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Decision Making Under Uncertainty in Electricity Markets

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To Olaia, Mireia and Núria

To Mayte, Toño, Miguel and Toñi

To my parents, Cloti and Pepe

Preface

Most relevant decisions to be made by market agents within an energy market involve a significant level of data *uncertainty*. For agents to make informed decisions within such a context, it is fundamental to model properly the nature and consequences of the uncertainty involved. *Stochastic programming* provides an appropriate mathematical modeling framework both to characterize the uncertainty and to derive informed decisions. Additionally, decision outcomes need to be characterized not only by their expected values but also by their variability levels, thus risk control of outcome volatility is needed and can be achieved using appropriate risk measures.

This book provides a number of stochastic programming models for optimal decision making under uncertainty in electricity markets. Risk control is embedded into most of the presented models, which involve producers, retailers, consumers, and the market/system operator. Particular attention is paid to electric energy systems with a large integration of non-dispatchable sources, such as wind power plants.

The contents of this book are briefly summarized below.

Chapter 1 describes the organization of electricity markets, which encompasses short-term trading in the pool and long-term commerce through the futures market and bilateral contracting. It provides a description of the role and aim of each of the market agents, including producers, retailers, consumers, the market operator, and the independent system operator. From this chapter, it is inferred that most decisions to be made by market agents involve a significant amount of uncertain data.

Chapter 2 provides a summary of stochastic programming techniques that enable informed decision making under a significant level of uncertainty on problem parameters. Both two-stage and multi-stage problems are addressed. Indexes to characterize the advantages of an stochastic programming approach vs. a deterministic one are defined and illustrated.

Chapter 3 describes diverse techniques to characterize via stochastic processes the uncertainty plaguing the data needed for decision making. Specifically, it provides procedures for scenario generation and scenario reduction, being a scenario the actual realization of uncertain parameters along the time. Techniques to take into account spatial correlations among uncertain data are also provided.

Chapter 4 presents and analyzes different risk measures that allow controlling the level of profit variability of producers and retailers, and the level of cost variability of consumers. In other words, it provides techniques to resolve the trade-off between the expected value of the profit/cost and the variability of such a profit/cost.

Chapter 5 provides techniques and procedures to construct offering curves for power producers participating in an electricity pool. Diverse trading floors within the pool are considered, in particular, the day-ahead market, the regulation market, and one adjustment market.

Chapter 6 complements Chapter 5 providing offering curves for producers of an intermittent non-dispatchable nature, such as a wind power producer. It describes strategies to build optimal offering curves for the day-ahead market and the adjustment markets, placing particular emphasis on the effect of the balancing mechanism. Such a mechanism is required to restore energy balance as a result of the intermittent nature of non-dispatchable production units.

Chapter 7 analyzes the involvement of power producers in long-term markets, such as a futures market. It describes how a producer can hedge its profit volatility using futures market products, e.g., forward contracts. It also analyzes the effect on forward contracting of the unexpected failure of any production unit. Specifically, it provides algorithms to select optimally which forward contracts to sign.

Chapter 8 considers an electricity retailer that buys energy through the futures market and in the pool to supply the electricity needs of its clients. It analyzes the complex task of any retailer that needs to offer its clients the lowest possible prices while facing both the volatility of pool prices and the inflexibility of the futures market commitments. It provides algorithms to calculate client prices and to select the forward contracts to be signed.

Chapter 9 addresses the electricity procurement problem faced by a large consumer that may buy energy in the pool at volatile prices or through bilateral contracts. Bilateral contracts are long-term agreements that eliminate price volatility, but prevent taking advantage of particularly low pool prices. This chapter provides algorithms to make informed decisions on contract signing and pool involvement.

Chapter 10 describes and analyzes a multi-commodity market-clearing procedure based on stochastic programming to clear a day-ahead electricity market involving both energy and reserve. The reserve determination is based on an endogenous probabilistic criterion. Equipment failures are taken into account.

Chapter 11 complements Chapter 10 by considering a multi-commodity market-clearing procedure particularly adapted for an electric energy system embedding a large amount of non-dispatchable producers. The uncertainty pertaining to these intermittent producers is comprehensively treated.

It is relevant to note the interrelation of the chapters in this book. Chapter 1 describes the electricity marketplace that allows the interaction of producers, consumers, and retailers under the coordinating role of both the market and the system operators. The producer decision making is analyzed in Chapters 5, 6 and 7. Complementarily, the retailer decision-making process is examined in Chapter 8, while the decision making of the consumer is studied in Chapter 9. Chapters 10 and 11 describe the tools needed by the market/sytem operator to clear pool markets. Providing an appropriate background, Chapters 2, 3 and 4 give a detailed introduction to decision making under uncertainty using stochastic programming.

Finally, Appendix A provides GAMS codes for some of the examples considered throughout the book, Appendix B describes the 24-bus system used in Chapters 10 and 11, and Appendix C provides solutions to selected end-of-chapter exercises.

For both teaching and learning purposes, the material in this book can be arranged in different manners depending on the interest of the reader, as indicated in the following:

1. Chapter 1 is recommended for readers not particularly familiar with *electricity markets* and their features.
2. Chapters 2-4 providing introductory material on stochastic programming and risk analysis are relevant for readers interested in the *mathematical tools* for decision making under uncertainty in electricity markets.
3. Chapters 5-7 on producer strategies for selling energy (and some ancillary services) in both short- and medium-term markets are relevant for readers interested in decision making under uncertainty for *producers.*
4. Chapter 8 on retailer trading strategies in futures markets is relevant for readers interested in decision making under uncertainty for *retailers.*
5. Chapter 9 on strategies for energy procurement by consumers in a medium-term time frame is relevant for readers interested in decision making under uncertainty for *consumers.*
6. Chapters 10 and 11 providing background and models to develop market-clearing tools to be used by market and system operators are relevant for readers interested in *market-clearing algorithms.*
7. Chapters 5, 6, 10 and 11 on short-term decision making problems involving the pool and pertaining to all market agents are relevant for readers interested in decision making in the *pool.*
8. Chapters 7-9 on medium-term problems involving futures markets and pertaining to all market agents are relevant for readers interested in decision making in *futures markets.*

A course on decision making for producers may include Chapters 1-4 and Chapters 5-7, for retailers Chapters 1-4 and Chapter 8, and for consumers Chapters 1-4 and Chapter 9. A course on market-clearing algorithms may comprise Chapters 1-4 and Chapters 10 and 11. A course on short-term decision making (pool involvement) may include Chapters 1-4 and Chapters 5, 6, 10 and 11, and on medium-term decision making (futures market involvement) may encompass Chapters 1-4 and Chapters 7-9.

The exposition of the material in this book relies on many elaborated examples that enhance its clarity and facilitates the always hard learning process. Theoretical developments and formulations are motivated by these examples of illustrative, clarifying, or computational nature.

Due to the widespread implementation of electricity markets worldwide, we feel that a book like this one is much needed within the energy engineering and economics communities. It may help graduate students and practitioners to learn the fundamentals of decision making under uncertainty in energy markets.

We are grateful to a number of colleagues that have provided insightful comments and inspiring observations for the different chapters of this book. Particularly, we are grateful to Prof. Baldick from the University of Texas at Austin, US; Prof. Bouffard from the University of Manchester, UK; Dr. Barroso from PSR, Inc., Brazil; Prof. Fleten from the Norwegian University of Science and Technology, Norway; Prof. Kirschen from the University of Manchester, UK; Prof. Rosehart from the University of Calgary, Canada; Prof. Shahidehpour from the Illinois Institute of Technology, US; and Prof. Schultz from the University of Duisburg-Essen, Germany.

We are particularly thankful to Antonio Canoyra, Ángel Caballero and Antonio de Andrés from Unión Fenosa Generación for many years of fruitful industry-university collaboration and for providing us with insightful comments, relevant observations, and pertinent suggestions throughout these years. We also thank our colleagues and students interested in decision making and electricity markets for providing us with a stimulating and creative interaction.

We are grateful to Camille Price for encouraging us to write this book and to Fred Hillier for supporting this endeavor. Neil Levine from Springer, New York, has been most helpful with all practical matters.

We are thankful to the Government of Castilla-La Mancha, Spain, for providing us with an outstanding research environment.

We have dedicated a significant amount of resources, time and enthusiasm to create this book in the hope that it will be useful for the next generation of graduate students and practitioners interested in the fascinating topic of decision making under uncertainty in electricity markets.

Ciudad Real and Toledo, Spain
June 2010.

A. J. Conejo
M. Carrión
J. M. Morales

Contents

Chapter 1
Electricity Markets

1.1 Introduction

This chapter provides an overview of the organization and agents of a typical fully-fledged electricity market, and outlines the decision-making problems faced by these agents. The time framework and the uncertainty of these decision-making processes are also discussed.

This chapter is intended to provide the basic framework required to understand the context in which the decision-making problems addressed in other chapters of the book take place.

For further information, relevant references analyzing electricity markets, their organization and agents, include [49, 88, 133, 135, 138].

This chapter is organized as follows. Section 1.2 provides a general description of the most common electricity market organization and the roles of the market agents. Section 1.3 presents the time frameworks for market clearing and decision making, and provides some insight into the sources of uncertainty involved. Section 1.4 outlines the problems faced by the different market agents. Finally, Section 1.5 concludes summarizing the chapter.

1.2 Organization and Agents

Over the last decades, the electric energy industry has evolved from a centralized operational paradigm to a competitive one in many countries all over the world. In 1996, the Federal Energy Regulatory Commission (FERC) [50] enacted *Order 888*, a legal framework to increase competition in the US wholesale electricity markets by promoting open access to transmission networks. In the same year, the European Union sponsored Directive 96/92/EC [48], aimed at the liberalization of the purchase of energy by *qualified* consumers in

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member states to establish the basic rules for a seminal European electricity market.

This new competitive framework is intended to promote an increase in the operational efficiency of power systems while guaranteeing an acceptable quality of the electricity supply and achieving minimum cost for electricity end users. In addition, it is aimed to provide better incentives for capital formation, better incentives for consumers to not consume when costs exceed their benefits, and better incentives for research and development.

This restructuring process has enabled the liberalization of the electricity sector and the emergence of electricity markets worldwide. The pioneering liberalization process corresponds to what took place in Chile in 1982. In the early 1990s, the electricity pool created in England and Wales was the first such an experience in Europe. In the US, in 1998, the California Independent System Operator established a pool-based electricity market, which dramatically failed in 2000 due to design flaws. US East Coast markets including PJM, ISO New England, and New York ISO started operation in 1997, 1999 and 1999, respectively, and have been successfully operating since them. The electricity markets in New Zealand and Australia started operating in 1996 and 1998, respectively.

1.2.1 Market Organization

Two different trading arenas are usually available to facilitate energy commerce between producers and consumers and are called pools and futures markets.

The pool is a marketplace where the energy is traded on a short-term basis. It typically includes:

1. A day-ahead market.
2. Several adjustment markets (not in US markets).
3. Balancing markets (also called real-time markets).

These markets are described in Subsection 1.2.3 below.

The day-ahead and adjustment markets cover generally the bulk of energy transactions within a day. The adjustment markets are similar to the day-ahead market but are cleared closer to power delivery and may cover a shorter trading horizon. Complementarily, the balancing market allows the short-term covering of dispatched power that does not materialize due to equipment failures or the intermittent nature of some sources (e.g., wind or solar-thermal power plants). It also allows covering load deviations and sometimes deals with transmission constraints.

On the other hand, the futures market allows electricity trading on a medium- or long-term horizon by means of purchases and sales of standard products, called derivatives or derivative products. This market is described in Subsection 1.2.4 below.

There also exists the possibility of signing bilateral contracts between suppliers and consumers. A bilateral contract is a free arrangement between a supplier and a consumer defined outside an organized marketplace.

Fig. 1.1 illustrates how bilateral contracts take place. Producers sign bilateral contracts with consumers or retailers. Consumers buy energy for their own consumption, while retailers buy energy to supply their clients' demands. Consumers deal directly with producers while retailers' clients deal with producers through their retailers. The arrows in this figure indicate the flow of energy.

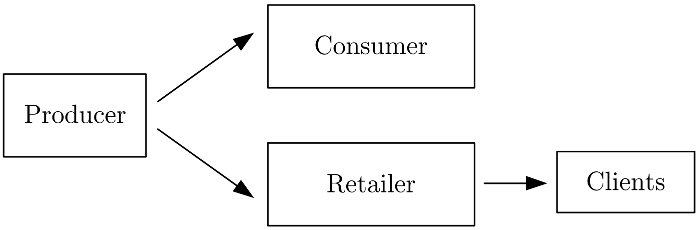


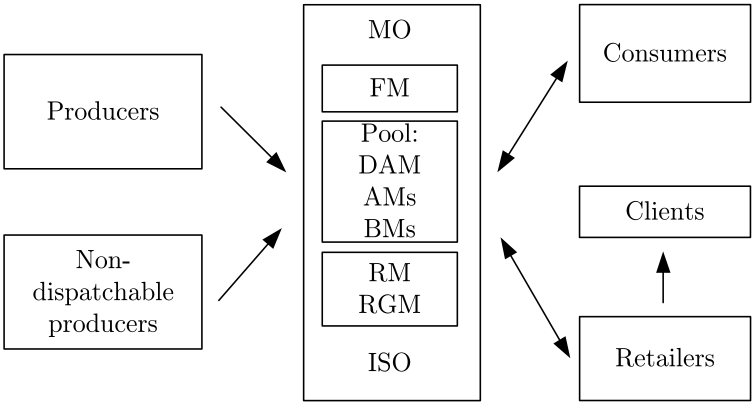
Fig. 1.1 Bilateral contracting of electricity

Other markets are also needed to ensure the secure system operation and energy delivery, namely, reserve and regulation markets.

The reserve market, cleared once a day, provides standby power (spinning and non-spinning) to cover the failure of facilities in operation (production units or transmission lines), large fluctuations of demand, and the intermittent energy generation from non-dispatchable sources, such as wind farms and solar-thermal production facilities. In most US market (e.g., MISO, `www.midwestiso.org`), energy and reserve are co-optimized by using a single clearing procedure involving both energy and reserve. This criterion is used in Chapters 10 and 11.

The regulation (automatic generation control, AGC) market provides up and down real-time load-following capability to enforce continuously the balance between production and consumption (and to keep fixed the system frequency), a hard technical requirement of electric energy systems. The regulation market is typically cleared once a day on an hourly basis and assigns to production units the power bands to be used in real-time operation for load following.

Fig. 1.2 illustrates the organization of a fully-fledged electricity marketplace, including the futures market and the pool, which are energy markets, and the reserve and regulation markets, which are markets to acquire capacity commitments. Generally, market operators (MO) clear the futures market and the pool, while the independent system operator (ISO) clears the reserve and the regulation markets. In most US markets, futures markets are managed and cleared by for-profit entities such as APX (www.apx.com) or ICE (www.theice.com). Producers sell energy, reserve and regulation, while consumers and retailers buy energy and may sell reserve as well. Arrows indicate the flows of energy, reserve power, and balancing energy.



FM: futures market, DAM: day-ahead market, AMs: adjustment markets, BMs: balancing markets, RM: reserve market, RGM: regulation market.

Fig. 1.2 Electricity marketplace

Services required for the appropriate functioning of the electric energy system and not provided generally via markets include reactive power management and voltage control, system restoration after a blackout, etc. These services are not studied in this book.

1.2.2 Agents

Agents participating in electricity markets are briefly described below. These market agents include consumers, retailers, producers, and non-dispatchable producers:

1. Consumers. They are the end users of the electricity. They may purchase energy in the pool or in the futures market, or may sign bilateral contracts with producers or be supplied by retailers. A consumer aims to either minimize its procurement cost or to maximize the utility it obtains from electricity usage.

Additionally, a consumer may participate in the reserve market if it is willing to change its consumption within pre-specified limits at the command of the independent system operator.

A consumer may need to participate in the balancing market if its consumption pattern deviates from that settled in the pool.

2. Retailers. They provide electricity to those consumers that do not participate directly in the electricity markets. Retailers do not generally own production units and they purchase the electricity to be supplied to their clients through bilateral contracts, in the futures market, and in the pool. The objective of a retailer is to maximize the profit it obtains from selling to its costumers. Its profit margin is generally narrow as it should buy as cheap as possible to provide its clients with the lowest possible prices; otherwise these clients may change retailer. Marketers play the same role as retailers but may also intermediate between producers and retailers.
3. Producers. They are the entities owning the production units that are in charge of the electricity generation. A producer may sell electric energy either to the electricity markets (pool and futures market) or directly to the consumers and the retailers through bilateral contracts. The objective of a producer is to attain maximum profit from the sale of electricity and eventually reserve and regulation.

   A producer may participate in both the reserve and the regulation markets, providing, respectively, reserve power and load following capacity within pre-specified power bounds.

   If beneficial, a producer may also participate in the balancing market to cover the excess/deficit of generation/demand.
4. Non-dispatchable producers. They are producers with non-dispatchable sources, such as wind or solar-thermal power plants. All market agents must cope with the intermittency and time-dependent nature of non-dispatchable sources. A non-dispatchable producer strives to maximize the profit from selling in the pool the energy it produces in an intermittent manner.

   A non-dispatchable producer needs to participate in the balancing market to cover its deviations from the production pattern settled in the pool.

The illustrative example below describes a variety of market agents.

Illustrative Example 1.1 (Market agents).

Typical market agents are:

1. An industrial consumer with a demand varying from 200 to 500 MW, depending on the time of the day.
2. A retailer serving an aggregate client load varying from 1000 to 2000 MW, depending on the time of the day.
3. A producer owning 3 combined cycle gas turbines (CCGT) of 200 MW each.

4. A hydro-thermal producer owning a coal power plant of 300 MW and a hydroelectric development involving 3 hydro power plants of 100, 150 and 200 MW, respectively.
5. A wind producer owning a wind farm including 50 wind turbines of 2 MW each.

□

Institutional market agents include the Market Operator (MO), the Independent System Operator (ISO), and the Regulator:

1. Market Operator (MO). It is generally a nonprofit entity responsible for the economic management of the marketplace as a whole. In addition, the market operator administers the market rules and determines the prices and quantities of energy traded in the market. In some cases, the MO is a for-profit regulated entity.
2. Independent System Operator (ISO). It is a nonprofit entity in charge of the technical management of the electric energy system pertaining to the electricity market. The independent system operator should provide equal access to the grid to all consumers, retailers and producers, and strive to facilitate the commerce among buying and selling agents.
   The independent system operator manages generally the reserve and the regulation markets, and assists the market operator to clear the balancing market.
3. Market Regulator. It is a government-independent entity whose function is to oversee the market and to ensure its competitive and adequate functioning. Additionally, the regulator promotes and enforces orders and regulations.

In some markets, such as the PJM Interconnection and ISO New England, the functions performed by the independent system operator and the market operator are carried out by a single entity. In this case, the independent system operator is in charge of both the technical control of the system and the economic management of the market. However, futures markets are managed by independent for-profit entities, e.g., APX or ICE.

1.2.3 Pool

The pool comprises a day-ahead market and several shorter-term markets known as adjustment markets. It also includes the balancing market that ensures the real-time balance between supply and demand.

The pool organization and its functioning are illustrated in Fig. 1.3. Producers submit production offers while consumers and retailers submit consumption bids to the day-ahead, adjustment, and balancing markets, and in turn, the market operator clears these markets and determines prices and traded quantities. Thin arrows indicate the flows of offers and bids, while the thick arrow indicates market outcomes.

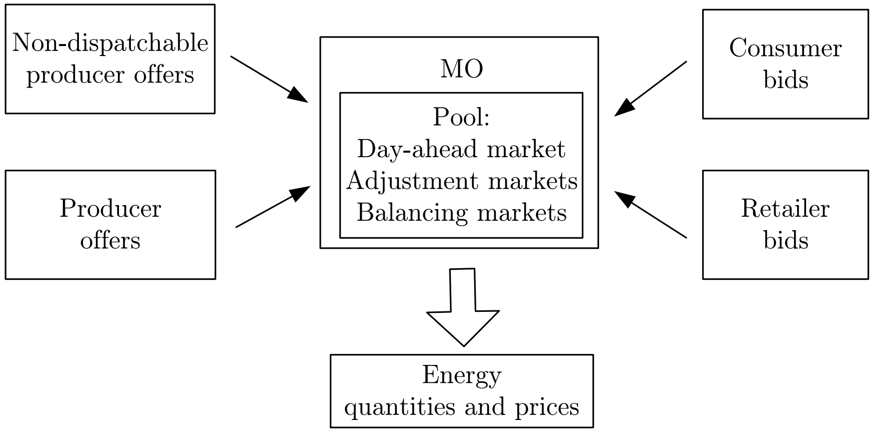


Fig. 1.3 Pool organization and functioning

The energy traded in the pool is mostly negotiated in the day-ahead market, while adjustment markets are used to make *adjustments* to the energy cleared in the day-ahead market. The balancing market allows *last minute* energy adjustments.

Since trading mechanisms closer in time to the power delivery allows a higher accuracy on actual power production forecasts by intermittent sources, non-dispatchable producers tend to rely more on adjustment markets than conventional producers.

In the day-ahead and adjustment markets, producers submit energy blocks and their corresponding minimum selling prices for every hour of the market horizon and every production unit. At the same time, retailers and consumers submit energy blocks and their corresponding maximum buying prices for every hour of the market horizon.

The market operator collects purchase bids and sale offers, and clears the market (both day-ahead and adjustment) using a market-clearing procedure. Market-clearing procedures are comprehensively analyzed in Chapters 10 and 11. A market-clearing procedure results in market-clearing prices, as well as production and consumption schedules. If the transmission grid is not considered in the market-clearing procedure, the resulting market-clearing price is identical for all market agents. On the other hand, if the transmission network is taken into account for clearing the market, instead of a single market-clearing price, a *locational marginal price* (LMP) is associated with each node of the power system. LMPs differ across nodes due to line losses and line congestion. If a transmission line is congested, more expensive generation is needed to be dispatched on the *downstream* side of the congested line. This increase in expensive generation yields an increase in the market-clearing prices in those nodes placed on the *downstream* side of the congested line.

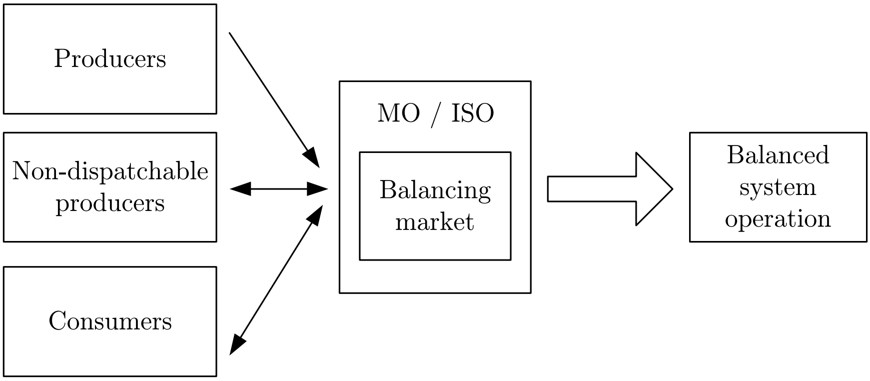


Fig. 1.4 Organization and functioning of the balancing market

The balancing (or real-time) market, cleared on a hourly basis (or several times within each hour) through an auction, provides energy to cover both generation excess and deficit, and constitutes the last market prior to power delivery to balance production and consumption. Producers/consumers submit balancing offers that are accepted by the market operator on an increasing price basis until balance is guaranteed in the case of deficit of generation. Alternatively, for the case of excess of generation, offers to reduce production are accepted on a decreasing price basis until balance is ensured.

Fig. 1.4 illustrates the organization and functioning of the balancing market. Producers participate providing balancing (up and down) energy, while non-dispatchable producers and consumers use this market to self-balance their energy productions and consumptions, respectively, to those values agreed in previous pool markets. Retailers, which behave as consumers, are not represented in this figure for the sake of simplicity. The balancing market ensures a balanced system operation. In Fig. 1.4, thin arrows indicate the flows of balancing energy, while the thick arrow indicates market outcomes.

Producers and consumers participate in this market by changing their respective production and consumption dispatches if profitable. Consumers may need to participate in the balancing market if they cannot control their consumption patterns and these patterns deviate from previous pool agreements. Non-dispatchable producers do need to participate in the balancing market due to the intermittent and uncertain nature of their production sources, which makes generally impossible to comply with a previously agreed production pattern.

Finally, it is relevant to note that in some pool-based markets, the time span is divided into periods shorter than one hour. For example, in the New Zealand electricity market, offers are submitted on a 30-minute basis.

In most electricity markets, the main characteristics of pool prices are: non-stationary mean and variance, multiple seasonality, calendar effect, high volatility, and high percentage of outliers [27,30]. Due to these characteristics, pool prices are hard to forecast. However, information about future pool prices is crucial for market agents to bid in the pool and to trade in the futures market.

Electricity pools in Europe include:

1. The Electricity Pool in the UK (www.apxgroup.com),
2. Nordpool in Scandinavia (www.nordpool.no), and
3. OMEL in the Iberian Penninsula (www.omel.es).

Pools in the US include:

1. ISO New England (www.iso-ne.com) and
2. PJM Interconnection (www.pjm.com).

The example below illustrates the organization of the pool, its markets and its functioning.

Illustrative Example 1.2 (Pool).

A typical pool includes the following markets:

1. A day-ahead market for day d cleared day d−1 at 10am.
2. Three adjustment markets involving energy transactions for day d and cleared day d−1 at 3pm, 7pm, and 11pm, respectively.
3. Three additional adjustment markets involving energy transactions for a part of day d and cleared the same day d at 3am, 7am, and 11am, respectively.
4. A balancing market for each hour of day d, which is cleared ten minutes in advance.

□

1.2.4 Futures Market

A futures market is an auction market in which participants buy and sell physical or financial products for delivery on a specified future date. These products are called derivatives or derivative products [36]. The most salient feature of futures markets is that they allow trading physical or financial products in the future at today prices. Thus, futures markets are useful if the price of electricity is highly uncertain in the pool, which is the case in pool-based electricity markets.

Pool prices (day-ahead, adjustment, and balancing) exhibit a set of characteristics such as high volatility, high percentage of outliers, etc., and such characteristics make pool prices highly uncertain. Uncertainty in the pool is undesirable since it is the main cause of volatility of profits or costs achieved by the agents participating in this market. Within this scene, electricity futures markets emerge as a tool to hedge against pool price uncertainty.

Futures markets with electricity derivatives in Europe include:

1. Nordpool in Scandinavia (www.nordpool.no),
2. EEX in Germany (www.eex.de), and
3. OMIP in the Iberian Peninsula (www.omip.pt).

Nordpool, EEX, and OMIP were launched in 1993, 2001, and 2006, respectively.

The derivative electricity products of ISO New England and PJM markets are traded at the New York Mercantile Exchange (NYMEX, www.nymex.com). NYMEX trading for PJM and ISO New England started in 2003 and 2004, respectively.

In Australia, the exchange group ASX (www.asx.com.au) trades with electricity derivatives.

In summary, futures markets provide derivative products (financial and physical) that span from one week to several years and allow consumers, retailers, and producers to hedge against the financial risk inherent to pool prices.

Products available in the futures market include, among others, forward contracts and options:

1. A forward contract is an agreement of delivering (consuming) a specified amount of energy in a future time period at a fixed price.
2. An option is an agreement for having the choice of delivering (consuming) a specified amount of energy in a future time period. Signing an option agreement involves a payment, denominated premium, regardless of whether or not energy is eventually delivered (consumed).

Examples of derivative products are given below. *Base* indicate all hours of the time span of the corresponding derivative product, while *peak* indicate peak hours, typically from 8am to 7pm, of the derivative time span.

1. Base and peak weekly forward contracts.
2. Base and peak monthly forward contracts.
3. Base and peak quarterly forward contracts.
4. Base and peak yearly forward contracts.
5. Options over the above products.

The futures market organization and functioning are illustrated in Fig. 1.5. A producer often uses this market to sell part of its production at stable prices, and conversely, consumers and retailers typically use this market to buy energy at stable prices. Thin arrows indicate the flows of offers and bids, while the thick arrow indicates market outcomes.

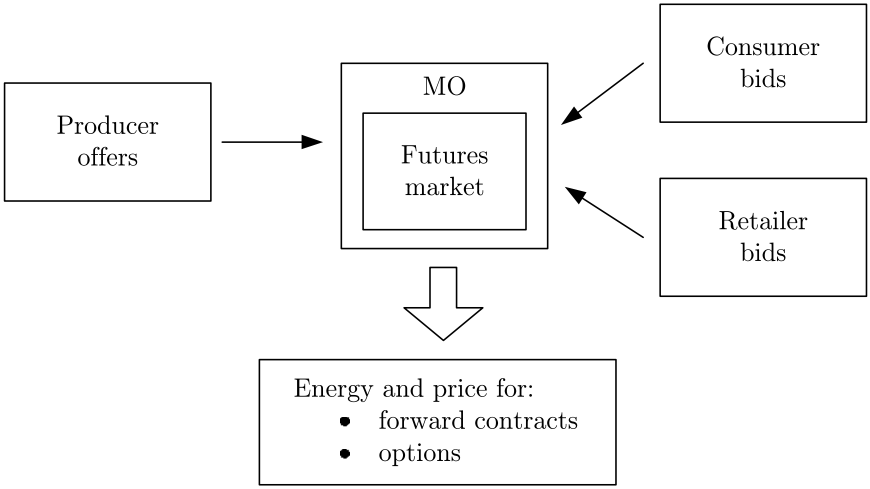


Fig. 1.5 Futures market organization and functioning

1.2.5 Reserve and Regulation Markets

Electricity markets are multi-commodity markets including at least four products: energy, reserve, regulation (load following capability), and balancing energy. Energy is the main product as assumed and explained in the previous sections of this chapter.

However, the reserve is an important product that guarantees that enough back-up generation is available in case of equipment failure, drastic fluctuations of production from intermittent sources, and sudden demand changes. The reserve market is cleared either jointly with the day-ahead market or immediately following it by the independent system operator. It is cleared using an auction algorithm with complexity varying from market to market.

Fig. 1.6 illustrates the organization and functioning of the reserve market. Mostly, producers provide reserve, but consumers/retailers may provide up/down reserve by reducing/increasing their consumptions. This market ensures a secure short-term system operation in terms of reserve availability. Thin arrows indicate the flows of reserve power (and offers), while the thick arrow indicates market outcomes. In some markets energy and reserve are co-optimized, i.e., they are cleared simultaneously. This criterion is used in Chapters 10 and 11.

The regulation market, cleared several hours prior to power delivery, allocates load following bands among production units with capability to provide this service and interest in providing it. Power bands are allocated based on

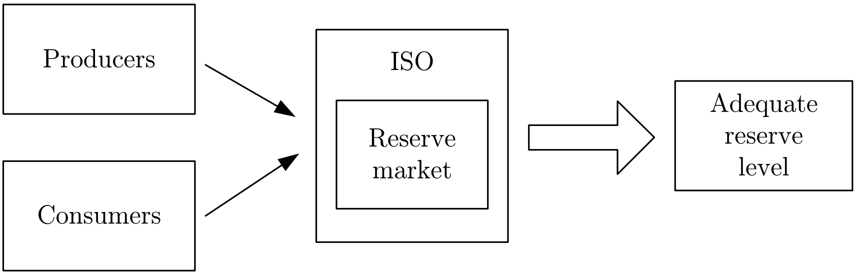


Fig. 1.6 Organization and functioning of the reserve market

an auction following an increasing price rule until enough regulating power is attained.

Fig. 1.7 illustrates the organization and functioning of the regulation market. Load following capability is provided by selected generation units that can and will. This market ensures that the system frequency is maintained within a narrow band. Thin arrows indicate the flows of regulating power (and offers), while the thick arrow indicates market outcomes.

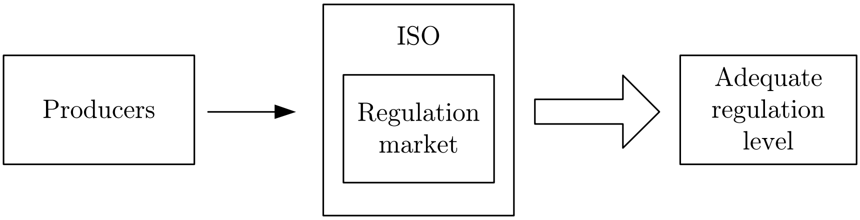


Fig. 1.7 Organization and functioning of the regulation market

The example below illustrates how reserve and regulation markets work.

Illustrative Example 1.3 (Reserve and regulation markets).

The reserve market allows allocating to each one of a set of producers (those willing to contribute to the reserve service) a power quantity to be realized at the command of the ISO. For instance, it may allocate 20 MW of reserve power to a production unit from 10pm to 11pm, which means that this production unit should be ready for actually producing these 20 MW at any time during that hour. The producer receives a payment for this service equal to 20 times the resulting reserve price.

The regulation market allows allocating to each one of a set of producers (those with the appropriate capability and willingness) power bands for load following. For instance, it may allocate 30 MW of regulation power from

6pm to 7pm to a particular unit, which means that the independent system operator seize control of that unit from 6pm to 7pm and can change its operating point $\pm(30/2)$ MW. The producer receives a payment for this service equal to 30 times the resulting regulation price. In some markets, regulating-up and regulating-down reserves are different products, which are traded independently. □

Since this book considers short- and medium-term horizons spanning from real time to one year, no *capacity markets* are considered. These important markets are intended to ensure that sufficient capacity (production and transmission) is added to the system so that the market can operate free of rationing due to production scarcity or network bottlenecks, i.e., so that conditions of demand higher than available supply capacity do not occur.

1.3 Time Framework and Uncertainty

1.3.1 Decision Sequence

The time framework of the different trading floors are described below, including the futures market, the pool, and the reserve and regulation markets.

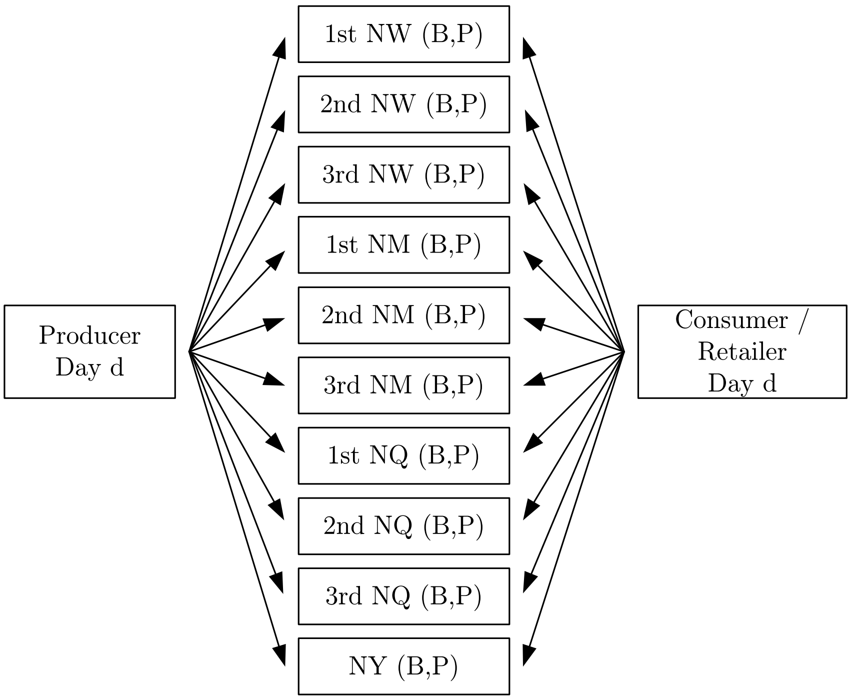
The futures market allows, at any given time and at a given price, selling or buying a quantity of energy during a future period (forward contract) spanning from one week to several years. The futures market also allows trading the option of selling or buying a quantity of energy during a future period (option) covering from one week to several years.

Fig. 1.8 illustrates the time framework of the futures market. As illustrated in this figure, this market allows a producer to sell energy today to be delivered throughout a future time period, and it allows a consumer/retailer to buy energy today to be supplied during a future time period. Arrows indicate the flows of offers and bids.

On a daily basis, market agents trade in the pool and in the reserve and regulation markets.

The day-ahead market pertaining to day d is typically cleared before noon of the previous day, day d$-$1. Adjustment markets for day d are cleared every few hours (e.g., every four hours) once the day-ahead market has been cleared. Reserve and regulation markets are typically cleared once the day-ahead market is closed. The balancing market is cleared just minutes before the actual power delivery by the producers.

Fig. 1.9 illustrates the day-ahead and adjustment market-clearing times, as well as clearing times of the reserve, regulation, and balancing markets. The day-ahead, reserve, and regulation markets are cleared on day d$-$1, while



NW: next week, NM: next month, NQ: next quarter, NY: next year, B: base product, P: peak product.

Fig. 1.8 Futures market time framework

the balancing market is cleared on a hourly basis throughout day d. The adjustment markets may be organized on both day d and day d−1.

Market agents trade in these markets following their respective clearing sequences. The example below clarifies the decision time framework faced by market agents.

Illustrative Example 1.4 (Time framework).

A typical decision framework for consumers/retailers is as follows:

1. On a yearly/quarterly/monthly/weekly basis the considered consumer or retailer decides on both forward contracts and options spanning one year/quarter/month/week to be negotiated in the futures market.
2. Bilateral contracting may entail any time framework, but typical time frameworks include from one week to one year.
3. Every day the consumer or retailer decides its bidding in the day-ahead, adjustment and balancing markets (pool involvement).

A typical decision framework for a power producer is as follows:

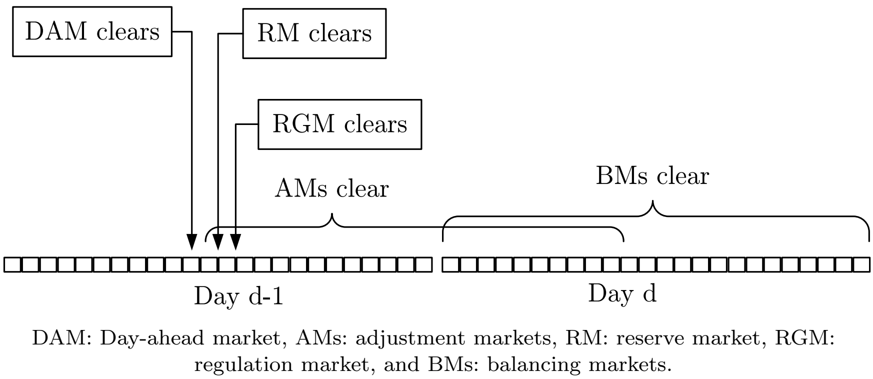


Fig. 1.9 Clearing sequence for daily markets

1. On a yearly/quarterly/monthly/weekly basis the producer decides on forward contracts and options spanning one year/quarter/month/week to be negotiated in the futures market.
2. Bilateral contracting may entail any time framework, but typical time frameworks include from one week to one year.
3. Every day the producer decides its offering strategies in the day-ahead market, the adjustment markets, and the balancing markets (pool involvement).
4. Every day the producer decides its possible involvement in the reserve and the regulation markets.

□

1.3.2 Uncertainty

Decision-making problems in electricity markets are plagued with uncertainty, which affects price, demand, intermittent production, equipment availability, etc. These problems include producer offering in pool markets, energy procurements for consumers and retailers, futures market trading for producers and consumers, etc.

Stochastic programming provides an adequate modeling framework in which problems of decision making under uncertainty are properly formulated [14]. Within an electricity market framework, the fundamentals of stochastic programming are provided in Chapter 2. Stochastic programming relies on the knowledge of the stochastic processes describing the uncertain parameters, e.g., the pool price. Once the uncertain parameters are modeled using stochastic processes, it is possible to formulate a mathematical programming problem that takes into account the uncertainty of these parameters. For modeling purposes, each uncertain parameter is modeled by a set of finite outcomes or scenarios, where each outcome represents a plausible realization of the uncertain parameters with an associated probability of occurrence.

The number of outcomes needed to properly represent an uncertain parameter is usually very large. Traditionally, scenario-reduction techniques are used to reduce the cardinality of the set of outcomes while retaining the statistical properties of the uncertain parameters.

Scenario-generation and scenario-reduction procedures are studied in Chapter 3.

In addition to maximizing profit or minimizing cost, market agents may wish to control the risk of profit/cost variability. Thus, it is appropriate to explicitly model the risk associated with the decisions made by the market agents. This can be done by including risk control in the stochastic programming problem through risk measures. Risk management is used by the decision makers to avoid implementing strategies that entail the possibility of low profits or high costs. However, risk reduction results in a lower expected profit or a higher expected cost with respect to the risk-neutral solution. Therefore, the decision maker faces a tradeoff between expected profit/cost and risk aversion.

Risk measures and risk management are studied in Chapter 4.

In electricity markets, specifically, uncertainty sources include:

1. Availability of production units and network components.
2. Power production for non-dispatchable producers, such as wind and solar-thermal power plants.
3. Day-ahead energy prices, reserve market prices, prices for all the adjustment markets, and balancing market prices.
4. Client demand for retailers.
5. Own demand for consumers

The market decision sequence and the uncertainty involved is schematically described below:

1. At the time of trading in the futures market, producers, consumers, and retailers need an appropriate description (typically via scenarios) of the stochastic processes constituted by the energy prices in the pool (mostly the day-ahead prices). That description needs to span the time horizon of the targeted futures products: forward contracts or options.
2. At the time of trading in both the day-ahead and the reserve markets, producers, consumers, and retailers need an appropriate description of the stochastic processes constituted by next-day energy and reserve prices.
3. For trading in the regulation market, producers need an adequate description of the stochastic process constituted by next-day regulation prices.
4. For trading in adjustment markets, prices for the time span of the corresponding market horizon need to be properly characterized.

5. Finally, balancing market prices need to be adequately described to asses the economic impacts of deviations that affect particularly non-dispatchable producers, consumers, and retailers.

The example below illustrates the uncertainty influencing decision making within electricity markets.

Illustrative Example 1.5 (Uncertainty).

Several examples of uncertainty affecting decision making are described below:

1. A consumer procuring energy for next year through a bilateral contract needs to model the uncertainty of the pool prices during the year spanned by the considered bilateral contract.
2. A retailer procuring energy through a bilateral contract and the pool (day-ahead market) needs to model the uncertainty of the pool prices during the year spanned by the considered bilateral contract, as well as the uncertainty related to client demands during such a year.
3. A producer participating in the futures market and targeting a monthly forward contract needs to model the uncertainty related to pool prices during the month spanned by the considered forward contract.
4. A producer offering in the day-ahead market needs to consider the uncertainty pertaining to tomorrow's day-ahead, adjustment, and balancing market prices. If this producer also plans to participate in the regulation and reserve markets, the price uncertainty in these markets needs to be accounted for as well.

□

1.4 Decision Making

This book provides decision-making tools for consumers, retailers, and producers that allow making informed decisions within a short- and medium-term planning horizon while explicitly considering uncertain prices, demands and productions. It provides also dispatching tools for the market operator considering uncertainty in equipment availability and intermittent sources. Specifically, the decision-making problems tackled in this book are summarized below.

1.4.1 Consumer

We consider a large consumer of electricity that seeks to select the bilateral contracts to sign in order to minimize its total expected procurement cost.

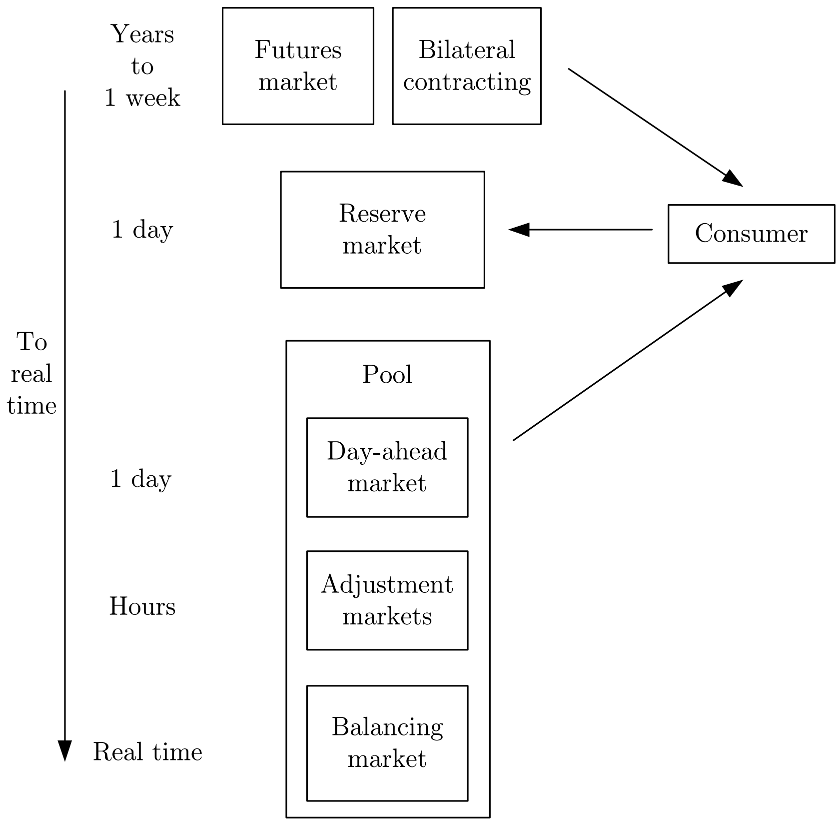


Fig. 1.10 Consumer decision-making problems

This problem is analyzed in Chapter 9. No futures-market involvement by consumers is explicitly considered in this book; however, models for decision making in futures markets are similar to those needed for bilateral contracting.

Once bilateral contracting and futures-market decisions are made, the consumer decides its involvement in the pool on a daily basis. Pool prices are considered uncertain and independent of the consumer actions.

The consumer may also participate in the reserve market, provided that it is willing to make a power block for reserve available. Models for optimal decision making in the pool by consumers are not explicitly tackled in this book. However, these models are similar to those designed for producers, but simpler. Producer models for optimal pool involvement are treated in Chapter 5.

Fig. 1.10 illustrates the decision-making problem of a consumer to procure the electrical energy it needs. As indicated in this figure, the consumer can buy through bilateral contracts, the futures market, and the pool, and may eventually sell reserve. Right-hand-side arrows indicate the interaction of the consumer with the market: buying energy and selling reserve.

The example below illustrates the decision-making problem faced by a consumer.

Illustrative Example 1.6 (Decision making: consumer).

At the beginning of February, a consumer may buy 50 MW during all peak hours (8am to 7pm) of the last week of February through a forward buying contract. To make that decision, it has to solve a complex optimization problem similar to that described in Chapter 9.

This consumer may also decide during the current year to subscribe an option (and pay the corresponding premium) to have the choice of eventually buying 300 MW during all the peak hours of next year. Again, to make this decision, it has to solve a complex optimization problem similar to that described in Chapter 9.

Additionally, this consumer needs to get involved in the pool (day-ahead, adjustment, and balancing markets) to fully supply its energy needs. To get involved in the pool, it has to solve a complex optimization problem similar to those described in Chapters 5 and 6 for producers.

It may also get involved in the reserve market provided it has the capability of reducing/increasing consumption as required by the ISO. □

1.4.2 Retailer

We consider a retailer that has to determine its forward contract portfolio and the selling price to be offered to its clients. The retailer must cope with uncertain pool prices and client demands, as well as the possibility that clients might choose a different supplier if the selling price offered by the retailer is not sufficiently competitive. This problem is analyzed in Chapter 8. A retailer may also consider signing bilateral contracts to procure its energy needs.

After deciding on bilateral contracting and futures market involvement, and selecting the selling price to its clients, the retailer must determine its purchases and sales in the pool. To get involved in the pool, it has to solve a complex optimization problem similar to those described in Chapters 5 and 6 for producers.

Fig. 1.11 illustrates the decision-making problem of a retailer to procure the electrical energy it needs to supply its clients. The retailer buys energy through bilateral contracts, the futures market, and the pool. Right-hand-side arrows indicate the interactions of the retailer: buying energy in the market and selling energy to clients.

The example below illustrates the decision-making problem faced by a retailer.

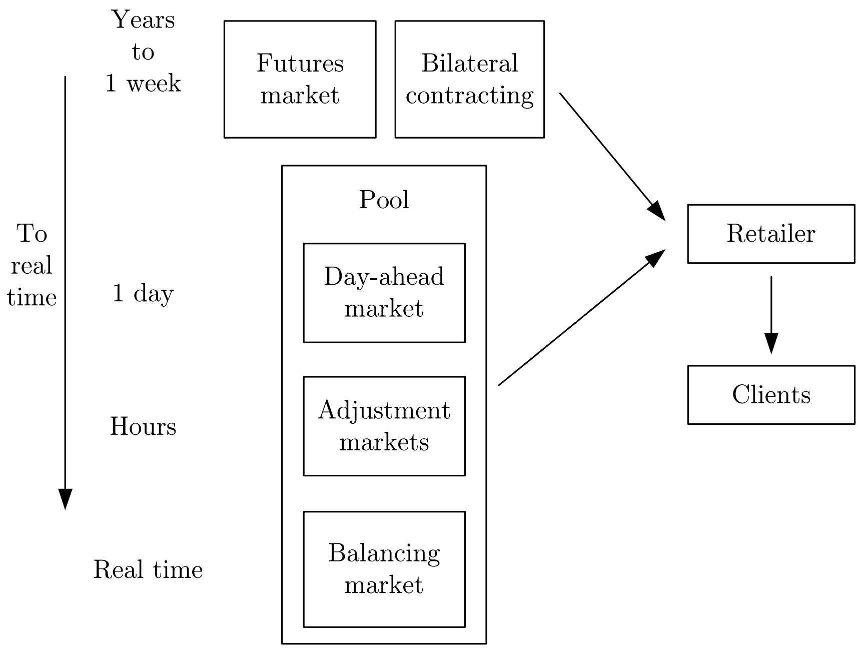


Fig. 1.11 Retailer decision-making problems

Illustrative Example 1.7 (Decision making: retailer).

At the beginning of February, a retailer may buy 50 MW during all peak hours (8am to 7pm) of the last week of February through a forward buying contract. This retailer may also decide during the current year to subscribe an option (and pay the corresponding premium) to have the choice of eventually buying 300 MW during all the peak hours of next year. To make these decisions, the retailer has to solve a complex optimization problem similar to that described in Chapter 8.

Additionally, this retailer needs to get involved in the pool (day-ahead, adjustment, and balancing markets) to fully supply the energy that its clients demand. To get involved in the pool, it has to solve a complex optimization problem similar to those described in Chapters 5 and 6 for producers. □

1.4.3 Producer

We consider a producer that seeks to determine its forward contract portfolio considering uncertain future pool prices. This problem is analyzed in Chapter

5. Bilateral contracting requires solving a similar problem to that described in Chapter 9.

Once futures market positions are determined, the producer must decide how to operate its production units and its involvement in the pool on a daily basis, as well as in the reserve and regulation markets. Pool and regulation market involvement by producers is analyzed in Chapter 5.

Fig. 1.12 illustrates the decision-making problem of a producer to sell its energy production. It may sell energy through bilateral contracts, the futures market, and the pool. Additionally, it may sell reserve and regulation power. Left-hand-side arrows indicate the interactions of the producer with the market: selling energy, reserve power, regulating power, and balancing energy.

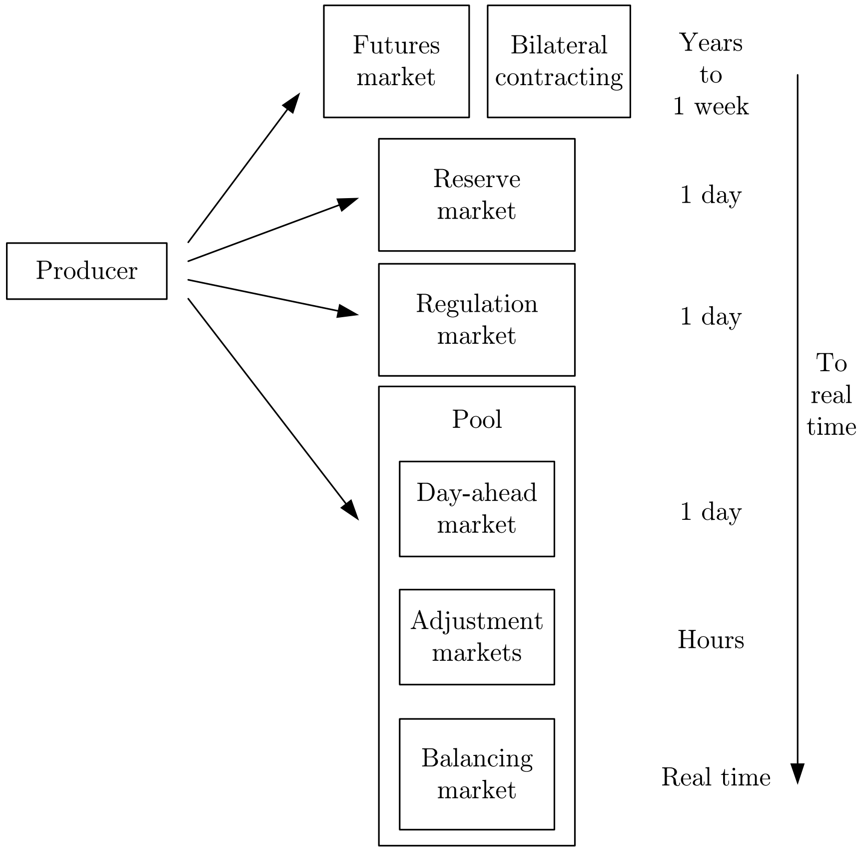


Fig. 1.12 Producer decision-making problems

The example below illustrates the decision-making problem faced by a producer.

Illustrative Example 1.8 (Decision making: producer).

At the beginning of January, a producer may sell 100 MW during all non-peak hours (8 pm to 7am) of February through a forward selling contract. At the beginning of March, this producer may decide to subscribe an option (and pay the corresponding premium) to have the possibility of selling 500 MW during all hours of the second quarter of the year. To make these decisions, it has to solve a complex optimization problem similar to that described in Chapter 7.

Additionally, this producer may need to get involved in the pool (day-ahead market, adjustment markets, and balancing markets) to take fully advantage of its production facilities. To get involved in the pool, it has to solve a complex optimization problem similar to those described in Chapters 5 and 6.

It may also get involved in the reserve market provided it has the capability of increasing/reducing production as required by the ISO; and in the regulation market if it has load following capability. To get involved in the regulation market, it has to solve a complex optimization problem similar to that described in Chapter 5. □

1.4.4 Non-Dispatchable Producer

A non-dispatchable producer may participate in the day-ahead and adjustment markets, and needs to participate in the balancing market to cope with its uncertain power output. Such a producer participates mostly in the adjustment markets, which are closer in time to power delivery and therefore, allow the producer to know its production with a reduced level of uncertainty. Pool involvement by non-dispatchable producers is analyzed in Chapter 6.

Fig. 1.13 illustrates the decision-making problem of a non-dispatchable producer. As indicated in this figure, such a producer sells energy in the pool and typically needs to buy/sell a significant amount of balancing energy in the balancing market. Left-hand-side arrows indicate the interactions of the non-dispatchable producer with the market: selling or purchasing energy depending on its power production expectations.

The example below illustrates the decision-making problem faced by a non-dispatchable producer.

Illustrative Example 1.9 (Non-dispatchable producer).

A wind producer owning a farm of 100 MW may submit offers to the day-ahead market and to all adjustment markets, mostly to those closer to power delivery.

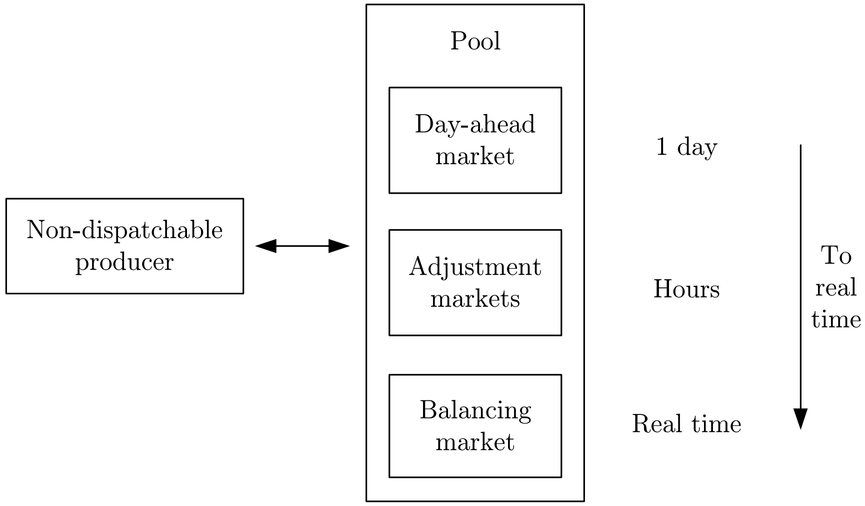


Fig. 1.13 Non-dispatchable producer decision-making problems

If this producer sells 50 MW in the day-ahead market and 30 MW in the last adjustment market during the time period 5pm-6pm, but actually produces 70 MW in that hour, it needs to buy 10 MW in the balancing market so that it can meet its pool obligation. To get involved in the pool, this non-dispatchable producer has to solve a complex optimization problem similar to that described in Chapter 6. □

1.4.5 Market Operator

The market operator clears the day-ahead and the adjustment markets, and with the assistance of the independent system operator clears the reserve and the balancing markets. It also clears the futures market using appropriate auction algorithms.

The market operator uses suitable market-clearing tools that may or may not include a description of the stochastic nature of some of the parameters involved, e.g., load, equipment failures, and non-dispatchable power productions.

Day-ahead markets are cleared using different type of auctions [49], among them:

1. Single-period auction: each hourly auction is cleared independently of the rest. Thus, no inter-temporal constraints affecting the production units are taken into account.

2. Multi-period auction: the 24-hour market is cleared considering simultaneously the 24 hourly auctions. Inter-temporal constraints affecting production units are thus accounted for.
3. Network-constrained multi-period auction: the 24-hour market is cleared considering simultaneously the 24 hourly auctions while enforcing network constraints. Thus, both inter-temporal and network constraints are considered.

Most European electricity markets use either single- or multi-period auctions that do not include network constraints, while markets in the East Cost of the US use mostly network-constrained auctions.

The adjustment markets and the balancing market are mostly cleared using simple auctions. However, balancing (real-time) markets in the US are generally cleared including network constraints.

Chapters 10 and 11 provide detailed descriptions of clearing procedures for the market operator based on stochastic programming.

The example below illustrates how the market operator clears the pool markets.

Illustrative Example 1.10 (Market operator).

An example of a clearing process for the day-ahead market is as follows. The five producers in the market send energy selling offers to the market operator for each one of their production units and each hour of the next day. The offer of each unit for each hour consists of up to 25 energy-price blocks in increasing price order. Assuming constant demand, the market operator clears the market in such a way that the cost of supplying the demand is minimal. Then, the market operator provides hourly prices and the accepted production blocks for each producer and hour. The market operator uses a market-clearing procedure similar to those explained in detail in Chapters 10 and 11. □

1.4.6 Independent System Operator

The independent system operator clears the reserve and regulation markets, and assists the market operator to clear the balancing market.

The reserve market can be cleared independently using an appropriate auction or simultaneously with the day-ahead market. Market clearing procedures for simultaneously clearing the day-ahead and the reserve markets are analyzed in Chapters 10 and 11.

To clear the regulation market an auction procedure is generally used so that regulation offers by generators are accepted in increasing price order until sufficient regulation power is allocated.

The example below illustrates how the independent system operator clears the regulation market.

Illustrative Example 1.11 (Regulation clearing: Independent System Operator).

Four of the five producers in the market offer 10, 50, 80, and 30 MW of load following power for the regulation market from 3am to 4am at prices 50, 55, 60 and $70/MWh, respectively. The independent system operator estimates that 70 MW of regulation power are sufficient and thus, accepts 10, 50 and 10 MW from the first, second, and third producers, respectively. □

1.5 Summary

This chapter provides an overview of a typical electricity marketplace, including its organization, its different trading floors, its agents, and its sequential organization. The sequence in which the different trading floors are cleared is explained, emphasizing the uncertainties affecting each specific market (day-ahead, adjustment, balancing, reserve, and regulation markets). The decision problems faced by each market agent, namely, producers, non-dispatchable producers, consumers, and retailers, are described and put into context, indicating the uncertainty involved. Clearing procedures for the market operator (or the independent system operator) are briefly described.

Relevant monographs describing the organization of the electricity marketplace are [49, 133, 135].

Basic references on uncertainty treatment and stochastic programming for decision making are [14, 78].

1.6 Exercises

Exercise 1.1. Scan FERC Order 888 (`www.ferc.gov`) and describe its most relevant features in what respect to market organization. FERC, The Federal Energy Regulatory Commission, is the regulator for the energy markets in the US.

Exercise 1.2. Scan Directive 96/92/EC of the European Union and describe its most relevant features in what respect to market organization. The text of this directive can be found at `ec.europa.eu/energy`.

Exercise 1.3. Describe the organization of the electricity pool of the UK and the trading floors it comprises.

Relevant information is available at `www.apxgroup.com`.

Exercise 1.4. Describe the derivative products available (forward contract and options) at the European Energy Exchange futures market, EEX.

Relevant information is available at www.eex.com.

Exercise 1.5. Describe the organization of the OMIP futures market in the Iberian Peninsula.

Relevant information is available at www.omip.pt.

Exercise 1.6. Describe the pool organization and the trading floors of the PJM Interconnection.

Relevant information on the markets of the PJM Interconnection is available at www.pjm.com.

Exercise 1.7. Describe the pool organization and the trading floors of the ISO New England.

Relevant information on the ISO New England electricity market is available at www.iso-ne.com.

Exercise 1.8. Describe the Australian Securities Exchange futures electricity market, ASX.

Relevant information on this Australian futures market is available at www.asx.com.au.

Exercise 1.9. Describe the balancing market of Scandinavia's Nordpool.

Relevant information on the markets pertaining to Nordpool is available at www.nordpool.com.

Exercise 1.10. Describe the reserve and regulation markets run by Spain's ISO, *Red Eléctrica de España*, REE, for the Spanish zone of the electricity market of the Iberian Peninsula.

Relevant information on the markets cleared by REE is available at www.ree.es.

Chapter 2
Stochastic Programming Fundamentals

2.1 Introduction

Unknown data abound in decision-making problems in the real world. This lack of *perfect information* is common in problems belonging to different knowledge areas such as engineering, economics, finances, etc.

Decision-making problems in electricity markets are no exception. In fact, uncertainty is present in most decision-making problems faced by electricity market agents. For example, electricity prices are unknown when agents have to submit their offers or bids to the pool. Similarly, at the time of procuring the energy needed to supply client loads, retailers do not know precisely the electricity demands of these clients.

However, decisions need to be made even with lack of perfect information. This is what motivates the use of *stochastic programming* models for decision making under uncertainty.

Most decision-making problems can be adequately formulated as optimization problems. If the input data of an optimization problem are well-defined and *deterministic*, its optimal solution (decision) is achieved by solving the problem. The decision is then implemented to attain the best outcome.

However, more often than not, the input data are uncertain but describable through probability functions. In such a situation, it is not clear how the decision-making problem should be formulated. One possibility is to substitute the uncertain input data (describable through probability functions) by their corresponding expected values, which results in a well defined and deterministic optimization problem. However, solving such a problem may lead to an solution that once implemented does not result in the best outcome.

Alternatively, the probability distribution of input data can be approximated by a collection of plausible sets of input data with associated probabilities of occurrence. For instance, three sets of input data with three values of probability of occurrence adding to 1.

A. J. Conejo et al., *Decision Making Under Uncertainty in Electricity Markets*,
International Series in Operations Research & Management Science 153,
DOI 10.1007/978-1-4419-7421-1_2,

Then a *stochastic* optimization problem can be formulated implicitly weighting (with the probabilities of occurrence) the individual solutions associated with each set of input data to achieve a single solution that is the best in some sense for all sets of input data. That is, we achieve a solution that is adequately pre-positioned with respect to all the sets of input data, but not to any one of them particularly. How to formulate meaningfully stochastic problems is the main topic of this chapter.

As a result of the uncertain input data being described by a collection of different sets of data, the resulting objective function is uncertain and needs to be characterized as a random variable. Since such objective function is not not a real-valued function but a random variable, the problem of establishing a specific objective for the decision-making problem arises. One alternative is to maximize the expected value of the objective function, other one, to maximize the expected value of such function but limiting its variance, etc.

Implementing the solution obtained by solving the stochastic problem above pre-positions the decision-maker in the best possible manner if considering all possible input data sets duly weighted by their respective probabilities. This solution is not the best for each individual set of input data but it is the best if all of them, weighted with their probabilities of occurrence, are simultaneously considered.

The price to be paid for using a stochastic programming approach is a dramatic increase in the size of the problem to be solved, which if handled without care may lead to intractability.

A wealth of motivating and clarifying examples can be found in tutorial references [78, 131].

Realistic case studies of interest for electricity market researchers and practitioners are scattered throughout the book:

1. For producer in Chapters 5-7.
2. For retailers in Chapter 8.
3. For consumers in Chapter 9.
4. For market/system operators in Chapters 10-11.

This chapter describes the basics of stochastic programming. Additional information can be found in [14,84,119,127]. Particularly, solution procedures are examined in [80], and friendly tutorials are available in [78, 131].

It should be emphasized that this chapter provides a basic introduction to stochastic programming. It is thus of interest for readers with no previous knowledge of stochastic programming. Those readers familiar with stochastic optimization should skip this chapter.

This chapter is organized as follows. Section 2.2 and 2.3 provide brief descriptions of random variables and stochastic processes, respectively, while Section 2.4 clarifies how to handle large scenario sets. In Section 2.5, formulations for both two-stage and multi-stage stochastic programming problems are presented. Section 2.6 describes two relevant metrics to characterize stochastic programming problems, as well as the concept of *out-of-sample* assessment. Section 2.7 briefly introduces the concepts of risk and risk management. Section 2.8 provides some general guidelines to solve large-scale stochastic programming problems and finally, Section 2.9 summarizes the chapter and lists some relevant conclusions.

2.2 Random Variables

Stochastic programming is used to formulate and solve problems with uncertain parameters. Within a stochastic programming context, each uncertain parameter is modeled as a random variable.

In stochastic programming, random variables are usually represented by a finite set of realizations or scenarios [14]. For instance, random variable λ can be represented by $\lambda(\omega),\ \omega = 1, \ldots, N_\Omega$, where ω is the scenario index, N_Ω is the number of scenarios considered and Ω is the set of scenarios. We denote by λ_Ω the set of possible realizations of random variable λ, i.e., $\lambda_\Omega = \{\lambda(1), \ldots, \lambda(N_\Omega)\}$.

Each realization $\lambda(\omega)$ is associated with a probability $\pi(\omega)$ defined as

$$\pi(\omega) = P(\omega|\lambda = \lambda(\omega)), \quad \text{where} \quad \sum_{\omega \in \Omega} \pi(\omega) = 1. \tag{2.1}$$

Illustrative Example 2.1 (Random variable: pool price at noon).

The pool price at noon on a future day of a particular electricity market (e.g., ISO New England) is a random variable.

As an example, suppose that the distribution of the pool price at noon can be characterized by the discrete distribution below:

$$\begin{aligned}
&\$50/\text{MWh with probability } 0.2\\
&\$46/\text{MWh with probability } 0.6\\
&\$44/\text{MWh with probability } 0.2\ .
\end{aligned}$$

□

Assuming a finite set of scenarios, random variable λ can be characterized by its cumulative distribution function (cdf),

$$F_\lambda(\eta) = P(\omega|\lambda(\omega) \le \eta) = \sum_{\omega \in \Omega | \lambda(\omega) \le \eta} \pi(\omega),\ \forall \eta \in \mathbb{R}, \tag{2.2}$$

where $\mathbb{R}$ is the set of real numbers.

A random variable can also be characterized by its statistical moments. Two useful moments are the mean (expected value) and the variance. The mean and the variance of random variable λ, $\overline{\lambda}$ and σ^2_λ, respectively, are

$$\overline{\lambda} = \mathcal{E}\{\lambda\} = \sum_{\omega \in \Omega} \pi(\omega)\lambda(\omega) \tag{2.3}$$

$$\sigma_\lambda^2 = \mathcal{V}\{\lambda\} = \sum_{\omega \in \Omega} \pi(\omega)\Big(\lambda(\omega) - \mathcal{E}\{\lambda\}\Big)^2, \tag{2.4}$$

being $\mathcal{E}$ and $\mathcal{V}$ the expectation and variance operators, respectively.

The mean is the expected value of the random variable, whereas the variance is a dispersion measure, whose square root is the standard deviation. These moments are particularly relevant if random variables represent profits or costs. For instance, it is desirable to achieve a profit with large mean and small variance, which indicates that the expected value of the profit is high and the probability of obtaining profit values different from the expected profit is low. Since the mean and the standard deviation are expressed in the same units (e.g., US dollars), hereinafter we use the standard deviation instead of the variance to characterize the dispersion of a random variable.

Illustrative Example 2.2 (Pool price at noon: distribution and moments).

The mean, the variance, and the standard deviation of the pool price at noon, considered in Illustrative Example 2.1, are, respectively,

$$\begin{aligned}
\overline{\lambda} &= 0.2 \times 50 + 0.6 \times 46 + 0.2 \times 44 = \$46.40/\text{MWh} \\
\sigma_\lambda^2 &= 0.2 \times (50 - 46.4)^2 + 0.6 \times (46 - 46.4)^2 + 0.2 \times (44 - 46.4)^2 \\
&= 3.84\ (\$/\text{MWh})^2 \\
\sigma_\lambda &= \sqrt{0.2 \times (50 - 46.4)^2 + 0.6 \times (46 - 46.4)^2 + 0.2 \times (44 - 46.4)^2} \\
&= \$1.9596/\text{MWh}\ .
\end{aligned}$$

□

A discrete random variable is also statistically described by its probability mass function (pmf). The pmf gives the probability of a discrete random variable being equal to a given value. The equivalent of the pmf for continuous variables is the probability density function (pdf). The integral of the pdf over an interval gives the probability of the random variable falling within that interval.

From a graphical point of view, the pmf is not appropriate for representing discrete random variables with many realizations, as the resulting graphic may be difficult to interpret. Therefore, we propose the use of an alternative representation referred to as *adjusted pdf*. The adjusted pdf of a discrete random variable λ is a bar chart built as follows:

1. The base of each bar represents a range of values of the random variable.
2. The height of each bar is equal to the sum of the probabilities of all realizations falling within the interval spanned by the base of the bar divided by the width of the bar.

The main advantage of this representation is that the area under the adjusted pdf is equal to 1. This property allows comparing the adjusted pdf plots of several discrete random variables, while pmf plots are not graphically comparable. This is shown in Illustrative Example 2.3 below.

Illustrative Example 2.3 (Random variable: adjusted pdf).
The random variable λ is represented by one thousand scenarios that have been randomly generated from a normal distribution $N(0, 1)$. In Fig. 2.1, in addition to the pmf, the adjusted pdf is illustrated using 15 intervals and depicted together with the pdf of the normal distribution $N(0, 1)$. □

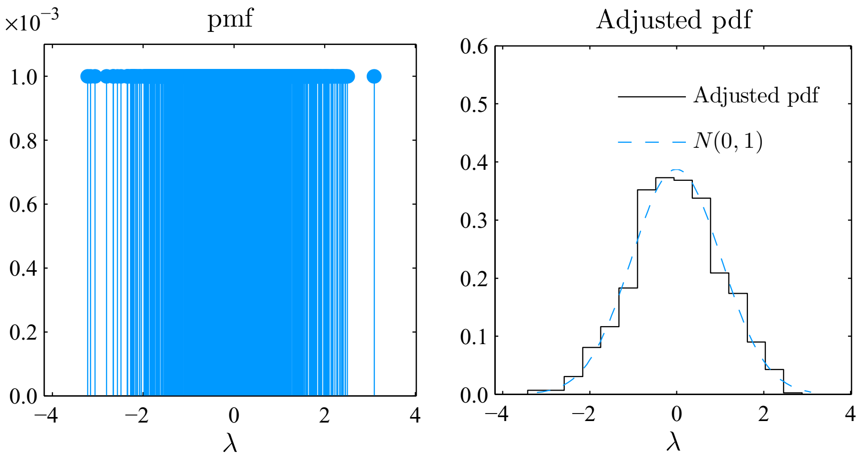


Fig. 2.1 Illustrative Example 2.3: pmf and adjusted pdf

2.3 Stochastic Processes

A random variable whose value evolves over time is known as a stochastic process. The pool price over one week is an example of a stochastic process, as well as the electricity demand over a month. For the purpose of this book, no further formality is required to define a stochastic process. For the interested reader, a more formal treatment of stochastic processes is given in [119].

Thus, a stochastic process is constituted by a set of dependent random variables sequentially arranged in time. For each time period, the corresponding random variable (e.g., the price at noon) depends on the other random variables (e.g., the prices in other hours of the day). In other words, the price at noon is a random variable and the collection of random variables corresponding to the hourly prices of the day constitutes a stochastic process.

Stochastic processes spanning a given time horizon can be represented by scenarios. For instance, stochastic process $\boldsymbol{\lambda}$ can be represented by vectors $\boldsymbol{\lambda}(\omega)$, $\omega = 1, \ldots, N_\Omega$, where ω is the scenario index and N_Ω is the number of scenarios considered. Note that $\boldsymbol{\lambda}$ contains the set of dependent random variables constituting the stochastic process. We denote by $\boldsymbol{\lambda}_\Omega$ the set of possible realizations of stochastic process $\boldsymbol{\lambda}$, i.e., $\boldsymbol{\lambda}_\Omega = \{\boldsymbol{\lambda}(1), \ldots, \boldsymbol{\lambda}(N_\Omega)\}$.

Each realization $\boldsymbol{\lambda}(\omega)$ is associated with a probability $\pi(\omega)$ defined as

$$\pi(\omega) = P(\omega|\boldsymbol{\lambda} = \boldsymbol{\lambda}(\omega)), \quad \text{where} \quad \sum_{\omega\in\Omega} \pi(\omega) = 1. \tag{2.5}$$

For example, if $\boldsymbol{\lambda}$ represents the 24 hourly electricity prices of tomorrow, $\boldsymbol{\lambda}(\omega)$ is a 24×1 vector representing one possible realization of these prices.

Illustrative Example 2.4 (Stochastic process: hourly pool prices at two consecutive hours).

We consider that the random variable representing the pool price at noon is characterized as

$$\begin{aligned}
&\$50/\text{MWh with probability } 0.2\\
&\$46/\text{MWh with probability } 0.6\\
&\$44/\text{MWh with probability } 0.2\ .
\end{aligned}$$

The random variable representing the pool price at 1pm is described in the following:

1. If the pool price at noon is \$50/MWh, the probabilities of the price at 1 pm being 52, 48 or \$45/MWh are, respectively, 0.8, 0.1 or 0.1.
2. If, on the other hand, the pool price at noon is \$46/MWh, the probabilities of the price at 1 pm being 52, 48 or \$45/MWh are, respectively, 0.2, 0.7 or 0.1.
3. Finally, if the pool price at noon is \$44/MWh, the probabilities of the price at 1 pm being 52, 48 or \$45/MWh are, respectively, 0.1, 0.4 or 0.5.

These two dependent random variables *price at noon* and *price at 1 pm* constitute a stochastic process. □

2.4 Scenarios

From a computational viewpoint, a convenient manner to characterize stochastic processes is through scenarios. As explained in Section 2.3, a scenario is a single realization of a stochastic process.

To adequately describe a stochastic process, it is critical to generate a sufficient number of scenarios so that these scenarios cover the most plausible realizations of the considered stochastic process. To achieve this, it is generally required to generate a very large number of scenarios, which may render the associated stochastic programming problem computationally intractable.

It is thus required to develop procedures to reduce the number of scenarios initially generated. These procedures should retain most of the relevant information on the stochastic process contained in the original scenario set while reducing significantly its cardinality.

Scenario-generation and scenario-reduction procedures are studied in Chapter 3.

Illustrative Example 2.5 (Stochastic process: scenarios).

All the scenarios describing the stochastic process *prices at noon and at 1 pm*, described in Illustrative Example 2.4, are provided in Table 2.1.

Table 2.1 Illustrative Example 2.5: price scenarios

Scenario #	Price at noon ($/MWh)	Price at 1pm ($/MWh)	Probability (per unit)
1	50	52	$0.2 \times 0.8 = 0.16$
2	50	48	$0.2 \times 0.1 = 0.02$
3	50	45	$0.2 \times 0.1 = 0.02$
4	46	52	$0.6 \times 0.2 = 0.12$
5	46	48	$0.6 \times 0.7 = 0.42$
6	46	45	$0.6 \times 0.1 = 0.06$
7	44	52	$0.2 \times 0.1 = 0.02$
8	44	48	$0.2 \times 0.4 = 0.08$
9	44	45	$0.2 \times 0.5 = 0.10$

In this simple example the number of scenarios is small (specifically nine) and thus no scenario reduction is needed. However, this is not generally the case when addressing realistic decision-making problems under uncertainty. □

Realistic descriptions of actual market prices (day-ahead, adjustment and balancing) are provided in Chapters 3 and 5-9, of demand in Chapters 8 and 9, and of wind production in Chapters 6 and 11.

2.5 Stochastic Programming Problems

In decision making under uncertainty, the decision maker has to make optimal decisions throughout a decision horizon with incomplete information. Over the considered decision horizon, a number of stages is defined. Each *stage* represents a point in time where decisions are made or where uncertainty partially or totally vanishes. The amount of information available to the decision maker is usually different from stage to stage. According to the number of stages considered we can distinguish between two-stage and multi-stage stochastic programming problems.

2.5.1 Two-Stage Problems

We consider a decision-making problem where decisions are made at two stages and there exists a stochastic process $\boldsymbol{\lambda}$ represented by a set of scenarios $\boldsymbol{\lambda}_{\Omega}$. We assume that two different decision variable vectors, namely $\boldsymbol{x}$ and $\boldsymbol{y}$, are involved in this problem. Decision $\boldsymbol{x}$ is made before knowing the actual value of the stochastic process $\boldsymbol{\lambda}$, while $\boldsymbol{y}$ is determined after the realization of $\boldsymbol{\lambda}$. Consequently, decision $\boldsymbol{y}$ depends on the decision $\boldsymbol{x}$ previously made and on the realization $\boldsymbol{\lambda}(\omega)$ of the stochastic process $\boldsymbol{\lambda}$. Thus, we can express $\boldsymbol{y}$ as $\boldsymbol{y}(\boldsymbol{x}, \omega)$. The decision-making process is as follows:

1. Decision $\boldsymbol{x}$ is made.
2. The stochastic process $\boldsymbol{\lambda}$ is realized as $\boldsymbol{\lambda}(\omega)$.
3. Decision $\boldsymbol{y}(\boldsymbol{x}, \omega)$ is made.

In this decision-making process, two different kinds of decisions are distinguished:

1. *First-stage* or *here-and-now* decisions. These decisions are made before the realization of the stochastic process. Hence, variables representing here-and-now decisions do not depend on each realization of the stochastic process.
2. *Second-stage* or *wait-and-see* decisions. These decisions are made after knowing the actual realization of the stochastic process. Consequently, these decisions depend on each realization vector of the stochastic process. If the stochastic process is represented by a set of scenarios, a second-stage decision variable is defined for each single scenario considered.

The decision framework above is conveniently visualized through a scenario tree, as the one in Fig. 2.2. Graphically, a scenario tree comprises a set of nodes and branches. The nodes represent states of the problem at a particular instant, i.e., the points where decisions are made. Each node has a single predecessor and can have several successors. The first node is called the root node, and it corresponds to the beginning of the planning horizon. In the root node, first-stage decisions are made. The nodes connected to the root node are the second-stage nodes and represent the points where the second-stage decisions are made. For a two-stage problem, the second-stage nodes are equal to the number of scenarios and are referred to as leaves. In a scenario tree, the branches represent different realizations of the random variables.

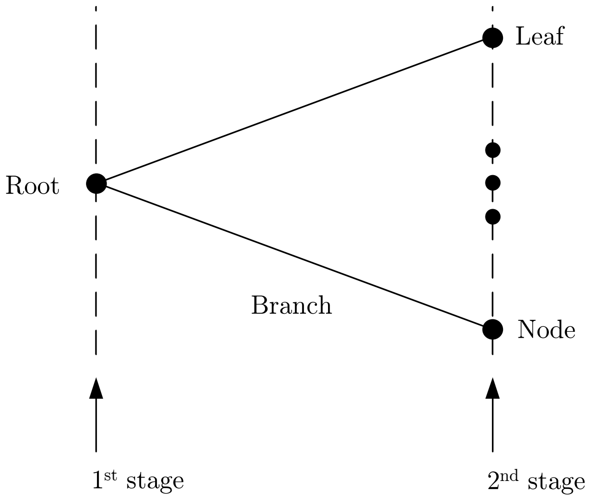


Fig. 2.2 Scenario tree for a two-stage problem

Note that for all these decisions to be optimal, they need to be derived simultaneously by solving a single optimization problem, so that the relationships among the decision variables are properly accounted for.

The general expression of a two-stage stochastic linear programming problem is the following:

$$\text{Minimize}_{\boldsymbol{x}} \;\; z = \boldsymbol{c}^\top \boldsymbol{x} + \mathcal{E}\{Q(\omega)\} \tag{2.6}$$

$$\text{subject to } \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{2.7}$$

$$\boldsymbol{x} \in X \tag{2.8}$$

where

$$Q(\omega) = \left\{ \begin{aligned} &\text{Minimize}_{\boldsymbol{y}(\omega)} \;\; \boldsymbol{q}(\omega)^\top \boldsymbol{y}(\omega) && (2.9)\\ &\text{subject to } \boldsymbol{T}(\omega)\boldsymbol{x} + \boldsymbol{W}(\omega)\boldsymbol{y}(\omega) = \boldsymbol{h}(\omega) && (2.10)\\ &\boldsymbol{y}(\omega) \in Y \end{aligned} \right\}, \; \forall \omega \in \Omega, \qquad (2.11)$$

where $\boldsymbol{x}$ and $\boldsymbol{y}(\omega)$ are the first- and second-stage decision variable vector, respectively, and $\boldsymbol{c}$, $\boldsymbol{q}(\omega)$, $\boldsymbol{b}$, $\boldsymbol{h}(\omega)$, $\boldsymbol{A}$, $\boldsymbol{T}(\omega)$, and $\boldsymbol{W}(\omega)$ are known vectors and matrices of appropriate size. Note that any of the vectors and matrices in problem (2.6)-(2.11) representing input data can depend (or not) on the set of stochastic process realizations $\boldsymbol{\lambda}_{\Omega}$.

Problem (2.9)-(2.11), wherein the decisions are made after uncertainty is cleared, is referred to as *recourse* problem.

It should be noted that for the sake of clarity, problem (2.6)-(2.11) is linear. However, a nonlinear version of (2.6)-(2.11) can be straightforwardly derived.

Under rather general assumptions [14], the two-stage stochastic programming problem (2.6)-(2.11) can be equivalently expressed as follows:

$$\text{Minimize}_{\boldsymbol{x},\boldsymbol{y}(\omega)} \quad z = \boldsymbol{c}^{\top}\boldsymbol{x} + \sum_{\omega \in \Omega} \pi(\omega)\boldsymbol{q}(\omega)^{\top}\boldsymbol{y}(\omega) \tag{2.12}$$

$$\text{subject to} \quad \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{2.13}$$

$$\boldsymbol{T}(\omega)\boldsymbol{x} + \boldsymbol{W}(\omega)\boldsymbol{y}(\omega) = \boldsymbol{h}(\omega), \ \forall \omega \in \Omega \tag{2.14}$$

$$\boldsymbol{x} \in X, \ \boldsymbol{y}(\omega) \in Y, \ \forall \omega \in \Omega. \tag{2.15}$$

Problem (2.12)-(2.15) is the *deterministic* equivalent problem of the stochastic programming problem (2.6)-(2.11).

It is important to note that stochastic programming problems can be mathematically formulated using either a *node-variable* formulation or a *scenario-variable* formulation. The first formulation relies on variables associated with decision points while the second one relies on variables associated with scenarios. The first formulation is comparatively more compact than the second one and is particularly well suited for a direct solution approach; the second one requires a larger number of variables and constraints than the first one but presents an exploitable structure that is well suited for decomposition. These two formulations are exemplified below.

Illustrative Example 2.6 (Two-stage problem. Node-variable formulation).

An electricity consumer is facing both uncertain electricity demand and price for next week. For simplicity, we consider that both price and demand are uncertain but constant throughout the week. Scenario data pertaining to demand and price are provided in Table 2.2.

Additionally, this consumer has the possibility of buying up to 90 MW at \$45/MWh throughout next week, by signing a bilateral contract before next week, i.e., before knowing the actual demand and pool price it has to face.

The decision-making problem of this consumer can be formulated as a two-stage stochastic programming problem. At the first stage, the consumer has to decide how much to buy from the contract, and the second stage reproduces pool purchases for each of the three considered demand/price realizations (scenarios).

The corresponding scenario tree is provided in Fig. 2.3.

Table 2.2 Illustrative Example 2.6: scenario data for the consumer

Scenario #	Probability (per unit)	Demand (MW)	Price (\$/MWh)
1	0.2	110	50
2	0.6	100	46
3	0.2	80	44

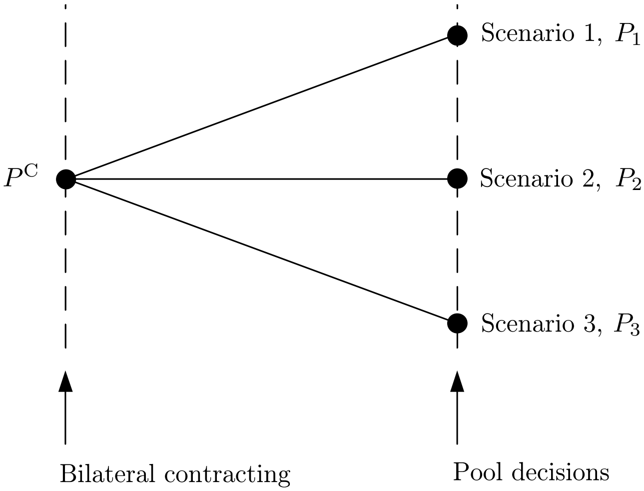


Fig. 2.3 Illustrative Example 2.6: scenario tree

The node-variable formulation of this two-stage stochastic programming problem is as follows:

$$
\begin{aligned}
&\text{Minimize}_{P^{\mathrm{C}},P_1,P_2,P_3} \\
&C^{\mathrm{S}} = 168(45P^{\mathrm{C}} + 0.2 \times 50P_1 + 0.6 \times 46P_2 + 0.2 \times 44P_3) \\
&\text{subject to} \\
&P^{\mathrm{C}} + P_1 \geq 110 \\
&P^{\mathrm{C}} + P_2 \geq 100 \\
&P^{\mathrm{C}} + P_3 \geq 80 \\
&0 \leq P^{\mathrm{C}} \leq 90 \\
&0 \leq P_1, P_2, P_3 \ .
\end{aligned}
$$

Variable P^{C} represents the power bought through the bilateral contract, while variables P_1, P_2, and P_3 represent the power bought in the pool for scenarios 1, 2, and 3, respectively.

The objective function is the expected cost faced by the consumer to supply its uncertain demand. Powers are multiplied by the number of hours in a week (168) to obtain the energy consumed throughout the week.

The first three constraints enforce energy supply for the three scenarios, while the remaining constraints are the contract bounds and non-negativity declarations for all the variables.

Note that the variables of this problem are associated with the nodes of the scenario tree (see Fig. 2.3) and so the denomination *node-variable* formulation.

The solution to this problem is $P^{\mathrm{C}*} = 80$, $P_1^* = 30$, $P_2^* = 20$, $P_3^* = 0$, which means that, before the week, the consumer buys 80 MW using the bilateral contract, and during the week, 30 , 20 or 0 MW for demands (prices) 110 (50), 100 (46) or 80 (44) MW ($/MWh), respectively. □

Complementing Illustrative Example 2.6, the example below illustrates the scenario-variable formulation of a stochastic programming problem.

Illustrative Example 2.7 (Two-stage problem. Scenario-variable formulation).

The problem in Illustrative Example 2.6 expressed according to a scenario-variable representation is formulated as follows:

$$\text{Minimize}_{P_1^{\mathrm{C}},P_2^{\mathrm{C}},P_3^{\mathrm{C}},P_1,P_2,P_3}$$

$$0.2 \times 168(45P_1^{\mathrm{C}} + 50P_1) + 0.6 \times 168(45P_2^{\mathrm{C}} + 46P_2) + 0.2 \times 168(45P_3^{\mathrm{C}} + 44P_3)$$

subject to

$$P_1^{\mathrm{C}} + P_1 \geq 110$$

$$P_2^{\mathrm{C}} + P_2 \geq 100$$

$$P_3^{\mathrm{C}} + P_3 \geq 80$$

$$0 \leq P_1^{\mathrm{C}}, P_2^{\mathrm{C}}, P_3^{\mathrm{C}} \leq 90$$

$$0 \leq P_1, P_2, P_3$$

$$P_1^{\mathrm{C}} = P_2^{\mathrm{C}} = P_3^{\mathrm{C}}\ .$$

Variables P_1^{C}, P_2^{C}, and P_3^{C} represent the power bought through the bilateral contract under scenarios 1, 2, and 3, respectively, while variables P_1, P_2, and P_3 represent the power bought in the pool also in scenarios 1, 2, and 3, respectively.

The objective function is the expected cost faced by the consumer to supply its uncertain demand.

The first three constraints enforce energy supply for the three scenarios, while the following constraints are the contract bounds and non-negativity declarations for all the variables. The last row of constraints contains the *nonanticipativity* conditions. These conditions guarantee that the decision on the bilateral contract cannot be dependent on the scenario realization. Note that these are logical constraints related to the availability of information at any decision point in time. Nonanticipativity conditions are further explained in the next subsection.

