{r} = \frac{\sqrt{2}}{2}$$

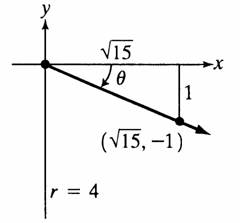
54. Let $\theta = \sin^{-1}\frac{1}{2}$, then $\sin\theta = \frac{1}{2}$.

Because $\sin\theta$ is positive, θ is in the first quadrant.



$$x^2 + y^2 = r^2$$
$$x^2 + 1^2 = 2^2$$
$$x^2 = 3$$
$$x = \sqrt{3}$$
$$\cos\left(\sin^{-1}\frac{1}{2}\right) = \cos\theta = \frac{x}{r} = \frac{\sqrt{3}}{2}$$

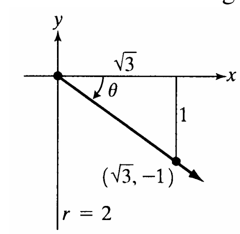
55. Let $\theta = \sin^{-1}\left(-\frac{1}{4}\right)$, then $\sin\theta = -\frac{1}{4}$. Because $\sin\theta$ is negative, θ is in quadrant IV.



$$x^2 + y^2 = r^2$$
$$x^2 + (-1)^2 = 4^2$$
$$x^2 = 15$$
$$x = \sqrt{15}$$
$$\sec\left[\sin^{-1}\left(-\frac{1}{4}\right)\right] = \sec\theta = \frac{r}{x} = \frac{4}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$$

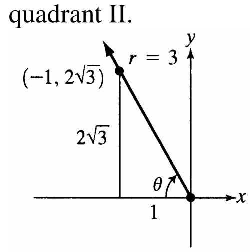
56. Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$, then $\sin\theta = -\frac{1}{2}$.

Because $\sin\theta$ is negative, θ is in quadrant IV.



$$x^2 + y^2 = r^2$$
$$x^2 + (-1)^2 = 2^2$$
$$x^2 = 3$$
$$x = \sqrt{3}$$
$$\sec\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = \sec\theta = \frac{r}{x} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

57. Let $\theta = \cos^{-1}\left(-\frac{1}{3}\right)$, then $\cos\theta = -\frac{1}{3}$. Because $\cos\theta$ is negative, θ is in quadrant II.



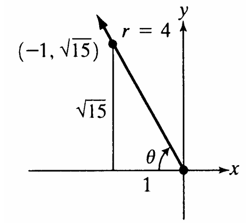
$$x^2 + y^2 = r^2$$
$$(-1)^2 + y^2 = 3^2$$
$$y^2 = 8$$
$$y = \sqrt{8}$$
$$y = 2\sqrt{2}$$

Use the right triangle to find the exact value.

$$\tan\left[\cos^{-1}\left(-\frac{1}{3}\right)\right] = \tan\theta = \frac{y}{x} = \frac{2\sqrt{2}}{-1} = -2\sqrt{2}$$

58. Let $\theta = \cos^{-1}\left(-\frac{1}{4}\right)$, then $\cos\theta = -\frac{1}{4}$.

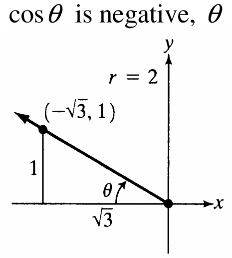
Because $\cos\theta$ is negative, θ is in quadrant II.



$$x^2 + y^2 = r^2$$
$$(-1)^2 + y^2 = 4^2$$
$$y^2 = 15$$
$$y = \sqrt{15}$$

$$\tan\left[\cos^{-1}\left(-\frac{1}{4}\right)\right] = \tan\theta = \frac{y}{x} = \frac{\sqrt{15}}{-1} = -\sqrt{15}$$

59. Let $\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, then $\cos\theta = -\frac{\sqrt{3}}{2}$. Because $\cos\theta$ is negative, θ is in quadrant II.

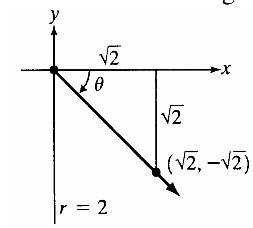


$$x^2 + y^2 = r^2$$
$$\left(-\sqrt{3}\right)^2 + y^2 = 2^2$$
$$y^2 = 1$$
$$y = 1$$

$$\csc\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \csc\theta = \frac{r}{y} = \frac{2}{1} = 2$$

60. Let $\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$, then $\sin\theta = -\frac{\sqrt{2}}{2}$.

Because $\sin\theta$ is negative, θ is in quadrant IV.

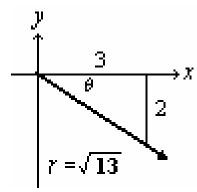


$$x^2 + y^2 = r^2$$
$$x^2 + \left(-\sqrt{2}\right)^2 = 2^2$$
$$x^2 = 2$$
$$x = \sqrt{2}$$

$$\sec\left[\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right] = \sec\theta = \frac{r}{x} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

61. Let $\theta = \tan^{-1}\left(-\frac{2}{3}\right)$, then $\tan\theta = -\frac{2}{3}$.

Because $\tan\theta$ is negative, θ is in quadrant IV.

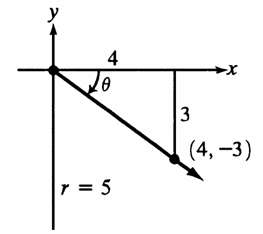


$$r^2 = x^2 + y^2$$
$$r^2 = 3^2 + (-2)^2$$
$$r^2 = 9 + 4$$
$$r^2 = 13$$
$$r = \sqrt{13}$$

$$\cos\left[\tan^{-1}\left(-\frac{2}{3}\right)\right] = \cos\theta = \frac{x}{r} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

62. Let $\theta = \tan^{-1}\left(-\frac{3}{4}\right)$, then $\tan\theta = -\frac{3}{4}$.

Because $\tan\theta$ is negative, θ is in quadrant IV.



$r^2 = x^2 + y^2$

$r^2 = 4^2 + (-3)^2$

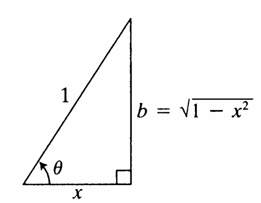
$r^2 = 16 + 9$

$r^2 = 25$

$r = 5$

$$\sin\left[\tan^{-1}\left(-\frac{3}{4}\right)\right] = \sin\theta = \frac{y}{r} = \frac{-3}{5} = -\frac{3}{5}$$

63. Let $\theta = \cos^{-1} x$, then $\cos\theta = x = \frac{x}{1}$.



Use the Pythagorean Theorem to find the third side, b.

$x^2 + b^2 = 1^2$

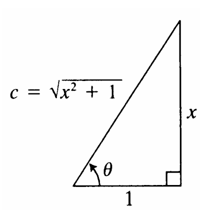
$b^2 = 1 - x^2$

$b = \sqrt{1 - x^2}$

Use the right triangle to write the algebraic expression.

$$\tan\left(\cos^{-1} x\right) = \tan\theta = \frac{\sqrt{1-x^2}}{x}$$

64. Let $\theta = \tan^{-1} x$, then $\tan\theta = x = \frac{x}{1}$.



Use the Pythagorean Theorem to find the third side, c.

$c^2 = x^2 + 1^2$

$c = \sqrt{x^2 + 1}$

Use the right triangle to write the algebraic expression.

$\sin(\tan^{-1}) = \sin\theta$

$$= \frac{x}{\sqrt{x^2+1}}$$

$$= \frac{x}{\sqrt{x^2+1}} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$$

$$= \frac{x\sqrt{x^2+1}}{x^2+1}$$

65. Let $\theta = \sin^{-1} 2x$, then $\sin\theta = 2x$

$y = 2x$, $r = 1$

Use the Pythagorean Theorem to find x.

$x^2 + (2x)^2 = 1^2$

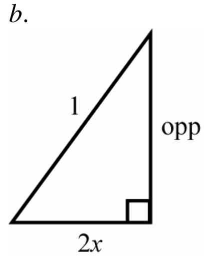
$x^2 = 1 - 4x^2$

$x = \sqrt{1 - 4x^2}$

$\cos(\sin^{-1} 2x) = \sqrt{1 - 4x^2}$

66. Let $\theta = \cos^{-1} 2x$.

Use the Pythagorean Theorem to find the third side, b.



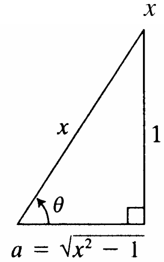
$(2x)^2 + b^2 = 1^2$

$b^2 = 1 - 4x^2$

$b = \sqrt{1 - 4x^2}$

$$\sin\left(\cos^{-1} 2x\right) = \frac{\sqrt{1-4x^2}}{1} = \sqrt{1-4x^2}$$

67. Let $\theta = \sin^{-1}\frac{1}{x}$, then $\sin\theta = \frac{1}{x}$.



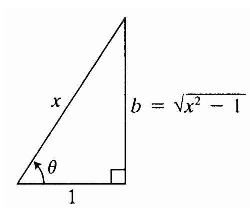
Use the Pythagorean Theorem to find the third side, a.

$$a^2 + 1^2 = x^2$$
$$a^2 = x^2 - 1$$
$$a = \sqrt{x^2 - 1}$$

Use the right triangle to write the algebraic expression.

$$\cos\left(\sin^{-1}\frac{1}{x}\right) = \cos\theta = \frac{\sqrt{x^2-1}}{x}$$

68. Let $\theta = \cos^{-1}\frac{1}{x}$, then $\cos\theta = \frac{1}{x}$.



Use the Pythagorean Theorem to find the third side, *b*.

$$1^2 + b^2 = x^2$$
$$b^2 = x^2 - 1$$
$$b = \sqrt{x^2 - 1}$$

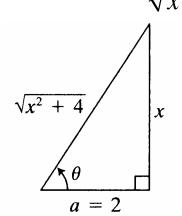
Use the right triangle to write the algebraic expression.

$$\sec\left(\cos^{-1}\frac{1}{x}\right) = \sec\theta = \frac{x}{1} = x$$

69. $\cot\left(\tan^{-1}\frac{x}{\sqrt{3}}\right) = \frac{\sqrt{3}}{x}$

70. $\cot\left(\tan^{-1}\frac{x}{\sqrt{2}}\right) = \frac{\sqrt{2}}{x}$

71. Let $\theta = \sin^{-1}\frac{x}{\sqrt{x^2+4}}$, then $\sin\theta = \frac{x}{\sqrt{x^2+4}}$.



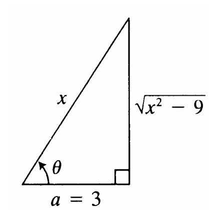
Use the Pythagorean Theorem to find the third side, *a*.

$$a^2 + x^2 = \left(\sqrt{x^2+4}\right)^2$$
$$a^2 = x^2 + 4 - x^2 = 4$$
$$a = 2$$

Use the right triangle to write the algebraic expression.

$$\sec\left(\sin^{-1}\frac{x}{\sqrt{x^2+4}}\right) = \sec\theta = \frac{\sqrt{x^2+4}}{2}$$

72. Let $\theta = \sin^{-1}\frac{\sqrt{x^2-9}}{x}$, then $\sin\theta = \frac{\sqrt{x^2-9}}{x}$.



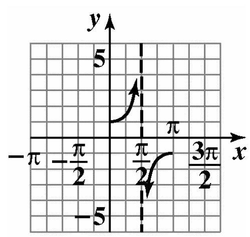
Use the Pythagorean Theorem to find the third side, *a*.

$$a^2 + \left(\sqrt{x^2-9}\right)^2 = x^2$$
$$a^2 = x^2 - x^2 + 9 = 9$$
$$a = 3$$

Use the right triangle to write the algebraic expression.

$$\cot\left(\sin^{-1}\frac{\sqrt{x^2-9}}{x}\right) = \frac{3}{\sqrt{x^2-9}}$$
$$= \frac{3}{\sqrt{x^2-9}} \cdot \frac{\sqrt{x^2-9}}{\sqrt{x^2-9}} = \frac{3\sqrt{x^2-9}}{x^2-9}$$

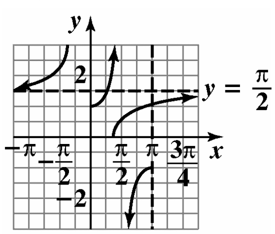
73. a. $y = \sec x$ is the reciprocal of $y = \cos x$. The x-values for the key points in the interval $[0, \pi]$ are $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4},$ and π. The key points are $(0, 1), \left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right), \left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{4}, -\frac{\sqrt{2}}{2}\right),$ and $(\pi, -1)$, Draw a vertical asymptote at $x = \frac{\pi}{2}$. Now draw our graph from (0, 1) through $\left(\frac{\pi}{4}, \sqrt{2}\right)$ to ∞ on the left side of the asymptote. From $-\infty$ on the right side of the asymptote through $\left(\frac{3\pi}{4}, -\sqrt{2}\right)$ to $(\pi, -1)$.



y = sec x

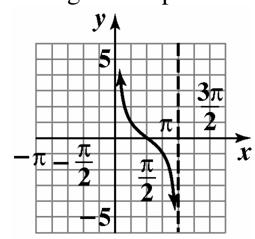
b. With this restricted domain, no horizontal line intersects the graph of $y = \sec x$ more than once, so the function is one-to-one and has an inverse function.

c. Reflecting the graph of the restricted secant function about the line $y = x$, we get the graph of $y = \sec^{-1} x$.



y = sec x
$y = \sec^{-1} x$

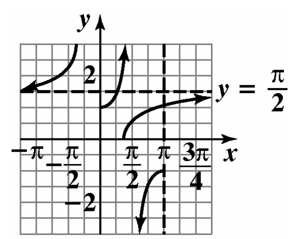
74. a. Two consecutive asymptotes occur at $x = 0$ and $x = \pi$. Midway between $x = 0$ and $x = \pi$ is $x = 0$ and $x = \pi$. An x-intercept for the graph is $\left(\frac{\pi}{2}, 0\right)$. The graph goes through the points $\left(\frac{\pi}{4}, 1\right)$ and $\left(\frac{3\pi}{4}, -1\right)$. Now graph the function through these points and using the asymptotes.



y = cot x

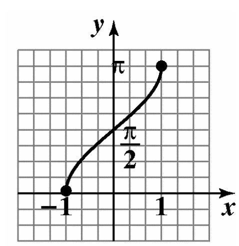
b. With this restricted domain no horizontal line intersects the graph of $y = \cot x$ more than once, so the function is one-to-one and has an inverse function. Reflecting the graph of the restricted cotangent function about the line $y = x$, we get the graph of $y = \cot^{-1} x$.

c.



y = sec x
$y = \sec^{-1} x$

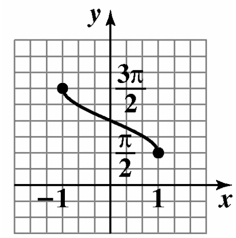
75.



$f(x) = \sin^{-1} x + \frac{\pi}{2}$

domain: $[-1, 1]$;
range: $[0, \pi]$

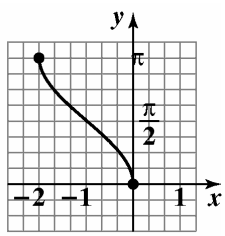
76.



$f(x) = \cos^{-1} x + \frac{\pi}{2}$

domain: $[-1, 1]$;
range: $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

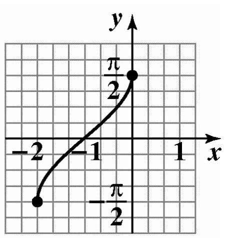
77.



$g(x) = \cos^{-1}(x + 1)$

domain: $[-2, 0]$;
range: $[0, \pi]$

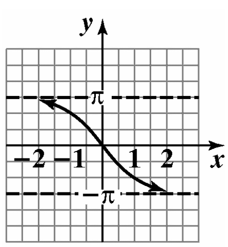
78.



$g(x) = \sin^{-1}(x + 1)$

domain: $[-2, 0]$;
range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

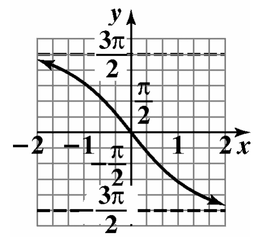
79.



$f(x) = -2\tan^{-1} x$

domain: $(-\infty, \infty)$;
range: $(-\pi, \pi)$

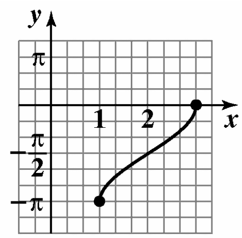
80.



$h(x) = -3\tan^{-1} x$

domain: $(-\infty, \infty)$;
range: $\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

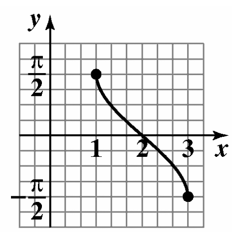
81.



$f(x) = \sin^{-1}(x - 2) - \frac{\pi}{2}$

domain: $(1, 3]$;
range: $[-\pi, 0]$

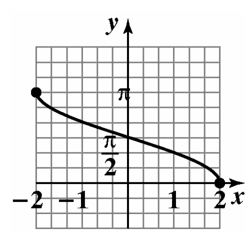
82.



$f(x) = \cos^{-1}(x - 2) - \frac{\pi}{2}$

domain: $[1, 3]$;
range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

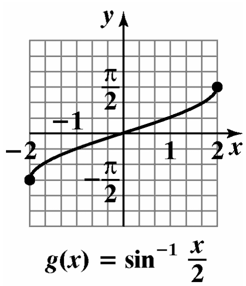
83.



$g(x) = \cos^{-1} \frac{\pi}{2}$

domain: $[-2, 2]$;
range: $[0, \pi]$

84.



domain: [–2, 2];
range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

85. The inner function, $\sin^{-1} x$, accepts values on the interval $[-1,1]$. Since the inner and outer functions are inverses of each other, the domain and range are as follows.
domain: $[-1,1]$; range: $[-1,1]$

86. The inner function, $\cos^{-1} x$, accepts values on the interval $[-1,1]$. Since the inner and outer functions are inverses of each other, the domain and range are as follows.
domain: $[-1,1]$; range: $[-1,1]$

87. The inner function, $\cos x$, accepts values on the interval $(-\infty, \infty)$. The outer function returns values on the interval $[0, \pi]$
domain: $(-\infty, \infty)$; range: $[0, \pi]$

88. The inner function, $\sin x$, accepts values on the interval $(-\infty, \infty)$. The outer function returns values on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
domain: $(-\infty, \infty)$; range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

89. The inner function, $\cos x$, accepts values on the interval $(-\infty, \infty)$. The outer function returns values on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
domain: $(-\infty, \infty)$; range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

90. The inner function, $\sin x$, accepts values on the interval $(-\infty, \infty)$. The outer function returns values on the interval $[0, \pi]$
domain: $(-\infty, \infty)$; range: $[0, \pi]$

91. The functions $\sin^{-1} x$ and $\cos^{-1} x$ accept values on the interval $[-1,1]$. The sum of these values is always $\frac{\pi}{2}$.
domain: $[-1,1]$; range: $\left\{\frac{\pi}{2}\right\}$

92. The functions $\sin^{-1} x$ and $\cos^{-1} x$ accept values on the interval $[-1,1]$. The difference of these values range from $-\frac{\pi}{2}$ to $\frac{3\pi}{2}$
domain: $[-1,1]$; range: $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$

93. $\theta = \tan^{-1}\frac{33}{x} - \tan^{-1}\frac{8}{x}$

x	θ
5	$\tan^{-1}\frac{33}{5} - \tan^{-1}\frac{8}{5} \approx 0.408$ radians
10	$\tan^{-1}\frac{33}{10} - \tan^{-1}\frac{8}{10} \approx 0.602$ radians
15	$\tan^{-1}\frac{33}{15} - \tan^{-1}\frac{8}{15} \approx 0.654$ radians
20	$\tan^{-1}\frac{33}{20} - \tan^{-1}\frac{8}{20} \approx 0.645$ radians
25	$\tan^{-1}\frac{33}{25} - \tan^{-1}\frac{8}{25} \approx 0.613$ radians

94. The viewing angle increases rapidly up to about 16 feet, then it decreases less rapidly; about 16 feet; when $x = 15$, $\theta = 0.6542$ radians; when $x = 17$, $\theta = 0.6553$ radians.

95. $\theta = 2\tan^{-1}\frac{21.634}{28} \approx 1.3157$ radians;
$1.3157\left(\frac{180}{\pi}\right) \approx 75.4°$

96. $\theta = 2\tan^{-1}\frac{21.634}{300} \approx 0.1440$ radians;

$0.1440\left(\frac{180}{\pi}\right) \approx 8.3°$

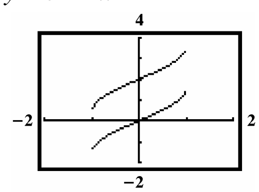
97. $\tan^{-1} b - \tan^{-1} a = \tan^{-1} 2 - \tan^{-1} 0$
≈ 1.1071 square units

98. $\tan^{-1} b - \tan^{-1} a = \tan^{-1} 1 - \tan^{-1}(-2)$
≈ 1.8925 square units

99. – 109. Answers may vary.

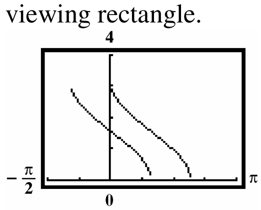
110. $y = \sin^{-1} x$

$y = \sin^{-1} x + 2$



The graph of the second equation is the graph of the first equation shifted up 2 units.

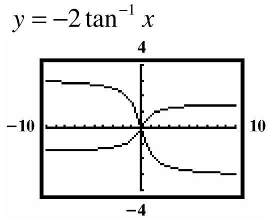
111. The domain of $y = \cos^{-1} x$ is the interval [–1, 1], and the range is the interval $[0, \pi]$. Because the second equation is the first equation with 1 subtracted from the variable, we will move our x max to π, and graph in a $\left[-\frac{\pi}{2}, \pi, \frac{\pi}{4}\right]$ by [0, 4, 1] viewing rectangle.



The graph of the second equation is the graph of the first equation shifted right 1 unit.

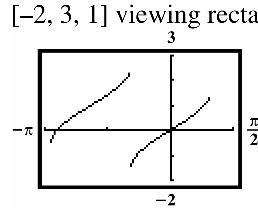
112. $y = \tan^{-1} x$

$y = -2\tan^{-1} x$



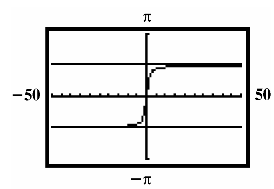
The graph of the second equation is the graph of the first equation reversed and stretched.

113. The domain of $y = \sin^{-1} x$ is the interval [–1, 1], and the range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Because the second equation is the first equation plus 1, and with 2 added to the variable, we will move our y max to 3, and move our x min to $-\pi$, and graph in a $\left[-\pi, \frac{\pi}{2}, \frac{\pi}{2}\right]$ by [–2, 3, 1] viewing rectangle.



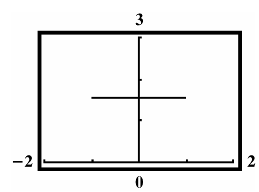
The graph of the second equation is the graph of the first equation shifted left 2 units and up 1 unit.

114. $y = \tan^{-1} x$



Observations may vary.

115.



It seems $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for $-1 \le x \le 1$.

116. does not make sense; Explanations will vary. Sample explanation: The cosine's inverse is not a function over that interval.

117. does not make sense; Explanations will vary. Sample explanation: Though this restriction works for tangent, it is not selected simply because it is easier to remember. Rather the restrictions are based on which intervals will have inverses.

118. makes sense

119. does not make sense; Explanations will vary. Sample explanation:

$$\sin^{-1}\left(\sin\frac{5\pi}{4}\right) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$$

120. $y = 2\sin^{-1}(x-5)$

$\frac{y}{2} = \sin^{-1}(x-5)$

$\sin\frac{y}{2} = x-5$

$x = \sin\frac{y}{2}+5$

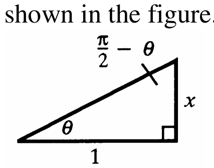
121. $2\sin^{-1}x = \frac{\pi}{4}$

$\sin^{-1}x = \frac{\pi}{8}$

$x = \sin\frac{\pi}{8}$

122. Prove: If $x > 0$, $\tan^{-1}x+\tan^{-1}\frac{1}{x} = \frac{\pi}{2}$

Since $x > 0$, there is an angle θ with $0 < \theta < \frac{\pi}{2}$ as shown in the figure.



$\tan\theta = x$ and $\tan\left(\frac{\pi}{2}-\theta\right) = \frac{1}{x}$ thus

$\tan^{-1}x = \theta$ and $\tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}-\theta$ so

$\tan^{-1}x+\tan^{-1}\frac{1}{x} = \theta+\frac{\pi}{2}-\theta = \frac{\pi}{2}$

123. Let α equal the acute angle in the smaller right triangle.

$\tan\alpha = \frac{8}{x}$

so $\tan^{-1}\frac{8}{x} = \alpha$

$\tan(\alpha+\theta) = \frac{33}{x}$

so $\tan^{-1}\frac{33}{x} = \alpha+\theta$

$\theta = \alpha+\theta-\alpha = \tan^{-1}\frac{33}{x}-\tan^{-1}\frac{8}{x}$

124. $\tan A = \frac{a}{b}$

$\tan 22.3^\circ = \frac{a}{12.1}$

$a = 12.1\tan 22.3^\circ$

$a \approx 4.96$

$\cos A = \frac{b}{c}$

$\cos 22.3^\circ = \frac{12.1}{c}$

$c = \frac{12.1}{\cos 22.3^\circ}$

$c \approx 13.08$

125. $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$\tan\theta = \frac{18}{25}$

$\theta = \tan^{-1}\left(\frac{18}{25}\right)$

$\theta \approx 35.8^\circ$

126. $10\cos\left(\frac{\pi}{6}x\right)$

amplitude: $|10| = 10$

period: $\frac{2\pi}{\frac{\pi}{6}} = 2\pi\cdot\frac{6}{\pi} = 12$

Section 5.8

Check Point Exercises

1. We begin by finding the measure of angle B. Because $C = 90°$ and the sum of a triangle's angles is 180°, we see that $A + B = 90°$. Thus, $B = 90° – A = 90° – 62.7° = 27.3°$.
Now we find b. Because we have a known angle, a known opposite side, and an unknown adjacent side, use the tangent function.

$$\tan 62.7° = \frac{8.4}{b}$$
$$b = \frac{8.4}{\tan 62.7°} \approx 4.34$$

Finally, we need to find c. Because we have a known angle, a known opposite side and an unknown hypotenuse, use the sine function.

$$\sin 62.7° = \frac{8.4}{c}$$
$$c = \frac{8.4}{\sin 62.7} \approx 9.45$$

In summary, $B = 27.3°$, $b \approx 4.34$, and $c \approx 9.45$.

2. Using a right triangle, we have a known angle, an unknown opposite side, a, and a known adjacent side. Therefore, use the tangent function.

$$\tan 85.4° = \frac{a}{80}$$
$$a = 80 \tan 85.4° \approx 994$$

The Eiffel tower is approximately 994 feet high.

3. Using a right triangle, we have an unknown angle, A, a known opposite side, and a known hypotenuse. Therefore, use the sine function.

$$\sin A = \frac{6.7}{13.8}$$
$$A = \sin^{-1} \frac{6.7}{13.8} \approx 29.0°$$

The wire makes an angle of approximately 29.0° with the ground.

4. Using two right triangles, a smaller right triangle corresponding to the smaller angle of elevation drawn inside a larger right triangle corresponding to the larger angle of elevation, we have a known angle, an unknown opposite side, a in the smaller triangle, b in the larger triangle, and a known adjacent side in each triangle. Therefore, use the tangent function.

$$\tan 32° = \frac{a}{800}$$
$$a = 800 \tan 32° \approx 499.9$$
$$\tan 35° = \frac{b}{800}$$
$$b = 800 \tan 35° \approx 560.2$$

The height of the sculpture of Lincoln's face is 560.2 – 499.9, or approximately 60.3 feet.

5. **a.** We need the acute angle between ray OD and the north-south line through O.
The measurement of this angle is given to be 25°. The angle is measured from the south side of the north-south line and lies east of the north-south line. Thus, the bearing from O to D is S 25°E.

b. We need the acute angle between ray OC and the north-south line through O.
This angle measures $90° - 75° = 15°$.
This angle is measured from the south side of the north-south line and lies west of the north-south line. Thus the bearing from O to C is S 15° W.

6. **a.** Your distance from the entrance to the trail system is represented by the hypotenuse, c, of a right triangle. Because we know the length of the two sides of the right triangle, we find c using the Pythagorean Theorem.
We have

$$c^2 = a^2 + b^2 = (2.3)^2 + (3.5)^2 = 17.54$$
$$c = \sqrt{17.54} \approx 4.2$$

You are approximately 4.2 miles from the entrance to the trail system.

b. To find your bearing from the entrance to the trail system, consider a north-south line passing through the entrance. The acute angle from this line to the ray on which you lie is $31° + \theta$. Because we are measuring the angle from the south side of the line and you are west of the entrance, your bearing from the entrance is S $(31° + \theta)$ W. To find θ, Use a right triangle and the tangent function.

$$\tan\theta = \frac{3.5}{2.3}$$

$$\theta = \tan^{-1}\frac{3.5}{2.3} \approx 56.7°$$

Thus, $31° + \theta = 31° + 56.7° = 87.7°$. Your bearing from the entrance to the trail system is S 87.7° W.

7. When the object is released $(t = 0)$, the ball's distance, d, from its rest position is 6 inches down. Because it is down, d is negative: when $t = 0$, $d = -6$. Notice the greatest distance from rest position occurs at $t = 0$. Thus, we will use the equation with the cosine function, $y = a\cos\omega t$, to model the ball's motion. Recall that $|a|$ is the maximum distance. Because the ball initially moves down, $a = -6$. The value of ω can be found using the formula for the period.

$$\text{period} = \frac{2\pi}{\omega} = 4$$

$$2\pi = 4\omega$$

$$\omega = \frac{2\pi}{4} = \frac{\pi}{2}$$

Substitute these values into $d = a\cos wt$. The equation for the ball's simple harmonic motion is

$$d = -6\cos\frac{\pi}{2}t.$$

8. We begin by identifying values for a and ω.

$$d = 12\cos\frac{\pi}{4}t,\ a = 12 \text{ and } \omega = \frac{\pi}{4}.$$

a. The maximum displacement from the rest position is the amplitude. Because $a = 12$, the maximum displacement is 12 centimeters.

b. The frequency, f, is

$$f = \frac{\omega}{2\pi} = \frac{\frac{\pi}{4}}{2\pi} = \frac{\pi}{4}\cdot\frac{1}{2\pi} = \frac{1}{8}$$

The frequency is $\frac{1}{8}$ cycle per second.

c. The time required for one cycle is the period.

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{4}} = 2\pi\cdot\frac{4}{\pi} = 8$$

The time required for one cycle is 8 seconds.

Exercise Set 5.8

1. Find the measure of angle B. Because $C = 90°$, $A + B = 90°$. Thus, $B = 90° - A = 90° - 23.5° = 66.5°$.
Because we have a known angle, a known adjacent side, and an unknown opposite side, use the tangent function.

$$\tan 23.5° = \frac{a}{10}$$

$$a = 10\tan 23.5° \approx 4.35$$

Because we have a known angle, a known adjacent side, and an unknown hypotenuse, use the cosine function.

$$\cos 23.5° = \frac{10}{c}$$

$$c = \frac{10}{\cos 23.5°} \approx 10.90$$

In summary, $B = 66.5°$, $a \approx 4.35$, and $c \approx 10.90$.

2. Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A = 90° - 41.5° = 48.5°$.
Because we have a known angle, a known adjacent side, and an unknown opposite side, use the tangent function.

$$\tan 41.5° = \frac{a}{20}$$

$$a = 20\tan 41.5° \approx 17.69$$

Because we have a known angle, a known adjacent side, and an unknown hypotenuse, use the cosine function.

$$\cos 41.5° = \frac{20}{c}$$

$$c = \frac{20}{\cos 41.5°} \approx 26.70$$

In summary, $B = 48.5°$, $a \approx 17.69$, and $c \approx 26.70$.

3. Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A = 90° - 52.6° = 37.4°$.
Because we have a known angle, a known hypotenuse, and an unknown opposite side, use the sine function.

$$\sin 52.6 = \frac{a}{54}$$
$$a = 54 \sin 52.6° \approx 42.90$$

Because we have a known angle, a known hypotenuse, and an unknown adjacent side, use the cosine function.

$$\cos 52.6° = \frac{b}{54}$$
$$b = 54 \cos 52.6° \approx 32.80$$

In summary, $B = 37.4°$, $a \approx 42.90$, and $b \approx 32.80$.

4. Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A = 90° - 54.8° = 35.2°$.
Because we have a known angle, a known hypotenuse, and an unknown opposite side, use the sine function.

$$\sin 54.8° = \frac{a}{80}$$
$$a = 80 \sin 54.8° \approx 65.37$$

Because we have a known angle, a known hypotenuse, and an unknown adjacent side, use the cosine function.

$$\cos 54.8 = \frac{b}{80}$$
$$b = 80 \cos 54.8° \approx 46.11$$

In summary, $B = 35.2°$, $a \approx 65.37$, and $c \approx 46.11$.

5. Find the measure of angle A. Because $C = 90°$, $A + B = 90°$.
Thus, $A = 90° - B = 90° - 16.8° = 73.2°$.
Because we have a known angle, a known opposite side and an unknown adjacent side, use the tangent function.

$$\tan 16.8° = \frac{30.5}{a}$$
$$a = \frac{30.5}{\tan 16.8°} \approx 101.02$$

Because we have a known angle, a known opposite side, and an unknown hypotenuse, use the sine function.

$$\sin 16.8° = \frac{30.5}{c}$$
$$c = \frac{30.5}{\sin 16.8°} \approx 105.52$$

In summary, $A = 73.2°$, $a \approx 101.02$, and $c \approx 105.52$.

6. Find the measure of angle A. Because $C = 90°$, $A + B = 90°$.
Thus, $A = 90° - B = 90° - 23.8° = 66.2°$.
Because we have a known angle, a known opposite side, and an unknown adjacent side, use the tangent function.

$$\tan 23.8° = \frac{40.5}{a}$$
$$a = \frac{40.5}{\tan 23.8°} \approx 91.83$$

Because we have a known angle, a known opposite side, and an unknown hypotenuse, use the sine function.

$$\sin 23.8° = \frac{40.5}{c}$$
$$c = \frac{40.5}{\sin 23.8°} \approx 100.36$$

In summary, $A = 66.2°$, $a \approx 91.83$, and $c \approx 100.36$.

7. Find the measure of angle A. Because we have a known hypotenuse, a known opposite side, and an unknown angle, use the sine function.

$$\sin A = \frac{30.4}{50.2}$$
$$A = \sin^{-1}\left(\frac{30.4}{50.2}\right) \approx 37.3°$$

Find the measure of angle B. Because $C = 90°$, $A + B = 90°$. Thus, $B = 90° - A \approx 90° - 37.3° = 52.7°$.
Use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$(30.4)^2 + b^2 = (50.2)^2$$
$$b^2 = (50.2)^2 - (30.4)^2 = 1595.88$$
$$b = \sqrt{1595.88} \approx 39.95$$

In summary, $A \approx 37.3°$, $B \approx 52.7°$, and $b \approx 39.95$.

8. Find the measure of angle A. Because we have a known hypotenuse, a known opposite side, and an unknown angle, use the sine function.

$$\sin A = \frac{11.2}{65.8}$$
$$A = \sin^{-1}\left(\frac{11.2}{65.8}\right) \approx 9.8°$$

Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A \approx 90° - 9.8° = 80.2°$.
Use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$(11.2)^2 + b^2 = (65.8)^2$$
$$b^2 = (65.8)^2 - (11.2)^2 = 4204.2$$
$$b = \sqrt{4204.2} \approx 64.84$$

9. Find the measure of angle A. Because we have a known opposite side, a known adjacent side, and an unknown angle, use the tangent function.

$$\tan A = \frac{10.8}{24.7}$$
$$A = \tan^{-1}\left(\frac{10.8}{24.7}\right) \approx 23.6°$$

Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A \approx 90° - 23.6° = 66.4°$.
Use the Pythagorean Theorem.

$$c^2 = a^2 + b^2 = (10.8)^2 + (24.7)^2 = 726.73$$
$$c = \sqrt{726.73} \approx 26.96$$

In summary, $A \approx 23.6°$, $B \approx 66.4°$, and $c \approx 26.96$.

10. Find the measure of angle A. Because we have a known opposite side, a known adjacent side, and an unknown angle, use the tangent function.

$$\tan A = \frac{15.3}{17.6}$$
$$A = \tan^{-1}\left(\frac{15.3}{17.6}\right) \approx 41.0°$$

Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A \approx 90° - 41.0° = 49.0°$.
Use the Pythagorean Theorem.

$$c^2 = a^2 + b^2 = (15.3)^2 + (17.6)^2 = 543.85$$
$$c = \sqrt{543.85} \approx 23.32$$

In summary, $A \approx 41.0°$, $B \approx 49.0°$, and $c \approx 23.32$.

11. Find the measure of angle A. Because we have a known hypotenuse, a known adjacent side, and unknown angle, use the cosine function.

$$\cos A = \frac{2}{7}$$
$$A = \cos^{-1}\left(\frac{2}{7}\right) \approx 73.4°$$

Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A \approx 90° - 73.4° = 16.6°$.
Use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$a^2 + (2)^2 = (7)^2$$
$$a^2 = (7)^2 - (2)^2 = 45$$
$$a = \sqrt{45} \approx 6.71$$

In summary, $A \approx 73.4°$, $B \approx 16.6°$, and $a \approx 6.71$.

12. Find the measure of angle A. Because we have a known hypotenuse, a known adjacent side, and an unknown angle, use the cosine function.

$$\cos A = \frac{4}{9}$$
$$A = \cos^{-1}\left(\frac{4}{9}\right) \approx 63.6°$$

Find the measure of angle B. Because $C = 90°$, $A + B = 90°$.
Thus, $B = 90° - A \approx 90° - 63.6° = 26.4°$.
Use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$a^2 + (4)^2 = (9)^2$$
$$a^2 = (9)^2 - (4)^2 = 65$$
$$a = \sqrt{65} \approx 8.06$$

In summary, $A \approx 63.6°$, $B \approx 26.4°$, and $a \approx 8.06$.

13. We need the acute angle between ray OA and the north-south line through O. This angle measure $90° - 75° = 15°$. This angle is measured from the north side of the north-south line and lies east of the north-south line. Thus, the bearing from O and A is N 15° E.

14. We need the acute angle between ray OB and the north-south line through O. This angle measures $90° - 60° = 30°$. This angle is measured from the north side of the north-south line and lies west of the north-south line. Thus, the bearing from O to B is N 30° W.

15. The measurement of this angle is given to be 80°. The angle is measured from the south side of the north-south line and lies west of the north-south line. Thus, the bearing from *O* to *C* is S 80° W.

16. We need the acute angle between ray OD and the north-south line through *O*. This angle measures $90° - 35° = 55°$. This angle is measured from the south side of the north-south line and lies east of the north-south line. Thus, the bearing from *O* to *D* is S 55° E.

17. When the object is released $(t = 0)$, the object's distance, *d*, from its rest position is 6 centimeters down. Because it is down, *d* is negative: When $t = 0,\ d = -6.$ Notice the greatest distance from rest position occurs at $t = 0.$ Thus, we will use the equation with the cosine function, $y = a\cos\omega t$ to model the object's motion. Recall that $|a|$ is the maximum distance. Because the object initially moves down, $a = -6$. The value of ω can be found using the formula for the period.

$$\text{period} = \frac{2\pi}{\omega} = 4$$
$$2\pi = 4\omega$$
$$\omega = \frac{2\pi}{4} = \frac{\pi}{2}$$

Substitute these values into $d = a\cos\omega t.$ The equation for the object's simple harmonic motion is $d = -6\cos\frac{\pi}{2}t.$

18. When the object is released $(t = 0)$, the object's distance, *d*, from its rest position is 8 inches down. Because it is down, *d*, is negative: When $t = 0$, $d = -8$. Notice the greatest distance from rest position occurs at $t = 0$. Thus, we will use the equation with the cosine function, $y = a\cos\omega t$, to model the object's motion. Recall that $|a|$ is the maximum distance. Because the object initially moves down, $a = -8$. The value of ω can be found using the formula for the period.

$$\text{period} = \frac{2\pi}{\omega} = 2$$
$$2\pi = 2\omega$$
$$\omega = \frac{2\pi}{2} = \pi$$

Substitute these values into $d = a\cos\omega t$.
The equation for the object's simple harmonic motion is $d = -8\cos\pi t$.

19. When $t = 0$, $d = 0$. Therefore, we will use the equation with the sine function, $y = a\sin\omega t,$ to model the object's motion. Recall that $|a|$ is the maximum distance. Because the object initially moves down, and has an amplitude of 3 inches, $a = -3$. The value of ω can be found using the formula for the period.

$$\text{period} = \frac{2\pi}{\omega} = 1.5$$
$$2\pi = 1.5\omega$$
$$\omega = \frac{2\pi}{1.5} = \frac{4\pi}{3}$$

Substitute these values into $d = a\sin\omega t.$ The equation for the object's simple harmonic motion is $d = -3\sin\frac{4\pi}{3}t.$

20. When $t = 0$, $d = 0$. Therefore, we will use the equation with the sine function, $y = a\sin\omega t,$ to model the object's motion. Recall that $|a|$ is the maximum distance. Because the object initially moves down, and has an amplitude of 5 centimeters, $a = -5$. The value of ω can be found using the formula for the period.

$$\text{period} = \frac{2\pi}{\omega} = 2.5$$
$$2\pi = 2.5\omega$$
$$\omega = \frac{2\pi}{2.5} = \frac{4\pi}{5}$$

Substitute these values into $d = a\sin\omega t$. The equation for the object's simple harmonic motion is $d = -5\sin\frac{4\pi}{5}t$.

21. We begin by identifying values for *a* and ω.

$d = 5\cos\frac{\pi}{2}t,\ a = 5$ and $\omega = \frac{\pi}{2}$

a. The maximum displacement from the rest position is the amplitude. Because $a = 5$, the maximum displacement is 5 inches.

b. The frequency, *f*, is

$$f = \frac{\omega}{2\pi} = \frac{\frac{\pi}{2}}{2\pi} = \frac{\pi}{2}\cdot\frac{1}{2\pi} = \frac{1}{4}.$$

The frequency is $\frac{1}{4}$ inch per second.

c. The time required for one cycle is the period.

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{2}} = 2\pi\cdot\frac{2}{\pi} = 4$$

The time required for one cycle is 4 seconds.

22. We begin by identifying values for a and ω.
$d = 10\cos 2\pi t,\ a = 10$ and $\omega = 2\pi$

a. The maximum displacement from the rest position is the amplitude.
Because $a = 10$, the maximum displacement is 10 inches.

b. The frequency, f, is $f = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 1$.
The frequency is 1 inch per second.

c. The time required for one cycle is the period.
period $= \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1$
The time required for one cycle is 1 second.

23. We begin by identifying values for a and ω.
$d = -6\cos 2\pi t,\ a = -6$ and $\omega = 2\pi$

a. The maximum displacement from the rest position is the amplitude.
Because $a = -6$, the maximum displacement is 6 inches.

b. The frequency, f, is
$f = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 1.$
The frequency is 1 inch per second.

c. The time required for one cycle is the period.
period $= \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1$
The time required for one cycle is 1 second.

24. We begin by identifying values for a and ω.
$d = -8\cos \frac{\pi}{2}t,\ a = -8$ and $\omega = \frac{\pi}{2}$

a. The maximum displacement from the rest position is the amplitude.
Because $a = -8$, the maximum displacement is 8 inches.

b. The frequency, f, is $f = \frac{\omega}{2\pi} = \frac{\frac{\pi}{2}}{2\pi} = \frac{1}{4}$.
The frequency is $\frac{1}{4}$ inch per second.

c. The time required for one cycle is the period.
period $= \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{2}} = 2\pi \cdot \frac{2}{\pi} = 4$
The time required for one cycle is 4 seconds.

25. We begin by identifying values for a and ω.
$d = \frac{1}{2}\sin 2t,\ a = \frac{1}{2}$ and $\omega = 2$

a. The maximum displacement from the rest position is the amplitude.
Because $a = \frac{1}{2}$, the maximum displacement is $\frac{1}{2}$ inch.

b. The frequency, f, is
$f = \frac{\omega}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi} \approx 0.32.$
The frequency is approximately 0.32 cycle per second.

c. The time required for one cycle is the period.
period $= \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi \approx 3.14$
The time required for one cycle is approximately 3.14 seconds.

26. We begin by identifying values for a and ω.
$d = \frac{1}{3}\sin 2t,\ a = \frac{1}{3}$ and $\omega = 2$

a. The maximum displacement from the rest position is the amplitude.
Because $a = \frac{1}{3}$, the maximum displacement is $\frac{1}{3}$ inch.

b. The frequency, f, is $f = \frac{\omega}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi} \approx 0.32$.
The frequency is approximately 0.32 cycle per second.

c. The time required for one cycle is the period.
period $= \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi \approx 3.14$
The time required for one cycle is approximately 3.14 seconds.

27. We begin by identifying values for a and ω.

$d = -5\sin\frac{2\pi}{3}t,\ a = -5$ and $\omega = \frac{2\pi}{3}$

a. The maximum displacement from the rest position is the amplitude.
Because $a = -5$, the maximum displacement is 5 inches.

b. The frequency, f, is

$$f = \frac{\omega}{2\pi} = \frac{\frac{2\pi}{3}}{2\pi} = \frac{2\pi}{3}\cdot\frac{1}{2\pi} = \frac{1}{3}.$$

The frequency is $\frac{1}{3}$ cycle per second.

c. The time required for one cycle is the period.

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{2\pi}{3}} = 2\pi\cdot\frac{3}{2\pi} = 3$$

The time required for one cycle is 3 seconds.

28. We begin by identifying values for a and ω.

$d = -4\sin\frac{3\pi}{2}t,\ a = -4$ and $\omega = \frac{3\pi}{2}$

a. The maximum displacement from the rest position is the amplitude.
Because $a = -4$, the maximum displacement is 4 inches.

b. The frequency, f, is

$$f = \frac{\omega}{2\pi} = \frac{\frac{3\pi}{2}}{2\pi} = \frac{3\pi}{2}\cdot\frac{1}{2\pi} = \frac{3}{4}.$$

The frequency is $\frac{3}{4}$ cycle per second.

c. The time required for one cycle is the period.

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{3\pi}{2}} = 2\pi\cdot\frac{2}{3\pi} = \frac{4}{3}$$

The required time for one cycle is $\frac{4}{3}$ seconds.

29. $x = 500\tan 40° + 500\tan 25°$
$x \approx 653$

30. $x = 100\tan 20° + 100\tan 8°$
$x \approx 50$

31. $x = 600\tan 28° - 600\tan 25°$
$x \approx 39$

32. $x = 400\tan 40° - 400\tan 28°$
$x \approx 123$

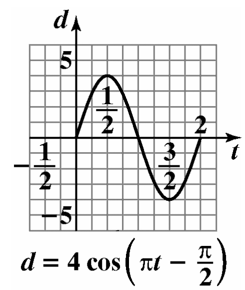
33. $x = \frac{300}{\tan 34°} - \frac{300}{\tan 64°}$
$x \approx 298$

34. $x = \frac{500}{\tan 20°} - \frac{500}{\tan 48°}$
$x \approx 924$

35. $x = \frac{400\tan 40°\tan 20°}{\tan 40° - \tan 20°}$
$x \approx 257$

36. $x = \frac{100\tan 43°\tan 38°}{\tan 43° - \tan 38°}$
$x \approx 482$

37. $d = 4\cos\left(\pi t - \frac{\pi}{2}\right)$



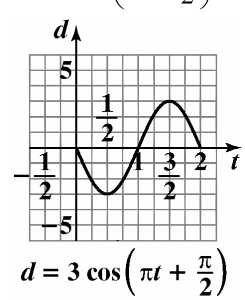
a. 4 in.

b. $\frac{1}{2}$ in. per sec

c. 2 sec

d. $\frac{1}{2}$

38. $d = 3\cos\left(\pi t + \frac{\pi}{2}\right)$



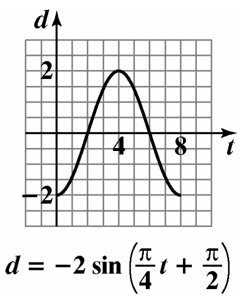
a. 3 in.

b. $\frac{1}{2}$ in. per sec

c. 2 sec

d. $-\frac{1}{2}$

39. $d = -2\sin\left(\frac{\pi t}{4} + \frac{\pi}{2}\right)$



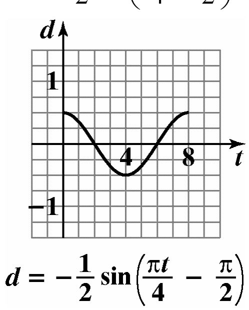
a. 2 in.

b. $\frac{1}{8}$ in. per sec

c. 8 sec

d. −2

40. $d = -\frac{1}{2}\sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right)$



a. $\frac{1}{2}$ in.

b. $\frac{1}{8}$ in. per sec

c. 8 sec

d. 2

41. Using a right triangle, we have a known angle, an unknown opposite side, *a*, and a known adjacent side. Therefore, use tangent function.

$$\tan 21.3° = \frac{a}{5280}$$
$$a = 5280\tan 21.3° \approx 2059$$

The height of the tower is approximately 2059 feet.

42. $30 \text{ yd} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} = 90 \text{ ft}$

Using a right triangle, we have a known angle, an unknown opposite side, *a*, and a known adjacent side. Therefore, use the tangent function.

$$\tan 38.7° = \frac{a}{90}$$
$$a = 90\tan 38.7° \approx 72$$

The height of the building is approximately 72 feet.

43. Using a right triangle, we have a known angle, a known opposite side, and an unknown adjacent side, *a*. Therefore, use the tangent function.

$$\tan 23.7° = \frac{305}{a}$$
$$a = \frac{305}{\tan 23.7°} \approx 695$$

The ship is approximately 695 feet from the statue's base.

44. Using a right triangle, we have a known angle, a known opposite side, and an unknown adjacent side, *a*. Therefore, use the tangent function.

$$\tan 22.3° = \frac{200}{a}$$
$$a = \frac{200}{\tan 22.3°} \approx 488$$

The ship is about 488 feet offshore.

45. The angle of depression from the helicopter to point *P* is equal to the angle of elevation from point *P* to the helicopter. Using a right triangle, we have a known angle, a known opposite side, and an unknown adjacent side, *d*. Therefore, use the tangent function.

$$\tan 36° = \frac{1000}{d}$$
$$d = \frac{1000}{\tan 36°} \approx 1376$$

The island is approximately 1376 feet off the coast.

46. The angle of depression from the helicopter to the stolen car is equal to the angle of elevation from the stolen car to the helicopter. Using a right triangle, we have a known angle, a known opposite side, and an unknown adjacent side, *d*. Therefore, use the tangent function.

$$\tan 72° = \frac{800}{d}$$
$$d = \frac{800}{\tan 72°} \approx 260$$

The stolen car is approximately 260 feet from a point directly below the helicopter.

47. Using a right triangle, we have an unknown angle, *A*, a known opposite side, and a known hypotenuse. Therefore, use the sine function.

$$\sin A = \frac{6}{23}$$
$$A = \sin^{-1}\left(\frac{6}{23}\right) \approx 15.1°$$

The ramp makes an angle of approximately 15.1° with the ground.

48. Using a right triangle, we have an unknown angle, *A*, a known opposite side, and a known adjacent side. Therefore, use the tangent function.

$$\tan A = \frac{250}{40}$$
$$A = \tan^{-1}\left(\frac{250}{40}\right) \approx 80.9°$$

The angle of elevation of the sun is approximately 80.9°.

49. Using the two right triangles, we have a known angle, an unknown opposite side, *a* in the smaller triangle, *b* in the larger triangle, and a known adjacent side in each triangle. Therefore, use the tangent function.

$$\tan 19.2° = \frac{a}{125}$$
$$a = 125 \tan 19.2° \approx 43.5$$
$$\tan 31.7° = \frac{b}{125}$$
$$b = 125 \tan 31.7° \approx 77.2$$

The balloon rises approximately 77.2 – 43.5 or 33.7 feet.

50. Using two right triangles, a smaller right triangle corresponding to the smaller angle of elevation drawn inside a larger right triangle corresponding to the larger angle of elevation, we have a known angle, an unknown opposite side, *a* in the smaller triangle, *b* in the larger triangle, and a known adjacent side in each triangle. Therefore, use the tangent function.

$$\tan 53° = \frac{a}{330}$$
$$a = 330 \tan 53° = 437.9$$
$$\tan 63° = \frac{b}{330}$$
$$b = 330 \tan 63° \approx 647.7$$

The height of the flagpole is approximately 647.7 – 437.9, or 209.8 feet.

51. Using a right triangle, we have a known angle, a known hypotenuse, and unknown sides. To find the opposite side, *a*, use the sine function.

$$\sin 53° = \frac{a}{150}$$
$$a = 150 \sin 53° \approx 120$$

To find the adjacent side, *b*, use the cosine function.

$$\cos 53° = \frac{b}{150}$$
$$b = 150 \cos 53° \approx 90$$

The boat has traveled approximately 90 miles north and 120 miles east.

52. Using a right triangle, we have a known angle, a known hypotenuse, and unknown sides. To find the opposite side, *a*, use the sine function.

$$\sin 64° = \frac{a}{40}$$
$$a = 40 \sin 64° \approx 36$$

To find the adjacent side, *b*, use the cosine function.

$$\cos 64° = \frac{b}{40}$$
$$b = 40 \cos 64° \approx 17.5$$

The boat has traveled about 17.5 mi south and 36 mi east.

53. The bearing from the fire to the second ranger is N 28° E. Using a right triangle, we have a known angle, a known opposite side, and an unknown adjacent side, *b*. Therefore, use the tangent function.

$$\tan 28° = \frac{7}{b}$$
$$b = \frac{7}{\tan 28°} \approx 13.2$$

The first ranger is 13.2 miles from the fire, to the nearest tenth of a mile.

54. The bearing from the lighthouse to the second ship is N 34° E. Using a right triangle, we have a known angle, a known opposite side, and an unknown adjacent side, *b*. Therefore, use the tangent function.

$$\tan 34° = \frac{9}{b}$$

$$b = \frac{9}{\tan 34°} \approx 13.3$$

The first ship is about 13.3 miles from the lighthouse, to the nearest tenth of a mile.

55. Using a right triangle, we have a known adjacent side, a known opposite side, and an unknown angle, *A*. Therefore, use the tangent function.

$$\tan A = \frac{1.5}{2}$$

$$A = \tan\left(\frac{1.5}{2}\right) \approx 37°$$

We need the acute angle between the ray that runs from your house through your location, and the north-south line through your house. This angle measures approximately $90° - 37° = 53°$. This angle is measured from the north side of the north-south line and lies west of the north-south line. Thus, the bearing from your house to you is N 53° W.

56. Using a right triangle, we have a known adjacent side, a known opposite side, and an unknown angle, *A*. Therefore, use the tangent function.

$$\tan A = \frac{6}{9}$$

$$A = \tan^{-1}\left(\frac{6}{9}\right) \approx 34°$$

We need the acute angle between the ray that runs from the ship through the harbor, and the north-south line through the ship. This angle measures $90° - 34° = 56°$. This angle is measured from the north side of the north-south line and lies west of the north-south line. Thus, the bearing from the ship to the harbor is
N 56° W. The ship should use a bearing of
N 56° W to sail directly to the harbor.

57. To find the jet's bearing from the control tower, consider a north-south line passing through the tower. The acute angle from this line to the ray on which the jet lies is $35° + \theta$. Because we are measuring the angle from the north side of the line and the jet is east of the tower, the jet's bearing from the tower is N $(35° + \theta)$ E. To find θ, use a right triangle and the tangent function.

$$\tan \theta = \frac{7}{5}$$

$$\theta = \tan^{-1}\left(\frac{7}{5}\right) \approx 54.5°$$

Thus, $35° + \theta = 35° + 54.5° = 89.5°$.
The jet's bearing from the control tower is
N 89.5° E.

58. To find the ship's bearing from the port, consider a north-south line passing through the port. The acute angle from this line to the ray on which the ship lies is $40° + \theta$. Because we are measuring the angle from the south side of the line and the ship is west of the port, the ship's bearing from the port is
$S\ (40° + \theta)\ W$. To find θ, use a right triangle and the tangent function.

$$\tan \theta = \frac{11}{7}$$

$$\theta = \tan^{-1}\left(\frac{11}{7}\right) \approx 57.5°$$

Thus, $40° + \theta = 40° + 57.5 = 97.5°$. Because this angle is over 90° we subtract this angle from 180° to find the bearing from the north side of the north-south line. The bearing of the ship
from the port is N 82.5° W.

59. The frequency, *f*, is $f = \frac{\omega}{2\pi}$, so

$$\frac{1}{2} = \frac{\omega}{2\pi}$$

$$\omega = \frac{1}{2} \cdot 2\pi = \pi$$

Because the amplitude is 6 feet, $a = 6$. Thus, the equation for the object's simple harmonic motion is $d = 6 \sin \pi t$.

60. The frequency, *f*, is $f = \frac{\omega}{2\pi}$, so

$$\frac{1}{4} = \frac{\omega}{2\pi}$$

$$\omega = \frac{1}{4} \cdot 2\pi = \frac{\pi}{2}$$

Because the amplitude is 8 feet, $a = 8$. Thus, the equation for the object's simple harmonic motion is

$$d = 8 \sin \frac{\pi}{2} t .$$

61. The frequency, f, is $f = \frac{\omega}{2\pi}$, so

$$264 = \frac{\omega}{2\pi}$$
$$\omega = 264 \cdot 2\pi = 528\pi$$

Thus, the equation for the tuning fork's simple harmonic motion is $d = \sin 528\pi t$.

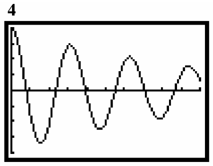
62. The frequency, f, is $f = \frac{\omega}{2\pi}$, so

$$98{,}100{,}000 = \frac{\omega}{2\pi}$$
$$\omega = 98{,}100{,}000 \cdot 2\pi = 196{,}200{,}000\pi$$

Thus, the equation for the radio waves' simple harmonic motion is $d = \sin 196{,}200{,}000\pi t$.

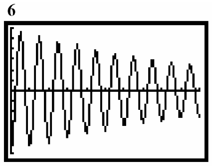
63. – 69. Answers may vary.

70. $y = 4e^{-0.1x} \cos 2x$



3 complete oscillations occur.

71. $y = -6e^{-0.09x} \cos 2\pi x$



10 complete oscillations occur.

72. makes sense

73. does not make sense; Explanations will vary. Sample explanation: When using bearings, the angle must be less than 90°.

74. does not make sense; Explanations will vary. Sample explanation: When using bearings, north and south are listed before east and west.

75. does not make sense; Explanations will vary. Sample explanation: Frequency and Period are inverses of each other. If the period is 10 seconds then the frequency is $\frac{1}{10} = 0.1$ oscillations per second.

76. Using the right triangle, we have a known angle, an unknown opposite side, r, and an unknown hypotenuse, $r + 112$. Because both sides are in terms of the variable r, we can find r by using the sine function.

$$\sin 76.6^\circ = \frac{r}{r+112}$$
$$\sin 76.6^\circ(r+112) = r$$
$$r\sin 76.6^\circ + 112\sin 76.6^\circ = r$$
$$r - r\sin 76.6^\circ = 112\sin 76.6^\circ$$
$$r\left(1-\sin 76.6^\circ\right) = 112\sin 76.6^\circ$$
$$r = \frac{112\sin 76.6^\circ}{1-\sin 76.6^\circ} \approx 4002$$

The Earth's radius is approximately 4002 miles.

77. Let d be the adjacent side to the 40° angle. Using the right triangles, we have a known angle and unknown sides in both triangles. Use the tangent function.

$$\tan 20^\circ = \frac{h}{75+d}$$
$$h = (75+d)\tan 20^\circ$$

Also, $\tan 40^\circ = \frac{h}{d}$

$$h = d\tan 40^\circ$$

Using the transitive property we have

$$(75+d)\tan 20^\circ = d\tan 40^\circ$$
$$75\tan 20^\circ + d\tan 20^\circ = d\tan 40^\circ$$
$$d\tan 40^\circ - d\tan 20^\circ = 75\tan 20^\circ$$
$$d(\tan 40^\circ - \tan 20^\circ) = 75\tan 20^\circ$$
$$d = \frac{75\tan 20^\circ}{\tan 40^\circ - \tan 20^\circ}$$

Thus, $h = d\tan 40^\circ$

$$= \frac{75\tan 20^\circ}{\tan 40^\circ - \tan 20^\circ}\tan 40^\circ \approx 48$$

The height of the building is approximately 48 feet.

78. Answers may vary.

79. $\sec x \cot x = \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} = \frac{1}{\sin x}$ or $\csc x$

80. $\tan x \csc x \cos x = \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \cdot \frac{\cos x}{1} = 1$

81. $\sec x + \tan x = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \frac{1+\sin x}{\cos x}$

Chapter 5 Review Exercises

1. The radian measure of a central angle is the length of the intercepted arc divided by the circle's radius.

$\theta = \frac{27}{6} = 4.5$ radians

2. $15° = 15° \cdot \frac{\pi \text{ radians}}{180°} = \frac{15\pi}{180}$ radian

$= \frac{\pi}{12}$ radian

3. $120° = 120° \cdot \frac{\pi \text{ radians}}{180°} = \frac{120\pi}{180}$ radians

$= \frac{2\pi}{3}$ radians

4. $315° = 315° \cdot \frac{\pi \text{ radians}}{180°} = \frac{315\pi}{180}$ radians

$= \frac{7\pi}{4}$ radians

5. $\frac{5\pi}{3}$ radians $= \frac{5\pi}{3}$ radians $\cdot \frac{180°}{\pi \text{ radians}}$

$= \frac{5 \cdot 180°}{3} = 300°$

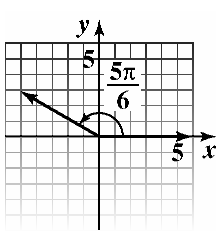
6. $\frac{7\pi}{5}$ radians $= \frac{7\pi}{5}$ radians $\cdot \frac{180°}{\pi \text{ radians}}$

$= \frac{7 \cdot 180°}{5} = 252°$

7. $-\frac{5\pi}{6}$ radians $= -\frac{5\pi}{6}$ radians $\cdot \frac{180°}{\pi \text{ radians}}$

$= -\frac{5 \cdot 180°}{6} = -150°$

8.



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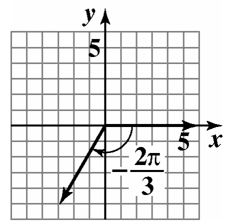

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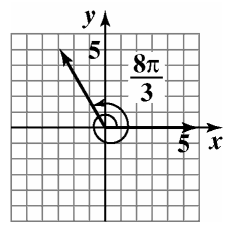

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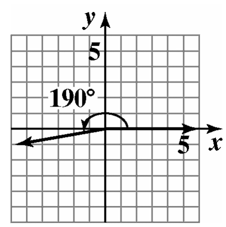

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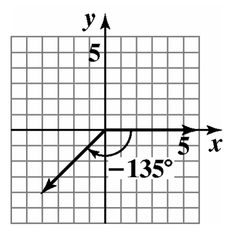


13. $400° - 360° = 40°$

14. $-445° + (2)360° = 275°$

15. $\frac{13\pi}{4} - 2\pi = \frac{13\pi}{4} - \frac{8\pi}{4} = \frac{5\pi}{4}$

16. $\frac{31\pi}{6} - (2)2\pi = \frac{31\pi}{6} - \frac{24\pi}{6} = \frac{7\pi}{6}$

17. $-\frac{8\pi}{3} + (2)2\pi = -\frac{8\pi}{3} + \frac{12\pi}{3} = \frac{4\pi}{3}$

18. $135° = 135° \cdot \frac{\pi \text{ radians}}{180°} = \frac{135 \cdot \pi}{180}$ radians

$= \frac{3\pi}{4}$ radians

$s = r\theta$

$s = (10 \text{ ft})\left(\frac{3\pi}{4}\right) = \frac{15\pi}{2} \text{ ft} \approx 23.56 \text{ ft}$

19. $\dfrac{10.3 \text{ revolutions}}{1 \text{ minute}} \cdot \dfrac{2\pi \text{ radians}}{1 \text{ revolution}}$
$= \dfrac{20.6\pi \text{ radians}}{1 \text{ minute}} = 20.6\pi$ radians per minute

20. Use $v = r\omega$ where v is the linear speed and ω is the angular speed in radians per minute.
$\omega = \dfrac{2250 \text{ revolutions}}{1 \text{ minute}} \cdot \dfrac{2\pi \text{ radians}}{1 \text{ revolution}}$
$= 4500\pi$ radians per minute
$v = 3 \text{ feet } \dfrac{4500\pi}{\text{minute}} = \dfrac{13{,}500\pi \text{ feet}}{\text{min}}$
$\approx 42{,}412$ ft per min

21. Use the Pythagorean Theorem to find the hypotenuse, c.
$c^2 = a^2 + b^2$
$c = \sqrt{8^2 + 5^2} = \sqrt{64 + 25} = \sqrt{89}$
$\sin\theta = \dfrac{5}{\sqrt{89}} = \dfrac{5\sqrt{89}}{\sqrt{89}}$
$\cos\theta = \dfrac{8}{\sqrt{89}} = \dfrac{8\sqrt{89}}{\sqrt{89}}$
$\tan\theta = \dfrac{5}{8}$
$\csc\theta = \dfrac{\sqrt{89}}{5}$
$\sec\theta = \dfrac{\sqrt{89}}{8}$
$\cot\theta = \dfrac{3}{5}$

22. $\sin\dfrac{\pi}{6} + \tan^2\dfrac{\pi}{3} = \dfrac{1}{2} + \left(\sqrt{3}\right)^2$
$= \dfrac{1}{2} + 3$
$= \dfrac{7}{2}$

23. $\cos^2\dfrac{\pi}{4} + \tan^2\dfrac{\pi}{4} = \left(\dfrac{\sqrt{2}}{2}\right)^2 - (1)^2$
$= \dfrac{1}{2} - 1$
$= -\dfrac{1}{2}$

24. $\sec^2\dfrac{\pi}{5} - \tan^2\dfrac{\pi}{5} = 1$

25. $\cos\dfrac{2\pi}{9}\sec\dfrac{2\pi}{9} = 1$

26. We can find the value of $\cos\theta$ by using the Pythagorean identity.
$\sin^2\theta + \cos^2\theta = 1$
$\left(\dfrac{2\sqrt{7}}{7}\right)^2 + \cos^2\theta = 1$
$\dfrac{4}{7} + \cos^2\theta = 1$
$\cos^2\theta = 1 - \dfrac{4}{7}$
$\cos^2\theta = \dfrac{3}{7}$
$\cos\theta = \dfrac{\sqrt{3}}{\sqrt{7}} = \dfrac{\sqrt{3}}{\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{\sqrt{21}}{7}$
Thus, $\cos\theta = \dfrac{\sqrt{21}}{7}$.

27. $\sin 70° = \cos(90° - 70°) = \cos 20°$

28. $\cos\dfrac{\pi}{2} = \sin\left(\dfrac{\pi}{2} - \dfrac{\pi}{2}\right) = \sin 0$

29. $\tan 23° = \dfrac{a}{100}$
$a = 100\tan 23°$
$a \approx 100(0.4245) \approx 42\,\text{mm}$

30. $\sin 61° = \dfrac{20}{c}$
$c = \dfrac{20}{\sin 61°}$
$c \approx \dfrac{20}{0.8746} \approx 23\,\text{cm}$

31. $\sin 48° = \dfrac{a}{50}$
$a = 50\sin 48°$
$a \approx 50(0.7431) \approx 37\,\text{in.}$

32. $\sin\theta = \frac{y}{r} = \frac{1}{4}$

$x^2 + y^2 = r^2$

$x^2 + 1^2 = 4^2$

$x^2 = 15$

$x = \sqrt{15}$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta = \frac{x}{y} = \frac{\sqrt{15}}{1} = \sqrt{15}$

33. $\frac{1}{2}$ mi. $= \frac{1}{2} \cdot 5280$ ft $= 2640$ ft

$\sin 17° = \frac{a}{2640}$

$a = 2640 \cdot \sin 17°$

$a \approx 2640(0.2924) \approx 772$

The hiker gains 772 feet of altitude.

34. $\tan 32° = \frac{d}{50}$

$d = 50\tan 32°$

$d \approx 50(0.6249) \approx 31$

The distance across the lake is about 31 meters.

35. $\tan\theta = \frac{6}{4}$

Use a calculator in degree mode to find θ.

Scientific Calculator
6 [÷] 4 [=] [TAN^{-1}]

Graphing Calculator
[TAN^{-1}] [(] 6 [÷] 4 [)] [ENTER]

The display should show approximately 56. Thus, the angle of elevation of the sun is approximately 56°.

36. We need values for x, y, and r. Because $P = (-1, -5)$ is a point on the terminal side of θ, $x = -1$ and $y = -5$. Furthermore,

$r = \sqrt{(-1)^2 + (-5)^2}$

$= \sqrt{1+25} = \sqrt{26}$

Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{26}} = \frac{-5\sqrt{26}}{\sqrt{26}\cdot\sqrt{26}} = -\frac{5\sqrt{26}}{26}$

$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{26}} = \frac{-1\sqrt{26}}{\sqrt{26}\cdot\sqrt{26}} = -\frac{\sqrt{26}}{26}$

$\tan\theta = \frac{y}{x} = \frac{-5}{-1} = 5$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{26}}{-5} = -\frac{\sqrt{26}}{5}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{26}}{-1} = -\sqrt{26}$

$\cot\theta = \frac{x}{y} = \frac{-1}{-5} = \frac{1}{5}$

37. We need values for x, y, and r. Because $P = (0, -1)$ is a point on the terminal side of θ, $x = 0$ and $y = -1$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{0^2 + (-1)^2}$

$= \sqrt{0+1} = \sqrt{1} = 1$

Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{-1}{1} = -1$

$\cos\theta = \frac{x}{r} = \frac{0}{1} = 0$

$\tan\theta = \frac{y}{x} = \frac{-1}{0}$, undefined

$\csc\theta = \frac{r}{y} = \frac{1}{-1} = -1$

$\sec\theta = \frac{r}{x} = \frac{1}{0}$, undefined

$\cot\theta = \frac{x}{y} = \frac{0}{-1} = 0$

38. Because $\tan\theta > 0$, θ cannot lie in quadrant II and quadrant IV; the tangent function is negative in those two quadrants. Thus, with $\tan\theta > 0$, θ lies in quadrant I or quadrant III. We are also given that $\sec\theta > 0$. Because quadrant I is the only quadrant in which the tangent is positive and the secant is positive, we conclude that θ lies in quadrant I.

39. Because $\tan\theta > 0$, θ cannot lie in quadrant II and quadrant IV; the tangent function is negative in those two quadrants. Thus, with $\tan\theta > 0$, θ lies in quadrant I or quadrant III. We are also given that $\cos\theta < 0$. Because quadrant III is the only quadrant in which the tangent is positive and the cosine is negative, we conclude that θ lies in quadrant III.

40. Because the cosine is positive and the sine is negative, θ lies in quadrant IV. In quadrant IV, x is positive and y is negative. Thus, $\cos\theta = \frac{2}{5} = \frac{x}{r}$, $x = 2$, $r = 5$. Furthermore,

$$x^2 + y^2 = r^2$$
$$2^2 + y^2 = 5^2$$
$$y^2 = 25 - 4 = 21$$
$$y = -\sqrt{21}$$

Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-\sqrt{21}}{5} = -\frac{\sqrt{21}}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{-\sqrt{21}}{2} = -\frac{\sqrt{21}}{2}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{-\sqrt{21}} = -\frac{5\cdot\sqrt{21}}{\sqrt{21}\cdot\sqrt{21}} = -\frac{5\sqrt{21}}{21}$$

$$\sec\theta = \frac{r}{x} = \frac{5}{2}$$

$$\cot\theta = \frac{x}{y} = \frac{2}{-\sqrt{21}} = -\frac{2\sqrt{21}}{\sqrt{21}\cdot\sqrt{21}} = -\frac{2\sqrt{21}}{21}$$

41. Because the tangent is negative and the sine is positive, θ lies in quadrant II. In quadrant II x is negative and y is positive. Thus,

$\tan\theta = -\frac{1}{3} = \frac{y}{x} = \frac{1}{-3}$, $x = -3$, $y = 1$.

Furthermore,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{1\cdot\sqrt{10}}{\sqrt{10}\cdot\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{10}} = -\frac{3\sqrt{10}}{\sqrt{10}\cdot\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{1} = \sqrt{10}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-3}{1} = -3$$

42. Because the cotangent is positive and the cosine is negative, θ lies in quadrant III. In quadrant III x and y are both negative. Thus,

$\cot\theta = \frac{3}{1} = \frac{x}{y} = \frac{-3}{-1}$, $x = -3$, $y = -1$.

Furthermore,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$

$$\tan\theta = \frac{y}{x} = \frac{-1}{-3} = \frac{1}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{-1} = -\sqrt{10}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3}$$

43. Because $265°$ lies between 180° and 270°, it is in quadrant III.
The reference angle is $\theta' = 265° - 180° = 85°$.

44. Because $\frac{5\pi}{8}$ lies between $\frac{\pi}{2} = \frac{4\pi}{8}$ and $\pi = \frac{8\pi}{8}$, it is in quadrant II.
The reference angle is $\theta' = \pi - \frac{5\pi}{8} = \frac{8\pi}{8} - \frac{5\pi}{8} = \frac{3\pi}{8}$.

45. Find the coterminal angle: $-410^\circ + (2)360^\circ = 310^\circ$
Find the reference angle: $360^\circ - 310^\circ = 50^\circ$

46. Find the coterminal angle: $\frac{17\pi}{6} - 2\pi = \frac{5\pi}{6}$
Find the reference angle: $2\pi - \frac{5\pi}{6} = \frac{\pi}{6}$

47. Find the coterminal angle: $-\frac{11\pi}{3} + 4\pi = \frac{\pi}{3}$
Find the reference angle: $\frac{\pi}{3}$

48. 240° lies in quadrant III.
The reference angle is
$\theta' = 240° - 180° = 60°$.
$\sin 60° = \frac{\sqrt{3}}{2}$
In quadrant III, $\sin\theta < 0$, so
$\sin 240° = -\sin 60° = -\frac{\sqrt{3}}{2}$.

49. 120° lies in quadrant II.
The reference angle is
$\theta' = 180° - 120° = 60°$.
$\tan 60° = \sqrt{3}$
In quadrant II, $\tan\theta < 0$, so
$\tan 120° = -\tan 60° = -\sqrt{3}$.

50. $\frac{7\pi}{4}$ lies in quadrant IV.
The reference angle is
$\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$.
$\sec\frac{\pi}{4} = \sqrt{2}$
In quadrant IV, $\sec\theta > 0$, so
$\sec\frac{7\pi}{4} = \sec\frac{\pi}{4} = \sqrt{2}$.

51. $\frac{11\pi}{6}$ lies in quadrant IV.
The reference angle is
$\theta' = 2\pi - \frac{11\pi}{6} = \frac{12\pi}{6} - \frac{11\pi}{6} = \frac{\pi}{6}$.
$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$
In quadrant IV, $\cos\theta > 0$, so $\cos\frac{11\pi}{6} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

52. –210° lies in quadrant II.
The reference angle is
$\theta' = 210° - 180° = 30°$.
$\cot 30° = \sqrt{3}$
In quadrant II, $\cot\theta < 0$, so
$\cot(-210°) = -\cot 30° = -\sqrt{3}$.

53. $-\frac{2\pi}{3}$ lies in quadrant III.
The reference angle is
$\theta' = \pi + \frac{-2\pi}{3} = \frac{3\pi}{3} - \frac{2\pi}{3} = \frac{\pi}{3}$.
$\csc\left(\frac{\pi}{3}\right) = \frac{2\sqrt{3}}{3}$
In quadrant III, $\csc\theta < 0$, so
$\csc\left(-\frac{2\pi}{3}\right) = -\csc\left(\frac{\pi}{3}\right) = -\frac{2\sqrt{3}}{3}$.

54. $-\frac{\pi}{3}$ lies in quadrant IV.
The reference angle is
$\theta' = \frac{\pi}{3}$.
$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$
In quadrant IV, $\sin\theta < 0$, so
$\sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$.

55. 495° lies in quadrant II.
$495° - 360° = 135°$
The reference angle is
$\theta' = 180° - 135° = 45°$.

$\sin 45° = \frac{\sqrt{2}}{2}$

In quadrant II, $\sin\theta > 0$, so

$\sin 495° = \sin 45° = \frac{\sqrt{2}}{2}$.

56. $\frac{13\pi}{4}$ lies in quadrant III.

$\frac{13\pi}{4} - 2\pi = \frac{13\pi}{4} - \frac{8\pi}{4} = \frac{5\pi}{4}$

The reference angle is

$\theta' = \frac{5\pi}{4} - \pi = \frac{5\pi}{4} - \frac{4\pi}{4} = \frac{\pi}{4}$.

$\tan\frac{\pi}{4} = 1$

In quadrant III, $\tan\theta > 0$, so $\tan\frac{13\pi}{4} = \tan\frac{\pi}{4} = 1$.

57.
$$\sin\frac{22\pi}{3} = \sin\left(\frac{22\pi}{3} - 6\pi\right) = \sin\frac{4\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

58.
$$\cos\left(-\frac{35\pi}{6}\right) = \cos\left(-\frac{35\pi}{6} + 6\pi\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

59. The equation $y = 3\sin 4x$ is of the form $y = A\sin Bx$ with $A = 3$ and $B = 4$. The amplitude is $|A| = |3| = 3$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{4} = \frac{\pi}{2}$. The quarter-period is $\frac{\frac{\pi}{2}}{4} = \frac{\pi}{2}\cdot\frac{1}{4} = \frac{\pi}{8}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{8} = \frac{\pi}{8}$

$x = \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi}{4}$

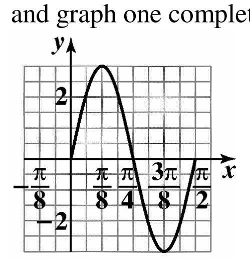
$x = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$

$x = \frac{3\pi}{8} + \frac{\pi}{8} = \frac{\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
0	(0, 0)
$\frac{\pi}{8}$	$\left(\frac{\pi}{8}, 3\right)$
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{3\pi}{8}$	$\left(\frac{3\pi}{8}, -3\right)$
$\frac{\pi}{2}$	$(2\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = 3 \sin 4x$

60. The equation $y=-2\cos 2x$ is of the form $y=A\cos Bx$ with $A = -2$ and $B = 2$. The amplitude is $|A|=|-2|=2$. The period is $\frac{2\pi}{B}=\frac{2\pi}{2}=\pi$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x=0$

$x=0+\frac{\pi}{4}=\frac{\pi}{4}$

$x=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$

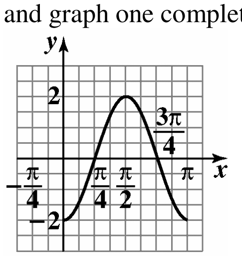
$x=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$

$x=\frac{3\pi}{4}+\frac{\pi}{4}=\pi$

Evaluate the function at each value of x.

x	coordinates
0	$(0, -2)$
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 2\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, 0\right)$
π	$(\pi, -2)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



61. The equation $y=2\cos\frac{1}{2}x$ is of the form $y=A\cos Bx$ with $A = 2$ and $B=\frac{1}{2}$. The amplitude is $|A|=|2|=2$. The period is $\frac{2\pi}{B}=\frac{2\pi}{\frac{1}{2}}=2\pi\cdot 2=4\pi$.

The quarter-period is $\frac{4\pi}{4}=\pi$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x=0$

$x=0+\pi=\pi$

$x=\pi+\pi=2\pi$

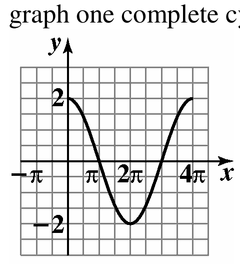
$x=2\pi+\pi=3\pi$

$x=3\pi+\pi=4\pi$

Evaluate the function at each value of x.

x	coordinates
0	$(0, 2)$
π	$(\pi, 0)$
2π	$(2\pi, -2)$
3π	$(3\pi, 0)$
4π	$(4\pi, 2)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



62. The equation $y = \frac{1}{2}\sin\frac{\pi}{3}x$ is of the form $y = A\sin Bx$ with $A = \frac{1}{2}$ and $B = \frac{\pi}{3}$. The amplitude is $|A| = \left|\frac{1}{2}\right| = \frac{1}{2}$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{3}} = 2\pi \cdot \frac{3}{\pi} = 6.$ The quarter-period is $\frac{6}{4} = \frac{3}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3}{2} = \frac{3}{2}$

$x = \frac{3}{2} + \frac{3}{2} = 3$

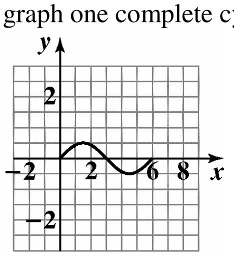
$x = 3 + \frac{3}{2} = \frac{9}{2}$

$x = \frac{9}{2} + \frac{3}{2} = 6$

Evaluate the function at each value of x.

x	coordinates
0	(0, 0)
$\frac{3}{2}$	$\left(\frac{3}{2}, \frac{1}{2}\right)$
3	(3, 0)
$\frac{9}{2}$	$\left(\frac{9}{2}, -\frac{1}{2}\right)$
6	(6, 0)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = \frac{1}{2}\sin\frac{\pi}{3}x$

63. The equation $y = -\sin\pi x$ is of the form $y = A\sin Bx$ with $A = -1$ and $B = \pi$. The amplitude is $|A| = |-1| = 1$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$. The quarter-period is $\frac{2}{4} = \frac{1}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{1}{2} = \frac{1}{2}$

$x = \frac{1}{2} + \frac{1}{2} = 1$

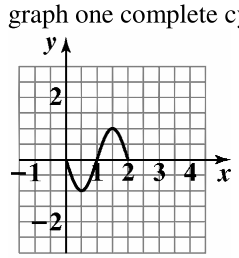
$x = 1 + \frac{1}{2} = \frac{3}{2}$

$x = \frac{3}{2} + \frac{1}{2} = 2$

Evaluate the function at each value of x.

x	coordinates
0	(0, 0)
$\frac{1}{2}$	$\left(\frac{1}{2}, -1\right)$
1	(1, 0)
$\frac{3}{2}$	$\left(\frac{3}{2}, 1\right)$
2	(2, 0)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -\sin\pi x$

64. The equation $y = 3\cos\frac{x}{3}$ is of the form $y = A\cos Bx$ with $A = 3$ and $B = \frac{1}{3}$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{3}} = 2\pi \cdot 3 = 6\pi$. The quarter-period is $\frac{6\pi}{4} = \frac{3\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{3\pi}{2} = 3\pi$

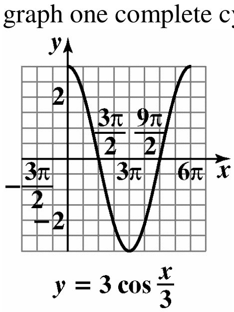
$x = 3\pi + \frac{3\pi}{2} = \frac{9\pi}{2}$

$x = \frac{9\pi}{2} + \frac{3\pi}{2} = 6\pi$

Evaluate the function at each value of x.

x	coordinates
0	(0, 3)
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, 0\right)$
3π	$(3\pi, -3)$
$\frac{9\pi}{2}$	$\left(\frac{9\pi}{2}, 0\right)$
6π	$(6\pi, 3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



65. The equation $y = 2\sin(x - \pi)$ is of the form $y = A\sin(Bx - C)$ with $A = 2$, $B = 1$, and $C = \pi$. The amplitude is $|A| = |2| = 2$. The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{1} = \pi$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$.

The cycle begins at $x = \pi$. Add quarter-periods to generate x-values for the key points.

$x = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

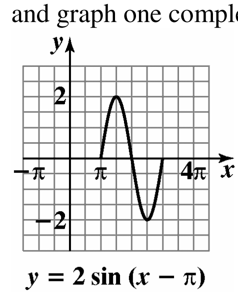
$x = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$

$x = \frac{5\pi}{2} + \frac{\pi}{2} = 3\pi$

Evaluate the function at each value of x.

x	coordinates
π	$(\pi, 0)$
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, 2\right)$
2π	$(2\pi, 0)$
$\frac{5\pi}{2}$	$\left(\frac{5\pi}{2}, -2\right)$
3π	$(3\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



66. $y = -3\cos(x+\pi) = -3\cos(x-(-\pi))$

The equation $y = -3\cos(x-(-\pi))$ is of the form $y = A\cos(Bx - C)$ with $A = -3$, $B = 1$, and $C = -\pi$. The amplitude is $|A| = |-3| = 3$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{-\pi}{1} = -\pi$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The cycle begins at $x = -\pi$. Add quarter-periods to generate x-values for the key points.

$x = -\pi$

$x = -\pi + \frac{\pi}{2} = -\frac{\pi}{2}$

$x = -\frac{\pi}{2} + \frac{\pi}{2} = 0$

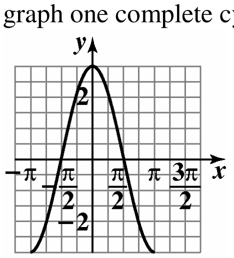
$x = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

Evaluate the function at each value of x.

x	coordinates
$-\pi$	$(-\pi, -3)$
$-\frac{\pi}{2}$	$\left(-\frac{\pi}{2}, 0\right)$
0	(0, 3)
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 0\right)$
π	$(\pi, -3)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -3\cos(x+\pi)$

67. $y = \frac{3}{2}\cos\left(2x + \frac{\pi}{4}\right) = \frac{3}{2}\cos\left(2x - \left(-\frac{\pi}{4}\right)\right)$

The equation $y = \frac{3}{2}\cos\left(2x - \left(-\frac{\pi}{4}\right)\right)$ is of the form $y = A\cos(Bx - C)$ with $A = \frac{3}{2}$, $B = 2$, and $C = -\frac{\pi}{4}$. The amplitude is $|A| = \left|\frac{3}{2}\right| = \frac{3}{2}$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{-\frac{\pi}{4}}{2} = -\frac{\pi}{4}\cdot\frac{1}{2} = -\frac{\pi}{8}$. The quarter-period is $\frac{\pi}{4}$.

The cycle begins at $x = -\frac{\pi}{8}$. Add quarter-periods to generate x-values for the key points.

$x = -\frac{\pi}{8}$

$x = -\frac{\pi}{8} + \frac{\pi}{4} = \frac{\pi}{8}$

$x = \frac{\pi}{8} + \frac{\pi}{4} = \frac{3\pi}{8}$

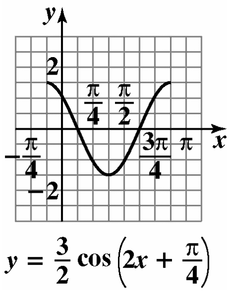
$x = \frac{3\pi}{8} + \frac{\pi}{4} = \frac{5\pi}{8}$

$x = \frac{5\pi}{8} + \frac{\pi}{4} = \frac{7\pi}{8}$

Evaluate the function at each value of x.

x	coordinates
$-\frac{\pi}{8}$	$\left(-\frac{\pi}{8}, \frac{3}{2}\right)$
$\frac{\pi}{8}$	$\left(\frac{\pi}{8}, 0\right)$
$\frac{3\pi}{8}$	$\left(\frac{3\pi}{8}, -\frac{3}{2}\right)$
$\frac{5\pi}{8}$	$\left(\frac{5\pi}{8}, 0\right)$
$\frac{7\pi}{8}$	$\left(\frac{7\pi}{8}, \frac{3}{2}\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



68. $y=\frac{5}{2}\sin\left(2x+\frac{\pi}{2}\right)=\frac{5}{2}\sin\left(2x-\left(-\frac{\pi}{2}\right)\right)$

The equation $y=\frac{5}{2}\sin\left(2x-\left(-\frac{\pi}{2}\right)\right)$ is of the form $y=A\sin(Bx-C)$ with $A=\frac{5}{2}$, $B=2$, and $C=-\frac{\pi}{2}$. The amplitude is $|A|=\left|\frac{5}{2}\right|=\frac{5}{2}$.

The period is $\frac{2\pi}{B}=\frac{2\pi}{2}=\pi$. The phase shift is $\frac{C}{B}=\frac{-\frac{\pi}{2}}{2}=-\frac{\pi}{2}\cdot\frac{1}{2}=-\frac{\pi}{4}$. The quarter-period is $\frac{\pi}{4}$.

The cycle begins at $x=-\frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x=-\frac{\pi}{4}$

$x=-\frac{\pi}{4}+\frac{\pi}{4}=0$

$x=0+\frac{\pi}{4}=\frac{\pi}{4}$

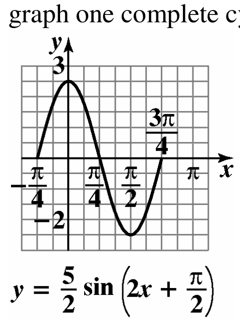
$x=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2}$

$x=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$

Evaluate the function at each value of x.

x	coordinates
$-\frac{\pi}{4}$	$(-\frac{\pi}{4}, 0)$
0	$\left(0, \frac{5}{2}\right)$
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, -\frac{5}{2}\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, 0\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



69. The equation $y=-3\sin\left(\frac{\pi}{3}x-3\pi\right)$ is of the form $y=A\sin(Bx-C)$ with $A=-3$, $B=\frac{\pi}{3}$, and $C=3\pi$. The amplitude is $|A|=|-3|=3$.

The period is $\frac{2\pi}{B}=\frac{2\pi}{\frac{\pi}{3}}=2\pi\cdot\frac{3}{\pi}=6$. The phase shift is $\frac{C}{B}=\frac{3\pi}{\frac{\pi}{3}}=3\pi\cdot\frac{3}{\pi}=9$. The quarter-period is $\frac{6}{4}=\frac{3}{2}$. The cycle begins at $x=9$. Add quarter-periods to generate x-values for the key points.

$x = 9$

$x = 9 + \frac{3}{2} = \frac{21}{2}$

$x = \frac{21}{2} + \frac{3}{2} = 12$

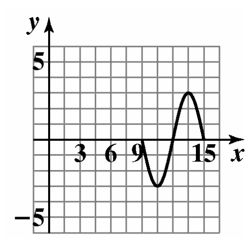
$x = 12 + \frac{3}{2} = \frac{27}{2}$

$x = \frac{27}{2} + \frac{3}{2} = 15$

Evaluate the function at each value of *x*.

x	coordinates
9	(9, 0)
$\frac{21}{2}$	$\left(\frac{21}{2}, -3\right)$
12	(12, 0)
$\frac{27}{2}$	$\left(\frac{27}{2}, 3\right)$
15	(15, 0)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



$y = -3\sin\left(\frac{\pi}{3}x - 3\pi\right)$

70. The graph of $y = \sin 2x + 1$ is the graph of $y = \sin 2x$ shifted one unit upward. The period for both functions is $\frac{2\pi}{2} = \pi$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = 0$. Add quarter-periods to generate *x*-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

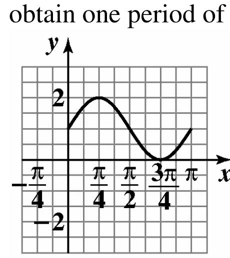
$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

Evaluate the function at each value of *x*.

x	coordinates
0	(0, 1)
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, 2\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 1\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, 0\right)$
π	$(\pi, 1)$

By connecting the points with a smooth curve we obtain one period of the graph.



$y = \sin 2x + 1$

71. The graph of $y = 2\cos\frac{1}{3}x - 2$ is the graph of $y = 2\cos\frac{1}{3}x$ shifted two units downward. The period for both functions is $\frac{2\pi}{\frac{1}{3}} = 2\pi \cdot 3 = 6\pi$. The quarter-period is $\frac{6\pi}{4} = \frac{3\pi}{2}$. The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{3\pi}{2} = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + \frac{3\pi}{2} = 3\pi$

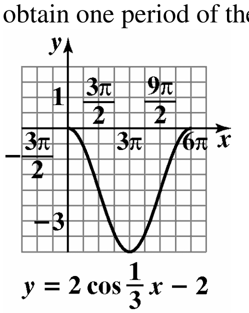
$x = 3\pi + \frac{3\pi}{2} = \frac{9\pi}{2}$

$x = \frac{9\pi}{2} + \frac{3\pi}{2} = 6\pi$

Evaluate the function at each value of x.

x	coordinates
0	(0, 0)
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, -2\right)$
3π	$(3\pi, -4)$
$\frac{9\pi}{2}$	$\left(\frac{9\pi}{2}, -2\right)$
6π	$(6\pi, 0)$

By connecting the points with a smooth curve we obtain one period of the graph.



72. a. At midnight $x = 0$. Thus,

$$y = 98.6 + 0.3\sin\left(\frac{\pi}{12}\cdot 0 - \frac{11\pi}{12}\right)$$
$$= 98.6 + 0.3\sin\left(-\frac{11\pi}{12}\right)$$
$$\approx 98.6 + 0.3(-0.2588) \approx 98.52$$

The body temperature is about 98.52°F.

b. period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{12}} = 2\pi \cdot \frac{12}{\pi} = 24$ hours

c. Solve the equation

$$\frac{\pi}{12}x - \frac{11\pi}{12} = \frac{\pi}{2}$$
$$\frac{\pi}{12}x = \frac{\pi}{2} + \frac{11\pi}{12} = \frac{6\pi}{12} + \frac{11\pi}{12} = \frac{17\pi}{12}$$
$$x = \frac{17\pi}{12}\cdot\frac{12}{\pi} = 17$$

The body temperature is highest for $x = 17$.

$$y = 98.6 + 0.3\sin\left(\frac{\pi}{12}\cdot 17 - \frac{11\pi}{12}\right)$$
$$= 98.6 + 0.3\sin\frac{\pi}{2} = 98.6 + 0.3 = 98.9$$

17 hours after midnight, which is 5 P.M., the body temperature is 98.9°F.

d. Solve the equation

$$\frac{\pi}{12}x - \frac{11\pi}{12} = \frac{3\pi}{2}$$
$$\frac{\pi}{12}x = \frac{3\pi}{2} + \frac{11\pi}{12} = \frac{18\pi}{12} + \frac{11\pi}{12} = \frac{29\pi}{12}$$
$$x = \frac{29\pi}{12}\cdot\frac{12}{\pi} = 29$$

The body temperature is lowest for $x = 29$.

$$y = 98.6 + 0.3\sin\left(\frac{\pi}{12}\cdot 29 - \frac{11\pi}{12}\right)$$
$$= 98.6 + 0.3\sin\left(\frac{3\pi}{2}\right)$$
$$= 98.6 + 0.3(-1) = 98.3°$$

29 hours after midnight or 5 hours after midnight, at 5 A.M., the body temperature is 98.3°F.

e. The graph of $y = 98.6 + 0.3\sin\left(\frac{\pi}{12}x - \frac{11\pi}{12}\right)$ is of the form $y = D + A\sin(Bx - C)$ with $A = 0.3$, $B = \frac{\pi}{12}$, $C = \frac{11\pi}{12}$, and $D = 98.6$. The amplitude is $|A| = |0.3| = 0.3$. The period from part (b) is 24. The quarter-period is $\frac{24}{4} = 6$. The phase shift is $\frac{C}{B} = \frac{\frac{11\pi}{12}}{\frac{\pi}{12}} = \frac{11\pi}{12} \cdot \frac{12}{\pi} = 11$. The cycle begins at $x = 11$. Add quarter-periods to generate x-values for the key points.

$x = 11$

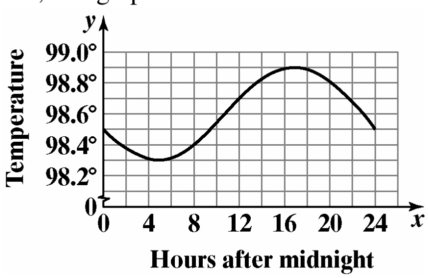
$x = 11 + 6 = 17$

$x = 17 + 6 = 23$

$x = 23 + 6 = 29$

$x = 29 + 6 = 35$

Evaluate the function at each value of x. The key points are (11, 98.6), (17, 98.9), (23, 98.6), (29, 98.3), (35, 98.6). Extend the pattern to the left, and graph the function for $0 \le x \le 24$.



73. Blue:
This is a sine wave with a period of 480.
Since the amplitude is 1, $A = 1$.

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{480} = \frac{\pi}{240}$$

The equation is $y = \sin\frac{\pi}{240}x$.

Red:
This is a sine wave with a period of 640.
Since the amplitude is 1, $A = 1$.

$$B = \frac{2\pi}{\text{period}} = \frac{2\pi}{640} = \frac{\pi}{320}$$

The equation is $y = \sin\frac{\pi}{320}x$.

74. Solve the equations

$$2x = -\frac{\pi}{2} \quad \text{and} \quad 2x = \frac{\pi}{2}$$

$$x = \frac{-\frac{\pi}{2}}{2} \qquad x = \frac{\frac{\pi}{2}}{2}$$

$$x = -\frac{\pi}{4} \qquad x = \frac{\pi}{4}$$

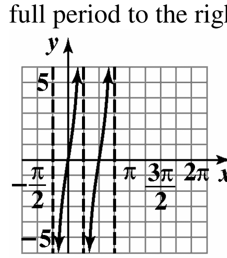
Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$.

$$x\text{-intercept} = \frac{-\frac{\pi}{4} + \frac{\pi}{4}}{2} = \frac{0}{2} = 0$$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 4, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –4 and 4.

Use the two consecutive asymptotes. $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4}$, to graph one full period of $y = 4\tan 2x$ from $-\frac{\pi}{4}$ to $\frac{\pi}{4}$.

Continue the pattern and extend the graph another full period to the right.



$y = 4\tan 2x$

75. Solve the equations

$\frac{\pi}{4}x = -\frac{\pi}{2}$ and $\frac{\pi}{4}x = \frac{\pi}{2}$

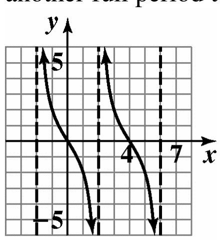
$x = -\frac{\pi}{2} \cdot \frac{4}{\pi}$ $\quad$ $x = \frac{\pi}{2} \cdot \frac{4}{\pi}$

$x = -2$ $\quad$ $x = 2$

Thus, two consecutive asymptotes occur at $x = -2$ and $x = 2$.

x-intercept $= \frac{-2+2}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is –2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 2 and –2. Use the two consecutive asymptotes, $x = -2$ and $x = 2$, to graph one full period of $y = -2\tan\frac{\pi}{4}x$ from –2 to 2. Continue the pattern and extend the graph another full period to the right.



$y = -2\tan\frac{\pi}{4}x$

76. Solve the equations

$x + \pi = -\frac{\pi}{2}$ and $x + \pi = \frac{\pi}{2}$

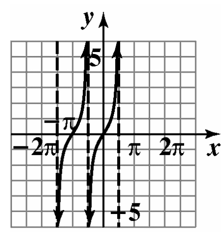
$x = -\frac{\pi}{2} - \pi$ $\quad$ $x = \frac{\pi}{2} - \pi$

$x = -\frac{3\pi}{2}$ $\quad$ $x = -\frac{\pi}{2}$

Thus, two consecutive asymptotes occur at $x = -\frac{3\pi}{2}$ and $x = -\frac{\pi}{2}$.

x-intercept $= \frac{-\frac{3\pi}{2} - \frac{\pi}{2}}{2} = \frac{-2\pi}{2} = -\pi$

An x-intercept is $-\pi$ and the graph passes through $(-\pi, 0)$. Because the coefficient of the tangent is 1, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –1 and 1. Use the two consecutive asymptotes, $x = -\frac{3\pi}{2}$ and $x = -\frac{\pi}{2}$, to graph one full period of $y = \tan(x + \pi)$ from $-\frac{3\pi}{2}$ to $-\frac{\pi}{2}$. Continue the pattern and extend the graph another full period to the right.



$y = \tan(x + \pi)$

77. Solve the equations

$x - \frac{\pi}{4} = -\frac{\pi}{2}$ and $x - \frac{\pi}{4} = \frac{\pi}{2}$

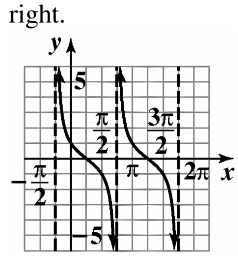
$x = -\frac{\pi}{2} + \frac{\pi}{4}$ $\quad$ $x = \frac{\pi}{2} + \frac{\pi}{4}$

$x = -\frac{\pi}{4}$ $\quad$ $x = \frac{3\pi}{4}$

Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{4}$ and $x = -\frac{3\pi}{4}$.

x-intercept $= \frac{-\frac{\pi}{4} - \frac{3\pi}{4}}{2} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$

An x-intercept is $\frac{\pi}{4}$ and the graph passes through $\left(\frac{\pi}{4}, 0\right)$. Because the coefficient of the tangent is –1, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 1 and –1. Use the two consecutive asymptotes, $x = -\frac{\pi}{4}$ and $x = \frac{3\pi}{4}$, to graph one full period of $y = -\tan\left(x - \frac{\pi}{4}\right)$ from $-\frac{\pi}{4}$ to $\frac{3\pi}{4}$. Continue the pattern and extend the graph another full period to the right.



$y = -\tan\left(x - \frac{\pi}{4}\right)$

78. Solve the equations

$3x = 0$ and $3x = \pi$

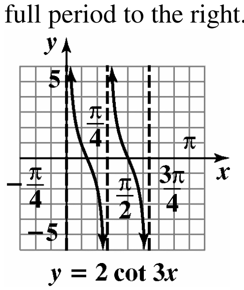
$x = 0$ $\quad x = \frac{\pi}{3}$

Thus, two consecutive asymptotes occur at $x = 0$ and $x = \frac{\pi}{3}$.

$x\text{-intercept} = \frac{0+\frac{\pi}{3}}{2} = \frac{\frac{\pi}{3}}{2} = \frac{\pi}{6}$

An x-intercept is $\frac{\pi}{6}$ and the graph passes through $\left(\frac{\pi}{6}, 0\right)$.

Because the coefficient of the tangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 2 and –2. Use the two consecutive asymptotes, $x = 0$ and $x = \frac{\pi}{3}$, to graph one full period of $y = 2\cot 3x$ from 0 to $\frac{\pi}{3}$. Continue the pattern and extend the graph another full period to the right.



79. Solve the equations

$\frac{\pi}{2}x = 0$ and $\frac{\pi}{2}x = \pi$

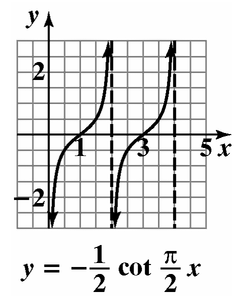
$x = 0$ $\quad x = \pi \cdot \frac{2}{\pi}$

$x = 2$

Thus, two consecutive asymptotes occur at $x = 0$ and $x = 2$.

$x\text{-intercept} = \frac{0+2}{2} = \frac{2}{2} = 1$

An x-intercept is 1 and the graph passes through (1, 0). Because the coefficient of the cotangent is $-\frac{1}{2}$, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of $-\frac{1}{2}$ and $\frac{1}{2}$. Use the two consecutive asymptotes, $x = 0$ and $x = 2$, to graph one full period of $y = -\frac{1}{2}\cot\frac{\pi}{2}x$ from 0 to 2. Continue the pattern and extend the graph another full period to the right.



80. Solve the equations

$x + \frac{\pi}{2} = 0$ and $x + \frac{\pi}{2} = \pi$

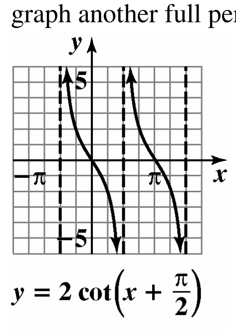
$x = 0 - \frac{\pi}{2}$ $\quad x = \pi - \frac{\pi}{2}$

$x = -\frac{\pi}{2}$ $\quad x = \frac{\pi}{2}$

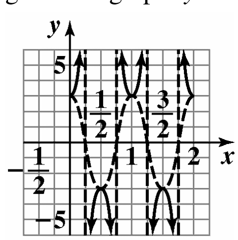
Thus, two consecutive asymptotes occur at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$.

$x\text{-intercept} = \frac{-\frac{\pi}{2}+\frac{\pi}{2}}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the cotangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of 2 and –2. Use the two consecutive asymptotes, $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$, to graph one full period of $y = 2\cot\left(x + \frac{\pi}{2}\right)$ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$. Continue the pattern and extend the graph another full period to the right.

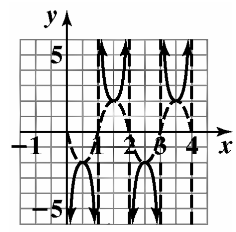


81. Graph the reciprocal cosine function, $y = 3\cos 2\pi x$.
The equation is of the form $y = A\cos Bx$ with $A = 3$ and $B = 2\pi$.
amplitude: $|A| = |3| = 3$
period: $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$
Use quarter-periods, $\frac{1}{4}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1. Evaluating the function at each value of x, the key points are (0, 3), $\left(\frac{1}{4}, 0\right)$, $\left(\frac{1}{2}, -3\right)$, $\left(\frac{3}{4}, 0\right)$, (1, 3) Use these key points to graph $y = 3\cos 2\pi x$ from 0 to 1. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 3\sec 2\pi x$.



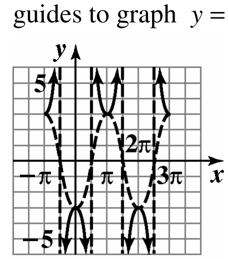
$y = 3 \sec 2\pi x$

82. Graph the reciprocal sine function, $y = -2\sin \pi x$.
The equation is of the form $y = A\sin Bx$ with $A = -2$ and $B = \pi$.
amplitude: $|A| = |-2| = 2$
period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$
Use quarter-periods, $\frac{2}{4} = \frac{1}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2. Evaluating the function at each value of x, the key points are (0, 0), $\left(\frac{1}{2}, -2\right)$, $(1, 0)$, $\left(\frac{3}{2}, 2\right)$, $(2, 0)$. Use these key points to graph $y = -2\sin \pi x$ from 0 to 2. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = -2\csc \pi x$.



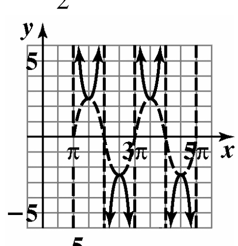
$y = -2 \csc \pi x$

83. Graph the reciprocal cosine function, $y = 3\cos(x + \pi)$. The equation is of the form $y = A\cos(Bx - C)$ with $A = 3$, $B = 1$, and $C = -\pi$.
amplitude: $|A| = |3| = 3$
period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$
phase shift: $\frac{C}{B} = \frac{-\pi}{1} = -\pi$
Use quarter-periods, $\frac{2\pi}{4} = \frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = -\pi$, the x-values are $-\pi$, $-\frac{\pi}{2}$, 0, $\frac{\pi}{2}$, π.
Evaluating the function at each value of x, the key points are $(-\pi, 3)$, $\left(-\frac{\pi}{2}, 0\right)$, $(0, -3)$, $\left(\frac{\pi}{2}, 0\right)$, $(\pi, 3)$. Use these key points to graph $y = 3\cos(x + \pi)$ from $-\pi$ to π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = 3\sec(x + \pi)$.



$y = 3 \sec (x + \pi)$

84. Graph the reciprocal sine function, $y = \frac{5}{2}\sin(x-\pi)$.
The equation is of the form $y = A\sin(Bx - C)$ with
$A = \frac{5}{2}$, $B = 1$, and $C = \pi$.
amplitude: $|A| = \left|\frac{5}{2}\right| = \frac{5}{2}$
period: $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$
phase shift: $\frac{C}{B} = \frac{\pi}{1} = \pi$
Use quarter-periods, $\frac{2\pi}{4} = \frac{\pi}{2}$, to find x-values for the five key points. Starting with $x = \pi$, the x-values are $\pi, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}, 3\pi$. Evaluating the function at each value of x, the key points are $(\pi, 0)$, $\left(\frac{3\pi}{2}, \frac{5}{2}\right)$, $(2\pi, 0)$, $\left(\frac{5\pi}{2}, -\frac{5}{2}\right)$, $(3\pi, 0)$.
Use these key points to graph $y = \frac{5}{2}\sin(x-\pi)$ from π to 3π. Extend the graph one cycle to the right. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = \frac{5}{2}\csc(x-\pi)$.



$y = \frac{5}{2}\csc(x-\pi)$

85. Let $\theta = \sin^{-1} 1$, then $\sin\theta = 1$.
The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = 1$ is $\frac{\pi}{2}$. Thus $\theta = \frac{\pi}{2}$, or $\sin^{-1} 1 = \frac{\pi}{2}$.

86. Let $\theta = \cos^{-1} 1$, then $\cos\theta = 1$.
The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = 1$ is 0. Thus $\theta = 0$, or $\cos^{-1} 1 = 0$.

87. Let $\theta = \tan^{-1} 1$, then $\tan\theta = 1$.
The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = 1$ is $\frac{\pi}{4}$. Thus $\theta = \frac{\pi}{4}$, or $\tan^{-1} 1 = \frac{\pi}{4}$.

88. Let $\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, then $\sin\theta = -\frac{\sqrt{3}}{2}$.
The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\sin\theta = -\frac{\sqrt{3}}{2}$ is $-\frac{\pi}{3}$. Thus $\theta = -\frac{\pi}{3}$, or $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$.

89. Let $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$, then $\cos\theta = -\frac{1}{2}$.
The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = -\frac{1}{2}$ is $\frac{2\pi}{3}$. Thus $\theta = \frac{2\pi}{3}$, or $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$.

90. Let $\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$, then $\tan\theta = -\frac{\sqrt{3}}{3}$.
The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = -\frac{\sqrt{3}}{3}$ is $-\frac{\pi}{6}$.
Thus $\theta = -\frac{\pi}{6}$, or $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$.

91. Let $\theta = \sin^{-1}\frac{\sqrt{2}}{2}$, then $\sin\theta = \frac{\sqrt{2}}{2}$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = \frac{\sqrt{2}}{2}$ is $\frac{\pi}{4}$.
Thus, $\cos\left(\sin^{-1}\frac{\sqrt{2}}{2}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

92. Let $\theta = \cos^{-1} 0$, then $\cos\theta = 0$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = 0$ is $\frac{\pi}{2}$.

Thus, $\sin\left(\cos^{-1} 0\right) = \sin\frac{\pi}{2} = 1$.

93. Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$, then $\sin\theta = -\frac{1}{2}$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = -\frac{1}{2}$ is $-\frac{\pi}{6}$.

Thus, $\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = \tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$.

94. Let $\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, then $\cos\theta = -\frac{\sqrt{3}}{2}$. The only angle in the interval $[0, \pi]$ that satisfies $\cos\theta = -\frac{\sqrt{3}}{2}$ is $\frac{5\pi}{6}$.

Thus, $\tan\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \tan\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$.

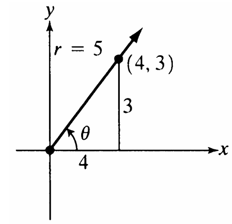
95. Let $\theta = \tan^{-1}\frac{\sqrt{3}}{3}$, then $\tan\theta = \frac{\sqrt{3}}{3}$.

The only angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ that satisfies $\tan\theta = \frac{\sqrt{3}}{3}$ is $\frac{\pi}{6}$.

Thus $\csc\left(\tan^{-1}\frac{\sqrt{3}}{3}\right) = \csc\frac{\pi}{6} = 2$.

96. Let $\theta = \tan^{-1}\frac{3}{4}$, then $\tan\theta = \frac{3}{4}$

Because $\tan\theta$ is positive, θ is in the first quadrant.



$r^2 = x^2 + y^2$

$r^2 = 4^2 + 3^2$

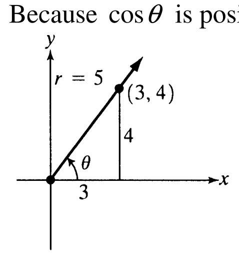
$r^2 = 25$

$r = 5$

$\cos\left(\tan^{-1}\frac{3}{4}\right) = \cos\theta = \frac{x}{r} = \frac{4}{5}$

97. Let $\theta = \cos^{-1}\frac{3}{5}$, then $\cos\theta = \frac{3}{5}$.

Because $\cos\theta$ is positive, θ is in the first quadrant.



$x^2 + y^2 = r^2$

$3^2 + y^2 = 5^2$

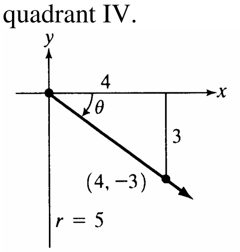
$y^2 = 25 - 9 = 16$

$y = \sqrt{16} = 4$

$\sin\left(\cos^{-1}\frac{3}{5}\right) = \sin\theta = \frac{y}{r} = \frac{4}{5}$

98. Let $\theta = \sin^{-1}\left(-\frac{3}{5}\right)$, then $\sin\theta = -\frac{3}{5}$.

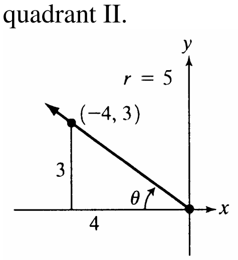
Because $\sin\theta$ is negative, θ is in quadrant IV.



$$x^2 + (-3)^2 = 5^2$$
$$x^2 + y^2 = r^2$$
$$x^2 = 25 - 9 = 16$$
$$x = \sqrt{16} = 4$$
$$\tan\left[\sin^{-1}\left(-\frac{3}{5}\right)\right] = \tan\theta = \frac{y}{x} = -\frac{3}{4}$$

99. Let $\theta = \cos^{-1}\left(-\frac{4}{5}\right)$, then $\cos\theta = -\frac{4}{5}$.

Because $\cos\theta$ is negative, θ is in quadrant II.



$$x^2 + y^2 = r^2$$
$$(-4)^2 + y^2 = 5^2$$
$$y^2 = 25 - 16 = 9$$
$$y = \sqrt{9} = 3$$

Use the right triangle to find the exact value.

$$\tan\left[\cos^{-1}\left(-\frac{4}{5}\right)\right] = \tan\theta = -\frac{3}{4}$$

100. Let $\theta = \tan^{-1}\left(-\frac{1}{3}\right)$,

Because $\tan\theta$ is negative, θ is in quadrant IV and $x = 3$ and $y = -1$.

$$r^2 = x^2 + y^2$$
$$r^2 = 3^2 + (-1)^2$$
$$r^2 = 10$$
$$r = \sqrt{10}$$
$$\sin\left[\tan^{-1}\left(-\frac{4}{5}\right)\right] = \sin\theta = \frac{y}{r} = \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$

101. $x = \frac{\pi}{3}$, x is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so $\sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

102. $x = \frac{2\pi}{3}$, x is not in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. x is in the domain of $\sin x$, so

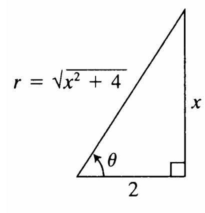
$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

103. $\sin^{-1}\left(\cos\frac{2\pi}{3}\right) = \sin^{-1}\left(-\frac{1}{2}\right)$

Let $\theta = \sin^{-1}\left(-\frac{1}{2}\right)$, then $\sin\theta = -\frac{1}{2}$. The only angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ that satisfies $\sin\theta = -\frac{1}{2}$ is $-\frac{\pi}{6}$. Thus, $\theta = -\frac{\pi}{6}$, or

$$\sin^{-1}\left(\cos\frac{2\pi}{3}\right) = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}.$$

104. Let $\theta = \tan^{-1}\dfrac{x}{2}$, then $\tan\theta = \dfrac{x}{2}$.



$r^2 = x^2 + 2^2$

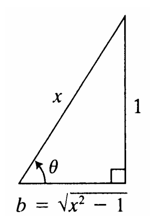
$r^2 = x^2 + y^2$

$r = \sqrt{x^2+4}$

Use the right triangle to write the algebraic expression.

$$\cos\left(\tan^{-1}\frac{x}{2}\right) = \cos\theta = \frac{2}{\sqrt{x^2+4}} = \frac{2\sqrt{x^2+4}}{x^2+4}$$

105. Let $\theta = \sin^{-1}\dfrac{1}{x}$, then $\sin\theta = \dfrac{1}{x}$.



Use the Pythagorean theorem to find the third side, b.

$1^2 + b^2 = x^2$

$b^2 = x^2 - 1$

$b = \sqrt{x^2-1}$

Use the right triangle to write the algebraic expression.

$$\sec\left(\sin^{-1}\frac{1}{x}\right) = \sec\theta = \frac{x}{\sqrt{x^2-1}} = \frac{x\sqrt{x^2-1}}{x^2-1}$$

106. Find the measure of angle B. Because $C = 90°$, $A + B = 90°$. Thus, $B = 90° - A = 90° - 22.3° = 67.7°$

We have a known angle, a known hypotenuse, and an unknown opposite side. Use the sine function.

$\sin 22.3° = \dfrac{a}{10}$

$a = 10\sin 22.3° \approx 3.79$

We have a known angle, a known hypotenuse, and an unknown adjacent side. Use the cosine function.

$\cos 22.3° = \dfrac{b}{10}$

$b = 10\cos 22.3° \approx 9.25$

In summary, $B = 67.7°$, $a \approx 3.79$, and $b \approx 9.25$.

107. Find the measure of angle A. Because $C = 90°$, $A + B = 90°$. Thus, $A = 90° - B = 90° - 37.4° = 52.6°$

We have a known angle, a known opposite side, and an unknown adjacent side. Use the tangent function.

$\tan 37.4° = \dfrac{6}{a}$

$a = \dfrac{6}{\tan 37.4°} \approx 7.85$

We have a known angle, a known opposite side, and an unknown hypotenuse. Use the sine function.

$\sin 37.4° = \dfrac{6}{c}$

$c = \dfrac{6}{\sin 37.4°} \approx 9.88$

In summary, $A = 52.6°$, $a \approx 7.85$, and $c \approx 9.88$.

108. Find the measure of angle A. We have a known hypotenuse, a known opposite side, and an unknown angle. Use the sine function.

$\sin A = \dfrac{2}{7}$

$A = \sin^{-1}\left(\dfrac{2}{7}\right) \approx 16.6°$

Find the measure of angle B. Because $C = 90°$, $A + B = 90°$. Thus, $B = 90° - A \approx 90° - 16.6° = 73.4°$

We have a known hypotenuse, a known opposite side, and an unknown adjacent side. Use the Pythagorean theorem.

$a^2 + b^2 = c^2$

$2^2 + b^2 = 7^2$

$b^2 = 7^2 - 2^2 = 45$

$b = \sqrt{45} \approx 6.71$

In summary, $A \approx 16.6°$, $B \approx 73.4°$, and $b \approx 6.71$.

109. Find the measure of angle A. We have a known opposite side, a known adjacent side, and an unknown angle. Use the tangent function.

$\tan A = \dfrac{1.4}{3.6}$

$A = \tan^{-1}\left(\dfrac{1.4}{3.6}\right) \approx 21.3°$

Find the measure of angle B. Because $C = 90°$, $A + B = 90°$. Thus, $B = 90° - A \approx 90° - 21.3° = 68.7°$

We have a known opposite side, a known adjacent side, and an unknown hypotenuse.

Use the Pythagorean theorem.

$c^2 = a^2 + b^2 = (1.4)^2 + (3.6)^2 = 14.92$

$c = \sqrt{14.92} \approx 3.86$

In summary, $A \approx 21.3°$, $B \approx 68.7°$, and $c \approx 3.86$.

110. Using a right triangle, we have a known angle, an unknown opposite side, h, and a known adjacent side. Therefore, use the tangent function.

$$\tan 25.6° = \frac{h}{80}$$
$$h = 80\tan 25.6° \approx 38.3$$

The building is about 38 feet high.

111. Using a right triangle, we have a known angle, an unknown opposite side, h, and a known adjacent side. Therefore, use the tangent function.

$$\tan 40° = \frac{h}{60}$$
$$h = 60\tan 40° \approx 50 \text{ yd}$$

The second building is 50 yds taller than the first. Total height = $40 + 50 = 90$ yd.

112. Using two right triangles, a smaller right triangle corresponding to the smaller angle of elevation drawn inside a larger right triangle corresponding to the larger angle of elevation, we have a known angle, a known opposite side, and an unknown adjacent side, d, in the smaller triangle. Therefore, use the tangent function.

$$\tan 68° = \frac{125}{d}$$
$$d = \frac{125}{\tan 68°} \approx 50.5$$

We now have a known angle, a known adjacent side, and an unknown opposite side, h, in the larger triangle. Again, use the tangent function.

$$\tan 71° = \frac{h}{50.5}$$
$$h = 50.5\tan 71° \approx 146.7$$

The height of the antenna is $146.7 - 125$, or 21.7 ft, to the nearest tenth of a foot.

113. We need the acute angle between ray OA and the north-south line through O. This angle measures $90° - 55° = 35°$. This angle measured from the north side of the north-south line and lies east of the north-south line. Thus the bearing from O to A is N35°E.

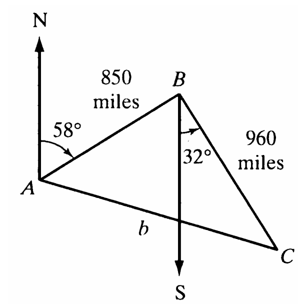
114. We need the acute angle between ray OA and the north-south line through O. This angle measures $90° - 55° = 35°$. This angle measured from the south side of the north-south line and lies west of the north-south line. Thus the bearing from O to A is S35°W.

115. Using a right triangle, we have a known angle, a known adjacent side, and an unknown opposite side, d. Therefore, use the tangent function.

$$\tan 64° = \frac{d}{12}$$
$$d = 12\tan 64° \approx 24.6$$

The ship is about 24.6 miles from the lighthouse.

116.



a. Using the figure,
$B = 58° + 32° = 90°$
Thus, use the Pythagorean Theorem to find the distance from city A to city C.

$$850^2 + 960^2 = b^2$$
$$b^2 = 722{,}500 + 921{,}600$$
$$b^2 = 1{,}644{,}100$$
$$b = \sqrt{1{,}644{,}100} \approx 1282.2$$

The distance from city A to city B is about 1282.2 miles.

b. Using the figure,

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{960}{850} \approx 1.1294$$
$$A \approx \tan^{-1}(1.1294) \approx 48°$$
$$180° - 58° - 48° = 74°$$

The bearing from city A to city C is S74°E.

117. $d = 20\cos\frac{\pi}{4}t$

$a = 20$ and $\omega = \frac{\pi}{4}$

a. maximum displacement:
$|a| = |20| = 20\text{cm}$

b. $f = \frac{\omega}{2\pi} = \frac{\frac{\pi}{4}}{2\pi} = \frac{\pi}{4}\cdot\frac{1}{2\pi} = \frac{1}{8}$

frequency: $\frac{1}{8}$ cycle per second

c. period: $\frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{4}} = 2\pi\cdot\frac{4}{\pi} = 8$

The time required for one cycle is 8 seconds.

118. $d = \frac{1}{2}\sin 4t$

$a = \frac{1}{2}$ and $\omega = 4$

a. maximum displacement:

$|a| = \left|\frac{1}{2}\right| = \frac{1}{2}$cm

b. $f = \frac{\omega}{2\pi} = \frac{4}{2\pi} = \frac{2}{\pi} \approx 0.64$

frequency: 0.64 cycle per second

c. period: $\frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2} \approx 1.57$

The time required for one cycle is about 1.57 seconds.

119. Because the distance of the object from the rest position at $t = 0$ is a maximum, use the form $d = a\cos\omega t$. The period is $\frac{2\pi}{\omega}$ so,

$2 = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{2} = \pi$

Because the amplitude is 30 inches, $|a| = 30$. because the object starts below its rest position $a = -30$. the equation for the object's simple harmonic motion is $d = -30\cos \pi t$.

120. Because the distance of the object from the rest position at $t = 0$ is 0, use the form $d = a\sin\omega t$. The period is $\frac{2\pi}{\omega}$ so

$5 = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{5}$

Because the amplitude is $\frac{1}{4}$ inch, $|a| = \frac{1}{4}$. a is negative since the object begins pulled down. The equation for the object's simple harmonic motion is

$d = -\frac{1}{4}\sin\frac{2\pi}{5}t.$

Chapter 5 Test

1. $135° = 135° \cdot \frac{\pi \text{ radians}}{180°}$

$= \frac{135\pi}{180}$ radians

$= \frac{3\pi}{4}$ radians

2. $75° = 75° \cdot \frac{\pi \text{ radians}}{180°} = \frac{75\pi}{180}$ radians

$= \frac{5\pi}{12}$ radians

$s = r\theta$

$s = 20\left(\frac{5\pi}{12}\right) = \frac{25\pi}{3}$ ft ≈ 26.18 ft

3. **a.** $\frac{16\pi}{3} - 4\pi = \frac{16\pi}{3} - \frac{12\pi}{3} = \frac{4\pi}{3}$

b. $\frac{16\pi}{3}$ is coterminal with $\frac{4\pi}{3}$.

$\frac{4\pi}{3} - \pi = \frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}$

4. P = (–2, 5) is a point on the terminal side of θ, $x = -2$ and $y = 5$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (5)^2}$

$= \sqrt{4 + 25} = \sqrt{29}$

Use x, y, and r, to find the six trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{\sqrt{29}\sqrt{29}} = \frac{5\sqrt{29}}{29}$

$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{29}} = -\frac{2\sqrt{29}}{\sqrt{29}\sqrt{29}} = -\frac{2\sqrt{29}}{29}$

$\tan\theta = \frac{y}{x} = \frac{5}{-2} = -\frac{5}{2}$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{5}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{29}}{-2} = -\frac{\sqrt{29}}{2}$

$\cot\theta = \frac{x}{y} = \frac{-2}{5} = -\frac{2}{5}$

5. Because $\cos\theta < 0,\ \theta$ cannot lie in quadrant I and quadrant IV; the cosine function is positive in those two quadrants. Thus, with $\cos\theta < 0,\ \theta$ lies in quadrant II or quadrant III. We are also given that $\cot\theta > 0.$ Because quadrant III is the only quadrant in which the cosine is negative and the cotangent is positive, θ lies in quadrant III.

6. Because the cosine is positive and the tangent is negative, θ lies in quadrant IV. In quadrant IV x is positive and y is negative. Thus,

$\cos\theta = \frac{1}{3} = \frac{x}{r},\ x = 1,\ r = 3.$ Furthermore,

$$x^2 + y^2 = r^3$$
$$1^2 + y^2 = 3^2$$
$$y^2 = 9 - 1 = 8$$
$$y = -\sqrt{8} = -2\sqrt{2}$$

Use x, y, and r, to find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{2}}{3} = -\frac{2\sqrt{2}}{3}$$
$$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{2}}{1} = -2\sqrt{2}$$
$$\csc\theta = \frac{r}{y} = \frac{3}{-2\sqrt{2}} = -\frac{3\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}} = -\frac{3\sqrt{2}}{4}$$
$$\sec\theta = \frac{r}{x} = \frac{3}{1} = 3$$
$$\cot\theta = \frac{x}{y} = \frac{1}{-2\sqrt{2}} = -\frac{1\cdot\sqrt{2}}{2\sqrt{2}\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

7. $\tan\frac{\pi}{6}\cos\frac{\pi}{3} - \cos\frac{\pi}{2} = \frac{\sqrt{3}}{3}\cdot\frac{1}{2} - 0 = \frac{\sqrt{3}}{6}$

8. 300° lies in quadrant IV.
The reference angle is

$$\theta' = 360° - 300° = 60°$$

$\tan 60° = \sqrt{3}$

In quadrant IV, $\tan\theta < 0$, so

$\tan 300° = -\tan 60 = -\sqrt{3}$.

9. $\frac{7\pi}{4}$ lies in quadrant IV.
The reference angle is

$$\theta' = 2\pi - \frac{7\pi}{4} = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

In quadrant IV, $\sin\theta < 0,$ so

$\sin\frac{7\pi}{4} = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$.

10.
$$\sec\frac{22\pi}{3} = \sec\frac{4\pi}{3} = -\sec\frac{\pi}{3} = \frac{1}{-\cos\frac{\pi}{3}} = \frac{1}{-\frac{1}{2}} = -2$$

11.
$$\cot\left(-\frac{8\pi}{3}\right) = \cot\left(\frac{4\pi}{3}\right) = \cot\frac{\pi}{3} = \frac{1}{\tan\frac{\pi}{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

12. $\tan\left(\frac{7\pi}{3} + n\pi\right) = \tan\frac{7\pi}{3} = \tan\frac{\pi}{3} = \sqrt{3}$

13. **a.**
$$\sin(-\theta) + \cos(-\theta) = -\sin(\theta) + \cos(\theta) = -a + b$$

b.
$$\tan\theta - \sec\theta = \frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta} = \frac{a}{b} - \frac{1}{b} = \frac{a-1}{b}$$

14. The equation $y = 3\sin 2x$ is of the form $y = A\sin Bx$ with $A = 3$ and $B = 2$. The amplitude is $|A| = |3| = 3$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The quarter-period is $\frac{\pi}{4}$.
The cycle begins at $x = 0$. Add quarter-periods to generate x-values for the key points.

$x = 0$

$x = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

$x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

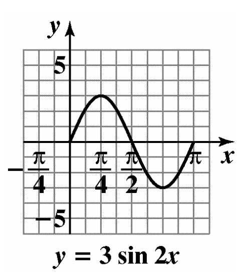
$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$

Evaluate the function at each value of x.

x	coordinates
0	$(0, 0)$
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, 3\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, -3\right)$
π	$(\pi, 0)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



15. The equation $y = -2\cos\left(x - \frac{\pi}{2}\right)$ is of the form $y = A\cos(Bx - C)$ with $A = -2$, $B = 1$, and $C = \frac{\pi}{2}$. The amplitude is $|A| = |-2| = 2$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi$. The phase shift is $\frac{C}{B} = \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$. The quarter-period is $\frac{2\pi}{4} = \frac{\pi}{2}$.

The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

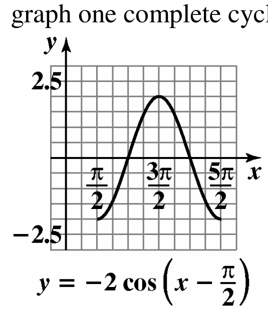
$x = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

$x = 2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$

Evaluate the function at each value of x.

x	coordinates
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, -2\right)$
π	$(\pi, 0)$
$\frac{3\pi}{2}$	$\left(\frac{3\pi}{2}, 2\right)$
2π	$(2\pi, 0)$
$\frac{5\pi}{2}$	$\left(\frac{5\pi}{2}, -2\right)$

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



16. Solve the equations

$\frac{x}{2} = -\frac{\pi}{2}$ and $\frac{x}{2} = \frac{\pi}{2}$

$x = -\frac{\pi}{2} \cdot 2$ $x = \frac{\pi}{2} \cdot 2$

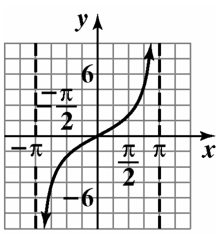
$x = -\pi$ $x = \pi$

Thus, two consecutive asymptotes occur at $x = -\pi$ and $x = \pi$.

x-intercept $= \frac{-\pi + \pi}{2} = \frac{0}{2} = 0$

An x-intercept is 0 and the graph passes through (0, 0). Because the coefficient of the tangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –2 and 2. Use the two consecutive asymptotes, $x = -\pi$ and $x = \pi$, to graph one

full period of $y = 2\tan\frac{x}{2}$ from $-\pi$ to π.



$y = 2\tan\frac{x}{2}$

17. Graph the reciprocal sine function, $y = -\frac{1}{2}\sin\pi x$.

The equation is of the form $y = A\sin Bx$ with $A = -\frac{1}{2}$ and $B = \pi$.

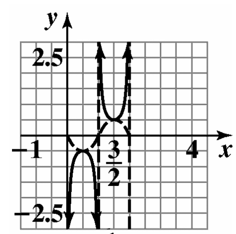
amplitude: $|A| = \left| -\frac{1}{2} \right| = \frac{1}{2}$

period: $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$

Use quarter-periods, $\frac{2}{4} = \frac{1}{2}$, to find x-values for the five key points. Starting with $x = 0$, the x-values are $0, \frac{1}{2}, 1, \frac{3}{2}, 2$. Evaluating the function at each value of x, the key points are

$(0, 3), \left(\frac{1}{2}, -\frac{1}{2}\right), (1, 0), \left(\frac{3}{2}, \frac{1}{2}\right), (2, 0)$.

Use these key points to graph $y = -\frac{1}{2}\sin\pi x$ from 0 to 2. Use the graph to obtain the graph of the reciprocal function. Draw vertical asymptotes through the x-intercepts, and use them as guides to graph $y = -\frac{1}{2}\csc\pi x$.



$y = -\frac{1}{2}\csc\pi x$

18. Let $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$, then $\cos\theta = -\frac{1}{2}$.

Because $\cos\theta$ is negative, θ is in quadrant II.

$x^2 + y^2 = r^2$

$(-1)^2 + y^2 = 2^2$

$y^2 = 4 - 1 = 3$

$y = \sqrt{3}$

$\tan\left[\cos^{-1}\left(-\frac{1}{2}\right)\right] = \tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

19. Let $\theta = \cos^{-1}\left(\frac{x}{3}\right)$, then $\cos\theta = \frac{x}{3}$.

Because $\cos\theta$ is positive, θ is in quadrant I.

$x^2 + y^2 = r^2$

$x^2 + y^2 = 3^2$

$y^2 = 9 - x^2$

$y = \sqrt{9 - x^2}$

$\sin\left[\cos^{-1}\left(\frac{x}{3}\right)\right] = \sin\theta = \frac{y}{r} = \frac{\sqrt{9 - x^2}}{3}$

20. Find the measure of angle *B*. Because $C = 90°, A + B = 90°$.
Thus, $B = 90° - A = 90° - 21° = 69°$.
We have a known angle, a known hypotenuse, and an unknown opposite side. Use the sine function.

$$\sin 21° = \frac{a}{13}$$
$$a = 13\sin 21° \approx 4.7$$

We have a known angle, a known hypotenuse, and an unknown adjacent side. Use the cosine function.

$$\cos 21° = \frac{b}{13}$$
$$b = 13\cos 21° \approx 12.1$$

In summary, $B = 69°$, $a \approx 4.7$, and $b \approx 12.1$.

21. Using a right triangle, we have a known angle, an unknown opposite side, *h,* and a known adjacent side. Therefore, use the tangent function.

$$\tan 37° = \frac{h}{30}$$
$$h = 30\tan 37° \approx 23$$

The building is about 23 yards high.

22. Using a right triangle, we have a known hypotenuse, a known opposite side, and an unknown angle. Therefore, use the sine function.

$$\sin\theta = \frac{43}{73}$$
$$\theta = \sin^{-1}\left(\frac{43}{73}\right) \approx 36.1°$$

The rope makes an angle of about 36.1° with the pole.

23. We need the acute angle between ray *OP* and the north-south line through *O.* This angle measures 90° – 10°. This angle is measured from the north side of the north-south line and lies west of the north-south line. Thus the bearing from *O* to *P* is N80°W.

24. $d = -6\cos\pi t$

$a = -6$ and $\omega = \pi$

a. maximum displacement: $|\ a\ | = |\ -6\ | = 6$ in.

b. $f = \dfrac{\omega}{2\pi} = \dfrac{\pi}{2\pi} = \dfrac{1}{2}$

frequency: $\dfrac{1}{2}$ cycle per second

c. period = $\dfrac{2\pi}{\omega} = \dfrac{2\pi}{\pi} = 2$

The time required for one cycle is 2 seconds.

25. Trigonometric functions are periodic.

Cumulative Review Exercises (Chapters 1-5)

1.
$$x^2 = 18 + 3x$$
$$x^2 - 3x - 18 = 0$$
$$(x-6)(x+3) = 0$$
$$x - 6 = 0 \quad\text{or}\quad x + 3 = 0$$
$$x = 6 \qquad\qquad x = -3$$

The solution set is {–3,6 }.

2.
$$x^3 + 5x^2 - 4x - 20 = 0$$
$$x^2(x+5) - 4(x+5) = 0$$
$$(x^2 - 4)(x+5) = 0$$
$$(x-2)(x+2)(x+5) = 0$$
$$x - 2 = 0 \quad\text{or}\quad x + 2 = 0 \quad\text{or}\quad x + 5 = 0$$
$$x = 2 \qquad\qquad x = -2 \qquad\qquad x = -5$$

The solution set is {–5,–2, 2}.

3.
$$\log_2 x + \log_2(x-2) = 3$$
$$\log_2 x(x-2) = 3$$
$$x(x-2) = 2^3$$
$$x^2 - 2x = 2^3$$
$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x - 4 = 0 \quad\text{or}\quad x + 2 = 0$$
$$x = 4 \qquad\qquad x = -2$$

$x = -2$ is extraneous

The solution set is {4}

4.
$$\sqrt{x-3} + 5 = x$$
$$\sqrt{x-3} = x - 5$$
$$\left(\sqrt{x-3}\right)^2 = (x-5)^2$$
$$x - 3 = x^2 - 10x + 25$$
$$x^2 - 11x + 28 = 0$$
$$(x-4)(x-7) = 0$$
$$x - 4 = 0 \quad\text{or}\quad x - 7 = 0$$
$$x = 4 \qquad\qquad x = 7$$
$$\sqrt{4-3} + 5 = 4$$
$$\sqrt{1} + 5 = 4$$
$$1 + 5 = 4 \quad \text{false}$$

$x = 4$ is not a solution

$$\sqrt{7-3} + 5 = 7$$
$$\sqrt{4} + 5 = 7$$
$$2 + 5 = 7 \quad \text{true}$$

The solution set is {7}.

5. $x^3-4x^2+x+6=0$

$p: \pm1, \pm2, \pm3, \pm6$

$q: \pm1$

$\frac{p}{q}: \pm1, \pm2, \pm3, \pm6$

2	1	–4	1	6
		2	–4	–6
	1	–2	–3	0

$x^3-4x^2+x+6=(x-2)\left(x^2-2x-3\right)$

Thus,

$$x^3-4x^2+x+6=0$$
$$(x-2)\left(x^2-2x-3\right)=0$$
$$(x-2)(x-3)(x+1)=0$$
$$x-2=0 \quad \text{or} \quad x-3=0 \quad \text{or} \quad x+1=0$$
$$x=2 \qquad x=3 \qquad x=-1$$

The solution set is $\{-1,2,3\}$

6. $|\,2x-5\,|\le 11$

$-11\le 2x-5\le 11$

$-6\le 2x\le 16$

$-3\le x\le 8$

The solution set is $\{x\,|-3\le x\le 8\}$

7. $f(x)=\sqrt{x-6}$

$x=\sqrt{y-6}$

$x^2=y-6$

$y=x^2+6$

$f^{-1}(x)=x^2+6$

8.

$$\begin{array}{r} 4x^2-\frac{14}{5}x-\frac{17}{25} \\ 5x+2\overline{\smash{)}20x^3-6x^2-9x+10} \\ \underline{20x^3+8x^2} \\ -14x^2-9x \\ \underline{-14x^2-\frac{28}{5}x} \\ -\frac{17}{5}x+10 \\ \underline{-\frac{17}{5}x-\frac{34}{25}} \\ \frac{284}{25} \end{array}$$

The quotient is $4x^2-\frac{14}{5}x-\frac{17}{25}+\frac{284}{125x+50}$.

9. $\log 25+\log 40=\log(25\cdot 40)$

$=\log 1000$

$=\log 10^3$

$=3$

10. $\frac{14\pi}{9}\text{ radians}=\frac{14\pi}{9}\text{ radians}\cdot\frac{180°}{\pi\text{ radians}}$

$=\frac{14\cdot 180°}{9}=280°$

11. $3x^4-2x^3+5x^2+x-9=0$

The sign changes 3 times so the equation has at most 3 positive real roots;

$f(-x)=3x^4+2x^3+5x^2-x-9$

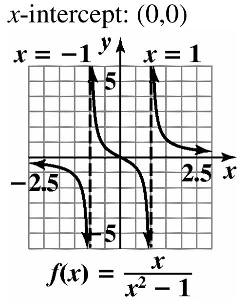
The sign changes 1 time, so the equation has at most 1 negative real root.

12. $f(x)=\frac{x}{x^2-1}$

vertical asymptotes: $x^2-1=0$, $x=1$ and $x=-1$

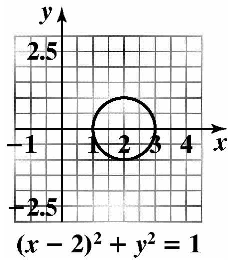
horizontal asymptote: $m=1$ and $n=2$ so $m<n$ and the x-axis is a horizontal asymptote.

x-intercept: (0,0)



13. $(x-2)^2+y^2=1$

The graph is a circle with center (2,0) and $r=1$.

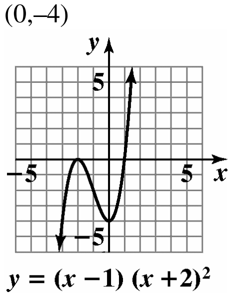


14. $y=(x-1)(x+2)^2$

x-intercepts: (1,0) and (–2,0)

y-intercept: $y=(-1)(2)^2=-4$

(0,–4)



15. $y=\sin\left(2x+\frac{\pi}{2}\right)=\sin\left(2x-\left(-\frac{\pi}{2}\right)\right)$

The equation $y=\sin\left(2x-\left(-\frac{\pi}{2}\right)\right)$ is of the form

$y=A\sin(Bx-C)$ with $A=1$, $B=2$, and $C=-\frac{\pi}{2}$.

The amplitude is $|A|=|1|=1$

The period is $\frac{2\pi}{B}=\frac{2\pi}{2}=\pi$. The phase shift is

$\frac{C}{B}=\frac{-\frac{\pi}{2}}{2}=-\frac{\pi}{2}\cdot\frac{1}{2}=-\frac{\pi}{4}$. The quarter-period is $\frac{\pi}{4}$.

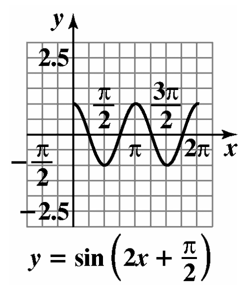
The cycle begins at $x=-\frac{\pi}{4}$. Add quarter-periods to generate x-values for the key points.

$x=-\frac{\pi}{4},\ x=-\frac{\pi}{4}+\frac{\pi}{4}=0,\ x=0+\frac{\pi}{4}=\frac{\pi}{4},$

$x=\frac{\pi}{4}+\frac{\pi}{4}=\frac{\pi}{2},\ x=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$ To graph from 0 to π, evaluate the function at the last four key points and at $x=\pi$.

x	coordinates
0	(0, 1)
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$\left(\frac{\pi}{2}, -1\right)$
$\frac{3\pi}{4}$	$\left(\frac{3\pi}{4}, 0\right)$
π	$(\pi, 1)$

Connect the points with a smooth curve and extend the graph one cycle to the right to graph from 0 to 2π.



16. Solve the equations

$3x=-\frac{\pi}{2}$ and $3x=\frac{\pi}{2}$

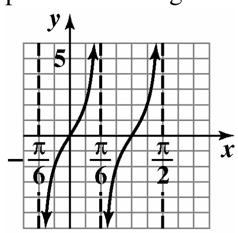
$x=\frac{-\frac{\pi}{2}}{3}$ $\quad x=\frac{\frac{\pi}{2}}{3}$

$x=-\frac{\pi}{6}$ $\quad x=\frac{\pi}{6}$

Thus, two consecutive asymptotes occur at $x=-\frac{\pi}{6}$ and $x=\frac{\pi}{6}$.

x-intercept = $\frac{-\frac{\pi}{6}+\frac{\pi}{6}}{2}=\frac{0}{2}=0$

An x-intercept is 0 and the graph passes through $(0,0)$. Because the coefficient of the tangent is 2, the points on the graph midway between an x-intercept and the asymptotes have y-coordinates of –2 and 2. Use the two consecutive asymptotes, $x=-\frac{\pi}{6}$ and $x=\frac{\pi}{6}$, to graph one full period of $y=2\tan 3x$ from $-\frac{\pi}{6}$ to $\frac{\pi}{6}$. Extend the pattern to the right to graph two complete cycles.



$y = 2\tan 3x$

17. $C(p)=30{,}000+2500p$

$R(p)=3125p$

$30{,}000+2500p=3125p$

$30{,}000=625p$

$p=48$

48 performances must be played for you to break even.

18. **a.** Let t be the number of years after 2000.

$$A=A_0e^{kt}$$
$$A=110e^{kt}$$
$$233=110e^{k(6)}$$
$$\frac{233}{110}=e^{k(6)}$$
$$\ln\frac{233}{110}=\ln e^{k(6)}$$
$$\ln\frac{233}{110}=6k$$
$$\frac{\ln\frac{233}{110}}{6}=k$$
$$k\approx 0.1251$$

Thus, $A=110e^{0.1251t}$

b.
$$A=110e^{0.2151t}$$
$$300=110e^{0.1251t}$$
$$\frac{300}{110}=e^{0.1251t}$$
$$\ln\frac{300}{110}=\ln e^{0.1251t}$$
$$\ln\frac{300}{110}=0.1251t$$
$$\frac{\ln\frac{300}{110}}{0.1251}=t$$
$$t\approx 8$$

There will be 300 million cell phone subscribers in the United States 8 years after 2000, or 2008.

19. $2200=\frac{k}{3.5}$

$k=7700$

$h=\frac{7700}{5}=1540$

The rate of heat loss is 1540 Btu per hour.

20. Using a right triangle, we have a known opposite side, a known adjacent side, and an unknown angle. Therefore, use the tangent function.

$\tan\theta=\frac{200}{50}=4$

$\theta=\tan^{-1}(4)\approx 76°$

The angle of elevation is about $76°$.

Chapter 6
Analytic Trigonometry

Section 6.1

Check Point Exercises

1. $$\csc x \tan x = \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} = \frac{1}{\cos x} = \sec x$$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

2. $$\cos x \cot x + \sin x = \cos x \cdot \frac{\cos x}{\sin x} + \sin x = \frac{\cos^2 x}{\sin x} + \sin x = \frac{\cos^2 x}{\sin x} + \sin x \cdot \frac{\sin x}{\sin x} = \frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x} = \frac{\cos^2 x + \sin^2 x}{\sin x} = \frac{1}{\sin x} = \csc x$$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

3. $$\sin x - \sin x \cos^2 x = \sin x\left(1 - \cos^2 x\right) = \sin x \cdot \sin^2 x = \sin^3 x$$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

4. $$\frac{1+\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \csc\theta + \cot\theta$$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

5. $$\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x}$$
$$= \frac{\sin x(\sin x)}{(1+\cos x)\sin x} + \frac{(1+\cos x)(1+\cos x)}{\sin x(1+\cos x)}$$
$$= \frac{\sin^2 x}{(1+\cos x)\sin x} + \frac{1+2\cos x+\cos^2 x}{(1+\cos x)\sin x}$$
$$= \frac{\sin^2 x+\cos^2 x+2\cos x+1}{(1+\cos x)\sin x}$$
$$= \frac{1+1+2\cos x}{(1+\cos x)\sin x}$$
$$= \frac{2+2\cos x}{(1+\cos x)\sin x}$$
$$= \frac{2\left(1+\cos x\right)}{(1+\cos x)\sin x}$$
$$= \frac{2}{\sin x}$$
$$= 2\csc x$$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

6. $$\frac{\cos x}{1+\sin x} = \frac{\cos x}{(1+\sin x)} \cdot \frac{1-\sin x}{1-\sin x} = \frac{\cos x(1-\sin x)}{1-\sin^2 x} = \frac{\cos x(1-\sin x)}{\cos^2 x} = \frac{1-\sin x}{\cos x}$$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

7. $\frac{\sin x}{1+\cos x}+\frac{1+\cos}{\sin x}$

$=\frac{\sin x(\sin x)}{(1+\cos x)\sin x}+\frac{(1+\cos x)(1+\cos x)}{\sin x(1+\cos x)}$

$=\frac{\sin^2 x}{(1+\cos x)\sin x}+\frac{1+2\cos x+\cos^2 x}{(1+\cos x)\sin x}$

$=\frac{\sin^2 x+\cos^2 x+2\cos x+1}{(1+\cos x)\sin x}$

$=\frac{1+1+2\cos x}{(1+\cos x)\sin x}$

$=\frac{2+2+\cos x}{(1+\cos x)\sin x}$

$=\frac{2(1+\cos x)}{(1+\cos x)\sin x}$

$=\frac{2}{\sin x}$

$=2\csc x$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

8. Left side:

$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$

$=\frac{1(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}+\frac{1(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$=\frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

$=\frac{2}{(1+\sin\theta)(1-\sin\theta)}$

$=\frac{2}{1-\sin^2\theta}$

Right side:

$2+2\tan^2\theta=2+2\left(\frac{\sin^2\theta}{\cos^2\theta}\right)$

$=\frac{2\cos^2\theta}{\cos^2\theta}+\frac{2\sin^2\theta}{\cos^2\theta}$

$=\frac{2\cos^2\theta+2\sin^2\theta}{\cos^2\theta}$

$=\frac{2}{\cos^2\theta}=\frac{2}{1-\sin^2\theta}$

The identity is verified because both sides are equal to $\frac{2}{1-\sin^2\theta}$.

Exercise Set 6.1

1. $\sin x\sec x=\sin x\cdot\frac{1}{\cos x}$

$=\frac{\sin x}{\cos x}$

$=\tan x$

2. $2\cos x\csc x=\cos x\cdot\frac{1}{\sin x}$

$=\frac{\cos x}{\sin x}$

$=\cot x$

3. $\tan(-x)\cdot\cos x=-\tan x\cdot\cos x$

$=-\frac{\sin x}{\cos x}\cdot\cos x$

$=-\sin x$

4. $\cot(-x)\sin x=-\cot x\sin x$

$=-\frac{\cos x}{\sin x}\cdot\sin x$

$=-\cos x$

5. $\tan x\csc x\cos x=\frac{\sin x}{\cos x}\cdot\frac{1}{\sin x}\cos x$

$=1$

6. $\cot x\sec x\sin x=\frac{\cos x}{\sin x}\cdot\frac{1}{\cos x}\cdot\sin x$

$=1$

7. $\sec x-\sec x\sin^2 x=\sec x(1-\sin^2 x)$

$=\frac{1}{\cos x}\cdot\cos^2 x$

$=\cos x$

8. $\csc x-\csc x\cos^2 x=\csc x(1-\cos^2 x)$

$=\frac{1}{\sin x}\cdot\sin x^2$

$=\sin x$

9. $\cos^2 x-\sin^2 x=\left(1-\sin^2 x\right)-\sin^2 x$

$=1-\sin^2 x-\sin^2 x$

$=1-2\sin^2 x$

10. $\cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x$

$= 2\cos^2 x - 1$

11. $\csc\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta$

$= \dfrac{1}{\sin\theta} - \dfrac{\sin^2\theta}{\sin\theta}$

$= \dfrac{1 - \sin^2\theta}{\sin\theta}$

$= \dfrac{\cos^2\theta}{\sin\theta}$

$= \dfrac{\cos\theta}{\sin\theta} \cdot \cos\theta$

$= \cot\theta\cos\theta$

12. $\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\sin\theta}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{\cos\theta}{\cos\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta\sin\theta} + \dfrac{\cos^2\theta}{\cos\theta\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$= \dfrac{1}{\cos\theta\sin\theta}$

$= \dfrac{1}{\cos\theta} \cdot \dfrac{1}{\sin\theta}$

$= \sec\theta\csc\theta$

13. $\dfrac{\tan\theta\cot\theta}{\csc\theta} = \dfrac{\dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}}$

$= \dfrac{1}{\dfrac{1}{\sin\theta}}$

$= 1 \div \dfrac{1}{\sin\theta}$

$= 1 \cdot \dfrac{\sin\theta}{1}$

$= \sin\theta$

14. $\dfrac{\cos\theta\sec\theta}{\cot\theta} = \dfrac{\dfrac{\cos\theta}{1} \cdot \dfrac{1}{\cos\theta}}{\dfrac{\cos\theta}{\sin\theta}}$

$= \dfrac{1}{\dfrac{\cos\theta}{\sin\theta}}$

$= 1 \div \dfrac{\cos\theta}{\sin\theta}$

$= 1 \cdot \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

15. $\sin^2\theta(1 + \cot^2\theta) = \sin^2\theta(\csc^2\theta)$

$= \sin^2\theta \cdot \dfrac{1}{\sin^2\theta}$

$= 1$

16. $\cos^2\theta(1 + \tan^2\theta) = \cos^2\theta(\sec^2\theta)$

$= \cos^2\theta \cdot \dfrac{1}{\cos^2\theta}$

$= 1$

17. $\dfrac{1 - \cos^2 t}{\cos t} = \dfrac{\sin^2 t}{\cos t}$

$= \sin t \cdot \dfrac{\sin t}{\cos t}$

$= \sin t\tan t$

18. $\dfrac{1 - \sin^2 t}{\sin t} = \dfrac{\cos^2 t}{\sin t}$

$= \cos t \cdot \dfrac{\cos t}{\sin t}$

$= \cos t\cot t$

19. $\dfrac{\csc^2 t}{\cot t} = \dfrac{\dfrac{1}{\sin^2 t}}{\dfrac{\cos t}{\sin t}}$

$= \dfrac{1}{\sin^2 t} \div \dfrac{\cos t}{\sin t}$

$= \dfrac{1}{\sin^2 t} \cdot \dfrac{\sin t}{\cos t}$

$= \dfrac{1}{\sin t} \cdot \dfrac{1}{\cos t}$

$= \csc t\sec t$

20. $$\frac{\sec^2 t}{\tan t} = \frac{\frac{1}{\cos^2 t}}{\frac{\sin t}{\cos t}} = \frac{1}{\cos^2 t} \div \frac{\sin t}{\cos t} = \frac{1}{\cos^2 t} \cdot \frac{\cos t}{\sin t} = \frac{1}{\cos t} \cdot \frac{1}{\sin t} = \sec t \csc t$$

21. $$\frac{\tan^2 t}{\sec t} = \frac{\sec^2 t - 1}{\sec t} = \frac{\sec^2 t}{\sec t} - \frac{1}{\sec t} = \sec t - \cos t$$

22. $$\frac{\cot^2 t}{\csc t} = \frac{\csc^2 t - 1}{\csc t} = \frac{\csc^2 t}{\csc t} - \frac{1}{\csc t} = \csc t - \sin t$$

23. $$\frac{1-\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \csc\theta - \cot\theta$$

24. $$\frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \sec\theta - \tan\theta$$

25. $$\frac{\sin t}{\csc t} + \frac{\cos t}{\sec t} = \frac{\sin t}{\frac{1}{\sin t}} + \frac{\cos t}{\frac{1}{\cos t}} = \sin t \div \frac{1}{\sin t} + \cos t \div \frac{1}{\cos t} = \sin t \cdot \frac{\sin t}{1} + \cos t \cdot \frac{\cos t}{1} = \sin^2 t + \cos^2 t = 1$$

26. $$\frac{\sin t}{\tan t} + \frac{\cos t}{\cot t} = \frac{\sin t}{\frac{\sin t}{\cos t}} + \frac{\cos t}{\frac{\cos t}{\sin t}} = \sin t \div \frac{\sin t}{\cos t} + \cos t \div \frac{\cos t}{\sin t} = \sin t \cdot \frac{\cos t}{\sin t} + \cos t \cdot \frac{\sin t}{\cos t} = \cos t + \sin t = \sin t + \cos t$$

27. $$\tan t + \frac{\cos t}{1+\sin t}$$
$$= \frac{\sin t}{\cos t} + \frac{\cos t}{1+\sin t}$$
$$= \frac{\sin t}{\cos t} \cdot \frac{1+\sin t}{1+\sin t} + \frac{\cos t}{1+\sin t} \cdot \frac{\cos t}{\cos t}$$
$$= \frac{\sin t + \sin^2 t}{\cos t(1+\sin t)} + \frac{\cos^2 t}{\cos t(1+\sin t)}$$
$$\frac{\sin t + \sin^2 t + \cos^2 t}{\cos t(1+\sin t)}$$
$$= \frac{1+\sin t}{\cos t(1+\sin t)}$$
$$= \frac{1}{\cos t}$$
$$= \sec t$$

28. $$\cot t + \frac{\sin t}{1+\cos t} = \frac{\cos t}{\sin t} + \frac{\sin t}{1+\cos t}$$
$$= \frac{\cos t}{\sin t} \cdot \frac{1+\cos t}{1+\cos t} + \frac{\sin t}{1+\cos t} \cdot \frac{\sin t}{\sin t}$$
$$= \frac{\cos t + \cos^2 t}{\sin t(1+\cos t)} + \frac{\sin^2 t}{\sin t(1+\cos t)}$$
$$= \frac{\cos t + \cos^2 t + \sin^2 t}{\sin t(1+\cos t)}$$
$$= \frac{\cos t + 1}{\sin t(1+\cos t)}$$
$$= \frac{1}{\sin t}$$
$$= \csc t$$

29. $$1-\frac{\sin^2 x}{1+\cos x}=1-\frac{\sin^2 x}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}$$
$$=1-\frac{\sin^2 x(1-\cos x)}{1-\cos^2 x}$$
$$=1-\frac{\sin^2 x(1-\cos x)}{\sin^2 x}$$
$$=1-1+\cos x$$
$$=\cos x$$

30. $$1-\frac{\cos^2 x}{1+\sin x}=1-\frac{\cos^2}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x}$$
$$=1-\frac{\cos^2 x(1-\sin x)}{1-\sin^2 x}$$
$$=1-\frac{\cos^2 x(1-\sin x)}{\cos^2 x}$$
$$=1-1+\sin x$$
$$=\sin x$$

31. $$\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}$$
$$=\frac{\cos x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x}+\frac{1-\sin x}{\cos x}$$
$$=\frac{\cos x(1+\sin x)}{1-\sin^2 x}+\frac{1-\sin x}{\cos x}$$
$$==\frac{\cos x(1+\sin x)}{\cos^2 x}+\frac{1-\sin x}{\cos x}$$
$$=\frac{1+\sin x}{\cos x}+\frac{1-\sin x}{\cos x}$$
$$=\frac{2}{\cos x}$$
$$=2\cdot\frac{1}{\cos x}$$
$$=2\sec x$$

32. $$\frac{\sin x}{\cos x+1}+\frac{\cos x-1}{\sin x}$$
$$=\frac{\sin x}{\cos x+1}\cdot\frac{\cos x-1}{\cos x-1}+\frac{\cos x-1}{\sin x}$$
$$=\frac{\sin x(\cos x-1)}{\cos^2 x-1}+\frac{\cos x-1}{\sin x}$$
$$=\frac{\sin x(\cos x-1)}{-\sin^2 x}+\frac{\cos x-1}{\sin x}$$
$$=\frac{\sin x(1-\cos x)}{\sin^2 x}+\frac{\cos x-1}{\sin x}$$
$$=\frac{1-\cos x}{\sin x}+\frac{\cos x-1}{\sin x}$$
$$=\frac{0}{\sin x}$$
$$=0$$

33. $$\sec^2 x\csc^2 x=(1+\tan^2 x)\csc^2 x$$
$$=\csc^2 x+\tan^2 x\csc^2 x$$
$$=\csc^2 x+\frac{\sin^2 x}{\cos^2 x}\cdot\frac{1}{\sin^2 x}$$
$$=\csc^2 x+\frac{1}{\cos^2 x}$$
$$=\csc^2 x+\sec^2 x$$
$$=\sec^2 x+\csc^2 x$$

34. $$\csc^2 x\sec x=(1+\cot^2 x)\sec x$$
$$=\sec x+\cot^2 x\sec x$$
$$=\sec x+\frac{\cos^2 x}{\sin^2 x}\cdot\frac{1}{\cos x}$$
$$=\sec x+\frac{\cos x}{\sin^2 x}$$
$$=\sec x+\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x}$$
$$=\sec x+\csc x\cot x$$

35. $$\frac{\sec x-\csc x}{\sec x+\csc x}=\frac{\frac{1}{\cos x}-\frac{1}{\sin x}}{\frac{1}{\cos x}+\frac{1}{\sin x}}=\frac{\frac{1}{\cos x}-\frac{1}{\sin x}}{\frac{1}{\cos x}+\frac{1}{\sin x}}\cdot\frac{\sin x}{\sin x}=\frac{\frac{\sin x}{\cos x}-1}{\frac{\sin x}{\cos x}+1}=\frac{\tan x-1}{\tan x+1}$$

36. $$\frac{\csc x-\sec x}{\csc x+\sec x}=\frac{\frac{1}{\sin x}-\frac{1}{\cos x}}{\frac{1}{\sin x}+\frac{1}{\cos x}}=\frac{\frac{1}{\sin x}-\frac{1}{\cos x}}{\frac{1}{\sin x}+\frac{1}{\cos x}}\cdot\frac{\cos x}{\cos x}=\frac{\frac{\cos x}{\sin x}-1}{\frac{\cos x}{\sin x}+1}=\frac{\cot x-1}{\cot x+1}$$

37. $$\frac{\sin^2 x-\cos^2 x}{\sin x+\cos x}=\frac{(\sin x+\cos x)(\sin x-\cos x)}{\sin x+\cos x}=\sin x-\cos x$$

38. $$\frac{\tan^2 x-\cot^2 x}{\tan x+\cot x}=\frac{(\tan x-\cot x)(\tan x+\cot x)}{\tan x+\cot x}=\tan x-\cot x$$

39. $$\tan^2 2x+\sin^2 2x+\cos^2 2x=\tan^2 2x+1=\sec^2 2x$$

40. $$\cot^2 2x+\cos^2 2x+\sin^2 2x=\cot^2 2x+1=\csc^2 2x$$

41. $$\frac{\tan 2\theta+\cot 2\theta}{\csc 2\theta}=\frac{\frac{\sin 2\theta}{\cos 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}}{\frac{1}{\sin 2\theta}}=\frac{\frac{\sin 2\theta}{\cos 2\theta}\cdot\frac{\sin 2\theta}{\sin 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}\cdot\frac{\cos 2\theta}{\cos 2\theta}}{\frac{1}{\sin 2\theta}}=\frac{\frac{\sin^2 2\theta+\cos 2\theta}{\cos 2\theta\sin 2\theta}}{\frac{1}{\sin 2\theta}}=\frac{1}{\cos 2\theta\sin 2\theta}\div\frac{1}{\sin 2\theta}=\frac{1}{\cos 2\theta\sin 2\theta}\cdot\frac{\sin 2\theta}{1}=\frac{1}{\cos 2\theta}=\sec 2\theta$$

42. $$\frac{\tan 2\theta+\cot 2\theta}{\sec 2\theta}=\frac{\frac{\sin 2\theta}{\cos 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}}{\frac{1}{\cos 2\theta}}=\frac{\frac{\sin 2\theta}{\cos 2\theta}\cdot\frac{\sin 2\theta}{\sin 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}\cdot\frac{\cos 2\theta}{\cos 2\theta}}{\frac{1}{\cos 2\theta}}=\frac{\frac{\sin^2 2\theta}{\cos 2\theta\sin 2\theta}+\frac{\cos^2 2\theta}{\cos 2\theta\sin 2\theta}}{\frac{1}{\cos 2\theta}}=\frac{\frac{\sin^2 2\theta+\cos^2 2\theta}{\cos 2\theta\sin 2\theta}}{\frac{1}{\cos 2\theta}}=\frac{1}{\cos 2\theta\sin 2\theta}\div\frac{1}{\cos 2\theta}=\frac{1}{\cos 2\theta\sin 2\theta}\cdot\frac{\cos 2\theta}{1}=\frac{1}{\sin 2\theta}=\csc 2\theta$$

43.
$$\frac{\tan x+\tan y}{1-\tan x\tan y}=\frac{\frac{\sin x}{\cos x}+\frac{\sin y}{\cos y}}{1-\frac{\sin x}{\cos y}\cdot\frac{\sin y}{\cos y}}\cdot\frac{\cos x\cos y}{\cos x\cos y}$$
$$=\frac{\sin x\cos y+\cos x\sin y}{\cos x\cos y-\sin x\sin y}$$

44.
$$\frac{\cot x+\cot y}{1-\cot x\cot y}=\frac{\frac{\cos x}{\sin x}+\frac{\cos y}{\sin y}}{1-\frac{\cos x}{\sin x}\cdot\frac{\cos y}{\sin y}}$$
$$=\frac{\frac{\cos x}{\sin x}+\frac{\cos y}{\sin y}}{1-\frac{\cos x}{\sin x}\cdot\frac{\cos y}{\sin y}}\cdot\frac{\sin x\sin y}{\sin x\sin y}$$
$$=\frac{\frac{\cos x}{\sin x}\cdot\frac{\sin x\sin y}{1}+\frac{\cos y}{\sin y}\cdot\frac{\sin x\sin y}{1}}{\sin x\sin y-\frac{\cos x}{\sin x}\cdot\frac{\cos y}{\sin y}\cdot\frac{\sin x\sin y}{1}}$$
$$=\frac{\cos x\sin y+\sin x\cos y}{\sin x\sin y-\cos x\cos y}$$

45. Left side:
$$(\sec x-\tan x)^2=\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)^2$$
$$=\left(\frac{1-\sin x}{\cos x}\right)^2$$
$$=\frac{(1-\sin x)^2}{\cos^2 x}$$
Right side:
$$\frac{1-\sin x}{1+\sin x}=\frac{1-\sin x}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x}$$
$$=\frac{(1-\sin x)^2}{1-\sin^2 x}$$
$$=\frac{(1-\sin x)^2}{\cos^2 x}$$
The identity is verified because both sides are equal to $\frac{(1-\sin x)^2}{\cos^2 x}$.

46. Left side: $(\csc x-\cot x)^2=\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)^2=\left(\frac{1-\cos x}{\sin x}\right)^2=\frac{(1-\cos x)^2}{\sin^2 x}$

Right side: $\frac{1-\cos x}{1+\cos x}=\frac{1-\cos x}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}=\frac{(1-\cos x)^2}{1-\cos^2 x}=\frac{(1-\cos x)^2}{\sin^2 x}$

The identity is verified because both sides are equal to $\frac{(1-\cos x)^2}{\sin^2 x}$.

47. $$\frac{\tan t}{\sec t-1}=\frac{\tan t}{\sec t-1}\cdot\frac{\sec t+1}{\sec t+1}=\frac{\tan t(\sec t+1)}{\sec^2 t-1}=\frac{\tan t(\sec t+1)}{\tan^2 t}=\frac{\sec t+1}{\tan t}$$

48. $$\frac{\cot t}{\csc t+1}=\frac{\cot t}{\csc t+1}\cdot\frac{\csc t-1}{\csc t-1}=\frac{\cot t(\csc t-1)}{\csc^2 t-1}=\frac{\cot t(\csc t-1)}{\cot^2 t}=\frac{\csc t-1}{\cot t}$$

49. Left side:
$$\frac{1+\cos t}{1-\cos t}=\frac{1+\cos t}{1-\cos t}\cdot\frac{1+\cos t}{1+\cos t}=\frac{(1+\cos t)^2}{1-\cos^2 t}=\frac{(1+\cos t)^2}{\sin^2 t}$$
Right side:
$$(\csc t+\cot t)^2=\left(\frac{1}{\sin t}+\frac{\cos t}{\sin t}\right)^2=\left(\frac{1+\cos t}{\sin t}\right)^2=\frac{(1+\cos t)^2}{\sin^2 t}$$
The identity is verified because both sides are equal to $\frac{(1+\cos t)^2}{\sin^2 t}$.

50. Left side:
$$\frac{\cos^2 t+4\cos t+4}{\cos t+2}=\frac{(\cos t+2)(\cos t+2)}{\cos t+2}=\cos t+2$$
Right side:
$$\frac{2\sec t+1}{\sec t}=\frac{2\sec t}{\sec t}+\frac{1}{\sec t}=2+\cos t=\cos t+2$$
The identity is verified because both sides are equal to $\cos t+2$.

51. $$\cos^4 t-\sin^4 t=\left(\cos^2 t-\sin^2 t\right)\left(\cos^2 t+\sin^2 t\right)=\left(\cos^2 t-\sin^2 t\right)\cdot 1=1-\sin^2 t-\sin^2 t=1-2\sin^2 t$$

52. $$\sin^4 t-\cos^4 t=\left(\sin^2 t-\cos^2 t\right)\left(\sin^2 t+\cos^2 t\right)=\left(\sin^2 t-\cos^2 t\right)\cdot 1=1-\cos^2 t-\cos^2 t=1-2\cos^2 t$$

53. $$\frac{\sin\theta-\cos\theta}{\sin\theta}+\frac{\cos\theta-\sin\theta}{\cos\theta}$$
$$=\frac{(\sin\theta-\cos\theta)\cos\theta}{\cos\theta\sin\theta}+\frac{(\cos\theta-\sin\theta)\sin\theta}{\cos\theta\sin\theta}$$
$$=\frac{\sin\theta\cos\theta-\cos^2\theta+\sin\theta\cos\theta-\sin^2\theta}{\sin\theta\cos\theta}$$
$$=\frac{2\sin\theta\cos\theta-\left(\cos^2\theta+\sin^2\theta\right)}{\sin\theta\cos\theta}$$
$$=\frac{2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$
$$=\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}-\frac{1}{\sin\theta\cos\theta}$$
$$=2-\frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta}$$
$$=2-\csc\theta\sec\theta$$
$$=2-\sec\theta\csc\theta$$

54. $\dfrac{\sin\theta}{1-\cot\theta}-\dfrac{\cos\theta}{\tan\theta-1}$

$=\dfrac{\sin\theta}{1-\dfrac{\cos\theta}{\sin\theta}}-\dfrac{\cos\theta}{\dfrac{\sin\theta}{\cos\theta}-1}$

$=\dfrac{\sin\theta}{\dfrac{\sin\theta}{\sin\theta}-\dfrac{\cos\theta}{\sin\theta}}-\dfrac{\cos\theta}{\dfrac{\sin\theta}{\cos\theta}-\dfrac{\cos\theta}{\cos\theta}}$

$=\dfrac{\sin\theta}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}}-\dfrac{\cos\theta}{\dfrac{\sin\theta-\cos\theta}{\cos\theta}}$

$=\sin\theta\div\dfrac{\sin\theta-\cos\theta}{\sin\theta}-\cos\theta\div\dfrac{\sin\theta-\cos\theta}{\cos\theta}$

$=\sin\theta\cdot\dfrac{\sin\theta}{\sin\theta-\cos\theta}-\cos\theta\cdot\dfrac{\cos\theta}{\sin\theta-\cos\theta}$

$=\dfrac{\sin^2\theta}{\sin\theta-\cos\theta}-\dfrac{\cos^2\theta}{\sin\theta-\cos\theta}$

$=\dfrac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$

$\dfrac{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}{\sin\theta-\cos\theta}$

$=\sin\theta+\cos\theta$

55. $\left(\tan^2\theta+1\right)\left(\cos^2\theta+1\right)$

$=\tan^2\theta\cos^2\theta+\tan^2\theta+\cos^2\theta+1$

$=\dfrac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta+\tan^2\theta+\cos^2\theta+1$

$=\sin^2\theta+\tan^2\theta+\cos^2\theta+1$

$=\sin^2\theta+\cos^2\theta+\tan^2\theta+1$

$=1+\tan^2\theta+1$

$=\tan^2\theta+2$

56. $(\cot^2\theta+1)(\sin^2+1)$

$=\cot^2\theta\sin^2\theta+\cot^2\theta+\sin^2\theta+1$

$=\dfrac{\cos^2\theta}{\sin^2\theta}\sin^2\theta+\cot^2\theta+\sin^2\theta+1$

$=\cos^2\theta+\sin^2\theta+\cot^2\theta+1$

$=1+\cot^2\theta+1$

$=\cot^2\theta+2$

57. $(\cos\theta-\sin\theta)^2+(\cos\theta+\sin\theta)^2$

$=\cos^2\theta-2\cos\theta\sin\theta+\sin^2\theta+\cos^2\theta+2\cos\theta\sin\theta+\sin^2\theta$

$=\cos^2\theta+\sin^2\theta+\cos^2\theta+\sin^2\theta$

$=1+1=2$

58. $(3\cos\theta - 4\sin\theta)^2 + (4\cos\theta + 3\sin\theta)^2$

$= 9\cos^2\theta - 24\cos\theta\sin\theta + 16\sin^2\theta +$

$+16\cos^2\theta + 24\cos\theta\sin\theta + 9\sin^2\theta$

$= 9\cos^2\theta + 9\sin^2\theta + 16\sin^2\theta + 16\cos^2\theta$

$= 9(\cos^2\theta + \sin^2\theta) + 16(\sin^2\theta + \cos^2\theta)$

$= 9(1) + 16(1)$

$= 25$

59. $\dfrac{\cos^2 x - \sin^2 x}{1 - \tan^2 x}$

$= \dfrac{\cos^2 x - \sin^2 x}{1 - \dfrac{\sin^2 x}{\cos^2 x}} = \dfrac{\cos^2 x - \sin^2 x}{\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x}}$

$= \dfrac{\cos^2 x - \sin^2 x}{1} \div \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x}$

$= \dfrac{\cos^2 x - \sin^2 x}{1} \cdot \dfrac{\cos^2 x}{\cos^2 x - \sin^2 x} = \cos^2 x$

60. $\dfrac{\sin x + \cos x}{\sin x} - \dfrac{\cos x - \sin x}{\cos x}$

$= \dfrac{(\sin x + \cos x)\cos x}{\sin x \cos x} - \dfrac{(\cos x - \sin x)\sin x}{\sin x \cos x}$

$= \dfrac{\sin x \cos x + \cos^2 x - \cos x \sin x + \sin^2 x}{\sin x \cos x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\sin x \cos x}$

$= \dfrac{1}{\sin x \cos x}$

$= \dfrac{1}{\sin x} \cdot \dfrac{1}{\cos x}$

$= \csc x \sec x$

$= \sec x \csc x$

61. Conjecture: left side is equal to $\cos x$

$\dfrac{(\sec x + \tan x)(\sec x - \tan x)}{\sec x} = \dfrac{\sec^2 x - \tan^2 x}{\sec x}$

$= \dfrac{1}{\sec x}$

$= \cos x$

62. Conjecture: left side is equal to $\sin x$

$$\frac{\sec^2 x\csc x}{\sec^2 x+\csc^2 x}=\frac{\frac{1}{\cos^2 x}\cdot\frac{1}{\sin x}}{\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}}\cdot\frac{\cos^2 x\sin^2 x}{\cos^2 x\sin^2 x}$$
$$=\frac{\sin x}{\sin^2 x+\cos^2 x}$$
$$=\frac{\sin x}{1}$$
$$=\sin x$$

63. Conjecture: left side is equal to $2\sin x$

$$\frac{\cos x+\cot x\sin x}{\cot x}=\frac{\cos x}{\cot x}+\frac{\cot x\sin x}{\cot x}$$
$$=\frac{\cos x}{\frac{\cos x}{\sin x}}+\frac{\cot x\sin x}{\cot x}$$
$$=\frac{\cos x\sin x}{\cos x}+\sin x$$
$$=\sin x+\sin x$$
$$=2\sin x$$

64. Conjecture: left side is equal to $\cos x-1$

$$\frac{\cos x\tan x-\tan x+2\cos x-2}{\tan x+2}=\frac{\cos x\tan x+2\cos x-\tan x-2}{\tan x+2}$$
$$=\frac{\cos x\tan x+2\cos x}{\tan x+2}+\frac{-\tan x-2}{\tan x+2}$$
$$=\frac{\cos x(\tan x+2)}{\tan x+2}-1$$
$$=\cos x-1$$

65. Conjecture: left side is equal to $2\sec x$

$$\frac{1}{\sec x+\tan x}+\frac{1}{\sec x-\tan x}=\frac{\sec^2 x-\tan^2 x}{\sec x+\tan x}+\frac{\sec^2 x-\tan^2 x}{\sec x-\tan x}$$
$$=\frac{(\sec x+\tan x)(\sec x-\tan x)}{\sec x+\tan x}+\frac{(\sec x+\tan x)(\sec x-\tan x)}{\sec x-\tan x}$$
$$=\sec x-\tan x+\sec x+\tan x$$
$$=2\sec x$$

66. Conjecture: left side is equal to $2\csc x$

$$\frac{1+\cos x}{\sin x}+\frac{\sin x}{1+\cos x}=\frac{1+\cos x}{\sin x}\cdot\frac{1+\cos x}{1+\cos x}+\frac{\sin x}{1+\cos x}\cdot\frac{\sin x}{\sin x}$$
$$=\frac{1+2\cos x+\cos^2 x}{(\sin x)(1+\cos x)}+\frac{\sin^2 x}{(\sin x)(1+\cos x)}$$
$$=\frac{1+2\cos x+\cos^2 x+\sin^2 x}{(\sin x)(1+\cos x)}$$
$$=\frac{1+2\cos x+1}{(\sin x)(1+\cos x)}$$
$$=\frac{2+2\cos x}{(\sin x)(1+\cos x)}$$
$$=\frac{2(1+\cos x)}{(\sin x)(1+\cos x)}$$
$$=\frac{2}{\sin x}$$
$$=2\csc x$$

67. $$\frac{\tan x+\cot x}{\csc x}=\frac{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$$
$$=\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)\frac{\sin x}{1}$$
$$=\frac{\sin^2 x}{\cos x}+\frac{\sin x\cos x}{\sin x}$$
$$=\frac{1-\cos^2 x}{\cos x}+\frac{\cos x}{1}$$
$$=\frac{1-\cos^2 x}{\cos x}+\frac{\cos^2 x}{\cos x}$$
$$=\frac{1}{\cos x}$$

68. $$\frac{\sec x+\csc x}{1+\tan x}=\left(\frac{\frac{1}{\cos x}+\frac{1}{\sin x}}{1+\frac{\sin x}{\cos x}}\right)\frac{\sin x\cos x}{\sin x\cos x}$$
$$=\frac{\frac{\sin x\cos x}{\cos x}+\frac{\sin x\cos x}{\sin x}}{\sin x\cos x+\frac{\sin^2 x\cos x}{\cos x}}$$
$$=\frac{\sin x+\cos x}{\sin x\cos x+\sin^2 x}$$
$$=\frac{\sin x+\cos x}{\sin x(\cos x+\sin x)}$$
$$=\frac{1}{\sin x}$$

69. $$\begin{aligned}\frac{\cos x}{1+\sin x}+\tan x &= \frac{\cos x}{1+\sin x}\cdot\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}\cdot\frac{1+\sin x}{1+\sin x}\\ &= \frac{\cos^2 x}{(1+\sin x)(\cos x)}+\frac{\sin x+\sin^2 x}{(1+\sin x)(\cos x)}\\ &= \frac{\cos^2 x+\sin x+\sin^2 x}{(1+\sin x)(\cos x)}\\ &= \frac{\sin x+\cos^2 x+\sin^2 x}{(1+\sin x)(\cos x)}\\ &= \frac{\sin x+1}{(1+\sin x)(\cos x)}\\ &= \frac{1}{\cos x}\end{aligned}$$

70. $$\begin{aligned}\frac{1}{\sin x\cos x}-\cot x &= \frac{1}{\sin x\cos x}-\frac{\sin x}{\cos x}\\ &= \frac{1}{\sin x\cos x}-\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}\\ &= \frac{1-\cos^2 x}{\sin x\cos x}\\ &= \frac{\sin^2 x}{\sin x\cos x}\\ &= \frac{\sin x}{\cos x}\\ &= \tan x\\ &= \frac{1}{\cot x}\end{aligned}$$

71. $$\begin{aligned}\frac{1}{1-\cos x}-\frac{\cos x}{1+\cos x} &= \frac{1}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x}-\frac{\cos x}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}\\ &= \frac{1+\cos x}{1-\cos^2 x}-\frac{\cos x-\cos^2 x}{1-\cos^2 x}\\ &= \frac{1+\cos x-\cos x+\cos^2 x}{1-\cos^2 x}\\ &= \frac{1+\cos^2 x}{\sin^2 x}\\ &= \frac{1}{\sin^2 x}+\frac{\cos^2 x}{\sin^2 x}\\ &= \csc^2 x+\cot^2 x\\ &= \csc^2 x+\csc^2 x-1\\ &= 2\csc^2 x-1\end{aligned}$$

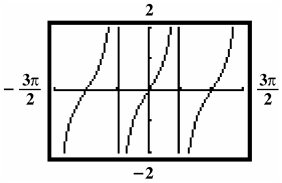
72. $$\begin{aligned}(\sec x+\csc x)(\sin x+\cos x)-2-\cot x &= \sec x\sin x+\sec x\cos x+\csc x\sin x+\csc x\cos x-2-\cot x\\ &= \sec x\sin x+\sec x\cos x+\csc x\sin x+\csc x\cos x-2-\cot x\\ &= \tan x+1+1+\cot x-2-\cot x\\ &= \tan x\end{aligned}$$

73. $$\begin{aligned}\frac{1}{\csc x-\sin x} &= \frac{1}{\frac{1}{\sin x}-\sin x}\\ &= \frac{1}{\frac{1}{\sin x}-\sin x}\\ &= \frac{1}{\frac{1}{\sin x}-\frac{\sin^2 x}{\sin x}}\\ &= \frac{1}{\frac{1-\sin^2 x}{\sin x}}\\ &= \frac{1}{\frac{\cos^2 x}{\sin x}}\\ &= \frac{\sin x}{\cos^2 x}\\ &= \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}\\ &= \sec x\tan x\end{aligned}$$

74. $$\begin{aligned}\frac{1-\sin x}{1+\sin x}-\frac{1+\sin x}{1-\sin x} &= \frac{(1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)}-\frac{(1+\sin x)(1+\sin x)}{(1-\sin x)(1+\sin x)}\\ &= \frac{(1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)}-\frac{(1+\sin x)(1+\sin x)}{(1-\sin x)(1+\sin x)}\\ &= \frac{1-2\sin x+\sin^2 x}{1-\sin^2 x}-\frac{1+2\sin x+\sin^2 x}{1-\sin^2 x}\\ &= \frac{-4\sin x}{1-\sin^2 x}\\ &= \frac{-4\sin x}{\cos^2 x}\\ &= \frac{-4}{\cos x}\cdot\frac{\sin x}{\cos x}\\ &= -4\sec x\tan x\end{aligned}$$

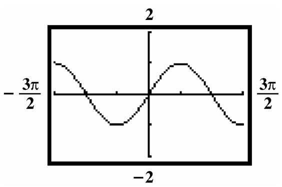
75. – 78. Answers may vary.

79.



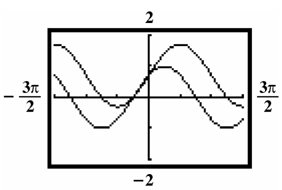
$$\begin{aligned}\sec x(\sin x - \cos x) + 1 &= \frac{1}{\cos x}(\sin x - \cos x) + 1\\ &= \frac{\sin x}{\cos x} - \frac{\cos x}{\cos x} + 1\\ &= \tan x - 1 + 1\\ &= \tan x\end{aligned}$$

80.



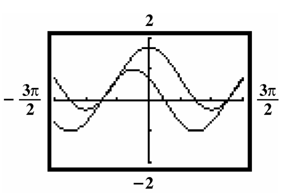
$$\begin{aligned}-\cos x \tan(-x) &= -\cos x \cdot \frac{\sin(-x)}{\cos(-x)}\\ &= -\cos x \cdot \frac{-\sin x}{\cos x} = \sin x\end{aligned}$$

81.



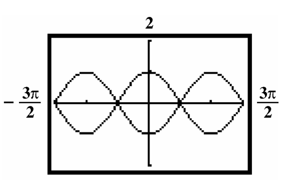
The graphs do not coincide.
Values for x may vary.

82.



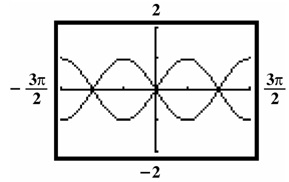
The graphs do not coincide.
Values for x may vary.

83.



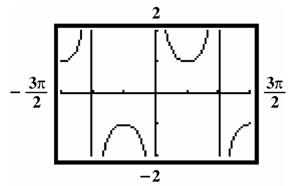
The graphs do not coincide.
Values for x may vary.

84.



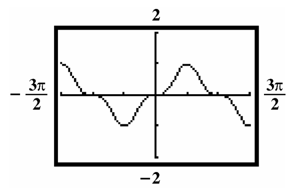
The graphs do not coincide.
Values for x may vary.

85.



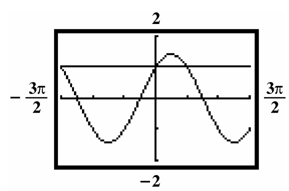
$$\frac{\sin x}{1-\cos^2 x} = \frac{\sin x}{\sin^2 x} = \frac{1}{\sin x} = \csc x$$

86.



$$\begin{aligned}\sin x - \sin x \cos^2 x &= \sin x(1-\cos^2 x)\\ &= \sin x\left(\sin^2 x\right) = \sin^3 x\end{aligned}$$

87.



The graphs do not coincide.
Values for x may vary.

88. makes sense

89. makes sense

90. makes sense

91. does not make sense; Explanations will vary. Sample explanation: The most efficient way to simplify the identity is to multiply out the numerator and then use a Pythagorean identity.

92. $\dfrac{\sin^3 x - \cos^3 x}{\sin x - \cos x}$

$= \dfrac{(\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)}{\sin x - \cos x}$

$= \sin^2 x + \cos^2 x + \sin x \cos x$

$= 1 + \sin x \cos x$

93. $\dfrac{\sin x - \cos x + 1}{\sin x + \cos x - 1}$

$= \dfrac{\sin x - \cos x + 1}{\sin x + \cos x - 1} \cdot \dfrac{\sin x - \cos x - 1}{\sin x - \cos x - 1}$

$= \dfrac{\sin^2 x - 2\cos x \sin x + \cos^2 x - 1}{\sin^2 x - 2\sin x - \cos^2 x + 1}$

$= \dfrac{\sin^2 x + \cos^2 x - 2\cos x \sin x - 1}{\sin x^2 - 2\sin x - (1 - \sin^2 x) + 1}$

$= \dfrac{1 - 2\cos x \sin x - 1}{\sin^2 x - 2\sin x + \sin x^2}$

$= \dfrac{-2\cos x \sin x}{2\sin^2 x - 2\sin x}$

$= \dfrac{-2\sin x \cos x}{2\sin x(\sin x - 1)}$

$= \dfrac{-\cos x}{\sin x - \cos x}$

$= \dfrac{-\cos x}{\sin x - 1} \cdot \dfrac{\sin x + 1}{\sin x + 1}$

$= \dfrac{-\cos x(\sin x + 1)}{\sin^2 x - 1}$

$= \dfrac{-\cos x(\sin x + 1)}{\cos^2 x - 1 - \cos^2 x}$

$= \dfrac{-\cos x(\sin x + 1)}{\cos^2 x}$

$= \dfrac{\sin x + 1}{\cos x}$

94. $-\ln|\cos x| = \ln|\cos x|^{-1} = \ln \dfrac{1}{|\cos x|}$

$= \ln\left|\dfrac{1}{\cos x}\right| = \ln|\sec x|$

95. $\ln e^{\tan^2 x - \sec^2 x}$

$= \tan^2 x - \sec^2 x$

$= -(-\tan^2 x + \sec^2 x)$

$= -(\sec^2 x - \tan^2 x)$

$= -1$

96. Answers may vary.

97. Answers may vary.

98. $\cos 30^\circ = \dfrac{\sqrt{3}}{2}$

$\sin 30^\circ = \dfrac{1}{2}$

$\cos 60^\circ = \dfrac{1}{2}$

$\sin 60^\circ = \dfrac{\sqrt{3}}{2}$

$\cos 90^\circ = 0$

$\sin 90^\circ = 1$

99. **a.** No, they are not equal.

$\cos(30^\circ + 60^\circ) \neq \cos 30^\circ + \cos 60^\circ$

$\cos 90^\circ \neq \dfrac{\sqrt{3}}{2} + \dfrac{1}{2}$

$0 \neq \dfrac{1 + \sqrt{3}}{2}$

b. Yes, they are equal.

$\cos(30^\circ + 60^\circ) = \cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ$

$\cos 90^\circ = \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{2}\right) - \left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)$

$0 = \dfrac{\sqrt{3}}{4} - \dfrac{\sqrt{3}}{4}$

$0 = 0$

100. **a.** No, they are not equal.

$$\sin(30^\circ + 60^\circ) \neq \sin 30^\circ + \sin 60^\circ$$
$$\sin 90^\circ \neq \frac{1}{2} + \frac{\sqrt{3}}{2}$$
$$1 \neq \frac{1+\sqrt{3}}{2}$$

b. Yes, they are equal.

$$\sin(30^\circ + 60^\circ) = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$$
$$\sin 90^\circ = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$
$$1 = \frac{1}{4} + \frac{3}{4}$$
$$1 = 1$$

Section 6.2

Check Point Exercises

1.
$$\begin{aligned}\cos 30^\circ &= \cos(90^\circ - 60^\circ)\\ &= \cos 90^\circ \cos 60^\circ + \sin 90^\circ \sin 60^\circ\\ &= 0 \cdot \frac{1}{2} + 1 \cdot \frac{\sqrt{3}}{2}\\ &= 0 + \frac{\sqrt{3}}{2}\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

2.
$$\begin{aligned}&\cos 70^\circ \cos 40^\circ + \sin 70^\circ \sin 40^\circ\\ &= \cos(70 - 40^\circ)\\ &= \cos 30^\circ\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

3.
$$\begin{aligned}\frac{\cos(\alpha - \beta)}{\cos\alpha\cos\beta} &= \frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha\cos\beta}\\ &= \frac{\cos\alpha}{\cos\alpha} \cdot \frac{\cos\beta}{\cos\beta} + \frac{\sin\alpha}{\cos\alpha} \cdot \frac{\sin\beta}{\cos\beta}\\ &= 1 \cdot 1 + \tan\alpha \cdot \tan\beta\\ &= 1 + \tan\alpha\tan\beta\end{aligned}$$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

4.
$$\begin{aligned}\sin\frac{5\pi}{12} &= \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)\\ &= \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}\\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}\\ &= \frac{\sqrt{2} + \sqrt{6}}{4}\end{aligned}$$

5. **a.** $\sin\alpha = \frac{4}{5} = \frac{y}{r}$

Find *x:*

$$x^2 + y^2 = r^2$$
$$x^2 + 4^2 = 5^2$$
$$x^2 + 16 = 25$$
$$x^2 = 9$$

Because α is in Quadrant II, *x* is negative.

$$x = -\sqrt{9} = -3$$
$$\cos\alpha = \frac{x}{r} = \frac{-3}{5} = -\frac{3}{5}$$

b. $\sin\beta = \frac{1}{2} = \frac{y}{r}$

Find *x:*

$$x^2 + y^2 = r^2$$
$$x^2 + 1^2 = 2^2$$
$$x^2 + 1 = 4$$
$$x^2 = 3$$

Because β is in Quadrant I, *x* is positive.

$$x = \sqrt{3}$$
$$\cos\beta = \frac{x}{r} = \frac{\sqrt{3}}{2}$$

c.
$$\begin{aligned}\cos(\alpha + \beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta\\ &= -\frac{3}{5} \cdot \frac{\sqrt{3}}{2} - \frac{4}{5} \cdot \frac{1}{2}\\ &= \frac{-3\sqrt{3}}{10} - \frac{4}{10}\\ &= \frac{-3\sqrt{3} - 4}{10}\end{aligned}$$

d. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$$=\frac{4}{5}\cdot\frac{\sqrt{3}}{2}+\frac{-3}{5}\cdot\frac{1}{2}$$
$$=\frac{4\sqrt{3}}{10}+\frac{-3}{10}$$
$$=\frac{4\sqrt{3}-3}{10}$$

6. a. The graph appears to be the sine curve, $y=\sin x.$
It cycles through intercept, maximum, intercept, minimum and back to intercept. Thus, $y=\sin x$ also describes the graph.

b. $\cos\left(x+\frac{3\pi}{2}\right)=\cos x\cos\frac{3\pi}{2}-\sin x\sin\frac{3\pi}{2}$

$$=\cos x\cdot 0-\sin x\cdot(-1)$$
$$=\sin x$$

This verifies our observation that $y=\cos\left(x+\frac{3\pi}{2}\right)$ and $y=\sin x$ describe the same graph.

7. $\tan(x+\pi)=\frac{\tan x+\tan\pi}{1-\tan x\tan\pi}$

$$=\frac{\tan x+0}{1-\tan x\cdot 0}$$
$$=\frac{\tan x}{1}$$
$$=\tan x$$

Exercise Set 6.2

1. $\cos(45°-30°)=\cos 45°\cos 30°+\sin 45°\sin 30°$

$$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$
$$=\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}$$
$$=\frac{\sqrt{6}+\sqrt{2}}{4}$$

2. $\cos(120^{\text{o}}-45^{\text{o}})$

$$=\cos 120^{\text{o}}\cos 45^{\text{o}}+\sin 120^{\text{o}}\sin 45^{\text{o}}$$
$$=-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$
$$=-\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$$
$$=\frac{-\sqrt{2}+\sqrt{6}}{4}$$

3. $\cos\left(\frac{3\pi}{4}-\frac{\pi}{6}\right)=\cos\frac{3\pi}{4}\cos\frac{\pi}{6}+\sin\frac{3\pi}{4}\sin\frac{\pi}{6}$

$$=-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$
$$=-\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}$$
$$=\frac{\sqrt{2}-\sqrt{6}}{4}$$

4. $\cos\left(\frac{2\pi}{3}-\frac{\pi}{6}\right)=\cos\frac{2\pi}{3}\cos\frac{\pi}{6}+\sin\frac{2\pi}{3}\sin\frac{\pi}{6}$

$$=-\frac{1}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\frac{1}{2}$$
$$=\frac{-\sqrt{3}}{4}+\frac{\sqrt{3}}{4}$$
$$=0$$

5. a. $\cos 50°\cos 20°+\sin 50°\sin 20°$
$=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
Thus, $\alpha=50°$ and $\beta=20°$.

b. $\cos 50°\cos 20°+\sin 50°\sin 20°$
$=\cos(50°-20°)$
$=\cos 30°$

c. $\cos 30°=\frac{\sqrt{3}}{2}$

6. **a.** $\cos 50^{\circ}\cos 5^{\circ}+\sin 50^{\circ}\sin 5^{\circ}$

$=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

Thus, $\alpha=50^{\circ}$ and $\beta=5^{\circ}$

b. $\cos 50^{\circ}\cos 5^{\circ}+\sin 50^{\circ}\sin 5^{\circ}$

$=\cos(50^{\circ}-5^{\circ})$

$=\cos 45^{\circ}$

c. $\cos 45^{\circ}=\dfrac{\sqrt{2}}{2}$

7. **a.** $\cos\dfrac{5\pi}{12}\cos\dfrac{\pi}{12}+\sin\dfrac{5\pi}{12}\sin\dfrac{\pi}{12}$

$=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

Thus, $\alpha=\dfrac{5\pi}{12}$ and $\beta=\dfrac{\pi}{12}$.

b. $\cos\dfrac{5\pi}{12}\cos\dfrac{\pi}{12}+\sin\dfrac{5\pi}{12}\sin\dfrac{\pi}{12}$

$=\cos\left(\dfrac{5\pi}{12}-\dfrac{\pi}{12}\right)$

$=\cos\dfrac{4\pi}{12}$

$=\cos\dfrac{\pi}{3}$

c. $\cos\dfrac{\pi}{3}=\dfrac{1}{2}$

8. **a.** $\cos\dfrac{5\pi}{18}\cos\dfrac{\pi}{9}+\sin\dfrac{5\pi}{18}\sin\dfrac{\pi}{9}$

$=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$\alpha=\dfrac{5\pi}{18}$ and $\beta=\dfrac{\pi}{9}$

b. $\cos\dfrac{5\pi}{18}\cos\dfrac{\pi}{9}+\sin\dfrac{5\pi}{18}\sin\dfrac{\pi}{9}$

$=\cos\left(\dfrac{5\pi}{18}-\dfrac{\pi}{9}\right)$

$=\cos\dfrac{3\pi}{18}$

$=\cos\dfrac{\pi}{6}$

c. $\cos\dfrac{\pi}{6}=\dfrac{\sqrt{3}}{2}$

9.

$$\begin{aligned}\frac{\cos(\alpha-\beta)}{\cos\alpha\sin\beta}&=\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\sin\beta}\\&=\frac{\cos\alpha}{\cos\alpha}\cdot\frac{\cos\beta}{\sin\beta}-\frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\sin\beta}\\&=1\cdot\cot\beta+\tan\alpha\cdot 1\\&=\tan\alpha+\cot\beta\end{aligned}$$

10.

$$\begin{aligned}\frac{\cos(\alpha-\beta)}{\sin\alpha\sin\beta}&=\frac{\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\sin\alpha\sin\beta}\\&=\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}+\frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}\\&=\frac{\cos\alpha}{\sin\alpha}\cdot\frac{\cos\beta}{\sin\beta}+1\\&=\cot\alpha\cot\beta+1\end{aligned}$$

11.

$$\begin{aligned}\cos\left(x-\frac{\pi}{4}\right)&=\cos x\cos\frac{\pi}{4}+\sin x\sin\frac{\pi}{4}\\&=\cos x\cdot\frac{\sqrt{2}}{2}+\sin x\cdot\frac{\sqrt{2}}{2}\\&=\frac{\sqrt{2}}{2}(\cos x+\sin x)\end{aligned}$$

12.

$$\begin{aligned}\cos\left(x-\frac{5\pi}{4}\right)&=\cos x\cos\frac{5\pi}{4}+\sin x\sin\frac{5\pi}{4}\\&=\cos x\cdot-\frac{\sqrt{2}}{2}+\sin x\cdot-\frac{\sqrt{2}}{2}\\&=-\frac{\sqrt{2}}{2}(\cos x+\sin x)\end{aligned}$$

13.

$$\begin{aligned}\sin(45^{\circ}-30^{\circ})&=\sin 45^{\circ}\cos 30^{\circ}-\cos 45^{\circ}\sin 30^{\circ}\\&=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}\\&=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}\\&=\frac{\sqrt{6}-\sqrt{2}}{4}\end{aligned}$$

14. $\sin(60^{\text{o}} - 45^{\text{o}})$

$= \sin 60^{\text{o}} \cos 45^{\text{o}} - \cos 60^{\text{o}} \sin 45^{\text{o}}$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

15. $\sin(105°) = \sin(60° + 45°)$

$= \sin 60° \cos 45° + \cos 60° \sin 45°$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6} + \sqrt{2}}{4}$

16. $\sin 75^{\text{o}} = \sin(30^{\text{o}} + 45^{\text{o}})$

$= \sin 30^{\text{o}} \cos 45^{\text{o}} + \cos 30^{\text{o}} \sin 45^{\text{o}}$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2} + \sqrt{6}}{4}$

17. $\cos(135° + 30°) = \cos 135° \cos 30° - \sin 135° \sin 30°$

$= \cos(90° + 45°) \cos 30° - \sin(90° + 45°) \sin 30°$

$= (\cos 90° \cos 45° - \sin 90° \sin 45°) \cos 30° - (\sin 90° \cos 45° + \cos 90° \sin 45°) \sin 30°$

$= \left(0 \cdot \frac{\sqrt{2}}{2} - 1 \cdot \frac{\sqrt{2}}{2}\right) \frac{\sqrt{3}}{2} - \left(1 \cdot \frac{\sqrt{2}}{2} + 0 \cdot \frac{\sqrt{2}}{2}\right) \frac{1}{2}$

$= \left(-\frac{\sqrt{2}}{2}\right) \frac{\sqrt{3}}{2} - \left(\frac{\sqrt{2}}{2}\right) \frac{1}{2}$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= -\frac{\sqrt{6} + \sqrt{2}}{4}$

18. $\cos(240° + 45°) = \cos 240° \cos 45° - \sin 240° \sin 45°$
$$= \cos(180° + 60°)\cos 45° - \sin(180° + 60°)\sin 45°$$
$$= (\cos 180° \cos 60° - \sin 180° \sin 60°)\cos 45° - (\sin 180° \cos 60° + \cos 180° \sin 60°)\sin 45°$$
$$= \left(-1 \cdot \frac{1}{2} - 0 \cdot \frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2} - \left(0 \cdot \frac{1}{2} + (-1)\frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2}$$
$$= \left(-\frac{1}{2}\right)\frac{\sqrt{2}}{2} + \left(\frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2}$$
$$= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$
$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

19. $\cos 75° = \cos(45° + 30°)$
$$= \cos 45° \cos 30° - \sin 45° \sin 30°$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$
$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$
$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

20. $\cos 105° = \cos(45° + 60°)$
$$= \cos 45° \cos 60° - \sin 45° \sin 60°$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$
$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

21. $\tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right)=\frac{\tan\frac{\pi}{6}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{6}\tan\frac{\pi}{4}}$

$=\frac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}\cdot 1}$

$=\frac{\frac{\sqrt{3}}{3}+\frac{3}{3}}{\frac{3}{3}-\frac{\sqrt{3}}{3}}$

$=\frac{\sqrt{3}+3}{3-\sqrt{3}}$

$=\frac{3+\sqrt{3}}{3-\sqrt{3}}\cdot\frac{3+\sqrt{3}}{3+\sqrt{3}}$

$=\frac{9+6\sqrt{3}+3}{9-3}$

$=\frac{12+6\sqrt{3}}{6}$

$=2+\sqrt{3}$

22. $\tan\left(\frac{\pi}{3}+\frac{\pi}{4}\right)=\frac{\tan\frac{\pi}{3}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$

$=\frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1}$

$=\frac{1+\sqrt{3}}{1-\sqrt{3}}$

$=\frac{1+\sqrt{3}}{1-\sqrt{3}}\cdot\frac{1+\sqrt{3}}{1+\sqrt{3}}$

$=\frac{1+2\sqrt{3}+3}{1-3}$

$=\frac{4+2\sqrt{3}}{-2}$

$=-2-\sqrt{3}$

23. $\tan\left(\frac{4\pi}{3}-\frac{\pi}{4}\right)=\frac{\tan\frac{4\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{4\pi}{3}\tan\frac{\pi}{4}}$

$=\frac{\sqrt{3}-1}{1+\sqrt{3}\cdot 1}$

$=\frac{-1+\sqrt{3}}{1+\sqrt{3}}$

$=\frac{-1+\sqrt{3}}{1+\sqrt{3}}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}}$

$=\frac{-1+2\sqrt{3}-3}{1-3}$

$=\frac{-4+2\sqrt{3}}{-2}$

$=2-\sqrt{3}$

24. $\tan\left(\frac{5\pi}{3}-\frac{\pi}{4}\right)=\frac{\tan\frac{5\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{5\pi}{3}\tan\frac{\pi}{4}}$

$=\frac{\tan\frac{5\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{5\pi}{3}\tan\frac{\pi}{4}}$

$=\frac{-\sqrt{3}-1}{1+\left(-\sqrt{3}\right)\cdot 1}$

$=\frac{-1-\sqrt{3}}{1-\sqrt{3}}$

$=\frac{-1-\sqrt{3}}{1-\sqrt{3}}\cdot\frac{1+\sqrt{3}}{1+\sqrt{3}}$

$=\frac{-1-2\sqrt{3}-3}{1-3}$

$=\frac{-4-2\sqrt{3}}{-2}$

$=2+\sqrt{3}$

25. $\sin 25°\cos 5°+\cos 25°\sin 5°=\sin(25°+5°)$

$=\sin 30°$

$=\frac{1}{2}$

26. $\sin 40^\circ \cos 20^\circ + \cos 40^\circ \sin 20^\circ$
$= \sin(40^\circ + 20^\circ)$
$= \sin 60^\circ$
$= \dfrac{\sqrt{3}}{2}$

27. $\dfrac{\tan 10^\circ + \tan 35^\circ}{1 - \tan 10^\circ \tan 35^\circ} = \tan(10^\circ + 35^\circ)$
$= \tan 45^\circ$
$= 1$

28. $\dfrac{\tan 50^\circ - \tan 20^\circ}{1 + \tan 50^\circ \tan 20^\circ} = \tan(50^\circ - 20^\circ)$
$= \tan 30^\circ$
$= \dfrac{\sqrt{3}}{3}$

29. $\sin\dfrac{5\pi}{12}\cos\dfrac{\pi}{4} - \cos\dfrac{5\pi}{12}\sin\dfrac{\pi}{4} = \sin\left(\dfrac{5\pi}{12} - \dfrac{\pi}{4}\right)$
$= \sin\left(\dfrac{2\pi}{12}\right)$
$= \sin\left(\dfrac{\pi}{6}\right)$
$= \dfrac{1}{2}$

30. $\sin\dfrac{7\pi}{12}\cos\dfrac{\pi}{12} - \cos\dfrac{7\pi}{12}\sin\dfrac{\pi}{12} = \sin\left(\dfrac{7\pi}{12} - \dfrac{\pi}{12}\right)$
$= \sin\dfrac{6\pi}{12}$
$= \sin\dfrac{\pi}{2}$
$= 1$

31. $\dfrac{\tan\frac{\pi}{5} - \tan\frac{\pi}{30}}{1 + \tan\frac{\pi}{5}\tan\frac{\pi}{30}} = \tan\left(\dfrac{\pi}{5} - \dfrac{\pi}{30}\right)$
$= \tan\left(\dfrac{5\pi}{30}\right) = \tan\left(\dfrac{\pi}{6}\right)$
$= \dfrac{\sqrt{3}}{3}$

32. $\dfrac{\tan\frac{\pi}{5} + \tan\frac{4\pi}{5}}{1 - \tan\frac{\pi}{5}\tan\frac{4\pi}{5}} = \tan\left(\dfrac{\pi}{5} + \dfrac{4\pi}{5}\right)$
$= \tan\dfrac{5\pi}{5}$
$= \tan\pi$
$= 0$

33. $\sin\left(x + \dfrac{\pi}{2}\right) = \sin x \cos\dfrac{\pi}{2} + \cos x \sin\dfrac{\pi}{2}$
$= \sin x \cdot 0 + \cos x \cdot 1$
$= \cos x$

34. $\sin\left(x + \dfrac{3\pi}{2}\right) = \sin x \cos\dfrac{3\pi}{2} + \cos x \sin\dfrac{3\pi}{2}$
$= \sin x \cdot 0 + \cos x \cdot (-1)$
$= -\cos x$

35. $\cos\left(x - \dfrac{\pi}{2}\right) = \cos x \cos\dfrac{\pi}{2} + \sin x \sin\dfrac{\pi}{2}$
$= \cos x \cdot 0 + \sin x \cdot 1$
$= \sin x$

36. $\cos(\pi - x) = \cos\pi \cos x + \sin\pi \sin x$
$= -1 \cdot \cos x + 0 \cdot \sin x$
$= -\cos x$

37. $\tan(2\pi - x) = \dfrac{\tan 2\pi - \tan x}{1 + \tan 2\pi \tan x}$
$= \dfrac{0 - \tan x}{1 + 0 \cdot \tan x}$
$= -\tan x$

38. $\tan(\pi - x) = \dfrac{\tan\pi - \tan x}{1 + \tan\pi \tan x}$
$= \dfrac{0 - \tan x}{1 + 0 \cdot \tan x}$
$= -\tan x$

39. $\sin(\alpha + \beta) + \sin(\alpha - \beta)$
$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$+ \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$= 2\sin\alpha\cos\beta$

40. $\cos(\alpha+\beta)+\cos(\alpha-\beta)$
$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$+ \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$= 2\cos\alpha\cos\beta$

41. $\dfrac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta} = \dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$
$= \dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} - \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$
$= \tan\alpha\cdot 1 - 1\cdot\tan\beta$
$= \tan\alpha - \tan\beta$

42. $\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$
$= \dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$
$= \tan\alpha + \tan\beta$

43. $\tan\left(\theta+\dfrac{\pi}{4}\right) = \dfrac{\tan\theta + \tan\frac{\pi}{4}}{1-\tan\theta\tan\frac{\pi}{4}}$
$= \dfrac{\tan\theta + 1}{1-\tan\theta}$
$= \dfrac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}$
$= \dfrac{\frac{\sin\theta+\cos\theta}{\cos\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$
$= \dfrac{\sin\theta+\cos\theta}{\cos\theta}\cdot\dfrac{\cos\theta}{\cos\theta-\sin\theta}$
$= \dfrac{\sin\theta+\cos\theta}{\cos\theta-\sin\theta}$
$= \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

44. $\tan\left(\dfrac{\pi}{4}-\theta\right) = \dfrac{\tan\frac{\pi}{4} - \tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}$
$= \dfrac{1-\tan\theta}{1+1\cdot\tan\theta}$
$= \dfrac{1-\tan\theta}{1+\tan\theta}$
$= \dfrac{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$
$= \dfrac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\cos\theta+\sin\theta}{\cos\theta}}$
$= \dfrac{\cos\theta-\sin\theta}{\cos\theta} \div \dfrac{\cos\theta+\sin\theta}{\cos\theta}$
$= \dfrac{\cos\theta-\sin\theta}{\cos\theta}\cdot\dfrac{\cos\theta}{\cos\theta+\sin\theta}$
$= \dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

45. $\cos(\alpha+\beta)\cos(\alpha-\beta)$
$= (\cos\alpha\cos\beta - \sin\alpha\sin\beta)$
$\cdot\ (\cos\alpha\cos\beta + \sin\alpha\sin\beta)$
$= \cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta$
$= \left(1-\sin^2\alpha\right)\cos^2\beta - \sin^2\alpha\left(1-\cos^2\beta\right)$
$= \cos^2\beta - \sin^2\alpha\cos^2\beta$
$-\sin^2\alpha + \sin^2\alpha\cos^2\beta$
$= \cos^2\beta - \sin^2\alpha$

46. $\sin(a+\beta)\sin(\alpha-\beta)$
$= (\sin\alpha\cos\beta + \cos\alpha\sin\beta)$
$\cdot(\sin\alpha\cos\beta - \cos\alpha\sin\beta)$
$= \sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta$
$= (1-\cos^2\alpha)\cos^2\beta$
$-\cos^2\alpha(1-\cos^2\beta)$
$= \cos^2\beta - \cos^2\alpha\cos^2\beta$
$-\cos^2\alpha + \cos^2\alpha\cos^2\beta$
$= \cos^2\beta - \cos^2\alpha$

47. $\dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$

$= \dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\sin\alpha\cos\beta-\cos\alpha\sin\beta}$

$= \dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\sin\alpha\cos\beta-\cos\alpha\sin\beta}\cdot\dfrac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}$

$= \dfrac{\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}$

$= \dfrac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}$

$= \dfrac{\tan\alpha\cdot 1+1\cdot\tan\beta}{\tan\alpha\cdot 1-1\cdot\tan\beta}$

$= \dfrac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}$

48. $\dfrac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$

$= \dfrac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta}$

$= \dfrac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta}\cdot\dfrac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}$

$= \dfrac{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$

$= \dfrac{1-\tan\alpha\tan\beta}{1+\tan\alpha\tan\beta}$

49. $\dfrac{\cos(x+h)-\cos x}{h}$

$= \dfrac{\cos x\cos h-\sin x\sin h-\cos x}{h}$

$= \dfrac{\cos x\cos h-\cos x-\sin x\sin h}{h}$

$= \dfrac{\cos x(\cos h-1)-\sin x\sin h}{h}$

$= \cos x\cdot\dfrac{\cos h-1}{h}-\sin x\cdot\dfrac{\sin h}{h}$

50. $\dfrac{\sin(x+h)-\sin x}{h}$

$= \dfrac{\sin x\cos h+\cos x\sin h-\sin x}{h}$

$= \dfrac{\sin x\cos h-\sin x+\cos x\sin h}{h}$

$= \dfrac{\sin x(\cos h-1)+\cos x\sin h}{h}$

$= \sin x\dfrac{\cos h-1}{h}+\cos x\dfrac{\sin h}{h}$

51. $\sin 2\alpha = \sin(\alpha+\alpha)$

$= \sin\alpha\cos\alpha+\cos\alpha\sin\alpha$

$= 2\sin\alpha\cos\alpha$

52. $\cos 2\alpha = \cos(\alpha+\alpha)$

$= \cos\alpha\cos\alpha+\sin\alpha\sin\alpha$

$= \cos^2\alpha+\sin^2\alpha$

53. $\tan 2\alpha = \tan(\alpha+\alpha)$

$= \dfrac{\tan\alpha+\tan\alpha}{1-\tan\alpha\tan\alpha}$

$= \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

54. $\tan\left(\dfrac{\pi}{4}+\alpha\right)-\tan\left(\dfrac{\pi}{4}-\alpha\right)$

$= \dfrac{\tan\frac{\pi}{4}+\tan\alpha}{1-\tan\frac{\pi}{4}\tan\alpha}-\dfrac{\tan\frac{\pi}{4}-\tan\alpha}{1+\tan\frac{\pi}{4}\tan\alpha}$

$= \dfrac{1+\tan\alpha}{1-\tan\alpha}-\dfrac{1-\tan\alpha}{1+\tan\alpha}$

$= \dfrac{(1+\tan\alpha)(1+\tan\alpha)}{(1-\tan\alpha)(1+\tan\alpha)}-\dfrac{(1-\tan\alpha)(1-\tan\alpha)}{(1+\tan\alpha)(1-\tan\alpha)}$

$= \dfrac{1+2\tan\alpha+\tan^2\alpha-(1-2\tan\alpha+\tan^2\alpha)}{1-\tan^2\alpha}$

$= \dfrac{4\tan\alpha}{1-\tan^2\alpha}$

$= \dfrac{2(2\tan\alpha)}{1-\tan^2\alpha}$

$= 2\dfrac{\tan\alpha+\tan\alpha}{1-\tan\alpha\tan\alpha}$

$= 2\tan(\alpha+\alpha)$

$= 2\tan 2\alpha$

55. $\tan(\alpha+\beta)=\dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$

$=\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$

$=\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}\cdot\dfrac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}$

$=\dfrac{\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$

$=\dfrac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$

$=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

56. $\tan(\alpha-\beta)=\tan(\alpha+(-\beta))$

$=\dfrac{\tan\alpha+\tan(-\beta)}{1-\tan\alpha\tan(-\beta)}$

$=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

57. $\sin\alpha=\dfrac{3}{5}=\dfrac{y}{r}$

$x^2+y^2=r^2$

$x^2+3^2=5^2$

$x^2+9=25$

$x^2=16$

Because α lies in quadrant I, x is positive.

$x=4$

Thus, $\cos\alpha=\dfrac{x}{r}=\dfrac{4}{5}$, and

$\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}=\dfrac{\frac{3}{5}}{\frac{4}{5}}=\dfrac{3}{4}.$

$\sin\beta=\dfrac{5}{13}=\dfrac{y}{r}$

$x^2+y^2=r^2$

$x^2+5^2=13^2$

$x^2+25=169$

$x^2=144$

Because β lies in quadrant II, x is negative.

$x=-12$

Thus, $\cos\beta=\dfrac{x}{r}=\dfrac{-12}{13}=-\dfrac{12}{13}$, and

$\tan\beta=\dfrac{\sin\beta}{\cos\beta}=\dfrac{\frac{5}{13}}{-\frac{12}{13}}=-\dfrac{5}{12}.$

a. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$=\dfrac{4}{5}\cdot\left(-\dfrac{12}{13}\right)-\dfrac{3}{5}\cdot\dfrac{5}{13}=-\dfrac{63}{65}$

b. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$=\dfrac{3}{5}\cdot\left(-\dfrac{12}{13}\right)+\dfrac{4}{5}\cdot\dfrac{5}{13}=-\dfrac{16}{65}$

c. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$=\dfrac{\frac{3}{4}+\left(-\frac{5}{12}\right)}{1-\frac{3}{4}\cdot\left(-\frac{5}{12}\right)}=\dfrac{\frac{4}{12}}{\frac{63}{48}}=\dfrac{16}{63}$

58. $\sin\alpha=\dfrac{4}{5}=\dfrac{y}{r}$

$x^2+y^2=r^2$

$x^2+4^2=5^2$

$x^2+16=25$

$x^2=9$

Because α lies in quadrant I, x is positive.

$x=3$

$\cos\alpha=\dfrac{x}{r}=\dfrac{3}{5}$

$\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}=\dfrac{\frac{4}{5}}{\frac{3}{5}}=\dfrac{4}{3}$

$\sin\beta=\dfrac{7}{25}=\dfrac{y}{r}$

$x^2+y^2=r^2$

$x^2+7^2=25^2$

$x^2+49=625$

$x^2=576$

Because β lies in quadrant II, x is negative.

$x = -24$

$\cos\beta = \frac{x}{r} = \frac{-24}{25}$

$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{\frac{7}{25}}{\frac{-24}{25}} = -\frac{7}{24}$

a. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{3}{5}\cdot\left(-\frac{24}{25}\right) - \frac{4}{5}\cdot\frac{7}{25} = -\frac{4}{5}$

b. $\sin\left(\alpha+\beta\right) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{4}{5}\cdot\left(-\frac{24}{25}\right) + \frac{3}{5}\cdot\frac{4}{25} = -\frac{3}{5}$

c. $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{\frac{4}{3} - \frac{7}{24}}{1 - \frac{4}{3}\cdot\left(-\frac{7}{24}\right)} = \frac{\frac{25}{24}}{\frac{25}{18}} = \frac{3}{4}$

59. $\tan\alpha = -\frac{3}{4} = \frac{3}{-4} = \frac{y}{x}$

$x^2 + y^2 = r^2$

$(-4)^2 + 3^2 = r^2$

$16 + 9 = r^2$

$25 = r^2$

Because r is a distance, it is positive.

$r = 5$

Thus, $\cos\alpha = \frac{x}{r} = \frac{-4}{5} = -\frac{4}{5}$, and

$\sin\alpha = \frac{y}{r} = \frac{3}{5}.$

$\cos\beta = \frac{1}{3} = \frac{x}{r}$

$x^2 + y^2 = r^2$

$1^2 + y^2 = 3^2$

$1 + y^2 = 9$

$y^2 = 8$

Because β lies in quadrant I, y is positive.

$y = \sqrt{8} = 2\sqrt{2}$

Thus, $\sin\beta = \frac{y}{r} = \frac{2\sqrt{2}}{3}$, and

$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = 2\sqrt{2}.$

a. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(-\frac{4}{5}\right)\cdot\frac{1}{3} - \frac{3}{5}\cdot\frac{2\sqrt{2}}{3}$

$= -\frac{4}{15} - \frac{6\sqrt{2}}{15}$

$= \frac{-4 - 6\sqrt{2}}{15}$

$= -\frac{4 + 6\sqrt{2}}{15}$

b. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{3}{5}\cdot\frac{1}{3} + \left(-\frac{4}{5}\right)\cdot\frac{2\sqrt{2}}{3}$

$= \frac{3}{15} - \frac{8\sqrt{2}}{15}$

$= \frac{3 - 8\sqrt{2}}{15}$

c. $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{-\frac{3}{4} + 2\sqrt{2}}{1 - \left(-\frac{3}{4}\right)\left(2\sqrt{2}\right)}$

$= \frac{\frac{-3+8\sqrt{2}}{4}}{\frac{4+6\sqrt{2}}{4}}$

$= \frac{-3 + 8\sqrt{2}}{4 + 6\sqrt{2}}\cdot\frac{\left(4 - 6\sqrt{2}\right)}{\left(4 - 6\sqrt{2}\right)}$

$= \frac{-108 + 50\sqrt{2}}{-56}$

$= \frac{54 - 25\sqrt{2}}{28}$

60. $\tan\alpha = \frac{-4}{3} = \frac{4}{-3} = \frac{y}{x}$

$x^2 + y^2 = r^2$

$(-3)^2 + 4^2 = r^2$

$9 + 16 = r^2$

$25 = r^2$

Because r is a distance, it is positive.

$r = 5$

$\cos\alpha = \frac{x}{r} = \frac{-3}{5}$

$\sin\alpha = \frac{y}{r} = \frac{4}{5}$

$\cos\beta = \frac{2}{3} = \frac{x}{r}$

$x^2 + y^2 = r^2$

$2^2 + y^2 = 3^2$

$4 + y^2 = 9$

$y^2 = 5$

Because β lies in quadrant I, y is positive.

$\sin\beta = \frac{y}{r} = \frac{\sqrt{5}}{3}$

$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}} = \frac{\sqrt{5}}{2}$

$y = \sqrt{5}$

a. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= -\frac{3}{5}\cdot\frac{2}{3} - \frac{4}{5}\cdot\frac{\sqrt{5}}{3} = \frac{-6-4\sqrt{5}}{15}$

b. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{4}{5}\cdot\frac{2}{3} + \left(-\frac{3}{5}\right)\cdot\frac{\sqrt{5}}{3} = \frac{8-3\sqrt{5}}{15}$

c. $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{-\frac{4}{3} + \frac{\sqrt{5}}{2}}{1 - \left(-\frac{4}{3}\right)\cdot\frac{\sqrt{5}}{2}}$

$= \frac{\frac{-8+3\sqrt{5}}{6}}{\frac{6+4\sqrt{5}}{6}}$

$= \frac{-8+3\sqrt{5}}{6+4\sqrt{5}}$

$= \frac{-8+3\sqrt{5}}{6+4\sqrt{5}}\cdot\frac{6-4\sqrt{5}}{6-+4\sqrt{5}}$

$= \frac{-108+50\sqrt{5}}{-44}$

$= \frac{54-25\sqrt{5}}{22}$

61. $\cos\alpha = \frac{8}{17} = \frac{x}{r}$

$x^2 + y^2 = r^2$

$8^2 + y^2 = 17^2$

$64 + y^2 = 289$

$y^2 = 225$

Because α lies in quadrant IV, y is negative.

$y = -15$

Thus, $\sin\alpha = \frac{y}{r} = \frac{-15}{17} = -\frac{15}{17}$, and

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{15}{17}}{\frac{8}{17}} = -\frac{15}{8}$.

$\sin\beta = -\frac{1}{2} = \frac{-1}{2} = \frac{y}{r}$

$x^2 + y^2 = r^2$

$x^2 + (-1)^2 = 2^2$

$x^2 + 1 = 4$

$x^2 = 3$

Because β lies in quadrant III, x is negative.

$x = -\sqrt{3}$

Thus, $\cos\beta = \frac{x}{r} = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$, and

$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

a. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$$=\frac{8}{17}\cdot\left(-\frac{\sqrt{3}}{2}\right)-\left(-\frac{15}{17}\right)\cdot\left(-\frac{1}{2}\right)$$
$$=\frac{-8\sqrt{3}-15}{34}$$
$$=-\frac{8\sqrt{3}+15}{34}$$

b. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$$=\left(-\frac{15}{17}\right)\cdot\left(-\frac{\sqrt{3}}{2}\right)+\frac{8}{17}\cdot\left(-\frac{1}{2}\right)$$
$$=\frac{15\sqrt{3}-8}{34}$$

c.
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\frac{-\frac{15}{8}+\frac{\sqrt{3}}{3}}{1-\left(-\frac{15}{8}\right)\left(\frac{\sqrt{3}}{3}\right)}$$
$$=\frac{\frac{-45+8\sqrt{3}}{24}}{\frac{24+15\sqrt{3}}{24}}$$
$$=\frac{-45+8\sqrt{3}}{24+15\sqrt{3}}\cdot\frac{24-15\sqrt{3}}{24-15\sqrt{3}}$$
$$=\frac{-1440+867\sqrt{3}}{-99}$$
$$=\frac{489-289\sqrt{3}}{33}$$

62. $\cos\alpha=\frac{1}{2}=\frac{x}{r}$

$$x^2+y^2=r^2$$
$$1^2+y^2=2^2$$
$$1+y^2=4$$
$$y^2=3$$

Because α lies in quadrant IV, y is negative.

$y=-\sqrt{3}$

$$\sin\alpha=\frac{y}{r}=\frac{\sqrt{3}}{2}$$
$$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}}=-\sqrt{3}$$

$$\sin\beta=-\frac{1}{3}=\frac{-1}{3}=\frac{y}{r}$$
$$x^2+y^2=r^2$$
$$x^2+(-1)^2=3^2$$
$$x^2+1=9$$
$$x^2=8$$

Because β lies in quadrant III, x is negative.

$x=-\sqrt{8}=-2\sqrt{2}$

$$\cos\beta=\frac{x}{r}=\frac{-2\sqrt{2}}{3}$$
$$\tan\beta=\frac{\sin\beta}{\cos\beta}=\frac{-\frac{1}{3}}{-\frac{2\sqrt{2}}{3}}=\frac{1}{2\sqrt{2}}=\frac{\sqrt{2}}{4}$$

a. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$$=\frac{1}{2}\cdot\left(-\frac{2\sqrt{2}}{3}\right)-\left(\frac{-\sqrt{3}}{2}\right)\left(-\frac{1}{3}\right)$$
$$=\frac{-2\sqrt{6}-\sqrt{3}}{6}$$

b. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$$=\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{2\sqrt{2}}{3}\right)+\frac{1}{2}\left(-\frac{1}{3}\right)$$
$$=\frac{2\sqrt{6}-1}{6}$$

c. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$=\frac{-\sqrt{3}+\frac{\sqrt{2}}{4}}{1-\left(-\sqrt{3}\right)\cdot\frac{\sqrt{2}}{4}}$$

$$=\frac{\frac{-4\sqrt{3}+\sqrt{2}}{4}}{\frac{4+\sqrt{6}}{4}}$$

$$=\frac{-4\sqrt{3}+\sqrt{2}}{4+\sqrt{6}}$$

$$=\frac{-4\sqrt{3}+\sqrt{2}}{4+\sqrt{6}}\cdot\frac{4-\sqrt{6}}{4+\sqrt{6}}$$

$$=\frac{-16\sqrt{3}+4\sqrt{18}+4\sqrt{2}-\sqrt{12}}{10}$$

$$=\frac{-16\sqrt{3}+12\sqrt{2}+4\sqrt{2}-2\sqrt{3}}{10}$$

$$=\frac{-18\sqrt{3}+16\sqrt{2}}{10}$$

$$=\frac{8\sqrt{2}-9\sqrt{3}}{5}$$

63. $\tan\alpha=\dfrac{3}{4}=\dfrac{y}{x}$

Because α lies in quadrant III, x and y are negative.

$r^2=x^2+y^2$

$r^2=(-4)^2+(-3)^2$

$r^2=25$

$r=5$

$\sin\alpha=\dfrac{y}{r}=\dfrac{-3}{5}=-\dfrac{3}{5}$

$\cos\alpha=\dfrac{x}{r}=\dfrac{-4}{5}=-\dfrac{4}{5}$

$\cos\beta=\dfrac{1}{4}=\dfrac{x}{r}$

Because β lies in quadrant IV, y is negative.

$x^2+y^2=r^2$

$1^2+y^2=4^2$

$y^2=15$

$y=-\sqrt{15}$

$\sin\beta=\dfrac{y}{r}=\dfrac{-\sqrt{15}}{4}=-\dfrac{\sqrt{15}}{4}$

$\tan\beta=\dfrac{y}{x}=\dfrac{-\sqrt{15}}{1}=-\sqrt{15}$

a. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$$=-\frac{4}{5}\cdot\left(\frac{1}{4}\right)-\left(-\frac{3}{5}\right)\left(-\frac{\sqrt{15}}{4}\right)$$

$$=-\frac{4}{20}-\frac{3\sqrt{15}}{20}$$

$$=-\frac{4+3\sqrt{15}}{20}$$

b. $\sin\left(\alpha+\beta\right)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$$=\left(-\frac{3}{5}\right)\left(\frac{1}{4}\right)+\left(-\frac{4}{5}\right)\left(-\frac{\sqrt{15}}{4}\right)$$

$$=-\frac{3}{20}+\frac{4\sqrt{15}}{20}$$

$$=\frac{-3+4\sqrt{15}}{20}$$

c. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$=\frac{\frac{3}{4}+\left(-\sqrt{15}\right)}{1-\frac{3}{4}\left(-\sqrt{15}\right)}$$

$$=\frac{\frac{3}{4}-\frac{4\sqrt{15}}{4}}{\frac{4}{4}+\frac{3\sqrt{15}}{4}}$$

$$=\frac{3-4\sqrt{15}}{4+3\sqrt{15}}$$

$$=\frac{3-4\sqrt{15}}{4+3\sqrt{15}}\cdot\frac{4-3\sqrt{15}}{4-3\sqrt{15}}$$

$$=\frac{12-9\sqrt{15}-16\sqrt{15}+180}{16-135}$$

$$=\frac{192-25\sqrt{15}}{-119}$$

$$=\frac{-192+25\sqrt{15}}{119}$$

64. $\sin\alpha = \frac{5}{6} = \frac{y}{r}$

Because α lies in quadrant II, x is negative.

$$x^2 + y^2 = r^2$$
$$x^2 + 5^2 = 6^2$$
$$x^2 = 11$$
$$x = -\sqrt{11}$$
$$\cos\alpha = \frac{x}{r} = \frac{-\sqrt{11}}{6} = -\frac{\sqrt{11}}{6}$$
$$\tan\alpha = \frac{y}{x} = \frac{5}{-\sqrt{11}} = -\frac{5\sqrt{11}}{11}$$

$\tan\beta = \frac{3}{7} = \frac{y}{x}$

Because β lies in quadrant III, x and y are both negative.

$$r^2 = x^2 + y^2$$
$$r^2 = 7^2 + 3^2$$
$$r^2 = 58$$
$$r = \sqrt{58}$$
$$\sin\beta = \frac{y}{r} = \frac{-3}{\sqrt{58}} = -\frac{3\sqrt{58}}{58}$$
$$\cos\beta = \frac{x}{r} = \frac{-7}{\sqrt{58}} = -\frac{7\sqrt{58}}{58}$$

a.
$$\begin{aligned}\cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta\\ &= -\frac{\sqrt{11}}{6}\cdot\left(-\frac{7\sqrt{58}}{58}\right) - \left(\frac{5}{6}\right)\left(-\frac{3\sqrt{58}}{58}\right)\\ &= \frac{7\sqrt{638}}{348} + \frac{15\sqrt{58}}{348}\\ &= \frac{7\sqrt{638}+15\sqrt{58}}{348}\end{aligned}$$

b.
$$\begin{aligned}\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta\\ &= \sin\alpha\cos\beta + \cos\alpha\sin\beta\\ &= \left(\frac{5}{6}\right)\left(-\frac{7\sqrt{58}}{58}\right) + \left(-\frac{\sqrt{11}}{6}\right)\left(-\frac{3\sqrt{58}}{58}\right)\\ &= -\frac{35\sqrt{58}}{348} + \frac{3\sqrt{638}}{348}\\ &= \frac{3\sqrt{638}-35\sqrt{58}}{348}\end{aligned}$$

c.
$$\begin{aligned}\tan(\alpha+\beta) &= \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\\ &= \frac{-\frac{5\sqrt{11}}{11}+\frac{3}{7}}{1-\left(-\frac{5\sqrt{11}}{11}\right)\left(\frac{3}{7}\right)}\\ &= \frac{-\frac{35\sqrt{11}}{77}+\frac{33}{77}}{\frac{77}{77}+\frac{15\sqrt{11}}{77}}\\ &= \frac{-35\sqrt{11}+33}{77+15\sqrt{11}}\\ &= \frac{33-35\sqrt{11}}{77+15\sqrt{11}}\cdot\frac{77-15\sqrt{11}}{77-15\sqrt{11}}\\ &= \frac{2541-495\sqrt{11}-2695\sqrt{11}+5775}{5929-2475}\\ &= \frac{8316-3190\sqrt{11}}{3454}\\ &= \frac{22\left(378-145\sqrt{11}\right)}{22\cdot 157}\\ &= \frac{378-145\sqrt{11}}{157}\end{aligned}$$

65. a. The graph appears to be the sine curve, $y = \sin x$. It cycles through intercept, maximum, minimum and back to intercept. Thus, $y = \sin x$ also describes the graph.

b.
$$\begin{aligned}\sin(\pi - x) &= \sin\pi\cos x - \cos\pi\sin x\\ &= 0\cdot\cos x - (-1)\cdot\sin x\\ &= \sin x\end{aligned}$$

This verifies our observation that $y = \sin(\pi - x)$ and $y = \sin x$ describe the same graph.

66. a. The graph appears to be the cosine curve, $y = \cos x$. It cycles through maximum, intercept, minimum, intercept and back to maximum. Thus, $y = \cos x$ also describes the graph.

b.
$$\begin{aligned}\cos(x-2\pi) &= \cos x\cos 2\pi + \sin x\sin 2\pi\\ &= \cos x\cdot 1 + \sin x\cdot 0\\ &= \cos x\end{aligned}$$

This verifies our observation that $y = \cos(x-2\pi)$ and $y = \cos x$ describe the same graph.

67. **a.** The graph appears to be 2 times the cosine curve, $y = 2\cos x$. It cycles through maximum, intercept, minimum, intercept and back to maximum. Thus $y = 2\cos x$ also describes the graph.

b. $\sin\left(x+\frac{\pi}{2}\right)+\sin\left(\frac{\pi}{2}-x\right)$

$= \sin x\cos\frac{\pi}{2}+\cos x\sin\frac{\pi}{2}+\sin\frac{\pi}{2}\cos x$

$-\cos\frac{\pi}{2}\sin x$

$= \sin x\cdot 0+\cos x\cdot 1+1\cdot\cos x-0\cdot\sin x$

$= \cos x+\cos x$

$= 2\cos x$

This verifies our observation that

$y=\sin\left(x+\frac{\pi}{2}\right)+\sin\left(\frac{\pi}{2}-x\right)$ and $y = 2\cos x$

describe the same graph.

68. **a.** The graph appears to be 2 times the sine curve, $y = 2\sin x$. It cycles through intercept, maximum, intercept, minimum and back to intercept. Thus, $y = 2\sin x$ also describes the graph.

b. $\cos\left(x-\frac{\pi}{2}\right)-\cos\left(x+\frac{\pi}{2}\right)$

$= \cos x\cos\frac{\pi}{2}+\sin x\sin\frac{\pi}{2}$

$-\left(\cos x\cos x\frac{\pi}{2}-\sin x\sin\frac{\pi}{2}\right)$

$= 2\sin x\sin\frac{\pi}{2}$

$= 2\sin x\cdot 1$

$= 2\sin x$

This verifies our observation that

$\cos\left(x-\frac{\pi}{2}\right)-\cos\left(x+\frac{\pi}{2}\right)$ and $y = 2\sin x$

describe the same graph.

69. $\cos(\alpha+\beta)\cos\beta+\sin(\alpha+\beta)\sin\beta$

$= \cos\left[(\alpha+\beta)-\beta\right]$

$= \cos\alpha$

70. $\sin(\alpha-\beta)\cos\beta+\cos(\alpha-\beta)\sin\beta$

$= \sin\left[(\alpha-\beta)+\beta\right]$

$= \sin\alpha$

71. $\dfrac{\sin(\alpha+\beta)-\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}$

$= \dfrac{(\sin\alpha\cos\beta+\cos\alpha\sin\beta)-(\sin\alpha\cos\beta-\cos\alpha\sin\beta)}{(\cos\alpha\cos\beta-\sin\alpha\sin\beta)+(\cos\alpha\cos\beta+\sin\alpha\sin\beta)}$

$= \dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta-\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta+\cos\alpha\cos\beta+\sin\alpha\sin\beta}$

$= \dfrac{\cos\alpha\sin\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta+\cos\alpha\cos\beta}$

$= \dfrac{2\cos\alpha\sin\beta}{2\cos\alpha\cos\beta}$

$= \dfrac{\sin\beta}{\cos\beta}$

$= \tan\beta$

72. $\dfrac{\cos(\alpha-\beta)+\cos(\alpha+\beta)}{-\sin(\alpha-\beta)+\sin(\alpha+\beta)}$

$= \dfrac{(\cos\alpha\cos\beta+\sin\alpha\sin\beta)+(\cos\alpha\cos\beta-\sin\alpha\sin\beta)}{-(\sin\alpha\cos\beta-\cos\alpha\sin\beta)+(\sin\alpha\cos\beta+\cos\alpha\sin\beta)}$

$= \dfrac{\cos\alpha\cos\beta+\sin\alpha\sin\beta+\cos\alpha\cos\beta-\sin\alpha\sin\beta}{-\sin\alpha\cos\beta+\cos\alpha\sin\beta+\sin\alpha\cos\beta+\cos\alpha\sin\beta}$

$= \dfrac{\cos\alpha\cos\beta+\cos\alpha\cos\beta}{\cos\alpha\sin\beta+\cos\alpha\sin\beta}$

$= \dfrac{2\cos\alpha\cos\beta}{2\cos\alpha\sin\beta}$

$= \dfrac{\cos\beta}{\sin\beta}$

$= \cot\beta$

73. $\cos\left(\frac{\pi}{6}+\alpha\right)\cos\left(\frac{\pi}{6}-\alpha\right)-\sin\left(\frac{\pi}{6}+\alpha\right)\sin\left(\frac{\pi}{6}-\alpha\right)$

$= \cos\left[\left(\frac{\pi}{6}+\alpha\right)+\left(\frac{\pi}{6}-\alpha\right)\right]$

$= \cos\left[\frac{\pi}{6}+\alpha+\frac{\pi}{6}-\alpha\right]$

$= \cos\frac{\pi}{3}$

$= \frac{1}{2}$

74. $\sin\left(\frac{\pi}{3}-\alpha\right)\cos\left(\frac{\pi}{3}+\alpha\right)+\cos\left(\frac{\pi}{3}-\alpha\right)\sin\left(\frac{\pi}{3}+\alpha\right)$

$=\sin\left[\left(\frac{\pi}{3}+\alpha\right)+\left(\frac{\pi}{3}-\alpha\right)\right]$

$=\sin\left[\frac{\pi}{3}+\alpha+\frac{\pi}{3}-\alpha\right]$

$=\sin\frac{2\pi}{3}$

$=\frac{\sqrt{3}}{2}$

75. Conjecture: the left side is equal to $\cos 3x$.

$\cos 2x\cos 5x+\sin 2x\sin 5x$

$=\cos(2x-5x)$

$=\cos(-3x)$

$=\cos 3x$

76. Conjecture: the left side is equal to $\sin 3x$.

$\sin 5x\cos 2x-\cos 5x\sin 2x$

$=\sin(5x-2x)$

$=\sin 3x$

77. Conjecture: the left side is equal to $\sin\frac{x}{2}$.

$\sin\frac{5x}{2}\cos 2x-\cos\frac{5x}{2}\sin 2x$

$=\sin\left(\frac{5x}{2}-2x\right)$

$=\sin\left(\frac{5x}{2}-\frac{4x}{2}\right)$

$=\sin\frac{x}{2}$

78. Conjecture: the left side is equal to $\cos\frac{x}{2}$.

$\cos\frac{5x}{2}\cos 2x+\sin\frac{5x}{2}\sin 2x$

$=\cos\left(\frac{5x}{2}-2x\right)$

$=\cos\left(\frac{5x}{2}-\frac{4x}{2}\right)$

$=\cos\frac{x}{2}$

79. $\tan\theta=\frac{3}{2}=\frac{y}{x}$

$x^2+y^2=r^2$

$2^2+3^3=r^2$

$4+9=r^2$

$13=r^2$

Because r is a distance, it is positive.

$r=\sqrt{13}$

Thus, $\sin\theta=\frac{y}{r}=\frac{3}{\sqrt{13}}$

and $\cos\theta=\frac{x}{r}=\frac{2}{\sqrt{13}}$

$\sqrt{13}\cos(t-\theta)=\sqrt{13}(\cos t\cos\theta+\sin t\sin\theta)$

$=\sqrt{13}\left(\cos t\cdot\frac{2}{\sqrt{13}}+\sin t\cdot\frac{3}{\sqrt{13}}\right)$

$=\cos t\cdot 2+\sin t\cdot 3$

$=2\cos t+3\sin t$

For the equation $y=\sqrt{13}\cos(t-\theta)$, the amplitude is $\left|\sqrt{13}\right|=\sqrt{13}$, and the period is $\frac{2\pi}{1}=2\pi$.

80. **a.** $\rho=3\sin 2t+2\sin(2t+\pi)$

$=3\sin 2t+2(\sin 2t\cos\pi+\cos 2t\cos\pi)$

$=3\sin 2t+2(\sin 2t\cdot(-1)+\cos 2t\cdot 0)$

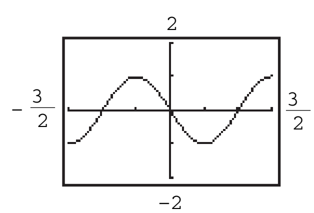
$=3\sin 2t-2\sin 2t$

$=\sin 2t$

b. No. The amplitude of p is 1.

81. – 87. Answers may vary.

88.

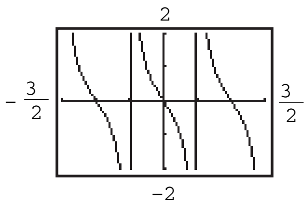


$\cos\left(\frac{3\pi}{2}-x\right)=\cos\frac{3\pi}{2}\cos x+\sin\frac{3\pi}{2}\sin x$

$=0\cdot\cos x+(-1)\sin x$

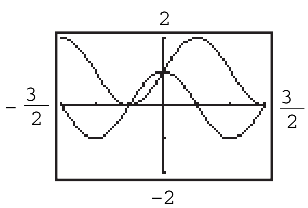
$=-\sin x$

89.



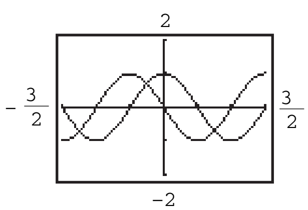
$$\tan(\pi - x) = \frac{\tan \pi - \tan x}{1 + \tan \pi \tan x}$$
$$= \frac{0 - \tan x}{1 + 0 \cdot \tan x}$$
$$= \frac{-\tan x}{1}$$
$$= -\tan x$$

90.



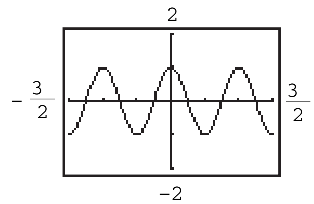
The graphs do not coincide.
Values for x may vary.

91.



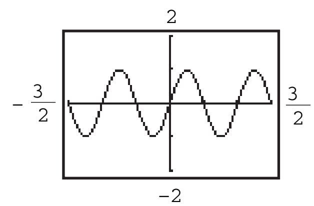
The graphs do not coincide.
Values for x may vary.

92.



$\cos 1.2x \cos 0.8x - \sin 1.2x \sin 0.8x$
$= \cos(1.2 + 0.8) = \cos 2x$

93.



$\sin 1.2x \cos 0.8x + \cos 1.2x \sin 0.8x$
$\sin(1.2x + 0.8x)$
$\sin 2x$

94. makes sense

95. makes sense

96. does not make sense; Explanations will vary. Sample explanation: The sum and difference formulas allow you to find exact values only for certain angles.

97. makes sense

98. $\dfrac{\sin(x-y)}{\cos x\cos y}+\dfrac{\sin(y-z)}{\cos y\cos z}+\dfrac{\sin(z-x)}{\cos z\cos x}$

$=\dfrac{\sin x\cos y-\cos x\sin y}{\cos x\cos y}+\dfrac{\sin y\cos z-\cos y\sin z}{\cos y\cos z}+\dfrac{\sin z\cos x-\cos z\sin x}{\cos z\cos x}$

$=\dfrac{\sin x\cos y}{\cos x\cos y}-\dfrac{\cos x\sin y}{\cos y\cos y}+\dfrac{\sin y\cos z}{\cos y\cos z}-\dfrac{\cos y\sin z}{\cos y\cos z}+\dfrac{\sin z\cos x}{\cos z\cos x}-\dfrac{\cos z\sin x}{\cos z\cos x}$

$=\dfrac{\sin x}{\cos x}-\dfrac{\sin y}{\cos y}+\dfrac{\sin y}{\cos y}-\dfrac{\sin z}{\cos z}+\dfrac{\sin z}{\cos z}-\dfrac{\sin x}{\cos x}$

$=0$

99. $\cos^{-1}\dfrac{1}{2}$

$x=1$

$y=\sqrt{3}$

$r=2$

$\sin^{-1}\dfrac{3}{5}$

$x=4$

$y=3$

$r=5$

$\sin\left(\cos^{-1}\dfrac{1}{2}+\sin^{-1}\dfrac{3}{5}\right)$

$=\sin\cos^{-1}\dfrac{1}{2}\cos\sin^{-1}\dfrac{3}{5}$

$\quad+\cos\cos^{-1}\dfrac{1}{2}\sin\sin^{-1}\dfrac{3}{5}$

$=\dfrac{\sqrt{3}}{2}\cdot\dfrac{4}{5}+\dfrac{1}{2}\cdot\dfrac{3}{5}$

$=\dfrac{4\sqrt{3}+3}{10}$

100. $\sin^{-1}\frac{3}{5}$

$y=3, r=5, x=4$

$\cos^{-1}\left(-\frac{4}{5}\right)$

$x=-4, y=3, r=5$

$$\sin\left(\sin^{-1}\frac{3}{5}-\cos^{-1}\left(-\frac{4}{5}\right)\right)$$
$$=\sin\sin^{-1}\frac{3}{5}\cos\cos^{-1}\left(-\frac{4}{5}\right)-\cos\sin^{-1}\frac{3}{5}\sin\cos^{-1}\left(-\frac{4}{5}\right)$$
$$=\frac{3}{5}\left(\frac{-4}{5}\right)-\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$
$$=\frac{-12}{25}-\frac{12}{25}$$
$$=-\frac{24}{25}$$

101. $\tan^{-1}\frac{4}{3}$

$x=3$

$y=4$

$r=5$

$\cos^{-1}\frac{5}{13}$

$x=5$

$y=12$

$r=13$

$$\cos\left(\tan^{-1}\frac{4}{3}+\cos^{-1}\frac{5}{13}\right)$$
$$=\cos\tan^{-1}\frac{4}{3}\cos\cos^{-1}\frac{5}{13}$$
$$-\sin\tan^{-1}\frac{4}{3}\sin\cos^{-1}\frac{5}{13}$$
$$=\frac{3}{5}\cdot\frac{5}{13}-\frac{4}{5}\cdot\frac{12}{13}$$
$$=-\frac{33}{65}$$

102. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\frac{5\pi}{6}\quad \sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$

$$\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)-\sin^{-1}\left(-\frac{1}{2}\right)\right]$$
$$=\cos\left[\frac{5\pi}{6}-\left(-\frac{\pi}{6}\right)\right]$$
$$=\cos\pi$$
$$=-1$$

103. Let $\alpha=\sin^{-1}x$, where $-\frac{\pi}{2}\le\alpha\le\frac{\pi}{2}$. $\sin\alpha=x$

Because x is positive, sin α is positive. Thus α is in quadrant I. Using a right triangle in quadrant I with sin α = x, the third side can be found using the Pythagorean Theorem.

$$a^2+x^2=1^2$$
$$a^2=1-x^2$$
$$a=\sqrt{1-x^2}$$
$$\text{Thus}\cos\alpha=\frac{\sqrt{1-x^2}}{1}=\sqrt{1-x^2}$$

Because y is positive, cos β is positive. Thus β is in quadrant I. Using a right triangle in quadrant I with cos β = y, the third side can be found using the Pythagorean Theorem.

$$b^2+y^2=1^2$$
$$b^2=1-x^2$$
$$a=\sqrt{1-y^2}$$
$$\text{Thus}\cos\alpha=\frac{\sqrt{1-y^2}}{1}=\sqrt{1-y^2}$$
$$\cos(\sin^{-1}x-\cos^{-1}y)=\cos(\alpha-\beta)$$
$$=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$
$$=\sqrt{1-x^2}\,y+x\sqrt{1-y^2}$$
$$=y\sqrt{1-x^2}+x\sqrt{1-y^2}$$

104. $\tan^{-1}x$ $\quad$ $\sin^{-1}x$

$y=x$ $\quad$ $y=y$

$x=1$ $\quad$ $r=1$

$r=\sqrt{x^2+1}$ $\quad$ $x=\sqrt{1-y^2}$

$$\sin(\tan^{-1}x-\sin^{-1}y)$$
$$=\sin\tan^{-1}x\cos\sin^{-1}y$$
$$-\cos\tan^{-1}x\sin\sin^{-1}y$$
$$=\frac{x}{\sqrt{x^2+1}}\cdot\frac{\sqrt{1-y^2}}{1}-\frac{1}{\sqrt{x^2+1}}\cdot y$$
$$=\frac{x\sqrt{1-y^2}-y}{\sqrt{x^2+1}}$$

105. $\sin^{-1}$

$x=\sqrt{1-x^2}$

$y=x$

$r=1$

$\cos^{-1}y$

$x=y$

$y=\sqrt{1-y^2}$

$r=1$

$$\tan(\sin^{-1}x+\cos^{-1}y)$$
$$=\frac{\tan\sin^{-1}x+\tan\cos^{-1}y}{1-\tan\sin^{-1}x\cdot\tan\cos^{-1}y}$$
$$=\frac{\frac{x}{\sqrt{1-x^2}}+\frac{\sqrt{1-y^2}}{y}}{1-\frac{x}{\sqrt{1-x^2}}\cdot\frac{\sqrt{1-y^2}}{y}}$$
$$=\frac{xy+\sqrt{1-y^2}\sqrt{1-x^2}}{y\sqrt{1-x^2}-x\sqrt{1-y^2}}$$

106. Answers may vary.

107. $\sin 30^\circ = \frac{1}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 60^\circ = \frac{1}{2}$

108. **a.** No, they are not equal.

$$\sin(2\cdot 30^\circ) \neq 2\sin 30^\circ$$
$$\sin 60^\circ \neq 2\cdot\frac{1}{2}$$
$$\frac{\sqrt{3}}{2} \neq 1$$

b. Yes, they are equal.

$$\sin(2\cdot 30^\circ) = 2\sin 30^\circ \cos 30^\circ$$
$$\sin 60^\circ = 2\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}$$
$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

109. **a.** No, they are not equal.

$$\cos(2\cdot 30^\circ) \neq 2\cos 30^\circ$$
$$\cos 60^\circ \neq 2\cdot\frac{\sqrt{3}}{2}$$
$$\frac{1}{2} \neq \sqrt{3}$$

b. Yes, they are equal.

$$\cos(2\cdot 30^\circ) = \cos^2 30^\circ - \sin^2 30^\circ$$
$$\cos 60^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$
$$\frac{1}{2} = \frac{3}{4} - \frac{1}{4}$$
$$\frac{1}{2} = \frac{1}{2}$$

Section 6.3

Check Point Exercises

1. $\sin\theta = \frac{4}{5} = \frac{y}{r}$

Because θ lies in quadrant II, x is negative.

$$x^2 + y^2 = r^2$$
$$x^2 + 4^2 = 5^2$$
$$x^2 = 5^2 - 4^2 = 9$$
$$x = -\sqrt{9} = -3$$

Now we use values for x, y, and r to find the required values.

a. $$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = -\frac{24}{25}$$

b. $$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$$

c. $$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(-\frac{4}{3}\right)}{1-\left(-\frac{4}{3}\right)^2} = \frac{-\frac{8}{3}}{1-\frac{16}{9}} = \frac{-\frac{8}{3}}{-\frac{7}{9}} = \left(-\frac{8}{3}\right)\left(-\frac{9}{7}\right) = \frac{24}{7}$$

2. The given expression is the right side of the formula for $\cos 2\theta$ with $\theta = 15°$.

$$\cos^2 15° - \sin^2 15° = \cos(2\cdot 15°) = \cos 30° = \frac{\sqrt{3}}{2}$$

3. $\sin 3\theta = \sin(2\theta + \theta)$
$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$
$= 2\sin\theta\cos\theta\cos\theta + (2\cos^2\theta - 1)\sin\theta$
$= 2\sin\theta\cos^2\theta + 2\sin\theta\cos^2\theta - \sin\theta$
$= 4\sin\theta\cos^2\theta - \sin\theta$
$= 4\sin\theta(1-\sin^2\theta) - \sin\theta$
$= 4\sin\theta - 4\sin^3\theta - \sin\theta$
$= 3\sin\theta - 4\sin^3\theta$

By working with the left side and expressing it in a form identical to the right side, we have verified the identity.

4. $\sin^4 x = \left(\sin^2 x\right)^2$
$= \left(\dfrac{1-\cos 2x}{2}\right)^2$
$= \dfrac{1 - 2\cos 2x + \cos^2 2x}{4}$
$= \dfrac{1}{4} - \dfrac{1}{2}\cos 2x + \dfrac{1}{4}\cos^2 2x$
$= \dfrac{1}{4} - \dfrac{1}{2}\cos 2x + \dfrac{1}{4}\left(\dfrac{1+\cos 2(2x)}{2}\right)$
$= \dfrac{1}{4} - \dfrac{1}{2}\cos 2x + \dfrac{1}{8} + \dfrac{1}{8}\cos 4x$
$= \dfrac{3}{8} - \dfrac{1}{2}\cos 2x + \dfrac{1}{8}\cos 4x$

5. Because 105° lies in quadrant II, $\cos 105° < 0$.

$\cos 105° = \cos\left(\dfrac{210°}{2}\right)$
$= -\sqrt{\dfrac{1+\cos 210°}{2}}$
$= -\sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{2}}$
$= -\sqrt{\dfrac{2-\sqrt{3}}{4}}$
$= -\dfrac{\sqrt{2-\sqrt{3}}}{2}$

6. $\dfrac{\sin 2\theta}{1+\cos 2\theta} = \dfrac{2\sin\theta\cos\theta}{1+\left(1-2\sin^2\theta\right)}$
$= \dfrac{2\sin\theta\cos\theta}{2-2\sin^2\theta}$
$= \dfrac{2\sin\theta\cos\theta}{2\left(1-\sin^2\theta\right)}$
$= \dfrac{2\sin\theta\cos\theta}{2\cos^2\theta}$
$= \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

The right side simplifies to $\tan\theta$, the expression on the left side. Thus, the identity is verified.

7. $\dfrac{\sec\alpha}{\sec\alpha\csc\alpha + \csc\alpha} = \dfrac{\frac{1}{\cos\alpha}}{\frac{1}{\cos\alpha}\cdot\frac{1}{\sin\alpha} + \frac{1}{\sin\alpha}}$
$= \dfrac{\frac{1}{\cos\alpha}}{\frac{1}{\cos\alpha\sin\alpha} + \frac{\cos\alpha}{\cos\alpha\sin\alpha}}$
$= \dfrac{\frac{1}{\cos\alpha}}{\frac{1+\cos\alpha}{\cos\alpha\sin\alpha}}$
$= \dfrac{1}{\cos\alpha}\cdot\dfrac{\cos\alpha\sin\alpha}{1+\cos\alpha}$
$= \dfrac{\sin\alpha}{1+\cos\alpha}$
$= \tan\dfrac{\alpha}{2}$

We worked with the right side and arrived at the left side. Thus, the identity is verified.

Exercise Set 6.3

1. $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\dfrac{3}{5}\right)\left(\dfrac{4}{5}\right) = \dfrac{24}{25}$

2. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(\dfrac{4}{5}\right)^2 - \left(\dfrac{3}{5}\right)^2 = \dfrac{16}{25} - \dfrac{9}{25} = \dfrac{7}{25}$

3. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$= \dfrac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \dfrac{\frac{3}{2}}{1-\frac{9}{16}}$

$= \dfrac{\frac{3}{2}}{\frac{7}{16}} = \left(\dfrac{3}{2}\right)\left(\dfrac{16}{7}\right) = \dfrac{24}{7}$

Use this information to solve problems 4, 5, and 6.

$\tan\alpha = \dfrac{7}{24} = \dfrac{y}{x}$

Because r is a distance it is positive.

$x^2 + y^2 = r^2$

$24^2 + 7^2 = r^2$

$576 + 49 = r^2$

$625 = r^2$

$r = 25$

$\sin\alpha = \dfrac{y}{r} = \dfrac{7}{25}$

$\cos\alpha = \dfrac{x}{r} = \dfrac{24}{25}$

4. $\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\left(\dfrac{7}{25}\right)\left(\dfrac{24}{25}\right) = \dfrac{336}{625}$

5. $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= \left(\dfrac{24}{25}\right)^2 - \left(\dfrac{7}{25}\right)^2 = \dfrac{576}{625} - \dfrac{49}{625}$

$= \dfrac{527}{625}$

6. $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

$= \dfrac{2\left(\frac{7}{24}\right)}{1-\left(\frac{7}{24}\right)^2} = \dfrac{\frac{14}{24}}{1-\frac{49}{576}} = \dfrac{\frac{14}{24}}{\frac{527}{576}}$

$= \left(\dfrac{14}{24}\right)\left(\dfrac{576}{527}\right) = \dfrac{336}{527}$

7. $\sin\theta = \dfrac{15}{17} = \dfrac{y}{r}$

Because θ lies in quadrant II, x is negative.

$x^2 + y^2 = r^2$

$x^2 + 15^2 = 17^2$

$x^2 = 17^2 - 15^2 = 64$

$x = -\sqrt{64} = -8$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\dfrac{15}{17}\right)\left(-\dfrac{8}{17}\right) = -\dfrac{240}{289}$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\dfrac{8}{17}\right)^2 - \left(\dfrac{15}{17}\right)^2 = \dfrac{64}{289} - \dfrac{225}{289}$

$= -\dfrac{161}{289}$

c. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$= \dfrac{2\left(-\frac{15}{8}\right)}{1-\left(-\frac{15}{8}\right)^2} = \dfrac{-\frac{15}{4}}{1-\frac{225}{64}} = \dfrac{-\frac{15}{4}}{-\frac{161}{64}}$

$= \left(-\dfrac{15}{4}\right)\left(-\dfrac{64}{161}\right) = \dfrac{240}{161}$

8. $\sin\theta = \dfrac{12}{13} = \dfrac{y}{r}$

Because θ lies in quadrant II, x is negative.

$x^2 + y^2 = r^2$

$x^2 + 12^2 = 13^2$

$x^2 = 25$

$x = -\sqrt{25} = -5$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cdot\dfrac{12}{13}\cdot\left(-\dfrac{5}{13}\right)$

$= -\dfrac{120}{169}$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= \left(-\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$$

$$= \frac{25}{169} - \frac{144}{169}$$

$$= -\frac{119}{169}$$

c. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$$= \frac{2\left(-\frac{12}{5}\right)}{1-\left(-\frac{12}{5}\right)^2} = \frac{\frac{-24}{5}}{1-\frac{144}{25}}$$

$$= \frac{-\frac{24}{5}}{-\frac{119}{5}} = \frac{120}{119}$$

9. $\cos\theta = \dfrac{24}{25} = \dfrac{x}{r}$

Because θ lies in quadrant IV, y is negative.

$$x^2 + y^2 = r^2$$

$$24^2 + y^2 = 25^2$$

$$y^2 = 25^2 - 24^2 = 49$$

$$y = -\sqrt{49} = -7$$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\left(-\frac{7}{25}\right)\left(\frac{24}{25}\right) = -\frac{336}{625}$$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= \left(\frac{24}{25}\right)^2 - \left(-\frac{7}{25}\right)^2$$

$$= \frac{576}{625} - \frac{49}{625} = \frac{527}{625}$$

c. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$$= \frac{2\left(-\frac{7}{24}\right)}{1-\left(-\frac{7}{24}\right)^2} = \frac{-\frac{7}{12}}{1-\frac{49}{576}} = \frac{-\frac{7}{12}}{\frac{527}{576}}$$

$$= \left(-\frac{7}{12}\right)\left(\frac{576}{527}\right) = -\frac{336}{527}$$

10. $\cos\theta = \dfrac{40}{41} = \dfrac{x}{r}$

Because θ lies in quadrant IV, y is negative.

$$x^2 + y^2 = r^2$$

$$40^2 + y^2 = 41^2$$

$$y^2 = 81$$

$$y = -\sqrt{81} = -9$$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\left(-\frac{9}{41}\right)\left(\frac{40}{41}\right) = -\frac{720}{1681}$$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= \left(\frac{40}{41}\right) - \left(-\frac{9}{41}\right)^2 = \frac{1600}{1681} - \frac{81}{1681}$$

$$= \frac{1519}{1681}$$

c. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$$= \frac{2\left(-\frac{9}{40}\right)}{1-\left(-\frac{9}{40}\right)^2} = \frac{-\frac{9}{20}}{1-\frac{81}{1600}}$$

$$= \frac{-\frac{9}{20}}{\frac{1519}{1600}} = \left(-\frac{9}{20}\right)\left(\frac{1600}{1519}\right) = -\frac{720}{1519}$$

11. $\cot\theta = 2 = \frac{-2}{-1} = \frac{x}{y}$

Because r is a distance, it is positive.

$r^2 = x^2 + y^2$

$r^2 = (-2)^2 + (-1)^2$

$r^2 = 5$

$r = \sqrt{5}$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(-\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right) = \frac{4}{5}$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{2}{\sqrt{5}}\right)^2 - \left(-\frac{1}{\sqrt{5}}\right)^2$

$= \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2} = \frac{1}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}}$

$= (1)\left(\frac{4}{3}\right) = \frac{4}{3}$

12. $\cot\theta = 3 = \frac{-3}{-1} = \frac{x}{y}$

Because r is a distance, it is positive.

$r^2 = x^2 + y^2$

$r^2 = (-3)^2 + (-1)^2$

$r^2 = 10$

$r = \sqrt{10}$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(-\frac{1}{\sqrt{10}}\right)\left(-\frac{3}{\sqrt{10}}\right) = \frac{6}{10} = \frac{3}{5}$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{3}{\sqrt{10}}\right)^2 - \left(-\frac{1}{\sqrt{10}}\right)^2$

$= \frac{9}{10} - \frac{1}{10} = \frac{8}{10} = \frac{4}{5}$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2} = \frac{\frac{2}{3}}{1-\frac{1}{9}}$

$= \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{2}{3}\cdot\frac{9}{8} = \frac{3}{4}$

13. $\sin\theta = -\frac{9}{41} = \frac{-9}{41} = \frac{y}{r}$

Because θ lies in quadrant III, x is negative.

$x^2 + y^2 = r^2$

$x^2 + (-9)^2 = 41^2$

$x^2 = 1600$

$x = -\sqrt{1600}$

$x = -40$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(-\frac{9}{41}\right)\left(-\frac{40}{41}\right) = \frac{720}{1681}$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{40}{41}\right)^2 - \left(-\frac{9}{41}\right)^2$

$= \frac{1600}{1681} - \frac{81}{1681}$

$= \frac{1519}{1681}$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{9}{40}\right)}{1-\left(\frac{9}{40}\right)^2} = \frac{\frac{9}{20}}{1-\frac{81}{1600}} = \frac{\frac{9}{20}}{\frac{1519}{1600}}$

$= \left(\frac{9}{20}\right)\left(\frac{1600}{1519}\right) = \frac{720}{1519}$

14. $\sin\theta = -\frac{2}{3} = \frac{-2}{3} = \frac{y}{r}$

Because θ lies in quadrant III, x is negative.

$$x^2 + y^2 = r^2$$
$$x^2 + (-2)^2 = 3^2$$
$$x^2 = 5$$
$$x = -\sqrt{5}$$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\left(-\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right) = \frac{4\sqrt{5}}{9}$$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= \left(-\frac{\sqrt{5}}{3}\right)^2 - \left(-\frac{2}{3}\right)^2$$
$$= \frac{5}{9} - \frac{4}{6} = \frac{1}{9}$$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$$= \frac{2\cdot\frac{2}{\sqrt{5}}}{1-\left(\frac{2}{\sqrt{5}}\right)^2} = \frac{\frac{4}{\sqrt{5}}}{1-\frac{4}{5}}$$
$$= \frac{\frac{4}{\sqrt{5}}}{\frac{1}{5}} = \frac{4\sqrt{5}}{4}\cdot\frac{5}{1} = 4\sqrt{5}$$

15. The given expression is the right side of the formula for $\sin 2\theta$ with $\theta = 15°$.

$$2\sin 15°\cos 15° = \sin(2\cdot 15°)$$
$$= \sin 30° = \frac{1}{2}$$

16. The given expression is the right side of the formula for sin 2θ with θ = 22.5°.

$$2\sin 22.5^{\circ}\cos 22.5^{\circ} = \sin(2\cdot 22.5^{\circ})$$
$$= \sin 45^{\circ} = \frac{\sqrt{2}}{2}$$

17. The given expression is the right side of the formula for $\cos 2\theta$ with $\theta = 75°$.

$$\cos^2 75° - \sin^2 75° = \cos(2\cdot 75°)$$
$$= \cos 150° = -\frac{\sqrt{3}}{2}$$

18. The given expression is the right side of the formula for cos 2θ with θ = 105°

$$\cos^2 105^{\circ} - \sin^2 105^{\circ} = \cos(2\cdot 105^{\circ})$$
$$= \cos 210^{\circ} = -\frac{\sqrt{3}}{2}$$

19. The given expression is the right side of the formula for $\cos 2\theta$ with $\theta = \frac{\pi}{8}$.

$$2\cos^2\frac{\pi}{8} - 1 = \cos\left(2\cdot\frac{\pi}{8}\right)$$
$$= \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

20. The given expression is the right side of the formula for cos 2θ with θ = $\frac{\pi}{12}$.

$$1 - 2\sin^2\frac{\pi}{12} = \cos\left(2\cdot\frac{\pi}{12}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

21. The given expression is the right side of the formula for $\tan 2\theta$ with $\theta = \frac{\pi}{12}$.

$$\frac{2\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}} = \tan\left(2\cdot\frac{\pi}{12}\right) = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

22. The given expression is the right side of the formula for tan 2θ with θ = $\frac{\pi}{8}$.

$$\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}} = \tan\left(2\cdot\frac{\pi}{8}\right) = \tan\frac{\pi}{4} = 1$$

23. $$\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\cdot\frac{\sin\theta}{\cos\theta}}{\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}}$$
$$= \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$$
$$= \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}$$
$$= \frac{2\sin\theta}{\cos\theta}\cdot\frac{\cos^2\theta}{1}$$
$$= 2\sin\theta\cos\theta$$
$$= \sin 2\theta$$

24. $$\frac{2\cot\theta}{1+\cot^2\theta} = \frac{2\cdot\frac{\cos\theta}{\sin\theta}}{\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}}$$
$$= \frac{\frac{2\cos\theta}{\sin\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}$$
$$= \frac{\frac{2\cos\theta}{\sin\theta}}{\frac{1}{\sin^2\theta}}$$
$$= \frac{2\cos\theta}{\sin\theta}\cdot\frac{\sin^2\theta}{1}$$
$$= 2\cos\theta\sin\theta$$
$$= \sin 2\theta$$

25. $$(\sin\theta+\cos\theta)^2 = \sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta$$
$$= \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta$$
$$= 1+2\sin\theta\cos\theta$$
$$= 1+\sin 2\theta$$

26. $$(\sin\theta-\cos\theta)^2$$
$$= \sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta$$
$$= \sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta$$
$$= 1-2\sin\theta\cos\theta$$
$$= 1-\sin 2\theta$$

27. $$\sin^2 x+\cos 2x = \sin^2 x+\cos^2 x-\sin^2 x$$
$$= \cos^2 x$$

28. $$\frac{\cos 2x}{\cos^2 x} = \frac{1-2\sin^2 x}{\cos^2 x}$$
$$= \frac{1-\sin^2 x-\sin^2 x}{\cos^2 x}$$
$$= \frac{\cos^2 x-\sin^2 x}{\cos^2 x}$$
$$= \frac{\cos^2 x}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}$$
$$= 1-\tan^2 x$$

29. $$\frac{\sin 2x}{1-\cos 2x} = \frac{2\sin x\cos x}{1-\left(\cos^2 x-\sin^2 x\right)}$$
$$= \frac{2\sin x\cos x}{1-\cos^2 x+\sin^2 x}$$
$$= \frac{2\sin x\cos x}{\sin^2+\sin^2 x}$$
$$= \frac{2\sin x\cos x}{2\sin^2 x}$$
$$= \frac{\cos x}{\sin x}$$
$$= \cot x$$

30. $$\frac{1+\cos 2x}{\sin 2x} = \frac{1+\cos^2 x-\sin^2 x}{2\sin x\cos x}$$
$$= \frac{1-\sin^2 x+\cos^2 x}{2\sin x\cos x}$$
$$= \frac{\cos^2 x+\cos^2 x}{2\sin x\cos x}$$
$$= \frac{2\cos^2 x}{2\sin x\cos x}$$
$$= \frac{\cos x}{\sin x}$$
$$= \cot x$$

31. $$\tan t\cos 2t = \frac{\sin t}{\cos t}\cdot\left(2\cos^2 t-1\right)$$
$$= \frac{2\sin t\cos^2 t}{\cos t}-\frac{\sin t}{\cos t}$$
$$= 2\sin t\cos t-\tan t$$
$$= \sin 2t-\tan t$$

32. $-\cos t\cos 2t = \dfrac{-\cos t}{\sin t}\left(1-2\sin^2 t\right)$

$= -\dfrac{\cos t}{\sin t} + \dfrac{2\cos t\sin^2 t}{\sin t}$

$= -\cot t + 2\cos t\sin t$

$= 2\cos t\sin t - \cot t$

$= \sin 2t - \cot t$

33. $\sin 4t = \sin(2t+2t)$

$= \sin 2t\cos 2t + \cos 2t\sin 2t$

$= \cos 2t(\sin 2t + \sin 2t)$

$= \cos 2t \cdot 2\sin 2t$

$= \left(\cos^2 t - \sin^2 t\right)\cdot 2\cdot 2\sin t\cos t$

$= 4\sin t\cos^3 t - 4\sin^3 t\cos t$

34. $\cos 4t = \cos 2(2t)$

$= 2\cos^2 2t - 1$

$= 2(2\cos^2 t - 1)^2 - 1$

$= 2(4\cos^4 t - 4\cos^2 t + 1) - 1$

$= 8\cos^4 t - 8\cos^2 t + 2 - 1$

$= 8\cos^4 t - 8\cos^2 t + 1$

35. $6\sin^4 x$

$= 6\left(\dfrac{1-\cos 2x}{2}\right)^2$

$= 6\left(\dfrac{1-2\cos 2x+\cos^2 2x}{4}\right)$

$= \dfrac{6-12\cos 2x+6\cos^2 2x}{4}$

$= \dfrac{3}{4} - 3\cos 2x + \dfrac{3}{2}\cos^2 2x$

$= \dfrac{3}{4} - 3\cos 2x + \dfrac{3}{2}\left(\dfrac{1+\cos 4x}{2}\right)$

$= \dfrac{3}{4} - 3\cos 2x + \dfrac{3}{2}\left(\dfrac{1}{2} + \dfrac{\cos 4x}{2}\right)$

$= \dfrac{3}{4} - 3\cos 2x + \dfrac{3}{4} + \dfrac{3}{4}\cos 4x$

$= \dfrac{9}{4} - 3\cos 2x + \dfrac{3}{4}\cos 4x$

36. $10\cos^2 x\cos^2 x = 10\left(\dfrac{1+\cos 2x}{2}\right)\left(\dfrac{1+\cos 2x}{2}\right)$

$= \dfrac{10(1+2\cos 2x+\cos^2 2x)}{4}$

$= \dfrac{10+20\cos 2x+10\cos^2 2x}{4}$

$= \dfrac{10}{4} + \dfrac{20\cos 2x}{4} + \dfrac{10\cos^2 2x}{4}$

$= \dfrac{5}{2} + 5\cos 2x + \dfrac{5}{4}\left(1+\cos 4x\right)$

$= \dfrac{5}{2} + 5\cos 2x + \dfrac{5}{4} + \dfrac{5}{4}\cos 4x$

$= \dfrac{15}{4} + 5\cos 2x + \dfrac{5}{4}\cos 4x$

37. $\sin^2 x\cos^2 x = \left(\dfrac{1-\cos 2x}{2}\right)\left(\dfrac{1+\cos 2x}{2}\right)$

$= \dfrac{1-\cos^2 2x}{4}$

$= \dfrac{1}{4} - \dfrac{1}{4}\cos^2 2x$

$= \dfrac{1}{4} - \dfrac{1}{4}\left(\dfrac{1+\cos(2\cdot 2x)}{2}\right)$

$= \dfrac{1}{4} - \dfrac{1}{8}(1+\cos 4x)$

$= \dfrac{1}{4} - \dfrac{1}{8} - \dfrac{1}{8}\cos 4x$

$= \dfrac{1}{8} - \dfrac{1}{8}\cos 4x$

38. $8\sin^2 x\cos^2 x = 8\left(\dfrac{1-\cos 2x}{2}\right)\left(\dfrac{1+\cos 2x}{2}\right)$

$= \dfrac{8(1-\cos^2 2x)}{4}$

$= \dfrac{8}{4} - \dfrac{8(\cos^2 2x)}{4}$

$= 2 - 2\left(\dfrac{1+\cos 2\cdot 2x}{2}\right)$

$= 2 - 1 - \cos 4x$

$= 1 - \cos 4x$

39. Because 15° lies in quadrant I, $\sin 15° > 0$.

$$\sin 15° = \sin\frac{30°}{2} = \sqrt{\frac{1-\cos 30°}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

40. Because 22.5° lies in quadrant I, $\cos 22.5° > 0$.

$$\cos 22.5° = \cos\frac{45°}{2} = \sqrt{\frac{1+\cos 45°}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

41. Because 157.5° lies in quadrant II, $\cos 157.5° < 0$.

$$\cos 157.5° = \cos\frac{315°}{2} = -\sqrt{\frac{1+\cos 315°}{2}} = -\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{2}}{4}} = -\frac{\sqrt{2+\sqrt{2}}}{2}$$

42. Because 105° lies in quadrant II, $\sin 105° > 0$.

$$\sin 105° = \sin\frac{210°}{2} = \sqrt{\frac{1-\cos 210°}{2}} = \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

43. Because 75° lies in quadrant I, $\tan 75° > 0$.

$$\tan 75° = \tan\frac{150°}{2} = \frac{1-\cos 150°}{\sin 150°} = \frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}} = 2+\sqrt{3}$$

44. Because 112.5° lies in quadrant II, $\tan 112.5^0 < 0$.

$$\tan 125° = \tan\frac{225°}{2} = \frac{1-\cos 225°}{\sin 225°} = \frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{-\frac{\sqrt{2}}{2}} = \frac{2+\sqrt{2}}{-\sqrt{2}} = -\frac{2}{\sqrt{2}}-1 = -\sqrt{2}-1$$

45. Because $\frac{7\pi}{8}$ lies in quadrant II, $\tan\frac{7\pi}{8} < 0$.

$$\tan\frac{7\pi}{8} = \tan\left(\frac{\frac{7\pi}{4}}{2}\right) = \frac{1-\cos\frac{7\pi}{4}}{\sin\frac{7\pi}{4}} = \frac{1-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}}+1 = -\sqrt{2}+1$$

46. Because $\frac{3\pi}{8}$ lies in quadrant I, $\tan\frac{3\pi}{8} > 0$.

$$\tan\frac{3\pi}{8} = \tan\frac{\frac{3\pi}{4}}{2} = \frac{1-\cos\frac{3\pi}{4}}{\sin\frac{3\pi}{4}} = \frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}+1 = \sqrt{2}+1$$

47. $\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$

$= \sqrt{\frac{1-\frac{4}{5}}{2}} = \sqrt{\frac{1}{10}}$

$= \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

48. $\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$

$= \sqrt{\frac{1+\frac{4}{5}}{2}}$

$= \sqrt{\frac{\frac{9}{5}}{2}}$

$= \sqrt{\frac{9}{10}}$

$= \frac{3}{\sqrt{10}}$

$= \frac{3\sqrt{10}}{10}$

49. $\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$

$= \frac{1-\frac{4}{5}}{\frac{3}{5}}$

$= \frac{1}{3}$

Use this information to solve problems 50, 51, 52 and 54.

$\tan\alpha = \frac{7}{24} = \frac{y}{x}$

Because r is a distance, it is positive.

$r^2 = x^2 + y^2$

$r^2 = 24^2 + 7^2$

$r^2 = 625$

$r = 25$

$\sin\alpha = \frac{y}{r} = \frac{7}{25}$

$\cos\alpha = \frac{x}{r} = \frac{24}{25}$

50. $\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}}$

$= \sqrt{\frac{1-\frac{24}{25}}{2}}$

$= \sqrt{\frac{\frac{1}{25}}{2}}$

$= \sqrt{\frac{1}{50}}$

$= \frac{1}{5\sqrt{2}}$

$= \frac{\sqrt{2}}{10}$

51. $\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\frac{24}{25}}{2}} = \sqrt{\frac{49}{50}}$

$= \frac{7}{5\sqrt{2}} = \frac{7\sqrt{2}}{10}$

52. $\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$

$= \frac{1-\frac{24}{25}}{\frac{7}{25}}$

$= \frac{\frac{1}{25}}{\frac{7}{25}}$

$= \frac{1}{7}$

53. $2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2\cdot\sqrt{\frac{1-\cos\theta}{2}}\cdot\sqrt{\frac{1+\cos\theta}{2}}$

$= 2\sqrt{\frac{1-\frac{4}{5}}{2}}\cdot\sqrt{\frac{1+\frac{4}{5}}{2}}$

$= 2\cdot\sqrt{\frac{1}{10}}\cdot\sqrt{\frac{9}{10}}$

$= 2\cdot\frac{1}{\sqrt{10}}\cdot\frac{3}{\sqrt{10}}$

$= \frac{6}{10} = \frac{3}{5}$

54. $2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = 2\sqrt{\frac{1-\cos\alpha}{2}}\cdot\sqrt{\frac{1+\cos\alpha}{2}}$

$= 2\sqrt{\frac{1-\frac{24}{25}}{2}}\cdot\sqrt{\frac{1+\frac{24}{25}}{2}}$

$= 2\sqrt{\frac{1}{50}}\cdot\sqrt{\frac{49}{50}}$

$= 2\cdot\frac{1}{\sqrt{50}}\cdot\frac{7}{\sqrt{50}}$

$= \frac{7}{25}$

55. $\tan\alpha = \frac{4}{3} = \frac{-4}{-3} = \frac{y}{x}$

Because r is a distance, it is positive.

$r^2 = x^2 + y^2$

$r^2 = (-4)^2 + (-3)^2$

$r^2 = 25$

$r = 5$

Since $180^\circ < \alpha < 270^\circ$, then $90^\circ < \frac{\alpha}{2} < 135^\circ$.

Therefore $\frac{\alpha}{2}$ lies in quadrant II.

Thus, $\sin\frac{\alpha}{2} > 0$, $\cos\frac{\alpha}{2} < 0$, and $\tan\frac{\alpha}{2} < 0$.

a. $\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}}$

$= \sqrt{\frac{\frac{8}{5}}{2}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

b. $\cos\frac{\alpha}{2} = -\sqrt{\frac{1+\cos\alpha}{2}} = -\sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}}$

$= -\sqrt{\frac{\frac{2}{5}}{2}} = -\sqrt{\frac{1}{5}} = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

c. $\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{1-\left(-\frac{3}{5}\right)}{-\frac{4}{5}}$

$= \frac{\frac{8}{5}}{-\frac{4}{5}} = \frac{8}{-4} = -2$

56. $\tan\alpha = \frac{8}{15} = \frac{-8}{-15} = \frac{y}{x}$

Because r is a distance, it is positive.

$r^2 = x^2 + y^2$

$r^2 = (-15)^2 + (-8)^2$

$r^2 = 289$

$r = \sqrt{289}$

$r = 17$

Since $180^\circ < \alpha < 270^\circ$, then $90^\circ < \frac{\alpha}{2} < 135^\circ$.

Therefore $\frac{\alpha}{2}$ lies in quadrant II.

Thus, $\sin\frac{\alpha}{2} > 0$, $\cos\frac{\alpha}{2} < 0$ and $\tan\frac{\alpha}{2} < 0$.

a. $\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}}$

$= \sqrt{\frac{1-\frac{-15}{17}}{2}}$

$= \sqrt{\frac{\frac{32}{17}}{2}}$

$= \sqrt{\frac{16}{17}}$

$= \frac{4}{\sqrt{17}}$

$= \frac{4\sqrt{17}}{17}$

b. $\cos\frac{\alpha}{2} = -\sqrt{\frac{1+\left(-\frac{15}{17}\right)}{2}}$

$= -\sqrt{\frac{\frac{2}{17}}{2}}$

$= -\sqrt{\frac{1}{17}}$

$= -\frac{1}{\sqrt{17}}$

$= -\frac{\sqrt{17}}{17}$

c. $\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$

$= \frac{1-\left(-\frac{15}{17}\right)}{-\frac{8}{17}}$

$= \frac{\frac{32}{17}}{-\frac{8}{17}} = \frac{32}{-8} = -4$

57. $\sec\alpha = -\frac{13}{5} = \frac{13}{-5} = \frac{r}{x}$

Because α lies in quadrant II, y is positive.

$x^2 + y^2 = r^2$

$(-5)^2 + y^2 = (13)^2$

$y^2 = 144$

$y = 12$

Since $\frac{\pi}{2} < \alpha < \pi$, then $\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$. Therefore $\frac{\alpha}{2}$ lies in quadrant I.

Thus, $\sin\frac{\alpha}{2} > 0$, $\cos\frac{\alpha}{2} > 0$, and $\tan\frac{\alpha}{2} > 0$.

a. $\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\left(-\frac{5}{13}\right)}{2}}$

$= \sqrt{\frac{18}{26}} = \sqrt{\frac{9}{13}} = \frac{3}{\sqrt{13}}$

$= \frac{3\sqrt{13}}{13}$

b. $\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\left(-\frac{5}{13}\right)}{2}}$

$= \sqrt{\frac{8}{26}} = \sqrt{\frac{4}{13}} = \frac{2}{\sqrt{13}}$

$= \frac{2\sqrt{13}}{13}$

c. $\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{1-\left(-\frac{5}{13}\right)}{\frac{12}{13}}$

$= \frac{13+5}{12} = \frac{18}{12} = \frac{3}{2}$

58. $\sec\alpha = -3 = \frac{3}{-1} = \frac{r}{x}$

Because α lies in quadrant II, y is positive.

