Physics Research and Technology

Quantum Gravity

Theory and Research

Brandon Mitchell
Editor

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PHYSICS RESEARCH AND TECHNOLOGY

QUANTUM GRAVITY

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BRANDON MITCHELL
EDITOR

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CONTENTS

Preface		**vii**
Chapter 1	Semi-Classically Corrected Gravity and Numerical Relativity *Anindita Dutta and Arundhati Dasgupta*	**1**
Chapter 2	Generalized Quantum Entanglement Family in Connection to Black Holes and Nanotechnology *Leila Marek Crnjac*	**33**
Chapter 3	A Particle-Like Description of Planckian Black Holes *Euro Spallucci and Anais Smailagic*	**49**
Chapter 4	Quantum Gravity Corrections to Gauge Theories with a Cutoff Regularization *G. Cynolter and E. Lendvai*	**73**
Related Nova Publications		**95**
Bibliography		**99**
Index		**157**

PREFACE

This book provides new research on quantum gravity. Chapter One reviews the semi-classical corrections to the metric of a spherically symmetric static black hole space-time. Chapter Two discusses quantum entanglement in connection to black holes and nanotechnology. Chapter Three provides a particle-like description of Planckian black holes. Chapter Four analyzes quantum gravity corrections to gauge theories with a cutoff regularization.

Chapter 1 – The authors review the semi-classical corrections to the metric of a spherically symmetric static black hole space-time. They observe that at first order, a twist vector is introduced breaking the static nature of the metric. Spherical symmetry is also broken. The authors investigate the effect if any, and the magnitude of the effect on astrophysical phenomena using numerical relativity. For this purpose, they review the methods and results for scalar gravitational collapse and binary black hole collisions. Motivated from this we investigate consequences of the non-static metrics for a scalar field in a black hole background and demonstrate numerical simulation of a toy model.

Chapter 2 – The authors present a new entanglement relativity theory by dividing Hardy's entanglement *P(H)*= $\phi^3\ \phi^n$ into two parts, a global part given by ϕ^3 and a local part ϕ^n. For different *n* we obtain a generalized quantum entanglement family ϕ^3, ϕ^4, ϕ^5, ϕ^6. They introduce the Fibonacci-like dimension sequence as an infinite geometric sequence and we extend the Fibonacci-like dimension sequence into the negative side.

The present work makes a leap from E-Infinity dissection of Einstein's equation into two parts, the ordinary energy $E(O) \approx mc^2/22$ plus the dark energy $E(D) \approx mc^2(21/22)$, to the connection by the E-Infinity scenario of the

Kerr black hole. The connection between the E-Infinity theory with the spinning Kerr black hole leads to a paradox. The ordinary and dark energy of the universe could be used as a guiding principle in the design of a nano-Casimir dark energy reactor.

Chapter 3 – In this paper the authors abandon the idea that even a "quantum" black hole, of Planck size, can still be described as a *classical*, more or less complicated, geometry. Rather, we consider a genuine quantum mechanical approach where a Planckian black hole is, by all means, just another "particle", even if with a distinguishing property: its wavelength increases with the energy. The horizon dynamics is equivalently described in terms of a particle moving in gravitational potential derived from the horizon equation itself in a self-consistent manner. The particle turning-points match the radius of the inner and outer horizons of a charged black hole. This classical model pave the way towards the wave equation for a truly quantum black hole. The authors compute the exact form of the wave function and determine the energy spectrum. Finally, they describe the classical limit in which the quantum picture correctly approaches the classical geometric formulation. The authors find that the quantum-to-classical transition occurs far above the Planck scale.

Chapter 4 – The gravitational waves recently observed by the LIGO collaboration is an experimental evidence that the weak field approximation of general relativity is a viable, calculable scenario. As a non-renormalizable theory, gravity can be successfully considered as an effective quantum field theory with reliable, but limited predictions. Though the influence of gravity on gauge and other interactions of elementary particles is still an open question. In this chapter the authors calculate the lowest order quantum gravity contributions to the QED beta function in an effective field theory picture with a momentum cutoff. They use a recently proposed 4 dimensional improved momentum cutoff that preserves gauge and Lorentz symmetries.

The authors find that there is a non-vanishing quadratic contribution to the photon 2-point function but after renormalization that does not lead to the running of the original coupling. They comment on corrections to the other gauge interactions and Yukawa couplings of heavy fermions. They argue that gravity cannot turn gauge interactions asymptotically free.

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Chapter 1

SEMI-CLASSICALLY CORRECTED GRAVITY AND NUMERICAL RELATIVITY

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Abstract

We review the semi-classical corrections to the metric of a spherically symmetric static black hole space-time. We observe that at first order, a twist vector is introduced breaking the static nature of the metric. Spherical symmetry is also broken. We investigate the effect if any, and the magnitude of the effect on astrophysical phenomena using numerical relativity. For this purpose, we review the methods and results for scalar gravitational collapse and binary black hole collisions. Motivated from this we investigate consequences of the non-static metrics for a scalar field in a black hole background and demonstrate numerical simulation of a toy model.

1. Introduction

With the discovery of gravity waves, the importance of numerical methods in real observational phenomena has been proved. With the increasing focus on complexity in nature, numerical method will acquire further importance in

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providing solutions to non-linear phenomena. In this paper we explore some numerical methods, review some aspects of numerical relativity for [1] classical gravity. However, our main aim is to find quantum gravity corrections to non-linear phenomena in astrophysics. It has been a challenging task to find observational phenomena for quantum gravity. The length scale for quantum gravity effects to acquire importance in a physical process is $l_p = 10^{-33}$ cm, much smaller than the length scales probed in present day accelerators. However, there are searches for indirect evidences and analogue models of gravity which simulate quantum gravity effects [2]. In particular in [3], it was pointed out that quantum gravity effects can produce observable effects in astrophysical phenomena. Semiclassical fluctuations around unstable orbits in Schwarzschild space-time can grow to observable scales. Whereas such instabilities with quantum gravity origins are difficult to verify in an accretion disk scenario, some other consequences of the quantum gravity corrections found in [3] have to be further examined.

Quantum gravity has substantially developed at this time. In particular Loop Quantum Gravity (LQG) , a generalization of the canonical quantization techniques to new variables [4] has yielded quite a few new results. The semiclassical states in this theory can be used to study classical phenomena. The typical semiclassical state used in quantum theory are the coherent states, used in Electrodynamics to represent classical light propagation. These states are thus useful semiclassical states in a quantized theory. In [3] LQG coherent states were taken and studied. These coherent states are peaked at the classical expectation values of the operators. Due to the non-abelian nature of the coherent states the operator expectation values are corrected beyond the classical values. These corrections are origins of ‘quantum corrections’ of the metric. Even though the classical solution is spherically symmetric, the coherent state is not constrained to be spherically symmetric, and thus the quantum fluctuations can bring in an asymmetry. In this article we discuss the corrections, and the implications of these corrections for collapsing black holes. In fact all metrics get spontaneously corrected due to semiclassical fluctuations. To be precise the corrections are proportional to l_p^2/r_g^2 where r_g is the Schwarzschild radius of the event horizon for a black hole. Thus corrections are rather infinitesimal, but due to non-linearity of Einstein’s evolution equations, these might have non-trivial effects on the physics of the system.

The theoretical collapse of matter in a spherical symmetric way to form such a black hole using numerical methods has been studied by various physicists [5, 6]. In a realistic situation, the collapse of a star is not spherically symmetric, and non-spherical collapse has been studied in the context of rotating axisymmetric collapse [7]. For detailed introduction to numerical relativity methods see [8, 9, 10] We discuss the nature of the 'semi-classically' corrected metric in [3] and how quantum fluctuations might induce non-spherical collapse. Perturbative and semiclassical corrections have been studied in [11] and non-perturbative corrections in reduced phase space have been studied in [12, 13, 14]. What is interesting is a common emergence of mass gap in the semiclassical/quantum corrected collapse, though the nature of the Choptuik scaling retains its universality. Our corrections are different, [3], and we find difficult to implement as the corrections break spherical symmetry.

In this article, we review three aspects of this approach to studying astrophysics (i) Quantum Corrections to classical gravity, (ii) Numerical relativity (iii) Quantum corrected numerical relativity. In addition we probe the nature of the modification of the metric motivated from the particular corrections obtained in [3]. In section II we discuss the corrections as that obtained in [3] and the motivations for studying such quantum corrected metrics. In section III we discuss the quantum corrections as computed in the spherically symmetric sector of the theory. In section IV we discuss numerical relativity, in section V we discuss computation of scalar field propagation in quantum corrected metrics and in quantum corrected gravitational collapse. We also discuss the difficulties in numerical computation of non-spherical collapse. Finally in section V we present some new results in scalar field propagation due to quantum corrections as observed in [3] in a toy model.

2. Quantum Corrections

In the study of quantum corrected collapse early efforts include the work by [11] where 'back reaction' effects of quantum energy momentum tensors and semiclassical corrections were studied. It was found in the 1970s that the black hole radiates particles in a thermal spectrum [15]. This process was then related to the conformal anomaly of a scalar field in the black hole background. The anomaly appears due to creation of a trace term of the energy momentum tensor; a violation of the scale invariance of the massless scalar field action. This is a quantum breaking of the classical conformal symmetry of the scalar action. Using this 'quantum' energy momentum tensor, and changed initial data, the numerical evolutions revealed that there was a 'minimum mass' of the black hole, identified as a mass gap. Further computations involved quantum corrected time evolution equations of scalar fields obtained from LQG (Loop Quantum Gravity) motivated quantization of spherically symmetric gravity [12, 13]. Again the presence of a mass gap was observed.

2.1. Loop Quantum Gravity Coherent States

The coherent states are useful semiclassical states in a quantum theory. In case of the simple harmonic oscillator, the coherent state can be formulated as the eigenstate of the annihilation operator ($a|z>= z|z>$). The same coherent states also appear as the Kernel of a transformation from the Hilbert space $L^2(R)$ to the Segal-Bergmann representation of the wave functions [16].

Using this latter definition of the coherent state as a Kernel, Hall [16] identified generalized coherent states for any SU(2) Hilbert space. In [17] these SU(2) coherent states were used to describe semiclassical states in quantum gravity, as the loop quantum gravity has a SU(2) Hilbert space.

Quantum gravity is a difficult theory, with various technical problems. A straightforward path-integral quantization or a canonical quantization of gravity yields a highly constrained system with computational problems. Loop Quantum Gravity (LQG) is a derived version of canonical gravity, where the theory has advanced towards a complete quantization of space-time. In a typical canonical slicing of the space-time metric a fiducial 'time-like' coordinate is used. The intrinsic metric of the three slices is described as q_{ab} (a, b =1..3) and the time-time metric component labeled as the Lapse N and the time-space components are labelled as the shift N_a. The lapse and the shift are Lagrange multipliers and are used to impose constraints, the Hamiltonian and the diffeomorphism constraints, on the phase space. The dynamics lies in the q_{ab} and its corresponding momentum π_{ab}, the canonical variables of the theory. However, LQG redefines these usual canonical variables using the tangent space to the manifold. LQG uses triads e_a^I (I is a tangent space index, I=1..3) which are also known as soldering forms, such that $e_a^I e_{bI} = q_{ab}$, and the corresponding affine connection Γ_a^I.

The phase space of loop quantum gravity is described thus [4]

$$A_a^I = \Gamma_a^I - \tilde{\beta}\, K_{ab}\, e^{Ib} \quad E_I^a = \frac{1}{\tilde{\beta}}\,(\det e)\, e_I^a \tag{1}$$

(e_a^I are the usual triads, K_{ab} is the extrinsic curvature, Γ_a^I the associated spin connection, $\tilde{\beta}$ the one parameter ambiguity in this redefinition and is known as the Immirzi parameter which we set to 1 for this paper.) The A_a^I is known as the Gauge field, and has an 'internal index' I which transforms in the tangent space symmetry group $SO(3)$, isomorphic to the group SU(2). This would be typical for a Yang-Mills gauge field. The E_a^I is an electric field and this transforms in the fundamental representation of the gauge group. The quantization of the Poisson algebra of these variables is done by smearing the connection along one dimensional edges e of length δ_e of a graph Γ to get holonomies $h_e(A)$. The triads are smeared in a set of 2-surface decomposition of the three dimensional spatial slice to get the corresponding momentum P_e^I.The 'holonomy' $h_e(A) = \mathcal{P}\exp(\int_e A.dx)$ and the momentum $P_e^I = \int_S *E^I$ as integrals over an edge e and 2-surface S are defined as functions of the gauge connection and the triads. These variables have a well defined Poisson bracket.

The algebra is then represented in a kinematic 'Hilbert space', in which the physical constraints can be 'formally' realized. Once the phase space variables have been identified, one can write a coherent state for these [16] i.e. minimum uncertainty states peaked at classical values of h_e, P_e^I for one edge of the graph [17]. The h_e is a unitary matrix and the $e^{iT^I P_e^I}$ form a Hermitian matrix with T^I being a SU(2) generator matrix (I=1..3). A complexified phase space for the SU(2) variables is built using the matrices $e^{iT^I P_e^I} h_e$. The SU(2) Hilbert space states can be functions of the 'position' h_e or 'momentum' P_e^I. The coherent state which appears as the Kernel of the transformation of the Hilbert space to the Segal Bergman representation (defined on the complexified phase space) is a function of the complexified matrices [17].

The coherent state in the momentum representation for one edge is defined to be

$$|\psi^{\tilde{t}}(g_e)> = \sum_{jmn} e^{-\tilde{t}j(j+1)/2}\pi_j(g_e)_{mn}|jmn> \tag{2}$$

In the above g_e is a complexified classical phase space element $e^{iT^I P_e^{I\mathrm{cl}}/2}\, h_e^{\mathrm{cl}}$, (the $P_e^{I\mathrm{cl}}$ and the h_e^{cl} represent classical momenta and holonomy obtained by embedding the edge in the classical metric). The $|jmn>$ are the basis spin network states given by $\pi_j(h)_{mn}$, which is the jth representation of the SU(2) element h_e [4]. The j is the quantum number of the SU(2) Casimir operator in that representation, and m, n represent azimuthal quantum numbers which run from $-j..j$. Similarly, $(2j+1)\times(2j+1)$ dimensional representations of the 2×2 matrix g_e are denoted as $\pi_j(g_e)_{mn}$. The coherent state is precisely peaked with maximum probability at the h_e^{cl} for the variable h_e as well as the classical momentum $P_e^{I\mathrm{cl}}$ for the variable P_e^I. The fluctuations

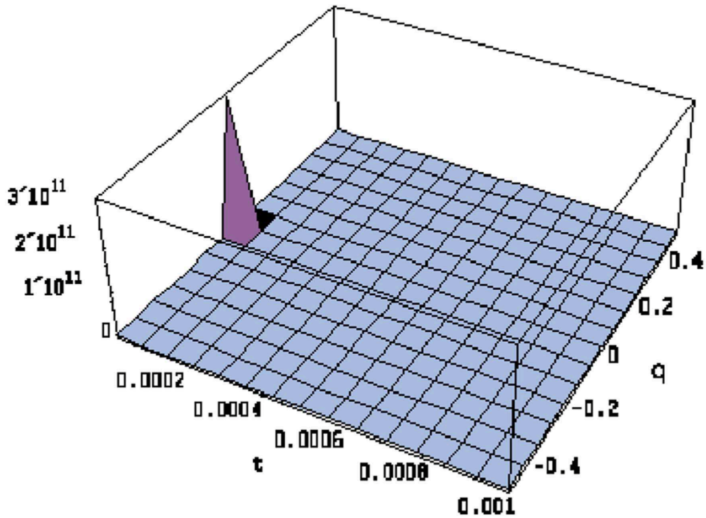


Figure 1. A SU(2) Coherent State.

about the classical value are controlled by the parameter $\tilde{t}$ (the semi classicality parameter). This parameter is given by l_p^2/a where l_p is Planck's constant and a a dimensional constant which characterizes the system. The coherent state for an entire slice can be obtained by taking the tensor product of the coherent state for each edge which form a graph Γ,

$$\Psi_\Gamma = \prod_e \psi_e^{\tilde{t}}. \tag{3}$$

Thus we are considering a semiclassical state, which is a state such that expectation values of operators are closest to their classical values. The information of the classical phase space variables are encoded in the complexified SU(2) elements labeled as g_e. The fluctuations over the classical values are controlled by the semiclassical parameter $\tilde{t}$.

For the purposes of this discussion the semiclassical parameter is taken as $\tilde{t} = \frac{l_p^2}{r_g^2} = 10^{-2n}$, where $r_g = 10^n l_p$ is the radius of the horizon of the Schwarzschild black hole. $\tilde{t} \to 0$ represents the 'classical limit' and the wave function is nicely peaked at the classical values of h_e and P_e^I. The fluctuations over the classical values can be obtained as a series in powers of $\tilde{t}$. (We use the $\tilde{t}$ notation to avoid confusing this with the time coordinate.) Thus

$$< \psi^{\tilde{t}}|\hat{P}_e^I|\psi^{\tilde{t}} >= P_e^I \left(1 + \tilde{t}\tilde{f}(P)\right) \tag{4}$$

where $\tilde{f}(P)$ is a function and is the first order correction to the classical value. The details of the function $\tilde{f}(P)$ can be found in [3] and as a function of the gauge invariant momentum $P_e = \sqrt{P_e^I P_e^I}$ it is:

$$\tilde{f}(P_e) = \frac{1}{P_e}\left(\frac{1}{P_e} - \coth(P_e)\right) \tag{5}$$

In [3], a coherent state was defined on a flat slicing of the Schwarzschild metric, and one can use the same formulation to estimate the corrections to the metric in Schwarzschild coordinates. For this, one starts with a metric defined in Lemaitre coordinates:

$$ds^2 = -d\tau^2 + \frac{dR^2}{\left[\frac{3}{2r_g}(R-\tau)\right]^{2/3}} + \left[\frac{3}{2}(R-\tau)\right]^{4/3} r_g^{2/3}(d\theta^2 + \sin^2\theta d\phi^2) \tag{6}$$

These are related to the Schwarzschild coordinates t, r using the following transformations:

$$\sqrt{\frac{r}{r_g}}dr = (dR \pm d\tau) \tag{7}$$

$$dt = \frac{1}{1-f^2}(d\tau \pm f dR) \quad f = \left(\frac{2r_g}{3(R-\tau)}\right)^{2/3} \tag{8}$$

The constant $\tau = \tau_c$ surface then has a flat metric, we use the coordinate r' as defined in [3] to obtain the induced metric. This r' coordinate is such that $dr'/dR = 1/[(3/2r_g)(R-\tau_c)]^{1/3}$. In that set of coordinates, the momenta $P_r'^I$ (momentum in the radial direction), P_θ^I (momentum in the θ direction) and P_ϕ^I (momentum in the ϕ direction) were calculated in [18]. The two surface bits used to compute the momenta were bits of 2-spheres, and in the limit the area of these bits went to zero,

$$P_e^I = S_e E_e^I \tag{9}$$

where S_e is the area of the two surface. Given that,

$$qq^{ab} = E^{aI}E^{bI} = \frac{P_{e_a}^I}{S_{e_a}}\frac{P_{e_b}^I}{S_{e_b}} \tag{10}$$

we get

$$q = \det\frac{P^I_{e_a}}{S_{e_a}} = P \tag{11}$$

Thus

$$q^{ab} = \frac{1}{P}\frac{P^I_{e_a}}{S_{e_a}}\frac{P^I_{e_b}}{S_{e_b}} \tag{12}$$

Thus calculating

$$< \psi|\frac{\hat{P}^I_{e_a}}{S_{e_a}}\frac{\hat{P}^I_{e_b}}{S_{e_b}}|\psi > \tag{13}$$

should be enough to calculate corrections to the metric.

For the specific purpose of calculating corrections to the unstable orbits, we find the corrections to the radial metric.

$$q^{r'r'} = \frac{1}{P}\left[\left(\frac{P_{e'_r}}{S_{e'_r}}\right)^2 + 2\tilde{t}\tilde{f}\left(\frac{P_{e'_r}}{S_{e'_r}}\right)\frac{P_{e'_r}}{S_{e'_r}}\right] \tag{14}$$

where $P_{e'_r} = \sqrt{P^I_{e'_r}P^I_{e'_r}}$ is the gauge invariant momentum. In the limit the $S_{e'_r} \to 0$, the

$$P_{e'_r} = \frac{2r'^2\sin\theta\delta\theta\delta\phi}{r_g^2} \tag{15}$$

and $S_{e'_r} = 2\delta\theta\delta\phi$. Needless to say in this approximation, we correctly recover $q^{r'r'} = 1 + O(\tilde{t})$. Further, as we consider regions $r > r_g$, the $\tilde{f}(P_{r'}/S_{e'_r}) = 1/(P_{r'}/'S_{e'_r}) + ..$ [3] gives a fractional contribution to the formulas. The density can be integrated or smeared over a small surface, the results of the smeared value of P_{e_r} can be found in [3]. The surfaces over which the density is smeared are two dimensional pieces of a sphere, intersected by an edge of the original graph. As $\tilde{f}(P)$ is a non-linear function of $\sin\theta$, the quantum fluctuations' dependence on $\sin\theta$ is non-trivial, and breaks spherical symmetry. Note that only the $q^{r'r'}$ component of the metric gets corrected, as the inner product of the momenta $P_{e_a} \cdot P_{e_b} = 0$ even at the quantum level and thus the cross terms such as $q^{r'\theta} = 0$ even at the quantum level. As in this quantization process, the degrees of freedom, or the quantum variables are the intrinsic metrics on the three slice, only these get their expectation values evaluated in the coherent states. The $q^{R\tau}$ represent the Shift metric in ADM parlance and thus remain as unquantized gauge degrees, or Lagrange multipliers in the system. We have thus discussed the correction to the induced metric in the $\tau = \tau_c$ slice in the Lemaitre slicing of the Schwarzschild space-time. We then perform a coordinate transformation to find the corrections to the metric in the Schwarzschild coordinates. In this case the coordinate transformations are

$$g^{tt} = \frac{dt}{d\tau}\frac{dt}{d\tau}g^{\tau\tau} + \frac{dt}{dR}\frac{dt}{dR}g^{RR} \tag{16}$$

$$g^{rr} = \frac{dr}{d\tau}\frac{dr}{d\tau}g^{\tau\tau} + \frac{dr}{dR}\frac{dr}{dR}g^{RR} \tag{17}$$

$$g^{rt} = \frac{dr}{d\tau}\frac{dt}{d\tau}g^{\tau\tau} + \frac{dr}{dR}\frac{dt}{dR}g^{RR} \tag{18}$$

We have

$$\frac{dt}{d\tau} = \frac{1}{1-f^2} \qquad \frac{dt}{dR} = \pm\frac{1}{1-f^2}\left(\frac{2r_g}{3(R-\tau)}\right)^{2/3} \tag{19}$$

$$\frac{dr}{dR} = \sqrt{\frac{r_g}{r}} \qquad \frac{dr}{d\tau} = \pm\sqrt{\frac{r_g}{r}} \tag{20}$$

This gives, in particular the corrections to g^{tr} as

$$g^{tr} = \pm 2\frac{1}{1-f^2}\left(\frac{r_g}{r}\right)^{3/2}\ \tilde{t}\,\tilde{f}\left(\frac{P_{e_r}}{S_{e_r}}\right) \tag{21}$$

We find that a non-zero g^{rt} term is created due to the non-cancellation of the two terms in equation (18). This is due to the new semi-classical corrections $\tilde{f}(P)$ not present in the classical Lemaitre metric. The reason we are giving this in details is because the quantum gravity effects have created a g^{rt} term in the corrected metric which normally wouldn't have been there. The g_{rt} of the inverse metric is $-g^{tr}/(g^{tt}g^{rr})$ and is thus given by the same rhs of (21). Clearly the cross term diverges at the horizon, but that is also a signature of the failure of the coordinates at that point. The corrections to the cross terms in the metric $g_{t\phi}, g_{t\theta}, g_{r\phi}, g_{r\theta}, g_{\theta\phi}$ are not there, as by choice of gauge in the internal directions the cross terms like $P^I_{e_r}P^I_{e_\theta} = 0$ to order $\tilde{t}$ in the quantum fluctuations.

2.2. Static Metrics, Spherical Metrics

In most discussions of quantum corrected collapse one addresses the spherically symmetric metrics, and those which are static. Let us recollect what the static metric is and what we mean by spherical symmetry. A metric is said to be stationary if the metric has isometries whose orbits are asymptotically time-like. This signifies the existence of a Killing vector ξ which generates these isometries.

$$\mathcal{L}_\xi g_{ab} = 0 \tag{22}$$

If in addition there exists spatial hyper surfaces Σ which are orthogonal to the Killing orbits, the space-time is said to be static. This also translates to the condition of hyper surface orthogonality using Frobenius theorem [19].

$$\xi_{[a} \nabla_b \xi_{c]} = 0 \tag{23}$$

If the Killing parameter is used as a time coordinate 't', ($\xi = \frac{\partial}{\partial t}$) and the space-like hyper surfaces orthogonal to the orbits of the Killing vector are described using coordinates of x^1, x^2, x^3. The metric appears as

$$ds^2 = -g_{tt}\, dt^2 + g_{ij}\, dx^i dx^j \tag{24}$$

The spherical symmetry is imposed by requiring that the metric has SO(3) isometries, and that implies component g_{tt} is a function of r . The g_{ij} can be written in spherical coordinates as:

$$g_{rr}(r) dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{25}$$

2.3. Unstatic

Relaxing the requirement of hyper surface orthogonality, one obtains the 'stationary metric'

$$ds^2 = -\left(g_{tt} dt - \omega_i dx^i\right)^2 + h_{ij} dx^i dx^j \tag{26}$$

with the introduction of a 'twist vector' ω_i. The Kerr metric has a non-zero ω_ϕ and this shows the origin of rotation.

It is therefore interesting that in [3], the first order correction to the metric due to semiclassical corrections was found to generate a twist term in the metric. In [3] the g_{tr} term in the metric was exactly shown to have a form:

$$g_{tr} = \pm \frac{1}{1 - \frac{r_g}{r}} \left(\frac{r_g}{r}\right)^{3/2} \frac{l_p^2}{r_g^2} \tilde{f} \tag{27}$$

where $r_g = 2GM$, l_p is the Planck length and $\tilde{f}$ is a function of densities triads used to define the LQG variables.

Though the metric correction diverges at the horizon, this is a reflection of the badness of the coordinates at the horizon. As the metric correction seems to be infinite at the horizon, the fluctuation in this coordinates is finite at the horizon. Curvature invariants though are expected to receive infinitesimal corrections.

Thus we can sufficiently conclude that the first order fluctuations of the metric break the static nature of the Schwarzschild metric.

2.4. Unspherical

The function $\tilde{f}$ in [3] was found to be

$$\tilde{f} = \frac{1}{P_e}\left(\frac{1}{P_e} - \coth(P_e)\right) \tag{28}$$

where $P_e = \frac{r^2}{r_g^2}\sin\theta$ and thus a function of θ. The origin of this area of two sphere dependent term comes from the densitized triads of the LQG variables. The θ dependence in (27) thus breaks the spherical symmetry of the original metric. At this order of the semiclassical fluctuations no other term is created [3], however this single correction term spontaneously breaks the spherical symmetry and static nature of the metric.

2.5. Scalar Propagation in Unstatic, Unspherical Metrics

Non-spherical collapse has been considered previously, e.g. oblate symmetry etc [20]. Whereas we are quite used to the study of non-static metrics, mainly in the study of stationary axisymmetric metrics, the above new term does not correspond to the usual Kerr metric. This term corrects the metric as well as the curvature (27). Thus the correction is a tangible gauge invariant quantity. However as noticed in [3], this correction makes a difference to particles propagating along unstable orbits in the Schwarzschild space-time. We study the effect on a scalar field propagating in a 'quantum corrected' black hole background. For this we use the Klein-Gordon equation in a Schwarzschild black hole back ground with the corrected g_{tr} term (27). If we introduce the g_{tr} term in the metric, the determinant of the metric is

$$g = g_{tt}g_{rr}g_{\theta\theta}g_{\phi\phi} - g_{rt}^2 g_{\theta\theta}g_{\phi\phi}.$$

As the correction is $O(g_{rt}^2)$ we can neglect this correction as it is second order in the semiclassical parameter $\tilde{t}$.

$$
\begin{aligned}
\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) &= 0 \quad (29)\\
\frac{1}{\sqrt{-g}}\left[\partial_t\left(\sqrt{-g}g^{tt}\partial_t\phi\right)+\partial_r\left(\sqrt{-g}g^{rr}\partial_r\phi\right)+\partial_t(\sqrt{-g}g^{tr}\partial_r\phi)\right.&\\
\left.+\partial_r(\sqrt{-g}g^{rt}\partial_t\phi)+\partial_\theta(\sqrt{-g}g^{\theta\theta}\partial_\theta\phi)+\partial_\phi(\sqrt{-g}g^{\phi\phi}\partial_\phi\phi)\right] &= 0 \quad (30)\\
-\partial_t^2\phi+2g^{tr}\partial_t\partial_r^*\phi+\frac{1}{r^2}\partial_r^*(g^{tr}r^2)\partial_t\phi+\partial_{r^*}^2\phi+\left((1-\frac{2GM}{r})\right)\nabla_{\theta\phi}\phi &= 0 \quad (31)
\end{aligned}
$$

where r^* is the Eddington-Finkelstein coordinate such that

$$\frac{dr^*}{dr}=\frac{1}{(1-2GM/r)},$$

and we have used $g^{tt}=1/(1-2GM/r)$ and $\nabla_{\theta\phi}$ has the angular derivative terms.

Thus as $r\rightarrow 2GM$ the equation of motion of a scalar field is similar to a scalar field in a flat background with a 'shift' term, which we label as β as the angular derivatives drop out. It is interesting that as $r\rightarrow 2GM$ the g^{tr} as in (27) becomes finite, as the ratio of $\tilde{t}/(1-2GM/r)$ is ratio of two small numbers. Note that as $\tilde{t}=l_p^2/(2GM)^2$ the ratio is of order 1 as $1-2GM/r\approx l_p^2/(2GM)^2$ or $r\approx(2GM)^3/((2GM)^2-l_p^2)$, or $r\approx 2GM(1+l_p^2/(2GM)^2)$. The non-spherical θ dependent $\tilde{f}$ derivative does not make a difference at this order of the approximation. If we observe equation (31) then we find that the ∂_θ terms are proportional to $(1-2GM/r)$, and thus vanish in the near horizon limit. If we keep the θ dependence in $\tilde{f}(P)$ then it contributes algebraically in the solution to the equation of (31) through the g^{tr} term as the derivative in θ of the function will similarly vanish. Thus we ignore this angular dependence in the next discussion.

2.6. The Strain

It can be shown that the above correction (27) causes an effective strain in the metric. This is similar to the 'strain' caused when a gravity wave passes through a given background. The strain due to a 'fluctuation' is:

$$e_{ij}=\frac{1}{2}\left(g'_{ij}-g_{ij}\right) \tag{32}$$

Given that the g^{tr} computed in (27) can be inverted, the 'fluctuation strain' is:

$$e_{tr} = \pm\frac{1}{2}g^{tr} \tag{33}$$

as $g_{tr} = g^{tr}g_{tt}g_{rr}$. If we are far away from the black hole, as would be on Earth. location of LIGO, $\tilde{f}(P_e) = \frac{1}{P_e} = \frac{r_g^2}{r^2\sin\theta}$. Using the specifications of the 'merged black hole' in LIGO; $r = 1.3$ billion light years$=1.23\times10^{25}$m, $r_g = 2GM = 1.57\times10^{22}$ m, this strain is computed to be

$$e_{tr} = 7.67\times10^{-125}\mathrm{cosec}\theta \tag{34}$$

(The order 10^{-125} suggests a relation to the cosmological constant, and it might be that the cosmological constant arises due to semiclassical fluctuations of the cosmological metric. This interesting aspect has to be investigated further.)

This would correspond to the H_1 mode in the linear 'even' perturbations about a Schwarzschild black hole [24]. This number is way smaller than the observed strain 10^{-21}, the amplitude of the gravity wave strain observed in LIGO. However, it has the same magnitude as observed in similar calculations of quantum corrections to gravity wave dispersion [14].

As we see that due to the nature of the correction, there is a $\mathrm{cosec}\theta$ in the strain, which is rather strange. This term appears due to the $1/P_e$ form in the correction, which cannot be expanded in a spherical harmonic anymore. Whereas, this might be due to the non-abelian nature of the coherent states, in a actual physical computation, this would be a peculiar correction. We thus use in the next discussion the quantization, which is in the spherically reduced sector of gravity, and examine the corrections in that sector.

2.7. Holonomy Correction

The corrections to the holonomy operator as seen from [17], is explicitly

$$\delta h_{AB} = e^{-\tilde{t}/16}e^{-p^2/\tilde{t}}\frac{z_0}{\sinh(z_0)}\frac{\sinh p}{p}\left[g_{AB}\cosh\left(\frac{z_0}{2}\right) + (\tau_j g)_{AB}\frac{tr(\tau_j g g^\dagger)}{2\sinh(z_0)}\sinh\left(\frac{z_0}{2}\right)\right] \tag{35}$$

where p is a momentum, and $z_0 = e\,\tilde{t}\,p$, where p corresponds to invariant momentum for a given edge of length e. The corrections tend to zero exponentially as $\tilde{t}\to 0$, nevertheless, a correction exists, and contributes for a possible probe for quantum gravity.

3. The Spherical Coherent State

We take the spherically symmetric reduced phase space used in [25] to describe the phase space of this system. We then derive the complexifier coherent states using the Hall-Thiemann prescription. These are also naturally eigenstates of the annihilation operator. We then see if the system is a solution to the Hamiltonian constraint.