The variables of this problem are associated with the scenarios and thus the denomination *scenario-variable* formulation. This is clarified through Fig. 2.4. Generally, a scenario-variable formulation entails a larger number of variables and constraints than a node-variable formulation, but the scenario-variable formulation allows using efficiently solution procedures based on decomposition (see Section 2.8).

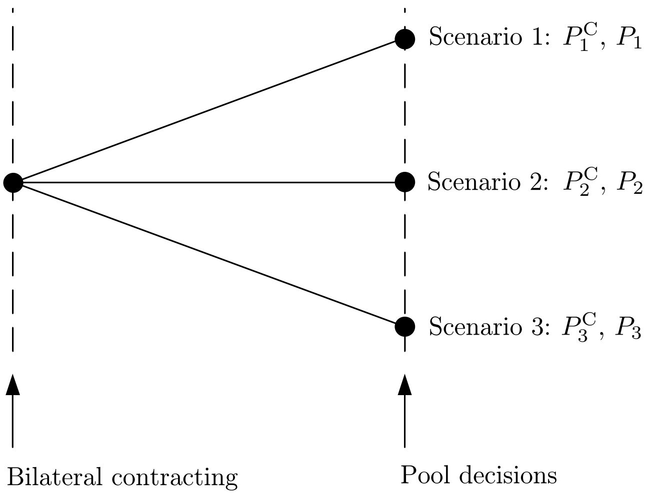


Fig. 2.4 Illustrative Example 2.7: scenario tree

The solution to this problem is identical to that of the problem in the Illustrative Example 2.6 above.

□

2.5.2 Multi-Stage Problems

In some cases, decision-making problems comprise more than two stages, and the two-stage stochastic programming problem presented in Subsection 2.5.1 is not appropriate. This fact motivates the use of multi-stage stochastic programming problems [14]. An example of a multi-stage decision-making process with r stages is the following:

1. Decisions $\boldsymbol{x}^1$ are made.
2. Stochastic process $\boldsymbol{\lambda}^1$ is realized as $\boldsymbol{\lambda}^1(\omega^1)$.
3. Decisions $\boldsymbol{x}^2(\boldsymbol{x}^1, \omega^1)$ are made.
4. Stochastic process $\boldsymbol{\lambda}^2$ is realized as $\boldsymbol{\lambda}^2(\omega^2)$.
5. Decisions $\boldsymbol{x}^3(\boldsymbol{x}^1, \omega^1, \boldsymbol{x}^2, \omega^2)$ are made.

...

2r-2. Stochastic process $\boldsymbol{\lambda}^{r-1}$ is realized as $\boldsymbol{\lambda}^{r-1}(\omega^{r-1})$.
2r-1. Decisions $\boldsymbol{x}^r(\boldsymbol{x}^1, \omega^1, \ldots, \boldsymbol{x}^{r-1}, \omega^{r-1})$ are made.

This decision framework is conveniently visualized through a scenario tree, as the one in Fig. 2.5. As explained in the previous subsection, graphically, a scenario tree is depicted as a set of nodes and branches. The nodes represent states of the problem at a particular instant, i.e., the points where decisions are made. In the first node, called the root, the first-stage decisions are made. The nodes connected to the root node are the second-stage nodes and represent the points where the second-stage decisions are made. The number of nodes in the last stage equals the number of scenarios. These nodes are referred to as leaves. In a scenario tree, the branches are different realizations of the random variables.

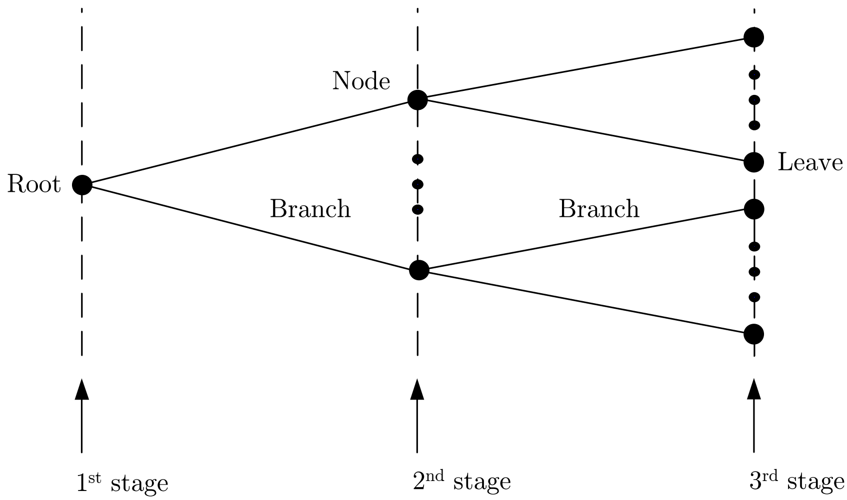


Fig. 2.5 Scenario tree for a three-stage problem

In all stochastic programming problems, and particularly in multi-stage problems, it is important to establish the *non-anticipativity* of the decisions

[14]. That is, if the realizations of the stochastic processes are identical up to stage k, the values of the decision variables must be then identical up to stage k.

The non-anticipativity of the decisions is implicitly taken into account in the previous decision sequence. That is, decisions $\boldsymbol{x}^1$ are independent of each future realization of the set of stochastic processes $\{\boldsymbol{\lambda}^1, \dots, \boldsymbol{\lambda}^{r-1}\}$. In the next stage, decisions $\boldsymbol{x}^2$ depend on each realization of the stochastic process in the first stage, $\boldsymbol{\lambda}^1$, but they are unique for all possible values of the stochastic processes that are realized in the future, i.e., $\{\boldsymbol{\lambda}^2, \dots, \boldsymbol{\lambda}^{r-1}\}$. Therefore, we can say that $\boldsymbol{x}^2$ are wait-and-see decisions with respect to $\boldsymbol{\lambda}^1$ and here-and-now decisions with respect to $\{\boldsymbol{\lambda}^2, \dots, \boldsymbol{\lambda}^{r-1}\}$.

The general expression of a multi-stage stochastic linear programming problem with r stages is as follows:

$$\text{Minimize}_{\boldsymbol{x}^1} \quad z = \boldsymbol{c}^{1,\top}\boldsymbol{x}^1 + \mathcal{E}_{\omega^1}\{Q^1(\boldsymbol{x}^1, \omega^1)\} \tag{2.16}$$

$$\text{subject to} \quad \boldsymbol{A}^1\boldsymbol{x}^1 = \boldsymbol{b}^1 \tag{2.17}$$

$$\boldsymbol{x}^1 \in X^1, \tag{2.18}$$

where

$$Q^1(\boldsymbol{x}^1, \omega^1) = \left\{ \begin{array}{rl} \text{Minimize}_{\boldsymbol{x}^2(\omega^1)} & \boldsymbol{c}^{2,\top}(\omega^1)\boldsymbol{x}^2(\omega^1) + \mathcal{E}_{\omega^2}\{Q^2(\boldsymbol{x}^1, \omega^1, \boldsymbol{x}^2(\omega^1), \omega^2)\} \\ \text{subject to} & \boldsymbol{T}^{1,1}(\omega^1)\boldsymbol{x}^1 + \boldsymbol{T}^{1,2}(\omega^1)\boldsymbol{x}^2(\omega^1) = \boldsymbol{h}^1(\omega^1) \\ & \boldsymbol{x}^2(\omega^1) \in X^2 \end{array} \right\}, \ \forall \omega^1 \in \Omega^1, \tag{2.19}$$

where

$$Q^2(\boldsymbol{x}^1, \omega^1, \boldsymbol{x}^2(\omega^1), \omega^2) = \left\{ \begin{array}{rl} \text{Minimize}_{\boldsymbol{x}^3(\omega^1,\omega^2)} & \boldsymbol{c}^{3,\top}(\omega^1,\omega^2)\boldsymbol{x}^3(\omega^1,\omega^2) + \\ & \mathcal{E}_{\omega^3}\{Q^3(\boldsymbol{x}^1, \omega^1, \boldsymbol{x}^2(\omega^1), \omega^2, \boldsymbol{x}^3(\omega^1,\omega^2), \omega^3)\} \\ \text{subject to} & \boldsymbol{T}^{2,1}(\omega^1,\omega^2)\boldsymbol{x}^1 + \boldsymbol{T}^{2,2}(\omega^1,\omega^2)\boldsymbol{x}^2(\omega^1) + \\ & \boldsymbol{T}^{2,3}(\omega^1,\omega^2)\boldsymbol{x}^3(\omega^1,\omega^2) = \boldsymbol{h}^2(\omega^1,\omega^2) \\ & \boldsymbol{x}^3(\omega^1,\omega^2) \in X^3 \end{array} \right\}, \ \forall \omega^1 \in \Omega^1, \forall \omega^2 \in \Omega^2, \tag{2.20}$$

...

$$Q^{r-1}(\boldsymbol{x}^1, \omega^1, \ldots, \omega^{r-1}) = \left\{ \begin{array}{rl} \text{Minimize}_{\boldsymbol{x}^r(\omega^1,\ldots,\omega^{r-1})} & \boldsymbol{c}^{r,\top}(\omega^1, \ldots, \omega^{r-1})\boldsymbol{x}^r(\omega^1, \ldots, \omega^{r-1}) \\ \text{subject to} & \boldsymbol{T}^{r-1,1}(\omega^1, \ldots, \omega^{r-1})\boldsymbol{x}^1 + \\ & \boldsymbol{T}^{r-1,2}(\omega^1, \ldots, \omega^{r-1})\boldsymbol{x}^2(\omega^1) + \ldots + \\ & \boldsymbol{T}^{r-1,r}(\omega^1, \ldots, \omega^{r-1})\boldsymbol{x}^r(\omega^1, \ldots, \omega^{r-1}) = \\ & \boldsymbol{h}^{r-1}(\omega^1, \ldots, \omega^{r-1}) \\ & \boldsymbol{x}^r(\omega^1, \ldots, \omega^{r-1}) \in X^r \end{array} \right\}, \\ \forall \omega^1 \in \Omega^1, \ldots, \forall \omega^{r-1} \in \Omega^{r-1}. \tag{2.21}$$

Under rather general assumptions [14], the multi-stage stochastic programming problem above can be equivalently expressed as the following deterministic problem,

$$\begin{aligned} &\text{Minimize}_{\boldsymbol{x}^1, \boldsymbol{x}^2(\omega^1), \ldots, \boldsymbol{x}^{\mathrm{r}}(\omega^1, \ldots, \omega^{\mathrm{r}-1})} \\ &\quad z = \\ &\quad \boldsymbol{c}^{1,\top}\boldsymbol{x}^1 + \\ &\quad \sum_{\omega^1 \in \Omega^1} \pi(\omega^1) \bigg(\boldsymbol{c}^{2,\top}(\omega^1)\boldsymbol{x}^2(\omega^1) + \\ &\quad \sum_{\omega^2 \in \Omega^2} \pi(\omega^2) \big(\boldsymbol{c}^{3,\top}(\omega^1, \omega^2)\boldsymbol{x}^3(\omega^1, \omega^2) \big) + \ldots + \\ &\quad \sum_{\omega^{\mathrm{r}-1} \in \Omega^{\mathrm{r}-1}} \pi(\omega^{\mathrm{r}-1}) \big(\boldsymbol{c}^{\mathrm{r},\top}(\omega^1, \ldots, \omega^{\mathrm{r}-1})\boldsymbol{x}^{\mathrm{r}}(\omega^1, \ldots, \omega^{\mathrm{r}-1}) \big) \bigg) \end{aligned} \tag{2.22}$$

subject to

$$\boldsymbol{A}^1\boldsymbol{x}^1 = \boldsymbol{b}^1 \tag{2.23}$$

$$\boldsymbol{x}^1 \in X^1 \tag{2.24}$$

$$\boldsymbol{T}^{1,1}(\omega^1)\boldsymbol{x}^1 + \boldsymbol{T}^{1,2}(\omega^1)\boldsymbol{x}^2(\omega^1) = \boldsymbol{h}^1(\omega^1);\ \forall \omega^1 \in \Omega^1 \tag{2.25}$$

$$\boldsymbol{x}^2(\omega^1) \in X^2;\ \forall \omega^1 \in \Omega^1 \tag{2.26}$$

$$\begin{aligned} &\boldsymbol{T}^{2,1}(\omega^1, \omega^2)\boldsymbol{x}^1 + \boldsymbol{T}^{2,2}(\omega^1, \omega^2)\boldsymbol{x}^2(\omega^1) + \\ &\boldsymbol{T}^{2,3}(\omega^1, \omega^2)\boldsymbol{x}^3(\omega^1, \omega^2) = \boldsymbol{h}^2(\omega^1, \omega^2);\ \forall \omega^1 \in \Omega^1, \forall \omega^2 \in \Omega^2 \end{aligned} \tag{2.27}$$

$$\boldsymbol{x}^3(\omega^1, \omega^2) \in X^3,\ \forall \omega^1 \in \Omega^1, \forall \omega^2 \in \Omega^2 \tag{2.28}$$

...

$$\begin{aligned}
&\boldsymbol{T}^{r-1,1}(\omega^1,\ldots,\omega^{r-1})\boldsymbol{x}^1+\boldsymbol{T}^{r-1,2}(\omega^1,\ldots,\omega^{r-1})\boldsymbol{x}^2(\omega^1)+\ldots+\\
&\boldsymbol{T}^{r-1,r}(\omega^1,\ldots,\omega^{r-1})\boldsymbol{x}^r(\omega^1,\ldots,\omega^{r-1})=\\
&\boldsymbol{h}^{r-1}(\omega^1,\ldots,\omega^{r-1});\ \forall\omega^1\in\Omega^1,\ldots,\forall\omega^{r-1}\in\Omega^{r-1} \qquad (2.29)\\
&\boldsymbol{x}^r(\omega^1,\ldots,\omega^{r-1})\in X^r,\ \forall\omega^1\in\Omega^1,\ldots,\forall\omega^{r-1}\in\Omega^{r-1}. \qquad (2.30)
\end{aligned}$$

Illustrative Examples 2.8 and 2.9 below clarify the above multi-stage formulation.

Illustrative Example 2.8 (Three-stage problem. Scenario-variable formulation).

Consider an electric energy producer with a production capacity of 120 MW.

This producer needs to determine its selling strategy during a future market horizon of 6 days, divided into two 3-day periods.

The bilateral contracts available to this producer are the following:

1. A selling contract of up to 50 MW at \$25/MWh at the beginning of the 6-day period and spanning the entire 6-day period.
2. A selling contract of up to 40 MW at \$26/MWh just after the first 3-day period and covering the last 3-day period.

Contract data are summarized in Table 2.3.

Table 2.3 Illustrative Example 2.8: contract data for the producer

Contract #	Duration	Hours	Price (\$/MWh)	Power Cap (MW)
A	Entire 6-day period	144	25	50
B	2nd 3-day period	72	26	40

Complementarily to bilateral contracting, the producer can sell energy in the pool at constant power during the first and second 3-day periods.

Pool prices scenarios are described below:

1. Pool prices for the first 3-day period are \$27/MWh with probability 0.4 and \$23/MWh with probability 0.6.
2. If the price during the first 3-day period is \$27/MWh, prices during the second 3-day period are \$28/MWh with probability 0.4 and \$26/MWh with probability 0.6.
3. Alternatively, if the price during the first 3-day period is \$23/MWh, prices during the second 3-day period are \$24/MWh with probability 0.4 and \$22/MWh with probability 0.6.

Scenario price data are given in Table 2.4 and the corresponding scenario tree is provided in Fig. 2.6.

Table 2.4 Illustrative Example 2.8: scenario data for the producer

Scenario #	1st 3-day price ($/MWh)	2nd 3-day price ($/MWh)	Probability (per unit)
1	27	28	$0.4 \times 0.4 = 0.16$
2	27	26	$0.4 \times 0.6 = 0.24$
3	23	24	$0.6 \times 0.4 = 0.24$
4	23	22	$0.6 \times 0.6 = 0.36$

The decision-making problem faced by the producer can be formulated as a three-stage stochastic programming problem. The decision process is as follows:

1. Before the 6-day period the producer needs to decide the quantity to be sold through the contract spanning the whole 6-day period. This is the first-stage decision.
2. For each price realization of the first 3-day period, the producer has to decide both the amount of energy to be sold through the contract spanning the second 3-day period and the amount of energy to be sold in the pool during the first 3-day period. These are the second-stage decisions, which depend on the pool price realization in the first 3-day period.
3. Finally, the producer needs to decide the power to be sold in the pool, which spans the second 3-day period, for each one of the four price combinations involving first and second 3-day periods.

This three-stage decision-making problem is formulated as

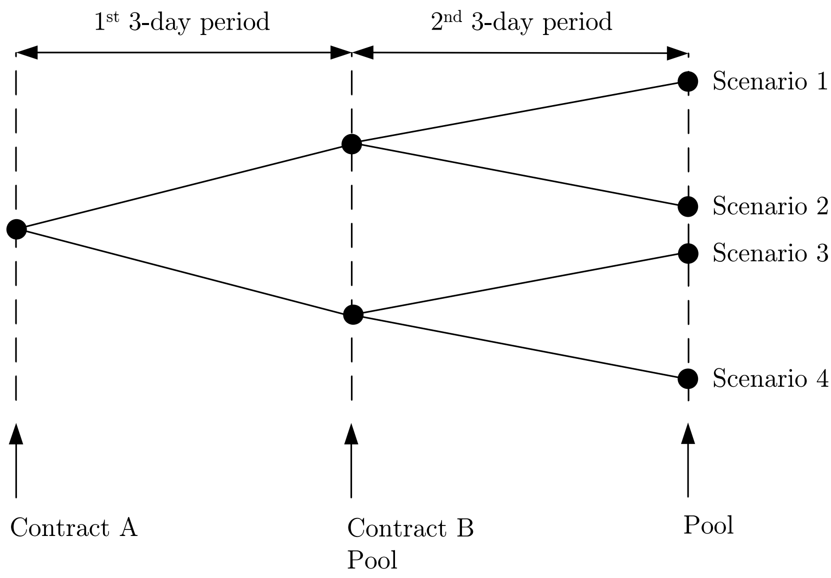


Fig. 2.6 Illustrative Example 2.8: scenario tree

$$
\begin{aligned}
&\text{Maximize}_{P_1^{\mathrm{A}},P_2^{\mathrm{A}},P_3^{\mathrm{A}},P_4^{\mathrm{A}},P_1^{\mathrm{B}},P_2^{\mathrm{B}},P_3^{\mathrm{B}},P_4^{\mathrm{B}},P_1^{t_1},P_2^{t_1},P_3^{t_1},P_4^{t_1},P_1^{t_2},P_2^{t_2},P_3^{t_2},P_4^{t_2}} \\
&\Pi^{\mathrm{S}} = \\
&0.16 \times 72(2 \times 25P_1^{\mathrm{A}} + 26P_1^{\mathrm{B}} + 27P_1^{t_1} + 28P_1^{t_2})+ \\
&0.24 \times 72(2 \times 25P_2^{\mathrm{A}} + 26P_2^{\mathrm{B}} + 27P_2^{t_1} + 26P_2^{t_2})+ \\
&0.24 \times 72(2 \times 25P_3^{\mathrm{A}} + 26P_3^{\mathrm{B}} + 23P_3^{t_1} + 24P_3^{t_2})+ \\
&0.36 \times 72(2 \times 25P_4^{\mathrm{A}} + 26P_4^{\mathrm{B}} + 23P_4^{t_1} + 22P_4^{t_2})
\end{aligned}
$$

subject to

$$
\begin{aligned}
&P_1^{\mathrm{A}} + P_1^{t_1} \leq 120 \\
&P_2^{\mathrm{A}} + P_2^{t_1} \leq 120 \\
&P_3^{\mathrm{A}} + P_3^{t_1} \leq 120 \\
&P_4^{\mathrm{A}} + P_4^{t_1} \leq 120
\end{aligned}
$$

$$
\begin{aligned}
&P_1^{\mathrm{A}} + P_1^{\mathrm{B}} + P_1^{t_2} \leq 120 \\
&P_2^{\mathrm{A}} + P_2^{\mathrm{B}} + P_2^{t_2} \leq 120 \\
&P_3^{\mathrm{A}} + P_3^{\mathrm{B}} + P_3^{t_2} \leq 120 \\
&P_4^{\mathrm{A}} + P_4^{\mathrm{B}} + P_4^{t_2} \leq 120
\end{aligned}
$$

$$
\begin{aligned}
&0 \le P_1^{\mathrm{A}}, P_2^{\mathrm{A}}, P_3^{\mathrm{A}}, P_4^{\mathrm{A}} \le 50 \\
&0 \le P_1^{\mathrm{B}}, P_2^{\mathrm{B}}, P_3^{\mathrm{B}}, P_4^{\mathrm{B}} \le 40 \\
&0 \le P_1^{t_1}, P_2^{t_1}, P_3^{t_1}, P_4^{t_1}, P_1^{t_2}, P_2^{t_2}, P_3^{t_2}, P_4^{t_2}
\end{aligned}
$$

$$
\begin{aligned}
&P_1^{\mathrm{A}} = P_2^{\mathrm{A}} = P_3^{\mathrm{A}} = P_4^{\mathrm{A}} \\
&P_1^{\mathrm{B}} = P_2^{\mathrm{B}}, P_3^{\mathrm{B}} = P_4^{\mathrm{B}} \\
&P_1^{t_1} = P_2^{t_1}, P_3^{t_1} = P_4^{t_1} \ .
\end{aligned}
$$

Variables are described below:

1. Variable Π^{S} is the expected profit. Superscript S stands for stochastic.
2. Variables $P_1^{\mathrm{A}}, P_2^{\mathrm{A}}, P_3^{\mathrm{A}}$, and P_4^{A} represent the power to be sold through the contract spanning the whole 6-day period, and correspond to scenarios $1, 2, 3$, and 4, respectively.
3. Variables $P_1^{\mathrm{B}}, P_2^{\mathrm{B}}, P_3^{\mathrm{B}}$, and P_4^{B} represent the power to be sold through the contract spanning the second 3-day period, and correspond to scenarios $1, 2, 3$, and 4, respectively.
4. Variables $P_1^{t_1}, P_2^{t_1}, P_3^{t_1}$, and $P_4^{t_1}$ represent the power to be sold in the pool that spans the first 3-day period, and correspond to scenarios $1, 2, 3$, and 4, respectively.
5. Variables $P_1^{t_2}, P_2^{t_2}, P_3^{t_2}$, and $P_4^{t_2}$ represent the power to be sold in the pool during the second 3-day period, and correspond to scenarios $1, 2, 3$, and 4, respectively.

The objective function is the expected profit computed by adding the profit in each scenario multiplied by its probability.

Constraints are described below:

1. The first block of constraints enforce the capacity limit of the producer (120 MW) in the first 3-day period for each of the four scenarios.
2. The second block of constraints enforce the capacity limit of the producer (120 MW) in the second 3-day period for each of the four scenarios.
3. The third block of constraints enforce contract bounds and declare all variables as non-negative.
4. The last block of constraints constitute non-anticipativity conditions. The first of these constraints corresponds to first-stage decisions while the last four ones (two final rows) correspond to second-stage decisions.

The solution to this problem is:

1. $P_1^{\mathrm{A}*} = P_2^{\mathrm{A}*} = P_3^{\mathrm{A}*} = P_4^{\mathrm{A}*} = 50$
2. $P_1^{\mathrm{B}*} = P_2^{\mathrm{B}*} = 0$, $P_3^{\mathrm{B}*} = P_4^{\mathrm{B}*} = 40$
3. $P_1^{t_1*} = P_2^{t_1*} = 70$, $P_3^{t_1*} = P_4^{t_1*} = 70$

4. $P_1^{t_2*} = 70$, $P_2^{t_2*} = 70$, $P_3^{t_2*} = 30$, $P_4^{t_2*} = 30$.

This means that before the 6-day period, the producer sells 50 MW during the whole period using the available bilateral contract, and it sells 0 or 40 MW during the second 3-day period depending on whether the price during the first 3-day period is 27 or \$23/MWh.

Through the pool and spanning the first 3-day period, the producer sells 70 MW irrespective of the pool price realization in this period. In contrast, it sells 70 or 30 MW during the second 3-day period depending on whether the pool price realization in the first one is 27 or \$23/MWh, respectively.

□

The example below provides an example of the node-variable formulation for a multi-stage stochastic programming problem.

Illustrative Example 2.9 (Three-stage problem. Node-variable formulation).

For the sake of completeness, the problem in the illustrative Example 2.8 is formulated below using a node-variable formulation,

$$
\begin{aligned}
&\text{Maximize}_{P^{\text{A}}, P_1^{\text{B}}, P_3^{\text{B}}, P_1^{t_1}, P_3^{t_1}, P_1^{t_2}, P_2^{t_2}, P_3^{t_2}, P_4^{t_2}} \\
&\Pi^{\text{S}} = 25 \times 144 P^{\text{A}} + \\
&0.4 \times 72(26 P_1^{\text{B}} + 27 P_1^{t_1} + 0.4 \times 28 P_1^{t_2} + 0.6 \times 26 P_2^{t_2}) + \\
&0.6 \times 72(26 P_3^{\text{B}} + 23 P_3^{t_1} + 0.4 \times 24 P_3^{t_2} + 0.6 \times 22 P_4^{t_2}) \\
&\text{subject to} \\
&P^{\text{A}} + P_1^{t_1} \le 120 \\
&P^{\text{A}} + P_3^{t_1} \le 120 \\
&P^{\text{A}} + P_1^{\text{B}} + P_1^{t_2} \le 120 \\
&P^{\text{A}} + P_1^{\text{B}} + P_2^{t_2} \le 120 \\
&P^{\text{A}} + P_3^{\text{B}} + P_3^{t_2} \le 120 \\
&P^{\text{A}} + P_3^{\text{B}} + P_4^{t_2} \le 120 \\
&0 \le P^{\text{A}} \le 50 \\
&0 \le P_1^{\text{B}}, P_3^{\text{B}} \le 40 \\
&0 \le P_1^{t_1}, P_3^{t_1}, P_1^{t_2}, P_2^{t_2}, P_3^{t_2}, P_4^{t_2} \ .
\end{aligned}
$$

Variables are described next:

1. Variable P^{A} represents the power to be sold through the contract spanning the whole 6-day period.
2. Variables P_1^{B} and P_3^{B} represent the power to be sold through the contract spanning the second 3-day period, and correspond to the two possible price realizations during the first 3-day period, 27 or \$23/MWh,

respectively. Subscripts 1 and 3 are used for variable consistency with Illustrative Example 2.8.

3. Variables $P_1^{t_1}$ and $P_3^{t_1}$ represent the power to be sold in the pool during the first 3-day period, and correspond to the two possible price realizations in this period, 27 or \$23/MWh, respectively. Subscripts 1 and 3 are used for variable consistency with Illustrative Example 2.8.
4. Variables $P_1^{t_2}, P_2^{t_2}, P_3^{t_2}$, and $P_4^{t_2}$ represent the power to be sold in the pool that spans the second 3-day period, and correspond to scenarios $1, 2, 3$, and 4, respectively.

The objective function is the expected profit computed according to the scenario tree in Fig. 2.6 and the scenario data in Table 2.4.

Constraints include a production capacity limit per scenario (first six constraints), bounds on contracts, and non-negativity declarations for all variables.

The solution to this problem is identical to that of the problem in Illustrative Example 2.8.

□

2.6 Quality Metrics

It is always a good practice to justify adequately why stochastic programming problems are solved instead of deterministic ones. Deterministic problems are obtained out of stochastic ones by replacing the random variables of the considered stochastic processes by their expected or forecast values. Thus, deterministic problems are simpler and easier to solve than stochastic ones. It is also important to assess how relevant is to have accurate forecasting procedures to identify the most likely scenarios.

Two metrics are usually used to appraise the interest of using stochastic programming models: the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS). These measures, which are widely used in two-stage stochastic programming problems [14], are explained next. The VSS and the EVPI for three-stage problems are also dealt with below through illustrative examples.

The extension of the VSS metric to multi-stage stochastic programming problems is thoroughly studied in [46].

Additionally, *out-of-sample* techniques should be used to asses the quality of the outcomes provided by the solutions obtained using stochastic programming models. In short, an out-of-sample assessment consists in evaluating these optimal solutions in terms of the actual outcomes that they provide using real-world data or data from a comprehensive external model.

2.6.1 Expected Value of Perfect Information

The expected value of perfect information represents the quantity that a decision maker is *willing to pay* for obtaining perfect information about the future. It constitutes a proxy for the value of accurate forecasts.

Assuming a maximization two-stage stochastic programming problem, we denote by $z^{\mathrm{S}*}$ its objective function optimal value, which represents the expected value of profit over all scenarios. Superscript S stands for stochastic.

On the other hand, $z^{\mathrm{P}*}$ is the objective function optimal value of the two-stage stochastic programming problem in which non-anticipativity constraints are relaxed (a scenario-variable formulation is assumed). Superscript P stands for perfect information. In other words, decisions are made with perfect information. This solution is usually known as the wait-and-see solution [14].

The expected value of perfect information (EVPI) is then computed as

$$\mathrm{EVPI}_{\mathrm{max}} = z^{\mathrm{P}*} - z^{\mathrm{S}*}. \tag{2.31}$$

Note that for minimization problems EVPI is given by

$$\mathrm{EVPI}_{\mathrm{min}} = z^{\mathrm{S}*} - z^{\mathrm{P}*}. \tag{2.32}$$

EVPI calculations are clarified through Illustrative Examples 2.10 and 2.11. The two-stage case corresponds to Illustrative Example 2.10 below.

Illustrative Example 2.10 (EVPI for a two-stage problem).

We consider the consumer problem stated in Illustrative Example 2.7. By relaxing the non-anticipativity constraints, the perfect information problem to be solved is

$$\begin{aligned}
&\mathrm{Minimize}_{P_1^{\mathrm{C}},P_2^{\mathrm{C}},P_3^{\mathrm{C}},P_1,P_2,P_3}\\
&C^{\mathrm{P}} = 0.2 \times 168(45P_1^{\mathrm{C}} + 50P_1)\\
&\qquad + 0.6 \times 168(45P_2^{\mathrm{C}} + 46P_2) + 0.2 \times 168(45P_3^{\mathrm{C}} + 44P_3)\\
&\text{subject to}\\
&P_1^{\mathrm{C}} + P_1 \geq 110\\
&P_2^{\mathrm{C}} + P_2 \geq 100\\
&P_3^{\mathrm{C}} + P_3 \geq 80\\
&0 \leq P_1^{\mathrm{C}}, P_2^{\mathrm{C}}, P_3^{\mathrm{C}} \leq 90\\
&0 \leq P_1, P_2, P_3\ ,
\end{aligned}$$

whose solution is $C^{\mathrm{P}*} = 742{,}560$, $P_1^{\mathrm{C}*} = 90$, $P_2^{\mathrm{C}*} = 90$, $P_3^{\mathrm{C}*} = 0$, and $P_1^* = 20$, $P_2^* = 10$, $P_3^* = 80$.

The value of EVPI is thus

$$\text{EVPI} = C^{\text{S}*} - C^{\text{P}*} = 747{,}936 - 742{,}560 = \$5376$$

where $C^{\text{S}*}$ is the objective function optimal value obtained from the solution of the stochastic programming problem in Illustrative Example 2.7. □

Illustrative Example 2.11 below clarifies EVPI calculations for a three-stage stochastic programming problem.

Illustrative Example 2.11 (EVPI for a three-stage problem).
We consider the producer problem stated in Illustrative Example 2.8.

Assuming that there is no uncertainty pertaining to the first stage, the perfect information problem to be solved becomes

$$\begin{aligned}
&\text{Maximize}_{P_1^{\text{A}},P_2^{\text{A}},P_3^{\text{A}},P_4^{\text{A}},P_1^{\text{B}},P_2^{\text{B}},P_3^{\text{B}},P_4^{\text{B}},P_1^{t_1},P_2^{t_1},P_3^{t_1},P_4^{t_1},P_1^{t_2},P_2^{t_2},P_3^{t_2},P_4^{t_2}}\\
&\Pi^{\text{P}_1} = \\
&0.16 \times 72(2 \times 25P_1^{\text{A}} + 26P_1^{\text{B}} + 27P_1^{t_1} + 28P_1^{t_2})+\\
&0.24 \times 72(2 \times 25P_2^{\text{A}} + 26P_2^{\text{B}} + 27P_2^{t_1} + 26P_2^{t_2})+\\
&0.24 \times 72(2 \times 25P_3^{\text{A}} + 26P_3^{\text{B}} + 23P_3^{t_1} + 24P_3^{t_2})+\\
&0.36 \times 72(2 \times 25P_4^{\text{A}} + 26P_4^{\text{B}} + 23P_4^{t_1} + 22P_4^{t_2})\\
&\text{subject to}\\
&P_1^{\text{A}} + P_1^{t_1} \le 120\\
&P_2^{\text{A}} + P_2^{t_1} \le 120\\
&P_3^{\text{A}} + P_3^{t_1} \le 120\\
&P_4^{\text{A}} + P_4^{t_1} \le 120\\
&P_1^{\text{A}} + P_1^{\text{B}} + P_1^{t_2} \le 120\\
&P_2^{\text{A}} + P_2^{\text{B}} + P_2^{t_2} \le 120\\
&P_3^{\text{A}} + P_3^{\text{B}} + P_3^{t_2} \le 120\\
&P_4^{\text{A}} + P_4^{\text{B}} + P_4^{t_2} \le 120\\
&0 \le P_1^{\text{A}}, P_2^{\text{A}}, P_3^{\text{A}}, P_4^{\text{A}} \le 50\\
&0 \le P_1^{\text{B}}, P_2^{\text{B}}, P_3^{\text{B}}, P_4^{\text{B}} \le 40\\
&0 \le P_1^{t_1}, P_2^{t_1}, P_3^{t_1}, P_4^{t_1}, P_1^{t_2}, P_2^{t_2}, P_3^{t_2}, P_4^{t_2}\\
&P_1^{\text{A}} = P_2^{\text{A}}, P_3^{\text{A}} = P_4^{\text{A}}\\
&P_1^{\text{B}} = P_2^{\text{B}}, P_3^{\text{B}} = P_4^{\text{B}}\\
&P_1^{t_1} = P_2^{t_1}, P_3^{t_1} = P_4^{t_1}\ .
\end{aligned}$$

Note that the non-anticipativity constraints have been modified to enforce that there is no uncertainty in the first stage. The objective function optimal value of the problem above is $\Pi^{\text{P}1*} = \$437{,}961.6$.

If we further assume that there is no uncertainty in both the first and second stages, then the perfect information problem to be solved becomes

$$
\begin{aligned}
&\text{Maximize}_{P_1^{\mathrm{A}},P_2^{\mathrm{A}},P_3^{\mathrm{A}},P_4^{\mathrm{A}},P_1^{\mathrm{B}},P_2^{\mathrm{B}},P_3^{\mathrm{B}},P_4^{\mathrm{B}},P_1^{t_1},P_2^{t_1},P_3^{t_1},P_4^{t_1},P_1^{t_2},P_2^{t_2},P_3^{t_2},P_4^{t_2}}\\
&\Pi^{\mathrm{P}_2} =\\
&0.16 \times 72(2 \times 25P_1^{\mathrm{A}} + 26P_1^{\mathrm{B}} + 27P_1^{t_1} + 28P_1^{t_2})+\\
&0.24 \times 72(2 \times 25P_2^{\mathrm{A}} + 26P_2^{\mathrm{B}} + 27P_2^{t_1} + 26P_2^{t_2})+\\
&0.24 \times 72(2 \times 25P_3^{\mathrm{A}} + 26P_3^{\mathrm{B}} + 23P_3^{t_1} + 24P_3^{t_2})+\\
&0.36 \times 72(2 \times 25P_4^{\mathrm{A}} + 26P_4^{\mathrm{B}} + 23P_4^{t_1} + 22P_4^{t_2})\\
&\text{subject to}\\
&P_1^{\mathrm{A}} + P_1^{t_1} \le 120\\
&P_2^{\mathrm{A}} + P_2^{t_1} \le 120\\
&P_3^{\mathrm{A}} + P_3^{t_1} \le 120\\
&P_4^{\mathrm{A}} + P_4^{t_1} \le 120\\
&P_1^{\mathrm{A}} + P_1^{\mathrm{B}} + P_1^{t_2} \le 120\\
&P_2^{\mathrm{A}} + P_2^{\mathrm{B}} + P_2^{t_2} \le 120\\
&P_3^{\mathrm{A}} + P_3^{\mathrm{B}} + P_3^{t_2} \le 120\\
&P_4^{\mathrm{A}} + P_4^{\mathrm{B}} + P_4^{t_2} \le 120\\
&0 \le P_1^{\mathrm{A}}, P_2^{\mathrm{A}}, P_3^{\mathrm{A}}, P_4^{\mathrm{A}} \le 50\\
&0 \le P_1^{\mathrm{B}}, P_2^{\mathrm{B}}, P_3^{\mathrm{B}}, P_4^{\mathrm{B}} \le 40\\
&0 \le P_1^{t_1}, P_2^{t_1}, P_3^{t_1}, P_4^{t_1}, P_1^{t_2}, P_2^{t_2}, P_3^{t_2}, P_4^{t_2}\ .
\end{aligned}
$$

The objective function optimal value of the problem above is $\Pi^{\mathrm{P2}*} =$ \$437,961.6.

Given the maximization nature of the two previous problems, it holds that

$$\Pi^{\mathrm{P2}*} \ge \Pi^{\mathrm{P1}*} \ge \Pi^{\mathrm{S}*},$$

where $\Pi^{\mathrm{S}*} = \$432{,}489.6$ is the objective function optimal value for the stochastic problem in Illustrative Example 2.8.

The EVPIs pertaining to the uncertainties affecting stages 1 and 2 are

$$
\begin{aligned}
\mathrm{EVPI}_1 &= \Pi^{\mathrm{P2}*} - \Pi^{\mathrm{P1}*} = 437{,}961.6 - 437{,}961.6 = 0\\
\mathrm{EVPI}_2 &= \Pi^{\mathrm{P1}*} - \Pi^{\mathrm{S}*} = 437{,}961.6 - 432{,}489.6 = \$5472.
\end{aligned}
$$

Finally,

$$\mathrm{EVPI} = \mathrm{EVPI}_1 + \mathrm{EVPI}_2 = \$5472\ .$$

□

2.6.2 Value of the Stochastic Solution

The value of the stochastic solution is a measure to quantify the advantage of using a stochastic programming approach over a deterministic one.

In the deterministic problem associated with a stochastic programming problem, the random variables of the considered stochastic processes are replaced by their respective expected values. The solution to this deterministic problem provides optimal values for the first-stage variables. The original stochastic programming problem can then be solved fixing the values of the first-stage variables to those provided by the deterministic one. This modified problem decomposes by scenario and is generally easy to solve. The optimal objective function value of the modified stochastic problem (with fixed first-stage decisions) is denoted by $z^{\mathrm{D}*}$. Superscript D stands for deterministic.

The value of the stochastic solution (VSS) is then calculated as

$$\mathrm{VSS}_{\mathrm{max}} = z^{\mathrm{S}*} - z^{\mathrm{D}*}, \tag{2.33}$$

where $z^{\mathrm{S}*}$ is the optimal value of the objective function of the problem defined in Subsection 2.6.1. The VSS provides a measure of the gain obtained from modeling random variables as such, avoiding to replace them with average values.

For minimization problems, the VSS becomes

$$\mathrm{VSS}_{\mathrm{min}} = z^{\mathrm{D}*} - z^{\mathrm{S}*}. \tag{2.34}$$

Illustrative Example 2.12 below clarifies the VSS computation for a two-stage stochastic programming problem.

Illustrative Example 2.12 (VSS for a two-stage problem).

Consider the consumer problem stated in Illustrative Example 2.6. The average scenario is characterized as

$$0.2 \times 110 + 0.6 \times 100 + 0.2 \times 80 = 98 \text{ MWh}$$
$$0.2 \times 50 + 0.6 \times 46 + 0.2 \times 44 = \$46.4/\text{MWh} .$$

The problem related to this average scenario is

$$
\begin{aligned}
&\text{Minimize}_{P^{\mathrm{C}},P}\\
&168(45P^{\mathrm{C}} + 46.4P)\\
&\text{subject to}\\
&P^{\mathrm{C}} + P \geq 98\\
&0 \leq P^{\mathrm{C}} \leq 90\\
&0 \leq P\ ,
\end{aligned}
$$

being the optimal value of the first-stage variable $P_{\mathrm{D}}^{\mathrm{C}} = 90$ MW, where the additional subscript D stands for deterministic. This is the quantity to be bought by the consumer at the beginning of the week using the bilateral contract. The subsequent multi-scenario problem to be solved with fixed first-stage decision is

$$
\begin{aligned}
&\text{Minimize}_{P_1,P_2,P_3}\\
&C^{\mathrm{D}} = 168(45P_{\mathrm{D}}^{\mathrm{C}} + 0.2 \times 50P_1 + 0.6 \times 46P_2 + 0.2 \times 44P_3)\\
&\text{subject to}\\
&P_1 \geq 110 - P_{\mathrm{D}}^{\mathrm{C}}\\
&P_2 \geq 100 - P_{\mathrm{D}}^{\mathrm{C}}\\
&P_3 \geq 80 - P_{\mathrm{D}}^{\mathrm{C}}\\
&0 \leq P_1, P_2, P_3\ .
\end{aligned}
$$

The solution to this problem is $P_1^* = 20$, $P_2^* = 10$, $P_3^* = 0$, which means that the consumer buys during the week 20, 10 and 0 MW for demands 110, 100 and 80 MW, respectively.

Note that the problem above decomposes by scenario and renders the trivial problems below:

$$
\begin{aligned}
&\text{Minimize}_{P_1}\\
&C_1^{\mathrm{D}} = 168(0.2 \times 50P_1)\\
&\text{subject to}\\
&P_1 \geq 110 - P_{\mathrm{D}}^{\mathrm{C}}\\
&0 \leq P_1,
\end{aligned}
$$

$$
\begin{aligned}
&\text{Minimize}_{P_2}\\
&C_2^{\mathrm{D}} = 168(0.6 \times 46P_2)\\
&\text{subject to}\\
&P_2 \geq 100 - P_{\mathrm{D}}^{\mathrm{C}}\\
&0 \leq P_2,
\end{aligned}
$$

$$
\begin{aligned}
&\text{Minimize}_{P_3}\\
&C_3^{\mathrm{D}} = 168(0.2 \times 44 P_3)\\
&\text{subject to}\\
&P_3 \geq 80 - P_{\mathrm{D}}^{\mathrm{C}}\\
&0 \leq P_3,
\end{aligned}
$$

being $C^{\mathrm{D}} = 168(45 P_{\mathrm{D}}^{\mathrm{C}}) + C_1^{\mathrm{D}} + C_2^{\mathrm{D}} + C_3^{\mathrm{D}}$.

Finally, the VSS is computed as

$$\mathrm{VSS} = C^{\mathrm{D}*} - C^{\mathrm{S}*} = 760{,}368 - 747{,}936 = \$12{,}432\ .$$

where $C^{\mathrm{S}*} = \$747{,}936$ is the objective function optimal value obtained from the solution to the stochastic programming problem of Illustrative Example 2.6. □

Illustrative Example 2.13 below clarifies the VSS computation for a three-stage stochastic programming problem.

Illustrative Example 2.13 (VSS for a three-stage problem).

We consider the producer problem stated in Illustrative Example 2.9.

If the price during the first 3-day period is \$27/MWh, the average price for the second 3-day period is

$$0.4 \times 28 + 0.6 \times 26 = \$26.8/\mathrm{MWh}.$$

On the other hand, if the price during the first 3-day period is \$23/MWh, the average price for the second 3-day period is

$$0.4 \times 24 + 0.6 \times 22 = \$22.8/\mathrm{MWh}.$$

The resulting scenario tree is depicted in Fig. 2.7

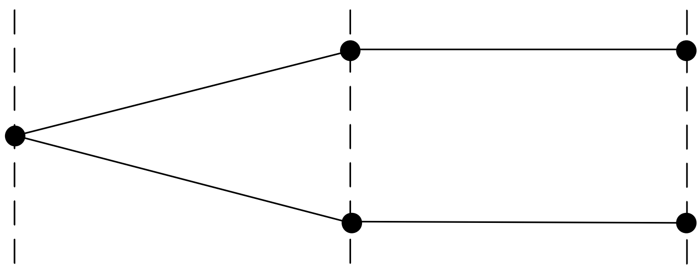


Fig. 2.7 Illustrative Example 2.13: tree for the VSS first calculation

Taking into account these average prices, the decision-making problem becomes

$$\begin{aligned}
&\text{Maximize}_{P^{\mathrm{A}},P_1^{\mathrm{B}},P_3^{\mathrm{B}},P_1^{t_1},P_3^{t_1},P_1^{t_2},P_2^{t_2}}\\
&25 \times 144 P^{\mathrm{A}} +\\
&0.4 \times 72(26 P_1^{\mathrm{B}} + 27 P_1^{t_1} + 26.8 P_1^{t_2}) +\\
&0.6 \times 72(26 P_3^{\mathrm{B}} + 23 P_3^{t_1} + 22.8 P_2^{t_2})\\
&\text{subject to}\\
&P^{\mathrm{A}} + P_1^{t_1} \le 120\\
&P^{\mathrm{A}} + P_3^{t_1} \le 120\\
&P^{\mathrm{A}} + P_1^{\mathrm{B}} + P_1^{t_2} \le 120\\
&P^{\mathrm{A}} + P_3^{\mathrm{B}} + P_2^{t_2} \le 120\\
&0 \le P^{\mathrm{A}} \le 50\\
&0 \le P_1^{\mathrm{B}}, P_3^{\mathrm{B}} \le 40\\
&0 \le P_1^{t_1}, P_3^{t_1}, P_1^{t_2}, P_2^{t_2}\ .
\end{aligned}$$

The solution to this problem provides the selling quantity through the 6-period contract, which is $P_{\mathrm{D1}}^{\mathrm{A}} = 50$ MW (subscript D stands for deterministic and subscript 1 stands for first VSS problem). This value is fixed in the multi-scenario problem below:

$$\begin{aligned}
&\text{Maximize}_{P_1^{\mathrm{B}},P_3^{\mathrm{B}},P_1^{t_1},P_3^{t_1},P_1^{t_2},P_2^{t_2},P_3^{t_2},P_4^{t_2}}\\
&\Pi^{\mathrm{D1}} = 25 \times 144 P_{\mathrm{D1}}^{\mathrm{A}} +\\
&0.4 \times 72(26 P_1^{\mathrm{B}} + 27 P_1^{t_1} + 0.4 \times 28 P_1^{t_2} + 0.6 \times 26 P_2^{t_2}) +\\
&0.6 \times 72(26 P_3^{\mathrm{B}} + 23 P_3^{t_1} + 0.4 \times 24 P_3^{t_2} + 0.6 \times 22 P_4^{t_2})\\
&\text{subject to}\\
&P_1^{t_1} \le 120 - P_{\mathrm{D1}}^{\mathrm{A}}\\
&P_3^{t_1} \le 120 - P_{\mathrm{D1}}^{\mathrm{A}}\\
&P_1^{\mathrm{B}} + P_1^{t_2} \le 120 - P_{\mathrm{D1}}^{\mathrm{A}}\\
&P_1^{\mathrm{B}} + P_2^{t_2} \le 120 - P_{\mathrm{D1}}^{\mathrm{A}}\\
&P_3^{\mathrm{B}} + P_3^{t_2} \le 120 - P_{\mathrm{D1}}^{\mathrm{A}}\\
&P_3^{\mathrm{B}} + P_4^{t_2} \le 120 - P_{\mathrm{D1}}^{\mathrm{A}}\\
&0 \le P_1^{\mathrm{B}}, P_3^{\mathrm{B}} \le 40\\
&0 \le P_1^{t_1}, P_3^{t_1}, P_1^{t_2}, P_2^{t_2}, P_3^{t_2}, P_4^{t_2}\ .
\end{aligned}$$

The objective function optimal value of the problem above is $\Pi^{\mathrm{D1}*} =$ \$432,489.6.

Next, let us consider the average prices in both the first and second stages,

$$0.4 \times 27 + 0.6 \times 23 = \$24.6/\text{MWh}$$
$$0.16 \times 28 + 0.24 \times 26 + 0.24 \times 24 + 0.36 \times 22 = \$24.4/\text{MWh} .$$

The corresponding scenario tree is depicted in Fig. 2.8.

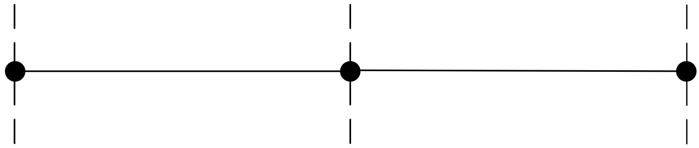


Fig. 2.8 Illustrative Example 2.13: tree for the VSS second calculation

For these average prices, the decision-making problem becomes

$$\begin{aligned}
&\text{Maximize}_{P^{\mathrm{A}}, P^{\mathrm{B}}, P^{t_1}, P^{t_2}} \\
&25 \times 144 P^{\mathrm{A}} + 72(26 P^{\mathrm{B}} + 24.6 P^{t_1} + 24.4 P^{t_2}) \\
&\text{subject to} \\
&P^{\mathrm{A}} + P^{t_1} \leq 120 \\
&P^{\mathrm{A}} + P^{\mathrm{B}} + P^{t_2} \leq 120 \\
&0 \leq P^{\mathrm{A}} \leq 50 \\
&0 \leq P^{\mathrm{B}} \leq 40 \\
&0 \leq P^{t_1}, P^{t_2} .
\end{aligned}$$

The solution to this problem provides the selling quantities through the 6-period and the 3-period contracts, which are $P^{\mathrm{A}}_{\mathrm{D2}} = 50$ and $P^{\mathrm{B}}_{\mathrm{D2}} = 40$, and the selling quantity in the pool during the first 3-period, which is $P^{t_1}_{\mathrm{D2}} = 70$ (subscript D stands for deterministic and subscript 2 stands for second VSS problem). These values are fixed in the multi-scenario problem below:

$$
\begin{aligned}
&\text{Maximize}_{P_1^{t_2}, P_2^{t_2}, P_3^{t_2}, P_4^{t_2}}\\
&\Pi^{\mathrm{D2}} = 25 \times 144 P_{\mathrm{D2}}^{\mathrm{A}} +\\
&0.4 \times 72(26 P_{\mathrm{D2}}^{B} + 27 P_{\mathrm{D2}}^{t_1} + 0.4 \times 28 P_1^{t_2} + 0.6 \times 26 P_2^{t_2}) +\\
&0.6 \times 72(26 P_{\mathrm{D2}}^{\mathrm{B}} + 23 P_{\mathrm{D2}}^{t_1} + 0.4 \times 24 P_3^{t_2} + 0.6 \times 22 P_4^{t_2})\\
&\text{subject to}\\
&P_1^{t_2} \le 120 - P_{\mathrm{D2}}^{\mathrm{A}} - P_{\mathrm{D2}}^{B}\\
&P_2^{t_2} \le 120 - P_{\mathrm{D2}}^{\mathrm{A}} - P_{\mathrm{D2}}^{B}\\
&P_3^{t_2} \le 120 - P_{\mathrm{D2}}^{\mathrm{A}} - P_{\mathrm{D2}}^{B}\\
&P_4^{t_2} \le 120 - P_{\mathrm{D2}}^{\mathrm{A}} - P_{\mathrm{D2}}^{B}\\
&0 \le P_1^{t_2}, P_2^{t_2}, P_3^{t_2}, P_4^{t_2} \ .
\end{aligned}
$$

The objective function optimal value of the problem above is $\Pi^{\mathrm{D2}*} = \$431{,}568$.

Given the maximization nature of the problem considered, it follows that

$$\Pi^{\mathrm{S}*} \ge \Pi^{\mathrm{D1}*} \ge \Pi^{\mathrm{D2}*} \ ,$$

where $\Pi^{\mathrm{S}*} = \$432{,}489.6$ is the objective function optimal value of the stochastic programming problem of Illustrative Example 2.9. Thus, the VSS values are

$$
\begin{aligned}
\mathrm{VSS}_1 &= \Pi^{\mathrm{D1}} - \Pi^{\mathrm{D2}} = 432{,}489.6 - 431{,}568 = \$921.6\\
\mathrm{VSS}_2 &= \Pi^{\mathrm{S}} - \Pi^{\mathrm{D1}} = 432{,}489.6 - 432{,}489.6 = 0
\end{aligned}
$$

and

$$\mathrm{VSS} = \mathrm{VSS}_1 + \mathrm{VSS}_2 = \$921.6 \ .$$

□

2.6.3 Out-of-Sample Assessment

The purpose of an out-of-sample assessment is to evaluate the effectiveness of the decisions obtained by solving a stochastic programming problem in terms of actual real-world outcomes. If real-world data are not available, a comprehensive simulation model can be used instead.

Additionally, out-of-sample simulations can be used to compare the effectiveness in terms of actual outcomes of alternative decisions, e.g., the decision obtained by solving a stochastic programming model and that obtained via a simplified deterministic one.

Roughly speaking, an out-of-sample simulation works as follows:

1. Solve a stochastic programming model to obtain the decision to be made at time t with all the available information up to time $t-1$. For instance, a producer derives offering curves for the day-ahead market (of certain electricity market) pertaining to day d with all the data available prior to the clearing of that market. For that, it uses a stochastic programming model.
2. Implement the decision obtained by solving the stochastic programming problem of item 1 above, and observe the outcome obtained that corresponds to time t. For instance, the producer does offer in the market the curves obtained in step 1 above and observes the actual profit it obtains.
3. Steps 1 and 2 above are repeated for times $t = 1, \ldots, T$ to statistically characterize the resulting outcome. T should be large enough. This way the producer in the considered example characterizes its profit.

Note that the above procedure can be carried out using historical data, which allows evaluating off-line the effectiveness of alternative offering strategies. For the example above, the producer can use (price) data pertaining to the considered day-ahead market of 2005 in order to compare the effectiveness in terms of expected profit and profit standard deviation of two offering strategies, one obtained by solving a stochastic programming model and another one by solving a simplified deterministic model.

Since out-of-sample techniques allow characterizing the consequences of decisions in terms of outcome distributions, they provide a most effective way to evaluate the quality of a decision and of the decision tool used to generate it.

2.7 Risk

Typically, stochastic programming problems maximize (minimize) an objective function representing an expected profit (cost). These problems constitute *risk-neutral* models. That is, the decision maker only focuses on the expected value of the profit (cost), ignoring the rest of parameters characterizing the distribution of the profit (cost).

If risk is accounted for, the decision maker also assesses the profit values in the worst scenarios, in addition to the expected value of the profit. In this case, the decision maker becomes a *risk-averse* agent.

The concept of risk is illustrated in Fig. 2.9. This figure shows the probability mass functions of two profit variables denoted by Profit 1 and Profit 2. The expected value of both profit variables is the same, \$0.7. Therefore, both profits are equally good for a risk-neutral decision maker. However, note that Profit 1 is always positive, i.e., the decision maker does not experience loss in any scenario. On the other hand, the probability of having a negative profit

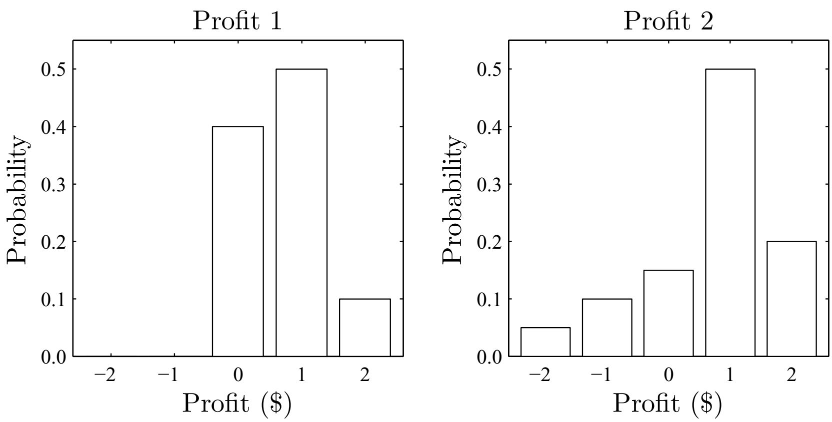


Fig. 2.9 Illustration of the concept of risk

is 0.15 in Profit 2. Hence, we may conclude that Profit 2 is riskier than Profit 1.

It is important to note that a variety of stochastic programming models can be set up depending on the relevance of the expected objective function value vs. its variability. As a consequence, the different decisions obtained by solving these models result in different outcomes in term of expected objective function value and objective function variability.

For example, a producer may derive market offering strategies pursuing maximum expected profit and disregarding profit variability. Alternatively, this producer may pursue maximum profit but limiting the variance of the profit distribution it achieves. Surely, the two strategies obtained by solving the two models above would generally be significantly different.

Since the risk-aversion level is an input parameter for all the considered models, a decision maker should know in advance the level of risk he/she is willing to assume.

Risk, risk measures and risk management procedures are comprehensively studied in Chapter 4.

2.8 Solving Stochastic Programming Problems

Since the size of a stochastic programming problem, measured in terms of its number of variables and constraints, grows with the number of scenarios, stochastic programming problems become easily large-scale involving millions of variables and constraints.

It is thus important to select carefully the number of scenarios to properly represent the stochastic processes involved. Chapter 3 considers the generation and reduction of scenarios to represent adequately stochastic processes. The interested reader is referred to that chapter.

However, stochastic programming problems present often an exploitable structure. Typically, the number of constraints involving variables across scenarios is comparatively small with respect to the number of variables and constraints pertaining to any particular scenario. These linking constraints are mostly non-anticipativity conditions. Note that this structure is particularly exploitable if a scenario-variable formulation (instead of a node-variable one) is used (see Illustrative Examples 2.8 and 2.9).

Thus, stochastic programming problems generally include complicating constraints, i.e., constraints that if relaxed render the resulting problem easy to solve as it decomposes by scenario. Therefore, decomposition techniques are particularly suited to tackle stochastic programming problems. A description of decomposition procedures for mathematical programming problems can be found in [26]. Decomposition procedures particularly adapted to linear stochastic programming problems are described in [80].

Some general rules on which solution technique to be used follow:

1. If the problem under consideration is linear and involves no discrete variables, its solution can be addressed directly for sizes up to a few millions variables/constrains.
2. If the problem is linear but involves discrete variables, depending on the number of discrete variables and the structure of the problem, either a direct solution technique or a decomposition procedure would be advisable.
3. Similarly, if the problem is nonlinear and continuous, depending on the number of variables and the structure of the problem, either a direct solution technique or a decomposition procedure would be advisable.
4. Finally, if the problem is nonlinear and involves continuous as well as discrete variables, a decomposition procedure is generally advisable.

As a consequence of the broad and versatile capabilities of currently available optimization solvers [141], throughout this book, the only solution technique actually considered is direct solution. For this reason, all models formulated are linear, mixed-integer linear or mixed-integer quadratic. Other solution approaches are outside the scope of this book. Nevertheless, the reader should be aware that a variety of effective solution techniques based on decomposition are reported in the technical literature [26, 79].

2.9 Summary and Conclusions

This chapter provides an overview of the fundamentals of stochastic programming. It is intended to build a stochastic programming base to be used throughout the following chapters.

First, random variables and stochastic processes are introduced and their differences clarified. Scenario generation and reduction are then briefly considered. Stochastic processes and scenario analysis are further examined in Chapter 3.

Second, stochastic programming problems are characterized and formulated. Emphasis is given to the commonly used two-stage problems.

Third, the expected value of perfect information (EVPI) and the value of the stochastic solution (VSS) are defined in order to assess the interest of using stochastic programming models. The convenience of using out-of-sample simulations is also pointed out.

The chapter closes illustrating the concept of risk (to be further treated in Chapter 4) and stating some basic ideas pertaining to solution approaches for stochastic programming problems.

Finally, the concluding remarks below are worth to mention:

1. Stochastic programming constitutes a useful tool to make decisions under uncertainty.
2. Two-stage stochastic programming models are particularly relevant for their practical interest, relative simplicity, and high versatility.
3. Uncertainty is properly described through stochastic processes, which in turn are conveniently characterized using scenarios.
4. A large enough number of scenarios needs to be generated to accurately represent a stochastic process.
5. More often than not the number of scenarios adequately describing a stochastic process is large, and thus the associated stochastic programming problem becomes computationally intractable.
6. Therefore, scenario-reduction techniques are needed to attain tractability while keeping as much as possible the stochastic information embedded in the original scenario set.

2.10 Exercises

Exercise 2.1. Characterize the random variable *price at 3 pm* in the electricity market of PJM considering historical data spanning one month of 2009.

PJM web: www.pjm.com.

Exercise 2.2. Characterize the stochastic process *hourly prices on Monday* in the electricity market of ISO New England considering historical data spanning one month of 2009.

ISO New England web: www.iso-ne.com.

Exercise 2.3. Formulate a two-stage problem involving a producer that sells energy through contracts and the pool, similar to the consumer problem in Illustrative Example 2.6. Discuss the results obtained.

Exercise 2.4. Draw a scenario tree and explain the decision process of the problem in Exercise 2.3.

Exercise 2.5. Compute the EVPI and the VSS for a two-stage problem involving a producer that sells energy through bilateral contracts and the pool (as in Exercise 2.3 above). Discuss the results obtained.

Exercise 2.6. Formulate a three-stage problem involving a consumer that buys energy through contracts and the pool, similar to the producer problem in Illustrative Example 2.8. Additionally, consider that the consumer owns a self-production facility of small size (with respect to its average demand). Discuss the results obtained.

Exercise 2.7. Draw a scenario tree and explain the decision process of the problem in Exercise 2.6.

Exercise 2.8. Compute the EVPIs and the VSSs for a three-stage problem involving a consumer that buys energy through bilateral contracts and the pool (as in Exercise 2.6 above). Discuss the results obtained.

Exercise 2.9. Describe two examples similar to that in Section 2.7 to illustrate the risk associated with the profit of a producer, and the risk associated with the cost of a consumer.

Exercise 2.10. Discuss possible mathematical programming decomposition alternatives for the problem in Illustrative Example 2.8.

Chapter 3
Uncertainty Characterization via Scenarios

3.1 Introduction

Many engineering and science problems are subject to uncertainty due to the inherent randomness of natural phenomena and/or to the imperfect knowledge of the variables determining the functional state of the human-created structures. In this context, computational methods that tackle uncertainty allow engineers and scientists to propose solutions less sensitive to environmental influences, while achieving simultaneously cost reduction, profit gains, and/or reliability improvement.

Decision-making problems related to electricity markets are not exempt from uncertainty. The own rules governing the functioning of these markets can be deemed responsible for the existence of uncertainties conditioning market agents' behavior. For instance, energy prices are known after producers and consumers submit their selling offers and purchasing bids, respectively, to the electricity market. As a result, decisions on the amount and price of the energy to be sold or purchased are irremediably made with inaccurate knowledge of the final market outcome. Likewise, the time gap existing between agreements on energy transactions and their physical implementation causes that a producer must face the trading process with a certain degree of uncertainty about the availability of its power sources.

In Chapter 2, stochastic programming was introduced as a efficient tool to optimize under uncertainty, that is, to find optimal decisions in problems involving uncertain data. Within a modeling framework based on stochastic programming, input data affected by uncertainty are conceptually described as stochastic processes. A stochastic process $\boldsymbol{\lambda}$ is defined as a collection of dependent random variables $\boldsymbol{\lambda} = \{\lambda_t, t \in T\}$. That is, for each t in the index set T, λ_t is a random variable. We often interpret t as time and call λ_t the state of the process at time t. A stochastic process $\boldsymbol{\lambda}$ is said to be *continuos* or *discrete* depending on whether its component random variables $\{\lambda_t, t \in T\}$ are continuos or discrete, respectively.

A. J. Conejo et al., *Decision Making Under Uncertainty in Electricity Markets*, International Series in Operations Research & Management Science 153, DOI 10.1007/978-1-4419-7421-1_3,

We present below examples of both a continuous and a discrete stochastic process.

Illustrative Example 3.1 (A continuos stochastic process). If we denote the wind speed in hour t at a given site A by v_t^A, then the set of random variables $\boldsymbol{V}^A = \{v_t^A, t = 1, 2, \ldots, 24\}$ describing wind speed at site A throughout a day is a continuous stochastic process. □

Illustrative Example 3.2 (A discrete stochastic process). Consider the binary variable u_{it}, which is equal to 1 if generating unit i is available in hour t and 0 otherwise. The collection of random variables $\boldsymbol{U}_i = \{u_{it}, t = 1, 2, \ldots, 168\}$ representing the availability of unit i throughout a week is a discrete stochastic process. □

Note that a discrete stochastic process can be represented by a finite set of actual vectors, referred to as *scenarios*, resulting from the combinations of all the discrete values that its component random variables can adopt. In mathematical terms, if $\boldsymbol{\lambda}$ is a discrete stochastic process, it can be expressed as $\boldsymbol{\lambda} = \{\boldsymbol{\lambda}(\omega), \omega = 1, 2, \ldots, N_\Omega\}$, where ω is the scenario index and N_Ω is the number of possible scenarios. In order for the discrete stochastic process to be perfectly determined, a probability of occurrence $\pi(\omega)$ needs to be associated with each realization $\boldsymbol{\lambda}(\omega)$ such that $\sum_{\omega=1}^{N_\Omega} \pi(\omega) = 1$.

The following example illustrates the mathematical representation of a discrete stochastic process.

Illustrative Example 3.3 (Mathematical representation of a discrete stochastic process). Let us consider the discrete stochastic process $\boldsymbol{U}_i$ representing the availability of generating unit i throughout a time horizon comprising two hourly periods. Process $\boldsymbol{U}_i$ can be mathematically expressed as $\boldsymbol{U}_i = \{\boldsymbol{U}_i(\omega) = \left[u_{i1}(\omega), u_{i2}(\omega)\right], \omega = 1, \ldots, 4\}$, where binary variable $u_{it}(\omega), t = 1, 2$, is equal to 1 if unit i is available in time period t and scenario ω, and 0 otherwise. Specifically, each scenario $\boldsymbol{U}_i(\omega)$ can be written in the form

$$
\begin{aligned}
\boldsymbol{U}_i(1) &= [1, 1], \text{ with a probability of occurrence } \pi(1) = 0.5 \\
\boldsymbol{U}_i(2) &= [0, 1], \text{ with } \pi(2) = 0.2 \\
\boldsymbol{U}_i(3) &= [1, 0], \text{ with } \pi(3) = 0.2 \\
\boldsymbol{U}_i(4) &= [0, 0], \text{ with } \pi(4) = 0.1.
\end{aligned}
$$

Note that the above probabilities of occurrence are merely illustrative. In practice, these probabilities are obtained through a rigorous analysis that accounts for the failure ratio of the generating unit under consideration. Further details on this issue can be found in Subsection 3.2.3.

□

Solving optimization problems where uncertainty on input data is modeled by continuous stochastic processes is very difficult, or even impossible in many cases. In contrast, discrete stochastic processes can be easily embedded into an optimization problem using the *deterministic equivalent problem* studied in Subsection 2.5.1 of Chapter 2. For this reason, prior to finding the solution of a stochatic programming problem, every continuous stochastic process is to be replaced by an approximate discrete one. For example, a continuos stochastic process $\boldsymbol{\gamma}$ can be approximated by a discrete one $\boldsymbol{\lambda}$ such that $\boldsymbol{\gamma} \approx \boldsymbol{\lambda} = \{\boldsymbol{\lambda}(\omega), \omega = 1, 2, \ldots, N_\Omega\}$, with each scenario $\boldsymbol{\lambda}(\omega)$ having an associated probability of occurrence equal to $\pi(\omega)$. Intuitively, a scenario can be understood as a plausible realization of the stochastic process that it represents. For simplicity, it is customary to simply state that the continuous stochastic process $\boldsymbol{\gamma}$ is approximated or represented through a set of scenarios $\{\boldsymbol{\gamma}(\omega), \omega = 1, 2, \ldots, N_\Omega\}$, where, in fact, $\boldsymbol{\gamma}(\omega) = \boldsymbol{\lambda}(\omega)$ for all ω.

The following example serves us to illustrate the discrete approximation of a continuous stochastic process.

Illustrative Example 3.4 (Approximation of a continuous process with a discrete one). After a comprehensive statistical analysis, we come to the conclusion that the continuous stochastic process $\boldsymbol{V}^A$, representing wind speed at site A throughout a time horizon of two hours, can be well approximated by the discrete process $\boldsymbol{\nu}^A = \{\boldsymbol{\nu}^A(\omega) = [v_1^A(\omega), v_2^A(\omega)], \omega = 1, 2, 3\}$, where variable $v_t^A(\omega), t = 1, 2$, represents wind speed (in m/s) at site A in period t and scenario ω. Specifically, each scenario $\boldsymbol{\nu}^A(\omega)$ is given by

$$
\begin{aligned}
\boldsymbol{\nu}^A(1) &= [5.5, 7.8] \text{ m/s, with a probability of occurrence } \pi(1) = 0.4 \\
\boldsymbol{\nu}^A(2) &= [10.0, 13.7] \text{ m/s, with } \pi(2) = 0.3 \\
\boldsymbol{\nu}^A(3) &= [1.2, 2.4] \text{ m/s, with } \pi(3) = 0.3.
\end{aligned}
$$

□

This chapter is specifically intended to provide general guidelines on how to build appropriate scenario sets representing the typical stochastic processes involved in electricity market problems. In particular, the scenario-generation techniques described in Section 3.2 yield discrete approximations whose accuracy is contingent on the number N_Ω of the scenarios considered. In fact, from a theoretical viewpoint, as this number approaches infinity ($N_\Omega \rightarrow \infty$), the discrete process $\boldsymbol{\lambda}$ used as an approximation converges to the original continuous one $\boldsymbol{\gamma}$ ($\boldsymbol{\lambda} \rightarrow \boldsymbol{\gamma}$). Consequently, good discrete approximations often requires the generation of a large number of scenarios, which may render the underlying optimization problem intractable. To regain tractability, two efficient scenario-reduction techniques are provided in Section 3.3 to trim down the number of scenarios deteriorating, as least as possible, the accuracy of the approximation.

In Section 3.4, some extensions of the ARIMA-based scenario-generation technique described in Subsection 3.2.2 are proposed to produce scenario sets for multiple stochastic processes that are statistically dependent. These extensions are tested on different case studies in Section 3.5. Finally, Section 3.6 summarizes and concludes this chapter.

3.2 Scenario Generation

3.2.1 Overview

In stochastic programming, stochastic processes can be represented using continuous or discrete random variables. In the best case, stochastic programming problems with continuous random variables can only be solved in small or illustrative instances. In point of fact, evaluating a possible solution in this kind of problems is frequently impossible. For this reason, the discrete representation of random variables using a finite set of possible outcomes becomes indispensable in actual decision-making problems under uncertainty.

Nevertheless, the appropriate representation of a continuous random variable using a finite set of values can be more difficult and time consuming than formulating and solving the resulting stochastic programming problem. For this reason, significant effort has been paid by the research community on this issue.

The set of finite values used to model a random variable is usually arranged in a so-called *scenario tree*. Graphically, a scenario tree comprises a set of nodes and arcs. The nodes represent states of the “world” at a particular instant, and they constitute the points where decisions are made. Each node has a single predecessor and can have several successors. The first node is called the root node, and it corresponds to the beginning of the planning horizon. In the root node the first-stage decisions are made. The nodes connected to the root node are the second-stage nodes and represent the points where the second-stage decisions are made. The nodes in the last stage are referred to as leaves. Each single path between the root node and a leave is named scenario.

In a scenario tree, the arcs represent different realizations of the random variables. Each arc has associated a probability of occurrence. In this way, the probability of a scenario is the product of all arcs probabilities associated with that scenario.

Illustrative Example 3.5 (Scenario tree). Fig. 3.1 shows an example of a scenario tree representing a stochastic process, e.g. the pool price, with four scenarios and three stages. Each arc has associated a realization of the stochastic process and a probability of occurrence. For instance, observe that two arcs are leaving the root node. These two arcs have associated the values 20 and 22, with probabilities 0.7 and 0.3, respectively.

Scenario 1, for example, comprises the values 20 and 18, and its probability of occurrence is equal to $0.7 \times 0.4 = 0.28$. Observe that the sum of the probabilities over all scenarios is equal to 1: $0.7 \times 0.4 + 0.7 \times 0.6 + 0.3 \times 0.5 + 0.3 \times 0.5 = 1$.

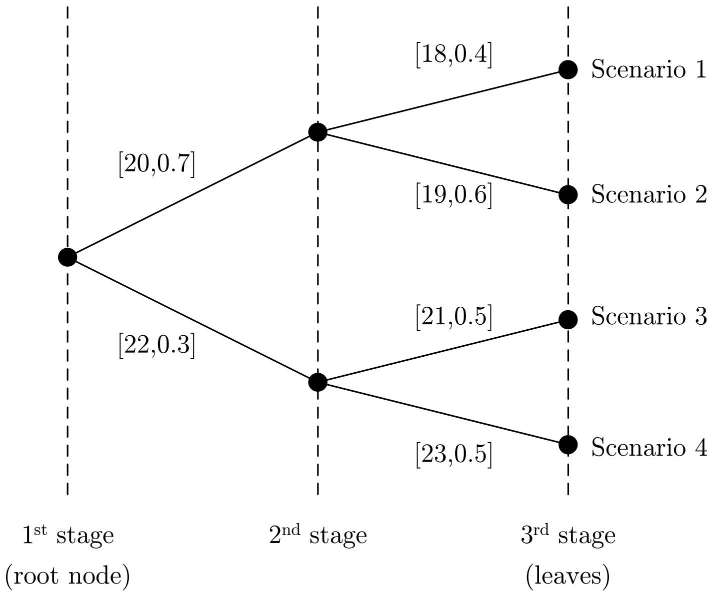


Fig. 3.1 Example of a three-stage scenario tree

□

Different techniques have been proposed in the technical literature to build scenario trees. Some relevant methods are the following:

1. *Generation of data trajectories or path-based methods:* These methods generate complete paths or scenarios by means of econometric and time series models. The set of scenarios obtained by these methods is called a *fan*. Once the fan is generated, the scenarios are clustered to build the scenario tree. A relevant reference using path-based methods is [43].
2. *Moment matching:* These methods generate discrete distributions that satisfy a prefixed set of statistical properties (e.g., based on moments, correlation matrix, percentiles, etc.) characterizing the original distributions of the random variables. Relevant references on moment matching are [82, 83].
3. *Internal sampling:* The internal sampling consists in a continuous sampling process from the original distribution functions of the random variables during the solution procedure. References on internal sampling methods are [79, 80].
4. *Scenario reduction:* These methods begin from a large set of randomly generated scenarios. This original set is reduced to a new set of prescribed cardinality, whose final distribution function is close enough to

the original one according to a given probability metric. References using scenario-reduction methods include [44, 74].

In this book, a procedure combining both path-based methods and scenario-reduction techniques is used to generate scenario trees. A large enough number of scenarios is first generated by ARIMA models. Subsequently, a scenario-reduction technique based on the Kantorovich distance between probability distributions is used to obtain a sufficiently small number of scenarios.

3.2.2 Scenario Generation using ARIMA Models

The probabilistic structure of a stochastic process $\boldsymbol{Y}$ is determined by identifying the joint distribution of its random variables $\{y_t, t \in T\}$. Roughly speaking, this distribution explains both the probabilistic behavior of each random variable y_t on its own (marginal distributions), and the interrelations existing among all of them (statistical dependencies).

In practice, the determination of the joint distribution is usually a complex and cumbersome endeavor. However, this estimation becomes much easier under the following two assumptions:

1. The joint distribution is a multivariate Gaussian distribution and therefore, is determined by just specifying the mean vector and the variance-covariance matrix of the random variables constituting the stochastic process.
2. The stochastic process under analysis is *stationary*, which means that neither the mean vector nor the variance-covariance matrix depend on time t.

Note that the two assumptions above imply that the marginal distributions of the random variables involved are all identical univariate Gaussian distributions. Being so and for simplicity, hereafter we will just refer to the marginal distribution associated with the considered stochastic process.

With reference to assumption 1, the non-diagonal elements of a variance-covariance matrix are generally referred to as *autocovariances*. Through a simple standardization process consisting in just dividing the autocovariances by the corresponding product of standard deviations, the *autocorrelations* are obtained. These are also known as *temporal correlations* and quantify the statistical interdependencies among the random variables.

The time series theory based on *autoregressive moving average* (ARMA) models relies on these two premises [18]. An ARMA(p, q) process $\boldsymbol{Y}$ is mathematically expressed as

$$y_t = \sum_{j=1}^{p} \phi_j y_{t-j} + \varepsilon_t - \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} \tag{3.1}$$

with p autoregressive parameters ϕ_1, ϕ_2, ..., ϕ_p, and q moving average parameters θ_1, θ_2, ..., θ_q. The term ε_t in equation (3.1) stands for an uncorrelated normal stochastic process with mean zero and variance σ_ε^2, and is also uncorrelated with $y_{t-1}, y_{t-2}, \dots, y_{t-p}$. Stochastic process ε_t is also referred to as *white noise*, *innovation term*, or *error term*.

The estimation and adjustment of ARMA models is outside of the scope of this book, but we refer the interested reader to [18] for specific information on the adjustment of time series models.

The procedure to generate a set of scenarios for stochastic process $\boldsymbol{Y}$ is based on the sampling of the error terms from their distribution $\varepsilon_t \sim N(0, \sigma)$. The proposed scenario-generation algorithm using an ARMA model is outlined below.

3.2.2.1 Algorithm

- *Step 1*: Initialize the scenario counter: $\omega \leftarrow 0$.
- *Step 2*: Update the scenario counter and initialize the time period counter: $\omega \leftarrow \omega + 1$, $t \leftarrow 0$.
- *Step 3*: Update the time period counter: $t \leftarrow t + 1$.
- *Step 4*: Randomly generate $\varepsilon_t \sim \mathrm{N}(0, \sigma)$.
- *Step 5*: Evaluate expression (3.1) to obtain $y_{t\omega}$.
- *Step 6*: If $t < N_{\mathrm{T}}$ go to Step 3), else go to Step 7)
- *Step 7*: If $\omega < N_{\Omega}$ go to Step 2), else the scenario-generation process concludes.

The example below illustrates this algorithm.

Illustrative Example 3.6 (Scenario generation based on ARMA models).

Consider the following ARMA(1,1) model for characterizing a certain variable y_t,

$$y_t = \phi_1 y_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1}. \tag{3.2}$$

The parameters of the model are $\phi_1 = 0.9914$ and $\theta_1 = 0.1131$. The value of y_0 is 21.3567 and $\varepsilon_0 = -0.0124$. The standard deviation of the error term is 0.021.

The aim of this example is to generate a set of two scenarios for a three-period horizon. The resulting set of scenarios is denoted as $\{y_{t\omega}, t = 1, 2, 3; \omega = 1, 2\}$.

The first step consists in generating random values for the error term. As the time horizon comprises three time periods, three random values of the error term are generated: 0.0060, -0.0241 and 0.0250. Considering these values, expression (3.2) is used to generate scenario $\omega = 1$ of $y_{t\omega}$ as follows:

$$\begin{aligned}
y_{11} &= \phi_1 y_0 + \varepsilon_{11} - \theta_1 \varepsilon_0 \\
&= 0.9914 \times 21.3567 + 0.0060 - 0.1131 \times (-0.0124) = 21.1804 \\
y_{21} &= \phi_1 y_{11} + \varepsilon_{21} - \theta_1 \varepsilon_{11} \\
&= 0.9914 \times 21.1804 - 0.0241 - 0.1131 \times 0.0060 = 20.9735 \\
y_{31} &= \phi_1 y_{21} + \varepsilon_{31} - \theta_1 \varepsilon_{21} \\
&= 0.9914 \times 20.9735 + 0.0250 - 0.1131 \times (-0.0241) = 20.8209.
\end{aligned}$$

The procedure for obtaining the second and last scenario is equivalent to that used for the first scenario. Again, three random values for the error term are generated: 0.0250, -0.0008 and 0.0069. In this case scenario $\omega = 2$ of $y_{t\omega}$ is obtained as follows:

$$\begin{aligned}
y_{12} &= \phi_1 y_0 + \varepsilon_{12} - \theta_1 \varepsilon_0 \\
&= 0.9914 \times 21.3567 + 0.0250 - 0.1131 \times (-0.0124) = 21.1994 \\
y_{22} &= \phi_1 y_{12} + \varepsilon_{22} - \theta_1 \varepsilon_{12} \\
&= 0.9914 \times 21.1994 - 0.0008 - 0.1131 \times 0.0250 = 21.0135 \\
y_{32} &= \phi_1 y_{22} + \varepsilon_{32} - \theta_1 \varepsilon_{22} \\
&= 0.9914 \times 21.0135 + 0.0069 - 0.1131 \times (-0.0008) = 20.8398.
\end{aligned}$$

□

3.2.2.2 Questioning the stationarity assumption. Integrated and seasonal ARMA processes

One of the major hypotheses concerning the use of ARMA models is to assume that the stochastic process under study is stationary (see Subsection (3.2.2)). That is, the mean and the variance of the process must remain stable throughout the time. However, this property does not always hold and it is necessary to make some transformations to the process in order to achieve stationarity. For instance, let us consider the process y_t represented in Fig. 3.2. It is clear that neither the mean nor the variance of the process are stable.

In order to attain stationarity for the variance, the so-called *Box-Cox transformations* are widely used. Basically, these transformations consist in applying either the logarithm or the square root functions to the original process. Fig. 3.3 represents the stochastic process in Fig. 3.2 after applying the logarithm function. Observe that the mean remains unstable, but the

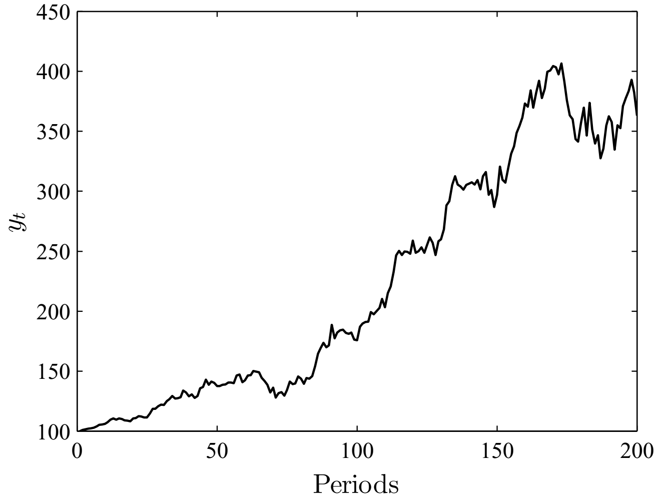


Fig. 3.2 Non-stationary process.

large changes in the time series, specially at the end of the time horizon, are eliminated. This fact results in a more stable variance.

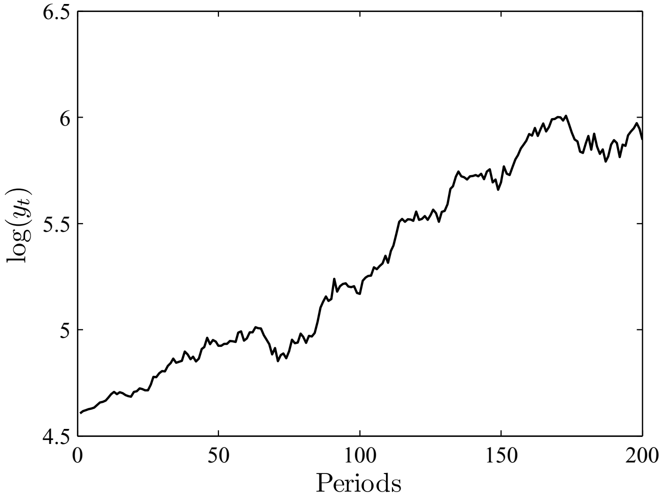


Fig. 3.3 Logarithm transformation of the non-stationary process.

The typical manner of obtaining a stable process in terms of the mean is to differentiate the process. For instance, differencing process y_t with order 1, denoted as $\nabla^1 y_t$, is expressed as

$$\nabla^1 y_t = (1 - B)y_t = y_t - y_{t-1},$$

where B is the backshift operator, i.e., $B^j y_t = y_{t-j}$.

Usually, a difference order equal to 1 is enough to obtain a stable mean. In general, differencing a process with order d is indicated as

$$\nabla^d y_t = (1 - B)^d y_t.$$

Fig. 3.4 represents the process in Fig. 3.3 differentiated with order 1. Observe that differencing the process is useful to remove the trend of the series and to obtain a stationary mean.

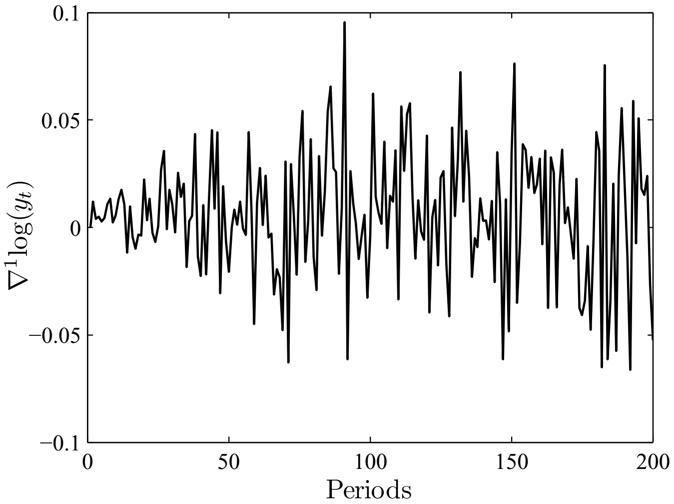


Fig. 3.4 Differentiation of the non-stationary process.

This differencing procedure together with the ARMA models previously presented gives rise to ARIMA models for non-stationary time series. ARIMA stands for *autoregressive integrated moving average*, where *integrated* indicates that the original non-stationary process is obtained by integrating the stationary process resulting after differencing. In this way, ARIMA models are defined by three parameters (p, d, q) corresponding to the number of autoregressive terms, the differencing order, and the number of moving-average terms, respectively. The general expression of an ARIMA model with parameters (p, d, q) is

$$\left(1-\sum_{j=1}^{p}\phi_j B^j\right)(1-B)^d y_t = \left(1-\sum_{j=1}^{q}\theta_j B^j\right)\varepsilon_t. \tag{3.3}$$

One special kind of non-stationarity appears when the processes present a periodic or seasonal pattern. This fact can be observed in Fig. 3.5, where hourly pool prices during a month are represented. Observe that hourly pool prices exhibit similar behavior every day and every week, constituting an example of both daily and weekly seasonalities. For instance, the daily seasonality indicates that a seasonal pattern of order equal to 24 can be identified in the series of hourly pool prices. This fact means that the price at hour h in day d is *similar* to the price at hour h in day d-1. Equivalent reasoning can be applied to explain the weekly seasonality, where the seasonality order is equal to $24 \times 7 = 168$.

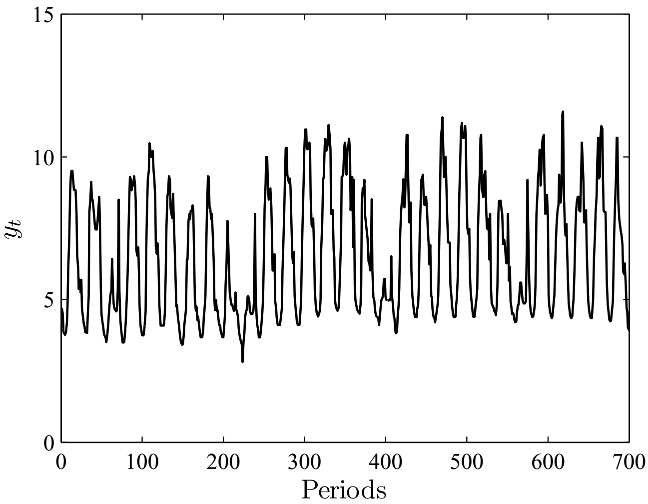


Fig. 3.5 Example of seasonal series.

In this context, seasonal ARIMA models, also known as SARIMA, are required. Let us consider a stochastic process with a seasonality of order S. The general expression of a seasonal ARIMA model with parameters $(p, d, q) \times (P, D, Q)_S$ is

$$\left(1-\sum_{j=1}^{p}\phi_j B^j\right)\left(1-\sum_{j=1}^{P}\Phi_j B^{jS}\right)(1-B)^d(1-B^S)^D y_t = \left(1-\sum_{j=1}^{q}\theta_j B^j\right)\left(1-\sum_{j=1}^{Q}\Theta_j B^{jS}\right)\varepsilon_t \tag{3.4}$$

with a seasonal component of P autoregressive parameters Φ_1, Φ_2, ..., Φ_P, Q moving average parameters Θ_1, Θ_2, ..., Θ_Q, and a differentiation order D.

3.2.2.3 Questioning the normality assumption. Normal Transformation

Observe in (3.1) that y_t boils down to a linear combination of white noises, and as such, the marginal distribution associated with the stochastic process $\boldsymbol{Y}$ is necessarily Gaussian, which is in accordance with the normality assumption on which ARMA processes are based. However, for a number of stochastic processes, the normality assumption does not hold. As an example, it is widely accepted that the marginal distribution characterizing the speed of local winds is not Gaussian, but follows a Weibull distribution [60, 76].

In order to preserve the original marginal distribution associated with the stochastic process $\boldsymbol{Y}$ while making use of the modeling capability of the ARMA models, a new stochastic process, $\boldsymbol{Z}$, with a standard normal marginal distribution is defined through the transformation

$$\boldsymbol{Z} = \Phi^{-1}\left[F_Y(\boldsymbol{Y})\right], \tag{3.5}$$

where F_Y is the cumulative distribution function (cdf) of the marginal distribution associated with the original stochastic process $\boldsymbol{Y}$ and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal random variable.