$x^2 + y^2 = r^2$

$(-1)^2 + y^2 = 3^2$

$y^2 = 8$

$y = \sqrt{8}$

$y = 2\sqrt{2}$

Since $\frac{\pi}{2} < \alpha < \pi$, then $\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$. Therefore $\frac{\alpha}{2}$ lies in quadrant I.

Thus, $\sin\frac{\alpha}{2} > 0$, $\cos\frac{\alpha}{2} > 0$, and $\tan\frac{\alpha}{2} > 0$.

a. $\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}}$

$= \sqrt{\frac{1-\left(-\frac{1}{3}\right)}{\frac{1}{3}}}$

$= \sqrt{\frac{\frac{4}{3}}{2}}$

$= \sqrt{\frac{2}{3}}$

$= \frac{\sqrt{2}}{\sqrt{3}}$

$= \frac{\sqrt{6}}{3}$

b. $\cos\frac{\alpha}{2}=\sqrt{\frac{1+\cos\alpha}{2}}$

$=\sqrt{\frac{1+\left(-\frac{1}{3}\right)}{\frac{1}{3}}}$

$=\sqrt{\frac{\frac{2}{3}}{2}}$

$=\sqrt{\frac{2}{3}}$

$=\frac{1}{\sqrt{3}}$

$=\frac{\sqrt{3}}{3}$

c. $\tan\frac{\alpha}{2}=\frac{1-\cos\alpha}{\sin\alpha}$

$=\frac{1-\left(-\frac{1}{3}\right)}{\frac{2\sqrt{2}}{3}}$

$=\frac{3+1}{2\sqrt{2}}$

$=\frac{4}{2\sqrt{2}}$

$=\sqrt{2}$

59. $\sin^2\frac{\theta}{2}=\frac{1-\cos 2\left(\frac{\theta}{2}\right)}{2}$

$=\frac{1-\cos\theta}{2}\cdot\frac{\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$

$=\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{2}{\cos\theta}}$

$=\frac{\frac{1}{\cos\theta}-\frac{\cos\theta}{\cos\theta}}{2\cdot\frac{1}{\cos\theta}}$

$=\frac{\sec\theta-1}{2\sec\theta}$

60. $\sin^2\frac{\alpha}{2}=\frac{1-\cos 2\left(\frac{\alpha}{2}\right)}{2}$

$=\frac{1-\cos\alpha}{2}\cdot\frac{\frac{1}{\sin\alpha}}{\frac{1}{\sin\alpha}}$

$=\frac{\frac{1-\cos\alpha}{\sin\alpha}}{\frac{2}{\sin\alpha}}$

$=\frac{\frac{1}{\sin\alpha}-\frac{\cos\alpha}{\sin\alpha}}{2\cdot\frac{1}{\sin\alpha}}$

$=\frac{\csc\alpha-\cot\alpha}{2\csc\alpha}$

61. $\cos^2\frac{\theta}{2}=\frac{1+\cos 2\left(\frac{\theta}{2}\right)}{2}$

$=\frac{1+\cos\theta}{2}$

$=\frac{1+\cos\theta}{2}\cdot\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$

$=\frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{2\cdot\frac{\sin\theta}{\cos\theta}}$

$=\frac{\tan\theta+\sin\theta}{2\tan\theta}$

$=\frac{\sin\theta+\tan\theta}{2\tan\theta}$

62. $\cos^2\frac{\theta}{2}=\frac{1+\cos 2\left(\frac{\theta}{2}\right)}{2}$

$=\frac{1+\cos\theta}{2}\cdot\frac{\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$

$=\frac{\frac{1}{\cos\theta}+1}{2\cdot\frac{1}{\cos\theta}}$

$=\frac{\sec\theta+1}{2\sec\theta}$

63. $\tan\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}$
$=\frac{\sin\alpha}{1+\cos\alpha}\cdot\frac{\frac{1}{\cos\alpha}}{\frac{1}{\cos\alpha}}$
$=\frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{1+\cos\alpha}{\cos\alpha}}$
$=\frac{\tan\alpha}{\frac{1}{\cos\alpha}+\frac{\cos\alpha}{\cos\alpha}}$
$=\frac{\tan\alpha}{\sec\alpha+1}$

64. $\frac{\sin^2\alpha+1-\cos^2\alpha}{\sin\alpha(1+\cos\alpha)}=\frac{\sin^2\alpha+\sin^2\alpha}{\sin\alpha(1+\cos\alpha)}$
$=\frac{2\sin^2\alpha}{\sin\alpha(1+\cos\alpha)}$
$=\frac{2\sin\alpha}{1+\cos\alpha}$
$=2\left(\frac{\sin\alpha}{1+\cos a}\right)$
$=2\tan\frac{\alpha}{2}$

65. $\frac{\sin x}{1-\cos x}=\frac{\sin x}{1-\cos x}\cdot\frac{\frac{1}{\sin x}}{\frac{1}{\sin x}}$
$=\frac{\frac{\sin x}{\sin x}}{\frac{1-\cos x}{\sin x}}$
$=\frac{1}{\tan\frac{x}{2}}$
$=\cot\frac{x}{2}$

66. $\frac{1+\cos x}{\sin x}=\frac{1+\cos x}{\sin x}\cdot\frac{\frac{1}{1+\cos x}}{\frac{1}{1+\cos x}}$
$=\frac{1}{\frac{\sin x}{1+\cos x}}$
$=\frac{1}{\tan\frac{x}{2}}$
$=\cot\frac{x}{2}$

67. $\tan\frac{x}{2}+\cot\frac{x}{2}=\frac{1-\cos x}{\sin x}+\frac{1}{\tan\frac{x}{2}}$
$=\frac{1-\cos x}{\sin x}+\frac{1}{\frac{\sin x}{1+\cos x}}$
$=\frac{1-\cos x}{\sin x}+\frac{1+\cos x}{\sin x}$
$=\frac{1-\cos x+1+\cos x}{\sin x}$
$=\frac{2}{\sin x}=2\csc x$

68. $\tan\frac{x}{2}-\cot\frac{x}{2}=\frac{1-\cos x}{\sin x}-\frac{1}{\tan\frac{x}{2}}$
$=\frac{1-\cos x}{\sin x}-\frac{1}{\frac{\sin x}{1+\cos x}}$
$=\frac{1-\cos x}{\sin x}-\frac{1+\cos x}{\sin x}$
$=\frac{-2\cos x}{\sin x}$
$=-2\cdot\frac{\cos x}{\sin x}$
$=-2\cot x$

69. Conjecture: The left side is equal to $\cos 2x$.

$\frac{\cot x-\tan x}{\cot x+\tan x}=\frac{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}}$
$=\frac{\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}}$
$=\frac{\frac{\cos^2 x}{\sin x\cos x}-\frac{\sin^2 x}{\sin x\cos x}}{\frac{\cos^2 x}{\sin x\cos x}+\frac{\sin^2 x}{\sin x\cos x}}$
$=\frac{\cos^2 x-\sin^2 x}{\cos^2 x+\sin^2 x}$
$=\frac{\cos 2x}{1}$
$=\cos 2x$

70. Conjecture: The left side is equal to $\sin 2x$.

$$\begin{aligned}\frac{2(\tan x-\cot x)}{\tan^2 x-\cot^2 x}&=\frac{2(\tan x-\cot x)}{(\tan x+\cot x)(\tan x-\cot x)}\\&=\frac{2}{\tan x+\cot x}\\&=\frac{2}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}\\&=\frac{2}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}\cdot\frac{\sin x\cos x}{\sin x\cos x}\\&=\frac{2\sin x\cos x}{\sin^2 x+\cos^2 x}\\&=\frac{\sin 2x}{1}\\&=\sin 2x\end{aligned}$$

71. Conjecture: The left side is equal to $\sin x+1$.

$$\begin{aligned}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2&=\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}+\cos^2\frac{x}{2}\\&=2\sin\frac{x}{2}\cos\frac{x}{2}+\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\\&=\left[2\sin\frac{x}{2}\cos\frac{x}{2}\right]+\left[\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right]\\&=\sin\left(2\cdot\frac{x}{2}\right)+1\\&=\sin x+1\end{aligned}$$

72. Conjecture: The left side is equal to $-\cos x$.

$$\begin{aligned}\sin^2\frac{x}{2}-\cos^2\frac{x}{2}&=-\left(-\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right)\\&=-\cos\left(2\cdot\frac{x}{2}\right)\\&=-\cos x\end{aligned}$$

73. Conjecture: The left side is equal to $\sec x$.

$$\begin{aligned}\frac{\sin 2x}{\sin x}-\frac{\cos 2x}{\cos x}&=\frac{2\sin x\cos x}{\sin x}-\frac{2\cos^2 x-1}{\cos x}\\&=2\cos x-\frac{2\cos^2 x}{\cos x}+\frac{1}{\cos x}\\&=2\cos x-2\cos x+\sec x\\&=\sec x\end{aligned}$$

74. Conjecture: The left side is equal to $2\sin x$.

$$\begin{aligned}\sin 2x\sec x&=2\sin x\cos x\cdot\frac{1}{\cos x}\\&=2\sin x\end{aligned}$$

75. Conjecture: The left side is equal to $2\csc 2x$.

$$\begin{aligned}\frac{\csc^2 x}{\cot x}&=\frac{\frac{1}{\sin^2 x}}{\frac{\cos x}{\sin x}}\\&=\frac{1}{\sin^2 x}\frac{\sin x}{\cos x}\\&=\frac{1}{\sin x\cos x}\\&=\frac{2}{2\sin x\cos x}\\&=\frac{2}{\sin 2x}\\&=2\csc 2x\end{aligned}$$

76. Conjecture: The left side is equal to $2\csc 2x$.

$$\begin{aligned}\tan x+\cot x&=\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\\&=\frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}+\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}\\&=\frac{\sin^2 x}{\sin x\cos x}+\frac{\cos^2 x}{\sin x\cos x}\\&=\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\\&=\frac{1}{\sin x\cos x}\\&=\frac{2}{2\sin x\cos x}\\&=\frac{2}{\sin 2x}\\&=2\csc 2x\end{aligned}$$

77. Conjecture: The left side is equal to $\sin 3x$.

$$\begin{aligned}\sin x(4\cos^2 x-1)&=\sin x(2\cos^2 x+2\cos^2 x-1)\\&=\sin x(2\cos^2 x+\cos 2x)\\&=2\sin x\cos^2 x+\sin x\cos 2x\\&=2\sin x\cos x\cos x+\sin x\cos 2x\\&=\sin 2x\cos x+\sin x\cos 2x\\&=\sin(2x+x)\\&=\sin 3x\end{aligned}$$

78. Conjecture: The left side is equal to $\cos 4x$.

$$\begin{aligned}1-8\sin^2 x\cos^2 x &= 1-(2\cdot 2\sin x\cos x\cdot 2\sin x\cos x)\\ &= 1-2\sin 2x\cdot\sin 2x\\ &= 1-2\sin^2 2x\\ &= \cos^2 2x-\sin^2 2x\\ &= \cos 2x\cos 2x-\sin 2x\sin 2x\\ &= \cos(2x+2x)\\ &= \cos 4x\end{aligned}$$

79. **a.** $$\begin{aligned}\frac{v_o^2}{16}\sin\theta\cos\theta &= \frac{v_o^2}{32}\cdot 2\sin\theta\cos\theta\\ &= \frac{v_o^2}{32}\cdot\sin 2\theta\end{aligned}$$

b. $\sin\alpha$ is at a maximum in the interval $[0, 2\pi]$ when $\alpha=\frac{\pi}{2}$, so $\sin 2\theta$ is at a maximum when $2\theta=\frac{\pi}{2}$ or $\theta=\frac{\pi}{4}$.

80. $$\theta=\frac{\pi}{6}$$

$$\begin{aligned}\sin\frac{\theta}{2} &= \sqrt{\frac{1-\cos\theta}{2}}\\ &= \sqrt{\frac{1-\cos\frac{\pi}{6}}{2}}\\ &= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}\\ &= \sqrt{\frac{2-\sqrt{3}}{4}}\\ &= \frac{\sqrt{2-\sqrt{3}}}{2}\end{aligned}$$

$$\sin\frac{\theta}{2}=\frac{1}{M}$$

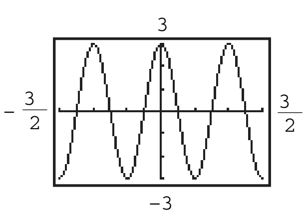
$$\frac{\sqrt{2-\sqrt{3}}}{2}=\frac{1}{M}$$

$$\begin{aligned}M &= \frac{2}{\sqrt{2-\sqrt{3}}}\\ &= \frac{2\sqrt{2-\sqrt{3}}}{2-\sqrt{3}}\\ &= \frac{2\sqrt{2-\sqrt{3}}}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}\\ &= \frac{4\sqrt{2-\sqrt{3}}+2\sqrt{3}\sqrt{2-\sqrt{3}}}{4-3}\\ &= 4\sqrt{2-\sqrt{3}}+2\sqrt{3}\sqrt{2-\sqrt{3}}\approx 3.9\end{aligned}$$

81.
$$\theta = \frac{\pi}{4}$$
$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$$
$$= \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}}$$
$$= \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$$
$$= \sqrt{\frac{2-\sqrt{2}}{4}}$$
$$= \frac{\sqrt{2-\sqrt{2}}}{2}$$
$$\sin\frac{\theta}{2} = \frac{1}{M}$$
$$\frac{\sqrt{2-\sqrt{2}}}{2} = \frac{1}{M}$$
$$M = \frac{2}{\sqrt{2-\sqrt{2}}}$$
$$= \frac{2\sqrt{2-\sqrt{2}}}{2-\sqrt{2}}$$
$$= \frac{2\sqrt{2-\sqrt{2}}}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}}$$
$$= \frac{4\sqrt{2-\sqrt{2}} + 2\sqrt{2}\sqrt{2-\sqrt{2}}}{4-2}$$
$$= \frac{2\left(2\sqrt{2-\sqrt{2}} + \sqrt{2}\sqrt{2-\sqrt{2}}\right)}{2}$$
$$= 2\sqrt{2-\sqrt{2}} + \sqrt{2}\sqrt{2-\sqrt{2}}$$
$$= \sqrt{2-\sqrt{2}} \cdot \left(2+\sqrt{2}\right) \approx 2.6$$

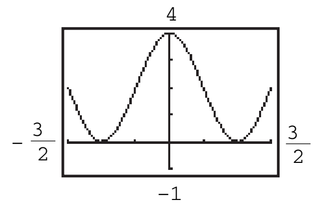
82. – 94. Answers may vary.

95.



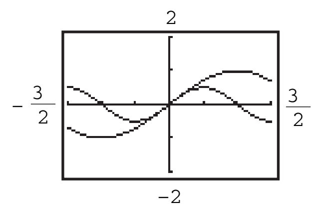
$$3-6\sin^2 x = 3-6\left(\frac{1-\cos 2x}{2}\right)$$
$$= 3-3(1-\cos 2x)$$
$$= 3-3+3\cos 2x$$
$$= 3\cos 2x$$

96.



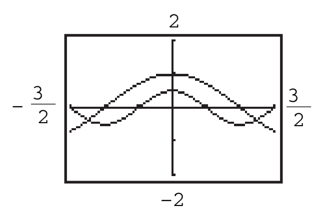
$$4\cos^2\frac{x}{2} = 4\left(\frac{1+\cos 2\left(\frac{x}{2}\right)}{2}\right)$$
$$= 2(1+\cos x)$$
$$= 2+2\cos x$$

97.



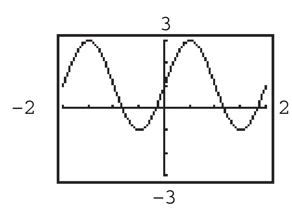
The graphs do not coincide.
Values for x may vary.

98.



The graphs do not coincide.
Values for x may vary.

99.



a. The graph appears to be the sum of 1 and 2 times the sine curve, $y = 1+2\sin x$. If you subtract 1 from the graph, it cycles through intercept, maximum, intercept, minimum, and back to intercept. Thus, $y = 1+2\sin x$ also describes the graph.

b. $\dfrac{1-2\cos 2x}{2\sin x-1}$

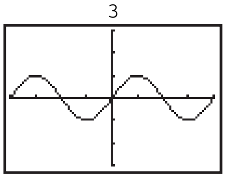
$= \dfrac{1-2\left(1-2\sin^2 x\right)}{2\sin x-1} = \dfrac{1-2+4\sin^2 x}{2\sin x-1}$

$= \dfrac{4\sin^2 x-1}{2\sin x-1} = \dfrac{(2\sin x-1)(2\sin x+1)}{2\sin x-1}$

$= 2\sin x+1 = 1+2\sin x$

This verifies our observation that $y = \dfrac{1-2\cos 2x}{2\sin x-1}$ and $y = 1+2\sin x$ describe the same graph.

100.



a. The graph appears to be the sine curve, y = sin *x*. It cycles through intercept, maximum, intercept, minimum and back to intercept. Thus, $y = \sin x$ also describes the graph.

b. $\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \dfrac{2\left(\dfrac{\sin x}{1+\cos x}\right)}{1+\left(\dfrac{1-\cos 2\left(\frac{x}{2}\right)}{1+\cos 2\left(\frac{x}{2}\right)}\right)}$

$= \dfrac{\dfrac{2\sin x}{1+\cos x}}{\dfrac{1+\cos x}{1+\cos x}+\dfrac{1-\cos x}{1+\cos x}}$

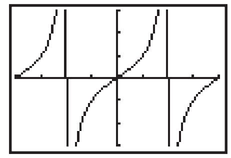
$= \dfrac{\dfrac{2\sin x}{1+\cos x}}{\dfrac{2}{1+\cos x}}$

$= \dfrac{2\sin x}{1+\cos x}\cdot\dfrac{1+\cos x}{2}$

$= \sin x$

This verifies our observation that $y = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and $y = \sin x$ describe the same graph.

101.



a. The graph appears to be the tangent of half the angle. It cycles from negative infinity through intercept to positive infinity. Thus, $y = \tan\dfrac{x}{2}$ also describes the graph.

b. $\tan\dfrac{x}{2} = \dfrac{1-\cos x}{\sin x} = \dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x} = \csc x - \cot x$

This verifies our observation that $y = \csc x - \cot x$ and $y = \tan\dfrac{x}{2}$ describe the same graph.

102. makes sense

103. does not make sense; Explanations will vary. Sample explanation: That procedure is not algebraically sound.

104. does not make sense; Explanations will vary. Sample explanation: An angle and its half-angle do not necessarily lie in the same quadrant.

105. does not make sense; Explanations will vary. Sample explanation: That method will not work well because 200° is not an angle with known trigonometric values.

106. $(\sin x+\cos x)\left(1-\dfrac{\sin 2x}{2}\right)$

$= (\sin x+\cos x)\left(1-\dfrac{2\sin x\cos x}{2}\right)$

$= (\sin x+\cos x)(1-\sin x\cos x)$

$= \sin x+\cos x-\sin^2 x\cos x-\sin x\cos^2 x$

$= \sin x+\cos x-(1-\cos^2 x)\cos x-\sin x\left(1-\sin^2 x\right)$

$= \sin x+\cos x-\cos x+\cos^3 x-\sin x+\sin^3 x$

$= \sin^3 x+\cos^3 x$

107. $\sin\left(2\sin^{-1}\frac{\sqrt{3}}{2}\right)=\sin\left(2\cdot\frac{\pi}{3}\right)$

$$=\sin\frac{2\pi}{3}$$

$$=\frac{\sqrt{3}}{2}$$

108. $\cos\left[2\tan^{-1}\left(-\frac{4}{3}\right)\right]$

Let $\theta=\tan^{-1}\left(-\frac{4}{3}\right)$

Since θ is in quadrant IV, x is positive and y is negative.

$\tan\theta=\frac{y}{x}=\frac{-4}{3}$

$r^2=x^2+y^2$

$r^2=3^2+(-4)^2$

$r^2=25$

$r=5$

$$\cos\left[2\tan^{-1}\left(-\frac{4}{3}\right)\right]=\cos 2\theta$$

$$=1-2\sin^2\theta$$

$$=1-2\left(\frac{y}{r}\right)^2$$

$$=1-2\left(\frac{-4}{5}\right)^2$$

$$=1-2\cdot\frac{16}{25}$$

$$=-\frac{7}{25}$$

109. $\cos^2\left[\frac{1}{2}\sin^{-1}\frac{3}{5}\right]$

Let $\theta=\sin^{-1}\frac{3}{5}$, then $\sin\theta=\frac{y}{r}=\frac{3}{5}$

Since θ is in quadrant I, x is positive.

$x^2+y^2=r^2$

$x^2+3^2=5^2$

$x^2=16$

$x=4$

$$\cos^2\left[\frac{1}{2}\sin^{-1}\frac{3}{5}\right]=\cos^2\left[\frac{1}{2}\theta\right]$$

$$=\frac{1+\cos\left(2\cdot\frac{1}{2}\theta\right)}{2}$$

$$=\frac{1+\cos\theta}{2}$$

$$=\frac{1+\frac{4}{5}}{2}$$

$$=\frac{1+\frac{x}{r}}{2}$$

$$=\frac{9}{10}$$

110. $\sin^2\left[\frac{1}{2}\cos^{-1}\frac{3}{5}\right]$

Let $\theta=\cos^{-1}\frac{3}{5}$, then $\cos\theta=\frac{3}{5}$

$$\sin^2\left[\frac{1}{2}\cos^{-1}\frac{3}{5}\right]=\sin^2\left[\frac{1}{2}\theta\right]$$

$$=\frac{1-\cos\left(2\cdot\frac{1}{2}\theta\right)}{2}$$

$$=\frac{1-\cos\theta}{2}$$

$$=\frac{1-\frac{3}{5}}{2}$$

$$=\frac{1}{5}$$

111 Let $\alpha = \sin^{-1} x$ where $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$.

$\sin\alpha = x$

Because x is positive, sin α is positive. Thus, α is in quadrant I. Using a right triangle in quadrant I with $\sin\alpha = x = \frac{x}{1}$ the third side a can be found using the Pythagorean Theorem.

$\alpha^2 + x^2 = 1^2$

$a^2 = 1 - x^2$

$a = \sqrt{1-x^2}$

$\cos\alpha = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

$\sin(2\sin^{-1} x) = \sin 2\alpha = 2x\sqrt{1-x^2}$

112. $\sin^6 x = \left(\sin^2 x\right)^3 = \left(\frac{1-\cos 2x}{2}\right)^3$

$$= \frac{1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x}{8}$$

$$= \frac{1}{8} - \frac{3}{8}\cos 2x + \frac{3}{8}\cos^2 2x - \frac{1}{8}\cos 2x \cos^2 2x$$

$$= \frac{1}{8} - \frac{3}{8}\cos 2x + \frac{3}{8}\left[\frac{1+\cos(2\cdot 2x)}{2}\right] - \frac{1}{8}\cos 2x\left[\frac{1+\cos(2\cdot 2x)}{2}\right]$$

$$= \frac{1}{8} - \frac{3}{8}\cos 2x + \frac{3}{16}(1+\cos 4x) - \frac{1}{16}\cos 2x(1+\cos 4x)$$

$$= \frac{1}{8} - \frac{3}{8}\cos 2x + \frac{3}{16} + \frac{3}{16}\cos 4x - \frac{1}{16}\cos 2x - \frac{1}{16}\cos 2x\cos 4x$$

$$= \frac{5}{16} - \frac{7}{16}\cos 2x + \frac{3}{16}\cos 4x - \frac{1}{16}\cos 2x\cos 4x$$

113. $\sin 60^\circ \sin 30^\circ = \frac{1}{2}\left[\cos(60^\circ - 30^\circ) - \cos(60^\circ + 30^\circ)\right]$

$$\frac{\sqrt{3}}{2}\cdot\frac{1}{2} = \frac{1}{2}\left[\cos 30^\circ - \cos 90^\circ\right]$$

$$\frac{\sqrt{3}}{4} = \frac{1}{2}\left[\frac{\sqrt{3}}{2} - 0\right]$$

$$\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{4}$$

114. $\cos\frac{\pi}{2}\cos\frac{\pi}{3}=\frac{1}{2}\left[\cos\left(\frac{\pi}{2}-\frac{\pi}{3}\right)+\cos\left(\frac{\pi}{2}+\frac{\pi}{3}\right)\right]$

$$0\cdot\frac{1}{2}=\frac{1}{2}\left[\cos\left(\frac{\pi}{6}\right)+\cos\left(\frac{5\pi}{6}\right)\right]$$
$$0=\frac{1}{2}\left[\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right]$$
$$0=\frac{1}{2}[0]$$
$$0=0$$

115. $\sin\pi\cos\frac{\pi}{2}=\frac{1}{2}\left[\sin\left(\pi+\frac{\pi}{2}\right)+\sin\left(\pi-\frac{\pi}{2}\right)\right]$

$$0\cdot 0=\frac{1}{2}\left[\sin\left(\frac{3\pi}{2}\right)+\sin\left(\frac{\pi}{2}\right)\right]$$
$$0\cdot 0=\frac{1}{2}[-1+1]$$
$$0=\frac{1}{2}[0]$$
$$0=0$$

Mid-Chapter 6 Check Point

1. $\cos x(\tan x+\cot x)$

$$=\cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$$
$$=\cos x\left(\frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}+\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}\right)$$
$$=\cos x\left(\frac{\sin^2 x}{\sin x\cos x}+\frac{\cos^2 x}{\sin x\cos x}\right)$$
$$=\cos x\left(\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right)$$
$$=\cos x\left(\frac{1}{\sin x\cos x}\right)$$
$$=\frac{\cos x}{\sin x\cos x}$$
$$=\frac{1}{\sin x}$$
$$=\csc x$$

2. $\frac{\sin(x+\pi)}{\cos\left(x+\frac{3\pi}{2}\right)}=\frac{-\sin x}{\cos\left((x+\pi)+\frac{\pi}{2}\right)}$

$$=\frac{-\sin x}{-\sin(x+\pi)}$$
$$=\frac{-\sin x}{\sin x}$$
$$=-1$$
$$=-\left(\sec^2 x-\tan^2 x\right)$$
$$=\tan^2 x-\sec^2 x$$

3. $(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2$

$$=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta$$
$$+\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta$$
$$=\sin^2\theta+\cos^2\theta+\sin^2\theta+\cos^2\theta$$
$$=1+1$$
$$=2$$

4. $\frac{\sin t-1}{\cos t}=\frac{\sin t-1}{\cos t}\cdot\frac{\cot t}{\cot t}$

$$=\frac{\sin t\cot t-\cot t}{\cos t\cot t}$$
$$=\frac{\sin t\cdot\frac{\cos t}{\sin t}-\cot t}{\cos t\cot t}$$
$$=\frac{\cos t-\cot t}{\cos t\cot t}$$

5. $\frac{1-\cos 2x}{\sin 2x}=\frac{1-2\cos^2 x-1}{2\sin x\cos x}$

$$=\frac{2\cos^2 x}{2\sin x\cos x}$$
$$=\frac{\cos x}{\sin x}$$
$$=\tan x$$

6. $\sin\theta\cos\theta+\cos^2\theta$
$=\cos\theta(\sin\theta+\cos\theta)$
$=\cos\theta(\sin\theta+\cos\theta)\cdot\frac{\csc\theta}{\csc\theta}$
$=\frac{\cos\theta(\sin\theta\csc\theta+\cos\theta\csc\theta)}{\csc\theta}$
$=\frac{\cos\theta\left(\sin\theta\cdot\frac{1}{\sin\theta}+\cos\theta\cdot\frac{1}{\sin\theta}\right)}{\csc\theta}$
$=\frac{\cos\theta(1+\tan\theta)}{\csc\theta}$

7. $\frac{\sin x}{\tan x}+\frac{\cos x}{\cot x}=\frac{\sin x}{\frac{\sin x}{\cos x}}+\frac{\cos x}{\frac{\cos x}{\sin x}}$
$=\sin x\cdot\frac{\cos x}{\sin x}+\cos x\cdot\frac{\sin x}{\cos x}$
$=\cos x+\sin x$
$=\sin x+\cos x$

8. $\sin^2\frac{t}{2}=\left(\sin\frac{t}{2}\right)^2$
$=\left(\pm\sqrt{\frac{1-\cos t}{2}}\right)^2$
$=\frac{1-\cos t}{2}$
$=\frac{1-\cos t}{2}\cdot\frac{\tan t}{\tan t}$
$=\frac{\tan t-\cos t\tan t}{2\tan t}$
$=\frac{\tan t-\cos t\cdot\frac{\sin t}{\cos t}}{2\tan t}$
$=\frac{\tan t-\sin t}{2\tan t}$

9. $\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$
$=\frac{1}{2}[\sin\alpha\cos\beta+\cos\alpha\sin\beta+$
$\sin\alpha\cos\beta-\cos\alpha\sin\beta]$
$=\frac{1}{2}[2\sin\alpha\cos\beta]$
$=\sin\alpha\cos\beta$

10. $\frac{1+\csc x}{\sec x}-\cot x=\frac{1+\frac{1}{\sin x}}{\frac{1}{\cos x}}-\frac{\cos x}{\sin x}$
$=\cos x\left(1+\frac{1}{\sin x}\right)-\frac{\cos x}{\sin x}$
$=\cos x+\frac{\cos x}{\sin x}-\frac{\cos x}{\sin x}$
$=\cos x$

11. $\frac{\cot x-1}{\cot x+1}=\frac{\frac{\cot x}{\cot x}-\frac{1}{\cot x}}{\frac{\cot x}{\cot x}+\frac{1}{\cot x}}$
$=\frac{1-\tan x}{1+\tan x}$

12. $2\sin^3\theta\cos\theta+2\sin\theta\cos^3\theta$
$=2\sin\theta\cos\theta(\sin^2\theta+\cos^2\theta)$
$=2\sin\theta\cos\theta$
$=\sin 2\theta$

13. $\frac{\sin t+\cos t}{\sec t+\csc t}=\frac{\sin t+\cos t}{\frac{1}{\cos t}+\frac{1}{\sin t}}$
$=\frac{\sin t+\cos t}{\frac{1}{\cos t}\cdot\frac{\sin t}{\sin t}+\frac{1}{\sin t}\cdot\frac{\cos t}{\cos t}}$
$=\frac{\sin t+\cos t}{\frac{\sin t+\cos t}{\sin t\cos t}}$
$=(\sin t+\cos t)\frac{\sin t\cos t}{\sin t+\cos t}$
$=\sin t\cos t$
$=\sin t\frac{1}{\sec t}$
$=\frac{\sin t}{\sec t}$

14. $\frac{\sec^2 x}{2-\sec^2 x}=\frac{\frac{\sec^2 x}{\sec^2 x}}{\frac{2}{\sec^2 x}-\frac{\sec^2 x}{\sec^2 x}}$
$=\frac{1}{2\cos^2 x-1}$
$=\frac{1}{\cos 2x}$
$=\sec 2x$

15. $\tan(\alpha+\beta)\tan(\alpha-\beta)$

$$= \tan(\alpha+\beta)\tan(\alpha-\beta)$$
$$= \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\cdot\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$
$$= \frac{\tan^2\alpha-\tan^2\beta}{1-\tan^2\alpha\tan^2\beta}$$

16. $\frac{\sin\theta}{1-\cos\theta} = \frac{\sin\theta}{1-\cos\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}$

$$= \frac{\sin\theta+\sin\theta\cos\theta}{1-\cos^2\theta}$$
$$= \frac{\sin\theta+\sin\theta\cos\theta}{\sin^2\theta}$$
$$= \frac{\sin\theta}{\sin^2\theta}+\frac{\sin\theta\cos\theta}{\sin^2\theta}$$
$$= \frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}$$
$$= \csc\theta+\cot\theta$$

17. $\frac{1}{\csc 2x} = \sin 2x$

$$= 2\sin x\cos x$$
$$= 2\sin x\cos x\cdot\frac{\cos x}{\cos x}$$
$$= \frac{2\sin x\cos^2 x}{\cos x}$$
$$= \frac{2\sin x}{\cos x}\cdot\cos^2 x$$
$$= 2\tan x\cdot\frac{1}{\sec^2 x}$$
$$= \frac{2\tan x}{\sec^2 x}$$
$$= \frac{2\tan x}{1+\tan^2 x}$$

18. $\frac{\sec t-1}{t\sec t} = \frac{\sec t-1}{t\sec t}\cdot\frac{\cos t}{\cos t}$

$$= \frac{\sec t\cos t-\cos t}{t\sec t\cos t}$$
$$= \frac{\frac{1}{\cos t}\cos t-\cos t}{t\frac{1}{\cos t}\cos t}$$
$$= \frac{1-\cos t}{t}$$

19. Use $\sin\alpha = \frac{3}{5} = \frac{y}{r}$ to find $\cos\alpha$ and $\tan\alpha$.

Because α is in Quadrant II, x is negative.

$$x^2+y^2=r^2$$
$$x^2+3^2=5^2$$
$$x^2=16$$
$$x=-\sqrt{16}$$
$$x=-4$$

Thus, $\cos\alpha = \frac{-4}{5} = -\frac{4}{5}$ and $\tan\alpha = \frac{-3}{4} = -\frac{3}{4}$.

Use $\cos\beta = \frac{-12}{13} = \frac{x}{r}$ to find $\sin\beta$ and $\tan\beta$.

Because β is in Quadrant III, x and y are negative.

$$x^2+y^2=r^2$$
$$(-12)^2+y^2=13^2$$
$$y^2=25$$
$$y=-\sqrt{25}$$
$$y=-5$$

Thus, $\sin\beta = \frac{-5}{13} = -\frac{5}{13}$ and $\tan\beta = \frac{-5}{-12} = \frac{5}{12}$.

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta+\sin\alpha\sin\beta$$
$$= \left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right)+\left(\frac{3}{5}\right)\left(-\frac{5}{13}\right)$$
$$= \frac{33}{65}$$

20. In exercise 19 it was shown that

$\tan\alpha = -\frac{3}{4}$ and $\tan\beta = \frac{5}{12}$. Thus,

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$= \frac{-\frac{3}{4}+\frac{5}{12}}{1-\left(-\frac{3}{4}\right)\frac{5}{12}}$$
$$= -\frac{16}{63}$$

21. In exercise 19 it was shown that

$\sin\alpha = \frac{3}{5}$ and $\cos\alpha = -\frac{4}{5}$.

Thus, $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$= 2\cdot\frac{3}{5}\left(-\frac{4}{5}\right)$$

$$= -\frac{24}{25}$$

22. $\cos\beta = -\frac{12}{13}$.

Since β is in quadrant III, $\frac{\beta}{2}$ is in quadrant II.

The cosine is negative in quadrant II.

$$\cos\frac{\beta}{2} = -\sqrt{\frac{1+\cos\beta}{2}}$$

$$= -\sqrt{\frac{1+\left(-\frac{12}{13}\right)}{2}}$$

$$= -\sqrt{\frac{1}{26}}$$

$$= -\frac{1}{\sqrt{26}}$$

$$= -\frac{\sqrt{26}}{26}$$

23. $\sin\left(\frac{3\pi}{4}+\frac{5\pi}{6}\right)$

$$= \sin\frac{3\pi}{4}\cos\frac{5\pi}{6} + \cos\frac{3\pi}{4}\sin\frac{5\pi}{6}$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= -\frac{\sqrt{6}+\sqrt{2}}{4}$$

24. $\cos^2 15° - \sin^2 15° = \cos(2\cdot 15°)$

$$= \cos 30°$$

$$= \frac{\sqrt{3}}{2}$$

25. $\cos\frac{5\pi}{12}\cos\frac{\pi}{12} + \sin\frac{5\pi}{12}\sin\frac{\pi}{12} = \cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$

$$= \cos\frac{4\pi}{12}$$

$$= \cos\frac{\pi}{3}$$

$$= \frac{1}{2}$$

26. $\tan 22.5° = \tan\frac{45°}{2}$

$$= \frac{\sin 45°}{1+\cos 45°}$$

$$= \frac{\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}$$

$$= \frac{\sqrt{2}}{2+\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2+\sqrt{2}}\cdot\frac{2-\sqrt{2}}{2-\sqrt{2}}$$

$$= \frac{2\sqrt{2}-2}{4-2}$$

$$= \frac{2\sqrt{2}-2}{2}$$

$$= \sqrt{2}-1$$

Section 6.4

Check Point Exercises

1. **a.** $\sin 5x\sin 2x$

$$= \frac{1}{2}[\cos(5x-2x) - \cos(5x+2x)]$$

$$= \frac{1}{2}[\cos 3x - \cos 7x]$$

b. $\cos 7x\cos x$

$$= \frac{1}{2}[\cos(7x-x) + \cos(7x+x)]$$

$$= \frac{1}{2}[\cos 6x + \cos 8x]$$

2. **a.**
$$\begin{aligned}&\sin 7x+\sin 3x\\&=2\sin\left(\frac{7x+3x}{2}\right)\cos\left(\frac{7x-3x}{2}\right)\\&=2\sin\left(\frac{10x}{2}\right)\cos\left(\frac{4x}{2}\right)\\&=2\sin 5x\cos 2x\end{aligned}$$

b.
$$\begin{aligned}&\cos 3x+\cos 2x\\&=2\cos\left(\frac{3x+2x}{2}\right)\cos\left(\frac{3x-2x}{2}\right)\\&=2\cos\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right)\end{aligned}$$

3.
$$\begin{aligned}\frac{\cos 3x-\cos x}{\sin 3x+\sin x}&=\frac{-2\sin\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-2x}{2}\right)}{2\sin\frac{3x+x}{2}\cos\left(\frac{3x-x}{2}\right)}\\&=\frac{-2\sin 2x\sin x}{2\sin 2x\cos x}\\&=\frac{-\sin x}{\cos x}\\&=-\tan x\end{aligned}$$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

Exercise Set 6.4

1. The given formula can be used to change a product of two sines into the difference of two cosine expressions.

2. The given formula can be used to change a product of two cosines into the sum of two cosine expressions.

3. The given formula can be used to change a product of a sine and a cosine into the sum of two sine expressions.

4. The given formula can be used to change a product of a cosine and a sine into the difference of two sine expressions.

5.
$$\begin{aligned}\sin 6x\sin 2x&=\frac{1}{2}\left[\cos(6x-2x)-\cos(6x+2x)\right]\\&=\frac{1}{2}\left[\cos 4x-\cos 8x\right]\end{aligned}$$

6.
$$\begin{aligned}\sin 8x\sin 4x&=\frac{1}{2}[\cos(8x-4x)-\cos(8x+4x)]\\&=\frac{1}{2}(\cos 4x-\cos 12x)\end{aligned}$$

7.
$$\begin{aligned}\cos 7x\cos 3x&=\frac{1}{2}\left[\cos(7x-3x)+\cos(7x+3x)\right]\\&=\frac{1}{2}\left[\cos 4x+\cos 10x\right]\end{aligned}$$

8.
$$\begin{aligned}\cos 9x\cos 2x&=\frac{1}{2}[\cos(9x-2x)+\cos(9x+2x)]\\&=\frac{1}{2}[\cos 7x+\cos 11x]\end{aligned}$$

9.
$$\begin{aligned}\sin x\cos 2x&=\frac{1}{2}\left[\sin(x+2x)+\sin(x-2x)\right]\\&=\frac{1}{2}\left[\sin 3x+\sin(-x)\right]\\&=\frac{1}{2}\left[\sin 3x-\sin x\right]\end{aligned}$$

10.
$$\begin{aligned}\sin 2x\cos 3x&=\frac{1}{2}[\cos(2x+3x)+\sin(2x-3x)]\\&=\frac{1}{2}[\cos 5x+\sin(-x)]\\&=\frac{1}{2}(\cos 5x-\sin x)\end{aligned}$$

11.
$$\begin{aligned}\cos\frac{3x}{2}\sin\frac{x}{2}&=\frac{1}{2}\left[\sin\left(\frac{3x}{2}+\frac{x}{2}\right)-\sin\left(\frac{3x}{2}-\frac{x}{2}\right)\right]\\&=\frac{1}{2}\left[\sin\left(\frac{4x}{2}\right)-\sin\left(\frac{2x}{2}\right)\right]\\&=\frac{1}{2}\left[\sin 2x-\sin x\right]\end{aligned}$$

12.
$$\begin{aligned}\cos\frac{5x}{2}\sin\frac{x}{2}&=\frac{1}{2}\left[\sin\left(\frac{5x}{2}+\frac{x}{2}\right)-\sin\left(\frac{5x}{2}-\frac{x}{2}\right)\right]\\&=\frac{1}{2}\left[\sin\left(\frac{6x}{2}\right)-\sin\left(\frac{4x}{2}\right)\right]\\&=\frac{1}{2}\left[\sin 3x-\sin 2x\right]\end{aligned}$$

13. The given formula can be used to change a sum of two sines into the product of a sine and a cosine expression.

14. The given formula can be used to change a difference of two sines into the product of a sine and a cosine expression.

15. The given formula can be used to change a sum of two cosines into the product of two cosine the expressions.

16. The given formula can be used to change a difference of two cosines into the product of two sine the expressions.

17. $$\sin 6x + \sin 2x = 2\sin\left(\frac{6x+2x}{2}\right)\cos\left(\frac{6x-2x}{2}\right) = 2\sin\left(\frac{8x}{2}\right)\cos\left(\frac{4x}{2}\right) = 2\sin 4x\cos 2x$$

18. $$\sin 8x + \sin 2x = 2\sin\left(\frac{8x+2x}{2}\right)\cos\left(\frac{8x-2x}{2}\right) = 2\sin\left(\frac{10x}{2}\right)\cos\left(\frac{6x}{2}\right) = 2\sin 5x\cos 3x$$

19. $$\sin 7x - \sin 3x = 2\sin\left(\frac{7x-3x}{2}\right)\cos\left(\frac{7x+3x}{2}\right) = 2\sin\left(\frac{4x}{2}\right)\cos\left(\frac{10x}{2}\right) = 2\sin 2x\cos 5x$$

20. $$\sin 11x - \sin 5x = 2\sin\left(\frac{11x-5x}{2}\right)\cos\left(\frac{11x+5x}{2}\right) = 2\sin\left(\frac{6x}{2}\right)\cos\left(\frac{16x}{2}\right) = 2\sin 3x\cos 8x$$

21. $$\cos 4x + \cos 2x = 2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) = 2\cos\left(\frac{6x}{2}\right)\cos\left(\frac{2x}{2}\right) = 2\cos 3x\cos x$$

22. $$\cos 9x - \cos 7x = -2\sin\left(\frac{9x+7x}{2}\right)\sin\left(\frac{9x-7x}{2}\right) = -2\sin\left(\frac{16x}{2}\right)\sin\left(\frac{2x}{2}\right) = -2\sin 8x\sin x$$

23. $$\sin x + \sin 2x = 2\sin\left(\frac{x+2x}{2}\right)\cos\left(\frac{x-2x}{2}\right) = 2\sin\left(\frac{3x}{2}\right)\cos\left(\frac{-x}{2}\right) = 2\sin\frac{3x}{2}\cos\frac{x}{2}$$

24. $$\sin x - \sin 2x = 2\sin\left(\frac{x-2x}{2}\right)\cos\left(\frac{x+2x}{2}\right) = 2\sin\left(\frac{-x}{2}\right)\cos\left(\frac{3x}{2}\right) = -2\sin\frac{x}{2}\cos\frac{3x}{2}$$

25. $$\cos\frac{3x}{2} + \cos\frac{x}{2} = 2\cos\left(\frac{\frac{3x}{2}+\frac{x}{2}}{2}\right)\cos\left(\frac{\frac{3x}{2}-\frac{x}{2}}{2}\right) = 2\cos\left(\frac{4x}{4}\right)\cos\left(\frac{2x}{4}\right) = 2\cos x\cos\frac{x}{2}$$

26. $$\sin\frac{3x}{2} + \sin\frac{x}{2} = 2\sin\left(\frac{\frac{3x}{2}+\frac{x}{2}}{2}\right)\cos\left(\frac{\frac{3x}{2}-\frac{x}{2}}{2}\right) = 2\sin\left(\frac{4x}{4}\right)\cos\left(\frac{2x}{4}\right) = 2\sin x\cos\frac{x}{2}$$

27. $$\sin 75° + \sin 15° = 2\sin\left(\frac{75°+15°}{2}\right)\cos\left(\frac{75°-15°}{2}\right) = 2\sin(45°)\cos(30°) = 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6}}{2}$$

28. $\cos 75° - \cos 15°$

$= -2\sin\left(\frac{75°+15°}{2}\right)\sin\left(\frac{75°-15°}{2}\right)$

$= -2\sin(45°)\sin(30°)$

$= -2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = -\frac{\sqrt{2}}{2}$

29. $\sin\frac{\pi}{12} - \sin\frac{5\pi}{12}$

$= 2\sin\left(\frac{\frac{\pi}{12}-\frac{5\pi}{12}}{2}\right)\cos\left(\frac{\frac{\pi}{12}+\frac{5\pi}{12}}{2}\right)$

$= 2\sin\left(-\frac{4\pi}{24}\right)\cos\left(\frac{6\pi}{24}\right)$

$= -2\sin\frac{\pi}{6}\cos\frac{\pi}{4}$

$= -2\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{2}}{2}$

30. $\cos\frac{\pi}{12} - \cos\frac{5\pi}{12}$

$= -2\sin\left(\frac{\frac{\pi}{12}+\frac{5\pi}{12}}{2}\right)\sin\left(\frac{\frac{\pi}{12}-\frac{5\pi}{12}}{2}\right)$

$= -2\sin\left(\frac{6\pi}{24}\right)\sin\left(-\frac{4\pi}{24}\right)$

$= 2\sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}}{2}$

31. $\frac{\sin 3x - \sin x}{\cos 3x - \cos x}$

$= \frac{2\sin\left(\frac{3x-x}{2}\right)\cos\left(\frac{3x+x}{2}\right)}{-2\sin\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}$

$= \frac{2\sin\left(\frac{2x}{2}\right)\cos\left(\frac{4x}{2}\right)}{-2\sin\left(\frac{4x}{2}\right)\sin\left(\frac{2x}{2}\right)}$

$= \frac{2\sin x\cos 2x}{-2\sin 2x\sin x}$

$= -\frac{\cos 2x}{\sin 2x} = -\cot 2x$

32. $\frac{\sin x + \sin 3x}{\cos x + \cos 3x}$

$= \frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$

$= \frac{2\sin\left(\frac{4x}{2}\right)\cos\left(\frac{-2x}{2}\right)}{2\cos\left(\frac{4x}{2}\right)\cos\left(\frac{-2x}{2}\right)}$

$= \frac{2\sin 2x\cos(-x)}{2\cos 2x\cos(-x)}$

$= \frac{\sin 2x}{\cos 2x} = \tan 2x$

33. $\frac{\sin 2x + \sin 4x}{\cos 2x + \cos 4x}$

$= \frac{2\sin\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right)}{2\cos\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right)}$

$= \frac{2\sin\left(\frac{6x}{2}\right)\cos\left(\frac{-2x}{2}\right)}{2\cos\left(\frac{6x}{2}\right)\cos\left(\frac{-2x}{2}\right)}$

$= \frac{2\sin 3x\cos(-x)}{2\cos 3x\cos(-x)}$

$= \frac{\sin 3x}{\cos 3x}$

$= \tan 3x$

34. $\dfrac{\cos 4x - \cos 2x}{\sin 2x - \sin 4x}$

$= \dfrac{-2\sin\left(\dfrac{4x+2x}{2}\right)\sin\left(\dfrac{4x-2x}{2}\right)}{2\sin\left(\dfrac{2x-4x}{2}\right)\cos\left(\dfrac{2x+4x}{2}\right)}$

$= \dfrac{-2\sin\left(\dfrac{6x}{2}\right)\sin\left(\dfrac{2x}{2}\right)}{2\sin\left(\dfrac{-2x}{2}\right)\cos\left(\dfrac{6x}{2}\right)}$

$= \dfrac{-2\sin 3x \sin x}{2\sin(-x)\cos 3x}$

$= \dfrac{-\sin x \sin 3x}{-\sin x \cos 3x}$

$= \dfrac{\sin 3x}{\cos 3x}$

$= \tan 3x$

35. $\dfrac{\sin x - \sin y}{\sin x + \sin y} = \dfrac{2\sin\left(\dfrac{x-y}{2}\right)\cos\left(\dfrac{x+y}{2}\right)}{2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)}$

$= \dfrac{\sin\left(\dfrac{x-y}{2}\right)}{\cos\left(\dfrac{x-y}{2}\right)} \cdot \dfrac{\cos\left(\dfrac{x+y}{2}\right)}{\sin\left(\dfrac{x+y}{2}\right)}$

$= \tan\dfrac{x-y}{2}\cot\dfrac{x+y}{2}$

36. $\dfrac{\sin x + \sin y}{\sin x - \sin y} = \dfrac{2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)}{2\sin\left(\dfrac{x-y}{2}\right)\cos\left(\dfrac{x+y}{2}\right)}$

$= \dfrac{\sin\left(\dfrac{x+y}{2}\right)}{\cos\left(\dfrac{x+y}{2}\right)} \cdot \dfrac{\cos\left(\dfrac{x-y}{2}\right)}{\sin\left(\dfrac{x-y}{2}\right)}$

$= \tan\dfrac{x+y}{2}\cot\dfrac{x-y}{2}$

37. $\dfrac{\sin x + \sin y}{\cos x + \cos y} = \dfrac{2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)}{2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)}$

$= \dfrac{\sin\left(\dfrac{x+y}{2}\right)}{\cos\left(\dfrac{x+y}{2}\right)}$

$= \tan\dfrac{x+y}{2}$

38. $\dfrac{\sin x - \sin y}{\cos x - \cos y} = \dfrac{2\sin\left(\dfrac{x-y}{2}\right)\cos\left(\dfrac{x+y}{2}\right)}{-2\sin\left(\dfrac{x+y}{2}\right)\sin\left(\dfrac{x-y}{2}\right)}$

$= -\dfrac{\sin\left(\dfrac{x-y}{2}\right)}{\sin\left(\dfrac{x-y}{2}\right)} \cdot \dfrac{\cos\left(\dfrac{x+y}{2}\right)}{\sin\left(\dfrac{x+y}{2}\right)}$

$= -\cot\dfrac{x+y}{2}$

39. **a.** $y = \cos x$ also describes the graph.

b. $$\frac{\sin x + \sin 3x}{2\sin 2x} = \frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\sin 2x} = \frac{2\sin\left(\frac{4x}{2}\right)\cos\left(\frac{-2x}{2}\right)}{2\sin 2x}$$
$$= \frac{2\sin 2x\cos(-x)}{2\sin 2x} = \cos(-x) = \cos x$$

40. **a.** $y = \tan x$ also describes the graph.

b. $$\frac{\cos x - \cos 3x}{\sin x + \sin 3x} = \frac{-2\sin\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$
$$= \frac{-2\sin 3x\sin(-x)}{2\sin 3x\cos(-x)}$$
$$= \frac{-\sin(-x)}{\cos x}$$
$$= \frac{\sin x}{\cos x}$$
$$= \tan x$$

41. **a.** $y = \tan 2x$ also describes the graph.

b. $$\frac{\cos x - \cos 5x}{\sin x + \sin 5x} = \frac{-2\sin\left(\frac{x+5x}{2}\right)\sin\left(\frac{x-5x}{2}\right)}{2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)} = \frac{-2\sin\left(\frac{6x}{2}\right)\sin\left(\frac{-4x}{2}\right)}{2\sin\left(\frac{6x}{2}\right)\cos\left(\frac{-4x}{2}\right)}$$
$$= \frac{-2\sin 3x\sin(-2x)}{2\sin 3x\cos(-2x)} = \frac{-\sin(-2x)}{\cos 2x} = \frac{\sin 2x}{\cos 2x} = \tan 2x$$

42. **a.** $y = -\tan x$ also describes the graph.

b. $$\frac{\cos 5x - \cos 3x}{\sin 5x + \sin 3x} = \frac{-2\sin\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)}{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$$
$$= \frac{-2\sin 4x\sin x}{2\sin 4x\cos x}$$
$$= \frac{-\sin x}{\cos x}$$
$$= -\tan x$$

43. a. $y = -\cot 2x$ also describes the graph.

b. $$\frac{\sin x - \sin 3x}{\cos x - \cos 3x} = \frac{2\sin\left(\frac{x-3x}{2}\right)\cos\left(\frac{x+3x}{2}\right)}{-2\sin\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}$$
$$= \frac{2\sin(-x)\cos 2x}{-2\sin 2x \sin(-x)}$$
$$= \frac{\cos 2x}{-\sin 2x}$$
$$= -\cot 2x$$

44. a. $y = -\cot 2x$ also describes the graph.

b. $$\frac{\sin 2x + \sin 6x}{\cos 6x - \cos 2x} = \frac{2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)}{-2\sin\left(\frac{6x+2x}{2}\right)\sin\left(\frac{6x-2x}{2}\right)}$$
$$= \frac{2\sin 4x \cos(-2x)}{-2\sin 4x \sin 2x}$$
$$= \frac{\cos 2x}{-\sin 2x}$$
$$= -\cot 2x$$

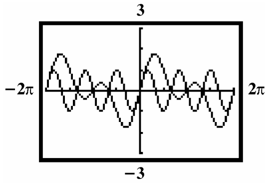
45. a. The low frequency is l = 852 cycles per second and the high frequency is h = 1209 cycles per second. The sound produced by touching 7 is described by $y = \sin 2\pi(852)t + \sin 2\pi(1209)t$, or $y = \sin 1704\pi t + \sin 2418\pi t$.

b. $$y = \sin 1704\pi t + \sin 2418\pi t$$
$$= 2\sin\left(\frac{1704\pi t + 2418\pi t}{2}\right)\cdot\cos\left(\frac{1704\pi t - 2418\pi t}{2}\right)$$
$$= 2\sin 2061\pi t \cdot \cos(-357\pi t)$$
$$= 2\sin 2061\pi t \cdot \cos 357\pi t$$

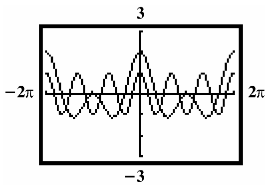
46. a. The low frequency is l = 697 cycles per second and the high frequency is h = 1477 cycles per second. The sound produced by touching 3 is described by $y = \sin 2\pi(697)t + \sin 2\pi(1477)t$, or $y = \sin 1394\pi t + \sin 2954\pi t$.

b. $$y = \sin 2954\pi t + \sin 1394\pi t$$
$$= 2\sin\left(\frac{2954\pi t + 1394\pi t}{2}\right)\cdot\cos\left(\frac{2954\pi t - 1394\pi t}{2}\right)$$
$$= 2\sin 2174\pi t \cdot \cos 780\pi t$$

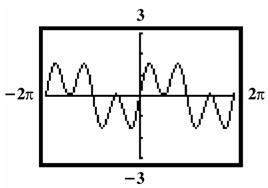
47. – 52. Answers may vary.

53. 

The graphs do not coincide.
Values for x may vary.

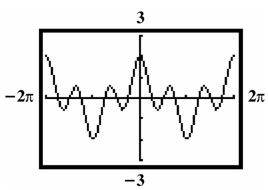
54. 

The graphs do not coincide. Values for x may vary.

55. 

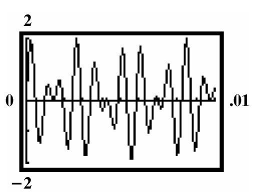
$$\sin x + \sin 3x = 2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)$$
$$= 2\sin 2x\cos(-x)$$
$$= 2\sin 2x\cos x$$

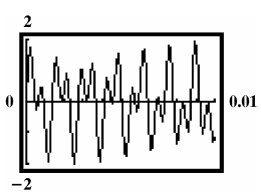
We worked with the left side and arrived at the right side. Thus, the identity is verified.

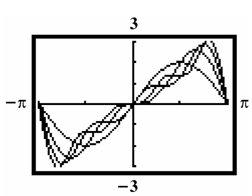
56. 

$$\cos x + \cos 3x = 2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)$$
$$= 2\cos 2x\cos(-x)$$
$$= 2\cos 2x\cos x$$

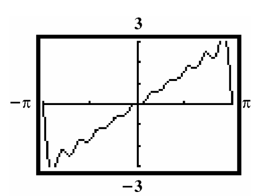
We worked with the left side and arrived at the right side. Thus, the identity is verified.

57. 

58. 

59. a. 

Answers may vary.

b. 

Answers may vary.

c. When $x = \frac{\pi}{2}$,

$$\frac{\pi}{2} = 2\left(\frac{\sin\frac{\pi}{2}}{1} - \frac{\sin\left(2\cdot\frac{\pi}{2}\right)}{2} + \frac{\sin\left(3\cdot\frac{\pi}{2}\right)}{3} - \frac{\sin\left(4\cdot\frac{\pi}{2}\right)}{4} + \frac{\sin\left(5\cdot\frac{\pi}{2}\right)}{5} - \frac{\sin\left(6\cdot\frac{\pi}{2}\right)}{6} + \frac{\sin\left(7\cdot\frac{\pi}{2}\right)}{7} - \frac{\sin\left(8\cdot\frac{\pi}{2}\right)}{8} + \cdots\right)$$

$$= 2\left(1 - 0 + \left(-\frac{1}{3}\right) - 0 + \frac{1}{5} - 0 + \left(-\frac{1}{7}\right) + \cdots\right)$$

$$= 2 - \frac{2}{3} + \frac{2}{5} - \frac{2}{7} + \cdots$$

Multiplying both sides by 2 gives: $\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \cdots$

60. makes sense

61. makes sense

62. makes sense

63. makes sense

64.

$$\begin{aligned} \sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta \\ +\ [\sin(\alpha-\beta) &= \sin\alpha\cos\beta - \cos\alpha\sin\beta] \\ \hline \sin(\alpha+\beta) + \sin(\alpha-\beta) &= 2\sin\alpha\cos\beta \end{aligned}$$

Solve for $\sin\alpha\cos\beta$ by multiplying both sides by $\frac{1}{2}$: $\frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right] = \sin\alpha\cos\beta$

65.

$$\begin{aligned} \sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta \\ -[\sin(\alpha-\beta) &= \sin\alpha\cos\beta - \cos\alpha\sin\beta] \\ \hline \sin(\alpha+\beta) - \sin(\alpha-\beta) &= 2\cos\alpha\sin\beta \end{aligned}$$

Solve for $\cos\alpha\sin\beta$ by multiplying both sides by $\frac{1}{2}$: $\frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right] = \cos\alpha\sin\beta$

66.

$$\begin{aligned} 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2} &= 2\cdot\frac{1}{2}\left[\sin\left(\frac{\alpha-\beta}{2} + \frac{\alpha+\beta}{2}\right) + \sin\left(\frac{\alpha-\beta}{2} - \frac{\alpha+\beta}{2}\right)\right] \\ &= \sin\left(\frac{2\alpha}{2}\right) + \sin\left(\frac{-2\beta}{2}\right) \\ &= \sin\alpha + \sin(-\beta) \\ &= \sin\alpha - \sin\beta \end{aligned}$$

67.

$$\begin{aligned} 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} &= 2\cdot\frac{1}{2}\left[\cos\left(\frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2}\right) + \cos\left(\frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2}\right)\right] \\ &= \cos\left(\frac{2\beta}{2}\right) + \cos\left(\frac{2\alpha}{2}\right) \\ &= \cos\beta + \cos\alpha \\ &= \cos\alpha + \cos\beta \end{aligned}$$

68. $$\frac{\sin 2x+(\sin 3x+\sin x)}{\cos 2x+(\cos 3x+\cos x)}=\frac{\sin 2x+2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)}{\cos 2x+2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)}$$
$$=\frac{\sin 2x+2\sin\left(\frac{4x}{2}\right)\cos\left(\frac{2x}{2}\right)}{\cos 2x+2\cos\left(\frac{4x}{2}\right)\cos\left(\frac{2x}{2}\right)}$$
$$=\frac{\sin 2x+2\sin 2x\cos x}{\cos 2x+2\cos 2x\cos x}$$
$$=\frac{\sin 2x(1+2\cos x)}{\cos 2x(1+2\cos x)}$$
$$=\frac{\sin 2x}{\cos 2x}$$
$$=\tan 2x$$

69. $$\sin 2x+\sin 4x+\sin 6x=\sin 4x+(\sin 2x+\sin 6x)$$
$$=\sin 4x+2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)$$
$$=\sin 4x+2\sin\left(\frac{8x}{2}\right)\cos\left(\frac{-4x}{2}\right)$$
$$=\sin 4x+2\sin 4x\cos(-2x)$$
$$=\sin 4x+2\sin 4x\cos 2x$$
$$=\sin(2\cdot 2x)+2\sin 4x\cos 2x$$
$$=2\sin 2x\cos 2x+2\sin 4x\cos 2x$$
$$=2\cos 2x(\sin 2x+\sin 4x)$$
$$=2\cos 2x\left(2\sin\left(\frac{2x+4x}{2}\right)\cos\left(\frac{2x-4x}{2}\right)\right)$$
$$=2\cos 2x\cdot 2\sin\left(\frac{6x}{2}\right)\cos\left(\frac{-2x}{2}\right)$$
$$=2\cos 2x\cdot 2\sin 3x\cos(-x)$$
$$=4\cos 2x\sin 3x\cos x$$
$$=4\cos x\cos 2x\sin 3x$$

70. Answers may vary.

71. $2(1-u^2)+3u=0$

$2-2u^2+3u=0$

$2u^2-3u-2=0$

$(2u+1)(u-2)=0$

$2u+1=0 \quad \text{and} \quad u-2=0$

$2u=-1 \qquad u=2$

$u=-\frac{1}{2}$

The solution set is $\left\{-\frac{1}{2}, 2\right\}$.

72. $u^3-3u=0$

$u(u^2-3)=0$

$u=0 \quad \text{or} \quad u^2-3=0$

$u^2=3$

$u=\pm\sqrt{3}$

The solution set is $\left\{-\sqrt{3}, 0, \sqrt{3}\right\}$.

73. $u^2-u-1=0$

$a=1, \quad b=-1, \quad c=-1$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-1)}}{2(1)}$

$x=\frac{1\pm\sqrt{5}}{2}$

The solution set is $\left\{\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right\}$.

Section 6.5

Check Point Exercises

1. $5\sin x=3\sin x+\sqrt{3}$

$5\sin x-3\sin x=3\sin x-3\sin x+\sqrt{3}$

$2\sin x=\sqrt{3}$

$\sin x=\frac{\sqrt{3}}{2}$

Because $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$, the solutions for $\sin x=\frac{\sqrt{3}}{2}$ in $[0, 2\pi)$ are

$x=\frac{\pi}{3}$

$x=\pi-\frac{\pi}{3}=\frac{3\pi}{3}-\frac{\pi}{3}=\frac{2\pi}{3}.$

Because the period of the sine function is 2π, the solutions are given by

$x=\frac{\pi}{3}+2n\pi \quad \text{or}$

$x=\frac{2\pi}{3}+2n\pi$

where n is any integer.

2. The period of the tangent function is π. In the interval $[0, \pi)$, the only value for which the tangent function is $\sqrt{3}$ is $\frac{\pi}{3}$. All the solutions to $\tan 2x=\sqrt{3}$ are given by

$2x=\frac{\pi}{3}+n\pi$

$x=\frac{\pi}{6}+\frac{n\pi}{2}$

where n is any integer. In the interval $[0, 2\pi)$, we obtain solutions as follows:

Let $n=0$. $x=\frac{\pi}{6}+\frac{0\pi}{2}$

$=\frac{\pi}{6}$

Let $n=1$. $x=\frac{\pi}{6}+\frac{1\pi}{2}$

$=\frac{\pi}{6}+\frac{3\pi}{6}=\frac{2\pi}{3}$

Let $n=2$. $x=\frac{\pi}{6}+\frac{2\pi}{2}$

$=\frac{\pi}{6}+\frac{6\pi}{6}=\frac{7\pi}{6}$

Let $n=3$. $x=\frac{\pi}{6}+\frac{3\pi}{2}$

$=\frac{\pi}{6}+\frac{9\pi}{6}=\frac{5\pi}{3}$

In the interval $[0, 2\pi)$, the solutions are $\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6},$ and $\frac{5\pi}{3}$.

3. The period of the sine function is 2π.
In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{1}{2}$. One is $\frac{\pi}{6}$. The sine is positive in quadrant II. Thus, the other value is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$. All the solutions to $\sin\frac{x}{3} = \frac{1}{2}$ are given by

$$\frac{x}{3} = \frac{\pi}{6} + 2n\pi$$
$$x = \frac{\pi}{2} + 6n\pi$$

or

$$\frac{x}{3} = \frac{5\pi}{6} + 2n\pi$$
$$x = \frac{5\pi}{2} + 6n\pi$$

where n is any integer. In the interval $[0, 2\pi)$, we obtain solutions as follows:

Let $n = 0$. $x = \frac{\pi}{2}$ or $x = \frac{5\pi}{2}$

If we let $n = 1$, we are adding 6π to each of these expressions. These values of x exceed 2π. Thus in the interval $[0, 2\pi)$, the solution set is $\left\{\frac{\pi}{2}, \frac{5\pi}{2}\right\}$

4. The given equation is in quadratic form
$2t^2 - 3t + 1 = 0$ with $t = \sin x$.

$$2\sin^2 x - 3\sin x + 1 = 0$$
$$(2\sin x - 1)(\sin x - 1) = 0$$
$$2\sin x - 1 = 0 \quad \text{or} \quad \sin x - 1 = 0$$
$$2\sin x = 1 \qquad \sin x = 1$$
$$\sin x = \frac{1}{2}$$
$$x = \frac{\pi}{6} \qquad x = \frac{\pi}{2}$$
$$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{\pi}{2}$, and $\frac{5\pi}{6}$.

5.
$$4\cos^2 x - 3 = 0$$
$$\cos^2 x = \frac{3}{4}$$
$$\cos x = \pm\sqrt{\frac{3}{4}}$$
$$\cos x = \pm\frac{\sqrt{3}}{2}$$
$$\cos x = \frac{\sqrt{3}}{2} \quad \text{or} \quad \cos x = -\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{6}, \frac{11\pi}{6} \qquad x = \frac{5\pi}{6}, \frac{7\pi}{6}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

6.
$$\sin x \tan x = \sin x$$
$$\sin x \tan x - \sin x = 0$$
$$\sin x(\tan x - 1) = 0$$
$$\sin x = 0 \quad \text{or} \quad \tan x - 1 = 0$$
$$x = 0 \quad x = \pi \qquad \tan x = 1$$
$$x = \frac{\pi}{4}$$
$$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

The solutions in the interval $[0, 2\pi)$ are $0, \frac{\pi}{4}, \pi$, and $\frac{5\pi}{4}$.

7.
$$2\sin^2 x - 3\cos x = 0$$
$$2\left(1 - \cos^2 x\right) - 3\cos x = 0$$
$$2 - 2\cos^2 x - 3\cos x = 0$$
$$-2\cos^2 x - 3\cos x + 2 = 0$$
$$2\cos^2 x + 3\cos x - 2 = 0$$
$$(2\cos x - 1)(\cos x + 2) = 0$$
$$2\cos x - 1 = 0 \quad \text{or} \quad \cos x + 2 = 0$$
$$\cos x = \frac{1}{2} \qquad \cos x = -2$$
$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

~~$\cos x = -2$~~ This equation has no solution.

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

8.
$$\cos 2x + \sin x = 0$$
$$1 - 2\sin^2 x + \sin x = 0$$
$$-2\sin^2 x + \sin x + 1 = 0$$
$$2\sin^2 x - \sin x - 1 = 0$$
$$(2\sin x + 1)(\sin x - 1) = 0$$
$$2\sin x + 1 = 0 \quad \text{or} \quad \sin x - 1 = 0$$
$$2\sin x = -1 \qquad \sin x = 1$$
$$\sin x = -\frac{1}{2} \qquad x = \frac{\pi}{2}$$
$$x = \pi + \frac{\pi}{6} = \frac{7\pi}{6} \text{ or}$$
$$x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{2}, \frac{7\pi}{6},$ and $\frac{11\pi}{6}.$

9.
$$\sin x \cos x = -\frac{1}{2}$$
$$2\sin x \cos x = -1$$
$$\sin 2x = -1$$

The period of the sine function is $2\pi.$ In the interval $[0, 2\pi),$ the sine function is –1 at $\frac{3\pi}{2}.$ All the solutions to $\sin 2x$ are given by

$$2x = \frac{3\pi}{2} + 2n\pi$$
$$x = \frac{3\pi}{4} + n\pi,$$

where n is any integer. The solutions in the interval $[0, 2\pi)$ are obtained by letting n = 0 and n = 1. The solutions are $\frac{3\pi}{4}$ and $\frac{7\pi}{4}.$

10.
$$\cos x - \sin x = -1$$
$$(\cos x - \sin x)^2 = (-1)^2$$
$$\cos^2 x - 2\cos x \sin x + \sin^2 x = 1$$
$$\cos^2 x + \sin^2 x - 2\cos x \sin x = 1$$
$$1 - 2\cos x \sin x = 1$$
$$-2\cos x \sin x = 0$$
$$\cos x \sin x = 0$$
$$\cos x = 0 \quad \text{or} \quad \sin x = 0$$
$$x = \frac{\pi}{2} \qquad x = 0$$
$$x = \frac{3\pi}{2} \qquad x = \pi$$

We check these proposed solutions to see if any are extraneous.

Check 0: $\cos 0 - \sin 0 = -1$
$$1 - 0 = -1$$
$$1 = -1, \text{ false}$$

Check $\frac{\pi}{2}$: $\cos\frac{\pi}{2} - \sin\frac{\pi}{2} = -1$
$$0 - 1 = -1$$
$$-1 = -1, \text{ true}$$

Check π: $\cos\frac{\pi}{2} - \sin\frac{\pi}{2} = -1$
$$-1 - 0 = -1$$
$$-1 = -1, \text{ true}$$

Check $\frac{3\pi}{2}$: $\cos\frac{3\pi}{2} - \sin\frac{3\pi}{2} = -1$
$$0 - (-1) = -1$$
$$1 = -1, \text{ false}$$

The actual solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{2}$ and $\pi.$

11. a. $\tan x = 3.1044$

Be sure calculator is in radian mode and find the inverse tangent of 3.1044. This gives the first quadrant reference angle.

$\theta = \tan^{-1} 3.1044 \approx 1.2592$

The tangent is positive in quadrants I and III thus,

$x \approx 1.2592$ or $x \approx \pi + 1.2592$

$x \approx 4.4008$

b. $\sin x = -0.2315$

Be sure calculator is in radian mode and find the inverse sine of +0.2315. This gives the first quadrant reference angle.

$\theta = \sin^{-1}(0.2315) \approx 0.2336$

The sine is negative in quadrants III and IV thus,

$x \approx \pi + 0.2336$ or $x \approx 2\pi - 1.2592$

$x \approx 3.3752$ $\quad$ $x \approx 6.0496$

12. $\cos^2 x + 5\cos x + 3 = 0$

Use the quadratic formula to solve for cos*x*.

$$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\cos x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(3)}}{2(1)}$$

$$\cos x = \frac{-5 \pm \sqrt{13}}{2}$$

$\cos x \approx -0.6972$ or $\cos x \approx -4.3028$

~~$\cos x \approx -4.3028$~~

This equation has no solution.

Be sure calculator is in radian mode and find the inverse cosine of +0.6972. This gives the first quadrant reference angle.

$\theta = \cos^{-1} 0.6972 \approx 0.7993$

The cosine is negative in quadrants II and III thus,

$x \approx \pi - 0.7993$ or $x \approx \pi + 0.7993$

$x \approx 2.3423$ $\quad$ $x \approx 3.9409$

Exercise Set 6.5

1. $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$ is true.

Thus, $\frac{\pi}{4}$ is a solution.

2. $\tan \frac{\pi}{3} = \sqrt{3}$

$\sqrt{3} = \sqrt{3}$ is true.

Thus, $\frac{\pi}{3}$ is a solution.

3. $\sin \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\frac{1}{2} = \frac{\sqrt{3}}{2}$ is false.

Thus, $\frac{\pi}{6}$ is not a solution.

4. $\sin \frac{\pi}{3} = \frac{\sqrt{2}}{2}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{2}$ is false.

Thus, $\frac{\pi}{3}$ is not a solution.

5. $\cos \frac{2\pi}{3} = -\frac{1}{2}$

$-\frac{1}{2} = -\frac{1}{2}$ is true.

Thus, $\frac{2\pi}{3}$ is a solution.

6. $\cos \frac{4\pi}{3} = -\frac{1}{2}$

$-\frac{1}{2} = -\frac{1}{2}$ is true.

Thus, $\frac{4\pi}{3}$ is a solution.

7. $\tan\left(2\cdot\frac{5\pi}{12}\right)=-\frac{\sqrt{3}}{3}$

$\tan\frac{5\pi}{6}=-\frac{\sqrt{3}}{3}$

$-\frac{\sqrt{3}}{3}=-\frac{\sqrt{3}}{3}$ is true.

Thus, $\frac{5\pi}{12}$ is a solution.

8. $\cos\frac{2\pi}{3}=-\frac{1}{2}$

$-\frac{1}{2}=-\frac{1}{2}$ is true.

Thus, $\frac{2\pi}{3}$ is a solution.

9. $\cos\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

$\frac{1}{2}=\frac{\sqrt{3}}{2}$ is false.

Thus, $\frac{\pi}{3}$ is not a solution.

10. $\cos\frac{\pi}{6}+2=\sqrt{3}\cdot\sin\frac{\pi}{6}$

$\frac{\sqrt{3}}{2}+2=\sqrt{3}\cdot\frac{1}{2}$

$\frac{4+\sqrt{3}}{2}=\frac{\sqrt{3}}{2}$ is false.

Thus, $\frac{\pi}{6}$ is not a solution.

11. $\sin x=\frac{\sqrt{3}}{2}$

Because $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$, the solutions for $\sin x=\frac{\sqrt{3}}{2}$ in $[0,2\pi)$ are

$x=\frac{\pi}{3}$

$x=\pi-\frac{\pi}{3}=\frac{3\pi}{3}-\frac{\pi}{3}=\frac{2\pi}{3}$.

Because the period of the sine function is 2π, the solutions are given by

$x=\frac{\pi}{3}+2n\pi$ or $x=\frac{2\pi}{3}+2n\pi$

where n is any integer.

12. $\cos x=\frac{\sqrt{3}}{2}$

Because $\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$, the solutions for $\cos x=\frac{\sqrt{3}}{2}$ in $[0,2\pi)$ are

$x=\frac{\pi}{6}$

$x=2\pi-\frac{\pi}{6}=\frac{12\pi}{6}-\frac{\pi}{6}=\frac{11\pi}{6}$.

Because the period of the cosine function is 2π, the solutions are given by

$x=\frac{\pi}{6}+2n\pi$ or $x=\frac{11\pi}{6}+2n\pi$

where n is any integer.

13. $\tan x=1$

Because $\tan\frac{\pi}{4}=1$, the solution for $\tan x=1$ in $[0,\pi)$ is

$x=\frac{\pi}{4}$.

Because the period of the tangent function is π, the solutions are given by

$x=\frac{\pi}{4}+n\pi$

where n is any integer.

14. $\tan x=\sqrt{3}$

Because $\tan\frac{\pi}{3}=\sqrt{3}$, the solution for $\tan x=\sqrt{3}$ in $[0,\pi)$ is

$x=\frac{\pi}{3}$.

Because the period of the tangent function is π, the solutions are given by

$x=\frac{\pi}{3}+n\pi$ where n is any integer.

15. $\cos x = -\frac{1}{2}$

Because $\cos\frac{\pi}{3} = \frac{1}{2}$, the solutions for $\cos x = -\frac{1}{2}$ in $[0, 2\pi)$ are

$x = \pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$

$x = \pi + \frac{\pi}{3} = \frac{3\pi}{3} + \frac{\pi}{3} = \frac{4\pi}{3}$.

Because the period of the cosine function is 2π, the solutions are given by

$x = \frac{2\pi}{3} + 2n\pi$ or $x = \frac{4\pi}{3} + 2n\pi$

where n is any integer.

16. $\sin x = -\frac{\sqrt{2}}{2}$

Because $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, the solutions for $\sin x = -\frac{\sqrt{2}}{2}$ in $[0, 2\pi)$ are

$x = \pi + \frac{\pi}{4} = \frac{4\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$

$x = 2\pi - \frac{\pi}{4} = \frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}$.

Because the period of the sine function is 2π, the solutions are given by

$x = \frac{5\pi}{4} + 2n\pi$ or $x = \frac{7\pi}{4} + 2n\pi$

where n is any integer.

17. $\tan x = 0$

Because $\tan 0 = 0$, the solution for $\tan x = 0$ in $[0, \pi)$ is

$x = 0$.

Because the period of the tangent function is π, the solutions are given by

$x = 0 + n\pi = n\pi$

where n is any integer.

18. $\sin x = 0$

Because $\sin 0 = 0$, the solutions for $\sin x = 0$ in $[0, 2\pi)$ are

$x = 0$

$x = \pi + 0 = \pi$.

Because the period of the sine function is 2π, the solutions are given by

$x = 0 + n\pi = n\pi$ or $x = \pi + 2n\pi$

where n is any integer.

19. $2\cos x + \sqrt{3} = 0$

$2\cos x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$

Because $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, the solutions for $\cos x = -\frac{\sqrt{3}}{2}$ in $[0, 2\pi)$ are

$x = \pi - \frac{\pi}{6} = \frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$

$x = \pi + \frac{\pi}{6} = \frac{6\pi}{6} + \frac{\pi}{6} = \frac{7\pi}{6}$.

Because the period of the cosine function is 2π, the solutions are given by

$x = \frac{5\pi}{6} + 2n\pi$ or $x = \frac{7\pi}{6} + 2n\pi$

where n is any integer.

20. $2\sin x + \sqrt{3} = 0$

$2\sin x = -\sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$

Because $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, the solutions for $\sin x = -\frac{\sqrt{3}}{2}$ in $[0, 2\pi)$ are

$x = \pi + \frac{\pi}{3} = \frac{3\pi}{3} + \frac{\pi}{3} = \frac{4\pi}{3}$

$x = 2\pi - \frac{\pi}{3} = \frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$.

Because the period of the sine function is 2π, the solutions are given by

$x = \frac{4\pi}{3} + 2n\pi$ or $x = \frac{5\pi}{3} + 2n\pi$

where n is any integer.

21. $4\sin\theta - 1 = 2\sin\theta$
$4\sin\theta - 2\sin\theta = 1$
$2\sin\theta = 1$
$\sin\theta = \frac{1}{2}$

Because $\sin\frac{\pi}{6} = \frac{1}{2}$, the solutions

for $\sin\theta = \frac{1}{2}$ in $[0, 2\pi)$ are

$\theta = \frac{\pi}{6}$

$\theta = \pi - \frac{\pi}{6} = \frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$.

Because the period of the sine function is 2π, the solutions are given by

$\theta = \frac{\pi}{6} + 2n\pi$ or $\theta = \frac{5\pi}{6} + 2n\pi$

where n is any integer.

22. $5\sin\theta + 1 = 3\sin\theta$
$5\sin\theta - 3\sin\theta = -1$
$2\sin\theta = -1$
$\sin\theta = -\frac{1}{2}$

Because $\sin\frac{\pi}{6} = \frac{1}{2}$, the solutions

for $\sin\theta = -\frac{1}{2}$ in $[0, 2\pi)$ are

$\theta = \pi + \frac{\pi}{6} = \frac{6\pi}{6} + \frac{\pi}{6} = \frac{7\pi}{6}$

$\theta = 2\pi - \frac{\pi}{6} = \frac{12\pi}{6} - \frac{\pi}{6} = \frac{11\pi}{6}$.

Because the period of the sine function is 2π, the solutions are given by

$\theta = \frac{7\pi}{6} + 2n\pi$ or $\theta = \frac{11\pi}{6} + 2n\pi$

where n is any integer.

23. $3\sin\theta + 5 = -2\sin\theta$
$3\sin\theta + 2\sin\theta = -5$
$5\sin\theta = -5$
$\sin\theta = -1$

Because $\sin\frac{\pi}{2} = 1$, the solutions

for $\sin\theta = -1$ in $[0, 2\pi)$ are

$\theta = \pi + \frac{\pi}{2} = \frac{2\pi}{2} + \frac{\pi}{2} = \frac{3\pi}{2}$

$\theta = 2\pi - \frac{\pi}{2} = \frac{4\pi}{2} - \frac{\pi}{2} = \frac{3\pi}{2}$.

Because the period of the sine function is 2π, the solutions are given by

$\theta = \frac{3\pi}{2} + 2n\pi$

where n is any integer.

24. $7\cos\theta + 9 = -2\cos\theta$
$7\cos\theta + 2\cos\theta = -9$
$9\cos\theta = -9$
$\cos\theta = -1$

Because $\cos\pi = -1$, the solution

for $\cos\theta = -1$ in $[0, 2\pi)$ is

$x = \pi$.

Because the period of the cosine function is 2π, the solutions are given by

$\theta = \pi + 2n\pi$

where n is any integer.

25. The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{\sqrt{3}}{2}$. One is $\frac{\pi}{3}$. The sine is positive in quadrant II; thus, the other value is $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$. All the solutions to $\sin 2x = \frac{\sqrt{3}}{2}$ are given by

$2x = \frac{\pi}{3} + 2n\pi$ or $2x = \frac{2\pi}{3} + 2n\pi$

$x = \frac{\pi}{6} + n\pi$ $\quad$ $x = \frac{\pi}{3} + n\pi$

Where n is any integer.

The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$ and $n = 1$.

The solutions are $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}$, and $\frac{4\pi}{3}$.

26. The period of the cosine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the cosine function is $\frac{\sqrt{2}}{2}$. One is $\frac{\pi}{4}$. The cosine is positive in quadrant IV; thus, the other value is $2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$. All the solutions to $\cos 2x = \frac{\sqrt{2}}{2}$ are given by

$2x = \frac{\pi}{4} + 2n\pi$ or $2x = \frac{7\pi}{4} + 2n\pi$

$x = \frac{\pi}{8} + n\pi$ $\quad x = \frac{7\pi}{8} + n\pi$

where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$ and $n = 1$.
The solutions are $\frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}$, and $\frac{15\pi}{8}$.

27. The period of the cosine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the cosine function is $-\frac{\sqrt{3}}{2}$. One is $\frac{5\pi}{6}$. The cosine is negative in quadrant III; thus, the other value is $2\pi - \frac{5\pi}{6} = \frac{7\pi}{6}$. All the solutions to $\cos 4x = -\frac{\sqrt{3}}{2}$ are given by

$4x = \frac{5\pi}{6} + 2n\pi$ or $4x = \frac{7\pi}{6} + 2n\pi$

$x = \frac{5\pi}{24} + \frac{n\pi}{2}$ $\quad x = \frac{7\pi}{24} + \frac{n\pi}{2}$

where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$, $n = 1$, $n = 2$, and $n = 3$.
The solutions are $\frac{5\pi}{24}, \frac{7\pi}{24}, \frac{17\pi}{24}, \frac{19\pi}{24}$, $\frac{29\pi}{24}, \frac{31\pi}{24}, \frac{41\pi}{24}$ and $\frac{43\pi}{24}$.

28. The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $-\frac{\sqrt{2}}{2}$. One is $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$. The sine is negative in quadrant IV; thus, the other value is $2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$. All solutions to $\sin 4x = -\frac{\sqrt{2}}{2}$ are given by

$4x = \frac{5\pi}{4} + 2n\pi$ or $4x = \frac{7\pi}{4} + 2n\pi$

$x = \frac{5\pi}{16} + \frac{n\pi}{2}$ $\quad x = \frac{7\pi}{16} + \frac{n\pi}{2}$

where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$, $n = 1$,
$n = 2$, and $n = 3$. The solutions are
$\frac{5\pi}{16}, \frac{7\pi}{16}, \frac{13\pi}{16}, \frac{15\pi}{16}, \frac{21\pi}{16}, \frac{23\pi}{16}, \frac{29\pi}{16}$ and $\frac{31\pi}{16}$.

29. The period of the tangent function is π. In the interval $[0, \pi)$, the only value for which the tangent function is $\frac{\sqrt{3}}{3}$ is $\frac{\pi}{6}$.

All the solutions to $\tan 3x = \frac{\sqrt{3}}{3}$ are given by

$3x = \frac{\pi}{6} + n\pi$

$x = \frac{\pi}{18} + \frac{n\pi}{3}$

where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$, $n = 1$, $n = 2$, $n = 3$, $n = 4$, and $n = 5$.
The solutions are
$\frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}$, and $\frac{31\pi}{18}$.

30. The period of the tangent function is π. In the interval $[0, \pi)$, the only value for which the tangent function is $\sqrt{3}$ is $\frac{\pi}{3}$.

All the solutions to $\tan 3x = \sqrt{3}$ are given by

$3x = \frac{\pi}{3} + n\pi$

$x = \frac{\pi}{9} + \frac{n\pi}{3}$

where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$, $n = 1$, $n = 2$, $n = 3$, $n = 4$, and $n = 5$.
The solutions are $\frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{10\pi}{9}, \frac{13\pi}{9}$, and $\frac{16\pi}{9}$.

31. The period of the tangent function is π. In the interval $[0, \pi)$, the only value for which the tangent function is $\sqrt{3}$ is $\frac{\pi}{3}$.

All the solutions to $\tan\frac{x}{2} = \sqrt{3}$ are given by

$$\frac{x}{2} = \frac{\pi}{3} + n\pi$$

$x = \frac{2\pi}{3} + 2n\pi$ where n is any integer. The solution in the interval $[0, 2\pi)$ is obtained by letting $n = 0$.

The only solution is $\frac{2\pi}{3}$.

32. The period of the tangent function is π. In the interval $[0, \pi)$, the only value for which the tangent function is $\frac{\sqrt{3}}{3}$ is $\frac{\pi}{6}$.

All the solutions to $\tan\frac{x}{2} = \frac{\sqrt{3}}{3}$ are given by

$$\frac{x}{2} = \frac{\pi}{6} + n\pi$$

$$x = \frac{\pi}{3} + 2n\pi$$

where n is any integer.
The solution in the interval $[0, 2\pi)$ is obtained by letting $n = 0$.

The only solution is $\frac{\pi}{3}$.

33. The period of the sine function is 2π. In the interval $[0, 2\pi)$, the only value for which the sine function is -1 is $\frac{3\pi}{2}$.

All the solutions to $\sin\frac{2\theta}{3} = -1$ are given by

$$\frac{2\theta}{3} = \frac{3\pi}{2} + 2n\pi$$

$\theta = \frac{9\pi}{4} + 3n\pi$ where n is any integer. All values of θ exceed 2π or are less than zero.
Thus, in the interval $[0, 2\pi)$ there is no solution.

34. The period of the cosine function is 2π. In the interval $[0, 2\pi)$, the only value for which the cosine function is –1 is π. All the solutions to $\cos\frac{2\theta}{3} = -1$ are given by

$$\frac{2\theta}{3} = \pi + 2n\pi$$

$$\theta = \frac{3\pi}{2} + 3n\pi$$

where n is any integer.
The solution in the interval $[0, 2\pi)$ is obtained by letting $n = 0$.

The only solution is $\frac{3\pi}{2}$.

35. The period of the secant function is 2π. In the interval $[0, 2\pi)$, there are two values at which the secant function is –2. One is $\frac{2\pi}{3}$. The secant is negative in quadrant III; thus, the other value is $2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$. All the solutions to $\sec\frac{3\theta}{2} = -2$ are given by

$$\frac{3\theta}{2} = \frac{2\pi}{3} + 2n\pi \quad \text{or} \quad \frac{3\theta}{2} = \frac{4\pi}{3} + 2n\pi$$

$$\theta = \frac{4\pi}{9} + \frac{4n\pi}{3} \qquad \theta = \frac{8\pi}{9} + \frac{4n\pi}{3}$$

where n is any integer. The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$ and $n = 1$.

Since $\frac{20\pi}{9}$ is not in $[0, 2\pi)$, the solutions are

$\frac{4\pi}{9}, \frac{8\pi}{9}$, and $\frac{16\pi}{9}$.

36. The period of the cotangent function is π. In the interval $[0, \pi)$, the only value for which the cotangent function is $-\sqrt{3}$ is $\frac{5\pi}{6}$

All the solutions to $\cot\frac{3\theta}{2} = -\sqrt{3}$ are given by

$$\frac{3\theta}{2} = \frac{5\pi}{6} + n\pi$$

$$\theta = \frac{5\pi}{9} + \frac{2n\pi}{3}$$

where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$, $n = 1$, and $n = 2$.

The solutions are $\frac{5\pi}{9}, \frac{11\pi}{9}$, and $\frac{17\pi}{9}$.

37. The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{1}{2}$. One is $\frac{\pi}{6}$. The sine is positive in quadrant II; Thus, the other value is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

All the solutions to $\sin\left(2x + \frac{\pi}{6}\right) = \frac{1}{2}$ are given by

$$2x + \frac{\pi}{6} = \frac{\pi}{6} + 2n\pi$$
$$2x = 2n\pi$$
$$x = n\pi \qquad \text{or}$$
$$2x + \frac{\pi}{6} = \frac{5\pi}{6} + 2n\pi$$
$$2x = \frac{4\pi}{6} + 2n\pi$$
$$x = \frac{2\pi}{6} + n\pi$$
$$x = \frac{\pi}{3} + n\pi$$

where n is any integer. The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$ and $n = 1$.

The solutions are $0, \frac{\pi}{3}, \pi,$ and $\frac{4\pi}{3}$.

38. The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{\sqrt{2}}{2}$. One is $\frac{\pi}{4}$. The sine is positive in quadrant II; Thus, the other value is $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$. All the solutions to $\sin\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ are given by

$$2x - \frac{\pi}{4} = \frac{\pi}{4} + 2n\pi$$
$$2x = \frac{2\pi}{4} + 2n\pi \qquad \text{or}$$
$$x = \frac{\pi}{4} + n\pi$$
$$2x - \frac{\pi}{4} = \frac{3\pi}{4} + 2n\pi \text{ where } n \text{ is any integer.}$$
$$2x = \frac{4\pi}{4} + 2n\pi$$
$$x = \frac{\pi}{2} + n\pi$$

The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$ and $n = 1$.

The solutions are $\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}$, and $\frac{3\pi}{2}$.

39.
$$2\sin^2 x - \sin x - 1 = 0$$
$$(2\sin x + 1)(\sin x - 1) = 0$$
$$2\sin x + 1 = 0 \qquad \text{or} \qquad \sin x - 1 = 0$$
$$2\sin x = -1 \qquad\qquad \sin x = 1$$
$$\sin x = -\frac{1}{2}$$
$$x = \frac{7\pi}{6} \quad x = \frac{11\pi}{6} \qquad\qquad x = \frac{\pi}{2}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{2}, \frac{7\pi}{6},$ and $\frac{11\pi}{6}$.

40.
$$2\sin^2 x + \sin x - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$
$$2\sin x - 1 = 0 \qquad \text{or} \qquad \sin x + 1 = 0$$
$$2\sin x = 1 \qquad\qquad \sin x = -1$$
$$\sin x = \frac{1}{2}$$
$$x = \frac{\pi}{6} \quad x = \frac{5\pi}{6} \qquad\qquad x = \frac{3\pi}{2}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6},$ and $\frac{3\pi}{2}$.

41.
$$2\cos^2 x + 3\cos x + 1 = 0$$
$$(2\cos x + 1)(\cos x + 1) = 0$$
$$2\cos x + 1 = 0 \qquad \text{or} \qquad \cos x + 1 = 0$$
$$2\cos x = -1 \qquad\qquad \cos x = -1$$
$$\cos x = -\frac{1}{2}$$
$$x = \frac{2\pi}{3} \quad x = \frac{4\pi}{3} \qquad\qquad x = \pi$$

The solutions in the interval $[0, 2\pi)$ are $\frac{2\pi}{3}, \pi,$ and $\frac{4\pi}{3}$.

42. $\cos^2 x + 2\cos x - 3 = 0$
$(\cos x - 1)(\cos x + 3) = 0$
$\cos x - 1 = 0$ or $\cos x + 3 = 0$
$\cos x = 1$ $\cos x = -3$
$x = 0$
$\cos x$ cannot be less than –1.
The solution in the interval $[0, 2\pi)$ is 0.

43. $2\sin^2 x = \sin x + 3$
$2\sin^2 x - \sin x - 3 = 0$
$(2\sin x - 3)(\sin x + 1) = 0$
$2\sin x - 3 = 0$ or $\sin x + 1 = 0$
$2\sin x = 3$ $\sin x = -1$
$\sin x = \frac{3}{2}$ $x = \frac{3\pi}{2}$
$\sin x$ cannot be greater than 1.
The solution in the interval $[0, 2\pi)$ is $\frac{3\pi}{2}$.

44. $2\sin^2 x = 4\sin x + 6$
$2\sin^2 x - 4\sin x - 6 = 0$
$(2\sin x + 2)(\sin x - 3) = 0$
$2\sin x + 2 = 0$ or $\sin x - 3 = 0$
$2\sin x = -2$ $\sin x = 3$
$\sin x = -1$
$x = \frac{3\pi}{2}$
$\sin x$ cannot be greater than 1.
The solution in the interval $[0, 2\pi)$ is $\frac{3\pi}{2}$.

45. $\sin^2\theta - 1 = 0$
$(\sin\theta - 1)(\sin\theta + 1) = 0$
$\sin\theta - 1 = 0$ or $\sin\theta + 1 = 0$
$\sin\theta = 1$ $\sin\theta = -1$
$\theta = \frac{\pi}{2}$ $\theta = \frac{3\pi}{2}$
The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

46. $\cos^2\theta - 1 = 0$
$(\cos\theta - 1)(\cos\theta + 1) = 1$
$\cos\theta - 1 = 0$ or $\cos\theta + 1 = 0$
$\cos\theta = 1$ $\cos\theta = -1$
$\theta = 0$ $\theta = \pi$
The solutions in the interval $[0, 2\pi)$ are 0 and π.

47. $4\cos^2 x - 1 = 0$
$(2\cos x + 1)(2\cos x - 1) = 1$
$2\cos x + 1 = 0$ or $2\cos x - 1 = 0$
$\cos x = -\frac{1}{2}$ $\cos x = \frac{1}{2}$
$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$
The solutions in the interval $[0, 2\pi)$ are
$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3},$ and $\frac{5\pi}{3}$.

48. $4\sin^2 x - 3 = 0$
$\sin^2 x = \frac{3}{4}$
$\sin x = \pm\sqrt{\frac{3}{4}}$
$\sin x = \pm\frac{\sqrt{3}}{2}$
$\sin x = \frac{\sqrt{3}}{2}$ or $\sin x = -\frac{\sqrt{3}}{2}$
$x = \frac{\pi}{3}, \frac{2\pi}{3}$ $x = \frac{4\pi}{3}, \frac{5\pi}{3}$
The solutions in the interval $[0, 2\pi)$ are
$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3},$ and $\frac{5\pi}{3}$.

49. $9\tan^2 x - 3 = 0$

$$\tan^2 x = \frac{3}{9}$$
$$\tan x = \pm\sqrt{\frac{3}{9}}$$
$$\tan x = \pm\frac{\sqrt{3}}{3}$$
$$\tan x = \frac{\sqrt{3}}{3} \quad \text{or} \quad \tan x = -\frac{\sqrt{3}}{3}$$
$$x = \frac{\pi}{6}, \frac{7\pi}{6} \qquad x = \frac{5\pi}{6}, \frac{11\pi}{6}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6},$ and $\frac{11\pi}{6}$.

50. $3\tan^2 x - 9 = 0$

$$\tan^2 x = \frac{9}{3}$$
$$\tan^2 x = 3$$
$$\tan x = \pm\sqrt{3}$$
$$\tan x = \sqrt{3} \quad \text{or} \quad \tan x = -\sqrt{3}$$
$$x = \frac{\pi}{3}, \frac{4\pi}{3} \qquad x = \frac{2\pi}{3}, \frac{5\pi}{3}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3},$ and $\frac{5\pi}{3}$.

51. $\sec^2 x - 2 = 0$

$$\sec^2 x = 2$$
$$\cos^2 x = \frac{1}{2}$$
$$\cos x = \pm\sqrt{\frac{1}{2}}$$
$$\cos x = \pm\frac{\sqrt{2}}{2}$$
$$\cos x = \frac{\sqrt{2}}{2} \quad \text{or} \quad \cos x = -\frac{\sqrt{2}}{2}$$
$$x = \frac{\pi}{4}, \frac{7\pi}{4} \qquad x = \frac{3\pi}{4}, \frac{5\pi}{4}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4},$ and $\frac{7\pi}{4}$.

52. $4\sec^2 x - 2 = 0$

$$\sec^2 x = \frac{2}{4}$$
$$\cos^2 x = 2$$
$$\cos x = \pm\sqrt{2}$$

No solution.

53. $(\tan x - 1)(\cos x + 1) = 0$

$$\tan x - 1 = 0 \quad \text{or} \quad \cos x + 1 = 0$$
$$\tan x = 1 \qquad \cos x = -1$$
$$x = \frac{\pi}{4} \quad x = \frac{5\pi}{4} \qquad x = \pi$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{4}, \pi,$ and $\frac{5\pi}{4}$

54. $(\tan x + 1)(\sin x - 1) = 0$

$$\tan x + 1 = 0 \quad \text{or} \quad \sin x - 1 = 0$$
$$\tan x = -1 \qquad \sin x = 1$$
$$x = \frac{3\pi}{4} \quad x = \frac{7\pi}{4} \qquad x = \frac{\pi}{2}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$ since tan is undefined at $\frac{\pi}{2}$.

55. $\left(2\cos x + \sqrt{3}\right)(2\sin x + 1) = 0$

$$2\cos x + \sqrt{3} = 0 \quad \text{or} \quad 2\sin x + 1 = 0$$
$$2\cos x = -\sqrt{3} \qquad 2\sin x = -1$$
$$\cos x = -\frac{\sqrt{3}}{2} \qquad \sin x = -\frac{1}{2}$$
$$x = \frac{5\pi}{6} \quad x = \frac{7\pi}{6} \qquad x = \frac{7\pi}{6} \quad x = \frac{11\pi}{6}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{5\pi}{6}, \frac{7\pi}{6},$ and $\frac{11\pi}{6}$.

56. $\left(2\cos x-\sqrt{3}\right)(2\sin x-1)=0$

$2\cos x-\sqrt{3}=0$ or $2\sin x-1=0$

$2\cos x=\sqrt{3}$ $\quad 2\sin x=1$

$\cos x=\frac{\sqrt{3}}{2}$ $\quad \sin x=\frac{1}{2}$

$x=\frac{\pi}{6}\quad x=\frac{11\pi}{6}\qquad x=\frac{\pi}{6}\quad x=\frac{5\pi}{6}$

The solutions in the interval $[0,2\pi)$ are

$\frac{\pi}{6},\frac{5\pi}{6}$, and $\frac{11\pi}{6}$.

57. $\cot x(\tan x-1)=0$

$\cot x=0$ or $\tan x-1=0$

$\tan x=1$

$x=\frac{\pi}{2}\quad x=\frac{3\pi}{2}\quad x=\frac{\pi}{4}\quad x=\frac{5\pi}{4}$

The solutions in the interval $[0,2\pi)$ are $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

since tan is undefined for $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

58. $\cot x(\tan x+1)=0$

$\cot x=0$ or $\tan x+1=0$

$\tan x=-1$

$x=\frac{\pi}{2}\quad x=\frac{3\pi}{2}\quad x=\frac{3\pi}{4}\quad x=\frac{7\pi}{4}$

The solutions in the interval $[0,2\pi)$

are

$\frac{3\pi}{4}$ and $\frac{7\pi}{4}$ since tan is undefined at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

59. $\sin x+2\sin x\cos x=0$

$\sin x(1+2\cos x)=0$

$\sin x=0$ or $1+2\cos x=0$

$2\cos x=-1$

$\cos x=-\frac{1}{2}$

$x=0\quad x=\pi\quad x=\frac{2\pi}{3}\quad x=\frac{4\pi}{3}$

The solutions in the interval $[0,2\pi)$ are

$0,\frac{2\pi}{3},\pi$, and $\frac{4\pi}{3}$.

60. $\cos x-2\sin x\cos x=0$

$\cos x(1-2\sin x)=0$

$\cos x=0$ or $1-2\sin x=0$

$-2\sin x=-1$

$\sin x=\frac{1}{2}$

$x=\frac{\pi}{2}\quad x=\frac{3\pi}{2}\quad x=\frac{\pi}{6}\quad x=\frac{5\pi}{6}$

The solutions in the interval $[0,2\pi)$ are

$\frac{\pi}{6},\frac{\pi}{2},\frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

61. $\tan^2 x\cos x=\tan^2 x$

$\tan^2 x\cos x-\tan^2 x=0$

$\tan^2 x(\cos x-1)=0$

$\tan^2 x=0$ or $\cos x-1=0$

$\tan x=0$ $\quad \cos x=1$

$x=0\quad x=\pi\qquad x=0$

The solutions in the interval $[0,2\pi)$ are 0 and π.

62. $\cot^2 x\sin x=\cot^2 x$

$\cot^2 x\sin x-\cot^2 x=0$

$\cot 2(\sin x-1)=0$

$\cot^2 x=0$ or $\sin x-1=0$

$\cot x=0$ $\quad \sin x=1$

$x=\frac{\pi}{2}\quad x=\frac{3\pi}{2}\qquad x=\frac{\pi}{2}$

The solutions in the interval $[0,2\pi)$ are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

63.

$$2\cos^2 x + \sin x - 1 = 0$$
$$2\left(1 - \sin^2 x\right) + \sin x - 1 = 0$$
$$2 - 2\sin^2 x + \sin x - 1 = 0$$
$$-2\sin^2 x + \sin x + 1 = 0$$
$$2\sin^2 x - \sin x - 1 = 0$$
$$(2\sin x + 1)(\sin x - 1) = 0$$
$$2\sin x + 1 = 0 \quad \text{or} \quad \sin x - 1 = 0$$
$$2\sin x = -1 \qquad \sin x = 1$$
$$\sin x = -\frac{1}{2}$$
$$x = \frac{7\pi}{6} \quad x = \frac{11\pi}{6} \qquad x = \frac{\pi}{2}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{2}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

64.

$$2\cos^2 x - \sin x - 1 = 0$$
$$2\left(1 - \sin^2 x\right) - \sin x - 1 = 0$$
$$2 - 2\sin^2 x - \sin x - 1 = 0$$
$$-2\sin^2 x - \sin x + 1 = 0$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$
$$2\sin x - 1 = 0 \quad \text{or} \quad \sin x + 1 = 0$$
$$2\sin x = 1 \qquad \sin x = -1$$
$$\sin x = \frac{1}{2}$$
$$x = \frac{\pi}{6} \quad x = \frac{5\pi}{6} \qquad x = \frac{3\pi}{2}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

65.

$$\sin^2 x - 2\cos x - 2 = 0$$
$$1 - \cos^2 x - 2\cos x - 2 = 0$$
$$-\cos^2 x - 2\cos x - 1 = 0$$
$$\cos^2 x + 2\cos x + 1 = 0$$
$$(\cos x + 1)(\cos x + 1) = 0$$
$$\cos x + 1 = 0$$
$$\cos x = -1$$
$$x = \pi$$

The solution in the interval $[0, 2\pi)$ is π.

66.

$$4\sin^2 x + 4\cos x - 5 = 0$$
$$4\left(1 - \cos^2 x\right) + 4\cos x - 5 = 0$$
$$4 - 4\cos^2 x + 4\cos x - 5 = 0$$
$$-4\cos^2 x + 4\cos x - 1 = 0$$
$$4\cos^2 x - 4\cos x + 1 = 0$$
$$(2\cos x - 1)(2\cos x - 1) = 0$$
$$2\cos x - 1 = 0$$
$$2\cos x = 1$$
$$\cos x = \frac{1}{2}$$
$$x = \frac{\pi}{3} \quad x = \frac{5\pi}{3}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

67.

$$4\cos^2 x = 5 - 4\sin x$$
$$4\cos^2 x + 4\sin x - 5 = 0$$
$$4\left(1 - \sin^2 x\right) + 4\sin x - 5 = 0$$
$$4 - 4\sin^2 x + 4\sin x - 5 = 0$$
$$-4\sin^2 x + 4\sin x - 1 = 0$$
$$4\sin^2 x - 4\sin x + 1 = 0$$
$$(2\sin x - 1)(2\sin x - 1) = 0$$
$$2\sin x - 1 = 0$$
$$2\sin x = 1$$
$$\sin x = \frac{1}{2}$$
$$x = \frac{\pi}{6} \quad x = \frac{5\pi}{6}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}$ and $\frac{5\pi}{6}$.

68.
$$3\cos^2 x = \sin^2 x$$
$$3(1-\sin^2 x) = \sin^2 x$$
$$3 - 3\sin^2 x - \sin^2 x = 0$$
$$-4\sin^2 x = -3$$
$$\sin^2 x = \frac{3}{4}$$
$$\sin x = \pm\sqrt{\frac{3}{4}}$$
$$\sin x = \pm\frac{\sqrt{3}}{2}$$
$$\sin x = \frac{\sqrt{3}}{2} \quad \text{or} \quad \sin x = -\frac{\sqrt{3}}{2}$$
$$x = \frac{\pi}{3} \quad x = \frac{2\pi}{3} \quad x = \frac{4\pi}{3} \quad x = \frac{5\pi}{3}$$
The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$, and $\frac{5\pi}{3}$.

69.
$$\sin 2x = \cos x$$
$$2\sin x\cos x = \cos x$$
$$2\sin x\cos x - \cos x = 0$$
$$\cos x(2\sin x - 1) = 0$$
$$\cos x = 0 \quad \text{or} \quad 2\sin x - 1 = 0$$
$$2\sin x = 1$$
$$\sin x = \frac{1}{2}$$
$$x = \frac{\pi}{2} \quad x = \frac{3\pi}{2} \qquad x = \frac{\pi}{6} \quad x = \frac{5\pi}{6}$$
The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

70.
$$\sin 2x = \sin x$$
$$2\sin x\cos x = \sin x$$
$$2\sin x\cos x - \sin x = 0$$
$$\sin x(2\cos x - 1) = 0$$
$$\sin x = 0 \quad \text{or} \quad 2\cos x - 1 = 0$$
$$2\cos x = 1$$
$$\cos x = \frac{1}{2}$$
$$x = 0 \quad x = \pi \quad x = \frac{\pi}{3} \quad x = \frac{5\pi}{3}$$
The solutions in the interval $[0, 2\pi)$ are $0, \frac{\pi}{3}, \pi$, and $\frac{5\pi}{3}$.

71.
$$\cos 2x = \cos x$$
$$2\cos^2 x - 1 = \cos x$$
$$2\cos^2 x - 1 - \cos x = 0$$
$$2\cos^2 x - \cos x - 1 = 0$$
$$(2\cos x + 1)(\cos x - 1) = 0$$
$$2\cos x + 1 = 0 \quad \text{or} \quad \cos x - 1 = 0$$
$$2\cos x = -1 \qquad \cos x = 1$$
$$\cos x = -\frac{1}{2} \qquad \cos x = 1$$
$$x = \frac{2\pi}{3} \quad x = \frac{4\pi}{3} \qquad x = 0$$
The solutions in the interval $[0, 2\pi)$ are $0, \frac{2\pi}{3}$, and $\frac{4\pi}{3}$.

72.
$$\cos 2x = \sin x$$
$$1 - 2\sin^2 x = \sin x$$
$$1 - 2\sin^2 x - \sin x = 0$$
$$-2\sin^2 x - \sin x + 1 = 0$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$
$$2\sin x - 1 = 0 \quad \text{or} \quad \sin x + 1 = 0$$
$$2\sin x = 1 \qquad \sin x = -1$$
$$\sin x = \frac{1}{2}$$
$$x = \frac{\pi}{6} \quad x = \frac{5\pi}{6} \qquad x = \frac{3\pi}{2}$$
The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

73.
$$\cos 2x + 5\cos x + 3 = 0$$
$$2\cos^2 x - 1 + 5\cos x + 3 = 0$$
$$2\cos^2 x + 5\cos x + 2 = 0$$
$$(2\cos x + 1)(\cos x + 2) = 0$$
$$2\cos x + 1 = 0 \quad \text{or} \quad \cos x + 2 = 0$$
$$2\cos x = -1 \qquad \cos x = -2$$
$$\cos x = -\frac{1}{2}$$
$$x = \frac{2\pi}{3} \quad x = \frac{4\pi}{3} \qquad \cos x \text{ cannot be less than } -1$$

The solutions in the interval $[0, 2\pi)$ are $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$.

74.
$$\cos 2x + \cos x + 1 = 0$$
$$2\cos^2 x - 1 + \cos x + 1 = 0$$
$$2\cos^2 x + \cos x = 0$$
$$\cos x(2\cos x + 1) = 0$$
$$\cos x = 0 \quad \text{or} \quad 2\cos x + 1 = 0$$
$$2\cos x = -1$$
$$\cos x = -\frac{1}{2}$$
$$x = \frac{\pi}{2} \quad x = \frac{3\pi}{2} \quad x = \frac{2\pi}{3} \quad x = \frac{4\pi}{3}$$

The solutions in the interval $[0, 2\pi)$ are

$\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$, and $\frac{3\pi}{2}$.

75.
$$\sin x \cos x = \frac{\sqrt{2}}{4}$$
$$2\sin x \cos x = \frac{\sqrt{2}}{2}$$
$$\sin 2x = \frac{\sqrt{2}}{2}$$

The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{\sqrt{2}}{2}$. One is $\frac{\pi}{4}$. The sine is positive in quadrant II; thus, the other value is $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

All the solutions to $\sin 2x = \frac{\sqrt{2}}{2}$ are given by
$$2x = \frac{\pi}{4} + 2n\pi \quad \text{or} \quad 2x = \frac{3\pi}{4} + 2n\pi$$
$$x = \frac{\pi}{8} + n\pi \qquad x = \frac{3\pi}{8} + n\pi$$
where n is any integer.

The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$ and $n = 1$.

The solutions are $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}$, and $\frac{11\pi}{8}$.

76.
$$\sin x \cos x = \frac{\sqrt{3}}{4}$$
$$2\sin x \cos x = \frac{\sqrt{3}}{2}$$
$$\sin 2x = \frac{\sqrt{3}}{2}$$

The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{\sqrt{3}}{2}$. One is $\frac{\pi}{3}$. The sine is positive in quadrant II; thus, the other value is $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

All the solutions to $\sin 2x = \frac{\sqrt{3}}{2}$ are given by
$$2x = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad 2x = \frac{2\pi}{3} + 2n\pi$$
$$x = \frac{\pi}{6} + n\pi \qquad x = \frac{\pi}{3} + n\pi$$
where n is any integer. The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$ and $n = 1$.

The solutions are $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}$, and $\frac{4\pi}{3}$.

77.
$$\sin x + \cos x = 1$$
$$(\sin x + \cos x)^2 = 1^2$$
$$\sin^2 x + 2\sin x \cos x + \cos^2 x = 1$$
$$\sin^2 x + \cos^2 x + 2\sin x \cos x = 1$$
$$1 + 2\sin x \cos x = 1$$
$$2\sin x \cos x = 0$$
$$\sin x \cos x = 0$$
$$\sin x = 0 \quad \text{or} \quad \cos x = 0$$
$$x = 0 \qquad x = \frac{\pi}{2}$$
$$x = \pi \qquad x = \frac{3\pi}{2}$$

After checking these proposed solutions, the actual solutions in the interval $[0, 2\pi)$ are 0 and $\frac{\pi}{2}$.

78.

$$\sin x + \cos x = -1$$
$$(\sin x + \cos x)^2 = (-1)^2$$
$$\sin^2 x + 2\sin x \cos x + \cos^2 x = 1$$
$$\sin^2 x + \cos^2 x + 2\sin x \cos x = 1$$
$$1 + 2\sin x \cos x = 1$$
$$2\sin x \cos x = 0$$
$$\sin x \cos x = 0$$
$$\sin x = 0 \quad \text{or} \quad \cos x = 0$$
$$x = 0 \qquad x = \frac{\pi}{2}$$
$$x = \pi \qquad x = \frac{3\pi}{2}$$

After checking these proposed solutions, the actual solutions in the interval $[0, 2\pi)$ are π and $\frac{3\pi}{2}$.

79.

$$\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = 1$$
$$\frac{1}{2}\left[\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right)\right] = 1 \cdot \frac{1}{2}$$
$$\sin x \cos \frac{\pi}{4} = \frac{1}{2}$$
$$\sin x \cdot \frac{\sqrt{2}}{2} = \frac{1}{2}$$
$$\sin x = \frac{1}{\sqrt{2}}$$
$$\sin x = \frac{\sqrt{2}}{2}$$
$$x = \frac{\pi}{4} \quad \text{or} \quad x = \frac{3\pi}{4}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{4}$ and $\frac{3\pi}{4}$.

80.

$$\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$$
$$\frac{1}{2}\left[\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right)\right] = 1 \cdot \frac{1}{2}$$
$$\sin x \cos \frac{\pi}{3} = \frac{1}{2}$$
$$\sin x \cdot \frac{1}{2} = \frac{1}{2}$$
$$\sin x = 1$$
$$x = \frac{\pi}{2}$$

The solution in the interval $[0, 2\pi)$ is $\frac{\pi}{2}$.

81.

$$\sin 2x \cos x + \cos 2x \sin x = \frac{\sqrt{2}}{2}$$
$$\sin(2x + x) = \frac{\sqrt{2}}{2}$$
$$\sin 3x = \frac{\sqrt{2}}{2}$$

The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{\sqrt{2}}{2}$. One is $\frac{\pi}{4}$. The sine function is positive in quadrant II; thus, the other value is $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

All the solutions to $\sin 3x = \frac{\sqrt{2}}{2}$ are given by

$$3x = \frac{\pi}{4} + 2n\pi \quad \text{or} \quad 3x = \frac{3\pi}{4} + 2n\pi$$
$$x = \frac{\pi}{12} + \frac{2n\pi}{3} \qquad x = \frac{\pi}{4} + \frac{2n\pi}{3}$$

where *n* is any integer. The solutions in the interval $[0, 2\pi)$ are obtained by letting *n* = 0, *n* = 1, and *n* = 2.

The solutions are $\frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}, \frac{17\pi}{12}$, and $\frac{19\pi}{12}$.

82. $\sin 3x\cos 2x + \cos 3x \sin 2x = 1$

$$\sin(3x+2x)=1$$
$$\sin 5x = 1$$

The period of the sine function is 2π. In the interval $[0, 2\pi)$, the only value at which the sine function is 1 is $\frac{\pi}{2}$. All the solutions to $\sin 5x = 1$ are given by

$$5x = \frac{\pi}{2} + 2n\pi$$
$$x = \frac{\pi}{10} + \frac{2n\pi}{5}$$

where n is any integer.
The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$, $n = 1$, $n = 2$, $n = 3$, and $n = 4$.
The solutions are $\frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}, \frac{13\pi}{10}$, and $\frac{17\pi}{10}$.

83. $\tan x + \sec x = 1$

$$\tan x - 1 = -\sec x$$
$$(\tan x - 1)^2 = (-\sec x)^2$$
$$\tan^2 x - 2\tan x + 1 = \sec^2 x$$
$$\tan^2 x - 2\tan x + 1 = 1 + \tan^2 x$$
$$-2\tan x = 0$$
$$\tan x = 0$$
$$x = 0 \quad x = \pi$$

We check these proposed solutions to see if any are extraneous.

Check 0: $\tan 0 \;+\; \sec 0 \;\; 0 \;\; 1$

$0 \;+\; 1 \;\; 0 \;\; 1$ True

Check π: $\tan\pi \;+\; \sec\pi \;\; 0 \;\; 1$

$0 \;+\; (-1) \;\; 0 \;\; 1$ False

The actual solution in the interval $[0, 2\pi)$ is 0.

84. $\tan x - \sec x = 1$

$$\tan x - 1 = \sec x$$
$$(\tan x - 1)^2 = \sec^2 x$$
$$\tan^2 x - 2\tan x + 1 = \sec^2 x$$
$$\tan^2 x - 2\tan x + 1 = 1 + \tan^2 x$$
$$-2\tan x = 0$$
$$\tan x = 0$$
$$x = 0 \quad x = \pi$$

We check these proposed solutions to see if any are extraneous.

Check 0: $\tan 0 - \sec 0 \;\; 0 \;\; 1$

$0 - 1 \;\; 0 \;\; 1$

False

Check π: $\tan\pi - \sec\pi \;\; 0 \;\; 1$

$0 - (-1) \;\; 0 \;\; 1$

True

The actual solution in the interval $[0, 2\pi)$ is π.

85. $\sin x = 0.8246$

Be sure calculator is in radian mode and find the inverse sine of 0.8246. This gives the first quadrant reference angle.

$\theta = \sin^{-1} 0.8246 \approx 0.9695$

The sine is positive in quadrants I and II thus,

$x \approx 0.9695$ or $x \approx \pi - 0.9695$

$x \approx 2.1721$

86. $\sin x = 0.7392$

Be sure calculator is in radian mode and find the inverse sine of 0.7392. This gives the first quadrant reference angle.

$\theta = \sin^{-1} 0.7392 \approx 0.8319$

The sine is positive in quadrants I and II thus,

$x \approx 0.8319$ or $x \approx \pi - 0.8319$

$x \approx 2.3097$

87. $\cos x = -\frac{2}{5}$

Be sure calculator is in radian mode and find the inverse cosine of $+\frac{2}{5}$. This gives the first quadrant reference angle.

$\theta = \cos^{-1}\frac{2}{5} \approx 1.1593$

The cosine is negative in quadrants II and III thus,

$x \approx \pi - 1.1593$ or $x \approx \pi + 1.1593$

$x \approx 1.9823$ $\quad$ $x \approx 4.3009$

88. $\cos x = -\frac{4}{7}$

Be sure calculator is in radian mode and find the inverse cosine of $+\frac{4}{7}$. This gives the first quadrant reference angle.

$\theta = \cos^{-1}\frac{4}{7} \approx 0.9626$

The cosine is negative in quadrants II and III thus,

$x \approx \pi - 0.9626$ or $x \approx \pi + 0.9626$

$x \approx 2.1790$ $x \approx 4.1041$

89. $\tan x = -3$

Be sure calculator is in radian mode and find the inverse tangent of +3. This gives the first quadrant reference angle.

$\theta = \tan^{-1} 3 \approx 1.2490$

The tangent is negative in quadrants II and IV thus,

$x \approx \pi - 1.2490$ or $x \approx 2\pi - 1.2490$

$x \approx 1.8925$ $x \approx 5.0341$

90. $\tan x = -5$

Be sure calculator is in radian mode and find the inverse tangent of +5. This gives the first quadrant reference angle.

$\theta = \tan^{-1} 5 \approx 1.3734$

The tangent is negative in quadrants II and IV thus,

$x \approx \pi - 1.3734$ or $x \approx 2\pi - 1.3734$

$x \approx 1.7682$ $x \approx 4.9098$

91. $\cos^2 x - \cos x - 1 = 0$

Use the quadratic formula to solve for cos*x*.

$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\cos x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$

$\cos x = \frac{1 \pm \sqrt{5}}{2}$

$\cos x \approx -0.6180$ or $\cos x \approx 1.6180$

~~$\cos x \approx 1.6180$~~

This equation has no solution.

Be sure calculator is in radian mode and find the inverse cosine of +0.6180. This gives the first quadrant reference angle.

$\theta = \cos^{-1} 0.6180 \approx 0.9046$

The cosine is negative in quadrants II and III thus,

$x \approx \pi - 0.9046$ or $x \approx \pi + 0.9046$

$x \approx 2.2370$ $x \approx 4.0462$

92. $3\cos^2 x - 8\cos x - 3 = 0$

$(3\cos x + 1)(\cos x - 3) = 0$

$3\cos x + 1 = 0$ or $\cos x - 3 = 0$

$\cos x = -\frac{1}{3}$ $\cos x = 3$

~~$\cos x = 3$~~

This equation has no solution.

Be sure calculator is in radian mode and find the inverse cosine of $+\frac{1}{3}$. This gives the first quadrant reference angle.

$\theta = \cos^{-1}\frac{1}{3} \approx 1.2310$

The cosine is negative in quadrants II and III thus,

$x \approx \pi - 1.2310$ or $x \approx \pi + 1.2310$

$x \approx 1.9106$ $x \approx 4.3726$

93. $4\tan^2 x - 8\tan x + 3 = 0$

$(2\tan x - 1)(2\tan x - 3) = 0$

$2\tan x - 1 = 0$ or $2\tan x - 3 = 0$

$\tan x = \frac{1}{2}$ $\tan x = \frac{3}{2}$

$x \approx 0.4636,\ 3.6052$ $x \approx 0.9828,\ 4.1244$

94. $\tan^2 x - 3\cos x + 1 = 0$

$\tan x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\tan x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$

$\tan x = \frac{3 \pm \sqrt{5}}{2}$

$\tan x \approx 0.3820$ or $\tan x \approx 2.6180$

$x \approx 0.3649,\ 3.5065$ $x \approx 1.2059,\ 4.3475$

95. $7\sin^2 x - 1 = 0$

$\sin^2 x = \frac{1}{7}$

$\sin x = \pm\sqrt{\frac{1}{7}}$

$\sin x = \pm\frac{\sqrt{7}}{7}$

$\sin x \approx 0.3780$ or $\sin x \approx -0.3780$

$x \approx 0.3876,\ 2.7540$ $\quad x \approx 3.5292,\ 5.8956$

96. $5\sin^2 x - 1 = 0$

$\sin^2 x = \frac{1}{5}$

$\sin x = \pm\sqrt{\frac{1}{5}}$

$\sin x = \pm\frac{\sqrt{5}}{5}$

$\sin x \approx 0.4472$ or $\sin x \approx -0.4472$

$x \approx 0.4636,\ 2.6780$ $\quad x \approx 3.6052,\ 5.8195$

97. $2\cos 2x + 1 = 0$

$\cos 2x = -\frac{1}{2}$

The period of the cosine function is 2π. On the interval $[0, 2\pi)$ the cosine function equals $-\frac{1}{2}$ at $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. This means that $2x = \frac{2\pi}{3}$ or $2x = \frac{4\pi}{3}$. Because the period is 2π, all the solutions of the equation are given by

$2x = \frac{2\pi}{3} + 2n\pi$ or $2x = \frac{4\pi}{3} + 2n\pi$

$x = \frac{\pi}{3} + n\pi$ $\quad x = \frac{2\pi}{3} + n\pi$

Use all values of n that result in x values on the interval $[0, 2\pi)$. Thus,

$x = \frac{\pi}{3},\ \frac{5\pi}{3}$ or $x = \frac{2\pi}{3},\ \frac{4\pi}{3}$

98. $2\sin 3x + \sqrt{3} = 0$

$\sin 3x = \frac{-\sqrt{3}}{2}$

The period of the sine function is 2π. On the interval $[0, 2\pi)$ the sine function equals $\frac{-\sqrt{3}}{2}$ at $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$. This means that $3x = \frac{4\pi}{3}$ or $3x = \frac{5\pi}{3}$. Because the period is 2π, all the solutions of the equation are given by

$3x = \frac{4\pi}{3} + 2n\pi$ or $3x = \frac{5\pi}{3} + 2n\pi$

$x = \frac{4\pi}{9} + \frac{2n\pi}{3}$ $\quad x = \frac{5\pi}{9} + \frac{2n\pi}{3}$

Use all values of n that result in x values on the interval $[0, 2\pi)$. Thus,

$x = \frac{4\pi}{9},\ \frac{5\pi}{9},\ \frac{10\pi}{9},\ \frac{11\pi}{9},\ \frac{16\pi}{9},\ \frac{17\pi}{9}$

99. $\sin 2x + \sin x = 0$

$2\sin x\cos x + \sin x = 0$

$\sin x(2\cos x + 1) = 0$

$\sin x = 0$ or $2\cos x + 1 = 0$

$x = 0, \pi$ $\quad \cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

Thus, $x = 0, \frac{2\pi}{3}, \pi$, and $\frac{4\pi}{3}$.

100. $\sin 2x + \cos x = 0$

$2\sin x\cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ or $2\sin x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad \sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

Thus, $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$, and $\frac{11\pi}{6}$.

101. $3\cos x - 6\sqrt{3} = \cos x - 5\sqrt{3}$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$

102. $\cos x - 5 = 3\cos x + 6$

$-2\cos x = 11$

$\cos x = -\frac{11}{2}$

The cosine function cannot be less than –1, thus the equation has no solution.

103. $\tan x = -4.7143$

Be sure calculator is in radian mode and find the inverse tangent of +4.7143. This gives the first quadrant reference angle. $\theta = \tan^{-1} 4.7143 \approx 1.3618$

The tangent is negative in quadrants II and IV thus,

$x \approx \pi - 1.3618$ or $x \approx 2\pi - 1.3618$

$x \approx 1.7798$ $\quad$ $x \approx 4.9214$

104. $\tan x = -6.2154$

Be sure calculator is in radian mode and find the inverse tangent of +6.2154. This gives the first quadrant reference angle. $\theta = \tan^{-1} 6.2154 \approx 1.4113$

The tangent is negative in quadrants II and IV thus,

$x \approx \pi - 1.4113$ or $x \approx 2\pi - 1.4113$

$x \approx 1.7303$ $\quad$ $x \approx 4.8719$

105. $2\sin^2 x = 3 - \sin x$

$2\sin^2 x + \sin x - 3 = 0$

$(\sin x - 1)(2\sin x + 3) = 0$

$\sin x - 1 = 0$ or $2\sin x + 3 = 0$

$\sin x = 1$ $\quad$ $\sin x = -\frac{3}{2}$

$x = \frac{\pi}{2}$ $\quad$ ~~$\sin x = -\frac{3}{2}$~~

106. $2\sin^2 x = 2 - 3\sin x$

$2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x - 1)(\sin x + 2) = 0$

$2\sin x - 1 = 0$ or $\sin x + 2 = 0$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -2$

~~$\sin x = -2$~~

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

107. $\cos x \csc x = 2\cos x$

$\cos x \csc x - 2\cos x = 0$

$\cos x(\csc x - 2) = 0$

$\cos x = 0$ or $\csc x - 2 = 0$

$\sin x = \frac{1}{2}$ $\quad$ $\csc x = 2$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Thus, $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

108. $\tan x \sec x = 2\tan x$

$\tan x \sec x - 2\tan x = 0$

$\tan x(\sec x - 2) = 0$

$\tan x = 0$ or $\sec x - 2 = 0$

$x = 0, \pi$ $\quad$ $\sec x = 2$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

Thus, $x = 0, \frac{\pi}{3}, \pi$, and $\frac{5\pi}{3}$.

109. $5\cot^2 x - 15 = 0$

$\cot^2 x = 3$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\sqrt{\frac{1}{3}}$

$\tan x = \pm\frac{\sqrt{3}}{3}$

$\tan x = \frac{\sqrt{3}}{3}$ or $\tan x = -\frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$ $\quad$ $x = \frac{5\pi}{6}, \frac{11\pi}{6}$

Thus, $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

110. $5\sec^2 x - 10 = 0$

$$\sec^2 x = 2$$

$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \pm\sqrt{\frac{1}{2}}$$

$$\cos x = \pm\frac{\sqrt{2}}{2}$$

$\cos x = \frac{\sqrt{2}}{2}$ or $\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{7\pi}{4}$ $\qquad$ $x = \frac{3\pi}{4}, \frac{5\pi}{4}$

Thus, $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, and $\frac{7\pi}{4}$.

111. $\cos^2 x + 2\cos x - 2 = 0$

Use the quadratic formula to solve for cos*x*.

$$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\cos x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$$

$$\cos x = \frac{-2 \pm \sqrt{12}}{2}$$

$$\cos x = \frac{-2 \pm 2\sqrt{3}}{2}$$

$$\cos x = -1 \pm \sqrt{3}$$

$\cos x \approx 0.7321$ or $\cos x \approx -2.7321$

~~$\cos x \approx -2.7321$~~

This equation has no solution.

Be sure calculator is in radian mode and find the inverse cosine of 0.7321. This gives the first quadrant reference angle.

$\theta = \cos^{-1} 0.7321 \approx 0.7495$

The cosine is positive in quadrants I and IV thus,

$x \approx 0.7495$ or $x \approx 2\pi - 0.7495$

$x \approx 5.5337$

112. $\cos^2 x + 5\cos x - 1 = 0$

Use the quadratic formula to solve for cos*x*.

$$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\cos x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-1)}}{2(1)}$$

$$\cos x = \frac{-5 \pm \sqrt{29}}{2}$$

$\cos x \approx 0.1926$ or $\cos x \approx -5.1926$

~~$\cos x \approx -5.1926$~~

This equation has no solution.

Be sure calculator is in radian mode and find the inverse cosine of 0.1926. This gives the first quadrant reference angle.

$\theta = \cos^{-1} 0.1926 \approx 1.3770$

The cosine is positive in quadrants I and IV thus,

$x \approx 1.3770$ or $x \approx 2\pi - 1.3770$

$x \approx 4.9062$

113.

$$5\sin x = 2\cos^2 x - 4$$

$$5\sin x = 2\left(1 - \sin^2 x\right) - 4$$

$$5\sin x = 2 - 2\sin^2 x - 4$$

$$2\sin^2 x + 5\sin x + 2 = 0$$

$$\left(2\sin x + 1\right)\left(\sin x + 2\right) = 0$$

$2\sin x + 1 = 0$ or $\sin x + 2 = 0$

$\sin x = -\frac{1}{2}$ $\qquad$ $\sin x = -2$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\qquad$ ~~$\sin x = -2$~~

114. $7\cos x = 4 - 2\sin^2 x$

$7\cos x = 4 - 2\left(1 - \cos^2 x\right)$

$7\cos x = 4 - 2 + 2\cos^2 x$

$-2\cos^2 x + 7\cos x - 2 = 0$

$2\cos^2 x - 7\cos x + 2 = 0$

$\cos x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\cos x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(2)}}{2(2)}$

$\cos x = \dfrac{7 \pm \sqrt{33}}{4}$

$\cos x \approx 0.3139$ or $\cos x \approx 3.1861$

~~$\cos x \approx 3.1861$~~
This equation has no solution.

Be sure calculator is in radian mode and find the inverse cosine of 0.3139. This gives the first quadrant reference angle.

$\theta = \cos^{-1} 0.3139 \approx 1.2515$

The cosine is positive in quadrants I and IV thus,

$x \approx 1.2515$ or $x \approx 2\pi - 1.2515$

$x \approx 5.0316$

115. $2\tan^2 x + 5\tan x + 3 = 0$

$\left(\tan x + 1\right)\left(2\tan x + 3\right) = 0$

$\tan x + 1 = 0$ or $2\tan x + 3 = 0$

$\tan x = -1$ $\qquad \tan x = -\dfrac{3}{2}$

$x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$ $\qquad x \approx 2.1588, 5.3004$

116. $3\tan^2 x - \tan x - 2 = 0$

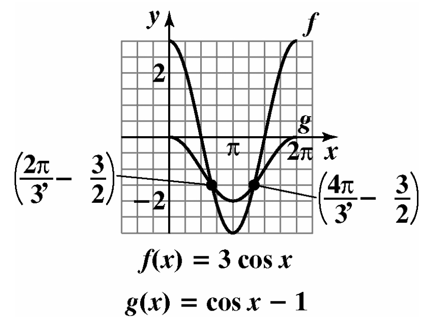
$\left(\tan x - 1\right)\left(3\tan x + 2\right) = 0$

$\tan x - 1 = 0$ or $3\tan x + 2 = 0$

$\tan x = 1$ $\qquad \tan x = -\dfrac{2}{3}$

$x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$ $\qquad x \approx 2.5536, 5.6952$

117.



$3\cos x = \cos x - 1$

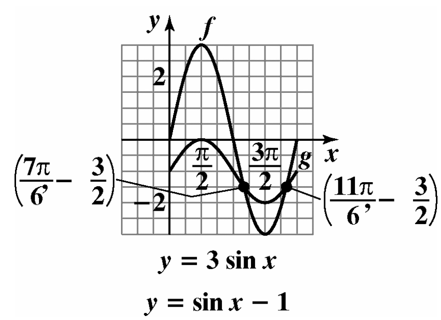
$2\cos x = -1$

$\cos x = -\dfrac{1}{2}$

$x = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

$\left(\dfrac{2\pi}{3}, -\dfrac{3}{2}\right), \left(\dfrac{4\pi}{3}, -\dfrac{3}{2}\right)$

118.



$3\sin x = \sin x - 1$

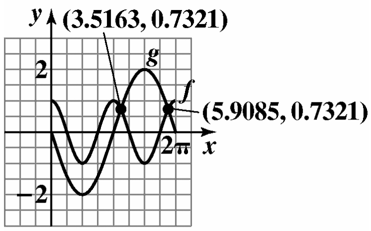
$2\sin x = -1$

$\sin x = -\dfrac{1}{2}$

$x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

$\left(\dfrac{7\pi}{6}, -\dfrac{3}{2}\right), \left(\dfrac{11\pi}{6}, -\dfrac{3}{2}\right)$

119.



$f(x) = \cos 2x$
$g(x) = -2\sin x$

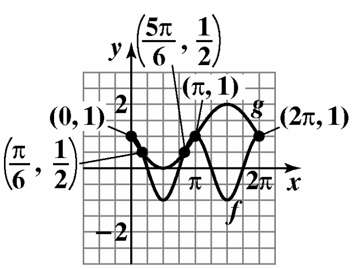
$$\cos 2x = -2\sin x$$
$$1 - 2\sin^2 x = -2\sin x$$
$$-2\sin^2 x + 2\sin x + 1 = 0$$
$$2\sin^2 x - 2\sin x - 1 = 0$$
$$\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\sin x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$$
$$\sin x = \frac{2 \pm \sqrt{12}}{4}$$
$$\sin x = \frac{2 \pm 2\sqrt{3}}{4}$$
$$\sin x = \frac{1 \pm \sqrt{3}}{2}$$

$\sin x \approx -0.3660$ or $\sin x \approx 1.3660$

$x \approx 3.5163,\ 5.9085$ ~~$\sin x \approx 1.3660$~~ This equation has no solution.

$(3.5163, 0.7321), (5.9085, 0.7321)$

120.



$f(x) = \cos 2x$
$g(x) = 1 - \sin x$

$$\cos 2x = 1 - \sin x$$
$$1 - 2\sin^2 x = 1 - \sin x$$
$$-2\sin^2 x + \sin x = 0$$
$$\sin x(-2\sin x + 1) = 0$$

$\sin x = 0$ or $-2\sin x + 1 = 0$

$x = 0, \pi, 2\pi$ $\quad \sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$$(0,1), \left(\frac{\pi}{6}, \frac{1}{2}\right), \left(\frac{5\pi}{6}, \frac{1}{2}\right), (\pi, 1), (2\pi, 1)$$

121. $|\cos x| = \frac{\sqrt{3}}{2}$

$\cos x = \frac{\sqrt{3}}{2}$ or $\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$ $\quad x = \frac{5\pi}{6}, \frac{7\pi}{6}$

122. $|\sin x| = \frac{1}{2}$

$\sin x = \frac{1}{2}$ or $\sin x = -\frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad x = \frac{7\pi}{6}, \frac{11\pi}{6}$

123.
$$10\cos^2 x + 3\sin x - 9 = 0$$
$$10(1 - \sin^2 x) + 3\sin x - 9 = 0$$
$$10 - 10\sin^2 x + 3\sin x - 9 = 0$$
$$-10\sin^2 x + 3\sin x + 1 = 0$$
$$10\sin^2 x - 3\sin x - 1 = 0$$
$$(2\sin x - 1)(5\sin x + 1) = 0$$

$2\sin x - 1 = 0$ or $5\sin x + 1 = 0$

$\sin x = \frac{1}{2}$ $\quad \sin x = -\frac{1}{5}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad x = 3.3430,\ 6.0818$

124.
$$3\cos^2 x - \sin x = \cos^2 x$$
$$2\cos^2 x - \sin x = 0$$
$$2\left(1-\sin^2 x\right) - \sin x = 0$$
$$2 - 2\sin^2 x - \sin x = 0$$
$$2\sin^2 x + \sin x - 2 = 0$$
$$\sin x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$\sin x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-2)}}{2(2)}$$
$$\sin x = \frac{-1 \pm \sqrt{17}}{4}$$
$\sin x = 0.7808$ or $\sin x = -1.2808$

$x = 0.8959,\ 2.2457$ ~~$\sin x = -1.2808$~~

125. $2\cos^3 x + \cos^2 x - 2\cos x - 1 = 0$
$$\cos^2 x\left(2\cos x + 1\right) - 1\left(2\cos x + 1\right) = 0$$
$$\left(2\cos x + 1\right)\left(\cos^2 x - 1\right) = 0$$
$$\left(2\cos x + 1\right)\left(\cos x + 1\right)\left(\cos x - 1\right) = 0$$
$$\cos x = -\frac{1}{2} \quad \text{or} \quad \cos x = -1 \quad \text{or} \quad \cos x = 1$$
$$x = 0,\ \frac{2\pi}{3},\ \pi,\ \frac{4\pi}{3}$$

126. $2\sin^3 x - \sin^2 x - 2\sin x + 1 = 0$
$$\sin^2 x\left(2\sin x - 1\right) - 1\left(2\sin x - 1\right) = 0$$
$$\left(2\sin x - 1\right)\left(\sin^2 x - 1\right) = 0$$
$$\left(2\sin x - 1\right)\left(\sin x + 1\right)\left(\sin x - 1\right) = 0$$
$$\sin x = \frac{1}{2} \quad \text{or} \quad \sin x = -1 \quad \text{or} \quad \sin x = 1$$
$$x = \frac{\pi}{6},\ \frac{\pi}{2},\ \frac{5\pi}{6},\ \frac{3\pi}{2}$$

127. 0.3649, 1.2059, 3.5064, 4.3475
This matches graph a.

128. 0.4636, 0.9828, 3.6052, 4.1244
This matches graph b.

129. Substitute $y = 0.3$ into the equation and solve for x:
$$0.3 = 0.6\sin\frac{2\pi}{5}x$$
$$\frac{0.3}{0.6} = \frac{0.6\sin\frac{2\pi}{5}x}{0.6}$$
$$\frac{1}{2} = \sin\frac{2\pi}{5}x$$
$$\sin\frac{2\pi}{5}x = \frac{1}{2}$$
The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{1}{2}$. One is $\frac{\pi}{6}$. The sine is positive in quadrant II; thus, the other value is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$. All of the solutions to $\sin\frac{2\pi}{5}x = \frac{1}{2}$ are given by
$$\frac{2\pi}{5}x = \frac{\pi}{6} + 2n\pi$$
$$x = \frac{5}{12} + 5n$$
or
$$\frac{2\pi}{5}x = \frac{5\pi}{6} + 2n\pi$$
$$x = \frac{25}{12} + 5n$$
where n is any integer. In the interval $[0, 5]$ we obtain solutions when $n = 0$. The solutions are $\frac{5}{12}$ and $\frac{25}{12}$.
Therefore, we are inhaling at 0.3 liter per second at $x = \frac{5}{12} \approx 0.4$ second and at $x = \frac{25}{12} \approx 2.1$ seconds.

130. When we exhale, velocity of air flow is negative, so $y = -0.3$ liters per second.
Solve:

$$-0.3 = 0.6\sin\frac{2\pi}{5}x$$

$$\frac{-0.3}{0.6} = \frac{0.6\sin\frac{2\pi}{5}x}{0.6}$$

$$-\frac{1}{2} = \sin\frac{2\pi}{5}x$$

$$\sin\frac{2\pi}{5}x = -\frac{1}{2}$$

The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $-\frac{1}{2}$. One is $\pi + \frac{\pi}{6} = \frac{7\pi}{6}$. The other is $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$. All the solutions to $\sin\frac{2\pi}{5}x = -\frac{1}{2}$ are given by

$$\frac{2\pi}{5}x = \frac{7\pi}{6} + 2n\pi$$

$$x = \frac{35}{12} + 5n$$

or $$\frac{2\pi}{5}x = \frac{11\pi}{6} + 2n\pi$$

$$x = \frac{55}{12} + 5n$$

where n is any integer.
In the interval $[0, 5]$ we obtain solutions when $n = 0$. The solutions are $\frac{35}{12}$ and $\frac{55}{12}$.
Therefore, we are exhaling at 0.3 liters per second at $x = \frac{35}{12} \approx 2.9$ seconds and at $x = \frac{55}{12} \approx 4.6$ seconds.

131. Substitute $y = 10.5$ into the equation and solve for x:

$$10.5 = 3\sin\left[\frac{2\pi}{365}(x-79)\right] + 12$$

$$-1.5 = 3\sin\left[\frac{2\pi}{365}(x-79)\right]$$

$$\frac{-1.5}{3} = \frac{3\sin\left[\frac{2\pi}{365}(x-79)\right]}{3}$$

$$-\frac{1}{2} = \sin\left[\frac{2\pi}{365}(x-79)\right]$$

$$\sin\left[\frac{2\pi}{365}(x-79)\right] = -\frac{1}{2}$$

The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $-\frac{1}{2}$. One is $\pi + \frac{\pi}{6} = \frac{7\pi}{6}$. The other is $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$. All the solutions to sin $\left[\frac{2\pi}{365}(x-79)\right] = -\frac{1}{2}$ are given by

$$\frac{2\pi}{365}(x-79) = \frac{7\pi}{6} + 2n\pi$$

$$x - 79 = \frac{2555}{12} + 365n$$

$$x = \frac{3503}{12} + 365n$$

or

$$\frac{2\pi}{365}(x-79) = \frac{11\pi}{6} + 2n\pi$$

$$x - 79 = \frac{4015}{12} + 365n$$

$$x = \frac{4963}{12} + 365n$$

where n is any integer.
Substitute various integers for n in the two equations. In the interval $[0, 365]$ we obtain values of 49 and 292 days. Thus, Boston has 10.5 hours of daylight 49 and 292 days after January 1.

132. Substitute $y = 13.5$ into the equation and solve for x:

$$13.5 = 3\sin\left[\frac{2\pi}{365}(x-79)\right] + 12$$

$$1.5 = 3\sin\left[\frac{2\pi}{365}(x-79)\right]$$

$$\frac{1.5}{3} = \frac{3\sin\left[\frac{2\pi}{365}(x-79)\right]}{3}$$

$$\frac{1}{2} = \sin\left[\frac{2\pi}{365}(x-79)\right]$$

$$\sin\left[\frac{2\pi}{365}(x-79)\right] = \frac{1}{2}$$

The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{1}{2}$. One is $\frac{\pi}{6}$. The sine function is positive in quadrant II; thus, the other value is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$. All the solutions to

$\sin\left[\frac{2\pi}{365}(x-79)\right] = \frac{1}{2}$ are given by

$$\frac{2\pi}{365}(x-79)=\frac{\pi}{6}+2n\pi$$
$$x-79=\frac{365}{12}+365n$$
$$x=\frac{1313}{12}+365n$$

or

$$\frac{2\pi}{365}(x-79)=\frac{5\pi}{6}+2n\pi$$
$$x-79=\frac{1825}{12}+365n$$
$$x=\frac{2773}{12}+365n$$

where n is any integer.
Substitute various integers for n in the two equations. In the interval $[0,365]$ we obtain values of 109 and 231 days. Thus, Boston has 13.5 hours of daylight 109 and 231 days after January 1.

133. Substitute $d = 2$ into the equation and solve for t:

$$2=-4\cos\frac{\pi}{3}t$$
$$\frac{2}{-4}=\frac{-4\cos\frac{\pi}{3}t}{-4}$$
$$-\frac{1}{2}=\cos\frac{\pi}{3}t$$
$$\cos\frac{\pi}{3}t=-\frac{1}{2}$$

The period of the cosine function is 2π. In the interval $[0,2\pi)$, there are two values at which the cosine function is $-\frac{1}{2}$. One is $\frac{2\pi}{3}$. The cosine function is negative in quadrant III; thus, the other value is $2\pi-\frac{2\pi}{3}=\frac{4\pi}{3}$. All solutions to $\cos\frac{\pi}{3}t=-\frac{1}{2}$ are given by

$$\frac{\pi}{3}t=\frac{2\pi}{3}+2n\pi$$
$$t=2+6n$$

or

$$\frac{\pi}{3}t=\frac{4\pi}{3}+2n\pi$$
$$t=4+6n$$

where n is any nonnegative integer.

134. Substitute $d = -2$ into the equation and solve for t:

$$-2=-4\cos\frac{\pi}{3}t$$
$$\frac{-2}{-4}=\frac{-4\cos\frac{\pi}{3}t}{-4}$$
$$\frac{1}{2}=\cos\frac{\pi}{3}t$$
$$\cos\frac{\pi}{3}t=\frac{1}{2}$$

The period of the cosine function is 2π. In the interval $[0,2\pi)$, there are two values at which the cosine function is $\frac{1}{2}$. One is $\frac{\pi}{3}$. The cosine function is positive in quadrant IV; thus, the other value is $2\pi-\frac{\pi}{3}=\frac{5\pi}{3}$. All the solutions to $\cos\frac{\pi}{3}t=\frac{1}{2}$ are given by

$$\frac{\pi}{3}t=\frac{\pi}{3}+2n\pi$$
$$t=1+6n$$

or

$$\frac{\pi}{3}t=\frac{5\pi}{3}+2n\pi$$
$$t=5+6n$$

where n is any nonnegative integer.

135. Substitute $v_0=90$ and $d = 170$, and solve for θ:

$$170=\frac{90^2}{16}\sin\theta\cos\theta$$
$$\frac{136}{405}=\sin\theta\cos\theta$$
$$2\cdot\frac{136}{405}=2\sin\theta\cos\theta$$
$$\frac{272}{405}=\sin 2\theta$$
$$\sin 2\theta=\frac{272}{405}$$

The period of the sine function is 360°. In the interval $[0,360°]$, there are two values at which the sine function is $\frac{272}{405}$.

One is $\sin^{-1}\left(\frac{272}{405}\right) \approx 42.19°$. The sine function is positive in quadrant II; Thus, the other value is $180° - 42.19° = 137.81°$. All solutions to

$\sin 2\theta = \frac{272}{405}$ are given by

$$2\theta = 42.19° + 360°n$$
$$\theta = 21.095° + 180°n$$

or

$$2\theta = 137.81° + 360°n$$
$$\theta = 68.905° + 180°n$$

where n is any integer.

In the interval $[0, 90°)$ we obtain the solutions by letting n = 0. The solutions are approximately 21° and 69°. Therefore, the angle of elevation should be 21° or 69°.

136. Substitute $v_o = 90$ and d = 200, and solve for θ:

$$200 = \frac{90^2}{16}\sin\theta\cos\theta$$
$$\frac{32}{81} = \sin\theta\cos\theta$$
$$2 \cdot \frac{32}{81} = 2\sin\theta\cos\theta$$
$$\frac{64}{81} = \sin 2\theta$$
$$\sin 2\theta = \frac{64}{81}$$

The period of the sine function is 360°. In the interval $[0, 360°]$, there are two values at which the sine function is $\frac{64}{81}$. One is $\sin^{-1}\left(\frac{64}{81}\right) \approx 52.20°$.

The sine function is positive in quadrant II; thus, the other value is $180° - 52.20° = 127.80°$. All solutions to $\sin 2\theta = \frac{64}{81}$ are given by

$$2\theta = 52.20° + 360°n$$
$$\theta = 26.10° + 180°n$$

or

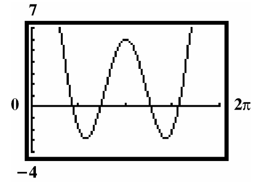
$$2\theta = 127.80° + 360°n$$
$$\theta = 63.90° + 180°n$$

where n is any integer.

In the interval $[0, 90°)$ we obtain the solutions by letting n = 0. The solutions are approximately 26° and 64°. Therefore, the angle of elevation should be 26° or 64°.

137. – 146. Answers may vary.

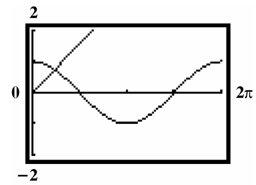
147.



$x = 1.37$, $x = 2.30$

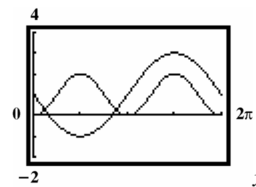
$x = 3.98$, or x=4.91

148.



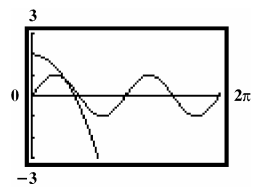
$x = 0.74$

149.



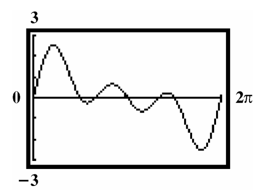
x = 0.37, 2.77

150.



$x = 1.08$

151.



$x = 0$, $x = 1.57$, $x = 2.09$,

$x = 3.14$, $x = 4.19$, or $x = 4.71$

152. does not make sense; Explanations will vary. Sample explanation: You did not attempt to "solve." You attempted to "verifiy" as if this was an identity.

153. makes sense

154. does not make sense; Explanations will vary. Sample explanation: It is more efficient to solve as follows.

$$\cos\left(x-\frac{\pi}{3}\right)=-1$$
$$x-\frac{\pi}{3}=\cos^{-1}(-1)$$
$$x-\frac{\pi}{3}=\pi$$
$$x=\pi+\frac{\pi}{3}$$
$$x=\frac{4\pi}{3}$$

155. does not make sense; Explanations will vary. Sample explanation: You do not need to solve a trigonometric equation. You nee to find a trigonometric value of an angle ang simplify using arithmetic.

156. true

157. false; Changes to make the statement true will vary. A sample change is: The equation has an infinite number of solutions

158. false; Changes to make the statement true will vary. A sample change is: To be an identity, the equation must be true for all defined values of the variable.

159. false; Changes to make the statement true will vary. A sample change is: Over this interval, the first equation has two solutions and the second equation has 4 solutions.

160. $2\cos x-1+3\sec x=0$

$$2\cos x-1=-3\sec x$$
$$2\cos x-1=\frac{-3}{\cos x}$$
$$\cos x\cdot(2\cos x-1)=\frac{-3}{\cos x}$$
$$2\cos^2 x-\cos x=-3$$
$$2\cos^2 x-\cos x+3=0$$

The equation is now in quadratic form $2t^2-t+3=0.$ Use the quadratic formula to solve.

$$\cos x=\frac{-(-1)\pm\sqrt{(-1)^2-4(2)(3)}}{2(2)}$$
$$\cos x=\frac{1\pm\sqrt{1-24}}{4}$$
$$\cos x=\frac{1\pm\sqrt{-23}}{4}$$

Since $\frac{1\pm\sqrt{-23}}{4}$ are not real numbers, the equation has no solution.

161. $\sin 3x+\sin x+\cos x=0$

$$2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)+\cos x=0$$
$$2\sin 2x\cos x+\cos x=0$$
$$\cos x(2\sin 2x+1)=0$$

$\cos x=0$ or $2\sin 2x+1=0$

$x=\frac{\pi}{2}$ $\quad x=\frac{3\pi}{2}$ $\qquad 2\sin 2x=-1$

$$\sin 2x=-\frac{1}{2}$$

The period of the sine function is $2\pi.$ In the interval $[0,2\pi),$ there are two values at which the sine function is $-\frac{1}{2}.$ One is $\pi+\frac{\pi}{6}=\frac{7\pi}{6}.$ The other is $2\pi-\frac{\pi}{6}=\frac{11\pi}{6}.$ All the solutions to $\sin 2x=-\frac{1}{2}$ are given by

$2x=\frac{7\pi}{6}+2n\pi$ or $2x=\frac{11\pi}{6}+2n\pi$

$x=\frac{7\pi}{12}+n\pi$ $\qquad x=\frac{11\pi}{12}+n\pi$

where n is any integer. The solutions in the interval $[0,2\pi)$ are obtained by letting n = 0 and n = 1.The solutions are $\frac{\pi}{2},\frac{3\pi}{2},\frac{7\pi}{12},\frac{11\pi}{12},\frac{19\pi}{12},$ and $\frac{23\pi}{12}.$

162.
$$\sin x + 2\sin\frac{x}{2} = \cos\frac{x}{2} + 1$$
$$\sin\left(2\cdot\frac{x}{2}\right) + 2\sin\frac{x}{2} = \cos\frac{x}{2} + 1$$
$$2\sin\frac{x}{2}\cos\frac{x}{2} + 2\sin\frac{x}{2} = \cos\frac{x}{2} + 1$$
$$2\sin\frac{x}{2}\cos\frac{x}{2} + 2\sin\frac{x}{2} - \cos\frac{x}{2} - 1 = 0$$
$$2\sin\frac{x}{2}\left(\cos\frac{x}{2} + 1\right) - \left(\cos\frac{x}{2} + 1\right) = 0$$
$$\left(\cos\frac{x}{2} + 1\right)\left(2\sin\frac{x}{2} - 1\right) = 0$$
$$\cos\frac{x}{2} + 1 = 0 \quad \text{or} \quad 2\sin\frac{x}{2} - 1 = 0$$
$$\cos\frac{x}{2} = -1 \qquad 2\sin\frac{x}{2} = 1$$
$$\sin\frac{x}{2} = \frac{1}{2}$$

The period of the sine function and cosine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $\frac{1}{2}$. One is $\frac{\pi}{6}$. The other is $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

In the interval $[0, 2\pi)$, the only value at which the cosine function is –1 is π. All of the solutions to $\cos\frac{x}{2} = -1$ are given by

$$\frac{x}{2} = \pi + 2n\pi$$
$$x = 2\pi + 4n\pi$$

where n is any integer. All of the solutions to $\sin\frac{x}{2} = \frac{1}{2}$ are given by

$$\frac{x}{2} = \frac{\pi}{6} + 2n\pi$$
$$x = \frac{\pi}{3} + 4n\pi$$

or

$$\frac{x}{2} = \frac{5\pi}{6} + 2n\pi$$
$$x = \frac{5\pi}{3} + 4n\pi$$

where n is any integer.

The solutions in the interval $[0, 2\pi)$, are obtained by letting n = 0. The solutions are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

163. $\dfrac{a}{\sin 46^\circ} = \dfrac{56}{\sin 63^\circ}$

$$a\sin 63^\circ = 56\sin 46^\circ$$
$$a = \frac{56\sin 46^\circ}{\sin 63^\circ}$$
$$a \approx 45.2^\circ$$

164. $\dfrac{81}{\sin 43^\circ} = \dfrac{62}{\sin B}$

$$81\sin B = 62\sin 43^\circ$$
$$\sin B = \frac{62\sin 43^\circ}{81}$$
$$\sin B \approx 0.522023436$$
$$B \approx \sin^{-1}(0.522023436)$$
$$B \approx 31.5^\circ$$

165. $\dfrac{51}{\sin 75^\circ} = \dfrac{71}{\sin B}$

$$51\sin B = 71\sin 75^\circ$$
$$\sin B = \frac{71\sin 75^\circ}{51}$$
$$\sin B \approx 1.344720268$$

No solution.

Chapter 6 Review Exercises

1.
$$\sec x - \cos x = \frac{1}{\cos x} - \cos x$$
$$= \frac{1}{\cos x} - \frac{\cos x}{1}\cdot\frac{\cos x}{\cos x}$$
$$= \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$$
$$= \frac{1-\cos^2 x}{\cos x}$$
$$= \frac{\sin^2 x}{\cos x}$$
$$= \frac{\sin x}{\cos x}\cdot\sin x$$
$$= \tan x\sin x$$

2. $\cos x + \sin x \tan x$

$= \dfrac{\cos x}{\cos x} \cdot \cos x + \sin x \cdot \dfrac{\sin x}{\cos x}$

$= \dfrac{\cos^2 x}{\cos x} + \dfrac{\sin^2 x}{\cos x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos x}$

$= \dfrac{1}{\cos x} = \sec x$

3. $\sin^2 \theta(1+\cot^2 \theta) = \sin^2 \theta + \sin^2 \theta \cot^2 \theta$

$= \sin^2 \theta + \sin^2 \theta \cdot \dfrac{\cos^2 \theta}{\sin^2 \theta}$

$= \sin^2 \theta + \cos^2 \theta$

$= 1$

4. $(\sec\theta - 1)(\sec\theta + 1) = \sec^2 \theta - 1$

$= 1 + \tan^2 \theta - 1$

$= \tan^2 \theta$

5. $\dfrac{1-\tan x}{\sin x} = \dfrac{1}{\sin x} - \dfrac{\tan x}{\sin x}$

$= \csc x - \dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\sin x}$

$= \csc x - \dfrac{1}{\cos x}$

$= \csc x - \sec x$

6. $\dfrac{1}{\sin t - 1} + \dfrac{1}{\sin t + 1}$

$= \dfrac{1}{\sin t - 1} \cdot \dfrac{\sin t + 1}{\sin t + 1} + \dfrac{1}{\sin t + 1} \cdot \dfrac{\sin t - 1}{\sin t - 1}$

$= \dfrac{\sin t + 1}{\sin^2 t - 1} + \dfrac{\sin t - 1}{\sin^2 t - 1}$

$= \dfrac{\sin t + 1 + \sin t - 1}{\sin^2 t - 1}$

$= \dfrac{2\sin t}{\sin^2 t - 1}$

$= \dfrac{2\sin t}{-\cos^2 t}$

$= -2 \cdot \dfrac{\sin t}{\cos t} \cdot \dfrac{1}{\cos t}$

$= -2\tan t \sec t$

7. $\dfrac{1+\sin t}{\cos^2 t} = \dfrac{1}{\cos^2 t} + \dfrac{\sin t}{\cos^2 t}$

$= \sec^2 t + \dfrac{\sin t}{\cos t} \cdot \dfrac{1}{\cos t}$

$= \tan^2 t + 1 + \tan t \sec t$

8. $\dfrac{\cos x}{1-\sin x} = \dfrac{\cos x}{1-\sin x} \cdot \dfrac{1+\sin x}{1+\sin x}$

$= \dfrac{\cos x(1+\sin x)}{1-\sin^2 x}$

$= \dfrac{\cos x(1+\sin x)}{\cos^2 x}$

$= \dfrac{1+\sin x}{\cos x}$

9. $1 - \dfrac{\sin^2 x}{1+\cos x} = 1 - \dfrac{1-\cos^2 x}{1+\cos x}$

$= 1 - \dfrac{(1+\cos x)(1-\cos x)}{1+\cos x}$

$= 1 - (1-\cos x)$

$= 1 - 1 + \cos x$

$= \cos x$

10. $(\tan\theta + \cot\theta)^2$

$= \tan^2 \theta + 2\tan\theta\cot\theta + \cot^2 \theta$

$= \sec^2 \theta - 1 + 2\dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\cos\theta}{\sin\theta} + \csc^2 \theta - 1$

$= \sec^2 \theta - 1 + 2 + \csc^2 \theta - 1$

$= \sec^2 \theta + \csc^2 \theta$

11. $\dfrac{1}{\sin\theta + \cos\theta} + \dfrac{1}{\sin\theta - \cos\theta}$

$= \dfrac{\sin\theta - \cos\theta}{\sin\theta - \cos\theta} \cdot \dfrac{1}{\sin\theta + \cos\theta}$

$+ \dfrac{\sin\theta + \cos\theta}{\sin\theta + \cos\theta} \cdot \dfrac{1}{\sin\theta - \cos\theta}$

$= \dfrac{\sin\theta - \cos\theta}{\sin^2 \theta - \cos^2 \theta} + \dfrac{\sin\theta + \cos\theta}{\sin^2 \theta - \cos^2 \theta}$

$= \dfrac{2\sin\theta}{\sin^2 \theta - \cos^2 \theta}$

$= \dfrac{2\sin\theta}{\sin^2 \theta - \cos^2 \theta} \cdot \dfrac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta + \cos^2 \theta}$

$= \dfrac{2\sin\theta \cdot 1}{\sin^4 \theta - \cos^4 \theta}$

$= \dfrac{2\sin\theta}{\sin^4 \theta - \cos^4 \theta}$

12. $$\frac{\cos t}{\cot t-5\cos t}=\frac{\cos t}{\cot t-5\cos t}\cdot\frac{\frac{1}{\cos t}}{\frac{1}{\cos t}}$$
$$=\frac{\frac{\cos t}{\cos t}}{\frac{\cot t-5\cos t}{\cos t}}$$
$$=\frac{1}{\frac{\cot t}{\cos t}-5}$$
$$=\frac{1}{\frac{\frac{\cos t}{\sin t}}{\cos t}-5}$$
$$=\frac{1}{\frac{\cos t}{\sin t}\cdot\frac{1}{\cos t}-5}$$
$$=\frac{1}{\frac{1}{\sin t}-5}$$
$$=\frac{1}{\csc t-5}$$

13. $$\frac{1-\cos t}{1+\cos t}=\frac{1-\cos t}{1+\cos t}\cdot\frac{1-\cos t}{1-\cos t}$$
$$=\frac{(1-\cos t)^2}{1-\cos^2 t}$$
$$=\frac{(1-\cos t)^2}{\sin^2 t}$$
$$=\left(\frac{1-\cos t}{\sin t}\right)^2$$
$$=\left(\frac{1}{\sin t}-\frac{\cos t}{\sin t}\right)^2$$
$$=(\csc t-\cot t)^2$$

14. $\cos(45°+30°)$
$$=\cos 45°\cos 30°-\sin 45°\sin 30°$$
$$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$
$$=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$$
$$=\frac{\sqrt{6}-\sqrt{2}}{4}$$

15. $\sin 195°=\sin(135°+60°)$
$$=\sin 135°\cos 60°+\cos 135°\sin 60°$$
$$=\frac{\sqrt{2}}{2}\cdot\frac{1}{2}+\left(-\frac{\sqrt{2}}{2}\right)\cdot\frac{\sqrt{3}}{2}$$
$$=\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}$$
$$=\frac{\sqrt{2}-\sqrt{6}}{4}$$

16. $$\tan\left(\frac{4\pi}{3}-\frac{\pi}{4}\right)=\frac{\tan\frac{4\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{4\pi}{3}\cdot\tan\frac{\pi}{4}}=\frac{\sqrt{3}-1}{1+\sqrt{3}\cdot(1)}$$
$$=\frac{\left(\sqrt{3}-1\right)\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)\left(1-\sqrt{3}\right)}=\frac{-\left(1-\sqrt{3}\right)^2}{1-3}$$
$$\frac{-\left(1-2\sqrt{3}+3\right)}{-2}=\frac{1-2\sqrt{3}+3}{2}$$
$$=\frac{4-2\sqrt{3}}{2}=\frac{2\left(2-\sqrt{3}\right)}{2}=2-\sqrt{3}$$

17. $$\tan\frac{5\pi}{12}=\tan\left(\frac{2\pi}{12}+\frac{3\pi}{12}\right)$$
$$=\tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$$
$$=\frac{\tan\frac{\pi}{6}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{6}\tan\frac{\pi}{4}}$$
$$=\frac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}\cdot 1}=\frac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}}\cdot\frac{\left(1+\frac{\sqrt{3}}{3}\right)}{\left(1+\frac{\sqrt{3}}{3}\right)}$$
$$=\frac{\frac{2\sqrt{3}}{3}+1+\frac{1}{3}}{1-\frac{1}{3}}=\frac{\frac{2\sqrt{3}}{3}+\frac{4}{3}}{\frac{2}{3}}$$
$$=\left(\frac{2\sqrt{3}}{3}+\frac{4}{3}\right)\cdot\frac{3}{2}=\sqrt{3}+2$$

18. $\cos 65°\cos 5°+\sin 65°\sin 5°$
$$=\cos(65°-5°)$$
$$=\cos 60°$$
$$=\frac{1}{2}$$

19. $\sin 80° \cos 50° - \cos 80° \sin 50°$
$= \sin(80° - 50°)$
$= \sin 30°$
$= \frac{1}{2}$

20. $\sin\left(x + \frac{\pi}{6}\right) - \cos\left(x + \frac{\pi}{3}\right)$
$= \sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6} - \left(\cos x \cos\frac{\pi}{3} - \sin x \sin\frac{\pi}{3}\right)$
$= \sin x \cdot \frac{\sqrt{3}}{2} + \cos x \cdot \frac{1}{2} - \left(\cos x \cdot \frac{1}{2} - \sin x \cdot \frac{\sqrt{3}}{2}\right)$
$= 2 \cdot \frac{\sqrt{3}}{2} \cdot \sin x$
$= \sqrt{3} \sin x$

21. $\tan\left(x + \frac{3\pi}{4}\right) = \frac{\tan x + \tan\frac{3\pi}{4}}{1 - \tan x \tan\frac{3\pi}{4}}$
$= \frac{\tan x + (-1)}{1 - \tan x(-1)}$
$= \frac{\tan x - 1}{1 + \tan x}$

22. $\sec(\alpha + \beta) = \frac{1}{\cos(\alpha + \beta)}$
$= \frac{1}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$
$= \frac{1}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} \cdot \frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}$
$= \frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$
$= \frac{\frac{1}{\cos\alpha} \cdot \frac{1}{\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$
$= \frac{\sec\alpha\sec\beta}{1 - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$
$= \frac{\sec\alpha\sec\beta}{1 - \tan\alpha\tan\beta}$

23. $\frac{\cos(\alpha - \beta)}{\cos\alpha\cos\beta} = \frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha\cos\beta}$
$= \frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}$
$= 1 + \tan\alpha\tan\beta$

24. $\cos^4 t - \sin^4 t = \left(\cos^2 t - \sin^2 t\right)\left(\cos^2 t + \sin^2 t\right)$
$= (\cos 2t) \cdot (1)$
$= \cos 2t$

25. $\sin t - \cos 2t = \sin t - \left(1 - 2\sin^2 t\right)$
$= \sin t - 1 + 2\sin^2 t$
$= 2\sin^2 t + \sin t - 1$
$= (2\sin t - 1)(\sin t + 1)$

26. $\frac{\sin 2\theta - \sin\theta}{\cos 2\theta + \cos\theta} = \frac{2\sin\theta\cos\theta - \sin\theta}{2\cos^2\theta - 1 + \cos\theta}$
$= \frac{\sin\theta(2\cos\theta - 1)}{2\cos^2\theta + \cos\theta - 1}$
$= \frac{\sin\theta(2\cos\theta - 1)}{(2\cos\theta - 1)(\cos\theta + 1)}$
$= \frac{\sin\theta}{\cos\theta + 1}$
$= \frac{\sin\theta}{\cos\theta + 1} \frac{\cos\theta - 1}{\cos\theta - 1}$
$= \frac{\sin\theta(\cos\theta - 1)}{\cos^2\theta - 1}$
$= \frac{\sin\theta(\cos\theta - 1)}{-\sin^2\theta}$
$= \frac{-(\cos\theta - 1)}{\sin\theta}$
$= \frac{1 - \cos\theta}{\sin\theta}$

27. $\frac{\sin 2\theta}{1 - \sin^2\theta} = \frac{2\sin\theta\cos\theta}{\cos^2\theta}$
$= \frac{2\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{\cos\theta}$
$= 2\tan\theta$

28. $2\sin t\cos t\sec 2t = \sin 2t \cdot \sec 2t$
$= \sin 2t \cdot \frac{1}{\cos 2t}$
$= \frac{\sin 2t}{\cos 2t}$
$= \tan 2t$

29. $\cos 4t = \cos(2\cdot 2t)$.

$$\begin{aligned}&= 1-2\sin^2 2t\\&= 1-2(\sin 2t)^2\\&= 1-2\cdot(2\sin t\cos t)^2\\&= 1-2\cdot 4\sin^2 t\cos^2 t\\&= 1-8\sin^2 t\cos^2 t\end{aligned}$$

30. $\tan\frac{x}{2}(1+\cos x) = \frac{\sin x}{1+\cos x}\cdot(1+\cos x) = \sin x$

31. $\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$

$$\begin{aligned}&= \frac{1-\cos x}{\sin x}\cdot\frac{\frac{1}{\cos x}}{\frac{1}{\cos x}}\\&= \frac{\frac{1-\cos x}{\cos x}}{\frac{\sin x}{\cos x}}\\&= \frac{\frac{1}{\cos x}-\frac{\cos x}{\cos x}}{\tan x}\\&= \frac{\sec x-1}{\tan x}\end{aligned}$$

32. **a.** The graph appears to be the cosine curve, $y=\cos x$. It cycles through maximum, intercept, minimum, intercept and back to maximum. Thus, $y=\cos x$ also describes the graph.

b. $\sin\left(x-\frac{3\pi}{2}\right) = \sin x\cos\frac{3\pi}{2} - \cos x\sin\frac{3\pi}{2}$

$$\begin{aligned}&= \sin x\cdot 0-\cos x\cdot(-1)\\&= \cos x\end{aligned}$$

33. **a.** The graph appears to be the negative of the sine curve, $y=-\sin x$. It cycles through intercept, minimum, intercept, maximum and back to intercept. Thus, $y=-\sin x$ also describes the graph.

b. $\cos\left(x+\frac{\pi}{2}\right) = \cos x\cos\frac{\pi}{2} - \sin x\sin\frac{\pi}{2}$

$$\begin{aligned}&= \cos x\cdot 0-\sin x\cdot 1\\&= -\sin x\end{aligned}$$

34. **a.** The graph appears to be the tangent curve, $y=\tan x$. It cycles through intercept to positive infinity, then from negative infinity through the intercept. Thus, $y=\tan x$ also describes the graph.

b. $y = \frac{\tan x-1}{1-\cot x}$

$$\begin{aligned}&= \frac{\frac{\sin x}{\cos x}-1}{1-\frac{\cos x}{\sin x}}\\&= \frac{\frac{\sin x-\cos x}{\cos x}}{\frac{\sin x-\cos x}{\sin x}}\\&= \frac{\sin x-\cos x}{\cos x}\cdot\frac{\sin x}{\sin x-\cos x}\\&= \frac{\sin x}{\cos x}\\&= \tan x\end{aligned}$$

35. $\sin\alpha = \frac{3}{5} = \frac{y}{r}$

Because α lies in quadrant I, x is positive.

$$\begin{aligned}x^2+3^2 &= 5^2\\x^2 &= 5^2-3^2 = 16\\x &= \sqrt{16} = 4\end{aligned}$$

Thus, $\cos\alpha = \frac{x}{r} = \frac{4}{5}$, and $\tan\alpha = \frac{y}{x} = \frac{3}{4}$.

$\sin\beta = \frac{12}{13} = \frac{y}{r}$

Because β lies in quadrant II, x is negative.

$$\begin{aligned}x^2+12^2 &= 13^2\\x^2 &= 13^2-12^2 = 25\\x &= -\sqrt{25} = -5\end{aligned}$$

Thus, $\cos\beta = \frac{x}{r} = \frac{-5}{13} = -\frac{5}{13}$, and

$\tan\beta = \frac{y}{x} = \frac{12}{-5} = -\frac{12}{5}$.

a. $\sin(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\beta$

$$= \frac{3}{5}\cdot\left(-\frac{5}{13}\right)+\frac{4}{5}\cdot\frac{12}{13} = \frac{33}{65}$$

b. $\cos(\alpha-\beta) = \cos\alpha\cos\beta+\sin\alpha\sin\beta$

$$= \frac{4}{5}\cdot\left(-\frac{5}{13}\right)+\frac{3}{5}\cdot\frac{12}{13} = \frac{16}{65}$$

c. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$=\dfrac{\frac{3}{4}+\left(-\frac{12}{5}\right)}{1-\frac{3}{4}\left(-\frac{12}{5}\right)}$

$=\dfrac{-\frac{33}{20}}{1+\frac{36}{20}}=\dfrac{-\frac{33}{20}}{\frac{56}{20}}$

$=-\dfrac{33}{56}$

d. $\sin 2\alpha=2\sin\alpha\cos\alpha=2\cdot\dfrac{3}{5}\cdot\dfrac{4}{5}=\dfrac{24}{25}$

e. $\cos\dfrac{\beta}{2}=\sqrt{\frac{1+\cos\beta}{2}}=\sqrt{\frac{1-\frac{5}{13}}{2}}$

$=\sqrt{\frac{8}{26}}=\sqrt{\frac{4}{13}}=\dfrac{2}{\sqrt{13}}=\dfrac{2\sqrt{13}}{13}$

36. $\tan\alpha=\dfrac{4}{3}=\dfrac{-4}{-3}=\dfrac{y}{x}$

Because r is a distance, it is positive.

$r^2=(-4)^2+(-3)^2=25$

$r=\sqrt{25}=5$

Thus, $\sin\alpha=\dfrac{y}{r}=\dfrac{-4}{5}=-\dfrac{4}{5}$, and

$\cos\alpha=\dfrac{x}{r}=\dfrac{-3}{5}=-\dfrac{3}{5}.$

$\tan\beta=\dfrac{5}{12}=\dfrac{y}{x}$

Because r is a distance, it is positive.

$r^2=5^2+12^2=169$

$r=\sqrt{169}=13$

Thus, $\sin\beta=\dfrac{y}{r}=\dfrac{5}{13}$, and $\cos\beta=\dfrac{x}{r}=\dfrac{12}{13}.$

a. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$=-\dfrac{4}{5}\cdot\dfrac{12}{13}+\left(-\dfrac{3}{5}\right)\cdot\dfrac{5}{13}$

$=-\dfrac{63}{65}$

b. $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$=-\dfrac{3}{5}\cdot\dfrac{12}{13}+\left(-\dfrac{4}{5}\right)\cdot\dfrac{5}{13}$

$=-\dfrac{56}{65}$

c. $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$=\dfrac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\cdot\frac{5}{12}}=\dfrac{\frac{21}{12}}{1-\frac{20}{36}}=\dfrac{\frac{21}{12}}{\frac{16}{36}}$

$=\dfrac{21}{12}\cdot\dfrac{36}{16}=\dfrac{63}{16}$

d. $\sin 2\alpha=2\sin\alpha\sin\alpha$

$=2\left(-\dfrac{4}{5}\right)\left(-\dfrac{4}{5}\right)=\dfrac{24}{25}$

e. $\cos\dfrac{\beta}{2}=\sqrt{\dfrac{1+\cos\beta}{2}}=\sqrt{\dfrac{1+\frac{12}{13}}{2}}$

$=\sqrt{\dfrac{25}{26}}=\dfrac{5}{\sqrt{26}}=\dfrac{5\sqrt{26}}{26}$

37.. $\tan\alpha=-3=\dfrac{3}{-1}=\dfrac{y}{x}$

Because r is a distance, it is positive.

$r^2=3^2+(-1)^2$

$r^2=10$

$r=\sqrt{10}$

$\sin\alpha=\dfrac{3}{\sqrt{10}}=\dfrac{3\sqrt{10}}{10}$

$\cos\alpha=\dfrac{-1}{\sqrt{10}}=-\dfrac{\sqrt{10}}{10}$

$\cot\beta=-3=\dfrac{3}{-1}=\dfrac{x}{y}$

Because r is a distance, it is positive.

$r^2=3^2+(-1)^2$

$r^2=10$

$r=\sqrt{10}$

$\sin\beta=\dfrac{-1}{\sqrt{10}}=-\dfrac{\sqrt{10}}{10}$

$\cos\beta=\dfrac{3}{\sqrt{10}}=\dfrac{3\sqrt{10}}{10}$

a. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
$=\frac{3\sqrt{10}}{10}\cdot\frac{3\sqrt{10}}{10}+\left(-\frac{\sqrt{10}}{10}\right)\left(-\frac{\sqrt{10}}{10}\right)$
$=\frac{90}{100}+\frac{10}{100}$
$=\frac{100}{100}$
$=1$

b.

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
$=\left(-\frac{\sqrt{10}}{10}\right)\left(\frac{3\sqrt{10}}{10}\right)+\frac{3\sqrt{10}}{10}\left(\frac{-\sqrt{10}}{10}\right)$
$=-\frac{60}{100}$
$=-\frac{3}{5}$

c. $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
$=\frac{-3+\left(\frac{-1}{3}\right)}{1-(-3)\left(-\frac{1}{3}\right)}$
$=\frac{\frac{-10}{3}}{0}$
Since this value is undefined, the tangent function is undefined at $\alpha+\beta$.

d. $\sin 2\alpha=2\sin\alpha\cos\alpha$
$=2\left(\frac{3\sqrt{10}}{10}\right)\left(\frac{-\sqrt{10}}{10}\right)$
$=-\frac{3}{5}$

e. $\cos\frac{\beta}{2}=\sqrt{\frac{1+\cos\beta}{2}}$
$=\sqrt{\frac{1+\frac{3\sqrt{10}}{10}}{2}}$
$=\sqrt{\frac{10+3\sqrt{10}}{20}}$
$=\frac{\sqrt{10+3\sqrt{10}}}{2\sqrt{5}}$

38. $\sin\alpha=-\frac{1}{3}=\frac{-1}{3}=\frac{y}{r}$
Because α is in quadrant II, x is negative.
$x^2+(-1)^2=3^2$
$x^2+1=9$
$x^2=8$
$x=-\sqrt{8}=-2\sqrt{2}$
$\cos\alpha=\frac{-2\sqrt{2}}{3}$
$\tan\alpha=\frac{-1}{-2\sqrt{2}}=\frac{\sqrt{2}}{4}$
$\cos\beta=-\frac{1}{3}=\frac{-1}{3}=\frac{x}{r}$
Because β is in quadrant III, y is negative.
$(-1)^2+y^2=3^2$
$y^2=8$
$y=-\sqrt{8}=-2\sqrt{2}$
$\sin\beta=\frac{-2\sqrt{2}}{3}$
$\tan\beta=\frac{-2\sqrt{2}}{-1}=2\sqrt{2}$

a. $\sin(\alpha+\beta)$
$=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
$=-\frac{1}{3}\cdot-\frac{1}{3}+\left(-\frac{2\sqrt{2}}{3}\right)\cdot\left(-\frac{2\sqrt{2}}{3}\right)$
$=\frac{9}{9}=1$

b. $\cos(\alpha-\beta)$
$=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
$=-\frac{2\sqrt{2}}{3}\cdot\left(-\frac{1}{3}\right)+\left(-\frac{1}{3}\right)\cdot\left(-\frac{2\sqrt{2}}{3}\right)$
$=\frac{4\sqrt{2}}{9}$

c. $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan B}$
$=\frac{\frac{\sqrt{2}}{4}+2\sqrt{2}}{1-\left(\frac{\sqrt{2}}{4}\right)\left(2\sqrt{2}\right)}$
$=\frac{\frac{9\sqrt{2}}{4}}{0}$
Since this value is undefined, the tangent function is undefined at $\alpha+\beta$.

d. $\sin 2\alpha=2\sin\alpha\cos\alpha$
$=2\left(-\frac{1}{3}\right)\left(-\frac{2\sqrt{2}}{3}\right)=\frac{4\sqrt{2}}{9}$

e. $\cos\frac{\beta}{2}=-\sqrt{\frac{1+\cos\beta}{2}}$
$=-\sqrt{\frac{1+\left(-\frac{1}{3}\right)}{2}}$
$=-\sqrt{\frac{\frac{2}{3}}{2}}=-\sqrt{\frac{1}{3}}$
$=-\frac{1}{\sqrt{3}}$
$=-\frac{\sqrt{3}}{3}$

39. The given expression is the right side of the formula for $\cos 2\theta$ with $\theta=15°$.
$\cos^2 15°-\sin^2 15°=\cos(2\cdot 15°)$
$=\cos 30°$
$=\frac{\sqrt{3}}{2}$

40. The given expression is the right side of the formula for $\tan 2\theta$ with $\theta=\frac{5\pi}{12}$.
$\frac{2\tan\frac{5\pi}{12}}{1-\tan^2\frac{5\pi}{12}}=\tan\left(2\cdot\frac{5\pi}{12}\right)$
$=\tan\frac{5\pi}{6}$
$=-\frac{\sqrt{3}}{3}$

41. Because 22.5° lies in quadrant I, $\sin 22.5°>0$.
$\sin 22.5°=\sin\frac{45°}{2}$
$=\sqrt{\frac{1-\cos 45°}{2}}=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$
$=\sqrt{\frac{2-\sqrt{2}}{4}}=\frac{\sqrt{2-\sqrt{2}}}{2}$

42. Because $\frac{\pi}{12}$ lies in quadrant I, $\tan\frac{\pi}{12}>0$.
$\tan\frac{\pi}{12}=\tan\frac{\frac{\pi}{6}}{2}$
$=\frac{1-\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}}=\frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$
$=2-\sqrt{3}$

43. $\sin 6x\sin 4x$
$=\frac{1}{2}\left[\cos(6x-4x)-\cos(6x+4x)\right]$
$=\frac{1}{2}\left[\cos 2x-\cos 10x\right]$

44. $\sin 7x\cos 3x$
$=\frac{1}{2}\left[\sin(7x+3x)+\sin(7x-3x)\right]$
$=\frac{1}{2}\left[\sin 10x+\sin 4x\right]$

45. $\sin 2x-\sin 4x$
$=2\sin\left(\frac{2x-4x}{2}\right)\cos\left(\frac{2x+4x}{2}\right)$
$=2\sin(-x)\cos 3x$
$=-2\sin x\cos 3x$

46. $\cos 75° + \cos 15°$

$= 2\cos\left(\frac{75° + 15°}{2}\right)\cos\left(\frac{75° - 15°}{2}\right)$

$= 2\cos 45° \cos 30°$

$= 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6}}{2}$

47. $\frac{\cos 3x + \cos 5x}{\cos 3x - \cos 5x} = \frac{2\cos\left(\frac{3x+5x}{2}\right)\cos\left(\frac{3x-5x}{2}\right)}{-2\sin\left(\frac{3x+5x}{2}\right)\sin\left(\frac{3x-5x}{2}\right)}$

$= \frac{2\cos\left(\frac{8x}{2}\right)\cos\left(\frac{-2x}{2}\right)}{-2\sin\left(\frac{8x}{2}\right)\sin\left(\frac{-2x}{2}\right)}$

$= \frac{2\cos 4x\cos(-x)}{-2\sin 4x\sin(-x)}$

$= \frac{2\cos 4x\cos x}{2\sin 4x\sin x}$

$= \frac{\cos 4x}{\sin 4x}\cdot\frac{\cos x}{\sin x}$

$= \cot 4x\cot x$

$= \cot x\cot 4x$

48. $\frac{\sin 2x + \sin 6x}{\sin 2x - \sin 6x} = \frac{2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)}{2\sin\left(\frac{2x-6x}{2}\right)\cos\left(\frac{2x+6x}{2}\right)}$

$= \frac{2\sin\left(\frac{8x}{2}\right)\cos\left(\frac{-4x}{2}\right)}{2\sin\left(\frac{-4x}{2}\right)\cos\left(\frac{8x}{2}\right)}$

$= \frac{\sin 4x\cos(-2x)}{\sin(-2x)\cos 4x}$

$= -\frac{\sin 4x\cos 2x}{\sin 2x\cos 4x}$

$= -\frac{\sin 4x}{\cos 4x}\cdot\frac{\cos 2x}{\sin 2x}$

$= -\tan 4x\cot 2x$

49. **a.** The graph appears to be the cotangent curve, $y = \cot x$. It cycles from positive infinity through the intercept to negative infinity. Thus, $y = \cot x$ also describes the graph.

b. $\frac{\cos 3x + \cos x}{\sin 3x - \sin x} = \frac{2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)}{2\sin\left(\frac{3x-x}{2}\right)\cos\left(\frac{3x+x}{2}\right)}$

$= \frac{2\cos\left(\frac{4x}{2}\right)\cos\left(\frac{2x}{2}\right)}{2\sin\left(\frac{2x}{2}\right)\cos\left(\frac{4x}{2}\right)}$

$= \frac{2\cos 2x\cos x}{2\sin x\cos 2x}$

$= \frac{\cos x}{\sin x}$

$= \cot x$

This verifies our observation that $y = \frac{\cos 3x + \cos x}{\sin 3x - \sin x}$ and $y = \cot x$ describe the same graph.

50. $\cos x = -\frac{1}{2}$

Because $\cos\frac{\pi}{3} = \frac{1}{2}$, the solutions for $\cos x = -\frac{1}{2}$ in $[0, 2\pi)$ are $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$.

Because the period of the cosine function is 2π, the solutions are given by

$x = \frac{2\pi}{3} + 2n\pi$ or $x = \frac{4\pi}{3} + 2n\pi$ where n is any integer.

51. $\sin x = \frac{\sqrt{2}}{2}$

Because $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, the solutions for $\sin x = \frac{\sqrt{2}}{2}$ in $[0, 2\pi)$ are $x = \frac{\pi}{4}$ and $x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$. Because the period of the cosine function is 2π, the solutions are given by $x = \frac{\pi}{4} + 2n\pi$ or $x = \frac{3\pi}{4} + 2n\pi$ where n is any integer.

52. $2\sin x + 1 = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

Because $\sin\frac{\pi}{6} = \frac{1}{2}$, the solutions for

$\sin x = -\frac{1}{2}$ in $[0, 2\pi)$ are

$x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}.$

Because the period of the sine function is 2π, the solutions are given by

$x = \frac{7\pi}{6} + 2n\pi$ or $x = \frac{11\pi}{6} + 2n\pi$

where n is any integer

53. $\sqrt{3}\tan x - 1 = 0$

$\sqrt{3}\tan x = 1$

$\tan x = \frac{1}{\sqrt{3}}$

Because $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, the solution for

$\tan x = \frac{1}{\sqrt{3}}$ in $[0, \pi)$ is $x = \frac{\pi}{6}$.

Because the period of the tangent function is π, the solutions are given by

$x = \frac{\pi}{6} + n\pi$ where n is any integer.

54. The period of the cosine function is 2π. In the interval $[0, 2\pi)$, the only value at which the cosine function is –1 is π. All the solutions to $\cos 2x = -1$ are given by

$2x = \pi + 2n\pi$

$x = \frac{\pi}{2} + n\pi$ where n is any integer.

The solutions in the interval $[0, 2\pi)$ are obtained by letting n = 0 and n = 1.

The solutions are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

55. The period of the sine function is 2π. In the interval $[0, 2\pi)$, the only value at which the sine function is 1 is $\frac{\pi}{2}$. All the solutions to $\sin 3x = 1$ are given by

$3x = \frac{\pi}{2} + 2n\pi$

$x = \frac{\pi}{6} + \frac{2n\pi}{3}$

where n is any integer. The solutions in the interval $[0, 2\pi)$ are obtained by letting n = 0, n = 1, and n =2.

The solutions are $\frac{\pi}{6}, \frac{5\pi}{6}$, and $\frac{9\pi}{6}$.

56. The period of the tangent function is π. In the interval $[0, \pi)$, the only value for which the tangent function is –1 is $\frac{3\pi}{4}$. All the solutions to

$\tan\frac{x}{2} = -1$ are given by

$\frac{x}{2} = \frac{3\pi}{4} + n\pi$

$x = \frac{3\pi}{2} + 2n\pi$

where n is any integer. The solution in the interval $[0, 2\pi)$ is obtained by letting n = 0.

The solution is $\frac{3\pi}{2}$.

57. $\tan x = 2\cos x \tan x$

$\tan x - 2\cos x \tan x = 0$

$\tan x(1 - 2\cos x) = 0$

$\tan x = 0$ or $1 - 2\cos x = 0$

$x = 0$ $x = \pi$ $\quad -2\cos x = -1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$ $\quad x = \frac{5\pi}{3}$

The solutions in the interval $[0, 2\pi)$ are 0, $\frac{\pi}{3}$, π, and $\frac{5\pi}{3}$.

58. The given equation is in quadratic form $t^2 - 2t = 3$ with $t = \cos x$.

$\cos^2 x - 2\cos x = 3$

$\cos^2 x - 2\cos x - 3 = 0$

$(\cos x + 1)(\cos x - 3) = 0$

$\cos x + 1 = 0$ or $\cos x - 3 = 0$

$\cos x = -1$ $\quad$ $\cos x = 3$

$x = \pi$ $\quad$ $\cos x$ cannot be greater than 1.

The solution in the interval $[0, 2\pi)$ is π.

59. $2\cos^2 x - \sin x = 1$

$2\left(1 - \sin^2 x\right) - \sin x = 1$

$2 - 2\sin^2 x - \sin x - 1 = 0$

$-2\sin^2 x - \sin x + 1 = 0$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$2\sin x = 1$ $\quad$ $\sin x = -1$

$\sin x = \frac{1}{2}$ $\quad$ $x = \frac{3\pi}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

60. The given equation is in quadratic form $4t^2 = 1$ with $t = \sin x$.

$4\sin^2 x = 1$

$4\sin^2 x - 1 = 0$

$(2\sin x - 1)(2\sin x + 1) = 0$

$2\sin x - 1 = 0$ or $2\sin x + 1 = 0$

$2\sin x = 1$ $\quad$ $2\sin x = -1$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -\frac{1}{2}$

$x = \frac{\pi}{6}$ $\quad x = \frac{5\pi}{6}$ $\quad x = \frac{7\pi}{6}$ $\quad x = \frac{11\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

61. $\cos 2x - \sin x = 1$

$2\cos^2 x - 1 - \sin x = 1$

$2\left(1 - \sin^2 x\right) - \sin x - 2 = 0$

$2 - 2\sin^2 x - \sin x - 2 = 0$

$-2\sin^2 x - \sin x = 0$

$2\sin^2 x + \sin x = 0$

$\sin x\left(2\sin x + 1\right) = 0$

$\sin x = 0$ $\quad$ $2\sin x + 1 = 0$

$x = 0, \pi$ $\quad$ $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are $0, \pi, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$..

62. $\sin 2x = \sqrt{3}\sin x$

$2\sin x\cos x = \sqrt{3}\sin x$

$2\sin x\cos x - \sqrt{3}\sin x = 0$

$\sin x\left(2\cos x - \sqrt{3}\right) = 0$

$\sin x = 0$ or $2\cos x - \sqrt{3} = 0$

$x = 0$ $\;x = \pi$ $\quad$ $2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}$ $\quad x = \frac{11\pi}{6}$

The solutions in the interval $[0, 2\pi)$ are 0, $\frac{\pi}{6}$, π, and $\frac{11\pi}{6}$.

63. $\sin x = \tan x$

$\sin x = \frac{\sin x}{\cos x}$

$\sin x \bullet \cos x = \sin x$

$\sin x\cos x - \sin x = 0$

$\sin x(\cos x - 1) = 0$

$\sin x = 0$ or $\cos x - 1 = 0$

$x = 0$ $\;x = \pi$ $\quad$ $\cos x = 1$

$x = 0$

The solutions in the interval $[0, 2\pi)$ are 0 and π.

64. $\sin x = -0.6031$
Be sure calculator is in radian mode and find the inverse sine of +0.6031. This gives the first quadrant reference angle.
$\theta = \sin^{-1}(0.6031) \approx 0.6474$
The sine is negative in quadrants III and IV thus,
$x \approx \pi + 0.6474$ or $x \approx 2\pi - 0.6474$
$x \approx 3.7890$ $x \approx 5.6358$

65. $5\cos^2 x - 3 = 0$
$$\cos^2 x = \frac{3}{5}$$
$$\cos x = \pm\sqrt{\frac{3}{5}}$$
$$\cos x = \pm\frac{\sqrt{15}}{5}$$
$\cos x \approx 0.7746$ or $\cos x \approx -0.7746$
$x \approx 0.6847,\ 5.5985$ $x \approx 2.4569,\ 3.8263$

66. $1 + \tan^2 x = 4\tan x - 2$
$$\tan^2 x - 4\tan x + 3 = 0$$
$$(\tan x - 1)(\tan x - 3) = 0$$
$\tan x = 1$ or $\tan x = 3$
$x = \frac{\pi}{4}, \frac{5\pi}{4}$ $x \approx 1.2490,\ 4.3906$

67. $2\sin^2 x + \sin x - 2 = 0$
$$\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\sin x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-2)}}{2(2)}$$
$$\sin x = \frac{-1 \pm \sqrt{17}}{4}$$
$\sin x = 0.7808$ or $\sin x = -1.2808$
$x = 0.8959,\ 2.2457$ ~~$\sin x = -1.2808$~~

68. Substitute $d = -3$ into the equation and solve for *t:*
$$-3 = -6\cos\frac{\pi}{2}t$$
$$\frac{-3}{-6} = \frac{-6\cos\frac{\pi}{2}t}{-6}$$
$$\frac{1}{2} = \cos\frac{\pi}{2}t$$
$$\cos\frac{\pi}{2}t = \frac{1}{2}$$
The period of the cosine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the cosine function is $\frac{1}{2}$. One is $\frac{\pi}{3}$. The cosine function is positive in quadrant IV. Thus, the other value is $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$.

All solutions to $\cos\frac{\pi}{2}t = \frac{1}{2}$ are given by
$$\frac{\pi}{2}t = \frac{\pi}{3} + 2n\pi$$
$$\frac{\pi}{2}t = \frac{5\pi}{3} + 2n\pi$$
$$t = \frac{2}{3} + 4n \text{ or}$$
$$t = \frac{10}{3} + 4n$$
where *n* is any integer.

69. Substitute $v_0 = 90$ and $d = 100$, and solve for θ:
$$100 = \frac{90^2}{16}\sin\theta\cos\theta$$
$$\frac{16}{81} = \sin\theta\cos\theta$$
$$2\cdot\frac{16}{81} = 2\sin\theta\cos\theta$$
$$\frac{32}{81} = \sin 2\theta$$
$$\sin 2\theta = \frac{32}{81}$$
The period of the sine function is 360°. In the interval $[0, 360°)$, there are two values at which the sine function is $\frac{32}{81}$. One is $\sin^{-1}\left(\frac{32}{81}\right) \approx 23.27°$. The sine function is positive in quadrant II. Thus, the other value is 180° – 23.27° = 156.73°. All solutions to $\sin 2\theta = \frac{32}{81}$ are given by
$$2\theta = 23.27° + 360°n$$
$$\theta = 11.635° + 180°n$$
or
$$2\theta = 156.73° + 360°n$$
$$\theta = 78.365° + 180°n$$
where *n* is any integer.
In the interval $[0, 90°)$ we obtain the solutions by letting *n* = 0. The solutions are approximately 12° and 78°. Therefore, the angle of elevation should be 12° or 78°.

Chapter 6 Test

For problems 1–4: $\sin\alpha = \frac{4}{5} = \frac{y}{r}$

Because α lies in quadrant II, x is negative.

$$x^2 + 4^2 = 5^2$$
$$x^2 = 5^2 - 4^2 = 9$$
$$x = -\sqrt{9} = -3$$

Thus, $\cos\alpha = \frac{x}{r} = \frac{-3}{5} = -\frac{3}{5}$, and

$$\tan\alpha = \frac{y}{x} = \frac{4}{-3} = -\frac{4}{3}.$$

$$\cos\beta = \frac{5}{13} = \frac{x}{r}$$

Because β lies in quadrant I, y is positive.

$$5^2 + y^2 = 13^2$$
$$y^2 = 13^2 - 5^2 = 144$$
$$y = \sqrt{144} = 12$$

Thus, $\sin\beta = \frac{y}{r} = \frac{12}{13}$, and $\tan\beta = \frac{y}{x} = \frac{12}{5}.$

1. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= -\frac{3}{5}\cdot\frac{5}{13} - \frac{4}{5}\cdot\frac{12}{13} = -\frac{63}{65}$$

2. $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

$$= \frac{-\frac{4}{3}-\frac{12}{5}}{1+\left(-\frac{4}{3}\right)\cdot\frac{12}{5}} = \frac{-\frac{56}{15}}{-\frac{33}{15}} = \frac{56}{33}$$

3. $\sin 2\alpha = 2\sin\alpha\cos\alpha \; = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = -\frac{24}{25}$

4. $\cos\frac{\beta}{2} = \sqrt{\frac{1+\cos\beta}{2}} = \sqrt{\frac{1+\frac{5}{13}}{2}} = \sqrt{\frac{18}{26}}$

$$= \frac{3\sqrt{2}}{\sqrt{26}} = \frac{3\sqrt{52}}{26} = \frac{3\cdot 2\sqrt{13}}{26}$$
$$= \frac{3\sqrt{13}}{13}$$

5. $\sin 105^\circ = \sin(135^\circ - 30^\circ)$

$$= \sin 135^\circ \cos 30^\circ - \cos 135^\circ \sin 30^\circ$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\cdot\frac{1}{2}$$
$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

6. $\cos x \csc x = \cos x \cdot \frac{1}{\sin x} = \frac{\cos x}{\sin x} = \cot x$

7. $\frac{\sec x}{\cot x + \tan x} = \frac{\frac{1}{\cos x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}$

$$= \frac{\frac{1}{\cos x}}{\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}}$$
$$= \frac{\frac{1}{\cos x}}{\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}} = \frac{\frac{1}{\cos x}}{\frac{1}{\sin x \cos x}}$$
$$= \frac{1}{\cos x}\cdot\frac{\sin x\cos x}{1}$$
$$= \sin x$$

8. $1 - \frac{\cos^2 x}{1+\sin x} = 1 - \frac{\left(1-\sin^2 x\right)}{1+\sin x}$

$$= 1 - \frac{(1+\sin x)(1-\sin x)}{1+\sin x}$$
$$= 1 - (1-\sin x)$$
$$= \sin x$$

9. $\cos\left(\theta + \frac{\pi}{2}\right) = \cos\theta\cos\frac{\pi}{2} - \sin\theta\sin\frac{\pi}{2}$

$$= \cos\theta \cdot 0 - \sin\theta \cdot 1$$
$$= -\sin\theta$$

10. $\frac{\sin(\alpha-\beta)}{\sin\alpha\cos\beta} = \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\sin\alpha\cos\beta}$

$$= \frac{\sin\alpha\cos\beta}{\sin\alpha\cos\beta} - \frac{\cos\alpha\sin\beta}{\sin\alpha\cos\beta}$$
$$= 1 - \cot\alpha\tan\beta$$

11. $\sin t\cos t(\tan t + \cot t) \; = \sin t \cos t\left(\frac{\sin t}{\cos t} + \frac{\cos t}{\sin t}\right)$

$$= \frac{\sin^2 t\cos t}{\cos t} + \frac{\sin t\cos^2 t}{\sin t}$$
$$= \sin^2 t + \cos^2 t$$
$$= 1$$

12. The period of the sine function is 2π. In the interval $[0, 2\pi)$, there are two values at which the sine function is $-\frac{1}{2}$. One is $\pi + \frac{\pi}{6} = \frac{7\pi}{6}$. The other is $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$. All the solutions to $\sin 3x = -\frac{1}{2}$ are given by $3x = \frac{7\pi}{6} + 2n\pi$

$$x = \frac{7\pi}{18} + \frac{2n\pi}{3}$$

or

$$3x = \frac{11\pi}{6} + 2n\pi$$

$$x = \frac{11\pi}{18} + \frac{2n\pi}{3}$$

where n is any integer. The solutions in the interval $[0, 2\pi)$ are obtained by letting $n = 0$, $n = 1$, and $n = 2$.

The solutions are $\frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}$, and $\frac{35\pi}{18}$.

13.

$$\sin 2x + \cos x = 0$$

$$2\sin x \cos x + \cos x = 0$$

$$\cos x(2\sin x + 1) = 0$$

$\cos x = 0$ or $2\sin x + 1 = 0$

$x = \frac{\pi}{2}$ $x = \frac{3\pi}{2}$ $\qquad 2\sin x = -1$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{7\pi}{6} \quad x = \frac{11\pi}{6}$$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$, and $\frac{11\pi}{6}$.

14.

$$2\cos^2 x - 3\cos x + 1 = 0$$

$$(2\cos x - 1)(\cos x - 1) = 0$$

$2\cos x - 1 = 0$ or $\cos x - 1 = 0$

$2\cos x = 1$ $\qquad \cos x = 1$

$\cos x = \frac{1}{2}$ $\qquad x = 0$

$x = \frac{\pi}{3}$ $\quad x = \frac{5\pi}{3}$

The solutions in the interval $[0, 2\pi)$ are $0, \frac{\pi}{3}$, and $\frac{5\pi}{3}$.

15.

$$2\sin^2 x + \cos x = 1$$

$$2\left(1 - \cos^2 x\right) + \cos x - 1 = 0$$

$$2 - 2\cos^2 x + \cos x - 1 = 0$$

$$-2\cos^2 x + \cos x + 1 = 0$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$2\cos x + 1 = 0$ or $\cos x - 1 = 0$

$2\cos x = -1$ $\qquad \cos x = 1$

$\cos x = -\frac{1}{2}$ $\qquad x = 0$

$x = \frac{2\pi}{3}$ $\quad x = \frac{4\pi}{3}$

The solutions in the interval $[0, 2\pi)$ are $0, \frac{2\pi}{3}$, and $\frac{4\pi}{3}$.

16. $\cos x = -0.8092$

Be sure calculator is in radian mode and find the inverse cosine of +0.8092. This gives the first quadrant reference angle.

$\theta = \cos^{-1}(0.8092) \approx 0.6280$

The cosine is negative in quadrants II and III thus,

$x \approx \pi - 0.6280$ or $x \approx \pi + 0.6280$

$x \approx 2.5136$ $\qquad x \approx 3.7696$

17.

$$\tan x \sec x = 3\tan x$$

$$\tan x \sec x - 3\tan x = 0$$

$$\tan x\left(\sec x - 3\right) = 0$$

$\tan x = 0$ or $\sec x - 3 = 0$

$\sec x = 3$

$x = 0, \ \pi$

$\cos x = \frac{1}{3}$

$x \approx 1.2310, \ 5.0522$

18. $\tan^2 x - 3\tan x - 2 = 0$

$$\tan x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\tan x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2(1)}$$

$$\tan x = \frac{3 \pm \sqrt{17}}{2}$$

$\tan x \approx 3.5616$ or $\tan x \approx -0.5616$

$x \approx 1.2971,\ 4.4387$ $\quad x \approx 2.6299,\ 5.7715$

Cumulative Review Exercises (Chapters 1–6)

1. $x^3 + x^2 - x + 15 = 0$

The possible rational zeros are: ±1, ±3, ±5, ±15.

Synthetic division shows that –3 is a zero:

–3	1	1	–1	15
		–3	6	–15
	1	–2	5	0

The quotient is $x^2 - 2x + 5$. The remaining zeros are found using the quadratic formula:

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 - 20}}{2}$$

$$= \frac{2 \pm \sqrt{-16}}{2}$$

$$= \frac{2 \pm 4i}{2}$$

$$= 1 \pm 2i$$

All solutions are: –3, 1 + 2*i* and 1 – 2*i*.

2.

$$11^{x-1} = 125$$

$$\log 11^{x-1} = \log 125$$

$$(x-1)\log 11 = \log 125$$

$$x - 1 = \frac{\log 125}{\log 11}$$

$$x = \frac{\log 125}{\log 11} + 1$$

or $x \approx 3.01$

3. $x^2 + 2x - 8 > 0$
$(x-2)(x+4) > 0$
zero points are $x = 2$ and $x = -4$.

Test Interval	Representative Number	Substitute into $x^2 + 2x - 8 > 0$	Conclusion
$(-\infty, -4)$	-5	$(-5)^2 + 2(-5) - 8 = 25 - 10 - 8 = 7 > 0$	$(-\infty, -4)$ belongs to the solution set.
$(-4, 2)$	0	$0^2 + 2(0) - 8 = -8 > 0$	$(-4, 2)$ does not belong to the solution set.
$(2, \infty)$	3	$3^2 + 2(3) - 8 = 9 + 6 - 8 = 7 > 0$	$(2, \infty)$ belongs to the solution set.

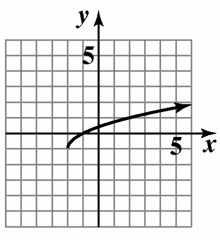
The solution intervals are $(-\infty, -4) \cup (2, \infty)$.

4. $\cos 2x + 3 = 5\cos x$
$2\cos^2 x - 1 + 3 = 5\cos x$
$2\cos^2 x - 5\cos x + 2 = 0$
$(2\cos x - 1)(\cos x - 2) = 0$
$2\cos x - 1 = 0$ or $\cos x - 2 = 0$
$2\cos x = 1$ $\cos x = 2$
$\cos x = \frac{1}{2}$ $\cos x$ cannot be greater than 1.
$x = \frac{\pi}{3}$ $x = \frac{5\pi}{3}$

The solutions in the interval $[0, 2\pi)$ are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

5. $\tan x + \sec^2 x = 3$
$\tan x + 1 + \tan^2 x = 3$
$\tan^2 x + \tan x - 2 = 0$
$(\tan x - 1)(\tan x + 2) = 0$
$\tan x - 1 = 0$ or $\tan x + 2 = 0$
$\tan x = 1$ $\tan x = -2$
$x = \frac{\pi}{4}, \frac{5\pi}{4}$ $x \approx 2.0344, 5.1760$

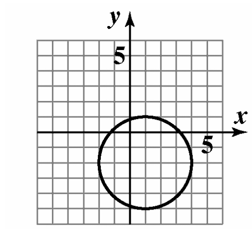
6.



$y = \sqrt{x+2} - 1$

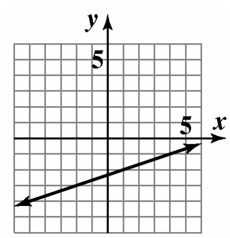
Shift the graph of $y = \sqrt{x}$ left 2 units and down 1 unit.

7.



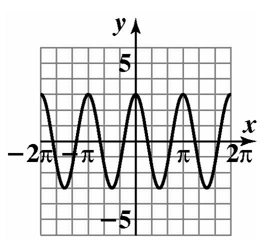
$(x-1)^2 + (y+2)^2 = 9$

8.



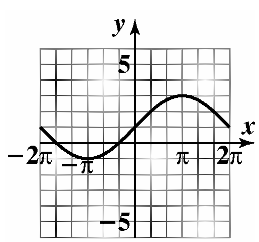
$y + 2 = \frac{1}{3}(x-1)$

9.



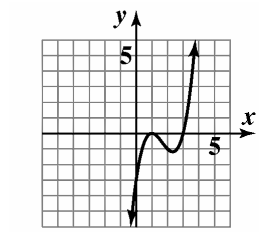
$y = 3\cos 2x$

10.



$y = 2\sin\frac{x}{2} + 1$

11.



$f(x) = (x-1)^2(x-3)$

12. $f(x) = x^2 + 3x - 1$

$$\frac{f(a+h) - f(a)}{h}$$
$$= \frac{(a+h)^2 + 3(a+h) - 1 - \left(a^2 + 3a - 1\right)}{h}$$
$$= \frac{a^2 + 2ah + h^2 + 3a + 3h - 1 - a^2 - 3a + 1}{h}$$
$$= \frac{2ah + h^2 + 3h}{h}$$
$$= 2a + h + 3$$

13. $\sin 225° = \sin(180° + 45°)$

$$= \sin 180° \cos 45° + \cos 180° \sin 45°$$
$$= 0 \cdot \frac{\sqrt{2}}{2} + (-1) \cdot \frac{\sqrt{2}}{2}$$
$$= -\frac{\sqrt{2}}{2}$$

14. $\sec^4 x - \sec^2 x$

$$= \sec^2 x \cdot \sec^2 x - \sec^2 x$$
$$= \left(1 + \tan^2 x\right)\left(1 + \tan^2 x\right) - \left(1 + \tan^2 x\right)$$
$$= 1 + 2\tan^2 x + \tan^4 x - 1 - \tan^2 x$$
$$= \tan^4 x + \tan^2 x$$

We worked with the left side and arrived at the right side. Thus, the identity is verified.

15. $320° \times \frac{\pi}{180°} = \frac{16}{9}\pi$ or 5.59 radians

16.
$$A = Pe^{rt}$$
$$3P = Pe^{0.0575t}$$
$$3 = e^{0.0575t}$$
$$\ln 3 = \ln e^{0.0575t}$$
$$\ln 3 = 0.0575t$$
$$\frac{\ln 3}{0.0575} = t$$
$$t \approx 19.1 \text{ years}$$

17.
$$f(x) = \frac{2x+1}{x-3}$$
$$y = \frac{2x+1}{x-3}$$
$$x = \frac{2y+1}{y-3}$$
$$x(y-3) = 2y+1$$
$$xy - 3x = 2y+1$$
$$xy - 2y = 3x+1$$
$$y(x-2) = 3x+1$$
$$y = \frac{3x+1}{x-2}$$
$$f^{-1}(x) = \frac{3x+1}{x-2}$$

18. The third angle is:
$B = 180° - 90° - 23° = 67°$.

Since $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$,
$$\sin A = \sin 23° = \frac{12}{c}$$
$$c = \frac{12}{\sin 23°} \approx 30.71 \text{ and}$$
$$\sin B = \sin 67° = \frac{b}{30.71}$$
$$b = 30.71 \cdot \sin 67° \approx 28.27$$
The angles are 90°, 23°, and 67°.
The sides are 12, 30.71, and 28.27.

19. Solve $8.5 = \frac{12}{150} \cdot a$
where a is the adult dose.
$$a = \frac{(8.5)\cdot 150}{12}$$
$$= 106.25 \text{ mg}$$
$$a \approx 106 \text{ mg}$$

20.. Let h be the height of the flagpole.
Then $\tan 53° = \frac{h}{12}$
$$h = 12 \cdot \tan 53°$$
$$h \approx 15.9 \text{ feet}$$

Chapter 7
Additional Topics in Trigonometry

Section 7.1

Check Point Exercises

1. Begin by finding B, the third angle of the triangle.

$$A + B + C = 180°$$
$$64° + B + 82° = 180°$$
$$146° + B = 180°$$
$$B = 34°$$

In this problem, we are given c and C: $c = 14$ and $C = 82°$. Thus, use the ratio $\frac{c}{\sin C}$, or $\frac{14}{\sin 82°}$, to find the other two sides. Use the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 64°} = \frac{14}{\sin 82°}$$
$$a = \frac{14 \sin 64°}{\sin 82°}$$
$$a \approx 13 \text{ centimeters}$$

Use the Law of Sines again, this time to find b.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 34°} = \frac{14}{\sin 82°}$$
$$b = \frac{14 \sin 34°}{\sin 82°}$$
$$b \approx 8 \text{ centimeters}$$

The solution is $B = 34°$, $a \approx 13$ centimeters, and $b \approx 8$ centimeters.

2. Begin by finding B.

$$A + B + C = 180°$$
$$40° + B + 22.5° = 180°$$
$$62.5° + B = 180°$$
$$B = 117.5°$$

In this problem, we are given that $b = 12$ and we find that $B = 117.5°$. Thus, use the ratio $\frac{b}{\sin B}$, or $\frac{12}{\sin 117.5°}$, to find the other two sides. Use the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 40°} = \frac{12}{\sin 117.5°}$$
$$a = \frac{12 \sin 40°}{\sin 117.5°} \approx 9$$

Use the Law of Sines again, this time to find c.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 22.5°} = \frac{12}{\sin 117.5°}$$
$$c = \frac{12 \sin 22.5°}{\sin 117.5°} \approx 5$$

The solution is $B = 117.5°$, $a \approx 9$, and $c \approx 5$.

3. The known ratio is $\frac{a}{\sin A}$, or $\frac{33}{\sin 57°}$. Because side b is given, Use the Law of Sines to find angle B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{33}{\sin 57°} = \frac{26}{\sin B}$$
$$33 \sin B = 26 \sin 57°$$
$$\sin B = \frac{26 \sin 57°}{33} \approx 0.6608$$
$$\sin B \approx 0.6608$$
$$B \approx 41°$$

$180° - 41° = 139°$ also has this sine value, but, the sum of $57°$ and $139°$ exceeds $180°$, so B cannot have this value.

$C = 180° - B - A = 180° - 41° - 57° = 82°$.

Use the law of sines to find C.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{33}{\sin 57°} = \frac{c}{\sin 82°}$$
$$c = \frac{33 \sin 82°}{\sin 57°}$$
$$c \approx 39$$

Thus, $B \approx 41°$, $C \approx 82°$, $c \approx 39$.

4. The known ratio is $\frac{a}{\sin A}$, or $\frac{10}{\sin 50°}$. Because side b is given, Use the Law of Sines to find angle B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{10}{\sin 50°} = \frac{20}{\sin B}$$
$$10\sin B = 20\sin 50°$$
$$\sin B = \frac{20\sin 50°}{10} \approx 1.53$$

Because the sine can never exceed 1, there is no angle B for which $\sin B \approx 1.53$. There is no triangle with the given measurements.

5. The known ratio is $\frac{a}{\sin A}$, or $\frac{12}{\sin 35°}$. Because side b is given, Use the Law of Sines to find angle B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{12}{\sin 35°} = \frac{16}{\sin B}$$
$$12\sin B = 16\sin 35°$$
$$\sin B = \frac{16\sin 35°}{12} \approx 0.7648$$

There are two angles possible:
$B_1 \approx 50°,\ B_2 \approx 180° - 50° = 130°$

There are two triangles:
$C_1 = 180° - A - B_1 \approx 180° - 35° - 50° = 95°$
$C_2 = 180° - A - B_2 \approx 180° - 35° - 130° = 15°$

Use the Law of Sines to find c_1 and c_2.

$$\frac{c_1}{\sin C_1} = \frac{a}{\sin A}$$
$$\frac{c_1}{\sin 95°} = \frac{12}{\sin 35°}$$
$$c_1 = \frac{12\sin 95°}{\sin 35°} \approx 21$$
$$\frac{c_2}{\sin C_2} = \frac{a}{\sin A}$$
$$\frac{c_2}{\sin 15°} = \frac{12}{\sin 35°}$$
$$c_2 = \frac{12\sin 15°}{\sin 35°} \approx 5$$

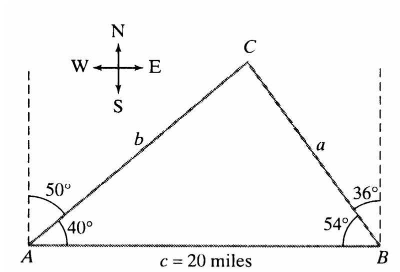
In one triangle, the solution is $B_1 \approx 50°$, $C_1 \approx 95°$, and $c_1 \approx 21$. In the other triangle, $B_2 \approx 130°$, $C_2 \approx 15°$, and $c_2 \approx 5$.

6. The area of the triangle is half the product of the lengths of the two sides times the sine of the included angle.

$$\text{Area} = \frac{1}{2}(8)(12)(\sin 135°) \approx 34$$

The area of the triangle is approximately 34 square meters.

7.



Using a north-south line, the interior angles are found as follows:
$A = 90° - 35° = 55°$
$B = 90° - 49° = 41°$

Find angle C using a 180° angle sum in the triangle.
$C = 180° - A - B = 180° - 55° - 41° = 84°$

The ratio $\frac{c}{\sin C}$, or $\frac{13}{\sin 84°}$ is now known. Use this ratio and the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 55°} = \frac{13}{\sin 84°}$$
$$a = \frac{13\sin 55°}{\sin 84°} \approx 11$$

The fire is approximately 11 miles from station B.

Exercise Set 7.1

1. Begin by finding B.

$$A + B + C = 180°$$
$$42° + B + 96° = 180°$$
$$138° + B = 180°$$
$$B = 42°$$

Use the ratio $\frac{c}{\sin C}$, or $\frac{12}{\sin 96°}$, to find the other two sides. Use the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 42°} = \frac{12}{\sin 96°}$$
$$a = \frac{12 \sin 42°}{\sin 96°}$$
$$a \approx 8.1$$

Use the Law of Sines again, this time to find b.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 42°} = \frac{12}{\sin 96°}$$
$$b = \frac{12 \sin 42°}{\sin 96°}$$
$$b \approx 8.1$$

The solution is $B = 42°$, $a \approx 8.1$, and $b \approx 8.1$.

2. Begin by finding C.

$$A + B + C = 180°$$
$$42° + 48° + C = 180°$$
$$90° + C = 180°$$
$$C = 90°$$

Use the ratio $\frac{c}{\sin C}$, or $\frac{12}{\sin 90°}$, to find the other two sides. Use the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 42°} = \frac{12}{\sin 90°}$$
$$a = \frac{12 \sin 42°}{\sin 90°}$$
$$a \approx 8.0$$

Use the Law of Sines again, this time to find b.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 48°} = \frac{12}{\sin 90°}$$
$$b = \frac{12 \sin 48°}{\sin 90°}$$
$$b \approx 8.9$$

The solution is $C = 90°$, $a \approx 8.0$, and $b \approx 8.9$.

3. Begin by finding A.

$$A + B + C = 180°$$
$$A + 54° + 82° = 180°$$
$$A + 136° = 180°$$
$$A = 44°$$

Use the ratio $\frac{a}{\sin A}$, or $\frac{16}{\sin 44°}$, to find the other two sides. Use the Law of Sines to find b.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 54°} = \frac{16}{\sin 44°}$$
$$b = \frac{16 \sin 54°}{\sin 44°}$$
$$b \approx 18.6$$

Use the Law of Sines again, this time to find c.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{c}{\sin 82°} = \frac{16}{\sin 44°}$$
$$c = \frac{16 \sin 82°}{\sin 44°}$$
$$c \approx 22.8$$

The solution is $A = 44°$, $b \approx 18.6$, and $c \approx 22.8$.

4. Begin by finding B.

$$A + B + C = 180°$$
$$33° + B + 128° = 180°$$
$$B + 161° = 180°$$
$$B = 19°$$

Use the ratio $\frac{a}{\sin A}$, or $\frac{16}{\sin 33°}$, to find the other two sides. Use the Law of Sines to find b.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 19°} = \frac{16}{\sin 33°}$$
$$b = \frac{16 \sin 19°}{\sin 33°}$$
$$b \approx 9.6$$

Use the Law of Sines again, this time to find c.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{c}{\sin 128°} = \frac{16}{\sin 33°}$$
$$c = \frac{16 \sin 128°}{\sin 33°}$$
$$c \approx 23.1$$

The solution is $B = 19°$, $b \approx 9.6$, and $c \approx 23.1$.

5. Begin by finding C.

$$A + B + C = 180°$$
$$48° + 37° + C = 180°$$
$$85° + C = 180°$$
$$C = 95°$$

Use the ratio $\frac{a}{\sin A}$, or $\frac{100}{\sin 48°}$, to find the other two sides. Use the Law of Sines to find b.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 37°} = \frac{100}{\sin 48°}$$
$$b = \frac{100 \sin 37°}{\sin 48°}$$
$$b \approx 81.0$$

Use the Law of Sines again, this time to find c.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{c}{\sin 95°} = \frac{100}{\sin 48°}$$
$$c = \frac{100 \sin 95°}{\sin 48°}$$
$$c \approx 134.1$$

The solution is $C = 95°$, $b \approx 81.0$, and $c \approx 134.1$.

6. Begin by finding C.

$$A + B + C = 180°$$
$$6° + 12° + C = 180°$$
$$18° + C = 180°$$
$$C = 162°$$

Use the ratio $\frac{c}{\sin C}$, or $\frac{100}{\sin 162°}$, to find the other two sides. Use the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 6°} = \frac{100}{\sin 162°}$$
$$a = \frac{100 \sin 6°}{\sin 162°}$$
$$a \approx 33.8$$

Use the Law of Sines again, this time to find b.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 12°} = \frac{100}{\sin 162°}$$
$$b = \frac{100 \sin 12°}{\sin 162°}$$
$$b \approx 67.3$$

The solution is $C = 162°$, $a \approx 33.8$, and $b \approx 67.3$.

7. Begin by finding B.

$$A+B+C=180°$$
$$38°+B+102°=180°$$
$$B+140°=180°$$
$$B=40°$$

Use the ratio $\frac{a}{\sin A}$, or $\frac{20}{\sin 38°}$, to find the other two sides. Use the Law of Sines to find b.

$$\frac{b}{\sin B}=\frac{a}{\sin A}$$
$$\frac{b}{\sin 40°}=\frac{20}{\sin 38°}$$
$$b=\frac{20\sin 40°}{\sin 38°}$$
$$b\approx 20.9$$

Use the Law of Sines again, this time to find c.

$$\frac{c}{\sin C}=\frac{a}{\sin A}$$
$$\frac{c}{\sin 102°}=\frac{20}{\sin 38°}$$
$$c=\frac{20\sin 102°}{\sin 38°}$$
$$c\approx 31.8$$

The solution is $B=40°, b\approx 20.9,$ and $c\approx 31.8$.

8. Begin by finding C.

$$A+B+C=180°$$
$$38°+102°+C=180°$$
$$140°+C=180°$$
$$C=40°$$

Use the ratio $\frac{a}{\sin A}$, or $\frac{20}{\sin 38°}$, to find the other two sides. Use the Law of Sines to find b.

$$\frac{b}{\sin B}=\frac{a}{\sin A}$$
$$\frac{b}{\sin 102°}=\frac{20}{\sin 38°}$$
$$b=\frac{20\sin 102°}{\sin 38°}$$
$$b\approx 31.8$$

Use the Law of Sines again, this time to find c.

$$\frac{c}{\sin C}=\frac{a}{\sin A}$$
$$\frac{c}{\sin 40°}=\frac{20}{\sin 38°}$$
$$c=\frac{20\sin 40°}{\sin 38°}$$
$$c\approx 20.9$$

The solution is $C=40°, b\approx 31.8,$ and $c\approx 20.9$.

9. Begin by finding C.

$$A+B+C=180°$$
$$44°+25°+C=180°$$
$$69°+C=180°$$
$$C=111°$$

Use the ratio $\frac{a}{\sin A}$, or $\frac{12}{\sin 44°}$, to find the other two sides. Use the Law of Sines to find b.

$$\frac{b}{\sin B}=\frac{a}{\sin A}$$
$$\frac{b}{\sin 25°}=\frac{12}{\sin 44°}$$
$$b=\frac{12\sin 25°}{\sin 44°}$$
$$b\approx 7.3$$

Use the Law of Sines again, this time to find c.

$$\frac{c}{\sin C}=\frac{a}{\sin A}$$
$$\frac{c}{\sin 111°}=\frac{12}{\sin 44°}$$
$$c=\frac{12\sin 111°}{\sin 44°}$$
$$c\approx 16.1$$

The solution is $C=111°, b\approx 7.3,$ and $c\approx 16.1$.

10. Begin by finding *B*.

$$A + B + C = 180°$$
$$56° + B + 24° = 180°$$
$$B + 80° = 180°$$
$$B = 100°$$

Use the ratio $\frac{a}{\sin A}$, or $\frac{22}{\sin 56°}$, to find the other two sides. Use the Law of Sines to find *b*.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 100°} = \frac{22}{\sin 56°}$$
$$b = \frac{22 \sin 100°}{\sin 56°}$$
$$b \approx 26.1$$

Use the Law of Sines again, this time to find *c*.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{c}{\sin 24°} = \frac{22}{\sin 56°}$$
$$c = \frac{22 \sin 24°}{\sin 56°}$$
$$c \approx 10.8$$

The solution is $B = 100°$, $b \approx 26.1$, and $c \approx 10.8$.

11. Begin by finding *A*.

$$A + B + C = 180°$$
$$A + 85° + 15° = 180°$$
$$A + 100° = 180°$$
$$A = 80°$$

Use the ratio $\frac{b}{\sin B}$, or $\frac{40}{\sin 85°}$, to find the other two sides. Use the Law of Sines to find *a*.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 80°} = \frac{40}{\sin 85°}$$
$$a = \frac{40 \sin 80°}{\sin 85°}$$
$$a \approx 39.5$$

Use the Law of Sines again, this time to find *c*.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 15°} = \frac{40}{\sin 85°}$$
$$c = \frac{40 \sin 15°}{\sin 85°}$$
$$c \approx 10.4$$

The solution is $A = 80°$, $a \approx 39.5$, and $c \approx 10.4$.

12. Begin by finding *C*.

$$A + B + C = 180°$$
$$85° + 35° + C = 180°$$
$$120° + C = 180°$$
$$C = 60°$$

Use the ratio $\frac{c}{\sin C}$, or $\frac{30}{\sin 60°}$, to find the other two sides. Use the Law of Sines to find *a*.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 85°} = \frac{30}{\sin 60°}$$
$$a = \frac{30 \sin 85°}{\sin 60°}$$
$$a \approx 34.5$$

Use the Law of Sines again, this time to find *b*.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 35°} = \frac{30}{\sin 60°}$$
$$b = \frac{30 \sin 35°}{\sin 60°}$$
$$b \approx 19.9$$

The solution is $C = 60°$, $a \approx 34.5$, and $b \approx 19.9$.

13. Begin by finding B.

$$A + B + C = 180°$$
$$115° + B + 35° = 180°$$
$$B + 150° = 180°$$
$$B = 30°$$

Use the ratio $\frac{c}{\sin C}$, or $\frac{200}{\sin 35°}$, to find the other two sides. Use the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 115°} = \frac{200}{\sin 35°}$$
$$a = \frac{200 \sin 115°}{\sin 35°}$$
$$a \approx 316.0$$

Use the Law of Sines again, this time to find b.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 30°} = \frac{200}{\sin 35°}$$
$$b = \frac{200 \sin 30°}{\sin 35°}$$
$$b \approx 174.3$$

The solution is $B = 30°$, $a \approx 316.0$, and $b \approx 174.3$.

14. Begin by finding A.

$$A + B + C = 180°$$
$$A + 5° + 125° = 180°$$
$$A + 130° = 180°$$
$$A = 50°$$

Use the ratio $\frac{b}{\sin B}$, or $\frac{200}{\sin 5°}$, to find the other two sides. Use the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 50°} = \frac{200}{\sin 5°}$$
$$a = \frac{200 \sin 50°}{\sin 5°}$$
$$a \approx 1757.9$$

Use the Law of Sines again, this time to find c.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 125°} = \frac{200}{\sin 5°}$$
$$c = \frac{200 \sin 125°}{\sin 5°}$$
$$c \approx 1879.7$$

The solution is $A = 50°$, $a \approx 1757.9$, and $c \approx 1879.7$.

15. Begin by finding C.

$$A + B + C = 180°$$
$$65° + 65° + C = 180°$$
$$130° + C = 180°$$
$$C = 50°$$

Use the ratio $\frac{c}{\sin C}$, or $\frac{6}{\sin 50°}$, to find the other two sides. Use the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 65°} = \frac{6}{\sin 50°}$$
$$a = \frac{6 \sin 65°}{\sin 50°}$$
$$a \approx 7.1$$

Use the Law of Sines to find angle B.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 65°} = \frac{6}{\sin 50°}$$
$$b = \frac{6 \sin 65°}{\sin 50°}$$
$$b \approx 7.1$$

The solution is $C = 50°$, $a \approx 7.1$, and $b \approx 7.1$.

16. Begin by finding A.

$$A+B+C=180°$$
$$A+80°+10°=180°$$
$$A+90°=180°$$
$$A=90°$$

Use the ratio $\frac{a}{\sin A}$, or $\frac{8}{\sin 90°}$, to find the other two sides. Use the Law of Sines to find b.

$$\frac{b}{\sin B}=\frac{a}{\sin A}$$
$$\frac{b}{\sin 80°}=\frac{8}{\sin 90°}$$
$$b=\frac{8\sin 80°}{\sin 90°}$$
$$b\approx 7.9$$

Use the Law of Sines again, this time to find c.

$$\frac{c}{\sin C}=\frac{a}{\sin A}$$
$$\frac{c}{\sin 10°}=\frac{8}{\sin 90°}$$
$$c=\frac{8\sin 10°}{\sin 90°}$$
$$c\approx 1.4$$

The solution is $A=90°,\ b\approx 7.9,$ and $c\approx 1.4$.

17. The known ratio is $\frac{a}{\sin A}$, or $\frac{20}{\sin 40°}$.

Use the Law of Sines to find angle B.

$$\frac{a}{\sin A}=\frac{b}{\sin B}$$
$$\frac{20}{\sin 40°}=\frac{15}{\sin B}$$
$$20\sin B=15\sin 40°$$
$$\sin B=\frac{15\sin 40°}{20}$$
$$\sin B\approx 0.4821$$

There are two angles possible:

$B_1\approx 29°,\ B_2\approx 180°-29°=151°$

B_2 is impossible, since $40°+151°=191°$.

We find C using B_1 and the given information $A =$ 40°.

$C=180°-B_1-A\approx 180°-29°-40°=111°$

Use the Law of Sines to find side c.

$$\frac{c}{\sin C}=\frac{a}{\sin A}$$
$$\frac{c}{\sin 111°}=\frac{20}{\sin 40°}$$
$$c=\frac{20\sin 111°}{\sin 40°}\approx 29.0$$

There is one triangle and the solution is B_1 (or B) $\approx 29°,\ C\approx 111°,$ and $c\approx 29.0$.

18. The known ratio is $\frac{a}{\sin A}$, or $\frac{30}{\sin 50°}$. Use the Law of Sines to find angle B.

$$\frac{a}{\sin A}=\frac{b}{\sin B}$$
$$\frac{30}{\sin 50°}=\frac{20}{\sin B}$$
$$30\sin B=20\sin 50°$$
$$\sin B=\frac{20\sin 50°}{30}$$
$$\sin B\approx 0.5107$$

There are two angles possible:

$B_1\approx 31°,\ B_2\approx 180°-31°=149°$

B_2 is impossible, since $50°+149°=199°$.

We find C using B_1 and the given information $A=50°$.

$C=180°-B_1-A\approx 180°-31°-50°=99°$

Use the Law of Sines to find c.

$$\frac{c}{\sin C}=\frac{a}{\sin A}$$
$$\frac{c}{\sin 99°}=\frac{30}{\sin 50°}$$
$$c=\frac{30\sin 99°}{\sin 50°}\approx 38.7$$

There is one triangle and the solution is B_1 (or B) $\approx 31°,\ C\approx 99°,$ and $c\approx 38.7$.

19. The known ratio is $\frac{a}{\sin A}$, or $\frac{10}{\sin 63°}$.

Use the Law of Sines to find angle *C*.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{10}{\sin 63°} = \frac{8.9}{\sin C}$$
$$10 \sin C = 8.9 \sin 63°$$
$$\sin C = \frac{8.9 \sin 63°}{10}$$
$$\sin C \approx 0.7930$$

There are two angles possible:

$C_1 \approx 52°, C_2 \approx 180° - 52° = 128°$

C_2 is impossible, since $63° + 128° = 191°$.

We find *B* using C_1 and the given information *A* = 63°.

$B = 180° - C_1 - A \approx 180° - 52° - 63° = 65°$

Use the Law of Sines to find side *b*.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 65°} = \frac{10}{\sin 63°}$$
$$b = \frac{10 \sin 65°}{\sin 63°} \approx 10.2$$

There is one triangle and the solution is

C_1 (or *C*) $\approx 52°$, $B \approx 65°$, and $b \approx 10.2$.

20. The known ratio is $\frac{a}{\sin A}$, or $\frac{57.5}{\sin 136°}$.

Use the Law of Sines to find angle *C*.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{57.5}{\sin 136°} = \frac{49.8}{\sin C}$$
$$57.5 \sin C = 49.8 \sin 136°$$
$$\sin C = \frac{49.8 \sin 136°}{57.5}$$
$$\sin C \approx 0.6016$$

There are two angles possible:

$C_1 \approx 37°, C_2 \approx 180° - 37° = 143°$

C_2 is impossible, since $136° + 143° = 279°$.

We find *B* using C_1 and the given information $A = 136°$.

$B = 180° - C_1 - A \approx 180° - 37° - 136° = 7°$

Use the Law of Sines to find *b*.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 7°} = \frac{57.5}{\sin 136°}$$
$$b = \frac{57.5 \sin 7°}{\sin 136°} \approx 10.1$$

There is one triangle and the solution is

C_1 (or *C*) $\approx 37°$, $B \approx 7°$, and $b \approx 10.1$.

21. The known ratio is $\frac{a}{\sin A}$, or $\frac{42.1}{\sin 112°}$.

Use the Law of Sines to find angle *C*.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{42.1}{\sin 112°} = \frac{37}{\sin C}$$
$$42.1 \sin C = 37 \sin 112°$$
$$\sin C = \frac{37 \sin 112°}{42.1}$$
$$\sin C \approx 0.8149$$

There are two angles possible:

$C_1 \approx 55°, C_2 \approx 180° - 55° = 125°$

C_2 is impossible, since $112° + 125° = 237°$.

We find *B* using C_1 and the given information *A* = 112°.

$B = 180° - C_1 - A \approx 180° - 55° - 112° = 13°$

Use the Law of Sines to find *b*.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 13°} = \frac{42.1}{\sin 112°}$$
$$b = \frac{42.1 \sin 13°}{\sin 112°} \approx 10.2$$

There is one triangle and the solution is

C_1 (or *C*) $\approx 55°$, $B \approx 13°$, and $b \approx 10.2$.

22. The known ratio is $\frac{a}{\sin A}$, or $\frac{6.1}{\sin 162°}$.

Use the Law of Sines to find angle *B*.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{6.1}{\sin 162°} = \frac{4}{\sin B}$$
$$6.1 \sin B = 4 \sin 162°$$
$$\sin B = \frac{4 \sin 162°}{6.1}$$
$$\sin B \approx 0.2026$$

There are two angles possible:

$B_1 \approx 12°,\ B_2 \approx 180° - 12° = 168°$

B_2 is impossible, since $162° + 168° = 330°$.

We find *C* using B_1 and the given information $A = 162°$.

$C = 180° - B_1 - A \approx 180° - 12° - 162° = 6°$

Use the Law of Sines to find *c*.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{c}{\sin 6°} = \frac{6.1}{\sin 162°}$$
$$c = \frac{6.1 \sin 6°}{\sin 162°} \approx 2.1$$

There is one triangle and the solution is $B_1 \text{ (or } B) \approx 12°,\ C \approx 6°,$ and $c \approx 2.1$.

23. The known ratio is $\frac{a}{\sin A}$, or $\frac{10}{\sin 30°}$.

Use the Law of Sines to find angle *B*.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{10}{\sin 30°} = \frac{40}{\sin B}$$
$$10 \sin B = 40 \sin 30°$$
$$\sin B = \frac{40 \sin 30°}{10} = 2$$

Because the sine can never exceed 1, there is no angle *B* for which sin *B* = 2. There is no triangle with the given measurements.

24. The known ratio is $\frac{a}{\sin A}$, or $\frac{10}{\sin 150°}$.

Use the Law of Sines to find angle *B*.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{10}{\sin 150°} = \frac{30}{\sin B}$$
$$10 \sin B = 30 \sin 150°$$
$$\sin B = \frac{30 \sin 150°}{10} = 1.5$$

Because the sine can never exceed 1, there is no angle *B* for which $\sin B = 1.5$. There is no triangle with the given measurements.

25. The known ratio is $\frac{a}{\sin A}$, or $\frac{16}{\sin 60°}$.

Use the Law of Sines to find angle *B*.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{16}{\sin 60°} = \frac{18}{\sin B}$$
$$16 \sin B = 18 \sin 60°$$
$$\sin B = \frac{18 \sin 60°}{16}$$
$$\sin B \approx 0.9743$$

There are two angles possible:

$B_1 \approx 77°,\ B_2 \approx 180° - 77° = 103°$

There are two triangles:

$C_1 = 180° - B_1 - A \approx 180° - 77° - 60° = 43°$

$C_2 = 180° - B_2 - A \approx 180° - 103° - 60° = 17°$ Use the Law of Sines to find c_1 and c_2.

$$\frac{c_1}{\sin C_1} = \frac{a}{\sin A}$$
$$\frac{c_1}{\sin 43°} = \frac{16}{\sin 60°}$$
$$c_1 = \frac{16 \sin 43°}{\sin 60°} \approx 12.6$$
$$\frac{c_2}{\sin C_2} = \frac{a}{\sin A}$$
$$\frac{c_2}{\sin 17°} = \frac{16}{\sin 60°}$$
$$c_2 = \frac{16 \sin 17°}{\sin 60°} \approx 5.4$$

In one triangle, the solution is $B_1 \approx 77°,\ C_1 \approx 43°,$ and $c_1 \approx 12.6$.

In the other triangle, $B_2 \approx 103°,\ C_2 \approx 17°,$ and $c_2 \approx 5.4$.

26. The known ratio is $\frac{a}{\sin A}$, or $\frac{30}{\sin 20°}$.
Use the Law of Sines to find angle B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{30}{\sin 20°} = \frac{40}{\sin B}$$
$$30\sin B = 40\sin 20°$$
$$\sin B = \frac{40\sin 20°}{30}$$
$$\sin B \approx 0.4560$$

There are two angles possible:
$B_1 \approx 27°,\ B_2 \approx 180° - 27° = 153°$
There are two triangles:
$C_1 = 180° - B_1 - A \approx 180° - 27° - 20° = 133°$
$C_2 = 180° - B_2 - A \approx 180° - 153° - 20° = 7°$
Use the Law of Sines to find c_1 and c_2.

$$\frac{c_1}{\sin C_1} = \frac{a}{\sin A}$$
$$\frac{c_1}{\sin 133°} = \frac{30}{\sin 20°}$$
$$c_1 = \frac{30\sin 133°}{\sin 20°} \approx 64.2$$
$$\frac{c_2}{\sin C_2} = \frac{a}{\sin A}$$
$$\frac{c_2}{\sin 7°} = \frac{30}{\sin 20°}$$
$$c_2 = \frac{30\sin 7°}{\sin 20°} \approx 10.7$$

In one triangle, the solution is
$B_1 \approx 27°,\ C_1 \approx 133°,$ and $c_1 \approx 64.2$.
In the other triangle,
$B_2 \approx 153°,\ C_2 \approx 7°,$ and $c_2 \approx 10.7$.

27. The known ratio is $\frac{a}{\sin A}$, or $\frac{12}{\sin 37°}$.
Use the Law of Sines to find angle B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{12}{\sin 37°} = \frac{16.1}{\sin B}$$
$$12\sin B = 16.1\sin 37°$$
$$\sin B = \frac{16.1\sin 37°}{12}$$
$$\sin B \approx 0.8074$$

There are two angles possible:
$B_1 \approx 54°,\ B_2 \approx 180° - 54° = 126°$
There are two triangles:
$C_1 = 180° - B_1 - A \approx 180° - 54° - 37° = 89°$
$C_2 = 180° - B_2 - A \approx 180° - 126° - 37° = 17°$ Use the Law of Sines to find c_1 and c_2.

$$\frac{c_1}{\sin C_1} = \frac{a}{\sin A}$$
$$\frac{c_1}{\sin 89°} = \frac{12}{\sin 37°}$$
$$c_1 = \frac{12\sin 89°}{\sin 37°} \approx 19.9$$
$$\frac{c_2}{\sin C_2} = \frac{a}{\sin A}$$
$$\frac{c_2}{\sin 17°} = \frac{12}{\sin 37°}$$
$$c_2 = \frac{12\sin 17°}{\sin 37°} \approx 5.8$$

In one triangle, the solution is
$B_1 \approx 54°,\ C_1 \approx 89°,$ and $c_1 \approx 19.9$.
In the other triangle,
$B_2 \approx 126°,\ C_2 \approx 17°,$ and $c_2 \approx 5.8$.

28. The known ratio is $\frac{a}{\sin A}$, or $\frac{7}{\sin 12°}$.
Use the Law of Sines to find angle B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{7}{\sin 12°} = \frac{28}{\sin B}$$
$$7\sin B = 28\sin 12°$$
$$\sin B = \frac{28\sin 12°}{7}$$
$$\sin B \approx 0.8316$$

There are two angles possible:
$B_1 \approx 56°,\ B_2 \approx 180° - 56° = 124°$
There are two triangles:
$C_1 = 180° - B_1 - A \approx 180° - 56° - 12° = 112°$
$C_2 = 180° - B_2 - A \approx 180° - 124° - 12° = 44°$
Use the Law of Sines to find c_1 and c_2.

$$\frac{c_1}{\sin C_1} = \frac{a}{\sin A}$$
$$\frac{c_1}{\sin 112°} = \frac{7}{\sin 12°}$$
$$c_1 = \frac{7\sin 112°}{\sin 12°} \approx 31.2$$
$$\frac{c_2}{\sin C_2} = \frac{a}{\sin A}$$
$$\frac{c_2}{\sin 44°} = \frac{7}{\sin 12°}$$
$$c_2 = \frac{7\sin 44°}{\sin 12°} \approx 23.4$$

In one triangle, the solution is
$B_1 \approx 56°,\ C_1 \approx 112°,$ and $c_1 \approx 31.2$.
In the other triangle,
$B_2 \approx 124°,\ C_2 \approx 44°,$ and $c_2 \approx 23.4$.

29. The known ratio is $\frac{a}{\sin A}$, or $\frac{22}{\sin 58°}$.
Use the Law of Sines to find angle C.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{22}{\sin 58°} = \frac{24.1}{\sin C}$$
$$22\sin C = 24.1\sin 58°$$
$$\sin C = \frac{24.1\sin 58°}{22}$$
$$\sin C \approx 0.9290$$

There are two angles possible:
$C_1 \approx 68°,\ C_2 \approx 180° - 68° = 112°$
There are two triangles:
$B_1 = 180° - C_1 - A \approx 180° - 68° - 58° = 54°$
$B_2 = 180° - C_2 - A \approx 180° - 112° - 58° = 10°$ Use the Law of Sines to find b_1 and b_2.

$$\frac{b_1}{\sin B_1} = \frac{a}{\sin A}$$
$$\frac{b_1}{\sin 54°} = \frac{22}{\sin 58°}$$
$$b_1 = \frac{22\sin 54°}{\sin 58°} \approx 21.0$$
$$\frac{b_2}{\sin B_2} = \frac{a}{\sin A}$$
$$\frac{b_2}{\sin 10°} = \frac{22}{\sin 58°}$$
$$b_2 = \frac{22\sin 10°}{\sin 58°} \approx 4.5$$

In one triangle, the solution is
$C_1 \approx 68°,\ B_1 \approx 54°,$ and $b_1 \approx 21.0$.
In the other triangle,
$C_2 \approx 112°,\ B_2 \approx 10°,$ and $b_2 \approx 4.5$.

30. The known ratio is $\frac{a}{\sin A}$, or $\frac{95}{\sin 49°}$.
Use the Law of Sines to find angle C.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{95}{\sin 49°} = \frac{125}{\sin C}$$
$$95 \sin C = 125 \sin 49°$$
$$\sin C = \frac{125 \sin 49°}{95}$$
$$\sin C \approx 0.9930$$

There are two angles possible:
$C_1 \approx 83°,\ C_2 \approx 180° - 83° = 97°$
There are two triangles:
$B_1 = 180° - C_1 - A \approx 180° - 83° - 49° = 48°$
$B_2 = 180° - C_2 - A \approx 180° - 97° - 49° = 34°$
Use the Law of Sines to find b_1 and b_2.

$$\frac{b_1}{\sin B_1} = \frac{a}{\sin A}$$
$$\frac{b_1}{\sin 48°} = \frac{95}{\sin 49°}$$
$$b_1 = \frac{95 \sin 48°}{\sin 49°} \approx 93.5$$
$$\frac{b_2}{\sin B_2} = \frac{a}{\sin A}$$
$$\frac{b_2}{\sin 34°} = \frac{95}{\sin 49°}$$
$$b_2 = \frac{95 \sin 34°}{\sin 49°} \approx 70.4$$

In one triangle, the solution is
$C_1 \approx 83°,\ B_1 \approx 48°,$ and $b_1 \approx 93.5$.
In the other triangle,
$C_2 \approx 97°,\ B_2 \approx 34°,$ and $b_2 \approx 70.4$.

31. The known ratio is $\frac{a}{\sin A}$, or $\frac{9.3}{\sin 18°}$.
Use the Law of Sines to find angle B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{9.3}{\sin 18°} = \frac{41}{\sin B}$$
$$9.3 \sin B = 41 \sin 18°$$
$$\sin B = \frac{41 \sin 18°}{9.3} \approx 1.36$$

Because the sine can never exceed 1, there is no angle B for which $\sin B = 1.36$. There is no triangle with the given measurements.

32. The known ratio is $\frac{a}{\sin A}$, or $\frac{1.4}{\sin 142°}$.
Use the Law of Sines to find angle B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{1.4}{\sin 142°} = \frac{2.9}{\sin B}$$
$$1.4 \sin B = 2.9 \sin 142°$$
$$\sin B = \frac{2.9 \sin 142°}{1.4} \approx 1.28$$

Because the sine can never exceed 1, there is no angle B for which $\sin B = 1.28$. There is no triangle with the given measurements.

33. Area $= \frac{1}{2} bc \sin A = \frac{1}{2}(20)(40)(\sin 48°) \approx 297$
The area of the triangle is approximately 297 square feet.

34. Area $= \frac{1}{2} bc \sin A = \frac{1}{2}(20)(50)(\sin 22°) \approx 187$
The area of the triangle is approximately 187 square feet.

35. Area $= \frac{1}{2} ac \sin B = \frac{1}{2}(3)(6)(\sin 36°) \approx 5$
The area of the triangle is approximately 5 square yards.

36. Area $= \frac{1}{2} ac \sin B = \frac{1}{2}(8)(5)(\sin 125°) \approx 16$
The area of the triangle is approximately 16 square yards.

37. Area $= \frac{1}{2} ab \sin C = \frac{1}{2}(4)(6)(\sin 124°) \approx 10$
The area of the triangle is approximately 10 square meters.

38. Area $= \frac{1}{2} ab \sin C = \frac{1}{2}(16)(20)(\sin 102°) \approx 157$
The area of the triangle is approximately 157 square meters.

39. $\angle ABC = 180° - 67° = 113°$
$\angle ACB = 180° - 43° - 113° = 24°$

Use the law of sines to find $\overline{BC}$.

$$\frac{\overline{BC}}{\sin 43°} = \frac{312}{\sin 24°}$$
$$\overline{BC} = \frac{312 \sin 43°}{\sin 24°}$$
$$\overline{BC} \approx 523.1$$

Use the law of sines to find *h*.

$$\frac{h}{\sin 67°} = \frac{523.1}{\sin 90°}$$
$$h = \frac{523.1 \sin 67°}{\sin 90°}$$
$$h \approx 481.5$$

40. $\angle ABC = 180° - 29° = 151°$
$\angle ACB = 180° - 25° - 151° = 4°$

Use the law of sines to find $\overline{BC}$.

$$\frac{\overline{BC}}{\sin 25°} = \frac{238}{\sin 4°}$$
$$\overline{BC} = \frac{238 \sin 25°}{\sin 4°}$$
$$\overline{BC} \approx 1441.9$$

Use the law of sines to find *h*.

$$\frac{h}{\sin 29°} = \frac{1441.9}{\sin 90°}$$
$$h = \frac{1441.9 \sin 29°}{\sin 90°}$$
$$h \approx 699.1$$

41. Begin by finding the six angles inside the two triangles. Then use the law of sines.

$$\frac{a}{\sin 4°} = \frac{\frac{450 \sin 145°}{\sin 34°}}{\sin 30°}$$
$$a \approx 64.4$$

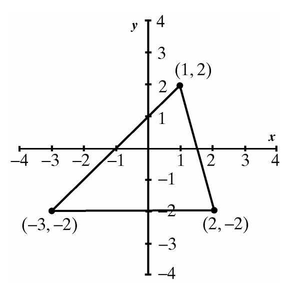
42. Begin by finding the six angles inside the two triangles. Then use the law of sines.

$$\frac{a}{\sin 22°} = \frac{\frac{120}{\sin 58°}}{\sin 100°}$$
$$a \approx 53.8$$

43.
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{300}{\sin 2\theta} = \frac{200}{\sin \theta}$$
$$200 \sin 2\theta = 300 \sin \theta$$
$$400 \sin \theta \cos \theta = 300 \sin \theta$$
$$\cos \theta = \frac{300 \sin \theta}{400 \sin \theta}$$
$$\cos \theta = \frac{3}{4}$$
$$\theta \approx 41°$$
$$2\theta \approx 82°$$
$$A \approx 82°, B \approx 41°, C \approx 57°, c \approx 255.7$$

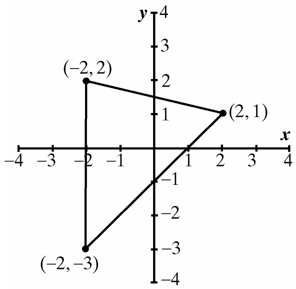
44.
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{400}{\sin 2\theta} = \frac{300}{\sin \theta}$$
$$300 \sin 2\theta = 400 \sin \theta$$
$$600 \sin \theta \cos \theta = 400 \sin \theta$$
$$\cos \theta = \frac{400 \sin \theta}{600 \sin \theta}$$
$$\cos \theta = \frac{2}{3}$$
$$\theta \approx 48°$$
$$2\theta \approx 96°$$
$$A \approx 96°, B \approx 48°, C \approx 36°, c \approx 237.3$$

45.



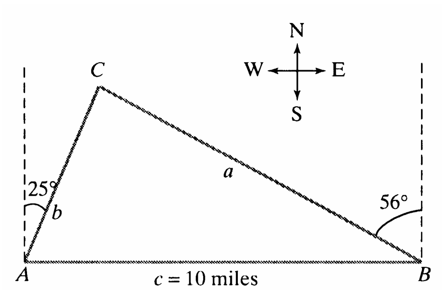
$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(5)(4)$$
$$= 10$$

46.



$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(5)(4)$$
$$= 10$$

47.



Using a north-south line, the interior angles are found as follows:

$A = 90° - 25° = 65°$

$B = 90° - 56° = 34°$

Find angle C using a 180° angle sum in the triangle.

$C = 180° - A - B = 180° - 65° - 34° = 81°$

The ratio $\frac{c}{\sin C}$, or $\frac{10}{\sin 81°}$, is now known. Use this ratio and the Law of Sines to find b and a.

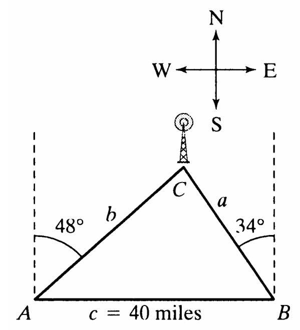
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 34°} = \frac{10}{\sin 81°}$$
$$b = \frac{10 \sin 34°}{\sin 81°} \approx 6$$

Station A is about 6 miles from the fire.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 65°} = \frac{10}{\sin 81°}$$
$$a = \frac{10 \sin 65°}{\sin 81°} \approx 9$$

Station B is about 9 miles from the fire.

48.



Using a north-south line, the interior angles are found as follows:

$A = 90° - 48° = 42°$

$B = 90° - 34° = 56°$

Find angle C using a 180º angle sum in the triangle.

$C = 180° - A - B = 180° - 42° - 56° = 82°$

The ratio $\frac{c}{\sin C}$, or $\frac{40}{\sin 82°}$, is now known.

Use this ratio and the Law of Sines to find b and a.

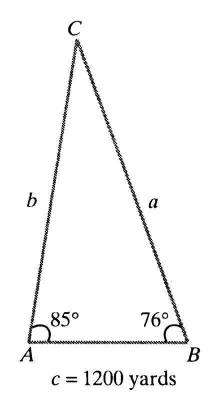
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 56°} = \frac{40}{\sin 82°}$$
$$b = \frac{40 \sin 56°}{\sin 82°} \approx 33$$

Station A is about 33 miles from the illegal station.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 42°} = \frac{40}{\sin 82°}$$
$$a = \frac{40 \sin 42°}{\sin 82°} \approx 27$$

Station B is about 27 miles from the illegal station.

49.



Using the figure,
$C = 180° - A - B = 180° - 85° - 76° = 19°$

The ratio $\dfrac{c}{\sin C}$, or $\dfrac{1200}{\sin 19°}$, is now known. Use this ratio and the Law of Sines to find *a* and *b*.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 85°} = \frac{1200}{\sin 19°}$$
$$a = \frac{1200 \sin 85°}{\sin 19°} \approx 3672$$
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 76°} = \frac{1200}{\sin 19°}$$
$$b = \frac{1200 \sin 76°}{\sin 19°} \approx 3576$$

The platform is about 3672 yards from one end of the beach and 3576 yards from the other.

50. Let c = distance from A to B.
Using the figure,
$B = 180° - A - C = 180° - 62° - 53° = 65°$

The ratio $\dfrac{b}{\sin B}$, or $\dfrac{300}{\sin 65°}$, is now known.

Use this ratio and the Law of Sines to find *c*.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 53°} = \frac{300}{\sin 65°}$$
$$c = \frac{300 \sin 53°}{\sin 65°} \approx 264$$

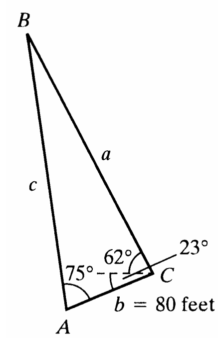
The distance between *A* and *B* is about 264.4 yards or 793 feet.

51. According to the figure,
$C = 180° - A - B = 180° - 84.7° - 50° = 45.3°$ The ratio $\dfrac{c}{\sin C}$, or $\dfrac{171}{\sin 45.3°}$, is now known. Use this ratio and the Law of Sines to find *b*.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 50°} = \frac{171}{\sin 45.3°}$$
$$b = \frac{171 \sin 50°}{\sin 45.3°} \approx 184$$

The distance is about 184 feet.

52.



Using the figure,
$C = 62° + 23° = 85°$
$B = 180° - A - C = 180° - 75° - 85° = 20°$

The ratio $\dfrac{b}{\sin B}$, or $\dfrac{80}{\sin 20°}$, is now known.

Use this ratio and the Law of Sines to find *c*.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 85°} = \frac{80}{\sin 20°}$$
$$c = \frac{80 \sin 85°}{\sin 20°} \approx 233$$

The height of the tree is about 233 feet.

53. The ratio $\dfrac{b}{\sin B}$, or $\dfrac{562}{\sin 85.3°}$, is known.

Use this ratio, the figure, and the Law of Sines to find *c*.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 5.7°} = \frac{562}{\sin 85.3°}$$
$$c = \frac{562 \sin 5.7°}{\sin 85.3°} \approx 56$$

The toss was about 56 feet.

54. Using the figure,

$$B + 85° = 180°$$
$$B = 95°$$
$$A + B + C = 180°$$
$$37° + 95° + C = 180°$$
$$132° + C = 180°$$
$$C = 48°$$

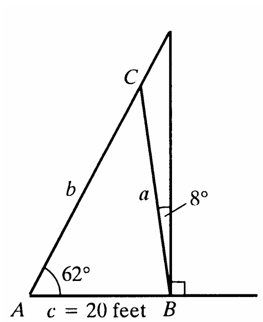
The ratio $\frac{c}{\sin C}$, or $\frac{100}{\sin 48°}$, is now known.

Use this ratio and the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 37°} = \frac{100}{\sin 48°}$$
$$a = \frac{100 \sin 37°}{\sin 48°} \approx 81$$

The pier is about 81 feet long.

55.



Using the figure,

$$B = 90° - 8° = 82°$$
$$C = 180° - A - B = 180° - 62° - 82° = 36°$$

The ratio $\frac{c}{\sin C}$, or $\frac{20}{\sin 36°}$, is now known. Use this ratio and the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 62°} = \frac{20}{\sin 36°}$$
$$a = \frac{20 \sin 62°}{\sin 36°} \approx 30$$

The length of the pole is about 30 feet.

56. Using the figure,

$$A = 90° - 6° = 84°$$
$$C = 180° - A - B = 180° - 84° - 22° = 74°$$

The ratio $\frac{c}{\sin C}$, or $\frac{40}{\sin 74°}$, is now known.

Use this ratio and the Law of Sines to find b.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 22°} = \frac{40}{\sin 74°}$$
$$b = \frac{40 \sin 22°}{\sin 74°} \approx 16$$

The height of the wall is about 16 feet.

57. a. Using the figure and the measurements shown,

$$B = 180° - 44° = 136°$$
$$C = 180° - B - A = 180° - 136° - 37° = 7°$$

The ratio $\frac{c}{\sin C}$, or $\frac{100}{\sin 7°}$, is now known. Use this ratio and the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 37°} = \frac{100}{\sin 7°}$$
$$a = \frac{100 \sin 37°}{\sin 7°} \approx 494$$

To the nearest foot, a = 494 feet.

Let a = 494 be the hypotenuse of the right triangle. Then if h represents the height of the tree,

$$\frac{h}{\sin 44°} = \frac{494}{\sin 90°}$$
$$h = \frac{494 \sin 44°}{\sin 90°} \approx 343$$

A typical redwood tree is about 343 feet.

58. a. Using the figure,

$$B = 180° - 66° = 114°$$
$$C = 180° - A - B = 180° - 22° - 114° = 44°$$

The ratio $\frac{c}{\sin C}$, or $\frac{1.6}{\sin 44°}$, is now known.

Use this ratio and the Law of Sines to find b.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 114°} = \frac{1.6}{\sin 44°}$$
$$b = \frac{1.6 \sin 114°}{\sin 44°} \approx 2.104$$

The cable car covers about 2.104 miles, or about 11,110 feet.

b. The known ratio is $\frac{c}{\sin C}$, or $\frac{1.6}{\sin 44°}$.
Use the Law of Sines to find *a*.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 22°} = \frac{1.6}{\sin 44°}$$
$$a = \frac{1.6 \sin 22°}{\sin 44°} \approx 0.863$$

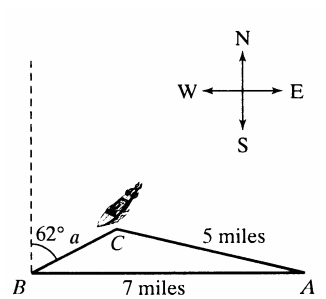
0.863 miles ≈ 4556 feet
$a \approx 4556$ feet

c. Let *a* = 4557, to the nearest foot, be the hypotenuse of the right triangle. Then if *h* represents the height of the mountain,

$$\frac{h}{\sin 66°} = \frac{a}{\sin 90°}$$
$$\frac{h}{\sin 66°} = \frac{4556}{\sin 90°}$$
$$h = \frac{4556 \sin 66°}{\sin 90°} \approx 4162$$

The mountain is about 4163 feet high.

59.



Using the figure,
$B = 90° - 62° = 28°$

The known ratio is $\frac{b}{\sin B}$, or $\frac{5}{\sin 28°}$.
Use the Law of Sines to find angle *C*.

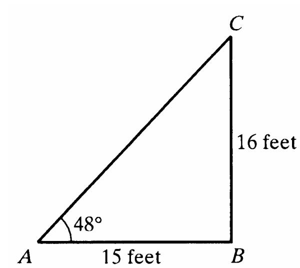
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{5}{\sin 28°} = \frac{7}{\sin C}$$
$$5 \sin C = 7 \sin 28°$$
$$\sin C = \frac{7 \sin 28°}{5} \approx 0.6573$$

There are two angles possible:
$C_1 \approx 41°, C_2 \approx 180° - 41° = 139°$
There are two triangles:
$A_1 = 180° - C_1 - B \approx 180° - 41° - 28° = 111°$
$A_2 = 180° - C_2 - B \approx 180° - 139° - 28° = 13°$
Use the Law of Sines to find a_1 and a_2.

$$\frac{a_1}{\sin A_1} = \frac{b}{\sin B}$$
$$\frac{a_1}{\sin 111°} = \frac{5}{\sin 28°}$$
$$a_1 = \frac{5 \sin 111°}{\sin 28°} \approx 9.9$$
$$\frac{a_2}{\sin A_2} = \frac{b}{\sin B}$$
$$\frac{a_2}{\sin 13°} = \frac{5}{\sin 28°}$$
$$a_2 = \frac{5 \sin 13°}{\sin 28°} \approx 2.4$$

The boat is either 9.9 miles or 2.4 miles from lighthouse *B*, to the nearest tenth of a mile.

60.



Using the figure, the known ratio is $\frac{a}{\sin A}$, or $\frac{16}{\sin 48°}$. Use this ratio and the Law of Sines to find *C*.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{16}{\sin 48°} = \frac{15}{\sin C}$$
$$16 \sin C = 15 \sin 48°$$
$$\sin C = \frac{15 \sin 48°}{16} \approx 0.6967$$

There are two angles possible:
$C_1 \approx 44°, C_2 \approx 180° - 44° = 136°$
C_2 is impossible, since $48° + 136° = 184°$
$B = 180° - 48° - 44° = 88°$
The flagpole is leaning because it makes an $88°$ angle with the ground.

61. – 70. Answers may vary.

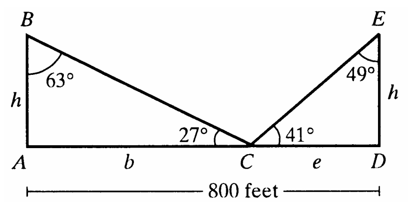
71. does not make sense; Explanations will vary. Sample explanation: The law of cosines would be appropriate for this situation.

72. makes sense

73. does not make sense; Explanations will vary. Sample explanation: The calculator will give you the acute angle. The obtuse angle is the supplement of the acute angle.

74. makes sense

75. No. Explanations may vary.

76. 

Let h = the height of the buildings. Using the figure,

$b + e = 800$

$e = 800 - b$

Now the law of sines gives the following equations:

$$\frac{b}{\sin 63°} = \frac{h}{\sin 27°} \qquad (1)$$

$$\frac{800 - b}{\sin 49°} = \frac{h}{\sin 41°} \qquad (2)$$

Solve (1) for b:

$$\frac{b}{\sin 63°} = \frac{h}{\sin 27°}$$

$$b = \frac{h \sin 63°}{\sin 27°}$$

Now substitute into (2):

$$\frac{800 - b}{\sin 49°} = \frac{h}{\sin 41°}$$

$$\frac{800 - \frac{h \sin 63°}{\sin 27°}}{\sin 49°} = \frac{h}{\sin 41°}$$

$$\frac{800 \sin 27° - h \sin 63°}{\sin 27° \sin 49°} = \frac{h}{\sin 41°}$$

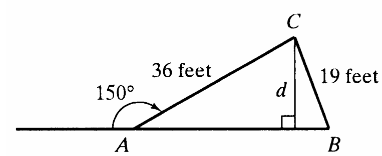
$$h \sin 27° \sin 49° = \sin 41°(800 \sin 27°) - h \sin 63° \text{s}$$

$$h \sin 27° \sin 49° + h \sin 63° \sin 41° = \sin 41°(800 \sin 27°)$$

$$h(\sin 27° \sin 49° + \sin 63° \sin 41°) = 800 \sin 41° \sin 27°$$

$$h = \frac{800 \sin 41° \sin 27°}{\sin 27° \sin 49° + \sin 63° \sin 41°} \approx 257$$

The buildings are about 257 feet high.

77. 

Using the figure,

$A = 180° - 150° = 30°$

Using the Law of Sines we have,

$$\frac{d}{\sin A} = \frac{36}{\sin 90°}$$

$$\frac{d}{\sin 30°} = \frac{36}{\sin 90°}$$

$$d = \frac{36 \sin 30°}{\sin 90°} = 18$$

$CC' = 18 + 5 + 18 = 41$

The wingspan CC' is 41 feet.

78.

$$\cos B = \frac{6^2 + 4^2 - 9^2}{2 \cdot 6 \cdot 4}$$

$$\cos B = \frac{-29}{48}$$

$$\cos B = \frac{-29}{48}$$

$$B = \cos^{-1}\left(\frac{-29}{48}\right)$$

$$B \approx 127^{\circ}$$

79.

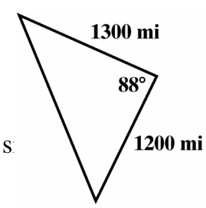
$$\sqrt{26(26-12)(26-16)(26-24)}$$

$$= \sqrt{26(14)(10)(2)}$$

$$= \sqrt{7280}$$

$$= 4\sqrt{455}$$

$$\approx 85$$

80. 

Section 7.2

Check Point Exercises

1. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle.
Thus, we will find a.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 7^2 + 8^2 - 2(7)(8)\cos 120°$$
$$= 49 + 64 - 112(-0.5)$$
$$= 169$$
$$a = \sqrt{169} = 13$$

Use the Law of Sines to find the angle opposite the shorter of the two sides. Thus, we will find acute angle B.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{7}{\sin B} = \frac{13}{\sin 120°}$$
$$13\sin B = 7\sin 120°$$
$$\sin B = \frac{7\sin 120°}{13} \approx 0.4663$$
$$B \approx 28°$$

Find the third angle.
$C = 180° - A - B \approx 180° - 120° - 28° = 32°$
The solution is $a = 13, B \approx 28°,$ and $C \approx 32°$.

2. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find the angle opposite the longest side.
Thus, we will find angle B.

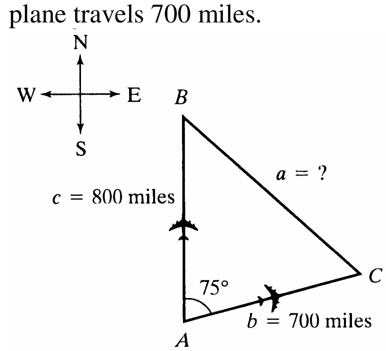
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$2ac\cos B = a^2 + c^2 - b^2$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos B = \frac{8^2 + 5^2 - 10^2}{2 \cdot 8 \cdot 5} = -\frac{11}{80}$$
$$\cos^{-1}\left(\frac{11}{80}\right) \approx 82.1°$$

B is obtuse, since cos B is negative.
$B \approx 180° - 82.1° = 97.9°$
Use the Law of Sines to find either of the two remaining acute angles. We will find angle A.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{8}{\sin A} = \frac{10}{\sin 97.9°}$$
$$10\sin A = 8\sin 97.9°$$
$$\sin A = \frac{8\sin 97.9°}{10} \approx 0.7924$$
$$A \approx 52.4°$$

Find the third angle.
$C = 180° - A - B \approx 180° - 52.4° - 97.9°$
$= 29.7°$
The solution is $B \approx 97.9°, A \approx 52.4°,$ and $C \approx 29.7°$

3. The plane flying 400 miles per hour travels $400 \cdot 2 = 800$ miles in 2 hours. Similarly, the other plane travels 700 miles.



Use the figure and the Law of Cosines to find a in this SAS situation.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 700^2 + 800^2 - 2(700)(800)\cos 75°$$
$$\approx 840{,}123$$
$$a \approx \sqrt{840{,}123} \approx 917$$

After 2 hours, the planes are approximately 917 miles apart.

4. Begin by calculating one-half the perimeter:

$$s = \frac{1}{2}(a + b + c) = \frac{1}{2}(6 + 16 + 18) = 20$$

Use Heron's formula to find the area.

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{20(20-6)(20-16)(20-18)}$$
$$= \sqrt{2240} \approx 47$$

The area of the triangle is approximately 47 square meters.

Exercise Set 7.2

1. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle.
Thus, we will find a.

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 4^2 + 8^2 - 2(4)(8)\cos 46°$

$a^2 = 16 + 64 - 64(\cos 46°)$

$a^2 \approx 35.54$

$a \approx \sqrt{35.54} \approx 6.0$

Use the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle B.

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{4}{\sin B} = \frac{\sqrt{35.54}}{\sin 46°}$

$\sqrt{35.54}\sin B = 4\sin 46°$

$\sin B = \frac{4\sin 46°}{\sqrt{35.54}} \approx 0.4827$

$B \approx 29°$

Find the third angle.

$C = 180° - A - B \approx 180° - 46° - 29° = 105°$

The solution is $a \approx 6.0, B \approx 29°,$ and $C \approx 105°$.

2. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle. Thus, we will find b.

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 6^2 + 8^2 - 2(6)(8)\cos 32°$

$b^2 = 36 + 64 - 96\cos 32°$

$b^2 \approx 18.59$

$b \approx \sqrt{18.59} \approx 4.3$

Use the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle A.

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{6}{\sin A} = \frac{\sqrt{18.59}}{\sin 32°}$

$\sqrt{18.59}\sin A = 6\sin 32°$

$\sin A = \frac{6\sin 32°}{\sqrt{18.59}} \approx 0.7374$

$A \approx 48°$

Find the third angle.

$C = 180° - A - B \approx 180° - 48° - 32° = 100°$

The solution is $b \approx 4.3, A \approx 48°,$ and $C \approx 100°$.

3. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle.
Thus, we will find c.

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 6^2 + 4^2 - 2(6)(4)\cos 96°$

$c^2 = 36 + 16 - 48(\cos 96°)$

$c^2 \approx 57.02$

$c \approx \sqrt{57.02} \approx 7.6$

Use the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle B.

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{4}{\sin B} = \frac{\sqrt{57.02}}{\sin 96°}$

$\sqrt{57.02}\sin B = 4\sin 96°$

$\sin B = \frac{4\sin 96°}{\sqrt{57.02}} \approx 0.5268$

$B \approx 32°$

Find the third angle.

$A = 180° - B - C \approx 180° - 32° - 96° = 52°$

The solution is $c \approx 7.6, A \approx 52°,$ and $B \approx 32°$.

4. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle. Thus, we will find a.

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 6^2 + 15^2 - 2(6)(15)\cos 22°$

$a^2 = 36 + 225 - 180(\cos 22°)$

$a^2 \approx 94.11$

$a \approx \sqrt{94.11} \approx 9.7$

Use the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle B.

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{6}{\sin B} = \frac{\sqrt{94.11}}{\sin 22°}$

$\sqrt{94.11}\sin B = 6\sin 22°$

$\sin B = \frac{6\sin 22°}{\sqrt{94.11}} \approx 0.2317$

$B \approx 13°$

Find the third angle.

$C = 180° - A - B \approx 180° - 22° - 13° = 145°$

The solution is $a \approx 9.7, B \approx 13°,$ and $C \approx 145°$.

5. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find the angle opposite the longest side. Since two sides have length 8, we can begin by finding angle *B* or *C*.

$b^2 = a^2 + c^2 - 2ac\cos B$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \dfrac{6^2 + 8^2 - 8^2}{2 \cdot 6 \cdot 8} = \dfrac{36}{96} = \dfrac{3}{8}$

$B \approx 68°$

Use the Law of Sines to find either of the two remaining acute angles. We will find angle *A*.

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\dfrac{6}{\sin A} = \dfrac{8}{\sin 68°}$

$8\sin A = 6\sin 68°$

$\sin A = \dfrac{6\sin 68°}{8} \approx 0.6954$

$A \approx 44°$

Find the third angle.

$C = 180° - B - A \approx 180° - 68° - 44° = 68°$

The solution is $A \approx 44°, B \approx 68°,$ and $C \approx 68°$.

6. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find the angle opposite the longest side. Thus, we will find *C*.

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \dfrac{10^2 + 12^2 - 16^2}{2 \cdot 10 \cdot 12} = -\dfrac{12}{240}$

C is obtuse, since cos *C* is negative.

$\cos^{-1}\left(\dfrac{12}{240}\right) \approx 87°$

$C \approx 180° - 87° = 93°$

Use the Law of Sines to find either of the two remaining acute angles. We will find angle *B*.

$\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\dfrac{12}{\sin B} = \dfrac{16}{\sin 93°}$

$16\sin B = 12\sin 93°$

$\sin B = \dfrac{12\sin 93°}{16} \approx 0.7490$

$B \approx 49°$

Find the third angle.

$A = 180° - B - C \approx 180° - 49° - 93° = 38°$

The solution is $A \approx 38°, B \approx 49°,$ and $C \approx 93°$.

7. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find the angle opposite the longest side. Thus, we will find angle *A*

$a^2 = b^2 + c^2 - 2bc\cos A$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{4^2 + 3^2 - 6^2}{2 \cdot 4 \cdot 3} = -\dfrac{11}{24}$

A is obtuse, since cos *A* is negative.

$\cos^{-1}\left(\dfrac{11}{24}\right) \approx 63°$

$A \approx 180° - 63° = 117°$

Use the Law of Sines to find either of the two remaining acute angles. We will find angle *B*.

$\dfrac{b}{\sin B} = \dfrac{a}{\sin A}$

$\dfrac{4}{\sin B} = \dfrac{6}{\sin 117°}$

$6\sin B = 4\sin 117°$

$\sin B = \dfrac{4\sin 117°}{6} \approx 0.5940$

$B \approx 36°$

Find the third angle.

$C = 180° - B - A \approx 180° - 36° - 117° = 27°$

The solution is $A \approx 117°, B \approx 36°,$ and $C \approx 27°$.

8. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find the angle opposite the longest side. Thus, we will find *B*.

$b^2 = a^2 + c^2 - 2ac\cos B$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \dfrac{10^2 + 8^2 - 16^2}{2 \cdot 10 \cdot 8} = -\dfrac{23}{40}$

B is obtuse, since cos *B* is negative.

$\cos^{-1}\left(\dfrac{23}{40}\right) \approx 55°$

$B \approx 180° - 55° = 125°$

Use the Law of Sines to find either of the two remaining acute angles. We will find angle A.

$$\frac{a}{\sin A}=\frac{b}{\sin B}$$

$$\frac{10}{\sin A}=\frac{16}{\sin 125°}$$

$$16\sin A=10\sin 125°$$

$$\sin A=\frac{10\sin 125°}{16}\approx 0.5120$$

$$A\approx 31°$$

Find the third angle.

$C=180°-B-A\approx 180°-125°-31°=24°$

The solution is $B\approx 125°, A\approx 31°,$ and $C\approx 24°$.

9. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle.
Thus, we will find c.

$$c^2=a^2+b^2-2ab\cos C$$

$$c^2=5^2+7^2-2(5)(7)\cos 42°$$

$$c^2=25+49-70(\cos 42°)$$

$$c^2\approx 21.98$$

$$c\approx\sqrt{21.98}\approx 4.7$$

Use the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle A.

$$\frac{a}{\sin A}=\frac{c}{\sin C}$$

$$\frac{5}{\sin A}=\frac{\sqrt{21.98}}{\sin 42°}$$

$$\sqrt{21.98}\sin A=5\sin 42°$$

$$\sin A=\frac{5\sin 42°}{\sqrt{21.98}}\approx 0.7136$$

$$A\approx 46°$$

Find the third angle.

$B=180°-C-A\approx 180°-42°-46°=92°$

The solution is $c\approx 4.7, A\approx 46°,$ and $B\approx 92°$.

10. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle. Thus, we will find c.

$$c^2=a^2+b^2-2ab\cos C$$

$$c^2=10^2+3^2-2(10)(3)\cos 15°$$

$$c^2=100+9-60(\cos 15°)$$

$$c^2\approx 51.04$$

$$c\approx\sqrt{51.04}\approx 7.1$$

Use the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle B.

$$\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$\frac{3}{\sin B}=\frac{\sqrt{51.04}}{\sin 15°}$$

$$\sqrt{51.04}\sin B=3\sin 15°$$

$$\sin B=\frac{3\sin 15°}{\sqrt{51.04}}\approx 0.1087$$

$$B\approx 6°$$

Find the third angle.

$A=180°-C-B\approx 180°-15°-6°=159°$

The solution is $c\approx 7.1, B\approx 6°,$ and $A\approx 159°$.

11. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle.
Thus, we will find a.

$$a^2=b^2+c^2-2bc\cos A$$

$$a^2=5^2+3^2-2(5)(3)\cos 102°$$

$$a^2=25+9-30(\cos 102°)$$

$$a^2\approx 40.24$$

$$a\approx\sqrt{40.24}\approx 6.3$$

Use the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle C.

$$\frac{c}{\sin C}=\frac{a}{\sin A}$$

$$\frac{3}{\sin C}=\frac{\sqrt{40.24}}{\sin 102°}$$

$$\sqrt{40.24}\sin C=3\sin 102°$$

$$\sin C=\frac{3\sin 102°}{\sqrt{40.24}}\approx 0.4626$$

$$C\approx 28°$$

Find the third angle.

$B=180°-C-A\approx 180°-28°-102°=50°$

The solution is $a\approx 6.3, C\approx 28°,$ and $B\approx 50°$.

12. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle. Thus, we will find a.

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 4^2 + 1^2 - 2(4)(1)\cos 100°$

$a^2 = 16 + 1 - 8(\cos 100°)$

$a^2 \approx 18.39$

$a \approx \sqrt{18.39} \approx 4.3$

Use the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle C.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{1}{\sin C} = \frac{\sqrt{18.39}}{\sin 100°}$$

$$\sqrt{18.39}\sin C = \sin 100°$$

$$\sin C = \frac{\sin 100°}{\sqrt{18.39}} \approx 0.2296$$

$$C \approx 13°$$

Find the third angle.

$B = 180° - C - A \approx 180° - 13° - 100° = 67°$

The solution is $a \approx 4.3, C \approx 13°,$ and $B \approx 67°$.

13. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle.
Thus, we will find b.

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 6^2 + 5^2 - 2(6)(5)\cos 50°$

$b^2 = 36 + 25 - 60(\cos 50°)$

$b^2 \approx 22.43$

$b \approx \sqrt{22.43} \approx 4.7$

Use the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle C.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{5}{\sin C} = \frac{\sqrt{22.43}}{\sin 50°}$$

$$\sqrt{22.43}\sin C = 5\sin 50°$$

$$\sin C = \frac{5\sin 50°}{\sqrt{22.43}} \approx 0.8087$$

$$C \approx 54°$$

Find the third angle.

$A = 180° - C - B \approx 180° - 54° - 50° = 76°$

The solution is $b \approx 4.7, C \approx 54°,$ and $A \approx 76°$.

14. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle. Thus, we will find b.

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 4^2 + 7^2 - 2(4)(7)\cos 55°$

$b^2 = 16 + 49 - 56(\cos 55°)$

$b^2 \approx 32.88$

$b \approx \sqrt{32.88} \approx 5.7$

Use the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle A.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{4}{\sin A} = \frac{\sqrt{32.88}}{\sin 55°}$$

$$\sqrt{32.88}\sin A = 4\sin 55°$$

$$\sin A = \frac{4\sin 55°}{\sqrt{32.88}} \approx 0.5714$$

$$A \approx 35°$$

Find the third angle.

$C = 180° - B - A \approx 180° - 55° - 35° = 90°$

The solution is $b \approx 5.7, A \approx 35°,$ and $C \approx 90°$.

15. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle.
Thus, we will find b.

$b^2 = a^2 + c^2 - 2ac\cos 90°$

$b^2 = 5^2 + 2^2 - 2(5)(2)\cos 90°$

$b^2 = 25 + 4 - 20\cos 90°$

$b^2 = 29$

$b = \sqrt{29} \approx 5.4$

(use exact value of b from previous step) Use the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle C.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{2}{\sin C} = \frac{\sqrt{29}}{\sin 90°}$$

$$\sqrt{29}\sin C = 2\sin 90°$$

$$\sin C = \frac{2\sin 90°}{\sqrt{29}} \approx 0.3714$$

$$C \approx 22°$$

Find the third angle.

$A = 180° - C - B \approx 180° - 22° - 90° = 68°$

The solution is $b \approx 5.4, C \approx 22°,$ and $A \approx 68°$.

16. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle. Thus, we will find b.

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 7^2 + 3^2 - 2(7)(3)\cos 90°$$
$$b^2 = 49 + 9 - 42\cos 90°$$
$$b^2 = 58$$
$$b = \sqrt{58} \approx 7.6$$

(use exact value of b from previous step)
Use the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle C.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{3}{\sin C} = \frac{\sqrt{58}}{\sin 90°}$$
$$\sqrt{58}\sin C = 3\sin 90°$$
$$\sin C = \frac{3\sin 90°}{\sqrt{58}} \approx 0.3939$$
$$C \approx 23°$$

Find the third angle.
$A = 180° - C - B \approx 180° - 23° - 90° = 67°$
The solution is $b \approx 7.6, C \approx 23°,$ and $A \approx 67°$.

17. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find the angle opposite the longest side. Thus, we will find C.

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$\cos C = \frac{5^2 + 7^2 - 10^2}{2\cdot 5\cdot 7} = -\frac{13}{35}$$

C is obtuse, since cos C is negative.

$$\cos^{-1}\left(\frac{13}{35}\right) \approx 68°$$
$$C \approx 180° - 68° = 112°$$

Use the Law of Sines to find either of the two remaining angles. We will find angle A.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{5}{\sin A} = \frac{10}{\sin 112°}$$
$$10\sin A = 5\sin 112°$$
$$\sin A = \frac{5\sin 112°}{10} \approx 0.4636$$
$$A \approx 28°$$

Find the third angle.
$B = 180° - C - A \approx 180° - 112° - 28° = 40°$
The solution is $C \approx 112°, A \approx 28°,$ and $B \approx 40°$.

18. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find the angle opposite the longest side. Thus, we will find C.

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$\cos C = \frac{4^2 + 6^2 - 9^2}{2\cdot 4\cdot 6} = -\frac{29}{48}$$

C is obtuse, since cos C is negative.

$$\cos^{-1}\left(\frac{29}{48}\right) \approx 53°$$
$$C \approx 180° - 53° = 127°$$

Use the Law of Sines to find either of the two remaining angles. We will find angle A.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{4}{\sin A} = \frac{9}{\sin 127°}$$
$$9\sin A = 4\sin 127°$$
$$\sin A = \frac{4\sin 127°}{9} \approx 0.3549$$
$$A \approx 21°$$

Find the third angle.
$B = 180° - C - A \approx 180° - 127° - 21° = 32°$
The solution is $C \approx 127°, A \approx 21°,$ and $B \approx 32°$.

19. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find the angle opposite the longest side. Thus, we will find *B*.

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{3^2 + 8^2 - 9^2}{2 \cdot 3 \cdot 8} = -\frac{1}{6}$$

B is obtuse, since cos *B* is negative.

$$\cos^{-1}\left(\frac{1}{6}\right) \approx 80°$$

$$B \approx 180° - 80° = 100°$$

Use the Law of Sines to find either of the two remaining angles. We will find angle *A*.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{3}{\sin A} = \frac{9}{\sin 100°}$$

$$9\sin A = 3\sin 100°$$

$$\sin A = \frac{3\sin 100°}{9} \approx 0.3283$$

$$A \approx 19°$$

Find the third angle.

$$C = 180° - B - A \approx 180° - 100° - 19° = 61°$$

The solution is $B \approx 100°, A \approx 19°,$ and $C \approx 61°$.

20. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find the angle opposite the longest side. Thus, we will find *B*.

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{4^2 + 6^2 - 7^2}{2 \cdot 4 \cdot 6} = \frac{1}{16}$$

$$B \approx 86°$$

Use the Law of Sines to find either of the two remaining angles. We will find angle *A*.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{4}{\sin A} = \frac{7}{\sin 86°}$$

$$7\sin A = 4\sin 86°$$

$$\sin A = \frac{4\sin 86°}{7} \approx 0.5700$$

$$A \approx 35°$$

Find the third angle.

$$C = 180° - B - A \approx 180° - 86° - 35° = 59°$$

The solution is $B \approx 86°, A \approx 35°,$ and $C \approx 59°$.

21. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find any of the three angles, since each side has the same measure.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{3^2 + 3^2 - 3^2}{2 \cdot 3 \cdot 3} = \frac{1}{2}$$

$$A = 60°$$

Use the Law of Sines to find either of the two remaining angles. We will find angle *B*.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{3}{\sin B} = \frac{3}{\sin 60°}$$

$$3\sin B = 3\sin 60°$$

$$\sin B = \sin 60°$$

$$B = 60°$$

Find the third angle.

$$C = 180° - A - B = 180° - 60° - 60° = 60°$$

The solution is $A = 60°, B = 60°,$ and $C = 60°$.

22. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find any of the three angles, since each side has the same measure.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{5^2 + 5^2 - 5^2}{2 \cdot 5 \cdot 5} = \frac{1}{2}$$

$$A = 60°$$

Use the Law of Sines to find either of the two remaining angles. We will find angle *B*.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{5}{\sin B} = \frac{5}{\sin 60°}$$

$$5\sin B = 5\sin 60°$$

$$\sin B = \sin 60°$$

$$B = 60°$$

Find the third angle.

$$C = 180° - A - B = 180° - 60° - 60° = 60°$$

The solution is $A = 60°, B = 60°,$ and $C = 60°$.

23. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find the angle opposite the longest side. Thus, we will find *A*.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{22^2 + 50^2 - 63^2}{2 \cdot 22 \cdot 50} = -\frac{985}{2200}$$

$$A \approx 117°$$

Use the Law of Sines to find either of the two remaining angles. We will find angle *B*.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{22}{\sin B} = \frac{63}{\sin 117°}$$

$$63\sin B = 22\sin 117°$$

$$\sin B = \frac{22\sin 117°}{63}$$

$$B = 18°$$

Find the third angle.

$C = 180° - A - B = 180° - 117° - 18° = 45°$

The solution is $A = 117°, B = 18°,$ and $C = 45°$.

24. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find the angle opposite the longest side. Thus, we will find *A*.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{25^2 + 45^2 - 66^2}{2 \cdot 25 \cdot 45} = -\frac{853}{1125}$$

A is obtuse, since cos *A* is negative.

$$\cos^{-1}\left(\frac{853}{1125}\right) \approx 41°$$

$$A \approx 180° - 41° = 139°$$

Use the Law of Sines to find either of the two remaining angles. We will find angle *B*.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{25}{\sin B} = \frac{66}{\sin 139°}$$

$$66\sin B = 25\sin 139°$$

$$\sin B = \frac{25\sin 139°}{66} \approx 0.2485$$

$$B \approx 14°$$

Find the third angle.

$C = 180° - A - B \approx 180° - 139° - 14° = 27°$

The solution is $A \approx 139°, B \approx 14°,$ and $C \approx 27°$.

25. $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(4+4+2) = 5$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{5(5-4)(5-4)(5-2)} = \sqrt{15} \approx 4$$

The area of the triangle is approximately 4 square feet.

26. $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(5+5+4) = 7$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{7(7-5)(7-5)(7-4)} = \sqrt{84} \approx 9$$

The area of the triangle is approximately 9 square feet.

27. $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(14+12+4) = 15$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{15(15-14)(15-12)(15-4)} = \sqrt{495} \approx 22$$

The area of the triangle is approximately 22 square meters.

28. $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(16+10+8) = 17$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{17(17-16)(17-10)(17-8)} = \sqrt{1071} \approx 33$$

The area of the triangle is approximately 33 square meters.

29. $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(11+9+7) = 13.5$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{13.5(13.5-11)(13.5-9)(13.5-7)} = \sqrt{987.1875} \approx 31$$

The area of the triangle is approximately 31 square yards.

30. $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(13+9+5) = 13.5$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{13.5(13.5-13)(13.5-9)(13.5-5)} = \sqrt{258.1875} \approx 16$$

The area of the triangle is approximately 16 square yards.

31. $C = 180° - 15° - 35° = 130°$

$c^2 = b^2 + c^2 - 2bc\cos C$

$c^2 = 8^2 + 13^2 - 2(8)(13)\cos 130°$

$c^2 \approx 366.6998$

$c \approx 19.1$

Use the law of sines to find the solution is $A \approx 31°, B \approx 19°, C \approx 130°,$ and $c \approx 19.1$. .

32. $C = 180° - 35° - 50° = 95°$

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 3^2 + 2^2 - 2(3)(2)\cos 95°$

$c^2 \approx 14.0459$

$c \approx 3.7$

Use the law of sines to find the solution is $A \approx 54°, B \approx 31°, C \approx 95°,$ and $c \approx 3.7$. .

33. Use the given radii to determine that $a = 7.5, b = 8.5,$ and $c = 9.0$.

$c^2 = a^2 + b^2 - 2ab\cos C$

$9^2 = 7.5^2 + 8.5^2 - 2(7.5)(8.5)\cos C$

$\cos C \approx 0.3725$

$C \approx 68°$

Use the law of sines to find the solution is $A \approx 51°, B \approx 61°,$ and $C \approx 68°$.

34. Use the given radii to determine that $a = 7.3, b = 10.5,$ and $c = 11.8$.

$c^2 = a^2 + b^2 - 2ab\cos C$

$11.8^2 = 7.3^2 + 10.5^2 - 2(7.3)(10.5)\cos C$

$\cos C \approx 0.1585$

$C \approx 81°$

Use the law of sines to find the solution is $A \approx 38°, B \approx 61°,$ and $C \approx 81°$.

35. Use the distance formula to determine that $a = \sqrt{61} \approx 7.8, b = \sqrt{10} \approx 3.2,$ and $c = 5$.

$a^2 = b^2 + c^2 - 2bc\cos A$

$\sqrt{61}^2 = \sqrt{10}^2 + 5^2 - 2\left(\sqrt{10}\right)(5)\cos A$

$\cos A \approx -0.8222$

$A \approx 145°$

Use the law of sines to find the solution is $A \approx 145°, B \approx 13°,$ and $C \approx 22°$.

36. Use the distance formula to determine that $a = \sqrt{13} \approx 3.6, b = \sqrt{26} \approx 5.1,$ and $c = 5$.

$b^2 = a^2 + c^2 - 2ac\cos B$

$\sqrt{26}^2 = \sqrt{13}^2 + 5^2 - 2\left(\sqrt{13}\right)(5)\cos B$

$\cos B \approx 0.3328$

$B \approx 71°$

Use the law of sines to find the solution is $A \approx 42°, B \approx 71°,$ and $C \approx 67°$.

37. Use the law of cosines.

$c^2 = a^2 + b^2 - 2ab\cos C$

$5.78^2 = 2.9^2 + 3.0^2 - 2(2.9)(3.0)\cos\theta$

$\cos\theta \approx -0.9194$

$\theta \approx 157°$

This dinosaur was an efficient walker.

38. Use the law of cosines.

$c^2 = a^2 + b^2 - 2ab\cos C$

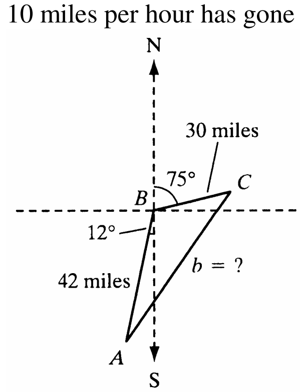
$5.2^2 = 3.6^2 + 3.2^2 - 2(3.6)(3.2)\cos\theta$

$\cos\theta \approx -0.1667$

$\theta \approx 100°$

This dinosaur was not an efficient walker.

39. Let b = the distance between the ships after three hours.
After three hours, the ship traveling 14 miles per hour has gone $3 \cdot 14$ or 42 miles. Similarly, the ship traveling 10 miles per hour has gone 30 miles.



Using the figure,

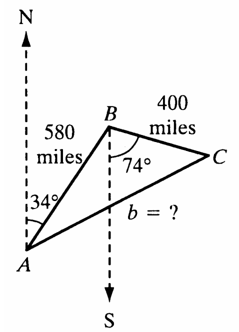
$B = 180° - 75° + 12° = 117°$

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 30^2 + 42^2 - 2(30)(42)\cos 117° \approx 3808$

$b \approx 61.7$

After three hours, the ships will be about 61.7 miles apart.

40. 

Using the figure,

$B = 74° + 34° = 108°$

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 400^2 + 580^2 - 2(400)(580)\cos 108°$

$\approx 639,784$

$b \approx \sqrt{639,784} \approx 799.9$

The distance from airport A to airport B is about 799.9 miles.

41. Let b = the distance across the lake.

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 160^2 + 140^2 - 2(160)(140)\cos 80°$

$\approx 37,421$

$b \approx \sqrt{37,421} \approx 193$

The distance across the lake is about 193 yards.

42. Let c = the distance from A to B.

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 105^2 + 65^2 - 2(105)(65)\cos 80° \approx 12,880$

$c \approx \sqrt{12,880} \approx 113$

The distance from A to B is about 113 yards.

43. Assume that Island B is due east of Island A. Let A = angle at Island A.

$a^2 = b^2 + c^2 - 2bc\cos A$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{5^2 + 6^2 - 7^2}{2 \cdot 5 \cdot 6} = \dfrac{1}{5}$

$A \approx 78°$

Since $90° - 78° = 12°$, you should navigate on a bearing of N12°E.

44. Assume that Island A is due west of Island B. Let B = angle at Island B.

$b^2 = a^2 + c^2 - 2ac\cos B$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \dfrac{7^2 + 6^2 - 5^2}{2 \cdot 7 \cdot 6} = \dfrac{5}{7}$

$B \approx 44°$

Since $90° - 44° = 46°$, you should navigate on a bearing of N46°W.

45. **a.** Using the figure,

$B = 90° - 40° = 50°$

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 13.5^2 + 25^2 - 2(13.5)(25)\cos 50°$

≈ 373

$b \approx \sqrt{373} \approx 19.3$

You are about 19.3 miles from the pier.

b. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

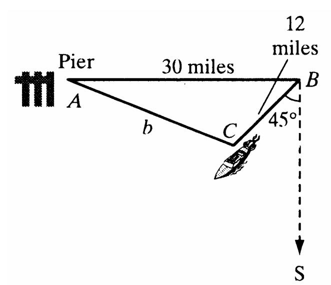
$\dfrac{13.5}{\sin A} = \dfrac{\sqrt{373}}{\sin 50°}$

$\sqrt{373}\sin A = 13.5\sin 50°$

$\sin A = \dfrac{13.5\sin 50°}{\sqrt{373}} \approx 0.5355$

$A \approx 32°$

Since 90° – 32° = 58°, the original bearing could have been S58ºE.

46. 

a. Using the figure,

$B = 90° - 45° = 45°$

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 12^2 + 30^2 - 2(12)(30)\cos 45° \approx 535$

$b \approx \sqrt{535} \approx 23.1$

You are about 23.1 miles from the pier.

b. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\dfrac{12}{\sin A} = \dfrac{\sqrt{535}}{\sin 45°}$

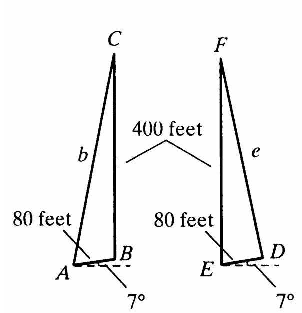
$\sqrt{535}\sin A = 12\sin 45°$

$\sin A = \dfrac{12\sin 45°}{\sqrt{535}} \approx 0.3669$

$A \approx 22°$

Since $90° - 22° = 68°$, the original bearing could have been S68°E.

47.



In the figure, b = the guy wire anchored downhill, e = the guy wire anchored uphill.

$B = 90° + 7° = 97°$

$E = 90° - 7° = 83°$

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 400^2 + 80^2 - 2(400)(80)\cos 97°$

$\approx 174,200$

$b \approx \sqrt{174,200} \approx 417.4$

$e^2 = d^2 + f^2 - 2df\cos E$

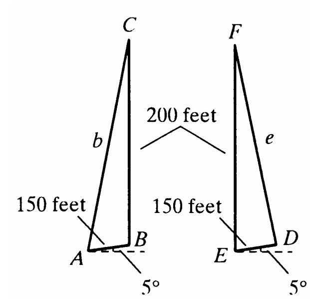
$e^2 = 400^2 + 80^2 - 2(400)(80)\cos 83°$

$\approx 158,600$

$e \approx \sqrt{158.600} \approx 398.2$

The guy wire anchored downhill is about 417.4 feet long. The one anchored uphill is about 398.2 feet long.

48.



In the figure, b = the guy wire anchored downhill, e = the guy wire anchored uphill.

$B = 90° + 5° = 95°$

$E = 90° - 5° = 85°$

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 200^2 + 150^2 - 2(200)(150)\cos 95° \approx 67,729$

$b \approx \sqrt{67,729} \approx 260.2$

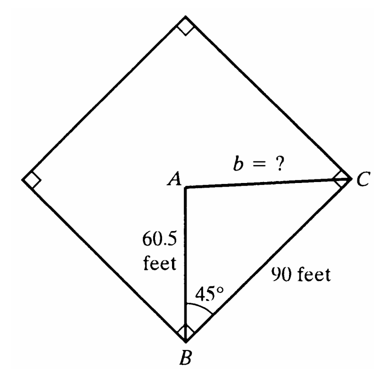
$e^2 = d^2 + f^2 - 2df\cos E$

$e^2 = 200^2 + 150^2 - 2(200)(150)\cos 85° \approx 57,271$

$e \approx \sqrt{57,271} \approx 239.3$

The guy wire anchored downhill is about 260.2 feet long. The one anchored uphill is about 239.3 feet long.

49.



Using the figure,

$B = 90° \div 2 = 45°$ (using symmetry)

$b^2 = a^2 + c^2 - 2ac\cos B$

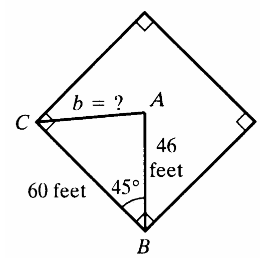
$b^2 = 90^2 + 60.5^2 - 2(90)(60.5)\cos 45°$

≈ 4060

$b \approx \sqrt{4060} \approx 63.7$

It is about 63.7 feet from the pitcher's mound to first base.

50.



Using the figure,

$B = 90° \div 2 = 45°$ (using symmetry)

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 60^2 + 46^2 - 2(60)(46)\cos 45° \approx 1813$

$b \approx \sqrt{1813} \approx 42.6$

It is about 42.6 feet from the pitcher's mound to third base.

51. First, find the area using Heron's formula.

$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(240+300+420) = 480$

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{480(480-240)(480-300)(480-420)}$

$= \sqrt{1,244,160,000} \approx 35,272.65$

Now multiply by the price per square foot.

$(35,272.65)(3.50) \approx 123,454$

The cost is $123,454, to the nearest dollar.

52. First, find the area using Heron's formula.

$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(320+510+410) = 620$

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{620(620-320)(620-510)(620-410)}$

$= \sqrt{4,296,600,000} \approx 65,548.46$

Now multiply by the price per square foot.

$(65,548.46)(4.50) \approx 294,968$

The cost is $294,968, to the nearest dollar.

53. – 59. Answers may vary.

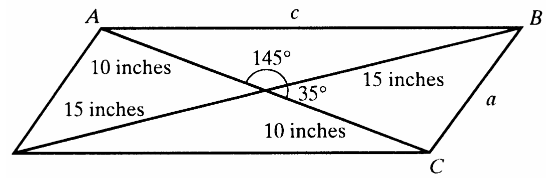
60. does not make sense; Explanations will vary. Sample explanation: The Law of Cosines is not simply the negative of the Law of Sines.

61. makes sense

62. makes sense

63. makes sense

64.



Using the given information and the hint, we arrive at the figure above. Let a = the side opposite the 35° angle, c = the side opposite the 145° angle.

$a^2 = 15^2 + 10^2 - 2(15)(10)\cos 35° \approx 79.3$

$a \approx \sqrt{79.3} \approx 8.9$

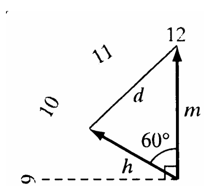
$c^2 = 15^2 + 10^2 - 2(15)(10)\cos 145° \approx 570.7$

$b \approx \sqrt{570.7} \approx 23.9$

The lengths of the parallelogram's sides are about 8.9 inches and 23.9 inches.

65. If we call the lower left point D, and the lower right point E, then the Law of Cosines will give all three angles in triangle ADE and triangle ABE. That allows us find $A \approx 29°$, $B \approx 87°$, and $C \approx 64°$. The Law of Sines will then allow us to find $a \approx 11.6$ and $b \approx 23.9$.

66.



The angle between the minute and hour hand is $\frac{2}{3}$ of the 90° angle from 9 to 12, or 60°.

Let d = the distance between the tips of the hands.

$d^2 = m^2 + h^2 - 2mh\cos 60°$

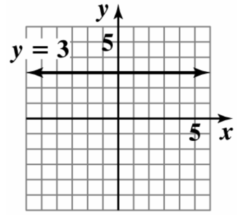
$= m^2 + h^2 - 2mh\left(\frac{1}{2}\right)$

$= m^2 + h^2 - mh$

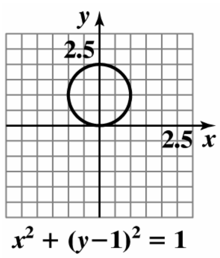
$d = \sqrt{m^2 + h^2 - mh}$

67. Answers may vary.

68. $y = 3$ is a horizontal line through (0, 3).



69. $x^2 + (y-1)^2 = 1$ is a circle centered at (0, 1) with a radius of 1.



70.
$$x^2 + 6x + y^2 = 0$$
$$x^2 + 6x \quad + y^2 = 0$$
$$x^2 + 6x + 9 + y^2 = 0 + 9$$
$$(x+3)^2 + y^2 = 9$$

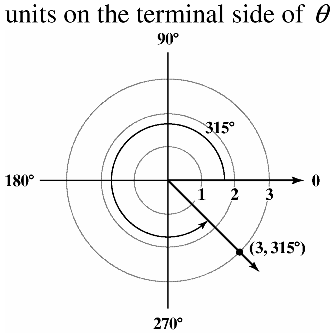
$(x+3)^2 + y^2 = 9$ is a circle centered at $(-3, 0)$ with a radius of 3.

Section 7.3

Check Point Exercises

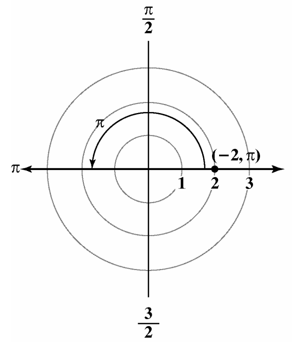
1. **a.** $(r, \theta) = (3, 315°)$

Because 315° is a positive angle, draw $\theta = 315°$ counterclockwise from the polar axis. Because $r > 0$, plot the point by going out 3 units on the terminal side of θ.



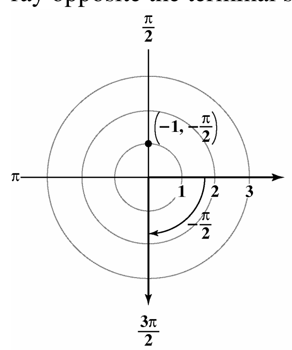
b. $(r, \theta) = (-2, \pi)$

Because π is a positive angle, draw $\theta = \pi$ counterclockwise from the polar axis. Because $r < 0$, plot the point by going out 2 units along the ray opposite the terminal side of θ.



c. $(r, \theta) = \left(-1, -\frac{\pi}{2}\right)$

Because $-\frac{\pi}{2}$ is a negative angle, draw $\theta = -\frac{\pi}{2}$ clockwise from the polar axis. Because $r < 0$, plot the point by going out one unit along the ray opposite the terminal side of θ.



2. **a.** Add 2π to the angle and do not change r.

$$\left(5, \frac{\pi}{4}\right) = \left(5, \frac{\pi}{4} + 2\pi\right) = \left(5, \frac{\pi}{4} + \frac{8\pi}{4}\right)$$
$$= \left(5, \frac{9\pi}{4}\right)$$

b. Add π to the angle and replace r by $-r$.

$$\left(5, \frac{\pi}{4}\right) = \left(-5, \frac{\pi}{4} + \pi\right) = \left(-5, \frac{\pi}{4} + \frac{4\pi}{4}\right)$$
$$= \left(-5, \frac{5\pi}{4}\right)$$

c. Subtract 2π from the angle and do not change r.

$$\left(5,\frac{\pi}{4}\right)=\left(5,\frac{\pi}{4}-2\pi\right)=\left(5,\frac{\pi}{4}-\frac{8\pi}{4}\right)$$
$$=\left(5,-\frac{7\pi}{4}\right)$$

3. **a.** $(r,\theta)=(3,\pi)$

$x=r\cos\theta=3\cos\pi=3(-1)=-3$

$y=r\sin\theta=3\sin\pi=3(0)=0$

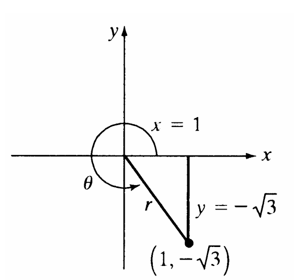
The rectangular coordinates of $(3,\pi)$ are $(-3, 0)$.

b. $(r,\theta)=\left(-10,\frac{\pi}{6}\right)$

$$x=r\cos\theta=-10\cos\frac{\pi}{6}=-10\left(\frac{\sqrt{3}}{2}\right)$$
$$=-5\sqrt{3}$$
$$y=r\sin\theta=-10\sin\frac{\pi}{6}=-10\left(\frac{1}{2}\right)=-5$$

The rectangular coordinates of $\left(-10,\frac{\pi}{6}\right)$ are $\left(-5\sqrt{3},-5\right)$.

4.



$$r=\sqrt{x^2+y^2}=\sqrt{1^2+\left(-\sqrt{3}\right)^2}$$
$$=\sqrt{1+3}=\sqrt{4}=2$$
$$\tan\theta=\frac{y}{x}=\frac{-\sqrt{3}}{1}=-\sqrt{3}$$

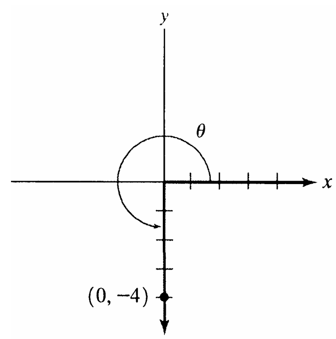
Because $\tan\frac{\pi}{3}=\sqrt{3}$ and θ lies in quadrant IV,

$$\theta=2\pi-\frac{\pi}{3}=\frac{6\pi}{3}-\frac{\pi}{3}=\frac{5\pi}{3}$$

The polar coordinates of $\left(1,-\sqrt{3}\right)$ are

$$(r,\theta)=\left(2,\frac{5\pi}{3}\right)$$

5.



$$r=\sqrt{x^2+y^2}=\sqrt{(0)^2+(-4)^2}=\sqrt{16}=4$$

The point (0, –4) is on the negative y-axis. Thus, $\theta=\frac{3\pi}{2}$. Polar coordinates of (0, –4) are $\left(4,\frac{3\pi}{2}\right)$.

6. **a.**

$$3x-y=6$$
$$3r\cos\theta-r\sin\theta=6$$
$$r(3\cos\theta-\sin\theta)=6$$
$$r=\frac{6}{3\cos\theta-\sin\theta}$$

b.

$$x^2+(y+1)^2=1$$
$$\left(r\cos\theta\right)^2+(r\sin\theta+1)^2=1$$
$$r^2\cos^2\theta+r^2\sin^2\theta+2r\sin\theta+1=1$$
$$r^2+2r\sin\theta=0$$
$$r\left(r+2\sin\theta\right)=0$$
$$r=0\quad\text{or}\quad r+2\sin\theta=0$$
$$r=-2\sin\theta$$

7. **a.** Use $r^2=x^2+y^2$ to convert to a rectangular equation.

$r=4$

$r^2=16$

$x^2+y^2=16$

The rectangular equation for $r = 4$ is $x^2+y^2=16$.

b. Use $\tan\theta = \frac{y}{x}$ to convert to a rectangular equation in x and y.

$$\theta = \frac{3\pi}{4}$$
$$\tan\theta = \tan\frac{3\pi}{4}$$
$$\tan\theta = -1$$
$$\frac{y}{x} = -1$$
$$y = -x$$

The rectangular equation for $\theta = \frac{3\pi}{4}$ is $y = -x$.

c.
$$r = -2\sec\theta$$
$$r = \frac{-2}{\cos\theta}$$
$$r\cos\theta = -2$$
$$x = -2$$

d.
$$r = 10\sin\theta$$
$$r^2 = 10r\sin\theta$$
$$x^2 + y^2 = 10y$$
$$x^2 + y^2 - 10y = 0$$
$$x^2 + y^2 - 10y + 25 = 25$$
$$x^2 + (y-5)^2 = 25$$

Exercise Set 7.3

1. 225° is in the third quadrant.
C

2. 315° is in the fourth quadrant.
D

3. $\frac{5\pi}{4} = 225°$ is in the third quadrant. Since r is negative, the point lies along the ray opposite the terminal side of θ, in the first quadrant.
A

4. $\frac{\pi}{4} = 45°$ is in the first quadrant. Since r is negative, the point lies along the ray opposite the terminal side of θ, in the third quadrant.
C

5. $\pi = 180°$ lies on the negative x-axis.
B

6. 0 = 0° lies on the positive x-axis. Since r is negative, the point lies along the ray opposite the terminal side of θ, on the negative x-axis.
B

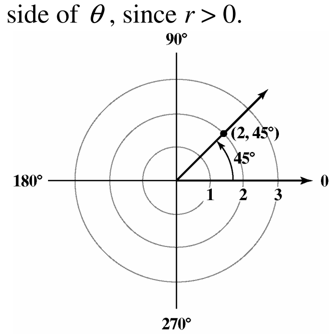
7. –135° is measured clockwise 135° from the positive x-axis. The point lies in the third quadrant.
C

8. –315° is measured clockwise 315° from the positive x-axis. The point lies in the first quadrant.
A

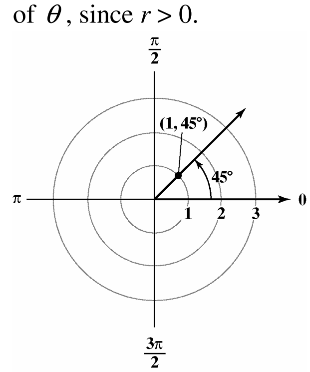
9. $-\frac{3\pi}{4} = -135°$ is measured clockwise 135° from the positive x-axis. Since r is negative, the point lies along the ray opposite the terminal side of θ, in the first quadrant.
A

10. $-\frac{5\pi}{4} = -225°$ is measured clockwise 225° from the positive x-axis. Since r is negative, the point lies along the ray opposite the terminal side of θ, in the fourth quadrant.
D

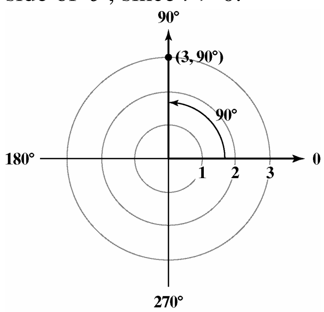
11. Draw $\theta = 45°$ counterclockwise, since θ is positive, from the polar axis. Go out 2 units on the terminal side of θ, since $r > 0$.



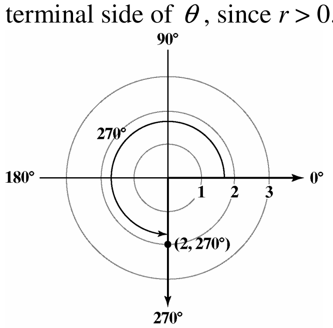
12. Draw $\theta = 45°$ counterclockwise, since θ is positive, from the polar axis. Go out 1 unit on the terminal side of θ, since $r > 0$.



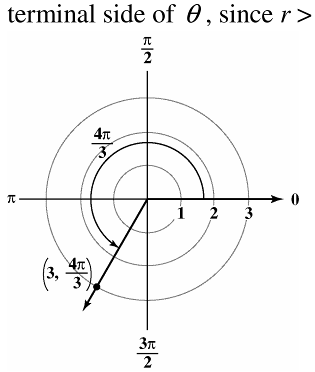
13. Draw $\theta = 90°$ counterclockwise, since θ is positive, from the polar axis. Go out 3 units on the terminal side of θ, since $r > 0$.



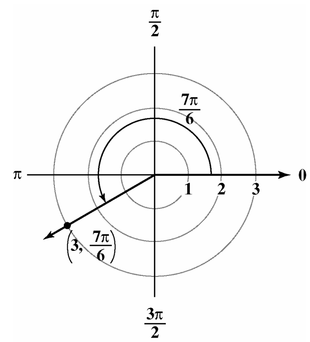
14. Draw $\theta = 270°$ counterclockwise, since θ is positive, from the polar axis. Go out 2 units on the terminal side of θ, since $r > 0$.



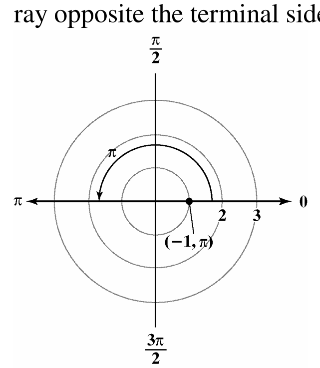
15. Draw $\theta = \frac{4\pi}{3} = 240°$ counterclockwise, since θ is positive, from polar axis. Go out 3 units on the terminal side of θ, since $r > 0$.



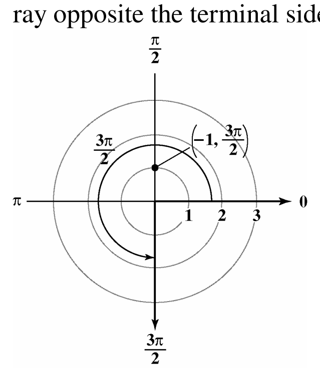
16. Draw $\theta = \frac{7\pi}{6} = 210°$ counterclockwise, since θ is positive, from the polar axis. Go out 3 units on the terminal side of θ, since $r > 0$.



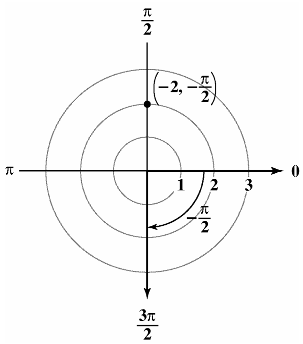
17. Draw $\theta = \pi = 180°$ counterclockwise, since θ is positive, from the polar axis. Go one unit out on the ray opposite the terminal side of θ, since $r < 0$.



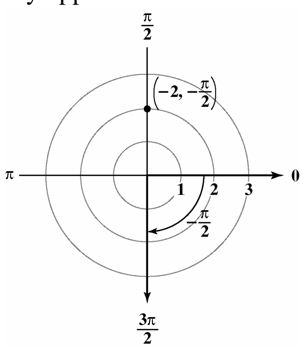
18. Draw $\theta = \frac{3\pi}{2} = 270°$ counterclockwise, since θ is positive, from the polar axis. Go one unit out on the ray opposite the terminal side of θ, since $r < 0$.



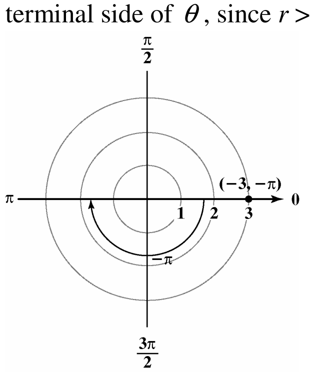
19. Draw $\theta = -\dfrac{\pi}{2} = -90°$ clockwise, since θ is positive, from the polar axis. Go 2 units out on the ray opposite the terminal side of θ, since $r < 0$.



20. Draw $\theta = -\pi = -180°$ clockwise, since θ is negative, from the polar axis. Go 3 units out on the ray opposite the terminal side of θ, since $r < 0$.



21. Draw $\theta = \dfrac{\pi}{6} = 30°$ counterclockwise, since θ is positive, from the polar axis. Go 5 units out on the terminal side of θ, since $r > 0$.



a. Add 2π to the angle and do not change r.

$$\left(5, \frac{\pi}{6}\right) = \left(5, \frac{\pi}{6} + 2\pi\right) = \left(5, \frac{13\pi}{6}\right)$$

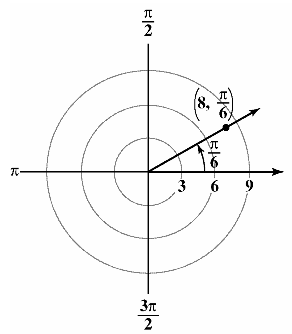
b. Add π to the angle and replace r by $-r$.

$$\left(5, \frac{\pi}{6}\right) = \left(-5, \frac{\pi}{6} + \pi\right) = \left(-5, \frac{7\pi}{6}\right)$$

c. Subtract 2π from the angle and do not change r.

$$\left(5, \frac{\pi}{6}\right) = \left(5, \frac{\pi}{6} - 2\pi\right) = \left(5, -\frac{11\pi}{6}\right)$$

22. Draw $\theta = \dfrac{\pi}{6} = 30°$ counterclockwise, since θ is positive, from the polar axis. Go out 8 units on the terminal side of θ, since $r > 0$.



a. Add 2π to the angle and do not change r.

$$\left(8, \frac{\pi}{6}\right) = \left(8, \frac{\pi}{6} + 2\pi\right) = \left(8, \frac{13\pi}{6}\right)$$

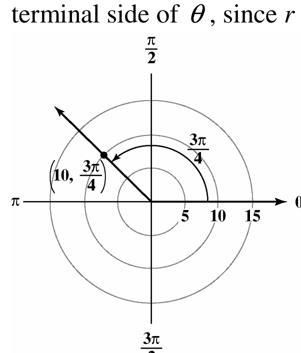
b. Add π to the angle and replace r by $-r$.

$$\left(8, \frac{\pi}{6}\right) = \left(-8, \frac{\pi}{6} + \pi\right) = \left(-8, \frac{7\pi}{6}\right)$$

c. Subtract 2π from the angle and do not change r.

$$\left(8, \frac{\pi}{6}\right) = \left(8, \frac{\pi}{6} - 2\pi\right) = \left(8, -\frac{11\pi}{6}\right)$$

23. Draw $\theta = \dfrac{3\pi}{4} = 135°$ counterclockwise, since θ is positive, from the polar axis. Go out 10 units on the terminal side of θ, since $r > 0$.



a. Add 2π to the angle and do not change r.

$$\left(10, \frac{3\pi}{4}\right) = \left(10, \frac{3\pi}{4} + 2\pi\right) = \left(10, \frac{11\pi}{4}\right)$$

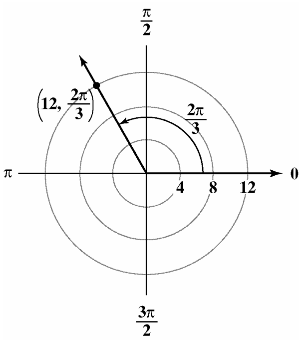
b. Add π to the angle and replace r by $-r$.

$$\left(10, \frac{3\pi}{4}\right) = \left(-10, \frac{3\pi}{4} + \pi\right) = \left(-10, \frac{7\pi}{4}\right)$$

c. Subtract 2π from the angle and do not change r.

$$\left(10, \frac{3\pi}{4}\right) = \left(10, \frac{3\pi}{4} - 2\pi\right) = \left(10, \frac{-5\pi}{4}\right)$$

24. Draw $\theta = \frac{2\pi}{3} = 120°$ counterclockwise, since θ is positive, from the polar axis. Go out 12 units on the terminal side of θ, since $r > 0$.



a. Add 2π to the angle and do not change r.

$$\left(12, \frac{2\pi}{3}\right) = \left(12, \frac{2\pi}{3} + 2\pi\right) = \left(12, \frac{8\pi}{3}\right)$$

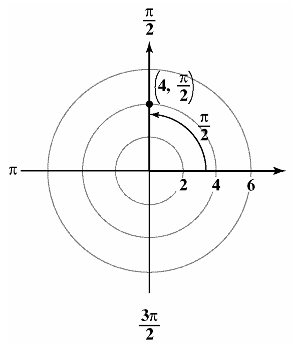
b. Add π to the angle and replace r by $-r$.

$$\left(12, \frac{2\pi}{3}\right) = \left(-12, \frac{2\pi}{3} + \pi\right) = \left(-12, \frac{5\pi}{3}\right)$$

c. Subtract 2π from the angle and do not change r.

$$\left(12, \frac{2\pi}{3}\right) = \left(12, \frac{2\pi}{3} - 2\pi\right) = \left(12, -\frac{4\pi}{3}\right)$$

25. Draw $\theta = \frac{\pi}{2} = 90°$ counterclockwise, since θ is positive, from the polar axis. Go 4 units out on the terminal side of θ, since $r > 0$.



a. Add 2π to the angle and do not change r.

$$\left(4, \frac{\pi}{2}\right) = \left(4, \frac{\pi}{2} + 2\pi\right) = \left(4, \frac{5\pi}{2}\right)$$

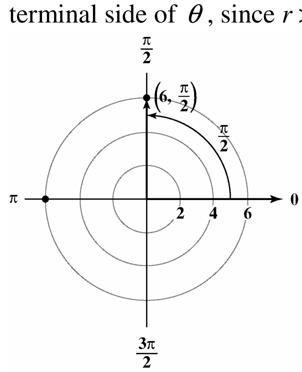
b. Add π to the angle and replace r by $-r$.

$$\left(4, \frac{\pi}{2}\right) = \left(-4, \frac{\pi}{2} + \pi\right) = \left(-4, \frac{3\pi}{2}\right)$$

c. Subtract 2π from the angle and do not change r.

$$\left(4, \frac{\pi}{2}\right) = \left(4, \frac{\pi}{2} - 2\pi\right) = \left(4, -\frac{3\pi}{2}\right)$$

26. Draw $\theta = \pi = 90°$ counterclockwise, since θ is positive, from the polar axis. Go 6 units out on the terminal side of θ, since $r > 0$.



a. Add 2π to the angle and do not change r.

$$\left(6, \frac{\pi}{2}\right) = \left(6, \frac{\pi}{2} + 2\pi\right) = \left(6, \frac{5\pi}{2}\right)$$

b. Add π to the angle and replace r by $-r$.

$$\left(6, \frac{\pi}{2}\right) = \left(-6, \frac{\pi}{2} + \pi\right) = \left(-6, \frac{3\pi}{2}\right)$$

c. Subtract 2π from the angle and do not change r.
$$\left(6, \frac{\pi}{2}\right) = \left(6, \frac{\pi}{2} - 2\pi\right) = \left(6, -\frac{3\pi}{2}\right)$$

27. a, b, d

28. a, c, d

29. b, d

30. a, d

31. a, b

32. a, c

33. The rectangular coordinates of (4, 90°) are (0, 4).

34. $x = r\cos\theta = 6\cos 180° = 6(-1) = -6$
$y = r\sin\theta = 6\sin 180° = 6\cdot 0 = 0$
The rectangular coordinates of (6, 180°) are (–6, 0)

35. $x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\left(\frac{1}{2}\right) = 1$

$y = r\sin\theta = 2\sin\frac{\pi}{3} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

The rectangular coordinates of $\left(2, \frac{\pi}{3}\right)$ are $\left(1, \sqrt{3}\right)$.

36. $x = r\cos\theta = 2\cos\frac{\pi}{6} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

$y = r\sin\theta = 2\sin\frac{\pi}{6} = 2\left(\frac{1}{2}\right) = 1$

The rectangular coordinates of $\left(2, \frac{\pi}{6}\right)$ are $\left(\sqrt{3}, 1\right)$.

37. $x = r\cos\theta = -4\cos\frac{\pi}{2} = -4\cdot 0 = 0$

$y = r\sin\theta = -4\sin\frac{\pi}{2} = -4(1) = -4$

The rectangular coordinates of $\left(-4, \frac{\pi}{2}\right)$ are (0, –4).

38. $x = r\cos\theta = -6\cos\frac{3\pi}{2} = -6\cdot 0 = 0$

$y = r\sin\theta = -6\sin\frac{3\pi}{2} = -6(-1) = 6$

The rectangular coordinates of $\left(-6, \frac{3\pi}{2}\right)$ are (0, 6).

39. $x = r\cos\theta = 7.4\cos 2.5 \approx 7.4(-0.80) \approx -5.9$
$y = r\sin\theta = 7.4\sin 2.5 \approx 7.4(0.60) \approx 4.4$
The rectangular coordinates of (7.4, 2.5) are approximately (–5.9, 4.4).

40. $x = r\cos\theta = 8.3\cos 4.6 \approx 8.3(-0.11) \approx -0.9$
$y = r\sin\theta = 8.3\sin 4.6 \approx 8.3(-0.99) \approx -8.2$
The rectangular coordinates of (8.3, 4.6) are approximately (–0.9, –8.2).

41. $r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + 2^2}$
$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \frac{y}{x} = \frac{2}{-2} = -1$

Because $\tan\theta = -1$ and θ lies in quadrant II, $\theta = \frac{3\pi}{4}$.

The polar coordinates of $(-2, 2)$ are $(r, \theta) = \left(\sqrt{8}, \frac{3\pi}{4}\right)$.

42. $r = \sqrt{x^2 + y^2} = \sqrt{(2)^2 + (-2)^2}$
$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \frac{y}{x} = \frac{-2}{2} = -1$

Because $\tan\frac{\pi}{4} = 1$ and θ lies in quadrant IV, $\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$.

The polar coordinates of (2, –2) are
$(r, \theta) = \left(\sqrt{8}, \frac{7\pi}{4}\right)$ or $\left(2\sqrt{2}, \frac{7\pi}{4}\right)$.

43. $r = \sqrt{x^2 + y^2} = \sqrt{(2)^2 + \left(-2\sqrt{3}\right)^2}$

$= \sqrt{4+12} = \sqrt{16} = 4$

$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{3}}{2} = -\sqrt{3}$

Because $\tan\frac{\pi}{3} = \sqrt{3}$ and θ lies in quadrant IV,

$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$.

The polar coordinates of $\left(2, -2\sqrt{3}\right)$ are

$(r, \theta) = \left(4, \frac{5\pi}{3}\right)$.

44. $r = \sqrt{x^2 + y^2} = \sqrt{\left(-2\sqrt{3}\right)^2 + (2)^2}$

$= \sqrt{12+4} = \sqrt{16} = 4$

$\tan\theta = \frac{y}{x} = \frac{2}{-2\sqrt{3}} = -\frac{1}{\sqrt{3}}$

Because $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ and θ lies in quadrant II,

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

The polar coordinates of $\left(-2\sqrt{3}, 2\right)$ are

$(r, \theta) = \left(4, \frac{5\pi}{6}\right)$.

45. $r = \sqrt{x^2 + y^2} = \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2}$

$= \sqrt{3+1} = \sqrt{4} = 2$

$\tan\theta = \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$

Because $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ and θ lies in quadrant III,

$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$.

The polar coordinates of $\left(-\sqrt{3}, -1\right)$ are

$(r, \theta) = \left(2, \frac{7\pi}{6}\right)$.

46. $r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + \left(-\sqrt{3}\right)^2}$

$= \sqrt{1+3} = \sqrt{4} = 2$

$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

Because $\tan\frac{\pi}{3} = \sqrt{3}$ and θ lies in quadrant III,

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$.

The polar coordinates of $\left(-1, -\sqrt{3}\right)$ are

$(r, \theta) = \left(2, \frac{4\pi}{3}\right)$.

47. $r = \sqrt{x^2 + y^2} = \sqrt{(5)^2 + (0)^2} = \sqrt{25} = 5$

$\tan\theta = \frac{y}{x} = \frac{0}{5} = 0$

Because $\tan 0 = 0$ and θ lies on the polar axis,
$\theta = 0$.
The polar coordinates of (5, 0) are
$(r, \theta) = (5, 0)$.

48. $r = \sqrt{x^2 + y^2} = \sqrt{(0)^2 + (-6)^2} = \sqrt{36} = 6$

$\tan\theta = \frac{y}{x} = \frac{-6}{0} = \text{undefined}$

Because $\tan\frac{\pi}{2}$ is undefined and θ lies on the

negative *y* axis, $\theta = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$.

The polar coordinates of (0, –6) are $(r, \theta) = \left(6, \frac{3\pi}{2}\right)$.

49.

$$3x + y = 7$$
$$3r\cos\theta + r\sin\theta = 7$$
$$r(3\cos\theta + \sin\theta) = 7$$
$$r = \frac{7}{3\cos\theta + \sin\theta}$$

50.

$$x + 5y = 8$$
$$r\cos\theta + 5r\sin\theta = 8$$
$$r(\cos\theta + 5\sin\theta) = 8$$
$$r = \frac{8}{\cos\theta + 5\sin\theta}$$

51.
$$x = 7$$
$$r\cos\theta = 7$$
$$r = \frac{7}{\cos\theta}$$

52.
$$y = 3$$
$$r\sin\theta = 3$$
$$r = \frac{3}{\sin\theta}$$

53.
$$x^2 + y^2 = 9$$
$$r^2 = 9$$
$$r = 3$$

54.
$$x^2 + y^2 = 16$$
$$r^2 = 16$$
$$r = 4$$

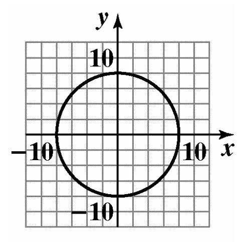
55.
$$(x-2)^2 + y^2 = 4$$
$$(r\cos\theta - 2)^2 + (r\sin\theta)^2 = 4$$
$$r^2\cos^2\theta - 4r\cos\theta + 4 + r^2\sin^2\theta^2 = 4$$
$$r^2 - 4r\cos\theta = 0$$
$$r^2 = 4r\cos\theta$$
$$r = 4\cos\theta$$

56.
$$x^2 + (y+3)^2 = 9$$
$$(r\cos\theta)^2 + (r\sin\theta + 3)^2 = 9$$
$$r^2\cos^2\theta + r^2\sin^2\theta + 6r\sin\theta + 9 = 9$$
$$r^2 + 6r\sin\theta = 0$$
$$r^2 = -6r\sin\theta$$
$$r = -6\sin\theta$$

57.
$$y^2 = 6x$$
$$(r\sin\theta)^2 = 6r\cos\theta$$
$$r^2\sin^2\theta = 6r\cos\theta$$
$$r\sin^2\theta = 6\cos\theta$$
$$r = \frac{6\cos\theta}{\sin^2\theta}$$

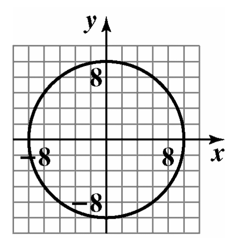
58.
$$x^2 = 6y$$
$$(r\cos\theta)^2 = 6r\sin\theta$$
$$r^2\cos^2\theta = 6r\sin\theta$$
$$r\cos^2\theta = 6\sin\theta$$
$$r = \frac{6\sin\theta}{\cos^2\theta}$$

59.
$$r = 8$$
$$r^2 = 64$$
$$x^2 + y^2 = 64$$



$x^2 + y^2 = 64$

60.
$$r = 10$$
$$r^2 = 100$$
$$x^2 + y^2 = 100$$

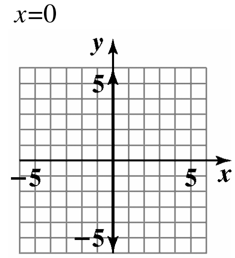


$x^2 + y^2 = 100$

61.
$$\theta = \frac{\pi}{2}$$
$$\tan\theta = \tan\frac{\pi}{2}$$
$\tan\theta$ is undefined

$\frac{y}{x}$ is undefined

$x=0$



$x = 0$

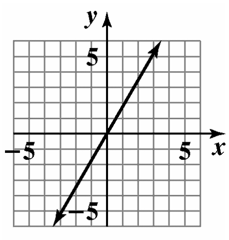
62. $\theta = \frac{\pi}{3}$

$\tan\theta = \tan\frac{\pi}{3}$

$\tan\theta = \sqrt{3}$

$\frac{y}{x} = \sqrt{3}$

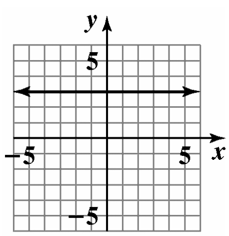
$y = \sqrt{3}x$



$y = \sqrt{3}\,x$

63. $r\sin\theta = 3$

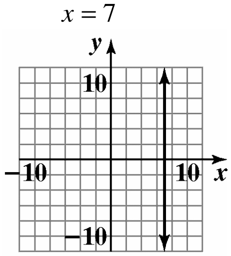
$y = 3$



$y = 3$

64. $r\cos\theta = 7$

$x = 7$



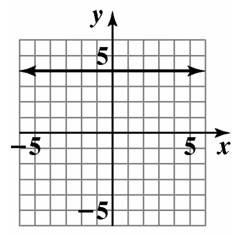
$x = 7$

65. $r = 4\csc\theta$

$r = \frac{4}{\sin\theta}$

$r\sin\theta = 4$

$y = 4$



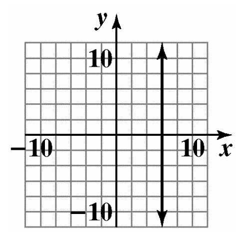
$y = 4$

66. $r = 6\sec\theta$

$r = \frac{6}{\cos\theta}$

$r\cos\theta = 6$

$x = 6$



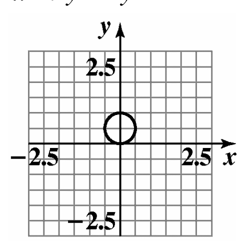
$x = 6$

67. $r = \sin\theta$

$r \cdot r = r \cdot \sin\theta$

$r^2 = r\sin\theta$

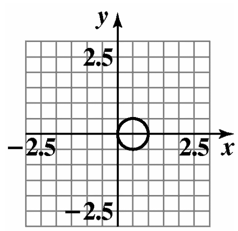
$x^2 + y^2 = y$



$x^2 + y^2 = y$

68.

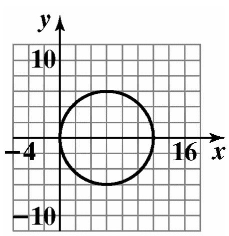
$$\begin{aligned} r &= \cos\theta \\ r \cdot r &= r \cdot \cos\theta \\ r^2 &= r\cos\theta \\ x^2 + y^2 &= x \end{aligned}$$



$x^2 + y^2 = x$

69.

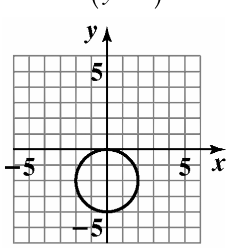
$$\begin{aligned} r &= 12\cos\theta \\ r^2 &= 12r\cos\theta \\ x^2 + y^2 &= 12x \\ x^2 - 12x + y^2 &= 0 \\ x^2 - 12x + 36 + y^2 &= 36 \\ (x-6)^2 + y^2 &= 36 \end{aligned}$$



$(x - 6)^2 + y^2 = 36$

70.

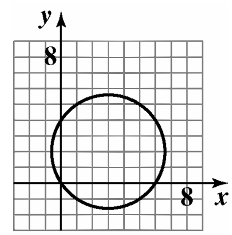
$$\begin{aligned} r &= -4\sin\theta \\ r^2 &= -4r\sin\theta \\ x^2 + y^2 &= -4y \\ x^2 + y^2 + 4y &= 0 \\ x^2 + y^2 + 4y + 4 &= 4 \\ x^2 + (y+2)^2 &= 4 \end{aligned}$$



$x^2 + (y + 2)^2 = 4$

71.

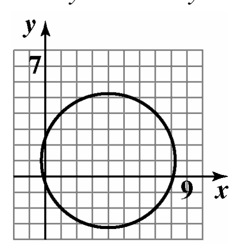
$$\begin{aligned} r &= 6\cos\theta + 4\sin\theta \\ r \cdot r &= r(6\cos\theta + 4\sin\theta) \\ r^2 &= 6r\cos\theta + 4r\sin\theta \\ x^2 + y^2 &= 6x + 4y \end{aligned}$$



$x^2 + y^2 = 6x + 4y$

72.

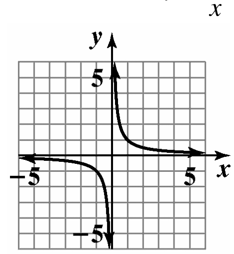
$$\begin{aligned} r &= 8\cos\theta + 2\sin\theta \\ r \cdot r &= r(8\cos\theta + 2\sin\theta) \\ r^2 &= 8r\cos\theta + 2r\sin\theta \\ x^2 + y^2 &= 8x + 2y \end{aligned}$$



$x^2 + y^2 = 8x + 2y$

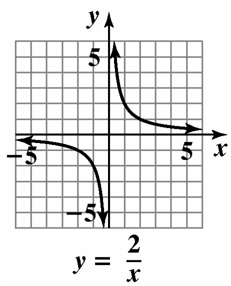
73.

$$\begin{aligned} r^2 \sin 2\theta &= 2 \\ r^2(2\sin\theta\cos\theta) &= 2 \\ 2r\sin\theta\, r\cos\theta &= 2 \\ 2yx &= 2 \\ xy &= 1 \\ y &= \frac{1}{x} \end{aligned}$$



$y = \dfrac{1}{x}$

74.
$$r^2\cos 2\theta = 2$$
$$r^2(\cos^2\theta - \sin^2\theta) = 2$$
$$r^2\cos^2\theta - r^2\sin^2\theta = 2$$
$$(r\cos\theta)^2 - (r\sin\theta)^2 = 2$$
$$x^2 - y^2 = 2$$



75.
$$r = a\sec\theta$$
$$r = \frac{a}{\cos\theta}$$
$$r\cos\theta = a$$
$$x = a$$
This is the equation of a vertical line.

76.
$$r = a\csc\theta$$
$$r = \frac{a}{\sin\theta}$$
$$r\sin\theta = a$$
$$y = a$$
This is the equation of a horizontal line.

77.
$$r = a\sin\theta$$
$$r^2 = ar\sin\theta$$
$$x^2 + y^2 = ay$$
$$x^2 + y^2 - ay = 0$$
$$x^2 + y^2 - ay + \frac{a^2}{4} = \frac{a^2}{4}$$
$$x^2 + \left(y - \frac{a}{2}\right)^2 = \left(\frac{a}{2}\right)^2$$

This is the equation of a circle of radius $\frac{a}{2}$ centered at $\left(0, \frac{a}{2}\right)$.

78.
$$r = a\cos\theta$$
$$r^2 = ar\cos\theta$$
$$x^2 + y^2 = ax$$
$$x^2 - ax + y^2 = 0$$
$$x^2 - ax + \frac{a^2}{4} + y^2 = \frac{a^2}{4}$$
$$\left(x - \frac{a}{2}\right)^2 + y^2 = \left(\frac{a}{2}\right)^2$$

This is the equation of a circle of radius $\frac{a}{2}$ centered at $\left(\frac{a}{2}, 0\right)$.

79.
$$r\sin\left(\theta - \frac{\pi}{4}\right) = 2$$
$$r\left(\sin\theta\cos\frac{\pi}{4} - \cos\theta\sin\frac{\pi}{4}\right) = 2$$
$$r\sin\theta\cdot\frac{\sqrt{2}}{2} - r\cos\theta\cdot\frac{\sqrt{2}}{2} = 2$$
$$y\cdot\frac{\sqrt{2}}{2} - x\cdot\frac{\sqrt{2}}{2} = 2$$
$$y = x + 2\sqrt{2}$$
$y = x + 2\sqrt{2}$ has slope of 1 and y-intercept of $2\sqrt{2}$.

80.
$$r\cos\left(\theta + \frac{\pi}{6}\right) = 8$$
$$r\left(\cos\theta\cos\frac{\pi}{6} - \sin\theta\sin\frac{\pi}{6}\right) = 8$$
$$r\cos\theta\cos\frac{\pi}{6} - r\sin\theta\sin\frac{\pi}{6} = 8$$
$$x\cdot\frac{\sqrt{3}}{2} - y\cdot\frac{1}{2} = 8$$
$$x\sqrt{3} - y = 16$$
$$-y = -x\sqrt{3} + 16$$
$$y = x\sqrt{3} - 16$$
$y = x\sqrt{3} - 16$ has slope of $\sqrt{3}$ and y-intercept of –16.

81. $x_1 = r\cos\theta = 2\cos\frac{2\pi}{3} = -1$

$y_1 = r\sin\theta = 2\sin\frac{2\pi}{3} = \sqrt{3}$

$\left(-1, \sqrt{3}\right)$

$x_2 = r\cos\theta = 4\cos\frac{\pi}{6} = 2\sqrt{3}$

$y_2 = r\sin\theta = 4\sin\frac{\pi}{6} = 2$

$\left(2\sqrt{3}, 2\right)$

$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

$d = \sqrt{\left(2\sqrt{3} + 1\right)^2 + \left(2 - \sqrt{3}\right)^2}$

$d = 2\sqrt{5}$

82. $x_1 = r\cos\theta = 6\cos\pi = -6$

$y_1 = r\sin\theta = 6\sin\pi = 0$

$\left(-6, 0\right)$

$x_2 = r\cos\theta = 5\cos\frac{7\pi}{4} = \frac{5\sqrt{2}}{2}$

$y_2 = r\sin\theta = 5\sin\frac{7\pi}{4} = -\frac{5\sqrt{2}}{2}$

$\left(\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)$

$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

$d = \sqrt{\left(\frac{5\sqrt{2}}{2} + 6\right)^2 + \left(-\frac{5\sqrt{2}}{2} - 0\right)^2}$

$d = \sqrt{61 + 30\sqrt{2}}$

83. The angle is measured counterclockwise from the polar axis.

$\theta = \frac{2}{3}(360°) = 240°$ or $\frac{4\pi}{3}$.

The distance from the inner circle's center to the outer circle is

$r = 6 + 3(3) = 6 + 9 = 15$

The polar coordinates are $(r, \theta) = \left(15, \frac{4\pi}{3}\right)$.

84. The angle is measured counterclockwise from the polar axis. $\theta = \frac{5}{6}(360°) = 300°$ or $\frac{5\pi}{3}$.

On the inner circle, r = 6.

The polar coordinates are $(r, \theta) = \left(6, \frac{5\pi}{3}\right)$.

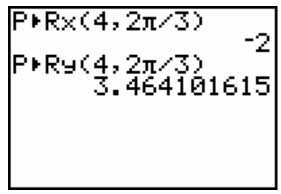
85. (6.3, 50°) represents a sailing speed of 6.3 knots at an angle of 50° to the wind.

86. (7.4, 85°) represents a sailing speed of 7.4 knots at an angle of 85° to the wind.

87. Out of the four points in this 10-knot-wind situation, you would recommend a sailing angle of 105°. A sailing speed of 7.5 knots is achieved at this angle.

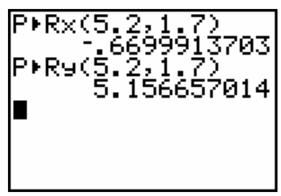
88. – 96. Answers may vary.

97.



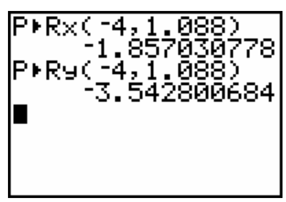
To three decimal places, the rectangular coordinates are (–2, 3.464).

98.



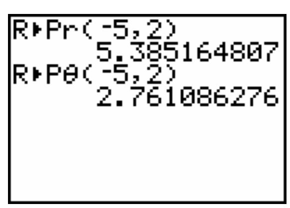
To three decimal places, the rectangular coordinates are (–0.670, 5.157).

99.

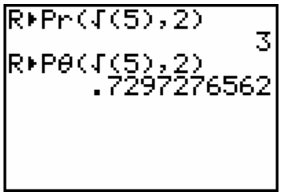


To three decimal places, the rectangular coordinates are (–1.857, –3.543).

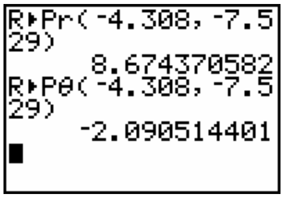
100.



To three decimal places, the polar coordinates are (r, θ) = (5.385, 2.761).

101. 

To three decimal places, the polar coordinates are (r, θ) = (3, 0.730).

102. 

To three decimal places, the polar coordinates are (r, θ) = (8.674, –2.091).

103. does not make sense; Explanations will vary. Sample explanation: There are multiple polar representations for a given point.

104. does not make sense; Explanations will vary. Sample explanation: There is only one rectangular representation for a given point.

105. makes sense

106. makes sense

107. Use the distance formula for rectangular coordinates, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Let $x_1 = r_1 \cos\theta_1, y_1 = r_1 \sin\theta_1,$

$x_2 = r_2 \cos\theta_2, y_2 = r_2 \sin\theta_2$

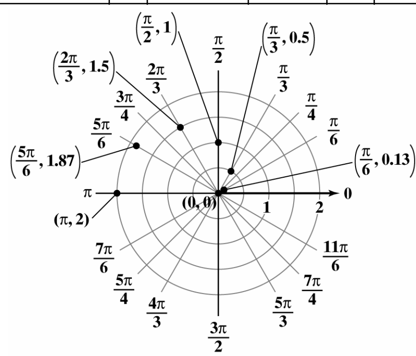
$$d = \sqrt{\left(r_2 \cos\theta_2 - r_1 \cos\theta_1\right)^2 + \left(r_2 \sin\theta_2 - r_1 \sin\theta_1\right)^2}$$
$$= \sqrt{r_2^2 \cos^2\theta_2 - 2r_1r_2 \cos\theta_1 \cos\theta_2 + r_1^2 \cos^2\theta_1 + r_2^2 \sin^2\theta_2 - 2r_1r_2 \sin\theta_1 \sin\theta_2 + r_1^2 \sin^2\theta_1}$$
$$= \sqrt{r_2^2\left(\cos^2\theta_2 + \sin^2\theta_2\right) + r_1^2\left(\cos^2\theta_1 + \sin^2\theta_1\right) - 2r_1r_2\left(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\right)}$$
$$= \sqrt{r_2^2(1) + r_1^2(1) - 2r_1r_2\left(\cos\left(\theta_2 - \theta_1\right)\right)}$$
$$= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\left(\theta_2 - \theta_1\right)}$$

108. Let $(r_1, \theta_1) = \left(2, \frac{5\pi}{6}\right), (r_2, \theta_2) = \left(4, \frac{\pi}{6}\right)$

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$$
$$= \sqrt{2^2 + 4^2 - 2(2)(4)\cos\left(\frac{\pi}{6} - \frac{5\pi}{6}\right)}$$
$$= \sqrt{20 - 16\cos\left(\frac{-2\pi}{3}\right)}$$
$$= \sqrt{20 - 16\left(-\frac{1}{2}\right)} = \sqrt{20 + 8} = \sqrt{28} \text{ or } 2\sqrt{7}$$

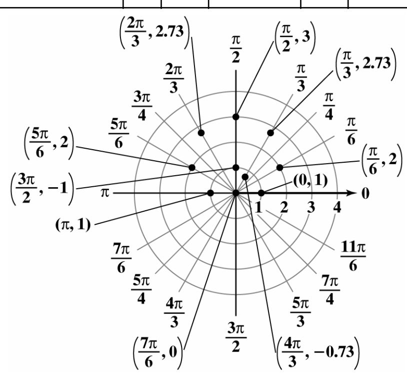
109.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$r = 1 - \cos\theta$	0	$\frac{2-\sqrt{3}}{2}$ ≈ 0.13	$\frac{1}{2}$ $= 0.5$	1	$\frac{3}{2}$ $= 1.5$	$\frac{2+\sqrt{3}}{2}$ ≈ 1.87	0



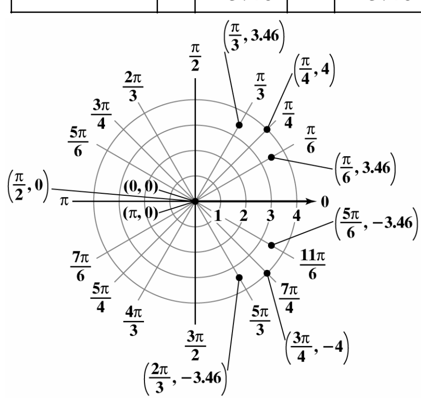
110.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$
$r = 1 + 2\sin\theta$	1	2	$1+\sqrt{3}$ ≈ 2.73	3	$1+\sqrt{3}$ ≈ 2.73	2	1	0	$1-\sqrt{3}$ ≈ -0.73	-1



111.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$r = 4\sin 2\theta$	0	$2\sqrt{3}$ ≈ 3.46	4	$2\sqrt{3}$ ≈ 3.46	0	$-2\sqrt{3}$ ≈ -3.46	-4	$-2\sqrt{3}$ ≈ -3.46	0



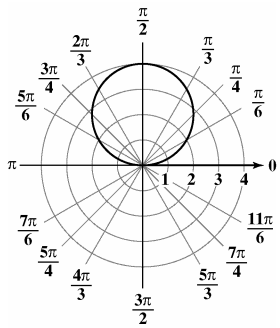
Section 7.4

Check Point Exercises

1. Construct a partial table of coordinates using multiples of $\frac{\pi}{6}$. Then plot the points and join them with a smooth curve.

$r = 4\sin\theta$

θ	r	(r, θ)
0	$4\sin 0 = 4\cdot 0 = 0$	$(0, 0)$
$\frac{\pi}{6}$	$4\sin\frac{\pi}{6} = 4\cdot\frac{1}{2} = 2$	$\left(2, \frac{\pi}{6}\right)$
$\frac{\pi}{3}$	$4\sin\frac{\pi}{3} = 4\cdot\frac{\sqrt{3}}{2} = 2\sqrt{3}$	$\left(2\sqrt{3}, \frac{\pi}{3}\right)$
$\frac{\pi}{2}$	$4\sin\frac{\pi}{2} = 4\cdot 1 = 4$	$\left(4, \frac{\pi}{2}\right)$
$\frac{2\pi}{3}$	$4\sin\frac{2\pi}{3} = 4\cdot\frac{\sqrt{3}}{2} = 2\sqrt{3}$	$\left(2\sqrt{3}, \frac{2\pi}{3}\right)$
$\frac{5\pi}{6}$	$4\sin\frac{5\pi}{6} = 4\cdot\frac{1}{2} = 2$	$\left(2, \frac{5\pi}{6}\right)$
π	$4\sin\pi = 4\cdot 0 = 0$	$(0, \pi)$



$r = 4\sin\theta$

2. **Polar Axis:** Replace θ by $-\theta$ in $r = 1 + \cos\theta$.

$$r = 1 + \cos(-\theta)$$
$$r = 1 + \cos\theta$$

Because the polar equation does not change when θ is replaced by $-\theta$, the graph is symmetric with respect to the polar axis.

The Line $\boldsymbol{\theta = \frac{\pi}{2}}$**:** Replace (r, θ) by $(-r, -\theta)$ in $r = 1 + \cos\theta$.

$$-r = 1 + \cos(-\theta)$$
$$-r = 1 + \cos\theta$$
$$r = -1 - \cos\theta$$

Because the polar equation changes when (r, θ) is replaced by $(-r, -\theta)$, the equation fails the symmetry test. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$.

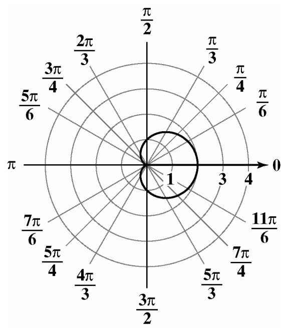
The Pole: Replace r by $-r$ in $r = 1 + \cos\theta$.

$$-r = 1 + \cos\theta$$
$$r = -1 - \cos\theta$$

Because the polar equation changes when r is replaced by $-r$, the equation fails the symmetry test. The graph may or may not be symmetric with respect to the pole.

Because the period of the cosine function is 2π, and the graph is symmetric with respect to the polar axis, begin by finding vales of r for values of θ from 0 to π. then graph $r = 1 + \cos\theta$ for these values and reflect the graph about the polar axis.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	2	1.87	1.5	1	0.5	0.13	0



$r = 1 + \cos\theta$

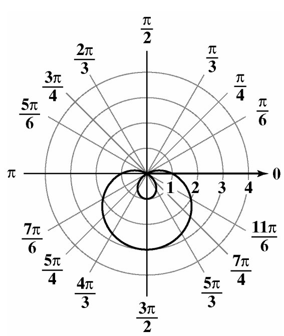
3. $r = 1 - 2\sin\theta$

Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 1 - 2\sin(-\theta)$	$-r = 1 - 2\sin(-\theta)$	$-r = 1 - 2\sin\theta$
$r = 1 - 2(-\sin\theta)$	$-r = 1 + 2\sin\theta$	
$r = 1 + 2\sin\theta$	$r = -1 - 2\sin\theta$	$r = -1 + 2\sin\theta$

There may be no symmetry, since each equation is not equivalent to $r = 1 - 2\sin\theta$. Because the period of the sine function is 2π, we need not consider values of θ beyond 2π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{6}$	$\frac{11\pi}{6}$	2π
r	1	0	-0.73	-1	-0.73	0	1	2	2.73	3	2.73	2	1



$r = 1 - 2\sin\theta$

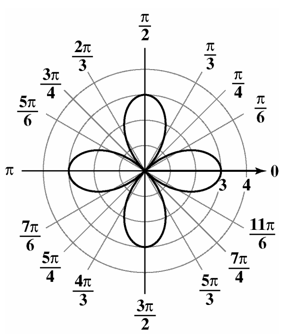
4. $r = 3\cos 2\theta$
Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 3\cos 2(-\theta)$	$-r = 3\cos 2(-\theta)$	$-r = 3\cos 2\theta$
	$-r = 3\cos 2\theta$	
$r = 3\cos 2\theta$	$r = -3\cos 2\theta$	$r = -3\cos 2\theta$

The graph has symmetry with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole.

Since the graph is symmetric with respect to the polar axis, calculate values of *r* for θ from 0 to π. Then, graph $r = 3\cos 2\theta$ for these values and reflect the graph about the polar axis.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	3	1.5	-1.5	-3	-1.5	1.5	3



5. $r^2 = 4\cos 2\theta$
Check for symmetry:

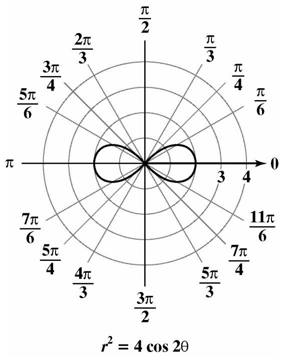
Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r^2 = 4\cos 2(-\theta)$	$(-r)^2 = 4\cos 2(-\theta)$	$(-r)^2 = 4\cos 2\theta$
$r^2 = 4\cos(-2\theta)$	$r^2 = 4\cos(-2\theta)$	
$r^2 = 4\cos 2\theta$	$r^2 = 4\cos 2\theta$	$r^2 = 4\cos 2\theta$

The graph has symmetry with respect to the polar axis, the line $\theta = \frac{\pi}{2}$, and the pole.

Calculate values of *r* for θ from 0 to $\frac{\pi}{2}$, and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
r	± 2	± 1.41	0	undef.	undef.

Use symmetry to obtain the graph.



Exercise Set 7.4

1. heart-shaped limaçon or cardioid

$\frac{a}{b} = 1$

$r = 0$ when $\theta = \frac{\pi}{2}$

The polar equation is $r = 1 - \sin\theta$.

2. rose curve

4 petals $\Rightarrow n = 2$

The polar equation is $r = 3\sin 2\theta$.

3. circle

$r = 2$ when $\theta = 0$

The polar equation is $r = 2\cos\theta$.

4. heart-shaped limaçon or cardioid

$\frac{a}{b} = 1$

$r = 2$ when $\theta = \frac{\pi}{2}$

The polar equation is $r = 1 + \sin\theta$.

5. rose curve

3 petals $\Rightarrow n = 3$

The polar equation is $r = 3\sin 3\theta$.

6. circle

$r = 2$ when $\theta = \frac{\pi}{2}$

The polar equation is $r = 2\sin\theta$.

7. **a.** $r = \sin\theta$

Replace θ with $-\theta$.

$r = \sin(-\theta)$

$r = -\sin\theta$

The graph may or may not have symmetry with respect to the polar axis.

b. $r = \sin\theta$

Replace (r, θ) with $(-r, -\theta)$.

$-r = \sin(-\theta)$

$-r = -\sin\theta$

$r = \sin\theta$

The graph has symmetry with respect to the line $\theta = \frac{\pi}{2}$.

c. $r = \sin\theta$

Replace r with $-r$.

$-r = \sin\theta$

$r = -\sin\theta$

The graph may or may not have symmetry about the pole.

8. **a.** $r = \cos\theta$

Replace θ with $-\theta$.

$r = \cos(-\theta)$

$r = \cos(\theta)$

The graph has symmetry with respect to the polar axis.

b. $r = \cos\theta$

Replace (r, θ) with $(-r, -\theta)$.

$-r = \cos(-\theta)$

$-r = \cos\theta$

$r = -\cos\theta$

The graph may or may not have symmetry with respect to the line $\theta = \frac{\pi}{2}$.

c. $r = \cos\theta$

Replace r with $-r$.

$-r = \cos\theta$

$r = -\cos\theta$

The graph may or may not have symmetry about the pole.

9. **a.** $r = 4 + 3\cos\theta$
Replace θ with $-\theta$.
$r = 4 + 3\cos(-\theta)$
$r = 4 + 3\cos\theta$
The graph has symmetry with respect to the polar axis.

b. $r = 4 + 3\cos\theta$
Replace (r, θ) with $(-r, -\theta)$.
$-r = 4 + 3\cos(-\theta)$
$-r = 4 + 3\cos\theta$
$r = -4 - 3\cos\theta$
The graph may or may not have symmetry with respect to the line $\theta = \frac{\pi}{2}$.

c. $r = 4 + 3\cos\theta$
Replace r with $-r$.
$-r = 4 + 3\cos\theta$
$r = -4 - 3\cos\theta$
The graph may or may not have symmetry about the pole.

10. **a.** $r = 2\cos 2\theta$
Replace θ with $-\theta$.
$r = 2\cos 2(-\theta)$
$r = 2\cos(-2\theta)$
$r = 2\cos 2\theta$
The graph has symmetry with respect to the polar axis.

b. $r = 2\cos 2\theta$
Replace (r, θ) with $(-r, -\theta)$.
$-r = 2\cos 2(-\theta)$
$-r = 2\cos 2\theta$
$r = -2\cos 2\theta$
The graph may or may not have symmetry with respect to the line $\theta = \frac{\pi}{2}$.

c. $r = 2\cos 2\theta$
Replace r with $-r$.
$-r = 2\cos 2\theta$
$r = -2\cos 2\theta$
The graph may or may not have symmetry about the pole.

11. **a.** $r^2 = 16\cos 2\theta$
Replace θ with $-\theta$.
$r^2 = 16\cos 2(-\theta)$
$r^2 = 16\cos(-2\theta)$
$r^2 = 16\cos 2\theta$
The graph has symmetry with respect to the polar axis.

b. $r^2 = 16\cos 2\theta$
Replace (r, θ) with $(-r, -\theta)$.
$(-r)^2 = 16\cos 2(-\theta)$
$r^2 = 16\cos 2\theta$
The graph has symmetry with respect to the line $\theta = \frac{\pi}{2}$.

c. $r^2 = 16\cos 2\theta$
Replace r with $-r$.
$(-r)^2 = 16\cos 2\theta$
$r^2 = 16\cos 2\theta$
The graph has symmetry about the pole.

12. **a.** $r^2 = 16\sin 2\theta$
Replace θ with $-\theta$.
$r^2 = 16\sin 2(-\theta)$
$r^2 = 16\sin(-2\theta)$
$r^2 = -16\sin 2\theta$
The graph may or may not have symmetry with respect to the polar axis.

b. $r^2 = 16\sin 2\theta$
Replace (r, θ) with $(-r, -\theta)$.
$(-r)^2 = 16\sin 2(-\theta)$
$r^2 = -16\sin 2\theta$
The graph may or may not have symmetry with respect to the line $\theta = \frac{\pi}{2}$.

c. $r^2 = 16\sin 2\theta$
Replace r with $-r$.
$(-r)^2 = 16\sin 2\theta$
$r^2 = 16\sin 2\theta$
The graph has symmetry with respect to the pole.

13. $r = 2\cos\theta$

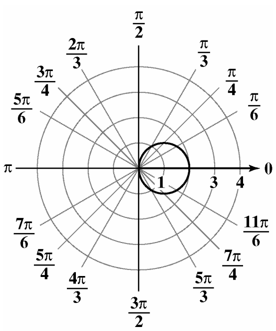
Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 2\cos(-\theta)$	$-r = 2\cos(-\theta)$	$-r = 2\cos\theta$
	$-r = 2\cos\theta$	
$r = 2\cos\theta$	$r = -2\cos\theta$	$r = -2\cos\theta$

The graph is symmetric with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole.

Calculate values of r for θ from 0 to π and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
r	2	1.73	1	0



$r = 2\cos\theta$

Notice that there are no points in quadrants II or III. Because the cosine is negative in quadrants II and III, r is negative here. This places the points in quadrants IV and I respectively.

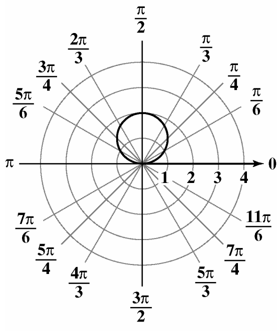
14. $r = 2\sin\theta$

Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 2\sin(-\theta)$	$-r = 2\sin(-\theta)$	$-r = 2\sin\theta$
	$-r = -2\sin\theta$	
$r = -2\sin\theta$	$r = 2\sin\theta$	

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$. The graph may or may not be symmetric with respect to the polar axis or the pole. Calculate values of r for θ from 0 to $\frac{\pi}{2}$ and for θ from π to $\frac{3\pi}{2}$. Then, use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$
r	0	1	1.41	1.73	2	0	–1	–1.41	–1.73	–2



$r = 2\sin\theta$

Notice that there are no points in quadrants III or IV. Because the sine is negative in quadrants III and IV, r is negative here. This places the points in quadrants I and II respectively.

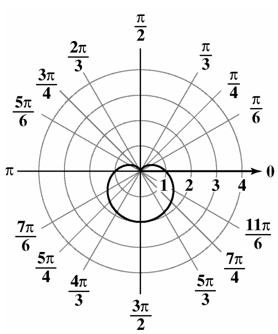
15. $r = 1 - \sin\theta$

Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = 1 - \sin(-\theta)$	$-r = 1 - \sin(-\theta)$	$-r = 1 - \sin\theta$
	$-r = 1 + \sin\theta$	
$r = 1 + \sin\theta$	$r = -1 - \sin\theta$	$r = -1 + \sin\theta$

There may be no symmetry since each equation is not equivalent to $r = 1 - \sin\theta$. Because the period of the sine function is 2π, we need not consider values of θ beyond 2π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{6}$	$\frac{11\pi}{6}$	2π
r	1	0.5	0.13	0	0.13	0.5	1	1.5	1.87	2	1.87	1.5	1



$r = 1 - \sin\theta$

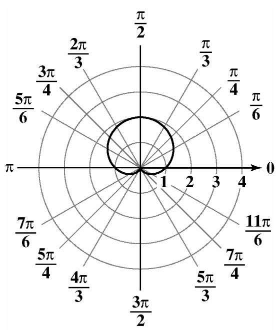
16. $r = 1 + \sin\theta$

Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = 1 + \sin(-\theta)$	$-r = 1 + \sin(-\theta)$	
	$-r = 1 - \sin\theta$	
$r = 1 - \sin\theta$	$r = -1 + \sin\theta$	

There may be no symmetry, since each equation is not equivalent to $r = 1 + \sin\theta$. Because the period of the sine function is 2π, we need not consider values of θ beyond 2π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
r	1	1.5	1.87	2	1.87	1.5	1	0.5	0.13	0	0.13	0.5	1



$r = 1 + \sin\theta$

17. $r = 2 + 2\cos\theta$

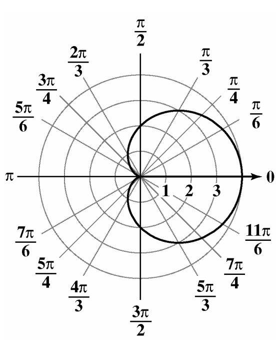
Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = 2 + 2\cos(-\theta)$	$-r = 2 + 2\cos(-\theta)$	$-r = 2 + 2\cos\theta$
	$-r = 2 + 2\cos\theta$	
$r = 2 + 2\cos\theta$	$r = -2 - 2\cos\theta$	$r = -2 - 2\cos\theta$

The graph is symmetric with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole.

Calculate values of r for θ from 0 to π and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	4	3.73	3	2	1	0.27	0



$r = 2 + 2\cos\theta$

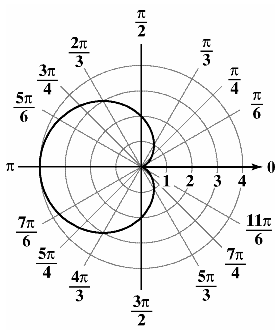
18. $r = 2 - 2\cos\theta$

Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 2 - 2\cos(-\theta)$	$-r = 2 - 2\cos(-\theta)$	
	$-r = 2 - 2\cos\theta$	
$r = 2 - 2\cos\theta$	$r = -2 + 2\cos\theta$	

The graph is symmetric with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole. Calculate values of r for θ from 0 to π and use the symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	0	0.27	1	2	3	3.73	4



$r = 2 - 2\cos\theta$

19. $r = 2 + \cos\theta$

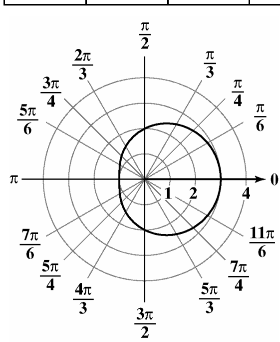
Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 2 + \cos(-\theta)$	$-r = 2 + \cos(-\theta)$	$-r = 2 + \cos\theta$
	$-r = 2 + \cos\theta$	
$r = 2 + \cos\theta$	$r = -2 - \cos\theta$	$r = -2 - \cos\theta$

The graph is symmetric with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole.

Calculate values of r for θ from 0 to π and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	3	2.87	2.5	2	1.5	1.13	1



$r = 2 + \cos\theta$

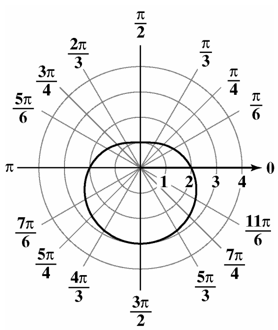
20. $r = 2 - \sin\theta$

Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = 2 - \sin(-\theta)$	$-r = 2 - \sin(-\theta)$	
	$-r = 2 + \sin\theta$	
$r = 2 + \sin\theta$	$r = -2 - \sin\theta$	

There may be no symmetry, since each equation is not equivalent to $r = 2 - \sin\theta$.
Because the period of the sine function is 2π, we need not consider values of θ beyond 2π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
r	2	1.5	1.13	1	1.13	1.5	2	2.5	2.87	3	2.87	2.5	2



$r = 2 - \sin\theta$

21. $r = 1 + 2\cos\theta$

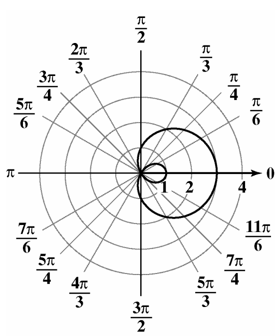
Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = 1 + 2\cos(-\theta)$	$-r = 1 + 2\cos(-\theta)$	$-r = 1 + 2\cos\theta$
	$-r = 1 + 2\cos\theta$	
$r = 1 + 2\cos\theta$	$r = -1 - 2\cos\theta$	$r = -1 - 2\cos\theta$

The graph is symmetric with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole.

Calculate values of *r* for θ from 0 to π and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	3	2.73	2	1	0	– 0.73	–1



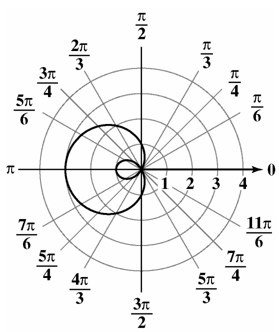
$r = 1 + 2\cos\theta$

22. $r = 1 - 2\cos\theta$
Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 1 - 2\cos(-\theta)$	$-r = 1 - 2\cos(-\theta)$	
	$-r = 1 - 2\cos\theta$	
$r = 1 - 2\cos\theta$	$r = -1 + 2\cos\theta$	

The graph is symmetric with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole. Calculate values of *r* for θ from 0 to π and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	–1	–0.73	0	1	2	2.73	3



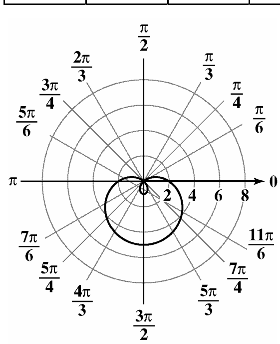
$r = 1 - 2\cos\theta$

23. $r = 2 - 3\sin\theta$
Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 2 - 3\sin(-\theta)$	$-r = 2 - 3\sin(-\theta)$	$-r = 2 - 3\sin\theta$
	$-r = 2 + 3\sin\theta$	
$r = 2 + 3\sin\theta$	$r = -2 - 3\sin\theta$	$r = -2 + 3\sin\theta$

There may be no symmetry since each equation is not equivalent to $r = 2 - 3\sin\theta$. Because the period of the sine function is 2π, we need not consider values of θ beyond 2π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{6}$	$\frac{11\pi}{6}$	2π
r	2	0.5	–0.60	–1	–0.60	0.5	2	3.5	4.6	5	4.6	3.5	2



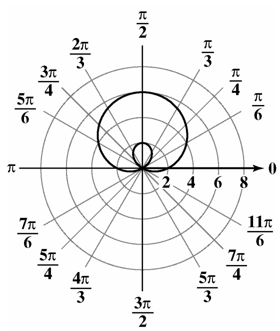
$r = 2 - 3\sin\theta$

24. $r = 2 + 4\sin\theta$
Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = 2 + 4\sin(-\theta)$	$-r = 2 + 4\sin(-\theta)$	
	$-r = 2 - 4\sin\theta$	
$r = 2 - 4\sin\theta$	$r = -2 + 4\sin\theta$	

There may be no symmetry, since each equation is not equivalent to $r = 2 + 4\sin\theta$.
Because the period of the sine function is 2π, we need not consider values of θ beyond 2π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
r	2	4	5.46	6	5.46	4	2	0	–1.46	–2	–1.46	0	2



r = 2 + 4 sin θ

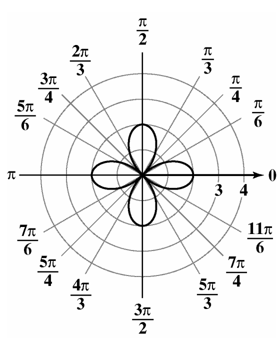
25. $r = 2\cos 2\theta$
Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = 2\cos 2(-\theta)$	$-r = 2\cos 2(-\theta)$	$-r = 2\cos 2\theta$
$r = 2\cos(-2\theta)$	$-r = 2\cos(-2\theta)$	
	$-r = 2\cos 2\theta$	
$r = 2\cos 2\theta$	$r = -2\cos 2\theta$	$r = -2\cos 2\theta$

The graph is symmetric with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole.

Calculate values of *r* for θ from 0 to π and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	2	1	–1	–2	–1	1	2



r = 2 cos 2θ

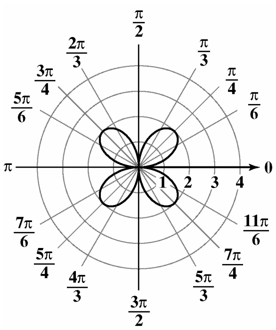
26. $r = 2\sin 2\theta$

Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 2\sin 2(-\theta)$	$-r = 2\sin 2(-\theta)$	
$r = 2\sin(-2\theta)$	$-r = 2\sin(-2\theta)$	
	$-r = -2\sin 2\theta$	
$r = -2\sin 2\theta$	$r = 2\sin 2\theta$	

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$. The graph may or may not be symmetric with respect to the polar axis or the pole. Calculate values of r for θ from 0 to $\frac{\pi}{2}$ and for θ from π to $\frac{3\pi}{2}$. Then, use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$
r	0	1.73	2	1.73	0	0	1.73	2	1.73	0



27. $r = 4\sin 3\theta$

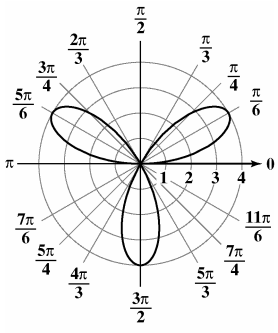
Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 4\sin 3(-\theta)$	$-r = 4\sin 3(-\theta)$	$-r = 4\sin 3\theta$
$r = 4\sin(-3\theta)$	$-r = 4\sin(-3\theta)$	
	$-r = -4\sin 3\theta$	
$r = -4\sin 3\theta$	$r = 4\sin 3\theta$	$r = -4\sin 3\theta$

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$. The graph may or may not be symmetric with respect to the polar axis or the poles.

Calculate values of r for θ from 0 to $\frac{\pi}{2}$ and for θ from π to $\frac{3\pi}{2}$. Then, use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$
r	0	4	2.83	0	–4	0	–4	–2.83	0	4



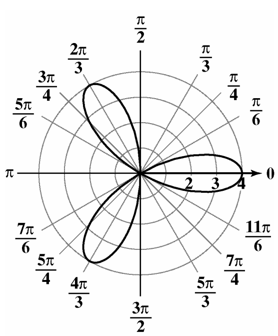
28. $r = 4\cos 3\theta$

Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = 4\cos 3(-\theta)$	$-r = 4\cos 3(-\theta)$	
$r = 4\cos(-3\theta)$	$-r = 4\cos(-3\theta)$	
	$-r = 4\cos(3\theta)$	
$r = 4\cos 3\theta$	$r = -4\cos 3\theta$	

The graph is symmetric with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole. Calculate values for r from θ for 0 to π and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	4	0	–4	0	4	0	–4



29. $r^2 = 9\cos 2\theta$
Check for symmetry:

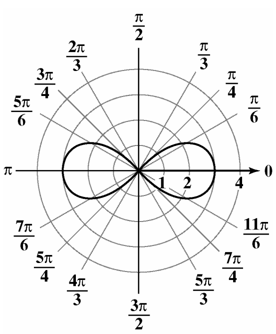
Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r^2 = 9\cos 2(-\theta)$	$(-r)^2 = 9\cos 2(-\theta)$	$(-r)^2 = 9\cos 2\theta$
$r^2 = 9\cos(-2\theta)$	$r^2 = 9\cos(-2\theta)$	
$r^2 = 9\cos 2\theta$	$r^2 = 9\cos 2\theta$	$r^2 = 9\cos 2\theta$

The graph is symmetric with respect to the polar axis, the line $\theta = \frac{\pi}{2}$, and the pole.

Note that since $\cos 2\theta$ is negative for $\frac{\pi}{4} < \theta < \frac{3\pi}{4}$, there is no graph there.

Calculate values of r for θ from 0 to $\frac{\pi}{4}$ and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$
r	±3	±2.12	0

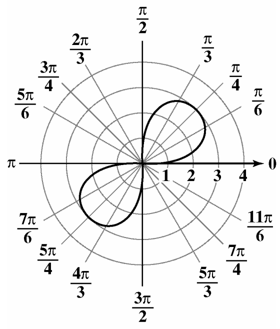


30. $r^2 = 9\sin 2\theta$
Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r^2 = 9\sin 2(-\theta)$	$(-r)^2 = 9\sin 2(-\theta)$	
$r^2 = 9\sin(-2\theta)$	$r^2 = 9\sin(-2\theta)$	
$r^2 = -9\sin 2\theta$	$r^2 = -9\sin 2\theta$	

The graph is symmetric with respect to the pole. The graph may or may not be symmetric with respect to the polar axis or the line $\theta = \frac{\pi}{2}$. Note that since $\sin 2\theta$ is negative for $\frac{\pi}{2} < \theta < \pi$, there is no graph in quadrant II or IV. Calculate values of r for θ from 0 to $\frac{\pi}{2}$ and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
r	0	±2.79	±3	±2.79	0



$r^2 = 9 \sin 2\theta$

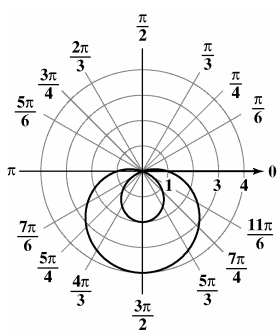
31. $r = 1 - 3\sin\theta$

Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = 1 - 3\sin(-\theta)$	$-r = 1 - 3\sin(-\theta)$	$-r = 1 - 3\sin\theta$
	$-r = 1 + 3\sin\theta$	
$r = 1 + 3\sin\theta$	$r = -1 - 3\sin\theta$	$r = -1 + 3\sin\theta$

There may be no symmetry. Since each equation is not equivalent to $r = 1 - 3\sin\theta$. Because the period of the sine function is 2π, we need not consider values of θ beyond 2π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{6}$	$\frac{11\pi}{6}$	2π
r	1	–0.5	–1.6	–2	–1.6	–0.5	1	2.5	3.6	4	3.6	2.5	1



$r = 1 - 3\sin\theta$

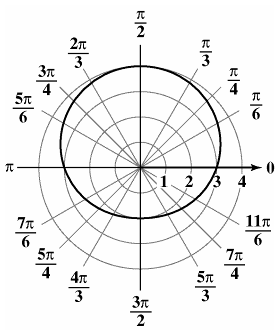
32. $r = 3 + \sin\theta$

Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 3 + \sin(-\theta)$	$-r = 3 + \sin(-\theta)$	
	$-r = 3 - \sin\theta$	
$r = 3 - \sin\theta$	$r = -3 + \sin\theta$	

There may be no symmetry, since each equation is not equivalent to $r = 3 + \sin\theta$. Because the period of the sine function is 2π, we need not consider values of θ beyond 2π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
r	3	3.5	3.87	4	3.87	3.5	3	2.5	2.13	2	2.13	2.5	3



$r = 3 + \sin\theta$

33. $r\cos\theta = -3$

$$r = \frac{-3}{\cos\theta} = -3\sec\theta$$

Check for symmetry:

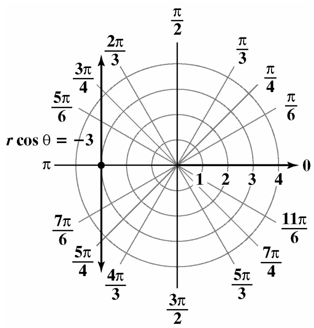
Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = -3\sec(-\theta)$	$-r = -3\sec(-\theta)$	$-r = -3\sec\theta$
	$-r = -3\sec\theta$	
$r = -3\sec\theta$	$r = 3\sec\theta$	$r = 3\sec\theta$

The graph is symmetric with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole.

Calculate values of r for θ from 0 to π. Then, use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	–3	–3.46	–6	Undef.	6	3.46	3

Note that since $\sec\theta$ is undefined when $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$, r increases without bound as θ approaches these angles.
$r\cos\theta = -3$ is equivalent to $x = -3$.



34. $r\sin\theta = 2$

$$r = \frac{2}{\sin\theta} = 2\csc\theta$$

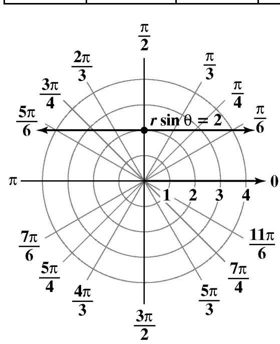
Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = 2\csc(-\theta)$	$-r = 2\csc(-\theta)$	
	$-r = -2\csc(\theta)$	
$r = -2\csc\theta$	$r = 2\csc\theta$	

The graph is symmetric with respect to the line $\theta = \frac{\pi}{2}$. The graph may or may not be symmetric with respect to the polar axis or the pole. Calculate values of r for θ from 0 to $\frac{\pi}{2}$ and for θ from π to $\frac{3\pi}{2}$. Then, use symmetry to obtain the graph.

Note that since $\csc\theta$ is undefined when $\theta = 0$ and $\theta = \pi$, r increases without bound as θ approaches these angles.
$r\sin\theta = 2$ is equivalent to $y = 2$.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$
r	undef	4	2.83	2.31	2	undef	–4	–2.83	–2.31	–2

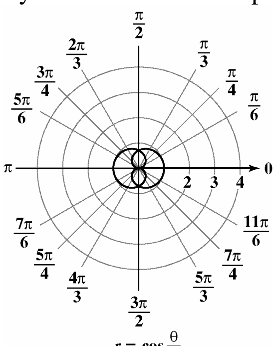


35. $r = \cos\dfrac{\theta}{2}$

Symmetrical about the polar axis: yes

Symmetrical about the line $\theta = \dfrac{\pi}{2}$: yes

Symmetrical about the pole: yes



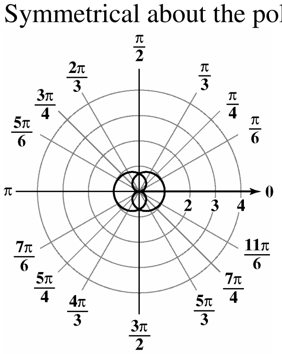
$r = \cos\frac{\theta}{2}$

36. $r = \sin\dfrac{\theta}{2}$

Symmetrical about the polar axis: yes

Symmetrical about the line $\theta = \dfrac{\pi}{2}$: yes

Symmetrical about the pole: yes



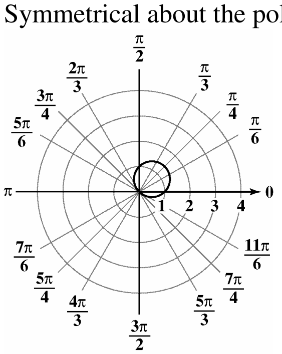
$r = \sin\frac{\theta}{2}$

37. $r = \sin\theta + \cos\theta$

Symmetrical about the polar axis: maybe

Symmetrical about the line $\theta = \dfrac{\pi}{2}$: maybe

Symmetrical about the pole: maybe



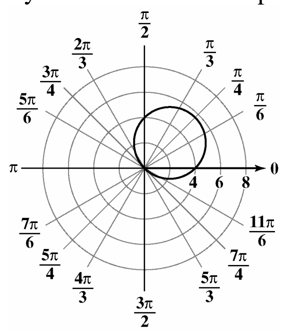
$r = \sin\theta + \cos\theta$

38. $r = 4\cos\theta + 4\sin\theta$

Symmetrical about the polar axis: maybe

Symmetrical about the line $\theta = \dfrac{\pi}{2}$: maybe

Symmetrical about the pole: maybe



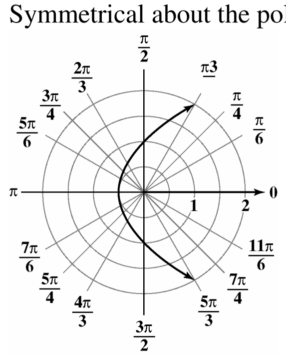
$r = 4\cos\theta + 4\sin\theta$

39. $r = \dfrac{1}{1-\cos\theta}$

Symmetrical about the polar axis: yes

Symmetrical about the line $\theta = \dfrac{\pi}{2}$: maybe

Symmetrical about the pole: maybe



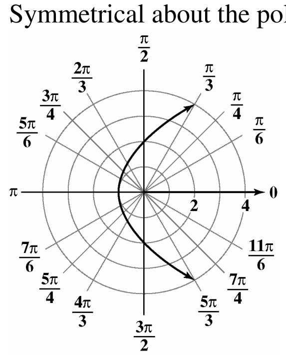
$r = \frac{1}{1-\cos\theta}$

40. $r = \dfrac{2}{1-\cos\theta}$

Symmetrical about the polar axis: yes

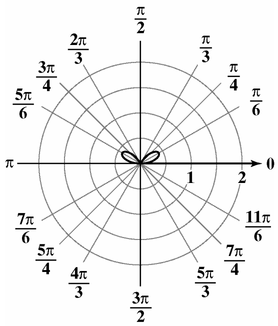
Symmetrical about the line $\theta = \dfrac{\pi}{2}$: maybe

Symmetrical about the pole: maybe



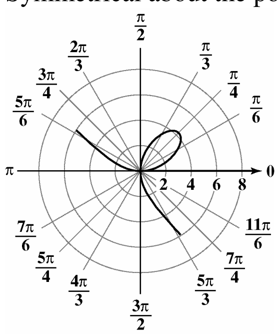
$r = \frac{2}{1-\cos\theta}$

41. $r = \sin\theta\cos^2\theta$
Symmetrical about the polar axis: maybe
Symmetrical about the line $\theta = \frac{\pi}{2}$: yes
Symmetrical about the pole: maybe



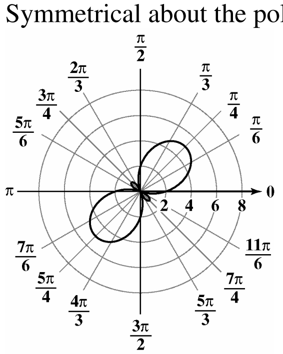
$r = \sin\theta\cos^2\theta$

42. $r = \dfrac{3\sin 2\theta}{\sin^3\theta + \cos^3\theta}$
Symmetrical about the polar axis: maybe
Symmetrical about the line $\theta = \frac{\pi}{2}$: maybe
Symmetrical about the pole: maybe



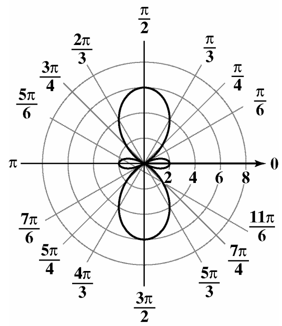
$r = \dfrac{3\sin 2\theta}{\sin^3\theta + \cos^3\theta}$

43. $r = 2 + 3\sin 2\theta$
Symmetrical about the polar axis: maybe
Symmetrical about the line $\theta = \frac{\pi}{2}$: maybe
Symmetrical about the pole: yes



$r = 2 + 3\sin 2\theta$

44. $r = 2 - 4\cos 2\theta$
Symmetrical about the polar axis: yes
Symmetrical about the line $\theta = \frac{\pi}{2}$: yes
Symmetrical about the pole: yes



$r = 2 - 4\cos 2\theta$

45. Using the graph, sailing at a 60° angle to the wind gives a speed of about 6 knots (to the nearest knot).

46. Using the graph, sailing at a 120º angle to the wind gives a speed of about 7 knots (to the nearest knot).

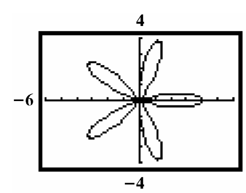
47. Using the graph, sailing at a 90° angle to the wind gives a speed of about 8 knots (to the nearest knot).

48. Using the graph, sailing at a 180º angle to the wind gives a speed of about 4 knots (to the nearest knot).

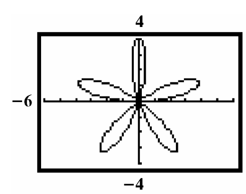
49. It appears that an angle of 90° gives a maximum speed of about $7\frac{1}{2}$ knots.

50. – 57. Answers may vary.

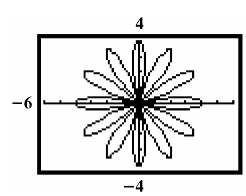
58.



59.



60.



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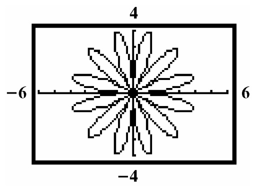
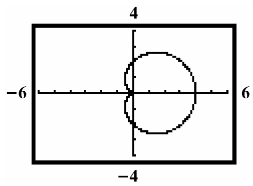


62.



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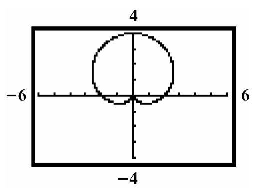

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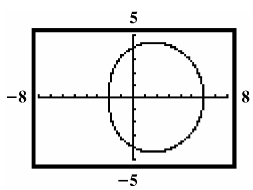

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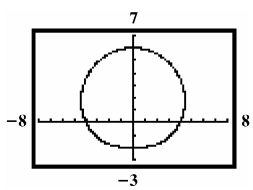

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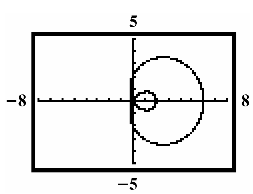

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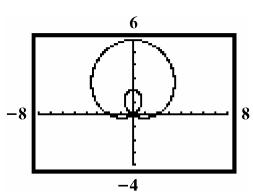
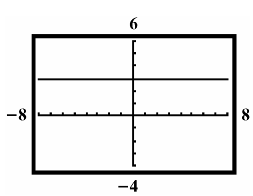
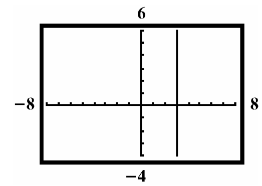


68.



69.



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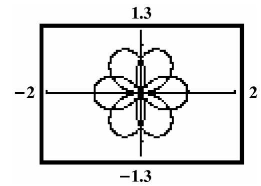

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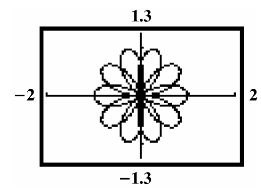

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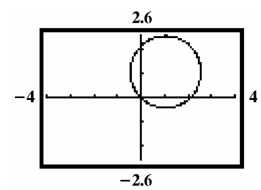
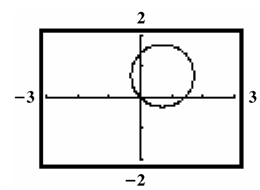


73.



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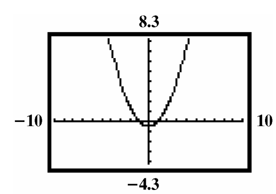

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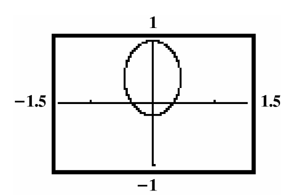
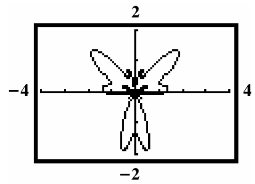


76. If $\theta \max = \pi$, the graph is drawn once.

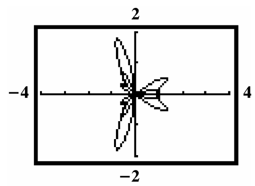
77. If $\theta \max = 2\pi$, the graph is drawn once.

78. If $\theta \max = \pi$, the graph is drawn once.

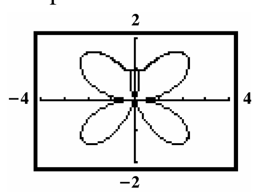
79. θ step $= 0.1$



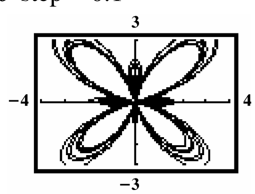
80. θ step $= 0.1$



81. θ step $= 0.1$



82. θ step $= 0.1$



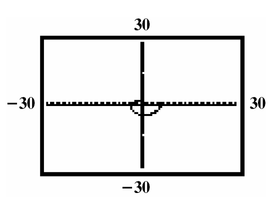
83. As n increases, $\sin n\theta$ increases its number of loops. If n is odd, there are n loops and $\theta\max = \pi$ traces the graph once, while if n is even, there are $2n$ loops and $\theta\max = 2\pi$ traces the graph once.

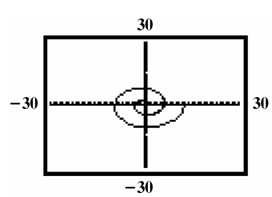
84. As n increases, $\cos n\theta$ increases it number of loops. If n is odd, there are n loops and $\theta\max = \pi$ traces the graph once, while if n is even, there are $2n$ loops and $\theta\max = 2\pi$ traces the graph once. Yes, the conclusions are the same as for $\sin n\theta$.

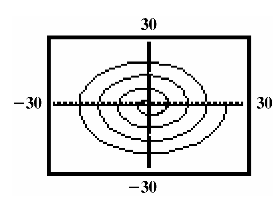
85. There are n small petals and n large petals for each value of n. For odd values of n, the small petals are inside the large petals. For even n, they are between the large petals.

86. There are n small petals and n large petals for each value of n. For odd values of n, the small petals are inside the large petals. For even n, they are between the large petals. Yes, the conclusions are the same as for $1+2\sin n\theta$.

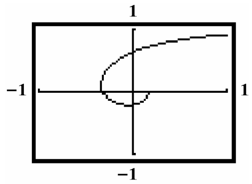
87. $\theta\min = 0, \theta\max = 2\pi$ $\theta\min = 0, \theta\max = 4\pi$ $\theta\min = 0, \theta\max = 8\pi$



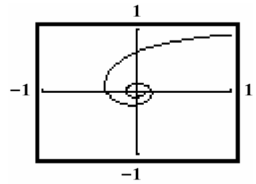




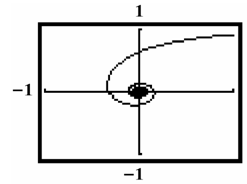
88. $\theta\min = 0,\ \theta\max = 2\pi$



$\theta\min = 0,\ \theta\max = 4\pi$



$\theta\min = 0,\ \theta\max = 8\pi$



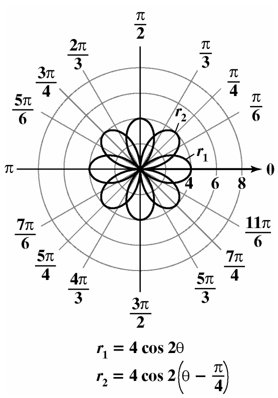
89. does not make sense; Explanations will vary. Sample explanation: If a polar equation fails a symmetry test, its graph may still have that kind of symmetry.

90. does not make sense; Explanations will vary. Sample explanation: A limaçon always has symmetry.

91. makes sense

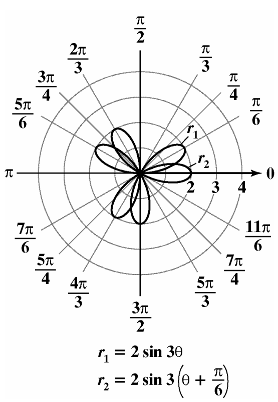
92. makes sense

93.



The graph of r_2 is the graph of r_1 rotated $\frac{\pi}{4}$ or $45°$.

94.



The graph of r_2 is the graph of r_1 rotated $-\frac{\pi}{6}$ or $-30°$.

95. Answers may vary.

96. $(1+i)(2+2i) = 2+2i+2i+2i^2$
$= 2+4i-2$
$= 4i$

97. $(-1+i\sqrt{3})(-1+i\sqrt{3})(-1+i\sqrt{3}) = (-1+i\sqrt{3})(1-i\sqrt{3}-i\sqrt{3}+3i^2)$
$= (-1+i\sqrt{3})(1-2i\sqrt{3}-3)$
$= (-1+i\sqrt{3})(-2-2i\sqrt{3})$
$= 2+2i\sqrt{3}-2i\sqrt{3}-6i^2$
$= 2+6$
$= 8$

98. $\frac{2+2i}{1+i} = \frac{2+2i}{1+i}\cdot\frac{1-i}{1-i}$
$= \frac{2-2i+2i-2i^2}{1-i+i-i^2}$
$= \frac{2+2}{1+1}$
$= 2$

Mid-Chapter 7 Check Point

1. $C = 180° - 32° - 41° = 107°$
Use the Law of Sines to find b.
$\frac{a}{\sin A} = \frac{b}{\sin B}$
$\frac{20}{\sin 32°} = \frac{b}{\sin 41°}$
$b = \frac{20\sin 41°}{\sin 32°}$
$b \approx 24.8$
Use the Law of Sines to find c.
$\frac{a}{\sin A} = \frac{c}{\sin C}$
$\frac{20}{\sin 32°} = \frac{c}{\sin 107°}$
$c = \frac{20\sin 107°}{\sin 32°}$
$c \approx 36.1$
The solution is $C = 107°$, $b \approx 24.8$, and $c \approx 36.1$.

2. Use the Law of Sines to find B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{63}{\sin 42^\circ} = \frac{57}{\sin B}$$
$$\sin B = \frac{57\sin 42^\circ}{63}$$
$$\sin B \approx 0.6054$$

There are two angles possible:

$B_1 \approx 37^\circ,\ B_2 \approx 180^\circ - 37^\circ = 143^\circ$

B_2 is impossible, since $42^\circ + 143^\circ = 185^\circ$.

$C = 180^\circ - B_1 - A \approx 180^\circ - 37^\circ - 42^\circ = 101^\circ$

Use the Law of Sines to find c.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{c}{\sin 101^\circ} = \frac{63}{\sin 42^\circ}$$
$$c = \frac{63\sin 101^\circ}{\sin 42^\circ}$$
$$c \approx 92.4$$

There is one triangle and the solution is $B_1 \text{ (or } B) \approx 37^\circ,\ C \approx 101^\circ,$ and $c \approx 92.4$.

3. Use the Law of Sines to find angle B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{6}{\sin 65^\circ} = \frac{7}{\sin B}$$
$$\sin B = \frac{7\sin 65^\circ}{6}$$
$$\sin B \approx 1.0574$$

The sine can never exceed 1. There is no triangle with the given measurements.

4. Use the Law of Cosines to find b.

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 10^2 + 16^2 - 2(10)(16)\cos 110^\circ$$
$$b^2 \approx 465.4464$$
$$b \approx 21.6$$

Use the Law of Sines to find A.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{21.6}{\sin 110^\circ} = \frac{10}{\sin A}$$
$$\sin A = \frac{10\sin 110^\circ}{21.6}$$
$$\sin A \approx 0.4350$$
$$A \approx 26^\circ$$

Find the third angle.

$C = 180^\circ - A - B \approx 180^\circ - 26^\circ - 110^\circ = 44^\circ$

The solution is $A \approx 26^\circ,\ C \approx 44^\circ,$ and $b \approx 21.6$.

5. Use the Law of Sines to find angle A.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{16}{\sin A} = \frac{13}{\sin 42^\circ}$$
$$\sin A = \frac{16\sin 42^\circ}{13}$$
$$\sin A \approx 0.8235$$

There are two angles possible:

$A_1 \approx 55^\circ,\ A_2 \approx 180^\circ - 55^\circ = 125^\circ$

There are two triangles:

$B_1 = 180^\circ - C - A_1 \approx 180^\circ - 42^\circ - 55^\circ = 83^\circ$

$B_2 = 180^\circ - C - A_2 \approx 180^\circ - 42^\circ - 125^\circ = 13^\circ$

Use the Law of Sines to find b_1 and b_2.

$$\frac{b_1}{\sin B_1} = \frac{c}{\sin C}$$
$$\frac{b_1}{\sin 83^\circ} = \frac{13}{\sin 42^\circ}$$
$$b_1 = \frac{13\sin 83^\circ}{\sin 42^\circ} \approx 19.3$$
$$\frac{b_2}{\sin B_2} = \frac{c}{\sin C}$$
$$\frac{b_2}{\sin 13^\circ} = \frac{13}{\sin 42^\circ}$$
$$b_2 = \frac{13\sin 13^\circ}{\sin 42^\circ} \approx 4.4$$

In one triangle, the solution is $A_1 \approx 55^\circ,\ B_1 \approx 83^\circ,\ b_1 \approx 19.3$.

In the other triangle, $A_2 \approx 125^\circ,\ B_2 \approx 13^\circ,\ b_2 \approx 4.4$.

6. Use the Law of Cosines to find the angle opposite the longest side.
Thus, find angle C.

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \dfrac{5^2 + 7.2^2 - 10.1^2}{2 \cdot 5 \cdot 7.2}$

$\cos C \approx -0.3496$

$C \approx 110°$

Use the Law of Sines to find angle A.

$\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

$\dfrac{5}{\sin A} = \dfrac{10.1}{\sin 110°}$

$\sin A = \dfrac{5\sin 110°}{10.1}$

$\sin A \approx 0.4652$

$A \approx 28°$

Find the third angle.

$B = 180° - A - C = 180° - 28° - 110° = 42°$

The solution is $A \approx 28°, B \approx 42°,$ and $C \approx 110°$

7. The area of the triangle is half the product of the lengths of the two sides times the sine of the included angle.

$\text{Area} = \frac{1}{2}(5)(7)(\sin 36°) \approx 10$

The area of the triangle is approximately 10 square feet.

8. Begin by calculating one-half the perimeter:

$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(7+9+12) = 14$

Use Heron's formula to find the area.

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{14(14-7)(14-9)(14-12)}$

$= \sqrt{980} \approx 31$

The area of the triangle is approximately 31 square meters.

9. The first train traveled 100 miles, the second train traveled 80 miles.
Use the Law of Cosines to find the distance.

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 100^2 + 80^2 - 2(100)(80)\cos 110°$

$c^2 \approx 21872.32229$

$c \approx 148$

The two trains are 148 miles apart.

10. Let the fire be at point C.

$A = 90° - 56° = 34°$

$B = 90° - 23° = 67°$

$C = 180° - 34° - 67° = 79°$

Use the law of sines to find b.

$\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\dfrac{b}{\sin 67°} = \dfrac{16}{\sin 79°}$

$b = \dfrac{16\sin 67°}{\sin 79°}$

$b \approx 15.0$

The fire is 15.0 miles from station A

11. Let point A be where the angle of elevation is $66°$.
Let point B be where the angle of elevation is $50°$.
Let point C be at the top of the tree.

$C = 180° - A - B = 180° - 66° - 50° = 64°$

Use the law of sines to find a.

$\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

$\dfrac{a}{\sin 66°} = \dfrac{420}{\sin 64°}$

$a = \dfrac{420\sin 66°}{\sin 64°}$

$a \approx 427$

The height of the tree, h, is given by

$h = a\sin B$

$h = 427\sin 50°$

$h \approx 327$

The tree is 327 feet tall.

12. $x = r\cos\theta = -3\cos\dfrac{5\pi}{4} = \dfrac{3\sqrt{2}}{2}$

$y = r\sin\theta = -3\sin\dfrac{5\pi}{4} = -\dfrac{3\sqrt{2}}{2}$

Ordered pair: $\left(\dfrac{3\sqrt{2}}{2}, -\dfrac{3\sqrt{2}}{2}\right)$

13. $x = r\cos\theta = 6\cos\left(-\dfrac{\pi}{2}\right) = 0$

$y = r\sin\theta = 6\sin\left(-\dfrac{\pi}{2}\right) = -6$

Ordered pair: $(0, -6)$

14. $r = \sqrt{x^2 + y^2} = \sqrt{2^2 + \left(-2\sqrt{3}\right)^2} = 4$

$\tan\theta = \frac{y}{x} = \frac{-2\sqrt{3}}{2} = -\sqrt{3}$

Because θ lies in quadrant IV, $\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Polar coordinates: $\left(4, \frac{5\pi}{3}\right)$

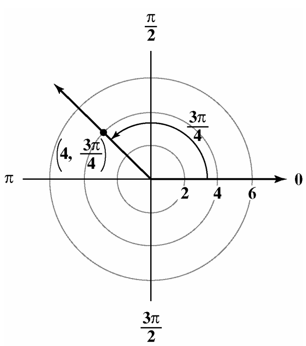
15. $r = \sqrt{x^2 + y^2} = \sqrt{(-6)^2 + 0^2} = 6$

$\tan\theta = \frac{y}{x} = \frac{0}{-6} = 0$

$\theta = \pi$

Polar coordinates: $(6, \pi)$

16.

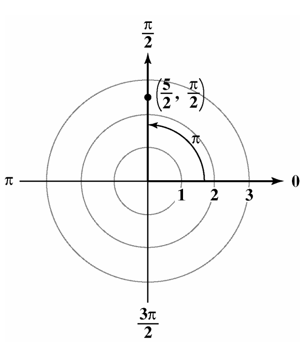


a. $\left(4, \frac{11\pi}{4}\right)$

b. $\left(-4, \frac{7\pi}{4}\right)$

c. $\left(4, \frac{5\pi}{4}\right)$

17.



a. $\left(\frac{5}{2}, \frac{5\pi}{2}\right)$

b. $\left(-\frac{5}{2}, \frac{3\pi}{2}\right)$

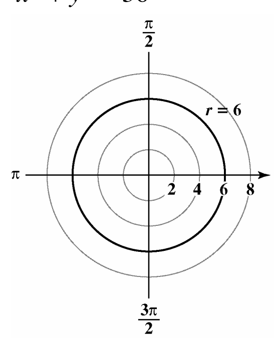
c. $\left(\frac{5}{2}, -\frac{3\pi}{2}\right)$

18.
$$5x - y = 7$$
$$5r\cos\theta - r\sin\theta = 7$$
$$r\left(5\cos\theta - \sin\theta\right) = 7$$
$$r = \frac{7}{5\cos\theta - \sin\theta}$$

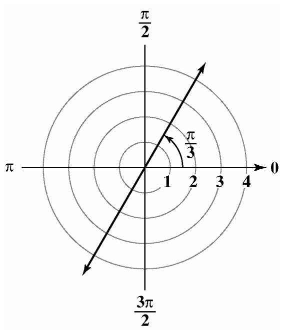
19.
$$y = -7$$
$$r\sin\theta = -7$$
$$r = \frac{-7}{\sin\theta}$$
$$r = -7\csc\theta$$

20.
$$(x+1)^2 + y^2 = 1$$
$$(r\cos\theta + 1)^2 + (r\sin\theta)^2 = 1$$
$$r^2\cos^2\theta + 2r\cos\theta + 1 + r^2\sin^2\theta^2 = 1$$
$$r^2 + 2r\cos\theta = 0$$
$$r^2 = -2r\cos\theta$$
$$r = -2\cos\theta$$

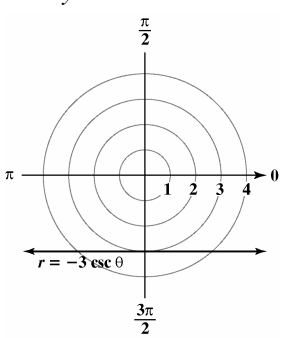
21.
$$r = 6$$
$$r^2 = 36$$
$$x^2 + y^2 = 36$$



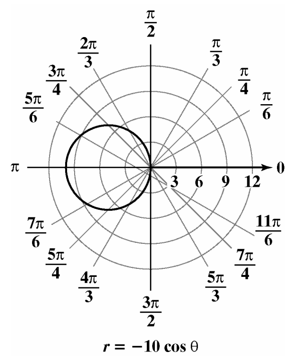
22.
$$\theta = \frac{\pi}{3}$$
$$\tan\theta = \tan\frac{\pi}{3}$$
$$\tan\theta = \sqrt{3}$$
$$\frac{y}{x} = \sqrt{3}$$
$$y = \sqrt{3}x$$



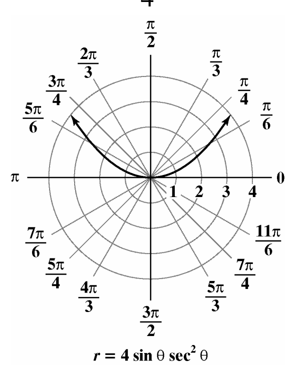
23.
$$r = -3\csc\theta$$
$$r = \frac{-3}{\sin\theta}$$
$$r\sin\theta = -3$$
$$y = -3$$



24.
$$r = -10\cos\theta$$
$$r^2 = -10r\cos\theta$$
$$x^2 + y^2 = -10x$$
$$x^2 + 10x + y^2 = 0$$
$$x^2 + 10x + 25 + y^2 = 25$$
$$(x+5)^2 + y^2 = 25$$



25.
$$r = 4\sin\theta\sec^2\theta$$
$$r = \frac{4\sin\theta}{\cos^2\theta}$$
$$r\cos^2\theta = 4\sin\theta$$
$$r^2\cos^2\theta = 4r\sin\theta$$
$$x^2 = 4y$$
$$y = \frac{1}{4}x^2$$



26. $r = 1 - 4\cos\theta$

a. Replace θ with $-\theta$.

$r = 1 - 4\cos(-\theta)$

$r = 1 - 4\cos\theta$

The graph is symmetric with respect to the polar axis.

b. Replace (r, θ) with $(-r, -\theta)$.

$-r = 1 - 4\cos(-\theta)$

$r = -1 + 4\cos\theta$

The graph may or may not be symmetric with respect to the line $\theta = \dfrac{\pi}{2}$.

c. Replace r with $-r$.

$-r = 1 - 4\cos\theta$

$r = -1 + 4\cos\theta$

The graph may or may not be symmetric with respect to the polar axis.

27. $r^2 = 4\cos 2\theta$

a. Replace θ with $-\theta$.

$r^2 = 4\cos(-2\theta)$

$r^2 = 4\cos 2\theta$

The graph is symmetric with respect to the polar axis.

b. Replace (r, θ) with $(-r, -\theta)$.

$(-r)^2 = 4\cos(-2\theta)$

$r^2 = 4\cos 2\theta$

The graph is symmetric with respect to the line $\theta = \dfrac{\pi}{2}$.

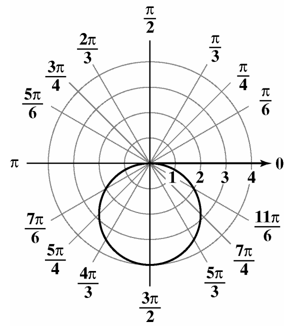
c. Replace r with $-r$.

$(-r)^2 = 4\cos 2\theta$

$r^2 = 4\cos 2\theta$

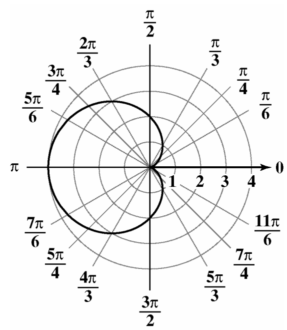
The graph is symmetric with respect to the polar axis.

28.



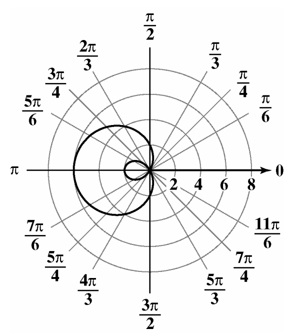
$r = -4\sin\theta$

29.



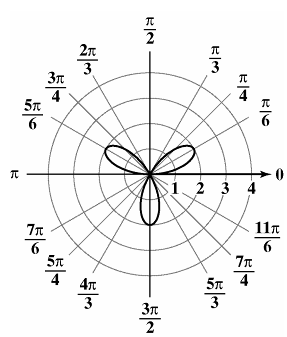
$r = 2 - 2\cos\theta$

30.



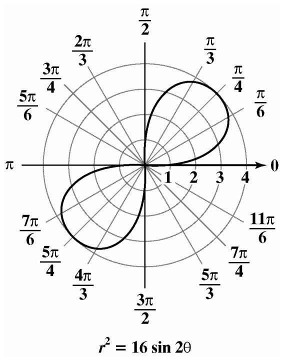
$r = 2 - 4\cos\theta$

31.



$r = 2\sin 3\theta$

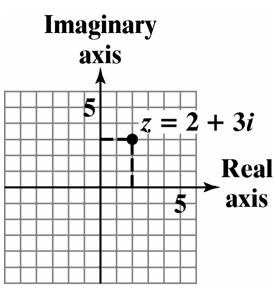
32.



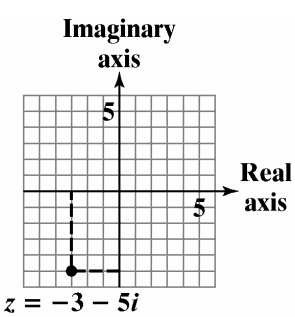
Section 7.5

Check Point Exercises

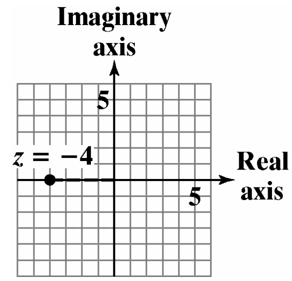
1. a. $z = 2 + 3i$ corresponds to the point (2, 3). Plot the complex number by moving two units to the right on the real axis and 3 units up parallel to the imaginary axis.



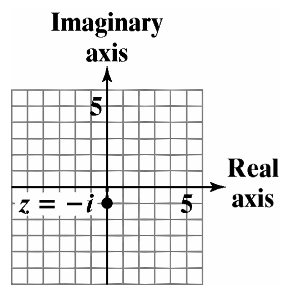
b. $z = –3 – 5i$ corresponds to the point (–3, –5). Plot the complex number by moving three units to the left on the real axis and five units down parallel to the imaginary axis.



c. Because $z = -4 = -4 + 0i$, this complex number corresponds to the point (–4, 0). Plot the complex number by moving four units to the left on the real axis.



d. Because $z = -i = 0 - i$, this complex number corresponds to the point (0, –1). Plot the complex number by moving one unit down on the imaginary axis.



2 a. $z = 5 + 12i$

$a = 5, b = 12$

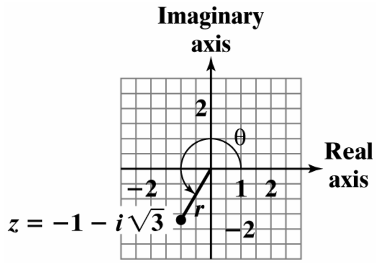
$|z| = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

b. $z = 2 - 3i$

$a = 2, b = -3$

$|z| = \sqrt{2^2 + (-3)^2} = \sqrt{4 + 9} = \sqrt{13}$

3. $z=-1-i\sqrt{3}$ corresponds to the point $\left(-1,-\sqrt{3}\right)$.



Use $r=\sqrt{a^2+b^2}$ with $a=-1$ and $b=-\sqrt{3}$ to find *r*.

$r=\sqrt{a^2+b^2}=\sqrt{(-1)^2+\left(-\sqrt{3}\right)^2}$

$=\sqrt{1+3}=\sqrt{4}=2$

Use $\tan\theta=\dfrac{b}{a}$ with $a=-1$ and $b=-\sqrt{3}$ to find θ.

$\tan\theta=\dfrac{b}{a}=\dfrac{-\sqrt{3}}{-1}=\sqrt{3}$

Because $\tan\dfrac{\pi}{3}=\sqrt{3}$ and θ lies in quadrant III, $\theta=\pi+\dfrac{\pi}{3}=\dfrac{3\pi}{3}+\dfrac{\pi}{3}=\dfrac{4\pi}{3}$.

The polar form of $z=-1-i\sqrt{3}$ is

$z=r(\cos\theta+i\sin\theta)=2\left(\cos\dfrac{4\pi}{3}+i\sin\dfrac{4\pi}{3}\right)$.

4. The complex number $z=4(\cos 30°+i\sin 30°)$ is in polar form, with $r=4$ and $\theta=30°$. We use exact values for $\cos 30°$ and $\sin 30°$ to write the number in rectangular form.

$4(\cos 30°+i\sin 30°)=4\left(\dfrac{\sqrt{3}}{2}+i\dfrac{1}{2}\right)=2\sqrt{3}+2i$

The rectangular form of $z=4(\cos 30°+i\sin 30°)$ is $z=2\sqrt{3}+2i$.

5. $z_1z_2=[6(\cos 40°+i\sin 40°)][5(\cos 20°+i\sin 20°)]=(6\cdot 5)[(\cos(40°+20°)+i\sin(40°+20°)]$
$=30(\cos 60°+i\sin 60°)$

6. $$\frac{z_1}{z_2}=\frac{50\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)}{5\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}=\frac{50}{5}\left[\cos\left(\frac{4\pi}{3}-\frac{\pi}{3}\right)+i\sin\left(\frac{4\pi}{3}-\frac{\pi}{3}\right)\right]=10(\cos\pi+i\sin\pi)$$

7. $$\left[2(\cos 30°+i\sin 30°)\right]^5=2^5\left[\cos(5\cdot 30°)+i\sin(5\cdot 30°)\right]=32(\cos 150°+i\sin 150°)=32\left(-\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)$$
$$=-16\sqrt{3}+16i$$

8. Write $1+i$ in $r(\cos\theta + i\sin\theta)$ form.

$$r = \sqrt{a^2+b^2} = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\tan\theta = \frac{b}{a} = \frac{1}{1} = 1 \text{ and } \theta = \frac{\pi}{4}$$

$$1+i = r(\cos\theta + i\sin\theta) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

Use DeMoivre's Theorem to raise 1 + i to the fourth power.

$$(1+i)^4 = \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^4 = \left(\sqrt{2}\right)^4\left[\cos\left(4\cdot\frac{\pi}{4}\right) + i\sin\left(4\cdot\frac{\pi}{4}\right)\right] = 4(\cos\pi + i\sin\pi) = 4(-1+0i) = -4$$

9. From DeMoivre's Theorem for finding complex roots, the fourth roots of $16(\cos 60° + i\sin 60°)$ are

$$z_k = \sqrt[4]{16}\left[\cos\left(\frac{60°+360°k}{4}\right) + i\sin\left(\frac{60°+360°k}{4}\right)\right],\ k = 0,1,2,3.$$

Substitute 0, 1, 2, and 3 for k in the above expression for z_k.

$$z_0 = \sqrt[4]{16}\left[\cos\left(\frac{60°+360°\cdot 0}{4}\right) + i\sin\left(\frac{60°+360°\cdot 0}{4}\right)\right] = \sqrt[4]{16}\left[\cos\frac{60°}{4} + i\sin\frac{60°}{4}\right] = 2(\cos 15° + i\sin 15°)$$

$$z_1 = \sqrt[4]{16}\left[\cos\left(\frac{60°+360°\cdot 1}{4}\right) + i\sin\left(\frac{60°+360°\cdot 1}{4}\right)\right] = \sqrt[4]{16}\left[\cos\frac{420°}{4} + i\sin\frac{420°}{4}\right] = 2(\cos 105° + i\sin 105°)$$

$$z_2 = \sqrt[4]{16}\left[\cos\left(\frac{60°+360°\cdot 2}{4}\right) + i\sin\left(\frac{60°+360°\cdot 2}{4}\right)\right] = \sqrt[4]{16}\left[\cos\frac{780°}{4} + i\sin\frac{780°}{4}\right] = 2(\cos 195° + i\sin 195°)$$

$$z_3 = \sqrt[4]{16}\left[\cos\left(\frac{60°+360°\cdot 3}{4}\right) + i\sin\left(\frac{60°+360°\cdot 3}{4}\right)\right] = \sqrt[4]{16}\left[\cos\frac{1140°}{4} + i\sin\frac{1140°}{4}\right] = 2(\cos 285° + i\sin 285°)$$

10. First, write 27 in polar form. $27 = r(\cos\theta + i\sin\theta) = 27(\cos 0 + \sin 0)$. From DeMoivre's theorem for finding complex roots, the cube roots of 27 are

$$z_k = \sqrt[3]{27}\left[\cos\left(\frac{0+2\pi k}{3}\right) + i\sin\left(\frac{0+2\pi k}{3}\right)\right],\ k = 0,1,2.$$

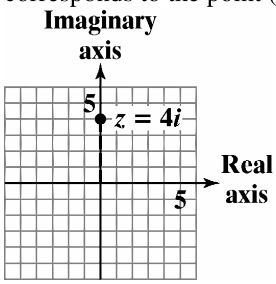
$$z_0 = \sqrt[3]{27}\left[\cos\left(\frac{0+2\pi\cdot 0}{3}\right) + i\sin\left(\frac{0+2\pi\cdot 0}{3}\right)\right] = 3(\cos 0 + i\sin 0) = 3(1+i\cdot 0) = 3$$

$$z_1 = \sqrt[3]{27}\left[\cos\left(\frac{0+2\pi\cdot 1}{3}\right) + i\sin\left(\frac{0+2\pi\cdot 1}{3}\right)\right] = 3\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 3\left(-\frac{1}{2} + i\cdot\frac{\sqrt{3}}{2}\right) = -\frac{3}{2} + \frac{3\sqrt{3}}{2}i$$

$$z_2 = \sqrt[3]{27}\left[\cos\left(\frac{0+2\pi\cdot 2}{3}\right) + i\sin\left(\frac{0+2\pi\cdot 2}{3}\right)\right] = 3\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right) = 3\left(-\frac{1}{2} + i\cdot\left(-\frac{\sqrt{3}}{2}\right)\right) = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$$

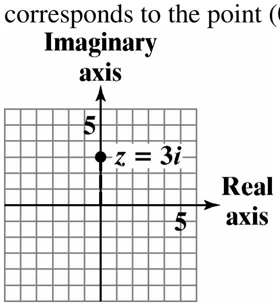
Exercise Set 7.5

1. Because $z = 4i = 0 + 4i$, this complex number corresponds to the point (0, 4).



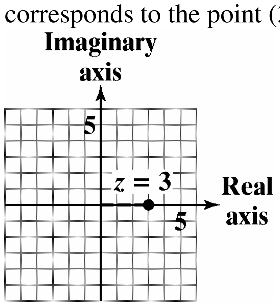
With $a = 0$ and $b = 4$,
$|z| = \sqrt{0^2 + 4^2} = \sqrt{16} = 4$.

2. Because $z = 3i = 0 + 3i$, this complex number corresponds to the point (0, 3).



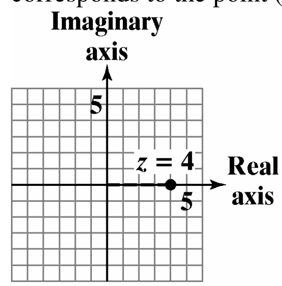
With $a = 0$ and $b = 3$, $|z| = \sqrt{0^2 + 3^2} = \sqrt{9} = 3$.

3. Because $z = 3 = 3 + 0i$, this complex number corresponds to the point (3, 0).



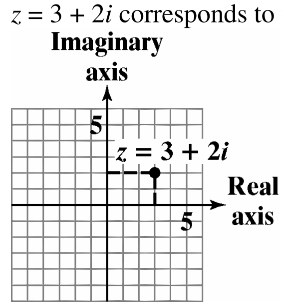
With $a = 3$ and $b = 0$, $|z| = \sqrt{3^2 + 0^2} = \sqrt{9} = 3$.

4. Because $z = 4 = 4 + 0i$, this complex number corresponds to the point (4, 0).



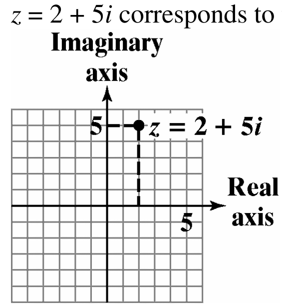
With $a = 4$ and $b = 0$, $|z| = \sqrt{4^2 + 0^2} = \sqrt{16} = 4$.

5. $z = 3 + 2i$ corresponds to the point (3, 2).



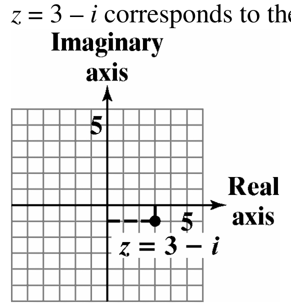
With $a = 3$ and $b = 2$,
$|z| = \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}$.

6. $z = 2 + 5i$ corresponds to the point (2, 5).



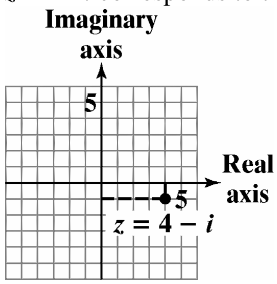
With $a = 2$ and $b = 5$,
$|z| = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$.

7. $z = 3 - i$ corresponds to the point (3, –1).



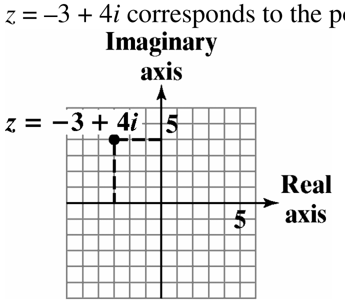
With $a = 3$ and $b = -1$,
$|z| = \sqrt{3^2 + (-1)^2} = \sqrt{9 + 1} = \sqrt{10}$.

8. $z = 4 - i$ corresponds to the point (4, –1).



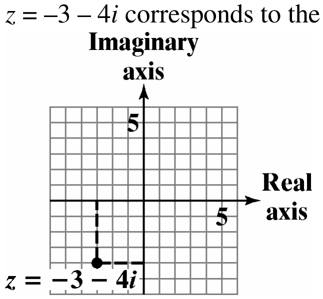
With $a = 4$ and $b = -1$,
$|z| = \sqrt{4^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}$.

9. $z = -3 + 4i$ corresponds to the point (–3, 4).



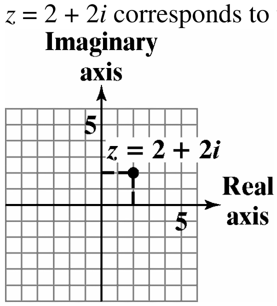
With $a = -3$ and $b = 4$,
$|z| = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$.

10. $z = -3 - 4i$ corresponds to the point (–3, –4).



With $a = -3$ and $b = -4$,
$|z| = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$.

11. $z = 2 + 2i$ corresponds to the point (2, 2).



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \dfrac{b}{a}$, with $a = 2$ and $b = 2$, to find r and θ.

$r = \sqrt{2^2 + 2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

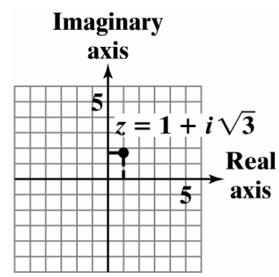
$\tan\theta = \dfrac{2}{2} = 1$

Because $\tan\dfrac{\pi}{4} = 1$ and θ lies in quadrant I,

$\theta = \dfrac{\pi}{4}$.

$z = 2 + 2i = r(\cos\theta + i\sin\theta)$
$= 2\sqrt{2}\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$
or $2\sqrt{2}(\cos 45° + i\sin 45°)$

12. $z = 1 + i\sqrt{3}$ corresponds to the point $\left(1, \sqrt{3}\right)$.



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \dfrac{b}{a}$, with $a = 1$ and $b = \sqrt{3}$, to find r and θ.

$r = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{4} = 2$

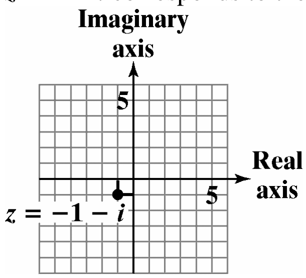
$\tan\theta = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

Because $\tan\dfrac{\pi}{3} = \sqrt{3}$ and θ lies in quadrant I,

$\theta = \dfrac{\pi}{3}$.

$z = 1 + i\sqrt{3} = r(\cos\theta + i\sin\theta) = 2\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$ or $2(\cos 60° + i\sin 60°)$

13. $z = -1 - i$ corresponds to the point $(-1, -1)$.



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \dfrac{b}{a}$, with $a = -1$ and $b = -1$, to find r and θ.

$r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$

$\tan\theta = \dfrac{-1}{-1} = 1$

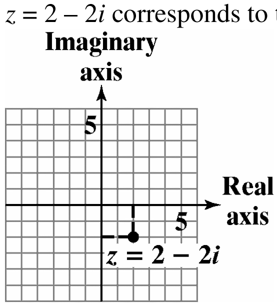
Because $\tan\dfrac{\pi}{4} = 1$ and θ lies in quadrant III, $\theta = \pi + \dfrac{\pi}{4} = \dfrac{5\pi}{4}$.

$z = -1 - i = r(\cos\theta + i\sin\theta)$

$= \sqrt{2}\left(\cos\dfrac{5\pi}{4} + i\sin\dfrac{5\pi}{4}\right)$

or $\sqrt{2}(\cos 225° + i\sin 225°)$

14. $z = 2 - 2i$ corresponds to the point $(2, -2)$.



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \dfrac{b}{a}$, with $a = 2$ and $b = -2$, to find r and θ.

$r = \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \dfrac{-2}{2} = -1$

Because $\tan\dfrac{\pi}{4} = 1$ and θ lies in quadrant IV,

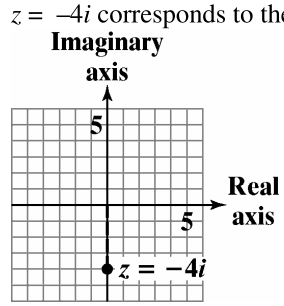
$\theta = 2\pi - \dfrac{\pi}{4} = \dfrac{7\pi}{4}$.

$z = 2 - 2i = r(\cos\theta + i\sin\theta)$

$= 2\sqrt{2}\left(\cos\dfrac{7\pi}{4} + i\sin\dfrac{7\pi}{4}\right)$

or $2\sqrt{2}(\cos 315° + i\sin 315°)$

15. $z = -4i$ corresponds to the point $(0, -4)$.



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \dfrac{b}{a}$, with $a = 0$ and $b = -4$, to find r and θ.

$r = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$

$\tan\theta = \dfrac{-4}{0}$ is undefined.

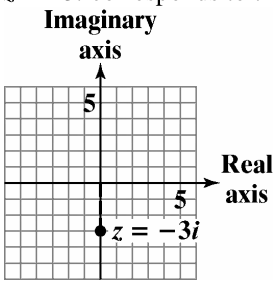
Because $\tan\dfrac{\pi}{2}$ is undefined and θ lies on the negative y-axis, $\theta = \dfrac{\pi}{2} + \pi = \dfrac{3\pi}{2}$.

$z = -4i = r(\cos\theta + i\sin\theta)$

$= 4\left(\cos\dfrac{3\pi}{2} + i\sin\dfrac{3\pi}{2}\right)$

or $4(\cos 270° + i\sin 270°)$

16. $z = -3i$ corresponds to the point (0, –3).



Use $r=\sqrt{a^2+b^2}$ and $\tan\theta=\frac{b}{a}$, with $a = 0$ and $b = -3$, to find r and θ.

$$r=\sqrt{0^2+(-3)^2}=\sqrt{9}=3$$

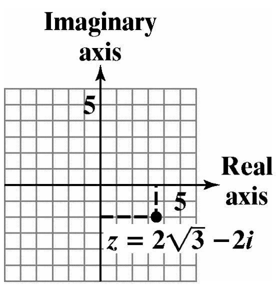
$\tan\theta=\frac{-3}{0}$ is undefined.

Because $\tan\frac{\pi}{2}$ is undefined and θ lies on the negative y-axis, $\theta=\frac{\pi}{2}+\pi=\frac{3\pi}{2}$.

$$z=-3i=r(\cos\theta+i\sin\theta)=3\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)$$

or $3(\cos 270°+i\sin 270°)$

17. $z=2\sqrt{3}-2i$ corresponds to the point $\left(2\sqrt{3},-2\right)$.



Use $r=\sqrt{a^2+b^2}$ and $\tan\theta=\frac{b}{a}$, with $a=2\sqrt{3}$ and $b = -2$, to find r and θ.

$$r=\sqrt{\left(2\sqrt{3}\right)^2+(-2)^2}=\sqrt{12+4}=\sqrt{16}=4$$

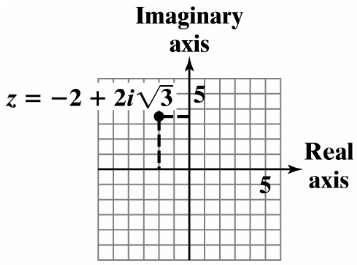
$$\tan\theta=\frac{-2}{2\sqrt{3}}=-\frac{1}{\sqrt{3}}$$

Because $\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$ and θ lies in quadrant IV, $\theta=2\pi-\frac{\pi}{6}=\frac{11\pi}{6}$.

$$z=2\sqrt{3}-2i=r(\cos\theta+i\sin\theta)$$
$$=4\left(\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right)$$

or $4(\cos 330°+i\sin 330°)$

18. $z=-2+2i\sqrt{3}$ corresponds to the point $\left(-2,\, 2\sqrt{3}\right)$.



Use $r=\sqrt{a^2+b^2}$ and $\tan\theta=\dfrac{b}{a}$, with $a=-2$ and $b=2\sqrt{3}$, to find r and θ.

$r=\sqrt{(-2)^2+\left(2\sqrt{3}\right)^2}=\sqrt{4+12}=\sqrt{16}=4$

$\tan\theta=\dfrac{2\sqrt{3}}{-2}=-\sqrt{3}$

Because $\tan\dfrac{\pi}{3}=\sqrt{3}$ and θ lies in

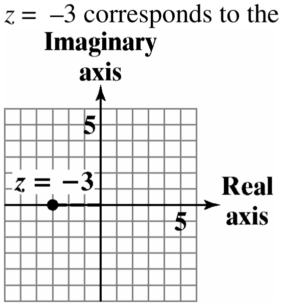
quadrant II, $\theta=\pi-\dfrac{\pi}{3}=\dfrac{2\pi}{3}$.

$z=-2+2i\sqrt{3}=r(\cos\theta+i\sin\theta)$

$=4\left(\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}\right)$

or $4(\cos 120^\circ+i\sin 120^\circ)$

19. $z=-3$ corresponds to the point $(-3, 0)$.



Use $r=\sqrt{a^2+b^2}$ and $\tan\theta=\dfrac{b}{a}$, with

$a=-3$ and $b=0$, to find r and θ.

$r=\sqrt{(-3)^2+0^2}=\sqrt{9}=3$

$\tan\theta=\dfrac{0}{-3}=0$

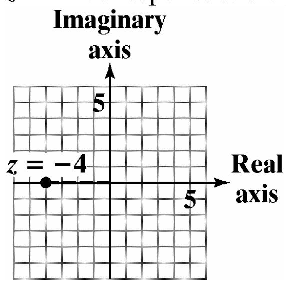
Because $\tan 0=0$ and θ lies on the negative x-axis, $\theta=0+\pi=\pi$.

$z=-3=r(\cos\theta+i\sin\theta)$

$=3\left(\cos\pi+i\sin\pi\right)$

or $3(\cos 180^\circ+i\sin 180^\circ)$

20. $z = -4$ corresponds to the point (–4, 0).



Use $r=\sqrt{a^2+b^2}$ and $\tan\theta=\frac{b}{a}$, with $a = -4$ and $b = 0$, to find r and θ.

$r=\sqrt{(-4)^2+0^2}=\sqrt{16}=4$

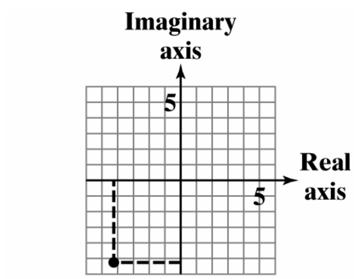
$\tan\theta=\frac{0}{-4}=0$

Because $\tan 0=0$ and θ lies on the negative x-axis, $\theta=0+\pi=\pi$.

$z=-4=r(\cos\theta+i\sin\theta)=4(\cos\pi+i\sin\pi)$

or $4(\cos 180°+i\sin 180°)$

21. $z=-3\sqrt{2}-3i\sqrt{3}$ corresponds to the point $\left(-3\sqrt{2},-3\sqrt{3}\right)$.



$z=-3\sqrt{2}-3i\sqrt{3}$

Use $r=\sqrt{a^2+b^2}$ and $\tan\theta=\frac{b}{a}$, with $a=-3\sqrt{2}$ and $b=-3\sqrt{3}$, to find r and θ.

$r=\sqrt{\left(-3\sqrt{2}\right)^2+\left(-3\sqrt{3}\right)^2}=\sqrt{18+27}$

$=\sqrt{45}=3\sqrt{5}$

$\tan\theta=\frac{-3\sqrt{3}}{-3\sqrt{2}}=\frac{\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{6}}{2}$

Because θ lies in quadrant III,

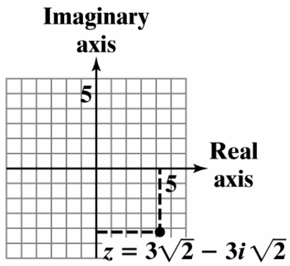
$\theta=180°+\tan^{-1}\left(\frac{\sqrt{6}}{2}\right)\approx 180°+50.8°$

$=230.8°$

$z=-3\sqrt{2}-3i\sqrt{3}=r(\cos\theta+i\sin\theta)$

$\approx 3\sqrt{5}(\cos 230.8°+i\sin 230.8°)$

22. $z = 3\sqrt{2} - 3i\sqrt{2}$ corresponds to the point $\left(3\sqrt{2}, -3\sqrt{2}\right)$.



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$, with $a = 3\sqrt{2}$ and $b = -3\sqrt{2}$, to find r and θ.

$r = \sqrt{\left(3\sqrt{2}\right)^2 + \left(-3\sqrt{2}\right)^2} = \sqrt{18+18} = \sqrt{36} = 6$

$\tan\theta = \frac{-3\sqrt{2}}{3\sqrt{2}} = -1$

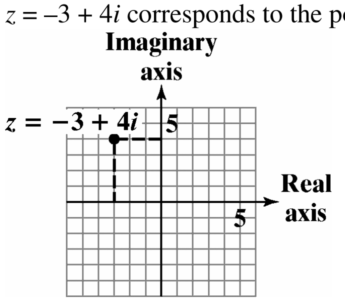
Because $\tan\frac{\pi}{4} = 1$ and θ lies in quadrant IV, $\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$.

$z = 3\sqrt{2} - 3i\sqrt{2} = r(\cos\theta + i\sin\theta)$

$= 6\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

or $6(\cos 315° + i\sin 315°)$

23. $z = -3 + 4i$ corresponds to the point $(-3, 4)$.



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$, with $a = -3$ and $b = 4$, to find r and θ.

$r = \sqrt{(-3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

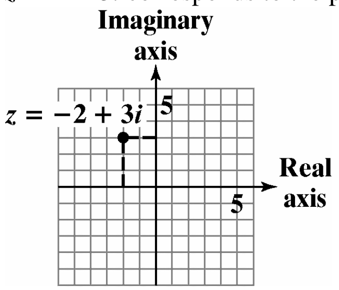
$\tan\theta = \frac{4}{-3} = -\frac{4}{3}$

Because θ lies in quadrant II, $\theta = 180° - \tan^{-1}\left(\frac{4}{3}\right) \approx 180° - 53.1° = 126.9°$.

$z = -3 + 4i = r(\cos\theta + i\sin\theta)$

$\approx 5(\cos 126.9° + i\sin 126.9°)$

24. $z = -2 + 3i$ corresponds to the point $(-2, 3)$.



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$, with $a = -2$ and $b = 3$, to find r and θ.

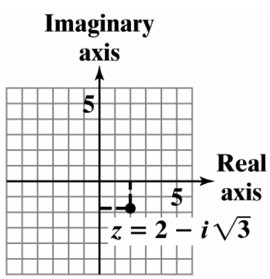
$r = \sqrt{(-2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13}$

$\tan\theta = \frac{3}{-2} = -\frac{3}{2}$

Because θ lies in quadrant II, $\theta = 180° - \tan^{-1}\left(\frac{3}{2}\right) \approx 180° - 56.3° = 123.7°$.

$$\begin{aligned} z = -2 + 3i &= r(\cos\theta + i\sin\theta) \\ &\approx \sqrt{13}(\cos 123.7° + i\sin 123.7°) \end{aligned}$$

25. $z = 2 - i\sqrt{3}$ corresponds to the point $\left(2, -\sqrt{3}\right)$.



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$, with

$a = 2$ and $b = -\sqrt{3}$, to find r and θ.

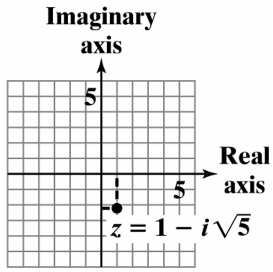
$r = \sqrt{2^2 + \left(-\sqrt{3}\right)^2} = \sqrt{4+3} = \sqrt{7}$

$\tan\theta = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$

Because θ lies in quadrant IV, $\theta = 360° - \tan^{-1}\left(\frac{\sqrt{3}}{2}\right) \approx 360° - 40.9° = 319.1°$

$$\begin{aligned} z = 2 - i\sqrt{3} &= r(\cos\theta + i\sin\theta) \\ &\approx \sqrt{7}(\cos 319.1° + i\sin 319.1°) \end{aligned}$$

26. $z = 1 - i\sqrt{5}$ corresponds to the point $\left(1, -\sqrt{5}\right)$



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \dfrac{b}{a}$, with $a = 1$ and $b = -\sqrt{5}$, to find r and θ.

$r = \sqrt{1^2 + \left(-\sqrt{5}\right)^2} = \sqrt{1+5} = \sqrt{6}$

$\tan\theta = \dfrac{-\sqrt{5}}{1} = -\sqrt{5}$

Because θ lies in quadrant IV, $\theta = 360° - \tan^{-1}\left(\sqrt{5}\right) \approx 360° - 65.9° = 294.1°$

$z = 1 - i\sqrt{5} = r(\cos\theta + i\sin\theta)$
$\approx \sqrt{6}(\cos 294.1° + i\sin 294.1°)$

27. $6(\cos 30° + i\sin 30°) = 6\left(\dfrac{\sqrt{3}}{2} + i\dfrac{1}{2}\right)$
$= 3\sqrt{3} + 3i$

The rectangular form of $z = 6(\cos 30° + i\sin 30°)$ is $z = 3\sqrt{3} + 3i$.

28. $12(\cos 60° + i\sin 60°) = 12\left(\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right) = 6 + 6i\sqrt{3}$

The rectangular form of $z = 12(\cos 60° + i\sin 60°)$ is $z = 6 + 6i\sqrt{3}$.

29. $4(\cos 240° + i\sin 240°) = 4\left(-\dfrac{1}{2} + i\left(-\dfrac{\sqrt{3}}{2}\right)\right)$
$= -2 - 2i\sqrt{3}$

The rectangular form of
$z = 4(\cos 240° + i\sin 240°)$ is $z = -2 - 2i\sqrt{3}$.

30. $10(\cos 210° + i\sin 210°) = 10\left(-\dfrac{\sqrt{3}}{2} + i\left(-\dfrac{1}{2}\right)\right)$
$= -5\sqrt{3} - 5i$

The rectangular form of
$z = 10(\cos 210° + i\sin 210°)$ is $z = -5\sqrt{3} - 5i$.

31. $8\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)=8\left(\frac{\sqrt{2}}{2}+i\left(-\frac{\sqrt{2}}{2}\right)\right)$

$=4\sqrt{2}-4i\sqrt{2}$

The rectangular form of
$8\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)$ is $z=4\sqrt{2}-4i\sqrt{2}$.

32. $4\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)=4\left(-\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)$

$=-2\sqrt{3}+2i$

The rectangular form of
$z=4\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$ is $z=-2\sqrt{3}+2i$.

33. $5\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)=5\left(0+i(1)\right)$

$=5i$

The rectangular form of
$z=5\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$ is $z=5i$.

34. $7\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)=7\left(0+i(-1)\right)=-7i$

The rectangular form of
$z=7\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)$ is $z=-7i$.

35. $20\left(\cos 205°+i\sin 205°\right)$

$\approx 20\left(-0.91+i(-0.42)\right)=-18.2-8.4i$

The rectangular form of
$z=20\left(\cos 205°+i\sin 205°\right)$ is $z\approx -18.2-8.5i$.

36. $30\left(\cos 2.3+i\sin 2.3\right)\approx -20.0+22.4i$

The rectangular form of
$z=30\left(\cos 2.3+i\sin 2.3\right)$ is $z\approx -20.0+22.4i$

37. z_1z_2

$=\left[6(\cos 20°+i\sin 20°)\right]\left[5(\cos 50°+i\sin 50°)\right]$

$=(6\cdot 5)\left[\cos(20°+50°)+i\sin(20°+50°)\right]$

$=30(\cos 70°+i\sin 70°)$

38. $z_1 z_2$

$= \left[4(\cos 15° + i \sin 15°)\right]\left[7(\cos 25° + i \sin 25°)\right]$

$= (4 \cdot 7)\left[\cos(15° + 25°) + i \sin(15° + 25°)\right]$

$= 28(\cos 40° + i \sin 40°)$

39. $z_1 z_2 = \left[3\left(\cos\frac{\pi}{5} + i \sin\frac{\pi}{5}\right)\right]\left[4\left(\cos\frac{\pi}{10} + i \sin\frac{\pi}{10}\right)\right] = (3 \cdot 4)\left[\cos\left(\frac{\pi}{5} + \frac{\pi}{10}\right) + i \sin\left(\frac{\pi}{5} + \frac{\pi}{10}\right)\right]$

$= 12\left(\cos\frac{3\pi}{10} + i \sin\frac{3\pi}{10}\right)$

40. $z_1 z_2 = \left[3\left(\cos\frac{5\pi}{8} + i \sin\frac{5\pi}{8}\right)\right]\left[10\left(\cos\frac{\pi}{16} + i \sin\frac{\pi}{16}\right)\right] = (3 \cdot 10)\left[\cos\left(\frac{5\pi}{8} + \frac{\pi}{16}\right) + i \sin\left(\frac{5\pi}{8} + \frac{\pi}{16}\right)\right]$

$= 30\left(\cos\frac{11\pi}{16} + i \sin\frac{11\pi}{16}\right)$

41. $z_1 z_2 = \left[\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right]\left[\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right] = \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \cos\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) + i \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$= \cos\frac{7\pi}{12} + i \sin\frac{7\pi}{12}$

42. $z_1 z_2 = \left[\cos\frac{\pi}{6} + i \sin\frac{\pi}{6}\right]\left[\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right] = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \cos\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) + i \sin\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right)$

$= \cos\frac{5\pi}{12} + i \sin\frac{5\pi}{12}$

43. Begin by converting $z_1 = 1 + i$ and $z_2 = -1 + i$ to polar form.

For z_1 : $a = 1$ and $b = 1$

$r = \sqrt{a^2 + b^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\tan\theta = \frac{b}{a} = \frac{1}{1} = 1$ and $\theta = \frac{\pi}{4}$.

$z_1 = r(\cos\theta + i \sin\theta) = \sqrt{2}\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$

For z_2 : $a = -1$ and $b = 1$

$r = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$\tan\theta = \frac{b}{a} = \frac{1}{-1} = -1$

Because $\tan\frac{\pi}{4} = 1$ and θ lies in quadrant II, $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

$z_2 = r(\cos\theta + i \sin\theta) = \sqrt{2}\left(\cos\frac{3\pi}{4} + i \sin\frac{3\pi}{4}\right)$

Now, find the product.

$z_1 z_2 = (1 + i)(-1 + i)$

$= \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)\right]\left[\sqrt{2}\left(\cos\frac{3\pi}{4} + i \sin\frac{3\pi}{4}\right)\right] = \left(\sqrt{2} \cdot \sqrt{2}\right)\left[\cos\left(\frac{\pi}{4} + \frac{3\pi}{4}\right) + i \sin\left(\frac{\pi}{4} + \frac{3\pi}{4}\right)\right]$

$= 2\left(\cos\pi + i \sin\pi\right)$

44. Begin by converting $z_1 = 1 + i$ and $z_2 = 2 + 2i$ to polar form.

For z_1 : $a = 1$ and $b = 1$

$r = \sqrt{a^2 + b^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\tan\theta = \frac{b}{a} = \frac{1}{1} = 1$ and $\theta = \frac{\pi}{4}$

$z_1 = r(\cos\theta + i\sin\theta) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

For z_2 : $a = 2$ and $b = 2$

$r = \sqrt{a^2 + b^2} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \frac{b}{a} = \frac{2}{2} = 1$ and $\theta = \frac{\pi}{4}$

$z_2 = r(\cos\theta + i\sin\theta) = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Now, find the product.

$$z_1 z_2 = (1+i)(2+2i)$$
$$= \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]\left[2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right] = \left(\sqrt{2}\cdot 2\sqrt{2}\right)\left[\cos\left(\frac{\pi}{4} + \frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4} + \frac{\pi}{4}\right)\right]$$
$$= 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

45. $$\frac{z_1}{z_2} = \frac{20(\cos 75° + i\sin 75°)}{4(\cos 25° + i\sin 25°)} = \frac{20}{4}\left[\cos(75° - 25°) + i\sin(75° - 25°)\right]$$
$$= 5(\cos 50° + i\sin 50°)$$

46. $$\frac{z_1}{z_2} = \frac{50(\cos 80° + i\sin 80°)}{10(\cos 20° + i\sin 20°)} = \frac{50}{10}\left[\cos(80° - 20°) + i\sin(80° - 20°)\right]$$
$$= 5(\cos 60° + i\sin 60°)$$

47. $$\frac{z_1}{z_2} = \frac{3\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)}{4\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)} = \frac{3}{4}\left[\cos\left(\frac{\pi}{5} - \frac{\pi}{10}\right) + i\sin\left(\frac{\pi}{5} - \frac{\pi}{10}\right)\right] = \frac{3}{4}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$$

48. $$\frac{z_1}{z_2} = \frac{3\left(\cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18}\right)}{10\left(\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}\right)} = \frac{3}{10}\left[\cos\left(\frac{5\pi}{18} - \frac{\pi}{16}\right) + i\sin\left(\frac{5\pi}{18} - \frac{\pi}{16}\right)\right] = \frac{3}{10}\left[\cos\left(\frac{40\pi}{144} - \frac{9\pi}{144}\right) + i\sin\left(\frac{40\pi}{144} - \frac{9\pi}{144}\right)\right]$$
$$= \frac{3}{10}\left(\cos\frac{31\pi}{144} + i\sin\frac{31\pi}{144}\right)$$

49. $$\frac{z_1}{z_2} = \frac{\cos 80° + i\sin 80°}{\cos 200° + i\sin 200°} = \cos(80° - 200°) + i\sin(80° - 200°) = \cos(-120°) + i\sin(-120°)$$
$$= \cos 240° + i\sin 240°$$

50. $\frac{z_1}{z_2}=\frac{\cos 70°+i\sin 70°}{\cos 230°+i\sin 230°}=\cos(70°-230°)+i\sin(70°-230°)=\cos(-160°)+i\sin(-160°)$
$=\cos 200°+i\sin 200°$

51. Begin by converting $z_1=2+2i$ and $z_2=1+i$ to polar form.
For z_1 : $a=2$ and $b=2$
$r=\sqrt{a^2+b^2}=\sqrt{2^2+2^2}=\sqrt{8}=2\sqrt{2}$
$\tan\theta=\frac{b}{a}=\frac{2}{2}=1$ and $\theta=\frac{\pi}{4}$
$z_1=r(\cos\theta+i\sin\theta)=2\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$
For z_2 : $a=1$ and $b=1$
$r=\sqrt{a^2+b^2}=\sqrt{1^2+1^2}=\sqrt{2}$
$\tan\theta=\frac{b}{a}=\frac{1}{1}=1$ and $\theta=\frac{\pi}{4}$
$z_2=r(\cos\theta+i\sin\theta)=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$.
Now, find the quotient.
$$\frac{z_1}{z_2}=\frac{2+2i}{1+i}=\frac{2\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)}{\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)}=2\left[\cos\left(\frac{\pi}{4}-\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}-\frac{\pi}{4}\right)\right]=2(\cos 0+i\sin 0)$$

52. Begin by converting $z_1=2-2i$ and $z_2=1-i$ to polar form. For z_1 : $a=2$ and $b=-2$
$r=\sqrt{a^2+b^2}=\sqrt{2^2+(-2)^2}=\sqrt{8}=2\sqrt{2}$
$\tan\theta=\frac{b}{a}=\frac{-2}{2}=-1$
Because $\tan\frac{\pi}{4}=1$ and θ lies in quadrant IV,
$\theta=2\pi-\frac{\pi}{4}=\frac{7\pi}{4}$.
$z_1=r(\cos\theta+i\sin\theta)=2\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)$
For z_2 : $a=1$ and $b=-1$
$r=\sqrt{a^2+b^2}=\sqrt{1^2+(-1)^2}=\sqrt{2}$
$\tan\theta=\frac{b}{a}=\frac{-1}{1}=-1$

Because $\tan\frac{\pi}{4} = 1$ and θ lies in quadrant IV,

$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$.

$z_2 = r(\cos\theta + i\sin\theta) = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

Now, find the quotient.

$$\frac{z_1}{z_2} = \frac{2-2i}{1-i} = \frac{2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)}{\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)} = 2\left[\cos\left(\frac{7\pi}{4} - \frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4} - \frac{7\pi}{4}\right)\right]$$

$$= 2(\cos 0 + i\sin 0)$$

53. $\left[4(\cos 15° + i\sin 15°)\right]^3 = (4)^3\left[\cos(3\cdot 15°) + i\sin(3\cdot 15°)\right] = 64(\cos 45° + i\sin 45°) = 64\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

$= 32\sqrt{2} + 32i\sqrt{2}$

54. $\left[2(\cos 10° + i\sin 10°)\right]^3 = (2)^3\left[\cos(3\cdot 10°) + i\sin(3\cdot 10°)\right] = 8(\cos 30° + i\sin 30°) = 8\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = 4\sqrt{3} + 4i$

55. $\left[2(\cos 80° + i\sin 80°)\right]^3 = (2)^3\left[\cos(3\cdot 80°) + i\sin(3\cdot 80°)\right] = 8(\cos 240° + i\sin 240°)$

$= 8\left(-\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) = -4 - 4i\sqrt{3}$

56. $\left[2(\cos 40° + i\sin 40°)\right]^3 = (2)^3\left[\cos(3\cdot 40°) + i\sin(3\cdot 40°)\right] = 8(\cos 120° + i\sin 120°)$

$= 8\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -4 + 4i\sqrt{3}$

57. $\left[\frac{1}{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\right]^6 = \left(\frac{1}{2}\right)^6\left[\cos\left(6\cdot\frac{\pi}{12}\right) + i\sin\left(6\cdot\frac{\pi}{12}\right)\right] = \frac{1}{64}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = \frac{1}{64}(0+i) = \frac{1}{64}i$

58. $\left[\frac{1}{2}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)\right]^5 = \left(\frac{1}{2}\right)^5\left[\cos\left(5\cdot\frac{\pi}{10}\right) + i\sin\left(5\cdot\frac{\pi}{10}\right)\right] = \frac{1}{32}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$= \frac{1}{32}(0+i) = \frac{1}{32}i$

59. $\left[\sqrt{2}\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)\right]^4 = \left(\sqrt{2}\right)^4\left[\cos\left(4\cdot\frac{5\pi}{6}\right) + i\sin\left(4\cdot\frac{5\pi}{6}\right)\right]$

$= 4\left(\cos\frac{20\pi}{6} + i\sin\frac{20\pi}{6}\right) = 4\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$= 4\left(-\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) = -2 - 2i\sqrt{3}$

60. $\left[\sqrt{3}\left(\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}\right)\right]^6=\left(\sqrt{3}\right)^6\left[\cos\left(6\cdot\frac{5\pi}{18}\right)+i\sin\left(6\cdot\frac{5\pi}{18}\right)\right]=27\left(\cos\frac{30\pi}{18}+i\sin\frac{30\pi}{18}\right)$

$$=27\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$$

$$=27\left(\frac{1}{2}+i\left(-\frac{\sqrt{3}}{2}\right)\right)=\frac{27}{2}-\frac{27\sqrt{3}}{2}i$$

61. Write $1+i$ in $r(\cos\theta+i\sin\theta)$ form.

$r=\sqrt{a^2+b^2}=\sqrt{1^2+1^2}=\sqrt{2}$

$\tan\theta=\frac{b}{a}=\frac{1}{1}=1$ and $\theta=\frac{\pi}{4}$

$1+i=r(\cos\theta+i\sin\theta)=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

Use DeMoivre's Theorem to raise $1+i$ to the fifth power.

$(1+i)^5=\left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^5=\left(\sqrt{2}\right)^5\left[\cos\left(5\cdot\frac{\pi}{4}\right)+i\sin\left(5\cdot\frac{\pi}{4}\right)\right]$

$$=4\sqrt{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)=4\sqrt{2}\left(-\frac{\sqrt{2}}{2}+i\left(-\frac{\sqrt{2}}{2}\right)\right)=-4-4i$$

62. Write $1-i$ in $r(\cos\theta+i\sin\theta)$ form.

$r=\sqrt{a^2+b^2}=\sqrt{1^2+(-1)^2}$

$=\sqrt{2}$

$\tan\theta=\frac{b}{a}=\frac{-1}{1}=-1$

Because $\tan\frac{\pi}{4}=1$ and θ lies in quadrant IV,

$\theta=2\pi-\frac{\pi}{4}=\frac{7\pi}{4}$.

$1-i=r(\cos\theta+i\sin\theta)$

$=\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)$

Use DeMoivre's Theorem to raise $1-i$ to the fifth power.

$(1-i)^5=\left[\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)\right]^5=\left(\sqrt{2}\right)^5\left[\cos\left(5\cdot\frac{7\pi}{4}\right)+i\sin\left(5\cdot\frac{7\pi}{4}\right)\right]=4\sqrt{2}\left(\cos\frac{35\pi}{4}+i\sin\frac{35\pi}{4}\right)$

$$=4\sqrt{2}\left(-\frac{\sqrt{2}}{2}+i\left(\frac{\sqrt{2}}{2}\right)\right)$$

$$=-4+4i$$

63. Write $\sqrt{3}-i$ in $r(\cos\theta+i\sin\theta)$ form.

$r=\sqrt{a^2+b^2}=\sqrt{\left(\sqrt{3}\right)^2+\left(-1\right)^2}=\sqrt{4}=2$

$\tan\theta=\frac{b}{a}=\frac{-1}{\sqrt{3}}=-\frac{1}{\sqrt{3}}$

Because $\tan 30°=\frac{1}{\sqrt{3}}$ and θ lies in quadrant IV, $\theta=360°-30°=330°$.

$\sqrt{3}-i=r(\cos\theta+i\sin\theta)=2(\cos 330°+i\sin 330°)$

Use DeMoivre's Theorem to raise $\sqrt{3}-i$ to the sixth power.

$$\begin{aligned}(\sqrt{3}-i)^6&=\left[2(\cos 330°+i\sin 330°)\right]^6=(2)^6\left[\cos(6\cdot 330°)+i\sin(6\cdot 330°)\right]\\&=64(\cos 1980°+i\sin 1980°)=64(\cos 180°+i\sin 180°)\\&=64(-1+0i)=-64\end{aligned}$$

64. Write $\sqrt{2}-i$ in $r(\cos\theta+i\sin\theta)$ form.

$r=\sqrt{a^2+b^2}=\sqrt{\left(\sqrt{2}\right)^2+(-1)^2}=\sqrt{3}$

$\tan\theta=\frac{b}{a}=\frac{-1}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$

Because θ lies in quadrant IV, $\theta=360°-\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)\approx 360°-35.3°=324.7°$.

$\sqrt{2}-i=r(\cos\theta+i\sin\theta)=\sqrt{3}(\cos 324.7°+i\sin 324.7°)$

Use DeMoivre's Theorem to raise $\sqrt{2}-i$ to the fourth power.

$$\begin{aligned}\left(\sqrt{2}-i\right)^4&\approx\left[\sqrt{3}(\cos 324.7°+i\sin 324.7°)\right]^4\\&=\left(\sqrt{3}\right)^4\left[\cos(4\cdot 324.7°)+i\sin(4\cdot 324.7°)\right]\\&=9(\cos 1298.8°+i\sin 1298.8°)\\&\approx -7-5.7i\end{aligned}$$

65. $9(\cos 30°+i\sin 30°)$

$$z_k=\sqrt[2]{9}\left[\cos\left(\frac{30°+360°k}{2}\right)+i\sin\left(\frac{30°+360°k}{2}\right)\right],\ k=0,1$$

$$z_0=\sqrt{9}\left[\cos\left(\frac{30°+360°\cdot 0}{2}\right)+i\sin\left(\frac{30°+360°\cdot 0}{2}\right)\right]=\sqrt{9}\left[\cos\left(\frac{30°}{2}\right)+i\sin\left(\frac{30°}{2}\right)\right]=3(\cos 15°+i\sin 15°)$$

$$z_1=\sqrt{9}\left[\cos\left(\frac{30°+360°\cdot 1}{2}\right)+i\sin\left(\frac{30°+360°\cdot 1}{2}\right)\right]=\sqrt{9}\left[\cos\left(\frac{390°}{2}\right)+i\sin\left(\frac{390°}{2}\right)\right]=3(\cos 195°+i\sin 195°)$$

66. $25(\cos 210° + i\sin 210°)$

$$z_k = \sqrt[2]{25}\left[\cos\left(\frac{210° + 360°k}{2}\right) + i\sin\left(\frac{210° + 360°k}{2}\right)\right],\ k = 0, 1$$

$$z_0 = \sqrt{25}\left[\cos\left(\frac{210° + 360°\cdot 0}{2}\right) + i\sin\left(\frac{210° + 360°\cdot 0}{2}\right)\right] = \sqrt{25}\left[\cos\left(\frac{210°}{2}\right) + i\sin\left(\frac{210°}{2}\right)\right]$$

$$= 5(\cos 105° + i\sin 105°)$$

$$z_1 = \sqrt{25}\left[\cos\left(\frac{210° + 360°\cdot 1}{2}\right) + i\sin\left(\frac{210° + 360°\cdot 1}{2}\right)\right] = \sqrt{25}\left[\cos\left(\frac{570°}{2}\right) + i\sin\left(\frac{570°}{2}\right)\right]$$

$$= 5(\cos 285° + i\sin 285°)$$

67. $8(\cos 210° + i\sin 210°)$

$$z_k = \sqrt[3]{8}\left[\cos\left(\frac{210° + 360°k}{3}\right) + i\sin\left(\frac{210° + 360°k}{3}\right)\right],\ k = 0, 1, 2$$

$$z_0 = \sqrt[3]{8}\left[\cos\left(\frac{210° + 360°\cdot 0}{3}\right) + i\sin\left(\frac{210° + 360°\cdot 0}{3}\right)\right] = \sqrt[3]{8}\left[\cos\left(\frac{210°}{3}\right) + i\sin\left(\frac{210°}{3}\right)\right]$$

$$= 2(\cos 70° + i\sin 70°)$$

$$z_1 = \sqrt[3]{8}\left[\cos\left(\frac{210° + 360°\cdot 1}{3}\right) + i\sin\left(\frac{210° + 360°\cdot 1}{3}\right)\right] = \sqrt[3]{8}\left[\cos\left(\frac{570°}{3}\right) + i\sin\left(\frac{570°}{3}\right)\right]$$

$$= 2(\cos 190° + i\sin 190°)$$

$$z_2 = \sqrt[3]{8}\left[\cos\left(\frac{210° + 360°\cdot 2}{3}\right) + i\sin\left(\frac{210° + 360°\cdot 2}{3}\right)\right] = \sqrt[3]{8}\left[\cos\left(\frac{930°}{3}\right) + i\sin\left(\frac{930°}{3}\right)\right]$$

$$= 2(\cos 310° + i\sin 310°)$$

68. $27(\cos 306° + i\sin 306°)$

$$z_k = \sqrt[3]{27}\left[\cos\left(\frac{306° + 360°k}{3}\right) + i\sin\left(\frac{306° + 360°k}{3}\right)\right],\ k = 0, 1, 2$$

$$z_0 = \sqrt[3]{27}\left[\cos\left(\frac{306° + 360°\cdot 0}{3}\right) + i\sin\left(\frac{306° + 360°\cdot 0}{3}\right)\right] = \sqrt[3]{27}\left[\cos\left(\frac{306°}{3}\right) + i\sin\left(\frac{306°}{3}\right)\right]$$

$$= 3(\cos 102° + i\sin 102°)$$

$$z_1 = \sqrt[3]{27}\left[\cos\left(\frac{306° + 360°\cdot 1}{3}\right) + i\sin\left(\frac{306° + 360°\cdot 1}{3}\right)\right] = \sqrt[3]{27}\left[\cos\left(\frac{666°}{3}\right) + i\sin\left(\frac{666°}{3}\right)\right]$$

$$= 3(\cos 222° + i\sin 222°)$$

$$z_2 = \sqrt[3]{27}\left[\cos\left(\frac{306° + 360°\cdot 2}{3}\right) + i\sin\left(\frac{306° + 360°\cdot 2}{3}\right)\right] = \sqrt[3]{27}\left[\cos\left(\frac{1026°}{3}\right) + i\sin\left(\frac{1026°}{3}\right)\right]$$

$$= 3(\cos 342° + i\sin 342°)$$

69. $81\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)$

$$z_k=\sqrt[4]{81}\left[\cos\left(\frac{\frac{4\pi}{3}+2\pi k}{4}\right)+i\sin\left(\frac{\frac{4\pi}{3}+2\pi k}{4}\right)\right],\ k=0,1,2,3$$

$$z_0=\sqrt[4]{81}\left[\cos\left(\frac{\frac{4\pi}{3}+2\pi\cdot 0}{4}\right)+i\sin\left(\frac{\frac{4\pi}{3}+2\pi\cdot 0}{4}\right)\right]=\sqrt[4]{81}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)=3\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=\frac{3}{2}+\frac{3\sqrt{3}}{2}i$$

$$z_1=\sqrt[4]{81}\left[\cos\left(\frac{\frac{4\pi}{3}+2\pi\cdot 1}{4}\right)+i\sin\left(\frac{\frac{4\pi}{3}+2\pi\cdot 1}{4}\right)\right]=\sqrt[4]{81}\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)=3\left(-\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)=-\frac{3\sqrt{3}}{2}+\frac{3}{2}i$$

$$z_2=\sqrt[4]{81}\left[\cos\left(\frac{\frac{4\pi}{3}+2\pi\cdot 2}{4}\right)+i\sin\left(\frac{\frac{4\pi}{3}+2\pi\cdot 2}{4}\right)\right]=\sqrt[4]{81}\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)$$

$$=3\left(-\frac{1}{2}+i\left(-\frac{\sqrt{3}}{2}\right)\right)=-\frac{3}{2}-\frac{3\sqrt{3}}{2}i$$

$$z_3=\sqrt[4]{81}\left[\cos\left(\frac{\frac{4\pi}{3}+2\pi\cdot 3}{4}\right)+i\sin\left(\frac{\frac{4\pi}{3}+2\pi\cdot 3}{4}\right)\right]=\sqrt[4]{81}\left(\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right)$$

$$=3\left(\frac{\sqrt{3}}{2}+i\left(-\frac{1}{2}\right)\right)=\frac{3\sqrt{3}}{2}-\frac{3}{2}i$$

70. $32\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$

$$z_k=\sqrt[5]{32}\left[\cos\left(\frac{\frac{5\pi}{3}+2\pi k}{5}\right)+i\sin\left(\frac{\frac{5\pi}{3}+2\pi k}{5}\right)\right],\ k=0,1,2,3,4$$

$$z_0=\sqrt[5]{32}\left[\cos\left(\frac{\frac{5\pi}{3}+2\pi\cdot 0}{5}\right)+i\sin\left(\frac{\frac{5\pi}{3}+2\pi\cdot 0}{5}\right)\right]=\sqrt[5]{32}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)=2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=1+i\sqrt{3}$$

$$z_1=\sqrt[5]{32}\left[\cos\left(\frac{\frac{5\pi}{3}+2\pi\cdot 1}{5}\right)+i\sin\left(\frac{\frac{5\pi}{3}+2\pi\cdot 1}{5}\right)\right]=\sqrt[5]{32}\left(\cos\frac{11\pi}{15}+i\sin\frac{11\pi}{15}\right)$$

$$\approx 2\left(-0.67+i(0.74)\right)\approx -1.3+1.5i$$

$$z_2=\sqrt[5]{32}\left[\cos\left(\frac{\frac{5\pi}{3}+2\pi\cdot 2}{5}\right)+i\sin\left(\frac{\frac{5\pi}{3}+2\pi\cdot 2}{5}\right)\right]=\sqrt[5]{32}\left(\cos\frac{17\pi}{15}+i\sin\frac{17\pi}{15}\right)$$

$$\approx 2(-0.91+i(-0.41))\approx -1.8-0.8i$$

$$z_3=\sqrt[5]{32}\left[\cos\left(\frac{\frac{5\pi}{3}+2\pi\cdot 3}{5}\right)+i\sin\left(\frac{\frac{5\pi}{3}+2\pi\cdot 3}{5}\right)\right]=\sqrt[5]{32}\left(\cos\frac{23\pi}{15}+i\sin\frac{23\pi}{15}\right)$$

$$\approx 2(0.10+i(-0.99))\approx 0.2-2.0i$$

$$z_4=\sqrt[5]{32}\left[\cos\left(\frac{\frac{5\pi}{3}+2\pi\cdot 4}{5}\right)+i\sin\left(\frac{\frac{5\pi}{3}+2\pi\cdot 4}{5}\right)\right]=\sqrt[5]{32}\left(\cos\frac{29\pi}{15}+i\sin\frac{29\pi}{15}\right)$$

$$\approx 2(0.98+i(-0.21))\approx 2.0-0.4i$$

71. $32 = 32(\cos 0° + i\sin 0°)$

$$z_k = \sqrt[5]{32}\left[\cos\left(\frac{0°+360°k}{5}\right) + i\sin\left(\frac{0°+360°k}{5}\right)\right],\ k = 0,1,2,3,4$$

$$z_0 = \sqrt[5]{32}\left[\cos\left(\frac{0°+360°\cdot 0}{5}\right) + i\sin\left(\frac{0°+360°\cdot 0}{5}\right)\right] = \sqrt[5]{32}(\cos 0° + i\sin 0°) = 2(1+0i) = 2$$

$$z_1 = \sqrt[5]{32}\left[\cos\left(\frac{0°+360°\cdot 1}{5}\right) + i\sin\left(\frac{0°+360°\cdot 1}{5}\right)\right] = \sqrt[5]{32}(\cos 72° + i\sin 72°) \approx 2(0.31 + i(0.95))$$

$$\approx 0.6 + 1.9i$$

$$z_2 = \sqrt[5]{32}\left[\cos\left(\frac{0°+360°\cdot 2}{5}\right) + i\sin\left(\frac{0°+360°\cdot 2}{5}\right)\right] = \sqrt[5]{32}(\cos 144° + i\sin 144°) \approx 2(-0.81 + i(0.59))$$

$$\approx -1.6 + 1.2i$$

$$z_3 = \sqrt[5]{32}\left[\cos\left(\frac{0°+360°\cdot 3}{5}\right) + i\sin\left(\frac{0°+360°\cdot 3}{5}\right)\right] = \sqrt[5]{32}(\cos 216° + i\sin 216°) \approx 2(-0.81 + i(-0.59))$$

$$\approx -1.6 - 1.2i$$

$$z_4 = \sqrt[5]{32}\left[\cos\left(\frac{0°+360°\cdot 4}{5}\right) + i\sin\left(\frac{0°+360°\cdot 4}{5}\right)\right] = \sqrt[5]{32}(\cos 288° + i\sin 288°) \approx 2(0.31 + i(-0.95))$$

$$\approx 0.6 - 1.9i$$

72. $64 = 64(\cos 0° + i\sin 0°)$

$$z_k = \sqrt[6]{64}\left[\cos\left(\frac{0°+360°\cdot k}{6}\right) + i\sin\left(\frac{0°+360°\cdot k}{6}\right)\right],\ k = 0,1,2,3,4,5$$

$$z_0 = \sqrt[6]{64}\left[\cos\left(\frac{0°+360°\cdot 0}{6}\right) + i\sin\left(\frac{0°+360°\cdot 0}{6}\right)\right] = \sqrt[6]{64}(\cos 0° + i\sin 0°) = 2(1+0i) = 2$$

$$z_1 = \sqrt[6]{64}\left[\cos\left(\frac{0°+360°\cdot 1}{6}\right) + i\sin\left(\frac{0°+360°\cdot 1}{6}\right)\right] = \sqrt[6]{64}(\cos 60° + i\sin 60°) = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 1 + i\sqrt{3}$$

$$z_2 = \sqrt[6]{64}\left[\cos\left(\frac{0°+360°\cdot 2}{6}\right) + i\sin\left(\frac{0°+360°\cdot 2}{6}\right)\right] = \sqrt[6]{64}(\cos 120° + i\sin 120°) = 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -1 + i\sqrt{3}$$

$$z_3 = \sqrt[6]{64}\left[\cos\left(\frac{0°+360°\cdot 3}{6}\right) + i\sin\left(\frac{0°+360°\cdot 3}{6}\right)\right] = \sqrt[6]{64}(\cos 180° + i\sin 180°) = 2(-1+0i) = -2$$

$$z_4 = \sqrt[6]{64}\left[\cos\left(\frac{0°+360°\cdot 4}{6}\right) + i\sin\left(\frac{0°+360°\cdot 4}{6}\right)\right] = \sqrt[6]{64}(\cos 240° + i\sin 240°) = 2\left(-\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) = -1 - i\sqrt{3}$$

$$z_5 = \sqrt[6]{64}\left[\cos\left(\frac{0°+360°\cdot 5}{6}\right) + i\sin\left(\frac{0°+360°\cdot 5}{6}\right)\right] = \sqrt[6]{64}(\cos 300° + i\sin 300°) = 2\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) = 1 - i\sqrt{3}$$

73. $1 = 1(\cos 0° + i\sin 0°)$

$$z_k = \sqrt[3]{1}\left[\cos\left(\frac{0° + 360°k}{3}\right) + i\sin\left(\frac{0° + 360°k}{3}\right)\right],\ k = 0, 1, 2$$

$$z_0 = \sqrt[3]{1}\left[\cos\left(\frac{0° + 360°\cdot 0}{3}\right) + i\sin\left(\frac{0° + 360°\cdot 0}{3}\right)\right] = \sqrt[3]{1}\left(\cos 0° + i\sin 0°\right) = 1(1 + 0i) = 1$$

$$z_1 = \sqrt[3]{1}\left[\cos\left(\frac{0° + 360°\cdot 1}{3}\right) + i\sin\left(\frac{0° + 360°\cdot 1}{3}\right)\right] = \sqrt[3]{1}\left(\cos 120° + i\sin 120°\right) = 1\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z_2 = \sqrt[3]{1}\left[\cos\left(\frac{0° + 360°\cdot 2}{3}\right) + i\sin\left(\frac{0° + 360°\cdot 2}{3}\right)\right] = \sqrt[3]{1}\left(\cos 240° + i\sin 240°\right) = 1\left(-\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$$

$$= -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

74. $i = 1(\cos 90° + i\sin 90°)$

$$z_k = \sqrt[3]{1}\left[\cos\left(\frac{90° + 360°k}{3}\right) + i\sin\left(\frac{90° + 360°k}{3}\right)\right],\ k = 0, 1, 2$$

$$z_0 = \sqrt[3]{1}\left[\cos\left(\frac{90° + 360°\cdot 0}{3}\right) + i\sin\left(\frac{90° + 360°\cdot 0}{3}\right)\right] = \sqrt[3]{1}\left(\cos 30° + i\sin 30°\right) = 1\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$

$$z_1 = \sqrt[3]{1}\left[\cos\left(\frac{90° + 360°\cdot 1}{3}\right) + i\sin\left(\frac{90° + 360°\cdot 1}{3}\right)\right] = \sqrt[3]{1}\left(\cos 150° + i\sin 150°\right) = 1\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$$

$$z_2 = \sqrt[3]{1}\left[\cos\left(\frac{90° + 360°\cdot 2}{3}\right) + i\sin\left(\frac{90° + 360°\cdot 2}{3}\right)\right] = \sqrt[3]{1}\left(\cos 270° + i\sin 270°\right) = 1(0 + i(-1)) = -i$$

75. $1 + i = \sqrt{2}\left(\cos 45° + i\sin 45°\right)$

$$z_k = \sqrt[4]{\sqrt{2}}\left[\cos\left(\frac{45° + 360°k}{4}\right) + i\sin\left(\frac{45° + 360°k}{4}\right)\right],\ k = 0, 1, 2, 3$$

$$z_0 = \sqrt[4]{\sqrt{2}}\left[\cos\left(\frac{45° + 360°\cdot 0}{4}\right) + i\sin\left(\frac{45° + 360°\cdot 0}{4}\right)\right] = \sqrt[4]{\sqrt{2}}\left(\cos 11.25° + i\sin 11.25°\right) \approx 1.1 + 0.2i$$

$$z_1 = \sqrt[4]{\sqrt{2}}\left[\cos\left(\frac{45° + 360°\cdot 1}{4}\right) + i\sin\left(\frac{45° + 360°\cdot 1}{4}\right)\right] = \sqrt[4]{\sqrt{2}}\left(\cos 101.25° + i\sin 101.25°\right) \approx -0.2 + 1.1i$$

$$z_2 = \sqrt[4]{\sqrt{2}}\left[\cos\left(\frac{45° + 360°\cdot 2}{4}\right) + i\sin\left(\frac{45° + 360°\cdot 2}{4}\right)\right] = \sqrt[4]{\sqrt{2}}\left(\cos 191.25° + i\sin 191.25°\right) \approx -1.1 - 0.2i$$

$$z_3 = \sqrt[4]{\sqrt{2}}\left[\cos\left(\frac{45° + 360°\cdot 3}{4}\right) + i\sin\left(\frac{45° + 360°\cdot 3}{4}\right)\right] = \sqrt[4]{\sqrt{2}}\left(\cos 281.25° + i\sin 281.25°\right) \approx 0.2 - 1.1i$$

76. $-1+i=\sqrt{2}\left(\cos 135°+i\sin 135°\right)$

$$z_k=\sqrt[5]{\sqrt{2}}\left[\cos\left(\frac{135°+360°k}{5}\right)+i\sin\left(\frac{135°+360°k}{5}\right)\right],\ k=0,1,2,3,4$$

$$z_0=\sqrt[5]{\sqrt{2}}\left[\cos\left(\frac{135°+360°\cdot 0}{5}\right)+i\sin\left(\frac{135°+360°\cdot 0}{5}\right)\right]=\sqrt[5]{\sqrt{2}}\left(\cos 27°+i\sin 27°\right)\approx 0.95+0.49i$$

$$z_1=\sqrt[5]{\sqrt{2}}\left[\cos\left(\frac{135°+360°\cdot 1}{5}\right)+i\sin\left(\frac{135°+360°\cdot 1}{5}\right)\right]=\sqrt[5]{\sqrt{2}}\left(\cos 99°+i\sin 99°\right)\approx -0.17+1.06i$$

$$z_2=\sqrt[5]{\sqrt{2}}\left[\cos\left(\frac{135°+360°\cdot 2}{5}\right)+i\sin\left(\frac{135°+360°\cdot 2}{5}\right)\right]=\sqrt[5]{\sqrt{2}}\left(\cos 171°+i\sin 171°\right)\approx -1.06+0.17i$$

$$z_3=\sqrt[5]{\sqrt{2}}\left[\cos\left(\frac{135°+360°\cdot 3}{5}\right)+i\sin\left(\frac{135°+360°\cdot 3}{5}\right)\right]=\sqrt[5]{\sqrt{2}}\left(\cos 243°+i\sin 243°\right)\approx -0.49-0.95i$$

$$z_4=\sqrt[5]{\sqrt{2}}\left[\cos\left(\frac{135°+360°\cdot 4}{5}\right)+i\sin\left(\frac{135°+360°\cdot 4}{5}\right)\right]=\sqrt[5]{\sqrt{2}}\left(\cos 315°+i\sin 315°\right)\approx 0.76-0.76i$$

77. $i\left(2+2i\right)\left(-\sqrt{3}+i\right)$

$$=\left[1\left(\cos 90°+i\sin 90°\right)\right]\left[2\sqrt{2}\left(\cos 45°+i\sin 45°\right)\right]\left[2\left(\cos 150°+i\sin 150°\right)\right]$$
$$=4\sqrt{2}\left(\cos 285°+i\sin 285°\right)$$
$$\approx 1.4641-5.4641i$$

78. $\left(1+i\right)\left(1-i\sqrt{3}\right)\left(-\sqrt{3}+i\right)$

$$=\left[\sqrt{2}\left(\cos 45°+i\sin 45°\right)\right]\left[2\left(\cos 300°+i\sin 300°\right)\right]\left[2\left(\cos 150°+i\sin 150°\right)\right]$$
$$=4\sqrt{2}\left(\cos 495°+i\sin 495°\right)$$
$$=4\sqrt{2}\left(\cos 135°+i\sin 135°\right)$$
$$=-4+4i$$

79. $\dfrac{\left(1+i\sqrt{3}\right)\left(1-i\right)}{2\sqrt{3}-2i}$

$$=\frac{\left[2\left(\cos 60°+i\sin 60°\right)\right]\left[\sqrt{2}\left(\cos 315°+i\sin 315°\right)\right]}{\left[4\left(\cos 330°+i\sin 330°\right)\right]}$$
$$=\frac{\sqrt{2}}{2}\left(\cos 45°+i\sin 45°\right)$$
$$=\frac{1}{2}+\frac{1}{2}i$$

80. $\dfrac{\left(-1+i\sqrt{3}\right)\left(2-2i\sqrt{3}\right)}{4\sqrt{3}-4i}$

$$= \frac{\left[2\left(\cos 120° + i\sin 120°\right)\right]\left[4\left(\cos 300° + i\sin 300°\right)\right]}{\left[8\left(\cos 330° + i\sin 330°\right)\right]}$$
$$= 1\left(\cos 90° + i\sin 90°\right)$$
$$= \cos 90° + i\sin 90°$$
$$= i$$

81. $x^6 - 1 = 0$

$$x^6 = 1$$
$$x = \sqrt[6]{1}$$
$$x = \sqrt[6]{1+0i}$$
$$x = \sqrt[6]{\cos 0° + i\sin 0°}$$

$$z_k = \sqrt[6]{1}\left[\cos\left(\frac{0° + 360° \cdot k}{6}\right) + i\sin\left(\frac{0° + 360° \cdot k}{6}\right)\right],\ k = 0,1,2,3,4,5$$

$$z_0 = \sqrt[6]{1}\left[\cos\left(\frac{0° + 360° \cdot 0}{6}\right) + i\sin\left(\frac{0° + 360° \cdot 0}{6}\right)\right] = \cos 0° + i\sin 0° = 1 + 0i = 1$$

$$z_1 = \sqrt[6]{1}\left[\cos\left(\frac{0° + 360° \cdot 1}{6}\right) + i\sin\left(\frac{0° + 360° \cdot 1}{6}\right)\right] = \cos 60° + i\sin 60° = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z_2 = \sqrt[6]{1}\left[\cos\left(\frac{0° + 360° \cdot 2}{6}\right) + i\sin\left(\frac{0° + 360° \cdot 2}{6}\right)\right] = \cos 120° + i\sin 120° = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z_3 = \sqrt[6]{1}\left[\cos\left(\frac{0° + 360° \cdot 3}{6}\right) + i\sin\left(\frac{0° + 360° \cdot 3}{6}\right)\right] = \cos 180° + i\sin 180° = -1 + 0i = -1$$

$$z_4 = \sqrt[6]{1}\left[\cos\left(\frac{0° + 360° \cdot 4}{6}\right) + i\sin\left(\frac{0° + 360° \cdot 4}{6}\right)\right] = \cos 240° + i\sin 240° = -\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$z_5 = \sqrt[6]{1}\left[\cos\left(\frac{0° + 360° \cdot 5}{6}\right) + i\sin\left(\frac{0° + 360° \cdot 5}{6}\right)\right] = \cos 300° + i\sin 300° = \frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right) = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

82. $x^6+1=0$

$$x^6=-1$$
$$x=\sqrt[6]{-1}$$
$$x=\sqrt[6]{-1+0i}$$
$$x=\sqrt[6]{1(\cos 180°+i\sin 180°)}$$

$$z_k=\sqrt[6]{1}\left[\cos\left(\frac{180°+360°\cdot k}{6}\right)+i\sin\left(\frac{180°+360°\cdot k}{6}\right)\right],\ k=0,1,2,3,4,5$$

$$z_0=\sqrt[6]{1}\left[\cos\left(\frac{180°+360°\cdot 0}{6}\right)+i\sin\left(\frac{180°+360°\cdot 0}{6}\right)\right]=\cos 30°+i\sin 30°=\frac{\sqrt{3}}{2}+\frac{1}{2}i$$

$$z_1=\sqrt[6]{1}\left[\cos\left(\frac{180°+360°\cdot 1}{6}\right)+i\sin\left(\frac{180°+360°\cdot 1}{6}\right)\right]=\cos 90°+i\sin 90°=0+i=i$$

$$z_2=\sqrt[6]{1}\left[\cos\left(\frac{180°+360°\cdot 2}{6}\right)+i\sin\left(\frac{180°+360°\cdot 2}{6}\right)\right]=\cos 150°+i\sin 150°=-\frac{\sqrt{3}}{2}+\frac{1}{2}i$$

$$z_3=\sqrt[6]{1}\left[\cos\left(\frac{180°+360°\cdot 3}{6}\right)+i\sin\left(\frac{180°+360°\cdot 3}{6}\right)\right]=\cos 210°+i\sin 210°=-\frac{\sqrt{3}}{2}-\frac{1}{2}i$$

$$z_4=\sqrt[6]{1}\left[\cos\left(\frac{180°+360°\cdot 4}{6}\right)+i\sin\left(\frac{180°+360°\cdot 4}{6}\right)\right]=\cos 270°+i\sin 270°=0-i=-i$$

$$z_5=\sqrt[6]{1}\left[\cos\left(\frac{180°+360°\cdot 5}{6}\right)+i\sin\left(\frac{180°+360°\cdot 5}{6}\right)\right]=\cos 330°+i\sin 330°=\frac{\sqrt{3}}{2}-\frac{1}{2}i$$

83. $x^4+16i=0$

$$x^4=-16i$$
$$x=\sqrt[4]{-16i}$$
$$x=\sqrt[4]{0-16i}$$
$$x=\sqrt[4]{16(\cos 270°+i\sin 270°)}$$

$$z_k=\sqrt[4]{16}\left[\cos\left(\frac{270°+360°k}{4}\right)+i\sin\left(\frac{270°+360°k}{4}\right)\right],\ k=0,1,2,3$$

$$z_0=\sqrt[4]{16}\left[\cos\left(\frac{270°+360°\cdot 0}{4}\right)+i\sin\left(\frac{270°+360°\cdot 0}{4}\right)\right]=2(\cos 67.5°+i\sin 67.5°)\approx 0.7654+1.8478i$$

$$z_1=\sqrt[4]{16}\left[\cos\left(\frac{270°+360°\cdot 1}{4}\right)+i\sin\left(\frac{270°+360°\cdot 1}{4}\right)\right]=2(\cos 157.5°+i\sin 157.5°)\approx -1.8478+0.7654i$$

$$z_2=\sqrt[4]{16}\left[\cos\left(\frac{270°+360°\cdot 2}{4}\right)+i\sin\left(\frac{270°+360°\cdot 2}{4}\right)\right]=2(\cos 247.5°+i\sin 247.5°)\approx -0.7654-1.8478i$$

$$z_3=\sqrt[4]{16}\left[\cos\left(\frac{270°+360°\cdot 3}{4}\right)+i\sin\left(\frac{270°+360°\cdot 3}{4}\right)\right]=2(\cos 337.5°+i\sin 337.5°)\approx 1.8478-0.7654i$$

84. $x^5 - 32i = 0$

$$x^5 = 32i$$
$$x = \sqrt[5]{32i}$$
$$x = \sqrt[5]{0 + 32i}$$
$$x = \sqrt[5]{32(\cos 90° + i\sin 90°)}$$

$$z_k = \sqrt[5]{32}\left[\cos\left(\frac{90° + 360°k}{5}\right) + i\sin\left(\frac{90° + 360°k}{5}\right)\right],\ k = 0, 1, 2, 3, 4$$

$$z_0 = \sqrt[5]{32}\left[\cos\left(\frac{90° + 360°\cdot 0}{5}\right) + i\sin\left(\frac{90° + 360°\cdot 0}{5}\right)\right] = 2(\cos 18° + i\sin 18°) \approx 1.9021 + 0.6180i$$

$$z_1 = \sqrt[5]{32}\left[\cos\left(\frac{90° + 360°\cdot 1}{5}\right) + i\sin\left(\frac{90° + 360°\cdot 1}{5}\right)\right] = 2(\cos 90° + i\sin 90°) = 0 + 2i = 2i$$

$$z_2 = \sqrt[5]{32}\left[\cos\left(\frac{90° + 360°\cdot 2}{5}\right) + i\sin\left(\frac{90° + 360°\cdot 2}{5}\right)\right] = 2(\cos 162° + i\sin 162°) \approx -1.9021 + 0.6180i$$

$$z_3 = \sqrt[5]{32}\left[\cos\left(\frac{90° + 360°\cdot 3}{5}\right) + i\sin\left(\frac{90° + 360°\cdot 3}{5}\right)\right] = 2(\cos 234° + i\sin 234°) \approx -1.1756 - 1.6180i$$

$$z_4 = \sqrt[5]{32}\left[\cos\left(\frac{90° + 360°\cdot 4}{5}\right) + i\sin\left(\frac{90° + 360°\cdot 4}{5}\right)\right] = 2(\cos 306° + i\sin 306°) \approx 1.1756 - 1.6180i$$

85. $x^3 - \left(1 + i\sqrt{3}\right) = 0$

$$x^3 = 1 + i\sqrt{3}$$
$$x = \sqrt[3]{1 + i\sqrt{3}}$$
$$x = \sqrt[3]{2(\cos 60° + i\sin 60°)}$$

$$z_k = \sqrt[3]{2}\left[\cos\left(\frac{60° + 360°k}{3}\right) + i\sin\left(\frac{60° + 360°k}{3}\right)\right],\ k = 0, 1, 2$$

$$z_0 = \sqrt[3]{2}\left[\cos\left(\frac{60° + 360°\cdot 0}{3}\right) + i\sin\left(\frac{60° + 360°\cdot 0}{3}\right)\right] = \sqrt[3]{2}(\cos 20° + i\sin 20°) \approx 1.1839 + 0.4309i$$

$$z_1 = \sqrt[3]{2}\left[\cos\left(\frac{60° + 360°\cdot 1}{3}\right) + i\sin\left(\frac{60° + 360°\cdot 1}{3}\right)\right] = \sqrt[3]{2}(\cos 140° + i\sin 140°) \approx -0.9652 + 0.8099i$$

$$z_2 = \sqrt[3]{2}\left[\cos\left(\frac{60° + 360°\cdot 2}{3}\right) + i\sin\left(\frac{60° + 360°\cdot 2}{3}\right)\right] = \sqrt[3]{2}(\cos 260° + i\sin 260°) \approx -0.2188 - 1.2408i$$

86. $x^3 - \left(1 - i\sqrt{3}\right) = 0$

$$x^3 = 1 - i\sqrt{3}$$

$$x = \sqrt[3]{1 - i\sqrt{3}}$$

$$x = \sqrt[3]{2(\cos 300° + i\sin 300°)}$$

$$z_k = \sqrt[3]{2}\left[\cos\left(\frac{300° + 360°k}{3}\right) + i\sin\left(\frac{300° + 360°k}{3}\right)\right],\ k = 0, 1, 2$$

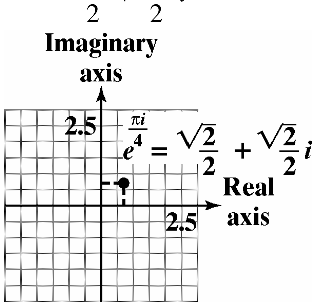
$$z_0 = \sqrt[3]{2}\left[\cos\left(\frac{300° + 360° \cdot 0}{3}\right) + i\sin\left(\frac{300° + 360° \cdot 0}{3}\right)\right] = \sqrt[3]{2}\left(\cos 100° + i\sin 100°\right) \approx -0.2188 + 1.2408i$$

$$z_1 = \sqrt[3]{2}\left[\cos\left(\frac{300° + 360° \cdot 1}{3}\right) + i\sin\left(\frac{300° + 360° \cdot 1}{3}\right)\right] = \sqrt[3]{2}\left(\cos 220° + i\sin 220°\right) \approx -0.9652 - 0.8099i$$

$$z_2 = \sqrt[3]{2}\left[\cos\left(\frac{300° + 360° \cdot 2}{3}\right) + i\sin\left(\frac{300° + 360° \cdot 2}{3}\right)\right] = \sqrt[3]{2}\left(\cos 340° + i\sin 340°\right) \approx 1.1839 - 0.4309i$$

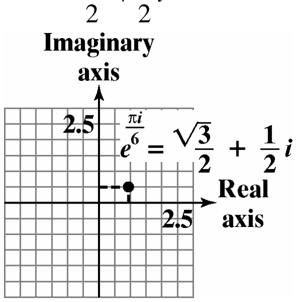
87. $e^{\frac{\pi i}{4}} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$

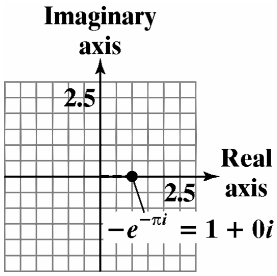


88. $e^{\frac{\pi i}{6}} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

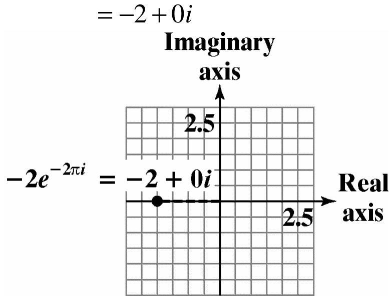
$$= \frac{\sqrt{3}}{2} + \frac{1}{2}i$$



89. $-e^{-\pi i} = -1\left(\cos(-\pi) + i\sin(-\pi)\right)$
$= -(-1) - i(0)$
$= 1 + 0i$



90. $-2e^{-2\pi i} = -2\left(\cos(-2\pi) + i\sin(-2\pi)\right)$
$= -2\left(\cos 0 + i\sin 0\right)$
$= -2 + 0i$



91. $z = i$

a. $z_1 = z = i$
$z_2 = z^2 + z = (i)^2 + i = -1 + i$
$z_3 = \left(z^2 + z\right)^2 + z = z_2^2 + z = \left(-1 + i\right)^2 + i = -i$
$z_4 = \left[\left(z^2 + z\right)^2 + z\right]^2 + z = z_3^2 + z = (-i)^2 + i = -1 + i$
$z_5 = z_4^2 + z = (-1 + i)^2 + i = -i$
$z_6 = z_5^2 + z = (-i)^2 + i = -1 + i$

b. $\left|-1 + i\right| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$
$\left|i\right| = \sqrt{0^2 + 1^2} = 1$

The absolute values of the terms in the sequence are 1 and $\sqrt{2}$.
Choose a complex number with absolute value less than 1, and another with absolute value greater than $\sqrt{2}$.
Complex numbers may vary.

92. $z = -i$

a. $z_1 = z = -i$

$z_2 = z^2 + z = (-i)^2 + (-i) = -1 - i$

$z_3 = \left(z^2 + z\right)^2 + z = z_2^2 + z = \left(-1 - i\right)^2 + (-i) = i$

$z_4 = \left[\left(z^2 + z\right)^2 + z\right]^2 + z = z_3^2 + z = i^2 + (-i) = -1 - i$

$z_5 = z_4^2 + z = (-1 - i)^2 + (-i) = i$

$z_6 = z_5^2 + z = i^2 + (-i) = -1 - i$

b. $|\ -1 - i\ | = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

$|\ i\ | = \sqrt{0^2 + 1^2} = 1$

The absolute values of the terms of the sequence are 1 and $\sqrt{2}$.
Choose a complex number with absolute value less than 1, and another with absolute value greater than $\sqrt{2}$.
Complex numbers may vary.

93. – 105. Answers may vary.

106. makes sense

107. does not make sense; Explanations will vary. Sample explanation: This process involves four multiplications.

108. makes sense

109. does not make sense; Explanations will vary. Sample explanation: $-1 - i\sqrt{3}$ and $-1 + i\sqrt{3}$ are the other 2 cube roots of 8.

110.
$$\frac{r_1\left(\cos\theta_1 + i\sin\theta_1\right)}{r_2\left(\cos\theta_2 + i\sin\theta_2\right)} = \frac{r_1\left(\cos\theta_1 + i\sin\theta_1\right)}{r_2\left(\cos\theta_2 + i\sin\theta_2\right)} \cdot \frac{\left(\cos\theta_2 - i\sin\theta_2\right)}{\left(\cos\theta_2 - i\sin\theta_2\right)}$$
$$= \frac{r_1\left[\cos\theta_1\cos\theta_2 + i\left(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2\right) + \sin\theta_1\sin\theta_2\right]}{r_2\left[\cos\theta_2\cos\theta_2 + i\left(\sin\theta_2\cos\theta_2 - \sin\theta_2\cos\theta_2\right) + \sin\theta_2\sin\theta_2\right]}$$
$$= \frac{r_1\left[\left(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\right) + i\left(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2\right)\right]}{r_2\left[\left(\cos^2\theta_2 + \sin^2\theta_2\right) + i(0)\right]}$$
$$= \frac{r_1\left[\cos\left(\theta_1 - \theta_2\right) + i\sin\left(\theta_1 - \theta_2\right)\right]}{r_2\left[1 + 0i\right]}$$
$$= \frac{r_1}{r_2}\left[\cos\left(\theta_1 - \theta_2\right) + i\sin\left(\theta_1 - \theta_2\right)\right]$$

111. $1 = 1\left(\cos 0° + i\sin 0°\right)$

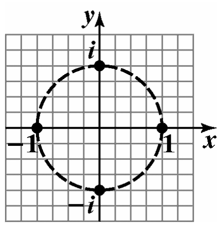
$$z_k = \sqrt[4]{1}\left[\cos\left(\frac{0° + 360°k}{4}\right) + i\sin\left(\frac{0° + 360°k}{4}\right)\right],\ k = 0,1,2,3$$

$$z_0 = \sqrt[4]{1}\left[\cos\left(\frac{0° + 360°\cdot 0}{4}\right) + i\sin\left(\frac{0° + 360°\cdot 0}{4}\right)\right] = \sqrt[4]{1}\left(\cos 0° + i\sin 0\right) = 1(1 + 0i) = 1$$

$$z_1 = \sqrt[4]{1}\left[\cos\left(\frac{0° + 360°\cdot 1}{4}\right) + i\sin\left(\frac{0° + 360°\cdot 1}{4}\right)\right] = \sqrt[4]{1}\left(\cos 90° + i\sin 90°\right) = 1(0 + i(1)) = i$$

$$z_2 = \sqrt[4]{1}\left[\cos\left(\frac{0° + 360°\cdot 2}{4}\right) + i\sin\left(\frac{0° + 360°\cdot 2}{4}\right)\right] = \sqrt[4]{1}\left(\cos 180° + i\sin 180°\right) = 1(-1 + 0i) = -1$$

$$z_3 = \sqrt[4]{1}\left[\cos\left(\frac{0° + 360°\cdot 3}{4}\right) + i\sin\left(\frac{0° + 360°\cdot 3}{4}\right)\right] = \sqrt[4]{1}\left(\cos 270° + i\sin 270°\right) = 1(0 + i(-1)) = -i$$



112. Answers may vary.

113. Find the distance from $(-3,-3)$ and $(0,3)$.

$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(3+3)^2 + (0+3)^2} \\ &= \sqrt{45} \\ &= 3\sqrt{5} \end{aligned}$$

Find the distance from $(0,0)$ and $(3,6)$.

$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(6-0)^2 + (3-0)^2} \\ &= \sqrt{45} \\ &= 3\sqrt{5} \end{aligned}$$

The line segments have the same length.

Section 7.6

Check Point Exercises

1. First, we show that **u** and **v** have the same magnitude.

$$\|\mathbf{u}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-2-(-5))^2 + (6-2)^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$

$$\|\mathbf{v}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5-2)^2 + (6-2)^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$

Thus, **u** and **v** have the same magnitude: $\|\mathbf{u}\| = \|\mathbf{v}\|$.

Next, we show that **u** and **v** have the same direction. the line on which **u** lies has slope

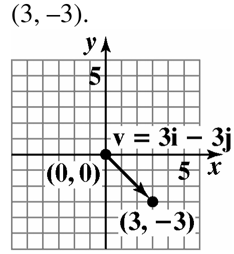
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{-2-(-5)} = \frac{4}{3}.$$

The line on which **v** lies has slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{5-2} = \frac{4}{3}.$$

Because **u** and **v** are both directed toward the upper right on lines having the same slope, $\frac{4}{3}$, they have the same direction. Thus, **u** and **v** have the same magnitude and direction, and **u = v.**

2. For the given vector $\mathbf{v} = 3\mathbf{i} - 3\mathbf{j}$, $a = 3$ and $b = -3$. The vector's initial point is the origin, (0, 0). The vector's terminal point is $(a, b) = (3, -3)$. We sketch the vector by drawing an arrow from (0, 0) to (3, –3).



We determine the magnitude of the vector by using the distance formula. Thus, the magnitude is

$$\|\mathbf{v}\| = \sqrt{a^2 + b^2}$$
$$= \sqrt{3^2 + (-3)^2}$$
$$= \sqrt{9+9}$$
$$= \sqrt{18}$$
$$= 3\sqrt{2}.$$

3. We identify the values for the variables in the formula.

$$P_1 = (-1, 3) \quad P_2 = (2, 7)$$
$$\uparrow\ \uparrow \qquad \uparrow\ \uparrow$$
$$x_1\ y_1 \qquad x_2\ y_2$$

Using these values, we write **v** in terms of **i** and **j** as follows:

$$\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$$
$$= (2-(-1))\mathbf{i} + (7-3)\mathbf{j}$$
$$= 3\mathbf{i} + 4\mathbf{j}$$

4. **a.**
$$\mathbf{v} + \mathbf{w} = (7\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - 5\mathbf{j})$$
$$= (7+4)\mathbf{i} + (3-5)\mathbf{j}$$
$$= 11\mathbf{i} - 2\mathbf{j}$$

b.
$$\mathbf{v} - \mathbf{w} = (7\mathbf{i} + 3\mathbf{j}) - (4\mathbf{i} - 5\mathbf{j})$$
$$= (7-4)\mathbf{i} + (3-(-5))\mathbf{j}$$
$$= 3\mathbf{i} + 8\mathbf{j}$$

5. **a.** $8\mathbf{v} = 8(7\mathbf{i} + 10\mathbf{j})$
$= (8?7)\mathbf{i} + (8?10)\mathbf{j}$
$= 56\mathbf{i} + 80\mathbf{j}$

b. $-5\mathbf{v} = -5(7\mathbf{i} + 10\mathbf{j})$
$= (-5?7)\mathbf{i} + (-5?10)\mathbf{j}$
$= -35\mathbf{i} - 50\mathbf{j}$

6. $6\mathbf{v} - 3\mathbf{w} = 6(7\mathbf{i} + 3\mathbf{j}) - 3(4\mathbf{i} - 5\mathbf{j})$
$= 42\mathbf{i} + 18\mathbf{j} - 12\mathbf{i} + 15\mathbf{j}$
$= (42 - 12)\mathbf{i} + (18 + 15)\mathbf{j}$
$= 30\mathbf{i} + 33\mathbf{j}$

7. First, find the magnitude of **v.**

$$\|\mathbf{v}\| = \sqrt{a^2 + b^2} = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

A unit vector in the same direction as **v** is

$$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{4\mathbf{i} - 3\mathbf{j}}{5} = \frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$$

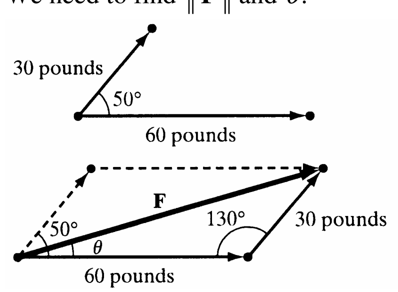
Now, we must verify that the magnitude of the vector is 1. The magnitude of $\frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$ is

$$\sqrt{\left(\frac{4}{5}\right)^2 + \left(-\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25} + \frac{9}{25}} = \sqrt{\frac{25}{25}} = 1.$$

8. $60 \cos 45^\circ\, \mathbf{i} + 60 \sin 45^\circ\, \mathbf{j}$

$$= 60 \cdot \frac{\sqrt{2}}{2}\mathbf{i} + 60 \cdot \frac{\sqrt{2}}{2}\mathbf{j}$$
$$= 30\sqrt{2}\mathbf{i} + 30\sqrt{2}\mathbf{j}$$

9. We need to find $\|\mathbf{F}\|$ and θ.



Use the Law of Cosines to find the magnitude of **F**.

$$\|\mathbf{F}\|^2 = 60^2 + 30^2 - 2(60)(30)\cos 130^\circ \approx 6814$$ The
$$\|\mathbf{F}\| \approx \sqrt{6814} \approx 82.5$$
magnitude of the resultant force is about 82.5 pounds.

To find θ, the direction of the resultant force, we use the Law of Sines.

$$\frac{82.5}{\sin 130^\circ} = \frac{30}{\sin \theta}$$
$$82.5 \sin \theta = 30 \sin 130^\circ$$
$$\sin \theta = \frac{30 \sin 130^\circ}{82.5}$$
$$\theta = \sin^{-1}\left(\frac{30 \sin 130^\circ}{82.5}\right) \approx 16.2^\circ$$

The two given forces are equivalent to a single force of about 82.5 pounds in the direction of approximately 16.2° relative to the 60-pound force.

Exercise Set 7.6

1. **a.** $$\|\mathbf{u}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4 - (-1))^2 + (6 - 2)^2} = \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41}$$

b. $$\|\mathbf{v}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5 - 0)^2 + (4 - 0)^2} = \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41}$$

c. Since $\|\mathbf{u}\| = \|\mathbf{v}\|$, and **u** and **v** have the same direction, we can conclude that **u** = **v**.

2. a.
$$\begin{aligned}\|\mathbf{u}\| &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-4-0)^2+(6-0)^2} \\ &= \sqrt{(-4)^2+(6)^2} \\ &= \sqrt{16+36} \\ &= \sqrt{52} \\ &= 2\sqrt{13}\end{aligned}$$

b.
$$\begin{aligned}\|\mathbf{v}\| &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-2-2)^2+(5-(-1))^2} \\ &= \sqrt{(-4)^2+(6)^2} \\ &= \sqrt{16+36} \\ &= \sqrt{52} \\ &= 2\sqrt{13}\end{aligned}$$

c. Since $\|\mathbf{u}\| = \|\mathbf{v}\|$, and **u** and **v** have the same direction, we can conclude that **u** = **v**.

3. a.
$$\begin{aligned}\|\mathbf{u}\| &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(5-(-1))^2+(1-1)^2} \\ &= \sqrt{6^2+0^2} \\ &= \sqrt{36+0} \\ &= \sqrt{36} \\ &= 6\end{aligned}$$

b.
$$\begin{aligned}\|\mathbf{v}\| &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(4-(-2))^2+(-1-(-1))^2} \\ &= \sqrt{6^2+0^2} \\ &= \sqrt{36+0} \\ &= \sqrt{36} \\ &= 6\end{aligned}$$

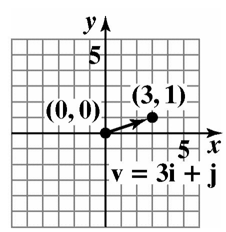
c. Since $\|\mathbf{u}\| = \|\mathbf{v}\|$, and **u** and **v** have the same direction, we can conclude that
u = **v**.

4. a.
$$\begin{aligned}\|\mathbf{u}\| &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-3-(-3))^2+(-2-3)^2} \\ &= \sqrt{0^2+(-5)^2} \\ &= \sqrt{0+25} \\ &= \sqrt{25} \\ &= 5\end{aligned}$$

$$\begin{aligned}\|\mathbf{v}\| &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(3-3)^2+(-4-1)^2} \\ &= \sqrt{0^2+(-5)^2} \\ &= \sqrt{0+25} \\ &= \sqrt{25} \\ &= 5\end{aligned}$$

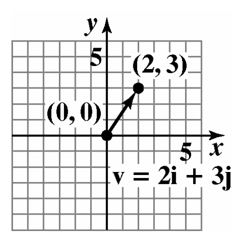
c. Since $\|\mathbf{u}\| = \|\mathbf{v}\|$, and **u** and **v** have the same direction, we can conclude that **u** = **v**.

5.



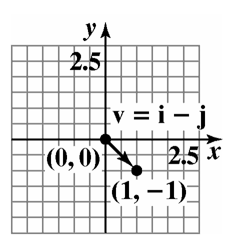
$\|\mathbf{v}\| = \sqrt{3^2+1^2} = \sqrt{9+1} = \sqrt{10}$

6.



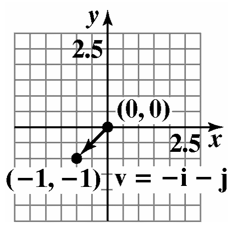
$\|\mathbf{v}\| = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$

7.



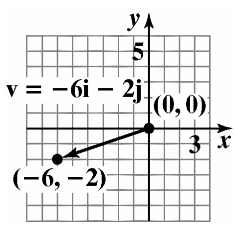
$\|\mathbf{v}\| = \sqrt{1^2+(-1)^2} = \sqrt{1+1} = \sqrt{2}$

8.



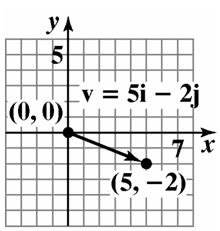
$$\|\mathbf{v}\| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

9.



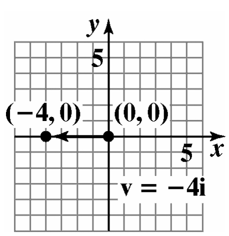
$$\|\mathbf{v}\| = \sqrt{(-6)^2 + (-2)^2}$$
$$= \sqrt{36+4}$$
$$= \sqrt{40}$$
$$= 2\sqrt{10}$$

10.



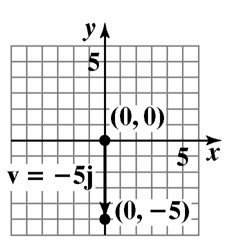
$$\|\mathbf{v}\| = \sqrt{5^2 + (-2)^2} = \sqrt{25+4} = \sqrt{29}$$

11.



$$\|\mathbf{v}\| = \sqrt{(-4)^2 + 0^2} = \sqrt{16+0} = \sqrt{16} = 4$$

12.



$$\|\mathbf{v}\| = \sqrt{0^2 + (-5)^2} = \sqrt{0+25} = \sqrt{25} = 5$$

13. $\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$
$\mathbf{v} = (6-(-4))\mathbf{i} + (2-(-4))\mathbf{j} = 10\mathbf{i} + 6\mathbf{j}$

14. $\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$
$\mathbf{v} = (-6-2)\mathbf{i} + (6-(-5))\mathbf{j} = -8\mathbf{i} + 11\mathbf{j}$

15. $\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$
$\mathbf{v} = (-2-(-8))\mathbf{i} + (3-6)\mathbf{j} = 6\mathbf{i} - 3\mathbf{j}$

16. $\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$
$\mathbf{v} = (0-(-7))\mathbf{i} + (-2-(-4))\mathbf{j} = 7\mathbf{i} + 2\mathbf{j}$

17. $\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$
$\mathbf{v} = (-7-(-1))\mathbf{i} + (-7-7)\mathbf{j} = -6\mathbf{i} - 14\mathbf{j}$

18. $\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$
$\mathbf{v} = (7-(-1))\mathbf{i} + (-5-6)\mathbf{j} = 8\mathbf{i} - 11\mathbf{j}$

19. $\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$
$\mathbf{v} = (6-(-3))\mathbf{i} + (4-4)\mathbf{j} = 9\mathbf{i} + 0\mathbf{j} = 9\mathbf{i}$

20. $\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$
$\mathbf{v} = (4-4)\mathbf{i} + (3-(-5))\mathbf{j} = 0\mathbf{i} + 8\mathbf{j} = 8\mathbf{j}$

21. $\mathbf{u} + \mathbf{v} = (2\mathbf{i} - 5\mathbf{j}) + (-3\mathbf{i} + 7\mathbf{j})$
$= (2-3)\mathbf{i} + (-5+7)\mathbf{j}$
$= -\mathbf{i} + 2\mathbf{j}$

22. $\mathbf{v} + \mathbf{w} = (-3\mathbf{i} + 7\mathbf{j}) + (-\mathbf{i} - 6\mathbf{j})$
$= (-3-1)\mathbf{i} + (7-6)\mathbf{j}$
$= -4\mathbf{i} + \mathbf{j}$

23. $\mathbf{u} - \mathbf{v} = (2\mathbf{i} - 5\mathbf{j}) - (-3\mathbf{i} + 7\mathbf{j})$
$= 2\mathbf{i} - 5\mathbf{j} + 3\mathbf{i} - 7\mathbf{j}$
$= (2+3)\mathbf{i} + (-5-7)\mathbf{j}$
$= 5\mathbf{i} - 12\mathbf{j}$

24. $\mathbf{v} - \mathbf{w} = (-3\mathbf{i} + 7\mathbf{j}) - (-\mathbf{i} - 6\mathbf{j})$
$= -3\mathbf{i} + 7\mathbf{j} + \mathbf{i} + 6\mathbf{j}$
$= (-3+1)\mathbf{i} + (7+6)\mathbf{j}$
$= -2\mathbf{i} + 13\mathbf{j}$

25. $\mathbf{v} - \mathbf{u} = (-3\mathbf{i} + 7\mathbf{j}) - (2\mathbf{i} - 5\mathbf{j})$
$= -3\mathbf{i} + 7\mathbf{j} - 2\mathbf{i} + 5\mathbf{j}$
$= (-3-2)\mathbf{i} + (7+5)\mathbf{j}$
$= -5\mathbf{i} + 12\mathbf{j}$

26. $\mathbf{w}-\mathbf{v} = (-\mathbf{i}-6\mathbf{j})-(-3\mathbf{i}+7\mathbf{j})$
$= -\mathbf{i}-6\mathbf{j}+3\mathbf{i}-7\mathbf{j}$
$= (-1+3)\mathbf{i}+(-6-7)\mathbf{j}$
$= 2\mathbf{i}-13\mathbf{j}$

27. $5\mathbf{v} = 5(-3\mathbf{i}+7\mathbf{j}) = -15\mathbf{i}+35\mathbf{j}$

28. $6\mathbf{v} = 6(-3\mathbf{i}+7\mathbf{j}) = -18\mathbf{i}+42\mathbf{j}$

29. $-4\mathbf{w} = -4(-\mathbf{i}-6\mathbf{j}) = 4\mathbf{i}+24\mathbf{j}$

30. $-7\mathbf{w} = -7(-\mathbf{i}-6\mathbf{j}) = 7\mathbf{i}+42\mathbf{j}$

31. $3\mathbf{w}+2\mathbf{v} = 3(-\mathbf{i}-6\mathbf{j})+2(-3\mathbf{i}+7\mathbf{j})$
$= -3\mathbf{i}-18\mathbf{j}-6\mathbf{i}+14\mathbf{j}$
$= (-3-6)\mathbf{i}+(-18+14)\mathbf{j}$
$= -9\mathbf{i}-4\mathbf{j}$

32. $3\mathbf{u}+4\mathbf{v} = 3(2\mathbf{i}-5\mathbf{j})+4(-3\mathbf{i}+7\mathbf{j})$
$= 6\mathbf{i}-15\mathbf{j}-12\mathbf{i}+28\mathbf{j}$
$= (6-12)\mathbf{i}+(-15+28)\mathbf{j}$
$= -6\mathbf{i}+13\mathbf{j}$

33. $3\mathbf{v}-4\mathbf{w} = 3(-3\mathbf{i}+7\mathbf{j})-4(-\mathbf{i}-6\mathbf{j})$
$= -9\mathbf{i}+21\mathbf{j}+4\mathbf{i}+24\mathbf{j}$
$= (-9+4)\mathbf{i}+(21+24)\mathbf{j}$
$= -5\mathbf{i}+45\mathbf{j}$

34. $4\mathbf{w}-3\mathbf{v} = 4(-\mathbf{i}-6\mathbf{j})-3(-3\mathbf{i}+7\mathbf{j})$
$= -4\mathbf{i}-24\mathbf{j}+9\mathbf{i}-21\mathbf{j}$
$= (-4+9)\mathbf{i}+(-24-21)\mathbf{j}$
$= 5\mathbf{i}-45\mathbf{j}$

35. $\|2\mathbf{u}\| = \|2(2\mathbf{i}-5\mathbf{j})\|$
$= \|4\mathbf{i}-10\mathbf{j}\|$
$= \sqrt{4^2+(-10)^2}$
$= \sqrt{16+100}$
$= \sqrt{116}$
$= 2\sqrt{29}$

36. $\|-2\mathbf{u}\| = \|-2(2\mathbf{i}-5\mathbf{j})\|$
$= \|-4\mathbf{i}+10\mathbf{j}\|$
$= \sqrt{(-4)^2+(10)^2}$
$= \sqrt{16+100}$
$= \sqrt{116}$
$= 2\sqrt{29}$

37. $\|\mathbf{w}-\mathbf{u}\| = \|(-\mathbf{i}-6\mathbf{j})-(2\mathbf{i}-5\mathbf{j})\|$
$= \|-\mathbf{i}-6\mathbf{j}-2\mathbf{i}+5\mathbf{j}\|$
$= \|(-1-2)\mathbf{i}+(-6+5)\mathbf{j}\|$
$= \|-3\mathbf{i}-\mathbf{j}\|$
$= \sqrt{(-3)^2+(-1)^2}$
$= \sqrt{9+1}$
$= \sqrt{10}$

38. $\|\mathbf{u}-\mathbf{w}\| = \|(2\mathbf{i}-5\mathbf{j})-(-\mathbf{i}-6\mathbf{j})\|$
$= \|2\mathbf{i}-5\mathbf{j}+\mathbf{i}+6\mathbf{j}\|$
$= \|(2+1)\mathbf{i}+(-5+6)\mathbf{j}\|$
$= \|3\mathbf{i}+\mathbf{j}\|$
$= \sqrt{3^2+1^2}$
$= \sqrt{9+1}$
$= \sqrt{10}$

39. $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{6\mathbf{i}}{\sqrt{6^2+0^2}} = \dfrac{6\mathbf{i}}{\sqrt{36}} = \dfrac{6\mathbf{i}}{6} = \mathbf{i}$

40. $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{-5\mathbf{j}}{\sqrt{0^2+(-5)^2}} = \dfrac{-5\mathbf{j}}{\sqrt{25}} = \dfrac{-5\mathbf{j}}{5} = -\mathbf{j}$

41. $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{3\mathbf{i}-4\mathbf{j}}{\sqrt{3^2+(-4)^2}}$
$= \dfrac{3\mathbf{i}-4\mathbf{j}}{\sqrt{9+16}}$
$= \dfrac{3\mathbf{i}-4\mathbf{j}}{\sqrt{25}}$
$= \dfrac{3\mathbf{i}-4\mathbf{j}}{5}$
$= \dfrac{3}{5}\mathbf{i}-\dfrac{4}{5}\mathbf{j}$

42. $\frac{\mathbf{v}}{\|\mathbf{v}\|}=\frac{8\mathbf{i}-6\mathbf{j}}{\sqrt{8^2+(-6)^2}}$
$=\frac{8\mathbf{i}-6\mathbf{j}}{\sqrt{64+36}}$
$=\frac{8\mathbf{i}-6\mathbf{j}}{\sqrt{100}}$
$=\frac{8\mathbf{i}-6\mathbf{j}}{10}$
$=\frac{8}{10}\mathbf{i}-\frac{6}{10}\mathbf{j}$
$=\frac{4}{5}\mathbf{i}-\frac{3}{5}\mathbf{j}$

43. $\frac{\mathbf{v}}{\|\mathbf{v}\|}=\frac{3\mathbf{i}-2\mathbf{j}}{\sqrt{3^2+(-2)^2}}$
$=\frac{3\mathbf{i}-2\mathbf{j}}{\sqrt{9+4}}$
$=\frac{3\mathbf{i}-2\mathbf{j}}{\sqrt{13}}$
$=\frac{3}{\sqrt{13}}\mathbf{i}-\frac{2}{\sqrt{13}}\mathbf{j}$

44. $\frac{\mathbf{v}}{\|\mathbf{v}\|}=\frac{4\mathbf{i}-2\mathbf{j}}{\sqrt{4^2+(-2)^2}}$
$=\frac{4\mathbf{i}-2\mathbf{j}}{\sqrt{16+4}}$
$=\frac{4\mathbf{i}-2\mathbf{j}}{\sqrt{20}}$
$=\frac{4\mathbf{i}-2\mathbf{j}}{2\sqrt{5}}$
$=\frac{4\mathbf{i}}{2\sqrt{5}}-\frac{2\mathbf{j}}{2\sqrt{5}}$
$=\frac{2\sqrt{5}}{5}\mathbf{i}-\frac{\sqrt{5}}{5}\mathbf{j}$

45. $\frac{\mathbf{v}}{\|\mathbf{v}\|}=\frac{\mathbf{i}+\mathbf{j}}{\sqrt{1^2+1^2}}$
$=\frac{\mathbf{i}+\mathbf{j}}{\sqrt{2}}$
$=\frac{\mathbf{i}}{\sqrt{2}}+\frac{\mathbf{j}}{\sqrt{2}}$
$=\frac{\sqrt{2}}{2}\mathbf{i}+\frac{\sqrt{2}}{2}\mathbf{j}$

46. $\frac{\mathbf{v}}{\|\mathbf{v}\|}=\frac{\mathbf{i}-\mathbf{j}}{\sqrt{1^2+(-1)^2}}=\frac{\mathbf{i}-\mathbf{j}}{\sqrt{2}}$
$=\frac{\mathbf{i}}{\sqrt{2}}-\frac{\mathbf{j}}{\sqrt{2}}$
$=\frac{\sqrt{2}}{2}\mathbf{i}-\frac{\sqrt{2}}{2}\mathbf{j}$

47. $\mathbf{v}=\|\mathbf{v}\|\cos\theta\mathbf{i}+\|\mathrm{v}\|\sin\theta\mathbf{j}$
$=6\cos 30°\mathbf{i}+6\sin 30°\mathbf{j}$
$=6\left(\frac{\sqrt{3}}{2}\right)\mathbf{i}+6\left(\frac{1}{2}\right)\mathbf{j}$
$=3\sqrt{3}\mathbf{i}+3\mathbf{j}$

48. $\mathbf{v}=\|\mathbf{v}\|\cos\theta\mathbf{i}+\|\mathbf{v}\|\sin\theta\mathbf{j}$
$=8\cos 45°\mathbf{i}+8\sin 45°\mathbf{j}$
$=8\left(\frac{\sqrt{2}}{2}\right)\mathbf{i}+8\left(\frac{\sqrt{2}}{2}\right)\mathbf{j}$
$=4\sqrt{2}\mathbf{i}+4\sqrt{2}\mathbf{j}$

49. $\mathbf{v}=\|\mathbf{v}\|\cos\theta\mathbf{i}+\|\mathbf{v}\|\sin\theta\mathbf{j}$
$=12\cos 225°\mathbf{i}+12\sin 225°\mathbf{j}$
$=12\left(-\frac{\sqrt{2}}{2}\right)\mathbf{i}+12\left(-\frac{\sqrt{2}}{2}\right)\mathbf{j}$
$=-6\sqrt{2}\mathbf{i}-6\sqrt{2}\mathbf{j}$

50. $\mathbf{v}=\|\mathbf{v}\|\cos\theta\mathbf{i}+\|\mathbf{v}\|\sin\theta\mathbf{j}$
$=10\cos 330°\mathbf{i}+10\sin 330°\mathbf{j}$
$=10\left(\frac{\sqrt{3}}{2}\right)\mathbf{i}+10\left(-\frac{1}{2}\right)\mathbf{j}$
$=5\sqrt{3}\mathbf{i}-5\mathbf{j}$

51. $\mathbf{v}=\|\mathbf{v}\|\cos\theta\mathbf{i}+\|\mathbf{v}\|\sin\theta\mathbf{j}$
$=\frac{1}{2}\cos 113°\mathbf{i}+\frac{1}{2}\sin 113°\mathbf{j}$
$\approx\frac{1}{2}(-0.39)\mathbf{i}+\frac{1}{2}(0.92)\mathbf{j}$
$\approx-0.20\mathbf{i}+0.46\mathbf{j}$

52. $\mathbf{v} = \|\mathbf{v}\|\cos\theta\mathbf{i} + \|\mathbf{v}\|\sin\theta\mathbf{j}$

$= \frac{1}{4}\cos 200°\mathbf{i} + \frac{1}{4}\sin 200°\mathbf{j}$

$\approx \frac{1}{4}(-0.94)\mathbf{i} + \frac{1}{4}(-0.34)\mathbf{j}$

$\approx -0.24\mathbf{i} - 0.09\mathbf{j}$

53. $4\mathbf{u} - (2\mathbf{v} - \mathbf{w}) = 4(-2\mathbf{i} + 3\mathbf{j}) - \left[2(6\mathbf{i} - \mathbf{j}) - (-3\mathbf{i})\right]$

$= -8\mathbf{i} + 12\mathbf{j} - \left[12\mathbf{i} - 2\mathbf{j} + 3\mathbf{i})\right]$

$= -8\mathbf{i} + 12\mathbf{j} - 12\mathbf{i} + 2\mathbf{j} - 3\mathbf{i})$

$= -23\mathbf{i} + 14\mathbf{j}$

54. $3\mathbf{u} - (4\mathbf{v} - \mathbf{w}) = 3(-2\mathbf{i} + 3\mathbf{j}) - \left[4(6\mathbf{i} - \mathbf{j}) - (-3\mathbf{i})\right]$

$= -6\mathbf{i} + 9\mathbf{j} - \left[24\mathbf{i} - 4\mathbf{j} + 3\mathbf{i})\right]$

$= -6\mathbf{i} + 9\mathbf{j} - 24\mathbf{i} + 4\mathbf{j} - 3\mathbf{i})$

$= -33\mathbf{i} + 13\mathbf{j}$

55. $\|\mathbf{u} + \mathbf{v}\|^2 - \|\mathbf{u} - \mathbf{v}\|^2$

$= \|-2\mathbf{i} + 3\mathbf{j} + 6\mathbf{i} - \mathbf{j}\|^2 - \|-2\mathbf{i} + 3\mathbf{j} - (6\mathbf{i} - \mathbf{j})\|^2$

$= \|4\mathbf{i} + 2\mathbf{j}\|^2 - \|-8\mathbf{i} + 4\mathbf{j}\|^2$

$= \left(\sqrt{4^2 + 2^2}\right)^2 - \left(\sqrt{(-8)^2 + 4^2}\right)^2$

$= 16 + 4 - (64 + 16)$

$= 20 - 80$

$= -60$

56. $\|\mathbf{v} + \mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2$

$= \|6\mathbf{i} - \mathbf{j} - 3\mathbf{i}\|^2 - \|6\mathbf{i} - \mathbf{j} + 3\mathbf{i}\|^2$

$= \|3\mathbf{i} - \mathbf{j}\|^2 - \|9\mathbf{i} - \mathbf{j}\|^2$

$= \left(\sqrt{3^2 + (-1)^2}\right)^2 - \left(\sqrt{9^2 + (-1)^2}\right)^2$

$= 9 + 1 - (81 + 1)$

$= 10 - 82$

$= -72$

57. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

$(a_1\mathbf{i} + b_1\mathbf{j}) + (a_2\mathbf{i} + b_2\mathbf{j}) = (a_2\mathbf{i} + b_2\mathbf{j}) + (a_1\mathbf{i} + b_1\mathbf{j})$

$(a_1 + a_2)\mathbf{i} + (b_1 + b_2)\mathbf{j} = (a_1 + a_2)\mathbf{i} + (b_1 + b_2)\mathbf{j}$

This demonstrates the commutative property of vectors.

58.
$$(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$$
$$\left((a_1\mathbf{i}+b_1\mathbf{j})+(a_2\mathbf{i}+b_2\mathbf{j})\right)+(a_3\mathbf{i}+b_3\mathbf{j})=(a_1\mathbf{i}+b_1\mathbf{j})+\left((a_2\mathbf{i}+b_2\mathbf{j})+(a_3\mathbf{i}+b_3\mathbf{j})\right)$$
$$(a_1+a_2+a_3)\mathbf{i}+(b_1+b_2+b_3)\mathbf{j}=(a_1+a_2+a_3)\mathbf{i}+(b_1+b_2+b_3)\mathbf{j}$$
This demonstrates the associative property of vectors.

59.
$$c(\mathbf{u}+\mathbf{v})=c\mathbf{u}+c\mathbf{v}$$
$$c\left((a_1\mathbf{i}+b_1\mathbf{j})+(a_2\mathbf{i}+b_2\mathbf{j})\right)=c(a_1\mathbf{i}+b_1\mathbf{j})+c(a_2\mathbf{i}+b_2\mathbf{j})$$
$$(ca_1+ca_2)\mathbf{i}+(cb_1+cb_2)\mathbf{j}=(ca_1+ca_2)\mathbf{i}+(cb_1+cb_2)\mathbf{j}$$
This demonstrates a distributive property of vectors.

60.
$$(c+d)\mathbf{u}=c\mathbf{u}+d\mathbf{u}$$
$$(c+d)(a_1\mathbf{i}+b_1\mathbf{j})=c(a_1\mathbf{i}+b_1\mathbf{j})+d(a_1\mathbf{i}+b_1\mathbf{j})$$
$$ca_1\mathbf{i}+cb_1\mathbf{j}+da_1\mathbf{i}+db_1\mathbf{j}=ca_1\mathbf{i}+cb_1\mathbf{j}+da_1\mathbf{i}+db_1\mathbf{j}$$
$$(ca_1+da_1)\mathbf{i}+(cb_1+db_1)\mathbf{j}=(ca_1+da_1)\mathbf{i}+(cb_1+db_1)\mathbf{j}$$
This demonstrates a distributive property of vectors.

61. $\|\mathbf{v}\|=\sqrt{(-10)^2+15^2}=\sqrt{325}\approx 18.03$

$\theta=\tan^{-1}\left(\dfrac{15}{-10}\right)\approx 123.7°$

62. $\|\mathbf{v}\|=\sqrt{2^2+(-8)^2}=\sqrt{68}\approx 8.25$

$\theta=\tan^{-1}\left(\dfrac{-8}{2}\right)\approx 284.0°$

63. $\mathbf{v}=(4\mathbf{i}-2\mathbf{j})-(4\mathbf{i}-8\mathbf{j})=6\mathbf{j}$

$\|\mathbf{v}\|=6$

$\theta=90°$

64. $\mathbf{v}=(7\mathbf{i}-3\mathbf{j})-(10\mathbf{i}-3\mathbf{j})=-3\mathbf{i}$

$\|\mathbf{v}\|=3$

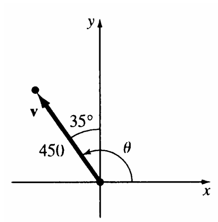
$\theta=180°$

65.
$$\begin{aligned}\mathbf{v}&=\|\mathbf{v}\|\cos\theta\mathbf{i}+\|\mathbf{v}\|\sin\theta\mathbf{j}\\&=44\cos30°\mathbf{i}+44\sin30°\mathbf{j}\\&=44\left(\frac{\sqrt{3}}{2}\right)\mathbf{i}+44\left(\frac{1}{2}\right)\mathbf{j}\\&=22\sqrt{3}\mathbf{i}+22\mathbf{j}\end{aligned}$$

66.
$$\begin{aligned}\mathbf{v}&=\|\mathbf{v}\|\cos\theta\mathbf{i}+\|\mathbf{v}\|\sin\theta\mathbf{j}\\&=30\cos45°\mathbf{i}+30\sin45°\mathbf{j}\\&=30\left(\frac{\sqrt{2}}{2}\right)\mathbf{i}+30\left(\frac{\sqrt{2}}{2}\right)\mathbf{j}\\&=15\sqrt{2}\mathbf{i}+15\sqrt{2}\mathbf{j}\end{aligned}$$

67. $\mathbf{v} = \|\mathbf{v}\|\cos\theta\mathbf{i} + \|\mathbf{v}\|\sin\theta\mathbf{j}$
$= 150\cos 8°\mathbf{i} + 150\sin 8°\mathbf{j}$
$\approx 148.5\mathbf{i} + 20.9\mathbf{j}$

68.



The vector's direction angle, from the positive x-axis to $\boldsymbol{v}$, is $\theta = 90° + 35° = 125°$.
Because the plane is traveling at 450 miles per hour, $\|\mathbf{v}\| = 450$. Thus,
$\mathbf{v} = \|\mathbf{v}\|\cos\theta\mathbf{i} + \|\mathbf{v}\|\sin\theta\mathbf{j}$
$= 450\cos 125°\mathbf{i} + 450\sin 125°\mathbf{j}$
$\approx 450(-0.57)\mathbf{i} + 450(0.82)\mathbf{j}$
$\approx -258.1\mathbf{i} + 368.6\mathbf{j}$

69. $\mathbf{v} = \|\mathbf{v}\|\cos\theta\mathbf{i} + \|\mathbf{v}\|\sin\theta\mathbf{j}$
$= 1.5\cos 25°\mathbf{i} + 1.5\sin 25°\mathbf{j}$
$\approx 1.4\mathbf{i} + 0.6\mathbf{j}$
The length of the shadow is $|1.4| = 1.4$ inches.

70. $\mathbf{v} = \|\mathbf{v}\|\cos\theta\mathbf{i} + \|\mathbf{v}\|\sin\theta\mathbf{j}$
$= 1.8\cos 40°\mathbf{i} + 1.8\sin 40°\mathbf{j}$
$= 1.4\mathbf{i} + 1.2\mathbf{j}$
The length of the shadow is $|1.4| = 1.4$ inches.

71. $\mathbf{F_1} = \|\mathbf{F_1}\|\cos\theta\mathbf{i} + \|\mathbf{F_1}\|\sin\theta\mathbf{j}$
$= 70\cos 326^{\text{O}}\mathbf{i} + 70\sin 326^{0}\mathbf{j}$
$= 58\mathbf{i} - 39.1\mathbf{j}$

$\mathbf{F_2} = \|\mathbf{F_2}\|\cos\theta\mathbf{i} + \|\mathbf{F_2}\|\sin\theta\mathbf{j}$
$= 50\cos 18^{0}\mathbf{i} + 50\sin 18^{0}\mathbf{j}$
$= 47.6\mathbf{i} + 15.5\mathbf{j}$

$\mathbf{F} = \mathbf{F_1} + \mathbf{F_2} = (58\mathbf{i} - 39.1\mathbf{j}) + (47.6\mathbf{i} + 15.5\mathbf{j})$
$= 105.6\mathbf{i} - 23.6\mathbf{j}$

$\|F\| = \sqrt{105.6^2 + (-23.6)^2} = 108.2$ pounds

$\cos\theta = \dfrac{a}{\|F\|}$

$\theta = \cos^{-1}\dfrac{105.6}{108.2} = 12.6^{\text{o}}, 90 - 12.6 = \text{S}77.4^{\text{o}}\text{E}$

72. $\mathbf{F_1} = 4200\cos 25^{\text{o}}\,\mathbf{i} + 4200\sin 25^{\text{o}}\,\mathbf{j}$
$= 3806.49\mathbf{i} + 1775\mathbf{j}$

$\mathbf{F_2} = 3000\cos 328^{\text{o}}\,\mathbf{i} + 3000\sin 328^{\text{o}}\,\mathbf{j}$
$= 2544.14\mathbf{i} - 1589.76\mathbf{j}$

$\mathbf{F} = (3806.49 + 2544.14)\,\mathbf{i} + (1775 - 1589.76)\mathbf{j}$
$= 6350.63\mathbf{i} + 185.24\mathbf{j}$
$\|\mathbf{F}\| = \sqrt{6350.63^2 + 185.24^2} \approx 6353.33$
6353 pounds
$\cos\theta = \dfrac{6350.63}{6353.33}$
$\theta = 1.7^{\text{O}}$

$\|F\| = \sqrt{105.6^2 + (-23.6)^2} = 108.2$ pounds

$\cos\theta = \dfrac{a}{\|F\|}$

$\theta = \cos^{-1}\dfrac{105.6}{108.2} = 12.6^{\text{o}}, 90 - 12.6 = \text{S}77.4^{\text{o}}\text{E}$

73. $\mathbf{F_1} = 1610\cos 125^{\text{o}}\,\mathbf{i} + 1610\sin 125^{\text{o}}\,\mathbf{j}$
$= -923.46\mathbf{i} + 1318.83\mathbf{j}$

$\mathbf{F_2} = 1250\cos 215^{\text{o}}\,\mathbf{i} + 1250\sin 215^{\text{o}}\,\mathbf{j}$
$= -1023.94\mathbf{i} - 716.97\mathbf{j}$

$\mathbf{F} = (-923.46 - 1023.94)\,\mathbf{i} + (1318.83 - 716.97)\mathbf{j}$
$= -1947.40\mathbf{i} + 601.86\mathbf{j}$

$\|\mathbf{F}\| = \sqrt{(-1947.40)^2 + 601.86^2} \approx 2038.28$
2038 kilograms
$\cos\theta = \dfrac{-1947.40}{2038.28}$
$\theta = 162.8^{\text{O}}$

74. $\mathbf{F_1} = 64\cos 129^{\text{o}}\,\mathbf{i} + 64\sin 129^{\text{o}}\,\mathbf{j}$
$= -40.28\mathbf{i} + 49.74\mathbf{j}$

$\mathbf{F_2} = 48\cos 211^{\text{o}}\,\mathbf{i} + 48\sin 211^{\text{o}}\,\mathbf{j}$
$= -41.14\mathbf{i} - 24.72\mathbf{j}$

$\mathbf{F} = (40.28 \; - 41.14)\,\mathbf{i} + (49.74 - 24.72)\mathbf{j}$
$= -81.42\mathbf{i} + 25.02\mathbf{j}$
$\|\mathbf{F}\| = \sqrt{(-81.42)^2 + 25.02^2} \approx 85.18$
85.18 kilograms
$\cos\theta = \dfrac{-81.42}{85.18}$
$\theta = 162.9^{\text{O}}$

75. $\mathbf{F}_1 = 70\cos 326^{o}\,\mathbf{i} + 70\sin 326^{0}\,\mathbf{j}$
$= -100\mathbf{j}$

To find the length of the BC: $\cos 18^{o} = \dfrac{a}{100}$

$a \approx 95$

$\mathbf{F}_2 = 95\cos 2888^{0} + 95\sin 288^{0}$
$= 29.4\mathbf{i} - 90.4\mathbf{j}$

$\mathbf{F} = \mathbf{F}_1 - \mathbf{F}_2 = (-100\mathbf{j}) - (29.4\mathbf{i} - 90.4\mathbf{j})$
$= -29.4\,\mathbf{i} - 9.6\,\mathbf{j}$

$\sqrt{(-29.4)^2 + (-9.6)^2} \approx 30.9$

The force required to pull the weight is 30.9 pounds.

76. $\sin 18° = \dfrac{30}{x}$

$x\sin 18° = 30$

$x = \dfrac{30}{\sin 18°}$

$x = 97.1$ pounds

77. **a.** 335 lb

b. 3484 lb

78. **a.** 61 lb

b. 273 lb

79. **a.** $\mathbf{F}_1 + \mathbf{F}_2 = (3 + 6)\mathbf{i} + (-5 + 2)\mathbf{j} = 9\mathbf{i} - 3\mathbf{j}$

b. $-9\mathbf{i} + 3\mathbf{j}$

80. **a.** $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = (-2 + 1 + 5)\mathbf{i} + (3 - 1 - 12)\mathbf{j}$
$= 4\mathbf{i} - 10\mathbf{j}$

b. $-4\mathbf{i} + 10\mathbf{j}$

81. **a.** $\mathbf{F}_1 = -3\mathbf{i}$ $(-3, 0)$
$\mathbf{F}_2 = -\mathbf{i} + 4\mathbf{j}$ $(-1, 4)$
$\mathbf{F}_3 = 4\mathbf{i} - 2\mathbf{i}$ $(4, -2)$
$\mathbf{F}_4 = -4\mathbf{j}$ $(0, -4)$
$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_2 + \mathbf{F}_2 = (-3 - 1 + 4)\mathbf{i}$
$+ (4 - 2 - 4)\mathbf{j} = -2\mathbf{j}$

b. $2\mathbf{j}$

82. **a.** $\mathbf{F}_1 = 8\cos 70^{o}\,\mathbf{i} + 8\sin {}^{o}\,\mathbf{j}$
$= 2.74\mathbf{i} + 7.52\mathbf{j}$

$\mathbf{F}_2 = 6\cos 140^{o}\,\mathbf{i} + 6\sin 140^{o}\,\mathbf{j}$
$= -4.60\mathbf{i} + 3.86\mathbf{j}$

$\mathbf{F}_3 = 4\cos 200^{o}\,\mathbf{i} + 4\sin 200^{o}\,\mathbf{j}$
$= -3.76\mathbf{i} - 1.37\mathbf{j}$

$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = (2.74 - 4.60 - 3.76)\mathbf{i} + (7.52 + 3.86 - 1.37)\mathbf{j} = -5.62\mathbf{i} + 10.01\mathbf{j}$

b. $5.62\mathbf{i} - 10.01\mathbf{j}$

83. **a.** $\mathbf{v} = 180\cos 40^{o}\,\mathbf{i} + 180\sin 40^{o}\,\mathbf{j}$
$= 137.88\mathbf{i} + 115.7\mathbf{j}$
$\mathbf{w} = 40\cos 0^{o}\,\mathbf{i} + 40\sin 0^{0}\,\mathbf{j}$
$= 40\mathbf{i}$

b. $\mathbf{v} + \mathbf{w} = (137.88 + 40)\mathrm{i} + 115.7\mathbf{j}$
$= 177.88\mathbf{i} + 115.7\mathbf{j}$

c. $\sqrt{177.88^2 + 115.7^2} \approx 212$ mph

d. $\cos\theta = \dfrac{177.88}{212}$

$\theta = 33^{o}$

$90^{o} - 33^{o} = \mathrm{N}57^{o}\,\mathrm{E}$

84. **a.** $\mathbf{v} = 400\cos 140^{o}\,\mathbf{i} + 400\sin 140^{o}\,\mathbf{j}$
$= -306.4\mathbf{i} + 257.1\mathbf{j}$

$\mathbf{w} = 30\cos 65^{o}\,\mathbf{i} + 30\sin 65^{o}\,\mathbf{j}$
$= 12.7\mathbf{i} + 27.2\mathbf{j}$

$\mathbf{v} + \mathbf{w} = (-306.4 + 12.7)\,\mathbf{i} + (257.1 + 27.2)\mathbf{j}$
$= -293.7\mathbf{i} + 284.3\mathbf{j}$

$\|\mathbf{v} + \mathbf{w}\| = \sqrt{(-293.7)^2 + 284.3^2} \approx 408.8$

409 mph

b. $\cos\theta = \dfrac{-306.4}{408.8}$

$\theta = 139^{o} = \mathrm{N}49^{o}\,\mathrm{W}$

85. $\mathbf{v} = 320\cos 20^\circ\,\mathbf{i} + 320\sin 20^\circ\,\mathbf{j}$
$= 300.7\mathbf{i} + 109.5\mathbf{j}$
$\mathbf{w} = 370\cos 30^\circ\,\mathbf{i} + 370\sin 30^0\,\mathbf{j}$
$= 320.4\mathbf{i} + 185\mathbf{j}$
$\mathbf{w} - \mathbf{v} = (320.4 - 300.7)\mathbf{i}$
$+ (115.7 - 109.5)\mathbf{j}$
$= 19.7\mathbf{i} + 75.6\mathbf{j}$
$\sqrt{19.7^2 + 75.6^2} \approx 78$ mph
$$\cos\theta = \frac{19.7}{78}$$
$\theta = 75.4^\circ$

86. $\mathbf{v} = 540\cos 306^\circ\,\mathbf{i} + 540\sin 306^\circ\,\mathbf{j}$
$= 317.4\mathbf{i} - 436.9\mathbf{j}$

$\mathbf{a} = 500\cos 314^\circ\,\mathbf{i} + 500\sin 314^\circ\,\mathbf{j}$
$= 347.3\mathbf{i} - 359.7\mathbf{j}$

$\mathbf{w} = \mathbf{a} - \mathbf{v} = (347.3 - 317.4)\,\mathbf{i} + (-359.7 + 436.9)\mathbf{j}$
$= 29.9\mathbf{i} + 77.2\mathbf{j}$
$\|\mathbf{w}\| = \sqrt{29.9^2 + 77.2^2} \approx 82.8$
83 mph
$$\cos\theta = \frac{29.9}{8.28}$$
$\theta = 68.8^\circ$

87. – 103. Answers may vary.

104. makes sense

105. does not make sense; Explanations will vary. Sample explanation: A vector represents a distance and a direction. A rate of change does not represent a distance and a direction.

106. makes sense

107. does not make sense; Explanations will vary. Sample explanation: The resultant force will have a magnitude less than two pounds unless both forces are in the same direction.

108. false; Changes to make the statement true will vary. A sample change is: $\mathbf{A} + \mathbf{B} = -\mathbf{E} \neq \mathbf{E}$

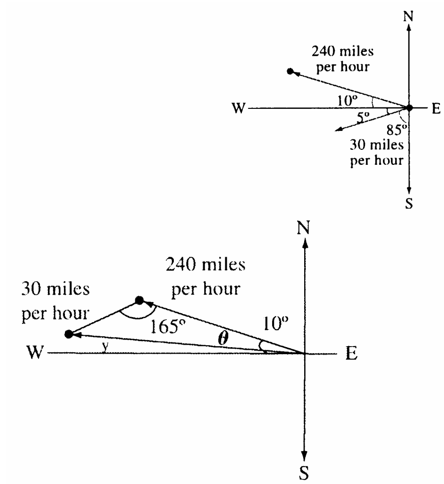
109. true

110. false; Changes to make the statement true will vary. A sample change is: $\mathbf{B} - \mathbf{E} \neq \mathbf{G} - \mathbf{F}$

111. true

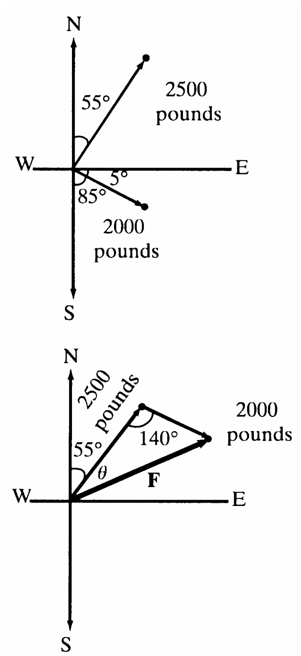
112.
$$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{a\mathbf{i} + b\mathbf{j}}{\|a\mathbf{i} + b\mathbf{j}\|} = \frac{a\mathbf{i} + b\mathbf{j}}{\sqrt{a^2 + b^2}}$$
$$= \frac{a}{\sqrt{a^2 + b^2}}\mathbf{i} + \frac{b}{\sqrt{a^2 + b^2}}\mathbf{j}$$
$$\left\|\frac{\mathbf{v}}{\|\mathbf{v}\|}\right\|^2 = \left(\frac{a}{\sqrt{a^2 + b^2}}\right)^2 + \left(\frac{b}{\sqrt{a^2 + b^2}}\right)^2$$
$$= \frac{a^2}{a^2 + b^2} + \frac{b^2}{a^2 + b^2} = \frac{a^2 + b^2}{a^2 + b^2} = 1$$
$$\left\|\frac{\mathbf{v}}{\|\mathbf{v}\|}\right\| = 1$$
Since $\left\|\frac{\mathbf{v}}{\|\mathbf{v}\|}\right\|$ is 1, $\left\|\frac{\mathbf{v}}{\|\mathbf{v}\|}\right\|$ is a unit vector.

113.



To find the magnitude of **v,** we use the Law of Cosines.
$$\|\mathbf{v}\|^2 = 30^2 + 240^2 - 2(30)(240)\cos 165^\circ$$
$$\approx 72{,}409.3$$
$$\|\mathbf{v}\| \approx \sqrt{72{,}409.3} \approx 269.1$$
The plane's true speed relative to the ground is about 269.1 miles per hour. To find the compass heading, relative to the ground, use the Law of Sines.
$$\frac{269.1}{\sin 165^\circ} = \frac{30}{\sin\theta}$$
$$269.1\sin\theta = 30\sin 165^\circ$$
$$\sin\theta = \frac{30\sin 165^\circ}{269.1}$$
$$\theta = \sin^{-1}\left(\frac{30\sin 165^\circ}{269.1}\right)$$
$$\theta \approx 1.7$$
The compass heading relative to the ground, is approximately 270° + (10° – 1.7°) = 278.3°

114.



To find the magnitude of **F**, we use the Law of Cosines.

$$\|\mathbf{F}\|^2 = 2500^2 + 2000^2 - 2(2500)(2000)\cos 140°$$
$$\approx 17{,}910{,}444.4$$
$$\|\mathbf{F}\| \approx \sqrt{17{,}910{,}444.4} \approx 4232.1$$

To find the compass direction of the resultant force, use the Law of Sines.

$$\frac{4232.1}{\sin 140°} = \frac{2000°}{\sin\theta}$$
$$4232.1\sin\theta = 2000\sin 140°$$
$$\sin\theta = \frac{2000\sin 140°}{4232.1}$$
$$\theta = \sin^{-1}\left(\frac{2000\sin 140°}{4232.1}\right)$$
$$\theta \approx 17.7°$$

The compass direction of the resultant force is 55° + 17.7° = 72.7°.

115. **a.** $\mathbf{a} = 310\cos\theta^{o}\,\mathbf{i} + 310\sin\theta^{o}\,\mathbf{j}$

$\mathbf{w} = 75\cos 0^{o}\,\mathbf{i} + 75\sin 0^{o}\,\mathbf{j}$

$= 75\mathbf{i}$

$$310\cos\theta + 75 = 0$$
$$\cos\theta = \frac{-75}{310}$$
$$\theta = 104^{o}$$
$$180^{o} - 104^{o} = 76^{o}$$

b. increase

116. $$\cos\theta = \frac{3(-1)+(-2)(4)}{\|\mathbf{v}\|\|\mathbf{w}\|}$$
$$\cos\theta = \frac{-3-8}{\sqrt{3^2+(-2)^2}\sqrt{(-1)^2+4^2}}$$
$$\cos\theta = \frac{-11}{\sqrt{13}\sqrt{17}}$$
$$\cos\theta = \frac{-11}{\sqrt{221}}$$
$$\theta = \cos^{-1}\left(\frac{-11}{\sqrt{221}}\right)$$
$$\theta \approx 137.7°$$

117. $$\frac{2(-2)+4(-6)}{\|\mathbf{w}\|^2}\mathbf{w} = \frac{-4-24}{\sqrt{(-2)^2+6^2}^{\,2}}(-2\mathbf{i}+6\mathbf{j})$$
$$= \frac{-28}{40}(-2\mathbf{i}+6\mathbf{j})$$
$$= \frac{7}{5}\mathbf{i} - \frac{21}{5}\mathbf{j}$$

118. **a.** $\|\mathbf{u}\|^2 = \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - 2\|\mathbf{v}\|\|\mathbf{w}\|\cos\theta$

b. $\|\mathbf{u}\| = \sqrt{(a_1-a_2)^2+(b_1-b_2)^2}$

$\|\mathbf{u}\|^2 = (a_1-a_2)^2+(b_1-b_2)^2$

$\|\mathbf{v}\| = \sqrt{(a_1-0)^2+(b_1-0)^2} = \sqrt{a_1^{\,2}+b_1^{\,2}}$

$\|\mathbf{v}\|^2 = a_1^{\,2}+b_1^{\,2}$

$\|\mathbf{w}\| = \sqrt{(0-a_2)^2+(0-b_2)^2} = \sqrt{a_2^{\,2}+b_2^{\,2}}$

$\|\mathbf{w}\|^2 = a_2^{\,2}+b_2^{\,2}$

Section 7.7

Check Point Exercises

1. **a.** $\mathbf{v}\cdot\mathbf{w} = 7(2) + (-4)(-1) = 14 + 4 = 18$

b. $\mathbf{w}\cdot\mathbf{v} = 2(7) + (-1)(-4) = 14 + 4 = 18$

c. $\mathbf{w}\cdot\mathbf{w} = 2(2) + (-1)(-1) = 4 + 1 = 5$

2.
$$\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\cdot\|\mathbf{w}\|} = \frac{(4\mathbf{i}-3\mathbf{j})\cdot(\mathbf{i}+2\mathbf{j})}{\sqrt{4^2+(-3)^2}\sqrt{1^2+2^2}} = \frac{4(1)+(-3)(2)}{\sqrt{25}\sqrt{5}} = \frac{2}{\sqrt{125}}$$

The angle θ between the vectors is
$\theta = \cos^{-1}\left(-\frac{2}{\sqrt{125}}\right) \approx 100.3°.$

3.
$$\mathbf{v}\cdot\mathbf{w} = (6\mathbf{i}-3\mathbf{j})\cdot(\mathbf{i}+2\mathbf{j}) = 6(1)+(-3)(2) = 6-6 = 0$$
The dot product is zero.
Thus, the given vectors are orthogonal.

4.
$$\text{proj}_{\mathbf{w}}\mathbf{v} = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w} = \frac{(2\mathbf{i}-5\mathbf{j})\cdot(\mathbf{i}-\mathbf{j})}{\left(\sqrt{1^2+(-1)^2}\right)^2}\mathbf{w} = \frac{2(1)+(-5)(-1)}{\left(\sqrt{2}\right)^2}\mathbf{w} = \frac{7}{2}\mathbf{w} = \frac{7}{2}(\mathbf{i}-\mathbf{j}) = \frac{7}{2}\mathbf{i}-\frac{7}{2}\mathbf{j}$$

5.
$$\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = \frac{7}{2}\mathbf{i}-\frac{7}{2}\mathbf{j}$$
$$\mathbf{v}_2 = \mathbf{v}-\mathbf{v}_1 = (2\mathbf{i}-5\mathbf{j})-\left(\frac{7}{2}\mathbf{i}-\frac{7}{2}\mathbf{j}\right) = -\frac{3}{2}\mathbf{i}-\frac{3}{2}\mathbf{j}$$

6.
$$W = \|\mathbf{F}\|\,\|\overrightarrow{AB}\|\cos\theta = (20)(150)\cos 30° \approx 2598$$
The work done is approximately 2598 foot-pounds.

Exercise Set 7.7

1. $\mathbf{v}\cdot\mathbf{w} = 3(1) + 1(3) = 3 + 3 = 6$
$\mathbf{v}\cdot\mathbf{v} = 3(3) + 1(1) = 9 + 1 = 10$

2. $\mathbf{v}\cdot\mathbf{w} = 3(1) + (3)(4) = 3 + 12 = 15$
$\mathbf{v}\cdot\mathbf{v} = 3(3) + 3(3) = 9 + 9 = 18$

3. $\mathbf{v}\cdot\mathbf{w} = 5(-2) + (-4)(-1) = -10 + 4 = -6$
$\mathbf{v}\cdot\mathbf{v} = 5(5) + (-4)(-4) = 25 + 16 = 41$

4. $\mathbf{v}\cdot\mathbf{w} = 4(-3) + (-2)(-1) = -21 + 2 = -19$
$\mathbf{v}\cdot\mathbf{v} = 7(7) + (-2)(-2) = 49 + 4 = 53$

5. $\mathbf{v}\cdot\mathbf{w} = -6(-10) + (-5)(-8) = 60 + 40 = 100$
$\mathbf{v}\cdot\mathbf{v} = -6(-6) + (-5)(-5) = 36 + 25 = 61$

6. $\mathbf{v}\cdot\mathbf{w} = -8(-10) + (-3)(-5) = 80 + 15 = 95$
$\mathbf{v}\cdot\mathbf{v} = -8(-8) + (-3)(-3) = 64 + 9 = 73$

7. $\mathbf{v}\cdot\mathbf{w} = 5(0) + 0(1) = 0 + 0 = 0$
$\mathbf{v}\cdot\mathbf{v} = 5(5) + 0(0) = 25 + 0 = 25$

8. $\mathbf{v} = \mathbf{i}+0\mathbf{j}$ and $\mathbf{w} = 0\mathbf{i}-5\mathbf{j}$
$\mathbf{v}\cdot\mathbf{w} = 1(0) + 0(-5) = 0 + 0 = 0$
$\mathbf{v}\cdot\mathbf{v} = 1(1) + 0(0) = 1 + 0 = 1$

9.
$$\begin{aligned}\mathbf{v}\cdot(\mathbf{v}+\mathbf{w}) &= (2\mathbf{i}-\mathbf{j})[(3\mathbf{i}+\mathbf{j})+(\mathbf{i}+4\mathbf{j})]\\ &= (2\mathbf{i}-\mathbf{j})[(3+1)\mathbf{i}+(1+4)\mathbf{j}]\\ &= (2\mathbf{i}-\mathbf{j})(4\mathbf{i}+5\mathbf{j})\\ &= 2(4)+(-1)(5)\\ &= 8-5\\ &= 3\end{aligned}$$

10.
$$\begin{aligned}\mathbf{v}\cdot(\mathbf{u}+\mathbf{w}) &= (3\mathbf{i}+\mathbf{j})\cdot[(2\mathbf{i}-\mathbf{j})+(\mathbf{i}+4\mathbf{j})]\\ &= (3\mathbf{i}+\mathbf{j})\cdot[(2+1)\mathbf{i}+(-1+4)\mathbf{j}]\\ &= (3\mathbf{i}+\mathbf{j})\cdot[3\mathbf{i}+3\mathbf{j}]\\ &= 3(3)+1(3)\\ &= 9+3\\ &= 12\end{aligned}$$

11. **u·v + u·w**
$$\begin{aligned}&= (2\mathbf{i}-\mathbf{j})\cdot(3\mathbf{i}+\mathbf{j})+(2\mathbf{i}-\mathbf{j})(\mathbf{i}+4\mathbf{j})\\ &= (2)(3)+(-1)(1)+2(1)+(-1)(4)\\ &= 6-1+2-4\\ &= 3\end{aligned}$$

12. **v·u + v·w**
$$\begin{aligned}&= (3\mathbf{i}+\mathbf{j})\cdot(2\mathbf{i}-\mathbf{j})+(3\mathbf{i}+\mathbf{j})\cdot(\mathbf{i}+4\mathbf{j})\\ &= [3(2)+1(-1)]+[3(1)+1(4)]\\ &= [6-1]+[3+4]\\ &= 5+7\\ &= 12\end{aligned}$$

13. $(4\mathbf{u})\cdot\mathbf{v}$
$$\begin{aligned}&= [(4(2\mathbf{i}-\mathbf{j})]\cdot(3\mathbf{i}+\mathbf{j})\\ &= (8\mathbf{i}-4\mathbf{j})\cdot(3\mathbf{i}+\mathbf{j})\\ &= (8)(3)+(-4)(1)\\ &= 24-4\\ &= 20\end{aligned}$$

14. (5**v**)·**w**
$$\begin{aligned}&= [5(3\mathbf{i}+\mathbf{j})]\cdot(\mathbf{i}+4\mathbf{j})\\ &= (15\mathbf{i}+5\mathbf{j})\cdot(\mathbf{i}+4\mathbf{j})\\ &= 15(1)+5(4)\\ &= 15+20\\ &= 35\end{aligned}$$

15. 4(**u·v**)
$$\begin{aligned}&= 4[(2\mathbf{i}-\mathbf{j})\cdot(3\mathbf{i}+\mathbf{j})]\\ &= 4[2(3)+(-1)1]\\ &= 4[6-1]\\ &= 4[5]\\ &= 20\end{aligned}$$

16. 5(**v·w**)
$$\begin{aligned}&= 5[(3\mathbf{i}+\mathbf{j})\cdot(\mathbf{i}+4\mathbf{j})]\\ &= 5[3(1)+1(4)]\\ &= 5[3+4]\\ &= 5[7]\\ &= 35\end{aligned}$$

17.
$$\begin{aligned}\cos\theta &\frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\ \|\mathbf{w}\|}\\ &= \frac{(2\mathbf{i}-\mathbf{j})\cdot(3\mathbf{i}+4\mathbf{j})}{\sqrt{2^2+(-1)^2}\sqrt{3^2+4^2}}\\ &= \frac{2(3)+(-1)(4)}{\sqrt{5}\sqrt{25}}\\ &= \frac{6-4}{\sqrt{125}}\\ &= \frac{2}{\sqrt{125}}\end{aligned}$$

The angle θ between the vectors is

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{125}}\right) \approx 79.7°.$$

18.
$$\begin{aligned}\cos\theta &= \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\ \|\mathbf{w}\|}\\ &= \frac{(-2\mathbf{i}+5\mathbf{j})\cdot(3\mathbf{i}+6\mathbf{j})}{\sqrt{(-2)^2+5^2}\sqrt{3^2+6^2}}\\ &= \frac{-2(3)+5(6)}{\sqrt{29}\sqrt{45}}\\ &= \frac{24}{\sqrt{1305}}\end{aligned}$$

The angle θ between the vectors is

$$\theta = \cos^{-1}\left(\frac{24}{\sqrt{1305}}\right) \approx 48.4°.$$

19.
$$\begin{aligned}\cos\theta &\frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\ \|\mathbf{w}\|}\\ &= \frac{(-3\mathbf{i}+2\mathbf{j})\cdot(4\mathbf{i}-\mathbf{j})}{\sqrt{(-3)^2+2^2}\sqrt{4^2+(-1)^2}}\\ &= \frac{-3(4)+2(-1)}{\sqrt{13}\sqrt{17}}\\ &= \frac{-14}{\sqrt{221}}\end{aligned}$$

The angle θ between the vectors is

$$\theta = \cos^{-1}\left(-\frac{14}{\sqrt{221}}\right) \approx 160.3°.$$

20. $\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\ \|\mathbf{w}\|}$

$= \frac{(\mathbf{i}+2\mathbf{j})\cdot(4\mathbf{i}-3\mathbf{j})}{\sqrt{1^2+2^2}\sqrt{4^2+(-3)^2}}$

$= \frac{1(4)+2(-3)}{\sqrt{5}\sqrt{25}}$

$= \frac{-2}{\sqrt{125}}$

The angle θ between the vectors is

$\theta = \cos^{-1}\left(-\frac{2}{\sqrt{125}}\right) \approx 100.3°.$

21. $\cos\theta \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\ \|\mathbf{w}\|}$

$= \frac{(6\mathbf{i}+0\mathbf{j})\cdot(5\mathbf{i}+4\mathbf{j})}{\sqrt{6^2+0^2}\sqrt{5^2+4^2}}$

$= \frac{6(5)+0(4)}{\sqrt{36}\sqrt{41}}$

$= \frac{30}{\sqrt{1476}}$

The angle θ between the vectors is

$\theta = \cos^{-1}\left(\frac{30}{\sqrt{1476}}\right) \approx 38.7°.$

22. $\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\ \|\mathbf{w}\|}$

$= \frac{(0\mathbf{i}+3\mathbf{j})\cdot(4\mathbf{i}+5\mathbf{j})}{\sqrt{0^2+3^2}\sqrt{4^2+5^2}}$

$= \frac{0(4)+3(5)}{\sqrt{9}\sqrt{41}}$

$= \frac{15}{\sqrt{369}}$

The angle θ between the vectors is

$\theta = \cos^{-1}\left(\frac{15}{\sqrt{369}}\right) \approx 38.7°.$

23. $\mathbf{v}\cdot\mathbf{w} = (\mathbf{i}+\mathbf{j})\cdot(\mathbf{i}-\mathbf{j}) = (1)(1)+1(-1) = 1-1 = 0$

The dot product is zero. Thus, the given vectors are orthogonal.

24. $\mathbf{v}\cdot\mathbf{w} = (\mathbf{i}+\mathbf{j})\cdot(-\mathbf{i}+\mathbf{j}) = 1(-1)+1(1) = -1 = 1 = 0$

The dot product is zero. Thus, the given vectors are orthogonal.

25. $\mathbf{v}\cdot\mathbf{w} = (2\mathbf{i}+8\mathbf{j})\cdot(4\mathbf{i}-\mathbf{j})$

$= 2(4)+(8)(-1)$

$= 8-8$

$= 0$

The dot product is zero. Thus, the given vectors are orthogonal.

26. $\mathbf{v}\cdot\mathbf{w} = (8\mathbf{i}-4\mathbf{j})\cdot(-6\mathbf{i}-12\mathbf{j}) = 8(-6)+(-4)(-12)$

$= -48 + 48 = 0$

The dot product is zero. Thus, the given vectors are orthogonal.

27. $\mathbf{v}\cdot\mathbf{w} = (2\mathbf{i}-2\mathbf{j})\cdot(-\mathbf{i}+\mathbf{j})$

$= 2(-1)+(-2)(1)$

$= -2-2$

$= -4$

The dot product is not zero. Thus, the given vectors are not orthogonal.

28. $\mathbf{v}\cdot\mathbf{w} = (5\mathbf{i}-5\mathbf{j})\cdot(\mathbf{i}-\mathbf{j}) = 5(1)+(-5)(-1)$

$= 5 + 5 = 10$

The dot product is not zero. Thus, the given vectors are not orthogonal.

29. $\mathbf{v}\cdot\mathbf{w} = (3\mathbf{i}+0\mathbf{j})\cdot(-4\mathbf{i}+0\mathbf{j})$

$= 3(-4)+0(0)$

$= -12+0$

$= -12$

The dot product is not zero. Thus, the given vectors are not orthogonal.

30. $\mathbf{v}\cdot\mathbf{w} = (5\mathbf{i}-0\mathbf{j})\cdot(-6\mathbf{i}+0\mathbf{j}) = 5(-6)+0(0)$

$= -30 + 0 = -30$

The dot product is not zero. Thus, the given vectors are not orthogonal.

31. $\mathbf{v}\cdot\mathbf{w} = (3\mathbf{i}+0\mathbf{j})\cdot(0\mathbf{i}-4\mathbf{j})$

$= 3(0)+(0)(-4)$

$= 0+0$

$= 0$

The dot product is zero. Thus, the given vectors are orthogonal.

32. $\mathbf{v}\cdot\mathbf{w} = (5\mathbf{i}+0\mathbf{j})\cdot(0\mathbf{i}-6\mathbf{j}) = 5(0)+0(-6)$

$= 0 + 0 = 0$

The dot product is zero. Thus, the given vectors are orthogonal.