The phase space for the spherically symmetric sector is:

$$\vec{A} = A_3\tau_3 dx + (A_1\tau_1 + A_2\tau_2)d\theta + (-A_2\tau_1 + A_1\tau_2)\sin\theta d\phi + \tau_3\cos\theta d\phi \quad (36)$$

$$\vec{E} = E_3\tau_3 \sin\theta\frac{\partial}{\partial x} + (E_1\tau_1 + E_2\tau_2)\sin\theta\frac{\partial}{\partial\theta} + (-E_2\tau_1 + E_1\tau_2)\frac{\partial}{\partial\phi} \quad (37)$$

where we have used the form described in [25]. $\tau^i = -\frac{\imath}{2}\sigma^i$ where σ^i are Pauli matrices. These forms are not unique and the choice of gauge in the internal directions can give different forms of the above, e.g. see [14, 3]. The Symplectic structure for this is [25]

$$\Omega = \frac{L_0}{2G}\left(2dA_1 \wedge dE_1 + 2dA_2 \wedge dE_2 + dA_3 \wedge dE_3\right) \quad (38)$$

where L_0 is the length of the fiducial 'radius' of the sphere. We have set the Barbero-Immirzi parameter to 1. The Gauss Constraint which preserves the internal gauge symmetry is

$$G_{ab} = K^j_{[a}e^j_{b]} = A_1E_2 - E_1A_2 = 0$$

This was solved in [25] by setting $E_1 = 0$, and $A_1 = 0$. Classically the non-zero components of the solution is found to be

$$A_2 = \sqrt{\frac{(2m-s)}{s}} \quad E_2 = \sqrt{s(2m-s)} \quad A_3 = -\frac{m}{s^2} \quad E_3 = s^2 \quad (39)$$

where s is along the Hamiltonian flow lines and can be identified as r in our notation. This represents the 'interior' of a black hole.

The holonomy for these can be defined by taking a graph with edges along the radial direction, e_r, and then along the latitudes and longitudes of a sphere for each r. The graph was used in [3], and also in [25]. However, we use the conventions of [25],

$$
\begin{aligned}
h_{e_x}(A) &= \exp(\int dx A_3\tau_3) = \cos\left(\frac{e_x A_3}{2}\right) + 2\tau_3 \sin\left(\frac{e_x A_3}{2}\right) &(40)\\
h_{e_\theta}(A) &= \exp(-\int d\phi A_2\tau_1) = \cos\left(\frac{e A_2}{2}\right) - 2\tau_1 \sin\left(\frac{e A_2}{2}\right) &(41)\\
h_{e_\phi}(A) &= \exp(\int d\theta A_2\tau_2) = \cos\left(\frac{e A_2}{2}\right) + 2\tau_2 \sin\left(\frac{e A_2}{2}\right) &(42)
\end{aligned}
$$

Exactly in the same way, we identify two smeared momentum variables $P_{e_x} = \int_{S_{e_x}} *E_3$ and $P_{e_\theta} = \int_{S_{e_\theta}} *E_2, P_{e_\phi} = \int_{S_{e_\phi}} *E_2$ These momenta P_e are equated to E up to the areas S_e which we can take to be constants. The quantum operator $E_{3(2)} = \imath l_{pl}^2 \frac{\partial}{\partial A_{3(2)}}$ has an easy representation. It is also easy to show that

$$
[\hat{h}_e, \hat{P}_e] \propto -\imath\tau_{1(2,3)} e\hat{h}_e \tag{43}
$$

We then complexify the holonomy to obtain a complexified phase space element which identifies for us the 'annihilation operator'. The closest analogy will be for the Harmonic oscillator, the annihilation operator $\hat{a} = \hat{x} - \imath\hat{p}$ and thus is a complexification of the phase space (x, p). This transformation is implemented using a 'complexifier' which is identified as

$$
\nabla_\gamma = \frac{1}{a}\sum_e P_e P_e
$$

where a is a area scale, which can be set to r_g^2 where r_g is radius of the black hole we are studying. Using that, the complexified SU(2) annihilation operator is found to be

$$
\hat{g} = e^{\tilde{t}\nabla_\gamma}\hat{h}_e e^{-\tilde{t}\nabla_\gamma} \tag{44}
$$

The parameter t is a semiclassical parameter, which could be as in [3] l_p^2/r_g^2. The state which is an eigenstate of the above operator, as shown in [17] is in the holonomy representation:

$$
\psi^t = \sum d_\pi e^{-\tilde{t}j(j+1)/2}\chi_j(gh^{-1}) \tag{45}
$$

Where $\chi_j(h)$ is the character of the element h in the j-th representation. g is the complexified phase space point corresponding to the classical holonomy, and can be shown to be $g = e^{-\imath \tilde{t} e P_e^i \tau^i} h_e$. The Momentum corrections in these coherent states have been computed in [3], and we shall use the same here

$$\delta P_e^I = P_e^I \left[\frac{\tilde{t}}{P_e} \left(\frac{1}{P_e} - \coth(P_e) \right) \right] = P_e^I F(P_e) \tag{46}$$

where I is the internal SU(2) index. If we use this, the corrections to the metric can be calculated. As the corrections are proportional to the classical momentum, no cross terms appear and spherical symmetry is maintained. This is in contrast to the results in [3], where the coherent state for the full sector of quantum gravity gets corrections which break the spherical symmetry. Further, due to the definitions of P_e^I, using (37), the corrections are in the expectation values of E_3 and E_2, (which makes $\hat{P}_{e_\theta} \propto \hat{P}_{e_\phi}$), unlike the example in [3], , where all the three 'densitized' momenta were corrected without any restrictions on them. In this paper, we also compute the corrections to the holonomy and hence the Gauge connection in the coherent states.

3.1. The Geometric Interpretations of the Semiclassical Corrections

We see that the momenta get corrected, and we then re-interpret this in terms of the metric and the extrinsic curvature. By definition (1, 10):

$$q q^{ab} = E_I^a E_I^b \tag{47}$$

If we add quantum corrections, then

$$q q^{ab} = (E^a + \delta E^a)_I (E^b + \delta E^b)_I \tag{48}$$

Note that in the above, as unlike the quantization in [3], the quantized operator E_3 does not have the $\sin\theta$, and thus the fluctuation is $\tilde{\delta} E = \delta E \sin\theta$.

Taking determinant of both sides, one obtains:

$$q^3 q^{-1} = \det(E + \delta E)^2 \rightarrow q = \det(E + \delta E) \tag{49}$$

Thus

$$q^{ab} = \frac{1}{\det(E + \delta E)} (E^a + \delta E^a)_I (E^b + \delta E^b)_I \tag{50}$$

In our example using [25]:

$$E_3^x = r^2 \sin\theta, \quad E_2^\theta = \sqrt{r(2m - r)} \sin\theta, \quad E_1^\phi = -\sqrt{r(2m - r)}$$

and therefore,

$$q = r^3(2m - r)\sin^2\theta \tag{51}$$

and

$$q^{xx} = \left(\frac{2m}{r} - 1\right)^{-1} \qquad q^{\theta\theta} = \frac{1}{r^2} \qquad q^{\phi\phi} = \frac{1}{r^2\sin^2\theta} \tag{52}$$

As the P^I_e vectors are related to the E vectors up to constants of area bits, we get

$$q^{xx} = \left(\frac{2m}{r} - 1\right)^{-1}\left(1 + 2F\left(\frac{r^2}{a}\right)\right) \tag{53}$$

$$q^{\theta\theta} = \frac{1}{r^2}\left(1 + 2F(\sqrt{r(2m-r)})\right) \quad q^{\phi\phi} = \frac{1}{r^2\sin^2\theta}\left(1 + 2F(\sqrt{r(2m-r)})\right) \tag{54}$$

There are no cross terms generated in the process, and the 'strain' discussed in the previous subsection does not arise in the process. However, this 'reduced phase' space approach has to be probed further, and a more robust discussion on coherent states in LQG will appear in [27].

We now see how holonomy corrections as in (35), will change the 'classical geometry'.

We can write the holonomy + fluctuation as:

$$h_{AB} + \delta h_{AB} = (\cos(eA) + \delta h_1)\delta_{AB} + \tau^i_{AB}(\pm 2\sin(eA) + \delta h_2) \tag{55}$$

then, we can interpret a change in the gauge $eA \rightarrow eA + \delta A$ (where we define $\delta h_1 = \sin(\delta A)\sin A, \delta h_2/2 = \sin(\delta A)\cos A$.). This is using the assumption that we can set $\cos\delta A \approx 1$.

The Extrinsic curvature then gets corrected. Using $K_{ab} = K^i_{(a}e^i_{b)}$, one can find the corrections to the extrinsic curvature as:

$$\delta K_{xx} = \left[\delta A_3 - 2\frac{m}{r^2}F\left(r^2/a\right)\right]\frac{\sqrt{r}}{\sqrt{2m-r}} \tag{56}$$

where $\delta A_3 \equiv \delta A_3(A_3, P)$ and we have used $K = A - \Gamma$, and $\Gamma = \cos\theta\,\tau_3 d\phi$ is the spin connection on the sphere and the Immirzi parameter $\beta = 1$.

In the propagation for the Gravity wave this will generate an effective $T_{\mu\nu}$.

4. Numerical Relativity in Classical Gravity

Numerical methods are important to solve non-linear systems, differential equations which cannot be solved analytically. Einstein equations are non-linear, the few exact solutions which exist are often simplified due to reduction in symmetry. Thus numerical methods and finite difference equations have been use in General relativity particularly to study dynamical solutions such as gravitational collapse, and then binary black hole collapse. We shall briefly review these.

4.1. Scalar Gravitational Collapse

Scalar gravitational collapse has been studied using numerical relativity techniques by various groups in the world reviewed in [8]. In particular rather interesting results were obtained in [5]. In this paper initial data of scalar field configurations were evolved numerically, and the resultant collapse to form black hole showed some universal behaviours.
(i) As functions of a parameter p which characterized the initial data, there emerged a critical value of that parameter p^*. Scalar time evolution showed that the scalar fields escaped to infinity for $p < p*$ and, collapsed to form black holes for $p > p*$.
(ii) A self similar behaviour of the fields was observed near the critical point.
(iii) The Mass of black holes formed showed a universal scaling behaviour near the critical value of the parameter space, in particular $M_{\rm BH} \sim |p - p^*|^\gamma$.
The numerical techniques include a finite difference method, and the use of a RNPL (Rapid Numerical Prototyping Language) code developed by the Choptuik group. A very comprehensive review of the numerical methods can be found in [8, 9]. Here we briefly describe the Choptuik scaling results and discuss the nature of the collapse. In Choptuik's original calculation a metric is taken

$$ds^2 = -\alpha^2 dt^2 + a^2 dr^2 + r^2 d\Omega \tag{57}$$

where $d\Omega = d\theta^2 + \sin^2\theta\, d\phi^2$ is the angular part of the metric. This metric is spherically reduced and the α, a are functions of r only in one time slice. When one evolves this numerically, the α, a are functions of time.

We couple this to a scalar field ϕ with action

$$S = \int dt\, dr\, d\theta\, d\phi\, r^2\, \sin\theta\, \alpha\, a \left(-\frac{1}{\alpha^2} (\partial_t \phi)^2 + (\nabla\phi)^2 \right) \tag{58}$$

where ∇ is the three dimensional gradient operator. From the above, the momentum can be defined for an effective two dimensional field (as the spherically symmetric ϕ can be taken independent of the angular coordinates)

$$\Pi = \frac{a}{\alpha}\partial_t \phi \tag{59}$$

and following the conventions of [5], define $\Phi = \partial_r \phi = \phi'$ and $\Pi = a\partial_t\phi/\alpha = a\dot{\phi}/\alpha$,(the ϕ' denotes the space derivative and $\dot{\phi}$ the time derivative) the equation of motion becomes

$$\dot{\Phi} = \left(\frac{\alpha}{a}\Pi\right)' \quad \dot{\Pi} = \frac{1}{r^2}\left(r^2\frac{\alpha}{a}\Phi\right)' \tag{60}$$

The Einstein's equation coupled with the scalar energy momentum tensor $T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial^\lambda\phi\partial_\lambda\phi$ gives the following

$$\frac{\alpha'}{\alpha} - \frac{a'}{a} + \frac{1-a^2}{r} = 0 \tag{61}$$

$$\frac{a'}{a} + \frac{a^2-1}{2r} - 2\pi r\left(\Pi^2 + \Phi^2\right) = 0 \tag{62}$$

Though GR is diffeomorphism invariant the technique of numerically solving the PDE's requires the equations to be framed in particular coordinates. The partial differential equations are solved by using finite difference methods. The discretization involves defining a discrete labelling of lattice points with lattice spacing h which can be varied for different runs of the numerical program. Features of the results which are independent of the lattice spacing are taken as robust.

Let us say we are trying to solve an equation in the variable (x, t). The 'mesh points' of x coordinate are labelled as $x_0, x_0{+}h, ..., x_0{+}jh$ with x_0 being the initial value of x. A function of x , $\phi(x)$ also exists defined at these mesh points, e.g. $\phi(x_0 + jh), \phi(x_0 + (j+1)h)$ etc. The derivative is approximated as

$$\frac{\phi(x+h) - \phi(x-h)}{2h} = \partial_x\phi + O(h) \tag{63}$$

Similarly the second derivative is introduced as

$$\frac{\phi(x+h) - 2\phi(x) + \phi(x-h)}{h^2} = \partial_x^2\phi + O(h^2) \tag{64}$$

The time axis is also discretized similarly, the discrete time step is $\Delta t = \lambda h$, where λ is labelled as the Courant number. Once the difference equations are framed, a further approximation known as Crank-Nicholson scheme is used. The finite difference equations are then iteratively solved using numerical coding in the form of RNPL. The iterations are implemented such that the 'residual' or the error after the 'l' th iteration goes to zero. We shall describe in some details the RNPL coding in section (IV) when we discuss a one dimensional scalar field toy model. The numerical techniques are very interesting and permit the time evolutions of non-linear of Einstein's equations. For interesting results on collapse of gravitational waves to form black holes see [10].

4.2. Binary Black Hole Collapse

Binary black hole collapse and emission of gravity waves from such an event has been studied in great details using analytic and numerical techniques. The observation of gravity waves have now been confirmed and the predicted waveform is real. This is a milestone discovery for physics, as General Relativity, and the early predictions of gravity waves in 1916 add to the status of GR as an experimentally verified theory. Binary black holes, which are usually spinning are modelled using the effective one body problem. The entire calculation is set up in harmonic gauge, and the numerical code was first obtained by Pretorius [7]. The black holes collapse due to emission of energy in gravity waves, collapse into one black hole and then settle down after what is known as the 'ring-down phase'.
The process can be classified into three phases

(i) The initial **inspiral** orbital phase where two black holes orbit each other,
(ii) The non-linear **merger** phase where the two black holes collapse into each other, travelling almost at the speed of light.
(iii) The **ringdown** phase where the collapsed state gives away energy in the form of gravitational waves to eventually settle down from a distorted form to a regular spinning black hole.

The process is computed using the time evolution of a scalar field with two source fields collapsing to form black holes, and eventually merging into one.

The initial metric is taken in ADM form as in [7]

$$ds^2 = -\alpha^2 dt^2 + h_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right) \tag{65}$$

The spatial metric is conformally flat in the initial slice

$$h_{ij} \quad = \quad \psi\eta_{ij} \tag{66}$$

$$\partial_t h_{ij} = \partial_t \psi \eta_{ij} \tag{67}$$

(ψ is the conformal factor and η_{ij} is the flat Minkowski metric) and the slice is maximally embedded with extrinsic curvature $K = 0, \partial_t K = 0$.

The Einstein equations are written in Harmonic gauge

$$\nabla x^\nu = H^\nu \tag{68}$$

(∇ is the Laplacian) H^ν is an arbitrary function. In the initial slice, $H^\mu = 0$. The Field equations get modified according to:

$$\frac{1}{2}g^{\mu\lambda}g_{\alpha\beta,\mu\lambda}+g^{\mu\lambda}_{,(\alpha}g_{\beta),\mu,\lambda}+H_{\alpha,\beta}-H_\lambda\Gamma^\lambda_{\alpha\beta}+\Gamma^\mu_{\beta\lambda}\Gamma^\lambda_{\alpha\mu} = -8\pi\left(T_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}T\right) \tag{69}$$

To this equation, the constraints $C^a = \nabla x^a - H^a$ which preserve the Gauge are added, and the system is put on a discretized grid after solving the initial data elliptical equations for ψ, α, β^i, $\partial_t\psi$, $\partial_t\alpha$, $\partial_t\beta^i$, on the initial slice which is flat.

The scalar field evolves according to the usual

$$\nabla\Phi = 0 \tag{70}$$

and in the initial slice is taken to be a Gaussian $\Phi = A\exp(-r^2/\Delta)$ and given a boost with a velocity v. The Harmonic vector H^a also evolves in time, a particular form of which can be found in [7] Thus the discretized versions of (69,70) and the time evolution of H^a are numerically coded using adaptive mesh refinement techniques [26]. Apparent horizons are detected using the apparent horizon finder equation, and singularity excision is implemented, by including grid points within the horizons up to a particular boundary.

The Gravity wave perturbation is studied at each stage of the binary evolution using the Newman-Penrose scalar Ψ_4 which is built from the Weyl tensor C_{abcd} using the Newman Penrose complex null tetrads m^a, n^a $\Psi_4 = C_{abcd}n^a\bar{m}^b n^c\bar{m}^d$. The energy radiated is

$$\frac{dE}{dt} = \frac{R^2}{4\pi}\int_0^t \Psi_4 dt \int_0^t \bar{\Psi}_4 dt \tag{71}$$

Detailed approach to binary black hole collision can also be found in various other Numerical Relativity groups. The particular emphasis is on numerical coding, and the scientific visualization of the effect. A more recent updated catalogue of gravitational waveforms can be found at the website: https://www.black-holes.org/for-researchers/waveform-catalog (SXS: Simulating eXtreme Space-times).

It is remarkable that the predicted gravitational wave form, and the amount of energy radiated out should completely agree with the LIGO data. The data can be found publicly at the website: https://losc.ligo.org/events/GW150914/. The Fig 2 graph is quoted from that website.

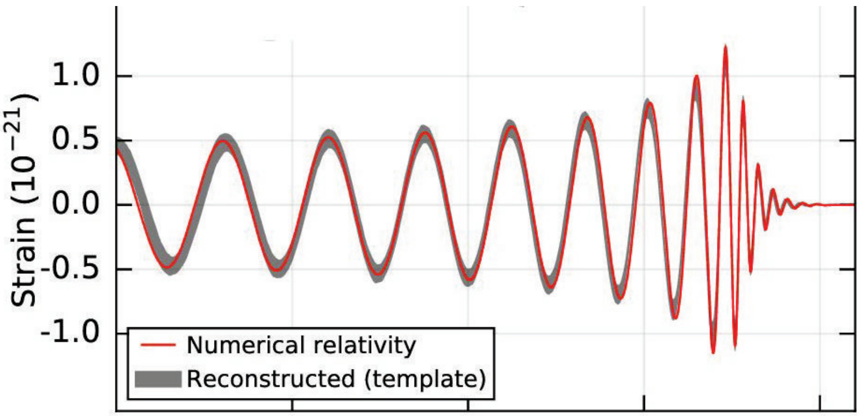


Figure 2. The LIGO gravity wave data vs prediction from numerical relativity.

5. Scalar Field in Corrected Black Hole Background

We now try to ascertain how such a spontaneously generated term due to quantum effects might affect the time evolution of space-times. At the level of calculations, this metric term yields a new contribution to the extrinsic curvature of the spatial slicing. Apart from the usual $K_{\theta\theta}$, $K_{\phi\phi}$ non-zero components of the Extrinsic curvature, a new term $K_{r\theta} \propto \partial_\theta \tilde{f}$ arises. This will contribute to the time evolution equations of dust collapse or scalar collapse systems theoretically.

As we know from Birkoff's theorem, the only spherically symmetric solution of Einstein's equation is the Schwarzschild solution. Therefore in the initial slice, the metric can be taken to be corrected exactly as in (27) in the Lemaitre coordinates, which is subsequently shown to give rise to a g_{tr} term in the Schwarzschild metric. A typical study of modified scalar collapse would be to start with a β term in the initial slice, and solve the time evolution equations. In particular we are asking the question, if a semi classically corrected scalar collapse has a drastic difference from a regular classical collapse.

The equation of motion including the shift are slightly modified:

$$\partial_t \Phi = \partial_r \left(\beta \Phi + \frac{\alpha}{a} \Pi \right) \tag{72}$$

$$\partial_t \Pi = \frac{1}{r^2} \partial_r \left(r^2 \left(\beta\, \Pi + \frac{\alpha}{a} \Phi \right) \right) \tag{73}$$

Where

$$\Phi(r,t) = \partial_r \phi \tag{74}$$

$$\Pi(r,t) = \frac{a}{\alpha} \left(\partial_t \phi - \beta\, \partial_r \phi \right) \tag{75}$$

This set of equations are similar to those set as a project for PSI students in 2010 [21]. The metric time evolution equations are modified versions of (62).

$$\dot{a} = -\alpha a K_r^r + (a\beta)' \tag{76}$$

$$\dot{K}_r^r = \beta K_r^{r\prime} - \frac{1}{a} \left(\frac{\alpha'}{a} \right)' + \alpha \left(\left(\frac{-2}{ra^2} \right)' + K K_r^r - 8\pi \frac{\Phi^2}{a^2} \right) \tag{77}$$

$$\dot{K}_\theta^\theta = \beta K_\theta^{\theta\prime} + \frac{\alpha}{(rb)^2} - \frac{1}{a(rb)^2} \left(\frac{\alpha r}{a} \right)' + \alpha K K_\theta^\theta \tag{78}$$

Where K_i^i are the components of the extrinsic curvature and K is the trace of the extrinsic curvature. In addition [9] the requirement of 'polar' 'areal' slicing and spherical symmetry, sets $\beta = 0$ and $K_{\theta\theta} = 0$. However in our quantum corrected metric with the g^{tr} term, there is a non-trivial correction to the effective curvature of the space-time, and this will affect the equations of evolution. At the level of adding a r dependent shift vector, we do not expect radical changes to the collapse equations. However, on adding the explicit θ dependent fluctuation to the geometry, we have to solve a three dimensional grid, and the code for that is quite difficult. We expect to discuss this in details in an upcoming publication [23]. Relaxing spherical symmetry will also generate a $K_{r\theta}$ term [23].

Instead of studying the entire collapse of the scalar system, which we expect to complete in the near future [23], we restrict ourselves to a more simpler problem: the solution to scalar propagation in the semi-classically corrected black hole metric. For the purposes of the paper, we simply study the scalar wave equation and any changes that might occur due to the introduction of a twist term in the metric. This is a preliminary toy model for a more in depth analysis of the behaviour of the scalar field in a forthcoming publication [23]. The question we are asking is: does the introduction of a β result in non-trivial changes of the time evolution of the scalar field. Our eventual aim is to study the scalar field collapse using a quantum corrected β in the time evolution equations (73, 78). To begin with, we observe the behaviour of a one dimensional scalar wave equation and the modified behaviour due to a cross term in the metric.

5.1. Numerical Calculations

As the numerical calculations involve finite difference method discretization and RNPL programming, we begin with a simple test model. We begin with a one dimensional scalar field wave with the following equation:

$$\partial_t^2 \phi = \partial_x^2 \phi \tag{79}$$

Using redefinition of

$$\Phi = \partial_x \phi \qquad \Pi = \partial_t \phi \tag{80}$$

The above (79) can be written as a first order differential equation:

$$\partial_t \Pi = \partial_x \Phi \quad \partial_x \Pi = \partial_t \Phi \tag{81}$$

In addition to this, there are boundary conditions. Following [22], we take the boundary in x to be finite [0,1] and the time to be arbitrary [0,T]. The boundary conditions are

$$\Pi(0,t) = \Phi(0,t) \quad \Pi(1,t) = -\Phi(1,t) \tag{82}$$

which ensure that no wave enters from the left or right ends of the boundary. The time and space are discretized, and using the same scheme as that in [22], we write the above equations in the form:

$$\frac{\Phi_j^{n+1} - \Phi_j^n}{\Delta t} = \frac{1}{2}\left[\frac{\Pi_{j+1}^{n+1} - \Pi_{j-1}^{n+1}}{2\Delta x} + \frac{\Pi_{j+1}^n - \Pi_{j-1}^n}{2\Delta x}\right]. \tag{83}$$

The LHS of this equation has a time derivative written as a finite difference, and the RHS of the equation has a space derivative 'averaged' over two time steps. This decreases the errors in the finite difference approximation of the derivative. This is also known as the Crank-Nicholson method.

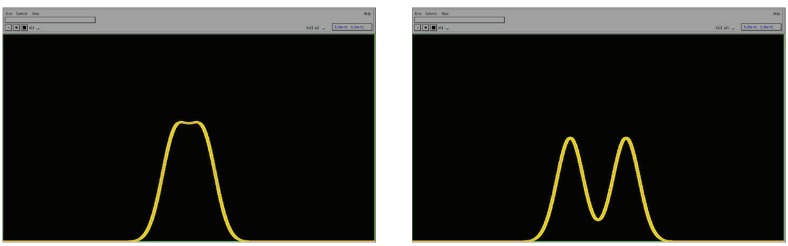
In the above the time steps are discretized using the n index, u^n, u^{n+1}, u^{n+2}... to label the field u at the n^{th}, $(n+1)^{th}$, $(n+2)^{th}$ time steps, similarly u_j, u_{j+1}, u_{j+2}.. labels the space discretization.

And using the fact that $\Delta t = \lambda \Delta x$, and a time step averaging operation $\mu_t = \frac{1}{2}\left(\phi_j^{n+1} + \phi_j^n\right)$ and the central derivative operation $D_x \Pi_j^n = \frac{1}{2\Delta x}\left(\Pi_{j+1}^n - \Pi_{j-1}^n\right)$

$$\frac{\Phi_j^{n+1} - \Phi_j^n}{\Delta t} = \mu_t \left(D_x \Pi_j^n\right) \tag{84}$$

$$\frac{\Pi_j^{n+1} - \Pi_j^n}{\Delta t} = \mu_t \left(D_x \Phi_j^n\right) \tag{85}$$

Here, Φ_j^{n+1} and Π_j^{n+1} are the unknown future variables and if we have all the initial conditions for Φ and Π, then the future form of Φ and Π can be found by using these equations. Also, it is to be noted that at the initial x, the first spatial derivative should be taken to be forward derivative as we do not have past information, similarly, at maximum x, we should take the backward derivative in order to maintain the boundary condition. These difference equations are then programmed into a fortran based RNPL code [22] and the time evolution for a Gaussian shaped initial scalar field is obtained in 128 time steps, and we show the following graphs



(a) t=3 (b) t=6

Figure 3. Time t=3 and t=6; Φ is the vertical axis, and x-the horizontal axis.

As expected, the Gaussian breaks into one left moving and right moving pulse as time progresses. The pulses eventually exit the x=0, and x=1 boundary respectively.

To simulate the addition of a twist field, we modify the flat metric of the previous one dimension as

$$ds^2 = -dt^2 + dx^2 + \beta dtdx \tag{86}$$

The β is a 'twist field', in the first approximation we assume that it is independent of time.

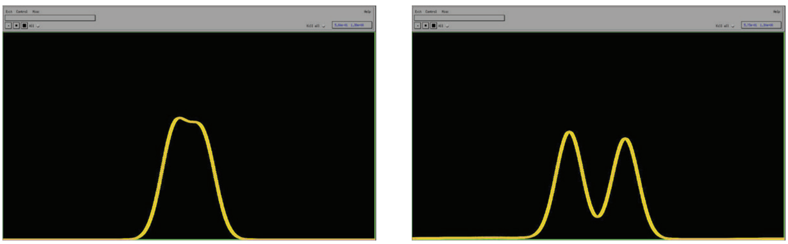
The scalar Lagrangian is

$$L = \frac{1}{2}\int d^2x \left[-(\partial_t\phi)^2 + (\partial_x\phi)^2 + \beta(\partial_t\phi)(\partial_r\phi)\right] \tag{87}$$

The equation of motion derived from this Lagrangian is

$$-\partial_t^2\phi + \partial_x^2\phi + \beta\partial_t\partial_x\phi = 0 \tag{88}$$

The above is similar to the near horizon behaviour of a scalar field.



(a) t=3 (b) t=6

Figure 4. Time t=3 and t=6; Φ is the vertical axis, and x-the horizontal axis.

This can be re-written as

$$\partial_t\left(\partial_t\phi - \beta\partial_x\phi\right) = \partial_x^2\phi \tag{89}$$

This as per new definition $\Pi = \partial_t\phi - \beta\partial_x\phi$ and $\Phi = \partial_x\phi$ has the same form as

$$\partial_t\Pi = \partial_x\Phi \tag{90}$$

However the reverse equation is considerably modified as

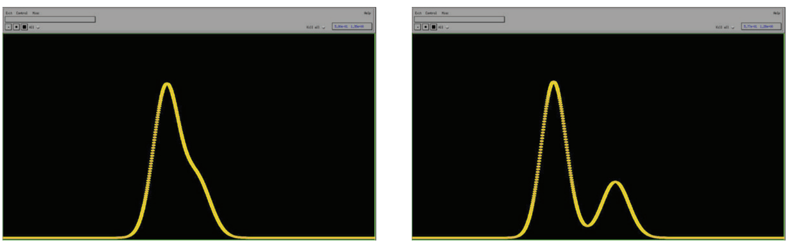
$$\partial_t \Phi = \partial_x(\partial_t \phi) = \partial_x \left(\Pi + \beta\Phi\right) = \partial_x \Pi + \partial_x \beta \Phi + \beta \partial_x \Phi \tag{91}$$

We thus use the discretized form of (91) to evolve the system as

$$\frac{\Phi_j^{n+1} - \Phi_j^n}{\Delta t} = \mu_t(D_x \Pi_j^n) + \mu_t((D_x \beta_j^n)\Phi_j^n) + \mu_t(\beta_j^n(D_x \Phi_j^n)) \tag{92}$$

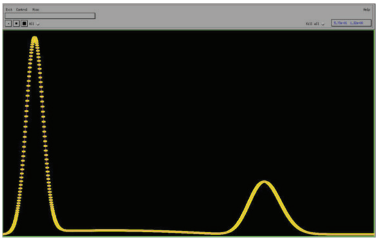
We use the boundary conditions $\beta(x = 0) = 0$ and $\beta(x = 1) = 0$. In the first example we take $\beta = x^2(x-1)^2$ and obtain the time evolution of the system. A clear space asymmetry emerges as the initial Gaussian splits into two (Fig 3). This asymmetry is not surprising as the input β function breaks the left and right symmetry of the wave equation through the time evolution (92). The speed of the right moving wave is modified as in [21]. As the equations show this is expected, and the numerical solutions confirm this. A more interesting $\beta = \sin^2(x^2(x-1)^2)$ is also used to study the behaviour. The Gaussian wave splits into two with greater asymmetry and the progress of the left and right waves towards the boundaries also happen asymmetrically. The time steps (t=3, 6, 25,40) are shown as evidence.

As is evident the introduction of a β causes an asymmetry in the left and right wave modes of a scalar field. In particular, the speeds of propagation change, particularly in the example of Figure (5). This might have non-trivial consequences for Hawking radiation, and emergence of matter flux from the horizon.

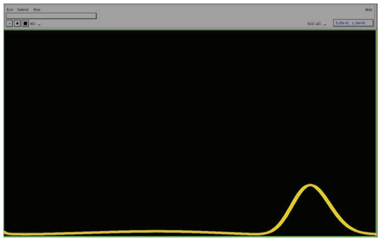


(a) t=3 (b) t=6

Figure 5. Time t=3 and t=6; Φ is the vertical axis, and x-the horizontal axis.



(a) t=25



(b) t=40

Figure 6. Time t=25 and t=40; Φ is the vertical axis, and x-the horizontal axis.

The convergence can be tested by taking the discretized equation of the following form,

$$L^h u^h - f^h = 0 \tag{93}$$

where L is the differential operator, u is the output function and f is the input function, h is the discretization width.

As the discretization unit width $h \to 0$, we expect convergence, i.e. $u^h \to u$. We have checked the program for convergence by taking different values of h, each at $h/2$, $h/4$, $h/8$ value of the zero-eth value. The value of h is taken as $1/64$ in the program. As can be seen the graphs are almost same, with the differences converging to zero. The courant number which relates the time discretization to space discretization dt/dx is 0.8 in all the graphs.

6. Conclusion

We discuss nature of semiclassical corrections to the metric, as computed using LQG coherent states. We show that the strain generated from the correction is too weak to be detected by the gravitational wave detector. We then try to obtain the same corrections in the spherically reduced sector of the theory, and observe that such a strain is missing in the reduced phase space. We however, try to obtain the effect of our correction on scalar waves near the horizon of a black hole using a numerical code for a toy model. We observe that the introduction of the shift vector term in the flat space wave equation changes the behaviour of the wave considerably by introducing a left right asymmetry in the propagation of the scalar wave. Though we tested the numerical program for a toy model

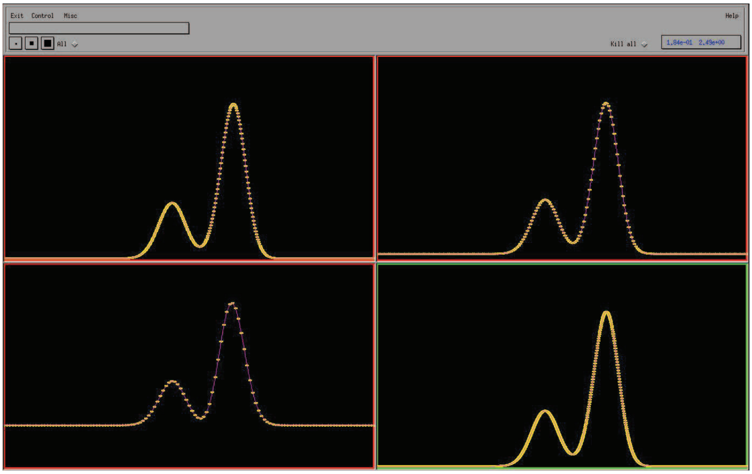


Figure 7. Time t=6 snapshots of the graphs at four different values of h.

of wave propagation, the answers for scalar wave propagation in a black hole background near the horizon would be similar. As the g^{tr} correction in (27) can become order 1 near the horizon we as the β functions in the test model, we will see tangible effects. We will be observing the behaviour of the Einstein equations with such corrections in details in a work in progress [23].

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Chapter 2

GENERALIZED QUANTUM ENTANGLEMENT FAMILY IN CONNECTION TO BLACK HOLES AND NANOTECHNOLOGY

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ABSTRACT

We present a new entanglement relativity theory by dividing Hardy's entanglement *P(H)*= $\phi^3 \phi^n$ into two parts, a global part given by ϕ^3 and a local part ϕ^n. For different n we obtain a generalized quantum entanglement family ϕ^3, ϕ^4, ϕ^5, ϕ^6.

We introduce the Fibonacci-like dimension sequence as an infinite geometric sequence and we extend the Fibonacci-like dimension sequence into the negative side.