Transformation (3.5) comes from [89] and is graphically illustrated in Fig. 3.6, where the path in direction d (direct) represents the referred transformation.

The ARMA model (3.1) is adjusted to the transformed stochastic process $\boldsymbol{Z}$. From the fitted model, a *transformed* set of scenarios Ω_Z is first generated, and then *untransformed* in order to produce a set of scenarios, Ω_Y, characterizing the original stochastic process, $\boldsymbol{Y}$. The inverse transformation can be written as

$$\boldsymbol{Y} = F_Y^{-1}\left[\Phi(\boldsymbol{Z})\right], \tag{3.6}$$

and is graphically represented by the path in direction i (inverse) of Fig. 3.6.

The following example illustrates the practical implementation of the normal transformation.

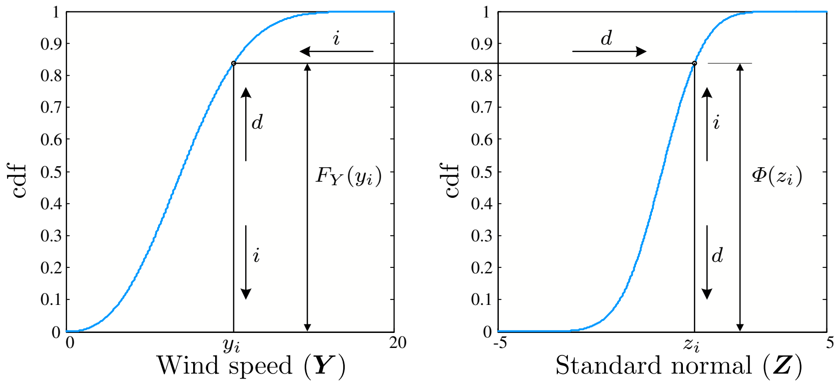


Fig. 3.6 Graphic interpretation of the normal transformation in equation (3.5).

Illustrative Example 3.7 (Normal transformation). Suppose that $y_1 = 1.32$ is a sample from an uniformly distributed random variable Y defined within the interval $[1, 2]$. Consequently, its cumulative distribution function (cdf), $F_Y(y)$, is given by

$$F_Y(y) = \begin{cases} 0, & \text{for } y < 1; \\ y - 1, & \text{for } 1 \leq y < 2; \\ 1, & \text{for } y \geq 2. \end{cases}$$

The direct transformation (3.5) is performed as follows:

$$F_Y(y_1) = 1.32 - 1 = 0.32 \rightarrow z_1 = \Phi^{-1}(0.32) = -0.47,$$

where $z_1 = -0.47$ is a sample from a standard normal variable Z.

Next, consider a new sample $z_2 = 0.7$ from such a random variable Z. The inverse transformation (3.6) is carried out as

$$\Phi(z_2) = 0.76 \rightarrow y_2 = F_Y^{-1}(0.76) = 1.76,$$

where $y_2 = 1.76$ is a sample from the uniformly distributed variable Y. □

3.2.3 Generating Scenarios for Unit Availability

One way of reducing the financial risk faced by a producer that participates in the pool is using forward contracts. However, selling electricity through forward contracts entails a different risk caused by a likely unavailability of the units of the producer. For instance, consider a producer owning a single unit that has signed a forward contract for a given time period. If the unit is not available when the contract is exercised, the producer is forced to buy energy in the pool in order to provide the energy previously committed in the forward contract. In this situation two cases are possible. If the pool prices are smaller than the price associated with the forward contract, the producer will attain a profit. However, if the pool prices are greater than the forward contract price, the producer will incur financial losses. For this reason, the unavailability of the generation units should be accounted for when modeling medium-term problems involving forward contracts.

The availability of a unit is usually characterized by the time to failure (t_{F}) and the time to repair (t_{R}), which constitute random variables following exponential distributions, namely

$$t_{\mathrm{F}} = -\mathrm{MTTF} \times \ln(u_1) \tag{3.7}$$

$$t_{\mathrm{R}} = -\mathrm{MTTR} \times \ln(u_2), \tag{3.8}$$

where MTTF and MTTR are the mean time to failure and the mean time to repair, respectively, and u_1 and u_2 are random variables that are uniformly distributed between 0 and 1.

The time to failure and time to repair distribution functions are exemplified below.

Illustrative Example 3.8 (Time to failure and time to repair distribution functions).

In this example we consider two generating units with parameters MTTF and MTTR equal to 100 and 10 hours for unit 1, and 150 and 20 hours for unit 2, respectively.

We assume that the times to failure and to repair for both units are given by expressions (3.7) and (3.8). The time evolution of these parameters are depicted in Fig. 3.7. Observe that the expected values of the times to failure and to repair coincide with parameters MTTF and MTTR, respectively.

□

Expressions (3.7) and (3.8) can be used to build availability scenarios. For a given generating unit, an availability scenario is a vector of 1s and 0s whose length is equal to the number of considered periods. If the t-element of the availability scenario is equal to 1, the unit is then available in period t, being unavailable if that element is equal to 0. The availability of unit g in period t and scenario ω is denoted by $a_{gt\omega}$.

The scenario-generation procedure is stated below.

3.2.3.1 Algorithm

- *Step 1*: Initialize the scenario counter: $\omega \leftarrow 0$.
- *Step 2*: Update the scenario counter and initialize the time period counter: $\omega \leftarrow \omega + 1$, $t \leftarrow 0$.

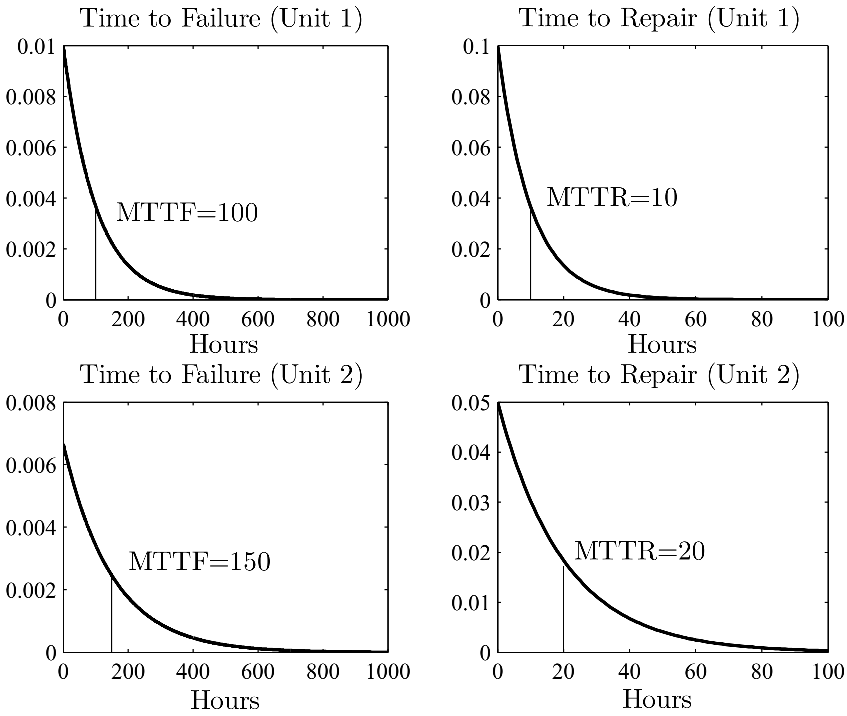


Fig. 3.7 Examples of time to failure and time to repair distributions.

- *Step 3*: Randomly generate $u_1 \sim \mathrm{U}(0,1)$ and $u_2 \sim \mathrm{U}(0,1)$.
- *Step 4*: Evaluate expressions (3.7) and (3.8) to obtain t_{F} and t_{R}.
- *Step 5*: From $\min(N_{\mathrm{T}}, t+1)$ to $\min(N_{\mathrm{T}}, \mathrm{round}(t+t_{\mathrm{F}}))$, $a_{gt\omega} = 1$.
- *Step 6*: From $\min(N_{\mathrm{T}}, \mathrm{round}(t+t_{\mathrm{F}}+1))$ to $\min(N_{\mathrm{T}}, \mathrm{round}(t+t_{\mathrm{F}}+t_{\mathrm{R}}))$, $a_{gt\omega} = 0$.
- *Step 7*: Update the time period counter: $t \leftarrow \min(round(t+t_{\mathrm{F}}+t_{\mathrm{R}}+1),\ N_{\mathrm{T}})$.
- *Step 8*: If $t < N_{\mathrm{T}}$ go to Step 3), else go to Step 9)
- *Step 9*: If $\omega < N_{\Omega}$ go to Step 2), else the scenario-generation process concludes.

The function denoted as $\mathrm{round}(x)$ yields the integer value closest to real number x. Note that the use of this function is needed since the values resulting from (3.7) and (3.8) are real, whereas the number of periods in the planning horizon is an integer.

The next example illustrates the procedure of building a single availability scenario.

Illustrative Example 3.9 (Availability scenarios). Suppose a planning horizon of 2000 hours, and a generating unit with MTTF and MTTR equal to 1000 and 50 hours, respectively.

The first step consists in randomly generating values from uniform random variables u_1 and u_2 in order to evaluate expressions (3.7) and (3.8) for obtaining the time to failure and the time to repair, respectively. In this case, we obtain $u_1 = 0.6324$ and $u_2 = 0.0975$, resulting in times to failure and to repair equal to 458.3 and 116.4 hours, respectively. Since we are using a discrete number of periods (2000 hours), we need to round those real numbers to integer values, resulting in 458 and 116 hours, respectively. This way, during the first 458 hours the unit is available. In hour 459 the unit breaks down and remains unavailable during 116 hours.

The procedure continues by generating new values for u_1 and u_2 until the planning horizon is covered. Thus, we generate $u_1 = 0.2785$ and $u_2 = 0.5469$, which translates into times to failure and repair equal to 1278 and 30 hours, in that order. At this point, $458 + 116 + 1278 + 30 = 1882$ hours of the planning horizon have been completed, while $2000 - 1882 = 118$ hours are still uncovered.

Next, we obtain $u_1 = 0.8003$ and $u_2 = 0.1419$, which yield times to failure and repair equal to 223 and 98, respectively. Observe that the time to failure, 223, is greater than the number of remaining hours, 118. Consequently, the unit remains available until the end of the planning horizon.

Graphically, the scenario generated is depicted in Fig. 3.8.

□

3.2.4 Quality of Scenario Subsets

Stochastic processes are represented in stochastic programming problems using a finite set of possible realizations arranged in a so-called scenario tree. It should be clear that scenario trees are needed because realistic stochastic programming problems including continuous expressions of the stochastic processes are usually impossible to solve. In fact, only illustrative instances or small examples can be solved directly.

For this reason, how to build appropriate scenario trees has become an active research field for stochastic programming theorists and practitioners during the last decades. The question that arises here is, *what is an appropriate scenario tree?* In theory, the answer is easy: a good scenario tree is that which included in the stochastic programming problem results in the same solution than that obtained by using the continuous representation of the stochastic processes. However, verifying this condition is not trivial since,

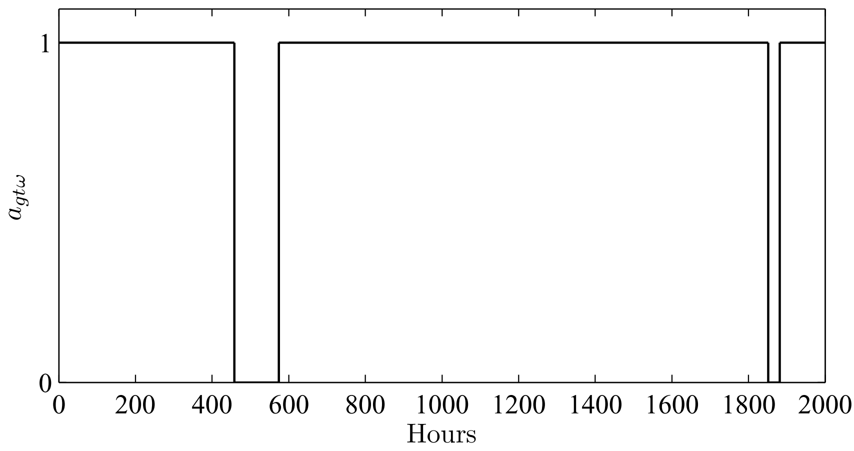


Fig. 3.8 Example of an availability scenario.

as we mentioned above, stochastic programming problems with continuous stochastic processes cannot be solved in general.

Significant effort has been made in order to measure the error associated with the usage of a discrete representation of the stochastic process instead of the continuous one. This error is usually expressed in terms of the difference between the objective functions of the discrete and continuous stochastic problems. Note that this error could be also expressed in terms of the resulting optimal decisions. Notwithstanding this, measuring the error by using objective function values is preferred because objective functions of stochastic programming problems are usually flat and different decisions may yield similar objective functions.

In [114] an upper bound of the error of using discrete representations for the stochastic processes is derived. This upper bound is obtained under some mild Lipschitz conditions of the objective function of the underlying problem.

Additionally, in [86] some hints are given on which properties *good* scenario trees should meet. The first requirement is the stability of the scenario trees. In other words, several trees randomly obtained using the same scenario generation procedure should provide (almost) the same solution to the stochastic problem considered. This property can be easily tested by generating several scenario sets and studying the optimal solutions achieved. If these solutions are (almost) the same, we can say that there exists *in-sample stability*. If, in addition, we evaluate the *true* objective function for the set of optimal decisions that results from each scenario tree and we obtain (almost) the same values, then we are in an *out-of-sample stability* case. Clearly, since in this case it is necessary to compute the true objective function, testing out-of-sample stability is more complicated than testing in-sample stability.

The second requirement is that the solution obtained with the scenario tree should be unbiased with respect to the true solution obtained with the continuous process. This requirement is difficult to check because, as we already know, the true solution is unknown. For this reason, an approximate method consists in building a *reference tree* representing the continuous (or true) process as well as possible. One important issue is that this reference tree must ensure unbiased solutions. The optimal decisions obtained from the *trial* tree can be tested on the reference tree and the resulting value of the objective function can be compared with that obtained from solving the problem with the reference tree directly. The reference tree should be as large as possible and it should not be generated using the same procedure than that used for generating the trial tree.

Another relevant issue for discussion is the shape of the scenario tree. That is, it must be determined how many stages the tree comprises and how many branches leave each node. Observe that the number of stages results directly from the considered decision-making problem, whereas the number of branches leaving each node is a decision that the modeler has to make. However, if a scenario-reduction procedure is used, the final number of branches leaving each node results automatically from using this procedure.

3.3 Scenario Reduction

3.3.1 Motivation

The scenario-generation technique described in Subsection 3.2.2 is based on a sampling approach. That is, after identifying the time series model that best represents the continuous stochastic process under study, a repeated random generation of white noises is performed to produce a discrete approximation in the form of a scenario set. Consequently, in order for this approximation to be accurate, a high number of scenarios is usually required. Given that the computational burden of a stochastic programming model rapidly increases with the number of scenarios, a mathematical tool aimed to *shrewdly* reduce such a number becomes a need. In other words, the necessity of reconciling scenario generation and computational tractability is what justifies the existence of *scenario-reduction techniques*.

In more detail, a scenario-reduction methodology seeks to downsize a scenario set while still keeping as intact as possible the stochastic information embedded in it. Logically, the quality of the reduction process is measured in terms of the solution to the underlying optimization problem. In this line, a *good* reduction process results in a reduced scenario set that yields an optimal solution close in value to the solution obtained from the initial one.

Next, we explain in detail two scenario-reduction procedures, both relying on the concept of *probability distance*. Roughly speaking, a probability distance allows us to quantify the *closeness* of two different scenario sets that represents the same stochastic process.

3.3.2 Scenario Reduction Using a Probability Distance

In two-stage stochastic linear programming problems, it is possible to reduce a large scenario set to a simpler one that is close to the original one if measured by a so-called *probability distance*. Under mild conditions on the problem data, it can be shown that the optimal value of the simpler problem (the one involving the reduced scenario set) is close to the value of the solution to the original problem (the one considering the original scenario set) if the scenario sets are sufficiently close in terms of such a probability distance. Reference [120] constitutes a comprehensive review of the theoretical bases behind the notion of probability distance. Likewise, its application to scenario reduction is analyzed in detail in [44].

Consider a probability distribution Q defined over the scenario set Ω. The problem of how to reduce optimally the set Ω can be rigourously stated as follows: determine a scenario subset $\Omega_S \subset \Omega$ of prescribed cardinality N_{Ω_S} or accuracy and assign new probabilities to the preserved scenarios such that the corresponding reduced probability distribution Q' defined over the subset Ω_S is the closest to the original distribution Q in terms of a probability distance.

The most common probability distance used in stochastic programming is the *Kantorovich distance*, $D_K(\cdot)$, defined between two discrete probability distributions Q and Q' by the problem

$$
\begin{aligned}
D_K(Q,Q') = \min_{\eta}\Bigg\{ \sum_{\substack{\omega\in\Omega\\ \omega'\in\Omega_S}} \nu(\omega,\omega')\eta(\omega,\omega') :\ & \\
\eta(\omega,\omega') \geq 0,\ \forall\omega\in\Omega, \forall\omega'\in\Omega_S,& \\
\sum_{\omega'\in\Omega_S} \eta(\omega,\omega') = \pi_\omega,\ \forall\omega\in\Omega,& \\
\sum_{\omega\in\Omega} \eta(\omega,\omega') = \tau_{\omega'},\ \forall\omega'\in\Omega_S \Bigg\},& \qquad (3.9)
\end{aligned}
$$

where π_ω and $\tau_{\omega'}$ represent the probabilities of scenarios ω and ω' in sets Ω and Ω_S according to probability distributions Q and Q', respectively.

Problem (3.9) is known as *Monge-Kantorovich mass transportation problem*. Further details on this problem are provided in [120]. In (3.9), $\nu(\omega,\omega')$

is a nonnegative, continuous, symmetric function, often referred to as *cost function*, and the minimum is taken over all joint probability distributions defined on $\Omega \times \Omega$. In addition, $D_K(\cdot)$ can only be properly called Kantorovich distance if function $\nu(\cdot)$ is given by a norm.

Given that the electricity-market problems addressed in this book solely present stochasticity in the objective function and right-hand sides, the Kantorovich distance can be equivalently determined as

$$D_K(Q, Q') = \sum_{\omega \in \Omega \setminus \Omega_S} \pi_\omega \min_{\omega' \in \Omega_S} \nu(\omega, \omega'). \tag{3.10}$$

Further information on this equivalency can be found, e.g., in [44].

Expression (3.10) can be used to derive several heuristics for generating reduced scenario sets that are close enough to an original set. Specifically, two different heuristic algorithms can be developed, namely, the *backward reduction* or the *forward selection*, depending on whether the subset $\Omega_S \subset \Omega$ is built by *eliminating* or *selecting* scenarios from the initial set Ω.

Electricity-market problems tackled via stochastic programming are usually computationally intensive, and as such, the scenario sets representing the stochastic processes involved often require a preliminar drastic reduction so as to enable tractability and/or enhance efficiency. In these cases, the forward selection algorithm exhibits a better performance. For this reason, we focus our study on this algorithm hereinafter.

In more detail, the forward selection algorithm is an iterative greedy process starting with an empty set ($\Omega_S = \emptyset$). In each iteration, from the set of non-selected scenarios ($\Omega \setminus \Omega_S$), the scenario which minimizes the Kantorovich distance between the reduced and original sets is selected. Then, this scenario is included in the reduced set Ω_S. The algorithm stops if either a specified number of scenarios or a certain Kantorovich distance is attained.

Note that this scenario-reduction technique is a heuristic, with no known performance guarantee. The reduced scenario set generated by the forward selection algorithm is not necessarily the closest in the Kantorovich distance to the original set (over all reduced sets of the same cardinality). Moreover, we have no guarantee that the reduced set gives a good approximation to the optimal value of the original problem. Nevertheless, empirical results reported in the literature (e.g., in [44, 74, 97]) indicate that the reduced sets defined by the forward selection algorithm perform well in practice.

We describe below the forward selection algorithm step by step.

3.3.3 Algorithm

Let us denote the original scenario set as Ω. The forward selection algorithm produces a succession of reduced sets $\Omega_S^{[0]}$, $\Omega_S^{[1]}$, …, $\Omega_S^{[i]}$, …, Ω_S^{*}, of increasing cardinality, where the set Ω_S^* is the target of the search. To this end, the forward selection algorithm works as follows:

- *Step 0*: The initial step consists in computing the function $\nu(\cdot)$ defining the Kantorovich distance (3.10) for each pair of scenarios. Two different variants of this algorithm are defined depending on how the cost function $\nu(\cdot)$ is calculated. These variants are explained below in detail.
- *Step 1*: The iterative process begins with the choice of a starting scenario, that is, with the scenario from which the reduced scenario set is built. Mathematically, the starting scenario (ω_1) is obtained from

$$\omega_1 = \arg\{\min_{\omega' \in \Omega} \sum_{\omega \in \Omega} \pi_\omega \nu(\omega, \omega')\}. \tag{3.11}$$

  The starting scenario can be interpreted as the most equidistant one from the rest. In other words, this first scenario can be seen as the *average* scenario.
- *Step i*: Starting from the scenario selected in step 1, in each iteration a new scenario is added to the reduced scenario set until it is considered to be close enough to the original one. For this purpose, this selection is carried out using

$$\omega_i = \arg\left\{ \min_{\omega' \in \Omega_J^{[i-1]}} \sum_{\omega \in \Omega_J^{[i-1]} \setminus \{\omega'\}} \pi_\omega \min_{\omega'' \in \Omega_S^{[i-1]} \bigcup \{\omega\}} \nu(\omega, \omega'') \right\}, \tag{3.12}$$

  where $\Omega_J^{[i]}$ represents the set composed of those scenarios which have not been selected in the first i steps of the algorithm and $\Omega_S^{[i]}$ represents the set comprising the selected scenarios until step i. Note that $\Omega_J^{[i]} \cup \Omega_S^{[i]} = \Omega$, $\Omega_J^{[0]} = \Omega$, $\Omega_S^{[0]} = \emptyset$, $\Omega_J^{[i]} = \Omega_J^{[i-1]} \setminus \{\omega_i\}$ and $\Omega_S^{[i]} = \Omega_S^{[i-1]} \cup \{\omega_i\}$.
  This step is repeated $N_{\Omega_S^*} - 1$ times, where $N_{\Omega_S^*}$ is the number of scenarios comprising the reduced set Ω_S^*.
- *Step* $N_{\Omega_S^*} + 1$: In this step, an optimal redistribution of probabilities is carried out. It consists of adding the probabilities of those scenarios which have not been finally selected $\big(\omega \in \Omega_J^*$, with $\Omega_J^* \cup \Omega_S^* = \Omega\big)$ to the probabilities of those comprising the reduced set $(\omega \in \Omega_S^*)$.
  Mathematically, this redistribution of probabilities can be accomplished as follows,

$$\pi_\omega^* = \pi_\omega + \sum_{\omega' \in J(\omega)} \pi_{\omega'}, \tag{3.13}$$

  where $J(\omega)$ is defined as the set of scenarios $\omega' \in \Omega_J^*$ such that $\omega = \arg \min_{\omega'' \in \Omega_S^*} \nu(\omega'', \omega')$.

In other words, the probability of each non-selected scenario is aggregated to the probability of the closest selected scenario according to the function $\nu(\cdot)$. Thus, the reduced scenario set is made up of the scenarios $w \in \Omega_S^*$ with associated probability π_ω^*.

A particularly efficient implementation of the algorithm described above is presented next and is referred to as *fast forward selection* algorithm, [74].

3.3.3.1 Algorithm

- *Step 0*: Compute cost function $\nu(\omega, \omega')$ for each pair of scenarios ω and ω' in Ω.
- *Step 1*: Compute $d_\omega = \sum_{\substack{\omega'=1 \\ \omega' \neq \omega}}^{N_\Omega} \pi_{\omega'} \nu(\omega, \omega'), \;\; \forall \omega \in \Omega$.
  Select $\omega_1 \in \arg \min_{\omega \in \Omega} d_\omega$.
  Set $\Omega_J^{[1]} = \{1, \ldots, N_\Omega\} \setminus \omega_1$.
- *Step i*: Compute
  $\nu^{[i]}(\omega, \omega') = \min\{\nu^{[i-1]}(\omega, \omega'), \nu^{[i-1]}(\omega, \omega_{i-1})\}, \forall \omega, \omega' \in \Omega_J^{[i-1]}$
  $d_\omega^{[i]} = \sum_{\omega' \in \Omega_J^{[i-1]} \setminus \{\omega\}} \pi_{\omega'} \nu^{[i]}(\omega', \omega), \;\; \forall \omega \in \Omega_J^{[i-1]}$.
  Select $\omega_i \in \arg \min_{\omega \in \Omega_J^{[i-1]}} d_\omega^{[i]}$.
  Set $\Omega_J^{[i]} = \Omega_J^{[i-1]} \setminus \omega_i$.
- *Step* $N_{\Omega_S^*} + 1$:
  $\Omega_J^* = \Omega_J^{[N_{\Omega_J^*}]}$ is the final set of indices of discarded scenarios.
  $\Omega_S^* = \Omega \setminus \Omega_J^*$ is the set of indices of selected scenarios.
  The probabilities of selected scenarios $\omega \in \Omega_S^*$ are computed as
  $\pi_\omega^* \leftarrow \pi_\omega + \sum_{\omega' \in J(\omega)} \pi_{\omega'}, \forall \omega \in \Omega_S^*$, where
  $J(\omega) = \{\omega' \in \Omega_J^* | \omega = j(\omega')\}, \;\; j(\omega') \in \arg \min_{\omega'' \in \Omega_S^*} \nu(\omega'', \omega')$.

Next, we present two variants of this scenario-reduction algorithm based on two different manners of computing the cost function $\nu(\cdot)$ in Step 0 above.

3.3.3.2 Variant 1: Cost function based on the norm of the difference between pairs of random vectors

Consider a stochastic process $\boldsymbol{\lambda}$ characterized by an initial set of scenarios $\boldsymbol{\lambda}(\omega)$, for all $\omega \in \Omega$, i.e., $\boldsymbol{\lambda} = \{\boldsymbol{\lambda}(\omega), \omega = 1, 2, \ldots, N_\Omega\}$.

Cost function $\nu(\cdot)$ can be computed as the norm of the difference between pairs of scenarios, i.e.,

$$\nu(\omega, \omega') = \|\boldsymbol{\lambda}(\omega) - \boldsymbol{\lambda}(\omega')\|, \tag{3.14}$$

where $\|\cdot\|$ stands for the norm operator.

Note that, in order for cost function $\nu(\cdot)$ to be computed according to (3.14), just the original scenario discretization $\{\boldsymbol{\lambda}(\omega), \forall \omega \in \Omega\}$ of the stochastic process $\boldsymbol{\lambda}$ is required, and therefore, this first variant of the forward selection algorithm yields the same reduced set Ω_S irrespective of the stochastic programming problem to be solved.

Further details on this variant can be found in [44, 74].

3.3.3.3 Variant 2: Cost function based on the objective function of a single-scenario problem

Consider the deterministic equivalent formulation (3.15)–(3.18) of a two-stage stochastic programming problem (see Subsection 2.5.1 in Chapter 2), namely

$$\begin{aligned} &\text{Minimize}_{\boldsymbol{x}, \boldsymbol{y}(\omega)} \\ z = \ & \boldsymbol{c}^\top \boldsymbol{x} + \sum_{\omega \in \Omega} \pi(\omega) \boldsymbol{q}(\omega)^\top \boldsymbol{y}(\omega) \end{aligned} \tag{3.15}$$

subject to

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{3.16}$$

$$\boldsymbol{T}(\omega)\boldsymbol{x} + \boldsymbol{W}(\omega)\boldsymbol{y}(\omega) = \boldsymbol{h}(\omega), \forall \omega \in \Omega \tag{3.17}$$

$$\boldsymbol{x} \in X, \ \boldsymbol{y}(\omega) \in Y; \ \forall \omega \in \Omega, \tag{3.18}$$

where, for simplicity, we just write $\boldsymbol{q}(\omega)$, $\boldsymbol{y}(\omega)$, $\boldsymbol{T}(\omega)$, $\boldsymbol{W}(\omega)$, and $\boldsymbol{h}(\omega)$, instead of $\boldsymbol{q}\big(\boldsymbol{\lambda}(\omega)\big)$, $\boldsymbol{y}\big(\boldsymbol{\lambda}(\omega)\big)$, $\boldsymbol{T}\big(\boldsymbol{\lambda}(\omega)\big)$, $\boldsymbol{W}\big(\boldsymbol{\lambda}(\omega)\big)$, and $\boldsymbol{h}\big(\boldsymbol{\lambda}(\omega)\big)$. For convenience, this problem is denoted by SP henceforth.

We also define $\text{DP}_{\mathcal{E}}$ as the deterministic expected value problem associated with SP, i.e., $\text{DP}_{\mathcal{E}}$ is equal to problem SP in which the random vector $\boldsymbol{\lambda}$ is replaced by its expected value, $\bar{\boldsymbol{\lambda}} = \mathcal{E}\{\boldsymbol{\lambda}\}$. Mathematically, this problem can be formulated as

$$\text{Minimize}_{\boldsymbol{x}, \boldsymbol{y}(\bar{\boldsymbol{\lambda}})} \tag{3.19}$$

$$z_{\mathcal{E}} = \ \boldsymbol{c}^\top \boldsymbol{x} + \boldsymbol{q}(\bar{\boldsymbol{\lambda}})^\top \boldsymbol{y}(\bar{\boldsymbol{\lambda}}) \tag{3.20}$$

subject to

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{3.21}$$

$$\boldsymbol{T}(\bar{\boldsymbol{\lambda}})\boldsymbol{x} + \boldsymbol{W}(\bar{\boldsymbol{\lambda}})\boldsymbol{y}(\bar{\boldsymbol{\lambda}}) = \boldsymbol{h}(\bar{\boldsymbol{\lambda}}) \tag{3.22}$$

$$\boldsymbol{x} \in X, \ \boldsymbol{y}(\bar{\boldsymbol{\lambda}}) \in Y. \tag{3.23}$$

Let us denote the optimal value of the first-stage variables in problem $\text{DP}_{\mathcal{E}}$ as $\bar{\boldsymbol{x}}$.

Next, we build the single-scenario problem DP_ω as the one resulting from problem SP if the stochastic process $\boldsymbol{\lambda}$ is replaced by its realization in scenario ω, $\boldsymbol{\lambda}(\omega)$, and its first-stage variables are fixed to $\bar{\boldsymbol{x}}$, i.e.,

$$\text{Minimize}_{\boldsymbol{y}(\omega)} \tag{3.24}$$

$$z_\omega = \boldsymbol{c}^\top \bar{\boldsymbol{x}} + \boldsymbol{q}(\omega)^\top \boldsymbol{y}(\omega) \tag{3.25}$$

subject to

$$\boldsymbol{T}(\omega)\bar{\boldsymbol{x}} + \boldsymbol{W}(\omega)\boldsymbol{y}(\omega) = \boldsymbol{h}(\omega) \tag{3.26}$$

$$\boldsymbol{y}(\omega) \in Y. \tag{3.27}$$

Finally, cost function $\nu(\cdot)$ is defined as

$$\nu(\omega, \omega') = |z_\omega - z_{\omega'}|, \tag{3.28}$$

where z_ω is the optimal value of the objective function (3.25) of problem DP_ω.

From expression (3.28), it can be inferred that, unlike in variant 1, the reduction process in this second variant of the forward selection algorithm is dependent on the stochastic programming problem SP to be solved to obtain z_ω. In other words, this variant incorporates information on the structure of the problem into the cost function $\nu(\cdot)$, which, in general, results in reduced sets that yield better solutions in practice. However, even though each single-scenario problem DP_ω is much easier to solve than the target problem SP, it is, by far, more involved than the calculation of a norm. Consequently, this second variant is computationally more demanding than the first one.

The following two examples are aimed to illustrate the two described variants of the *fast* forward selection algorithm.

Illustrative Example 3.10 (Fast forward selection: Variant 1). Assume that the power production P of a 100-MW wind farm in a given time period can be statistically represented by the four wind power scenarios $P_\omega, \omega = 1, \ldots, 4$, listed in Table 3.1, with associated probabilities $\pi_\omega, \omega = 1, \ldots, 4$. It follows that $N_\Omega = 4$. Likewise, note that, since just one period is considered, the wind power stochastic process boils down to a random variable.

Table 3.1 Illustrative Example 3.10: wind power scenarios

Scenario #	1	2	3	4
P_ω (MW)	5	40	60	100
π_ω	0.25	0.20	0.20	0.35

Suppose that we need to reduce the number of wind power scenarios to two, i.e., $N_{\Omega_S^*} = 2$. For this purpose, we decide to use variant 1 of the fast forward selection algorithm, which works as follows:

- *Step 0*: Before starting the iterative process, we compute cost function $\nu(\omega, \omega') = \|P_\omega - P_{\omega'}\|, \forall \omega, \omega' \in \Omega$, where $\Omega = \{1, 2, 3, 4\}$. The values of function ν can be conveniently arranged into a symmetric matrix with zero diagonal elements,

$$\nu = \begin{pmatrix} 0 & 35 & 55 & 95 \\ 35 & 0 & 20 & 60 \\ 55 & 20 & 0 & 40 \\ 95 & 60 & 40 & 0 \end{pmatrix} \text{MW}.$$

- *Step 1*: For each scenario ω candidate to be selected from Ω, we choose the one that minimizes the resulting Kantorovich distance between the reduced and original sets, Ω_S and Ω, respectively,

$$\begin{aligned} d_1 &= \pi_2\ \nu(1,2) + \pi_3\ \nu(1,3) + \pi_4\ \nu(1,4) = 51.25 \text{ MW} \\ d_2 &= \pi_1\ \nu(2,1) + \pi_3\ \nu(2,3) + \pi_4\ \nu(2,4) = 33.75 \text{ MW} \\ d_3 &= \pi_1\ \nu(3,1) + \pi_2\ \nu(3,2) + \pi_4\ \nu(3,4) = 31.75 \text{ MW} \\ d_4 &= \pi_1\ \nu(4,1) + \pi_2\ \nu(4,2) + \pi_3\ \nu(4,3) = 43.75 \text{ MW}. \end{aligned}$$

Therefore,

$$\begin{aligned} \Omega_S^{[1]} &= \{3\}, \\ \Omega_J^{[1]} &= \{1, 2, 4\}. \end{aligned}$$

- *Step 2*: Next, we update the cost matrix as follows:

$$\begin{aligned} \nu^{[2]}(1,2) &= \min\{\nu(1,2), \nu(1,3)\} = 35 \text{ MW} \\ \nu^{[2]}(1,4) &= \min\{\nu(1,4), \nu(1,3)\} = 55 \text{ MW} \\ \nu^{[2]}(2,1) &= \min\{\nu(2,1), \nu(2,3)\} = 20 \text{ MW} \\ \nu^{[2]}(2,4) &= \min\{\nu(2,4), \nu(2,3)\} = 20 \text{ MW} \\ \nu^{[2]}(4,1) &= \min\{\nu(4,1), \nu(4,3)\} = 40 \text{ MW} \\ \nu^{[2]}(4,2) &= \min\{\nu(4,2), \nu(4,3)\} = 40 \text{ MW}. \end{aligned}$$

Consequently,

$$\nu^{[2]} = \begin{pmatrix} 0 & 35 & 55 & 55 \\ 20 & 0 & 20 & 20 \\ 55 & 20 & 0 & 40 \\ 40 & 40 & 40 & 0 \end{pmatrix} \text{MW}.$$

Considering the new cost matrix $\nu^{[2]}$, we select from $\Omega_J^{[1]}$ the scenario ω that, if introduced into the subsequent reduced set $\Omega_S^{[2]}$, minimizes the Kantorovich distance between this set and the original one, Ω,

$$d_1^{[2]} = \pi_2\ \nu^{[2]}(2,1) + \pi_4\ \nu^{[2]}(4,1) = 18.00 \text{ MW}$$

$$d_2^{[2]} = \pi_1 \ \nu^{[2]}(1,2) + \pi_4 \ \nu^{[2]}(4,2) = 22.75 \text{ MW}$$
$$d_4^{[2]} = \pi_1 \ \nu^{[2]}(1,4) + \pi_2 \ \nu^{[2]}(2,4) = 17.75 \text{ MW}.$$

Hence,

$$\Omega_S^{[2]} = \Omega_S^* = \{3,4\}$$
$$\Omega_J^{[2]} = \Omega_J^* = \{1,2\}.$$

- *Step 3*: The scenario-reduction algorithm ends with the optimal *transfer* of probabilities from the set of non-selected scenarios, Ω_J^*, to the set of selected ones, Ω_S^*.
  Given that

  1. Scenario 3 in Ω_S^* is the *closest* one to scenario 1 in Ω_J^* ($\nu(1,3) = 55$, while $\nu(1,4) = 95$), and
  2. Scenario 3 in Ω_S^* is the *closest* one to scenario 2 in Ω_J^* ($\nu(2,3) = 20$, while $\nu(2,4) = 60$),

  it follows

$$\pi_3^* = \pi_3 + \pi_2 + \pi_1 = 0.65$$
$$\pi_4^* = \pi_4 = 0.35.$$

In short, the variant 1 of the forward selection algorithm provides a reduced scenario set $\Omega_S^* = \{3,4\}$ with associated probabilities $\pi_3^* = 0.65$ and $\pi_4^* = 0.35$. □

Illustrative Example 3.11 (Fast forward selection: Variant 2). In this example, we aim to reduce the wind power scenario set provided in Table 3.1 to two scenarios by using the variant 2 of the fast forward selection algorithm. Previous to tackling the reduction process itself, this variant requires to define the problem to be solved considering this scenario set (problem SP).

Let us assume that the producer owning the wind farm is interested in selling its wind power production to the day-ahead electricity market. Since the final power produced by its wind farm is uncertain, the producer has no option other than to submit an energy offer (P^D) to the day-ahead market, subsequently *fixing* its likely excess (Δ^+) or deficit of generation (Δ^-) through a real-time balancing process. Resorting to this balancing mechanism entails a cost for the wind producer, usually referred to as *imbalance cost*.

The decision on the amount of energy to be sold in the day-ahead market with a view to minimize the imbalance cost can be properly made by means of the following optimization problem,

$$\text{Maximize}_{P^{\text{D}};\Delta_\omega^+,\forall\omega;\Delta_\omega^-,\forall\omega}$$
$$z = \sum_{\omega=1}^{N_\Omega} \pi_\omega \big(\lambda^{\text{D}} P_\omega - \lambda^{\text{D}}(1 - r^+)\Delta_\omega^+ - \lambda^{\text{D}}(r^- - 1)\Delta_\omega^-\big)$$

subject to

$$0 \le P^{\text{D}} \le P^{\max}$$
$$\Delta_\omega^+ - \Delta_\omega^- = P_\omega - P^{\text{D}}, \forall\omega$$
$$0 \le \Delta_\omega^+ \le P_\omega, \forall\omega$$
$$0 \le \Delta_\omega^- \le P^{\max}, \forall\omega,$$

where λ^{D} and $P^{\max}$ are, respectively, the day-ahead market price and the installed capacity of the wind farm (100 MW). Parameters r^+ and r^- are the ratios between the positive and negative imbalance prices and the day-ahead market price. As the wind power production P, constants λ^{D}, r^+ and r^- are indeed stochastic processes. Nevertheless, for simplicity, we consider them perfectly known and equal to \$20/MWh, 0.95, and 1.4, respectively.

The problem formulated above constitutes a simplified version of the short-term trading problem of a wind power producer, comprehensively analyzed in Chapter 6 of this book.

Once the target problem SP is rightly defined, we can proceed with the scenario-reduction process:

- *Step 0*: Calculate cost function $\nu(\omega, \omega') = |z_\omega - z_{\omega'}|, \forall\omega, \omega' \in \Omega$.
  In order to determine the objective function values $z_\omega, \forall\omega \in \Omega$, we need to solve first the following deterministic expected value problem $\text{DP}_{\mathcal{E}}$,

  $$\text{Maximize}_{\bar{P}^{\text{D}},\bar{\Delta}^+,\bar{\Delta}^-}$$
  $$z_{\mathcal{E}} = \big(1125 - \bar{\Delta}^+ - 8\bar{\Delta}^-\big)$$

  subject to

  $$0 \le \bar{P}^{\text{D}} \le 100$$
  $$\bar{\Delta}^+ - \bar{\Delta}^- = 56.25 - \bar{P}^{\text{D}}$$
  $$0 \le \bar{\Delta}^+ \le 56.25$$
  $$0 \le \bar{\Delta}^- \le 100,$$

  where the wind power random variable P has been replaced by its expected value: $\mathcal{E}\{P\} = 5 \times 0.25 + 40 \times 0.20 + 60 \times 0.20 + 100 \times 0.35 = 56.25$ MW. Likewise, parameters $P^{\max}, \lambda^{\text{D}}$, r^+, and r^- have been replaced by their numerical values.
  The solution to this problem is trivial: $\bar{P}^{\text{D}} = 56.25$ MW, $\bar{\Delta}^+ = \bar{\Delta}^- = 0$, and $z_{\mathcal{E}} = \$1125$.

Finally, the objective function values $z_\omega, \forall \omega \in \Omega$, are obtained from the family of single-scenario problems $\mathrm{DP}_\omega, \omega = 1, \ldots, 4$, which result from problem SP if the wind power stochastic process is replaced by its realization in scenario ω, P_ω, and its first-stage variable P^{D} is fixed to $\bar{P}^{\mathrm{D}} = 56.25$ MW. Specifically, each problem DP_ω is formulated as follows:

$$\text{Maximize}_{\Delta_\omega^+;\Delta_\omega^-}$$
$$z_\omega = \left(20P_\omega - \Delta_\omega^+ - 8\Delta_\omega^-\right)$$

subject to

$$\Delta_\omega^+ - \Delta_\omega^- = P_\omega - 56.25$$
$$0 \le \Delta_\omega^+ \le P_\omega$$
$$0 \le \Delta_\omega^- \le 100.$$

The solution to this problem is also straightforward and is given by

$$\Delta_\omega^+ \text{ (MW)} = \begin{cases} 0, & \text{if } \omega = 1,2 \\ P_\omega - 56.25, & \text{if } \omega = 3,4. \end{cases}$$

$$\Delta_\omega^- \text{ (MW)} = \begin{cases} 56.25 - P_\omega, & \text{if } \omega = 1,2 \\ 0, & \text{if } \omega = 3,4. \end{cases}$$

$$z_\omega \text{ (\$)} = \begin{cases} 28P_\omega - 450, & \text{if } \omega = 1,2 \\ 19P_\omega + 56.25, & \text{if } \omega = 3,4. \end{cases}$$

Table 3.2 lists the numerical value of each objective function variable $z_\omega, \omega \in \Omega$, and its associated probability π_ω, i.e., the probability pertaining to the wind power realization P_ω from which the single-scenario problem DP_ω is built.

Table 3.2 Illustrative Example 3.11: objective function values z_ω of the single-scenario problems DP_ω

Scenario #	1	2	3	4
z_ω (\$)	-310	670	1196.25	1956.25
π_ω	0.25	0.20	0.20	0.35

Therefore, we already have the required information to compute cost matrix ν, which results in

$$\nu = \$ \begin{pmatrix} 0 & 980 & 1506.25 & 2266.25 \\ 980 & 0 & 526.25 & 1286.25 \\ 1506.25 & 526.25 & 0 & 760 \\ 2266.25 & 1286.25 & 760 & 0 \end{pmatrix}.$$

From this point on, this second variant of the fast forward selection algorithm proceeds similarly to variant 1 (see Illustrative Example 3.10).

- *Step 1*: Determine the scenario that, if selected, minimizes the resulting probability distance between the reduced and original sets, Ω_S and Ω, respectively,

$$\begin{aligned}
d_1 &= \pi_2\ \nu(1,2) + \pi_3\ \nu(1,3) + \pi_4\ \nu(1,4) = \$1290.4375\\
d_2 &= \pi_1\ \nu(2,1) + \pi_3\ \nu(2,3) + \pi_4\ \nu(2,4) = \$800.4375\\
d_3 &= \pi_1\ \nu(3,1) + \pi_2\ \nu(3,2) + \pi_4\ \nu(3,4) = \$747.8125\\
d_4 &= \pi_1\ \nu(4,1) + \pi_2\ \nu(4,2) + \pi_3\ \nu(4,3) = \$975.8125.
\end{aligned}$$

Consequently,

$$\begin{aligned}
\Omega_S^{[1]} &= \{3\},\\
\Omega_J^{[1]} &= \{1,2,4\}.
\end{aligned}$$

- *Step 2*: Update cost matrix and select the new scenario to be introduced into the reduced set,

$$\begin{aligned}
\nu^{[2]}(1,2) &= \min\{\nu(1,2),\nu(1,3)\} = \$980\\
\nu^{[2]}(1,4) &= \min\{\nu(1,4),\nu(1,3)\} = \$1506.25\\
\nu^{[2]}(2,1) &= \min\{\nu(2,1),\nu(2,3)\} = \$526.25\\
\nu^{[2]}(2,4) &= \min\{\nu(2,4),\nu(2,3)\} = \$526.25\\
\nu^{[2]}(4,1) &= \min\{\nu(4,1),\nu(4,3)\} = \$760\\
\nu^{[2]}(4,2) &= \min\{\nu(4,2),\nu(4,3)\} = \$760.
\end{aligned}$$

Therefore,

$$\nu^{[2]} = \$ \begin{pmatrix} 0 & 980 & 1506.25 & 1506.25\\ 526.25 & 0 & 526.25 & 526.25\\ 1506.25 & 526.25 & 0 & 760\\ 760 & 760 & 760 & 0 \end{pmatrix},$$

$$\begin{aligned}
d_1^{[2]} &= \pi_2\ \nu^{[2]}(2,1) + \pi_4\ \nu^{[2]}(4,1) = \$371.25\\
d_2^{[2]} &= \pi_1\ \nu^{[2]}(1,2) + \pi_4\ \nu^{[2]}(4,2) = \$511\\
d_4^{[2]} &= \pi_1\ \nu^{[2]}(1,4) + \pi_2\ \nu^{[2]}(2,4) = \$481.8125.
\end{aligned}$$

Hence,

$$\begin{aligned}
\Omega_S^{[2]} &= \Omega_S^* = \{1,3\}\\
\Omega_J^{[2]} &= \Omega_J^* = \{2,4\}.
\end{aligned}$$

- *Step 3*: Optimal redistribution of probabilities.
Considering that:

1. Scenario 3 in Ω_S^* is the *closest* one to scenario 2 in Ω_J^* ($\nu(2,3) = 526.25$, while $\nu(2,1) = 980$), and

2. Scenario 3 in Ω_S^* is the *closest* one to scenario 4 in Ω_J^* ($\nu(4,3) = 760$, while $\nu(4,1) = 2266.25$),

it follows,

$$\pi_1^* = \pi_1 = 0.25$$
$$\pi_3^* = \pi_3 + \pi_2 + \pi_4 = 0.75.$$

In summary, the variant 2 of the forward selection algorithm provides a reduced scenario set $\Omega_S^* = \{1, 3\}$ with associated probabilities $\pi_1^* = 0.25$ and $\pi_3^* = 0.75$.

□

Finally, observe that the original structure of the tree must be preserved after applying the scenario-reduction technique. Since two-stage trees can be viewed as a fan of scenarios in which each branch belongs to a single scenario, the former consideration only affects to multi-stage trees. Note that in multi-stage trees some branches belong simultaneously to several scenarios. In order to preserve the structure of the original tree in a multi-stage stochastic problem, caution should be exercised in the formulation of the non-anticipativity constraints presented in Subsection 2.5.2 of Chapter 2. As stated in that chapter, these constraints contain the information of the structure of the tree, and therefore they must be updated to take into account the elimination of some scenarios in the scenario-reduction process.

3.4 Scenario Generation for Dependent Stochastic Processes

3.4.1 Overview

Decision-making problems related to electricity markets may be affected by stochastic processes that are statistically dependent. For instance, the role of a retailer in the medium term, comprehensively analyzed in Chapter 8 of this book, boils down to buying energy in electricity markets at uncertain prices with the intention of reselling it to an uncertain demand. It is well-known that electricity prices and demand are strongly interrelated in such a way that periods of high/low prices often coincide with periods of high/low demand. This interrelation, if conveniently accounted for, can be used by a retailer to improve its profit margin. In the same vein, the power productions from spatially close wind farms frequently follow similar patterns. Recognizing and considering this fact can be of the utmost importance for producers owning wind farms at several geographical sites so as to make informed decisions concerning electricity trading and risk management.

In short, dependencies among the stochastic processes modeling uncertainty in optimization problems may have a significant impact on decision variables, and therefore, procedures capable of generating scenario sets incorporating such dependencies are required. With this aim in mind, we introduce in this section two different strategies designed to produce statistically correlated scenarios according to the features of the dependency structure that characterizes the stochastic processes under study.

Let us consider two stochastic processes, $\boldsymbol{Y}^a$ and $\boldsymbol{Y}^b$, that can be separately described by means of the following ARMA(p, q) models:

$$y_t^a = \sum_{j=1}^{p^a} \phi_j^a y_{t-j}^a + \varepsilon_t^a - \sum_{j=1}^{q^a} \theta_j^a \varepsilon_{t-j}^a \tag{3.29}$$

$$y_t^b = \sum_{j=1}^{p^b} \phi_j^b y_{t-j}^b + \varepsilon_t^b - \sum_{j=1}^{q^b} \theta_j^b \varepsilon_{t-j}^b \; . \tag{3.30}$$

As stated in Subsection 3.2.2, the series of errors $\boldsymbol{\varepsilon}^a$ and $\boldsymbol{\varepsilon}^b$ are not autocorrelated, i.e., $\mathcal{E}\{\varepsilon_t^a \varepsilon_{t-j}^a\} = 0$ and $\mathcal{E}\{\varepsilon_t^b \varepsilon_{t-j}^b\} = 0, \forall j \neq 0$. However, if stochastic processes $\boldsymbol{Y}^a$ and $\boldsymbol{Y}^b$ are statistically dependent, such a dependency is *transferred* to the series of residuals $\boldsymbol{\varepsilon}^a$ and $\boldsymbol{\varepsilon}^b$ through equations (3.29) and (3.30). Consequently, $\boldsymbol{\varepsilon}^a$ and $\boldsymbol{\varepsilon}^b$ should be indeed *cross-correlated*, i.e., $\mathcal{E}\{\varepsilon_t^a \varepsilon_{t-j}^b\} \neq 0$ and/or $\mathcal{E}\{\varepsilon_t^b \varepsilon_{t-j}^a\} \neq 0$ for some j. The dependency structure of the residual series $\boldsymbol{\varepsilon}^a$ and $\boldsymbol{\varepsilon}^b$ can be analyzed by means of their *cross-correlogram*, in which the cross-correlation coefficients for different time lags are represented [18]. Specifically, the positive and negative lag-k cross-correlation coefficients, ρ_k and ρ_{-k}, of $\boldsymbol{\varepsilon}^a$ and $\boldsymbol{\varepsilon}^b$ are defined, respectively, as $\mathcal{E}\{\varepsilon_t^a \varepsilon_{t-k}^b\} = \rho_k$ and $\mathcal{E}\{\varepsilon_{t-k}^a \varepsilon_t^b\} = \rho_{-k}$, where the equality holds for any t by appealing to the stationarity assumption. The cross-correlation coefficient corresponding to the lag zero, ρ_0, is usually referred to as *contemporaneous correlation*.

According to the main features characterizing the shape of the residual cross-correlogram, three different types of dependent stochastic processes can be distinguished:

Contemporaneous stochastic processes. The residual cross-correlogram of two contemporaneous stochastic processes is characterized by a single significative correlation coefficient placed at lag zero, that is, by a significative contemporaneous correlation. The rest of coefficients can be considered statistically zero. Contemporaneous stochastic processes are usually *driven* by a common physical cause. Fig. 3.9(a) illustrates a residual cross-correlogram typical of this type of dependent processes.

Quasi-contemporaneous stochastic processes. The residual cross-correlogram of two quasi-contemporaneous stochastic processes presents a triangular structure in which the most significant correlation coefficients form a group around the lag zero, with the lag-0 correlation coefficient being precisely the largest one. This structure is typical of stochastic processes with a common, but delayed, physical cause [96]. The triangle base length gives an idea of the delay magnitude. The narrower and the more pointed this triangle is, the more contemporaneous the stochastic processes are. Fig. 3.9(b) shows an example of the residual cross-correlogram of two quasi-contemporaneous stochastic processes.

Non-contemporaneous stochastic processes. With this name, we refer to any other pair of correlated stochastic processes that does not fit into any of the two previous categories. The modeling of the dependencies existing between two non-contemporaneous stochastic processes is generally more complex and cumbersome. Fig. 3.9(c) constitutes an example of a possible residual cross-correlogram of this kind of processes.

In the following section, we present a procedure to generate statistically correlated scenarios for contemporaneous or quasi-contemporaneous stochastic processes.

3.4.2 Scenarios for contemporaneous or quasi-contemporaneous stochastic processes

The methodology described below seeks to produce correlated scenarios by simulating a series of errors, $\boldsymbol{\varepsilon}^a$ and $\boldsymbol{\varepsilon}^b$, that reproduces the residual cross-correlogram of the stochastic processes, $\boldsymbol{Y}^a$ and $\boldsymbol{Y}^b$, under consideration. This methodology is equally valid for both contemporaneous and quasi-contemporaneous stochastic processes. In fact, the former should be considered as a simplified case of the latter.

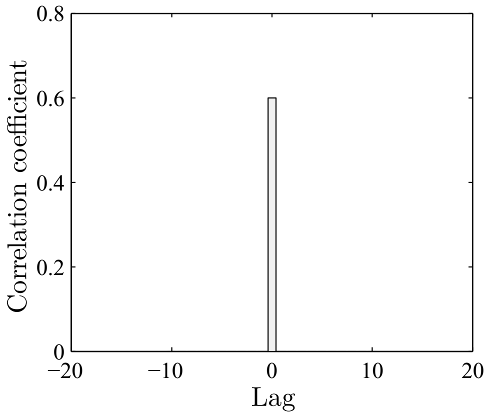
Mathematically, the cross-correlations between the series of errors $\boldsymbol{\varepsilon}^{\boldsymbol{a}}$ and $\boldsymbol{\varepsilon}^{\boldsymbol{b}}$ can be described by means of a variance-covariance matrix $\boldsymbol{G}$. This matrix is symmetric by definition, and therefore, can be always diagonalized. In other words, an orthogonal transformation can be used to model such series of errors as

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \boldsymbol{\varepsilon}^{\boldsymbol{a}} \\ \boldsymbol{\varepsilon}^{\boldsymbol{b}} \end{pmatrix} = \boldsymbol{B}\boldsymbol{\xi} = \boldsymbol{B} \begin{pmatrix} \boldsymbol{\xi}^{\boldsymbol{a}} \\ \boldsymbol{\xi}^{\boldsymbol{b}} \end{pmatrix}, \tag{3.31}$$

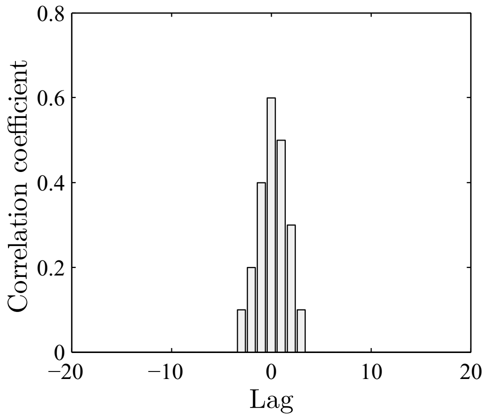
in which $\boldsymbol{B}$ is the orthogonal matrix of the transformation, and $\boldsymbol{\xi}$ is normal and satisfies that $E[\boldsymbol{\xi}\boldsymbol{\xi}^T] = \boldsymbol{I}$, being $\boldsymbol{I}$ the identity matrix. That is, $\boldsymbol{\xi}$ is neither autocorrelated, nor cross-correlated.

This way, white noises (independent standard normal errors) can be generated, and then, cross-correlated according to the variance-covariance matrix $\boldsymbol{G}$ by using the orthogonal transformation as expressed in (3.31).

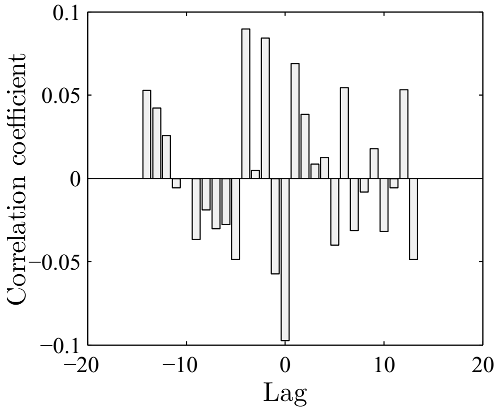
In most engineering applications, matrix $\boldsymbol{G}$, besides being symmetric, is also positive definite, and as such, can be decomposed through the computationally advantageous Cholesky decomposition, which avoids the calculation



(a) Contemporaneous



(b) Quasi-contemporaneous



(c) Non-contemporaneous

Fig. 3.9 Types of dependent stochastic processes according to their residual cross-correlogram: examples of eigenvalues and eigenvectors. The Cholesky decomposition can be stated as

$$\boldsymbol{G} = \boldsymbol{L}\boldsymbol{L}^T, \tag{3.32}$$

where $\boldsymbol{L}$ is an inferior triangular matrix that turns out to be the orthogonal matrix required for transformation (3.31), i.e., $\boldsymbol{B} = \boldsymbol{L}$, as shown below.

The variance-covariance matrix $\boldsymbol{G}$ can be computed as

$$\begin{aligned} \boldsymbol{G} &= cov(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^T) = cov(\boldsymbol{B}\boldsymbol{\xi}, \boldsymbol{\xi}^T\boldsymbol{B}^T) = \boldsymbol{B}\, cov(\boldsymbol{\xi}, \boldsymbol{\xi}^T)\boldsymbol{B}^T \\ &= \boldsymbol{B}\, E[\boldsymbol{\xi}\boldsymbol{\xi}^T]\boldsymbol{B}^T = \boldsymbol{B}\boldsymbol{I}\boldsymbol{B}^T = \boldsymbol{B}\boldsymbol{B}^T, \end{aligned} \tag{3.33}$$

where $cov(\cdot,\cdot)$ stands for the covariance operator.

By using the Cholesky decomposition (3.32), equation (3.33) yields

$$\boldsymbol{G} = \boldsymbol{L}\boldsymbol{L}^T = \boldsymbol{B}\boldsymbol{B}^T \Rightarrow \left(\boldsymbol{B}^{-1}\boldsymbol{L}\right)\left(\boldsymbol{L}^T(\boldsymbol{B}^T)^{-1}\right) = \boldsymbol{I} \Rightarrow \boldsymbol{B} = \boldsymbol{L}.$$

The simulation of the cross-correlated error $\boldsymbol{\varepsilon}$ becomes more costly as the number of significative cross-correlation coefficients increases. In this sense, the proposed methodology is efficacious provided that the statistical dependencies among residuals can be explained by a finite set of cross-correlation coefficients, and its efficaciousness is dependent on the cardinality of this set. In other words, this methodology is suitable for contemporaneous or quasi-contemporaneous stochastic processes.

Next, the procedure to generate a scenario set for a number N_S of quasi-contemporaneous stochastic processes is described step by step. The algorithm is also illustrated through the flow chart depicted in Fig. 3.10.

3.4.2.1 Algorithm

- *Step 1*: For each stochastic process, fit an ARMA model to the corresponding series of historical values. This fitting process yields non-autocorrelated normal residuals (historical errors).
- *Step 2*: Although the historical error series obtained in the preceding step are not *autocorrelated*, they should be *cross-correlated* if the stochastic processes under analysis are indeed interrelated. In this step, the cross-correlations are computed and, consequently, the variance-covariance matrix $\boldsymbol{G}$ is built. Considering for the sake of simplicity just two stochastic processes, $\boldsymbol{Y}^a$ and $\boldsymbol{Y}^b$, the structure of matrix $\boldsymbol{G}$ is as

$$\boldsymbol{G} = \begin{array}{cc} \begin{array}{cc} \boldsymbol{\varepsilon}^a & \boldsymbol{\varepsilon}^b \end{array} & \\ \left(\begin{array}{c|c} \boldsymbol{G}_{11} & \boldsymbol{G}_{12} \\ \hline \boldsymbol{G}_{21} & \boldsymbol{G}_{22} \end{array}\right) & \begin{array}{c} \boldsymbol{\varepsilon}^a \\ \boldsymbol{\varepsilon}^b \end{array} \end{array} \tag{3.34}$$

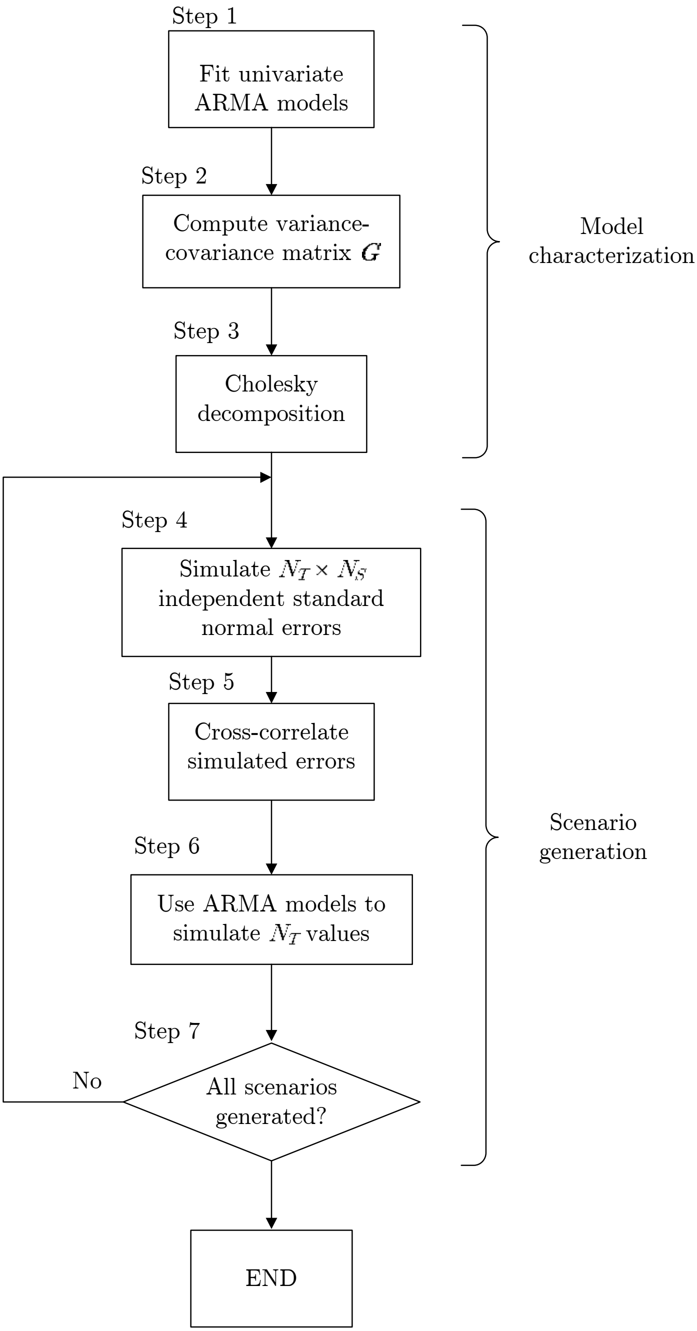
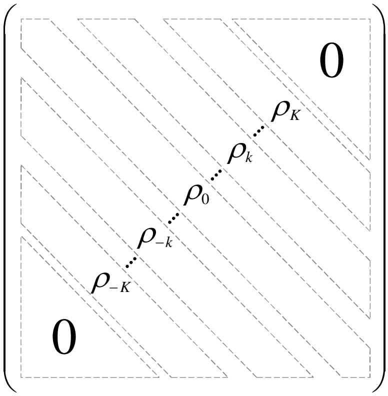


Fig. 3.10 Contemporaneous or quasi-contemporaneous stochastic processes: flow diagram of the algorithm

where $\boldsymbol{G}_{11} = \sigma^2_{\varepsilon^a} \boldsymbol{I}_{K+N_T}$, $\boldsymbol{G}_{22} = \sigma^2_{\varepsilon^b} \boldsymbol{I}_{K+N_T}$, and

$$\boldsymbol{G}_{12} = \boldsymbol{G}_{21}^T = \sigma_{\varepsilon^a}\sigma_{\varepsilon^b}$$



ρ_k is the positive lag-k cross-correlation coefficient, K is the maximum lag (positive or negative) with a significative cross-correlation coefficient, and N_T is the number of time periods to be covered in the decision-making process.
The size of matrix $\boldsymbol{G}$ is $(K + N_T) \times N_S$, where N_S is the number of stochastic processes under consideration. If $K \ll K + N_T$, then it is computationally advantageous to treat $\boldsymbol{G}$ as a sparse matrix.

- *Step 3*: Apply Cholesky decomposition to $\boldsymbol{G}$ (i.e., compute $\boldsymbol{L}$ such that $\boldsymbol{G} = \boldsymbol{L}\boldsymbol{L}^{\boldsymbol{T}}$) so as to obtain the matrix $\boldsymbol{B}$ of the orthogonal transformation (3.31).
- *Step 4*: Simulate $N_T \times N_S$ independent standard normal errors, $\boldsymbol{\xi}$.
- *Step 5*: Cross-correlate the $N_T \times N_S$ standard normal errors generated in the previous step by using the orthogonal transformation (3.31). Specifically, for two stochastic processes, $\boldsymbol{Y}^a$ and $\boldsymbol{Y}^b$, this cross-correlation process is performed as

$$\begin{pmatrix} \hat{\varepsilon}^a_{n-K+1} \\ \vdots \\ \hat{\varepsilon}^a_n \\ \varepsilon^a_{n+1} \\ \vdots \\ \varepsilon^a_{n+N_T} \\ \hline \hat{\varepsilon}^b_{n-K+1} \\ \vdots \\ \hat{\varepsilon}^b_n \\ \varepsilon^b_{n+1} \\ \vdots \\ \varepsilon^b_{n+N_T} \end{pmatrix} = \boldsymbol{L} \begin{pmatrix} \varepsilon^a_{n-K+1} \\ \vdots \\ \varepsilon^a_n \\ \xi^a_{n+1} \\ \vdots \\ \xi^a_{n+N_T} \\ \hline \varepsilon^b_{n-K+1} \\ \vdots \\ \varepsilon^b_n \\ \xi^b_{n+1} \\ \vdots \\ \xi^b_{n+N_T} \end{pmatrix}$$

where n is the sample size of the available historical data set, ε^a_{n-K+1}, $\ldots$, ε^a_n, ε^b_{n-K+1}, $\ldots$, ε^b_n are the last K error terms of the historical series, $\hat{\varepsilon}^a_{n-K+1}$, $\ldots$, $\hat{\varepsilon}^a_n$, $\hat{\varepsilon}^b_{n-K+1}$, $\ldots$, $\hat{\varepsilon}^b_n$ are their corresponding transformed values, and ξ^a_{n+1}, $\ldots$, $\xi^a_{n+N_T}$, ξ^b_{n+1}, $\ldots$, $\xi^b_{n+N_T}$ are the $N_T \times N_S$ ($N_S = 2$) independent standard normal errors simulated in Step 4 above.
Finally, the error series ε^a_{n+1}, $\ldots$, $\varepsilon^a_{n+N_T}$ and ε^b_{n+1}, $\ldots$, $\varepsilon^b_{n+N_T}$ are those to be introduced in the ARMA models fitted in Step 1.

- *Step 6*: For each stochastic process, use, as explained in Subsection 3.2.2, the corresponding N_T cross-correlated normal errors to simulate N_T values by means of the corresponding univariate ARMA model fitted in Step 1.
- *Step 7*: Repeat Steps 4-6 until the desired number (N_Ω) of scenarios is obtained.

The following example is intended to illustrate the algorithm described above.

Illustrative Example 3.12 (Contemporaneous or quasi-contemporaneous stochastic processes. Algorithm). Consider two contemporaneous stochastic processes, $\boldsymbol{Y}^a$ and $\boldsymbol{Y}^b$, with a contemporaneous correlation coefficient ρ_0 equal to 0.9. Suppose that $y^a_0 = 4.10$ and $y^b_0 = 2.95$. The procedure to generate a pair of correlated two-period scenarios for these stochastic processes is as follows:

- *Step 1*: We fit an ARMA model to each stochastic process by using available historical data. Let us assume that the fitting process results in the following two AR(1) models

$$
\begin{aligned}
y^a_t &= 0.910 y^a_{t-1} + \varepsilon^a_t, \\
y^b_t &= 0.898 y^b_{t-1} + \varepsilon^b_t,
\end{aligned}
$$

which can be equivalently expressed in matricial form as

$$
\begin{pmatrix} y^a_t \\ y^b_t \end{pmatrix} = \begin{pmatrix} 0.910 & 0 \\ 0 & 0.898 \end{pmatrix} \begin{pmatrix} y^a_{t-1} \\ y^b_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon^a_t \\ \varepsilon^b_t \end{pmatrix},
$$

where the estimated standard deviations of residuals, σ_{ε^a} and σ_{ε^b}, are 0.1 and 0.05, respectively.
- *Step 2*: Next, we build the variance-covariance matrix $\boldsymbol{G}$,

$$
\boldsymbol{G}_{11} = 0.1^2 \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.01 \end{pmatrix},
$$

$$
\boldsymbol{G}_{22} = 0.05^2 \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.0025 & 0 \\ 0 & 0.0025 \end{pmatrix},
$$

$$
\boldsymbol{G}_{12} = \boldsymbol{G}^T_{21} = 0.1 \times 0.05 \times \begin{pmatrix} 0.9 & 0 \\ 0 & 0.9 \end{pmatrix} = \begin{pmatrix} 0.0045 & 0 \\ 0 & 0.0045 \end{pmatrix}.
$$

Therefore,

$$\boldsymbol{G} = \left(\begin{array}{cc|cc} 0.01 & 0 & 0.0045 & 0 \\ 0 & 0.01 & 0 & 0.0045 \\ \hline 0.0045 & 0 & 0.0025 & 0 \\ 0 & 0.0045 & 0 & 0.0025 \end{array}\right).$$

Note that $N_T = 2$ and $K = 0$.

- *Step 3*: We apply the Cholesky decomposition to matrix $\boldsymbol{G}$ in order to obtain the orthogonal matrix $\boldsymbol{L}$,

$$\boldsymbol{L} = \left(\begin{array}{cc|cc} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ \hline 0.045 & 0 & 0.0218 & 0 \\ 0 & 0.045 & 0 & 0.0218 \end{array}\right).$$

- *Step 4*: The next step is to simulate $N_T \times N_S = 2 \times 2 = 4$ independent standard normal errors, $\boldsymbol{\xi}$,

$$\boldsymbol{\xi} = \left(\begin{array}{c} \boldsymbol{\xi}^a \\ \hline \boldsymbol{\xi}^b \end{array}\right) = \left(\begin{array}{c} \xi_1^a \\ \xi_2^a \\ \hline \xi_1^b \\ \xi_2^b \end{array}\right) = \left(\begin{array}{c} -0.4326 \\ -1.6656 \\ \hline 0.1253 \\ 0.2877 \end{array}\right).$$

- *Step 5*: We cross-correlate the four standard normal errors generated in the previous step by using the orthogonal transformation (3.31),

$$\left(\begin{array}{c} \varepsilon_1^a \\ \varepsilon_2^a \\ \hline \varepsilon_1^b \\ \varepsilon_2^b \end{array}\right) = \left(\begin{array}{cc|cc} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ \hline 0.045 & 0 & 0.0218 & 0 \\ 0 & 0.045 & 0 & 0.0218 \end{array}\right) \left(\begin{array}{c} -0.4326 \\ -1.6656 \\ \hline 0.1253 \\ 0.2877 \end{array}\right) = \left(\begin{array}{c} -0.0433 \\ -0.1666 \\ \hline -0.0167 \\ -0.0687 \end{array}\right).$$

- *Step 6*: Finally, we use the cross-correlated normal errors generated in Step 5 above to simulate two values of stochastic processes $\boldsymbol{Y}^a$ and $\boldsymbol{Y}^b$ by means of the ARMA models fitted in Step 1, that is,

$$\begin{aligned} \begin{pmatrix} y_1^a \\ y_1^b \end{pmatrix} &= \begin{pmatrix} 0.910 & 0 \\ 0 & 0.898 \end{pmatrix} \begin{pmatrix} y_0^a \\ y_0^b \end{pmatrix} + \begin{pmatrix} \varepsilon_1^a \\ \varepsilon_1^b \end{pmatrix} \\ &= \begin{pmatrix} 0.910 & 0 \\ 0 & 0.898 \end{pmatrix} \begin{pmatrix} 4.10 \\ 2.95 \end{pmatrix} + \begin{pmatrix} -0.0433 \\ -0.0167 \end{pmatrix} = \begin{pmatrix} 3.69 \\ 2.63 \end{pmatrix}; \end{aligned}$$

$$\begin{aligned} \begin{pmatrix} y_2^a \\ y_2^b \end{pmatrix} &= \begin{pmatrix} 0.910 & 0 \\ 0 & 0.898 \end{pmatrix} \begin{pmatrix} y_1^a \\ y_1^b \end{pmatrix} + \begin{pmatrix} \varepsilon_2^a \\ \varepsilon_2^b \end{pmatrix} \\ &= \begin{pmatrix} 0.910 & 0 \\ 0 & 0.898 \end{pmatrix} \begin{pmatrix} 3.69 \\ 2.63 \end{pmatrix} + \begin{pmatrix} -0.1666 \\ -0.0687 \end{pmatrix} = \begin{pmatrix} 3.19 \\ 2.29 \end{pmatrix}. \end{aligned}$$

In summary, the pair of correlated scenarios $\left(\hat{\boldsymbol{Y}}^a, \hat{\boldsymbol{Y}}^b\right)^T$ covering two periods are

$$\begin{pmatrix} \hat{\boldsymbol{Y}}^a \\ \hat{\boldsymbol{Y}}^b \end{pmatrix} = \begin{pmatrix} 3.69 & 3.19 \\ 2.63 & 2.29 \end{pmatrix} .$$

□

In the following section, we provide some general guidelines to tackle the problem of how to generate dependent scenarios for non-contemporaneous stochastic processes.

3.4.3 Scenarios for non-contemporaneous stochastic processes

In this subsection we focus on the generation of scenarios pertaining to correlated stochastic processes that do not exhibit contemporaneous or quasi-contemporaneous correlation. This is the case, for instance, of the pool price and the system demand. In fact, pool price and demand are correlated contemporaneously, but they also present a non-negligible cross-correlation in lags significantly larger than 1. In order to take into account these cross-correlations, the variance-covariance matrix $\boldsymbol{G}$ defined in the previous subsection would be too large to be managed. For this reason the methodology presented in the previous section is not appropriate for the case with non-contemporaneous processes.

In this subsection we propose a methodology based on dynamic regression models that can be efficiently used to generate scenarios for non-contemporaneous stochastic processes. We refer the interested reader to [18, 72] for further information on dynamic regression models.

We define $\boldsymbol{Y}$ $\{y_t,\ t \in T\}$ and $\boldsymbol{Y}^e$ $\{y_t^e,\ t \in T\}$ as two non-contemporaneous processes, where $\boldsymbol{Y}^e$ is considered an explanatory process of $\boldsymbol{Y}$. This means that the behavior of $\boldsymbol{Y}$ can be explained in part by the knowledge of $\boldsymbol{Y}^e$. Process $\boldsymbol{Y}$ is usually referred to as the forecast process. A dynamic regression model allows studying the effect of changes in the explanatory process on the forecast process. For instance, the system demand can be used as a explanatory process of the pool price.

The stochastic process $\boldsymbol{Y}$ can be expressed using a dynamic regression model as follows,

$$y_t = c + \sum_{j=1}^{p_1} u_j y_{t-j} + \sum_{j=0}^{p_2} v_j y_{t-j}^e + \varepsilon_t, \tag{3.35}$$

where c is a constant, and parameters $u_1, u_2, \ldots, u_{p_1}$ relate y_t to process $\boldsymbol{Y}$ in previous time periods, $y_{t-1}, y_{t-2}, \ldots, y_{t-p_1}$. Likewise, parameters

$v_0, v_1, \ldots, v_{p_2}$ link the explanatory variable $\boldsymbol{Y}^e$ in period t, y_t^e, and periods previous to t, $y_{t-1}^e, y_{t-2}^e, \ldots, y_{t-p_2}^e$ with y_t. Finally, the series of errors ε_t is a white noise process with mean zero and standard deviation σ_ε.

Note that obtaining the value of y_t using expression (3.35) requires to know the value of the explanatory variable $\boldsymbol{Y}^e$ in future period t, y_t^e. This means that is necessary to use a forecasting method for obtaining y_t^e before using expression (3.35). Therefore, the dynamic regression model (3.35) is only useful if the explanatory process can be predicted with a small forecast error. Observe that this is the case of the demand acting as an explanatory variable of the pool price. In the technical literature it can be checked that forecast errors of demands are sensibly smaller than those of pool prices. Assuming that the explanatory process can be efficiently described using an ARMA model, $\boldsymbol{Y}^e$ is given by

$$y_t^e = c^e + \sum_{j=1}^{p_e} \phi_j^e y_{t-j}^e + \varepsilon_t^e - \sum_{q=1}^{q_e} \theta_j^e \varepsilon_{t-j}^e, \tag{3.36}$$

with p_e autoregressive parameters $\phi_1^e, \phi_2^e, \ldots, \phi_{p_e}^e$, and q_e moving average parameters $\theta_1^e, \theta_2^e, \ldots, \theta_{q_e}^e$. The error term ε_t^e is a white noise process with mean equal to zero and standard deviation σ_ε^e.

In the same manner as in the ARMA models explained in Subsection 3.2.2, it may be necessary to make some initial transformations to the forecast and explanatory processes in order to attain normality and stationarity, which are necessary for building model (3.35) and for estimating its parameters. The estimation of the parameters can be done with specific software, using a minimum square method, or minimizing a likelihood function.

Given that the ARMA model for the explanatory process and the dynamic regression model for the forecast process are already established, the scenario-generation process is outlined below.

3.4.3.1 Algorithm

- *Step 1*: Initialize the scenario counter: $\omega \leftarrow 0$.
- *Step 2*: Update the scenario counter and initialize the time period counter: $\omega \leftarrow \omega + 1$, $t \leftarrow 0$.
- *Step 3*: Update the time period counter: $t \leftarrow t + 1$.
- *Step 4*: Randomly generate $\varepsilon_t^e(\omega) \sim \mathrm{N}(0, \sigma_\varepsilon^e)$.
- *Step 5*: Evaluate expression (3.36) to obtain $y_t^e(\omega)$.
- *Step 6*: Randomly generate $\varepsilon_t(\omega) \sim \mathrm{N}(0, \sigma_\varepsilon)$.
- *Step 7*: Evaluate expression (3.35) to obtain $y_t(\omega)$.
- *Step 8*: If $t < N_T$ go to Step 3), else go to Step 9).

- *Step 9*: If $\omega < N_\Omega$ go to Step 2), else the scenario-generation process concludes.

3.5 Case Studies

In this section two realistic case studies are discussed and analyzed. In the first one, a set of scenarios for electricity prices and demands is built. This case constitutes an example of scenario generation for non-contemporaneous stochastic processes. In the second case study, scenarios for wind speeds located at multiple sites are generated. This last case illustrates an example of scenario generation for quasi-contemporaneous stochastic processes.

3.5.1 Scenario Generation Using ARIMA and Dynamic Regression models: Electricity Prices and Demand

The objective of this case study is to build a set of scenarios modeling the pool price and the system demand in a short-term horizon for a given electricity market. Two cases are presented, which result from considering or not the correlation between prices and demand. Specifically, in the first instance, this correlation is accounted for by using a dynamic regression model, while in the second one, two independent ARIMA models are fitted for electricity prices and demand.

The historical data corresponding to pool prices and demand used in this case study come from the electricity market of the Iberian Peninsula and correspond to Spain. These data are publicly available in [106]. Hereinafter, pool prices refer to prices in the day-ahead market.

Two historical series of prices and demand throughout July 2005 are considered. These two series are depicted in Fig. 3.11.

Observe that daily and weekly seasonalities can be identified in the historical data of both series. Moreover, at first sight, pool prices and demand are strongly correlated. Fig. 3.12 represents demand and pool prices in axes x and y, respectively. In this figure it is easy to appreciate a positive correlation between both series.

Next, pool price and demand scenarios for one week are generated independently using ARIMA models without modeling explicitly the correlation between both series. Thus, the historical data represented in Fig. 3.11 are used to adjust two ARIMA models representing future pool prices and demand. Note that these two ARIMA models are fitted in an independent manner. The ARIMA model for the pool price, adjusted following the procedure presented in [18], is

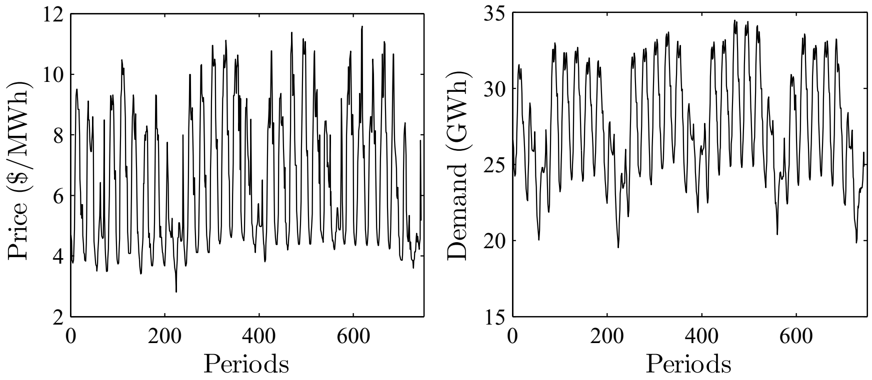


Fig. 3.11 Scenario-generation case studies: historic pool prices and demands

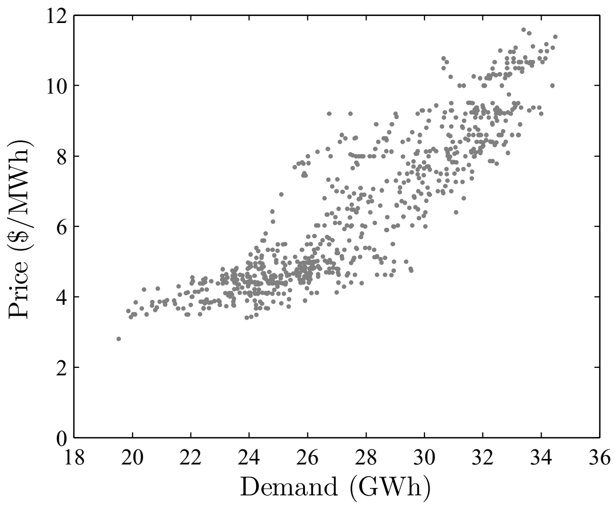


Fig. 3.12 Scenario-generation case studies: correlation between pool prices and demand

$$\left(1-\phi_1^{\mathrm{P}} B-\phi_2^{\mathrm{P}} B^2\right)\left(1-\phi_{24}^{\mathrm{P}} B^{24}\right)\left(1-\phi_{168}^{\mathrm{P}} B^{168}\right) \log \left(\lambda_t^{\mathrm{P}}\right)= \left(1-\theta_1^{\mathrm{P}} B-\theta_2^{\mathrm{P}} B^2\right)\left(1-\theta_{168}^{\mathrm{P}} B^{168}\right) \varepsilon_t^{\mathrm{P}}, \tag{3.37}$$

where λ_t^{P} represents the pool price in period t, and $\varepsilon_t^{\mathrm{P}} \sim N(0, \sigma^{\mathrm{P}})$ is the error term. Observe that the logarithm function is used to stabilize the variance of the series.

In a similar manner, the ARIMA model describing the demand is

$$\left(1-\phi_1^{\mathrm{D}}B\right)\left(1-\phi_{24}^{\mathrm{D}}B^{24}\right)\left(1-\phi_{168}^{\mathrm{D}}B^{168}\right)\log\left(E_t^{\mathrm{D}}\right) = \left(1-\theta_{168}^{\mathrm{D}}B^{168}\right)\varepsilon_t^{\mathrm{D}}, \qquad (3.38)$$

where E_t^{D} represents the demand in period t, and $\varepsilon_t^{\mathrm{D}} \sim N(0, \sigma^{\mathrm{D}})$ is the error term.

The adjusted parameters of models (3.37) and (3.38) are provided in Table 3.3. The standard deviations of the price and demand error terms, σ^{P} and σ^{D}, are 0.0334 and 0.0048, respectively. Observe that the standard deviation corresponding to the error term of the demand is lower than that of the price. This fact is a consequence of the higher volatility of the price series.

Table 3.3 Scenario-generation case studies: parameters for the price and demand ARIMA models

Price		Demand	
ϕ_1^{P}=0.3135	θ_1^{P}=-0.4495	ϕ_1^{D}=0.9470	$\theta_{168}^{\mathrm{D}}$= 0.6027
ϕ_2^{P}=0.6580	θ_2^{P}=0.1356	ϕ_{24}^{D}=0.3449	
ϕ_{24}^{P}=0.2599	$\theta_{168}^{\mathrm{P}}$=0.4717	ϕ_{168}^{D}=0.9813	
ϕ_{168}^{P} = 0.8674			

Using the scenario-generation algorithm described in Subsection 3.2.2 for both pool price and demand series, a set of 1000 scenarios for each series is obtained. These two sets are depicted in Fig. 3.13. In this figure it is easy to observe that the volatility of price scenarios is clearly higher than that of the demand scenarios. This fact can be explained by the higher standard deviation of the error term in the ARIMA model fitted to explain pool prices.

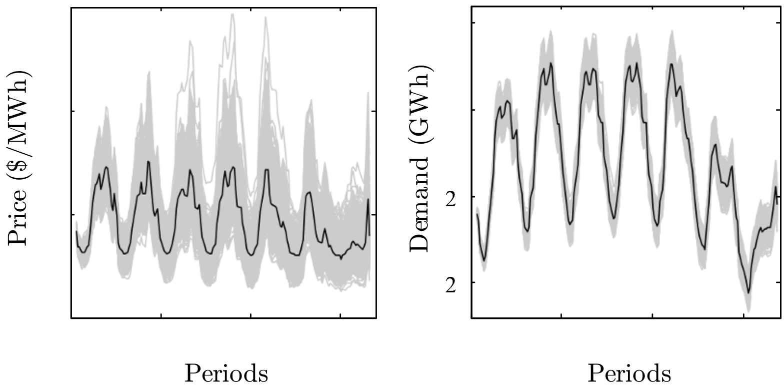


Fig. 3.13 Scenario-generation case studies: price and demand scenarios using ARIMA models

Once the scenarios of price and demand are obtained, it is possible to plot conveniently their values to observe the correlation between both series. This is represented in Fig. 3.14. Black points represent the pairs price-demand observed in the historical data, whereas grey points correspond to the pairs resulting from the generated scenarios.

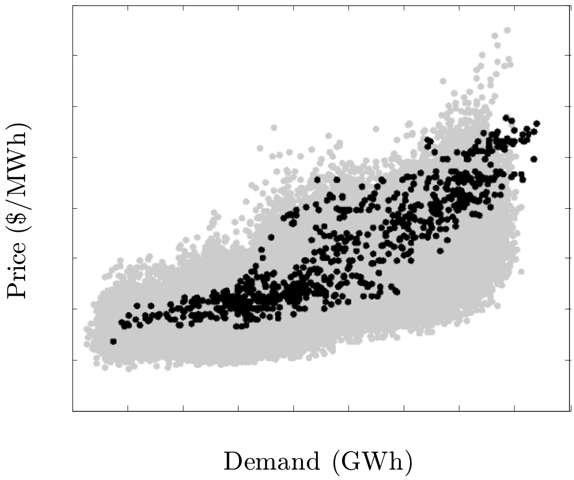


Fig. 3.14 Scenario-generation case studies: correlation of pool price and demand scenarios using ARIMA models

Observe that the points obtained from the historical data are within the area covered by the simulated points. However, some points generated are far away from the historical ones. This result indicates that the correlation between price and demand might not be properly modeled. To check this, it is appropriate to represent the correlation between the series of residuals resulting from the ARIMA fits for prices and demand. If these two series of residuals are not correlated, then the correlation between price and demand is being already captured by the independent ARIMA models. However, if the residuals are correlated, then it would be necessary to account for the correlation between prices and demand using a dedicated procedure. The correlation of the error terms of prices and demand is depicted in Fig 3.15. As observed, residuals from the ARIMA models of pool prices and demand are effectively correlated. Moreover, this correlation is non-contemporaneous. For this reason, it is appropriate to account explicitly for this correlation by means of a dynamic regression model as the one explained in Subsection 3.4.3.