33.
$$\begin{aligned}\text{proj}_{\mathbf{w}}\mathbf{v} &= \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}\\ &= \frac{(3\mathbf{i}-2\mathbf{j})\cdot(\mathbf{i}-\mathbf{j})}{\left(\sqrt{1^2+(-1)^2}\right)^2}\mathbf{w}\\ &= \frac{3(1)+(-2)(-1)}{\left(\sqrt{2}\right)^2}\\ &= \frac{5}{2}\mathbf{w}\\ &= \frac{5}{2}(\mathbf{i}-\mathbf{j})\\ &= \frac{5}{2}\mathbf{i}-\frac{5}{2}\mathbf{j}\end{aligned}$$

$$\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = \frac{5}{2}\mathbf{i}-\frac{5}{2}\mathbf{j}$$

$$\mathbf{v}_2 = \mathbf{v}-\mathbf{v}_1 = (3\mathbf{i}-2\mathbf{j})-\left(\frac{5}{2}\mathbf{i}-\frac{5}{2}\mathbf{j}\right)$$

$$= \frac{1}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}$$

34.
$$\begin{aligned}\text{proj}_{\mathbf{w}}\mathbf{v} &= \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}\\ &= \frac{(3\mathbf{i}-2\mathbf{j})\cdot(2\mathbf{i}+\mathbf{j})}{\left(\sqrt{2^2+1^2}\right)^2}\mathbf{w}\\ &= \frac{3(2)+(-2)(1)}{\left(\sqrt{5}\right)^2}\mathbf{w}\\ &= \frac{4}{5}(2\mathbf{i}+\mathbf{j})\\ &= \frac{8}{5}\mathbf{i}+\frac{4}{5}\mathbf{j}\end{aligned}$$

$$\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = \frac{8}{5}\mathbf{i}+\frac{4}{5}\mathbf{j}$$

$$\mathbf{v}_2 = \mathbf{v}-\mathbf{v}_1 = (3\mathbf{i}-2\mathbf{j})-\left(\frac{8}{5}\mathbf{i}+\frac{4}{5}\mathbf{j}\right)$$

$$= \frac{7}{5}\mathbf{i}-\frac{14}{5}\mathbf{j}$$

35.
$$\begin{aligned}\text{proj}_{\mathbf{w}}\mathbf{v} &= \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|}\mathbf{w}\\ &= \frac{(\mathbf{i}+3\mathbf{j})\cdot(-2\mathbf{i}+5\mathbf{j})}{\sqrt{1^2+(-1)^2}}\mathbf{w}\\ &= \frac{1(-2)+3(5)}{\left(\sqrt{(-2)^2+5^2}\right)^2}\mathbf{w}\\ &= \frac{13}{\left(\sqrt{29}\right)^2}\mathbf{w}\\ &= \frac{13}{29}\mathbf{w}\\ &= \frac{13}{29}(-2\mathbf{i}+5\mathbf{j})\\ &= \frac{-26}{29}\mathbf{i}+\frac{65}{29}\mathbf{j}\end{aligned}$$

$$\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = -\frac{26}{29}\mathbf{i}+\frac{65}{29}\mathbf{j}$$

$$\begin{aligned}\mathbf{v}_2 &= \mathbf{v}-\mathbf{v}_1\\ &= (\mathbf{i}+3\mathbf{j})-\left(-\frac{26}{29}\mathbf{i}+\frac{65}{29}\mathbf{j}\right)\\ &= \frac{55}{29}\mathbf{i}+\frac{22}{29}\mathbf{j}\end{aligned}$$

36.
$$\begin{aligned}\text{proj}_{\mathbf{w}}\mathbf{v} &= \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}\\ &= \frac{(2\mathbf{i}+4\mathbf{j})\cdot(-3\mathbf{i}+6\mathbf{j})}{\left(\sqrt{(-3)^2+6^2}\right)^2}\mathbf{w}\\ &= \frac{2(-3)+4(6)}{\left(\sqrt{45}\right)^2}\mathbf{w}\\ &= \frac{18}{45}\mathbf{w}\\ &= \frac{2}{5}(-3\mathbf{i}+6\mathbf{j})\\ &= \frac{-6}{5}\mathbf{i}+\frac{12}{5}\mathbf{j}\end{aligned}$$

$$\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = -\frac{6}{5}\mathbf{i}+\frac{12}{5}\mathbf{j}$$

$$\mathbf{v}_2 = \mathbf{v}-\mathbf{v}_1 = (2\mathbf{i}+4\mathbf{j})-\left(-\frac{6}{5}\mathbf{i}+\frac{12}{5}\mathbf{j}\right)$$

$$= \frac{16}{5}\mathbf{i}+\frac{8}{5}\mathbf{j}$$

37. $\text{proj}_{\mathbf{w}}\mathbf{v} = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}$

$= \dfrac{(\mathbf{i}+2\mathbf{j})\cdot(3\mathbf{i}+6\mathbf{j})}{\left(\sqrt{3^2+6^2}\right)^2}\mathbf{w}$

$= \dfrac{1(3)+2(6)}{\sqrt{45}}$

$= \dfrac{15}{45}\mathbf{w}$

$= \dfrac{1}{3}\mathbf{w}$

$= \dfrac{1}{3}(3\mathbf{i}+6\mathbf{j})$

$= \mathbf{i}+2\mathbf{j}$

$\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = \mathbf{i}+2\mathbf{j}$

$\mathbf{v}_2 = \mathbf{v}-\mathbf{v}_1$

$= (\mathbf{i}+2\mathbf{j})-(\mathbf{i}+2\mathbf{j})$

$= 0\mathbf{i}+0\mathbf{j}$

$= \mathbf{0}$

38. $\text{proj}_{\mathbf{w}}\mathbf{v} = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}$

$= \dfrac{(2\mathbf{i}+\mathbf{j})\cdot(6\mathbf{i}+3\mathbf{j})}{\left(\sqrt{6^2+3^2}\right)^2}\mathbf{w}$

$= \dfrac{2(6)+1(3)}{\left(\sqrt{45}\right)^2}\mathbf{w}$

$= \dfrac{15}{45}\mathbf{w}$

$= \dfrac{1}{3}(6\mathbf{i}+3\mathbf{j})$

$= 2\mathbf{i}+\mathbf{j}$

$\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = 2\mathbf{i}+\mathbf{j}$

$\mathbf{v}_2 = \mathbf{v}-\mathbf{v}_1 = (2\mathbf{i}+\mathbf{j})-(2\mathbf{i}+\mathbf{j})$

$= 0\mathbf{i}+0\mathbf{j} = \mathbf{0}$

39. $5\mathbf{u}\cdot(3\mathbf{v}-4\mathbf{w}) = 15\mathbf{u}\cdot\mathbf{v}-20\mathbf{u}\cdot\mathbf{w}$

$= 15\left[(-1)(3)+(1)(-2)\right]-20\left[(-1)(0)+(1)(-5)\right]$

$= 15\left[-5\right]-20\left[-5\right]$

$= 25$

40. $$\begin{aligned}4\mathbf{u}\cdot(5\mathbf{v}-3\mathbf{w}) &= 20\mathbf{u}\cdot\mathbf{v}-12\mathbf{u}\cdot\mathbf{w}\\ &= 20\left[(-1)(3)+(1)(-2)\right]-12\left[(-1)(0)+(1)(-5)\right]\\ &= 20\left[-5\right]-12\left[-5\right]\\ &= -40\end{aligned}$$

41. $$\begin{aligned}\text{proj}_{\mathbf{u}}(\mathbf{v}+\mathbf{w}) &= \frac{(\mathbf{v}+\mathbf{w})\cdot\mathbf{u}}{\|u\|^2}\mathbf{u}\\ &= \frac{(3\mathbf{i}-2\mathbf{j}-5\mathbf{j})\cdot(-\mathbf{i}+\mathbf{j})}{\|-\mathbf{i}+\mathbf{j}\|^2}(-\mathbf{i}+\mathbf{j})\\ &= \frac{(3\mathbf{i}-7\mathbf{j})\cdot(-\mathbf{i}+\mathbf{j})}{\left(\sqrt{(-1)^2+1^2}\right)^2}(-\mathbf{i}+\mathbf{j})\\ &= \frac{-3-7}{2}(-\mathbf{i}+\mathbf{j})\\ &= -5(-\mathbf{i}+\mathbf{j})\\ &= 5\mathbf{i}-5\mathbf{j}\end{aligned}$$

42. $$\begin{aligned}\text{proj}_{\mathbf{u}}(\mathbf{v}-\mathbf{w}) &= \frac{(\mathbf{v}-\mathbf{w})\cdot\mathbf{u}}{\|\mathbf{u}\|^2}\mathbf{u}\\ &= \frac{(3\mathbf{i}-2\mathbf{j}+5\mathbf{j})\cdot(-\mathbf{i}+\mathbf{j})}{\|-\mathbf{i}+\mathbf{j}\|^2}(-\mathbf{i}+\mathbf{j})\\ &= \frac{(3\mathbf{i}+3\mathbf{j})\cdot(-\mathbf{i}+\mathbf{j})}{\left(\sqrt{(-1)^2+1^2}\right)^2}(-\mathbf{i}+\mathbf{j})\\ &= \frac{-3+3}{2}(-\mathbf{i}+\mathbf{j})\\ &= 0(-\mathbf{i}+\mathbf{j})\\ &= 0\end{aligned}$$

43. $$\cos\theta = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|}$$

$$\cos\theta = \frac{\left(2\cos\frac{4\pi}{3}\right)\left(3\cos\frac{3\pi}{2}\right)+\left(2\sin\frac{4\pi}{3}\right)\left(3\sin\frac{3\pi}{2}\right)}{\sqrt{\left(2\cos\frac{4\pi}{3}\right)^2+\left(2\sin\frac{4\pi}{3}\right)^2}\sqrt{\left(3\cos\frac{3\pi}{2}\right)^2+\left(3\sin\frac{3\pi}{2}\right)^2}}$$

$$\cos\theta = \frac{3\sqrt{3}}{6}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 30^\circ$$

44. $\cos\theta = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\,\|\mathbf{w}\|}$

$$\cos\theta = \frac{\left(3\cos\frac{5\pi}{3}\right)(2\cos\pi) + \left(3\sin\frac{5\pi}{3}\right)(2\sin\pi)}{\sqrt{\left(3\cos\frac{5\pi}{3}\right)^2 + \left(3\sin\frac{5\pi}{3}\right)^2}\sqrt{(2\cos\pi)^2 + (2\sin\pi)^2}}$$

$$\cos\theta = \frac{-3}{6}$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = 120°$$

45. $\cos\theta = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|}$

$$\cos\theta = \frac{(3)(6) + (-5)(-10)}{\sqrt{(3)^2 + (-5)^2}\sqrt{(6)^2 + (-10)^2}}$$

$$\cos\theta = \frac{68}{68}$$

$$\cos\theta = 1$$

$$\theta = 0°$$

The vectors are parallel.

$v \cdot w = 68$

$v \cdot w \neq 0$

The vectors are not orthogonal.

46. $\cos\theta = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|}$

$$\cos\theta = \frac{(-2)(-6) + (3)(9)}{\sqrt{(-2)^2 + (3)^2}\sqrt{(-6)^2 + (9)^2}}$$

$$\cos\theta = \frac{39}{39}$$

$$\cos\theta = 1$$

$$\theta = 0°$$

The vectors are parallel.

$\mathbf{v}\cdot\mathbf{w} = 39$

$\mathbf{v}\cdot\mathbf{w} \neq 0$

The vectors are not orthogonal.

47. $\cos\theta = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|}$

$\cos\theta = \dfrac{(3)(6)+(-5)(10)}{\sqrt{(3)^2+(-5)^2}\sqrt{(6)^2+(10)^2}}$

$\cos\theta = \dfrac{-32}{68}$

$\theta \approx 118°$

The vectors are not parallel.

$\mathbf{v}\cdot\mathbf{w} = -32$

$\mathbf{v}\cdot\mathbf{w} \neq 0$

The vectors are not orthogonal.

48. $\cos\theta = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|}$

$\cos\theta = \dfrac{(-2)(-6)+(3)(-9)}{\sqrt{(-2)^2+(3)^2}\sqrt{(-6)^2+(-9)^2}}$

$\cos\theta = \dfrac{-15}{39}$

$\theta \approx 113°$

The vectors are not parallel.

$\mathbf{v}\cdot\mathbf{w} = -15$

$\mathbf{v}\cdot\mathbf{w} \neq 0$

The vectors are not orthogonal.

49. $\mathbf{v}\cdot\mathbf{w} = (3)(6)+(-5)\left(\dfrac{18}{5}\right) = 0$

The vectors are orthogonal.

50. $\mathbf{v}\cdot\mathbf{w} = (-2)(-6)+(3)(-4) = 0$

The vectors are orthogonal.

51. $\mathbf{v}\cdot\mathbf{w} = (240\mathbf{i}+300\mathbf{j})\cdot(2.90\mathbf{i}+3.07\mathbf{j})$

$= 240(2.90)+300(3.07)$

$= 696+921$

$= 1617$

$\mathbf{v}\cdot\mathbf{w} = 1617$ means \$1617 in revenue was generated on Monday by the sale of 240 gallons of regular gas at \$2.90 per gallon and 300 gallons of premium gas at \$3.07 per gallon.

52. $\mathbf{v}\cdot\mathbf{w} = (180\mathbf{i}+450\mathbf{j})\cdot(3\mathbf{i}+2\mathbf{j})$

$= 180(3)+450(2)$

$= 540+900$

$= 1440$

$\mathbf{v}\cdot\mathbf{w} = 1440$ means that the total collected by the video store when it rented 180 one-day videos at \$3 each and 450 three-day videos at \$2 each is \$1440.

53. Since the car is pushed along a level road, the angle between the force and the direction of motion is $\theta = 0$. The work done

$$\begin{aligned} W &= \|\mathbf{F}\| \left\|\overrightarrow{AB}\right\| \cos\theta \\ &= (95)(80)\cos 0° \\ &= 7600. \end{aligned}$$

The work done is 7600 foot-pounds.

54. Since the crane is vertically lifting the boulder through a vertical distance, the angle between the force and the direction of motion is $\theta = 0$. The work done is $W = \|\mathbf{F}\| \left\|\overrightarrow{AB}\right\| \cos\theta = (6000)(12)\cos 0°$

$$= 72,000.$$

The work done is 72,000 foot-pounds.

55. $$\begin{aligned} W &= \|\mathbf{F}\| \left\|\overrightarrow{AB}\right\| \cos\theta \\ &= (40)(100)\cos 32° \\ &\approx 3392 \end{aligned}$$

The work done is approximately 3392 foot-pounds.

56. $$W = \|\mathbf{F}\| \left\|\overrightarrow{AB}\right\| \cos\theta = (25)(100)\cos 38°$$

$$\approx 1970$$

The work done is approximately 1970 foot-pounds.

57. $$\begin{aligned} \mathbf{w} &= \mathbf{F} \cdot \overrightarrow{\mathbf{AB}} \\ &= 60(20)\cos(38^{\circ} - 12^{\circ}) \\ &= 1200\cos 26^{\circ} \\ &\approx 1079 \text{ foot-pounds} \end{aligned}$$

58. $$\begin{aligned} \mathbf{w} &= \mathbf{F} \cdot \overrightarrow{\mathbf{AB}} \\ &= 80(25)\cos(33^{\circ} - 10^{\circ}) \\ &= 2000\cos 23^{0} \\ &\approx 1841 \text{ foot-pounds} \end{aligned}$$

59. $$\begin{aligned} \mathbf{w} &= \mathbf{F} \cdot \overrightarrow{\mathbf{AB}} \\ &= (3,2) \cdot [(10,20) - (4,9)] \\ &= (3,2) \cdot (6,11) \\ &= 18 + 22 \\ &= 40 \text{ foot-pounds} \end{aligned}$$

60. $$\begin{aligned} \mathbf{w} &= \mathbf{F} \cdot \overrightarrow{\mathbf{AB}} \\ &= (5,7) \cdot [(18,20) - (8,11)] \\ &= (5,7) \cdot (10,9) \\ &= 5(10) + 7(9) \\ &= 113 \text{ meter-newtons} \end{aligned}$$

61. $\mathbf{w} = \mathbf{F} \cdot \overrightarrow{\mathbf{AB}}$

$= (4\cos 50^{\circ}, 4\sin 50^{\circ}) \cdot [(8,10) - (3,7)]$

$= (4\cos 50^{\circ}, 4\sin 50^{\circ}) \cdot (5,3)$

$= 20\cos 50^{\circ} + 12\sin 50^{\circ}$

≈ 22.05 foot-pounds

62. $\mathbf{w} = \mathbf{F} \cdot \overrightarrow{\mathbf{AB}}$

$= (6\cos 40^{\circ}, 6\sin 40^{\circ}) \cdot [(8,20) - (5,9)]$

$= (6\cos 40^{\circ}, 6\sin 40^{\circ}) \cdot [(3,11)]$

$= 18\cos 40^{\circ} + 66\sin 40^{\circ}$

≈ 56.21 foot pounds

63. a. $\cos 30^{\circ}\mathbf{i} + \sin 30^{\circ}\mathbf{j} = \frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}$

b. $\text{proj}_{\mathbf{u}}\mathbf{F} = \frac{(0,-700) \cdot \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)}{\|\mathbf{u}\|^2}\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$= -350\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) = -175\sqrt{3}\mathbf{i} - 175\mathbf{j}$

c. $\sqrt{(-175\sqrt{3})^2 + (-175)^2}$

$= \sqrt{122,500} = 350$

A force of 350 pounds is required to keep the boat from rolling down the ramp.

64. a. $\cos 30^{\circ}\mathbf{i} + \sin 30^{\circ}\mathbf{j} = \frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}$

b. $\text{proj}_{\mathbf{u}}\mathbf{F} = \frac{(0,-700) \cdot \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)}{\|\mathbf{u}\|}\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$= -325\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) = -\frac{325\sqrt{3}}{2}\mathbf{i} - \frac{325}{2}\mathbf{j}$

c. $\sqrt{\left(-\frac{325\sqrt{3}}{2}\right)^2 + \left(-\frac{325}{2}\right)^2} = 325$

A force of 325 pounds is required to keep the boat from rolling down the ramp.

65. – 74. Answers may vary.

75. makes sense

76. makes sense

77. makes sense

78. makes sense

79.
$$\begin{aligned}\mathbf{u}\cdot\mathbf{v} &= (a_1\mathbf{i}+b_1\mathbf{j})\cdot(a_2\mathbf{i}+b_2\mathbf{j})\\ &= a_1a_2+b_1b_2\\ &= a_2a_1+b_2b_1\\ &= (a_2\mathbf{i}+b_2\mathbf{j})\cdot(a_1\mathbf{i}+b_1\mathbf{j})\\ &= \mathbf{v}\cdot\mathbf{u}\end{aligned}$$

Thus $\mathbf{u}\cdot\mathbf{v}=\mathbf{v}\cdot\mathbf{u}$.

80.
$$\begin{aligned}(c\mathbf{u})\cdot\mathbf{v} &\stackrel{?}{=} c(\mathbf{u}\cdot\mathbf{v})\\ [c(a_1\mathbf{i}+b_1\mathbf{j})]\cdot(a_2\mathbf{i}+b_2\mathbf{j}) &\stackrel{?}{=} c[(a_1\mathbf{i}+b_1\mathbf{j})\cdot(a_2\mathbf{i}+b_2\mathbf{j})]\\ [ca_1\mathbf{i}+cb_1\mathbf{j}]\cdot(a_2\mathbf{i}+b_2\mathbf{j}) &\stackrel{?}{=} c[a_1a_2+b_1b_2]\\ ca_1a_2+cb_1b_2 &= ca_1a_2+cb_1b_2\end{aligned}$$

81.
$$\begin{aligned}\mathbf{u}\cdot(\mathbf{v}+\mathbf{w}) &= (a_1\mathbf{i}+b_1\mathbf{j})\cdot[(a_2\mathbf{i}+b_2\mathbf{j})+\left(a_3\mathbf{i}+a_3\mathbf{j}\right)]\\ &= (a_1\mathbf{i}+b_1\mathbf{j})\cdot[(a_2+a_3)\mathbf{i}+(b_2+b_3)\mathbf{j}]\\ &= a_1(a_2+a_3)+b_1(b_2+b_3)\\ &= a_1a_2+a_1a_3+b_1b_2+b_1b_3\\ &= a_1a_2+b_1b_2+a_1a_3+b_1b_3\\ &= (a_1\mathbf{i}+b_1\mathbf{j})\cdot(a_2\mathbf{i}+b_2\mathbf{j})+(a_1\mathbf{i}+b_1\mathbf{j})\cdot(a_3\mathbf{i}+b_3\mathbf{j})\\ &= \mathbf{u}\cdot\mathbf{v}+\mathbf{u}\cdot\mathbf{w}\end{aligned}$$

82. Answers may vary. One possible answer is $\mathbf{u} = 5\mathbf{i} + 2\mathbf{j}$.

83. Let $\mathbf{v}=15\mathbf{i}-3\mathbf{j}$ and $\mathbf{w}=-4\mathbf{i}+b\mathbf{j}$. The vectors $\mathbf{v}$ and $\mathbf{w}$ are orthogonal if $\mathbf{u}\cdot\mathbf{w} = 0$.
$\mathbf{v}\cdot\mathbf{w}=(15\mathbf{i}-3\mathbf{j})\cdot(-4\mathbf{i}+b\mathbf{j})=15(-4)+(-3)b=-60-3b$
$\mathbf{v}\cdot\mathbf{w} = 0$ if $-60-3b=0$. Solving the equation for b, we find $b=-20$.

84. Since the projection of $\mathbf{v}$ onto $\mathbf{i}$ is given by $\text{proj}_{\mathbf{i}}\mathbf{v}=\dfrac{\mathbf{v}\cdot\mathbf{i}}{\|\mathbf{i}\|^2}\mathbf{i}$. Let $\mathbf{v} = a\mathbf{i}+b\mathbf{j}$. Then

$$\begin{aligned}\text{proj}_{\mathbf{i}}\mathbf{v} &= \frac{(a\mathbf{i}+b\mathbf{j})\cdot(\mathbf{i}+0\mathbf{j})}{\left(\sqrt{1^2+0^2}\right)^2}(\mathbf{i}+0\mathbf{j})\\ &= \frac{a(1)+b(0)}{\left(\sqrt{1}\right)^2}(\mathbf{i}+0\mathbf{j})=a(\mathbf{i}+0\mathbf{j})=a\mathbf{i}\end{aligned}$$

$$\begin{aligned}(\mathbf{v}\cdot\mathbf{i})\mathbf{i} &= [(a\mathbf{i}+b\mathbf{j})\cdot(\mathbf{i}+0\mathbf{j})](\mathbf{i}+0\mathbf{j})\\ &= [a(1)+b(0)](\mathbf{i}+0\mathbf{j})=a(\mathbf{i}+0\mathbf{j})=a\mathbf{i}\end{aligned}$$

Since $\text{proj}_{\mathbf{i}}\mathbf{v} = a\mathbf{i}$ and $(\mathbf{v}\cdot\mathbf{i})\mathbf{i} = a\mathbf{i}$ as well, we can conclude that $\text{proj}_{\mathbf{i}}\mathbf{v}=(\mathbf{v}\cdot\mathbf{i})\mathbf{i}$ Thus, the projection of $\mathbf{v}$ onto $\mathbf{i}$ is $(\mathbf{v}\cdot\mathbf{i})\mathbf{i}$.

85. We know that $\text{proj}_{\mathbf{w}}\mathbf{v} = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}$ If the projection of **v** onto **w** is **v**, then $\mathbf{v} = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}$.

Since $\frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}$ is a scalar for all **v** and **w**, let $k = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}$. Substituting, we have $\mathbf{v} = k\mathbf{w}$.

When one vector can be expressed as a scalar multiple of another, the vectors have the same direction. Thus, the projection of **v** onto **w** is **v** only if **v** and **w** have the same direction. Thus, any two vectors, **v** and **w**, having the same direction will satisfy the condition that the projection of **v** onto **w** is **v**.

86. Answers may vary.

87. **a.**

$$\begin{aligned} x+2y &= 2 \\ 4+2(-1) &= 2 \\ 2 &= 2, \text{ true} \end{aligned}$$

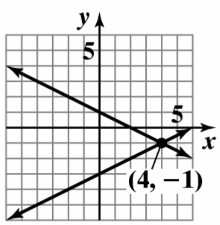
Yes, $(4,-1)$ satisfies the equation.

b.

$$\begin{aligned} x-2y &= 6 \\ 4-2(-1) &= 6 \\ 6 &= 6, \text{ true} \end{aligned}$$

Yes, $(4,-1)$ satisfies the equation.

88. The graphs intersect at $(4,-1)$.



89.

$$\begin{aligned} 5(2x-3)-4x &= 9 \\ 10x-15-4x &= 9 \\ 6x-15 &= 9 \\ 6x &= 24 \\ x &= 4 \end{aligned}$$

The solution set is $\{4\}$.

Chapter 7 Review Exercises

1. Begin by finding C.

$$A + B + C = 180°$$
$$70° + 55° + C = 180°$$
$$125° + C = 180°$$
$$C = 55°$$

Use the ratio $\frac{a}{\sin A}$, or $\frac{12}{\sin 70°}$, to find the other two sides. Use the Law of Sines to find b.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 55°} = \frac{12}{\sin 70°}$$
$$b = \frac{12 \sin 55°}{\sin 70°} \approx 10.5$$

Use the Law of Sines again, this time to find c.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$
$$\frac{c}{\sin 55°} = \frac{12}{\sin 70°}$$
$$c = \frac{12 \sin 55°}{\sin 70°} \approx 10.5$$

The solution is $C = 55°, b \approx 10.5, c \approx 10.5$.

2. Begin by finding A.

$$A + B + C = 180°$$
$$A + 107° + 30° = 180°$$
$$A + 137° = 180°$$
$$A = 43°$$

Use the ratio $\frac{c}{\sin C}$, or $\frac{126}{\sin 30°}$, to find the other two sides.

Use the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 43°} = \frac{126}{\sin 30°}$$
$$a = \frac{126 \sin 43°}{\sin 30°} \approx 171.9$$

Use the Law of Sines again, this time to find b.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 107°} = \frac{126}{\sin 30°}$$
$$b = \frac{126 \sin 107°}{\sin 30°} \approx 241.0$$

The solution is $A = 43°, a \approx 171.9,$ and $b \approx 241.0$.

3. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle. Thus, we will find b.

$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$b^2 = 17^2 + 12^2 - 2(17)(12) \cos 66°$$
$$b^2 = 289 + 144 - 408(\cos 66°)$$
$$b^2 \approx 267.05$$
$$b \approx \sqrt{267.05} \approx 16.3$$

Use the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle C.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{12}{\sin C} = \frac{\sqrt{267.05}}{\sin 66°}$$
$$\sqrt{267.05} \sin C = 12 \sin 66°$$
$$\sin C = \frac{12 \sin 66°}{\sqrt{267.05}} \approx 0.6708$$
$$C \approx 42°$$
$$A = 180° - B - C = 180° - 66° - 42° = 72°$$

The solution is $b \approx 16.3, A \approx 72°,$ and $C \approx 42°$.

4. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find the angle opposite the longest side. Thus, we will find angle C.

$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$= \frac{117^2 + 66^2 - 142^2}{2 \cdot 117 \cdot 66} \approx -0.1372$$

C is obtuse because $\cos C$ is negative.

$$\cos^{-1}(0.1372) \approx 82°$$
$$C \approx 180° - 82° = 98°$$

Use the Law of Sines to find either of the two remaining acute angles. We will find angle A.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{117}{\sin A} = \frac{142}{\sin 98°}$$
$$142 \sin A = 117 \sin 98°$$
$$\sin A = \frac{117 \sin 98°}{142} \approx 0.8159$$
$$A \approx 55°$$
$$B = 180° - A - C \approx 180° - 55° - 98° = 27°$$

The solution is $C \approx 98°, A \approx 55°,$ and $B \approx 27°$.

5. Begin by finding C.

$$A+B+C=180°$$
$$35°+25°+C=180°$$
$$60°+C=180°$$
$$C=120°$$

Use the ratio $\frac{c}{\sin C}$, or $\frac{68}{\sin 120°}$, to find the other two sides. Use the Law of Sines to find a.

$$\frac{a}{\sin A}=\frac{c}{\sin C}$$
$$\frac{a}{\sin 35°}=\frac{68}{\sin 120°}$$
$$a=\frac{68\sin 35°}{\sin 120°}\approx 45.0$$

Use the Law of Sines again, this time to find b.

$$\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$\frac{b}{\sin 25°}=\frac{68°}{\sin 120°}$$
$$b=\frac{68\sin 25°}{\sin 120°}\approx 33.2$$

The solution is $C=120°$, $a\approx 45.0$, and $b\approx 33.2$.

6. The known ratio is $\frac{a}{\sin A}$, or $\frac{20}{\sin 39°}$. Because side b is given, we used the Law of Sines to find angle B.

$$\frac{b}{\sin B}=\frac{a}{\sin A}$$
$$\frac{26}{\sin B}=\frac{20}{\sin 39°}$$
$$\sin B=\frac{26\sin 39°}{20}\approx 0.8181$$
$$B_1\approx 55°,\ B_2\approx 180°-55°=125°$$
$$C_1=180°-A-B_1\approx 180°-39°-55°=86°$$
$$C_2=180°-A-B_2\approx 180°-39°-125°=16°$$

Use the Law of Sines to find c_1 and c_2.

$$\frac{c_1}{\sin C_1}=\frac{a}{\sin A}$$
$$\frac{c_1}{\sin 86°}=\frac{20}{\sin 39°}$$
$$c_1=\frac{20\sin 86°}{\sin 39°}\approx 31.7$$
$$\frac{c_2}{\sin C_2}=\frac{a}{\sin A}$$
$$\frac{c_2}{\sin 16°}=\frac{20}{\sin 39°}$$
$$c_2=\frac{20\sin 16°}{\sin 39°}\approx 8.8$$

There are two triangles. In one triangle, the solution is $B_1\approx 55°$, $C_1\approx 86°$, and $c_1\approx 31.7$. In the other triangle, $B_2\approx 125°$, $C_2\approx 16°$, and $c_2\approx 8.8$.

7. The known ration is $\frac{c}{\sin C}$, or $\frac{1}{\sin 50°}$. Because side a is given, we used the Law of Sines to find angle A.

$$\frac{a}{\sin A}=\frac{c}{\sin C}$$
$$\frac{3}{\sin A}=\frac{1}{\sin 50°}$$
$$\sin A=\frac{3\sin 50°}{1}\approx 2.30$$

Because the sine can never exceed 1, there is no triangle with the given measurements.

8. Apply the three-step procedure for solving a SAS triangle. Use the Law of Cosines to find the side opposite the given angle. Thus, we will find *a*.

$a^2 = b^2 + c^2 - 2bc\cos A$ Use

$a^2 = (11.2)^2 + (48.2)^2 - 2(11.2)(48.2)\cos 162°$

≈ 3475.5

$a \approx \sqrt{3475.5} \approx 59.0$

the Law of Sines to find the angle opposite the shorter of the two given sides. Thus, we will find acute angle *B*.

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{11.2}{\sin B} = \frac{\sqrt{3475.5}}{\sin 162°}$

$\sin B = \frac{11.2\sin 162°}{\sqrt{3475.5}} \approx 0.0587$

$B \approx 3°$

$C = 180° - A - B \approx 180° - 162° - 3° = 15°$

The solution is $a \approx 59.0$, $B \approx 3°$, and $C \approx 15°$.

9. Apply the three-step procedure for solving a SSS triangle. Use the Law of Cosines to find the angle opposite the longest side. Thus, we will find angle *B*.

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \frac{(26.1)^2 + (36.5)^2 - (40.2)^2}{2 \cdot 26.1 \cdot 36.5}$

≈ 0.2086

$B \approx 78°$

Use the Law of Sines to find either of the two remaining acute angles. We will find angle *A*.

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{26.1}{\sin A} = \frac{40.2}{\sin 78°}$

$\sin A = \frac{26.1\sin 78°}{40.2} \approx 0.6351$

$A \approx 39°$

$C = 180° - A - B \approx 180° - 39° - 78° = 63°$

The solution is $B \approx 78°$, $A \approx 39°$, and $C \approx 63°$.

10. The known ratio is $\frac{a}{\sin A}$, or $\frac{6}{\sin 40°}$. Because side *b* is given, we used the Law of Sines to find angle *B*.

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{4}{\sin B} = \frac{6}{\sin 40°}$

$\sin B = \frac{4\sin 40°}{6} \approx 0.4285$

$B_1 \approx 25°$, $B_2 \approx 180° - 25° = 155°$

B_2 is impossible, since $40° + 155° = 195°$.

$C = 180° - A - B_1 \approx 180° - 40° - 25° = 115°$

Use the Law of Sines to find *c*.

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{c}{\sin 115°} = \frac{6}{\sin 40°}$

$c = \frac{6\sin 115°}{\sin 40°} \approx 8.5$

The solution is B_1 (or B) $\approx 25°$, $C \approx 115°$, and $c \approx 8.5$.

11. The known ratio is $\frac{b}{\sin B}$, or $\frac{8.7}{\sin 37°}$. Because side *a* is given, we use the Law of Sines to find angle *A*.

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{12.4}{\sin A} = \frac{8.7}{\sin 37°}$

$\sin A = \frac{12.4\sin 37°}{8.7} \approx 0.8578$

$A_1 \approx 59°$, $A_2 \approx 180° - 59° = 121°$

$C_1 = 180° - A_1 - B$

$\approx 180° - 59° - 37° - 84°$

$C_2 = 180° - A_2 - B$

$\approx 180° - 121° - 37° = 22°$

Use the Law of Sines to find c_1 and c_2.

$\frac{c_1}{\sin C_1} = \frac{b}{\sin B}$

$\frac{c_1}{\sin 84°} = \frac{8.7}{\sin 37°}$

$c_1 = \frac{8.7 \sin 84°}{\sin 37°} \approx 14.4$

$\frac{c_2}{\sin C_2} = \frac{b}{\sin B}$

$\frac{c_2}{\sin 22°} = \frac{8.7}{\sin 37°}$

$c_2 = \frac{8.7 \sin 22°}{\sin 37°} \approx 5.4$

There are two triangles. In one triangle, the solution is $A_1 \approx 59°$, $C_1 \approx 84°$, and $c_1 \approx 14.4$. In the other triangle, $A_2 \approx 121°$, $C_2 \approx 22°$, and $c_2 \approx 5.4$.

12. The known ratio is $\frac{a}{\sin A}$, or $\frac{54.3}{\sin 23°}$. Because side b is given, we used the Law of Sines to find angle B.

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{22.1}{\sin B} = \frac{54.3}{\sin 23°}$

$\sin B = \frac{22.1 \sin 23°}{54.3} \approx 0.1590$

$B_1 \approx 9°$, $B_2 \approx 180° - 9° = 171°$

B_2 is impossible, since $23° + 171° = 194°$.

$C = 180° - A - B_1 \approx 180° - 23° - 9° = 148°$

Use the Law of Sines to find c.

$\frac{c}{\sin C} = \frac{a}{\sin A}$

$\frac{c}{\sin 148°} = \frac{54.3}{\sin 23°}$

$c = \frac{54.3 \sin 148°}{\sin 23°} \approx 73.6$

The solution is B_1 (or B) $\approx 9°$, $C \approx 148°$, and $c \approx 73.6$.

13. $\text{Area} = \frac{1}{2} ab \sin C$

$= \frac{1}{2}(4)(6)\sin 42°$

≈ 8

The area of the triangle is approximately 8 square feet.

14. $\text{Area} = \frac{1}{2} bc \sin A$

$= \frac{1}{2}(4)(5)\sin 22°$

≈ 4

The area of the triangle is approximately 4 square feet.

15.

$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(2+4+5) = \frac{11}{2}$

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\frac{11}{2}\left(\frac{11}{2}-2\right)\left(\frac{11}{2}-4\right)\left(\frac{11}{2}-5\right)}$

$= \sqrt{\frac{231}{16}} \approx 4$

The area of the triangle is approximately 4 square meters.

16. $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(2+2+2) = 3$

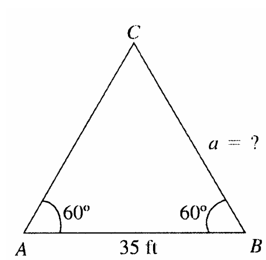
$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{3(3-2)(3-2)(3-2)}$

$= \sqrt{3} \approx 2$

The area of the triangle is approximately 2 square meters.

17.



Using the figure, $C = 180° - 60° - 60° = 60°$

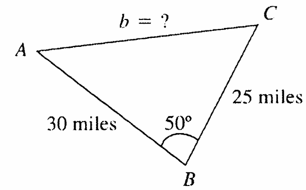
Use the Law of Sines to find a.

$\frac{a}{\sin 60°} = \frac{35}{\sin 60°}$

$a = 35$

The length of the roof is 35 feet.

18. One car travels 60 miles per hour for 30 minutes (half an hour), or $60\left(\frac{1}{2}\right)=30$ miles. Similarly, the other car travels 25 miles.

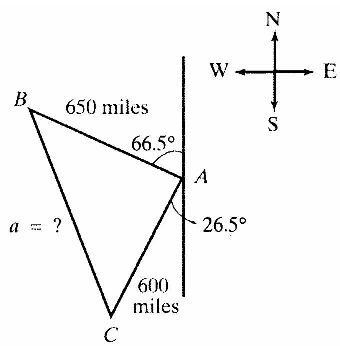


Using the figure,

$$b^2=a^2+c^2-2ac\cos B$$
$$=25^2+30^2-2(25)(30)\cos 80°\approx 1264.53$$
$$b\approx\sqrt{1264.53}\approx 35.6$$

The cars will be about 35.6 miles apart.

19. The first plane travels 325 miles per hour for 2 hours, or $325\cdot 2=650$ miles. Similarly, the other plane travels $300\cdot 2=600$ miles.

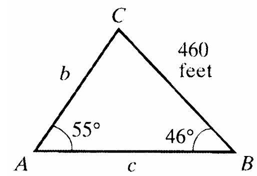


Using the figure,
$A=180°-66.5°-26.5°=87°$
Use the Law of Cosines to find a.

$$a^2=b^2+c^2-2bc\cos A$$
$$=600^2+650^2-2(600)(650)\cos 87°$$
$$\approx 741,678$$
$$a\approx\sqrt{741,678}\approx 861$$

The planes are about 861 miles apart.

20.



Using the figure,
$C=180°-A-B=180°-55°-46°=79°$
Use the Law of Sines to find b.

$$\frac{b}{\sin B}=\frac{a}{\sin A}$$
$$\frac{b}{\sin 46°}=\frac{460}{\sin 55°}$$
$$b=\frac{460\sin 46°}{\sin 55°}\approx 404$$

Use the Law of Sines again, this time to find c.

$$\frac{c}{\sin C}=\frac{a}{\sin A}$$
$$\frac{c}{\sin 79°}=\frac{460}{\sin 55°}$$
$$c=\frac{460\sin 79°}{\sin 55°}\approx 551$$

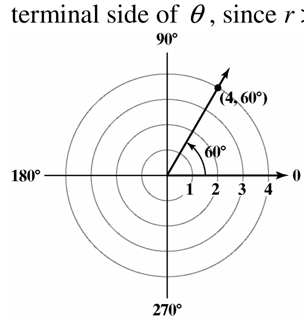
The lengths are about 404 feet and 551 feet.

21. $s=\frac{1}{2}(a+b+c)=\frac{1}{2}(260+320+450)=515$

$$\text{Area}=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{515(515-260)(515-320)(515-450)}$$
$$=\sqrt{1,664,544,375}\approx 40,798.83$$
$$\text{cost}\approx(5.25)(40,798.83)\approx 214,194$$

The cost is approximately \$214,194.

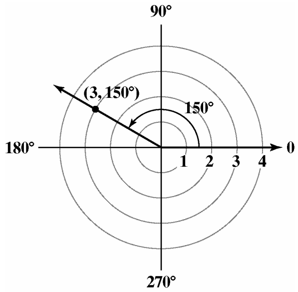
22. Draw $\theta=60°$ counterclockwise, since, θ is positive, from the polar axis. Go 4 units out on the terminal side of θ, since $r>0$.



$$x=r\cos\theta=4\cos 60°=4\left(\frac{1}{2}\right)=2$$
$$y=r\sin\theta=4\sin 60°=4\left(\frac{\sqrt{3}}{2}\right)=2\sqrt{3}$$

The rectangular coordinates of $(4,60°)$ are $\left(2,2\sqrt{3}\right)$.

23. Draw $\theta = 150°$ counterclockwise, since θ is positive, from the polar axis. Go 3 units out on the terminal side of θ, since $r > 0$.

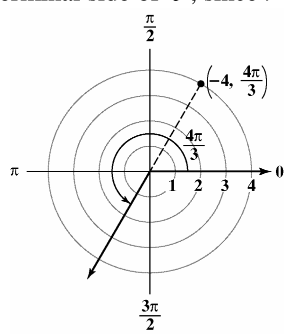


$$x = r\cos\theta = 3\cos 150° = 3\left(-\frac{\sqrt{3}}{2}\right) = -\frac{3\sqrt{3}}{2}$$

$$y = r\sin\theta = 3\sin 150° = 3\left(\frac{1}{2}\right) = \frac{3}{2}$$

The rectangular coordinates of $(3,150°)$ are $\left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$.

24. Draw $\theta = \frac{4\pi}{3} = 240°$ counterclockwise, since θ is positive, from the polar axis. Go 4 units out opposite the terminal side of θ, since $r < 0$.

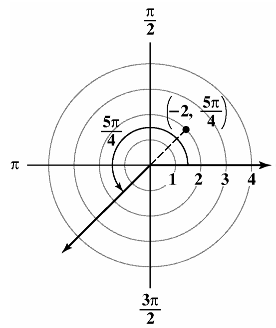


$$x = r\cos\theta = -4\cos\frac{4\pi}{3} = -4\left(-\frac{1}{2}\right) = 2$$

$$y = r\sin\theta = -4\sin\frac{4\pi}{3} = -4\left(-\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$$

The rectangular coordinates of $\left(-4, \frac{4\pi}{3}\right)$ are $\left(2, 2\sqrt{3}\right)$.

25. Draw $\theta = \frac{5\pi}{4} = 225°$ counterclockwise, since θ is positive from the polar axis. Go 2 units out opposite the terminal side of θ, since $r < 0$.

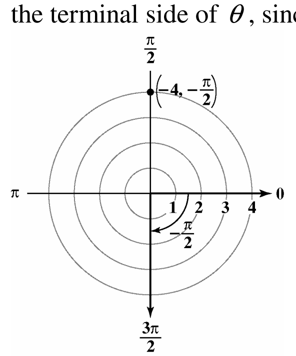


$$x = r\cos\theta = -2\cos\frac{5\pi}{4} = -2\left(-\frac{\sqrt{2}}{2}\right) = \sqrt{2}$$

$$y = r\sin\theta = -2\sin\frac{5\pi}{4} = -2\left(-\frac{\sqrt{2}}{2}\right) = \sqrt{2}$$

The rectangular coordinates of $\left(-2, \frac{5\pi}{4}\right)$ are $\left(\sqrt{2}, \sqrt{2}\right)$.

26. Draw $\theta = -\frac{\pi}{2} = -90°$ clockwise, since θ is negative, from the polar axis. Go 4 units out opposite the terminal side of θ, since $r < 0$.

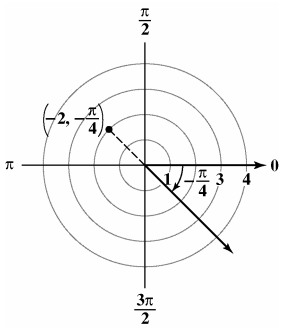


$$x = r\cos\theta = -4\cos\left(-\frac{\pi}{2}\right) = -4(0) = 0$$

$$y = r\sin\theta = -4\sin\left(-\frac{\pi}{2}\right) = -4(-1) = 4$$

The rectangular coordinates of $\left(-4, -\frac{\pi}{2}\right)$ are $(0, 4)$.

27. Draw $\theta = -\frac{\pi}{4} = -45°$ clockwise, since θ is negative, from the polar axis. Plot the point out 2 units opposite the terminal side of θ, since $r < 0$.

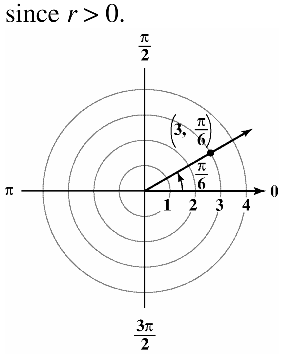


$$x = r\cos\theta = -2\cos\left(-\frac{\pi}{4}\right) = -2\left(\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$$

$$y = r\sin\theta = -2\sin\left(-\frac{\pi}{4}\right) = -2\left(-\frac{\sqrt{2}}{2}\right) = \sqrt{2}$$

The rectangular coordinates of $\left(-2, -\frac{\pi}{4}\right)$ are $\left(-\sqrt{2}, \sqrt{2}\right)$.

28. Draw $\theta = \frac{\pi}{6} = 30°$ counterclockwise, since θ is positive, from the polar axis. Go out 3 units on the terminal side of θ, since $r > 0$.

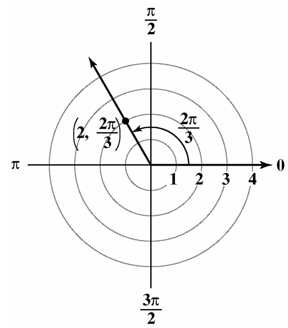


a. $\left(3, \frac{\pi}{6}\right) = \left(3, \frac{\pi}{6} + 2\pi\right) = \left(3, \frac{13\pi}{6}\right)$

b. $\left(3, \frac{\pi}{6}\right) = \left(-3, \frac{\pi}{6} + \pi\right) = \left(-3, \frac{7\pi}{6}\right)$

c. $\left(3, \frac{\pi}{6}\right) = \left(3, \frac{\pi}{6} - 2\pi\right) = \left(3, -\frac{11\pi}{6}\right)$

29. Draw $\theta = \frac{2\pi}{3} = 120°$ counterclockwise, since θ is positive, from the polar axis. Go out 3 units on the terminal side of θ, since $r > 0$.

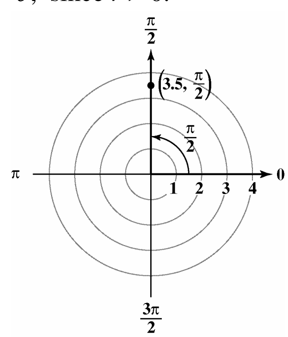


a. $\left(2, \frac{2\pi}{3}\right) = \left(2, \frac{2\pi}{3} + 2\pi\right) = \left(2, \frac{8\pi}{3}\right)$

b. $\left(2, \frac{2\pi}{3}\right) = \left(-2, \frac{2\pi}{3} + \pi\right) = \left(-2, \frac{5\pi}{3}\right)$

c. $\left(2, \frac{2\pi}{3}\right) = \left(2, \frac{2\pi}{3} - 2\pi\right) = \left(2, -\frac{4\pi}{3}\right)$

30. Draw $\theta = \frac{\pi}{2} = 90°$ counterclockwise, since θ is positive, from the polar axis. Go out 3.5 units on the terminal side of θ, since $r > 0$.



a. $\left(3, \frac{\pi}{2} + 2\pi\right) = \left(3, \frac{5\pi}{2}\right)$

b. $\left(-3.5, \frac{\pi}{2} + \pi\right) = \left(-3.5, \frac{3\pi}{2}\right)$

c. $\left(3.5, \frac{\pi}{2} - 2\pi\right) = \left(3.5, -\frac{3\pi}{2}\right)$

31. (–4,4)

$r=\sqrt{(-4)^2+(-4)^2}=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$

$\tan\theta=\frac{4}{-4}=-1$

Because $\tan\frac{\pi}{4}=1$ and θ lies in quadrant II,

$\theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$.

The polar coordinates of (–4,4) are $\left(4\sqrt{2},\frac{3\pi}{4}\right)$.

32. (3,–3)

$r=\sqrt{3^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$

$\tan\theta=\frac{-3}{3}=-1$

Because $\tan\frac{\pi}{4}=1$, and θ lies in quadrant IV, $\theta=2\pi-\frac{\pi}{4}=\frac{7\pi}{4}$.

The polar coordinates of (3, –3) are $\left(3\sqrt{2},\frac{7\pi}{4}\right)$

33. (5,12)

$r=\sqrt{5^2+12^2}=\sqrt{25+144}=\sqrt{169}=13$

$\tan\theta=\frac{12}{5}$

Because $\tan^{-1}\left(\frac{12}{5}\right)\approx 67°$ and θ lies in quadrant I, $\theta\approx 67°$.

The polar coordinates of (5,12) are approximately $(13, 67°)$.

34. (–3,4)

$r=\sqrt{(-3)^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$

$\tan\theta=\frac{4}{-3}=-\frac{4}{3}$

Because $\tan^{-1}\left(\frac{4}{3}\right)\approx 53°$ and θ lies in quadrant II, $\theta\approx 180°-53°=127°$. The polar coordinates of (–3, 4) are (5, 127°).

35. (0,–5)

$r=\sqrt{0^2+(-5)^2}=\sqrt{25}=5$

$\tan\theta=\frac{-5}{0}$ is undefined

Because $\tan\frac{\pi}{2}$ is undefined and θ lies on the negative y-axis, $\theta=\frac{\pi}{2}+\pi=\frac{3\pi}{2}$. The polar coordinates of (0,–5) are $\left(5,\frac{3\pi}{2}\right)$.

36. (1,0)

$r=\sqrt{1^2+0^2}=\sqrt{1}=1$

$\tan\theta=\frac{0}{1}=0$

Because $\tan 0=0$ and θ lies on the positive x-axis, $\theta=0$.

The polar coordinates of (1,0) are (1,0).

37.

$$2x+3y=8$$
$$2r\cos\theta+3r\sin\theta=8$$
$$r(2\cos\theta+3\sin\theta)=8$$
$$r=\frac{8}{2\cos\theta+3\sin\theta}$$

38.

$$x^2+y^2=100$$
$$r^2=100$$
$$r=10$$

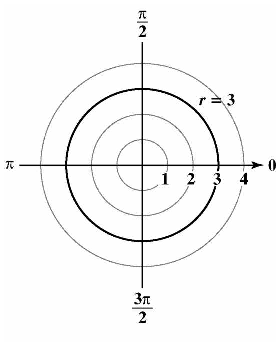
39.

$$(x-6)^2+y^2=36$$
$$(r\cos\theta-6)^2+(r\sin\theta)^2=36$$
$$r^2\cos^2\theta-12r\cos\theta+36+r^2\sin^2\theta=36$$
$$r^2-12r\cos\theta=0$$
$$r^2=12r\cos\theta$$
$$r=12\cos\theta$$

40. $r = 3$

$r^2 = 3^2$

$x^2 + y^2 = 9$

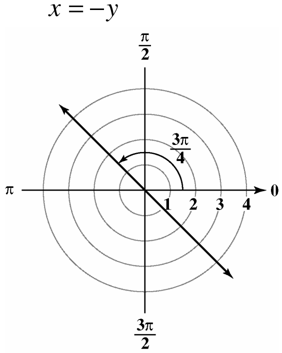


41. $\theta = \dfrac{3\pi}{4}$

$\tan\theta = \tan\dfrac{3\pi}{4}$

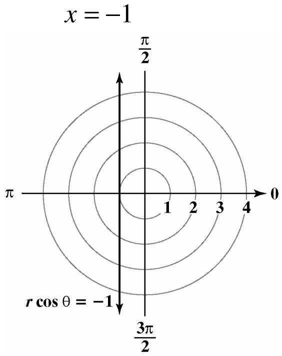
$\dfrac{x}{y} = -1$

$x = -y$



42. $r\cos\theta = -1$

$x = -1$

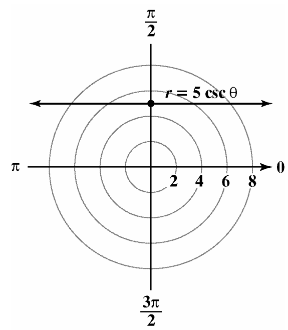


43. $r = 5\csc\theta$

$r = \dfrac{5}{\sin\theta}$

$r\sin\theta = 5$

$y = 5$



44. $r = 3\cos\theta$

$r \cdot r = r \cdot 3\cos\theta$

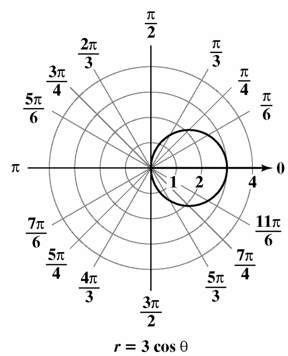
$r^2 = 3r\cos\theta$

$x^2 + y^2 = 3x$

$x^2 - 3x + y^2 = 0$

$x^2 - 3x + \dfrac{9}{4} + y^2 = \dfrac{9}{4}$

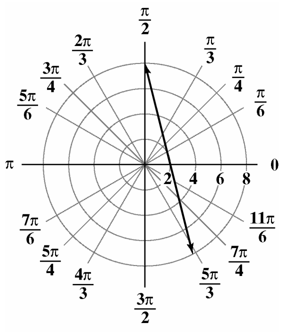
$\left(x - \dfrac{3}{2}\right)^2 + y^2 = \dfrac{9}{4}$



45. $4r\cos\theta + r\sin\theta = 8$

$4x + y = 8$

$y = -4x + 8$



46. $r^2 \sin 2\theta = -2$

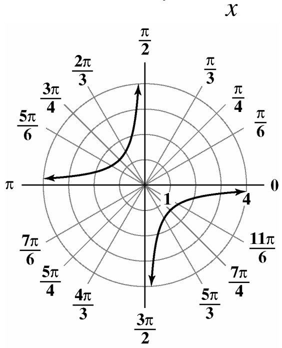
$r^2(2\sin\theta\cos\theta) = -2$

$2r\sin\theta r\cos\theta = -2$

$2yx = -2$

$y = \frac{-2}{2x}$

$y = -\frac{1}{x}$



47. $r = 5 + 3\cos\theta$

a. $r = 5 + 3\cos(-\theta)$

$r = 5 + 3\cos\theta$

The graph has symmetry about the polar axis.

b. $-r = 5 + 3\cos(-\theta)$

$-r = 5 + 3\cos\theta$

$r = -5 - 3\cos\theta$

The graph may or may not have symmetry with respect to the line $\theta = \frac{\pi}{2}$.

c. $-r = 5 + 3\cos\theta$

$r = -5 - 3\cos\theta$

The graph may or may not have symmetry with respect to the pole.

48. $r = 3\sin\theta$

a. $r = 3\sin(-\theta)$

$r = -3\sin\theta$

The graph may or may not have symmetry with respect to the polar axis.

b. $-r = 3\sin(-\theta)$

$-r = -3\sin\theta$

$r = 3\sin\theta$

The graph has symmetry with respect to line $\theta = \frac{\pi}{2}$.

c. $-r = 3\sin\theta$

$r = -3\sin\theta$

The graph may or may not have symmetry with respect to the pole.

49. $r^2 = 9\cos 2\theta$

a. $r^2 = 9\cos 2(-\theta)$

$r^2 = 9\cos(-2\theta)$

$r^2 = 9\cos 2\theta$

The graph has symmetry with respect to the polar axis.

b. $(-r)^2 = 9\cos 2(-\theta)$

$r^2 = 9\cos(-2\theta)$

$r^2 = 9\cos 2\theta$

The graph has symmetry with respect to the line $\theta = \frac{\pi}{2}$.

c. $(-r)^2 = 9\cos 2\theta$

$r^2 = 9\cos 2\theta$

The graph has symmetry with respect to the pole.

50. $r = 3\cos\theta$

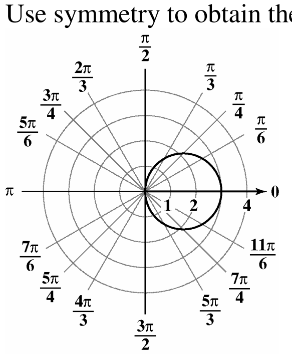
Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 3\cos(-\theta)$	$-r = 3\cos(-\theta)$	
	$-r = 3\cos\theta$	
$r = 3\cos\theta$	$r = -3\cos\theta$	

The graph has symmetry with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole. Calculate values of r for θ from 0 to π and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	3	2.6	1.5	0	–1.5	–2.6	–3

Use symmetry to obtain the graph.



$r = 3\cos\theta$

Notice that there are no points in quadrants II or III. Because the cosine is negative in quadrants II and III, r is negative here. This places the points in quadrants IV and I respectively.

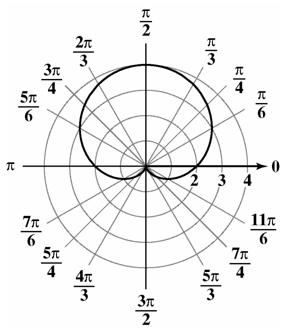
51. $r = 2 + 2\sin\theta$

Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 2 + 2\sin(-\theta)$	$-r = 2 + 2\sin(-\theta)$	$-r = 2 + 2\sin\theta$
	$-r = 2 - 2\sin\theta$	
$r = 2 - 2\sin\theta$	$r = -2 + 2\sin\theta$	$r = -2 - 2\sin\theta$

There may be no symmetry, since each equation is not equivalent to $r = 2 + 2\sin\theta$. Calculate values of r for θ from 0 to 2π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
r	2	3	3.73	4	3.73	3	2	1	0.27	0	0.27	1	2

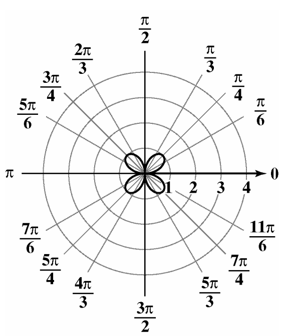


52. $r = \sin 2\theta$
Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = \sin 2(-\theta)$	$-r = \sin 2(-\theta)$	$-r = \sin 2\theta$
$r = \sin(-2\theta)$	$-r = \sin(-2\theta)$	
	$-r = -\sin 2\theta$	
$r = -\sin 2\theta$	$r = \sin 2\theta$	$r = -\sin 2\theta$

The graph has symmetry with respect to the line $\theta = \frac{\pi}{2}$. The graph may or may not be symmetric with respect to the polar axis or the pole. Calculate values of r for θ from 0 to $\frac{\pi}{2}$ and for θ from π to $\frac{3\pi}{2}$. Then, use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$
r	0	0.87	1	0.87	0	0	0.87	1	0.87	0

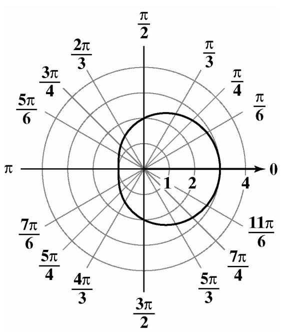


53. $r = 2 + \cos\theta$
Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 2 + \cos(-\theta)$	$-r = 2 + \cos(-\theta)$	$-r = 2 + \cos\theta$
	$-r = 2 + \cos\theta$	
$r = 2 + \cos\theta$	$r = -2 - \cos\theta$	$r = -2 - \cos\theta$

The graph is symmetric with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole. Calculate values of r for θ from 0 to π and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	3	2.87	2.5	2	1.5	1.13	1



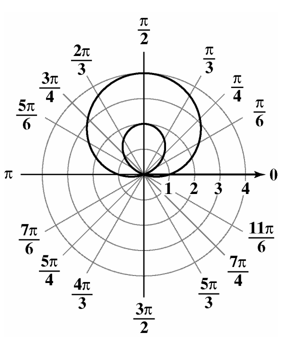
$r = 2 + \cos\theta$

54. $r = 1 + 3\sin\theta$
Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 1 + 3\sin(-\theta)$	$-r = 1 + 3\sin(-\theta)$	$-r = 1 + 3\sin\theta$
	$-r = 1 - 3\sin\theta$	
$r = 1 - 3\sin\theta$	$r = -1 + 3\sin\theta$	$r = -1 - 3\sin\theta$

There may be no symmetry, since each equation is not equivalent to $r = 1 + 3\sin\theta$. Calculate values of r for θ from 0 to 2π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
r	1	2.5	3.6	4	3.6	2.5	1	–0.5	–1.6	–2	–1.6	–0.5	1



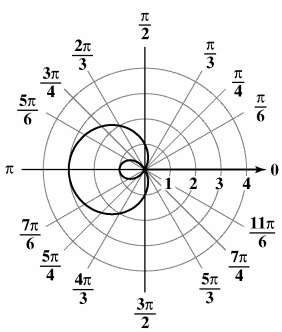
$r = 1 + 3\sin\theta$

55. $r = 1 - 2\cos\theta$
Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = 1 - 2\cos(-\theta)$	$-r = 1 - 2\cos(-\theta)$	$-r = 1 - 2\cos\theta$
	$-r = 1 - 2\cos\theta$	
$r = 1 - 2\cos\theta$	$r = -1 + 2\cos\theta$	$r = -1 + 2\cos\theta$

The graph is symmetric with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole. Calculate values of r for θ from 0 to π and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	–1	–0.73	0	1	2	2.73	3



$r = 1 - 2\cos\theta$

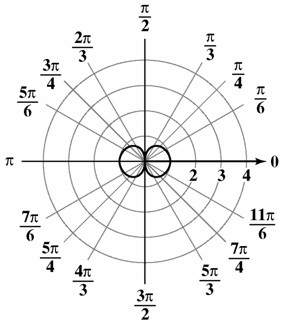
56. $r^2 = \cos 2\theta$
Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r^2 = \cos 2(-\theta)$	$(-r)^2 = \cos 2(-\theta)$	$(-r)^2 = \cos 2\theta$
$r^2 = \cos(-2\theta)$	$r^2 = \cos(-2\theta)$	
$r^2 = \cos 2\theta$	$r^2 = \cos 2\theta$	$r^2 = \cos 2\theta$

The graph has symmetry with respect to the polar axis, the line $\theta = \frac{\pi}{2}$, and the pole.

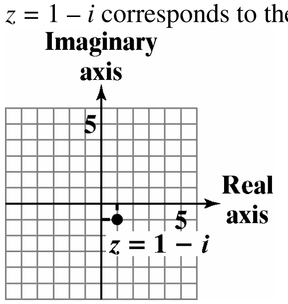
Calculate values of r for θ from 0 to $\frac{\pi}{2}$ and use symmetry to obtain the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
r	±1	±0.71	0	undef	undef



$r^2 = \cos 2\theta$

57. $z = 1 - i$ corresponds to the point $(1, -1)$.



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$, with $a = 1$ and $b = -1$, to find r and θ.

$$r = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$\tan\theta = \frac{-1}{1} = -1$$

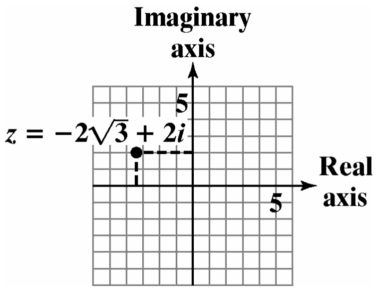
Because $\tan\frac{\pi}{4} = 1$ and θ lies in quadrant IV, $\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$.

$$z = 1 - i = r(\cos\theta + i\sin\theta)$$

$$= \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

or $\sqrt{2}(\cos 315° + i\sin 315°)$

58. $z = -2\sqrt{3} + 2i$ corresponds to the point $(-2\sqrt{3},\ 2)$.



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \dfrac{b}{a}$, with $a = -2\sqrt{3}$ and $b = 2$, to find r and θ.

$r = \sqrt{\left(-2\sqrt{3}\right)^2 + 2^2} = \sqrt{12+4} = \sqrt{16} = 4$

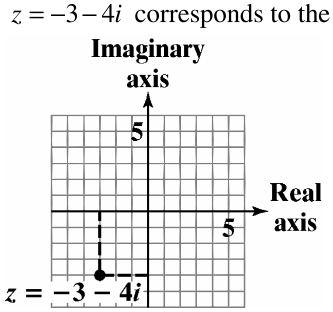
$\tan\theta = \dfrac{2}{-2\sqrt{3}} = -\dfrac{1}{\sqrt{3}}$

Because $\tan 30° = \dfrac{1}{\sqrt{3}}$ and θ lies in quadrant II, $\theta = 180° - 30° = 150°$.

$$\begin{aligned} z &= -2\sqrt{3} + 2i \\ &= r(\cos\theta + i\sin\theta) \\ &= 4(\cos 150° + i\sin 150°) \end{aligned}$$

or $4\left(\cos\dfrac{5\pi}{6} + i\sin\dfrac{5\pi}{6}\right)$

59. $z = -3 - 4i$ corresponds to the point $(-3, -4)$.



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \dfrac{b}{a}$, with $a = -3$ and $b = -4$, to find r and θ.

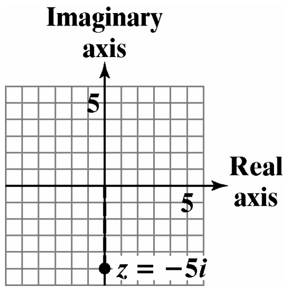
$r = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\tan\theta = \dfrac{-4}{-3} = \dfrac{4}{3}$

Because $\tan^{-1}\left(\dfrac{4}{3}\right) \approx 53°$ and θ lies in quadrant III, $\theta \approx 180° + 53° = 233°$.

$$\begin{aligned} z &= -3 - 4i = r(\cos\theta + i\sin\theta) \\ &\approx 5(\cos 233° + i\sin 233°) \end{aligned}$$

60. $z = -5i = 0 - 5i$ corresponds to the point $(0, -5)$.



Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$, with $a = 0$ and $b = -5$, to find r and θ.

$r = \sqrt{0^2 + (-5)^2} = \sqrt{25} = 5$

$\tan\theta = \frac{-5}{0}$ is undefined

Because $\tan\frac{\pi}{2}$ is undefined and θ lies on the negative y-axis, $\theta = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$.

$z = -5i = r(\cos\theta + i\sin\theta) = 5\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$ or $5(\cos 270° + i\sin 270°)$

61. $8(\cos 60° + i\sin 60°) = 8\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 4 + 4i\sqrt{3}$

The rectangular form of $z = 8(\cos 60° + i\sin 60°)$ is $z = 4 + 4i\sqrt{3}$.

62. $4(\cos 210° + i\sin 210°) = 4\left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right) = -2\sqrt{3} - 2i$

The rectangular form of $z = 4(\cos 210° + i\sin 210°)$ is $z = -2\sqrt{3} - 2i$.

63. $6\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 6\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -3 + 3i\sqrt{3}$

The rectangular form of $z = 6\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ is $z = -3 + 3i\sqrt{3}$.

64. $0.6(\cos 100° + i\sin 100°) \approx 0.6\left(-0.17 + i(0.98)\right) \approx -0.1 + 0.6i$

The rectangular form of $z = 0.6(\cos 100° + i\sin 100°)$ is $z \approx -0.1 + 0.6i$.

65.
$$\begin{aligned} z_1 z_2 &= \left[3(\cos 40° + i\sin 40°)\right]\left[5\left(\cos 70° + i\sin 70°\right)\right] \\ &= (3\cdot 5)\left[\cos(40° + 70°) + i\sin(40° + 70°)\right] \\ &= 15(\cos 110° + i\sin 110°) \end{aligned}$$

66.
$$\begin{aligned} z_1 z_2 &= \left[\cos 210° + i\sin 210°\right]\left[\cos 55° + i\sin 55°\right] \\ &= \cos\left(210° + 55°\right) + i\sin(210° + 55°) \\ &= \cos 265° + i\sin 265° \end{aligned}$$

67. $$z_1z_2 = \left[4\left(\cos\frac{3\pi}{7} + i\sin\frac{3\pi}{7}\right)\right]\left[10\left(\cos\frac{4\pi}{7} + i\frac{4\pi}{7}\right)\right]$$
$$= (4\cdot 10)\left[\cos\left(\frac{3\pi}{7} + \frac{4\pi}{7}\right) + i\sin\left(\frac{3\pi}{7} + \frac{4\pi}{7}\right)\right]$$
$$= 40(\cos\pi + i\sin\pi)$$

68. $$\frac{z_1}{z_2} = \frac{10(\cos 10° + i\sin 10°)}{5(\cos 5° + i\sin 5°)} = \frac{10}{5}\left[\cos(10° - 5°) + i\sin(10° - 5°)\right] = 2(\cos 5° + i\sin 5°)$$

69. $$\frac{z_1}{z_2} = \frac{5\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)}{10\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)} = \frac{5}{10}\left[\cos\left(\frac{4\pi}{3} - \frac{\pi}{3}\right) + i\sin\left(\frac{4\pi}{3} - \frac{\pi}{3}\right)\right] = \frac{1}{2}(\cos\pi + i\sin\pi)$$

70. $$\frac{z_1}{z_2} = \frac{2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)}{\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}} = 2\left[\cos\left(\frac{5\pi}{3} - \frac{\pi}{2}\right) + i\sin\left(\frac{5\pi}{3} - \frac{\pi}{2}\right)\right]$$
$$= 2\left[\cos\left(\frac{10\pi}{6} - \frac{3\pi}{6}\right) + i\sin\left(\frac{10\pi}{6} - \frac{3\pi}{6}\right)\right] = 2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$$

71. $$\left[2(\cos 20° + i\sin 20°)\right]^3 = (2)^3\left[\cos(3\cdot 20°) + i\sin(3\cdot 20°)\right] = 8(\cos 60° + i\sin 60°)$$
$$= 8\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 4 + 4\sqrt{3}i$$

72. $$\left[4(\cos 50° + i\sin 50°)\right]^3 = (4)^3\left[\cos(3\cdot 50°) + i\sin(3\cdot 50°)\right] = 64(\cos 150° + i\sin 150°) = 64\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$$
$$= -32\sqrt{3} + 32i$$

73. $$\left[\frac{1}{2}\left(\cos\frac{\pi}{14} + i\sin\frac{\pi}{14}\right)\right]^7 = \left(\frac{1}{2}\right)^7\left[\cos\left(7\cdot\frac{\pi}{14}\right) + i\sin\left(7\cdot\frac{\pi}{14}\right)\right] = \frac{1}{128}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$
$$= \frac{1}{128}(0 + i1) = \frac{1}{128}i$$

74. Write $1-i\sqrt{3}$ in $r(\cos\theta+i\sin\theta)$ form.

$r=\sqrt{a^2+b^2}=\sqrt{1^2+\left(-\sqrt{3}\right)^2}=\sqrt{1+3}=2$

$\tan\theta=\dfrac{b}{a}=\dfrac{-\sqrt{3}}{1}=-\sqrt{3}$

Because $\tan 60°=\sqrt{3}$ and θ lies in quadrant IV, $\theta=360°-60°=300°$.

$1-i\sqrt{3}=r(\cos\theta+i\sin\theta)=2(\cos 300°+i\sin 300°)$

Use DeMoivre's Theorem to raise $1-i\sqrt{3}$ to the seventh power.

$$\begin{aligned}\left(1-i\sqrt{3}\right)^7&=\left[2(\cos 300°+i\sin 300°)\right]^7\\&=(2)^7\left[\cos(7\cdot 300°)+i\sin(7\cdot 300°)\right]\\&=128(\cos 2100°+i\sin 2100°)\\&=128(\cos 300°+i\sin 300°)\\&=128\left(\frac{1}{2}+i\left(-\frac{\sqrt{3}}{2}\right)\right)\\&=64-64i\sqrt{3}\end{aligned}$$

75. Write $-2-2i$ in $r(\cos\theta+i\sin\theta)$ form.

$r=\sqrt{a^2+b^2}=\sqrt{(-2)^2+(-2)^2}=\sqrt{4+4}=2\sqrt{2}$

$\tan\theta=\dfrac{b}{a}=\dfrac{-2}{-2}=1$

Because $\tan\dfrac{\pi}{4}=1$ and θ lies in quadrant III, $\theta=\pi+\dfrac{\pi}{4}=\dfrac{5\pi}{4}$.

$-2-2i=r(\cos\theta+i\sin\theta)=2\sqrt{2}\left(\cos\dfrac{5\pi}{4}+i\sin\dfrac{5\pi}{4}\right)$

Use DeMoivre's Theorem to raise $-2-2i$ to the fifth power.

$$\begin{aligned}(-2-2i)^5&=\left[2\sqrt{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)\right]^5\\&=\left(2\sqrt{2}\right)^5\left[\cos\left(5\cdot\frac{5\pi}{4}\right)+i\sin\left(5\cdot\frac{5\pi}{4}\right)\right]\\&=128\sqrt{2}\left(\cos\frac{25\pi}{4}+i\sin\frac{25\pi}{4}\right)\\&=128\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\\&=128\sqrt{2}\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)\\&=128+128i\end{aligned}$$

76. $49(\cos 50° + i\sin 50°)$

$$z_k = \sqrt[2]{49}\left[\cos\left(\frac{50° + 360°k}{2}\right) + i\sin\left(\frac{50° + 360°k}{2}\right)\right],\ k = 0, 1$$

$$z_0 = \sqrt{49}\left[\cos\left(\frac{50° + 360° \cdot 0}{2}\right) + i\sin\left(\frac{50° + 360° \cdot 0}{2}\right)\right] = \sqrt{49}\left[\cos\left(\frac{50°}{2}\right) + i\sin\left(\frac{50°}{2}\right)\right]$$

$$= 7(\cos 25° + i\sin 25°)$$

$$z_1 = \sqrt{49}\left[\cos\left(\frac{50° + 360° \cdot 1}{2}\right) + i\sin\left(\frac{50° + 360° \cdot 1}{2}\right)\right] = \sqrt{49}\left[\cos\left(\frac{410°}{2}\right) + i\sin\left(\frac{410°}{2}\right)\right]$$

$$= 7(\cos 205° + i\sin 205°)$$

77. $125(\cos 165° + i\sin 165°)$

$$z_k = \sqrt[3]{125}\left[\cos\left(\frac{165° + 360°k}{3}\right) + i\sin\left(\frac{165° + 360°k}{3}\right)\right],\ k = 0, 1, 2$$

$$z_0 = \sqrt[3]{125}\left[\cos\left(\frac{165° + 360° \cdot 0}{3}\right) + i\sin\left(\frac{165° + 360° \cdot 0}{3}\right)\right] = \sqrt[3]{125}\left[\cos\left(\frac{165°}{3}\right) + i\sin\left(\frac{165°}{3}\right)\right]$$

$$= 5(\cos 55° + i\sin 55°)$$

$$z_1 = \sqrt[3]{125}\left[\cos\left(\frac{165° + 360° \cdot 1}{3}\right) + i\sin\left(\frac{165° + 360° \cdot 1}{3}\right)\right] = \sqrt[3]{125}\left[\cos\left(\frac{525°}{3}\right) + i\sin\left(\frac{525°}{3}\right)\right]$$

$$= 5(\cos 175° + i\sin 175°)$$

$$z_2 = \sqrt[3]{125}\left[\cos\left(\frac{165° + 360° \cdot 2}{3}\right) + i\sin\left(\frac{165° + 360° \cdot 2}{3}\right)\right] = \sqrt[3]{125}\left[\cos\left(\frac{885°}{3}\right) + i\sin\left(\frac{885°}{3}\right)\right]$$

$$= 5(\cos 295° + i\sin 295°)$$

78. $16\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$$z_k = \sqrt[4]{16}\left[\cos\left(\frac{\frac{2\pi}{3} + 2\pi k}{4}\right) + i\sin\left(\frac{\frac{2\pi}{3} + 2\pi k}{4}\right)\right],\ k = 0, 1, 2, 3$$

$$z_0 = \sqrt[4]{16}\left[\cos\left(\frac{\frac{2\pi}{3} + 2\pi \cdot 0}{4}\right) + i\sin\left(\frac{\frac{2\pi}{3} + 2\pi \cdot 0}{4}\right)\right] = \sqrt[4]{16}\left[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right] = 2\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \sqrt{3} + i$$

$$z_1 = \sqrt[4]{16}\left[\cos\left(\frac{\frac{2\pi}{3} + 2\pi \cdot 1}{4}\right) + i\sin\left(\frac{\frac{2\pi}{3} + 2\pi \cdot 1}{4}\right)\right] = \sqrt[4]{16}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -1 + i\sqrt{3}$$

$$z_2 = \sqrt[4]{16}\left[\cos\left(\frac{\frac{2\pi}{3} + 2\pi \cdot 2}{4}\right) + i\sin\left(\frac{\frac{2\pi}{3} + 2\pi \cdot 2}{4}\right)\right] = \sqrt[4]{16}\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right) = -\sqrt{3} - i$$

$$z_3 = \sqrt[4]{16}\left[\cos\left(\frac{\frac{2\pi}{3} + 2\pi \cdot 3}{4}\right) + i\sin\left(\frac{\frac{2\pi}{3} + 2\pi \cdot 3}{4}\right)\right] = \sqrt[4]{16}\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) = 2\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) = 1 - i\sqrt{3}$$

79. $8i = 8(\cos 90° + i\sin 90°)$

$$z_k = \sqrt[3]{8}\left[\cos\left(\frac{90° + 360°k}{3}\right) + i\sin\left(\frac{90° + 360°k}{3}\right)\right],\ k = 0, 1, 2$$

$$z_0 = \sqrt[3]{8}\left[\cos\left(\frac{90° + 360°\cdot 0}{3}\right) + i\sin\left(\frac{90° + 360°\cdot 0}{3}\right)\right] = \sqrt[3]{8}(\cos 30° + i\sin 30°) = 2\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \sqrt{3} + i$$

$$z_1 = \sqrt[3]{8}\left[\cos\left(\frac{90° + 360°\cdot 1}{3}\right) + i\sin\left(\frac{90° + 360°\cdot 1}{3}\right)\right] = \sqrt[3]{8}(\cos 150° + i\sin 150°) = 2\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = -\sqrt{3} + i$$

$$z_2 = \sqrt[3]{8}\left[\cos\left(\frac{90° + 360°\cdot 2}{3}\right) + i\sin\left(\frac{90° + 360°\cdot 2}{3}\right)\right] = \sqrt[3]{8}(\cos 270° + i\sin 270°) = 2(0 + i(-1)) = -2i$$

80. $-1 = \cos 180° + i\sin 180°$

$$z_k = \sqrt[3]{1}\left[\cos\left(\frac{180° + 360°k}{3}\right) + i\sin\left(\frac{180° + 360°k}{3}\right)\right],\ k = 0, 1, 2$$

$$z_0 = \sqrt[3]{1}\left[\cos\left(\frac{180° + 360°\cdot 0}{3}\right) + i\sin\left(\frac{180° + 360°\cdot 0}{3}\right)\right] = \sqrt[3]{1}(\cos 60° + i\sin 60°) = 1\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z_1 = \sqrt[3]{1}\left[\cos\left(\frac{180° + 360°\cdot 1}{3}\right) + i\sin\left(\frac{180° + 360°\cdot 1}{3}\right)\right] = \sqrt[3]{1}(\cos 180° + i\sin 180°) = 1(-1 + i0) = -1$$

$$z_2 = \sqrt[3]{1}\left[\cos\left(\frac{180° + 360°\cdot 2}{3}\right) + i\left(\frac{180° + 360°\cdot 2}{3}\right)\right] = \sqrt[3]{1}(\cos 300° + i\sin 300°) = 1\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

81. $-1 - i = \sqrt{2}(\cos 225° + i\sin 225°)$

$$z_k = \sqrt[5]{\sqrt{2}}\left[\cos\left(\frac{225° + 360°k}{5}\right) + i\sin\left(\frac{225° + 360°k}{5}\right)\right],\ k = 0, 1, 2, 3, 4$$

$$z_0 = \sqrt[5]{\sqrt{2}}\left[\cos\left(\frac{225° + 360°\cdot 0}{5}\right) + i\sin\left(\frac{225° + 360°\cdot 0}{5}\right)\right] = \sqrt[5]{\sqrt{2}}(\cos 45° + i\sin 45°) = \sqrt[5]{\sqrt{2}}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt[5]{8}}{2} + \frac{\sqrt[5]{8}}{2}i$$

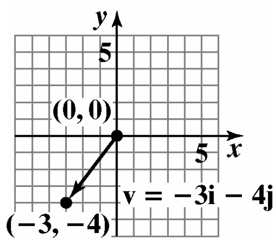
$$z_1 = \sqrt[5]{\sqrt{2}}\left[\cos\left(\frac{225° + 360°\cdot 1}{5}\right) + i\sin\left(\frac{225° + 360°\cdot 1}{5}\right)\right] = \sqrt[5]{\sqrt{2}}(\cos 117° + i\sin 117°) \approx -0.49 + 0.95i$$

$$z_2 = \sqrt[5]{\sqrt{2}}\left[\cos\left(\frac{225° + 360°\cdot 2}{5}\right) + i\sin\left(\frac{225° + 360°\cdot 2}{5}\right)\right] = \sqrt[5]{\sqrt{2}}(\cos 189° + i\sin 189°) \approx -1.06 - 0.17i$$

$$z_3 = \sqrt[5]{\sqrt{2}}\left[\cos\left(\frac{225° + 360°\cdot 3}{5}\right) + i\sin\left(\frac{225° + 360°\cdot 3}{5}\right)\right] = \sqrt[5]{\sqrt{2}}(\cos 261° + i\sin 261°) \approx -0.17 - 1.06i$$

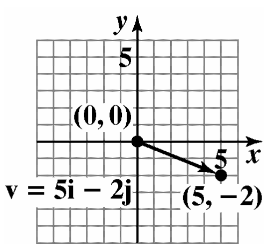
$$z_4 = \sqrt[5]{\sqrt{2}}\left[\cos\left(\frac{225° + 360°\cdot 4}{5}\right) + i\sin\left(\frac{225° + 360°\cdot 4}{5}\right)\right] = \sqrt[5]{\sqrt{2}}(\cos 333° + i\sin 333°) \approx 0.95 - 0.49i$$

82.



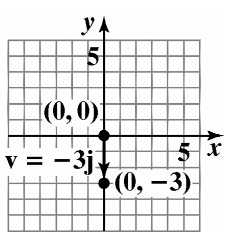
$$\|\mathbf{v}\| = \sqrt{a^2+b^2}$$
$$= \sqrt{(-3)^2+(-4)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$

83.



$$\|\mathbf{v}\| = \sqrt{a^2+b^2}$$
$$= \sqrt{5^2+(-2)^2}$$
$$= \sqrt{25+4}$$
$$= \sqrt{29}$$

84.



$$\|\mathbf{v}\| = \sqrt{a^2+b^2}$$
$$= \sqrt{0^2+(-3)^2}$$
$$= \sqrt{0+9}$$
$$= \sqrt{9}$$
$$= 3$$

85. $\mathbf{v} = (x_2-x_1)\mathbf{i}+(y_2-y_1)\mathbf{j}$
$$= (5-2)\mathbf{i}+[-3-(-1)]\mathbf{j}$$
$$= 3\mathbf{i}-2\mathbf{j}$$

86. $\mathbf{v} = (x_2-x_1)\mathbf{i}+(y_2-y_1)\mathbf{j}$
$$= [-2-(-3)\mathbf{i}]+(-2-0)\mathbf{j}$$
$$= \mathbf{i}-2\mathbf{j}$$

87. $\mathbf{v}+\mathbf{w} = (\mathbf{i}-5\mathbf{j})+(-2\mathbf{i}+7\mathbf{j})$
$$= [1+(-2)]\mathbf{i}+[-5+7]\mathbf{j}$$
$$= -\mathbf{i}+2\mathbf{j}$$

88. $\mathbf{w}-\mathbf{v} = (-2\mathbf{i}+7\mathbf{j})-(\mathbf{i}-5\mathbf{j})$
$$= (-2-1)\mathbf{i}+[7-(-5)]\mathbf{j}$$
$$= -3\mathbf{i}+12\mathbf{j}$$

89. $6\mathbf{v}-3\mathbf{w} = 6(\mathbf{i}-5\mathbf{j})-3(-2\mathbf{i}+7\mathbf{j})$
$$= 6\mathbf{i}-30\mathbf{j}+6\mathbf{i}-21\mathbf{j}$$
$$= 12\mathbf{i}-51\mathbf{j}$$

90. $\|-2\mathbf{v}\| = |-2|\,\|\mathbf{v}\|$
$$= 2\|\mathbf{v}\|$$
$$= 2\sqrt{a^2+b^2}$$
$$= 2\sqrt{1^2+(-5)^2}$$
$$= 2\sqrt{1+25}$$
$$= 2\sqrt{26}$$

91. First, find the magnitude of $\mathbf{v}$.
$$\|\mathbf{v}\| = \sqrt{a^2+b^2}$$
$$= \sqrt{8^2+(-6)^2}$$
$$= \sqrt{64+36}$$
$$= \sqrt{100}$$
$$= 10$$
A unit vector in the same direction as $\mathbf{v}$ is
$$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{8\mathbf{i}-6\mathbf{j}}{10} = \frac{4}{5}\mathbf{i}-\frac{3}{5}\mathbf{j}.$$

92. First, find the magnitude of $\mathbf{v}$.
$$\|\mathbf{v}\| = \sqrt{a^2+b^2} = \sqrt{(-1)^2+(2)^2} = \sqrt{1+4} = \sqrt{5}$$
A unit vector in the same direction as $\mathbf{v}$ is
$$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{-\mathbf{i}+2\mathbf{j}}{\sqrt{5}} = -\frac{1}{\sqrt{5}}\mathbf{i}+\frac{2}{\sqrt{5}}\mathbf{j}.$$

93. $\mathbf{v} = \|\mathbf{v}\|\cos\theta\,\mathbf{i} + \|\mathbf{v}\|\sin\theta\,\mathbf{j}$

$= 12\cos 60°\mathbf{i} + 12\sin 60°\mathbf{j}$

$= 12\left(\frac{1}{2}\right)\mathbf{i} + 12\left(\frac{\sqrt{3}}{2}\right)\mathbf{j}$

$= 6\mathbf{i} + 6\sqrt{3}\mathbf{j}$

94. $\mathbf{F_1} = 100\cos 65^\circ\,\mathbf{i} + 100\sin 65^\circ\,\mathbf{j}$

$= 42.3\mathbf{i} + 90.6\mathbf{j}$

$\mathbf{F_2} = 200\cos 10^\circ\,\mathbf{i} + 200\sin 10^\circ\,\mathbf{j}$

$= 197\mathbf{i} + 34.7\mathbf{j}$

$\mathbf{F_1} + \mathbf{F_2} = (42.3 + 197)\mathbf{i} + (90.6 + 34.7)\mathbf{j}$

$= 239.3\mathbf{i} + 125.3\mathbf{j}$

$\sqrt{239.3^2 + 125.3^2} \approx 270$ pounds

$\cos\theta = \frac{239.3}{270}$

$\theta = 27.6^\circ$

95. $\mathbf{v} = 15\cos 25^\circ\,\mathbf{i} + 15\sin 25^\circ\,\mathbf{j} = 13.6\mathbf{i} + 6.3\mathbf{j}$
$\mathrm{w} = 4\cos 270^\circ\,\mathbf{i} + 4\sin 270^\circ\,\mathbf{j} = -4\mathbf{j}$

a. $13.6\mathbf{i} + (6.3 - 4)\mathbf{j} = 13.6\mathbf{i} + 2.3\mathbf{j}$

b. $\sqrt{13.6^2 + 2.3^2} \approx 14$ mph

c. $\cos\theta = \frac{13.6}{14}; \theta = 13.7^\circ$

96. $\mathbf{v}\cdot(\mathbf{v} + \mathbf{w}) = (5\mathbf{i} + 2\mathbf{j})[(\mathbf{i} - \mathbf{j}) + (3\mathbf{i} - 7\mathbf{j})]$

$= (5\mathbf{i} + 2\mathbf{j})\cdot[4\mathbf{i} - 8\mathbf{j}]$

$= 5(4) + 2(-8)$

$= 20 - 16$

$= 4$

97. $\mathbf{v}\cdot\mathbf{w} = (2\mathbf{i} + 3\mathbf{j})\cdot(7\mathbf{i} - 4\mathbf{j}) = 2(7) + 3(-4) = 2$

$\cos\theta = \frac{2}{\sqrt{2^2 + 3^2}\sqrt{7^2 + (-4)^2}}$

$= \frac{2}{\sqrt{13}\sqrt{65}}$

$= \frac{2}{\sqrt{845}}$

The angle θ between the vectors is

$\theta = \cos^{-1}\left(\frac{2}{\sqrt{845}}\right) \approx 86.1°$.

98. $\mathbf{v}\cdot\mathbf{w} = (2\mathbf{i} + 4\mathbf{j})\cdot(6\mathbf{i} - 11\mathbf{j}) = 2(6) + 4(-11)$

$= 12 - 44$

$= -32$

$\cos\theta = \frac{-32}{\sqrt{2^2 + 4^2}\sqrt{6^2 + (-11)^2}}$

$= \frac{-32}{\sqrt{20}\sqrt{157}}$

$= \frac{-32}{\sqrt{3140}}$

The angle θ between the vectors is

$\theta = \cos^{-1}\left(-\frac{32}{\sqrt{3140}}\right) \approx 124.8°$.

99. $\mathbf{v}\cdot\mathbf{w} = (2\mathbf{i} + \mathbf{j})\cdot(\mathbf{i} - \mathbf{j}) = 2(1) + 1(-1)$

$= 2 - 1 = 1$

$\cos\theta = \frac{1}{\sqrt{2^2 + 1^2}\sqrt{1^2 + (-1)^2}}$

$= \frac{1}{\sqrt{5}\sqrt{2}}$

$= \frac{1}{\sqrt{10}}$

The angle θ between the vectors is

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{10}}\right) \approx 71.6°$.

100. $\mathbf{v}\cdot\mathbf{w} = (12\mathbf{i} - 8\mathbf{j})\cdot(2\mathbf{i} + 3\mathbf{j})$

$= 12(2) + (-8)(3)$

$= 24 - 24$

$= 0$

The dot product is zero. Thus, the given vectors are orthogonal.

101. $\mathbf{v}\cdot\mathbf{w} = (\mathbf{i} + 3\mathbf{j})\cdot(-3\mathbf{i} - \mathbf{j})$

$= 1(-3) + 3(-1)$

$= -3 - 3$

$= -6$

The dot product is not zero. Thus, the given vectors are not orthogonal.

102. $\text{proj}_{\mathbf{w}}\mathbf{v} = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}$

$$= \frac{(-2\mathbf{i}+5\mathbf{j})\cdot(5\mathbf{i}+4\mathbf{j})}{\left(\sqrt{5^2+4^2}\right)^2}\mathbf{w}$$

$$= \frac{-2(5)+5(4)}{\left(\sqrt{41}\right)^2}\mathbf{w}$$

$$= \frac{10}{41}(5\mathbf{i}+4\mathbf{j})$$

$$= \frac{50}{41}\mathbf{i}+\frac{40}{41}\mathbf{j}$$

$$\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = \frac{50}{41}\mathbf{i}+\frac{40}{41}\mathbf{j}$$

$$\mathbf{v}_2 = \mathbf{v}-\mathbf{v}_1 = (-2\mathbf{i}+5\mathbf{j})-\left(\frac{50}{41}\mathbf{i}+\frac{40}{41}\mathbf{j}\right)$$

$$= -\frac{132}{41}\mathbf{i}+\frac{165}{41}\mathbf{j}$$

103. $\text{proj}_{\mathbf{w}}\mathbf{v} = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}$

$$= \frac{(-\mathbf{i}+2\mathbf{j})\cdot(3\mathbf{i}-\mathbf{j})}{\left(\sqrt{3^2+(-1)^2}\right)^2}\mathbf{w}$$

$$= \frac{-1(3)+2(-1)}{\left(\sqrt{10}\right)^2}\mathbf{w}$$

$$= \frac{-5}{10}\mathbf{w}$$

$$= -\frac{1}{2}(3\mathbf{i}-1\mathbf{j})$$

$$= \frac{-3}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}$$

$$\mathbf{v}_1 = \text{proj}_{\mathbf{w}}\mathbf{v} = \frac{-3}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}$$

$$\mathbf{v}_2 = \mathbf{v}-\mathbf{v}_1 = (-\mathbf{i}+2\mathbf{j})-\left(\frac{-3}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}\right)$$

$$= \frac{1}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}$$

104. $W = \|\mathbf{F}\|\left\|\overrightarrow{AB}\right\|\cos\theta$

$$= (30)(50)\cos 42^\circ$$

$$\approx 1115$$

The work done is approximately 1115 foot-pounds.

Chapter 7 Test

1. The known ratio is $\dfrac{a}{\sin A}$, or $\dfrac{4.8}{\sin 34^\circ}$. Because angle B is given, we use the Law of Sines to find side b

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin 68^\circ} = \frac{4.8}{\sin 34^\circ}$$

$$b = \frac{4.8\sin 68^\circ}{\sin 34^\circ} \approx 8.0$$

2. Use the Law of Cosines to find c.

$$c^2 = a^2+b^2-2ab\cos C$$

$$c^2 = 5^2+6^2-2(5)(6)\cos 68^\circ$$

$$= 61-60\cos 68^\circ$$

$$\approx 38.52$$

$$c \approx \sqrt{38.52} \approx 6.2$$

3. $s = \dfrac{1}{2}(a+b+c) = \dfrac{1}{2}(17+45+32) = 47$

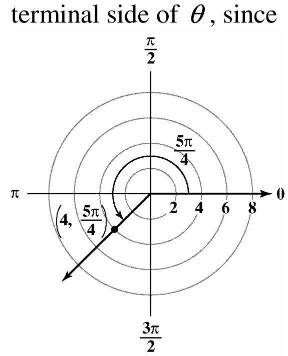
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{47(47-17)(47-45)(47-32)}$$

$$= \sqrt{42{,}300} \approx 206$$

The area of the triangle is approximately 206 square inches.

4. Draw $\theta = \dfrac{5\pi}{4} = 225^\circ$ counterclockwise, since θ is positive, from the polar axis. Go 4 units out on the terminal side of θ, since $r > 0$.



Ordered pairs may vary.

5. (1, –1)

$r = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$

$\tan\theta = \frac{-1}{1} = -1$

Because $\tan\frac{\pi}{4} = 1$ and θ lies in quadrant IV, $\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$.

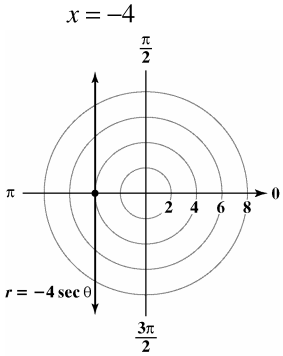
The polar coordinated of (1, –1) are $(r, \theta) = \left(\sqrt{2}, \frac{7\pi}{4}\right)$.

6.

$$x^2 + (y+8)^2 = 64$$
$$(r\cos\theta)^2 + (r\sin\theta + 8)^2 = 64$$
$$r^2\cos^2\theta + r^2\sin^2\theta + 16r\sin\theta + 64 = 64$$
$$r^2 + 16r\sin\theta = 0$$
$$r^2 = -16r\sin\theta$$
$$r = -16\sin\theta$$

7.

$$r = -4\sec\theta$$
$$r = \frac{-4}{\cos\theta}$$
$$r\cos\theta = -4$$
$$x = -4$$



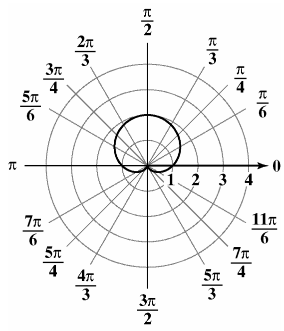
8. $r = 1 + \sin\theta$

Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = 1 + \sin(-\theta)$	$-r = 1 + \sin(-\theta)$	$-r = 1 + \sin\theta$
	$-r = 1 - \sin\theta$	
$r = 1 - \sin\theta$	$r = -1 + \sin\theta$	$r = -1 - \sin\theta$

There may be no symmetry, since each equation is not equivalent to $r = 1 + \sin\theta$.
Calculate values of *r* for θ from 0 to 2π.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
r	1	1.5	1.87	2	1.87	1.5	1	0.5	0.13	0	0.13	0.5	1



$r = 1 + \sin\theta$

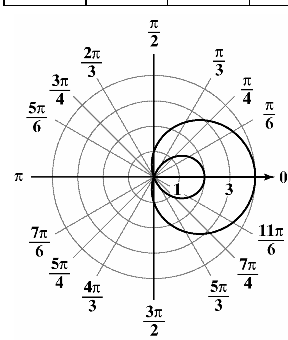
9. $r = 1 + 3\cos\theta$

Check for symmetry:

Polar Axis	**The Line** $\theta = \frac{\pi}{2}$	**The Pole**
$r = 1 + 3\cos(-\theta)$	$-r = 1 + 3\cos(-\theta)$	$-r = 1 + 3\cos\theta$
	$-r = 1 + 3\cos\theta$	
$r = 1 + 3\cos\theta$	$r = -1 - 3\cos\theta$	$r = -1 - 3\cos\theta$

The graph has symmetry with respect to the polar axis. The graph may or may not be symmetric with respect to the line $\theta = \frac{\pi}{2}$ or the pole. Calculate values of r for θ from 0 to π and use symmetry to complete the graph.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	4	3.6	2.5	1	–0.5	–1.6	–2



$r = 1 + 3\cos\theta$

10. Use $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$, with $a = -\sqrt{3}$ and $b = 1$, to find r and θ.

$r = \sqrt{\left(-\sqrt{3}\right)^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

$\tan\theta = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}}$

Because $\tan 30° = \frac{1}{\sqrt{3}}$ and θ lies in quadrant II, $\theta = 180° - 30° = 150°$.

The polar form of $z = -\sqrt{3} + i$ is $z = r(\cos\theta + i\sin\theta) = 2(\cos 150° + i\sin 150°)$ or $2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$.

11. $5(\cos 15° + i\sin 15°)\cdot 10(\cos 5° + i\sin 5°) = (5\cdot 10)\left[\cos(15°+5°) + i\sin(15°+5°)\right]$
$= 50(\cos 20° + i\sin 20°)$

12. $\dfrac{2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)}{4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)} = \dfrac{2}{4}\left[\cos\left(\frac{\pi}{2} - \frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right)\right] = \dfrac{2}{4}\left[\cos\left(\frac{3\pi}{6} - \frac{2\pi}{6}\right) + i\sin\left(\frac{3\pi}{6} - \frac{2\pi}{6}\right)\right]$
$= \dfrac{1}{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

13. $\left[2(\cos 10° + i\sin 10°)\right]^5 = (2)^5\left[\cos(5\cdot 10°) + i\sin(5\cdot 10°)\right] = 32(\cos 50° + i\sin 50°)$

14. $27 = 27(\cos 0° + i\sin 0°)$

$z_k = \sqrt[3]{27}\left[\cos\left(\dfrac{0° + 360°k}{3}\right) + i\sin\left(\dfrac{0° + 360°k}{3}\right)\right],\ k = 0, 1, 2$

$z_0 = \sqrt[3]{27}\left[\cos\left(\dfrac{0° + 360°\cdot 0}{3}\right) + i\sin\left(\dfrac{0° + 360°\cdot 0}{3}\right)\right] = \sqrt[3]{27}(\cos 0° + i\sin 0°) = 3(1 + 0i) = 3$

$z_1 = \sqrt[3]{27}\left[\cos\left(\dfrac{0° + 360°\cdot 1}{3}\right) + i\sin\left(\dfrac{0° + 360°\cdot 1}{3}\right)\right] = \sqrt[3]{27}\left(\cos 120° + i\sin 120°\right) = 3\left(-\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right) = -\dfrac{3}{2} + \dfrac{3\sqrt{3}}{2}i$

$z_2 = \sqrt[3]{27}\left[\cos\left(\dfrac{0° + 360°\cdot 2}{3}\right) + i\sin\left(\dfrac{0° + 360°\cdot 2}{3}\right)\right] = \sqrt[3]{27}(\cos 240° + i\sin 240°) = 3\left(-\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i\right) = -\dfrac{3}{2} - \dfrac{3\sqrt{3}}{2}i$

15. **a.** $\mathbf{v} = \left(x_2 - x_1\right)\mathbf{i} + \left(y_2 - y_1\right)\mathbf{j}$
$\mathbf{v} = [-1 - (-2)]\mathbf{i} + (5 - 3)\mathbf{j} = \mathbf{i} + 2\mathbf{j}$

b. $\|\mathbf{v}\| = \sqrt{a^2 + b^2} = \sqrt{1^2 + 2^2} = \sqrt{1 + 4} = \sqrt{5}$

16. $3\mathbf{v} - 4\mathbf{w} = 3(-5\mathbf{i} + 2\mathbf{j}) - 4(2\mathbf{i} - 4\mathbf{j}) = -15\mathbf{i} + 6\mathbf{j} - 8\mathbf{i} + 16\mathbf{j}$
$= (-15 - 8)\mathbf{i} + (6 + 16)\mathbf{j} = -23\mathbf{i} + 22\mathbf{j}$

17. $\mathbf{v}\cdot\mathbf{w} = (-5\mathbf{i} + 2\mathbf{j})\cdot(2\mathbf{i} - 4\mathbf{j}) = -5(2) + 2(-4) = -10 - 8 = -18$

18. $\cos\theta = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|}\mathbf{w}$

$= \dfrac{(-5\mathbf{i} + 2\mathbf{j})\cdot(2\mathbf{i} - 4\mathbf{j})}{\sqrt{(-5)^2 + 2^2}\sqrt{2^2 + (-4)^2}}$

$= \dfrac{-5(2) + 2(-4)}{\sqrt{29}\sqrt{20}}$

$= -\dfrac{18}{\sqrt{580}}$

The angle θ between the vectors is $\theta = \cos^{-1}\left(-\dfrac{18}{\sqrt{580}}\right) \approx 138°$.

19. $\text{proj}_{\mathbf{w}}\mathbf{v} = \frac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{w}\|^2}\mathbf{w}$

$$= \frac{(-5\mathbf{i}+2\mathbf{j})\cdot(2\mathbf{i}-4\mathbf{j})}{\left(\sqrt{2^2+(-4)^2}\right)^2}\mathbf{w}$$

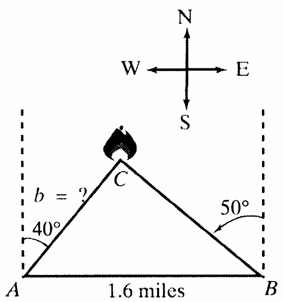
$$= \frac{-5(2)+2(-4)}{\left(\sqrt{20}\right)^2}\mathbf{w}$$

$$= -\frac{18}{20}\mathbf{w}$$

$$= -\frac{9}{10}(2\mathbf{i}-4\mathbf{j})$$

$$= -\frac{9}{5}\mathbf{i}+\frac{18}{5}\mathbf{j}$$

20.



Using the figure,

$B = 90° - 50° = 40°$

$A = 90° - 40° = 50°$

$C = 180° - B - A = 180° - 40° - 50° = 90°$

Use the Law of Sines to find *b*. $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\frac{b}{\sin 40°} = \frac{1.6}{\sin 90°}$$

$$b = \frac{1.6\sin 40°}{\sin 90°} \approx 1.0$$

The fire is about 1.0 mile from the station.

21. $\mathbf{F_1} = 250\cos 30°\,\mathbf{i} + 250\sin 30°\,\mathbf{j}$

$= 216.5\mathbf{i}+125\mathbf{j}$

$\mathbf{F}_2 = 150\cos 315^{\text{o}}\,\mathbf{i}+150\sin 315^{\text{o}}\,\mathbf{j}$

$= 106\mathbf{i}-106\mathbf{j}$

$\mathbf{F}_1+\mathbf{F}_2 = (216.5+106)\mathbf{i}+(125-106)\mathbf{j}$

$= 322.5\mathbf{i}+19\mathbf{j}$

$\|\mathbf{F}_1+\mathbf{F}_2\| = \sqrt{322.5^2+19^2} \approx 323$ pounds

$\cos\theta = \frac{322.5}{323} = 3.4^{\text{o}}$

22. $W = \|\mathbf{F}\| \|\overrightarrow{AB}\| \cos\theta$

$= (40)(60)\cos 35° \approx 1966$

The work done is approximately 1966 foot-pounds.

Cumulative Review Exercises (Chapters 1–7)

1. $x^4 - x^3 - x^2 - x - 2 = 0$

$\frac{p}{q}: \pm\frac{2}{1}, \pm\frac{1}{1}$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & -1 & -1 & -2 \\ & & -1 & 2 & -1 & 2 \\ \hline & 1 & -2 & 1 & -2 & 0 \end{array}$$

$$x^4 - x^3 - x^2 - x - 2 = 0$$
$$(x+1)(x^3 - 2x^2 + x - 2) = 0$$
$$(x+1)[x^2(x-2) + 1(x-2)] = 0$$
$$(x+1)(x-2)(x^2+1) = 0$$

$x+1=0 \quad x-2=0 \quad x^2+1=0$

$x=-1 \quad x=2 \quad x^2=-1$

$x = \pm i$

The solution set is $\{-1, 2\ i, -i\}$.

2. $2\sin^2\theta - 3\sin\theta + 1 = 0,\ 0 \le \theta < 2\pi$

$(2\sin\theta - 1)(\sin\theta - 1) = 0$

$2\sin\theta - 1 = 0$ or $\sin\theta - 1 = 0$

$2\sin\theta = 1 \quad \sin\theta = 1$

$\sin\theta = \frac{1}{2}$

The solutions in the interval $\left[0, 2\pi\right)$ are $\frac{\pi}{6}, \frac{5\pi}{6}$, and $\frac{\pi}{2}$.

3. Begin by solving the related quadratic equation. Thus, we will solve $x^2 + 2x + 3 = 11$.

$x^2 + 2x + 3 = 11$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x+4=0$ or $x-2=0$

$x=-4$ or $x=2$

The boundary points are –4 and 2. The boundary points divide the number line into three test intervals, namely $(-\infty, -4)$, (–4, 2), and $(2, \infty)$. Take one representative number within each test interval and substitute that number into the original inequality.

Test Interval	Representative Number	Substitute into $x^2+2x+3>11$	Conclusion
$(-\infty, -4)$	-5	$(-5)^2+2(-5)+3>11$ $18>11$ True	$(-\infty, -4)$ belongs to the solution set.
$(-4, -2)$	0	$0^2+2(0)+3>11$ $3>11$ False	$(-4, -2)$ does not belong to the solution set.
$(2, \infty)$	3	$3^2+2(3)+3>11$ $18>11$ True	$(2, \infty)$ belongs to the solution set.

The solution set is $\{x \mid x<-4 \text{ or } x>2\}$.

4. $\sin\theta\cos\theta = -\frac{1}{2}$

$$\frac{\sin 2\theta}{2} = -\frac{1}{2}$$

$$\sin 2\theta = -1$$

The period of the sine function is 2π. In the interval $[0, 2\pi)$, the only value for which the sine function is –1 is $\frac{3\pi}{2}$.

This means that $2\theta = \frac{3\pi}{2}$. Since the period is 2π, all the solutions to $\sin 2\theta = -1$ are given by $2\theta = \frac{3\pi}{2} + 2n\pi$

$$\theta = \frac{3\pi}{4} + n\pi$$

where n is any integer.

The solution in the interval $[0, 2\pi)$ is obtained by letting $n = 0$ and $n = 1$. The solutions are $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$.

5. The equation $y = 3\sin(2x-\pi)$ is of the form $y = A\sin(Bx-C)$ with $A = 3$, $B = 2$, and $C = \pi$. The amplitude is $|A| = |3| = 3$. The period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$. The phase shift is $\frac{C}{B} = \frac{\pi}{2}$. The quarter-period is $\frac{\pi}{4}$. The cycle begins at $x = \frac{\pi}{2}$. Add quarter-periods to generate x-values for the key points.

$$x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

$$x = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$$

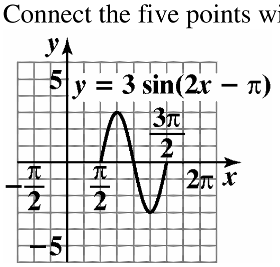
$$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$x = \frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$$

We evaluate the function at each value of x.

x	$y = 3\sin(2x-\pi)$	coordinates
$\frac{\pi}{2}$	$y = 3\sin\left(2\cdot\frac{\pi}{2}-\pi\right)$ $= 3\sin(\pi-\pi)$ $= 3\sin 0 = 3\cdot 0 = 0$	$\left(\frac{\pi}{2}, 0\right)$
$\frac{3\pi}{4}$	$y = 3\sin\left(2\cdot\frac{3\pi}{4}-\pi\right)$ $= 3\sin\left(\frac{3\pi}{2}-\pi\right)$ $= 3\sin\frac{\pi}{2} = 3\cdot 1 = 3$	$\left(\frac{3\pi}{4}, 3\right)$
π	$y = 3\sin(2\cdot\pi-\pi)$ $= 3\sin(2\pi-\pi)$ $= 3\sin\pi = 3\cdot 0 = 0$	$(\pi, 0)$
$\frac{5\pi}{4}$	$y = 3\sin\left(2\cdot\frac{5\pi}{4}-\pi\right)$ $= 3\sin\left(\frac{5\pi}{2}-\pi\right)$ $= 3\sin\frac{3\pi}{2}$ $= 3(-1) = -3$	$\left(\frac{5\pi}{4}, -3\right)$
$\frac{3\pi}{2}$	$y = 3\sin\left(2\cdot\frac{3\pi}{2}-\pi\right)$ $= 3\sin(3\pi-\pi)$ $= 3\sin 2\pi = 3\cdot 0 = 0$	$\left(\frac{3\pi}{2}, 0\right)$

Connect the five points with a smooth curve and graph one complete cycle of the given function.



6. The equation $y = -4\cos \pi x$ is of the form $y = A\cos Bx$ with $A = -4$, and $B = \pi$. Thus, the amplitude is $|A| = |-4| = 4$.

The period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$.

Find the x-values for the five key points by dividing the period, 2, by 4, $\frac{\text{period}}{4} = \frac{2}{4} = \frac{1}{2}$, then by adding quarter- periods to the value of x where the cycle begins, $x = 0$. The five x-values are

$x = 0$

$x = 0 + \frac{1}{2} = \frac{1}{2}$

$x = \frac{1}{2} + \frac{1}{2} = 1$

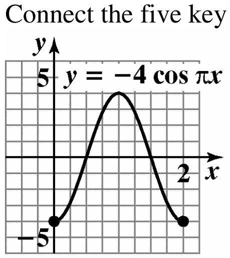
$x = 1 + \frac{1}{2} = \frac{3}{2}$

$x = \frac{3}{2} + \frac{1}{2} = 2$

We evaluate the function at each value of x.

x	$y = -4\cos \pi x$	coordinates
0	$y = -4\cos(\pi \cdot 0)$ $= -4\cos 0$ $= -4 \cdot 1 = -4$	(0, –4)
$\frac{1}{2}$	$y = -4\cos\left(\pi \cdot \frac{1}{2}\right)$ $= -4\cos\frac{\pi}{2}$ $= -4 \cdot 0 = 0$	$\left(\frac{1}{2}, 0\right)$
1	$y = -4\cos(\pi \cdot 1)$ $= -4\cos \pi$ $= -4 \cdot (-1) = 4$	(1, 4)
$\frac{3}{2}$	$y = -4\cos\left(\pi \cdot \frac{3}{2}\right)$ $= -4\cos\frac{3\pi}{2}$ $= -4 \cdot 0 = 0$	$\left(\frac{3}{2}, 0\right)$
2	$y = -4\cos(\pi \cdot 2)$ $= -4\cos 2\pi$ $= -4 \cdot 1 = -4$	(2, –4)

Connect the five key points with a smooth curve and graph one complete cycle of the given function.



7. $\sin\theta\csc\theta-\cos^2\theta=\sin\theta\left(\dfrac{1}{\sin\theta}\right)-\cos^2\theta$

$$=1-\cos^2\theta$$
$$=\sin^2\theta$$

8. $\cos\left(\theta+\dfrac{3\pi}{2}\right)=\cos\theta\cos\dfrac{3\pi}{2}-\sin\theta\sin\dfrac{3\pi}{2}$

$$=\cos\theta(0)-\sin\theta(-1)$$
$$=\sin\theta$$

9. $2x+4y-8=0$

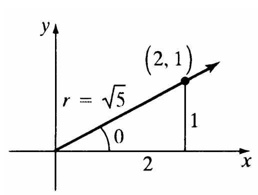
$$4y=-2x+8$$
$$\frac{4y}{4}=\frac{-2x+8}{4}$$
$$y=-\frac{1}{2}x+2$$

The slope is $-\dfrac{1}{2}$, and the y-intercept is 2.

10. $2\sin\dfrac{\pi}{3}-3\tan\dfrac{\pi}{6}=2\left(\dfrac{\sqrt{3}}{2}\right)-3\left(\dfrac{1}{\sqrt{3}}\right)$

$$=\sqrt{3}-\frac{3}{\sqrt{3}}$$
$$=\sqrt{3}-\sqrt{3}$$
$$=0$$

11. Let $\theta=\tan^{-1}\left(\dfrac{1}{2}\right)$, then $\tan\theta=\dfrac{1}{2}$. Because $\tan\theta$ is positive, θ is in the first quadrant.
Use the Pythagorean Theorem to find r.
$r=\sqrt{1^2+2^2}=\sqrt{1+4}=\sqrt{5}$



Use the right triangle to find the exact value.
$\sin\left(\tan^{-1}\dfrac{1}{2}\right)=\sin\theta=\dfrac{1}{\sqrt{5}}=\dfrac{\sqrt{5}}{5}$

12. $f(x)=\sqrt{5-x}$

$$5-x\ge 0$$
$$-x\ge -5$$
$$x\le 5$$

The domain of the function is $\{x\,|\,x\le 5\}$.

13. $g(x)=\dfrac{x-3}{x^2-9}$

$$x^2-9=0$$
$$(x-3)(x+3)=0$$
$$x-3=0 \quad\text{or}\quad x+3=0$$
$$x=3 \qquad x=-3$$

The domain of the function is $\{x\,|\,x\ne 3 \text{ and } x\ne -3\}$.

14. $s(t)=-16t^2+48t+8$

$$=-16\left(t^2-3t-\frac{1}{2}\right)$$
$$=-16\left(t^2-3t+\frac{9}{4}-\frac{1}{2}-\frac{9}{4}\right)$$
$$=-16\left[\left(t-\frac{3}{2}\right)^2-\frac{1}{2}-\frac{9}{4}\right]$$
$$=-16\left(t-\frac{3}{2}\right)^2+44$$

The ball reaches its maximum height after the first 1.5 seconds. The maximum height is 44 feet.

15. $d=4\sin 5t$ is of the form $d=a\sin\omega t$ with $a=4$ and $\omega=5$.

a. $|a|=|4|=4$
The maximum displacement is 4 meters.

b. $f=\dfrac{\omega}{2\pi}=\dfrac{5}{2\pi}$
The frequency is $\dfrac{5}{2\pi}$ cycle per second.

c. $\text{period}=\dfrac{2\pi}{\omega}=\dfrac{2\pi}{5}$
$\dfrac{2\pi}{5}$ seconds are required for one cycle.

16. Because 22.5° lies in quadrant I, $\cos 22.5° > 0$.

$$\cos 22.5° = \cos\frac{45°}{2} = \sqrt{\frac{1+\cos 45°}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

17. a. $3\mathbf{v}-\mathbf{w} = 3(2\mathbf{i}+7\mathbf{j})-(\mathbf{i}-2\mathbf{j})$
$= 6\mathbf{i}+21\mathbf{j}-\mathbf{i}+2\mathbf{j}$
$= 5\mathbf{i}+23\mathbf{j}$

b. $\mathbf{v}\cdot\mathbf{w} = (2\mathbf{i}+7\mathbf{j})\cdot(\mathbf{i}-2\mathbf{j})$
$= 2(1)+7(-2) = 2-14$
$= -12$

18. $\frac{1}{2}\log_b x - \log_b(x^2+1)$
$= \log_b x^{1/2} - \log_b(x^2+1)$
$= \log_b \sqrt{x} - \log_b(x^2+1)$
$= \log_b \frac{\sqrt{x}}{x^2+1}$

19. (4, –1) and (–8, 5)

$m = \frac{5-(-1)}{-8-4} = \frac{6}{-12} = -\frac{1}{2}$

$y-(-1) = -\frac{1}{2}(x-4)$

$y+1 = -\frac{1}{2}x+2$

$y = -\frac{1}{2}x+1$

20. $L = A\left(1-e^{-kt}\right)$

a. $20 = 300\left(1-e^{-k(5)}\right)$

$20 = 300-300e^{-5k}$

$300e^{-5k} = 280$

$e^{-5k} = \frac{14}{15}$

$\ln\left(e^{-5k}\right) = \ln\left(\frac{14}{15}\right)$

$-5k = \ln\left(\frac{14}{15}\right)$

$k = -\frac{\ln\left(\frac{14}{15}\right)}{5} \approx 0.014$

b. $L = 300\left(1-e^{-0.014(20)}\right) \approx 73$

After 20 minutes, the student will have learned approximately 73 words.

$260 = 300\left(1-e^{-0.014t}\right)$

$\frac{13}{15} = 1-e^{-0.014t}$

$-\frac{2}{15} = -e^{-0.014t}$

$\frac{2}{15} = e^{-0.014t}$

$\ln\left(\frac{2}{15}\right) = \ln\left(e^{-0.014t}\right)$

$\ln\left(\frac{2}{15}\right) = -0.014t$

$t = -\frac{\ln\left(\frac{2}{15}\right)}{0.014} \approx 144$

It will take about 144 minutes.

Chapter 8
Systems of Equations and Inequalities

Section 8.1

Check Point Exercises

1. **a.**

$$2x = 3y = -4$$
$$2(1) - 3(2) = -4$$
$$2 - 6 = -4$$
$$-4 = -4 \text{ true}$$
$$2x + y = 4$$
$$2(1) + 2 = 4$$
$$2 + 2 = 4$$
$$4 = 4 \text{ true}$$

(1, 2) is a solution of the system.

b.

$$2x = 3y = -4$$
$$2(7) - 3(6) = -4$$
$$14 - 18 = -4$$
$$-4 = -4 \text{ true}$$
$$2x + y = 4$$
$$2(7) + 6 = 4$$
$$14 + 6 = 4$$
$$20 = 4 \text{ false}$$

(7, 6) is not a solution of the system.

2.

$$3x + 2y = 4$$
$$2x + y = 1$$

Solve $2x + y = 1$ for y.

$$2x + y = 1$$
$$y = 1 - 2x$$

Substitute $1 - 2x$ for y in the other equation and solve.

$$3x + 2\overbrace{(1 - 2x)}^{y} = 4$$
$$3x + 2 - 4x = 4$$
$$-x = 2$$
$$x = -2$$

Back-substitute the obtained value:

$$3x + 2y = 4$$
$$3(-2) + 2y = 4$$
$$-6 + 2y = 4$$
$$2y = 10$$
$$y = 5$$

Checking these values in both equations shows that (–2, 5) is the solution of the system.

3. Rewrite one or both equations:

$$4x + 5y = 3 \xrightarrow{\text{No change}} 4x + 5y = 3$$
$$2x - 3y = 7 \xrightarrow{\text{Mult. by } -2} \underline{-4x + 6y = -14}$$
$$11y = -11$$
$$y = -1$$

Back-substitute into either equation:

$$4x + 5y = 3$$
$$4x + 5(-1) = 3$$
$$4x - 5 = 3$$
$$4x = 8$$
$$x = 2$$

Checking confirms the solution set is {(2, –1}.

4. Rewrite both equations in the form $Ax + By = C$:

$$2x = 9 + 3y \;\rightarrow\; 2x - 3y = 9$$
$$4y = 8 - 3x \;\rightarrow\; 3x + 4y = 8$$

Rewrite with opposite coefficients, then add and solve:

$$2x - 3y = 9 \xrightarrow{\text{Mult. by 4}} 8x - 12y = 36$$
$$3x + 4y = 8 \xrightarrow{\text{Mult. by 3}} \underline{9x + 12y = 24}$$
$$17x = 60$$
$$x = \frac{60}{17}$$

Back-substitute into either equation:

$$4y = 8 - 3x$$
$$4y = 8 - 3\left(\frac{60}{17}\right)$$
$$4y = -\frac{44}{17}$$
$$y = -\frac{11}{17}$$

Checking confirms the solution is $\left(\frac{60}{17}, -\frac{11}{17}\right)$.

5. Rewrite with a pair of opposite coefficients, then add:

$$\begin{array}{rcl} 5x-2y=4 & \xrightarrow{\text{Mult. by 2}} & 10x-4y=8 \\ -10x+4y=7 & \xrightarrow{\text{No change}} & \underline{-10x+4y=7} \\ & & 0=15 \end{array}$$

The statement 0 = 15 is false which indicates that the system has no solution. The solution set is the empty set, $\varnothing$.

6. Substitute $4y-8$ for x in the other equation:

$$5\overbrace{(4y-8)}^{x}-20y=-40$$
$$20y-40-20y=-40$$
$$-40=-40$$

The statement –40 = –40 is true which indicates that the system has infinitely many solutions. The solution set is $\{(x, y)|x=4y-8\}$ or $\{(x, y)|5x-20y=-40\}$.

7. **a.** $C(x)=300,000+30x$

b. $R(x)=80x$

c. $R(x)=C(x)$
$$80x=300,000+30x$$
$$50x=300,000$$
$$x=6000$$
$$C(6000)=300,000+30(6000)=480,000$$

Break even point (6000, 480000)
The company will need to make 6000 pairs of shoes and earn $480,000 to break even.

Exercise Set 8.1

1.
$x+3y=11$
$2+3(3)=11$
$2+9=11$
$11=11$ true
$x-5y=-13$
$2-5(3)=-13$
$2-15=-13$
$-13=-13$ true
(2, 3) is a solution.

2.
$9x+7y=8$
$9(-3)+7(5)=8$
$-27+35=8$
$8=8$ true
$8x-9y=-69$
$8(-3)-9(5)=-69$
$-24-45=-69$
$-69=-69$ true
(–3, 5) is a solution.

3.
$2x+3y=17$
$2(2)+3(5)=17$
$4+15=17$
$19=17$ false
(2, 5) is not a solution.

4.
$5x-4y=20$
$5(8)-4(5)=20$
$40-20=20$ true
$3y=2x+1$
$3(5)=2(8)+1$
$15=16+1$
$15=17$ false
(8, 5) is not a solution.

5.
$x+y=4$
$y=3x$
Substitute the expression $3x$ for y in the first equation and solve for x.
$x+3x=4$
$4x=4$
$x=1$
Substitute 1 for x in the second equation.
$y=3(1)=3$
The solution set is $\{(1, 3)\}$.

6.
$x+y=6$
$y=2x$
Substitute the expression $2x$ for y in the first equation and solve for x.
$x+2x=6$
$3x=6$
$x=2$
Substitute 2 for x in the second equation.
$y=2(2)=4$
The solution set is $\{(2, 4)\}$.

7. $x + 3y = 8$
$y = 2x - 9$
Substitute the expression $2x - 9$ for y in the first equation and solve for x.
$x + 3(2x - 9) = 8$
$x + 6x - 27 = 8$
$7x = 35$
$x = 5$
Substitute 5 for x in the second equation.
$y = 2(5) - 9 = 10 - 9 = 1$
The solution set is $\{(5, 1)\}$.

8. $2x - 3y = -13$
$y = 2x + 7$
Substitute the expression $2x + 7$ for y in the first equation and solve for x.
$2x - 3(2x + 7) = -13$
$2x - 6x - 21 = -13$
$-4x = 8$
$x = -2$
Substitute -2 for x in the second equation.
$y = 2(-2) + 7 = -4 + 7 = 3$
The solution set is $\{(-2, 3)\}$.

9. $x = 4y - 2$
$x = 6y + 8$
Substitute the expression $4y - 2$ for x in the second equation and solve for y.
$4y - 2 = 6y + 8$
$-10 = 2y$
$-5 = y$
Substitute -5 for y in the equation $x = 4y - 2$.
$x = 4(-5) - 2 = -22$
The solution set is $\{(-22, -5)\}$.

10. $x = 3y + 7$
$x = 2y - 1$
Substitute the expression $3y + 7$ for x in the second equation and solve for y.
$3y + 7 = 2y - 1$
$y = -8$
Substitute -8 for y in the equation $x = 3y + 7$.
$x = 3(-8) + 7 = -24 + 7 = -17$
The solution set is $\{(-17, -8)\}$.

11. $5x + 2y = 0$
$x - 3y = 0$
Solve the second equation for x.
$x = 3y$
Substitute the expression $3y$ for x in the first equation and solve for y.
$5(3y) + 2y = 0$
$15y + 2y = 0$
$17y = 0$
$y = 0$
Substitute 0 for y in the equation $x = 3y$
$y = 3(0) = 0$
The solution set is $\{(0, 0)\}$.

12. $4x + 3y = 0$
$2x - y = 0$
Solve the second equation for y.
$2x = y$
Substitute the expression $2x$ for y in the first equation and solve for x.
$4x + 3(2x) = 0$
$4x + 6x = 0$
$10x = 0$
$x = 0$
Substitute 0 for x in the equation $y = 2x$.
$y = 2(0) = 0$
The solution set is $\{(0, 0)\}$.

13. $2x + 5y = -4$
$3x - y = 11$
Solve the second equation for y.
$-y = -3x + 11$
$y = 3x - 11$
Substitute the expression $3x - 11$ for y in the first equation and solve for x.
$2x + 5(3x - 11) = -4$
$2x + 15x - 55 = -4$
$17x = 51$
$x = 3$
Substitute 3 for x in the equation $y = 3x - 11$.
$y = 3(3) - 11 = 9 - 11 = -2$
The solution set is $\{(3, -2)\}$.

14. $2x + 5y = 1$
$-x + 6y = 8$
Solve the second equation for x.
$-x+6y = 8$
$-x = -6y + 8$
$x = 6y - 8$
Substitute the expression $6y - 8$ for x in the first equation and solve for y.
$2(6y-8)+5y=1$
$12y-16+5y=1$
$17y=17$
$y=1$
Substitute 1 for y in the equation $x = 6y - 8$.
$x = 6(1) - 8 = -2$
The solution set is $\{(-2, 1)\}$.

15. $2x - 3y = 8 - 2x$
$2x + 4y = x + 3y + 14$
Solve the second equation for y.
$y = -2x + 14$
Substitute the expression $-2x + 14$ for y in the first equation and solve for x.
$2x-3(-2x+14)=8-2x$
$2x+6x-42=8-2x$
$8x-42=8-2x$
$10x=50$
$x=5$
Substitute 5 for x in the equation $y = -2x + 14$.
$y=-2(5)+14=-10+14=4$
The solution set is $\{(5, 4)\}$.

16. $3x - 4y = x - y + 4$
$2x + 6y = 5y - 4$
Solve the second equation for y.
$2x + 6y = 5y - 4$

$y = -2x - 4$
Substitute the expression $-2x - 4$ for y in the first equation and solve for x.
$3x - 4(-2x - 4) = x - (-2x - 4) + 4$
$3x + 8x + 16 = x + 2x + 4 + 4$
$8x = -8$
$x = -1$
Substitute -1 for x in the equation $y = -2x - 4$.
$y = -2(-1) - 4 = -2$
The solution set is $\{(-1, -2)\}$.

17. $y=\frac{1}{3}x+\frac{2}{3}$
$y=\frac{5}{7}x-2$
Substitute the expression $y=\frac{1}{3}x+\frac{2}{3}$ for y in the second equation and solve for x.
$\frac{1}{3}x+\frac{2}{3}=\frac{5}{7}x-2$
$7x+14=15x-42$
$56=8x$
$7=x$
Substitute 7 for x in the equation $y=\frac{1}{3}x+\frac{2}{3}$ and solve for y.
$y=\frac{1}{3}(7)+\frac{2}{3}=\frac{7}{3}+\frac{2}{3}=\frac{9}{3}=3$
The solution set is $\{(7, 3)\}$.

18. $y=-\frac{1}{2}x+2$
$y=\frac{3}{4}x+7$
Substitute the expression $y=-\frac{1}{2}x+2$ for y in the second equation. Multiply the equation by 4 to eliminate the fractions and then solve for x.
$-\frac{1}{2}x+2=\frac{3}{4}x+7$
$-2x+8=3x+28$
$-20=5x$
$-4=x$
Substitute -4 in the expression $y=-\frac{1}{2}x+2$ and solve for y.
$y=\frac{-1}{2}(-4)+2=2+4=4$
The solution set is $\{(-4, 4)\}$.

19. Eliminate y by adding the equations.
$x+y=1$
$\underline{x-y=3}$
$2x=4$
$x=2$
Substitute 2 for x in the first equation.
$2 + y = 1$
$y = -1$
The solution set is $\{(2, -1)\}$.

20. Eliminate y by adding the equations.

$$\begin{array}{r} x+y=6 \\ \underline{x-y=-2} \\ 2x=4 \\ x=2 \end{array}$$

Substitute 2 for x in the first equation.

$2+y=6$
$y=4$

The solution set is $\{(2, 4)\}$.

21. Eliminate y by adding the equations.

$$\begin{array}{r} 2x+3y=6 \\ \underline{2x-3y=6} \\ 4x=12 \\ x=3 \end{array}$$

Substitute 3 for x in the first equation.

$2(3)+3y=6$
$6+3y=6$
$3y=0$
$y=0$

The solution set is $\{(3, 0)\}$.

22. Eliminate y by adding the equations.

$$\begin{array}{r} 3x+2y=14 \\ \underline{3x-2y=10} \\ 6x=24 \\ x=4 \end{array}$$

Substitute 4 for x in the first equation.

$3(4)+2y=14$
$12+2y=14$
$2y=2$
$y=1$

The solution set is $\{(4, 1)\}$.

23. $x+2y=2$
$-4x+3y=25$

Eliminate x by multiplying the first equation by 4 and adding the resulting equations.

$$\begin{array}{r} 4x+8y=8 \\ \underline{-4x+3y=25} \\ 11y=33 \\ y=3 \end{array}$$

Substitute 3 for y in the first equation.

$x+2(3)=2$
$x+6=2$
$x=-4$

The solution set is $\{(-4, 3)\}$.

24. $2x-7y=2$
$3x+y=-20$

Eliminate y by multiplying the second equation by 7 and adding the resulting equations.

$$\begin{array}{r} 2x-7y=2 \\ \underline{21x+7y=-140} \\ 23x=-138 \\ x=-6 \end{array}$$

Substitute –6 for x in the second equation.

$3(-6)+y=-20$
$-18+y=-20$
$y=-2$

The solution set is $\{(-6, -2)\}$.

25. $4x+3y=15$
$2x-5y=1$

Eliminate x by multiplying the second equation by –2 and adding the resulting equations.

$$\begin{array}{r} 4x+3y=15 \\ \underline{-4x+10y=-2} \\ 13y=13 \\ y=1 \end{array}$$

Substitute 1 for y in the second equation.

$2x-5(1)=1$
$2x=6$
$x=3$

The solution set is $\{(3, 1)\}$.

26. $3x-7y=13$
$6x+5y=7$

Eliminate x by multiplying the first equation by –2 and adding the resulting equations.

$$\begin{array}{r} -6x+14y=-26 \\ \underline{6x+5y=7} \\ 19y=19 \\ y=-1 \end{array}$$

Substitute –1 for y in the first equation.

$3x-7(-1)=13$
$3x+7=13$
$3x=6$
$x=2$

The solution set is $\{(2, -1)\}$.

27. $3x - 4y = 11$
$2x + 3y = -4$
Eliminate x by multiplying the first equation by 2 and the second equation by –3. Add the resulting equations.

$$\begin{aligned} 6x - 8y &= 22 \\ -6x - 9y &= 12 \\ \hline -17y &= 34 \\ y &= -2 \end{aligned}$$

Substitute –2 for y in the second equation.

$$\begin{aligned} 2x + 3(-2) &= -4 \\ 2x - 6 &= -4 \\ 2x &= 2 \\ x &= 1 \end{aligned}$$

The solution set is $\{(1, -2)\}$.

28. $2x + 3y = -16$
$5x - 10y = 30$
Eliminate x by multiplying the first equation by –5 and the second equation by 2. Add the resulting equations.

$$\begin{aligned} -10x - 15y &= 80 \\ 10x - 20y &= 60 \\ \hline -35y &= 140 \\ y &= -4 \end{aligned}$$

Substitute –4 for y in the first equation.

$$\begin{aligned} 2x + 3(-4) &= -16 \\ 2x - 12 &= -16 \\ 2x &= -4 \\ x &= -2 \end{aligned}$$

The solution set is $\{(-2, -4)\}$.

29. $3x = 4y + 1$
$3y = 1 - 4x$
Arrange the system so that variable terms appear on the left and constants appear on the right.
$3x - 4y = 1$
$4x + 3y = 1$
Eliminate y by multiplying the first equation by 3 and the second equation by 4. Add the resulting equations.

$$\begin{aligned} 9x - 12y &= 3 \\ 16x + 12y &= 4 \\ \hline 25x &= 7 \\ x &= \frac{7}{25} \end{aligned}$$

Substitute $\frac{7}{25}$ for x in the second equation.

$$\begin{aligned} 3y &= 1 - 4\left(\frac{7}{25}\right) \\ 3y &= \frac{-3}{25} \\ y &= \frac{-1}{25} \end{aligned}$$

The solution set is $\left\{\left(\frac{7}{25}, -\frac{1}{25}\right)\right\}$.

30. $5x = 6y + 40$
$2y = 8 - 3x$
Arrange the system so that variable terms appear on the left and constants appear on the right.
$5x - 6y = 40$
$3x + 2y = 8$
Eliminate y by multiplying the second equation by 3 and adding the resulting equations.

$$\begin{aligned} 5x - 6y &= 40 \\ 9x + 6y &= 24 \\ \hline 14x &= 64 \\ x &= \frac{64}{14} = \frac{32}{7} \end{aligned}$$

Substitute $\frac{32}{7}$ for x in the second equation.

$$\begin{aligned} 2y &= 8 - 3\left(\frac{32}{7}\right) \\ 2y &= -\frac{40}{7} \\ y &= -\frac{20}{7} \end{aligned}$$

The solution set is $\left\{\left(\frac{32}{7}, -\frac{20}{7}\right)\right\}$.

31. The substitution method is used here to solve the system.
$x = 9 - 2y$
$x + 2y = 13$
Substitute the expression $9 - 2y$ for x in the second equation and solve for y.

$$\begin{aligned} 9 - 2y + 2y &= 13 \\ 9 &= 13 \end{aligned}$$

The false statement 9 = 13 indicates that the system has no solution.
The solution set is the empty set, $\varnothing$.

32. The substitution method is used here to solve the system.
$6x + 2y = 7$
$y = 2 - 3x$
Substitute the expression $2 - 3x$ for y in the first equation and solve for x.
$6x + 2(2 - 3x) = 7$
$6x + 4 - 6x = 7$
$4 = 7$
The false statement $4 = 7$ indicates that the system has no solution.
The solution set is the empty set, $\varnothing$.

33. The substitution method is used here to solve the system.
$y = 3x - 5$
$21x - 35 = 7y$
Substitute the expression $3x-5$ for y in the second equation and solve for x.
$21x - 35 = 7(3x - 5)$
$21x - 35 = 21x - 35$
$-35 = -35$
This true statement indicates that the system has infinitely many solutions.
The solution set is $\{(x, y) \mid y = 3x-5\}$

34. The substitution method is used here to solve the system.
$9x - 3y = 12$
$y = 3x - 4$
Substitute the expression $3x - 4$ for y in the first equation and solve for x.
$9x - 3(3x - 4) = 12$
$9x - 9x + 12 = 12$
$12 = 12$
This true statement indicates that the system has infinitely many solutions.
The solution set is $\{(x, y) \mid y = 3x-4\}$.

35. The elimination method is used here to solve the system.
$3x - 2y = -5$
$4x + y = 8$
Eliminate y by multiplying the second equation by 2 and adding the resulting equations.
$$\begin{aligned} 3x-2y&=-5\\ 8x+2y&=16\\ \hline 11x&=11\\ x&=1 \end{aligned}$$
Substitute 1 for x in the second equation.
$4(1) + y = 8$
$y = 4$
The solution set is $\{(1, 4)\}$.

36. The elimination method is used here to solve the system.
$2x+5y=-4$
$3x-y=11$
Eliminate y by multiplying the second equation by 5 and adding the resulting equations.
$$\begin{aligned} 2x+5y&=-4\\ 15x-5y&=55\\ \hline 17x&=51\\ x&=3 \end{aligned}$$
Substitute 3 for x in the second equation.
$3(3) - y = 11$
$9 - 11 = y$
$y = -2$
The solution set is $\{(3, -2)\}$.

37. The elimination method is used here to solve the system.
$x + 3y = 2$
$3x + 9y = 6$
Eliminate x by multiplying the first equation by -3 and adding the resulting equations.
$$\begin{aligned} -3x-9y&=-6\\ 3x+9y&=6\\ \hline 0&=0 \end{aligned}$$
This true statement indicates that the system has infinitely many solutions.
The solution set is $\{(x, y) \mid x+3y=2\}$.

38. The elimination method is used here to solve the system.
$4x - 2y = 2$
$2x - y = 1$
Eliminate x by multiplying the second equation by -2.
$$\begin{aligned} 4x-2y&=2\\ -4x+2y&=-2\\ \hline 0&=0 \end{aligned}$$
This true statement indicates that the system has infinitely many solutions.
The solution set is $\{(x, y) \mid 2x - y = 1\}$.

39. First multiply each term in the first equation by 4 to eliminate the fractions.

$\frac{x}{4} - \frac{y}{4} = -1$

$x - y = -4$

Multiply the first equation by –1 and add to the second equation and solve for *y*.

$-x + y = 4$

$x + 4y = -9$

$5y = -5$

$y = -1$

Substitute –1 for *y* in the equation $x - y = -4$ and solve for *x*.

$x - (-1) = -4$

$x + 1 = -4$

$x = -5$

The solution set is {(–5, –1)}.

40. Multiply the first equation by 6 to eliminate the fractions.

$x - 3y = 2$

$x + 2y = -3$

Eliminate *x* by multiplying the first equation by –1 and solve for *y*.

$-x + 3y = -2$

$x + 2y = -3$

$5y = -5$

$y = -1$

Substitute –1 for y into the equation $x - 3y = 2$ and solve for *x*.

$x - 3(-1) = 2$

$x\_ + 3 = 2$

$x = -1$

The solution set is {(–1, –1)}.

41. Rearrange the equations to get in the standard form.

$2x - 3y = 4$

$4x + 5y = 3$

Multiply the first equation by –2 and add to the second equation. Solve for *y*.

$-4x + 6y = -8$

$4x + 5y = 3$

$11y = -5$

$y = -\frac{5}{11}$

Multiply the first equation by 5 and the second equation by 3 and add the equations. Solve for *x*.

$10x - 15y = 20$

$12x + 15y = 9$

$22x = 29$

$x = \frac{29}{22}$

The solution set is $\left\{\left(\frac{29}{22}, -\frac{5}{11}\right)\right\}$.

42. Rearrange the equation so that the *x*'s and the *y*'s line up.

$4x - 3y = 8$

$2x - 5y = -14$

Multiply the second equation by -2 and add the result to the first equation and solve for *y*.

$4x - 3y = 8$

$-4x + 10y = 28$

$7y = 36$

$y = \frac{36}{7}$

Substitute *y* = 36/7 into the first equation and solve for *x*.

$4x - 3\left(\frac{36}{7}\right) = 8$

$4x - \frac{108}{7} = 8$

$28x - 108 = 56$

$28x = 164$

$x = \frac{164}{28}$

$x = \frac{41}{7}$

The solution set is $\left\{\left(\frac{41}{7}, \frac{36}{7}\right)\right\}$.

43. Add the equations to eliminate *y*.

$x + y = 7$

$x - y = -1$

$2x = 6$

$x = 3$

Substitute 3 for *x* in the first equation.

$3 + y = 7$

$y = 4$

The numbers are 3 and 4.

44. Add the equations to eliminate y.

$$\begin{aligned} x+y&=2\\ x-y&=8\\ \hline 2x&=10\\ x&=5 \end{aligned}$$

Substitute 5 for x in the first equation.

$5+y=2$

$y=-3$

The numbers are 5 and –3.

45. $3x-y=1$

$x+2y=12$

Eliminate y by multiplying the first equation by 2 and adding the resulting equations.

$$\begin{aligned} 6x-2y&=2\\ x+2y&=12\\ \hline 7x&=14\\ x&=2 \end{aligned}$$

Substitute 2 for x in the first equation.

$3(2)-y=1$

$6-y=1$

$-y=-5$

$y=5$

The numbers are 2 and 5.

46. $3x+2y=8$

$2x-y=3$

Eliminate y by multiplying the second equation by 2 and adding the resulting equations.

$$\begin{aligned} 3x+2y&=8\\ 4x-2y&=6\\ \hline 7x&=14\\ x&=2 \end{aligned}$$

Substitute 2 for x in the first equation.

$3(2)+2y=8$

$6+2y=8$

$2y=2$

$y=1$

The numbers are 2 and 1.

47. $\frac{x+2}{2}-\frac{y+4}{3}=3$

$\frac{x+y}{5}=\frac{x-y}{2}-\frac{5}{2}$

Start by multiplying each equation by its LCD and simplifying to clear the fractions.

$\frac{x+2}{2}-\frac{y+4}{3}=3$

$\frac{x+y}{5}=\frac{x-y}{2}-\frac{5}{2}$

Start by multiplying each equation by its LCD and simplifying to clear the fractions.

$$\begin{aligned} 6\left(\frac{x+2}{2}-\frac{y+4}{3}\right)&=6(3)\\ 3(x+2)-2(y+4)&=18\\ 3x+6-2y-8&=18\\ 3x-2y&=20 \end{aligned}$$

$$\begin{aligned} 10\left(\frac{x+y}{5}\right)&=10\left(\frac{x-y}{2}-\frac{5}{2}\right)\\ 2(x+y)&=5(x-y)-5(5)\\ 2x+2y&=5x-5y-25\\ 3x-7y&=25 \end{aligned}$$

We now need to solve the equivalent system of equations:

$3x-2y=20$

$3x-7y=25$

Subtract the two equations:

$$\begin{aligned} 3x-2y&=20\\ -(3x-7y&=25)\\ \hline 5y&=-5\\ y&=-1 \end{aligned}$$

Back-substitute this value for y and solve for x.

$$\begin{aligned} 3x-2y&=20\\ 3x-2(-1)&=20\\ 3x+2&=20\\ 3x&=18\\ x&=6 \end{aligned}$$

The solution is $(6,-1)$.

48. $\frac{x-y}{3}=\frac{x+y}{2}-\frac{1}{2}$

$\frac{x+2}{2}-4=\frac{y+4}{3}$

Start by multiplying each equation by its LCD and simplifying to clear the fractions.

$$6\left(\frac{x-y}{3}\right)=6\left(\frac{x+y}{2}-\frac{1}{2}\right)$$
$$2(x-y)=3(x+y)-3(1)$$
$$2x-2y=3x+3y-3$$
$$x+5y=3$$

$$6\left(\frac{x+2}{2}-4\right)=6\left(\frac{y+4}{3}\right)$$
$$3(x+2)-6(4)=2(y+4)$$
$$3x+6-24=2y+8$$
$$3x-2y=26$$

We now need to solve the equivalent system of equations:

$$x+5y=3$$
$$3x-2y=26$$

Multiply the first equation by -3 and then add the equations.

$$-3x-15y=-9$$
$$3x-\ 2y=26$$
$$-17y=17$$
$$y=-1$$

Back-substitute this value for *y* into one of the above equations and solve for *x*.

$$x+5(-1)=3$$
$$x-5=3$$
$$x=8$$

The solution is $(8,-1)$.

49. $5ax+4y=17$

$ax+7y=22$

Multiply the second equation by -5 and add the equations.

$$5ax+4y=17$$
$$-5ax-35y=-110$$
$$-31y=-93$$
$$y=3$$

Back-substitute into one of the original equations to solve for *x*.

$$ax+7y=22$$
$$ax+7(3)=22$$
$$ax+21=22$$
$$ax=1$$
$$x=\frac{1}{a}$$

The solution is $\left(\frac{1}{a},3\right)$.

50. $4ax+by=3$

$6ax+5by=8$

Multiply the first equation by -5 and add the equations.

$$-20ax-5by=-15$$
$$6ax+5by=8$$
$$-14ax=-7$$
$$x=\frac{1}{2a}$$

Back-substitute into one of the original equations to solve for *y*.

$$4a\left(\frac{1}{2a}\right)+by=3$$
$$2+by=3$$
$$by=1$$
$$y=\frac{1}{b}$$

The solution is $\left(\frac{1}{2a},\frac{1}{b}\right)$.

51. $f(-2)=11 \quad \rightarrow \quad -2m+b=11$

$f(3)=-9 \quad \rightarrow \quad 3m+b=-9$

We need to solve the resulting system of equations:

$$-2m+b=11$$
$$3m+b=-9$$

Subtract the two equations:

$$\begin{aligned} -2m+b&=11 \\ 3m+b&=-9 \\ \hline -5m&=20 \\ m&=-4 \end{aligned}$$

Back-substitute into one of the original equations to solve for *b*.

$$\begin{aligned} -2m+b&=11 \\ -2(-4)+b&=11 \\ 8+b&=11 \\ b&=3 \end{aligned}$$

Therefore, $m=-4$ and $b=3$.

52. $f(-3)=23 \quad \rightarrow \quad -3m+b=23$

$f(2)=-7 \quad \rightarrow \quad 2m+b=-7$

We need to solve the resulting system of equations:

$$-3m+b=23$$
$$2m+b=-7$$

Subtract the two equations:

$$\begin{aligned} -3m+b&=23 \\ 2m+b&=-7 \\ \hline -5m&=30 \\ m&=-6 \end{aligned}$$

Back-substitute into one of the original equations to solve for *b*.

$$\begin{aligned} -3m+b&=23 \\ -3(-6)+b&=23 \\ 18+b&=23 \\ b&=5 \end{aligned}$$

Therefore, $m=-6$ and $b=5$.

53. The solution to a system of linear equations is the point of intersection of the graphs of the equations in the system. If $(6,2)$ is a solution, then we need to find the lines that intersect at that point.
Looking at the graph, we see that the graphs of $x+3y=12$ and $x-y=4$ intersect at the point $(6,2)$. Therefore, the desired system of equations is

$$x+3y=12 \quad \text{or} \quad y=-\frac{1}{3}x+4$$
$$x-y=4 \qquad\qquad y=x-4$$

54. A system whose solution set is the empty set consists of parallel lines (assuming there are only two equations in the system). Therefore, we check the graph for two parallel lines.
From the graph, the desired system is

$$x-3y=-6 \quad \text{or} \quad y=\frac{1}{3}x+2$$
$$x-3y=6 \qquad\qquad y=\frac{1}{3}x-2$$

55. At the break-even point, $R(x)=C(x)$.

$$\begin{aligned} 10000+30x&=50x \\ 10000&=20x \\ 10000&=20x \\ 500&=x \end{aligned}$$

Five hundred radios must be produced and sold to break-even.

56. At the break-even point, $R(x)=C(x)$.

$$\begin{aligned} 10000+30x&=50x \\ 10000&=20x \\ 10000&=20x \\ 500&=x \end{aligned}$$

Five hundred radios must be produced and sold to break-even. So more than 500 radios must be produced and sold to have a profit.

57.
$$\begin{aligned} R(x)&=50x \\ R(200)&=50(200)=10000 \\ C(x)&=10000+30x \\ C(200)&=10000+30(200) \\ &=10000+6000=16000 \\ R(200)-C(200)&=10000-16000 \\ &=-6000 \end{aligned}$$

This means that if 200 radios are produced and sold the company will lose \$6,000.

58. $R(300) = 50(300) = 15000$
$C(300) = 10000 + 30(300)$
$= 10000 + 9000 = 19000$
$R(300) - C(300) = 15000 - 19000$
$= -4000$
This means that if 300 radios are produced and sold the company will lose $4,000.

59. **a.** $P(x) = R(x) - C(x)$
$= 50x - (10000 + 30x)$
$= 50x - 10000 - 30x$
$= 20x - 10000$
$P(x) = 20x - 10000$

b. $P(10000) = 20(10000) - 10000$
$= 200000 - 10000 = 190000$
If 10,0000 radios are produced and sold the profit will be $190,000.

60. **a.** $P(x) = R(x) - C(x)$
$= 50x - (10000 + 30x)$
$= 50x - 10000 - 30x$
$= 20x - 10000$

b. $P(20000) = 20(20000) - 10000$
$= 400000 - 10000$
$= 390000$
If 20,000 radios are produced and sold the profit will be $390,000.

61. **a.** The cost function is:
$C(x) = 18,000 + 20x$

b. The revenue function is:
$R(x) = 80x$

c. At the break-even point, $R(x) = C(x)$.
$80x = 18000 + 20x$
$60x = 18000$
$x = 300$
$R(x) = 80x$
$R(300) = 80(300)$
$= 24,000$
When approximately 300 canoes are produced the company will break-even with cost and revenue at $24,000.

62. **a.** The cost function is
$C(x) = 100,000 + 100x$.

b. The revenue function is $R(x) = 300x$.

c. At the break-even point, $R(x) = C(x)$.
$300x = 100,000 + 100x$
$200x = 100,000$
$x = 500$
$R(x) = 300x$
$R(500) = 300(500) = 150,000$
When 500 bicycles are produced and sold, both cost and revenue are $150,000.

63. **a.** The cost function is:
$C(x) = 30000 + 2500x$

b. The revenue function is:
$R(x) = 3125x$

c. At the break-even point, $R(x) = C(x)$.
$3125x = 30000 + 2500x$
$625x = 30000$
$x = 48$
After 48 sold out performances, the investor will break-even. ($150,000)

64. **a.** The cost function is $C(x) = 30,000 + 0.02x$

b. The revenue function is $R(x) = 0.5x$

c. At the break-even point,
$R(x) = C(x)$
$0.5x = 30,000 + 0.02x$
$0.48x = 30,000$
$x = 62,500$
$R(x) = 0.5x$
$R(62,500) = 0.5(62,500) = 31,250$
For 62,500 cards, both cost and revenue are $31,250.

65. **a.** Substitute $0.375x+3$ for p in the first equation.

$$p=-0.325x+5.8$$

$$\overbrace{0.375x+3}^{p}=-0.325x+5.8$$

$$0.375x+3=-0.325x+5.8$$

$$0.375x+0.325x+3=-0.325x+0.325x+5.8$$

$$0.7x+3=5.8$$

$$0.7x+3-3=5.8-3$$

$$0.7x=2.8$$

$$\frac{0.7x}{0.7}=\frac{2.8}{0.7}$$

$$x=4$$

Back-substitute to find p.

$$p=-0.325x+5.8$$

$$p=-0.325(4)+5.8=4.5$$

The ordered pair is (4,4.5).
Equilibrium number of workers: 4 million
Equilibrium hourly wage: $4.50

b. If workers are paid <u>$4.50</u> per hour, there will be <u>4</u> million available workers and <u>4</u> million workers will be hired. In this state of market equilibrium, there is no unemployment.

c.

$$p=-0.325x+5.8$$

$$5.15=-0.325x+5.8$$

$$0.65=-0.325x$$

$$\frac{-0.65}{-0.325}=\frac{-0.325x}{-0.325}$$

$$2=x$$

At $5.15 per hour, 2 million workers will be hired.

d.

$$p=0.375x+3$$

$$5.15=0.375x+3$$

$$2.15=0.375x$$

$$\frac{2.15}{0.375}=\frac{0.375x}{0.375}$$

$$x\approx 5.7$$

At $5.15 per hour, there will be about 5.7 million available workers.

e. $5.7-2=3.7$

At $5.15 per hour, there will be about 3.7 million more people looking for work than employers are willing to hire.

66. **a.** Demand model: $p=-50x+2000$

Supply model: $p=50x$

Use the substitution method.

$$p=-50x+2000$$

$$\overbrace{50x}^{p}=-50x+2000$$

$$50x=-50x+2000$$

$$100x=2000$$

$$x=20$$

Back-substitute 20 for x and find p.

$$p=50x$$

$$p=50(20)=1000$$

The solution set is $\{(20, 1000)\}$.
The equilibrium quantity is 20,000 and the equilibrium price is $1000.

b. When rents are <u>$1000</u> per month, consumers will demand <u>20,000</u> apartments and suppliers will offer <u>20,000</u> apartments for rent.

67.

$$x+2y=50$$

$$-x+2y=24$$

Add the equations.

$$x+2y=50$$

$$\underline{-x+2y=24}$$

$$4y=74$$

$$y=18.5$$

Back-substitute 18.5 for y and solve for x.

$$x+2y=50$$

$$x+2(18.5)=50$$

$$x+37=50$$

$$x=13$$

The solution is (13, 18.5). This means that 13 years after 1996, or 2009, the percentage who will be pro-life and pro-choice will be the same at 18.5%.

68. $13x+12y=992$
$-x+y=16$
Solve second equation for *y*.
$-x+y=16$
$y=x+16$
Substitute $x+16$ for *y* into first equation.
$13x+12y=992$
$13x+12(\overbrace{x+16}^{y})=992$
$13x+12(x+16)=992$
$13x+12x+192=992$
$25x=800$
$x=32$
Back-substitute to find *y*.
$y=x+16=32+16=48$
The solution is (32,48). This means that 32 years after 1988, or 2020, the percentage of Americans who are in favor of the death penalty and the percentage of Americans who oppose it will be the same at 48%.

69. **a.** $y=0.45x+0.8$

b. $y=0.15x+2.6$

c. To find the week in the semester when both groups report the same number of symptoms, we set the two equations equal to each other and solve for *x*.
$0.45x+0.8=0.15x+2.6$
$0.3x=1.8$
$x=6$
The number of symptoms will be the same in week 6.
$y=0.15x+2.6$
$y=0.15(6)+2.6$
$y=3.5$
The number of symptoms in week 6 will be 3.5 for both groups. This is shown in the graph by the intersection point (6, 3.5).

70. **a.** $y=0.04x+5.48$

b. $y=0.17x+1.84$

c. To find the year when the costs will be the same, we set the two equations equal to each other and solve for *x*.
$0.04x+5.48=0.17x+1.84$
$-0.13x=-3.64$
$x=28$
The costs will be the same 28 years after 2000, or 2028.
$y=0.17x+1.84$
$y=0.17(28)+1.84$
$y=6.6$
The cost of each program in 2028 will be 6.6% of the GDP. After that year, Medicare will have the greater cost.

71. **a.** $m=\dfrac{27.3-38}{20-0}=\dfrac{-10.7}{20}\approx-0.54$
From the point $(0,38)$ we have that the *y*-intercept is $b=38$. Therefore, the equation of the line is $y=-0.54x+38$.

b. $m=\dfrac{24.2-40}{20-0}=\dfrac{-15.8}{20}=-0.79$
From the point $(0,40)$ we have that the *y*-intercept is $b=40$. Therefore, the equation of the line is $y=-0.79x+40$.

c. To find the year when cigarette use is the same, we set the two equations equal to each other and solve for *x*.
$-0.54x+38=-0.79x+40$
$0.25x=2$
$x=8$
Cigarette use was the same for African Americans and Hispanics 8 years after 1985, or 1993.
$y=-0.54x+38$
$y=-0.54(8)+38$
$y=33.68$
At that time about 33.68% of each group used cigarettes.

72. **a.** $m = \frac{27.3 - 38.9}{20 - 0} = \frac{-11.6}{20} = -0.58$

From the point $(0, 38.9)$ we have that the y-intercept is $b = 38.9$. Therefore, the equation of the line is $y = -0.58x + 38.9$.

b. $m = \frac{24.2 - 40}{20 - 0} = \frac{-15.8}{20} = -0.79$

From the point $(0, 40)$ we have that the y-intercept is $b = 40$. Therefore, the equation of the line is $y = -0.79x + 40$.

c. To find the year when cigarette use is the same, we set the two equations equal to each other and solve for x.

$$-0.58x + 38.9 = -0.79x + 40$$
$$0.21x = 1.1$$
$$x \approx 5$$

Cigarette use was the same for whites and Hispanics 5 years after 1985, or 1990.

$$y = -0.58x + 38.9$$
$$y = -0.58(5) + 38.9$$
$$y = 36$$

At that time about 36% of each group used cigarettes.

73. Let x = the number of calories in a Mr. Goodbar.
Let y = the number of calories in a Mounds bar.

$$x + 2y = 780$$
$$2x + y = 786$$

Multiply the bottom equation by –2 and then add the equations to eliminate y.

$$\begin{aligned} x + 2y &= 780 \\ -4x - 2y &= -1572 \\ \hline -3x \quad &= -792 \\ x &= 264 \end{aligned}$$

Back-substitute to find y.

$$x + 2y = 780$$
$$264 + 2y = 780$$
$$2y = 516$$
$$y = 258$$

There are 264 calories in a Mr. Goodbar and 258 calories in a Mounds bar.

74. Let x = the number of calories in a Snickers bar.
Let y = the number of calories in a Reese's Peanut Butter Cup.

$$x + 2y = 737$$
$$2x + y = 778$$

Multiply the bottom equation by –2 and then add the equations to eliminate y.

$$\begin{aligned} x + 2y &= 737 \\ -4x - 2y &= -1556 \\ \hline -3x \quad &= -819 \\ x &= 273 \end{aligned}$$

Back-substitute to find y.

$$x + 2y = 737$$
$$273 + 2y = 737$$
$$2y = 464$$
$$y = 232$$

There are 273 calories in a Snickers bar and 232 calories in a Reese's Peanut Butter Cup.

75. Let x = the number of Mr. Goodbars.
Let y = the number of Mounds bars.

$$x + y = 5$$
$$16.3x + 14.1y - 70 = 7.1$$

Solve the first equation for y in terms of x.

$$x + y = 5$$
$$y = -x + 5$$

Substitute $-x + 5$ for y in the second equation.

$$16.3x + 14.1y - 70 = 7.1$$
$$16.3x + 14.1\overbrace{(-x + 5)}^{y} - 70 = 7.1$$
$$16.3x + 14.1(-x + 5) - 70 = 7.1$$
$$16.3x - 14.1x + 70.5 - 70 = 7.1$$
$$2.2x + 0.5 = 7.1$$
$$2.2x = 6.6$$
$$x = 3$$

Back-substitute to find y.

$$x + y = 5$$
$$3 + y = 5$$
$$y = 2$$

There are 3 Mr. Goodbars and 2 Mounds bars.

76. Let x = the number of Snickers bars.
Let y = the number of Reese's Peanut Butter Cups.

$$x+y=12$$
$$14.0x+13.7y-2(70)=26.5$$

Solve the first equation for y in terms of x.

$$x+y=12$$
$$y=-x+12$$

Substitute $-x+12$ for y in the second equation.

$$14.0x+13.7y-2(70)=26.5$$
$$14.0x+13.7(\overbrace{-x+12}^{y})-2(70)=26.5$$
$$14.0x+13.7(-x+12)-2(70)=26.5$$
$$14.0x-13.7x+164.4-140=26.5$$
$$0.3x+24.4=26.5$$
$$0.3x=2.1$$
$$x=7$$

Back-substitute to find y.

$$x+y=12$$
$$7+y=12$$
$$y=5$$

There are 7 Snickers bars and 5 Reese's Peanut Butter Cups.

77. $x + y = 200$
$100x + 80y = 17000$
Multiply the first equation by –100 and add to the second equation. Solve for y.

$$-100x-100y=-20000$$
$$100x+80y=17000$$
$$-20y=-3000$$
$$y=150$$

Substitute 150 for y in the first equation and solve for x.

$$x+150=200$$
$$x=50$$

There are 50 rooms with kitchenettes and 150 rooms without.

78. Let x = the number of two-seat tables.
Let y = the number of four-seat tables.

$$2x+4y=56$$
$$x+y=17$$

Multiply the second equation by -2 and add the two equations.

$$2x+4y=56$$
$$\underline{-2x-2y=-34}$$
$$2y=22$$
$$y=11$$

Back-substitute to find x.

$$x+y=17$$
$$x+11=17$$
$$x=6$$

The owners should buy 6 two-seat tables and 11 four-seat tables.

79. $2x + 2y = 360$
$20x + 8(2y) = 3280$
Multiply the first equation by −10 and add to the second equation. Solve for y.

$$-20x-20y=-3600$$
$$20x+16y=3280$$
$$-4y=-320$$
$$y=80$$

Substitute 80 for y in the first equation and solve for x.

$$2x+2(80)=360$$
$$2x+160=360$$
$$2x=200$$
$$x=100$$

The lot is 100 feet long and 80 feet wide.

80. Let x = width and y = length

$$2x+2y=320$$
$$5(2x)+16y=2140$$

$$-10x-10y=-1600$$
$$10x+16y=2140$$
$$6y=540$$
$$y=90$$

$$2x+2(90)=320$$
$$2x=140$$
$$x=70$$

The length is 90 feet and the width is 70 feet.

81. $(x + y)2 = 16$
$(x - y)2 = 8$
Multiply to remove the parentheses and then add the two equations together. Solve for *x*.

$$2x+2y=16$$
$$2x-2y=8$$
$$4x=24$$
$$x=6$$

Substitute 6 for *x* in the first equation and solve for *y*.

$$2(6)+2y=16$$
$$12+2y=16$$
$$2y=4$$
$$y=2$$

The crew rows 6 mph and the current is 2 mph.

82.

	rate	time	Distance
With wind	$x + y$	4	800
Without wind	$x - y$	5	800

$$4x+4y=800$$
$$5x-5y=800$$

$$-20x-20y=-4000$$
$$20x-20y=3200$$
$$-40x=-800$$
$$x=20$$

$$4x+4(20)=800$$
$$4x+80=800$$
$$4x=720$$
$$x=180$$

The speed of the airplane is 180 mph and the speed of the wind is 20 mph.

83.
$$x+2y=180$$
$$(2x-30)+y=180$$

Rewrite the second equation in standard form.

$$x+2y=180$$
$$2x+y=210$$

Multiply the first equation by -2 and add the equations.

$$-2x-4y=-360$$
$$2x+y=210$$
$$-3y=-150$$
$$y=50$$

Back-substitute to solve for *x*.

$$x+2y=180$$
$$x+2(50)=180$$
$$x+100=180$$
$$x=80$$

The three interior angles measure $80°$, $50°$, and $50°$.

84.
$$x+2y=180$$
$$(3x+15)+y=180$$

Rewrite the second equation in standard form.

$$x+2y=180$$
$$3x+y=165$$

Multiply the first equation by -3 and add the equations.

$$-3x-6y=-540$$
$$3x+y=165$$
$$-5y=-375$$
$$y=75$$

Back-substitute to solve for *x*.

$$x+2y=180$$
$$x+2(75)=180$$
$$x+150=180$$
$$x=30$$

The three interior angles measure $30°$, $75°$, and $75°$.

85. – 93. Answers may vary.

94. makes sense

95. makes sense

96. does not make sense; Explanations will vary. Sample explanation: Some linear systems have no solutions or one solution.

97. makes sense

98. Answers may vary.

99. $a_1x + b_1y = c_1$
$a_2x + b_2y = c_2$
Solve the first equation for *x*.
$$x = \frac{c_1 - b_1y}{a_1}$$
Substitute the expression $\frac{c_1 - b_1y}{a_1}$ for *x* in the second equation and solve for *y*.
$$a_2\left(\frac{c_1 - b_1y}{a_1}\right) + b_2y = c_2$$
$$a_2\left(\frac{c_1 - b_1y}{a_1}\right) + \frac{a_1b_2y}{a_1} = c_2$$
$$\frac{a_2c_1 - a_2b_1y + a_1b_2y}{a_1} = c_2$$
$$a_2c_1 - a_2b_1y + a_1b_2y = a_1c_2$$
$$y(a_1b_2 - a_2b_1) = a_1c_2 - a_2c_1$$
$$y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$$
Substitute the expression $\frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$ for *y* in the first equation and solve for *x*.
$$a_1x + b_1\left(\frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}\right) = c_1$$
$$a_1x + \frac{a_1b_1c_2 - a_2b_1c_1}{a_1b_2 - a_2b_1} = c_1$$
$$a_1x = c_1 - \frac{a_1b_1c_2 - a_2b_1c_1}{a_1b_2 - a_2b_1}$$
$$= \frac{c_1(a_1b_2 - a_2b_1)}{a_1b_2 - a_2b_1} - \frac{a_1b_1c_2 - a_2b_1c_1}{a_1b_2 - a_2b_1}$$
$$= \frac{a_1b_2c_1 - a_1b_1c_2}{a_1b_2 - a_2b_1}$$
$$x = \frac{a_1b_2c_1 - a_1b_1c_2}{a_1b_2 - a_2b_1} \div a_1$$
$$= \frac{a_1b_2c_1 - a_1b_1c_2}{a_1(a_1b_2 - a_2b_1)} = \frac{a_1(b_2c_1 - b_1c_2)}{a_1(a_1b_2 - a_2b_1)}$$
$$x = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1}$$

100. *x* = first lucky number
y = second lucky number
$3x + 6y = 12$
$x + 2y = 5$
Eliminate *x* by multiplying the second equation by –3 and adding the resulting equations.
$$\begin{aligned} 3x + 6y &= 12 \\ \underline{-3x - 6y} &\underline{= -15} \\ 0 &= -3 \end{aligned}$$
The false statement 0 = –3 indicates that the system has no solution. Therefore, the twin who always lies is talking.

101. *x* = number of hexagons formed
y = number of squares formed
$6x + y = 52$
$x + 4y = 24$
Eliminate *x* by multiplying the second equation by –6 and adding the resulting equations.
$$\begin{aligned} 6x + y &= 52 \\ \underline{-6x - 24y} &\underline{= -144} \\ -23y &= -92 \\ y &= 4 \end{aligned}$$
Substitute 4 for *y* in the second equation.
$$x + 4(4) = 24$$
$$x + 16 = 24$$
$$x = 8$$
Yes, they should make 8 hexagons and 4 squares.

102. Answers may vary.

103. $2x - y + 4z = -8$
$$2(3) - (2) + 4(-3) = -8$$
$$-8 = -8, \text{ true}$$
Yes, the ordered triple satisfies the equation.

104. $5x - 2y - 4z = 3$
$3x + 3y + 2z = -3$
Multiply Equation 2 by 2.
$5x - 2y - 4z = 3$
$6x + 6y + 4z = -6$
Then add to eliminate *z*.
$$\begin{aligned} 5x - 2y - 4z &= 3 \\ \underline{6x + 6y + 4z} &\underline{= -6} \\ 11x + 4y \quad &= -3 \end{aligned}$$

105. $ax^2 + bx + c = y$
$$a(4)^2 + b(4) + c = 1682$$
$$16a + 4b + c = 1682$$

Section 8.2

Check Point Exercises

1. $x - 2y + 3z = 22$

$-1 - 2(-4) + 3(5) = 22$

$-1 + 8 + 15 = 22$

$22 = 22$ true

$2x - 3y - z = 5$

$2(-1) - 3(-4) - 5 = 5$

$-2 + 12 - 5 = 5$

$5 = 5$ true

$3x + y - 5z = -32$

$3(-1) - 4 - 5(5) = -32$

$-3 - 4 - 25 = -32$

$-32 = -32$ true

(–1, –4, 5) is a solution of the system.

2. $x + 4y - z = 20$

$3x + 2y + z = 8$

$2x - 3y + 2z = -16$

Eliminate z from Equations 1 and 2 by adding Equation 1 and Equation 2.

$x + 4y - z = 20$

$\underline{3x + 2y + z = 8}$

$4x + 6y = 28$ Equation 4

Eliminate z from Equations 2 and 3 by multiplying Equation 2 by –2 and adding the resulting equation to Equation 3.

$-6x - 4y - 2z = -16$

$\underline{2x - 3y + 2z = -16}$

$-4x - 7y = -32$ Equation 5

Solve Equations 4 and 5 for x and y by adding Equation 4 and Equation 5.

$4x + 6y = 28$

$\underline{-4x - 7y = -32}$

$-y = -4$

$y = 4$

Substitute 4 for y in Equation 4 and solve for x.

$4x + 6(4) = 28$

$4x + 24 = 28$

$4x = 4$

$x = 1$

Substitute 1 for x and 4 for y in Equation 2 and solve for z.

$3(1) + 2(4) + z = 8$

$3 + 8 + z = 8$

$11 + z = 8$

$z = -3$

The solution set is {(1, 4, –3)}.

3. $2y - z = 7$

$x + 2y + z = 17$

$2x - 3y + 2z = -1$

Eliminate x and z from Equations 2 and 3 by multiplying Equation 2 by –2 and adding the resulting equation to Equation 3.

$-2x - 4y - 2z = -34$

$\underline{2x - 3y + 2z = -1}$

$-7y = -35$

$y = 5$

Substitute 5 for y in Equation 1 and solve for z.

$2(5) - z = 7$

$10 - z = 7$

$-z = -3$

$z = 3$

Substitute 5 for y and 3 for z in Equation 2 and solve for x.

$x + 2(5) + 3 = 17$

$x + 10 + 3 = 17$

$x + 13 = 17$

$x = 4$

The solution set is {(4, 5, 3)}.

4. (1, 4), (2, 1), (3, 4)

$y = ax^2 + bx + c$

Substitute 1 for x and 4 for y in

$y = ax^2 + bx + c.$

$4 = a(1)^2 + b(1) + c$

$4 = a + b + c$ Equation 1

Substitute 2 for x and 1 for y in

$y = ax^2 + bx + c.$

$1 = a(2)^2 + b(2) + c$

$1 = 4a + 2b + c$ Equation 2

Substitute 3 for x and 4 for y in

$y = ax^2 + bx + c.$

$4 = a(3)^2 + b(3) + c$

$4 = 9a + 3b + c$ Equation 3

Eliminate c from Equations 1 and 2 by multiplying Equation 2 by –1 and adding the resulting equation to Equation 1.

$4 = a + b + c$

$\underline{-1 = -4a - 2b - c}$

$3 = -3a - b$ Equation 4

Eliminate c from Equation 2 and 3 by multiplying Equation 3 by –1 and adding the resulting equation to Equation 2.

$1 = 4a + 2b + c$

$\underline{-4 = -9a - 3b - c}$

$-3 = -5a - b$ Equation 5

Solve Equations 4 and 5 for a and b by multiplying Equation 5 by –1 and adding the resulting equation to Equation 4.

$3 = -3a - b$

$\underline{3 = 5a + b}$

$6 = 2a$

$a = 3$

Substitute 3 for a in Equation 4 and solve for b.

$3 = -3(3) - b$

$3 = -9 - b$

$12 = -b$

$b = -12$

Substitute 3 for a and –12 for b in Equation 1 and solve for c.

$4 = 3 - 12 + c$

$4 = -9 + c$

$c = 13$

Substituting 3 for a, –12 for b, and 13 for c in the quadratic equation $y = ax^2 + bx + c$ gives

$y = 3x^2 - 12x + 13$.

Exercise Set 8.2

1. $x + y + z = 4$

$2 - 1 + 3 = 4$

$4 = 4$ true

$x - 2y - z = 1$

$2(2) - 2(-1) - 3 = 1$

$4 + 2 - 3 = 1$

$1 = 1$ true

$2x - y - 2z = -1$

$2(2) - (-1) - 2(3) = -1$

$4 + 1 - 6 = -1$

$-1 = -1$ false

(2, –1, 3) is a solution.

2. $x + y + z = 0$

$5 - 3 - 2 = 0$

$0 = 0$ true

$x + 2y - 3z = 5$

$5 + 2(-3) - 3(-2) = 5$

$5 - 6 + 6 = 5$

$5 = 5$ true

$3x + 4y + 2z = -1$

$3(5) + 4(-3) + 2(-2) = -1$

$15 - 12 - 4 = -1$

$-1 = -1$ true

(5, –3, –2) is a solution.

3. $x - 2y = 2$

$4 - 2(1) = 2$

$4 - 2 = 2$

$2 = 2$ true

$2x + 3y = 11$

$2(4) + 3(1) = 11$

$8 + 3 = 11$

$11 = 11$ true

$y - 4z = -7$

$1 - 4(2) = -7$

$1 - 8 = -7$

$-7 = -7$ true

(4, 1, 2) is a solution.

4. $x - 2z = -5$
$-1 - 2(2) = -5$
$-1 - 4 = -5$
$-5 = -5$ true
$y - 3z = -3$
$3 - 3(2) = -3$
$3 - 6 = -3$
$-3 = -3$ true
$2x - z = -4$
$2(-1) - (2) = -4$
$-2 - 2 = -4$
$-4 = -4$ true
(–1, 3, 2) is a solution.

5. $x + y + 2z = 11$
$x + y + 3z = 14$
$x + 2y - z = 5$
Eliminate x and y from Equations 1 and 2 by multiplying Equation 2 by –1 and adding the resulting equation to Equation 1.

$$\begin{array}{r} -x - y - 3z = -14 \\ \underline{x + y + 2z = 11} \\ -z = -3 \\ z = 3 \end{array}$$

Substitute 3 for z in Equations 1 and 3.
$x + y + 2(3) = 11$
$x + 2y - (3) = 5$
Simplify:
$x + y = 5$ Equation 4
$x + 2y = 8$ Equation 5
Solve Equations 4 and 5 for x and y by multiplying Equation 5 by –1 and adding the resulting equation to Equation 4.

$$\begin{array}{r} x + y = 5 \\ \underline{-x - 2y = -8} \\ -y = -3 \\ y = 3 \end{array}$$

Substitute 3 for z and 3 for y in Equation 2 and solve for x.
$x + 3 + 3(3) = 14$
$x + 12 = 14$
$x = 2$
The solution set is {(2, 3, 3)}.

6. $2x + y - 2z = -1$
$3x - 3y - z = 5$
$x - 2y + 3z = 6$
Eliminate z from Equations 1 and 2 by multiplying Equation 2 by –2 and adding the resulting equation to Equation 1.

$$\begin{array}{r} 2x + y - 2z = -1 \\ \underline{-6x + 6y + 2z = -10} \\ -4x + 7y = -11 \end{array} \quad \text{Equation 4}$$

Eliminate z from Equation 2 and 3 by multiplying Equation 2 by 3 and adding the resulting equation to Equation 3.

$$\begin{array}{r} 9x - 9y - 3z = 15 \\ \underline{x - 2y + 3z = 6} \\ 10x - 11y = 21 \end{array} \quad \text{Equation 5}$$

Solve Equations 4 and 5 for x and y by multiplying Equation 4 by 5 and Equation 5 by 2. Add the resulting equations.

$$\begin{array}{r} -20x + 35y = -55 \\ \underline{20x - 22y = 42} \\ 13y = -13 \\ y = -1 \end{array}$$

Substitute –1 for y in Equation 4 and solve for x.
$-4x + 7(-1) = -11$
$-4x - 7 = -11$
$-4x = -4$
$x = 1$
Substitute 1 for x and –1 for y in Equation 2 and solve for z.
$3(1) - 3(-1) - z = 5$
$3 + 3 - z = 5$
$6 - z = 5$
$-z = -1$
$z = 1$
The solution set is {(1, –1, 1)}.

7. $4x-y+2z=11$
$x+2y-z=-1$
$2x+2y-3z=-1$
Eliminate *y* from Equation 1 and 2 by multiplying Equation 1 by 2 and adding the resulting equation to Equation 2 and 3.

$$\begin{array}{r} 8x-2y+4z=22 \\ \underline{x+2y-z=-1} \\ 9x+3z=21 \end{array} \quad \text{Equation 4}$$

Eliminate *y* from Equations 1 and 3 by multiplying Equation 1 by 2 and adding the resulting equation to Equation 3.

$$\begin{array}{r} 8x-2y+4z=22 \\ \underline{2x+2y-3z=-1} \\ 10x+z=21 \end{array} \quad \text{Equation 5}$$

Solve Equations 4 and 5 for *x* and *z* by multiplying Equation 5 by –3 and adding the resulting equation to Equation 4.

$$\begin{array}{r} 9x+3z=21 \\ \underline{-30x-3z=-63} \\ -21x=-42 \\ x=2 \end{array}$$

Substitute 2 for *x* in Equation 5 and solve for *z*. $10(2)+z=21$

$$\begin{array}{r} 20+z=21 \\ z=1 \end{array}$$

Substitute 2 for *x* and 1 for *z* in Equation 2 and solve for *y*.

$$\begin{array}{r} 2+2y-1=-1 \\ 2y+1=-1 \\ 2y=-2 \\ y=-1 \end{array}$$

The solution set is $\{(2,-1,1)\}$.

8. $x-y+3z=8$
$3x+y-2z=-2$
$2x+4y+z=0$
Eliminate *y* from Equations 1 and 2 by adding Equations 1 and 2.

$$\begin{array}{r} x-y+3z=8 \\ \underline{3x+y-2z=-2} \\ 4x+z=6 \end{array} \quad \text{Equation 4}$$

Eliminate *y* from Equations 1 and 3 by multiplying Equation 1 by 4 and adding the resulting equation to Equation 3.

$$\begin{array}{r} 4x-4y+12z=32 \\ \underline{2x+4y+z=0} \\ 6x+13z=32 \end{array} \quad \text{Equation 5}$$

Solve Equations 4 and 5 for *x* and *z* by multiplying Equation 4 by –13 and adding the resulting equation to Equation 5.

$$\begin{array}{r} -52x-13z=-78 \\ \underline{6x+13z=32} \\ -46x=-46 \\ x=1 \end{array}$$

Substitute 1 for *x* in Equation 4 and solve for *z*.

$$\begin{array}{r} 4(1)+z=6 \\ 4+z=6 \\ z=2 \end{array}$$

Substitute 1 for *x* and 2 for *z* in Equation 2 and solve for *y*.

$$\begin{array}{r} 3(1)+y-2(2)=-2 \\ 3+y-4=-2 \\ y-1=-2 \\ y=-1 \end{array}$$

The solution set is $\{(1,-1,2)\}$.

9. $3x + 2y - 3z = -2$
$2x - 5y + 2z = -2$
$4x - 3y + 4z = 10$
Eliminate z from Equations 1 and 2 by multiplying Equation 1 by 2 and Equation 2 by 3. Add the resulting equations.

$$\begin{array}{r} 6x + 4y - 6z = -4 \\ \underline{6x - 15y + 6z = -6} \\ 12x - 11y = -10 \end{array} \quad \text{Equation 4}$$

Eliminate z from Equations 2 and 3 by multiplying Equation 2 by –2.

$$\begin{array}{r} -4x + 10y - 4z = 4 \\ \underline{4x - 3y + 4z = 10} \\ 7y = 14 \end{array} \quad \text{Equation 5}$$

Solve Equation 5 for y

$$7y = 14$$
$$y = 2$$

Solve for x by substituting 7 for y in Equation 4.

$$12x - 11y = -10$$
$$12x - 11(2) = -10$$
$$12x - 22 = -10$$
$$12x = 12$$
$$x = 1$$

Substitute 2 for y and 1 for x in Equation 2 and solve for z.

$$2x - 5y + 2z = -2$$
$$2(1) - 5(2) + 2z = -2$$
$$2 - 10 + 2z = -2$$
$$2z = 6$$
$$z = 3$$

The solution set is {(1, 2, 3)}.

10. $2x + 3y + 7z = 13$
$3x + 2y - 5z = -22$
$5x + 7y - 3z = -28$
Eliminate x from Equations 1 and 2 by multiplying Equation 1 by 3 and Equation 2 by –2. Add the resulting equations.

$$\begin{array}{r} 6x + 9y + 21z = 39 \\ \underline{-6x - 4y + 10z = 44} \\ 5y + 31z = 83 \end{array} \quad \text{Equation 4}$$

Eliminate x from Equations 1 and 3 by multiplying Equation 1 by 5 and Equation 3 by –2. Add the resulting equations.

$$\begin{array}{r} 10x + 15y + 35z = 65 \\ \underline{-10x - 14y + 6z = 56} \\ y + 41z = 121 \end{array} \quad \text{Equation 5}$$

Solve Equations 4 and 5 for y and z by solving Equation 5 for y and substituting for y in Equation 4.

$$y = 121 - 41z$$
$$5(121 - 41z) + 31z = 83$$
$$605 - 205z + 31z = 83$$
$$-174z = -522$$
$$z = 3$$

Substitute 3 for z in Equation 5 and solve for y.

$$y + 41(3) = 121$$
$$y + 123 = 121$$
$$y = -2$$

Substitute –2 for y and 3 for z in Equation 1 and solve for x.

$$2x + 3(-2) + 7(3) = 13$$
$$2x - 6 + 21 = 13$$
$$2x + 15 = 13$$
$$2x = -2$$
$$x = -1$$

The solution set is {(–1, –2, 3)}.

11. $2x - 4y + 3z = 17$
$x + 2y - z = 0$
$4x - y - z = 6$
Eliminate z from Equations 1 and 2 by multiplying Equation 2 by 3 and adding the resulting equation to Equation 1.
$$\begin{array}{r} 2x - 4y + 3z = 17 \\ \underline{3x + 6y - 3z = 0} \\ 5x + 2y = 17 \end{array} \quad \text{Equation 4}$$
Eliminate z from Equations 2 and 3 by multiplying Equation 2 by –1 and adding the resulting equation to Equation 3.
$$\begin{array}{r} -x - 2y + z = 0 \\ \underline{4x - y - z = 6} \\ 3x - 3y = 6 \end{array} \quad \text{Equation 5}$$
Solve Equations 4 and 5 for x and y by multiplying Equation 5 by $\frac{2}{3}$ and adding the resulting equation to Equation 4.
$$\begin{array}{r} 5x + 2y = 17 \\ \underline{2x - 2y = 4} \\ 7x = 21 \\ x = 3 \end{array}$$
Substitute 3 for x in Equation 4 and solve for y.
$$\begin{aligned} 5(3) + 2y &= 17 \\ 15 + 2y &= 17 \\ 2y &= 2 \\ y &= 1 \end{aligned}$$
Substitute 3 for x and 1 for y in Equation 2 and solve for z.
$$\begin{aligned} 3 + 2(1) - z &= 0 \\ 3 + 2 - z &= 0 \\ 5 - z &= 0 \\ 5 &= z \end{aligned}$$
The solution set is $\{(3,1,5)\}$.

12. $x + z = 3$
$x + 2y - z = 1$
$2x - y + z = 3$
Eliminate z from Equations 1 and 2 by adding Equation 1 and Equation 2.
$$\begin{array}{r} x + z = 3 \\ \underline{x + 2y - z = 1} \\ 2x + 2y = 4 \end{array} \quad \text{Equation 4}$$
Eliminate z from Equations 2 and 3 by adding Equation 2 and Equation 3.
$$\begin{array}{r} x + 2y - z = 1 \\ \underline{2x - y + z = 3} \\ 3x + y = 4 \end{array} \quad \text{Equation 5}$$
Solve Equations 4 and 5 for x and y by multiplying Equation 5 by –2 and adding the resulting equation to Equation 4.
$$\begin{array}{r} 2x + 2y = 4 \\ \underline{-6x - 2y = -8} \\ -4x = -4 \\ x = 1 \end{array}$$
Substitute 1 for x in Equation 4 and solve for y.
$$\begin{aligned} 2(1) + 2y &= 4 \\ 2 + 2y &= 4 \\ 2y &= 2 \\ y &= 1 \end{aligned}$$
Substitute 1 for x in Equation 1 and solve for z.
$$\begin{aligned} 1 + z &= 3 \\ z &= 2 \end{aligned}$$
The solution set is $\{(1, 1, 2)\}$.

13. $2x + y = 2$
$x + y - z = 4$
$3x + 2y + z = 0$
Eliminate z from Equations 2 and 3 by adding Equation 2 and Equation 3.
$$\begin{array}{r} x + y - z = 4 \\ \underline{3x + 2y + z = 0} \\ 4x + 3y = 4 \end{array} \quad \text{Equation 4}$$
Solve Equations 1 and 4 for x and y by multiplying Equation 1 by –3 and adding the resulting equation to Equation 4.
$$\begin{array}{r} -6x - 3y = -6 \\ \underline{4x + 3y = 4} \\ -2x = -2 \\ x = 1 \end{array}$$
Substitute 1 for x in Equation 1 and solve for y.
$$\begin{aligned} 2(1) + y &= 2 \\ 2 + y &= 2 \\ y &= 0 \end{aligned}$$
Substitute 1 for x and 0 for y in Equation 2 and solve for z.
$$\begin{aligned} 1 + 0 - z &= 4 \\ 1 - z &= 4 \\ -z &= 3 \\ z &= -3 \end{aligned}$$
The solution set is $\{(1, 0, -3)\}$.

14. $x + 3y + 5z = 20$
$y - 4z = -16$
$3x - 2y + 9z = 36$
Eliminate x from Equations 1 and 3 by multiplying Equation 1 by –3 and adding the resulting equation to Equation 3.
$$\begin{array}{r} -3x - 9y - 15z = -60 \\ \underline{3x - 2y + 9z = 36} \\ -11y - 6z = -24 \end{array} \quad \text{Equation 4}$$
Solve Equations 2 and 4 for y and z by multiplying Equation 2 by 11 and adding the resulting equation to Equation 4.
$$\begin{array}{r} 11y - 44z = -176 \\ \underline{-11y - 6z = -24} \\ -50z = -200 \\ z = 4 \end{array}$$

Substitute 4 for z in Equation 2 and solve for y.
$y - 4(4) = -16$
$y - 16 = -16$
$y = 0$
Substitute 0 for y and 4 for z in Equation 1 and solve for x.
$x + 3(0) + 5(4) = 20$
$x + 20 = 20$
$x = 0$
The solution set is {(0, 0, 4)}.

15. $x + y = -4$
$y - z = 1$
$2x + y + 3z = -21$
Eliminate y from Equations 1 and 2 by multiplying Equation 1 by –1 and adding the resulting equation to Equation 2.
$$\begin{array}{r} -x - y = 4 \\ \underline{y - z = 1} \\ -x - z = 5 \end{array} \quad \text{Equation 4}$$
Eliminate y from Equations 2 and 3 by multiplying Equation 2 by –1 and adding the resulting equation to Equation 3.
$$\begin{array}{r} -y + z = -1 \\ \underline{2x + y + 3z = -21} \\ 2x + 4z = -22 \end{array} \quad \text{Equation 5}$$
Solve Equations 4 and 5 for x and z by multiplying Equation 4 by 2 and adding the resulting equation to Equation 5.
$$\begin{array}{r} -2x - 2z = 10 \\ \underline{2x + 4z = -22} \\ 2z = -12 \\ z = -6 \end{array}$$
Substitute –6 for z in Equation 2 and solve for y.
$y - (-6) = 1$
$y + 6 = 1$
$y = -5$
Substitute –5 for y in Equation 1 and solve for x
$x + (-5) = -4$
$x = 1$
The solution set is {(1, –5, –6)}.

16. $x + y = 4$
$x + z = 4$
$y + z = 4$
Eliminate x from Equations 1 and 2 by multiplying Equation 1 by –1 and adding the resulting equation to Equation 2.
$$\begin{array}{r} -x - y = -4 \\ \underline{x + z = 4} \\ -y + z = 0 \end{array} \quad \text{Equation 4}$$
Solve Equations 3 and 4 for y and z by adding Equation 4 to Equation 3.
$$\begin{array}{r} -y + z = 0 \\ \underline{y + z = 4} \\ 2z = 4 \\ z = 2 \end{array}$$
Substitute 2 for z in Equation 2 and solve for x.
$x + 2 = 4$
$x = 2$
Substitute 2 for z in Equation 3 and solve for y.
$y + 2 = 4$
$y = 2$
The solution set is {(2, 2, 2)}.

17. $3(2x + y) + 5z = -1$
$2(x-3y+4z)=-9$
$4(1+x)=-3(z-3y)$
Simplify each equation.
$6x+3y+5z=-1$ Equation 4
$2x-6y+8z=-9$ Equation 5
$4+4x=-3z+9y$
$4x-9y+3z=-4$ Equation 6
Eliminate x from Equations 4 and 5 by multiplying Equation 5 by –3 and adding the resulting equation to Equation 4.

$$\begin{array}{r} -6x+3y+5z=-1 \\ \underline{-6x+18y-24z=27} \\ 21y-19z=26 \end{array} \quad \text{Equation 7}$$

Eliminate x from Equations 5 and 6 by multiplying Equation 5 by –2 and adding the resulting equation to Equation 6.

$$\begin{array}{r} -4x+12y-16z=18 \\ \underline{4x-9y+3z=-4} \\ 3y-13z=14 \end{array} \quad \text{Equation 8}$$

Solve Equations 7 and 8 for y and z by multiplying Equation 8 by –7 and adding the resulting equation to Equation 7.

$$\begin{array}{r} 21y-19z=26 \\ \underline{-21y+91z=-98} \\ 72z=-72 \\ z=-1 \end{array}$$

Substitute -1 for z in Equation 8 and solve for y.

$$\begin{aligned} 3y-13(-1)&=14 \\ 3y+13&=14 \\ 3y&=1 \\ y&=\frac{1}{3} \end{aligned}$$

Substitute $\frac{1}{3}$ for y and –1 for z in Equation 5 and solve for x.

$$\begin{aligned} 2x-6\left(\frac{1}{3}\right)+8(-1)&=-9 \\ 2x-2-8&=-9 \\ 2x-10&=-9 \\ 2x&=1 \\ x&=\frac{1}{2} \end{aligned}$$

The solution set is $\left\{\left(\frac{1}{2},\frac{1}{3},-1\right)\right\}$.

18. $7z - 3 = 2(x - 3y)$
$5y + 3z - 7 = 4x$
$4 + 5z = 3(2x - y)$
Simplify each equation.
$7z - 3 = 2x - 6y$
$-2x + 6y + 7z = 3$ Equation 4
$4x - 5y - 3z = -7$ Equation 5
$4 + 5z = 6x - 3y$
$6x - 3y - 5z = 4$ Equation 6
Eliminate x from Equations 4 and 5 by multiplying Equation 4 by 2 and adding the resulting equation to Equation 5.

$$\begin{array}{r} -4x+12y+14z=6 \\ \underline{4x-5y-3z=-7} \\ 7y+11z=-1 \end{array} \quad \text{Equation 7}$$

Eliminate x from Equations 4 and 6 by multiplying Equation 4 by 3 and adding the resulting equation to Equation 6.

$$\begin{array}{r} -6x+18y+21z=9 \\ \underline{6x-3y-5z=4} \\ 15y+16z=13 \end{array} \quad \text{Equation 8}$$

Solve Equations 7 and 8 for y and z by multiplying Equation 7 by 15 and Equation 8 by –7. Add the resulting equations.

$$\begin{array}{r} 105y+165z=-15 \\ \underline{-105y-112z=-91} \\ 53z=-106 \\ z=-2 \end{array}$$

Substitute –2 for z in Equation 7 and solve for y.

$$\begin{aligned} 7y + 11(-2) &= -1 \\ 7y - 22 &= -1 \\ 7y &= 21 \\ y &= 3 \end{aligned}$$

Substitute 3 for y and –2 for z in Equation 2 and solve for x.

$$\begin{aligned} 5(3) + 3(-2) - 7 &= 4x \\ 15 - 6 - 7 &= 4x \\ 2 &= 4x \\ x &= \frac{1}{2} \end{aligned}$$

The solution set is $\left\{\left(\frac{1}{2},\ 3,\ -2\right)\right\}$.

19. (–1, 6), (1, 4), (2, 9)

$y = ax^2 + bx + c$

Substitute –1 for x and 6 for y in $y = ax^2 + bx + c$.

$6 = a(-1)^2 + b(-1) + c$

$6 = a - b + c$ Equation 1

Substitute 1 for x and 4 for y in $y = ax^2 + bx + c$.

$4 = a(1)^2 + b(1) + c$

$4 = a + b + c$ Equation 2

Substitute 2 for x and 9 for y in $y = ax^2 + bx + c$.

$9 = a(2)^2 + b(2) + c$

$9 = 4a + 2b + c$ Equation 3

Eliminate b from Equations 1 and 2 by adding Equation 1 and Equation 2.

$6 = a - b + c$

$\underline{4 = a + b + c}$

$10 = 2a + 2c$ Equation 4

Eliminate b from Equations 1 and 3 by multiplying Equation 1 by 2 and adding the resulting equation to Equation 3.

$12 = 2a - 2b + 2c$

$\underline{9 = 4a + 2b + c}$

$21 = 6a + 3c$ Equation 5

Solve Equations 4 and 5 for a and c by multiplying Equation 4 by –3 and adding the resulting equation to Equation 5.

$-30 = -6a - 6c$

$\underline{21 = 6a + 3c}$

$-9 = -3c$

$c = 3$

Substitute 3 for c in Equation 4 and solve for a.

$10 = 2a + 2(3)$

$10 = 2a + 6$

$4 = 2a$

$a = 2$

Substitute 2 for a and 3 for c in Equation 2 and solve for b.

$4 = 2 + b + 3$

$4 = b + 5$

$b = -1$

Substituting 2 for a, –1 for b, and 3 for c in the quadratic equation $y = ax^2 + bx + c$ gives

$y = 2x^2 - x + 3$.

20. (–2, 7), (1, –2), (2, 3)

$y = ax^2 + bx + c$

Substitute –2 for x and 7 for y in $y = ax^2 + bx + c$.

$7 = a(-2)2 + b(-2) + c$

$7 = 4a - 2b + c$ Equation 1

Substitute 1 for x and –2 for y in $y = ax^2 + bx + c$.

$-2 = a(1)^2 + b(1) + c$

$-2 = a + b + c$ Equation 2

Substitute 2 for x and 3 for y in $y = ax^2 + bx + c$.

$3 = a(2)^2 + b(2) + c$

$3 = 4a + 2b + c$ Equation 3

Eliminate b from Equations 1 and 2 by multiplying Equation 2 by 2 and adding the resulting equation to Equation 1.

$7 = 4a - 2b + c$

$\underline{-4 = 2a + 2b + 2c}$

$3 = 6a + 3c$ Equation 4

Eliminate b from Equations 1 and 3 by adding Equation 1 and Equation 3.

$7 = 4a - 2b + c$

$\underline{3 = 4a + 2b + c}$

$10 = 8a + 2c$ Equation 5

Solve Equations 4 and 5 for a and c by multiplying Equation 4 by $\frac{1}{3}$ and Equation 5 by $-\frac{1}{2}$. Add the resulting equations.

$1 = 2a + c$

$\underline{-5 = -4a - c}$

$-4 = -2a$

$a = 2$

Substitute 2 for a in Equation 5 and solve for c.

$10 = 8(2) + 2c$

$10 = 16 + 2c$

$-6 = 2c$

$c = -3$

Substitute 2 for a and –3 for c in Equation 2 and solve for b.

$-2 = 2 + b - 3$

$-2 = b - 1$

$b = -1$

Substituting 2 for a, –1 for b, and –3 for c in the quadratic equation $y = ax^2 + bx + c$ gives

$y = 2x^2 - x - 3$.

21. (–1, –4), (1, –2), (2, 5)

Substitute –1 for x and –4 for y in $y = ax^2 + bx + c$.

$-4 = a(-1)^2 + b(-1) + c$

$-4 = a - b + c$ Equation 1

Substitute 1 for x and –2 for y in $y = ax^2 + bx + c$.

$-2 = a(1)^2 + b(1) + c$

$-2 = a + b + c$ Equation 2

Substitute 2 for x and 5 for y in $y = ax^2 + bx + c$.

$5 = a(2)^2 + b(2) + c$

$5 = 4a + 2b + c$ Equation 3

Eliminate a and b from Equations 1 and 2 by multiplying Equation 1 by –1 and adding the resulting equation to Equation 2.

$4 = -a + b - c$

$\underline{-2 = a + b + c}$

$2 = 2b$

$b = 1$

Eliminate c from Equations 1 and 3 by multiplying Equation 1 by –1 and adding
the resulting equation to Equation 3.

$4 = -a + b - c$

$\underline{5 = 4a + 2b + c}$

$9 = 3a + 3b$ Equation 4

Substitute 1 for b in Equation 4 and solve for a.

$9 = 3a + 3(1)$

$9 = 3a + 3$

$6 = 3a$

$a = 2$

Substitute 2 for a and 1 for b in Equation 2 and solve for c.

$-2 = 2 + 1 + c$

$-2 = c + 3$

$c = -5$

Substituting 2 for a, 1 for b, and –5 for c in quadratic equation $y = ax^2 + bx + c$ gives $y = 2x^2 + x - 5$.

22. (1, 3), (3, –1), (4, 0)

$y = ax^2 + bx + c$

Substitute 1 for x and 3 for y in $y = ax^2 + bx + c$.

$3 = a(1)^2 + b(1) + c$

$3 = a + b + c$ Equation 1

Substitute 3 for x and –1 for y in $y = ax^2 + bx + c$.

$-1 = a(3)^2 + b(3) + c$

$-1 = 9a + 3b + c$ Equation 2

Substitute 4 for x and 0 for y in $y = ax^2 + bx + c$.

$0 = a(4)^2 + b(4) + c$

$0 = 16a + 4b + c$ Equation 3

Eliminate c from Equations 1 and 2 by multiplying Equation 3 by –1 and adding the resulting equation to Equation 1.

$3 = a + b + c$

$\underline{0 = -16a - 4b - c}$

$3 = -15a - 3b$ Equation 4

Eliminate c from Equations 2 and 3 by multiplying Equation 3 by –1 and adding the resulting equation to Equation 2.

$-1 = 9a + 3b + c$

$\underline{0 = -16a - 4b - c}$

$-1 = -7a - b$ Equation 5

Solve Equations 4 and 5 for a and b by multiplying Equation 5 by –3 and adding the resulting equation to Equation 4.

$3 = -15a - 3b$

$\underline{3 = 21a + 3b}$

$6 = 6a$

$a = 1$

Substitute 1 for a in Equation 5 and solve for b.

$-1 = -7(1) - b$

$-1 = -7 - b$

$6 = -b$

$b = -6$

Substitute 1 for a and –6 for b in Equation 1 and solve for c.

$3 = 1 - 6 + c$

$3 = c - 5$

$c = 8$

Substituting 1 for a, –6 for b, and 8 for c in the quadratic equation $y = ax^2 + bx + c$ gives $y = x^2 - 6x + 8$.

23. $x + y + z = 16$

$2x+3y+4z=46$

$5x-y=31$

Eliminate z from Equations 1 and 2 by multiplying Equation 1 by –4 and adding the resulting equation to Equation 2.

$$\begin{array}{r} -4x-4y-4z=-64 \\ 2x+3y+4z=46 \\ \hline -2x-y=-18 \end{array} \quad \text{Equation 4}$$

Solve Equations 3 and 4 for x and y by multiplying Equation 4 by –1 and adding the resulting equation to Equation 3.

$$\begin{array}{r} 5x-y=31 \\ 2x+y=18 \\ \hline 7x=49 \\ x=7 \end{array}$$

Substitute 7 for x in Equation 3 and solve for y.

$$\begin{aligned} 5(7)-y&=31 \\ 35-y&=31 \\ -y&=-4 \\ y&=4 \end{aligned}$$

Substitute 7 for x and 4 for y in Equation 1 and solve for z.

$$\begin{aligned} 7+4+z&=16 \\ z+11&=16 \\ z&=5 \end{aligned}$$

The numbers are 7, 4 and 5.

24. $3x + y + 2z = 5$

$x - 3y + 3z = 2$

$2x + 3y - z = 1$

Eliminate y from Equations 1 and 2 by multiplying Equation 1 by 3 and adding the resulting equation to Equation 2.

$$\begin{array}{r} 9x+3y+6z=15 \\ x-3y+3z=2 \\ \hline 10x+9z=17 \end{array} \quad \text{Equation 4}$$

Eliminate y from Equations 2 and 3 by adding Equation 2 and Equation 3.

$$\begin{array}{r} x-3y+3z=2 \\ 2x+3y-z=1 \\ \hline 3x+2z=3 \end{array} \quad \text{Equation 5}$$

Solve Equations 4 and 5 for x and z by multiplying Equation 4 by 2 and Equation 5 by –9. Add the resulting equations.

$$\begin{array}{r} 20x+18z=34 \\ -27x-18z=-27 \\ \hline -7x=7 \\ x=-1 \end{array}$$

Substitute –1 for x in Equation 5 and solve for z.

$$\begin{aligned} 3(-1) + 2z &= 3 \\ -3 + 2z &= 3 \\ 2z &= 6 \\ z &= 3 \end{aligned}$$

Substitute –1 for x and 3 for z in Equation 1 and solve for y.

$$\begin{aligned} 3(-1) + y + 2(3) &= 5 \\ -3 + y + 6 &= 5 \\ y + 3 &= 5 \\ y &= 2 \end{aligned}$$

The numbers are –1, 2, and 3.

25.

$$\begin{aligned} \frac{x+2}{6}-\frac{y+4}{3}+\frac{z}{2}&=0 \\ 6\left(\frac{x+2}{6}-\frac{y+4}{3}+\frac{z}{2}\right)&=6(0) \\ (x+2)-2(y+4)+3z&=0 \\ x+2-2y-8+3z&=0 \\ x-2y+3z&=6 \end{aligned}$$

$$\begin{aligned} \frac{x+1}{2}+\frac{y-1}{2}-\frac{z}{4}&=\frac{9}{2} \\ 4\left(\frac{x+1}{2}+\frac{y-1}{2}-\frac{z}{4}\right)&=4\left(\frac{9}{2}\right) \\ 2(x+1)+2(y-1)-z&=18 \\ 2x+2+2y-2-z&=18 \\ 2x+2y-z&=18 \end{aligned}$$

$$\begin{aligned} \frac{x-5}{4}+\frac{y+1}{3}+\frac{z-2}{2}&=\frac{19}{4} \\ 12\left(\frac{x-5}{4}+\frac{y+1}{3}+\frac{z-2}{2}\right)&=12\left(\frac{19}{4}\right) \\ 3(x-5)+4(y+1)+6(z-2)&=57 \\ 3x-15+4y+4+6z-12&=57 \\ 3x+4y+6z&=80 \end{aligned}$$

We need to solve the equivalent system:

$x-2y+3z=6$

$2x+2y-z=18$

$3x+4y+6z=80$

Add the first two equations together.

$$\begin{aligned} x-2y+3z&=6\\ 2x+2y-z&=18\\ \hline 3x+2z&=24 \end{aligned}$$

Multiply the second equation by -2 and add it to the third equation.

$$\begin{aligned} -4x-4y+2z&=-36\\ 3x+4y+6z&=80\\ \hline -x+8z&=44 \end{aligned}$$

Using the two reduced equations, we solve the system

$$3x+2z=24$$
$$-x+8z=44$$

Multiply the second equation by 3 and add the equations.

$$\begin{aligned} 3x+2z&=24\\ -3x+24z&=132\\ \hline 26z&=156\\ z&=6 \end{aligned}$$

Back-substitute to find *x*.

$$\begin{aligned} -x+8(6)&=44\\ -x+48&=44\\ -x&=-4\\ x&=4 \end{aligned}$$

Back substitute to find *y*.

$$\begin{aligned} x-2y+3z&=6\\ 4-2y+3(6)&=6\\ -2y&=-16\\ y&=8 \end{aligned}$$

The solution is $(4,8,6)$.

26.

$$\begin{aligned} \frac{x+3}{2}-\frac{y-1}{2}+\frac{z+2}{4}&=\frac{3}{2}\\ 4\left(\frac{x+3}{2}-\frac{y-1}{2}+\frac{z+2}{4}\right)&=4\left(\frac{3}{2}\right)\\ 2(x+3)-2(y-1)+(z+2)&=6\\ 2x+6-2y+2+z+2&=6\\ 2x-2y+z&=-4 \end{aligned}$$

$$\begin{aligned} \frac{x-5}{2}+\frac{y+1}{3}-\frac{z}{4}&=-\frac{25}{6}\\ 12\left(\frac{x-5}{2}+\frac{y+1}{3}-\frac{z}{4}\right)&=12\left(-\frac{25}{6}\right)\\ 6(x-5)+4(y+1)-3z&=-50\\ 6x-30+4y+4-3z&=-50\\ 6x+4y-3z&=-24 \end{aligned}$$

$$\begin{aligned} \frac{x-3}{4}-\frac{y+1}{2}+\frac{z-3}{2}&=-\frac{5}{2}\\ 4\left(\frac{x-3}{4}-\frac{y+1}{2}+\frac{z-3}{2}\right)&=4\left(-\frac{5}{2}\right)\\ (x-3)-2(y+1)+2(z-3)&=-10\\ x-3-2y-2+2z-6&=-10\\ x-2y+2z&=1 \end{aligned}$$

We need to solve the equivalent system:

$$2x-2y+z=-4$$
$$6x+4y-3z=-24$$
$$x-2y+2z=1$$

Multiply the first equation by 2 and add it to the second equation.

$$\begin{aligned} 4x-4y+2z&=-8\\ 6x+4y-3z&=-24\\ \hline 10x-z&=-32 \end{aligned}$$

Multiply the third equation by -1 and add to the first equation.

$$\begin{aligned} 2x-2y+z&=-4\\ -x+2y-2z&=-1\\ \hline x-z&=-5 \end{aligned}$$

Using the two reduced equations, we solve the system

$$10x-z=-32$$
$$x-z=-5$$

Multiply the second equation by -1 and add it to the first.

$$\begin{aligned} 10x - z &= -32 \\ -x + z &= 5 \\ \hline 9x &= -27 \\ x &= -3 \end{aligned}$$

Back-substitute to solve for z.

$$\begin{aligned} x - z &= -5 \\ -3 - z &= -5 \\ -z &= -2 \\ z &= 2 \end{aligned}$$

Back-substitute to solve for y.

$$\begin{aligned} x - 2y + 2z &= 1 \\ -3 - 2y + 2(2) &= 1 \\ -3 - 2y + 4 &= 1 \\ -2y &= 0 \\ y &= 0 \end{aligned}$$

The solution is $(-3, 0, 2)$.

27. Selected points may vary, but the equation will be the same.

$y = ax^2 + bx + c$

Use the points $(2,-2)$, $(4,1)$, and $(6,-2)$ to get the system

$$\begin{aligned} 4a + 2b + c &= -2 \\ 16a + 4b + c &= 1 \\ 36a + 6b + c &= -2 \end{aligned}$$

Multiply the first equation by -1 and add to the second equation.

$$\begin{aligned} -4a - 2b - c &= 2 \\ 16a + 4b + c &= 1 \\ \hline 12a + 2b &= 3 \end{aligned}$$

Multiply the first equation by -1 and add to the third equation.

$$\begin{aligned} -4a - 2b - c &= 2 \\ 36a + 6b + c &= -2 \\ \hline 32a + 4b &= 0 \end{aligned}$$

Using the two reduced equations, we get the system

$$\begin{aligned} 12a + 2b &= 3 \\ 32a + 4b &= 0 \end{aligned}$$

Multiply the first equation by -2 and add to the second equation.

$$\begin{aligned} -24a - 4b &= -6 \\ 32a + 4b &= 0 \\ \hline 8a &= -6 \\ a &= -\frac{3}{4} \end{aligned}$$

Back-substitute to solve for b.

$$\begin{aligned} 12a + 2b &= 3 \\ 12\left(-\frac{3}{4}\right) + 2b &= 3 \\ -9 + 2b &= 3 \\ 2b &= 12 \\ b &= 6 \end{aligned}$$

Back-substitute to solve for c.

$$\begin{aligned} 4a + 2b + c &= -2 \\ 4\left(-\frac{3}{4}\right) + 2(6) + c &= -2 \\ -3 + 12 + c &= -2 \\ c &= -11 \end{aligned}$$

The equation is:

$y = -\frac{3}{4}x^2 + 6x - 11$

28. Selected points may vary, but the equation will be the same.

$y = ax^2 + bx + c$

Use the points $(3,4)$, $(4,2)$, and $(5,2)$ to get the system

$$\begin{aligned} 9a + 3b + c &= 4 \\ 16a + 4b + c &= 2 \\ 25a + 5b + c &= 2 \end{aligned}$$

Multiply the first equation by -1 and add to the second equation.

$$\begin{aligned} -9a - 3b - c &= -4 \\ 16a + 4b + c &= 2 \\ \hline 7a + b &= -2 \end{aligned}$$

Multiply the first equation by -1 and add to the third equation.

$$\begin{aligned} -9a - 3b - c &= -4 \\ 25a + 5b + c &= 2 \\ \hline 16a + 2b &= -2 \end{aligned}$$

Use the two reduced equations to get the system

$$\begin{aligned} 7a + b &= -2 \\ 16a + 2b &= -2 \end{aligned}$$

Multiply the first equation by -2 and add to the second equation.

$$\begin{aligned} -14a - 2b &= 4 \\ 16a + 2b &= -2 \\ \hline 2a &= 2 \\ a &= 1 \end{aligned}$$

Back-substitute to solve for *b*.

$$7a+b=-2$$
$$7(1)+b=-2$$
$$7+b=-2$$
$$b=-9$$

Back-substitute to solve for *c*.

$$9a+3b+c=4$$
$$9(1)+3(-9)+c=4$$
$$9-27+c=4$$
$$c=22$$

The equation is:

$$y=x^2-9x+22$$

29. $ax-by-2cz=21$

$ax+by+cz=0$

$2ax-by+cz=14$

Add the first two equations.

$$ax-by-2cz=21$$
$$\underline{ax+by+cz=0}$$
$$2ax-cz=21$$

Multiply the first equation by -1 and add to the third equation.

$$-ax+by+2cz=-21$$
$$\underline{2ax-by+cz=14}$$
$$ax+3cz=-7$$

Use the two reduced equations to get the following system:

$2ax-cz=21$

$ax+3cz=-7$

Multiply the second equation by -2 and add the equations.

$$2ax-cz=21$$
$$\underline{-2ax-6cz=14}$$
$$-7cz=35$$
$$z=-\frac{5}{c}$$

Back-substitute to solve for *x*.

$$ax+3cz=-7$$
$$ax+3c\left(-\frac{5}{c}\right)=-7$$
$$ax-15=-7$$
$$ax=8$$
$$x=\frac{8}{a}$$

Back-substitute to solve for *y*.

$$ax+by+cz=0$$
$$a\left(\frac{8}{a}\right)+by+c\left(-\frac{5}{c}\right)=0$$
$$8+by-5=0$$
$$by=-3$$
$$y=-\frac{3}{b}$$

The solution is $\left(\frac{8}{a},-\frac{3}{b},-\frac{5}{c}\right)$.

30. $ax-by+2cz=-4$

$ax+3by-cz=1$

$2ax+by+3cz=2$

Multiply the first equation by -1 and add to the second equation.

$$-ax+by-2cz=4$$
$$\underline{ax+3by-cz=1}$$
$$4by-3cz=5$$

Multiply the first equation by -2 and add to the third equation.

$$-2ax+2by-4cz=8$$
$$\underline{2ax+by+3cz=2}$$
$$3by-cz=10$$

Use the two reduced equations to get the following system:

$4by-3cz=5$

$3by-cz=10$

Multiply the second equation by -3 and add to the first equation.

$$4by-3cz=5$$
$$\underline{-9by+3cz=-30}$$
$$-5by=-25$$
$$y=\frac{5}{b}$$

Back-substitute to solve for z.

$$4by - 3cz = 5$$
$$4b\left(\frac{5}{b}\right) - 3cz = 5$$
$$20 - 3cz = 5$$
$$-3cz = -15$$
$$z = \frac{5}{c}$$

Back-substitute to solve for x.

$$ax - by + 2cz = -4$$
$$ax - b\left(\frac{5}{b}\right) + 2c\left(\frac{5}{c}\right) = -4$$
$$ax - 5 + 10 = -4$$
$$ax = -9$$
$$x = -\frac{9}{a}$$

The solution is $\left(-\frac{9}{a}, \frac{5}{b}, \frac{5}{c}\right)$.

31. a. Substitute the values for x and y into the quadratic form.

$$224 = a(1)^2 + b(1) + c$$
$$a + b + c = 224$$

$$176 = a(3)^2 + b(3) + c$$
$$9a + 3b + c = 176$$

$$104 = a(4)^2 + b(4) + c$$
$$16a + 4b + c = 104$$

Multiply the first equation by –1 and add to both the second and the third equations to obtain 2 new equations with 2 variables.

$$-a - b - c = -224$$
$$9a + 3b + c = 176$$
$$8a + 2b = -48$$

$$-a - b - c = -224$$
$$16a + 4b + c = 104$$
$$15a + 3b = -120$$

Use the two new equations to solve for a and b. Multiply the first equation by –3 and the second equation by 2 and add the results together. Solve for a. Substitute that value in $8a + 2b = -48$ and solve for b.

$$-24a - 6b = 144$$
$$30a + 6b = -240$$
$$6a = -96$$
$$a = -16$$

$$8(-16) + 2b = -48$$
$$-128 + 2b = -48$$
$$2b = 80$$
$$b = 40$$

Substitute –16 for a and 40 for b into the equation $a + b + c = 224$ and solve for c.

$$-16 + 40 + c = 224$$
$$c = 200$$

The equation is $y = -16x^2 + 40x + 200$.

b. $y = -16(5)^2 + 40(5) + 200 = 0$

The ball hit the ground after 5 seconds.

32. a. (1, 46) (2, 84) (3, 114)

$$a(1)^2 + b(1) + c = 46$$
$$a + b + c = 46$$

$$a(2)^2 + b(2) + c = 84$$
$$4a + 2b + c = 84$$

$$a(3)^2 + b(3) + c = 114$$
$$9a + 3b + c = 114$$

$$-4a - 4b - 4c = -184$$
$$4a + 2b + c = 84$$
$$-2b - 3c = -100$$

$$-9a - 9b - 9c = -414$$
$$9a + 3b + c = 114$$
$$-6b - 8c = -300$$

$$6b + 9c = 300$$
$$-6b - 8c = 300$$
$$c = 0$$

$$6b + 9(0) = 300$$
$$b = 50$$

$$a + 50 + 0 = 46$$
$$a = -4$$

$$y = -4x^2 + 50x$$

b. $y = -4x^2 + 50x$

$$y = -4(6)^2 + 50(6) = 156$$

A car that is in motion 6 seconds after the brakes are applied will travel 156 feet after the brakes are applied.

33. Let $w =$ the percent of body weight that consists of water.
Let $f =$ the percent of body weight that consists of fat.
Let $p =$ the percent of body weight that consists of protein.
The information is represented by the following system of equations.

$$w + f + p = 95$$
$$w - f = 35$$
$$f - p = 9$$

Solving $w - f = 35$ for w gives $w = f + 35$.
Solving $f - p = 9$ for p gives $p = f - 9$.
Use substitution to find *f*.

$$w + f + p = 95$$
$$\overbrace{f + 35}^{w} + f + \overbrace{f - 9}^{p} = 95$$
$$f + 35 + f + f - 9 = 95$$
$$3f + 26 = 95$$
$$3f = 69$$
$$f = 23$$

Find *w*.
$$w = f + 35$$
$$w = 23 + 35 = 58$$
Find *p*.
$$p = f - 9$$
$$p = 23 - 9 = 14$$
The total body weight consists of 58% water, 23% fat, and 14% protein.

34. Let $w =$ the percent of body weight that consists of water.
Let $f =$ the percent of body weight that consists of fat.
Let $p =$ the percent of body weight that consists of protein.
The information is represented by the following system of equations.

$$w + f + p = 95$$
$$w - f = 47$$
$$p - f = 2$$

Solving $w - f = 47$ for w gives $w = f + 47$.
Solving $p - f = 2$ for p gives $p = f + 2$.
Use substitution to find *f*.

$$w + f + p = 94$$
$$\overbrace{f + 47}^{w} + f + \overbrace{f + 2}^{p} = 94$$
$$f + 47 + f + f + 2 = 94$$
$$3f + 49 = 94$$
$$3f = 45$$
$$f = 15$$

Find *w*.
$$w = f + 47$$
$$w = 15 + 47 = 62$$
Find *p*.
$$p = f + 2$$
$$p = 15 + 2 = 17$$
The total body weight consists of 62% water, 15% fat, and 17% protein.

35. x = number of \$8 tickets sold
y = number of \$10 tickets sold
z = number of \$12 tickets sold
From the given conditions we have the following system of equations.

$$x + y + z = 400$$
$$8x + 10y + 12z = 3700$$
$$x + y = 7z \text{ or } x + y - 7z = 0$$

Eliminate z from Equations 1 and 2 multiplying Equation 1 by –12 and adding the resulting equation to Equation 2.

$$\begin{array}{r} -12x-12y-12z=-4800 \\ 8x+10y+12z=3700 \\ \hline -4x-2y=-1100 \end{array} \quad \text{Equation 4}$$

Eliminate z from Equations 1 and 3 by multiplying Equation 1 by 7 and adding the resulting equation to Equation 3.

$$\begin{array}{r} 7x+7y+7z=2800 \\ x+y-7z=0 \\ \hline 8x+8y=2800 \end{array} \quad \text{Equation 5}$$

Solve Equations 4 and 5 for x and y by multiplying Equation 4 by 2 and adding the resulting equation to Equation 5.

$$\begin{array}{r} -8x-4y=-2200 \\ 8x+8y=2800 \\ \hline 4y=600 \\ y=150 \end{array}$$

Substitute 150 for y in Equation 5 and solve for x.

$$8x + 8(150) = 2800$$
$$8x = 2800 - 1200$$
$$8x = 1600$$
$$x = 200$$

Substitute 200 for x and 150 for y in Equation 1 and solve for z.

$$200 + 150 + z = 400$$
$$350 + z = 400$$
$$z = 50$$

The number of \$8 tickets sold was 200.
The number of \$10 tickets sold was 150.
The number of \$12 tickets sold was 50.

36. x = number of packages of 6 blades
y = number of packages of 12 blades
z = number of packages of 24 blades.

$$x + y + z = 12$$
$$6x + 12y + 24z = 162$$
$$2x + 3y + 4z = 35$$

Eliminate x from Equations 1 and 2 by multiplying Equation 1 by –6 and adding the resulting equation to Equation 2.

$$\begin{array}{r} -6x-6y-6z=-72 \\ 6x+12y+24z=162 \\ \hline 6y+18z=90 \end{array} \quad \text{Equation 4}$$

Eliminate x from Equations 1 and 3 by multiplying Equation 1 by –2 and adding the resulting equation to Equation 3.

$$\begin{array}{r} -2x-2y-2z=-24 \\ 2x+3y+4z=35 \\ \hline y+2z=11 \end{array} \quad \text{Equation 5}$$

Solve Equations 4 and 5 for y and z by multiplying Equations 5 by –6 and adding the resulting equation to Equation 4.

$$\begin{array}{r} 6y+18z=90 \\ -6y-12z=-66 \\ \hline 6z=24 \\ z=4 \end{array}$$

Substitute 4 for z in Equation 5 and solve for y.

$$y + 2(4) = 11$$
$$y + 8 = 11$$
$$y = 3$$

Substitute 3 for y and 4 for z in Equation 1 and solve for x.

$$x + 3 + 4 = 12$$
$$x + 7 = 12$$
$$x = 5$$

The store sold 5 packages of 6 blades, 3 packages of 12 blades, and 4 packages of 24 blades.

37. x = amount of money invested at 10%
y = amount of money invested at 12%
z = amount of money invested at 15%

$$x + y + z = 6700$$
$$0.08x + 0.10y + 0.12z = 716$$
$$z = x + y + 300$$

Arrange Equation 3 so that variable terms appear on the left and constants appear on the right.

$-x - y + z = 300$ Equation 4

Eliminate x and y from Equations 1 and 4 by adding Equations 1 and 4.

$$x + y + z = 6700$$
$$-x - y + z = 300$$
$$2z = 7000$$
$$z = 3500$$

Substitute 3500 for z in Equation 1 and Equation 2 and simplify.

$$x + y + 3500 = 6700$$
$x + y = 3200$ Equation 5
$$0.08x + 0.10y + 0.12(3500) = 716$$
$$0.08x + 0.10y + 420 = 716$$ Solve
$0.08x + 10y = 296$ Equation 6

Equations 5 and 6 for x and y by multiplying Equation 5 by –0.10 and adding the resulting equation to Equation 6.

$$-0.10x - 0.10y = -320$$
$$0.08x + 0.10y = 296$$
$$-0.02x = 24$$
$$x = 1200$$

Substitute 1200 for x and 3,500 for z in Equation 1 and solve for y.

$$1200 + y + 3500 = 6700$$
$$y + 4700 = 6700$$
$$y = 2000$$

The person invested \$1200 at 8%, \$2000 at 10%, and \$3500 at 12%.

38. x = amount of money invested at 10%
y = amount of money invested at 12%
z = amount of money invested at 15%

$$x + y + z = 17{,}000$$
$$0.10x + 0.12y + 0.15z = 2110$$
$$y = x + z - 1000$$

Arrange Equation 3 so that variable terms appear on the left and constants appear on the right.

$-x + y - z = -1000$ Equation 4

Eliminate x and z from Equations 1 and 4 by adding Equation 1 and Equation 4.

$$x + y + z = 1700$$
$$-x + y - z = -1000$$
$$2y = 16000$$
$$y = 8000$$

Substitute 8000 for y in Equation 1 and Equation 2.

$$x + 8000 + z = 17{,}000$$
$$x + z = 9000$$
$$0.10x + 0.12(8000) + 0.15z = 2110$$
$0.10x + 960 + 0.15z = 2110$ Equation 5
$0.10x + 0.15z = 1150$ Equation 6

Solve Equations 5 and 6 for x and z by multiplying Equation 5 by –0.10 and adding the resulting equation to Equation 6.

$$-0.10x - 0.10z = -900$$
$$0.10x + 0.15z = 1150$$
$$0.05z = 250$$
$$z = 5000$$

Substitute 8000 for y and 5000 for z in Equation 1 and solve for x.

$$x + 8000 + 5000 = 17{,}000$$
$$x + 13{,}000 = 17{,}000$$
$$x = 4000$$

The person invested \$4000 at 10%, \$8000 at 12%, and \$5000 at 15%.

39. $x + y + z = 180$

$2x - 5 + z = 180$
$2x + z = 185$

$2x + 5 + y = 180$
$2x + y = 175$

Multiply the second equation by –1 and add to the first equation. Use the new equation and the third equation to solve for x and z.

$-2x - z = -185$
$x + y + z = 180$
$-x + y = -5$

Multiply the new equation by -1.

$x - y = 5$
$2x + y = 175$
$3x = 180$
$x = 60$

$60 - y = 5$
$-y = -55$
$y = 55$

Substitute 60 for x and 55 for y in the first equation and solve for z.

$60 + 55 + z = 180$
$z = 65$

40. – 45. Answers may vary.

46. does not make sense; Explanations will vary. Sample explanation: The third variable could possibly have the same variable as one of the other two.

47. does not make sense; Explanations will vary. Sample explanation: A system of linear equations in three variables can contain an equation of the form $y = mx + b.$ For this equation, the coefficient of z is 0.

48. makes sense

49. makes sense

50. Answers may vary.

51. x = number of triangles
y = number of rectangles
z = number of pentagons

$x + y + z = 40$
$3x + 4y + 5z = 153$
$2y + 5z = 72$

Eliminate x from Equations 1 and 2 by multiplying Equation 1 by –3 and adding the resulting equation to Equation 2.

$-3x - 3y - 3z = -120$
$3x + 4y + 5z = 153$
$y + 2z = 33$ Equation 4

Solve for z by multiplying Equation 4 by –2 and adding the resulting equation to Equation 3.

$2y + 5z = 72$
$-2y - 4z = -66$
$z = 6$

Substitute 6 for z in Equation 4 and solve for y.

$y + 2(6) = 33$
$y + 12 = 33$
$y = 21$

Substitute 21 for y and 6 for z in Equation 1 and solve for x.

$x + 21 + 6 = 40$
$x + 27 = 40$
$x = 13$

The painting has 13 triangles, 21 rectangles, and 6 pentagons.

52. Answers may vary.

53.
$$\frac{3}{x-4} - \frac{2}{x+2} = \frac{3(x+2)}{(x-4)(x+2)} - \frac{2(x-4)}{(x-4)(x+2)}$$
$$= \frac{3x+6}{(x-4)(x+2)} - \frac{2x-8}{(x-4)(x+2)}$$
$$= \frac{3x+6-2x+8}{(x-4)(x+2)}$$
$$= \frac{x+14}{(x-4)(x+2)}$$

54. $$\frac{5x-3}{x^2+1}+\frac{2x}{(x^2+1)^2}=\frac{(5x-3)(x^2+1)}{(x^2+1)(x^2+1)}+\frac{2x}{(x^2+1)^2}$$
$$=\frac{5x^3-3x^2+5x-3}{(x^2+1)^2}+\frac{2x}{(x^2+1)^2}$$
$$=\frac{5x^3-3x^2+5x-3+2x}{(x^2+1)^2}$$
$$=\frac{5x^3-3x^2+7x-3}{(x^2+1)^2}$$

55. $$A+B=3$$
$$2A-2B+C=17$$
$$4A-2C=14$$
Solving $A+B=3$ for B gives $B=3-A$.
Solving $4A-2C=14$ for A gives $C=2A-7$.
Use substitution to find *f*.
$$2A-2B+C=17$$
$$2A-2\overbrace{(3-A)}^{B}+\overbrace{2A-7}^{C}=17$$
$$2A-2(3-A)+2A-7=17$$
$$2A-6+2A+2A-7=17$$
$$6A-13=17$$
$$6A=30$$
$$A=5$$
Find *B*.
$$B=3-A$$
$$B=3-5=-2$$
Find *C*.
$$C=2A-7$$
$$C=2(5)-7=3$$
The solution set is {(5, –2, 3)}.

Section 8.3

Check Point Exercises

1. $$\frac{5x-1}{(x-3)(x+4)}=\frac{A}{x-3}+\frac{B}{x+4}$$
Multiply both sides of the equation by the least common denominator $(x-3)(x+4)$ and divide out common factors.
$$5x-1=A(x+4)+B(x-3)$$
$$5x-1=Ax+4A+Bx-3B$$
$$5x-1=(A+B)x+4A-3B$$

Equate coefficients of like powers of x and equate constant terms.
$$A+B=5$$
$$4A-3B=-1$$
Solving the above system for A and B we find $A = 2$ and $B = 3$.
$$\frac{5x-1}{(x-3)(x+4)}=\frac{2}{x-3}+\frac{3}{x+4}$$

2. $$\frac{x+2}{x(x-1)^2}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$$
Multiply both sides of the equation by the least common denominator $x(x-1)^2$ and divide out common factors.

$$x+2=A(x-1)^2+Bx(x-1)+Cx$$
$$x+2=A\left(x^2-2x+1\right)+Bx^2-Bx+Cx$$
$$x+2=Ax^2-2Ax+A+Bx^2-Bx+Cx$$
$$x+2=Ax^2+Bx^2-2Ax-Bx+Cx+A$$
$$x+2=(A+B)x^2+(-2A-B+C)x+A$$
Equate coefficients of like powers of x and equate constant terms.
$$A+B=0$$
$$-2A-B+C=1$$
$$A=2$$
Since $A = 2$, we find that $B = -2$ and $C = 3$ by substitution.
$$\frac{x+2}{x(x-1)^2}=\frac{2}{x}-\frac{2}{x-1}+\frac{3}{(x-1)^2}$$

3. $\dfrac{8x^2+12x-20}{(x+3)(x^2+x+2)}=\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+x+2}$

Multiply both sides of the equation by the least common denominator $(x+3)(x^2+x+2)$ and divide out
common factors.

$8x^2+12x-20=A(x^2+x+2)+(Bx+C)(x+3)$

$8x^2+12x-20=Ax^2+Ax+2A+Bx^2+3Bx+Cx+3C$

$8x^2+12x-20=Ax^2+Bx^2+Ax+3Bx+Cx+2A+3C$

$8x^2+12x-20=(A+B)x^2+(A+3B+C)x+2A+3C$

Equate coefficients of like powers of *x* and equate constant terms.

$A+B=8$

$A+3B+C=12$

$2A+3C=-20$

Solving the above system for *A*, *B*, and *C* we find *A* = 2, *B* = 6, and *C* = –8.

$\dfrac{8x^2+12x-20}{(x+3)(x^2+x+2)}=\dfrac{2}{x+3}+\dfrac{6x-8}{x^2+x+2}$

4. $\dfrac{2x^3+x+3}{(x^2+1)^2}=\dfrac{Ax+B}{x^2+1}+\dfrac{Cx+D}{(x^2+1)^2}$

Multiply both sides of the equation by the common denominator $(x^2+1)^2$ and divide out common factors.

$2x^3+x+3=(Ax+B)(x^2+1)+Cx+D$

$2x^3+x+3=Ax^3+Bx^2+Ax+B+Cx+D$

$2x^3+x+3=Ax^3+Bx^2+Ax+Cx+B+D$

$2x^3+x+3=Ax^3+Bx^2+(A+C)x+B+D$

Equate coefficients of like powers of *x* and equate constant terms.

$A=2$

$B=0$

$A+C=1$

$B+D=3$

Since *A* = 2 and *B* = 0 we find that *C* = –1 and *D* = 3 by substitution.

$\dfrac{2x^3+x+3}{(x^2+1)^2}=\dfrac{2x}{x^2+1}+\dfrac{-x+3}{(x^2+1)^2}=\dfrac{2x}{x^2+1}-\dfrac{x-3}{(x^2+1)^2}$

Exercise Set 8.3

1. $\dfrac{11x-10}{(x-2)(x+1)}=\dfrac{A}{x-2}+\dfrac{B}{x+1}$

2. $\dfrac{5x+7}{(x-1)(x+3)}=\dfrac{A}{x-1}+\dfrac{B}{x+3}$

3. $\dfrac{6x^2-14x-27}{(x+2)(x-3)^2}=\dfrac{A}{x+2}+\dfrac{B}{x-3}+\dfrac{C}{(x-3)^2}$

4. $\dfrac{3x+16}{(x+1)(x-2)^2}=\dfrac{A}{x+1}+\dfrac{B}{x-2}+\dfrac{C}{(x-2)^2}$

5. $\dfrac{5x^2-6x+7}{(x-1)(x^2+1)}=\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+1}$

6. $\dfrac{5x^2-9x+19}{(x-4)(x^2+5)}=\dfrac{A}{x-4}+\dfrac{Bx+C}{x^2+5}$

7. $\dfrac{x^3+x^2}{(x^2+4)^2}=\dfrac{Ax+B}{x^2+4}+\dfrac{Cx+D}{(x^2+4)^2}$

8. $\dfrac{7x^2-9x+3}{(x^2+7)^2}=\dfrac{Ax+B}{x^2+7}+\dfrac{Cx+D}{(x^2+7)^2}$

9. $\dfrac{x}{(x-3)(x-2)}=\dfrac{A}{x-3}+\dfrac{B}{x-2}$

Multiply both sides of the equation by the least common denominator $(x-3)(x-2)$ and divide out common factors.

$x=A(x-2)+B(x-3)$

$x=A\,x-2A+Bx-3B$

$x=Ax+Bx-2A-3B$

$x=(A+B)x-(2A+3B)$

Equate coefficients of like powers of *x*, and equate constant terms.

$A+B=1$

$2A+3B=0$

Solving the above system for *A* and *B*, we find *A* = 3 and *B* = –2.

$\dfrac{x}{(x-3)(x-2)}=\dfrac{3}{x-3}-\dfrac{2}{x-2}$

10. $\dfrac{1}{x(x-1)}=\dfrac{A}{x}+\dfrac{B}{x-1}$

Multiply both sides of the equation by the least common denominator $x(x - 1)$ and divide out common factors.

$1=A(x-1)+Bx$

$1=Ax-A+Bx$

$1=Ax+Bx-A$

$1=x(A+B)-A$

Equate coefficients of like powers of x and equate constant terms.

$A+B=0$

$-A=1$

Solving for A and B gives $A = –1$ and $B = 1$.

$\dfrac{1}{x(x-1)}=-\dfrac{1}{x}+\dfrac{1}{x-1}$

11. $\dfrac{3x+50}{(x-9)(x+2)}=\dfrac{\text{A}}{x-9}+\dfrac{\text{B}}{x+2}$

Multiply both sides of the equation by the least common denominator $(x-9)(x+2)$ and divide out common factors.

$3x+50=A(x+2)+B(x-9)$

$3x+50=Ax+2A+Bx-9B$

$3x+50=Ax+Bx+2A-9B$

$3x+50=(A+B)x+(2A-9B)$

Equate coefficients of like powers of x, and equate constant terms.

$A+B=3$

$2A-9B=50$

Solving the above system for A and B, we find $A = 7$ and $B = –4$.

$\dfrac{3x+50}{(x-9)(x+2)}=\dfrac{7}{x-9}-\dfrac{4}{x+2}$

12. $\dfrac{5x-1}{(x-2)(x+1)}=\dfrac{A}{x-2}+\dfrac{B}{x+1}$

Multiply both sides of the equation by the common denominator $(x – 2)(x + 1)$ and divide out common factors.

$5x-1=A(x+1)+B(x-2)$

$5x-1=Ax+A+Bx-2B$

$5x-1=Ax+Bx+A-2B$

$5x-1=(A+B)x+(A-2B)$

Equate coefficients of like powers of x and equate constant terms.

$A+B=5$

$A-2B=-1$

$A+B=5$

Solving the above system for A and B we find $A = 3$ and $B = 2$.

$\dfrac{5x-1}{(x-2)(x+1)}=\dfrac{3}{x-2}+\dfrac{2}{x+1}$

13. $\dfrac{7x-4}{x^2-x-12}=\dfrac{7x-4}{(x-4)(x+3)}=\dfrac{\text{A}}{x-4}+\dfrac{\text{B}}{x+3}$

Multiply both sides of the last equation by the least common denominator $(x-4)(x-3)$ and divide out common factors.

$7x-4=A(x+3)+B(x-4)$

$7x-4=A\,x+3\,A+Bx-4B$

$7x-4=Ax+Bx+3A-4B$

$7x-4=(A+B)x+(3A-4B)$

Equate coefficients of like powers of x, and equate constant terms.

$A+B=7$

$3A-4B=-4$

Solving the above system for A and B, we find $A = \dfrac{24}{7}$ and $B = \dfrac{25}{7}$.

$\dfrac{7x-4}{x^2-x-12}=\dfrac{24}{7(x-4)}+\dfrac{25}{7(x+3)}$

14. $\dfrac{9x+21}{x^2+2x-15}=\dfrac{9x+21}{(x-3)(x+5)}=\dfrac{A}{x-3}+\dfrac{B}{x+5}$

Multiply both sides of the last equation by the common denominator $(x-3)(x+5)$ and divide out common factors.

$9x+21=A(x+5)+B(x-3)$

$9x+21=Ax+5A+Bx-3B$

$9x+21=Ax+Bx+5A-3B$

$9x+21=(A+B)x+(5A-3B)$

Equate coefficients of like powers of x, and equate constant terms.

$A+B=9$

$5A-3B=21$

Solving the above system for A and B we find $A=6$ and $B=3$.

$\dfrac{9x+21}{x^2+2x-15}=\dfrac{6}{x-3}+\dfrac{3}{x+5}$

15. $\dfrac{4}{(2x+1)(x-3)}=\dfrac{A}{2x+1}+\dfrac{B}{x-3}$

Multiply both sides of the equation by the least common denominator $(2x+1)(x-3)$ and divide out common factors.

$4=A(x-3)+B(2x+1)$

$4=Ax-A3+B2x+B$

$4=(A+2B)x+(-3A+B)$

Equate coefficients of like powers of x and equate the constant terms. Solve for A and B.

$A+2B=0$

$-3A+B=4$

$3A+6B=0$

$-3A+B=4$

$7B=4$

$B=\dfrac{4}{7}$

$A+2B=0$

$6A-2B=-8$

$7A=-8$

$A=-\dfrac{8}{7}$

$\dfrac{4}{(2x+1)(x-3)}=\dfrac{-8}{7(2x+1)}+\dfrac{4}{7(x-3)}$

16. $\dfrac{x}{(x+3)(x-1)}=\dfrac{A}{x+3}+\dfrac{B}{x-1}$

Multiply both sides of the last equation by the common denominator $(x-1)(x+3)$ and divide out common factors.

$x=A(x-1)+B(x+3)$

$x=Ax-A+Bx+3B$

$x=Ax+Bx-A+3B$

$x=(A+B)x-A+3B$

Equate coefficients of like powers of x, and equate constant terms.

$A+B=1$

$A-3B=0$

Solving the above system for A and B we find $A=3/4$ and $B=1/4$.

$\dfrac{x}{x^2+2x-3}=\dfrac{\frac{3}{4}}{x+3}+\dfrac{\frac{1}{4}}{x-1}$

Multiply the numerators and the denominators of the left side of the equal sign to simplify the complex fractions.

$\dfrac{x}{x^2+2x-3}=\dfrac{3}{4x+12}+\dfrac{1}{4x-4}$

17. $\dfrac{4x^2+13x-9}{x(x-1)(x+3)}=\dfrac{A}{x}+\dfrac{B}{x-3}+\dfrac{C}{x+3}$

Multiply both sides of the equation by the least common denominator $x(x-1)(x+3)$ and divide out common factors.

$4x^2+13x-9=A(x\text{-}1)(x+3)+Bx(x+3)+Cx(x-1)$

$4x^2+13x-9=A(x^2+2x-3)+Bx^2+3Bx+Cx^2-Cx$

$4x^2+13x-9=Ax^2+2Ax-3A+Bx^2+3Bx+Cx^2-Cx$

$4x^2+13x-9=Ax^2+Bx^2+Cx^2+2Ax+3Bx-Cx-3A$

$4x^2+13x-9=(A+B+C)x^2+(2A+3B-C)x-3A$

Equate coefficients of like powers of x, and equate constant terms.

$A+B+C=4$

$2A+3B-C=13$

$-3A=-9$

Solving the above system for A, B, and C, we find $A=3$ and $B=2$, and $C=-1$.

$\dfrac{4x^2+13x-9}{x(x-1)(x+3)}=\dfrac{3}{x}+\dfrac{2}{x-1}-\dfrac{1}{x+3}$

18. $\dfrac{4x^2-5x-15}{x(x+1)(x-5)}=\dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{x-5}$

Multiply both sides of the equation by the common denominator $x(x + 1)(x - 5)$ and divide out common factors.

$$4x^2-5x-15=A(x+1)(x-5)+Bx(x-5)+Cx(x+1)$$
$$4x^2-5x-15=A(x^2-4x-5)+Bx^2-5Bx+Cx^2+Cx$$
$$4x^2-5x-15=Ax^2-4Ax-5A+Bx^2-5Bx+Cx^2+Cx$$
$$4x^2-5x-15=Ax^2+Bx^2+Cx^2-4Ax-5Bx+Cx-5A$$
$$4x^2-5x-15=(A+B+C)x^2+(-4A-5B+C)x-5A$$

Equate coefficients of like powers of x and equate constant terms.

$$A+B+C=4$$
$$-4A-5B+C=-5$$
$$-5A=-15$$

Solving the above system for A, B, C, we find $A = 3$, $B = -1$, and $C = 2$.

$$\frac{4x^2-5x-15}{x(x+1)(x-5)}=\frac{3}{x}-\frac{1}{x+1}+\frac{2}{x-5}$$

19. $\dfrac{4x^2-7x-3}{x^3-x}=\dfrac{4x^2-7x-3}{x(x+1)(x-1)}=\dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{x-1}$

Multiply both sides of the last equation by the least common denominator $x(x+1)(x-1)$ and divide out common factors.

$$4x^2-7x-3=A(x+1)(x-1)+Bx(x-1)+Cx(x+1)$$
$$4x^2-7x-3=A(x^2-1)+Bx^2-Bx+Cx^2+Cx$$
$$4x^2-7x-3=Ax^2-A+Bx^2-Bx+Cx^2+Cx$$
$$4x^2-7x-3=Ax^2+Bx^2+Cx^2-Bx+Cx-A$$
$$4x^2-7x-3=(A+B+C)x^2+(-B+C)x-A$$

Equate coefficients of like powers of x, and equate constant terms.

$$A+B+C=4$$
$$-B+C=-7$$
$$-A=-3$$

Solving the above system for A, B, and C, we find $A = 3$ and $B = 4$, and $C = -3$.

$$\frac{4x^2-7x-3}{x^3-x}=\frac{3}{x}+\frac{4}{x+1}-\frac{3}{x-1}$$

20. $\dfrac{2x^2-18x-12}{x^3-4x}=\dfrac{2x^2-18x-12}{x(x-2)(x+2)}=\dfrac{A}{x}+\dfrac{B}{x-2}+\dfrac{C}{x+2}$

Multiply both sides of the last equation by the common denominator $x(x - 2)(x + 2)$ and divide out common factors.

$$2x^2-18x-12=A(x-2)(x+2)+Bx(x+2)+Cx(x-2)$$
$$2x^2-18x-12=A(x^2-4)+Bx^2+2Bx+Cx^2-2Cx$$
$$2x^2-18x-12=Ax^2-4A+Bx^2+2Bx+Cx^2-2Cx$$
$$2x^2-18x-12=Ax^2+Bx^2+Cx^2+2Bx-2Cx-4A$$
$$2x^2-18x-12=(A+B+C)x^2+(2B-2C)x-4A$$

Equate coefficients of like powers of x and equate constant terms.

$$A + B + C = 2$$
$$2B - 2C = -18$$
$$-4A = -12$$

Solving the above system for A, B, C, we find $A = 3$, $B = -5$, and $C = 4$.

$$\frac{2x^2-18x-12}{x^3-4x}=\frac{3}{x}-\frac{5}{x-2}+\frac{4}{x+2}$$

21. $\dfrac{6x-11}{(x-1)^2}=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}$

Multiply both sides of the equation by the least common denominator $(x-1)^2$ and divide out common factors.

$6x - 11 = A(x - 1) + B$

$6x - 11 = Ax - A + B$

Equate coefficients of like powers of x, and equate constant terms.

$$A = 6$$
$$-A + B = -11$$

Since $A = 6$, we find that $B = -5$ by substitution.

$$\frac{6x-11}{(x-1)^2}=\frac{6}{x-1}-\frac{5}{(x-1)^2}$$

22. $\dfrac{x}{(x+1)^2}=\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}$

Multiply both sides of the equation by the common denominator $(x+1)^2$ and divide out common factors.

$x = A(x + 1) + B$

$x = Ax + A + B$

Equate coefficients of like powers of x and equate constant terms.

$$A = 1$$
$$A + B = 0$$

Since $A = 1$, we find that $B = -1$ by substitution.

$$\frac{x}{(x+1)^2}=\frac{1}{x+1}-\frac{1}{(x+1)^2}$$

23. $\dfrac{x^2-6x+3}{(x-2)^3}=\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{(x-2)^3}$

Multiply both sides of the equation by the least common denominator $(x-2)^3$ and divide out common factors.

$x^2-6x+3=A(x-2)^2+B(x-2)+C$

$x^2-6x+3=A(x^2-4x+4)+Bx-2B+C$

$x^2-6x+3=Ax^2-4Ax+4A+Bx-2B+C$

$x^2-6x+3=Ax^2-4Ax+Bx+4A-2B+C$

$x^2-6x+3=Ax^2+(-4A+B)x+4A-2B+C$

Equate coefficients of like powers of *x*, and equate constant terms.

$A=1$
$-4A+B=-6$
$4A-2B+C=3$

Since $A = 1$, we find that $B = -2$ and $C = -5$ by substitution. $\dfrac{x^2-6x+3}{(x-2)^3}=\dfrac{1}{x-2}-\dfrac{2}{(x-2)^2}-\dfrac{5}{(x-2)^3}$

24. $\dfrac{2x^2+8x+3}{(x+1)^3}=\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{C}{(x+1)^3}$

Multiply both sides of the equation by the common denominator $(x+1)^3$ and divide out common factors.

$2x^2+8x+3=A(x+1)^2+B(x+1)+C$

$2x^2+8x+3=A(x^2+2x+1)+Bx+B+C$

$2x^2+8x+3=Ax^2+2Ax+A+Bx+B+C$

$2x^2+8x+3=Ax^2+2Ax+Bx+A+B+C$

$2x^2+8x+3=Ax^2+(2A+B)x+A+B+C$

Equate coefficients of like powers of *x* and equate constant terms.

$A=2$
$2A+B=8$
$A+B+C=3$

Since $A = 2$, we find that $B = 4$ and $C = -3$ by substitution.

$\dfrac{2x^2+8x+3}{(x+1)^3}=\dfrac{2}{x+1}+\dfrac{4}{(x+1)^2}-\dfrac{3}{(x+1)^3}$

25. $\dfrac{x^2+2x+7}{x(x-1)^2}=\dfrac{A}{x}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$

Multiply both sides of the equation by the least common denominator $x(x-1)^2$ and divide out common factors.

$x^2+2x+7=A(x-1)^2+Bx(x-1)+Cx$

$x^2+2x+7=A(x^2-2x+1)+Bx^2-Bx+Cx$

$x^2+2x+7=Ax^2-2Ax+A+Bx^2-Bx+Cx$

$x^2+2x+7=Ax^2+Bx^2-2Ax-Bx+Cx+A$

$x^2+2x+7=(A+B)x^2+(-2A-B+C)x+A$

$A+B=1$
$-2A-B+C=2$
$A=7$

Since $A = 7$, we find that $B = -6$ and $C = 10$ by substitution. $\dfrac{x^2+2x+7}{x(x-1)^2}=\dfrac{7}{x}-\dfrac{6}{x-1}+\dfrac{10}{(x-1)^2}$

26. $\dfrac{3x^2+49}{x(x+7)^2}=\dfrac{A}{x}+\dfrac{B}{x+7}+\dfrac{C}{(x+7)^2}$

Multiply both sides of the equation by the common denominator $x(x+7)^2$ and divide out common factors.

$3x^2+49=A(x+7)^2+Bx(x+7)+Cx$

$3x^2+49=A(x^2+14x+49)+Bx^2+7Bx+Cx$

$3x^2+49=Ax^2+14Ax+49A+Bx^2+7Bx+Cx$

$3x^2+49=Ax^2+Bx^2+14Ax+7Bx+Cx+49A$

$3x^2+49=(A+B)x^2+(14A+7B+C)x+49A$

Equate coefficients of like powers of *x* and equate constant terms.

$A+B=3$
$14A+7B+C=0$
$49A=49$

Solving the above system for *A*, *B*, and *C*, we find $A = 1$, $B = 2$, and $C = -28$.

$\dfrac{3x^2+49}{x(x+7)^2}=\dfrac{1}{x}+\dfrac{2}{x+7}-\dfrac{28}{(x+7)^2}$

27. $\dfrac{x^2}{(x+1)(x-1)^2} = \dfrac{A}{x+1} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$

Multiply both sides of the equation by the least common denominator $(x+1)(x-1)^2$ and divide out common factors.

$x^2 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$

$x^2 = x^2A - 2xA + A + Bx^2 - B + Cx + C$

$x^2 = (A+B)x^2 + (-2A+C)x + (A-B+C)$

Equate coefficients of like powers of x, and equate constant terms.

$A + B = 1$
$-2A + C = 0$
$A - B + C = 0$

Solving the above system for A, B, and C, we find $A = \dfrac{1}{4}$, $B = \dfrac{3}{4}$, and $C = \dfrac{1}{2}$.

$\dfrac{x^2}{(x+1)(x-1)^2} = \dfrac{1}{4(x+1)} + \dfrac{3}{4(x-1)} + \dfrac{1}{2(x-1)^2}$

28. $\dfrac{x^2}{(x-1)^2(x+1)^2} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+1} + \dfrac{D}{(x+1)^2}$

Multiply both sides of the equation by the common denominator $(x-1)^2(x+1)^2$ and divide out common factors.

$x^2 = A(x-1)(x+1)^2 + B(x+1)^2 + C(x-1)^2(x+1) + D(x-1)^2$

$x^2 = Ax^3 + Ax^2 - Ax - A + Bx^2 + 2Bx + B + Cx^3 - Cx^2 - Cx + C + Dx^2 - 2Dx + D$ Equate coefficients of like powers of x

$x^2 = (A+C)x^3 + (A+B-C+D)x^2 + (-A+2B-C-2D)x + (-A+B+C+D)$

and equate constant terms.

$A + C = 0$
$A + B - C + D = 1$
$-A + 2B - C - 2D = 0$
$-A + B + C + D = 0$

Since $A + C = 0$, $C = -A$. Substitute $-A$ for C, and get three equations with three unknowns.

$2A + B + D = 1$
$2B - 2D = 0$
$-2A + B + D = 0$

Solving the above system for A, B, and C, we find $A = \dfrac{1}{4}$, $B = \dfrac{1}{4}$, $C = -\dfrac{1}{4}$, and $D = \dfrac{1}{4}$.

$\dfrac{x^2}{(x-1)^2(x+1)^2} = \dfrac{\frac{1}{4}}{x-1} + \dfrac{\frac{1}{4}}{(x-1)^2} + \dfrac{-\frac{1}{4}}{x+1} = \dfrac{\frac{1}{4}}{(x+1)^2}$

Multiply the numerators and denominators of the fractions on the right side of the equal to simplify the complex fractions.

$\dfrac{x^2}{(x-1)^2(x+1)^2} = \dfrac{1}{4(x-1)} + \dfrac{1}{4(x-1)^2} - \dfrac{1}{4(x+1)} + \dfrac{1}{4(x+1)^2}$

29. $\frac{5x^2-6x+7}{(x-1)(x^2+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$

Multiply both sides of the equation by the least common denominator $(x-1)(x^2+1)$ and divide out common factors.

$5x^2-6x+7=A(x^2+1)+(Bx+C)(x-1)$

$5x^2-6x+7=Ax^2+A+Bx^2-Bx+Cx-C$

$5x^2-6x+7=Ax^2+Bx^2-Bx+Cx+A-C$

$5x^2-6x+7=(A+B)x^2+(-B+C)x+A-C$

Equate coefficients of like powers of *x*, and equate constant terms.

$A+B=5$
$-B+C=-6$
$A-C=7$

Solving the above system for *A*, *B*, and *C*, we find $A=3$, $B=2$, and $C=-4$.

$\frac{5x^2-6x+7}{(x-1)(x^2+1)}=\frac{3}{x-1}+\frac{2x-4}{x^2+1}$

30. $\frac{5x^2-9x+19}{(x-4)(x^2+5)}=\frac{A}{x-4}+\frac{Bx+C}{x^2+5}$

Multiply both sides of the equation by the common denominator $(x-4)(x^2+5)$ and divide out common factors.

$5x^2-9x+19=A(x^2+5)+(Bx+C)(x-4)$

$5x^2-9x+19=Ax^2+5A+Bx^2-4Bx+Cx-4C$

$5x^2-9x+19=Ax^2+Bx^2-4Bx+Cx+5A-4C$

$5x^2-9x+19=(A+B)x^2+(-4B+C)x+5A-4C$

Equate coefficients of like powers of *x* and equate constant terms.

$A+B=5$
$-4B+C=-9$
$5A-4C=19$

Solving the above system for *A*, *B*, and *C*, we find $A=3$, $B=2$, and $C=-1$.

$\frac{5x^2-9x+19}{(x-4)(x^2+5)}=\frac{3}{x-4}+\frac{2x-1}{x^2+5}$

31. $\frac{5x^2+6x+3}{(x+1)(x^2+2x+2)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+2x+2}$

Multiply both sides of the equation by the least common denominator $(x+1)(x^2+2x+2)$ and divide out common factors.

$5x^2+6x+3=A(x^2+2x+2)+(Bx+C)(x+1)$

$5x^2+6x+3=Ax^2+2Ax+2A+Bx^2+Bx+Cx+C$

$5x^2+6x+3=Ax^2+Bx^2+2Ax+Bx+Cx+2A+C$

$5x^2+6x+3=(A+B)x^2+(2A+B+C)x+2A+C$

Equate coefficients of like powers of *x*, and equate constant terms.

$A+B=5$
$2A+B+C=6$
$2A+C=3$

Solving the above system for *A*, *B*, and *C*, we find $A=2$, $B=3$, and $C=-1$.

$\frac{5x^2+6x+3}{(x+1)(x^2+2x+2)}=\frac{2}{x+1}+\frac{3x-1}{x^2+2x+2}$

32. $\dfrac{9x+2}{(x-2)(x^2+2x+2)}=\dfrac{A}{x-2}+\dfrac{Bx+C}{x^2+2x+2}$

Multiply both sides of the equation by the common denominator $(x-2)(x^2+2x+2)$ and divide out common factors.

$9x+2=A(x^2+2x+2)+(Bx+C)(x-2)$

$9x+2=Ax^2+2Ax+2A+Bx^2-2Bx+Cx-2C$

$9x+2=Ax^2+Bx^2+2Ax-2Bx+Cx+2A-2C$

$9x+2=(A+B)x^2+(2A-2B+C)x+2A-2C$

Equate coefficients of like powers of *x* and equate constant terms.

$A+B=0$
$2A-2B+C=9$
$2A-2C=2$

Solving the above system for *A*, *B*, and *C*, we find $A=2$, $B=-2$, and $C=1$.

$\dfrac{9x+2}{(x-2)(x^2+2x+2)}=\dfrac{2}{x-2}-\dfrac{2x-1}{x^2+2x+2}$

33. $\dfrac{x+4}{x^2(x^2+4)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{Cx+D}{x^2+4}$

Multiply both sides of the equation by the least common denominator $x^2(x^2+4)$ and divide out common factors.

$x+4=Ax(x^2+4)+B(x^2+4)+(Cx+D)x^2$

$x+4=Ax^3+4Ax+Bx^2+4B+Cx^3+Dx^2$

$x+4=(A+C)x^3+(B+D)x^2+4Ax+4B$

Equate coefficients of like powers of *x*, and equate constant terms

$A+C=0$

$B+D=0$

$4A=1$

$4B=4$

Solving the above system for *A*, *B*, and *C*, we find $A=\dfrac{1}{4},\ B=1,\ C=-\dfrac{1}{4},$ and $D=-1$.

$\dfrac{x+4}{x^2(x^2+4)}=\dfrac{1}{4x}+\dfrac{1}{x^2}+\dfrac{-1x-4}{4\left(x^2+4\right)}$

34. $\dfrac{10x^2+2x}{(x-1)^2(x^2+2)}=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{Cx+D}{x^2+2}$

Multiply both sides of the equation by the common denominator $(x-1)^2(x^2+2)$ and divide out common factors.

$10x^2+2x=A(x-1)(x^2+2)+B(x^2+2)+(Cx+D)(x-1)^2$

$10x^2+2x=Ax^3-Ax^2+2Ax-2A+Bx^2+2B+Cx^3-2Cx^2+Cx+Dx^2-2Dx+D$

$10x^2+2x=(A+C)x^3+(-A+B-2C+D)x^2+(2A+C-2D)x+(-2A+2B+D)$

Equate coefficients of like powers of *x* and equate constant terms.

$A+C=0$

$-A+B-2C+D=10$

$2A+C-2D=2$

$-2A+2B+D=0$

Since $A + C = 0$, $C = -A$. Substitute $-A$ for C to get three equations with three unknowns.

$$A+B+D=10$$
$$A-2D=2$$
$$-2A+2B+D=0$$

Solving the above system for A, B, C, and D we find $A=\frac{14}{3}, B=4, C=-\frac{14}{3},$ and $D=\frac{4}{3}$.

$$\frac{10x^2+2x}{(x-1)^2(x^2+2)}=\frac{\frac{14}{3}}{x-1}+\frac{4}{(x-1)^2}+\frac{\frac{-14}{3}x+\frac{4}{3}}{x^2+2}$$

Multiply the numerators and denominators of the complex fractions to simplify.

$$\frac{10x^2+2x}{(x-1)^2(x^2+2)}=\frac{14}{3(x-1)}+\frac{4}{(x-1)^2}+\frac{-14x+4}{3(x^2+2)}$$

35. $\frac{6x^2-x+1}{x^3+x^2+x+1}=\frac{6x^2-x+1}{(x+1)(x^2+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+1}$

Multiply both sides of the last equation by the least common denominator $(x+1)(x^2+1)$ and divide out common factors.

$$6x^2-x+1=A(x^2+1)+(Bx+C)(x+1)$$
$$6x^2-x+1=Ax^2+A+Bx^2+Bx+Cx+C$$
$$6x^2-x+1=Ax^2+Bx^2+Bx+Cx+A+C$$
$$6x^2-x+1=(A+B)x^2+(B+C)x+A+C$$

Equate coefficients of like powers of x, and equate constant terms.

$A + B = 6$
$B + C = -1$
$A + C = 1$

Solving the above system for A, B, and C, we find $A = 4$, $B = 2$, and $C = -3$.

$$\frac{6x^2-x+1}{x^3+x^2+x+1}=\frac{4}{x+1}+\frac{2x-3}{x^2+1}$$

36. $\frac{3x^2-2x+8}{x^3+2x^2+4x+8}=\frac{3x^2-2x+8}{(x+2)(x^2+4)}=\frac{A}{x+2}+\frac{Bx+C}{x^2+4}$

Multiply both sides of the last equation by the common denominator $(x+2)(x^2+4)$ and divide out common factors.

$$3x^2-2x+8=A(x^2+4)+(Bx+C)(x+2)$$
$$3x^2-2x+8=Ax^2+4A+Bx^2+2Bx+Cx+2C$$
$$3x^2-2x+8=Ax^2+Bx^2+2Bx+Cx+4A+2C$$
$$3x^2-2x+8=(A+B)x^2+(2B+C)x+4A+2C$$

Equate coefficients of like powers of x and equate constant terms.

$A + B = 3$
$2B + C = -2$
$4A + 2C = 8$

Solving the above system for A, B, and C, we find $A = 3$, $B = 0$, and $C = -2$.

$$\frac{3x^2-2x+8}{x^3+2x^2+4x+8}=\frac{3}{x+2}-\frac{2}{x^2+4}$$

37. $\dfrac{x^3+x^2+2}{\left(x^2+2\right)^2}=\dfrac{\text{A}x+\text{B}}{x^2+2}+\dfrac{\text{C}x+\text{D}}{\left(x^2+2\right)^2}$

Multiply both sides of the last equation by the least common denominator $(x^2+2)^2$ and divide out common factors.

$x^3+x^2+2=(Ax+B)(x^2+2)+Cx+D$

$x^3+x^2+2=Ax^3+Bx^2+2Ax+2B+Cx+D$

$x^3+x^2+2=Ax^3+Bx^2+2Ax+Cx+2B+D$

$x^3+x^2+2=Ax^3+Bx^2+(2A+C)x+(2B+D)$

Equate coefficients of like powers of *x*, and equate constant terms.

$A=1$

$B=1$

$2A+C=0$

$2B+D=2$

Since A = 1 and B = 1, we find that C = –2 and D = 0 by substitution.

$\dfrac{x^3+x^2+2}{\left(x^2+2\right)^2}=\dfrac{x+1}{x^2+2}-\dfrac{2x}{\left(x^2+2\right)^2}$

38. $\dfrac{x^2+2x+3}{(x^2+4)^2}=\dfrac{Ax+B}{x^2+4}+\dfrac{Cx+D}{(x^2+4)^2}$

Multiply both sides of the equation by the common denominator $\left(x^2+4\right)^2$ and divide out common factors.

$x^2+2x+3=(Ax+B)(x^2+4)+Cx+D$

$x^2+2x+3=Ax^3+Bx^2+4Ax+4B+Cx+D$

$x^2+2x+3=Ax^3+Bx^2+4Ax+Cx+4B+D$

$x^2+2x+3=Ax^3+Bx^2+(4A+C)x+(4B+D)$

Equate coefficients of like powers of *x* and equate constant terms.

$A=0$

$B=1$

$4A+C=2$

$4B+D=3$

Since A = 0 and B = 1, we find that and C = 2 and D = –1 by substitution.

$\dfrac{x^2+2x+3}{(x^2+4)^2}=\dfrac{1}{x^2+4}+\dfrac{2x-1}{(x^2+4)^2}$

39. $\dfrac{x^3-4x^2+9x-5}{(x^2-2x+3)^2}=\dfrac{Ax+B}{x^2-2x+3}+\dfrac{Cx+D}{(x^2-2x+3)^2}$

Multiply both sides of the equation by the least common denominator $(x^2-2x+3)^2$ and divide out common factors.

$x^3-4x^2+9x-5=(Ax+B)(x^2-2x+3)+Cx+D$

$x^3-4x^2+9x-5=Ax^3-2Ax^2+3Ax+Bx^2-2Bx+3B+Cx+D$

$x^3-4x^2+9x-5=Ax^3-2Ax^2+Bx^2+3Ax-2Bx+Cx+3B+D$

$x^3-4x^2+9x-5=Ax^3+(-2A+B)x^2+(3A-2B+C)x+3B+D$

Equate coefficients of like powers of *x*, and equate constant terms.

$A=1$
$-2A+B=-4$
$3A-2B+C=9$
$3B+D=-5$

Since $A = 1$, we find that $B = -2$, $C = 2$, and $D = 1$ by substitution.

$\dfrac{x^3-4x^2+9x-5}{(x^2-2x+3)^2}=\dfrac{x-2}{x^2-2x+3}+\dfrac{2x+1}{(x^2-2x+3)^2}$

40. $\dfrac{3x^3-6x^2+7x-2}{(x^2-2x+2)^2}=\dfrac{Ax+B}{x^2-2x+2}+\dfrac{Cx+D}{(x^2-2x+2)^2}$

Multiply both sides of the equation by the common denominator $\left(x^2-2x+2\right)^2$ and divide out common factors.

$3x^3-6x^2+7x-2=(Ax+B)(x^2-2x+2)+Cx+D$

$3x^3-6x^2+7x-2=Ax^3-2Ax^2+2Ax+Bx^2-2Bx+2B+Cx+D$

$3x^3-6x^2+7x-2=Ax^3-2Ax^2+Bx^2+2Ax-2Bx+Cx+2B+D$

$3x^3-6x^2+7x-2=Ax^3+(-2A+B)x^2+(2A-2B+C)x+2B+D$

Equate coefficients of like powers of *x* and equate constant terms.

$A=3$
$-2A+B=-6$
$2A-2B+C=7$
$2B+D=-2$

Since $A = 0$ we find that $B = 0$, $C = 1$, and $D = -2$ by substitution.

$\dfrac{3x^3-6x^2+7x-2}{(x^2-2x+2)^2}=\dfrac{3x}{x^2-2x+2}+\dfrac{x-2}{(x^2-2x+2)^2}$

41. $\dfrac{4x^2+3x+14}{x^3-8}=\dfrac{4x^2+3x+14}{(x-2)(x^2+2x+4)}=\dfrac{A}{x-2}+\dfrac{Bx+C}{x^2+2x+4}$

Multiply both sides of the last equation by the least common denominator $(x-2)(x^2+2x+4)$ and divide out common factors.

$4x^2+3x+14=A(x^2+2x+4)+(Bx+C)(x-2)$

$4x^2+3x+14=A^2+2Ax+4A+Bx^2-2Bx+Cx-2C$

$4x^2+3x+14=Ax^2+Bx^2+2Ax-2Bx+Cx+4A-2C$

$4x^2+3x+14=(A+B)x^2+(2A-2B+C)x+(4A-2C)$

Equate coefficients of like powers of *x*, and equate constant terms.

$A+B=4$

$2A-2B+C=3$

$4A-2C=14$

Solving the above system for *A*, *B*, and *C*, we find *A* = 3, *B* = 1, and *C* = –1.

$\dfrac{4x^2+3x+4}{x^3-8}=\dfrac{3}{x-2}+\dfrac{x-1}{x^2+2x+4}$

42. $\dfrac{3x-5}{(x-1)(x^2+x+1)}=\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+x+1}$

Multiply both sides of the last equation by the common denominator $(x-1)(x^2+x+1)$ and divide out common factors.

$3x-5=A(x^2+x+1)+(Bx+C)(x-1)$

$3x-5=Ax^2+Ax+A+Bx^2+Cx-Bx-C$

$3x-5=(A+B)x^2+(A-B+C)x+(A-C)$

Equate coefficients of like powers of *x* and equate constant terms.

$A+B=0$

$A-B+C=3$

$A-C=-5$

Solving the above system for *A*, *B*, and *C*, we find $A=-\dfrac{2}{3}, B=\dfrac{2}{3}, C=\dfrac{13}{3}$.

$\dfrac{3x-5}{(x-1)(x^2+x+1)}=\dfrac{-2}{3(x-1)}+\dfrac{2x+13}{3(x^2+x+1)}$

43. Divide x^5+2 by x^2-1.

$$\begin{array}{r l}
 & x^3 + x \\
x^2-1 \,\big) & x^5 \qquad\qquad + 2 \\
 & \underline{x^5 - x^3} \\
 & \quad x^3 \\
 & \quad \underline{x^3 - x} \\
 & \qquad\quad x + 2
\end{array}$$

$\dfrac{x^5+2}{x^2-1}=x^3+x+\dfrac{x+2}{x^2-1}$

Decompose $\dfrac{x+2}{x^2-1}$.

$$\frac{x+2}{x^2-1}=\frac{x+2}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}$$

$$(x+1)(x-1)\frac{x+2}{(x+1)(x-1)}=(x+1)(x-1)\left(\frac{A}{x+1}+\frac{B}{x-1}\right)$$

$$x+2=(x-1)A+(x+1)B$$

$$x+2=Ax-A+Bx+B$$

$$x+2=(A+B)x+(-A+B)$$

Equate coefficients:

$A+B=1$

$-A+B=2$

Solving this system results in $A=-\frac{1}{2}$ and $B=\frac{3}{2}$.

$$\frac{x^5+2}{x^2-1}=x^3+x+\frac{-\frac{1}{2}}{x+1}+\frac{\frac{3}{2}}{x-1} \quad \text{or} \quad x^3+x+\frac{-1}{2(x+1)}+\frac{3}{2(x-1)}$$

44. Divide x^5 by x^2-4x+4.

$$\begin{array}{r|l} & x^3+4x^2+12x+32 \\ x^2-4x+4 & x^5 \\ & \underline{x^5-4x^4+4x^3} \\ & 4x^4-4x^3 \\ & \underline{4x^4-16x^3+16x^2} \\ & 12x^3-16x^2 \\ & \underline{12x^3-48x^2+48x} \\ & 32x^2-48x \\ & \underline{32x^2-128x+128} \\ & 80x-128 \end{array}$$

$$\frac{x^5}{x^2-4x+4}=x^3+4x^2+12x+32+\frac{80x-128}{x^2-4x+4}$$

Decompose $\frac{80x-128}{x^2-4x+4}$.

$$\frac{80x-128}{x^2-4x+4}=\frac{80x-128}{(x-2)^2}=\frac{A}{x-2}+\frac{B}{(x-2)^2}$$

$$(x-2)^2\frac{80x-128}{(x-2)^2}=(x-2)^2\left(\frac{A}{x-2}+\frac{B}{(x-2)^2}\right)$$

$$80x-128=Ax-2A+B$$

$$80x-128=Ax+(-2A+B)$$

Equate coefficients:

$A=80$

$-2A+B=-128$

Solving this system results in $A=80$ and $B=32$.

$$\frac{x^5}{x^2-4x+4}=x^3+4x^2+12x+32+\frac{80}{x-2}+\frac{32}{(x-2)^2}$$

45. Divide $x^4 - x^2 + 2$ by $x^3 - x^2$.

$$\begin{array}{r} x+1 \\ x^3 - x^2 \overline{\smash{)}\, x^4 \qquad - x^2 + 2} \\ \underline{x^4 - x^3} \qquad\qquad \\ x^3 - x^2 \qquad \\ \underline{x^3 - x^2} \qquad \\ 2 \end{array}$$

$$\frac{x^4 - x^2 + 2}{x^3 - x^2} = x + 1 + \frac{2}{x^3 - x^2}$$

Decompose $\frac{2}{x^3 - x^2}$.

$$\frac{2}{x^3 - x^2} = \frac{2}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$$x^2(x-1)\frac{2}{x^2(x-1)} = x^2(x-1)\left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}\right)$$

$$2 = x(x-1)A + (x-1)B + x^2C$$

$$2 = Ax^2 - Ax + Bx - B + Cx^2$$

$$2 = (A+C)x^2 + (-A+B)x + (-B)$$

Equate coefficients:

$$A + C = 0$$

$$-A + B = 0$$

$$-B = 2$$

Solving this system results in $A = -2$, $B = -2$, and $C = 2$.

$$\frac{x^4 - x^2 + 2}{x^3 - x^2} = x + 1 + \frac{-2}{x} + \frac{-2}{x^2} + \frac{2}{x-1}$$

46. Divide $x^4 + 2x^3 - 4x^2 + x - 3$ by $x^2 - x - 2$.

$$\begin{array}{r} x^2 + 3x + 1 \\ x^2 - x - 2 \overline{\smash{)}\, x^4 + 2x^3 - 4x^2 + x - 3} \\ \underline{x^4 - x^3 \; - 2x^2} \qquad\qquad \\ 3x^3 - 2x^2 + x \qquad \\ \underline{3x^3 - 3x^2 - 6x} \qquad \\ x^2 + 7x - 3 \\ \underline{x^2 \; - x - 2} \\ 8x - 1 \end{array}$$

$$\frac{x^4 + 2x^3 - 4x^2 + x - 3}{x^2 - x - 2} = x^2 + 3x + 1 + \frac{8x - 1}{x^2 - x - 2}$$

Decompose $\frac{8x-1}{x^2-x-2}$.

$$\frac{8x-1}{x^2-x-2}=\frac{8x-1}{(x+1)(x-2)}=\frac{A}{x+1}+\frac{B}{x-2}$$

$$\frac{8x-1}{(x+1)(x-2)}=\frac{A}{x+1}+\frac{B}{x-2}$$

$$(x+1)(x-2)\frac{8x-1}{(x+1)(x-2)}=(x+1)(x-2)\left(\frac{A}{x+1}+\frac{B}{x-2}\right)$$

$$8x-1=(x-2)A+(x+1)B$$

$$8x-1=Ax-2A+Bx+B$$

$$8x-1=(A+B)x+(-2A+B)$$

Equate coefficients:

$$A+B=8$$

$$-2A+B=-1$$

Solving this system results in $A=3$ and $B=5$.

$$\frac{x^4+2x^3-4x^2+x-3}{x^2-x-2}=x^2+3x+1+\frac{3}{x+1}+\frac{5}{x-2}$$

47. $$\frac{1}{x^2-c^2}=\frac{1}{(x+c)(x-c)}=\frac{A}{x+c}+\frac{B}{x-c}$$

$$(x+c)(x-c)\frac{1}{(x+c)(x-c)}=(x+c)(x-c)\left(\frac{A}{x+c}+\frac{B}{x-c}\right)$$

$$1=(x-c)A+(x+c)B$$

$$1=Ax-Ac+Bx+Bc$$

$$1=(A+B)x+(-Ac+Bc)$$

Equate coefficients:

$$A+B=0$$

$$-Ac+Bc=1$$

Solving this system results in $A=\frac{-1}{2c}$ and $B=\frac{1}{2c}$.

$$\frac{1}{x^2-c^2}=\frac{\frac{-1}{2c}}{x+c}+\frac{\frac{1}{2c}}{x-c}$$

48. $\dfrac{ax+b}{x^2-c^2}=\dfrac{ax+b}{(x+c)(x-c)}=\dfrac{D}{x+c}+\dfrac{E}{x-c}$

$$\frac{ax+b}{(x+c)(x-c)}=\frac{D}{x+c}+\frac{E}{x-c}$$

$$(x+c)(x-c)\frac{ax+b}{(x+c)(x-c)}=(x+c)(x-c)\left(\frac{D}{x+c}+\frac{E}{x-c}\right)$$

$$ax+b=(x-c)D+(x+c)E$$

$$ax+b=Dx-Dc+Ex+Ec$$

$$ax+b=(D+E)x+(-Dc+Ec)$$

Equate coefficients:

$D+E=a$

$-Dc+Ec=b$

Solving this system results in $D=\dfrac{ac-b}{2c}$ and $E=\dfrac{ac+b}{2c}$.

$$\frac{ax+b}{x^2-c^2}=\frac{\frac{ac-b}{2c}}{x+c}+\frac{\frac{ac+b}{2c}}{x-c}$$

49. $\dfrac{ax+b}{(x-c)^2}=\dfrac{D}{x-c}+\dfrac{E}{(x-c)^2}$

$$\frac{ax+b}{(x-c)^2}=\frac{D}{x-c}+\frac{E}{(x-c)^2}$$

$$(x-c)^2\frac{ax+b}{(x-c)^2}=(x-c)^2\left(\frac{D}{x-c}+\frac{E}{(x-c)^2}\right)$$

$$ax+b=(x-c)D+E$$

$$ax+b=Dx-Dc+E$$

$$ax+b=(D)x+(-Dc+E)$$

Equate coefficients:

$D=a$

$-Dc+E=b$

Solving this system results in $D=a$ and $E=ac+b$.

$$\frac{ax+b}{(x-c)^2}=\frac{a}{x-c}+\frac{ac+b}{(x-c)^2}$$

50. $\dfrac{1}{x^2-ax-bx+ab}=\dfrac{1}{(x-a)(x-b)}=\dfrac{C}{x-a}+\dfrac{D}{x-b}$

$$\frac{1}{(x-a)(x-b)}=\frac{C}{x-a}+\frac{D}{x-b}$$
$$(x-a)(x-b)\frac{1}{(x-a)(x-b)}=(x-a)(x-b)\left(\frac{C}{x-a}+\frac{D}{x-b}\right)$$
$$1=(x-b)C+(x-a)D$$
$$1=Cx-Cb+Dx-Da$$
$$1=(C+D)x+(-Cb-Da)$$

Equate coefficients:

$$C+D=0$$
$$-Cb-Da=1$$

Solving this system results in $C=\dfrac{1}{a-b}$ and $D=\dfrac{1}{b-a}$.

$$\frac{1}{x^2-ax-bx+ab}=\frac{\frac{1}{a-b}}{x-a}+\frac{\frac{1}{b-a}}{x-b}$$

51. $\dfrac{1}{x(x+1)}=\dfrac{A}{x}+\dfrac{B}{x+1}$

Multiply both sides of the equation by the least common denominator $x(x+1)$ and divide out common factors.

$$1=A(x+1)+Bx$$
$$1=Ax+A+Bx$$
$$1=Ax+Bx+A$$
$$1=(A+B)x+A$$

Equate coefficients of like powers of *x*, and equate constant terms.

$$A+B=0$$
$$A=1$$

Since $A = 1$ we find that $B = -1$ by substitution.

$$\frac{1}{x(x+1)}=\frac{1}{x}-\frac{1}{x+1}$$

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots\frac{1}{99\cdot 100}=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots\left(\frac{1}{99}-\frac{1}{100}\right)$$
$$=\frac{1}{1}-\frac{1}{100}$$
$$=\frac{99}{100}$$

52. $\frac{2}{x(x+2)}=\frac{A}{x}+\frac{B}{x+2}$

Multiply both sides of the equation by the common denominator $x(x+2)$ and divide out common factors.

$2=A(x+2)+Bx$

$2=Ax+2A+Bx$

$2=Ax+Bx+2A$

$2=(A+B)x+2A$

Equate coefficients of like powers of *x* and equate constant terms.

$A+B=0$

$2A=2$

Solving the above system for *A* and *B* we find *A* = 1, and *B* = –1.

$\frac{2}{x(x+2)}=\frac{1}{x}-\frac{1}{x+2}$

$\frac{2}{1\cdot 3}+\frac{2}{3\cdot 5}+\frac{2}{5\cdot 7}+\cdots+\frac{2}{99\cdot 101}=\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\cdots\left(\frac{1}{99}-\frac{1}{101}\right)=\frac{1}{1}-\frac{1}{101}=\frac{100}{101}$

53. – 59. Answers may vary.

60. does not make sense; Explanations will vary. Sample explanation: Partial fraction decomposition involves finding a sum of two or more rational expressions.

61. does not make sense; Explanations will vary. Sample explanation: To perform partial fraction decomposition, the degree in the numerator must be less than the degree in the denominator.

62. does not make sense; Explanations will vary. Sample explanation: The quadratic factor in the denominator must be factored.

63. does not make sense; Explanations will vary. Sample explanation: The second denominator should be $(x+3)^2$.

64. Answers may vary.

65. $\frac{4x^2+5x-9}{x^3-6x-9}=\frac{4x^2+5x-9}{(x-3)(x^2+3x+3)}=\frac{A}{x-3}+\frac{Bx+C}{x^2+3x+3}$

Multiply both sides of the last equation by the common denominator $(x-3)(x^2+3x+3)$ and divide out common factors.

$4x^2+5x-9=A(x^2+3x+3)+(Bx+C)(x-3)$

$4x^2+5x-9=Ax^2+3Ax+3A+Bx^2-3Bx+Cx-3C$

$4x^2+5x-9=Ax^2+Bx^2+3Ax-3Bx+Cx+3A-3C$

$4x^2+5x-9=(A+B)x^2+(3A-3B+C)x+3A-3C$

Equate coefficients of like powers of *x* and equate constant terms.

$A+B=4$

$3A-3B+C=5$

$3A-3C=-9$

Solving the above system for *A*, *B*, and *C*, we find *A* = 2, and *B* = 2, and *C* = 5.

$\frac{4x^2+5x-9}{x^3-6x-9}=\frac{2}{x-3}+\frac{2x+5}{x^2+3x+3}$

66. $4x + 3y = 4$

$y = 2x - 7$

Substitute.

$$4x + 3y = 4$$

$$4x + 3\overbrace{(2x-7)}^{y} = 4$$

$$4x + 3(2x - 7) = 4$$

$$4x + 6x - 21 = 4$$

$$10x = 25$$

$$x = 2.5$$

Back-substitute.

$$y = 2x - 7$$

$$y = 2(2.5) - 7 = -2$$

The solution set is {(2.5, –2)}.

67. $2x + 4y = -4$

$3x + 5y = -3$

Multiply the first equation by –5 and the second equation by 4.

$$-10x - 20y = 20$$

$$12x + 20y = -12$$

Add the equations and solve for x.

$$2x = 8$$

$$x = 4$$

Back-substitute to find y.

$$2x + 4y = -4$$

$$2(4) + 4y = -4$$

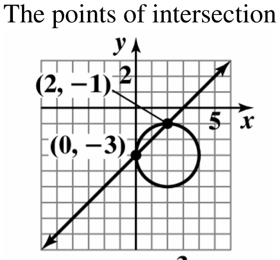
$$8 + 4y = -4$$

$$4y = -12$$

$$y = -3$$

The solution set is {(4, –3)}.

68. The points of intersection are (0, –3) and (2, –1).



$x - y = 3$

$(x-2)^2 + (y+3)^2 = 4$

Point (0, –3):

$$x - y = 3$$

$$0 - (-3) = 3$$

$$3 = 3, \text{ true}$$

$$(x-2)^2 + (y+3)^2 = 4$$

$$(0-2)^2 + (-3+3)^2 = 4$$

$$(-2)^2 + (0)^2 = 4$$

$$4 = 4, \text{ true}$$

Point (2, –1):

$$x - y = 3$$

$$2 - (-1) = 3$$

$$3 = 3, \text{ true}$$

$$(x-2)^2 + (y+3)^2 = 4$$

$$(2-2)^2 + (-1+3)^2 = 4$$

$$(0)^2 + (2)^2 = 4$$

$$4 = 4, \text{ true}$$

Section 8.4

Check Point Exercises

1. $x^2 = y - 1$

$4x - y = -1$

Solve the first equation for y.

$$y = x^2 + 1$$

Substitute the expression $x^2 + 1$ for y in the second equation and solve for x.

$$4x - (x^2 + 1) = -1$$

$$4x - x^2 - 1 = -1$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$x = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = 4$$

If $x = 0,\ y = (0)^2 + 1 = 1.$

If $x = 4,\ y = (4)^2 + 1 = 17.$

The solution set is {(0, 1), (4, 17)}.

2. $x+2y=0$
$(x-1)^2+(y-1)^2=5$
Solve the first equation for *x*.
$x=-2y$
Substitute the expression $-2y$ for *x* in the second equation and solve for *y*.

$$(-2y-1)^2+(y-1)^2=5$$
$$4y^2+4y+1+y^2-2y+1=5$$
$$5y^2+2y-3=0$$
$$(5y-3)(y+1)=0$$
$$5y-3=0 \quad \text{or} \quad y+1=0$$
$$y=\frac{3}{5} \quad \text{or} \quad y=-1$$

If $y=\frac{3}{5}$, $x=-2\left(\frac{3}{5}\right)=-\frac{6}{5}$.
If $y=-1$, $x=-2(-1)=2$.
The solution set is $\left\{\left(-\frac{6}{5}, \frac{3}{5}\right), (2, -1)\right\}$.

3. $3x^2+2y^2=35$
$4x^2+3y^2=48$
Eliminate the y^2-term by multiplying the first equation by –3 and the second equation by 2. Add the resulting equations.

$$-9x^2-6y=-105$$
$$\underline{8x^2+6y^2=96}$$
$$-x^2=-9$$
$$x^2=9$$
$$x=\pm 3$$

If $x=3$,
$$3(3)^2+2y^2=35$$
$$y^2=4$$
$$y=\pm 2$$
If $x=-3$,
$$3(-3)^2+2y^2=35$$
$$y^2=4$$
$$y=\pm 2$$
The solution set is $\{(3,2),(3,-2),(-3,2),(-3,-2)\}$.

4. $y=x^2+5$
$x^2+y^2=25$
Arrange the first equation so that variable terms appear on the left, and constants appear on the right. Add the resulting equations to eliminate the x^2-terms and solve for *y*.

$$-x^2+y=5$$
$$\underline{x^2+y^2=25}$$
$$y^2+y=30$$
$$y^2+y-30=0$$
$$(y+6)(y-5)=0$$
$$y+6=0 \quad \text{or} \quad y-5=0$$
$$y=-6 \quad \text{or} \quad y=5$$

If $y=-6$,
$$x^2+(-6)^2=25$$
$$x^2=-11$$
no real solution
If $y=5$,
$$x^2+(5)^2=25$$
$$x^2=0$$
$$x=0$$
The solution set is $\{(0, 5)\}$.

5. $2x+2y=20$
$xy=21$
Solve the second equation for *x*.
$$x=\frac{21}{7}$$
Substitute the expression $\frac{21}{y}$ for *x* in the first equation and solve for *y*.

$$2\left(\frac{21}{y}\right)+2y=20$$
$$\frac{42}{y}+2y=20$$
$$y^2-10y+21=0$$
$$(y-7)(y-3)=0$$
$$y-7=0 \quad \text{or} \quad y-3=0$$
$$y=7 \quad \text{or} \quad y=3$$

If $y=7$, $x=\frac{21}{7}=3$.
If $y=3$, $x=\frac{21}{3}=7$.
The dimensions are 7 feet by 3 feet.