The present work makes a leap from E-Infinity dissection of Einstein's equation into two parts, the ordinary energy $E(O) \approx mc^2/22$ plus the dark energy $E(D) \approx mc^2(21/22)$, to the connection by the E-Infinity scenario of the Kerr black hole.

The connection between the E-Infinity theory with the spinning Kerr black hole leads to a paradox. The ordinary and dark energy of the universe could be used as a guiding principle in the design of a nano-Casimir dark energy reactor.

1. Introduction

Quantum entanglement is a physical phenomenon that occurs when pairs (or groups) of particles are generated or interact in a way that the quantum state of each member must subsequently be described relative to each other.

There are two different and equally important facets to Hardy's classical work on entanglement [1, 2, 3]. He demonstrates in an almost perfect way that quantum mechanics is non-local [1-8]. This is what most researchers concentrated upon [4-8]. However, Hardy's probability of 9.0169945% for quantum entanglement must be looked upon as an incredible result as soon as one realizes that 9.0169945% is exactly equal to the inverse of the golden mean $\phi = \frac{\sqrt{5}-1}{2}$ to the power of five ϕ^5 [5, 6, 7]. This value is not profound because it is the most irrational number ϕ which is ubiquitous in art, science and natural forms [9, 10] but because it stands for the Hausdorff-Besicovitch dimension of a zero measure random Cantor set [9, 10].

In the Cantorian space-time theory the quantum particle is represented by a Cantor zero set while the quantum wave is represented by an empty Cantor set.

2. Missing Dark Energy of the Universe

Dark energy or the missing energy in the universe constitutes the most challenging problem in physics and cosmology [14-18]. Accurate measurement has shown that only 4.5% of the total energy thought to be contained in the universe is detectable. The simple conclusion for these results, which were awarded the 2011 Nobel Prize in Physics, is that either Einstein's equation contains some errors, or 95.5% of the energy in the universe is due to the mysterious dark matter and dark energy which cannot be detected with any known methods. Einstein's famous equation $E = mc^2$ consists of two parts and is the sum of the ordinary energy $E(O) \approx mc^2/22$ and the missing dark energy $E(D) \approx mc^2(21/22)$ [9, 14-18].

Adding both expressions we find that

$$E = E(O) + E(D) = E(Einstein) = mc^2. \tag{1}$$

By dividing Hardy's entanglement into two parts *P(H)* $=\phi^3\,\phi^n$, a global counterfactual part given by ϕ^3 (where $\phi=\frac{\sqrt{5}-1}{2}$) and a local part ϕ^n where n is the number of quantum particles, Hardy's quantum topological entanglement ϕ^5 is found for n = 2. It is therefore closely related to the Unruh temperature ϕ^4 where n = 1 and the Immirzi parameter ϕ^6 for n = 3 [19, 20]. We obtain a generalized quantum entanglement family ϕ^3, ϕ^4, ϕ^5, ϕ^6, for n = 0, 1, 2, 3.

The global part ϕ^3 of Hardy's entanglement *P(H)* $=\phi^3\,\phi^n$ leads to the ordinary part of the space-time topological energy $E_T(O)=(\phi^3)(\phi^2)/2=\phi^5/2$ and this leads further to the ordinary energy density $E(O)=(\phi^5/2)mc^2\approx mc^2/22$.

Similarly, dark energy is clearly the part of the topological energy of the space-time and is equal to $E_T(D)=1-(\phi^5/2)=5\phi^2/2$ which leads to $E(D)=(5\phi^2/2)mc^2\approx mc^2(21/22)$ [9, 14, 15].

We obtain

$$E=E(O)+E(D)=(\phi^5/2)mc^2+(5\phi^2/2)mc^2=mc^2. \tag{2}$$

3. The Relation between Neumann-Connes' Non-Commutative Geometry Dimension Function and E-Infinity Bijection Formula

Consider the dimension function of the non-commutative quotient space representing the well-known Penrose tiling [6], $D(a,b)=a+b\phi$; where a, b $\in$ Ƶ and $\phi=\frac{\sqrt{5}-1}{2}$.

This is necessarily a fractal universe resembling a compactified holographic boundary. Our aim is to show that under certain conditions this dimension function will yield the bijection formula of E-infinity [9, 10, 21-24], $d_c^{(n)}=(1/\phi)^{n-1}$. Let us set D_n (a_n, b_n) to be first $D(0)\equiv D_0$ (0, 1) and $D(1)\equiv$

D_1 (1, 0). Subsequently we add a_i and b_i following the Fibonacci scheme: $a_n = a_{n-1} + a_{n-2}$ and $b_n = b_{n-1} + b_{n-2}$

$$D(0) = D_0\,(0, 1) = 0 + \phi = \phi$$

$$D(1) = D_1\,(1, 0) = 1 + (0)\,\phi = 1$$

$$D(2) = D_2\,(0 + 1, 1 + 0) = 1 + \phi = 1/\phi$$

$$D(3) = D_3\,(1 + 1, 0 + 1) = 2 + \phi = (1/\phi)^2 \qquad (3)$$

$$D(4) = D_4\,(1 + 2, 1 + 1) = 3 + 2\phi = (1/\phi)^3$$

$$D(5) = D_5\,(2 + 3, 1 + 2) = 5 + 3\phi = (1/\phi)^4$$

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$$D(n) = D_n\,(a_n, b_n) = (a_{n-1} + a_{n-2}) + (b_{n-1} + b_{n-2})\,\phi = (1/\phi)^{n-1}$$

By induction we conclude that

$$D(n) = (1/\phi)^{n-1}. \qquad (4)$$

We obtain a Fibonacci- like dimension sequence $F_\phi(n)$

$$F_\phi(n) = \{\phi, 1, 1+\phi, 2+\phi, 3+2\phi, 5+3\phi, \ldots\}. \qquad (5)$$

The classical Fibonacci sequence F_n is defined by the recurrence relation

$$F_{n+1} = F_n + F_{n-1},\ \ n \geq 1 \qquad (6)$$

where $F_0 = 0$, $F_1 = 1$, $F_2 = 1$. The first few Fibonacci numbers of the classical Fibonacci sequence are given {0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ... }.

The *n*-th Fibonacci number is given by the formula which is called the Binet form, named after Jaques Binet

$$F_n = \frac{\left(\phi^{-1}\right)^n - \left(-\phi\right)^n}{\phi^{-1} + \phi} \tag{7}$$

where ϕ^{-1} and $-\phi$ are the solutions of the quadratic equation $x^2 = x + 1$. The solutions we can write as $x_1 = \frac{\sqrt{5}+1}{2} = \frac{1}{\phi}$ and $x_2 = \frac{1-\sqrt{5}}{2} = -\phi$.

The Binet form of the n-th Fibonacci-like number of the $F_\phi(n)$ sequence can be expressed similarly to the classical Fibonacci sequence [21, 22]

$$F_\phi(n) = \frac{\left(\phi^{-1}\right)^n - \left(-\phi\right)^n}{\phi^{-1} + \phi} + \frac{\left(\phi^{-1}\right)^{n-1} - \left(-\phi\right)^{n-1}}{\phi^{-1} + \phi}\phi, \qquad n \geq 0$$

$$F_\phi(n) = \frac{1}{\phi^{-1} + \phi}\left(\left(\phi^{-1}\right)^n - \left(-\phi\right)^n + \left(\left(\phi^{-1}\right)^{n-1}\phi - \left(-\phi\right)^{n-1}\phi\right)\right)$$

$$F_\phi(n) = \frac{1}{\phi^{-1} + \phi}\left(\left(\phi^{-1}\right)^n + \left(\phi^{-1}\right)^{n-1}\phi - \left(-\phi\right)^n - \left(-\phi\right)^{n-1}\phi\right) \tag{8}$$

$$F_\phi(n) = \frac{\phi}{1+\phi^2}\left(\left(\phi^{-1}\right)^n\left(1+\phi^2\right) - \left(-\phi\right)^n\left(1+\left(-\phi\right)^{-1}\phi\right)\right)$$

$$F_\phi(n) = \left(\phi^{-1}\right)^{n-1}.$$

The Fibonacci-like dimension sequence $F_\phi(n)$ can be presented as an infinite geometric sequence

$$\{\phi, 1, 1+\phi, 2+\phi, 3+2\phi, 5+3\phi, \ldots\} = \left\{\phi, \frac{1}{\phi^0}, \frac{1}{\phi}, \frac{1}{\phi^2}, \frac{1}{\phi^3}, \ldots\right\} \tag{9}$$

The Golden Section Principle that connects the adjacent powers of the golden mean is seen from the infinite geometric sequence. The formula for the n-th Fibonacci number and the bijection formula are the same. This is the bijection formula of E-infinity theory [9, 10], as shown in $d_c^{(n)} = (1/\phi)^{n-1}$, where $d_c^{(0)} = \phi$. However, we see that the bijection notation is more compact and economical and we recognize two dimensions at once; the n is the Menger-Urysohn dimension while $d_c^{(n)}$ is the Hausdorff-Besicovitch dimension. Our Fibonacci-like dimension series could be extended into the negative side using the same logic as before [25]

$$D(1) = D_1\,(1, 0) = 1 + (0)\,\phi = 1$$

$$D(0) = D_0\,(0, 1) = 0 + \phi = \phi$$

$$D(-1) = D_{-1}(1-0, 0-1) = 1 - \phi = \phi^2$$

$$D(-2) = D_{-2}(0-1, 1-(-1)) = -1 + 2\phi = \phi^3$$

$$D(-3) = D_{-3}(1-(-1), -1-2) = 2 - 3\phi = \phi^4 \tag{10}$$

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$$D(-n) = D_{-n}(a_n, b_n) = (a_{n-1} - a_{n-2}) + (b_{n-1} - b_{n-2})\,\phi = \phi^{n+1}$$

By induction we conclude that

$$D(-n) = \phi^{n+1}. \tag{11}$$

The Binet form of the n-th Fibonacci-like number of the $F_{-\phi}(n)$ sequence can also be expressed similarly to the classical Fibonacci sequence [21, 22]

$$F_{-\phi}(n) = (-1)^{n-1}\frac{\left(\phi^{-1}\right)^n - (-\phi)^n}{\phi^{-1}+\phi} + (-1)^n\frac{\left(\phi^{-1}\right)^{n+1} - (-\phi)^{n+1}}{\phi^{-1}+\phi}\phi, \quad n\geq 0$$

$$F_{-\phi}(n) = \frac{1}{\phi^{-1}+\phi}\left((-1)^{n-1}\left(\left(\phi^{-1}\right)^n - (-\phi)^n\right) + (-1)^n\left(\left(\phi^{-1}\right)^{n+1}\phi - (-\phi)^{n+1}\phi\right)\right)$$

$$F_{-\phi}(n) = \frac{1}{\phi^{-1}+\phi}\left((-1)^{n-1}\left((\phi^{-1})^n + (-1)(\phi^{-1})^{n+1}\phi\right) - (-1)^{n-1}\left((-\phi)^n + (-1)(-\phi)^{n+1}\phi\right)\right)$$

$$F_{-\phi}(n) = \frac{\phi}{1+\phi^2}\left((-1)^{n-1}(\phi^{-1})^n(1-1) + (-1)^n(-\phi)^n(1+\phi^2)\right) \tag{12}$$

$$F_{-\phi}(n) = \phi^{n+1}.$$

We obtain a Fibonacci-like dimension sequence $F_{-\phi}(n)$

$$\{1,\ \phi,\ 1-\phi,\ -1+2\phi,\ 2-3\phi,\ ...\} = \{1,\ \phi,\ \phi^2,\ \phi^3,\ \phi^4,\ ...\} \tag{13}$$

Consequently, it is easy to extend the bijection formula $d_c^{(n)} = (1/\phi)^{n-1}$ to negative dimensions so that we would have for instance [26-30]

$$d_c^{(1)} = (1/\phi)^{-1-1} = (1/\phi)^{-2} = \phi^2 \tag{14}$$

which is the empty set dimension binary and it can be written with the Connes-El Naschie bi-dimension formula in the following way $D(-n) = D(-n,\ \phi^{n+1})$. The empty set models the quantum wave and is given as [24]

$$D(-1) = (-1,\ \phi^2) \tag{15}$$

where –1 is the topologically invariant Menger-Urysohn dimension while ϕ^2 is the Hausdorff-Besicovitch dimension which is not topologically invariant but extremely useful.

The zero set on the other hand models the quantum particle [31-36]

$$d_c^{(0)} = (1/\phi)^{0-1} = (1/\phi)^{-1} = \phi \tag{16}$$

and can be written as

$$D(0) = (0,\ \phi) \tag{17}$$

which is well known and in full agreement with the dimensional function of non-commutative geometry [23].

The zero set $d_c^{(0)}$ separates the sets $d_c^{(n)}$ from the empty sets $d_c^{(-n)}$ and we can determine the degree of emptiness of an empty set as we move from $n = -1$, $n = -2$ … to $n = -\infty$ which leads to zero. We see clearly that the totally empty set, by a short verification, must be [24, 26-30]

$$d_c^{(-\infty)} = (1/\phi)^{-\infty -1} = 0 \tag{18}$$

4. The Possibility of a Correspondence between a Rotating Kerr Black Hole and E-infinity Conception to Ordinary and Dark Energy

The E-infinity model of dark energy relies on the dissection of $E = mc^2$ into $E(O) = mc^2/22$ plus dark energy $E(D) = mc^2(21/22)$ where $E(O)$ is the ordinary measurable cosmic energy of the quantum particle modelled by the zero set, while $E(D)$ is the dark cosmic energy density of the quantum wave modelled by the empty set. Further, dark energy $E(D)$ can be divided into two parts: dark matter $E(DM)$ and pure dark energy $E(PD)$. It has been shown [9, 14, 15, 19] that $E(D) = mc^2(21/22)$ which constitutes 95.5% of total dark energy, consists of $E(DM) \approx 5/22 \approx 22.7\%$ dark matter and $E(PD) \approx 16/22 \approx 72.7\%$ of pure dark energy.

Einstein's equation $E = mc^2$ can be divided into three parts

$$E = mc^2/22 + mc^2(21/22) = mc^2/22 + mc^2(5/22) + mc^2(16/22) = mc^2 \tag{19}$$

or with the expression of the golden mean [15]

$$E = (\phi^5/2)mc^2 + (5\phi^5/2)mc^2 + (10\phi^4/2)mc^2 = mc^2. \tag{20}$$

The connection from E-infinity scenario to the spinning Kerr black hole is presented in the following way. The spinning Kerr black hole has three regions; it has two event horizons and not only one as the static black hole. There is an inner event horizon surrounding the circular black hole pipe at the core and a second outer event horizon separating the ergosphere from the rest of the Kerr black hole [19, 37, 38]. The horizon is the region from which no signal can escape.

In the present work we rely heavily on the Kerr space-time geometry of rotating black holes. Kerr's geometry and its ergosphere tie almost perfectly with our dark energy theory. As a direct consequence of this new insight $E = mc^2$ can be written as $E = E(O) + E(D)$, where the rational approximation $E(O) = mc^2/22$ is the ordinary energy density of the cosmos and $E(D) = mc^2(21/22)$ is the corresponding dark energy of the ergosphere of the Kerr energy [9, 14, 15, 19]. In this sense we have a Kerr black hole nucleus having all the ordinary energy in it and that could be seen as a mini black hole model for elementary particles.

The paradox of the black holes is leading to the satisfactory resolution confirming that at the minimum of 95.5% of energy and information of the ergosphere will never be lost while 4.5% in the Kerr black hole nucleus will not be directly accessible for us. We can conclude that 95.5% of the information of a black hole is the ordinary information and the remaining 4.5% is the dark information [39].

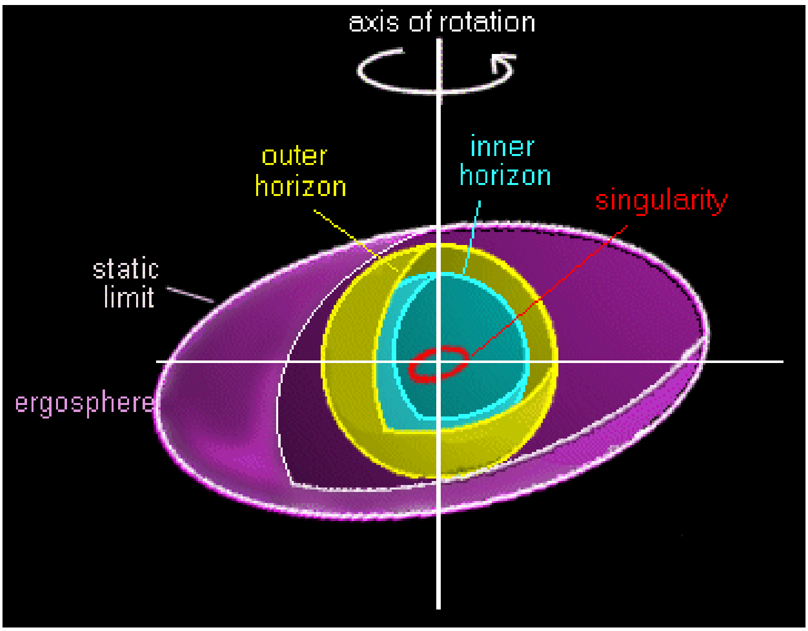

Figure1. Black hole regions.

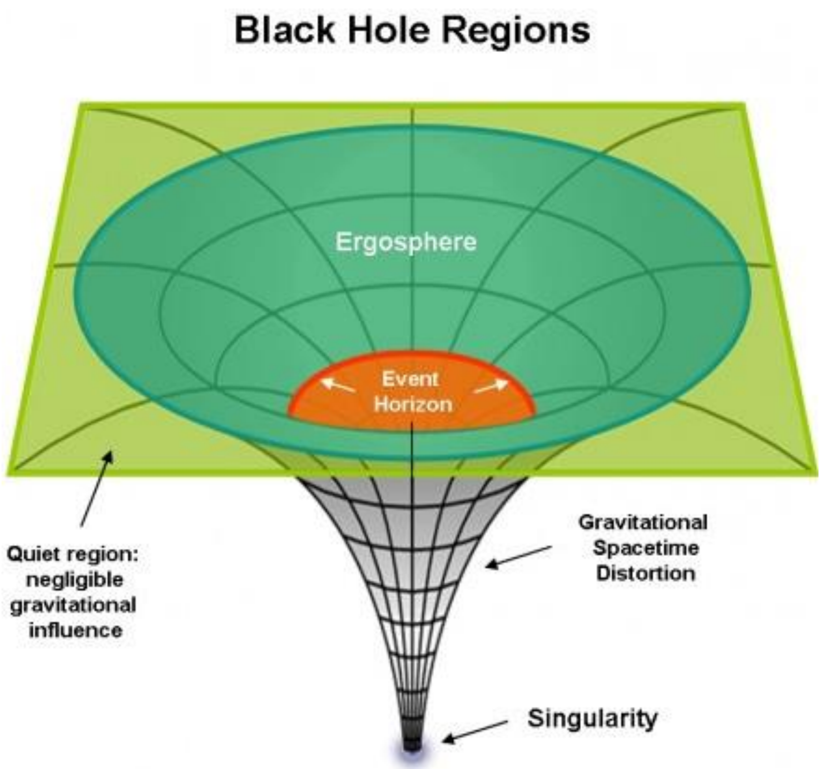

Figure 2. Black hole regions.

5. Topological Interpretation of the Casimir Effect as a Property of the Geometrical Topological Structure of the Quantum-Cantorian Micro Space-Time

The Casimir effect is a small attractive force that acts between two close parallel uncharged conducting plates. It is due to quantum vacuum fluctuations of the electromagnetic field.

The effect was predicted by the Dutch physicist Hendrick Casimir in 1948. According to the quantum theory, the vacuum contains virtual particles which are in a continuous state of fluctuation. Casimir realised that between two plates, only those virtual photons whose wavelengths fit a whole number of times into the gap should be counted when calculating the vacuum energy.

The energy density decreases as the plates are moved closer, which implies that there is a small force drawing them together. Although, the Casimir effect can be expressed in terms of virtual particles interacting with the objects, it is best described and more easily calculated in terms of the zero-point energy of a quantized field in the intervening space between the objects.

The Casimir effect is a natural consequence of the quantum field theory. There are at least two fundamental interpretations of this effect. The first is connected to boundary conditions and the zero-point quantum vacuum fluctuation which may be the common way of looking at the Casimir effect. The second is to see the Casimir effect as a source in the mould of Schwinger's way of thinking [20, 37-41].

In the present paper we opted for a rather different point of viewing the Casimir effect as a natural necessity of a Cantorian space-time fabric that was woven from an infinite number of zero Cantor sets and empty Cantor sets. The zero set is taken following von Neumann- Connes' dimensional function to model the quantum particle while the empty set models the quantum wave.

The quintessence of the present theory is easily explained as the ϕ^3 intrinsic topological energy, where $\phi = \frac{\sqrt{5}-1}{2}$ is produced from the zero set ϕ of the quantum particle when we extract the empty set quantum wave ϕ^2 from it.

The Casimir energy, the universal fluctuation ϕ^3, is the difference between the Hausdorff dimension of the particle zero set ϕ and the empty set ϕ^2. The result is almost equal to double the value found by Zee [42]. He used an imaginative modification of the classical Casimir experiment and found the dimensionless Casimir energy equal to $\pi/24 \approx 0.1308$

Surrounding the zero set quantum particle we have the quantum wave empty set with the Connes-El Naschie bi-dimension $D(-1) = (-1, \phi^2)$ acting as a surface of the quantum particle, i.e. zero set $D(0) = (0, \phi)$. The infinite number of zero and empty sets have an average bi-dimension $D(-2) = (-2, \phi^3)$. This triadic picture of a quantum particle zero set wrapped in a propagating quantum wave empty set and floating in a quantum space-time, which has ϕ^3 average topological Casimir pressure, is more satisfactory than any previous picture which was presented in the past [9, 20, 40, 41].

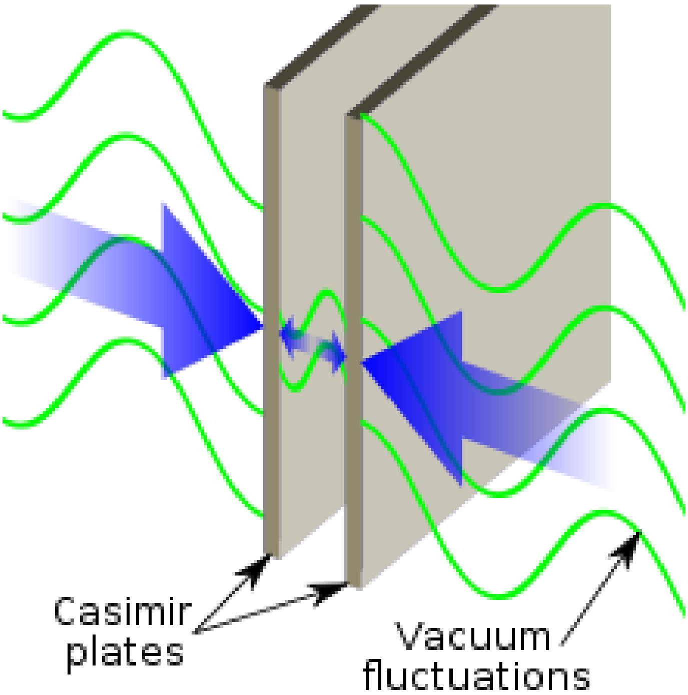

Figure 3. Casimir effect.

6. Nanotechnology

In recent years nanotechnology invaded all scientific fields and played a significant role in Casimir effect experiments. We know, thanks to E-infinity theory, that there exists a physical-mathematical connection between dark energy and ordinary measurable energy on the one side and the Casimir effect on the other side. A natural consequence of this discovered reality of the quantum wave is rendering it a relatively simple task to find a way to harness dark energy or Casimir energy. The difference between Casimir energy and dark energy is a difference of boundary condition where the boundary of the holographic boundary of the universe is a one sided Möbius-like manifold [20]. This seems simple but it is extremely difficult and in the moment impossible. There are many ideas about how to start, irrespective of the connection to Kerr black holes.

We can start for instance with a highly complex sub-structuring of space using nano-tubes and nano-particles and in that way create fractal-like nano-spheres packing. We stress in this connection that we have a clear model for our nano-reactor based on two important facts. The first is the equivalence between branching polymer clusters and Cantorian-fractal space-time [25, 40, 41]. The second is that we replace the Casimir plates of our model with Casimir spheres and model these spheres with real nano-particles and in principle this is our reactor. The main idea is a construction of a nano-universe and extracting dark energy from its nano-boundary of its holographic boundary. That means extracting energy from such a nano-reactor. It is at the edge of the universe that 95.5% of the energy resides as dark energy. This follows from the incredible measure theoretical theorem of Dvoretzky [18] which explains why energy is concentrated at the edge of the universe. The Dvoretzky theorem states that the volume of a sphere is concentrated at the surface, more accurately, 95.5% of the volume would be at the surface while in the so called bulk we have only 4.5%. However, we could create many nano-universes from which its 95.5% energy concentration could be extracted without actually reaching to the boundary of our universe which is of course factually impossible [16, 18]. On the other hand if we could produce nano-Kerr black holes, then a Penrose process could be feasible after all following broadly the preceding lines of speculation.

CONCLUSION

We introduced the generalized entanglement family and the Fibonacci-like dimension sequence which was extended into the negative side.

Our model of the universe is very simple. Applying the Dvoretzky theorem we can reason that $E = mc^2$ can be split into a quantum wave energy density $E(D) = mc^2(21/22)$ concentrated at the holographic boundary. This is the surface of the universe which we call dark energy. $E(D)$ cannot be measured in any direct way with the present-time technology. The ordinary energy density $E(O) = mc^2/22$, the core of the quantum particle universe can be measured directly. The connection from E-infinity scenario to the spinning Kerr black hole leads to a paradox. The dark energy and information in the ergosphere of the black holes is accessible because the ordinary energy in the horizon, where no information can escape, is lost. We can conclude that 95.5% of the information of a black hole is the ordinary information and the remaining 4.5% is dark information. The situation is analogous to that of the ordinary and dark energy of the universe and could be used as a guiding principle in the design of a nano-Casimir dark-energy reactor.

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Chapter 3

A PARTICLE-LIKE DESCRIPTION OF PLANCKIAN BLACK HOLES

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Abstract

In this paper we abandon the idea that even a "quantum" black hole, of Planck size, can still be described in terms of a *classical*, more or less complicated, geometry. Rather, we consider a genuine quantum mechanical approach, where a Planckian black hole is just another "particle", but with a distinguishing property: its wavelength increases with the energy. The horizon dynamics is described in terms of a particle moving in gravitational potential derived from the horizon equation in a self-consistent manner. The particle turning-points match the radius of the inner and outer horizons of a charged black hole. This classical model pave the way towards the wave equation of a truly quantum black hole. We compute the exact form of the wave function and determine the energy spectrum. Finally, we describe the classical limit in which the quantum picture correctly approaches the classical geometric formulation. We find that the quantum-to-classical transition occurs far above the Planck scale.

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1. Introduction

Since the introduction of the concept of radiating , "mini" black holes by Hawking [1], there has been an increasing interest in the study of black holes (BHs) which are not produced by stellar gravitational collapse, but for those that have linear size comparable, or even smaller, than an elementary particle. Despite the "*abyssal*" difference in size and mass between a galactic center BH and a theoretical quantum BH smaller than an atomic nucleus, the formal description of such two very different objects remained the same. In both cases deal classical solutions of Einstein equations, i.e. a classical geometrical description, the only difference is that cosmic objects interact with classical matter, while micro BHs interacts with quantum particles.

This state of mind has led to various models of quantum BHs in which the "quantum" nature is simulated through non-trivial geometrical and topological distortions, e.g. "large" or "warped" extra-dimensions. In such framework, the restriction to find "imprints" of mini-BHs existence in the early universe only, can be circumvented by the exciting possibility to study them in the lab through high energy particle collisions.

The standard approach to "quantum" BHs is motivated by the generally accepted idea that true quantum gravity effects will manifest themselves only near the Planck energy scale. Thus, BHs much smaller than a proton, can still be considered "*classical*" objects, as long as their size is large with respect the Planck length $l_P = 10^{-33}\,cm$. The main shortcoming of this "*scale downgrading*" approach is that it breaks down just near the Planck scale where it is supposed that these objects should be appear!

A clear example of this failure, is that the final stage of the BH thermal decay cannot be defined except for BHs admitting an *extremal* configuration. Even in this case, the third law of thermodynamics seems to be violated, since the temperature is zero, but the entropy is given by the *non-vanishing* area of the degenerate horizon. Last but not the least, the truly statistical description in terms of micro-states remains confined to a limited number of special very super-symmetric models.

Against this background, we would like to propose the idea of "*energy scale upgrade*" in the sense that we start from elementary particles below the Planck scale and gradually approach the Planck phase from below. This line of reasoning is inspired by the UV self-complete quantum gravity program introduced in [2, 3]. In this picture hadronic collisions at Planckian energy [4, 5, 6, 7, 8, 9, 10, 11, 12], [13, 14, 15, 16, 17, 18, 19] can result in the production of "non-geometrical" BHs described as Bose-Einstein graviton condensates[20, 21, 22, 24, 25].

Stimulated by the hope that this new scenario can cure previously described limitations of the "scale downgrading" approach, and give new insight into the quantum nature of BHs, we build a quantum model "from scratch" by considering the evolution of an elementary particle when its energy approaches the Planck scale from below. In this sub-Planckian regime the increase of particle energy leads to diminishing wave-length. However, when Planck energy is reached, a "phase transition" takes place corresponding to an *increase* of wavelength with the energy. This non-standard behavior can be seen as the quantum manifestation of the relation between mass and radius of a classical BH. In other words, the quantum particle changes its nature by crossing the Planck barrier. Once it is given additional energy, it will increase in size and eventually reach a semi-classical regime where the geometrical description can be properly applied.

In the spirit of the above discussion, one concludes that the quantum BH should be considered just as another quantum particle, though with a particular relation between its energy and size.

In recent papers [26, 27] we have made a first step towards the formulation of a quantum theory of BHs by starting with a simple one-dimensional model of a neutral BH. This toy-model has shown nice and simple quantization features, as well as, a natural limit towards a classical Schwarzschild BH for large principal quantum number.

In this work we would like to extend the toy-model to a realistic three dimensional, charged BH, hopefully to be produced in the proton-proton collision at LHC. To realize this project we are guided by the *Holographic Principle* [28, 29, 30] asserting that the dynamics of a quantum BH is the dynamics of its horizon.

At first glance, this statement is in clear contradiction with the purely geometric, and static, nature of a classical horizon. Thus, the first problem one encounters in trying to implement the Holographic Principle is to introduce an intrinsic dynamics for the horizon. In the simplest case of a spherically symmetric BH, we are guided by the analogy with the two-body problem in the central potential where the relative dynamics can be described in terms of a "fictitious" particle of reduced mass moving in a suitable one-dimensional *effective potential*. Following the same line of reasoning, we start by noting that the equation for the horizon(s) in the Reissner-Nordstrom geometry can be interpreted as the equation for the turning-points of a particle of energy $E = M$ moving between $r = r_-$ and $r = r_+$ (where $r_\pm$ are the inner and outer horizons for a BH of mass M and charge Q). Accordingly, we propose to assign the horizon an effective dynamics described by the motion of such a representative particle. The motion of the particle in the interval $r_- \leq r \leq r_+$ corresponds to the "deformations" of the horizon.

In Section(2) we give an Hamiltonian formulation of the particle motion and solve the equation for the orbits. Each orbit is characterized by a fixed value of the energy $E(= M$ mass of the BH), the charge Q (= charge of the BH) and angular momentum L. The motion of the particle is always bounded, but the orbits are not always closed.

This particle-like model has the advantage to allow a straightforward quantization leading to the corresponding quantum horizon model.

In Section(3), we solve the horizon wave equation and determine the energy spectrum. As it can be expected from the classical motion analysis, we find *discrete* energy levels depending on the radial quantum number n and the orbital quantum number l. Contrary to the classical description, the BH mass in the neutral case $Q = 0$, cannot be arbitrarily small, but is bounded from below by the ground-state energy $E \simeq 1.22 \times M_{Pl}$.

Finally, we find that in the classical limit $n >> 1$, the absolute maximum of the probability density approaches the classical value for the horizon radius.

In the concluding Section (4) we stress the modification our model introduces in the current picture of gravitational "classicalization" at the Planck scale.

2. Particle Analogue of a Charged BH

The quantization of mechanical system, say a "particle", starts from a classical Hamiltonian encoding its motion. On the other hand, a classical BH is defined as a particular solution of the Einstein equations. We give up such a starting point and replace it with a particle-like formulation translating in a mechanical language the key features of a geometrical BH :

1. BHs are intrinsically *generally relativistic* objects, in the sense of strong gravitational fields. Thus, the equivalent particle model should start with a

relativistic-like dispersion relation for energy and momentum rather than a Newtonian one;

2. the particle model must share the same spherical symmetry of the RNBH and the classical motion will be described in terms of a radial and an angular degree of freedom;
3. the “mass” to be assigned to the horizon is the ADM mass;
4. The equation for the horizons,$r_\pm$, of a charged BH, becomes the equation for the turning points of a particle with total energy $E = M$ in a suitable potential.

$$M = \frac{r_\pm}{2\,G_N}\left(1+\frac{Q^2 G_N}{r_\pm^2}\right) \longleftrightarrow E = V(r_\pm) \tag{1}$$

This identification allows us to map the problem of finding the horizons in a given metric into the problem of determining the turning points for the bounded motion of a classical, relativistic, particle.