The dynamic regression model proposed is

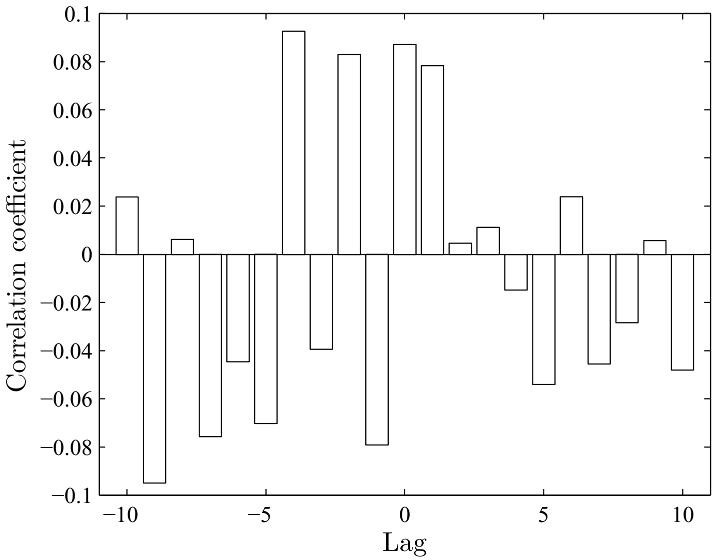


Fig. 3.15 Scenario-generation case studies: residual cross-correlogram

$$\left(1 - u_1^{\mathrm{R}} B - u_2^{\mathrm{R}} B^2\right)\left(1 - u_{24}^{\mathrm{R}} B^{24}\right)\left(1 - u_{168}^{\mathrm{R}} B^{168}\right)\log\left(\lambda_t^{\mathrm{P}}\right) = \left(v_0^{\mathrm{R}} + v_1^{\mathrm{R}} B + v_2^{\mathrm{R}} B^2\right)\log\left(E_t^{\mathrm{D}}\right) + \varepsilon_t^{\mathrm{R}}, \tag{3.39}$$

where $\varepsilon_t^{\mathrm{R}} \sim N(0, \sigma^{\mathrm{R}})$ is the error term. Observe that it is necessary to know the value of the demand in period t, E_t^{D}, to forecast the price in that period, λ_t^{P}. For this reason, if using dynamic regression models an additional model is required to forecast the explanatory variable. To this end, we use the ARIMA model (3.38) above to forecast the demand.

The parameters of the dynamic regression model (3.39) are listed in Table 3.4. The standard deviation of the error term, σ^{R}, is 0.0295. Observe that this value is smaller than the standard deviation of the error term resulting from the ARIMA model of the pool prices, which suggests that the use of the demand as an explanatory variable is helpful to reduce the value of the error term in the model of the pool price.

Table 3.4 Scenario-generation case studies: price dynamic regression model parameters

u_1^{R}=0.7073	v_0^{R}= 1.5514
u_2^{R}=0.1306	v_1^{R}=-1.0658
u_{24}^{R}= 0.1998	v_2^{R}=-0.3172
u_{168}^{R} = 0.2901	

Once the dynamic regression model is adjusted, a set of 1000 scenarios of pool prices is generated using the procedure explained in Subsection 3.4.3. As previously stated, ARIMA model (3.38) is used first to generate 1000-demand scenarios. For this example, the same demand scenario set obtained in the preceding subsection has been used to obtain the pool price set. Fig. 3.16 depicts the resulting set of pool price scenarios. Note that the volatility of price scenarios is much smaller than that pertaining to the set obtained by means of the ARIMA model (3.37), as a result of the smaller standard deviation of the error term in the dynamic regression model (3.39).

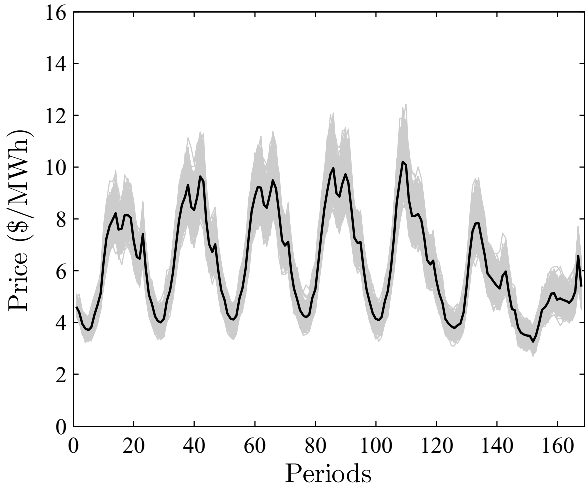


Fig. 3.16 Scenario-generation case studies: pool price scenarios using a dynamic regression model

Finally, Fig. 3.17 depicts the resulting set of demand scenarios versus the set of pool price scenarios (grey points) together with the historical data (black points). Observe that the coincidence between simulated and historical data is apparent.

3.5.2 Scenario Generation for Quasi-contemporaneous Stochastic Processes: Wind Speeds at Multiple Sites

Wind speeds at two different sites that are geographically close constitute an example of quasi-contemporaneous stochastic processes. Hence, the statistical dependence existing between both sites can be embedded into a scenario set

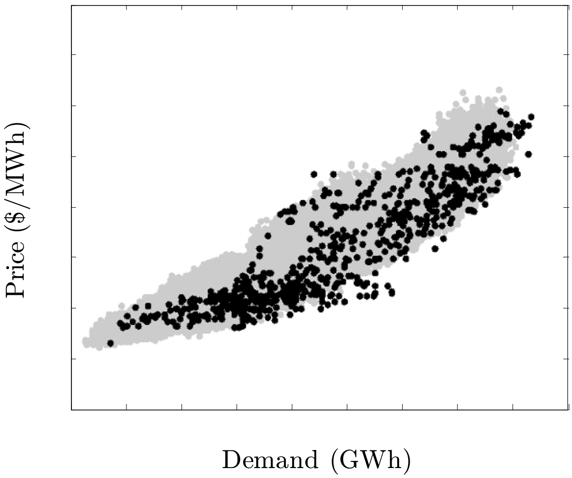


Fig. 3.17 Scenario-generation case studies: correlation of pool price and demand scenarios using a dynamic regression model

by means of the methodology explained in Subsection 3.4.2. The proximity between sites is required for their winds to be spatially correlated.

In this case study, the referred methodology is applied to model wind speed stochastic processes, $\boldsymbol{Y}^A$ and $\boldsymbol{Y}^B$, at two sites, A and B, in Massachusetts, US. Specifically, site A is located at Bishop and Clerks, within the three-mile state limit of Massachusetts's waters, at 41° 34' 27.6" North and 70° 14' 59.5" West. Site B is situated at the Water Treatment Plant in Falmouth, at 41° 36' 21.6" North, 70° 37' 15.60" West. The distance between sites is 31.05 km. All the historical wind speed series used in this case study are publicly available in [123]. The wind data sets comprise ten-minute averaged speed measurements that have been aggregated into hourly time intervals. These data were collected between June 1 and December 30, 2004.

Prior to using the algorithm illustrated in Fig. 3.10, we should deal with the fact that wind speed is not usually normally distributed, but follows a Weibull distribution. As an example, Fig. 3.18 plots both the wind speed frequency distribution of site A and the corresponding probability density function (pdf) of the fitted Weibull.

Consequently, in order to reconcile the normality assumption on which ARMA models are based with the non-Gaussian nature of local winds, we make use of the marginal transformation (3.5) to *normalize* the wind speed historical data, i.e,

$$\boldsymbol{Z}^A = \Phi^{-1}\left[F_{Y^A}(\boldsymbol{Y^A})\right],$$
$$\boldsymbol{Z}^B = \Phi^{-1}\left[F_{Y^B}(\boldsymbol{Y^B})\right],$$

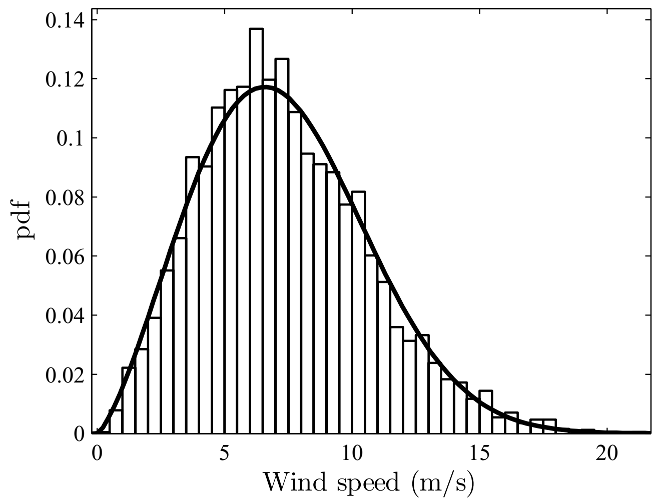


Fig. 3.18 Scenario-generation case studies: wind speed frequency distribution for site A and the pdf of the Weibull fit

where F_{Y^A} and F_{Y^B} are the cumulative distribution functions (cdf) of the Weibull distributions fitted for wind speeds at sites A and B, respectively, and Φ is the cdf of the standard normal random variable (with zero mean and unit standard deviation).

The procedure described through the flowchart in Fig. 3.10 is then applied to the resulting *transformed* series.

Fig. 3.19 is the normality plot of the transformed data corresponding to site A. Needless to say, the normal transformation is not perfect because neither is the Weibull fit. However, note that it constitutes a good enough approximation.

The next step of the algorithm consists in modeling the autocorrelations inferred from the historical data series by adjusting univariate ARMA(p, q) models. This fitting process is briefly described in Subsection 3.2.2 and provides the following time series models:

Site A: ARMA(1,2)

$$z_t^A = 0.925 z_{t-1}^A + \varepsilon_t^A + 0.183 \varepsilon_{t-1}^A - 0.025 \varepsilon_{t-2}^A.$$

Site B: ARMA(1,2)

$$z_t^B = 0.915 z_{t-1}^B + \varepsilon_t^B + 0.092 \varepsilon_{t-1}^B - 0.040 \varepsilon_{t-2}^B.$$

The residual errors derived from the univariate time series analysis should not be autocorrelated, but they are cross-correlated if there exists a spatial

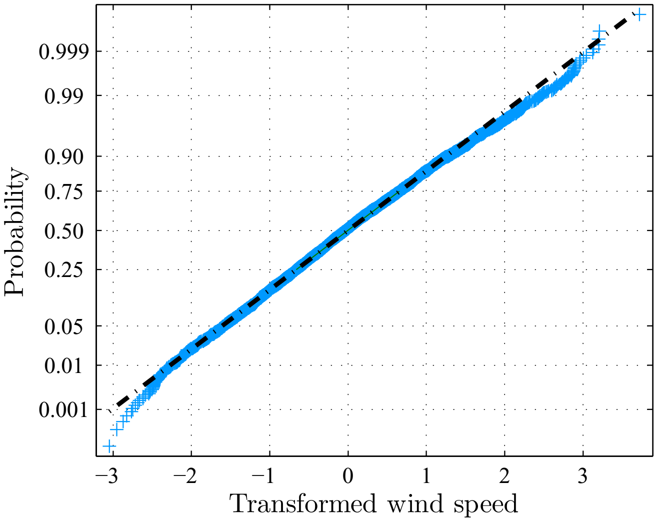


Fig. 3.19 Scenario-generation case studies: normality plot of the transformed data for site A

interaction between wind sites A and B. It is important to remark that the cross-correlation among wind speeds at multiple sites stems from their geographical closeness. This is the reason why this type of cross-correlation is also referred to as *spatial correlation*. In Fig. 3.20, the cross-correlogram of the residuals (which we often call *residual cross-correlogram*) is depicted. Observe the triangular structure, typical of quasi-contemporaneous stochastic processes, with the most significant correlation coefficients around the lag zero.

From this point, the algorithm focuses on simulating errors that reproduce the cross-correlogram of Fig. 3.20. Nevertheless, not all the cross-correlation coefficients should be reproduced, but just those considered significant enough from a statistical point of view. The significance of a given cross-correlation coefficient is appraised by means of a *p-value* testing the hypothesis of no correlation [128]. This p-value is the probability of obtaining a correlation as large as the observed one by random chance, when the true correlation is zero. In Fig. 3.21(a), only those coefficients with a p-value smaller than a *significance level* $\alpha = 0.05$ are represented.

The simulation process starts generating $2 \times N_T$ independent standard normal errors, which are subsequently cross-correlated according to the variance-covariance matrix $\boldsymbol{G}$ defined by the cross-correlations coefficients of Fig. 3.21(a) and built as explained in Step 2 of the algorithm described in Subsection 3.4.2. For comparative purposes, the cross-correlogram of the

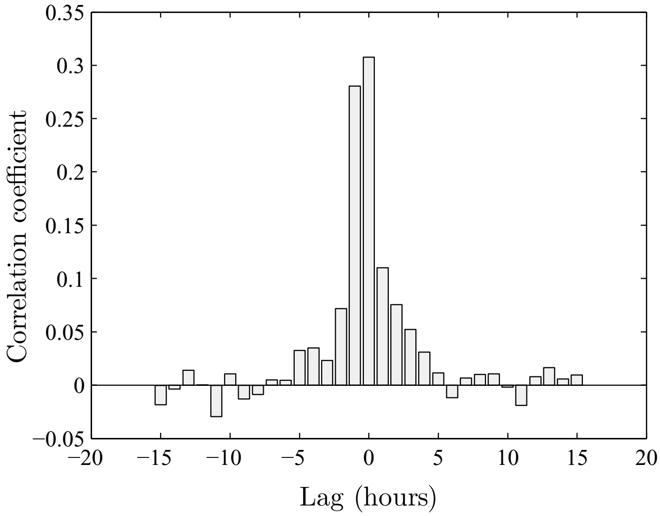


Fig. 3.20 Scenario-generation case studies: cross-correlogram of the residuals obtained after the ARMA fit

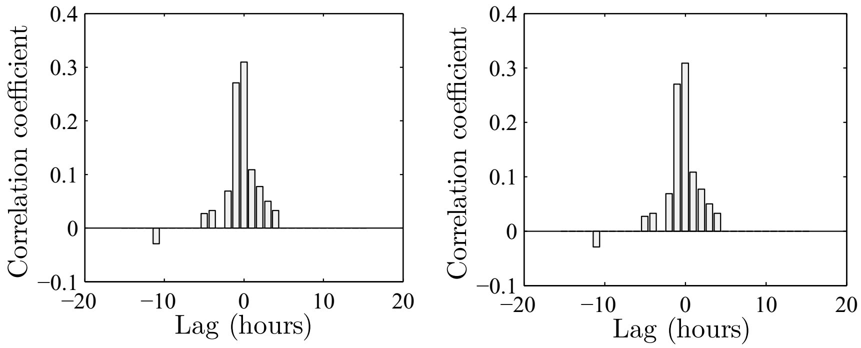


Fig. 3.21 Scenario-generation case studies: cross-correlograms of errors with a significance level α= 0.05

simulated errors for $N_T = 50{,}000$ is depicted in Fig. 3.21(b). The time span to cover is selected long enough to properly observe and assess the stationary properties of the simulated processes. The fidelity of the reproduction is apparent, which highlights the effectiveness of the cross-correlation procedure.

Once the cross-correlated errors have been generated, they are introduced into the previously fitted ARMA models in order to obtain the simulated series of *normalized* wind speeds for sites A and B. Finally, we apply the inverse marginal transformation (3.6) to these normalized series in order to produce wind speed values distributed according to the Weibull fits, i.e.,

$$\boldsymbol{Y}^A = F_{Y^A}^{-1}\left[\Phi(\boldsymbol{Z^A})\right]$$
$$\boldsymbol{Y}^B = F_{Y^B}^{-1}\left[\Phi(\boldsymbol{Z^B})\right].$$

In Fig. 3.22, the cross-correlogram of the simulated wind speed series, obtained using the above methodology, is compared with that estimated from the historical data. The 95% confidence limits are also shown. The lower and

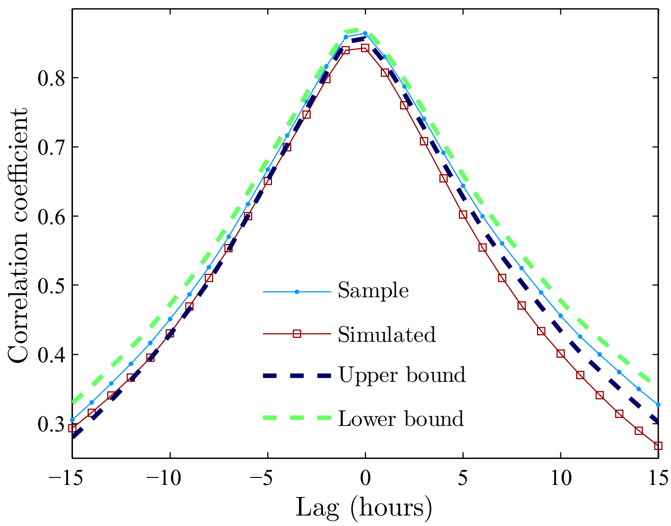


Fig. 3.22 Scenario-generation case studies: comparison between sample and simulated cross-correlograms

upper confidence bounds of the population lag-k cross-correlation coefficient, r_k^{lo} and r_k^{up}, respectively, are approximately computed by applying the Fisher Transformation [81] to the sample cross-correlation coefficient ρ_k, that is,

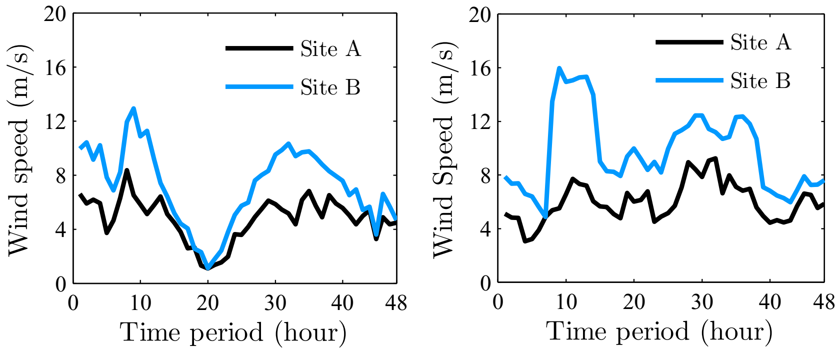
$$r_k^{\text{lo}} = \tanh\left[\frac{1}{2}\log\left(\frac{1+\rho_k}{1-\rho_k}\right) + \frac{\text{erfinv}\,(\beta-1)\,\sqrt{2}}{\sqrt{n-k-3}}\right] \tag{3.40}$$

$$r_k^{\text{up}} = \tanh\left[\frac{1}{2}\log\left(\frac{1+\rho_k}{1-\rho_k}\right) - \frac{\text{erfinv}\,(\beta-1)\,\sqrt{2}}{\sqrt{n-k-3}}\right] \tag{3.41}$$

where erfinv$(\cdot)$ is the value of the inverse error function, n is the sample size of the available historical data set, and β is the confidence level (equal to 0.05 in this case).

Note that the methodology designed to deal with quasi-contemporaneous stochastic processes succeeds in capturing the shape of the sample cross-correlogram with a remarkable degree of precision, albeit some cross-correlation coefficients fall out of the confidence limits. This is mostly due to the irremediable inaccuracies associated with the fitting processes (both ARMA and Weibull fits) and with the cross-correlogram reproduction task. It is also important to underline the significant spatial correlation existing between wind speeds at sites A and B, with a high contemporaneous correlation coefficient equal to 0.8643, which is due to the geographical proximity of both sites.

This strong correlation can be graphically appreciated in Fig. 3.23(a), where a 48-h sample from the historical time series is depicted. Fig. 3.23(b) illustrates a 48-h extract from the simulated wind speed series. By simple inspection, one can intuitively realize that the simulation replicates the cross-correlated behavior.



(a) Sample (b) Simulated

Fig. 3.23 Scenario-generation case studies: a 48-hour extract from the historical and simulated wind speed series at sites A and B

Finally, in order to illustrate the usefulness of transformation (3.5)–(3.6) to preserve the original marginal distribution of the wind stochastic processes, Fig. 3.24 shows, for wind site A, a comparison between the empirical cdf of the simulated series and the cdf of the Weibull distribution estimated from the historical data. Their similarity is clear from this figure. So as to complement this graphical contrast by giving simulation error values, Table 3.5 offers a numerical comparison between the 5, 25, 50, 75, and 95% percentiles computed from both the simulated and the historical wind series. The last

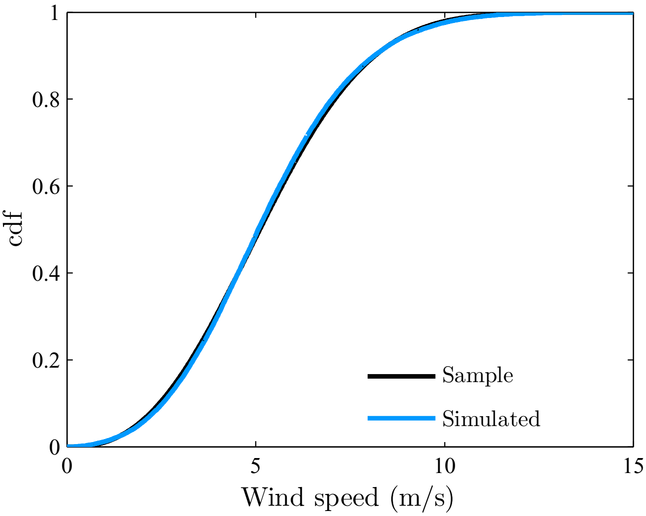


Fig. 3.24 Scenario-generation case studies: comparison between fitted and simulated cumulative probability functions for wind site A

row of this table provides the error of the simulation relative to the sample percentile. Note that this error keeps reasonably low.

Table 3.5 Scenario-generation case studies: percentile comparison for wind site A

	Percentile				
	5	25	50	75	95
Sample	1.85	3.62	5.11	6.69	9.04
Simulated	1.91	3.68	5.06	6.61	9.09
Error (%)	3.31	1.63	0.85	1.22	0.56

3.6 Summary and Conclusions

Stochastic programming constitutes a powerful modeling framework to solve decision-making problems affected by uncertain input data. Previous to undertaking the solution of a stochastic programming problem, uncertainties on the problem parameters are to be modeled as stochastic processes. In most cases, such a modeling endeavor includes the generation of a set of scenarios representing plausible realizations of the stochastic processes throughout the decision-making horizon. This chapter provides several methodologies to produce scenario sets using time series analysis.

The number of scenarios needed to properly represent stochastic processes in electricity-market problems is generally very large, which may render the associated optimization problem computationally intractable. For this reason, and as a complement to the scenario-generation topic, this chapter describes two efficient scenario-reduction procedures to trim down significantly the number of scenarios maintaining, as much as possible, the statistical information embedded in them.

In some electricity-market problems, the stochastic processes involved may be statistically dependent, and this dependency may have a non-negligible impact on the decisions to be made. This is particularly true, for example, in the case of producers with a generation portfolio including wind farms at several locations. Therefore, in such instances, the scenario-generation methodology to be employed should be able to recognize and incorporate dependencies among stochastic processes, and produce scenarios in consequence. This chapter presents a variety of strategies designed to this aim.

The chapter ends with a series of case studies pointing up the intricacies of the proposed scenario-generation techniques for typical stochastic processes in problems related to electricity markets (energy prices, demand, and wind speed).

In short, we conclude that:

1. Uncertainties in optimization problems need to be often represented via scenarios.
2. Tools for the analysis and modeling of time series can be exploited to design user-friendly scenario-generation techniques.
3. For the sake of computational tractability, procedures intended to reduce the number of scenarios required to conveniently represent stochastic processes become a need in large-scale decision-making problems.
4. Generally, those scenario-reduction techniques that use information of the optimization problem to be solved allow achieving a higher level of reduction for the same degree of accuracy.
5. Dependencies among stochastic processes may have a significant influence on decision making, and therefore, such dependencies must be accounted for in the corresponding scenario-generation process. The type of methodology to be used for this purpose is contingent on the shape of the cross-correlogram of residuals derived from the univariate analysis of each stochastic process under consideration.

3.7 Exercises

Exercise 3.1. Obtain a set of three scenarios for a time horizon of four periods for the process y_t expressed by means of an ARMA(2,0,1) model. The parameters of the model are $\phi_1 = 0.8124$, $\phi_2 = 0.2514$, and $\theta_1 = 0.8714$; the initial values of y_t are $y_0 = 125.2134$ and $y_{-1} = 120.1054$; the error term is a white-noise process with standard deviation equal to 0.1018, and initial value $\varepsilon_0 = 0.0124$.

Exercise 3.2. Obtain the series of prices in the electricity market of PJM considering historical data spanning one month of 2009 and make the needed transformations in order to obtain a stationary series.

Relevant information on the PJM market is available at `www.pjm.com`.

Exercise 3.3. Represent graphically the time to failure and the time to repair for two units with MTTF equal to 500 and 600 h, and MTTR equal to 25 and 35 h, respectively.

Exercise 3.4. Consider a generating unit with MTTF and MTTR equal to 2500 and 150 hours, respectively. Assuming a planning horizon of one year, obtain three scenarios of availability for the considered unit.

Exercise 3.5. Obtain three samples of a Weibull distribution with shape parameters $\alpha = 2$ and $\beta = 5$ from three samples of a standard normal random variable. To this end, use the marginal transformation (3.6).

Exercise 3.6. Consider that the wind power production of a 50-MW wind farm can be statistically represented by the set of scenarios Ω listed in Table 3.6.

Table 3.6 Exercise 3.6: wind power scenarios

Scenario #	1	2	3	4	5
P_ω (MW)	15	20	37	45	50
π_ω	0.25	0.20	0.20	0.25	0.10

Use the variant 1 of the fast forward selection algorithm to reduce the number of wind power scenarios to two.

Exercise 3.7. Consider the short-term trading problem of a wind power producer briefly outlined in Illustrative Example 3.11. Use the variant 2 of the fast forward selection algorithm to build a set of three scenarios from the set Ω provided in Table 3.6 of Exercise 3.6. Suppose that λ^{D}, r^+ and r^- are equal to \$40/MWh, 0.2, and 1.05, respectively.

Exercise 3.8. Two stochastic processes, $\boldsymbol{Y}^a$ and $\boldsymbol{Y}^b$, are known to be quasi-contemporaneous, with the residual cross-correlogram depicted in Fig. 3.25. Construct the variance-covariance matrix $\boldsymbol{G}$ determining the statistical dependencies between $\boldsymbol{Y}^a$ and $\boldsymbol{Y}^b$. Table 3.7 constitutes the numerical translation of Fig. 3.25.

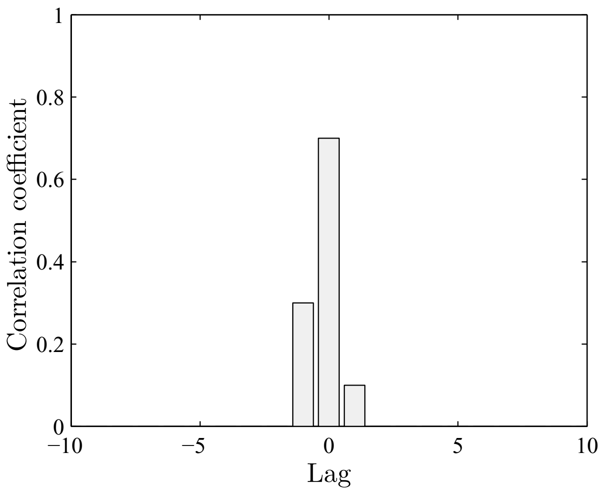


Fig. 3.25 Exercise 3.8: cross-correlogram of residuals

Table 3.7 Exercise 3.8: cross-correlation coefficients

Lag k	≤ -2	-1	0	1	≥ 2
ρ_k	0	0.3	0.7	0.1	0

Exercise 3.9. Assume that stochastic processes $\boldsymbol{Y}^a$ and $\boldsymbol{Y}^b$ in Exercise 3.8 can be properly modeled by the following two ARMA$(1, 1)$ models:

$$y_t^a = 0.908 y_{t-1}^a + \varepsilon_t^a + 0.182 \varepsilon_{t-1}^a,$$
$$y_t^b = 0.850 y_{t-1}^b + \varepsilon_t^b + 0.203 \varepsilon_{t-1}^b.$$

Generate a correlated pair of two-period scenarios for stochastic processes $\boldsymbol{Y}^a$ and $\boldsymbol{Y}^b$ according to the variance-covariance matrix $\boldsymbol{G}$ built in Exercise 3.8. Suppose that $y_0^a = 9.25$, $y_0^b = 5.60$, $\varepsilon_0^a = -0.30$, $\varepsilon_0^b = -0.12$, $\sigma_{\varepsilon^a} = 0.50$, and $\sigma_{\varepsilon^b} = 0.20$.

Exercise 3.10. Consider two stochastic processes, $\boldsymbol{Y}$ and $\boldsymbol{Y}^e$, that are related according to the following dynamic regression model,

$$y_t = 0.985 y_{t-1} + 0.083 y_t^e + \varepsilon_t,$$
$$y_t^e = 0.912 y_{t-1}^e + \varepsilon_t^e + 0.102 \varepsilon_{t-1}^e.$$

Generate three correlated two-period scenarios for stochastic processes $\boldsymbol{Y}$ and $\boldsymbol{Y}^e$. Suppose that $y_0 = 12.36$, $y_0^e = 4.13$, $\varepsilon_0^e = -0.13$, $\sigma_\varepsilon = 0.40$, and $\sigma_{\varepsilon^e} = 0.15$.

Chapter 4
Risk management

4.1 Introduction

In Chapter 2 we study the basic formulation of decision-making problems using stochastic programming. In these problems we consider that the objective of the decision-making agents is either maximizing profit (e.g., the financial profit of an electricity producer) or minimizing cost (e.g., the electricity procurement cost of an industrial consumer). In stochastic programming, where uncertain data are modeled as stochastic processes, the profit or cost is a random variable that can be characterized by a probability distribution. In an optimization problem involving a random objective function it is necessary to optimize a function characterizing the distribution of this random variable, for instance, its expected value. This is the criterion that is generally used in stochastic programming problems. Therefore, the problem consisting in maximizing "the profit" obtained by a decision-making agent results in maximizing the expected profit achieved by this agent.

Despite the numerous advantages of representing a random variable by its expected value, its main drawback is that the remaining parameters characterizing the distribution associated with the random variable are neglected. For instance, a random variable representing a profit with an expected value acceptable for the decision maker could also present a non-negligible probability of experiencing negative profits or losses.

In order to control the risk of experiencing profit distributions with non-desirable properties, e.g., with a high probability of low profit, risk control constitutes an important issue when formulating stochastic programming models. The most usual way of managing risk is to include in the formulation of the problem a term measuring the risk associated with a profit distribution. This term is usually referred to as risk functional or risk measure. Examples of risk measures are the variance of the profit, the probability of falling behind a target value, or the expected value of the profit being inferior to a specified value.

A. J. Conejo et al., *Decision Making Under Uncertainty in Electricity Markets*,
International Series in Operations Research & Management Science 153,
DOI 10.1007/978-1-4419-7421-1_4,

In this chapter we define and formulate some of the most usual risk measures regarding stochastic programming problems in electricity markets and financial optimization. For the sake of simplicity, all measures considered in this chapter are formulated using a two-stage stochastic programming problem whose objective function consists in maximizing the expected profit attained by a particular decision maker. Specifically, we consider the following risk measures:

1. Variance
2. Shortfall probability
3. Expected shortage
4. Value-at-Risk (VaR)
5. Conditional Value-at-Risk (CVaR).

We also deal with another risk management strategy based on imposing stochastic dominance constraints in the formulation of stochastic programming problems.

The rest of the chapter is organized as follows. Section 4.2 defines concepts related to risk-neutral and risk-averse decision-making problems. In Section 4.3 the risk measures enumerated above are defined, formulated, and tested by means of illustrative examples. Lastly, both a summary of the chapter and some relevant conclusions are provided in Section 4.4.

4.2 Risk Control in Stochastic Programming Problems

4.2.1 Risk-Neutral Decision Making

In Chapter 2 we present the general formulation of a two-stage stochastic programming problem,

$$\text{Maximize}_{\boldsymbol{x},\boldsymbol{y}(\omega)}$$

$$\boldsymbol{c}^\top \boldsymbol{x} + \sum_{\omega \in \Omega} \pi(\omega) \boldsymbol{q}(\omega)^\top \boldsymbol{y}(\omega) \tag{4.1}$$

subject to

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{4.2}$$

$$\boldsymbol{T}(\omega)\boldsymbol{x} + \boldsymbol{W}(\omega)\boldsymbol{y}(\omega) = \boldsymbol{h}(\omega),\ \forall \omega \in \Omega \tag{4.3}$$

$$\boldsymbol{x} \in X,\ \boldsymbol{y}(\omega) \in Y,\ \forall \omega \in \Omega, \tag{4.4}$$

where $\boldsymbol{x}$ and $\boldsymbol{y} = \{\boldsymbol{y}(\omega)\,; \forall\omega \in \Omega\}$ are the first- and second-stage decision variable vectors, respectively, and c, $\boldsymbol{q}(\omega)$, $\boldsymbol{b}$, $\boldsymbol{h}(\omega)$, $\boldsymbol{A}$, $\boldsymbol{T}(\omega)$, and $\boldsymbol{W}(\omega)$ are known vectors and matrices of appropriate size.

Defining

$$f(\boldsymbol{x},\omega) = \tag{4.5}$$
$$\boldsymbol{c}^\top\boldsymbol{x} + \max_{\boldsymbol{y}(\omega)}\{\boldsymbol{q}(\omega)^\top\boldsymbol{y}(\omega) : \boldsymbol{T}(\omega)\boldsymbol{x} + \boldsymbol{W}(\omega)\boldsymbol{y}(\omega) = \boldsymbol{h}(\omega),\ \boldsymbol{y}(\omega) \in Y\},$$

problem (4.1)-(4.4) can be equivalently expressed in compact form as

$$\text{Maximize}_{\boldsymbol{x}}$$
$$\mathcal{E}_\omega\{f(\boldsymbol{x},\omega)\} \tag{4.6}$$

subject to

$$\boldsymbol{x} \in X,\ \forall\omega \in \Omega. \tag{4.7}$$

The objective of problem (4.6)-(4.7) is to maximize the expected value of the function $f(\boldsymbol{x},\omega)$, which can represent, for instance, the profit achieved by a power producer within a given planning horizon.

From a more abstract perspective, it is embedded in the definition of $f(\boldsymbol{x},\omega)$ that, after the decision on $\boldsymbol{x}$ and the observation of ω, the second-stage decision $\boldsymbol{y}(\omega)$ has to be an optimal solution to the remaining optimization problem

$$\text{Maximize}_{\boldsymbol{y}(\omega)}$$
$$\boldsymbol{q}(\omega)^\top\boldsymbol{y}(\omega) \tag{4.8}$$

subject to

$$\boldsymbol{W}(\omega)\boldsymbol{y}(\omega) = \boldsymbol{h}(\omega) - \boldsymbol{T}(\omega)\boldsymbol{x} \tag{4.9}$$
$$\boldsymbol{y}(\omega) \in Y. \tag{4.10}$$

Conceptually, it is convenient to view $f(\boldsymbol{x},\omega)$ as the value of the random value $f(\boldsymbol{x},\cdot)$ at the argument ω. Then, for $\boldsymbol{x}$ varying in X, a family $\{f(\boldsymbol{x},\cdot) : \boldsymbol{x} \in X\}$ of random variables is induced by the two-stage decision problem under uncertainty. Finding a best $\boldsymbol{x}$ now corresponds to finding a best random variable in this family. In (4.6)-(4.7) this is accomplished by ranking the random variables according to their expectations and picking the biggest. Certainly, other modes of ranking are possible too. They will lead to other types of stochastic problems which we will study later on.

Since vectors of variables $\boldsymbol{x}$ and $\boldsymbol{y}(\omega)$ are obtained by maximizing the expected profit without modeling the risk, we denote (4.1)-(4.4) and the equivalent problem (4.6)-(4.7) as *risk-neutral problems*.

The following example illustrates the concept of risk-neutral problem.

Illustrative Example 4.1 (Risk-neutral problem).

Consider the problem faced by an electricity retailer that seeks to determine the purchases in the futures market in order to maximize the profit resulting from selling energy to a group of clients. In doing so, the retailer buys energy in both the pool and a futures market. The price of the energy in the pool in each period t is assumed to be unknown and is characterized as a random variable, which is modeled using a set of scenarios, $\lambda^{\mathrm{P}}_{t\omega}$. The retailer participates in the futures market by buying energy through three different contracts, $f = 1, 2, 3$, defined by a purchasing price, λ^{F}_{f}, and a maximum quantity of power that can be purchased, $X^{\max}_{f}$. The power purchased in contract f, x_f, is delivered in each period of the planning horizon. The demand of the consumers in each period t, P^{C}_{t}, is assumed to be known (deterministic), while the selling price of the energy to the clients, λ^{C}, is fixed to \$35/MWh. For this illustrative example, the considered planning horizon comprises 3 hourly periods.

The client demands for the three periods are 150, 225, and 175 MW. The data concerning the forward contracts available in the futures market are given in Table 4.1. The price of the electricity in the pool in every period is represented by a set of 10 equiprobable scenarios. Pool price data are provided in Table 4.2.

Table 4.1 Illustrative Example 4.1: forward contract data

Contract #	Price (\$/MWh)	Maximum quantity (MW)
1	34	50
2	35	30
3	36	25

Table 4.2 Illustrative Example 4.1: pool price data (\$/MWh)

Scenario #	Period #			Scenario #	Period #		
	1	2	3		1	2	3
1	28.5	36.3	31.4	6	29.2	34.8	31.2
2	27.3	37.5	29.6	7	34.1	36.9	35.4
3	29.4	35.7	31.3	8	33.4	35.4	34.9
4	33.9	35.4	35.1	9	28.4	36.3	32.9
5	34.5	38.9	37.5	10	27.6	38.9	32.1

The formulation of the risk-neutral profit maximization problem is

$$\text{Maximize}_{x_f, y_{t\omega}}$$
$$\sum_{t=1}^{3} \lambda^{\mathrm{C}} P_t^{\mathrm{C}} - \sum_{f=1}^{3} \sum_{t=1}^{3} \lambda_f^{\mathrm{F}} x_f - \sum_{\omega=1}^{10} \pi_\omega \sum_{t=1}^{3} \lambda_{t\omega}^{\mathrm{P}} y_{t\omega} \tag{4.11}$$

subject to

$$0 \le x_f \le X_f^{\max}, \quad f = 1, 2, 3 \tag{4.12}$$

$$\sum_{f=1}^{3} x_f + y_{t\omega} = P_t^{\mathrm{C}}, \quad t = 1, 2, 3; \ \omega = 1, \dots, 10 \tag{4.13}$$

$$y_{t\omega} \ge 0, \quad t = 1, 2, 3; \ \omega = 1, \dots, 10, \tag{4.14}$$

where x_f is the power purchased from forward contract f and $y_{t\omega}$ is the power purchased from the pool in period t and scenario ω. Note that x_f is a here-and-now decision variable, whereas $y_{t\omega}$ is wait-and-see.

The objective function (4.11) represents the expected profit achieved by the retailer. This expected profit is equal to the revenue obtained from selling energy to clients minus the cost of purchasing in both the futures and the pool markets. Observe that the two first terms concerning the revenue and the cost of purchasing in the futures market are deterministic, whereas the third term corresponding to the expected cost of purchasing in the pool is evaluated over all pool price scenarios. Parameter π_ω in this last term refers to the probability of scenario ω. Note that the term modeling the revenue of the retailer is constant in this problem and thus it can be removed from the objective fucntion.

Constraints (4.12) establish the lower and upper bounds for the power purchased from the futures market in each contract f. The power balance in each period and scenario is stated in (4.13). Finally, constraints (4.14) impose the non-negativity of variables $y_{t\omega}$.

The optimal solution to problem (4.11)-(4.14) in terms of the optimal purchases in the futures market is $\{x_1^*, x_2^*, x_3^*\} = \{0, 0, 0\}$. That is, the retailer prefers to supply the demand of its clients only buying from the pool. This result is reasonable since the mean value of the pool prices provided in Table 4.2 is \$33.46/MWh, which is smaller than the prices of the three available forward contracts (Table 4.1).

The resulting expected profit is \$618.75. Table 4.3 provides the value of the profit for each pool price scenario. Observe that the profit exhibits high volatility varying from minus \$1240 to plus \$1580.

Fig. 4.1 represents the cumulative distribution function (cdf) of the profit. Since risk-neutral problem (4.11)-(4.14) only focuses on maximizing the expected profit, the risk of experiencing low profits in some scenarios is not avoided. Observe that there exists a probability equal to 0.2 of experiencing losses.

□

Table 4.3 Illustrative Example 4.1: profit per scenario

Scenario #	Profit ($)	Scenario #	Profit ($)
1	1312.50	6	1580.00
2	1537.50	7	-362.50
3	1330.00	8	167.50
4	57.50	9	1065.00
5	-1240.00	10	740.00

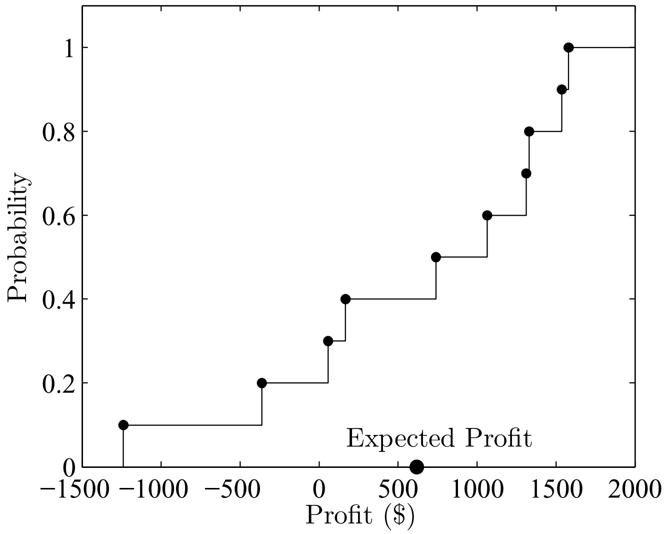


Fig. 4.1 Illustrative Example 4.1: profit cdf for the risk-neutral case

4.2.2 Risk-Averse Decision Making

The main disadvantage of ignoring risk in problem (4.1)-(4.4) is that the optimal values of variables $\boldsymbol{x}$ and $\boldsymbol{y}(\omega)$ may lead to the maximum expected profit at the expense of experiencing very low profits in some unfavorable scenarios. In order to avoid such situations, it is advisable to include in the formulation of the problem a term modeling the risk of variability associated with the profit $f(\boldsymbol{x}, \omega)$. Thus, we introduce the function $r_\omega\big\{f(\boldsymbol{x}, \omega)\big\}$ that assigns to a given random variable representing profit, $f(\boldsymbol{x}, \omega)$, $\forall \omega \in \Omega$, a real number characterizing the risk associated with that profit. The function $r_\omega\big\{f(\boldsymbol{x}, \omega)\big\}$ is referred to as *risk measure*.

Risk measures can be incorporated either into the objective function of the problem, as the mean-risk approach proposed in [91], or as an additional set of constraints in the problem formulation.

Consider the two-stage stochastic programming problem

$$\text{Maximize}_{\boldsymbol{x}}$$
$$\mathcal{E}_\omega\big\{f(\boldsymbol{x},\omega)\big\} - \beta r_\omega\big\{f(\boldsymbol{x},\omega)\big\} \tag{4.15}$$

subject to

$$\boldsymbol{x} \in X, \tag{4.16}$$

where $\beta \in [0,\infty)$ is a weighting parameter used to materialize the tradeoff between expected profit and risk aversion. If $\beta = 0$, the risk term in the objective function is neglected and the resulting problem becomes the risk-neutral one. As β increases, the expected profit term becomes less significant with respect to the risk term.

Observe that the risk faced by the decision maker can be also controlled by including the risk measure as an additional constraint, i.e.,

$$\text{Maximize}_{\boldsymbol{x},\boldsymbol{y}(\omega)}$$
$$\mathcal{E}_\omega\{f(\boldsymbol{x},\omega)\} \tag{4.17}$$

subject to

$$\boldsymbol{x} \in X \tag{4.18}$$
$$r_\omega\big\{f(\boldsymbol{x},\omega)\big\} \leq \delta, \tag{4.19}$$

where δ represents the maximum risk that the decision maker is willing to take.

The optimal solution obtained from solving problem (4.15)-(4.16) or (4.17)-(4.19) depends on the value of parameters β/δ. This optimal solution, in terms of expected profit and risk for a given value of parameters β/δ, defines an efficient point. In other words, an efficient point is a pair expected profit/risk in such a way that it is impossible to find a set of decision variables yielding simultaneously greater expected profit and lower risk. This way, a solution with greater expected profit than that of an efficient point can only be obtained at the cost of experiencing a higher risk, and viceversa. A collection of efficient points (obtained for different values of β/δ) defines an efficient frontier [91]. Fig. 4.2 illustrates an example of efficient frontier for problem (4.15)-(4.16). Observe that a small value of β yields a solution with high expected profit and also high risk. On the contrary, a large value of β achieves a solution with smaller expected profit and smaller risk. Thus, efficient frontiers are relevant instruments used by decision makers to resolve the tradeoff between expected profit and risk. Finally, note that efficient frontiers are composed of a finite set of efficient points (thus, they are not continuous) and the line resulting from joining efficient points is not necessarily either convex or concave.

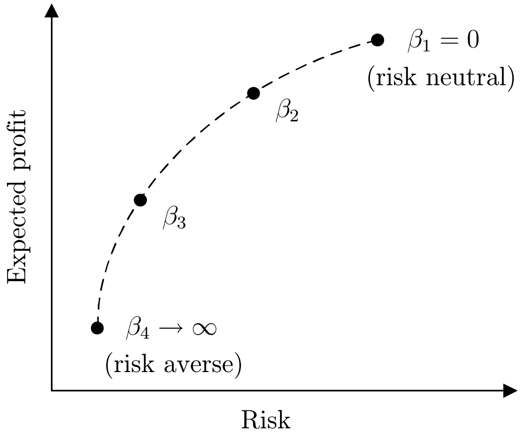


Fig. 4.2 Example of efficient frontier

4.3 Risk Measures

Risk measures are needed for characterizing the risk associated with a given decision. This way, risk measures enable us to compare two different decisions in terms of the risk involved.

In the technical literature it is possible to find a wide set of risk measures used for different applications. Artzner et al. present in [7] a set of desirable properties that risk measures should fulfill. These properties are the following:

1. Translation invariance
2. Subadditivity
3. Positive homogeneity
4. Monotonicity.

Measures satisfying these four properties are defined as *coherent risk measures.*

Let us consider two possible random outcomes (e.g., profits) $f_1(\omega)$ and $f_2(\omega) \in F$ and a risk measure $r_\omega\{f(\omega)\}$, that is a function of the random outcome $f(\omega) \in F$, where F is the set of all possible random profit outcomes. If the monetary units of random outcomes $f(\omega)$ are identical to those of the risk measure $r_\omega\{f(\omega)\}$, the properties of coherent risk measures are formulated as follows:

1. Translation invariance. For all $f_1(\omega) \in F$ and all real numbers a, it holds that $r_\omega\{f_1(\omega) + a\} = r_\omega\{f_1(\omega)\} + a$.
2. Subadditivity. For all $f_1(\omega)$, $f_2(\omega) \in F$, it is satisfied that $r_\omega\{f_1(\omega) + f_2(\omega)\} \leq r_\omega\{f_1(\omega)\} + r_\omega\{f_2(\omega)\}$.

3. Positive homogeneity. For all $f_1(\omega) \in F$ and all real numbers a, it is verified that $r_\omega\{a \times f_1(\omega)\} = a \times r_\omega\{f_1(\omega)\}$.
4. Monotonicity. For all $f_1(\omega), f_2(\omega) \in F$, if $f_1(\omega) \geq f_2(\omega)$, then $r_\omega\{f_1(\omega)\} \leq r_\omega\{f_2(\omega)\}$.

Next, we introduce and analyze several risk measures used in stochastic programming. For illustrative purposes, we consider a two-stage problem where the stochastic processes involved are represented by a discrete set of scenarios.

4.3.1 Variance

The use of the variance of a profit/cost distribution as a risk measure was first proposed by Nobel laureate H. M. Markowitz. Due to the important mean-variance model first proposed in [91], Markowitz is known as the father of the modern portfolio theory and is considered to be responsible for the upgrade of financial theory to a scientific discipline.

Basically, the mean-variance model considers that a decision (or position) can be characterized by two parameters: the expected return (expected profit/cost) and the variance of this return, which, as a dispersion measure, is used to model the risk faced by the decision maker. Therefore, a large variance indicates that there exists a high risk of experiencing a profit different from the expected one.

Considering the profit $f(\boldsymbol{x}, \omega)$, the variance can be formulated as

$$\mathrm{V}(\boldsymbol{x}) = \mathcal{E}_\omega \left\{ \Big(f(\boldsymbol{x}, \omega) - \mathcal{E}_\omega\{f(\boldsymbol{x}, \omega)\} \Big)^2 \right\}. \tag{4.20}$$

Observe that the decision maker desires to obtain a variance as low as possible in order to avoid the variability of the profit.

The variance can be incorporated into the risk-neutral problem (4.1) as shown below:

$$\begin{aligned} &\text{Maximize}_{\boldsymbol{x}, \boldsymbol{y}(\omega)} \\ &(1-\beta) \left(\boldsymbol{c}^\top \boldsymbol{x} + \sum_{\omega \in \Omega} \pi(\omega) \boldsymbol{q}(\omega)^\top \boldsymbol{y}(\omega) \right) - \\ &\beta \sum_{\omega \in \Omega} \pi(\omega) \left(f(\boldsymbol{x}, \omega) - \sum_{\omega' \in \Omega} \pi(\omega') f(\boldsymbol{x}, \omega') \right)^2 \end{aligned} \tag{4.21}$$

subject to

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{4.22}$$

$$\boldsymbol{T}(\omega)\boldsymbol{x} + \boldsymbol{W}(\omega)\boldsymbol{y}(\omega) = \boldsymbol{h}(\omega),\ \forall \omega \in \Omega \tag{4.23}$$

$$\boldsymbol{x} \in X,\ \boldsymbol{y}(\omega) \in Y,\ \forall \omega \in \Omega. \tag{4.24}$$

Note that problem (4.21)-(4.24) is a quadratic problem, which can turn into a quadratic mixed-integer problem if there exists any integer requirement in sets X or Y. Observe that the variance of the profit is multiplied by the parameter β, whereas the expected profit is multiplied by $(1-\beta)$. Here, β lies in the interval [0,1]. Thus, if $\beta = 0$, the variance is neglected, while the expected profit is disregarded if $\beta = 1$. Observe that the formulation with $\beta \in [0,1]$ is equivalent to (4.15)-(4.16), where $\beta \in [0,\infty)$. It should be noted that both formulations obtain the same efficient points. However, the advantage of using the formulation with $\beta \in [0,1]$ is that the value of this parameter is limited to a finite interval.

The usage of the variance of the profit as a risk-measure is illustrated in the following example.

Illustrative Example 4.2 (Variance).

Let us consider the risk-neutral problem (4.11)-(4.14) presented in Illustrative Example 4.1. If the variance of the profit is included in this problem to hedge against profit variability, the resulting formulation is

$$\text{Maximize}_{x_f, y_{t\omega}}$$

$$(1-\beta)\left(\sum_{t=1}^{3} \lambda^{\mathrm{C}} P_t^{\mathrm{C}} - \sum_{f=1}^{3}\sum_{t=1}^{3} \lambda_f^{\mathrm{F}} x_f - \sum_{\omega=1}^{10} \pi_\omega \sum_{t=1}^{3} \lambda_{t\omega}^{\mathrm{P}} y_{t\omega}\right) -$$

$$\beta \sum_{\omega=1}^{10} \pi_\omega \left(\lambda_{t\omega}^{\mathrm{P}} y_{t\omega} - \sum_{\omega'=1}^{10} \pi_{\omega'} \sum_{t=1}^{3} \lambda_{t\omega'}^{\mathrm{P}} y_{t\omega'}\right)^2 \tag{4.25}$$

subject to

$$0 \le x_f \le X_f^{\max}, \quad f = 1,2,3 \tag{4.26}$$

$$\sum_{f=1}^{3} x_f + y_{t\omega} = P_t^{\mathrm{C}}, \quad t = 1,2,3;\ \omega = 1,\ldots,10 \tag{4.27}$$

$$y_{t\omega} \ge 0, \quad t = 1,2,3;\ \omega = 1,\ldots,10. \tag{4.28}$$

Observe that since the pool price is the only stochastic process, the variance of the profit coincides with the variance of the cost of purchasing in the pool.

The resulting cumulative distribution functions for the profit in cases $\beta = \{0,1\}$ are represented in Fig. 4.3. The optimal purchases in the futures market for $\beta = 1$ are $\{x_1^*, x_2^*, x_3^*\} = \{50, 30, 25\}$. That is, the minimum variance is obtained if the retailer purchases as much as possible in the futures market.

For this situation, the expected profit is equal to \$208.65, that is, a value much smaller than that obtained for the case $\beta = 0$ (\$618.75). However, this reduction of 66.27% in the expected profit leads to a decrease of 56% in the profit of the worst scenario, from minus \$1240 to minus \$545.50. This result indicates that the futures market is an effective tool to reduce the risk associated with profit variability.

Note that one of the worst features of using the variance as risk measure is that in addition to penalizing the "worst" zone of the profit distribution (values smaller than the expected profit), it also penalizes those scenarios with profit higher than the expected profit. For this reason, a high reduction of the profit in the best scenarios can be also observed in Fig. 4.3. In other words, altering the variance affects both tails of the profit distribution.

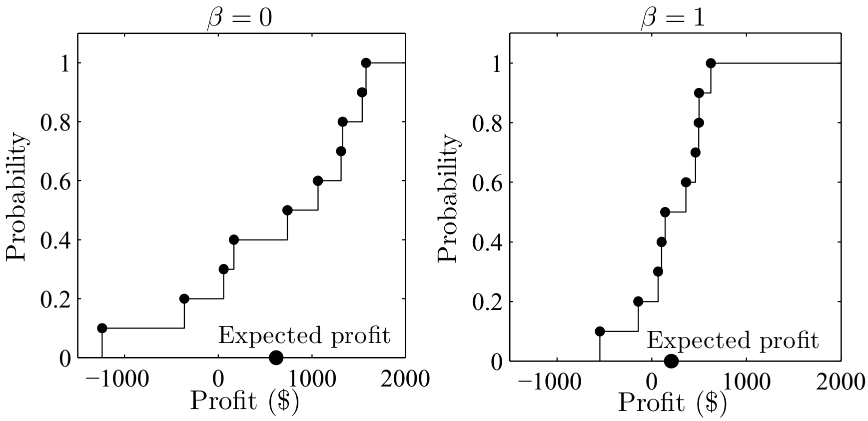


Fig. 4.3 Illustrative Example 4.2: cdfs for $\beta = 0$ and $\beta = 1$

The profit per scenario for $\beta = 1$ is provided in Table 4.4. As previously stated, the expected profit is equal to \$208.65. Given this value and the profit per scenario provided in Table 4.4, the variance of the profit is computed as

$$
\begin{gathered}
0.1 \times (463.50 - 208.65)^2 + 0.1 \times (499.50 - 208.65)^2 + \\
0.1 \times (502.00 - 208.65)^2 + 0.1 \times (69.50 - 208.65)^2 + \\
0.1 \times (-545.50 - 208.65)^2 + 0.1 \times (626.00 - 208.65)^2 + \\
0.1 \times (-140.50 - 208.65)^2 + 0.1 \times (106.00 - 208.65)^2 + \\
0.1 \times (363.00 - 208.65)^2 + 0.1 \times (143.00 - 208.65)^2 = \\
(\$^2)128{,}720.
\end{gathered} \tag{4.29}
$$

Table 4.4 Illustrative Example 4.2: profit per scenario for $\beta = 1$

Scenario #	Profit ($)	Scenario #	Profit ($)
1	463.50	6	626.00
2	499.50	7	-140.50
3	502.00	8	106.00
4	69.50	9	363.00
5	-545.50	10	143.00

Finally, Fig. 4.4 provides the efficient frontier of problem (4.25)-(4.28). As expected, the variance of the profit decreases as the value of β increases. As a consequence of the decrease in the variance, the expected profit also diminishes.

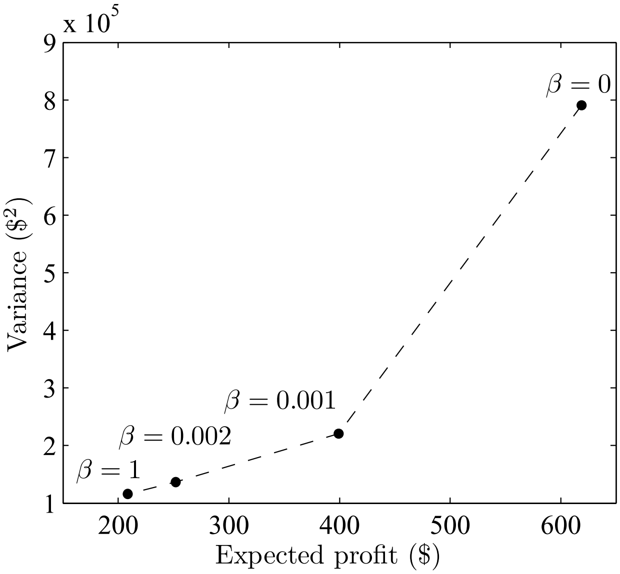


Fig. 4.4 Illustrative Example 4.2: efficient frontier

□

4.3.2 Shortfall Probability

The shortfall probability, $\mathrm{SP}(\eta, \boldsymbol{x})$, is equal to the probability of the profit being less than a pre-fixed value η. Mathematically the shortfall probability is defined as

$$\text{SP}(\eta, \boldsymbol{x}) = P\Big(\omega | f(\boldsymbol{x}, \omega) < \eta\Big), \ \forall \eta \in \mathbb{R}. \tag{4.30}$$

To reduce the risk faced by the decision maker, it is desirable that the shortfall probability of the profit is as low as possible.

The shortfall probability is equal to the sum of the probabilities of those profit scenarios on the left of η. Regarding the cumulative distribution function, the shortfall probability is equal to the value of that distribution for a profit less than η.

The shortfall probability can be incorporated into the risk-neutral problem (4.1)-(4.4) as shown below:

$$\text{Maximize}_{\boldsymbol{x}, \boldsymbol{y}(\omega), \theta(\omega)}$$

$$(1-\beta)\left(\boldsymbol{c}^\top \boldsymbol{x} + \sum_{\omega \in \Omega} \pi(\omega) \boldsymbol{q}(\omega)^\top \boldsymbol{y}(\omega)\right) - \beta \sum_{\omega \in \Omega} \pi(\omega)\theta(\omega) \tag{4.31}$$

subject to

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{4.32}$$

$$\boldsymbol{T}(\omega)\boldsymbol{x} + \boldsymbol{W}(\omega)\boldsymbol{y}(\omega) = \boldsymbol{h}(\omega), \ \forall \omega \in \Omega \tag{4.33}$$

$$\eta - \Big(\boldsymbol{c}^\top \boldsymbol{x} + \boldsymbol{q}(\omega)^\top \boldsymbol{y}(\omega)\Big) \le M\theta(\omega), \ \forall \omega \in \Omega \tag{4.34}$$

$$\theta(\omega) \in \{0, 1\}, \ \forall \omega \in \Omega \tag{4.35}$$

$$\boldsymbol{x} \in X, \ \boldsymbol{y}(\omega) \in Y, \ \forall \omega \in \Omega, \tag{4.36}$$

where $\theta(\omega)$ is a binary variable that is equal to 1 if the profit in scenario ω is smaller than η (and equal to 0 otherwise) and M is a sufficiently large constant. Thus, the term $\sum_{\omega \in \Omega} \pi(\omega)\theta(\omega)$ represents the cumulative probability of all scenarios whose profit is less than η, i.e., it is equal to the shortfall probability.

A drawback of the shortfall probability is that it gives no information about the profit distribution beyond the parameter η. Thus, a "fat tail" that may appear in profit distributions is not detected by this risk measure. Moreover, the use of the fixed target η in addition to the fact that this measure is not expressed in profit units causes the shortfall probability not to satisfy the properties of coherent risk measures.

The following example illustrates the formulation of the shortfall probability.

Illustrative Example 4.3 (Shortfall probability).

Consider the risk-neutral problem (4.11)-(4.14) presented in Illustrative Example 4.1. The formulation of this problem including the shortfall probability in its objective function is

$$\text{Maximize}_{x_f, y_{t\omega}, \theta_\omega}$$
$$(1-\beta)\left(\sum_{t=1}^{3} \lambda^{\mathrm{C}} P_t^{\mathrm{C}} - \sum_{f=1}^{3}\sum_{t=1}^{3} \lambda_f^{\mathrm{F}} x_f - \sum_{\omega=1}^{10} \pi_\omega \sum_{t=1}^{3} \lambda_{t\omega}^{\mathrm{P}} y_{t\omega}\right) - \beta \sum_{\omega=1}^{10} \pi_\omega \theta_\omega \tag{4.37}$$

subject to

$$0 \le x_f \le X_f^{\max}, \quad f = 1,2,3 \tag{4.38}$$

$$\sum_{f=1}^{3} x_f + y_{t\omega} = P_t^{\mathrm{C}}, \quad t = 1,2,3;\ \omega = 1,\ldots,10 \tag{4.39}$$

$$y_{t\omega} \ge 0, \quad t = 1,2,3;\ \omega = 1,\ldots,10 \tag{4.40}$$

$$\eta - \left(\sum_{t=1}^{3} \lambda^{\mathrm{C}} P_t^{\mathrm{C}} - \sum_{f=1}^{3}\sum_{t=1}^{3} \lambda_f^{\mathrm{F}} x_f - \sum_{t=1}^{3} \lambda_{t\omega}^{\mathrm{P}} y_{t\omega}\right) \le M\theta_\omega, \quad \forall \omega = 1,\ldots,10 \tag{4.41}$$

$$\theta_\omega \in \{0,1\}, \quad \forall \omega = 1,\ldots,10. \tag{4.42}$$

For this example, we use $\eta = \$175$ and M=10,000.

Fig. 4.5 represents the cumulative distribution functions of the profit for $\beta = \{0,1\}$. In the case $\beta = 0$, the shortfall probability is equal to 0.4. That is, there is a probability of 0.4 of experiencing a profit less than $\eta = \$175$. For $\beta = 1$, the shortfall probability is reduced up to 0.2. In this case, the expected profit is \$539.44, i.e, a 13% smaller than that corresponding to the risk-neutral case (\$618.75). The optimal purchases in the futures market for this case are $\{x_1^*, x_2^*, x_3^*\} = \{49, 0, 0\}$.

The profit and the optimal value of θ_ω per scenario for $\beta = 1$ are provided in Table 4.5. Observe that variable θ_ω is equal to 1 in all scenarios with a profit smaller than $\eta = \$175$, being 0 otherwise. Variable θ_ω is 1 for scenarios 5 and 7, with profits equal to minus \$804.27 and minus \$147.08, respectively. The shortfall probability is calculated as

$$\sum_{\omega=1}^{10} \pi_\omega \theta_\omega^* = 0.1 + 0.1 = 0.2. \tag{4.43}$$

The efficient frontier of the shortfall probability versus the expected profit is depicted in Fig. 4.6. In this figure we can observe that the shortfall probability decreases as the weighting parameter β increases. Note that the difference in magnitude between the shortfall probability and the expected profit causes that large values of β are needed to reduce effectively the shortfall probability. □

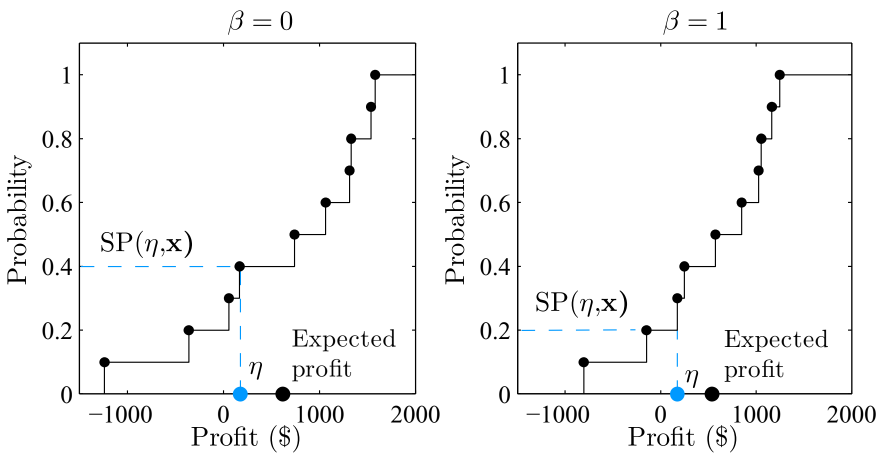


Fig. 4.5 Illustrative Example 4.3: cdfs for $\beta = 0$ and $\beta = 1$

Table 4.5 Illustrative Example 4.3: profit and variable θ_ω per scenario for $\beta = 1$

Scenario #	Profit ($)	θ_ω	Scenario #	Profit ($)	θ_ω
1	1028.54	0	6	1247.08	0
2	1165.42	0	7	-147.08	1
3	1055.83	0	8	250.73	0
4	175.00	0	9	849.58	0
5	-804.27	1	10	573.54	0

4.3.3 Expected Shortage

The expected shortage, $\mathrm{ES}(\eta, \boldsymbol{x})$, is the expectation of the profit in those scenarios with a profit smaller than a pre-fixed value η. The expected shortage is defined as

$$\mathrm{ES}(\eta, \boldsymbol{x}) = \eta - \frac{1}{\mathrm{SP}(\eta, \boldsymbol{x})} \mathcal{E}_\omega \left\{ \max\Big\{ \eta - f(\boldsymbol{x}, \omega), 0 \Big\} \right\}, \ \forall \eta \in \mathbb{R}. \tag{4.44}$$

Expression $\max\{\eta - f(\boldsymbol{x}, \omega), 0\}$ is different from zero in all scenarios in which the profit is smaller than η, being zero otherwise. In order to properly calculate the expected value of the profit over such scenarios, it is necessary not to take into account the probability of those scenarios with a profit greater than η from $\mathcal{E}_\omega\big\{\max\{\eta - f(\boldsymbol{x}, \omega), 0\}\big\}$. For this reason the expectation expression above must be divided by the sum of the probabilities of all scenarios with a profit smaller than η. The sum of these probabilities is equal to the shortfall probability $\mathrm{SP}(\eta, \boldsymbol{x})$, defined in Subsection 4.3.2.

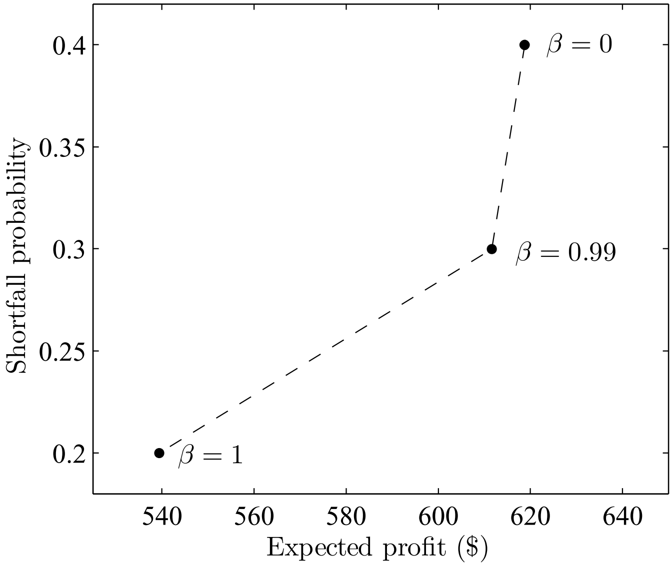


Fig. 4.6 Illustrative Example 4.3: efficient frontier

It should be noted that some authors define the expected shortage as $\mathcal{E}_\omega\{\max\{\eta - f(\boldsymbol{x},\omega), 0\}\}$, [105]. Observe that if the expected shortage is defined as in (4.44), it is equivalent to the conditional expectation of $f(\boldsymbol{x},\omega)$, i.e.,

$$\mathrm{ES}(\eta, \boldsymbol{x}) = \mathcal{E}_\omega\{f(\boldsymbol{x},\omega) | f(\boldsymbol{x},\omega) < \eta\}. \tag{4.45}$$

Profit distributions with a high expected shortage are desirable to reduce the risk of experiencing low profits in the worst scenarios. Observe that while the shortage probability represents a probability, the expected shortage is measured in profit units.

The expected shortage can be incorporated into the risk-neutral problem (4.1)-(4.4) as

$$\text{Maximize}_{\boldsymbol{x},\boldsymbol{y}(\omega),s(\omega)}$$
$$(1-\beta)\left(\boldsymbol{c}^\top \boldsymbol{x} + \sum_{\omega\in\Omega} \pi(\omega)\boldsymbol{q}(\omega)^\top \boldsymbol{y}(\omega)\right) - \beta \sum_{\omega\in\Omega} \pi(\omega) s(\omega) \tag{4.46}$$

subject to

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{4.47}$$

$$\boldsymbol{T}(\omega)\boldsymbol{x} + \boldsymbol{W}(\omega)\boldsymbol{y}(\omega) = \boldsymbol{h}(\omega), \ \forall \omega \in \Omega \tag{4.48}$$

$$\eta - \left(\boldsymbol{c}^{\top}\boldsymbol{x} + \boldsymbol{q}(\omega)^{\top}\boldsymbol{y}(\omega)\right) \leq s(\omega), \ \forall \omega \in \Omega \tag{4.49}$$

$$s(\omega) \geq 0, \ \forall \omega \in \Omega \tag{4.50}$$

$$\boldsymbol{x} \in X, \ \boldsymbol{y}(\omega) \in Y, \ \forall \omega \in \Omega, \tag{4.51}$$

where $s(\omega)$ is a continuous and non-negative variable equal to the maximum value between $\eta - \left(\boldsymbol{c}^{\top}\boldsymbol{x} + \boldsymbol{q}(\omega)^{\top}\boldsymbol{y}(\omega)\right)$ and 0.

Once problem (4.46)-(4.51) is solved and the optimal values of variables $s(\omega)$ are obtained, the expected shortage is equal to

$$\eta - \frac{1}{\sum_{\omega \in \Omega | s(w) \geq 0} \pi(\omega)} \sum_{\omega \in \Omega} \pi(\omega) s(\omega).$$

Note that the use of the fixed target η causes the expected shortage not to satisfy the properties of coherence as described in Section 4.3.

The formulation of the expected shortage is illustrated in the following example.

Illustrative Example 4.4 (Expected shortage).

The formulation of problem (4.11)-(4.14) in Illustrative Example 4.1 if the expected shortage is included in its objective function is

$$\text{Maximize}_{x_f, y_{t\omega}, s_\omega}$$

$$(1-\beta)\left(\sum_{t=1}^{3} \lambda^{\text{C}} P_t^{\text{C}} - \sum_{f=1}^{3}\sum_{t=1}^{3} \lambda_f^{\text{F}} x_f - \sum_{\omega=1}^{10} \pi_\omega \sum_{t=1}^{3} \lambda_{t\omega}^{\text{P}} y_{t\omega}\right) - \beta \sum_{\omega \in \Omega} \pi_\omega s_\omega \tag{4.52}$$

subject to

$$0 \leq x_f \leq X_f^{\max}, \quad f = 1,2,3 \tag{4.53}$$

$$\sum_{f=1}^{3} x_f + y_{t\omega} = P_t^{\text{C}}, \quad t = 1,2,3; \ \omega = 1,\dots,10 \tag{4.54}$$

$$y_{t\omega} \geq 0, \quad t = 1,2,3; \ \omega = 1,\dots,10 \tag{4.55}$$

$$\eta - \left(\sum_{t=1}^{3} \lambda^{\text{C}} P_t^{\text{C}} - \sum_{f=1}^{3}\sum_{t=1}^{3} \lambda_f^{\text{F}} x_f - \sum_{t=1}^{3} \lambda_{t\omega}^{\text{P}} y_{t\omega}\right) \leq s_\omega, \quad \forall \omega = 1,\dots,10 \tag{4.56}$$

$$s_\omega \geq 0, \quad \forall \omega = 1,\dots,10. \tag{4.57}$$

For this example we consider $\eta = \$0$.

In the case $\beta = 0$, the expected shortage is equal to minus \$801.25. That is, the expected value of the profit below $\eta = \$0$ is equal to minus \$801.25. For $\beta = 1$, the expected shortage increases, being equal to minus \$342.75. For this case, the expected profit is equal to \$208.65, and the optimal decision vector is $\{x_1^*, x_2^*, x_3^*\} = \{50, 30, 25\}$.

The cumulative distribution functions of the profit for $\beta = \{0, 1\}$ are represented in Fig. 4.7. The efficient frontier is provided in Fig. 4.8. Observe that the increase of the expected shortage is obtained by means of a significant reduction of the expected profit.

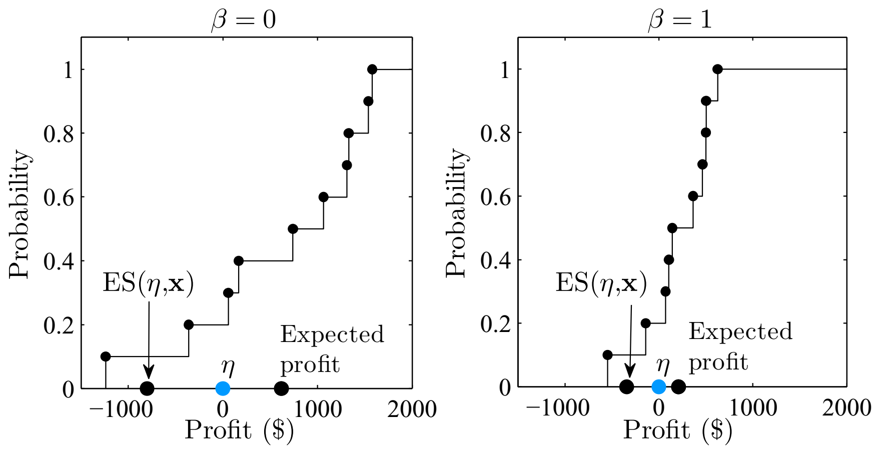


Fig. 4.7 Illustrative Example 4.4: cdfs for $\beta = 0$ and $\beta = 1$

The profit and the optimal value of s_ω per scenario for $\beta = 1$ are provided in Table 4.6. Variable s_ω is equal to the difference between $\eta = \$0$ and the profit in scenario ω if this difference is positive, being 0 otherwise. In Table 4.6 we can observe that s_ω is different from zero in scenarios 5 and 7. Given these values of variable s_ω the expected shortage is computed as

$$\eta - \frac{1}{\sum_{\omega=1|s_\omega^* \geq 0}^{10} \pi_\omega} \sum_{\omega=1}^{10} \pi_\omega s_\omega^* =$$

$$0 - \frac{1}{0.1 + 0.1}(0.1 \times 545.50 + 0.1 \times 140.50) =$$

$$= -\$342.75.$$

□

Table 4.6 Illustrative Example 4.4: profit and variable s_ω per scenario for $\beta = 1$

Scenario #	Profit (\$)	s_ω	Scenario #	Profit (\$)	s_ω
1	463.50	0	6	626.00	0
2	499.50	0	7	-140.50	140.50
3	502.00	0	8	106.00	0
4	69.50	0	9	363.00	0
5	-545.50	545.50	10	143.00	0

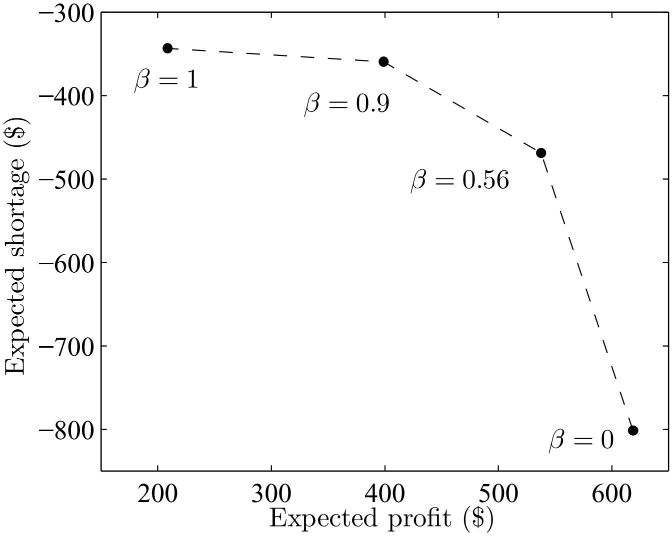


Fig. 4.8 Illustrative Example 4.4: efficient frontier

4.3.4 Value-at-Risk

For a given $\alpha \in (0, 1)$, the value-at-risk, VaR, is equal to the largest value η ensuring that the probability of obtaining a profit less than η is lower than $1 - \alpha$. In other words, the $\mathrm{VaR}(\alpha, \boldsymbol{x})$ is the $(1 - \alpha)$-quantile of the profit distribution. Mathematically, the $\mathrm{VaR}(\alpha, \boldsymbol{x})$ is defined as

$$\mathrm{VaR}(\alpha, \boldsymbol{x}) = \max\Big\{\eta : P\big(\omega | f(\boldsymbol{x}, \omega) < \eta\big) \leq 1 - \alpha\Big\}, \ \forall \alpha \in (0, 1). \qquad (4.58)$$

Note that η is not a given parameter, but the risk measure associated with the random variable representing the profit.

The $\mathrm{VaR}(\alpha, \boldsymbol{x})$ can be incorporated into the risk-neutral problem (4.1)-(4.4) as

$$\text{Maximize}_{\boldsymbol{x},\boldsymbol{y}(\omega),\eta,\theta(\omega)}$$
$$(1-\beta)\left(\boldsymbol{c}^{\top}\boldsymbol{x}+\sum_{\omega\in\Omega}\pi(\omega)\boldsymbol{q}(\omega)^{\top}\boldsymbol{y}(\omega)\right)+\beta\eta \tag{4.59}$$

subject to

$$\boldsymbol{A}\boldsymbol{x}=\boldsymbol{b} \tag{4.60}$$
$$\boldsymbol{T}(\omega)x+\boldsymbol{W}(\omega)\boldsymbol{y}(\omega)=\boldsymbol{h}(\omega),\ \forall\omega\in\Omega \tag{4.61}$$
$$\sum_{\omega\in\Omega}\pi(\omega)\theta(\omega)\leq 1-\alpha \tag{4.62}$$
$$\eta-\left(\boldsymbol{c}^{\top}\boldsymbol{x}+\boldsymbol{q}(\omega)^{\top}\boldsymbol{y}(\omega)\right)\leq M\theta(\omega),\ \forall\omega\in\Omega \tag{4.63}$$
$$\theta(\omega)\in\{0,1\},\ \forall\omega\in\Omega \tag{4.64}$$
$$\boldsymbol{x}\in X,\ \boldsymbol{y}(\omega)\in Y,\ \forall\omega\in\Omega, \tag{4.65}$$

where η is a variable whose optimal value is equal to the $\mathrm{VaR}(\alpha,\boldsymbol{x})$, $\theta(\omega)$ is a binary variable which is equal to 1 if the profit in scenario ω is less than η (and equal to 0 otherwise) and M is a large enough constant.

A serious shortcoming of the VaR is that it gives no information about the profit distribution beyond its value. Thus, a "fat tail" appearing in profit distributions is not detected by the VaR. On the other hand, the VaR satisfies all coherence properties except subadditivity.

The usage of the VaR as a risk measure is illustrated by means of the following example.

Illustrative Example 4.5 (VaR). The formulation of problem (4.11)-(4.14) in Illustrative Example 4.1 incorporating the VaR as a risk-control mechanism is

$$\text{Maximize}_{x_f,y_{t\omega},\theta_\omega,\eta}$$
$$(1-\beta)\left(\sum_{t=1}^{3}\lambda^{\mathrm{C}}P_t^{\mathrm{C}}-\sum_{f=1}^{3}\sum_{t=1}^{3}\lambda_f^{\mathrm{F}}x_f-\sum_{\omega=1}^{10}\pi_\omega\sum_{t=1}^{3}\lambda_{t\omega}^{\mathrm{P}}y_{t\omega}\right)+\beta\eta \tag{4.66}$$

subject to

$$0 \le x_f \le X_f^{\max}, \quad f = 1, 2, 3 \tag{4.67}$$

$$\sum_{f=1}^{3} x_f + y_{t\omega} = P_t^{\mathrm{C}}, \quad t = 1, 2, 3;\ \omega = 1, \ldots, 10 \tag{4.68}$$

$$y_{t\omega} \ge 0, \quad t = 1, 2, 3;\ \omega = 1, \ldots, 10 \tag{4.69}$$

$$\eta - \left(\sum_{t=1}^{3} \lambda^{\mathrm{C}} P_t^{\mathrm{C}} - \sum_{f=1}^{3} \sum_{t=1}^{3} \lambda_f^{\mathrm{F}} x_f - \sum_{t=1}^{3} \lambda_{t\omega}^{\mathrm{P}} y_{t\omega} \right) \le M\theta_\omega, \quad \forall \omega = 1, \ldots, 10 \tag{4.70}$$

$$\sum_{\omega=1}^{10} \pi_\omega \theta_\omega \le 1 - \alpha \tag{4.71}$$

$$\theta_\omega \in \{0, 1\}, \quad \forall \omega = 1, \ldots, 10, \tag{4.72}$$

where $\alpha = 0.8$ and M=10,000.

Fig. 4.9 provides the cumulative distribution functions of the profit for $\beta = \{0, 1\}$. The VaR for $\beta = 0$ is \$57.50, which is equivalent to say that the 0.2-quantil of the profit distribution is equal to \$57.50. Observe that the VaR coincides with the value of the third scenario with smallest profit. Thus, the probability of the profit being less than the profit under this scenario is equal to 0.2, which is the value of $1 - \alpha = 1 - 0.8 = 0.2$.

The VaR and the expected profit for $\beta = 1$ are \$177.50 and \$535.75, respectively, which correspond to the decision vector $\{x_1^*, x_2^*, x_3^*\} = \{50, 0, 0\}$. This result indicates that the highest VaR is not obtained by means of the maximum participation in the futures market, $\{x_1, x_2, x_3\} = \{50, 30, 25\}$. In fact, for the maximum participation in the futures market the resulting VaR is equal to \$69.50 and the expected profit is equal to \$208.65. Observe that this pair of VaR and expected profit is outperformed by the solution for $\beta = 1$. For this reason, the solution attained with $\{x_1, x_2, x_3\} = \{50, 30, 25\}$ (maximum participation in the futures market) is not an efficient point.

The profit and the optimal value of θ_ω per scenario for $\beta = 1$ are provided in Table 4.7. The variable θ_ω is equal to 1 if the profit in scenario ω is smaller than the optimal value of η. For $\beta = 1$, the optimal value of η is \$177.50, which is precisely the value of the VaR. In Table 4.7, we observe that θ_ω is different from zero in scenarios 5 and 7. That is, the variable θ_ω is equal to 1 in all scenarios with a profit smaller than \$177.50.

The efficient frontier is depicted in Fig. 4.10. In this case, only two efficient points are obtained, which correspond to the extreme values of the parameter β (0 and 1).

□

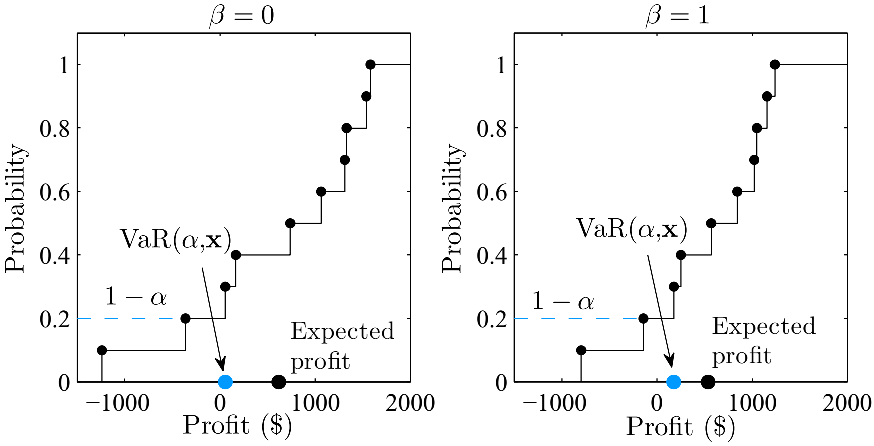


Fig. 4.9 Illustrative Example 4.5: cdfs for $\beta = 0$ and $\beta = 1$

Table 4.7 Illustrative Example 4.5: profit and variable θ_ω per scenario for $\beta = 1$

Scenario #	Profit ($)	θ_ω	Scenario #	Profit ($)	θ_ω
1	1022.50	0	6	1240.00	0
2	1157.50	0	7	-142.50	1
3	1050.00	0	8	252.50	0
4	177.50	0	9	845.00	0
5	-795.00	1	10	570.00	0

4.3.5 Conditional Value-at-Risk

For a given $\alpha \in (0,1)$, the conditional value-at-risk, CVaR, is defined as the expected value of the profit smaller than the $(1-\alpha)$-quantile of the profit distribution. If all profit scenarios are equiprobable, $\mathrm{CVaR}(\alpha, \boldsymbol{x})$ is computed as the expected profit in the $(1-\alpha) \times 100\%$ worst scenarios. The CVaR is also known as mean excess loss or average value-at-risk.

Mathematically, the $\mathrm{CVaR}(\alpha, \boldsymbol{x})$ for a discrete distribution is defined as, [124, 125],

$$\mathrm{CVaR}(\alpha, \boldsymbol{x}) = \max\left\{\eta - \frac{1}{1-\alpha}\mathcal{E}_\omega\Big\{\max\{\eta - f(\boldsymbol{x}, \omega), 0\}\Big\}\right\}, \ \forall \alpha \in (0,1). \tag{4.73}$$

The $\mathrm{CVaR}(\alpha, \boldsymbol{x})$ can be incorporated into the risk-neutral problem (4.1) as

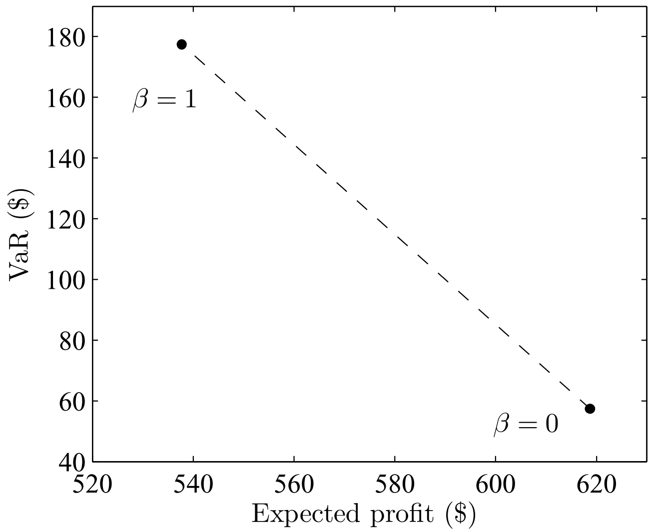


Fig. 4.10 Illustrative Example 4.5: efficient frontier

$$\text{Maximize}_{\boldsymbol{x},\boldsymbol{y}(\omega),\eta,s(\omega)}$$
$$(1-\beta)\left(\boldsymbol{c}^\top\boldsymbol{x} + \sum_{\omega\in\Omega}\pi(\omega)\boldsymbol{q}(\omega)^\top\boldsymbol{y}(\omega)\right) + \beta\left(\eta - \frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi(\omega)s(\omega)\right) \tag{4.74}$$

subject to

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{4.75}$$
$$\boldsymbol{T}(\omega)\boldsymbol{x} + \boldsymbol{W}(\omega)\boldsymbol{y}(\omega) = \boldsymbol{h}(\omega),\ \forall\omega\in\Omega \tag{4.76}$$
$$\eta - (\boldsymbol{c}^\top\boldsymbol{x} + \boldsymbol{q}(\omega)^\top\boldsymbol{y}(\omega)) \le s(\omega),\ \forall\omega\in\Omega \tag{4.77}$$
$$s(\omega)\ge 0,\ \forall\omega\in\Omega \tag{4.78}$$
$$\boldsymbol{x}\in X,\ \boldsymbol{y}(\omega)\in Y,\ \forall\omega\in\Omega, \tag{4.79}$$

where η is an auxiliary variable and $s(\omega)$ is a continuous non-negative variable equal to the maximum of $\eta - (\boldsymbol{c}^\top\boldsymbol{x} + \boldsymbol{q}(\omega)^\top\mathbf{y}(\omega))$ and 0.

One of the most important advantages of the CVaR is its ability to quantify fat tails beyond the VaR, apart from being a coherent risk measure as established in Section 4.3, [7]. Moreover no binary variables are needed for its calculation.

The following example illustrates the formulation of the CVaR.

Illustrative Example 4.6 (CVaR).

The formulation of problem (4.11)-(4.14) in Illustrative Example 4.1 including the CVaR in the objective function is

$$\begin{aligned}
&\text{Maximize}_{x_f, y_{t\omega}, s_\omega, \eta} \\
&(1-\beta)\left(\sum_{t=1}^{3} \lambda^{\mathrm{C}} P_t^{\mathrm{C}} - \sum_{f=1}^{3}\sum_{t=1}^{3} \lambda_f^{\mathrm{F}} x_f - \sum_{\omega=1}^{10} \pi_\omega \sum_{t=1}^{3} \lambda_{t\omega}^{\mathrm{P}} y_{t\omega} \right) + \\
&\beta \left(\eta - \frac{1}{1-\alpha} \sum_{\omega=1}^{10} \pi_\omega s_\omega \right)
\end{aligned} \tag{4.80}$$

subject to

$$0 \le x_f \le X_f^{\max}, \quad f = 1,2,3 \tag{4.81}$$

$$\sum_{f=1}^{3} x_f + y_{t\omega} = P_t^{\mathrm{C}}, \quad t = 1,2,3;\ \omega = 1,\ldots,10 \tag{4.82}$$

$$y_{t\omega} \ge 0, \quad t = 1,2,3;\ \omega = 1,\ldots,10 \tag{4.83}$$

$$\eta - \left(\sum_{t=1}^{3} \lambda^{\mathrm{C}} P_t^{\mathrm{C}} - \sum_{f=1}^{3}\sum_{t=1}^{3} \lambda_f^{\mathrm{F}} x_f - \sum_{t=1}^{3} \lambda_{t\omega}^{\mathrm{P}} y_{t\omega} \right) \le s_\omega, \quad \forall \omega = 1,\ldots,10 \tag{4.84}$$

$$s_\omega \ge 0, \quad \forall \omega = 1,\ldots,10. \tag{4.85}$$

Fig. 4.11 represents the cumulative distribution functions for $\beta = \{0,1\}$. For $\beta = 0$, the CVaR is equal to minus \$801.25, whereas it is equal to minus \$343.00 for $\beta = 1$. In this last case, the expected profit is equal to \$208.65. The optimal decision vector is $\{x_1^*, x_2^*, x_3^*\} = \{0,0,0\}$ for $\beta = 0$, and $\{x_1^*, x_2^*, x_3^*\} = \{50,30,25\}$ for $\beta = 1$.