Exercise Set 8.4

1. $x + y = 2$
$y = x^2 - 4$
Solve the first equation for *y*. $y = 2 - x$.
Substitute the expression $2 - x$ for *y* in the second equation and solve for *x*.
$$2 - x = x^2 - 4$$
$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x + 3 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -3 \quad \text{or} \quad x = 2$$
If $x = -3$, $y = 2 - (-3) = 5$.
If $x = 2$, $y = 2 - 2 = 0$.
The solution set is {(–3, 5), (2, 0)}.

2. $x - y = -1$
$y = x^2 + 1$
Substitute the expression $x^2 + 1$ for *y* in the first equation and solve for *x*.
$$x - (x^2 + 1) = -1$$
$$x - x^2 - 1 = -1$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = 1$$
If $x = 0$, $y = (0)^2 + 1 = 1$.
If $x = 1$, $y = (1)^2 + 1 = 2$.
The solution set is {(0, 1), (1, 2)}.

3. $x + y = 2$
$y = x^2 - 4x + 4$
Substitute the expression $x^2 - 4x + 4$ for *y* in the first equation and solve for *x*.
$$x + x^2 - 4x + 4 = 2$$
$$x^2 - 3x + 2 = 0$$
$$(x-1)(x-2) = 0$$
$$x - 1 = 0 \qquad x - 2 = 0$$
$$x = 1 \qquad x = 2$$
Substitute $x = 1$ and then $x = 2$ into the equation $x + y = 2$ and solve for each value of *y*.
$1 + y = 2 \quad 2 + y = 2$
$y = 1 \qquad y = 0$
The solution set is {(1, 1), (2, 0)}.

4. $2x + y = -5$
$y = x^2 + 6x + 7$
Substitute the expression $x^2 + 6x + 7$ for *y* in the first equation and solve for *x*.
$$2x + (x^2 + 6x + 7) = -5$$
$$x^2 + 8x + 12 = 0$$
$$(x+6)(x+2) = 0$$
$$x + 6 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = -6 \quad \text{or} \quad x = -2$$
Solve the first equation for *y*.
$y = -2x - 5$
If $x = -6$, $y = -2(-6) - 5 = 7$.
If $x = -2$, $y = -2(-2) - 5 = -1$.
The solution set is {(–6, 7), (–2, –1)}.

5. $y = x^2 - 4x - 10$
$y = -x^2 - 2x + 14$
Substitute the expression $x^2 - 4x - 10$ for *y* in the second equation and solve for *x*.
$$x^2 - 4x - 10 = -x^2 - 2x + 14$$
$$2x^2 - 2x - 24 = 0$$
$$x^2 - x - 12 = 0$$
$$(x-4)(x+3) = 0$$
$$x - 4 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = 4 \quad \text{or} \quad x = -3$$
If $x = 4$, $y = (4)^2 - 4(4) - 10 = -10$.
If $x = -3$, $y = (-3)^2 - 4(-3) - 10 = 11$.
The solution set is $\{(4, -10), (-3, 11)\}$.

6. $y = x^2 + 4x + 5$
$y = x^2 + 2x - 1$
Substitute the expression $x^2 + 4x + 5$ for *y* in the first equation and solve for *x*.
$$y = x^2 + 2x - 1$$
$$x^2 + 4x + 5 = x^2 + 2x - 1$$
$$2x = -6$$
$$x = -3$$
$$y = (-3)^2 + 4(-3) + 5 = 2$$
The solution set is {(–3, 2)}.

7. $x^2 + y^2 = 25$
$x - y = 1$
Solve the second equation for *y*. $y = x - 1$
Substitute the expression $x-1$ for *y* in the first equation and solve for *x*.

$$x^2 + (x-1)^2 = 25$$
$$x^2 + x^2 - 2x + 1 = 25$$
$$2x^2 - 2x - 24 = 0$$
$$x^2 - x - 12 = 0$$
$$(x-4)(x+3) = 0$$
$$x-4=0 \quad \text{or} \quad x+3=0$$
$$x=4 \quad \text{or} \quad x=-3$$

If $x = 4, y = 4 - 1 = 3$.
If $x = -3, y = -3 - 1 = -4$.
The solution set is {(4, 3), (–3, –4)}.

8. $x^2 + y^2 = 5$
$3x - y = 5$
Solve the second equation for *y*.
$y = 3x - 5$
Substitute the expression $3x-5$ for *y* in the first equation and solve for *x*.

$$x^2 + (3x-5)^2 = 5$$
$$x^2 + 9x^2 - 30x + 25 = 5$$
$$10x^2 - 30x + 20 = 0$$
$$x^2 - 3x + 2 = 0$$
$$(x-2)(x-1) = 0$$
$$x-2=0 \quad \text{or} \quad x-1=0$$
$$x=2 \quad \text{or} \quad x=1$$

If $x = 2,\ y = 3(2) - 5 = 1$.
If $x = 1,\ y = 3(1) - 5 = -2$.
The solution set is {(2, 1), (1, –2)}.

9. $xy = 6$
$2x - y = 1$
Solve the first equation for *y*.
$y = \frac{6}{x}$
Substitute the expression $\frac{6}{x}$ for *y* in the second equation and solve for *x*.

$$2x - \frac{6}{x} = 1$$
$$2x^2 - 6 = x$$
$$2x^2 - x - 6 = 0$$
$$(2x+3)(x-2) = 0$$
$$2x+3=0 \quad \text{or} \quad x-2=0$$
$$x = -\frac{3}{2} \quad \text{or} \quad x=2$$

If $x = -\frac{3}{2},\ y = \frac{6}{-\frac{3}{2}} = -4$.
If $x = 2,\ y = \frac{6}{2} = 3$.
The solution set is $\left\{\left(-\frac{3}{2}, -4\right), (2, 3)\right\}$.

10. $xy = -12$
$x - 2y + 14 = 0$
Solve the first equation for *y*.
$y = -\frac{12}{x}$
Substitute the expression $-\frac{12}{x}$ for *y* in the second equation and solve for *x*.

$$x - 2\left(-\frac{12}{x}\right) + 14 = 0$$
$$x^2 + 14x + 24 = 0$$
$$(x+2)(x+12) = 0$$
$$x+2=0 \quad \text{or} \quad x+12=0$$
$$x=-2 \quad \text{or} \quad x=-12$$

If $x = -2,\ y = \frac{-12}{-2} = 6$.
If $x = -12,\ y = \frac{-12}{-12} = 1$.
The solution set is {(–2, 6), (–12, 1)}.

11. $y^2 = x^2 - 9$
$2y = x - 3$
Solve the second equation for *y*.
$y = \frac{x-3}{2}$
Substitute the expression $\frac{x-3}{2}$ for *y* in the first equation and solve for *x*.
$$\left(\frac{x-3}{2}\right)^2 = x^2 - 9$$
$$\frac{x^2-6x+9}{4} = x^2 - 9$$
$$x^2 - 6x + 9 = 4x^2 - 36$$
$$3x^2 + 6x - 45 = 0$$
$$x^2 + 2x - 15 = 0$$
$$(x+5)(x-3) = 0$$
$x + 5 = 0$ or $x - 3 = 0$
$x = -5$ or $x = 3$
If $x = -5$, $y = \frac{-5-3}{2} = -4$.
If $x = 3$, $y = \frac{3-3}{2} = 0$.
The solution set is $\{(-5,-4),(3,0)\}$.

12. $x^2 + y = 4$
$2x + y = 1$
Solve the second equation for *y*.
$y = 1 - 2x$
Substitute the expression $1 - 2x$ for *y* in the first equation and solve for *x*.
$$x^2 + (1 - 2x) = 4$$
$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$
$x - 3 = 0$ or $x + 1 = 0$
$x = 3$ or $x = -1$
If $x = 3$, $y = 1 - 2(3) = -5$.
If $x = -1$, $y = 1 - 2(-1) = 3$.
The solution set is {(3, –5)}, {(–1, 3)}.

13. $xy = 3$
$x^2 + y^2 = 10$
Solve the second equation for *y*.
$y = \frac{3}{x}$
Substitute the expression $\frac{3}{x}$ for *y* in the second equation and solve for *x*.
$$x^2 + \left(\frac{3}{x}\right)^2 = 10$$
$$x^2 + \frac{9}{x^2} - 10 = 0$$
$$x^4 - 10x^2 + 9 = 0$$
$$(x^2 - 9)(x^2 - 1) = 0$$
$$(x-3)(x+3)(x-1)(x+1) = 0$$
$x - 3 = 0$ or $x + 3 = 0$ or $x - 1 = 0$ or $x + 1 = 0$
$x = 3$ or $x = -3$ or $x = 1$ or $x = -1$
If $x = 3$, $y = \frac{3}{3} = 1$.
If $x = -3$, $y = \frac{3}{-3} = -1$.
If $x = 1$, $y = \frac{3}{1} = 3$.
If $x = -1$, $y = \frac{3}{-1} = -3$.
The solution set is $\{(3,1),(-3,-1),(1,3),(-1,-3)\}$.

14. $xy = 4$
$x^2 + y^2 = 8$
Solve the first equation for *y*.
$y = \frac{4}{x}$ Substitute the expression $\frac{4}{x}$ for *y* in the second equation and solve for *x*.
$$x^2 + \left(\frac{4}{x}\right)^2 = 8$$
$$x^2 + \frac{16}{x^2} - 8 = 0$$
$$x^4 - 8x^2 + 16 = 0$$
$$(x^2 - 4)(x^2 - 4) = 0$$
$$(x^2 - 4)^2 = 0$$
$$x^2 - 4 = 0$$
$$(x-2)(x+2) = 0$$
$x - 2 = 0$ or $x + 2 = 0$
$x = 2$ or $x = -2$
If $x = 2$, $y = \frac{4}{2} = 2$.
If $x = -2$, $y = \frac{4}{-2} = -2$.
The solution set is {(2, 2), (–2, –2)}.

15. $x + y = 1$

$x^2 + xy - y^2 = -5$

Solve the first equation for *y*. $y = 1 - x$

Substitute the expression $1 - x$ for *y* in the second equation and solve for *x*.

$$x^2 + x(1-x) - (1-x)^2 = -5$$
$$x^2 + x - x^2 - (1 - 2x + x^2) = -5$$
$$x - 1 + 2x - x^2 = -5$$
$$x^2 - 3x - 4 = 0$$
$$(x-4)(x+1) = 0$$
$$x - 4 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 4 \quad \text{or} \quad x = -1$$

If $x = 4$, $y = 1 - 4 = -3$.

If $x = -1$, $y = 1 - (-1) = 2$.

The solution set is $\{(4,-3),(-1,2)\}$.

16. $x + y = -3$

$x^2 + 2y^2 = 12y + 18$

Solve the first equation for *x*.

$x = -y - 3$

Substitute the expression $-y - 3$ for *x* in the second equation and solve for *y*.

$$(-y-3)^2 + 2y^2 = 12y + 18$$
$$y^2 + 6y + 9 + 2y^2 - 12y - 18 = 0$$
$$3y^2 - 6y - 9 = 0$$
$$y^2 - 2y - 3 = 0$$
$$(y-3)(y+1) = 0$$
$$y - 3 = 0 \quad \text{or} \quad y + 1 = 0$$
$$y = 3 \quad \text{or} \quad y = -1$$

If $y = 3$, $x = -3 - 3 = -6$.

If $y = -1$, $x = -(-1) - 3 = -2$.

The solution set is $\{(-6, 3), (-2, -1)\}$.

17. $x + y = 1$

$(x-1)^2 + (y+2)^2 = 10$

Solve the first equation for *y*.

$y = 1 - x$

Substitute the expression $1 - x$ for *y* in the second equation and solve for *x*.

$$(x-1)^2 + (1-x+2)^2 = 10$$
$$(x-1)^2 + (3-x)^2 = 10$$
$$x^2 - 2x + 1 + 9 - 6x + x^2 - 10 = 0$$
$$2x^2 - 8x = 0$$
$$x^2 - 4x = 0$$
$$x(x-4) = 0$$
$$x = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = 4$$

If $x = 0$, $y = 1 - 0 = 1$.

If $x = 4$, $y = 1 - 4 = -3$.

The solution set is $\{(0,1),(4,-3)\}$.

18. $2x + y = 4$

$(x+1)^2 + (y-2)^2 = 4$

Solve the first equation for *y*.

$y = 4 - 2x$

Substitute the expression $4 - 2x$ for *y* in the second equation and solve for *x*.

$$(x+1)^2 + (4 - 2x - 2)^2 = 4$$
$$(x+1)^2 + (2 - 2x)^2 = 4$$
$$x^2 + 2x + 1 + 4 - 8x + 4x^2 - 4 = 0$$
$$5x^2 - 6x + 1 = 0$$
$$(5x-1)(x-1) = 0$$
$$5x - 1 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = \frac{1}{5} \quad \text{or} \quad x = 1$$

If $x = \frac{1}{5}$, $y = 4 - 2\left(\frac{1}{5}\right) = \frac{18}{5}$.

If $x = 1$, $y = 4 - 2(1) = 2$.

The solution set is $\left\{\left(\frac{1}{5}, \frac{18}{5}\right), (1, 2)\right\}$.

19. Eliminate the y^2 –terms by adding the equations.

$$\begin{aligned} x^2 + y^2 &= 13 \\ x^2 - y^2 &= 5 \\ \hline 2x^2 &= 18 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

If $x = 3$,

$$\begin{aligned} (3)^2 + y^2 &= 13 \\ y^2 &= 4 \\ y &= \pm 2 \end{aligned}$$

If $x = -3$,

$$\begin{aligned} (-3)^2 + y^2 &= 13 \\ y^2 &= 4 \\ y &= \pm 2 \end{aligned}$$

The solution set is
{(3, 2), (3, –2), (–3, 2), (–3, –2)}.

20. Eliminate the y^2-terms by adding the equations.

$$\begin{aligned} 4x^2 - y^2 &= 4 \\ 4x^2 + y^2 &= 4 \\ \hline 8x^2 &= 8 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

If $x = 1$,

$$\begin{aligned} 4(1) + y^2 &= 4 \\ y^2 &= 0 \\ y &= 0 \end{aligned}$$

If $x = -1$,

$$\begin{aligned} 4(-1)^2 + y^2 &= 4 \\ y^2 &= 0 \\ y &= 0 \end{aligned}$$

The solution set is {(1, 0), (–1, 0)}.

21.

$$\begin{aligned} x^2 - 4y^2 &= -7 \\ 3x^2 + y^2 &= 31 \end{aligned}$$

Eliminate the x^2–terms by multiplying the first equation by -3 and adding the resulting equations.

$$\begin{aligned} -3x^2 + 12y^2 &= 21 \\ 3x^2 + y^2 &= 31 \\ \hline 13y^2 &= 52 \\ y^2 &= 4 \\ y &= \pm 2 \end{aligned}$$

If $y = 2$,

$$\begin{aligned} x^2 - 4(2)^2 &= -7 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

If $y = -2$,

$$\begin{aligned} x^2 - 4(-2)^2 &= -7 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

The solution set is $\{(3,2),(3,-2),(-3,2),(-3,-2)\}$.

22.

$$\begin{aligned} 3x^2 - 2y^2 &= -5 \\ 2x^2 - y^2 &= -2 \end{aligned}$$

Eliminate the y^2-terms by multiplying the second equation by –2 and adding the resulting equations.

$$\begin{aligned} 3x^2 - 2y^2 &= -5 \\ -4x^2 + 2y^2 &= 4 \\ \hline -x^2 &= -1 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

If $x = 1$,

$$\begin{aligned} 2(1)^2 - y^2 &= -2 \\ y^2 &= 4 \\ y &= \pm 2 \end{aligned}$$

If $x = -1$,

$$\begin{aligned} 2(-1)^2 - y &= -2 \\ y^2 &= 4 \\ y &= \pm 2 \end{aligned}$$

The solution set is
{(1, 2), (1, –2), (–1, 2), (–1, –2)}.

23. Arrange the equations so that variable terms appear on the left and constants appear on the right.

$3x^2+4y^2=16$

$2x^2-3y^2=5$

Eliminate the y^2–terms by multiplying the first equation by 3 and the second equation by 4. Add the resulting equations.

$$\begin{aligned}9x^2+12y^2&=48\\ 8x^2-12y^2&=20\\ \hline 17x^2&=68\\ x^2&=4\\ x&=\pm 2\end{aligned}$$

If $x = 2$,

$$\begin{aligned}3(2)^2+4y^2&=16\\ y^2&=1\\ y&=\pm 1\end{aligned}$$

If $x = -2$,

$$\begin{aligned}3(-2)^2+4y^2&=16\\ y&=\pm 1\end{aligned}$$

The solution set is $\{(2, 1), (2, -1), (-2, 1), (-2, -1)\}$.

24. Arrange the equations so that variable terms appear on the left and constants appear on the right.

$16x^2-4y^2=72$

$x^2-y^2=3$

Eliminate the y^2-terms by multiplying the second equation by – 4 and adding the resulting equations.

$$\begin{aligned}16x^2-4y^2&=72\\ -4x^2+4y^2&=-12\\ 12x^2&=60\\ x^2&=5\\ x&=\pm\sqrt{5}\end{aligned}$$

If $x=\sqrt{5}$,

$$\begin{aligned}\left(\sqrt{5}\right)^2-y^2&=3\\ 5-y^2&=3\\ -y^2&=-2\\ y^2&=2\\ y&=\pm\sqrt{2}\end{aligned}$$

If $x=-\sqrt{5}$,

$$\begin{aligned}\left(-\sqrt{5}\right)^2-y^2&=3\\ 5-y^2&=3\\ -y^2&=-2\\ y^2&=2\\ y&=\pm\sqrt{2}\end{aligned}$$

The solution set is

$\left\{\left(\sqrt{5},\sqrt{2}\right),\left(\sqrt{5},-\sqrt{2}\right),\left(-\sqrt{5},\sqrt{2}\right),\left(-\sqrt{5},-\sqrt{2}\right)\right\}$

25. $x^2+y^2=25$

$(x-8)^2+y^2=41$

Expand the second equation and eliminate x^2 and y^2–terms by multiplying the first equation by –1 and adding the resulting equations.

$$\begin{aligned}x^2-16x+64+y^2&=41\\ -x^2-y^2&=-25\\ \hline -16x+64&=16\\ -16x&=-48\\ x&=3\end{aligned}$$

If $x = 3$,

$$\begin{aligned}(3)^2+y^2&=25\\ y^2&=16\\ y&=\pm 4\end{aligned}$$

The solution set is $\{(3,4),\ (3,-4)\}$.

26. $x^2 + y^2 = 5$

$x^2 + (y-8)^2 = 41$

Expand the second equation and eliminate the x^2 and y^2-terms by multiplying the first equation by –1 and adding the resulting equations.

$$\begin{aligned} x^2 + y^2 - 16y + 64 &= 41 \\ \underline{-x^2 - y^2 = -5} & \\ -16y + 64 &= 36 \\ -16y &= -28 \\ y &= \frac{28}{16} = \frac{7}{4} \end{aligned}$$

If $y = \frac{7}{4}$,

$$x^2 + \left(\frac{7}{4}\right)^2 = 5$$

$$x^2 = \frac{31}{16}$$

$$x = \pm\frac{\sqrt{31}}{4}$$

The solution set is $\left\{\left(\frac{\sqrt{31}}{4}, \frac{7}{4}\right), \left(-\frac{\sqrt{31}}{4}, \frac{7}{4}\right)\right\}$.

27. $y^2 - x = 4$

$x^2 + y^2 = 4$

Eliminate the y^2–terms by multiplying the first equation by –1 and adding the resulting equations.

$$\begin{aligned} x - y^2 &= -4 \\ \underline{x^2 + y^2 = 4} & \\ x^2 + x &= 0 \end{aligned}$$

$x(x+1) = 0$

$x = 0$ or $x + 1 = 0$

$x = -1$

If $x = 0$,

$y^2 = 4$

$y = \pm 2$

If $x = -1$,

$y^2 - (-1) = 4$

$y^2 = 3$

$y = \pm\sqrt{3}$

The solution set is

$\left\{(0,2), (0,-2), \left(-1, \sqrt{3}\right), \left(-1, -\sqrt{3}\right)\right\}$.

28. $x^2 - 2y = 8$

$x^2 + y^2 = 16$

Eliminate the x^2-terms by multiplying the first equation by –1 and adding the resulting equations.

$$\begin{aligned} -x^2 + 2y &= -8 \\ \underline{x^2 + y^2 = 16} & \\ y^2 + 2y &= 8 \\ y^2 + 2y - 8 &= 0 \end{aligned}$$

$(y+4)(y-2) = 0$

$y + 4 = 0$ or $y - 2 = 0$

$y = -4$ or $y = 2$

If $y = -4$,

$x^2 - 2(-4) = 8$

$x^2 = 0$

$x = 0$

If $y = 2$,

$x^2 - 2(2) = 8$

$x^2 = 12$

$x = \pm\sqrt{12} = \pm 2\sqrt{3}$

The solution set is $\left\{(0, -4), \left(2\sqrt{3}, 2\right), \left(-2\sqrt{3}, 2\right)\right\}$.

29. The addition method is used here to solve the system.

$3x^2 + 4y^2 = 16$

$2x^2 - 3y^2 = 5$

Eliminate the y^2–terms by multiplying the first equation by 3 and the second equation by 4. Add the resulting equations.

$$\begin{aligned} 9x^2 + 12y^2 &= 48 \\ \underline{8x^2 - 12y^2 = 20} & \\ 17x^2 &= 68 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

If $x = 2$,

$3(2)^2 + 4y^2 = 16$

$y^2 = 1$

$y = \pm 1$

If $x = -2$,

$3(-2)^2 + 4y^2 = 16$

$y = \pm 1$

The solution set is

{(2, 1), (2, –1), (–2, 1), (–2, –1)}.

30. The addition method is used here to solve the system.

$x+y^2=4$

$x^2+y^2=16$

Eliminate the y^2-terms by multiplying the first equation by –1 and adding the resulting equations.

$-x-y^2=-4$

$\underline{x^2+y^2=16}$

$x^2-x=12$

$x^2-x-12=0$

$(x-4)(x+3)=0$

$x-4=0$ or $x+3=0$

$x=4$ or $x=-3$

If $x=4$,

$4+y^2=4$

$y^2=0$

$y=0$

If $x=-3$,

$-3+y^2=4$

$y^2=7$

$y=\pm\sqrt{7}$

The solution set is $\left\{(4,0),\left(-3,\sqrt{7}\right),\left(-3,-\sqrt{7}\right)\right\}$.

31. The substitution method is used here to solve the system.

$2x^2+y^2=18$

$xy=4$

Solve the second equation for *y*.

$y=\frac{4}{x}$

Substitute the expression $\frac{4}{x}$ for *y* in the first equation and solve for *x*.

$2x^2+\left(\frac{4}{x}\right)^2=18$

$2x^2+\frac{16}{x^2}=18$

$2x^4+16=18x^2$

$x^4-9x^2+8=0$

$\left(x^2-8\right)\left(x^2-1\right)=0$

$x^2-8=0$ or $x^2-1=0$

$x^2=8$ or $x^2=1$

$x=\pm2\sqrt{2}$ or $x=\pm1$

If $x=2\sqrt{2}$, $y=\frac{4}{2\sqrt{2}}=\sqrt{2}$.

If $x=-2\sqrt{2}$, $y=\frac{4}{-2\sqrt{2}}=-\sqrt{2}$.

If $x=1$, $y=\frac{4}{1}=4$.

If $x=-1$, $y=\frac{4}{-1}=-4$.

The solution set is

$\left\{\left(2\sqrt{2},\sqrt{2}\right),\left(-2\sqrt{2},-\sqrt{2}\right),(1,4),(-1,-4)\right\}$.

32. The substitution method is used here to solve the system.

$x^2+4y^2=20$

$xy=4$

Solve the second equation for *y*.

$y=\frac{4}{x}$

Substitute the expression $\frac{4}{x}$ for *y* in the first equation and solve for *x*.

$x^2+4\left(\frac{4}{x}\right)^2=20$

$x^2+\frac{64}{x^2}=20$

$x^4+64=20x^2$

$x^4-20x^2+64=0$

$\left(x^2-4\right)\left(x^2-16\right)=0$

$x^2-4=0$ or $x^2-16=0$

$x^2=4$ or $x^2=16$

$x=\pm2$ or $x=\pm4$

If $x=2$,

$y=\frac{4}{2}=2$

If $x=-2$,

$y=\frac{4}{-2}=-2$.

If $x=4$, $y=\frac{4}{4}=1$.

If $x=-4$, $y=\frac{4}{-4}=-1$.

The solution set is

{(2, 2), (–2, –2), (4, 1), (–4, –1)}.

33. The substitution method is used here to solve the system.

$x^2 + 4y^2 = 20$
$x + 2y = 6$

Solve the second equation for *x*.

$x = 6 - 2y$

Substitute the expression $6 - 2y$ for *x* in the first equation and solve for *y*.

$$(6-2y)^2 + 4y^2 = 20$$
$$36 - 24y + 4y^2 + 4y^2 - 20 = 0$$
$$8y^2 - 24y + 16 = 0$$
$$y^2 - 3y + 2 = 0$$
$$(y-2)(y-1) = 0$$

$y - 2 = 0$ or $y - 1 = 0$
$y = 2$ or $y = 1$

If $y = 2, x = 6 - 2(2) = 2.$

If $y = 1, x = 6 - 2(1) = 4.$

The solution set is $\{(2,2),(4,1)\}$.

34. The substitution method is used here to solve the system.

$3x^2 - 2y^2 = 1$
$4x - y = 3$

Solve the second equation for *y*.

$y = 4x - 3$

Substitute the expression $4x - 3$ for *y* in the first equation and solve for *x*.

$$3x^2 - 2(4x-3)^2 = 1$$
$$3x^2 - 2(16x^2 - 24x + 9) - 1 = 0$$
$$3x^2 - 32x^2 + 48x - 18 - 1 = 0$$
$$-29x^2 + 48x - 19 = 0$$
$$29x^2 - 48x + 19 = 0$$
$$(29x - 19)(x - 1) = 0$$

$29x - 19 = 0$ or $x - 1 = 0$
$x = \frac{19}{29}$ or $x = 1$

If $x = \frac{19}{29}, y = 4\left(\frac{19}{29}\right) - 3 = -\frac{11}{29}$

If $x = 1,\ y = 4(1) - 3 = 1.$

The solution set is $\left\{\left(\frac{19}{29}, -\frac{11}{29}\right), (1,1)\right\}$.

35. Eliminate *y* by adding the equations.

$$x^3 + y = 0$$
$$x^2 - y = 0$$
$$\overline{x^3 + x^2 = 0}$$
$$x^2(x+1) = 0$$

$x^2 = 0$ or $x + 1 = 0$
$x = 0$ or $x = -1$

If $x = 0$,
$(0)^3 + y = 0$
$y = 0$

If $x = -1$,
$(-1)^3 + y = 0$
$y = 1$

The solution set is $\{(0,0),(-1,1)\}$.

36. Eliminate *y* by adding the equations.

$$x^3 + y = 0$$
$$2x^2 - y = 0$$
$$\overline{x^3 + 2x^2 = 0}$$
$$x^2(x+2) = 0$$

$x^2 = 0$ or $x + 2 = 0$
$x = 0$ or $x = -2$

If $x = 0$,
$(0)^3 + y = 0$
$y = 0$

If $x = -2$,
$(-2)^3 + y = 0$
$y = 8$

The solution set is $\{(0, 0), (-2, 8)\}$.

37. The substitution method is used here to solve the system.

$x^2+(y-2)^2=4$

$x^2-2y=0$

Solve the second equation for x^2.

$x^2=2y$

Substitute the expression 2*y* for x^2 in the first equation and solve for *y*.

$$\begin{aligned}2y+(y-2)^2&=4\\2y+y^2-4y+4&=4\\y^2-2y&=0\\y(y-2)&=0\end{aligned}$$

$y=0$ or $y-2=0$

$y=2$

If *y* = 0,

$x^2=2(0)$

$x^2=0$

$x=0$

If *y* = 2,

$x^2=2(2)$

$x^2=4$

$x=\pm 2$

The solution set is {(0, 0), (–2, 2), (2, 2)}.

38. Eliminate the *y* and y^2-terms by adding the equations.

$$\begin{aligned}x^2-y^2-4x+6y-4&=0\\x^2+y^2-4x-6y+12&=0\\\hline 2x^2-8x+8&=0\\x^2-4x+4&=0\\(x-2)(x-2)&=0\\(x-2)^2&=0\\x-2&=0\\x&=2\end{aligned}$$

If *x* = 2,

$$\begin{aligned}(2)^2-y^2-4(2)+6(y)-4&=0\\-y^2+6y-8&=0\\y^2-6y+8&=0\\(y-4)(y-2)&=0\end{aligned}$$

$y-4=0$ or $y-2=0$

$y=4$ or $y=2$

The solution set is {(2, 2), (2, 4)}.

39. The substitution method is used here to solve the system.

$y=(x+3)^2$

$x+2y=-2$

Solve the first equation for *x*.

$x=-2y-2$

Substitute the expression –2*y*–2 for x in the first equation and solve for *y*.

$$\begin{aligned}y&=(-2y-2+3)^2=(-2y+1)^2\\y&=4y^2-4y+1\end{aligned}$$

$4y^2-5y+1=0$

$(4y-1)(y-1)=0$

$4y-1=0$ or $y-1=0$

$y=\frac{1}{4}$ or $y=1$

If $y=\frac{1}{4}$, $x=-2\left(\frac{1}{4}\right)-2=-\frac{5}{2}$.

If *y* = 1, *x* = –2(1) – 2 = –4.

The solution set is $\left\{(-4, 1), \left(-\frac{5}{2}, \frac{1}{4}\right)\right\}$.

40. The substitution method is used here to solve the system.

$(x-1)^2+(y+1)^2=5$

$2x-y=3$

Solve the second equation for *y*.

$y=2x-3$

Substitute the expression $2x-3$ for *y* in the first equation and solve for *x*.

$$\begin{aligned}(x-1)^2+(2x-3+1)^2&=5\\(x-1)^2+(2x-2)^2&=5\\x^2-2x+1+4x^2-8x+4-5&=0\\5x^2-10x&=0\\x^2-2x&=0\\x(x-2)&=0\end{aligned}$$

$x=0$ or $x-2=0$

$x=2$

If $x=0$, $y=2(0)-3=-3$.

If $x=2$, $y=2(2)-3=1$.

The solution set is {(0, –3), (2, 1)}.

41. The substitution method is used here to solve the system.

$x^2 + y^2 + 3y = 22$

$2x + y = -1$

Solve the second equation for *y*.

$y = -2x - 1$

Substitute the expression $-2x-1$ for *y* in the first equation and solve for *x*.

$$x^2 + (-2x-1)^2 + 3(-2x-1) - 22 = 0$$
$$x^2 + 4x^2 + 4x + 1 - 6x - 3 - 22 = 0$$
$$5x^2 - 2x - 24 = 0$$
$$(5x-12)(x+2) = 0$$

$5x - 12 = 0$ or $x + 2 = 0$

$x = \frac{12}{5}$ or $x = -2$

If $x = \frac{12}{5}$, $y = -2\left(\frac{12}{5}\right) - 1 = -\frac{29}{5}$.

If $x = -2$, $y = -2(-2) - 1 = 3$.

The solution set is $\left\{\left(\frac{12}{5}, -\frac{29}{5}\right), (-2, 3)\right\}$.

42. The substitution method is used here to solve the system.

Solve the first equation for *x*.

$x = 3y - 5$

Substitute the expression $3y - 5$ for *x* in the second equation and solve for *x*.

$$(3y-5)^2 + y^2 - 25 = 0$$
$$9y^2 - 30y + 25 + y^2 - 25 = 0$$
$$10y^2 - 30y = 0$$
$$10y(y-3) = 0$$

$10y = 0$ $\quad y - 3 = 0$

$y = 0$ $\quad y = 3$

Since $x = 3y - 5$, for $y = 0$, $x = -5$ and for $y = 3$, $x = 3(3) - 5 = 4$.

The solution set is $\{(-5, 0), (4, 3)\}$

43. The substitution method is used here to solve the system.

$x + y = 10$

$xy = 24$

Solve the first equation for *y*.

$y = 10 - x$

Substitute the expression $10 - x$ for *y* in the second equation and solve for *x*.

$$x(10 - x) = 24$$
$$10x - x^2 = 24$$
$$x^2 - 10x + 24 = 0$$
$$(x-4)(x-6) = 0$$

$x - 4 = 0$ or $x - 6 = 0$

$x = 4$ or $x = 6$

If $x = 4$, $y = 10 - 4 = 6$.

If $x = 6$, $y = 10 - 6 = 4$.

The numbers are 4 and 6.

44. The substitution method is used here to solve the system.

$x + y = 20$

$xy = 96$

Solve the first equation for *y*.

$y = 20 - x$

Substitute the expression $20 - x$ for *y* in the second equation and solve for *x*.

$$x(20 - x) = 96$$
$$20x - x^2 = 96$$
$$x^2 - 20x + 96 = 0$$
$$(x-8)(x-12) = 0$$

$x - 8 = 0$ or $x - 12 = 0$

$x = 8$ or $x = 12$

If $x = 8$, $y = 20 - 8 = 12$.

If $x = 12$, $y = 20 - 12 = 8$.

The numbers are 8 and 12.

45. Eliminate the y^2–terms by adding the equations.

$$\begin{aligned} x^2 - y^2 &= 3 \\ 2x^2 + y^2 &= 9 \\ \hline 3x^2 &= 12 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

If $x = 2$,

$$\begin{aligned} 2(2)^2 + y^2 &= 9 \\ y^2 &= 1 \\ y &= \pm 1 \end{aligned}$$

If $x = -2$,

$$\begin{aligned} 2(-2)^2 + y^2 &= 9 \\ y^2 &= 1 \\ y &= \pm 1 \end{aligned}$$

The numbers are 2 and 1, 2 and –1, –2 and 1, or –2 and –1.

46. The addition method is used here to solve the system.

$$\begin{aligned} x^2 - y^2 &= 5 \\ 3x^2 - 2y^2 &= 19 \end{aligned}$$

Eliminate the y^2-terms by multiplying the first equation by –2 and adding the resulting equations.

$$\begin{aligned} -2x^2 + 2y^2 &= -10 \\ 3x^2 - 2y^2 &= 19 \\ \hline x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

If $x = 3$,

$$\begin{aligned} (3)^2 - y^2 &= 5 \\ y^2 &= 4 \\ y &= \pm 2 \end{aligned}$$

If $x = -3$,

$$\begin{aligned} (-3)^2 - y^2 &= 5 \\ y^2 &= 4 \\ y &= \pm 2 \end{aligned}$$

The numbers are, 3 and 2, 3 and –2, –3 and 2, or –3 and –2.

47.

$$\begin{aligned} 2x^2 + xy &= 6 \\ x^2 + 2xy &= 0 \end{aligned}$$

Multiply the first equation by -2 and add the two equations.

$$\begin{aligned} -4x^2 - 2xy &= -12 \\ x^2 + 2xy &= 0 \\ \hline -3x^2 &= -12 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

Back-substitute these values for *x* in the second equation and solve for *y*.

For $x = -2$: $(-2)^2 + 2(-2)y = 0$

$$\begin{aligned} 4 - 4y &= 0 \\ -4y &= -4 \\ y &= 1 \end{aligned}$$

For $x = 2$: $(2)^2 + 2(2)y = 0$

$$\begin{aligned} 4 + 4y &= 0 \\ 4y &= -4 \\ y &= -1 \end{aligned}$$

The solution set is $\{(-2,1),(2,-1)\}$.

48.

$$\begin{aligned} 4x^2 + xy &= 30 \\ x^2 + 3xy &= -9 \end{aligned}$$

Multiply the first equation by -3 and add the equations.

$$\begin{aligned} -12x^2 - 3xy &= -90 \\ x^2 + 3xy &= -9 \\ \hline -11x^2 &= -99 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

Back-substitute these values for *x* in the second equation and solve for *y*.

For $x = -3$: $(-3)^2 + 3(-3)y = -9$

$$\begin{aligned} 9 - 9y &= -9 \\ -9y &= -18 \\ y &= 2 \end{aligned}$$

For $x = 3$: $(3)^2 + 3(3)y = 9$

$$\begin{aligned} 9 + 9y &= -9 \\ 9y &= -18 \\ y &= -2 \end{aligned}$$

The solution set is $\{(-3,2),(3,-2)\}$.

49. $-4x + y = 12$

$y = x^3 + 3x^2$

Substitute $x^3 + 3x^2$ for y in the first equation and solve for x.

$$-4x + \left(x^3 + 3x^2\right) = 12$$
$$x^3 + 3x^2 - 4x - 12 = 0$$
$$x^2(x+3) - 4(x+3) = 0$$
$$(x+3)\left(x^2 - 4\right) = 0$$
$$(x+3)(x-2)(x+2) = 0$$

$x = -3$, $x = 2$, or $x = -2$

Substitute these values for x in the second equation and solve for y.

For $x = -3$: $y = (-3)^3 + 3(-3)^2 = -27 + 27 = 0$

For $x = 2$: $y = (2)^3 + 3(2)^2 = 8 + 12 = 20$

For $x = -2$: $y = (-2)^3 + 3(-2)^2 = -8 + 12 = 4$

The solution set is $\{(-3,0),(2,20),(-2,4)\}$.

50. $-9x + y = 45$

$y = x^3 + 5x^2$

Substitute $x^3 + 5x^2$ for y in the first equation and solve for x.

$$-9x + \left(x^3 + 5x^2\right) = 45$$
$$x^3 + 5x^2 - 9x - 45 = 0$$
$$x^2(x+5) - 9(x+5) = 0$$
$$(x+5)\left(x^2 - 9\right) = 0$$
$$(x+5)(x-3)(x+3) = 0$$

$x = -5$, $x = 3$, or $x = -3$

Substitute these values for x in the second equation and solve for y.

For $x = -5$: $y = (-5)^3 + 5(-5)^2 = -125 + 125 = 0$

For $x = 3$: $y = (3)^3 + 5(3)^2 = 27 + 45 = 72$

For $x = -3$: $y = (-3)^3 + 5(-3)^2 = -27 + 45 = 18$

The solution set is $\{(-5,0),(3,72),(-3,18)\}$.

51. $\dfrac{3}{x^2} + \dfrac{1}{y^2} = 7$

$\dfrac{5}{x^2} - \dfrac{2}{y^2} = -3$

Multiply the first equation by 2 and add the equations.

$$\frac{6}{x^2} + \frac{2}{y^2} = 14$$
$$\frac{5}{x^2} - \frac{2}{y^2} = -3$$
$$\frac{11}{x^2} = 11$$
$$x^2 = 1$$
$$x = \pm 1$$

Back-substitute these values for x in the first equation and solve for y.

For $x = -1$:

$$\frac{3}{(-1)^2} + \frac{1}{y^2} = 7$$
$$3 + \frac{1}{y^2} = 7$$
$$\frac{1}{y^2} = 4$$
$$y^2 = \frac{1}{4}$$
$$y = \pm\frac{1}{2}$$

For $x = 1$:

$$\frac{3}{(1)^2} + \frac{1}{y^2} = 7$$
$$3 + \frac{1}{y^2} = 7$$
$$\frac{1}{y^2} = 4$$
$$y^2 = \frac{1}{4}$$
$$y = \pm\frac{1}{2}$$

The solution set is

$\left\{\left(-1,-\frac{1}{2}\right),\left(-1,\frac{1}{2}\right),\left(1,-\frac{1}{2}\right),\left(1,\frac{1}{2}\right)\right\}$.

52. $\frac{2}{x^2}+\frac{1}{y^2}=11$

$\frac{4}{x^2}-\frac{2}{y^2}=-14$

Multiply the first equation by 2 and add the two equations.

$\frac{4}{x^2}+\frac{2}{y^2}=22$

$\frac{4}{x^2}-\frac{2}{y^2}=-14$

$\frac{8}{x^2}=8$

$x^2=1$

$x=\pm 1$

Back-substitute these values for x in the first equation and solve for y.

For $x=-1$:

$\frac{2}{(-1)^2}+\frac{1}{y^2}=11$

$2+\frac{1}{y^2}=11$

$\frac{1}{y^2}=9$

$y^2=\frac{1}{9}$

$y=\pm\frac{1}{3}$

For $x=1$:

$\frac{2}{(1)^2}+\frac{1}{y^2}=11$

$2+\frac{1}{y^2}=11$

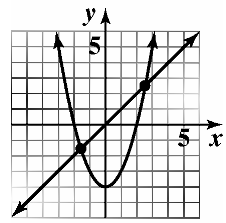
$\frac{1}{y^2}=9$

$y^2=\frac{1}{9}$

$y=\pm\frac{1}{3}$

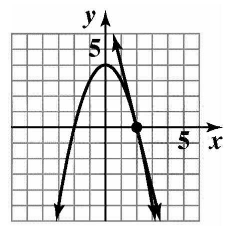
The solution set is $\left\{\left(-1,-\frac{1}{3}\right), \left(-1,\frac{1}{3}\right), \left(1,-\frac{1}{3}\right), \left(1,\frac{1}{3}\right)\right\}$.

53.



$y = x^2 - 4$
$y = x$

54.



$y = 4 - x^2$
$y = -4x + 8$

55. $16x^2+4y^2=64$

$y=x^2-4$

Substitute the expression x^2-4 for y in the first equation and solve for x.

$16x^2+4\left(x^2-4\right)^2=64$

$16x^2+4\left(x^4-8x^2+16\right)=64$

$16x^2+4x^4-32x^2+64=64$

$4x^4-16x^2=0$

$x^4-4x^2=0$

$x^2\left(x^2-4\right)=0$

$x^2=0$ or $x^2-4=0$

$x=0$ or $x^2=4$

$x=\pm 2$

If $x = 0$, $y=(0)^2-4=-4$.

If $x = 2$, $y=(2)^2-4=0$.

If $x = -2$, $y=(-2)^2-4=0$.

It is possible for the comet to intersect the orbiting body at (0, –4), (–2, 0), (2, 0).

56. Rearrange the equation so that the like terms are aligned.

$-x^2 + 2y^2 = 1$

$2x^2 - y^2 = 1$

Eliminate the x^2-terms by multiplying the first equation by 2 and adding the resulting equations.

$-2x^2 + 4y^2 = 2$

$2x^2 - y^2 = 1$

$3y^2 = 3$

$y^2 = 1$

$y = \pm 1$

Since the ship is located in the first quadrant, we only use the value $y = 1$. If $y = 1$, $2x^2 - 1 = 1$

$2x^2 = 2$

$x^2 = 1$

$x = \pm 1$

Since the ship is in the first quadrant, we only use $x = 1$. The location of the ship is (1, 1).

57. $2L + 2W = 36$

$LW = 77$

Divide each term in the first equation by 2 and solve L.

$L + W = 18$

$L = 18 - W$

Substitute the expression 18 – W for L in the second equation and solve for W.

$(18 - W)W = 77$

$18W - W^2 = 77$

$W^2 - 18W + 77 = 0$

$(W - 11)(W - 7) = 0$

$W - 11 = 0$ or $W - 7 = 0$

$W = 11$ or $W = 7$

If $W = 11, L = 18 - 11 = 7$.

If $W = 7, L = 18 - 7 = 11$.

The dimensions are 11 feet by 7 feet.

58. $2L + 2W = 40$

$LW = 96$

Divide each term in the first equation by 2 and solve for L.

$L + W = 20$

$L = 20 - W$

Substitute the expression 20 – W for L in the second equation and solve for W.

$(20 - W)W = 96$

$20W - W^2 = 96$

$W^2 - 20W + 96 = 0$

$(W - 8)(W - 12) = 0$

$W - 8 = 0$ or $W - 12 = 0$

$W = 8$ or $W = 12$

If $W = 8, L = 20 - 8 = 12$.

If $W = 12, L = 20 - 12 = 8$.

The dimensions are 12 feet by 8 feet.

59. $L^2 + W^2 = 10^2 = 100$

$LW = 48$

Solve the second equation for L. $L = \frac{48}{W}$

Substitute the expression $\frac{48}{W}$ for L in the first equation and solve for W.

$\left(\frac{48}{W}\right)^2 + W^2 = 100$

$\frac{2304}{W^2} + W^2 - 100 = 0$

$2304 + W^4 - 100W^2 = 0$

$W^4 - 100W^2 + 2304 = 0$

$(W^2 - 36)(W^2 - 64) = 0$

$W^2 - 36 = 0$ or $W^2 - 64 = 0$

$W^2 = 36$ or $W^2 = 64$

$W = \pm 6$ or $W = \pm 8$

The width cannot be –6 or –8 inches.

If $W = 6$,

$L = \frac{48}{6} = 8$

If $W = 8$,

$L = \frac{48}{8} = 6$

The dimensions are 8 inches by 6 inches.

60. $LW = 108$

$L^2 + W^2 = 15^2 = 225$

Solve the first equation for *L*.

$L = \frac{108}{W}$

Substitute the expression $\frac{108}{W}$ for *L* in the second equation and solve for *W*.

$$\left(\frac{108}{W}\right)^2 + W^2 = 225$$

$$\frac{11,664}{W^2} + W^2 - 225 = 0$$

$$11,664 + W^4 - 225W^2 = 0$$

$$W^4 - 225W^2 + 11,664 = 0$$

$$\left(W^2 - 81\right)\left(W^2 - 144\right) = 0$$

$W^2 - 81 = 0$ or $W^2 - 144 = 0$

$W^2 = 81$ or $W^2 = 144$

$W = \pm 9$ or $W = \pm 12$

If $W = 9$,

$L = \frac{108}{9} = 12$

The length is 12 feet and the width is 9 feet.

61. $x^2 - y^2 = 21$

$4x + 2y = 24$

Divide each term in the second equation by 2 and solve for *y*.

$2x + y = 12$

$y = 12 - 2x$

Substitute the expression $12 - 2x$ for *y* in the first equation and solve for *x*.

$$x^2 - (12 - 2x)^2 = 21$$

$$x^2 - \left(144 - 48x + 4x^2\right) = 21$$

$$3x^2 - 48x + 165 = 0$$

$$x^2 - 16x + 55 = 0$$

$$(x - 5)(x - 11) = 0$$

$x - 5 = 0$ or $x - 11 = 0$

$x = 5$ or $x = 11$

If $x = 11$, $y = 12 - 2(11) = -10$.

If $x = 5$, $y = 12 - 2(5) = 2$.

The dimensions of the floor are 5 meters by 5 meters and the dimensions of the square that will accommodate the pool are 2 meters by 2 meters.

62. $LW = 216$

$2(L - 4)(W - 4) = 224$

Expand and simplify the second equation.

$$2(LW - 4L - 4W + 16) = 224$$

$$LW - 4L - 4W + 16 = 112$$

$$LW - 4L - 4W - 96 = 0$$

Solve the first equation for *L*.

$L = \frac{216}{W}$

Substitute $\frac{216}{W}$ for L in the equation $LW - 4L - 4W - 96 = 0$ and solve for *W*.

$$\frac{216}{W} \cdot W - 4\left(\frac{216}{W}\right) - 4W - 96 = 0$$

$$216 - \frac{864}{W} - 4W - 96 = 0$$

$$120W - 864 - 4W^2 = 0$$

$$4W^2 - 120W + 864 = 0$$

$$W^2 - 30W + 216 = 0$$

$$(W - 12)(W - 18) = 0$$

$W - 12 = 0$ or $W - 18 = 0$

$W = 12$ or $W = 18$

If $W = 12$,

$L = \frac{216}{12} = 18$

The length is 18 inches and the width is 12 inches.

63. **a.** It appears from the graphs that the percentage of white-collar workers was the same as blue-collar workers between the 1940s and 1960s.

b. $0.5x - y = -18$

$y = -0.004x^2 + 0.23x + 41$

Substitute $-0.004x^2 + 0.23x + 41$ in the first equation and solve for *x*.

$$0.5x - y = -18$$

$$0.5x - (\overbrace{-0.004x^2 + 0.23x + 41}^{y}) = -18$$

$$0.5x + 0.004x^2 - 0.23x - 41 = -18$$

$$0.004x^2 + 0.27x - 23 = 0$$

Use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(0.27) \pm \sqrt{0.27^2 - 4(0.004)(-23)}}{2(0.004)}$$

$x \approx 49$ or -117

The percentage of white-collar workers was the same as blue-collar workers 49 years after 1900, or 1949.

Let $x = 49$ and solve for y in the white-collar model.

$$\begin{aligned} 0.5x - y &= -18 \\ 0.5(49) - y &= -18 \\ 24.5 - y &= -18 \\ -y &= -42.5 \\ y &\approx 43 \end{aligned}$$

The percentage of white-collar workers in 1949 was about 43%

Let $x = 49$ and solve for y in the blue-collar model.

$y = -0.004(49)^2 + 0.23(49) + 41 \approx 43\%$

The percentage of blue-collar workers in 1949 was about 43%.

c. According to the graph, the percentage of white-collar workers was the same as farmers in 1920.
The percentages of white-collar workers and farmers in 1920 were both 28%.

d.

$$\begin{aligned} 0.5x - y &= -18 \\ 0.4x + y &= 35 \\ \hline 0.9x &= 17 \\ x &= \frac{17}{0.9} \\ x &\approx 19 \end{aligned}$$

According to the models, the percentage of white-collar workers was the same as farmers 19 years after 1900, or 1919.

Let $x = 19$ and solve for y in the white-collar model.

$$\begin{aligned} 0.5x - y &= -18 \\ 0.5(19) - y &= -18 \\ 9.5 - y &= -18 \\ -y &= -27.5 \\ y &= 27.5 \end{aligned}$$

The percentage of white-collar workers in 1919 was about 27.5%

Let $x = 19$ and solve for y in the farming model.

$$\begin{aligned} 0.4x + y &= 35 \\ 0.4(19) + y &= 35 \\ 0.4(19) + y &= 35 \\ 7.6 + y &= 35 \\ y &= 27.4 \end{aligned}$$

The percentage of farm workers in 1919 was about 27.4%

These answers model the actual data from part c (the graph) fairly well.

64. – 68. Answers may vary.

69. makes sense

70. does not make sense; Explanations will vary. Sample explanation: Since the orbits of earth and Mars do not intersect, their system of equations will have no solution.

71. makes sense

72. makes sense

73. false; Changes to make the statement true will vary. A sample change is: A circle and a line can intersect in at most two points, and therefore such a system has at most two real solutions.

74. true

75. false; Changes to make the statement true will vary. A sample change is: It is possible that a system of two equations in two variables whose graphs represent circles do not intersect, or intersect in a single point. This means that the system would have no solution, or a single solution, respectively.

76. false; Changes to make the statement true will vary. A sample change is: It is possible that a system of two equations in two variables whose graphs represent a parabola and a circle to have one real ordered-pair solution. This will occur if the graphs intersect in a single point.

77. First determine the solution to the following system of equations.

$xy = 20$

$x^2 + y^2 = 41$

Solve the first equation for *x*.

$x = \frac{20}{y}$

Substitute the expression $\frac{20}{y}$ for *x* in the second equation and solve for in the second equation and solve for *y*.

$\left(\frac{20}{y}\right)^2 + y^2 = 41$

$\frac{400}{y^2} + y^2 - 41 = 0$

$y^4 - 41y^2 + 400 = 0$

$\left(y^2 - 25\right)\left(y^2 - 16\right) = 0$

$y^2 - 25 = 0$ or $y^2 - 16 = 0$

$y^2 = 25$ or $y^2 = 16$

$y = \pm 5$ or $y = \pm 4$

If $y = 5$, $x = \frac{20}{5} = 4$.

If $y = -5$, $x = \frac{20}{-5} = -4$.

If $y = 4$, $x = \frac{20}{4} = 5$.

If $y = -4$, $x = \frac{20}{-4} = -5$.

The rectangle formed by joining the points of intersection has sides *a* and *b*. The length of *a* is $\sqrt{(5-4)^2 + (4-5)^2} = \sqrt{2}$. The length of *b* is $\sqrt{(4-(-5))^2 + (5-(-4))^2} = 9\sqrt{2}$.

The area of the rectangle is $a \cdot b = \left(\sqrt{2}\right)\left(9\sqrt{2}\right) = 18$ square units.

78. By the Pythagorean Theorem:

$a^2 + b^2 = 10^2 = 100$

$a^2 + (b+9)^2 = 17^2$

Expand the second equation.

$a^2 + b^2 + 18b + 81 = 289$

Eliminate the a^2 and b^2–terms by multiplying the first equation by –1 and adding the resulting equations.

$$\begin{aligned} a^2 + b^2 + 18b &= 208 \\ -a^2 - b^2 &= -100 \\ \hline 18b &= 108 \\ b &= 6 \end{aligned}$$

$a^2 + (6)^2 = 100$

$a^2 = 64$

$a = 8$

79. $\log_y x = 3$

$\log_y (4x) = 5$

$x = y^3$

$4x = y^5$

Substitute y^3 for *x* in the equation $4x = y^5$ and solve for *y*.

$4(y^3) = y^5$

$4 = y^2$

$y = \pm 2$

Keep the positive base.

$x = 2^3 = 8$

The solution set is {(8, 2)}.

80. $\log x^2 = y + 3$

$\log x = y - 1$

$10^{y+3} = x^2$

$10^{y-1} = x$

Substitute the expression 10^{y-1} for *x* in the equation $10^{y+3} = x^2$ and solve for *y*.

$10^{y+3} = \left(10^{y-1}\right)^2$

$10^{y+3} = 10^{2y-2}$

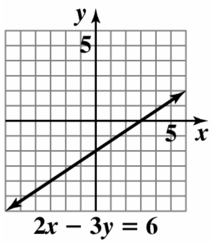
$y + 3 = 2y - 2$

$y = 5$

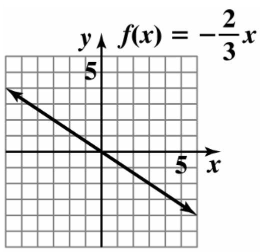
$x = 10^{5-1} = 10{,}000$

The solution set is $\{(10{,}000, 5)\}$.

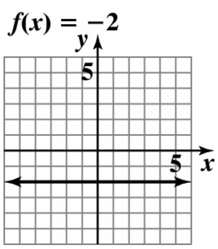
81.



82.



83.



Mid-Chapter 8 Check Point

1. $x = 3y - 7$

$4x + 3y = 2$

Since the first equation is solved for *x* already, we will use substitution.
Let $x = 3y - 7$ in the second equation and solve for *y*.

$$4(3y-7)+3y = 2$$
$$12y - 28 + 3y = 2$$
$$15y = 30$$
$$y = 2$$

Substitute this value for *y* in the first equation.

$x = 3(2) - 7 = 6 - 7 = -1$

The solution is $(-1, 2)$.

2. $3x + 4y = -5$

$2x - 3y = 8$

Multiply the first equation by 3 and the second equation by 4, then add the equations.

$$9x + 12y = -15$$
$$\underline{8x - 12y = 32}$$
$$17x = 17$$
$$x = 1$$

Back-substitute to solve for *y*.

$$3x + 4y = -5$$
$$3(1) + 4y = -5$$
$$3 + 4y = -5$$
$$4y = -8$$
$$y = -2$$

The solution is $(1, -2)$.

3. $\frac{2x}{3} + \frac{y}{5} = 6$

$\frac{x}{6} - \frac{y}{2} = -4$

Multiply the first equation by 15 and the second equation by 6 to eliminate the fractions.

$$15\left(\frac{2x}{3} + \frac{y}{5}\right) = 15(6)$$
$$10x + 3y = 90$$
$$6\left(\frac{x}{6} - \frac{y}{2}\right) = 6(-4)$$
$$x - 3y = -24$$

We now need to solve the equivalent system

$10x + 3y = 90$

$x - 3y = -24$

Add the two equations to eliminate *y*.

$$10x + 3y = 90$$
$$\underline{x - 3y = -24}$$
$$11x = 66$$
$$x = 6$$

Back-substitute to solve for *y*.

$$x - 3y = -24$$
$$6 - 3y = -24$$
$$-3y = -30$$
$$y = 10$$

The solution is $(6, 10)$.

4. $y = 4x - 5$
$8x - 2y = 10$

Since the first equation is already solved for *y*, we will use substitution.
Let $y = 4x - 5$ in the second equation and solve for *x*.

$$8x - 2(4x - 5) = 10$$
$$8x - 8x + 10 = 10$$
$$10 = 10$$

This statement is an identity. The system is dependent so there are an infinite number of solutions. The solution set is $\{(x, y) \mid y = 4x - 5\}$.

5. $2x + 5y = 3$
$3x - 2y = 1$

Multiply the first equation by 3 and the second equation by -2, then add the equations.

$$6x + 15y = 9$$
$$-6x + 4y = -2$$
$$19y = 7$$
$$y = \frac{7}{19}$$

Back-substitute to solve for *x*.

$$2x + 5y = 3$$
$$2x + 5\left(\frac{7}{19}\right) = 3$$
$$2x + \frac{35}{19} = 3$$
$$2x = \frac{22}{19}$$
$$x = \frac{11}{19}$$

The solution is $\left(\frac{11}{19}, \frac{7}{19}\right)$.

6. $\frac{x}{12} - y = \frac{1}{4}$
$4x - 48y = 16$

Solve the first equation for *y*.

$$\frac{x}{12} - y = \frac{1}{4}$$
$$-y = -\frac{x}{12} + \frac{1}{4}$$
$$y = \frac{x}{12} - \frac{1}{4}$$

Let $y = \frac{x}{12} - \frac{1}{4}$ in the second equation and solve for *x*.

$$4x - 48\left(\frac{x}{12} - \frac{1}{4}\right) = 16$$
$$4x - 4x + 12 = 16$$
$$12 = 16$$

This statement is a contradiction. The system is inconsistent so there is no solution. The solution is $\{\ \}$ or $\varnothing$.

7. $2x - y + 2z = -8$
$x + 2y - 3z = 9$
$3x - y - 4z = 3$

Multiply the first equation by 2 and add to the second equation.

$$4x - 2y + 4z = -16$$
$$x + 2y - 3z = 9$$
$$5x + z = -7$$

Multiply the first equation by -1 and add to the third equation.

$$-2x + y - 2z = 8$$
$$3x - y - 4z = 3$$
$$x - 6z = 11$$

Use the two reduced equations to get the following system:
$5x + z = -7$
$x - 6z = 11$

Multiply the first equation by 6 and add to the second equation.

$$30x + 6z = -42$$
$$x - 6z = 11$$
$$31x = -31$$
$$x = -1$$

Back-substitute to solve for z.

$$5x+z=-7$$
$$5(-1)+z=-7$$
$$-5+z=-7$$
$$z=-2$$

Back-substitute to solve for y.

$$2x-y+2z=-8$$
$$2(-1)-y+2(-2)=-8$$
$$-2-y-4=-8$$
$$-y=-2$$
$$y=2$$

The solution is $(-1,2,-2)$.

8. $x \quad -3z=-5$

$2x-\ y+2z=16$

$7x-3y-5z=19$

Multiply the second equation by -3 and add to the third equation.

$$-6x+3y-6z=-48$$
$$7x-3y-5z=19$$
$$x-11z=-29$$

Use this reduced equation and the original first equation to obtain the following system:

$$x-3z=-5$$
$$x-11z=-29$$

Multiply the second equation by -1 and add to the first equation.

$$x-3z=-5$$
$$-x+11z=29$$
$$8z=24$$
$$z=3$$

Back-substitute to solve for x.

$$x-3z=-5$$
$$x-3(3)=-5$$
$$x-9=-5$$
$$x=4$$

Back-substitute to solve for y.

$$2x-y+2z=16$$
$$2(4)-y+2(3)=16$$
$$8-y+6=16$$
$$-y=2$$
$$y=-2$$

The solution is $(4,-2,3)$.

9. Solve $x+2y-3=0$ for x and substitute into the other equation.

$$(\overbrace{-2y+3}^{x})^2+y^2=9$$
$$4y^2-12y+9+y^2=9$$
$$5y^2-12y=0$$
$$y(5y-12)=0$$
$$y=0 \text{ or } y=\frac{12}{5}$$

Back-substitute these values to find x.

When $y=0$, $x=-2(0)+3=3$.

When $y=\frac{12}{5}$, $x=-2\left(\frac{12}{5}\right)+3=-\frac{9}{5}$.

The solution set is $\left\{(3,0),\left(-\frac{9}{5},\frac{12}{5}\right)\right\}$

10. $3x^2+2y^2=14$

$2x^2-\ y^2=7$

Multiply the second equation by 2 and add.

$$3x^2+2y^2=14$$
$$4x^2-2y^2=14$$
$$7x^2=28$$
$$x^2=4$$
$$x=\pm 2$$

Back-substitute these values to find y.

$$3(2)^2+2y^2=14$$
$$12+2y^2=14$$
$$2y^2=2$$
$$y^2=1$$
$$y=\pm 1$$
$$3(-2)^2+2y^2=14$$
$$12+2y^2=14$$
$$2y^2=2$$
$$y^2=1$$
$$y=\pm 1$$

The solution set is $\{(2,1),(2,-1),(-2,1),(-2,-1)\}$.

11. Use the first equation to substitute for *y* in the second equation.

$$x^2+(\overbrace{x^2-6}^{y})^2=8$$
$$x^2+(x^2-6)^2=8$$
$$x^2+x^4-12x^2+36=8$$
$$x^4-11x^2+28=0$$
$$(x^2-4)(x^2-7)=0$$
$$x=\pm 2 \text{ or } x=\pm\sqrt{7}$$

Back-substitute these values to find *x*.

$y=(2)^2-6$ $\quad y=(-2)^2-6$
$y=-2$ $\quad y=-2$
$y=(\sqrt{7})^2-6$ $\quad y=(-\sqrt{7})^2-6$
$y=1$ $\quad y=1$

The solution set is $\left\{(2,-2),(-2,-2),\left(\sqrt{7},1\right),\left(-\sqrt{7},1\right)\right\}$.

12. Use the first equation to substitute for *x* in the second equation.

$$2y^2+(\overbrace{2y+4}^{x})y=8$$
$$2y^2+(2y+4)y=8$$
$$2y^2+2y^2+4y=8$$
$$4y^2+4y-8=0$$
$$y^2+y-2=0$$
$$(y+2)(y-1)=0$$
$$y=-2 \text{ or } y=1$$

Back-substitute these values to find *x*.

$x=2(-2)+4$ $\quad x=2(1)+4$
$x=0$ $\quad y=6$

The solution set is $\left\{(0,-2),(6,1)\right\}$.

13. $\dfrac{x^2-6x+3}{(x-2)^3}=\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{(x-2)^3}$

Multiply both sides of the equation by the common denominator $(x-2)^3$.

$$(x-2)^3\frac{x^2-6x+3}{(x-2)^3}=(x-2)^3\left(\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)^3}\right)$$

$$x^2-6x+3=A(x-2)^2+B(x-2)+C$$

$$x^2-6x+3=A(x^2-4x+4)+Bx-2B+C$$

$$x^2-6x+3=Ax^2-4Ax+4A+Bx-2B+C$$

$$x^2-6x+3=(A)x^2+(-4A+B)x+(4A-2B+C)$$

Equate coefficients of like powers of *x* and equate constant terms.

$$A=1$$

$$-4A+B=-6$$

$$4A-2B+C=3$$

Since A = 1, we find that B = –2 and C = –5 by substitution.

$$\frac{2x^2-6x+3}{(x-2)^3}=\frac{1}{x-2}+\frac{-2}{(x-2)^2}+\frac{-5}{(x-2)^3}$$

14. $\dfrac{10x^2+9x-7}{(x+2)(x^2-1)}=\dfrac{10x^2+9x-7}{(x+2)(x+1)(x-1)}=\dfrac{A}{x+2}+\dfrac{B}{x+1}+\dfrac{C}{x-1}$

Multiply both sides of the equation by the least common denominator.

$$(x+2)(x+1)(x-1)\frac{10x^2+9x-7}{(x+2)(x+1)(x-1)}=(x+2)(x+1)(x-1)\left(\frac{A}{x+2}+\frac{B}{x+1}+\frac{C}{x-1}\right)$$

$$10x^2+9x-7=(x+1)(x-1)A+(x+2)(x-1)B+(x+2)(x+1)C$$

$$10x^2+9x-7=(x^2-1)A+(x^2+x-2)B+(x^2+3x+2)C$$

$$10x^2+9x-7=x^2A-A+x^2B+xB-2B+x^2C+3xC+2C$$

$$10x^2+9x-7=(A+B+C)x^2+(B+3C)x+(-A-2B+2C)$$

Equate coefficients of like powers of *x*, and equate constant terms.

$$A+B+C=10$$

$$B+3C=9$$

$$-A-2B+2C=-7$$

Solving the above system for *A*, *B*, and *C*, we find $A=5,\ B=3,$ and $C=2$.

$$\frac{10x^2+9x-7}{(x+2)(x^2-1)}=\frac{5}{x+2}+\frac{3}{x+1}+\frac{2}{x-1}$$

15. $\dfrac{x^2+4x-23}{(x+3)(x^2+4)}=\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+4}$

Multiply both sides of the equation by the least common denominator.

$$(x+3)(x^2+4)\frac{x^2+4x-23}{(x+3)(x^2+4)}=(x+3)(x^2+4)\left(\frac{A}{x+3}+\frac{Bx+C}{x^2+4}\right)$$

$$x^2+4x-23=(x^2+4)A+(x+3)(Bx+C)$$

$$x^2+4x-23=x^2A+4A+x^2B+3xB+xC+3C$$

$$x^2+4x-23=(A+B)x^2+(3B+C)x+(4A+3C)$$

Equate coefficients of like powers of *x*, and equate constant terms.

$A+B=1$

$3B+C=4$

$4A+3C=-23$

Solving the above system for *A*, *B*, and *C*, we find $A=-2,\ B=3,$ and $C=-5.$

$$\frac{x^2+4x-23}{(x+3)(x^2+4)}=\frac{-2}{x+3}+\frac{3x-5}{x^2+4}$$

16. $\dfrac{x^3}{\left(x^2+4\right)^2}=\dfrac{\mathrm{A}x+\mathrm{B}}{x^2+4}+\dfrac{\mathrm{C}x+\mathrm{D}}{\left(x^2+4\right)^2}$

Multiply both sides of the equation by the least common denominator.

$$\left(x^2+4\right)^2\frac{x^3}{\left(x^2+4\right)^2}=\left(x^2+4\right)^2\left(\frac{\mathrm{A}x+\mathrm{B}}{x^2+4}+\frac{\mathrm{C}x+\mathrm{D}}{\left(x^2+4\right)^2}\right)$$

$$x^3=\left(x^2+4\right)(\mathrm{A}x+\mathrm{B})+(\mathrm{C}x+\mathrm{D})$$

$$x^3=x^3A+x^2B+4xA+4B+\mathrm{C}x+\mathrm{D}$$

$$x^3=(A)x^3+(B)\mathrm{x}^2+(4A+\mathrm{C})x+(4B+\mathrm{D})$$

Equate coefficients of like powers of *x*, and equate constant terms.

$A=1$

$B=0$

$4A+\mathrm{C}=0$

$4B+\mathrm{D}=0$

Since $A = 1$ and $B = 0$, we find that $C = -4$ and $D = 0$ by substitution.

$$\frac{x^3}{\left(x^2+4\right)^2}=\frac{1x+0}{x^2+4}+\frac{-4x+0}{\left(x^2+4\right)^2}=\frac{x}{x^2+4}+\frac{-4x}{\left(x^2+4\right)^2}$$

17. **a.** $C(x) = 400{,}000 + 20x$

b. $R(x) = 100x$

c. $P(x) = R(x) - C(x)$
$= 100x - (400{,}000 + 20x)$
$= 80x - 400{,}000$

d. The break even point is the point where cost and revenue are the same. We need to solve the following system.
$y = 400{,}000 + 20x$
$y = 100x$
Let $y = 400{,}000 + 20x$ in the second equation and solve for *x*.
$400{,}000 + 20x = 100x$
$400{,}000 = 80x$
$5000 = x$
Back-substitute to solve for *y*.
$y = 100x$
$= 100(5000)$
$= 500{,}000$
Thus, the break-even point is $(5000,\ 500{,}000)$.
The company will break even when it produces and sells 5000 PDAs. At this level, the cost and revenue will both be \$500,000.

18. Let *x* = the number of roses.
Let *y* = the number of carnations.
$x + y = 20$
$3x + 1.5y = 39$
Solve the first equation for *x*.
$x + y = 20$
$x = 20 - y$
Substitute this expression for *x* in the second equation and solve for *y*.
$3(20 - y) + 1.5y = 39$
$60 - 3y + 1.5y = 39$
$-1.5y = -21$
$y = 14$
Back-substitute to solve for *x*.
$x = 20 - y = 20 - 14 = 6$
There are 6 roses and 14 carnations in the bouquet.

19. Because the sum of the measures of the angles of any triangle is 180°, or $x + y + 90 = 180$
$$x + y = 90.$$
Because the angle with measures *x* and $(3y + 20)$ are supplementary, $x + (3y + 20) = 180$.
Simplify this equation.
$x + 3y + 20 = 180$
$x + 3y = 160$
We now have the system
$x + y = 90$
$x + 3y = 160.$
To solve this system by the addition method, multiply the first equation by −1 and add to the second equation.
$-x - y = -90$
$\underline{x + 3y = 160}$
$2y = 70$
$y = 35$
Back-substitute.
$x + y = 90$
$x + 35 = 90$
$x = 55$
Thus, *x* = 55°, *y* = 35°, and $(3y + 20) = 125°$.

20. Using the points $(-1, 0)$, $(1, 4)$, and $(2, 3)$ in the equation $y = ax^2 + bx + c$, we get the following system of equations:
$a - b + c = 0$
$a + b + c = 4$
$4a + 2b + c = 3$
Add the first two equations.
$a - b + c = 0$
$\underline{a + b + c = 4}$
$2a + 2c = 4$
Multiply the first equation by 2 and add to the third equation.
$2a - 2b + 2c = 0$
$\underline{4a + 2b + c = 3}$
$6a + 3c = 3$
Using the two reduced equations, we get the following system of equations:
$2a + 2c = 4$
$6a + 3c = 3$

Multiply the first equation by -3 and add to the second equation.

$$\begin{aligned} -6a-6c&=-12\\ 6a+3c&=3\\ \hline -3c&=-9\\ c&=3 \end{aligned}$$

Back-substitute to solve for *a*.

$$\begin{aligned} 2a+2c&=4\\ 2a+2(3)&=4\\ 2a+6&=4\\ 2a&=-2\\ a&=-1 \end{aligned}$$

Back-substitute to solve for *b*.

$$\begin{aligned} a+b+c&=4\\ -1+b+3&=4\\ b&=2 \end{aligned}$$

The equation is $y=-x^2+2x+3$.

21. $2l+2w=21$
$lw=20$

Solving $lw=20$ for *l* gives $l=\frac{20}{w}$.

Substitute into the other equation:

$$\begin{aligned} 2\overbrace{\left(\frac{20}{w}\right)}^{l}+2w&=21\\ \frac{40}{w}+2w&=21\\ w\left(\frac{40}{w}+2w\right)&=w(21)\\ 40+2w^2&=21w \end{aligned}$$

$$2w^2-21w+40=0$$

$$(2w-5)(w-8)=0$$

$w=\frac{5}{2}$ or $w=8$.

Back-substitute to find *l*.

When $w=\frac{5}{2}$, $l=\frac{20}{\frac{5}{2}}=8$.

When $w=8$, $l=\frac{20}{8}=\frac{5}{2}$.

The dimensions of the rectangle are $\frac{5}{2}$ m by 8 m.

Section 8.5

Check Point Exercises

1. $4x-2y\ge 8$

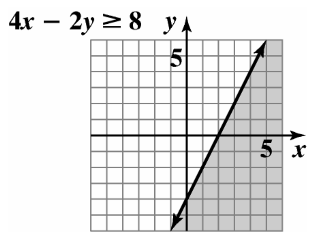
Graph the equation $4x-2y=8$ as a solid line.

Choose a test point that is not on the line.

Test $(0,0)$

$$\begin{aligned} 4x-2y&\ge 8\\ 4(0)-2(0)&\ge 8\\ 0&\ge 8, \text{ false} \end{aligned}$$

Since the statement is false, shade the other half-plane.



2. $y>-\frac{3}{4}x$

Graph the equation $y=-\frac{3}{4}x$ as a dashed line.

Choose a test point that is not on the line.

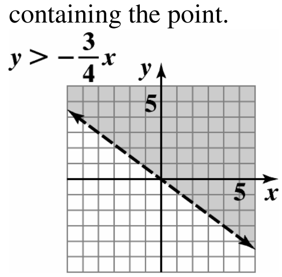
Test $(1,1)$

$$y>-\frac{3}{4}x$$

$$1>-\frac{3}{4}(1)$$

$$1>-\frac{3}{4}, \text{ true}$$

Since the statement is true, shade the half-plane containing the point.



3. **a.** $y > 1$

Graph the equation $y = 1$ as a dashed line.

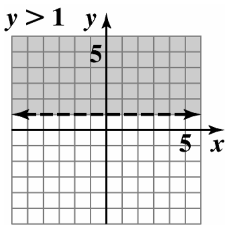
Choose a test point that is not on the line.

Test $(0,0)$

$y > 1$

$0 > 1$, false

Since the statement is false, shade the other half-plane.



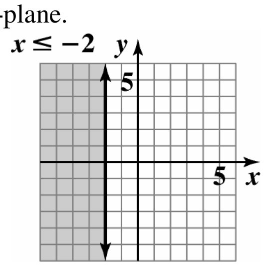
b. Graph the equation $x = -2$ as a solid line.
Choose a test point that is not on the line.

Test $(0,0)$

$x \le -2$

$0 \le -2$, false

Since the statement is false, shade the other half-plane.



4. Graph $x^2 + y^2 = 16$ as a solid circle with radius 4 and center (0, 0).

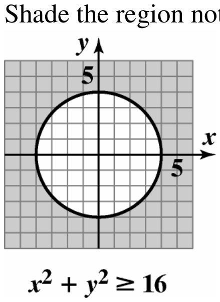
Test $(0,0)$

$x^2 + y^2 \ge 16$

$(0)^2 + (0)^2 \ge 16$

$0 \ge 16$, false

Shade the region not containing (0, 0).



5. Point $B = (66,130)$

$4.9x - y \ge 165$

$4.9(66) - 130 \ge 165$

$193.4 \ge 165$, true

$3.7x - y \le 125$

$3.7(66) - 130 \le 125$

$114.2 \le 125$, true

Point B is a solution of the system.

6. Graph the equation $x - 3y = 6$ as a dashed line.

Test $(0,0)$

$x - 3y < 6$

$(0) - 3(0) < 6$

$0 < 6$, true

Since the statement is true, shade the half-plane containing the point.
Graph the equation $2x + 3y = -6$ as a solid line.

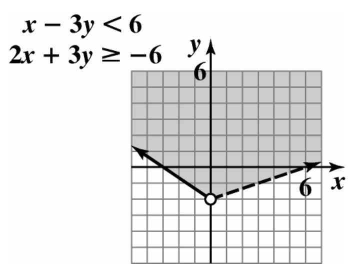
Test $(0,0)$

$2x + 3y \ge -6$

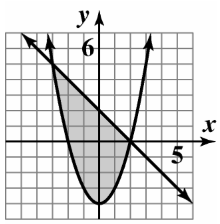
$2(0) + 3(0) \ge -6$

$0 \ge -6$, true

Since the statement is true, shade the half-plane containing the point.
For the solution graph, place an open circle at the point of intersection and shade the region that satisfies both inequalities.



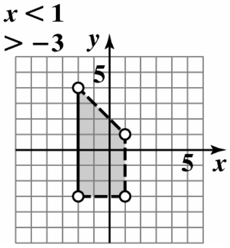
7. Begin by graphing $y = x^2 - 4$ as a solid parabola with vertex (0, –4) and x-intercepts (–2, 0) and (2, 0). Since (0, 0) makes the inequality $y \geq x^2 - 4$ true, shade the region containing (0, 0). Graph $x + y = 2$ as a solid line by using its x-intercept, (2, 0), and its y-intercept (0, 2). Since (0, 0) makes the inequality $x + y \leq 2$ true, shade the region containing (0, 0).



$y \geq x^2 - 4$
$x + y \leq 2$

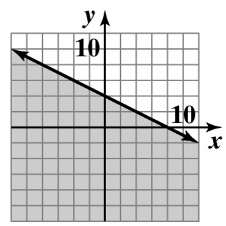
8. Graph the lines $x + y = 2$, $x = 1$, and $y = -3$ with dashed lines.
Graph the line $x = -2$ with a solid line.
Test points indicate that the solution contains the region to the right of –2, to the left of 1, above –3, and below the line $x + y = 2$. The corner points are represented as open circles because none satisfy all three inequalities.

$x + y < 2$
$-2 \leq x < 1$
$y > -3$



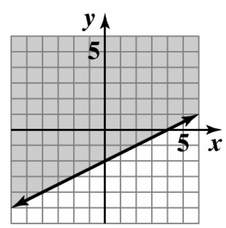
Exercise Set 8.5

1. Graph $x + 2y = 8$ as a solid line using its x-intercept, (8, 0), and its y-intercept, (0, 4).
Test (0, 0):
$0 + 2(0) \leq 8$?
$0 \leq 8$ true
Shade the half-plane containing (0, 0).



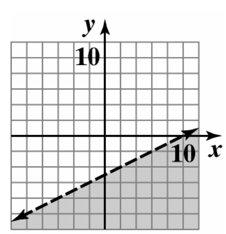
$x + 2y \leq 8$

2. Graph $3x - 6y = 12$ as a solid line using its x-intercept, (4, 0), and its y-intercept (0, –2).
Test (0, 0):
$3(0) - 6(0) \leq 12$?
$0 \leq 12$ true
Shade the half-plane containing (0, 0).



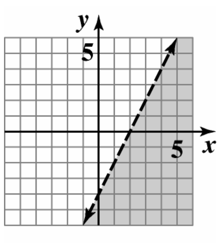
$3x - 6y \leq 12$

3. Graph $x - 2y = 10$ as a dashed line using its x-intercept, (10, 0), and its y-intercept, (0, –5).
Test (0, 0):
$0 - 2(0) > 10$?
$0 > 10$ false
Shade the half-plane not containing (0, 0).



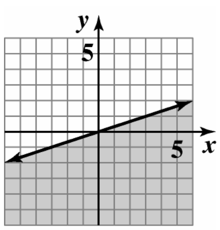
$x - 2y > 10$

4. Graph $2x - y = 4$ as a dashed line using its x-intercept, (2, 0), and its y-intercept (0, –4).
Test (0, 0):
$2(0) - 0 > 4?$
$0 > 4$ false
Shade the half-plane not containing (0, 0).



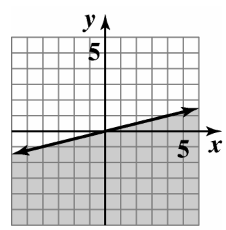
$2x - y > 4$

5. Graph $y = \frac{1}{3}x$ as a solid line using its slope, $\frac{1}{3}$, and its y-intercept (0, 0).
Test (1, 1):
$1 \le \frac{1}{3}(1)?$
$1 \le \frac{1}{3}$ false
Shade the half-plane not containing (1, 1).



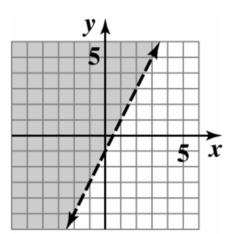
$y \le \frac{1}{3}x$

6. Graph $y = \frac{1}{4}x$ as a solid line using its slope, $\frac{1}{4}$, and its y-intercept (0, 0).
Test (1, 1):
$1 \le \frac{1}{4}(1)?$
$1 \le \frac{1}{4}$ false
Shade the half-plane not containing (1, 1).



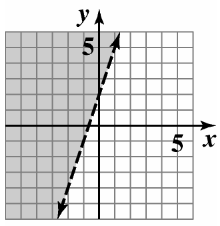
$y \le \frac{1}{4}x$

7. Graph $y = 2x - 1$ as a dashed line using its x-intercept, $\left(\frac{1}{2}, 0\right)$ and its y-intercept, (0, –1).
Test (0,0):
$0 > 2(0) - 1$?
$0 > -1$ true
Shade the half-plane containing (0, 0).



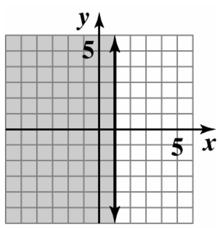
$y > 2x - 1$

8. Graph $y = 3x + 2$ as a dashed line using its x-intercept, $\left(-\frac{2}{3}, 0\right)$, and its y-intercept (0, 2).
Test (0, 0):
$0 > 3(0) + 2?$
$0 > 2$ false
Shade the half-plane not containing (0, 0).



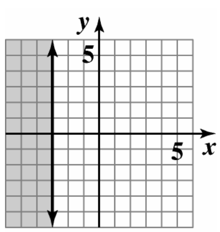
$y > 3x + 2$

9. Graph $x = 1$ as a solid vertical line.
Test (0, 0):
$0 \le 1$ true
Shade the half-plane containing (0, 0).



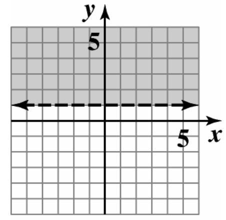
$x \le 1$

10. Graph $x = -3$ as a solid vertical line.
Test (0, 0):
$0 \le -3$ false
Shade the half-plane not containing (0, 0).



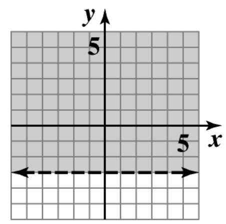
$x \le -3$

11. Graph $y = 1$ as a dashed horizontal line.
Test (0, 0):
$0 > 1$ false
Shade the half-plane not containing (0, 0).



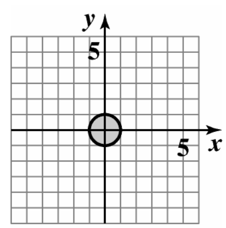
$y > 1$

12. Graph $y = -3$ as a dashed horizontal line.
Test (0, 0):
$0 > -3$ true
Shade the half-plane containing (0, 0).



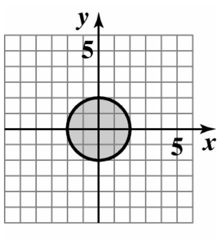
$y > -3$

13. Graph $x^2 + y^2 = 1$ as a solid circle with radius 1 and center (0, 0).
Test (0, 0):
$(0)^2 + (0)^2 \le 1?$
$0 \le 1$ true
Shade the region containing (0, 0).



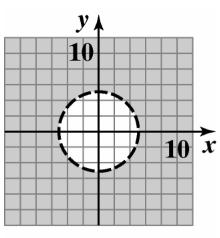
$x^2 + y^2 \le 1$

14. Graph $x^2 + y^2 = 4$ as a solid circle with radius 2 and center (0, 0).
Test (0, 0):
$(0)^2 + (0)^2 \le 4?$
$0 \le 4$ true
Shade the region containing (0, 0).



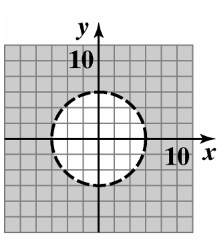
$x^2 + y^2 \le 4$

15. Graph $x^2 + y^2 = 25$ as a dashed circle with radius 5 and center (0, 0).
Test (0, 0):
$(0)^2 + (0)^2 > 25?$
$0 > 25$ false
Shade the region not containing (0, 0).



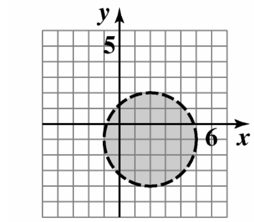
$x^2 + y^2 > 25$

16. Graph $x^2 + y^2 = 36$ as a dashed circle with radius 6 and center (0, 0).
Test (0, 0):
$(0)^2 + (0)^2 > 36$
$0 > 36$ false
Shade the region not containing (0, 0).



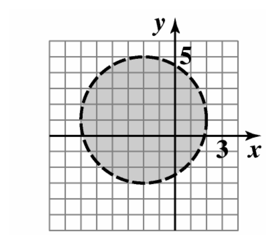
$x^2 + y^2 > 36$

17. Graph $(x-2)^2 + (y+1)^2 = 9$ as a dashed circle.
Test $(0, 0)$
$(x-2)^2 + (y+1)^2 < 9$
$(0-2)^2 + (0+1)^2 < 9$
$5 < 9$, true
Shade the region containing (0, 0).



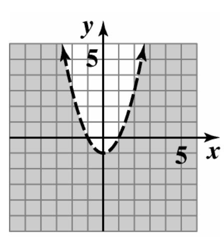
$(x - 2)^2 + (y + 1)^2 < 9$

18. Graph $(x+2)^2 + (y-1)^2 = 16$ as a dashed circle.
Test $(0, 0)$
$(x+2)^2 + (y-1)^2 < 16$
$(0+2)^2 + (0-1)^2 < 16$
$5 < 16$, true
Shade the region containing (0, 0).



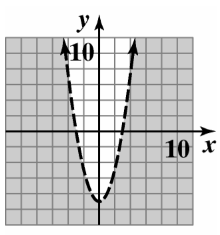
$(x + 2)^2 + (y - 1)^2 < 16$

19. Graph $y = x^2 - 1$ as a dashed parabola with vertex (0, –1) and x-intercepts (1, 0) and (–1, 0).
Test (0, 0):
$0 < (0)^2 - 1?$
$0 < -1$ false
Shade the region not containing (0, 0).



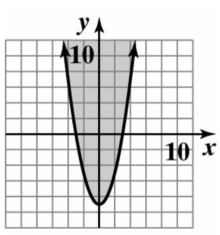
$y < x^2 - 1$

20. Graph $y = x^2 - 9$ as a dashed parabola with vertex (0, –9) and x-intercepts (3, 0) and (–3, 0).
Test (0, 0):
$0 < (0)^2 - 9$?
$0 < -9$ false
Shade the region not containing (0, 0).



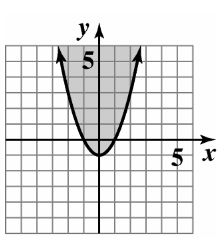
$y < x^2 - 9$

21. Graph $y = x^2 - 9$ as a solid parabola with vertex (0, –9) and x-intercepts (3, 0) and (–3, 0).
Test (0, 0):
$0 \geq (0)^2 - 9$?
$0 \geq -9$ true
Shade the region containing (0, 0).



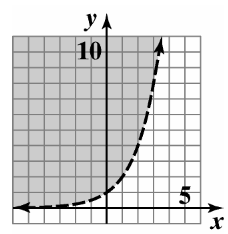
$y \geq x^2 - 9$

22. Graph $y = x^2 - 1$ as a solid parabola with vertex (0, –1) and x-intercepts (1, 0) and (–1, 0).
Test (0, 0):
$0 \geq (0)^2 - 1$?
$0 \geq -1$ true
Shade the region containing (0, 0).



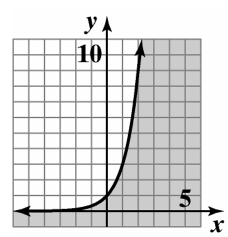
$y \geq x^2 - 1$

23. Graph $y = 2^x$ as a dashed exponential function with base 2 that passes through the point (0, 1).
Test (0, 0):
$0 > 2^0$?
$0 > 1$ false
Shade the region not containing (0, 0).



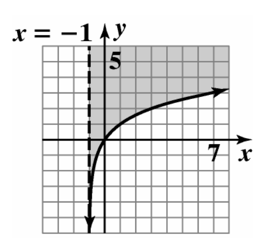
$y > 2^x$

24. Graph $y = 3^x$ as a solid exponential function with base 3 that passes through the point (0, 1).
Test (0, 0):
$0 \leq 3^0$?
$0 \leq 1$ true
Shade the region containing (0, 0).



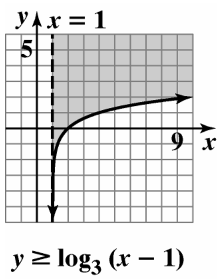
$y \leq 3^x$

25. Graph $y = \log_2(x+1)$ as a solid logarithmic function.
Test (0, 0):
$y \geq \log_2(x+1)$
$0 \geq \log_2(0+1)$
$0 \geq 0$, true
Shade the region containing (0, 0).

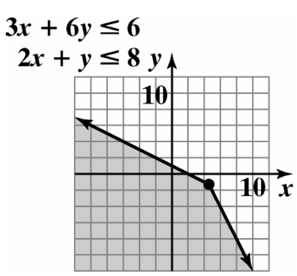


$y \geq \log_2(x + 1)$

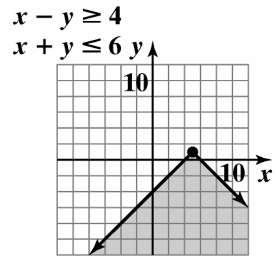
26. Graph $y = \log_3(x-1)$ as a solid logarithmic function.
Test (2, 2):
$y \geq \log_3(x-1)$
$2 \geq \log_3(2-1)$
$2 \geq 0$, true
Shade the region containing (2, 2).



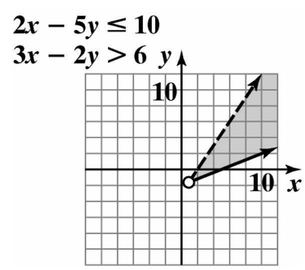
27. Begin by graphing $3x+6y=6$ as a solid line using its x-intercept, (2, 0), and its y-intercept, (0, 1). Since (0, 0) makes the inequality $3x+6y \leq 6$ true, shade the half-plane containing (0, 0). Graph $2x+y=8$ as a solid line using its x-intercept, (4, 0), and its y-intercept, (0, 8). Since (0, 0) makes the inequality $2x+y \leq 8$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above shaded half-planes, and is shown as the shaded region in the following graph.



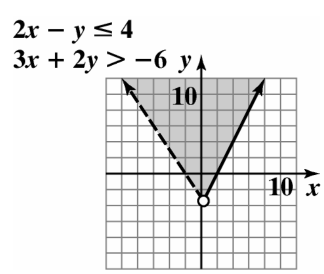
28. Begin by graphing $x-y=4$ as a solid line using its x-intercept, (4, 0), and its y-intercept (0, –4). Since (0, 0) makes the inequality $x-y \geq 4$ false, shade the half-plane not containing (0, 0). Graph $x+y=6$ as a solid line using its x-intercept, (6, 0), and its y-intercept (0, 6). Since (0, 0) makes the inequality $x+y \leq 6$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above shaded half-planes, and is shown as the shaded region in the following graph.



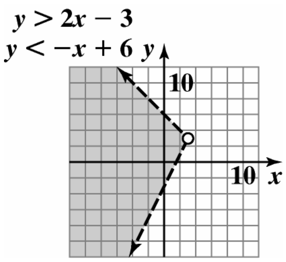
29. Begin by graphing $2x-5y=10$ as a solid line using its x-intercept, (5, 0), and its y-intercept, (0, –2). Since (0, 0) makes the inequality $2x-5y \leq 10$ true, shade the half-plane containing (0, 0). Graph $3x-2y=6$ as a dashed line using its x-intercept, (2, 0), and its y-intercept, (0, –3). Since (0, 0) makes the inequality $3x-2y>6$ false, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above shaded half-planes, and is shown as the shaded region in the following graph.



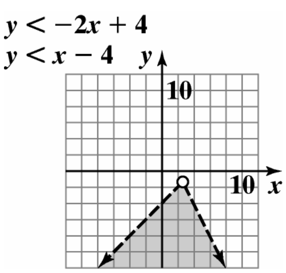
30. Begin by graphing $2x-y=4$ as a solid line using its x-intercept, (2, 0), and its y-intercept (0, –4). Since (0, 0) makes the inequality $2x-y \leq 4$ true, shade the half-plane containing (0, 0). Graph $3x+2y=-6$ as a dashed line using its x-intercept, (–2, 0), and its y-intercept (0, –3). Since (0, 0) makes the inequality $3x+2y>-6$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above shaded half-plane, and is shown as the shaded region in the following graph.



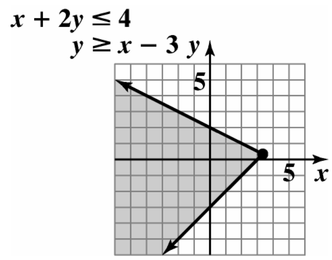
31. Begin by graphing $y = 2x - 3$ as a dashed line using its slope, 2, and its y-intercept, (0, –3). Since (0, 0) makes the inequality $y > 2x - 3$ true, shade the half-plane containing (0, 0). Graph $y = -x + 6$ as a dashed line using its slope, –1, and its y-intercept, (0, 6). Since (0, 0) makes the inequality $y < -x + 6$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above shaded half-planes, and is shown as the shaded region in the following graph.



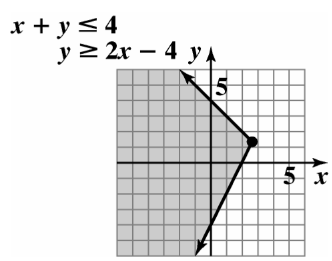
32. Begin by graphing $y = -2x + 4$ as a dashed line using its slope, –2, and y-intercept (0, 4). Since (0, 0) makes the inequality $y < -2x + 4$ true, shade the half-plane containing (0, 0). Graph $y = x - 4$ as a dashed line using its slope, 1, and y-intercept (0, –4). Since (0, 0) makes the inequality $y < x - 4$ false, shade the half-plane not containing (0, 0). The solution set of a system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.



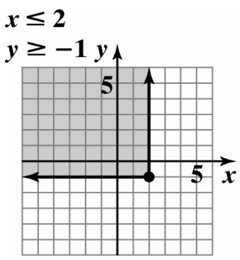
33. Begin by graphing $x + 2y = 4$ as a solid line using its x-intercept, (4, 0), and its y-intercept, (0, 2). Since (0, 0) makes the inequality $x + 2y \le 4$ true, shade the half-plane containing (0, 0). Graph $y = x - 3$ as a solid line using its slope, 1, and its y-intercept, (0, –3). Since (0, 0) makes the inequality $y \ge x - 3$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above shaded half-planes, and is shown as the shaded region in the following graph.



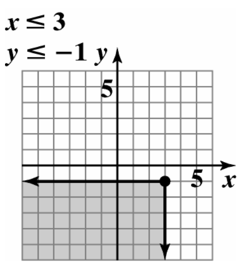
34. Begin by graphing $x + y = 4$ as a solid line using its x-intercept, (4, 0), and its y-intercept (0, 4). Since (0, 0) makes the inequality $x + y \le 4$ true, shade the half-plane containing (0, 0). Graph $y = 2x - 4$ as a solid line using its slope, 2, and its y-intercept (0, 4). Since (0, 0) makes the inequality $y \ge 2x - 4$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.



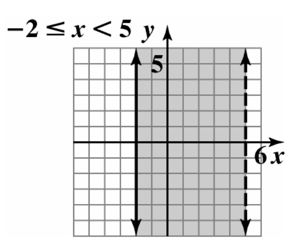
35. Begin by graphing $x = 2$ as a solid vertical line . Since (0, 0) makes the inequality $x \le 2$ true, shade the half-plane containing (0, 0). Graph $y = 1$ as a solid horizontal line. Since (0, 0) makes the inequality $y \ge -1$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above shaded half-planes, and is shown as the shaded region in the following graph.



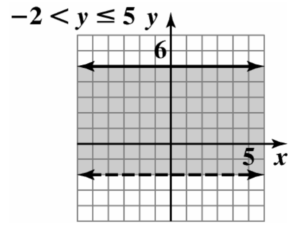
36. Begin by graphing $x = 3$ as a solid vertical line. Since (0, 0) makes the inequality $x \le 3$ true, shade the half-plane containing (0, 0). Graph $y = 1$ as a solid horizontal line. Since (0, 0) makes the inequality $y \le -1$ false, shade the half-plane not containing (0, 0). The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graphs.



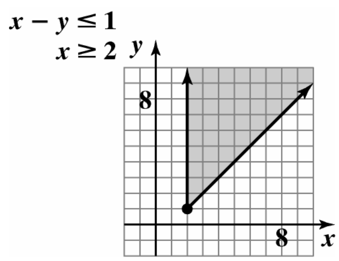
37. Graph $x = -2$ as a solid vertical line and $x = 5$ as a dashed vertical line. Since (0, 0) makes the inequality $-2 \le x < 5$ true, shade the region between the two lines.



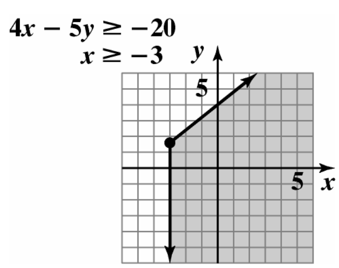
38. Graph $y = -2$ as a dashed horizontal line and $y = 5$ as a solid horizontal line. Since (0, 0) makes the inequality $-2 \le y \le 5$ true, shade the region between the two lines.



39. Begin by graphing $x - y = 1$ as a solid line using its x-intercept, (1, 0), and its y-intercept (0, -1). Since (0, 0) makes the inequality $x - y \le 1$ true, shade the half-plane containing (0, 0). Graph $x = 2$ as a solid horizontal line. Since (0, 0) makes the inequality $x \ge 2$ false, shade the half-plane not containing (0, 0). The solution set of the system is the intersection of the above shaded half-planes, and is shown as the shaded region in the following graph.



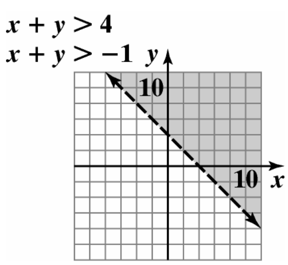
40. Begin by graphing $4x - 5y = -20$ as a solid line using its x-intercept, (–5, 0) and its y-intercept (0, 4). Since (0, 0) makes the inequality $4x - 5y \ge -20$ true, shade the half-plane containing (0, 0). Graph $x = 3$ as a solid vertical line. Since (0, 0) makes the inequality $x \ge -3$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.



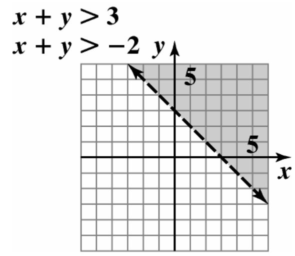
41. $x + y > 4$
$x + y < -1$
Begin by graphing $x + y = 4$ as a dashed line using its x-intercept, (4, 0), and its y-intercept (0, 4). Since (0, 0) makes the inequality $x + y > 4$ false, shade the half-plane not containing (0, 0). Graph $x + y = -1$ as a dashed line using its x-intercept, (–1, 0), and its y-intercept, (0, –1). Since (0, 0) makes the inequality $x + y < -1$ false, shade the half-plane not containing (0, 0). Since these half-planes do not intersect the system has no solution.

42. Begin by graphing $x + y = 3$ as a dashed line using its x-intercept, (3, 0), and its y-intercept (0, 3). Since (0, 0) makes the inequality $x + y > 3$ false, shade the half-plane not containing (0, 0). Graph $x + y = -2$ as a dashed line using its x-intercept, (–2, 0), and its y-intercept (0, –2). Since (0, 0) makes the inequality $x + y < -2$ false, shade the half-plane not containing (0, 0). Since these half-planes do not intersect the system has no solution.

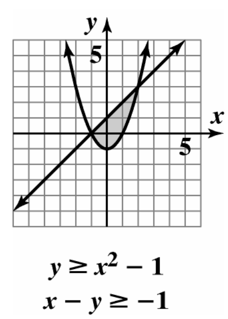
43. Begin by graphing $x + y = 4$ as a dashed line using its x-intercept, (4, 0), and its y-intercept, (0, 4). Since (0, 0) makes the inequality $x + y > 4$ false, shade the half-plane not containing (0, 0). Graph $x + y = -1$ as a dashed line using its x-intercept, (-1, 0), and its y-intercept, (0, –1). Since (0, 0) makes the inequality $x + y > -1$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.



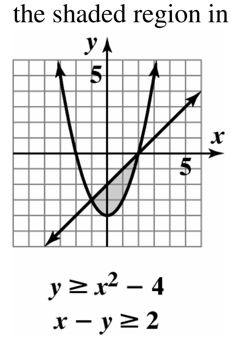
44. Begin by graphing $x + y = 3$ as a dashed line using its x-intercept, (3, 0), and its y-intercept (0, 3). Since (0, 0) makes the inequality $x + y > 3$ false, shade the half-plane not containing (0, 0). Graph $x + y = -2$ as a dashed line using its x-intercept, (–2, 0), and its y-intercept (0, –2). Since (0, 0) makes the inequality $x + y > -2$ true, shade the half-plane containing (0, 0). The solution of the above half-planes, and is shown as the shaded region in the following graph.



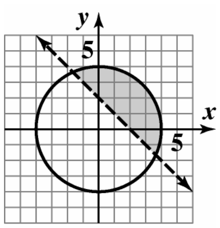
45. Begin by graphing $y = x^2 - 1$ as a solid parabola with vertex (0, –1) and x-intercepts, (–1, 0), and (1, 0). Since (0, 0) makes the inequality $y \geq x^2 - 1$ true, shade the half-plane containing (0, 0). Graph $x - y = -1$ as a solid line using its x-intercept, (–1, 0), and its y-intercept, (0, 1). Since (0, 0) makes the inequality $x - y \geq -1$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.



46. Begin by graphing $y = x^2 - 4$ as a solid parabola with vertex (0, –4) and x-intercepts (–2, 0) and (2, 0). Since (0, 0) makes the inequality $y \geq x^2 - 4$ true, shade the half-plane containing (0, 0). Graph $x - y = 2$ as a solid line using its x-intercept, (2, 0), and its y-intercept (0, –2). Since (0, 0) makes the inequality $x - y \geq 2$ false, shade the half-plane not containing (0, 0). The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.

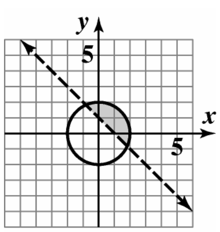


47. Begin by graphing $x^2 + y^2 = 16$ as a solid circle with radius 4 and center, (0, 0). Since (0, 0) makes the inequality $x^2 + y^2 \le 16$ true, shade the half-plane containing (0, 0). Graph $x + y = 2$ as a dashed line using its x-intercept, (2, 0), and its y-intercept, (0, 2). Since (0, 0) makes the inequality $x + y > 2$ false, shade the half-plane not containing (0, 0). The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.



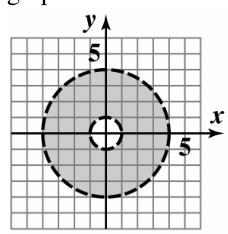
$x^2 + y^2 \le 16$
$x + y > 2$

48. Begin by graphing $x^2 + y^2 = 4$ as a solid circle with radius 2 and center (0, 0). Since (0, 0) makes the inequality $x^2 + y^2 \le 4$ true, shade the half-plane containing (0, 0). Graph $x + y = 1$ as a dashed line using its x-intercept, (1, 0), and its y-intercept (0, 1). Since (0, 0) makes the inequality $x + y > 1$ false, shade the half-plane not containing (0, 0). The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.



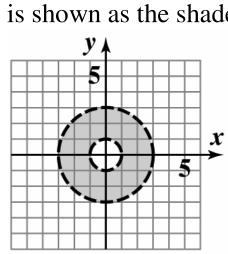
$x^2 + y^2 \le 4$
$x + y > 1$

49. Begin by graphing $x^2 + y^2 = 1$ as a dashed circle with radius 1 and center, (0, 0). Since (0, 0) makes the inequality $x^2 + y^2 > 1$ false, shade the half-plane not containing (0, 0). Graph $x^2 + y^2 = 16$ as a dashed circle with radius 4 and center (0, 0). Since (0, 0) makes the inequality $x^2 + y^2 < 16$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.



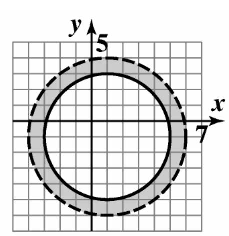
$x^2 + y^2 > 1$
$x^2 + y^2 < 16$

50. Begin by graphing $x^2 + y^2 = 1$ as a dashed circle with radius 1 and center (0, 0). Since (0, 0) makes the inequality $x^2 + y^2 > 1$ false, shade the half-plane not containing (0, 0). Graph $x^2 + y^2 = 9$ as a dashed circle with radius 3 and center (0, 0). Since (0, 0) makes the inequality $x^2 + y^2 < 9$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.



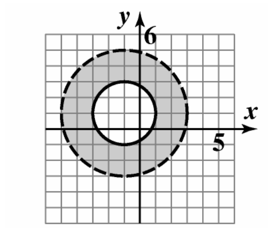
$x^2 + y^2 > 1$
$x^2 + y^2 < 9$

51.



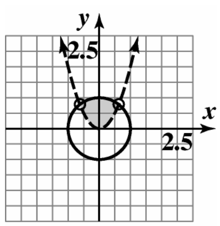
$(x - 1)^2 + (y + 1)^2 < 25$
$(x - 1) + (y + 1)^2 \ge 16$

52.



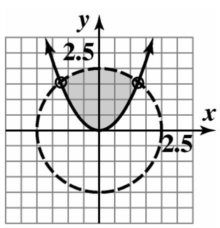
$(x + 1)^2 + (y - 1)^2 < 16$
$(x + 1)^2 + (y - 1)^2 \geq 4$

53.



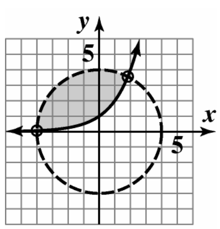
$x^2 + y^2 \leq 1$
$y - x^2 > 0$

54.



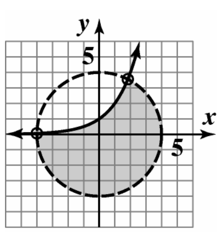
$x^2 + y^2 < 4$
$y - x^2 \geq 0$

55.



$x^2 + y^2 < 16$
$y \geq 2^x$

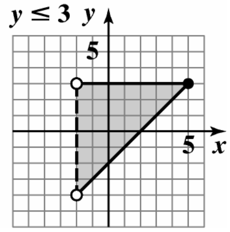
56.



$x^2 + y^2 \leq 16$
$y < 2^x$

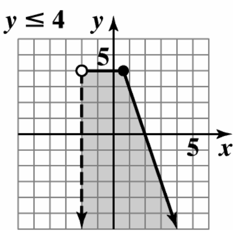
57. Begin by graphing $x - y = 2$ as a solid line using its x-intercept, (2, 0), and its y-intercept, (0, –2). Since (0, 0) makes the inequality $x - y \leq 2$ true, shade the half-plane containing (0, 0). Graph $x = -2$ as a solid vertical line. Since (0, 0) makes the inequality $x \geq -2$ true, shade the half-plane containing (0, 0). Graph $y = 3$ as a solid horizontal line. Since (0, 0) makes the inequality $y \leq 3$ true, shade the half-plane containing (0, 0).The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.

$x - y \leq 2$
$x > -2$
$y \leq 3$

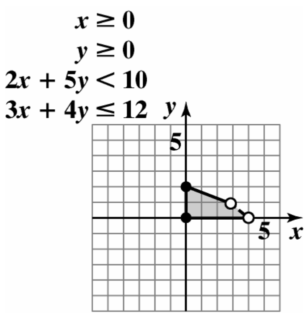


58. Begin by graphing $3x + y = 6$ as a solid line using its x-intercept, (2, 0), and its y-intercept (0, 6). Since (0, 0) makes the inequality $3x + y \leq 6$ true, shade the half-plane containing (0, 0). Graph $x = -2$ as a solid vertical line.
Since (0, 0) makes the inequality $x \geq -2$ true, shade the half-plane containing (0, 0). Graph $y = 4$ as a solid horizontal line. Since (0, 0) makes the inequality $y \leq 4$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.

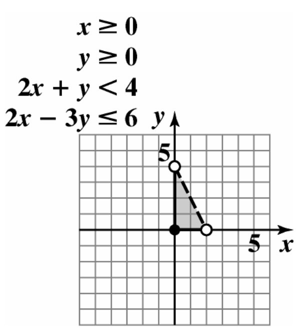
$3x + y \leq 6$
$x > -2$
$y \leq 4$



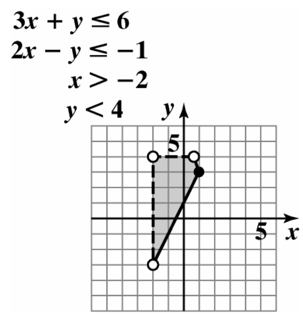
59. Since $x \geq 0$ and $y \geq 0$ the solution to the system lies in the first quadrant. Graph $2x + 5y = 10$ as a solid line using its x-intercept, (5, 0), and its y-intercept, (0, 2). Since (0, 0) makes the inequality $2x + 5y \leq 10$ true, shade the half-plane containing (0, 0). Graph $3x + 4y = 12$ as a solid line by using its x-intercept, (4, 0), and its y-intercept, (0, 3). Since (0, 0) makes the inequality $3x + 4y \leq 12$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above half-planes which lies in the first quadrant, and is shown as the shaded region in the following graph.



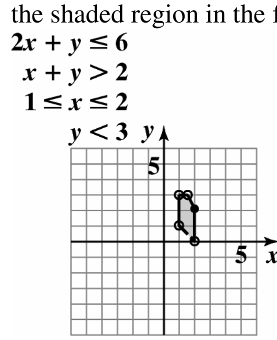
60. Since $x \geq 0$ and $y \geq 0$ the solution to the system lies in the first quadrant. Graph $2x + y = 4$ as a solid line using its x-intercept, (2, 0), and y-intercept (0, 4). Since (0, 0) makes the inequality $2x + y \leq 4$ true, shade the half-plane containing (0, 0). Graph $2x - 3y = 6$ as a solid line using its x-intercept, (3, 0), and its y-intercept (0, –2). Since (0, 0) makes the inequality $2x - 3y \leq 6$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above half-planes which lies in the first quadrant, and is shown as the shaded region in the following graph.



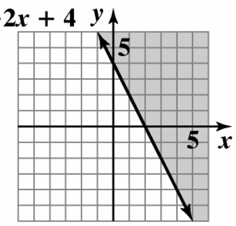
61. Begin by graphing $3x + y = 6$ as a solid line using its x-intercept, (2, 0), and its y-intercept, (0, 6). Since (0, 0) makes the inequality $3x + y \leq 6$ true, shade the half-plane containing (0, 0). Graph $2x - y = -1$ as a solid line using its x-intercept, $\left(-\frac{1}{2}, 0\right)$, and its y-intercept, (0, 1). Since (0, 0) makes the inequality $2x - y \leq -1$ false, shade the half-plane not containing (0, 0). Graph $x = -2$ as a solid vertical line. Since (0, 0) makes the inequality $x \geq -2$ true, shade the half-plane containing (0, 0). Graph $y = 4$ as a solid horizontal line. Since (0, 0) makes the inequality $y \leq 4$ true, shade the half-plane containing (0, 0).The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.



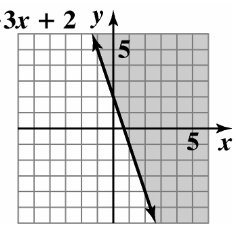
62. Begin by graphing $2x + y = 6$ as a solid line by using its x-intercept, (3, 0) and its y-intercept (0, 6). Since (0, 0) makes the inequality $2x + y \leq 6$ true, shade the half-plane containing (0, 0). Graph $x + y = 2$ as a solid line using its x-intercept, (2, 0), and its y-intercept (0, 2). Since (0, 0) makes the inequality $x + y \geq 2$ false, shade the half-plane not containing (0, 0). Graph $x = 1$ and $x = 2$ as solid vertical lines. Since (0, 0) makes the inequality $1 \leq x \leq 2$ false, shade the region between the two vertical lines. Graph $y = 3$ as a solid horizontal line. Since (0, 0) makes the inequality $y \leq 3$ true, shade the half-plane containing (0, 0). The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.



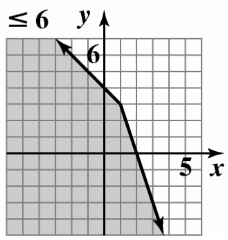
63. $y \geq -2x + 4$



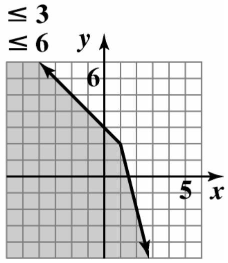
64. $y \geq -3x + 2$



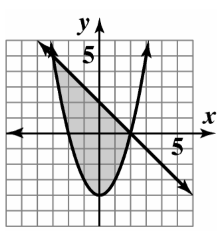
65. $x + y \leq 4$
$3x + y \leq 6$



66. $x + y \leq 3$
$4x + y \leq 6$



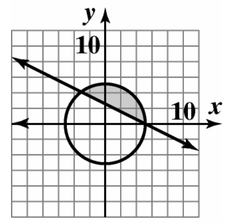
67.



$x + y \leq 2$

$y \geq x^2 - 4$

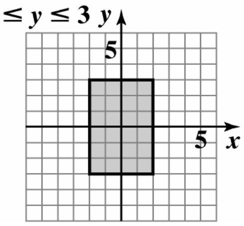
68.



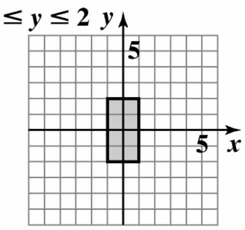
$x^2 + y^2 \leq 25$

$2y + x \geq 5$

69. $-2 \leq x \leq 2$
$-3 \leq y \leq 3$

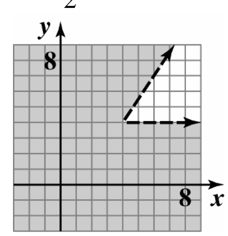


70. $-1 \leq x \leq 1$
$-2 \leq y \leq 2$

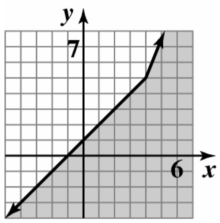


71. Find the union of solutions of

$y > \frac{3}{2}x - 2$ and $y < 4$.



72. Find the union of solutions of $x - y \ge -1$ and $5x - 2y \le 10$.



73. The system $\begin{matrix} 3x + 3y < 9 \\ 3x + 3y > 9 \end{matrix}$ has no solution.

The number $3x + 3y$ cannot both be less than 9 and greater than 9 at the same time.

74. The system $\begin{matrix} 6x - y \le 24 \\ 6x - y > 24 \end{matrix}$ has no solution.

The number $6x - y$ cannot both be less than or equal to 24 and greater than 24 at the same time.

75. The system has an infinite number of solutions. The solution is the set of points that make up the circle $(x+4)^2 + (y-3)^2 = 9$

76. The system has an infinite number of solutions. The solution is the set of points that make up the circle $(x-4)^2 + (y+3)^2 = 24$

77. Point $A = (66,160)$

$$5.3x - y \ge 180$$
$$5.3(66) - 160 \ge 180$$
$$189.8 \ge 180, \text{ true}$$

$$4.1x - y \le 14$$
$$4.1(66) - 160 \le 140$$
$$110.6 \le 140, \text{ true}$$

Point A is a solution of the system.

78. Point $B = (76,220)$

$$5.3x - y \ge 180$$
$$5.3(76) - 220 \ge 180$$
$$182.8 \ge 180, \text{ true}$$

$$4.1x - y \le 14$$
$$4.1(76) - 220 \le 140$$
$$91.6 \le 140, \text{ true}$$

Point B is a solution of the system.

79. Point = (72,205)

$$5.3x - y \ge 180$$
$$5.3(72) - 205 \ge 180$$
$$176.6 \ge 180, \text{ false}$$

$$4.1x - y \le 14$$
$$4.1(72) - 205 \le 140$$
$$90.2 \le 140, \text{ true}$$

The data does not satisfy both inequalities. The person is not within the healthy weight region.

80. Point = (68,135)

$$5.3x - y \ge 180$$
$$5.3(68) - 135 \ge 180$$
$$225.4 \ge 180, \text{ true}$$

$$4.1x - y \le 14$$
$$4.1(68) - 135 \le 140$$
$$143.8 \le 140, \text{ false}$$

The data does not satisfy both inequalities. The person is not within the healthy weight region.

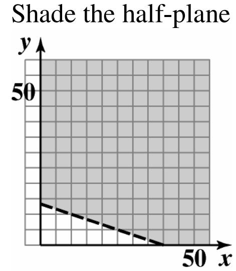
81. **a.** $50x + 150y > 2000$

b. Graph $50x + 150y$ as a dashed line using its x-intercept, (40, 0), and its y-intercept, $\left(0, \frac{40}{3}\right)$.

Test (0, 0):

$$50(0) + 150(0) > 2000?$$
$$0 > 2000 \text{ false}$$

Shade the half-plane not containing (0, 0).

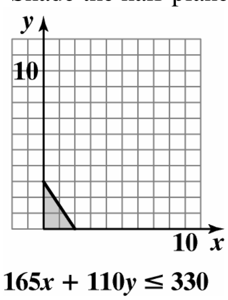


$50x + 150y > 2000$

c. Ordered pairs may vary.

82. a. $165x+110y \le 330$

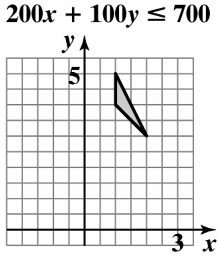
b. Graph $165x+110y = 330$ as a solid line by using its x-intercept, (2, 0), and its y-intercept (0, 3).
Test (0, 0):
$165(0)+110(0) \le 330$?
$0 \le 330$ true
Shade the half-plane containing (0, 0).



c. Ordered pairs may vary.

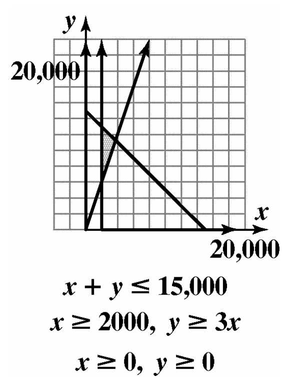
83. a.
$$y \ge 0$$
$$x+y \ge 5$$
$$x \ge 1$$
$$200x+100y \le 700$$

b. $\mathbf{y \ge 0}$
$\mathbf{x + y \ge 5}$
$\mathbf{x \ge 1}$
$\mathbf{200x + 100y \le 700}$



c. 2 nights

84. x = amount invested at high risk.
y = amount invested at high risk.
$x+y \le 15{,}000$
$x \ge 2000$
$y \ge 3x$
$x \ge 0$
$y \ge 0$
Since $x \ge 0$ and $y \ge 0$ the solution set to the system lies in the first quadrant. Graph $x+y=15{,}000$ as a solid line using its x-intercept, (15,000, 0), and its y-intercept, (0, 15,000). Since (0, 0) makes the inequality $x+y \le 15{,}000$ true, shade the half-plane containing (0, 0). Graph $y = 3x$ as a solid line by using its slope, 3, and its y-intercept, (0, 0). Since (1, 1) makes the inequality $y \ge 3x$ false, shade the half-plane not containing (0, 0). Graph $x = 2000$ as a solid vertical line. Since (0, 0) makes the inequality $x \ge 2000$ false, shade the half- plane not containing (0, 0). The solution set of the system is the intersection of the above half-planes, and is shown as the shaded region in the following graph.



85. a. $\text{BMI} = \dfrac{703W}{H^2} = \dfrac{703(200)}{72^2} \approx 27.1$

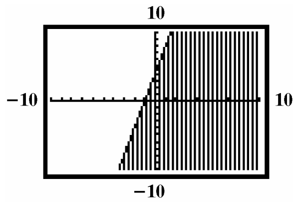
b. A 20 year old man with a BMI of 27.1 is classified as overweight.

86. a. $\text{BMI} = \dfrac{703W}{H^2} = \dfrac{703(105)}{66^2} \approx 16.9$

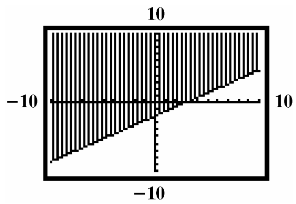
b. A 25 year old woman with a BMI of 16.9 is classified as underweight.

87. – 96. Answers may vary.

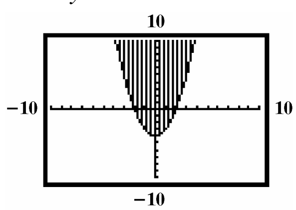
97.



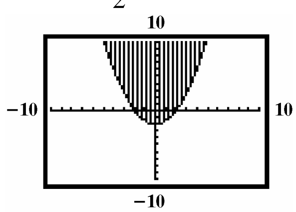
98. $y \geq \frac{2}{3}x - 2$



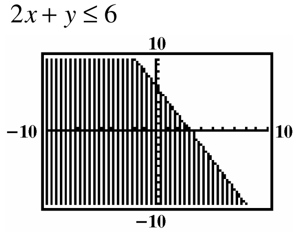
99. $y \geq x^2 - 4$



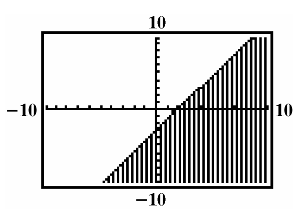
100. $y \geq \frac{1}{2}x^2 - 2$



101. $2x + y \leq 6$



102. $3x - 2y \geq 6$



103. – 105. Answers may vary.

106. does not make sense; Explanations will vary. Sample explanation: (0, 0) can not be used as a test point when it lies on the related equation.

107. does not make sense; Explanations will vary. Sample explanation: It is necessary to graph the linear equation with a dashed line to represent its role as a borderline.

108. makes sense

109. makes sense

110. $x \geq -2, y > -1$

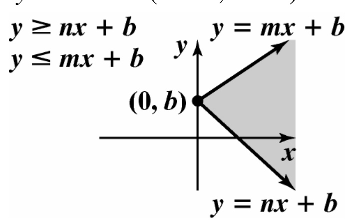
111. $y > x - 3$
$y \leq x$

112. $x^2 + y^2 \leq 9$
$y < x^2$

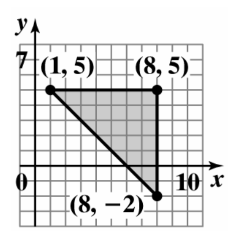
113. $x + 2y \leq 6$ or $2x + y \leq 6$

114. Answers may vary.

115. $y \geq nx + b \quad (n < 0,\ b > 0)$
$y \leq mx + b \quad (m > 0,\ b > 0)$



116. **a.** $x + y \geq 6$
$x \leq 8$
$y \leq 5$

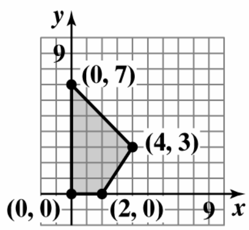


b. The corner points are $(1,5)$, $(8,5)$, and $(8,-2)$.

c. At $(1,5)$, $3x + 2y = 3(1) + 2(5) = 13$.
At $(8,5)$, $3x + 2y = 3(8) + 2(5) = 34$.
At $(8,-2)$, $3x + 2y = 3(8) + 2(-2) = 20$.

117. a.

$$x \ge 0$$
$$y \ge 0$$
$$3x - 2y \le 6$$
$$y \le -x + 7$$



b. The corner points are $(0,0)$, $(2,0)$, $(4,3)$, and $(0,7)$.

c. At $(0,0)$, $2x + 5y = 2(0) + 5(0) = 0.$
At $(2,0)$, $2x + 5y = 2(2) + 5(0) = 4.$
At $(4,3)$, $2x + 5y = 2(4) + 5(3) = 23.$
At $(0,7)$, $2x + 5y = 2(0) + 5(7) = 35.$

118. $20x + 10y \le 80{,}000$

Section 8.6

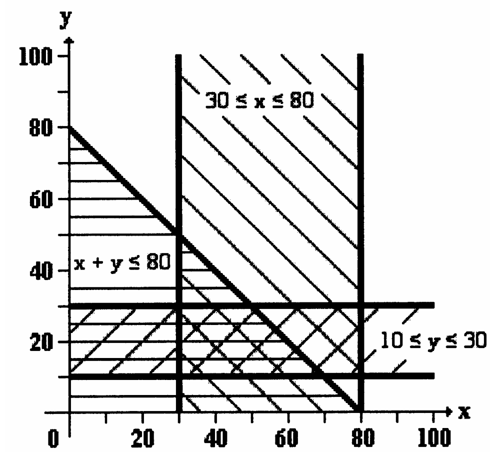
Check Point Exercises

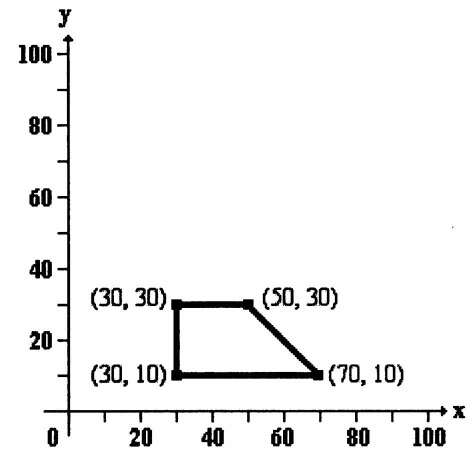
1. The total profit is 25 times the number of bookshelves, *x*, plus 55 times the number of desks, *y*. The objective function is $z = 25x + 55y$

2. Not more than a total of 80 bookshelves and desks can be manufactured per day. This is represented by the inequality $x + y \le 80.$

3. Objective function: $z = 25x + 55y$
Constraints: $x + y \le 80$
$30 \le x \le 80$
$10 \le y \le 30$

4. Graph the constraints and find the corners, or vertices, of the region of intersection.



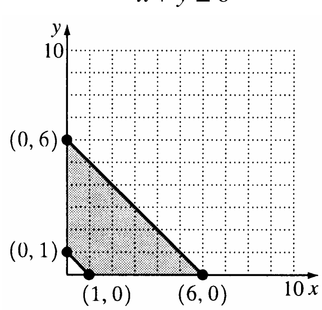


Find the value of the objective function at each corner of the graphed region.

Corner (x, y)	Objective Function $z = 25x + 55y$ z
(30, 10)	$z = 25(30) + 55(10)$ $= 750 + 550 = 1300$
(30, 30)	$z = 25(30) + 55(30)$ $= 750 + 1650 = 2400$
(50, 30)	$z = 25(50) + 55(30)$ $= 1250 + 1650 = 2900 \leftarrow$ Maximum
(70, 10)	$z = 25(70) + 55(10)$ $= 1750 + 550 = 2300$

The maximum value of z is 2900 and it occurs at the point (50, 30).
In order to maximize profit, 50 bookshelves and 30 desks must be produced each day for a profit of $2900.

5. objective function: $z = 3x + 5y$
constraints: $x \ge 0,\ y \ge 0$
$x + y \ge 1$
$x + y \le 6$



Evaluate the objective function at the four vertices of the region shown:
$(1, 0): 3(1) + 5(0) = 3$
$(0, 1): 3(0) + 5(1) = 5$
$(0, 6): 3(0) + 5(6) = 30$
$(6, 0): 3(6) + 5(0) = 18$
The maximum value of z is 30 and this occurs when $x = 0$ and $y = 6$.

Exercise Set 8.6

1. $z = 5x + 6y$
(1, 2): 5(1) + 6(2) = 5 + 12 = 17
(2, 10): 5(2) + 6(10) = 10 + 60 = 70
(7, 5): 5(7) + 6(5) = 35 + 30 = 65
(8, 3): 5(8) + 6(3) = 40 + 18 = 58
The maximum value is $z = 70$; the minimum value is $z = 17$.

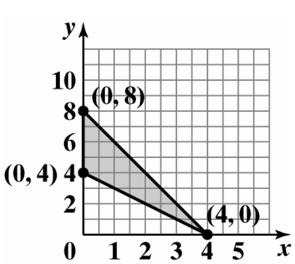
2. $z = 3x + 2y$
(3, 2): 3(3) + 2(2) = 9 + 4 = 13
(4, 10): 3(4) + 2(10) = 12 + 20 = 32
(5, 12): 3(5) + 2(12) = 15 + 24 = 39
(8, 6): 3(8) + 2(6) = 24 + 12 = 36
(7, 4): 3(7) + 2(4) = 21 + 8 = 29
The maximum value is $z = 39$; the minimum value is $z = 13$.

3. $z = 40x + 50y$
(0, 0): 40(0) + 50(0) = 0 + 0 = 0
(0, 8): 40(0) + 50(8) = 0 + 400 = 400
(4, 9): 40(4) + 50(9) = 160 + 450 = 610
(8, 0): 40(8) + 50(0) = 320 + 0 = 320
The maximum value is $z = 610$; the minimum value is $z = 0$.

4. $z = 30x + 45y$
(0, 0): 30(0) + 45(0) = 0 + 0 = 0
(0, 9): 30(0) + 45(9) = 0 + 405 = 405
(4, 4): 30(4) + 45(4) = 120 + 180 = 300
(3, 0): 30(3) + 45(0) = 90 + 0 = 90
The maximum value is $z = 405$; the minimum value is $z = 0$.

5. $z = 3x + 2y$
$x \ge 0, y \ge 0$
$2x + y \le 8$
$x + y \ge 4$

a.

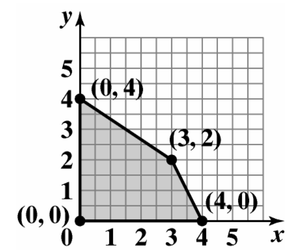


b. $(0,8): z = 3(0) + 2(8) = 16$
$(0,4): z = 3(0) + 2(4) = 8$
$(4,0): z = 3(4) + 2(0) = 12$

c. The maximum value is 16 at $x = 0$ and $y = 8$.

6. $z = 2x + 3y$
$x \ge 0, y \ge 0$
$2x + y \le 8$
$2x + 3y \le 12$

a.

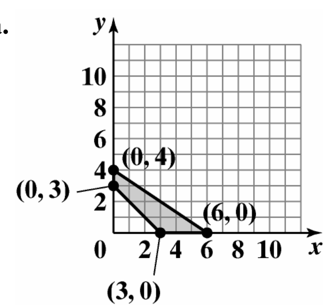


b. $(0,0): 2(0) + 3(0) = 0$
(0,4): 2(0) + 3(4) = 12
(3,2): 2(3) + 3(2) = 12
(4,0): 2(4) + 3(0) = 8

c. The maximum value is 12 at $x = 0$ and $y = 4$ or $x = 3$ and $y = 2$.

7. $z = 4x + y$
$x \ge 0, y \ge 0$
$2x + 3y \le 12$
$x + y \ge 3$

a.



b. (0, 4): $z = 4(0) + 4 = 4$
(0, 3): $z = 4(0) + 3 = 3$
(3, 0): $z = 4(3) + 0 = 12$
(6, 0): $z = 4(6) + 0 = 24$

c. The maximum value is 24 at $x = 6$ and $y = 0$.

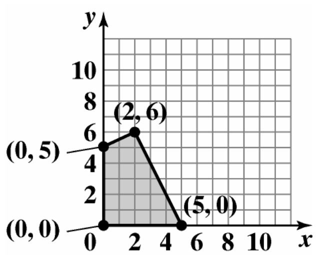
8. $z = x + 6y$

$x \geq 0, y \geq 0$

$2x + y \leq 10$

$x - 2y \geq -10$

a.



b. $(0,5): z = 0 + 6(5) = 30$

$(0,0): z = 0 + 6(0) = 0$

$(2,6): z = 2 + 6(6) = 38$

$(5,0): z = 5 + 6(0) = 5$

c. The maximum value is 38 at $x = 2$ and $y = 6$.

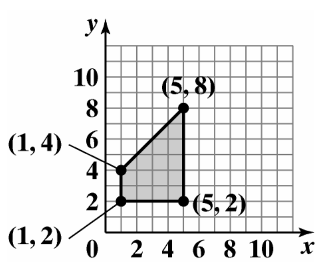
9. $z = 3x - 2y$

$1 \leq x \leq 5$

$y \geq 2$

$x - y \geq -3$

a.



b. (1, 2): $z = 3(1) - 2(2) = -1$

(1, 4): $z = 3(1) - 2(4) = -5$

(5, 8): $z = 3(5) - 2(8) = -1$

(5, 2): $z = 3(5) - 2(2) = 11$

c. Maximum value is 11 at $x = 5$ and $y = 2$.

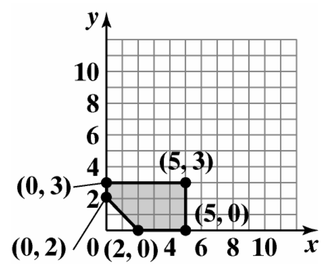
10. $z = 5x - 2y$

$0 \leq x \leq 5$

$0 \leq y \leq 3$

$x + y \geq 2$

a.



b. $(0,3): z = 5(0) - 2(3) = -6$

$(0,2): z = 5(0) - 2(2) = -4$

$(2,0): z = 5(2) - 2(0) = 10$

$(5,0): z = 5(5) - 2(0) = 25$

$(5,3): z = 5(5) - 2(3) = 19$

c. The maximum value is 25 at $x = 5$ and $y = 0$.

11. $z = 4x + 2y$

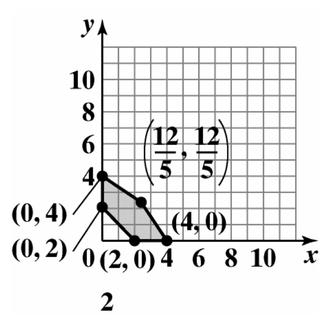
$x \geq 0, y \geq 0$

$2x + 3y \leq 12$

$3x + 2y \leq 12$

$x + y \geq 2$

a.



b. (0, 4): $z = 4(0) + 2(4) = 8$

(0, 2): $z = 4(0) + 2(2) = 4$

(2, 0): $z = 4(2) + 2(0) = 8$

(4, 0): $z = 4(4) + 2(0) = 16$

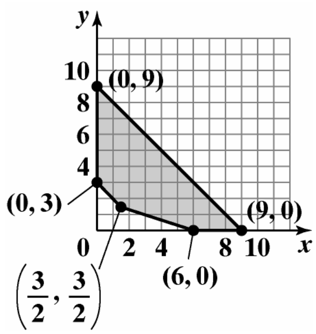
$$\left(\frac{12}{5}, \frac{12}{5}\right): z = 4\left(\frac{12}{5}\right) + 2\left(\frac{12}{5}\right)$$

$$= \frac{48}{5} + \frac{24}{5} = \frac{72}{5}$$

c. The maximum value is 16 at $x = 4$ and $y = 0$.

12. $z = 2x + 4y$
$x \geq 0, y \geq 0$
$x + 3y \geq 6$
$x + y \geq 3$
$x + y \leq 9$

a.

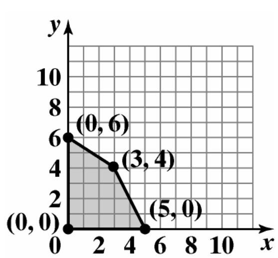


b. $(0,9): z = 2(0) + 4(9) = 36$
$(0,3): z = 2(0) + 4(3) = 12$
$(6,0): z = 2(6) + 4(0) = 12$
$(9,0): z = 2(9) + 4(0) = 18$
$\left(\frac{3}{2},\frac{3}{2}\right): z = 2\left(\frac{3}{2}\right) + 4\left(\frac{3}{2}\right) = 3 + 6 = 9$

c. The maximum value is 36 at $x = 0$ and $y = 9$.

13. $z = 10x + 12y$
$x \geq 0, y \geq 0$
$x + y \leq 7$
$2x + y \leq 10$
$2x + 3y \leq 18$

a.

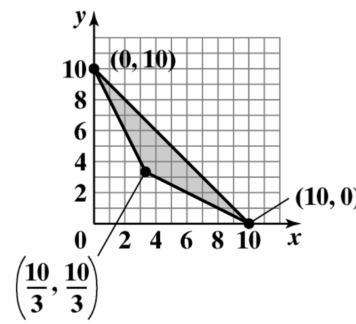


b. (0, 6): $z = 10(0) + 12(6) = 72$
(0, 0): $z = 10(0) + 12(0) = 0$
(5, 0): $z = 10(5) + 12(0) = 50$
(3, 4): $z = 10(3) + 12(4)$
$= 30 + 48 = 78$

c. The maximum value is 78 at $x = 3$ and $y = 4$.

14. $z = 5x + 6y$
$x \geq 0, y \geq 0$
$2x + y \geq 10$
$x + 2y \geq 10$
$x + y \leq 10$

a.



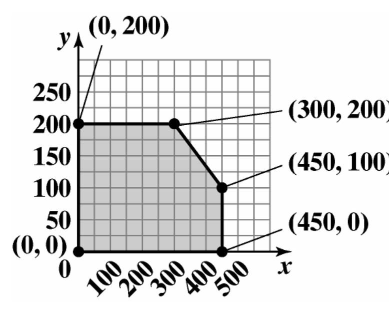
b. $(0,10): z = 5(0) + 6(10) = 60$
$(10,0): z = 5(10) + 6(0) = 50$
$\left(\frac{10}{3},\frac{10}{3}\right): z = 5\left(\frac{10}{3}\right) + 6\left(\frac{10}{3}\right) = \frac{50}{3} + \frac{60}{3}$
$= \frac{110}{3}$

c. The maximum value is 60 at $x = 0$ and $y = 10$.

15. a. $z = 125x + 200y$

b. $x \leq 450$
$y \leq 200$
$600x + 900y \leq 360{,}000$

c. Simplify the third inequality by dividing by 300 to get $2x + 3y \leq 1200$.



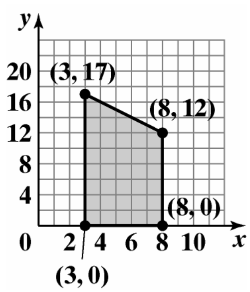
d. (0, 0): 125(0) + 200(0) = 0 + 0 = 0
(0, 200): 125(0) + 200(200)
= 0 + 40,000 = 40,000
(300, 200): 125(300) + 200(200)
= 37,500 + 40,000 = 77,500
(450, 100): 125(450) + 200(100)
= 56,250 + 20,000 = 76,250
(450, 0): 125(450) + 200(0)
= 56,250 + 0 = 56,250

e. The television manufacturer will make the greatest profit by manufacturing 300 rear-projection televisions each month and 200 plasma televisions each month. The maximum monthly profit is $77,500.

16. a. Let x = number of hours spent tutoring and y = number of hours spent as a teacher's aid. The objective is to maximize $z = 10x + 7y$.

b. The constraints are:
$x + y \le 20$
$x \ge 3$
$x \le 8$

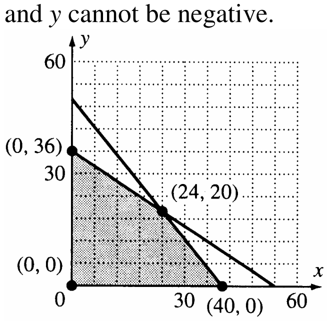
c.



d. (3, 0): 10(3) + 7(0) = 30 + 0 = 30
(3, 17): 10(3) + 7(17) = 30 + 119 = 149
(8, 12): 10(8) + 7(12) = 80 + 84 = 164
(8, 0): 10(8) + 7(0) = 80 + 0 = 80

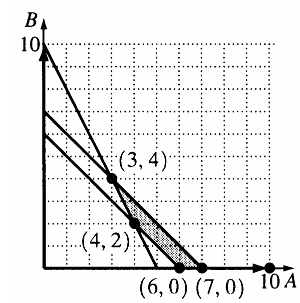
e. The student can earn the maximum amount per week by tutoring for 8 hours a week and working as a teacher's aid for 12 hours a week. The maximum that the student can earn each week is $164.

17. Let x = number of model A bicycles and y = number of model B bicycles.
The constraints are
$5x + 4y \le 200$
$2x + 3y \le 108$
Graph these inequalities in the first quadrant, since x and y cannot be negative.



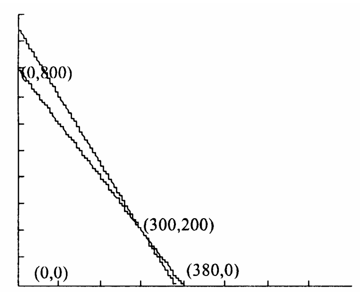
The quantity to be maximized is the profit, which is $25x + 15y$.
(0, 0): 25(0) + 15(0) = 0 + 0 = 0
(0, 36): 25(0) + 15(36) = 0 + 540 = 540
(24, 20): 25(24) + 15(20) = 600 + 300 = 900
(40, 0): 25(40) + 15(0) = 1000 + 0 = 1000
40 model A bicycles and no model B bicycles should be produced.

18. Let x = the number of ounces of food A, y = the number of ounces of food B.
Minimize: $z = 0.12A + 0.08B$
Constraints: $A + B \ge 6$
$A + B \le 7$
$2A + B \ge 10$
$A \ge 0, B \ge 0$



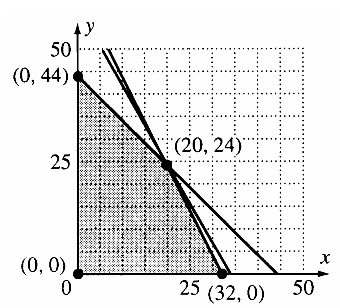
$(6,0): 0.12(6) + 0.08(0) = 0.72$
$(7,0): 0.12(7) + 0.08(0) = 0.84$
$(3,4): 0.12(3) + 0.08(4) = 0.36 + 0.32 = 0.68$
$(4,2): 0.12(4) + 0.08(2) = 0.48 + 0.16 = 0.64$
The institution should include 4 ounces of food A and 2 ounces of food B in a serving to minimize cost.

19. Let x = the number of cartons of food and y = the number of cartons of clothing.
The constraints are:
$20x + 10y \leq 8{,}000$ or $2x + y \leq 8000$
$50x + 20y \leq 19{,}000$ or $5x + 2y \leq 1900$
Graph these inequalities in the first quadrant, since x and y cannot be negative.



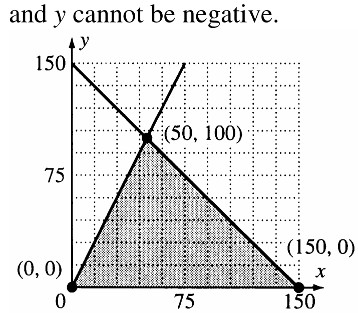
The quantity to be maximized is the number of people helped, which is $12x + 5y$.
(0, 0): $12(0) + 5(0) = 0 + 0 = 0$
(0, 800): $12(0) + 5(800) = 0 + 4000 = 4000$
(300, 200): $12(300) + 5(200) = 4600$
(380, 0): $12(380) + 5(0) = 4500$
300 cartons of food and 200 cartons of clothing should be shipped. This will help 4600 people.

20. Let x = the number of American planes and y = the number of British planes.
The constraints are:
$$x + y \leq 44$$
$$16x + 8y \leq 512$$
$$9000x + 5000y \leq 300{,}000$$
or
$$x + y \leq 44$$
$$2x + y \leq 64$$
$$9x + 5y \leq 300$$
Graph these inequalities in the first quadrant, since x and y cannot be negative.



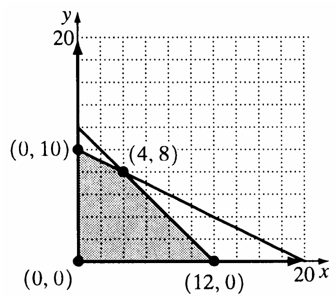
The quantity to be maximized is cargo capacity, which is $30{,}000x + 20{,}000y$.
(0, 0): $30{,}000(0) + 20{,}000(0) = 0 + 0 = 0$
(0, 44): $30{,}000(0) + 20{,}000(44) = 0 + 880{,}000 = 880{,}000$
(20, 24): $30{,}000(20) + 20{,}000(24) = 600{,}000 + 480{,}000 = 1{,}080{,}000$
(32, 0): $30{,}000(32) + 20{,}000(0) = 960{,}000 + 0 = 960{,}000$
To maximize cargo capacity, 20 American planes and 24 British planes should be used.

21. Let x = number of students attending and y = number of parents attending.
The constraints are
$$x + y \leq 150$$
$$2x \geq y$$
or
$$x + y \leq 150$$
$$2x - y \geq 0$$
Graph these inequalities in the first quadrant, since x and y cannot be negative.



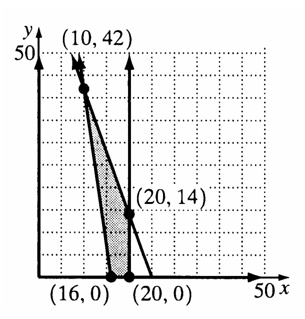
The quantity to be maximized is the amount of money raised, which is $x + 2y$.
(0, 0): $0 + 2(0) = 0 + 0 = 0$
(50, 100): $50 + 2(100) = 50 + 200 = 250$
(150, 0): $150 + 2(0) = 150 + 0 = 150$
50 students and 100 parents should attend.

22. Let x = number of computation problems,
y = number of word problems.
Maximize: $z = 6x + 10y$
Constraints:
$2x + 4y \le 40$
$x + y \le 12$
$x \ge 0, y \ge 0$



$(0,10): 6(0) + 10(10) = 100$
$(12,0): 6(12) + 10(0) = 72$
$(0,0): 6(0) + 10(0) = 0$
$(4,8): 6(4) + 10(8) = 24 + 80 = 104$
You should answer 4 computation problems and 8 word problems to get a maximum score of 104.

23. Let x = number of Boeing 727s, y = number of Falcon 20s.
Maximize $z = x + y$ with the following constraints:
$1400x + 500y \le 35,000$ or $14x + 5y \le 350$
$42,000x + 6000y \ge 672,000$ or $7x + y \ge 112$
$x \le 20$
$x \ge 0, y \ge 0$



$(16,0): z = 16$
$(20,0): z = 20$
$(20,14): z = 34$
$(10,42): z = 52$
Federal Express should have purchased 10 Boeing 727s and 42 Falcon 20s.

24. – 28. Answers may vary.

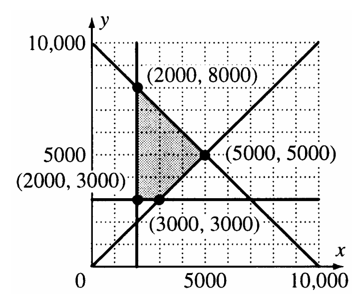
29. does not make sense; Explanations will vary. Sample explanation: Solving a linear programming problem does not require graphing the objective function.

30. makes sense

31. makes sense

32. makes sense

33. Let x = amount invested in stocks and
y = amount invested in bonds.
The constraints are:
$x + y \le 10,000$
$y \ge 3000$
$x \ge 2000$
$y \ge x$
Graph these inequalities in the first quadrant, since x and y cannot be negative.



The quantity to be maximized is the return on the investment, which is $0.12x + 0.08y$.
(2000, 3000):
0.12(2000) + 0.08(3000) = 240 + 240 = 480
(2000, 8000):
0.12(2000) + 0.08(8000) = 240 + 640 = 880
(5000, 5000):
0.12(5000) + 0.08(5000) = 600 + 400 = 1000
(3000, 3000):
0.12(3000) + 0.08(3000) = 360 + 240 = 600
The greatest return occurs when \$5000 is invested in stocks and \$5000 is invested in bonds.

34. The vertices of the region are (0, 0), (0, 3), (3, 1), and (2, 0). If $A = \frac{2}{3}B,$ then the objective function is $z = \frac{2}{3}Bx + By.$

(0, 0): $\frac{2}{3}B(0) + B(0) = 0 + 0 = 0$

(0, 3): $\frac{2}{3}B(0) + B(3) = 0 + 3B = 3B$

(3, 1): $\frac{2}{3}B(3) + B(1) = 2B + B = 3B$

(2, 0): $\frac{2}{3}B(2) + B(0) = \frac{4}{3}B + = \frac{4}{3}B$

Since *A* and *B* are positive, then the objective function has the same maximum value (3*B*) at the vertices are (3, 1) and (0, 3).

35. – 36. Answers may vary.

37. Back-substitute $z = 5$ to find *y* in the second equation.

$$y + 2z = 13$$
$$y + 2(5) = 13$$
$$y + 10 = 13$$
$$y = 3$$

Back-substitute to find *x* in the first equation.

$$x + y + 2z = 19$$
$$x + 3 + 2(5) = 19$$
$$x + 13 = 19$$
$$x = 6$$

The solution set is {(6, 3, 5)}.
Explanations may vary.

38. Back-substitute $z = 3$ to find *y* in the third equation.

$$y - z = 1$$
$$y - 3 = 1$$
$$y = 4$$

Back-substitute to find *x* in the second equation.

$$x - \frac{1}{3}y + z = \frac{8}{3}$$
$$x - \frac{1}{3}(4) + 3 = \frac{8}{3}$$
$$x - \frac{4}{3} + \frac{9}{3} = \frac{8}{3}$$
$$x + \frac{5}{3} = \frac{8}{3}$$
$$x = \frac{3}{3}$$
$$x = 1$$

Back-substitute to find *w* in the first equation.

$$w - x + 2y - 2z = -1$$
$$w - 1 + 2(4) - 2(3) = -1$$
$$w - 1 + 8 - 6 = -1$$
$$w + 1 = -1$$
$$w = -2$$

The solution set is {(–2, 1, 4, 3)}.
Explanations may vary.

39.
$$\begin{bmatrix} 1 & 2 & -1 \\ 1(-4)+4 & 2(-4)+(-3) & -1(-4)+(-15) \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & -11 & -11 \end{bmatrix}$$

Chapter 8 Review Exercises

1.
$$y = 4x + 1$$
$$3x + 2y = 13$$

Substitute $4x + 1$ for *y* in the second equation:

$$3x + 2(4x + 1) = 13$$
$$3x + 8x + 2 = 13$$
$$11x = 11$$
$$x = 1$$
$$y = 4(1) + 1 = 5$$

The solution set is $\{(1,5)\}$.

2.
$$x + 4y = 14$$
$$2x - y = 1$$

Multiply the second equation by 4 and add to the first equation.

$$x + 4y = 14$$
$$\underline{8x - 4y = 4}$$
$$9x = 18$$
$$x = 2$$
$$2(2) - y = 1$$
$$-y = -3$$
$$y = 3$$

The solution set is $\{(2,3)\}$.

3. $5x+3y=1$

$3x+4y=-6$

Multiply the first equation by 4 and the second equation by –3.
Then add.

$$\begin{aligned} 20x+12y&=4\\ -9x-12y&=18\\ \hline 11x&=22\\ x&=2 \end{aligned}$$

$$\begin{aligned} 5(2)+3y&=1\\ 3y&=-9\\ y&=-3 \end{aligned}$$

The solution set is $\{(2,-3)\}$.

4. $2y-6x=7$
$3x-y=9$
The second equation can be written as $y=3x-9$.
Substitute:

$$\begin{aligned} 2(3x-9)-6x&=7\\ 6x-18-6x&=7\\ -18&=7 \end{aligned}$$

Since this is false, the system has no solution.
The solution set is the empty set, $\varnothing$.

5. $4x-8y=16$
$3x-6y=12$
Divide the first equation by 4 and the second equation by 3.
$x-2y=4$
$x-2y=4$
Since these equations are identical, the system has an infinite number of solutions.
The solution set is $\left\{(x, y) \mid 3x-6y=12\right\}$.

6. **a.** $C(x)=60,000+200x$

b. $R(x)=450x$

c. $450x = 60000 + 200x$
$250x = 60000$
$x = 240$
$450(240) = 108,000$
The company must make 240 desks at a cost of $108,000 to break even.

7. Let $x=$ the selling price for Klint's work.
Let $y=$ the selling price for Picasso's work.

$x+y=239$

$x-y=31$

Add the equations to eliminate y and solve for x.

$$\begin{aligned} x+y&=239\\ x-y&=31\\ \hline 2x\quad&=270\\ x&=135 \end{aligned}$$

Back-substitute to find y.

$$\begin{aligned} x+y&=239\\ 135+y&=239\\ y&=104 \end{aligned}$$

Klint's work sold for $135 million and Picasso's work sold for $104 million.

8. **a.** Answers will vary. Approximate point is $(2004,180)$. This means that in 2004 the number of cellphones and land-lines were both 180 million.

b. $y=19.8x+98$

c. Using substitution,

$$\begin{aligned} 4.3x+y&=198\\ 4.3x+\overbrace{(19.8x+98)}^{y}&=198\\ 4.3x+19.8x+98&=198\\ 24.1x+98&=198\\ 24.1x&=100\\ x&\approx 4 \end{aligned}$$

The number of cellphone and land-line customers will be they same 4 years after 2000, or 2004.

$$\begin{aligned} y&=19.8x+98\\ y&=19.8(4)+98\\ y&=177.2\\ y&\approx 180 \end{aligned}$$

The number of customers that each will have in 2004 is about 180 million.

d. The models describe the point of intersection quite well.

9. Let l = the length of the table.
Let w = the width of the table.
$2l + 2w = 34$
$4l - 3w = 33$
Multiply the first equation by –2 and solve by addition.
$$\begin{aligned} -4l - 4w &= -68 \\ 4l - 3w &= 33 \\ \hline -7w &= -35 \\ w &= 5 \end{aligned}$$
Back-substitute 5 for w to find l.
$$\begin{aligned} 2l + 2w &= 34 \\ 2l + 2(5) &= 34 \\ 2l + 10 &= 34 \\ 2l &= 24 \\ l &= 12 \end{aligned}$$
The dimensions of the table are 12 feet by 5 feet.

10. Let x = the cost of the hotel
y = the cost of the car
$3x + 2y = 360$
$4x + 3y = 500$
Solve the system.
$$\begin{aligned} 12x + 8y &= 1440 \\ -12x - 9y &= -1500 \\ -y &= -60 \\ y &= 60 \end{aligned}$$
$$\begin{aligned} 3x + 2(60) &= 360 \\ 3x &= 240 \\ x &= 80 \end{aligned}$$
The room costs \$80 a day and the car rents for \$60 a day.

11. x = number of apples
y = number of avocados
$$\begin{aligned} 100x + 350y &= 1000 \\ 24x + 14y &= 100 \\ 100x + 350y &= 1000 \\ -600x - 350y &= -2500 \\ \hline -500x &= -1500 \\ x &= 3 \\ 100(3) + 350y &= 1000 \\ 350y &= 700 \\ y &= 2 \end{aligned}$$
3 apples and 2 avocados supply 1000 calories and 100 grams of carbohydrates.

12. $2x - y + z = 1 \, (1)$
$3x - 3y + 4z = 5 \, (2)$
$4x - 2y + 3z = 4 \, (3)$
Eliminate y from (1) and (2) by multiplying (1) by –3 and adding the result to (2).
$$\begin{aligned} -6x + 3y - 3z &= -3 \\ 3x - 3y + 4z &= 5 \\ \hline -3x + z &= 2 \quad (4) \end{aligned}$$
Eliminate y from (1) and (3) by multiplying (1) by –2 and adding the result to (3).
$$\begin{aligned} -4x + 2y - 2z &= -2 \\ 4x - 2y + 3z &= 4 \\ \hline z &= 2 \end{aligned}$$
Substituting $z = 2$ into (4), we get:
$-3x + 2 = 2$
$-3x = 0$
$x = 0$
Substituting $x = 0$ and $z = 2$ into (1), we have:
$2(0) - y + 2 = 1$
$-y = -1$
$y = 1$
The solution set is $\{(0, 1, 2)\}$.

13. $x + 2y - z = 5 \quad (1)$
$2x - y + 3z = 0 \quad (2)$
$2y + z = 1 \quad (3)$
Eliminate x from (1) and (2) by multiplying (1) by –2 and adding the result to (2).
$$\begin{aligned} -2x - 4y + 2z &= -10 \\ 2x - y + 3z &= 0 \\ \hline -5y + 5z &= -10 \\ y - z &= 2 \quad (4) \end{aligned}$$
Adding (3) and (4), we get:
$$\begin{aligned} 2y + z &= 1 \\ y - z &= 2 \\ \hline 3y &= 3 \\ y &= 1 \end{aligned}$$
Substituting $y = 1$ into (3), we have:
$2(1) + z = 1$
$z = -1$
Substituting $y = 1$ and $z = -1$ into (1), we obtain:
$x + 2(1) - (-1) = 5$
$x + 3 = 5$
$x = 2$
The solution set is $\{(2, 1, -1)\}$.

14. $y = ax^2 + bx + c$

$(1,4): 4 = a + b + c \quad (1)$

$(3,20): 20 = 9a + 3b + c \quad (2)$

$(-2,25): 25 = 4a - 2b + c \quad (3)$

Multiply (1) by –1 and add to (2).

$$\begin{aligned} 20 &= 9a + 3b + c \\ -4 &= -a - b - c \\ \hline 16 &= 8a + 2b \\ 8 &= 4a + b \\ 8 &= 4a + b \quad (4) \end{aligned}$$

Multiply (1) by –1 and add to (3).

$$\begin{aligned} 25 &= 4a - 2b + c \\ -4 &= -a - b - c \\ \hline 21 &= 3a - 3b \\ 7 &= a - b \quad (5) \end{aligned}$$

Add (4) and (5).

$$\begin{aligned} 8 &= 4a + b \\ 7 &= a - b \\ \hline 15 &= 5a \\ a &= 3 \end{aligned}$$

$$8 = 4(3) + b$$
$$b = -4$$
$$3 - 4 + c = 4$$
$$c = 5$$

Hence, the quadratic function is $y = 3x^2 - 4x + 5$.

15. Let x = average debt for the 18 – 29 age group in the U.S.
Let y = average debt for the 30 – 39 age group in the U.S.
Let z = average debt for the 40 – 49 age group in the U.S.

$$x + y + z = 44,200$$
$$y - x = 8100$$
$$z - y = 3100$$

Solve the second equation for x.

$$\begin{aligned} y - x &= 8100 \\ -x &= -y + 8100 \\ x &= y - 8100 \end{aligned}$$

Solve the third equation for z.

$$\begin{aligned} z - y &= 3100 \\ z &= y + 3100 \end{aligned}$$

Substitute the expressions for x and z into the first equation and solve for y.

$$\begin{aligned} x + y + z &= 44,200 \\ \overbrace{(y - 8100)}^{x} + y + \overbrace{(y + 3100)}^{z} &= 44,200 \\ y - 8100 + y + y + 3100 &= 44,200 \\ 3y - 5000 &= 44,200 \\ 3y &= 49,200 \\ y &= 16,400 \end{aligned}$$

Back-substitute to solve for x and z.

$$\begin{aligned} x &= y - 8100 \\ &= 16,400 - 8100 \\ &= 8300 \end{aligned}$$

$$\begin{aligned} z &= y + 3100 \\ &= 16,400 + 3100 \\ &= 19,500 \end{aligned}$$

The average debt for the 18 – 29 age group in the U.S. is \$8300, for the 30 – 39 age group is \$16,400, and for the 40 – 49 age group is \$19,500.

16. $\dfrac{x}{(x-3)(x+2)} = \dfrac{A}{x-3} + \dfrac{B}{x+2}$

$$\begin{aligned} x &= A(x+2) + B(x-3) \\ &= (A+B)x + (2A - 3B) \end{aligned}$$

$$A + B = 1$$
$$2A - 3B = 0$$

Multiply first equation by 3, then add to second equation.

$$\begin{aligned} 3A + 3B &= 3 \\ 2A - 3B &= 0 \\ \hline 5A &= 3 \end{aligned}$$

$$A = \frac{3}{5},\ B = \frac{2}{5}$$

$$\frac{x}{(x-3)(x+2)} = \frac{3}{5(x-3)} + \frac{2}{5(x+2)}$$

17. $\dfrac{11x-2}{x^2-x-12}=\dfrac{11x-2}{(x-4)(x+3)}=\dfrac{A}{x-4}+\dfrac{B}{x+3}$

$$\begin{aligned}11x-2&=A(x+3)+B(x-4)\\&=Ax+3A+Bx-4B\\&=(A+B)x+(3A-4B)\end{aligned}$$

$A+B=11$
$3A-4B=-2$

Multiply first equation by 4, then add to second equation.

$$\begin{aligned}3A-4B&=-2\\4A+4B&=44\\\hline 7A&=42\end{aligned}$$

$A=6,\ B=5$

$\dfrac{11x-2}{x^2-x-12}=\dfrac{6}{x-4}+\dfrac{5}{x+3}$

18. $\dfrac{4x^2-3x-4}{x^3+x^2-2x}=\dfrac{4x^2-3x-4}{x(x+2)(x-1)}$

$=\dfrac{A}{x}+\dfrac{B}{x+2}+\dfrac{C}{x-1}$

$$\begin{aligned}4x^2-3x-4&=A(x+2)(x-1)+Bx(x-1)+Cx(x+2)\\&=A(x^2+x-2)+Bx^2-Bx+Cx^2+2Cx\\&=Ax^2+Ax-2A+Bx^2-Bx+Cx^2+2Cx\\&=(A+B+C)x^2+(A-B+2C)x-2A\end{aligned}$$

$A+B+C=4$
$A-B+2C=-3$
$-2A=-4$
$A=2$
$B+C=2$
$\underline{-B+2C=-5}$
$3C=-3$
$C=-1$
$B-1=2$
$B=3$

$\dfrac{4x^2-3x-4}{x^3+x^2-2x}=\dfrac{2}{x}+\dfrac{3}{x+2}-\dfrac{1}{x-1}$

19. $\dfrac{2x+1}{(x-2)^2}=\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}$

$2x+1=A(x-2)+B=Ax-2A+B$

$A=2$
$-2A+B=1$
$-2(2)+B=1$
$B=5$

$\dfrac{2x+1}{(x-2)^2}=\dfrac{2}{x-2}+\dfrac{5}{(x-2)^2}$

20. $\dfrac{2x-6}{(x-1)(x-2)^2}=\dfrac{A}{x-1}+\dfrac{B}{x-2}+\dfrac{C}{(x-2)^2}$

$$\begin{aligned}2x-6&=A(x-2)^2+B(x-1)(x-2)+C(x-1)\\&=A(x^2-4x+4)+B(x^2-3x+2)+C(x-1)\\&=Ax^2-4Ax+4A+Bx^2-3Bx+2B+Cx-C\\&=(A+B)x^2+(-4A-3B+C)x+(4A+2B-C)\end{aligned}$$

$A+B=0$
$-4A-3B+C=2$
$\underline{4A+2B-C=-6}$
$-B=-4$
$B=4$
$A=-4$
$4(-4)+2(4)-C=-6$
$-16+8-C=-6$
$-C-8=-6$
$-C=2$
$C=-2$

$\dfrac{2x-6}{(x-1)(x-2)^2}=-\dfrac{4}{x-1}+\dfrac{4}{x-2}-\dfrac{2}{(x-2)^2}$

21. $\dfrac{3x}{(x-2)(x^2+1)} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+1}$

$3x = A(x^2+1) + (Bx+C)(x-2)$

$= Ax^2 + A + Bx^2 - 2Bx + Cx - 2C$

$= (A+B)x^2 + (-2B+C)x - (2C-A)$

$A+B=0$

$-2B+C=3$

$2C-A=0$

$A=2C$

$B+2C=0$

$\underline{4B-2C=-6}$

$5B=-6$

$B=-\dfrac{6}{5}$

$A=\dfrac{6}{5}$

$C=\dfrac{6}{10}=\dfrac{3}{5}$

$\dfrac{3x}{(x-2)(x^2+1)} = \dfrac{6}{5(x-2)} + \dfrac{-6x+3}{5(x^2+1)}$

22. $\dfrac{7x^2-7x+23}{(x-3)(x^2+4)} = \dfrac{A}{x-3} + \dfrac{Bx+C}{x^2+4}$

$7x^2-7x+23 = A(x^2+4) + (Bx+C)(x-3)$

$= Ax^2 + 4A + Bx^2 - 3Bx + Cx - 3C$

$= (A+B)x^2 + (-3B+C)x + (4A-3C)$

$A+B=7$

$-3B+C=-7$

$4A-3C=23$

$3A+3B=21$

$\underline{-3B+C=-7}$

$3A+C=14$

$9A+3C=42$

$\underline{4A-3C=23}$

$13A=65$

$A=5$

$5+B=7$

$B=7-5=2$

$-3(2)+C=-7$

$C=-7+6=-1$

$\dfrac{7x^2-7x+23}{(x-3)(x^2+4)} = \dfrac{5}{x-3} + \dfrac{2x-1}{x^2+4}$

23. $\dfrac{x^3}{(x^2+4)^2} = \dfrac{Ax+B}{x^2+4} + \dfrac{Cx+D}{(x^2+4)^2}$

$x^3 = (Ax+B)(x^2+4) + Cx + D$

$= Ax^3 + 4Ax + Bx^2 + 4B + Cx + D$

$= Ax^3 + Bx^2 + (4A+C)x + (4B+D)$

$A=1$

$B=0$

$4A+C=0$

$4B+D=0$

$C=-4$

$0+D=0, D=0$

$\dfrac{x^2}{(x^2+4)^2} = \dfrac{x}{x^2+4} - \dfrac{4x}{(x^2+4)^2}$

24. $\dfrac{4x^3+5x^2+7x-1}{(x^2+x+1)^2} = \dfrac{Ax+B}{x^2+x+1} + \dfrac{Cx+D}{(x^2+x+1)^2}$

$4x^3+5x^2+7x-1$

$= (Ax+B)(x^2+x+1) + Cx + D$

$= Ax^3 + Ax^2 + Ax + Bx^2 + Bx + B + Cx + D$

$= Ax^3 + (A+B)x^2(A+B+C)x + (B+D)$

$A=4$

$A+B=5$

$A+B+C=7$

$B+D=-1$

$4+B=5, B=1$

$4+1+C=7, C=2$

$1+D=-1, D=-2$

$\dfrac{4x^3+5x^2+7x-1}{(x^2+x+1)^2} = \dfrac{4x+1}{x^2+x+1} + \dfrac{2x-2}{(x^2+x+1)^2}$

25. $5y = x^2-1$

$x-y=1$

$y=x-1$

$5(x-1)=x^2-1$

$5x-5=x^2-1$

$x^2-5x+4=0$

$(x-4)(x-1)=0$

$x=4,1$

If $x=4$, $y=4-1=3$.

If $x=1$, $y=1-1=0$.

The solution set is $\{(4,3),(1,0)\}$.

26. $y = x^2 + 2x + 1$
$x + y = 1$
$y = 1 - x$
$1 - x = x^2 + 2x + 1$
$x^2 + 3x = 0$
$x(x+3) = 0$
$x = 0, -3$
If $x = 0, y = 1 - 0 = 1.$
If $x = -3, y = 1 - (-3) = 4.$
The solution set is $\{(0,1), (-3,4)\}$.

27. $x^2 + y^2 = 2$
$x + y = 0$
$x = -y$
$(-y)^2 + y^2 = 2$
$2y^2 = 2$
$y^2 = 1$
$y = 1, -1$
If $y = 1, x = -1.$
If $y = -1, x = 1.$
The solution set is $\{(1,-1), (-1,1)\}$.

28. $2x^2 + y^2 = 24$
$x^2 + y^2 = 15$

$$\begin{array}{r} 2x^2 + y^2 = 24 \\ \underline{-x^2 - y^2 = -15} \\ x^2 = 9 \end{array}$$

$x = 3, -3$
If $x = 3, 3^2 + y^2 = 15, y^2 = 6$ and $y = \pm\sqrt{6}.$
If $x = -3, y = \pm\sqrt{6}$.
The solution set is
$\left\{\left(3,\sqrt{6}\right),\left(3,-\sqrt{6}\right),\left(-3,\sqrt{6}\right),\left(-3,-\sqrt{6}\right)\right\}$.

29. $xy - 4 = 0$
$y - x = 0$
$y = x$
$xy = 4$
$x^2 = 4$
$x = 2, -2$
If $x = 2, y = 2.$
If $x = -2, y = -2.$
The solution set is $\{(2,2), (-2,-2)\}$.

30. $y^2 = 4x$
$x - 2y + 3 = 0$
$x = \frac{y^2}{4}$
$\frac{y^2}{4} - 2y + 3 = 0$
$y^2 - 8y + 12 = 0$
$(y-6)(y-2) = 0$
$y = 6, 2$
If $y = 6, x = \frac{36}{4} = 9.$
If $y = 2, x = \frac{4}{4} = 1.$
The solution set is $\{(9,6), (1,2)\}$.

31. $x^2 + y^2 = 10$
$y = x + 2$
$x^2 + (x+2)^2 = 10$
$x^2 + x^2 + 4x + 4 - 10 = 0$
$2x^2 + 4x - 6 = 0$
$x^2 + 2x - 3 = 0$
$(x+3)(x-1) = 0$
$x = -3, 1$
If $x = -3, y = -3 + 2 = -1.$
If $x = 1, y = 1 + 2 = 3.$
The solution set is $\{(-3,-1), (1,3)\}$.

32. $xy = 1$
$y = 2x + 1$
$x(2x+1) = 1$
$2x^2 + x - 1 = 0$
$(2x-1)(x+1) = 0$
$x = \frac{1}{2}, -1$
If $x = \frac{1}{2}, y = 2\left(\frac{1}{2}\right) + 1 = 2.$
If $x = -1, y = 2(-1) + 1 = -1.$
The solution set is $\left\{\left(\frac{1}{2}, 2\right), (-1,-1)\right\}$.

33. $$x + y + 1 = 0$$
$$x^2 + y^2 + 6y - x = -5$$
$$x = -y - 1$$
$$(-y-1)^2 + y^2 + 6y - (-y-1) + 5 = 0$$
$$y^2 + 2y + 1 + y^2 + 6y + y + 1 + 5 = 0$$
$$2y^2 + 9y + 7 = 0$$
$$(2y+7)(y+1) = 0$$
$$y = -\frac{7}{2}, -1$$

If $y = -\frac{7}{2}, x = \frac{7}{2} - 1 = \frac{5}{2}$.

If $y = -1, x = 1 - 1 = 0$.

The solution set is $\left\{\left(\frac{5}{2}, -\frac{7}{2}\right), (0, -1)\right\}$.

34. $$x^2 + y^2 = 13$$
$$x^2 - y = 7$$
$$x^2 + y^2 = 13$$
$$\underline{-x^2 + y = -7}$$
$$y^2 + y = 6$$
$$y^2 + y - 6 = 0$$
$$(y+3)(y-2) = 0$$
$$y = -3, 2$$

If $y = -3, x^2 + 3 = 7$

$x^2 = 4, x = 2, -2$

If $y = 2, x^2 - 2 = 7, x^2 = 9, x = 3, -3$.

The solution set is $\{(2,-3), (-2,-3), (3,2), (-3,2)\}$.

35. $$2x^2 + 3y^2 = 21$$
$$3x^2 - 4y^2 = 23$$
$$8x^2 + 12y^2 = 84$$
$$\underline{9x^2 - 12y^2 = 69}$$
$$17x^2 = 153$$
$$x^2 = \frac{153}{17} = 9$$
$$x = 3, -3$$

If $x = 3, 2(3)^2 + 3y^2 = 21$.

$3y^2 = 21 - 18 = 3$

$y^2 = 1, y = 1, -1$

If $x = -3, y = 1, -1$.

The solution set is $\{(3,1), (3,-1), (-3,1), (-3,-1)\}$.

36. $$2L + 2W = 26$$
$$LW = 40$$
$$L = \frac{40}{W}$$
$$2\left(\frac{40}{W}\right) + 2W = 26$$
$$\frac{80}{W} + 2W = 26$$
$$80 + 2W^2 = 26W$$
$$2W^2 - 26W + 80 = 0$$
$$W^2 - 13W + 40 = 0$$
$$(W-8)(W-5) = 0$$
$$W = 8, 5$$

If $W = 5, L = \frac{40}{5} = 8$

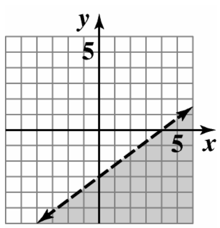
The dimensions are 8 m by 5 m.

37. $$xy = 6$$
$$y = \frac{6}{x}$$
$$2x + y = 8$$
$$2x + \frac{6}{x} = 8$$
$$2x^2 + 6 = 8x$$
$$2x^2 - 8x + 6 = 0$$
$$x^2 - 4x + 3 = 0$$
$$(x-1)(x-3) = 0$$
$$x = 1, 3$$

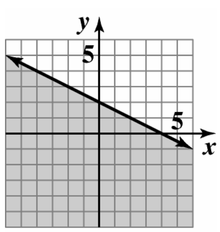
If $x = 1$, $y = 6$.
If $x = 3$, $y = 2$.
The solution set is $\{(1, 6), (3, 2)\}$.

38.
$$x^2 + y^2 = 2900$$
$$4x + 2y = 240$$
$$2x + y = 120$$
$$y = 120 - 2x$$
$$x^2 + (120 - 2x)^2 = 2900$$
$$x^2 + 14,400 - 480x + 4x^2 - 2900 = 0$$
$$5x^2 - 480x + 11,500 = 0$$
$$x^2 - 96x + 2300 = 0$$
$$(x - 46)(x - 50) = 0$$
$$x = 46, 50$$

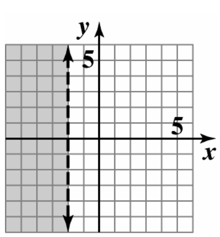
If $x = 46$, $y = 120 - 2(46) = 28$.
If $x = 50$, $y = 120 - 2(50) = 20$.
$x = 46$ ft and $y = 28$ ft or
$x = 50$ ft and $y = 20$ ft

39.


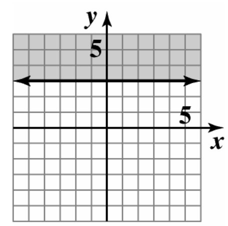
$3x - 4y > 12$

40.


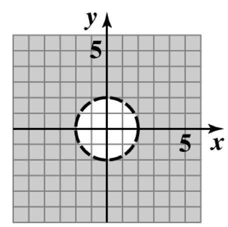
$y \le -\frac{1}{2}x + 2$

41.


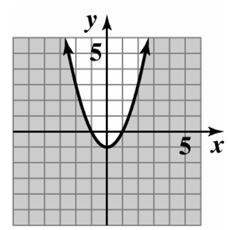
$x < -2$

42.


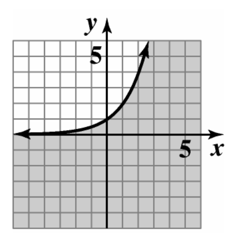
$y \ge 3$

43.


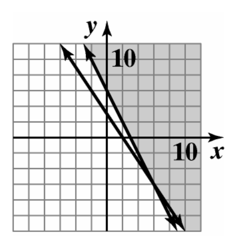
$x^2 + y^2 > 4$

44.


$y \le x^2 - 1$

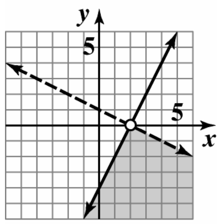
45.


$y \le 2^x$

46.


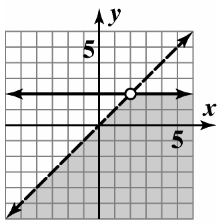
$3x + 2y \ge 6$
$2x + y \ge 6$

47.



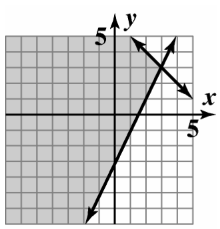
$2x - y \geq 4$
$x + 2y < 2$

48.



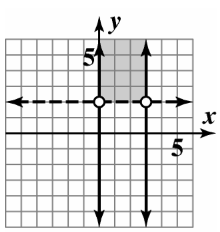
$y < x$
$y \leq 2$

49.



$x + y \leq 6$
$y \geq 2x - 3$

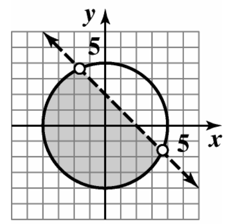
50.



$0 \leq x \leq 3$
$y > 2$

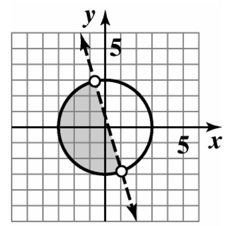
51. No solution

52.



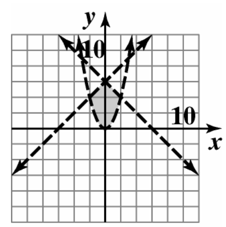
$x^2 + y^2 \leq 16$
$x + y < 2$

53.



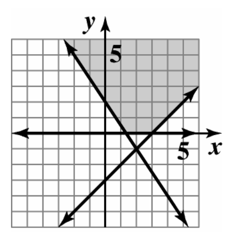
$x^2 + y^2 \leq 9$
$y < -3x + 1$

54.



$y > x^2$
$x + y < 6$
$y < x + 6$

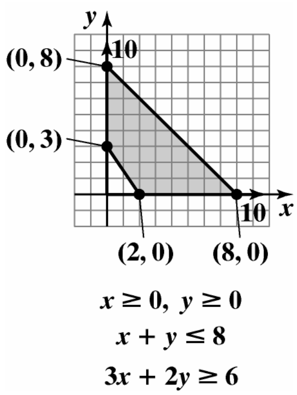
55.



$y \geq 0$
$3x + 2y \geq 4$
$x - y \leq 3$

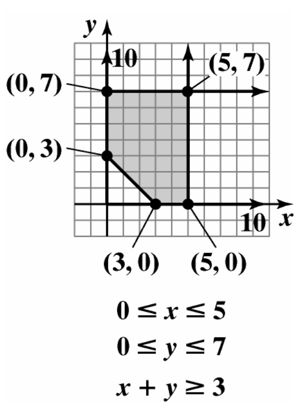
56. $z = 2x + 3y$
$(2, 2): z = 2(2) + 3(2) = 10$
$(4, 0): z = 2(4) + 3(0) = 8$
$\left(\frac{1}{2}, \frac{1}{2}\right): z = 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right) = \frac{5}{2}$
$(1, 0): z = 2(1) + 3(0) = 2$
The maximum value is 10 and the minimum value is 2.

57. $z = 2x + 3y$



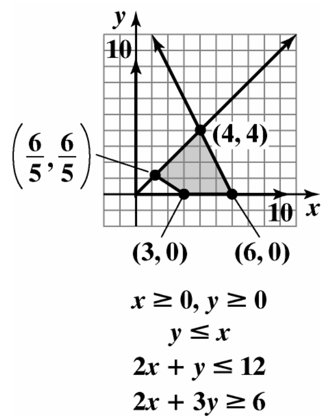
$(0,8): z = 2(0) + 3(8) = 24$
$(8,0): z = 2(8) + 3(0) = 16$
$(0,3): z = 2(0) + 3(3) = 9$
$(2,0): z = 2(2) + 3(0) = 6$
Maximum value is 24 at (0, 8).

58. $z = x + 4y$



$(0,3): z = 0 + 4(3) = 12$
$(3,0): z = 3 + 4(0) = 3$
$(0,7): z = 0 + 4(7) = 28$
$(5,0): z = 5 + 4(0) = 5$
$(5,7): z = 5 + 4(7) = 33$
Maximum value is 33 at (5,7).

59. $z = 5x + 6y$

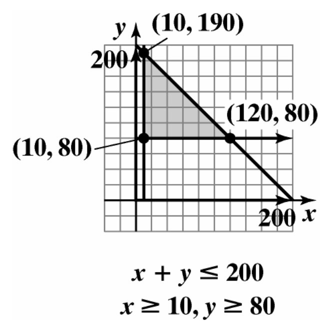


$(3,0): z = 5(3) + 6(0) = 15$
$(6,0): z = 5(6) + 6(0) = 30$
$\left(\frac{6}{5}, \frac{6}{5}\right): z = 5\left(\frac{6}{5}\right) + 6\left(\frac{6}{5}\right) = \frac{66}{5} = 13.2$
$(4,4): 5(4) + 6(4) = 44$
The maximum value is 44.

60. **a.** $z = 500x + 350y$

b. $x + y \le 200$
$x \ge 10$
$y \ge 80$

c.

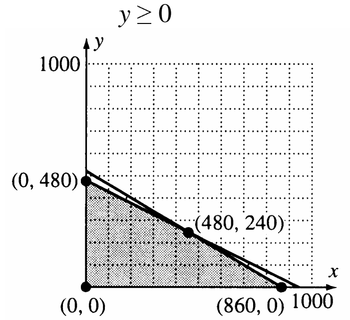


d.

Vertex	Objective Function $z = 500x + 350y$
(10, 80)	$z = 500(10) + 350(80)$ $= 33,000$
(10, 190)	$z = 500(10) + 350(190)$ $= 71,500$
(120, 80)	$z = 500(120) + 350(80)$ $= 88,000$

e. The company will make the greatest profit by producing 120 units of writing paper and 80 units of newsprint each day. The maximum daily profit is $88,000.

61. Let x = number of model A tents produced and y = number of model B tents produced.
The constraints are:
$0.9x + 1.8y \le 864$
$0.8x + 1.2y \le 672$
$x \ge 0$
$y \ge 0$



The vertices of the region are (0, 0), (0, 480), (480, 240), and (840, 0).
The objective is to maximize $25x + 40y$.
(0, 0): 25(0) + 40(0) = 0 + 0 = 0
(0, 480): 25(0) + 40(480) = 0 + 19,200 = 19,200
(480,240): 25(480) + 40(240) = 12,000 + 9600
= 21,600
(840, 0): 25(840) + 40(0) = 21,000 + 0 = 21,000
The manufacturer should make 480 of model A and 240 of model B.

Chapter 8 Test

1. $x = y + 4$
$3x + 7y = -18$
Substitute $y + 4$ for x into second equation.
$3(y+4) + 7y = -18$
$3y + 12 + 7y = -18$
$10y = -30$
$y = -3$
$x = -3 + 4 = 1$
The solution set to the system is $\{(1, -3)\}$.

2. $2x + 5y = -2$
$3x - 4y = 20$
Multiply the first equation by 3 and the second equation by –2 and add the result.
$6 + 15y = -6$
$-6x + 8y = -40$
$23y = -46$
$y = -2$
Substitute $y = -2$ into the first equation:
$2x + 5(-2) = -2$
$2x - 10 = -2$
$2x = 8$
$x = 4$
The solution to the system is $\{(4, -2)\}$.

3. $x + y + z = 6(1)$
$3x + 4y - 7z = 1(2)$
$2x - y + 3z = 5(3)$
Eliminate x by multiplying (1) by –3 and adding the result to (2) and by multiplying (1) by –2 and adding the result to (3).
$-3x - 3y - 3z = -18$
$3x + 4y - 7z = 1$
$y - 10z = -17\ (4)$
$-2x - 2y - 2z = -12$
$2x - y + 3z = 5$
$-3y + z = -7\ \ (5)$
Multiply (4) by 3 and add the result to (5) to eliminate y.
$3y - 30z = -51$
$-3y + z = -7$
$-29z = -58$
$z = 2$
Substitute $z = 2$ into (5).
$-3y + 2 = -7$
$-3y = -9$
$y = 3$
Substitute $z = 2$ and $y = 3$ into (1).
$x + 3 + 2 = 6$
$x = 1$
The solution to the system is $\{(1, 3, 2)\}$.

4. $x^2 + y^2 = 25$

$x + y = 1$

$y = 1 - x$

Substitute $1 - x$ for y in the first equation.

$$x^2 + (1-x)^2 = 25$$
$$x^2 + 1 - 2x + x^2 = 25$$
$$2x^2 - 2x - 24 = 0$$
$$x^2 - x - 12 = 0$$
$$(x-4)(x+3) = 0$$
$$x = 4,\ -3$$

If $x = 4$, $y = 1 - 4 = -3$.

If $x = -3$, $y = 1 - (-3) = 4$.

The solution set is $\{(4, -3), (-3, 4)\}$.

5. $2x^2 - 5y^2 = -2$

$3x^2 + 2y^2 = 35$

Multiply first equation by 2 and the second equation by 5. Then add.

$$4x^2 - 10y^2 = -4$$
$$\underline{15x^2 + 10y^2 = 175}$$
$$19x^2 = 171$$
$$x^2 = 9$$
$$x = 3, -3$$

If $x = 3, 2(3)^2 - 5y^2 = -2$.

$$18 - 5y^2 = -2$$
$$-5y^2 = -20$$
$$y^2 = 4$$
$$y = 2, -2$$

If $x = -3$, $y = -2$.

The solution to the system is $\{(3,\ 2), (3,\ -2), (-3,\ 2), (-3,\ -2)\}$.

6. $\dfrac{x}{(x+1)(x^2+9)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+9}$

$$x = A(x^2+9) + (Bx+C)(x+1)$$
$$= Ax^2 + 9A + Bx^2 + Bx + Cx + C$$
$$= (A+B)x^2 + (B+C)x + (9A+C)$$

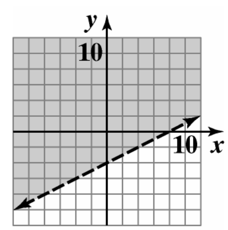
$A + B = 0 \rightarrow A = -B$

$$B + C = 1$$
$$9A + C = 0$$
$$-9B + C = 0$$
$$9B - C = 0$$
$$\underline{B + C = 1}$$
$$10B = 1$$
$$B = \frac{1}{10}$$
$$A = -\frac{1}{10}$$

$\dfrac{1}{10} + C = 1,\ C = \dfrac{9}{10}$

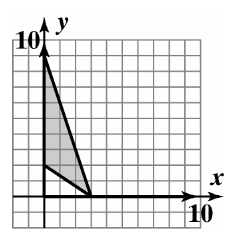
$$\frac{x}{(x+1)(x^2+9)} = \frac{-1}{10(x+1)} + \frac{x+9}{10(x^2+9)}$$

7.

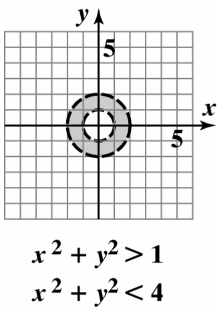


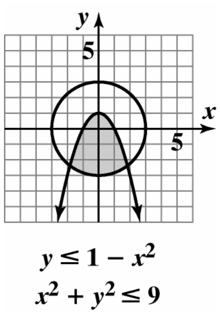
x − 2y < 8

8.



x ≥ 0, y ≥ 0
3x + y ≤ 9
2x + 3y ≥ 6

9. 

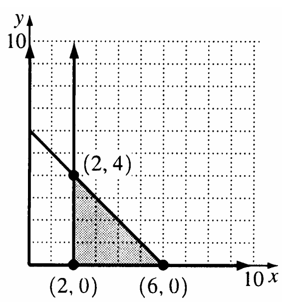
10. 

11. $z = 3x + 5y$

$x \geq 0, y \geq 0$

$x + y \leq 6$

$x \geq 2$



$(2,0): z = 3(2) + 5(0) = 6$

$(6,0): z = 3(6) + 5(0) = 18$

$(2,4): z = 3(2) + 5(4) = 26$

Maximum value is 26.

12. x = mg of cholesterol in one ounce of shrimp
y = mg of cholesterol in one ounce of scallops
$3x + 2y = 156$
$5x + 3y = 255$
Multiply the first equation by –3 and multiply the second equation by 2.
Add the resulting equations together.

$$\begin{array}{r} -9x - 6y = -468 \\ \underline{10x + 6y = \ 510} \\ x = \ 42 \end{array}$$

$3(42) + 2y = 156$
$126 + 2y = 156$
$2y = 30$
$y = 15$

$3(42) + 2y = 156$

$126 + 2y = 156$

$2y = 30$

$y = 15$

Shrimp: 42 mg of cholesterol per ounce
Scallops: 15 mg of cholesterol per ounce

13. **a.** $C(x) = 360,000 + 850x$

b. $R(x) = 1150x$

c. $1150x = 360000 + 850x$
$300x = 360000$
$x = 1200$
$1150(1200) = 1,380,000$
1200 computers need to be sold to make $1,380,00 for the company to break even.

14. $y = ax^2 + bx + c$

$(-1,-2): -2 = a - b + c$

$(2,1): 1 = 4a + 2b + c$

$(-2,1): 1 = 4a - 2b + c$

$$\begin{array}{r} 4a + 2b + c = 1 \\ \underline{-4a + 2b - c = -1} \\ 4b = 0 \\ b = 0 \end{array}$$

$a + c = -2$

$$\begin{array}{r} 4a + c = 1 \\ \underline{-a - c = 2} \\ 3a = 3 \\ a = 1 \end{array}$$

$a + c = -2$

$c = -3$

The quadratic function is $y = x^2 - 3$.

15. $2x + y = 39$

$xy = 180$

$y = 39 - 2x$

$x(39 - 2x) = 180$

$39x - 2x^2 = 180$

$2x^2 - 39x + 180 = 0$

$(2x - 15)(x - 12) = 0$

$x = \frac{15}{2}, 12$

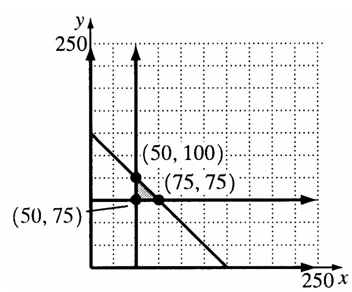
If $x = \frac{15}{2}, \frac{15}{2}y = 180$ and $y = 24.$

If $x = 12, 12y = 180$ and $y = 15.$

The dimensions are 7.5 ft by 24 ft
or 12 ft by 15 ft

16. Let x = regular, y = deluxe.
objective function: $z = 200x + 250y$
constraints: $x \geq 50, y \geq 75$

$x + y \leq 150$



$(50, 75): z = 200(50) + 250(75) = 28,750$

$(50, 100): z = 200(50) + 250(100) = 35,000$

$(75, 75): z = 200(75) + 250(75) = 33,750$

For a maximum profit of $35,000 a week, the company should manufacture 50 regular and 100 deluxe jet skis.

Cumulative Review Exercises (Chapters 1–8)

1. Domain: $(-\infty, \infty)$ Range: $[-\infty, 3)$

2. –1 and 1 are the zeros.

3. The relative maximum is $y = 3$ and it occurs at $x = 0.$

4. $f(x)$ is decreasing on the interval $(0, 2).$

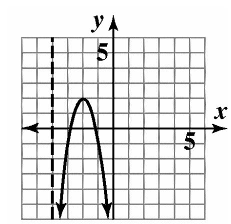
5. At $x = -0.7,$ the curve is above the x-axis and thus $f(x)$ is positive.

6. $(f \circ f)(-1) = f(f(-1)) = f(0) = 3$

7. $f(x) \to -\infty$ as $x \to -2^+$ or as $x \to 2^-.$

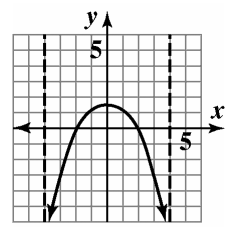
8. $f(-x) = f(x)$ thus the function is even.

9. The graph of $g(x) = f(x+2) - 1$ can be obtained by shifting $f(x)$ 2 units left and 1 unit down.



$g(x) = f(x + 2) - 1$

10. The graph of $h(x) = \frac{1}{2} f\left(\frac{1}{2}x\right)$ can be obtained by shrinking the graph of $f(x)$ horizontally by a factor of $\frac{1}{2}$ and vertically by a factor of $\frac{1}{2}$.



$h(x) = \frac{1}{2} f(\frac{1}{2} x)$

11. $\sqrt{x^2 - 3x} = 2x - 6$

$x^2 - 3x = 4x^2 - 24x + 36$

$3x^2 - 21x + 36 = 0$

$x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

$x = 3, 4$

The solution set is $\{3, 4\}$.

12. $4x^2 = 8x - 7$

$4x^2 - 8x + 7 = 0$

$$x = \frac{8 \pm \sqrt{64-112}}{8} = \frac{8 \pm \sqrt{-48}}{8}$$

$$= \frac{8 \pm 4\sqrt{3}i}{8} = \frac{2 \pm \sqrt{3}i}{2}$$

The solution set is $\left\{\frac{2+i\sqrt{3}}{2}, \frac{2-i\sqrt{3}}{2}\right\}$.

13. $\left|\frac{x}{3}+2\right| < 4$

$-4 < \frac{x}{3} + 2 < 4$

$-6 < \frac{x}{3} < 2$

$-18 < x < 6$

The solution is $\{x \mid -18 < x < 6\}$ or $(-18, 6)$.

14. $\frac{x+5}{x-1} > 2$

$\frac{x+5}{x-1} - 2 > 0$

$\frac{x+5-2(x-1)}{x-1} > 0$

$\frac{x+5-2x+2}{x-1} > 0$

$\frac{-x+7}{x-1} > 0$

$\frac{-x+7}{x-1} = 0$ when $x = 7$ and is undefined when $x = 1$.

Test $x = 0$:

$\frac{0+5}{0-1} > 2?$

$\frac{5}{-1} > 2?$

$-5 \not> 2$

Test $x = 2$:

$\frac{2+5}{2-1} > 2?$

$\frac{7}{1} > 2?$

$7 > 2$

Test $x = 8$:

$\frac{8+5}{8-1} > 2?$

$\frac{13}{7} > 2?$

$\frac{13}{7} \geq \frac{14}{7}$

The solution is $\{x \mid 1 < x < 7\}$ or $(1, 7)$.

15. $2x^3 + x^2 - 13x + 6 = 0$

$f(x) = 2x^3 + x^2 - 13x + 6$ has 2 sign changes: 2 or 0 positive real roots.

$f(-x) = -2x^3 + x^2 + 13x + 6$ has 1 sign change: 1 negative real root.

p: ±1, ±2, ±3, ±6

q: ±1, ±2

$\frac{p}{q}: \pm 1, \pm\frac{1}{2}, \pm 2, \pm 3, \pm\frac{3}{2}, \pm 6$

–3	2	1	–13	6
		–6	15	–6
	2	–5	2	0

$$2x^3 + x^2 - 13x + 6 = (x+3)(2x^2 - 5x + 2)$$
$$= (x+3)(2x-1)(x-2)$$

$x = -3,\ x = \frac{1}{2},\ x = 2$

The solution set is $\left\{-3, \frac{1}{2}, 2\right\}$.

16. $6x - 3(5x+2) = 4(1-x)$

$6x - 15x - 6 = 4 - 4x$

$-9x - 6 = 4 - 4x$

$-5x = 10$

$x = -2$

The solution set is $\{-2\}$.

17. $\log(x+3)+\log x=1$

$\log x(x+3)=1$

$x(x+3)=10$

$x^2+3x-10=0$

$(x+5)(x-2)=0$

$x=-5$ or $x=2$

$x=-5$ is extraneous.

$x=2$

The solution set is $\{2\}$.

18. $3^{x+2}=11$

$\log_3 3^{x+2}=\log_3 11$

$x+2=\log_3 11$

$x=-2+\log_3 11$

$x=-2+\frac{\log 11}{\log 3}\approx 0.18$

The solution set is $\{-2+\log_3 11\}$.

19. $x^{\frac{1}{2}}-2x^{\frac{1}{4}}-15=0$

$\sqrt[4]{x}^2-2\sqrt[4]{x}-15=0$

$\left(\sqrt[4]{x}+3\right)\left(\sqrt[4]{x}-5\right)=0$

$\sqrt[4]{x}+3=0$ or $\sqrt[4]{x}-5=0$

$\sqrt[4]{x}=-3$ or $\sqrt[4]{x}=5$

$x=(-3)^4$ or $x=5^4$

$x=81$ or $x=625$

81 does not check. The solution set is $\{625\}$.

20. $3x-y=-2$

$2x^2-y=0$

Solve the first equation for *y*.

$3x-y=-2$

$y=3x+2$

Use this equation to substitute into the other equation.

$2x^2-(\overbrace{3x+2}^{y})=0$

$2x^2-3x-2=0$

$(2x+1)(x-2)=0$

$x=-\frac{1}{2}$ or $x=2$.

Back-substitute to find *y*.

$y=3x+2$ or $y=3x+2$

$y=3(2)+2$ $\quad y=3\left(-\frac{1}{2}\right)+2$

$y=8$ $\quad y=\frac{1}{2}$

$y=3x+2$

The solution set is $\left\{(2,8),\left(-\frac{1}{2},\frac{1}{2}\right)\right\}$.

21. $x+2y+3z=-2$

$3x+3y+10z=-2$

$2y-5z=6$

Multiply equation 1 by –3 and add to equation 2.

$-3x-6y-9z=6$

$3x+3y+10z=-2$

$-3y+z=4$ Equation 4

Multiply equation 4 by 5 and add to equation 3 and solve for *y*.

$-15y+5z=20$

$2y-5z=6$

$-13y=26$

$y=-2$

Back-substitute to find *z*.

$-3y+z=4$

$-3(-2)+z=4$

$z=-2$

Back-substitute to find *x*.

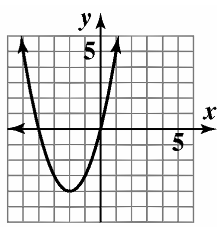
$x+2y+3z=-2$

$x+2(-2)+3(-2)=-2$

$x=8$

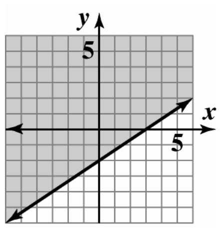
The solution set is $\{8,-2,-2\}$.

22. vertex: (–2, –4)
y-intercept:
$f(0) = (0+2)^2 - 4 = 0$
x-intercepts:
$(x+2)^2 - 4 = 0$
$x^2 + 4x + 4 - 4 = 0$
$x^2 + 4x = 0$
$x(x+4) = 0$
$x = 0, x - 4$



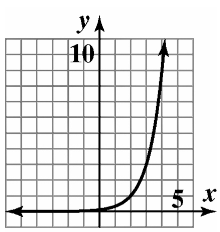
$f(x) = (x + 2)^2 - 4$

23.



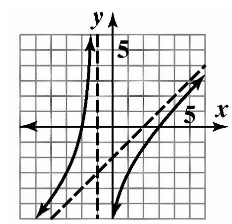
$2x - 3y \le 6$

24.



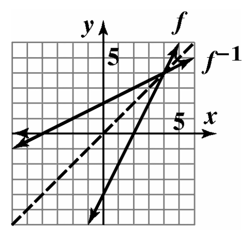
$y = 3^{x-2}$

25. vertical asymptote: $x = -1$
horizontal asymptote: $m > n$, none
x-intercepts:
$x^2 - x - 6 = 0$
$(x-3)(x+2) = 0$
$x = 3, x = -2$
y-intercept:
$f(0) = \dfrac{0^2 - 0 - 6}{0+1} = -6$



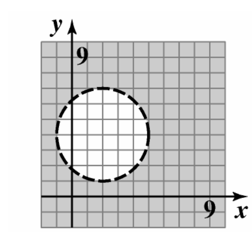
$f(x) = \dfrac{x^2 - x - 6}{x + 1}$

26.



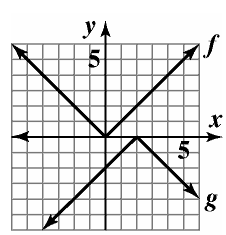
$f(x) = 2x - 4$
$f^{-1}(x) = \dfrac{x + 4}{2}$

27.



$(x - 2)^2 + (y - 4)^2 > 9$

28.



$f(x) = |x|$
$g(x) = -|x - 2|$

29. $(f \circ g)(x) = f(g(x))$

$= 2(1-x)^2 - (1-x) - 1$

$= 2x^2 - 3x$

$(g \circ f) = g(f(x))$

$= 1 - (2x^2 - x - 1)$

$= 1 - 2x^2 + x + 1$

$= -2x^2 + x + 2$

30. $\dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{\left[2(x+h)^2 - (x+h) - 1\right] - \left[2x^2 - x - 1\right]}{h}$

$= \dfrac{2x^2 + 4hx + 2h^2 - x - h - 1 - 2x^2 + x + 1}{h}$

$= \dfrac{4hx + 2h^2 - h}{h}$

$= 4x + 2h - 1$

31. Find slope: $m = \dfrac{4-(-2)}{2-4} = \dfrac{6}{-2} = -3$

Use point slope form to find an equation.

$y - y_1 = m(x - x_1)$

$y - 4 = -3(x - 2)$

Put in slope-intercept form.

$y - 4 = -3(x - 2)$

$y - 4 = -3x + 6$

$y = -3x + 10$

32. Find the slope of the perpendicular line by putting in slope-intercept form.

$x + 3y - 6 = 0$

$3y = -x + 6$

$y = -\dfrac{1}{3}x + 2$

The slope of the perpendicular line is $-\dfrac{1}{3}$ so the slope of the desired line is the negative reciprocal, or 3.

Use point slope form to find an equation.

$y - y_1 = m(x - x_1)$

$y - 0 = 3(x + 1)$

Put in slope-intercept form.

$y - 0 = 3(x + 1)$

$y = 3x + 3$

33. Let x = the amount invested at 12%

Let $4000 - x$ = the amount invested at 14%

$0.12x + 0.14(4000 - x) = 508$

$0.12x + 560 - 0.14x = 508$

$-0.02x = -52$

$x = \dfrac{-52}{-0.02}$

$x = 2600$

$4000 - x = 4000 - 2600 = 1400$

Thus, \$2600 was invested at 12% and \$1400 was invested at 14%.

34. $L = 2W + 1$

$LW = 36$

$W(2W + 1) = 36$

$2W^2 + W - 36 = 0$

$(2W + 9)(W - 4) = 0$

$W = -\dfrac{9}{2} \text{ or } 4$

Length cannot be negative. If $W = 4$, $4L = 36$, $L = 9$. The dimensions are 4 m by 9 m.

35. $A = Pe^{rt}$

$18{,}000 = 6000e^{10r}$

$3 = e^{10r}$

$\ln 3 = \ln e^{10r}$

$\ln 3 = 10r$

$r = \dfrac{\ln 3}{10} \approx 0.1099$

10.99%

36. $\sec\theta - \cos\theta = \dfrac{1}{\cos\theta} - \cos\theta$

$= \dfrac{1 - \cos^2\theta}{\cos\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} \cdot \sin\theta$

$= \tan\theta \cdot \sin\theta$

37. $\tan x + \tan y = \dfrac{\sin x}{\cos x} + \dfrac{\sin y}{\cos y}$

$= \dfrac{\sin x \cdot \cos y + \sin y \cdot \cos x}{\cos x \cdot \cos y}$

$= \dfrac{\sin(x + y)}{\cos x \cdot \cos y}$

38.

$$\sin\theta = \tan\theta$$
$$\sin\theta = \frac{\sin\theta}{\cos\theta}$$
$$\sin\theta - \frac{\sin\theta}{\cos\theta} = 0$$
$$\sin\theta\left(1 - \frac{1}{\cos\theta}\right) = 0$$
$$\sin\theta = 0 \quad \text{or} \quad 1 - \frac{1}{\cos\theta} = 0$$
$$\theta = 0,\ \pi \quad \text{or} \quad 1 = \frac{1}{\cos\theta}$$
$$\cos\theta = 1$$
$$\theta = 0$$

The solutions in the interval $[0, 2\pi)$ are 0 and π.

39.

$$2 + \cos 2\theta = 3\cos\theta$$
$$2 + (2\cos^2\theta - 1) = 3\cos\theta$$
$$2\cos^2\theta + 1 = 3\cos\theta$$
$$2\cos^2\theta - 3\cos\theta + 1 = 0$$
$$(2\cos\theta - 1)(\cos\theta - 1) = 0$$
$$2\cos\theta - 1 = 0 \quad \text{or} \quad \cos\theta - 1 = 0$$
$$2\cos\theta = 1 \qquad \cos\theta = 1$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{3}, \frac{5\pi}{3} \quad \text{or} \quad \theta = 0$$

The solutions in the interval $[0, 2\pi)$ are $0, \frac{\pi}{3}$, and $\frac{5\pi}{3}$.

40.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 75^\circ} = \frac{20}{\sin 12^\circ}$$
$$b = \frac{20\sin 75^\circ}{\sin 12^\circ}$$
$$b \approx 92.9$$

Chapter 9
Matrices and Determinants

Section 9.1

Check Point Exercises

1. **a.** The notation $R_1 \leftrightarrow R_2$ means to interchange the elements in row 1 and row 2. This results in the row-equivalent matrix

$$\left[\begin{array}{ccc|c} 1 & 6 & -3 & 7 \\ 4 & 12 & -20 & 8 \\ -3 & -2 & 1 & -9 \end{array}\right].$$

b. The notation $\frac{1}{4}R_1$ means to multiply each element in row 1 by $\frac{1}{4}$. This results in the row-equivalent matrix

$$\left[\begin{array}{ccc|c} \frac{1}{4}(4) & \frac{1}{4}(12) & \frac{1}{4}(-20) & \frac{1}{4}(8) \\ 1 & 6 & -3 & 7 \\ -3 & -2 & 1 & -9 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 3 & -5 & 2 \\ 1 & 6 & -3 & 7 \\ -3 & -2 & 1 & -9 \end{array}\right]$$

c. The notation $3R_2 + R_3$ means to add 3 times the elements in row 2 to the corresponding elements in row 3. Replace the elements in row 3 by these sums. First, we find 3 times the elements in row 2: $3(1) = 3,\ 3(6) = 18,\ 3(-3) = -9,\ 3(7) = 21.$ Now we add these products to the corresponding elements in row 3. This results in the row equivalent matrix

$$\left[\begin{array}{ccc|c} 4 & 12 & -20 & 8 \\ 1 & 6 & -3 & 7 \\ -3+3=0 & -2+18=16 & 1-9=-8 & -9+21=12 \end{array}\right] = \left[\begin{array}{ccc|c} 4 & 12 & -20 & 8 \\ 1 & 6 & -3 & 7 \\ 0 & 16 & -8 & 12 \end{array}\right].$$

2.
$$\begin{array}{r} 2x+y+2z=18 \\ x-y+2z=9 \\ x+2y-z=6 \end{array} \rightarrow \left[\begin{array}{ccc|c} 2 & 1 & 2 & 18 \\ 1 & -1 & 2 & 9 \\ 1 & 2 & -1 & 6 \end{array}\right]$$

Interchange row 1 with row 2 to get 1 in the top position of the first column.

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 2 & 1 & 2 & 18 \\ 1 & 2 & -1 & 6 \end{array}\right]$$

Multiply the first row by –2 and add these products to row 2.

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 2+-2=0 & 1+2=3 & 2+-4=-2 & -18+18=0 \\ 1 & 2 & -1 & 6 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -2 & 0 \\ 1 & 2 & -1 & 6 \end{array}\right]$$

Next, multiply the top row by –1 and add these products to row 3.

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -2 & 0 \\ 1+-1=0 & 2+1=3 & -1-2=-3 & 6-9=-3 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -2 & 0 \\ 0 & 3 & -3 & -3 \end{array}\right]$$

Next, to obtain a 1 in the second row, second column, multiply 3 by its reciprocal, $\frac{1}{3}$. Therefore, we multiply all the numbers in the second row by $\frac{1}{3}$ to get

$$\left[\begin{array}{rrr|r} 1 & -1 & 2 & 9 \\ 0 & 1 & -\frac{2}{3} & 0 \\ 0 & 3 & -3 & -3 \end{array}\right].$$

Next, to obtain a 0 in the third row, second column, multiply the second row by –3 and add the products to row three. The resulting matrix is

$$\left[\begin{array}{rrr|r} 1 & -1 & 2 & 9 \\ 0 & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & -1 & -3 \end{array}\right].$$

To get 1 in the third row, third column, multiply –1 by its reciprocal, –1. Multiply all numbers in the third row by –1 to obtain the resulting matrix

$$\left[\begin{array}{rrr|r} 1 & -1 & 2 & 9 \\ 0 & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 1 & 3 \end{array}\right].$$

The system represented by this matrix is:

$$\begin{aligned} x - y + 2z &= 9 \\ y - \frac{2}{3}z &= 0 \\ z &= 3 \end{aligned}$$

Use back substitution to find y and x.

$$\begin{aligned} y - \frac{2}{3}(3) &= 0 \\ y - 2 &= 0 \\ y &= 2 \end{aligned} \qquad \begin{aligned} x - 2 + 6 &= 9 \\ x + 4 &= 9 \\ x &= 5 \end{aligned}$$

The solution set for the original system is {(5, 2, 3)}.

3.

$$\begin{aligned} w - 3x - 2y + z &= -3 \\ 2w - 7x - y + 2z &= 1 \\ 3w - 7x - 3y + 3z &= -5 \\ 5w + x + 4y - 2z &= 18 \end{aligned}$$

The augmented matrix is

$$\left[\begin{array}{rrrr|r} 1 & -3 & -2 & 1 & -3 \\ 2 & -7 & -1 & 2 & 1 \\ 3 & -7 & -3 & 3 & -5 \\ 5 & 1 & 4 & -2 & 18 \end{array}\right].$$

Multiply the top row by –2 and add the products to the second row. Multiply the top row by –3 and add the pro-ducts to the third row. Multiply the top row by –5 and add the products to the fourth row. The resulting matrix is

$$\left[\begin{array}{rrrr|r} 1 & -3 & -2 & 1 & -3 \\ 0 & -1 & 3 & 0 & 7 \\ 0 & 2 & 3 & 0 & 4 \\ 0 & 16 & 14 & -7 & 33 \end{array}\right].$$

Next, multiply the second row by –1 to obtain a 1 in the second row, second column.

$$\left[\begin{array}{cccc|c} 1 & -3 & -2 & 1 & -3 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 2 & 3 & 0 & 4 \\ 0 & 16 & 14 & -7 & 33 \end{array}\right]$$

Next, multiply the second row by –2 and add the products to the third row. Multiply the second row by –16 and add the products to the fourth row. The resulting matrix is

$$\left[\begin{array}{cccc|c} 1 & -3 & -2 & 1 & -3 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 9 & 0 & 18 \\ 0 & 0 & 62 & -7 & 145 \end{array}\right].$$

Next, multiply the third row by $\frac{1}{9}$ to obtain a 1 in the third row, third column. The resulting matrix is

$$\left[\begin{array}{cccc|c} 1 & -3 & -2 & 1 & -3 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 62 & -7 & 145 \end{array}\right].$$

Multiply the third row by –62 and add the products to the fourth row to obtain the resulting matrix

$$\left[\begin{array}{cccc|c} 1 & -3 & -2 & 1 & -3 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & -7 & 21 \end{array}\right].$$

Multiply the fourth row by $-\frac{1}{7}$, the reciprocal of –7. The resulting matrix is

$$\left[\begin{array}{cccc|c} 1 & -3 & -2 & 1 & -3 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right].$$

The system of linear equations corresponding to the resulting matrix is

$$\begin{aligned} w-3x-2y+z &= -3 \\ x-3y &= -7 \\ y &= 2 \\ z &= -3 \end{aligned}$$

Using back-substitution solve for *x* and *w*.

$$\begin{aligned} x-3(2) &= -7 \\ x &= -1 \end{aligned}$$

$$\begin{aligned} w-3(-1)-2(2)-3 &= -3 \\ w-4 &= -3 \\ w &= 1 \end{aligned}$$

The solution set is {(1, –1, 2, –3)}.

4. The matrix obtained in 3 will be the starting point.

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Next, multiply the third row by $\frac{2}{3}$ and add the products to the second row. Multiply the third row by 2 and add the products to the first row. The resulting matrix is

$$\left[\begin{array}{cccc} 1 & -1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right].$$

Add the second row to the first row and replace the first row.

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

This matrix corresponds to $x = 5$, $y = 2$ and $z = 3$. The solution set is $\{(5, 2, 3)\}$.

Exercise Set 9.1

1. $\left[\begin{array}{ccc|c} 2 & 1 & 2 & 2 \\ 3 & -5 & -1 & 4 \\ 1 & -2 & -3 & -6 \end{array}\right]$

2. $\left[\begin{array}{ccc|c} 3 & -2 & 5 & 31 \\ 1 & 3 & -3 & -12 \\ -2 & -5 & 3 & 11 \end{array}\right]$

3. $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 0 & 1 & -12 & -15 \\ 0 & 0 & 1 & 1 \end{array}\right]$

4. $\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$

5. $\left[\begin{array}{ccc|c} 5 & -2 & -3 & 0 \\ 1 & 1 & 0 & 5 \\ 2 & 0 & -3 & 4 \end{array}\right]$

6. $\left[\begin{array}{ccc|c} 1 & -2 & 1 & 10 \\ 3 & 1 & 0 & 5 \\ 7 & 0 & 2 & 2 \end{array}\right]$

7. $\left[\begin{array}{cccc|c} 2 & 5 & -3 & 1 & 2 \\ 0 & 3 & 1 & 0 & 4 \\ 1 & -1 & 5 & 0 & 9 \\ 5 & -5 & -2 & 0 & 1 \end{array}\right]$

8. $\left[\begin{array}{cccc|c} 4 & 7 & -8 & 1 & 3 \\ 0 & 5 & 1 & 0 & 5 \\ 1 & -1 & -1 & 0 & 17 \\ 2 & -2 & 11 & 0 & 4 \end{array}\right]$

9.
$$\begin{aligned} 5x+3z&=-11 \\ y-4z&=12 \\ 7x+2y&=3 \end{aligned}$$

10.
$$\begin{aligned} 7x+4z&=-13 \\ y-5z&=11 \\ 2x+7y&=6 \end{aligned}$$

11.
$$\begin{aligned} w+x+4y+z&=3 \\ -w+x-y&=7 \\ 12w+5z&=11 \\ 12y+4z&=5 \end{aligned}$$

12.
$$\begin{aligned} 4w+x+5y+z&=6 \\ w-x-z&=8 \\ 3w+7z&=4 \\ 11y+5z&=3 \end{aligned}$$

13. $\left[\begin{array}{ccc|c} 2\left(\frac{1}{2}\right) & -6\left(\frac{1}{2}\right) & 4\left(\frac{1}{2}\right) & 10\left(\frac{1}{2}\right) \\ 1 & 5 & -5 & 0 \\ 3 & 0 & 4 & 7 \end{array}\right]\frac{1}{2}R_1$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 5 \\ 1 & 5 & -5 & 0 \\ 3 & 0 & 4 & 7 \end{array}\right]$$

14. $\left[\begin{array}{ccc|c} 3\left(\frac{1}{3}\right) & -12\left(\frac{1}{3}\right) & 6\left(\frac{1}{3}\right) & 9\left(\frac{1}{3}\right) \\ 1 & -4 & 4 & 0 \\ 2 & 0 & 7 & 4 \end{array}\right]\frac{1}{3}R_1$

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 1 & -4 & 4 & 0 \\ 2 & 0 & 7 & 4 \end{array}\right]$$

15. $\left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ -3(1)+3 & -3(-3)+1 & -3(2)+-1 & -3(0)+7 \\ 2 & -2 & 1 & 3 \end{array}\right] -3R_1 + R_2$

$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & 10 & -7 & 7 \\ 2 & -2 & 1 & 3 \end{array}\right]$

16. $\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ -3(1)+3 & -3(-1)+3 & -3(5)+(-1) & -3(-6)+10 \\ 1 & 3 & 2 & 5 \end{array}\right] -3R_1 + R_2$

$\left[\begin{array}{ccc|c} 1 & -1 & 5 & -6 \\ 0 & 6 & -16 & 28 \\ 1 & 3 & 2 & 5 \end{array}\right]$

17. $\left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 3 \\ 0 & 1 & -2 & -1 & 0 \\ 2 & 0 & 3 & 4 & 11 \\ 5 & 1 & 2 & 4 & 6 \end{array}\right] \begin{array}{l} \\ -2R_1 + R_3 \\ -5R_1 + R_4 \\ \end{array}$

$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 3 \\ 0 & 1 & -2 & -1 & 0 \\ -2(1)+2 & -2(-1)+0 & -2(1)+3 & -2(1)+4 & -2(3)+11 \\ -5(1)+5 & -5(-1)+1 & -5(1)+2 & -5(1)+4 & -5(3)+6 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & -1 & 1 & 1 & 3 \\ 0 & 1 & -2 & -1 & 0 \\ 0 & 2 & 1 & 2 & 5 \\ 0 & 6 & -3 & -1 & -9 \end{array}\right]$

18. $\left[\begin{array}{cccc|c} 1 & -5 & 2 & -2 & 4 \\ 0 & 1 & -3 & -1 & 0 \\ 3 & 0 & 2 & -1 & 6 \\ -4 & 1 & 4 & 2 & -3 \end{array}\right] \begin{array}{l} \\ -3R_1 + R_3 \\ 4R_1 + R_4 \\ \end{array}$

$\left[\begin{array}{cccc|c} 1 & -5 & 2 & -2 & 4 \\ 0 & 1 & -3 & -1 & 0 \\ -3(1)+3 & -3(-5)+0 & -3(2)+2 & -3(-2)+(-1) & -3(4)+6 \\ 4(1)+(-4) & 4(-5)+1 & 4(2)+4 & 4(-2)+2 & 4(4)+(-3) \end{array}\right] = \left[\begin{array}{cccc|c} 1 & -5 & 2 & -2 & 4 \\ 0 & 1 & -3 & -1 & 0 \\ 0 & 15 & -4 & 5 & -6 \\ 0 & -19 & 12 & -6 & 13 \end{array}\right]$

19. $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 2 & 3 & -1 & -2 \\ 3 & -2 & -9 & 9 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ -2(1)+2 & -2(-1)+3 & -2(1)-1 & -2(8)-2 \\ -3(1)+3 & -3(-1)-2 & -3(1)-9 & -3(8)+9 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 0 & 5 & \boxed{-3} & \boxed{-18} \\ 0 & 1 & \boxed{-12} & \boxed{-15} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 0\left(\frac{1}{5}\right) & 1\left(\frac{1}{5}\right) & -3\left(\frac{1}{5}\right) & -18\left(\frac{1}{5}\right) \\ 0 & 1 & -12 & -15 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 0 & 1 & \boxed{-\frac{3}{5}} & \boxed{-\frac{18}{5}} \\ 0 & 1 & \boxed{-12} & \boxed{-15} \end{array}\right]$$

20. $\left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ -3 & 4 & -1 & -2 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 5 & \boxed{-10} & \boxed{-5} \\ 0 & -2 & \boxed{8} & \boxed{10} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 1 & \boxed{-2} & \boxed{-1} \\ 0 & -2 & \boxed{8} & \boxed{10} \end{array}\right]$$

21. $x+y-z=-2$

$2x-y+z=5$

$-x+2y+2z=1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right] -2R_1+R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ -1 & 2 & 2 & 1 \end{array}\right] 1R_1+R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 3 & 1 & -1 \end{array}\right] -\tfrac{1}{3}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 3 & 1 & -1 \end{array}\right] -3R_2+R_3$$

$$=\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

$4z=8$

$z=2$

$y-z=-3$

$y-2=-3$

$y=-1$

$x+y-z=-2$

$x-1-2=-2$

$x-3=-2$

$x=1$

The solution set is $\{(1, -1, 2)\}$.

22. $x-2y-z=2$

$2x-y+z=4$

$-x+y-2z=-4$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 2 \\ 2 & -1 & 1 & 4 \\ -1 & 1 & -2 & -4 \end{array}\right] -2R_1+R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 2 \\ 0 & 3 & 3 & 0 \\ -1 & 1 & -2 & -4 \end{array}\right] 1R_1+R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 2 \\ 0 & 3 & 3 & 0 \\ 0 & -1 & -3 & -2 \end{array}\right] \tfrac{1}{3}R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & -3 & -2 \end{array}\right] 1R_2+R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -2 & -2 \end{array}\right] -\tfrac{1}{2}R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$z=1$

$y+z=0$

$y+1=0$

$y=-1$

$x-2y-z=2$

$x+2-1=2$

$x=1$

The solution set is $\{(1, -1, 1)\}$.

23.
$$x+3y=0$$
$$x+y+z=1$$
$$3x-y-z=11$$

$$\left[\begin{array}{ccc|c}1&3&0&0\\1&1&1&1\\3&-1&-1&11\end{array}\right]-1R_1+R_2$$

$$\left[\begin{array}{ccc|c}1&3&0&0\\0&-2&1&1\\3&-1&-1&11\end{array}\right]-3R_1+R_3$$

$$\left[\begin{array}{ccc|c}1&3&0&0\\0&-2&1&1\\0&-10&-1&11\end{array}\right]-\tfrac{1}{2}R_2$$

$$\left[\begin{array}{ccc|c}1&3&0&0\\0&1&-\tfrac{1}{2}&-\tfrac{1}{2}\\0&-10&-1&11\end{array}\right]10R_2+R_3$$

$$\left[\begin{array}{ccc|c}1&3&0&0\\0&1&-\tfrac{1}{2}&-\tfrac{1}{2}\\0&0&-6&6\end{array}\right]-\tfrac{1}{6}R_3$$

$$\left[\begin{array}{ccc|c}1&3&0&0\\0&1&-\tfrac{1}{2}&-\tfrac{1}{2}\\0&0&1&-1\end{array}\right]$$

$z=-1$

$$y-\frac{1}{2}z=-\frac{1}{2}$$
$$y-\frac{1}{2}(-1)=-\frac{1}{2}$$
$$y+\frac{1}{2}=-\frac{1}{2}$$
$$y=-1$$

Interchange row one and row two.
$$x+3y=0$$
$$x+3(-1)=0$$
$$x=3$$
The solution set is $\{(3,-1,-1)\}$.

24.
$$3y-z=-1$$
$$x+5y-z=-4$$
$$-3x+6y+2z=11$$

$$\left[\begin{array}{ccc|c}0&3&-1&-1\\1&5&-1&-4\\-3&6&2&11\end{array}\right]R_1\leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c}1&5&-1&-4\\0&3&-1&-1\\-3&6&2&11\end{array}\right]3R_1+R_3$$

$$\left[\begin{array}{ccc|c}1&5&-1&-4\\0&3&-1&-1\\0&21&-1&-1\end{array}\right]\tfrac{1}{3}R_2$$

$$\left[\begin{array}{ccc|c}1&5&-1&-4\\0&1&-\tfrac{1}{3}&-\tfrac{1}{3}\\0&21&-1&-1\end{array}\right]-21R_2+R_3$$

$$\left[\begin{array}{ccc|c}1&5&-1&-4\\0&1&-\tfrac{1}{3}&-\tfrac{1}{3}\\0&0&6&6\end{array}\right]\tfrac{1}{6}R_3$$

$$\left[\begin{array}{ccc|c}1&5&-1&-4\\0&1&-\tfrac{1}{3}&-\tfrac{1}{3}\\0&0&1&1\end{array}\right]$$

$z=1$

$$y-\frac{1}{3}z=-\frac{1}{3}$$
$$y-\frac{1}{3}(1)=-\frac{1}{3}$$
$$y=0$$
$$x+5y-z=-4$$
$$x+5(0)-1=-4$$
$$x-1=-4$$
$$x=-3$$
The solution set is $\{(-3,0,1)\}$.

25. $2x - y - z = 4$
$x + y - 5z = -4$
$x - 2y = 4$

$$\begin{bmatrix} 2 & -1 & -1 & 4 \\ 1 & 1 & -5 & -4 \\ 1 & -2 & 0 & 4 \end{bmatrix}$$

Interchange rows one and two.

$$\begin{bmatrix} 1 & 1 & -5 & -4 \\ 2 & -1 & -1 & 4 \\ 1 & -2 & 0 & 4 \end{bmatrix}$$

Replace row two with $-2R_1 + R_2$.
Replace row three with $-R_1 + R_3$.

$$\begin{bmatrix} 1 & 1 & -5 & -4 \\ 0 & -3 & 9 & 12 \\ 0 & -3 & 5 & 8 \end{bmatrix}$$

Replace row two with $-\frac{1}{3}R_2$.

$$\begin{bmatrix} 1 & 1 & -5 & -4 \\ 0 & 1 & -3 & -4 \\ 0 & -3 & 5 & 8 \end{bmatrix}$$

Replace row three with $3R_2 + R_3$.

$$\begin{bmatrix} 1 & 1 & -5 & -4 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & -4 & -4 \end{bmatrix}$$

Replace row three with $-\frac{1}{4}R_3$.

$$\begin{bmatrix} 1 & 1 & -5 & -4 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$z = 1$

$$y - 3z = -4$$
$$y - 3(1) = -4$$
$$y = -1$$
$$x + y - 5z = -4$$
$$x - 1 - 5(1) = -4$$
$$x - 6 = -4$$
$$x = 2$$

The solution set is $\{(2, -1, 1)\}$.

26. $x \quad - 3z = -2$
$2x + 2y + z = 4$
$3x + y - 2z = 5$

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 2 & 2 & 1 & 4 \\ 3 & 1 & -2 & 5 \end{bmatrix}$$

Replace Row two with $-2R_1 + R_2$.
Replace Row three with $-3R_1 + R_3$.

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 2 & 7 & 8 \\ 0 & 1 & 7 & 11 \end{bmatrix}$$

Interchange Row two and Row three.

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 2 & 7 & 8 \end{bmatrix}$$

Replace Row three with $-2R_2 + R_3$.

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & -7 & -14 \end{bmatrix}$$

Replace Row three with $-\frac{1}{7}R_3$.

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$z = 2$
$y + 7(2) = 11$
$y = -3$
$x - 3(2) = -2$
$x - 6 = -2$
$x = 4$

The solution set is $\{(4, -3, 2)\}$.

27. $x + y + z = 4$
$x - y - z = 0$
$x - y + z = 2$

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 1 & -1 & -1 & 0 \\ 1 & -1 & 1 & 2 \end{bmatrix}$$

Replace row two with $-R_1 + R_2$.
Replace row three with $-R_1 + R_3$.

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & -2 & -2 & -4 \\ 0 & -2 & 0 & -2 \end{bmatrix}$$

Replace row two with $-\frac{1}{2}R_2$.

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & -2 & 0 & -2 \end{bmatrix}$$

Replace row 3 with $2R_2 + R_3$.

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

Replace row 3 with $\frac{1}{2}R_3$.

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$z = 1$
$y + 1 = 2$
$y = 1$
$x + 1 + 1 = 4$
$x = 2$
The solution set is {(2, 1, 1)}.

28. $3x + y - z = 0$
$x + y + 2z = 6$
$2x + 2y + 3z = 10$

$$\begin{bmatrix} 3 & 1 & -1 & 0 \\ 1 & 1 & 2 & 6 \\ 2 & 2 & 3 & 10 \end{bmatrix}$$

Interchange Row 1 and Row 2.

$$\begin{bmatrix} 1 & 1 & 2 & 6 \\ 3 & 1 & -1 & 0 \\ 2 & 2 & 3 & 10 \end{bmatrix}$$

Replace Row 2 with $-3R_1 + R_2$.
Replace Row 3 with $-2R_1 + R_3$.

$$\begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & -2 & -7 & -18 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

Replace Row two with $-\frac{1}{2}R_2$.
Replace Row three with $-R_3$.

$$\begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & 1 & \frac{7}{2} & 9 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$z = 2$
$y + \frac{7}{2}(2) = 9$
$y + 7 = 9$
$y = 2$
$x + 2 + 2(2) = 6$
$x + 6 = 6$
$x = 0$
The solution set is {(0, 2, 2)}.

29. Write the equations in standard form.

$x + 2y - z = -1$
$x - y + z = 4$
$x + y - 3z = -2$

$$\begin{bmatrix} 1 & 2 & -1 & -1 \\ 1 & -1 & 1 & 4 \\ 1 & 1 & -3 & -2 \end{bmatrix}$$

Replace row two with $-R_1 + R_2$.
Replace row three with $-R_1 + R_3$.

$$\begin{bmatrix} 1 & 2 & -1 & -1 \\ 0 & -3 & 2 & 5 \\ 0 & -1 & -2 & -1 \end{bmatrix}$$

Replace row two with $-R_3$.
Replace row three with R_2.

$$\begin{bmatrix} 1 & 2 & -1 & -1 \\ 0 & 1 & 2 & 1 \\ 0 & -3 & 2 & 5 \end{bmatrix}$$

Replace row 3 with $3R_2 + R_3$.

$$\begin{bmatrix} 1 & 2 & -1 & -1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 8 & 8 \end{bmatrix}$$

Replace row 3 with $\frac{1}{8}R_3$.

$$\begin{bmatrix} 1 & 2 & -1 & -1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$z = 1$
$y + 2(1) = 1$
$y = -1$
$x + 2(-1) - 1 = -1$
$x = 2$
The solution set is $\{(2, -1, 1)\}$.

30. Rearrange equation to standard form.

$2x + y - z = 1$
$2x - 3y + z = 1$
$x + y + z = 4$

$$\begin{bmatrix} 2 & 1 & -1 & 1 \\ 2 & -3 & 1 & 1 \\ 1 & 1 & 1 & 4 \end{bmatrix}$$

Interchange Row 1 and Row 3.

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 2 & -3 & 1 & 1 \\ 2 & 1 & -1 & 1 \end{bmatrix}$$

Replace Row 2 with $-2R_1 + R_2$.
Replace Row 3 with $-2R_1 + R_3$.

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & -5 & -1 & -7 \\ 0 & -1 & -3 & -7 \end{bmatrix}$$

Replace Row 2 with $-R_3$.
Replace Row 3 with R_2.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 7 \\ 0 & -5 & -1 & -7 \end{bmatrix}$$

Replace Row 3 with $5R_2 + R_3$.

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 14 & 28 \end{bmatrix}$$

Replace Row 3 with $\frac{1}{14}\mathrm{R}_3$.

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$z = 2$
$y + 3(2) = 7$
$y = 1$
$x + y + z = 4$
$x + 1 + 2 = 4$
$x = 1$
The solution set is $\{(1, 1, 2)\}$.

31. $3a - b - 4c = 3$
$2a - b + 2c = -8$
$a + 2b - 3c = 9$
Interchange equations 1 and 3.
$$\begin{bmatrix} 1 & 2 & -3 & 9 \\ 2 & -1 & 2 & -8 \\ 3 & -1 & -4 & 3 \end{bmatrix}$$
Replace row two with $-2R_1 + R_2$.
Replace row three with $-3R_1 + R_3$.
$$\begin{bmatrix} 1 & 2 & -3 & 9 \\ 0 & -5 & 8 & -26 \\ 0 & -7 & 5 & -24 \end{bmatrix}$$
Replace row two with $-\frac{1}{5}R_2$
$$\begin{bmatrix} 1 & 2 & -3 & 9 \\ 0 & 1 & -\frac{8}{5} & \frac{26}{5} \\ 0 & -7 & 5 & -24 \end{bmatrix}$$
Replace row three with $7R_2 + R_3$.
$$\begin{bmatrix} 1 & 2 & -3 & 9 \\ 0 & 1 & -\frac{8}{5} & \frac{26}{5} \\ 0 & 0 & -\frac{31}{5} & \frac{62}{5} \end{bmatrix}$$
Replace row 3 with $-\frac{5}{31}R_3$.
$$\begin{bmatrix} 1 & 2 & -3 & 9 \\ 0 & 1 & -\frac{8}{5} & \frac{26}{5} \\ 0 & 0 & 1 & -2 \end{bmatrix}$$
$z = -2$
$$y - \frac{8}{5}(-2) = \frac{26}{5}$$
$$y + \frac{16}{5} = \frac{26}{5}$$
$$y = 2$$
$$x + 2(2) - 3(-2) = 9$$
$$x + 4 + 6 = 9$$
$$x = -1$$
The solution set is {(–1, 2, –2)}.

32. $3a + b - c = 0$
$2a + 3b - 5c = 1$
$a - 2b + 3c = -4$
Interchange equations 1 and 3.
$$\begin{bmatrix} 1 & -2 & 3 & -4 \\ 2 & 3 & -5 & 1 \\ 3 & 1 & -1 & 0 \end{bmatrix}$$
Replace Row 2 with $-2R_1 + R_2$.
Replace Row 3 with $-3R_1 + R_3$.
$$\begin{bmatrix} 1 & -2 & 3 & -4 \\ 0 & 7 & -11 & 9 \\ 0 & 7 & -10 & 12 \end{bmatrix}$$
Replace Row 3 with $-R_2 + R_3$.
$$\begin{bmatrix} 1 & -2 & 3 & -4 \\ 0 & 7 & -11 & 9 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$
$z = 3$
$7y - 11(3) = 9$
$7y = 42$
$y = 6$
$x - 2(6) + 3(3) = -4$
$x - 3 = -4$
$x = -1$
The solution set is {(–1, 6, 3)}.

33. $2x+2y+7z=-1$
$2x+y+2z=2$
$4x+6y+z=15$

$$\left[\begin{array}{ccc|c}2&2&7&-1\\2&1&2&2\\4&6&1&15\end{array}\right]\tfrac{1}{2}R_1$$

$$\left[\begin{array}{ccc|c}1&1&\tfrac{7}{2}&-\tfrac{1}{2}\\2&1&2&2\\4&6&1&15\end{array}\right]-2R_1+R_2$$

$$\left[\begin{array}{ccc|c}1&1&\tfrac{7}{2}&-\tfrac{1}{2}\\0&-1&-5&3\\4&6&1&15\end{array}\right]-4R_1+R_3$$

$$\left[\begin{array}{ccc|c}1&1&\tfrac{7}{2}&-\tfrac{1}{2}\\0&-1&-5&3\\0&2&-13&17\end{array}\right]-1R_2$$

$$\left[\begin{array}{ccc|c}1&1&\tfrac{7}{2}&-\tfrac{1}{2}\\0&1&5&-3\\0&2&-13&17\end{array}\right]-2R_2+R_3$$

$$\left[\begin{array}{ccc|c}1&1&\tfrac{7}{2}&-\tfrac{1}{2}\\0&1&5&-3\\0&0&-23&23\end{array}\right]-\tfrac{1}{23}R_3$$

$$\left[\begin{array}{ccc|c}1&1&\tfrac{7}{2}&-\tfrac{1}{2}\\0&1&5&-3\\0&0&1&-1\end{array}\right]$$

$z=-1$
$y+5z=-3$
$y+5(-1)=-3$
$y-5=-3$
$y=2$

$x+y+\frac{7}{2}z=-\frac{1}{2}$
$x+2+\frac{7}{2}(-1)=-\frac{1}{2}$
$x-\frac{3}{2}=-\frac{1}{2}$
$x=1$

The solution set is {(1, 2, –1)}.

34. $3x+2y+3z=3$
$4x-5y+7z=1$
$2x+3y-2z=6$
leads to:

$$\left[\begin{array}{ccc|c}3&2&3&3\\4&-5&7&1\\2&3&-2&6\end{array}\right]\tfrac{1}{3}R_1$$

$$\left[\begin{array}{ccc|c}1&\tfrac{2}{3}&1&1\\4&-5&7&1\\2&3&-2&6\end{array}\right]-4R_1+R_2$$

$$\left[\begin{array}{ccc|c}1&\tfrac{2}{3}&1&1\\0&-\tfrac{23}{3}&3&-3\\2&3&-2&6\end{array}\right]-2R_1+R_3$$

$$=\left[\begin{array}{ccc|c}1&\tfrac{2}{3}&1&1\\0&-\tfrac{23}{3}&3&-3\\0&\tfrac{5}{3}&-4&4\end{array}\right]-\tfrac{3}{23}R_2$$

$$=\left[\begin{array}{ccc|c}1&\tfrac{2}{3}&1&1\\0&1&-\tfrac{9}{23}&\tfrac{9}{23}\\0&\tfrac{5}{3}&-4&4\end{array}\right]-\tfrac{5}{3}R_2+R_3$$

$$\left[\begin{array}{ccc|c}1&\tfrac{2}{3}&1&1\\0&1&-\tfrac{9}{23}&\tfrac{9}{23}\\0&0&-\tfrac{77}{23}&\tfrac{77}{23}\end{array}\right]-\tfrac{23}{77}R_3$$

$$\left[\begin{array}{ccc|c}1&\tfrac{2}{3}&1&1\\0&1&-\tfrac{9}{23}&\tfrac{9}{23}\\0&0&1&-1\end{array}\right]$$

$z=-1$
$y-\frac{9}{23}z=\frac{9}{23}$
$y-\frac{9}{23}(-1)=\frac{9}{23}$
$y+\frac{9}{23}=\frac{9}{23}$
$y=0$

$x+\frac{2}{3}y+z=1$
$x+\frac{2}{3}(0)+(-1)=1$
$x-1=1$
$x=2$

The solution set is {(2, 0, –1)}.

35.
$$\begin{aligned} w+x+y+z&=4\\ 2w+x-2y-z&=0\\ w-2x-y-2z&=-2\\ 3w+2x+y+3z&=4 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4\\ 2 & 1 & -2 & -1 & 0\\ 1 & -2 & -1 & -2 & -2\\ 3 & 2 & 1 & 3 & 4 \end{array}\right] -2R_1+R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4\\ 0 & -1 & -4 & -3 & -8\\ 1 & -2 & -1 & -2 & -2\\ 3 & 2 & 1 & 3 & 4 \end{array}\right] -1R_1+R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4\\ 0 & -1 & -4 & -3 & -8\\ 0 & -3 & -2 & -3 & -6\\ 3 & 2 & 1 & 3 & 4 \end{array}\right] -3R_1+R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4\\ 0 & -1 & -4 & -3 & -8\\ 0 & -3 & -2 & -3 & -6\\ 0 & -1 & -2 & 0 & -8 \end{array}\right] -1R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4\\ 0 & 1 & 4 & 3 & 8\\ 0 & -3 & -2 & -3 & -6\\ 0 & -1 & -2 & 0 & -8 \end{array}\right] 3R_2+R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4\\ 0 & 1 & 4 & 3 & 8\\ 0 & 0 & 10 & 6 & 18\\ 0 & -1 & -2 & 0 & -8 \end{array}\right] 1R_2+R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4\\ 0 & 1 & 4 & 3 & 8\\ 0 & 0 & 10 & 6 & 18\\ 0 & 0 & 2 & 3 & 0 \end{array}\right] \tfrac{1}{10}R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4\\ 0 & 1 & 4 & 3 & 8\\ 0 & 0 & 1 & \frac{3}{5} & \frac{9}{5}\\ 0 & 0 & 2 & 3 & 0 \end{array}\right] -2R_3+R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4\\ 0 & 1 & 4 & 3 & 8\\ 0 & 0 & 1 & \frac{3}{5} & \frac{9}{5}\\ 0 & 0 & 0 & \frac{9}{5} & -\frac{18}{5} \end{array}\right] \tfrac{5}{9}R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4\\ 0 & 1 & 4 & 3 & 8\\ 0 & 0 & 1 & \frac{3}{5} & \frac{9}{5}\\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$z=-2$

$$\begin{aligned} y+\frac{3}{5}z&=\frac{9}{5}\\ y+\frac{3}{5}(-2)&=\frac{9}{5}\\ y-\frac{6}{5}&=\frac{9}{5}\\ y&=3 \end{aligned}$$

$$\begin{aligned} x+4y+3z&=8\\ x+4(3)+3(-2)&=8\\ x+6&=8\\ x&=2 \end{aligned}$$

$$\begin{aligned} w+x+y+z&=4\\ w+2+3-2&=4\\ w+3&=4\\ w&=1 \end{aligned}$$

The solution set is {(1, 2, 3, –2)}.

36. $w+x+y+z=5$
$w+2x-y-2z=-1$
$w-3x-3y-z=-1$
$2w-x+2y-z=-2$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 1 & 2 & -1 & -2 & -1 \\ 1 & -3 & -3 & -1 & -1 \\ 2 & -1 & 2 & -1 & -2 \end{array}\right] -1R_1+R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -3 & -6 \\ 1 & -3 & -3 & -1 & -1 \\ 2 & -1 & 2 & -1 & -2 \end{array}\right] -1R_1+R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -3 & -6 \\ 0 & -4 & -4 & -2 & -6 \\ 2 & -1 & 2 & -1 & -2 \end{array}\right] -2R_1+R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -3 & -6 \\ 0 & -4 & -4 & -2 & -6 \\ 0 & -3 & 0 & -3 & -12 \end{array}\right] 4R_2+R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -3 & -6 \\ 0 & 0 & -12 & -14 & -30 \\ 0 & -3 & 0 & -3 & -12 \end{array}\right] 3R_2+R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -3 & -6 \\ 0 & 0 & -12 & -14 & -30 \\ 0 & 0 & -6 & -12 & -30 \end{array}\right] -\tfrac{1}{12}R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -3 & -6 \\ 0 & 0 & 1 & \frac{7}{6} & \frac{5}{2} \\ 0 & 0 & -6 & -12 & -30 \end{array}\right] 6R_3+R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -3 & -6 \\ 0 & 0 & 1 & \frac{7}{6} & \frac{5}{2} \\ 0 & 0 & 0 & -5 & -15 \end{array}\right] -\tfrac{1}{5}R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -3 & -6 \\ 0 & 0 & 1 & \frac{7}{6} & \frac{5}{2} \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$z=3$

$y+\frac{7}{6}(3)=\frac{5}{2}$

$y+\frac{7}{2}=\frac{5}{2}$

$y=-1$

$x-2(-1)-3(3)=-6$

$x+2-9=-6$

$x-7=-6$

$x=1$

$w+1-1+3=5$

$w+3=5$

$w=2$

The solution set is $\{(2, 1, -1, 3)\}$.

37.

$$\begin{aligned} 3w-4x+y+z&=9\\ w+x-y-z&=0\\ 2w+x+4y-2z&=3\\ -w+2x+y-3z&=3 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 3 & -4 & 1 & 1 & 9\\ 1 & 1 & -1 & -1 & 0\\ 2 & 1 & 4 & -2 & 3\\ -1 & 2 & 1 & -3 & 3 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0\\ 3 & -4 & 1 & 1 & 9\\ 2 & 1 & 4 & -2 & 3\\ -1 & 2 & 1 & -3 & 3 \end{array}\right] -3R_1 + R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0\\ 0 & -7 & 4 & 4 & 9\\ 2 & 1 & 4 & -2 & 3\\ -1 & 2 & 1 & -3 & 3 \end{array}\right] -2R_1 + R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0\\ 0 & -7 & 4 & 4 & 9\\ 0 & -1 & 6 & 0 & 3\\ -1 & 2 & 1 & -3 & 3 \end{array}\right] 1R_1 + R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0\\ 0 & -7 & 4 & 4 & 9\\ 0 & -1 & 6 & 0 & 3\\ 0 & 3 & 0 & -4 & 3 \end{array}\right] R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0\\ 0 & -1 & 6 & 0 & 3\\ 0 & -7 & 4 & 4 & 9\\ 0 & 3 & 0 & -4 & 3 \end{array}\right] -R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0\\ 0 & 1 & -6 & 0 & -3\\ 0 & -7 & 4 & 4 & 9\\ 0 & 3 & 0 & -4 & 3 \end{array}\right] 7R_2 + R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0\\ 0 & 1 & -6 & 0 & -3\\ 0 & 0 & -38 & 4 & -12\\ 0 & 3 & 0 & -4 & 3 \end{array}\right] -3R_2 + R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0\\ 0 & 1 & -6 & 0 & -3\\ 0 & 0 & -38 & 4 & -12\\ 0 & 0 & 18 & -4 & 12 \end{array}\right] -\tfrac{1}{38}R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0\\ 0 & 1 & -6 & 0 & -3\\ 0 & 0 & 1 & -\frac{2}{19} & \frac{6}{19}\\ 0 & 0 & 18 & -4 & 12 \end{array}\right] -18R_3 + R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0\\ 0 & 1 & -6 & 0 & -3\\ 0 & 0 & 1 & -\frac{2}{19} & \frac{6}{19}\\ 0 & 0 & 0 & -\frac{40}{19} & \frac{120}{19} \end{array}\right] -\tfrac{19}{40}R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & 0\\ 0 & 1 & -6 & 0 & -3\\ 0 & 0 & 1 & -\frac{2}{19} & \frac{6}{19}\\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$$

$z=-3$

$$\begin{aligned} y-\frac{2}{19}z&=\frac{6}{19}\\ y-\frac{2}{19}(-3)&=\frac{6}{19}\\ y+\frac{6}{19}&=\frac{6}{19}\\ y&=0 \end{aligned}$$

$$\begin{aligned} x-6y&=-3\\ x-6(0)&=-3\\ x&=-3 \end{aligned}$$

$$\begin{aligned} w+x-y-z&=0\\ w-3+0+3&=0\\ w&=0 \end{aligned}$$

The solution set is {(0, –3, 0, –3)}.

38. $2w + y - 3z = 8$
$w - x + 4z = -10$
$3w + 5x - y - z = 20$
$w + x - y - z = 6$

$$\left[\begin{array}{cccc|c} 2 & 0 & 1 & -3 & 8 \\ 1 & -1 & 0 & 4 & -10 \\ 3 & 5 & -1 & -1 & 20 \\ 1 & 1 & -1 & -1 & 6 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 4 & -10 \\ 2 & 0 & 1 & -3 & 8 \\ 3 & 5 & -1 & -1 & 20 \\ 1 & 1 & -1 & -1 & 6 \end{array}\right] -2R_1 + R_2$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 4 & -10 \\ 0 & 2 & 1 & -11 & 28 \\ 3 & 5 & -1 & -1 & 20 \\ 1 & 1 & -1 & -1 & 6 \end{array}\right] -3R_1 + R_3$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 4 & -10 \\ 0 & 2 & 1 & -11 & 28 \\ 0 & 8 & -1 & -13 & 50 \\ 1 & 1 & -1 & -1 & 6 \end{array}\right] -1R_1 + R_4$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 4 & -10 \\ 0 & 2 & 1 & -11 & 28 \\ 0 & 8 & -1 & -13 & 50 \\ 0 & 2 & -1 & -5 & 16 \end{array}\right] \tfrac{1}{2}R_2$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 4 & -10 \\ 0 & 1 & \frac{1}{2} & -\frac{11}{2} & 14 \\ 0 & 8 & -1 & -13 & 50 \\ 0 & 2 & -1 & -5 & 16 \end{array}\right] -8R_2 + R_3$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 4 & -10 \\ 0 & 1 & \frac{1}{2} & -\frac{11}{2} & 14 \\ 0 & 0 & -5 & 31 & -62 \\ 0 & 2 & -1 & -5 & 16 \end{array}\right] -2R_2 + R_4$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 4 & -10 \\ 0 & 1 & \frac{1}{2} & -\frac{11}{2} & 14 \\ 0 & 0 & -5 & 31 & -62 \\ 0 & 0 & -2 & 6 & -12 \end{array}\right] -\tfrac{1}{5}R_3$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 4 & -10 \\ 0 & 1 & \frac{1}{2} & -\frac{11}{2} & 14 \\ 0 & 0 & 1 & -\frac{31}{5} & \frac{62}{5} \\ 0 & 0 & -2 & 6 & -12 \end{array}\right] 2R_3 + R_4$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 4 & -10 \\ 0 & 1 & \frac{1}{2} & -\frac{11}{2} & 14 \\ 0 & 0 & 1 & -\frac{31}{5} & \frac{62}{5} \\ 0 & 0 & 0 & -\frac{32}{5} & \frac{64}{5} \end{array}\right] -\tfrac{5}{32}R_4$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 4 & -10 \\ 0 & 1 & \frac{1}{2} & -\frac{11}{2} & 14 \\ 0 & 0 & 1 & -\frac{31}{5} & \frac{62}{5} \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$z = -2$

$$y - \frac{31}{5}(-2) = \frac{62}{5}$$

$$y + \frac{62}{5} = \frac{62}{5}$$

$$y = 0$$

$$x + \frac{1}{2}y - \frac{11}{2}z = 14$$

$$x + \frac{1}{2}(0) - \frac{11}{2}(-2) = 14$$

$$x + 11 = 14$$

$$x = 3$$

$w - x + 4z = -10$
$w - 3 + 4(-2) = -10$
$w - 11 = -10$
$w = 1$

The solution set is $\{(1, 3, 0, -2)\}$.

39. $f(x) = ax^2 + bx + c$

Use the given function values to find three equations in terms of a, b, and c.

$f(-2) = a(-2)^2 + b(-2) + c = -4$
$4a - 2b + c = -4$
$f(1) = a(1)^2 + b(1) + c = 2$
$a + b + c = 2$
$f(2) = a(2)^2 + b(2) + c = 0$
$4a + 2b + c = 0$

System of equations:
$4a - 2b + c = -4$
$a + b + c = 2$
$4a + 2b + c = 0$

Matrix:

$$\left[\begin{array}{ccc|c} 4 & -2 & 1 & -4 \\ 1 & 1 & 1 & 2 \\ 4 & 2 & 1 & 0 \end{array}\right]$$

This gives $a = -1$, $b = 1$, and $c = 2$.

Thus, $f(x) = -x^2 + x + 2$.

40. $f(x) = ax^2 + bx + c$

Use the given function values to find three equations in terms of *a*, *b*, and *c*.

$f(-1) = a(-1)^2 + b(-1) + c = 5$
$a - b + c = 5$
$f(1) = a(1)^2 + b(1) + c = 3$
$a + b + c = 3$
$f(2) = a(2)^2 + b(2) + c = 5$
$4a + 2b + c = 5$

System of equations:

$$\begin{aligned} a - b + c &= 5 \\ a + b + c &= 3 \\ 4a + 2b + c &= 5 \end{aligned}$$

Matrix:

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 5 \\ 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 5 \end{array}\right]$$

This gives $a = 1$, $b = -1$, and $c = 3$.

Thus, $f(x) = x^2 - x + 3$.

41. $f(x) = ax^3 + bx^2 + cx + d$

Use the given function values to find four equations in terms of *a*, *b*, *c*, and *d*.

$f(-1) = a(-1)^3 + b(-1)^2 + c(-1) + d = 0$
$-a + b - c + d = 0$
$f(1) = a(1)^3 + b(1)^2 + c(1) + d = 2$
$a + b + c + d = 2$
$f(2) = a(2)^3 + b(2)^2 + c(2) + d = 3$
$8a + 4b + 2c + d = 3$
$f(3) = a(3)^3 + b(3)^2 + c(3) + d = 12$
$27a + 9b + 3c + d = 12$

System of equations:

$$\begin{aligned} -a + b - c + d &= 0 \\ a + b + c + d &= 2 \\ 8a + 4b + 2c + d &= 3 \\ 27a + 9b + 3c + d &= 12 \end{aligned}$$

Matrix:

$$\left[\begin{array}{cccc|c} -1 & 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 2 \\ 8 & 4 & 2 & 1 & 3 \\ 27 & 9 & 3 & 1 & 12 \end{array}\right]$$

This gives $a = 1$, $b = -2$, $c = 0$, and $d = 3$.

Thus, $f(x) = x^3 - 2x^2 + 3$.

42. $f(x) = ax^3 + bx^2 + cx + d$

Use the given function values to find four equations in terms of *a*, *b*, *c*, and *d*.

$f(-1) = a(-1)^3 + b(-1)^2 + c(-1) + d = 3$
$-a + b - c + d = 3$
$f(1) = a(1)^3 + b(1)^2 + c(1) + d = 1$
$a + b + c + d = 1$
$f(2) = a(2)^3 + b(2)^2 + c(2) + d = 6$
$8a + 4b + 2c + d = 6$
$f(3) = a(3)^3 + b(3)^2 + c(3) + d = 7$
$27a + 9b + 3c + d = 7$

System of equations:

$$\begin{aligned} -a + b - c + d &= 3 \\ a + b + c + d &= 1 \\ 8a + 4b + 2c + d &= 6 \\ 27a + 9b + 3c + d &= 7 \end{aligned}$$

Matrix:

$$\left[\begin{array}{cccc|c} -1 & 1 & -1 & 1 & 3 \\ 1 & 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 & 6 \\ 27 & 9 & 3 & 1 & 7 \end{array}\right]$$

This gives $a = -1$, $b = 4$, $c = 0$, and $d = -2$.

Thus, $f(x) = -x^3 + 4x^2 - 2$.

43. Let $A = \ln w$, $B = \ln x$, $C = \ln y$, and $D = \ln z$.

System of equations:

$$\begin{aligned} 2A + B + 3C - 2D &= -6 \\ 4A + 3B + C - D &= -2 \\ A + B + C + D &= -5 \\ A + B - C - D &= 5 \end{aligned}$$

Matrix:

$$\left[\begin{array}{cccc|c} 2 & 1 & 3 & -2 & -6 \\ 4 & 3 & 1 & -1 & -2 \\ 1 & 1 & 1 & 1 & -5 \\ 1 & 1 & -1 & -1 & 5 \end{array}\right]$$

This gives $A = -1$, $B = 1$, $C = -3$, and $D = -2$.

Substitute back to find *w*, *x*, *y*, and *z*.

$A = -1$	$B = 1$
$\ln w = -1$	$\ln x = 1$
$w = e^{-1}$	$x = e^1$
$w \approx 0.37$	$x \approx 2.72$
$C = -3$	$D = -2$
$\ln y = -3$	$\ln z = -2$
$y = e^{-3}$	$z = e^{-2}$
$y \approx 0.05$	$z \approx 0.14$

44. Let $A = \ln w,\ B = \ln x,\ C = \ln y,$ and $D = \ln z.$

System of equations:

$$A + B + C + D = -1$$
$$-A + 4B + C - D = 0$$
$$A - 2B + C - 2D = 11$$
$$-A - 2B + C + 2D = -3$$

Matrix:

$$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -1 \\ -1 & 4 & 1 & -1 & 0 \\ 1 & -2 & 1 & -2 & 11 \\ -1 & -2 & 1 & 2 & -3 \end{array}\right]$$

This gives $A = 1,\ B = -1,\ C = 2,$ and $D = -3.$

Substitute back to find w, x, y, and z.

$A = 1 \qquad B = -1$

$\ln w = 1 \qquad \ln x = -1$

$w = e^1 \qquad x = e^{-1}$

$w \approx 2.72 \qquad x \approx 0.37$

$C = 2 \qquad D = -3$

$\ln y = 2 \qquad \ln z = -3$

$y = e^2 \qquad z = e^{-3}$

$y \approx 7.39 \qquad z \approx 0.05$

45. a. $s(t) = \frac{1}{2}at^2 + v_0 t + s_0$

Use the given function values to find three equations in terms of a, v_0, and s_0.

$$s(1) = \frac{1}{2}a(1)^2 + v_0(1) + s_0 = 40$$
$$\frac{1}{2}a + v_0 + s_0 = 40$$
$$s(2) = \frac{1}{2}a(2)^2 + v_0(2) + s_0 = 48$$
$$2a + 2v_0 + s_0 = 48$$
$$s(3) = \frac{1}{2}a(3)^2 + v_0(3) + s_0 = 24$$
$$\frac{9}{2}a + 3v_0 + s_0 = 24$$

System of equations:

$$\frac{1}{2}a + v_0 + s_0 = 40$$
$$2a + 2v_0 + s_0 = 48$$
$$\frac{9}{2}a + 3v_0 + s_0 = 24$$

Matrix:

$$\left[\begin{array}{ccc|c} \frac{1}{2} & 1 & 1 & 40 \\ 2 & 2 & 1 & 48 \\ \frac{9}{2} & 3 & 1 & 24 \end{array}\right]$$

This gives $a = -32,\ v_0 = 56,$ and $s_0 = 0.$

Thus, $s(t) = \frac{1}{2}(-32)t^2 + (56)t + (0)$

$s(t) = -16t^2 + 56t$

b. $s(t) = -16t^2 + 56t$

$s(3.5) = -16(3.5)^2 + 56(3.5) = 0$

This is the point (3.5, 0).
The ball's height is 0 feet after 3.5 seconds.
This is the point (3.5, 0).

c. The maximum occurs when $x = -\frac{b}{2a}$.

$$x = -\frac{b}{2a} = -\frac{v_0}{2a} = -\frac{56}{2(-16)} = 1.75$$

$s(1.75) = -16(1.75)^2 + 56(1.75) = 49$

At 1.75 seconds the ball will reach its maximum height of 49 feet.

46. a. $s(t) = \frac{1}{2}at^2 + v_0 t + s_0$

Use the given function values to find three equations in terms of a, v_0, and s_0.

$$s(2) = \frac{1}{2}a(2)^2 + v_0(2) + s_0 = 198$$
$$2a + 2v_0 + s_0 = 198$$
$$s(5) = \frac{1}{2}a(5)^2 + v_0(5) + s_0 = 246$$
$$\frac{25}{2}a + 5v_0 + s_0 = 246$$
$$s(8) = \frac{1}{2}a(8)^2 + v_0(8) + s_0 = 6$$
$$32a + 8v_0 + s_0 = 6$$

System of equations:

$$2a + 2v_0 + s_0 = 198$$

$$\frac{25}{2}a + 5v_0 + s_0 = 246$$

$$32a + 8v_0 + s_0 = 6$$

Matrix:

$$\left[\begin{array}{ccc|c} 2 & 2 & 1 & 198 \\ \frac{25}{2} & 5 & 1 & 246 \\ 32 & 8 & 1 & 6 \end{array}\right]$$

This gives $a = -32$, $v_0 = 128$, and $s_0 = 6$.

Thus, $s(t) = \frac{1}{2}(-32)t^2 + (128)t + (6)$

$s(t) = -16t^2 + 128t + 6$

b. $s(t) = -16t^2 + 128t + 6$

$s(7) = -16(7)^2 + 128(7) + 6 = 118$

The ball's height is 118 feet after 7 seconds. This is the point (7, 118).

c. The maximum occurs when $x = -\frac{b}{2a}$.

$$x = -\frac{b}{2a} = -\frac{v_0}{2a} = -\frac{128}{2(-16)} = 4$$

$s(4) = -16(4)^2 + 128(4) + 6 = 262$

At 4 seconds the ball will reach its maximum height of 262 feet.

47. Let x = Food *A*
Let y = Food *B*
Let z = Food *C*

$$40x + 200y + 400z = 660$$
$$5x + 2y + 4z = 25$$
$$30x + 10y + 300z = 425$$
$$2x + 10y + 20z = 33$$
$$5x + 2y + 4z = 25$$
$$6x + 2y + 60z = 85$$

$$\left[\begin{array}{ccc|c} 2 & 10 & 20 & 33 \\ 5 & 2 & 4 & 25 \\ 6 & 2 & 60 & 85 \end{array}\right] \tfrac{1}{2}R_1$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 10 & \frac{33}{2} \\ 5 & 2 & 4 & 25 \\ 6 & 2 & 60 & 85 \end{array}\right] -5R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 10 & \frac{33}{2} \\ 0 & -23 & -46 & -\frac{115}{2} \\ 6 & 2 & 60 & 85 \end{array}\right] -6R_1 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 10 & \frac{33}{2} \\ 0 & -23 & -46 & -\frac{115}{2} \\ 0 & -28 & 0 & -14 \end{array}\right] -\tfrac{1}{23}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 10 & \frac{33}{2} \\ 0 & 1 & 2 & \frac{5}{2} \\ 0 & -28 & 0 & -14 \end{array}\right] 28R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 10 & \frac{33}{2} \\ 0 & 1 & 2 & \frac{5}{2} \\ 0 & 0 & 56 & 56 \end{array}\right] \tfrac{1}{56}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 10 & \frac{33}{2} \\ 0 & 1 & 2 & \frac{5}{2} \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$z = 1$$
$$y + 2z = \frac{5}{2}$$
$$y + 2 = \frac{5}{2}$$
$$2y + 4 = 5$$
$$2y = 1$$
$$y = \frac{1}{2}$$
$$x + 5y + 10z = \frac{33}{2}$$
$$x + \frac{5}{2} + 10 = \frac{33}{2}$$
$$2x + 5 + 20 = 33$$
$$2x + 25 = 33$$
$$2x = 8$$
$$x = 4$$

4 ounces of Food *A*

$\frac{1}{2}$ ounce of Food *B*

1 ounce of Food *C*

48. Let x = Children's model
Let y = Office model
Let z = Deluxe model

$$2x+3y+2z=100$$
$$2x+y+3z=100$$
$$x+y+2z=65$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 2 & 100 \\ 2 & 1 & 3 & 100 \\ 1 & 1 & 2 & 65 \end{array}\right] R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 65 \\ 2 & 1 & 3 & 100 \\ 2 & 3 & 2 & 100 \end{array}\right] -2R_1+R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 65 \\ 0 & -1 & -1 & -30 \\ 2 & 3 & 2 & 100 \end{array}\right] -2R_1+R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 65 \\ 0 & -1 & -1 & -30 \\ 0 & 1 & -2 & -30 \end{array}\right] -1R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 65 \\ 0 & 1 & 1 & 30 \\ 0 & 1 & -2 & -30 \end{array}\right] -1R_2+R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 65 \\ 0 & 1 & 1 & 30 \\ 0 & 0 & -3 & -60 \end{array}\right] -\tfrac{1}{3}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 65 \\ 0 & 1 & 1 & 30 \\ 0 & 0 & 1 & 20 \end{array}\right]$$

$$z=20$$
$$y+z=30$$
$$y+20=30$$
$$y=10$$

$$x+y+2z=65$$
$$x+10+2(20)=65$$
$$x+50=65$$
$$x=15$$

15 children's models
10 office models
20 deluxe models

49. Let w = number of Asians
Let x = number of Africans
Let y = number of Europeans
Let z = number of Americans
Use the variables to model each sentence.

$$w+x+y+z=183$$
$$w-x-y=70$$
$$y-z=15$$
$$2x-y-z=23$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 183 \\ 1 & -1 & -1 & 0 & 70 \\ 0 & 0 & 1 & -1 & 15 \\ 0 & 2 & -1 & -1 & 23 \end{array}\right] -R_1+R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 183 \\ 0 & -2 & -2 & -1 & -113 \\ 0 & 0 & 1 & -1 & 15 \\ 0 & 2 & -1 & -1 & 23 \end{array}\right] R_4+R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 183 \\ 0 & -2 & -2 & -1 & -113 \\ 0 & 0 & 1 & -1 & 15 \\ 0 & 0 & -3 & -2 & -90 \end{array}\right] -\frac{1}{2}R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 183 \\ 0 & 1 & 1 & \frac{1}{2} & \frac{113}{2} \\ 0 & 0 & 1 & -1 & 15 \\ 0 & 0 & -3 & -2 & -90 \end{array}\right] 3R_3+R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 183 \\ 0 & 1 & 1 & \frac{1}{2} & \frac{113}{2} \\ 0 & 0 & 1 & -1 & 15 \\ 0 & 0 & 0 & -5 & -45 \end{array}\right] -\frac{1}{5}R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 183 \\ 0 & 1 & 1 & \frac{1}{2} & \frac{113}{2} \\ 0 & 0 & 1 & -1 & 15 \\ 0 & 0 & 0 & 1 & 9 \end{array}\right]$$

Back-substitute $z=9$ to find y.

$$y-z=15$$
$$y-9=15$$
$$y=24$$

Back-substitute to find x.

$$x + y + \frac{1}{2}z = \frac{113}{2}$$
$$x + 24 + \frac{1}{2}(9) = \frac{113}{2}$$
$$x + 24 + \frac{1}{2}(9) = \frac{113}{2}$$
$$x + 28\frac{1}{2} = 56\frac{1}{2}$$
$$x = 28$$

Back-substitute to find w.

$$w + x + y + z = 183$$
$$w + 28 + 24 + 9 = 183$$
$$w + 28 + 24 + 9 = 183$$
$$w + 61 = 183$$
$$w = 122$$

The number of Asians, Africans, Europeans, and Americans are 122, 28, 24, and 9, respectively.

50. Let w = number of rooms
Let x = number of bathrooms
Let y = number of fireplaces
Let z = number of elevators
Use the variables to model each sentence.

$$w + x + y + z = 198$$
$$w - x - y = 69$$
$$y - z = 25$$
$$2x - y - z = 39$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 198 \\ 1 & -1 & -1 & 0 & 69 \\ 0 & 0 & 1 & -1 & 25 \\ 0 & 2 & -1 & -1 & 39 \end{array}\right] -R_1 + R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 198 \\ 0 & -2 & -2 & -1 & -129 \\ 0 & 0 & 1 & -1 & 25 \\ 0 & 2 & -1 & -1 & 39 \end{array}\right] R_4 + R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 198 \\ 0 & -2 & -2 & -1 & -129 \\ 0 & 0 & 1 & -1 & 25 \\ 0 & 0 & -3 & -2 & -90 \end{array}\right] -\frac{1}{2}R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 198 \\ 0 & 1 & 1 & \frac{1}{2} & \frac{129}{2} \\ 0 & 0 & 1 & -1 & 25 \\ 0 & 0 & -3 & -2 & -90 \end{array}\right] 3R_3 + R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 198 \\ 0 & 1 & 1 & \frac{1}{2} & \frac{129}{2} \\ 0 & 0 & 1 & -1 & 25 \\ 0 & 0 & 0 & -5 & -15 \end{array}\right] -\frac{1}{5}R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 198 \\ 0 & 1 & 1 & \frac{1}{2} & \frac{129}{2} \\ 0 & 0 & 1 & -1 & 25 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

Back-substitute $z = 3$ to find y.

$$y - z = 25$$
$$y - 3 = 25$$
$$y = 28$$

Back-substitute to find x.

$$x + y + \frac{1}{2}z = \frac{129}{2}$$
$$x + 28 + \frac{1}{2}(3) = \frac{129}{2}$$
$$x + 28 + \frac{1}{2}(3) = \frac{129}{2}$$
$$x + 29\frac{1}{2} = 64\frac{1}{2}$$
$$x = 35$$

Back-substitute to find w.

$$w + x + y + z = 198$$
$$w + 35 + 28 + 3 = 198$$
$$w + 66 = 198$$
$$w = 132$$

The number of rooms, bathrooms, fireplaces, and elevators are 132, 35, 28, and 3, respectively.

51. – 57. Answers may vary.

58. {(1, –1, 2, –2, 0)}

59. makes sense

60. makes sense

61. makes sense

62. makes sense

63. false; Changes to make the statement true will vary. A sample change is: Multiplying a row by a negative fraction is permitted.

64. false; Changes to make the statement true will vary. A sample change is: The augmented matrix should be $\left[\begin{array}{rrr|r} 1 & -3 & 0 & 5 \\ 0 & 1 & -2 & 7 \\ 2 & 0 & 1 & 4 \end{array}\right]$.

65. false; Changes to make the statement true will vary. A sample change is: When solving a system of three equations in three variables, we use row operations to obtain ones along the diagonal and zeros below the ones.

66. false; Changes to make the statement true will vary. A sample change is: This indicates you should add k times the elements in row i to the corresponding elements in row j.

67.
$$y = ax^2 + bx + c$$
$$5900 = a(30)^2 + b(30) + c$$
$$5900 = 900a + 30b + c$$
$$7500 = a(50)^2 + b(50) + c$$
$$7500 = 2500a + 50b + c$$
$$4500 = a(100)^2 + b(100) + c$$
$$4500 = 10{,}000a + 100b + c$$
$$900a + 30b + c = 5900$$
$$2500a + 50b + c = 7500$$
$$10{,}000a + 100b + c = 4500$$
$$\left[\begin{array}{rrr|r} 900 & 30 & 1 & 5900 \\ 2500 & 50 & 1 & 7500 \\ 10{,}000 & 100 & 1 & 4500 \end{array}\right] R_1 \leftrightarrow R_2$$
$$\left[\begin{array}{rrr|r} 2500 & 50 & 1 & 7500 \\ 900 & 30 & 1 & 5900 \\ 10{,}000 & 100 & 1 & 4500 \end{array}\right] \frac{1}{2500} R_1$$
$$\left[\begin{array}{rrr|r} 1 & \frac{1}{50} & \frac{1}{2500} & 3 \\ 900 & 30 & 1 & 5900 \\ 10{,}000 & 100 & 1 & 4500 \end{array}\right] \begin{array}{l} -900R_1 + R_2 \\ -10{,}000R_1 + R_3 \end{array}$$
$$\left[\begin{array}{rrr|r} 1 & \frac{1}{50} & \frac{1}{2500} & 3 \\ 0 & 12 & \frac{16}{25} & 3200 \\ 0 & -100 & -3 & -25{,}500 \end{array}\right] R_2 \leftrightarrow R_3$$
$$\left[\begin{array}{rrr|r} 1 & \frac{1}{50} & \frac{1}{2500} & 3 \\ 0 & -100 & -3 & -25{,}500 \\ 0 & 12 & \frac{16}{25} & 3200 \end{array}\right] \frac{-1}{100} R_2$$
$$\left[\begin{array}{rrr|r} 1 & \frac{1}{50} & \frac{1}{2500} & 3 \\ 0 & 1 & \frac{3}{100} & 255 \\ 0 & 12 & \frac{16}{25} & 3200 \end{array}\right] \begin{array}{l} -12R_2 + R_3 \\ -\frac{1}{50} R_2 + R_1 \end{array}$$
$$\left[\begin{array}{rrr|r} 1 & 0 & \frac{-1}{5000} & \frac{-21}{10} \\ 0 & 1 & \frac{3}{100} & 255 \\ 0 & 0 & \frac{7}{25} & 140 \end{array}\right] \frac{25}{7} R_3$$
$$\left[\begin{array}{rrr|r} 1 & 0 & \frac{-1}{5000} & \frac{-21}{10} \\ 0 & 1 & \frac{3}{100} & 255 \\ 0 & 0 & 1 & 500 \end{array}\right] \begin{array}{l} \frac{-3}{100} R_3 + R_2 \\ \frac{1}{5000} R_3 + R_1 \end{array}$$
$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 240 \\ 0 & 0 & 1 & 500 \end{array}\right]$$
$$y = -2x^2 + 240x + 500$$
$$y = -2(x^2 - 120x) + 500$$
$$y = -2(x^2 - 120x + 3600) + 500 + 7200$$
$$y = -2(x - 60)^2 + 7700$$
60 units produce $7700.

68. When $z = 0$, $(12z + 1,\ 10z - 1,\ z)$ is equivalent to $(12(0) + 1,\ 10(0) - 1,\ 0)$ or $(1,\ -1,\ 0)$.
Check $(1,\ -1,\ 0)$ in each equation.
$$3x - 4y + 4z = 7$$
$$3(1) - 4(-1) + 4(0) = 7$$
$$7 = 7, \text{ true}$$
$$x - y - 2z = 2$$
$$1 - (-1) - 2(0) = 2$$
$$2 = 2, \text{ true}$$
$$2x - 3y - 2z = 5$$
$$2(1) - 3(-1) + 6(0) = 5$$
$$5 = 5, \text{ true}$$
$(1,\ -1,\ 0)$ satisfies each equation and, therefore, satisfies the system.

69. When $z = 1$, $(12z + 1,\ 10z - 1,\ z)$ is equivalent to $(12(1) + 1,\ 10(1) - 1,\ 1)$ or $(13,\ 9,\ 1)$.
Check $(13,\ 9,\ 1)$ in each equation.
$$3x - 4y + 4z = 7$$
$$3(13) - 4(9) + 4(1) = 7$$
$$7 = 7, \text{ true}$$
$$x - y - 2z = 2$$
$$13 - (9) - 2(1) = 2$$
$$2 = 2, \text{ true}$$
$$2x - 3y - 2z = 5$$
$$2(13) - 3(9) + 6(1) = 5$$
$$5 = 5, \text{ true}$$
$(13,\ 9,\ 1)$ satisfies each equation and, therefore, satisfies the system.

70. **a.** Answers may vary. A sample answer is given selecting $z = 10$.

When $z = 10$, $(12z+1,\ 10z-1,\ z)$ is equivalent to $(12(10)+1,\ 10(10)-1,\ 10)$ or $(121,\ 99,\ 10)$.

Check $(121,\ 99,\ 10)$ in each equation.

$$3x - 4y + 4z = 7$$
$$3(121) - 4(99) + 4(10) = 7$$
$$7 = 7, \text{ true}$$
$$x - y - 2z = 2$$
$$121 - (99) - 2(10) = 2$$
$$2 = 2, \text{ true}$$
$$2x - 3y - 2z = 5$$
$$2(121) - 3(99) + 6(10) = 5$$
$$5 = 5, \text{ true}$$

$(121,\ 99,\ 10)$ satisfies each equation and, therefore, satisfies the system.

b. This system has more than one solution.

Section 9.2

Check Point Exercises

1.
$$\begin{aligned} x-2y-z&=5\\ 2x-3y-z&=0\\ 3x-4y-z&=1 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 2 & -3 & -1 & 0 \\ 3 & -4 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 2 & -3 & -1 & 0 \\ 3 & -4 & -1 & 1 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \\ -3R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 1 & 1 & 10 \\ 0 & 2 & 2 & 16 \end{array}\right] -2R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -5 \\ 0 & 1 & -1 & -10 \\ 0 & 0 & 0 & -4 \end{array}\right]$$

$0x + 0y + 0z = -4$ This equation can never be a true statement. Consequently, the system has no solution. The solution set is $\varnothing$, the empty set.

2.
$$\begin{aligned} x-2y-z&=5\\ 2x-5y+3z&=16\\ x-3y+4z&=1 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -1 & 5 \\ 2 & -5 & 3 & 6 \\ 1 & -3 & 4 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 5 \\ 2 & -5 & 3 & 6 \\ 1 & -3 & 4 & 1 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \\ -1R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 5 \\ 0 & -1 & 5 & -4 \\ 0 & -1 & 5 & -4 \end{array}\right] -1R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 5 \\ 0 & 1 & -5 & 4 \\ 0 & -1 & 5 & -4 \end{array}\right] 1R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 5 \\ 0 & 1 & -5 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$0x + 0y + 0z = 0$ or $0 = 0$

This equation, $0x + 0y + 0z = 0$ is *dependent* on the other two equations. Thus, it can be dropped from the system which can now be expressed in the form

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 5 \\ 0 & 1 & -5 & 4 \end{array}\right]$$

The original system is equivalent to the system

$$x - 2y - z = 5$$
$$y - 5z = 4$$

Solve for *x* and *y* in terms of *z*.

$$y = 5z + 4$$

Use back-substitution for *y* in the previous equation.

$$x - 2(5z+4) - z = 5$$
$$x - 10z - 8 - z = 5$$
$$x = 11z + 13$$

Finally, letting $z = t$ (or any letter of your choice), the solutions to the system are all of the form $x = 11t + 13$, $y = 5t + 4$, $z = t$,
where *t* is a real number. The solution set of the system with dependent equations can be written as $\{(11t+13,\ 5t+4,\ t)\}$.

3. $\begin{matrix} x+2y+3z=70 \\ x+y+z=60 \end{matrix} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 70 \\ 1 & 1 & 1 & 60 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 70 \\ 1 & 1 & 1 & 60 \end{array}\right] -1R_1 + R_2$

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 70 \\ 0 & -1 & -2 & -10 \end{array}\right] -1R_2$

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 70 \\ 0 & 1 & 2 & 10 \end{array}\right] \rightarrow \begin{matrix} x+2y+3z=70 \\ y+2z=10 \end{matrix}$

Express x and y in terms of z using back-substitution.

$y=-2z+10$

$x+2(-2z+10)+3z=70$

$x-4z+20+3z=70$

$x=z+50$

With $z = t$, the ordered solution (x, y, z) enables us to express the system's solution set as $\{(t+50, -2t+10, t)\}$.

4. **a.** I_1 : 10 + 5 = 15 cars enter I_1, and $w + z$ cars leave I_1, then $w + z = 15$.

I_2 : 20 + 10 = 30 cars enter I_2 and $w + x$ cars leave I_2, then $w + x = 30$.

I_3 : 15 + 30 = 45 cars enter I_3 and $x + y$ cars leave I_3, then $x + y = 45$.

I_4 : 10 + 20 = 30 cars enter I_4 and $y + z$ cars leave I_4, then $y + z = 30$.

The system of equations that describes this situation is given by

$w+z=15$

$w+x=30$

$x+y=45$

$y+z=30$

b. $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 15 \\ 1 & 1 & 0 & 0 & 30 \\ 0 & 1 & 1 & 0 & 45 \\ 0 & 0 & 1 & 1 & 30 \end{array}\right] -1R_1 + R_2$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 15 \\ 0 & 1 & 0 & -1 & 15 \\ 0 & 1 & 1 & 0 & 45 \\ 0 & 0 & 1 & 1 & 30 \end{array}\right] -1R_2 + R_3$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 15 \\ 0 & 1 & 0 & -1 & 15 \\ 0 & 0 & 1 & 1 & 30 \\ 0 & 0 & 1 & 1 & 30 \end{array}\right] -1R_3 + R_4$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 15 \\ 0 & 1 & 0 & -1 & 15 \\ 0 & 0 & 1 & 1 & 30 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$x+w=15$

$y-w=15$

$z+w=30$

The last row of the matrix shows that the system has dependent equations and infinitely many solutions.

Let z be any real number.

Express w, x and y in terms of z:

$w=15-z$

$x=15+z$

$y=30-z$

With $w = t$, the ordered solution (w, x, y, z) enables us to express the system's solution set as $\{(-t+15, t+15, -t+30, t)\}$

Exercise Set 9.2

1. $\left[\begin{array}{ccc|c} 5 & 12 & 1 & 10 \\ 2 & 5 & 2 & -1 \\ 1 & 2 & -3 & 5 \end{array}\right] R_1 \leftrightarrow R_3$

$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 2 & 5 & 2 & -1 \\ 5 & 12 & 1 & 10 \end{array}\right] \begin{matrix} \\ -2R_1 + R_2 \\ -5R_1 + R_3 \end{matrix}$

$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 1 & 8 & -11 \\ 0 & 2 & 16 & -15 \end{array}\right] -2R_2 + R_3$

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 8 & -11 \\ 0 & 0 & 0 & 7 \end{array}\right]$

From the last row, we see that the system has no solution. The solution set is $\varnothing$, the empty set.

2.

$$\left[\begin{array}{ccc|c} 2 & -4 & 1 & 3 \\ 1 & -3 & 1 & 5 \\ 3 & -7 & 2 & 12 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 5 \\ 2 & -4 & 1 & 3 \\ 3 & -7 & 2 & 12 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \\ -3R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 5 \\ 0 & 2 & -1 & -7 \\ 0 & 2 & -1 & -3 \end{array}\right] -R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 5 \\ 0 & 2 & -1 & -7 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

From the last row, we see that the system has no solution. Then solution set is $\varnothing$.

3.

$$\left[\begin{array}{ccc|c} 5 & 8 & -6 & 14 \\ 3 & 4 & -2 & 8 \\ 1 & 2 & -2 & 3 \end{array}\right] R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 3 & 4 & -2 & 8 \\ 5 & 8 & -6 & 14 \end{array}\right] \begin{array}{l} -3R_1 + R_2 \\ -5R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & -2 & 4 & -1 \\ 0 & -2 & 4 & -1 \end{array}\right] -1R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & -2 & 4 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] -\frac{1}{2}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & 1 & -2 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The system $\begin{array}{l} x + 2y - 2z = 3 \\ y - 2z = \frac{1}{2} \end{array}$ has no unique solution.

Express x and y in terms of z:

$y = 2z + \frac{1}{2}$

$x + 2\left(2z + \frac{1}{2}\right) - 2z = 3$

$x + 4z + 1 - 2z = 3$

$x + 2z + 1 = 3$

$x = -2z + 2$

With $z = t$, the complete solution to the system is $\left\{\left(-2t + 2,\ 2t + \frac{1}{2},\ t\right)\right\}$.

4.

$$\left[\begin{array}{ccc|c} 5 & -11 & 6 & 12 \\ -1 & 3 & -2 & -4 \\ 3 & -5 & 2 & 4 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} -1 & 3 & -2 & -4 \\ 5 & -11 & 6 & 12 \\ 3 & -5 & 2 & 4 \end{array}\right] -1R_1$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 5 & -11 & 6 & 12 \\ 3 & -5 & 2 & 4 \end{array}\right] \begin{array}{l} 5R_1 + R_2 \\ -3R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 0 & 4 & -4 & -8 \\ 0 & 4 & -4 & -8 \end{array}\right] -1R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 0 & 4 & -4 & -8 \\ 0 & 0 & 0 & 0 \end{array}\right] \tfrac{1}{4}R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 4 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The system $\begin{array}{l} x - 3y + 2z = 4 \\ y - z = -2 \end{array}$ has no unique solution.

Express x and y in terms of z:

$y = -2 + z$

$x - 3(-2 + z) + 2z = 4$

$x + 6 - 3z + 2z = 4$

$x + 6 - z = 4$

$x = -2 + z$

With $z = t$, the complete solution to the system is $\{(-2 + t, -2 + t, t)\}$.

5. $\left[\begin{array}{ccc|c} 3 & 4 & 2 & 3 \\ 4 & -2 & -8 & -4 \\ 1 & 1 & -1 & 3 \end{array}\right] R_1 \leftrightarrow R_3$

$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 4 & -2 & -8 & -4 \\ 3 & 4 & 2 & 3 \end{array}\right] \begin{array}{l} -4R_1 + R_2 \\ -3R_1 + R_3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -6 & -4 & -16 \\ 0 & 1 & 5 & -6 \end{array}\right] R_2 \leftrightarrow R_3$

$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 1 & 5 & -6 \\ 0 & -6 & -4 & -16 \end{array}\right] 6R_2 + R_3$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 26 & -52 \end{array}\right] \frac{1}{26} R_3$

$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 1 & -2 \end{array}\right]$

This corresponds to the system

$x + y - z = 3$

$y + 5z = -6$

$z = -2$

Use back-substitution to find the values of x and y:

$y + 5(-2) = -6$

$y - 10 = -6$

$y = 4$

$x + 4 + 2 = 3$

$x + 6 = 3$

$x = -3$

The solution to the system is $\{(-3, 4, -2)\}$.

6. $\left[\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ 1 & 2 & 1 & 3 \\ 3 & 4 & 2 & 8 \end{array}\right] R_1 \leftrightarrow R_2$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & -1 & -1 & 0 \\ 3 & 4 & 2 & 8 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \\ -3R_1 + R_3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -5 & -3 & -6 \\ 0 & -2 & -1 & -1 \end{array}\right] -\tfrac{1}{5} R_2$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & \tfrac{3}{5} & \tfrac{6}{5} \\ 0 & -2 & -1 & -1 \end{array}\right] 2R_2 + R_3$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & \tfrac{3}{5} & \tfrac{6}{5} \\ 0 & 0 & \tfrac{1}{5} & \tfrac{7}{5} \end{array}\right] 5R_3$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & \tfrac{3}{5} & \tfrac{6}{5} \\ 0 & 0 & 1 & 7 \end{array}\right]$

This corresponds to the system

$x + 2y + z = 3$

$y + \frac{3}{5}z = \frac{6}{5}$

$z = 7$

Use back substitution to find the values of x and y:

$y + \frac{3}{5}(7) = \frac{6}{5}$

$y + \frac{21}{5} = \frac{6}{5}$

$y = -3$

$x + 2(-3) + 7 = 3$

$x - 6 + 7 = 3$

$x + 1 = 3$

$x = 2$

The solution to the system is $\{(2, -3, 7)\}$.