The above requirements are encoded in the following Hamiltonian

$$H \equiv \sqrt{\vec{p}^{\,2} + m^2\,(\,r\,)} = \sqrt{p_r^2 + \frac{p_\phi^2}{r^2} + \frac{r^2}{4\,G_N^2}\left(1+\frac{Q^2 G_N}{r^2}\right)^2} \tag{2}$$

Both the total energy and the angular momentum are constant of motion

$$\frac{\partial H}{\partial t} = 0 \longrightarrow H = const. \equiv E\,, \tag{3}$$

$$\frac{\partial H}{\partial \phi} = 0 \longrightarrow p_\phi = const. \equiv L \tag{4}$$

From the Hamilton equations we obtain the orbit parametric equations

$$\dot{r}^2 = 1 - \frac{L^2}{E^2 r^2} - \frac{r^2}{4\,G_N^2 E^2}\left(1+\frac{Q^2 G_N}{r^2}\right)^2\,, \tag{5}$$

$$\dot{\phi}^2 = \frac{L^2}{E^2 r^4} \tag{6}$$

The solutions of (5),(6) are:

$$r(t) = \sqrt{2}G_N E\left[1 - \frac{Q}{2G_N E^2} + \sqrt{1 - \frac{1}{G_N E^2}\left(Q^2 + \frac{L^2}{G_N E^2}\right)}\cos\left(\frac{t}{G_N E}\right)\right]^{1/2} \quad (7)$$

$$\phi(t) = \frac{1}{\sqrt{1+\frac{Q^4}{4L^2}}}\arctan\left[\frac{L}{G_N E^2}\frac{\sqrt{1+\frac{Q^4}{4L^2}}\ \tan\left(t/2G_N E\right)}{1 - \frac{Q}{2G_N E^2} + \sqrt{1 - \frac{1}{G_N E62}\left(Q^2 + \frac{L^2}{G_N E^2}\right)}}\right] \quad (8)$$

A qualitative description of the motion can be obtained by writing equation (5) as the equation of motion for a particle in the *effective potential*

$$\dot{r}^2 = 1 - V_{eff}(r)^2/E^2 \quad (9)$$

where

$$V_{eff}(r) = \left[\frac{L^2}{r^2} + \frac{r^2}{4\,G_N^2}\left(1 + \frac{Q^2 G_N}{r^2}\right)^2\right]^{1/2} \quad (10)$$

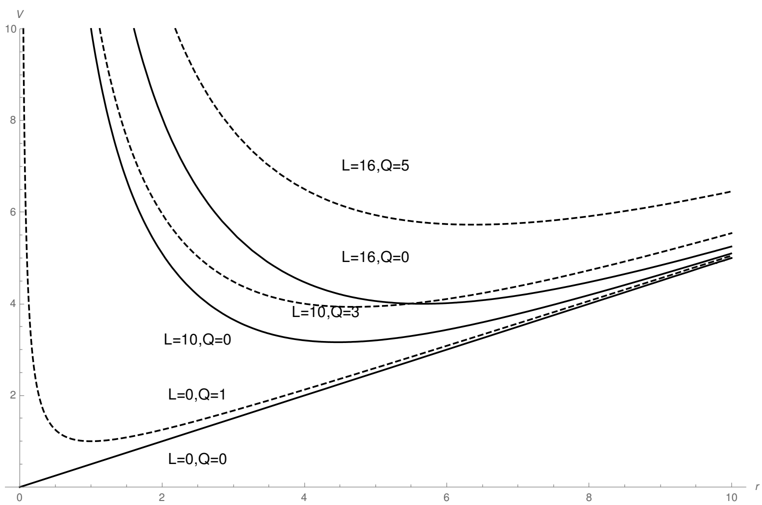

Figure 1. Plot of the equation (10) for different values of L and Q.

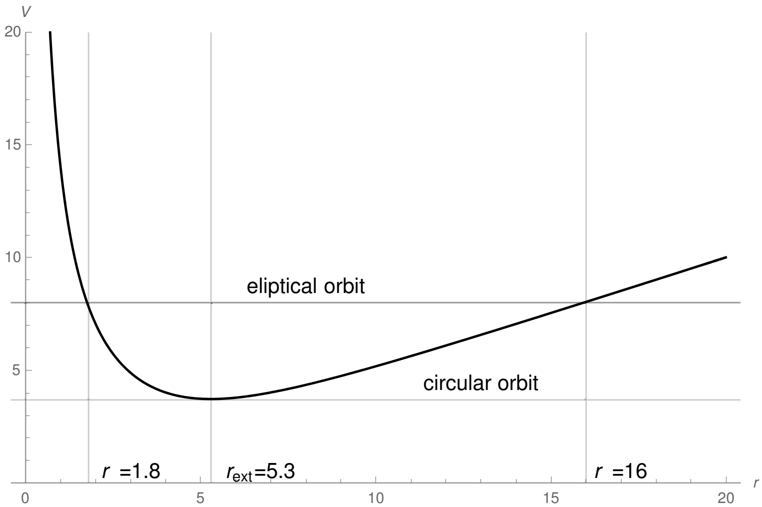

Figure 2. Plot of $V_{eff}(r)$, with $L = 14$, $Q = 0$, $r_+ = a$, $r_- = b$ are the turning-points corresponding to the maximum and and minimum distance from the origin. For $E = E_m$ the orbit degenerates into a circular orbit.

The charge introduces an additional repulsive effect, at short distance, adding up to the centrifugal barrier. Instead, at large distance the charge-independent harmonic term is the leading one.

It follows that we have only *bounded* orbits describing a bounded motion. This is in agreement with our purpose to model horizon vibrations around a stable equilibrium configuration in terms of the motion of a representative "particle". In order to substantiate this analogy, let us check, at first, the correspondence between turning-points and horizon positions.

$$\frac{dV_{eff}(r)^2}{dr} = 0 \longrightarrow r_m^2 = 2G_N L\sqrt{1 + \frac{Q^4}{4L^2}} \tag{11}$$

The existence of a minimum corresponds to a stable circular orbits of radius r_m, or a static horizon of radius $r_+ = r_m$

$$V_{eff}(r_m) = \frac{1}{2G_N}\left(Q^2 + \sqrt{Q^4 + 4L^2}\right) \tag{12}$$

The energy of the particle on the circular orbit is given by

$$E_m^2 = V_{eff}(r_m) = \frac{1}{2G_N}\left(Q^2 + \sqrt{Q^4 + 4L^2}\right) \tag{13}$$

and its angular frequency is

$$\dot{\phi}^2 = \frac{L^2}{E_m^2 r_m^4} = \frac{1}{2G_N}\sqrt{1 + \frac{Q^4 E_m^2}{4L^2}} \tag{14}$$

For $E > E_m$ there are two turning points which are the solutions of the equation $\dot{r} = 0$. By introducing the variable $x \equiv r^2$, one gets the algebraic quadratic equation

$$x^2 - 2\left(2G_N^2 E^2 - G_N Q^2\right) x + 4G_N^2 L^2\left(1 + \frac{Q^4}{4L^2}\right) = 0 \tag{15}$$

Thus,

$$r_\pm^2 = \left(2G_N^2 E^2 - G_N Q^2\right) \pm 2G_N E\sqrt{G_N^2 E^2 - Q^2 G_N - L^2/E^2} \tag{16}$$

where

$$E^2 \geq \frac{Q^2}{2G_N}\left(1 + \sqrt{1 + 4L^2/Q^4}\right) \tag{17}$$

For $L = 0$ the condition (17) reduces to the condition $G_N E^2 \geq Q^2$ for the existence of the static RN horizons. Furthermore, the turning-points equation (16) correctly gives the radius of both the inner (Cauchy) and outer (Killing) horizons.

$$r_\pm = G_N E \pm \sqrt{G_N^2 E^2 - Q^2 G_N} \tag{18}$$

From the Hamilton equations (5), (6) one obtains the orbit equation

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{E^2 r^4}{L^2}\left[1 - \frac{L^2}{E^2 r^2} - \frac{r^2}{4\,G_N^2 E^2}\left(1 + \frac{Q^2 G_N}{r^2}\right)^2\right] \tag{19}$$

which can be integrated:

$$r^2\,(\,\phi\,) = \frac{2L^2}{E^2}\left(1+\frac{Q^4}{4L^2}\right)\times$$
$$\frac{1}{1-\frac{Q^2}{2G_NE^2}+\sqrt{1-\frac{1}{G_NE^2}\left(Q^2+\frac{L^2}{G_NE^2}\right)}\sin\left[2\sqrt{1+\frac{Q^4}{4L^2}}\,(\,\phi-\phi_0\,)\right]} \tag{20}$$

where ϕ_0 is an arbitrary integration constant. The same solution can be obtained by eliminating time from equation (7), (8).
The orbit equation (20) can be conveniently re-written as

$$r^2\,(\,\phi\,) = \frac{2L^2\beta^2}{E^2}\frac{1}{1-\frac{Q^2}{2G_NE^2}-\sqrt{1-\frac{1}{G_NE^2}\left(Q^2+\frac{L^2}{G_NE^2}\right)}\cos\left[\,2\beta\,\phi\,\right]} \tag{21}$$

where

$$\beta \equiv \sqrt{1+\frac{Q^4}{4L^2}}\,, \tag{22}$$
$$\phi_0 = \pi/4\beta \tag{23}$$

To understand the property of the orbit, let us consider the neutral BH $Q=0$ first. This case describes the dynamics of the Schwarzschild horizon.

2.1. Neutral Orbits $Q = 0$

For $\beta = 1$ the orbits simplify to

$$r\,(\,\phi\,) = \frac{\sqrt{2}L}{E}\frac{1}{\left[1-\sqrt{1-L^2/G^2E^4}\,\cos\left(\,2\phi\right)\right]^{1/2}} \tag{24}$$

Equation (24) describes ellipses centered at the origin with major and minor semi-axis, a and b respectively, given by

$$a = \sqrt{2}G_NE\sqrt{1+\sqrt{1-L^2/G_N^2E^4}}\,, \tag{25}$$
$$b = \sqrt{2}G_NE\sqrt{1-\sqrt{1-L^2/G_N^2E^4}}\,, \tag{26}$$
$$L \leq G_NE^2 \tag{27}$$

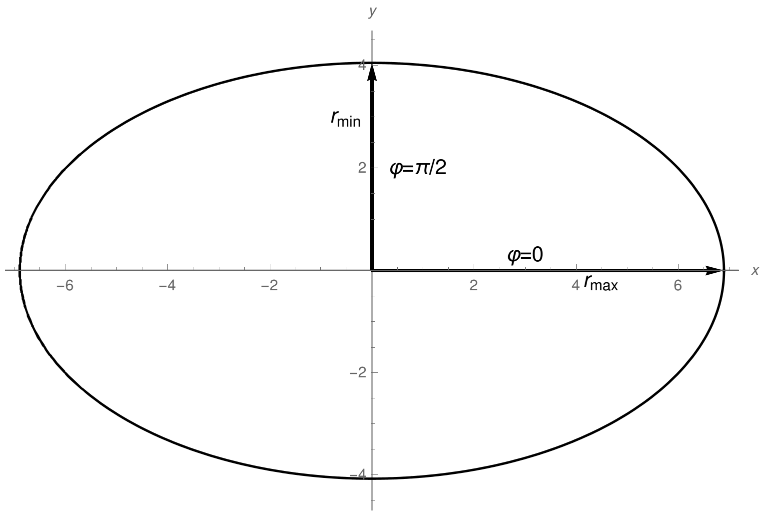

Figure 3. Plot of the equation (24) in terms of the rescaled variables $r/\sqrt{G_N}$ with $L = 14$, $\sqrt{G_N}E = 4$.

This type of orbits correspond to a radially "*breathing*" mode of the Schwarzschild horizon:

$$\frac{\sqrt{2}L}{E}\frac{1}{\left[1+\sqrt{1-L^2/G^2E^4}\right]^{1/2}} \le r(\phi) \le \frac{\sqrt{2}L}{E}\frac{1}{\left[1-\sqrt{1-L^2/G^2E^4}\right]^{1/2}} \quad (28)$$

Two limits are of special interest.

For $L \to 0$ ellipses degenerates into a segment and the motion becomes e one-dimensional oscillation between the origin and the Schwartzschild radius $a = 2G_NE$, while $b = 0$.

The other limiting case is $L = G_NE^2$. In this case, the ellipse degenerate into a circle of radius $r = \sqrt{2}G_NE$ and the horizon "freezes" into a static configuration. $E = \sqrt{L/G_N}$ is the ground state energy corresponding to the stable minimum of the effective potential.

We recall that r corresponds to the radius of the BH. The existence of r_{min} and

r_{max}, for $L \neq 0$, defines the range of radial vibrations of the Schwarzschild horizon. To clarify the role of angular momentum we plot below orbits for different L

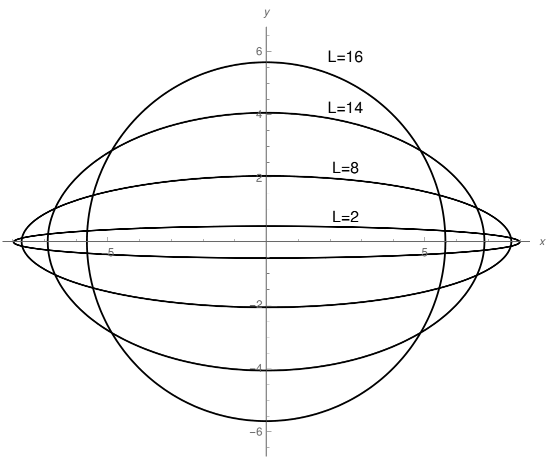

Figure 4. Plot of the equation (24) for different values of L. $L = 16$ is the limiting value corresponding to a circular orbit.

The figure (4) clearly shows that there exist a maximum value of $L = G_N E^2$, for any given E, corresponding to the circular orbit. Let us remark that, as it is expected, for $L = 0$ $r_{max} = r(\phi = 0) = 2G_N E$ is the Schwarzschild radius and $r_{min} = r(\phi = \pi/2) = 0$. In the absence of angular momentum the whole problem collapses into a one-dimensional harmonic motion.

2.2. Charged Orbits $Q \neq 0$

When $Q \neq 0$ the general solution of the orbit equation reads

$$r^2\left(\phi\right) = \frac{2L^2}{E^2} \frac{1 + Q^4/4L^2}{1 - \frac{Q^2}{2G_N E^2} - \sqrt{1 - \frac{1}{G_N E^2}\left(Q^2 + \frac{L^2}{G_N E^2}\right)} \cos\left[\,2\beta\,\phi\,\right]} \tag{29}$$

describing a bounded motion of the particle around the origin. Again orbits are not always closed.

2.3. Closed Orbits

Orbits are closed only if $\beta = n$, $n = 2\,,3\,,4\,\ldots.$

$$r^2_{closed}\,(\,\phi\,) = \frac{2L^2}{E^2}\frac{n^2}{1-\frac{L\sqrt{n^2-1}}{G_NE^2}-\sqrt{1-\frac{1}{G_NE^2}\left(2L\sqrt{n^2-1}+\frac{L^2}{G_NE^2}\right)}\cos\left[\,2n\,\phi\,\right]} \tag{30}$$

with

$$E^2 \geq \frac{L}{G_N}\left(\sqrt{n^2-1}+n\right) \tag{31}$$

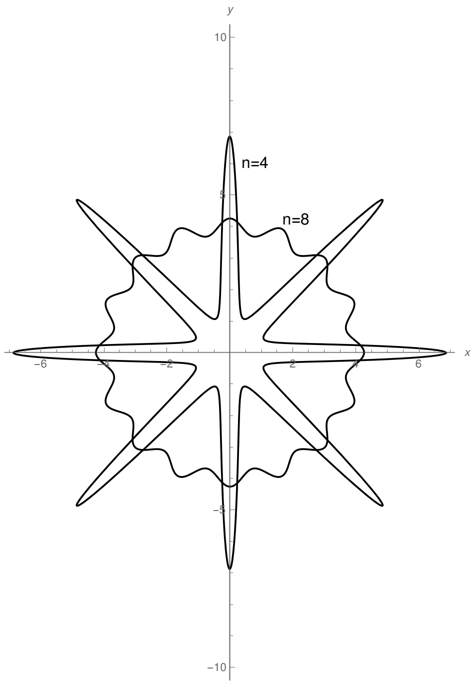

Figure 5. Plot of two closed orbits with $n = 4$ and $n = 8$.

2.4. Open Orbits

For $\beta \neq n$ orbits are *open* and rotate by an angle $\Delta\phi = \pi/\beta$ for every revolution Fig.(7).

$$r^2_{open}\,(\,\phi\,) = \frac{2L^2}{E^2}\frac{\beta^2}{1-\frac{L\sqrt{\beta^2-1}}{G_N E^2}-\sqrt{1-\frac{1}{G_N E^2}\left(2L\sqrt{\beta^2-1}+\frac{L^2}{G_N E^2}\right)}\cos\left[\,2\beta\,\phi\,\right]} \tag{32}$$

Whatever is the value of β, we can compute the maximum and minum distance from the origin.

$$\frac{dr^2}{d\phi} = 0 \longrightarrow \sin\left(\,2\beta\phi\,\right) = 0 \longrightarrow \phi_k = k\frac{\pi}{2\beta} \leq 2\pi \tag{33}$$

with $k = 0\,,1\,,2\,,3\,,\ldots.$

$$r^2\,(\,\phi_k\,) = r_k^2 = \frac{2L^2\beta^2}{E^2}\frac{1}{1-\frac{Q^2}{2G_N E^2}+(\,-1\,)^{k+1}\sqrt{1-\frac{1}{G_N E^2}\left(\,Q^2+\frac{L^2}{G_N^2 E^2}\,\right)}} \tag{34}$$

k odd gives minimum distance r_-, and k even gives maximum distance r_+. The limit $L \rightarrow 0$ is "singular" in the sense that $\beta \rightarrow \infty$ and the orbit degenerates in a one-dimensional motion over the interval $r_- \leq r \leq r_+$:

$$r^2\,(\,\phi_k\,) \rightarrow r_\pm^2 = 2G_N^2 E^2 - Q^2 G_N \pm 2G_N E\sqrt{G_N E^2 - Q^2} \tag{35}$$

For vanishing angular momentum the trajectory describes the oscillation of the horizon between the inner and outer Reissner-Nordstrom radii:

$$r_\pm = EG_N \pm \sqrt{E^2 G_N^2 - Q^2 G_N} \tag{36}$$

Finally, we notice that for

$$2G_N E^2 = Q^2\left[1+\sqrt{1+\frac{4L^2}{Q^4}}\right] \tag{37}$$

the orbit is ϕ independent, i.e. it is a circle

$$r^2\,(\,\phi\,) = \frac{2L^2\beta^2}{E^2}\frac{1}{1-\frac{Q^2}{2G_N E^2}} = 2G_N\beta L \tag{38}$$

For $L \rightarrow 0$ equation(37) gives the extremality condition for the RN black hole $G_N E^2 = Q^2$, and $r^2(\phi) \rightarrow G_N Q^2 = G_N^2 E^2$. Thus, the condition (37) represents a generalized *extremality* condition in the presence of the angular momentum L.

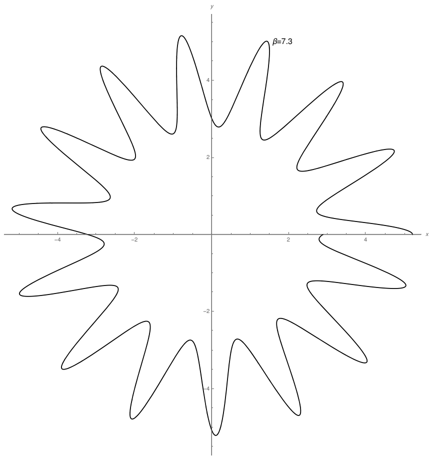

Figure 6. Plots of an open orbit with $L = 1$, $E = 4/\sqrt{G_N}$,$\beta = 7.3$.

3. Quantum Charged BH

In this section we shall quantize the classical model described previously. The quantization scheme contains the underlying idea to make the radius of the horizon(s) "*uncertain*" and thus, unavoidably, described only in terms of a probability amplitude, or "wave function". From this perspective the horizon radius looses its classical geometrical meaning. It acquires the role of wave-length of a Planckian BH. This description is motivated by the fact that in the vicinity of the Planck scale the wavelength of an ordinary quantum particle and the quantum mean radius of a Planckian BH merge and there is no distinction between the two. Therefore, it is important to remark that a Planckian BH is *very different* from a (semi)classical one! It is no more characterized by a one-way geometric boundary, but by a wave-length which is an increasing function of the energy. Only far above the Planck scale, where the quantum fluctuations "freeze-out", one can resume the concept of classical horizon.

Our quantum description has a two-fold motivation:

- it is generally accepted that the dynamics of a quantum gravitational system is completely encoded in its boundary. This is the celebrated Holographic Principle which seems to find its natural realization in the quantum dynamics of a BH, where the "boundary" is the horizon it-

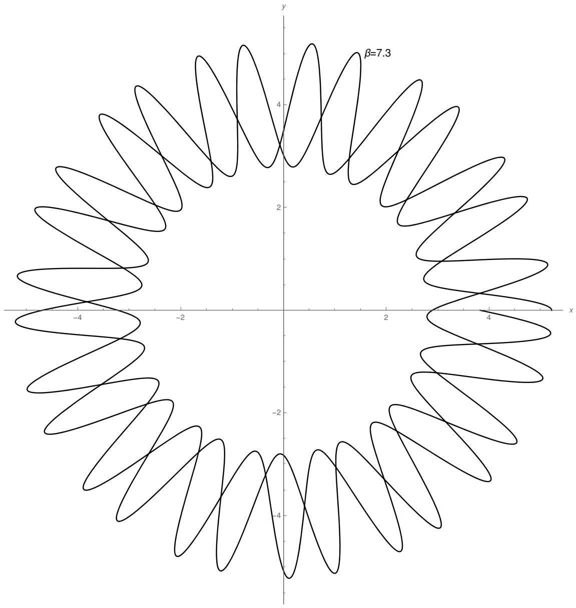

Figure 7. Precession of the open orbit with $L = 1$, $E = 4/\sqrt{G_N}$,$\beta = 7.3$ after two revolutions.

self. Already at the semi-classical level this principle is implied by the Bekenstein-Hawking "area law".

- As we have shown in the previous section, the classical horizon dynamics can be described in terms of a "particle" moving in a suitable self-gravitational potential. Thus, it is straightforward to proceed by looking for the horizon wave function as the solution of a quantum wave equation for the corresponding classical particle studied before.

Starting from the classical Hamiltonian (2), following the standard quantization procedure, one obtains the corresponding wave equation a

$$\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]\Psi\left(r\,,\theta\,,\phi\right)$$
$$+\left[E^2-\frac{r^2}{4\,G_N^2}\left(1+\frac{Q^2G_N}{r^2}\right)^2\right]\Psi\left(r\,,\theta\,,\phi\right)=0 \tag{39}$$

The $O(3)$ symmetry of the problem allows to express the angular dependence of the wave function in terms of spherical harmonics $Y_l^m(\theta, \phi)$ as:

$$\Psi(r, \theta, \phi) = \psi(r) Y_l^m(\theta, \phi) , \tag{40}$$

$$l = 0, 1, 2, \ldots \qquad -l \leq m \leq l \tag{41}$$

Thus, the radial wave equation reads:

$$\left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) \right] \psi(r) + \left[E^2 - \frac{r^2}{4G_N^2} \left(1 + \frac{Q^2 G_N}{r^2} \right)^2 - \frac{l(l+1)}{r^2} \right] \psi(r) = 0 \tag{42}$$

The radial wave-function is given in terms of generalized Laguerre polynomials $L_n^\alpha(x)$ as:

$$\psi_n(r) = N_n \frac{r^{2s}}{(2G_N)^s} e^{-r^2/4G_N} L_n^{2s+1/2}\left(r^2/2G_N \right) \tag{43}$$

where

$$L_n^\alpha(x) \equiv \sum_{k=0}^{n} \frac{\Gamma(n+\alpha+1)}{\Gamma(n-k+1)\Gamma(\alpha+k+1)} \frac{(-x)^k}{k!} \tag{44}$$

and

$$4s \equiv \sqrt{Q^4 + (2l+1)^2} - 1 \tag{45}$$

The normalization coefficient N_n is recovered from the unitarity condition

$$4\pi \int_0^\infty dr r^2 |\psi|^2 = 1 \longrightarrow N_n = \frac{1}{2} \frac{\sqrt{n!}}{\sqrt{\sqrt{2}\,\pi\, G_N^{3/2} \Gamma(n+2s+3/2)}} \tag{46}$$

As it is expected from the classical analysis of the particle motion, one obtains a discrete energy spectrum at the quantum level:

$$\begin{aligned} 2G_N E_n^2 - Q^2 &= 4n + 2 + \sqrt{Q^4 + (2l+1)^2} , \\ &= 4(n+s) + 3 , \qquad n = 0, 1, 2, \ldots \end{aligned} \tag{47}$$

Equation (47)is a concrete and simple realization of the general conjecture that mass spectrum of a quantum BH should be discrete [31, 32]. Furthermore, the result shows that a quantum BH is significantly different from its classical counterpart. In fact, even in the neutral case, $Q = 0$, a stable, *non-singular* ground state configuration with $n = 0$ does exist. The ground state energy is finite and close to the Planck energy

$$E_0 = \sqrt{\frac{3}{2}}\, M_P \approx 1.22 \times M_P \tag{48}$$

This is the *lightest*, stable, BH physically admissible, and no physical process can decrease its mass below this lower bound. The true ground state of a quantum BH is free from all the pathologies of semi-classical, geometrical, BHs, e.g. singularities, thermodynamical instability, etc.
This is to be expected since all the semi-classical arguments loose their meaning at the truly quantum level.
Having acquired the notion that Plankian BHs are quite different objects from their classical "cousins", we would like to address the question of how to consistently connect Planckian and semi-classical BHs. As usual, one assumes that the quantum system approaches the semi-classical one in the "large-n" limit in which the energy spectrum becomes continuous. Before doing so, let us first consider the radial density describing the probability of finding the particle at distance r from the origin, define as $p_n(r) \equiv 4\pi r^2 |\psi|^2$:

$$p_n(x) = \frac{2\, n!}{\Gamma\left(\, n + 2s + 3/2\,\right)}\, x^{4s+2}\, e^{-x^2} \left(L_n^{2s+1/2}\left(\, x^2\,\right) \right)^2 \, , \quad x \equiv r/\sqrt{2G_N} \tag{49}$$

The local maxima in figure (8) represent the most probable size of the Planckian BH. These maxima are solutions of the equation

$$\left(\, 2s + 1 - x^2 + 4n \,\right) \, L_n^{2s+1/2}\left(\, x^2\,\right) - 2\left(\, 2n + 2s + 1/2\,\right) L_{n-1}^{2s+1/2}\left(\, x^2\,\right) = 0 \tag{50}$$

Equation (50) cannot be solved analytically , but its large-n limit can be evaluated as follows. First, perform the division $L_n^{2s+1/2}/L_{n-1}^{2s+1/2}$, and then write

$$L_n^{2s+1/2}\left(\, x^2\,\right) = P_2\left(\, x^2\,\right)\, L_{n-1}^{2s+1/2}\left(\, x^2\,\right) + Q_{n-2}\left(\, x^2\,\right) \tag{51}$$

where,

$$P_2 = \frac{a_n}{b_{n-1}}\left(\, x^2 - 2n - 2s + 1/2\,\right) \, , \tag{52}$$

$$\begin{aligned} Q_{n-2}\left(\, x^2 , \right) &= c_{n-2} x^{2n-4} + \cdots \\ &= -\left(\, n-1\,\right)\left(\, n + 2s - 1/2\,\right) a_n\, x^{2n-4} + \cdots \end{aligned} \tag{53}$$

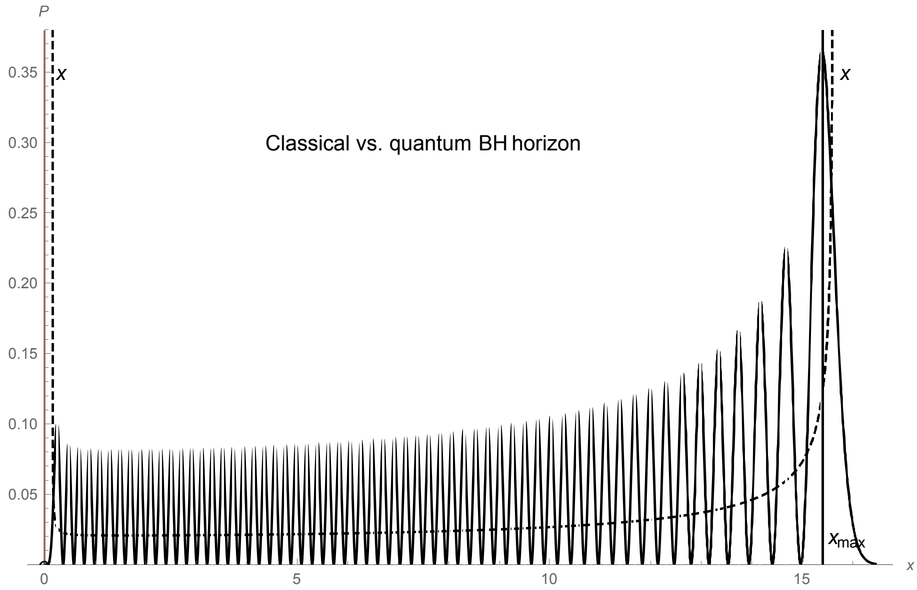

Figure 8. Plot of the function $p_{n=60}(x)$, $s = 1$ (continuous line) vs classical probability (dashed line). For large n the position of the first peak approaches r_-, while the last peak approaches r_+.

By inserting equation (51) in equation (50) and by keeping terms up order x^{2n-2}, the equation for maxima turns into

$$\left[x^2 - 2(n+s) - 1\right]\left[x^2 - 2(n+s) + 1/2\right] + \left[2n + 4s + 1\right]\frac{b_{n-1}}{a_n} = (n-1)(n+2s-1/2) \tag{54}$$

where the coefficients of the of $L_n^{2s+1/2}$ and $L_{n-1}^{2s+1/2}$ from (44) are given by

$$a_n = \frac{(-1)^n}{n!}\,, \tag{55}$$

$$b_{n-1} = \frac{(-1)^{n-1}}{(n-1)!} \tag{56}$$

Equation (54), for large n reduces to

$$3n\,(n+2s) = \left(x^2 - 2\,(n+s)\right)^2$$
$$x^2 = 2\,(n+s) + \sqrt{3n\,(n+2s)} \underbrace{\approx}_{s<<n} 2\,(n+s) + \sqrt{3}n\,(1+s/n) + \cdots$$
$$x^2 = (2+\sqrt{3})\,(n+s) = 3,73\,(n+s)$$

Thus, one finds the absolute maximum to be

$$x^2 = 3.73 \times (\,n+s\,) \tag{57}$$

while, the classical radius of the horizon, for $E >> Q/\sqrt{G_N}$, is obtained by expressing (16) in terms of s and (47)

$$\frac{r_+^2}{2G_N} \simeq 2G_N E^2 - Q^2 \simeq 4n + \sqrt{1+Q^4} \simeq 4\,(\,n+s\,) \tag{58}$$

which leads to

$$x_+^2 = 4\;(\,n+s\,) \tag{59}$$

Thus, we find that most probable value of r approaches the horizon radius r_+ for $E >> M_p$, restoring the (semi)classical picture of BH.

4. Discussion and Future Perspectives

n this closing section we would like to answer a couple of possible questions about our non geometric approach to quantum BHs.

First of all, why should one use a single particle-like formulation?

Before answering this question one needs to explain what does it mean "to quantize a BH". Naively, one could think of the amplitude as describing the probability to find the BH somewhere in space at a given instant of time. This is not the correct interpretation because we are not interested in the *global* quantum dynamics of the BH as a whole, but rather in its "*internal*" dynamics. At this point we face the problem to define what is this internal dynamics. To give the correct answer the Holographic Principle provide the road map. The internal dynamics is nothing else but the horizon dynamics. On the other hand, General Relativity does not provide any dynamics being the BH horizon a purely geometrical boundary. At the quantum, level one expects that the radius and the shape of the horizon become *uncertain*. Near the Planck scale the mean value of the horizon radius $< r_+ >$ becomes comparable, or even smaller, than the the uncertainty Δr_+ and the very concept of geometrical description of the horizon become meaningless. Thus, the first step towards a quantum BH is to move away from the safe land of General Relativity towards an uncharted territory.

In the case of a spherically symmetric BH, we exploited the analogy with the two-body problem in the central potential to describe the BH as a "fictitious" particle moving in a suitable radial *effective potential*. Following the same line of reasoning, we described the horizon equation like the equation for the turning-points of a particle of energy E moving between $r = r_-$ and $r = r_+$. Accordingly, we assign the horizon an effective dynamics described by the motion of such representative particle. The motion of the particle in the interval $r_- \leq r \leq r_+$ corresponds to the vibrational modes of the horizon. Thus, we conclude that our particle-like approach provides a simple and effective implementation of the Holographic Principle.

The second important question is how does a geometric picture of the horizon emerges from the quantum description.
The classical limit is, perhaps, the most delicate feature of any quantum theory. Nevertheless, in our case, the answer should be pretty clear. The wave function (41) is the probability amplitude to find the BH with an horizon of radius r_+. As the probability density (49) and the plot in Fig.(8) show, there are many possible values of the horizon radius for a given energy level E_n, but there is a single highest peak of the probability density. For $E_n >> M_P$, the peak approaches the classical classical radius $r = r_+$. This behavior is clearly shown in Eq.(59). Thus, the geometrical picture of the horizon is recovered in the sense that the most probable value of the horizon radius reduces to the classical value provided by General Relativity in a far trans-Planck regime.

Having clarified the two main points above, let us conclude this paper with a brief comment about elementary particles and Planckian BHs.
The underlying idea that motivated this paper is the generally accepted view that, at the Planck scale, a kind of " transition " between particles and micro-BHs takes place [33, 34, 35]. An elementary particle, in the sub-Planckian regime, has its wavelength inversely proportional to its energy, but when it crosses the "Planck energy barrier" this relation *suddenly* changes into a direct proportionality. This is due to the fact that the system has acquired energy

enough for the appearance of a micro-BH.

In recent, so-called UV, self-complete quantum gravity program, this transition has been called "classicalization" [36, 37] in the sense that a quantum particle turns at once into a *classical*, but microscopic, BH. Although we are in agreement, in general terms, with this picture, in this work we presented a refined version. In our view, classicalization does not take place abruptly at the Planck scale, but far above. The intermediate region, just above the Planck scale, is dominated by pure quantum objects which have all the characteristics of a quantum particle. The only difference is in the relation between its wavelength and energy. These objects could be tentatively called "quantum Planckian BHs" bearing in mind that they are very different from the their (semi)classical, geometrical counterparts. Nevertheless, they deserve the name "black holes" because, as we have shown, in the high energy limit they turn into (semi)classical BHs as we know them. The main difference between these two families bearing the same name resides in the fact that the Planckian BHs have no horizon in the classical sense and, therefore, no geometrical interpretation. In fact, they behave and interact as ordinary quantum particles. Even if there will be no available energy to produce them in high energy experiments, they should be taken into account as virtual intermediate states. From this point of view, it is possible to expect to measure their indirect effects in particle collisions even at energy much below the Planck scale. the most promising scenario for this effects to be seen is within *large extra-dimension* models [38], where the Planck scale can hopefully be lowered not far from the TeV scale.