The profit and the optimal value of s_ω per scenario for $\beta = 1$ are provided in Table 4.8. The variable s_ω is equal to the difference between η and the profit in scenario ω if this difference is positive, being 0 otherwise. In Table 4.8, we observe that s_ω is different from zero only in scenario 5. For the values of variable s_ω listed in this table and the optimal value of $\eta = -\$140.50$, the CVaR is computed as

$$\begin{aligned}
\eta - \frac{1}{1-\alpha} \sum_{\omega=1}^{10} \pi_\omega s_\omega &= \\
-140.50 - \frac{1}{1-0.8}(0.1 \times 405.00) &= \\
&= -\$343.
\end{aligned}$$

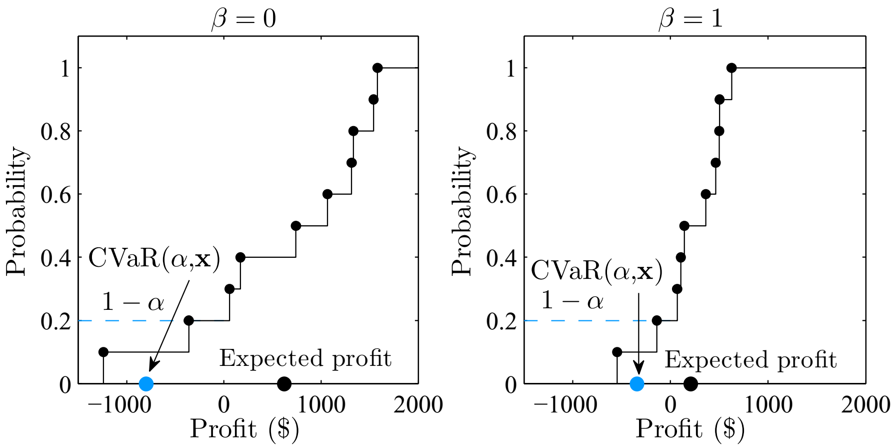


Fig. 4.11 Illustrative Example 4.6: cdfs for $\beta = 0$ and $\beta = 1$

Table 4.8 Illustrative Example 4.6: profit and variable s_ω per scenario for $\beta = 1$

Scenario #	Profit ($)	s_ω	Scenario #	Profit ($)	s_ω
1	463.50	0	6	626.00	0
2	499.50	0	7	-140.50	0
3	502.00	0	8	106.00	0
4	69.50	0	9	363.00	0
5	-545.50	405.00	10	143.00	0

The efficient frontier expressed in terms of the CVaR is provided in Fig. 4.12.

□

4.3.6 Stochastic Dominance

Stochastic dominance allows managing risk from a different point of view than that pertaining to the previously analyzed risk measures. Rather than searching for the "best" member of the profit distribution $f(\boldsymbol{x}, \omega)$, we seek "acceptable" members, and optimize over them. This leads to a recently introduced class of stochastic programming problems [39, 40, 98].

The stochastic dominance is a well-established concept in decision theory, which enables the comparison of two random variables in terms of the desired acceptability. In this section we deal with first- and second-order stochastic dominance.

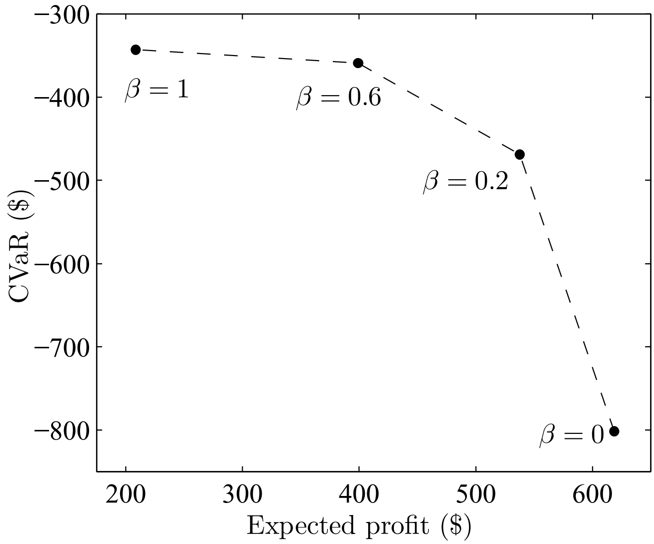


Fig. 4.12 Illustrative Example 4.6: efficient frontier

4.3.6.1 First-Order Stochastic Dominance Constraints (FOSDCs)

When preferring large realizations of random variables to small ones, a random variable $f_1(\omega)$ is said to dominate a random variable $f_2(\omega)$ to first order $\Big(f_1(\omega) \succeq_{(1)} f_2(\omega)\Big)$ if and only if [66]

$$P\Big(\omega | f_1(\omega) > \eta\Big) \geq P\Big(\omega | f_2(\omega) > \eta\Big). \tag{4.86}$$

Expression (4.86) means that, for all possible values of η, the probability of $f_1(\omega)$ being greater than η is higher than that of $f_2(\omega)$. For convenience, expression (4.86) can be also written as

$$P\Big(\omega | f_1(\omega) \leq \eta\Big) \leq P\Big(\omega | f_2(\omega) \leq \eta\Big). \tag{4.87}$$

Note that (4.87) can be equivalently expressed in terms of the cumulative distribution functions of variables $f_1(\omega)$ and $f_2(\omega)$,

$$F_1(\eta) \leq F_2(\eta), \ \forall \eta \in \mathbb{R}, \tag{4.88}$$

where $F(\eta) = P\Big(\omega | f(\omega) \leq \eta\Big)$ is the cumulative distribution function of random variable $f(\omega)$.

For example, consider a stochastic programming problem whose objective function consists in maximizing the expected profit. In this case, first-order stochastic dominance constraints can be easily incorporated to impose that the resulting profit dominates a pre-specified benchmark profit profile. Thus, we ensure that the resulting profit is "better" than a given benchmark that is acceptable for the decision maker. Clearly, caution should be exercised when selecting an appropriate benchmark in order to avoid unfeasible instances in which the resulting profit cannot outperform the pre-fixed benchmark. The formulation of this problem is

$$\text{Maximize}_{\boldsymbol{x},\boldsymbol{y}(\omega),\theta(\omega,\upsilon)} \quad \boldsymbol{c}^\top \boldsymbol{x} + \sum_{\omega\in\Omega} \pi(\omega)\boldsymbol{q}(\omega)^\top \boldsymbol{y}(\omega) \tag{4.89}$$

subject to

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{4.90}$$

$$\boldsymbol{T}(\omega)\boldsymbol{x} + \boldsymbol{W}(\omega)\boldsymbol{y}(\omega) = \boldsymbol{h}(\omega), \ \forall\omega\in\Omega \tag{4.91}$$

$$k(\upsilon) - \left(\boldsymbol{c}^\top \boldsymbol{x} - \boldsymbol{q}(\omega)^\top \boldsymbol{y}(\omega)\right) + \epsilon \le M\theta(\omega,\upsilon), \ \forall\omega\in\Omega, \forall\upsilon\in\Upsilon \tag{4.92}$$

$$\sum_{\omega\in\Omega} \pi(\omega)\theta(\omega,\upsilon) \le \sum_{\upsilon'\in\Upsilon | k(\upsilon')\le k(\upsilon)} \tau(\upsilon'), \ \forall\upsilon\in\Upsilon \tag{4.93}$$

$$\theta(\omega,\upsilon)\in\{0,1\}, \ \forall\omega\in\Omega, \forall\upsilon\in\Upsilon \tag{4.94}$$

$$\boldsymbol{x}\in X, \ \boldsymbol{y}(\omega)\in Y, \ \forall\omega\in\Omega, \tag{4.95}$$

where the random variable $\{k(\upsilon), \forall\upsilon\in\Upsilon\}$ provides the pre-fixed profit benchmark that is acceptable for the decision maker, $\theta(\omega,\upsilon)$ is a binary variable that is equal to 1 if the benchmark scenario $k(\upsilon)$ is greater than the profit in scenario ω (and 0 otherwise), $\tau(\upsilon)$ represents the probability associated with scenario υ of the benchmark random variable, M is a large enough constant, and ϵ is a positive, small constant, which is used to ensure that if $k(\upsilon)$ is equal to the profit in scenario ω the binary variable $\theta(\omega,\upsilon)$ is equal to 1.

Observe that imposing benchmark k in problem (4.89)-(4.95) ensures the acceptability of the resulting profit distribution in spite of maximizing the expected profit. For this reason, it is also possible to replace the expected profit in the objective function by another function $g(\boldsymbol{x})$ that may characterize the preferences of the decision maker for the resulting decision vector $\boldsymbol{x}$.

The following illustrative example characterizes the usage of first-order stochastic dominance constraints in a stochastic programming problem.

Illustrative Example 4.7 (First-order stochastic dominance constraints).

In order to examine the influence of using first-order stochastic dominance constraints on problem (4.11)-(4.14) in Illustrative Example 4.1, two

5-scenario benchmark profiles are used. The parameters of these benchmarks are provided in Table 4.9.

Table 4.9 Illustrative Example 4.7: benchmark data

Scenario #	Probability	Benchmark 1 ($)	Benchmark 2 ($)
1	0.05	-1000	-750
2	0.25	-500	100
3	0.30	0	200
4	0.25	500	400
5	0.15	1000	500

The formulation of problem (4.11)-(4.14) including first-order stochastic dominance constraints is

$$\text{Maximize}_{x_f, y_{t\omega}, \theta_{\omega\upsilon}}$$

$$\sum_{t=1}^{3} \lambda^{\mathrm{C}} P_t^{\mathrm{C}} - \sum_{f=1}^{3} \sum_{t=1}^{3} \lambda_f^{\mathrm{F}} x_f - \sum_{\omega=1}^{10} \pi_\omega \sum_{t=1}^{3} \lambda_{t\omega}^{\mathrm{P}} y_{t\omega} \tag{4.96}$$

subject to

$$0 \leq x_f \leq X_f^{\max}, \quad f = 1, 2, 3 \tag{4.97}$$

$$\sum_{f=1}^{3} x_f + y_{t\omega} = P_t^{\mathrm{C}}, \quad t = 1, 2, 3; \ \omega = 1, \ldots, 10 \tag{4.98}$$

$$y_{t\omega} \geq 0, \quad t = 1, 2, 3; \ \omega = 1, \ldots, 10 \tag{4.99}$$

$$k_\upsilon - \left(\sum_{t=1}^{3} \lambda^{\mathrm{C}} P_t^{\mathrm{C}} - \sum_{f=1}^{3} \sum_{t=1}^{3} \lambda_f^{\mathrm{F}} x_f - \sum_{t=1}^{3} \lambda_{t\omega}^{\mathrm{P}} y_{t\omega} \right) + \epsilon \leq M \theta_{\omega\upsilon},$$

$$\omega = 1, \ldots, 10; \ \upsilon = 1, \ldots, 5 \tag{4.100}$$

$$\sum_{\omega=1}^{10} \pi_\omega \theta_{\omega\upsilon} \leq \sum_{\upsilon'=1,\ldots,5 | k_{\upsilon'} \leq k_\upsilon} \tau_{\upsilon'}, \quad \upsilon = 1, \ldots, 5 \tag{4.101}$$

$$\theta_{\omega\upsilon} \in \{0, 1\}, \quad \forall \omega = 1, \ldots, 10; \ \upsilon = 1, \ldots, 5. \tag{4.102}$$

In this case we select M =10,000 and $\epsilon = 0.01$.

The cumulative distribution functions of the profit obtained by imposing the two benchmarks are provided in Fig. 4.13. Observe that benchmark 2 is comparatively more restrictive and allows reducing the worst scenario from minus \$1000, in the case considering benchmark 1, to minus \$750. The optimal decision vector and expected profit for the case with benchmark 1 are $\{x_1^*, x_2^*, x_3^*\} = \{26.97, 0, 0\}$ and \$575.06, respectively. For benchmark 2, the result is $\{x_1^*, x_2^*, x_3^*\} = \{50, 7.63, 0\}$ with an expected profit of \$502.51. Note that in order to meet the requirements enforced by benchmark 2, the retailer must purchase in the futures market a higher quantity of energy than in the case of benchmark 1.

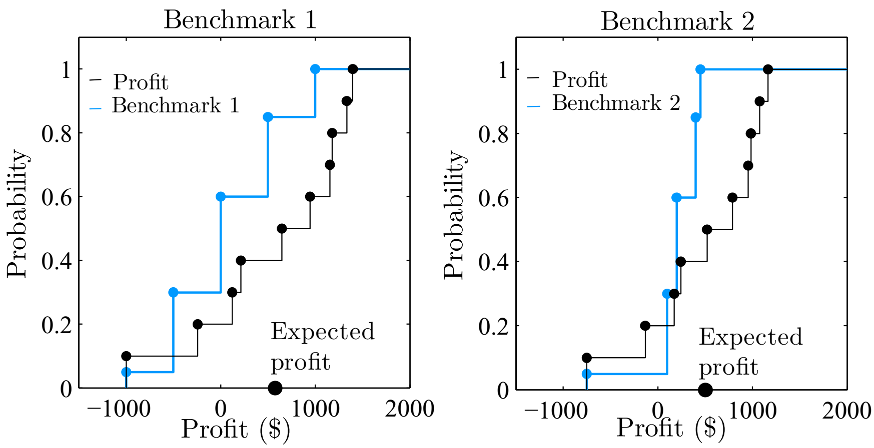


Fig. 4.13 Illustrative Example 4.7: cdfs imposing benchmarks 1 and 2

The profit per price scenario and the optimal value of $\theta_{\omega\upsilon}$ per each price scenario and benchmark scenario are provided in Table 4.10. The variable $\theta_{\omega\upsilon}$ is equal to 1 if the profit in scenario ω is less than the benchmark scenario υ. For instance, $\theta_{52} = 1$ because the profit in scenario 5, minus \$999.99, is smaller than that in scenario 2 of the benchmark, minus \$500. Observe that constraint (4.101) is satisfied for all benchmark scenarios.

□

4.3.6.2 Second-Order Stochastic Dominance Constraints (SOSDCs)

When preferring large realizations of random variables to small ones, a random variable $f_1(\omega)$ is said to dominate a random variable $f_2(\omega)$ to second order $\left(f_1(\omega) \succeq_{(2)} f_2(\omega)\right)$ if and only if, [65],

$$F_1^{(2)}(\eta) \le F_2^{(2)}(\eta), \ \forall \eta \in \mathbb{R}, \tag{4.103}$$

with

Table 4.10 Illustrative Example 4.7: solution for benchmark 1

Scenario #	Profit ($)	$\theta_{\omega 1}$	$\theta_{\omega 2}$	$\theta_{\omega 3}$	$\theta_{\omega 4}$	$\theta_{\omega 5}$
1	1156.09	0	0	0	0	1
2	1332.55	0	0	0	0	1
3	1178.98	0	0	0	0	1
4	122.22	0	0	0	1	1
5	-999.99	0	1	1	1	1
6	1396.62	0	0	0	0	1
7	-243.84	0	1	1	1	1
8	213.34	0	0	0	1	1
9	946.34	0	0	0	0	1
10	648.31	0	0	0	0	1
	$\sum_{\omega=1}^{10} \pi_\omega \theta_{\omega \upsilon}$	0.00	0.20	0.20	0.40	1.00
	$\sum_{\upsilon'=1,\ldots,5 \mid k_{\upsilon'} \leq k_\upsilon} \tau'_\upsilon$	0.05	0.30	0.60	0.85	1.00

$$F_1^{(2)}(\eta) = \int_{-\infty}^{\eta} F_1(t)\, dt, \ \forall \eta \in \mathbb{R}, \tag{4.104}$$

where $F_1(\eta)$ represents the cumulative distribution function of the random variable $f_1(\omega)$, i.e., $F_1(\eta) = P\Big(\omega | f_1(\omega) \leq \eta\Big)$. Therefore, $F_1^{(2)}(\eta)$ represents the area below $F_1(\eta)$ within the interval $(-\infty, \eta]$. Hence it can be said that $f_1(\omega) \succeq_{(2)} f_2(\omega)$ holds if and only if the area below $F_1(\eta)$ is less than or equal to the area below $F_2(\eta)$ in all intervals $(-\infty, \eta]$, $\forall \eta \in \mathbb{R}$.

A useful equivalent definition of second-order stochastic dominance of $f_1(\omega)$ above $f_2(\omega)$, when preferring greater returns, is

$$\mathcal{E}_\omega \Big\{ \big(\eta - f_1(\omega)\big)_+ \Big\} \leq \mathcal{E}_\omega \Big\{ \big(\eta - f_2(\omega)\big)_+ \Big\}, \ \forall \eta \in \mathbb{R}, \tag{4.105}$$

where operator $(a)_+$ denotes the greatest value between 0 and a.

Therefore, a random variable $f_1(\omega)$ stochastically dominates a random variable $f_2(\omega)$ to the second order if and only if, for any value η, the expected shortfall of $f_1(\omega)$ below η is less than or equal to the expected shortfall of $f_2(\omega)$ below η.

Second-order stochastic dominance can be incorporated into the risk-neutral problem (4.1)-(4.4) in order to assess the risk aversion with respect to a given benchmark profile k, which is selected by the decision maker. Hence, decision $\boldsymbol{x}$ is considered acceptable if $f(\boldsymbol{x}, \omega) \succeq_{(2)} k$.

Note that there might be many decisions $x \in X$ that are considered acceptable. Thus, over all "acceptable" $x \in X$, we select the decision achieving the maximum expected profit. This leads to the following stochastic programming problem with dominance constraints

$$\text{Maximize}_{\boldsymbol{x},\boldsymbol{y}(\omega),s(\omega,\upsilon)}$$
$$\boldsymbol{c}^\top \boldsymbol{x} + \sum_{\omega \in \Omega} \pi(\omega)\boldsymbol{q}(\omega)^\top \boldsymbol{y}(\omega) \tag{4.106}$$

subject to

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{4.107}$$
$$\boldsymbol{T}(\omega)\boldsymbol{x} + \boldsymbol{W}(\omega)\boldsymbol{y}(\omega) = \boldsymbol{h}(\omega),\ \forall \omega \in \Omega \tag{4.108}$$
$$k(\upsilon) - (\boldsymbol{c}^\top \boldsymbol{x} - \boldsymbol{q}(\omega)^\top \boldsymbol{y}(\omega)) \le s(\omega,\upsilon),\ \forall \omega \in \Omega,\ \forall \upsilon \in \Upsilon \tag{4.109}$$
$$\sum_{\omega \in \Omega} \pi(\omega) s(\omega,\upsilon) \le \sum_{\upsilon' \in \Upsilon} \tau(\upsilon')\max\Big(k(\upsilon) - k(\upsilon'), 0\Big),\ \forall \upsilon \in \Upsilon \tag{4.110}$$
$$s(\omega,\upsilon) \ge 0,\ \forall \omega \in \Omega, \forall \upsilon \in \Upsilon \tag{4.111}$$
$$\boldsymbol{x} \in X,\ \boldsymbol{y}(\omega) \in Y,\ \forall \omega \in \Omega, \tag{4.112}$$

where the random variable $\{k(\upsilon), \forall \upsilon \in \Upsilon\}$ provides the pre-fixed profit benchmark that is acceptable for the decision maker and $s(\omega,\upsilon)$ is a variable that measures the shortfall of the profit in scenario ω below the benchmark scenario υ.

The formulation of the second-order stochastic dominance constrains is illustrated by means of the following example.

Illustrative Example 4.8 (Second-order stochastic dominance constraints).

In order to study the effect of including second-order stochastic dominance constraints in problem (4.11)-(4.14) of Illustrative Example 4.1, two 5-scenario benchmark profiles are used. The parameters of these benchmarks are provided in Table 4.11.

Table 4.11 Illustrative Example 4.8: benchmark data

Scenario #	Probability	Benchmark 1 ($)	Benchmark 2 ($)
1	0.05	-1000	-750
2	0.10	-750	-500
3	0.20	50	50
4	0.35	500	500
5	0.30	1000	700

The formulation of problem (4.11)-(4.14) including second-order stochastic dominance constraints is

$$\text{Maximize}_{x_f, y_{t\omega}, s_{\omega\upsilon}}$$

$$\sum_{t=1}^{3} \lambda^{\mathrm{C}} P_t^{\mathrm{C}} - \sum_{f=1}^{3} \sum_{t=1}^{3} \lambda_f^{\mathrm{F}} x_f - \sum_{\omega=1}^{10} \pi_\omega \sum_{t=1}^{3} \lambda_{t\omega}^{\mathrm{P}} y_{t\omega} \tag{4.113}$$

subject to

$$0 \leq x_f \leq X_f^{\max}, \quad f = 1, 2, 3 \tag{4.114}$$

$$\sum_{f=1}^{3} x_f + y_{t\omega} = P_t^{\mathrm{C}}, \quad t = 1, 2, 3;\ \omega = 1, \ldots, 10 \tag{4.115}$$

$$y_{t\omega} \geq 0, \quad t = 1, 2, 3;\ \omega = 1, \ldots, 10 \tag{4.116}$$

$$k_\upsilon - \left(\sum_{t=1}^{3} \lambda^{\mathrm{C}} P_t^{\mathrm{C}} - \sum_{f=1}^{3} \sum_{t=1}^{3} \lambda_f^{\mathrm{F}} x_f - \sum_{t=1}^{3} \lambda_{t\omega}^{\mathrm{P}} y_{t\omega} \right) \leq s_{\omega\upsilon},$$

$$\omega = 1, \ldots, 10;\ \upsilon = 1, \ldots, 5 \tag{4.117}$$

$$\sum_{\omega=1}^{10} \pi_\omega s_{\omega\upsilon} \leq \sum_{\upsilon'=1}^{5} \tau'_\upsilon \max\left((k_\upsilon - k_{\upsilon'}, 0)\right), \quad \upsilon = 1, \ldots, 5 \tag{4.118}$$

$$s_{\omega\upsilon} \geq 0, \quad \forall \omega = 1, \ldots, 10;\ \upsilon = 1, \ldots, 5. \tag{4.119}$$

Fig. 4.14 provides the cumulative distribution functions of the profit obtained by imposing the two benchmarks in Table 4.11. The expected profit for the case with benchmark 1 is \$552.31, which corresponds to the optimal decision vector $\{x_1^*, x_2^*, x_3^*\} = \{41, 0, 0\}$. In this case, the worst scenario involves a profit of minus \$875. On the other hand, for benchmark 2, the optimal decision vector is $\{x_1^*, x_2^*, x_3^*\} = \{50, 28.8, 0\}$, with an associated expected profit of \$404.63, being the profit of the worst scenario minus \$625.

The profit per price scenario, and the optimal value of $s_{\omega\upsilon}$ per both price scenario and benchmark scenario are provided in Table 4.12. The variable $s_{\omega\upsilon}$ is equal to the difference between the k_υ scenario of the benchmark and the profit in scenario ω if this difference is positive, being 0 otherwise. For instance, $s_{53} = \$925.0$ because the value of the scenario 3 of the benchmark, \$50, minus the profit in scenario 5, minus \$875.00, is equal to \$925.00. Observe that constraint (4.118) is satisfied for all benchmark scenarios and is active for benchmark scenarios 1 and 2.

□

4.4 Summary and Conclusions

In this chapter we discuss the need of risk control in stochastic programming problems concerning electricity markets. We observe that accounting for risk

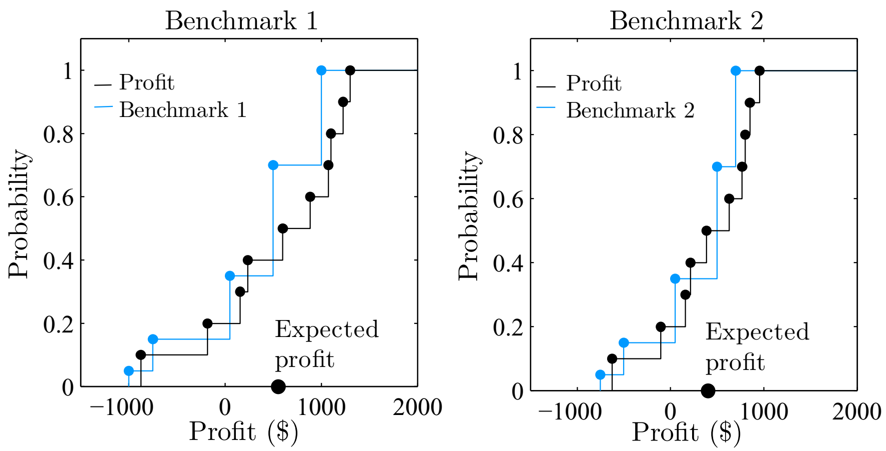


Fig. 4.14 Illustrative Example 4.8: cdfs imposing benchmarks 1 and 2

Table 4.12 Illustrative Example 4.8: solution for benchmark 1

Scenario #	Profit ($)	$s_{\omega 1}$	$s_{\omega 2}$	$s_{\omega 3}$	$s_{\omega 4}$	$s_{\omega 5}$
1	1074.63	0	0	0	0	0
2	1225.81	0	0	0	0	0
3	1100.34	0	0	0	0	0
4	155.93	0	0	0	344.1	844.1
5	-875.00	0	125.0	925.0	1375.0	1875.0
6	1301.12	0	0	0	0	0
7	-182.05	0	0	232.0	682.0	1182.0
8	237.22	0	0	0	262.8	762.8
9	884.55	0	0	0	0	115.4
10	600.56	0	0	0	0	399.4
	$\sum_{\omega=1}^{10} \pi_\omega s_{\omega\upsilon}$	0.00	12.50	115.71	266.39	517.88
	$\sum_{\upsilon'=1}^{5} \tau'_\upsilon \max(k_\upsilon - k_{\upsilon'}, 0)$	0.00	12.50	132.50	290.00	640.00

in the formulation of these problems enables to make informed decisions avoiding undesirable outcomes in the "worst" scenarios.

Usually, risk management is performed by means of the so-called risk measures. A risk measure is a function that associates a given random variable (profit or cost) with a real number characterizing the risk. This real number serves us to compare different decisions in terms of risk.

In this chapter, we analyze different risk measures characterizing the risk of a profit objective function. The first one is the profit variance, first proposed by Nobel laureate H. M. Markowitz in his pioneering work [91]. The variance is an intuitive dispersion measure of the profit that, if included in a stochastic programming problem, reduces the probability of experiencing a profit different from the expected one. However, its drawback is that in addition of penalizing low-profit scenarios, it also penalizes those scenarios with profits higher than the expected profit.

The shortfall probability and the expected shortfall are linear measures that characterize the risk as the probability of the profit being smaller of a given target value for profit, and the expected value of the profit being below that target, respectively [129]. These two measures are easy to implement, but require to specify an *arbitrary* target value for profit. Moreover, they are not coherent risk measures as defined in [7].

Other risk measures extensively used for characterizing risk are the value-at-risk (VaR) and the conditional value-at-risk (CVaR). These measures require the use of a probability, $(1-\alpha)$, that makes reference to the part of the profit distribution that is desired to be *managed* in order to hedge the risk. The VaR is the $(1-\alpha)$-quantile of the profit distribution, whereas the CVaR is the expected value of the profit distribution below that $(1-\alpha)$-quantile. Since these measures do not use a profit target value, they are more suitable for modeling risk. In addition, the CVaR is a coherent risk measure.

A different point of view for risk control consists in using stochastic dominance constraints in the formulation of the considered problem. Using a profit benchmark that is acceptable for the decision maker enables to impose that the resulting profit is "better" than this benchmark. We study first- and second-order stochastic dominance constraints. First-order constraints are enforced by means of a set of auxiliary binary variables, whereas second-order constraints are formulated with continuous variables. The main drawback of using stochastic dominance constraints are the need of specifying a benchmark profile and a significantly higher computational cost than that required by other risk measures.

Currently, the CVaR is widely used because, besides being a coherent risk measure, it can be expressed using a linear formulation. For these two reasons, the CVaR is the most used risk measure in problems pertaining to electricity markets.

4.5 Exercises

Exercise 4.1. Consider the problem faced by a producer owning a 100-MW power plant. This producer seeks to determine the on/off status of its unit and the selling of power in the futures market for a planning horizon of 4 hours. This producer can sell power in the futures market by means of three contracts with prices 40, 42, and \$45/MWh, each contract involving an amount of power up to 25 MW throughout the 4-hour market horizon. This producer can also sell energy in a pool, whose prices are unknown. Therefore, pool prices are characterized as random variables and described through a set of five equiprobable scenarios, which are provided in Table 4.13.

Formulate and solve the risk-neutral profit maximization model associated with the problem described above.

Table 4.13 Exercise 4.1: pool price data ($/MWh)

Scenario #	Period #			
	1	2	3	4
1	39.4	46.3	41.2	43.2
2	33.2	46.1	48.3	44.4
3	37.1	44.3	42.4	41.4
4	43.4	42.2	44.4	43.6
5	44.5	47.3	46.3	44.2

Exercise 4.2. Include the variance of the profit in the objective function of the problem described in Exercise 4.1. Solve the problem for different values of the weighting parameter β, and build the corresponding efficient frontier.

Exercise 4.3. Include the shortfall probability in the objective function of the problem described in Exercise 4.1. Solve the problem for $\eta = \$0$ and for different values of the weighting parameter β, and build the corresponding efficient frontier.

Exercise 4.4. Include the expected shortfall in the objective function of the problem described in Exercise 4.1. Solve the problem for $\eta = \$0$ and for different values of the weighting parameter β, and build the corresponding efficient frontier.

Exercise 4.5. Include the VaR in the objective function of the problem described in Exercise 4.1. Solve the problem for $\alpha = 0.75$ and for different values of the weighting parameter β and build the corresponding efficient frontier.

Exercise 4.6. Include the CVaR in the objective function of the problem described in Exercise 4.1. Solve the problem for $\alpha = 0.75$ and for different values of the weighting parameter β, and build the corresponding efficient frontier.

Exercise 4.7. Formulate the problem described in Exercise 4.1 including first-order stochastic dominance constraints. Build two profit benchmark profiles comprising 4 scenarios, solve the problem for these two benchmarks and compare the solutions.

Exercise 4.8. Formulate the problem described in Exercise 4.1 including second-order stochastic dominance constraints. Build two profit benchmark profiles comprising 4 scenarios, solve the problem for these two benchmarks and compare the solutions.

Exercise 4.9. Reformulate and solve problems in Exercises 4.2-4.6 limiting the value of the risk measure using an additional constrain instead of including the risk measure in the objective function.

Exercise 4.10. Derive the mathematical expressions of the risk measures described in this chapter for the case of cost minimization.

Chapter 5
Producer Pool Trading

5.1 Introduction

In this chapter we discuss the problem faced by a producer participating in a pool. This pool is assumed to be comprised of three markets that are sequentially cleared, namely, day-ahead, regulation, and adjustment markets. We consider that the producer owns a set of thermal units.

The objective of this chapter is to provide a methodology to determine the offering strategy for a producer in the day-ahead market considering that this producer has also the opportunity of participating in the regulation and adjustment markets. The planning horizon in this problem is one trading day divided into 24 hourly periods. The technical constraints of the thermal units of the producer are modeled in detail.

We consider that the thermal producer is a price-taker agent in the day-ahead and regulation markets. However, in the adjustment market, with less volume of negotiated energy, it is not realistic to neglect the influence of the producer actions on the resulting price. For this reason, we assume that the producer is a price-maker agent in this market.

The resulting prices in the day-ahead, regulation, and adjustment markets are unknown to the producer when offering in the day-ahead market. Thus, to deal with these uncertain parameters, a stochastic programming approach is used. Since the producer has available three sequential markets for trading, we propose to use a three-stage stochastic programming problem where decisions related to each market are made in each stage (see Chapter 2). The risk associated with profit variability is explicitly taken into account through the Conditional Value-at-Risk (see Chapter 4). The resulting deterministic equivalent model is expressed using a scenario-based formulation characterized as a mixed-integer quadratic programming problem that is solved using commercially available software.

Relevant references dealing with different problems faced by a producer in the short term include [3, 5, 29, 33, 135]. The optimal derivation of offering

A. J. Conejo et al., *Decision Making Under Uncertainty in Electricity Markets*, International Series in Operations Research & Management Science 153,
DOI 10.1007/978-1-4419-7421-1_5,

strategies has been studied, for instance, in [8,9,59,90,112,118]. In [51], offering curves are built for hydrothermal producers. In [32,69,102,132] risk management is taken into account for deriving optimal offering curves. Finally, in [126] the offering strategy is determined by means of solving a stochastic bilevel problem that considers a multi-period network-constrained market-clearing algorithm.

The rest of this chapter is organized as follows. Section 5.2 describes the decision framework for the considered problem. Section 5.3 provides the uncertainty characterization for the unknown parameters appearing in the problem. In Section 5.4, the structure of the pool-based market is described in detail. The technical constraints of the thermal units of the producer and the mathematical formulation of the profit achieved by the producer are provided in Section 5.5. Section 5.6 contains the complete mathematical formulation of the problem. In Section 5.7 an illustrative example is solved and comprehensively discussed in order to highlight the main features of the proposed model. In Section 5.8, the performance of the proposed model is tested on a realistic case study. Section 5.9 provides a summary for the chapter together with some relevant conclusions.

5.2 Decision Framework

As explained above, we focus on the problem faced by a thermal producer that desires to build its optimal offering strategy for the day-ahead market. Thus, the producer needs to determine an offering curve for each hour of the considered trading day. These curves are submitted by the producer to the day-ahead market that is cleared the day before in which the actual power delivery takes place. Once the day-ahead market is cleared, the producer submits offering curves for the regulation market. Finally, when the regulation market is cleared, the producer submits the offering curves for the adjustment market. The time sequence of the decisions made by the producer is represented in Fig. 5.1. Observe that, as in the day-ahead market, offering curves for the regulation and adjustment markets are submitted by the producer the day before to the trading day.

Day-ahead market decisions are made before knowing the resulting market-clearing prices in the day-ahead, regulation, and adjustment markets. These decisions are called first-stage decisions. Once the day-ahead market is cleared the producer makes the regulation market decisions, which constitute the second-stage decisions. Note that regulation market decisions are made after the day-ahead market is cleared, and thus they are made with perfect information on the amount of power not committed in the day-ahead market and that is available for trading in the regulation and adjustment markets. In this stage, regulation and adjustment prices are still unknown. Next, the regulation market is cleared and the producer makes the adjustment market

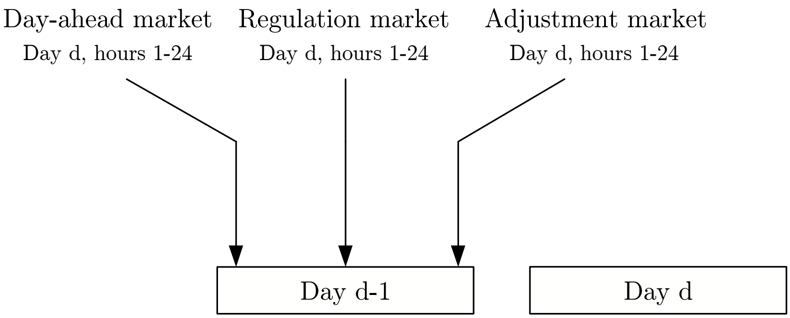


Fig. 5.1 Decision framework of the producer pool problem. Day-ahead, regulation and adjustment markets for day d are cleared successively in day d-1

decisions with perfect information on the quantity of power committed in the regulation market and under uncertain adjustment prices. The adjustment market decisions are thus third-stage decisions.

The example below illustrates the short-term decision-making process of a producer.

Illustrative Example 5.1 (Decision-making process). We consider a producer owning a single thermal unit of 100 MW and a planning horizon of one hour. Note that considering a period of one hour, the concepts of power and energy become equivalent. We assume that this producer participates in the day-ahead, regulation, and adjustment markets.

Given the decision framework depicted in Fig. 5.1, the first step is to submit an offering curve to the day-ahead market. Fig. 5.2 represents the offering curve submitted to the day-ahead market, which comprises three blocks.

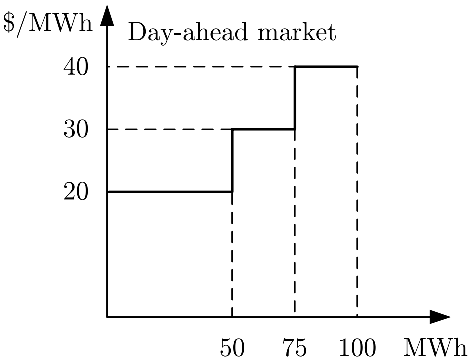


Fig. 5.2 Illustrative Example 5.1: offering curve for the day-ahead market

Once all agents submit their offering (producers) and bidding (consumers) curves, the market operator clears the market and publishes the market-clearing price and the set of accepted generation offers and demand bids. In this case, suppose that the market operator establishes a market-clearing price equal to \$35/MWh. According to this price, the first and second blocks of the offering curve submitted by the considered producer are accepted, and as a result 75 MW of its capacity are committed to the day-ahead market.

Given the quantity of remaining power, $100 - 75 = 25$ MW, the producer submits an offering curve to the regulation market, which is represented in Fig. 5.3.

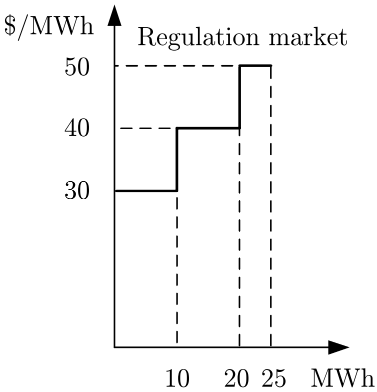


Fig. 5.3 Illustrative Example 5.1: offering curve for the regulation market

Assume that the market operator establishes a market-clearing price equal to \$45/MWh in the regulation market. Therefore, the first and second blocks of the offering curve in Fig. 5.3 are accepted and thus a power range of 20 MW is committed to the regulation market. We consider that units change production up or down a quantity equal to the half of the committed power range as explained later on in Subsection 5.4.2. Therefore, at most, the unit will be called to provide 10 MW for the regulation service in the considered period. Taking into account the 75 MW previously committed to the day-ahead market, the producer still has 15 MW available for trading in the adjustment market. With this information, the producer submits an offering curve for the adjustment market. Fig. 5.4 represents this curve.

Once all agents submit the offering and consumption curves to the adjustment market, the market operator clears it and establishes that the market-clearing price is \$35/MWh. According to this price, only the first block of 5 MW of the offering curve submitted by the producer is accepted.

Finally, note that, at most, 90 MW of the capacity of the producer is scheduled to be dispatched the next day.

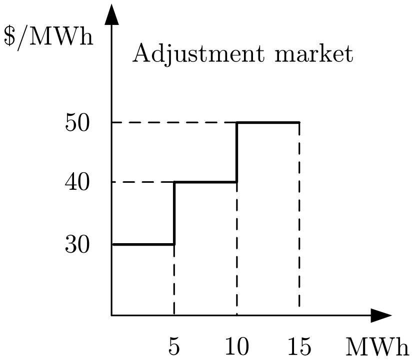


Fig. 5.4 Illustrative Example 5.1: offering curve for the adjustment market

□

5.3 Uncertainty Characterization

5.3.1 Day-ahead, Regulation, and Adjustment Prices

There are three main sources of uncertainty in the producer pool trading problem, which correspond to the electricity prices in the day-ahead, regulation, and adjustment markets for the next day. These prices are modeled as stochastic processes.

The prices in the day-ahead and regulation markets are considered independent of the producer actions. That is, the producer is assumed to be a price-taker agent in the day-ahead and regulation markets. The day-ahead and regulation prices in period t are denoted by λ_t^{D} and λ_t^{R}, respectively. Hereinafter, we consider that each period t corresponds to a single hour.

As previously stated, we assume that the producer is a price-maker agent in the adjustment market. Thus, the final hourly price of energy in this market depends on the quantity of power traded by the producer. We denote by P_t^{A} and λ_t^{A} the total quantity of power traded by the producer and the price in the adjustment market in period t, respectively. If we consider that the relationship between the power traded by the producer and the final price in the adjustment market is linear, the price λ_t^{A} is expressed as follows:

$$\lambda_t^{\mathrm{A}} = \lambda_t^{\mathrm{A0}} + \gamma_t^{\mathrm{A}} P_t^{\mathrm{A}}, \ \forall t, \tag{5.1}$$

where λ_t^{A0} represents the price of energy in the adjustment market in period t if the producer does not participate in this market. In other words, λ_t^{A0}

is the intercept of the inverse demand curve in period t. The parameter γ_t^{A} represents the slope of the inverse demand curve in period t. Expression (5.1) constitutes generally a reasonable approximation.

The following illustrative example shows the variation of the prices in the adjustment market with respect to the power traded by the producer in this market.

Illustrative Example 5.2 (Adjustment prices).

Consider a producer that participates in the adjustment market. This producer is a price-maker agent in this market and owns a single thermal unit of 100 MW.

Assuming a one-hour horizon, the price in the adjustment market, λ^{A}, is characterized as

$$\lambda^{\mathrm{A}} = 20 - 0.05P^{\mathrm{A}}. \tag{5.2}$$

Fig. 5.5 illustrates the variation of the clearing price with respect to the power traded by the producer in the adjustment market.

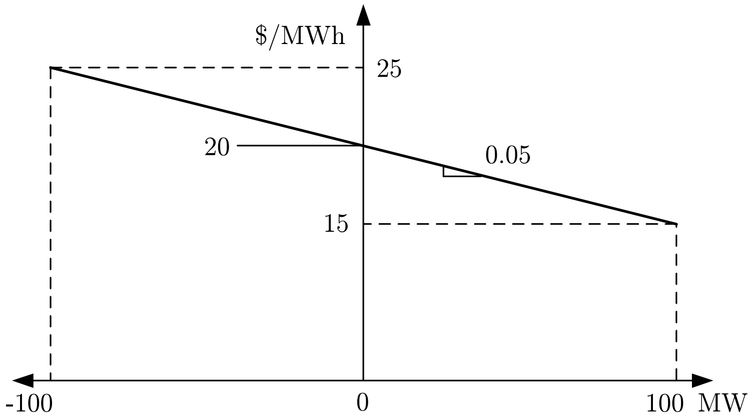


Fig. 5.5 Illustrative Example 5.2: adjustment price

If the producer does not participate in the adjustment market the resulting price is equal to \$20/MWh. However, if the producer sells power, the resulting price decreases linearly. The minimum price, \$15/MWh, corresponds to selling the capacity of the unit, 100 MW. On the other hand, if the producer buys power the price increases. For 100 MW bought in the adjustment market, the price increases up to \$25/MWh. □

As it is common practice in stochastic programming we use a finite set of scenarios Ω to represent the stochastic processes, being N_Ω the number of considered scenarios (see Chapter 3). Thus, each scenario $\omega \in \Omega$ is a vector of 24 day-ahead prices, 24 regulation prices, and 24 intercepts and 24 slopes characterizing adjustment prices, and a probability of occurrence π_ω. Note that the sum of the probabilities over all scenarios must be equal to 1, i.e., $\sum_{\omega=1}^{N_\Omega} \pi_\omega = 1$. An example of scenario is

$$\text{Scenario}\, \omega = \left\{ \lambda^{\mathrm{D}}_{t\omega}, \lambda^{\mathrm{R}}_{t\omega}, \lambda^{\mathrm{A0}}_{t\omega}, \gamma^{\mathrm{A}}_{t\omega}, \pi_\omega \right\}, \; \forall t. \tag{5.3}$$

5.3.2 Scenario Tree

The set of scenarios is arranged in a multi-stage scenario tree with three stages. The day-ahead decisions are determined at the first stage, regulation decisions are made in the second stage, and finally, the adjustment market trading is determined in the third stage. Fig. 5.6 shows the corresponding three-stage scenario tree.

In accordance with this tree, the day-ahead market decisions are here-and-now decisions, which are unique for all possible scenarios. However, since the objective of the considered problem is to derive offering curves in the day-ahead market, it is not adequate to determine a single quantity of power to be traded in this market for all possible scenarios. Instead, a producer is interested in determining the quantity of power to be sold for every possible price realization in the day-ahead market. This way, each different price yields an optimal quantity of power to be sold by the producer in the day-ahead market. Therefore, it is appropriate to relax the uniqueness of the day-ahead market decisions, making them dependent on the day-ahead market price scenarios. Thus, each scenario represents a block in the offering curve for the day-ahead market. Only if two scenarios have associated the same day-ahead prices, the first-stage decisions have to be identical.

Note that the same reasoning applies to the regulation and adjustment markets, where the producer also participates by submitting offering curves. However, since the objective of the considered problem is to derive the optimal offering strategy in the day-ahead market, for simplicity, we consider that the producer participates in the regulation and adjustment markets by offering single quantities instead of complete offering curves.

The multi-stage tree resulting from relaxing the first-stage decisions is depicted in Fig. 5.7.

At first sight, the scenario tree in Fig. 5.7 may be decomposed into so many trees as different first-stage scenarios. However, this decomposition is not possible since it is necessary to impose that the offering curves submitted by the producer must be non-decreasing. Note that this condition couples all scenarios together.

The following illustrative example provides an instance of scenario tree for the pool producer problem.

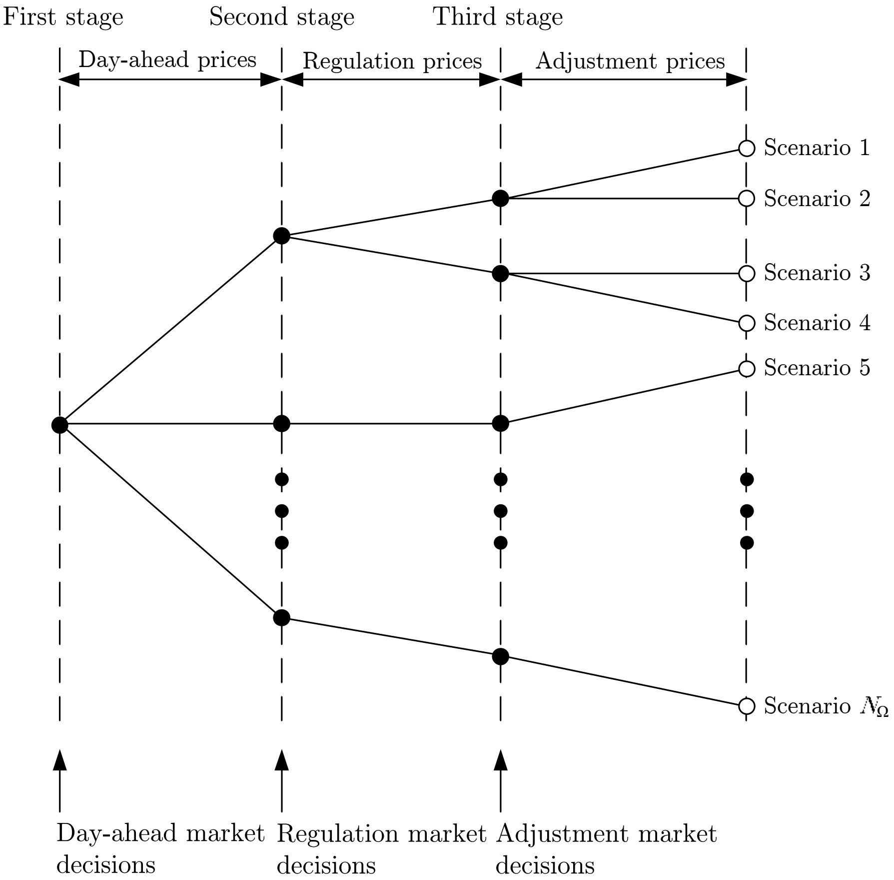


Fig. 5.6 Three-stage scenario tree of the producer pool problem

Illustrative Example 5.3 (Scenario tree).

Consider the scenario tree depicted in Fig. 5.8. This tree comprises three stages and five scenarios. The considered horizon is a single hour. The values of the stochastic processes in this hour are provided in Table 5.1.

Observe that some branches of the tree are common for several scenarios. For example scenarios 1-3 contain the same day-ahead prices, while scenarios 4-5 contain the same regulation prices.

Decisions are made at the nodes of the tree. Since day-ahead decisions are made in the first stage, note that only two different day-ahead decisions can be made, corresponding to the two nodes in the first stage. Therefore, the offering curve in the day-ahead market is composed of two blocks, each one related to one node.

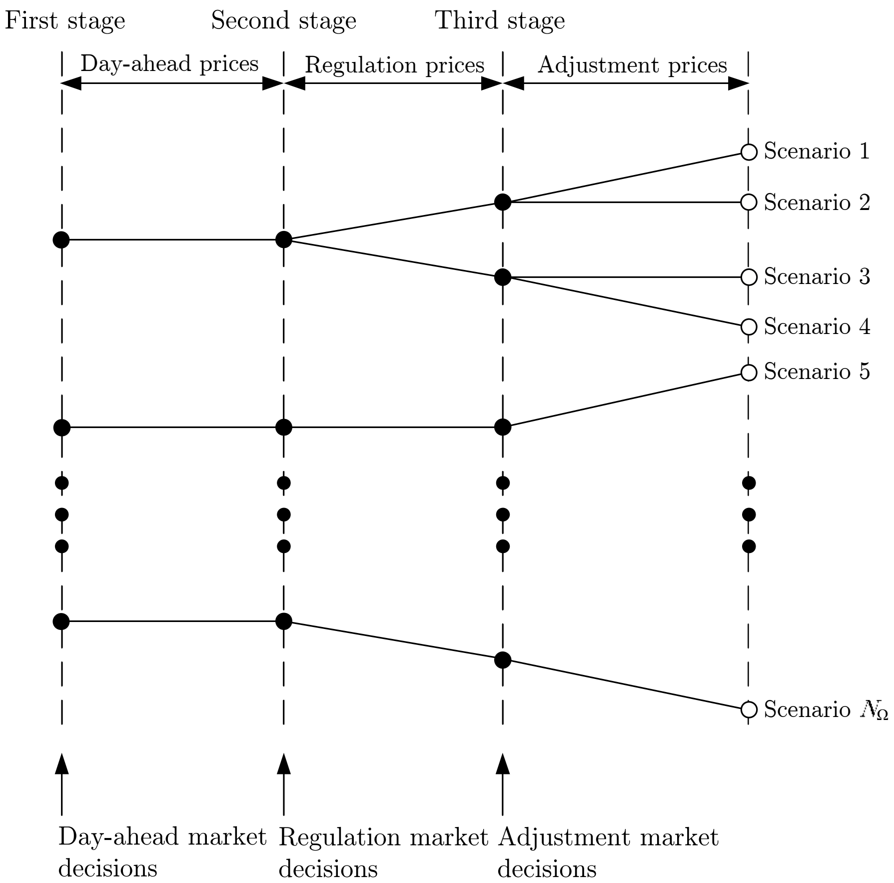


Fig. 5.7 Decoupled three-stage scenario tree of the producer pool problem

Observe that day-ahead decisions are made under complete information on day-ahead prices. On the other hand, regulation and adjustment market decisions are made with lack of knowledge of the prices in these two markets. For example, regulation decisions in scenarios 1-3 have to be unique for prices equal to 23 and \$18/MWh. The same reasoning can be made for the adjustment market. For instance, adjustment decisions for scenarios 4 and 5 have to be the same, although adjustment prices are different in these two scenarios.

□

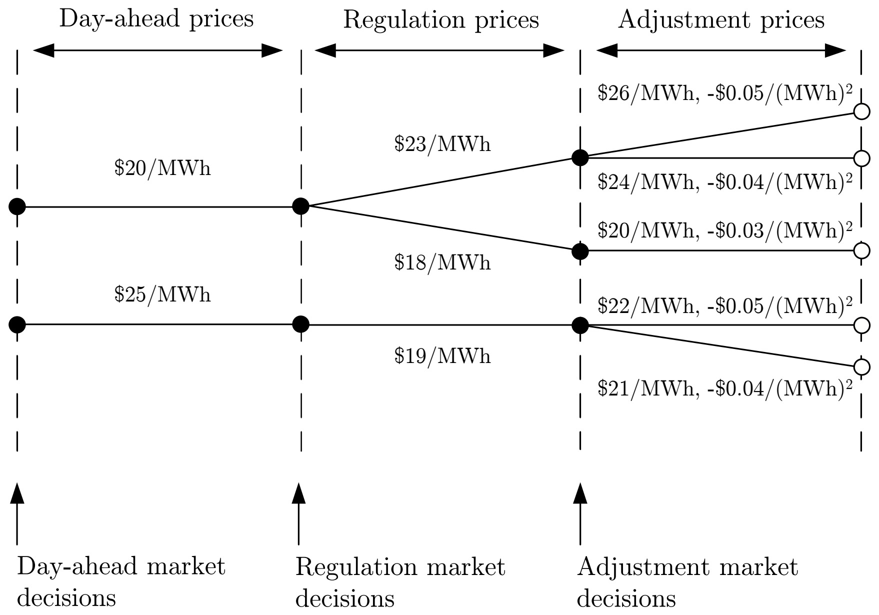


Fig. 5.8 Illustrative Example 5.3: scenario tree

Table 5.1 Illustrative Example 5.3: scenario tree data

Scenario #	Day-ahead price $\lambda^{\mathrm{D}}_{t\omega}$ (\$/MWh)	Regulation price $\lambda^{\mathrm{R}}_{t\omega}$ (\$/MWh)	Adjustment price $\lambda^{\mathrm{A0}}_{t\omega}$ (\$/MWh)	$\gamma^{\mathrm{A}}_{t\omega}$ (\$/(MWh)^2)
1	20	23	26	-0.05
2	20	23	24	-0.04
3	20	18	20	-0.03
4	25	19	22	-0.05
5	25	19	21	-0.04

5.4 Pool Structure

In this section, we discuss in detail the three different trading floors considered in the producer problem: day-ahead, regulation, and adjustment markets.

5.4.1 Day-Ahead Market

The day-ahead market is a trading floor that allows energy transactions between producers and consumers. It is an auction that takes place the day prior to energy delivery. Generally, it constitutes the most important trading floor in a pool. A general framework for the day-ahead market is described in Chapter 1.

Producers and consumers submit production offers and consumption bids, respectively. Afterwards, the market operator clears the market and, for every hour, determines the market clearing price and the sets of accepted production offers and consumption bids.

The variable $P^{\mathrm{D}}_{gt\omega}$ represents the quantity of power that unit g is willing to sell for time period t in the day-ahead market for day-ahead price $\lambda^{\mathrm{D}}_{t\omega}$. Subindex ω refers to the considered scenario. Note that different scenarios of day-ahead prices, $\left\{\lambda^{\mathrm{D}}_{t1}, \ldots, \lambda^{\mathrm{D}}_{tN_{\Omega}}\right\}$, will result in different quantities of power to be sold, $\left\{P^{\mathrm{D}}_{gt1}, \ldots, P^{\mathrm{D}}_{gtN_{\Omega}}\right\}$.

This way, for each period t it is possible to obtain a set of day-ahead prices and selling quantities in order to build an offering curve for each unit g of the considered producer in the day-ahead market. In this respect, it is necessary to enforce that offering curves must be non-decreasing, which is an offering requirement in most markets. This can be accomplished as follows:

$$P^{\mathrm{D}}_{gt\omega} - P^{\mathrm{D}}_{gt\omega'} \leq 0, \ \forall g, \forall t, \ \forall \omega, \omega'; \ O^{\mathrm{D}}(t,\omega) + 1 = O^{\mathrm{D}}(t,\omega'), \tag{5.4}$$

where matrix $\boldsymbol{O}^{\mathbf{D}}$ is used to sort the day-ahead prices associated with each period in an increasingly manner for each scenario ω. Therefore, element $O^{\mathrm{D}}(t,\omega)$ represents the position of day-ahead price $\lambda^{\mathrm{D}}_{t\omega}$ over all scenarios $\omega \in \Omega$. If this price is the smallest one, then $O^{\mathrm{D}}(t,\omega) = 1$. On the contrary, if $\lambda^{\mathrm{D}}_{t\omega}$ corresponds to the largest price, $O^{\mathrm{D}}(t,\omega)$ is equal to the number of different day-ahead prices in period t. Considering a given time period, identical day-ahead prices have associated equal values in matrix $\boldsymbol{O}^{\mathbf{D}}$, i.e., if $\lambda^{\mathrm{D}}_{t\omega} = \lambda^{\mathrm{D}}_{t\omega'}$, then $O^{\mathrm{D}}(t,\omega) = O^{\mathrm{D}}(t,\omega')$.

The non-anticipativity of day-ahead market decisions has to be accounted for in addition to enforcing non-decreasing offering curves. The non-anticipativity establishes that if the realizations of the random variables are identical up to a given stage, then the decisions made by the producer have to be equal up to that stage. In other words, if two scenarios contain identical day-ahead prices, the power blocks of the offering curve associated with these two scenarios have to be the same. The non-anticipativity in the day-ahead market is formulated as follows:

$$P^{\mathrm{D}}_{gt\omega} - P^{\mathrm{D}}_{gt\omega+1} = 0, \ \forall g, \forall t, \forall \omega; \text{ if } A^{\mathrm{D}}(\omega) = 1, \tag{5.5}$$

where $\boldsymbol{A}^{\mathbf{D}}$ is a vector whose element $A^{\mathrm{D}}(\omega) = 1$ if scenarios ω and $\omega + 1$ have identical day-ahead prices, and 0 otherwise. Observe that this formulation requires that scenarios with equal day-ahead prices must be ordered consecutively (as done in Figs. 5.6 and 5.7).

Illustrative Example 5.4 (Non-anticipativity and non-decreasing offering curves in the day-ahead market).

Consider the scenario tree depicted in Fig. 5.8. The non-anticipativity constraints impose that identical day-ahead decisions have to be made in all scenarios with equal day-ahead prices. In Fig. 5.8 it is observed that scenarios 1-3 and 4-5 comprise the same day-ahead prices. Thus, the blocks in the offering curve associated with these scenarios have to be identical. This is enforced by the non-anticipativity constraints (5.5). These constraints are formulated using the previously defined vector $\boldsymbol{A}^{\mathbf{D}}$. For the tree shown in Fig. 5.8, vector $\boldsymbol{A}^{\mathbf{D}}$ is

$$\boldsymbol{A}^{\mathbf{D}} = (1,\ 1,\ 0,\ 1,\ 0)\,. \tag{5.6}$$

Note that element $A^{\mathrm{D}}(\omega)$ is equal to 1 if scenarios ω and $\omega{+}1$ have identical day-ahead prices. For instance, element 1 is equal to 1 because scenarios 1 and 2 have the same day-ahead prices. However, element 3 is equal to 0 because scenarios 3 and 4 have different day-ahead prices. Considering the value of $\boldsymbol{A}^{\mathbf{D}}$, the non-anticipativity constraints in the day-ahead market for the tree in Fig. 5.8 are

$$P^{\mathrm{D}}_{gt1} - P^{\mathrm{D}}_{gt2} = 0,\ \forall g, \forall t \tag{5.7}$$

$$P^{\mathrm{D}}_{gt2} - P^{\mathrm{D}}_{gt3} = 0,\ \forall g, \forall t \tag{5.8}$$

$$P^{\mathrm{D}}_{gt4} - P^{\mathrm{D}}_{gt5} = 0,\ \forall g, \forall t. \tag{5.9}$$

Observe that constraints (5.7)-(5.9) impose $P^{\mathrm{D}}_{gt1} = P^{\mathrm{D}}_{gt2} = P^{\mathrm{D}}_{gt3}$ and $P^{\mathrm{D}}_{gt4} = P^{\mathrm{D}}_{gt5}$.

The fact that the offering curves have to be non-decreasing is established by constraints (5.4). As explained above, matrix $\boldsymbol{O}^{\mathbf{D}}$ sorts the day-ahead prices associated with each period in an increasingly manner for each scenario ω. According to the tree in Fig. 5.8, and considering that the planning horizon covers a single period, matrix $\boldsymbol{O}^{\mathbf{D}}$ contains a single row and is expressed as

$$\boldsymbol{O}^{\mathbf{D}} = (1,\ 1,\ 1,\ 2,\ 2)\,. \tag{5.10}$$

Observe that the first three scenarios have identical day-ahead prices, and therefore they have associated the same value in $\boldsymbol{O}^{\mathbf{D}}$. The same happens for scenarios 4 and 5. Moreover, the order associated with scenarios 1-3 is smaller than that of scenarios 4-5, because the day-ahead price in scenarios 1-3 is lower than in scenarios 4 and 5. Considering the value of $\boldsymbol{O}^{\mathbf{D}}$, the constraints enforcing non-decreasing offering curves in the day-ahead market are

$$P^{\mathrm{D}}_{gt1} - P^{\mathrm{D}}_{gt4} \leq 0, \ \forall g, \forall t \tag{5.11}$$

$$P^{\mathrm{D}}_{gt1} - P^{\mathrm{D}}_{gt5} \leq 0, \ \forall g, \forall t \tag{5.12}$$

$$P^{\mathrm{D}}_{gt2} - P^{\mathrm{D}}_{gt4} \leq 0, \ \forall g, \forall t \tag{5.13}$$

$$P^{\mathrm{D}}_{gt2} - P^{\mathrm{D}}_{gt5} \leq 0, \ \forall g, \forall t \tag{5.14}$$

$$P^{\mathrm{D}}_{gt3} - P^{\mathrm{D}}_{gt4} \leq 0, \ \forall g, \forall t \tag{5.15}$$

$$P^{\mathrm{D}}_{gt3} - P^{\mathrm{D}}_{gt5} \leq 0, \ \forall g, \forall t. \tag{5.16}$$

Given that the non-anticipativity constraints (5.7)-(5.9) make equal those power blocks associated with identical day-ahead prices, it is possible to reduce the number of constraints in (5.11)-(5.16). In fact, from all subsets of scenarios with identical day-ahead prices, it is only necessary to impose constraints (5.4) to one scenario in each subset. That is, the six sets of constraints in (5.11)-(5.16) can be equivalently replaced by

$$P^{\mathrm{D}}_{gt3} - P^{\mathrm{D}}_{gt5} \leq 0, \ \forall g, \forall t. \tag{5.17}$$

□

Taking into account the simplifying considerations in Illustrative Example 5.4 pertaining to the non-anticipativity constraints and those constraints enforcing non-decreasing offering curves, constraints (5.4) can be equivalently recast as

$$\begin{aligned} &P^{\mathrm{D}}_{gt\omega} - P^{\mathrm{D}}_{gt\omega'} \leq 0, \ \forall g, \forall t, \ \forall \omega, \omega'; \\ &\qquad\qquad O^{\mathrm{D}}(t,\omega) + 1 = O^{\mathrm{D}}(t,\omega'), \text{if } A^{\mathrm{D}}(\omega) = A^{\mathrm{D}}(\omega') = 0. \end{aligned} \tag{5.18}$$

Finally, the net revenue attained by the producer as a result of participating in the day-ahead market, $R^{\mathrm{D}}_{t\omega}$, is expressed as

$$R^{\mathrm{D}}_{t\omega} = \sum_{g=1}^{N_{\mathrm{G}}} \lambda^{\mathrm{D}}_{t\omega} P^{\mathrm{D}}_{gt\omega}, \ \forall t, \forall \omega. \tag{5.19}$$

The example below illustrates the formulation of the non-anticipativity and non-decreasing offering curves constraints for the day-ahead market.

5.4.2 Regulation Market

The regulation service is used to balance both scheduled and actual electricity demands throughout the electric energy system. In electricity markets, determining which generators provide the regulation service can be done through an auction where generators offer a regulation band every hour. The market operator selects the regulation offers in an increasing price order until enough regulation power has been allocated to meet the system requirements. Note that the price of the regulation market is unknown to the generators at the time of submitting their offers.

In the regulation market, producers offer a range of power. During the period in which the producer provides the regulation service, the system operator controls automatically the output of the units that provide the service to follow the load within the limits of the range accepted in the regulation market. We consider that a unit with regulation capability can follow the load up or down a quantity equal to half of the total power range committed.

It is relevant to note that not all units can participate in the regulation market. The requirement for a unit to provide regulation service is to be able to follow load within a given regulation range in a specific time. These generators are automatically controlled by the system operator in such a way that a continuous generation-demand balance is achieved. This way, generators automatically increase or decrease their outputs to obtain a continuous balance between generation and demand in the system.

The range of power offered in the regulation market by unit g in period t and scenario ω is denoted as $P^{\mathrm{R}}_{gt\omega}$. It is assumed that this unit can increase or decrease its output at most $\frac{1}{2}P^{\mathrm{R}}_{gt\omega}$, being its offered power range limited by $P^{\mathrm{R,max}}_{g}$.

If a unit provides regulation service during hour t, its output is automatically controlled by the system operator. It is thus assumed that the unit cannot ramp up or down its production during this hour. Therefore, it is necessary to impose that if a unit provides regulation in a given hour it cannot ramp up or down during that hour. To this end, we use the binary variable $w_{gt\omega}$, which is equal to 1 if unit g provides regulation in period t and scenario ω, and 0 otherwise. Thus, the power range offered in the regulation market is bounded as

$$0 \leq P^{\mathrm{R}}_{gt\omega} \leq P^{\mathrm{R,max}}_{g} w_{gt\omega}, \ \forall g, \forall t, \forall \omega \tag{5.20}$$

$$w_{gt\omega} \in \{0, 1\}, \ \forall g, \forall t, \forall \omega. \tag{5.21}$$

Analogously to the day-ahead market, it is necessary to impose non-anticipativity in the regulation market decisions. The non-anticipativity forces regulation decisions to be equal in all scenarios that are identical up to the regulation decision time. This is stated as

$$P^{\mathrm{R}}_{gt\omega} - P^{\mathrm{R}}_{gt\omega+1} = 0, \ \forall g, \forall t, \ \forall \omega; \ \text{if}\, A^{\mathrm{R}}(\omega) = 1, \tag{5.22}$$

where $\boldsymbol{A}^{\mathbf{R}}$ is a vector whose element $A^{\mathrm{R}}(\omega)$ is equal to 1 if scenarios ω and $\omega + 1$ are identical up to the second stage, and 0 otherwise.

The formulation of the non-anticipativity constraints in the regulation market are illustrated by means of the following example.

Illustrative Example 5.5 (Non-anticipativity constraints in the regulation market).

Consider the scenario tree shown in Fig. 5.8. The non-anticipativity constraints state that decisions in the regulation market have to be the same for all scenarios that are identical up to the point where regulation decisions are made.

Scenarios 1-3 and 4-5 are identical up to the second stage. Thus, the decisions in the regulation market have to be the same for every scenario in these two sets. This is enforced by the non-anticipativity constraints (5.22). These constraints are implemented with the help of vector $\boldsymbol{A}^{\mathbf{R}}$. According to Fig. 5.8, the vector $\boldsymbol{A}^{\mathbf{R}}$ is

$$\boldsymbol{A}^{\mathbf{R}} = (1,\, 1,\, 0,\, 1,\, 0)\,. \tag{5.23}$$

Note that element $\boldsymbol{A}^{\mathbf{R}}(\omega)$ is equal to 1 if scenarios ω and $\omega+1$ are identical up to stage 2. Observe that vector $\boldsymbol{A}^{\mathbf{R}}$ is equal to vector $\boldsymbol{A}^{\mathbf{D}}$, which is used to impose the non-anticipativity constraints in the day-ahead market. This is a consequence of the fact that day-ahead and regulation decisions are both made with perfect information on day-ahead prices and under uncertainty on regulation and adjustment prices. This results from considering that the producer submits an offering curve in the day-ahead market (a block for each scenario), while the participation in the regulation market is associated with a single power quantity.

According to the value of $\boldsymbol{A}^{\mathbf{R}}$, the non-anticipativity constraints for the regulation market are

$$P^{\mathrm{R}}_{gt1} - P^{\mathrm{R}}_{gt2} = 0,\ \forall g, \forall t \tag{5.24}$$

$$P^{\mathrm{R}}_{gt2} - P^{\mathrm{R}}_{gt3} = 0,\ \forall g, \forall t \tag{5.25}$$

$$P^{\mathrm{R}}_{gt4} - P^{\mathrm{R}}_{gt5} = 0,\ \forall g, \forall t. \tag{5.26}$$

Observe that the constraints above impose $P^{\mathrm{R}}_{gt1} = P^{\mathrm{R}}_{gt2} = P^{\mathrm{R}}_{gt3}$ and $P^{\mathrm{R}}_{gt4} = P^{\mathrm{R}}_{gt5}$. □

Finally, note that the net revenue attained by the producer by participating in the regulation market, $R^{\mathrm{R}}_{t\omega}$, is given by

$$R^{\mathrm{R}}_{t\omega} = \sum_{g=1}^{N_{\mathrm{G}}} \lambda^{\mathrm{R}}_{t\omega} P^{\mathrm{R}}_{gt\omega},\ \forall t, \forall \omega. \tag{5.27}$$

5.4.3 Adjustment Market

The adjustment market is a trading floor with a planning horizon similar to that of the day-ahead and regulation markets, where the agents participate in order to make final adjustments to the energy previously traded for each period of the next day. The production offers and consumption bids are submitted to the market operator once the regulation market is cleared. Thus, the market agents have complete information on the quantities committed in the day-ahead and regulation markets when bidding in the adjustment market. Note that, typically, the adjustment market is the most important trading floor available in a pool after the day-ahead market has been cleared.

For simplicity, we assume that the producer only participates in the adjustment market by selling power, being $P^{\mathrm{A}}_{gt\omega}$ the power sold by unit g in the adjustment market during period t and scenario ω.

In line with the decisions made in the day-ahead and regulation markets, it is also necessary to impose the non-anticipativity on the adjustment decisions. Non-anticipativity conditions enforce that adjustment market decisions should be equal in all scenarios that are identical up to the adjustment market decision stage. This is formulated as

$$P^{\mathrm{A}}_{gt\omega} - P^{\mathrm{A}}_{gt\omega+1} = 0, \ \forall g, \forall t, \ \forall \omega; \text{ if } A^{\mathrm{A}}(\omega) = 1, \tag{5.28}$$

where $\boldsymbol{A}^{\mathbf{A}}$ is a vector whose element $A^{\mathrm{A}}(\omega)$ is equal to 1 if scenarios ω and $\omega + 1$ are identical up to the third stage, and 0 otherwise.

The following example illustrates the formulation of the non-anticipativity constraints in the adjustment market.

Illustrative Example 5.6 (Non-anticipativity constraints in the adjustment market).

Consider the scenario tree shown in Fig. 5.8. The non-anticipativity constraints state that decisions in the adjustment market have to be equal for all scenarios that are identical up to the stage where adjustment decisions are made.

We observe that scenarios 1-2 and 4-5 are identical up to the third stage where adjustment decisions are made. Thus, the decisions in the adjustment market have to be equal for scenarios 1 and 2, and also for scenarios 4 and 5. This is enforced by the non-anticipativity constraints (5.28), through vector $\boldsymbol{A}^{\mathbf{A}}$. According to Fig. 5.8, vector $\boldsymbol{A}^{\mathbf{A}}$ is

$$\boldsymbol{A}^{\mathbf{A}} = (1, \, 0, \, 0, \, 1, \, 0) \,. \tag{5.29}$$

Note that element $A^{\mathrm{A}}(\omega)$ is equal to 1 if scenarios ω and $\omega+1$ are identical up to stage 3. For instance, element 1 is equal to 1 because scenarios 1 and 2 have the same day-ahead and regulation prices. However, element 2 is equal to 0 because scenarios 2 and 3 have different regulation prices.

According to the value of $\boldsymbol{A}^{\mathbf{A}}$, the non-anticipativity constraints are

$$P^{\mathrm{A}}_{gt1} - P^{\mathrm{A}}_{gt2} = 0, \ \forall g, \forall t \tag{5.30}$$

$$P^{\mathrm{A}}_{gt4} - P^{\mathrm{A}}_{gt5} = 0, \ \forall g, \forall t. \tag{5.31}$$

Observe that constraints (5.30) and (5.31) impose $P^{\mathrm{A}}_{gt1} = P^{\mathrm{A}}_{gt2}$ and $P^{\mathrm{A}}_{gt4} = P^{\mathrm{A}}_{gt5}$. □

The volume of energy traded in the adjustment market is generally significantly smaller than that in the day-ahead market. Therefore, assuming the producer as a price-taker agent in this market is not realistic. As previously explained in Section 5.3, we assume that the final price in the adjustment market changes linearly with the power traded by the producer. The power traded by the producer is denoted by $P^{\mathrm{A}}_{t\omega}$, and

$$P^{\mathrm{A}}_{t\omega} = \sum_{g=1}^{N_{\mathrm{G}}} P^{\mathrm{A}}_{gt\omega}, \ \forall t, \forall \omega. \tag{5.32}$$

Therefore, the adjustment price in each period and scenario, $\lambda^{\mathrm{A}}_{t\omega}$, is given by

$$\lambda^{\mathrm{A}}_{t\omega} = \lambda^{\mathrm{A0}}_{t\omega} + \gamma^{\mathrm{A}}_{t\omega} P^{\mathrm{A}}_{t\omega}, \ \forall t, \forall \omega. \tag{5.33}$$

Fig. 5.9 represents the adjustment price as a function of the power traded by the producer for a given hour in the adjustment market. The parameter $P^{\mathrm{A,max}}$ refers to the maximum amount of power that can be sold by the producer. Observe that the slope of the adjustment price is negative. That is, the adjustment price decreases as the producer sells more and more energy.

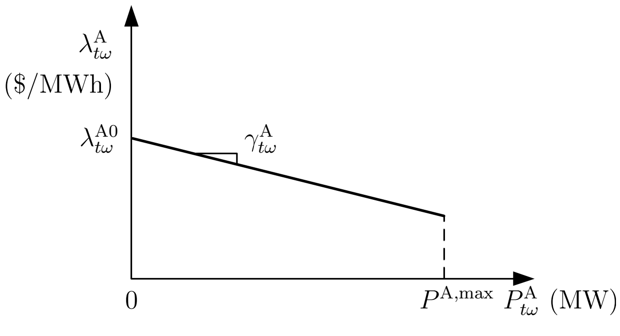


Fig. 5.9 Adjustment market price

Since the adjustment prices depend linearly on the power traded by the producer, the final net revenue obtained from selling in the adjustment market, $R^{\mathrm{A}}_{t\omega}$, is a quadratic function of the traded power, which is formulated as

$$R^{\mathrm{A}}_{t\omega} = \lambda^{\mathrm{A}}_{t\omega} P^{\mathrm{A}}_{t\omega} = \lambda^{\mathrm{A0}}_{t\omega} P^{\mathrm{A}}_{t\omega} + \gamma^{\mathrm{A}}_{t\omega} \left(P^{\mathrm{A}}_{t\omega}\right)^2, \ \forall t, \forall \omega. \tag{5.34}$$

Fig. 5.10 illustrates an example of the quadratic revenue of the producer in the adjustment market.

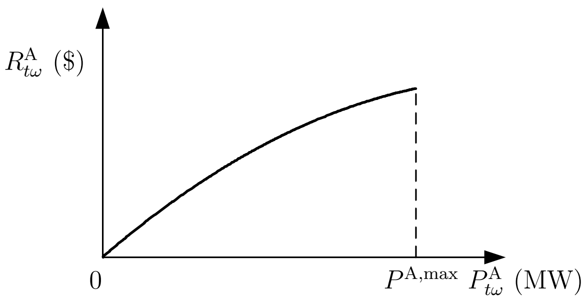


Fig. 5.10 Quadratic formulation of the producer revenue in the adjustment market

The calculation of the revenue in the adjustment market is illustrated by means of the illustrative example below.

Illustrative Example 5.7 (Revenue in the adjustment market).
This example is based on the characterization of the adjustment prices provided in Illustrative Example 5.2. In that example we consider a producer owning a single thermal unit of 100 MW. The horizon comprises one hour, and the price in the adjustment market is given by

$$\lambda^{\mathrm{A}} = 20 - 0.05P^{\mathrm{A}}. \tag{5.35}$$

Therefore, the revenue attained by the producer in the adjustment market can be expressed as

$$R^{\mathrm{A}} = 20P^{\mathrm{A}} - 0.05(P^{\mathrm{A}})^2. \tag{5.36}$$

Fig. 5.11 represents the quadratic revenue R^{A} as a function of the energy sold during one hour in the adjustment market. If the producer sells 50 MW, it achieves a revenue equal to $20 \times 50 - 0.05 \times (50)^2 =$ \$875. Moreover, if the producer sells its complete power capacity, 100 MW, it attains a maximum revenue equal to \$1500.

□

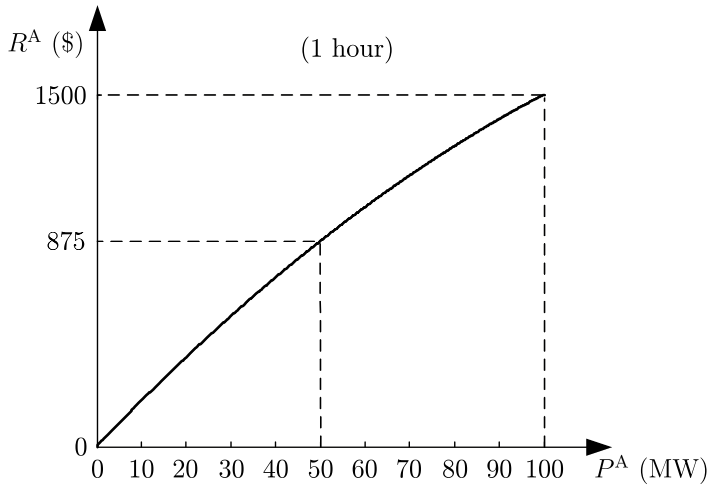


Fig. 5.11 Illustrative Example 5.7: quadratic revenue in the adjustment market

5.5 Producer Model

5.5.1 Unit Constraints

In this section we describe the technical constraints of the production units owned by the producer. We consider that the producer owns only thermal units. However, note that the proposed approach can be straightforwardly extended to include hydro units.

Since we model a 24-hour horizon, start-up and shut-down decisions are not considered. These decisions are typically made within a weekly horizon. For this reason, the on/off status of the thermal units are considered as input data in this model.

The unit model is mathematically expressed as

$$P^{\mathrm{D}}_{gt\omega} + \frac{1}{2}P^{\mathrm{R}}_{gt\omega} + P^{\mathrm{A}}_{gt\omega} \leq P^{\mathrm{G,max}}_{g}, \ \forall g, \forall t, \forall \omega \tag{5.37}$$

$$P^{\mathrm{D}}_{gt\omega} - \frac{1}{2}P^{\mathrm{R}}_{gt\omega} + P^{\mathrm{A}}_{gt\omega} \geq P^{\mathrm{G,min}}_{g}, \ \forall g, \forall t, \forall \omega \tag{5.38}$$

$$P^{\mathrm{G}}_{gt\omega} - P^{\mathrm{G}}_{gt-1\omega} \leq \left(1 - w_{gt\omega}\right) R^{\mathrm{U}}_{g}, \ \forall g, \forall t, \forall \omega \tag{5.39}$$

$$P^{\mathrm{G}}_{gt-1\omega} - P^{\mathrm{G}}_{gt\omega} \leq \left(1 - w_{gt\omega}\right) R^{\mathrm{D}}_{g}, \ \forall g, \forall t, \forall \omega \tag{5.40}$$

$$P^{\mathrm{D}}_{gt\omega} + P^{\mathrm{A}}_{gt\omega} = \frac{1}{2}\left(P^{\mathrm{G}}_{gt-1\omega} + P^{\mathrm{G}}_{gt\omega}\right), \ \forall g, \forall t, \forall \omega. \tag{5.41}$$

Constraints (5.37) and (5.38) state that the total power committed in the day-ahead, regulation, and adjustment markets must lie between the minimum power output and the capacity of each unit.

Constraints (5.39) and (5.40) impose ramp-up and ramp-down limits to the power output of each unit. The variable $P^{\mathrm{G}}_{gt\omega}$ denotes the power output of unit g and scenario ω at the end of period t. Thus, if unit g provides regulation service in period t, it cannot ramp up or down during that period. This limitation is modeled by the binary variable $w_{gt\omega}$ and constraints (5.39) and (5.40). If $w_{gt\omega} = 1$, unit g is contributing to the regulation service in period t and scenario ω and it cannot ramp up or down during that period, i.e., $P^{\mathrm{G}}_{gt\omega} = P^{\mathrm{G}}_{gt-1\omega}$. On the other hand, if $w_{gt\omega} = 0$, the power output of unit g is only bounded by power and ramping limits.

Constraints (5.41) express the power sold in the day-ahead and adjustment markets in each period as the mean value of the power supplied at the beginning and at the end of the period. Note that the range of regulation committed by the producer is not included in the energy balance since the participation in the regulation market does not imply additional energy supply, but a fluctuation over the scheduled power, $P^{\mathrm{D}}_{gt\omega} + P^{\mathrm{A}}_{gt\omega}$. Recall that hourly periods are considered and thus power and energy are equivalent.

Finally, we consider a linear cost function to represent the production cost of the thermal units. Since the start-ups and shut-downs are not considered, the fixed cost is not modeled. The mathematical formulation of the production cost, $C^{\mathrm{G}}_{t\omega}$, is

$$C^{\mathrm{G}}_{t\omega} = \sum_{g=1}^{N_{\mathrm{G}}} C^{\mathrm{P}}_{g} (P^{\mathrm{D}}_{gt\omega} + P^{\mathrm{A}}_{gt\omega}), \ \forall t, \forall \omega, \tag{5.42}$$

where C^{P}_{g} is the linear production cost of unit g. In case that the production cost of a thermal unit is given by a convex, but non-linear function, expression (5.42) should be and can be easily converted into a piecewise linear one.

5.5.2 Expected Profit

The net profit of the producer is equal to the net revenues obtained from trading in the day-ahead, regulation, and adjustment markets, minus the production cost. Mathematically, the profit associated with scenario ω is equal to

$$\sum_{t=1}^{N_{\mathrm{T}}} \left(R_{t\omega}^{\mathrm{D}} + R_{t\omega}^{\mathrm{R}} + R_{t\omega}^{\mathrm{A}} - C_{t\omega}^{\mathrm{G}} \right) =$$
$$\sum_{t=1}^{N_{\mathrm{T}}} \left(\sum_{g=1}^{N_{\mathrm{G}}} \lambda_{t\omega}^{\mathrm{D}} P_{gt\omega}^{\mathrm{D}} + \sum_{g=1}^{N_{\mathrm{G}}} \lambda_{t\omega}^{\mathrm{R}} P_{gt\omega}^{\mathrm{R}} + \lambda_{t\omega}^{\mathrm{A0}} P_{t\omega}^{\mathrm{A}} + \gamma_{t\omega}^{\mathrm{A}} \left(P_{t\omega}^{\mathrm{A}} \right)^2 - \right.$$
$$\left. \sum_{g=1}^{N_{\mathrm{G}}} C_g^{\mathrm{P}} (P_{gt\omega}^{\mathrm{D}} + P_{gt\omega}^{\mathrm{A}}) \right), \ \forall \omega. \tag{5.43}$$

The expected profit is computed as the sum over all scenarios of the profit in each scenario (5.43) multiplied by its probability of occurrence. The expected profit is thus calculated as

$$\sum_{\omega=1}^{N_{\Omega}} \pi_{\omega} \sum_{t=1}^{N_{\mathrm{T}}} \left(R_{t\omega}^{\mathrm{D}} + R_{t\omega}^{\mathrm{R}} + R_{t\omega}^{\mathrm{A}} - C_{t\omega}^{\mathrm{G}} \right). \tag{5.44}$$

5.5.3 Risk Modeling

We use the Conditional Value-at-Risk (CVaR) to account for the risk of profit variability faced by the producer.

The CVaR represents approximately the expected profit of the $(1-\alpha)\times 100$ scenarios yielding the lowest profits.

Using expression (5.43) for the profit, the CVaR for a confidence level α is given by

$$\alpha - \mathrm{CVaR} = \mathrm{Maximize}_{\zeta, \eta_{\omega}} \qquad \zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_{\Omega}} \pi_{\omega} \, \eta_{\omega} \tag{5.45}$$

subject to

$$\zeta - \sum_{t=1}^{N_{\mathrm{T}}} \left(R_{t\omega}^{\mathrm{D}} + R_{t\omega}^{\mathrm{R}} + R_{t\omega}^{\mathrm{A}} - C_{t\omega}^{\mathrm{G}} \right) \le \eta_{\omega}, \ \forall \omega \tag{5.46}$$

$$\eta_{\omega} \ge 0, \ \forall \omega. \tag{5.47}$$

As explained in Section 4.3.5 of Chapter 4, the CVaR can be easily included in the formulation of the problem.

5.6 Formulation

The formulation of the pool trading problem for a power producer is provided below,

$$\begin{aligned}&\text{Maximize}_{P^{\text{A}}_{t\omega}, P^{\text{A}}_{gt\omega}, P^{\text{D}}_{gt\omega}, P^{\text{G}}_{gt\omega}, P^{\text{R}}_{gt\omega}, w_{gt\omega}, \zeta, \eta_\omega}\\
&\sum_{\omega=1}^{N_\Omega} \pi_\omega \sum_{t=1}^{N_{\text{T}}} \left(\sum_{g=1}^{N_{\text{G}}} \lambda^{\text{D}}_{t\omega} P^{\text{D}}_{gt\omega} + \sum_{g=1}^{N_{\text{G}}} \lambda^{\text{R}}_{t\omega} P^{\text{R}}_{gt\omega} + \lambda^{\text{A0}}_{t\omega} P^{\text{A}}_{t\omega} + \gamma^{\text{A}}_{t\omega} \left(P^{\text{A}}_{t\omega}\right)^2 \right.\\
&\left. - \sum_{g=1}^{N_{\text{G}}} C^{\text{P}}_g (P^{\text{D}}_{gt\omega} + P^{\text{A}}_{gt\omega}) \right) + \beta \left(\zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\Omega} \pi_\omega \eta_\omega \right) \qquad (5.48)\end{aligned}$$

subject to

$$\begin{aligned}&P^{\text{D}}_{gt\omega} - P^{\text{D}}_{gt\omega'} \leq 0,\ \forall g, \forall t,\\
&\forall \omega;\ O^{\text{D}}(t,\omega) + 1 = O^{\text{D}}(t,\omega'),\ \text{if}\, A^{\text{D}}(\omega) = A^{\text{D}}(\omega') = 0 \qquad (5.49)\end{aligned}$$

$$P^{\text{D}}_{gt\omega} - P^{\text{D}}_{gt\omega+1} = 0,\ \forall g, \forall t, \forall \omega;\ \text{if}\, A^{\text{D}}(\omega) = 1 \qquad (5.50)$$

$$0 \leq P^{\text{R}}_{gt\omega} \leq P^{\text{R,max}}_g w_{gt\omega},\ \forall g, \forall t, \forall \omega \qquad (5.51)$$

$$P^{\text{R}}_{gt\omega} - P^{\text{R}}_{gt\omega+1} = 0,\ \forall g, \forall t,\ \forall \omega;\ \text{if}\, A^{\text{R}}(\omega) = 1 \qquad (5.52)$$

$$P^{\text{A}}_{gt\omega} - P^{\text{A}}_{gt\omega+1} = 0,\ \forall g, \forall t,\ \forall \omega;\ \text{if}\, A^{\text{A}}(\omega) = 1 \qquad (5.53)$$

$$\sum_{g=1}^{N_{\text{G}}} P^{\text{A}}_{gt\omega} = P^{\text{A}}_{t\omega},\ \forall t, \forall \omega \qquad (5.54)$$

$$P^{\text{D}}_{gt\omega} + \frac{1}{2} P^{\text{R}}_{gt\omega} + P^{\text{A}}_{gt\omega} \leq P^{\text{G,max}}_g,\ \forall g, \forall t, \forall \omega \qquad (5.55)$$

$$P^{\text{D}}_{gt\omega} - \frac{1}{2} P^{\text{R}}_{gt\omega} + P^{\text{A}}_{gt\omega} \geq P^{\text{G,min}}_g,\ \forall g, \forall t, \forall \omega \qquad (5.56)$$

$$P^{\text{G}}_{gt\omega} - P^{\text{G}}_{gt-1\omega} \leq (1 - w_{gt\omega})\, R^{\text{U}}_g,\ \forall g, \forall t, \forall \omega \qquad (5.57)$$

$$P^{\text{G}}_{gt-1\omega} - P^{\text{G}}_{gt\omega} \leq (1 - w_{gt\omega})\, R^{\text{D}}_g,\ \forall g, \forall t, \forall \omega \qquad (5.58)$$

$$P^{\text{D}}_{gt\omega} + P^{\text{A}}_{gt\omega} = \frac{1}{2} \left(P^{\text{G}}_{gt-1\omega} + P^{\text{G}}_{gt\omega}\right),\ \forall g, \forall t, \forall \omega \qquad (5.59)$$

$$\begin{aligned}&\zeta - \sum_{t=1}^{N_{\text{T}}} \left(\sum_{g=1}^{N_{\text{G}}} \lambda^{\text{D}}_{t\omega} P^{\text{D}}_{gt\omega} + \sum_{g=1}^{N_{\text{G}}} \lambda^{\text{R}}_{t\omega} P^{\text{R}}_{gt\omega} + \lambda^{\text{A0}}_{t\omega} P^{\text{A}}_{t\omega} + \gamma^{\text{A}}_{t\omega} \left(P^{\text{A}}_{t\omega}\right)^2 - \right.\\
&\left. \sum_{g=1}^{N_{\text{G}}} C^{\text{P}}_g (P^{\text{D}}_{gt\omega} + P^{\text{A}}_{gt\omega}) \right) \leq \eta_\omega,\ \forall \omega \qquad (5.60)\end{aligned}$$

$$P^{\text{D}}_{gt\omega},\ P^{\text{R}}_{gt\omega},\ P^{\text{A}}_{gt\omega},\ P^{\text{G}}_{gt\omega} \geq 0,\ \forall g, \forall t, \forall \omega \qquad (5.61)$$

$$\eta_\omega \geq 0,\ \forall \omega \qquad (5.62)$$

$$w_{gt\omega} \in \{0, 1\},\ \forall g, \forall t, \forall \omega. \qquad (5.63)$$

The objective function (5.48) comprises two terms: i) the expected profit and ii) the CVaR multiplied by the weighting parameter β. The parameter β models the tradeoff between the expected profit and the profit variability (measured in terms of the CVaR).