7. $\left[\begin{array}{ccc|c} 8 & 5 & 11 & 30 \\ -1 & -4 & 2 & 3 \\ 2 & -1 & 5 & 12 \end{array}\right] R_1 \leftrightarrow R_2$

$\left[\begin{array}{ccc|c} -1 & -4 & 2 & 3 \\ 8 & 5 & 11 & 30 \\ 2 & -1 & 5 & 12 \end{array}\right] -1R_1$

$\left[\begin{array}{ccc|c} 1 & 4 & -2 & -3 \\ 8 & 5 & 11 & 30 \\ 2 & -1 & 5 & 12 \end{array}\right] \begin{array}{l} -8R_1 + R_2 \\ -2R_1 + R_3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 4 & -2 & -3 \\ 0 & -27 & 27 & 54 \\ 0 & -9 & 9 & 18 \end{array}\right] -\frac{1}{27}R_2$

$\left[\begin{array}{ccc|c} 1 & 4 & -2 & -3 \\ 0 & 1 & -1 & -2 \\ 0 & -9 & 9 & 18 \end{array}\right] 9R_2 + R_3$

$\left[\begin{array}{ccc|c} 1 & 4 & -2 & -3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

The system $\begin{array}{r} x + 4y - 2z = -3 \\ y - z = -2 \end{array}$ has no unique solution.

Express x and y in terms of z:

$y = -2 + z$

$x + 4(-2 + z) - 2z = -3$

$x - 8 + 4z - 2z = -3$

$x - 8 + 2z = -3$

$x = 5 - 2z$

With $z = t$, the complete solution to the system is $\{(5 - 2t, -2 + t, t)\}$.

8. $\left[\begin{array}{ccc|c} 1 & 1 & -10 & -4 \\ 1 & 0 & -7 & -5 \\ 3 & 5 & -36 & -10 \end{array}\right] \begin{array}{l} -R_1 + R_2 \\ \hline -3R_1 + R_3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 1 & -10 & 4 \\ 0 & -1 & 3 & -1 \\ 0 & 2 & -6 & 2 \end{array}\right] -1R_2$

$\left[\begin{array}{ccc|c} 1 & 1 & -10 & 4 \\ 0 & 1 & -3 & 1 \\ 0 & 2 & -6 & 2 \end{array}\right] -2R_2 + R_3$

$\left[\begin{array}{ccc|c} 1 & 1 & -10 & -4 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

The system $\begin{array}{r} x + y - 10z = -4 \\ y - 3z = 1 \end{array}$ has no unique solution.

Express x and y in terms of z:

$y = 1 + 3z$

$x + (1 + 3z) - 10z = -4$

$x + 1 - 7z = -4$

$x = -5 + 7z$

With $z = t$, the complete solution to the system is $\{(-5 + 7t, 1 + 3t, t)\}$.

9. $\left[\begin{array}{cccc|c} 1 & -2 & -1 & -3 & -9 \\ 1 & 1 & -1 & 0 & 0 \\ 3 & 4 & 0 & 1 & 6 \\ 0 & 2 & -2 & 1 & 3 \end{array}\right] \begin{array}{l} -1R_1 + R_2 \\ \hline -3R_1 + R_3 \end{array}$

$\left[\begin{array}{cccc|c} 1 & -2 & -1 & -3 & -9 \\ 0 & 3 & 0 & 3 & 9 \\ 0 & 10 & 3 & 10 & 33 \\ 0 & 2 & -2 & 1 & 3 \end{array}\right] \tfrac{1}{3}R_2$

$\left[\begin{array}{cccc|c} 1 & -2 & -1 & -3 & -9 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 10 & 3 & 10 & 33 \\ 0 & 2 & -2 & 1 & 3 \end{array}\right] \begin{array}{l} -10R_2 + R_3 \\ -2R_2 + R_4 \end{array}$

$\left[\begin{array}{cccc|c} 1 & -2 & -1 & -3 & -9 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 3 & 0 & 3 \\ 0 & 0 & -2 & -1 & -3 \end{array}\right] \tfrac{1}{3}R_3$

$\left[\begin{array}{cccc|c} 1 & -2 & -1 & -3 & -9 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & -2 & -1 & -3 \end{array}\right] 2R_3 + R_4$

$\left[\begin{array}{cccc|c} 1 & -2 & -1 & -3 & -9 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{array}\right] -1R_4$

$\left[\begin{array}{cccc|c} 1 & -2 & -1 & -3 & -9 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$

This corresponds to the system

$$\begin{aligned} w-2x-y-3z&=-9\\ x+z&=3\\ y&=1\\ z&=1 \end{aligned}$$

Use back-substitution to find the values of w and x:

$$\begin{aligned} x+1&=3\\ x&=2 \end{aligned}$$

$$\begin{aligned} w-2(2)-1-3(1)&=-9\\ w-4-1-3&=-9\\ w-8&=-9\\ w&=-1 \end{aligned}$$

The solution to the system is $\{(-1, 2, 1, 1)\}$.

10. $\left[\begin{array}{rrrr|r} 2 & 1 & -2 & -1 & 3\\ 1 & -2 & 1 & 1 & 4\\ -1 & -8 & 7 & 5 & 13\\ 3 & 1 & -2 & 2 & 6 \end{array}\right] R_1 \leftrightarrow R_2$

$\left[\begin{array}{rrrr|r} 1 & -2 & 1 & 1 & 4\\ 2 & 1 & -2 & -1 & 3\\ -1 & -8 & 7 & 5 & 13\\ 3 & 1 & -2 & 2 & 6 \end{array}\right] \begin{matrix} -2R_1+R_2\\ 1R_1+R_3\\ -3R_1+R_4 \end{matrix}$

$\left[\begin{array}{rrrr|r} 1 & -2 & 1 & 1 & 4\\ 0 & 5 & -4 & -3 & -5\\ 0 & -10 & 8 & 6 & 17\\ 0 & 7 & -5 & -1 & -6 \end{array}\right] \tfrac{1}{5}R_2$

$\left[\begin{array}{rrrr|r} 1 & -2 & 1 & 1 & 4\\ 0 & 1 & -\frac{4}{5} & -\frac{3}{5} & -1\\ 0 & -10 & 8 & 6 & 17\\ 0 & 7 & -5 & -1 & -6 \end{array}\right] \begin{matrix} 10R_2+R_3\\ -7R_2+R_4 \end{matrix}$

$\left[\begin{array}{rrrr|r} 1 & -2 & 1 & 1 & 4\\ 0 & 1 & -\frac{4}{5} & -\frac{3}{5} & -1\\ 0 & 0 & 0 & 0 & 7\\ 0 & 0 & \frac{3}{5} & \frac{16}{5} & 1 \end{array}\right] R_3 \leftrightarrow R_4$

$\left[\begin{array}{rrrr|r} 1 & -2 & 1 & 1 & 4\\ 0 & 1 & -\frac{4}{5} & -\frac{3}{5} & -1\\ 0 & 0 & \frac{3}{5} & \frac{16}{5} & 1\\ 0 & 0 & 0 & 0 & 7 \end{array}\right]$

From the last row, we see that the system has no solution. The solution set is $\varnothing$.

11. $\left[\begin{array}{rrrr|r} 2 & 1 & -1 & 0 & 3\\ 1 & -3 & 2 & 0 & -4\\ 3 & 1 & -3 & 1 & 1\\ 1 & 2 & -4 & -1 & -2 \end{array}\right] R_1 \leftrightarrow R_2$

$\left[\begin{array}{rrrr|r} 1 & -3 & 2 & 0 & -4\\ 2 & 1 & -1 & 0 & 3\\ 3 & 1 & -3 & 1 & 1\\ 1 & 2 & -4 & -1 & -2 \end{array}\right] \begin{matrix} -2R_1+R_2\\ -3R_1+R_3\\ -1R_1+R_4 \end{matrix}$

$\left[\begin{array}{rrrr|r} 1 & -3 & 2 & 0 & -4\\ 0 & 7 & -5 & 0 & 11\\ 0 & 10 & -9 & 1 & 13\\ 0 & 5 & -6 & -1 & 2 \end{array}\right] \tfrac{1}{7}R_2$

$\left[\begin{array}{rrrr|r} 1 & -3 & 2 & 0 & -4\\ 0 & 1 & -\frac{5}{7} & 0 & \frac{11}{7}\\ 0 & 10 & -9 & 1 & 13\\ 0 & 5 & -6 & -1 & 2 \end{array}\right] \begin{matrix} -10R_2+R_3\\ -5R_2+R_4 \end{matrix}$

$\left[\begin{array}{rrrr|r} 1 & -3 & 2 & 0 & -4\\ 0 & 1 & -\frac{5}{7} & 0 & \frac{11}{7}\\ 0 & 0 & -\frac{13}{7} & 1 & -\frac{19}{7}\\ 0 & 0 & -\frac{17}{7} & -1 & -\frac{41}{7} \end{array}\right] -\tfrac{7}{13}R_3$

$\left[\begin{array}{rrrr|r} 1 & -3 & 2 & 0 & -4\\ 0 & 1 & -\frac{5}{7} & 0 & \frac{11}{7}\\ 0 & 0 & 1 & -\frac{7}{13} & \frac{19}{13}\\ 0 & 0 & -\frac{17}{7} & -1 & -\frac{41}{7} \end{array}\right] \tfrac{17}{7}R_3+R_4$

$\left[\begin{array}{rrrr|r} 1 & -3 & 2 & 0 & -4\\ 0 & 1 & -\frac{5}{7} & 0 & \frac{11}{7}\\ 0 & 0 & 1 & -\frac{7}{13} & \frac{19}{13}\\ 0 & 0 & 0 & -\frac{30}{13} & -\frac{30}{13} \end{array}\right] -\tfrac{13}{30}R_4$

$\left[\begin{array}{rrrr|r} 1 & -3 & 2 & 0 & -4\\ 0 & 1 & -\frac{5}{7} & 0 & \frac{11}{7}\\ 0 & 0 & 1 & -\frac{7}{13} & \frac{19}{13}\\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$

This corresponds to the system

$$\begin{aligned} w-3x+2y&=-4\\ x-\frac{5}{7}y&=\frac{11}{7}\\ y-\frac{7}{13}z&=\frac{19}{13}\\ z&=1 \end{aligned}$$

Use back-substitution to find the values of w, x, and y:

$$y-\frac{7}{13}z=\frac{19}{13}$$
$$y-\frac{7}{13}(1)=\frac{19}{13}$$
$$y=2$$
$$x-\frac{5}{7}(2)=\frac{11}{7}$$
$$x-\frac{10}{7}=\frac{11}{7}$$
$$x=3$$
$$w-3(3)+2(2)=-4$$
$$w-9+4=-4$$
$$w-5=-4$$
$$w=1$$

The solution to the system is $\{(1, 3, 2, 1)\}$.

12. $\left[\begin{array}{cccc|c} 2 & -1 & 3 & 1 & 0 \\ 3 & 2 & 4 & -1 & 0 \\ 5 & -2 & -2 & -1 & 0 \\ 2 & 3 & -7 & -5 & 0 \end{array}\right] \frac{1}{2}R_1$

$\left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 \\ 3 & 2 & 4 & -1 & 0 \\ 5 & -2 & -2 & -1 & 0 \\ 2 & 3 & -7 & -5 & 0 \end{array}\right] \begin{array}{l} -3R_1+R_2 \\ -5R_1+R_3 \\ -2R_1+R_4 \end{array}$

$\left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & \frac{7}{2} & -\frac{1}{2} & -\frac{5}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{19}{2} & -\frac{7}{2} & 0 \\ 0 & 4 & -10 & -6 & 0 \end{array}\right] R_2 \leftrightarrow R_3$

$\left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{19}{2} & -\frac{7}{2} & 0 \\ 0 & \frac{7}{2} & -\frac{1}{2} & -\frac{5}{2} & 0 \\ 0 & 4 & -10 & -6 & 0 \end{array}\right] 2R_2$

$\left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -19 & -7 & 0 \\ 0 & \frac{7}{2} & -\frac{1}{2} & -\frac{5}{2} & 0 \\ 0 & 4 & -10 & -6 & 0 \end{array}\right] \begin{array}{l} -\frac{7}{2}R_2+R_3 \\ 4R_2+R_4 \end{array}$

$\left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -19 & -7 & 0 \\ 0 & 0 & 66 & 22 & 0 \\ 0 & 0 & 66 & 22 & 0 \end{array}\right] -1R_3+R_4$

$\left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -19 & -7 & 0 \\ 0 & 0 & 66 & 22 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \frac{1}{66}R_3$

$\left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -19 & -7 & 0 \\ 0 & 0 & 1 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

The system
$$w-\frac{1}{2}x+\frac{3}{2}y+\frac{1}{2}z=0$$
$$x-19y-7z=0$$
$$y+\frac{1}{3}z=0$$
has no unique solution. Express w, x, and y in terms of z:

$$y=-\frac{1}{3}z$$
$$x-19\left(-\frac{1}{3}z\right)-7z=0$$
$$x+\frac{19}{3}z-7z=0$$
$$x-\frac{2}{3}z=0$$
$$x=\frac{2}{3}z$$

$$w-\frac{1}{2}\left(\frac{2}{3}z\right)+\frac{3}{2}\left(-\frac{1}{3}z\right)+\frac{1}{2}z=0$$
$$w-\frac{1}{3}z-\frac{1}{2}z+\frac{1}{2}z=0$$
$$w-\frac{1}{3}z=0$$
$$w=\frac{1}{3}z$$

With $z = t$, the complete solution to the system is $\left\{\left(\frac{1}{3}t, \frac{2}{3}t, -\frac{1}{3}t, t\right)\right\}$.

13. $\left[\begin{array}{rrrr|r} 1 & -3 & 1 & -4 & 4 \\ -2 & 1 & 2 & 0 & -2 \\ 3 & -2 & 1 & -6 & 2 \\ -1 & 3 & 2 & -1 & -6 \end{array}\right]\begin{array}{l} 2R_1+R_2 \\ -3R_1+R_3 \\ R_1+R_4 \end{array}$

$\left[\begin{array}{rrrr|r} 1 & -3 & 1 & -4 & 4 \\ 0 & -5 & 4 & -8 & 6 \\ 0 & 7 & -2 & 6 & -10 \\ 0 & 0 & 3 & -5 & -2 \end{array}\right] -\tfrac{1}{5}R_2$

$\left[\begin{array}{rrrr|r} 1 & -3 & 1 & -4 & 4 \\ 0 & 1 & -\frac{4}{5} & \frac{8}{5} & -\frac{6}{5} \\ 0 & 7 & -2 & 6 & -10 \\ 0 & 0 & 3 & -5 & -2 \end{array}\right] -7R_2+R_3$

$\left[\begin{array}{rrrr|r} 1 & -3 & 1 & -4 & 4 \\ 0 & 1 & -\frac{4}{5} & \frac{8}{5} & -\frac{6}{5} \\ 0 & 0 & \frac{18}{5} & -\frac{26}{5} & -\frac{8}{5} \\ 0 & 0 & 3 & -5 & -2 \end{array}\right] \tfrac{5}{18}R_3$

$\left[\begin{array}{rrrr|r} 1 & -3 & 1 & -4 & 4 \\ 0 & 1 & -\frac{4}{5} & \frac{8}{5} & -\frac{6}{5} \\ 0 & 0 & 1 & -\frac{13}{9} & -\frac{4}{9} \\ 0 & 0 & 3 & -5 & -2 \end{array}\right] -3R_3+R_4$

$\left[\begin{array}{rrrr|r} 1 & -3 & 1 & -4 & 4 \\ 0 & 1 & -\frac{4}{5} & \frac{8}{5} & -\frac{6}{5} \\ 0 & 0 & 1 & -\frac{13}{9} & -\frac{4}{9} \\ 0 & 0 & 0 & -\frac{2}{3} & -\frac{2}{3} \end{array}\right] -\tfrac{3}{2}R_4$

$\left[\begin{array}{rrrr|r} 1 & -3 & 1 & -4 & 4 \\ 0 & 1 & -\frac{4}{5} & \frac{8}{5} & -\frac{6}{5} \\ 0 & 0 & 1 & -\frac{13}{9} & -\frac{4}{9} \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$

This corresponds to the system

$w-3x+y-4z=4$

$x-\dfrac{4}{5}y+\dfrac{8}{5}z=-\dfrac{6}{5}$

$y-\dfrac{13}{9}z=-\dfrac{4}{9}$

$z=1$

Use back-substitution to find the values of w, z, and y:

$y-\dfrac{13}{9}(1)=-\dfrac{4}{9}$

$y=1$

$x-\dfrac{4}{5}(1)+\dfrac{8}{5}(1)=-\dfrac{6}{5}$

$x+\dfrac{4}{5}=-\dfrac{6}{5}$

$x=-2$

$w-3(-2)+1-4=4$

$w+6-3=4$

$w=1$

The solution to the system is $\{(1, -2, 1, 1)\}$.

14. $\left[\begin{array}{rrrr|r} 3 & 2 & -1 & 2 & -12 \\ 4 & -1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & -2 \\ -2 & 3 & 2 & -3 & 10 \end{array}\right] R_1 \leftrightarrow R_3$

$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -2 \\ 4 & -1 & 1 & 2 & 1 \\ 3 & 2 & -1 & 2 & -12 \\ -2 & 3 & 2 & -3 & 10 \end{array}\right]\begin{array}{l} -4R_1+R_2 \\ -3R_1+R_3 \\ 2R_1+R_4 \end{array}$

$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -2 \\ 0 & -5 & -3 & -2 & 9 \\ 0 & -1 & -4 & -1 & -6 \\ 0 & 5 & 4 & -1 & 6 \end{array}\right] R_2 \leftrightarrow R_3$

$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -2 \\ 0 & -1 & -4 & -1 & -6 \\ 0 & -5 & -3 & -2 & 9 \\ 0 & 5 & 4 & -1 & 6 \end{array}\right] -1R_2$

$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & 4 & 1 & 6 \\ 0 & -5 & -3 & -2 & 9 \\ 0 & 5 & 4 & -1 & 6 \end{array}\right]\begin{array}{l} 5R_2+R_3 \\ -5R_2+R_4 \end{array}$

$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & 4 & 1 & 6 \\ 0 & 0 & 17 & 3 & 39 \\ 0 & 0 & -16 & -6 & -24 \end{array}\right] \tfrac{1}{17}R_3$

$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & 4 & 1 & 6 \\ 0 & 0 & 1 & \frac{3}{17} & \frac{39}{17} \\ 0 & 0 & -16 & -6 & -24 \end{array}\right] 16R_3+R_4$

$\left[\begin{array}{rrrr|r} 1 & 1 & 1 & 1 & -2 \\ 0 & 1 & 4 & 1 & 6 \\ 0 & 0 & 1 & \frac{3}{17} & \frac{39}{17} \\ 0 & 0 & 0 & -\frac{54}{17} & \frac{216}{17} \end{array}\right] -\tfrac{17}{54}R_4$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & | & -2 \\ 0 & 1 & 4 & 1 & | & 6 \\ 0 & 0 & 1 & \frac{3}{17} & | & \frac{39}{17} \\ 0 & 0 & 0 & 1 & | & -4 \end{bmatrix}$$

This corresponds to the system

$$w + x + y + z = -2$$
$$x + 4y + z = 6$$
$$y + \frac{3}{17}z = \frac{39}{7}$$
$$z = -4$$

Use back-substitution to find the values of w, x, and y:

$$y + \frac{3}{17}(-4) = \frac{39}{17}$$
$$y - \frac{12}{17} = \frac{39}{17}$$
$$y = 3$$
$$x + 4(3) - 4 = 6$$
$$x + 8 = 6$$
$$x = -2$$

$$w - 2 + 3 - 4 = -2$$
$$w - 3 = -2$$
$$w = 1$$

The solution to this system is $\{(1, -2, 3, -4)\}$.

15. $\begin{bmatrix} 2 & 1 & -1 & | & 2 \\ 3 & 3 & -2 & | & 3 \end{bmatrix} \frac{1}{2}R_1$ $\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} & | & 1 \\ 3 & 3 & -2 & | & 3 \end{bmatrix} -3R_1 + R_2$

$\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} & | & 1 \\ 0 & \frac{3}{2} & -\frac{1}{2} & | & 0 \end{bmatrix} \frac{2}{3}R_2$

$\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} & | & 1 \\ 0 & 1 & -\frac{1}{3} & | & 0 \end{bmatrix}$

The system $x + \frac{1}{2}y - \frac{1}{2}z = 1$, $y - \frac{1}{3}z = 0$ has no unique solution.

Express x and y in terms of z:

$$y = \frac{1}{3}z$$
$$x + \frac{1}{2}\left(\frac{1}{3}z\right) - \frac{1}{2}z = 1$$
$$x + \frac{1}{6}z - \frac{1}{2}z = 1$$
$$x - \frac{1}{3}z = 1$$
$$x = 1 + \frac{1}{3}z$$

With $z = t$, the complete solution to the system is $\left\{\left(1 + \frac{1}{3}t, \frac{1}{3}t, t\right)\right\}$.

16. $\begin{bmatrix} 3 & 2 & -1 & | & 5 \\ 1 & 2 & -1 & | & 1 \end{bmatrix} R_1 \leftrightarrow R_2$

$\begin{bmatrix} 1 & 2 & -1 & | & 1 \\ 3 & 2 & -1 & | & 5 \end{bmatrix} -3R_1 + R_2$

$\begin{bmatrix} 1 & 2 & -1 & | & 1 \\ 0 & -4 & 2 & | & 2 \end{bmatrix} -\frac{1}{4}R_2$

$\begin{bmatrix} 1 & 2 & -1 & | & 1 \\ 0 & 1 & -\frac{1}{2} & | & -\frac{1}{2} \end{bmatrix}$

The system $x + 2y - z = 1$, $y - \frac{1}{2}z = -\frac{1}{2}$ has no unique solution.

Express x and y in terms of z:

$$y - \frac{1}{2}z = -\frac{1}{2}$$
$$y = \frac{1}{2}z - \frac{1}{2}$$
$$x + 2\left(\frac{1}{2}z - \frac{1}{2}\right) - z = 1$$
$$x + z - 1 - z = 1$$
$$x - 1 = 1$$
$$x = 2$$

With $z = t$, the complete solution to the system is $\left\{\left(2, \frac{1}{2}t - \frac{1}{2}, t\right)\right\}$.

17. The system $x + 2y + 3z = 5$, $y - 5z = 0$ has no unique solution.

Express x and y in terms of z:

$$y = 5z$$
$$x + 2(5z) + 3z = 5$$
$$x + 10z + 3z = 5$$
$$x = -13z + 5$$

With $z = t$, the complete solution to the system is $\{(-13t + 5, 5t, t)\}$.

18. The system $\begin{aligned} 3x - y + 4z &= 8 \\ y + 2z &= 1 \end{aligned}$ has no unique solution.

Express *x* and *y* in terms of *z*:

$y = -2z + 1$

$3x - (-2z + 1) + 4z = 8$

$3x + 2z - 1 + 4z = 8$

$3x = -6z + 9$

$x = -2z + 3$

With $z = t$, the complete solution to the system is $\{(-2t + 3, -2t + 1, t)\}$.

19. $\left[\begin{array}{ccc|c} 1 & 1 & -2 & 2 \\ 3 & -1 & -6 & -7 \end{array}\right] -3R_1 + R_2$

$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 2 \\ 0 & -4 & 0 & -13 \end{array}\right] -\frac{1}{4}R_2$

$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 2 \\ 0 & 1 & 0 & \frac{13}{4} \end{array}\right]$

The system $\begin{aligned} x + y - 2z &= 2 \\ y &= \frac{13}{4} \end{aligned}$ has no unique solution.

Express *x* in terms of *z*:

$x + \frac{13}{4} - 2z = 2$

$x = 2z - \frac{5}{4}$

With $z = t$, the complete solution to the system is $\left\{\left(2t - \frac{5}{4}, \frac{13}{4}, t\right)\right\}$.

20. $\left[\begin{array}{ccc|c} -2 & -5 & 10 & 19 \\ 1 & 2 & -4 & 12 \end{array}\right] R_1 \leftrightarrow R_2$

$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 12 \\ -2 & -5 & 10 & 19 \end{array}\right] 2R_1 + R_2$

$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 12 \\ 0 & -1 & 2 & 43 \end{array}\right] -1R_2$

$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 12 \\ 0 & 1 & -2 & -43 \end{array}\right]$

The system $\begin{aligned} x + 2y - 4z &= 12 \\ y - 2z &= -43 \end{aligned}$ has no unique solution. Express *x* and *y* in terms of *z*:

$y = 2z - 43$

$x + 2(2z - 43) - 4z = 12$

$x + 4z - 86 - 4z = 12$

$x = 98$

With $z = t$, the complete solution to the system is $\{(98, 2t - 43, t)\}$.

21. $\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -2 \\ 2 & -1 & 2 & -1 & 7 \\ -1 & 2 & 1 & 2 & -1 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \\ 1R_1 + R_3 \end{array}$

$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -2 \\ 0 & -3 & 4 & -3 & 11 \\ 0 & 3 & 0 & 3 & -3 \end{array}\right] R_2 \leftrightarrow R_3$

$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -2 \\ 0 & 3 & 0 & 3 & -3 \\ 0 & -3 & 4 & -3 & 11 \end{array}\right] \frac{1}{3}R_2$

$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -2 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & -3 & 4 & -3 & 11 \end{array}\right] 3R_2 + R_3$

$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -2 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 4 & 0 & 8 \end{array}\right] \frac{1}{4}R_3$

$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -2 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 2 \end{array}\right]$

The system $\begin{aligned} x + y - z + w &= -2 \\ x + z &= -1 \\ y &= 2 \end{aligned}$ has no unique solution. Let $z = t$ and use back substitution to find remaining variables. The complete solution to the system is $\{(1, -t - 1, 2, t)\}$.

22. $\left[\begin{array}{cccc|c} 2 & -3 & 4 & 1 & 7 \\ 1 & -1 & 3 & -5 & 10 \\ 3 & 1 & -2 & -2 & 6 \end{array}\right] R_1 \leftrightarrow R_2$

$\left[\begin{array}{cccc|c} 1 & -1 & 3 & -5 & 10 \\ 2 & -3 & 4 & 1 & 7 \\ 3 & 1 & -2 & -2 & 6 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \\ -3R_1 + R_3 \end{array}$

$\left[\begin{array}{cccc|c} 1 & -1 & 3 & -5 & 10 \\ 0 & -1 & -2 & 11 & -13 \\ 0 & 4 & -11 & 13 & -24 \end{array}\right] -1R_2$

$\left[\begin{array}{cccc|c} 1 & -1 & 3 & -5 & 10 \\ 0 & 1 & 2 & -11 & 13 \\ 0 & 4 & -11 & 13 & -24 \end{array}\right] -4R_2 + R_3$

$\left[\begin{array}{cccc|c} 1 & -1 & 3 & -5 & 10 \\ 0 & 1 & 2 & -11 & 13 \\ 0 & 0 & -19 & 57 & -76 \end{array}\right] -\frac{1}{19}R_3$

$$\left[\begin{array}{cccc|c} 1 & -1 & 3 & -5 & 10 \\ 0 & 1 & 2 & -11 & 13 \\ 0 & 0 & 1 & -3 & 4 \end{array}\right]$$

The system $\begin{aligned} w-x+3y-5z&=10 \\ x+2y-11z&=13 \\ y-3z&=4 \end{aligned}$ has no unique solution. Let $z=t$ and use back substitution to find remaining variables. The complete solution to the system is $\{(t+3, 5t+5, 3t+4, t)\}$.

23. $\left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 7 \\ 0 & 2 & -3 & 1 & 4 \\ 1 & -4 & 1 & 0 & 3 \end{array}\right] -1R_1+R_3$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 7 \\ 0 & 2 & -3 & 1 & 4 \\ 0 & -6 & -2 & 1 & -4 \end{array}\right] \tfrac{1}{2}R_2$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 7 \\ 0 & 1 & -\frac{3}{2} & \frac{1}{2} & 2 \\ 0 & -6 & -2 & 1 & -4 \end{array}\right] 6R_2+R_3$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 7 \\ 0 & 1 & -\frac{3}{2} & \frac{1}{2} & 2 \\ 0 & 0 & -11 & 4 & 8 \end{array}\right] -\tfrac{1}{11}R_3$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & -1 & 7 \\ 0 & 1 & -\frac{3}{2} & \frac{1}{2} & 2 \\ 0 & 0 & 1 & -\frac{4}{11} & -\frac{8}{11} \end{array}\right]$$

The system has no unique solution. Let $z=t$ and use back substitution to find remaining variables. The complete solution to the system is

$$\left\{\left(-\frac{2}{11}t+\frac{81}{11}, \frac{1}{22}t+\frac{10}{11}, \frac{4}{11}t-\frac{8}{11}, t\right)\right\}.$$

24. $\left[\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 0 \\ 1 & -4 & 1 & 2 & 0 \\ 3 & 0 & -1 & 2 & 0 \end{array}\right] \begin{array}{l} -1R_1+R_2 \\ -3R_1+R_3 \end{array}$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 0 \\ 0 & -3 & 1 & 1 & 0 \\ 0 & 3 & -1 & -1 & 0 \end{array}\right] 1R_2+R_3$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 0 \\ 0 & -3 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] -\tfrac{1}{3}R_2$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -\frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The system $\begin{aligned} w-x+z&=0 \\ x-\frac{1}{3}y-\frac{1}{3}z&=0 \end{aligned}$ has no unique solution.

Let $y=t$ and $z=s$ and use back substitution to find remaining variables.
The complete solution to the system is
$\left\{\left(\frac{1}{3}t-\frac{2}{3}s, \frac{1}{3}t+\frac{1}{3}s, t, s\right)\right\}$.

25. **a.** System of equations:

$$\begin{aligned} 4w-2x+2y-3z&=0 \\ 7w-x-y-3z&=0 \\ w+x-y-z&=0 \end{aligned}$$

b. Reduced system:

$$\begin{aligned} w-0.5z&=0 \\ x&=0 \\ y-0.5z&=0 \end{aligned}$$

Find *w* and *y* in terms of *z*.

$$\begin{aligned} w-0.5z&=0 \\ w&=0.5z \\ y-0.5z&=0 \\ y&=0.5z \end{aligned}$$

The complete solution to the system is
$\{(0.5z, 0, 0.5z, z)\}$.

26. **a.** System of equations:

$$\begin{aligned} 2w+17x-23y+40z&=0 \\ 2w+5x+y+3z&=0 \\ x-2y+3z&=0 \end{aligned}$$

b. Reduced system:

$$w + 5.5y = 0$$
$$x - 2y = 0$$
$$z = 0$$

Find w and x in terms of y.

$$w + 5.5y = 0$$
$$w = -5.5y$$
$$x - 2y = 0$$
$$x = 2y$$

The complete solution to the system is $\{(-5.5y,\ 2y,\ y,\ 0)\}$.

27. a. System of equations:

$$w + 2x + 5y + 5z = 3$$
$$w + x + 3y + 4z = -1$$
$$w - x - y + 2z = 3$$

b. Reduced system:

$$w + y + 3z = 1$$
$$x + 2y + z = -2$$

Find w and x in terms of y and z.

$$w + y + 3z = 1$$
$$w = 1 - y - 3z$$
$$x + 2y + z = -2$$
$$x = -2 - 2y - z$$

The complete solution to the system is $\{(1 - y - 3z,\ -2 - 2y - z,\ y,\ z)\}$.

28. a. System of equations:

$$w + y + z = 0$$
$$w - x + 2y + 3z = 0$$
$$3w - 2x + 5y + 7z = 0$$

b. Reduced system:

$$w + y + z = 0$$
$$x - y - 2z = 0$$

Find w and x in terms of y and z.

$$w + y + z = 0$$
$$w = -y - z$$
$$x - y - 2z = 0$$
$$x = y + 2z$$

The complete solution to the system is $\{(-y - z,\ y + 2z,\ y,\ z)\}$.

29. $z + 12 = x + 6$

30. $y + 6 = z + 8$

31. $x - y = 4$

$$x - z = 6$$
$$y - z = 2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 4 \\ 1 & 0 & -1 & 6 \\ 0 & 1 & -1 & 2 \end{array}\right] \begin{array}{l} -1R_1 + R_2 \\ \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & -1 & 0 & 4 \\ 0 & -1 & 1 & -2 \\ 0 & 1 & -1 & 2 \end{bmatrix} \begin{array}{l} \\ -1R_2 \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & -1 & 0 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 1 & -1 & 2 \end{bmatrix} \begin{array}{l} \\ -1R_2 + R_3 \\ 1R_2 + R_1 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -1 & 6 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The system has no unique solution. Express x and y in terms of z:

$$x - z = 6$$
$$y - z = 2$$
$$x = z + 6$$
$$y = z + 2$$

With $z = t$, the complete solution to the system is $\{(t + 6,\ t + 2,\ t)\}$.

32. $z = 4$

$x = 4 + 6$, $x = 10$

$y = 4 + 2$, $y = 6$

To keep traffic flowing, 10 cars per minute must be rooted between I_1 and I_2, and 6 cars per minute must be rooted between I_1 and I_3.

33. a. From left to right along Palm Drive, then along Sunset Drive, we get the equations

$w + z = 200 + 180 = 380$;

$w + x = 400 + 200 = 600$;

$z + 70 = y + 20$ or $y - z = 50$;

$y + 200 = x + 30$ or $x - y = 170$.

The system is

$$w + z = 380$$
$$w + x = 600$$
$$y - z = 50$$
$$x - y = 170$$

b. $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 380 \\ 0 & 1 & 0 & -1 & 220 \\ 0 & 0 & 1 & -1 & 50 \\ 0 & 1 & -1 & 0 & 170 \end{array}\right] -1R_2 + R_4$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 380 \\ 0 & 1 & 0 & -1 & 220 \\ 0 & 0 & 1 & -1 & 50 \\ 0 & 0 & -1 & 1 & -50 \end{array}\right] 1R_3 + R_4$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 380 \\ 0 & 1 & 0 & -1 & 220 \\ 0 & 0 & 1 & -1 & 50 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

The system has no unique solution. Express x and y and z in terms of z:

$w = 380 - z$

$x = 220 + z$

$y = 50 + z$

With $z = t$, the complete solution to the system is $\{(380 - t, 220 + t, 50 + t, t)\}$.

c. Letting $z = 50$, the solution is

$w = 380 - 50 = 330$

$x = 220 + 50 = 270$

$y = 50 + 50 = 100$

34. Let x = the amount of Food A,
y = the amount of Food B, and
z = the amount of Food C, in ounces.
The amount of thiamin is $3x + y + 3z$;
the amount of riboflavin is $7x + 5y + 8z$;
the amount of niacin is $x + 3y + 2z$.

a. The corresponding system for these conditions is

$3x + y + 3z = 14$
$7x + 5y + 8z = 32$
$x + 3y + 2z = 9.$

$\left[\begin{array}{ccc|c} 3 & 1 & 3 & 14 \\ 7 & 5 & 8 & 32 \\ 1 & 3 & 2 & 9 \end{array}\right] R_1 \leftrightarrow R_3$

$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 9 \\ 7 & 5 & 8 & 32 \\ 3 & 1 & 3 & 14 \end{array}\right] \begin{array}{l} -7R_1 + R_2 \\ -3R_1 + R_3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 9 \\ 0 & -16 & -6 & -31 \\ 0 & -8 & -3 & -13 \end{array}\right] 2R_3$

$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 9 \\ 0 & -16 & -6 & -31 \\ 0 & -16 & -6 & -26 \end{array}\right] -1R_2 + R_3$

$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 9 \\ 0 & -16 & -6 & -31 \\ 0 & 0 & 0 & 5 \end{array}\right]$

From the last row, we see that the system has no solution. Thus, there is no combination of the foods that can satisfy the given requirements.

b. The new system is

$3x + y + 3z = 14$
$7x + 5y + 8z = 37$
$x + 3y + 2z = 9.$

$\left[\begin{array}{ccc|c} 3 & 1 & 3 & 14 \\ 7 & 5 & 8 & 37 \\ 1 & 3 & 2 & 9 \end{array}\right] R_1 \leftrightarrow R_3$

$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 9 \\ 7 & 5 & 8 & 37 \\ 3 & 1 & 3 & 14 \end{array}\right] \begin{array}{l} -7R_1 + R_2 \\ -3R_1 + R_3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 9 \\ 0 & -16 & -6 & -26 \\ 0 & -8 & -3 & -13 \end{array}\right] 2R_3$

$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 9 \\ 0 & -16 & -6 & -26 \\ 0 & -16 & -6 & -26 \end{array}\right] -1R_2 + R_3$

$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 9 \\ 0 & -16 & -6 & -26 \\ 0 & 0 & 0 & 0 \end{array}\right] -\tfrac{1}{16}R_2$

$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 9 \\ 0 & 1 & \frac{3}{8} & \frac{13}{8} \\ 0 & 0 & 0 & 0 \end{array}\right]$

When the riboflavin requirement is increased by 5 milligrams the system has infinitely many solutions so there are many combinations of the foods that satisfy the new requirements.

35. Let x = the number of ounces of Food 1, y = the number of ounces of Food 2, and z = the number of ounces of Food 3.
The amount of vitamin A is $20x + 30y + 10z$; the amount of iron is $20x + 10y + 10z$; the amount of calcium is $10x + 10y + 30z$.

a. Not having Food 1 means that all x terms are left out. The vitamin A requirement can then be represented by $30y + 10z = 220$; the iron requirement is $10y + 10z = 180$; the calcium requirement is $10y + 30z = 340$.
The corresponding system is
$30y + 10z = 220$
$10y + 10z = 180$
$10y + 30z = 340$.
Dividing all of the numbers by 10, the matrix for this system is

$$\left[\begin{array}{cc|c} 3 & 1 & 22 \\ 1 & 1 & 18 \\ 1 & 3 & 34 \end{array}\right] R_1 \leftrightarrow R_2 \left[\begin{array}{cc|c} 1 & 1 & 18 \\ 3 & 1 & 22 \\ 1 & 3 & 34 \end{array}\right] \begin{array}{l} -3R_1 + R_2 \\ -1R_1 + R_3 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 18 \\ 0 & -2 & -32 \\ 0 & 2 & 16 \end{array}\right] 1R_2 + R_3$$

$$\left[\begin{array}{cc|c} 1 & 1 & 18 \\ 0 & -2 & -32 \\ 0 & 0 & -16 \end{array}\right].$$

From the last row, we see that the system has no solution, so there is no way to satisfy these dietary requirements with no Food 1 available.

b. With Food 1 available, and dropping the vitamin A requirement, the system is
$20x + 10y + 10z = 180$
$10x + 10y + 30z = 340$.
Dividing all of the numbers by 10, the matrix for this system is

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 18 \\ 1 & 1 & 3 & 34 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 34 \\ 2 & 1 & 1 & 18 \end{array}\right] -2R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 34 \\ 0 & -1 & -5 & -50 \end{array}\right] -1R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 34 \\ 0 & 1 & 5 & 50 \end{array}\right].$$

The system $\begin{aligned} x + y + 3z &= 34 \\ y + 5z &= 50 \end{aligned}$ has no unique solution. Express x and y in terms of z:
$y = -5z + 50$
$x + (-5z + 50) + 3z = 34$
$x - 2z + 50 = 34$
$x = 2z - 16$
Now we can choose a value for z, i.e., an amount of Food 3, and find the corresponding values of x and y. Note that negative amounts of food are not realistic, so $z \geq 0$, $y = -5z + 50 \geq 0$, and $x = 2z - 16 \geq 0$. These conditions are equivalent to
$8 \leq z \leq 10$.
Using $z = 8$ and $z = 10$, two possibilities are 0 ounces of Food 1, 10 ounces of Food 2, and 8 ounces of Food 3 or 4 ounces of Food 1, 0 ounces of Food 2, and 10 ounces of Food 3. (Other answers are possible.)

36. Let x, y, and z be the amounts of Products A, B, and C, respectively. The corresponding system is
$7x + 6y + 3z = 67$
$2x + 2y + z = 20$.

$$\left[\begin{array}{ccc|c} 7 & 6 & 3 & 67 \\ 2 & 2 & 1 & 20 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 1 & 20 \\ 7 & 6 & 3 & 67 \end{array}\right] \tfrac{1}{2}R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & \frac{1}{2} & 10 \\ 7 & 6 & 3 & 67 \end{array}\right] -7R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & \frac{1}{2} & 10 \\ 0 & -1 & -\frac{1}{2} & -3 \end{array}\right] -1R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & \frac{1}{2} & 10 \\ 0 & 1 & \frac{1}{2} & 3 \end{array}\right]$$

The system $\begin{aligned} x + y + \tfrac{1}{2}z &= 10 \\ y + \tfrac{1}{2}z &= 3 \end{aligned}$ has no unique solution.

Express x and y in terms of z:

$$y = -\tfrac{1}{2}z + 3$$

$$x - \tfrac{1}{2}z + 3 + \tfrac{1}{2}z = 10$$

$$x = 7$$

With $z = 2$ and 4, the solutions are (7, 2, 2) and (7, 1, 4). These solutions correspond to the two solutions: 7 of product A, 2 of product B, 2 of product C; 7 of product A, 1 of product B, 4 of product C (other answers are possible). Note that negative numbers of products are impossible so $0 \leq z \leq 6$.

37. – 39. Answers may vary.

40. **a.** From left to right along 95th Street, then along 104th Street, we have the equations:
$x_1+800=x_6+900$ or $x_1-x_6=100$;
$x_6+600=x_2+x_7$ or $x_2-x_6+x_7=600$;
$x_3+x_7=200+700$ or $x_3+x_7=900$;
$x_1+600=x_4+400$ or $x_1-x_4=-200$;
$x_2+x_5=x_4+100$ or $x_2-x_4+x_5=100$;
$x_3+x_5=300+400$ or $x_3+x_5=700$.
The system is

$$\begin{aligned} x_1-x_6&=100\\ x_2-x_6+x_7&=600\\ x_3+x_7&=900\\ x_1-x_4&=-200\\ x_2-x_4+x_5&=100\\ x_3+x_5&=700. \end{aligned}$$

b. The matrix for the system is

$$\left[\begin{array}{ccccccc|c} 1&0&0&0&0&-1&0&100\\ 0&1&0&0&0&-1&1&600\\ 0&0&1&0&0&0&1&900\\ 1&0&0&-1&0&0&0&-200\\ 0&1&0&-1&1&0&0&100\\ 0&0&1&0&1&0&0&700 \end{array}\right].$$

Using the matrix capabilities of a graphing utility, this matrix can be transformed into the matrix

$$\left[\begin{array}{ccccccc|c} 1&0&0&0&0&-1&0&100\\ 0&1&0&0&0&-1&1&600\\ 0&0&1&0&0&0&1&900\\ 0&0&0&1&0&-1&0&300\\ 0&0&0&0&1&0&-1&-200\\ 0&0&0&0&0&0&0&0 \end{array}\right].$$

41. does not make sense; Explanations will vary. Sample explanation: Row 3 indicates that this system has no solution. If eliminated, it would falsely appear that the system has an infinite number of solutions.

42. makes sense

43. does not make sense; Explanations will vary. Sample explanation: In a nonsquare system, the number of equations differs from the number of variables.

44. makes sense

45.

$$\left[\begin{array}{ccc|c} 1&3&1&a^2\\ 2&5&2a&0\\ 1&1&a^2&-9 \end{array}\right] \begin{array}{l}-2R_1+R_2\\ -1R_1+R_3\end{array}$$

$$\left[\begin{array}{ccc|c} 1&3&1&a^2\\ 0&-1&2a-2&-2a^2\\ 0&-2&a^2-1&-9-a^2 \end{array}\right] -1R_2$$

$$\left[\begin{array}{ccc|c} 1&3&1&a^2\\ 0&1&2-2a&2a^2\\ 0&-2&a^2-1&-9-a^2 \end{array}\right] 2R_2+R_3$$

$$\left[\begin{array}{ccc|c} 1&3&1&a^2\\ 0&1&2-2a&2a^2\\ 0&0&a^2-4a+3&-9+3a^2 \end{array}\right]$$

The system will be inconsistent when $a^2-4a+3=0$ but $-9+3a^2\neq 0$.
$a^2-4a+3=(a-1)(a-3)=0$ when $a=1$ or $a=3$. $-9+3a^2=0$ when $a=\pm\sqrt{3}$.
Thus, the system is inconsistent when $a=1$ or $a=3$.

46. Answers may vary.

47. $-6-(-5)=-6+5=-1$

48. $1(-4)+2(5)+3(-6)=-4+10-18=-12$

49. $\frac{1}{2}[8-(-8)]=\frac{1}{2}[8+8]=\frac{1}{2}[16]=8$

Section 9.3

Check Point Exercises

1. **a.** The matrix $A=\begin{bmatrix}5&-2\\-3&\pi\\1&6\end{bmatrix}$ has 3 rows and 2 columns, so it is of order 3×2.

b. The element a_{12} is in the first row and second column. Thus, $a_{12}=-2$. The element a_{31} is in the third row and first column. Thus, $a_{31}=1$.

2. **a.**
$$\begin{bmatrix} -4 & 3 \\ 7 & -6 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 2 & -4 \end{bmatrix}$$
$$= \begin{bmatrix} -4+6 & 3+(-3) \\ 7+2 & -6+(-4) \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 9 & -10 \end{bmatrix}$$

b.
$$\begin{bmatrix} 5 & 4 \\ -3 & 7 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 8 \\ 6 & 0 \\ -5 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 5-(-4) & 4-8 \\ -3-6 & 7-0 \\ 0-(-5) & 1-3 \end{bmatrix}$$
$$= \begin{bmatrix} 9 & -4 \\ -9 & 7 \\ 5 & -2 \end{bmatrix}$$

3. **a.**
$$-6B = -6\begin{bmatrix} -1 & -2 \\ 8 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} -6(-1) & -6(-2) \\ -6(8) & -6(5) \end{bmatrix}$$
$$= \begin{bmatrix} -6 & 12 \\ -48 & -30 \end{bmatrix}$$

b.
$$3A + 2B = \begin{bmatrix} -4 & 1 \\ 3 & 0 \end{bmatrix} + 2\begin{bmatrix} -1 & -2 \\ 8 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} 3(-4) & 3(1) \\ 3(3) & 3(0) \end{bmatrix} + \begin{bmatrix} 2(-1) & 2(-2) \\ 2(8) & 2(5) \end{bmatrix}$$
$$= \begin{bmatrix} -12 & 3 \\ 9 & 0 \end{bmatrix} + \begin{bmatrix} -2 & -4 \\ 16 & 10 \end{bmatrix}$$
$$= \begin{bmatrix} -12+(-2) & 3+(-4) \\ 9+16 & 0+10 \end{bmatrix}$$
$$= \begin{bmatrix} -14 & -1 \\ 25 & 10 \end{bmatrix}$$

4. $3X + A = B$

$$3X = B - A$$

$$X = \frac{1}{3}(B - A)$$

$$X = \frac{1}{3}\left(\begin{bmatrix} -10 & 1 \\ -9 & 17 \end{bmatrix} - \begin{bmatrix} 2 & -8 \\ 0 & 4 \end{bmatrix}\right)$$

$$X = \frac{1}{3}\begin{bmatrix} -12 & 9 \\ -9 & 13 \end{bmatrix}$$

$$X = \begin{bmatrix} -4 & 3 \\ -3 & \frac{13}{3} \end{bmatrix}$$

5. Given $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix}$,

$$AB = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 4 & 6 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1(4)+3(1) & 1(6)+3(0) \\ 2(4)+5(1) & 2(6)+5(0) \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 6 \\ 13 & 12 \end{bmatrix}$$

6. If $A = \begin{bmatrix} 2 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix}$, then

$$AB = \begin{bmatrix} 2 & 0 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2(1)+0(3)+4(7) \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+28 \end{bmatrix}$$

$$= \begin{bmatrix} 30 \end{bmatrix}$$

and $BA = \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix}\begin{bmatrix} 2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1(2) & 1(0) & 1(4) \\ 3(2) & 3(0) & 3(4) \\ 7(2) & 7(0) & 7(4) \end{bmatrix} = \begin{bmatrix} 2 & 0 & 4 \\ 6 & 0 & 12 \\ 14 & 0 & 28 \end{bmatrix}$.

7. **a.** $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 & -1 & 6 \\ 0 & 5 & 4 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1(2)+3(0) & 1(3)+3(5) & 1(-1)+3(4) & 1(6)+3(1) \\ 0(2)+2(0) & 0(3)+2(5) & 0(-1)+2(4) & 0(6)+2(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 18 & 11 & 9 \\ 0 & 10 & 8 & 2 \end{bmatrix}$$

b. $\begin{bmatrix} 2 & 3 & -1 & 6 \\ 0 & 5 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$

The number of columns in the first matrix does not equal the number of rows in the second matrix. Thus, the product of these two matrices is undefined.

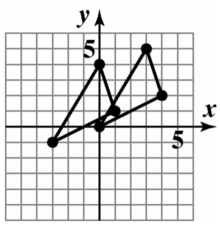
8. Because the L is dark gray and the background is light gray, the digital photograph can be represented by the matrix

$$\begin{bmatrix} 2 & 1 & 1 \\ 2 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

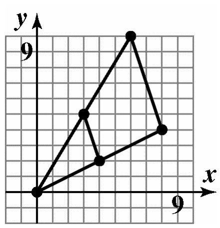
We can make the L light gray by decreasing each 2 in the above matrix to 1. We can make the background black by increasing each 1 in the matrix to 3. This is accomplished using the following matrix addition.

$$\begin{bmatrix} 2 & 1 & 1 \\ 2 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 2 \\ -1 & 2 & 2 \\ -1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 3 \\ 1 & 1 & 3 \end{bmatrix}$$

9. **a.** $\begin{bmatrix} 0 & 3 & 4 \\ 0 & 5 & 2 \end{bmatrix} + \begin{bmatrix} -3 & -3 & -3 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 1 \\ -1 & 4 & 1 \end{bmatrix}$



b. $2\begin{bmatrix} 0 & 3 & 4 \\ 0 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 8 \\ 0 & 10 & 4 \end{bmatrix}$



c. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 3 & 4 \\ 0 & 5 & 2 \end{bmatrix} = \begin{bmatrix} (-1)(0)+0(0) & (-1)(3)+0(5) & (-1)(4)+0(2) \\ 0(0)+1(0) & 0(3)+1(5) & 0(4)+1(2) \end{bmatrix}$

$= \begin{bmatrix} 0 & -3 & -4 \\ 0 & 5 & 2 \end{bmatrix}$

Multiplication with $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ reflects the triangle over the x-axis.

Exercise Set 9.3

1. **a.** 2×3

b. a_{32} does not exist (A only has 2 rows).
$a_{23} = -1$

2. **a.** 2×3

b. a_{32} does not exist (A only has 2 rows). $a_{23} = \frac{1}{2}$

3. **a.** 3×4

b. $a_{32} = \frac{1}{2};\ a_{23} = -6$

4. **a.** 3×4

b. $a_{32} = 0;\ a_{23} = \pi$

5. $\begin{bmatrix} x \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ y \end{bmatrix}$

$x = 6$
$y = 4$

6. $\begin{bmatrix} x \\ 7 \end{bmatrix} = \begin{bmatrix} 11 \\ y \end{bmatrix}$

$x = 11$

$y = 7$

7. $\begin{bmatrix} x & 2y \\ z & 9 \end{bmatrix} = \begin{bmatrix} 4 & 12 \\ 3 & 9 \end{bmatrix}$

$x = 4$
$2y = 12$
$y = 6$
$z = 3$

8. $\begin{bmatrix} x & y+3 \\ 2z & 8 \end{bmatrix} = \begin{bmatrix} 12 & 5 \\ 6 & 8 \end{bmatrix}$

$x = 12$
$y + 3 = 5$
$y = 2$
$2z = 6$
$z = 3$

9. **a.** $A + B = \begin{bmatrix} 4+5 & 1+9 \\ 3+0 & 2+7 \end{bmatrix} = \begin{bmatrix} 9 & 10 \\ 3 & 9 \end{bmatrix}$

b. $A - B = \begin{bmatrix} 4-5 & 1-9 \\ 3-0 & 2-7 \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 3 & -5 \end{bmatrix}$

c. $-4A = \begin{bmatrix} -16 & -4 \\ -12 & -8 \end{bmatrix}$

d. $3A + 2B = \begin{bmatrix} 12+10 & 3+18 \\ 9+0 & 6+14 \end{bmatrix} = \begin{bmatrix} 22 & 21 \\ 9 & 20 \end{bmatrix}$

10. a. $A+B=\begin{bmatrix}-2+8 & 3+1\\ 0+5 & 1+4\end{bmatrix}=\begin{bmatrix}6 & 4\\ 5 & 5\end{bmatrix}$

b. $A-B=\begin{bmatrix}-2-8 & 3-1\\ 0-5 & 1-4\end{bmatrix}=\begin{bmatrix}-10 & 2\\ -5 & -3\end{bmatrix}$

c. $-4A=\begin{bmatrix}-4(-2) & -4(3)\\ -4(0) & -4(1)\end{bmatrix}=\begin{bmatrix}8 & -12\\ 0 & -4\end{bmatrix}$

d. $3A+2B=\begin{bmatrix}3(-2)+2(8) & 3(3)+2(1)\\ 3(0)+2(5) & 3(1)+2(4)\end{bmatrix}$

$=\begin{bmatrix}-6+16 & 9+2\\ 0+10 & 3+8\end{bmatrix}$

$=\begin{bmatrix}10 & 11\\ 10 & 11\end{bmatrix}$

11. a. $A+B=\begin{bmatrix}1+2 & 3+(-1)\\ 3+3 & 4+(-2)\\ 5+0 & 6+1\end{bmatrix}=\begin{bmatrix}3 & 2\\ 6 & 2\\ 5 & 7\end{bmatrix}$

b. $A-B=\begin{bmatrix}1-2 & 3-(-1)\\ 3-3 & 4-(-2)\\ 5-0 & 6-1\end{bmatrix}=\begin{bmatrix}-1 & 4\\ 0 & 6\\ 5 & 5\end{bmatrix}$

c. $-4A=\begin{bmatrix}-4 & -12\\ -12 & -16\\ -20 & -24\end{bmatrix}$

d. $3A+2B=\begin{bmatrix}3+4 & 9-2\\ 9+6 & 12-4\\ 15+0 & 18+2\end{bmatrix}=\begin{bmatrix}7 & 7\\ 15 & 8\\ 15 & 20\end{bmatrix}$

12. a. $A+B=\begin{bmatrix}3+2 & 1+(-3) & 1+6\\ -1+(-3) & 2+1 & 5+(-4)\end{bmatrix}$

$=\begin{bmatrix}5 & -2 & 7\\ -4 & 3 & 1\end{bmatrix}$

b. $A-B=\begin{bmatrix}3-2 & 1-(-3) & 1-6\\ -1-(-3) & 2-1 & 5-(-4)\end{bmatrix}$

$=\begin{bmatrix}1 & 4 & -5\\ 2 & 1 & 9\end{bmatrix}$

c. $-4A=\begin{bmatrix}-12 & -4 & -4\\ 4 & -8 & -20\end{bmatrix}$

d. $3A-2B=\begin{bmatrix} 9+4 & 3-6 & 3+12 \\ -3+-6 & 6+2 & 15-8 \end{bmatrix}$

$=\begin{bmatrix} 13 & -3 & 15 \\ -9 & 8 & 7 \end{bmatrix}$

13. a. $A+B=\begin{bmatrix} 2+(-5) \\ -4+3 \\ 1+(-1) \end{bmatrix}=\begin{bmatrix} -3 \\ -1 \\ 0 \end{bmatrix}$

b. $A-B=\begin{bmatrix} 2-(-5) \\ -4-3 \\ 1-(-1) \end{bmatrix}=\begin{bmatrix} 7 \\ -7 \\ 2 \end{bmatrix}$

c. $-4A=\begin{bmatrix} -8 \\ 16 \\ -4 \end{bmatrix}$

d. $3A+2B=\begin{bmatrix} 6-10 \\ -12+6 \\ 3-2 \end{bmatrix}=\begin{bmatrix} -4 \\ -6 \\ 1 \end{bmatrix}$

14. a. $A+B=[6+4 \quad 2+(-2) \quad -3+3]$
$=[10 \quad 0 \quad 0]$

b. $A-B=[6-4 \quad 2-(-2) \quad -3-3]$
$=[2 \quad 4 \quad -6]$

c. $-4A=[-24 \quad -8 \quad 12]$

d. $3A+2B=\begin{bmatrix} 18+8 & 6-4 & -9+6 \end{bmatrix}$
$\begin{bmatrix} 26 & 2 & -3 \end{bmatrix}$

15. a. $A+B=\begin{bmatrix} 2+6 & -10+10 & -2+(-2) \\ 14+0 & 12+(-12) & 10+(-4) \\ 4+(-5) & -2+2 & 2+(-2) \end{bmatrix}$

$=\begin{bmatrix} 8 & 0 & -4 \\ 14 & 0 & 6 \\ -1 & 0 & 0 \end{bmatrix}$

b. $A-B=\begin{bmatrix} 2-6 & -10-10 & -2-(-2) \\ 14-0 & 12-(-12) & 10-(-4) \\ 4-(-5) & -2-2 & 2-(-2) \end{bmatrix}$

$=\begin{bmatrix} -4 & -20 & 0 \\ 14 & 24 & 14 \\ 9 & -4 & 4 \end{bmatrix}$

c. $-4A = \begin{bmatrix} -8 & 40 & 8 \\ -56 & -48 & -40 \\ -16 & 8 & -8 \end{bmatrix}$

d. $3A + 2B = \begin{bmatrix} 6+12 & -30+20 & -6-4 \\ 42+0 & 36-24 & 30-8 \\ 12-10 & -6+4 & 6-4 \end{bmatrix}$

$= \begin{bmatrix} 18 & -10 & -10 \\ 42 & 12 & 22 \\ 2 & -2 & 2 \end{bmatrix}$

16. a. $A + B = \begin{bmatrix} 6-3 & -3+5 & 5+1 \\ 6-1 & 0+2 & -2-6 \\ -4+2 & 2+0 & -1+4 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 5 & 2 & -8 \\ -2 & 2 & 3 \end{bmatrix}$

b. $A - B = \begin{bmatrix} 6+3 & -3-5 & 5-1 \\ 6+1 & 0-2 & -2+6 \\ -4-2 & 2-0 & -1-4 \end{bmatrix} = \begin{bmatrix} 9 & -8 & 4 \\ 7 & -2 & 4 \\ -6 & 2 & -5 \end{bmatrix}$

c. $-4A = \begin{bmatrix} -4(6) & -4(-3) & -4(5) \\ -4(6) & -4(0) & -4(-2) \\ -4(-4) & -4(2) & -4(-1) \end{bmatrix} = \begin{bmatrix} -24 & 12 & -20 \\ -24 & 0 & 8 \\ 16 & -8 & 4 \end{bmatrix}$

d. $3A + 2B = \begin{bmatrix} 3(6)+2(-3) & 3(-3)+2(5) & 3(5)+2(1) \\ 3(6)+2(-1) & 3(0)+2(2) & 3(-2)+2(-6) \\ 3(-4)+2(2) & 3(-2)+2(0) & 3(-1)+2(4) \end{bmatrix} = \begin{bmatrix} 18-6 & -9-10 & 15+2 \\ 18-2 & 0+4 & -6-12 \\ -12+4 & 6+0 & -3+8 \end{bmatrix} = \begin{bmatrix} 12 & 1 & 17 \\ 16 & 4 & -18 \\ -8 & 6 & 5 \end{bmatrix}$

17. $X - A = B$

$X = A + B$

$X = \begin{bmatrix} -3 & -7 \\ 2 & -9 \\ 5 & 0 \end{bmatrix} + \begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} -8 & -8 \\ 2 & -9 \\ 8 & -4 \end{bmatrix}$

18. $X - B = A$

$X = A + B$

$X = \begin{bmatrix} -3 & -7 \\ 2 & -9 \\ 5 & 0 \end{bmatrix} + \begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} -8 & -8 \\ 2 & -9 \\ 8 & -4 \end{bmatrix}$

19. $2X + A = B$

$$2X = B - A$$

$$X = \frac{1}{2}(B - A)$$

$$X = \frac{1}{2}\left(\begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix} - \begin{bmatrix} -3 & -7 \\ 2 & -9 \\ 5 & 0 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} -2 & 6 \\ -2 & 9 \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -1 & \frac{9}{2} \\ -1 & -2 \end{bmatrix}$$

20. $3X + A = B$

$$3X = B - A$$

$$X = \frac{1}{3}(B - A)$$

$$X = \frac{1}{3}\left(\begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix} - \begin{bmatrix} -3 & -7 \\ 2 & -9 \\ 5 & 0 \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} -2 & 6 \\ -2 & 9 \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & 2 \\ -\frac{2}{3} & 3 \\ -\frac{2}{3} & -\frac{4}{3} \end{bmatrix}$$

21. $3X + 2A = B$

$$3X = B - 2A$$

$$X = \frac{1}{3}(B - 2A)$$

$$X = \frac{1}{3}\left(\begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix} - 2\begin{bmatrix} -3 & -7 \\ 2 & -9 \\ 5 & 0 \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} 1 & 13 \\ -4 & 18 \\ -7 & -4 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{13}{3} \\ -\frac{4}{3} & 6 \\ -\frac{7}{3} & -\frac{4}{3} \end{bmatrix}$$

22. $2X + 5A = B$

$$2X = B - 5A$$

$$X = \frac{1}{2}(B - 5A)$$

$$X = \frac{1}{2}\left(\begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix} - 5\begin{bmatrix} -3 & -7 \\ 2 & -9 \\ 5 & 0 \end{bmatrix}\right) = \frac{1}{2}\left(\begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 15 & 35 \\ -10 & 45 \\ -25 & 0 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 10 & 34 \\ -10 & 45 \\ -22 & -4 \end{bmatrix} = \begin{bmatrix} 5 & 17 \\ -5 & \frac{45}{2} \\ -11 & -2 \end{bmatrix}$$

23. $B - X = 4A$

$$B - 4A = X$$

$$X = \begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix} - 4\begin{bmatrix} -3 & -7 \\ 2 & -9 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 12 & 28 \\ -8 & 36 \\ -20 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 27 \\ -8 & 36 \\ -17 & -4 \end{bmatrix}$$

24. $A - X = 4B$

$A - 4B = X$

$$X = \begin{bmatrix} -3 & -7 \\ 2 & -9 \\ 5 & 0 \end{bmatrix} - 4\begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} -3 & -7 \\ 2 & -9 \\ 5 & 0 \end{bmatrix} + \begin{bmatrix} 20 & 4 \\ 0 & 0 \\ -12 & 16 \end{bmatrix} = \begin{bmatrix} 17 & -3 \\ 2 & -9 \\ -7 & 16 \end{bmatrix}$$

25. $4A + 3B = -2X$

$-\frac{1}{2}(4A + 3B) = X$

$$X = -\frac{1}{2}\left(4\begin{bmatrix} -3 & -7 \\ 2 & -9 \\ 5 & 0 \end{bmatrix} + 3\begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix}\right) = -\frac{1}{2}\left(\begin{bmatrix} -12 & -28 \\ 8 & -36 \\ 20 & 0 \end{bmatrix} + \begin{bmatrix} -15 & -3 \\ 0 & 0 \\ 9 & -12 \end{bmatrix}\right) = -\frac{1}{2}\begin{bmatrix} -27 & -31 \\ 8 & -36 \\ 29 & -12 \end{bmatrix} = \begin{bmatrix} \frac{27}{2} & \frac{31}{2} \\ -4 & 18 \\ -\frac{29}{2} & 6 \end{bmatrix}$$

26. $4B + 3A = -2X$

$$X = -\frac{1}{2}\left(4\begin{bmatrix} -5 & -1 \\ 0 & 0 \\ 3 & -4 \end{bmatrix} + 3\begin{bmatrix} -3 & -7 \\ 2 & -9 \\ 5 & 0 \end{bmatrix}\right) = -\frac{1}{2}\left(\begin{bmatrix} -20 & -4 \\ 0 & 0 \\ 12 & -16 \end{bmatrix} + \begin{bmatrix} -9 & -21 \\ 6 & -27 \\ 15 & 0 \end{bmatrix}\right) = -\frac{1}{2}\begin{bmatrix} -29 & -25 \\ 6 & -27 \\ 27 & -16 \end{bmatrix} = \begin{bmatrix} \frac{29}{2} & \frac{25}{2} \\ -3 & \frac{27}{2} \\ -\frac{27}{2} & 8 \end{bmatrix}$$

27. a. $AB = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -1 & 6 \end{bmatrix} = \begin{bmatrix} (1)(3)+(3)(-1) & (1)(-2)+(3)(6) \\ (5)(3)+(3)(-1) & (5)(-2)+(3)(6) \end{bmatrix} = \begin{bmatrix} 3-3 & -2+18 \\ 15-3 & -10+18 \end{bmatrix} = \begin{bmatrix} 0 & 16 \\ 12 & 8 \end{bmatrix}$

b. $BA = \begin{bmatrix} 3 & -2 \\ -1 & 6 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} (3)(1)+(-2)(5) & (3)(3)+(-2)(3) \\ (-1)(1)+(6)(5) & (-1)(3)+(6)(3) \end{bmatrix} = \begin{bmatrix} 3-10 & 9-6 \\ -1+30 & -3+18 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 29 & 15 \end{bmatrix}$

28. a. $AB = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 5 & -6 \end{bmatrix} = \begin{bmatrix} (3)(0)+(-2)(5) & (3)(0)+(-2)(-6) \\ (1)(0)+(5)(5) & (1)(0)+(5)(-6) \end{bmatrix} = \begin{bmatrix} 0-10 & 0+12 \\ 0+25 & 0-30 \end{bmatrix} = \begin{bmatrix} -10 & 12 \\ 25 & -30 \end{bmatrix}$

b. $BA = \begin{bmatrix} 0 & 0 \\ 5 & -6 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} (0)(3)+(0)(1) & (0)(-2)+(0)(5) \\ (5)(3)+(-6)(1) & (5)(-2)+(-6)(5) \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 15-6 & -10-30 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 9 & -40 \end{bmatrix}$

29. a. $AB = [1\ \ 2\ \ 3\ \ 4]\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = [(1)(1) + (2)(2) + (3)(3) + (4)(4)] = [1 + 4 + 9 + 16] = [30]$

b. $BA = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}[1\ \ 2\ \ 3\ \ 4] = \begin{bmatrix} (1)(1) & (1)(2) & (1)(3) & (1)(4) \\ (2)(1) & (2)(2) & (2)(3) & (2)(4) \\ (3)(1) & (3)(2) & (3)(3) & (3)(4) \\ (4)(1) & (4)(2) & (4)(3) & (4)(4) \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$

30. **a.** $AB=\begin{bmatrix}-1\\-2\\-3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\end{bmatrix}=\begin{bmatrix}(-1)(1) & (-1)(2) & (-1)(3)\\(-2)(1) & (-2)(2) & (-2)(3)\\(-3)(1) & (-3)(2) & (-3)(3)\end{bmatrix}=\begin{bmatrix}-1 & -2 & -3\\-2 & -4 & -6\\-3 & -6 & -9\end{bmatrix}$

b. $BA=\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}-1\\-2\\-3\end{bmatrix}=[(1)(-1)+(2)(-2)+(3)(-3)]=[-1-4-9]=[-14]$

31. **a.** $AB=\begin{bmatrix}1 & -1 & 4\\4 & -1 & 3\\2 & 0 & -2\end{bmatrix}\begin{bmatrix}1 & 1 & 0\\1 & 2 & 4\\1 & -1 & 3\end{bmatrix}$

$=\begin{bmatrix}(1)(1)+(-1)(1)+(4)(1) & (1)(1)+(-1)(2)+(4)(-1) & (1)(0)+(-1)(4)+(4)(3)\\(4)(1)+(-1)(1)+(3)(1) & (4)(1)+(-1)(2)+(3)(-1) & (4)(0)+(-1)(4)+(3)(3)\\(2)(1)+(0)(1)+(-2)(1) & (2)(1)+(0)(2)+(-2)(-1) & (2)(0)+(0)(4)+(-2)(3)\end{bmatrix}$

$=\begin{bmatrix}1-1+4 & 1-2-4 & 0-4+12\\4-1+3 & 4-2-3 & 0-4+9\\2+0-2 & 2+0+2 & 0+0-6\end{bmatrix}=\begin{bmatrix}4 & -5 & 8\\6 & -1 & 5\\0 & 4 & -6\end{bmatrix}$

b. $BA=\begin{bmatrix}1 & 1 & 0\\1 & 2 & 4\\1 & -1 & 3\end{bmatrix}\begin{bmatrix}1 & -1 & 4\\4 & -1 & 3\\2 & 0 & -2\end{bmatrix}$

$=\begin{bmatrix}(1)(1)+(1)(4)+(0)(2) & (1)(-1)+(1)(-1)+(0)(0) & (1)(4)+(1)(3)+(0)(-2)\\(1)(1)+(2)(4)+(4)(2) & (1)(-1)+(2)(-1)+(4)(0) & (1)(4)+(2)(3)+(4)(-2)\\(1)(1)+(-1)(4)+(3)(2) & (1)(-1)+(-1)(-1)+(3)(0) & (1)(4)+(-1)(3)+(3)(-2)\end{bmatrix}$

$=\begin{bmatrix}1+4+0 & -1-1+0 & 4+3+0\\1+8+8 & -1-2+0 & 4+6-8\\1-4+6 & -1+1+0 & 4-3-6\end{bmatrix}=\begin{bmatrix}5 & -2 & 7\\17 & -3 & 2\\3 & 0 & -5\end{bmatrix}$

32. **a.** $AB=\begin{bmatrix}1 & -1 & 1\\5 & 0 & -2\\3 & -2 & 2\end{bmatrix}\begin{bmatrix}1 & 1 & 0\\1 & -4 & 5\\3 & -1 & 2\end{bmatrix}$

$=\begin{bmatrix}(1)(1)+(-1)(1)+(1)(3) & (1)(1)+(-1)(-4)+(1)(-1) & (1)(0)+(-1)(5)+(1)(2)\\(5)(1)+(0)(1)+(-2)(3) & (5)(1)+(0)(-4)+(-2)(-1) & (5)(0)+(0)(5)+(-2)(2)\\(3)(1)+(-2)(1)+(2)(3) & (3)(1)+(-2)(-4)+(2)(-1) & (3)(0)+(-2)(5)+(2)(2)\end{bmatrix}$

$=\begin{bmatrix}1-1+3 & 1+4-1 & 0-5+2\\5+0-6 & 5+0+2 & 0+0-4\\3-2+6 & 3+8-2 & 0-10+4\end{bmatrix}=\begin{bmatrix}3 & 4 & -3\\-1 & 7 & -4\\7 & 9 & -6\end{bmatrix}$

b. $BA=\begin{bmatrix}1&1&0\\1&-4&5\\3&-1&2\end{bmatrix}\begin{bmatrix}1&-1&1\\5&0&-2\\3&-2&2\end{bmatrix}$

$$=\begin{bmatrix}(1)(1)+(1)(5)+(0)(3)&(1)(-1)+(1)(0)+(0)(-2)&(1)(1)+(1)(-2)+(0)(2)\\(1)(1)+(-4)(5)+(5)(3)&(1)(-1)+(-4)(0)+(5)(-2)&(1)(1)+(-4)(-2)+(5)(2)\\(3)(1)+(-1)(5)+(2)(3)&(3)(-1)+(-1)(0)+(2)(-2)&(3)(1)+(-1)(-2)+(2)(2)\end{bmatrix}$$

$$=\begin{bmatrix}1+5+0&-1+0+0&1-2+0\\1-20+15&-1+0-10&1+8+10\\3-5+6&-3+0-4&3+2+4\end{bmatrix}=\begin{bmatrix}6&-1&-1\\-4&-11&19\\4&-7&9\end{bmatrix}$$

33. a. $AB=\begin{bmatrix}4&2\\6&1\\3&5\end{bmatrix}\begin{bmatrix}2&3&4\\-1&-2&0\end{bmatrix}=\begin{bmatrix}(4)(2)+(2)(-1)&(4)(3)+(2)(-2)&(4)(4)+(2)(0)\\(6)(2)+(1)(-1)&(6)(3)+(1)(-2)&(6)(4)+(1)(0)\\(3)(2)+(5)(-1)&(3)(3)+(5)(-2)&(3)(4)+(5)(0)\end{bmatrix}$

$$=\begin{bmatrix}8-2&12-4&16+0\\12-1&18-2&24+0\\6-5&9-10&12+0\end{bmatrix}=\begin{bmatrix}6&8&16\\11&16&24\\1&-1&12\end{bmatrix}$$

b. $BA=\begin{bmatrix}2&3&4\\-1&-2&0\end{bmatrix}\begin{bmatrix}4&2\\6&1\\3&5\end{bmatrix}=\begin{bmatrix}(2)(4)+(3)(6)+(4)(3)&(2)(2)+(3)(1)+(4)(5)\\(-1)(4)+(-2)(6)+(0)(3)&(-1)(2)+(-2)(1)+(0)(5)\end{bmatrix}$

$$=\begin{bmatrix}8+18+12&4+3+20\\-4-12+0&-2-2+0\end{bmatrix}=\begin{bmatrix}38&27\\-16&-4\end{bmatrix}$$

34. a. $AB=\begin{bmatrix}2&4\\3&1\\4&2\end{bmatrix}\begin{bmatrix}3&2&0\\-1&-3&5\end{bmatrix}=\begin{bmatrix}(2)(3)+(4)(-1)&(2)(2)+(4)(-3)&(2)(0)+(4)(5)\\(3)(3)+(1)(-1)&(3)(2)+(1)(-3)&(3)(0)+(1)(5)\\(4)(3)+(2)(-1)&(4)(2)+(2)(-3)&(4)(0)+(2)(5)\end{bmatrix}$

$$=\begin{bmatrix}6-4&4-12&0+20\\9-1&6-3&0+5\\12-2&8-6&0+10\end{bmatrix}=\begin{bmatrix}2&-8&20\\8&3&5\\10&2&10\end{bmatrix}$$

b. $BA=\begin{bmatrix}3&2&0\\-1&-3&5\end{bmatrix}\begin{bmatrix}2&4\\3&1\\4&2\end{bmatrix}=\begin{bmatrix}(3)(2)+(2)(3)+(0)(4)&(3)(4)+(2)(1)+(0)(2)\\(-1)(2)+(-3)(3)+(5)(4)&(-1)(4)+(-3)(1)+(5)(2)\end{bmatrix}$

$$=\begin{bmatrix}6+6+0&12+2+0\\-2-9+20&-4-3+10\end{bmatrix}=\begin{bmatrix}12&14\\9&3\end{bmatrix}$$

35. a. $AB=\begin{bmatrix}2&-3&1&-1\\1&1&-2&1\end{bmatrix}\begin{bmatrix}1&2\\-1&1\\5&4\\10&5\end{bmatrix}=\begin{bmatrix}(2)(1)+(-3)(-1)+(1)(5)+(-1)(10)&(2)(2)+(-3)(1)+(1)(4)+(-1)(5)\\(1)(1)+(1)(-1)+(-2)(5)+(1)(10)&(1)(2)+(1)(1)+(-2)(4)+(1)(5)\end{bmatrix}$

$$=\begin{bmatrix}2+3+5-10&4-3+4-5\\1-1-10+10&2+1-8+5\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

36. **a.** $AB = \begin{bmatrix} 2 & -1 & 3 & 2 \\ 1 & 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & 1 \\ 3 & -4 \\ 6 & 5 \end{bmatrix} = \begin{bmatrix} (2)(-1)+(-1)(1)+(3)(3)+(2)(6) & (2)(2)+(-1)(1)+(3)(-4)+(2)(5) \\ (1)(-1)+(0)(1)+(-2)(3)+(1)(6) & (1)(2)+(0)(1)+(-2)(-4)+(1)(5) \end{bmatrix}$

$= \begin{bmatrix} -2-1+9+12 & 4-1-12+10 \\ -1+0-6+6 & 2+0+8+5 \end{bmatrix} = \begin{bmatrix} 18 & 1 \\ -1 & 15 \end{bmatrix}$

b. $BA = \begin{bmatrix} -1 & 2 \\ 1 & 1 \\ 3 & -4 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 & 2 \\ 1 & 0 & -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} (-1)(2)+(2)(1) & (-1)(-1)+(2)(0) & (-1)(3)+(2)(-2) & (-1)(2)+(2)(1) \\ (1)(2)+(1)(1) & (1)(-1)+(1)(0) & (1)(3)+(1)(-2) & (1)(2)+(1)(1) \\ (3)(2)+(-4)(1) & (3)(-1)+(-4)(0) & (3)(3)+(-4)(-2) & (3)(2)+(-4)(1) \\ (6)(2)+(5)(1) & (6)(-1)+(5)(0) & (6)(3)+(5)(-2) & (6)(2)+(5)(1) \end{bmatrix}$

$= \begin{bmatrix} -2+2 & 1+0 & -3-4 & -2+2 \\ 2+1 & -1+0 & 3-2 & 2+1 \\ 6-4 & -3+0 & 9+8 & 6-4 \\ 12+5 & -6+0 & 18-10 & 12+5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -7 & 0 \\ 3 & -1 & 1 & 3 \\ 2 & -3 & 17 & 2 \\ 17 & -6 & 8 & 17 \end{bmatrix}$

37. $4B-3C = \begin{bmatrix} 20 & 4 \\ -8 & -8 \end{bmatrix} - \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 20-3 & 4-(-3) \\ -8-(-3) & -8-3 \end{bmatrix} = \begin{bmatrix} 17 & 7 \\ -5 & -11 \end{bmatrix}$

38. $5C-2B = \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix} - \begin{bmatrix} 10 & 2 \\ -4 & -4 \end{bmatrix} = \begin{bmatrix} -5 & -7 \\ -1 & 9 \end{bmatrix}$

39. $BC+CB = \begin{bmatrix} 5-1 & -5+1 \\ -2+2 & 2-2 \end{bmatrix} + \begin{bmatrix} 5+2 & 1+2 \\ -5-2 & -1-2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 7 & 3 \\ -7 & -3 \end{bmatrix} = \begin{bmatrix} 11 & -1 \\ -7 & -3 \end{bmatrix}$

40. $A(B+C) = \begin{bmatrix} 4 & 0 \\ -3 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 0 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} 24+0 & 0+0 \\ -18-15 & 0-5 \\ 0-3 & 0-1 \end{bmatrix} = \begin{bmatrix} 24 & 0 \\ -33 & -5 \\ -3 & -1 \end{bmatrix}$

41. $A-C$ is not defined because A is 3 x 2 and C is 2 x 2.

42. $B-A$ is not defined because B is 2 x 2 and A is 3 x 2.

43. $A(BC) = \begin{bmatrix} 4 & 0 \\ -3 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5-1 & -5+1 \\ -2+2 & 2-2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -3 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & -4 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 16+0 & -16+0 \\ -12+0 & 12+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 16 & -16 \\ -12 & 12 \\ 0 & 0 \end{bmatrix}$

44. $A(CB) = \begin{bmatrix} 4 & 0 \\ -3 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5+2 & 1+2 \\ -5-2 & -1-2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -3 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 7 & 3 \\ -7 & -3 \end{bmatrix} = \begin{bmatrix} 28+0 & 12+0 \\ -21-35 & -9-15 \\ 0-7 & 0-3 \end{bmatrix} = \begin{bmatrix} 28 & 12 \\ -56 & -24 \\ -7 & -3 \end{bmatrix}$

45. $(A+B)(C-D) = \left(\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\right)\left(\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}-\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}\right)=\left(\begin{bmatrix}2 & 0\\0 & 0\end{bmatrix}\right)\left(\begin{bmatrix}0 & 0\\0 & 2\end{bmatrix}\right)=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

46. $(A-B)(C+D) = \left(\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}-\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\right)\left(\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}\right)=\left(\begin{bmatrix}0 & 0\\0 & 2\end{bmatrix}\right)\left(\begin{bmatrix}-2 & 0\\0 & 0\end{bmatrix}\right)=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

47. – 48. Answers may vary.

49. $BZ=\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}x\\-y\end{bmatrix}$ This reflects the graphic about the x-axis because all y-coordinates are negated.

50. $CZ=\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-x\\y\end{bmatrix}$ This reflects the graphic about the y-axis because all x-coordinates are negated.

51. **a.** $\begin{bmatrix}1 & 3 & 1\\3 & 3 & 3\\1 & 3 & 1\end{bmatrix}$

b. $\begin{bmatrix}1 & 3 & 1\\3 & 3 & 3\\1 & 3 & 1\end{bmatrix}+\begin{bmatrix}-1 & -1 & -1\\-1 & -1 & -1\\-1 & -1 & -1\end{bmatrix}=\begin{bmatrix}0 & 2 & 0\\2 & 2 & 2\\0 & 2 & 0\end{bmatrix}$

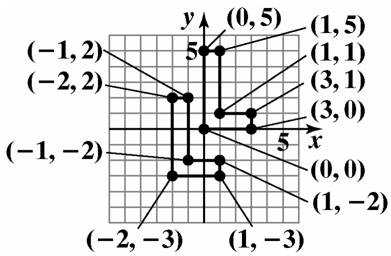
c. $\begin{bmatrix}1 & 3 & 1\\3 & 3 & 3\\1 & 3 & 1\end{bmatrix}+\begin{bmatrix}1 & -2 & 1\\-2 & -2 & -2\\1 & -2 & 1\end{bmatrix}=\begin{bmatrix}2 & 1 & 2\\1 & 1 & 1\\2 & 1 & 2\end{bmatrix}$

52. **a.** $\begin{bmatrix}1 & 3 & 1\\3 & 3 & 3\\1 & 3 & 1\end{bmatrix}$

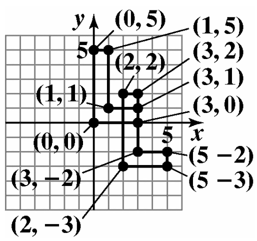
b. $\begin{bmatrix}1 & 3 & 1\\3 & 3 & 3\\1 & 3 & 1\end{bmatrix}+\begin{bmatrix}2 & -1 & 2\\-1 & -1 & -1\\2 & -1 & 2\end{bmatrix}=\begin{bmatrix}3 & 2 & 3\\2 & 2 & 2\\3 & 2 & 3\end{bmatrix}$

c. $\begin{bmatrix}1 & 3 & 1\\3 & 3 & 3\\1 & 3 & 1\end{bmatrix}+\begin{bmatrix}-1 & 0 & -1\\0 & 0 & 0\\-1 & 0 & -1\end{bmatrix}=\begin{bmatrix}0 & 3 & 0\\3 & 3 & 3\\0 & 3 & 0\end{bmatrix}$

53. $\begin{bmatrix} 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 5 & 5 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 & -2 & -2 & -2 \\ -3 & -3 & -3 & -3 & -3 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 1 & -1 & -1 & -2 \\ -3 & -3 & -2 & -2 & 2 & 2 \end{bmatrix}$

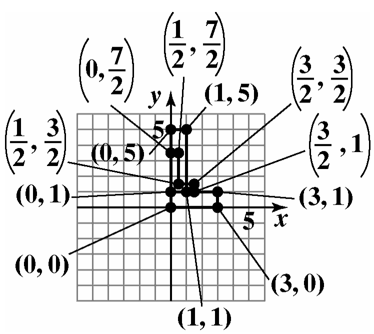


54. $\begin{bmatrix} 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 5 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ -3 & -3 & -3 & -3 & -3 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 5 & 3 & 3 & 2 \\ -3 & -3 & -2 & -2 & 2 & 2 \end{bmatrix}$



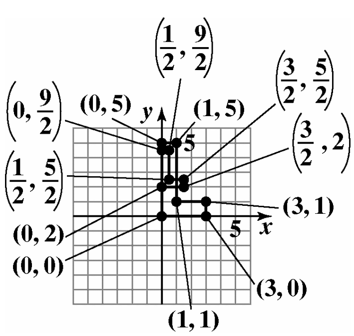
55. $0.5\begin{bmatrix} 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 5 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1.5 & 1.5 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0.5 & 2.5 & 2.5 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1.5 & 1.5 & 0.5 & 0.5 & 0 \\ 1 & 1 & 1.5 & 1.5 & 3.5 & 3.5 \end{bmatrix}$



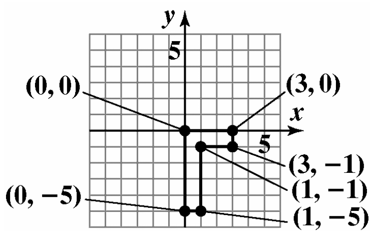
56. $0.5\begin{bmatrix} 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 5 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1.5 & 1.5 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0.5 & 2.5 & 2.5 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1.5 & 1.5 & 0.5 & 0.5 & 0 \\ 2 & 2 & 2.5 & 2.5 & 4.5 & 4.5 \end{bmatrix}$



57. a. $AB = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 5 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & -5 & -5 \end{bmatrix}$

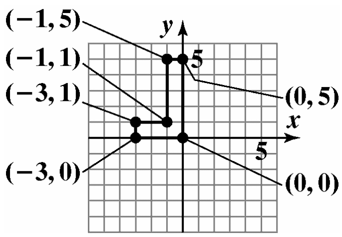
b.



Rotated L about the *x*-axis.

58. a. $AB = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 5 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -3 & -3 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 5 & 5 \end{bmatrix}$

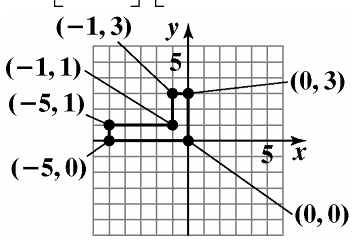
b.



Rotated L about the *y*-axis.

59. a. $AB = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 5 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & -1 & -5 & -5 \\ 0 & 3 & 3 & 1 & 1 & 0 \end{bmatrix}$

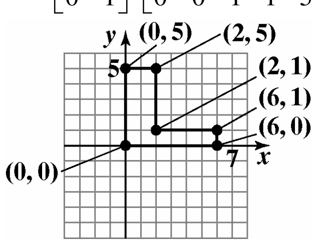
b.



Rotated L $90°$ counterclockwise about the origin.

60. a. $AB = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 5 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 6 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 & 5 & 5 \end{bmatrix}$

b.



Stretch L by a factor of 2 horizontally.

61. a. $A = \begin{bmatrix} 2 & 6 \\ 31 & 46 \end{bmatrix}$

b. $B = \begin{bmatrix} 9 & 29 \\ 65 & 77 \end{bmatrix}$

c. $B - A = \begin{bmatrix} 9 & 29 \\ 65 & 77 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 31 & 46 \end{bmatrix} = \begin{bmatrix} 7 & 23 \\ 34 & 31 \end{bmatrix}$

This matrix represents the percentages of people completing the transition to adulthood in 1960 and 2000 by age and gender.

62. **a.** $M = \begin{bmatrix} 2400 & 2700 & 3000 \\ 2200 & 2500 & 2900 \\ 2000 & 2300 & 2600 \end{bmatrix}$

b. $W = \begin{bmatrix} 2000 & 2100 & 2400 \\ 1800 & 2000 & 2200 \\ 1600 & 1800 & 2100 \end{bmatrix}$

c. $M - W = \begin{bmatrix} 2400 & 2700 & 3000 \\ 2200 & 2500 & 2900 \\ 2000 & 2300 & 2600 \end{bmatrix} - \begin{bmatrix} 2000 & 2100 & 2400 \\ 1800 & 2000 & 2200 \\ 1600 & 1800 & 2100 \end{bmatrix} = \begin{bmatrix} 400 & 600 & 600 \\ 400 & 500 & 700 \\ 400 & 500 & 500 \end{bmatrix}$

This matrix represents the differences between the basic caloric needs of men and women by age and activity level.

63. **a.** System 1: The midterm and final both count for 50% of the course grade.
System 2: The midterm counts for 30% of the course grade and the final counts for 70%

b. $AB = \begin{bmatrix} 84 & 87.2 \\ 79 & 81 \\ 90 & 88.4 \\ 73 & 68.6 \\ 69 & 73.4 \end{bmatrix}$

System 1 grades are listed first (if different).
Student 1: B; Student 2: C or B; Student 3: A or B; Student 4: C or D; Student 5: D or C

64. **a.** $AB = \begin{bmatrix} 0.40 & 0.30 & 0.70 \\ 0.30 & 0.60 & 0.25 \\ 0.30 & 0.10 & 0.05 \end{bmatrix} \begin{bmatrix} 6000 & 8000 \\ 12{,}000 & 14{,}000 \\ 14{,}000 & 16{,}000 \end{bmatrix} = \begin{bmatrix} 15{,}800 & 18{,}600 \\ 12{,}500 & 14{,}800 \\ 3700 & 4600 \end{bmatrix}$

b. *AB* represents the distribution of voters by gender and political party registration. In this county, there are 14,800 female Democrats.

c. *AB* represents the distribution of voters by gender and political party registration. In this county, there are 15,800 male Republicans.

65. – 76. Answers may vary.

77. makes sense

78. does not make sense; Explanations will vary. Sample explanation: Matrix multiplication is not accomplished by simply multiplying corresponding elements.

79. makes sense

80. makes sense

81. Answers may vary.

82. When a matrix of this type is multiplied by itself, each element of the product is the square of the corresponding element of the original..

83. $AB = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$-BA = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$AB = -BA$ so they are anticommutative.

84. Answers may vary.

85. $AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Nothing happens to the elements in the first matrix.

86. $\left[\begin{array}{ccc|c} -1 & -1 & -1 & 1 \\ 4 & 5 & 0 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right] 4R_1 + R_2$

$\left[\begin{array}{ccc|c} -1 & -1 & -1 & 1 \\ 0 & 1 & -4 & 4 \\ 0 & 1 & -3 & 0 \end{array}\right] \begin{array}{l} R_2 + R_1 \\ \\ -R_2 + R_3 \end{array}$

$\left[\begin{array}{ccc|c} -1 & 0 & -5 & 5 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & -4 \end{array}\right] -R_1$

$\left[\begin{array}{ccc|c} 1 & 0 & 5 & -5 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 1 & -4 \end{array}\right] \begin{array}{l} -5R_3 + R_1 \\ 4R_3 + R_2 \\ \\ \end{array}$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & -12 \\ 0 & 0 & 1 & -4 \end{array}\right]$

The solution set is $\{(15, -12, -4)\}$.

87. $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ x \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

$\begin{bmatrix} a_1x + b_1y + c_1x \\ a_2x + b_2y + c_2z \\ a_3x + b_3y + c_3z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

The linear system is written as follows.

$a_1x + b_1y + c_1x = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

Mid-Chapter 9 Check Point

1. $\left[\begin{array}{rrr|r} 1 & 2 & -3 & -7 \\ 3 & -1 & 2 & 8 \\ 2 & -1 & 1 & 5 \end{array}\right]\begin{array}{l} \\ -3R_1+R_2 \\ -2R_1+R_3 \end{array}$

$\left[\begin{array}{rrr|r} 1 & 2 & -3 & -7 \\ 0 & -7 & 11 & 29 \\ 0 & -5 & 7 & 19 \end{array}\right]\begin{array}{l} \\ -\frac{1}{7}R_2 \\ -\frac{1}{5}R_3 \end{array}$

$\left[\begin{array}{rrr|r} 1 & 2 & -3 & -7 \\ 0 & 1 & -\frac{11}{7} & -\frac{29}{7} \\ 0 & 1 & -\frac{7}{5} & -\frac{19}{5} \end{array}\right]\begin{array}{l} \\ \\ -R_2+R_3 \end{array}$

$\left[\begin{array}{rrr|r} 1 & 2 & -3 & -7 \\ 0 & 1 & -\frac{11}{7} & -\frac{29}{7} \\ 0 & 0 & \frac{6}{35} & \frac{12}{35} \end{array}\right]\begin{array}{l} \\ \\ \frac{35}{6}R_3 \end{array}$

$\left[\begin{array}{rrr|r} 1 & 2 & -3 & -7 \\ 0 & 1 & -\frac{11}{7} & -\frac{29}{7} \\ 0 & 0 & 1 & 2 \end{array}\right]$

Back-substitute to find y.

$$y-\frac{11}{7}z=-\frac{29}{7}$$
$$y-\frac{11}{7}(2)=-\frac{29}{7}$$
$$y-\frac{22}{7}=-\frac{29}{7}$$
$$y=-\frac{7}{7}$$
$$y=-1$$

Back-substitute to find x.

$$x+2y-3z=-7$$
$$x+2(-1)-3(2)=-7$$
$$x-2-6=-7$$
$$x-8=-7$$
$$x=1$$

The solution is $\{(1,-1,2)\}$.

2. $\left[\begin{array}{rrr|r} 2 & 4 & 5 & 2 \\ 1 & 1 & 2 & 1 \\ 3 & 5 & 7 & 4 \end{array}\right] R_1 \leftrightarrow R_3$

$\left[\begin{array}{rrr|r} 1 & 1 & 2 & 1 \\ 2 & 4 & 5 & 2 \\ 3 & 5 & 7 & 4 \end{array}\right]\begin{array}{l} \\ -2R_1+R_2 \\ -3R_1+R_3 \end{array}$

$\left[\begin{array}{rrr|r} 1 & 1 & 2 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 1 \end{array}\right]\begin{array}{l} \\ \\ -R_2+R_3 \end{array}$

$\left[\begin{array}{rrr|r} 1 & 1 & 2 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

The third row of the matrix is equivalent to $0x+0y+0z=1$ which is false.

The solution is $\varnothing$.

3. $\left[\begin{array}{rrr|r} 1 & -2 & 2 & -2 \\ 2 & 3 & -1 & 1 \end{array}\right]\begin{array}{l} \\ -2R_1+R_2 \end{array}$

$\left[\begin{array}{rrr|r} 1 & -2 & 2 & -2 \\ 0 & 7 & -5 & 5 \end{array}\right]\begin{array}{l} \\ \frac{1}{7}R_2 \end{array}$

$\left[\begin{array}{rrr|r} 1 & -2 & 2 & -2 \\ 0 & 1 & -\frac{5}{7} & \frac{5}{7} \end{array}\right]$

Back-substitute to find y in terms of z.

$$y-\tfrac{5}{7}z=\tfrac{5}{7}$$
$$y=\tfrac{5}{7}z+\tfrac{5}{7}$$

Back-substitute to find x in terms of z.

$$x-2y+2z=-2$$
$$x-2\left(\tfrac{5}{7}z+\tfrac{5}{7}\right)+2z=-2$$
$$x-\tfrac{10}{7}z-\tfrac{10}{7}+2z=-2$$
$$x+\tfrac{4}{7}z-\tfrac{10}{7}=-2$$
$$x=-\tfrac{4}{7}z-\tfrac{4}{7}$$

The solution is $\left\{\left(-\frac{4}{7}z-\frac{4}{7}, \frac{5}{7}z+\frac{5}{7}, z\right)\right\}$.

4. $\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 1 & -1 & 3 & 1 & -14 \\ 1 & 2 & 0 & -3 & 12 \\ 2 & 3 & 6 & 1 & 1 \end{array}\right] \begin{array}{l} \\ -R_1+R_2 \\ -R_1+R_3 \\ -2R_1+R_4 \end{array}$

$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & -2 & 2 & 0 & -20 \\ 0 & 1 & -1 & -4 & 6 \\ 0 & 1 & 4 & -1 & -11 \end{array}\right] R_2 \leftrightarrow R_3$

$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -4 & 6 \\ 0 & -2 & 2 & 0 & -20 \\ 0 & 1 & 4 & -1 & -11 \end{array}\right] \begin{array}{l} \\ \\ 2R_2+R_3 \\ -R_2+R_4 \end{array}$

$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -4 & 6 \\ 0 & 0 & 0 & -8 & -8 \\ 0 & 0 & 5 & 3 & -17 \end{array}\right] R_3 \leftrightarrow R_4$

$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -4 & 6 \\ 0 & 0 & 5 & 3 & -17 \\ 0 & 0 & 0 & -8 & -8 \end{array}\right] \begin{array}{l} \\ \\ \frac{1}{5}R_3 \\ -\frac{1}{8}R_4 \end{array}$

$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -4 & 6 \\ 0 & 0 & 1 & \frac{3}{5} & -\frac{17}{5} \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$

Back-substitute to find y in terms of z.

$$y+\frac{3}{5}z=-\frac{17}{5}$$
$$y+\frac{3}{5}(1)=-\frac{17}{5}$$
$$y+\frac{3}{5}=-\frac{17}{5}$$
$$y=-\frac{20}{5}$$
$$y=-4$$

Back-substitute to find x in terms of z.

$$x-y-4z=6$$
$$x-(-4)-4(1)=6$$
$$x+4-4=6$$
$$x=6$$

Back-substitute to find w in terms of z.

$$w+x+y+z=6$$
$$w+(6)+(-4)+(1)=6$$
$$w+3=6$$
$$w=3$$

The solution is $\{(3, 6, -4, 1)\}$.

5. $\left[\begin{array}{ccc|c} 2 & -2 & 2 & 5 \\ 1 & -1 & 1 & 2 \\ 2 & 1 & -1 & 1 \end{array}\right] \begin{array}{l} -R_2+R_1 \\ \\ -R_1+R_3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 1 & -1 & 1 & 2 \\ 0 & 3 & -3 & -4 \end{array}\right] -R_1+R_2$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 0 & 0 & -1 \\ 0 & 3 & -3 & -4 \end{array}\right]$

The second row of the matrix is equivalent to $0x+0y+0z=-1$ which is false.

The solution is $\varnothing$.

6. $2C-\frac{1}{2}B$

$$=2\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}-\frac{1}{2}\begin{bmatrix} 4 & 1 \\ -6 & -2 \end{bmatrix}$$
$$=\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}-\begin{bmatrix} 2 & \frac{1}{2} \\ -3 & -1 \end{bmatrix}$$
$$=\begin{bmatrix} -4 & -\frac{1}{2} \\ 3 & 3 \end{bmatrix}$$

7. $A(B+C)$

$$=\begin{bmatrix} 0 & 2 \\ -1 & 3 \\ 1 & 0 \end{bmatrix}\left(\begin{bmatrix} 4 & 1 \\ -6 & -2 \end{bmatrix}+\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$
$$=\begin{bmatrix} 0 & 2 \\ -1 & 3 \\ 1 & 0 \end{bmatrix}\left(\begin{bmatrix} 3 & 1 \\ -6 & -1 \end{bmatrix}\right)$$
$$=\begin{bmatrix} -12 & -2 \\ -21 & -4 \\ 3 & 1 \end{bmatrix}$$

8. $A(BC)$

$$=\begin{bmatrix} 0 & 2 \\ -1 & 3 \\ 1 & 0 \end{bmatrix}\left(\begin{bmatrix} 4 & 1 \\ -6 & -2 \end{bmatrix}\cdot\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$
$$=\begin{bmatrix} 0 & 2 \\ -1 & 3 \\ 1 & 0 \end{bmatrix}\left(\begin{bmatrix} -4 & 1 \\ 6 & -2 \end{bmatrix}\right)$$
$$=\begin{bmatrix} 12 & -4 \\ 22 & -7 \\ -4 & 1 \end{bmatrix}$$

9. The operation is not defined. Matrices must have the same dimensions in order to be added.

10. $2X - 3C = B$

$$2X = B + 3C$$
$$X = \tfrac{1}{2}(B + 3C)$$
$$= \tfrac{1}{2}\left(\begin{bmatrix} 4 & 1 \\ -6 & -2 \end{bmatrix} + 3\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$
$$= \tfrac{1}{2}\left(\begin{bmatrix} 4 & 1 \\ -6 & -2 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 0 & 3 \end{bmatrix}\right)$$
$$= \tfrac{1}{2}\left(\begin{bmatrix} 1 & 1 \\ -6 & 1 \end{bmatrix}\right)$$
$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -3 & \frac{1}{2} \end{bmatrix}$$

Section 9.4

Check Point Exercises

1. We must show that: $AB = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and

$$BA = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$
$$AB = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 2(1)+1(-1) & 2(-1)+1(2) \\ 1(1)+1(-1) & 1(-1)+1(2) \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$BA = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1(2)+-1(1) & 1(1)+-1(1) \\ -1(2)+2(1) & -1(1)+2(1) \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Both products (*AB* and *BA*) give the multiplicative identity matrix, I_2. Thus, *B* is the multiplicative inverse of *A*.

2. Let us denote the multiplicative inverse of *A* by $A^{-1} = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$. Because *A* is a 2×2 matrix, we use the equation $AA^{-1} = I_2$ to find values for *w*, *x*, *y* and *z*.

$$\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 5w+7y & 5x+7z \\ 2w+3y & 2x+3z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$5w + 7y = 1 \qquad 5x + 7z = 0$

$2w + 3y = 0 \qquad 2x + 3z = 1$

Each of these systems can be solved using the addition method.

Multiply by –2: $5w + 7y = 1 \to -10w - 14y = -2$

Multiply by 5: $2w + 3y = 0 \to \quad 10w + 15y = 0$

Use back substitution: *w* = 3, *y* = –2

Multiply by –2: $5x + 7z = 0 \to -10x - 14z = 0$

Multiply by 5: $2x + 3z = 1 \to 10x + 15z = 5$

Use back substitution: *x* = –7, *z* = 5

Using these values, we have

$$A^{-1} = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}.$$

3. $A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$= \frac{1}{3(1) - (-2)(-1)}\begin{bmatrix} 1 & -(-2) \\ -(-1) & 3 \end{bmatrix}$$
$$= \frac{1}{3-2}\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$
$$= \frac{1}{1}\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

4. The augmented matrix $[A \mid I_3]$ is

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right].$$

Perform row transformations on $[A \mid I_3]$ to obtain a matrix of the form $[I_3 \mid B]$.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right] 1R_1 / R_2$$

$$=\left[\begin{array}{ccc|ccc}1 & 0 & 2 & 1 & 0 & 0\\ 0 & 2 & 5 & 1 & 1 & 0\\ 1 & -1 & 0 & 0 & 0 & 1\end{array}\right]-1R_3$$

$$=\left[\begin{array}{ccc|ccc}1 & 0 & 2 & 1 & 0 & 0\\ 0 & 2 & 5 & 1 & 1 & 0\\ -1 & 1 & 0 & 0 & 0 & -1\end{array}\right]R_1+R_3$$

$$=\left[\begin{array}{ccc|ccc}1 & 0 & 2 & 1 & 0 & 0\\ 0 & 2 & 5 & 1 & 1 & 0\\ 0 & 1 & 2 & 1 & 0 & -1\end{array}\right]\tfrac{1}{2}R_2$$

$$=\left[\begin{array}{ccc|ccc}1 & 0 & 2 & 1 & 0 & 0\\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0\\ 0 & 1 & 2 & 1 & 0 & -1\end{array}\right]-1R_2+R_3$$

$$=\left[\begin{array}{ccc|ccc}1 & 0 & 2 & 1 & 0 & 0\\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0\\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -1\end{array}\right]-2R_3$$

$$=\left[\begin{array}{ccc|ccc}1 & 0 & 2 & 1 & 0 & 0\\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0\\ 0 & 0 & 1 & -1 & 1 & 2\end{array}\right]\begin{matrix}-2R_3+R_1\\ -\frac{5}{2}R_3+R_2\end{matrix}$$

$$=\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 3 & -2 & -4\\ 0 & 1 & 0 & 3 & -2 & -5\\ 0 & 0 & 1 & 1 & 1 & 2\end{array}\right]$$

Thus, the multiplicative inverse of A is

$$A^{-1}=\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}.$$

5. The linear system can be written as $AX = B$.

$$\begin{bmatrix}1 & 0 & 2\\ -1 & 2 & 3\\ 1 & -1 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}6\\ -5\\ 6\end{bmatrix}.$$

$$X=A^{-1}B=\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}\begin{bmatrix}6\\ -5\\ 6\end{bmatrix}$$

$$=\begin{bmatrix}3(6)+-2(-5)+-4(6)\\ 3(6)+-2(-5)+-5(6)\\ -1(6)+1(-5)+2(6)\end{bmatrix}$$

$$=\begin{bmatrix}18+10-24\\ 18+10-30\\ -6-5+12\end{bmatrix}=\begin{bmatrix}4\\ -2\\ 1\end{bmatrix}$$

Thus, $x = 4$, $y = -2$, and $z = 1$. The solution set is $\{(4, -2, 1)\}$.

6. The numerical representation of the word BASE is 2, 1, 19, 5. The 2×2 matrix formed is $\begin{bmatrix}2 & 19\\ 1 & 5\end{bmatrix}$.

$$\begin{bmatrix}-2 & -3\\ 3 & 4\end{bmatrix}\begin{bmatrix}2 & 19\\ 1 & 5\end{bmatrix}$$

$$=\begin{bmatrix}-2(2)+-3(1) & -2(19)+-3(5)\\ 3(2)+4(1) & 3(19)+4(5)\end{bmatrix}$$

$$=\begin{bmatrix}-4-3 & -38-15\\ 6+4 & 57+20\end{bmatrix}=\begin{bmatrix}-7 & -53\\ 10 & 77\end{bmatrix}$$

The encoded message is –7, 10, –53, 77.

7. Use the multiplicative inverse of the coding matrix. It is $\begin{bmatrix}4 & 3\\ -3 & -2\end{bmatrix}$.

$$\begin{bmatrix}4 & 3\\ -3 & -2\end{bmatrix}\begin{bmatrix}-7 & -53\\ 10 & 77\end{bmatrix}$$

$$=\begin{bmatrix}4(-7)+3(10) & 4(-53)+3(77)\\ -3(-7)+-2(10) & -3(-53)+-2(77)\end{bmatrix}$$

$$=\begin{bmatrix}-28+30 & -212+231\\ 21-20 & 159-154\end{bmatrix}=\begin{bmatrix}2 & 19\\ 1 & 5\end{bmatrix}$$

The numbers are 2, 1, 19, and 5. Using letters, the decoded message is BASE.

Exercise Set 9.4

1. $A=\begin{bmatrix}4 & -3\\ -5 & 4\end{bmatrix}\quad B=\begin{bmatrix}4 & 3\\ 5 & 4\end{bmatrix}$

$$AB=\begin{bmatrix}16-15 & 12-12\\ -20+20 & -15+16\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$BA=\begin{bmatrix}16-15 & -12+12\\ 20-20 & -15+16\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Since $AB = I_2$ $BA = I_2$, $B = A^{-1}$.

2. $A=\begin{bmatrix}-2 & -1\\ -1 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$

$$AB=\begin{bmatrix}-2-1 & -2-2\\ -1+1 & -1+2\end{bmatrix}=\begin{bmatrix}-3 & -4\\ 0 & 1\end{bmatrix}$$

$$BA=\begin{bmatrix}-2-1 & -1+1\\ -2-2 & -1+2\end{bmatrix}=\begin{bmatrix}-3 & 0\\ -4 & 1\end{bmatrix}$$

If B is the multiplicative inverse of A, both products (AB and BA) will be the multiplicative identity matrix, I_2. Therefore, B is not the multiplicative inverse of A. That is, $B \neq A^{-1}$.

3. $AB = \begin{bmatrix} 8+0 & -16+0 \\ -2+0 & 4+3 \end{bmatrix} = \begin{bmatrix} 8 & -16 \\ -2 & 7 \end{bmatrix}$

$BA = \begin{bmatrix} 8+4 & 0+12 \\ 0+1 & 0+3 \end{bmatrix} = \begin{bmatrix} 12 & 12 \\ 1 & 3 \end{bmatrix}$

If B is the multiplicative inverse of A, both products (AB and BA) will be the multiplicative identity matrix, I_2. Therefore, B is not the multiplicative inverse of A. That is, $B \neq A^{-1}$.

4. $AB = \begin{bmatrix} -2-4 & -4-8 \\ 1+2 & 2+4 \end{bmatrix} = \begin{bmatrix} -6 & -12 \\ 3 & 6 \end{bmatrix}$

$BA = \begin{bmatrix} -2+2 & 4-4 \\ 2-2 & -4+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

If B is the multiplicative inverse of A, both products (AB and BA) will be the multiplicative identity matrix, I_2. Therefore, B is not the multiplicative inverse of A. That is, $B \neq A^{-1}$.

5. $AB = \begin{bmatrix} -2+3 & -4+4 \\ \frac{3}{2}-\frac{3}{2} & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} -2+3 & 1-1 \\ -6+6 & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Since $AB = I_2$ and $BA = I_2$, $B = A^{-1}$.

6. $AB = \begin{bmatrix} 6-5 & -10+10 \\ 3-3 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

$BA = \begin{bmatrix} 6-5 & \frac{15}{2}-\frac{15}{2} \\ -4+4 & -6+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Since $AB = I_2$ and $BA = I_2$, $B = A^{-1}$.

7. $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 0+1+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+1 & 0+0+0 \\ 0+0+0 & 0+0+0 & 1+0+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $BA = \begin{bmatrix} 0+0+1 & 0+0+0 & 0+0+0 \\ 0+0+0 & 1+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Since $AB = I_3$ and $BA = I_3$, $B = A^{-1}$.

8. $A = \begin{bmatrix} -2 & 1 & -1 \\ -5 & 2 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} -2+2+1 & 0+1-1 & -2+3-1 \\ -5+4+1 & 0+2-1 & -5+6-1 \\ 3-2-1 & 0-1+1 & 3-3+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $BA = \begin{bmatrix} -2+0+3 & 1+0-1 & -1+0+1 \\ -4-5+9 & 2+2-3 & -2-16+3 \\ 2-5+3 & -1+2-1 & 1-1+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Since $AB = I_3$ and $BA = I_3$, $B = A^{-1}$.

9. $AB = \begin{bmatrix} \frac{7}{2}-1-\frac{3}{2} & -3+0+3 & \frac{1}{2}+1-\frac{3}{2} \\ \frac{7}{2}-\frac{3}{2}-2 & -3+0+4 & \frac{1}{2}+\frac{3}{2}-2 \\ \frac{7}{2}-2-\frac{3}{2} & -3+0+3 & \frac{1}{2}+2-\frac{3}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} \frac{7}{2}-3+\frac{1}{2} & 7-9+2 & \frac{21}{2}-12+\frac{3}{2} \\ -\frac{1}{2}+0+\frac{1}{2} & -1+0+2 & -\frac{3}{2}+0+\frac{3}{2} \\ -\frac{1}{2}+1-\frac{1}{2} & -1+3-2 & -\frac{3}{2}+4-\frac{3}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Since $AB = I_3$ and $BA = I_3$, $B = A^{-1}$.

10. $AB = \begin{bmatrix} 0+1+0 & 0+0+0 & 0+0+0 \\ -10.5+1.5+9 & -3+0+4 & 6+0-6 \\ -7+2.5+4.5 & -2+0+2 & 4+0-3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 0-3+4 & -7-3+10 & 0-2+2 \\ 0+0+0 & 1+0+0 & 0+0+0 \\ 0+6-6 & 9+6-15 & 0+4-3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Since $AB = I_3$ and $BA = I_3$, $B = A^{-1}$.

11. $AB = \begin{bmatrix} 0+0+0+1 & 0+0-2+2 & 0+0+0+0 & 0+0-2+2 \\ -1+0+0+1 & -2+0+1+2 & 0+0+0+0 & -3+0+1+2 \\ 0+0+0+0 & 0+1-1+0 & 0+1+0+0 & 0+1-1+0 \\ 1+0+0-1 & 2+0+0-2 & 0+0+0+0 & 3+0+0-2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 0-2+0+3 & 0+0+0+0 & -2+2+0+0 & 1+2+0-3 \\ 0-1+0+1 & 0+0+1+0 & 0+1-1+0 & 0+1+0-1 \\ 0-1+0+1 & 0+0+0+0 & 0+1+0+0 & 0+1+0-1 \\ 0-2+0+2 & 0+0+0+0 & -2+2+0+0 & 1+2+0-2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Since $AB = I_4$ and $BA = I_4$, $B = A^{-1}$.

12. $AB = \begin{bmatrix} 1+0+0+0 & 2-2+0+0 & 3-4+1+0 & 4-6+2+0 \\ 0+0+0+0 & 0+1+0+0 & 0+2-2+0 & 0+3-4+1 \\ 0+0+0+0 & 0+0+0+0 & 0+0+1+0 & 0+0+2-2 \\ 0+0+0+0 & 0+0+0+0 & 0+0+0+0 & 0+0+0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 1+0+0+0 & -2+2+0+0 & 1-4+3+0 & 0+2-6+4 \\ 0+0+0+0 & 0+1+0+0 & 0-2+2+0 & 0+1-4+3 \\ 0+0+0+0 & 0+0+0+0 & 0+0+1+0 & 0+0-2+2 \\ 0+0+0+0 & 0+0+0+0 & 0+0+0+0 & 0+0+0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Since $AB = I_4$ and $BA = I_4$, $B = A^{-1}$.

13. $ad - bc = (2)(2) - (3)(-1) = 4 + 3 = 7$

$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{7} & -\frac{3}{7} \\ \frac{1}{7} & \frac{2}{7} \end{bmatrix}$

$AA^{-1} = \begin{bmatrix} \frac{4}{7}+\frac{3}{7} & -\frac{6}{7}+\frac{6}{7} \\ -\frac{2}{7}+\frac{2}{7} & \frac{3}{7}+\frac{4}{7} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^{-1}A = \begin{bmatrix} \frac{4}{7}+\frac{3}{7} & \frac{6}{7}-\frac{6}{7} \\ \frac{2}{7}-\frac{2}{7} & \frac{3}{7}+\frac{4}{7} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

14. $ad - bc = (0)(-2) - (3)(4) = 0 - 12 = -12$

$$A^{-1} = \frac{1}{-12}\begin{bmatrix} -2 & -3 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & \frac{1}{4} \\ \frac{1}{3} & 0 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 0+\frac{3}{3} & 0+0 \\ \frac{4}{6}-\frac{2}{3} & \frac{4}{4}+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } A^{-1}A = \begin{bmatrix} 0+\frac{4}{4} & \frac{3}{6}-\frac{2}{4} \\ 0+0 & \frac{3}{3}+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

15. $ad - bc = (3)(2) - (-1)(-4) = 6 - 4 = 2$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 3-2 & \frac{3}{2}-\frac{3}{2} \\ -4+4 & -\frac{4}{2}+\frac{6}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } A^{-1}A = \begin{bmatrix} 3-\frac{4}{2} & -1+\frac{2}{2} \\ 6-\frac{12}{2} & -2+\frac{6}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

16. $ad - bc = (2)(-2) - (-6)(1) = -4 + 6 = 2$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 6 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} -2+\frac{6}{2} & 6-6 \\ -1+\frac{2}{2} & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } A^{-1}A = \begin{bmatrix} -2+3 & 6-6 \\ -\frac{2}{2}+1 & \frac{6}{2}-2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

17. $ad - bc = (10)(1) - (-2)(-5) = 10 - 10 = 0$

Since division by zero is undefined, A does not have an inverse.

For Problems 19–24, verification that $AA^{-1} = I$ and $A^{-1}A = I$ is left to the student.

18. $ad - bc = (6)(1) - (-3)(-2) = 6 - 6 = 0$

Since division by zero is undefined, A does not have an inverse.

For Problems 19–24, verification that $AA^{-1} = I$ and $A^{-1}A = I$ is left to the student.

19. $\begin{bmatrix} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 6 & 0 & 0 & 1 \end{bmatrix}$

Divide row 1 by 2, divide row 2 by 4 and divide row 4 by 6.

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{6} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{6} \end{bmatrix}$$

20. $\begin{bmatrix} 3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 6 & 0 & 0 & 1 & 0 \\ 0 & 0 & 9 & 0 & 0 & 1 \end{bmatrix}$

Replace row 1 with $\frac{1}{3}R_1$.

Replace row 2 with $\frac{1}{6}R_2$.

Replace row 3 with $\frac{1}{9}R_3$.

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{9} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{9} \end{bmatrix}$$

21. $\begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$

Replace row 2 with $2R_1 + R_2$.
Replace row 3 with $R_1 - R_3$.

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 4 & -1 & 2 & 1 & 0 \\ 0 & 3 & -1 & 1 & 0 & -1 \end{bmatrix}$$

Replace row 1 with $R_2 - 2R_1$.
Replace row 3 with $-3R_2 + 4R_3$.

$$\begin{bmatrix} -2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 4 & -1 & 2 & 1 & 0 \\ 0 & 0 & -1 & -2 & -3 & -4 \end{bmatrix}$$

Replace row 1 with $R_3 + R_1$.
Replace row 2 with $R_2 - R_3$.
Replace row 3 with $-R_3$.

$$\begin{bmatrix} -2 & 0 & 0 & -2 & -2 & -4 \\ 0 & 4 & 0 & 4 & 4 & 4 \\ 0 & 0 & 1 & 2 & 3 & 4 \end{bmatrix}$$

Divide row 1 by –2 and divide row 2 by 4.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 3 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

22. $\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 2 & 3 & 0 & 0 & 0 & 1 \end{array}\right] -2R_1 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 0 & 5 & -2 & -2 & 0 & 1 \end{array}\right] \tfrac{1}{2}R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 5 & -2 & -2 & 0 & 1 \end{array}\right] \begin{matrix} 1R_2 + R_1 \\ -5R_2 + R_3 \end{matrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & 1 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & -2 & -\frac{5}{2} & 1 \end{array}\right] \begin{matrix} -1R_3 + R_1 \\ 1R_3 + R_2 \end{matrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 3 & -1 \\ 0 & 1 & 0 & -2 & -2 & 1 \\ 0 & 0 & \frac{1}{2} & -2 & -\frac{5}{2} & 1 \end{array}\right] 2R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 3 & -1 \\ 0 & 1 & 0 & -2 & -2 & 1 \\ 0 & 0 & 1 & -4 & -5 & 2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

23. $\left[\begin{array}{rrr|rrr} 2 & 2 & -1 & 1 & 0 & 0 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ -1 & -2 & 1 & 0 & 0 & 1 \end{array}\right] R_1 \leftrightarrow R_3$

$\left[\begin{array}{rrr|rrr} -1 & -2 & 1 & 0 & 0 & 1 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ 2 & 2 & -1 & 1 & 0 & 0 \end{array}\right] -1R_1$

$\left[\begin{array}{rrr|rrr} 1 & 2 & -1 & 0 & 0 & -1 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ 2 & 2 & -1 & 1 & 0 & 0 \end{array}\right] -2R_1 + R_3$

$\left[\begin{array}{rrr|rrr} 1 & 2 & -1 & 0 & 0 & -1 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ 0 & -2 & 1 & 1 & 0 & 2 \end{array}\right] \frac{1}{3}R_2$

$\left[\begin{array}{rrr|rrr} 1 & 2 & -1 & 0 & 0 & -1 \\ 0 & 1 & -\frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & -2 & 1 & 1 & 0 & 2 \end{array}\right] \begin{array}{l} -2R_2 + R_1 \\ 2R_2 + R_3 \end{array}$

$\left[\begin{array}{rrr|rrr} 1 & 0 & -\frac{1}{3} & 0 & -\frac{2}{3} & -1 \\ 0 & 1 & -\frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 1 & \frac{2}{3} & 2 \end{array}\right] \begin{array}{l} 1R_3 + R_1 \\ 1R_2 + R_1 \end{array}$

$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 2 \\ 0 & 0 & \frac{1}{3} & 1 & \frac{2}{3} & 2 \end{array}\right] 3R_3$

$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 & 2 & 6 \end{array}\right]$

$A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 3 & 2 & 6 \end{bmatrix}$

24. $\begin{bmatrix} 2 & 4 & -4 & 1 & 0 & 0 \\ 1 & 3 & -4 & 0 & 1 & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{bmatrix}$

Interchange row 1 and row 2.

$\begin{bmatrix} 1 & 3 & -4 & 0 & 1 & 0 \\ 2 & 4 & -4 & 1 & 0 & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{bmatrix}$

Replace row 2 with $-2R_1 + R_2$.
Replace row 3 with $-2R_1 + R_3$.

$\begin{bmatrix} 1 & 3 & -4 & 0 & 1 & 0 \\ 0 & -2 & 4 & 1 & -2 & 0 \\ 0 & -2 & 5 & 0 & -2 & 1 \end{bmatrix}$

Replace row 1 with $2R_1 + 3R_2$.
Replace row 3 with $-R_2 + R_3$.

$\begin{bmatrix} 2 & 0 & 4 & 3 & -4 & 0 \\ 0 & -2 & 4 & 1 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$

Replace row 1 with $R_1 - 4R_3$.
Replace row 2 with $R_2 - 4R_3$.

$\begin{bmatrix} 2 & 0 & 0 & 7 & -4 & -4 \\ 0 & -2 & 0 & 5 & -2 & -4 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$

Divide row by 2 and row 2 by –2.

$\begin{bmatrix} 1 & 0 & 0 & \frac{7}{2} & -2 & -2 \\ 0 & 1 & 0 & -\frac{5}{2} & 1 & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{7}{2} & -2 & -2 \\ -\frac{5}{2} & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

25. $\left[\begin{array}{ccc|ccc} 5 & 0 & 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ -3 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \frac{1}{5}R_1$

$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{5} & \frac{1}{5} & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ -3 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \begin{matrix} -2R_1 + R_2 \\ 3R_1 + R_3 \end{matrix}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{5} & \frac{1}{5} & 0 & 0 \\ 0 & 2 & \frac{1}{5} & -\frac{2}{5} & 1 & 0 \\ 0 & 1 & \frac{1}{5} & \frac{3}{5} & 0 & 1 \end{array}\right] R_2 \leftrightarrow R_3$

$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{5} & \frac{1}{5} & 0 & 0 \\ 0 & 1 & \frac{1}{5} & \frac{3}{5} & 0 & 1 \\ 0 & 2 & \frac{1}{5} & -\frac{2}{5} & 1 & 0 \end{array}\right] -2R_2 + R_3$

$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{5} & \frac{1}{5} & 0 & 0 \\ 0 & 1 & \frac{1}{5} & \frac{3}{5} & 0 & 1 \\ 0 & 0 & -\frac{1}{5} & -\frac{8}{5} & 1 & -2 \end{array}\right] \begin{matrix} 2R_3 + R_1 \\ 1R_3 + R_2 \end{matrix}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & -4 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & -\frac{1}{5} & -\frac{8}{5} & 1 & -2 \end{array}\right] -5R_3$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & -4 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 8 & -5 & 10 \end{array}\right]$

$A^{-1} = \begin{bmatrix} -3 & 2 & -4 \\ -1 & 1 & -1 \\ 8 & -5 & 10 \end{bmatrix}$

26. $\left[\begin{array}{ccc|ccc} 3 & 2 & 6 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 2 & 2 & 5 & 0 & 0 & 1 \end{array}\right] R_1 \leftrightarrow R_2$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 3 & 2 & 6 & 1 & 0 & 0 \\ 2 & 2 & 5 & 0 & 0 & 1 \end{array}\right] \begin{matrix} -3R_1 + R_2 \\ -2R_1 + R_3 \end{matrix}$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right] -1R_2$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right] -1R_2 + R_1$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & -2 & 0 \\ 0 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right] -2R_3 + R_1$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & -2 \\ 0 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right]$

$A^{-1} = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

27. $\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] -1R_1 + R_4$

$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right] -1R_2$

$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \frac{1}{3}R_3$

$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$

$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$

28. $\left[\begin{array}{cccc|cccc} 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \frac{1}{2}R_1$

$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right] -1R_3$

$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \frac{1}{2}R_4$

$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{2} \end{array}\right] -\frac{1}{2}R_4 + R_1$

$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & -\frac{1}{4} \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{2} \end{array}\right]$

$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{4} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$

29. $\begin{bmatrix} 6 & 5 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 10 \end{bmatrix}$

30. $\begin{bmatrix} 7 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 23 \\ 10 \end{bmatrix}$

31. $\begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 1 & 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \\ -6 \end{bmatrix}$

32. $\begin{bmatrix} 1 & 4 & -1 \\ 1 & 3 & -2 \\ 2 & 7 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 12 \end{bmatrix}$

33. $4x - 7y = -3$
$2x - 3y = 1$

34. $3x = 6$
$-3x + y = -7$

35. $2x - z = 6$
$3y = 9$
$x + y = 5$

36. $-x + z = -4$
$-y = 2$
$y + z = 4$

37. a. $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 9 \end{bmatrix}$

$\begin{bmatrix} \frac{7}{2} & 0 & -3 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 8 \\ 10 \\ 9 \end{bmatrix} = \begin{bmatrix} 28+0-27 \\ -8+10+0 \\ 0-10+9 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

The solution to the system is {(1, 2, –1)}.

38. a. $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 8 \\ -1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$

b. $\begin{bmatrix} 2 & -1 & -1 \\ 12 & -7 & -2 \\ -5 & 3 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 4+(-3)+(-3) \\ 24-21-6 \\ -10+9+3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix}$

The solution set is {(–2, –3, 2)}.

39. a. $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \\ 1 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ -7 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 24-21-1 \\ -16+14+1 \\ -32+35+2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$

The solution to the system is
{(2, –1, 5)}.

40. a. $\begin{bmatrix} 1 & -6 & 3 \\ 2 & -7 & 3 \\ 4 & -12 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 14 \\ 25 \end{bmatrix}$

b. $\begin{bmatrix} 1 & -6 & 3 \\ 2 & -7 & 3 \\ 4 & -12 & 5 \end{bmatrix}\begin{bmatrix} 11 \\ 14 \\ 25 \end{bmatrix}\begin{bmatrix} 11-84+75 \\ 22-98+75 \\ 44-168+125 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

The solution set is {(2, –1, 1)}.

41. a. $\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 1 \\ -1 & 1 & -1 & 2 \\ 0 & -1 & 1 & -2 \end{bmatrix}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ 2 \\ -4 \end{bmatrix}$

b. $\begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 4 & 1 & 3 \\ 1 & 2 & 1 & 2 \\ 0 & -1 & 0 & -1 \end{bmatrix}\begin{bmatrix} -3 \\ 4 \\ 2 \\ -4 \end{bmatrix}$

$= \begin{bmatrix} 0+0-2+4 \\ -3+16+2-12 \\ -3+8+2-8 \\ 0-4+0+4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \\ 0 \end{bmatrix}$

The solution to the system is $\{(2, 3, -1, 0)\}$.

42. a. $\begin{bmatrix} 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 1 \\ -1 & 1 & -2 & 1 \\ 4 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ 4 \\ 6 \end{bmatrix}$

b. $\begin{bmatrix} -1 & 2 & -1 & -1 \\ -4 & 9 & -5 & -6 \\ 0 & 1 & -1 & -1 \\ 3 & -5 & 3 & 3 \end{bmatrix}\begin{bmatrix} 6 \\ 9 \\ 4 \\ 6 \end{bmatrix}$

$\begin{bmatrix} -6+18-4-6 \\ -24+81-20-36 \\ 0+9-4-6 \\ 18-45+12+18 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 3 \end{bmatrix}$

The solution set is $\{(2, 1, -1, 3)\}$.

43. $A = \begin{bmatrix} e^x & e^{3x} \\ -e^{3x} & e^{5x} \end{bmatrix}$

$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A^{-1} = \frac{1}{(e^x)(e^{5x})-(e^{3x})(-e^{3x})}\begin{bmatrix} e^{5x} & -e^{3x} \\ -(-e^{3x}) & e^x \end{bmatrix}$

$A^{-1} = \frac{1}{e^{6x}+e^{6x}}\begin{bmatrix} e^{5x} & -e^{3x} \\ e^{3x} & e^x \end{bmatrix}$

$A^{-1} = \frac{1}{2e^{6x}}\begin{bmatrix} e^{5x} & -e^{3x} \\ e^{3x} & e^x \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{e^{5x}}{2e^{6x}} & \frac{-e^{3x}}{2e^{6x}} \\ \frac{e^{3x}}{2e^{6x}} & \frac{e^x}{2e^{6x}} \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{2e^x} & -\frac{1}{2e^{3x}} \\ \frac{1}{2e^{3x}} & \frac{1}{2e^{5x}} \end{bmatrix}$ or $\begin{bmatrix} \frac{e^{-x}}{2} & -\frac{e^{-3x}}{2} \\ \frac{e^{-3x}}{2} & \frac{e^{-5x}}{2} \end{bmatrix}$

Check:

$\begin{bmatrix} e^x & e^{3x} \\ -e^{3x} & e^{5x} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2e^x} & -\frac{1}{2e^{3x}} \\ \frac{1}{2e^{3x}} & \frac{1}{2e^{5x}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

44. $A = \begin{bmatrix} e^{2x} & -e^x \\ e^{3x} & e^{2x} \end{bmatrix}$

$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$A^{-1} = \frac{1}{(e^{2x})(e^{2x})-(-e^x)(e^{3x})}\begin{bmatrix} e^{2x} & e^x \\ -(e^{3x}) & e^{2x} \end{bmatrix}$

$A^{-1} = \frac{1}{e^{4x}+e^{4x}}\begin{bmatrix} e^{2x} & e^x \\ -e^{3x} & e^{2x} \end{bmatrix}$

$A^{-1} = \frac{1}{2e^{4x}}\begin{bmatrix} e^{2x} & e^x \\ -e^{3x} & e^{2x} \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{e^{2x}}{2e^{4x}} & \frac{e^x}{2e^{4x}} \\ \frac{-e^{3x}}{2e^{4x}} & \frac{e^{2x}}{2e^{4x}} \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{2e^{2x}} & \frac{1}{2e^{3x}} \\ -\frac{1}{2e^x} & \frac{1}{2e^{2x}} \end{bmatrix}$ or $\begin{bmatrix} \frac{e^{-2x}}{2} & \frac{e^{-3x}}{2} \\ -\frac{e^{-x}}{2} & \frac{e^{-2x}}{2} \end{bmatrix}$

Check:

$\begin{bmatrix} e^{2x} & -e^x \\ e^{3x} & e^{2x} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2e^{2x}} & \frac{1}{2e^{3x}} \\ -\frac{1}{2e^x} & \frac{1}{2e^{2x}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

45. $A = \begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -7 & 5 \\ 3 & -1 \end{bmatrix}$

$(I - A)^{-1} = \dfrac{1}{(-7)(-1)-(5)(3)} \begin{bmatrix} -1 & -(5) \\ -(3) & -7 \end{bmatrix}$

$(I - A)^{-1} = \dfrac{1}{7-15} \begin{bmatrix} -1 & -5 \\ -3 & -7 \end{bmatrix}$

$(I - A)^{-1} = \dfrac{1}{-8} \begin{bmatrix} -1 & -5 \\ -3 & -7 \end{bmatrix}$

$(I - A)^{-1} = \begin{bmatrix} \frac{-1}{-8} & \frac{-5}{-8} \\ \frac{-3}{-8} & \frac{-7}{-8} \end{bmatrix}$

$(I - A)^{-1} = \begin{bmatrix} \frac{1}{8} & \frac{5}{8} \\ \frac{3}{8} & \frac{7}{8} \end{bmatrix}$

46. $A = \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} -6 & 5 \\ 4 & -2 \end{bmatrix}$

$(I - A)^{-1} = \dfrac{1}{(-6)(-2)-(5)(4)} \begin{bmatrix} -2 & -(5) \\ -(4) & -6 \end{bmatrix}$

$(I - A)^{-1} = \dfrac{1}{12-20} \begin{bmatrix} -2 & -5 \\ -4 & -6 \end{bmatrix}$

$(I - A)^{-1} = \dfrac{1}{-8} \begin{bmatrix} -2 & -5 \\ -4 & -6 \end{bmatrix}$

$(I - A)^{-1} = \begin{bmatrix} \frac{-2}{-8} & \frac{-5}{-8} \\ \frac{-4}{-8} & \frac{-6}{-8} \end{bmatrix}$

$(I - A)^{-1} = \begin{bmatrix} \frac{1}{4} & \frac{5}{8} \\ \frac{1}{2} & \frac{3}{4} \end{bmatrix}$

47. $A=\begin{bmatrix}2 & 1\\3 & 1\end{bmatrix}\quad B=\begin{bmatrix}4 & 7\\1 & 2\end{bmatrix}$

$A^{-1}=\begin{bmatrix}-1 & 1\\3 & -2\end{bmatrix}\quad B^{-1}=\begin{bmatrix}2 & -7\\-1 & 4\end{bmatrix}$

$AB=\begin{bmatrix}2 & 1\\3 & 1\end{bmatrix}\begin{bmatrix}4 & 7\\1 & 2\end{bmatrix}=\begin{bmatrix}9 & 16\\13 & 23\end{bmatrix}$

$(AB)^{-1}=\left(\begin{bmatrix}9 & 16\\13 & 23\end{bmatrix}\right)^{-1}=\begin{bmatrix}-23 & 16\\13 & -9\end{bmatrix}$

$A^{-1}B^{-1}=\begin{bmatrix}-1 & 1\\3 & -2\end{bmatrix}\begin{bmatrix}2 & -7\\-1 & 4\end{bmatrix}=\begin{bmatrix}-3 & 11\\8 & -29\end{bmatrix}$

$B^{-1}A^{-1}=\begin{bmatrix}2 & -7\\-1 & 4\end{bmatrix}\begin{bmatrix}-1 & 1\\3 & -2\end{bmatrix}=\begin{bmatrix}-23 & 16\\13 & -9\end{bmatrix}$

Observe that $(AB)^{-1}=B^{-1}A^{-1}$.

48. $A=\begin{bmatrix}2 & -9\\1 & -4\end{bmatrix}\quad B=\begin{bmatrix}9 & 5\\7 & 4\end{bmatrix}$

$A^{-1}=\begin{bmatrix}-4 & 9\\-1 & 2\end{bmatrix}\quad B^{-1}=\begin{bmatrix}4 & -5\\-7 & 9\end{bmatrix}$

$AB=\begin{bmatrix}2 & -9\\1 & -4\end{bmatrix}\begin{bmatrix}9 & 5\\7 & 4\end{bmatrix}=\begin{bmatrix}-45 & -26\\-19 & -11\end{bmatrix}$

$(AB)^{-1}=\left(\begin{bmatrix}-45 & -26\\-19 & -11\end{bmatrix}\right)^{-1}=\begin{bmatrix}-11 & 26\\19 & -45\end{bmatrix}$

$A^{-1}B^{-1}=\begin{bmatrix}-4 & 9\\-1 & 2\end{bmatrix}\begin{bmatrix}4 & -5\\-7 & 9\end{bmatrix}=\begin{bmatrix}-79 & 101\\-18 & 23\end{bmatrix}$

$B^{-1}A^{-1}=\begin{bmatrix}4 & -5\\-7 & 9\end{bmatrix}\begin{bmatrix}-4 & 9\\-1 & 2\end{bmatrix}=\begin{bmatrix}-11 & 26\\19 & -45\end{bmatrix}$

Observe that $(AB)^{-1}=B^{-1}A^{-1}$.

49. $\begin{bmatrix}a & 0 & 0\\0 & b & 0\\0 & 0 & c\end{bmatrix}\begin{bmatrix}\frac{1}{a} & 0 & 0\\0 & \frac{1}{b} & 0\\0 & 0 & \frac{1}{c}\end{bmatrix}$

$$=\begin{bmatrix}(a)(\frac{1}{a})+(0)(0)+(0)(0) & (a)(0)+(0)(\frac{1}{b})+(0)(0) & (a)(0)+(0)(0)+(0)(\frac{1}{c})\\(0)(\frac{1}{a})+(b)(0)+(0)(0) & (0)(0)+(b)(\frac{1}{b})+(0)(0) & (0)(0)+(b)(0)+(0)(\frac{1}{c})\\(0)(\frac{1}{a})+(0)(0)+(c)(0) & (0)(0)+(0)(\frac{1}{b})+(c)(0) & (0)(0)+(0)(0)+(c)(\frac{1}{c})\end{bmatrix}$$

$$=\begin{bmatrix}\frac{a}{a}+0+0 & 0+0+0 & 0+0+0\\0+0+0 & 0+\frac{b}{b}+0 & 0+0+0\\0+0+0 & 0+0+0 & 0+0+\frac{c}{c}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

50. Let $A^{-1} = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$

$$A \cdot A^{-1} = I$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} aw+by & ax+bz \\ cw+dy & cx+dz \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equate corresponding elements to obtain two systems of equations:

$$\begin{aligned} aw+by &= 1 \\ cw+dy &= 0 \end{aligned} \quad \text{and} \quad \begin{aligned} ax+bz &= 0 \\ cx+dz &= 1 \end{aligned}$$

Solve $cw+dy=0$ for y in terms of *c*, *d*, and *w*.

$$cw+dy=0$$
$$dy=-cw$$
$$y=-\frac{cw}{d}$$

Substitute to find *w* in terms of *a*, *b*, *c*, and *d*.

$$aw+by=1$$
$$aw+b\left(-\frac{cw}{d}\right)=1$$
$$aw-\frac{bcw}{d}=1$$
$$adw-bcw=d$$
$$w(ad-bc)=d$$
$$w=\frac{d}{ad-bc}$$

Substitute to find *y* in terms of *a*, *b*, *c*, and *d*.

$$y=-\frac{cw}{d}$$
$$y=-\frac{c\left(\frac{d}{ad-bc}\right)}{d}$$
$$y=-\frac{c}{ad-bc}$$

Use a similar process to find *x* and *z*.

$$x=-\frac{b}{ad-bc}$$
$$z=\frac{a}{ad-bc}$$

Thus, $A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

51. The numerical equivalent of HELP is 8, 5, 12, 16.

$$\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \end{bmatrix} = \begin{bmatrix} 27 \\ -19 \end{bmatrix},$$

$$\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 16 \end{bmatrix} = \begin{bmatrix} 32 \\ -20 \end{bmatrix}$$

The encoded message is 27, –19, 32, –20.

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 27 \\ -19 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 32 \\ -20 \end{bmatrix} = \begin{bmatrix} 12 \\ 16 \end{bmatrix}$$

The decoded message is 8, 5, 12, 16 or HELP.

52. The numerical equivalent of LOVE is 12, 15, 22, 5.

$$\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 33 \\ -21 \end{bmatrix}, \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 22 \\ 5 \end{bmatrix} = \begin{bmatrix} 83 \\ -61 \end{bmatrix}$$

The encoded message is 33, –21, 83, –61.

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 33 \\ -21 \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 83 \\ -61 \end{bmatrix} = \begin{bmatrix} 22 \\ 5 \end{bmatrix}$$

The decoded message is 12, 15, 22, 5 or LOVE.

53. $\begin{bmatrix} 1 & -1 & 0 \\ 3 & 0 & 2 \\ -1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 19 & 4 & 1 \\ 5 & 0 & 19 \\ 14 & 3 & 8 \end{bmatrix}$

$$= \begin{bmatrix} 19-5+0 & 4+0+0 & 1-19+0 \\ 57+0+28 & 12+0+6 & 3+0+16 \\ -19+0-14 & -4+0-3 & -1+0-8 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 4 & -18 \\ 85 & 18 & 19 \\ -33 & -7 & -9 \end{bmatrix}$$

The encoded message is 14, 85, –33, 4, 18, –7, –18, 19, –9.

$$\begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 2 \\ 0 & -1 & -3 \end{bmatrix}\begin{bmatrix} 14 & 4 & -18 \\ 85 & 18 & 19 \\ -33 & -7 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 0+85-66 & 0+18-14 & 0+19-18 \\ -14+85-66 & -4+18-14 & 18+19-18 \\ 0-85+99 & 0-18+21 & 0-19+27 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 4 & 1 \\ 5 & 0 & 19 \\ 14 & 3 & 8 \end{bmatrix}$$

The decoded message is 19, 5, 14, 4, 0, 3, 1, 19, 8 or SEND_CASH

54. $\begin{bmatrix} 1 & -1 & 0 \\ 3 & 0 & 2 \\ -1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 19 & 25 & 5 \\ 20 & 0 & 12 \\ 1 & 23 & 12 \end{bmatrix}$

$= \begin{bmatrix} 19-20+0 & 25+0+0 & 5-12+0 \\ 57+0+2 & 75+0+46 & 15+0+24 \\ -19+0-1 & -25+0-23 & -5+0-12 \end{bmatrix}$

$= \begin{bmatrix} -1 & 25 & -7 \\ 59 & 121 & 39 \\ -20 & -48 & -17 \end{bmatrix}$

The encoded message is –1, 59, –20, 25, 121, –48, –7, 39, –17.

$\begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 2 \\ 0 & -1 & -3 \end{bmatrix}\begin{bmatrix} -1 & 25 & -7 \\ 59 & 121 & 39 \\ -20 & -48 & -17 \end{bmatrix}$

$= \begin{bmatrix} 0+59-40 & 0+121-96 & 0+39-34 \\ 1+59-40 & -25+121-96 & 7+39-34 \\ 0-59+60 & 0-121+144 & 0-39+51 \end{bmatrix}$

$= \begin{bmatrix} 19 & 25 \\ 20 & 0 \\ 1 & 23 \end{bmatrix}$

The decoded message is 19, 20, 1, 25, 0, 23, 5, 12, 12. or STAY_WELL

55. – 64. Answers may vary.

65. Enter the matrix $\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$ as $[A]$, then use $[A]^{-1}$.

$[A]^{-1} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

Verify this result by showing that $[A][A]^{-1} = I_2$ and $[A]^{-1}[A] = I_2$.

66. Enter the matrix $\begin{bmatrix} -4 & 1 \\ 6 & -2 \end{bmatrix}$ as $[A]$, then use $[A]^{-1}$.

$[A]^{-1} = \begin{bmatrix} -1 & -\frac{1}{2} \\ -3 & -2 \end{bmatrix}$

Verify the result by showing $[A][A]^{-1} = I_2$ and $[A]^{-1}[A] = I_2$.

67. Enter the matrix $\begin{bmatrix} -2 & 1 & -1 \\ -5 & 2 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ as $[A]$, then use $[A]^{-1}$.

$[A]^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$

Verify this result by showing that $[A][A]^{-1} = I_3$ and $[A]^{-1}[A] = I_3$.

68. Enter the matrix $\begin{bmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{bmatrix}$ as $[A]$, then use $[A]^{-1}$.

$[A]^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$

Verify the result by showing $[A][A]^{-1} = I_3$ and $[A]^{-1}[A] = I_3$.

69. Enter the matrix $\begin{bmatrix} 7 & -3 & 0 & 2 \\ -2 & 1 & 0 & -1 \\ 4 & 0 & 1 & -2 \\ -1 & 1 & 0 & -1 \end{bmatrix}$ as $[A]$, then use $[A]^{-1}$. $[A]^{-1} = \begin{bmatrix} 0 & -1 & 0 & 1 \\ -1 & -5 & 0 & 3 \\ -2 & -4 & 1 & -2 \\ -1 & -4 & 0 & 1 \end{bmatrix}$

Verify this result by showing that $[A][A]^{-1} = I_4$ and $[A]^{-1}[A] = I_4$.

70. Enter the matrix $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 3 & 0 & 1 \\ 4 & 0 & 0 & 2 \end{bmatrix}$ as $[A]$, then use $[A]^{-1}$.

$[A]^{-1} = \begin{bmatrix} \frac{3}{5} & 0 & -\frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & 0 & \frac{1}{5} & -\frac{1}{10} \\ 0 & 1 & 0 & 0 \\ -\frac{6}{5} & 0 & \frac{4}{5} & \frac{1}{10} \end{bmatrix}$

Verify the result by showing $[A][A]^{-1} = I_4$ and $[A]^{-1}[A] = I_4$.

71. The system is $AX = B$ where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 4 & 2 & 1 \\ 4 & -2 & 1 \end{bmatrix},\quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ \text{and}\ B = \begin{bmatrix} -6 \\ 9 \\ -3 \end{bmatrix}.$$

$X = \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}$, so the solution to the system is {(2, 3, –5)}.

72. The system is $AX = B$ where $A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$,

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, and $B = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$. $X = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, so the solution to the system is {(1, 2, –1)}.

73. The system is $AX = B$ where

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 4 & -5 & 3 \\ 2 & -1 & 5 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ \text{and}\ B = \begin{bmatrix} -2 \\ -9 \\ -5 \end{bmatrix}.$$

$X = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ so the solution to the system is {(1, 2, –1)}.

74. The system is $AX = B$ where $A = \begin{bmatrix} 1 & -1 & 0 \\ 6 & 1 & 20 \\ 0 & 1 & 3 \end{bmatrix}$,

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, and $B = \begin{bmatrix} 1 \\ 14 \\ 1 \end{bmatrix}$. $X = \begin{bmatrix} 5 \\ 4 \\ -1 \end{bmatrix}$, so the solution to the system is {(5, 4, –1)}.

75. The system is $AX = B$ where

$$A = \begin{bmatrix} 1 & 0 & -3 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & 4 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix},\ X = \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix}\ \text{and}\ B = \begin{bmatrix} -3 \\ -1 \\ 7 \\ -8 \\ 8 \end{bmatrix}.$$

$X = \begin{bmatrix} 2 \\ 1 \\ 3 \\ -2 \\ 4 \end{bmatrix}$, so the solution to the system is {(2, 1, 3, –2, 4)}.

76. The system is $AX = B$ where

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & -2 & 2 \\ 2 & 2 & 1 & 1 \\ 1 & -1 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}\ \text{and}\ B = \begin{bmatrix} 4 \\ 7 \\ 3 \\ 5 \end{bmatrix}.$$

$X = \begin{bmatrix} -\frac{22}{5} \\ \frac{17}{5} \\ \frac{11}{5} \\ \frac{14}{5} \end{bmatrix}$, so the solution to the system is

$\left\{\left(-\frac{22}{5}, \frac{17}{5}, \frac{11}{5}, \frac{14}{5}\right)\right\}$.

77. Answers may vary.

78. Answers may vary.

79. does not make sense; Explanations will vary. Sample explanation: Only square matrices have inverses.

80. makes sense

81. makes sense

82. does not make sense; Explanations will vary. Sample explanation: As long as you use the inverse of that matrix, you will be able to decode the message.

83. false; Changes to make the statement true will vary. A sample change is: Not all square matrices have inverses.

84. true

85. false; Changes to make the statement true will vary. A sample change is: You need to multiply the inverse of A and B.

86. false; Changes to make the statement true will vary. A sample change is: $(AB)^{-1} = B^{-1}A^{-1}$

87. false; Changes to make the statement true will vary. A sample change is: $(A+B)^{-1} \neq A^{-1} + B^{-1}$

88. false; Changes to make the statement true will vary. A sample change is: The matrix is not invertible.

89. Answers may vary.

90. $A=\begin{bmatrix}3 & 5\\ 2 & 4\end{bmatrix}$

$$A^{-1}=\frac{1}{12-10}\begin{bmatrix}4 & -5\\ -2 & 3\end{bmatrix}$$
$$=\frac{1}{2}\begin{bmatrix}4 & -5\\ -2 & 3\end{bmatrix}=\begin{bmatrix}-2 & \frac{-5}{2}\\ -1 & \frac{3}{2}\end{bmatrix}$$
$$(A^{-1})^{-1}=\frac{1}{3-\frac{3}{2}}\begin{bmatrix}\frac{3}{2} & \frac{5}{2}\\ 1 & 2\end{bmatrix}$$
$$=2\begin{bmatrix}\frac{3}{2} & \frac{5}{2}\\ 1 & 2\end{bmatrix}=\begin{bmatrix}3 & 5\\ 2 & 4\end{bmatrix}$$

So, $(A^{-1})^{-1}=A$.

91. Using the statement before problems 9–14, we want to find values for *a* such that $(1)(4)-(a+1)(a-2)=0$.
$$(1)(4)-(a+1)(a-2)=4-(a^2-a-2)$$
$$=-a^2+a+6$$
$$0=-a^2+a+6$$
$$0=a^2-a-6$$
$$0=(a-3)(a+2)$$
$$a=3,\,-2$$

92. Answers may vary.

93. $2(-5)-(-3)(4)=-10+12=2$

94. $\dfrac{2(-5)-1(-4)}{5(-5)-6(-4)}=\dfrac{-10+4}{-25+24}=\dfrac{-6}{-1}=6$

95. $2(-30-(-3))-3(6-9)+(-1)(1-15)$
$$=2(-27)-3(-3)+(-1)(-14)$$
$$=-54+9+14$$
$$=-31$$

Section 9.5

Check Point Exercises

1. **a.** $\begin{vmatrix}10 & 9\\ 6 & 5\end{vmatrix}=10\cdot5-6\cdot9=50-54=-4$

b. $\begin{vmatrix}4 & 3\\ -5 & -8\end{vmatrix}=4\cdot(-8)-(-5)\cdot(3)$
$$=-32+15=-17$$

2. $5x+4y=12$
$3x-6y=24$
$$D=\begin{vmatrix}5 & 4\\ 3 & -6\end{vmatrix}=5\cdot(-6)-3\cdot4$$
$$=-30-12=-42$$
$$D_x=\begin{vmatrix}12 & 4\\ 24 & -6\end{vmatrix}=12(-6)-24(4)$$
$$=-72-96=-168$$
$$D_y=\begin{vmatrix}5 & 12\\ 3 & 24\end{vmatrix}=5(24)-3(12)$$
$$=120-36=84$$

Thus, $x=\dfrac{D_x}{D}=\dfrac{-168}{-42}=4$
$$y=\frac{D_y}{D}=\frac{84}{-42}=-2$$

The solution set is $\{(4,-2)\}$.

3. $\begin{bmatrix}2 & 1 & 7\\ -5 & 6 & 0\\ -4 & 3 & 1\end{bmatrix}$

The minor for 2 is $\begin{vmatrix}6 & 0\\ 3 & 1\end{vmatrix}$.

The minor for –5 is $\begin{vmatrix}1 & 7\\ 3 & 1\end{vmatrix}$.

The minor for –4 is $\begin{vmatrix}1 & 7\\ 6 & 0\end{vmatrix}$.

$$\begin{bmatrix}2 & 1 & 7\\ -5 & 6 & 0\\ -4 & 3 & 1\end{bmatrix}=2\begin{vmatrix}6 & 0\\ 3 & 1\end{vmatrix}-(-5)\begin{vmatrix}1 & 7\\ 3 & 1\end{vmatrix}-4\begin{vmatrix}1 & 7\\ 6 & 0\end{vmatrix}$$
$$=2(6\cdot1-3\cdot0)+5(1\cdot1-3\cdot7)-4(1\cdot0-6\cdot7)$$
$$=2(6-0)+5(1-21)-4(0-42)$$
$$=12-100+168$$
$$=80$$

4. $\begin{vmatrix} 6 & 4 & 0 \\ -3 & -5 & 3 \\ 1 & 2 & 0 \end{vmatrix} = 0\begin{vmatrix} -3 & -5 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 6 & 4 \\ 1 & 2 \end{vmatrix} + 0\begin{vmatrix} 6 & 4 \\ -3 & -5 \end{vmatrix}$

$= 0 - 3(6 \cdot 2 - 1 \cdot 4) + 0$
$= -3(12 - 4)$
$= -3(8)$
$= -24$

5. $3x - 2y + z = 16$
$2x + 3y - z = -9$
$x + 4y + 3z = 2$

$D = \begin{vmatrix} 3 & -2 & 1 \\ 2 & 3 & -1 \\ 1 & 4 & 3 \end{vmatrix}$; $D_x = \begin{vmatrix} 16 & -2 & 1 \\ -9 & 3 & -1 \\ 2 & 4 & 3 \end{vmatrix}$; $D_y = \begin{vmatrix} 3 & 16 & 1 \\ 2 & -9 & -1 \\ 1 & 2 & 3 \end{vmatrix}$; $D_z = \begin{vmatrix} 3 & -2 & 16 \\ 2 & 3 & -9 \\ 1 & 4 & 2 \end{vmatrix}$

$D = \begin{vmatrix} 3 & -2 & 1 \\ 2 & 3 & -1 \\ 1 & 4 & 3 \end{vmatrix} = 3\begin{vmatrix} 3 & -1 \\ 4 & 3 \end{vmatrix} - 2\begin{vmatrix} -2 & 1 \\ 4 & 3 \end{vmatrix} + 1\begin{vmatrix} -2 & 1 \\ 3 & -1 \end{vmatrix}$

$= 3[(3) \cdot 3 - 4 \cdot (-1)] - 2[(-2) \cdot 3 - 4 \cdot 1] + 1[(-2) \cdot (-1) - (3) \cdot 1]$
$= 3(9 + 4) - 2(-6 - 4) + 1(2 - 3)$
$= 39 + 20 - 1$
$= 58$

$D_x = \begin{vmatrix} 16 & -2 & 1 \\ -9 & 3 & -1 \\ 2 & 4 & 3 \end{vmatrix} = 1\begin{vmatrix} -9 & 3 \\ 2 & 4 \end{vmatrix} - (-1)\begin{vmatrix} 16 & -2 \\ 2 & 4 \end{vmatrix} + 3\begin{vmatrix} 16 & -2 \\ -9 & 3 \end{vmatrix}$

$= 1[(-9) \cdot 4 - 2 \cdot (3)] + 1[16 \cdot 4 - 2(-2)] + 3[16 \cdot (3) - (-9) \cdot (-2)]$
$= 1(-36 - 6) + 1(64 + 4) + 3(48 - 18)$
$= -42 + 68 + 90$
$= 116$

$D_y = \begin{vmatrix} 3 & 16 & 1 \\ 2 & -9 & -1 \\ 1 & 2 & 3 \end{vmatrix} = 3\begin{vmatrix} -9 & -1 \\ 2 & 3 \end{vmatrix} - 2\begin{vmatrix} 16 & 1 \\ 2 & 3 \end{vmatrix} + 1\begin{vmatrix} 16 & 1 \\ -9 & -1 \end{vmatrix}$

$= 3[(-9) \cdot 3 - 2 \cdot (-1)] - 2[16 \cdot 3 - 2 \cdot 1] + 1[16(-1) - (-9) \cdot 1]$
$= 3(-27 + 2) - 2(48 - 2) + 1(-16 + 9)$
$= -75 - 92 - 7$
$= -174$

$$D_z = \begin{vmatrix} 3 & -2 & 16 \\ 2 & 3 & -9 \\ 1 & 4 & 2 \end{vmatrix} = 3\begin{vmatrix} 3 & -9 \\ 4 & 2 \end{vmatrix} - 2\begin{vmatrix} -2 & 16 \\ 4 & 2 \end{vmatrix} + 1\begin{vmatrix} -2 & 16 \\ 3 & -9 \end{vmatrix}$$

$$= 3[(3)2 - 4(-9)] - 2[(-2)2 - 4\cdot 16] + 1[(-2)(-9) - (3)\cdot 16]$$

$$= 3(6+36) - 2(-4-64) + 1(18-48)$$

$$= 126 + 136 - 30$$

$$= 232$$

$$x = \frac{D_x}{D} = \frac{116}{58} = 2$$

$$y = \frac{D_y}{D} = \frac{-174}{58} = -3$$

$$z = \frac{D_z}{D} = \frac{232}{58} = 4$$

The solution to the system is {(2, –3, 4)}.

6. $|A| = \begin{vmatrix} 0 & 4 & 0 & -3 \\ -1 & 1 & 5 & 2 \\ 1 & -2 & 0 & 6 \\ 3 & 0 & 0 & 1 \end{vmatrix} = (-1)^{2+3}5\begin{vmatrix} 0 & 4 & -3 \\ 1 & -2 & 6 \\ 3 & 0 & 1 \end{vmatrix} = -5\begin{vmatrix} 0 & 4 & -3 \\ 1 & -2 & 6 \\ 3 & 0 & 1 \end{vmatrix}$

Evaluate the third-order determinant to get $|A| = -5(50) = -250$.

Exercise Set 9.5

1. $\begin{vmatrix} 5 & 7 \\ 2 & 3 \end{vmatrix} = 5\cdot 3 - 2\cdot 7 = 15 - 14 = 1$

2. $\begin{vmatrix} 4 & 8 \\ 5 & 6 \end{vmatrix} = 4\cdot 6 - 5\cdot 8 = 24 - 40 = -16$

3. $\begin{vmatrix} -4 & 1 \\ 5 & 6 \end{vmatrix} = (-4)6 - 5\cdot 1 = -24 - 5 = -29$

4. $\begin{vmatrix} 7 & 9 \\ -2 & -5 \end{vmatrix} = 7(-5) - (-2)9 = -35 + 18 = -17$

5. $\begin{vmatrix} -7 & 14 \\ 2 & -4 \end{vmatrix} = (-7)(-4) - 2(14) = 28 - 28 = 0$

6. $\begin{vmatrix} 1 & -3 \\ -8 & 2 \end{vmatrix} = 1\cdot 2 - (-8)(-3) = 2 - 24 = -22$

7. $\begin{vmatrix} -5 & -1 \\ -2 & -7 \end{vmatrix} = (-5)(-7) - (-2)(-1) = 35 - 2 = 33$

8. $\begin{vmatrix} \frac{1}{5} & \frac{1}{6} \\ -6 & 5 \end{vmatrix} = \frac{1}{5}(5) - (-6)\frac{1}{6} = 1 + 1 = 2$

9. $\begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{8} & -\frac{3}{4} \end{vmatrix} = \frac{1}{2}\left(-\frac{3}{4}\right) - \frac{1}{8} \cdot \frac{1}{2} = -\frac{3}{8} - \frac{1}{16} = -\frac{7}{16}$

10. $\begin{vmatrix} \frac{2}{3} & \frac{1}{3} \\ -\frac{1}{2} & \frac{3}{4} \end{vmatrix} = \frac{2}{3} \cdot \frac{3}{4} - \left(\frac{-1}{2}\right)\frac{1}{3} = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

11. $D = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2$

$D_x = \begin{vmatrix} 7 & 1 \\ 3 & -1 \end{vmatrix} = -7 - 3 = -10$

$D_y = \begin{vmatrix} 1 & 7 \\ 1 & 3 \end{vmatrix} = 3 - 7 = -4$

$x = \frac{D_x}{D} = \frac{-10}{-2} = 5$

$y = \frac{D_y}{D} = \frac{-4}{-2} = 2$

The solution set is {(5, 2)}.

12. $D = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2 - 1 = -3$

$D_x = \begin{vmatrix} 3 & 1 \\ 3 & -1 \end{vmatrix} = -3 - 3 = -6$

$D_y = \begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix} = 6 - 3 = 3$

$x = \frac{D_x}{D} = \frac{-6}{-3} = 2$

$y = \frac{D_y}{D} = \frac{3}{-3} = -1$

The solution set is {(2, –1)}.

13. $D = \begin{vmatrix} 12 & 3 \\ 2 & -3 \end{vmatrix} = -36 - 6 = -42$

$D_x = \begin{vmatrix} 15 & 3 \\ 13 & -3 \end{vmatrix} = -45 - 39 = -84$

$D_y = \begin{vmatrix} 12 & 15 \\ 2 & 13 \end{vmatrix} = 156 - 30 = 126$

$x = \frac{D_x}{D} = \frac{-84}{-42} = 2$

$y = \frac{D_y}{D} = \frac{126}{-42} = -3$

The solution set is {(2, –3)}.

14. $D = \begin{vmatrix} 1 & -2 \\ 5 & -1 \end{vmatrix} = -1 - (-10) = 9$

$D_x = \begin{vmatrix} 5 & -2 \\ -2 & -1 \end{vmatrix} = -5 - 4 = -9$

$D_y = \begin{vmatrix} 1 & 5 \\ 5 & -2 \end{vmatrix} = -2 - 25 = -27$

$x = \frac{D_x}{D} = \frac{-9}{9} = -1$

$y = \frac{D_y}{D} = \frac{-27}{9} = -3$

The solution set is {(–1, –3)}.

15. $D = \begin{vmatrix} 4 & -5 \\ 2 & 3 \end{vmatrix} = 12 - (-10) = 22$

$D_x = \begin{vmatrix} 17 & -5 \\ 3 & 3 \end{vmatrix} = 51 - (-15) = 66$

$D_y = \begin{vmatrix} 4 & 17 \\ 2 & 3 \end{vmatrix} = 12 - 34 = -22$

$x = \frac{D_x}{D} = \frac{66}{22} = 3$

$y = \frac{D_y}{D} = \frac{-22}{22} = -1$

The solution set is {(3, –1)}.

16. $D = \begin{vmatrix} 3 & 2 \\ 2 & 2 \end{vmatrix} = 6 - 4 = 2$

$D_x = \begin{vmatrix} 2 & 2 \\ 3 & 2 \end{vmatrix} = 4 - 6 = -2$

$D_y = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 9 - 4 = 5$

$x = \frac{D_x}{D} = \frac{-2}{2} = -1$

$y = \frac{D_y}{D} = \frac{5}{2} = \frac{5}{2}$

The solution set is $\{(-1, \frac{5}{2})\}$.

17. $D = \begin{vmatrix} 1 & 2 \\ 5 & 10 \end{vmatrix} = 10 - 10 = 0$

$D_x = \begin{vmatrix} 3 & 2 \\ 15 & 10 \end{vmatrix} = 30 - 30 = 0$

$D_y = \begin{vmatrix} 1 & 3 \\ 5 & 15 \end{vmatrix} = 15 - 15 = 0$

Because all 3 determinants equal zero, the system is dependent.

18. $D = \begin{vmatrix} 2 & -9 \\ 3 & -3 \end{vmatrix} = -6 - (-27) = 21$

$D_x = \begin{vmatrix} 5 & -9 \\ 11 & -3 \end{vmatrix} = -15 - (-99) = 84$

$D_y = \begin{vmatrix} 2 & 5 \\ 3 & 11 \end{vmatrix} = 22 - 15 = 7$

$x = \frac{D_x}{D} = \frac{84}{21} = 4$

$y = \frac{D_y}{D} = \frac{7}{21} = \frac{1}{3}$

The solution set is $\{(4, \frac{1}{3})\}$.

19. $D = \begin{vmatrix} 3 & -4 \\ 2 & 2 \end{vmatrix} = 6 - (-8) = 14$

$D_x = \begin{vmatrix} 4 & -4 \\ 12 & 2 \end{vmatrix} = 8 - (-48) = 56$

$D_y = \begin{vmatrix} 3 & 4 \\ 2 & 12 \end{vmatrix} = 36 - 8 = 28$

$x = \frac{D_x}{D} = \frac{56}{14} = 4$

$y = \frac{D_y}{D} = \frac{28}{14} = 2$

The solution set is $\{(4, 2)\}$.

20. $D = \begin{vmatrix} 3 & -7 \\ 2 & -3 \end{vmatrix} = -9 - (-14) = 5$

$D_x = \begin{vmatrix} 1 & -7 \\ -1 & -3 \end{vmatrix} = -3 - 7 = -10$

$D_y = \begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} = -3 - 2 = -5$

$x = \frac{D_x}{D} = \frac{-10}{5} = -2$

$y = \frac{D_y}{D} = \frac{-5}{5} = -1$

The solution set is $\{(-2, -1)\}$.

21. $D = \begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix} = 8 - (-15) = 23$

$D_x = \begin{vmatrix} 2 & -3 \\ 51 & 4 \end{vmatrix} = 8 - (-153) = 161$

$D_y = \begin{vmatrix} 2 & 2 \\ 5 & 51 \end{vmatrix} = 102 - 10 = 92$

$x = \frac{D_x}{D} = \frac{161}{23} = 7$

$y = \frac{D_y}{D} = \frac{92}{23} = 4$

The solution set is $\{(7, 4)\}$.

22. $y = -4x + 2$
$2x = 3y + 8$
Write the equations in standard form.
$4x + y = 2$
$2x - 3y = 8$

$$D = \begin{vmatrix} 4 & 1 \\ 2 & -3 \end{vmatrix} = -14$$

$$D_x = \begin{vmatrix} 2 & 1 \\ 8 & -3 \end{vmatrix} = -14$$

$$D_y = \begin{vmatrix} 4 & 2 \\ 2 & 8 \end{vmatrix} = 28$$

$$x = \frac{-14}{-14} = 1$$

$$y = \frac{28}{-14} = -2$$

The solution set is $\{(1, -2)\}$.

23. $$D = \begin{vmatrix} 3 & 3 \\ 2 & 2 \end{vmatrix} = 6 - 6 = 0$$

$$D_x = \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = 4 - 9 = -5$$

$$D_y = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 9 - 4 = 5$$

Because $D = 0$ but D_x or $D_y \neq 0$, the system is inconsistent.

24. $x + 2y - 3 = 0$
$12 = 8y + 4x$
Write the equations in standard form.
$x + 2y = 3$
$4x + 8y = 12$

$$D = \begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix} = 0$$

$$D_x = \begin{vmatrix} 3 & 2 \\ 12 & 8 \end{vmatrix} = 0$$

$$D_y = \begin{vmatrix} 1 & 3 \\ 4 & 12 \end{vmatrix} = 0$$

Since all determinants are zero, the system is dependent.

25. Write the equations in standard form.
$3x + 4y = 16$
$6x + 8y = 32$

$$D = \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix} = 24 - 24 = 0$$

$$D_x = \begin{bmatrix} 16 & 4 \\ 32 & 8 \end{bmatrix} = 128 - 128 = 0$$

$$D_y = \begin{bmatrix} 3 & 16 \\ 6 & 32 \end{bmatrix} = 96 - 69 = 0$$

Since all determinants are zero, the system is dependent.

26. $$D = \begin{vmatrix} 2 & -3 \\ 4 & -6 \end{vmatrix} = -12 - (-12) = 0$$

$$D_x = \begin{vmatrix} 7 & -3 \\ 3 & -6 \end{vmatrix} = -42 + 9 = -33$$

$$D_y = \begin{vmatrix} 2 & 7 \\ 4 & 3 \end{vmatrix} = 6 - 28 = -22$$

Because $D = 0$ but D_x and $D_y \neq 0$, the system is inconsistent.

27. $$\begin{vmatrix} 3 & 0 & 0 \\ 2 & 1 & -5 \\ -2 & 5 & -1 \end{vmatrix} = 3\begin{vmatrix} 1 & -5 \\ 5 & -1 \end{vmatrix} - 0\begin{vmatrix} 2 & -5 \\ -2 & -1 \end{vmatrix} + 0\begin{vmatrix} 2 & 1 \\ -2 & 5 \end{vmatrix}$$

$= 3[(1)(-1) - (5)(-5)]$
$= 3(-1 + 25) = 3(24)$
$= 72$

28. $$\begin{vmatrix} 4 & 0 & 0 \\ 3 & -1 & 4 \\ 2 & -3 & 5 \end{vmatrix} = 4\begin{vmatrix} -1 & 4 \\ -3 & 5 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix} + 0\begin{vmatrix} 3 & -1 \\ 2 & -3 \end{vmatrix}$$

$= 4[(-1)(5) - (-3)(4)]$
$= 4(-5 + 12)$
$= 4(7)$
$= 28$

29. $$\begin{vmatrix} 3 & 1 & 0 \\ -3 & 4 & 0 \\ -1 & 3 & -5 \end{vmatrix} = 0\begin{vmatrix} -3 & 4 \\ -1 & 3 \end{vmatrix} - 0\begin{vmatrix} 3 & 1 \\ -1 & 3 \end{vmatrix} + (-5)\begin{vmatrix} 3 & 1 \\ -3 & 4 \end{vmatrix}$$

$= -5[3 \cdot 4 - (-3)(1)]$
$= -5(12 + 3) = -5(15)$
$= -75$

30. $\begin{vmatrix} 2 & -4 & 2 \\ -1 & 0 & 5 \\ 3 & 0 & 4 \end{vmatrix} = -(-4)\begin{vmatrix} -1 & 5 \\ 3 & 4 \end{vmatrix} + 0\begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} - 0\begin{vmatrix} 2 & 2 \\ -1 & 5 \end{vmatrix}$

$= 4[(-1)(4) - 3 \cdot 5]$
$= 4(-4 - 15)$
$= 4(-19)$
$= -76$

31. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -3 & 4 & -5 \end{vmatrix} - 2R_1 + R_2$

$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -3 & 4 & -5 \end{vmatrix} = 0$

32. $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 2 & -3 \\ 3 & 2 & 1 \end{vmatrix}$

$= 1\begin{vmatrix} 2 & -3 \\ 2 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ 2 & -3 \end{vmatrix}$

$= 1[2 \cdot 1 - 2(-3)] - 2(2 \cdot 1 - 2 \cdot 3) + 3[2(-3) - 2 \cdot 3]$
$= (2 + 6) - 2(2 - 6) + 3(-6 - 6)$
$= 8 - 2(-4) + 3(-12)$
$= 8 + 8 - 36$
$= -20$

33. $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ -1 & 3 & -1 \end{vmatrix}$

$= \begin{vmatrix} -1 & 1 \\ 3 & -1 \end{vmatrix} - \begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix}$

$= (1 - 3) - [-2 - (-1)] + (6 - 1)$
$= -2 - (-1) + 5 = -2 + 1 + 5 = 4$

$D_x = \begin{vmatrix} 0 & 1 & 1 \\ -1 & -1 & 1 \\ -8 & 3 & -1 \end{vmatrix} = (-1)\begin{vmatrix} -1 & 1 \\ -8 & -1 \end{vmatrix} + \begin{vmatrix} -1 & -1 \\ -8 & 3 \end{vmatrix}$

$= (-1)[1 - (-8)] + (-3 - 8) = (-1)(9) - 11$
$= -20$

$D_y = \begin{vmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ -1 & -8 & -1 \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ -8 & -1 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & -8 \end{vmatrix}$

$= 1 - (-8) + (-16 - 1) = 1 + 8 - 17 = -8$

$D_z = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -1 & -1 \\ -1 & 3 & -8 \end{vmatrix} = 1\begin{vmatrix} -1 & -1 \\ 3 & -8 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ -1 & -8 \end{vmatrix}$

$= 8 - (-3) - 1(-16 - 1) = 11 + 17 = 28$

$x = \frac{D_x}{D} = \frac{-20}{4} = -5$

$y = \frac{D_y}{D} = \frac{-8}{4} = -2$

$z = \frac{D_z}{D} = \frac{28}{4} = 7$

The solution to the system is {(–5, –2, 7)}.

34. $D = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \\ -1 & -1 & 3 \end{vmatrix}$

$= 1\begin{vmatrix} 3 & 1 \\ -1 & 3 \end{vmatrix} - (-1)\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} + 2\begin{vmatrix} 2 & 3 \\ -1 & -1 \end{vmatrix}$

$= 1[9 - (-1)] + [6 - (-1)] + 2[-2 - (-3)]$
$= 10 + 7 + 2(1)$
$= 19$

$D_x = \begin{vmatrix} 3 & -1 & 2 \\ 9 & 3 & 1 \\ 11 & -1 & 3 \end{vmatrix}$

$= 3\begin{vmatrix} 3 & 1 \\ -1 & 3 \end{vmatrix} - (-1)\begin{vmatrix} 9 & 1 \\ 11 & 3 \end{vmatrix} + 2\begin{vmatrix} 9 & 3 \\ 11 & -1 \end{vmatrix}$

$= 3[9 - (-1)] + (27 - 11) + 2(-9 - 33)$
$= 3(10) + 16 + 2(-42) = 30 + 16 - 84$
$= -38$

$$D_y = \begin{vmatrix} 1 & 3 & 2 \\ 2 & 9 & 1 \\ -1 & 11 & 3 \end{vmatrix}$$

$$= 1\begin{vmatrix} 9 & 1 \\ 11 & 3 \end{vmatrix} - 3\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} + 2\begin{vmatrix} 2 & 9 \\ -1 & 11 \end{vmatrix}$$

$= 1(27-11) - 3[6-(-1)] + 2[22-(-9)]$
$= 16 - 3(7) + 2(31)$
$= 16 - 21 + 62$
$= 57$

$$D_z = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 3 & 9 \\ -1 & -1 & 11 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & 9 \\ -1 & 11 \end{vmatrix} - (-1)\begin{vmatrix} 2 & 9 \\ -1 & 11 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ -1 & -1 \end{vmatrix}$$

$= 1[33-(-9)] + [22-(-9)] + 3[-2-(-3)]$
$= 42 + 31 + 3(1)$
$= 76$

$$x = \frac{D_x}{D} = \frac{-38}{19} = -2, \quad y = \frac{D_y}{D} = \frac{57}{19} = 3,$$

$$z = \frac{D_z}{D} = \frac{76}{19} = 4$$

The solution set is {(–2, 3, 4)}.

35. $$D = \begin{vmatrix} 4 & -5 & -6 \\ 1 & -2 & -5 \\ 2 & -1 & 0 \end{vmatrix} = 2\begin{vmatrix} -5 & -6 \\ -2 & -5 \end{vmatrix} - (-1)\begin{vmatrix} 4 & -6 \\ 1 & -5 \end{vmatrix}$$

$= 2(25 - 12) + [-20 - (-6)] = 2(13) + (-14)$
$= 26 - 14 = 12$

$$D_x = \begin{vmatrix} -1 & -5 & -6 \\ -12 & -2 & -5 \\ 7 & -1 & 0 \end{vmatrix}$$

$$= 7\begin{vmatrix} -5 & -6 \\ -2 & -5 \end{vmatrix} - (-1)\begin{vmatrix} -1 & -6 \\ -12 & -5 \end{vmatrix}$$

$= 7(25 - 12) + (5 - 72) = 7(13) - 67$
$= 91 - 67 = 24$

$$D_y = \begin{vmatrix} 4 & -1 & -6 \\ 1 & -12 & -5 \\ 2 & 7 & 0 \end{vmatrix} = 2\begin{vmatrix} -1 & -6 \\ -12 & -5 \end{vmatrix} - 7\begin{vmatrix} 4 & -6 \\ 1 & -5 \end{vmatrix}$$

$= 2(5 - 72) - 7[-20 - (-6)]$
$= 2(-67) - 7(-14) = -134 + 98 = -36$

$$D_z = \begin{vmatrix} 4 & -5 & -1 \\ 1 & -2 & -12 \\ 2 & -1 & 7 \end{vmatrix}$$

$$= 4\begin{vmatrix} -2 & -12 \\ -1 & 7 \end{vmatrix} - (-5)\begin{vmatrix} 1 & -12 \\ 2 & 7 \end{vmatrix} + (-1)\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix}$$

$= 4(-14 - 12) + 5[7 - (-24)] - [-1 - (-4)]$
$= 4(-26) + 5(31) - (3) = -104 + 155 - 3 = 48$

$$x = \frac{D_x}{D} = \frac{24}{12} = 2, \; y = \frac{D_y}{D} = \frac{-36}{12} = -3,$$

$$z = \frac{D_z}{D} = \frac{48}{12} = 4$$

The solution set is {(2, –3, 4)}.

36. $$D = \begin{vmatrix} 1 & -3 & 1 \\ 1 & 2 & 0 \\ 2 & -1 & 0 \end{vmatrix} = 1\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1 - 4 = -5$$

$$D_x = \begin{vmatrix} -2 & -3 & 1 \\ 8 & 2 & 0 \\ 1 & -1 & 0 \end{vmatrix} = 1\begin{vmatrix} 8 & 2 \\ 1 & -1 \end{vmatrix} = -8 - 2 = -10$$

$$D_y = \begin{vmatrix} 1 & -2 & 1 \\ 1 & 8 & 0 \\ 2 & 1 & 0 \end{vmatrix} = 1\begin{vmatrix} 1 & 8 \\ 2 & 1 \end{vmatrix} = 1 - 16 = -15$$

$$D_z = \begin{vmatrix} 1 & -3 & -2 \\ 1 & 2 & 8 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & 8 \\ -1 & 1 \end{vmatrix} - 1\begin{vmatrix} -3 & -2 \\ -1 & 1 \end{vmatrix} + 2\begin{vmatrix} -3 & -2 \\ 2 & 8 \end{vmatrix}$$

$= 2 - (-8) - (-3 - 2) + 2[-24 - (-4)]$
$= 2 + 8 - (-5) + 2(-20)$
$= 10 + 5 - 40$
$= -25$

$$x = \frac{D_x}{D} = \frac{-10}{-5} = 2, \; y = \frac{D_y}{D} = \frac{-15}{-5} = 3,$$

$$z = \frac{D_z}{D} = \frac{-25}{-5} = 5$$

The solution set is {(2, 3, 5)}.

37. $D=\begin{vmatrix}1&1&1\\1&-2&1\\1&3&2\end{vmatrix}=1\begin{vmatrix}-2&1\\3&2\end{vmatrix}-1\begin{vmatrix}1&1\\1&2\end{vmatrix}+1\begin{vmatrix}1&-2\\1&3\end{vmatrix}$

$= -4-3-(2-1)+[3-(-2)]$
$= -7-1+5=-3$

$D_x=\begin{vmatrix}4&1&1\\7&-2&1\\4&3&2\end{vmatrix}=4\begin{vmatrix}-2&1\\3&2\end{vmatrix}-1\begin{vmatrix}7&1\\4&2\end{vmatrix}+1\begin{vmatrix}7&-2\\4&3\end{vmatrix}$

$= 4(-4-3)-(14-4)+[21-(-8)]$
$= 4(-7)-10+29=-28+19=-9$

$D_y=\begin{vmatrix}1&4&1\\1&7&1\\1&4&2\end{vmatrix}=1\begin{vmatrix}7&1\\4&2\end{vmatrix}-1\begin{vmatrix}4&1\\4&2\end{vmatrix}+1\begin{vmatrix}4&1\\7&1\end{vmatrix}$

$= 14-4-(8-4)+(4-7)=10-4-3=3$

$D_z=\begin{vmatrix}1&1&4\\1&-2&7\\1&3&4\end{vmatrix}=1\begin{vmatrix}-2&7\\3&4\end{vmatrix}-1\begin{vmatrix}1&4\\3&4\end{vmatrix}+1\begin{vmatrix}1&4\\-2&7\end{vmatrix}$

$= -8-21-(4-12)+[7-(-8)]$
$= -29+8+15=-6$

$x=\frac{D_x}{D}=\frac{-9}{-3}=3,\ y=\frac{D_y}{D}=\frac{3}{-3}=-1,$

$z=\frac{D_z}{D}=\frac{-6}{-3}=2$

The solution set is {3, –1, 2}.

38. $D=\begin{vmatrix}2&2&3\\4&-1&1\\5&-2&6\end{vmatrix}$

$=2\begin{vmatrix}-1&1\\-2&6\end{vmatrix}-2\begin{vmatrix}4&1\\5&6\end{vmatrix}+3\begin{vmatrix}4&-1\\5&-2\end{vmatrix}$

$=2[-6-(-2)]-2(24-5)+3[-8-(-5)]$
$=2(-4)-2(19)+3(-3)$
$=-8-38-9$
$=-55$

$D_x=\begin{vmatrix}10&2&3\\-5&-1&1\\1&-2&6\end{vmatrix}$

$=10\begin{vmatrix}-1&1\\-2&6\end{vmatrix}-2\begin{vmatrix}-5&1\\1&6\end{vmatrix}+3\begin{vmatrix}-5&-1\\1&-2\end{vmatrix}$

$=10[-6-(-2)]-2(-30-1)+3[10-(-1)]$
$=10(-4)-2(-31)+3(11)$
$=-40+62+33$
$=55$

$D_y=\begin{vmatrix}2&10&3\\4&-5&1\\5&1&6\end{vmatrix}$

$=2\begin{vmatrix}-5&1\\1&6\end{vmatrix}-10\begin{vmatrix}4&1\\5&6\end{vmatrix}+3\begin{vmatrix}4&-5\\5&1\end{vmatrix}$

$= 2(-30-1)-10(24-5)+3[4-(-25)]$
$= 2(-31)-10(19)+3(29)$
$= -62-190+87$
$= -165$

$D_z=\begin{vmatrix}2&2&10\\4&-1&-5\\5&-2&1\end{vmatrix}$

$=2\begin{vmatrix}-1&-5\\-2&1\end{vmatrix}-2\begin{vmatrix}4&-5\\5&1\end{vmatrix}+10\begin{vmatrix}4&-1\\5&-2\end{vmatrix}$

$= 2(-1-10)-2[4-(-25)]+10[-8-(-25)]$
$= 2(-11)-2(29)+10(-3)$
$= -22-58-30$
$= -110$

$x=\frac{D_x}{D}=\frac{55}{-55}=-1,\ y=\frac{D_y}{D}=\frac{-165}{-55}=3,$

$z=\frac{D_z}{D}=\frac{-110}{-55}=2$

The solution set is {(–1, 3, 2)}.

39. $D = \begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ 3 & 0 \end{vmatrix} + 2\begin{vmatrix} 0 & 2 \\ 2 & 3 \end{vmatrix}$

$= 0 - (-3) + 2(0 - 4) = 3 - 8 = -5$

$D_x = \begin{vmatrix} 4 & 0 & 2 \\ 5 & 2 & -1 \\ 13 & 3 & 0 \end{vmatrix} = 4\begin{vmatrix} 2 & -1 \\ 3 & 0 \end{vmatrix} + 2\begin{vmatrix} 5 & 2 \\ 13 & 3 \end{vmatrix}$

$= 4[0 - (-3)] + 2(15 - 26)$

$= 4(3) + 2(-11) = 12 - 22 = -10$

$D_y = \begin{vmatrix} 1 & 4 & 2 \\ 0 & 5 & -1 \\ 2 & 13 & 0 \end{vmatrix} = \begin{vmatrix} 5 & -1 \\ 13 & 0 \end{vmatrix} + 2\begin{vmatrix} 4 & 2 \\ 5 & -1 \end{vmatrix}$

$= 0 - (-13) + 2(-4 - 10)$

$= 13 + 2(-14) = 13 - 28 = -15$

$D_z = \begin{vmatrix} 1 & 0 & 4 \\ 0 & 2 & 5 \\ 2 & 3 & 13 \end{vmatrix} = \begin{vmatrix} 2 & 5 \\ 3 & 13 \end{vmatrix} + 4\begin{vmatrix} 0 & 2 \\ 2 & 3 \end{vmatrix}$

$= 26 - 15 + 4(0 - 4) = 11 + 4(-4)$

$= 11 - 16 = -5$

$x = \frac{D_x}{D} = \frac{-10}{-5} = 2,\ y = \frac{D_y}{D} = \frac{-15}{-5} = 3,$

$z = \frac{D_z}{D} = \frac{-5}{-5} = 1$

The solution set is {(2, 3, 1)}.

40. $D = \begin{vmatrix} 3 & 0 & 2 \\ 5 & -1 & 0 \\ 0 & 4 & 3 \end{vmatrix}$

$= 3\begin{vmatrix} -1 & 0 \\ 4 & 3 \end{vmatrix} + 2\begin{vmatrix} 5 & -1 \\ 0 & 4 \end{vmatrix}$

$= 3(-3 - 0) + 2(20 - 0) = 3(-3) + 2(20)$

$= -9 + 40$

$= 31$

$D_x = \begin{vmatrix} 4 & 0 & 2 \\ -4 & -1 & 0 \\ 22 & 4 & 3 \end{vmatrix}$

$= 4\begin{vmatrix} -1 & 0 \\ 4 & 3 \end{vmatrix} + 2\begin{vmatrix} -4 & -1 \\ 22 & 4 \end{vmatrix}$

$D_y = \begin{vmatrix} 3 & 4 & 2 \\ 5 & -4 & 0 \\ 0 & 22 & 3 \end{vmatrix}$

$= 3\begin{vmatrix} -4 & 0 \\ 22 & 3 \end{vmatrix} - 5\begin{vmatrix} 4 & 2 \\ 22 & 3 \end{vmatrix}$

$= 3(-12 - 0) - 5(12 - 44)$

$= 3(-12) - 5(-32)$

$= -36 + 160$

$= 124$

$D_z = \begin{vmatrix} 3 & 0 & 4 \\ 5 & -1 & -4 \\ 0 & 4 & 22 \end{vmatrix}$

$= 3\begin{vmatrix} -1 & -4 \\ 4 & 22 \end{vmatrix} + 4\begin{vmatrix} 5 & -1 \\ 0 & 4 \end{vmatrix}$

$= 3[-22 - (-16)] + 4(20 - 0)$

$= 3(-6) + 4(20)$

$= -18 + 80$

$= 62$

$x = \frac{D_x}{D} = \frac{0}{31} = 0,\ y = \frac{D_y}{D} = \frac{124}{31} = 4,$

$z = \frac{D_z}{D} = \frac{62}{31} = 2$

The solution set is {(0, 4, 2)}.

41. $\begin{vmatrix} 4 & 2 & 8 & -7 \\ -2 & 0 & 4 & 1 \\ 5 & 0 & 0 & 5 \\ 4 & 0 & 0 & -1 \end{vmatrix} = -2\begin{vmatrix} -2 & 4 & 1 \\ 5 & 0 & 5 \\ 4 & 0 & -1 \end{vmatrix} + 0\begin{vmatrix} 4 & 8 & -7 \\ 5 & 0 & 5 \\ 4 & 0 & -1 \end{vmatrix} - 0\begin{vmatrix} 4 & 8 & -7 \\ -2 & 4 & 1 \\ 4 & 0 & -1 \end{vmatrix} + 0\begin{vmatrix} 4 & 8 & -7 \\ -2 & 4 & 1 \\ 5 & 0 & 5 \end{vmatrix}$

$= (-2)\left[(-4)\begin{vmatrix} 5 & 5 \\ 4 & -1 \end{vmatrix} + 0\begin{vmatrix} -2 & 1 \\ 4 & -1 \end{vmatrix} - 0\begin{vmatrix} -2 & 1 \\ 5 & 5 \end{vmatrix}\right] = (-2)(-4)[5(-1) - 4 \cdot 5] = 8(-5 - 20) = 8(-25) = -200$

42. $\begin{vmatrix} 3 & -1 & 1 & 2 \\ -2 & 0 & 0 & 0 \\ 2 & -1 & -2 & 3 \\ 1 & 4 & 2 & 3 \end{vmatrix} = -(-2)\begin{vmatrix} -1 & 1 & 2 \\ -1 & -2 & 3 \\ 4 & 2 & 3 \end{vmatrix} + 0\begin{vmatrix} 3 & 1 & 2 \\ 2 & -2 & 3 \\ 1 & 2 & 3 \end{vmatrix} - 0\begin{vmatrix} 3 & -1 & 2 \\ 2 & -1 & 3 \\ 1 & 4 & 3 \end{vmatrix} + 0\begin{vmatrix} 3 & -1 & 1 \\ 2 & -1 & -2 \\ 1 & 4 & 2 \end{vmatrix}$

$= 2\left[(-1)\begin{vmatrix} -2 & 3 \\ 2 & 3 \end{vmatrix} - 1\begin{vmatrix} -1 & 3 \\ 4 & 3 \end{vmatrix} + 2\begin{vmatrix} -1 & -2 \\ 4 & 2 \end{vmatrix}\right] = 2\{(-1)(-2 \cdot 3 - 2 \cdot 3) - 1[(-1)(3) - 4 \cdot 3] + 2[(-1)(2) - 4(-2)]\}=78$

43. $\begin{vmatrix} -2 & -3 & 3 & 5 \\ 1 & -4 & 0 & 0 \\ 1 & 2 & 2 & -3 \\ 2 & 0 & 1 & 1 \end{vmatrix} = -1\begin{vmatrix} -3 & 3 & 5 \\ 2 & 2 & -3 \\ 0 & 1 & 1 \end{vmatrix} + (-4)\begin{vmatrix} -2 & 3 & 5 \\ 1 & 2 & -3 \\ 2 & 1 & 1 \end{vmatrix} - 0\begin{vmatrix} -2 & -3 & 5 \\ 1 & 2 & -3 \\ 2 & 0 & 1 \end{vmatrix} + 0\begin{vmatrix} -2 & -3 & 3 \\ 1 & 2 & 2 \\ 2 & 0 & 1 \end{vmatrix}$

$= (-1)\left[0\begin{vmatrix} 3 & 5 \\ 2 & -3 \end{vmatrix} - 1\begin{vmatrix} -3 & 5 \\ 2 & -3 \end{vmatrix} + 1\begin{vmatrix} -3 & 3 \\ 2 & 2 \end{vmatrix}\right] - 4\left[2\begin{vmatrix} 3 & 5 \\ 2 & -3 \end{vmatrix} - 1\begin{vmatrix} -2 & 5 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} -2 & 3 \\ 1 & 2 \end{vmatrix}\right]$

$= (-1)\{(-1)[(-3)(-3) - 2\cdot5] + [(-3)(2) - 2\cdot3]\} - 4\{2[3(-3) - 2\cdot5] - [(-2)(-3) - 1\cdot5] + [(-2)(2) - 1\cdot3]\}=195$

44. $\begin{vmatrix} 1 & -3 & 2 & 0 \\ -3 & -1 & 0 & -2 \\ 2 & 1 & 3 & 1 \\ 2 & 0 & -2 & 0 \end{vmatrix} = (-2)\begin{vmatrix} -3 & 2 & 0 \\ -1 & 0 & -2 \\ 1 & 3 & 1 \end{vmatrix} + 0\begin{vmatrix} 1 & 2 & 0 \\ -3 & 0 & -2 \\ 2 & 3 & 1 \end{vmatrix} - (-2)\begin{vmatrix} 1 & -3 & 0 \\ -3 & -1 & -2 \\ 2 & 1 & 1 \end{vmatrix} + 0\begin{vmatrix} 1 & -3 & 2 \\ -3 & -1 & 0 \\ 2 & 1 & 3 \end{vmatrix}$

$= (-2)\left[-(-1)\begin{vmatrix} 2 & 0 \\ 3 & 1 \end{vmatrix} + 0\begin{vmatrix} -3 & 0 \\ 1 & 1 \end{vmatrix} - (-2)\begin{vmatrix} -3 & 2 \\ 1 & 3 \end{vmatrix}\right] + 2\left[1\begin{vmatrix} -1 & -2 \\ 1 & 1 \end{vmatrix} - (-3)\begin{vmatrix} -3 & -2 \\ 2 & 1 \end{vmatrix} + 0\begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix}\right]$

$= -2\{(2 \cdot 1 - 3 \cdot 0) + 2[(-3)(3) - 1 \cdot 2]\} + 2\{[(-1)(1) - (1)(-2)] + 3[(-3)(1) - (2)(-2)]\}=48$

45. $\begin{vmatrix} \begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix} & \begin{vmatrix} 7 & 0 \\ 1 & 5 \end{vmatrix} \\ \begin{vmatrix} 3 & 0 \\ 0 & 7 \end{vmatrix} & \begin{vmatrix} 9 & -6 \\ 3 & 5 \end{vmatrix} \end{vmatrix} = \begin{vmatrix} 3(3)-(-2)(1) & 7(5)-1(0) \\ 3(7)-0(0) & 9(5)-3(-6) \end{vmatrix} = \begin{vmatrix} 9+2 & 35-0 \\ 21-0 & 45+18 \end{vmatrix} = \begin{vmatrix} 11 & 35 \\ 21 & 63 \end{vmatrix}$

$= 11(63) - 21(35) = 693 - 735 = -42$

46. $\begin{vmatrix} \begin{vmatrix} 5 & 0 \\ 4 & -3 \end{vmatrix} & \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} \\ \begin{vmatrix} 7 & -5 \\ 4 & 6 \end{vmatrix} & \begin{vmatrix} 4 & 1 \\ -3 & 5 \end{vmatrix} \end{vmatrix} = \begin{vmatrix} 5(-3)-4(0) & (-1)(-1)-0 \\ 7(6)-4(-5) & 4(5)-(-3)(1) \end{vmatrix} = \begin{vmatrix} -15-0 & 1+0 \\ 42+20 & 20+3 \end{vmatrix} = \begin{vmatrix} -15 & 1 \\ 62 & 23 \end{vmatrix}$

$= (-15)(23) - 62(1) = -345 - 62 = -407$

47. From $D = \begin{vmatrix} 2 & -4 \\ 3 & 5 \end{vmatrix}$ we obtain the coefficients of the variables in our equations:

$$2x - 4y = c_1$$
$$3x + 5y = c_2$$

From $D_x = \begin{vmatrix} 8 & -4 \\ -10 & 5 \end{vmatrix}$ we obtain the constant coefficients: 8 and -10

$$2x - 4y = 8$$
$$3x + 5y = -10$$

48. From $D = \begin{vmatrix} 2 & -3 \\ 5 & 6 \end{vmatrix}$ we obtain the coefficients of the variables in our equations:

$$2x - 3y = c_1$$
$$5x + 6y = c_2$$

From $D_x = \begin{vmatrix} 8 & -3 \\ 11 & 6 \end{vmatrix}$ we obtain the constant coefficients: 8 and 11

$$2x - 3y = 8$$
$$5x + 6y = 11$$

49.

$$\begin{vmatrix} -2 & x \\ 4 & 6 \end{vmatrix} = 32$$
$$-2(6) - 4(x) = 32$$
$$-12 - 4x = 32$$
$$-4x = 44$$
$$x = -11$$

The solution is -11.

50.

$$\begin{vmatrix} x+3 & -6 \\ x-2 & -4 \end{vmatrix} = 28$$
$$(x+3)(-4) - (x-2)(-6) = 28$$
$$-4x - 12 + 6x - 12 = 28$$
$$2x - 24 = 28$$
$$2x = 52$$
$$x = 26$$

The solution is 26.

51. $$\begin{vmatrix} 1 & x & -2 \\ 3 & 1 & 1 \\ 0 & -2 & 2 \end{vmatrix} = -8$$

$$0\begin{vmatrix} x & -2 \\ 1 & 1 \end{vmatrix} - (-2)\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & x \\ 3 & 1 \end{vmatrix} = -8$$

$$2[1(1)-3(-2)]+2[1(1)-3(x)] = -8$$
$$2(1+6)+2(1-3x) = -8$$
$$2(7)+2(1-3x) = -8$$
$$14+2-6x = -8$$
$$-6x = -24$$
$$x = 4$$

The solution is 4.

52. $$\begin{vmatrix} 2 & x & 1 \\ -3 & 1 & 0 \\ 2 & 1 & 4 \end{vmatrix} = 39$$

$$-(-3)\begin{vmatrix} x & 1 \\ 1 & 4 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 2 & 4 \end{vmatrix} - 0\begin{vmatrix} 2 & x \\ 2 & 1 \end{vmatrix} = 39$$

$$3(4x-1)+(8-2) = 39$$
$$12x-3+6 = 39$$
$$12x = 36$$
$$x = 3$$

The solution is 3.

53. $$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 3 & -5 & 1 \\ 2 & 6 & 1 \\ -3 & 5 & 1 \end{vmatrix} = \pm\frac{1}{2}\begin{vmatrix} 3 & -5 & 1 \\ -1 & 11 & 0 \\ -6 & 10 & 0 \end{vmatrix} = \pm\frac{1}{2}\begin{vmatrix} -1 & 11 \\ -6 & 10 \end{vmatrix} = \pm\frac{1}{2}[-10-(-66)] = \pm\frac{1}{2}(56) = 28$$

The area is 28 square units.

The slope of the line through (3, –5) and (–3, 5) is $m = \frac{5-(-5)}{-3-3} = \frac{10}{-6} = -\frac{5}{3}$.

The equation of the line is $y-(-5) = -\frac{5}{3}(x-3)$ or $y = -\frac{5}{3}x$.

The line perpendicular to $y = -\frac{5}{3}x$ through (2, 6) has equation $y-6 = \frac{3}{5}(x-2)$ or $y = \frac{3}{5}x+\frac{24}{5}$.

These lines intersect where $-\frac{5}{3}x = \frac{3}{5}x+\frac{24}{5}$.

$-\frac{36}{17} = x$ and $-\frac{24}{5} = \frac{34}{15}x$ $y = -\frac{5}{3}\left(-\frac{36}{17}\right) = \frac{60}{17}$

Using the side connecting (3, –5) and

(–3, 5) as the base, the height is the distance from (2, 6) to $\left(-\frac{36}{17}, \frac{60}{17}\right)$.

$$b = \sqrt{[3-(-3)]^2+(-5-5)^2}$$
$$= \sqrt{36+100} = \sqrt{136} = 2\sqrt{34}$$

$$h = \sqrt{\left[2 - \left(-\frac{36}{17}\right)\right]^2 + \left(6 - \frac{60}{17}\right)^2}$$

$$= \sqrt{\frac{4900}{289} + \frac{1764}{289}} = \frac{14\sqrt{34}}{17}$$

$$\frac{1}{2}bh = \frac{1}{2}\left(2\sqrt{34}\right)\left(\frac{14\sqrt{34}}{17}\right) = \frac{14(34)}{17}$$

= 14(2) = 28 square units

54. $\text{Area} = \pm\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ -2 & -3 & 1 \\ 11 & -3 & 1 \end{vmatrix} = \pm\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ -3 & -4 & 0 \\ 10 & -4 & 0 \end{vmatrix} = \pm\frac{1}{2}\begin{vmatrix} -3 & -4 \\ 10 & -4 \end{vmatrix} = \pm\frac{1}{2}[12 - (-40)] = \frac{1}{2}(52) = 26$

The area is 26 square units.

55. $\begin{vmatrix} 3 & -1 & 1 \\ 0 & -3 & 1 \\ 12 & 5 & 1 \end{vmatrix} = \begin{vmatrix} 3 & -1 & 1 \\ -3 & -2 & 0 \\ 9 & 6 & 0 \end{vmatrix} = \begin{vmatrix} -3 & -2 \\ 9 & 6 \end{vmatrix}$

$= -18 - (-18) = 0$

Yes, the points are collinear.

56. $\begin{vmatrix} -4 & -6 & 1 \\ 1 & 0 & 1 \\ 11 & 12 & 1 \end{vmatrix} = \begin{vmatrix} -4 & -6 & 1 \\ 5 & 6 & 0 \\ 15 & 18 & 0 \end{vmatrix} = \begin{vmatrix} 5 & 6 \\ 15 & 18 \end{vmatrix} = (5)(6)\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} = 30(3-3) = 0$

Yes, the points are collinear.

57. $\begin{vmatrix} x & y & 1 \\ 3 & -5 & 1 \\ -2 & 6 & 1 \end{vmatrix} = x\begin{vmatrix} -5 & 1 \\ 6 & 1 \end{vmatrix} - y\begin{vmatrix} 3 & 1 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 3 & -5 \\ -2 & 6 \end{vmatrix} = x(-5 - 6) - y[3 - (-2)] + (18 - 10)$

$= -11x - 5y + 8$

The equation of the line is $-11x - 5y + 8 = 0$. The equation of the line in slope-intercept form is $y = -\frac{11}{5}x + \frac{8}{5}$.

58. $\begin{vmatrix} x & y & 1 \\ -1 & 3 & 1 \\ 2 & 4 & 1 \end{vmatrix} = x\begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} - y\begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} -1 & 3 \\ 2 & 4 \end{vmatrix} = x(3 - 4) - y(-1 - 2) + (-4 - 6) = -x + 3y - 10$

The equation of the line is $-x + 3y - 10 = 0$.

The equation of the line in slope-intercept form is $y = \frac{1}{3}x + \frac{10}{3}$.

59. – 67. Answers may vary.

68. Input the matrix as [*A*], then use det[*A*] to find the determinant.

$$\begin{vmatrix} 3 & -2 & -1 & 4 \\ -5 & 1 & 2 & 7 \\ 2 & 4 & 5 & 0 \\ -1 & 3 & -6 & 5 \end{vmatrix} = -2100$$

69. Input the matrix as [*A*], then use det[*A*] to find the determinant.

$$\begin{vmatrix} 8 & 2 & 6 & -1 & 0 \\ 2 & 0 & -3 & 4 & 7 \\ 2 & 1 & -3 & 6 & -5 \\ -1 & 2 & 1 & 5 & -1 \\ 4 & 5 & -2 & 3 & -8 \end{vmatrix} = 13,200$$

70. Gauss-Jordan elimination.

71. does not make sense; Explanations will vary. Sample explanation: Determinants must be square.

72. does not make sense; Explanations will vary. Sample explanation: D does not necessarily equal D_x.

73. does not make sense; Explanations will vary. Sample explanation: The number of determinants needed is one greater than the number of variables.

74. does not make sense; Explanations will vary. Sample explanation: Linear systems with $D = 0$ are inconsistent if at least one of determinants in the numerator is not zero. If a linear system with $D = 0$ has all determinants in the numerator also equal to zero, then the system has infinitely many solutions.

75. In this exercise, expansions are all done about the first column of the matrix and the resulting products of 0 and a determinant are not shown.

a. $$\begin{vmatrix} a & a \\ 0 & a \end{vmatrix} = a^2 - 0 = a^2$$

b. $$\begin{vmatrix} a & a & a \\ 0 & a & a \\ 0 & 0 & a \end{vmatrix} = a\begin{vmatrix} a & a \\ 0 & a \end{vmatrix} - 0 + 0$$
$$= a\left(a^2\right) = a^3$$

c. $$\begin{vmatrix} a & a & a & a \\ 0 & a & a & a \\ 0 & 0 & a & a \\ 0 & 0 & 0 & a \end{vmatrix} = a\begin{vmatrix} a & a & a \\ 0 & a & a \\ 0 & 0 & a \end{vmatrix} - 0 + 0 - 0$$
$$= a\left(a^3\right) = a^4$$

d. Each determinant has zeros below the main diagonal and *a*'s everywhere else.

e. Each determinant equals *a* raised to the power equal to the order of the determinant.

76. $\begin{vmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{vmatrix} = 2\begin{vmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix}$

$= 2(3)\begin{vmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{vmatrix}$

$= 6(2)\begin{vmatrix} 1 & 0 \\ 0 & 4 \end{vmatrix}$

$= 12(1 \cdot 4 - 0 \cdot 0)$

$= 12(4)$

$= 48$

77. The sign of the value is changed when 2 columns are interchanged in a 2nd order determinant.

78. $a_1 x + b_1 y = c_1$

$a_2 x + b_2 y = c_2$

$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \; D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} \; D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

$x = \dfrac{D_x}{D} = \dfrac{a_1 c_2 - a_2 b_1}{a_1 b_2 - a_2 b_1}$

$y = \dfrac{D_y}{D} = \dfrac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1}$

$(a_1 + a_2)x + (b_1 + b_2)x = c_1 + c_2$

$a_2 x + b_2 y = c_2$

$D = \begin{vmatrix} a_1 + a_2 & b_1 + b_2 \\ a_2 & b_2 \end{vmatrix} = b_2(a_1 + a_2) - a_2(b_1 + b_2)$

$D_x = \begin{vmatrix} c_1 + c_2 & b_1 + b_2 \\ c_2 & b_2 \end{vmatrix} = b_2(c_1 + c_2) - c_2(b_1 + b_2)$

$D_y = \begin{vmatrix} a_1 + a_2 & c_1 + c_2 \\ a_2 & c_2 \end{vmatrix} = c_2(a_1 + a_2) - a_2(c_1 + c_2)$

$x = \dfrac{b_2(c_1 + c_2) - c_2(b_1 + b_2)}{b_2(a_1 + a_2) - a_2(b_1 + b_2)}$

$= \dfrac{b_2 c_1 + b_2 c_2 - c_2 b_1 - c_2 b_2}{b_2 a_1 + b_2 a_2 - a_2 b_1 - a_2 b_2}$

$= \dfrac{b_2 c_1 - c_2 b_1}{b_2 a_1 - a_2 b_1}$

$= \dfrac{c_1 b_2 - b_1 c_2}{a_1 b_2 - a_2 b_1}$

$y = \dfrac{c_2(a_1 + a_2) - a_2(c_1 + c_2)}{b_2(a_1 + a_2) - a_2(b_1 + b_2)}$

$= \dfrac{c_2 a_1 + c_2 a_2 - a_2 c_1 - a_2 c_2}{b_2 a_1 + b_2 a_2 - a_2 b_1 - a_2 b_2}$

$= \dfrac{c_2 a_1 - a_2 c_1}{b_2 a_1 - a_2 b_1}$

For both systems, $x = \dfrac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1}$ and

$y = \dfrac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1}$.

79. Evaluate the determinate and write the equation in slope intercept form.

$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

$x\begin{vmatrix} y_1 & 1 \\ y_2 & 1 \end{vmatrix} - y\begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix} + 1\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = 0$

$x(y_1 - y_2) - y(x_1 - x_2) + x_1 y_2 - x_2 y_1 = 0$

$-y(x_1 - x_2) = -x(y_1 - y_2) + x_2 y_1 - x_1 y_2$

$y(x_2 - x_1) = x(y_2 - y_1) + x_2 y_1 - x_1 y_2$

$y = \dfrac{y_2 - y_1}{x_2 - x_1}x + \dfrac{x_2 y_1 - x_1 y_2}{x_2 - x_1}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} \qquad b = \dfrac{x_2 y_1 - x_1 y_2}{x_2 - x_1}$

Write the slope-point equation of the line the in point slope form.

$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}x + \dfrac{-x_1 y_2 + x_1 y_1}{x_2 - x_1}$

$y = \dfrac{y_2 - y_1}{x_2 - x_1}x + \dfrac{-x_1 y_2 + x_1 y_1}{x_2 - x_1} + y_1$

$y = \dfrac{y_2 - y_1}{x_2 - x_1}x + \dfrac{-x_1 y_2 + x_1 y_1}{x_2 - x_1} + \dfrac{x_2 y_1 - x_1 y_1}{x_2 - x_1}$

$y = \dfrac{y_2 - y_1}{x_2 - x_1}x + \dfrac{x_2 y_1 - x_1 y_2}{x_2 - x_1}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} \qquad b = \dfrac{x_2 y_1 - x_1 y_2}{x_2 - x_1}$

Since both forms give the same slope and *y*-intercept, the determinant does give the equation of the line.

80. Answers may vary.

81. **a.** $\frac{x^2}{9}+\frac{y^2}{4}=1$

$\frac{x^2}{9}+\frac{0^2}{4}=1$

$\frac{x^2}{9}=1$

$x^2=9$

$x=\pm 3$

b. $\frac{x^2}{9}+\frac{y^2}{4}=1$

$\frac{0^2}{9}+\frac{y^2}{4}=1$

$\frac{y^2}{4}=1$

$y^2=4$

$y=\pm 2$

82. $25x^2+16y^2=400$

$\frac{25x^2}{400}+\frac{16y^2}{400}=\frac{400}{400}$

$\frac{x^2}{16}+\frac{y^2}{25}=1$

83. $x^2+y^2-2x+4y=4$

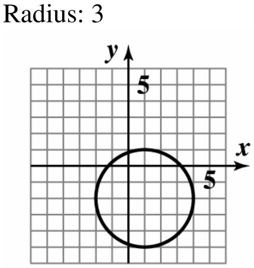
$x^2-2x \quad +y^2+4y \quad =4$

$x^2-2x+1+y^2+4y+4=4+1+4$

$(x-1)^2+(y+2)^2=9$

$(x-1)^2+(y+2)^2=3^2$

Center: (1, –2)
Radius: 3



$x^2+y^2-2x+4y=4$

Chapter 9 Review Exercises

1. $\left[\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & 1 & -1 & 2 \\ 0 & 5 & 4 & 1 \end{array}\right] -5R_2+R_3$

$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 9 & -9 \end{array}\right]$

2. $\left[\begin{array}{ccc|c} 2 & -2 & 1 & -1 \\ 1 & 2 & -1 & 2 \\ 6 & 4 & 3 & 5 \end{array}\right] \frac{1}{2}R_1$

$\left[\begin{array}{ccc|c} 1 & -1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 2 & -1 & 2 \\ 6 & 4 & 3 & 5 \end{array}\right]$

3. $\left[\begin{array}{ccc|c} 1 & 2 & 3 & -5 \\ 2 & 1 & 1 & 1 \\ 1 & 1 & -1 & 8 \end{array}\right] \begin{array}{l} -2R_1+R_2 \\ -1R_1+R_3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -5 \\ 0 & -3 & -5 & 11 \\ 0 & -1 & -4 & 13 \end{array}\right] R_2 \leftrightarrow R_3$

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -5 \\ 0 & -1 & -4 & 13 \\ 0 & -3 & -5 & 11 \end{array}\right] -1R_2$

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -5 \\ 0 & 1 & 4 & -13 \\ 0 & -3 & -5 & 11 \end{array}\right] 3R_2+R_3$

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -5 \\ 0 & 1 & 4 & -13 \\ 0 & 0 & 7 & -28 \end{array}\right] \frac{1}{7}R_3$

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -5 \\ 0 & 1 & 4 & -13 \\ 0 & 0 & 1 & -4 \end{array}\right] -2R_2+R_1$

$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 21 \\ 0 & 1 & 4 & -13 \\ 0 & 0 & 1 & -4 \end{array}\right] \begin{array}{l} 5R_3+R_1 \\ -4R_3+R_2 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -4 \end{array}\right]$

The solution set is {(1, 3, –4)}.

4. $\left[\begin{array}{rrr|r} 1 & -2 & 1 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 2 & 5 & -2 \end{array}\right] -2R_2 + R_3$

$\left[\begin{array}{rrrr} 1 & -2 & 1 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 11 & 0 \end{array}\right] \frac{1}{11}R_3$

$\left[\begin{array}{rrrr} 1 & -2 & 1 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right] 2R_2 + R_1$

$\left[\begin{array}{rrrr} 1 & 0 & -5 & -2 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \\ 3R_3 + R_2 \\ 5R_3 + R_1 \end{array}$

$\left[\begin{array}{rrrr} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$x = -2$; $y = -1$; $z = 0$

The solution set is $\{(-2, -1, 0)\}$.

5. $\left[\begin{array}{rrrr|r} 3 & 5 & -8 & 5 & -8 \\ 1 & 2 & -3 & 1 & -7 \\ 2 & 3 & -7 & 3 & -11 \\ 4 & 8 & -10 & 7 & -10 \end{array}\right] R_1 \leftrightarrow R_2$

$\left[\begin{array}{rrrr|r} 1 & 2 & -3 & 1 & -7 \\ 3 & 5 & -8 & 5 & -8 \\ 2 & 3 & -7 & 3 & -11 \\ 4 & 8 & -10 & 7 & -10 \end{array}\right] \begin{array}{l} -3R_1 + R_2 \\ -2R_1 + R_3 \\ -4R_1 + R_4 \end{array}$

$\left[\begin{array}{rrrr|r} 1 & 2 & -3 & 1 & -7 \\ 0 & -1 & 1 & 2 & 13 \\ 0 & -1 & -1 & 1 & 3 \\ 0 & 0 & 2 & 3 & 18 \end{array}\right] -1R_2$

$\left[\begin{array}{rrrr|r} 1 & 2 & -3 & 1 & -7 \\ 0 & 1 & -1 & -2 & -13 \\ 0 & -1 & -1 & 1 & 3 \\ 0 & 0 & 2 & 3 & 18 \end{array}\right] \begin{array}{l} -2R_2 + R_1 \\ 1R_2 + R_3 \end{array}$

$\left[\begin{array}{rrrr|r} 1 & 0 & -1 & 5 & 19 \\ 0 & 1 & -1 & -2 & -13 \\ 0 & 0 & -2 & -1 & -10 \\ 0 & 0 & 2 & 3 & 18 \end{array}\right] -\frac{1}{2}R_3$

$\left[\begin{array}{rrrr|r} 1 & 0 & -1 & 5 & 19 \\ 0 & 1 & -1 & -2 & -13 \\ 0 & 0 & 1 & \frac{1}{2} & 5 \\ 0 & 0 & 2 & 3 & 18 \end{array}\right] \begin{array}{l} 1R_3 + R_1 \\ 1R_3 + R_2 \\ -2R_3 + R_4 \end{array}$

$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & \frac{11}{2} & 24 \\ 0 & 1 & 0 & -\frac{3}{2} & -8 \\ 0 & 0 & 1 & \frac{1}{2} & 5 \\ 0 & 0 & 0 & 2 & 8 \end{array}\right] \frac{1}{2}R_4$

$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & \frac{11}{2} & 24 \\ 0 & 1 & 0 & -\frac{3}{2} & -8 \\ 0 & 0 & 1 & \frac{1}{2} & 5 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \begin{array}{l} -\frac{11}{2}R_4 + R_1 \\ \frac{3}{2}R_4 + R_2 \\ -\frac{1}{2}R_4 + R_3 \end{array}$

$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$

The solution set is $\{(2, -2, 3, 4)\}$.

6. **a.** The function must satisfy:

$98 = 4a = 2b + c$

$138 = 16a + 4b + c$

$162 = 100a + 10b + c.$

$\left[\begin{array}{rrr|r} 4 & 2 & 1 & 98 \\ 16 & 4 & 1 & 138 \\ 100 & 10 & 1 & 162 \end{array}\right] \frac{1}{4}R_1$

$\left[\begin{array}{rrr|r} 1 & \frac{1}{2} & \frac{1}{4} & \frac{49}{2} \\ 16 & 4 & 1 & 138 \\ 100 & 10 & 1 & 162 \end{array}\right] \begin{array}{l} -16R_1 + R_2 \\ -100R_1 + R_3 \end{array}$

$\left[\begin{array}{rrr|r} 1 & \frac{1}{2} & \frac{1}{4} & \frac{49}{2} \\ 0 & -4 & -3 & -254 \\ 0 & -40 & -24 & -2288 \end{array}\right] -\frac{1}{4}R_2$

$\left[\begin{array}{rrr|r} 1 & \frac{1}{2} & \frac{1}{4} & \frac{49}{2} \\ 0 & 1 & \frac{3}{4} & \frac{127}{2} \\ 0 & -40 & -24 & -2288 \end{array}\right] 40R_2 + R_3$

$\left[\begin{array}{rrr|r} 1 & \frac{1}{2} & \frac{1}{4} & \frac{49}{2} \\ 0 & 1 & \frac{3}{4} & \frac{127}{2} \\ 0 & 0 & 6 & 252 \end{array}\right] \frac{1}{6}R_3$

$\left[\begin{array}{rrr|r} 1 & \frac{1}{2} & \frac{1}{4} & \frac{49}{2} \\ 0 & 1 & \frac{3}{4} & \frac{127}{2} \\ 0 & 0 & 1 & 42 \end{array}\right] \begin{array}{l} -\frac{1}{4}R_3 + R_1 \\ -\frac{3}{4}R_3 + R_2 \end{array}$

$\left[\begin{array}{rrr|r} 1 & \frac{1}{2} & 0 & 14 \\ 0 & 1 & 0 & 32 \\ 0 & 0 & 1 & 42 \end{array}\right] -\frac{1}{2}R_3 + R_1$

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 32 \\ 0 & 0 & 1 & 42 \end{array}\right]$

The function is $y = -2x^2 + 32x + 42$ and $a = -2$, $b = 32$ and $c = 42$.

b. $y = -2x^2 + 32x + 42$ is a parabola.

The maximum occurs when

$$x = \frac{-32}{2(-2)} = \frac{-32}{-4} = 8.$$

The air pollution level is a maximum 8 hours after 6 A.M., which is 2 P.M.

When $x = 8$, $y = -2(64) + 32(8) + 42$
$= -128 + 256 + 42.$
$= 170.$

The maximum level is 170 parts per million at 2 P.M.

7. Write the equations.

$$w + x + y + z = 80$$
$$y - w - x = 18$$
$$y - z = 4$$
$$3x - w - y = 10$$

Rewrite the equations with terms in consistent order.

$$w + x + y + z = 80$$
$$-w - x + y = 18$$
$$y - z = 4$$
$$-w + 3x - y = 10$$

Write as a matrix and solve.

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 80 \\ -1 & -1 & 1 & 0 & 18 \\ 0 & 0 & 1 & -1 & 4 \\ -1 & 3 & -1 & 0 & 10 \end{array}\right] \begin{array}{l} \\ R_1 + R_2 \\ \\ R_1 + R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 80 \\ 0 & 0 & 2 & 1 & 98 \\ 0 & 0 & 1 & -1 & 4 \\ 0 & 4 & 0 & 1 & 90 \end{array}\right] R_2 \leftrightarrow R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 80 \\ 0 & 4 & 0 & 1 & 90 \\ 0 & 0 & 1 & -1 & 4 \\ 0 & 0 & 2 & 1 & 98 \end{array}\right] \frac{1}{4}R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 80 \\ 0 & 1 & 0 & \frac{1}{4} & \frac{45}{2} \\ 0 & 0 & 1 & -1 & 4 \\ 0 & 0 & 2 & 1 & 98 \end{array}\right] -2R_3 + R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 80 \\ 0 & 1 & 0 & \frac{1}{4} & \frac{45}{2} \\ 0 & 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & 3 & 90 \end{array}\right] \frac{1}{3}R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 80 \\ 0 & 1 & 0 & \frac{1}{4} & \frac{45}{2} \\ 0 & 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & 1 & 30 \end{array}\right]$$

Back-substitute $z = 30$ to find y.

$$y - z = 4$$
$$y - 30 = 4$$
$$y = 34$$

Back-substitute to find x.

$$x + \frac{1}{4}z = \frac{45}{2}$$
$$x + \frac{1}{4}(30) = \frac{45}{2}$$
$$x + \frac{15}{2} = \frac{45}{2}$$
$$x = 15$$

Back-substitute to find x.

$$w + x + y + z = 80$$
$$w + 15 + 34 + 30 = 80$$
$$w + 79 = 80$$
$$w = 1$$

Capitalist: 1%
Upper Middle: 15%
Lower Middle: 34%
Working: 30%

8. $\left[\begin{array}{ccc|c} 2 & -3 & 1 & 1 \\ 1 & -2 & 3 & 2 \\ 3 & -4 & -1 & 1 \end{array}\right] R_1 \leftrightarrow R_2$

$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 2 & -3 & 1 & 1 \\ 3 & -4 & -1 & 1 \end{array}\right] \begin{matrix} -2R_1 + R_2 \\ -3R_1 + R_3 \end{matrix}$

$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 1 & -5 & -3 \\ 0 & 2 & -10 & -5 \end{array}\right] -2R_2 + R_3$

$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 1 & -5 & -3 \\ 0 & 0 & 0 & 1 \end{array}\right]$

From the last line, we see that the system has no solution. Thus, the solution set is $\varnothing$.

9. $\left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ -2 & 1 & 3 & -7 \\ 1 & -4 & 2 & 0 \end{array}\right] \begin{matrix} 2R_1 + R_2 \\ -1R_1 + R_3 \end{matrix}$

$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & -5 & 5 & -5 \\ 0 & -1 & 1 & -1 \end{array}\right] -\frac{1}{5}R_2$

$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & -1 & 1 & -1 \end{array}\right] 1R_2 + R_3$

$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

The system $\begin{aligned} x - 3y + z &= 1 \\ y - z &= 1 \end{aligned}$ has no unique solution.

Express x and y in terms of z:

$y = z + 1$

$x - 3(z + 1) + z = 1$

$x - 3z - 3 + z = 1$

$x = 2z + 4$

With $z = t$, the complete solution to the system is $\{(2t + 4, t + 1, t)\}$.

10. $\left[\begin{array}{cccc|c} 1 & 4 & 3 & -6 & 5 \\ 1 & 3 & 1 & -4 & 3 \\ 2 & 8 & 7 & -5 & 11 \\ 2 & 5 & 0 & -6 & 4 \end{array}\right] \begin{matrix} -1R_1 + R_2 \\ -2R_1 + R_3 \\ -2R_1 + R_4 \end{matrix}$

$\left[\begin{array}{cccc|c} 1 & 4 & 3 & -6 & 5 \\ 0 & -1 & -2 & 2 & -2 \\ 0 & 0 & 1 & 7 & 1 \\ 0 & -3 & -6 & 6 & -6 \end{array}\right] -1R_2$

$\left[\begin{array}{cccc|c} 1 & 4 & 3 & -6 & 5 \\ 0 & 1 & 2 & -2 & 2 \\ 0 & 0 & 1 & 7 & 1 \\ 0 & -3 & -6 & 6 & -6 \end{array}\right] 3R_2 + R_4$

$\left[\begin{array}{cccc|c} 1 & 4 & 3 & -6 & 5 \\ 0 & 1 & 2 & -2 & 2 \\ 0 & 0 & 1 & 7 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

The system $\begin{aligned} x_1 + 4x_2 + 3x_3 - 6x_4 &= 5 \\ x_2 + 2x_3 - 2x_4 &= 2 \\ x_3 + 7x_4 &= 1 \end{aligned}$

does not have a unique solution.

Express x_1, x_2, and x_3 in terms of x_4:

$x_3 = -7x_4 + 1$

$x_2 + 2(-7x_4 + 1) - 2x_4 = 2$

$x_2 - 14x_4 + 2 - 2x_4 = 2$

$x_2 = 16x_4$

$x_1 + 4(16x_4) + 3(-7x_4 + 1) - 6x_4 = 5$

$x_1 + 64x_4 - 21x_4 + 3 - 6x_4 = 5$

$x_1 = -37x_4 + 2$

With $x_4 = t$, the complete solution to the system is $\{(-37t + 2, 16t, -7t + 1, t)\}$.

11. $\left[\begin{array}{ccc|c} 2 & 3 & -5 & 15 \\ 1 & 2 & -1 & 4 \end{array}\right] R_1 \leftrightarrow R_2$

$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 2 & 3 & -5 & 15 \end{array}\right] -2R_1 + R_2$

$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & -1 & -3 & 7 \end{array}\right] -1R_2$

$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 1 & 3 & -7 \end{array}\right]$

The system $\begin{array}{r} x+2y-z=4 \\ y+3z=-7 \end{array}$ has no unique solution.

Express x and y in terms of z:

$y = -3z - 7$

$x + 2(-3z - 7) - z = 4$

$x - 6z - 14 - z = 4$

$x = 7z + 18$

With $z = t$, the complete solution to the system is $\{(7t + 18, -3t - 7, t)\}$.

12. a. $350 + 400 = x + z$

$450 + z = y + 700$

$x + y = 300 + 200$

or

$x + z = 750$

$y - z = -250$

$x + y = 500$

b. $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 750 \\ 0 & 1 & -1 & -250 \\ 1 & 1 & 0 & 500 \end{array}\right] -1R_1 + R_3$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 750 \\ 0 & 1 & -1 & -250 \\ 0 & 1 & -1 & -250 \end{array}\right] -1R_2 + R_3$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 750 \\ 0 & 1 & -1 & -250 \\ 0 & 0 & 0 & 0 \end{array}\right]$

The system $\begin{array}{r} x+z=750 \\ y-z=-250 \end{array}$ has no unique solution.

Express x and y in terms of z:

$y = z - 250$

$x = -z + 750$

With $z = t$, the complete solution to the system is $\{(-t + 750, t - 250, t)\}$.

c. $x = -400 + 750 = 350$

$y = 400 - 250 = 150$

13. $2x = -10$

$x = -5$

$y + 7 = 13$

$y = 6$

$z = 6$

$x = -5$; $y = 6$; $z = 6$

14. $A + D = \begin{bmatrix} 2-2 & -1+3 & 2+1 \\ 5+3 & 3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 3 \\ 8 & 1 & 3 \end{bmatrix}$

15. $2B = \begin{bmatrix} 2(0) & 2(-2) \\ 2(3) & 2(2) \\ 2(1) & 2(-5) \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 6 & 4 \\ 2 & -10 \end{bmatrix}$

16. $D - A = \begin{bmatrix} -2-2 & 3+1 & 1-2 \\ 3-5 & -2-3 & 4+1 \end{bmatrix}$

$= \begin{bmatrix} -4 & 4 & -1 \\ -2 & -5 & 5 \end{bmatrix}$

17. Not possible since B is 3×2 and C is 3×3.

18. $3A + 2D = \begin{bmatrix} 6 & -3 & 6 \\ 15 & 9 & -3 \end{bmatrix} + \begin{bmatrix} -4 & 6 & 2 \\ 6 & -4 & 8 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 & 8 \\ 21 & 5 & 5 \end{bmatrix}$

19.

$-2A + 4D = \begin{bmatrix} -4 & 2 & -4 \\ -10 & -6 & 2 \end{bmatrix} + \begin{bmatrix} -8 & 12 & 4 \\ 12 & -8 & 16 \end{bmatrix}$

$= \begin{bmatrix} -12 & 14 & 0 \\ 2 & -14 & 18 \end{bmatrix}$

20. $-5(A + D) = -5\left(\begin{bmatrix} 0 & 2 & 3 \\ 8 & 1 & 3 \end{bmatrix}\right) = \begin{bmatrix} 0 & -10 & -15 \\ -40 & -5 & -15 \end{bmatrix}$

21. $AB = \begin{bmatrix} 0-3+2 & -4-2-10 \\ 0+9-1 & -10+6+5 \end{bmatrix} = \begin{bmatrix} -1 & -16 \\ 8 & 1 \end{bmatrix}$

22. $BA = \begin{bmatrix} 0-10 & 0-6 & 0+2 \\ 6+10 & -3+6 & 6-2 \\ 2-25 & -1-15 & 2+5 \end{bmatrix} = \begin{bmatrix} -10 & -6 & 2 \\ 16 & 3 & 4 \\ -23 & -16 & 7 \end{bmatrix}$

23. $BD = \begin{bmatrix} 0-6 & 0+4 & 0-8 \\ -6+6 & 9-4 & 3+8 \\ -2-15 & 3+10 & 1-20 \end{bmatrix} = \begin{bmatrix} -6 & 4 & -8 \\ 0 & 5 & 11 \\ -17 & 13 & -19 \end{bmatrix}$

24. $DB = \begin{bmatrix} 0+9+1 & 4+6-5 \\ 0-6+4 & -6-4-20 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ -2 & -30 \end{bmatrix}$

25. Not possible since *AB* is 2 x 2 and *BA* is 3 x 3.

26.

$$(A-D)C = \begin{bmatrix} 4 & -4 & 1 \\ 2 & 5 & -5 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4-1 & 8-4+2 & 12-8+1 \\ 2-5+5 & 4+5-10 & 6+10-5 \end{bmatrix} = \begin{bmatrix} 7 & 6 & 5 \\ 2 & -1 & 11 \end{bmatrix}$$

27. $B(AC) = \begin{bmatrix} 0 & -2 \\ 3 & 2 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 2+1-2 & 4-1+4 & 6-2+2 \\ 5-3+1 & 10+3-2 & 15+6-1 \end{bmatrix}$

$$= \begin{bmatrix} 0 & -2 \\ 3 & 2 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 1 & 7 & 6 \\ 3 & 11 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 0-6 & 0-22 & 0-40 \\ 3+6 & 21+22 & 18+40 \\ 1-15 & 7-55 & 6-100 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -22 & -40 \\ 9 & 43 & 58 \\ -14 & -48 & -94 \end{bmatrix}$$

28. $3X + A = B$

$3X = B - A$

$X = \frac{1}{3}(B - A)$

$X = \frac{1}{3}\left(\begin{bmatrix} -2 & -12 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -5 & 0 \end{bmatrix}\right)$

$X = \frac{1}{3}\begin{bmatrix} -6 & -18 \\ 9 & 1 \end{bmatrix}$

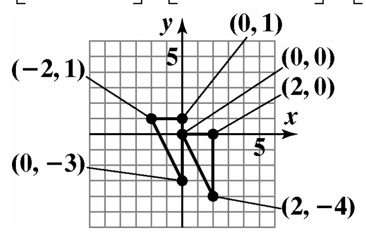
$X = \begin{bmatrix} -2 & -6 \\ 3 & \frac{1}{3} \end{bmatrix}$

29. $\begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

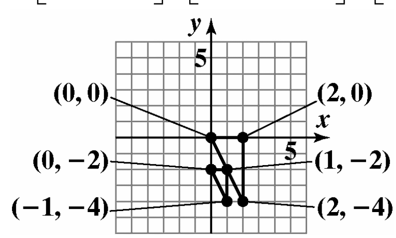
30. $\begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 3 \\ 0 & 3 & 0 \\ 0 & 3 & 0 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 3 & 3 \\ 0 & 3 & 0 \\ 0 & 3 & 0 \end{bmatrix}$

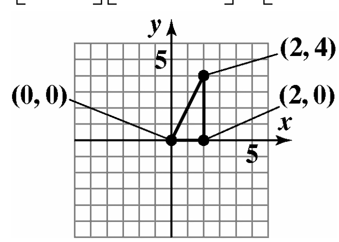
31. $\begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -4 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 1 & -3 \end{bmatrix}$



32. $\frac{1}{2}\begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -2 & -2 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -1 & -1 \\ -2 & -2 & -4 \end{bmatrix}$

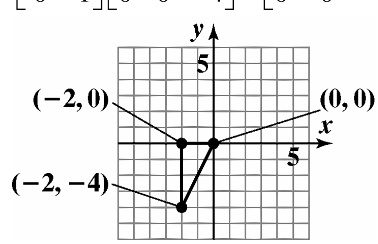


33. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & 4 \end{bmatrix}$



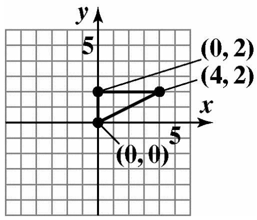
The triangle is reflected about the *x*-axis.

34. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -2 \\ 0 & 0 & -4 \end{bmatrix}$



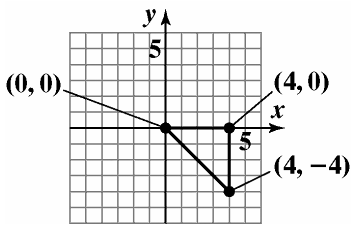
The triangle is reflected about the *y*-axis.

35. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -4 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 4 \\ 0 & 2 & 2 \end{bmatrix}$



The triangle is rotated 90° counterclockwise about the origin.

36. $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -4 \end{bmatrix}=\begin{bmatrix} 0 & 4 & 4 \\ 0 & 0 & -4 \end{bmatrix}$



The triangle is stretched by a factor of 2 horizontally.

37. $AB=\begin{bmatrix} 8-7 & -14+21 \\ 4-4 & -7+12 \end{bmatrix}=\begin{bmatrix} 1 & 7 \\ 0 & 5 \end{bmatrix}$

$BA=\begin{bmatrix} 8-7 & 28-28 \\ -2+3 & -7+12 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 1 & 5 \end{bmatrix}$

If *B* is the multiplicative inverse of *A,* both products (*AB* and *BA*) will be the multiplicative identity matrix, I_2. Therefore, *B* is not the multiplicative inverse of *A*.

38. $AB=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -7 \\ 0 & -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 7 \\ 0 & 1 & 2 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$BA=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 7 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -7 \\ 0 & -1 & 4 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

If *B* is the multiplicative inverse of *A,* both products (*AB* and *BA*) will be the multiplicative identity matrix, I_3. Therefore, *B* is the multiplicative inverse of *A*.

39. $A^{-1}=\frac{1}{3-2}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}=\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$AA^{-1}=\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

$=\begin{bmatrix} 3-2 & 1-1 \\ -6+6 & -2+3 \end{bmatrix}$

$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{-1}A=\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$

$=\begin{bmatrix} 3-2 & 3+3 \\ 2-2 & -2+3 \end{bmatrix}$

$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

40. $A^{-1}=\frac{1}{0-5}\begin{bmatrix} 3 & -1 \\ -5 & 0 \end{bmatrix}$

$=\frac{-1}{5}\begin{bmatrix} 3 & -1 \\ -5 & 0 \end{bmatrix}$

$=\begin{bmatrix} -\frac{3}{5} & \frac{1}{5} \\ 1 & 0 \end{bmatrix}$

$AA^{-1}=\begin{bmatrix} 0 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} -\frac{3}{5} & \frac{1}{5} \\ 1 & 0 \end{bmatrix}$

$=\begin{bmatrix} 0+1 & 0+0 \\ -3+3 & 1+0 \end{bmatrix}$

$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{-1}A=\begin{bmatrix} -\frac{3}{5} & \frac{1}{5} \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 5 & 3 \end{bmatrix}$

$=\begin{bmatrix} 0+1 & -\frac{3}{5}+\frac{3}{5} \\ 0+0 & 1+0 \end{bmatrix}$

$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

41. $\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & -3 & 0 & 0 & 1 \end{array}\right] \begin{matrix} -2R_1 + R_2 \\ -1R_1 + R_3 \end{matrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 4 & -2 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{array}\right] -1R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 4 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right] \begin{matrix} 2R_3 + R_1 \\ -4R_3 + R_2 \end{matrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 0 & -2 \\ 0 & 1 & 0 & -6 & 1 & 4 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & 0 & -2 \\ -6 & 1 & 4 \\ 1 & 0 & -1 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 0 \\ 1 & 0 & -3 \end{bmatrix} \begin{bmatrix} 3 & 0 & -2 \\ -6 & 1 & 4 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0-2 & 0+0+0 & -2+0+2 \\ 6-6+0 & 0+1+0 & -4+4+0 \\ 3+0-3 & 0+0+0 & -2+0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} 3 & 0 & -2 \\ -6 & 1 & 4 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 0 \\ 1 & 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0-2 & 0+0+0 & -6+0+6 \\ -6+2+4 & 0+1+0 & 12+0-12 \\ 1+0-1 & 0+0+0 & -2+0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

42. $\left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 4 & 13 & -7 & 0 & 1 & 0 \\ 5 & 16 & -8 & 0 & 0 & 1 \end{array}\right] \begin{matrix} -4R_1 + R_2 \\ -5R_1 + R_3 \\ \end{matrix}$

$\left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -4 & 1 & 0 \\ 0 & 1 & 2 & -5 & 0 & 1 \end{array}\right] \begin{matrix} -1R_2 + R_3 \\ -3R_2 + R_1 \end{matrix}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & -5 & 13 & -3 & 0 \\ 0 & 1 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right] \begin{matrix} -1R_3 + R_2 \\ 5R_3 + R_1 \end{matrix}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & -8 & 5 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right]$

$$A^{-1} = \begin{bmatrix} 8 & -8 & 5 \\ -3 & 2 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 3 & -2 \\ 4 & 13 & -7 \\ 5 & 16 & -8 \end{bmatrix} \begin{bmatrix} 8 & -8 & 5 \\ -3 & 2 & -1 \\ -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 8-9+2 & -8+6+2 & 5-3-2 \\ 32-39+7 & -32+26+7 & 20-13-7 \\ 40-48+8 & -40+32+8 & 25-16-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} 8 & -8 & 5 \\ -3 & 2 & -1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -2 \\ 4 & 13 & -7 \\ 5 & 16 & -8 \end{bmatrix} = \begin{bmatrix} 8-32+25 & 24-104+80 & -16+56-40 \\ -3+8-5 & -9+26-16 & 6-14+8 \\ -1-4+5 & -3-13+16 & 2+7-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

43. a. $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 3 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 0 \end{bmatrix}.$

b. $A^{-1}B = \begin{bmatrix} -2 & 2 & 1 \\ 9 & -8 & -3 \\ -3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -14-4+0 \\ 63+16+0 \\ -21-6+0 \end{bmatrix} = \begin{bmatrix} -18 \\ 79 \\ -27 \end{bmatrix}$

The solution to the system is {(–18, 79, –27)}.

44. a. $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -5 \\ 10 \end{bmatrix}$

b. $A^{-1}B = \begin{bmatrix} 2 & 2 & -1 \\ -1 & 0 & 1 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ -5 \\ 10 \end{bmatrix} = \begin{bmatrix} 24-10-10 \\ -12+10 \\ -12+5+10 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$

The solution to the system is {(4, –2, 3)}.

45. R U L E has a numerical equivalent of 18, 21, 12, 5.

$$\begin{bmatrix}3 & 2\\4 & 3\end{bmatrix}\begin{bmatrix}18 & 12\\21 & 5\end{bmatrix}=\begin{bmatrix}54+42 & 36+10\\72+63 & 48+15\end{bmatrix}=\begin{bmatrix}96 & 46\\135 & 63\end{bmatrix}$$

The encoded message is 96, 135, 46, 63.

$$\begin{bmatrix}3 & -2\\-4 & 3\end{bmatrix}\begin{bmatrix}96 & 46\\135 & 63\end{bmatrix}=\begin{bmatrix}288-270 & 138-126\\-384+405 & -184+189\end{bmatrix}=\begin{bmatrix}18 & 12\\21 & 5\end{bmatrix}$$

The decoded message is 18, 21, 12, 5 or RULE.

46. $\begin{vmatrix}3 & 2\\-1 & 5\end{vmatrix}=15-(-2)=17$

47. $\begin{vmatrix}-2 & -3\\-4 & -8\end{vmatrix}=16-12=4$

48. $\begin{vmatrix}2 & 4 & -3\\1 & -1 & 5\\-2 & 4 & 0\end{vmatrix}=-2\begin{vmatrix}4 & -3\\-1 & 5\end{vmatrix}-4\begin{vmatrix}2 & -3\\1 & 5\end{vmatrix}+0\begin{vmatrix}2 & 4\\1 & -1\end{vmatrix}$

$= -2(20 - 3) - 4[10 - (-3)] + 0$
$= -2(17) - 4(13)$
$= -34 - 52$
$= -86$

49. $\begin{vmatrix}4 & 7 & 0\\-5 & 6 & 0\\3 & 2 & -4\end{vmatrix}=4\begin{vmatrix}6 & 0\\2 & -4\end{vmatrix}+5\begin{vmatrix}7 & 0\\2 & -4\end{vmatrix}+3\begin{vmatrix}7 & 0\\6 & 0\end{vmatrix}$

$= 4(-24-0)+5(-28-0)+3(0-0)$
$= 4(-24)+5(-28)+0$
$= -236$

50. $\begin{vmatrix}1 & 1 & 0 & 2\\0 & 3 & 2 & 1\\0 & -2 & 4 & 0\\0 & 3 & 0 & 1\end{vmatrix}=\begin{vmatrix}3 & 2 & 1\\-2 & 4 & 0\\3 & 0 & 1\end{vmatrix}$

$= 3\begin{vmatrix}2 & 1\\4 & 0\end{vmatrix}+\begin{vmatrix}3 & 2\\-2 & 4\end{vmatrix}$

$= 3(0-4)+[12-(-4)]$
$= 3(-4) + 16$
$= -12 + 16$
$= 4$

51. $\begin{vmatrix} 2 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & 2 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{vmatrix}$

$= 2(2)\begin{vmatrix} 2 & 2 \\ 0 & 2 \end{vmatrix}$

$= 2(2)(4)$

$= 16$

52. $D = \begin{vmatrix} 1 & -2 \\ 3 & 2 \end{vmatrix} = 2-(-6) = 2+6 = 8$

$D_x = \begin{vmatrix} 8 & -2 \\ -1 & 2 \end{vmatrix} = 16-2 = 14$

$D_y = \begin{vmatrix} 1 & 8 \\ 3 & -1 \end{vmatrix} = -1-24 = -25$

$x = \frac{D_x}{D} = \frac{14}{8} = \frac{7}{4},\ y = \frac{D_y}{D} = \frac{-25}{8} = -\frac{25}{8}$

The solution to the system is $\left\{\left(\frac{7}{4}, -\frac{25}{8}\right)\right\}$.

53. $D = \begin{vmatrix} 7 & 2 \\ 2 & 1 \end{vmatrix} = 7-4 = 3$

$D = \begin{vmatrix} 7 & 2 \\ 2 & 1 \end{vmatrix} = 7-4 = 3$

$D_x = \begin{vmatrix} 0 & 2 \\ -3 & 1 \end{vmatrix} = 0-(-6) = 6$

$D_y = \begin{vmatrix} 7 & 0 \\ 2 & -3 \end{vmatrix} = -21-0 = -21$

$x = \frac{D_x}{D} = \frac{6}{3} = 2$

$y = \frac{D_y}{D} = \frac{-21}{3} = -7$

The solution to the system is {(2, –7)}.

54. $D = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 4 & 7 \\ -2 & -5 & -2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 2 & 2 \\ 0 & 0 & 3 \\ 0 & -1 & 2 \end{vmatrix}$

$= \begin{vmatrix} 0 & 3 \\ -1 & 2 \end{vmatrix}$

$= 0-(-3)$

$= 3$

$D_x = \begin{vmatrix} 5 & 2 & 2 \\ 19 & 4 & 7 \\ 8 & -5 & -2 \end{vmatrix}$

$= 5\begin{vmatrix} 4 & 7 \\ -5 & -2 \end{vmatrix} - 2\begin{vmatrix} 19 & 7 \\ 8 & -2 \end{vmatrix} + 2\begin{vmatrix} 19 & 4 \\ 8 & -5 \end{vmatrix}$

$= 5[-8-(-35)] - 2(-38-56) + 2(-95-32)$

$= 5(27) - 2(-94) - 2(127)$

$= 135 + 188 - 254$

$= 69$

$D_y = \begin{vmatrix} 1 & 5 & 2 \\ 2 & 19 & 7 \\ -2 & 8 & -2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 5 & 2 \\ 0 & 9 & 3 \\ 0 & 18 & 2 \end{vmatrix}$

$= \begin{vmatrix} 9 & 3 \\ 18 & 2 \end{vmatrix}$

$= 18-54$

$= -36$

$D_z = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 4 & 19 \\ -2 & -5 & 8 \end{vmatrix}$

$= \begin{vmatrix} 1 & 2 & 5 \\ 0 & 0 & 9 \\ 0 & -1 & 18 \end{vmatrix}$

$= \begin{vmatrix} 0 & 9 \\ -1 & 18 \end{vmatrix}$

$= 0-(-9)$

$= 9$

$x = \frac{D_x}{D} = \frac{69}{3} = 23,\ y = \frac{D_y}{D} = \frac{-36}{3} = -12,$

$z = \frac{D_z}{D} = \frac{9}{3} = 3$

The solution to the system is {(23, –12, 3)}.

55. $D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 1 & -2 \\ 3 & 0 & -2 \end{vmatrix}$

$= 2\begin{vmatrix} 1 & -2 \\ 0 & -2 \end{vmatrix} + 3\begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix}$
$= 2(-2-0)+3(-2-0)$
$= 2(-2)+3(-2)$
$= -4-6$
$= -10$

$D_x = \begin{vmatrix} -4 & 1 & 0 \\ 0 & 1 & -2 \\ -11 & 0 & -2 \end{vmatrix}$

$= -1\begin{vmatrix} 0 & -2 \\ -11 & -2 \end{vmatrix} + 1\begin{vmatrix} -4 & 0 \\ -11 & -2 \end{vmatrix}$
$= -1(0-22)+1(8-0)$
$= 22+8$
$= 30$

$D_y = \begin{vmatrix} 2 & -4 & 0 \\ 0 & 0 & -2 \\ 3 & -11 & -2 \end{vmatrix}$

$= 2\begin{vmatrix} 0 & -2 \\ -11 & -2 \end{vmatrix} + 3\begin{vmatrix} -4 & 0 \\ 0 & -2 \end{vmatrix}$
$= 2(0-22)+3(8-0)$
$= 2(-22)+3(8)$
$= -44+24$
$= -20$

$D_z = \begin{vmatrix} 2 & 1 & -4 \\ 0 & 1 & 0 \\ 3 & 0 & -11 \end{vmatrix}$

$= 2\begin{vmatrix} 1 & 0 \\ 0 & -11 \end{vmatrix} + 3\begin{vmatrix} 1 & -4 \\ 1 & 0 \end{vmatrix}$
$= 2(-11-0)+3(0+4)$
$= 2(-11)+3(+4) = -22+12$
$= -10$

$x = \frac{D_x}{D} = \frac{30}{-10} = -3$

$y = \frac{D_y}{D} = \frac{-20}{-10} = 2$

$z = \frac{D_z}{D} = \frac{-10}{-10} = 1$

The solution to the system is $\{(-3, 2, 1)\}$.

56. The quadratic function must satisfy
$f(20) = 400 = 400a + 20b + c$
$f(40) = 150 = 1600a + 40b + c$
$f(60) = 400 = 3600a + 60b + c$

$D = \begin{vmatrix} 400 & 20 & 1 \\ 1600 & 40 & 1 \\ 3600 & 60 & 1 \end{vmatrix}$

$= (400)(20)\begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{vmatrix}$

$= 8000\begin{vmatrix} 1 & 1 & 1 \\ 3 & 1 & 0 \\ 8 & 2 & 0 \end{vmatrix} = 8000\begin{vmatrix} 3 & 1 \\ 8 & 2 \end{vmatrix}$
$= 8000(6-8)$
$= 8000(-2)$
$= -16{,}000$

$D_a = \begin{vmatrix} 400 & 20 & 1 \\ 150 & 40 & 1 \\ 400 & 60 & 1 \end{vmatrix}$

$= (50)(20)\begin{vmatrix} 8 & 1 & 1 \\ 3 & 2 & 1 \\ 8 & 3 & 1 \end{vmatrix}$

$= 1000\begin{vmatrix} 8 & 1 & 1 \\ -5 & 1 & 0 \\ 0 & 2 & 0 \end{vmatrix}$

$= 1000\begin{vmatrix} -5 & 1 \\ 0 & 2 \end{vmatrix}$
$= 1000(-10 - 0)$
$= -10{,}000$

$D_b = \begin{vmatrix} 400 & 400 & 1 \\ 1600 & 150 & 1 \\ 3600 & 400 & 1 \end{vmatrix}$

$= (400)(50)\begin{vmatrix} 1 & 8 & 1 \\ 4 & 3 & 1 \\ 9 & 8 & 1 \end{vmatrix}$

$= 20{,}000\begin{vmatrix} 1 & 8 & 1 \\ 3 & -5 & 0 \\ 8 & 0 & 0 \end{vmatrix}$

$= 20{,}000\begin{vmatrix} 3 & -5 \\ 8 & 0 \end{vmatrix}$
$= 20{,}000[0 - (-40)]$
$= 20{,}000(40)$
$= 800{,}000$

$$D_c = \begin{vmatrix} 400 & 20 & 400 \\ 1600 & 40 & 150 \\ 3600 & 60 & 400 \end{vmatrix}$$

$$= (400)(20)(50)\begin{vmatrix} 1 & 1 & 8 \\ 4 & 2 & 3 \\ 9 & 3 & 8 \end{vmatrix}$$

$$= 400,000\begin{vmatrix} 1 & 0 & 0 \\ 4 & -2 & -29 \\ 2 & -6 & -64 \end{vmatrix}$$

$$= 400,000\begin{vmatrix} -2 & -29 \\ -6 & -64 \end{vmatrix}$$

= 400,000(128 – 174)
= 400,000(–46)
= –18,400,000

$$a = \frac{D_a}{D} = \frac{-10,000}{-16,000} = \frac{5}{8},$$

$$b = \frac{D_b}{D} = \frac{800,000}{-16,000} = -50,$$

$$c = \frac{D_c}{D} = \frac{-18,400,000}{-16,000} = 1150$$

The model is $f(x) = \frac{5}{8}x^2 - 50x + 1150$.

$$f(30) = \frac{5}{8}(900) - 50(30) + 1150$$
= 562.5 – 1500 + 1150
= 212.5

$$f(50) = \frac{5}{8}(2500) - 50(50) + 1150$$
= 1562.8 – 2500 + 1150
= 212.5

30- and 50-year-olds are involved in an average of 212.5 automobile accidents per day.

Chapter 9 Test

1. $\left[\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ 2 & -4 & 1 & -7 \\ -2 & 2 & -3 & 4 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \\ 2R_1 + R_3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ 0 & -8 & 3 & -1 \\ 0 & 6 & -5 & -2 \end{array}\right] -\frac{1}{8}R_2$

$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ 0 & 1 & -\frac{3}{8} & \frac{1}{8} \\ 0 & 6 & -5 & -2 \end{array}\right] -6R_2 + R_3$

$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ 0 & 1 & -\frac{3}{8} & \frac{1}{8} \\ 0 & 0 & -\frac{11}{4} & -\frac{11}{4} \end{array}\right] -\frac{4}{11}R_3$

$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ 0 & 1 & -\frac{3}{8} & \frac{1}{8} \\ 0 & 0 & 1 & 1 \end{array}\right]$

$$x + 2y - z = -3$$
$$y - \frac{3}{8}z = \frac{1}{8}$$
$$z = 1$$

Using back substitution,

$y - \frac{3}{8}(1) = \frac{1}{8}$ and $x + 2\left(\frac{1}{2}\right) - 1 = -3$.

$y = \frac{1}{2}$ $\quad$ $x + 1 - 1 = -3$

$x = -3$

The solution to the system is $\left\{\left(-3, \frac{1}{2}, 1\right)\right\}$.

2. $\left[\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ 2 & -1 & -1 & 1 \end{array}\right] -2R_1 + R_2$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ 0 & 3 & -3 & -3 \end{array}\right] \frac{1}{3}R_2$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ 0 & 1 & -1 & -1 \end{array}\right]$

The system $\begin{array}{r} x - 2y + z = 2 \\ y - z = -1 \end{array}$ has no unique solution.

Express x and y in terms of z:
$y = z - 1$
$x - 2(z - 1) + z = 2$
$x - 2z + 2 + z = 2$
$x = z$

With $z = t$, the complete solution to the system is $\{(t, t - 1, t)\}$.

3. $2B+3C=\begin{bmatrix}2 & -2\\ 4 & 2\end{bmatrix}+\begin{bmatrix}3 & 6\\ -3 & 9\end{bmatrix}=\begin{bmatrix}5 & 4\\ 1 & 11\end{bmatrix}$

4. $AB=\begin{bmatrix}3+2 & -3+1\\ 1+0 & -1+0\\ 2+2 & -2+1\end{bmatrix}=\begin{bmatrix}5 & -2\\ 1 & -1\\ 4 & -1\end{bmatrix}$

5. $C^{-1}=\frac{1}{(1)(3)-(2)(-1)}\begin{bmatrix}3 & -2\\ 1 & 1\end{bmatrix}$

$=\frac{1}{3+2}\begin{bmatrix}3 & -2\\ 1 & 1\end{bmatrix}=\begin{bmatrix}\frac{3}{5} & -\frac{2}{5}\\ \frac{1}{5} & \frac{1}{5}\end{bmatrix}$

6. $BC=\begin{bmatrix}1+1 & 2-3\\ 2-1 & 4+3\end{bmatrix}=\begin{bmatrix}2 & -1\\ 1 & 7\end{bmatrix}$

$BC-3B=\begin{bmatrix}2 & -1\\ 1 & 7\end{bmatrix}-\begin{bmatrix}3 & -3\\ 6 & 3\end{bmatrix}=\begin{bmatrix}-1 & 2\\ -5 & 4\end{bmatrix}$

7. $AB=\begin{bmatrix}-3+14-10 & 2-8+6 & 0+2-2\\ -6+21-15 & 4-12+9 & 0+3-3\\ -3-7+10 & 2+4-6 & 0-1+2\end{bmatrix}$

$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I_3$

$BA=\begin{bmatrix}-3+4+0 & -6+6+0 & -6+6+0\\ 7-8+1 & 14-12-1 & 14-12-2\\ -5+6-1 & -10+9+1 & -10+9+2\end{bmatrix}$

$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I_3$

8. **a.** $\begin{bmatrix}3 & 5\\ 2 & -3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}9\\ -13\end{bmatrix}$

b. $A^{-1}=\frac{1}{(3)(-3)-(5)(2)}\begin{bmatrix}-3 & -5\\ -2 & 3\end{bmatrix}$

$=\frac{1}{-19}\begin{bmatrix}-3 & -5\\ -2 & 3\end{bmatrix}=\begin{bmatrix}\frac{3}{19} & \frac{5}{19}\\ \frac{2}{19} & -\frac{3}{19}\end{bmatrix}$

c. $A^{-1}B=\begin{bmatrix}\frac{3}{19} & \frac{5}{19}\\ \frac{2}{19} & -\frac{3}{19}\end{bmatrix}\begin{bmatrix}9\\ -13\end{bmatrix}=\begin{bmatrix}\frac{27}{19}-\frac{65}{19}\\ \frac{18}{19}+\frac{39}{19}\end{bmatrix}=\begin{bmatrix}-2\\ 3\end{bmatrix}$

The solution to the system is {(–2, 3)}.

9. $\begin{vmatrix}4 & -1 & 3\\ 0 & 5 & -1\\ 5 & 2 & 4\end{vmatrix}=4\begin{vmatrix}5 & -1\\ 2 & 4\end{vmatrix}+5\begin{vmatrix}-1 & 3\\ 5 & -1\end{vmatrix}$

$=4[20-(-2)]+5(1-15)$

$=4(22)+5(-14)$

$=88-70$

$=18$

10. $D=\begin{vmatrix}3 & 1 & -2\\ 2 & 7 & 3\\ 4 & -3 & -1\end{vmatrix}=3\begin{vmatrix}7 & 3\\ -3 & -1\end{vmatrix}-1\begin{vmatrix}2 & 3\\ 4 & -1\end{vmatrix}-2\begin{vmatrix}2 & 7\\ 4 & -3\end{vmatrix}$

$=3[-7-(-9)]-1(-2-12)-2(-6-28)$

$=3(2)-1(-14)-2(-34)$

$=6+14+68$

$=88$

$D_x=\begin{vmatrix}-3 & 1 & -2\\ 9 & 7 & 3\\ 7 & -3 & -1\end{vmatrix}-3\begin{vmatrix}7 & 3\\ -3 & -1\end{vmatrix}-1\begin{vmatrix}9 & 3\\ 7 & -1\end{vmatrix}-2\begin{vmatrix}9 & 7\\ 7 & -3\end{vmatrix}$

$=-3[-7-(-9)]-1(-9-21)-2(-27-49)$

$=-3(2)-1(-30)-2(-76)$

$=-6+30+152$

$=176$

$x=\frac{D_x}{D}=\frac{176}{88}=2$

Cumulative Review Exercises (Chapters 1–9)

1. $2x^2=4-x$

$2x^2+x-4=0$

$x=\frac{-1\pm\sqrt{1^2-4(2)(-4)}}{2(2)}$

$x=\frac{-1\pm\sqrt{1-32}}{4}$

$x=\frac{-1\pm\sqrt{33}}{4}$

The solution set is $\left\{\frac{-1+\sqrt{33}}{4}, \frac{-1-\sqrt{33}}{4}\right\}$.

2. $5x+8 \le 7(1+x)$

$5x+8 \le 7+7x$

$-2x \le -1$

$x \ge \dfrac{1}{2}$

The solution set is $\left\{x \,\middle|\, x \ge \dfrac{1}{2}\right\}$ or $\left[\dfrac{1}{2}, \infty\right)$.

3. $\sqrt{2x+4}-\sqrt{x+3}-1=0$

$\sqrt{2x+4}=\sqrt{x+3}+1$

$2x+4=x+3+2\sqrt{x+3}+1$

$x=2\sqrt{x+3}$

$x^2=4(x+3)$

$x^2=4x+12$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$x=6$ or $x=-2$

$x=-2$ does not check. The solution set is $\{6\}$.

4. $3x^3 + 8x^2 - 15x + 4 = 0$

$p = 61, 62, 64$

$q = 61, 63$

$\dfrac{p}{q} = 61, 6\dfrac{1}{3}, 62, 6\dfrac{2}{3}, \; 64, 6\dfrac{4}{3}$

$$\begin{array}{r|rrrr} -4 & 3 & 8 & -15 & 4 \\ & & -12 & 16 & -4 \\ \hline & 3 & -4 & 1 & 0 \end{array}$$

$(x + 4)(3x^2 - 4x + 1) = 0$

$(x + 4)(3x - 1)(x - 1) = 0$

$x=-4, \; x=\dfrac{1}{3}, \; x=1$

The solution set is $\left\{-4, \dfrac{1}{3}, 1\right\}$.

5. $e^{2x}-14e^x+45=0 \;\; le+t=e^x$

$t^2-14t+45=0$

$(t-5)(t-9)=0$

$t=5 \;\; t=9$

$e^x=5 \quad e^x=9$

$\ln e^x=\ln 5 \quad \ln e^x=\ln 9$

$x=e^x=\ln 5 \quad x=\ln 9$

The solution set is $\{\ln 5, \ln 9\}$.

6. $\log_3 x+\log_3(x+2)=1$

$\log_3 x^2+2x=1$

$3^1=x^2+2x$

$x^2+2x-3=0$

$(x-1)(x+3)=0$

$x=1, \; x=-3$

$x=-3$ does not check. The solution set is $\{1\}$.

7. $\left[\begin{array}{rrr|r} 1 & -1 & 1 & 17 \\ 2 & 3 & 1 & 8 \\ -4 & 1 & 5 & -2 \end{array}\right] \begin{array}{l} \\ -2R_1+R_2 \\ 4R_1+R_3 \end{array}$

$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 17 \\ 0 & 5 & -1 & -26 \\ 0 & -3 & 9 & 66 \end{array}\right] -\tfrac{1}{3}R_3$

$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 17 \\ 0 & 1 & -3 & -22 \\ 0 & 5 & -1 & -26 \end{array}\right] \begin{array}{l} \\ -5R_2+R_3 \\ 1R_2+R_1 \end{array}$

$\left[\begin{array}{rrr|r} 1 & 0 & -2 & -5 \\ 0 & 1 & -3 & -22 \\ 0 & 0 & 14 & 84 \end{array}\right] \tfrac{1}{14}R_3$

$\left[\begin{array}{rrrr} 1 & 0 & -2 & -5 \\ 0 & 1 & -3 & -22 \\ 0 & 0 & 1 & 6 \end{array}\right] \begin{array}{l} \\ 3R_3+R_2 \\ 2R_3+R_1 \end{array}$

$\left[\begin{array}{rrrr} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 6 \end{array}\right]$

$x = 7 \;\; y = -4 \;\; z = 6$

The solution set is $\{(7, -4, 6)\}$.

8. $D = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & -2 \end{vmatrix}$

$= 1\begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix} - 2\begin{vmatrix} -2 & 1 \\ 2 & -2 \end{vmatrix} + 3\begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix}$

$= 1(-2+2) - 2(4-2) + 3(2-1)$

$= 0 - 4 + 3$

$= -1$

$D_y = \begin{vmatrix} 1 & 7 & 1 \\ 2 & 0 & -1 \\ 3 & -2 & -2 \end{vmatrix} = 7\begin{vmatrix} 2 & -1 \\ 3 & -2 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$

$= 7(-4+3) - 2(-1-2)$

$= -7 + 6 = 1$

$y = \dfrac{D_y}{D} = \dfrac{1}{-1} = -1$

$y = -1$

9. $y = \sqrt{4x-7}$

$x = \sqrt{4y-7}$

$x^2 = 4y - 7$

$x^2 + 7 - 4y$

$\dfrac{x^2+7}{4} = y$

$f^{-1}(x) = \dfrac{x^2+7}{4}\ (x \geq 0)$

10. $f(x) = \dfrac{x}{x^2-16}$

$f(0) = \dfrac{0}{-16} = 0$

y-intercept at 0

$0 = \dfrac{x}{x^2-16}$

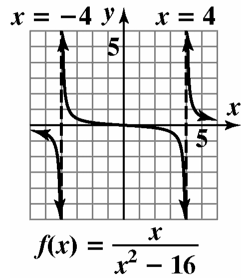
$0 = x$

x-intercept at 0

$f(x) = \dfrac{x}{(x+4)(x-4)}$

vertical asymptotes at 4, –4

horizontal asymptote at 0



11. $f(x) = 4x^4 - 4x^3 - 25x^2 + x + 6$

–2	4	–4	–25	1	6
		–8	24	2	–6
3	4	–12	–1	3	0
		12	0	–3	
	4	0	–1	0	

$f(x) = (x+2)(x-3)(4x^2-1)$

$f(x) = (x+2)(x-3)(2x+1)(2x-1)$

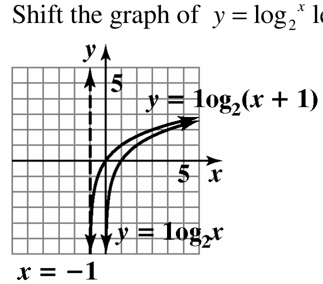
12. $y = \log_2 x$

$2^y = x$

x	y
1	0
2	1
$\frac{1}{2}$	–1

$y = \log_2(x+1)$

Shift the graph of $y = \log_2{}^x$ left one unit.



13. a. $A = A_0 e^{kt}$

$$450 = 900e^{k(40)}$$
$$\frac{1}{2} = e^{40k}$$
$$\ln\left(\frac{1}{2}\right) = \ln\left(e^{40k}\right)$$
$$\ln\left(\frac{1}{2}\right) = 40k$$
$$k = \frac{\ln\left(\frac{1}{2}\right)}{40} \approx -0.017$$
$$A = 900e^{-0.017t}$$

b. $A = 900e^{-0.017(10)}$

$$A = 900e^{-0.17}$$
$$A \approx 759.30 \text{ grams}$$

14. $$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 2 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4-2+0 & -1+0+0 \\ 8+2+3 & -2+0+3 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 13 & 1 \end{bmatrix}$$

15. $$\frac{3x^2+17x-38}{(x-3)(x-2)(x+2)} = \frac{A}{x-3}+\frac{B}{x-2}+\frac{C}{x+2}$$

$$3x^2+17x-38 = A(x^2-4)+B(x^2-x-6)+C(x^2-5x+6)$$
$$3x^2+17x-38 = Ax^2-4A+Bx^2-Bx-6B+Cx^2-5Cx+6c$$
$$3x^2+17x-38 = (A+B+C)x^2+(-B-5C)x-(4A+6B-6C)$$
$$A+B+C=3$$
$$-B-5C=17$$
$$4A+6B-6C=38$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & -5 & 17 \\ 4 & 6 & -6 & 38 \end{array}\right] -4R_1+R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & -5 & 17 \\ 0 & 2 & -10 & 26 \end{array}\right] -1R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 5 & -17 \\ 0 & 2 & -10 & 26 \end{array}\right] \begin{matrix} \\ -2R_2+R_3 \\ -1R_2+R_1 \end{matrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & 20 \\ 0 & 1 & 5 & -17 \\ 0 & 0 & -20 & 60 \end{array}\right] -\tfrac{1}{20}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & 20 \\ 0 & 1 & 5 & -17 \\ 0 & 0 & 1 & -3 \end{array}\right] \begin{matrix} \\ -5R_3+R_2 \\ 4R_3+R_1 \end{matrix}$$

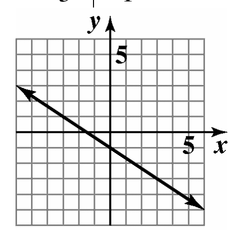
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

$A=8,\quad B=-2,\quad C=-3$

$$\frac{3x^2+17x-38}{(x-3)(x-2)(x+2)} = \frac{8}{x-3}+\frac{-2}{x-2}+\frac{-3}{x+2}$$

16. $y = -\frac{2}{3}x - 1$

x	y
0	–1
3	–3
–3	1

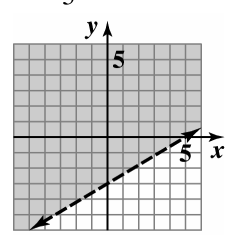


$y = -\frac{2}{3}x - 1$

17. $3x - 5y < 15$

$-5y < -3x + 15$

$y > \frac{3}{5}x - 3$

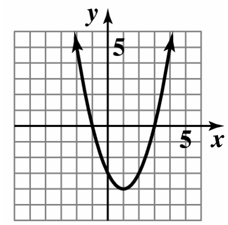


$3x - 5y < 15$

18. $f(x) = x^2 - 2x - 3$

$f(x) = (x^2 - 2x + 1) - 3 - 1$

$f(x) = (x-1)^2 - 4$

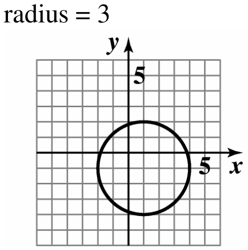


$f(x) = x^2 - 2x - 3$

19. $(x-1)^2 + (y+1)^2 = 9$

center $(1, -1)$

radius = 3



$(x-1)^2 + (y+1)^2 = 9$

20.

2	1	0	–6	4
		2	4	–4
	1	2	–2	0

$$\frac{x^3 - 6x + 4}{x-2} = x^2 + 2x - 2$$

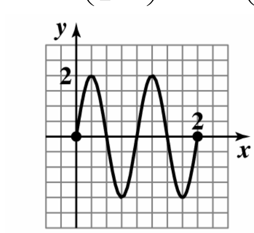
21. $y = 2\sin 2\pi x,\ 0 \le x \le 2$

Amplitude: $|A| = |2| = 2$

Period: $\frac{2\pi}{B} = \frac{2\pi}{2\pi} = 1$

x-intercepts:

$(0, 0), \left(\frac{1}{2}, 0\right), (1, 0), \left(\frac{3}{2}, 0\right), (2, 0)$



$y = 2\sin 2\pi x, 0 \le x \le 2$

22. $\cos\left[\tan^{-1}\left(-\frac{4}{3}\right)\right]$

If $\tan\theta = -\frac{4}{3}$, θ lies in quadrant IV.

$\tan\theta = -\frac{4}{3} = \frac{y}{x} = \frac{-4}{3}$

$r = \sqrt{(3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\cos\left[\tan^{-1}\left(-\frac{4}{3}\right)\right] = \frac{x}{r} = \frac{3}{5}$

23. $\frac{\cos 2x}{\cos x - \sin x} = \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$

$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$

$= \cos x + \sin x$

24. $\cos^2 x + \sin x + 1 = 0$

$(1 - \sin^2 x) + \sin x + 1 = 0$

$-\sin^2 x + \sin x + 2 = 0$

$\sin^2 x - \sin x - 2 = 0$

$(\sin x - 2)(\sin x + 1) = 0$

$\sin x - 2 = 0$ or $\sin x + 1 = 0$

$\sin x = 2$ $\quad$ $\sin x = -1$

no solution or $x = \frac{3\pi}{2}$

The solution in the interval $[0, 2\pi)$ is $\frac{3\pi}{2}$.

25. $4\mathbf{w} - 5\mathbf{v} = 4(-7\mathbf{i} + 3\mathbf{j}) - 5(-6\mathbf{i} + 5\mathbf{j})$

$= -28\mathbf{i} + 12\mathbf{j} + 30\mathbf{i} - 25\mathbf{j}$

$= 2\mathbf{i} - 13\mathbf{j}$

Chapter 10
Conic Sections

Section 10.1

Check Point Exercises

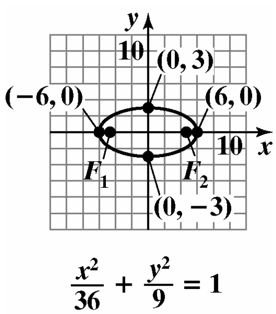
1. $a^2 = 36,\ a = 6$

$b^2 = 9,\ b = 3$

$c^2 = a^2 - b^2 = 36 - 9 = 27$

$c = \sqrt{27} = 3\sqrt{3}$

The foci are located at $(-3\sqrt{3},\ 0)$ and $(3\sqrt{3},\ 0)$.



2. $\dfrac{16x^2}{144} + \dfrac{9y^2}{144} = \dfrac{144}{144}$

$\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$

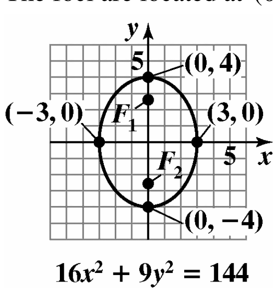
$a^2 = 16,\ a = 4$

$b^2 = 9,\ b = 3$

$c^2 = a^2 - b^2 = 16 - 9 = 7$

$c = \sqrt{7}$

The foci are located at $(0,\ -\sqrt{7})$ and $(0,\ \sqrt{7})$.



3. $c^2 = 4,\ a^2 = 9$

$b^2 = a^2 - c^2 = 9 - 4 = 5$

$\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$

4. $a^2 = 9, a = 3$

$b^2 = 4, b = 2$

center at (–1, 2)

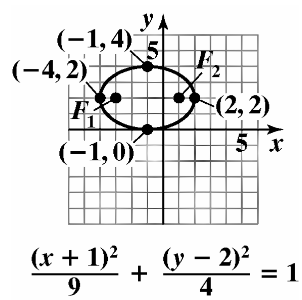
$c^2 = a^2 - b^2$

$c^2 = 9 - 4$

$c^2 = 5$

$c = \sqrt{5}$

The foci are located at $(-1 - \sqrt{5},\ 2)$ and $(-1 + \sqrt{5},\ 2)$.



5. $\dfrac{x^2}{20^2} + \dfrac{y^2}{10^2} = 1$

$\dfrac{x^2}{400} + \dfrac{y^2}{100} = 1$

Since the truck is 12 feet wide, substitute $x = 6$ into the equation to find y.

$$\frac{6^2}{400} + \frac{y^2}{100} = 1$$

$$400\left(\frac{36}{400} + \frac{y^2}{100}\right) = 400(1)$$

$$36 + 4y^2 = 400$$

$$4y^2 = 364$$

$$y^2 = 91$$

$$y = \sqrt{91}$$

$$y \approx 9.54$$

6 feet from the center, the height of the archway is 9.54 feet. Since the truck's height is 9 feet, it will fit under the archway.

Exercise Set 10.1

1. $\frac{x^2}{16}+\frac{y^2}{4}=1$

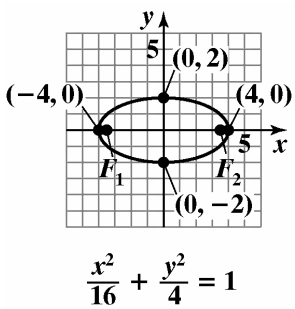
$a^2=16,\ a=4$

$b^2=4,\ b=2$

$c^2=a^2-b^2=16-4=12$

$c=\sqrt{12}=2\sqrt{3}$

The foci are located at $(-2\sqrt{3},\ 0)$ and $(2\sqrt{3},\ 0)$.

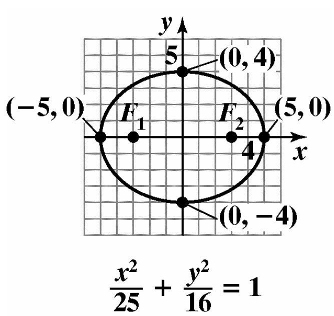


2. $a^2=25,\ a=5$

$b^2=16,\ b=4$

$c^2=a^2-b^2=25-16=9$, $c=3$

The foci are located at $(-3,\ 0)$ and $(3,\ 0)$.



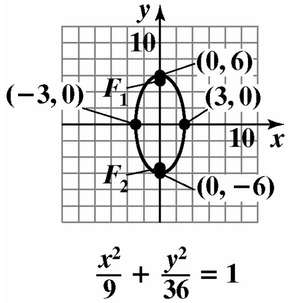
3. $a^2=36,\ a=6$

$b^2=9,\ b=3$

$c^2=a^2-b^2=36-9=27$

$c=\sqrt{27}=3\sqrt{3}$

The foci are located at $(0,\ -3\sqrt{3})$ and $(0,\ 3\sqrt{3})$.



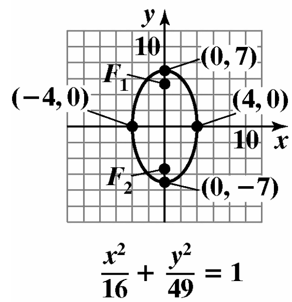
4. $\frac{x^2}{16}+\frac{y^2}{49}=1$

$a^2=49,\ a=7$

$b^2=16,\ b=4$

$c^2=a^2-b^2=49-16=33$, $c=\sqrt{33}$

The foci are located at $(0,\ -\sqrt{33})$ and $(0,\ \sqrt{33})$.



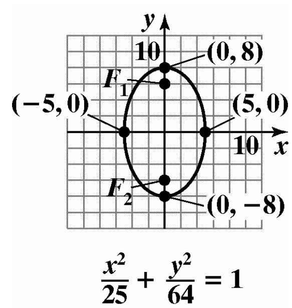
5. $a^2=64,\ a=8$

$b^2=25,\ b=5$

$c^2=a^2-b^2=64-25=39$

$c=\sqrt{39}$

The foci are located at $(0,\ -\sqrt{39})$ and $(0,\ \sqrt{39})$.

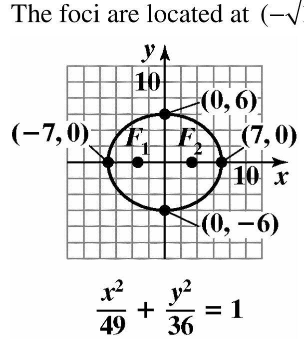


6. $a^2=49,\ a=7$

$b^2=36,\ b=6$

$c^2=a^2-b^2=49-36=13$, $c=\sqrt{13}$

The foci are located at $(-\sqrt{13},\ 0)$ and $(\sqrt{13},\ 0)$.



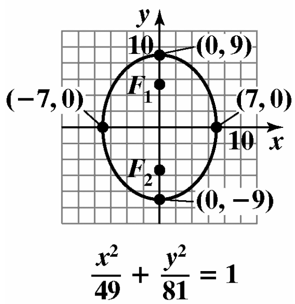
7. $a^2 = 81,\ a = 9$

$b^2 = 49,\ b = 7$

$c^2 = a^2 - b^2 = 81 - 49 = 32$

$c = \sqrt{32} = 4\sqrt{2}$

The foci are located at $(0, -4\sqrt{2})$ and $(0, 4\sqrt{2})$.

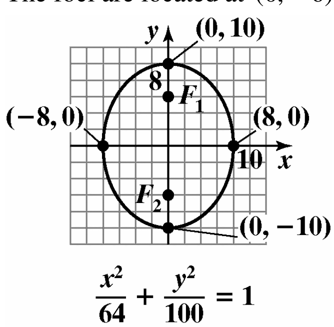


8. $a^2 = 100,\ a = 10$

$b^2 = 64,\ b = 8$

$c^2 = a^2 - b^2 = 100 - 64 = 36$

The foci are located at $(0, -6)$ and $(0, 6)$.



9. $\frac{x^2}{\frac{9}{4}} + \frac{y^2}{\frac{25}{4}} = 1$

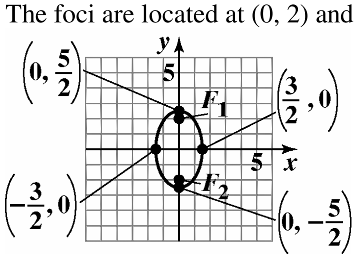
$c^2 = \frac{25}{4} - \frac{9}{4}$

$c^2 = \frac{16}{4}$

$c^2 = 4$

$c = 2$

The foci are located at (0, 2) and (0, –2).



10. $c^2 = \frac{81}{4} - \frac{25}{16}$

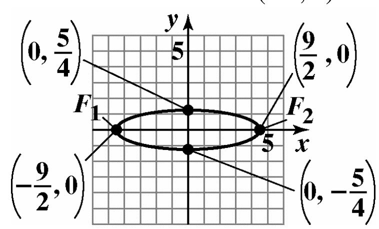
$c^2 = \frac{324}{16} - \frac{25}{16}$

$c^2 = \frac{299}{16}$

$c = \pm\frac{\sqrt{299}}{4}$

$c \approx \pm 4.3$

The foci are located at (4.3, 0) and (–4.3, 0).



11. $x^2 = 1 - 4y^2$

$x^2 + 4y^2 = 1$

$x^2 + \frac{y^2}{\frac{1}{4}} = 1$

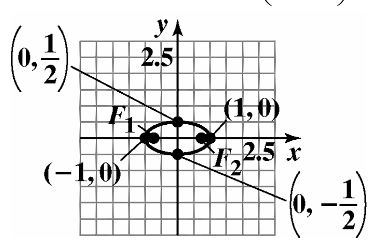
$c^2 = 1 - \frac{1}{4}$

$c^2 = \frac{3}{4}$

$c = \pm\frac{\sqrt{3}}{2}$

$c \approx \pm 0.9$

The foci are located at $\left(\frac{\sqrt{3}}{2}, 0\right)$ and $\left(-\frac{\sqrt{3}}{2}, 0\right)$.



12. $y^2 = 1 - 4x^2$

$4x^2 + y^2 = 1$

$\frac{x^2}{1/4} + \frac{y^2}{1} = 1$

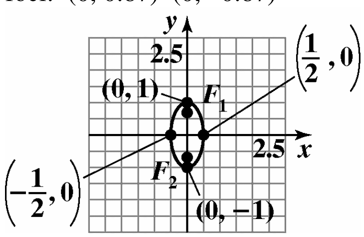
$c^2 = 1 - \frac{1}{4}$

$c^2 = \frac{3}{4}$

$c = \pm\frac{\sqrt{3}}{2}$

$c \approx \pm 0.87$

foci: (0, 0.87) (0, –0.87)



13. $25x^2 + 4y^2 = 100$

$\frac{25x^2}{100} + \frac{4y^2}{100} = \frac{100}{100}$

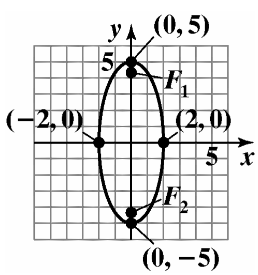
$\frac{x^2}{4} + \frac{y^2}{25} = 1$

$a^2 = 25,\ a = 5$

$b^2 = 4,\ b = 2$

$c^2 = a^2 = b^2 = 25 - 4 = 21$

The foci are located at $(0, -\sqrt{21})$ and $(0, \sqrt{21})$.



14. $9x^2 + 4y^2 = 36$

$\frac{9x^2}{36} + \frac{4y^2}{36} = \frac{36}{36}$

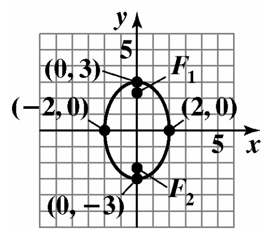
$\frac{x^2}{4} + \frac{y^2}{9} = 1$

$a^2 = 9,\ a = 3$

$b^2 = 4,\ b = 2$

$c^2 = a^2 - b^2 = 9 - 4 = 5$, $c = \sqrt{5}$

The foci are located at $(0, -\sqrt{5})$ and $(0, \sqrt{5})$.



15. $4x^2 + 16y^2 = 64$

$\frac{x^2}{16} + \frac{y^2}{4} = 1$

$a^2 = 16, a = 4$

$b^2 = 4, b = 2$

$c^2 = 16 - 4$

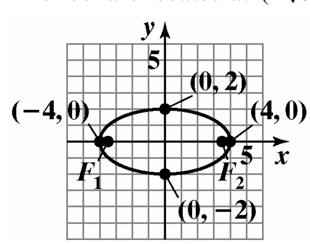
$c^2 = 12$

$c = \pm\sqrt{12}$

$c = \pm 2\sqrt{3}$

$c \approx \pm 3.5$

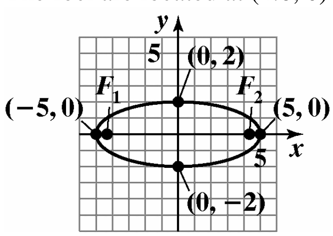
The foci are located at $(2\sqrt{3}, 0)$ and $(-2\sqrt{3}, 0)$.



16. $4x^2 + 25y^2 = 100$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$
$$c^2 = 25 - 4$$
$$c^2 = 21$$
$$c = \sqrt{21}$$
$$c \approx \pm 4.6$$

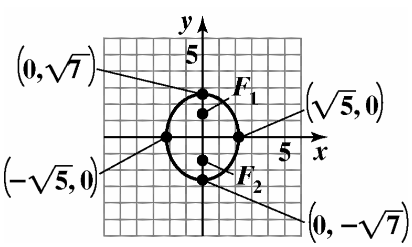
The foci are located at (4.6, 0) and (–4.6, 0).



17. $7x^2 = 35 - 5y^2$

$$7x^2 + 5y^2 = 35$$
$$\frac{x^2}{5} + \frac{y^2}{7} = 1$$
$$a^2 = 7, a = \sqrt{7}$$
$$b^2 = 5, b = \sqrt{5}$$
$$c^2 = 7 - 5$$
$$c^2 = 2$$
$$c = \pm\sqrt{2}$$
$$c \approx \pm 1.4$$

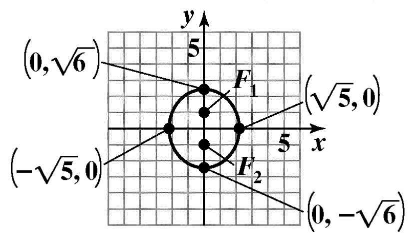
The foci are located at $(0, \sqrt{2})$ and $(0, -\sqrt{2})$.



18. $6x^2 = 30 - 5y^2$

$$6x^2 + 5y^2 = 30$$
$$\frac{x^2}{5} + \frac{y^2}{6} = 1$$
$$c^2 = 6 - 5$$
$$c^2 = 1$$
$$c = \pm 1$$

The foci are located at (0, 1) and (0, –1).



19. $a^2 = 4, b^2 = 1,$ center at $(0, 0)$

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$
$$c^2 = a^2 - b^2 = 4 - 1 = 3$$
$$c = \sqrt{3}$$

The foci are at $(-\sqrt{3}, 0)$ and $(\sqrt{3}, 0)$.

20. $a^2 = 16, b^2 = 4,$ center at (0, 0)

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$
$$c^2 = a^2 - b^2 = 16 - 4 = 12, \; c = \sqrt{12} = 2\sqrt{3}$$

The foci are at $(-2\sqrt{3}, 0)$ and $(2\sqrt{3}, 0)$.

21. $a^2 = 4, b^2 = 1,$

center: (0, 0)

$$\frac{x^2}{1} + \frac{y^2}{4} = 1$$
$$c^2 = a^2 - b^2 = 4 - 1 = 3$$
$$c = \sqrt{3}$$

The foci are at $(0, \sqrt{3})$ and $(0, -\sqrt{3})$.

22. $a^2 = 16, b^2 = 4$, center: (0, 0)

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$
$$c^2 = a^2 - b^2 = 16 - 4 = 12, \; c = \sqrt{12} = 2\sqrt{3}$$

The foci are at $(0, 2\sqrt{3})$ and $(0, -2\sqrt{3})$.

23. $\frac{(x+1)^2}{4}+\frac{(y-1)^2}{1}=1$

$a^2=4,\quad b^2=1$

$c^2=4-1$

$c^2=3$

$c=\pm\sqrt{3}$

The foci are located at $(-1+\sqrt{3},1)$ and $(-1-\sqrt{3},1)$.

24. $a^2=4, b^2=1$ center: (–1, –1)

$\frac{(x+1)^2}{1}+\frac{(y+1)^2}{4}=1$

$c^2=4-1$

$c^2=3$

$c=\pm\sqrt{3}$

$c\approx\pm1.7$

(0 – 1, 1.7 – 1), (0 – 1, – 1.7 – 1)

The foci are at (–1, 0.7) and (–1, –2.7).