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Chapter 4

QUANTUM GRAVITY CORRECTIONS TO GAUGE THEORIES WITH A CUTOFF REGULARIZATION

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Abstract

The gravitational waves recently observed by the LIGO collaboration is an experimental evidence that the weak field approximation of general relativity is a viable, calculable scenario. As a non-renormalizable theory, gravity can be successfully considered as an effective quantum field theory with reliable, but limited predictions. Though the influence of gravity on gauge and other interactions of elementary particles is still an open question. In this chapter we calculate the lowest order quantum gravity contributions to the QED beta function in an effective field theory picture with a momentum cutoff. We use a recently proposed 4 dimensional improved momentum cutoff that preserves gauge and Lorentz symmetries. We find that there is a non-vanishing quadratic contribution to the photon 2-point function but after renormalization that does not lead to the running of the original coupling. We comment on corrections to the other gauge interactions and Yukawa couplings of heavy fermions. We argue that gravity cannot turn gauge interactions asymptotically free.

1. Introduction

Recently, in the latest four-five years there were two outstanding discoveries in the area of physics of fundamental interactions. The upgraded LIGO experiment observed [1] gravitational waves in 2015 and published in 2016 and the LHC has announced the discovery of the Higgs boson in Run I in 2012. The observation of the gravitational waves traveling with the speed of light is a direct evidence that the weak field approximation of general relativity can be used reliably in high precision calculation. Furthermore the source of the event GW150914 is found to be consistent with merging of two black hole with mass approximately 39 and 32 solar masses and the LIGO collaboration found no evidence for violations of general relativity in this strong field regime of gravity. Despite this success perturbatively quantized general relativity is still considered to be a non-renormalizable theory due to its dimensionful coupling constant κ with negative mass dimension ($\kappa^2 = 32\pi G_{\mathrm{N}} = 1/M_P^2$). This way the naively quantized Eintein theory cannot be considered as a fundamental theory at the quantum level [2] as newer and newer counter terms have to be introduced at each order of the perturbative calculation and the cutoff cannot be taken to infinity. However Donoghue argued that assuming there is some yet unknown, well defined theory of quantum gravity that yields the observed general relativity as a low energy limit, then the Einstein-Hilbert action can be used to calculate gravitational correction in the framework of effective field theories (well) below the Planck mass $M_P \simeq 1.2 \times 10^{18}$ GeV[3, 4]. The subject was reviewed in details by Burgess in [5].

The other important recent achievement was the discovery of the SM (Standard Model) Higgs boson with a mass approximately 125 GeV by the ATLAS and CMS collaborations [6, 7]. So far the properties of the 125 GeV scalar are in complete agreement with the SM Higgs predictions, few sigma anomalies in the photon-photon and the lepton number violating mu-tau final states (at CMS) have disappeared. This value of the Higgs mass falls in a special region where not only several different decay channel are experimentally tested, but it implies that the SM is perturbatively renormalizable up to M_P. The complete Standard Model might be valid up to the Planck scale [8, 9]. In this case we live close to the stability region in the $(m_{top},\ M_H)$ plane in a metastable world [10], where the tunneling to the lower, real minimum is longer than the lifetime of our Universe. Considering the SM or its extensions valid up to the Planck scale gravity can influence the SM observables and running parameters at the loop-level. The gravitational corrections can be estimated in an effective field theory framework and may be important as they may modify the running of the various coupling, possibly alter the gauge coupling unification and the conclusions concerning the stability of the Standard Model. In the seventies the first attempts using dimensional regularization showed that only higher order operators get renormalized at one-loop order [11].

The effective field theory treatment of gravity was recently used to study quantum corrections to gauge and other theories. In the pioneering work, starting the new era, Robinson and Wilczek argued that the gravity contribution to the Yang-Mills beta function is quadratically divergent and negative, further the corrections point toward asymptotic freedom [12]. There were several controversial results about this claim in the literature. Pietrykowski showed in [13] that in the Maxwell-Einstein theory the result is gauge dependent and doubted the validity of the Robinson Wilczek result. Toms repeated the calculation in the gauge choice independent background field method using dimensional regularization and has found no quantum gravity contribution to the beta function [14]. Diagrammatic calculation employing dimensional regularization and naive momentum cutoff [15] found vanishing quadratic contribution. The authors showed that the logarithmic divergences renormalize the dimension-6 operators in agreement with the early results of Deser et al. [11]. Toms later applied proper time cutoff regularization and claimed that the quadratic dependence on the energy remains in the QED one-loop effective action [16]. Analysis using the background field method employing the gauge invariant Vilkovisky-DeWitt formalism [17, 18, 19] and special loop regularization that respects Ward identities both found non-vanishing quadratic contributions to the beta function, but [17] with sign opposite to [12, 16]. Nielsen showed that the quadratic divergences are generally still gauge dependent in the Vilkovisky-DeWitt formalism [20]. In the asymptotic safety scenario [21, 22] Reuter et al. has found going beyond naive perturbation theory that gravity contribution points towards asymptotic freedom of the Yang-Mills theory [23], later Litim et al. showed that gravity does not contribute to the running of the gauge coupling [24]. In a higher derivative renormalizable theory of gravity the authors [25] showed that the gravity correction vanishes in any gauge theory. There are many various results (for more complete list see the references in e.g., [18]), sometimes contradicting to each other and the physical reality of quadratic corrections to the gauge coupling was questioned [26, 27, 28, 29]. The situation could be clarified using a straightforward cutoff calculation respecting the symmetries of the models and correctly interpreting the divergences appearing in the calculations.

Earlier the present authors developed a new improved momentum cutoff regularization which by construction respects the gauge and Lorentz symmetries of gauge theories at one loop level [30]. In this chapter we discuss the application to the effective Maxwell-Einstein and Einstein-Yang-Mills systems to estimate the regularized gravitational corrections to the photon/gluon two and three point functions in the simplest possible model and later discuss more involved theories.

The paper is organized as follows. In section 2. the effective gravity contribution to quantum electrodynamics is calculated, in section 3. the renormalization is discussed. In chapter 4 corrections to a Yang-Mills theory is presented. The paper is closed with conclusions and an appendix summarizing the improved momentum cutoff method.

2. Effective Maxwell-Einstein Theory

In this section we present the calculation of the gravitational quantum corrections to the photon self energy in the simple Einstein-Maxwell theory, given by the Lagrangian [29]

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{2}{\kappa^2} R - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \right], \tag{1}$$

where R is the Ricci scalar, $\kappa^2 = 32\pi G_{\mathrm{N}}$ and $F_{\mu\nu}$ denotes the $U(1)$ field strength tensor. Quantum effects are calculated in the weak field expansion around the flat Minkowski metric ($\eta_{\mu\nu} = (1, -1, -1, -1)$)

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x). \tag{2}$$

This is considered an exact relation, but the inverse of the metric contains higher order terms

$$g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_{\alpha} h^{\nu\alpha} + \dots, \tag{3}$$

in an effective treatment it can be truncated at the second order. The photon propagator is defined in the Landau gauge

$$\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2 - i\epsilon},$$

and the graviton propagator in de Donder, or harmonic gauge, where the gauge condition is (with $h = h^{\alpha}_{\alpha}$)

$$\partial^{\nu} h_{\mu\nu} - \frac{1}{2}\partial_{\mu} h = 0. \tag{4}$$

Expanding the Lagrangian up to second order in the graviton field we get the following graviton propagator in d dimensions

$$G^{G}_{\alpha\beta\gamma\delta}(k) = i\frac{\frac{1}{2}\eta_{\alpha\gamma}\eta_{\beta\delta} + \frac{1}{2}\eta_{\alpha\delta}\eta_{\beta\gamma} - \frac{1}{d-2}\eta_{\alpha\beta}\eta_{\gamma\delta}}{k^2 - i\epsilon}. \tag{5}$$

There are two relevant vertices with two photons. The two photon-graviton vertex is

$$\begin{aligned} V_{\gamma\gamma G}(k_{1\mu}, k_{2\nu}, \alpha, \beta) \;=\; & -i\frac{\kappa}{2}\left[\eta_{\alpha\beta}\left(k_{1\nu}k_{2\mu} - \eta_{\mu\nu}(k_1 k_2)\right) + \right. \\ & + Q_{\mu\nu,\alpha\beta}(k_1 k_2) + Q_{k_1 k_2,\alpha\beta}\eta_{\mu\nu} \\ & \left. - Q_{\mu k_2,\alpha\beta}k_{1\nu} - Q_{k_1\nu,\alpha\beta}k_{2\mu}\right], \end{aligned} \tag{6}$$

and the two photon-two graviton vertex is even more complicated

$$\begin{aligned} V_{\gamma\gamma GG}(k_{1\mu}, k_{2\nu}, \alpha, \beta, \gamma, \delta) \;=\; & -i\frac{\kappa^2}{4}\left[P_{\alpha\beta\gamma\delta}\left(k_{1\nu}k_{2\mu} - \eta_{\mu\nu}(k_1 k_2)\right)\right. \\ & + U_{\mu\nu,\alpha\beta,\gamma\delta}(k_1 k_2) + \\ & + U_{k_1 k_2,\alpha\beta,\gamma\delta}\eta_{\mu\nu} \\ & - U_{\mu k_2,\alpha\beta,\gamma\delta}k_{1\nu} - U_{k_1\nu,\alpha\beta,\gamma\delta}k_{2\mu} + \\ & + Q_{\mu\nu,\alpha\beta}Q_{\gamma\delta,k_1 k_2} + Q_{\mu\nu,\gamma\delta}Q_{\alpha\beta,k_1 k_2} \\ & \left. - Q_{k_1\nu,\alpha\beta}Q_{\mu k_2,\gamma\delta} - Q_{\mu k_2,\alpha\beta}Q_{k_1\nu,\gamma\delta}\ \right]. \end{aligned} \tag{7}$$

For the sake of simplicity we have defined

$$U_{\mu\nu,\alpha\beta,\gamma\delta} = \eta_{\mu\alpha}P_{\nu\beta,\gamma\delta} + \eta_{\mu\beta}P_{\alpha\nu,\gamma\delta} + \eta_{\mu\gamma}P_{\alpha\beta,\nu\delta} + \eta_{\mu\delta}P_{\alpha\beta,\gamma\nu}, \tag{8}$$

$$P_{\alpha\beta,\mu\nu} = \eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}, \tag{9}$$

and finally

$$Q_{\alpha\beta,\mu\nu} = \eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}. \tag{10}$$

There are two graphs contributing to the photon self energy with two vertices (31) giving $\Pi^{(a)}$ (Fig. 1. left) and one 4-leg vertex (7) providing $\Pi^{(b)}$ (Fig.

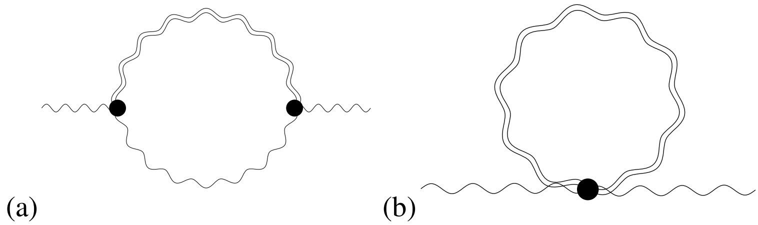

Figure 1. Feynman graphs with graviton (double) lines contributing to the photon two point function.

1. right). We calculated the finite and divergent parts of the 2-point function with improved cutoff, naive 4-dimensional momentum cutoff and dimensional regularization. The improved momentum cutoff is defined to respect gauge and Lorentz symmetries and allows for shifting the loop momentum under divergent loop-integrals. Compared to naive cutoff it changes the coefficient of the quadratic divergence and gives a finite shift in the presence of a universal logarithmic divergence. The details of the new regularization scheme with some example and outlook on the broad literature can be found in the Appendix. For comparison, using the technique of dimensional regularization with different assumptions about treating the number of dimensions d in the propagator and vertices various quadratically divergent cutoff results can be identified using the connection between cutoff and dimenisonal regularization results, see (39) in the Appendix. Each of these calculation defines a different regularization scheme.

The calculation of the diagrams is straightforward, we used the symbolic manipulation program *FORM* [31] to deal with the large number of terms. The quadratically divergent contributions of the two graphs with **improved cutoff** (I) do not cancel each other

$$\Pi^{\mathrm{I}(a)}_{\mu\nu}(p) = \frac{i}{16\pi^2}\kappa^2\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\left(-2\Lambda^2 - \frac{1}{6}p^2\left(\ln\left(\frac{\Lambda^2}{p^2}\right) + \frac{2}{3}\right)\right), \quad (11)$$

$$\Pi^{\mathrm{I}(b)}_{\mu\nu}(p) = \frac{i}{16\pi^2}\kappa^2\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\left(\frac{3}{2}\Lambda^2\right). \quad (12)$$

In the **naive cutoff** (N) calculation using (36) there is a cancellation of the Λ^2 terms, the finite term do not match the previous one, and it is remarkable that the result is transverse without any subtractions

$$\Pi^{\mathrm{N}(a)}_{\mu\nu}(p) = \frac{i}{16\pi^2}\kappa^2\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\left(-\frac{3}{2}\Lambda^2 - \frac{1}{6}p^2\ln\left(\frac{\Lambda^2}{p^2}\right) - \frac{7}{36}p^2\right), \quad (13)$$

$$\Pi^{\mathrm{N}(b)}_{\mu\nu}(p) = \frac{i}{16\pi^2}\kappa^2\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\left(\ \frac{3}{2}\Lambda^2\right). \quad (14)$$

In **dimensional regularization** (DR) the space-time dimension is continued in all terms originating from the gauge and gravitational part, too (e.g., $\eta^\mu_\mu = d = 4 - 2\epsilon$). The result (just as using the naive cutoff above) agrees with [15] (without the finite terms which are first given here)

$$\Pi^{DR1(a)}_{\mu\nu}(p) = \frac{i}{16\pi^2}\kappa^2\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\left(-\frac{1}{6}p^2\left(\frac{2}{\epsilon} + \ln\left(\frac{\mu^2}{p^2}\right) + \frac{1}{6}\right)\right), \quad (15)$$

$$\Pi^{DR1(b)}_{\mu\nu}(p) = 0, \quad (16)$$

where we have omitted the constants $-\gamma_E + \ln 4\pi$ beside $2/\epsilon$.

In what follows we present various "cutoff" results we arrived at using the technique of dimensional regularization based on different assumptions about the continuation of the dimension. Each result defines a different regularization scheme, and they are denoted by the superscript $DR1$, $DR2$, $DR3$ and the corresponding cutoff results by $\Lambda1$, $\Lambda2$, $\Lambda3$ based on the extension of dimensional regularization.

Now with the help of the equations in the appendix (39), (40) and (41) we can define three cutoff results based on the dimensional regularization one. In the first case the dimension is modified in each terms where d appears, also in the graviton propagator (5), though gravity is not a dynamical theory in $d = 2$. Each graph is quadratically divergent, even $1/(\epsilon - 1)^2$ type of singularities appear in single graphs, but they cancel in the sum of the graphs, like the $\frac{1}{\epsilon^2}$ terms in usual gauge theories (e.g., in QCD) at two loops.

$$\Pi^{\Lambda 1}_{\mu\nu}(p) = \frac{i}{16\pi^2}\kappa^2\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\left(-\frac{1}{4}\Lambda^2 - \frac{1}{6}p^2\left(\ln\left(\frac{\Lambda^2}{p^2}\right) - \frac{5}{6}\right)\right) \quad (17)$$

also quadratically divergent, but only the coefficient of the logarithmic term agrees with other results.

To find connection with existing, partially controversial literature, we have performed the calculation with weaker assumptions. First the term in the graviton propagator is set $\frac{1}{d-2} = \frac{1}{2}$ as is usually done in earlier results e.g., [27, 28].

The divergent part of the dimensional regularization result agrees with [15]. The contribution of the tadpole in Fig. 1b $\Pi^{\mathrm{DR2(b)}}$ vanishes, the sum is

$$\Pi_{\mu\nu}^{DR2}(p) = \frac{i}{16\pi^2}\kappa^2\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\left(-\frac{1}{6}p^2\left(\frac{2}{\epsilon} + \ln\left(\frac{\mu^2}{p^2}\right) + \frac{1}{6}\right)\right). \quad (18)$$

We can identify a cutoff result, Fig. 1b gives $\Pi_{\mu\nu}^{\Lambda 2(\mathrm{b})}(p) \sim \frac{1}{2}\Lambda^2$, the only quadratically divergent term and

$$\Pi_{\mu\nu}^{\Lambda 2}(p) = \frac{i}{16\pi^2}\kappa^2\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\left(\frac{1}{2}\Lambda^2 - \frac{1}{6}p^2\left(\ln\left(\frac{\Lambda^2}{p^2}\right) - \frac{5}{6}\right)\right). \quad (19)$$

Notice that this result differs from (17) only in the value and the sign of the coefficient of the first term, the change originates from the different treatment of the graviton propagator.

The result of the improved momentum cutoff can be reproduced applying dimensional regularization with care. The improved cutoff method works in four physical dimensions and special rules have to be applied only at the evaluation of the last tensor integrals. It is equivalent to setting $d = 4$ in the Einstein-Maxwell theory, e.g., both in the graviton propagator and in the trace of the metric tensor. Dimensional regularization is then applied at the last step evaluating the tensor and scalar momentum integrals. We have found that $\Pi_{\mu\nu}^{\mathrm{DR3(b)}} = 0$ and

$$\Pi_{\mu\nu}^{DR3}(p) = \frac{i}{16\pi^2}\kappa^2\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\left(-\frac{1}{6}p^2\left(\frac{2}{\epsilon} + \ln\left(\frac{\mu^2}{p^2}\right) + \frac{5}{3}\right)\right). \quad (20)$$

The corresponding cutoff result diverges quadratically and agrees with the improved cutoff calculation (11,12)

$$\Pi_{\mu\nu}^{\Lambda 3(a)}(p) = \frac{i}{16\pi^2}\kappa^2\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\left(-2\Lambda^2 - \frac{1}{6}p^2\left(\ln\left(\frac{\Lambda^2}{p^2}\right) + \frac{2}{3}\right)\right), \quad (21)$$

$$\Pi_{\mu\nu}^{\Lambda 3(b)}(p) = \frac{i}{16\pi^2}\kappa^2\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\left(\frac{3}{2}\Lambda^2\right). \quad (22)$$

The quadratic divergences $\left(\Lambda^2\right)$ here are identified with the $d = 2$ poles in the extension [48] of dimensional regularizations [51, 52]. There may appear an additional pole $1/(d-2)$ in the graviton propagator (5). It is coming from a non-physical point of the Einstein-Hilbert theory as this theory is not a dynamical one in $d = 2$, the Lagrangian reduces to a trivial surface integral. In the first case, in (17) we apply continuous d both in the propagator (5) and in the vertices during tracing. The second treatment sets $d = 4$ in the propagator (as usually done in the literature) while using continuous d during tracing the indices. This hybrid treatment looks not fully consistent as even in the loops one part of the theory feels the modified d dimensions the other part not, e.g., feels fixed number of dimensions $d = 4$ and gives (18). We prefer the third, conceptionally simple case, when the gravity algebra is performed in fixed $d = 4$ and the rest of the calculation is done using the standard dimensional regularization technique. Moreover, the third result (21) and (22) agrees completely with the improved cutoff calculation case.

In principle a theory is completely defined via specifying the Lagrangian and the method of calculation e.g., fixing the regularization and the treatment of the divergent terms, though the physical quantities must be independent of the details of the regularization scheme. It is remarkable that the transverse structure of the photon propagator is not violated in any of the previous schemes and the logarithmic term is universal in the three cases and agrees with earlier results [15, 11]. The question is whether the Λ^2 terms contribute to the running of the gauge coupling, or have any other effects on measurable physical quantities.

3. Quadratic Divergences

and Renormalization

In the previous section we have calculated the 1-loop radiative correction to the photon self energy from the effective theory of gravity in the simplest Maxwell-Einstein theory. We have found under various assumptions various quadratically divergent contributions (vanishing particularly using a naive momentum cutoff). The 1-loop corrections to the 2-point function generally modify the bare Lagrangian, the divergences have to be removed by the properly chosen counterterms via renormalization conditions.

Consider the QED action with the convention [32]

$$L_0 = -\frac{1}{4e_0^2} F_{\mu\nu}F^{\mu\nu} + \bar{\Psi} i D_\mu \gamma^\mu \Psi, \qquad D_\mu = \partial_\mu + iA_\mu. \tag{23}$$

The divergences calculated from the interaction (1) gives the 1-loop effective action, here we focus only on the gravitational, divergent contributions

$$L = -\frac{1 + a\kappa^2\Lambda^2}{4e_0^2} F_{\mu\nu}F^{\mu\nu} + a_2 \ln\frac{\Lambda^2}{p^2} \left(D_\mu F^{\mu\nu}\right)^2 + \left(\bar{\Psi} i D_\mu \gamma^\mu \Psi, \right) \tag{24}$$

where p^2 is the Euclidean momentum at which the 2-point function was calculated. The question is wheter should we interpret the coefficient of the usual kinetic term as a varying, i.e., running electric charge $\left(e^2(\Lambda) \simeq e_0^2\left(1 - a\kappa^2\Lambda^2\right)\right)$? The answer is no, because of the necessary wavefunction and charge renormalization.

In quantum field theories the divergent terms have to be canceled by the counterterms. New dimension-six term must be added to match the $p^2 \ln\left(\frac{\Lambda^2}{p^2}\right)$ term already shown in (24)

$$L_{\mathrm{ct}} = \frac{\delta Z_1}{4e_0^2} F_{\mu\nu}F^{\mu\nu} + \delta Z_2 \left(D_\mu F^{\mu\nu}\right)^2 . \tag{25}$$

In principle there are three possible dimension-six counterterms $\left(D_\mu F^{\mu\nu}\right)^2$, $\left(D_\mu F_{\nu\rho}\right)^2$ and $F_\mu^\nu F_\nu^\rho F_\rho^\mu$. Only two of them are linearly independent up to total derivatives and it turns out that the first, the $\left(D_\mu F^{\mu\nu}\right)^2$ term can cancel all divergences [15]. The coefficient of the first term in (24) cannot be understood as defining a running coupling but it is compensated by a counterterm through a renormalization condition. It can be fixed either by the Coulomb potential or Thomson scattering at low energy identifying the usual electric charge as

$$\frac{e_0^2}{4\pi(1 + a\kappa^2\Lambda^2)} = \frac{e^2}{4\pi} \simeq \frac{1}{137}. \tag{26}$$

Thus the quadratically divergent correction defines the relation between the bare charge $e_0(\Lambda)$ in a theory with the physical cutoff Λ and the physical charge effective at low energies. After fixing the parameters of the theory (e.g., by a measurement at low energy) and using e to calculate the predictions of the model the cutoff dependence completely disappears from the physical charge [27, 32]. The role of the quadratic correction is to define the relation(26) this way renormalizing the bare coupling constant $e_0(\Lambda)$ (and does not appear in the running of the physical charge).

Quadratic divergences are the main cause of the hierarchy problem and discussed with other regularization methods. In [33] the authors use Implicit Regularization, a general parametrization of the basic divergent integrals, which separates the divergences for a given problem in a process-independent way without referring to a specific regularization (see also the Appendix). They argue that their basic divergent integrals, thus the quadratic divergences can be absorbed in the renormalization constants without explicitly determining their value. Arbitrary parameters, such as the isolated quadratically divergent contribution to the

Higgs mass can be fixed by additional (in the Higgs case: conformal) symmetry. Similar conclusion is reached in [34] using Wilsonian renormalization group (RG). They argued that the additive (they call it subtractive) and multiplicative renormalization procedure and the corresponding quadratic and logarithmic divergences can be treated independently. They show that quadratic divergences are the artifact of the regularization procedure and in the Wilsonian RG they are naturally subtracted and simply define position of the critical surface in the theory space. It is in complete agreement with our claim in (26) that the quadratic divergence disappears from the physical quantities. The fate of the logarithmic divergence could have been different.

The logarithmically divergent contribution on the other hand defines the renormalization of the higher dimensional operator $(D_\mu F^{\mu\nu})^2$ and again not the running of the gauge coupling. After renormalization (at a point $p^2 = \mu^2$) the logarithmic coefficient of the dim-6 term in (24) changes to $a_2 \ln \frac{\Lambda^2}{p^2} - a_2 \ln \frac{\Lambda^2}{\mu^2} = -a_2 \ln \frac{p^2}{\mu^2}$ defining a would be running parameter. Furthermore note that this term can be removed [15, 26] by local field redefinition of A_μ up to higher dimensional operators

$$A_\mu \to A_\mu - c\nabla_\nu F^\nu_\mu, \tag{27}$$

where ∇_μ is the gravitational covariant derivative, as the new term is proportional to the tree level equation of motion

$$\nabla_\mu F^{\mu\nu} = 0. \tag{28}$$

The logarithmic corrections were found in the first papers discussing the gravitational contributions by Deser et al. [11] using dimensional regularization and this way neglecting the quadratically divergent contribution spotted by [12]. Generally it can be shown, that all photon propagator corrections can be removed by appropriate field redefinition which are bilinear in A_μ even if they contain arbitrary number of derivatives, on-shell scattering processes are not influenced by the presence of such effective terms [35].

4. Corrections to the Gauge Coupling in Yang-Mills Theories

We have discussed the simplest example including gravitational corrections in Chapter 2, but already Deser, Tsao and Nieuenhuizen [11] later Robinson

Figure 2. Feynman graphs with graviton (double) lines contributing to the gluon three point function.

and Wilczek [12] and many other authors performed their calculation in the Einstein-Yang-Mills system. Here we follow the presentation of [15] to show that after renormalization no meaningful running coupling can be defined even identifying quadratic divergences using cutoff regularization.

Consider the Einstein-Yang-Mills Lagrangian

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{2}{\kappa^2} R - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta} \mathrm{Tr} \left[F_{\mu\nu} F_{\alpha\beta} \right] \right], \tag{29}$$

where the field strength has a Yang-Mills index $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig \left[A_\mu ,\ A_\nu \right]$ and g is the Yang-Mills coupling.

In the Yang-Mills theory the bare gluon three-point functions get modified by gravity, too. Beside the corrections to the gluon two point functions, which are order κ^2 and are the same as presented in Chapter 2 see (13) and (14) and Figure 1, there are contributions to the gluon three-point functions at the $g\kappa^2$ order.

There are new vertices with three selfinteracting gluon with extra one- and two-graviton legs. The three gluon-one graviton vertex is

$$\begin{aligned} V_{gggG}(k_{1\mu}^{(a)}, k_{2\nu}^{(b)}, k_{3\rho}^{(c)}, \alpha\beta) \quad = \quad & -ig\kappa f^{abc} \Big[P^{\alpha\beta,\mu\nu} \left(k_1 - k_2 \right)^\rho + \eta^{\alpha\beta} \Big(\eta^{\rho\alpha} \left(k_1 - k_2 \right)^\beta \\ & + \eta^{\rho\alpha} \left(k_1 - k_2 \right)^\beta \Big) + \\ & + \text{cycl.perm.} \left\{ (\mu, k_1), (\nu, k_2), (\rho, k_3) \right\} \Big], \end{aligned} \tag{30}$$

where f^{abc} is the Yang-Mills structure constant. The three gluon-two graviton vertex is again rather complicated and lengthy

$$
\begin{aligned}
V_{gggGG}(k^{(a)}_{1\mu}, k^{(b)}_{2\nu}, k^{(c)}_{3\rho}, \alpha\beta, \gamma\delta) \;=\; & -ig\kappa^2 f^{abc}\Big[(k_1-k_2)^\rho \Big(I^{\mu\nu,\alpha\gamma}\eta^{\delta\beta} \\
& +I^{\mu\nu,\alpha\delta}\eta^{\gamma\beta} + \Big\{^{(\mu\nu)\longleftrightarrow(\alpha\beta)}\Big\} \\
& -\frac{1}{2}\Big(\eta^{\alpha\beta}I^{\mu\nu,\gamma\delta} + \eta^{\gamma\delta}I^{\mu u,\alpha\beta}\Big) - \eta^{\mu\nu}P^{\alpha\beta,\gamma\delta}\Big) \\
& +\Big(2\eta^{\mu\nu}P^{\gamma\delta,\alpha\beta} + I^{\mu\nu,\gamma\delta}\Big)(k_1-k_2)^\rho + \Big\{^{(\alpha)\longleftrightarrow(\beta)}\Big\} \\
& \Big\{^{(\gamma\delta)\longleftrightarrow(\alpha\beta)}\Big\} + \text{cycl.perm.}\,\{(\mu,k_1),(\nu,k_2),(\rho,k_3)\}\Big]\,.
\end{aligned} \tag{31}
$$

With these vertices there are three graphs contributing to the gluon three-point function at one-loop, Fig.2. The external gluons are labeled as in the vertices $\{(\mu,k_1),(\nu,k_2),(\rho,k_3)\}$ and the the 3-point function contributions must be symmetrized in these index-pairs. The graph (c) is only logarithmically divergent

$$
G_3^{(c)} \sim \frac{1}{16\pi^2} g\kappa^2 f^{abc} \log\Lambda^2 F_3^{\mu\nu\rho}\left(k_1^\mu, k_2^\nu, k_3^\rho\right), \tag{32}
$$

where the lengthy $F_3^{\mu\nu\rho}$ function scales with the third power of momenta. The graphs (d) and (e) are similar to (a) and (b) only with the exception of an additional gluon leg starting from the main vertex. Graph (d) has similar logarithmic correction as (32) and a quadratic divergence, while in (e) the divergence is purely quadratic.

$$
G_3^{(d)} \simeq \frac{1}{16\pi^2} g\kappa^2 f^{abc}\frac{3}{2}\left(\eta^{\mu\nu}(k_1-k_2)^\rho + \text{symmetrized}\right)\Lambda^2 + \text{log terms}, \tag{33}
$$

$$
G_3^{(e)} \simeq -\frac{1}{16\pi^2} g\kappa^2 f^{abc}\frac{3}{2}\left(\eta^{\mu\nu}(k_1-k_2)^\rho + \text{symmetrized}\right)\Lambda^2. \tag{34}
$$

The sum of the quadratic contributions from graphs (d) and (e) exactly cancel just as for the two point functions in Fig. 1. The remaining logarithmic divergence can be surprisingly canceled by only the second term in (25).

$$
L_{c.t.} \supset \frac{1}{16\pi^2}\frac{1}{6}\kappa^2 \log\frac{\Lambda^2}{\mu^2}\left(D_\mu F^{\mu\nu}\right), \tag{35}
$$

where μ is the renormalization scale in agreement with the result of [11] and later works. We emphasize that the counterterm in (35) corrects a higher dimensional operator, and the contribution can be removed by a non-linear field redefinitions of the gauge field (27) as discussed in Chapter 3 and does not lead to a change in the running of physical parameters.

5. Conclusion

We have calculated and presented in this chapter the gravitational corrections to gauge theories in the framework of effective field theories. The study was motivated by the various, sometime controversial results in the literature. Our method and the presented results were capable of identifying quadratically divergent contributions to the photon and generalized gluon two and three point functions, thanks to the gauge invariant construction. In the first, QED part, to test our calculation we defined the cutoff dependence employing (39), (40) and dimensional regularization with various assumptions about treating the number of dimensions d. We observed that the 1-loop gravity corrections to the two point function in all but one cases contain Λ^2 divergence with the exception of the naive momentum cutoff which violates gauge symmetries usually. Here all the corrections are transverse. The logarithmic term universally agrees with the literature starting from Deser et al. [11]. Then we presented the corrections in a more general Yang-Mills theory. We found that the logarithmically divergent terms contribute to the dimension-6 terms and can be removed by local field redefinitions this way do not affecting the running of the gauge coupling. Λ^2 corrections to the QED or Yang-Mills effective actions are absent using a naive cutoff regularizations and are present with more sophisticated methods, but those are proved to be non-physical.

The quadratically divergent corrections to the photon or gluon self-energy do not lead to the modification of the running of the gauge coupling. Robinson and Wilczek claimed that the $-a\kappa^2\Lambda^2$ correction could turn the beta function negative and make the Einstein-Maxwell and Einstein-Yang-Mills theory asymptotically free. This statement and the calculation was criticized in the literature. We showed in this chapter using explicit cutoff calculation that Λ^2 corrections may appear in the 2-point function, but those will define the renormalization connection between the cutoff dependent bare coupling and the physical coupling (26) and do not lead to a running coupling. This conclusion is in complete agreement with other results concerning quadratic divergences [27, 33, 34]. Indeed the Λ^2 correction can be absorbed into the physical charge and does not appear in physical processes. Donoghue et al. argue in [27] that an universal, i.e., process independent running coupling constant cannot be defined in the effective theory of gravity independently of the applied regularization. They demonstrate that because of the crossing symmetry in theories (except the $\lambda\Phi^4$) even the sign of the would be quadratic running is ambiguous and a running coupling would be process dependent, thus not useful. Generally the logarithmically divergent corrections could define the renormalization of higher dimensional operators. It turns out that even these logarithmic correction can be removed by appropriate field redefinitions and do not contribute to on-shell scattering processes. We note that the authors in [28] showed using their 4-dimensional implicit regularization method that the quadratic terms are coming from ambiguous surface terms, discussed in more details in [30, 43], and as such are non-physical. Interestingly those surface terms vanish if we evaluate them with our improved cutoff [36].

Finally we point out that we have found gravity corrections to the two and three-point functions in gauge theories. Using a momentum cutoff the quadratically divergent contributions define the renormalization of the bare charge and thus using the physical charge the Λ^2 corrections do not appear in physical processes. On the other hand logarithmic corrections are universal but merely define the renormalization of a dimension-6 term in the Lagrangian, which term can be eliminated by local field redefinition. We conclude that gravity corrections do not lead to the modification of the usual running of gauge coupling and cannot point towards asymptotic freedom in the case of gauge theories.

Appendix: Improved Momentum Cutoff

In this appendix we present the novel regularization of gauge theories, proposed in [30] and discussed with broader outlook on the literature in [36]. It is based on 4 dimensional momentum cutoff to evaluate 1-loop divergent integrals. The idea was to construct a cutoff regularization which does not brake gauge symmetries and the necessary shift of the loop-momentum is allowed as no surface terms are generated. The loop calculation starts with Wick rotation, Feynman-parametrization and loop-momentum shift. Only the treatment of free Lorentz indices under divergent integrals should be changed compared to the naive cutoff calculation.