Constraints (5.49) impose that the offering curves must be non-decreasing in the day-ahead market. Constraints (5.50) state the non-anticipativity of the decisions in the day-ahead market. Constraints (5.51) bound the maximum power range that can be offered in the regulation market. Constraints (5.52) and (5.53) formulate the non-anticipativity conditions for the decisions made in the regulation and adjustment markets, respectively. Constraints (5.54) are used to formulate the quadratic revenues in the adjustment market. Constraints (5.55)-(5.59) represent the technical limits of the generating units. Constraints (5.60) are used to compute the CVaR. Finally, (5.61)-(5.63) constitute variable declarations.

5.7 Producer Pool Example

In this section we present some results obtained from a simple example. Its purpose is to clarify the problem analyzed in the previous sections. We consider a power producer owning two thermal units, whose data are provided in Table 5.2.

Table 5.2 Producer pool example: thermal unit data

Unit #	$P_g^{\mathrm{G,min}}$ (MW)	$P_g^{\mathrm{G,max}}$ (MW)	R_g^{U} (MW/h)	R_g^{D} (MW/h)	C_g^{P} (\$/MWh)	$P_{g,0}^{\mathrm{G}}$ (MW)	$P_g^{\mathrm{R,max}}$ (MW)
1	10	100	25	25	12	50	15
2	10	50	15	15	16	25	20

Considering a 3-hour market horizon, we assume that the producer participates in the day-ahead, regulation, and adjustment markets. The price of the power traded in each market and period is characterized using a set of 12 scenarios. The data corresponding to these prices are provided in Table 5.3. The prices in the day-ahead and regulation markets are considered to be independent of the producer actions, while adjustment prices depend on the quantity of power traded by the producer in this market.

According to the data in Table 5.3, the scenario tree in this example has the structure represented in Fig. 5.12. For example, scenarios 1-4 have the same day-ahead prices, while scenarios 1-2 have similar regulation prices. Adjustment prices are different for all scenarios.

Table 5.3 Producer pool example: day-ahead, regulation, and adjustment prices

Scenario #	Day ahead $\lambda^{\mathrm{D}}_{t\omega}$ (\$/MWh) Period #			Regulation $\lambda^{\mathrm{R}}_{t\omega}$ (\$/MWh) Period #			Adjustment $\lambda^{\mathrm{A0}}_{t\omega}$ (\$/MWh) Period #			Adjustment $\gamma^{\mathrm{A}}_{t\omega}$ (\$/(MWh)^2) Period #		
	1	2	3	1	2	3	1	2	3	1	2	3
1	11.0	16.5	12.1	5.5	8.5	6.5	17.0	28.0	15.0	-0.01	-0.03	-0.02
2	11.0	16.5	12.1	5.5	8.5	6.5	10.0	17.0	13.0	-0.01	-0.03	-0.02
3	11.0	16.5	12.1	5.0	7.0	5.5	12.0	17.0	15.0	-0.01	-0.03	-0.02
4	11.0	16.5	12.1	5.0	7.0	5.5	9.0	14.0	11.0	-0.01	-0.03	-0.02
5	13.2	18.7	16.5	6.5	9.5	8.0	17.0	22.0	19.0	-0.01	-0.03	-0.02
6	13.2	18.7	16.5	6.5	9.5	8.0	14.0	19.0	15.0	-0.01	-0.03	-0.02
7	13.2	18.7	16.5	6.0	8.5	7.5	16.0	25.0	21.0	-0.01	-0.03	-0.02
8	13.2	18.7	16.5	6.0	8.5	7.5	13.0	17.0	14.0	-0.01	-0.03	-0.02
9	15.4	19.8	17.6	7.0	10.5	10.5	17.0	28.0	24.0	-0.01	-0.03	-0.02
1	15.4	19.8	17.6	7.0	10.5	10.5	14.0	23.0	20.0	-0.01	-0.03	-0.02
11	15.4	19.8	17.6	6.5	9.0	9.5	16.0	23.0	21.0	-0.01	-0.03	-0.02
12	15.4	19.8	17.6	6.5	9.0	9.5	13.0	19.0	18.0	-0.01	-0.03	-0.02

The structure of the tree provides the values of the vectors $\boldsymbol{A}^{\mathbf{D}}$, $\boldsymbol{A}^{\mathbf{R}}$, and $\boldsymbol{A}^{\mathbf{A}}$, which are used to establish the non-anticipativity of the decisions in the day-ahead, regulation, and adjustment markets, respectively. As previously stated, vectors $\boldsymbol{A}^{\mathbf{D}}$ and $\boldsymbol{A}^{\mathbf{R}}$ are identical, and their values for the tree in Fig. 5.12 are

$$\boldsymbol{A}^{\mathbf{D}} = \boldsymbol{A}^{\mathbf{R}} = (1,\ 1,\ 1,\ 0,\ 1,\ 1,\ 1,\ 0,\ 1,\ 1,\ 1,\ 0)\,. \tag{5.64}$$

The value of $\boldsymbol{A}^{\mathbf{A}}$ is

$$\boldsymbol{A}^{\mathbf{A}} = (1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 1,\ 0)\,. \tag{5.65}$$

Considering Table 5.3 and Fig. 5.12, the matrix $\boldsymbol{O}^{\mathbf{D}}$ used to sort the day-ahead prices for every period t in an increasing manner for each scenario ω and to enforce non-decreasing offering curves in the day-ahead market is

$$\boldsymbol{O}^{\mathbf{D}} = \begin{pmatrix} 1\,1\,1\,1\,2\,2\,2\,2\,3\,3\,3\,3 \\ 1\,1\,1\,1\,2\,2\,2\,2\,3\,3\,3\,3 \\ 1\,1\,1\,1\,2\,2\,2\,2\,3\,3\,3\,3 \end{pmatrix}. \tag{5.66}$$

Problem (5.48)-(5.63), which contains 637 constraints, 410 real variables, and 72 binary variables is solved for different values of the weighting parameter β and for a confidence level α equal to 0.95. Table 5.4 provides the expected profit, profit standard deviation, and CVaR for different values of

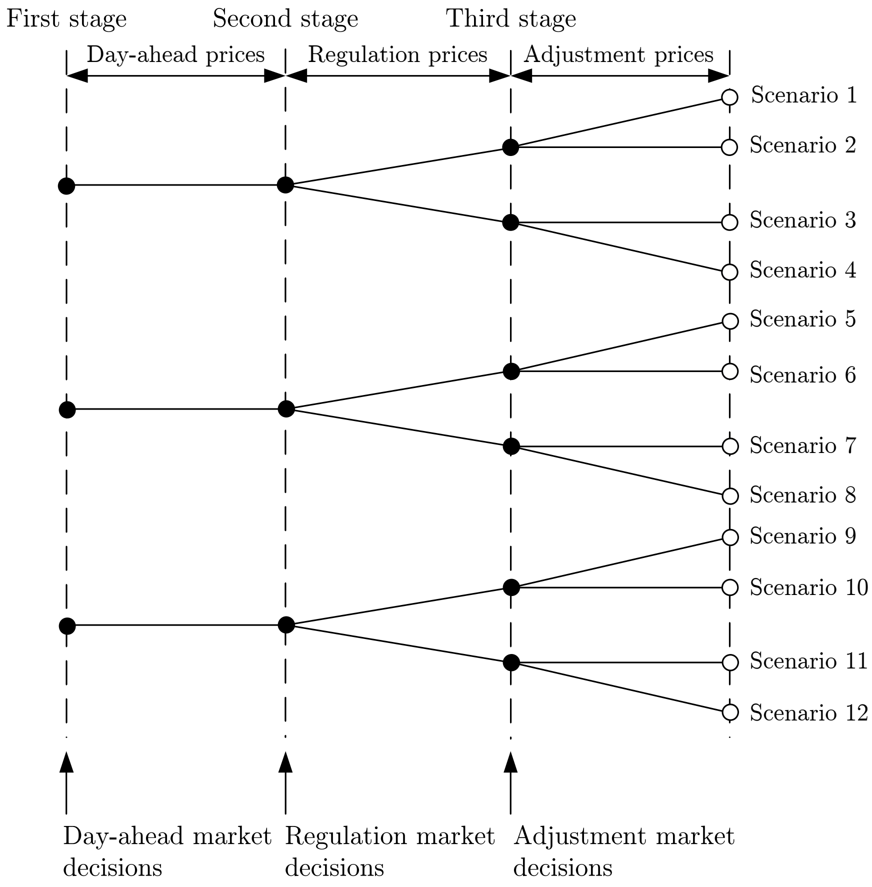


Fig. 5.12 Producer pool example: scenario tree

β. As expected, the optimal solution attained for $\beta = 0$ achieves the highest expected profit and the highest risk, measured in terms of both the standard deviation and the CVaR of the profit. The expected profit varies from \$1415 for $\beta = 0$ to \$1344.9 for $\beta = 1$, respectively. As can be observed, a reduction of 4.95% in the expected profit produces a reduction of 9.86% in the profit standard deviation and a 710.22% increase in the CVaR. Note that relatively small values of the profit standard deviation and large values of the CVaR indicate risk-averse solutions.

Fig. 5.13 represents the efficient frontier, i.e., the expected profit versus the CVaR for different values of β. Note that low-risk solutions, i.e., solutions with high CVaR, correspond to low expected profits.

Table 5.4 Producer pool example: results

β	Expected profit ($)	Profit standard deviation ($)	CVaR ($)
0.00	1415.0	760.8	54.8
0.10	1407.4	724.1	218.6
0.25	1389.9	698.4	327.5
0.30	1383.0	692.7	352.6
1.00	1344.9	685.8	444.0

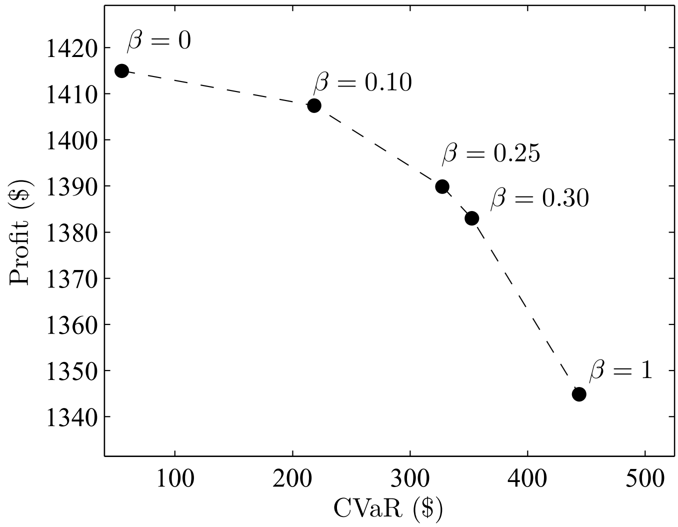


Fig. 5.13 Producer pool example: efficient frontier

The aggregated expected value of the power traded during the considered planning horizon in the day-ahead, regulation, and adjustment markets by each unit and for different values of β is provided in Table 5.5. Observe that the total quantity of power traded in the day-ahead market increases as the parameter β grows. This increase produces a decrease in the power traded in the adjustment market, as this market is more volatile than the day-ahead market. Therefore, the producer prefers to trade in the day-ahead market in order to attain a more stable profit distribution at the cost of reducing its average value. This option prevents the producer from obtaining high profits in some periods in the adjustment market.

The optimal offering curves derived by the producer in cases $\beta = \{0, 1\}$ are provided in Fig. 5.14. First, note that these curves are non-decreasing for all units, periods, and values of β. This behavior is enforced by constraints (5.49). Moreover, the power offered by unit 1 is always greater than that

Table 5.5 Producer pool example: aggregated expected value of power sold (MW)

β	Unit 1			Unit 2		
	Day ahead	Regulation	Adjustment	Day ahead	Regulation	Adjustment
0.00	120.5	25.0	94.3	38.2	23.3	41.0
0.10	145.5	25.0	71.3	39.8	23.3	39.4
0.25	174.7	25.0	45.4	44.2	23.3	36.5
0.30	182.2	25.0	38.7	46.1	23.3	34.9
1.00	212.4	20.0	17.5	54.1	23.3	25.6

offered by unit 2, a consequence of the higher production cost of unit 2. As an example, the interpretation of the offering curve for unit 1, period 1 and $\beta = 0$ is the following: if the selling price is smaller than \$11/MWh (smallest day-ahead price scenario for period 1), the producer prefers not to sell energy. If the price is equal to \$11/MWh, the producer is willing to sell 9.1 MW to the day-ahead market. This result corresponds to the first point of the offering curve. Observe that the second point has a price equal to \$13.2/MWh and it has associated the same power that the first point, i.e., 9.1 MW. This fact indicates that for prices between 11 and \$13.2/MWh the producer is willing to sell the same quantity of power, 9.1 MW. The third point of the offering curve corresponds to a price equal to \$15.4/MWh. For this price, the producer is willing to sell 62.5 MW. For prices between 13.2 and \$15.4/MWh, the power that the producer is willing to sell is straightforwardly obtained from the linear function that links points 2 and 3 of the offering curve. Since the third point of the offering curve is the last one, it is assumed that the producer is willing to sell the same quantity, 62.5 MW, for all prices greater than \$15.4/MWh. Finally, note that in the case that step-wise offering curves constitute a market requirement, the linear offering curves shown in Fig. 5.14 should be approximated by stepwise functions.

Observe that the effect of increasing parameter β is a shift to the right of the offering curves. That is, if risk is accounted for, the producer is willing to offer a greater quantity of power for the same price. This is consistent with the results provided in Table 5.5.

Tables 5.6-5.7 provide the power sold in the day-ahead, regulation, and adjustment markets, together with the power generated by unit 1 in cases $\beta = 0$ and $\beta = 1$. Recall that the generated power refers to the power output at the end of period t.

Observe that the non-anticipativity of day-ahead, regulation, and adjustment decisions is satisfied for both units and cases $\beta = 0$ and $\beta = 1$. Day-ahead and regulation decisions are equal in those scenarios that are identical up to stage 2. For instance, these decisions are equal in scenarios 1-4, 5-8, and 9-12. Likewise, adjustment decisions are equal in scenarios identical up to stage 3. In this way, adjustment decisions are identical in scenarios 1-2, 3-4, 5-6, 7-8, 9-10, and 11-12.

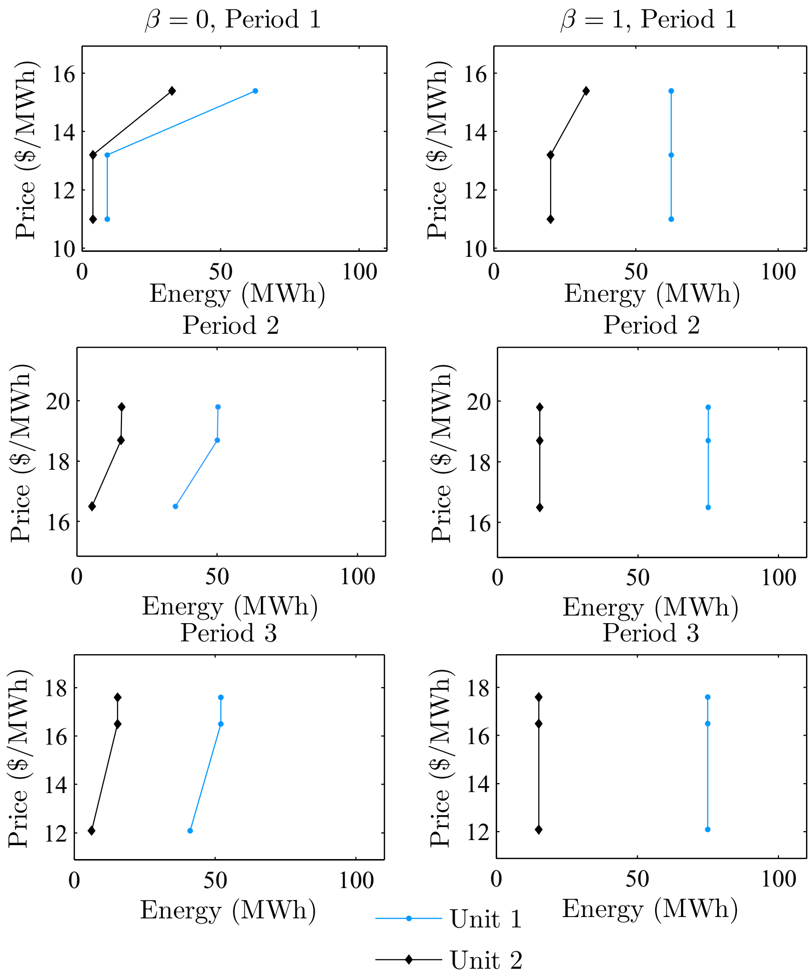


Fig. 5.14 Producer pool example: optimal offering curves in the day-ahead market

It is relevant to note that the relationship given by expressions (5.59) between the power sold in the day-ahead and adjustment markets and the power generation is met. For instance, for unit 1, scenario 1, and $\beta = 0$ equations (5.59) are satisfied as follows:

$$P^{\mathrm{D}}_{1t1} + P^{\mathrm{A}}_{1t1} = \frac{1}{2}\left(P^{\mathrm{G}}_{1t-1,1} + P^{\mathrm{G}}_{1t1}\right), \ \forall t \tag{5.67}$$

$$t = 1: \quad 9.11 + 53.39 = \frac{1}{2}(50 + 75) = 62.75 \tag{5.68}$$

$$t = 2: \quad 35.19 + 39.81 = \frac{1}{2}(75 + 75) = 75 \tag{5.69}$$

$$t = 3: \quad 41.18 + 33.82 = \frac{1}{2}(75 + 75) = 75. \tag{5.70}$$

The ramp-up and ramp-down limits are binding in the first period for both units in cases $\beta = 0$ and $\beta = 1$. Observe that unit 1 ramps up from 50 to 75 MW in scenarios 1-2 and 5-12. It should be noted that both units do not ramp in those periods in which they contribute to the regulation service. This is enforced by constraints (5.51), (5.57) and (5.58).

Finally, note that neither of the units reaches its capacity in any period and scenario. This is mainly due to the economic advantage of contributing to the regulation service. For example, in period 3 of scenarios 9-10, unit 1 generates 92.5 MW. Since the offered range of regulation power is 15 MW, the system operator could automatically increase the power output of the unit half of this power, which is 7.5 MW. Thus, the sum of the power generated at the beginning of period 3, 92.5 MW, plus 7.5 MW constitutes the capacity of unit 1, 100 MW.

Table 5.6 Producer pool example: power traded by unit 1 for $\beta = 0$ (MW)

Scenario #	Day ahead			Regulation			Adjustment			Generation		
	Period #			Period #			Period #			Period #		
	1	2	3	1	2	3	1	2	3	1	2	3
1	9.1	35.2	41.2	0.0	15.0	15.0	53.4	39.8	33.8	75.0	75.0	75.0
2	9.1	35.2	41.2	0.0	15.0	15.0	53.4	39.8	33.8	75.0	75.0	75.0
3	9.1	35.2	41.2	0.0	15.0	15.0	45.7	24.5	18.5	59.6	59.6	59.6
4	9.1	35.2	41.2	0.0	15.0	15.0	45.7	24.5	18.5	59.6	59.6	59.6
5	9.1	50.1	52.1	0.0	15.0	15.0	53.4	24.9	22.9	75.0	75.0	75.0
6	9.1	50.1	52.1	0.0	15.0	15.0	53.4	24.9	22.9	75.0	75.0	75.0
7	9.1	50.1	52.1	0.0	15.0	15.0	53.4	24.9	22.9	75.0	75.0	75.0
8	9.1	50.1	52.1	0.0	15.0	15.0	53.4	24.9	22.9	75.0	75.0	75.0
9	62.5	50.3	52.1	0.0	0.0	15.0	0.0	33.4	40.4	75.0	92.5	92.5
10	62.5	50.3	52.1	0.0	0.0	15.0	0.0	33.4	40.4	75.0	92.5	92.5
11	62.5	50.3	52.1	0.0	0.0	15.0	0.0	33.4	40.4	75.0	92.5	92.5
12	62.5	50.3	52.1	0.0	0.0	15.0	0.0	33.4	40.4	75.0	92.5	92.5

Table 5.7 Producer pool example: power traded by unit 1 for $\beta = 1$ (MW)

Scenario #	Day ahead			Regulation			Adjustment			Generation		
	Period #			Period #			Period #			Period #		
	1	2	3	1	2	3	1	2	3	1	2	3
1	62.5	75.0	75.0	0.0	15.0	15.0	0.0	0.0	0.0	75.0	75.0	75.0
2	62.5	75.0	75.0	0.0	15.0	15.0	0.0	0.0	0.0	75.0	75.0	75.0
3	62.5	75.0	75.0	0.0	15.0	15.0	0.0	0.0	0.0	75.0	75.0	75.0
4	62.5	75.0	75.0	0.0	15.0	15.0	0.0	0.0	0.0	75.0	75.0	75.0
5	62.5	75.0	75.0	0.0	0.0	15.0	0.0	8.75	17.5	75.0	92.5	92.5
6	62.5	75.0	75.0	0.0	0.0	15.0	0.0	8.75	17.5	75.0	92.5	92.5
7	62.5	75.0	75.0	0.0	0.0	15.0	0.0	8.75	17.5	75.0	92.5	92.5
8	62.5	75.0	75.0	0.0	0.0	15.0	0.0	8.75	17.5	75.0	92.5	92.5
9	62.5	75.0	75.0	0.0	0.0	15.0	0.0	8.75	17.5	75.0	92.5	92.5
10	62.5	75.0	75.0	0.0	0.0	15.0	0.0	8.75	17.5	75.0	92.5	92.5
11	62.5	75.0	75.0	0.0	0.0	15.0	0.0	8.75	17.5	75.0	92.5	92.5
12	62.5	75.0	75.0	0.0	0.0	15.0	0.0	8.75	17.5	75.0	92.5	92.5

5.8 Producer Pool Case Study

In this section, we present a case study based on the electricity market of the Iberian Peninsula [106]. We consider a power producer that seeks to obtain the optimal offering curves for the 24 hours of a typical trading day. This producer owns four units, whose technical data are provided in Table 5.8. Observe that only unit 4 is able to provide regulation service.

Table 5.8 Producer pool case study: thermal unit data

Unit #	$P_g^{\mathrm{G,min}}$ (MW)	$P_g^{\mathrm{G,max}}$ (MW)	R_g^{U} (MW/h)	R_g^{D} (MW/h)	C_g^{P} (\$/MWh)	$P_{g,0}^{\mathrm{G}}$ (MW)	$P_g^{\mathrm{R,max}}$ (MW)
1	75	140	65	65	19.81	130	0
2	150	350	200	200	23.40	335	0
3	160	380	220	220	25.17	185	0
4	180	390	210	210	25.51	205	50

The uncertain parameters (day-ahead and regulation prices, and intercepts and slopes of the adjustment prices) have been characterized using ARIMA models. For numerical reasons, the initial random variables have been transformed in order to properly adjust the ARIMA models provided in Table 5.9, where

$$\tilde{\lambda}_t^{\mathrm{D}} = \frac{\log_{10}(\lambda_t^{\mathrm{D}}) - \mathcal{E}\left(\log_{10}(\lambda_t^{\mathrm{D}})\right)}{\mathcal{STD}\left(\log_{10}(\lambda_t^{\mathrm{D}})\right)} \tag{5.71}$$

$$\tilde{\lambda}_t^{\mathrm{R}} = \frac{\log_{10}(\lambda_t^{\mathrm{R}}) - \mathcal{E}\left(\log_{10}(\lambda_t^{\mathrm{R}})\right)}{\mathcal{STD}\left(\log_{10}(\lambda_t^{\mathrm{R}})\right)} \tag{5.72}$$

$$\tilde{\lambda}_t^{\mathrm{A0}} = \frac{\left(\lambda_t^{\mathrm{A0}} - \lambda_t^{\mathrm{D}}\right) - \mathcal{E}\left(\lambda_t^{\mathrm{A0}} - \lambda_t^{\mathrm{D}}\right)}{\mathcal{STD}\left(\lambda_t^{\mathrm{A0}} - \lambda_t^{\mathrm{D}}\right)} \tag{5.73}$$

$$\tilde{\gamma}_t^{\mathrm{A}} = \frac{\log_{10}(10 + \gamma_t^{\mathrm{A}}) - \mathcal{E}\left(\log_{10}(10 + \gamma_t^{\mathrm{A}})\right)}{\mathcal{STD}\left(\log_{10}(10 + \gamma_t^{\mathrm{A}})\right)}, \tag{5.74}$$

where $\mathcal{E}$ indicates expected value and $\mathcal{STD}$ standard deviation.

Table 5.9 Producer pool case study: ARIMA models

Variable	ARIMA model
$\tilde{\lambda}_t^{\mathrm{D}}$	$\mathrm{ARIMA}(1,0,1)(2,0,1)_{24}$
$\tilde{\lambda}_t^{\mathrm{R}}$	$\mathrm{ARIMA}(5,0,1)(1,0,1)_{24}$
$\tilde{\lambda}_t^{\mathrm{A0}}$	$\mathrm{ARIMA}(1,0,2)(1,0,1)_{24}$
$\tilde{\gamma}_t^{\mathrm{A}}$	$\mathrm{ARIMA}(2,0,1)(1,0,1)_{24}$

The logarithm function is used in expressions (5.71), (5.72) and (5.73) in order to attain constant variance. Observe that because γ_t^{A} is a negative value, the logarithm transformation is made on $10 + \gamma_t^{\mathrm{A}}$, which is a value greater than 0. In expression (5.73), the logarithmic conversion is not necessary since the difference between the day-ahead price and the intercept of the inverse demand is considered. Additional information on typical variable transformations related to ARIMA models can be found in [18].

The values of the parameters and the variance of the error terms in the ARIMA models used in this case study are provided in Table 5.10.

These ARIMA models have been used to generate a scenario tree with 60 scenarios and a $(20\times1\times3)$ structure. The scenario generation procedure is the following: first, 20 day-ahead price scenarios are generated. Next, from each day-ahead scenario a single regulation price scenario is obtained. Finally, for each regulation price scenario, three intercept and slope scenarios modeling the adjustment price are generated. Fig. 5.15 represents the sets of generated scenarios. Observe that the volatility of the adjustment prices is greater than that of the day-ahead prices.

Table 5.10 Producer pool case study: ARIMA model data

$\tilde{\lambda}_t^{\mathrm{D}}$ (ARIMA$(1,0,1)(2,0,1)_{24}$)					
ϕ_1	θ_1	Φ_{24}	Φ_{48}	Θ_{24}	Error variance
2.874	0.157	1.088	-0.110	0.819	0.408

$\tilde{\lambda}_t^{\mathrm{R}}$ (ARIMA$(5,0,1)(1,0,1)_{24}$)								
ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	θ_1	Φ_{24}	Θ_{24}	Error variance
1.176	-0.250	-0.020	0.058	0.009	0.829	0.991	0.910	0.517

$\tilde{\lambda}_t^{\mathrm{A0}}$ (ARIMA$(1,0,2)(1,0,1)_{24}$)					
ϕ_1	θ_1	θ_2	Φ_{24}	Θ_{24}	Error variance
0.880	0.193	0.066	0.458	0.266	0.405

$\tilde{\gamma}_t^{\mathrm{A}}$ (ARIMA$(2,0,1)(1,0,1)_{24}$)					
ϕ_1	ϕ_2	θ_1	Φ_{24}	Θ_{24}	Error variance
1.163	-0.214	0.812	0.992	0.976	0.819

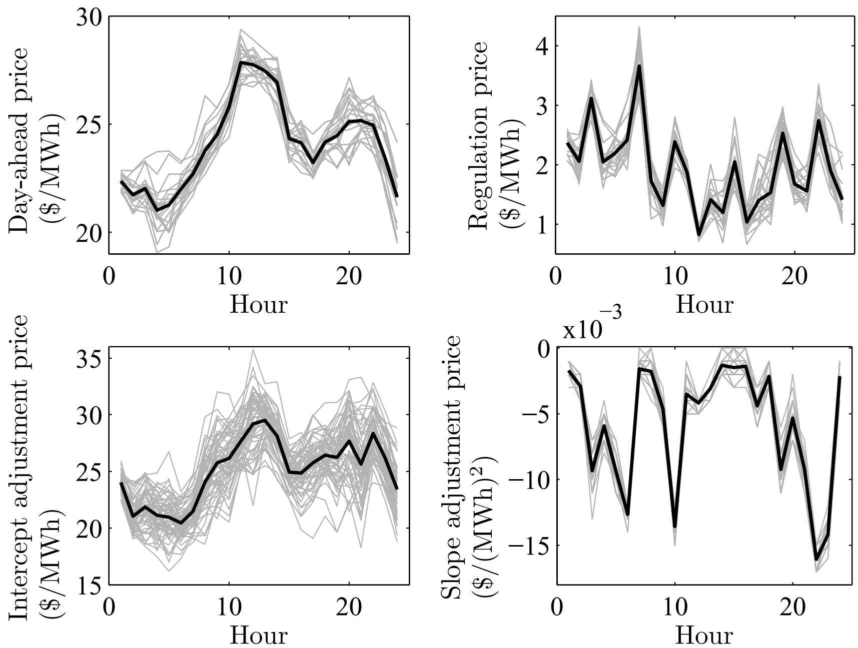


Fig. 5.15 Producer pool case study: uncertain parameter scenarios

For implementation purposes, the quadratic revenue of selling in the adjustment market has been linearized using a piecewise linear approximation involving four blocks as in [118].

The case study above is solved using model (5.48)-(5.63) in order to derive optimal offering curves for different values of the weighting parameter β. This problem, characterized by 55,165 constraints, 34,622 real variables, and 1440 binary variables, is solved at a confidence level $\alpha = 0.95$ using CPLEX 10.2 [142] under GAMS [141]. The optimality gap achieved is less than 0.6% in all instances solved.

Fig. 5.16 represents the efficient frontier in terms of the expected profit and the CVaR for different values of the parameter β. If β increases from 0 to 1, the expected profit decreases 1.9%, from \$19,992.65 to \$19,608.21. In other words, the expected profit is not highly dependent on the risk-aversion of the producer. However, the CVaR increases 78.3% if risk is accounted for ($\beta = 1$). This result means that a small decrease in the expected profit can be used to reduce efficiently the risk of profit variability.

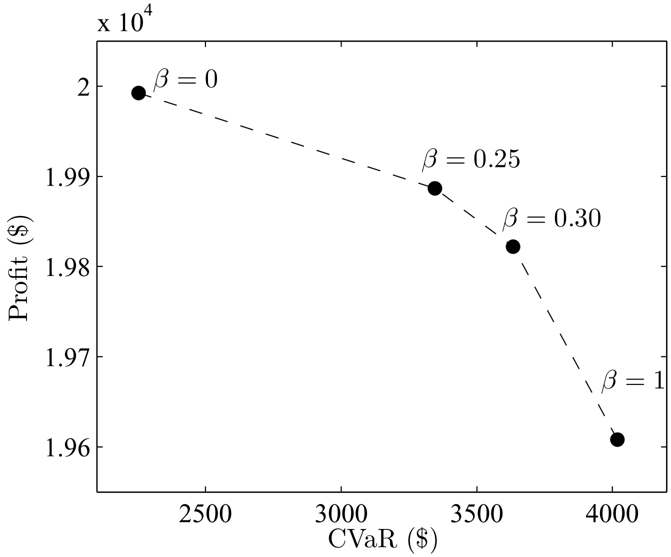


Fig. 5.16 Producer pool case study: efficient frontier

Finally, the resulting offering curves for all units, cases $\beta = 0$ and $\beta = 1$, and three different hours are depicted in Fig. 5.17. As expected, the quantity of power offered in the day-ahead market increases if risk is accounted for ($\beta = 1$) with respect to the risk-neutral case ($\beta = 0$). Note that a similar behavior takes place for the rest of the hours.

Finally, it is relevant to highlight that using currently available optimization software and current personal computer technology, the required com-

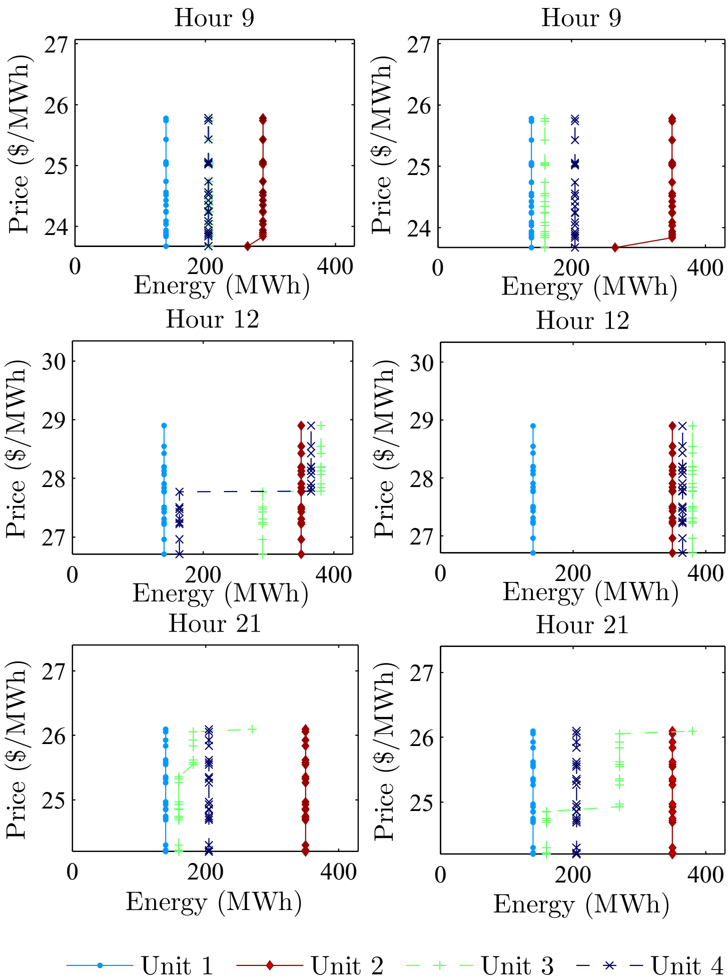


Fig. 5.17 Producer pool case study: optimal offering curves in the day-ahead market

puter time to solve problems similar to the one discussed above is moderate and compatible with the considered decision-time framework.

5.9 Summary and Conclusions

This chapter presents a stochastic programming model that allows a power producer to derive optimal offering curves for the day-ahead market. It considers that the producer participates also in the regulation and adjustment markets.

The electricity prices in the day-ahead, regulation, and adjustment markets are characterized as stochastic processes. The prices in the day-ahead and regulation markets are considered independent of the producer actions, whereas the producer is considered to be a price-maker agent in the adjustment market.

The risk-aversion is incorporated by means of the CVaR. The objective function of the model comprises the expected profit plus the CVaR multiplied by a weighting parameter. Solving the problem for different values of the weighting parameter allows the representation of the so-called efficient frontier.

The resulting stochastic programming model is recast as a mixed-integer quadratic programming problem that can be solved using commercial software. The proposed mixed-integer quadratic formulation is both efficient and robust as demonstrated via both small and real-world case studies.

The resulting offering curves are highly dependent on the risk-aversion faced by the producer. That is, the producer relies more in the day-ahead market than in the adjustment market as the risk-aversion increases. On the other hand, a risk-neutral producer is willing not to sell in the day-ahead market a portion of its capacity in the hope of achieving a large profit by selling it in the adjustment market if high prices happen in this market.

5.10 Notation

The notation used throughout this chapter is stated below for quick reference.

Indices and Numbers:

g	Index of generating units, running from 1 to N_{G}.
t	Index of time periods, running from 1 to N_{T}.
ω	Index of scenarios, running from 1 to N_{Ω}.

Real Variables:

$C^{\mathrm{G}}_{t\omega}$ Production cost incurred by the producer in period t and scenario ω (\$).

P^{A}_{t} Total power traded by the producer in the adjustment market (MWh). The value of this variable in scenario ω is denoted by $P^{\mathrm{A}}_{t\omega}$. P^{A}_{t} and $P^{\mathrm{A}}_{t\omega}$ are limited to $P^{\mathrm{A,max}}$.

$P^{\mathrm{A}}_{gt\omega}$ Power sold by unit g in the adjustment market in period t and scenario ω (MW).

$P^{\mathrm{G}}_{gt\omega}$ Power output of unit g in scenario ω at the end of period t (MW). Limited by $P^{\mathrm{G,min}}_{g}$ and $P^{\mathrm{G,max}}_{g}$.

$P^{\mathrm{D}}_{gt\omega}$ Power sold by unit g in the day-ahead market in period t and scenario ω (MW).

$P^{\mathrm{R}}_{gt\omega}$ Power sold by unit g in the regulation market in period t and scenario ω (MW). Limited to $P^{\mathrm{R,max}}_{g}$.

$R^{\mathrm{A}}_{t\omega}$ Revenue obtained from selling in the adjustment market in period t and scenario ω (\$).

$R^{\mathrm{D}}_{t\omega}$ Revenue obtained from selling in the day-ahead market in period t and scenario ω (\$).

$R^{\mathrm{R}}_{t\omega}$ Revenue obtained from selling in the regulation market in period t and scenario ω (\$).

λ^{A}_{t} Final energy price in the adjustment market in period t (\$/MWh). λ^{A}_{t} depends on the total quantity of power traded by the producer in the adjustment market.

ζ Auxiliary variable used to compute the CVaR.

η_{ω} Auxiliary variable related to scenario ω and used to compute the CVaR.

Binary Variables:

$w_{gt\omega}$ 0/1 variable that is equal to 1 if unit g provides regulation service in period t and scenario ω, and 0 otherwise.

Random Variables:

$\lambda^{\mathrm{A0}}_{t}$ Intercept of the the inverse demand curve of the producer in the adjustment market in period t (\$/MWh). $\lambda^{\mathrm{A0}}_{t\omega}$ represents the realization of this variable in scenario ω.

λ^{D}_{t} Day-ahead market price in period t (\$/MWh). $\lambda^{\mathrm{D}}_{t\omega}$ represents the realization of this variable in scenario ω.

λ_t^{R} Regulation market price in period t (\$/MWh). $\lambda_{t\omega}^{\mathrm{R}}$ represents the realization of this variable in scenario ω.

γ_t^{A} Slope of the inverse demand curve of the producer in the adjustment market in period t (\$/(MWh)^2). $\gamma_{t\omega}^{\mathrm{A}}$ represents the realization of this variable in scenario ω.

Constants:

$\boldsymbol{A}^{\mathbf{A}}$ Vector of 0s and 1s used to formulate non-anticipativity constraints in the adjustment market, whose element $A^{\mathrm{A}}(\omega)$ is equal to 1 if scenarios ω and $\omega+1$ are identical up to stage 3, being 0 otherwise.

$\boldsymbol{A}^{\mathbf{D}}$ Vector of 0s and 1s used to formulate non-anticipativity constraints in the day-ahead market, whose element $A^{\mathrm{D}}(\omega)$ is equal to 1 if scenarios ω and $\omega+1$ are identical up to stage 2, being 0 otherwise.

$\boldsymbol{A}^{\mathbf{R}}$ Vector of 0s and 1s used to formulate non-anticipativity constraints in the regulation market, whose element $A^{\mathrm{R}}(\omega)$ is equal to 1 if scenarios ω and $\omega+1$ are identical up to stage 2, being 0 otherwise.

C_g^{P} Linear production cost of unit g (\$/MWh).

$\boldsymbol{O}^{\mathbf{D}}$ Matrix containing the price ranking (increasing value) for day-ahead price scenarios. Element $O^{\mathrm{D}}(t,\omega)$ provides the ranking of price associated with scenario ω in period t. Therefore, $O^{\mathrm{D}}(t,\omega)=1$ if the smallest day-ahead price in period t realized in scenario ω.

$P_{g,0}^{\mathrm{G}}$ Initial value of the power output of unit g (MW).

R_g^{D} Ramp-down limit of unit g (MW/h).

R_g^{U} Ramp-up limit of unit g (MW/h).

α Confidence level used for the calculation of the CVaR.

β Weighting parameter used to model the tradeoff between expected profit and CVaR.

π_ω Probability of occurrence of scenario ω.

Sets:

Ω Set of scenarios.

5.11 Exercises

Exercise 5.1. Extend formulation (5.48)-(5.63) so that the producer can derive regulation and adjustment offering curves instead of single power quantities for each of these markets. Consider that all offering curves must be non-decreasing.

Exercise 5.2. Extend formulation (5.48)-(5.63) to take into account the start-ups and shut-downs of the units.

Exercise 5.3. Draw and describe a decision framework (similar to that in Fig. 5.1) for a producer that participates in a second adjustment market that takes place after the closing of the first adjustment market and whose trading horizon spans hours 12-24 of the next day.

Exercise 5.4. Draw and describe a scenario tree (similar to that in Fig. 5.2) for Exercise 5.3 above. Consider that 2 branches leave each node of the tree.

Exercise 5.5. Write non-anticipativity conditions for the producer problem in Exercises 5.1 and 5.3 above.

Exercise 5.6. Reformulate (5.48)-(5.63) avoiding parameter β while still modeling the tradeoff of expected profit versus CVaR.

Exercise 5.7. Extend formulation (5.48)-(5.63) in order to account for the possibility that the producer buys energy in the adjustment market.

Exercise 5.8. Add additional day-ahead price scenarios to the tree defined in the example of Section 5.7 and study their effect on the resulting offering curves.

Exercise 5.9. Include an additional thermal unit in the example of Section 5.7 and compare the obtained results with those pertaining to the original case.

Exercise 5.10. Calculate VSS and EVPI for the risk-neutral ($\beta = 0$) case of the example in Section 5.7.

Chapter 6
Pool Trading for Wind Power Producers

6.1 Introduction

The amount of wind generation is rapidly increasing in many electric energy systems across the world. The large-scale electricity production based on the exploitation of wind energy resources is currently feasible and is gradually becoming a profitable business attracting the attention of investors. Two main driving forces are responsible for the growing relevance of wind generation: the increasing trend of fossil fuel prices (natural gas, oil, and coal) as less and less reserves are available, and the need to curb carbon emissions to preserve a functioning atmosphere.

When it comes to the energy-selling business, three basic aspects differentiate the conventional producer (thermal and nuclear producer), comprehensively analyzed in Chapter 5, and the wind power producer, namely:

1. Wind power constitutes an emission-free energy source, and as such, its development is spurred on by governments willing to face the issue of global warming. Economically speaking, this usually translates into subsidies that increase the value of the MWh produced by a wind farm.
2. Wind energy production does not require fuel consumption, and as a result, its associated fuel cost is zero.
3. Wind power is non-dispatchable and plagued by the major uncertainty that constitutes wind availability. Decisions on the amount of wind energy to be traded are therefore risky.

While the features indicated in points 1 and 2 above clearly boost the integration of wind generation in power systems, the characteristic expressed in point 3, the wind availability uncertainty, constitutes the major obstacle to the natural incorporation of wind producers into a market framework in both technical and economic terms. How can a wind power producer *survive* by itself in a competitive environment especially designed for conventional producers?

A. J. Conejo et al., *Decision Making Under Uncertainty in Electricity Markets*,
International Series in Operations Research & Management Science 153,
DOI 10.1007/978-1-4419-7421-1_6,

From a technical viewpoint, the proper management of wind production uncertainty is generally carried out by means of a real-time market, hereafter referred to as *balancing market*, that allows covering the lack of production of uncertain sources (demand, production unit failures, and wind generation). The balancing actions are often provided by energy sources that are expensive, but dispatchable, such as combine-cycle gas turbines. The balancing energy is available in the form of reserves, which are dealt with in Chapters 10 and 11.

From an economic perspective, we assume that the cost of the energy required to maintain the system balance in real time is assumed by the market agents incurring energy deviations, that is, wind producers among others. Therefore, as a wind producer needs to rely more and more on the balancing mechanism to fix its energy production imbalances, its profitability and competitiveness deteriorates quickly. In plain words, the *invisible hand* governing market laws might bring about the downfall of the less sharp wind producers.

Roughly speaking, in response to this situation, three different attitudes can be adopted by regulators:

1. Wind generation is managed through the electricity market (as a negative demand), but wind power producers are paid a regulated tariff for their actual energy production. This way, wind producers are released from the risk stemming from wind availability at the expense of losing the opportunity of making a higher profit through a shrewd market participation.
2. Wind producers compete in the different trading floors available in the electricity market, including the balancing market, in which they must face the cost of their energy deviations. For each sold MWh, they are paid the price resulting from the market-clearing process plus a subsidy intended to strengthen the competitiveness of wind producers in the electricity market.
3. According to a pure competitive rationale, wind producers must bear the burden of the market as any other market participant. Consequently, this situation is achieved by eliminating the subsidies in the previous point.

Within a decision-making framework, situation 3 is the one of higher interest for the purpose of this book, and as such, its analysis and modeling from the wind producer's viewpoint constitutes the leitmotif of the present chapter. In order for a wind power producer to *stay afloat* in such a strictly competitive panorama, three main lines of action have been proposed in the literature. One of them is based on the combined and coordinated use of wind power and energy storage technologies (pumped-storage facilities, compressed air facilities, etc.), [2, 10, 22, 61]. However, plants including storage facilities have no clear incentive to associate with wind farms. Another relevant area of possibilities is the use of financial options as a tool for wind producers to hedge against wind generation uncertainty, [73]. Nevertheless, no clear financial instruments are available to hedge against the risk of production unavailability. The third line of action focuses on designing stochastic models intended to produce optimal offering strategies for a wind producer participating in an electricity market on its own, [11, 93, 95, 117].

This chapter explores this last area of research and, in particular, introduces a stochastic programming model to generate optimal offers for trading wind generation in short-term energy markets.

The rest of this chapter is organized as follows. Section 6.2 describes the decision framework for the short-term trading problem of a wind power producer and Section 6.3 unravels the key factors affecting the solution to this problem, namely, the mechanism to price energy deviations in the balancing market, the cost derived from such deviations, and the certainty gain effect linked to the existence of an adjustment market within the daily market structure. Section 6.4 tackles the characterization and modeling of the uncertainty sources in the wind producer problem via time series, and describes the sequential order in which decisions are made and information is revealed by means of a scenario tree. Section 6.5 provides a mathematical formulation based on multi-stage stochastic programming to solve the wind producer problem. Section 6.6 consists of a detailed example illustrating the most significant aspects of the proposed model. Section 6.7 reports and discusses results from a realistic case study. Lastly, both a summary of the chapter and some relevant conclusions drawn from the example and the case study are provided in Section 6.8.

6.2 Decision Framework

Without loss of generality, let us consider a wind power producer participating in a pool-based electricity market that includes three independent and successive short-term trading floors: the day-ahead market, the adjustment market, and the balancing market. We also assume that the wind producer has no market-power capability in either of the aforementioned markets. Its objective is to maximize its expected profits from trading energy in the day-ahead and adjustment markets, and minimizing the cost incurred in the balancing market due to energy deviations.

Each of these three markets is cleared through a single auction process. More information on these markets can be found in Chapters 1 and 5. Likewise, more details about market-clearing algorithms are provided in Chapter 10. The time framework for market clearing is as represented in Fig. 6.1. The day-ahead market, which involves energy transactions coming into effect during the whole day d, is cleared at a given time period t^{D} of day $\mathrm{d}-1$. Due to the usual delay between the closure of this market and the beginning of the energy delivery horizon (day d), the electricity pool includes an adjustment market, where it is possible to take corrective actions. These actions are intended to modify the outcomes of the day-ahead market in order to make them closer to the expected production for the day d. This adjustment

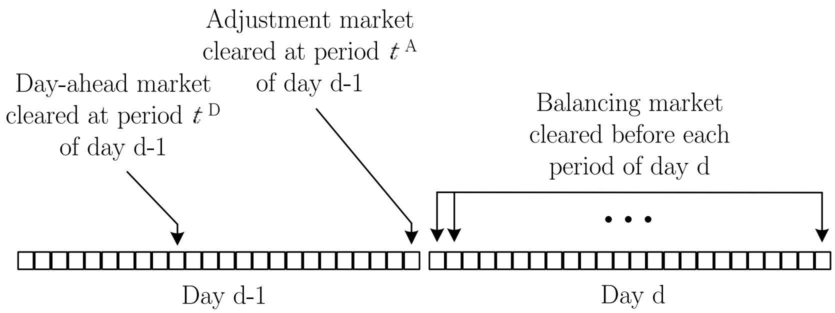


Fig. 6.1 Decision framework of the wind producer problem.

market is therefore of especial usefulness for wind producers as it allows them to incorporate into their offers the certainty gained on production availability during the time interval between the closure of both markets by updating their generation forecasts and thus, reducing the associated uncertainty. This phenomenon is formally stated in Subsection 6.3.3. For convenience, we consider that the adjustment market is cleared just before the delivery horizon (day d), as indicated in Fig. 6.1. Consequently, energy transactions in this market cover the whole day d. Nevertheless, additional adjustments markets can be sequentially arranged throughout the time span comprising the closure of the day-ahead market and the end of day d, and as a result, energy transactions resolved in them may cover the whole day d or just a part of it. Finally, the balancing market ensures the real-time balance between generation and demand by offsetting the differences between the real-time operation and the last energy program settled in the previous markets. For that reason, this market is cleared just before each time period of day d. Therefore, through this last market, energy imbalances are corrected and priced according to the settlement mechanism for imbalance prices explained in Subsection 6.3.1 below.

In short, a wind power producer competing within a market structure as the one described above and graphically represented in Fig. 6.1 should: i) decide the offering curve to be submitted to the day-ahead market for each time period of day d; ii) modify in the adjustment market the energy offers submitted to the day-ahead market according to the wind power forecast updates; and iii) amend its energy deviations in the balancing market for each time period of day d.

The short-term decision-making process of a wind power producer is analyzed in Subsection 6.4.3 from a stochastic programming approach. Hereinafter, the time horizon spanning this process is assumed to be comprised of hourly periods.

The following example serves us to illustrate the market structure and time framework described above.

Illustrative Example 6.1 (Market structure and time framework). Consider an electricity market for short-term energy transactions that includes one day-ahead market and one adjustment market arranged throughout the scheduling horizon as shown in Fig. 6.2.

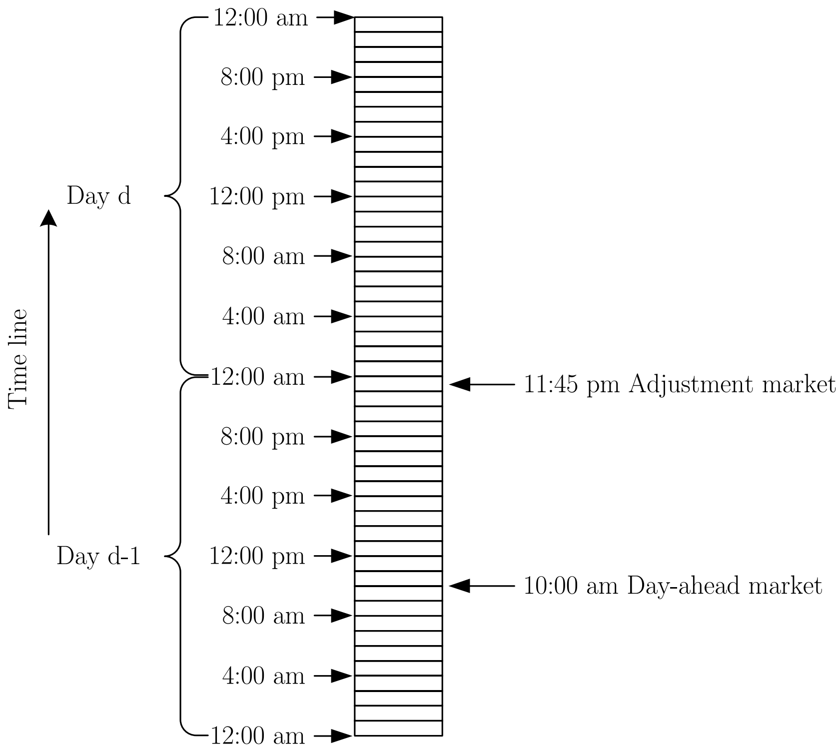


Fig. 6.2 Illustrative Example 6.1: time framework

Likewise, a balancing market is organized before each trading hour of the market horizon (day d) to guarantee the real-time energy balance between generation and demand. For this purpose, this market remains open until 10 minutes before the delivery hour under consideration. For instance, the balancing market intended to ensure the system energy balance during the hour 8:00-9:00 pm of day d is closed at 7:50 pm of this day. We assume therefore that 10 minutes prior to the energy delivery hour, energy deviations incurred by market participants in that hour are perfectly known. In this respect, we emphasize that, from a theoretical point of view, the balancing market should be understood just as a mechanism through which real-time energy deviations are covered and priced under competition.

The day-ahead market is closed at 10:00 am of day d − 1, i.e., 14 hours before the beginning of the energy delivery period (day d). The adjustment market, which is cleared at 11:45 pm of day d − 1, can be used by market agents (wind producers, among others) to alter their scheduled productions (or consumptions in the case of loads) for the trading periods after its closure, that is for the 24 hours of day d. □

6.3 The Key Issues

In this section, we discuss in detail the key factors influencing, in a particular manner, the short-term decision-making process of a wind power producer seeking to maximize its profit in an electricity market framework. Specifically, these factors are: the mechanism through which energy deviations incurred by market agents are priced in the balancing market, the cost entailed by such deviations (imbalance cost), and the beneficial impact of an adjustment market following the clearing of the day-ahead market.

6.3.1 Mechanism for Imbalance Prices

In this book, we call *balancing market* the trading floor available in an electricity market to guarantee the balance between energy supplied by producers and energy demanded by consumers on a real-time basis. The development of this function requires the balancing market to be cleared as close as possible to the period of physical energy delivery so that the available production means and the actual consumption needs are known. This allows us to assume that every wind power producer attends the balancing market with perfect information on its wind power production.

Any market agent expecting a final production below or above the last energy schedule resulting from trading in the day-ahead and adjustment markets is commanded to amend its energy deviations in the balancing market. These imbalances can be positive or negative. We consider a positive energy deviation as a higher production or lower consumption than scheduled. Analogously, a negative energy deviation is defined as a lower production or higher consumption than scheduled. In practice, energy imbalances in a power system are mostly caused by consumers, wind producers, and conventional producers (as the ones described in Chapter 5) suffering from an unexpected unit failure.

Thus, every *deviated* producer must sell its generation surplus or buy its generation deficit in the balancing market, at a price referred to as *imbalance price*. The selling and purchase prices are obtained from a single auction process. Without loss of generality, we describe below the general rationale behind such a process, a rationale fundamentally shared by many electricity markets worldwide, but with slight variants.

Let us start considering a day-ahead market price λ_t^{D} for a given time period t of the market horizon. This price is obtained as the intercept between the aggregated power supply and demand curves as shown in Fig. 6.3, where the behavior of consumers is supposed to be completely inelastic for simplicity. The aggregated supply function is built from the energy offers

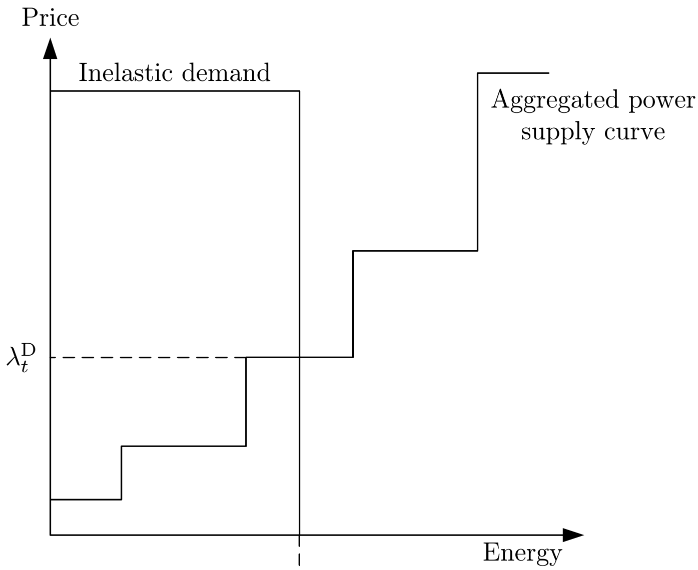


Fig. 6.3 Day-ahead market price formation

submitted by producers to the day-ahead market. If the functioning of this market follows the rules of perfect competition, which is undoubtedly an utopian paradigm in practice, energy offers by producers equal their marginal production costs, and so, the aggregated supply curve depicted in Fig. 6.3 constitutes the minimum cost function at which any level of demand can be satisfied. Likewise, if we extend the assumption of perfect competition to the balancing market, the cost of energy deviations around the intercept defining the day-ahead price in Fig. 6.3 should be given by the same aggregated supply curve as the one of the day-ahead market for the referred time period t. This line of argument constitutes the logic supporting the mechanism for imbalance price formation described below. However, electricity markets are far from being perfectly competitive, and thus, the aggregated supply curves resulting in both markets can be very different. Further, some generating units may not be able to alter their productions levels in the time frame required to provide balancing energy. As a result, some conventional producers may not participate in the balancing market.

In the balancing market, a price λ_t^+ for positive energy deviations (higher productions or lower consumptions than scheduled) and a price λ_t^- for negative energy deviations (lower productions or higher consumptions than scheduled) are settled for each time period. The basic idea is that these prices represent the cost of the energy required to counteract the unplanned total energy imbalance of the power system, and consequently, they depend on the sign, positive or negative, of the imbalance as a whole (system imbalance). Specifically:

1) If the system imbalance δ_t is negative (deficit of generation in the system), $\delta_t < 0$, then

$$\lambda_t^+ = \lambda_t^{\mathrm{D}} \tag{6.1}$$

$$\lambda_t^- = \max(\lambda_t^{\mathrm{D}}, \lambda_t^{\mathrm{UP}}). \tag{6.2}$$

where λ_t^{UP} is the price of the energy (upward energy) that needs to be added to the system. This situation is graphically represented in Fig. 6.4. Note that a generation shortage can be equivalently interpreted as an identical increase in the level of demand to be supplied.

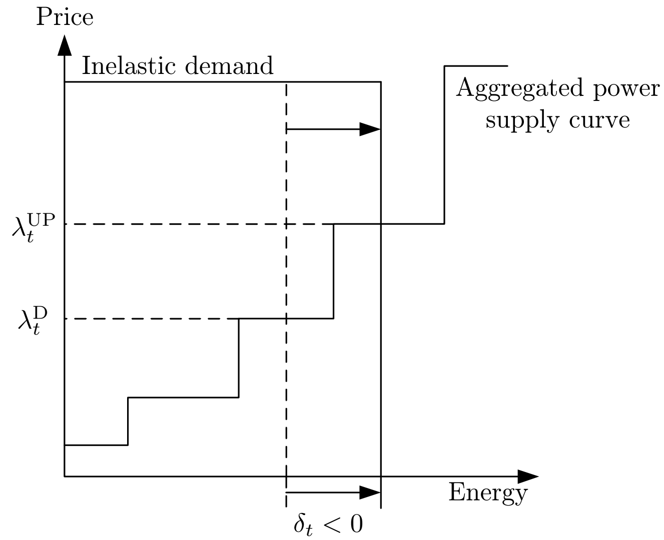


Fig. 6.4 Mechanism for imbalance price formation: negative system imbalance

In this case, the operator organizes a balancing market to which producers submit additional selling offers. Again assuming a rational behavior, producers are willing to produce the amount of energy required to cover the deficit of generation at a price (λ_t^{UP}) higher than the day-ahead market-clearing price (λ_t^{D}). The cost of this transaction falls on those market participants responsible for the system imbalance. In this sense, note that, as the price of this additional energy is higher than the actual price of the energy traded in the day-ahead market, the balancing process implies a profit loss for these market participants with respect to the profit they would have obtained by selling only their actual productions in the day-ahead market. On the other hand, those market participants producing more than agreed on the day-ahead market and, consequently, contributing to mitigate the negative system imbalance are paid at λ_t^{D} for their overproduction, and hence they do not incur imbalance cost.

Note that taking the maximum in equation (6.2) guarantees that the price of the energy bought in this market to cover generation deficits is higher than or equal to the day-ahead market price. This needs to be enforced owing to the fact that electricity markets are not perfectly competitive environments.

2) If the system imbalance δ_t is positive (excess of generation in the power system), $\delta_t \geq 0$, then

$$\lambda_t^{+} = \min(\lambda_t^{\mathrm{D}}, \lambda_t^{\mathrm{DN}}), \tag{6.3}$$

$$\lambda_t^{-} = \lambda_t^{\mathrm{D}}, \tag{6.4}$$

where λ_t^{DN} is the price of the energy (downward energy) that needs to be removed from the system. This situation is illustrated in Fig. 6.5. Note that a generation surplus can be equivalently understood as an identical decrease in the level of demand to be met.

In order to palliate the excess of generation in the system, the operator uses the balancing market where producers submit offers to repurchase some of the energy previously sold in the day-ahead market. Assuming a rational behavior, producers are only willing to repurchase the excess of energy at a price (λ_t^{DN}) smaller than the day-ahead market-clearing price (λ_t^{D}). Thus, those market participants contributing to the positive system imbalance are paid for their overproduction at λ_t^{DN}. As a result, they obtain a profit smaller than the profit they would have achieved if they had sold their overproduction in the day-ahead market. On the contrary, those market participants producing less energy than that resulting from the day-ahead market and, consequently, helping to alleviate the positive system imbalance are just paid at λ_t^{D} for their actual productions and so, they do not incur opportunity cost.

Note that taking the minimum in equation (6.3) guarantees that the generation surplus is sold at a price smaller than or equal to the day-ahead market price. Again, this needs to be imposed due to the fact that, in general, the rules of perfect competition do not hold in electricity markets.

From the discussion above, it follows that $\lambda_t^{+} \leq \lambda_t^{\mathrm{D}}$ and $\lambda_t^{-} \geq \lambda_t^{\mathrm{D}}$.

It is important to emphasize that, according to these criteria, those participants who offset the system imbalance, i.e., those participants who contribute

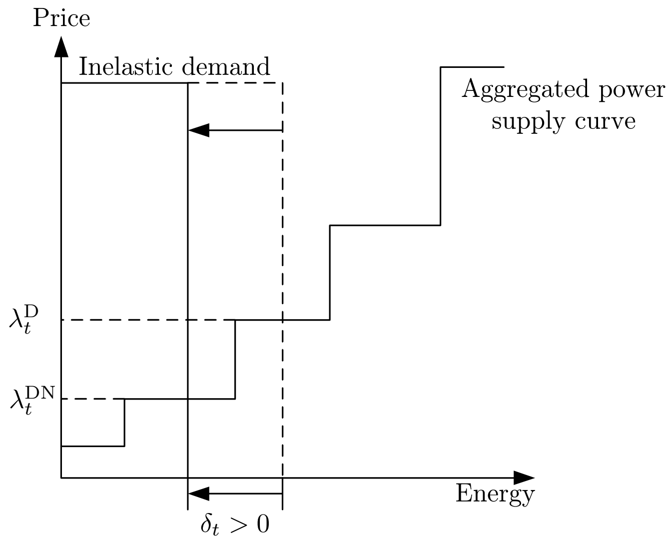


Fig. 6.5 Mechanism for imbalance price formation: positive system imbalance

to restore the system balance, are not *punished* by the market, which is a sound principle.

The following example illustrates the mechanism for imbalance price formation.

Illustrative Example 6.2 (Mechanism for imbalance price formation). Suppose that the energy offers submitted by producers to the day-ahead market for a given time period t make up the aggregated supply curve represented in Fig. 6.6(a). This market is cleared in such a period at a price $\lambda_t^{\mathrm{D}} = \$40/\mathrm{MWh}$, which corresponds to a level of inelastic demand equal to 400 MWh.

Next, assume that the referred aggregated supply curve also reflects the way producers offer in the balancing market, which is organized to correct energy deviations in period t. Being so, if the total system imbalance in this period is, for instance, negative and equal to -200 MWh, the imbalance prices for positive and negative deviations are given by

$$\lambda_t^{+} = \lambda_t^{\mathrm{D}} = \$40/\mathrm{MWh}$$
$$\lambda_t^{-} = \max(\lambda_t^{\mathrm{D}}, \lambda_t^{\mathrm{UP}}) = \max(40, 50) = \$50/\mathrm{MWh}.$$

The price formation process in this case is represented in Fig. 6.6(b). Note that the 200-MWh deficit of generation has to be provided by more expensive production units, and consequently, the energy price rises up to \$50/MWh

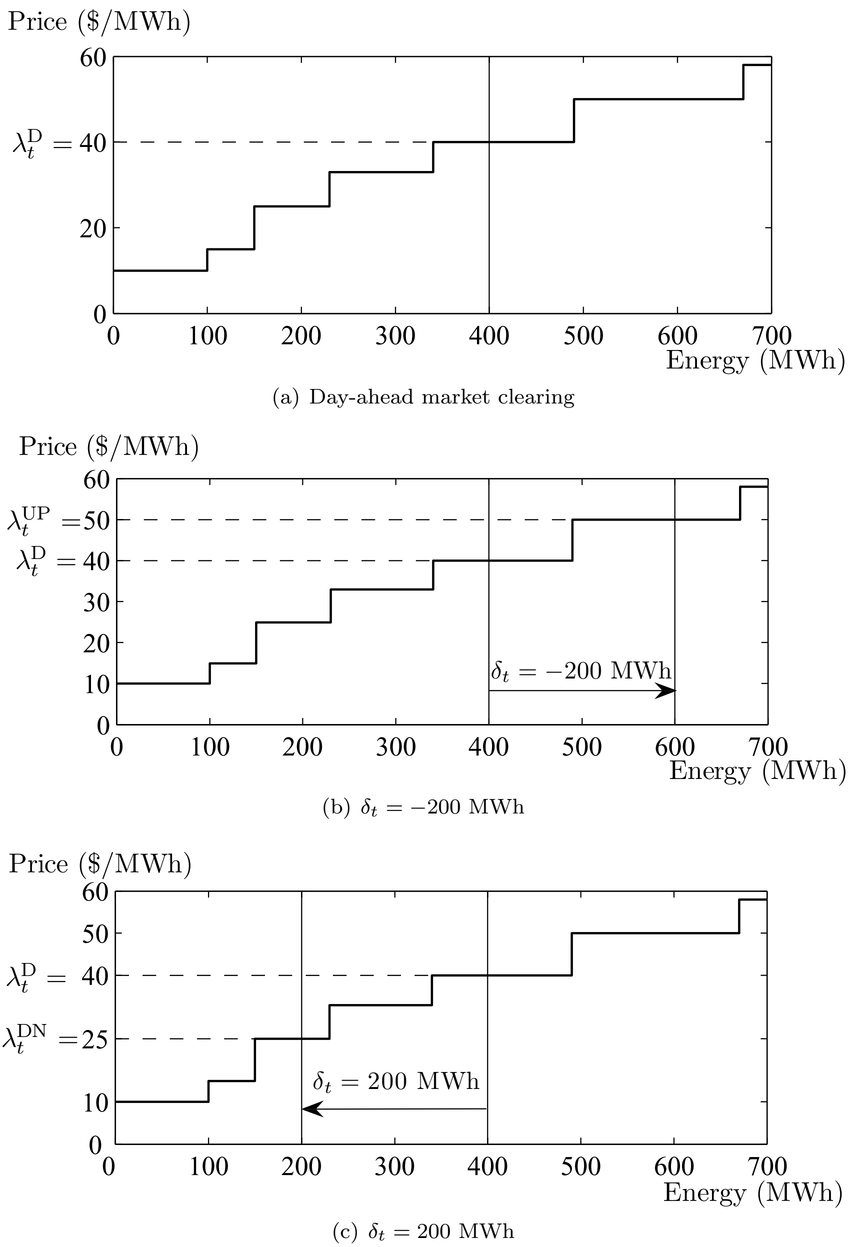


Fig. 6.6 Illustrative Example 6.2: mechanism for imbalance price formation

in the balancing market, i.e., $10/MWh higher than the value of the energy in the day-ahead market.

On the contrary, if the net system imbalance is, for example, positive and equal to 200 MWh, the imbalance prices for positive and negative deviations are

$$\lambda_t^+ = \min(\lambda_t^{\mathrm{D}}, \lambda_t^{\mathrm{DN}}) = \min(40, 25) = \$25/\mathrm{MWh},$$
$$\lambda_t^- = \lambda_t^{\mathrm{D}} = \$40/\mathrm{MWh}.$$

This situation is illustrated in Fig. 6.6(c). The overproduction, 200 MWh, has to be eliminated from the system and, to this end, is put up for sale in the balancing market. From a producer perspective, the acquisition of a certain quantity of this overproduction entails reducing its generation level the same amount accordingly, an amount previously sold in the day-ahead market at a price of \$40/MWh. Therefore, such a transaction makes sense from a purely economic viewpoint if the purchase price of the leftover energy is lower than \$40/MWh, specifically, equal to \$25/MWh for the particular instance represented in Fig. 6.6(c). □

6.3.2 Revenue and Imbalance Cost

Consider a wind power producer that offers and gets accepted a level of energy E_t^{D} into the day-ahead market for time period t, but actually generates E_t. For the sake of simplicity, no adjustment market is considered in this exposition. The revenue R_t of this wind power producer for period t is

$$R_t = \lambda_t^{\mathrm{D}} E_t^{\mathrm{D}} + I_t, \tag{6.5}$$

where I_t is the imbalance income resulting from the balancing process and may be negative, i.e., it may represent a cost. The total deviation Δ_t incurred by the wind producer is defined as

$$\Delta_t = E_t - E_t^{\mathrm{D}} = d_t(P_t - P_t^{\mathrm{D}}), \tag{6.6}$$

where d_t, P_t, and P_t^{D} are, respectively, the time span between two consecutive time periods (1 hour), the actual power produced by the wind power producer in period t, and the level of power it sells in the day-ahead market for such a period.

Consequently, I_t is given by

$$I_t = \begin{cases} \lambda_t^+ \Delta_t, & \Delta_t \geq 0 \\ \lambda_t^- \Delta_t, & \Delta_t < 0. \end{cases} \tag{6.7}$$

According to expression (6.6), a positive deviation means production surplus and a negative deviation represents a deficit of production. Therefore, λ_t^+ (λ_t^-) is the price at which the wind producer will be paid (charged) for its excess (deficit) of generation. Lastly, if we define

$$r_t^+ = \frac{\lambda_t^+}{\lambda_t^{\mathrm{D}}}, \quad r_t^+ \leq 1, \tag{6.8}$$

$$r_t^- = \frac{\lambda_t^-}{\lambda_t^{\mathrm{D}}}, \quad r_t^- \geq 1 \tag{6.9}$$

then

$$I_t = \begin{cases} \lambda_t^{\mathrm{D}} r_t^+ \varDelta_t, \ \varDelta_t \geq 0 \\ \lambda_t^{\mathrm{D}} r_t^- \varDelta_t, \ \varDelta_t < 0. \end{cases} \tag{6.10}$$

Note that definitions (6.8) and (6.9) are consistent under the assumption that the hourly electricity prices in the day-ahead market are necessarily positive, i.e., $\lambda_t^{\mathrm{D}} > 0$. This corresponds to the general behavior of energy prices in most electricity markets. If not, ratios r_t^+ and r_t^- need to be redefined in a different manner, or simply, the imbalance income function I_t should be expressed in the form of (6.7).

A wind power producer that needs to correct its energy deviations in the balancing market incurs an opportunity cost as it loses the chance of trading the deviated energy through the day-ahead market at a more competitive price. We can easily reformulate the revenue function (6.5) so that it explicitly reflects such an opportunity cost. We distinguish two cases:

1) The energy deviation incurred by the wind power producer is positive, i.e., $\varDelta_t > 0$. According to (6.5) and (6.10), we can write

$$R_t = \lambda_t^{\mathrm{D}} E_t^{\mathrm{D}} + \lambda_t^{\mathrm{D}} r_t^+ \varDelta_t. \tag{6.11}$$

By using the definition (6.6) of the total deviation $\varDelta_t$, we can replace E_t^{D} in (6.11) by $E_t - \varDelta_t$. Thus, we obtain

$$R_t = \lambda_t^{\mathrm{D}}(E_t - \varDelta_t) + \lambda_t^{\mathrm{D}} r_t^+ \varDelta_t = \lambda_t^{\mathrm{D}} E_t - \lambda_t^{\mathrm{D}}(1 - r_t^+)\varDelta_t, \tag{6.12}$$

with $\varDelta_t \geq 0$.

2) The energy deviation incurred by the wind power producer is negative, i.e., $\varDelta_t < 0$. Following a reasoning similar to that of the previous case, we arrive at the revenue expression below,

$$R_t = \lambda_t^{\mathrm{D}}(E_t - \varDelta_t) + \lambda_t^{\mathrm{D}} r_t^- \varDelta_t = \lambda_t^{\mathrm{D}} E_t + \lambda_t^{\mathrm{D}}(r_t^- - 1)\varDelta_t, \tag{6.13}$$

with $\varDelta_t < 0$.

Equations (6.12) and (6.13) can be expressed in the following general form

$$R_t = \lambda_t^{\mathrm{D}} E_t - C_t, \tag{6.14}$$

where

$$C_t = \begin{cases} \lambda_t^{\mathrm{D}}(1 - r_t^+)\Delta_t, & \Delta_t \geq 0 \\ -\lambda_t^{\mathrm{D}}(r_t^- - 1)\Delta_t, & \Delta_t < 0. \end{cases} \tag{6.15}$$

The term $\lambda_t^{\mathrm{D}} E_t$ in (6.14) constitutes the maximum level of revenue that the wind power producer could collect from trading its energy production in a situation free of wind uncertainty, that is, in a position with perfect information on its future wind generation. The term C_t represents the aforementioned opportunity cost, which results from trading the energy deviations in the balancing market at a less attractive price. This opportunity cost is usually referred to as *imbalance cost* and can be also interpreted as the cost of imperfect wind power forecasts. In the specific case for which a market participant is not charged for its deviation because this deviation helps to restore the system balance, it holds that C_t is zero. In addition, note that, since the term $\lambda_t^{\mathrm{D}} E_t$ is an uncontrollable component within the revenue expression (6.14), maximizing R_t boils down to minimizing the imbalance cost C_t.

The definition of imbalance cost becomes somewhat unclear if an adjustment market is taken into account in the previous analysis due to the price differences in the day-ahead and adjustment markets. Notwithstanding this complication, it is clear that, even if an adjustment market is available, every market agent forced to correct its energy deviations in the balancing market incurs an opportunity cost.

We conclude this section pointing out that, as the generation cost of a wind power producer is zero (if we neglect operating expenses), revenue equals profit.

The example below illustrates the computation of the imbalance cost incurred by a wind power producer and the revenues that it obtains from the electricity trading.

Illustrative Example 6.3 (Revenues and imbalance cost). Suppose a wind power producer that offers and gets accepted 150 MWh of energy in the day-ahead market at a price \$40/MWh for a given time period t. However, it finally produces 100 MWh in that period, so it needs to buy 50 MWh in the balancing market to cover its lack of generation. If the imbalance price ratios r_t^- and r_t^+ are equal to 1.5 and 1, respectively, the imbalance cost incurred by the wind power producer in period t is given by

$$C_t = -\lambda_t^{\mathrm{D}}(r_t^- - 1)\Delta_t = -40 \times (1.5 - 1) \times (-50) = \$1000,$$

and, according to (6.14), the revenues obtained from the energy transaction in this period is

$$R_t = 40 \times 150 - 1000 = \$5000.$$

We can deduce that the net imbalance of the system in period t is negative from the fact that r_t^- is greater than 1 and r_t^+ is equal to 1. □

6.3.3 Certainty Gain Effect

Adjustments markets are short-term trading floors intended to allow market agents to modify their production or consumption schedule in response to previously obtained market results that are unfavorable or unforseen events, e.g., unexpected equipment failures or sudden changes in loads' behavior. For this reason, these markets are organized after the clearing of the day-ahead market and may cover the whole energy delivery horizon or just a part of it. Apart from this service, adjustment markets provide market participants with additional trading opportunities to make more profit by taking advantage of the energy price differences existing between this market and the day-ahead market.

As indicated in Section 6.2, for simplicity, we solely work with one adjustment market cleared just before the beginning of the energy delivery horizon and covering all of it. Fig. 6.7 illustrates the temporal position of this trading floor within the considered market structure.

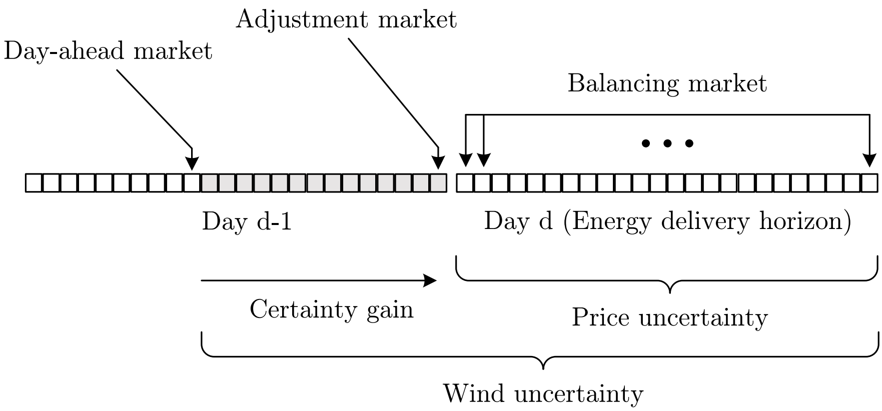


Fig. 6.7 Certainty gain on wind power stochastic behavior

Adjustment markets are especially useful for wind power producers, because these markets are cleared closer to the delivery time, and therefore, energy trading in them can be carried out with a diminished level of uncertainty as for the availability of wind energy. Wind production volatility is strongly dependent on the magnitude of the time interval up to its realization. For example, the forecasts of wind one hour ahead are much more accurate than the forecasts available forty hours ahead. In the particular case represented in Fig. 6.7, this means that the likely realization of wind during the hours comprising the energy delivery horizon is better known in the adjustment market than in the day-ahead market. We say that some certainty on wind power stochastic behavior is gained during the time periods in between the closures of these two markets. This certainty gain is a phenomenon inherent to the physics of the short-term trading problem of a wind power producer, and as such, it exists irrespective of the approach used to tackle its solution.

When it comes to trade in the day-ahead market, the wind power producer should account for this phenomenon to build its strategy offer, because its consideration is expected to have an impact on the decisions to be made in this market. This impact is what we call *certainty gain effect*.

Formally, the certainty gain effect can be defined as the economic surplus that the wind power producer obtains if it builds its offering strategy for the day-ahead market considering that the subsequent energy trading in the adjustment market can be done with a greater level of certainty on its future wind energy production.

From a mathematical point of view, the consideration of such a wind certainty gain affects the modeling of the wind producer problem in two aspects: i) the generation of wind power scenarios, which must reflect a level of wind volatility in the adjustment market lower than that in the day-ahead market (see Subsection 6.4.2), and ii) the constraints pertaining to the decision-making in the adjustment market (see Subsection 6.5.4).

The physical phenomenon described above can be also extended to the stochastic processes describing energy prices in the balancing market, although such an extension is not made in this chapter for simplicity. Notwithstanding this, we should point out that the modeling of the certainty gain effect related to both wind production and imbalance prices are mathematically analogous.

6.4 Uncertainty Characterization

The wind producer problem described above is subject to four sources of uncertainty, namely, the day-ahead market price, the adjustment market price, the prices for imbalances and the wind power generation. Among these sources of uncertainty, the one associated with the wind power production constitutes the keystone of the decision-making process. Given that the wind production is characterized by a zero generation cost, its unpredictable nature is directly responsible for the likely loss of gains incurred by the wind power producer through the imbalance cost.

In order to account for the impact of these sources of uncertainty on the wind producer problem, we propose to characterize them as stochastic processes, which are conveniently discretized in the form of scenarios as stated in Chapter 3.

6.4.1 Day-ahead, Adjustment, and Imbalance Prices

The day-ahead market price is mathematically treated as a stochastic process $\boldsymbol{\lambda}^{\mathrm{D}}$ spanning the N_{T} periods of the market horizon, i.e., $\boldsymbol{\lambda}^{\mathrm{D}} = \{\lambda_t^{\mathrm{D}}, t = 1, 2, \ldots, N_{\mathrm{T}}\}$. This stochastic process is represented through a finite set of N_{D} scenarios.

Analogously, the adjustment market price can be modeled as a stochastic process $\boldsymbol{\lambda}^{\mathrm{A}} = \{\lambda_t^{\mathrm{A}}, t = 1, 2, \ldots, N_{\mathrm{T}}\}$, which is dicretized in N_{A} scenarios.

In practice, prices $\boldsymbol{\lambda}^{\mathrm{D}}$ and $\boldsymbol{\lambda}^{\mathrm{A}}$ in the day-ahead and adjustment markets, respectively, are unquestionably correlated. The existing relationship is direct (or positive), that is, periods of high day-ahead price realizations often coincide with periods of high adjustment price values. This strong correlation may have a significant impact on the decisions to be made by market participants concerning energy trading and risk management, and therefore, should not be hastily ignored.

To deal with the statistical correlation between $\boldsymbol{\lambda}^{\mathrm{D}}$ and $\boldsymbol{\lambda}^{\mathrm{A}}$, one of the procedures described in Chapter 3 for the modeling and analysis of multivariate stochastic processes can be used. Nevertheless, a simple alternative to these more sophisticated methods consists in working on the price difference $\Delta\boldsymbol{\lambda}$ between markets, i.e.,

$$\boldsymbol{\lambda}^{\mathrm{A}} = \boldsymbol{\lambda}^{\mathrm{D}} + \Delta\boldsymbol{\lambda}, \tag{6.16}$$

where the price difference $\Delta\boldsymbol{\lambda}$ is an stochastic process $\Delta\boldsymbol{\lambda} = \{\Delta\lambda_t = \lambda_t^{\mathrm{A}} - \lambda_t^{\mathrm{D}}, t = 1, 2, \ldots, N_{\mathrm{T}}\}$ discretized in N_{Δ} scenarios.

Thus, the N_{A} adjustment market price scenarios are obtained from the scenarios corresponding to the day-ahead price and the price difference as expressed in (6.16). It holds that $N_{\mathrm{A}} = N_{\mathrm{D}} \times N_{\Delta}$. The following illustrative example is intended to clarify this issue.

Illustrative Example 6.4 (Day-ahead and adjustment market price scenarios). Consider an electricity market with the short-term trading structure represented in Fig. 6.8.

The market horizon only spans one hour. Consequently, subscript t can be omitted. The day-ahead market is cleared one hour in advance, while the adjustment market closes just before the single delivery period. Note that the name *day-ahead* market is somewhat misleading here. Rather, it should be called *1-hour ahead* market. Finally, in the balancing market, the energy deviations incurred by market participants during the 1-hour market horizon are covered and priced accordingly.

Let us assume that:

1. The day-ahead market price λ^{D} is equal to \$50/MWh with a probability of 0.40 and equal to \$20/MWh with a probability of 0.60.
2. The price difference $\Delta\lambda$ between markets is minus 10\$/MWh with a probability of 0.35 and \$3/MWh with a probability of 0.65.

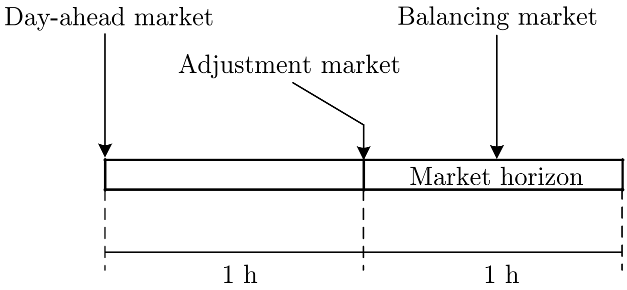


Fig. 6.8 Illustrative Example 6.4: market structure and time framework

Therefore, the adjustment market price λ^{A} can adopt the following four possible values:

1. λ^{A} = \$50/MWh - \$10/MWh = \$40/MWh with a probability $0.40 \times 0.35 = 0.14$.
2. λ^{A} = \$50/MWh + \$3/MWh = \$53/MWh with a probability $0.40 \times 0.65 = 0.26$.
3. λ^{A} = \$20/MWh - \$10/MWh = \$10/MWh with a probability $0.60 \times 0.35 = 0.21$.
4. λ^{A} = \$20/MWh + \$3/MWh = \$23/MWh with a probability $0.60 \times 0.65 = 0.39$.

□

For each period t of the market horizon, we have two imbalance prices, namely, λ_t^+ and λ_t^-, for positive and negative energy deviations incurred in that period, respectively. The two series of values describing the realizations of these prices throughout the market horizon constitute each a stochastic process. However, these series are deterministically interrelated due to the rules governing the mechanism for imbalance price formation explained in Subsection 6.3.1. These rules can be easily summarized by considering the imbalance price ratios r_t^+ and r_t^- instead of the actual imbalance prices λ_t^+ and λ_t^-:

1. If $r_t^+ < 1$, then the net system balance is positive, and as a result, it holds that $r_t^- = 1$.
2. If $r_t^- > 1$, then the net system imbalance is negative, and consequently, $r_t^+ = 1$.
3. If the net system imbalance is zero, then it follows that $r_t^- = r_t^+ = 1$.

We can create a new random variable r_t as

$$r_t = \begin{cases} r_t^+, & \text{if } r_t^+ \neq 1, \\ r_t^-, & \text{if } r_t^+ = 1, \end{cases} \tag{6.17}$$

which naturally incorporates the three previous rules in its definition. In addition, it turns out that random variable r_t can be expressed linearly as $r_t = r_t^+ + r_t^- - 1$. Such a deduction is straightforward and is graphically illustrated in Fig. 6.9. Moreover, note that random variable r_t does not share the pronounced discrete character of the imbalance prices ratios r_t^+ and r_t^-, which, by definition, are likely to exhibit "flat" behavior. Hence, stochastic process r_t is easier to model by employing the time-series-based techniques presented in Chapter 3.

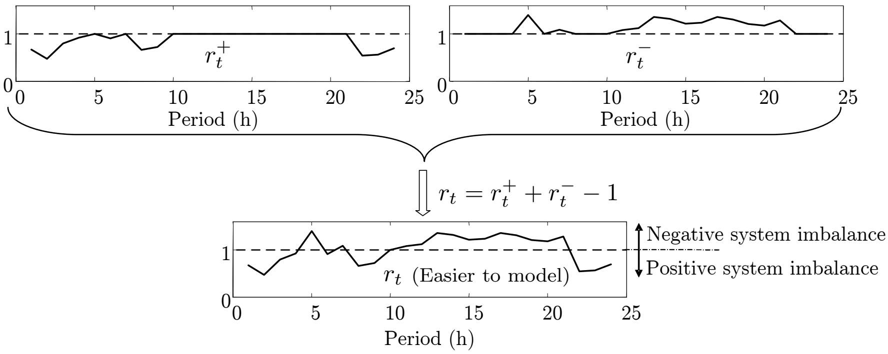


Fig. 6.9 Formation of random variable r_t from the imbalance price ratios r_t^+ and r_t^-

The stochastic process $\boldsymbol{r}$ defined as the collection of random variables r_t throughout the market horizon, i.e., $\boldsymbol{r} = \{r_t, t = 1, 2, \ldots, N_\mathrm{T}\}$, is represented by a set of N_I scenarios. From each one of these scenarios, a series of pairs (r_t^+, r_t^-), $\forall t = 1, 2, \ldots, N_\mathrm{T}$, is obtained by applying definition (6.17). The example below illustrates this issue.

Illustrative Example 6.5 (Imbalance prices). Consider again the market structure represented in Fig. 6.8.

Suppose that random variable r is conveniently discretized in the following two scenarios:

1. Variable r is equal to 1.5 with a probability of 0.7.
2. Variable r is equal to 0.9 with a probability of 0.3.

According to definition (6.17), the above description is equivalent to the next one:

1. Pair (r^+, r^-) is equal to $(1, 1.5)$ with a probability of 0.7.
2. Pair (r^+, r^-) is equal to $(0.9, 1)$ with a probability of 0.3.

$\square$

6.4.2 Wind Power Production

Similarly to market prices, wind power production is modeled as a stochastic process $\boldsymbol{P} = \{P_t, t = 1, 2, \ldots, N_\mathrm{T}\}$ represented by a finite set of N_P scenarios.

An appropriate modeling of the uncertainty characterizing the wind producer problem should include the knowledge of the wind generation stochastic process gained during the time interval between the closures of the day-ahead and adjustment markets. As explained in Subsection 6.3.3, this gain of certainty is due to the fact that the energy selling/buying offer to submit in the adjustment market can be decided taking into account the realized values of the wind generation stochastic process for the time periods after the day-ahead market clearing. In order to reflect this circumstance within the scenario representation, we propose to generate N_P wind power scenarios as follows:

1. Simulate N_{P_1} wind power scenarios for the periods making up the time interval between the closures of the day-ahead and adjustment markets. In other words, if such time periods are indexed by τ, our aim in this step is then to create N_{P_1} series of wind power values in the form $\{P_\tau, \tau = 1, 2, \ldots, N_{\mathrm{T}_1}\}$, where N_{T_1} is the number of periods comprising the time lapse between the day-ahead and adjustment market-clearing processes.
2. For each one of the wind power scenarios generated in the previous step, simulate N_{P_2} wind power scenarios for the time interval spanning from the closure of the adjustment market and the end of the market horizon. Therefore, this second step yields $N_\mathrm{P} = N_{\mathrm{P}_1} \times N_{\mathrm{P}_2}$ series of wind power values in the form $\{P_t, t = 1, 2, \ldots, N_\mathrm{T}\}$, where t is the index of time periods covering the market horizon.