25. $c^2=25,\ a^2=64$

$b^2=a^2-c^2=64-25=39$

$\frac{x^2}{64}+\frac{y^2}{39}=1$

26. $c^2=4\ a^2=36$

$b^2=a^2-c^2=36-4=32$

$\frac{x^2}{36}+\frac{y^2}{32}=1$

27. $c^2=16,\ a^2=49$

$b^2=a^2-c^2=49-16=33$

$\frac{x^2}{33}+\frac{y^2}{49}=1$

28. $c^2=9,\ a^2=16$

$b^2=a^2-c^2=16-9=7$

$\frac{x^2}{7}+\frac{y^2}{16}=1$

29. $c^2=4,\ b^2=9$

$a^2=b^2+c^2=9+4=13$

$\frac{x^2}{13}+\frac{y^2}{9}=1$

30. $c^2=4,\ b^2=4$

$a^2=b^2+c^2=4+4=8$

$\frac{x^2}{4}+\frac{y^2}{8}=1$

31. $2a=8,\ a=4,\ a^2=16$

$2b=4,\ b=2,\ b^2=4$

$\frac{x^2}{16}+\frac{y^2}{4}=1$

32. $2a=12,\ a=6,\ a^2=36$

$2b=6,\ b=3,\ b^2=9$

$\frac{x^2}{36}+\frac{y^2}{9}=1$

33. $2a=10,\ a=5,\ a^2=25$

$2b=4,\ b=2,\ b^2=4$

$\frac{(x+2)^2}{4}+\frac{(y-3)^2}{25}=1$

34. $2a=20,\ a=10,\ a^2=100$

$2b=10,\ b=5,\ b^2=25$

center (2, –3)

$\frac{(x-2)^2}{25}+\frac{(y+3)^2}{100}=1$

35. length of the major axis = 9 – 3 = 6

2*a* = 6, *a* = 3 major axis is vertical

length of the minor axis = 9 – 5 = 4

2*b* = 4, *b* = 2

Center is at (7, 6).

$\frac{(x-7)^2}{4}+\frac{(y-6)^2}{9}=1$

36. length of major axis = 8 – 2 = 6, 2*a* = 6, *a* = 3

length of minor axis = 5 – 3 = 2, 2*b* = 2, *b* = 1, center (5, 2)

$\frac{(x-5)^2}{9}+\frac{(y-2)^2}{1}=1$

37. $a^2 = 9,\ a = 3$

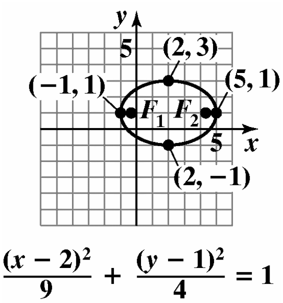
$b^2 = 4,\ b = 2$

center: (2, 1)

$c^2 = a^2 - b^2 = 9 - 4 = 5$

$c = \sqrt{5}$

The foci are at $(2-\sqrt{5},\ 1)$ and $(2+\sqrt{5},\ 1)$.



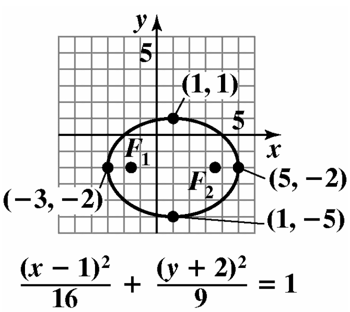
38. $a^2 = 16,\ a = 4$

$b^2 = 9,\ b = 3$

center: $(1,\ -2)$

$c^2 = a^2 - b^2 = 16 - 9 = 7$, $c = \sqrt{7}$

The foci are at $(1-\sqrt{7},\ -2)$ and $(1+\sqrt{7}, -2)$.



39. $\dfrac{(x+3)^2}{16} + \dfrac{4(y-2)^2}{16} = \dfrac{16}{16}$

$\dfrac{(x+3)^2}{16} + \dfrac{(y-2)^2}{4} = 1$

$a^2 = 16,\ a = 4$

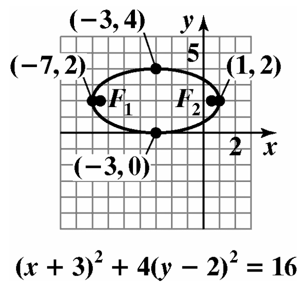
$b^2 = 4,\ b = 2$

center: $(-3,\ 2)$

$c^2 = a^2 - b^2 = 16 - 4 = 12$

$c = \sqrt{12} = 2\sqrt{3}$

The foci are at $(-3-2\sqrt{3},\ 2)$ and $(-3+2\sqrt{3},\ 2)$.



40. $\dfrac{(x-3)^2}{18} + \dfrac{9(y+2)^2}{18} = \dfrac{18}{18}$

$\dfrac{(x-3)^2}{18} + \dfrac{(y+2)^2}{2} = 1$

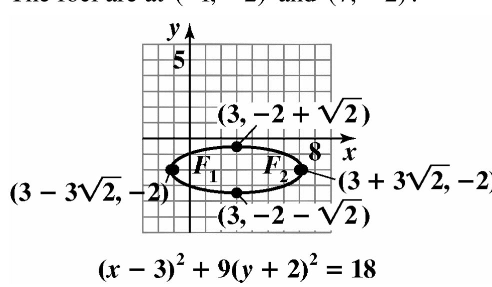
$a^2 = 18,\ a = \sqrt{18} = 3\sqrt{2}$

$b^2 = 2,\ b = \sqrt{2}$

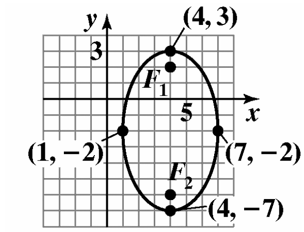
center: $(3,\ -2)$

$c^2 = a^2 - b^2 = 18 - 2 - 16,\ c = 4$

The foci are at $(-1,\ -2)$ and $(7,\ -2)$.

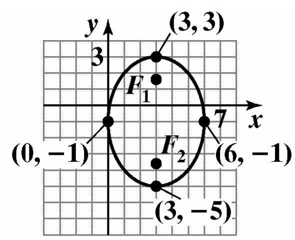


41. $a^2 = 25,\ a = 5$
$b^2 = 9,\ b = 3$
center: $(4, -2)$
$c^2 = a^2 - b^2 = 25 - 9 = 16$
$c = 4$
The foci are at $(4, 2)$ and $(4, -6)$.



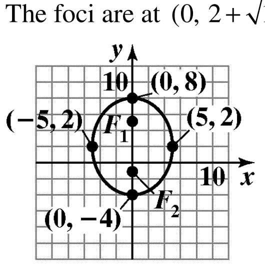
$$\frac{(x-4)^2}{9} + \frac{(y+2)^2}{25} = 1$$

42. $a^2 = 16,\ a = 4$
$b^2 = 9,\ b = 3$
center: $(3, -1)$
$c^2 = a^2 - b^2 = 16 - 9 = 7$, $c = \sqrt{7}$
The foci are at $(3, -1 + \sqrt{7})$ and $(3, -1 - \sqrt{7})$.



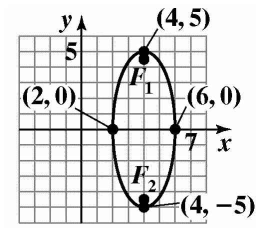
$$\frac{(x-3)^2}{9} + \frac{(y+1)^2}{16} = 1$$

43. $a^2 = 36,\ a = 6$
$b^2 = 25,\ b = 5$
center: $(0, 2)$
$c^2 = a^2 - b^2 = 36 - 25 = 11$
$c = \sqrt{11}$
The foci are at $(0, 2 + \sqrt{11})$ and $(0, 2 - \sqrt{11})$.



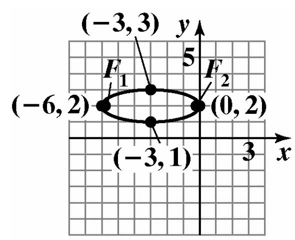
$$\frac{x^2}{25} + \frac{(y-2)^2}{36} = 1$$

44. $a^2 = 25$, a = 5
$b^2 = 4,\ b = 2$
center: (4, 0)
$c^2 = a^2 - b^2 = 25 - 4 = 21,\ c = \sqrt{21}$
The foci are at $(4, \sqrt{21})$ and $(4, -\sqrt{21})$.



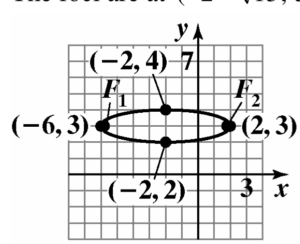
$$\frac{(x-4)^2}{4} + \frac{y^2}{25} = 1$$

45. $a^2 = 9,\ a = 3$
$b^2 = 1,\ b = 1$
center: (–3, 2)
$c^2 = a^2 - b^2 = 9 - 1 = 8$
$c = \sqrt{8} = 2\sqrt{2}$
The foci are at $(-3 - 2\sqrt{2},\ 2)$ and
$(-3 + 2\sqrt{2},\ 2)$.



$$\frac{(x+3)^2}{9} + (y-2)^2 = 1$$

46. $a^2 = 16,\ a = 4$
$b^2 = 1,\ b = 1$
center: (–2, 3)
$c^2 = a^2 - b^2 = 16 - 1 = 15,\ c = \sqrt{15}$
The foci are at $(-2 - \sqrt{15}, 3)$ and $(-2 + \sqrt{15}, 3)$.



$$\frac{(x+2)^2}{16} + (y-3)^2 = 1$$

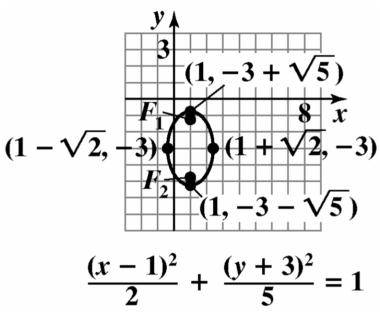
47. $c^2 = 5 - 2$

$c^2 = 3$

$c = \pm\sqrt{3}$

$c \approx \pm 1.7$

The foci are located at $(1, -3+\sqrt{3})$ and $(1, -3-\sqrt{3})$.



48. $c^2 = 5 - 2$

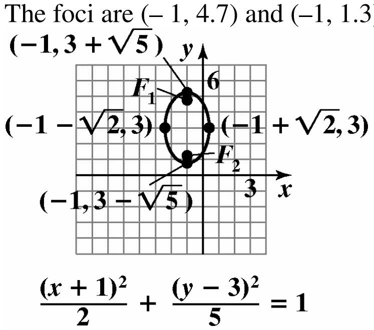
$c^2 = 3$

$c = \pm\sqrt{3}$

$c \approx \pm 1.7$

(–1, 3 + 1.7) (–1 , 3 – 1.7)

The foci are (– 1, 4.7) and (–1, 1.3).



49. $\dfrac{9(x-1)^2}{36} + \dfrac{4(y+3)^2}{36} = \dfrac{36}{36}$

$\dfrac{(x-1)^2}{4} + \dfrac{(y+3)^2}{9} = 1$

$a^2 = 9$, $a = 3$

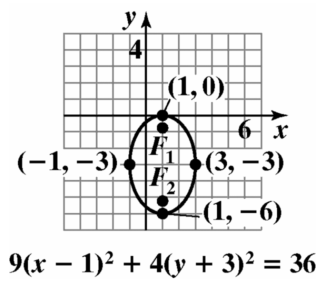
$b^2 = 4$, $b = 2$

center: (1, –3)

$c^2 = a^2 - b^2 = 9 - 4 = 5$

$c = \sqrt{5}$

The foci are at $(1, -3+\sqrt{5})$ and $(1, -3-\sqrt{5})$.



50. $\dfrac{36(x+4)^2}{36} + \dfrac{(y+3)^2}{36} = \dfrac{36}{36}$

$(x+4)^2 + \dfrac{(y+3)^2}{36} = 1$

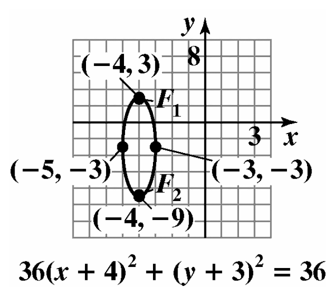
$a^2 = 36$, $a = 6$

$b^2 = 1$, $b = 1$

center: (–4, –3)

$c^2 = a^2 - b^2 = 36 - 1 = 35$, $c = \sqrt{35}$

The foci are at $(-4, -3+\sqrt{35})$ and $(-4, -3-\sqrt{35})$.



51. $(9x^2-36x)+(25y^2+50y)=164$

$9(x^2-4x)+25(y^2+2y)=164$

$9(x^2-4x+4)+25(y^2+2y+1)$

$=164+36+25$

$9(x-2)^2+25(y+1)^2=225$

$$\frac{9(x-2)^2}{225}+\frac{25(y+1)^2}{225}=\frac{225}{225}$$

$$\frac{(x-2)^2}{25}+\frac{(y+1)^2}{9}=1$$

center: (2, –1)

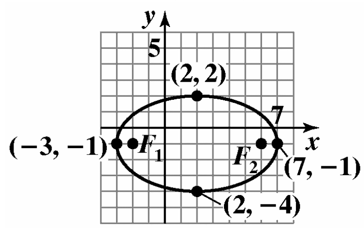
$a^2 = 25$, $a = 5$

$b^2 = 9$, $b = 3$

$c^2 = a^2 - b^2 = 25 - 9 = 16$

$c = 4$

The foci are at (–2, –1) and (6, –1).



$9x^2 + 25y^2 - 36x + 50y - 164 = 0$

52. $(4x^2-32x)+(9y^2+36y)=-64$

$4(x^2-8x)+9(y^2+4y)=-64$

$4(x^2-8x+16)+9(y^2+4y+4)=-64+64+36$

$4(x-4)^2+9(y+2)^2=36$

$$\frac{4(x-4)^2}{36}+\frac{9(y+2)^2}{36}=\frac{36}{36}$$

$$\frac{(x-4)^2}{9}+\frac{(y+2)^2}{4}=1$$

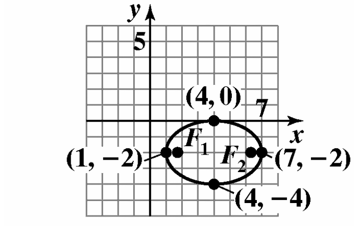
center: (4, –2)

$a^2=9, a=3$

$b^2=4, b=2$

$c^2=a^2-b^2=9-4=5$, $c=\sqrt{5}$

The foci are at $\left(4-\sqrt{5},-2\right)$ and $\left(4+\sqrt{5},-2\right)$.



$4x^2 + 9y^2 - 32x + 36y + 64 = 0$

53. $(9x^2 - 18x) + (16y^2 + 64y) = 71$

$9(x^2 - 2x) + 16(y^2 + 4y) = 71$

$9(x^2 - 2x + 1) + 16(y^2 + 4y + 4)$

$= 71 + 9 + 64$

$9(x - 1)^2 + 16(y + 2)^2 = 144$

$$\frac{9(x-1)^2}{144}+\frac{16(y+2)^2}{144}=\frac{144}{144}$$

$$\frac{(x-1)^2}{16}+\frac{(y+2)^2}{9}=1$$

center: (1, –2)

$a^2 = 16$, $a = 4$

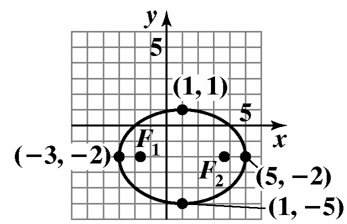
$b^2 = 9$, $b = 3$

$c^2 = a^2 - b^2 = 16 - 9 = 7$

$c = \sqrt{7}$

The foci are at

$(1 - \sqrt{7}, -2)$ and $(1+\sqrt{7}, -2)$.



$9x^2 + 16y^2 - 18x + 64y - 71 = 0$

54. $(x^2+10x)+(4y^2-8y)=-13$

$(x^2+10x+25)+4(y^2-2y)=-13+25$

$(x^2+10x+25)+4(y^2-2y+1)=-13+25+4$

$(x+5)^2+4(y-1)^2=16$

$$\frac{(x+5)^2}{16}+\frac{4(y-1)^2}{16}=\frac{16}{16}$$

$$\frac{(x+5)^2}{16}+\frac{(y-1)^2}{4}=1$$

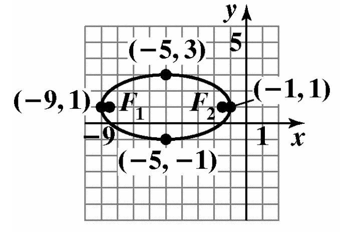
center: (–5, 1)

$a^2=16,\ a=4$

$b^2=4,\ b=2$

$c^2=a^2-b^2=16-4=12$, $c=\sqrt{12}=2\sqrt{3}$

The foci are at $(-5-2\sqrt{3},\ 1)$ and $(-5+2\sqrt{3},\ 1)$.



$x^2 + 4y^2 + 10x - 8y + 13 = 0$

55.

$$(4x^2+16x)+(y^2-6y)=39$$
$$4(x^2+4x)+(y^2-6y)=39$$
$$4(x^2+4x+4)+(y^2-6y+9)=39+16+9$$
$$4(x+2)^2+(y-3)^2=64$$
$$\frac{4(x+2)^2}{64}+\frac{(y-3)^2}{64}=\frac{64}{64}$$
$$\frac{(x+2)^2}{16}+\frac{(y-3)^2}{64}=1$$

center: (–2, 3)

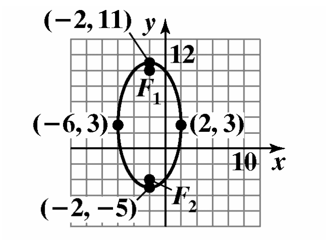
$a^2=64,\ a=8$

$b^2=16,\ b=4$

$c^2=a^2-b^2=64-16=48$

$c=\sqrt{48}=4\sqrt{3}$

The foci are at $(-2,\ 3+4\sqrt{3})$ and $(-2,\ 3-4\sqrt{3})$.



$4x^2+y^2+16x-6y-39=0$

56.

$$(4x^2-24x)+(25y^2+100y)=-36$$
$$4(x^2-6x)+25(y^2+4y)=-36$$
$$4(x^2-6x+9)+25(y^2+4y+4)=-36+36+100$$
$$4(x-3)^2+25(y+2)^2=100$$
$$\frac{4(x-3)^2}{100}+\frac{25(y+2)^2}{100}=\frac{100}{100}$$
$$\frac{(x-3)^2}{25}+\frac{(y+2)^2}{4}=1$$

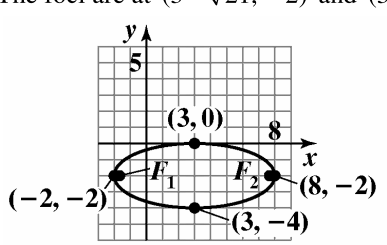
center: (3, –2)

$a^2=25,\ a=5$

$b^2=4,\ b=2$

$c^2=a^2-b^2=25-4=21$, $c=\sqrt{21}$

The foci are at $(3-\sqrt{21},\ -2)$ and $(3+\sqrt{21},\ -2)$.

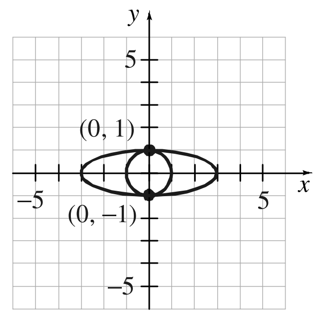


$4x^2+25y^2-24x+100y+36=0$

57. $x^2+y^2=1$ $\qquad$ $x^2+9y^2=9$

$$\frac{x^2}{9}+\frac{9y^2}{9}=\frac{9}{9}$$
$$\frac{x^2}{9}+\frac{y^2}{1}=1$$

The first equation is that of a circle with center at the origin and $r=1$. The second equation is that of an ellipse with center at the origin, horizontal major axis of length 6 units $(a=3)$, and vertical minor axis of length 2 units $(b=1)$.



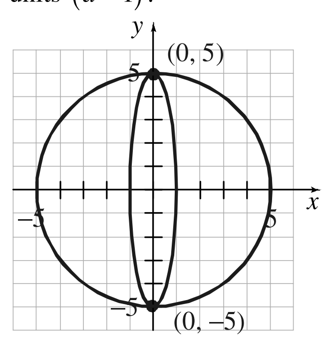
Check each intersection point.

The solution set is $\{(0,-1),(0,1)\}$.

58. $x^2+y^2=25$ $\qquad$ $25x^2+y^2=25$

$$\frac{25x^2}{25}+\frac{y^2}{25}=\frac{25}{25}$$
$$\frac{x^2}{1}+\frac{y^2}{25}=1$$

The first equation is for a circle with center at the origin and $r=5$. The second is for an ellipse with center at the origin, vertical major axis of length 10 units $(b=5)$, and horizontal minor axis of length 2 units $(a=1)$.

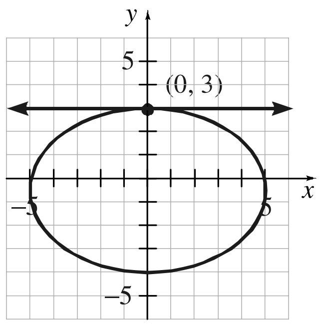


Check each intersection point.

The solution set is $\{(0,-5),(0,5)\}$.

59. $\frac{x^2}{25}+\frac{y^2}{9}=1 \qquad y=3$

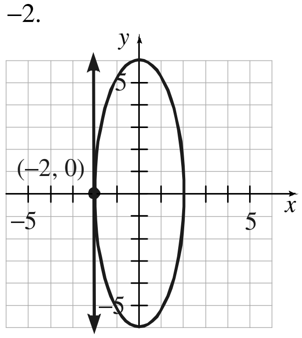
The first equation is for an ellipse centered at the origin with horizontal major axis of length 10 units and vertical minor axis of length 6 units. The second equation is for a horizontal line with a y-intercept of 3.



Check the intersection point.
The solution set is $\{(0,3)\}$.

60. $\frac{x^2}{4}+\frac{y^2}{36}=1 \qquad x=-2$

The first equation is for an ellipse centered at the origin with vertical major axis of length 12 units and horizontal minor axis of length 4 units. The second equation is for a horizontal line with an x-intercept of −2.



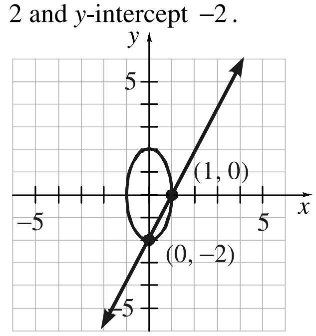
Check the intersection point.
The solution set is $\{(-2,0)\}$.

61. $4x^2+y^2=4 \qquad 2x-y=2$

$\frac{4x^2}{4}+\frac{y^2}{4}=\frac{4}{4} \qquad -y=-2x+2$

$\frac{x^2}{1}+\frac{y^2}{4}=1 \qquad y=2x-2$

The first equation is for an ellipse centered at the origin with vertical major axis of length 4 units ($b=2$) and horizontal minor axis of length 2 units $(a=1)$. The second equation is for a line with slope 2 and y-intercept -2.



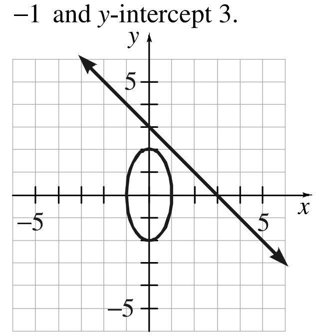
Check the intersection points.
The solution set is $\{(0,-2),(1,0)\}$.

62. $4x^2+y^2=4 \qquad x+y=3$

$\frac{4x^2}{4}+\frac{y^2}{4}=\frac{4}{4} \qquad y=-x+3$

$\frac{x^2}{1}+\frac{y^2}{4}=1$

The first equation is for an ellipse centered at the origin with vertical major axis of length 4 units ($b=2$) and horizontal minor axis of length 2 units $(a=1)$. The second equation is for a line with slope -1 and y-intercept 3.



The two graphs never cross, so there are no intersection points.
The solution set is $\{\ \}$ or $\varnothing$.

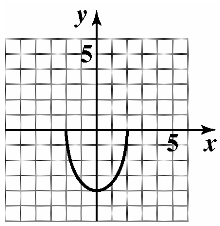
63. $y^2 = \left(-\sqrt{16-4x^2}\right)^2$

$y^2 = 16 - 4x^2$

$4x^2 + y^2 = 16$

$\dfrac{x^2}{4} + \dfrac{y^2}{16} = 1$

We want to graph the bottom half of an ellipse centered at the origin with a vertical major axis of length 8 units ($b = 4$) and horizontal minor axis of length 4 units $(a = 2)$.



$y = -\sqrt{16-4x^2}$

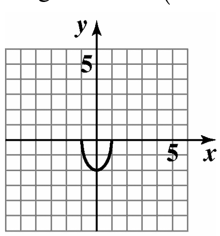
64. $y^2 = \left(-\sqrt{4-4x^2}\right)^2$

$y^2 = 4 - 4x^2$

$4x^2 + y^2 = 4$

$\dfrac{x^2}{1} + \dfrac{y^2}{4} = 1$

We want to graph the bottom half of an ellipse centered at the origin with a vertical major axis of length 4 units ($b = 2$) and horizontal minor axis of length 2 units $(a = 1)$.



$y = -\sqrt{4-4x^2}$

65. $a = 15$, $b = 10$

$\dfrac{x^2}{225} + \dfrac{y^2}{100} = 1$

Let $x = 4$

$\dfrac{4^2}{225} + \dfrac{y^2}{100} = 1$

$900\left(\dfrac{16}{225} + \dfrac{y^2}{100}\right) = 900(1)$

$64 + 9y^2 = 900$

$9y^2 = 836$

$y = \sqrt{\dfrac{836}{9}} \approx 9.64$

Yes, the truck only needs 7 feet so it will clear.

66. $a = 25$, $b = 20$

$\dfrac{x^2}{625} + \dfrac{y^2}{400} = 1$

Let $x = 10$

$\dfrac{(10)^2}{625} + \dfrac{y^2}{400} = 1$

$10{,}000\left(\dfrac{100}{625} + \dfrac{y^2}{400}\right) = 10{,}000(1)$

$1600 + 25y^2 = 10{,}000$

$25y^2 = 8400$

$y = \sqrt{336} \approx 18.3$

Yes, the truck only needs 14 feet so it will clear.

67. **a.** $a = 48$, $a^2 = 2304$
$b = 23$, $b^2 = 529$

$\dfrac{x^2}{2304} + \dfrac{y^2}{529} = 1$

b. $c^2 = a^2 - b^2 = 2304 - 529 = 1775$
$c = \sqrt{1775} \approx 42.13$
He situated his desk about 42 feet from the center of the ellipse, along the major axis.

68. $a = 50$, $b = 30$

$\dfrac{x^2}{50^2} + \dfrac{y^2}{30^2} = 1$

$c^2 = a^2 - b^2$

$= 50^2 - 30^2 = 2500 - 900 = 1600$

$c = 40$

The focus is 40 feet from the center of the room so one person should stand at 10 feet along the 100 foot width and the other person should stand at 90 feet.

69. – 77. Answers may vary.

78. $2a = 186,\ a = 93$

$2b = 185.8,\ a = 92.9$

Earth's orbit:

$$\frac{x^2}{(93)^2} + \frac{y^2}{(92.9)^2} = 1$$

$$y^2 = (92.9)^2\left(1 - \frac{x^2}{(93)^2}\right)$$

$$y = \pm 92.9\sqrt{1 - \frac{x^2}{(93)^2}}$$

$2a = 283.5,\ a = 141.75$

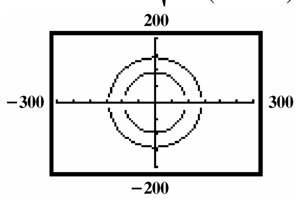
$2b = 278.5,\ b = 139.25$

Mar's orbit:

$$\frac{x^2}{(141.75)^2} + \frac{y^2}{(139.25)^2} = 1$$

$$y^2 = (139.25)^2\left(1 - \frac{x^2}{(141.75)^2}\right)$$

$$y = \pm 139.25\sqrt{1 - \frac{x^2}{(141.75)^2}}$$



79. does not make sense; Explanations will vary. Sample explanation: The foci are on the major axis.

80. does not make sense; Explanations will vary. Sample explanation: An ellipse is symmetrical about both its major and minor axes.

81. does not make sense; Explanations will vary. Sample explanation: We must also know the other vertices.

82. makes sense

83. $a = 6,\ a^2 = 36$

$$\frac{x^2}{b^2} + \frac{y^2}{36} = 1$$

When $x = 2$ and $y = -4$,

$$\frac{2^2}{b^2} + \frac{(-4)^2}{36} = 1$$

$$\frac{4}{b^2} + \frac{16}{36} = 1$$

$$\frac{4}{b^2} = \frac{5}{9}$$

$$36 = 5b^2$$

$$b^2 = \frac{36}{5}$$

$$\frac{x^2}{\frac{36}{5}} + \frac{y^2}{36} = 1$$

84. **a.** The perigee is at the point (5000, 0). If the center of the earth is at (16, 0), and the radius is 4000 miles, the right endpoint of the earth along the major axis is (4016, 0). The perigee is 5000 – 4016 = 984 miles above the earth's surface.

b. The apogee is at the point (–5000, 0). The left endpoint of the earth along the major axis is (–3984, 0). The apogee is $|-5000 - (-3984)|$ = 1016 miles above the earth's surface.

85. The large circle has radius 5 with center (0, 0). Its equation is $x^2 + y^2 = 25$. The small circle has radius 3 with center (0, 0). Its equation is $x^2 + y^2 = 9$.

86. $\frac{c}{a}$ is close to zero when c is very small. This happens when a and b are nearly equal, or when the shape of the graph is nearly circular.

87. $4x^2 - 9y^2 = 36$

$$\frac{4x^2}{36} - \frac{9y^2}{36} = \frac{36}{36}$$

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

The terms are separated by subtraction rather than by addition.

88. $\frac{x^2}{16}-\frac{y^2}{9}=1$

a. Substitute 0 for *y*.

$$\frac{x^2}{16}-\frac{0^2}{9}=1$$
$$\frac{x^2}{16}=1$$
$$x^2=16$$
$$x=\pm 4$$

The *x*-intercepts are -4 and 4.

b. $\frac{0^2}{16}-\frac{y^2}{9}=1$

$$-\frac{y^2}{9}=1$$
$$y^2=-9$$

The equation $y^2=-9$ has no real solutions.

89. $\frac{y^2}{9}-\frac{x^2}{16}=1$

a. Substitute 0 for *x*.

$$\frac{y^2}{9}-\frac{0^2}{16}=1$$
$$\frac{y^2}{9}=1$$
$$y^2=9$$
$$y=\pm 3$$

The *y*-intercepts are -3 and 3.

b. $\frac{0^2}{9}-\frac{x^2}{16}=1$

$$-\frac{x^2}{16}=1$$
$$x^2=-16$$

The equation $x^2=-16$ has no real solutions.

Section 10.2

Check Point Exercises

1. **a.** $a^2=25,\ a=5$

vertices: (5, 0) and (–5, 0)

$b^2=16$

$c^2=a^2+b^2=25+16=41$

$c=\sqrt{41}$

The foci are at $(\sqrt{41},\ 0)$ and $(-\sqrt{41},\ 0)$.

b. $a^2=25,\ a=5$

vertices: (0, 5) and (0, –5)

$b^2=16$

$c^2=a^2+b^2=25+16=41$

$c=\sqrt{41}$

The foci are at $(0,\ \sqrt{41})$ and $(0,\ -\sqrt{41})$.

2. $a=3,\ c=5$

$b^2=c^2-a^2=25-9=16$

$\frac{y^2}{9}-\frac{x^2}{16}=1$

3. $a^2=36,\ a=6$

The vertices are (6, 0) and (-6, 0).

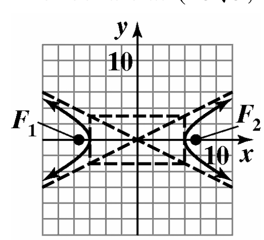
$b^2=9,\ b=3$

asymptotes: $y=\pm\frac{b}{a}x=\pm\frac{3}{6}x=\pm\frac{1}{2}x$

$c^2=a^2+b^2=36+9=45$

$c=\sqrt{45}=3\sqrt{5}$

The foci are at $(-3\sqrt{5},\ 0)$ and $(3\sqrt{5},\ 0)$.



$\frac{x^2}{36}-\frac{y^2}{9}=1$

4. $\frac{y^2}{4}-\frac{4x^2}{4}=\frac{4}{4}$

$\frac{y^2}{4}-x^2=1$

$a^2=4,\ a=2$

The vertices are (0, 2) and (0, –2).

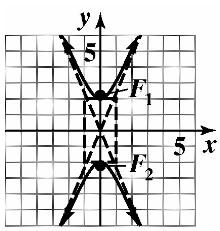
$b^2=1,\ b=1$

asymptotes: $y=\pm\frac{a}{b}x=\pm 2x$

$c^2=a^2+b^2=4+1=5$

$c=\sqrt{5}$

The foci are at $(0,\sqrt{5})$ and $(0,-\sqrt{5})$.



5. center at (3, 1)

$a^2=4,\ a=2$

$b^2=1,\ b=1$

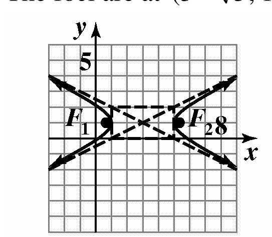
The vertices are (1, 1) and (5, 1).

asymptotes: $y-1=\pm\frac{1}{2}(x-3)$

$c^2=a^2+b^2=4+1=5$

$c=\sqrt{5}$

The foci are at $(3-\sqrt{5},1)$ and $(3+\sqrt{5},1)$.



6. $4\left(x^2-6x\quad\right)-9\left(y^2+10y\quad\right)=153$

$$4\left(x^2-6x+9\right)-9\left(y^2+10y+25\right)=153+36+(-225)$$

$$4(x-3)^2-9(y+5)^2=-36$$

$$\frac{4(x-3)^2}{-36}-\frac{9(y+5)^2}{-36}=\frac{-36}{-36}$$

$$-\frac{(x-3)^2}{9}+\frac{(y+5)^2}{4}=1$$

$$\frac{(y+5)^2}{4}-\frac{(x-3)^2}{9}=1$$

center at (3, –5)

$a^2=4,\ a=2$

$b^2=9,\ b=3$

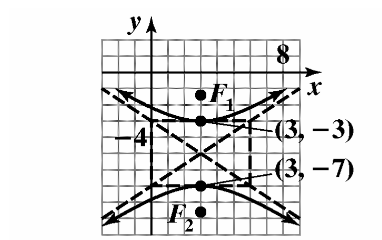
The vertices are (3, –3) and (3, –7).

asymptotes: $y+5=\pm\frac{2}{3}(x-3)$

$c^2=a^2+b^2=4+9=13$

$c=\sqrt{13}$

The foci are at $(3,-5-\sqrt{13})$ and $(3,-5+\sqrt{13})$.



7. $c=5280$

$2a=3300, a=1650$

$b^2=c^2-a^2=5280^2-1650^2=25{,}155{,}900$

The explosion occurred somewhere at the right branch of the hyperbola given by

$\frac{x^2}{2{,}722{,}500}-\frac{y^2}{25{,}155{,}900}=1.$

Exercise Set 10.2

1. $a^2 = 4$, a = 2
The vertices are (2, 0) and (–2, 0).
$b^2 = 1$
$c^2 = a^2 + b^2 = 4 + 1 = 5$
$c = \sqrt{5}$
The foci are located at $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$.
graph (b)

2. $a^2 = 1$, a = 1
The vertices are (1, 0) and (–1, 0).
$b^2 = 4$
$c^2 = a^2 + b^2 = 4 + 1 = 5$, $c = \sqrt{5}$
The foci are at $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$.
graph (d)

3. $a^2 = 4$, $a = 2$
The vertices are (0, 2) and (0, –2).
$b^2 = 1$
$c^2 = a^2 + b^2 = 4 + 1 = 5$
$c = \sqrt{5}$
The foci are located at
$(0, \sqrt{5})$ and $(0, -\sqrt{5})$.
graph (a)

4. $a^2 = 1$, a = 1
The vertices are (0, 1) and (0, –1).
$b^2 = 4$
$c^2 = a^2 + b^2 = 1 + 4 = 5$, $c = \sqrt{5}$
The foci are at $(0, \sqrt{5})$ and $(0, -\sqrt{5})$.
graph (c)

5. $a = 1$, $c = 3$
$b^2 = c^2 - a^2 = 9 - 1 = 8$
$$y^2 - \frac{x^2}{8} = 1$$

6. $a = 2$, $c = 6$
$b^2 = c^2 - a^2 = 36 - 4 = 32$
$$\frac{y^2}{4} - \frac{x^2}{32} = 1$$

7. $a = 3$, $c = 4$
$b^2 = c^2 - a^2 = 16 - 9 = 7$
$$\frac{x^2}{9} - \frac{y^2}{7} = 1$$

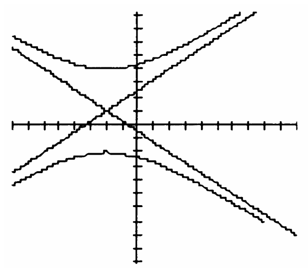
8. $a = 5$, $c = 7$
$b^2 = c^2 - a^2 = 49 - 25 = 24$
$$\frac{x^2}{25} - \frac{y^2}{24} = 1$$

9. $2a = 6 - (-6)$
$2a = 12$
$a = 6$
$\frac{a}{b} = 2$
$\frac{6}{b} = 2$
$6 = 2b$
$3 = b$
Transverse axis is vertical.
$$\frac{y^2}{36} - \frac{x^2}{9} = 1$$

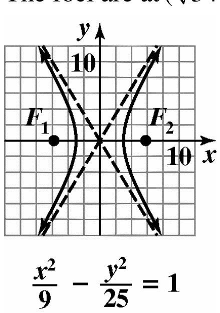
10. $a = 4$
$\frac{b}{a} = 2$
$\frac{b}{4} = 2$
$b = 8$
Transverse axis is horizontal.
$$\frac{x^2}{16} - \frac{y^2}{64} = 1$$

11. $a = 2$, c = 7 – 4 = 3
$2^2 + b^2 = 3^2$
$4 + b^2 = 9$
$b^2 = 5$
Transverse axis is horizontal.
$$\frac{(x-4)^2}{4} - \frac{(y+2)^2}{5} = 1$$

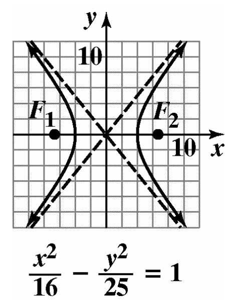
12. $a = 4 - 1 = 3$
$c = 6 - 1 = 5$
$3^2 + b^2 = 5^2$
$9 + b^2 = 25$
$b^2 = 16$
$b = 4$
$$\frac{(y-1)^2}{9} - \frac{(x+2)^2}{16} = 1$$
asymptotes: $y - 1 = \pm\frac{3}{4}(x+2)$
Transverse axis is vertical.



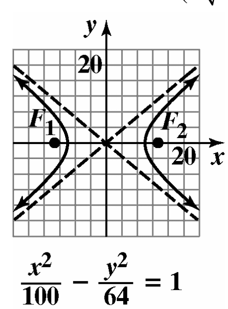
13. $a^2 = 9$, $a = 3$
$b^2 = 25$, $b = 5$
vertices: (3, 0) and (–3, 0)
asymptotes: $y = \pm\frac{b}{a}x = \pm\frac{5}{3}x$
$c^2 = a^2 + b^2 = 9 + 25 = 34$
$c = \sqrt{34}$ on x-axis
The foci are at $(\sqrt{34}, 0)$ and $(-\sqrt{34}, 0)$.



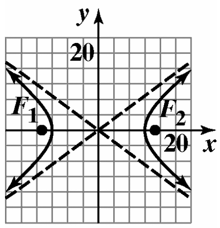
14. $a^2 = 16$, $a = 4$
The vertices are (4, 0) and (–4, 0).
$b^2 = 25$, $b = 5$
asymptotes: $y = \pm\frac{b}{a}x = \pm\frac{5}{4}x$
$c^2 = a^2 + b^2 = 16 + 25 = 41$, $c = \sqrt{41}$ on x-axis
The foci are at $(\sqrt{41}, 0)$ and $(-\sqrt{41}, 0)$.



15. $a^2 = 100$, $a = 10$
$b^2 = 64$, $b = 8$
vertices: (10, 0) and (–10, 0)
asymptotes: $y = \pm\frac{b}{a}x = \pm\frac{8}{10}x$
or $y = \pm\frac{4}{5}x$
$c^2 = a^2 + b^2 = 100 + 64 = 164$
$c = \sqrt{164} = 2\sqrt{41}$ on x-axis
The foci are at $(2\sqrt{41}, 0)$ and $(-2\sqrt{41}, 0)$.

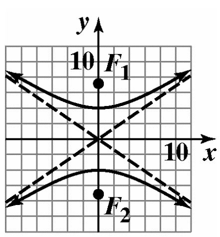


16. $a^2 = 144,\ a = 12$
$b^2 = 81,\ b = 9$
The vertices are (12, 0) and (–12, 0).
asymptotes: $y = \pm\frac{b}{a}x = \pm\frac{3}{4}x$
$c^2 = a^2 + b^2 = 144 + 81 = 225$
$c = 15$ on x-axis
The foci are at (15, 0) and (–15, 0).



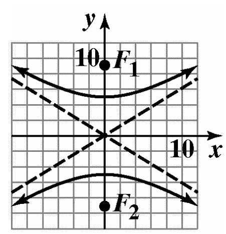
$$\frac{x^2}{144} - \frac{y^2}{81} = 1$$

17. $a^2 = 16,\ a = 4$
$b^2 = 36,\ b = 6$
vertices: (0, 4) and (0, –4)
asymptotes: $y = \pm\frac{a}{b}x = \pm\frac{4}{6}x = \pm\frac{2}{3}x$
or $y = \pm\frac{2}{3}x$
$c^2 = a^2 + b^2 = 16 + 36 = 52$
$c = \sqrt{52} = 2\sqrt{13}$ on y-axis
The foci are at $(0,\ 2\sqrt{13})$ and $(0,\ -2\sqrt{13})$.



$$\frac{y^2}{16} - \frac{x^2}{36} = 1$$

18. $a^2 = 25,\ a = 5$
$b^2 = 64,\ b = 8$
The vertices are (0, 5) and (0, –5).
asymptotes: y = $\pm\frac{a}{b}x = \pm\frac{5}{8}x$
$c^2 = a^2 + b^2 = 25 + 64 = 89$
$c = \sqrt{89}$ on y-axis
The foci are at $(0,\ \sqrt{89})$ and $(0,\ -\sqrt{89})$.



$$\frac{y^2}{25} - \frac{x^2}{64} = 1$$

19. $\dfrac{y^2}{\frac{1}{4}} - x^2 = 1$

$a^2 = \frac{1}{4}, a = \frac{1}{2}$

$b^2 = 1, b = 1$

$c^2 = a^2 + b^2$

$c^2 = \frac{1}{4} + 1$

$c^2 = \frac{5}{4}$

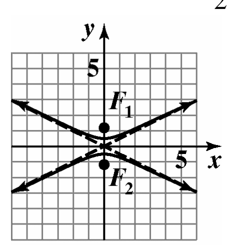
$c = \pm\frac{\sqrt{5}}{2}$

$c \approx \pm 1.1$

The foci are located at $\left(0, \frac{\sqrt{5}}{2}\right)$ and $\left(0, -\frac{\sqrt{5}}{2}\right)$.

asymptotes: $y = \pm\dfrac{\frac{1}{2}}{1}x$

$y = \pm\frac{1}{2}x$



$$4y^2 - x^2 = 1$$

20. $\dfrac{y^2}{\frac{1}{9}} - \dfrac{x^2}{1} = 1$

$a = \dfrac{1}{3}, b = 1$

$c^2 = \dfrac{1}{9} + 1$

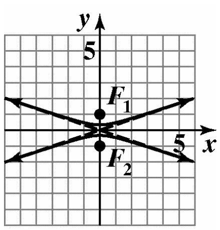
$c^2 = \dfrac{10}{9}$

$c = \pm\dfrac{\sqrt{10}}{3}$

$c = \pm 1.1$

The foci are (0, 1.1) and (0, –1.1).

Asymptote: $y = \pm\dfrac{\frac{1}{3}}{1}x = \pm\dfrac{1}{3}x$



$9y^2 - x^2 = 1$

21. $\dfrac{9x^2}{36} - \dfrac{4y^2}{36} = \dfrac{36}{36}$

$\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$

$a^2 = 4, a = 2$

$b^2 = 9, b = 3$

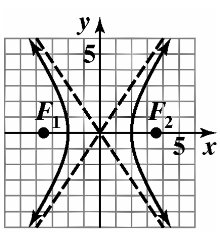
vertices: (2, 0) and (–2, 0)

asymptotes: $y = \pm\dfrac{b}{a}x = \pm\dfrac{3}{2}x$

$c^2 = a^2 + b^2 = 4 + 9 = 13$

$c = \sqrt{13}$ on x-axis

The foci are at $(\sqrt{13}, 0)$ and $(-\sqrt{13}, 0)$.



$9x^2 - 4y^2 = 36$

22. $\dfrac{4x^2}{100} - \dfrac{25y^2}{100} = \dfrac{100}{100}$

$\dfrac{x^2}{25} - \dfrac{y^2}{4} = 1$

$a^2 = 25, a = 5$

$b^2 = 4, b = 2$

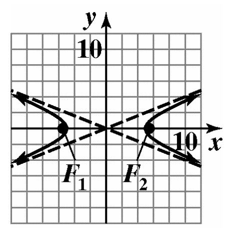
The vertices are (5, 0) and (–5, 0).

asymptotes: $y = \pm\dfrac{b}{a}x = \pm\dfrac{2}{5}x$

$c^2 = a^2 + b^2 = 25 + 4 = 29$

$c = \sqrt{29}$ on x-axis

The foci are at $(\sqrt{29}, 0)$ and $(-\sqrt{29}, 0)$.



$4x^2 - 25y^2 = 100$

23. $\dfrac{9y^2}{225} - \dfrac{25x^2}{225} = \dfrac{225}{225}$

$\dfrac{y^2}{25} - \dfrac{x^2}{9} = 1$

$a^2 = 25, a = 5$

$b^2 = 9, b = 3$

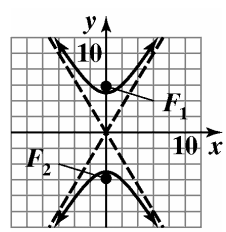
vertices: (0, 5) and (0, –5)

asymptotes: $y = \pm\dfrac{a}{b}x = \pm\dfrac{5}{3}x$

$c^2 = a^2 + b^2 = 25 + 9 = 34$

$c = \sqrt{34}$ on y-axis

The foci are at $(0, \sqrt{34})$ and $(0, -\sqrt{34})$.



$9y^2 - 25x^2 = 225$

24. $\frac{16y^2}{144}-\frac{9x^2}{144}=\frac{144}{144}$

$\frac{y^2}{9}-\frac{x^2}{16}=1$

$a^2=9,\ a=3$

$b^2=16,\ b=4$

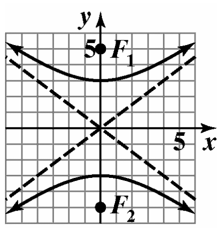
The vertices are (0, 3) and (0, –3).

asymptotes: $y=\pm\frac{a}{b}x=\pm\frac{3}{4}x$

$c^2=a^2+b^2=9+16=25$

$c=5$ on y-axis

The foci are at $(0,\ 5)$ and $(0,\ -5)$.



$16y^2-9x^2=144$

25. $y^2=x^2-2$

$2=x^2-y^2$

$1=\frac{x^2}{2}-\frac{y^2}{2}$

$a^2=2, a=\sqrt{2}$

$b^2=2, b=\sqrt{2}$

$c^2=2+2$

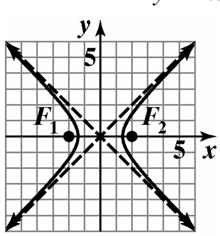
$c^2=4$

$c=2$

The foci are located at (2,0) and (–2, 0).

asymptotes: $y=\pm\frac{\sqrt{2}}{\sqrt{2}}x$

$y=\pm x$



$y=\pm\sqrt{x^2-2}$

26. $y^2=x^2-3$

$3=x^2-y^2$

$1=\frac{x^2}{3}-\frac{y^2}{3}$

Vertices: $(\sqrt{3},0)$ and $(-\sqrt{3},0)$.

Asymptotes: $y=\pm\frac{\sqrt{3}}{\sqrt{3}}x$

$y=\pm x$

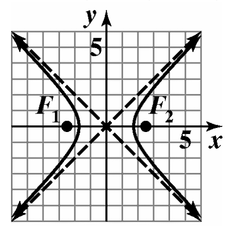
$c^2=3+3$

$c^2=6$

$c=\pm\sqrt{6}$

$c\approx\pm 2.4$

Foci: (2.4, 0) and (–2.4, 0).



$y=\pm\sqrt{x^2-3}$

27. $a=3,\ b=5$

$\frac{x^2}{9}-\frac{y^2}{25}=1$

28. $a=3,\ b=2$

$\frac{x^2}{9}-\frac{y^2}{4}=1$

29. $a=2,\ b=3$

$\frac{y^2}{4}-\frac{x^2}{9}=1$

30. $a=5,\ b=3$

$\frac{y^2}{25}-\frac{x^2}{9}=1$

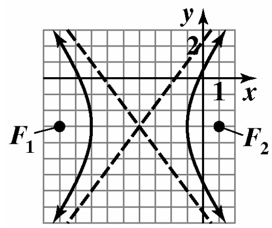
31. Center (2, –3), $a=2$, $b=3$

$\frac{(x-2)^2}{4}-\frac{(y+3)^2}{9}=1$

32. Center (–1, –2) $a=2$, $b=2$

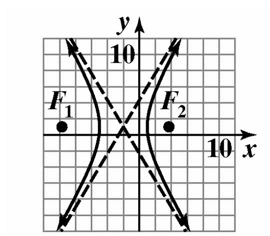
$\frac{(x+1)^2}{4}-\frac{(y+2)^2}{4}=1$

33. center: (–4, –3)
$a^2 = 9, a = 3$
$b^2 = 16, b = 4$
vertices: (–7, –3) and (–1, –3)
asymptotes: $y + 3 = \pm\frac{4}{3}(x+4)$
$c^2 = a^2 + b^2 = 9 + 16 = 25$
$c = \pm 5$ parallel to x-axis
The foci are at (–9, –3) and (1, –3).



$$\frac{(x+4)^2}{9} - \frac{(y+3)^2}{16} = 1$$

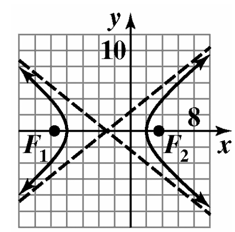
34. The center is located at (–2, 1).
$a^2 = 9, a = 3$
$b^2 = 25, b = 5$
The vertices are (–5, 1) and (1, 1).
asymptotes: $y - 1 = \pm\frac{5}{3}(x+2)$
$c^2 = a^2 + b^2 = 9 + 25 = 34$
$c = \sqrt{34}$ parallel to x-axis
The foci are located at
$(-2+\sqrt{34}, 1)$ and $(-2-\sqrt{34}, 1)$.



$$\frac{(x+2)^2}{9} - \frac{(y-1)^2}{25} = 1$$

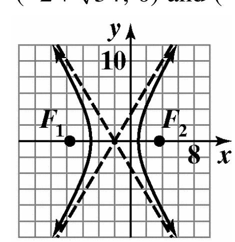
35. center: (–3, 0)
$a^2 = 25, a = 5$
$b^2 = 16, b = 4$
vertices: (2, 0) and (–8, 0)
asymptotes: $y = \pm\frac{4}{5}(x+3)$
$c^2 = a^2 + b^2 = 25 + 16 = 41$
$c = \sqrt{41}$

The foci are at $(-3+\sqrt{41}, 0)$ and $(-3-\sqrt{41}, 0)$.



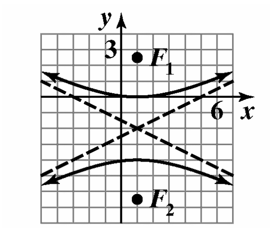
$$\frac{(x+3)^2}{25} - \frac{y^2}{16} = 1$$

36. The center is located at (–2, 0).
$a^2 = 9, a = 3$
$b^2 = 25, b = 5$
The vertices are (–5, 0) and (1, 0).
asymptotes: $y = \pm\frac{5}{3}(x+2)$
$c^2 = a^2 + b^2 = 9 + 25 = 34$
$c = \sqrt{34}$ parallel to x-axis
The foci are located at
$(-2+\sqrt{34}, 0)$ and $(-2-\sqrt{34}, 0)$.



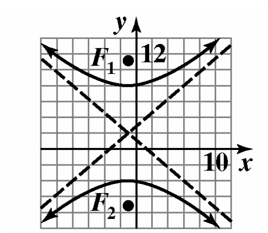
$$\frac{(x+2)^2}{9} - \frac{y^2}{25} = 1$$

37. center: (1, –2)
$a^2 = 4, a = 2$
$b^2 = 16, b = 4$
vertices: (1, 0) and (1, –4)
asymptotes: $y+2=\pm\frac{1}{2}(x-1)$
$c^2 = a^2 + b^2 = 4 + 16 = 20$
$c = \sqrt{20} = 2\sqrt{5}$ parallel to y-axis
The foci are at $(1, -2+2\sqrt{5})$ and $(1, -2-2\sqrt{5})$.



$$\frac{(y+2)^2}{4} - \frac{(x-1)^2}{16} = 1$$

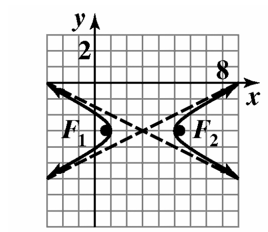
38. The center is located at (–1, 2).
$a^2 = 36, a = 6$
$b^2 = 49, b = 7$
The vertices are (–1, 8) and (–1, –4).
asymptotes: $y - 2 = \pm\frac{6}{7}(x+1)$
$c^2 = a^2 + b^2 = 36 + 49 = 85$
$c = \sqrt{85}$ parallel to y-axis
The foci are located at
$(-1, 2+\sqrt{85})$ and $(-1, 2-\sqrt{85})$.



$$\frac{(y-2)^2}{36} - \frac{(x+1)^2}{49} = 1$$

39. $\frac{(x-3)^2}{4} - \frac{4(y+3)^2}{4} = \frac{4}{4}$

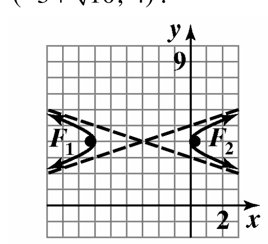
$\frac{(x-3)^2}{4} - (y+3)^2 = 1$
center: (3, –3)
$a^2 = 4, a = 2$
$b^2 = 1, b = 1$
vertices: (1, –3) and (5, –3)
asymptotes: $y+3=\pm\frac{1}{2}(x-3)$
$c^2 = a^2 + b^2 = 4 + 1 = 5$
$c = \sqrt{5}$
The foci are at $(3+\sqrt{5}, -3)$ and $(3-\sqrt{5}, -3)$.



$$(x-3)^2 - 4(y+3)^2 = 4$$

40. $\frac{(x+3)^2}{9} - \frac{9(y-4)^2}{9} = \frac{9}{9}$

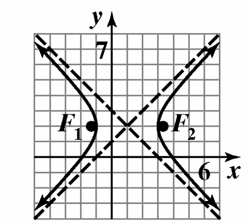
$\frac{(x+3)^2}{9} - (y-4)^2 = 1$
The center is located at (–3, 4).
$a^2 = 9, a = 3$
$b^2 = 1, b = 1$
The vertices are (–6, 4) and (0, 4).
asymptotes: $y - 4 = \pm\frac{1}{3}(x+3)$
$c^2 = a^2 + b^2 = 9 + 1 = 10$
$c = \sqrt{10}$ parallel to x-axis
The foci are located at $(-3-\sqrt{10}, 4)$ and
$(-3+\sqrt{10}, 4)$.



$$(x+3)^2 - 9(y-4)^2 = 9$$

41. $\frac{(x-1)^2}{3} - \frac{(y-2)^2}{3} = 1$

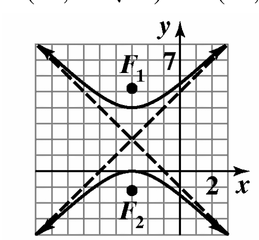
center: (1, 2)
$a^2 = 3,\ a = \sqrt{3}$
$b^2 = ,\ b = \sqrt{3}$
vertices: (–1, 2) and (3, 2)
asymptotes: $y - 2 = \pm(x - 1)$
$c^2 = a^2 + b^2 = 3 + 3 = 6$
$c = \sqrt{6}$ parallel to y-axis
The foci are at $(1+\sqrt{6}, 2)$ and $(1-\sqrt{6}, 2)$.



$(x - 1)^2 - (y - 2)^2 = 3$

42. $\frac{(y-2)^2}{5} - \frac{(x+3)^2}{5} = 1$

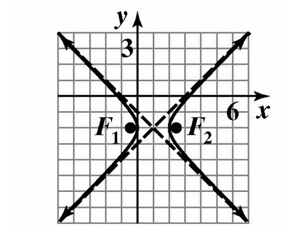
The center is located at (–3, 2).
$a^2 = 5,\ a = \sqrt{5}$
$b^2 = 5,\ b = \sqrt{5}$
The vertices are (–3, 0) and (–3, 4).
asymptotes: $y - 2 = \pm(x+3)$
$c^2 = a^2 + b^2 = 5 + 5 = 10$
$c = \sqrt{10}$ parallel to y-axis
The foci are located
at $(-3, 2+\sqrt{10})$ and $(-3, 2-\sqrt{10})$.



$(y - 2)^2 - (x + 3)^2 = 5$

43.
$$(x^2 - 2x) - (y^2 + 4y) = 4$$
$$(x^2 - 2x + 1) - (y^2 + 4y + 4) = 4 + 1 - 4$$
$$(x - 1)^2 - (y + 2)^2 = 1$$

center: (1, –2)
$a^2 = 1,\ a = 1$
$b^2 = 1,\ b = 1$
$c^2 = a^2 + b^2 = 1 + 1 = 2$
$c = \sqrt{2}$
asymptotes: $y + 2 = \pm(x - 1)$
The foci are at $(1+\sqrt{2}, -2)$ and $(1-\sqrt{2}, -2)$.

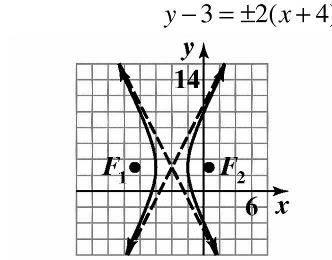


$x^2 - y^2 - 2x - 4y - 4 = 0$

44.
$$(4x^2 + 32x) - (y^2 - 6y) = -39$$
$$4(x^2 + 8x + 16) - (y^2 - 6y + 9) = -39 + 64 - 9$$
$$4(x + 4)^2 - (y - 3)^2 = 16$$
$$\frac{(x+4)^2}{4} - \frac{(y-3)^2}{16} = 1$$

center: (–4, 3)
$a^2 = 4,\ a = 2$
$b^2 = 16,\ b = 4$
$c^2 = a^2 + b^2 = 4 + 16 = 20$
$c = \sqrt{20} = 2\sqrt{5}$
The foci are at $(-4+2\sqrt{5}, 3)$ and $(-4-2\sqrt{5}, 3)$.

Asymptotes: $y - 3 = \pm\frac{4}{2}(x+4)$
$y - 3 = \pm 2(x+4)$



$4x^2 - y^2 + 32x + 6y + 39 = 0$

45. $(16x^2 + 64x) - (y^2 + 2y) = -67$

$16(x^2 + 4x + 4) - (y^2 + 2y + 1) = -67 + 64 - 1$

$16(x+2)^2 - (y+1)^2 = -4$

$$\frac{16(x+2)^2}{-4} - \frac{(y+1)^2}{-4} = \frac{-4}{-4}$$

$$\frac{(y+1)^2}{4} - \frac{(x+2)^2}{\frac{1}{4}} = 1$$

center: $(-2, -1)$

$a^2 = 4,\ a = 2$

$b^2 = \frac{1}{4},\ b = \frac{1}{2}$

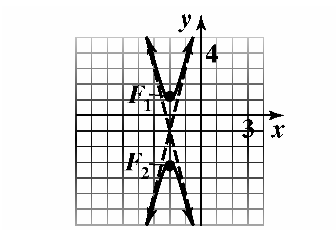
$c^2 = a^2 + b^2 = 4 + \frac{1}{4} = \frac{17}{4}$

$c = \sqrt{\frac{17}{4}} = \sqrt{4.25}$

asymptotes: $(y+1) = \pm\frac{2}{\frac{1}{2}}(x+2)$

$y+1 = \pm 4(x+2)$

The foci are at $\left(-2, -1+\sqrt{4.25}\right)$ and $\left(-2, -1-\sqrt{4.25}\right)$.



$16x^2 - y^2 + 64x - 2y + 67 = 0$

46. $(9y^2 - 18y) - (4x^2 - 24x) = 63$

$9(y^2 - 2y + 1) - 4(x^2 - 6x + 9) = 63 + 9 - 36$

$9(y-1)^2 - 4(x-3)^2 = 36$

$$\frac{(y-1)^2}{4} - \frac{(x-3)^2}{9} = 1$$

The center is located at $(3, 1)$.

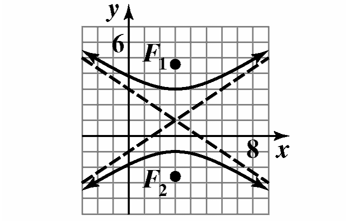
$a^2 = 4,\ a = 2$

$b^2 = 9,\ b = 3$

$c^2 = a^2 + b^2 = 4 + 9 = 13,\ c = \sqrt{13}$

The foci are at $(3, 1+\sqrt{13})$ and $(3, 1-\sqrt{13})$.

Asymptotes: $y - 1 = \pm\frac{2}{3}(x-3)$



$9y^2 - 4x^2 - 18y + 24x - 63 = 0$

47. $(4x^2 - 16x) - (9y^2 - 54y) = 101$

$4(x^2 - 4x + 4) - 9(y^2 - 6y + 9) = 101 + 16 - 81$

$4(x-2)^2 - 9(y-3)^2 = 36$

$$\frac{(x-2)^2}{9} - \frac{(y-3)^2}{4} = 1$$

center: $(2, 3)$

$a^2 = 9,\ a = 3$

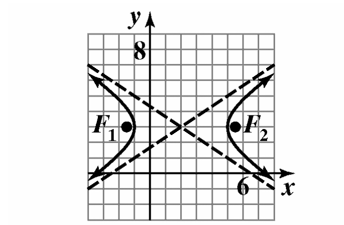
$b^2 = 4,\ b = 2$

$c^2 = a^2 + b^2 = 9 + 4 = 13$

$c = \sqrt{13}$

asymptotes: $y - 3 = \pm\frac{2}{3}(x-2)$

The foci are at $(2+\sqrt{13}, 3)$ and $(2-\sqrt{13}, 3)$.



$4x^2 - 9y^2 - 16x + 5y - 101 = 0$

48. $(4x^2 + 8x) - (9y^2 + 18y) = 6$

$4(x^2 + 2x + 1) - 9(y^2 + 2y + 1) = 6 + 4 - 9$

$4(x+1)^2 - 9(y+1)^2 = 1$

$$\frac{(x+1)^2}{\frac{1}{4}} - \frac{(y+1)^2}{\frac{1}{9}} = 1$$

The center is located at $(-1, -1)$.

$a^2 = \frac{1}{4},\ a = \frac{1}{2}$

$b^2 = \frac{1}{9},\ b = \frac{1}{3}$

$c^2 = a^2 + b^2 = \frac{1}{4} + \frac{1}{9} = \frac{13}{36},\ c = \frac{\sqrt{13}}{6}$

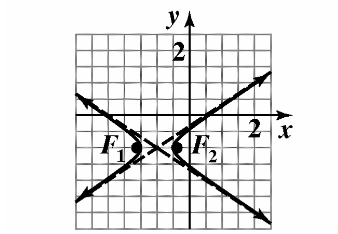
The foci are at

$\left(-1+\frac{\sqrt{13}}{6}, -1\right)$ and $\left(-1-\frac{\sqrt{13}}{6}, -1\right)$.

Asymptotes:

$$y+1=\pm\frac{\frac{1}{3}}{\frac{1}{2}}(x+1)$$

$$y+1=\pm\frac{2}{3}(x+1)$$



$\mathbf{4x^2 - 9y^2 + 8x - 18y - 6 = 0}$

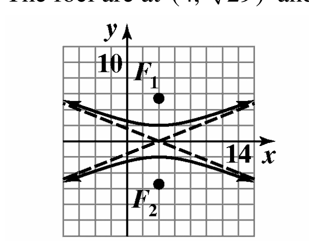
49.

$$(4x^2-32x)-25y^2=-164$$
$$4(x^2-8x+16)-25y^2=-164+64$$
$$4(x-4)^2-25y^2=-100$$
$$\frac{4(x-4)^2}{-100}-\frac{25y^2}{-100}=\frac{-100}{-100}$$
$$\frac{y^2}{4}-\frac{(x-4)^2}{25}=1$$

center: (4, 0)
$a^2 = 4$, $a = 2$
$b^2 = 25$, $b = 5$
$c^2 = a^2 + b^2 = 4 + 25 = 29$
$c=\sqrt{29}$

asymptotes: $y=\pm\frac{2}{5}(x-4)$

The foci are at $(4, \sqrt{29})$ and $(4, -\sqrt{29})$.



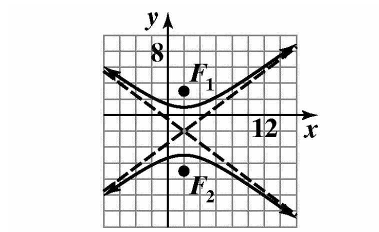
$\mathbf{4x^2 - 25y^2 - 32x + 164 = 0}$

50.

$$(9x^2-36x)-(16y^2+64y)=-116$$
$$9(x^2-4x+4)-16(y^2+4y+4)=-116+36-64$$
$$9(x-2)^2-16(y+2)^2=-144$$
$$\frac{9(x-2)^2}{-144}-\frac{16(y+2)^2}{-144}=\frac{-144}{-144}$$
$$\frac{(y+2)^2}{9}-\frac{(x-2)^2}{16}=1$$

The center is located at (2, –2).
$a^2 = 9$, $a = 3$
$b^2 = 16$, $b = 4$
$c^2 = a^2 + b^2 = 9 + 16 = 25$, c = 5
The foci are at (2, –7) and (2, 3).

Asymptotes: $y+2=\pm\frac{3}{4}(x-2)$



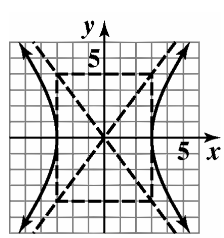
$\mathbf{9x^2 - 16y^2 - 36x - 64y + 116 = 0}$

51. $\frac{x^2}{9}-\frac{y^2}{16}=1$

The equation is for a hyperbola in standard form with the transverse axis on the x-axis. We have $a^2=9$ and $b^2=16$, so $a=3$ and $b=4$.

Therefore, the vertices are at $(\pm a, 0)$ or $(\pm 3, 0)$.

Using a dashed line, we construct a rectangle using the ± 3 on the x-axis and ± 4 on the y-axis. Then use dashed lines to draw extended diagonals for the rectangle. These represent the asymptotes of the graph.



$\mathbf{\frac{x^2}{9} - \frac{y^2}{16} = 1}$

From the graph we determine the following:

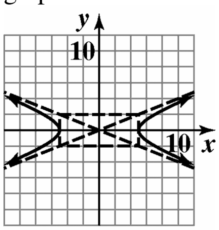
Domain: $\{x \mid x \le -3 \text{ or } x \ge 3\}$ or $(-\infty,-3] \cup [3,\infty)$

Range: $\{y \mid y \text{ is a real number}\}$ or $(-\infty,\infty)$

52. $\frac{x^2}{25}-\frac{y^2}{4}=1$

The equation is for a hyperbola in standard form with the transverse axis on the *x*-axis. We have $a^2=25$ and $b^2=4$, so $a=5$ and $b=2$. Therefore, the vertices are at $(\pm a,0)$ or $(\pm 5,0)$.

Using a dashed line, we construct a rectangle using the ± 5 on the *x*-axis and ± 2 on the *y*-axis. Then use dashed lines to draw extended diagonals for the rectangle. These represent the asymptotes of the graph.



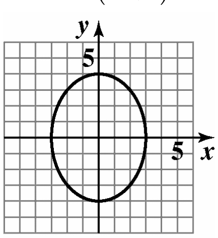
$\mathbf{\frac{x^2}{25}-\frac{y^2}{4}=1}$

From the graph we determine the following:
Domain: $\{x\,|\,x\le -5 \text{ or } x\ge 5\}$ or $(-\infty,-5]\cup[5,\infty)$

Range: $\{y\,|\,y \text{ is a real number}\}$ or $(-\infty,\infty)$

53. $\frac{x^2}{9}+\frac{y^2}{16}=1$

The equation is for an ellipse in standard form with major axis along the y-axis. We have $a^2=16$ and $b^2=9$, so $a=4$ and $b=3$. Therefore, the vertices are $(0,\pm a)$ or $(0,\pm 4)$. The endpoints of the minor axis are $(\pm b,0)$ or $(\pm 3,0)$.



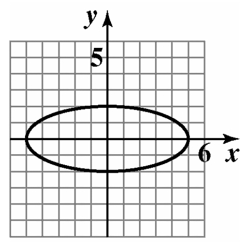
$\mathbf{\frac{x^2}{9}+\frac{y^2}{16}=1}$

From the graph we determine the following:
Domain: $\{x\,|-3\le x\le 3\}$ or $[-3,3]$

Range: $\{y\,|-4\le y\le 4\}$ or $[-4,4]$.

54. $\frac{x^2}{25}+\frac{y^2}{4}=1$

The equation is for an ellipse in standard form with major axis along the y-axis. We have $a^2=25$ and $b^2=4$, so $a=5$ and $b=2$. Therefore, the vertices are $(\pm a,0)$ or $(\pm 5,0)$. The endpoints of the minor axis are $(0,\pm b)$ or $(0,\pm 2)$.



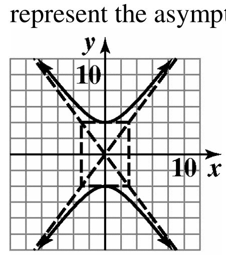
$\mathbf{\frac{x^2}{25}+\frac{y^2}{4}=1}$

From the graph we determine the following:
Domain: $\{x\,|-5\le x\le 5\}$ or $[-5,5]$

Range: $\{y\,|-2\le y\le 2\}$ or $[-2,2]$.

55. $\frac{y^2}{16}-\frac{x^2}{9}=1$

The equation is in standard form with the transverse axis on the *y*-axis. We have $a^2=16$ and $b^2=9$, so $a=4$ and $b=3$. Therefore, the vertices are at $(0,\pm a)$ or $(0,\pm 4)$. Using a dashed line, we construct a rectangle using the ± 4 on the *y*-axis and ± 3 on the *x*-axis. Then use dashed lines to draw extended diagonals for the rectangle. These represent the asymptotes of the graph.



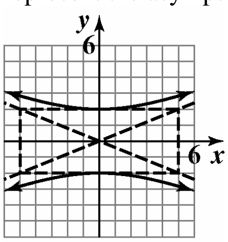
$\mathbf{\frac{y^2}{16}-\frac{x^2}{9}=1}$

From the graph we determine the following:
Domain: $\{x\,|\,x \text{ is a real number}\}$ or $(-\infty,\infty)$

Range: $\{y\,|\,y\le -4 \text{ or } y\ge 4\}$ or $(-\infty,-4]\cup[4,\infty)$

56. $\frac{y^2}{4}-\frac{x^2}{25}=1$

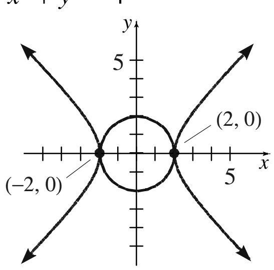
The equation is in standard form with the transverse axis on the *y*-axis. We have $a^2=4$ and $b^2=25$, so $a=2$ and $b=5$. Therefore, the vertices are at $(0,\pm a)$ or $(0,\pm 2)$. Using a dashed line, we construct a rectangle using the ± 2 on the *y*-axis and ± 5 on the *x*-axis. Then use dashed lines to draw extended diagonals for the rectangle. These represent the asymptotes of the graph.



$\frac{y^2}{4}-\frac{x^2}{25}=1$

From the graph we determine the following:
Domain: $\{x \mid x \text{ is a real number}\}$ or $(-\infty,\infty)$
Range: $\{y \mid y \le -2 \text{ or } y \ge 2\}$ or $(-\infty,-2]\cup[2,\infty)$

57. $x^2-y^2=4$
$x^2+y^2=4$



Check $(-2,0)$:

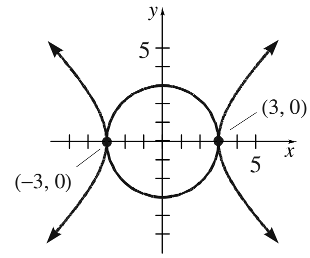
$(-2)^2-0^2=4$ $\quad$ $(-2)^2+0^2=4$
$4-0=4$ $\quad$ $4+0=4$
$4=4$ true $\quad$ $4=4$ true

Check $(2,0)$:

$(2)^2-0^2=4$ $\quad$ $(2)^2+0^2=4$
$4-0=4$ $\quad$ $4+0=4$
$4=4$ true $\quad$ $4=4$ true

The solution set is $\{(-2,0),(2,0)\}$.

58. $x^2-y^2=9$
$x^2+y^2=9$



Check $(-3,0)$:

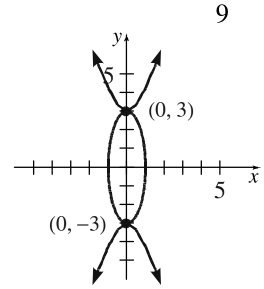
$(-3)^2-0^2=9$ $\quad$ $(-3)^2+0^2=9$
$9-0=9$ $\quad$ $9+0=9$
$9=9$ true $\quad$ $9=9$ true

Check $(3,0)$:

$(3)^2-0^2=9$ $\quad$ $(3)^2+0^2=9$
$9-0=9$ $\quad$ $9+0=9$
$9=9$ true $\quad$ $9=9$ true

The solution set is $\{(-3,0),(3,0)\}$.

59. $9x^2+y^2=9$ or $\frac{x^2}{1}+\frac{y^2}{9}=1$
$y^2-9x^2=9$ $\quad$ $\frac{y^2}{9}-\frac{x^2}{1}=1$



Check $(0,-3)$:

$9(0)^2+(-3)^2=9$ $\quad$ $(-3)^2-9(0)^2=9$
$0+9=9$ $\quad$ $9-0=9$
$9=9$ $\quad$ $9=9$
true $\quad$ true

Check $(0,3)$:

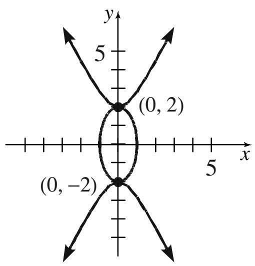
$9(0)^2+(3)^2=9$ $\quad$ $(3)^2-9(0)^2=9$
$0+9=9$ $\quad$ $9-0=9$
$9=9$ $\quad$ $9=9$
true $\quad$ true

The solution set is $\{(0,-3),(0,3)\}$.

60. $4x^2 + y^2 = 4$ or $\frac{x^2}{1} + \frac{y^2}{4} = 1$

$y^2 - 4x^2 = 4$

$\frac{y^2}{4} - \frac{x^2}{1} = 1$



Check $(0,-2)$:

$4(0)^2 + (-2)^2 = 4$ $\quad$ $(-2)^2 - 4(0)^2 = 4$

$0 + 4 = 4$ $\quad$ $4 - 0 = 4$

$4 = 4$ $\quad$ $4 = 4$

true $\quad$ true

Check $(0,2)$:

$4(0)^2 + (2)^2 = 4$ $\quad$ $(2)^2 - 4(0)^2 = 4$

$0 + 4 = 4$ $\quad$ $4 - 0 = 4$

$4 = 4$ $\quad$ $4 = 4$

true $\quad$ true

The solution set is $\{(0,-2),(0,2)\}$.

61. $|d_2 - d_1| = 2a = (2 \text{ s})(1100 \text{ ft / s}) = 2200 \text{ ft}$

$a = 1100$ ft

$2c = 5280$ ft, $c = 2640$ ft

$b^2 = c^2 - a^2 = (2640)^2 - (1100)^2$

$= 5{,}759{,}600$

$\frac{x^2}{(1100)^2} - \frac{y^2}{5{,}759{,}600} = 1$

$\frac{x^2}{1{,}210{,}000} - \frac{y^2}{5{,}759{,}600} = 1$

If M_1 is located 2640 feet to the right of the origin on the x-axis, the explosion is located on the right branch of the hyperbola given by the equation above.

62. **a.** $2c = 200$ km, $c = 100$ km

$|d_2 - d_1| = 2a = (500\mu\text{s})\left(300\frac{\text{m}}{\mu\text{s}}\right)$

$2a = 150{,}000$ m $= 150$ km

$a = 75$ km

$b^2 = c^2 - a^2 = (100)^2 - (75)^2 = 4375$

$\frac{x^2}{(75)^2} - \frac{y^2}{4375} = 1$

$\frac{x^2}{5625} - \frac{y^2}{4375} = 1$

b. The x-coordinate of the ship is 100 km:

$\frac{(100)^2}{5625} - \frac{y^2}{4375} = 1$

$\frac{y^2}{4375} = \frac{10{,}000}{5625} - 1$

$y = \pm\sqrt{4375}\sqrt{\frac{10{,}000}{5625} - 1} \approx \pm 58.3$

The ship is about 58.3 kilometers from the coast.

63. $625y^2 - 400x^2 = 250{,}000$

$\frac{625y^2}{250{,}000} - \frac{400x^2}{250{,}000} = \frac{250{,}000}{250{,}000}$

$\frac{y^2}{400} - \frac{x^2}{625} = 1$

$a^2 = 400$, $a = \sqrt{400} = 20$

$2a = 40$

The houses are 40 yards apart at their closest point.

64. $a = 3$

$\frac{x^2}{9} - \frac{y^2}{b^2} = 1$

To find b, use the equation of the slope of the asymptote,

$\frac{b}{a}$: $\quad \frac{b}{3} = \frac{1}{2}$

Solving for b: $2b = 3$, $b = \frac{3}{2}$.

$\frac{x^2}{9} - \frac{y^2}{\frac{9}{4}} = 1$

65. **a.** ellipse

b. $x^2 + 4y^2 = 4$

66. **a.** hyperbola

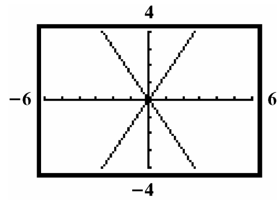
b. $x^2 - y^2 = 1$

67. – 76. Answers may vary.

77. $\frac{x^2}{4} - \frac{y^2}{9} = 0$

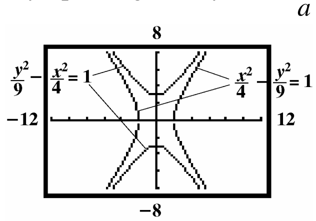
$y^2 = \frac{9}{4}x^2$

$y = \pm\frac{3}{2}x$



No; in general, the graph is two intersecting lines.

78. Answers may vary depending on the choice for *a* and *b*. For *a*= 2, *b* = 3, a graph is shown. The two graphs open right/left and up/down, sharing a common set of asymptotes given by $y = \pm\frac{b}{a}x$.



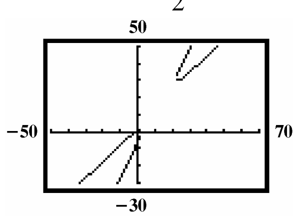
79. $4x^2 - 6xy + 2y^2 - 3x + 10y - 6 = 0$

$2y^2 + (10 - 6x)y + (4x^2 - 3x - 6) = 0$

$$y = \frac{6x - 10 \pm \sqrt{(10-6x)^2 - 8(4x^2 - 3x - 6)}}{4}$$

$$y = \frac{6x - 10 \pm \sqrt{4(x^2 - 24x + 37)}}{4}$$

$$y = \frac{3x - 5 \pm \sqrt{x^2 - 24x + 37}}{2}$$



The *xy*-term rotates the hyperbola. Separation of terms into ones containing only *x* or only *y* would not be possible.

80. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

$\frac{y^2}{9} = \frac{x^2}{16} - 1$

$y^2 = 9\left(\frac{x^2 - 16}{16}\right)$

$y = \pm\sqrt{9\left(\frac{x^2 - 16}{16}\right)}$

$y = \pm\frac{3}{4}\sqrt{x^2 - 16}$

$\frac{x|x|}{16} - \frac{y|y|}{9} = 1$

$\frac{y|y|}{9} = \frac{x|x|}{16} - 1$

$y|y| = 9\left(\frac{x|x| - 16}{16}\right)$

If $y \ge 0$, $y|y| = y^2$

$y^2 = 9\left(\frac{x|x| - 16}{16}\right)$

$y = \sqrt{9\left(\frac{x|x| - 16}{16}\right)}$

$y = \frac{3}{4}\sqrt{x|x| - 16}$ $(x \ge 4)$

If $y < 0$, $y|y| = -y^2$

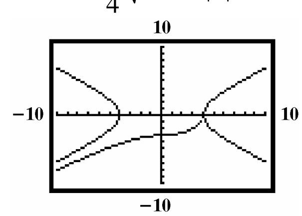
$-y^2 = 9\left(\frac{x|x| - 16}{16}\right)$

$y^2 = -9\left(\frac{x|x| - 16}{16}\right)$

$y = -\sqrt{-9\left(\frac{x|x| - 16}{16}\right)}$

$y = -\frac{3}{4}\sqrt{-(x|x| - 16)}$

$y = -\frac{3}{4}\sqrt{16 - x|x|}$ $(x \le 4)$



The second equation is a function with domain $(-\infty, \infty)$.

81. does not make sense; Explanations will vary. Sample explanation: This would change the ellipse to a hyperbola.

82. makes sense

83. makes sense

84. makes sense

85. false; Changes to make the statement true will vary. A sample change is: If a hyperbola has a transverse axis along the x–axis and one of the branches is removed, the remaining branch does not define a function of x.

86. false; Changes to make the statement true will vary. A sample change is: The points on the hyperbola's asymptotes do not satisfy the hyperbola's equation.

87. true

88. false; Changes to make the statement true will vary. A sample change is: It is possible for two different hyperbolas to share the same asymptotes. For example $\frac{x^2}{4}-\frac{y^2}{9}=1$ and $\frac{y^2}{9}-\frac{x^2}{4}=1$ share the same asymptotes.

89. $\frac{c}{a}$ will be large when a is small. When this happens, the asymptotes will be nearly vertical.

90. The center is at the midpoint of the line segment joining the vertices, so it is located at (5, 0). The standard form is:

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

$(h, k) = (5, 0)$, and $a = 6$, so $a^2 = 36$.

$$\frac{y^2}{36}-\frac{(x-5)^2}{b^2}=1.$$

Substitute $x = 0$ and $y = 9$:

$$\frac{9^2}{36}-\frac{(0-5)^2}{b^2}=1$$

$$-\frac{25}{b^2}=-\frac{5}{4}$$

$$-100=-5b^2$$

$$b^2=20$$

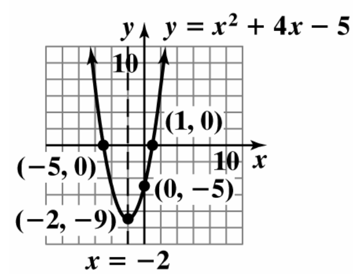
Standard form: $\frac{y^2}{36}-\frac{(x-5)^2}{20}=1$

91. If the asymptotes are perpendicular, then their slopes are negative reciprocals. For the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, the asymptotes are $y=\pm\frac{b}{a}x$. The slopes are negative reciprocals when $\frac{b}{a}=\frac{a}{b}$ (since one is already the negative of the other). This happens when $b^2=a^2$, so $a = b$. Any hyperbola where $a = b$, such as $\frac{x^2}{4}-\frac{y^2}{4}=1$, has perpendicular asymptotes.

92. $y=x^2+4x-5$

Since $a=1$ is positive, the parabola opens upward. The x-coordinate of the vertex is $x=-\frac{b}{2a}=-\frac{4}{2(1)}=-2$. The y-coordinate of the vertex is $y=(-2)^2+4(-2)-5=-9$.

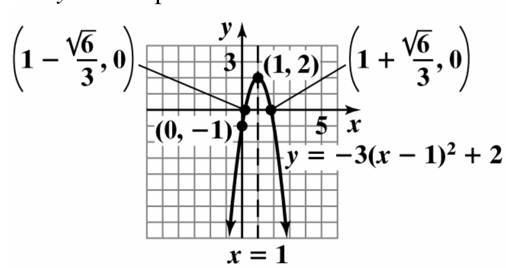
Vertex: $(-2,-9)$.



93. $y=-3(x-1)^2+2$

Since $a=-3$ is negative, the parabola opens downward. The vertex of the parabola is $(h,k)=(1,2)$.

The y–intercept is -1.



94. $y^2 + 2y + 12x - 23 = 0$

$$y^2 + 2y = -12x + 23$$

$$y^2 + 2y + 1 = -12x + 23 + 1$$

$$(y+1)^2 = -12x + 24$$

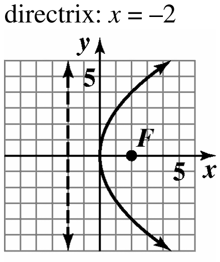
Section 10.3

Check Point Exercises

1. $4p = 8,\ p = 2$

focus: (2, 0)

directrix: $x = -2$



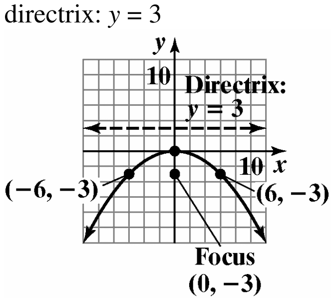
$y^2 = 8x$

2. $x^2 = -12y$

$4p = -12,\ p = 3$

focus: (0, –3)

directrix: $y = 3$



3. $p = 8$

$$y^2 = 4 \cdot 8x$$

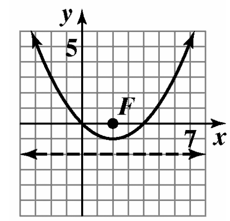
$$y^2 = 32x$$

4. $4p = 4,\ p = 1$

vertex: (2, –1)

focus: (2, 0)

directrix: $y = -2$



$(x - 2)^2 = 4(y + 1)$

5. $y^2 + 2y = -4x + 7$

$$y^2 + 2y + 1 = -4x + 7 + 1$$

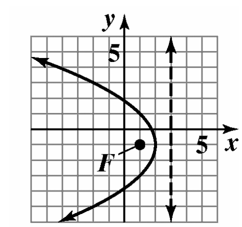
$$(y+1)^2 = -4(x-2)$$

$4p = -4,\ p = -1$

vertex: (2, –1)

focus: (1, –1)

directrix: $x = 3$



$y^2 + 2y + 4x - 7 = 0$

6. $x^2 = 4py$

Let $x = 3$ and $y = 4$.

$$3^2 = 4p \cdot 4$$

$$9 = 16p$$

$$p = \frac{9}{16}$$

$$x^2 = \frac{9}{4}y$$

The light should be placed at $\left(0, \frac{9}{16}\right)$ or $\frac{9}{16}$ inch above the vertex.

Exercise Set 10.3

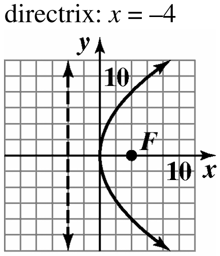
1. $y^2 = 4x$
$4p = 4,\ p = 1$
vertex: (0, 0)
focus: (1, 0)
directrix: $x = -1$
graph (c)

2. $x^2 = 4y$
$4p = 4,\ p = 1$
vertex: (0, 0)
focus: (0, 1)
directrix: $y = -1$
graph (a)

3. $x^2 = -4y$
$4p = -4,\ p = -1$
vertex: (0, 0)
focus: (0, –1)
directrix: $y = 1$
graph (b)

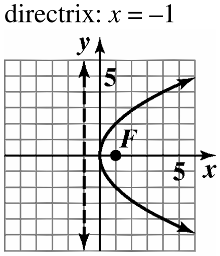
4. $y^2 = -4x$
$4p = -4,\ p = -1$
vertex: (0, 0)
focus: (–1, 0)
directrix: $x = 1$
graph (d)

5. $4p = 16,\ p = 4$
vertex: (0, 0)
focus: (4, 0)
directrix: $x = -4$



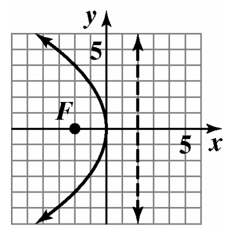
$y^2 = 16x$

6. $4p = 4,\ p = 1$
vertex: (0, 0)
focus: (1, 0)
directrix: $x = -1$



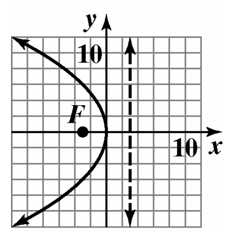
$y^2 = 4x$

7. $4p = -8,\ p = -2$
vertex: (0, 0)
focus: (–2, 0)
directrix: $x = 2$



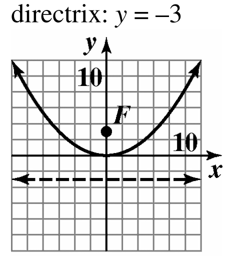
$y^2 = -8x$

8. $4p = -12,\ p = -3$
vertex: (0, 0)
focus: (–3, 0)
directrix: $x = 3$



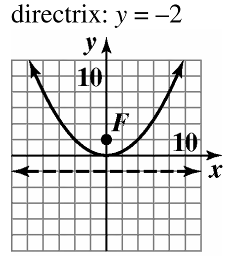
$y^2 = -12x$

9. $4p = 12,\ p = 3$
vertex: (0, 0)
focus: (0, 3)
directrix: $y = -3$



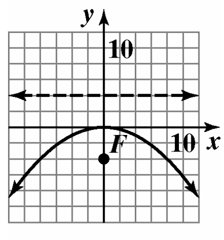
$x^2 = 12y$

10. $4p = 8,\ p = 2$
vertex: (0, 0)
focus: (0, 2)
directrix: $y = -2$



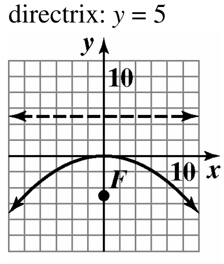
$x^2 = 8y$

11. $4p = -16,\ p = -4$
vertex: (0, 0)
focus: (0, –4)
directrix: $y = 4$



$x^2 = -16y$

12. $4p = -20,\ p = -5$
vertex: (0, 0)
focus: (0, –5)
directrix: $y = 5$

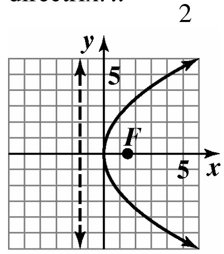


$x^2 = -20y$

13. $y^2 = 6x$

$4p = 6,\ p = \frac{6}{4} = \frac{3}{2}$
vertex: (0, 0)
focus: $\left(\frac{3}{2}, 0\right)$

directrix: $x = -\frac{3}{2}$

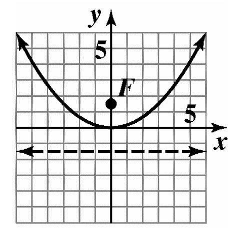


$y^2 - 6x = 0$

14. $x^2 = 6y$

$4p = 6,\ p = \frac{6}{4} = \frac{3}{2}$
vertex: (0, 0)
focus: $\left(0, \frac{3}{2}\right)$

directrix: $y = -\frac{3}{2}$



$x^2 - 6y = 0$

15.
$8x^2 = -4y$

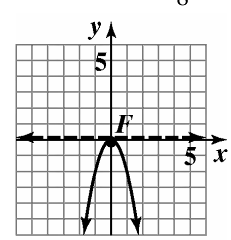
$x^2 = -\frac{1}{2}y$

$4p = -\frac{1}{2}$

$p = -\frac{1}{8}$

focus: $\left(0, -\frac{1}{8}\right)$

directrix: $y = \frac{1}{8}$



$8x^2 + 4y = 0$

16. $8y^2 = -4x$

$y^2 = -\frac{1}{2}x$

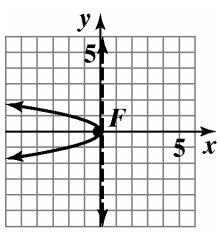
$4p = -\frac{1}{2}$

$p = -\frac{1}{8}$

vertex: (0, 0)

focus: $\left(-\frac{1}{8}, 0\right)$

directrix: $x = \frac{1}{8}$



$8y^2 + 4x = 0$

17. $p = 7$, $4p = 28$
$y^2 = 28x$

18. $p = 9$, $4p = 36$
$y^2 = 36x$

19. $p = -5$, $4p = -20$
$y^2 = -20x$

20. $p = -10$, $4p = -40$
$y^2 = -40x$

21. $p = 15$, $4p = 60$
$x^2 = 60y$

22. $p = 20$, $4p = 80$
$x^2 = 80y$

23. $p = -25$, $4p = -100$
$x^2 = -100y$

24. $p = -15$, $4p = -60$
$x^2 = -60y$

25. $p = -5 - (-3) = -2$ Vertex, (2, –3)
$(x-2)^2 = -8(y+3)$

26. vertex: (5, –2); $p = 7 - 5 = 2$

$y^2 = 4px$

$y^2 = 8x$

$(y+2)^2 = 8(x-5)^2$

27. vertex: (1, 2) $p = 2$
$(y-2)^2 = 8(x-1)$

28. vertex: (–1, 4) $p = 3$

$y^2 = 4px$

$y^2 = 4(3)x$

$y^2 = 12x$

$(y-4)^2 = 12(x+1)$

29. vertex: (–3, 3), $p = 1$
$(x+3)^2 = 4(y-3)$

30. vertex: (7, –5) $p = 4$
$(x-7)^2 = 16(y+5)$

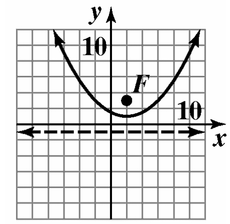
31. $(y-1)^2 = 4(x-1)$
$4p = 4$, $p = 1$
vertex: (1, 1)
focus: (2, 1)
directrix: $x = 0$
graph (c)

32. $(x+1)^2 = 4(y+1)$
$4p = 4$, $p = 1$
vertex: (–1, –1)
focus: (–1, 0)
directrix: $y = -2$
graph (a)

33. $(x+1)^2 = -4(y+1)$
$4p = -4$, $p = -1$
vertex: (–1, –1)
focus: (–1, –2)
directrix: $y = 0$
graph (d)

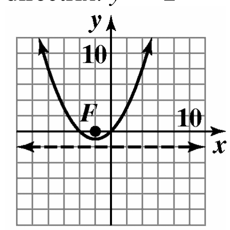
34. $(y-1)^2 = -4(x-1)$
$4p = -4$, $p = -1$
vertex: (1, 1)
focus: (0, 1)
directrix: $x = 2$
graph (b)

35. $4p = 8, p = 2$
vertex: (2, 1)
focus: (2, 3)
directrix: $y = -1$



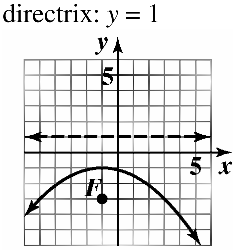
$\mathbf{(x - 2)^2 = 8(y - 1)}$

36. $4p = 4, p = 1$
vertex: (–2, –1)
focus: (–2, 0)
directrix: $y = -2$



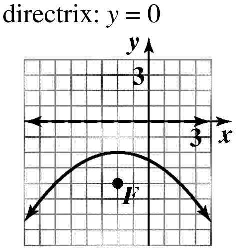
$\mathbf{(x + 2)^2 = 4(y + 1)}$

37. $4p = -8, p = -2$
vertex: (–1, –1)
focus: (–1, –3)
directrix: $y = 1$



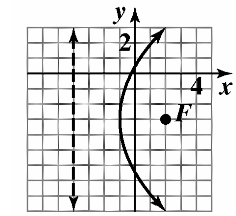
$\mathbf{(x + 1)^2 = -8(y + 1)}$

38. $4p = -8, p = -2$
vertex: (–2, –2)
focus: (–2, –4)
directrix: $y = 0$



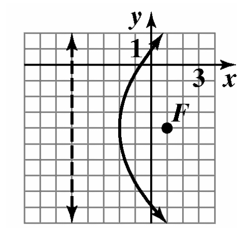
$\mathbf{(x + 2)^2 = -8(y + 2)}$

39. $4p = 12, p = 3$
vertex: (–1, –3)
focus: (2, –3)
directrix: $x = -4$



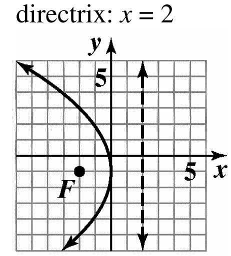
$\mathbf{(y + 3)^2 = 12(x + 1)}$

40. $4p = 12, p = 3$
vertex: (–2, –4)
focus: (1, –4)
directrix: $x = -5$



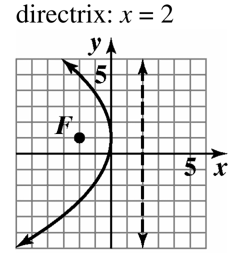
$\mathbf{(y + 4)^2 = 12(x + 2)}$

41. $(y + 1)^2 = -8(x - 0)$
$4p = -8, p = -2$
vertex: (0, –1)
focus: (–2, –1)
directrix: $x = 2$



$\mathbf{(y + 1)^2 = -8x}$

42. $(y - 1)^2 = -8(x - 0)$
$4p = -8, p = -2$
vertex: (0, 1)
focus: (–2, 1)
directrix: $x = 2$



$\mathbf{(y - 1)^2 = -8x}$

43. $x^2 - 2x + 1 = 4y - 9 + 1$

$(x-1)^2 = 4y - 8$

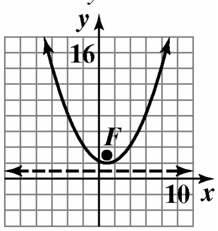
$(x-1)^2 = 4(y-2)$

$4p = 4, p = 1$

vertex: $(1, 2)$

focus: $(1, 3)$

directrix: $y = 1$



$x^2 - 2x - 4y + 9 = 0$

44. $x^2 + 6x = -8y - 1$

$x^2 + 6x + 9 = -8y - 1 + 9$

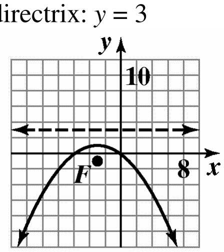
$(x+3)^2 = -8y + 8 = -8(y-1)$

$4p = -8, p = -2$

vertex: $(-3, 1)$

focus: $(-3, -1)$

directrix: $y = 3$



$x^2 + 6x + 8y + 1 = 0$

45. $y^2 - 2y + 1 = -12x + 35 + 1$

$(y-1)^2 = -12x + 36$

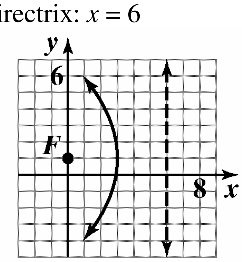
$(y-1)^2 = -12(x-3)$

$4p = -12, p = -3$

vertex: $(3, 1)$

focus: $(0, 1)$

directrix: $x = 6$



$y^2 - 2y + 12x - 35 = 0$

46. $y^2 - 2y = 8x - 1$

$y^2 - 2y + 1 = 8x - 1 + 1$

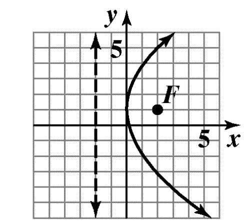
$(y-1)^2 = 8x$

$4p = 8, p = 2$

vertex: $(0, 1)$

focus: $(2, 1)$

directrix: $x = -2$



$y^2 - 2y - 8x + 1 = 0$

47. $x^2 + 6x = 4y - 1$

$x^2 + 6x + 9 = 4y - 1 + 9$

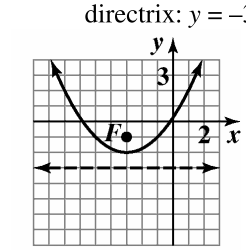
$(x+3)^2 = 4(y+2)$

$4p = 4, p = 1$

vertex: $(-3, -2)$

focus: $(-3, -1)$

directrix: $y = -3$



$x^2 + 6x - 4y + 1 = 0$

48. $x^2 + 8x + 16 = 4y - 8 + 16$

$(x+4)^2 = 4y + 8$

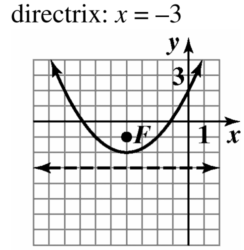
$(x+4)^2 = 4(y+2)$

$4p = 4, p = 1$

vertex: $(-4, -2)$

focus: $(-4, -1)$

directrix: $x = -3$



$x^2 + 8x - 4y + 8 = 0$

49. The y-coordinate of the vertex is

$y = -\frac{b}{2a} = -\frac{6}{2(1)} = -3$

The x-coordinate of the vertex is

$$\begin{aligned} x &= (-3)^2 + 6(-3) + 5 \\ &= 9 - 18 + 5 \\ &= -4 \end{aligned}$$

The vertex is $(-4, -3)$.

Since the squared term is y and $a > 0$, the graph opens to the right.

Domain: $\{x \mid x \geq -4\}$ or $[-4, \infty)$

Range: $\{y \mid y \text{ is a real number}\}$ or $(-\infty, \infty)$

The relation is not a function.

50. The y-coordinate of the vertex is

$y = -\frac{b}{2a} = -\frac{(-2)}{2(1)} = 1$

The x-coordinate of the vertex is

$$\begin{aligned} x &= (1)^2 - 2(1) - 5 \\ &= 1 - 2 - 5 \\ &= -6 \end{aligned}$$

The vertex is $(-6, 1)$.

Since the squared term is y and $a > 0$, the graph opens to the right.

Domain: $\{x \mid x \geq -6\}$ or $[-6, \infty)$

Range: $\{y \mid y \text{ is a real number}\}$ or $(-\infty, \infty)$

The relation is not a function.

51. The x-coordinate of the vertex is

$x = -\frac{b}{2a} = -\frac{(4)}{2(-1)} = 2$

The y-coordinate of the vertex is

$$\begin{aligned} y &= -(2)^2 + 4(2) - 3 \\ &= -4 + 8 - 3 \\ &= 1 \end{aligned}$$

The vertex is $(2, 1)$.

Since the squared term is x and $a < 0$, the graph opens down.

Domain: $\{x \mid x \text{ is a real number}\}$ or $(-\infty, \infty)$

Range: $\{y \mid y \leq 1\}$ or $(-\infty, 1]$

The relation is a function.

52. The x-coordinate of the vertex is

$x = -\frac{b}{2a} = -\frac{(-4)}{2(-1)} = -2$

The y-coordinate of the vertex is

$$\begin{aligned} y &= -(-2)^2 - 4(-2) + 4 \\ &= -4 + 8 + 4 \\ &= 8 \end{aligned}$$

The vertex is $(-2, 8)$.

Since the squared term is x and $a < 0$, the graph opens down.

Domain: $\{x \mid x \text{ is a real number}\}$ or $(-\infty, \infty)$

Range: $\{y \mid y \leq 8\}$ or $(-\infty, 8]$

The relation is a function.

53. The equation is in the form $x = a(y-k)^2 + h$

From the equation, we can see that the vertex is $(3, 1)$.

Since the squared term is y and $a < 0$, the graph opens to the left.

Domain: $\{x \mid x \leq 3\}$ or $(-\infty, 3]$

Range: $\{y \mid y \text{ is a real number}\}$ or $(-\infty, \infty)$

The relation is not a function.

54. The equation is in the form $x = a(y-k)^2 + h$

From the equation, we can see that the vertex is $(-2, 1)$.

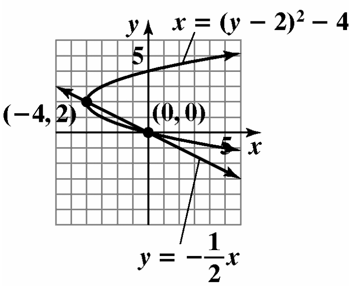
Since the squared term is y and $a < 0$, the graph opens to the left.

Domain: $\{x \mid x \leq -2\}$ or $(-\infty, -2]$

Range: $\{y \mid y \text{ is a real number}\}$ or $(-\infty, \infty)$

The relation is not a function.

55.



Check $(-4,2)$:

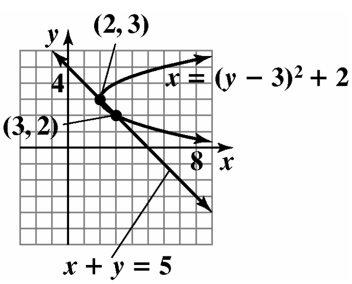
$-4 = (2-2)^2 - 4$ $\quad$ $2 = -\frac{1}{2}(-4)$
$-4 = 0 - 4$ $\quad$ $2 = 2$
$-4 = -4$ $\quad$ true
true

Check $(0,0)$:

$0 = (0-2)^2 - 4$ $\quad$ $0 = -\frac{1}{2}(0)$
$0 = 4 - 4$ $\quad$ $0 = 0$
$0 = 0$ $\quad$ true
true

The solution set is $\{(-4,2),(0,0)\}$.

56.



Check $(2,3)$:

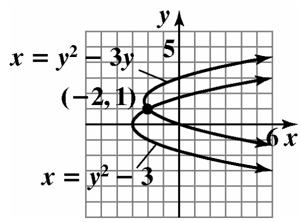
$2 = (3-3)^2 + 2$ $\quad$ $2 + 3 = 5$
$2 = 0 + 2$ $\quad$ $5 = 5$
$2 = 2$ $\quad$ true
true

Check $(3,2)$:

$3 = (2-3)^2 + 2$ $\quad$ $3 + 2 = 5$
$3 = 1 + 2$ $\quad$ $5 = 5$
$3 = 3$ $\quad$ true
true

The solution set is $\{(2,3),(3,2)\}$.

57.

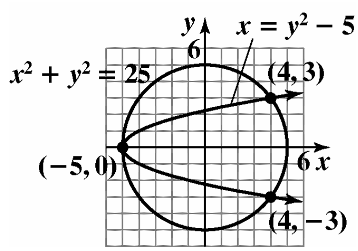


Check $(-2,1)$:

$-2 = (1)^2 - 3$ $\quad$ $-2 = (1)^2 - 3(1)$
$-2 = 1 - 3$ $\quad$ $-2 = 1 - 3$
$-2 = -2$ true $\quad$ $-2 = -2$ true

The solution set is $\{(-2,1)\}$.

58.



Check $(-5,0)$:

$-5 = 0^2 - 5$ $\quad$ $(-5)^2 + 0^2 = 25$
$-5 = 0 - 5$ $\quad$ $25 + 0 = 25$
$-5 = -5$ true $\quad$ $25 = 25$ true

Check $(4,-3)$:

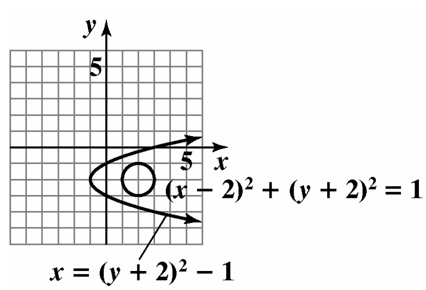
$4 = (-3)^2 - 5$ $\quad$ $(4)^2 + (-3)^2 = 25$
$4 = 9 - 5$ $\quad$ $16 + 9 = 25$
$4 = 4$ true $\quad$ $25 = 25$ true

Check $(4,3)$:

$4 = (3)^2 - 5$ $\quad$ $(4)^2 + (3)^2 = 25$
$4 = 9 - 5$ $\quad$ $16 + 9 = 25$
$4 = 4$ true $\quad$ $25 = 25$ true

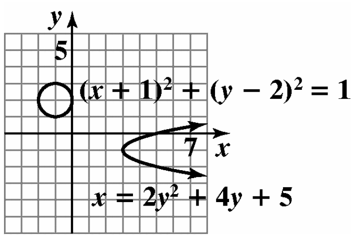
The solution set is $\{(-5,0),(4,-3),(4,3)\}$.

59.



The two graphs do not cross. Therefore, the solution set is the empty set, $\{\ \}$ or $\varnothing$.

60.



The two graphs do not cross. Therefore, the solution set is the empty set, $\{\ \}$ or $\varnothing$.

61. $x^2 = 4py$
$2^2 = 4p(1)$
$4 = 4$
$p = 1$
The light bulb should be placed 1 inch above the vertex.

62. $x^2 = 4py$
$4^2 = 4p(1$
$16 = 4p$
$p = 4$
The light bulb should be placed 4 inches above the vertex.

63. $x^2 = 4py$
$6^2 = 4p(2)$
$36 = 8p$
$p = \frac{36}{8} = \frac{9}{2} = 4.5$
The receiver should be located 4.5 feet from the base of the dish.

64. $x^2 = 4py$
$3^2 = 4p(2)$
$9 = 8p$
$p = \frac{9}{8} = 1.125$
The receiver should be placed 1.125 feet from the base of the smaller dish.

65. $x^2 = 4py$
$(640)^2 = 4p(160)$
$p = \frac{(640)^2}{640} = 640$
$x = 640 - 200 = 440$
$(440)^2 = 4(640)y$
$y = \frac{(440)^2}{4(640)} = 75.625$
The height is 76 meters.

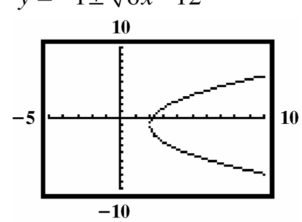
66. $x^2 = 4py$
$(400)^2 = 4p(160)$
$160{,}000 = 640p$
$p = \frac{160{,}000}{640} = 250$
$x = 400 - 100 = 300$
$300^2 = 4(250)y$
$y = \frac{300^2}{4(250)} = 90$
The height is 90 feet.

67. $x^2 = 4py$
$\left(\frac{200}{2}\right)^2 = 4p(-50)$
$\frac{10{,}000}{-50} = 4p$
$4p = -200$
$x^2 = -200y$
$(30)^2 = -200y$
$y = \frac{900}{-200} = -4.5$
(height of bridge) = 50 – 4.5 = 45.5 feet.
Yes, the boat will clear the arch.

68. $y^2 = 4px$
$25 = 4p(6)$
$\frac{25}{6} = 4p$
$\frac{25}{6}$ feet

69. – 76. Answers may vary.

77. $y^2 + 2y - 6x + 13 = 0$
$y^2 + 2y + (-6x + 13) = 0$
$y = \frac{-2 \pm \sqrt{2^2 - 4(-6x+13)}}{2}$
$y = \frac{-2 \pm \sqrt{24x - 48}}{2}$
$y = -1 \pm \sqrt{6x - 12}$



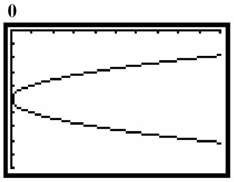
78. $y^2 + 10y - x + 25 = 0$

$y^2 + 10y + (-x + 25) = 0$

$y = \dfrac{-10 \pm \sqrt{10^2 - 4(-x+25)}}{2}$

$y = \dfrac{-10 \pm \sqrt{4x}}{2}$

$y = -5 \pm \sqrt{x}$



79. $16x^2 - 24xy + 9y^2 - 60x - 80y + 100 = 0$

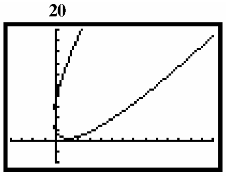
$9y^2 - (24x + 80)y + (16x^2 - 60x + 100) = 0$

$y = \dfrac{24x + 80 \pm \sqrt{(24x+80)^2 - 36(16x^2 - 60x + 100)}}{18}$

$y = \dfrac{24x + 80 \pm \sqrt{6000x + 2800}}{18}$

$y = \dfrac{24x + 80 \pm 20\sqrt{15x+7}}{18}$

$y = \dfrac{12x + 40 \pm 10\sqrt{15x+7}}{9}$



80. $x^2 + 2\sqrt{3}\,xy + 3y^2 + 8\sqrt{3}\,x - 8y + 32 = 0$

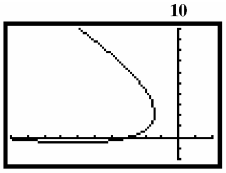
$3y^2 + (2\sqrt{3}\,x - 8)y + (x^2 + 8\sqrt{3}\,x + 32) = 0$

$y = \dfrac{-(2\sqrt{3}x - 8) \pm \sqrt{(2\sqrt{3}x-8)^2 - 12(x^2 + 8\sqrt{3}x + 32)}}{6}$

$y = \dfrac{-2\sqrt{3}x + 8 \pm \sqrt{-128\sqrt{3}x - 320}}{6}$

$y = \dfrac{-2\sqrt{3}x + 8 \pm 8\sqrt{-2\sqrt{3}x - 5}}{6}$

$y = \dfrac{-\sqrt{3}x + 4 \pm 4\sqrt{-2\sqrt{3}x - 5}}{3}$



81. does not make sense; Explanations will vary. Sample explanation: Horizontal parabolas will rise without limit.

82. does not make sense; Explanations will vary. Sample explanation: More information is necessary to determine how quickly it opens.

83. makes sense

84. makes sense

85. false; Changes to make the statement true will vary. A sample change is: Because $a = -1$, the parabola will open to the left.

86. true

87. false; Changes to make the statement true will vary. A sample change is: If a parabola defines y as a function of x, it will open up or down.

88. false; Changes to make the statement true will vary. A sample change is: $x = a(y-k) + h$ is not a parabola. There is no squared variable.

89. $Ax^2 + Ey = 0$

$Ax^2 = -Ey$ $\qquad 4p = -\dfrac{E}{A}y$

$x^2 = -\dfrac{E}{A}y$ $\qquad p = -\dfrac{E}{4A}y$

focus: $\left(0, -\dfrac{E}{4A}\right)$,

directrix: $y = \dfrac{E}{4A}$

90. $y = 4$ is the directrix and $(-1, 0)$ is the focus. The vertex must be located halfway between them at the point $(-1, 2)$. $p = -2$ and the parabola opens down.
$(x + 1)^2 = 4(-2)(y - 2)$
$(x + 1)^2 = -8(y - 2)$

91. Answers may vary.

92. $\left[\frac{\sqrt{2}}{2}(x'-y')\right]\left[\frac{\sqrt{2}}{2}(x'+y')\right]=1$

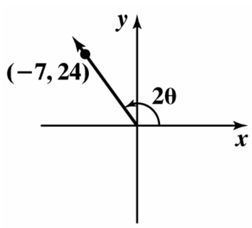
$$\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}(x'-y')(x'+y')=1$$

$$\frac{2}{4}\left((x')^2-(y')^2\right)=1$$

$$\frac{x'^2}{2}-\frac{y'^2}{2}=1$$

$$x'^2-y'^2=2$$

93. **a.**



b. $\cos 2\theta=-\frac{7}{25}$

c. $\sin\theta=\sqrt{\frac{1-\cos 2\theta}{2}}$

$$\sin\theta=\sqrt{\frac{1-\left(-\frac{7}{25}\right)}{2}}$$

$$\sin\theta=\sqrt{\frac{16}{25}}$$

$$\sin\theta=\frac{4}{5}$$

$$\cos\theta=\sqrt{\frac{1+\cos 2\theta}{2}}$$

$$\cos\theta=\sqrt{\frac{1+\left(-\frac{7}{25}\right)}{2}}$$

$$\cos\theta=\sqrt{\frac{9}{25}}$$

$$\cos\theta=\frac{3}{5}$$

d. Since $90^\circ<2\theta<180^\circ$, we have $45^\circ<\theta<90^\circ$. Both $\sin\theta$ and $\cos\theta$ are positive when $45^\circ<\theta<90^\circ$.

94. $B^2-4AC=(-2\sqrt{3})^2-4(3)(1)$

$$=12-12$$

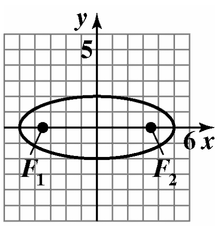
$$=0$$

Mid-Chapter 10 Check Point

1. Center: $(0,0)$

Because the denominator of the x^2 – term is greater than the denominator of the y^2 – term, the major axis is horizontal. Since $a^2 = 25$, $a = 5$ and the vertices are $(-5,0)$ and $(5,0)$. Since $b^2 = 4$, $b = 2$ and endpoints of the minor axis are $(0,-2)$ and $(0,2)$.

Foci: $\left(\pm\sqrt{21},0\right)$



$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

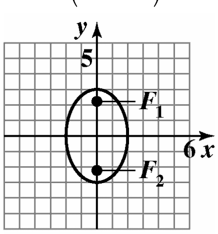
2. Divide both sides by 36 to get the standard form:

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

Center: $(0,0)$

Because the denominator of the y^2 – term is greater than the denominator of the x^2 – term, the major axis is vertical. Since $a^2 = 9$, $a = 3$ and the vertices are $(0,-3)$ and $(0,3)$. Since $b^2 = 4$, $b = 2$ and endpoints of the minor axis are $(-2,0)$ and $(2,0)$.

Foci: $\left(0,\pm\sqrt{5}\right)$



$$9x^2 + 4y^2 = 36$$

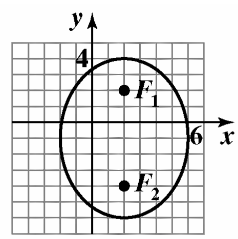
3. Center: $(2,-1)$

Because the denominator of the y^2 – term is greater than the denominator of the x^2 – term, the major axis is vertical. We have $a^2 = 25$ and $b^2 = 16$, so $a = 5$ and $b = 4$. The vertices lie 5 units above and below the center. The endpoints of the minor axis lie 4 units to the left and right of the center.

Vertices: $(2,4)$ and $(2,-6)$

Minor endpoints: $(-2,-1)$ and $(6,-1)$

Foci: $(2,2)$, $(2,-4)$



$$\frac{(x-2)^2}{16} + \frac{(y+1)^2}{25} = 1$$

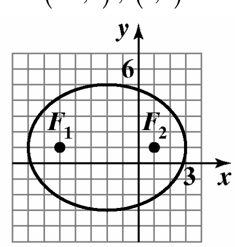
4. Center: $(-2,1)$

Because the denominator of the x^2 – term is greater than the denominator of the y^2 – term, the major axis is horizontal. We have $a^2 = 25$ and $b^2 = 16$, so $a = 5$ and $b = 4$. The vertices lie 5 units to the left and right of the center. The endpoints of the minor axis lie 4 units above and below the center.

Vertices: $(-7,1)$ and $(3,1)$

Minor endpoints: $(-2,5)$ and $(-2,-3)$

Foci: $(-5,1)$, $(1,1)$



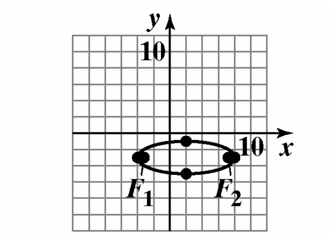
$$\frac{(x+2)^2}{25} + \frac{(y-1)^2}{16} = 1$$

5.

$$x^2 - 4x \quad + 9y^2 + 54y \quad = -49$$
$$\left(x^2 - 4x \quad\right) + 9\left(y^2 + 6y \quad\right) = -49$$
$$\left(x^2 - 4x + 4\right) + 9\left(y^2 + 6y + 9\right) = -49 + 4 + 81$$
$$\left(x-2\right)^2 + 9\left(y+3\right)^2 = 36$$
$$\frac{\left(x-2\right)^2}{36} + \frac{9\left(y+3\right)^2}{36} = \frac{36}{36}$$
$$\frac{\left(x-2\right)^2}{36} + \frac{\left(y+3\right)^2}{4} = 1$$

Center: $\left(2,-3\right)$

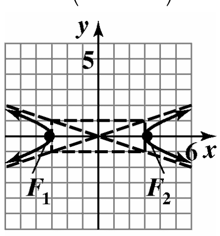
Foci: $\left(2 \pm 4\sqrt{2}, -3\right)$



$x^2 + 9y^2 - 4x + 54y + 49 = 0$

6. The equation is for a hyperbola in standard form with the transverse axis on the *x*-axis. We have $a^2 = 9$ and $b^2 = 1$, so $a = 3$ and $b = 1$. Therefore, the vertices are at $\left(\pm a, 0\right)$ or $\left(\pm 3, 0\right)$. Using a dashed line, we construct a rectangle using the ± 3 on the *x*-axis and ± 1 on the *y*-axis. Then use dashed lines to draw extended diagonals for the rectangle. These represent the asymptotes of the graph.

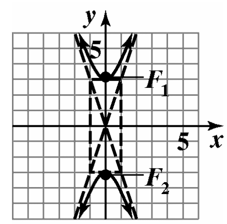
Foci: $\left(\pm\sqrt{10}, 0\right)$



$\frac{x^2}{9} - y^2 = 1$

7. The equation is in the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ with $a^2 = 9$, and $b^2 = 1$. We know the transverse axis lies on the *y*-axis and the vertices are $\left(0,-3\right)$ and $\left(0,3\right)$. Because $a^2 = 9$ and $b^2 = 1$, $a = 3$ and $b = 1$. Construct a rectangle using –1 and 1 on the *x*–axis, and –3 and 3 on the *y*–axis. Draw extended diagonals to obtain the asymptotes.

Foci: $\left(0, \pm\sqrt{10}\right)$

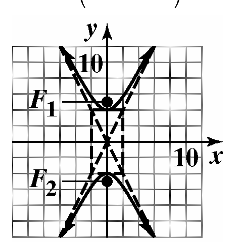


$\frac{y^2}{9} - x^2 = 1$

8. $\frac{y^2}{16} - \frac{x^2}{4} = 1$

The equation is in the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ with $a^2 = 16$, and $b^2 = 4$. We know the transverse axis lies on the *y*-axis and the vertices are $\left(0,-4\right)$ and $\left(0,4\right)$. Because $a^2 = 16$ and $b^2 = 4$, $a = 4$ and $b = 2$. Construct a rectangle using –2 and 2 on the *x*–axis, and –4 and 4 on the *y*–axis. Draw extended diagonals to obtain the asymptotes.

Foci: $\left(0, \pm 2\sqrt{5}\right)$



$y^2 - 4x^2 = 16$

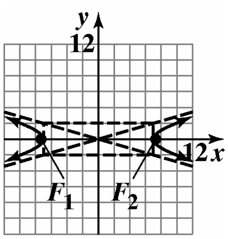
9. $\dfrac{x^2}{49}-\dfrac{y^2}{4}=1$

The equation is for a hyperbola in standard form with the transverse axis on the *x*-axis. We have $a^2=49$ and $b^2=4$, so $a=7$ and $b=2$.

Therefore, the vertices are at $(\pm a,0)$ or $(\pm 7,0)$.

Using a dashed line, we construct a rectangle using the ± 7 on the *x*-axis and ± 2 on the *y*-axis. Then use dashed lines to draw extended diagonals for the rectangle. These represent the asymptotes of the graph.

Foci: $\left(\pm\sqrt{53},0\right)$

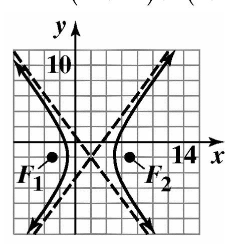


$4x^2 - 49y^2 = 196$

10. The equation is for a hyperbola in standard form with center (2, –2). We have $a^2=9$ and $b^2=16$, so $a=3$ and $b=4$.

Asymptotes: $y+2=\pm\dfrac{4}{3}(x-2)$

Foci: $(-3,-2)$, $(7,-2)$



$\dfrac{(x-2)^2}{9}-\dfrac{(y+2)^2}{16}=1$

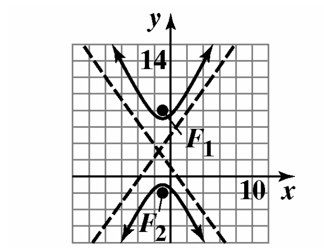
11. Write the equation for the hyperbola in standard form:

$$4x^2-y^2+8x+6y+11=0$$
$$4x^2+8x \quad -y^2+6y \quad =-11$$
$$4\left(x^2+2x \quad\right)-\left(y^2-6y \quad\right)=-11$$
$$4\left(x^2+2x+1\right)-\left(y^2-6y+9\right)=-11+4-9$$
$$4(x+1)^2-(y-3)^2=-16$$
$$\frac{4(x+1)^2}{-16}-\frac{(y-3)^2}{-16}=\frac{-16}{-16}$$
$$\frac{(y-3)^2}{16}-\frac{(x+1)^2}{4}=1$$

Center (–1, 3).

Asymptotes: $y-3=\pm 2(x+1)$

Foci: $\left(-1,3\pm 2\sqrt{5}\right)$, $(7,-2)$



$4x^2 - y^2 + 8x + 6y + 11 = 0$

12. $(x-2)^2=-12(y+1)$

$h=2$

$k=-1$

$4p=-12$

$p=-3$

Vertex: (h,k)

$(2,-1)$

Focus: $(h,k+p)$

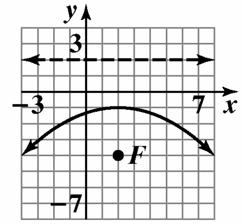
$(2,-1-3)$

$(2,-4)$

Directrix: $y=k-p$

$y=-1-(-3)$

$y=2$



$(x-2)^2 = -12(y+1)$

13. $y^2 - 2x - 2y - 5 = 0$

$$y^2 - 2y = 2x + 5$$
$$y^2 - 2y + 1 = 2x + 5 + 1$$
$$(y-1)^2 = 2x + 6$$
$$(y-1)^2 = 2(x+3)$$

$h = -3$

$k = 1$

$4p = 2$

$p = \frac{1}{2}$

Vertex: (h, k)

$(-3, 1)$

Focus: $(h + p, k)$

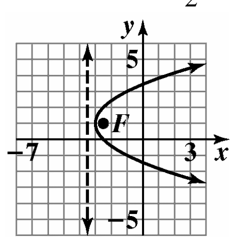
$\left(-3 + \frac{1}{2}, 1\right)$

$\left(-\frac{5}{2}, 1\right)$

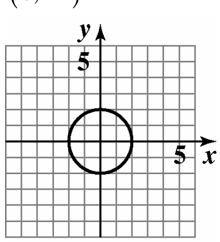
Directrix: $y = h - p$

$y = -3 - \left(\frac{1}{2}\right)$

$y = -\frac{7}{2}$



14. This is the equation of a circle centered at the origin with radius $r = \sqrt{4} = 2$.
We can plot points that are 2 units to the left, right, above, and below the origin and then graph the circle. The points are $(-2, 0)$, $(2, 0)$, $(0, 2)$, and $(0, -2)$.

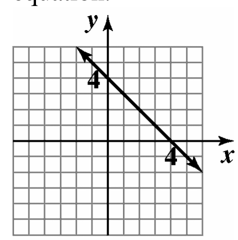


$x^2 + y^2 = 4$

15. $x + y = 4$

$y = -x + 4$

This is the equation of a line with slope $m = -1$ and a y-intercept of 4. We can plot the point $(0, 4)$, use the slope to get an additional point, connect the points with a straight line and then extend the line to represent the graph of the equation.



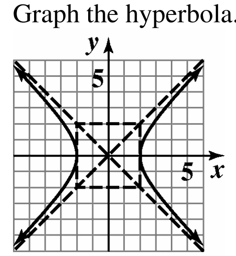
$x + y = 4$

16. $x^2 - y^2 = 4$

$\frac{x^2}{4} - \frac{y^2}{4} = 1$

The equation is for a hyperbola in standard form with the transverse axis on the x-axis. We have $a^2 = 4$ and $b^2 = 4$, so $a = 2$ and $b = 2$. Therefore, the vertices are at $(\pm a, 0)$ or $(\pm 2, 0)$. Using a dashed line, we construct a rectangle using the ± 2 on the x-axis and ± 2 on the y-axis. Then use dashed lines to draw extended diagonals for the rectangle. These represent the asymptotes of the graph.

Graph the hyperbola.



$x^2 - y^2 = 4$

17. $x^2+4y^2=4$

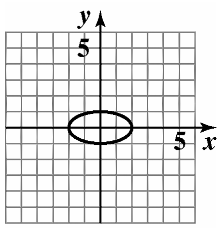
$\frac{x^2}{4}+\frac{y^2}{1}=1$

Center: $(0,0)$

Because the denominator of the x^2-term is greater than the denominator of the y^2-term, the major axis is horizontal. We have $a^2=4$ and $b^2=1$, so $a=2$ and $b=1$. The vertices lie 2 units to the left and right of the center. The endpoints of the minor axis lie 1 unit above and below the center.

Vertices: $(-2,0)$ and $(2,0)$

Minor endpoints: $(0,-1)$ and $(0,1)$



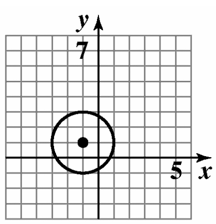
$x^2 + 4y^2 = 4$

18. Center: $(-1,1)$

Radius: $r=\sqrt{4}=2$

We plot the points that are 2 units to the left, right, above and below the center.

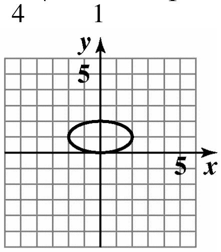
These points are $(-3,1)$, $(1,1)$, $(-1,3)$, and $(-1.-1)$.



$(x + 1)^2 + (y - 1)^2 = 4$

19. $x^2+4(y-1)^2=4$

$\frac{x^2}{4}+\frac{(y-1)^2}{1}=1$

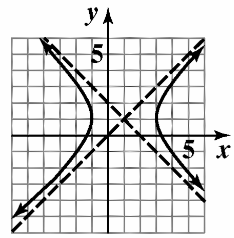


$x^2 + 4(y - 1)^2 = 4$

20. $(x-1)^2-(y-1)^2=4$

$\frac{(x-1)^2}{4}-\frac{(y-1)^2}{4}=1$

The equation is for a hyperbola in standard form centered at (1, 1). We have $a^2=4$ and $b^2=4$, so $a=2$ and $b=2$.



$(x-1)^2 - (y-1)^2 = 4$

21. $(y+1)^2=4(x-1)$

$h=1$

$k=-1$

$4p=4$

$p=1$

Vertex: (h,k)

$(1,-1)$

Focus: $(h+p,k)$

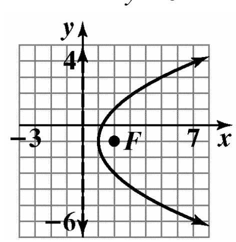
$(1+1,-1)$

$(2,-1)$

Directrix: $y=h-p$

$y=1-1$

$y=0$



22. The foci and vertices show that *c* is 4 and *a* is 5.

$c^2=a^2-b^2$

$4^2=5^2-b^2$

$b^2=25-16$

$b^2=9$

$\frac{x^2}{25}+\frac{y^2}{9}=1$

23. The endpoints show that the center is (1, 2).
Since $2a = 18,\ a = 9$ and $a^2 = 81$.
Since $2c = 10,\ c = 5$ and $c^2 = 25$.

$$c^2 = a^2 - b^2$$
$$25 = 81 - b^2$$
$$b^2 = 81 - 25$$
$$b^2 = 56$$
$$\frac{(x-1)^2}{81} + \frac{(y-2)^2}{56} = 1$$

24. The foci and vertices show that *c* is 3 and *a* is 2.

$$b^2 = c^2 - a^2$$
$$b^2 = 3^2 - 2^2$$
$$b^2 = 9 - 4$$
$$b^2 = 5$$
$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

25. The endpoints show that the center is (–1, 5).
Since $2a = 4,\ a = 2$ and $a^2 = 4$.
Since $2c = 6,\ c = 3$ and $c^2 = 9$.

$$b^2 = c^2 - a^2$$
$$b^2 = 9 - 4$$
$$b^2 = 5$$
$$\frac{(x+1)^2}{4} - \frac{(y-5)^2}{5} = 1$$

26. Focus: $(\overbrace{h}^{4}, \overbrace{k+p}^{5})$

Directrix: $y = \overbrace{k-p}^{-1}$

$$(k+p) + (k-p) = (5) + (-1)$$
$$2k = 4$$
$$k = 2$$
$$k + p = 5$$
$$p = 5 - k$$
$$p = 5 - 2$$
$$p = 3$$
$$(x-h)^2 = 4p(y-k)$$
$$(x-4)^2 = 4(3)(y-2)$$
$$(x-4)^2 = 12(y-2)$$

27. Focus: $(\overbrace{h+p}^{-2}, \overbrace{k}^{6})$

Directrix: $x = \overbrace{h-p}^{8}$

$$(h+p) + (h-p) = (-2) + (8)$$
$$2h = 6$$
$$h = 3$$
$$h + p = -2$$
$$p = -2 - h$$
$$p = -2 - 3$$
$$p = -5$$
$$(y-k)^2 = 4p(x-h)$$
$$(y-6)^2 = 4(-5)(x-3)$$
$$(y-6)^2 = -20(x-3)$$

28. $a = 15,\ b = 10$

$$\frac{x^2}{15^2} + \frac{y^2}{10^2} = 1$$
$$\frac{x^2}{225} + \frac{y^2}{100} = 1$$

Since the truck is 10 feet wide, substitute $x = 5$ into the equation to find *y*.

$$\frac{5^2}{225} + \frac{y^2}{100} = 1$$
$$\frac{25}{225} + \frac{y^2}{100} = 1$$
$$\frac{1}{9} + \frac{y^2}{100} = 1$$
$$900\left(\frac{1}{9} + \frac{y^2}{100}\right) = 900(1)$$
$$100 + 9y^2 = 900$$
$$9y^2 = 800$$
$$y^2 = 88.8889$$
$$y = \sqrt{88.8889}$$
$$y \approx 9.43$$

5 feet from the center, the height of the archway is 9.43 feet. Since the truck's height is 9.5 feet, it will not fit under the archway.

29. Find the distance between the foci.
Since $2a = 40$, $a = 20$ and $a^2 = 400$.
Since $2b = 20$, $b = 10$ and $b^2 = 100$.

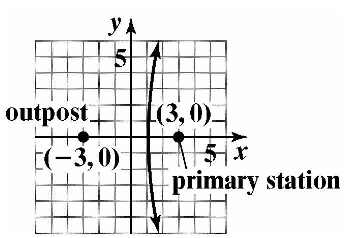
$$c^2 = a^2 - b^2$$
$$c^2 = 400 - 100$$
$$c^2 = 300$$
$$c = \sqrt{300}$$
$$= 10\sqrt{3}$$
$$2c = 20\sqrt{3}$$
$$2c \approx 34.64$$

The kidney stone should be 34.64 cm from the electrode that sends the ultrasound waves.

30. **a.** Since $2c = 6$, $c = 3$ and $c^2 = 9$.
The ranger at the primary station heard the explosion 6 seconds before the other ranger. This means that the explosion occurred $6 \times 0.35 = 2.1$ miles closer to the primary station.
Since $2a = 2.1$, $a = 1.05$ and $a^2 = 1.1025$.

$$b^2 = c^2 - a^2$$
$$b^2 = 9 - 1.1025$$
$$b^2 = 7.8975$$
$$\frac{x^2}{1.1025} - \frac{y^2}{7.8975} = 1$$

b.



31. Consider the peak of the dome as the point $(0,10)$. Since the width is 15 meters, the points $(\pm 7.5, 0)$ are on the parabola.
Use standard form to find *p*.

$$(x-h)^2 = 4p(y-k)$$
$$(7.5-0)^2 = 4p(0-10)$$
$$7.5^2 = -40p$$
$$\frac{56.25}{-40} = p$$
$$p \approx -1.4$$

The light should be about 1.4 meters below the peak of the ceiling.

Section 10.4

Check Point Exercises

1. **a.** $A = 3$ and $C = 2$.
$AC = 3(2) = 6$. Since $A \neq C$ and $AC > 0$, the graph is an ellipse.

b. $A = 1$ and $C = 1$. Since $A = C$, the graph is a circle.

c. $A = 0$ and $C = 1$. Since $AC = 0$, the graph is a parabola.

d. $A = 9$ and $C = -16$. Since $AC < 0$, the graph is a hyperbola.

2.

$$x = x'\cos 45° - y'\sin 45° = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' - y')$$

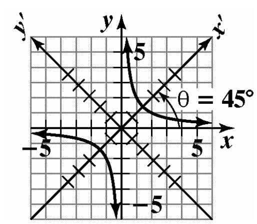
$$y = x'\sin 45° + y'\cos 45° = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' + y')$$

Substitute into the equation: $xy = 2$

$$\left[\frac{\sqrt{2}}{2}(x' - y')\right]\left[\frac{\sqrt{2}}{2}(x' + y')\right] = 2$$

$$\frac{1}{2}\left(x'^2 - y'^2\right) = 2$$

$$\frac{x'^2}{4} - \frac{y'^2}{4} = 1$$



3. $2x^2 + \sqrt{3}xy + y^2 - 2 = 0$

Step 1

$A = 2, B = \sqrt{3},$ and $C = 1$.

$$\cot 2\theta = \frac{A - C}{B} = \frac{2 - 1}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Step 2

Since $\cot 2\theta = \frac{\sqrt{3}}{3}, 2\theta = 60°$. Thus, $\theta = 30°$.

Step 3

$$x = x'\cos 30° - y'\sin 30° = x'\left(\frac{\sqrt{3}}{2}\right) - y'\left(\frac{1}{2}\right) = \frac{\sqrt{3}x' - y'}{2}$$

$$y = x'\sin 30° + y'\cos 30° = x'\left(\frac{1}{2}\right) + y'\left(\frac{\sqrt{3}}{2}\right) = \frac{x' + \sqrt{3}y'}{2}$$

Step 4
Substitute into the equation:

$$2x^2+\sqrt{3}xy+y^2-2=0$$

$$2\left(\frac{\sqrt{3}x'-y'}{2}\right)^2+\sqrt{3}\left(\frac{\sqrt{3}x'-y'}{2}\right)\left(\frac{x'+\sqrt{3}y'}{2}\right)+\left(\frac{x'+\sqrt{3}y'}{2}\right)^2-2=0$$

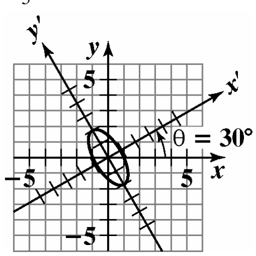
$$2\left(\frac{3x'^2-2\sqrt{3}x'y'+y'^2}{4}\right)+\sqrt{3}\left(\frac{\sqrt{3}x'^2+2x'y'-\sqrt{3}y'^2}{4}\right)+\frac{x'^2+2\sqrt{3}x'y'+3y'^2}{4}=2$$

$$6x'^2-4\sqrt{3}x'y'+2y'^2+3x'^2+2\sqrt{3}x'y'-3y'^2+x'^2+2\sqrt{3}x'y'+3y'^2=8$$

$$10x'^2+2y'^2=8$$

$$\frac{10x'^2}{8}+\frac{2y'^2}{8}=\frac{8}{8}$$

$$\frac{x'^2}{\frac{4}{5}}+\frac{y^2}{4}=1$$



4. $4x^2-4xy+y^2-8\sqrt{5}x-16\sqrt{5}y=0$

Step 1:
$A=4, B=-4,$ and $C=1.$

$$\cot 2\theta=\frac{A-C}{B}=\frac{4-1}{-4}=\frac{-3}{4}$$

Step 2: Since θ is always acute, and $\cot 2\theta$ is negative, 2θ is in quadrant II.
The third side of the right triangle is found using the Pythagorean theorem:

$$(-3)^2+4^2=r^2$$
$$25=r^2$$
$$r=5$$

So, $\cos 2\theta=\frac{\text{adjacent}}{\text{hypotenuse}}=\frac{-3}{5}.$

$$\sin\theta=\sqrt{\frac{1-\cos 2\theta}{2}}=\sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}}=\sqrt{\frac{8}{10}}=\sqrt{\frac{4}{5}}=\frac{2\sqrt{5}}{5}$$ and

$$\cos\theta=\sqrt{\frac{1+\cos 2\theta}{2}}=\sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}}=\sqrt{\frac{1}{5}}=\frac{\sqrt{5}}{5}$$

Step 3:

$$x=x'\cos\theta-y'\sin\theta=x'\left(\frac{\sqrt{5}}{5}\right)-y'\left(\frac{2\sqrt{5}}{5}\right)=\sqrt{5}\left(\frac{x'-2y'}{5}\right)$$

$$y=x'\sin\theta+y'\cos\theta=x'\left(\frac{2\sqrt{5}}{5}\right)+y'\left(\frac{\sqrt{5}}{5}\right)=\sqrt{5}\left(\frac{2x'+y'}{5}\right)$$

Step 4: Substitute into the equation: $4x^2-4xy+y^2-8\sqrt{5}x-16\sqrt{5}y=0$

$$4\left[\sqrt{5}\left(\frac{x'-2y'}{5}\right)\right]^2-4\left[\sqrt{5}\left(\frac{x'-2y'}{5}\right)\right]\left[\sqrt{5}\left(\frac{2x'+y'}{5}\right)\right]+\left[\sqrt{5}\left(\frac{2x'+y'}{5}\right)\right]^2-8\sqrt{5}\left[\sqrt{5}\left(\frac{x'-2y'}{5}\right)\right]$$

$$-16\sqrt{5}\left[\sqrt{5}\left(\frac{2x'+y'}{5}\right)\right]=0$$

$$20\left(\frac{x'^2-4x'y'+4y'^2}{25}\right)-20\left(\frac{2x'^2-3x'y'-2y'^2}{25}\right)+5\left(\frac{4x'^2+4x'y'+y'^2}{25}\right)-40\left(\frac{x'-2y'}{5}\right)$$

$$-80\left(\frac{2x'+y'}{5}\right)=0$$

Multiply both sides by 25:

$$20x'^2-80x'y'+80y'^2-40x'^2+60x'y'+40y'^2+20x'^2+20x'y'+5y'^2-200x'+400y'$$

$$-800x'-400y'=0$$

$$125y'^2-1000x'=0$$

$$y'^2-8x'=0$$

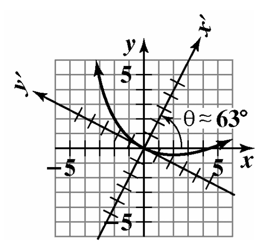
Step 5: This is a parabola, since it has only the y' squared.

$$y'^2-8x'=0$$

$$y'^2=8x'$$

The vertex of the parabola, relative to the $x'y'$ system, is (0, 0). Using a calculator to solve

$\sin\theta=\frac{2\sqrt{5}}{5}$, we find $\theta=\sin^{-1}\left(\frac{2\sqrt{5}}{5}\right)\approx 63°$. Rotate the axes through approximately 63°.



5. $3x^2-2\sqrt{3}xy+y^2+2x+2\sqrt{3}y=0$

$A=3, B=-2\sqrt{3},$ and $C=1$.

$B^2-4AC=\left(-2\sqrt{3}\right)^2-4(3)(1)=12-12=0$

Because $B^2-4AC=0,$ the graph of the equation is a parabola.

Exercise Set 10.4

1. $A = 0$ and $C = 1$. Since $AC = 0$, the graph is a parabola.

2. $A = 0$ and $C = 1$. Since $AC = 0$, the graph is a parabola.

3. $A = 4$ and $C = –9$. Since $AC = –36 < 0$, the graph is a hyperbola.

4. $A = 9$ and $C = 25$. Since $A \neq C$ and $AC = 225 > 0$, the graph is an ellipse.

5. $A = 4$ and $C = 4$. Since $A = C$, the graph is a circle.

6. $A = 9$ and $C = 4$. Since $A \neq C$ and $AC = 36 > 0$, the graph is an ellipse.

7. $A = 100$ and $C = -7$. Since $AC = -700 < 0$, the graph is a hyperbola.

8. $A = 0$ and $C = 1$. Since $AC = 0$, the graph is a parabola.

9. $$x = x'\cos 45° - y'\sin 45° = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' - y')$$

$$y = x'\sin 45° + y'\cos 45° = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' + y')$$

Substitute into the equation: $xy = -1$

$$\left[\frac{\sqrt{2}}{2}(x' - y')\right]\left[\frac{\sqrt{2}}{2}(x' + y')\right] = -1$$

$$\frac{1}{2}\left(x'^2 - y'^2\right) = -1$$

$$\frac{y'^2}{2} - \frac{x'^2}{2} = 1$$

10. $$x = x'\cos 45° - y'\sin 45° = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' - y')$$

$$y = x'\sin 45° + y'\cos 45° = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' + y')$$

Substitute into the equation: $xy = -4$

$$\left[\frac{\sqrt{2}}{2}(x' - y')\right]\left[\frac{\sqrt{2}}{2}(x' + y')\right] = -4$$

$$\frac{1}{2}\left(x'^2 - y'^2\right) = -4$$

$$\frac{y'^2}{8} - \frac{x'^2}{8} = 1$$

11. $x = x'\cos 45° - y'\sin 45° = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' - y')$

$y = x'\sin 45° + y'\cos 45° = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' + y')$

Substitute into the equation: $x^2 - 4xy + y^2 - 3 = 0$

$$\left[\frac{\sqrt{2}}{2}(x' - y')\right]^2 - 4\left[\frac{\sqrt{2}}{2}(x' - y')\right]\left[\frac{\sqrt{2}}{2}(x' + y')\right] + \left[\frac{\sqrt{2}}{2}(x' + y')\right]^2 - 3 = 0$$

$$\frac{1}{2}(x' - y')^2 - 4\left[\frac{1}{2}\left(x'^2 - y'^2\right)\right] + \frac{1}{2}(x' + y')^2 = 3$$

$$\frac{1}{2}\left(x'^2 - 2x'y' + y'^2\right) - 2x'^2 + 2y'^2 + \frac{1}{2}\left(x'^2 + 2x'y' + y'^2\right) = 3$$

$$\frac{1}{2}x'^2 - x'y' + \frac{1}{2}y'^2 - 2x'^2 + 2y'^2 + \frac{1}{2}x'^2 + x'y' + \frac{1}{2}y'^2 = 3$$

$$-x'^2 + 3y'^2 = 3$$

$$\frac{-x'^2}{3} + \frac{3y'^2}{3} = \frac{3}{3}$$

$$\frac{y'^2}{1} - \frac{x'^2}{3} = 1$$

12. $x = x'\cos 45° - y'\sin 45° = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' - y')$

$y = x'\sin 45° + y'\cos 45° = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' + y')$

Substitute into the equation: $13x^2 - 10xy + 13y^2 - 72 = 0$

$$3\left[\frac{\sqrt{2}}{2}(x' - y')\right]^2 - 10\left[\frac{\sqrt{2}}{2}(x' - y')\right]\left[\frac{\sqrt{2}}{2}(x' + y')\right] + 13\left[\frac{\sqrt{2}}{2}(x' + y')\right]^2 - 72 = 0$$

$$13\left[\frac{1}{2}(x' - y')^2\right] - 10\left[\frac{1}{2}\left(x'^2 - y'^2\right)\right] + 13\left[\frac{1}{2}(x' + y')^2\right] = 72$$

$$\frac{13}{2}\left(x'^2 - 2x'y' + y'^2\right) - 5x'^2 + 5y'^2 + \frac{13}{2}\left(x'^2 + 2x'y' + y'^2\right) = 72$$

$$\frac{13}{2}x'^2 - 13x'y' + \frac{13}{2}y'^2 - 5x'^2 + 5y'^2 + \frac{13}{2}x'^2 + 13x'y' + \frac{13}{2}y'^2 = 72$$

$$8x'^2 + 18y'^2 = 72$$

$$\frac{8x'^2}{72} + \frac{18y'^2}{72} = \frac{72}{72}$$

$$\frac{x'^2}{9} + \frac{y'^2}{4} = 1$$

13. $x = x'\cos 30° - y'\sin 30° = x'\left(\frac{\sqrt{3}}{2}\right) - y'\left(\frac{1}{2}\right) = \frac{\sqrt{3}x' - y'}{2}$

$y = x'\sin 30° + y'\cos 30° = x'\left(\frac{1}{2}\right) + y'\left(\frac{\sqrt{3}}{2}\right) = \frac{x' + \sqrt{3}y'}{2}$

Substitute into the equation: $23x^2 + 26\sqrt{3}xy - 3y^2 - 144 = 0$

$$23\left(\frac{\sqrt{3}x' - y'}{2}\right)^2 + 26\sqrt{3}\left(\frac{\sqrt{3}x' - y'}{2}\right)\left(\frac{x' + \sqrt{3}y'}{2}\right) - 3\left(\frac{x' + \sqrt{3}y'}{2}\right)^2 = 144$$

$$23\left(\frac{3x'^2 - 2\sqrt{3}x'y' + y'^2}{4}\right) + 26\sqrt{3}\left(\frac{\sqrt{3}x'^2 + 2x'y' - \sqrt{3}y'^2}{4}\right) - 3\left(\frac{x'^2 + 2\sqrt{3}x'y' + 3y'^2}{4}\right) = 144$$

$$69x'^2 - 46\sqrt{3}x'y' + 23y'^2 + 78x'^2 + 52\sqrt{3}x'y' - 78y'^2 - 3x'^2 - 6\sqrt{3}x'y' - 9y'^2 = 576$$

$$144x'^2 - 64y'^2 = 576$$

$$\frac{144x'^2}{576} - \frac{64y'^2}{576} = \frac{576}{576}$$

$$\frac{x'^2}{4} - \frac{y'^2}{9} = 1$$

14. $x = x'\cos 60° - y'\sin 60° = x'\left(\frac{1}{2}\right) - y'\left(\frac{\sqrt{3}}{2}\right) = \frac{x' - \sqrt{3}y'}{2}$

$y = x'\sin 60° + y'\cos 60° = x'\left(\frac{\sqrt{3}}{2}\right) + y'\left(\frac{1}{2}\right) = \frac{\sqrt{3}x' + y'}{2}$

Substitute into the equation: $13x^2 - 6\sqrt{3}xy + 7y^2 - 16 = 0$

$$13\left(\frac{x' - \sqrt{3}y'}{2}\right)^2 - 6\sqrt{3}\left(\frac{x' - \sqrt{3}y'}{2}\right)\left(\frac{\sqrt{3}x' + y'}{2}\right) + 7\left(\frac{\sqrt{3}x' + y'}{2}\right)^2 = 16$$

$$13\left(\frac{x'^2 - 2\sqrt{3}x'y' + 3y'^2}{4}\right) - 6\sqrt{3}\left(\frac{\sqrt{3}x'^2 - 2x'y' - \sqrt{3}y'^2}{4}\right) + 7\left(\frac{3x'^2 + 2\sqrt{3}x'y' + y'^2}{4}\right) = 16$$

$$13x'^2 - 26\sqrt{3}x'y' + 39y'^2 - 18x'^2 + 12\sqrt{3}x'y' + 18y'^2 + 21x'^2 + 14\sqrt{3}x'y' + 7y'^2 = 64$$

$$16x'^2 + 64y'^2 = 64$$

$$\frac{16x'^2}{64} + \frac{64y'^2}{64} = \frac{64}{64}$$

$$\frac{x'^2}{4} + \frac{y'^2}{1} = 1$$

15. $x^2 + xy + y^2 - 10 = 0$

$A = 1, B = 1, \text{ and } C = 1.$

$\cot 2\theta = \frac{A-C}{B} = \frac{1-1}{1} = 0$

$2\theta = 90°$

$\theta = 45°$

$x = x'\cos 45° - y'\sin 45° = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' - y')$

$y = x'\sin 45° + y'\cos 45° = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' + y')$

16. $x^2 + 4xy + y^2 - 3 = 0$

$A = 1, B = 4, \text{ and } C = 1.$

$\cot 2\theta = \frac{A-C}{B} = \frac{1-1}{4} = 0$

$2\theta = 90°$

$\theta = 45°$

$x = x'\cos 45° - y'\sin 45° = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' - y')$

$y = x'\sin 45° + y'\cos 45° = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' + y')$

17. $3x^2 - 10xy + 3y^2 - 32 = 0$

$A = 3, B = -10, \text{ and } C = 3.$

$\cot 2\theta = \frac{A-C}{B} = \frac{3-3}{-10} = 0$

$2\theta = 90°$

$\theta = 45°$

$x = x'\cos 45° - y'\sin 45° = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' - y')$

$y = x'\sin 45° + y'\cos 45° = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' + y')$

18. $5x^2 - 8xy + 5y^2 - 9 = 0$

$A = 5, B = -8,$ and $C = 5.$

$\cot 2\theta = \frac{A-C}{B} = \frac{5-5}{-8} = 0$

$2\theta = 90°$

$\theta = 45°$

$x = x'\cos 45° - y'\sin 45° = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' - y')$

$y = x'\sin 45° + y'\cos 45° = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' + y')$

19. $11x^2 + 10\sqrt{3}xy + y^2 - 4 = 0$

$A = 11, B = 10\sqrt{3},$ and $C = 1.$

$\cot 2\theta = \frac{A-C}{B} = \frac{11-1}{10\sqrt{3}} = \frac{10}{10\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$2\theta = 60°$

$\theta = 30°$

$x = x'\cos 30° - y'\sin 30°$

$x = x'\left(\frac{\sqrt{3}}{2}\right) - y'\left(\frac{1}{2}\right)$

$x = \frac{\sqrt{3}x' - y'}{2}$

$y = x'\sin 30° + y'\cos 30°$

$y = x'\left(\frac{1}{2}\right) + y'\left(\frac{\sqrt{3}}{2}\right)$

$y = \frac{x' + \sqrt{3}y'}{2}$

20. $7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0$

$A = 7, B = -6\sqrt{3},$ and $C = 13.$

$\cot 2\theta = \frac{A-C}{B} = \frac{7-13}{-6\sqrt{3}} = \frac{-6}{-6\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$2\theta = 60°$

$\theta = 30°$

$x = x'\cos 30° - y'\sin 30° = x'\left(\frac{\sqrt{3}}{2}\right) - y'\left(\frac{1}{2}\right) = \frac{\sqrt{3}x' - y'}{2}$

$y = x'\sin 30° + y'\cos 30° = x'\left(\frac{1}{2}\right) + y'\left(\frac{\sqrt{3}}{2}\right) = \frac{x' + \sqrt{3}y'}{2}$

21. $10x^2+24xy+17y^2-9=0$

$A=10, B=24,$ and $C=17.$

$\cot 2\theta=\frac{A-C}{B}=\frac{10-17}{24}=\frac{-7}{24}$

Since θ is always acute, and $\cot 2\theta$ is negative, 2θ is in quadrant II.
The third side of the right triangle is found by using the Pythagorean theorem:

$$(-7)^2+24^2=r^2$$
$$625=r^2$$
$$r=25$$

So, $\cos 2\theta=\frac{-7}{25}.$

$$\sin\theta=\sqrt{\frac{1-\cos 2\theta}{2}}=\sqrt{\frac{1-\left(\frac{-7}{25}\right)}{2}}=\frac{4}{5} \text{ and}$$

$$\cos\theta=\sqrt{\frac{1+\cos 2\theta}{2}}=\sqrt{\frac{1+\left(\frac{-7}{25}\right)}{2}}=\frac{3}{5}$$

$$x=x'\cos\theta-y'\sin\theta$$
$$x=x'\left(\frac{3}{5}\right)-y'\left(\frac{4}{5}\right)$$
$$x=\frac{3x'-4y'}{5}$$
$$y=x'\sin\theta+y'\cos\theta$$
$$y=x'\left(\frac{4}{5}\right)+y'\left(\frac{3}{5}\right)$$
$$y=\frac{4x'+3y'}{5}$$

22. $32x^2-48xy+18y^2-15x-20y=0$

$A=32, B=-48,$ and $C=18.$

$\cot 2\theta=\frac{A-C}{B}=\frac{32-18}{-48}=\frac{14}{-48}=\frac{-7}{24}$

Since θ is always acute, and $\cot 2\theta$ is negative, 2θ is in quadrant II.
The third side of the right triangle is found by using the Pythagorean theorem:

$$(-7)^2+24^2=r^2$$
$$625=r^2$$
$$r=25$$

So, $\cos 2\theta=\frac{-7}{25}.$ $\sin\theta=\sqrt{\frac{1-\cos 2\theta}{2}}=\sqrt{\frac{1-\left(\frac{-7}{25}\right)}{2}}=\frac{4}{5}$ and $\cos\theta=\sqrt{\frac{1+\cos 2\theta}{2}}=\sqrt{\frac{1+\left(\frac{-7}{25}\right)}{2}}=\frac{3}{5}$

$$x=x'\cos\theta-y'\sin\theta=x'\left(\frac{3}{5}\right)-y'\left(\frac{4}{5}\right)=\frac{3x'-4y'}{5}$$
$$y=x'\sin\theta+y'\cos\theta=x'\left(\frac{4}{5}\right)+y'\left(\frac{3}{5}\right)=\frac{4x'+3y'}{5}$$

23. $x^2 + 4xy - 2y^2 - 1 = 0$

$A = 1, B = 4,$ and $C = -2.$

$\cot 2\theta = \frac{A-C}{B} = \frac{1-(-2)}{4} = \frac{3}{4}$

Since θ is always acute, and $\cot 2\theta$ is positive, 2θ is in quadrant I.

The third side of the right triangle is found using the Pythagorean theorem:

$$3^2 + 4^2 = r^2$$
$$25 = r^2$$
$$r = 5$$

So, $\cos 2\theta = \frac{3}{5}.$

$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \frac{\sqrt{5}}{5}$ and

$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \frac{2\sqrt{5}}{5}$

$$x = x'\cos\theta - y'\sin\theta$$
$$x = x'\left(\frac{2\sqrt{5}}{5}\right) - y'\left(\frac{\sqrt{5}}{5}\right)$$
$$x = \sqrt{5}\left(\frac{2x'-y'}{5}\right)$$
$$y = x'\sin\theta + y'\cos\theta$$
$$y = x'\left(\frac{\sqrt{5}}{5}\right) + y'\left(\frac{2\sqrt{5}}{5}\right)$$
$$y = \sqrt{5}\left(\frac{x'+2y'}{5}\right)$$

24. $3xy - 4y^2 + 18 = 0$

$A = 0, B = 3,$ and $C = -4.$

$\cot 2\theta = \frac{A-C}{B} = \frac{0-(-4)}{3} = \frac{4}{3}$

Since θ is always acute, and $\cot 2\theta$ is positive, 2θ is in quadrant I.

The third side of the right triangle is found using the Pythagorean theorem:

$$4^2 + 3^2 = r^2$$
$$25 = r^2$$
$$r = 5$$

So, $\cos 2\theta = \frac{4}{5}.$

$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-\frac{4}{5}}{2}} = \frac{\sqrt{10}}{10}$ and $\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2}} = \frac{3\sqrt{10}}{10}$

$$x = x'\cos\theta - y'\sin\theta = x'\left(\frac{3\sqrt{10}}{10}\right) - y'\left(\frac{\sqrt{10}}{10}\right) = \sqrt{10}\left(\frac{3x'-y'}{10}\right)$$
$$y = x'\sin\theta + y'\cos\theta = x'\left(\frac{\sqrt{10}}{10}\right) + y'\left(\frac{3\sqrt{10}}{10}\right) = \sqrt{10}\left(\frac{x'+3y'}{10}\right)$$

25. $34x^2 - 24xy + 41y^2 - 25 = 0$

$A = 34, B = -24,$ and $C = 41.$

$$\cot 2\theta = \frac{A-C}{B} = \frac{34-41}{-24} = \frac{-7}{-24} = \frac{7}{24}$$

Since θ is always acute, and $\cot 2\theta$ is positive, 2θ is in quadrant I.
The third side of the right triangle is found using the Pythagorean theorem:

$$7^2 + 24^2 = r^2$$
$$625 = r^2$$
$$r = 25$$

So, $\cos 2\theta = \frac{7}{25}.$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-\frac{7}{25}}{2}} = \frac{3}{5} \text{ and } \cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+\frac{7}{25}}{2}} = \frac{4}{5}$$

$$x = x'\cos\theta - y'\sin\theta = x'\left(\frac{4}{5}\right) - y'\left(\frac{3}{5}\right) = \frac{4x' - 3y'}{5}$$

$$y = x'\sin\theta + y'\cos\theta = x'\left(\frac{3}{5}\right) + y'\left(\frac{4}{5}\right) = \frac{3x' + 4y'}{5}$$

26. $6x^2 - 6xy + 14y^2 - 45 = 0$

$A = 6, B = -6,$ and $C = 14.$

$$\cot 2\theta = \frac{A-C}{B} = \frac{6-14}{-6} = \frac{-8}{-6} = \frac{4}{3}$$

Since θ is always acute, and $\cot 2\theta$ is positive, 2θ is in quadrant I.
The third side of the right triangle is found using the Pythagorean theorem:

$$4^2 + 3^2 = r^2$$
$$25 = r^2$$
$$r = 5$$

So, $\cos 2\theta = \frac{4}{5}.$ $\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-\frac{4}{5}}{2}} = \frac{\sqrt{10}}{10}$ and $\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2}} = \frac{3\sqrt{10}}{10}$

$$x = x'\cos\theta - y'\sin\theta = x'\left(\frac{3\sqrt{10}}{10}\right) - y'\left(\frac{\sqrt{10}}{10}\right) = \sqrt{10}\left(\frac{3x' - y'}{10}\right)$$

$$y = x'\sin\theta + y'\cos\theta = x'\left(\frac{\sqrt{10}}{10}\right) + y'\left(\frac{3\sqrt{10}}{10}\right) = \sqrt{10}\left(\frac{x' + 3y'}{10}\right)$$

27. a. From Exercise 15,

$x=\frac{\sqrt{2}}{2}(x'-y')$ and $y=\frac{\sqrt{2}}{2}(x'+y')$.

Substitute into the equation: $x^2+xy+y^2-10=0$

$$\left[\frac{\sqrt{2}}{2}(x'-y')\right]^2+\left[\frac{\sqrt{2}}{2}(x'-y')\right]\left[\frac{\sqrt{2}}{2}(x'+y')\right]+\left[\frac{\sqrt{2}}{2}(x'+y')\right]^2-10=0$$

$$\frac{1}{2}\left(x'^2-2x'y'+y'^2\right)+\frac{1}{2}\left(x'^2-y'^2\right)+\frac{1}{2}\left(x'^2+2x'y'+y'^2\right)=10$$

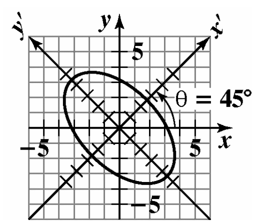
Multiply both sides by 2.

$x'^2-2x'y'+y'^2+x'^2-y'^2+x'^2+2x'y'+y'^2=20$

$3x'^2+y'^2=20$

b. $\frac{x'^2}{\frac{20}{3}}+\frac{y'^2}{20}=1$

c.



28. a. From Exercise 16,

$x=\frac{\sqrt{2}}{2}(x'-y')$ and $y=\frac{\sqrt{2}}{2}(x'+y')$.

Substitute into the equation: $x^2+4xy+y^2-3=0$

$$\left[\frac{\sqrt{2}}{2}(x'-y')\right]^2+4\left[\frac{\sqrt{2}}{2}(x'-y')\right]\left[\frac{\sqrt{2}}{2}(x'+y')\right]+\left[\frac{\sqrt{2}}{2}(x'+y')\right]^2-3=0$$

$$\frac{1}{2}\left(x'^2-2x'y'+y'^2\right)+2\left(x'^2-y'^2\right)+\frac{1}{2}\left(x'^2+2x'y'+y'^2\right)=3$$

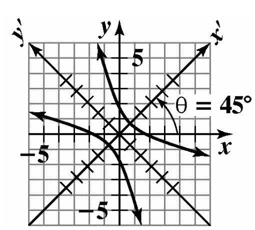
Multiply both sides by 2.

$x'^2-2x'y'+y'^2+4x'^2-4y'^2+x'^2+2x'y'+y'^2=6$

$6x'^2-2y'^2=6$

b. $\frac{x'^2}{1}-\frac{y'^2}{3}=1$

c.



29. **a.** From Exercise 17, $x = \frac{\sqrt{2}}{2}(x' - y')$ and $y = \frac{\sqrt{2}}{2}(x' + y')$.

Substitute into the equation: $3x^2 - 10xy + 3y^2 - 32 = 0$

$$3\left[\frac{\sqrt{2}}{2}(x' - y')\right]^2 - 10\left[\frac{\sqrt{2}}{2}(x' - y')\right]\left[\frac{\sqrt{2}}{2}(x' + y')\right] + 3\left[\frac{\sqrt{2}}{2}(x' + y')\right]^2 - 32 = 0$$

$$3\left[\frac{1}{2}\left(x'^2 - 2x'y' + y'^2\right)\right] - 10\left[\frac{1}{2}\left(x'^2 - y'^2\right)\right] + 3\left[\frac{1}{2}\left(x'^2 + 2x'y' + y'^2\right)\right] = 32$$

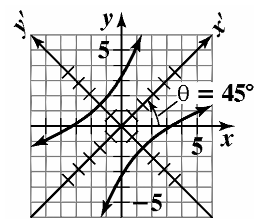
Multiply both sides by 2.

$$3x'^2 - 6x'y' + 3y'^2 - 10x'^2 + 10y'^2 + 3x'^2 + 6x'y' + 3y'^2 = 64$$

$$-4x'^2 + 16y'^2 = 64$$

b. $\frac{y'^2}{4} - \frac{x'^2}{16} = 1$

c.



30. **a.** From Exercise 18, $x = \frac{\sqrt{2}}{2}(x' - y')$ and $y = \frac{\sqrt{2}}{2}(x' + y')$.

Substitute into the equation: $5x^2 - 8xy + 5y^2 - 9 = 0$

$$5\left[\frac{\sqrt{2}}{2}(x' - y')\right]^2 - 8\left[\frac{\sqrt{2}}{2}(x' - y')\right]\left[\frac{\sqrt{2}}{2}(x' + y')\right] + 5\left[\frac{\sqrt{2}}{2}(x' + y')\right]^2 - 9 = 0$$

$$5\left[\frac{1}{2}\left(x'^2 - 2x'y' + y'^2\right)\right] - 8\left[\frac{1}{2}\left(x'^2 - y'^2\right)\right] + 5\left[\frac{1}{2}\left(x'^2 + 2x'y' + y'^2\right)\right] = 9$$

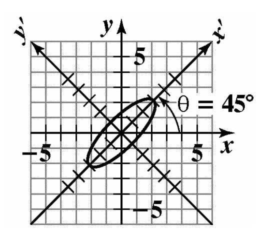
Multiply both sides by 2:

$$5x'^2 - 10x'y' + 5y'^2 - 8x'^2 + 8y'^2 + 5x'^2 + 10x'y' + 5y'^2 = 18$$

$$2x'^2 + 18y'^2 = 18$$

b. $\frac{x'^2}{9} + \frac{y'^2}{1} = 1$

c.



31. a. From Exercise 19,

$x=\dfrac{\sqrt{3}x'-y'}{2}$ and $y=\dfrac{x'+\sqrt{3}y'}{2}$.

Substitute into the equation: $11x^2+10\sqrt{3}xy+y^2-4=0$

$$11\left(\frac{\sqrt{3}x'-y'}{2}\right)^2+10\sqrt{3}\left(\frac{\sqrt{3}x'-y'}{2}\right)\left(\frac{x'+\sqrt{3}y'}{2}\right)+\left(\frac{x'+\sqrt{3}y'}{2}\right)^2-4=0$$

$$11\left(\frac{3x'^2-2\sqrt{3}x'y'+y'^2}{4}\right)+10\sqrt{3}\left(\frac{\sqrt{3}x'^2+2x'y'-\sqrt{3}y'^2}{4}\right)+\frac{x'^2+2\sqrt{3}x'y'+3y'^2}{4}=4$$

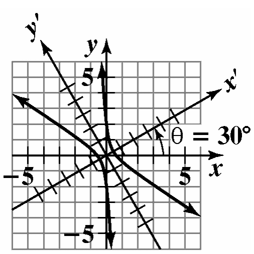
Multiply both sides by 4:

$$33x'^2-22\sqrt{3}x'y'+11y'^2+30x'^2+20\sqrt{3}x'y'-30y'^2+x'^2+2\sqrt{3}x'y'+3y'^2=16$$

$$64x'^2-16y'^2=16$$

b. $\dfrac{x'^2}{\frac{1}{4}}-\dfrac{y'^2}{1}=1$

c.



32. a. From Exercise 20, $x=\dfrac{\sqrt{3}x'-y'}{2}$ and $y=\dfrac{x'+\sqrt{3}y'}{2}$.

Substitute into the equation: $7x^2-6\sqrt{3}xy+13y^2-16=0$

$$7\left(\frac{\sqrt{3}x'-y'}{2}\right)^2-6\sqrt{3}\left(\frac{\sqrt{3}x'-y'}{2}\right)\left(\frac{x'+\sqrt{3}y'}{2}\right)+13\left(\frac{x'+\sqrt{3}y'}{2}\right)^2-16=0$$

$$7\left(\frac{3x'^2-2\sqrt{3}x'y'+y'^2}{4}\right)-6\sqrt{3}\left(\frac{\sqrt{3}x'^2+2x'y'-\sqrt{3}y'^2}{4}\right)+13\left(\frac{x'^2+2\sqrt{3}x'y'+3y'^2}{4}\right)=16$$

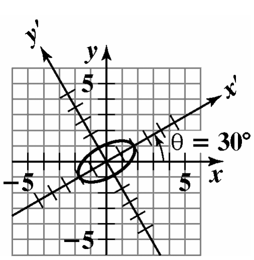
Multiply both sides by 4:

$$21x'^2-14\sqrt{3}x'y'+7y'^2-18x'^2-12\sqrt{3}x'y'+18y'^2+13x'^2+26\sqrt{3}x'y'+39y'^2=64$$

$$16x'^2+64y'^2=64$$

b. $\dfrac{x'^2}{4}+\dfrac{y'^2}{1}=1$

c.



33. a. From Exercise 21, $x=\dfrac{3x'-4y'}{5}$ and $y=\dfrac{4x'+3y'}{5}$.

Substitute into the equation: $10x^2+24xy+17y^2-9=0$

$$10\left(\frac{3x'-4y'}{5}\right)^2+24\left(\frac{3x'-4y'}{5}\right)\left(\frac{4x'+3y'}{5}\right)+17\left(\frac{4x'+3y'}{5}\right)^2-9=0$$

$$10\left(\frac{9x'^2-24x'y'+16y'^2}{25}\right)+24\left(\frac{12x'^2-7x'y'-12y'^2}{25}\right)+17\left(\frac{16x'^2+24x'y'+9y'^2}{25}\right)=9$$

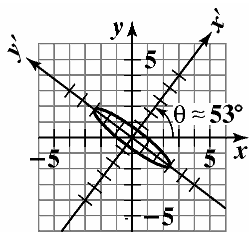
Multiply both sides by 25:

$$90x'^2-240x'y'+160y'^2+288x'^2-168x'y'-288y'^2+272x'^2+408x'y'+153y'^2=225$$

$$650x'^2+25y'^2=225$$

b. $\dfrac{x'^2}{\frac{9}{26}}+\dfrac{y'^2}{9}=1$

c.



The axes are rotated by $\theta=\sin^{-1}\left(\dfrac{4}{5}\right)\approx 53°$.

34. a. From Exercise 22,

$x=\dfrac{3x'-4y'}{5}$ and $y=\dfrac{4x'+3y'}{5}$.

Substitute into the equation: $32x^2-48xy+18y^2-15x-20y=0$

$$32\left(\frac{3x'-4y'}{5}\right)^2-48\left(\frac{3x'-4y'}{5}\right)\left(\frac{4x'+3y'}{5}\right)+18\left(\frac{4x'+3y'}{5}\right)^2-15\left(\frac{3x'-4y'}{5}\right)-20\left(\frac{4x'+3y'}{5}\right)=0$$

$$32\left(\frac{9x'^2-24x'y'+16y'^2}{25}\right)-48\left(\frac{12x'^2-7x'y'-12y'^2}{25}\right)+18\left(\frac{16x'^2+24x'y'+9y'^2}{25}\right)-25x'=0$$

Multiply both sides by 25:

$$288x'^2-768x'y'+512y'^2-576x'^2+336x'y'+576y'^2+288x'^2+432x'y'+162y'^2-625x'=0$$

$$1250y'^2-625x'=0$$

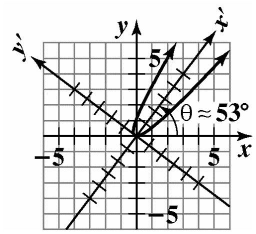
b.

$$1250y'^2=625x'$$

$$\frac{1250y'^2}{1250}=\frac{625x'}{1250}$$

$$y'^2=\frac{1}{2}x'$$

c.



The axes are rotated by $\theta = \sin^{-1}\left(\frac{4}{5}\right) \approx 53°$.

35. a. From Exercise 23,

$x = \sqrt{5}\left(\frac{2x' - y'}{5}\right)$ and $y = \sqrt{5}\left(\frac{x' + 2y'}{5}\right)$.

Substitute into the equation: $x^2 + 4xy - 2y^2 - 1 = 0$

$$\left[\sqrt{5}\left(\frac{2x' - y'}{5}\right)\right]^2 + 4\left[\sqrt{5}\left(\frac{2x' - y'}{5}\right)\right]\left[\sqrt{5}\left(\frac{x' + 2y'}{5}\right)\right] - 2\left[\sqrt{5}\left(\frac{x' + 2y'}{5}\right)\right]^2 = 1$$

$$5\left(\frac{4x'^2 - 4x'y' + y'^2}{25}\right) + 20\left(\frac{2x'^2 + 3x'y' - 2y'^2}{25}\right) - 10\left(\frac{x'^2 + 4x'y' + 4y'^2}{25}\right) = 1$$

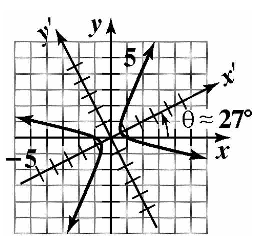
Multiply both sides by 25:

$$20x'^2 - 20x'y' + 5y'^2 + 40x'^2 + 60x'y' - 40y'^2 - 10x'^2 - 40x'y' - 40y'^2 = 25$$

$$50x'^2 - 75y'^2 = 25$$

b. $\frac{x'^2}{\frac{1}{2}} - \frac{y'^2}{\frac{1}{3}} = 1$

c.



The axes are rotated by $\theta = \sin^{-1}\left(\frac{\sqrt{5}}{5}\right) \approx 27°$.

36. a. From Exercise 24,

$x=\sqrt{10}\left(\frac{3x'-y'}{10}\right)$ and $y=\sqrt{10}\left(\frac{x'+3y'}{10}\right)$.

Substitute into the equation: $3xy-4y^2+18=0$

$$3\left[\sqrt{10}\left(\frac{3x'-y'}{10}\right)\right]\left[\sqrt{10}\left(\frac{x'+3y'}{10}\right)\right]-4\left[\sqrt{10}\left(\frac{x'+3y'}{10}\right)\right]^2=-18$$

$$30\left(\frac{3x'^2+8x'y'-3y'^2}{100}\right)-40\left(\frac{x'^2+6x'y'+9y'^2}{100}\right)=-18$$

Multiply both sides by 100:

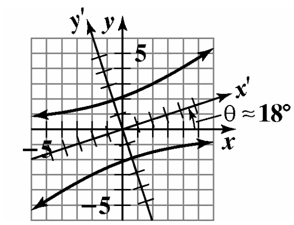
$$90x'^2+240x'y'-90y'^2-40x'^2-240x'y'-360y'^2=-1800$$

$$50x'^2-450y'^2=-1800$$

b. $$\frac{50x'^2-450y'^2}{-1800}=\frac{-1800}{-1800}$$

$$\frac{x'^2}{-36}+\frac{y'^2}{4}=1$$

$$\frac{y'^2}{4}-\frac{x'^2}{36}=1$$

c. 

The axes are rotated by $\theta=\sin^{-1}\left(\frac{\sqrt{10}}{10}\right)\approx 18^\circ$.

37. a. From Exercise 25,

$x=\frac{4x'-3y'}{5}$ and $y=\frac{3x'+4y'}{5}$.

Substitute into the equation: $34x^2-24xy+41y^2-25=0$

$$34\left(\frac{4x'-3y'}{5}\right)^2-24\left(\frac{4x'-3y'}{5}\right)\left(\frac{3x'+4y'}{5}\right)+41\left(\frac{3x'+4y'}{5}\right)^2=25$$

$$34\left(\frac{16x'^2-24x'y'+9y'^2}{25}\right)-24\left(\frac{12x'^2+7x'y'-12y'^2}{25}\right)+41\left(\frac{9x'^2+24x'y'+16y'^2}{25}\right)=25$$

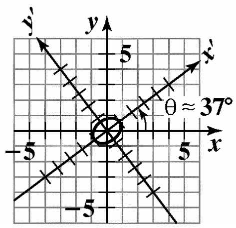
Multiply both sides by 25:

$$544x'^2-816x'y'+306y'^2-288x'^2-168x'y'+288y'^2+369x'^2+984x'y'+656y'^2=625$$

$$625x'^2+1250y'^2=625$$

b. $$\frac{625x'^2+1250y'^2}{625}=\frac{625}{625}$$

$$\frac{x'^2}{1}+\frac{y'^2}{\frac{1}{2}}=1$$

c. 

The axes are rotated by $\theta = \sin^{-1}\left(\frac{3}{5}\right) \approx 37°$.

38. a. From Exercise 26,

$x = \sqrt{10}\left(\frac{3x' - y'}{10}\right)$ and $y = \sqrt{10}\left(\frac{x' + 3y'}{10}\right)$.

Substitute into the equation: $6x^2 - 6xy + 14y^2 - 45 = 0$

$$6\left[\sqrt{10}\left(\frac{3x'-y'}{10}\right)\right]^2 - 6\left[\sqrt{10}\left(\frac{3x'-y'}{10}\right)\right]\left[\sqrt{10}\left(\frac{x'+3y'}{10}\right)\right] + 14\left[\sqrt{10}\left(\frac{x'+3y'}{10}\right)\right]^2 = 45$$

$$60\left(\frac{9x'^2 - 6x'y' + y'^2}{100}\right) - 60\left(\frac{3x'^2 + 8x'y' - 3y'^2}{100}\right) + 140\left(\frac{x'^2 + 6x'y' + 9y'^2}{100}\right) = 45$$

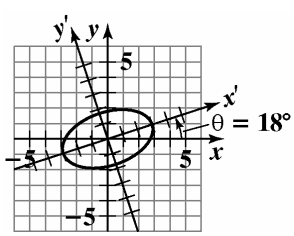
Multiply both sides by 100:

$$540x'^2 - 360x'y' + 60y'^2 - 180x'^2 - 480x'y' + 180y'^2 + 140x'^2 + 840x'y' + 1260y'^2 = 4500$$

$$500x'^2 + 1500y'^2 = 4500$$

b.
$$\frac{500x'^2 + 1500y'^2}{4500} = \frac{4500}{4500}$$

$$\frac{x'^2}{9} + \frac{y'^2}{3} = 1$$

c. 

The axes are rotated by $\theta = \sin^{-1}\left(\frac{\sqrt{10}}{10}\right) \approx 18°$.

39. $5x^2 - 2xy + 5y^2 - 12 = 0$

$A = 5, B = -2,$ and $C = 5.$

$B^2 - 4AC = (-2)^2 - 4(5)(5) = -96.$

Since $B^2 - 4AC < 0$, the graph is an ellipse or a circle.

40. $10x^2 + 24xy + 17y^2 - 9 = 0$
$A = 10, B = 24,$ and $C = 17.$
$B^2 - 4AC = 24^2 - 4(10)(17) = 576 - 680 = -104$
Since $B^2 - 4AC < 0$, the graph is an ellipse or a circle.

41. $24x^2 + 16\sqrt{3}xy + 8y^2 - x + \sqrt{3}y - 8 = 0$
$A = 24, B = 16\sqrt{3},$ and $C = 8.$
$B^2 - 4AC = \left(16\sqrt{3}\right)^2 - 4(24)(8) = 768 - 768 = 0$
Since $B^2 - 4AC = 0$, the graph is a parabola.

42. $3x^2 - 2\sqrt{3}xy + y^2 + 2x + 2\sqrt{3}y = 0$
$A = 3, B = -2\sqrt{3},$ and $C = 1.$
$B^2 - 4AC = \left(-2\sqrt{3}\right)^2 - 4(3)(1) = 12 - 12 = 0$
Since $B^2 - 4AC = 0$, the graph is a parabola.

43. $23x^2 + 26\sqrt{3}xy - 3y^2 - 144 = 0$
$A = 23, B = 26\sqrt{3},$ and $C = -3.$
$B^2 - 4AC = \left(26\sqrt{3}\right)^2 - 4(23)(-3) = 2028 + 276 = 2304$
Since $B^2 - 4AC > 0$, the graph is a hyperbola.

44. $4xy + 3y^2 + 4x + 6y - 1 = 0$
$A = 0, B = 4,$ and $C = 3.$
$B^2 - 4AC = 4^2 - 4(0)(3) = 16$
Since $B^2 - 4AC > 0$, the graph is a hyperbola.

45. Find θ:

$$\cot 2\theta = \frac{A - C}{B} = \frac{5 - 5}{-6} = 0$$
$$2\theta = 90°$$
$$\theta = 45°$$

Substitute θ into rotation formulas:

$$x = x'\cos\theta - y'\sin\theta = x'\cos 45° - y'\sin 45° = x'\frac{\sqrt{2}}{2} - y'\frac{\sqrt{2}}{2} = \frac{x'\sqrt{2} - y'\sqrt{2}}{2}$$
$$y = x'\sin\theta + y'\cos\theta = x'\sin 45° + y'\cos 45° = x'\frac{\sqrt{2}}{2} + y'\frac{\sqrt{2}}{2} = \frac{x'\sqrt{2} + y'\sqrt{2}}{2}$$

Substitute into equation:

$$5x^2 - 6xy + 5y^2 - 8 = 0$$
$$5\left(\frac{x'\sqrt{2} - y'\sqrt{2}}{2}\right)^2 - 6\left(\frac{x'\sqrt{2} - y'\sqrt{2}}{2}\right)\left(\frac{x'\sqrt{2} + y'\sqrt{2}}{2}\right) + 5\left(\frac{x'\sqrt{2} + y'\sqrt{2}}{2}\right)^2 - 8 = 0$$
$$\frac{x'^2}{4} + \frac{y'^2}{1} = 1$$

$\frac{x'^2}{4} + \frac{y'^2}{1} = 1$ is an ellipse with minor axis vertices of $(0, -1)$ and $(0, 1)$.

46. Find $\sin\theta$ and $\cos\theta$:

$$\cot 2\theta = \frac{A-C}{B} = \frac{2-5}{-4} = \frac{3}{4}$$

$$\cot 2\theta = \frac{x}{y} = \frac{3}{4}$$

$$r = \sqrt{x^2+y^2} = \sqrt{3^2+4^2} = 5$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-\left(\frac{3}{5}\right)}{2}} = \frac{\sqrt{5}}{5} \quad \text{and} \quad \cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+\left(\frac{3}{5}\right)}{2}} = \frac{2\sqrt{5}}{5}$$

Substitute into rotation formulas:

$$x = x'\cos\theta - y'\sin\theta = x'\frac{2\sqrt{5}}{5} - y'\frac{\sqrt{5}}{5} = \frac{x'2\sqrt{5} - y'\sqrt{5}}{5}$$

$$y = x'\sin\theta + y'\cos\theta = x'\frac{\sqrt{5}}{5} + y'\frac{2\sqrt{5}}{5} = \frac{x'\sqrt{5} - y'2\sqrt{5}}{5}$$

Substitute into equation:

$$2x^2 - 4xy + 5y^2 - 36 = 0$$

$$2\left(\frac{x'2\sqrt{5} - y'\sqrt{5}}{5}\right)^2 - 4\left(\frac{x'2\sqrt{5} - y'\sqrt{5}}{5}\right)\left(\frac{x'\sqrt{5} + y'2\sqrt{5}}{5}\right) + 5\left(\frac{x'\sqrt{5} + y'2\sqrt{5}}{5}\right)^2 - 36 = 0$$

$$x'^2 + 6y'^2 = 36$$

$$\frac{x'^2}{36} + \frac{y'^2}{6} = 1$$

$\frac{x'^2}{36} + \frac{y'^2}{6} = 1$ is an ellipse with minor axis vertices of $\left(0,-\sqrt{6}\right)$ and $\left(0,\sqrt{6}\right)$.

47. Find $\sin\theta$ and $\cos\theta$:

$$\cot 2\theta = \frac{A-C}{B} = \frac{1-4}{-4} = \frac{3}{4}$$

$$\cot 2\theta = \frac{x}{y} = \frac{3}{4}$$

$$r = \sqrt{x^2+y^2} = \sqrt{3^2+4^2} = 5$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-\left(\frac{3}{5}\right)}{2}} = \frac{\sqrt{5}}{5} \quad \text{and} \quad \cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+\left(\frac{3}{5}\right)}{2}} = \frac{2\sqrt{5}}{5}$$

Substitute into rotation formulas:

$$x = x'\cos\theta - y'\sin\theta = x'\frac{2\sqrt{5}}{5} - y'\frac{\sqrt{5}}{5} = \frac{x'2\sqrt{5} - y'\sqrt{5}}{5}$$

$$y = x'\sin\theta + y'\cos\theta = x'\frac{\sqrt{5}}{5} + y'\frac{2\sqrt{5}}{5} = \frac{x'\sqrt{5} - y'2\sqrt{5}}{5}$$

Substitute into equation:

$$x^2-4xy+4y^2+5\sqrt{5}y-10=0$$

$$\left(\frac{x'2\sqrt{5}-y'\sqrt{5}}{5}\right)^2-4\left(\frac{x'2\sqrt{5}-y'\sqrt{5}}{5}\right)\left(\frac{x'\sqrt{5}+y'2\sqrt{5}}{5}\right)+4\left(\frac{x'\sqrt{5}+y'2\sqrt{5}}{5}\right)^2+5\sqrt{5}\left(\frac{x'\sqrt{5}+y'2\sqrt{5}}{5}\right)-10=0$$

$$x'+y'^2+2y'-2=0$$
$$x'=-y'^2-2y'+2$$
$$x'=-\left(y'^2+2y'\right)+2$$
$$x'=-\left(y'^2+2y'+1\right)+2+1$$
$$x'=-\left(y'+1\right)^2+3$$

$x'=-\left(y'+1\right)^2+3$ is a parabola with vertex $(3,-1.)$

48. Find $\sin\theta$ and $\cos\theta$:

$\cot 2\theta=\frac{A-C}{B}=\frac{1-(-2)}{4}=\frac{3}{4}$

$\cot 2\theta=\frac{x}{y}=\frac{3}{4}$

$r=\sqrt{x^2+y^2}=\sqrt{3^2+4^2}=5$

$\sin\theta=\sqrt{\frac{1-\cos 2\theta}{2}}=\sqrt{\frac{1-\left(\frac{3}{5}\right)}{2}}=\frac{\sqrt{5}}{5}$ and $\cos\theta=\sqrt{\frac{1+\cos 2\theta}{2}}=\sqrt{\frac{1+\left(\frac{3}{5}\right)}{2}}=\frac{2\sqrt{5}}{5}$

Substitute into rotation formulas:

$x=x'\cos\theta-y'\sin\theta=x'\frac{2\sqrt{5}}{5}-y'\frac{\sqrt{5}}{5}=\frac{x'2\sqrt{5}-y'\sqrt{5}}{5}$

$y=x'\sin\theta+y'\cos\theta=x'\frac{\sqrt{5}}{5}+y'\frac{2\sqrt{5}}{5}=\frac{x'\sqrt{5}-y'2\sqrt{5}}{5}$

Substitute into equation:

$$x^2+4xy-2y^2-6=0$$

$$\left(\frac{x'2\sqrt{5}-y'\sqrt{5}}{5}\right)^2+4\left(\frac{x'2\sqrt{5}-y'\sqrt{5}}{5}\right)\left(\frac{x'\sqrt{5}+y'2\sqrt{5}}{5}\right)-2\left(\frac{x'\sqrt{5}+y'2\sqrt{5}}{5}\right)^2-6=0$$

$$2x'^2-3y'^2-6=0$$
$$2x'^2-3y'^2=6$$
$$\frac{2x'^2}{6}-\frac{3y'^2}{6}=\frac{6}{6}$$
$$\frac{x'^2}{3}-\frac{y'^2}{2}=1$$

$\frac{x'^2}{3}-\frac{y'^2}{2}=1$ is a hyperbola with $a=\sqrt{3}$ and $b=\sqrt{2}$.

The asymptotes are $y=\pm\frac{b}{a}x=\pm\frac{\sqrt{2}}{\sqrt{3}}x=\pm\frac{\sqrt{2}}{\sqrt{3}}x$

$y=\pm\frac{\sqrt{6}}{3}x$

49. – 53. Answers may vary.

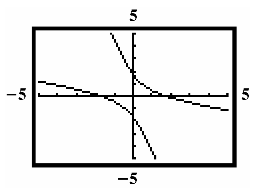
$$y_1 = \frac{-(Bx+E)+\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C} \text{ and } y_2 = \frac{-(Bx+E)-\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C}$$

for equations of the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$.

54. $$y_1 = \frac{-(Bx+E)+\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C} \text{ and } y_2 = \frac{-(Bx+E)-\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C}$$

for equations of the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$.
$A = 1$, $B = 4$, $C = 1$, $D = 0$, $E = 0$, and $F = -3$.

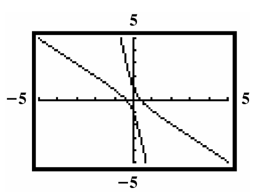
Graph $y_1 = \frac{-4x+\sqrt{16x^2-4(x^2-3)}}{2}$ and $y_2 = \frac{-4x-\sqrt{16x^2-4(x^2-3)}}{2}$



55. $$y_1 = \frac{-(Bx+E)+\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C} \text{ and } y_2 = \frac{-(Bx+E)-\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C}$$

for equations of the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$.
$A = 7$, $B = 8$, $C = 1$, $D = 0$, $E = 0$, and $F = -1$.

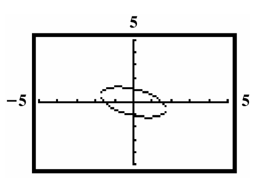
Graph $y_1 = \frac{-8x+\sqrt{64x^2-4(7x^2-1)}}{2}$ and $y_2 = \frac{-8x-\sqrt{64x^2-4(7x^2-1)}}{2}$.



56. $$y_1 = \frac{-(Bx+E)+\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C} \text{ and } y_2 = \frac{-(Bx+E)-\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C}$$

for equations of the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$.
$A = 3$, $B = 4$, $C = 6$, $D = 0$, $E = 0$, and $F = -7$.

Graph $y_1 = \frac{-4x+\sqrt{16x^2-24(3x^2-7)}}{12}$ and $y_2 = \frac{-4x-\sqrt{16x^2-24(3x^2-7)}}{12}$

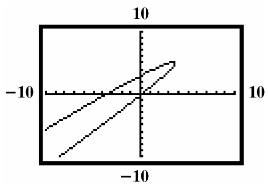


57. $y_1 = \dfrac{-(Bx+E)+\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C}$ and $y_2 = \dfrac{-(Bx+E)-\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C}$

for equations of the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$.

$A = 3$, $B = -6$, $C = 3$, $D = 10$, $E = -8$, and $F = -2$.

Graph $y_1 = \dfrac{-(-6x-8)+\sqrt{(-6x-8)^2-12(3x^2+10x-2)}}{6}$ and $y_2 = \dfrac{-(-6x-8)-\sqrt{(-6x-8)^2-12(3x^2+10x-2)}}{6}$.

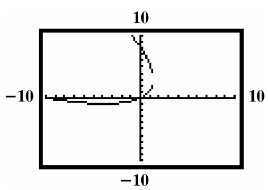


58. $y_1 = \dfrac{-(Bx+E)+\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C}$ and $y_2 = \dfrac{-(Bx+E)-\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C}$

for equations of the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$.

$A = 9$, $B = 24$, $C = 16$, $D = 90$, $E = -130$, and
$F = 0$.

Graph $y_1 = \dfrac{-(24x-130)+\sqrt{(24x-130)^2-64(9x^2+90x)}}{32}$ and $y_2 = \dfrac{-(24x-130)-\sqrt{(24x-130)^2-64(9x^2+90x)}}{32}$

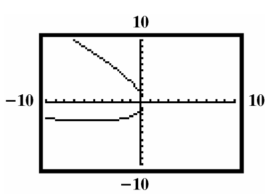


59. $y_1 = \dfrac{-(Bx+E)+\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C}$ and $y_2 = \dfrac{-(Bx+E)-\sqrt{(Bx+E)^2-4C(Ax^2+Dx+F)}}{2C}$

for equations of the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$.

$A = 1$, $B = 4$, $C = 4$, $D = 10\sqrt{5}$, $E = 0$, and $F = -9$.

Graph $y_1 = \dfrac{-4x+\sqrt{16x^2-16(x^2+10\sqrt{5}x-9)}}{8}$ and $y_2 = \dfrac{-4x-\sqrt{16x^2-16(x^2+10\sqrt{5}x-9)}}{8}$.

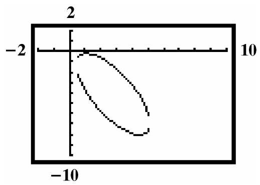


60. $y_1 = \dfrac{-\left(Bx+E\right)+\sqrt{\left(Bx+E\right)^2-4C\left(Ax^2+Dx+F\right)}}{2C}$ and $y_2 = \dfrac{-\left(Bx+E\right)-\sqrt{\left(Bx+E\right)^2-4C\left(Ax^2+Dx+F\right)}}{2C}$

for equations of the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$.

$A = 7$, $B = 6$, $C = 2.5$, $D = -14$, $E = 4$, and $F = 9$.

Graph $y_1 = \dfrac{-(6x+4)+\sqrt{(6x+4)^2-10\left(7x^2-14x+9\right)}}{5}$ and $y_2 = \dfrac{-(6x+4)-\sqrt{(6x+4)^2-10\left(7x^2-14x+9\right)}}{5}$



61. does not make sense; Explanations will vary. Sample explanation: This is not necessary because there is no *xy* term.

62. does not make sense; Explanations will vary. Sample explanation: If the original ellipse is not centered at the origin then the rotated ellipse will not be centered at the origin.

63. makes sense

64. makes sense

65. $A = 3, B = 2,$ and $C = 3.$

$$\cot 2\theta = \frac{A-C}{B} = \frac{3-3}{2} = 0$$
$$2\theta = 90°$$
$$\theta = 45°$$

$$x = x'\cos 45° - y'\sin 45° = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x'-y')$$

$$y = x'\sin 45° + y'\cos 45° = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x'+y')$$

Substitute into the equation: $3x^2-2xy+3y^2+2=0$

$$3\left[\frac{\sqrt{2}}{2}(x'-y')\right]^2 - 2\left[\frac{\sqrt{2}}{2}(x'-y')\right]\left[\frac{\sqrt{2}}{2}(x'+y')\right] + 3\left[\frac{\sqrt{2}}{2}(x'+y')\right]^2 + 2 = 0$$

$$\frac{3}{2}\left(x'^2-2x'y'+y'^2\right) - \left(x'^2-y'^2\right) + \frac{3}{2}\left(x'^2+2x'y'+y'^2\right) = -2$$

Multiply both sides by 2:

$$3x'^2-6x'y'+3y'^2-2x'^2+2y'^2+3x'^2+6x'y'+3y'^2 = -4$$

$$4x'^2+8y'^2 = -4$$

$$\frac{4x'^2+8y'^2}{-4} = \frac{-4}{-4}$$

$$-x'^2-2y'^2 = 1$$

There are no solutions to this equation since the left side of the equation is negative or 0 for all values of x' and y'. Thus, there are no points on the graph of this equation, just as one hand clapping makes no sound.

66. Let $x = x'\cos\theta - y'\sin\theta$ and $y = x'\sin\theta + y'\cos\theta$ in the equation $x^2 + y^2 = r^2$.

$$x^2 + y^2 = r^2$$
$$(x'\cos\theta - y'\sin\theta)^2 + (x'\sin\theta + y'\cos\theta)^2 = r^2$$
$$x'^2\cos^2\theta - 2x'y'\sin\theta\cos\theta + y'^2\sin^2\theta + x'^2\sin^2\theta + 2x'y'\sin\theta\cos\theta + y'^2\cos^2\theta = r^2$$
$$x'^2\left(\cos^2\theta + \sin^2\theta\right) + y'^2\left(\sin^2\theta + \cos^2\theta\right) = r^2$$
$$x'^2 + y'^2 = r^2$$

In a rotated $x'y'$-system, x and y are replaced with x' and y', respectively.

67.
$$A' = A\cos^2\theta + B\sin\theta\cos\theta + C\sin^2\theta$$
$$C' = A\sin^2\theta - B\sin\theta\cos\theta + C\cos^2\theta$$
$$\begin{aligned} A' + C' &= A\cos^2\theta + B\sin\theta\cos\theta + C\sin^2\theta + A\sin^2\theta - B\sin\theta\cos\theta + C\cos^2\theta \\ &= A\left(\cos^2\theta + \sin^2\theta\right) + B(\sin\theta\cos\theta - \sin\theta\cos\theta) + C\left(\sin^2\theta + \cos^2\theta\right) \\ &= A(1) + B(0) + C(1) \\ &= A + C \end{aligned}$$

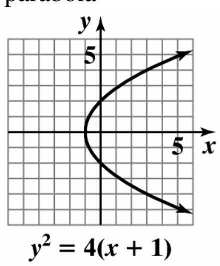
68.
$$A' = A\cos^2\theta + B\sin\theta\cos\theta + C\sin^2\theta$$
$$B' = B\left(\cos^2\theta - \sin^2\theta\right) + 2(C - A)(\sin\theta\cos\theta)$$
$$C' = A\sin^2\theta - B\sin\theta\cos\theta + C\cos^2\theta$$
$$\begin{aligned} B'^2 &= \left[B\left(\cos^2\theta - \sin^2\theta\right) + 2(C - A)(\sin\theta\cos\theta)\right]^2 \\ &= B^2\left(\cos^2\theta - \sin^2\theta\right)^2 + 4B(C - A)(\sin\theta\cos\theta)\left(\cos^2\theta - \sin^2\theta\right) + 4(C - A)^2(\sin\theta\cos\theta)^2 \\ &= B^2\left(\cos^4\theta - 2\cos^2\theta\sin^2\theta + \sin^4\theta\right) + 4BC(\sin\theta\cos\theta)\left(\cos^2\theta - \sin^2\theta\right) - 4AB(\sin\theta\cos\theta)\left(\cos^2\theta - \sin^2\theta\right) \\ &\quad + 4C^2(\sin\theta\cos\theta)^2 - 8AC(\sin\theta\cos\theta)^2 + 4A^2(\sin\theta\cos\theta)^2 \end{aligned}$$
$$\begin{aligned} 4A'C' &= 4\left[\left(A\cos^2\theta + B\sin\theta\cos\theta + C\sin^2\theta\right)\left(A\sin^2\theta - B\sin\theta\cos\theta + C\cos^2\theta\right)\right] \\ &= 4\left[A^2(\sin\theta\cos\theta)^2 - AB\cos^2\theta(\sin\theta\cos\theta) + AC\cos^4\theta\right] \\ &\quad + 4\left[AB\sin^2\theta(\sin\theta\cos\theta) - B^2(\sin\theta\cos\theta)^2 + BC\cos^2\theta(\sin\theta\cos\theta)\right] \\ &\quad + 4\left[AC\sin^4\theta - BC\sin^2\theta(\sin\theta\cos\theta) + C^2(\sin\theta\cos\theta)^2\right] \\ &= 4A^2(\sin\theta\cos\theta)^2 - 4B^2(\sin\theta\cos\theta)^2 + 4C^2(\sin\theta\cos\theta)^2 + 4AB\left(\sin^2\theta(\sin\theta\cos\theta) - \cos^2\theta(\sin\theta\cos\theta)\right) \\ &\quad + 4AC\left(\cos^4\theta + \sin^4\theta\right) + 4BC\left(\cos^2\theta(\sin\theta\cos\theta) - \sin^2\theta(\sin\theta\cos\theta)\right) \end{aligned}$$

Subtracting these results,

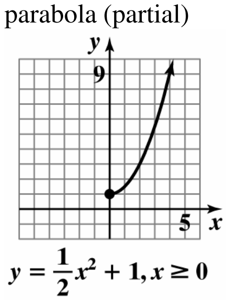
$$B'^2 - 4A'C'$$
$$= 4A^2\left[(\sin\theta\cos\theta)^2 - (\sin\theta\cos\theta)^2\right] + B^2\left[\cos^4\theta + 2\cos^2\theta\sin^2\theta + \sin^4\theta\right] + 4C^2\left[(\sin\theta\cos\theta)^2 - (\sin\theta\cos\theta)^2\right]$$
$$+4AB\left[\cos^2\theta(\sin\theta\cos\theta) - \sin^2\theta(\sin\theta\cos\theta) - (\sin\theta\cos\theta)\left(\cos^2\theta - \sin^2\theta\right)\right]$$
$$-4AC\left[\cos^4\theta + 2(\sin\theta\cos\theta)^2 + \sin^4\theta\right] + 4BC\left[(\sin\theta\cos\theta)\left(\cos^2\theta - \sin^2\theta\right) - (\sin\theta\cos\theta)\left(\cos^2\theta - \sin^2\theta\right)\right]$$
$$= 4A^2[0] + B^2\left[\left(\cos^2\theta + \sin^2\theta\right)^2\right] + 4C^2[0] + 4AB\left[\sin\theta\cos\theta\left(\cos^2\theta - \sin^2\theta - \cos^2\theta + \sin^2\theta\right)\right]$$
$$-4AC\left[\left(\cos^2\theta + \sin^2\theta\right)^2\right] + 4BC\left[\sin\theta\cos\theta\left(\cos^2\theta - \sin^2\theta - \cos^2\theta + \sin^2\theta\right)\right]$$
$$= B^2\left[(1)^2\right] + 4AB[0] - 4AC\left[(1)^2\right] + 4BC[0] = B^2 - 4AC$$

69. Answers may vary.

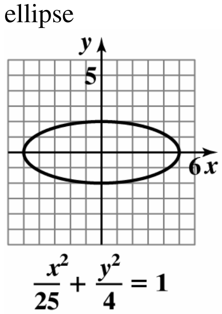
70. parabola



71. parabola (partial)



72. ellipse

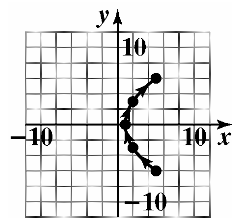


Section 10.5

Check Point Exercises

1.

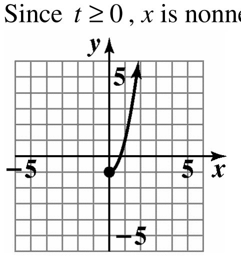
t	$x = t^2 + 1$	$y = 3t$	$(x,\ y)$
-2	$(-2)^2 + 1 = 5$	$3(-2) = -6$	$(5,\ -6)$
-1	$(-1)^2 + 1 = 2$	$3(-1) = -3$	$(2,\ -3)$
0	$0^2 + 1 = 1$	$3(0) = 0$	$(1,\ 0)$
1	$1^2 + 1 = 2$	$3(1) = 3$	$(2,\ 3)$
2	$2^2 + 1 = 5$	$3(2) = 6$	$(5,\ 6)$



2. $x = \sqrt{t} \quad \Rightarrow \quad t = x^2$

Using $t = x^2$ and $y = 2t - 1$, $y = 2x^2 - 1$.

Since $t \geq 0$, x is nonnegative.



3. $x = 6\cos t$ and $y = 4\sin t,\ \pi \leq t \leq 2\pi$

$\frac{x}{6} = \cos t \qquad \frac{y}{4} = \sin t$

Square and add the equations:

$$\frac{x^2}{36} = \cos^2 t$$

$$+ \quad \frac{y^2}{16} = \sin^2 t$$

$$\frac{x^2}{36} + \frac{y^2}{16} = \cos^2 t + \sin^2 t$$

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

Since t is in the interval $[\pi, 2\pi]$, we use $t=\pi,\ t=\frac{3\pi}{2},$ and $t=2\pi$:

$t=\pi$: $\quad x=6\cos\pi=-6$

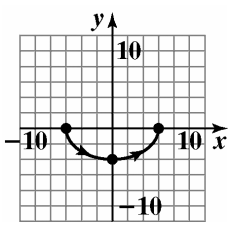
$\quad y=4\sin\pi=0$

$t=\frac{3\pi}{2}$: $\quad x=6\cos\frac{3\pi}{2}=0$

$\quad y=4\sin\frac{3\pi}{2}=-4$

$t=2\pi$: $\quad x=6\cos 2\pi=6$

$\quad y=4\cos 2\pi=0$



4. $y=x^2-25$

Let $x=t$. Then $y=t^2-25$.

The parametric equations are $x=t$ and $y=t^2-25$.

Exercise Set 10.5

1. $x=3-5(1)=-2$

$y=4+2(1)=6;$

$(-2, 6)$

2. $x=7-4(1)=3$

$y=5+6(1)=11;$

$(3, 11)$

3. $x=2^2+1=5$

$y=5-2^3=5-8=-3;$

$(5, -3)$

4. $x=2^2+3=4+3=7$

$y=6-2^3=6-8=-2;$

$(7, -2)$

5. $x=4+2\cos\frac{\pi}{2}=4+2(0)=4$

$y=3+5\sin\frac{\pi}{2}=3+5(1)=8;$

$(4, 8)$

6. $x = 2 + 3\cos\pi = 2 + 3(-1) = -1$

$y = 4 + 2\sin\pi = 4 + 2(0) = 4;$

$(-1,\ 4)$

7. $x = (60\cos 30^\circ)(2) = \left(60 \cdot \frac{\sqrt{3}}{2}\right)(2) = 60\sqrt{3}$

$y = 5 + (60\sin 30^\circ)(2) - 16(2)^2$

$= 5 + \left(60 \cdot \frac{1}{2}\right)(2) - 16 \cdot 4$

$= 5 + 60 - 64 = 1;$

$\left(60\sqrt{3},\ 1\right)$

8. $x = (80\cos 45^\circ)(2) = \left(80 \cdot \frac{\sqrt{2}}{2}\right)(2) = 80\sqrt{2}$

$y = 6 + (80\sin 45^\circ)(2) - 16(2)^2$

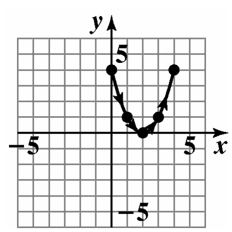
$= 6 + \left(80 \cdot \frac{\sqrt{2}}{2}\right)(2) - 16 \cdot 4$

$= 6 + 80\sqrt{2} - 64 = -58 + 80\sqrt{2};$

$\left(80\sqrt{2},\ -58 + 80\sqrt{2}\right)$

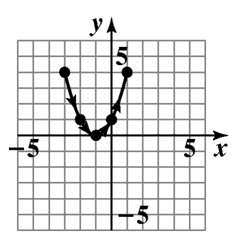
9.

t	$x = t + 2$	$y = t^2$	$(x,\ y)$
-2	$-2 + 2 = 0$	$(-2)^2 = 4$	$(0,\ 4)$
-1	$-1 + 2 = 1$	$(-1)^2 = 1$	$(1,\ 1)$
0	$0 + 2 = 2$	$0^2 = 0$	$(2,\ 0)$
1	$1 + 2 = 3$	$1^2 = 1$	$(3,\ 1)$
2	$2 + 2 = 4$	$2^2 = 4$	$(4,\ 4)$



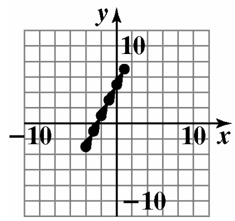
10.

t	$x = t - 1$	$y = t^2$	$(x,\ y)$
-2	$(-2) - 1 = -3$	$(-2)^2 = 4$	$(-3,\ 4)$
-1	$(-1) - 1 = -2$	$(-1)^2 = 1$	$(-2,\ 1)$
0	$0 - 1 = -1$	$0^2 = 0$	$(-1,\ 0)$
1	$1 - 1 = 0$	$1^2 = 1$	$(0,\ 1)$
2	$2 - 1 = 1$	$2^2 = 4$	$(1,\ 4)$



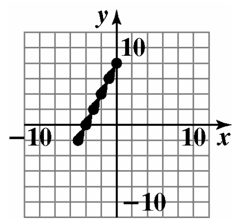
11.

t	$x=t-2$	$y=2t+1$	(x, y)
–2	$-2-2=-4$	$2(-2)+1=-3$	$(-4, -3)$
–1	$-1-2=-3$	$2(-1)+1=-1$	$(-3, -1)$
0	$0-2=-2$	$2(0)+1=1$	$(-2, 1)$
1	$1-2=-1$	$2(1)+1=3$	$(-1, 3)$
2	$2-2=0$	$2(2)+1=5$	$(0, 5)$
3	$3-2=1$	$2(3)+1=7$	$(1, 7)$



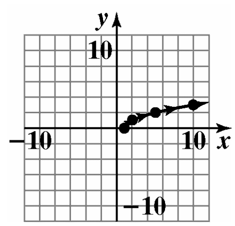
12.

t	$x=t-3$	$y=2t+2$	(x, y)
–2	$-2-3=-5$	$2(-2)+2=-2$	$(-5, -2)$
–1	$-1-3=-4$	$2(-1)+2=0$	$(-4, 0)$
0	$0-3=-3$	$2(0)+2=2$	$(-3, 2)$
1	$1-3=-2$	$2(1)+2=4$	$(-2, 4)$
2	$2-3=-1$	$2(2)+2=6$	$(-1, 6)$
3	$3-3=0$	$2(3)+2=8$	$(0, 8)$



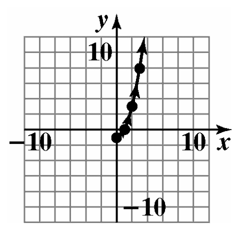
13.

t	$x=t+1$	$y=\sqrt{t}$	(x, y)
0	$0+1=1$	$\sqrt{0}=0$	$(1, 0)$
1	$1+1=2$	$\sqrt{1}=1$	$(2, 1)$
4	$4+1=5$	$\sqrt{4}=2$	$(5, 2)$
9	$9+1=10$	$\sqrt{9}=3$	$(10, 3)$



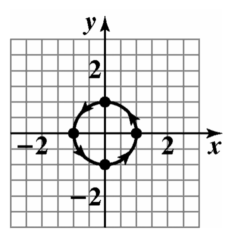
14.

t	$x=\sqrt{t}$	$y=t-1$	(x, y)
0	$\sqrt{0}=0$	$0-1=-1$	$(0, -1)$
1	$\sqrt{1}=1$	$1-1=0$	$(1, 0)$
4	$\sqrt{4}=2$	$4-1=3$	$(2, 3)$
9	$\sqrt{9}=3$	$9-1=8$	$(3, 8)$



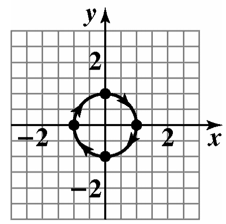
15.

t	$x=\cos t$	$y=\sin t$	(x, y)
0	$\cos 0=1$	$\sin 0=0$	$(1, 0)$
$\frac{\pi}{2}$	$\cos\frac{\pi}{2}=0$	$\sin\frac{\pi}{2}=1$	$(0, 1)$
π	$\cos\pi=-1$	$\sin\pi=0$	$(-1, 0)$
$\frac{3\pi}{2}$	$\cos\frac{3\pi}{2}=0$	$\sin\frac{3\pi}{2}=-1$	$(0, -1)$
2π	$\cos 2\pi=1$	$\sin 2\pi=0$	$(1, 0)$



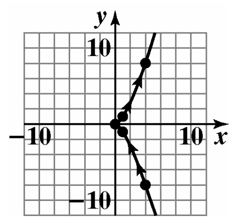
16.

t	$x = -\sin t$	$y = -\cos t$	(x, y)
0	$-\sin 0 = 0$	$-\cos 0 = -1$	$(0, -1)$
$\frac{\pi}{2}$	$-\sin\frac{\pi}{2} = -1$	$-\cos\frac{\pi}{2} = 0$	$(-1, 0)$
π	$-\sin\pi = 0$	$-\cos\pi = 1$	$(0, 1)$
$\frac{3\pi}{2}$	$-\sin\frac{3\pi}{2} = 1$	$-\cos\frac{3\pi}{2} = 0$	$(1, 0)$
2π	$-\sin 2\pi = 0$	$-\cos 2\pi = -1$	$(0, -1)$



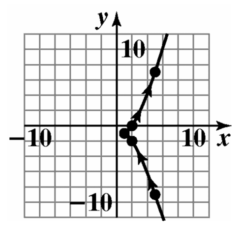
17.

t	$x = t^2$	$y = t^3$	(x, y)
-2	$(-2)^2 = 4$	$(-2)^3 = -8$	$(4, -8)$
-1	$(-1)^2 = 1$	$(-1)^3 = -1$	$(1, -1)$
0	$0^2 = 0$	$0^3 = 0$	$(0, 0)$
1	$1^2 = 1$	$1^3 = 1$	$(1, 1)$
2	$2^2 = 4$	$2^3 = 8$	$(4, 8)$



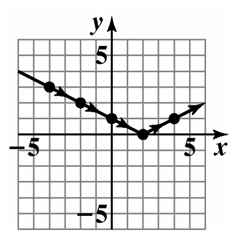
18.

t	$x = t^2 + 1$	$y = t^3 - 1$	(x, y)
-2	$(-2)^2 + 1 = 5$	$(-2)^3 - 1 = -9$	$(5, -9)$
-1	$(-1)^2 + 1 = 2$	$(-1)^3 - 1 = -2$	$(2, -2)$
0	$0^2 + 1 = 1$	$0^3 - 1 = -1$	$(1, -1)$
1	$1^2 + 1 = 2$	$1^3 - 1 = 0$	$(2, 0)$
2	$2^2 + 1 = 5$	$2^3 - 1 = 7$	$(5, 7)$



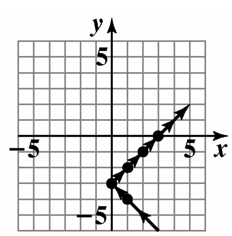
19.

t	$x = 2t$	$y = \lvert t-1 \rvert$	(x, y)
-2	$2(-2) = -4$	$\lvert -2-1 \rvert = 3$	$(-4, 3)$
-1	$2(-1) = -2$	$\lvert -1-1 \rvert = 2$	$(-2, 2)$
0	$2(0) = 0$	$\lvert 0-1 \rvert = 1$	$(0, 1)$
1	$2(1) = 2$	$\lvert 1-1 \rvert = 0$	$(2, 0)$
2	$2(2) = 4$	$\lvert 2-1 \rvert = 1$	$(4, 1)$

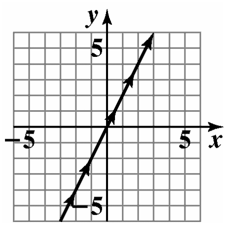


20.

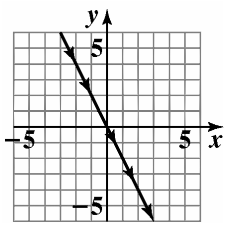
t	$x = \lvert t+1 \rvert$	$y = t - 2$	(x, y)
-2	$\lvert -2+1 \rvert = 1$	$-2-2 = -4$	$(1, -4)$
-1	$\lvert -1+1 \rvert = 0$	$-1-2 = -3$	$(0, -3)$
0	$\lvert 0+1 \rvert = 1$	$0-2 = -2$	$(1, -2)$
1	$\lvert 1+1 \rvert = 2$	$1-2 = -1$	$(2, -1)$
2	$\lvert 2+1 \rvert = 3$	$2-2 = 0$	$(3, 0)$



21. $x = t \quad \Rightarrow \quad y = 2x$



22. $x = t \quad \Rightarrow \quad y = -2x$



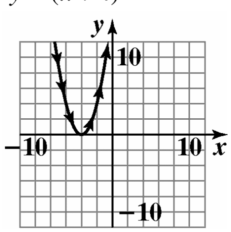
23. $x = 2t - 4$

$\frac{x+4}{2} = t$

Substitute into *y*:

$y = 4\left(\frac{x+4}{2}\right)^2 = (x+4)^2$

$y = (x+4)^2$

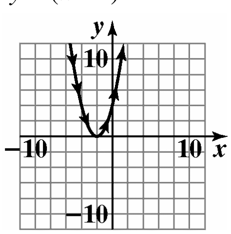


24. $x = t - 2$

$x + 2 = t$

Substitute into *y*:

$y = (x+2)^2$



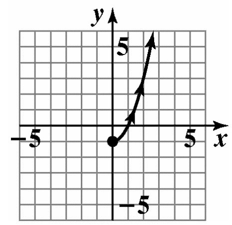
25. $x = \sqrt{t}$

$x^2 = t$

Substitute into *y*:

$y = x^2 - 1$

Since $t \geq 0$ in $x = \sqrt{t}$, $x \geq 0$.



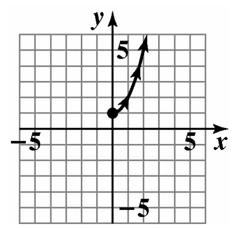
26. $x = \sqrt{t}$

$x^2 = t$

Substitute into *y*:

$y = x^2 + 1$

Since $t \geq 0$ in $x = \sqrt{t}$, $x \geq 0$.



27. $x = 2\sin t$ and $y = 2\cos t$

$\frac{x}{2} = \sin t \qquad \frac{y}{2} = \cos t$

Square and add the equations:

$$\frac{x^2}{4} = \sin^2 t$$

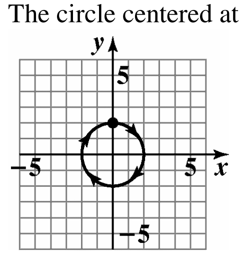
$$+ \quad \frac{y^2}{4} = \cos^2 t$$

$$\frac{x^2}{4} + \frac{y^2}{4} = \sin^2 t + \cos^2 t$$

$$\frac{x^2}{4} + \frac{y^2}{4} = 1$$

$$x^2 + y^2 = 4$$

The circle centered at (0, 0) with radius 2.



28. $x = 3\sin t$ and $y = 3\cos t$

$\frac{x}{3} = \sin t \qquad \frac{y}{3} = \cos t$

Square and add the equations:

$$\frac{x^2}{9} = \sin^2 t$$

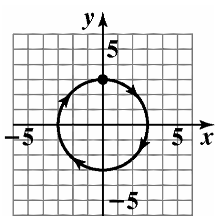
$$+ \quad \frac{y^2}{9} = \cos^2 t$$

$$\frac{x^2}{9} + \frac{y^2}{9} = \sin^2 t + \cos^2 t$$

$$\frac{x^2}{9} + \frac{y^2}{9} = 1$$

$$x^2 + y^2 = 9$$

This is a circle centered at (0, 0) with radius 3.



29. $x = 1 + 3\cos t$ and $y = 2 + 3\sin t$

$\frac{x-1}{3} = \cos t \qquad \frac{y-2}{3} = \sin t$

Square and add the equations:

$$\frac{(x-1)^2}{9} = \cos^2 t$$

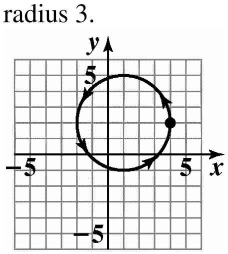
$$+ \quad \frac{(y-2)^2}{9} = \sin^2 t$$

$$\frac{(x-1)^2}{9} + \frac{(y-2)^2}{9} = \cos^2 t + \sin^2 t$$

$$\frac{(x-1)^2}{9} + \frac{(y-2)^2}{9} = 1$$

$$(x-1)^2 + (y-2)^2 = 9$$

This is a circle centered at (1, 2) with radius 3.



30. $x = -1 + 2\cos t$ and $y = 1 + 2\sin t$

$\frac{x+1}{2} = \cos t \qquad \frac{y-1}{2} = \sin t$

Square and add the equations:

$$\frac{(x+1)^2}{4} = \cos^2 t$$

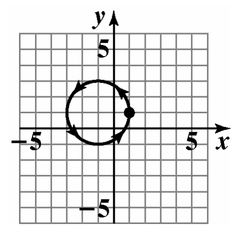
$$+ \quad \frac{(y-1)^2}{4} = \sin^2 t$$

$$\frac{(x+1)^2}{4} + \frac{(y-1)^2}{4} = \cos^2 t + \sin^2 t$$

$$\frac{(x+1)^2}{4} + \frac{(y-1)^2}{4} = 1$$

$$(x+1)^2 + (y-1)^2 = 4$$

This a circle centered at (–1, 1) with radius 2.



31. $x = 2\cos t$ and $y = 3\sin t$

$\frac{x}{2} = \cos t \qquad \frac{y}{3} = \sin t$

Square and add the equations:

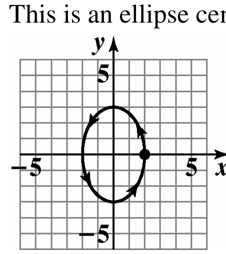
$$\frac{x^2}{4} = \cos^2 t$$

$$+ \quad \frac{y^2}{9} = \sin^2 t$$

$$\frac{x^2}{4} + \frac{y^2}{9} = \cos^2 t + \sin^2 t$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

This is an ellipse centered at (0, 0).



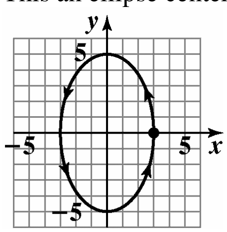
32. $x = 3\cos t$ and $y = 5\sin t$

$\frac{x}{3} = \cos t \qquad \frac{y}{5} = \sin t$

Square and add the equations:

$$\begin{array}{ll} & \frac{x^2}{9} = \cos^2 t \\ + & \frac{y^2}{25} = \sin^2 t \\ \hline & \frac{x^2}{9} + \frac{y^2}{25} = \cos^2 t + \sin^2 t \\ & \frac{x^2}{9} + \frac{y^2}{25} = 1 \end{array}$$

This an ellipse centered at (0, 0).



33. $x = 1 + 3\cos t$ and $y = -1 + 2\sin t$

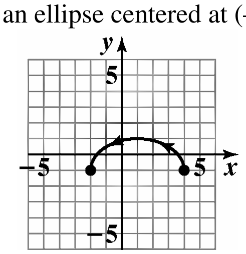
$\frac{x-1}{3} = \cos t \qquad \frac{y+1}{2} = \sin t$

Square and add the equations:

$$\begin{array}{ll} & \frac{(x-1)^2}{9} = \cos^2 t \\ + & \frac{(y+1)^2}{4} = \sin^2 t \\ \hline & \frac{(x-1)^2}{9} + \frac{(y+1)^2}{4} = \cos^2 t + \sin^2 t \\ & \frac{(x-1)^2}{9} + \frac{(y+1)^2}{4} = 1 \end{array}$$

Since $0 \le t \le \pi$, $-1 \le \cos t \le 1$ and $0 \le \sin t \le 1$.

Thus, $-1 \le \frac{x-1}{3} \le 1$ and $0 \le \frac{y+1}{2} \le 1$. Hence, $-2 \le x \le 4$ and $-1 \le y \le 1$. This is the upper half of an ellipse centered at (–1, 1).



34. $x = 2 + 4\cos t$ and $y = -1 + 3\sin t$

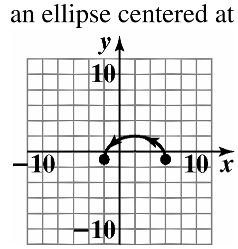
$\frac{x-2}{4} = \cos t \qquad \frac{y+1}{3} = \sin t$

Square and add the equations:

$$\begin{array}{ll} & \frac{(x-2)^2}{16} = \cos^2 t \\ + & \frac{(y+1)^2}{9} = \sin^2 t \\ \hline & \frac{(x-2)^2}{16} + \frac{(y+1)^2}{9} = \cos^2 t + \sin^2 t \\ & \frac{(x-2)^2}{16} + \frac{(y+1)^2}{9} = 1 \end{array}$$

Since $0 \le t \le \pi$, $-1 \le \cos t \le 1$ and $0 \le \sin t \le 1$.

Thus, $-1 \le \frac{x-2}{4} \le 1$ and $0 \le \frac{y+1}{3} \le 1$. Hence, $-2 \le x \le 6$ and $-1 \le y \le 2$. This is the upper half of an ellipse centered at (2, –1).

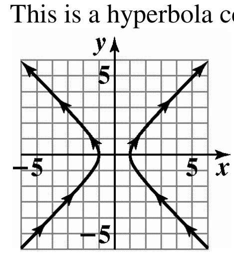


35. $x = \sec t$ and $y = \tan t$

Square and subtract the equations:

$$\begin{array}{ll} & x^2 = \sec^2 t \\ - & (y^2 = \tan^2 t) \\ \hline & x^2 - y^2 = \sec^2 t - \tan^2 t \\ & x^2 - y^2 = 1 \end{array}$$

This is a hyperbola centered at (0, 0).



36. $x = 5\sec t$ and $y = 3\tan t$

$\frac{x}{5} = \sec t \qquad \frac{y}{3} = \tan t$

Square and subtract the equations:

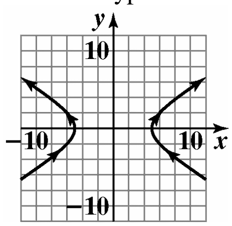
$$\frac{x^2}{25} = \sec^2 t$$

$$-\left(\frac{y^2}{9} = \tan^2 t\right)$$

$$\frac{x^2}{25} - \frac{y^2}{9} = \sec^2 t - \tan^2 t$$

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

This is a hyperbola centered at (0, 0).



37. $x = t^2 + 2$ and $y = t^2 - 2$

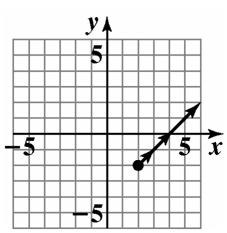
$x - 2 = t^2$

Substitute $x - 2$ into $y = t^2 - 2$ for t^2:

$y = (x-2) - 2$

$y = x - 4$

Since $t^2 \geq 0$ for all t, $x \geq 2$ and $y \geq -2$.



38. $x = \sqrt{t} + 2$ and $y = \sqrt{t} - 2$

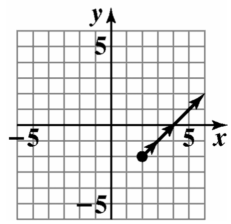
$x - 2 = \sqrt{t}$

Substitute $x - 2$ into $y = \sqrt{t} - 2$ for $\sqrt{t}$:

$y = (x-2) - 2$

$y = x - 4$

Since $t \geq 0$ in $\sqrt{t}$, $x \geq 2$ and $y \geq -2$.



39. $x = 2^t$ and $y = 2^{-t}$

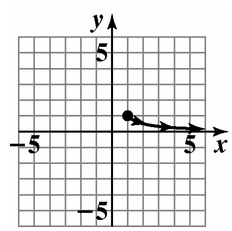
$$y = \left(2^t\right)^{-1}$$

Substitute x in $y = \left(2^t\right)^{-1}$ for 2^t:

$y = (x)^{-1}$

$y = \frac{1}{x}$

Since $t \geq 0$, $x \geq 1$ and $y \geq 0$.



40. $x = e^t$ and $y = e^{-t}$

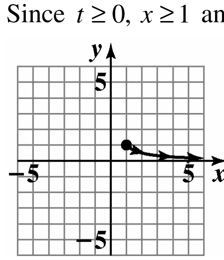
$$y = \left(e^t\right)^{-1}$$

Substitute x in $y = \left(e^t\right)^{-1}$ for e^t:

$y = (x)^{-1}$

$y = \frac{1}{x}$

Since $t \geq 0$, $x \geq 1$ and $y \geq 0$.



41. $x = h + r\cos t$ and $y = k + r\sin t$

$\frac{x-h}{r} = \cos t$ $\frac{y-k}{r} = \sin t$

Square and add the equations:

$$\frac{(x-h)^2}{r^2} = \cos^2 t$$

$$+ \quad \frac{(y-k)^2}{r^2} = \sin^2 t$$

$$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = \cos^2 t + \sin^2 t$$

$$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$$

$$(x-h)^2 + (y-k)^2 = r^2$$

42. $x = h + a\cos t$ and $y = k + b\sin t$

$\frac{x-h}{a} = \cos t$ $\frac{y-k}{b} = \sin t$

Square and add the equations:

$$\frac{(x-h)^2}{a^2} = \cos^2 t$$

$$+ \quad \frac{(y-k)^2}{b^2} = \sin^2 t$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = \cos^2 t + \sin^2 t$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

43. $x = h + a\sec t$ and $y = k + b\tan t$

$\frac{x-h}{a} = \sec t$ $\frac{y-k}{b} = \tan t$

Square and subtract the equations:

$$\frac{(x-h)^2}{a^2} = \sec^2 t$$

$$- \quad \left(\frac{(y-k)^2}{b^2} = \tan^2 t\right)$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = \sec^2 t - \tan^2 t$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

44. $x = x_1 + t(x_2 - x_1)$

$\frac{x - x_1}{x_2 - x_1} = t$

Substitute into the "y" equation:

$$y = y_1 + \left(\frac{x - x_1}{x_2 - x_1}\right)(y_2 - y_1)$$

$$y - y_1 = \frac{(x - x_1)}{(x_2 - x_1)} \cdot (y_2 - y_1)$$

$$y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)} \cdot (x - x_1)$$

$$y - y_1 = m(x - x_1) \text{ where } m\frac{y_2 - y_1}{x_2 - x_1}$$

45. $h = 3,\ k = 5,$ and $r = 6$

$x = h + r\cos t$ and $y = k + r\sin t$

$x = 3 + 6\cos t$ $y = 5 + 6\sin t$

46. $h = 4,\ k = 6,$ and $r = 9$

$x = h + r\cos t$ and $y = k + r\sin t$

$x = 4 + 9\cos t$ $y = 6 + 9\sin t$

47. $h = -2,\ k = 3,\ a = 5,\ b = 2$

$x = h + a\cos t$ and $y = k + b\sin t$

$x = -2 + 5\cos t$ $y = 3 + 2\sin t$

48. $h = 4,\ k = -1,\ a = 3,\ b = 5$

$x = h + a\cos t$ and $y = k + b\sin t$

$x = 4 + 3\cos t$ $y = -1 + 5\sin t$

49. $h = 0,\ k = 0,\ a = 4,\ c = 6$

$$c^2 = a^2 + b^2$$

$$6^2 = 4^2 + b^2$$

$$36 - 16 = b^2$$

$$b = \sqrt{20} = 2\sqrt{5}$$

$x = h + a\sec t$ and $y = k + b\tan t$

$x = 0 + 4\sec t$ $y = 0 + 2\sqrt{5}\tan t$

$x = 4\sec t$ $y = 2\sqrt{5}\tan t$

50. $h = 0,\ k = 0,\ b = 4,\ c = 5$

$c^2 = a^2 + b^2$

$5^2 = a^2 + 4^2$

$a = 3$

The sec*t* and tan*t* factors must be switched:

$x = h + a\tan t$ and $y = k + b\sec t$

$x = 0 + 3\tan t$ $\quad y = 0 + 4\sec t$

$x = 3\tan t$ $\quad y = 4\sec t$

51. $x_1 = -2,\ y_1 = 4,\ x_2 = 1,\ y_2 = 7$

52. $x_1 = 3,\ y_1 = -1,\ x_2 = 9,\ y_2 = 12$

$x = x_1 + t(x_2 - x_1)$ and $y = y_1 + t(y_2 - y_1)$

$x = 3 + t(9 - 3)$ $\quad y = -1 + t(12 - (-1))$

$x = 3 + 6t$ $\quad y = -1 + 13t$

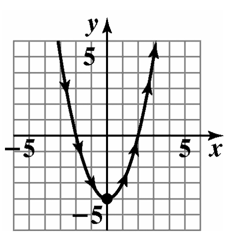
53. Answers may vary.
Sample answer:
$x = t$ and $y = 4t - 3$; $x = t + 1$ and $y = 4t + 1$

54. Answers may vary.
Sample answer:
$x = t$ and $y = 2t - 5$; $x = t + 1$ and $y = 2t - 3$

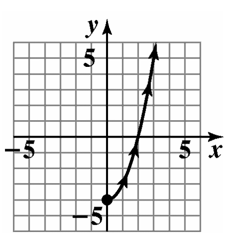
55. Answers may vary.
Sample answer:
$x = t$ and $y = t^2 + 4$; $x = t + 1$ and $y = t^2 + 2t + 5$

56. Answers may vary.
Sample answer:
$x = t$ and $y = t^2 - 3$; $x = t + 1$ and $y = t^2 + 2t - 2$

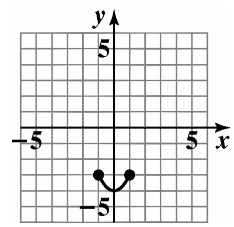
57. **a.**



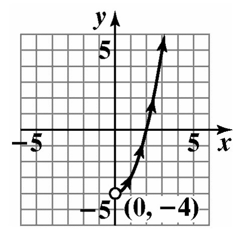
b.



c.

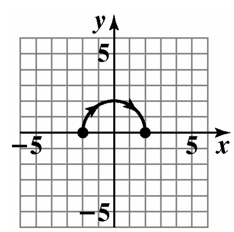


d.

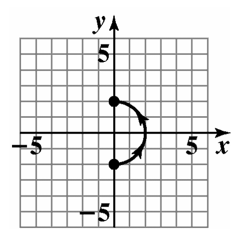


Explanations of how the curves differ from each other may vary.

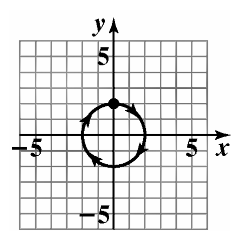
58. **a.**



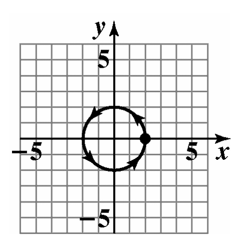
b.



c.

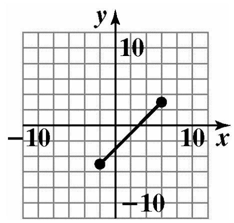


d.



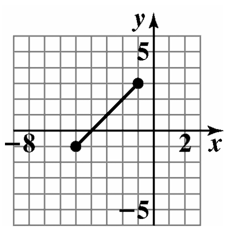
Explanations of how the curves differ from each other may vary.

59.



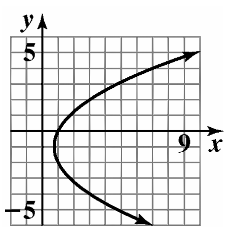
domain: [−2, 6];
range: [−5, 3]

60.



domain: [−5, −1];
range: [−1, 3]

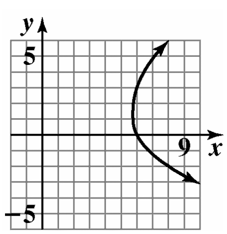
61.



domain: $\left[\frac{3}{4},\infty\right]$;

range: $[-\infty,\infty]$

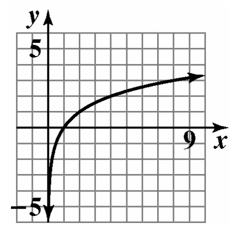
62.



domain: $\left[\frac{3}{4},\infty\right]$;

range: $[-\infty,\infty]$

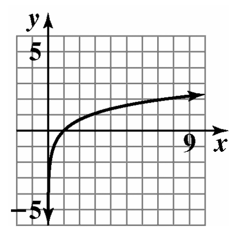
63.



a. increasing: $[-\infty,\infty]$

b. no maximum or minimum

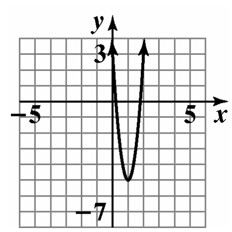
64.



a. increasing: $[-\infty,\infty]$

b. no maximum or minimum

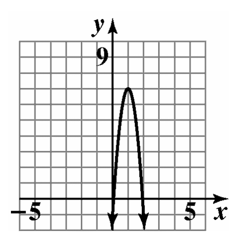
65.



a. decreasing: $[-\infty,1]$; increasing: $(1, \infty)$

b. minimum of −5 at $x = 1$

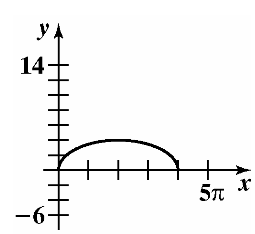
66.



a. increasing: $[-\infty,1]$; decreasing: $(1, \infty)$

b. maximum of 7 at $x = 1$

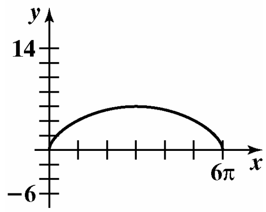
67.



a. increasing: $(1, 2\pi)$; decreasing: $(2\pi, 4\pi)$

b. maximum of 4 at $x = 2\pi$;
minimum of 0 at $x = 0$ and $x = 4\pi$

68.



a. increasing: $(0, 3\pi)$; decreasing: $(3\pi, 6\pi)$

b. maximum of 6 at $x = 3\pi$;
minimum of 0 at $x = 0$ and $x = 6\pi$

69. **a.** $x = (180\cos 40°)t$

$y = 3 + (180\sin 40°)t - 16t^2$

b. After 1 second:
$x = (180\cos 40°)\cdot 1$
≈ 137.9 feet in distance

$y = 3 + (180\sin 40°)1 - 16\cdot 1^2$
≈ 102.7 feet in height

After 2 seconds:
$x = (180\cos 40°)\cdot 2$
≈ 275.8 feet in distance

$y = 3 + (180\sin 40°)\cdot 2 - 16\cdot 2^2$
≈ 170.4 feet in height

After 3 seconds:
$x = (180\cos 40°)\cdot 3$
≈ 413.7 feet in distance

$y = 3 + (180\sin 40°)\cdot 3 - 16\cdot 3^2$
≈ 206.1 feet in height

The points on the curve are (137.9, 102.7), (275.8, 170.4), (413.7, 206.1).

c. The ball is no longer in flight when its height above ground is zero:

$0 = 3 + (180\sin 40°)t - 16t^2$

$0 = -16t^2 + (180\sin 40°)t + 3$

$$t = \frac{-(180\sin 40°) \pm \sqrt{(180\sin 40°)^2 - 4\cdot(-16)(3)}}{2(-16)}$$

$t \approx -.03$ or $t \approx 7.3$

Since we cannot use the negative time, the ball hits the ground at $t \approx 7.3$ seconds.
The total horizontal distance is:
$x = (180\cos 40°)\cdot(7.3) \approx 1006.6$ feet

d. Answers may vary.

70. **a.** $x = (150\cos 35°)t$

$y = 3 + (150\sin 35°)t - 16t^2$

b. After 1 second:
$x = (150\cos 35°)\cdot 1$
≈ 122.9 feet in distance

$y = 3 + (150\sin 35°)1 - 16\cdot 1^2$
≈ 73.0 feet in height

After 2 seconds:
$x = (150\cos 35°)\cdot 2$
≈ 245.7 feet in distance

$y = 3 + (150\sin 35°)\cdot 2 - 16\cdot 2^2$
≈ 111.1 feet in height

After 3 seconds:
$x = (150\cos 35°)\cdot 3$
≈ 368.6 feet in distance

$y = 3 + (150\sin 35°)\cdot 3 - 16\cdot 3^2$
≈ 117.1 feet in height

The points on the curve are (122.9, 73.0), (245.7, 111.1), (368.6, 117.1).

c. The ball is no longer in flight when its height is zero:

$0 = 3 + (150\sin 35°)t - 16t^2$

$0 = -16t^2 + (150\sin 35°)t + 3$

$$t = \frac{-(150\sin 35°) \pm \sqrt{(150\sin 35°)^2 - 4\cdot(-16)(3)}}{2(-16)}$$

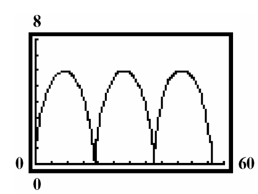
$t \approx -.03$ or $t \approx 5.4$

Since we cannot use the negative time, the ball hits the ground at $t = 5.4$ seconds.
The total horizontal distance is:
$x = (150\cos 35°)\cdot(5.4)$
≈ 663.5 feet

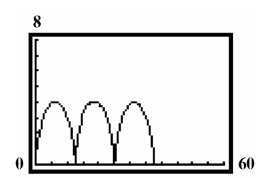
d. Answers may vary

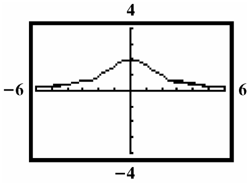
71. – 77. Answers may vary.

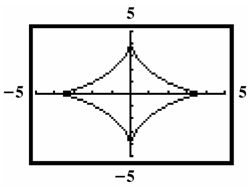
78.

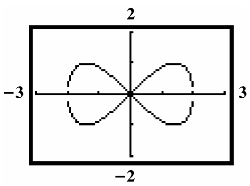


79.



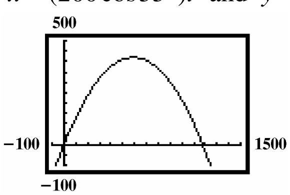
80. 

81. 

82. 

83. $x = (v_0 \cos\theta)t$ and $y = h + (v_0 \sin\theta)t - 16t^2$ where v_0 is the initial velocity, θ is the angle from horizontal, *h* is the height above the ground, and *t* is the time, in seconds.

$x = (200\cos 55^\circ)t$ and $y = (200\sin 55^\circ)t - 16t^2$

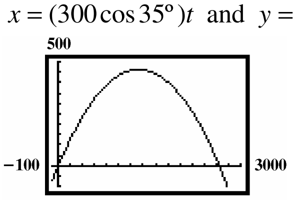


Window: [−100, 1500]×[−100, 500]

The maximum height is 419.4 feet at a time of 5.1 seconds. The range of the projective is 1174.6 feet horizontally. It hits the ground at 10.2 seconds.

84. $x = (v_0 \cos\theta)t$ and $y = h + (v_0 \sin\theta)t - 16t^2$ where v_0 is the initial velocity, θ is the angle from horizontal, *h* is the height above the ground, and *t* is the time, in seconds.

$x = (300\cos 35^\circ)t$ and $y = (300\sin 35^\circ)t - 16t^2$



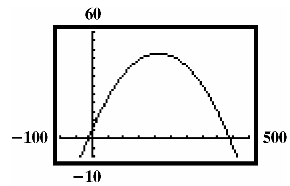
Window: [−100, 3000]×[−100, 500]

The maximum height is 462.6 feet at a time of 5.4 seconds.
The range of the projective is 2642.9 feet horizontally. It hits the ground at 10.8 seconds.

85. **a.** $x = (v_0 \cos\theta)t$ and $y = h + (v_0 \sin\theta)t - 16t^2$ where v_0 is the initial velocity, θ is the angle from horizontal, *h* is the height above the ground, and *t* is the time, in seconds.

$x = (140\cos 22^\circ)t$ and

$y = 5 + (140\sin 22^\circ)t - 16t^2$

b. 

Window: [−100, 500]×[−10, 60]

c. The maximum height is 48.0 feet. It occurs at 1.6 seconds.

d. The ball is in the air for 3.4 seconds.

e. The ball travels 437.5 feet.

86. makes sense

87. makes sense

88. does not make sense; Explanations will vary. Sample explanation: Those equations are not correct. One possible pair of equations is $x = t^2$ and $y = t^2 - 9$.

89. makes sense

90. $x = \cos^3 t$ and $y = \sin^3 t$

Raise both sides of each equation to the 2/3 power, and add the equations:

$$\begin{aligned} x^{2/3} &= \cos^2 t \\ + \quad y^{2/3} &= \sin^2 t \\ \hline x^{2/3} + y^{2/3} &= \cos^2 t + \sin^2 t \\ x^{2/3} + y^{2/3} &= 1 \\ \sqrt[3]{x^2} + \sqrt[3]{y^2} &= 1 \end{aligned}$$

91. $x = 3\sin t$ and $y = 3\cos t$

92. a. $\sin t = \frac{PB}{a} \Rightarrow \sin t = \frac{XA}{a} \Rightarrow a\sin t = XA$

OA is the same as the length of arc *PA*. Since the radius is *a*, and the central angle is *t*, the length of the arc is *at*.
So, *OA* = *at*. Therefore,

$$x = OA - XA$$
$$x = at - a\sin t$$
$$x = a(t - \sin t).$$

b. $\cos t = \frac{BC}{a} \Rightarrow a\sin t = BC$

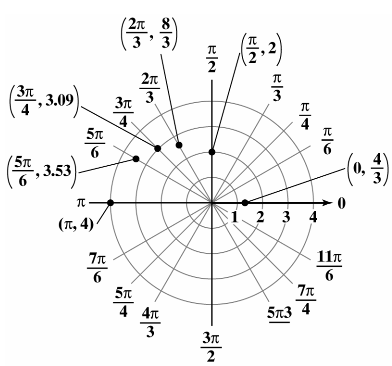
and *AC* = *a*, since *a* is the radius of the circle.
So,

$$y = AC - BC$$
$$y = a - a\cos t$$
$$y = a(1 - \cos t).$$

93. $r = \frac{2}{1 + \frac{1}{2}\cos\theta}$

94.

θ	0	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$r = \frac{4}{2 + \cos\theta}$	$\frac{4}{3}$ ≈ 1.33	2	$\frac{8}{3}$ ≈ 2.66	$\frac{16 + 4\sqrt{2}}{7}$ ≈ 3.09	$\frac{32 + 8\sqrt{3}}{13}$ ≈ 3.53	4



95. a.

$$r = \frac{1}{3 - 3\cos\theta}$$
$$r(3 - 3\cos\theta) = 1$$
$$3r - 3r\cos\theta = 1$$
$$3r = 1 + 3r\cos\theta$$
$$(3r)^2 = (1 + 3r\cos\theta)^2$$
$$9r^2 = (1 + 3r\cos\theta)^2$$

b.

$$9r^2 = (1 + 3r\cos\theta)^2$$
$$9(x^2 + y^2) = (1 + 3x)^2$$
$$9x^2 + 9y^2 = 1 + 6x + 9x^2$$
$$9y^2 = 1 + 6x$$

This is the equation of a parabola.

Section 10.6

Check Point Exercises

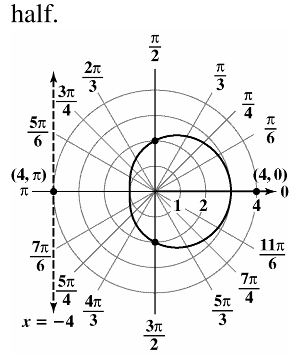
1. Graph $r = \frac{4}{2 - \cos\theta}$.

Step 1: Divide numerator and denominator by 2 to write the equation in standard form:
$r = \frac{2}{1 - \frac{1}{2}\cos\theta}$

Step 2: $e = \frac{1}{2}$ and $ep = \frac{1}{2}p = 2,$ so $p = 4.$ Since $e < 1,$ the graph is an ellipse.

Step 3: The graph has symmetry with respect to the polar axis. One focus is at the pole and the directrix is $x = -4$.
Find the vertices by selecting $\theta = 0$ and $\theta = \pi$: (4, 0) and $\left(\frac{4}{3}, \pi\right)$.

Sketch the upper half by plotting some points, then use the symmetry of the graph to sketch the lower half.



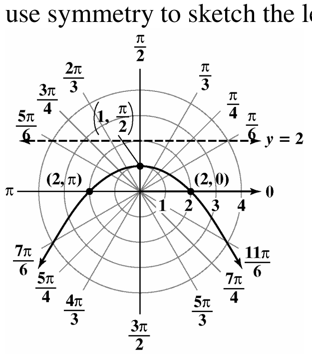
2. Graph $r = \dfrac{8}{4+4\sin\theta}$.

Step 1: Divide numerator and denominator by 4 to write the equation in standard form:
$r = \dfrac{2}{1+\sin\theta}$

Step 2: $e = 1$ and $ep = 1 \;\; p = 2,$ so $p = 2.$ Since $e = 1,$ the graph is parabola.

Step 3: The graph has symmetry with respect to $\theta = \dfrac{\pi}{2}$. The focus is at the pole and the directrix is $y = 2$. Since the vertex is on the line $\theta = \dfrac{\pi}{2}$ (y-axis) the vertex is at $\left(1, \dfrac{\pi}{2}\right)$. To find the intercepts on the polar axis, select $\theta = 0$ and $\theta = \pi$: (2, 0) and $(2, \pi)$.

Sketch the right half by plotting some points, then use symmetry to sketch the left half.



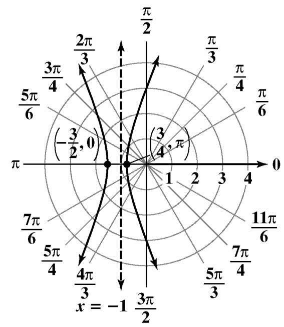
3. Graph $r = \dfrac{9}{3-9\cos\theta}$.

Step 1: Divide numerator and denominator by 3 to write the equation in standard form:
$r = \dfrac{3}{1-3\cos\theta}$

Step 2: $e = 3$ and $ep = 3p = 3,$ so $p = 1.$ Since $e > 1$, the graph is a hyperbola.

Step 3: The graph is symmetric with respect to the polar axis. One focus is at the pole and the directrix is at $x = -1$. The transverse axis is horizontal and the vertices lie on the polar axis. Find them by selecting $\theta = 0$ and $0 = \pi$:
$\left(-\dfrac{3}{2}, 0\right)$ and $\left(\dfrac{3}{4}, \pi\right)$.

Sketch the upper half of the hyperbola by plotting some points, then use symmetry to sketch the lower half.



Exercise Set 10.6

1. $r = \dfrac{3}{1+\sin\theta}$
$e = 1$ and $ep = 3$, so $p = 3$.

- **a.** The graph is a parabola.
- **b.** The directrix is 3 units above the pole, at $y = 3$.

2. $r = \dfrac{3}{1+\cos\theta}$
$e = 1$ and $ep = 3$, so $p = 3$.

- **a.** The graph is a parabola.
- **b.** The directrix is 3 units to the right of the pole, at $x = 3$.

3. $r = \dfrac{6}{3-2\cos\theta}$
Divide numerator and denominator by 3: $r = \dfrac{2}{1-\frac{2}{3}\cos\theta}$.

$e = \dfrac{2}{3}$ and $ep = 2,$ so $p = 3$.

- **a.** The graph is an ellipse.
- **b.** The directrix is 3 units to the left of the pole, at $x = -3$.

4. $r = \dfrac{6}{3+2\cos\theta}$

Divide numerator and denominator by 3: $r = \dfrac{2}{1+\frac{2}{3}\cos\theta}$.

$e = \dfrac{2}{3}$ and $ep = 2$, so $p = 3$.

a. The graph is an ellipse.

b. The directrix is 3 units to the right of the pole, at $x = 3$.

5. $r = \dfrac{8}{2+2\sin\theta}$

Divide numerator and denominator by 2: $r = \dfrac{4}{1+\sin\theta}$.

$e = 1$ and $ep = 4$, so $p = 4$.

a. The graph is a parabola.

b. The directrix is 4 units above the pole, at $y = 4$.

6. $r = \dfrac{8}{2-2\sin\theta}$

Divide numerator and denominator by 2: $r = \dfrac{4}{1-\sin\theta}$.

$e = 1$ and $ep = 4$, so $p = 4$.

a. The graph is a parabola.

b. The directrix is 4 units below the pole, at $y = -4$.

7. $r = \dfrac{12}{2-4\cos\theta}$

Divide numerator and denominator by 2: $r = \dfrac{6}{1-2\cos\theta}$.

$e = 2$ and $ep = 6$, so $p = 3$.

a. The graph is a hyperbola.

b. The directrix is 3 units to the left of the pole, at $x = -3$.

8. $r = \dfrac{12}{2+4\cos\theta}$

Divide numerator and denominator by 2: $r = \dfrac{6}{1+2\cos\theta}$.

$e = 2$ and $ep = 6$, so $p = 3$.

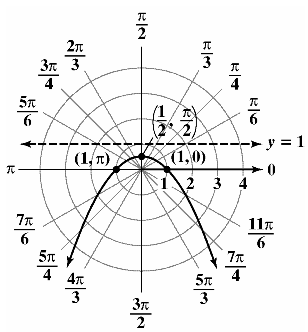
a. The graph is a hyperbola.

b. The directrix is 3 to the right of the pole, at $x = 3$.

9. $r = \dfrac{1}{1+\sin\theta}$

$e = 1$ and $ep = 1$, so $p = 1$.

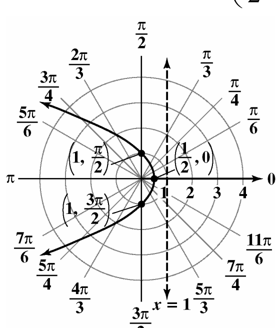
Since $e = 1$, the graph is a parabola. It is symmetric with respect to the y-axis and has a directrix at $y = 1$. The vertex is at $\left(\dfrac{1}{2}, \dfrac{\pi}{2}\right)$.



10. $r = \dfrac{1}{1+\cos\theta}$

$e = 1$ and $ep = 1$, so $p = 1$.

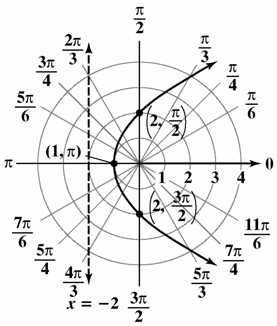
Since $e = 1$, the graph is a parabola. It is symmetric with respect to the polar axis and has a directrix at $x = 1$. The vertex is at $\left(\dfrac{1}{2}, 0\right)$.



11. $r = \dfrac{2}{1 - \cos\theta}$

$e = 1$ and $ep = 2$, so $p = 2$.

Since $e = 1$, the graph is a parabola. It is symmetric with respect to the polar axis and has a directrix at $x = -2$. The vertex is at $(1, \pi)$.

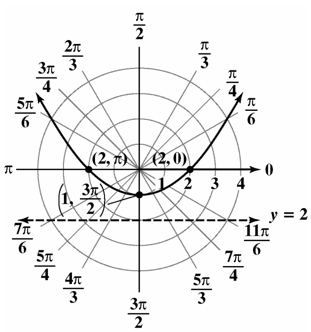


12. $r = \dfrac{2}{1 - \sin\theta}$

$e = 1$ and $ep = 2$, so $p = 2$.

Since $e = 1$, the graph is a parabola. It is symmetric with respect to the y-axis and has a directrix at $y = -2$.

The vertex is at $\left(1, \dfrac{3\pi}{2}\right)$.

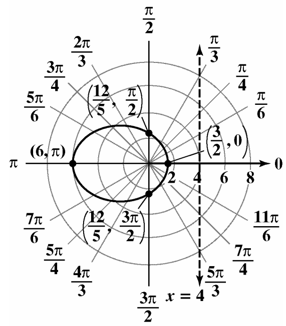


13. $r = \dfrac{12}{5 + 3\cos\theta}$

Write in standard form:

$r = \dfrac{\frac{12}{5}}{1 + \frac{3}{5}\cos\theta}$

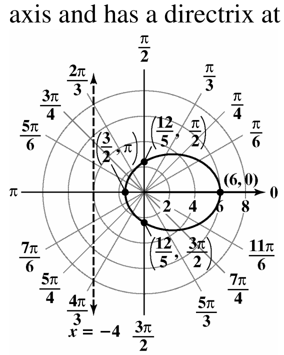
$e = \dfrac{3}{5}$ and $ep = \dfrac{12}{5}$, so $p = 4$. Since $e < 1$, the graph is an ellipse. It is symmetric with respect to the polar axis and has a directrix at $x = 4$.



14. $r = \dfrac{12}{5 - 3\cos\theta}$

Write in standard form: $r = \dfrac{\frac{12}{5}}{1 - \frac{3}{5}\cos\theta}$

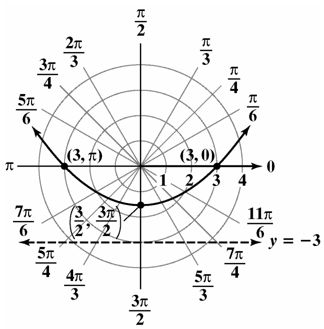
$e = \dfrac{3}{5}$ and $ep = \dfrac{12}{5}$, so $p = 4$. Since $e < 1$, the graph is an ellipse. It is symmetric with respect to the polar axis and has a directrix at $x = -4$.



15. $r = \dfrac{6}{2 - 2\sin\theta}$

Write in standard form: $r = \dfrac{3}{1 - \sin\theta}$

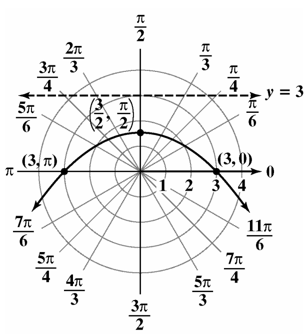
$e = 1$ and $ep = 3$, so $p = 3$. Since $e = 1$, the graph is a parabola. It is symmetric with respect to the y-axis and has a directrix at $y = -3$. The vertex is at $\left(\dfrac{3}{2}, \dfrac{3\pi}{2}\right)$.



16. $r = \dfrac{6}{2 + 2\sin\theta}$

Write in standard form: $r = \dfrac{3}{1 + \sin\theta}$

$e = 1$ and $ep = 3$, so $p = 3$. Since $e = 1$, the graph is a parabola. It is symmetric with respect to the y-axis and has a directrix at $y = 3$. The vertex is at $\left(\dfrac{3}{2}, \dfrac{\pi}{2}\right)$.

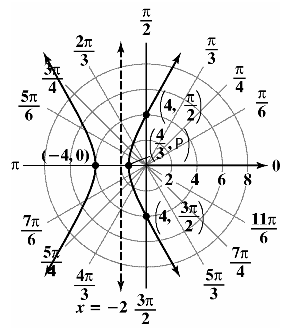


17. $r = \dfrac{8}{2 - 4\cos\theta}$

Write in standard form:

$r = \dfrac{4}{1 - 2\cos\theta}$

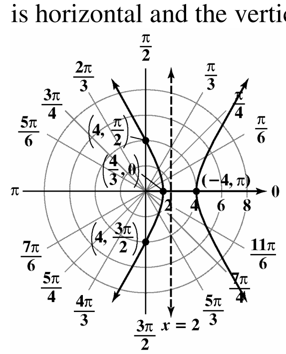
$e = 2$ and $ep = 4$, so $p = 2$. Since $e > 1$, the graph is a hyperbola. It is symmetric with respect to the polar axis and it has a directrix at $x = -2$. The transverse axis is horizontal and the vertices lie on the polar axis.



18. $r = \dfrac{8}{2 + 4\cos\theta}$

Write in standard form: $r = \dfrac{4}{1 + 2\cos\theta}$

$e = 2$ and $ep = 4$, so $p = 2$. Since $e > 1$, the graph is a hyperbola. It is symmetric with respect to the polar axis and it has a directrix at $x = 2$. The transverse axis is horizontal and the vertices lie on the polar axis.

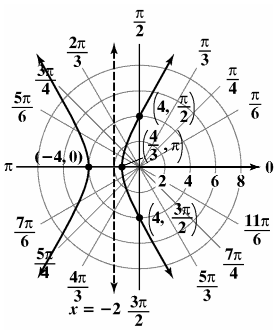


19. $r = \dfrac{12}{3 - 6\cos\theta}$

Write in standard form:

$r = \dfrac{4}{1 - 2\cos\theta}$

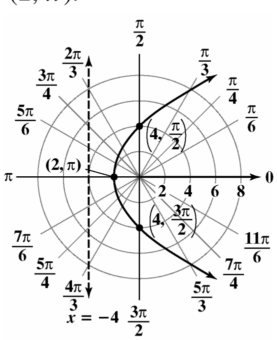
$e = 2$ and $ep = 4$, so $p = 2$. Since $e > 2$, the graph is a hyperbola. It is symmetric with respect to the polar axis and has a directrix at $x = -2$. The transverse axis is horizontal and the vertices lie on the polar axis.



20. $r = \dfrac{12}{3 - 3\cos\theta}$

Write in standard form: $r = \dfrac{4}{1 - \cos\theta}$

$e = 1$ and $ep = 4$, so $p = 4$. Since $e = 1$, the graph is a parabola. It is symmetric with respect to the polar axis and has a directrix at $x = -4$. The vertex is at $(2, \pi)$.



21. [–3, 15, 1] by [–7, 7, 1]

22. [–2, 8, 1] by [–4, 4, 1]

23. [–4, 2, 1] by [–10, 10, 1]

24. [–2, 4, 1] by [–10, 10, 1]

25. [–2, 5, 1] by [–10, 10, 1]

26. [0, 9, 1] by [–5, 5, 1]

27. [–4, 4, 1] by [–10, 0.4, 1]

28. [–4, 4, 1] by $\left[-10, \dfrac{1}{3}, 1\right]$

29. The shortest distance from the sun occurs on the positive *y*-axis, at $\theta = \dfrac{\pi}{2}$.

When $\theta = \dfrac{\pi}{2}$, $r = \dfrac{1.069}{1 + 0.967\sin\frac{\pi}{2}} = \dfrac{1.069}{1.967}$

≈ 0.54 astronomical units or about 51 million miles.

30. The greatest distance from the sun occurs on the negative *y*-axis, at $\theta = \dfrac{3\pi}{2}$. When

$\theta = \dfrac{3\pi}{2}$, $r = \dfrac{1.069}{1 + 0.967\sin\frac{3\pi}{2}} = \dfrac{1.069}{0.033} \approx 32.39$

astronomical units or about 3013 million miles.

31. His greatest distance from Earth's center occurred when $\theta = 0$:

$r = \dfrac{4090.76}{1 - 0.0076\cos 0} = \dfrac{4090.76}{0.9924}$

≈ 4122 miles from the center of the earth.

Assuming the earth to be perfectly spherical, he was 4122 – 3960 = 162 miles from the surface of the earth.

32. His closest distance from Earth's center occurred when $\theta = \pi$:

$r = \dfrac{4090.76}{1 - 0.0076\cos\pi}$

$= \dfrac{4090.76}{1.0076}$

≈ 4060 miles from the center of the earth.

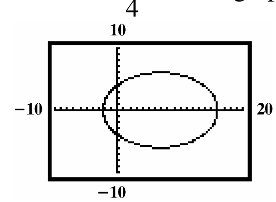
Assuming the earth to be perfectly spherical, he was 4060 – 3960 = 100 miles from the surface of the earth.

33. – 39. Answers may vary.

40. Write the equation in standard form:

$r = \dfrac{4}{1 - \frac{3}{4}\cos\theta}$

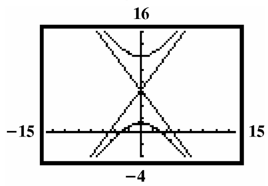
Since $e = \dfrac{3}{4} < 1$, the graph is an ellipse.



41. Write the equation in standard form:

$$r = \frac{3}{1+\frac{5}{4}\sin\theta}$$

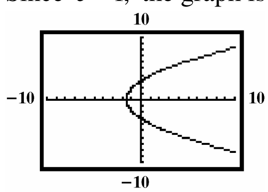
Since $e = \dfrac{5}{4} > 1,$ the graph is a hyperbola.



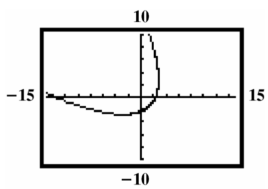
42. Write the equation in standard form:

$$r = \frac{3}{1-\cos\theta}$$

Since $e = 1,$ the graph is a parabola.

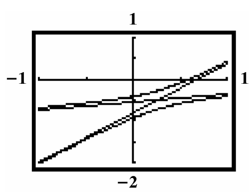


43.



The graph appears to be rotated counter- clockwise through an angle of $\dfrac{\pi}{4}$ radians.

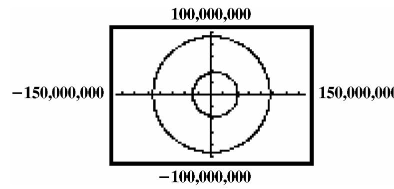
44.



The graph appears to be rotated clockwise through an angle of $\dfrac{\pi}{3}$ radians.

45. Mercury: $r = \dfrac{\left(1-0.2056^2\right)\left(36.0\times10^6\right)}{1-0.2056\cos\theta}$

Earth: $r = \dfrac{\left(1-0.0167^2\right)\left(92.96\times10^6\right)}{1-0.0167\cos\theta}$



$r: -150,000,000-150,000,000$

$y: -100,000,000, -100,000,000$

Observations may vary.

46. makes sense

47. does not make sense; Explanations will vary. Sample explanation: This form is not symmetrical with respect to the *y*-axis.

48. makes sense

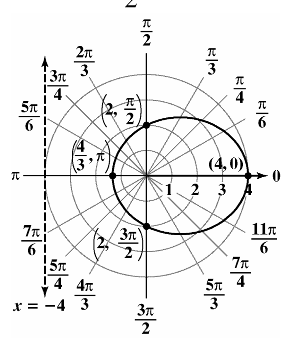
49. does not make sense; Explanations will vary. Sample explanation: A knowledge of conic sections is necessary to graph such equations.

50. $r = \dfrac{4\sec\theta}{2\sec\theta-1} = \dfrac{4\cdot\frac{1}{\cos\theta}}{2\cdot\frac{1}{\cos\theta}-1}$

$$= \frac{\left(4\cdot\frac{1}{\cos\theta}\right)\cdot\cos\theta}{\left(2\cdot\frac{1}{\cos\theta}-1\right)\cdot\cos\theta} = \frac{4}{2-\cos\theta}$$

In standard form, the equation is: $r = \dfrac{2}{1-\frac{1}{2}\cos\theta}$

Since $e = \dfrac{1}{2} < 1,$ the graph is an ellipse.



51. Since the equation is an ellipse with a vertex at (4, 0), the polar axis is the major axis. Since $\frac{PF}{PD} = e,$ then at the point (4, 0),
$\frac{4}{x} = \frac{1}{2},$ where x is the distance between the point (4, 0) and the directrix. Thus, $x = 8$ and the distance between the point (4, 0) and the directrix is 8.The directrix is either
$x = -4$ or $x = 12$. There are two polar equations that meet the given conditions. If the directrix is $x = -4$,

$$r = \frac{\frac{1}{2}(4)}{1 - \frac{1}{2}\cos\theta} = \frac{2}{1 - \frac{1}{2}\cos\theta}.$$

If the directrix is $x = 12$,

$$r = \frac{\frac{1}{2}(12)}{1 + \frac{1}{2}\cos\theta} = \frac{6}{1 + \frac{1}{2}\cos\theta}.$$

52. Since the equation is a hyperbola with directrix $x = -$ 1, the transverse axis lies on the polar axis. The standard form for a hyperbola with directrix $x = -p$ is $r = \frac{ep}{1 - e\cos\theta}$. In this case, since $x = -1$, $p = 1$. We know $e = \frac{3}{2},$ so the standard form is $r = \frac{\frac{3}{2}}{1 - \frac{3}{2}\cos\theta}.$

53. $r = \frac{1}{2 - 2\cos\theta}$

Write the equation in standard form: $r = \frac{\frac{1}{2}}{1 - \cos\theta}$

Since $e = 1$, the graph is a parabola.
Write in rectangular coordinates:

$$r = \frac{\frac{1}{2}}{1 - \cos\theta}$$

$$r(1 - \cos\theta) = \frac{1}{2}$$

$$r - r\cos\theta = \frac{1}{2}$$

$$r = r\cos\theta + \frac{1}{2}$$

$$r = x + \frac{1}{2}$$

Substitution: $x = r\cos\theta$

$$r^2 = \left(x + \frac{1}{2}\right)^2$$

Square both sides

$$x^2 + y^2 = \left(x + \frac{1}{2}\right)^2$$

Substitution: $r^2 = x^2 + y^2$

$$x^2 + y^2 = x^2 + x + \frac{1}{4}$$

$$y^2 = x + \frac{1}{4}$$

54. The standard form of the polar equation is $r = \frac{ep}{1 - e\cos\theta}$. For $\theta = 0,$ $r = \frac{ep}{1 - e}$.

For $\theta = \pi,$ $r = \frac{ep}{1 + e}$. Thus,

$$2a = \frac{ep}{1 - e} + \frac{ep}{1 + e}$$

$$2a = \frac{ep(1 + e) + ep(1 - e)}{(1 - e)(1 + e)}$$

$$2a = \frac{ep + e^2 p + ep - e^2 p}{1 - e^2}$$

$$2a = \frac{2ep}{1 - e^2}$$

$$2a\left(1 - e^2\right) = 2ep$$

$$\frac{2a\left(1 - e^2\right)}{2e} = p$$

$$\frac{a\left(1 - e^2\right)}{e} = p$$

Substituting into the standard form we get

$$r = \frac{e\left(\frac{a\left(1 - e^2\right)}{e}\right)}{1 - e\cos\theta} = \frac{a\left(1 - e^2\right)}{1 - e\cos\theta} = \frac{\left(1 - e^2\right)a}{1 - e\cos\theta}$$

55. For $n = 1$; $\frac{(-1)^n}{3^n - 1} = \frac{(-1)^1}{3^1 - 1} = \frac{-1}{3-1} = -\frac{1}{2}$

For $n = 2$; $\frac{(-1)^n}{3^n - 1} = \frac{(-1)^2}{3^2 - 1} = \frac{1}{9-1} = \frac{1}{8}$

For $n = 3$; $\frac{(-1)^n}{3^n - 1} = \frac{(-1)^3}{3^3 - 1} = \frac{-1}{27-1} = -\frac{1}{26}$

For $n = 4$; $\frac{(-1)^n}{3^n - 1} = \frac{(-1)^4}{3^4 - 1} = \frac{1}{81-1} = \frac{1}{80}$

56. $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

57. For $i = 1$; $i^2 + 1 = 1^2 + 1 = 1 + 1 = 2$

For $i = 2$; $i^2 + 1 = 2^2 + 1 = 4 + 1 = 5$

For $i = 3$; $i^2 + 1 = 3^2 + 1 = 9 + 1 = 10$

For $i = 4$; $i^2 + 1 = 4^2 + 1 = 16 + 1 = 17$

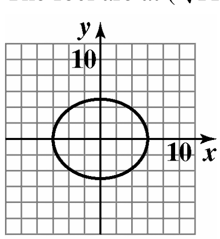
For $i = 5$; $i^2 + 1 = 5^2 + 1 = 25 + 1 = 26$

For $i = 6$; $i^2 + 1 = 6^2 + 1 = 36 + 1 = 37$

$2 + 5 + 10 + 17 + 26 + 37 = 97$

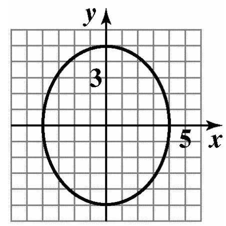
Chapter 10 Review Exercises

1. $a^2 = 36, a = 6$
$b^2 = 25, b = 5$
$c^2 = a^2 - b^2 = 36 - 25 = 11$
$c = \sqrt{11}$
The foci are at $(\sqrt{11}, 0)$ and $(-\sqrt{11}, 0)$



$\frac{x^2}{36} + \frac{y^2}{25} = 1$

2. $a^2 = 25, a = 5$
$b^2 = 16, b = 4$
$c^2 = a^2 - b^2$
$c^2 = 25 - 16$
$c^2 = 9$
$c = 3$
The foci are (0, 3) and (0, –3).

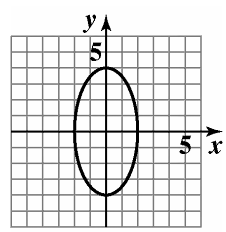


$\frac{y^2}{25} + \frac{x^2}{16} = 1$

3. $\frac{4x^2}{16} + \frac{y^2}{16} = \frac{16}{16}$

$\frac{x^2}{4} + \frac{y^2}{16} = 1$

$b^2 = 4, b = 2$
$a^2 = 16, a = 4$
$c^2 = a^2 - b^2 = 16 - 4 = 12$
$c = \sqrt{12} = 2\sqrt{3}$
The foci are at $(0, 2\sqrt{3})$ and $(0, -2\sqrt{3})$.

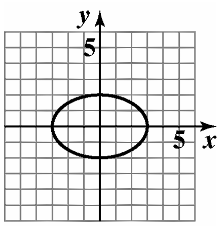


$4x^2 + y^2 = 16$

4. $\frac{4x^2}{36}+\frac{9y^2}{36}=\frac{36}{36}$

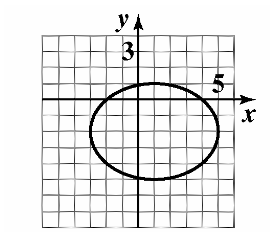
$\frac{x^2}{9}+\frac{y^2}{4}=1$

$a^2 = 9, a = 3$
$b^2 = 4,\ b = 2$
$c^2 = a^2 - b^2 = 9 - 4 = 5, c = \sqrt{5}$
The foci are at $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$.



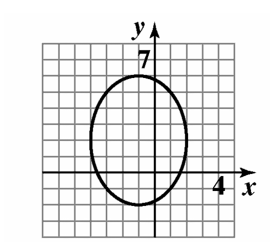
$\mathbf{4x^2 + 9y^2 = 36}$

5. $a^2 = 16\ a = 4$
$b^2 = 9\ b = 3$
$c^2 = 16 - 9 = 7, c = \sqrt{7}$
center: (1, –2)
The foci are at $(1+\sqrt{7}, -2)$ and $(1-\sqrt{7}, -2)$.



$\mathbf{\frac{(x-1)^2}{16}+\frac{(y+2)^2}{9}=1}$

6. $a^2 = 16, a = 4$
$b^2 = 9,\ b = 3$
$c^2 = a^2 - b^2 = 16 - 9 = 7,\ c = \sqrt{7}$
center: (–1, 2)
The foci are at $(-1, 2+\sqrt{7})$ and $(-1, 2-\sqrt{7})$.

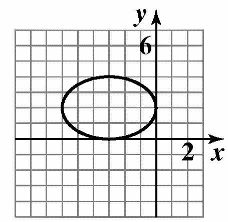


$\mathbf{\frac{(x+1)^2}{9}+\frac{(y-2)^2}{16}=1}$

7. $4x^2 + 24x + 9y^2 - 36y = -36$
$4(x^2 + 6x + 9) + 9(y^2 - 4y + 4)$
$= -36 + 36 + 36$
$= 4(x + 3)^2 + 9(y - 2)^2 = 36$

$\frac{(x+3)^2}{9}+\frac{(y-2)^2}{4}=1$

$c^2 = a^2 - b^2 = 5,\ c = \sqrt{5}$
center: (–3, 2)
The foci are at $(-3+\sqrt{5}, 2)$ and $(-3-\sqrt{5}, 2)$.

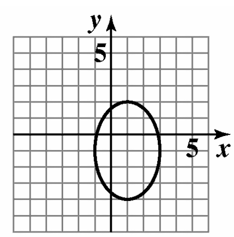


$\mathbf{4x^2 + 9y^2 + 24x - 36y + 36 = 0}$

8. $9x^2 - 18x + 4y^2 + 8y = 23$
$9(x^2 - 2x + 1) + 4(y^2 + 2y + 1) = 23 + 9 + 4$
$9(x - 1)^2 + 4(y + 1)^2 = 36$

$\frac{(x-1)^2}{4}+\frac{(y+1)^2}{9}=1$

$c^2 = a^2 - b^2 = 9 - 4 = 5,\ c = \sqrt{5}$
center: (1, –1)
The foci are at $(1, -1+\sqrt{5})$ and $(1, -1-\sqrt{5})$.



$\mathbf{9x^2 + 4y^2 - 18x + 8y - 23 = 0}$

9. $c = 4, c^2 = 16$
$a = 5, a^2 = 25$
$b^2 = a^2 - c^2 = 25 - 16 = 9$

$\frac{x^2}{25}+\frac{y^2}{9}=1$

10. $c = 3, c^2 = 9$
$a = 6, a^2 = 36$
$b^2 = a^2 - c^2 = 36 - 9 = 27$

$\frac{x^2}{27}+\frac{y^2}{36}=1$

11. $2a = 12,\ a = 6,\ a^2 = 36$
$2b = 4,\ b = 2,\ b^2 = 4$
$$\frac{(x+3)^2}{36} + \frac{(y-5)^2}{4} = 1$$

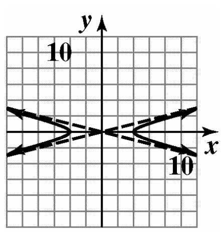
12. $2a = 20,\ a = 10,\ a^2 = 100$
$b = 6,\ b^2 = 36$
$$\frac{x^2}{100} + \frac{y^2}{36} = 1$$

13. $2a = 50,\ a = 25$
$b = 15$
$$\frac{x^2}{625} + \frac{y^2}{225} = 1$$
Let $x = 14$.
$$\frac{(14)^2}{625} + \frac{y^2}{225} = 1$$
$$y^2 = 225\left(1 - \frac{196}{625}\right)$$
$y \approx 15(0.8285) \approx 12.4 > 12$
Yes, the truck can drive under the archway.

14. The hit ball will collide with the other ball.

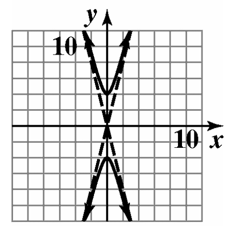
15. $c^2 = a^2 + b^2 = 16 + 1 = 17,\ c = \sqrt{17}$
The foci are at $(\sqrt{17}, 0)$ and $(-\sqrt{17}, 0)$.

Asymptotes: $y = \pm\frac{1}{4}x$



$$\frac{x^2}{16} - y^2 = 1$$

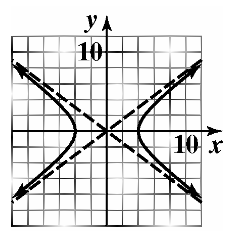
16. $c^2 = a^2 + b^2 = 16 + 1 = 17$
$c = \sqrt{17}$
The foci are at $(0, \sqrt{17})$ and $(0, -\sqrt{17})$.
Asymptotes: $y = \pm 4x$



$$\frac{y^2}{16} - x^2 = 1$$

17. $\frac{x^2}{16} - \frac{y^2}{9} = 1$
$c^2 = a^2 + b^2 = 16 + 9 = 25,\ c = 5$
The foci are at (5, 0) and (–5, 0).

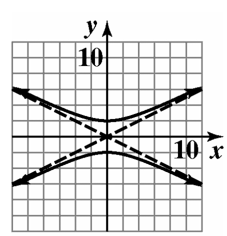
Asymptotes: $y = \pm\frac{3}{4}x$



$9x^2 - 16y^2 = 144$

18. $\frac{y^2}{4} - \frac{x^2}{16} = 1$
$c^2 = a^2 + b^2 = 4 + 16 = 20$
$c = \sqrt{20} = 2\sqrt{5}$
The foci are at $(0, 2\sqrt{5})$ and $(0, -2\sqrt{5})$.

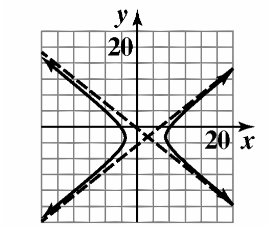
Asymptotes: $y = \pm\frac{1}{2}x$



$4y^2 - x^2 = 16$

19. $c^2 = a^2 + b^2 = 25 + 16 = 41$, $c = \sqrt{41}$
center: (2, –3)
The foci are at $(2+\sqrt{41}, -3)$ and $(2-\sqrt{41}, -3)$.

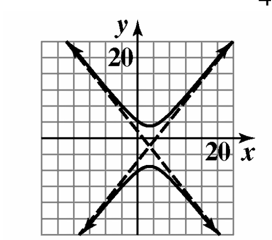
Asymptotes: $y+3 = \pm\frac{4}{5}(x-2)$



$$\frac{(x-2)^2}{25} - \frac{(y+3)^2}{16} = 1$$

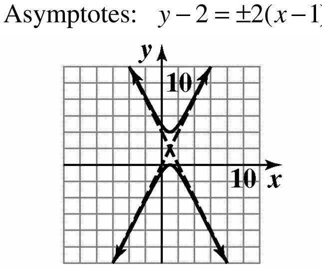
20. $c^2 = a^2 + b^2 = 25 + 16 = 41$, $c = \sqrt{41}$
center: (3, –2)
The foci are at $(3, -2+\sqrt{41})$ and $(3, -2-\sqrt{41})$.

Asymptotes: $y+2 = \pm\frac{5}{4}(x-3)$



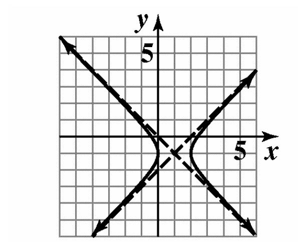
$$\frac{(y+2)^2}{25} - \frac{(x-3)^2}{16} = 1$$

21.
$$y^2 - 4y - 4x^2 + 8x - 4 = 0$$
$$(y^2 - 4y + 4) - 4(x^2 - 2x + 1) = 4 + 4 - 4$$
$$(y-2)^2 - 4(x-1)^2 = 4$$
$$\frac{(y-2)^2}{4} - (x-1)^2 = 1$$
$c^2 = a^2 + b^2 = 4 + 1 = 5$, $c = \sqrt{5}$
center: (1, 2)
The foci are at $(1, 2+\sqrt{5})$ and $(1, 2-\sqrt{5})$.
Asymptotes: $y-2 = \pm 2(x-1)$



$$y^2 - 4y - 4x^2 + 8x - 4 = 0$$

22.
$$x^2 - 2x - y^2 - 2y = 1$$
$$(x^2 - 2x + 1) - (y^2 + 2y + 1) = 1 + 1 - 1$$
$$(x-1)^2 - (y+1)^2 = 1$$
$c^2 = a^2 + b^2 = 1+1 = 2$, $c = \sqrt{2}$
center: (1, –1)
The foci are at $(1+\sqrt{2}, -1)$ and $(1-\sqrt{2}, -1)$.
asymptotes: $y+1 = \pm(x-1)$



$$x^2 - y^2 - 2x - 2y - 1 = 0$$

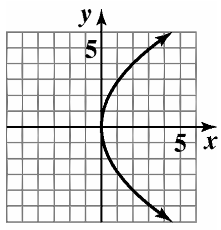
23. $c = 4$, $c^2 = 16$
$a = 2$, $a^2 = 4$
$b^2 = c^2 - a^2 = 16 - 4 = 12$
$$\frac{y^2}{4} - \frac{x^2}{12} = 1$$

24. $c = 8$, $c^2 = 64$
$a = 3$, $a^2 = 9$
$b^2 = c^2 - a^2 = 64 - 9 = 55$
$$\frac{x^2}{9} - \frac{y^2}{55} = 1$$

25. If the foci are at (0, –2) and (0, 2), then $c = 2$. If the vertices are at (0, –3) and (0, 3) then $a = 3$. This is not possible since c must be greater than a.