We start with the observation that the contraction with $\eta_{\mu\nu}$ (tracing) does not necessarily commute with loop-integration in divergent cases. Therefore the substitution of

$$k_\mu k_\nu \to \frac{1}{4}\eta_{\mu\nu}k^2 \tag{36}$$

is not valid under divergent integrals, where k is the loop-momentum[1]. The usual factor $1/4$ is the result of tracing both sides under the loop integral, e.g., changing the order of tracing and the integration. In the new approach the integrals with free Lorentz indices are defined using physical consistency conditions, such as gauge invariance or freedom of momentum routing. Based on the diagrammatic proof of gauge invariance it can be shown that the two conditions are related and both are in connection with the requirement of vanishing surface terms. It was proposed in [30] that instead of (36) the general identification of the cutoff regulated integrals in gauge theories

$$\int_{\Lambda\, reg} d^4 l_E \frac{l_{E\mu} l_{E\nu}}{\left(l_E^2+m^2\right)^{n+1}} = \frac{1}{2n}\eta_{\mu\nu}\int_{\Lambda\, reg} d^4 l_E \frac{1}{\left(l_E^2+m^2\right)^{n}}, \qquad n=1,2,... \tag{37}$$

will satisfy the Ward-Takahashi identities and gauge invariance at 1-loop (l_E is the shifted Euclidean loop-momentum). In case of divergent integrals it differs from (36), for non-divergent cases both substitutions give the same results at $\mathcal{O}(1/\Lambda^2)$ (the difference is a vanishing surface term). It is shown in [30] that this definition is robust in gauge theories, differently organized calculations of the 1-loop functions agree with each other using (37) and disagree using (36). For four free indices the gauge invariance dictates ($n=2,3,...$)

$$\int_{\Lambda\, reg} d^4 l_E \frac{l_{E\alpha} l_{E\beta} l_{E\mu} l_{E\rho}}{\left(l_E^2+m^2\right)^{n+1}} = \frac{1}{4n(n-1)}\int_{\Lambda\, reg} d^4 l_E \frac{\eta_{\alpha\beta}\eta_{\mu\rho}+\eta_{\alpha\mu}\eta_{\beta\rho}+\eta_{\alpha\rho}\eta_{\beta\mu}}{\left(l_E^2+m^2\right)^{n-1}}. \tag{38}$$

For 6 and more free indices appropriate rules can be derived (or (37) can be used recursively for each allowed pair). Finally the scalar integrals are evaluated with a simple Euclidean momentum cutoff. The method was successfully applied to an effective model to estimate oblique corrections [37].

There are similar attempts to define a regularization that respects the original gauge and Lorentz symmetries of the Lagrangian but work in four spacetime dimensions usually with a cutoff [38, 39]. Some methods can separate the divergences of the theories and does not rely on a physical cutoff [40, 41, 42] or even could be independent of it [44]. For further literature see references in [30].

[1] The metric tensor is denoted by $\eta_{\mu\nu}$ both in Minkowski and Euclidean space.

Under this modified cutoff regularization the terms with numerators proportional to the loop momentum are all defined by the possible tensor structures. Odd number of l_E's give zero as usual, but the integral of even number of l_E's is defined by (37), (38) and similarly for more indices, this guarantees that the symmetries are not violated. The calculation is performed in 4 dimensions, the finite terms are equivalent with the results of dimensional regularization. The method identifies quadratic divergences while gauge and Lorentz symmetries are respected. We stress that the method treats differently momenta with free ($k_\mu k_\nu$) and contracted Lorentz indices (k^2), the order of tracing and performing the regulated integral cannot be changed similarly to dimensional regularization. The famous triangle anomaly can be unambiguously defined and presented in [45] see also [46], [47].

However even using dimensional regularization one is able to define cutoff results in agreement with the present method. In dimensional regularization singularities are identified as $1/\epsilon$ poles, power counting shows that these are the logarithmic divergences of the theory. Naively quadratic divergences are set to zero in the process, but already Veltman noticed [48] that these divergences can be identified by calculating the poles in $d = 2$ $(\epsilon = 1)$. Careful calculation of the Veltman-Passarino 1-loop functions in dimensional regularization and with 4-momentum cutoff leads to the following identifications [30, 49, 50]

$$4\pi\mu^2\left(\frac{1}{\epsilon-1}+1\right) = \Lambda^2, \tag{39}$$

$$\frac{1}{\epsilon}-\gamma_E+\ln\left(4\pi\mu^2\right)+1 = \ln\Lambda^2. \tag{40}$$

The finite terms are unambiguously defined

$$f_{\text{finite}} = \lim_{\epsilon\to 0}\left[f(\epsilon) - R(0)\left(\frac{1}{\epsilon}-\gamma_E+\ln 4\pi+1\right) - R(1)\left(\frac{1}{\epsilon-1}+1\right)\right], \tag{41}$$

where $R(0),\ R(1)$ are the residues of the poles at $\epsilon = 0,\ 1$ respectively. Using (39), (40) and (41) at 1-loop the results of the improved cutoff can be reproduced using dimensional regularization without any ambiguous subtraction.

The loop integrals are calculated as follows. First the loop momentum (k) integral is Wick rotated (to k_E), with Feynman parameter(s) the denominators are combined, then the order of Feynman parameter and the momentum integrals are changed. After that the loop momentum ($k_E \to l_E$) is shifted to have a spherically symmetric denominator.

Finally we present two divergent integrals calculated by the new regularization. Δ can be any loop momentum independent expression depending on the Feynman x parameter, external momenta, masses, e.g., $\Delta(x, q_i, m)$. The integration is understood for Euclidean momenta with absolute value below the Λ cutoff ($|l_E| \leq \Lambda$).

The integral (42) is just given for comparison, it is calculated with a simple momentum cutoff. In (43) with the standard (36) substitution one would get a constant $-\frac{3}{2}$ instead of -1 [30].

$$\int_{\Lambda\, reg} \frac{d^4 l_E}{i(2\pi)^4} \frac{1}{\left(l_E^2 + \Delta^2\right)^2} = \frac{1}{(4\pi)^2} \left(\ln\left(\frac{\Lambda^2 + \Delta^2}{\Delta^2} \right) + \frac{\Delta^2}{\Lambda^2 + \Delta^2} - 1 \right). \tag{42}$$

$$\int_{\Lambda\, reg} \frac{d^4 k}{i(2\pi)^4} \frac{l_{E\mu} l_{E\nu}}{\left(l_E^2 + \Delta^2\right)^3} = \frac{1}{(4\pi)^2} \frac{g_{\mu\nu}}{4} \left(\ln\left(\frac{\Lambda^2 + \Delta^2}{\Delta^2} \right) + \frac{\Delta^2}{\Lambda^2 + \Delta^2} - 1 \right) \tag{43}$$

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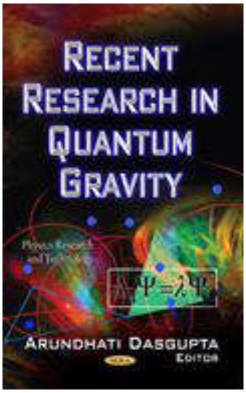

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COUPLING FUNCTIONS AND QUANTUM CORRECTIONS IN GAUSS-BONNET GRAVITY*

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We demonstrate how the changing of dilatonic coupling function changing in black-hole like solutions could provide some information about the model parameters. We work in the frames of the gravity model from Einstein-dilaton-Gauss-Bonnet (EDGB) string inspired theory. Our numerical results indicate that external structure of the new solution is also similar to the Schwarzschild one. On the other hand, new solution has different dependence of a horizon radius upon the black hole mass. We use the general form of a power-law correction to the Schwarzschild solution and analyze its impact on the event horizon and circular orbits for different parameter values. Such a correction does not contradict with existing observational data. On the other hand, an exact definition of model parameters requires precise measurements to distinguish between different possible cases. Having these values we

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suppose that the direct imaging of the event horizon area can shed light on the correction viability. In addition our results may be directly applied to future black hole observations.

BIBLIOGRAPHY

60 years of Yang-Mills gauge field theories: C.N. Yang's contributions to physics

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physics. It is seemingly the only fully consistent relativistic quantum many-body theory in four space-time dimensions. As such it is the underlying theoretical framework for the Standard Model of Particle Physics, which has been shown to be the correct theory at the energies we now can measure. It has been investigated also from many other perspectives, and many new and unexpected features have been uncovered from this theory. In recent decades, apart from high energy physics, the theory has been actively applied in other branches of physics, such as statistical physics, condensed matter physics, nonlinear systems, etc. This makes the theory an indispensable topic for all who are involved in physics. The conference celebrated the exceptional achievements using Yang-Mills theory over the years but also many other truly remarkable contributions to different branches of physics from Prof C N Yang. This volume collects the invaluable talks by Prof C N Yang and the invited speakers reviewing these remarkable contributions and their importance for the future of physics"-- Provided by publisher.

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Subjects	Yang, Chen Ning, 1922---Congresses. Yang-Mills theory--History--Congresses. Gauge fields (Physics)--Congresses. Quantum theory--Congresses.
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Type of material	Book
Personal name	Cianfrani, Francesco.
Main title	Canonical quantum gravity: fundamentals and recent developments/Francesco Cianfrani, University of Wroclaw, Poland, Orchidea Maria Lecian, "Sapienza," University of Rome, Italy, Max Planck Institute for Gravitational Physics, Germany & ICRA International Center for Relativistic Astrophysics, Italy, Matteo Lulli, "Sapienza," University of Rome, Italy, Giovanni Montani, ENEA - C. R. Frascati, UTFUS-MAG & "Sapienza," University of Rome, Italy.
Published/Produced	[Hackensack,] New Jersey: World Scientific, [2014] ©2014
Description	xxiii, 300 pages: illustrations; 24 cm
ISBN	9789814556644 (hardcover: alk. paper) 9814556645 (hardcover: alk. paper)
LC classification	QC178 .C35 2014

Portion of title	Quantum gravity
Related names	Lecian, Orchidea Maria, 1980- Lulli, Matteo. Montani, Giovanni, 1966-
Subjects	Quantum gravity. Gravitation. General relativity (Physics)
Notes	Includes bibliographical references (pages 275-293) and index.

Covariant loop quantum gravity: an elementary introduction to quantum gravity and spinfoam theory

LCCN	2014028593
Type of material	Book
Personal name	Rovelli, Carlo, 1956- author.
Main title	Covariant loop quantum gravity: an elementary introduction to quantum gravity and spinfoam theory/Carlo Rovelli, Université d' Aix-Marseille, Francesca Vidotto, Radboud Universiteit Nijmegen.
Published/Produced	Cambridge, United Kingdom; New York: Cambridge University Press, 2015. ©2015
Description	xii, 254 pages; 26 cm
Links	Contributor biographical information http://www.loc.gov/catdir/enhancements/fy1503/2014028593-b.html Publisher description http://www.loc.gov/catdir/enhancements/fy1503/2014028593-d.html Table of contents only http://www.loc.gov/catdir/enhancements/fy1503/2014028593-t.html
ISBN	9781107069626 (hbk.) 1107069629 (hbk.)
LC classification	QC178 .R68 2015
Related names	Vidotto, Francesca, 1980- author.
Contents	Spacetime as a quantum object -- Physics without time -- Gravity -- Classical discretization -- 3D Euclidean theory -- Bubbles and cosmological constant -- The real world: 4D Lorentzian theory --

	Classical limit -- Matter -- Black holes -- Cosmology -- Scattering -- Final remarks.
Subjects	Quantum gravity. Quantum cosmology.
Notes	Includes bibliographical references and index.

Discrete or continuous?: the quest for fundamental length in modern physics

LCCN	2014006277
Type of material	Book
Personal name	Hagar, Amit, 1969- author.
Main title	Discrete or continuous?: the quest for fundamental length in modern physics/Amit Hagar, Indiana University.
Published/Produced	Cambridge; New York: Cambridge University Press, 2014.
Description	xi, 267 pages: illustrations; 26 cm
ISBN	9781107062801 (hardback: alk. paper) 1107062802 (hardback: alk. paper)
LC classification	QC173.59.S65 H34 2014
Contents	Arguments from math -- Arguments from philosophy -- Electrodynamics, QED, and early QFT -- Quantum gravity: prehistory -- Einstein on the notion of length -- Quantum gravity: current approaches -- The proof is in the pudding -- Coda.
Subjects	Space and time--Philosophy. Length measurement.
Notes	Includes bibliographical references (pages 239-263) and index.

Einstein and Hilbert: dark matter

LCCN	2011017302
Type of material	Book
Main title	Einstein and Hilbert: dark matter/Valeriy V. Dvoeglazov, editor.
Published/Created	Hauppauge, N.Y.: Nova Science Publishers, c2012.
Description	xi, 199 p.: ill.; 26 cm.
ISBN	9781613248409 (hardcover: alk. paper)
LC classification	QC178 .E337 2012

Related names	Dvoeglazov, Valeri V.
Subjects	Quantum gravity. Einstein field equations. Dark matter (Astronomy) General relativity (Physics)
Notes	Includes bibliographical references and index.

Einstein and others: unification

LCCN	2014039702
Type of material	Book
Main title	Einstein and others: unification/Valeriy V. Dvoeglazov, editor.
Published/Produced	New York: Nova Publishers, [2015] ©2015
Description	x, 227 pages: illustrations; 26 cm.
ISBN	9781634632768 (hardcover) 1634632761 (hardcover)
LC classification	QC174.45 .E56 2015
Related names	Dvoeglazov, Valeri V., editor.
Subjects	Quantum field theory. Unified field theories. Quantum gravity.
Notes	Includes bibliographical references and index.
Series	Contemporary fundamental physics

Einstein and the changing worldviews of physics

LCCN	2011943090
Type of material	Book
Main title	Einstein and the changing worldviews of physics/edited by Christoph Lehner, Jürgen Renn, and Matthias Schemmel; in cooperation with John Beckman and Eric Stengler.
Published/Created	New York: Birkhäuser, c2012.
Description	xii, 363 p.: ill. (some col.); 25 cm.
Links	Publisher description http://www.loc.gov/catdir/enhancements/fy1316/2011943090-d.html Table of contents only http://www.loc.gov/catdir/enhancements/fy1316/2011943090-t.html
ISBN	9780817649395 (alk. paper)

	0817649395 (alk. paper) 9780817649401 (e-ISBN)
LC classification	QC173.6 .E375 2012
Related names	Lehner, Christoph, 1962- Renn, Jürgen, 1956- Schemmel, Matthias.
Contents	Part I: At the limits of the classical worldview. Theories of gravitation in the twilight of classical physics/Jürgen Renn and Matthias Schemmel; The Newtonian theory of light propagation/Jean Eisenstaedt; Mach and Einstein, or, clearing troubled waters in the history of science/Gereon Wolters -- Part II: Contexts of the relativity revolution. Tilling the seedbed of Einstein's politics: a pre-1905 harbinger?/Robert Schulmann; The early reception of Einstein's relativity among British philosophers/José M. Sánchez-Ron; Science and ideology in Einstein's visit to South America in 1925/Alfredo Tiomno Tolmasquim; The reception of Einstein's relativity theories in literature and the arts (1920-1950)/Hubert F. Goenner -- Part III: The emergence of the relativisticworldview. Hilbert's axiomatic method and his "foundations of physics": reconciling causality with the axiom of general invariance/Katherine A. Brading and Thomas A. Ryckman; Not only because of theory: Dyson, Eddington, and the competing myths of the 1919 Eclipse Expedition/Daniel Kennefick; Peter Havas (1916-2004)/Hubert F. Goenner; Peter Bergmann and the invention of constrained Hamiltonian dynamics/D.C. Salisbury; Thoughts about a conceptual framework for relativistic gravity/Bernard F. Schutz -- Part IV: A new worldview in the making. Observational tests of general relativity: an historical look at measurements prior to the advent of modern space-borne instruments/J.E. Beckman; Primordial magnetic fields and cosmic microwave background/Eduardo Battaner and Estrella Florido; Singularity theorems

	in general relativity: achievements and open questions/José M.M. Senovilla; The history and present status of quantum field theory in curved spacetime/Robert M. Wald; The border between relativity and quantum theory/Tevian Dray; The issue of the beginning in quantum gravity/Abhay Ashtekar.
Subjects	Einstein, Albert, 1879-1955. General relativity (Physics) General relativity (Physics)--History.
Notes	Includes bibliographical references.
Series	Einstein studies; v. 12 Einstein studies; v. 12.

Exactly solvable models for cluster and many-body condensed matter systems

LCCN	2016013314
Type of material	Book
Personal name	March, Norman H. (Norman Henry), 1927- author.
Main title	Exactly solvable models for cluster and many-body condensed matter systems/N.H. March (Oxford University), G.G.N. Angilella (University of Catania, Italy).
Published/Produced	Singapore; Hackensack, NJ: World Scientific Publishing Co. Pte. Ltd., [2016] ©2016
ISBN	9789813140141 (hardcover; alk. paper) 9813140143 (hardcover; alk. paper)
LC classification	QC173.454 .M365 2016
Related names	Angilella, G. G. N., author.
Summary	"The book reviews several theoretical, mostly exactly solvable, models for selected systems in condensed states of matter, including the solid, liquid, and disordered states, and for systems of few or many bodies, both with boson, fermion, or anyon statistics. Some attention is devoted to models for quantum liquids, including superconductors and superfluids. Open problems in relativistic fields and quantum gravity are also briefly reviewed. The book

	ranges almost comprehensively, but concisely, across several fields of theoretical physics of matter at various degrees of correlation and at different energy scales, with relevance to molecular, solid-state, and liquid-state physics, as well as to phase transitions, particularly for quantum liquids. Mostly exactly solvable models are presented, with attention also to their numerical approximation and, of course, to their relevance for experiments"-- Provided by publisher.
Contents	Low order density matrices -- Solvable models for small clusters of fermions -- Small clusters of bosons -- Anyon statistics with models -- Superconductivity and superfluidity -- Exact results for an isolated impurity in a solid -- Pair potential and many-body force models for liquids -- Anderson localization in disordered systems -- Statistical field theory: especially models of critical exponents -- Relativistic fields -- Towards quantum gravity.
Subjects	Condensed matter--Mathematical models. Many-body problem--Mathematical models. Microclusters--Mathematical models.

Exploring science through science fiction

LCCN	2013945865
Type of material	Book
Personal name	Luokkala, Barry B., author.
Main title	Exploring science through science fiction/Barry B. Luokkala.
Published/Produced	New York: Springer, [2014]
Description	xix, 241 pages: illustrations (some color); 23 cm.
ISBN	9781461478904 (pbk.: acid-free paper) 1461478901
Summary	"How does Einstein's description of space and time compare with Dr. Who? Can James Bond really escape from an armor-plated railroad car by cutting through the floor with a laser concealed in a wristwatch? What would it take to create a fully-intelligent android, such as Star Trek's Commander

Data? How might we discover intelligent civilizations on other planets in the galaxy? Is human teleportation possible? Will our technological society ever reach the point at which it becomes lawful to discriminate on the basis of genetic information, as in the movie GATTACA? Exploring Science Through Science Fiction addresses these and other interesting questions, using science fiction as a springboard for discussing fundamental science concepts and cutting-edge science research. The book is designed as a primary text for a college-level course which should appeal to students in the fine arts and humanities as well as to science and engineering students. It includes references to original research papers, landmark scientific publications and technical documents, as well as a broad range of science literature at a more popular level. With over 180 references to specific scenes in 130 sci-fi movies and TV episodes, spanning over 100 years of cinematic history, it should be an enjoyable read for anyone with an interest in science and science fiction."--Cover.

Contents Introduction: discerning the real, the possible and the impossible. The first sci-fi movie -- Exploration topic: is it safe to launch humans into space from a giant gun? -- The first literary work of science fiction -- Reference frames, revisited -- Roadmap to the rest of the book -- What is the nature of space and time? (The physics of space travel and time travel). Changing perspectives through history -- Newton's laws -- Einstein and relativity -- Stephen Hawking, black holes, wormholes, and quantum gravity -- Other time travel scenarios -- Exploration topics -- What is the universe made of? (Matter, energy and interactions). The standard model of particle physics -- The atomic nucleus: protons, neutrons, isotopes, and radioactivity -- Gases -- Solid-state materials -- Phase transitions -- Transparency and invisibility: optical properties of solids -- Energy and power -- Exploration topics -- Can a machine become self-aware? (The sciences of computing and cognition). Computer hardware performance specifications -- Analog computers -- Digital computers -- Beyond digital computers -- Information storage -- Robotics -- Robot behavior -- Toward the creation of artificial consciousness -- Exploration topics -- Are we alone in the universe? (The search for extraterrestrial intelligence). Major considerations -- Searching for ET: government agency or private industry? -- Listening for ET: what form of communication might we expect? -- Conditions necessary for intelligent life to arise -- Cinema and the science of the SETI Project -- Where might first contact occur and how will humans and aliens interact? -- Exploration topics -- What does it mean to be human? (Biological sciences, biotechnology and other considerations). Bodies with replaceable parts -- Resistance to disease -- Cell structure and radiation damage -- DNA and the human genome -- Cloning -- Human teleportation: a complex, interdisciplinary problem -- Teleportation estimations -- Beyond biology -- What can we learn from an android about what it means to be human? -- Exploration topics -- How do we solve our problems? (Science, technology and society). The public perception of science and scientists -- The methodology of science -- Science, pseudoscience, and nonsense -- Problems to be solved -- Exploration topics -- What lies ahead? (The future of our technological society). Accurate predictions -- Coming soon: possibilities for the not-too-distant future -- Science fiction in historical context -- Visions of the future -- Appendix A: catalog of movies cited -- Appendix B: television series episodes cited -- Appendix C: YouTube videos cited -- Appendix D: solutions to estimation problems.

Subjects	Science in motion pictures.

	Science fiction films.
	Science fiction television programs.
	Science in mass media.
	Literature and science.
	Literature and science.
	Science fiction films.
	Science fiction television programs.
	Science in mass media.
	Science in motion pictures.
Notes	Includes bibliographical references and index.
Series	Science and fiction, 2197-1188
	Science and fiction (Springer (Firm))

Foundations of quantum gravity

LCCN	2013001652
Type of material	Book
Personal name	Lindesay, James.
Main title	Foundations of quantum gravity/James Lindesay, Computational Physics Laboratory, Howard University.
Published/Produced	Cambridge: Cambridge University Press, [2013]
Description	416 p.: ill.; 25 cm.
Links	Cover image http://assets.cambridge.org/97811070/08403/cover/9781107008403.jpg
ISBN	9781107008403 (hardback)
LC classification	QC178 .L58 2013
Summary	"Exploring how the subtleties of quantum coherence can be consistently incorporated into Einstein's theory of gravitation, this book is ideal for researchers interested in the foundations of relativity and quantum physics. The book examines those properties of coherent gravitating systems that are most closely connected to experimental observations. Examples of consistent co-gravitating quantum systems whose overall effects upon the geometry are independent of the coherence state of each constituent are provided, and the properties of the trapping regions of non-singular black objects, black holes, and a dynamic de Sitter cosmology are

discussed analytically, numerically, and diagrammatically. The extensive use of diagrams to summarise the results of the mathematics enables readers to bypass the need for a detailed understanding of the steps involved. Assuming some knowledge of quantum physics and relativity, the book provides textboxes featuring supplementary information for readers particularly interested in the philosophy and foundations of the physics"--Provided by publisher.

Contents — Machine generated contents note: Introduction; Part I. Galilean and Special Relativity: 1. Classical special relativity; 2. Quantum mechanics, classical, and special relativity; 3. Microscopic formulations of particle interactions; 4. Group theory in quantum mechanics; Part II. General Relativity: 5. Fundamentals of general relativity; 6. Quantum mechanics in curved space-time backgrounds; 7. The physics of horizons and trapping regions; 8. Cosmology; 9. Gravitation of interacting systems; Appendixes; References; Index.

Subjects — Quantum gravity.
SCIENCE/Cosmology.

Notes — Includes bibliographical references and index.

Foundations of space and time: reflections on quantum gravity

LCCN — 2011000387

Type of material — Book

Main title — Foundations of space and time: reflections on quantum gravity/edited by Jeff Murugan, Amanda Weltman & George F.R. Ellis.

Published/Created — Cambridge; New York: Cambridge University Press, 2012.

Description — xiv, 437 p.: ill.; 25 cm.

ISBN — 9780521114400 (hardback)

LC classification — QC173.59.S65 F68 2012

Related names — Murugan, Jeff.
Weltman, Amanda.
Ellis, George F. R. (George Francis Rayner)

Summary "After almost a century, the field of quantum gravity remains as difficult and inspiring as ever. Today, it finds itself a field divided, with two major contenders dominating: string theory, the leading exemplification of the covariant quantization program; and loop quantum gravity, the canonical scheme based on Dirac's constrained Hamiltonian quantization. However, there are now a number of other innovative schemes providing promising new avenues. Encapsulating the latest debates on this topic, this book details the different approaches to understanding the very nature of space and time. It brings together leading researchers in each of these approaches to quantum gravity to explore these competing possibilities in an open way. Its comprehensive coverage explores all the current approaches to solving the problem of quantum gravity, addressing the strengths and weaknesses of each approach, to give researchers and graduate students an up-to-date view of the field"-- Provided by publisher.

Contents Machine generated contents note: 1. The problem with quantum gravity Jeff Murugan, Amanda Weltman and George F. R. Eliis; 2. A dialogue on the nature of gravity Thanu Padmanabhan; 3. Effective theories and modifications of gravity Cliff Burgess; 4. The small scale structure of spacetime Steve Carlip; 5. Ultraviolet divergences in supersymmetric theories Kellog Stelle; 6. Cosmological quantum billiards Axel Kleinschmidt and Hermann Nicolai; 7. Progress in RNS string theory and pure spinors Dimitri Polyakov; 8. Recent trends in superstring phenomenology Massimo Bianchi; 9. Emergent spacetime Robert de Mello Koch and Jeff Murugan; 10. Loop quantum gravity Hanno Sahlmann; 11. Loop quantum gravity and cosmology Martin Bojowald; 12. The microscopic dynamics of quantum space as a group field theory Daniele Oriti; 13. Causal dynamical triangulations

	and the quest for quantum gravity Jan Ambjørn, J. Jurkiewicz and Renate Loll; 14. Proper time is stochastic time in 2D quantum gravity Jan Ambjorn, Renate Loll, Y. Watabiki, W. Westra and S. Zohren; 15. Logic is to the quantum as geometry is to gravity Rafael Sorkin; 16. Causal sets: discreteness without symmetry breaking Joe Henson; 17. The Big Bang, quantum gravity, and black-hole information loss Roger Penrose; Index.
Subjects	Space and time. Quantum gravity. SCIENCE/Mathematical Physics
Notes	Includes bibliographical references and index.

General relativity and gravitation: a centennial perspective

LCCN	2014041670
Type of material	Book
Main title	General relativity and gravitation: a centennial perspective/edited by Abhay Ashtekar (editor in chief) The Pennsylvania State University, Beverly K. Berger, International Society for Relativity and Gravitation, James Isenberg, University of Oregon, Malcolm MacCallum, Queen Mary University of London.
Published/Produced	Cambridge: Cambridge University Press, 2015. ©2015
Description	xxi, 674 pages: illustrations (chiefly color); 26 cm
ISBN	9781107037311 (hardback)
LC classification	QC173.6 .G465 2015
Related names	Ashtekar, Abhay, editor. Berger, B. (Beverly), editor. Isenberg, James A., editor. MacCallum, M. A. H., editor.
Contents	100 years of general relativity/George F.R. Ellis -- Was Einstein right?: a centenary assessment/Clifford M. Will -- Cosmology/David Wands, Misao Sasaki, Eiichiro Komatsu, Roy Maartens and Malcolm A.H. MacCallum -- Relativistic astrophysics/Peter Schneider, Ramesh Narayan, Jeffrey E. McClintock,

Peter Mészáros and Martin J. Rees -- Receiving gravitational waves/Beverly K. Berger, Karsten Danzmann, Gabriela Gonzalez, Andrea Lommen, Guido Mueller, Albrecht Rüdiger and William Joseph Weber -- Sources of gravitational waves: theory and observations/Alessandra Buonanno and B.S. Sathyaprakash -- Probing strong field gravity through numerical simulations/Frans Pretorius, Matthew W. Choptuik and Luis Lehner -- The initial data and the Einstein constraint equations/Gregory J. Galloway, Pengzi Miao and Richard Schoen -- Global behavior of solutions to Einstein's equations/Stefanos Aretakis, James Isenberg, Vincent Moncrief and Igor Rodnianski -- Quantum fields in curved space-times/Stefan Hollands and Robert M. Wald -- From general relativity to quantum gravity/Abhay Ashtekar, Martin Reuter and Carlo Rovelli -- Quantum gravity via supersymmetry and holography/Henriette Elvang and Gary T. Horowitz.

Subjects General relativity (Physics)
Gravitation.

Notes Includes bibliographical references and index.

Gravity and strings

LCCN 2014017469

Type of material Book

Personal name Ortín, Tomás, 1964- author.

Main title Gravity and strings/Tomás Ortín, Spanish National Research Council (CSIC).

Edition Second edition.

Published/Produced Cambridge: Cambridge University Press, 2015.

Description xxvi, 1015 pages: illustrations; 26 cm.

ISBN 9780521768139 (hardback)
0521768136 (hardback)

LC classification QC178 .O78 2015

Contents Differential geometry -- Symmetries and Noether's theorems -- A perturbative introduction to general relativity -- Action principles for gravity -- Pure

	N=1,2,d=4 supergravities -- Matter-coupled N=1,d=4 supergravity -- Matter-coupled N=2,d=4 supergravity -- A generic description of all the N=2,d=4 SUEGRAS -- Matter-coupled N=1,d=5 supergravity -- Conserved charges in general relativity -- The Schwarzschild black hole -- The Reissner-Nordstrom black hole -- The Taub-NUT solution -- Gravitational pp-waves -- The Kaluza-Klein black hole -- Dilaton and dilaton/axion black holes -- Unbroken supersymmetry I: supersymmetric vacua -- Unbroken supersymmetry II: partially supersymmetric solutions -- Supersymmetric black holes from supergravity -- String theory -- The string effective action and T duality -- From eleven to four dimensions -- The type-IIB superstring and type-II T duality -- Extended objects -- The extended objects of string theory -- String black holes in four and five dimensions -- The FGK formalism for (single, static) black holes and branes.
Subjects	Quantum gravity. String models.
Notes	Includes bibliographical references (pages 969-1001) and index.
Series	Cambridge monographs on mathematical physics Cambridge monographs on mathematical physics.

Images of time: mind, science, reality

LCCN	2015949945
Type of material	Book
Personal name	Jaroszkiewicz, George, author.
Main title	Images of time: mind, science, reality/George Jaroszkiewicz, The University of Nottingham, UK.
Edition	First edition.
Published/Produced	Oxford: Oxford University Press, 2016.
Description	xvi, 305 pages: illustrations; 25 cm
ISBN	9780198718062 0198718063
LC classification	BD638 .J37 2016

Summary	Provides a broad survey of many of these views, these Images of Time, covering historical, cultural, philosophical, biological, mathematical and physical Images of Time, including classical and quantum mechanics, special and general relativity and cosmology.
Contents	1 Introduction: Plan of this book -- What is time? -- Paradigms -- Events in time -- Temporal ordering and time labels -- The contextuality of temporal ordering -- Arrows of time -- Arrows versus cycles. 2 Observers and time: Introduction -- metaphysics, physics, and validation -- Observers and the physical universe -- The contextuality of scientific truth -- Contextual completeness -- Time and contextuality -- The contextuality of the existential quantifier -- Primary observers -- experiments -- Heisenberg cuts -- Generalize propositions. 3 Subjective images of time: The Greeks -- The dogmatists -- The sceptics -- The Age of Reason -- The classical mechanists -- The metaphysicists -- The mathematical physicists. 4 Cultural images of time: Cosmogony and theogony -- Stone Age calendars -- The Mesopotamians -- The Middle east -- The Europeans -- Eastern philosophies -- Australian Aboriginal concepts of time -- The Central Americans -- The Far East. 5 Literary images of time: Introduction -- Literary varieties. 6 Objective images of time: Introduction -- Optics -- Other factors -- Time in the laboratory -- Laboratory frames and observer choruses -- Chorus protocol -- Superobservers. 7 Mathematical images of time: Introduction -- Some reasonable requirements -- Sets -- Temporal ordering and ordered sets -- Functions and the mathematical arrow of time -- Cardinality -- The natural numbers -- Linear continua -- Further concepts. 8 Illusionary images of time: The Block Universe -- Persistence and transtemporal identity -- The problem of time in cosmology. 9 Causal images of time: Introduction -- Causality -- Blame and

responsibility -- Determinism and known unknowns -- Teleology -- Chaos. 10 Physics and time: Physics and persistence -- Scales of time. 11 Biological time: Introduction -- The solar model -- Lifetimes of organisms -- Chronobiology -- Biological time travel. 12 The dimensions of time: Introduction -- Partial differential equations and the flow of information -- The signature of spacetime -- Empirical studies. 13 The architecture of time: What is temporal architecture? -- Examples of temporal architectures -- Architectural levels of observation. 14 Absolute time: Introduction -- Clocks -- The reparameterization of time -- Absolute space -- Aristotelian space-time versus Galilean-Newtonian space-time -- Particle worldlines -- The Newtonian mechanical paradigm -- The Euler-Lagrange mechanical paradigm -- Phase space -- Canonical transformation theory -- Infinitesimal transformations. 15 The reparametrization of time: Introduction -- Temporal parametrization -- Temporal reparametrization -- Action integrals -- Temporal reparametrization in detail -- Reparametrization form invariance -- The extended equations of motion -- Transformation to phase space -- Reparametrized primary identity. 16 Origins of relativity: Inertial frames -- The speed of light and Galilean transformations -- The Michelson-=Morley experiment -- FitzGerald length contraction -- Derivation of FitzGerald length contraction -- FitzGerald time dilation -- Lorentz transformations. 17 Special relativity: Lorentz transformations -- Simultaneity in special relativity -- Time dilation -- The clock hypothesis -- The Twin Paradox -- Lightcones -- The Klein-Gordon equation -- The causal propgation of special relativistic fields -- Fock-Kemmer front velocity and the memory field. 18 Generalized transformations: Introduction -- Constraints -- The Michelson-Morley constraint -- Some standard transformations -- The splitting of causality -- Empirical evidence for a preferred frame. 19 General relativistic time: Space-time versus spacetime -- Lorentzian signature metrics -- Pseudo-Riemannian manifolds -- The Schwarzschild metric -- Gravitational time dilation -- Black hole geometry -- The spinning disc. 20 Time travel: Introduction -- Information flow -- Tachyons -- Spreadsheet time travel -- The Gödel metric -- Timelike geodesics. 21 Imaginary time: Introduction -- Minkowski's imaginary time -- Application to wave mechanics -- Propagators and Green's functions -- Path integrals -- Quantum gravity -- Quantum thermodynamics -- Black home thermodynamics -- Quantum cosmology -- Conclusions. 22 Irreversible time: Introduction -- Glauber's correlations -- Probability -- The expansion of the universe -- Poincaré recurrence. 23 Discrete time: Introduction -- Difference equations -- The action sum -- Caldirola's proper time chronon -- Caldirola's microverse model -- Discrete-time classical electrodynamics. 24 Time and quanta: Introduction -- Schrödinger versus Heisenberg -- de Broglie waves -- The time-energy uncertainty relation -- The relativistic propagator. 25 temporal correlations: Introduction -- Classical bit temporal correlations -- Quantum bit temporal correlation -- Understanding the Leggett-Garg prediction. 26 Time reversal: Introduction -- classical active time reversal -- Schrodinger wave mechanics -- THe time-reversal operator -- The Pauli equation -- The Dirac wave equation -- TCP theorem -- Kaons. 27 Quantized spacetime: Introduction -- Mach's relationalism -- Einstein's relationalism -- Planck, quanta, photons, and existence -- Snyder's quantized spacetime. 28 Epilogue. Appendix: Sets -- Groups -- Metric spaces -- RIngs and fields -- Vector spaces -- Hilbert space -- Observables -- Antilinear and antiunitary operators -- Affine spaces -- Manifolds -- Signature -- Variational derivation of Einstein's field

	equations from the Hilbert action -- Doppler shifts. Bibliography -- Index.
Subjects	Time--Mathematical models.
	Metaphysics.
Notes	Includes bibliographical references (pages 289-299) and index(es).