The roles played by indexes τ and t in the above process are graphically clarified in Fig. 6.10.

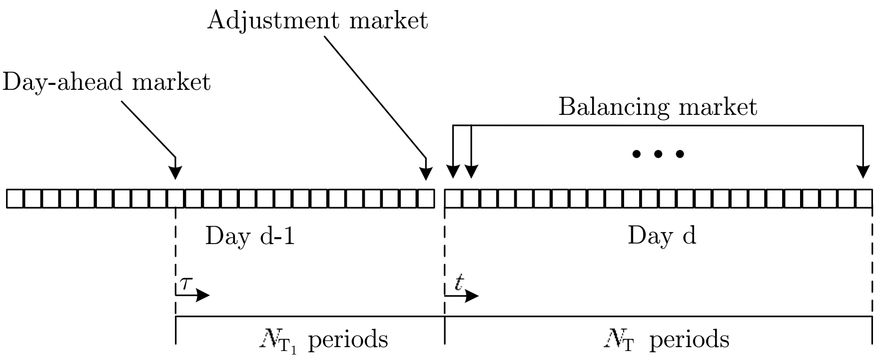


Fig. 6.10 Graphical illustration of the time periods covered by indexes τ and t

It is important to stress that, in order for the certainty gain effect to be properly embedded into the scenario representation of the wind power production, the scenario-generation process carried out in step 2 above must be somehow *fed* with the wind power scenarios previously built in step 1. The following three illustrative examples serve us to provide some insight into this relevant issue.

Illustrative Example 6.6 (Wind power scenarios. Experience-based approach). Consider the market structure represented in Fig. 6.8.

Suppose that the amount of power produced by a certain 100-MW wind farm during the hour between the closures of the day-ahead and adjustment markets ($\tau = 1$) can be described by means of two scenarios ($N_{\mathrm{P}_1} = 2$), namely, scenarios *high* and *low* corresponding to wind power values of 60 MW and 30 MW, respectively.

On the other hand, the amount of power that the wind farm is expected to produce during the hour comprising the market horizon ($t = 1$) is contingent on the scenario, *high* or *low*, realized in the previous hour. Such a dependency can be expressed as follows:

1. If the wind power production in $\tau = 1$ is *high* (60 MW), then the amount of power produced in $t = 1$ (market horizon) can be *extremely high* (100 MW) or *relatively high* (50 MW), with probabilities 0.1 and 0.3, respectively.
2. If the wind power production in $\tau = 1$ is *low* (30 MW), then the amount of power produced in $t = 1$ (market horizon) can be *extremely low* (0 MW) or *relatively low* (40 MW), with probabilities 0.35 and 0.25, respectively.

Therefore, the wind power production during the 1-hour market horizon can be statistically represented by means of four scenarios, namely, scenario *extremely high* (100 MW), scenario *relatively high* (50 MW), scenario *relatively low* (30 MW), and scenario *extremely low* (0 MW) with probabilities 0.1, 0.3, 0.25, and 0.35 in that order.

□

Illustrative Example 6.7 (Wind power scenarios. Time-series-based approach). Suppose in this case that the amount of power P_t (in MW) that the wind farm in the previous example produces during the 1-hour market horizon is given by the following autoregressive model,

$$P_t = C + 0.9P_\tau + \varepsilon_t,$$

where C is a constant equal to 5 MW, P_τ is the amount of power produced in between the closures of the day-ahead and adjustment markets, and ε_t is a white noise with standard deviation equal to 15 MW.

Let us assume that we need to describe the wind power production in period t with four scenarios. We distinguish two cases:

1. P_τ is *high* (60 MW). We produce two samples from ε_t, e.g., 10 MW and -16 MW. Thus:

   a. $P_t = 5 + 0.9 \times 60 + 10 = 69$ MW;
   b. $P_t = 5 + 0.9 \times 60 + -16 = 43$ MW.

2. P_τ is *low* (30 MW). Again, we generate two samples from ε_t, e.g., -8 MW and 3 MW. Thus:

   a. $P_t = 5 + 0.9 \times 30 - 8 = 24$ MW;
   b. $P_t = 5 + 0.9 \times 30 + 3 = 35$ MW.

Therefore, on the assumption that scenarios *high* and *low* describing the stochastic behavior of P_τ are equiprobable, the four scenarios modeling the wind power produced in the 1-hour market horizon, P_t, are 69, 43, 24, and 35 MW, all of them with a probability of 0.25. □

Illustrative Example 6.8 (Wind power scenarios. Graphic example). Fig. 6.11 shows a possible wind power scenario set for the market structure and time framework described in Fig. 6.2 of Illustrative Example 6.1. This set is made up of $N_{\mathrm{P}} = N_{\mathrm{P}_1} \times N_{\mathrm{P}_2} = 2 \times 50 = 100$ wind power scenarios generated using a time-series-based approach similar to that exemplified in Illustrative Example 6.7. Note that from each one of the 2 scenarios modeling the wind power production during the 14 hours in between the closures of the day-ahead and adjustment markets, a new bunch of 50 scenarios representing plausible wind power realizations throughout the market horizon springs up.

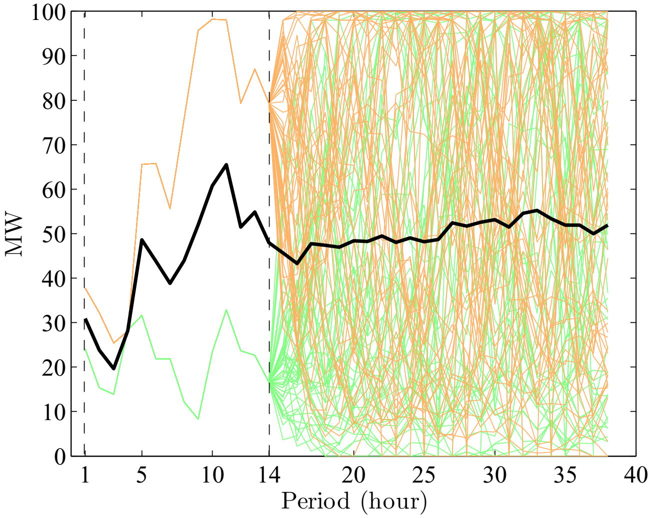
The bold lines in Figs. 6.11(a) and 6.11(b) represent the series of wind power expected values observed from the day-ahead and adjustment markets, respectively. Note that the view of the wind producer about its future production throughout the market horizon is more accurate in the adjustment market than in the day-ahead market, especially, for the first few hours. This is so because the wind power scenarios split up into two subsets at the clearing time of the adjustment market in order to incorporate the information on the wind power values realized during the periods in between both markets. □

6.4.3 Scenario Tree

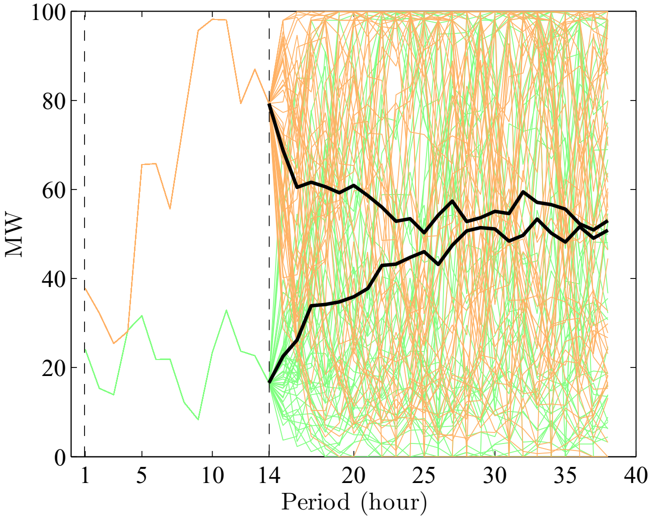
The sets of scenarios characterizing the uncertainty associated with market prices and wind generation in the wind producer problem can be arranged in a symmetric scenario tree with the form depicted in Fig. 6.12.

Specifically, this tree is built as follows:

1. Generate N_{D} price scenarios for the day-ahead market.



(a) Wind power average values observed from the day-ahead market



(b) Wind power average values observed from the adjustment market

Fig. 6.11 Illustrative Example 6.8: wind power scenario set with structure $N_{\mathrm{P}} = N_{\mathrm{P}_1} \times N_{\mathrm{P}_2} = 2 \times 50$

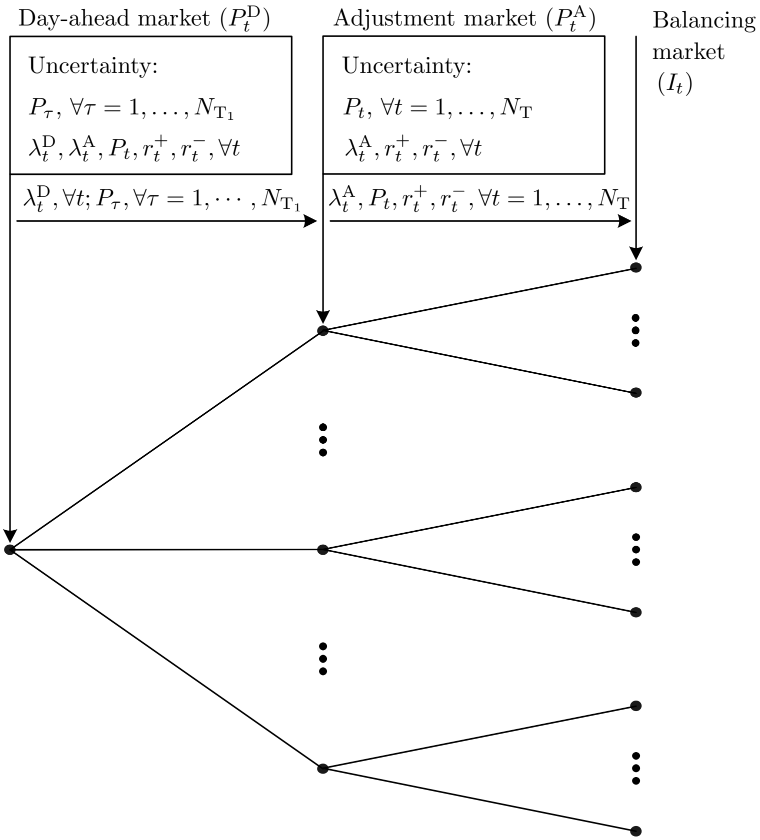


Fig. 6.12 Wind producer: scenario tree

2. For each realization of the day-ahead market prices, simulate N_Δ scenarios modeling the difference between the day-ahead and adjustment market prices.
3. For each scenario of the adjustment market prices, generate N_{P} wind power realizations.
4. Finally, for each wind power realization, simulate N_{I} imbalance price ratio scenarios.

Each scenario ω in the tree so built is made up of a set of vectors representing plausible realizations of the stochastic processes involved in the wind producer problem, namely, market prices and wind generation. Mathematically, this can be expressed as:

$$\text{Scenario } \omega = \{\lambda^{\mathrm{D}}_{t\omega}, P_{\tau\omega}, \lambda^{\mathrm{A}}_{t\omega}, P_{t\omega}, r^{+}_{t\omega}, r^{-}_{t\omega}\}, \qquad \forall t = 1, \ldots, N_{\mathrm{T}}, \forall \tau = 1, \ldots, N_{\mathrm{T}_1}, \quad (6.18)$$

where $\lambda^{\mathrm{A}}_{t\omega} = \lambda^{\mathrm{D}}_{t\omega} + \Delta\lambda_{t\omega}$.

Likewise, each scenario ω in the tree has a probability of occurrence π_ω computed as the product of the probabilities associated with vectors or pairs of vectors $\lambda^{\mathrm{D}}_{t\omega}$, $\Delta\lambda_{t\omega}$, $(P_{\tau\omega}, P_{t\omega})$, and $(r^{+}_{t\omega}, r^{-}_{t\omega})$. Needless to say, the summation of all these probabilities π_ω over the whole set of scenarios Ω must be equal to 1, i.e., $\sum_{\omega=1}^{N_\Omega} \pi_\omega = 1$. The total number of scenarios composing the tree is $N_\Omega = N_{\mathrm{D}} N_\Delta N_{\mathrm{P}} N_{\mathrm{I}}$.

We provide below an example of scenario tree for the wind producer problem.

Illustrative Example 6.9 (Scenario tree). The scenario tree built from the scenario representation of the day-ahead market price, the price difference between day-ahead and adjustment markets, the imbalance price ratios and the wind power production provided in Illustrative Examples 6.4–6.6 is shown in Fig. 6.13. Prices are in \$/MWh, powers in MW and ratios are non-dimensional numbers.

Some of the probabilities π_ω are computed below for the sake of illustration. The rest of them are shown in the aforementioned figure. The computed ones are

$$\begin{aligned}
\pi_1 &= 0.40 \times 0.35 \times 0.10 \times 0.70 = 0.0098 \\
\pi_2 &= 0.40 \times 0.35 \times 0.10 \times 0.30 = 0.0042 \\
&\vdots \\
\pi_{15} &= 0.40 \times 0.65 \times 0.25 \times 0.70 = 0.0455 \\
\pi_{16} &= 0.40 \times 0.65 \times 0.25 \times 0.30 = 0.0195 \\
&\vdots \\
\pi_{31} &= 0.60 \times 0.65 \times 0.25 \times 0.70 = 0.06825 \\
\pi_{32} &= 0.60 \times 0.65 \times 0.25 \times 0.30 = 0.02925.
\end{aligned}$$

□

It is important to point out that the procedure described above to build the scenario tree implicitly assumes that market prices and wind production are independent stochastic processes. If, on the contrary, they are correlated, the scenario tree construction calls for a more complex joint scenario-generation process for market prices and wind production. Chapter 3 provides further information on the type of methodologies able to create scenarios preserving dependencies among stochastic processes.

Lastly, the scenario tree is fed into a three-stage stochastic programming model wherein each stage represents a market. The sequence of stages and decisions is as follows:

1. Design the offer strategy for the day-ahead market and submit the resulting energy selling offers to this market for each period of the market

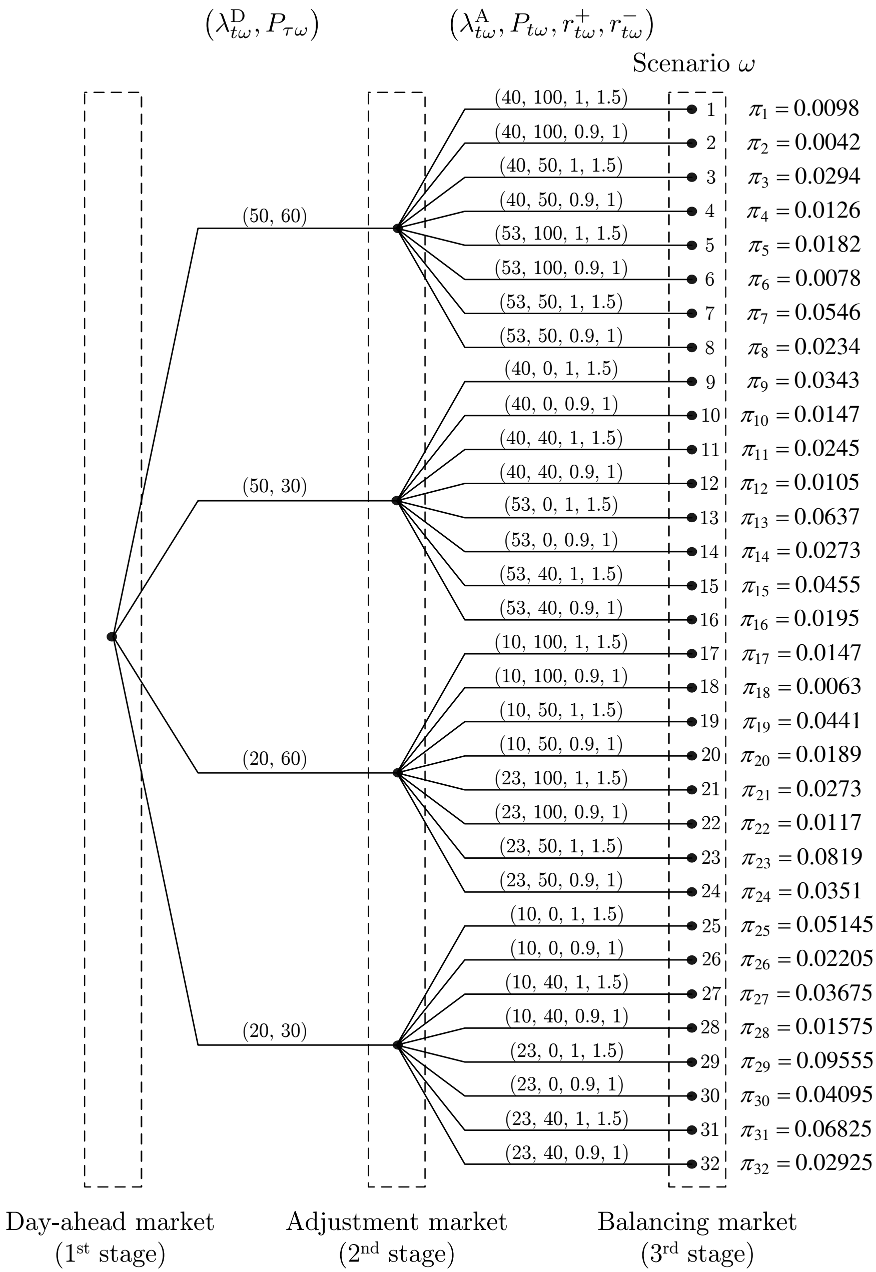


Fig. 6.13 Illustrative Example 6.9: scenario tree

horizon. In this stage, decisions are made based on plausible realizations of the stochastic processes involved, namely, market prices (day-ahead, adjustment, and balancing) and wind power production.

2. Once the day-ahead market price is known for each time period, decide the amount of energy to sell/buy in/from the adjustment market. Specifically, at the moment that this decision is made, the 24 hourly day-ahead market prices and the wind power generated in the time periods between the closures of the day-ahead and adjustment markets are known. On the contrary, the adjustment and balancing market prices, and the wind power production for the rest of the market horizon are still uncertain. Therefore, in this second stage, and for every day-ahead market price realization, decisions are made based on plausible scenarios of adjustment and balancing prices and of wind power production. Note that the wind production scenarios, spanning the whole market horizon and available at this second stage, are generated knowing the actual wind production in the periods between the closures of the day-ahead and adjustment markets.
3. The last stage of the stochastic programming approach is determined by the materialization of the adjustment market prices, the imbalance prices and the wind power generated in the time periods spanning the whole market horizon. Thus, the deviation incurred by the wind power producer in each one of these periods is known and the consequent cost for imbalance can be computed.

In the following section, we analyze in detail the three-stage stochastic programming model used in this chapter to solve the wind producer problem.

6.5 Wind Producer Model

In this section, we describe the mathematical modeling of the wind power producer problem in detail. We start with the basics and we finish with its complete formulation.

6.5.1 Basic Model

Firstly, no adjustment market is considered. Then, the optimization problem aimed to maximize the expected profit of a wind power producer trading its production in the day-ahead and balancing markets can be formulated as follows,

$$\text{Maximize}_{P_t^{\mathrm{D}},\forall t;\varDelta_{t\omega},\forall t,\forall\omega}$$
$$\xi\{R\} = \sum_{\omega=1}^{N_\Omega}\sum_{t=1}^{N_\mathrm{T}} \pi_\omega\big(\lambda_{t\omega}^{\mathrm{D}}\ P_t^{\mathrm{D}}\ d_t + I_{t\omega}\big) \tag{6.19}$$

subject to

$$I_{tw} = \begin{cases} \lambda_{t\omega}^{\mathrm{D}}\ r_{t\omega}^{+}\ \varDelta_{t\omega},\ \varDelta_{t\omega} \geq 0 \\ \lambda_{t\omega}^{\mathrm{D}}\ r_{t\omega}^{-}\ \varDelta_{t\omega},\ \varDelta_{t\omega} < 0 \end{cases},\ \forall t,\forall\omega \tag{6.20}$$

$$0 \leq P_t^{\mathrm{D}} \leq P^{\max},\ \forall t \tag{6.21}$$
$$\varDelta_{t\omega} = d_t(P_{t\omega} - P_t^{\mathrm{D}}),\ \forall t,\forall\omega. \tag{6.22}$$

The objective function (6.19) represents the expected profit obtained by the wind power producer from trading its production in the day-ahead and balancing markets. As explained in Subsection 6.3.2, the profit equals the revenue, and consequently, maximizing the expected profit boils down to maximizing the expected revenue. Equation (6.20) constitutes the definition, per period and scenario, of the imbalance income function. Constraints (6.21) limit the amount of power that can be sold in the day-ahead market to the installed capacity of the wind farm, and equation (6.22) defines the total deviation incurred by the wind producer in each period and scenario.

Problem (6.19)–(6.22) cannot be solved through optimization techniques as it stands, since the imbalance income $I_{t\omega}$ is a piecewise function. A simple way to circumvent this hurdle is to define a binary variable y_{tw}, per period and scenario, indicating the direction, positive or negative, of the energy deviation. Thus, problem (6.19)–(6.22) can be recast as

$$\text{Maximize}_{P_t^{\mathrm{D}},\forall t;\varDelta_{t\omega},\forall t,\forall\omega}$$
$$\xi\{R\} = \sum_{\omega=1}^{N_\Omega}\sum_{t=1}^{N_\mathrm{T}} \pi_\omega\big(\lambda_{t\omega}^{\mathrm{D}}\ P_t^{\mathrm{D}}\ d_t + \lambda_{t\omega}^{\mathrm{D}}\ r_{t\omega}^{+}\ \varDelta_{t\omega}(1-y_{t\omega}) + \lambda_{t\omega}^{\mathrm{D}}\ r_{t\omega}^{-}\ \varDelta_{t\omega}y_{t\omega}\big) \tag{6.23}$$

subject to

$$0 \leq P_t^{\mathrm{D}} \leq P^{\max},\ \forall t \tag{6.24}$$
$$\varDelta_{t\omega} = d_t(P_{t\omega} - P_t^{\mathrm{D}}),\ \forall t,\forall\omega \tag{6.25}$$
$$\varDelta_{t\omega} \leq M(1-y_{t\omega}),\ \forall t,\forall\omega \tag{6.26}$$
$$-\varDelta_{t\omega} \leq My_{t\omega},\ \forall t,\forall\omega \tag{6.27}$$
$$y_{t\omega} \in \{0,1\},\ \forall t,\forall\omega, \tag{6.28}$$

where M is a large positive number exceeding any maximum feasible value of $|\varDelta_{t\omega}|$. Note that $y_{t\omega}$ is equal to 1 if the energy deviation incurred by the wind power producer in period t under scenario ω is negative (generation shortage), and 0 otherwise.

Problem (6.23)–(6.28) is both integer, because of the use of binary variables $y_{t\omega}$, and non-linear, due to the product of variables $\Delta_{t\omega}(1-y_{t\omega})$ and $\Delta_{t\omega}y_{t\omega}$ appearing in the objective function. Finding the optimum of a mixed-integer non-linear programming problem is, in general, complex owing to the lack of theoretical results guaranteeing its existence and uniqueness. Fortunately for us, problem (6.23)–(6.28) can be easily transformed into a mixed-integer linear one. To this end, we decompose the total energy imbalance $\Delta_{t\omega}$ into the sum of a positive and a negative imbalances, $\Delta^{+}_{t\omega}$ and $\Delta^{-}_{t\omega}$, respectively. Thanks to this simple decomposition, the previous products of variables are not needed anymore, as illustrated in the formulation below,

$$\text{Maximize}_{P^{\mathrm{D}}_t,\forall t;\Delta^{+}_{t\omega},\forall t,\forall\omega;\Delta^{-}_{t\omega},\forall t,\forall\omega}$$

$$\xi\{R\} = \sum_{\omega=1}^{N_\Omega}\sum_{t=1}^{N_\mathrm{T}} \pi_\omega \left(\lambda^{\mathrm{D}}_{t\omega}P^{\mathrm{D}}_t d_t + \lambda^{\mathrm{D}}_{t\omega}r^{+}_{t\omega}\Delta^{+}_{t\omega} - \lambda^{\mathrm{D}}_{t\omega}r^{-}_{t\omega}\Delta^{-}_{t\omega}\right) \tag{6.29}$$

subject to

$$0 \le P^{\mathrm{D}}_t \le P^{\max},\ \forall t \tag{6.30}$$

$$\Delta_{t\omega} = d_t(P_{t\omega} - P^{\mathrm{D}}_t),\ \forall t,\forall\omega \tag{6.31}$$

$$\Delta_{t\omega} = \Delta^{+}_{t\omega} - \Delta^{-}_{t\omega},\ \forall t,\forall\omega \tag{6.32}$$

$$0 \le \Delta^{+}_{t\omega} \le M_1(1-y_{t\omega}),\ \forall t,\forall\omega \tag{6.33}$$

$$0 \le \Delta^{-}_{t\omega} \le M_2\ y_{t\omega},\ \forall t,\forall\omega \tag{6.34}$$

$$y_{t\omega} \in \{0,1\},\ \forall t,\forall\omega, \tag{6.35}$$

where M_1 and M_2 are large positive scalars that exceed any maximum feasible value of $\Delta^{+}_{t\omega}$ and $\Delta^{-}_{t\omega}$, respectively. Through simple reasoning, we can find appropriate values for constants M_1 and M_2: maximum positive energy deviations occur in scenarios where the wind power producer does not sell any amount of energy in the day-ahead market, $P^{\mathrm{D}}_t = 0$, for a given time period t, but it eventually produces $P_{t\omega}$ MW of wind power during that period. Therefore, we can fix M_1 to $P_{t\omega}$. Similarly, maximum negative energy deviations happen in scenarios where the wind power producer sells its full capacity in the day-ahead market, $P^{\mathrm{D}}_t = P^{\max}$, for a period t, but its final production in that period is nil, $P_{t\omega} = 0$. Hence, we can make M_2 equal to $P^{\max}$.

Optimization problem (6.29)–(6.35) naturally tends to minimize the imbalance cost as underlined in Subsection 6.3.2. As a result, given the total energy deviation $\Delta_{t\omega} = \Delta^{+}_{t\omega} - \Delta^{-}_{t\omega}$ incurred by the wind power producer in period t and scenario ω, the solution of problem (6.29)–(6.35) is guaranteed to be achieved always with one of the variables $\Delta^{+}_{t\omega}$ or $\Delta^{-}_{t\omega}$ equal to zero due to the fact that $r^{+}_{t\omega} \le 1$ and $r^{-}_{t\omega} \ge 1$. That is, any other feasible solution for which $\Delta_{t\omega}^{+}$ and $\Delta_{t\omega}^{-}$ are simultaneously different from zero in period t and scenario ω is economically less profitable. Consequently, binary variables $y_{t\omega}$ are in fact not necessary, and thus, the mixed-integer programming problem (6.29)–(6.35) can be equivalently transformed into a linear one with the consequent gain in robustness, simplicity, and computational efficiency. The basic linear formulation of the wind producer problem is presented below,

$$\text{Maximize}_{P_t^{\mathrm{D}},\forall t;\Delta_{t\omega}^{+},\forall t,\forall\omega;\Delta_{t\omega}^{-},\forall t,\forall\omega}$$

$$\xi\{R\} = \sum_{\omega=1}^{N_{\Omega}} \sum_{t=1}^{N_{\mathrm{T}}} \pi_{\omega}\left(\lambda_{t\omega}^{\mathrm{D}} P_t^{\mathrm{D}} d_t + \lambda_{t\omega}^{\mathrm{D}} r_{t\omega}^{+} \Delta_{t\omega}^{+} - \lambda_{t\omega}^{\mathrm{D}} r_{t\omega}^{-} \Delta_{t\omega}^{-}\right) \tag{6.36}$$

subject to

$$0 \leq P_t^{\mathrm{D}} \leq P^{\max}, \ \forall t \tag{6.37}$$

$$\Delta_{t\omega} = d_t(P_{t\omega} - P_t^{\mathrm{D}}), \ \forall t, \forall\omega \tag{6.38}$$

$$\Delta_{t\omega} = \Delta_{t\omega}^{+} - \Delta_{t\omega}^{-}, \ \forall t, \forall\omega \tag{6.39}$$

$$0 \leq \Delta_{t\omega}^{+} \leq P_{t\omega} d_t, \ \forall t, \forall\omega \tag{6.40}$$

$$0 \leq \Delta_{t\omega}^{-} \leq P^{\max} d_t, \ \forall t, \forall\omega. \tag{6.41}$$

Model (6.36)-(6.41) can be solved by decomposition into N_{T} optimization problems, one for each time period t, because it does not contain any constraint or any decision variable coupling different time periods, i.e., it does not include any *inter-temporal* constraint or variable.

Next, we introduce a small example to illustrate the linear formulation of the basic wind producer problem.

Illustrative Example 6.10 (linear formulation of the wind producer problem). Consider the market structure and time framework described in Fig. 6.8. For the scenario tree presented in Illustrative Example 6.9 and depicted in Fig. 6.13, model (6.36)-(6.41) becomes

$$
\begin{aligned}
&\text{Maximize}_{P^{\mathrm{D}};\Delta_\omega^+,\Delta_\omega^-,\forall\omega} \\
&\xi\{R\} = (0.4 \times \$50/\text{MWh} + 0.6 \times \$20/\text{MWh})P^{\mathrm{D}} + \\
&+ \$50/\text{MWh}\ [(0.0098\Delta_1^+ + 0.0294\Delta_3^+ + \ldots + 0.0455\Delta_{15}^+) \\
&- 1.5(0.0098\Delta_1^- + 0.0294\Delta_3^- + \ldots + 0.0455\Delta_{15}^-)] \\
&+ \$50/\text{MWh}\ [0.9(0.0042\Delta_2^+ + 0.0126\Delta_4^+ + \ldots + 0.0195\Delta_{16}^+) \\
&- (0.0042\Delta_2^- + 0.0126\Delta_4^- + \ldots + 0.0195\Delta_{16}^-)] \\
&+ \$20/\text{MWh}\ [(0.0147\Delta_{17}^+ + 0.0441\Delta_{19}^+ + \ldots + 0.06825\Delta_{31}^+) \\
&- 1.5(0.0147\Delta_{17}^- + 0.0441\Delta_{19}^- + \ldots + 0.06825\Delta_{31}^-)] \\
&+ \$20/\text{MWh}\ [0.9(0.0063\Delta_{18}^+ + 0.0189\Delta_{20}^+ + \ldots + 0.02925\Delta_{32}^+) \\
&- (0.0063\Delta_{18}^- + 0.0189\Delta_{20}^- + \ldots + 0.02925\Delta_{32}^-)]
\end{aligned}
$$

subject to

$$0 \le P^{\mathrm{D}} \le 100$$

$$
\begin{aligned}
\Delta_\omega &= 100 - P^{\mathrm{D}}, \omega = 1, 2, 5, 6, 17, 18, 21, \text{and } 22 \\
\Delta_\omega &= 50 - P^{\mathrm{D}}, \omega = 3, 4, 7, 8, 19, 20, 23, \text{and } 24 \\
\Delta_\omega &= 40 - P^{\mathrm{D}}, \omega = 11, 12, 15, 16, 27, 28, 31, \text{and } 32 \\
\Delta_\omega &= -P^{\mathrm{D}}, \omega = 9, 10, 13, 14, 25, 26, 29, \text{and } 30
\end{aligned}
$$

$$\Delta_\omega = \Delta_\omega^+ - \Delta_\omega^-,\ \forall\omega$$

$$
\begin{aligned}
0 \le \Delta_\omega^+ &\le 100, \omega = 1, 2, 5, 6, 17, 18, 21, \text{and } 22 \\
0 \le \Delta_\omega^+ &\le 50, \omega = 3, 4, 7, 8, 19, 20, 23, \text{and } 24 \\
0 \le \Delta_\omega^+ &\le 40, \omega = 11, 12, 15, 16, 27, 28, 31, \text{and } 32 \\
0 \le \Delta_\omega^+ &\le 0, \omega = 9, 10, 13, 14, 25, 26, 29, \text{and } 30
\end{aligned}
$$

$$0 \le \Delta_\omega^- \le 100\ \forall\omega.$$

□

6.5.2 Offering Curves

The mathematical model presented above is intended to obtain the optimal quantities (in a stochastic programming sense) to be sold in the day-ahead market (e.g., 20 MWh in hour 1, 65 MWh in hour 2, etc.). However, it may be convenient to derive optimal offering curves for every hour of the day-ahead market as done in Chapter 5 for a thermal producer. For this purpose, variables P_t^{D} representing the power traded in this market for each time period t are made dependent on scenarios ($P_t^{\mathrm{D}} \longrightarrow P_{tw}^{\mathrm{D}}$) and the following constraints are added to model (6.36)–(6.41),

$$P_{t\omega}^{\mathrm{D}} - P_{t\omega'}^{\mathrm{D}} \leq 0, \forall t, \forall \omega, \omega' : O(\lambda_{t\omega}^{\mathrm{D}}) + 1 = O(\lambda_{t\omega'}^{\mathrm{D}}) \tag{6.42}$$

$$P_{t\omega}^{\mathrm{D}} = P_{t\omega'}^{\mathrm{D}}, \forall t, \forall \omega, \omega' : \lambda_{t\omega'}^{\mathrm{D}} = \lambda_{t\omega}^{\mathrm{D}}. \tag{6.43}$$

Constraints (6.42) make offering curves non-decreasing, which is a requirement in most markets. Equations (6.43) are non-anticipativity constraints, which model the fact that only one offering curve can be submitted to the day-ahead market for each hour irrespective of the wind power and imbalance price realizations. That is, this equation enforces non-anticipativity for the wind power and imbalance price realizations. Note that this model is less constrained than the basic one. In fact, the basic model can be obtained from this one by rewriting the non-anticipativity constraints (6.43) as $P_{t\omega}^{\mathrm{D}} = P_{t\omega'}^{\mathrm{D}}, \forall t, \forall \omega, \omega'$.

Despite including the above constraints, the offer strategy model continues being decomposable by time period (hour).

The following example is intended to illustrate the constraints required to build offering curves for the day-ahead market.

Illustrative Example 6.11 (Offering curves). Let us consider the market structure and time framework shown in Fig. 6.8. For the scenario tree represented in Fig. 6.13, equations (6.42) and (6.43) render

$$P_{17}^{\mathrm{D}} - P_{1}^{\mathrm{D}} \leq 0,$$
$$P_{1}^{\mathrm{D}} = P_{2}^{\mathrm{D}} = P_{3}^{\mathrm{D}} = P_{4}^{\mathrm{D}} = P_{5}^{\mathrm{D}} = P_{6}^{\mathrm{D}} = P_{7}^{\mathrm{D}} = P_{8}^{\mathrm{D}} = P_{9}^{\mathrm{D}} = P_{10}^{\mathrm{D}} = P_{11}^{\mathrm{D}} = P_{12}^{\mathrm{D}} =$$
$$P_{13}^{\mathrm{D}} = P_{14}^{\mathrm{D}} = P_{15}^{\mathrm{D}} = P_{16}^{\mathrm{D}},$$
$$P_{17}^{\mathrm{D}} = P_{18}^{\mathrm{D}} = P_{19}^{\mathrm{D}} = P_{20}^{\mathrm{D}} = P_{21}^{\mathrm{D}} = P_{22}^{\mathrm{D}} = P_{23}^{\mathrm{D}} = P_{24}^{\mathrm{D}} = P_{25}^{\mathrm{D}} = P_{26}^{\mathrm{D}} = P_{27}^{\mathrm{D}} = P_{28}^{\mathrm{D}} =$$
$$P_{29}^{\mathrm{D}} = P_{30}^{\mathrm{D}} = P_{31}^{\mathrm{D}} = P_{32}^{\mathrm{D}}.$$

Note that we have made a smart use of the non-anticipativity constraints to implement the non-decreasing condition of the offering curves with just one constraint. □

6.5.3 Risk Modeling

A simple manner to control the risk of profit variability in the wind producer problem is to introduce the Conditional Value-at-Risk at the α confidence level (α-CVaR) in the problem formulation. As explained in Chapter 4 of this book, this metric can be expressed linearly within an optimization problem and exhibits good mathematical properties. If maximizing a discrete profit distribution, α-CVaR can be defined approximately as the expected profit of the $(1-\alpha)100\%$ scenarios with lowest profit.

The risk-constrained formulation of the problem faced by the wind producer is provided below,

$$\text{Maximize}_{P^{\mathrm{D}}_{t\omega},\forall t,\forall\omega;\Delta^{+}_{t\omega},\forall t,\forall\omega;\Delta^{-}_{t\omega},\forall t,\forall\omega;\eta_{\omega},\forall\omega;\zeta}$$

$$\xi\{R\}+\beta\left(\zeta-\frac{1}{1-\alpha}\sum_{\omega=1}^{N_{\Omega}}\pi_{\omega}\eta_{\omega}\right), \tag{6.44}$$

subject to constraints (6.37)–(6.43) plus the following ones required for the linear formulation of the CVaR,

$$-\sum_{t=1}^{N_{\mathrm{T}}}\left(\lambda^{\mathrm{D}}_{t\omega}P^{\mathrm{D}}_{t\omega}d_t+\lambda^{\mathrm{D}}_{t\omega}r^{+}_{t\omega}\Delta^{+}_{t\omega}-\lambda^{\mathrm{D}}_{t\omega}r^{-}_{t\omega}\Delta^{-}_{t\omega}\right)+\zeta-\eta_{\omega}\leq 0,\forall\omega \tag{6.45}$$

$$\eta_{\omega}\geq 0,\qquad \forall\omega. \tag{6.46}$$

The objective function (6.44) to be maximized includes the expected profit of the wind producer ($\xi\{R\}$ in (6.36)) and the CVaR of the profit multiplied by the weighting parameter $\beta\in[0,\infty)$. This weighting factor enforces the tradeoff between expected profit and risk in such a way that the higher the value of β, the more risk averse the wind producer is. Thus, if risk is not considered (risk-neutral case), the value of β is set to 0.

Since risk is considered throughout the entire market horizon, model (6.37)–(6.46) cannot be decomposed by time period. This is so because variables ζ and η_{ω}, used to compute the CVaR over the total revenue distribution, couple the decisions pertaining to different hours.

6.5.4 Adjustment Market

The consideration of an adjustment market in the wind producer problem entails three changes or additions to the mathematical model presented in Subsection 6.5.1:

1. First, a term accounting for the revenue obtained from trading energy in the adjustment market needs to be included in the objective function, i.e.,

$$\xi\{R\}=\sum_{\omega=1}^{N_{\Omega}}\sum_{t=1}^{N_{\mathrm{T}}}\pi_{\omega}\left(\lambda^{\mathrm{D}}_{t\omega}P^{\mathrm{D}}_{t\omega}d_t+\lambda^{\mathrm{A}}_{t\omega}P^{\mathrm{A}}_{t\omega}d_t+\lambda^{\mathrm{D}}_{t\omega}r^{+}_{t\omega}\Delta^{+}_{t\omega}-\lambda^{\mathrm{D}}_{t\omega}r^{-}_{t\omega}\Delta^{-}_{t\omega}\right), \tag{6.47}$$

where $P^{\mathrm{A}}_{t\omega}$ is the wind power traded (sold or purchased) in the adjustment market. Note that term $\lambda^{\mathrm{A}}_{t\omega} P^{\mathrm{A}}_{t\omega} d_t$ in (6.47) can be negative if the wind power producer decides to purchase energy from this market ($P^{\mathrm{A}}_{t\omega} < 0$).

2. Second, the total energy deviation $\Delta_{t\omega}$ is computed with respect to the final energy schedule ($P^{\mathrm{S}}_{tw} d_t$), that is, the summation of the amounts of energy traded in the day-ahead and adjustment markets,

$$P^{\mathrm{S}}_{t\omega} = P^{\mathrm{D}}_{t\omega} + P^{\mathrm{A}}_{t\omega}, \forall t, \forall \omega \tag{6.48}$$

$$0 \leq P^{\mathrm{S}}_{t\omega} \leq P^{\max}, \forall t, \forall \omega \tag{6.49}$$

$$\Delta_{t\omega} = d_t (P_{t\omega} - P^{\mathrm{S}}_{t\omega}), \forall t, \forall \omega. \tag{6.50}$$

3. Lastly, non-anticipativity constraints for the decisions made in the adjustment market have to be added,

$$\begin{aligned} P^{\mathrm{A}}_{t\omega} = P^{\mathrm{A}}_{t\omega'}, \forall t, \forall \omega, \omega' : (\lambda^{\mathrm{D}}_{t\omega'} = \lambda^{\mathrm{D}}_{t\omega},\ \forall t) \\ \text{and } (P_{\tau\omega'} = P_{\tau\omega}, \forall \tau = 1, 2, \ldots, N_{\mathrm{T}_1}), \end{aligned} \tag{6.51}$$

where N_{T_1} is the number of time periods between the closures of the day-ahead and adjustment markets.

Constraints (6.51) express that the amount of energy traded in the adjustment market may be different depending on the day-ahead market price realization and the wind power generated after and before the closure of the day-ahead and adjustment markets, respectively. Specifically, the second term of the "and" condition in (6.51) models the certainty gain effect explained in Subsection 6.3.3. Furthermore, these constraints also ensure non-anticipativity with respect to the adjustment market and imbalance price realizations, and the wind power realization after the closure of the adjustment market.

The following example is aimed to clarify the major additions to the offer strategy model of a wind power producer brought about by the possibility of trading in an adjustment market.

Illustrative Example 6.12 (Revenue from the adjustment market and non-anticipativity constraints). Let us consider the market structure and time framework indicated in Fig. 6.8. In this case, the wind power producer is willing to trade in the available adjustment market. Being so, the expected profit $\xi\{R'\}$ of the wind power producer for the scenario tree shown in Fig. 6.13 is given by

$$
\begin{aligned}
\xi\{R'\} = \xi\{R\} + &\\
&+ \$40/\mathrm{MWh} \times (0.0098P_1^{\mathrm{A}} + 0.0042P_2^{\mathrm{A}} + 0.0294P_3^{\mathrm{A}} + 0.0126P_4^{\mathrm{A}}\\
&+ 0.0343P_9^{\mathrm{A}} + 0.0147P_{10}^{\mathrm{A}} + 0.0245P_{11}^{\mathrm{A}} + 0.0105P_{12}^{\mathrm{A}})\\
&+ \$53/\mathrm{MWh} \times (0.0182P_5^{\mathrm{A}} + 0.0078P_6^{\mathrm{A}} + 0.0546P_7^{\mathrm{A}} + 0.0234P_8^{\mathrm{A}}\\
&+ 0.0637P_{13}^{\mathrm{A}} + 0.0273P_{14}^{\mathrm{A}} + 0.0455P_{15}^{\mathrm{A}} + 0.0195P_{16}^{\mathrm{A}})\\
&+ \$10/\mathrm{MWh} \times (0.0147P_{17}^{\mathrm{A}} + 0.0063P_{18}^{\mathrm{A}} + 0.0441P_{19}^{\mathrm{A}} + 0.0189P_{20}^{\mathrm{A}}\\
&+ 0.05145P_{25}^{\mathrm{A}} + 0.02205P_{26}^{\mathrm{A}} + 0.03675P_{27}^{\mathrm{A}} + 0.01575P_{28}^{\mathrm{A}})\\
&+ \$23/\mathrm{MWh} \times (0.0273P_{21}^{\mathrm{A}} + 0.0117P_{22}^{\mathrm{A}} + 0.0819P_{23}^{\mathrm{A}} + 0.0351P_{24}^{\mathrm{A}}\\
&+ 0.09555P_{29}^{\mathrm{A}} + 0.04095P_{30}^{\mathrm{A}} + 0.06825P_{31}^{\mathrm{A}} + 0.02925P_{32}^{\mathrm{A}}),
\end{aligned}
$$

where $\xi\{R\}$ is the expected profit from trading in the day-ahead and balancing markets (see Illustrative Example 6.10).

On the other hand, the non-anticipativity constraints linked to the decision-making in the adjustment market are as follows:

$$
\begin{aligned}
&P_1^{\mathrm{A}} = P_2^{\mathrm{A}} = P_3^{\mathrm{A}} = P_4^{\mathrm{A}} = P_5^{\mathrm{A}} = P_6^{\mathrm{A}} = P_7^{\mathrm{A}} = P_8^{\mathrm{A}}\\
&P_9^{\mathrm{A}} = P_{10}^{\mathrm{A}} = P_{11}^{\mathrm{A}} = P_{12}^{\mathrm{A}} = P_{13}^{\mathrm{A}} = P_{14}^{\mathrm{A}} = P_{15}^{\mathrm{A}} = P_{16}^{\mathrm{A}}\\
&P_{17}^{\mathrm{A}} = P_{18}^{\mathrm{A}} = P_{19}^{\mathrm{A}} = P_{20}^{\mathrm{A}} = P_{21}^{\mathrm{A}} = P_{22}^{\mathrm{A}} = P_{23}^{\mathrm{A}} = P_{24}^{\mathrm{A}}\\
&P_{25}^{\mathrm{A}} = P_{26}^{\mathrm{A}} = P_{27}^{\mathrm{A}} = P_{28}^{\mathrm{A}} = P_{29}^{\mathrm{A}} = P_{30}^{\mathrm{A}} = P_{31}^{\mathrm{A}} = P_{32}^{\mathrm{A}}.
\end{aligned}
$$

□

6.5.5 Formulation

The short-term trading problem of a wind power producer participating in an electricity market as the one represented in Fig. 6.1 (with one day-ahead market, one adjustment market and a balancing market) can be formulated as follows:

$$
\begin{aligned}
&\mathrm{Maximize}_{P_{t\omega}^{\mathrm{D}},\forall t,\forall\omega;P_{t\omega}^{\mathrm{A}},\forall t,\forall\omega;\Delta_{t\omega}^{+},\forall t,\forall\omega;\Delta_{t\omega}^{-},\forall t,\forall\omega;\eta_\omega,\forall\omega;\zeta}\\
&\sum_{\omega=1}^{N_\Omega}\sum_{t=1}^{N_{\mathrm{T}}}\pi_\omega\left[\lambda_{t\omega}^{\mathrm{D}}P_{t\omega}^{\mathrm{D}}d_t + \lambda_{t\omega}^{\mathrm{A}}P_{t\omega}^{\mathrm{A}}d_t + \lambda_{t\omega}^{\mathrm{D}}r_{t\omega}^{+}\Delta_{t\omega}^{+} - \lambda_{t\omega}^{\mathrm{D}}r_{t\omega}^{-}\Delta_{t\omega}^{-}\right]\\
&\qquad\qquad +\beta\left(\zeta - \frac{1}{1-\alpha}\sum_{\omega=1}^{N_\Omega}\pi_\omega\eta_\omega\right)
\end{aligned}
\tag{6.52}
$$

subject to

$$0 \leq P^{\mathrm{D}}_{t\omega} \leq P^{\max}, \forall t, \forall \omega \tag{6.53}$$

$$P^{\mathrm{S}}_{t\omega} = P^{\mathrm{D}}_{t\omega} + P^{\mathrm{A}}_{t\omega}, \forall t, \forall \omega \tag{6.54}$$

$$0 \leq P^{\mathrm{S}}_{t\omega} \leq P^{\max}, \forall t, \forall \omega \tag{6.55}$$

$$\Delta_{t\omega} = d_t(P_{t\omega} - P^{\mathrm{S}}_{t\omega}), \forall t, \forall \omega \tag{6.56}$$

$$\Delta_{t\omega} = \Delta^{+}_{t\omega} - \Delta^{-}_{t\omega}, \forall t, \forall \omega \tag{6.57}$$

$$0 \leq \Delta^{+}_{t\omega} \leq P_{t\omega} d_t, \forall t, \forall \omega \tag{6.58}$$

$$0 \leq \Delta^{-}_{t\omega} \leq P^{\max} d_t, \forall t, \forall \omega \tag{6.59}$$

$$P^{\mathrm{D}}_{t\omega} - P^{\mathrm{D}}_{t\omega'} \leq 0, \forall t, \forall \omega, \omega' : O(\lambda^{\mathrm{D}}_{t\omega}) + 1 = O(\lambda^{\mathrm{D}}_{t\omega'}) \tag{6.60}$$

$$P^{\mathrm{D}}_{t\omega} = P^{\mathrm{D}}_{t\omega'}, \forall t, \forall \omega, \omega' : \lambda^{\mathrm{D}}_{t\omega'} = \lambda^{\mathrm{D}}_{t\omega} \tag{6.61}$$

$$\begin{aligned} P^{\mathrm{A}}_{t\omega} = P^{\mathrm{A}}_{t\omega'}, \forall t, \forall \omega, \omega' : (\lambda^{\mathrm{D}}_{t\omega'} = \lambda^{\mathrm{D}}_{t\omega}\ \forall t) \\ \text{and } (P_{\tau\omega'} = P_{\tau\omega}, \forall \tau = 1, 2, \ldots, N_{\mathrm{T}_1}) \end{aligned} \tag{6.62}$$

$$-\sum_{t=1}^{N_{\mathrm{T}}} \left[\lambda^{\mathrm{D}}_{t\omega} P^{\mathrm{D}}_{t\omega} d_t + \lambda^{\mathrm{A}}_{t\omega} P^{\mathrm{A}}_{t\omega} d_t + \lambda^{\mathrm{D}}_{t\omega}(r^{+}_{t\omega}\Delta^{+}_{t\omega} - r^{-}_{t\omega}\Delta^{-}_{t\omega})\right] + \zeta - \eta_\omega \leq 0, \forall \omega \tag{6.63}$$

$$\eta_\omega \geq 0, \qquad \forall \omega. \tag{6.64}$$

The objective function (6.52) comprises two terms: i) the expected profit and ii) the CVaR multiplied by the weighting factor β, which allows controlling the risk-aversion degree of the wind producer.

Constraints (6.53)–(6.55) limit the amount of energy that can be traded in both the day-ahead and adjustment markets to the installed capacity of the wind farm. The set of equations (6.56)–(6.59) determines the total, positive and negative energy deviations incurred by the wind producer per period and scenario. Constraints (6.60) enforces the non-decreasing condition of the resulting offering curves. Constraints (6.61) and (6.62) constitute the non-anticipativity conditions related to the decisions made in the day-ahead and adjustment markets, respectively. Finally, equations (6.63)–(6.64) are required to compute the CVaR.

It is important to note that the three-stage model (6.52)–(6.64) above is intended to produce optimal offering strategies for the day-ahead market. Once the position in this market is fixed, a detailed two-stage stochastic programming formulation analogous to (6.52)–(6.64) can be built to generate optimal offers for the adjustment market.

6.6 Wind Producer Example

In this example, the offering strategy model formulated in Section 6.5 is applied to the scenario tree built in Illustrative Example 6.9 and depicted in Fig. 6.13. We recall that this scenario tree is specifically tailored to the market structure and time framework described in Illustrative Example 6.4 and represented in Fig. 6.8. We also emphasize that the referred stochastic programming model is intended to serve as a tool for wind power producers to generate optimal offering strategies for the day-ahead market taking into account the availability of an adjustment market and the existence of a balancing market.

To begin with the fundamentals, we first consider the basic model introduced in Subsection 6.5.1, which produces optimal energy quantities (not offering curves) to be offered in the day-ahead market at price zero by a risk-neutral wind power producer that ignores the presence of the adjustment market. Being so, in this case, the scenario tree in Fig. 6.13 can be reduced to half of its former size, i.e., it can be shrunk from 32 to 16 scenarios. Every pair of scenarios in the tree sharing the same realizations for the day-ahead market prices, imbalance price ratios and wind power production can be merged into a single one by adding their probabilities. The tree so condensed is shown in Fig. 6.14. As an example, probability π_1 in this tree is obtained as follows,

$$\pi_1 = 0.0098 + 0.0182 = 0.028,$$

where 0.0098 and 0.0182 are, respectively, the probabilities associated with scenarios $\omega = 1$ and $\omega = 5$ in the original tree.

When it comes to deal with uncertainties in an optimization problem, it is a good practice to ask oneself first about the convenience of using a stochastic solution approach instead of a simpler deterministic one. In order to settle this dilemma, we calculate the Value of the Stochastic Solution (VSS), which allows assessing the effectiveness of a stochastic programming approach. The VSS is comprehensively treated in Chapter 2. In a few words, it is defined as the difference between the expected profit of the stochastic model and the expected profit obtained from the problem where decisions variables are fixed to those resulting from the associated deterministic problem, i.e., from the problem in which stochastic processes are replaced by their respective expected values. In the considered case, the deterministic approach to tackle the short-term trading problem of a wind power producer is naïve and results in submitting the forecast wind power production to the day-ahead market. According to the scenario tree in Fig. 6.14, the predicted wind power produced during the single period comprising the market horizon, $\hat{P}$, can be calculated as

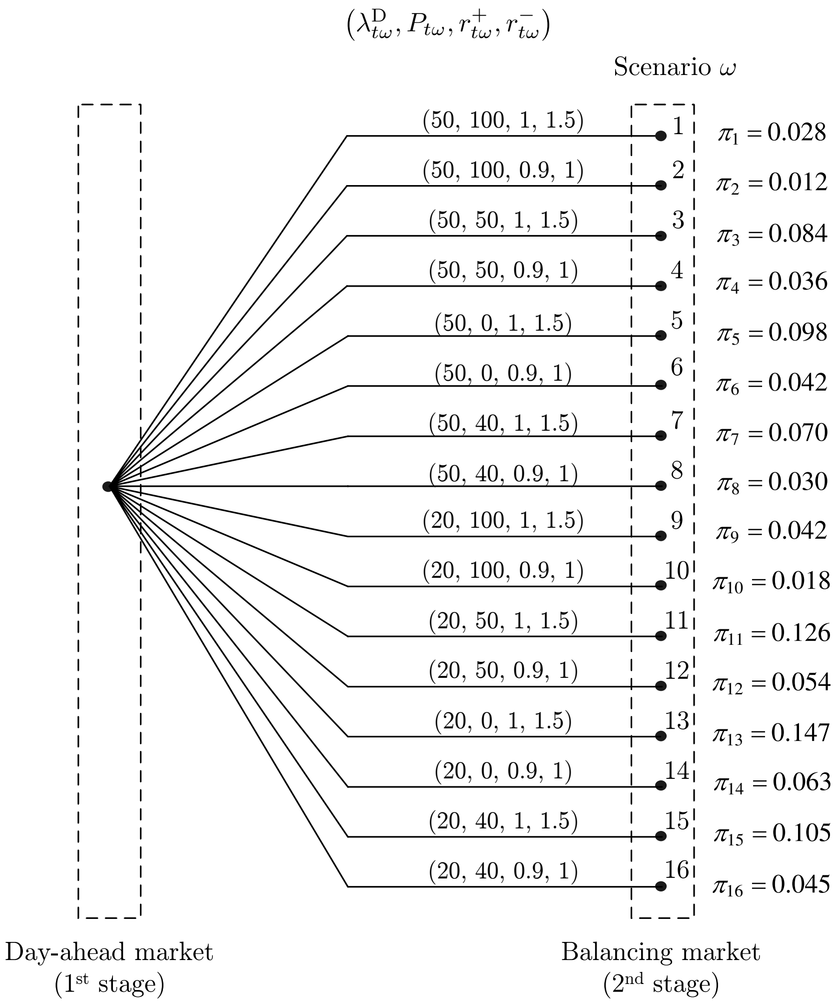


Fig. 6.14 Wind producer example: two-stage scenario tree (excluding the adjustment market)

$$
\begin{aligned}
\hat{P} = \xi\{P\} = \sum_{\omega=1}^{16} \pi_\omega P_\omega &= (0.028 + 0.012 + 0.042 + 0.018) \times 100 \\
&+ (0.084 + 0.036 + 0.126 + 0.054) \times 50 \\
&+ (0.098 + 0.042 + 0.147 + 0.063) \times 0 \\
&+ (0.070 + 0.030 + 0.105 + 0.045) \times 40 = 35\text{MW}.
\end{aligned}
$$

The expected profit resulting from model (6.36)-(6.41) if the first-stage variable P^{D} is fixed to 35 MW is equal to \$971.04. For convenience, we denote the expected profit so obtained as $z^{\mathrm{D}*}$, with superscript D standing for *deterministic*. On the other hand, the stochastic solution of this model yields an expected profit $z^{\mathrm{S}*}$ equal to \$1086.4, which results from trading 0 MW in the day-ahead market (i.e., $P^{\mathrm{D}} = 0$). Note that superscript S indicates *stochastic*. Consequently, the Value of Stochastic Solution is given by

$$\mathrm{VSS} = 1086.4 - 971.04 = \$115.36,$$

which approximately represents a percentage of 10.6% with respect to $z^{\mathrm{S}*}$. This significant value justifies the use of a stochastic approach to tackle the wind producer problem instead of a prediction-based one, and reveals that the short-term decision-making process of a wind power producer is highly influenced by the uncertainties involved, namely, day-ahead market prices, imbalance price ratios, and wind power production.

In this respect, it is relevant to quantify the relative economic impact of each source of uncertainty on the decision-making undertaken by a wind producer. To this end, we can make use of the Expected Value of Perfect Information (EVPI), which measures how much the wind producer would be willing to pay to know the future realizations of the stochastic processes under consideration (market prices and wind availability). It is computed as the difference between the expected profit resulting from the problem where the decisions to be made *anticipate* the subsequent outcomes of the random variables and the expected profit of the stochastic model. A more detailed explanation of the EVPI can be found in Chapter 2. In the example under study, we determine the EVPI by assuming perfect information on each stochastic process separately in order to isolate the economic effect of each uncertainty source from the rest.

Table 6.1 Wind producer example: expected values of perfect information

Perfect information on	λ^{D}	P	(r^+, r^-)	all
EVPI (%)	0	3.09	3.09	3.09

Table 6.1 shows the expected values of perfect information as improvements in percentage with respect to the expected profit obtained from model (6.36)–(6.41). In light of the results presented in this table, the following two comments are in order. First, the economic improvement produced by a perfect knowledge of day-ahead market prices is smaller than the other two, which evidences that the uncertainty associated with these prices plays a secondary role in the wind producer offering problem. Moreover, this economic improvement is nil. This is due to the fact that, as highlighted in Subsection 6.3.2, the maximization of the expected profit (6.36) boils down

to the minimization of the imbalance cost. This cost depends on r^+ and r^-, but is independent of λ^{D}. Second, the economic improvements owing to perfect information on wind production and imbalance price ratios are equal. The reason for this is that knowing the values of r^+ and r^- in advance is equivalent to know perfectly the future wind production availability from a strategic point of view. As a result, the total EVPI that is obtained assuming perfect information on all three stochastic processes, namely, day-ahead market prices, imbalance price ratios and wind power production, is also 3.09.

According to model (6.36)–(6.41), the optimal offering strategy of a wind power producer facing the two-stage scenario tree depicted in Fig. 6.14 comes down to not participating in the day-ahead market, and thus, trading all its actual production in the balancing market. That is, $P^{\mathrm{D}} = 0$. The ultimate purpose behind this strategy is to avoid producing less than settled in the day-ahead market because:

1. The probability that the net system imbalance is negative (deficit of generation) is high (0.7), while the probability of being positive (excess of generation) is comparatively low (0.3).
2. If the net system imbalance is indeed negative, the price at which the wind producer has to buy energy in the balancing market to cover a possible generation shortage is 50% greater than the day-ahead market price ($r^- = 1.5$). However, it can sell its possible overproduction at a price equal to the day-ahead market price ($r^+ = 1$).
3. In contrast, if the net system imbalance is positive, the price at which the wind power producer can sell its generation surplus is just 10% lower than the day-ahead market price ($r^+ = 0.9$).

Therefore, the wind producer has little to lose by trading all its actual production in the balancing market. This attitude is even easier to justify by appealing to the fact that the maximization of the expected profit (6.36) is equivalent to the minimization of the imbalance cost defined in (6.15).

In Fig. 6.15(a), the imbalance cost incurred by the wind producer is plotted as a function of the amount of power sold in the day-ahead market. Observe that the minimum imbalance cost is achieved for $P^{\mathrm{D}} = 0$. The shape of the imbalance cost function can be easily altered by changing the possible values of the imbalance price ratios r^+ and r^- as illustrated in Figs. 6.15(b), 6.15(c), and 6.15(d). Note that these figures represent situations in which the best energy offer to the day-ahead market turns out to be 40, 50, and 100 MW, respectively.

Another effective manner to alter the optimal offering strategy of a wind power producer consists in introducing a measure allowing to control the risk of profit variability into its offering model. The mathematical adjustments required to this end are explained in Subsection 6.5.3, where the referred measure is the Conditional Value-at-Risk (CVaR).

Let us consider the case represented in Fig. 6.15(d), in which the pair (r^+, r^-) is equal to $(1, 1.02)$ with a probability of 0.7 and equal to $(0.1, 1)$

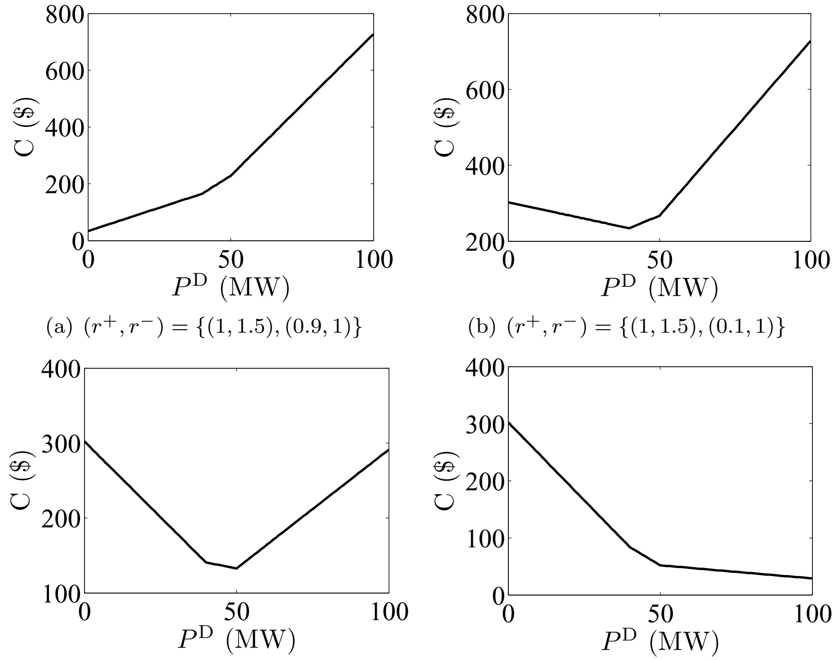


Fig. 6.15 Wind producer example: imbalance cost as a function of the amount of power sold in the day-ahead market

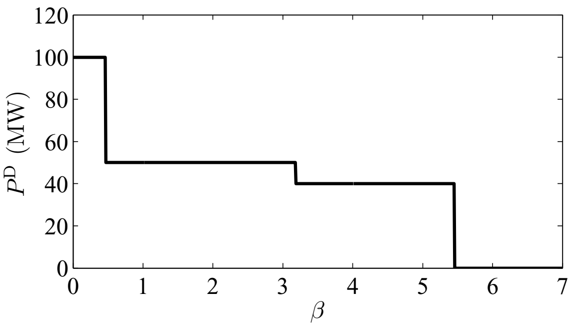


Fig. 6.16 Wind producer example: power sold in the day-ahead market as a function of the risk-aversion parameter β

with a probability of 0.3. As indicated above, the best offer to the day-ahead market in this case is 100 MW. Actually, it is 100 MW as long as the wind power producer adopts a risk-neutral attitude ($\beta = 0$). Fig. 6.16 illustrates the evolution of the best offer as the wind producer becomes more and more risk averse, i.e., as the parameter β increases. Observe that the amount of power sold in the day-ahead market abruptly diminishes for certain values of β determining a change in the risk perception of the wind producer. In fact, this evolution reflects the basic strategy that a wind power producer can adopt to hedge against profit volatility in the absence of an adjustment market: trading less energy in the day-ahead market in the hope that its overproduction can be sold in the balancing market at a still competitive price. In this sense, it is important to note that, according to the mechanism for imbalance price formation described in Subsection 6.3.1, where negative electricity prices are *prohibited*, negative energy imbalances can entail economic losses (profit below zero) for the wind producer, while positive energy imbalances can lead to nil profit at the most. Therefore, in general terms, suffering negative deviations is riskier than incurring positive deviations.

So far we have ignored the possibility of submitting an offering curve to the day-ahead market instead of just a single energy quantity. Although the key issues involved in the design of such a curve are comprehensively analyzed in Chapter 5, its mathematical modeling is provided in Subsection 6.5.2 of this chapter. It is based on adding constraints (6.42) and (6.43) to the offering strategy model of the wind power producer. As explained in the referred subsection, on the one hand, constraints (6.42) enforce the non-decreasing nature of the offering curve, required in most electricity markets while, on the other hand, constraints (6.43) constitute a *loosened* implementation of the non-anticipatory character of information on day-ahead market prices, which precisely allows us to obtain the optimal energy-price pairs making up the offering curve. In particular, for the scenario tree represented in Fig. 6.14, these constraints boil down to

$$
\begin{aligned}
&P_9^{\mathrm{D}} - P_1^{\mathrm{D}} \leq 0,\\
&P_1^{\mathrm{D}} = P_2^{\mathrm{D}} = P_3^{\mathrm{D}} = P_4^{\mathrm{D}} = P_5^{\mathrm{D}} = P_6^{\mathrm{D}} = P_7^{\mathrm{D}} = P_8^{\mathrm{D}},\\
&P_9^{\mathrm{D}} = P_{10}^{\mathrm{D}} = P_{11}^{\mathrm{D}} = P_{12}^{\mathrm{D}} = P_{13}^{\mathrm{D}} = P_{14}^{\mathrm{D}} = P_{15}^{\mathrm{D}} = P_{16}^{\mathrm{D}}.
\end{aligned}
$$

Therefore, the offering curve resulting from the wind producer model is simply composed of two different energy-price pairs $(1\ \mathrm{h} \times P_1^{\mathrm{D}}, \lambda_1^{\mathrm{D}})$ and $(1\ \mathrm{h} \times P_9^{\mathrm{D}}, \lambda_9^{\mathrm{D}})$ corresponding to the two possible values of the day-ahead market price, \$50/MWh and \$20/MWh, respectively. More realistic offering curves are obtained in the case study presented in Section 6.7.

Due to the fact that the generation cost of a wind farm is zero, offering curves are, a priori, less useful for a wind producer than they are for a thermal one. Actually, offering curves allow thermal producers to account somehow for the shape of their quadratic generation cost function in the trading process. Needless to say, this is not an issue for a wind producer. As an example of this, let us consider the four cases represented in Fig. 6.15. In all these instances, the optimal energy-price curve resulting from the offering strategy model including constraints (6.42) and (6.43) is such that $P_1^{\mathrm{D}} = P_9^{\mathrm{D}}$. In plain words, there is no curve, but just a point, a single optimal energy quantity to be offered at price zero.

Notwithstanding the remark above, there are some special circumstances in which submitting offering curves to the day-ahead market can be potentially advantageous for a wind producer. These circumstances occur in electricity markets where the value of the day-ahead market price is indicative of the direction, positive or negative, and/or the magnitude of the real-time net system imbalance. Statistically speaking, we refer to those situations in which the day-ahead market price and the imbalance price ratios are correlated. As an example, let us suppose the following two opposite and illustrative contexts:

A) In context A, if the day-ahead market price λ^{D} is high (\$50/MWh), the net system imbalance is negative and the pair of imbalance price ratios (r^+, r^-) resulting from the balancing market is equal to $(1, 1.5)$ with a probability of 0.7 and equal to $(1, 1.3)$ with a probability of 0.3. Conversely, if the day-ahead market price is low (\$20/MWh), the net system imbalance is positive and the imbalance price ratios (r^+, r^-) are $(0.6, 1)$ and $(0.9, 1)$ with probabilities 0.7 and 0.3, respectively.
   Submitting an offering curve to the day-ahead market is useless in this situation due to the enforced non-decreasing character of the curve. Actually, the optimal energy-price pairs making up the curve in this case are (0 MWh, \$20/MWh) and (0 MWh, \$50/MWh). Thus, all the wind power production is traded in the balancing market.
B) In context B, if the day-ahead market price λ^{D} is high (\$50/MWh), the net system imbalance is positive and the pair of imbalance price ratios (r^+, r^-) is equal to $(0.6, 1)$ with a probability of 0.7 and equal to $(0.9, 1)$ with a probability of 0.3. On the contrary, if the day-ahead market price λ^{D} is low (\$20/MWh), the net system imbalance is negative and the pair of imbalance price ratios (r^+, r^-) is equal to $(1, 1.5)$ with a probability of 0.7 and equal to $(1, 1.3)$ with a probability of 0.3. Note that context B is right the opposite of context A.
   Offering a curve results advantageous in this situation, where the optimal energy-price pairs turn out to be (0 MWh, \$20/MWh) and (100 MWh, \$50/MWh). In this second case, the wind producer can allow itself to trade a greater amount of energy (100 MWh) in the day-ahead market for a high price realization (\$50/MWh) without having to worry about the imbalance cost entailed by a possible generation shortage. Specifically, if the wind producer submits the previous optimal energy-price curve to the day-ahead market in this second instance, the achieved economic improvement in terms of expected profit amounts to over 13.1% with respect to the expected profit obtained if the opportunity of submitting a curve is wasted.

To conclude this example, we assess how much the expected profit of the wind power producer increases if it decides to make use of the adjustment market available within the short-term trading structure illustrated in Fig. 6.8. As pointed out in Subsection 6.3.3, such an increase is caused by two different effects, namely:

i) As a trading floor, the adjustment market provides the wind producer with an additional opportunity to buy or sell energy at a price different than the day-ahead market price. For convenience, hereinafter, we refer to this effect as *additional trading effect*.
ii) The clearing time of the adjustment market is closer to the energy delivery horizon, and consequently, trading in this market can be performed with a reduced level of uncertainty about the future wind power production. This is what we call *certainty gain effect*.

In order to isolate both effects, we manipulate the non-anticipativity constraints (6.51) related to the decisions made in the adjustment market. As indicated in Subsection 6.5.4, the certainty gain effect is mathematically implemented by means of the second term of the "and" condition appearing in these constraints. Therefore, if we ignore this term, the certainty gain effect is not accounted for in the offering strategy model. In particular, for the scenario tree depicted in Fig. 6.13, non-anticipativity constraints (6.51) are

$$
\begin{aligned}
&P_1^{\mathrm{A}} = P_2^{\mathrm{A}} = P_3^{\mathrm{A}} = P_4^{\mathrm{A}} = P_5^{\mathrm{A}} = P_6^{\mathrm{A}} = P_7^{\mathrm{A}} = P_8^{\mathrm{A}},\\
&P_9^{\mathrm{A}} = P_{10}^{\mathrm{A}} = P_{11}^{\mathrm{A}} = P_{12}^{\mathrm{A}} = P_{13}^{\mathrm{A}} = P_{14}^{\mathrm{A}} = P_{15}^{\mathrm{A}} = P_{16}^{\mathrm{A}},\\
&P_{17}^{\mathrm{A}} = P_{18}^{\mathrm{A}} = P_{19}^{\mathrm{A}} = P_{20}^{\mathrm{A}} = P_{21}^{\mathrm{A}} = P_{22}^{\mathrm{A}} = P_{23}^{\mathrm{A}} = P_{24}^{\mathrm{A}},\\
&P_{25}^{\mathrm{A}} = P_{26}^{\mathrm{A}} = P_{27}^{\mathrm{A}} = P_{28}^{\mathrm{A}} = P_{29}^{\mathrm{A}} = P_{30}^{\mathrm{A}} = P_{31}^{\mathrm{A}} = P_{32}^{\mathrm{A}}.
\end{aligned}
$$

However, if the certainty gain effect is disregarded, such non-anticipativity constraints become

$$
\begin{aligned}
&P_1^{\mathrm{A}} = P_2^{\mathrm{A}} = P_3^{\mathrm{A}} = P_4^{\mathrm{A}} = P_5^{\mathrm{A}} = P_6^{\mathrm{A}} = P_7^{\mathrm{A}} = P_8^{\mathrm{A}} = P_9^{\mathrm{A}} = P_{10}^{\mathrm{A}} =\\
&= P_{11}^{\mathrm{A}} = P_{12}^{\mathrm{A}} = P_{13}^{\mathrm{A}} = P_{14}^{\mathrm{A}} = P_{15}^{\mathrm{A}} = P_{16}^{\mathrm{A}},\\
&P_{17}^{\mathrm{A}} = P_{18}^{\mathrm{A}} = P_{19}^{\mathrm{A}} = P_{20}^{\mathrm{A}} = P_{21}^{\mathrm{A}} = P_{22}^{\mathrm{A}} = P_{23}^{\mathrm{A}} = P_{24}^{\mathrm{A}} = P_{25}^{\mathrm{A}} = P_{26}^{\mathrm{A}} =\\
&= P_{27}^{\mathrm{A}} = P_{28}^{\mathrm{A}} = P_{29}^{\mathrm{A}} = P_{30}^{\mathrm{A}} = P_{31}^{\mathrm{A}} = P_{32}^{\mathrm{A}}.
\end{aligned}
$$

Observe that this last version of the non-anticipativity constraints is *tighter* than the previous one, and therefore, the consideration of the certainty gain effect is expected to improve the solution to the wind producer problem in terms of profit.

Let us consider again the four scenario representations (a), (b), (c), and (d) of the imbalance price ratios (r^+, r^-) used as examples to build the imbalance cost functions in Figs. 6.15(a), 6.15(b), 6.15(c), and 6.15(d), respectively. Table 6.2 shows, for all these cases, the economic improvements due to the two effects indicated above in percentage with respect to the expected profit obtained if the adjustment market is not taken into account. It is interesting to underline that, according to the results provided in Table 6.2,

Table 6.2 Wind producer example: adjustment market effects on the expected profit of the wind producer (increases in percentage)

Effect	Case			
	(a)	(b)	(c)	(d)
Additional trading	14.27	11.83	8.64	4.99
Certainty gain	0	14.03	4.24	2.05
Total	14.27	25.86	12.88	7.04

the economic impact of the certainty gain effect may be as significant as the additional trading effect, and therefore, overlooking it in the wind producer problem should be seen as a mistake. In contrast, the certainty gain effect is nonexistent in the short-term trading problem of a conventional producer.

6.7 Wind Producer Case Study

In this section, the features of the proposed stochastic model aimed to produce optimal offering strategies for a wind power producer are further illustrated through a realistic case study based on the time framework and market structure described in Illustrative Example 6.1.

We consider a wind farm having 40 2.5-MW commercial wind generators. The total installed wind power capacity of the plant is therefore 100 MW. All the turbines making up the wind farm are assumed to be operating for the considered trading day.

Stochastic processes describing wind speed and market prices are modeled via the time-series-based approach comprehensively analyzed in Subsection 3.2.2 of Chapter 3. Specifically, to characterize wind speed uncertainty, the following second-order autoregressive model is used,

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + a_t, \tag{6.65}$$

where Z_t represents the *normalized* wind speed in time period t, a_t is a white noise term, and ϕ_1 and ϕ_2, with values equal to 1.175 and -0.244, respectively, are parameters estimated from the historical wind speed data collected by Dr. G. L. Johnson from a location in the state of Kansas (US). This set of data is publicly available in [45]. The historical records used comprise average hourly wind speed measurements from January 25 to March 31, 1996. As explained in Subsection 3.2.2 of Chapter 3, a normalization of the wind speed series is performed in order to preserve its marginal distribution, usually a Weibull. Fig. 6.17 represents both the wind speed frequency distribution of the considered site and the probability density function (pdf) of the fitted Weibull.

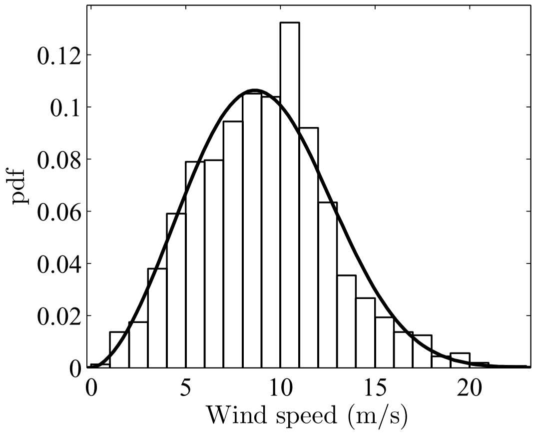


Fig. 6.17 Wind producer case study: wind speed frequency distribution of the considered location and the pdf of the Weibull fit

To convert wind speed scenarios into wind power scenarios, the power curve of the turbine model installed in the wind farm is used. Such a power curve is depicted in Fig. 6.18. Then, on the hypothesis that the characteristics of the wind are the same all over the plant at each instant, scenarios representing the power output of the wind farm can be obtained by just aggregating the production of each available turbine in the plant.

For simplicity, the stochastic processes describing price behavior in the day-ahead, adjustment, and balancing markets are modeled using simplified seasonal ARIMA models. The ARIMA models fitted for the day considered in this case study (April 1, 2008) are provided in Table 6.3. The historical data

Table 6.3 Wind producer case study: stochastic processes for prices and price functions

Variable	Time series model
$\log(\lambda_t^{\mathrm{D}})$	$\mathrm{ARIMA}(2,0,1)(1,1,1)_{24}$
$\Delta\lambda_t = \lambda_t^{\mathrm{A}} - \lambda_t^{\mathrm{D}}$	$\mathrm{ARIMA}(1,0,11)(1,0,1)_{24}$
$\sqrt{r_t} = \sqrt{r_t^+ + r_t^- - 1}$	$\mathrm{ARIMA}(2,0,1)(1,0,1)_{24}$

used for the fitting process belong to the electricity market of the Iberian Peninsula [106] and are publicly available in [122]. These data correspond to the working days between October 2007 and March 2008. Only working

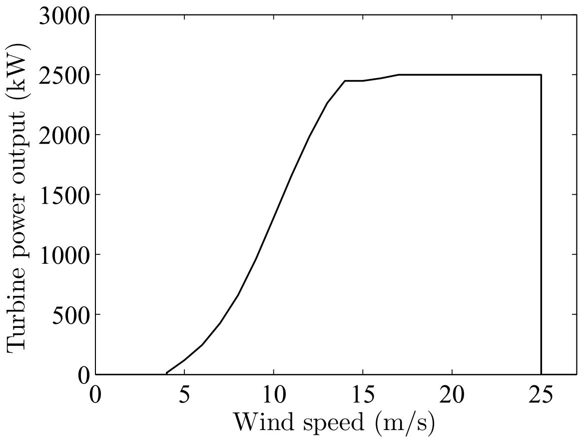


Fig. 6.18 Wind producer case study: power curve of the considered turbine model

days are considered to simplify the ARIMA models by eliminating the weekly seasonality.

The number of generated scenarios for the day-ahead, adjustment, and balancing markets are, respectively, 1000, 1000, and 1000. The number of scenarios simulated to characterize the wind generation uncertainty is also 1000, resulting from the combination of 20 (N_{P_1}) scenarios representing the wind power realizations in time periods between the closures of the day-ahead and adjustment markets, and 50 (N_{P_2}) scenarios to embrace the possible wind power realizations after the clearing of the adjustment market.

The total number of scenarios comprising the original scenario tree is therefore 10^9, which clearly yields an optimization problem that is intractable. Hence, to achieve tractability, the variant 1 of the scenario-reduction technique described in Subsection 3.3.3 of Chapter 3 is applied to each stochastic process, resulting in 10, 5, and 10 scenarios for the day-ahead, adjustment and balancing markets, respectively, and 20 scenarios for the wind farm production ($N_{\mathrm{P}_1} \times N_{\mathrm{P}_2} = 5 \times 4$). Thus, the reduced scenario tree includes 10,000 scenarios.

A confidence level $\alpha = 0.95$ is used to compute the conditional value-at-risk (CVaR) in all instances.

Firstly, no adjustment market is considered. Being so and for the risk-neutral case ($\beta = 0$), the value of the stochastic solution (VSS) is 1.87%, expressed as a percentage of the expected profit obtained from model (6.36)–(6.43). This value justifies the interest in using a stochastic approach for solving the wind producer problem instead of a naïve deterministic one. If the wind producer decides to adopt a risk-neutral attitude ($\beta = 0$), offers to the day-ahead market just consist of an optimal energy quantity submitted at zero price for each time period. However, these single quantities may turn into curves if the wind producer becomes risk averse ($\beta \neq 0$). Table 6.4 shows the energy quantities offered to the day-ahead market for different hours in the risk-neutral case, and Fig. 6.19 depicts the resulting offer curves in the same hours if $\beta = 1$.

Table 6.4 Wind producer case study: optimal energy bids for $\beta = 0$

Period #	7	15	20	21
Offer (MWh)	56.68	69.56	91.16	81.18

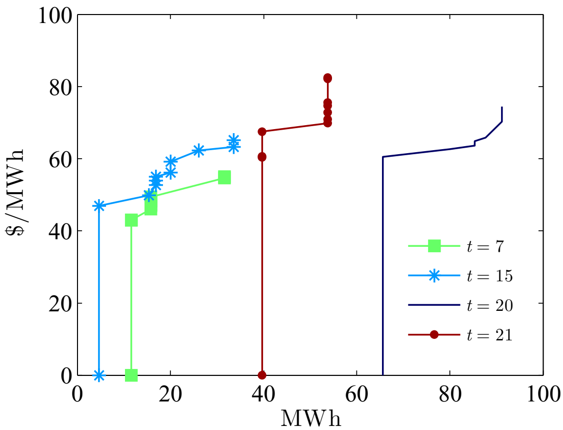


Fig. 6.19 Wind producer case study: optimal offering curves for $\beta = 1$

The changes undergone by the offering curves for increasing β values are intended to reduce the expected energy traded in the day-ahead market as the wind producer becomes more risk averse.

This trend is illustrated in Fig. 6.20, where the expected amount of energy sold in this market is represented as a function of the risk-aversion parameter β. As pointed out in the wind producer example of Section 6.6, the only measure that a wind power producer can take to hedge against profit variability in the absence of an adjustment market is to trade less energy in the day-ahead market in the hope of selling its generation surplus in the balancing market at a good price. Consequently, as the risk aversion increases, the expected negative deviation incurred by the wind producer all over the market horizon decreases at the expense of rising the expected positive deviation. This is shown in Fig. 6.21. Nevertheless, unlike submitting just a single energy value for each time period, offering curves allows reducing the expected

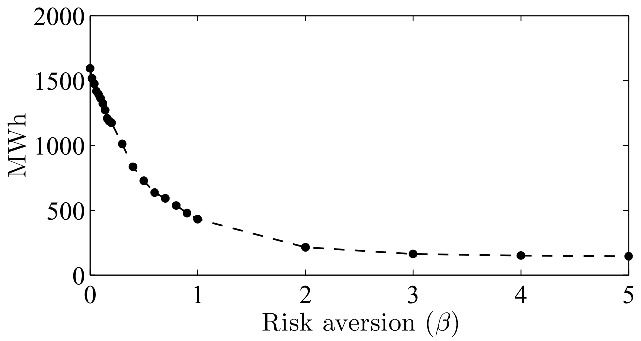


Fig. 6.20 Wind producer case study: expected energy traded in the day-ahead market as a function of the risk-aversion parameter β

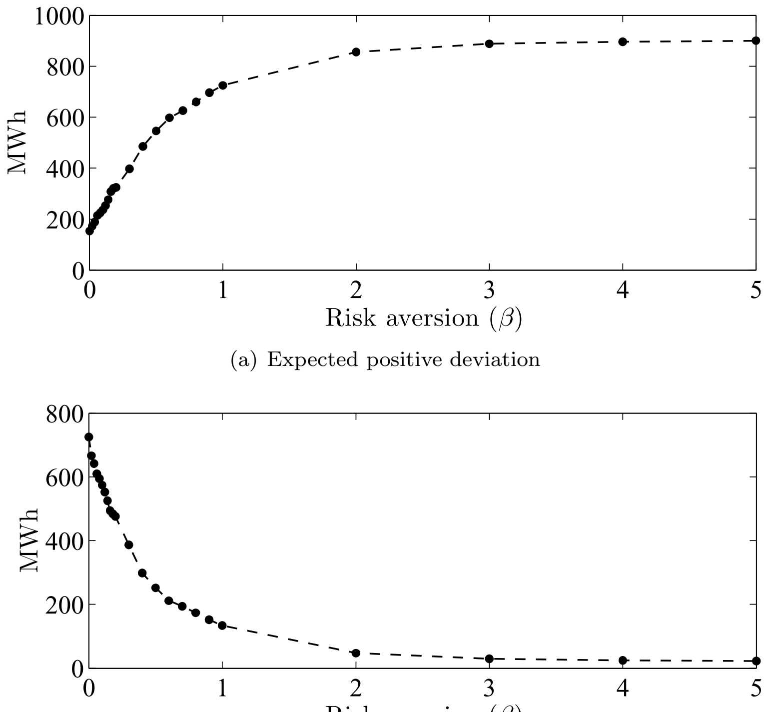
energy placed in the day-ahead market while still keeping the possibility of trading greater amounts of energy for high price realizations.

It is important to note that for a market requiring step-wise offer stacks, the linear blocks of the curves in Fig. 6.19 should be approximated by step-wise functions.

Next, we analyze the enhancement in competitiveness that the wind producer achieves if it takes full advantage of the certainty gain associated with the availability of an adjustment market within the market structure. For this purpose, we perform a comparison between the results provided by model (6.52)–(6.64) depending on whether the certainty gain introduced by the adjustment market is considered or not, i.e., depending on whether the second term of the "and" condition in constraints (6.62) is included or not.

To start with, if an adjustment market is taken into account in the offering strategy model, the value of the stochastic solution for the risk-neutral case increases up to 3.61%, i.e., 1.74% more than the value obtained if this market is ignored. However, the VSS is reduced to 2.58% (1.03% lower!) if the associated certainty gain effect is not accounted for. This result reveals that one of the most important advantages of using a stochastic approach to tackle the wind producer problem, instead of just a deterministic one, is indeed to consider this effect.

In Fig. 6.22 the expected total deviations incurred by the wind producer all over the market horizon are compared for different values of the weighting factor β. The reduction in the expected total amount of imbalances due to the certainty gain associated with trading in the adjustment market is apparent. Fig. 6.23 shows the value of this reduction per time periods and $\beta = 0$. Observe that the decrease in expected deviation is considerably more significant for the first few periods, in which the volatility pertaining to wind power generation is comparatively smaller. Reduction levels in energy deviations similar to those obtained in these first periods could be achieved for the



(a) Expected positive deviation

(b) Expected negative deviation

Fig. 6.21 Wind producer case study: impact of risk aversion on expected energy deviations

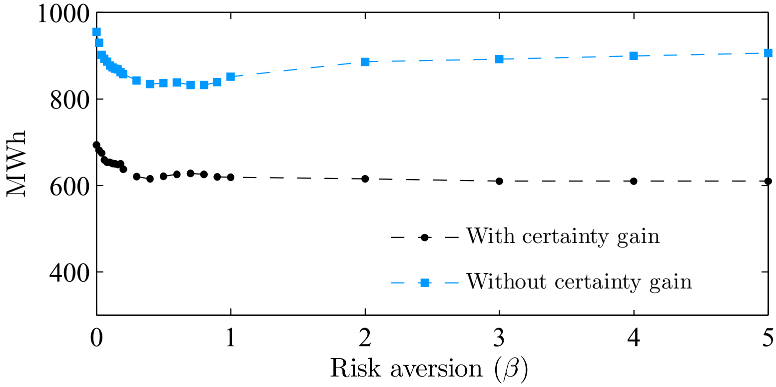


Fig. 6.22 Wind producer case study: impact of the certainty gain on expected imbalances

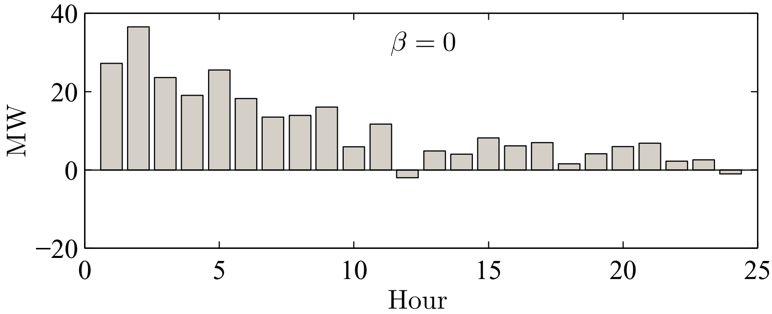


Fig. 6.23 Wind producer case study: hourly reduction in expected power deviation due to the certainty gain effect

rest of hours by including additional adjustment trading floors sequentially arranged all over the market horizon.

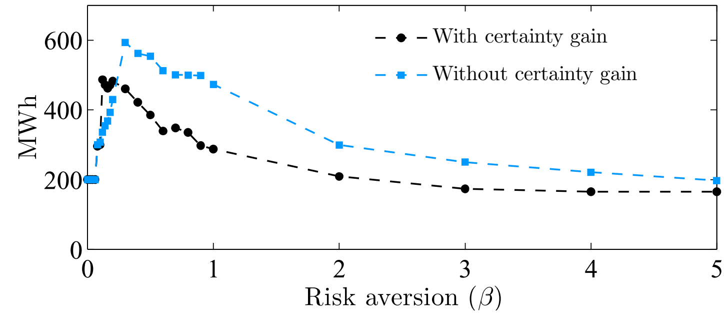
The certainty gain effect involves noticeable changes in the offering strategy of the wind producer as can be inferred from Fig. 6.24.

To grasp the reason for such changes, we must bear in mind that the certainty gain effect translates into a better knowledge on how to offer in the adjustment market in order to decrease the expected level of imbalances. Thus, for high enough levels of risk aversion ($\beta > 0.2$), there is a transfer of trading from the day-ahead market to the adjustment market, where the information on the future wind production realization is more accurate.