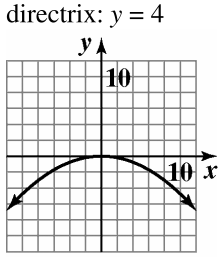
26. foci: (±100, 0), $c = 100$
$$|d_1 - d_2| = \left(0.186\frac{\text{mi}}{\mu\text{s}}\right)(500\mu\text{s}) = 93 \text{ mi} = 2a$$
$$a = \frac{93}{2}$$
$$b^2 = c^2 - a^2 = (100)^2 - \left(\frac{93}{2}\right)^2 = 7837.75$$
$$\frac{x^2}{\left(\frac{93}{2}\right)^2} - \frac{y^2}{7837.75} = 1$$
$$\frac{x^2}{2162.25} - \frac{y^2}{7837.75} = 1$$

27. $4p = 8, p = 2$
vertex: (0, 0)
focus: (2, 0)
directrix: $x = -2$



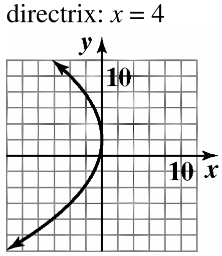
$y^2 = 8x$

28. $x^2 + 16y = 0$
$x^2 = -16y$
$4p = -16$
$p = -4$
vertex: (0, 0)
focus: (0, –4)
directrix: $y = 4$



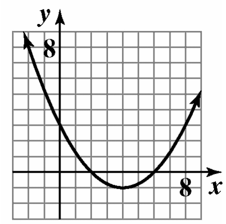
$x^2 + 16y = 0$

29. $4p = -16$
$p = -4$
vertex: (0, 2)
focus: (–4, 2)
directrix: $x = 4$



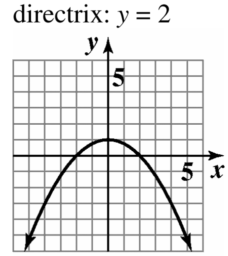
$(y - 2)^2 = -16x$

30. $4p = 4, p = 1$
vertex: (4, –1)
focus: (4, 0)
directrix: $y = -2$



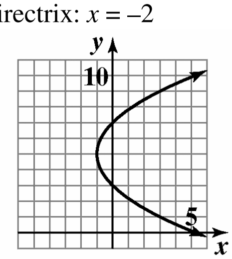
$(x - 4)^2 = 4(y + 1)$

31. $x^2 = -4y + 4$
$x^2 = -4(y - 1)$
$4p = -4, p = -1$
vertex: (0, 1)
focus: (0, 0)
directrix: $y = 2$



$x^2 + 4y = 4$

32.
$$y^2 - 10y = 4x - 21$$
$$y^2 - 10y + 25 = 4x - 21 + 25$$
$$(y - 5)^2 = 4(x + 1)$$
$4p = 4, p = 1$
vertex: (–1, 5)
focus: (0, 5)
directrix: $x = -2$



$y^2 - 4x - 10y + 21 = 0$

33. $x^2 - 4x - 2y = 0$

$x^2 - 4x = 2y$

$(x^2 - 4x + 4) = 2y + 4$

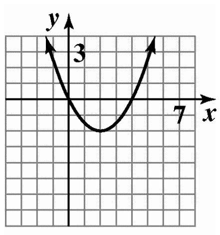
$(x-2)^2 = 2(y+2)$

$4p = 2,\ p = \frac{1}{2}$

vertex: $(2, -2)$

focus: $\left(2, -\frac{3}{2}\right)$

directrix: $y = -\frac{5}{2}$



$x^2 - 4x - 2y = 0$

34. $p = 12$

$y^2 = 48x$

35. $p = -11$

$x^2 = -44y$

36. $x^2 = 4py$

$(6)^2 = 4p(3)$

$p = 3$

$x^2 = 12y$

Place the light 3 inches from the vertex at (0, 3).

37. $x^2 = 4py$

$(1750)^2 = 4p(316)$

$4p \approx 9691$

$x^2 = 9691y$

Let $x = 1750 - 1000 = 750$.

$y = \frac{x^2}{9691} = \frac{(750)^2}{9691} \approx 58$

The height is approximately 58 feet.

38. $x^2 = 4py$

$(150)^2 = 4p(44)$

$22{,}500 = 176p$

$p \approx 128$

The receiver should be placed approximately 128 feet from the base of the dish.

39. $A = 0,\ C = 1.$

$AC = 0$, so the graph is a parabola.

40. $A = 1,\ C = 16.$

$AC = 16 > 0$ and $A \neq C$, so the graph is an ellipse.

41. $A = 16,\ C = 9.$

$AC = 16 \cdot 9 = 144 > 0$ and $A \neq C$, so the graph is an ellipse.

42. $A = 4,\ C = -9.$

$AC = 4(-9) = 36 < 0$, so the graph is a hyperbola.

43. $A = 5,\ B = 2\sqrt{3},\ C = 3.$

$B^2 - 4AC = \left(2\sqrt{3}\right)^2 - 4(5)(3) = 12 - 60 = -48$ Since $B^2 - 4AC < 0,$ the graph is an ellipse or a circle.

44. $A = 5,\ B = -8,\ C = 7.$

$B^2 - 4AC = (-8)^2 - 4(5)(7) = 64 - 140 = -76.$ Since $B^2 - 4AC < 0,$ the graph is an ellipse or a circle.

45. $A = 1,\ B = 6,\ C = 9.$

$B^2 - 4AC = 6^2 - 4(1)(9) = 36 - 36 = 0.$ Since $B^2 - 4AC = 0,$ the graph is a parabola.

46. $A = 1,\ B = -2,\ C = 3.$

$B^2 - 4AC = (-2)^2 - 4(1)(3) = 4 - 12 = -8$ Since $B^2 - 4AC < 0,$ the graph is an ellipse or a circle.

47. $xy - 4 = 0$

a. $A = 0, B = 1, C = 0.$

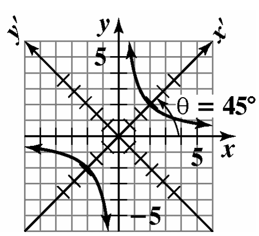
$$\cot 2\theta = \frac{A-C}{B} = \frac{0-0}{1} = 0$$
$$2\theta = 90°$$
$$\theta = 45°$$
$$x = x'\cos 45° - y'\sin 45° = x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' - y')$$
$$y = x'\sin 45° + y'\cos 45° = x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' + y')$$

Substitute into the equation: $xy - 4 = 0$

$$\left[\frac{\sqrt{2}}{2}(x' - y')\right]\left[\frac{\sqrt{2}}{2}(x' - y')\right] - 4 = 0$$
$$\frac{1}{2}\left(x'^2 - y'^2\right) - 4 = 0$$
$$x'^2 - y'^2 = 8$$

b. $\dfrac{x'^2}{8} - \dfrac{y'^2}{8} = 1$

c.



48. $x^2 + xy + y^2 - 1 = 0$

a. $A = 1, B = 1, C = 1.$

$$\cot 2\theta = \frac{A-C}{B} = \frac{1-1}{1} = \frac{0}{1} = 0$$
$$2\theta = 90°$$
$$\theta = 45°$$
$$x = x'\cos 45° - y'\sin 45° = \frac{\sqrt{2}}{2}(x' - y')$$
$$y = x'\sin 45° + y'\cos 45° = \frac{\sqrt{2}}{2}(x' + y')$$

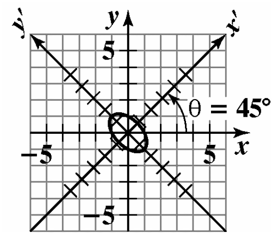
Substitute into the equation: $x^2 + xy + y^2 - 1 = 0$

$$\left[\frac{\sqrt{2}}{2}(x' - y')\right]^2 + \left[\frac{\sqrt{2}}{2}(x' - y')\right]\left[\frac{\sqrt{2}}{2}(x' + y')\right] + \left[\frac{\sqrt{2}}{2}(x' + y')\right]^2 - 1 = 0$$
$$\frac{1}{2}\left(x'^2 - 2x'y' + y'^2\right) + \frac{1}{2}\left(x'^2 - y'^2\right) + \frac{1}{2}\left(x'^2 + 2x'y' + y'^2\right) = 1$$

Multiply both sides by 2 and simplify: $3x'^2 + y'^2 = 2$

b. $\dfrac{x'^2}{\frac{2}{3}}+\dfrac{y'^2}{2}=1$

c.



49. $4x^2+10xy+4y^2-9=0$

a. $A=4$, $B=10$, $C=4$.

$$\cot 2\theta=\frac{A-C}{B}=\frac{4-4}{10}=\frac{0}{10}=0$$
$$2\theta=90^\circ$$
$$\theta=45^\circ$$
$$x=x'\cos 45^\circ-y'\sin 45^\circ=\frac{\sqrt{2}}{2}(x'-y')$$
$$y=x'\sin 45^\circ+y'\cos 45^\circ=\frac{\sqrt{2}}{2}(x'+y')$$

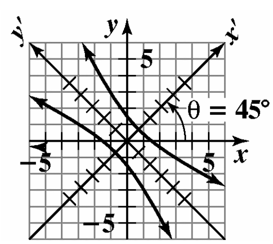
Substitute into the equation: $4x^2+10xy+4y^2-9=0$

$$4\left[\frac{\sqrt{2}}{2}(x'-y')\right]^2+10\left[\frac{\sqrt{2}}{2}(x'-y')\right]\left[\frac{\sqrt{2}}{2}(x'+y')\right]+4\left[\frac{\sqrt{2}}{2}(x'+y')\right]^2-9=0$$
$$4\cdot\frac{1}{2}\left(x'^2-2x'y'+y'^2\right)+10\cdot\frac{1}{2}\left(x'^2-y'^2\right)+4\cdot\frac{1}{2}\left(x'^2+2x'y'+y'^2\right)=9$$

Multiply both sides by 2 and simplify: $18x'^2-2y'^2=18$

b. $\dfrac{x'^2}{1}-\dfrac{y'^2}{9}=1$

c.



50. $6x^2 - 6xy + 14y^2 - 45 = 0$

a. $A = 6, B = -6\ C = 14$

$\cot 2\theta = \frac{A-C}{B} = \frac{6-14}{-6} = \frac{-8}{-6} = \frac{4}{3}$

Since θ is always acute, and $\cot 2\theta$ is positive, 2θ lies in quadrant I. The third side of the right triangle is found using the Pythagorean Theorem.

$$4^2 + 3^2 = r^2$$
$$r = 5$$

So, $\cos 2\theta = \frac{4}{5}$.

$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-\frac{4}{5}}{2}} = \frac{\sqrt{10}}{10}$ and $\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+\frac{4}{5}}{2}} = \frac{3\sqrt{10}}{10}$.

So, $x = x'\cos\theta - y'\sin\theta = x'\left(\frac{3\sqrt{10}}{10}\right) - y'\left(\frac{\sqrt{10}}{10}\right) = \frac{\sqrt{10}}{10}(3x' - y')$

and $y = x'\sin\theta - y'\cos\theta = x'\left(\frac{\sqrt{10}}{10}\right) + y'\left(\frac{3\sqrt{10}}{10}\right) = \frac{\sqrt{10}}{10}(x' + 3y')$.

Substitute into the equation: $6x^2 - 6xy + 14y^2 - 45 = 0$

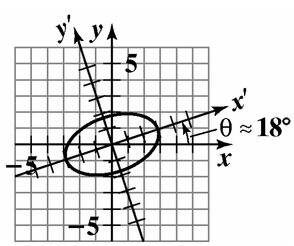
$$6\left[\frac{\sqrt{10}}{10}(3x'-y')\right]^2 - 6\left[\frac{\sqrt{10}}{10}(3x'-y')\right]\left[\frac{\sqrt{10}}{10}(x'+3y')\right] + 14\left[\frac{\sqrt{10}}{10}(x'+3y')\right]^2 - 45 = 0$$

$$6\left[\frac{1}{10}\left(9x'^2 - 6x'y' + y'^2\right)\right] - 6\left[\frac{1}{10}\left(3x'^2 + 8x'y' - 3y'^2\right)\right] + 14\left[\frac{1}{10}\left(x'^2 + 6x'y' + 9y'^2\right)\right] = 45$$

Multiply both sides by 10 and simplify: $50x'^2 + 150y'^2 = 450$

b. $\frac{x'^2}{9} + \frac{y'^2}{3} = 1$

c.



The axes are rotated by $\theta = \sin^{-1}\left(\frac{\sqrt{10}}{10}\right) \approx 18°$.

51. $x^2 + 2\sqrt{3}xy + 3y^2 - 12\sqrt{3}x + 12y = 0$

$A = 1,\ B = 2\sqrt{3},\ C = 3$

$\cot 2\theta = \dfrac{A-C}{B} = \dfrac{1-3}{2\sqrt{3}} = \dfrac{-2}{2\sqrt{3}} = \dfrac{-\sqrt{3}}{3}$

$2\theta = 120°$

$\theta = 60°$

$x = x'\cos 60° - y'\sin 60° = x'\left(\dfrac{1}{2}\right) - y'\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{1}{2}\left(x' - \sqrt{3}y'\right)$

and $y = x'\sin 60° + y'\cos 60° = x'\left(\dfrac{\sqrt{3}}{2}\right) + y'\left(\dfrac{1}{2}\right) = \dfrac{1}{2}\left(\sqrt{3}x' + y'\right)$

Substitute into the equation: $x^2 + 2\sqrt{3}xy + 3y^2 - 12\sqrt{3}x + 12y = 0$

$\left[\dfrac{1}{2}\left(x' - \sqrt{3}y'\right)\right]^2 + 2\sqrt{3}\left[\dfrac{1}{2}\left(x' - \sqrt{3}y'\right)\right]\left[\dfrac{1}{2}\left(\sqrt{3}x' + y'\right)\right] + 3\left[\dfrac{1}{2}\left(\sqrt{3}x' + y'\right)\right]^2 - 12\sqrt{3}\left[\dfrac{1}{2}\left(x' - \sqrt{3}y'\right)\right]$

$+12\left[\dfrac{1}{2}\left(\sqrt{3}x' + y'\right)\right] = 0$

$\dfrac{1}{4}\left(x'^2 - 2\sqrt{3}x'y' + 3y'^2\right) + 2\sqrt{3}\cdot\dfrac{1}{4}\left(\sqrt{3}x'^2 - 2x'y' - \sqrt{3}y'^2\right) + 3\cdot\dfrac{1}{4}\left(3x'^2 + 2\sqrt{3}x'y' + y'^2\right) - 6\sqrt{3}x' + 18y'$

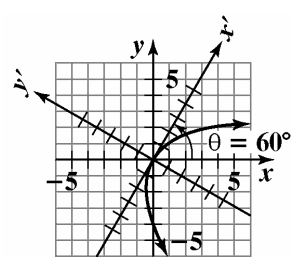
$+6\sqrt{3}x' + 6y' = 0$

Multiply both sides by 4 and simplify: $16x'^2 + 96y' = 0$

b. $16x'^2 = -96y'$

$x'^2 = -6y'$

c.

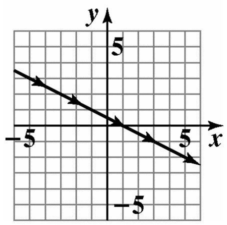


52. $x = 2t - 1$ and $y = 1 - t$; $-\infty < t < \infty$

$\frac{x+1}{2} = t$

Substitute into y: $y = 1 - \left(\frac{x+1}{2}\right)$

$y = -\frac{1}{2}x + \frac{1}{2}$



53. $x = t^2$ and $y = t - 1$; $-1 \le t \le 3$

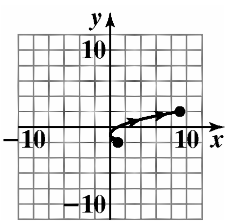
$y + 1 = t$

Substitute into x:

$x = (y+1)^2$

$(y+1)^2 = x$

$0 \le x \le 9$, $-2 \le y \le 2$



54. $x = 4t^2$ and $y = t + 1$; $-\infty < t < \infty$

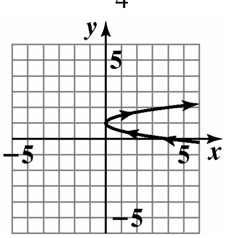
$y - 1 = t$

Substitute into x:

$x = 4(y-1)^2$

$\frac{1}{4}x = (y-1)^2$

$(y-1)^2 = \frac{1}{4}x$



55. $x = 4\sin t$, $y = 3\cos t$; $0 \le t < \pi$

$\frac{x}{4} = \sin t \quad \frac{y}{3} = \cos t$

Square and add the equations:

$\frac{x^2}{16} = \sin^2 t$

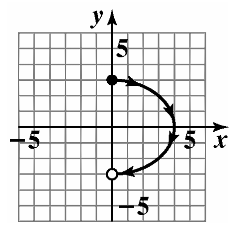
$+ \quad \frac{y^2}{9} = \cos^2 t$

$\frac{x^2}{16} + \frac{y^2}{9} = \sin^2 t + \cos^2 t$

$\frac{x^2}{16} + \frac{y^2}{9} = \sin^2 t + \cos^2 t$

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

$0 \le x \le 4$, $-3 \le y \le 3$



56. $x = 3 + 2\cos t$, $y = 1 + 2\sin t$; $0 \le t < 2\pi$

$\frac{x-3}{2} = \cos t \quad \frac{y-1}{2} = \sin t$

Square and add the equations:

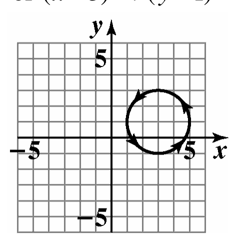
$\frac{(x-3)^2}{4} = \cos^2 t$

$+ \quad \frac{(y-1)^2}{4} = \sin^2 t$

$\frac{(x-3)^2}{4} + \frac{(y-1)^2}{4} = \cos^2 t + \sin^2 t$

$\frac{(x-3)^2}{4} + \frac{(y-1)^2}{4} = 1$

or $(x-3)^2 + (y-1)^2 = 4$



57. $x = 3\sec t,\ y = 3\tan t;\ 0 \le t \le \frac{\pi}{4}$

$\frac{x}{3} = \sec t \quad \frac{y}{3} = \tan t$

Square and subtract the equations:

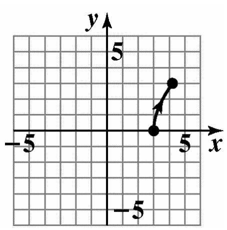
$$\frac{x^2}{9} = \sec^2 t$$

$$-\left(\frac{y^2}{9} = \tan^2 t\right)$$

$$\frac{x^2}{9} - \frac{y^2}{9} = \sec^2 t - \tan^2 t$$

$$\frac{x^2}{9} - \frac{y^2}{9} = 1$$

$3 \le x \le 3\sqrt{2},\ 0 \le y \le 3$



58. Answers may vary. Sample answer:
$x = t$ and $y = t^2 + 6;\ x = t + 1$ and $y = t^2 + 2t + 7$

59. a. $x = (100\cos 40°)t$

$y = 6 + (100\sin 40°)t - 16t^2$

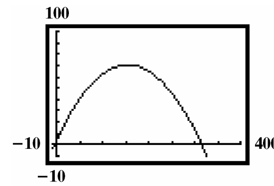
b. After 1 second:
$x = (100\cos 40°)\cdot 1$
≈ 76.6 feet in distance
$y = 6 + (100\sin 40°)\cdot 1 - 16(1)^2$
≈ 54.3 feet in height
After 2 seconds:
$x = (100\cos 40°)\cdot 2$
≈ 153.2 feet in distance
$y = 6 + (100\sin 40°)\cdot 2 - 16(2)^2$
≈ 70.6 feet in height
After 3 seconds:
$x = (100\cos 40°)\cdot 3$
≈ 229.8 feet in distance
$y = 6 + (100\sin 40°)\cdot 3 - 16(3)^2$
≈ 54.8 feet in height

c. $0 = 6t(100\sin 40°)t - 16t^2$
Using the quadratic formula with
$a = -16,\ b = 100\sin 40°,$ and $c = 6$,

$$t = \frac{-100\sin 40° \pm \sqrt{(100\sin 40°)^2 - 4(-16)(6)}}{2(-16)}$$

$t \approx -0.1$ or $t \approx 4.1$
Since t cannot be negative, we discard $t \approx -0.1$.
At $t \approx 4.1,\ x = (100\cos 40°)(4.1) \approx 314.1$
The ball is in flight for 4.1 seconds. It travels a total horizontal distance of 314.1 feet.

d.

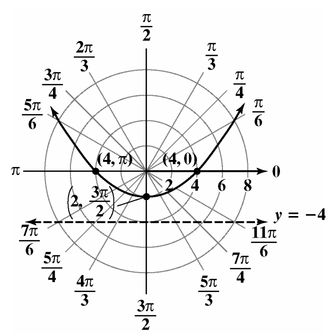


The ball is at its maximum height at 2.0 seconds. The maximum height is 70.6 feet.

60. a. $r = \frac{4}{1 - \sin\theta}$

b. $e = 1$ and $ep = 4$, so $p = 4$. Since $e = 1$, the graph is a parabola.

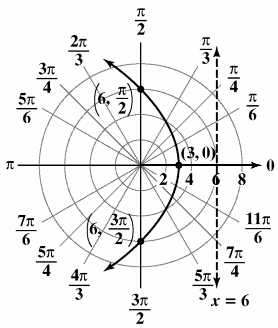
c.



61. a. $r = \dfrac{6}{1+\cos\theta}$

b. $e = 1$ and $ep = 6$, so $p = 6$. Since $e = 1$, the graph is a parabola.

c.

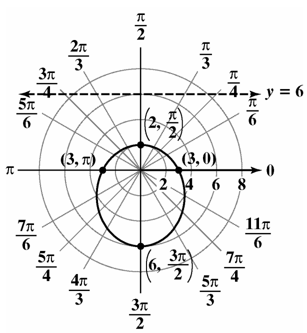


62. a. Divide numerator and denominator by 2:

$r = \dfrac{3}{1+\frac{1}{2}\sin\theta}$

b. $e = \dfrac{1}{2}$ and $ep = 3$, so $p = 6$. Since $e < 1$, the graph is an ellipse.

c.

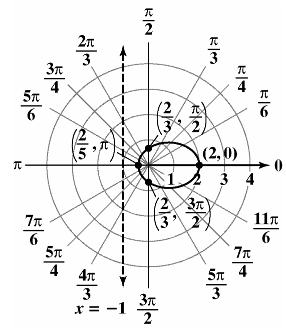


63. a. Divide the numerator and denominator by 3:

$r = \dfrac{\frac{2}{3}}{1-\frac{2}{3}\cos\theta}$

b. $e = \dfrac{2}{3}$ and $ep = \dfrac{2}{3}$, so $p = 1$. Since $e < 1$, the graph is an ellipse.

c.

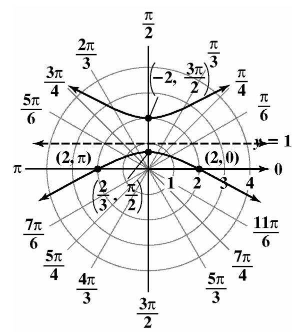


64. a. Divide the numerator and denominator by 3:

$r = \dfrac{2}{1+2\sin\theta}$

b. $e = 2$ and $ep = 2$, so $p = 1$. Since $e > 1$, the graph is a hyperbola.

c.

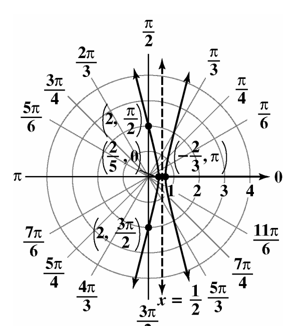


65. a. Divide the numerator and denominator by 4:

$r = \dfrac{2}{1+4\cos\theta}$

b. $e = 4$ and $ep = 2$, so $p = \dfrac{1}{2}$. Since $e > 1$, the graph is a hyperbola.

c.



Chapter 10 Test

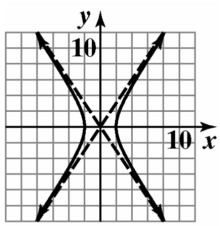
1. $\dfrac{x^2}{4}-\dfrac{y^2}{9}=1$

$c^2=a^2+b^2=4+9=13$, $c=\sqrt{13}$

hyperbola

The foci are at $\left(\sqrt{13},0\right)$ and $\left(-\sqrt{13},\,0\right)$.

Asymptotes: $y=\pm\dfrac{3}{2}x$



$9x^2-4y^2=36$

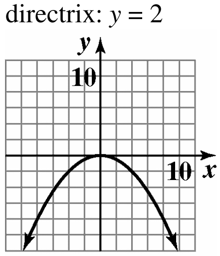
2. $4p=-8$, $p=-2$

parabola

vertex: (0, 0)

focus: (0, –2)

directrix: $y=2$



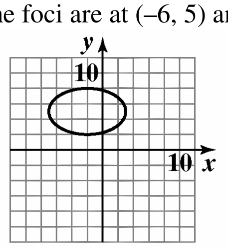
$x^2=-8y$

3. The center is at (–2, 5).

$c^2=a^2-b^2=25-9=16$, $c=4$

ellipse

The foci are at (–6, 5) and (2, 5).



$\dfrac{(x+2)^2}{25}+\dfrac{(y-5)^2}{9}=1$

4.
$$4x^2-y^2+8x+2y+7=0$$
$$\left(4x^2+8x\right)-\left(y^2-2y\right)=-7$$
$$4\left(x^2+2x+1\right)-\left(y^2-2y+1\right)=-7+4-1$$
$$4(x+1)^2-(y-1)^2=-4$$
$$(y-1)^2-4(x+1)^2=4$$
$$\frac{(y-1)^2}{4}-(x+1)^2=1$$

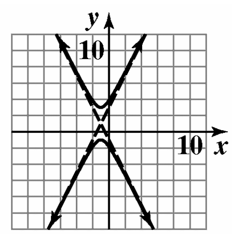
$c^2=a^2+b^2=4+1=5$, $c=\sqrt{5}$

The center is at (–1, 1).

Asymptotes: $y-1=\pm 2(x+1)$

hyperbola

The foci are at $\left(-1,1+\sqrt{5}\right)$ and $\left(-1,\,1-\sqrt{5}\right)$.



$4x^2-y^2+8x+2y+7=0$

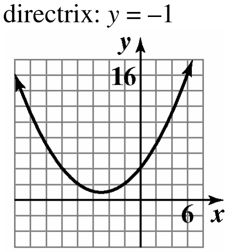
5. $4p=8$, $p=2$

parabola

vertex: (–5, 1)

focus: (–5, 3)

directrix: $y=-1$



$(x+5)^2=8(y-1)$

6. $c=7$, $c^2=49$

$a=10$, $a^2=100$

$b^2=a^2-c^2=100-49=51$

$\dfrac{x^2}{100}+\dfrac{y^2}{51}=1$

7. $c=10$, $c^2=100$

$a=7$, $a^2=49$

$b^2=c^2-a^2=100-49=51$

$\dfrac{y^2}{49}-\dfrac{x^2}{51}=1$

8. $p = 50$
$y^2 = 4px$
$y^2 = 200x$

9. $b = 24,\ b^2 = 576$
$2a = 80,\ a = 40,\ a^2 = 1600$
$c^2 = a^2 - b^2 = 1600 - 576 = 1024$
$c = \sqrt{1024} = 32$
The two people should each stand 32 feet from the center of the room, along the major axis.

10. a. $x^2 = 4py$
when $x = \pm 3$, $y = 3$
$9 = 4p(3)$
$3 = 4p$
$\frac{3}{4} = p$
$x^2 = 3y$

b. focus: $\left(0, \frac{3}{4}\right)$

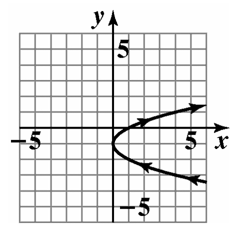
The light is placed $\frac{3}{4}$ inch above the vertex.

11. $A = 1,\ C = 9$
$AC = 1 \cdot 9 = 9 > 0$, so the graph is an ellipse.

12. $A = 1,\ B = 1,\ C = 1$
$B^2 - 4AC = 1^2 - 4(1)(1) = -3$.
Since $B^2 - 4AC < 0$, the graph is an ellipse or circle.

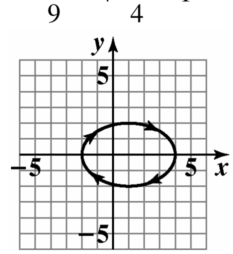
13. $7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0$
$A = 7,\ B = -6\sqrt{3},\ C = 13$
$\cot 2\theta = \frac{A - C}{B} = \frac{7 - 13}{-6\sqrt{3}}$
$= \frac{-6}{-6\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$2\theta = 60°$
$\theta = 30°$

14. $x = t^2, \quad y = t - 1;\ -\infty < t < \infty$
$y + 1 = t$
Substitute into x:
$x = (y+1)^2$
$(y+1)^2 = x$

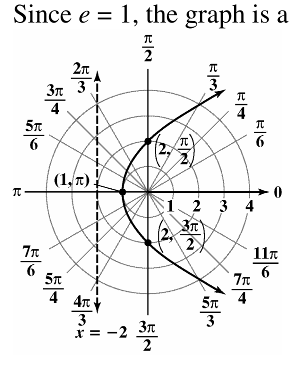


15. $x = 1 + 3\sin t, \quad y = 2\cos t;\ 0 \le t < 2\pi$
$\frac{x-1}{3} = \sin t \qquad \frac{y}{2} = \cos t$
Square and add the equations:

$$\frac{(x-1)^2}{9} = \sin^2 t$$
$$+ \quad \frac{y^2}{4} = \cos^2 t$$
$$\frac{(x-1)^2}{9} + \frac{y^2}{4} = \sin^2 t + \cos^2 t$$
$$\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$$



16. $r = \frac{2}{1 - \cos\theta}$
$e = 1$ and $ep = 2$, so $p = 2$.
Since $e = 1$, the graph is a parabola.

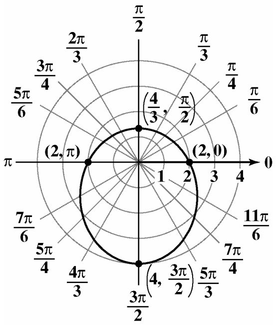


17. $r = \dfrac{4}{2+\sin\theta}$

Divide the numerator and denominator by 2:

$r = \dfrac{2}{1+\frac{1}{2}\sin\theta}$

$e = \dfrac{1}{2}$ and $ep = 2$, so $p = 4$. Since $e < 1$, the graph is an ellipse.



Cumulative Review Exercises (Chapters 1–10)

1. $2(x-3) + 5x = 8(x-1)$

$2x - 6 + 5x = 8x - 8$

$7x - 6 = 8x - 8$

$-x = -2$

$x = 2$

The solution set is $\{2\}$.

2. $-3(2x-4) > 2(6x-12)$

$-6x + 12 > 12x - 24$

$-18x > -36$

$x < 2$

The solution set is $\{x \mid x < 2\}$.

3. $x - 5 = \sqrt{x+7}$

$(x-5)^2 = x + 7$

$x^2 - 10x + 25 = x + 7$

$x^2 - 11x + 18 = 0$

$(x-2)(x-9) = 0$

$x = 2$ or $x = 9$

The solution $x = 2$ is extraneous, so the only solution is $x = 9$.

The solution set is $\{9\}$.

4. $(x-2)^2 = 20$

$x - 2 = \pm\sqrt{20}$

$x - 2 = \pm 2\sqrt{5}$

$x = 2 \pm 2\sqrt{5}$

The solution set is $\{2+2\sqrt{5},\ 2-2\sqrt{5}\}$.

5. $|2x-1| \ge 7$

$2x - 1 \ge 7$ or $2x - 1 \le -7$

$2x \ge 8$ $\quad$ $2x \le -6$

$x \ge 4$ or $x \le -3$

The solution set is $\{x \mid x \le -3 \text{ or } x \ge 4\}$

6. $3x^3 + 4x^2 - 7x + 2 = 0$

$p: \pm 1, \pm 2$

$q: \pm 1, \pm 3$

$\dfrac{p}{q}: \pm 1,\ \pm 2,\ \pm\dfrac{1}{3},\ \pm\dfrac{2}{3}$

Let $f(x) = 3x^3 + 4x^2 - 7x + 2.$

Evaluate *f* at the possible rational zeros to find $f\left(\dfrac{2}{3}\right) = 0.$

$\frac{2}{3}$	3	4	–7	2
		2	4	–2
	3	6	–3	0

$\left(x - \dfrac{2}{3}\right)(3x^2 + 6x - 3) = 0$

$(3x-2)(x^2 + 2x - 1) = 0$

$x = \dfrac{2}{3}$ or $x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-1)}}{2}$

$x = \dfrac{-2 \pm \sqrt{8}}{2}$

$x = -1 \pm \sqrt{2}$

The solution set is $\left\{\dfrac{2}{3},\ -1+\sqrt{2},\ -1-\sqrt{2}\right\}$.

7. $\log_2(x+1) + \log_2(x-1) = 3$

$\log_2(x^2 - 1) = 3$

$x^2 - 1 = 2^3$

$x^2 = 9$

$x = \pm 3$

$x = -3$ is not a solution of the original equation. The solution set is $\{3\}$.

8. $3x+4y=2$
$2x+5y=-1$

$$\begin{aligned} 6x+8y&=4\\ -6x-15y&=3\\ \hline -7y&=7\\ y&=-1 \end{aligned}$$

$3x+4(-1)=2$
$3x=6$
$x=2$

The solution set is $\{(2,-1)\}$.

9. $2x^2-y^2=-8$
$x-y=6$

$x-y=6$
$x=y+6$
$x^2=(y+6)^2=y^2+12y+36$

Substitute into first equation.

$2(y^2+12y+36)-y^2=-8$
$2y^2+24y+72-y^2=-8$
$y^2+24y+80=0$
$(y+4)(y+20)=0$
$y=-4$ or $y=-20$
$x=2 \quad x=-14$

The solution set is $\{(2,-4),(-14,-20)\}$.

10. Set up the augmented matrix and use Gauss-Jordan reduction.

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 17\\ -4 & 1 & 5 & -2\\ 2 & 3 & 1 & 8 \end{array}\right]$$

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 17\\ 0 & -3 & 9 & 66\\ 0 & 5 & -1 & -26 \end{array}\right]\begin{array}{l} \\ 4R_1+R_2\\ -2R_1+R_3 \end{array}$$

$$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 17\\ 0 & 1 & -3 & -22\\ 0 & 5 & -1 & -26 \end{array}\right]-\frac{1}{3}R_2$$

$$\left[\begin{array}{rrr|r} 1 & 0 & -2 & -5\\ 0 & 1 & -3 & -22\\ 0 & 0 & 14 & 84 \end{array}\right]\begin{array}{l} R_2+R_1\\ \\ -5R_2+R_3 \end{array}$$

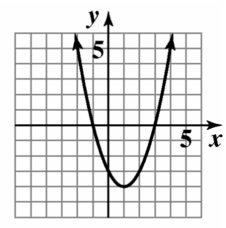
$$\left[\begin{array}{rrr|r} 1 & 0 & -2 & -5\\ 0 & 1 & -3 & -22\\ 0 & 0 & 1 & 6 \end{array}\right]\frac{1}{14}R_3$$

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 7\\ 0 & 1 & 0 & -4\\ 0 & 0 & 1 & 6 \end{array}\right]\begin{array}{l} 2R_3+R_1\\ 3R_3+R_2\\ \\ \end{array}$$

$x=7$, $y=-4$, $z=6$

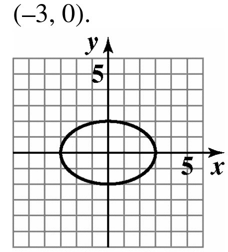
The solution set is $\{(7,-4,6)\}$.

11. Parabola with vertex at $(1,-4)$.



$f(x)=(x-1)^2-4$

12. Ellipse with center at $(0,0)$ and vertices at $(3,0)$ and $(-3,0)$.

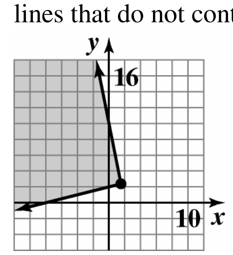


$\frac{x^2}{9}+\frac{y^2}{4}=1$

13. $5x+y\le 10 \qquad y\ge \frac{1}{4}x+2$

$y\le -5x+10$

Graph with solid line $y=-5x+10$ and $y=\frac{1}{4}x+2$. Shade the region that is below the line $y=-5x+10$ and above the line $y=\frac{1}{4}x+2$. Then dash the solid lines that do not contain the solution set.



14. a. $p:\pm1,\pm3$
$q:\pm1,\pm2,\pm4,\pm8,\pm16,\pm32$

$\frac{p}{q}: \pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{8},$
$\pm\frac{3}{8}, \pm\frac{1}{16}, \pm\frac{3}{16}, \pm\frac{1}{32}, \pm\frac{3}{32}$

b. $x = 1$ appears to be a root.

1	32	–52	17	3
		32	–20	–3
	32	–20	–3	0

$32x^3 - 52x^2 + 17x + 3 = 0$
$(x-1)(32x^2 - 20x - 3) = 0$
$(x-1)(4x-3)(8x+1) = 0$
$x = 1$ or $x = \frac{3}{4}$ or $x = -\frac{1}{8}$

The solution set is $\left\{-\frac{1}{8}, \frac{3}{4}, 1\right\}$.

15. **a.** domain: $(-2, 2)$
range: $[-3, \infty)$

b. the relative minimum of –3 occurs at $x = 0$.

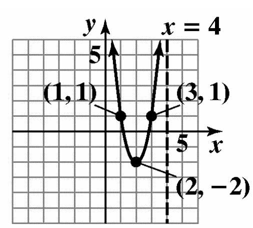
c. increasing: $(0, 2)$

d. $f(-1) - f(0) = 0 - (-3) = 3$

e. $(f \circ f)(1) = f(f(1)) = f(0) = -3$

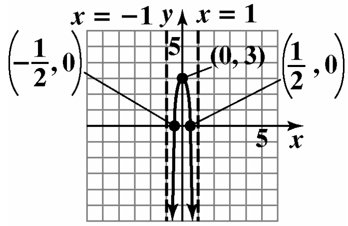
f. $f(x) \to \infty$ as $x \to -2^+$ or as $x \to 2^-$

g.



$g(x) = f(x-2) + 1$

h.



$h(x) = -f(2x)$

16. $f(x) = x^2 - 4, g(x) = x + 2$
$(g \circ f)(x) = g(x^2 - 4) = (x^2 - 4) + 2 = x^2 - 2$

17. $\log_5 \frac{x^3\sqrt{y}}{125} = \log_5 x^3\sqrt{y} - \log_5 125$
$= \log_5 x^3 + \log_5 \sqrt{y} - 3$
$= 3\log_5 x + \frac{1}{2}\log_5 y - 3$

18. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-4)}{-5 - 1} = \frac{12}{-6} = -2$
$y - y_1 = m(x - x_1)$
$y + 4 = -2(x - 1)$
$y = -2x - 2$

19. Let R = the cost of a rental at Rent-a-Truck and let A = the cost of a rental at Ace Truck Rentals.
$R = 39 + 0.16m$
$A = 25 + 0.24m$
where m is the number of miles.
$39 + 0.16m = 25 + 0.24m$
$14 = 0.08m$
$m = 175$
$R = 39 + 0.16(175) = 67$
The cost will be the same when the number of miles driven is 175 miles. The cost will be $67.

20. Let x = cost of basic cable,
Let y = cost of movie channel.
$x + y = 35$
$x + 2y = 45$
Multiply the first equation by –1 and then add the two equations.
$-x - y = -35$
$x + 2y = 45$
$y = 10$
Use back-substitution to find x.
$x + 10 = 35$
$x = 25$
Basic cable costs $25 and one movie channel costs $10.

21. $\dfrac{\csc\theta-\sin\theta}{\sin\theta}=\dfrac{\frac{1}{\sin\theta}-\sin\theta}{\sin\theta}\cdot\dfrac{\sin\theta}{\sin\theta}$

$=\dfrac{1-\sin^2\theta}{\sin^2\theta}$

$=\dfrac{\cos^2\theta}{\sin^2\theta}$

$=\left(\dfrac{\cos\theta}{\sin\theta}\right)^2$

$=\cot^2\theta$

22. $y=2\cos(2x+\pi)$

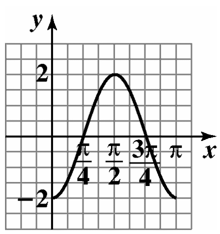
$A=2,\ B=2,\ C=-\pi$

Amplitude: $|A|=|2|=2$

Period: $\dfrac{2\pi}{B}=\dfrac{2\pi}{2}=\pi$

Phase Shift: $\dfrac{C}{B}=\dfrac{-\pi}{2}=-\dfrac{\pi}{2}$

$(0,-2),\left(\dfrac{\pi}{4},0\right),\left(\dfrac{\pi}{2},2\right),\left(\dfrac{3\pi}{4},0\right),(\pi,-2)$



23. $(\mathbf{v}\cdot\mathbf{w})\mathbf{w}=\left[(3\mathbf{i}-6\mathbf{j})\cdot(\mathbf{i}+\mathbf{j})\right](\mathbf{i}+\mathbf{j})$

$=\left[3(1)-6(1)\right](\mathbf{i}+\mathbf{j})$

$=(3-6)(\mathbf{i}+\mathbf{j})$

$=-3(\mathbf{i}+\mathbf{j})$

$=-3\mathbf{i}-3\mathbf{j}$

24. $\sin 2\theta=\sin\theta,\ 0\le\theta<2\pi$

$2\sin\theta\cos\theta=\sin\theta$

$2\sin\theta\cos\theta-\sin\theta=0$

$\sin\theta(2\cos\theta-1)=0$

$\sin\theta=0$ or $2\cos\theta-1=0$

$\theta=0,\ \pi$ $\qquad 2\cos\theta=1$

$\cos\theta=\dfrac{1}{2}$

$\theta=\dfrac{\pi}{3},\ \dfrac{5\pi}{3}$

The solutions in the interval $[0,\ 2\pi)$ are $0,\ \pi,\ \dfrac{\pi}{3}$, and $\dfrac{5\pi}{3}$.

25. $A+B+C=180°$

$64°+72°+C=180°$

$136°+C=180°$

$C=44°$

$\dfrac{b}{\sin B}=\dfrac{a}{\sin A}$

$\dfrac{b}{\sin 72°}=\dfrac{13.6}{\sin 64°}$

$b=\dfrac{13.6\sin 72°}{\sin 64°}\approx 14.4$

$\dfrac{c}{\sin C}=\dfrac{a}{\sin A}$

$\dfrac{c}{\sin 44°}=\dfrac{13.6}{\sin 64°}$

$c=\dfrac{13.6\sin 44°}{\sin 64°}\approx 10.5$

The solution is $C=44°$, $b\approx 14.4$, and $c\approx 10.5$.

Chapter 11
Sequences, Induction, and Probability

Section 11.1

Check Point Exercises

1. **a.** $a_n = 2n+5$

$a_1 = 2(1)+5 = 7$

$a_2 = 2(2)+5 = 9$

$a_3 = 2(3)+5 = 11$

$a_4 = 2(4)+5 = 13$

The first four terms are 7, 9, 11, and 13.

b. $a_n = \dfrac{(-1)^n}{2^n+1}$

$a_1 = \dfrac{(-1)^1}{2^1+1} == \dfrac{-1}{3} - \dfrac{1}{3}$

$a_2 = \dfrac{(-1)^2}{2^2+1} = \dfrac{1}{5}$

$a_3 = \dfrac{(-1)^3}{2^3+1} = \dfrac{-1}{9} = -\dfrac{1}{9}$

$a_4 = \dfrac{(-1)^4}{2^4+1} = \dfrac{1}{17}$

The first four terms are $-\frac{1}{3}, \frac{1}{5}, -\frac{1}{9}$, and $\frac{1}{17}$.

2. $a_1 = 3$ and $a_n = 2a_{n-1}+5$ for $n \ge 2$

$a_2 = 2a_1 + 5$

$= 2(3)+5 = 11$

$a_3 = 2a_2 + 5$

$= 2(11)+5 = 27$

$a_4 = 2a_3 + 5$

$= 2(27)+5 = 59$

The first four terms are 3, 11, 27, and 59.

3. $a_n = \dfrac{20}{(n+1)!}$

$a_1 = \dfrac{20}{(1+1)!} = \dfrac{20}{2!} = 10$

$a_2 = \dfrac{20}{(2+1)!} = \dfrac{20}{3!} = \dfrac{20}{6} = \dfrac{10}{3}$

$a_3 = \dfrac{20}{(3+1)!} = \dfrac{20}{4!} = \dfrac{20}{24} = \dfrac{5}{6}$

$a_4 = \dfrac{20}{(4+1)!} = \dfrac{20}{5!} = \dfrac{20}{120} = \dfrac{1}{6}$

The first four terms are $10, \frac{10}{3}, \frac{5}{6}$, and $\frac{1}{6}$.

4. **a.** $\dfrac{14!}{2!\,12!} = \dfrac{14\cdot 13\cdot 12!}{2!\,12!} = \dfrac{14\cdot 13}{2\cdot 1} = 91$

b. $\dfrac{n!}{(n-1)!} = \dfrac{n\cdot(n-1)!}{(n-1)!} = n$

5. **a.** $\sum_{i=1}^{6} 2i^2$

$= 2(1)^2 + 2(2)^2 + 2(3)^2$

$\quad + 2(4)^2 + 2(5)^2 + 2(6)^2$

$= 2+8+18+32+50+72$

$= 182$

b. $\sum_{k=3}^{5} \left(2^k - 3\right)$

$= \left(2^3-3\right) + \left(2^4-3\right) + \left(2^5-3\right)$

$= (8-3) + (16-3) + (32-3)$

$= 5+13+29$

$= 47$

c. $\sum_{i=1}^{5} 4 = 4+4+4+4+4 = 20$

6. **a.** The sum has nine terms, each of the form i^2, starting at $i=1$ and ending at $i=9$.

$1^2 + 2^2 + 3^2 + \cdots + 9^2 = \sum_{i=1}^{9} i^2$

b. The sum has *n* terms, each of the form $\frac{1}{2^{i-1}}$, starting at $i=1$ and ending at $i=n$.

$1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \cdots + \dfrac{1}{2^{n-1}} = \sum_{i=1}^{n} \dfrac{1}{2^{i-1}}$

Exercise Set 11.1

1. $a_n = 3n+2$

$a_1 = 3(1)+2 = 5$

$a_2 = 3(2)+2 = 8$

$a_3 = 3(3)+2 = 11$

$a_4 = 3(4)+2 = 14$

The first four terms are 5, 8, 11, and 14.

2. $a_n = 4n - 1$

$a_1 = 4(1) - 1 = 3$

$a_2 = 4(2) - 1 = 7$

$a_3 = 4(3) - 1 = 11$

$a_4 = 4(4) - 1 = 15$

The first four terms are 3, 7, 11, and 15.

3. $a_n = 3^n$

$a_1 = 3^1 = 3$

$a_2 = 3^2 = 9$

$a_3 = 3^3 = 27$

$a_4 = 3^4 = 81$

The first four terms are 3, 9, 27, and 81.

4. $a_n = \left(\frac{1}{3}\right)^n$

$a_1 = \left(\frac{1}{3}\right)^1 = \frac{1}{3}$

$a_2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$a_3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

$a_4 = \left(\frac{1}{3}\right)^4 = \frac{1}{81}$

The first four terms are $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$, and $\frac{1}{81}$.

5. $a_n = (-3)^n$

$a_1 = (-3)^1 = -3$

$a_2 = (-3)^2 = 9$

$a_3 = (-3)^3 = -27$

$a_4 = (-3)^4 = 81$

The first four terms are –3, 9, –27, and 81.

6. $a_n = \left(-\frac{1}{3}\right)^n$

$a_1 = \left(-\frac{1}{3}\right)^1 = -\frac{1}{3}$

$a_2 = \left(-\frac{1}{3}\right)^2 = \frac{1}{9}$

$a_3 = \left(-\frac{1}{3}\right)^3 = -\frac{1}{27}$

$a_4 = \left(-\frac{1}{3}\right)^4 = \frac{1}{81}$

The first four terms are $-\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}$, and $\frac{1}{81}$.

7. $a_n = (-1)^n (n+3)$

$a_1 = (-1)^1 (1+3) = -4$

$a_2 = (-1)^2 (2+3) = 5$

$a_3 = (-1)^3 (3+3) = -6$

$a_4 = (-1)^4 (4+3) = 7$

The first four terms are –4, 5, –6, and 7.

8. $a_n = (-1)^{n+1} (n+4)$

$a_1 = (-1)^{1+1} (1+4) = 5$

$a_2 = (-1)^{2+1} (2+4) = -6$

$a_3 = (-1)^{3+1} (3+4) = 7$

$a_4 = (-1)^{4+1} (4+4) = -8$

The first four terms are 5, –6, 7, and –8.

9. $a_n = \frac{2n}{n+4}$

$a_1 = \frac{2(1)}{1+4} = \frac{2}{5}$

$a_2 = \frac{2(2)}{2+4} = \frac{4}{6} = \frac{2}{3}$

$a_3 = \frac{2(3)}{3+4} = \frac{6}{7}$

$a_4 = \frac{2(4)}{4+4} = \frac{8}{8} = 1$

The first four terms are $\frac{2}{5}, \frac{2}{3}, \frac{6}{7}$, and 1.

10. $a_n = \dfrac{3n}{n+5}$

$a_1 = \dfrac{3(1)}{1+5} = \dfrac{3}{6} = \dfrac{1}{2}$

$a_2 = \dfrac{3(2)}{2+5} = \dfrac{6}{7}$

$a_3 = \dfrac{3(3)}{3+5} = \dfrac{9}{8}$

$a_4 = \dfrac{3(4)}{4+5} = \dfrac{12}{9} = \dfrac{4}{3}$

The first four terms are $\frac{1}{2}, \frac{6}{7}, \frac{9}{8}$, and $\frac{4}{3}$.

11. $a_n = \dfrac{(-1)^{n+1}}{2^n - 1}$

$a_1 = \dfrac{(-1)^{1+1}}{2^1 - 1} = \dfrac{1}{1}$ $n = 1$

$a_2 = \dfrac{(-1)^{2+1}}{2^2 - 1} = -\dfrac{1}{3}$

$a_3 = \dfrac{(-1)^{3+1}}{2^3 - 1} = \dfrac{1}{7}$

$a_4 = \dfrac{(-1)^{4+1}}{2^4 - 1} = -\dfrac{1}{15}$

The first four terms are $1, -\frac{1}{3}, \frac{1}{7}$, and $-\frac{1}{15}$.

12. $a_n = \dfrac{(-1)^{n+1}}{2^n + 1}$

$a_1 = \dfrac{(-1)^{1+1}}{2^1 + 1} = \dfrac{1}{3}$

$a_2 = \dfrac{(-1)^{2+1}}{2^2 + 1} = -\dfrac{1}{5}$

$a_3 = \dfrac{(-1)^{3+1}}{2^3 + 1} = \dfrac{1}{9}$

$a_4 = \dfrac{(-1)^{4+1}}{2^4 + 1} = -\dfrac{1}{17}$

The first four terms are $\frac{1}{3}, -\frac{1}{5}, \frac{1}{9}$, and $-\frac{1}{17}$.

13. $a_1 = 7$ and $a_n = a_{n-1} + 5$ for $n \ge 2$

$a_2 = a_1 + 5 = 7 + 5 = 12$

$a_3 = a_2 + 5 = 12 + 5 = 17$

$a_4 = a_3 + 5 = 17 + 5 = 22$

The first four terms are 7, 12, 17, and 22.

14. $a_1 = 12$ and $a_n = a_{n-1} + 4$ for $n \ge 2$

$a_2 = a_1 + 4 = 12 + 4 = 16$

$a_3 = a_2 + 4 = 16 + 4 = 20$

$a_4 = a_3 + 4 = 20 + 4 = 24$

The first four terms are 12, 16, 20, and 24.

15. $a_1 = 3$ and $a_n = 4a_{n-1}$ for $n \ge 2$

$a_2 = 4a_1 = 4(3) = 12$

$a_3 = 4a_2 = 4(12) = 48$

$a_4 = 4a_3 = 4(48) = 192$

The first four terms are 3, 12, 48, and 192.

16. $a_1 = 2$ and $a_n = 5a_{n-1}$ for $n \ge 2$

$a_2 = 5a_1 = 5(2) = 10$

$a_3 = 5a_2 = 5(10) = 50$

$a_4 = 5a_3 = 5(50) = 250$

The first four terms are 2, 10, 50, and 250.

17. $a_1 = 4$ and $a_n = 2a_{n-1} + 3$

$a_2 = 2(4) + 3 = 11$

$a_3 = 2(11) + 3 = 25$

$a_4 = 2(25) + 3 = 53$

The first four terms are 4, 11, 25, and 53.

18. $a_1 = 5$ and $a_n = 3a_{n-1} - 1$

$a_2 = 3(5) - 1 = 14$

$a_3 = 3(14) - 1 = 41$

$a_4 = 3(41) - 1 = 122$

The first four terms are 5, 14, 41, and 122.

19. $a_n = \dfrac{n^2}{n!}$

$a_1 = \dfrac{1^2}{1!} = 1$

$a_2 = \dfrac{2^2}{2!} = 2$

$a_3 = \dfrac{3^2}{3!} = \dfrac{9}{6} = \dfrac{3}{2}$

$a_4 = \dfrac{4^2}{4!} = \dfrac{16}{24} = \dfrac{2}{3}$

The first four terms are 1, 2, $\frac{3}{2}$, and $\frac{2}{3}$.

20. $a_n = \dfrac{(n+1)!}{n^2}$

$a_1 = \dfrac{(1+1)!}{1^2} = 2$

$a_2 = \dfrac{(2+1)!}{2^2} = \dfrac{3!}{4} = \dfrac{6}{4} = \dfrac{3}{2}$

$a_3 = \dfrac{(3+1)!}{3^2} = \dfrac{4!}{9} = \dfrac{24}{9} = \dfrac{8}{3}$

$a_4 = \dfrac{(4+1)!}{4^2} = \dfrac{5!}{16} = \dfrac{120}{16} = \dfrac{15}{2}$

The first four terms are $2, \frac{3}{2}, \frac{8}{3},$ and $\frac{15}{2}$.

21. $a_n = 2(n+1)!$

$a_1 = 2(1+1)! = 2(2) = 4$

$a_2 = 2(2+1)! = 2(6) = 12$

$a_3 = 2(3+1)! = 2(24) = 48$

$a_4 = 2(4+1)! = 2(120) = 240$

The first four terms are 4, 12, 48, and 240.

22. $a_n = -2(n-1)!$

$a_1 = -2(1-1)! = -2(1) = -2$

$a_2 = -2(2-1)! = -2(1) = -2$

$a_3 = -2(3-1)! = -2(2) = -4$

$a_4 = -2(4-1)! = -2(6) = -12$

The first four terms are –2, –2, –4, and –12.

23. $\dfrac{17!}{15!} = \dfrac{17 \cdot 16 \cdot 15!}{15!} = 17 \cdot 16 = 272$

24. $\dfrac{18!}{16!} = \dfrac{18 \cdot 17 \cdot 16!}{16!} = 18 \cdot 17 = 306$

25. $\dfrac{16!}{2!14!} = \dfrac{16 \cdot 15 \cdot 14!}{2!14!} = \dfrac{16 \cdot 15}{2 \cdot 1} = \dfrac{8 \cdot 15}{1} = 120$

26. $\dfrac{20!}{2!18!} = \dfrac{20 \cdot 19 \cdot 18!}{2!18!} = \dfrac{20 \cdot 19}{2 \cdot 1} = \dfrac{10 \cdot 19}{1} = 190$

27. $\dfrac{(n+2)!}{n!} = \dfrac{(n+2)(n+1)n!}{n!} = (n+2)(n+1)$

28. $\dfrac{(2n+1)!}{(2n)!} = \dfrac{(2n+1)(2n)!}{(2n)!} = 2n+1$

29. $\sum_{i=1}^{6} 5i = 5 \cdot 1 + 5 \cdot 2 + 5 \cdot 3 + 5 \cdot 4 + 5 \cdot 5 + 5 \cdot 6$

$= 5 + 10 + 15 + 20 + 25 + 30$

$= 105$

30. $\sum_{i=1}^{6} 7i = 7 \cdot 1 + 7 \cdot 2 + 7 \cdot 3 + 7 \cdot 4 + 7 \cdot 5 + 7 \cdot 6$

$= 7 + 14 + 21 + 28 + 35 + 42 = 147$

31. $\sum_{i=1}^{4} 2i^2 = 2 \cdot 1^2 + 2 \cdot 2^2 + 2 \cdot 3^2 + 2 \cdot 4^2$

$= 2 + 8 + 18 + 32$

$= 60$

32. $\sum_{i=1}^{5} i^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3$

$= 1 + 8 + 27 + 64 + 125 = 225$

33. $\sum_{k=1}^{5} k(k+4) = 1(5) + 2(6) + 3(7) + 4(8) + 5(9)$

$= 5 + 12 + 21 + 32 + 45$

$= 115$

34. $\sum_{k=1}^{4} (k-3)(k+2)$

$= (1-3)(1+2) + (2-3)(2+2)$

$\quad + (3-3)(3+2) + (4-3)(4+2)$

$= (-2)(3) + (-1)(4) + (0)(5) + (1)(6)$

$= -4$

35. $\sum_{i=1}^{4} \left(\dfrac{-1}{2}\right)^i = \left(-\dfrac{1}{2}\right)^1 + \left(-\dfrac{1}{2}\right)^2 + \left(-\dfrac{1}{2}\right)^3 + \left(-\dfrac{1}{2}\right)^4$

$= -\dfrac{1}{2} + \dfrac{1}{4} + -\dfrac{1}{8} + \dfrac{1}{16}$

$= -\dfrac{5}{16}$

36. $\sum_{i=2}^{4} \left(-\dfrac{1}{3}\right)^i = \left(-\dfrac{1}{3}\right)^2 + \left(-\dfrac{1}{3}\right)^3 + \left(-\dfrac{1}{3}\right)^4$

$= \dfrac{1}{9} - \dfrac{1}{27} + \dfrac{1}{81}$

$= \dfrac{7}{81}$

37. $\sum_{i=5}^{9} 11 = 11 + 11 + 11 + 11 + 11 = 55$

38. $\sum_{i=3}^{7} 12 = 12+12+12+12+12 = 60$

39. $\sum_{i=0}^{4} \frac{(-1)^i}{i!}$

$= \frac{(-1)^0}{0!} + \frac{(-1)^1}{1!} + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \frac{(-1)^4}{4!}$

$= 1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24}$

$= \frac{9}{24} = \frac{3}{8}$

40. $\sum_{i=0}^{4} \frac{(-1)^{i+1}}{(i+1)!}$

$= \frac{(-1)^1}{1!} + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \frac{(-1)^4}{4!} + \frac{(-1)^5}{5!}$

$= -1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} = -\frac{19}{30}$

41. $\sum_{i=1}^{5} \frac{i!}{(i-1)!} = \frac{1!}{0!} + \frac{2!}{1!} + \frac{3!}{2!} + \frac{4!}{3!} + \frac{5!}{4!}$

$= 1+2+3+4+5 = 15$

42. $\sum_{i=1}^{5} \frac{(i+2)!}{i!}$

$= \sum_{i=1}^{5} (i+2)(i+1) = 3\cdot 2 + 4\cdot 3 + 5\cdot 4 + 6\cdot 5 + 7\cdot 6$

$= 6+12+20+30+42 = 110$

43. $1^2 + 2^2 + 3^2 + \cdots + 15^2 = \sum_{i=1}^{15} i^2$

44. $1^4 + 2^4 + 3^4 + \cdots + 12^4 = \sum_{i=1}^{12} i^4$

45. $2 + 2^2 + 2^3 + 2^4 + \cdots + 2^{11} = \sum_{i=1}^{11} 2^i$

46. $5 + 5^2 + 5^3 + \cdots + 5^{12} = \sum_{i=1}^{12} 5^i$

47. $1+2+3+\cdots+30 = \sum_{i=1}^{30} i$

48. $1+2+3+\cdots+40 = \sum_{i=1}^{40} i$

49. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{14}{14+1} = \sum_{i=1}^{14} \frac{i}{i+1}$

50. $\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \cdots + \frac{16}{16+2} = \sum_{i=1}^{16} \frac{i}{i+2}$

51. $4 + \frac{4^2}{2} + \frac{4^3}{3} + \cdots + \frac{4^n}{n} = \sum_{i=1}^{n} \frac{4^i}{i}$

52. $\frac{1}{9} + \frac{2}{9^2} + \frac{3}{9^3} + \cdots + \frac{n}{9^n} = \sum_{i=1}^{n} \frac{i}{9^i}$

53. $1+3+5+\cdots+(2n-1) = \sum_{i=1}^{n} (2i-1)$

54. $a + ar + ar^2 + \cdots + ar^{n-1} = \sum_{i=1}^{n} \left(ar^{i-1}\right)$

55. $5+7+9+\cdots+31$

Possible answer: $\sum_{k=1}^{14} (2k+3)$

56. 6 + 8 + 10 + 12 + . . . + 32

Possible answer: $\sum_{k=3}^{16} 2k$

57. $a + ar + ar^2 + \cdots + ar^{12}$

Possible answer: $\sum_{k=0}^{12} ar^k$

58. $a + ar + ar^2 + \cdots + ar^{14}$

Possible answer: $\sum_{k=0}^{14} ar^k$

59. $a + (a+d) + (a+2d) + \cdots + (a+nd)$

Possible answer: $\sum_{k=0}^{n} (a+kd)$

60. $(a+d) + (a+d^2) + \cdots + (a+d^n)$

Possible answer: $\sum_{k=1}^{n} \left(a+d^k\right)$

61. $$\sum_{i=1}^{5}(a_i^2+1)=\left((-4)^2+1\right)+\left((-2)^2+1\right)+\left((0)^2+1\right)+\left((2)^2+1\right)+\left((4)^2+1\right)$$
$$=17+5+1+5+17$$
$$=45$$

62. $$\sum_{i=1}^{5}(b_i^2-1)=\left((4)^2-1\right)+\left((2)^2-1\right)+\left((0)^2-1\right)+\left((-2)^2-1\right)+\left((-4)^2-1\right)$$
$$=15+3+(-1)+3+15$$
$$=35$$

63. $$\sum_{i=1}^{5}(2a_i+b_i)=\left(2(-4)+4\right)+\left(2(-2)+2\right)+\left(2(0)+0\right)+\left(2(2)+(-2)\right)+\left(2(4)+(-4)\right)$$
$$=-4+(-2)+0+2+4=0$$

64. $$\sum_{i=1}^{5}(a_i+3b_i)=\left(-4+3(4)\right)+\left(-2+3(2)\right)+\left(0+3(0)\right)+\left(2+3(-2)\right)+\left(4+3(-4)\right)$$
$$=8+4+0-4-8=0$$

65. $$\sum_{i=4}^{5}\left(\frac{a_i}{b_i}\right)^2=\left(\frac{2}{-2}\right)^2+\left(\frac{4}{-4}\right)^2=(-1)^2+(-1)^2=1+1=2$$

66. $$\sum_{i=4}^{5}\left(\frac{a_i}{b_i}\right)^3=\left(\frac{2}{-2}\right)^3+\left(\frac{4}{-4}\right)^3=(-1)^3+(-1)^3=(-1)+(-1)=-2$$

67. $$\sum_{i=1}^{5}a_i^2+\sum_{i=1}^{5}b_i^2=\left((-4)^2+(-2)^2+0^2+2^2+4^2\right)+\left(4^2+2^2+0^2+(-2)^2+(-4)^2\right)$$
$$=\left(16+4+0+4+16\right)+\left(16+4+0+4+16\right)=80$$

68. $$\sum_{i=1}^{5}a_i^2-\sum_{i=3}^{5}b_i^2=\left((-4)^2+(-2)^2+0^2+2^2+4^2\right)-\left(0^2+(-2)^2+(-4)^2\right)$$
$$=\left(16+4+0+4+16\right)-\left(0+4+16\right)=40-20=20$$

69. **a.** $$\frac{1}{7}\sum_{i=1}^{7}a_i=\frac{1}{7}\left(8.1+7.2+6.1+8.1+10.0+13.1+16.7\right)=\frac{1}{7}(69.3)=9.9$$
From 2000 through 2006, Online ad spending averaged \$9.9 billion per year.

b. $a_n=0.5n^2-1.5n+8$ $\quad a_4=0.5(4)^2-1.5(4)+8=10$
$a_1=0.5(1)^2-1.5(2)+8=7$ $\quad a_5=0.5(5)^2-1.5(5)+8=13$
$a_2=0.5(2)^2-1.5(2)+8=7$ $\quad a_6=0.5(6)^2-1.5(6)+8=17$
$a_3=0.5(3)^2-1.5(3)+8=8$ $\quad a_7=0.5(7)^2-1.5(7)+8=22$
$$\frac{1}{7}\sum_{i=1}^{7}a_i=\frac{1}{7}\left(7+7+8+10+13+17+22\right)=\frac{1}{7}\left(84\right)=12$$
This overestimates the actual sum by \$2.1 billion.

70. **a.** $\frac{1}{5}\sum_{i=1}^{5} a_i = \frac{1}{5}\left(2.9+3.6+4.5+4.9+5.5\right) = \frac{1}{5}(21.4) = 4.28$

From 2002 through 2006, spending for consumer drug ads averaged $4.28 billion per year.

b. $a_n = 0.65n + 2.3$ $\quad$ $a_3 = 0.65(3) + 2.3 = 4.25$

$a_1 = 0.65(1) + 2.3 = 2.95$ $\quad$ $a_4 = 0.65(4) + 2.3 = 4.9$

$a_2 = 0.65(2) + 2.3 = 3.6$ $\quad$ $a_5 = 0.65(5) + 2.3 = 5.55$

$\frac{1}{5}\sum_{i=1}^{5} a_i = \frac{1}{5}\left(2.95+3.6+4.45+4.9+5.55\right) = \frac{1}{5}\left(21.25\right) = 4.25$

This is a reasonable model.

71. $a_n = 6000\left(1+\frac{0.06}{4}\right)^n, n = 1, 2, 3, \cdots$

$a_{20} = 6000 \quad 1 + \frac{0.06}{4}^{\quad 20} \approx 8081.13$

After five years, the balance is $8081.13.

72. $a_n = 10,000\left(1+\frac{0.08}{4}\right)^n, n = 1, 2, 3, \cdots$

$a_{24} = 10,000 \quad 1 + \frac{0.08}{4}^{\quad 24} \approx 16,084.37$

After six years, the balance is $16,084.37.

73. – 80. Answers may vary.

81. Most calculators give error message if the expression is entered directly.

However, $\frac{200!}{198!} = \frac{200 \cdot 199 \cdot 198!}{198!} = 200 \cdot 199 = 39,800$

82. $\frac{300}{20} \; ! = 15! = 1,307,674,368,000$

However, most calculators give a rounded answer in scientific notation.

83. $\frac{20!}{300} = 8,109,673,360,588,800$

However, most calculators give a rounded answer in scientific notation.

84. $\frac{20!}{(20-3)!} = 6840$

85. $\frac{54!}{(54-3)3!} = 24,804$

86. Answers may vary.

87. Answers may vary.

88. $a_n = \left(1+\frac{1}{n}\right)^n$

$a_{10} = \left(1+\frac{1}{10}\right)^{10} \approx 2.5937$

$a_{100} = \left(1+\frac{1}{100}\right)^{100} \approx 2.7048$

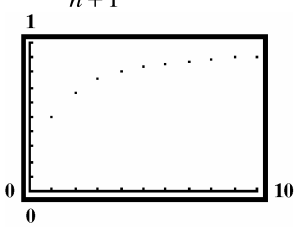
$a_{1000} = \left(1+\frac{1}{1000}\right)^{1000} \approx 2.7169$

$a_{10,000} = \left(1+\frac{1}{10,000}\right)^{10,000} \approx 2.7181$

$a_{100,000} = \left(1+\frac{1}{100,000}\right)^{100,000} \approx 2.7183$

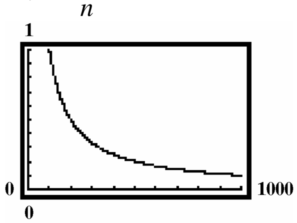
As n gets larger, a_n gets closer to $e \approx 2.7183$.

89. $a_n = \frac{n}{n+1}$



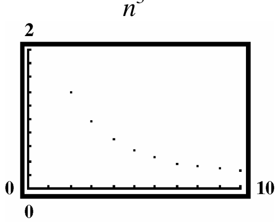
As n gets larger, a_n approaches 1.

90. $a_n = \frac{100}{n}$



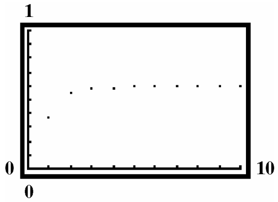
As n gets larger, a_n approaches 0.

91. $a_n = \frac{2n^2+5n-7}{n^3}$



As n gets larger, a_n approaches 0.

92. $a_n = \dfrac{3n^4 + n - 1}{5n^4 + 2n^2 + 1}$



As n gets larger, a_n approaches $\dfrac{3}{5}$.

93. does not make sense; Explanations will vary. Sample explanation: There is nothing that implies that there is a negative number of sheep.

94. does not make sense; Explanations will vary. Sample explanation: Any of the terms of this sequence could be negative and/or include non-integers.

95. makes sense

96. does not make sense; Explanations will vary. Sample explanation: Since $2n$ must be an even exponent, all the terms of the sequence will be positive.

97. false; Changes to make the statement true will vary. A sample change is: $\dfrac{n!}{(n-1)!} = \dfrac{n \cdot (n-1)!}{(n-1)!} = n$

98. true

99. false; Changes to make the statement true will vary. A sample change is:

$$\sum_{i=1}^{2} (-1)^i 2^i = (-1)^1 2^1 + (-1)^2 2^2 = -1(2) + 1(4) = -2 + 4 = 2$$

100. false; Changes to make the statement true will vary. A sample change is: $\sum_{i=1}^{2} a_i b_i \neq \sum_{i=1}^{2} a_i \sum_{i=1}^{2} b_i$

$$\sum_{i=1}^{2} a_i b_i = a_1 b_1 + a_2 b_2$$

$$\sum_{i=1}^{2} a_i \sum_{i=1}^{2} b_i = (a_1 + a_2)(b_1 + b_2) = a_1 b_1 + a_1 b_2 + a_2 b_1 + a_2 b_2$$

101. $a_n = \begin{cases} \dfrac{a_{n-1}}{2} & \text{if } a_{n-1} \text{ is even.} \\ 3a_n + 5 & \text{if } a_{n-1} \text{ is odd} \end{cases}$

for $n \geq 2$.

$a_1 = 9$

Since 9 is odd, $a_2 = 3(9) + 5 = 32$.

Since 32 is even, $a_3 = \dfrac{32}{2} = 16$.

Similarly, $a_4 = \dfrac{16}{2} = 8$, $a_5 = \dfrac{8}{2} = 4$.

The first five terms of the sequence are 9, 32, 16, 8, and 4.

102. Answers may vary.

103. $a_2 - a_1 = 3 - 8 = -5$
$a_3 - a_2 = -2 - 3 = -5$
$a_4 - a_3 = -7 - (-2) = -5$
$a_5 - a_4 = -12 - (-7) = -5$
The difference between consecutive terms is always -5.

104. $a_2 - a_1 = \left(4(2) - 3\right) - \left(4(1) - 3\right) = 4$
$a_3 - a_2 = \left(4(3) - 3\right) - \left(4(2) - 3\right) = 4$
$a_4 - a_3 = \left(4(4) - 3\right) - \left(4(3) - 3\right) = 4$
$a_5 - a_4 = \left(4(5) - 3\right) - \left(4(4) - 3\right) = 4$
The difference between consecutive terms is always 4.

105. $a_n = 4 + (n-1)(-7)$
$a_8 = 4 + (8-1)(-7) = 4 + (7)(-7) = 4 - 49 = -45$

Section 11.2

Check Point Exercises

1. $a_1 = 100$
$a_2 = a_1 - 30 = 100 - 30 = 70$
$a_3 = a_2 - 30 = 70 - 30 = 40$
$a_4 = a_3 - 30 = 40 - 30 = 10$
$a_5 = a_4 - 30 = 10 - 30 = -20$
$a_6 = a_5 - 30 = -20 - 30 = -50$
The first five terms are 100, 70, 40, 10, –20, –50.

2. $a_1 = 6,\ d = -5$
To find the ninth term, a_9, replace n in the formula with 9, a_1 with 6, and d with –5.
$$\begin{aligned} a_n &= a_1 + (n-1)d \\ a_9 &= 6 + (9-1)(-5) \\ &= 6 + 8(-5) \\ &= 6 + (-40) \\ &= -34 \end{aligned}$$

3. **a.** $$\begin{aligned} a_n &= a_1 + (n-1)d \\ &= 32 + (n-1)0.7 \\ &= 0.7n + 31.3 \end{aligned}$$

b. $a_n = 0.7n + 31.3$
$a_{11} = 0.7(11) + 31.3 = 39$
In 2014 Americans will average 39 car meals.

4. 3, 6, 9, 12, ...
To find the sum of the first 15 terms, S_{15}, replace n in the formula with 15.

$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$S_{15} = \frac{15}{2}(a_1 + a_{15})$$

Use the formula for the general term of a sequence to find a_{15}. The common difference, d, is 3, and the first term, a_1, is 3.

$$\begin{aligned} a_n &= a_1 + (n-1)d \\ a_{15} &= 3 + (15-1)(3) \\ &= 3 + 14(3) \\ &= 3 + 42 \\ &= 45 \end{aligned}$$

Thus, $S_{15} = \frac{15}{2}(3+45) = \frac{15}{2}(48) = 360$.

5. $\sum_{i=1}^{30}(6i-11) = (6\cdot 1-11)+(6\cdot 2-11)+(6\cdot 3-11)+\ldots+(6\cdot 30-11)$

$= -5+1+7+\ldots+169$

So the first term, a_1, is –5; the common difference, d, is $1-(-5)=6$; the last term, a_{30}, is 169.

Substitute $n = 30$, $a_1 = -5$, and $a_{30} = 169$ in the formula $S_n = \frac{n}{2}(a_1 + a_n)$.

$$S_{30} = \frac{30}{2}(-5+169) = 15(164) = 2460$$

Thus, $\sum_{i=1}^{30}(6i-11) = 2460$

6.
$$\begin{aligned} a_n &= 1800n + 64{,}130 \\ a_1 &= 1800(1) + 64{,}130 = 65{,}930 \\ a_{10} &= 1800(10) + 64{,}130 = 82{,}130 \end{aligned}$$

$$\begin{aligned} S_n &= \frac{n}{2}(a_1 + a_n) \\ S_{10} &= \frac{10}{2}(a_1 + a_{10}) \\ &= 5(65{,}930 + 82{,}130) \\ &= 5(148{,}060) \\ &= \$740{,}300 \end{aligned}$$

It would cost $740,300 for the ten-year period beginning in 2009.

Exercise Set 11.2

1. $a_1 = 200,\ d = 20$
The first six terms are 200, 220, 240, 260, 280, and 300.

2. $a_1 = 300$, $d = 50$
The first six terms are 300, 350, 400, 450, 500, and 550.

3. $a_1 = -7,\ d = 4$
The first six terms are –7, –3, 1, 5, 9, and 13.

4. $a_1 = -8,\ d = 5$
The first six terms are –8, –3, 2, 7, 12, and 17.

5. $a_1 = 300,\ d = -90$
The first six terms are 300, 210, 120, 30, –60, and –150.

6. $a_1 = 200,\ d = -60$
The first six terms are 200, 140, 80, 20, –40,
–100.

7. $a_1 = \frac{5}{2},\ d = -\frac{1}{2}$

The first six terms are $\frac{5}{2}, 2, \frac{3}{2}, 1, \frac{1}{2}$, and 0.

8. $a_1 = \frac{3}{4},\ d = -\frac{1}{4}$

The first six terms are $\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, 0, -\frac{1}{4}$,

and $-\frac{1}{2}$.

9. $a_n = a_{n-1} + 6,\ a_1 = -9$
The first six terms are –9, –3, 3, 9, 15, and 21.

10. $a_n = a_n + 4,\ a_1 = -7$
The first six terms are –7, –3, 1, 5, 9, and 13.

11. $a_n = a_{n-1} - 10,\ a_1 = 30$
The first six terms are 30, 20, 10, 0, –10, and
–20.

12. $a_n = a_{n-1} - 20,\ a_1 = 50$
The first six terms are 50, 30, 10, –10, –30, and –50.

13. $a_n = a_{n-1} - 0.4,\ a_1 = 1.6$
The first six terms are 1.6, 1.2, 0.8, 0.4, 0, and
–0.4.

14. $a_n = a_{n-1} - 0.3,\ a_1 = -1.7$
The first six terms are –1.7, –2.0, –2.3, –2.6, –2.9, and –3.2.

15. $a_1 = 13,\ d = -4$

$a_n = 13 + (n-1)4$

$a_6 = 13 + 5(4) = 13 + 20 = 33$

16. $a_1 = 9,\ d = 2$

$a_n = 9 + (n-1)2$

$a_{16} = 9 + (15)2 = 9 + 30 = 39$

17. $a_1 = 7,\ d = 5$

$a_n = 7 + (n-1)2$

$a_{50} = 7 + 49(5) = 252$

18. $a_1 = 8,\ d = 6$

$a_n = 8 + (n-1)6$

$a_{60} = 8 + (59)6 = 362$

19. $a_1 = -40,\ d = 5$

$a_n = -40 + (n-1)5$

$a_{200} = -40 + (199)5 = 955$

20. $a_1 = -60,\ d = 5$

$a_n = -60 + (n-1)5$

$a_{150} = -60 + (149)5 = 685$

21. $a_1 = 35,\ d = -3$

$a_n = 35 - 3(n-1)$

$a_{60} = 35 - 3(59) = -142$

22. $a_1 = -32,\ d = 4$

$a_n = -32 + (n-1)4$

$a_{70} = -32 + (69)4 = 244$

23. 1, 5, 9, 13, …

$d = 5 - 1,\ = 4$

$a_n = 1 + (n-1)4 = 1 + 4n - 4$

$a_n = 4n - 3$

$a_{20} = 4(20) - 3 = 77$

24. 2, 7, 12, 17, …

$d = 7 - 2 = 5$

$a_n = 2 + (n-1)5 = 2 + 5n - 5$

$a_n = 5n - 3$

$a_{20} = 5(20) - 3 = 97$

25. 7, 3, –1, –5, …

$d = 3 - 7 = -4$

$a_n = 7 + (n-1)(-4) = 7 - 4n + 4$

$a_n = 11 - 4n$

$a_{20} = 11 - 4(20) = -69$

26. 6, 1, –4, –9, …

$d = 1 - 6 = -5$

$a_n = 6 + (n-1)(-5) = 6 - 5n + 5$

$a_n = 11 - 5n$

$a_{20} = 11 - 5(20) = -89$

27. $a_1 = 9,\ d = 2$
$a_n = 9 + (n-1)(2)$
$a_n = 7 + 2n$
$a_{20} = 7 + 2(20) = 47$

28. $a_1 = 6,\ d = 3$
$a_n = 6 + (n-1)3 = 6 + 3n - 3 = 3n + 3$
$a_{20} = 3(20) + 3 = 63$

29. $a_1 = -20,\ d = -4$
$a_n = -20 + (n-1)(-4)$
$a_n = -20 - 4n + 4$
$a_n = -16 - 4n$
$a_{20} = -16 - 4(20) = -96$

30. $a_1 = -70,\ d = -5$
$a_n = -70 - 5(n-1) = -70 - 5n + 5 = -65 - 5n$
$a_{20} = -65 - 5(20) = -165$

31. $a_n = a_{n-1} + 3,\ a_1 = 4$
$d = 3$
$a_n = 4 + (n-1)(3)$
$a_n = 1 + 3n$
$a_{20} = 1 + 3(20) = 61$

32. $a_n = a_{n-1} + 5,\ a_1 = 6,\ d = 5$
$a_n = 6 + (n-1)5 = 6 + 5n - 5 = 5n + 1$
$a_{20} = 5(20) + 1 = 101$

33. $a_n = a_{n-1} - 10,\ a_1 = 30,\ d = -10$
$a_n = 30 - 10(n-1) = 30 - 10n + 10$
$a_n = 40 - 10n$
$a_{20} = 40 - 10(20) = -160$

34. $a_n = a_{n-1} - 12,\ a_1 = 24,\ d = -12$
$a_n = 24 - 12(n-1) = 24 - 12n + 12$
$a_n = 36 - 12n$
$a_{20} = 36 - 12(20) = -204$

35. 4, 10, 16, 22, . . .
$d = 10 - 4 = 6$
$a_n = 4 + (n-1)(6)$
$a_{20} = 4 + (19)(6) = 118$
$S_{20} = \frac{20}{2}(4 + 118) = 1220$

36. 7, 19, 31, 43, …

$d = 12$

$a_n = 7 + (n-1)(12)$

$a_{25} = 7 + (24)(12) = 295$

$S_{25} = \frac{25}{2}(7 + 295) = 3775$

37. –10, –6, –2, 2, . . .

$d = -6 - (-10) = -6 + 10 = 4$

$a_n = -10 + (n-1)4$

$a_{50} = -10 + (49)4 = 186$

$S_{50} = \frac{50}{2}(-10 + 186) = 4400$

38. –15, –9, –3, 3, . . .

$d = -9 - (-15) = -9 + 15 = 6$

$a_n = -15 + (n-1)6$

$a_{50} = -15 + (49)6 = 279$

$S_{50} = \frac{50}{2}(-15 + 279) = 6600$

39. $1 + 2 + 3 + 4 + \cdots + 100$

$S_{100} = \frac{100}{2}(1 + 100) = 5050$

40. $2 + 4 + 6 + \cdots + 200$

$S_{100} = \frac{100}{2}(2 + 200) = 10{,}100$

41. $2 + 4 + 6 + \cdots + 120$

$S_{60} = \frac{60}{2}(2 + 120) = 3660$

42. $a_{80} = 2 + 79(2) = 160$

$S_{80} = \frac{80}{2}(2 + 160) = 6480$

43. even integers between 21 and 45;

$22 + 24 + 26 + \cdots + 44$

$S_{12} = \frac{12}{2}(22 + 44) = 396$

44. odd integers between 30 and 54: $31 + 33 + 35 + \cdots + 53$

$S_{12} = \frac{12}{2}(31 + 53) = 504$

45. $\sum_{i=1}^{17}(5i + 3) = (5 + 3) + (10 + 3) + (15 + 3) + \cdots + (85 + 3) = 8 + 13 + 18 + \cdots + 88$

$S_{17} = \frac{17}{2}(8 + 88) = 816$

46. $\sum_{i=1}^{20}(6i-4)=(6-4)+(12-4)+(18-4)+\cdots+(120-4)=2+8+14+\cdots+116$

$S_{20}=\frac{20}{2}\left(2+116\right)=1180$

47. $\sum_{i=1}^{30}(-3i+5)=(-3+5)+(-6+5)+(-9+5)+\cdots+(-90+5)=2-1-4-\cdots-85$

$S_{30}=\frac{30}{2}(2-85)=-1245$

48. $\sum_{i=1}^{40}(-2i+6)=(-2+6)+(-4+6)+(-6+6)+\cdots+(-80+6)=4+2+0-\cdots-74$

$S_{40}=\frac{40}{2}(4-74)=-1400$

49. $\sum_{i=1}^{100}4i=4+8+12+\cdots+400$

$S_{100}=\frac{100}{2}\left(4+400\right)=20,200$

50. $\sum_{i=1}^{50}-4i=-4-8-12-\cdots-200$

$S_{50}=\frac{50}{2}\left(-4-200\right)=-5100$

51. First find a_{14} and b_{12}:

$a_{14}=a_1+(n-1)d$

$=1+(14-1)(-3-1)=-51$

$b_{12}=b_1+(n-1)d$

$=3+(12-1)(8-3)=58$

So, $a_{14}+b_{12}=-51+58=7$.

52. First find a_{16} and b_{18}:

$a_{16}=a_1+(n-1)d$

$=1+(16-1)(-3-1)=-59$

$b_{18}=b_1+(n-1)d$

$=3+(18-1)(8-3)=88$

So, $a_{16}+b_{18}=-59+88=29$.

53. $a_n=a_1+\left(n-1\right)d$

$-83=1+(n-1)(-3-1)$

$-83=1+-4(n-1)$

$-84=-4n+4$

$-88=-4n$

$n=22$

There are 22 terms.

54. $b_n = b_1 + (n-1)d$

$93 = 3 + (n-1)(8-3)$

$93 = 3 + 5(n-1)$

$93 = 5n - 2$

$95 = 5n$

$n = 19$

There are 19 terms.

55. $S_n = \frac{n}{2}(a_1 + a_n)$

For $\{a_n\}$: $S_{14} = \frac{14}{2}(a_1 + a_{14}) = 7(1+(-51)) = -350$ For $\{b_n\}$: $S_{14} = \frac{14}{2}(b_1 + b_{14}) = 7(3+68) = 497$

So $\sum_{n=1}^{14} b_n - \sum_{n=1}^{14} a_n = 497 - (-350) = 847$

56. First find a_{15} and b_{15}:

$a_{15} = a_1 + (n-1)d$

$= 1 + (15-1)(-3-1) = -55$

$b_{15} = b_1 + (n-1)d$

$= 3 + (15-1)(8-3) = 73$

Using $S_n = \frac{n}{2}(a_1 + a_n)$ for $\{a_n\}$: $S_{15} = \frac{15}{2}(a_1 + a_{15}) = 7.5(1+(-55))$

$= -405$

And then for $\{b_n\}$: $S_{15} = \frac{15}{2}(b_1 + b_{15}) = 7.5(3+73) = 570$ So $\sum_{n=1}^{15} b_n - \sum_{n=1}^{15} a_n = 570 - (-405) = 975$

57. Two points on the graph are (1, 1) and (2, –3). Finding the slope of the line; $m = \frac{y_2 - y_1}{x_2 - x_2} = \frac{-3-1}{2-1} = \frac{-4}{1} = -4$

Using the point-slope form of an equation of a line;

$y - y_2 = m(x - x_2)$

$y - 1 = -4(x-1)$

$y - 1 = -4x + 4$

$y = -4x + 5$

Thus, $f(x) = -4x + 5$.

58. Two points on the graph are (1, 3) and (2, 8). Finding the slope of the line; $m = \frac{y_2 - y_1}{x_2 - x_2} = \frac{8-3}{2-1} = \frac{5}{1} = 5$

Using the point-slope form of an equation of a line;

$y - y_2 = 5(x - x_2)$

$y - 3 = 5(x-1)$

$y - 3 = 5x - 5$

$y = 5x - 2$

Thus, $g(x) = 5x - 2$.

59. Using $a_n = a_1 + (n-1)d$ and $a_2 = 4$:

$a_2 = a_1 + (2-1)d$

$4 = a_1 + d$

And since $a_6 = 16$:

$a_6 = a_1 + (6-1)d$

$16 = a_1 + 5d$

The system of equations is

$4 = a_1 + d$

$16 = a_1 + 5d$

Solving the first equation for a_1:

$a_1 = 4 - d$

Substituting the value into the second equation and solving for *d*:

$16 = (4-d) + 5d$

$16 = 4 + 4d$

$12 = 4d$

$3 = d$

Back-substitute:

$a_1 = 4 - d$

$a_1 = 4 - 3)$

$a_1 = 1$

Then $a_n = a_1 + (n-1)d$

$a_n = 1 + (n-1)3$

$a_n = 1 + 3n - 3$

$a_n = 3n - 2$

60. Using $a_n = a_1 + (n-1)d$ and $a_3 = 7$:

$a_3 = a_1 + (3-1)d$

$7 = a_1 + 2d$

And since $a_8 = 17$:

$a_8 = a_1 + (8-1)d$

$17 = a_1 + 7d$

The system of equations is

$7 = a_1 + 2d$

$17 = a_1 + 7d$

Solving the first equation for a_1:

$a_1 = 7 - 2d$

Substituting the value into the second equation and solving for *d*:

$17 = a_1 + 7d$

$17 = (7-2d) + 7d$

$17 = 7 + 5d$

$10 = 5d$

$2 = d$

Back-substitute:

$a_1 = 7 - 2d$

$a_1 = 7 - 2(2)$

$a_1 = 3$

Then $a_n = a_1 + (n-1)d$

$a_n = 3 + (n-1)2$

$a_n = 3 + 2n - 2$

$a_n = 2n + 1$

61. **a.** $a_n = a_1 + (n-1)d$

$a_n = 10 + (n-1)(0.77)$

$a_n = 10 + 0.77n - 0.77$

$a_n = 0.77n + 9.23$

b. 2011 is 27 years after 1984.

$a_{27} = 0.77(27) + 9.23 \approx 30.0$

If trends continue, in 2011 the percentage of Americans with no close friends will be 30.0%.

62. **a.** $a_n = a_1 + (n-1)d$

$a_n = 17.6 + (n-1)(0.83)$

$a_n = 17.6 + 0.83n - 0.83$

$a_n = 0.83n + 16.77$

b. 2018 is 51 years after 1967.

$a_{51} = 0.83(51) + 16.77 = 59.1$

63. Company *A:*

$a_n = 24000 + (n-1)1600$

$= 24000 + 1600n - 1600$

$= 1600n + 22400$

$a_{10} = 1600(10) + 22400$

$= 16000 + 22400 = 38400$

Company *B:*

$a_n = 28000 + (n-1)1000$

$= 28000 + 1000n - 1000$

$= 1000n + 27000$

$a_{10} = 1000(10) + 27000$

$= 10000 + 27000 = 37000$

Company *A* will pay $1400 more in year 10.

64. Company *A*:

$$a_n = 23000 + (n-1)1200$$
$$= 23000 + 1200n - 1200 = 1200n + 21800$$
$$a_{10} = 1200(10) + 21800$$
$$= 12000 + 21800 = 33800$$

Company *B*:

$$a_n = 26000 + (n-1)800$$
$$= 26000 + 800n - 800 = 800n + 25200$$
$$a_{10} = 800(10) + 25200 = 8000 + 25200$$
$$= 33200$$

Company *A* will pay \$600 more in year 10.

65. **a.** Total cost: $\$4694 + \$5132 + \$5491 + \$5836 = \$21,153$

b. $a_1 = 379(1) + 4342 = 4721$

$a_4 = 379(4) + 4342 = 5858$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_4 = \frac{4}{2}(4721 + 5858) = 2(10,579) = \$21,158$$

The model overestimates the actual sum by \$5.

66. **a.** Total cost: $\$19,710 + \$20,082 + \$21,235 + \$22,218 = \$83,245$

b. $a_1 = 868(1) + 18,642 = 19,510$

$a_4 = 868(4) + 18,642 = 22,114$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_4 = \frac{4}{2}(19,510 + 22,114) = 2(41,624) = \$83,248$$

The model overestimates the actual sum by \$3.

67. Answers will vary.

68. $a_n = 33,000 + (n-1)(2500)$

$a_{10} = 33,000 + 9(2500) = 55,500$

$$S_n = \frac{10}{2}(33,000 + 55,500) = 442,500$$

The total ten year salary is \$442,500.

69. Company A:

$a_n = 19,000 + (n-1)2600$

$a_{10} = 19,000 + (9)2600 = \$42,200$

$S_{10} = \frac{10}{2}(19000 + 42400) = \$307,000$

Company B:

$a_n = 27,000 + (n-1)1200$

$a_{10} = 27,000 + (9)1200 = \$37,800$

$S_{10} = \frac{10}{2}(27,000 + 37,800) = \$324,000$

Company B pays the greater total amount.

70. $a_n = 30 + (n-1)2$

$a_{26} = 30 + (25)2 = 80$

$S_{26} = \frac{26}{2}(30 + 80) = 1430$

The theater has 1430 seats.

71. $a_n = 20 + (n+1)3$

$a_{38} = 20 + (37)3 = 131$

$S_{38} = \frac{38}{2}(20 + 131) = 2869$

The theater has 2869 seats.

72. – 77. Answers may vary.

78. does not make sense; Explanations will vary. Sample explanation: The difference between terms is not constant. Thus, this is not an arithmetic sequence.

79. makes sense

80. makes sense

81. makes sense

82. 21,700, 23,172, 24,644, 26,166, . . . , 314,628

$d = 23{,}172 - 21{,}700 = 1472$

$314{,}628 = 1472n + 20{,}228$

$1472n = 294{,}400$

$n = 200$

It is the 200th term.

83. Degree days: 23, 25, 27, …

$a_1 = 23, d = 2$

$a_{10} = 23 + 9(2) = 41$

$S_{10} = \frac{10}{2}(a_1 + a_{10})$

$S_{10} = \frac{10}{2}(23 + 41) = 320$

There are 320 degree-days.

84. $1 + 3 + 5 + \cdots + (2n-1)$

$$S_n = \frac{n}{2}(1 + 2n - 1) = \frac{n}{2}(2n) = n^2$$

85. $\frac{a_2}{a_1} = \frac{-2}{1} = -2$

$\frac{a_3}{a_2} = \frac{4}{-2} = -2$

$\frac{a_4}{a_3} = \frac{-8}{4} = -2$

$\frac{a_5}{a_4} = \frac{16}{-8} = -2$

The ratio of a term to the term that directly precedes it is always -2.

86. $\frac{a_2}{a_1} = \frac{3 \cdot 5^2}{3 \cdot 5^1} = 5$

$\frac{a_3}{a_2} = \frac{3 \cdot 5^3}{3 \cdot 5^2} = 5$

$\frac{a_4}{a_3} = \frac{3 \cdot 5^4}{3 \cdot 5^3} = 5$

$\frac{a_5}{a_4} = \frac{3 \cdot 5^5}{3 \cdot 5^4} = 5$

The ratio of a term to the term that directly precedes it is always 5.

87. $a_n = a_1 3^{n-1}$

$a_7 = 11 \cdot 3^{7-1} = 11 \cdot 3^6 = 11 \cdot 729 = 8019$

Section 11.3

Check Point Exercises

1. $a_1 = 12,\ r = \frac{1}{2}$

$a_2 = 12\left(\frac{1}{2}\right)^1 = 6$

$a_3 = 12\left(\frac{1}{2}\right)^2 = \frac{12}{4} = 3$

$a_4 = 12\left(\frac{1}{2}\right)^3 = \frac{12}{8} = \frac{3}{2}$

$a_5 = 12\left(\frac{1}{2}\right)^4 = \frac{12}{16} = \frac{3}{4}$

$a_6 = 12\left(\frac{1}{2}\right)^5 = \frac{12}{32} = \frac{3}{8}$

The first six terms are 12, 6, 3, $\frac{3}{2}, \frac{3}{4}$, and $\frac{3}{8}$.

2. $a_1 = 5,\ r = -3$

$a_n = 5r^{n-1}$

$a_7 = 5(-3)^{7-1} = 5(-3)^6 = 5(729) = 3645$

The seventh term is 3645.

3. 3, 6, 12, 24, 48, ...

$r = \frac{6}{3} = 2,\ a_1 = 3$

$a_n = 3(2)^{n-1}$

$a_8 = 3(2)^{8-1} = 3(2)^7 = 3(128) = 384$

The eighth term is 384.

4. $a_1 = 2,\ r = \frac{-6}{2} = -3$

$S_n = \frac{a_1(1-r^r)}{1-r}$

$S_9 = \frac{2\left(1-(-3)^9\right)}{1-(-3)} = \frac{2(19,684)}{4} = 9842$

The sum of the first nine terms is 9842.

5. $\sum_{i=1}^{8} 2\cdot 3^i$

$a_1 = 2\cdot(3)^1 = 6,\ r = 3$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_8 = \frac{6\left(1-3^8\right)}{1-3} = \frac{6(-6560)}{-2} = 19,680$

Thus, $\sum_{i=1}^{8} 2\cdot 3^i = 19,680$.

6. $a_1 = 30,000,\ r = 1.06$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_{30} = \frac{30,000\left(1-(1.06)^{30}\right)}{1-1.06} \approx 2,371,746$

The total lifetime salary is \$2,371,746.

7. **a.** $A = \frac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}}$

$P = 100,\ r = 0.095,\ n = 12,\ t = 35$

$A = \frac{100\left[\left(1+\frac{0.095}{12}\right)^{12\cdot 35} - 1\right]}{\frac{0.095}{12}} \approx 333,946$

The value of the IRA will be \$333,946.

b. Interest = Value of IRA − Total deposits

$\approx \$333,946 - \$100\cdot 12\cdot 35$

$\approx \$333,946 - \$42,000$

$\approx \$291,946$

8. $3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots$

$a_1 = 3,\ r = \frac{2}{3}$

$S = \frac{a_1}{1-r}$

$S = \frac{3}{1-\frac{2}{3}} = \frac{3}{\frac{1}{3}} = 9$

The sum of this infinite geometric series is 9.

9. $0.\overline{9} = 0.9999\cdots = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \cdots$

$a_1 = \frac{9}{10}, r = \frac{1}{10}$

$S = \frac{\frac{9}{10}}{1 - \frac{1}{10}} = \frac{\frac{9}{10}}{\frac{9}{10}} = 1$

An equivalent fraction for $0.\overline{9}$ is 1.

10. $a_1 = 1000(0.8) = 800,\ r = 0.8$

$S = \frac{800}{1 - 0.8} = 4000$

The total amount spent is $4000.

Exercise Set 11.3

1. $a_1 = 5,\ r = 3$

First five terms: 5, 15, 45, 135, 405.

2. $a_1 = 4,\ r = 3$

First five terms: 4, 12, 36, 108, 324.

3. $a_1 = 20,\ r = \frac{1}{2}$

First five terms: 20, 10, 5, $\frac{5}{2}$, $\frac{5}{4}$ $\frac{5}{4}$.

4. $a_1 = 24,\ r = \frac{1}{3}$

First five terms: 24, 8, $\frac{8}{3}, \frac{8}{9}, \frac{8}{27}$.

5. $a_n = -4a_{n-1},\ a_1 = 10$

First five terms: 10, –40, 160, –640, 2560.

6. $a_n = -3a_{n-1},\ a_1 = 10$

First five terms: 10, –30, 90, –270, 810.

7. $a_n = -5a_{n-1},\ a_1 = -6$

First five terms: –6, 30, –150, 750,–3750.

8. $a_n = -6a_{n-1},\ a_1 = -2$

First five terms: –2, 12, –72, 432,–2592.

9. $a_1 = 6,\ r = 2$

$a_n = 6 \cdot 2^{n-1}$

$a_8 = 6 \cdot 2^7 = 768$

10. $a_1 = 5,\ r = 3$

$a_n = 5 \cdot 3^{n-1}$

$a_8 = 5 \cdot 3^7 = 10{,}935$

11. $a_1 = 5,\ r = -2$

$a_n = 5 \cdot (-2)^{n-1}$

$a_{12} = 5 \cdot (-2)^{11} = -10{,}240$

12. $a_1 = 4,\ r = -2$

$a_n = 4 \cdot (-2)^{n-1}$

$a_{12} = 4 \cdot (-2)^{11} = -8192$

13. $a_1 = 1000,\ r = -\frac{1}{2}$

$a_n = 1000\left(-\frac{1}{2}\right)^{n-1}$

$a_{40} = 1000\left(-\frac{1}{2}\right)^{39}$

≈ 0.000000002

14. $a_1 = 8000,\ r = -\frac{1}{2}$

$a_n = 8000\left(-\frac{1}{2}\right)^{n-1}$

$a_{30} = 8000\left(-\frac{1}{2}\right)^{29} \approx -0.000014901$

15. $a_1 = 1{,}000{,}000,\ r = 0.1$

$a_n = 1{,}000{,}000(0.1)^{n-1}$

$a_8 = 1{,}000{,}000(0.1)^7 = 0.1$

16. $a_1 = 40{,}000,\ r = 0.1$

$a_n = 40{,}000(0.1)^{n-1}$

$a_8 = 40{,}000(0.1)^7 = 0.004$

17. 3, 12, 48, 192, . . .

$r = \frac{12}{3} = 4$

$a_n = 3(4)^{n-1}$

$a_7 = 3(4)^6 = 12{,}288$

18. 3, 15, 75, 375, ...

$r = \frac{15}{3} = 5$

$a_n = 3(5)^{n-1}$

$a_7 = 3(5)^6 = 46,875$

19. $19, 6, 2, \frac{2}{3}, \cdots$ $\quad r = \frac{6}{18} = \frac{1}{3}$

$a_n = 18\left(\frac{1}{3}\right)^{n-1}$

$a_7 = 18\left(\frac{1}{3}\right)^6 = \frac{2}{81}$

20. $12, 6, 3, \frac{3}{2}, \cdots$

$r = \frac{6}{12} = \frac{1}{2}$

$a_n = 12\left(\frac{1}{2}\right)^{n-1}$

$a_7 = 12\left(\frac{1}{2}\right)^6 = \frac{3}{16}$

21. 1.5, –3, 6, –12, . . .

$r = \frac{6}{-3} = -2$

$a_n = 1.5(-2)^{n-1}$

$a_7 = 1.5(-2)^6 = 96$

22. $5, -1, \frac{1}{5}, -\frac{1}{25}, \ \ldots$

$r = \frac{-1}{5} = -\frac{1}{5}$

$a_n = 5\left(-\frac{1}{5}\right)^{n-1}$

$a_7 = 5\left(-\frac{1}{5}\right)^6 = \frac{1}{3125}$

23. 0.0004, –0.004, 0.04, –0.4, . . .

$r = \frac{-0.004}{0.0004} = -10$

$a_n = 0.0004(-10)^{n-1}$

$a_7 = 0.0004(-10)^6 = 400$

24. 0.0007, –0.007, 0.07, –0.7, . .

$r = \frac{-0.007}{0.0007} = -10$

$a_n = 0.0007(-10)^{n-1}$

$a_7 = 0.0007(-10)^6 = 700$

25. 2, 6, 18, 54, . . .

$r = \frac{6}{2} = 3$

$S_{12} = \frac{2\left(1-3^{12}\right)}{1-3} = \frac{2(-531,440)}{-2} = 531,440$

26. 3, 6, 12, 24, …

$r = \frac{6}{3} = 2$

$S_{12} = \frac{3\left(1-2^{12}\right)}{1-2} = \frac{3(-4095)}{-1} = 12,285$

27. 3, –6, 12, –24, . . .

$r = \frac{-6}{3} = -2$

$S_{11} = \frac{3\left[1-(-2)^{11}\right]}{1-(-2)} = \frac{3(2049)}{3} = 2049$

28. 4, –12, 36, –108, …

$r = \frac{-12}{4} = -3$

$S_{11} = \frac{4\left[1-(-3)^{11}\right]}{1-(-3)} = \frac{4(177,148)}{4} = 177,148$

29. $-\frac{3}{2}, 3, -6, 12, \cdots$

$r = \frac{3}{\frac{-3}{2}} = -2$

$S_{14} = \frac{-\frac{3}{2}\left[1-(-2)^{14}\right]}{1-(-2)} = \frac{-\frac{3}{2}(-16,383)}{3} = \frac{16,383}{2}$

30. $-\frac{1}{24}, \frac{1}{12}, -\frac{1}{6}, \frac{1}{3}, \ \ldots$

$r = \frac{\frac{1}{12}}{-\frac{1}{24}} = -2$

$S_{14} = \frac{-\frac{1}{24}\left[1-(-2)^{14}\right]}{1-(-2)} = \frac{-\frac{1}{24}(-16,383)}{3} = \frac{5461}{24}$

31. $\sum_{i=1}^{8} 3^i$

$r = 3,\ \ a_1 = 3$

$S_8 = \dfrac{3\left(1-3^8\right)}{1-3} = \dfrac{3(-6560)}{-2} = 9840$

32. $\sum_{i=1}^{6} 4^i$

$r = 4,\ \ a_1 = 4$

$S_6 = \dfrac{4\left(1-4^6\right)}{1-4} = \dfrac{4(-4095)}{-3} = 5460$

33. $\sum_{i=1}^{10} 5\cdot 2^i$

$r = 2,\ \ a_1 = 10$

$S_{10} = \dfrac{10\left(1-2^{10}\right)}{1-2} = \dfrac{10(-1023)}{-1} = 10,230$

34. $\sum_{i=1}^{7} 4(-3)^i$

$r = -3,\ \ a_1 = -12$

$S_7 = \dfrac{-12\left(1-(-3)^7\right)}{1-(-3)} = \dfrac{-12(2188)}{4} = -6564$

35. $\sum_{i=1}^{6} \left(\dfrac{1}{2}\right)^{i+1}$

$r = \dfrac{1}{2},\ \ a_1 = \dfrac{1}{4}$

$S_6 = \dfrac{\frac{1}{4}\left(1-\left(\frac{1}{2}\right)^6\right)}{1-\frac{1}{2}} = \dfrac{\frac{1}{4}\left(\frac{63}{64}\right)}{\frac{1}{2}} = \dfrac{63}{128}$

36. $\sum_{i=1}^{6} \left(\dfrac{1}{3}\right)^{i+1}$

$r = \dfrac{1}{3},\ a_1 = \dfrac{1}{9}$

$S_6 = \dfrac{\frac{1}{9}\left(1-\left(\frac{1}{3}\right)^6\right)}{1-\frac{1}{3}} = \dfrac{\frac{1}{9}\left(\frac{728}{729}\right)}{\frac{2}{3}} = \dfrac{364}{2187}$

37. $r = \dfrac{1}{3}$

$S_\infty = \dfrac{1}{1-\frac{1}{3}} = \dfrac{1}{\frac{2}{3}} = \dfrac{3}{2}$

38. $1 + \dfrac{1}{4} + \dfrac{1}{16} + \dfrac{1}{64} + \cdots$

$r = \dfrac{1}{4}$

$S_\infty = \dfrac{1}{1-\frac{1}{4}} = \dfrac{1}{\frac{3}{4}} = \dfrac{4}{3}$

39. $r = \dfrac{1}{4}$

$S_\infty = \dfrac{3}{1-\frac{1}{4}} = \dfrac{3}{\frac{3}{4}} = 4$

40. $5 + \dfrac{5}{6} + \dfrac{5}{6^2} + \dfrac{5}{6^3} + \cdots$

$r = \dfrac{1}{6}, a_1 = 5$

$S_\infty = \dfrac{5}{1-\frac{1}{6}} = \dfrac{5}{\frac{5}{6}} = 6$

41. $r = -\dfrac{1}{2}$

$S_\infty = \dfrac{1}{1-\left(-\frac{1}{2}\right)} = \dfrac{1}{\frac{3}{2}} = \dfrac{2}{3}$

42. $3 - 1 + \dfrac{1}{3} - \dfrac{1}{9} + \ \ldots$

$r = -\dfrac{1}{3}$

$S_\infty = \dfrac{3}{1-\left(-\frac{1}{3}\right)} = \dfrac{3}{\frac{4}{3}} = \dfrac{9}{4}$

43. $r = -0.3$

$S_\infty = \dfrac{8}{1-(-0.3)} = \dfrac{8}{1.3} \approx 6.15385$

44. $\sum_{i=1}^{\infty} 12(-0.7)^{i-1} = 12 - 8.4 + \cdots$

$r = -0.7$

$S_\infty = \frac{12}{1-(-0.7)} = \frac{12}{1.7} \approx 7.05882$

45. $r = \frac{1}{10}$

$S_\infty = \frac{\frac{5}{10}}{1-\frac{1}{10}} = \frac{\frac{5}{10}}{\frac{9}{10}} = \frac{5}{9}$

46. $0.\bar{1} = \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \cdots$

$r = \frac{1}{10}$

$S_\infty = \frac{\frac{1}{10}}{1-\frac{1}{10}} = \frac{\frac{1}{10}}{\frac{9}{10}} = \frac{1}{9}$

47. $r = \frac{1}{100}$

$S_\infty = \frac{\frac{47}{100}}{1-\frac{1}{100}} = \frac{\frac{47}{100}}{\frac{99}{100}} = \frac{47}{99}$

48. $0.\overline{83} = \frac{83}{100} + \frac{83}{10,000} + \frac{83}{1,000,000} + \cdots$

$r = \frac{1}{100}$

$S_\infty = \frac{\frac{83}{100}}{1-\frac{1}{100}} = \frac{\frac{83}{100}}{\frac{99}{100}} = \frac{83}{99}$

49. $0.\overline{257} = \frac{257}{1000} + \frac{257}{10^6} + \frac{257}{10^9} + \cdots$

$r = \frac{1}{1000}$

$S_\infty = \frac{\frac{257}{1000}}{1-\frac{1}{1000}} = \frac{\frac{257}{1000}}{\frac{999}{1000}} = \frac{257}{999}$

50. $0.\overline{529} = \frac{529}{1000} + \frac{529}{10^6} + \frac{529}{10^9} + \cdots$

$r = \frac{1}{1000}$

$S_\infty = \frac{\frac{529}{1000}}{1-\frac{1}{1000}} = \frac{\frac{529}{1000}}{\frac{999}{1000}} = \frac{529}{999}$

51. $a_n = n + 5$

arithmetic, $d = 1$

52. $a_n = n - 3$

arithmetic, $d = 1$

53. $a_n = 2^n$

geometric, $r = 2$

54. $a_n = \frac{1}{2}^n$

geometric, $r = \frac{1}{2}$

55. $a_n = n^2 + 5$

neither

56. $a_n = n^2 - 3$

neither

57. First find a_{10} and b_{10}:

$a_{10} = a_1 r^{n-1}$

$= (-5)\left(\frac{10}{-5}\right)^{10-1} = (-5)(-2)^9$

$= 2560$

$b_{10} = b_1 + (n-1)d$

$= 10 + (10-1)(-5-10)$

$= 10 + (9)(-15) = -125$

So, $a_{10} + b_{10} = 2560 + (-125) = 2435$.

58. First find a_{11} and b_{11}:

$a_{11} = a_1 r^{n-1}$

$= (-5)\left(\frac{10}{-5}\right)^{11-1} = (-5)(-2)^{10}$

$= -5120$

$b_{11} = b_1 + (n-1)d$

$= 10 + (11-1)(-5-10)$

$= 10 + (10)(-15) = -140$

So, $a_{11} + b_{11} = -5120 + (-140) = -5260$.

59. For $\{a_n\}$, $r = \dfrac{10}{-5} = -2$ and

$$S_{10} = \frac{a_1(1-r^n)}{1-r} = \frac{(-5)\left(1-(-2)^{10}\right)}{1-(-2)} = \frac{(-5)(-1023)}{3} = 1705$$

For $\{b_n\}$,

$$b_{10} = b_1 + (n-1)d = 10 + (10-1)(-5-10) = 10 + (9)(-15) = -125$$

$$S_n = \frac{n}{2}(b_1 + b_n)$$

$$S_{10} = \frac{n}{2}(b_1 + b_{10}) = \frac{10}{2}(10 + (-125)) = 5(-115) = -575$$

60. For $\{a_n\}$, $r = \dfrac{10}{-5} = -2$ and :

$$S_{11} = \frac{a_1(1-r^n)}{1-r} = \frac{(-5)\left(1-(-2)^{11}\right)}{1-(-2)} = \frac{(-5)(2049)}{3} = -3415$$

For $\{b_n\}$,

$$b_{11} = b_1 + (n-1)d = 10 + (11-1)(-5-10) = 10 + (10)(-15) = -140$$

$$S_{11} = \frac{n}{2}(b_1 + b_n) = \frac{11}{2}(10 + (-140)) = 5.5(-130) = -715$$

So, $\displaystyle\sum_{n=1}^{11} a_n - \sum_{n=1}^{11} b_n = -3415 - (-715) = -2700$

61. For $\{a_n\}$,

$$S_6 = \frac{a_1(1-r^n)}{1-r} = \frac{(-5)\left(1-(-2)^{6}\right)}{1-(-2)} = \frac{(-5)(-63)}{3} = 105$$

For $\{c_n\}$,

$$S = \frac{a_1}{1-r} = \frac{-2}{1-\frac{1}{-2}} = \frac{-2}{\frac{3}{2}} = -\frac{4}{3}$$

So, $S_6 \cdot S = 105\left(-\dfrac{4}{3}\right) = -140$

62. For $\{a_n\}$,

$$S_9 = \frac{a_1(1-r^n)}{1-r} = \frac{(-5)\left(1-(-2)^{9}\right)}{1-(-2)} = \frac{(-5)(513)}{3} = -855$$

For $\{c_n\}$,

$$S = \frac{c_1}{1-r} = \frac{-2}{1-\frac{1}{-2}} = \frac{-2}{\frac{3}{2}} = -\frac{4}{3}$$

So, $S_9 \cdot S = -855\left(-\dfrac{4}{3}\right) = 1140$

63. It is given that $a_4 = 27$. Using the formula $a_n = a_1 r^{n-1}$ when $n = 4$ we have

$$27 = 8r^{4-1}$$

$$\frac{27}{8} = r^3$$

$$r = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$$

Thus,

$$a_n = a_1 r^{n-1}$$

$$a_2 = 8\left(\frac{3}{2}\right)^{2-1} = 8\left(\frac{3}{2}\right) = 12$$

$$a_3 = 8\left(\frac{3}{2}\right)^{3-1} = 8\left(\frac{3}{2}\right)^2 = 8\left(\frac{9}{4}\right) = 18$$

64. It is given that $a_4 = -54$. Using the formula $a_n = a_1 r^{n-1}$ when $n = 4$ we have

$-54 = 2r^{4-1}$

$-27 = r^3$

$r = \sqrt[3]{-27} = -3$

Thus,

$a_n = a_1 r^{n-1}$

$a_2 = 2(-3)^{2-1} = 2(-3) = -6$

$a_3 = 2(-3)^{3-1} = 2(-3)^2 = 2(9) = 18$

65. 1, 2, 4, 8, . . .

$r = 2$

$a_n = 2^{n-1}$

$a_{15} = 2^{14} = \$16,384$

66. $a_n = 2^{n-1}$

$a_{30} = 2^{29} = \$536,870,912$

67. $a_1 = 3,000,000$

$r = 1.04$

$a_n = 3,000,000(1.04)^{n-1}$

$a_7 = 3,000,000(1.04)^6 = \$3,795,957$

68. $a_1 = 30,000$

$r = 1.05$

$a_n = 30,000(1.05)^{n-1}$

$a_6 = 30,000(1.05)^5 = \$38,288.45$

69. **a.** $r_{2003 \text{ to } 2004} = \dfrac{35.89}{35.48} \approx 1.01$

$r_{2004 \text{ to } 2005} = \dfrac{36.13}{35.89} \approx 1.01$

$r_{2005 \text{ to } 2006} = \dfrac{36.46}{36.13} \approx 1.01$

r is approximately 1.01 for each division.

b. $a_n = a_1 r^{n-1}$

$a_n = 35.48(1.01)^{n-1}$

c. Since year 2010 is the 8th term, find a_8.

$a_n = 35.48(1.01)^{n-1}$

$a_8 = 35.48(1.01)^{8-1} \approx 38.04$

The population of California will be approximately 38.04 million in 2010.

70. **a.** $r_{2003 \text{ to } 2004} = \dfrac{22.49}{22.12} \approx 1.02$

$r_{2004 \text{ to } 2005} = \dfrac{22.86}{22.49} \approx 1.02$

$r_{2005 \text{ to } 2006} = \dfrac{23.41}{22.86} \approx 1.02$

r is approximately 1.02 for each division.

b. $a_n = a_1 r^{n-1}$

$a_n = 22.12(1.02)^{n-1}$

c. Since year 2010 is the 8th term, find a_8.

$a_n = 22.12(1.02)^{n-1}$

$a_8 = 22.12(1.02)^{8-1} \approx 25.41$

The population of Texas will be approximately 25.41 million in 2010.

71. 1, 2, 4, 8, . . .

$r = 2$

$S_{15} = \dfrac{1(1-2^{15})}{1-2} = 32,767$

The total savings is \$32,767.

72. $S_{30} = \dfrac{1-2^{30}}{1-2} = 1,073,741,823$

The total savings is \$1,073,741,823.

73. $a_1 = 24,000, \ r = 1.05$

$S_{20} = \dfrac{24,000\left[1-(1.05)^{20}\right]}{1-1.05} = 793,582.90$

The total salary is \$793,583.

74. Company A:

$a_1 = 30,000, \ r = 1.06$

$S_5 = \dfrac{30,000\left[1-(1.06)^5\right]}{1-1.06} = \$169,112.79$

Company B:

$a_1 = 32,000, \ r = 1.03$

$S_5 = \dfrac{32,000\left[1-(1.03)^5\right]}{1-1.03} = \$169,892.35$

Company B offers the better total salary by \$780.

75. $r = 0.9$

$S_{10} = \dfrac{20(1-0.9^{10})}{1-0.9} \approx 130.26$

The total length is 130.26 inches.

76. $26+0.96(16)+0.96^2(16)+\cdots$

$r=0.96$

$S_{10}=\dfrac{16\left(1-0.96^{10}\right)}{1-0.96}\approx 134.07$

The total length is 134.07 inches.

77. **a.** $A=\dfrac{P\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}=\dfrac{2000\left[\left(1+\frac{0.075}{1}\right)^{5}-1\right]}{\frac{0.075}{1}}\approx \$11,617$

b. $\$11,617-5\times\$2000=\$1617$

78. **a.** $A=\dfrac{P\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}=\dfrac{2500\left[\left(1+\frac{0.0625}{1}\right)^{5}-1\right]}{\frac{0.0625}{1}}\approx \$14,163$

b. $\$14,163-5\times\$2500=\$1663$

79. **a.** $A=\dfrac{P\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}=\dfrac{50\left[\left(1+\frac{0.055}{12}\right)^{12\times 40}-1\right]}{\frac{0.055}{12}}\approx \$87,052$

b. $\$87,052-\$50\cdot 12\cdot 40=\$63,052$

80. **a.** $A=\dfrac{P\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}=\dfrac{75\left[\left(1+\frac{0.065}{12}\right)^{12\times 40}-1\right]}{\frac{0.065}{12}}\approx \$171,271$

b. $\$171,271-\$75\cdot 12\cdot 40=\$135,271$

81. **a.** $A=\dfrac{P\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}=\dfrac{10,000\left[\left(1+\frac{0.105}{4}\right)^{4\times 10}-1\right]}{\frac{0.105}{4}}\approx \$693,031$

b. $\$693,031-\$10,000\cdot 4\cdot 10=\$293,031$

82. **a.** $A=\dfrac{P\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}=\dfrac{15,000\left[\left(1+\frac{0.09}{4}\right)^{4\times 10}-1\right]}{\frac{0.09}{4}}\approx \$956,793$

b. $\$956,793-\$15,000\cdot 4\cdot 10=\$356,793$

83. Find the total value of the lump-sum investment.

$A = P(1+r)^t = 30,000(1+0.05)^{20} \approx 79,599$

Find the total value of the annuity.

$$A = \frac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}} = \frac{1500\left[\left(1+\frac{0.05}{1}\right)^{20} - 1\right]}{\frac{0.05}{1}} \approx 49,599$$

$\$79,599 - \$49,599 = \$30,000$

You will have $30,000 more from the lump-sum investment.

84. Find the total value of the lump-sum investment.

$A = P(1+r)^t = 40,000(1+0.065)^{25} \approx 193,108$

Find the total value of the annuity.

$$A = \frac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}} = \frac{1600\left[\left(1+\frac{0.065}{1}\right)^{25} - 1\right]}{\frac{0.065}{1}} \approx 94,220$$

$\$193,108 - \$94,220 = \$98,888$

You will have $98,888 more from the lump-sum investment.

85. $r = 0.6$

$$S_\infty = \frac{6(0.6)}{1-0.6} = 9$$

The total economic impact is $9 million.

86. $10(0.6) + 10(0.6)^2 + 10(0.6)^3 + \cdots$

$a_1 = 10(0.6)$, $r = 0.6$

$$S_\infty = \frac{10(0.6)}{1-0.6} = 15$$

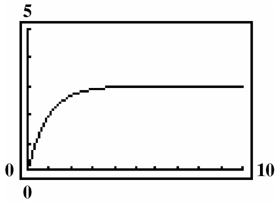
An additional $15 billion will be spent.

87. $r = \frac{1}{4}$

$$S_\infty = \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{1}{4}\cdot\frac{4}{3} = \frac{1}{3}$$

88. – 98. Answers may vary.

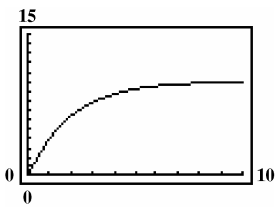
99. $f(x) = \dfrac{2\left[1-\left(\frac{1}{3}\right)^x\right]}{1-\frac{1}{3}}$



$S = \dfrac{2}{1-\frac{1}{3}} = \dfrac{2}{\frac{2}{3}} = 2 \div \dfrac{2}{3} = 2 \cdot \dfrac{3}{2} = 3$

The sum of the series is 3 and the asymptote of the function is $y = 3$.

100. $f(x) = \dfrac{4\left[1-(0.6)^x\right]}{1-0.6}$



$S = \dfrac{4}{1-0.6} = \dfrac{4}{0.4} = 10$

The sum of the series is 10 and the asymptote of the function is $y = 10$.

101. makes sense

102. makes sense

103. makes sense

104. does not make sense; Explanations will vary. Sample explanation: It is not possible to add each term individually of an infinite series.

105. false; Changes to make the statement true will vary. A sample change is: The sequence is not geometric. There is not a common ratio.

106. false; Changes to make the statement true will vary. A sample change is: We do not need to know the terms between $\frac{1}{8}$ and $\frac{1}{512}$, but we do need to know how many terms there are between $\frac{1}{8}$ and $\frac{1}{512}$.

107. false; Changes to make the statement true will vary. A sample change is: The sum of the sequence is $\dfrac{10}{1-\left(-\frac{1}{2}\right)}$.

108. true

109. Let a_1 equal the number of flies released each day. On any day, the total number of flies is the number released that day, plus 90% of those released the day before, plus 90% of 90% of those released two days before, etc.:

$$S = \frac{a_1}{1-r}$$
$$20,000 = \frac{a_1}{1-.9}$$
$$20,000 = \frac{a_1}{.1}$$
$$2000 = a_1$$

2000 flies to be released each day.

110. $1,000,000 = P\dfrac{\left(1+\frac{0.1}{12}\right)^{360} - 1}{\frac{0.1}{12}}$

$1,000,000 \approx 2260.49P$

$P \approx 442.38$

You must deposit $442 monthly.

111. Answers may vary.

112. $1+2+3 = \dfrac{3(3+1)}{2}$

$6 = \dfrac{3(4)}{2}$

$6 = 6$

113. $1+2+3+4+5 = \dfrac{5(5+1)}{2}$

$15 = \dfrac{5(6)}{2}$

$15 = 15$

114. $\frac{k(k+1)(2k+1)}{6}+(k+1)^2$

$$=\frac{k(k+1)(2k+1)}{6}+\frac{6(k+1)^2}{6}$$
$$=\frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$
$$=\frac{(k+1)[k(2k+1)+6(k+1)]}{6}$$
$$=\frac{(k+1)[2k^2+k+6k+6]}{6}$$
$$=\frac{(k+1)[2k^2+7k+6]}{6}$$
$$=\frac{(k+1)(k+2)(2k+3)}{6}$$

Mid-Chapter 11 Check Point

1. $a_n=(-1)^{n+1}\frac{n}{(n-1)!}$

$$a_1=(-1)^{1+1}\frac{1}{(1-1)!}=(-1)^2\frac{1}{0!}=1\cdot1=1$$
$$a_2=(-1)^{2+1}\frac{2}{(2-1)!}=(-1)^3\frac{2}{1!}=(-1)(2)=-2$$
$$a_3=(-1)^{3+1}\frac{3}{(3-1)!}=(-1)^4\frac{3}{2!}=1\cdot\frac{3}{2}=\frac{3}{2}$$
$$a_4=(-1)^{4+1}\frac{4}{(4-1)!}=(-1)^5\frac{4}{3!}=(-1)\frac{4}{6}=-\frac{2}{3}$$
$$a_5=(-1)^{5+1}\frac{5}{(5-1)!}=(-1)^6\frac{5}{4!}=1\cdot\frac{5}{24}=\frac{5}{24}$$

2. Using $a_n=a_1+(n-1)d$;

$$a_1=5$$
$$a_2=5+(2-1)(-3)=5+1(-3)=5-3=2$$
$$a_3=5+(3-1)(-3)=5+2(-3)=5-6=-1$$
$$a_4=5+(4-1)(-3)=5+3(-3)=5-9=-4$$
$$a_5=5+(5-1)(-3)=5+4(-3)=5-12=-7$$

3. Using $a_n=a_1r^{n-1}$;

$$a_1=5$$
$$a_2=5(-3)^{2-1}=5(-3)^1=5(-3)=-15$$
$$a_3=5(-3)^{3-1}=5(-3)^2=5(9)=45$$
$$a_4=5(-3)^{4-1}=5(-3)^3=5(-27)=-135$$
$$a_5=5(-3)^{5-1}=5(-3)^4=5(81)=405$$

4. $a_n=-a_{n-1}+4$

$$a_1=3$$
$$a_2=-a_1+4=-3+4=1$$
$$a_3=-a_2+4=-1+4=3$$
$$a_4=-a_3+4=-3+4=1$$
$$a_5=-a_4+4=-1+4=3$$

5. $d=a_2-a_1=6-2=4$

$$a_n=a_1+(n-1)d$$
$$=2+(n-1)4$$
$$=2+4n-4$$
$$=4n-2$$
$$a_{20}=4(20)-2=78$$

6. $r=\frac{a_2}{a_1}=\frac{6}{3}=2$

$$a_n=a_1r^{n-1}$$
$$=3(2)^{n-1}$$
$$a_{10}=3(2)^{10-1}$$
$$=3(2)^9$$
$$=1536$$

7. $d=a_2-a_1=1-\frac{3}{2}=-\frac{1}{2}$

$$a_n=a_1+(n-1)d$$
$$=\frac{3}{2}+(n-1)\left(-\frac{1}{2}\right)$$
$$=\frac{3}{2}-\frac{1}{2}n+\frac{1}{2}$$
$$=-\frac{1}{2}n+2$$

$$a_{30}=-\frac{1}{2}(30)+2$$
$$=-15+2$$
$$=-13$$

8. $S_n=\frac{a_1(1-r^n)}{1-r};\ r=\frac{a_2}{a_1}=\frac{10}{5}=2$

$$S_{10}=\frac{5(1-2^{10})}{1-2}=\frac{5(-1023)}{-1}=5115$$

9. First find a_{10};

$$d = a_2 - a_1 = 0-(-2) = 2$$

$$a_{50} = a_1 + (n-1)d = -2+(50-1)(2) = -2+49(2) = 96$$

$$S_{50} = \frac{n}{2}(a_1 + a_n) = \frac{50}{2}(-2+96) = 25(94) = 2350$$

10. $r = \dfrac{a_2}{a_1} = \dfrac{40}{-20} = -2$

$$S_{10} = \frac{a_1(1-r^n)}{1-r} = \frac{-20(-1-(-2)^{10})}{1-(-2)} = \frac{-20(-1023)}{3} = \frac{20460}{3} = 6820$$

11. First find a_{100};

$$d = a_2 - a_1 = -2-4 = -6$$

$$a_{100} = a_1 + (n-1)d = 4+(100-1)(-6) = 4+99(-6) = -590$$

$$S_{100} = \frac{n}{2}(a_1 + a_n) = \frac{100}{2}(4-590) = 50(-586) = -29,300$$

12. $\displaystyle\sum_{i=1}^{4}(i+4)(i-1) = (1+4)(1-1)+(2+4)(2-1)+(3+4)(3-1)+(4+4)(4-1)$

$$= 5(0)+6(1)+7(2)+8(3) = 0+6+14+24 = 44$$

13. $\displaystyle\sum_{i=1}^{50}(3i-2) = (3\cdot 1-2)+(3\cdot 2-2)+(3\cdot 3-3)+...+(3\cdot 50-2)$

$$= (3-2)+(6-2)+(9-3)+...+(150-2)$$

$$= 1+4+6+...+148$$

The sum of this arithmetic sequence is given by $S_n = \dfrac{n}{2}(a_1 + a_n)$;

$$S_{50} = \frac{50}{2}(1+148) = 25(149) = 3725$$

14. $\displaystyle\sum_{i=1}^{6}\left(\frac{3}{2}\right)^i = \left(\frac{3}{2}\right)^1 + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^3 + \left(\frac{3}{2}\right)^4 + \left(\frac{3}{2}\right)^5 + \left(\frac{3}{2}\right)^6$

$$= \frac{3}{2}+\frac{9}{4}+\frac{27}{8}+\frac{81}{16}+\frac{243}{32}+\frac{729}{64} = \frac{1995}{64}$$

15. $\displaystyle\sum_{i=1}^{\infty}\left(-\frac{2}{5}\right)^{i-1} = \left(-\frac{2}{5}\right)^{1-1} + \left(-\frac{2}{5}\right)^{2-1} + \left(-\frac{2}{5}\right)^{3-1} + ...$

$$= \left(-\frac{2}{5}\right)^{0} + \left(-\frac{2}{5}\right)^{1} + \left(-\frac{2}{5}\right)^{2} + ...$$

$$= 1 + \left(-\frac{2}{5}\right) + \frac{4}{25} + ...$$

This is an infinite geometric sequence with $r = \dfrac{a_2}{a_1} = \dfrac{-\frac{2}{5}}{1} = -\dfrac{2}{5}$.

$$S = \frac{a_1}{1-r} = \frac{1}{1-\left(-\frac{2}{5}\right)} = \frac{1}{\frac{7}{5}} = \frac{5}{7}$$

16. $0.\overline{45} = \frac{a_1}{1-r} = \frac{\frac{45}{100}}{1-\frac{1}{100}} = \frac{\frac{45}{100}}{\frac{99}{100}}$

$= \frac{45}{100} \div \frac{99}{100} = \frac{45}{100} \cdot \frac{100}{99} = \frac{45}{99} = \frac{5}{11}$

17. Answers may vary. An example is $\sum_{i=1}^{18} \frac{i}{i+2}$.

18. The arithmetic sequence is 16, 48, 80, 112, ….
First find a_{15} where $d = a_2 - a_1 = 48 - 16 = 32$.
$a_{15} = a_1 + (n-1)d = 16 + (15-1)(32) = 16 + 14(32) = 16 + 448 = 464$
The distance the skydiver falls during the 15th second is 464 feet.
$S_{15} = \frac{n}{2}(a_1 + a_n) = \frac{15}{2}(16 + 464) = 7.5(480) = 3600$
The total distance the skydiver falls in 15 seconds is 3600 feet.

19. $r = 0.10$
$A = P(1+r)^t$
$= 120000(1+0.10)^{10}$
≈ 311249
The value of the house after 10 years is $311,249.

Section 11.4

Check Point Exercises

1. **a.** $S_1 : 2 = 1(1+1)$
$S_k : 2 + 4 + 6 + \cdots 2k = k(k+1)$
$S_{k+1} : 2 + 4 + 6 + \cdots + 2(k+1) = (k+1)(k+2)$

b. $S_1 : 1^3 = \frac{1^2(1+1)^2}{4}$

$S_k = 1^3 + 2^3 + 3^3 + \cdots + k^3 = \frac{k^2(k+1)^2}{4}$

$S_{k+1} = 1^3 + 2^3 + 3^3 + \cdots + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4}$

2. $S_1 : 2 = 1(1+1)$
$2 = 2$ is true.
$S_k : 2 + 4 + 6 + \cdots + 2k = k(k+1)$
$S_{k+1} : 2 + 4 + 6 + \cdots + 2k + 2(k+1) = (k+1)(k+2)$
Add $2(k+1)$ to both sides of S_k:
$2 + 4 + 6 + \cdots + 2k + 2(k+1) = k(k+1) + 2(k+1)$
Simplify the right-hand side:
$k(k+1) + 2(k+1) = (k+1)(k+2)$
If S_k is true, then S_{k+1} is true. The statement is true for all *n*.

3. $S_1 : 1^3 = \frac{1^2(1+1)^2}{4}$

$1 = \frac{4}{4}$

$1 = 1$ is true.

$S_k : 1^3 + 2^3 + 3^3 + \cdots + k^3 = \frac{k^2(k+1)^2}{4}$

$S_{k+1} : 1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4}$

Add $(k + 1)^3$ to both sides of S_k:

$1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3$

Simplify the right hand side:

$$\frac{k^2(k+1)^2}{4} + (k+1)^3 = \frac{k^2(k+1)^2 + 4(k+1)^3}{4} = \frac{(k+1)^2\left[k^2 + 4(k+1)\right]}{4} = \frac{(k+1)^2(k^2+4k+4)}{4}$$

$$= \frac{(k+1)^2(k+2)^2}{4}$$

If S_k is true, then S_{k+1} is true. The statement is true for all n.

4. S_1: 2 is a factor of $1^2 + 1 = 2$, since $2 = 2 \cdot 1$.

S_k: 2 is a factor of $k^2 + k$

S_{k+1}: 2 is a factor of $(k + 1)^2 + (k + 1)$

Simplify:

$$\begin{aligned}(k+1)^2 + (k+1) &= k^2 + 2k + 1 + k + 1 \\ &= k^2 + 3k + 2 \\ &= k^2 + k + 2k + 2 \\ &= (k^2 + k) + 2(k+1)\end{aligned}$$

Because we assume S_k is true, we know 2 is a factor of $k^2 + k$. Since 2 is a factor of $2(k + 1)$, we conclude 2 is a factor of the sum $(k^2 + k) + 2(k + 1)$. If S_k is true, then S_{k+1} is true. The statement is true for all n.

Exercise Set 11.4

1. $S_n = 1 + 3 + 5 + \cdots + (n-1) = n^2$

$S_1 : 1 = 1^2$

$1 = 1$ true

$S_2 :\ 1 + 3 = 2^2$

$4 = 4$ true

$S_3 :\ 1 + 3 + 5 = 3^2$

$9 = 9$ true

2. $S_n : 3+4+5+\cdots+(n+2)=\dfrac{n(n+5)}{2}$

$S_1:\ 3=\dfrac{1(6)}{2}$

$3=3$ true

$S_2:\ 3+4=\dfrac{2(7)}{2}$

$7=7$ true

$S_3:\ 3+4+5=\dfrac{3(8)}{2}$

$12=12$ true

3. $S_n:$ 2 is a factor of n^2-n

$S_1:$ 2 is a factor of $1^2-1=0$

$0=0\cdot 2$ so 2 is a factor of 0 is true.

$S_2:$ 2 is a factor of $2^2-2=2$

$2=1\cdot 2$ so 2 is a factor of 2 is true.

$S_3:$ 2 is a factor of $3^2-3=6$

$6=3\cdot 2$ so 2 is a factor of 6 is true.

4. S_n: 3 is a factor of n^3-n.

$S_1:$ 3 is a factor of $1^3-1=0$

$0=0\cdot 3$ so 3 is a factor of 0 is true.

$S_2:$ 3 is a factor of $2^3-2=6$

$6=2\cdot 3$ so 3 is a factor of 6 is true.

$S_3:$ 3 is a factor of $3^3-3=24$

$24=8\cdot 3$ so 3 is a factor of 24 is true.

5. $S_n : 4+8+12+\cdots+4n=2n(n+1)$

$S_k : 4+8+12+\cdots+4k=2k(k+1)$

$S_{k+1} : 4+8+12+\cdots+4(k+1)=2(k+1)(k+1+1)$

$4+8+12+\cdots+4(k+1)=2(k+1)(k+2)$

6. $S_n : 3+4+5+\cdots+(n+2)=\dfrac{n(n+5)}{2}$

$S_k : 3+4+5+\cdots+(k+2)=\dfrac{k(k+5)}{2}$

$S_{k+1} : 3+4+5+\cdots+(k+1+2)=\dfrac{(k+1)(k+1+5)}{2}$

$3+4+5+\cdots+(k+3)=\dfrac{(k+1)(k+6)}{2}$

7. $S_n: 3+7+11+\cdots+(4n-1)=n(2n+1)$

$S_k: 3+7+11+\cdots+(4k-1)=k(2k+1)$

$S_{k+1}: 3+7+11+\cdots+[4(k+1)-1]=(k+1)[2(k+1)+1]$

$3+7+11+\cdots+(4k+3)=(k+1)(2k+3)$

8. $S_n: 2+7+12+\cdots+(5n-3)=\dfrac{n(5n-1)}{2}$

$S_k: 2+7+12+\cdots+(5k-3)=\dfrac{k(5k-1)}{2}$

$S_{k+1}: 2+7+12+\cdots+[5(k+1)-3]=\dfrac{(k+1)[5(k+1)-1]}{2}$

$2+7+12+\cdots+(5k+2)=\dfrac{(k+1)(5k+4)}{2}$

9. S_n: 2 is a factor of n^2-n+2

S_k: 2 is a factor of k^2-k+2

S_{k+1}: 2 is a factor of $(\text{k+1})^2-(k+1)+2$

$k^2+2k+1-k-1+2=k^2+k+2$

S_{k+1}: 2 is a factor of k^2+k+2.

10. S_n: 2 is a factor of n^2-n.

S_k: 2 is a factor of k^2-k.

S_{k+1}: 2 is a factor of $(k+1)^2-(k+1)$.

$(k+1)^2-(k+1)=k^2+2k+1-k-1=k^2+k$

S_{k+1}: 2 is a factor of k^2+k.

11. $S_1: 4=2(1)(1+1)$

$4=2(2)$

$4=4$ is true.

$S_k: 4+8+12+\cdots+4k=2k(k+1)$

$S_{k+1}: 4+8+12+\cdots 4(k+1)=2(k+1)(k+1+1)$

Add 4(*k* + 1) to both sides of S_k:

$4+8+12+\cdots+4(k+1)=2k(k+1)+4(k+1)$

Simplify the right-hand side:

$=2k(k+1)+4(k+1)=(2k+4)(k+1)$

$=2(k+2)(k+1)$

$=2(k+1)(k+1+1)$

If S_k is true, then S_{k+1} is true. The statement is true for all *n*.

12. $S_1:\ 3=\frac{1(1+5)}{2}$

$3=\frac{6}{2}$

$3=3$ is true.

$S_k: 3+4+5+\cdots+(k+2)=\frac{k(k+5)}{2}$

$S_{k+1}: 3+4+5+\cdots+(k+3)=\frac{(k+1)(k+6)}{2}$

Add $k+3$ to both sides of $S_k: 3+4+5+\cdots+(k+2)+(k+3)=\frac{k(k+5)}{2}+(k+3)$

Simplify right-hand side:

$$\frac{k(k+5)}{2}+(k+3)=\frac{k(k+5)+2(k+3)}{2}=\frac{k^2+5k+2k+6}{2}=\frac{k^2+7k+6}{2}$$
$$=\frac{(k+1)(k+6)}{2}$$

If S_k is true, then S_{k+1} is true. The statement is true for all n.

13. $S_1: 1=1^2$

$1=1$ is true.

$S_k: 1+3+5+\cdots+(2k-1)=k^2$

$S_{k+1}: 1+3+5+\cdots+(2k-1)+[2(k+1)-1]=(k+1)^2$

$1+3+5+\cdots+(2k-1)+(2k+1)=(k+1)^2$

Add $(2k+1)$ to both sides of S_k:

$1+3+5+\cdots+(2k-1)+(2k+1)=k^2+(2k+1)$

Simplify the right-hand side:

$=k^2+(2k+1)$

$=(k+1)^2$

If S_k is true, then S_{k+1} is true. The statement is true for all n.

14. $S_1:\ 3=\frac{3(1)(1+1)}{2}$

$3=\frac{3(2)}{2}$

$3=3$ is true

$S_k: 3+6+9+\cdots+3k=\frac{3k(k+1)}{2}$

$S_{k+1}: 3+6+9+\cdots+3k+3(k+1)=\frac{3(k+1)(k+2)}{2}$

Add $3(k+1)$ to both sides of $S_k: 3+6+9+\cdots+3k+3(k+1)=\frac{3k(k+1)}{2}+3(k+1)$

Simplify the right-hand side:

$$\begin{aligned}\frac{3k(k+1)}{2}+3(k+1)&=\frac{3k(k+1)+6(k+1)}{2}\\&=\frac{3k^2+3k+6k+6}{2}\\&=\frac{3(k^2+3k+2)}{2}\\&=\frac{3(k+1)(k+2)}{2}\end{aligned}$$

If S_k is true, then S_{k+1} is true. The statement is true for all n.

15. $S_1: 3=1[2(1)+1)]$

$3=3$ is true.

$S_k: 3+7+11+\cdots+(4k-1)=k(2k+1)$

$S_{k+1}: 3+7+11+\cdots+(4k-1)+[4(k+1)-1]=(k+1)[2(k+1)+1]$

$3+7+11+\cdots+(4k-1)+(4k+3)=(k+1)(2k+3)$

Add $(4k + 3)$ to both sides of S_k:

$3 + 7 + 11 + \ldots + (4k - 1) + (4k + 3) = k(2k + 1) + 4(k + 3)$

Simplify the right-hand side:

$=k(2k+1)+(4k+3)=2k^2+k+4k+3$

$=2k^2+5k+3$

$=(k+1)(2k+3)$

If S_k is true, then S_{k+1} is true. The statement is true for all n.

16. $S_1: \ 2 = \frac{1[5(1)-1]}{2}$

$2 = \frac{4}{2}$

$2 = 2$ is true.

$S_k: 2+7+12+\cdots+(5k-3) = \frac{k(5k-1)}{2}$

$S_{k+1}: 2+7+12+\cdots+[5(k+1)-3] = \frac{(k+1)[5(k+1)-1]}{2}$

$2+7+12+\cdots+(5k+2) = \frac{(k+1)(5k+4)}{2}$

Add $(5k+2)$ to both sides of S_k: $2+7+12+\cdots+(5k-3)+(5k+2) = \frac{k(5k-1)}{2}+(5k+2)$

Simplify the right-hand side:

$$\frac{k(5k-1)}{2}+(5k+2) = \frac{k(5k-1)+2(5k+2)}{2}$$
$$= \frac{5k^2-k+10k+4}{2}$$
$$= \frac{5k^2+9k+4}{2}$$
$$= \frac{(k+1)(5k+4)}{2}$$

If S_k is true, then S_{k+1} is true. The statement is true for all *n*.

17. S_1: $1 = 2^1 - 1$

$1 = 1$ is true.

$S_k: 1+2+2^2+\cdots+2^{k-1} = 2^k - 1$

$S_{k+1}: 1+2+2^2+\cdots+2^{k-1}+2^{k+1-1} = 2^{k+1}-1$

$1+2+2^2+\cdots+2^{k-1}+2^k = 2^{k+1}-1$

Add 2^k to both sides of S_k:

$1+2+2^2+\cdots+2^{k-1}+2^k = 2^k+2^k-1$

Simplify the right-hand side:

$= 2^k+2^k-1 = 2\left(2^k\right)-1$

$= 2^{k+1}-1$

If S_k is true, then S_{k+1} is true. The statement is true for all *n*.

18. $S_1:\ 1=\dfrac{3^1-1}{2}$

$1=1$ is true.

$S_k: 1+3+3^2+\cdots+3^{k-1}=\dfrac{3^k-1}{2}$

$S_{k+1}: 1+3+3^2+\cdots+3^{k-1}+3^k=\dfrac{3^{k+1}-1}{2}$

$1+3+3^2+\cdots+3^{k-1}+3^k=\dfrac{3^{k+1}-1}{2}$

Add 3^k to both sides of S_k:

$1+3+3^2+\cdots+3^{k-1}+3^k=\dfrac{3^k-1}{2}+3^k$

Simplify the right-hand side:

$$\frac{3^k-1}{2}+3^k=\frac{3^k-1+2(3^k)}{2}$$
$$=\frac{3(3^k)-1}{2}$$
$$=\frac{3^{k+1}-1}{2}$$

If S_k is true, then S_{k+1} is true. The statement is true for all *n*.

19. $S_1: 2=2^{1+1}-2$

$2=4-2$

$2=2$ is true.

$S_k: 2+4+8+\cdots+2^k=2^{k+1}-2$

$S_{k+1}: 2+4+8+\cdots+2^k+2^{k+1}=2^{k+2}-2$

Add 2^{k+1} to both sides of S_k:

$2+4+8+\cdots+2^k+2^{k+1}=2^{k+1}+2^{k+1}-2$

Simplify the right-hand side:

$=2^{k+1}+2^{k+1}-1=2\left(2^{k+1}\right)-2$

$=2^{k+2}-2$

If S_k is true, then S_{k+1} is true. The statement is true for all *n*.

20. $S_1 : \frac{1}{2} = 1 - \frac{1}{2^1}$

$\frac{1}{2} = \frac{1}{2}$ is true

$S_k : \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$

$S_{k+1} : \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^k} + \frac{1}{2^{k+1}} = 1 - \frac{1}{2^{k+1}}$

Add $\frac{1}{2^{k+1}}$ to both sides of S_k :

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^k} + \frac{1}{2^{k+1}} = 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}$

Simplify the right-hand side:

$$1 - \frac{1}{2^k} + \frac{1}{2^{k+1}} = 1 - \frac{2}{2^{k+1}} + \frac{1}{2^{k+1}}$$
$$= 1 - \frac{1}{2^{k+1}}$$

If S_k is true, then S_{k+1} is true. The statement is true for all n.

21. $S_1 : 1 \cdot 2 = \frac{1(1+1)(1+2)}{3}$

$2 = \frac{6}{3}$

$2 = 2$ is true.

$S_k : 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + k(k+1) = \frac{k(k+1)(k+2)}{3}$

$S_{k+1} : 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + k(k+1) + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$

Add $(k+1)(k+2)$ to both sided of S_k:

$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + k(k+1) + (k+1)(k+2) = \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$

Simplify the right-hand side:

$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2) = \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3}$$
$$= \frac{(k+1)(k+2)(k+3)}{3}$$

If S_k is true, then S_{k+1} is true. The statement is true for all n.

22. $S_1 : 1\cdot 3 = \frac{1(1+1)[2(1)+7]}{6}$

$3 = \frac{2\cdot 9}{6}$

$3 = 3$ is true.

$S_k : 1\cdot 3 + 2\cdot 4 + 3\cdot 5 + \cdots + k(k+2) = \frac{k(k+1)(2k+7)}{6}$

$S_{k+1} : 1\cdot 3 + 2\cdot 4 + 3\cdot 5 + \cdots + k(k+2) + (k+1)(k+3) = \frac{(k+1)(k+2)(2k+9)}{6}$

Add $(k+1)(k+3)$ to both sides of S_k :

$$1\cdot 3 + 2\cdot 4 + 3\cdot 5 + \cdots + k(k+2) + (k+1)(k+3) = \frac{k(k+1)(2k+7)}{6} + (k+1)(k+3)$$

Simplify the right-hand side:

$$\begin{aligned}\frac{k(k+1)(2k+7)}{6} + (k+1)(k+3) &= \frac{k(k+1)(2k+7) + 6(k+1)(k+3)}{6}\\ &= \frac{(k+1)[k(2k+7) + 6(k+3)]}{6}\\ &= \frac{(k+1)(2k^2+13k+18)}{6}\\ &= \frac{(k+1)(k+2)(2k+9)}{6}\end{aligned}$$

If S_k is true, then S_{k+1} is true. The statement is true for all n.

23. $S_1 : \frac{1}{1\cdot 2} = \frac{1}{1+1}$

$\frac{1}{2} = \frac{1}{2}$ is true.

$S_k : \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{k(k+1)} = \frac{k}{k+1}$

$S_{k+1} : \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$

Add $\frac{1}{(k+1)(k+2)}$ to both sides of S_k:

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$

Simplify the right-hand side:

$$\begin{aligned}\frac{k}{(k+1)} + \frac{1}{(k+1)(k+2)} &= \frac{k(k+2)+1}{(k+1)(k+2)}\\ &= \frac{k^2+2k+1}{(k+1)(k+2)}\\ &= \frac{(k+1)(k+1)}{(k+1)(k+2)}\\ &= \frac{k+1}{k+2}\end{aligned}$$

If S_k is true, then S_{k+1} is true. The statement is true for all n.

24. $S_1: \frac{1}{2\cdot 3} = \frac{1}{2(1)+4}$

$\frac{1}{6} = \frac{1}{6}$ is true.

$S_k: \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots + \frac{1}{(k+1)(k+2)} = \frac{k}{2k+4}$

$S_{k+1}: \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots + \frac{1}{(k+1)(k+2)} + \frac{1}{(k+2)(k+3)} = \frac{k+1}{2k+6}$

Add $\frac{1}{(k+2)(k+3)}$ to both sides of S_k:

$$\frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots + \frac{1}{(k+1)(k+2)} + \frac{1}{(k+2)(k+3)} = \frac{k}{2k+4} + \frac{1}{(k+2)(k+3)}$$

Simplify the right-hand side:

$$\begin{aligned}\frac{k}{2k+4} + \frac{1}{(k+2)(k+3)} &= \frac{k(k+3)}{2(k+2)(k+3)} + \frac{2}{2(k+2)(k+3)} \\ &= \frac{k(k+3)+2}{2(k+2)(k+3)} \\ &= \frac{k^2+3k+2}{2(k+2)(k+3)} \\ &= \frac{(k+1)(k+2)}{2(k+2)(k+3)} \\ &= \frac{k+1}{2k+6}\end{aligned}$$

If S_k is true, S_{k+1} is true. The statement is true for all *n*.

25. S_1 : 2 is a factor of $1^2 - 1 = 0$, since $0 = 2\cdot 0$.

S_k : 2 is a factor of $k^2 - k$

S_{k+1} : 2 is a factor of $\left(k+1\right)^2 - \left(k+1\right)$

$$\begin{aligned}(k+1)^2 - (k-1) &= k^2 + 2k + 1 - k - 1 \\ &= k^2 + k \\ &= k^2 - k + 2k \\ &= (k^2 - k) + 2k\end{aligned}$$

Because we assume S_k is true, we know 2 as a factor of $k^2 - k$. Since 2 is a factor of 2*k*, we conclude 2 is factor of the sum $(k^2 + k) + 2k$. If S_k is true, then S_{k+1} is true. The statement is true for all *n*.

26. S_1 : 2 is a factor of $1^2+3(1)=4$ since

$4=2\cdot 2$

S_k : 2 is a factor of k^2+3k

S_{k+1} : 2 is a factor of $(k+1)^2+3(k+1)$

$$\begin{aligned}(k+1)^2+3(k+1) &= k^2+2k+1+3k+3\\ &= k^2+3k+2k+4\\ &= k^2+5k+4\\ &= (k^2+3k)+2(k+2)\end{aligned}$$

Because we assume S_k is true, we know 2 is a factor of k^2+3k . Since 2 is a factor of the sum $(k^2+3k)+2(k+2)$.

If S_k is true, then S_{k+1} is true.

The statement is true for all *n*.

27. S_1: 6 is a factor of $1(1+1)(1+2)=6$, since $6 = 6 \cdot 1$.

S_k: 6 is a factor of $k(k+1)(k+2)$

S_{k+1}: 6 is a factor of $(k+1)(k+2)(k+3)$

$(k+1)(k+2)(k+3)=k(k+1)(k+2)+3(k+1)(k+2)$

Because we assume S_k is true, we know 6 as a factor of $k(k+1)(k+2)$. Since either $k+1$ or $k+2$ must be even, the product $(k+1)(k+2)$ is even. Thus 2 is a factor of $(k+1)(k+2)$, and we can conclude that 6 is factor of $3(k+1)(k+2)$ If S_k is true, then S_{k+1} is true.

The statement is true for all *n*.

28. S_1 : 3 is a factor of $1(1+1)(1-1)=0$

since $0 = 3\cdot 0$.

S_k : 3 is a factor of $k(k+1)(k-1)=k^3-k$

S_{k+1} : 3 is a factor of $(k+1)(k+2)k$

$$\begin{aligned}k(k+1)(k+2) &= k(k^2+3k+2)\\ &= k^3+3k^2+2k\\ &= (k^3-k)+3(k^2+k)\end{aligned}$$

Because we assume S_k is true, we know 3 is a factor of k^3-k . Since 3 is a factor of $3(k^2+k)$, we conclude 3 is a factor of the sum $(k^3-k)+(k^2+k)$.

If S_k is true, then S_{k+1} is true.

29. $\sum_{i=1}^{n} 5 \cdot 6^i = 6(6^n - 1)$

Show that S_1 *is true*: $\sum_{i=1}^{1} 5 \cdot 6^i = 6(6^1 - 1)$

$$5 \cdot 6^1 = 6(6-1)$$

$$5 \cdot 6 = 6 \cdot 5, \text{ True}$$

Show that if S_k *is true, then* S_{k+1} *is true*:

Assume $S_k : \sum_{i=1}^{k} 5 \cdot 6^i = 6(6^k - 1)$ is true. Then,

$$\sum_{i=1}^{k} 5 \cdot 6^i + 5 \cdot 6^{k+1} = 6(6^k - 1) + 5 \cdot 6^{k+1}$$

$$\sum_{i=1}^{k+1} 5 \cdot 6^i = 6^{k+1} - 6 + 5 \cdot 6^{k+1}$$

$$\sum_{i=1}^{k+1} 5 \cdot 6^i = 6 \cdot 6^{k+1} - 6$$

$$\sum_{i=1}^{k+1} 5 \cdot 6^i = 6(6^{k+1} - 1)$$

The final statement is S_{k+1}. Thus, by mathematical induction, we have proven that $\sum_{i=1}^{n} 5 \cdot 6^i = 6(6^n - 1)$.

30. $\sum_{i=1}^{n} 7 \cdot 8^i = 8(8^n - 1)$

Show that S_1 *is true*: $\sum_{i=1}^{1} 7 \cdot 8^i = 8(8^1 - 1)$

$$7 \cdot 8^1 = 8(8-1)$$

$$7 \cdot 8 = 8 \cdot 7, \text{ True}$$

Show that if S_k *is true, then* S_{k+1} *is true*:

Assume $S_k : \sum_{i=1}^{k} 7 \cdot 8^i = 8(8^k - 1)$ is true. Then,

$$\sum_{i=1}^{k} 7 \cdot 8^i + 7 \cdot 8^{k+1} = 8(8^k - 1) + 7 \cdot 8^{k+1}$$

$$\sum_{i=1}^{k+1} 7 \cdot 8^i = 8^{k+1} - 8 + 7 \cdot 8^{k+1}$$

$$\sum_{i=1}^{k+1} 7 \cdot 8^i = 8 \cdot 8^{k+1} - 8$$

$$\sum_{i=1}^{k+1} 7 \cdot 8^i = 8(8^{k+1} - 1)$$

The final statement is S_{k+1}. Thus, by mathematical induction, we have proven that $\sum_{i=1}^{n} 7 \cdot 8^i = 8(8^n - 1)$.

31. $n+2>n$

Show that S_1 *is true*: $1+2>1$

$3>1$, True

Show that if S_k *is true, then* S_{k+1} *is true*:

Assume $S_k : k+2>k$ is true. Then,

$k+2+1>k+1$

$(k+1)+2>k+1$

The final statement is S_{k+1}. Thus, by mathematical induction, we have proven that $n+2>n$.

32. If $0<x<1$, then $0<x^n<1$.

Show that S_1 *is true*: $0<x^1<1$ is true because it is equivalent to the if statement $0<x<1$.

Show that if S_k *is true, then* S_{k+1} *is true*:

Assume $S_k : 0<x^k<1$ is true. Then,

$0\cdot x<x^k\cdot x<1\cdot x$

$0<x^{k+1}<x$

Now we know that $x<1$, so $0<x^{k+1}<1$ is true.

The final statement is S_{k+1}. Thus, by mathematical induction, we have proven that $0<x^n<1$.

33. $S_1 : (ab)^1 = a^1b^1$

$ab=ab$ is true.

$S_k : (ab)^k = a^kb^k$

$S_{k+1} : (ab)^{k+1} = a^{k+1}b^{k+1}$

Multiply both sides of S_k by *ab*:

$(ab)^k(ab) = a^kb^k(ab)$

$(ab)^{k+1} = a^{k+1}b^{k+1}$

If S_k is true, then S_{k+1} is true.

The statement is true for all *n*.

34. $S_1 : \left(\frac{a}{b}\right)^1 = \frac{a^1}{b^1}$

$\frac{a}{b}=\frac{a}{b}$ is true.

$S_k : \left(\frac{a}{b}\right)^k = \frac{a^k}{b^k}$

$S_{k+1} : \left(\frac{a}{b}\right)^{k+1} = \frac{a^{k+1}}{b^{k+1}}$

Multiply both sides of S_k by $\left(\frac{a}{b}\right)$:

$\left(\frac{a}{b}\right)^k\cdot\left(\frac{a}{b}\right) = \frac{a^k}{b^k}\cdot\left(\frac{a}{b}\right)$

$\left(\frac{a}{b}\right)^{k+1} = \frac{a^{k+1}}{b^{k+1}}$

If S_k is true, then S_{k+1} is true. The statement is true for all *n*.

35. Answers may vary.

36. Answers may vary.

37. does not make sense; Explanations will vary. Sample explanation: We use mathematical induction to prove statements involving positive integers.

38. does not make sense; Explanations will vary. Sample explanation: When using mathematical induction, we do not assume S_{k+1} is true.

39. does not make sense; Explanations will vary. Sample explanation: It is necessary for all the dominoes to topple.

40. makes sense

41. $n^2 > 2n + 1$ for $n \geq 3$

S_3: $3^2 > 2 \cdot 3 + 1$

$9 > 7$

S_k: $k^2 > 2k + 1$ for $k \geq 3$

S_{k+1}: $(k + 1)^2 > 2k + 3$.

Add $2k + 1$ to both sides of S_k.

$k^2 + (2k+1) > 2k+1+(2k+1)$

Write the left side of the inequalities as the square of a binomial and simplify the right side. $(k+1)^2 > 4k+2$

Since $4k + 2 > 2k + 3$ for $k \geq 3$, we can conclude that $(k+1)^2 > 4k+2 > 2k+3$.

By the transitive property,

$(k+1)^2 > 2k+3$

$(k+1)^2 > 2(k+1)+1$

If S_k is true, then S_{k+1} is true.

The statement is true for all n.

42. $S_n : 2^n > n^2$ for $n \geq 5$

$S_5 : 2^5 > 5^2$

$32 > 25$

$S_k : 2^k > k^2$ for $k \geq 5$

$S_{k+1} : 2^{k+1} > (k+1)^2$

Multiply both sides of $2^k > k^2$ by 2 and simplify. $2 \cdot 2^k > 2 \cdot k^2$

$$2^{k+1} > k^2 + k^2$$

But for $k \geq 5$, $k^2 > 2k+1$, so

$$2^{k+1} > k^2 + k^2 > k^2 + (2k+1)$$
$$2^{k+1} > k^2 + k^2 > k^2 + 2k + 1$$
$$2^{k+1} > k^2 + k^2 > (k+1)(k+1)$$
$$2^{k+1} > k^2 + k^2 > (k+1)^2$$

Thus, $2^{k+1} > (k+1)^2$. If S_k is true, then S_{k+1} is true. The statement is true for all n.

43. $S_1 = \frac{1}{4} = \frac{1}{4}$

$S_2 = \frac{1}{4} + \frac{1}{12} = \frac{1}{3}$

$S_3 = \frac{1}{4} + \frac{1}{12} + \frac{1}{24} = \frac{3}{8}$

$S_4 = \frac{1}{4} + \frac{1}{12} + \frac{1}{24} + \frac{1}{40} = \frac{2}{5}$

$S_5 = \frac{1}{4} + \frac{1}{12} + \frac{1}{24} + \frac{1}{40} + \frac{1}{60} = \frac{5}{12}$

$S_n = \frac{1}{4} + \frac{1}{12} + \frac{1}{24} + \cdots + \frac{1}{2n(n+1)} = \frac{n}{2n+2}$

$S_k = \frac{1}{4} + \frac{1}{12} + \frac{1}{24} + \cdots + \frac{1}{2k(k+1)} = \frac{k}{2k+2}$

$S_{k+1} = \frac{1}{4} + \frac{1}{12} + \frac{1}{24} + \cdots + \frac{1}{2k(k+1)} + \frac{1}{2(k+1)(k+2)} = \frac{k+1}{2k+4}$

Add $\frac{1}{2(k+1)(k+2)}$ to both sides of S_k:

$$\frac{1}{4} + \frac{1}{12} + \frac{1}{24} + \cdots + \frac{1}{2k(k+1)} + \frac{1}{2(k+1)(k+2)} = \frac{k}{2k+2} + \frac{1}{2(k+1)(k+2)}$$

Simplify the right-hand side:

$$\frac{k}{2k+2} + \frac{1}{2(k+1)(k+2)} = \frac{k(k+2)+1}{2(k+1)(k+2)} = \frac{k^2+2k+1}{2(k+1)(k+2)} = \frac{(k+1)^2}{2(k+1)(k+2)} = \frac{k+1}{2k+4}$$

If S_k is true, then S_{k+1} is true. The conjecture is proven.

44. $S_1 = \left(1 - \frac{1}{2}\right) = \frac{1}{2}$

$S_2 = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right) = \frac{1}{2}\left(\frac{2}{3}\right) = \frac{1}{3}$

$S_3 = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) = \frac{1}{3}\left(\frac{3}{4}\right) = \frac{1}{4}$

$S_4 = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) = \frac{1}{4}\left(\frac{4}{5}\right) = \frac{1}{5}$

$S_5 = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{6}\right) = \frac{1}{5}\left(\frac{5}{6}\right) = \frac{1}{6}$

$S_n = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\ldots\left(1 - \frac{1}{n+1}\right) = \frac{1}{n+1}$

$S_k = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\ldots\left(1 - \frac{1}{k+1}\right) = \frac{1}{k+1}$

$S_{k+1} = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\ldots\left(1 - \frac{1}{k+1}\right)\left(1 - \frac{1}{k+2}\right) = \frac{1}{k+2}$

Multiply both sides of S_k by $\left(1 - \frac{1}{k+2}\right)$:

$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\ldots\left(1 - \frac{1}{k+1}\right)\left(1 - \frac{1}{k+2}\right) = \left(\frac{1}{k+1}\right)\left(1 - \frac{1}{k+2}\right)$$

Simplify the right-hand side:

$$\left(\frac{1}{k+1}\right)\left(1-\frac{1}{k+2}\right)=\frac{1}{k+1}-\frac{1}{(k+1)(k+2)}=\frac{k+2-1}{(k+1)(k+2)}=\frac{1}{k+2}$$

If S_k is true, S_{k+1} is true. The conjecture is proven.

45. Answers may vary.

46. The exponents begin with the exponent on $a+b$ and decrease by 1 in each successive term.

47. The exponents begin with 0, increase by 1 in each successive term, and end with the exponent on $a+b$.

48. The sum of the exponents is the exponent on $a+b$.

Section 11.5

Check Point Exercises

1. a. $\binom{6}{3}=\frac{6!}{3!(6-3)!}=\frac{6!}{3!3!}=\frac{5\cdot 4}{1}=20$

b. $\binom{6}{0}=\frac{6!}{0!(6-0)!}=\frac{6!}{6!}=1$

c. $\binom{8}{2}=\frac{8!}{2!(8-2)!}=\frac{8!}{2!6!}=\frac{8\cdot 7}{2}=28$

d. $\binom{3}{3}=\frac{3!}{3!(3-3)!}=\frac{3!}{3!0!}=\frac{3!}{3!}=1$

2. $(x+1)^4=\binom{4}{0}x^4+\binom{4}{1}x^3+\binom{4}{2}x^2+\binom{4}{1}x+\binom{4}{0}=x^4+4x^3+6x^2+4x+1$

3. $(x-2y)^5=\binom{5}{0}x^5(-2y)^0+\binom{5}{1}x^4(-2y)^1+\binom{5}{2}x^3(-2y)^2+\binom{5}{3}x^2(-2y)^3+\binom{5}{4}x(-2y)^4+\binom{5}{5}x^0(-2y)^5$

$=x^5-5x^4(2y)+10x^3(4y^2)-10x^2(8y^3)+5x(16y^4)-32y^5$

$=x^5-10x^4y+40x^3y^2-80x^2y^3+80xy^4-32y^5$

4. $(2x+y)^9$

fifth term $=\binom{9}{4}(2x)^5y^4=\frac{9!}{4!5!}(32x^5)y^4=4032x^5y^4$

Exercise Set 11.5

1. $\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$

2. $\binom{7}{2} = \frac{7!}{2!5!} = \frac{7 \cdot 6}{2} = 21$

3. $\binom{12}{1} = \frac{12!}{1!11!} = 12$

4. $\binom{11}{1} = \frac{11!}{1!10!} = 11$

5. $\binom{6}{6} = \frac{6!}{0!6!} = 1$

6. $\binom{15}{2} = \frac{15!}{2!13!} = \frac{15 \cdot 14}{2} = 105$

7. $\binom{100}{2} = \frac{100!}{2!98!} = \frac{100 \cdot 99}{2} = 4950$

8. $\binom{100}{98} = \frac{100!}{2!98!} = \frac{100 \cdot 99}{2} = 4950$

9. $(x+2)^3 = \binom{3}{0}x^3 + \binom{3}{1}2x^2 + \binom{3}{2}4x + \binom{3}{3}8 = x^3 + 3x^2 \cdot 2 + 3x \cdot 4 + 8 = x^3 + 6x^2 + 12x + 8$

10. $(x+4)^3 = \binom{3}{0}x^3 + \binom{3}{1}x^2(4) + \binom{3}{2}x(4)^2 + \binom{3}{3}(4)^3 = x^3 + 12x^2 + 48x + 64$

11. $(3x+y)^3 = \binom{3}{0}27x^3 + \binom{3}{1}9x^2y + \binom{3}{2}3xy^2 + \binom{3}{3}y^3 = 27x^3 + 27x^2y + 9xy^2 + y^3$

12. $(x+3y)^3 = \binom{3}{0}x^3 + \binom{3}{1}3x^2y + \binom{3}{2}9xy^2 + \binom{3}{3}27y^3 = x^3 + 9x^2y + 27xy^2 + 27y^3$

13. $(5x-1)^3 = \binom{3}{0}125x^3 - \binom{3}{1}25x^2 + \binom{3}{2}5x - \binom{3}{3} = 125x^3 - 75x^2 + 15x - 1$

14. $(4x-1)^3 = \binom{3}{0}(4x)^3 + \binom{3}{1}(4x)^2(-1) + \binom{3}{2}(4x)(-1)^2 + \binom{3}{3}(-1)^3 = 64x^3 - 48x^2 + 12x - 1$

15. $(2x+1)^4 = \binom{4}{0}16x^4 - \binom{4}{1}8x^3 + \binom{4}{2}4x^2 + \binom{4}{3}2x + \binom{4}{4} = 16x^4 + 32x^3 + 24x^2 + 8x + 1$

16. $(3x+1)^4 = \binom{4}{0}81x^4 + \binom{4}{1}27x^3 + \binom{4}{2}9x^2 + \binom{4}{3}3x + \binom{4}{4} = 81x^4 + 108x^3 + 54x^2 + 12x + 1$

17. $(x^2+2y)^4 = \binom{4}{0}(x^2)^4 + \binom{4}{1}(x^2)^3(2y) + \binom{4}{2}(x^2)^2(2y)^2 + \binom{4}{3}(x^2)^1(2y)^3 + \binom{4}{4}(2y)^4$

$= 1(x^8) + 4(x^6)(2y) + 6(x^4)(4y^2) + 4x^2(8y^3) + 1(16y^4)$

$= x^8 + 8x^6y + 24x^4y^2 + 32x^2y^3 + 16y^4$

18. $(x^2+y)^4 = \binom{4}{0}x^8 + \binom{4}{1}x^6y + \binom{4}{2}x^4y^2 + \binom{4}{3}x^2y^3 + \binom{4}{4}y^4 = x^8 + 4x^6y + 6x^4y^2 + 4x^2y^3 + y^4$

19. $(y-3)^4 = \binom{4}{0}y^4 + \binom{4}{1}y^3(-3) + \binom{4}{2}y^2(-3)^2 + \binom{4}{3}y(-3)^3 + \binom{4}{4}(-3)^4$

$= y^4 + 4(y^3)(-3) + 6(y^2)(9) + 4(y)(-27) + 81$

$= y^4 - 12y^3 + 54y^2 - 108y + 81$

20. $(y-4)^4 = \binom{4}{0}y^4 + \binom{4}{1}y^3(-4) + \binom{4}{2}y^2(-4)^2 + \binom{4}{3}y(-4)^3 + \binom{4}{4}(-4)^4$

$= y^4 + 4(y^3)(-4) + 6(y^2)(16) + 4(y)(-64) + 256$

$= y^4 - 16y^3 + 96y^2 - 256y + 256$

21. $(2x^3-1)^4 = \binom{4}{0}(2x^3)^4 + \binom{4}{1}(2x^3)^3(-1) + \binom{4}{2}(2x^3)^2(-1)^2 + \binom{4}{3}(2x^3)(-1)^3 + \binom{4}{4}(-1)^4$

$= 16x^{12} - 4(8x^9) + 6(4x^6) - 4(2x^3) + 1$

$= 16x^{12} - 32x^9 + 24x^6 - 8x^3 + 1$

22. $(2x^5-1)^4 = \binom{4}{0}(2x^5)^4 + \binom{4}{1}(2x^5)^3(-1) + \binom{4}{2}(2x^5)^2(-1)^2 + \binom{4}{3}(2x^5)(-1)^3 + \binom{4}{4}(-1)^4$

$= 16x^{20} - 4(8x^{15}) + 6(4x^{10}) - 4(2x^5) + 1$

$= 16x^{20} - 32x^{15} + 24x^{10} - 8x^5 + 1$

23. $(c+2)^5 = \binom{5}{0}c^5 + \binom{5}{1}c^4(2) + \binom{5}{2}c^3(2^2) + \binom{5}{3}c^2(2^3) + \binom{5}{4}c(2^4) + \binom{5}{5}(2^5)$

$= c^5 + 5c^4(2) + 10c^3(4) + 10c^2(8) + 5c(16) + 32$

$= c^5 + 10c^4 + 40c^3 + 80c^2 + 80c + 32$

24. $(c+3)^5 = \binom{5}{0}c^5 + \binom{5}{1}3c^4 + \binom{5}{2}3^2c^3 + \binom{5}{3}3^3c^2 + \binom{5}{4}3^4c + \binom{5}{5}3^5 = c^5 + 15c^4 + 90c^3 + 270c^2 + 405c + 243$

25. $(x-1)^5 = \binom{5}{0}x^5 - \binom{5}{1}x^4 + \binom{5}{2}x^3 - \binom{5}{3}x^2 + \binom{5}{4}x - \binom{5}{5} = x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$

26. $(x-2)^5 = \binom{5}{0}x^5 + \binom{5}{1}x^4(-2) + \binom{5}{2}x^3(-2)^2 + \binom{5}{3}x^2(-2)^3 + \binom{5}{4}x(-2)^4 + \binom{5}{5}(-2)^5$

$= x^5 + 5x^4(-2) + 10x^3(4) + 10x^2(-8) + 5x(16) + (-32)$

$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

27. $(3x-y)^5 = \binom{5}{0}(3x)^5 - \binom{5}{1}(3x)^4 y + \binom{5}{2}(3x)^3 y^2 - \binom{5}{3}(3x)^2 y^3 + \binom{5}{4}3xy^4 - \binom{5}{5}y^5$

$= (1)243x^5 - 5(81x^4)y + 10(27x^3)y^2 - 10(9x^2)y^3 + 5(3x)y^4 - (1)y^5$

$= 243x^5 - 405x^4 y + 270x^3 y^2 - 90x^2 y^3 + 15xy^4 - y^5$

28. $(x-3y)^5 = \binom{5}{0}x^5 - \binom{5}{1}x^4(3y) + \binom{5}{2}x^3(3y)^2 - \binom{5}{3}x^2(3y)^3 + \binom{5}{4}x(3y)^4 - \binom{5}{5}(3y)^5$

$= x^5 - 15x^4 y + 90x^3 y^2 - 270x^2 y^3 + 405xy^4 - 243y^5$

29. $(2a+b)^6 = \binom{6}{0}(2a)^6 + \binom{6}{1}(2a)^5 b + \binom{6}{2}(2a)^4 b^2 + \binom{6}{3}(2a)^3 b^3 + \binom{6}{4}(2a)^2 b^4 + \binom{6}{5}(2a)b^5 + \binom{6}{6}b^6$

$= 64a^6 + 6(32a^5)b + 15(16a^4)b^2 + 20(8a^3)b^3 + 15(4a^2)b^4 + 6(2a)b^5 + b^6$

$= 64a^6 + 192a^5b + 240a^4b^2 + 160a^3b^3 + 60a^2b^4 + 12ab^5 + b^6$

30. $(a+2b)^6 = \binom{6}{0}a^6 + \binom{6}{1}a^5(2b) + \binom{6}{2}a^4(2b)^2 + \binom{6}{3}a^3(2b)^3 + \binom{6}{4}a^2(2b)^4 + \binom{6}{5}a(2b)^5 + \binom{6}{6}(2b)^6$

$= a^6 + 6a^5(2b) + 15a^4(4b^2) + 20a^3(8b^3) + 15a^2(16b^4) + 6a(32b^5) + 64b^6$

$= a^6 + 12a^5b + 60a^4b^2 + 160a^3b^3 + 240a^2b^4 + 192ab^5 + 64b^6$

31. $(x+2)^8 = \binom{8}{0}x^8 + \binom{8}{1}x^7 2 + \binom{8}{3}x^6(2)^2 + \cdots$

$= x^8 + 16x^7 + 112x^6 + \cdots$

32. $(x+3)^8 = \binom{8}{0}x^8 + \binom{8}{1}3x^7 + \binom{8}{2}3^2 y^7 + \cdots$

$= x^8 + 24x^7 + 252x^6 + \cdots$

33. $(x-2y)^{10} = \binom{10}{0}x^{10} - \binom{10}{1}x^9(2y) + \binom{10}{2}x^8(2y)^2 - \cdots$

$= x^{10} - 20x^9 y + 180x^8 y^2 - \cdots$

34. $(x-2y)^8 = \binom{9}{0}x^9 - \binom{9}{1}x^8(2y) + \binom{9}{2}x^7(2y)^2 + \cdots$

$= x^9 - 18x^8 y + 144x^7 y^2 - \cdots$

35. $(x^2+1)^{16} = \binom{16}{0}(x^2)^{16} + \binom{16}{1}(x^2)^{15} + \binom{16}{2}(x^2)^{14} + \cdots$

$= x^{32} + 16x^{30} + 120x^{28} + \cdots$

36. $(x^2+1)^{17} = \binom{17}{0}\left(x^2\right)^{17} + \binom{17}{1}\left(x^2\right)^{16} + \binom{17}{2}\left(x^2\right)^{15} + \cdots$
$= x^{34} + 17x^{32} + 136x^{30} + \cdots$

37. $(y^3-1)^{20} = \binom{20}{0}\left(y^3\right)^{20} - \binom{20}{1}(y^3)^{19} + \binom{20}{2}(y^3)^{18} - \cdots$
$= y^{60} - 20y^{57} + 190y^{54} - \cdots$

38. $(y^3-1)^{21} = \binom{21}{0}\left(y^3\right)^{21} - \binom{21}{1}\left(y^3\right)^{20} + \binom{21}{2}\left(y^3\right)^{19} + \cdots$
$= y^{63} - 21x^{60} + 210y^{57} + \cdots$

39. $(2x+y)^6$; third term $= \binom{6}{2}(2x)^4(y)^2 = 15\left(16x^4y^2\right) = 240x^4y^2$

40. $\left(x+2y\right)^6$; third term $= \binom{6}{2}x^4\left(2y\right)^2 = 15x^4 \cdot 4y^2 = 60x^4y^2$

41. $\left(x-1\right)^9$; fifth term $= \binom{9}{4}x^5\left(-1\right)^4 = 126x^5$

42. $\left(x-1\right)^{10}$; fifth term $= \binom{10}{4}x^6\left(-1\right)^4 = 210x^6$

43. $\left(x^2+y^3\right)^8$; sixth term $= \binom{8}{5}\left(x^2\right)^3\left(y^3\right)^5 = 56x^6y^{15}$

44. $\left(x^3+y^2\right)^8$; sixth term $= \binom{8}{5}\left(x^3\right)^3\left(y^2\right)^5 = 56x^9y^{10}$

45. $\left(x-\frac{1}{2}\right)^9$; fourth term $= \binom{9}{3}x^6\left(-\frac{1}{2}\right)^3 = 84x^6\left(-\frac{1}{8}\right) = -\frac{21}{2}x^6$

46. $\left(x+\frac{1}{2}\right)^8$; fourth term $= \binom{8}{3}x^5\left(\frac{1}{2}\right)^3 = 56x^5 \cdot \frac{1}{8} = 7x^5$

47. $\binom{22}{14}(x^2)^8y^{14} = 319,770x^{16}y^{14}$

48. $\binom{10}{6}x^4(2y)^6 = 13,440x^4y^6$

49.

$$\left(x^3+x^{-2}\right)^4=\binom{4}{0}\left(x^3\right)^4+\binom{4}{1}\left(x^3\right)^3\left(x^{-2}\right)+\binom{4}{2}\left(x^3\right)^2\left(x^{-2}\right)^2+\binom{4}{3}\left(x^3\right)^1\left(x^{-2}\right)^3+\binom{4}{4}\left(x^{-2}\right)^4$$

$$=\frac{4!}{0!(4-0)!}x^{12}+\frac{4!}{1!(4-1)!}x^9x^{-2}+\frac{4!}{2!(4-2)!}x^6x^{-4}+\frac{4!}{3!(4-3)!}x^3x^{-6}+\frac{4!}{4!(4-4)!}x^{-8}$$

$$=\frac{\not{4}!}{0!\cdot\not{4}!}x^{12}+\frac{4\cdot\not{3}!}{1!\cdot\not{3}!}x^7+\frac{4\cdot3\cdot\not{2}!}{2\cdot1\cdot\not{2}!}x^2+\frac{4\cdot\not{3}!}{\not{3}!\cdot1!}x^{-3}+\frac{\not{4}!}{\not{4}!\cdot0!}x^{-8}$$

$$=x^{12}+4x^7+6x^2+\frac{4}{x^3}+\frac{1}{x^8}$$

50.

$$\left(x^2+x^{-3}\right)^4=\binom{4}{0}\left(x^2\right)^4+\binom{4}{1}\left(x^2\right)^3\left(x^{-3}\right)+\binom{4}{2}\left(x^2\right)^2\left(x^{-3}\right)^2+\binom{4}{3}\left(x^2\right)^1\left(x^{-3}\right)^3+\binom{4}{4}\left(x^{-3}\right)^4$$

$$=\frac{4!}{0!(4-0)!}x^8+\frac{4!}{1!(4-1)!}x^6x^{-3}+\frac{4!}{2!(4-2)!}x^4x^{-6}+\frac{4!}{3!(4-3)!}x^2x^{-9}+\frac{4!}{4!(4-4)!}x^{-12}$$

$$=\frac{\not{4}!}{0!\cdot\not{4}!}x^8+\frac{4\cdot\not{3}!}{1!\cdot\not{3}!}x^3+\frac{4\cdot3\cdot\not{2}!}{2\cdot1\cdot\not{2}!}x^{-2}+\frac{4\cdot\not{3}!}{\not{3}!\cdot1!}x^{-7}+\frac{\not{4}!}{\not{4}!\cdot0!}x^{-12}$$

$$=x^8+4x^3+\frac{6}{x^2}+\frac{4}{x^7}+\frac{1}{x^{12}}$$

51.

$$\left(x^{\frac{1}{3}}-x^{-\frac{1}{3}}\right)^3=\left(x^{\frac{1}{3}}+\left(-x^{-\frac{1}{3}}\right)\right)^3=\binom{3}{0}\left(x^{\frac{1}{3}}\right)^3+\binom{3}{1}\left(x^{\frac{1}{3}}\right)^2\left(-x^{-\frac{1}{3}}\right)+$$

$$+\binom{3}{2}\left(x^{\frac{1}{3}}\right)^1\left(-x^{-\frac{1}{3}}\right)^2+\binom{3}{3}\left(-x^{-\frac{1}{3}}\right)^3$$

$$=\frac{3!}{0!(3-0)!}x^1+\frac{3!}{1!(3-1)!}x^{\frac{2}{3}}\cdot-x^{-\frac{1}{3}}+\frac{3!}{2!(3-2)!}x^{\frac{1}{3}}x^{-\frac{2}{3}}+\frac{3!}{3!(3-3)!}\cdot-x^{-1}$$

$$=\frac{\not{3}!}{0!\cdot\not{3}!}x+\frac{3\cdot\not{2}!}{1!\cdot\not{2}!}\cdot-x^{\frac{1}{3}}+\frac{\cdot3\cdot\not{2}!}{\not{2}!\cdot1!}x^{-\frac{1}{3}}+\frac{\not{3}!}{\not{3}!\cdot0!}\cdot-x^{-1}$$

$$=x-3x^{\frac{1}{3}}+\frac{3}{x^{\frac{1}{3}}}-\frac{1}{x}$$

52.

$$\left(x^{\frac{2}{3}}-\frac{1}{\sqrt[3]{x}}\right)^3=\left(x^{\frac{2}{3}}+\left(-\frac{1}{x^{\frac{1}{3}}}\right)\right)^3=\left(x^{\frac{2}{3}}+\left(-x^{-\frac{1}{3}}\right)\right)^3=$$

$$=\left(x^{\frac{2}{3}}+\left(-x^{-\frac{1}{3}}\right)\right)^3=\binom{3}{0}\left(x^{\frac{2}{3}}\right)^3+\binom{3}{1}\left(x^{\frac{2}{3}}\right)^2\left(-x^{-\frac{1}{3}}\right)+$$

$$+\binom{3}{2}\left(x^{\frac{2}{3}}\right)^1\left(-x^{-\frac{1}{3}}\right)^2+\binom{3}{3}\left(-x^{-\frac{1}{3}}\right)^3$$

$$=\frac{3!}{0!(3-0)!}x^2+\frac{3!}{1!(3-1)!}x^{\frac{4}{3}}\cdot -x^{-\frac{1}{3}}+\frac{3!}{2!(3-2)!}x^{\frac{2}{3}}x^{-\frac{2}{3}}+\frac{3!}{3!(3-3)!}\cdot -x^{-1}$$

$$=\frac{\cancel{3!}}{0!\cdot\cancel{3!}}x^2+\frac{3\cdot\cancel{2!}}{1!\cdot\cancel{2!}}\cdot -x^1+\frac{\cdot 3\cdot\cancel{2!}}{\cancel{2!}\cdot 1!}x^0+\frac{\cancel{3!}}{\cancel{3!}\cdot 0!}\cdot -x^{-1}$$

$$=x^2-3x+3-\frac{1}{x}$$

53.

$f(x)=x^4+7$;

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{(x+h)^4+7-\left(x^4+7\right)}{h}$$

$$=\frac{\binom{4}{0}x^4+\binom{4}{1}x^3h+\binom{4}{2}x^2h^2+\binom{4}{3}xh^3+\binom{4}{4}h^4+7-x^4-7}{h}$$

$$=\frac{\frac{4!}{0!(4-0)!}x^4+\frac{4!}{1!(4-1)!}x^3h+\frac{4!}{2!(4-2)!}x^2h^2+\frac{4!}{3!(4-3)!}xh^3+\frac{4!}{4!(4-4)!}h^4-x^4}{h}$$

$$=\frac{\frac{\cancel{4!}}{0!\cancel{4!}}x^4+\frac{4\cdot\cancel{3!}}{1!\cancel{3!}}x^3h+\frac{4\cdot 3\cdot\cancel{2!}}{\cancel{2!}\cdot 2\cdot 1}x^2h^2+\frac{4\cdot\cancel{3!}}{\cancel{3!}1!}xh^3+\frac{\cancel{4!}}{\cancel{4!}0!}h^4-x^4}{h}$$

$$=\frac{\cancel{x^4}+4x^3h+6x^2h^2+4xh^3+h^4-\cancel{x^4}}{h}$$

$$=\frac{h(4x^3+6x^2h+4xh^2+h^3)}{h}$$

$$=4x^3+6x^2h+4xh^2+h^3$$

54. $f(x) = x^5 + 8$

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{(x+h)^5 + 8 - (x^5 - 8)}{h}$$

$$= \frac{\binom{5}{0}x^5 + \binom{5}{1}x^4h + \binom{5}{2}x^3h^2 + \binom{5}{3}x^2h^3 + \binom{5}{4}xh^4 + \binom{5}{5}h^5 + 8 - x^5 - 8}{h}$$

$$= \frac{\frac{5!}{0!(5-0)!}x^5 + \frac{5!}{1!(5-1)!}x^4h + \frac{5!}{2!(5-2)!}x^3h^2 + \frac{5!}{3!(5-3)!}x^2h^3 + \frac{5!}{4!(5-4)!}xh^4 + \frac{5!}{5!(5-5)!}h^5 - x^5}{h}$$

$$= \frac{\frac{\cancel{5!}}{0!\cancel{5!}}x^5 + \frac{5 \cdot \cancel{4!}}{1!\cancel{4!}}x^4h + \frac{5 \cdot 4 \cdot \cancel{3!}}{2!\cancel{3!}}x^3h^2 + \frac{5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}2!}x^2h^3 + \frac{5 \cdot \cancel{4!}}{\cancel{4!}1!}xh^4 + \frac{\cancel{5!}}{\cancel{5!}0!}h^5 - x^5}{h}$$

$$= \frac{x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5 - x^5}{h}$$

$$= \frac{5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5}{h}$$

$$= \frac{h(5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4)}{h}$$

$$= 5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4$$

55. Find the $(5+1) = 6^{th}$ term.

$$\binom{n}{r}a^{n-r}b^r = \binom{10}{5}\left(\frac{3}{x}\right)^{10-5}\left(\frac{x}{3}\right)^5 = \frac{10!}{5!(10-5)!}\left(\frac{3}{x}\right)^5\left(\frac{x}{3}\right)^5$$

$$= \frac{\cancel{10} \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!} \cdot \cancel{5} \cdot 4 \cdot 3 \cdot \cancel{2} \cdot 1}\left(\frac{3}{x}\right)^5\left(\frac{x}{3}\right)^5 = 252 \cdot \frac{3^5}{x^5} \cdot \frac{x^5}{3^5} = 252$$

56. Find the $(6+1) = 7^{th}$ term.

$$\binom{n}{r}a^{n-r}b^r = \binom{12}{6}\left(\frac{1}{x}\right)^{12-6}\left(-x^2\right)^6 = \frac{12!}{6!(12-6)!}\left(\frac{1}{x}\right)^6\left(-x^2\right)^6$$

$$= \frac{\cancel{12} \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot \cancel{6} \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot 1}\left(\frac{1}{x}\right)^6\left(-x^2\right)^6 = 924 \cdot \frac{1}{x^6} \cdot x^{12} = 924x^{12-6} = 924x^6$$

57. $(0.28 + 0.72)^5$

Third Term $(r = 2)$: $\binom{n}{r}a^{n-r}b^r = \binom{5}{2}0.28^{5-2}0.72^2 = \frac{5!}{2!(5-2)!}0.28^{5-2}0.72^2 = \frac{5!}{2!3!}0.28^3 0.72^2 \approx 0.1138$

58. $(0.12 + 0.88)^5$

Third Term $(r = 2)$: $\binom{n}{r}a^{n-r}b^r = \binom{5}{2}0.12^{5-2}0.88^2 = \frac{5!}{2!(5-2)!}0.12^{5-2}0.88^2 = \frac{5!}{2!3!}0.12^3 0.88^2 \approx 0.0134$

59. – 67. Answers may vary.

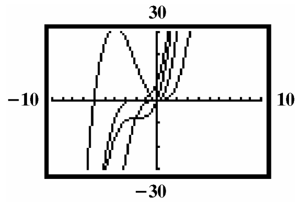
68. $f_1(x) = (x+2)^3$

$f_2(x) = x^3$

$f_3(x) = x^3 + 6x^2$

$f_4(x) = x^3 + 6x^2 + 12x$

$f_5(x) = x^3 + 6x^2 + 12x + 8$



f_2, f_3, and f_4 are approaching $f_1 = f_5$.

69. $f_1(x) = (x+1)^4$

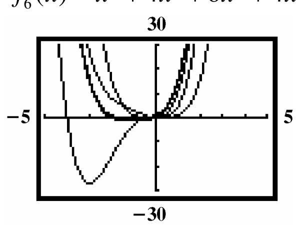
$f_2(x) = x^4$

$f_3(x) = x^4 + 4x^3$

$f_4(x) = x^4 + 4x^3 + 6x^2$

$f_5(x) = x^4 + 4x^3 + 6x^2 + 4x$

$f_6(x) = x^4 + 4x^3 + 6x^2 + 4x + 1$



f_2, f_3, f_4, and f_5 are approaching $f_1 = f_6$.

70. $f_1(x) = (x-1)^3 = \binom{3}{0}x^3 + \binom{3}{1}x^2(-1) + \binom{3}{2}x(-1)^2 + \binom{3}{3}(-1)^3 = x^3 + 3x^2(-1) + 3x(1) + (-1)$

$= x^3 - 3x^2 + 3x - 1$

71. $f_1(x) = (x-2)^4$

$= \binom{4}{0}x^4 + \binom{4}{1}x^3(-2) + \binom{4}{2}x^2(-2)^2 + \binom{4}{3}x(-2)^3 + \binom{4}{4}(-2)^4$

$= x^4 + 4x^3(-2) + 6x^2(4) + 4x(-8) + 16$

$= x^4 - 8x^3 + 24x^2 - 32x + 16$

72. $f_1(x) = (x+2)^6$

$= \binom{6}{0}x^6 + \binom{6}{1}x^5(2) + \binom{6}{2}x^4(2)^2 + \binom{6}{3}x^3(2)^3 + \binom{6}{4}x^2(2)^4 + \binom{6}{5}x(2)^5 + \binom{6}{6}2^6$

$= x^6 + 6x^5(2) + 15x^4(4) + 20x^3(8) + 15x^2(16) + 6x(32) + 64$

$= x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$

73. makes sense

74. makes sense

75. does not make sense; Explanations will vary. Sample explanation: $\binom{n}{0}$ and $\binom{n}{1}$ are the coefficients of the first and second term.

76. does not make sense; Explanations will vary. Sample explanation: $7 \neq 2+4$

77. false; Changes to make the statement true will vary. A sample change is: The binomial expansion for $(a+b)^n$ contains $n+1$ terms.

78. true

79. false; Changes to make the statement true will vary. A sample change is: The sum of the binomial coefficients in $(a+b)^n$ is 2^n.

80. false; Changes to make the statement true will vary. A sample change is: There are values of *a* and *b* for which $(a+b)^4 = a^4 + b^4$. Consider $a = 0$ and $b = 1$.

$$\begin{aligned}(0+1)^4 &= 0^4 + 1^4 \\ (1)^4 &= 0+1 \\ 1 &= 1\end{aligned}$$

81.

$$\begin{aligned}(x^2+x+1)^3 &= \left[x^2+(x+1)\right]^3 \\ &= \binom{3}{0}(x^2)^3 + \binom{3}{1}(x^2)^2(x+1) + \binom{3}{2}x^2(x+1)^2 + \binom{3}{3}(x+1)^3 \\ &= x^6 + 3x^4(x+1) + 3x^2(x^2+2x+1) + x^3 + 3x^2 + 3x + 1 \\ &= x^6 + 3x^5 + 3x^4 + 3x^4 + 6x^3 + 3x^2 + x^3 + 3x^2 + 3x + 1 \\ &= x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1\end{aligned}$$

82. $(x^2+y^2)^5$

Since $(x^2)^2 = x^4$, the fourth term will contain x^4: fourth term $= \binom{5}{3}(x^2)^2(y^2)^3 = 10x^4y^6$

83. $$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!r!} = \frac{n!}{(n-r)!\left[n-(n-r)\right]!} = \binom{n}{n-r}$$

84. $\binom{n}{r}+\binom{n}{r+1}=\frac{n!}{r!(n-r)!}+\frac{n!}{(r+1)!(n-(r+1))!}$

$$=\frac{n!}{r!(n-r)!}+\frac{n!}{(r+1)!(n-r-1)!}$$

$$=\frac{n!(r+1)}{(r+1)r!(n-r)!(n-r-1)!}+\frac{(n-r)n!}{(n-r)(r+1)!(n-r-1)!}$$

$$=\frac{n!(r+1)+n!(n-r)}{(r+1)!(n-r)!}$$

$$=\frac{n!(r+1+n-r)}{(r+1)!(n-r)!}$$

$$=\frac{n!(n+1)}{(r+1)!(n-r)!}$$

$$=\frac{(n+1)!}{(r+1)!(n+1-r-1)!}$$

$$=\frac{(n+1)!}{(r+1)!(n+1-(r+1))!}$$

$$=\binom{n+1}{r+1}$$

85. **a.** $S_1:(a+b)^1=\binom{1}{0}a^1+\binom{1}{1}a^{1-1}b=a+b$

b. $S_k:(a+b)^k=\binom{k}{0}a^k+\binom{k}{1}a^{k-1}b+\binom{k}{2}a^{k-2}b^2+\cdots+\binom{k}{k-1}ab^{k-1}+\binom{k}{k}b^k$

$S_{k+1}:(a+b)^{k+1}=\binom{k+1}{0}a^{k+1}+\binom{k+1}{1}a^kb+\binom{k+1}{2}a^{k-1}b^2+\cdots+\binom{k+1}{k}ab^k+\binom{k+1}{k+1}b^{k+1}$

c. $(a+b)(a+b)^k$

$$(a+b)^{k+1}=\binom{k}{0}a^{k+1}+\binom{k}{0}a^kb+\binom{k}{1}a^kb+\binom{k}{1}a^{k-1}b^2+\binom{k}{2}a^{k-1}b^2+\binom{k}{2}a^{k-2}b^3+\cdots$$
$$=\binom{k}{k-1}a^2b^{k-1}+\binom{k}{k-1}ab^k+\binom{k}{k}ab^k+\binom{k}{k}b^{k+1}$$

d. $(a+b)^{k+1}=\binom{k}{0}a^{k+1}+\left[\binom{k}{0}+\binom{k}{1}\right]a^kb+\left[\binom{k}{1}+\binom{k}{2}\right]a^{k-1}b^2+\left[\binom{k}{2}+\binom{k}{3}\right]a^{k-2}b^3+\cdots$
$$+\left[\binom{k}{k-1}+\binom{k}{k}\right]ab^k+\binom{k}{k}b^{k+1}$$

e. $(a+b)^{k+1}=\binom{k}{0}a^{k+1}+\binom{k+1}{1}a^kb+\binom{k+1}{2}a^{k-1}b^2+\binom{k+1}{3}a^{k-2}b^3+\cdots+\binom{k+1}{k}ab^k+\binom{k}{k}b^{k+1}$

f. $\binom{k}{0} = \binom{k+1}{0}$ because both equal 1. $\binom{k}{k} = \binom{k+1}{k+1}$ also because both equal 1.

$$S_{k+1} : (a+b)^{k+1} = \binom{k+1}{0}a^{k+1} + \binom{k+1}{1}a^k b + \binom{k+1}{2}a^{k-1}b^2 + \cdots + \binom{k+1}{k}ab^k + \binom{k+1}{k+1}b^{k+1}$$

86. $\dfrac{n!}{(n-r)!} = \dfrac{20!}{(20-3)!} = \dfrac{20!}{17!} = \dfrac{20 \cdot 19 \cdot 18 \cdot 17!}{17!} = 20 \cdot 19 \cdot 18 = 6840$

87. $\dfrac{n!}{(n-r)!r!} = \dfrac{8!}{(8-3)!3!} = \dfrac{8!}{5!3!} = \dfrac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2 \cdot 1 \cdot 5!} = \dfrac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$

88. true

Section 11.6

Check Point Exercises

1. We use the Fundamental Counting Principal to find the number of ways a one-topping pizza can be ordered.

<u>Size</u>: <u>Crust</u>: <u>Topping</u>:

3 × 4 × 6 = 72

There are 72 different ways of ordering a one-topping pizza.

2. We use the Fundamental Counting Principal to find the number of ways we can answer the questions.

<u>Question #1</u>:		<u>Question #2</u>:		<u>Question #3</u>:		<u>Question #4</u>:		<u>Question #5</u>:		<u>Question #6</u>:		
3	×	3	×	3	×	3	×	3	×	3	=	3^6 = 729

There are 729 ways of answering the questions.

3. We use the Fundamental Counting Principal to find the number of different license plates that can be manufactured. Multiply the number of different letters, 26, for the first two places and the number of different digits, 10, for the next three places. $26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 26^2 \cdot 1000 = 676{,}000$ plates There are 676,000 different license plates possible.

4. Your group is choosing $r = 4$ officers from a group of $n = 7$ people. The order in which the officers are chosen matters because the four officers to be chosen have different responsibilities. Thus, we are looking for the number of permutations of 7 things taken 4 at a time.

We use the formula ${}_nP_r = \dfrac{n!}{(n-r)!}$ with $n = 7$ and $r = 4$. ${}_7P_4 = \dfrac{7!}{(7-4)!} = \dfrac{7!}{3!} = 840.$

Thus, there are 840 different ways of filling the four offices.

5. Because you are using all six of your books in every possible arrangement, you are arranging $r = 6$ books from a group of $n = 6$ books. Thus, we are looking for the number of permutations of 6 things taken 6 at a time. We use the formula

${}_nP_r = \dfrac{n!}{(n-r)!}$ with $n = 6$ and $r = 6$.

${}_6P_6 = \dfrac{6!}{(6-6)!} = \dfrac{6!}{0!} = 6! = 720.$

There are 720 different possible permutations. Thus, you can arrange the books in 720 ways.

6. **a.** The order does not matter; this is a combination.

b. Since what place each runner finishes matters, this is a permutation.

7. The order in which the four people are selected does not matter. This is a problem of selecting $r = 4$ people from a group of $n = 10$ people. We are looking for the number of combinations of 10 things taken 4 at a time. We use the formula $_nC_r = \frac{n!}{(n-r)!\ r!}$ with $n = 10$ and $r = 4$.

$$_{10}C_4 = \frac{10!}{(10-4)!4!} = \frac{10!}{6!4!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6!}{6!\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1} = 210$$

Thus, 210 committees of 4 people each can be found from 10 people at the conference on acupuncture.

8. Because the order in which the 4 cards are dealt does not matter, this is a problem involving combinations. We are looking for the number of combinations of $n = 16$ cards drawn $r = 4$ at a time. We use the formula $_nC_r = \frac{n!}{(n-r)!\ r!}$ with $n = 16$ and $r = 4$.

$$_{16}C_4 = \frac{16!}{(16-4)!4!} = \frac{16!}{12!4!} = \frac{16\cdot 15\cdot 14\cdot 13\cdot 12!}{12!\cdot 4\cdot 3\cdot 2\cdot 1} = 1820$$

Thus, there are 1820 different 4-card hands possible.

Exercise Set 11.6

1. $_9P_4 = \frac{9!}{5!} = 3024$

2. $_7P_3 = \frac{7!}{4!} = 210$

3. $_8P_5 = \frac{8!}{3!} = 8\cdot 7\cdot 6\cdot 5\cdot 4 = 6720$

4. $_{10}P_4 = \frac{10!}{6!} = 10\cdot 9\cdot 8\cdot 7 = 5040$

5. $_6P_6 = \frac{6!}{0!} = 720$

6. $_9P_9 = \frac{9!}{0!} = 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 362{,}880$

7. $_8P_0 = \frac{8!}{8!} = 1$

8. $_6P_0 = \frac{6!}{6!} = 1$

9. $_9C_5 = \frac{9!}{4!5!} = \frac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2\cdot 1} = \frac{3\cdot 7\cdot 6}{1} = 126$

10. $_{10}C_6 = \frac{10!}{6!4!} = 210$

11. $_{11}C_4 = \frac{11!}{7!4!} = \frac{11\cdot 10\cdot 9\cdot 8}{4\cdot 3\cdot 2\cdot 1} = \frac{11\cdot 10\cdot 3}{1} = 330$

12. $_{12}C_5 = \frac{12!}{7!5!} = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8}{5\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{11\cdot 9\cdot 8}{1} = 792$

13. $_7C_7 = \frac{7!}{0!7!} = 1$

14. $_4C_4 = \frac{4!}{0!4!} = 1$

15. $_5C_0 = \frac{5!}{5!0!} = 1$

16. $_6C_0 = \frac{6!}{6!0!} = 1$

17. combinations; The order in which the volunteers are chosen does not matter.

18. permutations; The order in which the prizes are awarded matters since they are different amounts.

19. permutations; The order of the letters matters because ABCD is not the same as BADC.

20. combinations; The order in which the prizes are awarded does not matter since all are the same amount.

21. $\frac{_7P_3}{3!} - {_7C_3} = \frac{\frac{7!}{(7-3)!}}{3!} - \frac{7!}{(7-3)!3!} = \frac{\frac{7!}{4!}}{3!} - \frac{7!}{4!3!} = \frac{7!}{4!3!} - \frac{7!}{4!3!} = 0$

22. $\frac{_{20}P_2}{2!} - {_{20}C_2} = \frac{\frac{20!}{(20-2)!}}{2!} - \frac{20!}{(20-2)!2!} = \frac{20!}{(20-2)!2!} - \frac{20!}{(20-2)!2!} = 0$

23. $1 - \frac{_3P_2}{_4P_3} = 1 - \frac{\frac{3!}{(3-2)!}}{\frac{4!}{(4-3)!}} = \frac{\frac{3!}{1!}}{\frac{4!}{1!}} = 1 - \frac{3!}{4!} = 1 - \frac{3!}{4\cdot 3!} = 1 - \frac{1}{4} = \frac{3}{4}$

24. $1 - \frac{_5P_3}{_{10}P_4} = 1 - \frac{\frac{5!}{(5-3)!}}{\frac{10!}{(10-4)!}} = \frac{\frac{5!}{2!}}{\frac{10!}{6!}} = 1 - \frac{5!}{2!}\cdot\frac{6!}{10!} = 1 - \frac{5\cdot 4\cdot 3\cdot \cancel{2!}\cdot \cancel{6!}}{\cancel{2!}\cdot 10\cdot 9\cdot 8\cdot 7\cdot \cancel{6!}} = 1 - \frac{60}{5040} = \frac{83}{84}$

25. $\frac{_7C_3}{_5C_4} - \frac{98!}{96!} = \frac{\frac{7!}{(7-3)!3!}}{\frac{5!}{(5-4)!4!}} - \frac{98\cdot 97\cdot \cancel{96!}}{\cancel{96!}} = \frac{\frac{7!}{4!3!}}{\frac{5!}{1!4!}} - 95067 = \frac{\frac{7\cdot 6\cdot 5\cdot \cancel{4!}}{\cancel{4!}3\cdot 2\cdot 1}}{\frac{5\cdot \cancel{4!}}{1!\cancel{4!}}} - 9506 = \frac{35}{5} - 9506 = 7 - 9506 = -9499$

26. $\frac{{}_{10}C_3}{{}_6C_4} - \frac{46!}{44!} = \frac{\frac{10!}{(10-3)!3!}}{\frac{6!}{(6-4)!4!}} - \frac{46\cdot 45\cdot 44!}{44!} = \frac{\frac{10!}{7!3!}}{\frac{6!}{2!4!}} - 46\cdot 45 = \frac{\frac{10\cdot 9\cdot 8\cdot 7!}{7!3!}}{\frac{6\cdot 5\cdot 4!}{2!4!}} - 2070 = \frac{\frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1}}{\frac{6\cdot 5}{2\cdot 1}} - 2070$

$= \frac{10\cdot 9\cdot 8}{3\cdot 6\cdot 5} - 2070 = 8 - 2070 = -2062$

27. $\frac{{}_4C_2\cdot{}_6C_1}{{}_{18}C_3} = \frac{\frac{4!}{(4-2)!2!}\cdot\frac{6!}{(6-1)!1!}}{\frac{18!}{(18-3)!3!}} = \frac{\frac{4!}{2!2!}\cdot\frac{6!}{5!1!}}{\frac{18!}{15!3!}} = \frac{\frac{4\cdot 3\cdot \cancel{2!}}{\cancel{2!}2\cdot 1}\cdot\frac{6\cdot \cancel{5!}}{\cancel{5!}1!}}{\frac{18\cdot 17\cdot 16\cdot \cancel{15!}}{\cancel{15!}3\cdot 2\cdot 1}} = \frac{36}{816} = \frac{3}{68}$

28. $\frac{{}_5C_1\cdot{}_7C_2}{{}_{12}C_3} = \frac{\frac{5!}{(5-1)!1!}\cdot\frac{7!}{(7-2)!2!}}{\frac{12!}{(12-3)!3!}} = \frac{\frac{5!}{4!1!}\cdot\frac{7!}{5!2!}}{\frac{12!}{9!3!}} = \frac{\frac{5\cdot \cancel{4!}}{\cancel{4!}\cdot 1}\cdot\frac{7\cdot 6\cdot \cancel{5!}}{\cancel{5!}\cdot 2\cdot 1}}{\frac{12\cdot 11\cdot 10\cdot \cancel{9!}}{\cancel{9!}\cdot 3\cdot 2\cdot 1}} = \frac{1260}{2640} = \frac{21}{44}$

29. $9\cdot 3 = 27$ ways

30. $3\cdot 4 = 12$ choices

31. $2\cdot 4\cdot 5 = 40$ ways

32. $4\cdot 3\cdot 4\cdot 3 = 144$ ways
Meal descriptions may vary.

33. $3^5 = 243$ ways

34. $3^8 = 6561$ ways

35. $8\cdot 2\cdot 9 = 144$ area codes

36. $2\cdot 26\cdot 26\cdot 26 = 35{,}152$ call letters

37. $5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 1 = 120$ ways

38. $4\cdot 3\cdot 2\cdot 1\cdot 1 = 24$ ways

39. $1\cdot 3\cdot 2\cdot 1\cdot 1 = 6$ paragraphs

40. $2\cdot 1\cdot 2\cdot 1\cdot 1 = 4$ ways

41. ${}_{10}P_3 = \frac{10!}{7!3!} = 10\cdot 9\cdot 8 = 720$ ways

42. ${}_{10}P_4 = \frac{10!}{6!} = 10\cdot 9\cdot 8\cdot 7 = 5040$ ways

43. ${}_{13}P_7 = \frac{13!}{6!} = 13\cdot 12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7$
$=8{,}648{,}640$ ways

44. $_{20}P_3 = \frac{20!}{17!} = 20 \cdot 19 \cdot 18 = 6840$ ways

45. $_6P_3 = \frac{6!}{3!} = 6 \cdot 5 \cdot 4 = 120$ ways

46. $_8P_3 = \frac{8!}{5!} = 336$ ways

47. $_9P_5 = \frac{9!}{4!} = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 15{,}120$ lineups

48. $_7P_4 = \frac{7!}{3!} = 840$ arrangements

49. $_6C_3 = \frac{6!}{3!3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$ ways

50. $_{11}C_4 = \frac{11!}{7!4!} = \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2 \cdot 1} = 330$ committees

51. $_{12}C_4 = \frac{12!}{8!4!} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1}$
=495 collections

52. $_{14}C_6 = \frac{14!}{8!6!} = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 3003$ ways

53. $_{17}C_8 = \frac{17!}{9!8!} = \frac{17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
=24,310 groups

54. $_{100}C_{18} = \frac{100!}{82!18!} = 3.07 \times 10^{19}$ ways

55. $_{53}C_6 = \frac{53!}{47!6!} = 22{,}957{,}480$ selections

56. $_{59}C_6 = \frac{59!}{53! \cdot 6!} = 45{,}057{,}474$ selections

57. $_6P_4 = \frac{6!}{2!} = 6 \cdot 5 \cdot 4 \cdot 3 = 360$ ways

58. $_{40}C_8 = \frac{40!}{32!8!} = 76{,}904{,}685$ selections

59. $_{13}C_6 = \frac{13!}{7!6!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
=1716 ways

60. $_{50}P_3 = \frac{50!}{47!} = 50 \cdot 49 \cdot 48 = 177,600$ ways

61. $_{20}C_3 = \frac{20!}{17!3!} = \frac{20 \cdot 19 \cdot 18}{3 \cdot 2 \cdot 1} = 1140$ ways

62. $_{50}C_3 = \frac{50!}{47!3!} = \frac{50 \cdot 49 \cdot 48}{3 \cdot 2 \cdot 1} = 19,600$ ways

63. $_7P_4 = \frac{7!}{3!} = 840$ passwords

64. $_9P_5 = \frac{9!}{4!} = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 15,120$ ways

65. $_{15}P_3 = \frac{15!}{12!} = 15 \cdot 14 \cdot 13 = 2730$ cones

66. $_{31}C_3 = \frac{31!}{28!3!} = \frac{31 \cdot 30 \cdot 29}{3 \cdot 2 \cdot 1} = 4495$ bowls

67. $_6P_6 = \frac{6!}{0!} = 6! = 720$ rankings

68. $_5P_5 = \frac{5!}{0!} = 5! = 120$ rankings

69. $_6C_3 = \frac{6!}{3!3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$ ways

70. $_6C_2 = \frac{6!}{4!2!} = \frac{6 \cdot 5}{2 \cdot 1} = 15$ ways

71. $_4P_4 = \frac{4!}{0!} = 4! = 24$ ways

72. First select 1 man $\left(_5C_1\right)$, then order the remaining 5 jokes $\left(_5P_5\right)$.

$$_5C_1 \cdot {}_5P_5 = \frac{5!}{4!1!} \cdot \frac{5!}{0!} = 5 \cdot 5! = 5 \cdot 120 = 600 \text{ ways}$$

73. – 82. Answers may vary.

83. makes sense

84. makes sense

85. does not make sense; Explanations will vary. Sample explanation: Since order matters use permutations.

86. does not make sense; Explanations will vary. Sample explanation: Since order does not matter use combinations.

87. false; Changes to make the statement true will vary. A sample change is: The number of ways to choose four questions out of ten questions is $_{10}C_4$.

88. false; Changes to make the statement true will vary. A sample change is: If $r > 1$, ${}_nP_r$ is greater than ${}_nC_r$.

89. true

90. false; Changes to make the statement true will vary. A sample change is: The number of ways to pick a winner and first runner-up is ${}_{20}P_2$.

91. $5 \cdot 5 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 14,400$ ways

92. $2 \cdot 6 \cdot 6 \cdot 2 = 144$ numbers

93. $${}_{10}C_8 \cdot {}_5C_3 = \frac{10!}{(10-8)!8!} \cdot \frac{5!}{(5-3)!3!}$$
$$= 45 \cdot 10 = 450$$
They can be chosen in 450 ways.

94. Answers may vary.

95. $\frac{4}{6}$ or $\frac{2}{3}$ are less than 5.

96. $\frac{2}{6}$ or $\frac{1}{3}$ are not less than 5.

97. 2, 4, 5, and 6 are even or greater than three. This is a fraction of $\frac{4}{6}$ or $\frac{2}{3}$.

Section 11.7

Check Point Exercises

1. **a.** $P(\text{positive test}) = \frac{\text{\# women w/positive test}}{\text{total number of women}} = \frac{7664}{100,000} = \frac{479}{6250} \approx 0.077$

b. $P(\text{positive test}) = \frac{\text{\# women w/breast cancer and positive test}}{\text{total number of women w/breast cancer}} = \frac{720}{800} = \frac{9}{10} = 0.9$

c. $P(\text{breast cancer}) = \frac{\text{\# women w/breast cancer and positive test}}{\text{total number of women w/positive test}} = \frac{720}{7664} = \frac{45}{479} = 0.094$

2. The sample space of equally likely outcomes is $S = \{1, 2, 3, 4, 5, 6\}$. There are six outcomes in the sample space, so $n(S) = 6$. The event of getting a number greater than 4 can be represented by $E = \{5, 6\}$.
There are two outcomes in this event, so $n(E) = 2$.
The probability of rolling a number greater than 4 is $P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$.

3. We have $n(S) = 36$. The phrase "getting a sum of 5" describes the event $E = \{(1,4), (2,3), (3,2), (4,1)\}$. This event has 4 outcomes, so $n(E) = 4$. Thus, the probability of getting a sum of 5 is $P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$.

4. Let E be the event of being dealt a king. Because there are 4 kings in the deck, the event of being dealt a king can occur in 4 ways, i.e., $n(E) = 4$. With 52 cards in the deck, $n(S) = 52$.

The probability of being dealt a king is $P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$.

5. Because the order of the six numbers does not matter, this is a situation involving combinations. With one lottery ticket, there is only one way of winning so $n(E) = 1$. Using the combinations formula ${}_nC_r = \frac{n!}{(n-r)!r!}$ to find the number of outcomes in the sample space, we are selecting $r = 6$ numbers from a collection of $n = 49$ numbers.

${}_{49}C_6 = \frac{49!}{43!\cdot 6!} = 13{,}983{,}816$ So $n(S) = 13{,}983{,}816$. If a person bought one lottery ticket, the probability of winning was

$P(E) = \frac{n(E)}{n(S)} = \frac{1}{13{,}983{,}816}$

The probability of winning the state lottery was 0.0000000715.

6. $P(\text{not } 50-59) = 1 - P(50-59) = 1 - \frac{31}{191} = \frac{160}{191}$

7. We find the probability that either of these mutually exclusive events will occur by adding their individual probabilities.

$P(4 \text{ or } 5) = P(4) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

The probability of selecting a 4 or a 5 is $\frac{1}{3}$.

8. It is possible for the pointer to land on a number that is odd and less than 5. Two of the numbers , 1 and 3, are odd and less than 5. These events are not mutually exclusive. The probability of landing on a number that is odd and less than 5 is

P (odd or less than 5)

$= P$ (odd) + P (less than 5) – P (odd and less than 5) $= \frac{4}{8} + \frac{4}{8} - \frac{2}{8} = \frac{6}{8} = \frac{3}{4}$

The probability that the pointer will stop on an odd number or a number less than 5 is $\frac{3}{4}$.

9. **a.** These events are not mutually exclusive.

P(at least \$100,000 or was not audited)

$= P(\text{at least } \$100{,}000) + P(\text{was not audited}) - P(\text{at least } \$100{,}000 \text{ and was not audited})$

$= \frac{10{,}927{,}511}{120{,}851{,}273} + \frac{120{,}035{,}962}{120{,}851{,}273} - \frac{10{,}775{,}542}{120{,}851{,}273} = \frac{120{,}187{,}931}{120{,}851{,}273} \approx 0.99$

b. These events are mutually exclusive.

P(less than \$25,000 or between \$50,000 and \$99,999, inclusive)

$= P(\text{less than } \$25{,}000) + P(\text{between } \$50{,}000 \text{ and } \$99{,}999, \text{ inclusive})$

$= \frac{53{,}207{,}268}{120{,}851{,}273} + \frac{25{,}616{,}486}{120{,}851{,}273} = \frac{78{,}823{,}754}{120{,}851{,}273} \approx 0.65$

10. The wheel has 38 equally likely outcomes and 2 are green. Thus, the probability of a green occurring on a play is $\frac{2}{38}$, or $\frac{1}{19}$. The result that occurs on each play is independent of all previous results. Thus,

P (green and green) = P (green) · P (green) $=\frac{1}{19}\cdot\frac{1}{19}=\frac{1}{361}\approx 0.00277.$

The probability of green occurring on two consecutive plays is $\frac{1}{361}$.

11. If two or more events are independent, we can find the probability of them all occurring by multiplying the probabilities. The probability of a baby boy is $\frac{1}{2}$, so the probability of having four boys in a row is P (4 boys in a row) $=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{16}.$

Exercise Set 11.7

1. $P(\text{divorced})=\frac{\text{number of persons divorced}}{\text{total number of U.S. adults}}=\frac{21.7}{212.5}\approx 0.10$

2. $P(\text{never married})=\frac{\text{number of persons never married}}{\text{total number of U.S. adults}}=\frac{51.9}{212.5}\approx 0.24$

3. $P(\text{female})=\frac{\text{number of females}}{\text{total number of U.S. adults}}=\frac{110.1}{212.5}\approx 0.52$

4. $P(\text{male})=\frac{\text{number of males}}{\text{total number of U.S. adults}}=\frac{102.4}{212.5}\approx 0.48$

5. $P(\text{widowed male})=\frac{\text{number of widowed males}}{\text{total number of U.S. adults}}=\frac{2.7}{212.5}\approx 0.01$

6. $P(\text{widowed female})=\frac{\text{number of widowed females}}{\text{total number of U.S. adults}}=\frac{11.3}{212.5}\approx 0.05$

7. $P(\text{selecting a woman from the divorced population})=\frac{\text{number of divorced women}}{\text{number of persons divorced}}=\frac{12.7}{21.7}\approx 0.59$

8. $P(\text{selecting a man from the divorced population})=\frac{\text{number of divorced men}}{\text{number of persons divorced}}=\frac{9.0}{21.7}\approx 0.41$

9. $P(\text{selecting a married man from the adult male population})=\frac{\text{number of married men}}{\text{number of males}}=\frac{62.1}{102.4}\approx 0.61$

10. $P(\text{selecting a married woman from the adult female population})=\frac{\text{number of married women}}{\text{number of females}}=\frac{62.8}{110.1}\approx 0.57$

11. $P(R)=\frac{n(E)}{n(S)}=\frac{1}{6}$

12. $P(R)=\dfrac{n(E)}{n(S)}=\dfrac{1}{6}$

13. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{3}{6}=\dfrac{1}{2}$

14. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{3}{6}=\dfrac{1}{2}$

15. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{2}{6}=\dfrac{1}{3}$

16. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{0}{6}=0$

17. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{4}{52}=\dfrac{1}{13}$

18. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{13}{52}=\dfrac{1}{4}$

19. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{12}{52}=\dfrac{3}{13}$

20. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{12}{52}=\dfrac{3}{13}$

21. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{1}{4}$

22. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{2}{4}=\dfrac{1}{2}$

23. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{7}{8}$

24. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{4}{8}=\dfrac{1}{2}$

25. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{3}{36}=\dfrac{1}{12}$

26. $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{5}{36}$

27. Buying 1 ticket:

$$P(E)=\frac{n(E)}{n(S)}=\frac{1}{{}_{51}C_6}=\frac{1}{18,009,460}$$

Buying 100 tickets:

$$P(E)=\frac{100}{18,009,460}=\frac{5}{900,473}$$

28. $${}_{30}C_6=\frac{30!}{24!6!}=\frac{30\cdot 29\cdot 28\cdot 27\cdot 26\cdot 25}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}=593,775$$

For 1 ticket:

$$P(E)=\frac{n(E)}{n(S)}=\frac{1}{593,775}$$

For 100 tickets: $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{100}{593,775}=\dfrac{4}{23,751}$

29. **a.** $${}_{52}C_5=\frac{52!}{47!5!}=\frac{52\cdot 51\cdot 50\cdot 49\cdot 48}{5\cdot 4\cdot 3\cdot 2\cdot 1}=2,598,960$$

b. $${}_{13}C_5=\frac{13!}{8!5!}=\frac{13\cdot 12\cdot 11\cdot 10\cdot 9}{5\cdot 4\cdot 3\cdot 2\cdot 1}=1287$$

c. $$P(E)=\frac{n(E)}{n(S)}=\frac{1287}{2,598,960}\approx 0.0005$$

30. P(3 picture cards) = $\dfrac{{}_{12}C_3}{{}_{52}C_3}=\dfrac{220}{22,100}=\dfrac{11}{1105}\approx 0.00995$

31. $P(\text{not completed 4 years of college})=1-P(\text{completed 4 years of college})=1-\dfrac{45}{174}=\dfrac{43}{58}$

32. $P(\text{not completed 4 years high school})=1-P(\text{completed 4 years high school})=1-\dfrac{29}{174}=\dfrac{1}{6}$

33. P(completed H.S. or less than 4 yrs college) = P(completed H.S) + P(less than 4 yrs college)

$$=\frac{56}{174}+\frac{44}{174}=\frac{100}{174}=\frac{50}{87}$$

34. $P(\text{completed less than 4 yrs HS or 4 yrs HS})=\dfrac{29+56}{174}=\dfrac{85}{174}$

35. P(completed 4 yrs H.S. or man) = P(completed 4 yrs H.S.) + P(man) − P(man who completed 4 yrs H.S)

$$=\frac{56}{174}+\frac{82}{174}-\frac{25}{174}=\frac{113}{174}$$

36. $P(\text{completed 4 yrs HS or is a woman})$

$$= P(\text{completed 4 yrs HS}) + P(\text{woman}) - P(\text{woman completed 4 yrs HS})$$

$$= \frac{56}{174} + \frac{92}{174} - \frac{31}{174} = \frac{117}{174} = \frac{39}{58}$$

37. $P(\text{not king}) = 1 - P(\text{king}) = 1 - \frac{4}{52} = 1 - \frac{1}{13} = \frac{12}{13}$

38. $P(\text{not dealt picture card}) = 1 - P(\text{picture card}) = 1 - \frac{12}{52} = \frac{10}{13}$

39. $P(2 \text{ or } 3) = P(2) + P(3) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$

40. $P(\text{red 7 or black 8}) = P(\text{red 7}) + P(\text{black 8}) = \frac{2}{52} + \frac{2}{52} = \frac{4}{52} = \frac{1}{13}$

41. $P(\text{7 or red card}) = P(7) + P(\text{red card}) - P(\text{7 and red}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$

42. $P(\text{5 or black card}) = P(5) + P(\text{black card}) - P(\text{black 5}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{7}{13}$

43. $P(\text{odd or less than 6}) = P(\text{odd}) + P(\text{less than 6}) - P(\text{odd \# less than 6}) = \frac{4}{8} + \frac{5}{8} - \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$

44. $P(\text{odd or greater than 3}) = P(\text{odd}) + P(\text{greater than 3}) - P(\text{odd number greater than 3}) = \frac{4}{8} + \frac{5}{8} - \frac{2}{8} = \frac{7}{8}$

45. $P(\text{professor or male}) = P(\text{professor}) + P(\text{male}) - P(\text{male professor}) = \frac{19}{40} + \frac{22}{40} - \frac{8}{40} = \frac{33}{40}$

46. $P(\text{professor or female}) = P(\text{prof.}) + P(\text{female}) - P(\text{female prof.}) = \frac{19}{40} + \frac{18}{40} - \frac{11}{40} = \frac{26}{40} = \frac{13}{20}$

47. $P(2 \text{ and } 3) = P(2) \cdot P(3) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

48. $P(\text{5 and a 1}) = P(5) \cdot P(1) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

49. $P(\text{even and greater than 2}) = P(\text{even}) \cdot P\left(\text{greater than 2}\right) = \frac{3}{6} \cdot \frac{4}{6} = \frac{1}{3}$

50. $P(\text{odd and less than 3}) = P(\text{odd}) \cdot P\left(\text{less than 3}\right) = \frac{3}{6} \cdot \frac{2}{6} = \frac{6}{36} = \frac{1}{6}$

51. $P(\text{all heads}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$

52. $P(\text{all tails}) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \left(\frac{1}{2}\right)^7 = \frac{1}{128}$

53. **a.** $P(\text{hit 2 yrs in a row}) = P(\text{hit in }1^{\text{st}}\text{ year and }2^{\text{nd}}\text{ year}) = P(\text{hit }1^{\text{st}}\text{ year})\cdot P(\text{hit }2^{\text{nd}}\text{ year}) = \frac{1}{16}\cdot\frac{1}{16} = \frac{1}{256}$

b. $P(\text{hit 3 yrs in a row}) = \frac{1}{16}\cdot\frac{1}{16}\cdot\frac{1}{16} = \frac{1}{4096}$

c. First find the probability that South Florida will not be hit by a major hurricane in a single year:

$P(\text{not hit}) = 1 - \frac{1}{16} = \frac{15}{16}$

$P(\text{not hit in next 10 years}) = \left(\frac{15}{16}\right)^{10} \approx 0.524$

d. $P(\text{hit at least once}) = 1 - P(\text{hit none}) = 1 - \left(\frac{15}{16}\right)^{10} \approx 0.476$

54. – 63. Answers may vary.

64. does not make sense; Explanations will vary. Sample explanation: The two probabilities must add to 1.

65. does not make sense; Explanations will vary. Sample explanation: The probability of a Democrat winning and the probability of a Republican winning are not necessarily equal. For instance, it is possible that the probability of a Democrat winning is 0.6 and the probability of a Republican winning is 0.4.

66. does not make sense; Explanations will vary. Sample explanation: The probability of an event cannot be greater than 1.

67. makes sense

68. total area = $12^2 = 144\text{ in.}^2$

middle colored area = $3^2 = 9\text{ in.}^2$

other colored area = $9^2 - 6^2$

$= 81 - 36 = 45\text{ in.}^2$

$$P(E) = \frac{9}{144} + \frac{45}{144} = \frac{54}{144} = \frac{3}{8}$$

69. Answers may vary.

70. First find the total number of all three-digit numbers: $9\cdot 10\cdot 9 = 810$.
There are $1\cdot 9\cdot 1 = 9$ three-digit numbers beginning with 1 and ending with 1. There are $1\cdot 9\cdot 1 = 9$ three-digit numbers beginning with 2 and ending with 2. There will also be 9 three-digit numbers read the same forward and backward beginning with 3. The same will hold for the number of three-digit numbers read the same forward and backward beginning with 4, 5, 6, 7, 8, and 9.
So there are $9\cdot 9 = 81$ three-digit numbers read the same forward and backward. The probability of selecting one of these numbers is $\frac{81}{810} = \frac{1}{10}$.

71. **a.** $P(\text{Democrat who is not a business major})$

$$= \frac{\text{\# of students who are Democrats but not business majors}}{\text{\# of students}}$$

$$= \frac{29-5}{50} = \frac{24}{50} = \frac{12}{25}$$

b. $P(\text{neither Democrat nor business major})$

$$= 1 - P(\text{Democrat or business major})$$

$$= 1 - \left(P(\text{Democrat}) + P(\text{business major}) - P(\text{Democrat and business major})\right)$$

$$= 1 - \left(\frac{29}{50} + \frac{11}{50} - \frac{5}{50}\right) = 1 - \frac{35}{50} = \frac{15}{50} = \frac{3}{10}$$

72. $P(\text{driving while intoxicated or having a driving accident})$

$= P(\text{driving while intox.}) + P(\text{a driving accident.}) - P(\text{driving while intox. and a driving accident})$

Let $p = P(\text{having a driving accident while intoxicated})$.

Then $0.32 = 0.32 + 0.09 - p$

$0.35 = 0.41 - p$

$p = .06$

So, $P(\text{having a driving accident while intoxicated}) = 0.06$.

73. **a.** The first person can have any birthday in the year. The second person can have all but one birthday.

b. $\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \approx 0.99$

c. $100\% - 99\% = 0.01$

d. $\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdots \frac{346}{365} \approx 0.59$

$1 - 0.59 = 0.41$

e. With 23 people, the probability that at least two people have the same birthday is

$$P(E) = 1 - \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdots + \frac{342}{365}$$

$$\approx 1 - 0.4927 \approx 0.5073$$

74. Answers may vary.

Chapter 11 Review Exercises

1. $a_n = 7n - 4$

$a_1 = 7 - 4 = 3$

$a_2 = 14 - 4 = 10$

$a_3 = 21 - 4 = 17$

$a_4 = 28 - 4 = 24$

First four terms: 3, 10, 17, 24.

2. $a_n = (-1)^n \dfrac{n+2}{n+1}$

$a_1 = (-1)^1 \dfrac{1+2}{1+1} = -\dfrac{3}{2}$

$a_2 = (-1)^2 \dfrac{2+2}{2+1} = \dfrac{4}{3}$

$a_3 = (-1)^3 \dfrac{3+2}{3+1} = -\dfrac{5}{4}$

$a_4 = (-1)^4 \dfrac{4+2}{4+1} = \dfrac{6}{5}$

First four terms: $-\dfrac{3}{2}, \dfrac{4}{3}, -\dfrac{5}{4}, \dfrac{6}{5}$.

3. $a_n = \dfrac{1}{(n-1)!}$

$a_1 = \dfrac{1}{0!} = 1$

$a_2 = \dfrac{1}{1!} = 1$

$a_3 = \dfrac{1}{2!} = \dfrac{1}{2}$

$a_4 = \dfrac{1}{3!} = \dfrac{1}{6}$

First four terms: $1, 1, \dfrac{1}{2}, \dfrac{1}{6}$.

4. $a_n = \dfrac{(-1)^{n+1}}{2^n}$

$a_1 = \dfrac{(-1)^2}{2^1} = \dfrac{1}{2}$

$a_2 = \dfrac{(-1)^3}{2^2} = -\dfrac{1}{4}$

$a_3 = \dfrac{(-1)^4}{2^3} = \dfrac{1}{8}$

$a_4 = \dfrac{(-1)^5}{2^4} = -\dfrac{1}{16}$

First four terms: $\dfrac{1}{2}, -\dfrac{1}{4}, \dfrac{1}{8}, -\dfrac{1}{16}$.

5. $a_1 = 9$ and $a_n = \dfrac{2}{3a_{n-1}}$

$a_1 = 9$

$a_2 = \dfrac{2}{3 \cdot 9} = \dfrac{2}{27}$

$a_3 = \dfrac{2}{3} \cdot \dfrac{27}{2} = \dfrac{54}{6} = 9$

$a_4 = \dfrac{2}{3 \cdot 9} = \dfrac{2}{27}$

First four terms: $9, \dfrac{2}{27}, 9, \dfrac{2}{27}$.

6. $a_1 = 4$ and $a_n = 2a_{n-1} + 3$

$a_1 = 4$

$a_2 = 2 \cdot 4 + 3 = 8 + 3 = 11$

$a_3 = 2 \cdot 11 + 3 = 22 + 3 = 25$

$a_4 = 2 \cdot 25 + 3 = 50 + 3 = 53$

First four terms: 4, 11, 25, and 53.

7. $\dfrac{40!}{4! 38!} = \dfrac{40 \cdot 39 \cdot 38!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 38!} = 65$

8. $\sum_{i=1}^{5}\left(2i^2 - 3\right) = (2-3) + \left(2 \cdot 2^2 - 3\right) + \left(2 \cdot 3^2 - 3\right) + \left(2 \cdot 4^2 - 3\right) + \left(2 \cdot 5^2 - 3\right)$

$= -1 + 5 + 15 + 29 + 47$

$= 95$

9. $\sum_{i=0}^{4}(-1)^{i+1} i! = (-1)^1 0! + (-1)^2 1! + (-1)^3 3! + (-1)^4 4!$

$= -1 + 1 - 2 + 6 - 24$

$= -20$

10. $\dfrac{1}{3} + \dfrac{2}{4} + \dfrac{3}{5} + \cdots + \dfrac{15}{17} = \sum_{i=1}^{15} \dfrac{i}{i+2}$

11. $4^3 + 5^3 + 6^3 + \cdots + 13^3 = \sum_{i=1}^{10}(i+3)^3$

12. $a_1 = 7,\ d = 4$

First six terms: 7, 11, 15, 19, 23, 27.

13. $a_1 = -4,\ d = -5$

First six terms: $-4, -9, -14, -19, -24, -29$.

14. $a_1 = \dfrac{3}{2},\ d = -\dfrac{1}{2}$

First six terms: $\dfrac{3}{2}, 1, \dfrac{1}{2}, 0, -\dfrac{1}{2}, -1$.

15. $a_{n+1} = a_n + 5,\ a_1 = -2$
First six terms: $-2, 3, 8, 13, 18, 23$.

16. $a_1 = 5,\ d = 3$
$a_n = 5 + (n-1)3$
$a_6 = 5 + (5)3 = 20$

17. $a_1 = -8,\ d = -2$
$a_n = -8 + (n-1)(-2)$
$a_{12} = -8 + 11(-2) = -30$

18. $a_1 = 14,\ d = -4$
$a_n = 14 + (n-1)(-4)$
$a_{14} = 14 + (13)(-4) = -38$

19. $-7, -3, 1, 5, \ldots$
$d = -3 - (-7) = 4$
$a_n = -7 + (n-1)(4)$
$a_n = 4n - 11$
$a_{20} = 4(20) - 11$
$a_{20} = 69$

20. $a_1 = 200,\ d = -20$
$a_n = 200 + (n-1)(-20)$
$a_n = 220 - 20n$
$a_{20} = 220 - 20(20)$
$a_{20} = -180$

21. $a_n = a_{n-1} - 5,\ a_1 = 3$
$d = -5$
$a_n = 3 + (n-1)(-5) = 3 - 5n + 5$
$a_n = 8 - 5n$
$a_{20} = 8 - 5(20) = -92$

22. $5, 12, 19, 26, \ldots$
$d = 7$
$a_n = 5 + (n-1)(7)$
$a_{22} = 5 + 21(7) = 152$
$S_{22} = \frac{22}{2}(5 + 152) = 1727$

23. $-6, -3, 0, 3, \ldots$
$d = 3$
$a_n = -6 + (n-1)3$
$a_{15} = -6 + (14)3 = 36$
$S_{15} = \frac{15}{2}(-6 + 36) = 225$

24. $3 + 6 + 9 + \ldots + 300$
$S_{100} = \frac{100}{2}(3 + 300) = 15{,}150$

25. $\sum_{i=1}^{16}(3i + 2)$
$a_1 = 3 + 2 = 5$
$a_{16} = 3(16) + 2 = 50$
$S_{16} = \frac{16}{2}(5 + 50) = 440$

26. $\sum_{i=1}^{25}(-2i + 6)$
$a_1 = -2 + 6 = 4$
$a_{25} = -2(25) + 6 = -44$
$S_{25} = \frac{25}{2}(4 - 44) = -500$

27. $\sum_{i=1}^{30} -5i$
$a_1 = -5$
$a_{30} = -5(30) = -150$
$S_{30} = \frac{30}{2}(-5 - 150) = -2325$

28. **a.** $a_n = 39 + (n-1)(4.75)$
$= 39 + 4.75n - 4.75$
$= 4.75n + 34.25$

b. $a_n = 4.75n + 34.25$
$a_{12} = 4.75(13) + 34.25$
$= 96$
The percentage of students ages 12 – 18 who will report seeing security cameras at school in the year 2013 will be approximately 96%.

29. $a_n = 31{,}500 + (n-1)2300$

$a_{10} = 31{,}500 + (9)2300 = 52{,}200$

$S_{10} = \frac{10}{2}(31{,}500 + 52{,}200) = 418{,}500$

The total salary is \$418, 500.

30. $a_n = 25 + (n-1)$

$a_{35} = 25 + 34 = 59$

$S_{35} = \frac{35}{2}(25 + 59) = 1470$

There are 1470 seats.

31. $a_1 = 3,\ r = 2$

First five terms: 3, 6, 12, 24, 48.

32. $a_1 = \frac{1}{2},\ r = \frac{1}{2}$

First five terms: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$.

33. $a_1 = 16,\ r = -\frac{1}{2}$

First five terms: $16, -8, 4, -2, 1$.

34. $a_n = -5a_{n-1},\ a_1 = -1$

First five terms: –1, 5, –25, 125, –625.

35. $a_1 = 2,\ r = 3$

$a_n = 2 \cdot 3^{n-1}$

$a_7 = 2 \cdot 3^6 = 1458$

36. $a_1 = 16,\ r = \frac{1}{2}$

$a_n = 16\left(\frac{1}{2}\right)^{n-1}$

$a_6 = 16\left(\frac{1}{2}\right)^5 = \frac{16}{32} = \frac{1}{2}$

37. $a_1 = -3,\ r = 2$

$a_n = -3 \cdot 2^{n-1}$

$a_5 = -3 \cdot 2^4 = -48$

38. 1, 2, 4, 8, ...

$a_1 = 1,\ r = \frac{2}{1} = 2$

$a_n = 2^{n-1}$

$a_8 = 2^7 = 128$

39. 100, 10, 1, $\frac{1}{10}$, ...

$a_1 = 100,\ r = \frac{10}{100} = \frac{1}{10}$

$a_n = 100\left(\frac{1}{10}\right)^{n-1}$

$a_8 = 100\left(\frac{1}{10}\right)^7 = \frac{1}{100{,}000}$

40. $12, -4, \frac{4}{3}, -\frac{4}{9} \cdots$

$a_1 = 12,\ r = -\frac{4}{12} = -\frac{1}{3}$

$a_n = 12\left(-\frac{1}{3}\right)^{n-1}$

$a_8 = 12\left(-\frac{1}{3}\right)^7 = -\frac{4}{729}$

41. 5, –15, 45, –135, …

$r = \frac{-15}{5} = -3$

$S_{15} = \frac{5\left[1-(-3)^{15}\right]}{1-(-3)} = 17{,}936{,}135$

42. $r = \frac{1}{2}, a_1 = 8$

$S_{78} = \frac{8\left[1-\left(\frac{1}{2}\right)^7\right]}{1-\frac{1}{2}} = -16\left(1-\frac{1}{128}\right)$

$= -16\left(-\frac{127}{128}\right) = \frac{127}{8}$

43. $S_6 = \frac{5(1-5^6)}{1-5} = \frac{5(-15624)}{-4} = 19{,}530$

44. $\sum_{i=1}^{7} 3(-2)^i$

$a_1 = -6,\ r = -2$

$S_7 = \frac{-6\left[1-(-2)^7\right]}{1-(-2)} = \frac{-6(129)}{3} = -258$

45. $\sum_{i=1}^{5} 2\left(\frac{1}{4}\right)^{i-1}$

$a_1 = 2,\ r = \frac{1}{4}$

$S_5 = \frac{2\left[1-\left(\frac{1}{4}\right)^5\right]}{1-\frac{1}{4}} = \frac{2\left(\frac{1023}{1024}\right)}{\frac{3}{4}} = \frac{341}{128}$

46. $a_1 = 9,\ r = \frac{1}{3}$

$S_\infty = \frac{9}{1-\frac{1}{3}} = \frac{9}{\frac{2}{3}} = 9 \cdot \frac{3}{2} = \frac{27}{2}$

47. $a_1 = 2,\ r = -\frac{1}{2}$

$S_\infty = \frac{2}{1-\left(-\frac{1}{2}\right)} = \frac{2}{\frac{3}{2}} = \frac{4}{3}$

48. $a_1 = -6,\ r = -\frac{2}{3}$

$S_\infty = \frac{-6}{1-\left(-\frac{2}{3}\right)} = \frac{-6}{\frac{5}{3}} = -\frac{18}{5}$

49. $r = 0.8$

$S_\infty = \frac{4}{1-0.8} = 20$

50. $0.\overline{6} = 0.6 + 0.06 + 0.006 + \cdots$

$a_1 = \frac{6}{10},\ r = \frac{1}{10}$

$S_\infty = \frac{\frac{6}{10}}{1-\frac{1}{10}} = \frac{\frac{6}{10}}{\frac{9}{10}} = \frac{6}{9} = \frac{2}{3}$

51. $0.\overline{47} = 0.47 + 0.0047 + 0.000047 + \cdots$

$a_1 = \frac{47}{100},\ r = \frac{1}{100}$

$S_\infty = \frac{\frac{47}{100}}{1-\frac{1}{100}} = \frac{\frac{47}{100}}{\frac{99}{100}} = \frac{47}{99}$

52. a. Divide each value by the previous value:

$\frac{5.9}{4.2} = 1.405$

$\frac{8.3}{5.9} = 1.407$

$\frac{11.6}{8.3} = 1.398$

$\frac{16.2}{11.6} = 1.397$

$\frac{22.7}{16.2} = 1.401$

The population is increasing geometrically with $r = 1.4$.

b. $a_n = 4.2 \cdot 1.4^n$

c. 2080 is 8 decades after 2000 so $n = 8$.

$a_n = 4.2 \cdot 1.4^n$

$a_8 = 4.2 \cdot 1.4^8 \approx 62.0$

In 2080, the model predicts the U.S. population, ages 85 and older, will be 62.0 million

53. $a_1 = 32,000,\ r = 1.06$

$a_6 = 32,000(1.06)^5 \approx \$42,823$

The sixth year salary is $42, 823.

$S_6 = \frac{32,000\left(1-1.06^6\right)}{1-1.06}$

$= \frac{32,000\left(1-1.06^6\right)}{-0.06}$

$\approx 223,210$

The total salary paid is $223, 210.

54. **a.** $A = \dfrac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}}$

$P = \$520,\ \ r = 0.06,\ \ n = 1,\ \ t = 20$

$$A = \frac{\$520\left[\left(1+\dfrac{0.06}{1}\right)^{1\cdot 20} - 1\right]}{\dfrac{0.06}{1}} = \frac{\$520\left[(1.06)^{20} - 1\right]}{0.06} \approx \$19{,}129$$

The value of the annuity will be \$19,129.

b. Interest = Value of annuity − Total deposits

$\approx\ \ \$19{,}129\ -\ \$520\cdot 20$

$\approx\ \$8729$

55. **a.** $A = \dfrac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}}$

$P = 100,\ \ r = 0.055,\ \ n = 12,\ \ t = 30$

$$A = \frac{\$100\left[\left(1+\dfrac{0.055}{12}\right)^{12\cdot 30} - 1\right]}{\dfrac{0.055}{12}} \approx \$91{,}361$$

The value of the IRA will be \$91,361.

b. Interest = Value of IRA − Total deposits

$\approx\ \ \$91{,}361\ -\ \$100\cdot 12\cdot 30$

$\approx\ \$55{,}361$

56. $4(0.7) + 4(0.7)^2 + \cdots;\ r = 0.7$

$S_\infty = \dfrac{4(0.7)}{1-0.7} = 9.\overline{3}$

The total spending is $\$9\frac{1}{3}$ million.

57. $S_1: 5 = \frac{5(1)(1+1)}{2}$

$5 = \frac{5(2)}{2}$

$5 = 5$ is true.

$S_k: 5+10+15+\cdots+5k = \frac{5k(k+1)}{2}$

$S_{k+1}: 5+10+15+\cdots+5k+5(k+1)$
$= \frac{5(k+1)(k+2)}{2}$

Add $5(k+1)$ to both sides of S_k:

$5+10+15+\cdots+5k+5(k+1)$
$= \frac{5k(k+1)}{2} + 5k(k+1)$

Simplify the right-hand side:

$$\frac{5k(k+1)}{2} + 5(k+1) = \frac{5k(k+1)+10(k+1)}{2} = \frac{(5k+10)(k+1)}{2} = \frac{5(k+1)(k+2)}{2}$$

If S_k is true, then S_{k+1} is true.
The statement is true for all *n*.

58. $S_1: 1 = \frac{4^1 - 1}{3}$

$1 = \frac{3}{3}$

$1 = 1$ is true.

$S_k: 1+4+4^2+\cdots+4^{k-1} = \frac{4^k - 1}{3}$

$S_{k+1}: 1+4+4^2+\cdots+4^{k-1}+4^k = \frac{4^{k+1}-1}{3}$ Add 4^k to both sides of S_k:

$S_k: 1+4+4^2+\cdots+4^{k-1} = \frac{4^k - 1}{3}$

$1+4+4^2+\cdots+4^{k-1}+4^k = \frac{4^k-1}{3} + 4^k$

Simplify the right-hand side:

$$\frac{4^k-1}{3} + 4^k = \frac{4^k - 1 + 3\cdot 4^k}{3} = \frac{4\cdot 4^k - 1}{3} = \frac{4^{k+1}-1}{3}$$

If S_k is true, then S_{k+1} is true.
The statement is true for all *n*.

59. $S_1 : 2 = 2(1)^2$

$2 = 2$ is true.

$S_k : 2 + 6 + 10 + \cdots + (4k - 2) = 2k^2$

$S_{k+1} : 2 + 6 + 10 + \cdots + (4k - 2) + (4k + 2) = 2(k+1)^2$

Add $(4k+2)$ to both sides of S_k:

$2 + 6 + 10 + \cdots + (4k - 2) + (4k + 2) = 2k^2 + (4k + 2)$

Simplify the right-hand side:

$$\begin{aligned} 2k^2 + 4k + 2 &= 2(k^2 + 2k + 1) \\ &= 2(k+1)^2 \end{aligned}$$

If S_k is true, then S_{k+1} is true. The statement is true for all n.

60. $S_1 : 1 \cdot 3 = \dfrac{1(1+1)[2(1)+7]}{6}$

$3 = \dfrac{2 \cdot 9}{6}$

$3 = \dfrac{18}{6}$

$3 = 3$ is true.

$S_k : 1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + k(k+2) = \dfrac{k(k+1)(2k+7)}{6}$

$S_{k+1} : 1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + k(k+2) + (k+1)(k+3) = \dfrac{(k+1)(k+2)(2k+9)}{6}$

Add $(k+1)(k+3)$ to both sides of S_k:

$1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + k(k+2) + (k+1)(k+3) = \dfrac{k(k+1)(2k+7)}{6} + (k+1)(k+3)$

Simplify the right-hand side:

$$\begin{aligned} &= \frac{k(k+1)(2k+7)}{6} + (k+1)(k+3) \\ &= \frac{k(k+1)(2k+7) + 6(k+1)(k+3)}{6} \\ &= \frac{(k+1)[k(2k+7) + 6(k+3)]}{6} \\ &= \frac{(k+1)(2k^2 + 13k + 18)}{6} \\ &= \frac{(k+1)(k+2)(2k+9)}{6} \end{aligned}$$

If S_k is true, then S_{k+1} is true. The statement is true for all n.

61. S_1: 2 is a factor of $1^2+5(1)=6$ since $6 = 2 \cdot 3$.

S_k : 2 is a factor of k^2+5k.

S_{k+1} : 2 is a factor of $(k+1)^2+5(k+1)$.

$$\begin{aligned}(k+1)^2+5(k+1) &= k^2+2k+1+5k+5\\ &= k^2+7k+6\\ &= k^2+5k+2(k+3)\\ &= (k^2+5k)+2(k+3)\end{aligned}$$

Because we assume S_k is true, we know 2 is a factor of k^2+5k. Since 2 is a factor of $2(k+3)$, we conclude 2 is a factor of the sum $(k^2+5k)+2(k+3)$. If S_k is true, then S_{k+1} is true. The statement is true for all *n*.

62. $\binom{11}{8}=\frac{11!}{3!8!}=\frac{11\cdot 10\cdot 9}{3\cdot 2\cdot 1}=165$

63. $\binom{90}{2}=\frac{90!}{88!2!}=\frac{90\cdot 89}{2\cdot 1}=4005$

64. $$\begin{aligned}(2x+1)^3 &= \binom{3}{0}(2x)^3+\binom{3}{1}(2x)^2\cdot 1+\binom{3}{2}(2x)1^2+\binom{3}{3}1^3\\ &= 8x^3+3(4x^2)+3(2x)+1\\ &= 8x^3+12x^2+6x+1\end{aligned}$$

65. $$\begin{aligned}(x^2-1)^4 &= \binom{4}{0}(x^2)^4+\binom{4}{1}(x^2)^3(-1)+\binom{4}{2}(x^2)^2(-1)^2+\binom{4}{3}x^2(-1)^3+\binom{4}{4}(-1)^4\\ &= x^8-4x^6+6x^4-4x^2+1\end{aligned}$$

66. $$\begin{aligned}(x+2y)^5 &= \binom{5}{0}x^5+\binom{5}{1}x^4(2y)+\binom{5}{2}x^3(2y)^2+\binom{5}{3}x^2(2y)^3+\binom{5}{4}x(2y)^4+\binom{5}{5}(2y)^5\\ &= x^5+5(2)x^4y+10(4)x^3y^2+10(8)x^2y^3+5(16)xy^4+32y^5\\ &= x^5+10x^4y+40x^3y^2+80x^2y^3+80xy^4+32y^5\end{aligned}$$

67. $$\begin{aligned}(x-2)^6 &= \binom{6}{0}x^6+\binom{6}{1}x^5(-2)+\binom{6}{2}x^4(-2)^2+\binom{6}{3}x^3(-2)^3+\binom{6}{4}x^2(-2)^4+\binom{6}{5}x(-2)^5\binom{6}{6}(-2)^6\\ &= x^6+6x^5(-2)+15x^4(4)+20x^3(-8)+15x^2(16)+6x(-32)+64\\ &= x^6-12x^5+60x^4-160x^3+240x^2-192x+64\end{aligned}$$

68. $$\begin{aligned}(x^2+3)^8 &= \binom{8}{0}(x^2)^8+\binom{8}{1}(x^2)^7 3+\binom{8}{2}(x^2)^6 3^2+\cdots\\ &= x^{16}+8x^{14}3+28x^{12}9+\cdots\\ &= x^{16}+24x^{14}+252x^{12}+\cdots\end{aligned}$$

69. $(x-3)^9 = \binom{9}{0}x^9 + \binom{9}{1}x^8(-3) + \binom{9}{2}x^7(-3)^2 - \cdots$
$= x^9 + 9(-3)x^8 + 36(9)x^7 - \cdots$
$= x^9 - 27x^8 + 324x^7 - \cdots$

70. $(x+2)^5$

fourth term $= \binom{5}{3}x^2(2)^3$
$= 10(8)x^2 = 80x^2$

71. $(2x-3)^6$

fifth term $= \binom{6}{4}(2x)^2(-3)^4$
$= 15(4x^2)(81) = 4860x^2$

72. ${}_8P_3 = \frac{8!}{5!} = 8\cdot 7\cdot 6 = 336$

73. ${}_9P_5 = \frac{9!}{4!} = 9\cdot 8\cdot 7\cdot 6\cdot 5 = 15{,}120$

74. ${}_8C_3 = \frac{8!}{5!3!} = \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1} = 56$

75. ${}_{13}C_{11} = \frac{13!}{2!11!} = \frac{13\cdot 12}{2\cdot 1} = 78$

76. 4 · 5 = 20 choices

77. $3^5 =$ 243 possibilities

78. ${}_{15}P_4 = \frac{15!}{11!} = 15\cdot 14\cdot 13\cdot 12 = 32{,}760$ ways

79. ${}_{20}C_4 = \frac{20!}{16!4!} = \frac{20\cdot 19\cdot 18\cdot 17}{4\cdot 3\cdot 2\cdot 1} = 4845$ ways

80. ${}_{20}C_3 = \frac{20!}{17!3!} = \frac{20\cdot 19\cdot 18}{3\cdot 2\cdot 1} = 1140$ sets

81. ${}_{20}P_4 = \frac{20!}{16!}$
$= 20\cdot 19\cdot 18\cdot 17$
=116,280 ways

82. 5! = 120 ways

83. $P(\text{public college}) = \dfrac{252}{350} = \dfrac{18}{25}$

84. $P(\text{not from high-income family}) = 1 - P(\text{from high-income family}) = 1 - \dfrac{50}{350} = \dfrac{350}{350} - \dfrac{50}{350} = \dfrac{300}{350} = \dfrac{6}{7}$

85. $P(\text{from middle-income family or high-income family}) = \dfrac{160+50}{350} = \dfrac{210}{350} = \dfrac{3}{5}$

86. $P(\text{attended private college or is from a high income family})$

$= P(\text{private college}) + P(\text{high income family}) - P(\text{attended private college and is from a high income family})$

$= \dfrac{98}{350} + \dfrac{50}{350} - \dfrac{28}{350} = \dfrac{120}{350} = \dfrac{12}{35}$

87. $P(\text{selecting a student from a low-income family from among those attending public college}) = \dfrac{120}{252} = \dfrac{10}{21}$

88. $P(\text{selecting a student that attends a private college from among those in middle-income families}) = \dfrac{50}{160} = \dfrac{5}{16}$

89. $P(\text{less than 5}) = P(\text{rolling a 1, 2, 3, 4})$

$= \dfrac{4}{6} = \dfrac{2}{3}$

90. $P(\text{less than 3 or greater than 4})$

$= P(\text{less than 3}) + P(\text{greater than 4})$

$= P(\text{rolling a 1, 2}) + P(\text{rolling a 5, 6})$

$= \dfrac{2}{6} + \dfrac{2}{6} = \dfrac{4}{6} = \dfrac{2}{3}$

91. $P(E) = \dfrac{4}{52} + \dfrac{4}{52} = \dfrac{8}{52} = \dfrac{2}{13}$

92. $P(E) = \dfrac{4}{52} + \dfrac{26}{52} - \dfrac{2}{52} = \dfrac{28}{52} = \dfrac{7}{13}$

93. $P(\text{not yellow}) = 1 - P(\text{yellow}) = 1 - \dfrac{1}{6} = \dfrac{5}{6}$

94. $P(\text{red or greater than 3}) = \dfrac{3}{6} + \dfrac{3}{6} - \dfrac{1}{6} = \dfrac{5}{6}$

95. $P(\text{green, then less than 4}) = \dfrac{2}{6} \cdot \dfrac{3}{6} = \dfrac{6}{36} = \dfrac{1}{6}$

96. **a.** $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{1}{{}_{20}C_5} = \dfrac{1}{15{,}504}$

b. $P(E) = \dfrac{100}{15{,}504} = \dfrac{25}{3876}$

97. $P(E) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

98. **a.** $(0.2)^2 = 0.04$

b. $(0.2)^3 = 0.008$

c. $(1-0.2)^4 = (0.8)^4 = 0.4096$

Chapter 11 Test

1. $a_n = \frac{(-1)^{n+1}}{n^2}$

$a_1 = \frac{(-1)^2}{1^2} = 1$

$a_2 = \frac{(-1)^3}{2^2} = -\frac{1}{4}$

$a_3 = \frac{(-1)^4}{3^2} = \frac{1}{9}$

$a_4 = \frac{(-1)^5}{4^2} = -\frac{1}{16}$

$a_5 = \frac{(-1)^6}{5^2} = \frac{1}{25}$

First five terms: $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}$.

2. $\sum_{i=1}^{5}(i^2+10) = 11+14+19+26+35 = 105$

3. $\sum_{i=1}^{20}(3i-4)$

$a_1 = 3-4 = -1$

$d = 3$

$a_n = -1+(n-1)3$

$a_{20} = -1+(19)3 = 56$

$S_{20} = \frac{20}{2}(-1+56) = 550$

4. $\sum_{i=1}^{15}(-2)^i$

$a_1 = -2,\ r = -2$

$S_{15} = \frac{-2\left[1-(-2)^{15}\right]}{1-(-2)} = -21{,}846$

5. $\binom{9}{2} = \frac{9!}{7!2!} = \frac{9\cdot 8}{2\cdot 1} = 36$

6. ${}_{10}P_3 = \frac{10!}{7!} = 10\cdot 9\cdot 8 = 720$

7. ${}_{10}C_3 = \frac{10!}{7!3!} = \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1} = 120$

8. $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\cdots+\frac{21}{22} = \sum_{i=1}^{20}\frac{i+1}{i+2}$

9. 4, 9, 14, 19, . . .

$a_1 = 4,\ d = 5$

$a_n = 4+(n-1)\cdot 5 = 4+5n-1$

$a_n = 5n-1$

$a_{12} = 5(12)-1 = 59$

10. $16, 4, 1, \frac{1}{4}, \cdots$

$a_1 = 16,\ r = \frac{1}{4}$

$a_n = 16\left(\frac{1}{4}\right)^{n-1}$

$a_{12} = 16\left(\frac{1}{4}\right)^{11} = \frac{1}{262{,}144}$

11. 7, –14, 28, –56, . . .

$a_1 = 7,\ r = -2$

$S_{10} = \frac{7\left[1-(-2)^{10}\right]}{1-(-2)} = \frac{7(-1023)}{3} = -2387$

12. –7, –14, –21, –28, . . .

$a_1 = -7,\ d = -7$

$a_n = -7+(n-1)(-7)$

$a_{10} = -7+9(-7) = -70$

$S_{10} = \frac{10}{2}(-7-70) = -385$

13. $4+\frac{4}{2}+\frac{4}{2^2}+\frac{4}{2^3}+\cdots$

$r = \frac{1}{2}$

$S_\infty = \frac{4}{1-\frac{1}{2}} = 8$

14. $0.\overline{73} = 0.73 + 0.0073 + 0.000073 + \cdots$

$a_1 = \frac{73}{100},\ r = \frac{1}{100}$

$$S_\infty = \frac{\frac{73}{100}}{1-\frac{1}{100}} = \frac{\frac{73}{100}}{\frac{99}{100}} = \frac{73}{99}$$

15. $a_1 = 30,000,\ r = 1.04$

$$S_8 = \frac{30,000\left[1-(1.04)^8\right]}{1-1.04} \approx 276,426.79$$

The total salary is \$276,427.

16. $S_1 : 1 = \frac{1[3(1)-1]}{2}$

$1 = \frac{2}{2}$

$1 = 1$ is true.

$S_k : 1+4+7+\cdots+(3k-2) = \frac{k(3k-1)}{2}$

$S_{k+1} : 1+4+7+\cdots+(3k-2)+(3k+1) = \frac{(k+1)(3k+2)}{2}$

Add $(3k+1)$ to both sides of S_k :

$$1+4+7+\cdots+(3k-2)+(3k+1) = \frac{k(3k-1)}{2}+(3k+1)$$

Simplify the right-hand side:

$$\begin{aligned}\frac{k(3k-1)}{2}+(3k+1) &= \frac{k(3k-1)+2(3k+1)}{2} \\ &= \frac{3k^2+5k+2}{2} \\ &= \frac{(k+1)(3k+2)}{2}\end{aligned}$$

If S_k is true, then S_{k+1} is true. The statement is true for all n.

17. $$\begin{aligned}(x^2-1)^5 &= \binom{5}{0}(x^2)^5 + \binom{5}{1}(x^2)^4(-1) + \binom{5}{2}(x^2)^3(-1)^2 + \binom{5}{3}(x^2)^2(-1)^3 + \binom{5}{4}x^2(-1)^4 + \binom{5}{5}(-1)^5 \\ &= x^{10} - 5x^8 + 10x^6 - 10x^4 + 5x^2 - 1\end{aligned}$$

18. $\left(x+y^2\right)^8$

First Term $\binom{n}{r-1}a^{n-r+1}b^{r-1}=\binom{8}{1-1}x^{8-1+1}\left(y^2\right)^{1-1}=\binom{8}{0}x^8\left(y^2\right)^0=\frac{8!}{0!(8-0)!}x^8\cdot 1=\frac{\cancel{8!}}{0!\cancel{8!}}x^8$

$=x^8$

Second Term $\binom{n}{r-1}a^{n-r+1}b^{r-1}=\binom{8}{2-1}x^{8-2+1}\left(y^2\right)^{2-1}=\binom{8}{1}x^7\left(y^2\right)^1=\frac{8!}{1!(8-1)!}x^7y^2=\frac{8\cdot\cancel{7!}}{1\cdot\cancel{7!}}x^7y^2$

$=8x^7y^2$

Third Term $\binom{n}{r-1}a^{n-r+1}b^{r-1}=\binom{8}{3-1}x^{8-3+1}\left(y^2\right)^{3-1}=\binom{8}{2}x^6\left(y^2\right)^2=\frac{8!}{2!(8-2)!}x^6y^4=\frac{8\cdot 7\cdot\cancel{6!}}{2\cdot 1\cdot\cancel{6!}}x^6y^4$

$=28x^6y^4$

$x^8+8x^7y^2+28x^6y^4+\cdots$

19. ${}_{11}P_3=\frac{11!}{8!}=11\cdot 10\cdot 9=990$ ways

20. ${}_{10}C_4=\frac{10!}{6!4!}=\frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1}=210$ sets

21. Four digits are open: $10^4=10,000$

22. $P(\text{not brown eyes})=1-P(\text{brown eyes})$

$=1-\frac{40}{50}$

$=\frac{60}{100}=\frac{3}{5}$

23. $P(\text{brown eyes or blue eyes})$

$=P(\text{brown eyes})+P(\text{blue eyes})$

$=\frac{40}{100}+\frac{38}{100}=\frac{78}{100}=\frac{39}{50}$

24. $P(\text{female or green eyes})$

$=P(\text{female})+P(\text{green eyes})$

$-P(\text{female with green eyes})$

$=\frac{50}{100}+\frac{22}{100}-\frac{12}{100}=\frac{60}{100}=\frac{3}{5}$

25. $P(\text{male, given blue eyes})=\frac{18}{38}=\frac{9}{19}$

26. ${}_{15}C_6=\frac{15!}{9!6!}=\frac{15\cdot 14\cdot 13\cdot 12\cdot 11\cdot 10}{6\cdot 5\cdot 4\cdot 3\cdot 2}=5005$

$P(E)=\frac{50}{5005}=\frac{10}{1001}$

27. $P(E) = \frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52} = \frac{8}{13}$

28. $P(E) = \frac{25}{50} + \frac{20}{50} - \frac{15}{50} = \frac{30}{50} = \frac{3}{5}$

29. $P(E) = \left(\frac{1}{4}\right)^4 = \frac{1}{256}$

30. $P(E) = \frac{2}{8} \cdot \frac{2}{8} = \frac{1}{16}$

Cumulative Review Exercises (Chapters 1–11)

1. domain: $[-4,1)$; range: $(-\infty,2]$

2. maximum of 2 at $x = -2$

3. decreasing interval: $(-2,1)$

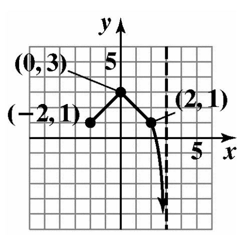
4. neither

5. $f(-3) = 1$ and $f(-1) = 1$

6. $(f \circ f)(-4) = f(f(-4)) = f(0) = 0$

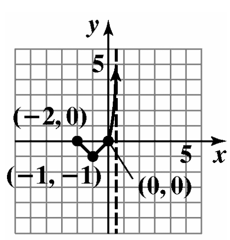
7. $f(x) \to -\infty$ as $x \to 1^-$

8.



$g(x) = f(x-2) + 1$

9.



$h(x) = -f(2x)$

10.
$$-2(x-5) + 10 = 3(x+2)$$
$$-2x + 10 + 10 = 3x + 6$$
$$14 = 5x$$
$$x = \frac{14}{5}$$

The solution set is $\left\{\frac{14}{5}\right\}$.

11. $3x^2 - 6x + 2 = 0$
$$x = \frac{6 \pm \sqrt{36-24}}{6} = \frac{6 \pm \sqrt{12}}{6} = \frac{6 \pm 2\sqrt{3}}{6} = \frac{3 \pm \sqrt{3}}{3}$$

The solution set is $\left\{\frac{3+\sqrt{3}}{3}, \frac{3-\sqrt{3}}{3}\right\}$.

12. $\log_2 x + \log_2 (2x-3) = 1$
$$\log_2 x(2x-3) = 1$$
$$x(2x-3) = 2$$
$$2x^2 - 3x - 2 = 0$$
$$(2x+1)(x-2) = 0$$
$$2x+1 = 0 \quad \text{or} \quad x-2 = 0$$
$$x = -\frac{1}{2} \qquad x = 2$$

$x = -\frac{1}{2}$ does not check since $\log_2\left(-\frac{1}{2}\right)$ does not exist.
The solution set is {2}.

13. $x^{1/2} - 6x^{1/4} + 8 = 0$
Let $t = x^{1/4}$.
$$t^2 - 6t + 8 = 0$$
$$(t-2)(t-4) = 0$$
$$t-2 = 0 \quad \text{or} \quad t-4 = 0$$
$$t = 2 \qquad t = 4$$
$$x^{1/4} = 2 \qquad x^{1/4} = 4$$
$$x = 16 \qquad x = 256$$

The solution set is {16, 256}.

14. $\sqrt{2x+4}-\sqrt{x+3}-1=0$

$$\left(\sqrt{2x+4}\right)^2=\left(\sqrt{x+3}+1\right)^2$$
$$2x+4=(x+3)+2\sqrt{x+3}+1$$
$$x=2\sqrt{x+3}$$
$$x^2=4(x+3)$$
$$x^2-4x-12=0$$
$$(x-6)(x+2)=0$$
$$x-6=0 \quad \text{or} \quad x+2=0$$
$$x=6 \qquad x=-2$$
$$x=-2 \text{ does not check.}$$

The solution set is {6}.

15. $|2x+1|\le 1$
$-1\le 2x+1\le 1$
$-2\le 2x\le 0$
$-1\le x\le 0$ or $[-1, 0]$
The solution set is $\{x|-1\le x\le 0\}$ or $[-1, 0]$.

16. $6x^2-6<5x$

$$6x^2-5x-6<0$$
$$6x^2-5x-6=0$$
$$(3x+2)(2x-3)=0$$
$$3x+2=0 \text{ or } 2x-3=0$$
$$x=-\frac{2}{3} \qquad x=\frac{3}{2}$$

The test intervals are $\left(-\infty, -\frac{2}{3}\right), \left(-\frac{2}{3}, \frac{3}{2}\right)$, and $\left(\frac{3}{2}, \infty\right)$. Testing a point in each interval shows that the solution is $\left(-\frac{2}{3}, \frac{3}{2}\right)$.

17. $\frac{x-1}{x+3}\le 0$
The test intervals are $(-\infty, -3)$, $(-3, 1)$ and $(1, \infty)$.
Testing a point in each interval shows that the solution is $(-3, 1]$.

18. $30e^{0.7x}=240$

$$e^{0.7x}=8$$
$$\ln e^{0.7x}=\ln 8$$
$$0.7x=\ln 8$$
$$x=\frac{\ln 8}{0.7}\approx 2.9706$$

The solution set is $\left\{\frac{\ln 8}{0.7}\right\}$ or $\{2.9706\}$.

19. $2x^3+3x^2-8x+3=0$
p: ±1, ±3
q: ±1, ±2
$\frac{p}{q}$: $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$

1	2	3	–8	3
		2	5	–3
	2	5	–3	0

$$(x-1)(2x^2+5x-3)=0$$
$$(x-1)(2x-1)(x+3)=0$$
$$x=1 \text{ or } x=\frac{1}{2} \text{ or } x=-3$$

The solution set is $\left\{-3, \frac{1}{2}, 1\right\}$.

20. $4x^2+3y^2=48$
$3x^2+2y^2=35$
Multiply equation 1 by –2.
Multiply equation 2 by 3.

$$-8x^2-6y^2=-96$$
$$9x^2+6y^2=105$$
$$\text{Add:} \quad x^2=9$$
$$x=\pm 3$$

Let $x=-3$:

$$4(-3)^2+3y^2=48$$
$$36+3y^2=48$$
$$3y^2=12$$
$$y^2=4$$
$$y=\pm 2$$

Let $x=3$:

$$4(3)^2+3y^2=48$$
$$36+3y^2=48$$
$$3y^2=12$$
$$y^2=4$$
$$y=\pm 2$$

The solution set is
{(3, 2), (3, –2), (–3, 2), (–3, –2)}.

21.
$$\begin{aligned} x - 2y + z &= 16 \\ 2x - y - z &= 14 \\ 3x + 5y - 4z &= -10 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 16 \\ 0 & -1 & -1 & 14 \\ 3 & 5 & -4 & -10 \end{array}\right] \begin{array}{l} \\ -2R_1 + R_2 \\ -3R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 16 \\ 0 & 3 & -3 & -18 \\ 0 & 11 & -7 & -58 \end{array}\right] \begin{array}{l} \\ \frac{1}{3}R_2 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 16 \\ 0 & 1 & -1 & -6 \\ 0 & 11 & -7 & -58 \end{array}\right] \begin{array}{l} \\ 2R_2 + R_1 \\ -11R_2 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & 4 & 8 \end{array}\right] \begin{array}{l} \\ \frac{1}{4}R_3 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} R_3 + R_1 \\ R_2 + R_1 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} R_3 + R_1 \\ R_2 + R_1 \\ \\ \end{array}$$

The solution set is {(6, –4, 2)}.

22.
$$\begin{cases} x - y = 1 \\ x^2 - x - y = 1 \end{cases}$$

Solving $x - y = 1$ for y gives $y = x - 1$.

Substitute:

$$x^2 - x - y = 1$$
$$x^2 - x - \overbrace{(x-1)}^{y} = 1$$
$$x^2 - x - x + 1 = 1$$
$$x^2 - 2x + 1 = 1$$
$$x^2 - 2x = 0$$
$$x(x-2) = 0$$

$x = 0$ or $x = 2$

If $x = 0$, $0 - y = 1$ so $y = -1$.

If $x = 2$, $2 - y = 1$ so $y = 1$.

The solution set is {(0, –1), (2, 1)}.

23. $100x^2 + y^2 = 25$

$$4x^2 + \frac{y^2}{25} = 1$$
$$\frac{x^2}{\left(\frac{1}{4}\right)} + \frac{y^2}{25} = 1$$

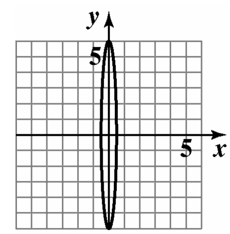
Ellipse

Foci on the y-axis

$a^2 = 25$ and $b^2 = \frac{1}{4}$, so $\frac{1}{4} = 25 - c^2$.

$$c^2 = \frac{99}{4}$$
$$c = \frac{3\sqrt{11}}{2}$$

Foci: $\left(0, -\frac{3\sqrt{11}}{2}\right), \left(0, \frac{3\sqrt{11}}{2}\right)$



$100x^2 + y^2 = 25$

24.
$$4x^2 - 9y^2 - 16x + 54y - 29 = 0$$
$$4(x^2 - 4x) - 9(y^2 - 6y) = 29$$
$$4(x^2 - 4x + 4) - 9(y^2 - 6y + 9) = 16 - 81 + 29$$
$$4(x-2)^2 - 9(y-3)^2 = -36$$
$$\frac{(y-3)^2}{4} - \frac{(x-2)^2}{9} = 1$$

Hyperbola with center at (2, 3)

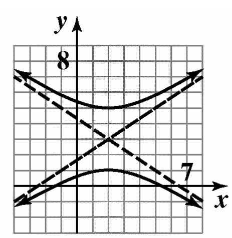
Transverse axis vertical

$a^2 = 4$ and $b^2 = 9$, so $9 = c^2 - 4$.

$c^2 = 13$

$c = \sqrt{13}$

Foci: $\left(2, 3 - \sqrt{13}\right), \left(2, 3 + \sqrt{13}\right)$



$4x^2 - 9y^2 - 16x + 54y - 29 = 0$

25. Symmetry:

$$f(-x) = \frac{x^2 - 1}{-x - 2}$$

No symmetry since $f(-x) \neq f(x)$ and $f(-x) \neq -f(-x)$.

x-intercepts:

$$x^2 - 1 = 0$$
$$x = \pm 1$$

y-intercept:

$$f(0) = \frac{1}{2}$$
$$y = \frac{1}{2}$$

Vertical asymptote:

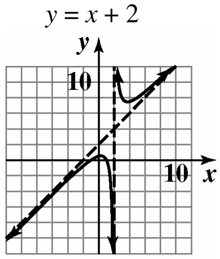
$$x - 2 = 0$$
$$x = 2$$

Horizontal asymptote:

$n > m$, so no horizontal asymptote.

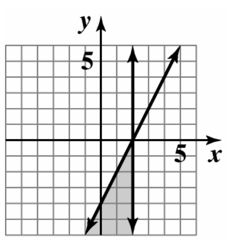
Slant asymptote: $n = m + 1$

$$f(x) = x + 2 + \frac{3}{x - 2}$$
$$y = x + 2$$



$$f(x) = \frac{x^2 - 1}{x - 2}$$

26.



$2x - y \geq 4$
$x \leq 2$

27. $f(x) = x^2 - 4x - 5$

$$x = \frac{-b}{2a} = \frac{4}{2} = 2$$

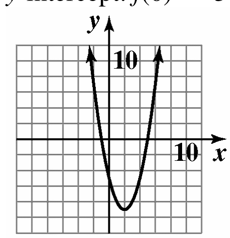
$f(2) = 2^2 - 8 - 5 = -9$

vertex: $(2, -9)$

x-intercepts:

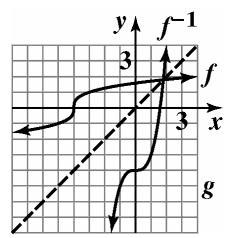
$$x^2 - 4x - 5 = 0$$
$$(x - 5)(x + 1) = 0$$
$$x = 5, -1$$

y-intercept: $f(0) = -5$

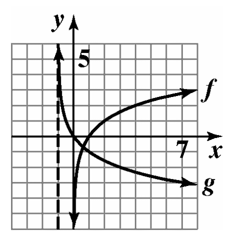


$f(x) = x^2 - 4x - 5$

28.



29.



30. $f(x) = -x^2 - 2x + 1, \quad g(x) = x - 1$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= -(x-1)^2 - 2(x-1) + 1 \\ &= -x^2 + 2x - 1 - 2x + 2 + 1 \\ &= -x^2 + 2\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= \left(-x^2 - 2x + 1\right) - 1 \\ &= -x^2 - 2x\end{aligned}$$

31. $f(x) = -x^2 - 2x + 1$

$$\frac{f(x+h)-f(x)}{h} = \frac{\left(-(x+h)^2 - 2(x+h)+1\right) - \left(-x^2 - 2x + 1\right)}{h}$$
$$= \frac{-x^2 - 2xh - h^2 - 2x - 2h + 1 + x^2 + 2x - 1}{h}$$
$$= \frac{-2xh - h^2 - 2h}{h}$$
$$= \frac{h(-2x - h - 2)}{h}$$
$$= -2x - h - 2$$

32. $AB - 4A = \begin{bmatrix} 4 & 2 \\ 1 & -1 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} - 4\begin{bmatrix} 4 & 2 \\ 1 & 1 \\ 0 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 14 & 18 \\ -1 & 3 \\ 15 & 5 \end{bmatrix} - \begin{bmatrix} 16 & 8 \\ 4 & -4 \\ 0 & 20 \end{bmatrix} = \begin{bmatrix} -2 & 10 \\ -5 & 7 \\ 15 & -15 \end{bmatrix}$$

33. $\dfrac{2x^2 - 10x + 2}{(x-2)(x^2+2x+2)} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+2x+2}$

$$2x^2 - 10x + 2 = A(x^2 + 2x + 2) + (Bx + C)(x - 2)$$
$$= Ax^2 + 2Ax + 2A + Bx^2 - 2Bx + Cx - 2C$$
$$= (A + B)x^2 + (2A - 2B + C)x + 2A - 2C$$

Thus we have the following system of equations.

$A + B = 2$
$2A - 2B + C = -10$
$2A - 2C = 2$

Add twice the first equation to the second equation.

$2A + 2B = 4$
$\underline{2A - 2B + C = -10}$
$4A + C = -6$

Add twice the resulting equation to the third equation.

$8A + 2C = -12$
$\underline{2A - 2C = 2}$
$10A = -10$
$A = -1$

Back-substitute to find B and C.

$2(-1) - 2C = 2$
$-2 - 2C = 2$
$-2C = 4$
$C = -2$
$-1 + B = 2$
$B = 3$

$$\frac{-1}{x-2} + \frac{3x-2}{x^2+2x+2}$$

34. $(x^3+2y)^5 = \binom{5}{0}(x^3)^5 + \binom{5}{1}(x^3)^4(2y) + \binom{5}{2}(x^3)^3(2y)^2 + \binom{5}{3}(x^3)^2(2y)^3 + \binom{5}{4}(x^3)(2y)^4 + \binom{5}{5}(2y)^5$

$= x^{15} + 5x^{12}(2y) + 10x^9(4y^2) + 10x^6(8y^3) + 5x^3(16y^4) + 32y^5$

$= x^{15} + 10x^{12}y + 40x^9y^2 + 80x^6y^3 + 80x^3y^4 + 32y^5$

35. $\sum_{i=1}^{50}(4i-25)$

$a_1 = 4(1) - 25 = -21$

$a_{50} = 4(50) - 25 = 175$

$S_{50} = \frac{50}{2}(-21+175) = 3850$

36. Find slope:

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{-2-6} = \frac{-2}{-8} = \frac{1}{4}$

Find equation:

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{4}(x - 6)$

$y - 3 = \frac{1}{4}x - \frac{3}{2}$

$y = \frac{1}{4}x + \frac{3}{2}$

37. Find the slope:

$x - 5y - 20 = 0$

$-5y = -x + 20$

$\frac{-5y}{-5} = \frac{-x}{-5} + \frac{20}{-5}$

$y = \frac{1}{5}x - 4$

$m = -\frac{1}{\frac{1}{5}} = -5$

Find the equation:

$y - y_1 = m(x - x_1)$

$y - (-2) = -5(x - 0)$

$y + 2 = -5x$

$y = -5x - 2$

38. $200 + 0.05x = 0.15x$

$200 = 0.1x$

$\frac{200}{0.1} = \frac{0.1x}{0.1}$

$2000 = x$

At $2000 in sales, the two earnings will be the same.

39. $2L + 2W = 300$
$L = W + 50$
Rearrange the equations and add:
$L + W = 150$
$\underline{L - W = 50}$
$2L = 200$
$L = 100$
$W = 50$
length: 100 yards, width 50 yards

40. $10x + 12y = 42$
$5x + 10y = 29$
Multiply second equation by –2 and add:
$10x + 12y = 42$
$\underline{-10x - 20y = -58}$
$-8y = -16$
$y = 2$
Back substitute:
$5x + 10(2) = 29$
$5x = 9$
$x = 1.8$
pen: \$1.80, pad: \$2

41. $s(t) = -16t^2 + 80t + 96$

a. $-16t^2 + 80t + 96 = 0$
$t^2 - 5t - 6 = 0$
$(t + 1)(t - 6) = 0$
$t = -1$ or $t = 6$
The ball will strike the ground after 6 seconds.

b. $t = \frac{-b}{2a} = \frac{-80}{-32} = \frac{5}{2}$ or 2.5
$S(2.5) = -16(2.5)^2 + 80(2.5) + 96 = 196$
The ball reaches a maximum height of 196 feet, 2.5 seconds after it is thrown.

42. $I = \frac{k}{R}$
$5 = \frac{k}{22}$
$k = 110$
$I = \frac{110}{10} = 11$
11 amperes

43. Let x represent the number of years after 1980. The data from 1980 and 2004 are represented as $(0, 33.2)$ and $(24, 20.9)$
Find slope: $m = \frac{20.9 - 33.2}{24 - 0} \approx -0.51$
Thus, $y = -0.51x + 33.2$
At this rate there may eventually be no smokers among U.S. adults.

44. $d = 10\sin\frac{3\pi}{4}t$

a. $|a| = |10| = 102a = 20$
The maximum displacement is 20 inches.

b. $f = \frac{\omega}{2\pi} = \frac{\frac{3\pi}{4}}{2\pi} = \frac{3}{8}$
The frequency is $\frac{3}{8}$ cycle per second.

c. period $= \frac{2\pi}{\omega} = \frac{2\pi}{\frac{3\pi}{4}} = \frac{8}{3}$
The time required for one oscillation is $\frac{8}{3}$ seconds.

45. $$\tan x + \frac{1}{\tan x} = \frac{\sin x}{\cos x} + \frac{1}{\frac{\sin x}{\cos x}} = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$
$$= \frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x}$$
$$= \frac{1}{\cos x \cdot \sin x}$$

46. $$\frac{1 - \tan^2 x}{1 + \tan^2 x} = \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}} \cdot \frac{\cos^2 x}{\cos^2 x}$$
$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$$
$$= \frac{\cos 2x}{1} = \cos 2x$$

47. $y = -2\cos(3x - \pi)$

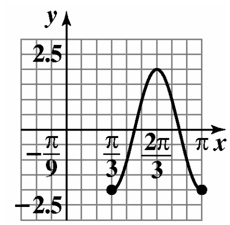
Amplitude: $|A| = |-2| = 2$

Period: $\frac{2\pi}{B} = \frac{2\pi}{3}$

Phase shift: $\frac{C}{B} = \frac{\pi}{3}$

$\left(\frac{\pi}{3}, -2\right)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{2\pi}{3}, 2\right)$, $\left(\frac{5\pi}{6}, 0\right)$,

$(\pi, -2)$



$y = -2\cos(3x - \pi)$

48. $4\cos^2 x = 3$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

The solutions in the interval $[0, 2\pi)$ are

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

49. $2\sin^2 x + 3\cos x - 3 = 0$

$$2(1 - \cos^2 x) + 3\cos x - 3 = 0$$

$$2 - 2\cos^2 x + 3\cos x - 3 = 0$$

$$2\cos^2 x - 3\cos x + 1 = 0$$

$$(2\cos x - 1)(\cos x - 1) = 0$$

$2\cos x - 1 = 0$ or $\cos x - 1 = 0$

$2\cos x = 1$ $\quad$ $\cos x = 1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ or $x = 0$

The solutions in the interval $[0, 2\pi)$ are

$0, \frac{\pi}{3}$, and $\frac{5\pi}{3}$.

50. $\cot \cos^{-1} -\frac{5}{6}$

If $\cos\theta = -\frac{5}{6}$, θ lies in quadrant II.

$$\cos\theta = -\frac{5}{6} = \frac{x}{r} = \frac{-5}{6}$$

$$x^2 + y^2 = r^2$$

$$(-5)^2 + y^2 = 6^2$$

$$25 + y^2 = 36$$

$$y^2 = 11$$

$$\cot \cos^{-1} -\frac{5}{6} = \frac{x}{y} = \frac{-5}{\sqrt{11}} = -\frac{5\sqrt{11}}{11}$$

51. $r = 1 + 2\cos\theta$

Check for symmetry:

Polar Axis	The Line $\theta = \frac{\pi}{2}$	The Pole
$r = 1 + 2\cos(-\theta)$	$-r = 1 + 2\cos(-\theta)$	$-r = 1 + 2\cos\theta$
$r = 1 + 2\cos\theta$	$r = -1 - 2\cos\theta$	$r = -1 - 2\cos\theta$

Graph is symmetric with respect to the polar axis.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	3	2.73	2	1	0	–0.73	–1

Use symmetry to obtain the graph.