One hundred years of general relativity: from genesis and empirical foundations to gravitational waves, cosmology and quantum gravity

LCCN	2015032705
Type of material	Book
Main title	One hundred years of general relativity: from genesis and empirical foundations to gravitational waves, cosmology and quantum gravity/editor, Wei-Tou Ni, National Tsing Hua University, Hsinchu.
Published/Produced	Singapore; Hackensack, NJ: World Scientific, [2015]
ISBN	9789814635127 (set: alk. paper)
	981463512X (set: alk. paper)
	9789814678483 (v.1: alk. paper)
	9814678481 (v.1: alk. paper)
	9789814678490 (v.2: alk. paper)
	981467849X (v.2: alk. paper)
LC classification	QC173.6 .O54 2015
Related names	Ni, Wei-Tou, 1944- editor.
Summary	"The aim of this two-volume title is to give a comprehensive review of one hundred years of development of general relativity and its scientific influences. This unique title provides a broad introduction and review to the fascinating and profound subject of general relativity, its historical development, its important theoretical consequences, gravitational wave detection and applications to astrophysics and cosmology. The series focuses on five aspects of the theory: Genesis, Solutions and Energy Empirical Foundations, Gravitational Waves, Cosmology, Quantum Gravity. The first three topics are covered in Volume 1 and the remaining two are covered in Volume 2. While this

is a two-volume title, it is designed so that each volume can be a stand-alone reference volume for the related topic"-- Provided by publisher.

Contents Volume 1: A genesis of special relativity/Valerie Messager and Christophe Letellier -- Genesis of general relativity: a concise exposition/W.-T. Ni -- Schwarzschild and Kerr solutions of Einstein's field equation: an introduction -- Christian Heinicke and Friederich W. Hehl -- Gravitational energy for GR and Poincaré gauge theories: a covariant Hamiltonian approach/Jiang-Mei Chen, James Nester and Roh-Suan Tung -- Equivalence principles, spacetime structure and the cosmic connection/W.-T. Ni -- Cosmic polarization rotation: an astrophysical test of fundamental physics/Sperello di Serego Alighieri -- Clock comparison based on laser ranging technologies/Étienne Samain -- Solar-system tests of relativistic gravity/W.-T. Ni -- Pulsars and gravity/R.N. Manchester -- GWs: classification, sources, methods of detection and sensitivities/K. Kuroda, W.-T. Ni and W.-P. Pan -- Introduction to ground based gravitational wave detectors/K. Kuroda -- GW detection in space/W.-T. Ni.
Volume 2: General relativity and cosmology/M. Bucher and W.-T. Ni -- Cosmic structure/Marc Davis -- Physics of the cosmic microwave background anisotropy/M. Bucher -- SNe Ia as a cosmological probe/Xiangcun Meng, Yan Gao and Zhanwen Han -- Gravitational lensing in cosmology/Toshifumi Futamase -- Inflationary cosmology: first 30+ years/K. Sato and J. Yokoyama -- Inflation, string theory and cosmic strings/David Chernoff and Henry Tye -- Quantum gravity: a brief history of ideas and some outlooks/S. Carlip, D.-W. Chiou, W.-T. Ni, R. Woodard -- Perturbative quantum gravity comes of age/R. Woodard -- Black hole thermodynamics/S. Carlip -- Loop quantum gravity/Dah-Wei Chiou.

Subjects	General relativity (Physics)--History. Gravitational waves. Cosmology. Quantum gravity.
Notes	Includes bibliographical references.

Particle physics at the tercentenary of Mikhail Lomonosov: proceedings of the Fifteenth Lomonosov Conference on Elementary Particle Physics, Moscow, Russia, 18-24 August 2011

LCCN	2014451473
Type of material	Book
Meeting name	Lomonosov Conference on Elementary Particle Physics (15th: 2011: Moscow, Russia)
Main title	Particle physics at the tercentenary of Mikhail Lomonosov: proceedings of the Fifteenth Lomonosov Conference on Elementary Particle Physics, Moscow, Russia, 18-24 August 2011/editor, Alexander I. Studenikin.
Published/Produced	Singapore; Hackensack, NJ: World Scientific Publishing Co. Pte. Ltd., [2013] ©2013
Description	xx, 460 pages: illustrations; 24 cm
ISBN	9814436828 (hardback) 9789814436823 (hardback)
LC classification	QC793 .L66 2011
Portion of title	Proceedings of the Fifteenth Lomonosov Conference on Elementary Particle Physics
Related names	Studenikin, A. I., editor. Moskovskiĭ gosudarstvennyĭ universitet im. M.V. Lomonosova. Mezhregionalʹnyĭ tsentr fundamentalʹnykh issledovaniĭ.
Summary	This volume is devoted to a wide variety of investigations, both in theory and experiment, of particle physics such as electroweak theory, fundamental symmetries, tests of the Standard Model and beyond, neutrino and astroparticle physics, heavy quark physics, non-perturbative QCD, quantum gravity effects, and present and

	future accelerator physics.-- Source other than Library of Congress.
Subjects	Particles (Nuclear physics)--Congresses.
	Particle acceleration--Congresses.
	Nuclear astrophysics--Congresses.
	Quantum field theory--Congresses.
	Nuclear astrophysics.
	Particle acceleration.
	Particles (Nuclear physics)
	Quantum field theory.
Form/Genre	Conference proceedings.
Notes	At head of title: Faculty of Physics of Moscow State University, Interregional Centre for Advanced Studies.
	Includes bibliographical references.

Progress in relativity, gravitation, cosmology

LCCN	2011024894
Type of material	Book
Main title	Progress in relativity, gravitation, cosmology/V.V. Dvoeglazov, A. Molgado, editors.
Published/Created	Hauppauge, N.Y.: Nova Science Publishers, c2012.
Description	x, 176 p.: ill.; 26 cm.
ISBN	9781613248119 (hardcover)
LC classification	QC173.55 .P765 2012
Related names	Dvoeglazov, Valeri V.
	Molgado, A.
Subjects	Relativity (Physics)
	Quantum theory.
	Quantum cosmology.
	Quantum gravity.
Notes	Includes bibliographical references and index.

Quantum field theories in two dimensions: collected works of Alexei Zamolodchikov

LCCN	2012554700
Type of material	Book
Personal name	Zamolodchikov, Alexei, 1952-2007.
Main title	Quantum field theories in two dimensions: collected

	works of Alexei Zamolodchikov/Alexander Belavin, Yaroslav Pugai, Alexander Zamolodchikov, editors.
Published/Created	Hackensack, N.J.: World Scientific, c2012.
Description	2 v. (xi, 1045 p.): ill.; 27 cm.
ISBN	9789814324069 (set) 981432406X (set) 9789814324076 (vol. 1) 9814324078 (vol. 1) 9789814324083 (vol. 2) 9814324086 (vol. 2)
LC classification	QC174.45.A2 Z36 2012
Related names	Belavin, A. A. (Aleksandr Abramovich), 1942- Pugai, Y. Zamolodchikov, A. B., 1952-
Contents	vol. 1. -- Conformal field theories -- Quantum gravity and Liouville theory -- vol. 2. -- Non-peturbative methods -- Integrable models and thermodynamic Bethe Ansatz.
Subjects	Quantum field theory.
Notes	Includes bibliographical references.

Quantum gravity and quantum cosmology

LCCN	2012952147
Type of material	Book
Main title	Quantum gravity and quantum cosmology/Gianluca Calcagni, Lefteris Papantonopoulos, George Siopsis, Nikos Tsamis, editors.
Published/Produced	Heidelberg; New York; Dordrecht; London: Springer, [2013]
Description	xii, 399 pages: illustrations; 24 cm.
ISBN	9783642330353 (pbk.) 3642330355 (pbk.)
LC classification	QC178 .Q363 2013
Related names	Calcagni, Gianluca, editor.
Subjects	Quantum gravity. Quantum cosmology. Quantum cosmology. Quantum gravity.
Subject keywords	Cosmology

Notes	Includes bibliographical references and index.
Series	Lecture notes in physics, 0075-8450; volume 863 Lecture notes in physics; 863. 0075-8450

Quantum gravity

LCCN	2012932071
Type of material	Book
Personal name	Kiefer, Claus, 1958-
Main title	Quantum gravity/Claus Kiefer.
Edition	3rd ed.
Published/Created	Oxford: Oxford University Press, 2012.
Description	xii, 393 p.: ill.; 26 cm.
ISBN	9780199585205 (hbk.) 0199585202 (hbk.)
LC classification	QC178 .K557 2012
Subjects	Quantum gravity.
Notes	Previous ed.: 2007. Includes bibliographical references (p. [554]-388) and index.
Series	International series of monographs on physics; 155

Quantum physics wthout quantum philosophy

LCCN	2012952411
Type of material	Book
Personal name	Dürr, Detlef, Prof. Dr., author.
Main title	Quantum physics wthout quantum philosophy/Detlef Dürr, Sheldon Goldstein, Nino Zanghì.
Published/Produced	Heidelberg; New York: Springer, [2013] ©2013
Description	xvii, 284 pages: illustrations; 24 cm
ISBN	9783642306891 (alk. paper) 3642306896 (alk. paper)
LC classification	QC174.12 .D873 2013
Related names	Goldstein, Sheldon, 1947- author. Zanghì, Nino, author.
Contents	Part 1. Quantum Equilibrium -- Quantum Equilibrium and the Origin of Absolute Uncertainty -- Quantum Equilibrium and the Role of Operators as Observables in Quantum Theory -- Quantum

	Philosophy: The Flight from Reason in Science -- Part 2. Quantum Motion -- Seven Steps Towards the Classical World -- On the Quantum Probability Flux through Surfaces -- On the Weak Measurement of Velocity in Bohmian Mechanics -- Topological Factors Derived From Bohmian Mechanics -- Part 3. Quantum Relativity -- Hypersurface Bohm-Dirac Models -- Bohmian Mechanics and Quantum Field Theory -- Quantum Spacetime without Observers: Ontological Clarity and the Conceptual Foundations of Quantum Gravity -- Reality and the Role of the Wave Function in Quantum Theory.
Subjects	Quantum theory--Philosophy. Quantum theory--Philosophy. Bohmsche Quantenmechanik.
Notes	Includes bibliographical references and index.

Recent research in quantum gravity

LCCN	2011050198
Type of material	Book
Main title	Recent research in quantum gravity/Arundhati Dasgupta, editor.
Published/Created	New York: Nova Science Publishers, c2013.
Description	174 p.: ill. (some col.); 24 cm.
ISBN	9781619423862 (hardcover) 1619423863 (hardcover)
LC classification	QC178 .R43 2013
Related names	Dasgupta, Arundhati.
Subjects	Quantum gravity.
Notes	Includes bibliographical references and index.
Series	Physics research and technology

Reflections on the next step for LHC: proceedings of the International School of Subnuclear Physics

LCCN	2015005052
Type of material	Book
Meeting name	International School of Subnuclear Physics (51st: 2013: Erice, Italy)
Main title	Reflections on the next step for LHC: proceedings of

	the International School of Subnuclear Physics/edited by Antonino Zichichi, European Physical Society, Geneva, Switzerland.
Published/Produced	Singapore; Hackensack, NJ: World Scientific, [2015] ©2015
Description	ix, 509 pages: illustrations; 26 cm.
ISBN	9789814678100 (alk. paper)
LC classification	QC793 .I555 2013
Portion of title	Proceedings of the International School of Subnuclear Physics
Related names	Zichichi, Antonino, editor.
Contents	From Planck to complexity/A. Zichichi -- Mass hierarchy and physics beyond the standard model/I. Antoniadis -- Status of the perturbative approach to supergravity/Z. Bern -- The pedagogic Higgs - or somebody's Boson/F. Close -- Magic supergravity from squaring Yang Mills/M.J. Duff -- Electric-magnetic duality and supersymmetry/P. Aschieri, S. Ferrara and A. Marrani -- Composite weak bosons at the LHC/H. Fritzsch -- Gauge forces: from QCD to quantum gravity/L. N. Lipatov -- Embedding oscillatory modes of quarks for baryons in QCD - looking to contstruct a bridge/P. Minkowski -- Quantum origin of the universe structure/V. Mukhanov -- No-scale supergravity in the light of LHC and Planck/T. Li, J.A. Maxin, D.V. Nanopoulos and J.W. Walker -- Hidden beauty in supersymmetric gauge theory/E. Sokatchev -- Three Erice lectures/G. 't Hooft -- Highlights from ATLAS - ALICE - CMS/S. Bertolucci -- LNGS: past, present and future/F. Ferroni -- Highlights from ALICE/P. Giubellino -- Planck highlights/A. Riazuelo -- Latest results from BNL and RHIC/M.J. Tannenbaum -- Status of the three neutrinos/A. Bettini -- What is the ontological status of the Higgs particle?/T.Y. Cao -- Roadmap at the LHC to the Higgs Boson and beyond/P. Jenni -- New spectroscopy with charm and beauty multiquarks

	states/L. Maiani -- Present status of the emc effect/K. Rith -- Reflections on the next step for LHC/H. Wenninger -- The problem of (CPT) invariance in experimental physics and the time of flight (TOF) world record/A. Zichichi -- Advances in fast timing up to 16 ps/K. Doroud, M.C.S. Williams and A. Zichichi.
Subjects	Particles (Nuclear physics)--Congresses. Colliders (Nuclear physics)--Congresses. Colliders (Nuclear physics)--Experiments--Congresses.
Notes	Includes index. Conference held June 24-July 3, 2013, in Erice, Italy.
Series	The subnuclear series; volume 51 Subnuclear series; v. 51.

Relativistic cosmology

LCCN	2011040518
Type of material	Book
Personal name	Ellis, George F. R. (George Francis Rayner)
Main title	Relativistic cosmology/George F. R. Ellis, Roy Maartens, Malcolm A. H. MacCallum.
Published/Created	Cambridge; New York: Cambridge University Press, 2012.
Description	xiv, 622 p.: ill.; 26 cm.
Links	Cover image http://assets.cambridge.org/97805213/81154/cover/9780521381154.jpg Contributor biographical information http://www.loc.gov/catdir/enhancements/fy1117/2011040518-b.html Publisher description http://www.loc.gov/catdir/enhancements/fy1117/2011040518-d.html Table of contents only http://www.loc.gov/catdir/enhancements/fy1117/2011040518-t.html
ISBN	9780521381154 (hbk.) 0521381150 (hbk.)
LC classification	QB981 .E4654 2012

Related names	Maartens, R. (Roy) MacCallum, M. A. H.
Summary	"Cosmology has been transformed by dramatic progress in high-precision observations and theoretical modelling. This book surveys key developments and open issues for graduate students and researchers. Using a relativistic geometric approach, it focuses on the general concepts and relations that underpin the standard model of the Universe. Part I covers foundations of relativistic cosmology whilst Part II develops the dynamical and observational relations for all models of the Universe based on general relativity. Part III focuses on the standard model of cosmology, including inflation, dark matter, dark energy, perturbation theory, the cosmic microwave background, structure formation and gravitational lensing. It also examines modified gravity and inhomogeneity as possible alternatives to dark energy. Anisotropic and inhomogeneous models are described in Part IV, and Part V reviews deeper issues, such as quantum cosmology, the start of the universe and the multiverse proposal. Colour versions of some figures are available at www.cambridge.org/9780521381154"-- Provided by publisher.
Contents	Machine generated contents note: Part I. Foundations: 1. The nature of cosmology; 2. Geometry; 3. Classical physics and gravity; Part II. Relativistic Cosmological Models: 4. Kinematics of cosmological models; 5. Matter in the Universe; 6. Dynamics of cosmological models; 7. Observations in cosmological models; 8. Light-cone approach to relativistic cosmology; Part III. The Standard Model and Extensions: 9. Homogeneous FLRW universes; 10. Perturbations of FLRW universes; 11. The cosmic background radiation; 12. Structure formation and gravitational lensing; 13. Confronting the Standard Model with observations; 14.

	Acceleration from dark energy or modified gravity; 15. 'Acceleration' from large scale inhomogeneity?; 16. 'Acceleration' from small scale inhomogeneity?; Part IV. Anisotropic and Inhomogeneous Models: 17. The space of cosmological models; 18. Spatially homogeneous anisotropic models; 19. Inhomogeneous models; Part V. Broader Perspective: 20. Quantum gravity and the start of the Universe; 21. Cosmology in a larger setting; 22. Conclusion: our picture of the Universe; Appendix; References; Index.
Subjects	Cosmology. Relativistic astrophysics. Relativistic quantum theory. SCIENCE/Cosmology.
Notes	Includes bibliographical references and index.

Relativity, gravitation, cosmology: foundations

LCCN	2015032800
Type of material	Book
Main title	Relativity, gravitation, cosmology: foundations/V.V. Dvoeglazov, editor.
Published/Produced	New York: Nova Publishers, [2016]
Description	x, 213 pages: illustrations; 26 cm.
ISBN	9781634837897 (hardcover) 1634837894 (hardcover)
LC classification	QC173.55 .R45 2016
Related names	Dvoeglazov, Valeri V., editor.
Subjects	Relativity (Physics) Quantum theory. Quantum cosmology. Quantum gravity.
Notes	Includes bibliographical references and index.
Series	Contemporary fundamental physics Contemporary fundamental physics.

Road to reality with Roger Penrose

LCCN	2015304990
Type of material	Book

Main title	Road to reality with Roger Penrose/edited by James Ladyman, Stuart Presnell, Gordon McCabe, Michał Eckstein, Sebastian J. Szybka.
Published/Produced	Kraków: Copernicus Center Press, [2015] ©2015
Description	xii, 279 pages: illustrations; 25 cm.
ISBN	9788378861690 8378861694
LC classification	Q175.32.R42 R63 2015
Related names	Ladyman, James, 1969- editor. Presnell, Stuart, editor. McCabe, Gordon, editor. Eckstein, Michał, editor. Szybka, Sebastian J., editor.
Contents	From geometric quantum mechanics to quantum information/Paolo Aniello, Jesús Clemente-Gallardo, Giuseppe Marmo, Georg F. Volkert -- Black holes in general relativity/Abhay Ashtekar -- Gravitational energy: a quasi-local, Hamiltonian approach/Katarzyna Grabowska & Jerzy Kijowski -- General relativity and von Neumann algebras/Michael Heller, Zdisław Odrzygóźdź, Leszek Pysiak & Wiesław Sasin -- Penrose's metalogical argument is unsound/Stanisław Krajewski -- Mach's principle within general relativity/Donald Lynden-Bell -- Algebraic approach to quantum gravity I: relative realism/Shahn Majid -- On the abuse of gravity theories in cosmology/Leszek M. Sokołowski -- Penrose's Weyl curvature hypothesis and conformally-cyclic cosmology/Paul Tod -- Can empirical facts become mathematical truths?/Krzysztof Wójtowicz -- Twistors and special functions/Nick Woodhouse.
Subjects	Penrose, Roger. Penrose, Roger. Quantum theory. Realism. Mathematical physics. Mathematical physics.

	Quantum theory.
	Realism.
Notes	Includes bibliographical references (pages 255-279).

Rocket science for the rest of us: cutting-edge concepts made simple

LCCN	2015297118
Type of material	Book
Personal name	Gilliland, Ben, author.
Main title	Rocket science for the rest of us: cutting-edge concepts made simple/written by Ben Gilliand; consultant, Jack Challoner.
Edition	First American edition.
Published/Produced	New York, New York: DK Publishing, 2015. ©2015
Description	192 pages: color illustrations; 24 cm
ISBN	9781465433657 (paperback) 1465433651 (paperback)
LC classification	TL782.5 .G468 2015
Related names	Challoner, Jack.
Summary	Want to understand black holes, antimatter, physics, and space exploration? Looking for a common sense guide to quantum physics that you can actually understand? Rocket Science for the Rest of Us is the book you're looking for! Get a grip on even the most mysterious and complex sciences with Ben Gilliland's guide to dark matter, exo-planets, Planck time, earth sciences, and more.-- Source other than Library of Congress.
Contents	Mysterious universe. How big is the universe? -- The star that redrew the cosmos -- Expanding universe -- Welcome to the multiverse -- We are all doomed! -- Catch up with the stellar speed demons -- Meet the smelly dwarf -- Mercury's secrets -- How to catch a comet -- Saturn's amazing rings -- The search for alien life -- The hostile blue planet -- The space rock that "killed" Pluto -- To boldly go. The first human in space -- Pioneer 10: the little spacecraft that could -- Voyager: our distant emissary -- Is there life on Mars? -- Colonizing Mars

	-- Mapping the Milky Way -- Detecting killer asteroids -- Looking beyond Mars for life -- A Webb to catch the oldest stars -- ESA's Rosetta comet chaser -- Gravity lensing to see the cosmos -- Engage warp drive! -- Space: the fatal frontier -- The appliance of science. It is only a theory -- Why does anything exist? -- Leap second -- A weird, almost perfect universe -- What is dark matter? -- Why is gravity so weak? -- Dark matter builds the universe -- We are all made of stars -- The story of the pulsar -- Doing the black hole twist -- Helium shortage -- Death rays from outer space -- Gravity slingshot -- Is glass a liquid? -- Curiosity: science's heart -- Teeny tiny, supersmall stuff. The story of the atom -- Discovering the neutron -- The world of the insanely tiny -- The certainty of uncertainty -- Seeking supersymmetry -- Higgs boson: a bluffer's guide -- Quantum gravity -- X-ray crystallography -- Particle accelerators -- Attack of the micro black holes.
Subjects	Rockets (Aeronautics)--Popular works. Quantum theory--Popular works. Earth sciences--Popular works. Dark matter (Astronomy)--Popular works. Outer space--Exploration--Popular works.
Notes	Includes index.

Seven brief lessons on physics

LCCN	2016304894
Type of material	Book
Personal name	Rovelli, Carlo, 1956- author.
Uniform title	Sette brevi lezioni di fisica. English
Main title	Seven brief lessons on physics/Carlo Rovelli; translated by Simon Carnell and Erica Segre.
Edition	First American edition.
Published/Produced	New York, New York: Riverhead Books, 2016. ©2015
Description	86 pages: illustrations; 20 cm
ISBN	9780399184413

	0399184414
LC classification	QC24.5 .R68313 2016
Related names	Carnell, Simon, 1962- translator. Segre, Erica, translator.
Summary	'Here, on the edge of what we know, in contact with the ocean of the unknown, shines the mystery and the beauty of the world. And it's breathtaking' These seven short, simple lessons guide us through the scientific revolution that shook physics in the twentieth century and still continues to shake us today. Theoretical physicist Carlo Rovelli, a founder of the loop quantum gravity theory. explains Einstein's theory of general relativity, quantum mechanics, black holes, the complex architecture of the universe, elementary particles, gravity, and the nature of the mind. In under eighty pages, readers will understand the most transformative scientific discoveries of the twentieth century and what they mean for us.
Contents	The most beautiful of theories -- Quanta -- The architecture of the cosmos -- Particles -- Grains of space -- Probability, time, and the heat of black holes -- Ourselves.
Subjects	Physics--Popular works. Physics--History--20th century. Physics.
Notes	Translation from the Italian of: Sette brevi lezioni di fisica. "English translation published in Great Britain by Allen Lane, an imprint of Penguin Random House UK"--Title page verso. Includes index.
Other edition	Reproduction of (manifestation): Rovelli, Carlo, 1956- Seven brief lessons on physics. [London]: Allen Lane, an imprint of Penguin Books, 2015 9780241235966

Skyrmions: a great finishing touch to classical Newtonian philosophy

LCCN	2012013597

Type of material	Book
Main title	Skyrmions: a great finishing touch to classical Newtonian philosophy/editors, Maricel Agop and Nicolae Mazilu.
Published/Created	Hauppauge, N.Y.: Nova Science Publisher, c2012.
Description	xvi, 230 p.; 27 cm.
ISBN	9781620816288 (hardcover)
LC classification	QC173.4.A87 S59 2012
Related names	Agop, Maricel. Mazilu, Nicolae.
Subjects	Skyrme, Tony Hilton Royle, 1922-1987. Atomic structure. Quantum gravity.
Notes	Includes bibliographical references (p. [217]-228) and index.

Space-time foliation in quantum gravity.

LCCN	2014939403
Type of material	Book
Main title	Space-time foliation in quantum gravity.
Published/Produced	New York: Springer, 2014.
Links	Contributor biographical information http://www.loc.gov/catdir/enhancements/fy1411/2014939403-b.html Publisher description http://www.loc.gov/catdir/enhancements/fy1411/2014939403-d.html Table of contents only http://www.loc.gov/catdir/enhancements/fy1411/2014939403-t.html
ISBN	9784431549468

The arrows of time: a debate in cosmology

LCCN	2012939849
Type of material	Book
Main title	The arrows of time: a debate in cosmology/Laura Mersini-Houghton, Rudy Vaas, editors.
Published/Created	Heidelberg; New York: Springer, ©2012.
Description	v, 221 pages: illustrations (some color); 24 cm.
ISBN	9783642232589 (hbk: acid-free paper)

	3642232582 (hbk: acid-free paper)
LC classification	QB209 .A75 2012
Related names	Mersini-Houghton, Laura. Vaas, Rüdiger.
Contents	Introduction -- Time After Time -- Big Bang Cosmology and the Arrows of Time/Rüdiger Vaas -- Fundamental Loss of Quantum Coherence from Quantum Gravity/Rodolfo Gambini, Rafael A. Porto and Jorge Pullin -- The Clock Ambiguity: Implications and New Developments/Andreas Albrecht and Alberto Iglesias -- Holographic Cosmology and the Arrow of Time/Tom Banks -- The Emergent Nature of Time and the Complex Numbers in Quantum Cosmology/Gary W. Gibbons -- The Phantom Bounce: A New Proposal for an Oscillating Cosmology/Katherine Freese, Matthew G. Brown and William H. Kinney -- Notes on Time's Enigma/Laura Mersini-Houghton -- A Momentous Arrow of Time/Martin Bojowald -- Can the Arrow of Time Be Understood from Quantum Cosmology?/Claus Kiefer -- Open Questions Regarding the Arrow of Time/H. Dieter Zeh.
Subjects	Time. Cosmology. Cosmology. Time. Zeit. Zeitrichtung. Zeitmessung.
Notes	Includes bibliographical references and index.
Series	Fundamental theories of physics; v. 172 Fundamental theories of physics; v. 172.

The Big bang theory and philosophy: rock, paper, scissors, Aristotle, Locke

LCCN	2011043333
Type of material	Book
Main title	The Big bang theory and philosophy: rock, paper, scissors, Aristotle, Locke/edited by Dean Kowalski.

Published/Produced	Hoboken, New Jersey: John Wiley & Sons, Inc., [2012]
Description	x, 278 pages; 23 cm.
ISBN	9781118074558 (pbk.)
LC classification	PN1992.77.B485 B54 2012
Related names	Kowalski, Dean A., editor of compilation.
Summary	"There are books that debate math, science, and history; there are books that help you build walls or even pyramids; there are even books that discuss Neanderthals with tools and autotrophs that drool. This book discusses philosophy. But you don't need an IQ of 187 to enjoy it. I swear to cow! As you'll see, the philosophy is theoretical, but the fun is real"-- Provided by publisher.
Contents	Machine generated contents note: Acknowledgments Introduction: "Unraveling the Mysteries" Part One. "It All Began on a Warm Summer's Evening in Greece": Aristotelian Insights 1. Aristotle on Sheldon Cooper: Ancient Greek Meets Modern Geek Greg Littmann 2. "You're a Sucky, Sucky Friend": Seeking Aristotelian Friendship in The Big Bang Dean A. Kowalski 3. The Big Bang Theory on the Use and Abuse of Modern Technology Kenneth Wayne Sayles III Part Two. "Is It Wrong to Say I Love Our Killer Robot?": Ethics and Virtue 4. Feeling Good about Feeling Good: Is It Morally Wrong to Laugh at Sheldon? W. Scott Clifton 5...But Is Wil Wheaton Evil? Donna Marie Smith 6. Do We Need a Roommate Agreement?: Pleasure, Selfishness, and Virtue in The Big Bang Gregory L. Bock and Jeffrey L. Bock Part Three. "Perhaps You Mean a Different Thing Than I Do When You Say "Science": Science, Scientism, and Religion 7. Getting Fundamental about Doing Physics in The Big Bang Jonathan Lawhead 8. Sheldon, Leonard, and Leslie: The Three Faces of Quantum Gravity Andrew Zimmerman Jones 9. The One Paradigm to Rule Them All: Scientism and The Big Bang Massimo Pigliucci 10. Cooper Considerations Adam

	Barkman and Dean A. Kowalski Part Four. "I Need Your Opinion on a Matter of Semiotics": Language and Meaning 11. Wittgenstein and Language Games in The Big Bang Theory Janelle Pötzsch 12. "I'm Afraid You Couldn't Be More Wrong!": Sheldon and Being Right about Being Wrong Adolfas Mackonis 13. The Cooper Conundrum: Good Lord, Who's Tolerating Who? Ruth E. Lowe 14. The Mendacity Bifurcation Don Fallis Part Five. "The Human Experience That has Always Eluded Me": The Human Condition 15. Mothers and Sons of The Big Bang Ashley Barkman 16. Penny, Sheldon, and Personal Growth through Difference Nicholas G. Evans 17. Deconstructing the Women of The Big Bang Theory: So Much More than Girlfriends Mark D. White and Maryanne L. Fisher The Episode Compendium:"Hey, It's a Big Menu--There's Two Pages Just for Desserts" Contributors. "But If We Were Part of the Team... We Could Drink for Free in Any Bar in Any College Town" Index. "Cornucopia...Let's Make that Our Word of the Day" .
Subjects	Big bang theory (Television program) Philosophy--Miscellanea. PHILOSOPHY/General.
Notes	Includes bibliographical references and index.
Series	The Blackwell philosophy and pop culture series; 44

The science of Interstellar

LCCN	2015304869
Type of material	Book
Personal name	Thorne, Kip S., author.
Main title	The science of Interstellar/Kip Thorne.
Edition	First edition.
Published/Produced	New York: W.W. Norton & Company, [2014] ©2014
Description	x, 324 pages: illustrations (chiefly color); 26 cm
ISBN	9780393351378 (pbk) 0393351378 (pbk)

LC classification	PN1995.9.S26 T49 2014 PN1997.2.I635 T49 2014
Related names	Nolan, Christopher, 1970- writer of foreword.
Summary	"Interstellar, from acclaimed filmmaker Christopher Nolan, takes us on a fantastic voyage far beyond our solar system. Yet in The Science of Interstellar, Kip Thorne, the physicist who assisted Nolan on the scientific aspects of Interstellar, shows us that the movie's jaw-dropping events and stunning, never-before-attempted visuals are grounded in real science. Thorne shares his experiences working as the science adviser on the film and then moves on to the science itself. In chapters on wormholes, black holes, interstellar travel, and much more, Thorne's scientific insights many of them triggered during the actual scripting and shooting of Interstellar, describe the physical laws that govern our universe and the truly astounding phenomena that those laws make possible."-- Publisher's description.
Contents	A scientist in Hollywood: the genesis of Interstellar -- Foundations. Our universe in brief; The laws that control the universe; Warped time and space, and tidal gravity; Black holes -- Gargantua. Gargantua's anatomy; Gravitational slingshots; Imaging Gargantua; Disks and jets; Accident is the first building block of evolution -- Disaster on earth. Blight; Gasping for oxygen; Interstellar travel -- The wormhole. Wormholes; Visualizing Interstellar's wormhole; Discovering the wormhole: gravitational waves -- Exploring Gargantua's environs. Miller's planet; Gargantua's vibrations; Mann's planet; The Endurance -- Extreme physics. The fourth and fifth dimensions; Bulk beings; Confining gravity; Gravitational anomalies; The professor's equation; Singularities and quantum gravity -- Climax. The volcano's rim; Into Gargantua; The tesseract; Messaging the past; Lifting colonies off Earth -- Where can you learn more?
Subjects	Interstellar (Motion picture)

	Science in motion pictures.
Notes	"Foreword by Christopher Nolan"--Cover. Includes bibliographical references (pages 305-309) and indexes.

The story of collapsing stars: black holes, naked singularities, and the cosmic play of quantum gravity

LCCN	2014948114
Type of material	Book
Personal name	Joshi, Pankaj S., author.
Main title	The story of collapsing stars: black holes, naked singularities, and the cosmic play of quantum gravity/Pankaj S. Joshi.
Published/Produced	New York, NY: Oxford University Press, [2015]
Description	xiii, 225 pages: illustrations; 22 cm
ISBN	9780199686766 (hbk.) 0199686769 (hbk.)
LC classification	QB806 .J67 2015
Portion of title	Black holes, naked singularities, and the cosmic play of quantum gravity
Summary	"This book journeys into one of the most fascinating intellectual adventures of recent decades - understanding and exploring the final fate of massive collapsing stars in the universe. The issue is of great interest in fundamental physics and cosmology today, from both the perspective of gravitation theory and of modern astrophysical observations. This is a revolution in the making and may be intimately connected to our search for a unified understanding of the basic forces of nature. According to the general theory of relativity, a massive star that collapses catastrophically under its own gravity when it runs out of its internal nuclear fuel must give rise to a space-time singularity. Such singularities are regions in the universe where all physical quantities take their extreme values and become arbitrarily large. The singularities may be covered within a black hole, or visible to faraway observers in the universe. Thus, the final fate of a

collapsing massive star is either a black hole or a visible naked singularity. We discuss here recent results and developments on the gravitational collapse of massive stars and possible observational implications when naked singularities happen in the universe."--Back cover.

Contents 1. Our universe (Microcosm, macrocosm, and forces of Nature; The role of gravity; Dynamical evolution in the universe; Black holes, singularities, and quantum gravity; Our trajectory) -- 2. The fabric of spacetime (The force of gravity; Spacetime continuum; Einstein's Theory of Relativity; Physical implications; Local and global aspects; Spacetime foam) -- 3. Black holes (Life of a star; Collapse of massive stars; A black hole is born; Gravitational collapse; The debate on horizon and singularity; Black hole physics) -- 4. Singularities (The existence; Can we avoid singularities?; Causality violations; Energy conditions and trapped surfaces; Fundamental challenges) -- 5. Cosmic censorship (What is a naked singularity?; Censoring the cosmos; Inhomogeneous dust collapse; The genericity aspects)

6. Naked singularities (Collapsing a massive star; Gravitational collapse studies; Non-spherical collapse; Numerical simulations; Event-like and object-like singularities; Collapse scenarios; Why a naked singularity forms; Observational aspects and quantum gravity) -- 7. Cosmic conundrums (Can we reformulate the censorship?; Are naked singularities stable and generic?; Structure of naked singularities; Questions on collapse and singularities) -- 8. Is our universe predictable? (Predictability defined; Is relativity a predictable theory ?; Singularities and predictability; Rabbits popping out of a hat?; Restoring the predictability) -- 9. A lab for quantum gravity (The quest for quantum gravity; Need for observational data; Singularity resolution in quantum gravity; Naked singularity and quantum

	gravity; Quantum stars?) -- 10. The frontiers (Observational frontiers; Testing censorship using astronomical observations; Super-Kerr geometries; Observable signatures of naked singularities; Distinguishing black holes and naked singularities; Shockwaves near a singularity?; Black hole paradoxes; Infall into a black hole versus naked singularity; Emerging perspective)
Subjects	Stars--Evolution. Quantum gravity. Quantum gravity. Stars--Evolution. Sternentwicklung. Quantengravitation. Schwarzes Loch. Gravitationskollaps.
Notes	Includes bibliographical references (page 219) and index.

The Thirteenth Marcel Grossmann Meeting on Recent Developments in Theoretical and Experimental General Relativity, Astrophysics, and Relativistic Field Theories: proceedings of the MG13 Meeting on General Relativity, Stockholm University, Sweden, 1-7 July 2012

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Related names	Rosquist, Kjell, 1948- editor. Jantzen, Robert T., editor. Ruffini, Remo.
Subjects	General relativity (Physics)--Congresses. Gravitation--Congresses. Quantum gravity--Congresses. Cosmology--Congresses.
Notes	Includes bibliographical references.

The timeless approach: frontier perspectives in 21st century physics

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Personal name	Fiscaletti, Davide, author.
Main title	The timeless approach: frontier perspectives in 21st century physics/Davide Fiscaletti, SpaceLife Institute, Italy.
Published/Produced	New Jersery: World Scientific, [2016]
Description	vi, 449 pages; 24 cm.
ISBN	9789814713153 (hardcover: alk. paper) 9814713155 (hardcover: alk. paper)
LC classification	QC21.3 .F565 2016
Variant title	Frontier perspectives in twenty-first century physics
Portion of title	Frontier perspectives in 21st century physics
Contents	About time as the numerical order of material changes -- Three-dimensional Euclid space and special relativity -- Three-dimensional non-Euclid space as a direct information medium and quantum phenomena -- About quantum cosmology in a background space as an immediate information medium -- The gravitational space in an a-temporal quantum-gravity space theory -- A three-

	dimensional timeless quantum vacuum as the fundamental bridge between gravitation and the quantum behavior.
Subjects	Physics. Space and time.
Notes	Includes bibliographical references and index.
Series	Series on the foundations of natural science and technology; v. 9 Series on the foundations of natural science and technology; v. 9.

The Twelfth Marcel Grossmann Meeting on recent developments in theoretical and experimental general relativity, astrophysics and relativistic field theories: proceedings of the MG12 Meeting on General Relativity, UNESCO headquarters, Paris, France, 12-18 July 2009

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Type of material	Book
Meeting name	Marcel Grossmann Meeting on General Relativity (12th: 2009: Paris)
Main title	The Twelfth Marcel Grossmann Meeting on recent developments in theoretical and experimental general relativity, astrophysics and relativistic field theories: proceedings of the MG12 Meeting on General Relativity, UNESCO headquarters, Paris, France, 12-18 July 2009/editors, Thibault Damour, Robert T. Jantzen, series editor, Remo Ruffini.
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Description	3 v.: ill.; 26 cm.
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Portion of title	Proceedings of the MG12 Meeting on General Relativity

	Recent developments in theoretical and experimental general relativity, astrophysics and relativistic field theories
Cover title	Proceedings of the Twelfth Marcel Grossmann Meeting on General Relativity
Related names	Damour, Thibault. Jantzen, Robert T. Ruffini, Remo.
Subjects	General relativity (Physics)--Congresses. Gravitation--Congresses. Quantum gravity--Congresses.

The universe: a view from classical and quantum gravity

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Personal name	Bojowald, Martin, author.
Main title	The universe: a view from classical and quantum gravity/Martin Bojowald.
Published/Produced	Weinheim, Germany: Wiley-VCH, [2013]
Description	viii, 362 pages: illustrations; 24 cm.
ISBN	9783527410187 (paperback: acid-free paper) 352741018X (paperback: acid-free paper)
LC classification	QB981 .B685 2013
Subjects	Cosmology--Textbooks. Quantum gravity--Textbooks. Gravitation--Textbooks.
Notes	Includes bibliographical references and index.
Series	Physics textbook

Traditions and transformations in the history of quantum physics: HQ-3, Third International Conference on the History of Quantum Physics, Berlin, June 28-July 2, 2010

LCCN	2015493218
Type of material	Book
Meeting name	International Conference on the History of Quantum Physics (3rd: 2010: Berlin, Germany)
Main title	Traditions and transformations in the history of quantum physics: HQ-3, Third International Conference on the History of Quantum Physics,

	Berlin, June 28-July 2, 2010/Shaul Katzir, Christoph Lehner, Jürgen Renn (eds.).
Published/Produced	[Berlin]: Edition Open Access, 2013.
Description	xi, 340 pages: illustrations; 21 cm
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LC classification	QC173.98 I58 2010
Related names	Katzir, Shaul, editor. Lehner, Christoph, 1962- editor. Renn, Jürgen, 1956- editor.
Contents	From classical to quantum physics. Theoretical challenges by experimental physics: radiation and its interaction with matter/Shaul Katzir. Challenging the boundaries between classical and quantum physics: the case of optical dispersion/Marta Jordi Taltavull. Putting the quantum to work: Otto Sackur's pioneering exploits in the quantum theory of gases/Massimiliano Badino and Bretislav Friedrich -- Quantum mechanics in the making. The concepts of light atoms and light molecules and their final interpretation/Dieter Fick and Horst Kant. Early interactions of quantum statistics and quantum mechanics/Daniela Monaldi. Pourparlers for amalgamation: some early sources of quantum gravity research/Dean Rickles -- Extending the framework of quantum physics. Superposing dynamos and electrons: electrical engineering and quantum physics in the case of Nishina Yoshio/Kenji Ito. The origins of Maria Göppert's dissertation on two-photon quantum transitions at Göttingen's Institutes of Physics, 1920-1933/Barry R. Masters. An act of creation: the Meitner-Frisch interpretation of nuclear fission/Roger H. Stuewer -- The challenges of quantum field theory. Tsung-Sui Chang's contribution to the quantization of constrained Hamiltonian systems/Xiaodong Yin, Zhongyuan Zhu, Donald C. Salisbury. Feynman's struggle and Dyson's surprise: the development and early application of a new means of representation/Adrian Wüthrich -- Traditions and

	debates in recent quantum physics. Orthodoxies on the interpretation of quantum theory: the case of the consistent history approach/Olival Freire. From do-it-yourself quantum mechanics to nanotechnology?: the history of experimental semiconductor physics, 1970-2000/Christian Kehrt.
Subjects	Quantum theory--History--Congresses.
Notes	Includes bibliographical references.
Series	Max Planck research library for the history and development of knowledge. Proceedings; 5

What we would like LHC to give us: proceedings of the International School of Subnuclear Physics

LCCN	2014026049
Type of material	Book
Meeting name	International School of Subnuclear Physics (50th: 2012: Erice, Italy)
Main title	What we would like LHC to give us: proceedings of the International School of Subnuclear Physics/edited by Antonino Zichichi, European Physical Society, Geneva, Switzerland.
Published/Produced	[Hackensack,] New Jersey: World Scientific, [2014] ©2014
Description	xiv, 581 pages: illustrations; 26 cm
ISBN	9789814603898 (hardcover: alk. paper) 9814603899 (hardcover: alk. paper)
LC classification	QC793 .I555 2012
Related names	Zichichi, Antonino, editor of compilation.
Contents	Some reminiscences of research leading to QCD and beyond/M. Gell-Mann -- The Erice Centre, Gell-Mann, QCD, the effective energy and complexity/A. Zichichi -- History of QCD/H. Fritzsch -- On the history of the strong interaction/H. Leutwyler -- Color transparency and saturation in QCD/D. Schildknecht -- Glue-mesons: their conception needs all of QCD in the infrared/P. Minkowski -- Quark masses in QCD/C.A. Dominguez -- The quark model and QCD/F. Close -- Key steps toward the creation of QCD--notes on the logic and history of

the genesis of QCD/T.Y. Cao -- Perturbative gravity from gauge theory/Z. Bern -- Black holes and supersymmetry/L. Andrianopoli, R. D'Auria and S. Ferrara -- Composite weak bosons at the LHC/H. Fritzsch -- High energy scattering in QCD and in quantum gravity/L.N. Lipatov -- One-parameter model for the superworld/D.V. Nanopoulos et al. -- Beyond relativistic quantum string theory/G. 't Hooft -- Borexino latest results/G. Bellini -- Highlights from LHC/P. Bloch -- Highlights from ATLAS/P. Jenni -- Origin and status of LUNA at Gran Sasso/C. Broggini -- Highlights from ALICE/P. Giubellino -- Highlights from BNL-RHIC/M.J. Tannenbaum -- Origin and status of the Gran Sasso INFN Laboratory/L. Votano -- Status of OPERA/D. Autiero -- The origin and status of the third neutrino/A. Bettini -- High energy physics and gravitational waves/E. Coccia -- Decades of computing in subnuclear physics--from bubble chamber to LHC/J. Knobloch -- The LAA Project and the consequences on LHC/H. Wenninger -- Complexity and the QGCW Project/A. Zichichi -- Patterns of flavour violation at the dawn of the LHC era/M.V. Carlucci -- Precise measurement of the W boson mass with the DO detector/R. Lopes de Sá -- QFT and unification of knot theories/A. Sleptsov -- Hunting in Daya Bay neutrino experiment/F. Zhang -- Vacuum stability in the SM and the three-loop [beta]-function for the Higgs self-interaction/M.F. Zoller.

Subjects	Particles (Nuclear physics)--Congresses. Colliders (Nuclear physics)--Congresses.
Notes	Conference held in June/July 2012 in Erice, Italy. Includes bibliographical references and index.
Series	The subnuclear series; volume 50 Subnuclear series; v. 50.

XVIIth International Congress on Mathematical Physics: Aalborg, Denmark, 6-11 August 2012

LCCN	2014395160
Type of material	Book
Meeting name	International Conference on Mathematical Physics (17th: 2012: Aalborg, Denmark)
Main title	XVIIth International Congress on Mathematical Physics: Aalborg, Denmark, 6-11 August 2012/edited by Arne Jensen.
Published/Produced	Singapore; Hackensack, N.J.: World Scientific Pub. Co., [2014] ©2014
Description	xvii, 724 pages illustrations (some color); 24 cm
ISBN	9789814449236 9814449245 (ebk.)
Variant title	17th International Congress on Mathematical Physics Seventeenth International Congress on Mathematical Physics
Related names	Jensen, A. (Arne), 1950-, editor of compilation.
Contents	Prizes -- pt. A. Plenary lectures. Integrable combinatorics/P. Di Francesco -- Piecewise smooth perturbations of integrable systems/D. Dolgopyat -- Applications of random matrices to operator algebra theory/U. Haagerup -- Reading in the brain/K. Hepp -- d = 4, N = 2 field theory and physical mathematics/G.W. Moore -- Microlocal singularities and scattering theory for Schrödinger equations on manifolds/S. Nakamura -- The Kardar-Parisi-Zhang equation and universality class/J.D. Quastel -- Generalized entropies/F. Dupuis ... [et al.] -- Associative algebraic approach to logarithmic conformal field theory/H. Saleur -- The method of concentration compactness and dispersive Hamiltonian evolution equations/W. Schlag -- Quantum dynamics, coherent states and Bogoliubov transformations/B. Schlein -- Spectral theory of orthogonal polynomials/B. Simon -- Quasilocal mass and surface Hamiltonian in spacetime/M.-T. Wang -- Surprises in the phase diagram of the Anderson model on the Bethe lattice/S. Warzel --

Randomness -- a computational complexity perspective/A. Wigderson -- pt. B. Topical sessions. Dynamical systems, classical and quantum. Periodic solutions of the planetary N-body problem/L. Chierchia -- Entropy, chaos and weak horseshoe for infinite dimensional random dynamical systems/W. Huang -- Instability in nearly integrable Hamiltonian systems: geometric methods/T.M. Seara -- Unstable manifolds and L[symbol] nonlinear instability of Euler equations/C. Zeng -- Contributed talks -- Posters -- Equilibrium and non-equilibrium statistical mechanics. Stochastic stability and the spin glass phase. The state of art for mean field and finite dimensional models/P. Contucci -- Macdonald processes/A. Borodin and I. Corwin -- Some simple questions from nonequilibrium physics/W. De Roeck -- Conformal invariance of Ising model correlations/C. Hongler -- Entropic functionals in quantum statistical mechanics/V. Jakšić and C.-A. Pillet -- Stochastic energy exchange models with degenerate rate functions/M. Sasada -- Quantum Heisenberg models and random loop representations/D. Ueltschi -- Contributed talks -- Posters.
PDE and general relativity. Black hole formation from a complete regular past for Vlasov matter/H. Andréasson -- Symmetries and hidden symmetries for fields outside black holes/P. Blue -- Existence of dynamical vacuum black holes/G. Holzegel -- The role of Liouville type systems in the study of non-topological Chern-Simons vortices/R. Fortini and G. Tarantello -- Local dynamics near unstable branches of NLS solitons/T.-P. Tsai -- Breakdown criteria of Einstein equations in CMC gauge/Q. Wang -- Contributed talks -- Posters -- Stochastic models and probability. Complexity of random energy landscapes/G. Ben Arous -- Bulk universality for one-dimensional log-gases/P. Bourgade -- Vacant set of random walk on finite graphs/J. Cerny --

Invariant measures and the soliton resolution conjecture/S. Chatterjee -- Solving the KPZ equation/M. Hairer -- The Brownian map: a universal limit for random planar maps/J.-F. Le Gall -- Contributed talks -- Operator algebras, exactly solvable models. Razumov-Stroganov type correspondences/L. Cantini -- The resolvent algebra of the canonical commutation relations/H. Grundling -- Fermionic basis of local operators in quantum integrable models/M. Jimbo -- Some universal properties of Levin-Wen models/L. Kong -- On the developments of Sklyanin's quantum separation of variables for integrable quantum field theories/G. Niccoli -- Construction of wedge-local QFT through Longo-Witten endomorphisms/Y. Tanimoto -- Contributed talks -- Posters -- Quantum mechanics and spectral theory. Ground state properties of multi-polaron systems/R.L. Frank ... [et al.] -- Dynamical localization of random quantum walks on the lattice/A. Joye -- Inverse problems, trace formulae for Schrödinger operators on the square lattice/H. Isozaki and E. Korotyaev -- On the number of electrons that a nucleus can bind/P.T. Nam -- A trace formula for eigenvalue clusters of the perturbed Landau Hamiltonian/A.B. Pushnitski, G.D. Raikov and C. Villegas-Blas -- Absence of positive eigenvalues for hard-core N-body systems/K. Ito and E. Skibsted -- Contributed talks -- Posters -- Quantum information and computation. An improved area-law for the ground states of 1D gapped Hamiltonians/I. Arad -- Is a random state entangled?/G. Aubrun -- Criticality without frustration for quantum spin-1 chains/S. Bravyi -- Inner approximations of the one-body quantum marginal polytope/D. Gross -- Towards the fast scrambling conjecture/P. Hayden -- Finite-dimensional approximations of quantum systems and Connes' embedding conjecture/V.B. Scholz -- Contributed talks -- Posters.

Quantum many-body theory and condensed matter physics. Mean-field electronic structure models for disordered materials/É. Cancès, S. Lahbabi and M. Lewin -- The nematic phase of a system of long hard rods/A. Giuliani -- Mean field limits for photons -- a way to establish the semiclassical Schrödinger equation/V. Matulevicius and P. Pickl -- Microscopic derivation of the Ginzburg-Landau model/R.L. Frank ... [et al.] -- 2D Coulomb gas, Abrikosov lattice and renormalized energy/S. Serfaty -- Real analyticity of solutions to Schrödinger equations involving a fractional Laplacian and other Fourier multipliers/A. Dall'Acqua ... [et al.] -- Disordered Bose Einstein condensates with interaction/R. Seiringer, J. Yngvason and V.A. Zagrebnov -- Contributed talks -- Posters -- Quantum field theory. New light on infrared problems: sectors, statistics, spectrum and all that/D. Buchholz -- Two-dimensional quantum field models (with applications to lattice statistical mechanics)/P. Falco -- Construction and properties of noncommutative quantum fields/H. Grosse and R. Wulkenhaar -- A review of the 1/N expansion in random tensor models/R. Gurau -- Fedosov quantization approach to QFT/S. Hollands -- A field theoretic approach to stochastic calculus: exploring rough paths/J. Unterberger -- Contributed talks -- Posters -- String theory and quantum gravity. On-shell physics and the positive Grassmannian/J. Bourjaily -- Is a graviton detectable?/F. Dyson -- Exact spectrum of 4D conformal gauge theories from integrability/N. Gromov -- Quantum Teichmüller theory and TQFT/J.E. Andersen and R.M. Kashaev -- Instanton partition functions of N = 2 quiver gauge theories and integrable systems/V. Pestun -- From groups and knots to black hole entropy -- mathematical aspects of loop quantum gravity/H. Sahlmann -- Superconformal indices and partition functions for supersymmetric field

	theories/I.B. Gahramanov and G.S. Vartanov -- Contributed talks -- Other Topics. Posters -- pt. C. Young researcher symposium. Plenary talks -- Contributed talks.
Subjects	Mathematical physics--Congresses.
Notes	Includes bibliographical references and index.

INDEX

#

20th century, 137
21st century, 146

A

accelerator, 101, 125
acid, 110, 138, 148
amplitude, 13, 67, 68
anatomy, 142
anisotropy, 123
annihilation, 4, 14, 15
antimatter, 134
Aristotle, 139, 140
arrow of time, 120
assessment, 117
assets, 113, 131
asteroids, 135
asymmetry, 2, 27, 28
ATLAS, 74, 90, 130, 151
atomic nucleus, 50, 112
atoms, 149
automata, 103

B

background radiation, 132
baryons, 130
Big Bang, 103, 116, 138, 140
biotechnology, 112
black hole entropy, 155
black hole space-time, vii, 1
black hole(s), vii, viii, 1, 2, 3, 6, 11, 13, 14, 18, 20, 22, 24, 29, 30, 33, 40, 41, 44, 45, 49, 50, 69, 70, 71, 74, 95, 96, 103, 104, 106, 111, 114, 118, 133, 134, 135, 136, 137, 141, 142, 143, 144, 150, 153, 155
boson(s), 101, 110, 129, 135, 150, 151
brain, 152
branching, 45
Britain, 137

C

calculus, 155
Cantor set, 34, 43, 48
causality, 108, 121
challenges, 29, 144, 149
chaos, 48, 152
Chicago, 31
China, 47, 48, 103
classical electrodynamics, 121
classification, 99, 102, 104, 105, 106, 107, 108, 109, 113, 115, 116, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 131, 133, 134, 136, 137, 138, 139, 141, 143, 145, 146, 148, 150
clusters, 45, 110, 154

CMC, 153
coding, 20, 22
cognition, 112
coherence, 113
collaboration, viii, 73, 74
collisions, vii, 1, 50
color, 110, 116, 134, 138, 141, 152
combinatorics, 152
communication, 112
compilation, 104, 139, 150, 152
complexity, 1, 129, 150, 152
computation, 3, 13, 154
computing, 112, 151
configuration, 50, 55, 65
congress, 125, 130, 135, 151, 152
construction, 45, 76, 86
contradiction, 51
controversial, 79, 86
convergence, 28
correlation(s), 110, 121, 153
cosmos, 41, 135, 137, 144
CPT, 130
critical value, 18
cycles, 102, 119

D

dark energy, vii, 33, 34, 35, 40, 41, 44, 45, 46, 47, 48, 131, 132
dark matter, 34, 40, 106, 131, 135
decay, 50, 74
decomposition, 5
degenerate, 50, 153
Denmark, 151
density matrices, 110
depth, 24, 95
derivatives, 12, 82, 83
detectable, 34, 155
detection, 123
differential equations, 120
dilation, 121
discreteness, 116
discretization, 19, 24, 25, 28, 105
disordered systems, 110
dispersion, 13, 53, 149
distortions, 50
divergence, 78, 83, 85, 86
DNA, 112
duality, 118, 129
dynamical systems, 152

E

early universe, 50
effective field theory, viii, 73, 75
e-infinity dissection, vii, 33
e-infinity theory, viii, 33
electric charge, 82
electric field, 5
electromagnetic, 42
electronic structure, 154
electrons, 149, 154
elementary particle, viii, 41, 50, 51, 68, 73, 136
emission, 20
encoding, 52
energy, vii, viii, 3, 19, 20, 21, 22, 33, 34, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 64, 65, 68, 69, 74, 75, 76, 77, 81, 82, 86, 100, 110, 112, 121, 123, 131, 132, 133, 150, 153, 155
energy density, 35, 40, 41, 43, 45
energy momentum tensor, 3, 19
engineering, 111, 149
entanglement relativity XE "relativity" theory, vii, 33
entropy, 50, 155
EPS, 70
equilibrium, 55, 101, 104, 153
estimation problems, 113
Euclidean space, 88
evaporation, 70
evidence, viii, 27, 73, 74, 121
evolution, 2, 4, 18, 20, 21, 22, 23, 24, 25, 27, 51, 108, 142, 143, 152
excision, 21

F

fermions, viii, 73, 110
Fibonacci-like dimension sequence, vii, 33, 37, 39, 45
field theory, viii, 43, 107, 109, 110, 116, 125, 126, 149, 152, 155
films, 113
fission, 149
fluctuations, 2, 5, 6, 9, 11, 13, 42
force, 42, 43, 110, 143
formation, 131, 132, 153
formula, 35, 36, 37, 39, 154
foundations, 108, 113, 122, 131, 132, 146
fractal space, 45
fractality, 46
France, 104, 147, 152
freedom, 8, 53, 75, 87, 88

G

galactic center, 50
gauge group, 5
gauge invariant, 7, 11, 75, 86
gauge theories, vii, 76, 79, 86, 87, 88, 123, 155
gauge theory, 75, 100, 130, 150
general relativity, 31, 68, 91, 114, 145, 147
genetic information, 111
geometry, viii, 17, 23, 40, 41, 48, 49, 52, 114, 116, 118, 121
Germany, 47, 48, 104, 148
gluons, 85
graduate students, 95, 115, 131
graph, 5, 6, 8, 14, 22, 79, 85
gravitation, 108, 113, 116, 125, 132, 143, 146
gravitational collapse, vii, 1, 3, 18, 30, 50, 143
gravitational field, 52
gravitational lensing, 131, 132
gravitational waves, viii, 20, 30, 73, 74, 117, 122, 124, 142, 151
gravity, vii, viii, 1, 2, 3, 4, 12, 13, 16, 20, 22, 29, 73, 74, 75, 76, 79, 81, 84, 86, 87, 96, 101, 102, 104, 105, 106, 107, 108, 110, 111, 113, 114, 115, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 142, 143, 144, 146, 148, 149, 150, 155
Great Britain, 137
Greece, 140
Greeks, 119

H

Hamiltonian, 4, 14, 52, 53, 108, 115, 123, 133, 149, 152, 154
Hausdorff dimension, 43, 47
Hausdorff-Besicovitch dimension, 34, 38, 39
Hawking radiation, 27
Hawking, Stephen, 111
Higgs boson, 74, 101, 135
Higgs particle, 100, 130
Hilbert space, 4, 5, 122
history, 108, 111, 124, 139, 148, 149, 150
hologram, 71
human, 111, 112, 135
human genome, 112
Hungary, 73
hybrid, 81
hypothesis, 121, 134

I

ICRA, 104
identification, 53, 88
identity, 120
ideology, 108
image(s), 113, 119, 131
indirect effect, 69
induction, 36, 38
industry, 112
inflation, 131
inhomogeneity, 131, 132

integration, 57, 87, 88, 101
intelligence, 112
interference, 46
invariants, 11, 101
Israel, 91
Italy, 49, 102, 104, 109, 129, 130, 145, 146, 150, 151
iteration, 20

K

Kerr black hole, viii, 33, 40, 41, 44, 45
Klein-Gordon equation, 11
knots, 155

L

Lagrange multipliers, 8
landscapes, 153
laws, 111, 142
lepton, 74
Lie group, 30
lifetime, 74
light, 2, 13, 97, 101, 108, 120, 130, 149, 155
LIGO collaboration, viii, 73, 74
linear function, 8
linear systems, 18
liquids, 110
localization, 103, 110, 154
Lorentz symmetries, viii, 73, 78, 88, 89

M

magnetic field, 109
magnitude, vii, 1, 13
manifolds, 121, 152
manipulation, 78
many-body theory, 100, 154
Mars, 104, 135
mass, 3, 4, 30, 46, 50, 51, 52, 53, 64, 65, 74, 83, 96, 113, 151, 152
mass media, 113
materials, 112, 154
mathematics, 114, 152
matrix, 5
matter, 2, 27, 30, 34, 40, 50, 100, 106, 107, 109, 110, 131, 135, 136, 149, 153, 154
measurement, 34, 48, 82, 96, 106, 109, 151
media, 113
mercury, 135
mesons, 150
methodology, 112
Milky Way, 135
modelling, 131
models, 2, 39, 43, 50, 69, 76, 103, 109, 110, 118, 122, 126, 131, 132, 153, 154
modifications, 115
molecules, 149
momentum, viii, 3, 4, 5, 7, 13, 15, 52, 53, 59, 61, 73, 75, 76, 78, 80, 82, 86, 87, 88, 89, 90
Moscow, 96, 124, 125
muon collider, 101

N

nanotechnology, v, vii, 33, 44, 149
National Research Council, 118
natural science, 146
neglect, 12
neutral, 51, 52, 57, 64
neutrinos, 130
neutrons, 112
Newtonian theory, 108
Nobel Prize, 34
nonequilibrium, 153
nonlinear systems, 100
nucleus, 41, 112, 154
null, 21

O

one dimension, 20, 24, 26
optical properties, 112
orbit, 20, 52, 53, 55, 56, 57, 59, 61
orthogonality, 10
oscillation, 61
oxygen, 142

P

parallel, 42
partial differential equations, 19
particle collisions, 50, 69
particle physics, 46, 112, 125
partition, 155
path integrals, 101
permit, 20
phase diagram, 152
phase transitions, 110
phenomenology, 29, 116
photons, 42, 77, 122, 154
physical laws, 142
physical theories, 102
physics, 2, 3, 20, 30, 31, 34, 46, 47, 48, 74, 95, 99, 100, 102, 106, 107, 108, 110, 111, 113, 114, 118, 119, 123, 124, 125, 127, 128, 129, 130, 132, 133, 134, 136, 137, 139, 142, 143, 144, 146, 148, 149, 151, 153, 154, 155
Planck scale, viii, 49, 50, 51, 68, 69, 70, 74
Planckian black hole, vii, viii, 49
planets, 111, 135
Poland, 104
polar, 23
polarization, 123
politics, 108
polymer clusters, 45
predictability, 144
principles, 118, 123
probability, 5, 34, 52, 65, 67, 68, 153
probe, 3, 13, 124
programming, 24
project, 23, 51
propagation, 2, 3, 17, 24, 27, 28, 29, 108
proportionality, 68
protons, 112

Q

QCD, 79, 125, 129, 150
QED, viii, 73, 75, 81, 86, 106
QED beta function, viii, 73
quanta, 121
quantization, 2, 4, 5, 8, 13, 16, 51, 52, 95, 115, 149, 155
quantum cosmology, 126, 131, 146
quantum electrodynamics, 76
quantum entanglement, vii, 33, 34, 35, 47
quantum field theory, viii, 43, 109, 149
quantum fields, 155
quantum fluctuations, 2, 3, 8
quantum gravity, vii, viii, 2, 4, 9, 13, 30, 50, 69, 73, 74, 75, 101, 102, 104, 105, 109, 110, 111, 113, 114, 115, 117, 122, 124, 125, 128, 129, 134, 136, 137, 138, 142, 143, 144, 148, 149, 150, 155
quantum gravity corrections, vii, 2
quantum groups, 101
quantum mechanics, 34, 46, 114, 119, 133, 136, 149
quantum objects, 69
quantum phenomena, 146
quantum state, 34, 102
quantum theory, 2, 4, 42, 51, 68, 102, 109, 132, 149
quarks, 130

R

radiation, 112, 132, 149
radiation damage, 112
radius, viii, 2, 6, 14, 15, 49, 51, 52, 56, 59, 67, 68, 96
random matrices, 152
random walk, 153
realism, 134
reality, 44, 48, 75, 102, 119, 133
reasoning, 50, 51, 68
reception, 108
reciprocity, 102
recurrence, 36, 121
relativity, vii, viii, 1, 2, 3, 18, 22, 30, 33, 46, 73, 74, 102, 104, 105, 107, 108, 109, 111, 113, 114, 116, 117, 118, 119, 120, 122, 123, 124, 125, 131, 133, 136, 143, 144, 145, 146, 147, 153
relevance, 110

renormalization, viii, 73, 81, 82, 83, 84, 85, 87, 100
requirement(s), 10, 23, 53, 88, 120
researchers, 22, 34, 95, 113, 115, 131
residues, 89
resolution, 41, 46, 48, 144, 153
restrictions, 16
Russia, 96, 124

S

safety, 75, 95
saturation, 150
Saturn, 135
scalar field, vii, 1, 3, 4, 11, 12, 18, 20, 21, 24, 25, 26, 27
scaling, 3, 18, 30
scattering, 82, 83, 87, 101, 150, 152
Schwarzschild solution, 23, 96
science, 34, 108, 110, 111, 113, 119, 134, 135, 139, 141, 146
scientific publications, 111
semiconductor, 149
set theory, 95
shortage, 135
SIC, 118
simulation(s), vii, 1, 117, 144
Singapore, 99, 102, 103, 109, 122, 124, 129, 147, 152
society, 111, 112
solar system, 141
solitons, 153
solution, 2, 12, 14, 24, 52, 57, 59, 96, 118
South America, 108
space-time, vii, 1, 34, 35, 41, 43, 45, 46, 47, 88, 100, 102, 104, 109, 114, 115, 117, 120, 123, 143, 152
special relativity, 114, 121, 123, 146
specifications, 13, 112
spectroscopy, 130
speculation, 45
speed of light, 20, 74, 120
spherical collapse, 3, 11, 144
spin, 4, 5, 17, 48, 153, 154
square lattice, 154
stability, 74, 75, 102, 151, 153
stabilization, 48
Standard Model, 75, 100, 125, 132
stars, 135, 142, 143, 144
state(s), 2, 4, 5, 6, 7, 8, 13, 14, 15, 16, 17, 20, 28, 30, 31, 34, 42, 45, 46, 50, 52, 65, 69, 74, 102, 109, 110, 112, 114, 130, 152, 153, 154
statistics, 110, 149, 155
string theory, 115, 118, 124, 151
strong interaction, 150
structure, 14, 81, 84, 96, 112, 115, 123, 130, 131, 137, 154
structure formation, 131
structuring, 44
substitution, 88, 90
superfluidity, 110
supergravity, 118, 129
supersymmetry, 117, 118, 129, 135, 150
Sweden, 99, 145
Switzerland, 129, 150
symmetry, vii, 1, 3, 5, 8, 10, 11, 14, 16, 23, 24, 27, 53, 64, 83, 86, 116

T

tau, 74
techniques, 18, 20, 21
technologies, 123
technology, 45, 112, 129, 146
temperature, 35, 50
territory, 68
thermodynamics, 50, 104, 121, 124
total energy, 34, 53
trajectory, 61, 143
transactions, 103
transformation(s), 4, 5, 9, 15, 120, 148, 152
translation, 137
transparency, 150
treatment, 75, 76, 80, 81, 87
tunneling, 74
twist, vii, 1, 10, 24, 26, 135

U

UNESCO, 147
unification, 75, 103, 107, 151
United Kingdom, 105
universality, 3, 152, 153
universe, viii, 33, 34, 35, 44, 45, 103, 112, 119, 130, 131, 135, 136, 142, 143, 144, 148
USA, 48, 145

V

vacuum, 42, 43, 103, 146, 153
validation, 119
variables, 2, 4, 5, 6, 8, 11, 15, 25, 154
vector, vii, 1, 10, 21, 23, 28
velocity, 21, 121
visualization, 22

W

wave propagation, 29
wavelengths, 42
worldview, 108

Y

Yang-Mills, 99, 100, 101
yield, 35
Yukawa couplings, viii, 73