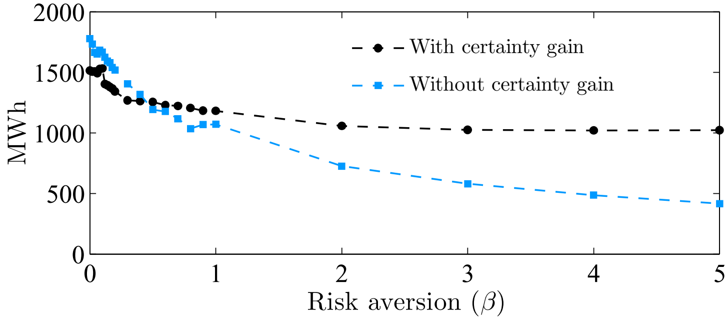
In terms of profit, the improvements due to the certainty gain effect can be summarized by means of Fig. 6.25, which represents the optimal pairs (CVaR, expected profit) obtained for different values of the risk aversion paramater β. The resulting curve is usually referred to as *efficient frontier*. Note that the certainty gain effect produces a significant displacement of the efficient frontier upward and to the right, that is, a displacement in the direction of increasing expected profits and increasing CVaRs.

This displacement can be amplified by adding to the market structure additional adjustment markets throughout the scheduling horizon in a sequential manner. Finally, observe that a significant CVaR gain is attained for a moderate expected profit decrease. For instance, if the weighting factor β is increased from 0 to 1, the ratio $-\frac{\Delta \text{CVaR}}{\Delta \xi\{R\}}$ is equal to 4.21 if the certainty gain effect is accounted for in the offering strategy model. However, this ratio drops to 3.14 if such an effect is disregarded. Therefore, adjustment trading floors are efficacious structural elements to enhance the competitiveness degree of wind producers in an electricity market.

Lastly, we point out that using currently available optimization software and personal computing technology, the calculation time required to solve problems similar to the one previously described is on the order of seconds.



(a) Energy traded in the day-ahead market



(b) Energy traded in the adjustment market

Fig. 6.24 Wind producer case study: impact of the certainty gain effect on the energy amounts traded in both the day-ahead and adjustment markets

6.8 Summary and Conclusions

The competitive sale of wind generation in current electricity markets constitutes a significant challenge for wind power producers due to the inherent randomness of the underlying energy resource: the wind. The market strategy of a wind power producer must be profit effective while able to damp as much as possible the profit variability caused by significant wind fluctuation. In addition to uncertainty on wind availability, wind producers must also cope with uncertain market prices as any other market agent.

As a result of the time lapse existing between the day-ahead market and the real-time operation of the power system, wind power producers have serious trouble to fulfill the production scheduling settled in this market. Consequently, they are forced to correct their energy deviations in a balancing market through which the real-time energy balance between generation and demand is ensured on a hourly basis.

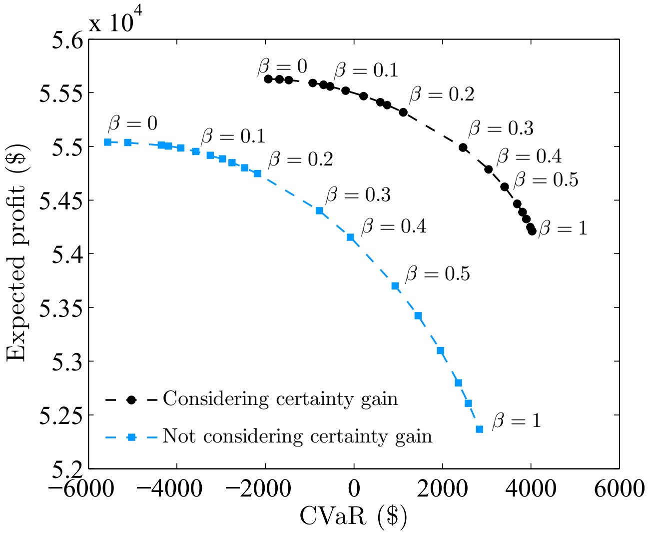


Fig. 6.25 Wind producer case study: impact of the certainty gain effect on risk management (efficient frontier)

However, the price formation mechanism in the balancing market guarantees that any amount of energy produced above the one stipulated in the day-ahead market is paid at a price equal to or lower than the day-ahead price. Similarly, any amount of energy bought in this market to cover a deficit of generation is charged at a price equal or higher than the day-ahead market price. Therefore, energy trading in the balancing market entails an opportunity cost for those market agents incurring deviations. This cost is usually referred to as *imbalance cost*. Hence, as a wind producer needs to resort to this balancing mechanism, its profitability diminishes.

This chapter focuses on the design of a multi-stage stochastic programming model to build energy offers allowing wind producers to shrewdly compete in the different short-term trading floors available in a fully-fledged electricity market, namely, the day-ahead, adjustment, and balancing markets. The strategy offer obtained from the proposed model is aimed at maximizing the expected profit of the short-term wind energy trading, while controlling profit variability and minimizing the need for balancing energy. Uncertainties affecting decision variables, i.e., wind availability and market prices, are modeled via a set of scenarios, whose cardinality is conveniently reduced to make the resulting linear optimization problem tractable.

The main technical aspects of the proposed model, as well as the characteristics of the decisions derived from its use, are highlighted and discussed by means of a detailed example and a realistic case study, which allow us to conclude that:

1. The short-term trading problem of a wind power producer is essentially stochastic and as such should be treated. This is inferred from the positive character of the VSS linked to the solution provided by the risk-neutral variant of the proposed stochastic programming approach.
2. A multi-stage stochastic modeling of the wind producer problem constitutes a natural way to reproduce the sequential order in which decisions are made and uncertain information is revealed.
3. Risk management in the wind producer problem is possible if understood as the possibility of achieving a high decrease in the risk of profit variability for a comparatively small reduction in expected profit.
4. Energy offers for the day-ahead market anticipating the improvement in wind power forecasts as the adjustment market clearing time approaches (certainty gain effect) enable the wind power producer to increase its expected profit while reducing the level of risk involved.
5. Adjustment markets can be seen as structural elements within electricity markets to increase the competitiveness degree of wind power producers.

6.9 Notation

The main notation used throughout this chapter is stated below for quick reference.

Indices and Numbers:

t	Index of time periods spanning the market horizon and running from 1 to N_{T}.
τ	Index of time periods spanning the time interval between the closures of the day-ahead and adjustment markets, and running from 1 to N_{T_1}.
ω	Index of scenarios running from 1 to N_{Ω}.

Continuous Variables:

C_t	Imbalance cost incurred by the wind producer in time period t [\$].
P_t^{D}	Power offered by the wind producer in the day-ahead market for time period t [MW].
$P_{t\omega}^{\mathrm{A}}$	Power offered by the wind producer in the adjustment market for time period t and scenario ω [MW].

$P^{\mathrm{S}}_{t\omega}$	Power production scheduled for the wind producer in time period t and scenario ω [MW].
$\Delta_{t\omega}$	Total deviation of energy incurred by the wind producer with respect to the schedule in time period t and scenario ω [MWh].
$\Delta^{+}_{t\omega}$	Excess of energy (positive deviation) incurred by the wind producer with respect to the schedule in time period t and scenario ω [MWh].
$\Delta^{-}_{t\omega}$	Deficit of energy (negative deviation) incurred by the wind producer with respect to the schedule in time period t and scenario ω [MWh].
ζ	Auxiliary variable used to compute the Conditional Value at Risk, CVaR [$].
η_{ω}	Auxiliary variable in scenario ω used to compute the CVaR [$].

Random Variables:

$\Delta\lambda_t$	Difference between adjustment and day-ahead market prices in time period t [$/MWh].
λ^{A}_t	Adjustment market price in time period t [$/MWh].
λ^{D}_t	Day-ahead market price in time period t [$/MWh].
λ^{UP}_t	Price for upward energy resulting from the balancing market in time period t [$/MWh].
λ^{DN}_t	Price for downward energy resulting from the balancing market in time period t [$/MWh].
λ^{+}_t	Price of positive imbalances incurred in period t [$/MWh].
λ^{-}_t	Price of negative imbalances incurred in period t [$/MWh].
P_t	Wind power production in time period t of the market horizon [MW].
P_{τ}	Wind power production in time period τ of the time interval between the closures of the day-ahead and adjustment markets [MW].
r^{+}_t	Ratio between positive imbalance price and day-ahead market price in time period t.
r^{-}_t	Ratio between negative imbalance price and day-ahead market price in time period t.

These variables, if augmented with the subscript ω, represent their realization in scenario ω.

Constants:

d_t	Duration of time period t [h].
$P^{\max}$	Installed capacity of the wind farm [MW].
α	Per unit confidence level.
β	Risk-aversion parameter.
π_ω	Probability of occurrence of scenario ω.

Set:

Ω	Set of scenarios.

6.10 Exercises

Exercise 6.1. Describe the short-term trading structure of the electricity market of the Iberian Peninsula. How many adjustment floors does this market include?

Information on this market can be found in `www.omel.com`.

Exercise 6.2. Explore the mechanism for imbalance price formation in the Nordic Power Market and point out the differences, if any, with respect to the procedure described in Subsection 6.3.1.

Information on this market can be found in `www.nordpool.com`.

Exercise 6.3. Calculate the imbalance cost incurred in a given time period t by a wind power producer that sells 75 MWh of energy in the day-ahead market at a price of \$24/MWh for that period, but eventually generates 100 MWh. Suppose that the positive and negative imbalance price ratios in period t turn out to be 0.85 and 1, respectively.

What is the profit obtained by this wind power producer in such a period?

Exercise 6.4. Consider a market horizon comprising six time periods. Given the series of ratios $\boldsymbol{r} = \{r_t, t = 1, \ldots, 6\} = \{1.65, 0.56, 0.90, 1.30, 1.10, 0.30\}$, deduce the series of imbalance price ratios $\boldsymbol{r}^+ = \{r_t^+, t = 1, \ldots, 6\}$ and $\boldsymbol{r}^- = \{r_t^-, t = 1, \ldots, 6\}$ from the rules governing the mechanism for imbalance price formation explained in Subsection 6.3.1 and represent them graphically.

Exercise 6.5. Build a scenario tree for the market organization and time framework represented in Fig. 6.8 (see Illustrative Example 6.4) according to the following information:

1. The day-ahead market price λ^{D} is equal to \$65/MWh with a probability of 0.3 and equal to \$30/MWh with a probability of 0.7.
2. The price difference $\Delta\lambda$ between markets is either -\$15/MWh or \$15/MWh, both with a probability of 0.5.
3. Pair (r^{+}, r^{-}) is equal to $(1, 1.3)$ with a probability of 0.45 and equal to $(0.6, 1)$ with a probability of 0.55.
4. The wind power production during the hour τ between the closures of the day-ahead and adjustment markets is *high* (70 MW) with a probability of 0.6 and *low* (30 MW) with a probability of 0.4.
5. If the wind power production in period τ is *high*, then the wind power produced during the single hour covering the market horizon is either *extremely high* (100 MW) or *relatively high* (60 MW), both with a probability of 0.5.
6. If the wind power production in period τ is *low*, then the wind power produced throughout the market horizon is *extremely low* (10 MW) with a probability of 0.35 and *relatively low* (45 MW) with a probability of 0.65.

Exercise 6.6. Formulate and solve model (6.36)–(6.41) for the scenario tree built in Exercise 6.5. Suppose that the installed capacity of the wind farm is 120 MW and discuss the obtained solution.

Exercise 6.7. Compute the Value of Stochastic Solution (VSS) for the offering strategy model formulated in Exercise 6.6. Also, calculate the Expected Value of Perfect Information (EVPI) to appraise the economic impact of each uncertainty source.

Exercise 6.8. Draw the imbalance cost function associated with the offering strategy model formulated in Exercise 6.6.

Exercise 6.9. Formulate and solve model (6.52)–(6.64) for the scenario tree built in Exercise 6.5. Assess the economic impact of the certainty gain effect.

Exercise 6.10. Obtain the efficient frontier for the offering strategy model formulated in Exercise 6.9.

Chapter 7
Futures Market Trading for Producers

7.1 Introduction

This chapter considers the problem of an electric energy producer that sells its production of energy in both the futures market and the pool. The futures market allows the producer to sell its energy production at fixed prices through forward contracts, which span a pre-specified time period, e.g., one month. On the other hand, the pool allows the power producer to sell energy on a short-term basis taking advantage of periods of high price, but at the cost of suffering high price volatility.

Needless to say, the actual production of the producer is constrained by the technical limitations of its generating units. Moreover, if any of these units fails, the producer may have to buy energy in the pool to meet its contracting obligations.

In a medium-term planning horizon, the objective of the producer is to determine its involvement in the futures market to maximize its expected profit over a given planning horizon, while controlling the risk of profit variability. Risk management is explicitly carried out by means of the Conditional Value-at-Risk, CVaR.

Relevant references on futures market tools for producers include [24, 28, 62, 87, 104, 116, 136, 140]. Futures markets currently in operation include EEX [47], OMIP [107] and NYMEX [101].

7.2 Decision Framework

We consider a power producer whose goal is to determine its optimal involvement in the futures market to hedge against the pool price volatility through forward contracts, which are used during a medium-term planning horizon, e.g., one year. The producer decides its involvement in the futures market

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at the beginning of the planning horizon, whereas pool trading decisions are made throughout it.

Decisions related to forward contracting, which are made on a monthly or quarterly basis, are decided prior to the knowledge of pool prices. These decisions are thus *here-and-now* decisions (see Chapter 2 for a detailed explanation of here-and-now decisions), and are identical for each possible realization of pool prices.

Once forward decisions are made, the operation of the generating units and the amount of power traded in the pool are settled for each period of the planning horizon. These decisions are made close to the time when pool prices are realized. By using adequate prediction tools that generally involve a high degree of accuracy, these price predictions can be considered exact. Thus, decisions pertaining to pool trading are assumed to be made with perfect information on pool prices. This is equivalent to say that these decisions are *wait-and-see* decisions (see Chapter 2 for a detailed explanation of wait-and-see decisions), which are dependent on each realization of the pool prices.

This decision framework is depicted in Fig. 7.1, where N_{T} represents the number of periods in the planning horizon, and is clarified by the following illustrative example.

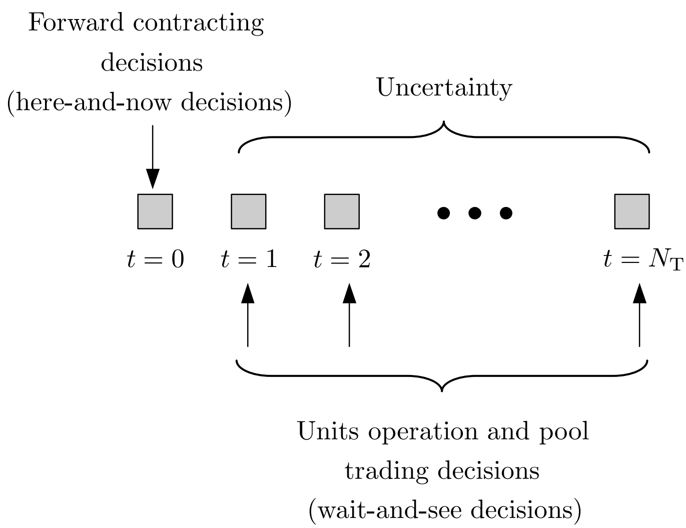

Fig. 7.1 Decision framework for a producer

Illustrative Example 7.1 (Two-stage decision framework).

Consider a one-month market horizon. Four weekly forward contracts are available for the producer to sell energy, each corresponding to each of the four weeks of the month, as well as one monthly contract. The pool allows trading on an hourly basis. The decision framework for the producer is as follows:

1. At the beginning of the month, the producer decides which weekly and monthly forward contracts to sign for the whole market horizon.
2. For each realization of the hourly pool prices of the month, the producer decides the amount of energy to be sold each hour in the pool.

□

Considering a yearly decision-making framework, the producer would implement optimal forward contract decisions pertaining to the whole year at the beginning of the first month. By the time that forward contract decisions at the commencement of the second month must be made, pool prices of the first month are already known. Thus, information on prices is updated and the model is re-run to determine forward contract decisions pertaining to the following twelve months (months two to thirteen). In order to make optimal decisions for forward contracts one year ahead, this procedure is repeated at the beginning of each month. Note that the one-year horizon and the monthly decision framework are arbitrarily selected and should be tailored to the needs of the considered producer.

Other decisions frameworks are possible, and particularly, multi-stage ones. A multi-stage stochastic programming problem can be used to account for the possibility of making successive forward contract decisions throughout the planning horizon. Note, however, that the use of multi-stage scenario trees increases the computational size of the problem, frequently making it intractable. See Chapter 2 for further details.

A five-stage decision framework is described below as an example.

Illustrative Example 7.2 (Five-stage decision framework).

Consider a one-month market horizon and the same forward contracts and pool arrangements as in Illustrative Example 7.1 above. A multi-stage (five-stage) decision framework for the producer is as follows:

1. At the beginning of the month, the producer decides whether or not to sign the monthly contract and the weekly contract pertaining to the first week of the month. These decisions should be made taking into account subsequent decisions throughout the considered month.
2. For each realization of the hourly pool prices during the first week of the month, the producer decides the amount of energy to be sold in the pool each hour of this first week.
3. At the beginning of the second week of the month and for each realization of the pool prices during the first week, the producer decides whether or not to sign the weekly contract pertaining to this second week. This decision should be made taking into account subsequent decisions throughout the remaining three weeks of the considered month.

4. For each realization of the hourly pool prices during the second week of the month, the producer decides the amount of energy to be sold in the pool each hour of this second week.
5. At the beginning of the third week and for each realization of the pool prices during the second week, the producer decides whether or not to sign the weekly contract pertaining to this third week. This decision should be made taking into account subsequent decisions throughout the remaining two weeks of the considered month.
6. For each realization of the hourly pool prices during the third week of the month, the producer decides the amount of energy to be sold in the pool each hour of this third week.
7. At the beginning of the fourth week and for each realization of the pool prices during the third week, the producer decides whether or not to sign the weekly contract pertaining to this fourth week.
8. For each realization of the hourly pool prices during the fourth week of the month, the producer decides the amount of energy to be sold in the pool each hour of this fourth week.

For the sake of simplicity, we consider just two-stage decision frameworks throughout this chapter. See Chapter 2 for further details.

□

7.3 Uncertainty Characterization

In this section, two sources of uncertainty, namely, pool prices and unit availability, are characterized.

7.3.1 Pool Prices

The main source of uncertainty faced by the producer is that related to pool prices, which are unknown when forward contract decisions need to be made. Pool prices result from the offers and bids submitted by pool agents and, for a price-taker producer (a producer with no capability of altering market-clearing prices), are independent of the producer actions. Hence, pool prices can be assumed to be exogenous to the decision-making process of the producer.

In order to tackle the uncertainty pertaining to future pool prices, we model the pool price in period t as a random variable denoted by λ_t^{P}. As it is customary, the random variables constituting the stochastic process associated with future pool prices are represented using a finite set of scenarios. If N_{T} is the number of time periods in the planning horizon, each scenario $\omega \in \Omega$ comprises a vector of pool prices with N_{T} components, i.e.,

$$\text{Scenario } \omega : \{\lambda^{\mathrm{P}}_{1\omega}, \ldots, \lambda^{\mathrm{P}}_{N_{\mathrm{T}}\omega}\}.$$

Note that each scenario ω has a probability of occurrence π_ω, such that the summation of the probabilities over all scenarios is equal to 1, i.e., $\sum_{\omega=1}^{N_\Omega} \pi_\omega = 1$.

Pool price scenarios are exemplified below.

Illustrative Example 7.3 (Pool price scenarios).

Table 7.1 Illustrative Example 7.3: pool price scenarios

Scenario #	Probability	Pool price ($/MWh) Period #	
		1	2
1	1/2	33	43
2	1/2	32	39

Table 7.1 provides an example of pool price scenarios. The market horizon spans two time periods and the number of pool price scenarios considered is just two. The probability of each scenario is 1/2. □

7.3.2 Unit Availability

The unavailability of the production units is modeled as stated below. The historical data of the time between two consecutive unit failures, as well as the time to repair a failure, follow exponential distributions [12]. In consequence, the time to failure, t_F, of the unit is given by

$$t_F = -\mathrm{MTTF} \times \ln(u_1) \;, \tag{7.1}$$

similarly, the time to repair, t_R, is obtained as

$$t_R = -\mathrm{MTTR} \times \ln(u_2) \;. \tag{7.2}$$

Note that t_F and t_R are contingent on random variables u_1 and u_2, which are uniformly distributed between 0 and 1.

Thus, the status of the generating unit is represented by a two-state model as shown in Fig. 7.2.

Every scenario representing the availability of the unit is then generated by evaluating successively equations (7.1) and (7.2) until the time horizon under consideration is completed. Additional details on modeling unit availability can be found in Chapter 10 and in [12, 116].

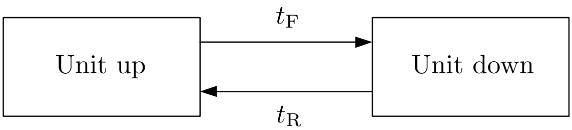

Fig. 7.2 Two-state (up and down) availability model of a generating unit.

In order to represent all possible realizations of the availability of the unit throughout the considered market horizon, a number of scenarios sufficiently large should be generated. The appropriate number of availability scenarios depends on the time horizon and the value of the parameters MTTF and MTTR. For example, if the time horizon is one month and the MTTF is one week, the number of scenarios to represent all possible realizations of the availability of the unit is larger than if the MTTF is equal to one year.

An example of a scenario set describing the availability of a given unit is provided in the Illustrative Example 7.4 below.

Illustrative Example 7.4 (Unit availability scenarios).

Table 7.2 Illustrative Example 7.4: unit availability scenarios (1: available, 0: unavailable)

Scenario #	Probability	Unit Availability Period #	
		1	2
1	0.95	1	1
2	0.02	0	1
3	0.02	1	0
4	0.01	0	0

Consider a producer that owns a single unit and faces a two-period market horizon. The possible scenarios of unit availability are indicated in Table 7.2. Note that the most likely scenario is the one in which the unit is available in both periods. □

7.3.3 Scenario Tree

The set of scenarios together with the decision timing of the problem are represented via a two-stage scenario tree as shown in Fig. 7.3. At the first stage (root node), the amount of energy sold through forward contracts in the planning horizon is decided. In contrast, the operation of the units and the amount of power traded in the pool in each time period are determined for each single realization of pool prices at the second-stage nodes (leaves).

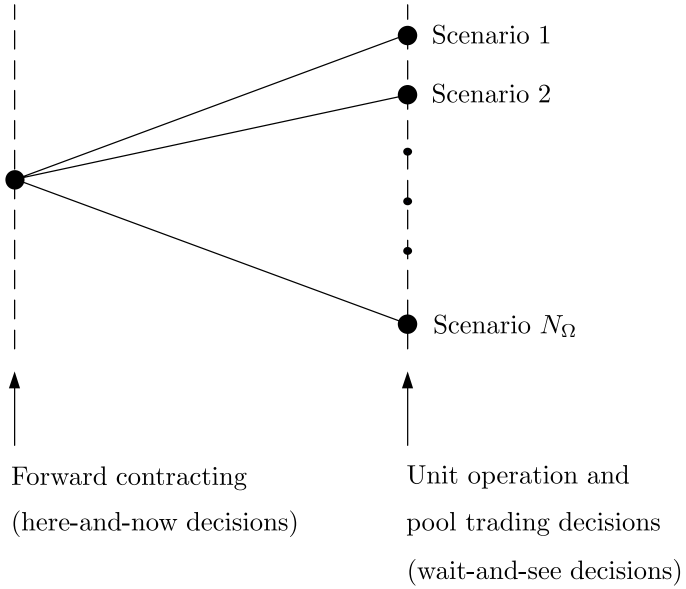

Fig. 7.3 Illustrative Example 7.5: scenario tree

An example of a scenario tree is provided in the following illustrative example.

Illustrative Example 7.5 (Scenario tree).
Considering that the price scenarios in Illustrative Example 7.3 and the availability scenarios in Illustrative Example 7.4 are independent, the final number of scenarios is 8, as shown in Table 7.3. These scenarios constitute the leaves of the tree in Fig. 7.3. Note that period 1 is an off-peak period whereas period 2 is a peak one. □

7.4 Market Structure

7.4.1 Futures Market

The futures market allows the producer to sell electricity in a future time span at a fixed price. Chapter 1 provides a general overview of the electricity marketplace including the futures market. Therefore, once a forward contract

Table 7.3 Illustrative Example 7.5: price and unit availability scenarios (1: available, 0: unavailable)

Scenario #	Probability	Availability (0/1) Period #		Price (\$/MWh) Period #	
		1 (off-peak)	2 (peak)	1 (off-peak)	2 (peak)
1	$0.95 \times 1/2$	1	1	33	43
2	$0.02 \times 1/2$	0	1	33	43
3	$0.02 \times 1/2$	1	0	33	43
4	$0.01 \times 1/2$	0	0	33	43
5	$0.95 \times 1/2$	1	1	32	39
6	$0.02 \times 1/2$	0	1	32	39
7	$0.02 \times 1/2$	1	0	32	39
8	$0.01 \times 1/2$	0	0	32	39

is signed, the revenue from selling the electricity associated with that contract is risk free. In other words, since forward contract prices are fixed quantities, no risk pertaining to profit variability is incurred while selling energy through forward contracts. Different products are available in a typical futures market. According to their duration, there exist weekly, monthly, quarterly, yearly, and multi-year forward contracts. We also distinguish between peak and base contracts. Peak contracts are used only during peak periods of weekdays. For instance, peak periods may comprise hours from 8am to 8pm (12 hours) on weekdays. On the other hand, base contracts are defined for all hours in the day. Typically, peak contracts have a higher price than base contracts. In addition to forward contracts, a producer may sell energy through an option contract. An option leaves the producer the choice to sell a quantity of energy during a future time period by paying a fee known as premium. A producer can also sign insurance contracts to protect itself against the failure of its production units if a significant number of forward contracts are signed. See Chapter 1 for further details on futures market products.

The power signed in a forward contract remains constant over the lifetime of the contract. Thus, the energy contracted in period t through forward contract f is equal to the product of the contracted power, P_f^{F}, and the number of hours, d_t.

We propose the use of forward contracting curves to model the price impact of the producer on the futures market [28]. For each contract, a forward contracting curve expresses the variation in the contract price with the amount of power traded by the producer. Forward contract prices are assumed to decrease in a stepwise manner as the amount of power sold grows. In other words, we consider that as the producer tries to sell more an more energy in the futures markets through a forward contract, the price of that contract goes down as a result of an excess of supply. This is not necessarily like that in all markets and thus these forward contracting curves should be tailored to the specific characteristics of the market under consideration.

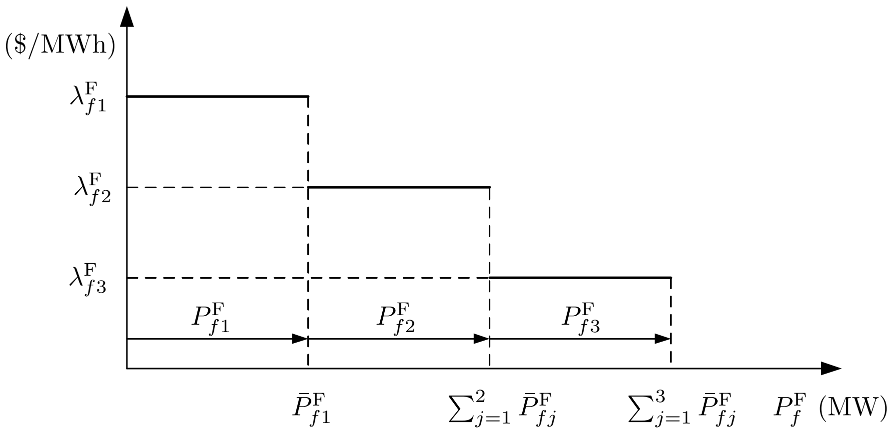

Fig. 7.4 Example of forward contracting curve for selling energy

Fig. 7.4 shows an example of a forward contracting curve with three blocks. The parameter $\bar{P}^{\mathrm{F}}_{fj}$ is the upper limit of the power sold through contract f in block j, and $\lambda^{\mathrm{F}}_{fj}$ is the price of block j in the forward contracting curve of this contract. We consider that forward contracting curves are estimated by the producer prior to the time of making decisions on trading in the futures market.

Given the forward contracting curves of the producer, the revenue obtained from selling through forward contracts in period t is formulated as

$$R^{\mathrm{F}}_t = \sum_{f \in F_t} \sum_{j=1}^{N_{\mathrm{J}}} \lambda^{\mathrm{F}}_{fj} P^{\mathrm{F}}_{fj} d_t, \ \forall t \tag{7.3}$$

$$0 \leq P^{\mathrm{F}}_{fj} \leq \bar{P}^{\mathrm{F}}_{fj}, \ \forall f \in F_t, j = 1, \ldots, N_{\mathrm{J}} \tag{7.4}$$

$$P^{\mathrm{F}}_{f} = \sum_{j=1}^{N_{\mathrm{J}}} P^{\mathrm{F}}_{fj}, \ \forall f \in F_t. \tag{7.5}$$

Expressions (7.3) provide the revenue obtained from selling energy through forward contracts in each time period. This revenue depends on the contracted power, the energy price in each block of the forward contracting curve, and the duration of each period. Constraints (7.4) state that the power sold in each block is non-negative and bounded above. Finally, expressions (7.5) state for each contract that the total power sold is the sum of the power sold through each block.

For the sake of simplicity, the producer is assumed not to buy energy in the futures market. Nevertheless, it is straightforward to include such a purchasing option in the proposed model.

Illustrative Example 7.6 below shows an example of a typical forward contract used by a producer to sell energy in the futures market.

Illustrative Example 7.6 (Forward contracts).

Table 7.4 provides the description of two forward contracts spanning the two periods of a market horizon. As indicated in this table, the forward contracting curve corresponding to each contract has two blocks.

Table 7.4 Illustrative Example 7.6: forward contracts for producers

Contract #	Usage period	Block 1 Price ($/MWh)	Block 2 Price ($/MWh)	Block 1 Power (MW)	Block 2 Power (MW)
1	1-2	35.0	34.0	100	100
2	1-2	34.5	32.0	110	110

□

7.4.2 Pool

The producer may participate in the pool to sell part of its electricity production. For the sake of simplicity, we assume that the producer does not purchase energy in this market unless some of its production units are out of order due to unexpected failures.

Future pool prices are unknown to the producer when it sells in the futures market. Thus, they are modeled as a stochastic process using a set of scenarios $\{\lambda^{\mathrm{P}}_{t\omega}, t = 1, \ldots, N_{\mathrm{T}}, \forall \omega \in \Omega\}$.

The total amount of energy sold in the pool by the producer per time period and scenario is denoted by the continuous and non-negative variable $E^{\mathrm{P}}_{t\omega}$. Since the objective is to determine the forward contract positions (i.e., which forward contracts to sign) for a medium-term planning horizon, we are only interested in the total energy traded in the pool by the producer, and not in the energy traded by each single generating unit. This issue does not alter the optimal involvement of the producer in the futures market and reduces the number of continuous variables in the problem formulation.

Variable $E^{\mathrm{P}}_{t\omega}$ depends on the time index t and the scenario index ω, and constitutes a wait-and-see decision that is made with perfect information about the pool price, $\lambda^{\mathrm{P}}_{t\omega}$. In other words, it is assumed for each scenario that the pool prices are known at the time of making decisions on pool trading.

The revenue obtained from selling in the pool is computed as

$$R^{\mathrm{P}}_{t\omega} = \lambda^{\mathrm{P}}_{t\omega} E^{\mathrm{P}}_{t\omega}, \ \forall t, \forall \omega, \tag{7.6}$$

with

$$E^{\mathrm{P}}_{t\omega} \geq 0, \ \forall t, \forall \omega. \tag{7.7}$$

7.5 Producer Model

7.5.1 Unit Constraints

Let g be the index of generation units and $E^{\mathrm{G}}_{gt\omega}$ the energy produced by unit g in period t and scenario ω. Since we study a medium-term planning horizon and time periods t are not necessarily hours, only production limits are considered in this problem,

$$0 \leq E^{\mathrm{G}}_{gt\omega} \leq P^{\mathrm{G,max}}_{g} d_t, \ \forall g, \forall t, \forall \omega, \tag{7.8}$$

where $P^{\mathrm{G,max}}_{g}$ is the capacity of unit g, and d_t is the duration of period t in hours. The minimum power output and power ramp limits are therefore not considered in the unit modeling.

In order to avoid solutions with multiple start-ups and shut-downs, we enforce that the generation during any off-peak period is greater than or equal to a percentage of the generation during the successive peak period. Mathematically, this is formulated as follows,

$$E^{\mathrm{G}}_{gt\omega} \geq H_g E^{\mathrm{G}}_{gt'\omega}, \ \forall g, \forall \omega, \ \forall t, t' \mid W(t,t') = 1, \tag{7.9}$$

where the parameter H_g represents the minimum percentage of the energy produced by unit g during peak periods that must be generated during off-peak periods, and W is a matrix whose element $W(t,t')$ is equal to 1 if t and t' are two consecutive off-peak and peak periods, respectively, and 0 otherwise. Constant H_g is producer dependent and its value should be tailored to the conditions of the considered producer.

The production cost incurred by unit g in period t and scenario ω, $C^{\mathrm{G}}_{gt\omega}$, is computed as

$$C^{\mathrm{G}}_{gt\omega} = C^{\mathrm{G}}_{g} E^{\mathrm{G}}_{gt\omega}, \ \forall g, \forall t, \forall \omega, \tag{7.10}$$

where C^{G}_{g} is the production cost coefficient of unit g.

Similar to the energy generated, $E^{\mathrm{G}}_{gt\omega}$, the production cost incurred by unit g in period t depends on the pool price scenario.

Illustrative Example 7.7 below exemplifies the restrictions that constrain the operation of the production units.

Illustrative Example 7.7 (Unit constraints).

Technical parameters of two typical production units are provided in Table 7.5.

Table 7.5 Illustrative Example 7.7: data for two typical production units

Unit #	Capacity (MW)	Production Cost (\$/MWh)	Peak/Off-Peak Factor (per unit)
1	300	31	0.33
2	250	32	0.33

Considering period 1 of scenario 1 of Table 7.3 (period 1 is off-peak whereas period 2 is peak) and unit 1 in Table 7.5, equations (7.8), (7.9) and (7.10) above become

$$0 \leq E^{\mathrm{G}}_{111} \leq 300$$

$$E^{\mathrm{G}}_{111} \geq 0.33 E^{\mathrm{G}}_{121}$$

$$C^{\mathrm{G}}_{111} = 31 E^{\mathrm{G}}_{111}.$$

□

7.5.2 Unit Availability

If a given unit is subject to unavailability, its maximum power output is either its rated maximum power or zero. This is modeled via scenarios and boils down to an updated version of equation (7.8) above, i.e.,

$$0 \leq E^{\mathrm{G}}_{gt\omega} \leq P^{\mathrm{G,max}}_{g} a_{gt\omega} d_t, \ \forall g, \forall t, \forall \omega, \tag{7.11}$$

where $a_{gt\omega}$ is a 0/1 constant indicating the availability of unit g during period t and scenario ω.

Illustrative Example 7.8 below exemplifies the availability restrictions that constrain the operation of the production units.

Illustrative Example 7.8 (Availability constraints).

Considering time period 1 in Table 7.3 and unit 1 in Table 7.5, equation (7.8) for scenarios 1 and 2 renders, respectively,

$$0 \leq E_{111}^{\mathrm{G}} \leq 300$$

$$0 \leq E_{112}^{\mathrm{G}} \leq 0.$$

□

7.5.3 Energy Balance

The electric energy balance of the producer is mathematically formulated as follows:

$$\sum_{g=1}^{N_{\mathrm{G}}} E_{gt\omega}^{\mathrm{G}} = E_{t\omega}^{\mathrm{P}} + \sum_{f \in F_t} \sum_{j=1}^{N_{\mathrm{J}}} P_{fj}^{\mathrm{F}} d_t + E_t^{\mathrm{PC}}, \ \forall t, \forall \omega, \tag{7.12}$$

where F_t is the set of forward contracts available in period t, and the parameter E_t^{PC} represents the previously contracted energy that must be supplied in period t. For instance, E_t^{PC} may represent the energy corresponding to a forward contract signed in the past, which is available for several years and is currently in force.

Illustrative Example 7.9 below shows an energy balance constraint.

Illustrative Example 7.9 (Energy balance).

Considering time period 1 of scenario 1 in Table 7.3, the two units in Table 7.5, the forward contracts in Table 7.4, and that 20 MW were sold trough an old forward contract still in force, equation (7.12) renders

$$E_{111}^{\mathrm{G}} + E_{211}^{\mathrm{G}} = E_{11}^{\mathrm{P}} + P_{11}^{\mathrm{F}} + P_{12}^{\mathrm{F}} + P_{21}^{\mathrm{F}} + P_{22}^{\mathrm{F}} + 20.$$

□

7.5.4 Expected Profit

The profit of the producer is expressed as the sum of the revenue obtained from selling through forward contracts and in the pool, minus the production cost of the generating units. The profit in scenario ω is mathematically expressed as

$$\sum_{t=1}^{N_{\mathrm{T}}}\left(R_t^{\mathrm{F}}+R_{t\omega}^{\mathrm{P}}-\sum_{g=1}^{N_{\mathrm{G}}}C_{gt\omega}^{\mathrm{G}}\right)=$$

$$\sum_{t=1}^{N_{\mathrm{T}}}\left(\sum_{f\in F_t}\sum_{j=1}^{N_{\mathrm{J}}}\lambda_{fj}^{\mathrm{F}}P_{fj}^{\mathrm{F}}d_t+\lambda_{t\omega}^{\mathrm{P}}E_{t\omega}^{\mathrm{P}}-\sum_{g=1}^{N_{\mathrm{G}}}C_g^{\mathrm{G}}E_{gt\omega}^{\mathrm{G}}\right),\ \forall\omega. \tag{7.13}$$

The three terms in expression (7.13) correspond to the revenue from selling in the futures market, the revenue from selling in the pool, and the production cost, respectively.

Illustrative Example 7.10 below shows the computation of the profit per scenario.

Illustrative Example 7.10 (Profit per scenario).

Considering scenario 1 in Table 7.3, the two units in Table 7.5, and the forward contracts in Table 7.4, equation (7.13) renders

$$2\left(35P_{11}^{\mathrm{F}}+34P_{12}^{\mathrm{F}}+34.5P_{21}^{\mathrm{F}}+32P_{22}^{\mathrm{F}}\right)+33E_{11}^{\mathrm{P}}+43E_{21}^{\mathrm{P}}$$
$$-\left(31E_{111}^{\mathrm{G}}+31E_{121}^{\mathrm{G}}+32E_{211}^{\mathrm{G}}+32E_{221}^{\mathrm{G}}\right).$$

□

The expected profit is equal to the sum over all scenarios of the profit in each scenario (7.13) multiplied by its probability of occurrence. The expected profit is expressed as

$$\sum_{\omega=1}^{N_\Omega}\pi_\omega\sum_{t=1}^{N_{\mathrm{T}}}\left(R_t^{\mathrm{F}}+R_{t\omega}^{\mathrm{P}}-\sum_{g=1}^{N_{\mathrm{G}}}C_{gt\omega}^{\mathrm{G}}\right). \tag{7.14}$$

7.5.5 Risk Modeling

We incorporate the conditional value-at-risk (CVaR) in the model to take into account the risk associated with the volatility of the profit. In this case, the CVaR is useful for limiting the probability of experiencing low profits in unfavorable scenarios. The conditional value-at-risk at α confidence level, $\alpha-$CVaR, is approximately equal to the expected profit of the $(1-\alpha)\times100\%$ scenarios with lowest profit. Given the expression of the profit in scenario ω, (7.13), the $\alpha-$CVaR is computed as

$$\alpha-\mathrm{CVaR}=\mathrm{maximum}_{\zeta,\eta_\omega}\qquad\zeta-\frac{1}{1-\alpha}\sum_{\omega=1}^{N_\Omega}\pi_\omega\eta_\omega \tag{7.15}$$

subject to

$$\zeta - \sum_{t=1}^{N_\mathrm{T}} \left(R_t^\mathrm{F} + R_{t\omega}^\mathrm{P} - \sum_{g=1}^{N_\mathrm{G}} C_{gt\omega}^\mathrm{G} \right) \leq \eta_\omega, \ \forall \omega \tag{7.16}$$

$$\eta_\omega \geq 0, \ \forall \omega. \tag{7.17}$$

The optimal value of ζ is the greatest value of the profit such that the probability of the actual profit being smaller than or equal to ζ is less than or equal to $1-\alpha$. The optimal value of ζ is the value-at-risk (VaR) of the profit at a confidence level equal to α. At the optimum, the auxiliary variable η_ω represents the excess of ζ over the profit of scenario ω, provided that this excess is positive.

7.6 Formulation

The formulation of the forward contracting problem for a producer is provided below.

$$\begin{aligned} & \text{Maximize}_{P_{fj}^\mathrm{F}, E_{t\omega}^\mathrm{P}, E_{gt\omega}^\mathrm{G}, \zeta, \eta_\omega} \\ & \sum_{\omega=1}^{N_\Omega} \pi_\omega \sum_{t=1}^{N_\mathrm{T}} \left(\sum_{f \in F_t} \sum_{j=1}^{N_\mathrm{J}} \lambda_{fj}^\mathrm{F} P_{fj}^\mathrm{F} d_t + \lambda_{t\omega}^\mathrm{P} E_{t\omega}^\mathrm{P} - \sum_{g=1}^{N_\mathrm{G}} C_g^\mathrm{G} E_{gt\omega}^\mathrm{G} \right) \\ & \qquad + \beta \left(\zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\Omega} \pi_\omega \eta_\omega \right) \end{aligned} \tag{7.18}$$

subject to

$$0 \leq P_{fj}^\mathrm{F} \leq \bar{P}_{fj}^\mathrm{F}, \ \forall f, j = 1, \ldots, N_\mathrm{J} \tag{7.19}$$

$$0 \leq E_{gt\omega}^\mathrm{G} \leq P_g^{\mathrm{G,max}} d_t, \ \forall g, \forall t, \forall \omega \tag{7.20}$$

$$E_{gt\omega}^\mathrm{G} \geq H_g E_{gt'\omega}^\mathrm{G}, \ \forall g, \forall \omega, \forall t, t' \mid W(t,t') = 1 \tag{7.21}$$

$$\sum_{g=1}^{N_\mathrm{G}} E_{gt\omega}^\mathrm{G} = E_{t\omega}^\mathrm{P} + \sum_{f \in F_t} \sum_{j=1}^{N_\mathrm{J}} P_{fj}^\mathrm{F} d_t + E_t^\mathrm{PC}, \ \forall t, \forall \omega \tag{7.22}$$

$$\zeta - \sum_{t=1}^{N_\mathrm{T}} \left(\sum_{f \in F_t} \sum_{j=1}^{N_\mathrm{J}} \lambda_{fj}^\mathrm{F} P_{fj}^\mathrm{F} d_t + \lambda_{t\omega}^\mathrm{P} E_{t\omega}^\mathrm{P}; - \sum_{g=1}^{N_\mathrm{G}} C_g^\mathrm{G} E_{gt\omega}^\mathrm{G} \right) \leq \eta_\omega, \forall \omega \tag{7.23}$$

$$\eta_\omega \geq 0, \ \forall \omega \tag{7.24}$$

$$E_{t\omega}^\mathrm{P} \geq 0, \ \forall t, \forall \omega. \tag{7.25}$$

The objective function (7.18) comprises two terms: i) the expected profit and ii) the CVaR multiplied by the weighting factor β. This factor allows modeling the tradeoff between the expected profit and the CVaR.

Constraints (7.19) bound the power contracted from each block of the forward contracting curve of each contract. Constraints (7.20) and (7.21) are the technical constraints of the units owned by the producer. Constraints (7.22) impose the energy balance in each period and scenario. Constraints (7.23) and (7.24) are used to compute the CVaR. Finally, (7.25) are variable declarations.

Problem (7.18)-(7.25) is a linear programming problem easy to solve.

7.7 Producer Futures Market Example. No Unit Unavailability

Table 7.6 Producer futures market example: thermal unit data

Unit #	Capacity (MW)	Cost ($/MWh)	Peak/Off-Peak Factor (per unit)
1	300	31	0.33
2	250	32	0.33

An example of reduced size is analyzed next to illustrate the previously presented formulation of the problem faced by a power producer to decide on forward contracting.

We consider a power producer that owns two thermal units. Technical parameters of both units are given in Table 7.6.

We assume a planning horizon of four hourly periods, where periods one and three are off-peak, while periods two and four are peak.

Table 7.7 Producer futures market example: forward contracting curve data

Contract #	Usage period	Block 1 Price ($/MWh)	Block 2 Price ($/MWh)	Block 1 Power (MW)	Block 2 Power (MW)
1	1-4	35.0	34.0	100	100
2	1-4	34.5	32.0	100	100

Two forward contracts spanning the four periods of the planning horizon are available. The forward contracting curves corresponding to each contract have two blocks. The parameters defining each contract are provided in Table 7.7.

Pool prices are modeled using a set of four equiprobable scenarios. Table 7.8 presents the pool price in each period and scenario.

Table 7.8 Producer futures market example: pool price scenarios

Scenario #	Pool price ($/MWh) Period #			
	1	2	3	4
1	33	43	29	36
2	32	39	26	33
3	34	48	28	37
4	33	45	26	39

Problem (7.18)-(7.25), characterized by 105 constraints and 58 real variables, is solved for different values of the weighting factor β and for a confidence level α equal to 0.95. The expected profit, the profit standard deviation, and the CVaR for different values of β are listed in Table 7.9. The optimal solution attained for $\beta = 0$ achieves the highest expected profit and the highest risk, measured in terms of both the standard deviation and the CVaR of the profit. The expected profit varies from \$9548.13 for $\beta = 0$ to \$8367.50 for $\beta = 10$, respectively. As can be seen, a reduction of 12.4% in the expected profit produces a reduction of 54.3% in the profit standard deviation and a 34.3% increase in the CVaR. Note that relatively small values of the profit standard deviation and large values of the CVaR indicate risk-averse solutions.

Table 7.9 Producer futures market example: results

β	Expected profit ($)	Profit standard deviation ($)	CVaR ($)
0	9548.13	3164.75	4555.00
0.5	9522.38	2543.33	5552.03
1	9278.50	2132.85	5962.00
5	9167.50	2025.77	6017.50
10	8367.50	1446.98	6117.50

The efficient frontiers, representing the expected profit versus both the profit standard deviation and the CVaR for different values of β, are shown in Fig. 7.5. Note that low-risk solutions, i.e., solutions with a low standard deviation and a high CVaR, correspond to low expected profits.

The power sold through forward contracts is given in Table 7.10. If $\beta = 0$ no contract is signed due to the fact that the expected pool prices are greater

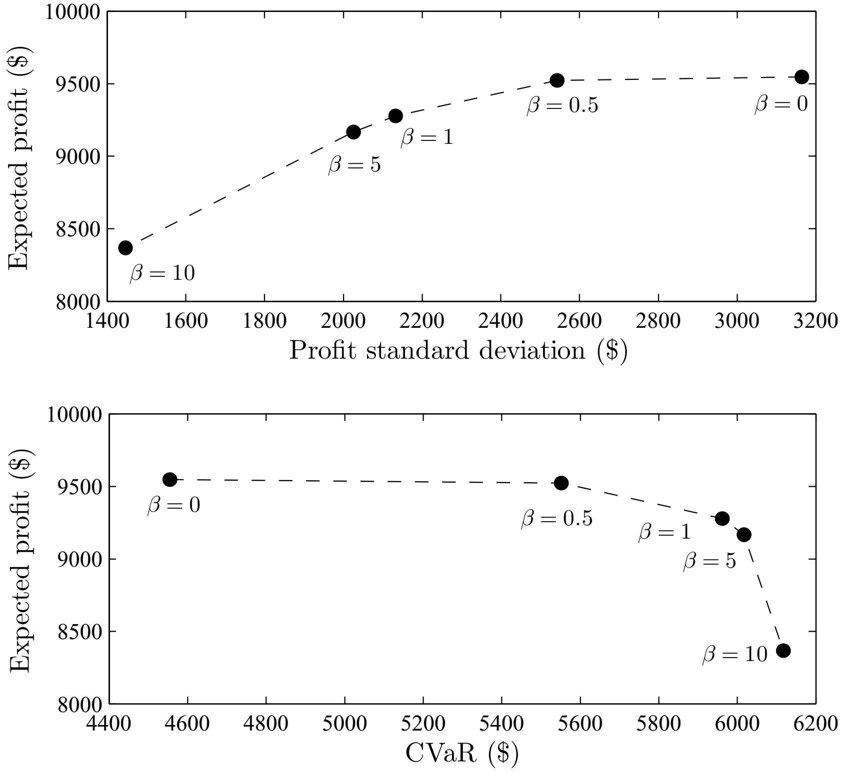

Fig. 7.5 Producer futures market example: efficient frontier

Table 7.10 Producer futures market example: power sold through forward contracts (MW)

	β				
Contract #	0	0.5	1	5	10
1	0	100	100	100	200
2	0	0	82	100	100

than the forward contract prices (see Tables 7.7 and 7.8). The energy sold in the futures market increases as β grows, which, together with the information in Table 7.9, indicates that forward contracts are efficiently used to hedge against the volatility of pool prices.

The expected electricity sales in the futures market and in the pool, as well as the expected energy generated, are provided in Table 7.11. The amount of electricity sold in the futures market increases with β. We conclude that the more risk averse the producer is, the higher the forward contract sales are. The opposite effect is observed for pool sales. That is, pool sales decrease as β increases. This table also shows that, in general, the energy generated by each unit increases with β. This fact is a consequence of the use of forward contracts. In other words, if only the pool is considered, each unit produces energy during those periods when pool prices are greater than its production cost and in those off-peak periods where production is forced by constraints (7.21). Note that the producer has to provide a fixed amount of power over the lifetime of its forward contracts, regardless of the pool prices.

Table 7.11 Producer futures market example: expected electricity generation and sales

β	Expected generation (MWh)	Forward contract sales (MWh)	Expected pool sales (MWh)
0	1748.38	0.00	1748.38
0.5	1707.51	400.00	1307.51
1	1789.63	726.00	1063.63
5	1808.13	800.00	1008.13
10	1908.13	1200.00	708.13

Table 7.12 provides the sales of energy in the futures market and in the pool, and the energy generation in each period and scenario for $\beta = 0$. In this case, no contract is signed and the total energy produced by units 1 and 2 is sold in the pool. Observe that both units are operating in periods 1 and 2 at their maximum power outputs in all scenarios. This result is due to the fact that pool prices in all scenarios are greater than the production costs of both units in periods 1 and 2. However, in off-peak period 3, pool prices in all scenarios are smaller than production costs. In this period, constraints (7.21) are active and consequently, the generated energy in period 3 is 33% of the energy production in peak period 4. Observe that constraints (7.21) are responsible for the shut-down of unit 2 in scenario 2 and period 4 despite the fact that the pool price in that period and scenario is greater than the production cost of unit 2.

Table 7.13 provides the energy dispatched in each period and scenario for $\beta = 10$. In this case, forward contracts 1 and 2 are used. As a result of the increase in the value of β, most of the energy generated by the producer is sold through forward contracts.

Table 7.12 Producer futures market example: energy dispatched for $\beta = 0$

	Period #							
	1				2			
Scenario #	F	P	G_1	G_2	F	P	G_1	G_2
1	0.0	550.0	300.0	250.0	0.0	550.0	300.0	250.0
2	0.0	550.0	300.0	250.0	0.0	550.0	300.0	250.0
3	0.0	550.0	300.0	250.0	0.0	550.0	300.0	250.0
4	0.0	550.0	300.0	250.0	0.0	550.0	300.0	250.0

	Period #							
	3				4			
Scenario #	F	P	G_1	G_2	F	P	G_1	G_2
1	0.0	181.5	99.0	82.5	0.0	550.0	300.0	250.0
2	0.0	99.0	99.0	0.0	0.0	300.0	300.0	0.0
3	0.0	181.5	99.0	82.5	0.0	550.0	300.0	250.0
4	0.0	181.5	99.0	82.5	0.0	550.0	300.0	250.0

F: energy sold through forward contracts (MWh), P: energy sold in the pool (MWh), and G_g: energy generated by unit g (MWh).

Table 7.13 Producer futures market example: energy dispatched for $\beta = 10$

	Period #							
	1				2			
Scenario #	F	P	G_1	G_2	F	P	G_1	G_2
1	300.0	250.0	300.0	250.0	300.0	250.0	300.0	250.0
2	300.0	82.5	300.0	82.5	300.0	250.0	300.0	250.0
3	300.0	250.0	300.0	250.0	300.0	250.0	300.0	250.0
4	300.0	250.0	300.0	250.0	300.0	250.0	300.0	250.0

	Period #							
	3				4			
Scenario #	F	P	G_1	G_2	F	P	G_1	G_2
1	300.0	0.0	217.5	82.5	300.0	250.0	300.0	250.0
2	300.0	0.0	217.5	82.5	300.0	250.0	300.0	250.0
3	300.0	0.0	217.5	82.5	300.0	250.0	300.0	250.0
4	300.0	0.0	217.5	82.5	300.0	250.0	300.0	250.0

F: energy sold through forward contracts (MWh), P: energy sold in the pool (MWh), and G_g: energy generated by unit g (MWh).

7.8 Producer Futures Market Example. Unit Unavailability

This example is identical to the one in Section 7.7 above, but considering unavailability scenarios. We assume that unit 1 is always available (it is highly reliable), but unit 2 may fail. To take into account the failures of unit 2, the five scenarios provided in Table 7.14 are used.

Table 7.14 Producer futures market example (unit unavailability): availability scenarios for unit 2 (1: on, 0: off)

Availability scenario #	Probability	Period #			
		1	2	3	4
1	0.96	1	1	1	1
2	0.01	0	1	1	1
3	0.01	1	0	1	1
4	0.01	1	1	0	1
5	0.01	1	1	1	0

Given the four pool price scenarios in Table 7.8, the total number of considered scenarios is thus twenty (5 availability scenarios $\times$ 4 pool price scenarios = 20 scenarios). Table 7.15 provides the data describing the pool price and availability scenarios. Observe that unit 2 is available in all periods in scenarios 1-4 ($a_{2t\omega} = 1$). In scenarios 5-8 it is unavailable in period 1 ($a_{2t\omega} = 0$), in scenarios 9-12 it is unavailable in period 2, and so on.

For the scenarios in which unit 2 is unavailable, the producer is allowed to buy energy in the pool to meet its contracting obligations.

The expected profit, the profit standard deviation, and the CVaR for different values of β are presented in Table 7.16. Comparing these results with those provided in the case with no unavailability (Table 7.9), it is observed that, for $\beta = 0$, the expected profit is reduced 0.8 % if the unavailability of unit 2 is accounted for. In addition, the CVaR is reduced 38.4 %, from \$4550 to \$2805. This result indicates that the consideration of unavailability scenarios causes a strong profit reduction in some unfavorable scenarios.

The efficient frontiers, representing the expected profit versus both the profit standard deviation and the CVaR for different values of β, are shown in Fig. 7.6. As expected, solutions with low profit correspond to low standard deviations and high CVaRs.

The power sold through forward contracts is given in Table 7.17. As in the case with no unavailability, the contracted power grows as the parameter β increases. However, the amount of power sold through contracts decreases if unavailability is accounted for. Observe that for $\beta = 10$ the producer sells in the futures market 100 + 100 = 200 MW in each period, while in the case with no unavailability the producer sells 200 + 100 = 300 MW for the

Table 7.15 Producer futures market example (unit unavailability): scenario data

		Period #							
		1		2		3		4	
Scenario #	Probability	$\lambda^{\mathrm{P}}_{1\omega}$	$a_{21\omega}$	$\lambda^{\mathrm{P}}_{2\omega}$	$a_{22\omega}$	$\lambda^{\mathrm{P}}_{3\omega}$	$a_{23\omega}$	$\lambda^{\mathrm{P}}_{4\omega}$	$a_{24\omega}$
1	0.24	33	1	43	1	29	1	36	1
2	0.24	32	1	39	1	26	1	33	1
3	0.24	34	1	48	1	28	1	37	1
4	0.24	33	1	45	1	26	1	39	1
5	0.025	33	0	43	1	29	1	36	1
6	0.025	32	0	39	1	26	1	33	1
7	0.025	34	0	48	1	28	1	37	1
8	0.025	33	0	45	1	26	1	39	1
9	0.025	33	1	43	0	29	1	36	1
10	0.025	32	1	39	0	26	1	33	1
11	0.025	34	1	48	0	28	1	37	1
12	0.025	33	1	45	0	26	1	39	1
13	0.025	33	1	43	1	29	0	36	1
14	0.025	32	1	39	1	26	0	33	1
15	0.025	34	1	48	1	28	0	37	1
16	0.025	33	1	45	1	26	0	39	1
17	0.025	33	1	43	1	29	1	36	0
18	0.025	32	1	39	1	26	1	33	0
19	0.025	34	1	48	1	28	1	37	0
20	0.025	33	1	45	1	26	1	39	0

Table 7.16 Producer futures market example (unit unavailability): results

β	Expected profit ($)	Profit standard deviation ($)	CVaR ($)
0	9472.24	3172.42	2805.00
0.5	9446.47	2559.19	5376.93
1	9200.74	2156.30	5776.03
10	9090.44	2051.65	5824.13
50	8692.31	1770.45	5836.63

same value of β (Table 7.10). Therefore, the modeling of the unavailability in the problem causes a decrease in the power sold through forward contracts. This result is consistent since the unavailability of the units causes that the producer must buy the power sold through forward contracts in the pool under uncertain prices if a unit failure occurs. However, note that since the parameter β is only a weighting factor without physical meaning, caution should be exercised when comparing solutions of different problems for the same value of β.

Table 7.18 provides the energy dispatched in each period and scenario for $\beta = 50$. In this case, forward contracts 1 and 2 are used (250 MW are sold

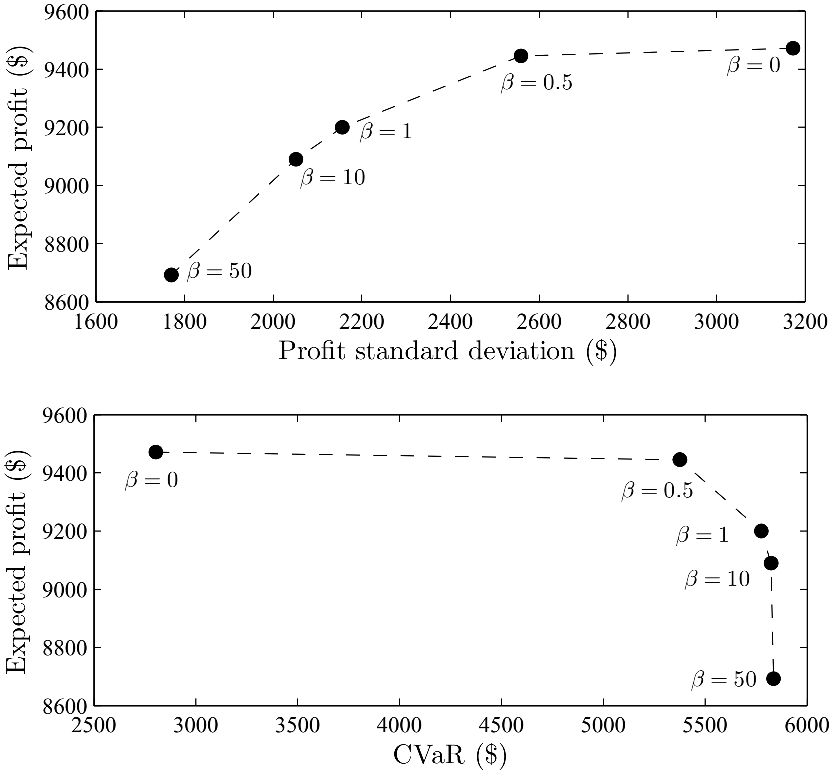

Fig. 7.6 Producer futures market example (unit unavailability): efficient frontier

Table 7.17 Producer futures market example (unit unavailability): power sold through forward contracts (MW)

	β				
Contract #	0	0.5	1	10	50
1	0	100	100	100	150
2	0	0	82	100	100

in the futures market in each period). Observe that unit 2 is available in scenarios 1-4 to produce power in all periods.

In scenarios 5-8 unit 2 is unavailable during period 1. In this case, the power sold in the futures market (250 MW) is entirely provided by unit 1, which is operating at full capacity, 300 MW. Observe that the power generated by unit 2 in period 2 is also equal to zero in these scenarios due to constraint (7.21), which relates the power produced in consecutive off-peak and peak periods. This result can also be observed in period 4 of scenarios 13-16.

It is worth highlighting that, in some scenarios, the producer prefers buying in the pool the energy committed in the futures market instead of generating it by itself. This behavior can be observed in period 3 of scenarios 13-16. In these scenarios the producer buys in the pool 151 MW of the 250 MW committed in the futures market. Note that in these scenarios pool prices are smaller than the production cost of unit 1, which is \$31/MWh.

Table 7.18 Producer futures market example (unit unavailability): energy dispatched for $\beta = 50$

	Period #															
	1				2				3				4			
Scenario #	F	P	G_1	G_2	F	P	G_1	G_2	F	P	G_1	G_2	F	P	G_1	G_2
1	250	300	300	250	250	300	300	250	250	0	167.5	82.5	250	300	300	250
2	250	300	300	250	250	300	300	250	250	0	167.5	82.5	250	300	300	250
3	250	300	300	250	250	300	300	250	250	0	167.5	82.5	250	300	300	250
4	250	300	300	250	250	300	300	250	250	0	167.5	82.5	250	300	300	250
5	250	50	300	0	250	50	300	0	250	0	167.5	82.5	250	300	300	250
6	250	50	300	0	250	50	300	0	250	0	167.5	82.5	250	300	300	250
7	250	50	300	0	250	50	300	0	250	0	167.5	82.5	250	300	300	250
8	250	50	300	0	250	50	300	0	250	0	167.5	82.5	250	300	300	250
9	250	300	300	250	250	50	300	0	250	0	167.5	82.5	250	300	300	250
10	250	300	300	250	250	50	300	0	250	0	167.5	82.5	250	300	300	250
11	250	300	300	250	250	50	300	0	250	0	167.5	82.5	250	300	300	250
12	250	300	300	250	250	50	300	0	250	0	167.5	82.5	250	300	300	250
13	250	300	300	250	250	300	300	250	250	-151	99	0	250	50	300	0
14	250	300	300	250	250	300	300	250	250	-151	99	0	250	50	300	0
15	250	300	300	250	250	300	300	250	250	-151	99	0	250	50	300	0
16	250	300	300	250	250	300	300	250	250	-151	99	0	250	50	300	0
17	250	300	300	250	250	300	300	250	250	0	250	0	250	50	300	0
18	250	300	300	250	250	300	300	250	250	0	250	0	250	50	300	0
19	250	300	300	250	250	300	300	250	250	0	250	0	250	50	300	0
20	250	300	300	250	250	300	300	250	250	0	250	0	250	50	300	0

F: energy sold through forward contracts (MWh), P: energy sold in the pool (MWh), and G_g: energy generated by unit g (MWh).

7.9 Producer Futures Market Case Study

Consider a producer owning six thermal units, whose technical parameters are provided in Table 7.19. For the sake of simplicity, no unit unavailability is accounted for.

Table 7.19 Producer futures market case study: thermal unit data

Unit #	Capacity (MW)	Cost (\$/MWh)	Peak/Off-Peak Factor (%)
1	240	24.0	33
2	240	24.8	33
3	300	25.6	33
4	300	26.4	33
5	300	21.6	33
6	300	23.2	33

A one-year planning horizon is considered, divided into 72 periods, i.e., six periods per month. These six periods correspond to the following sets of hours: *Monday off-peak, Monday peak, weekday off-peak* (other than Monday), *weekday peak* (other than Monday), *weekend off-peak* and *weekend peak*, where peak hours are $\{11, 12, 13, 14, 19, 20, 21, 22\}$, and all other hours are off-peak. For example, the pool price in the first period of the planning horizon is equal to the average of pool prices during off-peak hours on all Mondays in January. The ARIMA model below is used to characterize pool prices using a historical time series,

$$\begin{gathered}\left(1-\phi_4 B^4-\phi_5 B^5-\phi_{12} B^{12}-\phi_{66} B^{66}-\phi_{72} B^{72}-\phi_{144} B^{144}\right)\\ \left(1-B^1\right)\left(1-B^{72}\right)\ln\left(\lambda_t^{\mathrm{P}}\right)=\\ \left(1-\theta_1 B^1-\theta_2 B^2-\theta_3 B^3-\theta_6 B^6-\theta_7 B^7-\theta_{48} B^{48}-\theta_{60} B^{60}\right)\epsilon_t,\ \forall t.\end{gathered} \tag{7.26}$$

The standard deviation of the error term ϵ_t is equal to 2.38 ln(\$/MWh). Each realization or scenario is the result of a random generation of a 72-component vector of error terms, ϵ_t. The values of the parameters in (7.26) are provided in Table 7.20.

Table 7.20 Producer futures market case study: parameters of the ARIMA model (7.26)

$\phi_4 = 0.0989$	$\theta_1 = 0.4205$
$\phi_5 = -0.0637$	$\theta_2 = -0.1812$
$\phi_{12} = -0.1394$	$\theta_3 = 0.2325$
$\phi_{66} = 0.2076$	$\theta_6 = -0.3178$
$\phi_{72} = -0.8223$	$\theta_7 = 0.1877$
$\phi_{144} = -0.2240$	$\theta_{48} = 0.1137$
	$\theta_{60} = 0.1691$

Three monthly and four quarterly forward contracts for peak and base hours are available (fourteen contracts). Note that peak contracts are used during peak periods of weekdays, while base contracts are used in all periods. Table 7.21 provides the price and the upper bound for the first block of the forward contracting curve of each contract. Nine additional, equally-sized blocks are considered for each contract, with prices decreasing 5% in each subsequent block.

For the sake of simplicity, we assume that no previous forward contracts have been signed, i.e., $E_t^{\mathrm{PC}} = 0, \forall t$.

Table 7.21 Producer futures market case study: forward contracting curve data

	Peak contracts		Base contracts	
Contract #	First Price ($/MWh)	First Block (MW)	First Price ($/MWh)	First Block (MW)
Monthly 1	38.60	500	32.29	500
Monthly 2	36.12	400	29.99	400
Monthly 3	31.43	200	27.42	200
Quarterly 1	35.72	500	29.90	500
Quarterly 2	33.89	200	30.21	200
Quarterly 3	37.29	100	32.66	100
Quarterly 4	36.12	100	31.34	100

Using a time series (ARIMA) model, a set of 2000 scenarios of pool prices is generated. After applying a scenario-reduction technique, a final tree of 200 scenarios is used. Further details on scenario generation and reduction can be found in Chapter 3 and in [30, 97]. Fig. 7.7 depicts the set of the selected 200 pool price scenarios.

This case study is solved using the formulation (7.18)-(7.25), where the goal is to maximize a multi-objective function formed by the expected profit plus the CVaR multiplied by the weighting factor β.

This problem, characterized by 216,341 constraints and 101,142 real variables, is solved for different values of the risk parameter β and a confidence level $\alpha = 0.95$. Fig. 7.8 plots the expected profit versus both the profit standard deviation and the CVaR for different values of β (efficient frontier). The expected profit varies from \$108.62 million ($\beta = 0$) to \$84.85 million ($\beta = 100$). The profit standard deviation varies from \$78.47 million to \$39.14 million, whereas the CVaR ranges between \$28.69 million and \$50.18 million. Observe that a 21.9% decrease in the expected profit yields a 50.1% reduction in the profit standard deviation and a 74.9% increase in the CVaR.

Table 7.22 indicates the power produced by each thermal unit as a percentage of its maximum power output. This table shows that, in general, the energy generated by each unit increases with the parameter β, as a result of the use of forward contracts. That is, if only the pool is considered, each unit

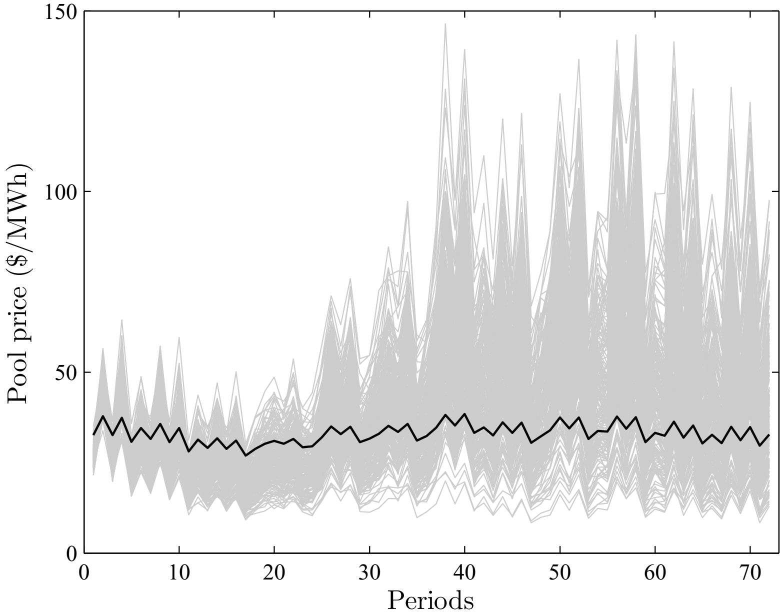

Fig. 7.7 Producer futures market case study: pool price scenarios

produces energy during those periods when pool prices exceed its production cost and in those off-peak periods when production is forced by constraints (7.21). In contrast, the futures market involvement entails that the producer has to provide a fixed amount of power over the lifetime of its forward contracts, regardless of the pool prices. Thus, an increase in the use of forward contracts leads to an increase in the power output of the units owned by the producer. Note that unit 5, which is the cheapest one, reaches 100% of its capacity for $\beta = 100$.

Table 7.22 Producer futures market case study: percentage of energy supplied by units

Unit	β						
#	0	0.5	1	2	5	10	100
1	72.2	74.6	74.6	78.9	81.4	81.4	81.2
2	68.2	70.9	70.9	70.9	74.4	74.9	74.9
3	58.4	61.8	63.0	64.6	65.4	65.8	66.8
4	50.9	51.6	52.4	52.4	56.0	57.4	58.2
5	89.8	94.5	97.5	99.1	99.8	99.9	100.0
6	78.7	83.8	85.3	89.6	92.8	94.2	96.7

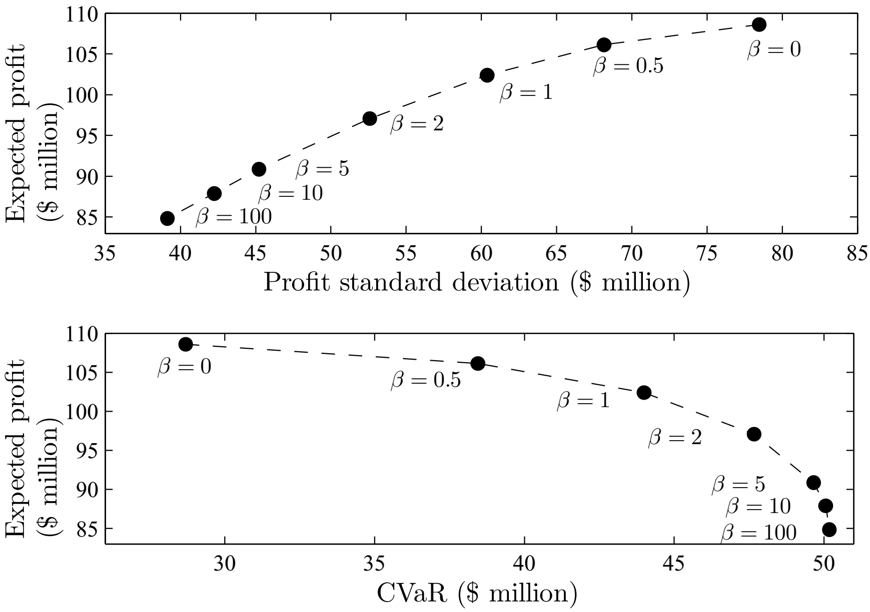

Fig. 7.8 Producer futures market case study: efficient frontier

Fig. 7.9 shows the pdfs of the profit obtained by the producer for $\beta = 0$ and $\beta = 100$. The profit for $\beta = 0$ ranges from \$27.5 million to \$564.8 million, with the expected profit being equal to \$108.62 million. For $\beta = 100$, the profit distribution varies from \$50.1 million to \$329.8 million, with an expected profit of \$84.85 million. Comparing the probability density functions for $\beta = 0$ and $\beta = 100$, we observe that the introduction of risk-aversion modeling causes an increase of \$22.6 million in the profit value for the worst scenario. However, the profit in the best scenario decreases in \$235 million, from \$564.8 million to \$329.8 million. Therefore, a risk-averse producer is willing to sacrifice high profits in the best scenarios in the hope of avoiding low profits in the worst scenarios.

Finally, Fig. 7.10 provides the percentages of the expected energy sold through forward contracts and the pool for different values of the factor β. For example, for $\beta = 0$, 61% of the energy produced is sold in the pool. As β increases, the percentage of energy sold in the pool becomes smaller, falling to 42% and 25% for $\beta = 1$ and $\beta = 100$, respectively. On the contrary, the energy sold through forward contracts increases with β. As Fig. 7.10 shows, the energy sold through forward contracts is assigned mostly to base contracts. This behavior is consistent, since the highest pool prices usually

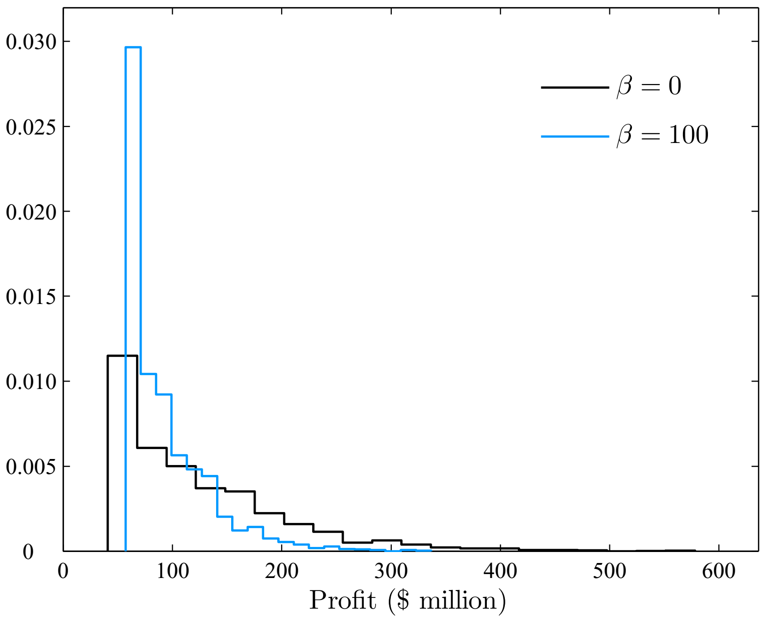

Fig. 7.9 Producer futures market case study: adjusted pdfs of the profit for $\beta = 0$ and $\beta = 100$

occur in peak periods. Thus, the producer is interested in leaving part of its production capacity available for periods in which the pool price is high.

Finally, it is relevant to note that using currently available optimization software and current personal computer technology, the required computation time to solve problems similar to the one discussed above is compatible with the considered decision-time framework.

7.10 Summary and Conclusions

In this chapter the problem faced by a power producer that seeks its optimal involvement in the futures market is presented. The futures market allows a power producer to hedge against the risk of profit volatility due to uncertain pool prices. Within a medium-term horizon, this chapter provides a stochastic programming model to determine how a power producer should select forward contracts so that its expected profit is maximized while taking into account the risk associated with profit variability. Pool prices and the availability of

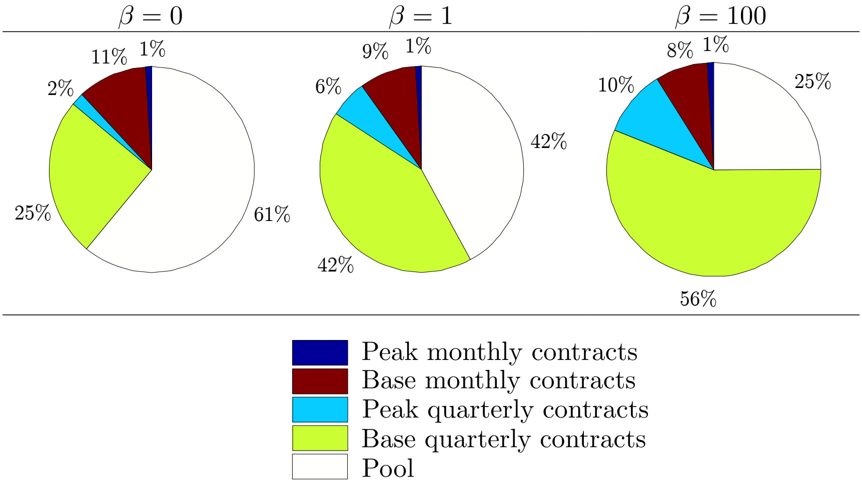

Fig. 7.10 Producer futures market case study: expected electricity sales

the production units are the sources of uncertainty in this problem, and they are modeled as stochastic processes using scenarios.

The influence of the producer actions on the forward contract prices is explicitly modeled using stepwise forward contracting curves, which express the price of the contracted energy as a function of the power sold through forward contracts.

The proposed stochastic programming model results in a large-scale linear programming problem that is efficiently solved using commercially available software.

The conclusions below are worth mentioning:

1. A power producer may advantageously sell its production not only in the pool but also in the futures market.
2. Forward contracting constitutes an efficacious risk hedging instrument for a power producer. Risk hedging results in a low variability of the profit at the cost of a small reduction in its average value.
3. Risk to be hedged arises from both pool prices and unit availability.
4. A detailed decision-making model pertaining to forward contracting translates into a linear programming problem, which is easy to solve using readily available optimization software.
5. The methodology proposed in this chapter is of interest for a power producer operating in a fully-fledged electricity market that includes both a pool and a futures market.

Finally, it should be noted that the proposed model can be easily extended to take into account additional features of the considered markets (other types of contracts, different decision frameworks, etc.), and the operating constraints of the producers (those pertaining to hydro producers, natural-gas producers, fully oligopolistic producers, etc.).

7.11 Notation

The notation used throughout this chapter is stated below for quick reference.

Indices and Numbers:

f	Index of forward contracts, running from 1 to N_{F}.
g	Index of generating units, running from 1 to N_{G}.
j	Index of blocks in the forward contracting curves, running from 1 to N_{J}.
t	Index of time periods, running from 1 to N_{T}.
ω	Index of scenarios, running from 1 to N_{Ω}.

Real Variables:

$C^{\mathrm{G}}_{gt\omega}$	Production cost incurred by generating unit g in period t and scenario ω ($).
$E^{\mathrm{P}}_{t\omega}$	Energy traded in the pool in period t and scenario ω (MWh).
P^{F}_{f}	Power contracted from forward contract f (MW).
P^{F}_{fj}	Power contracted from block j of the forward contracting curve of forward contract f (MW).
R^{F}_{t}	Revenue from selling in the futures market in period t ($).
$R^{\mathrm{P}}_{t\omega}$	Revenue from selling in the pool in period t and scenario ω ($).
ζ	Auxiliary variable used to calculate the CVaR ($).
η_{ω}	Auxiliary variable related to scenario ω and used to calculate the CVaR ($).

Random Variables:

t_{F}	Time to failure (h).
t_{R}	Time to repair (h).
u_1	Uniformly distributed random variable used to formulate the time to failure.

u_2 Uniformly distributed random variable used to formulate the time to repair.
ϵ_t Error term in the ARIMA model of pool prices (ln($/MWh)).
λ_t^{P} Random variable modeling the price of energy in the pool in period t ($/MWh). $\lambda_{t\omega}^{\mathrm{P}}$ represents the realization of this random variable in scenario ω.

Constants:

$a_{gt\omega}$ 0/1 constant that is equal to 1 if generating unit g is available in period t and scenario ω, being 0 otherwise.
C_g^{G} Production cost coefficient of generating unit g ($/MWh).
d_t Duration of period t (h).
E_t^{PC} Energy contracted prior to the beginning of the planning horizon that is used in period t (MWh).
H_g Minimum percentage of energy produced by generating unit g during peak periods that must be produced during off-peak periods.
MTTF Mean time to failure (MWh).
MTTR Mean time to repair (MWh).
$\bar{P}_{fj}^{\mathrm{F}}$ Upper limit of the power contracted from block j of the forward contracting curve of forward contract f (MW).
$P_g^{\mathrm{G,max}}$ Capacity of generating unit g (MW).
$\boldsymbol{W}$ Matrix whose element $W(t,t')$ is equal to 1 if t and t' are two consecutive off-peak and peak periods, respectively, and 0 otherwise.
$\lambda_{fj}^{\mathrm{F}}$ Price of block j of the forward contracting curve of forward contract f ($/MWh).
α Confidence level used in the calculation of the CVaR.
β Weighting factor.
θ_u Parameter related to delay u used in the ARIMA model of pool prices.
π_ω Probability of occurrence of scenario ω.
ϕ_u Parameter related to delay u used in the ARIMA model of pool prices.

Sets:

F_t Set of forward contracts available in period t.
Ω Set of scenarios.

Others:

B^u Backshift operator related to delay u.

7.12 Exercises

Exercise 7.1. Reformulate constraints (7.11) to include the minimum power output of the production units.

Exercise 7.2. Extend formulation (7.18)-(7.25) so that the producer can buy and sell in both the pool and the futures market.

Exercise 7.3. Considering a one-week market horizon, a pool that is cleared on a hourly basis, and different numbers of weekly forward contracts, reformulate problem (7.18)-(7.25) to consider the ramping limits of the production units as well as their minimum up-time and down-time constraints.

Exercise 7.4. Reformulate (7.18)-(7.25) avoiding parameter β while still modeling the tradeoff of expected profit versus profit variability.

Exercise 7.5. Replace CVaR in (7.18)-(7.25) by an alternative risk measure.

Exercise 7.6. Indicate ways to make formulation (7.18)-(7.25) more realistic.

Exercise 7.7. Solve the example in Section 7.7 (whose GAMS code is provided in Section A.4 of Appendix A) considering just two of the scenarios provided in Table 7.8. Compare all solutions involving just two arbitrarily selected scenarios of those in Table 7.8.

Exercise 7.8. Reduce the number of available forward contracts in the example of Section 7.7 and compare the results obtained as the number of available forward contracts decreases.

Exercise 7.9. Calculate VSS and EVPI ($\beta = 0$) for the example in Section 7.7.

Exercise 7.10. Consider a production unit whose time to repair is always above one week and a market horizon spanning three days with a pool cleared twice a day. Considering that the unit is available at the beginning of the market horizon, list all possible availability scenarios.

Chapter 8
Medium-Term Retailer Trading

8.1 Introduction

In this chapter we discuss the problem faced by a retailer of electrical energy in a medium-term horizon. A retailer (also called marketer) can be seen as an intermediary between producers and consumers (or retailers) that supplies energy to those consumers (or other retailers) not participating in the electricity markets. In general, retailers are not supposed to own generating units or to consume electricity.

The activity carried out by a retailer consists in buying energy from the electricity markets for selling it afterwards to consumers or other retailers at a fixed price. Thus, the profit gained by a retailer comes from the difference between the revenue from selling to consumers (its clients) and the cost of purchasing in the electricity markets.

The main trading floors available for a retailer are the futures market and the pool. Here, we consider the pool as a competitive day-ahead market where the final price of the energy does not depend on the retailer actions. On the other hand, we assume that the volume of energy traded in the futures market is smaller than that in the pool. Thus, the actions of the retailer might influence the final price of the energy in the futures market. Chapter 1 provides descriptions of both the futures market and the pool.

In the medium-term (e.g., one year) the retailer makes two different decisions at regular but sufficiently long intervals throughout the planning horizon: it selects the purchases in the futures market and the selling price offered to its clients. Together with the medium-term decisions, the retailer makes shorter-time decisions, like the trading in the pool, which are decided continuously throughout the planning horizon. In this context the retailer faces two main sources of uncertainty: the pool prices and the client demands.

Because of the existence of uncertainty and the fact that the retailer buys energy at a variable price (in the futures market and the pool) and sells it at a fixed price, risk management is a key issue in the modeling of the retailer

A. J. Conejo et al., *Decision Making Under Uncertainty in Electricity Markets*, International Series in Operations Research & Management Science 153,
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problem. The risk of profit variability is accounted for by including a risk term in the formulation of the problem.

In this chapter we propose a two-stage stochastic programming model in order to tackle the medium-term problem faced by a retailer where the sensitivity of the clients to the price offered by the retailer is mathematically expressed by means of price-quota curves. The resulting model is formulated as a mixed-integer linear programming problem.

The proposed framework is general enough to accommodate a wealth of practical situations; moreover it can be easily extended to alternative futures market and pool arrangements.

Relevant references dealing with electricity retailers include [31, 52, 55, 56, 113].

The example below illustrates the role of a retailer.

Illustrative Example 8.1 (Retailer). We consider the situation faced by a retailer illustrated in Fig. 8.1. This retailer participates in the futures market and the pool to provide the electricity demand of three consumers.

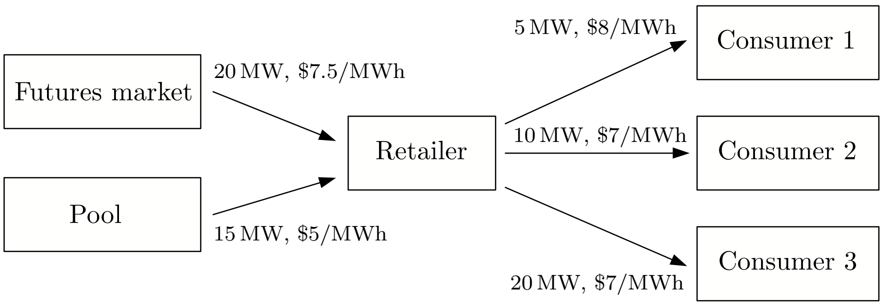

Fig. 8.1 Illustrative Example 8.1: retailer example

As Fig. 8.1 shows, the retailer purchases in the futures market and the pool 20 and 15 MW at prices 7.5 and \$5/MWh, respectively. Likewise, the retailer sells 5, 10, and 20 MW at prices 8, 7, and \$7/MWh to consumers 1, 2, and 3, respectively.

The procurement cost paid by the retailer is equal to $20\,\text{MW}\times\$7.5/\text{MWh} + 15\,\text{MW}\times\$5/\text{MWh} = \$225/\text{h}$. On the other hand, the revenue obtained by the retailer is $5\,\text{MW}\times\$8/\text{MWh} + 10\,\text{MW}\times\$7/\text{MWh} + 20\,\text{MW}\times\$7/\text{MWh} = \$250/\text{h}$. Therefore, the profit achieved by the retailer in this case is 250 - 225 = \$25/h.

□

The rest of this chapter is organized as follows. Section 8.2 describes the decision framework for the medium-term trading problem of a retailer of electrical energy. Section 8.3 provides the characterization of the uncertain parameters of this problem. The structure of the pool and the futures market is described in Section 8.4. The mathematical modeling of the retailer problem is presented and discussed in Section 8.5. Section 8.6 provides the complete problem formulation. In Section 8.7 an illustrative example is analyzed and discussed in detail. In Section 8.8, the performance of the proposed model is tested on a realistic case study. Finally, Section 8.9 summarizes and concludes the chapter.

8.2 Decision Framework

From a retailer point of view we can distinguish between medium-term and short-term decisions. The configuration of the futures market portfolio and the determination of the selling price offered to the clients are medium-term decisions, while the transactions in the pool are decided in the short-run. Medium-term decisions are made at the beginning of the planning horizon, whereas short-time decisions are made throughout it.

The main difference between these two kinds of decisions lies in the degree of uncertainty revealed at the moment of the decision making. For this reason we distinguish between *here-and-now* and *wait-and-see* decisions. Considering a two-stage stochastic programming model, here-and-now decisions are those that are made before uncertainty is disclosed. Considering a medium-term horizon, these decisions correspond to the futures market trading and the selling price determination. In contrast, the decisions referred to as wait-and-see are made after uncertainty is revealed. Typically, the pool trading is assumed to be made with perfect information on pool prices and client demands when a medium-term problem is considered. Chapter 2 provides additional information on here-and-now and wait-and-see decisions.

Considering a planning horizon of one year, the usage of the tool presented in this chapter is as follows. At the beginning of month 1, the user of the proposed tool would implement optimal forward contract decisions pertaining to the whole year. Moreover, the retailer uses the optimal selling prices determined by the proposed approach for the annual agreements signed during month 1 with each type of client. By the time that forward contract decisions at the beginning of the second month must be made, pool prices and demands of the first month are already known. Thus, information on prices and demands is updated and the tool is re-run to determine forward contract decisions pertaining to the following 12 months (months 2-13) and to identify optimal selling prices for new annual agreements signed with clients only during month 2. In order to make optimal decisions for forward contracts and selling prices a year ahead, this procedure is repeated at the beginning of each month. Note that a multi-stage stochastic programming problem can be used to formulate the retailer problem that accounts for the possibility of making successive forward contract decisions throughout the planning horizon. This multi-stage approach requires generating a multi-stage scenario tree that properly models the uncertainty of pool prices and client demands for each forward contract decision. The use of multi-stage scenario trees increases the computational size of the problem, frequently causing the problem to be intractable. Finally, note that the one-year horizon and the monthly decision framework are arbitrarily selected and should be tailored to the needs of the considered retailer.

The decision framework of this problem is illustrated in Fig. 8.2. The parameter N_{T} represents the number of time periods in the planning horizon. Note that the time periods are not necessarily hours.

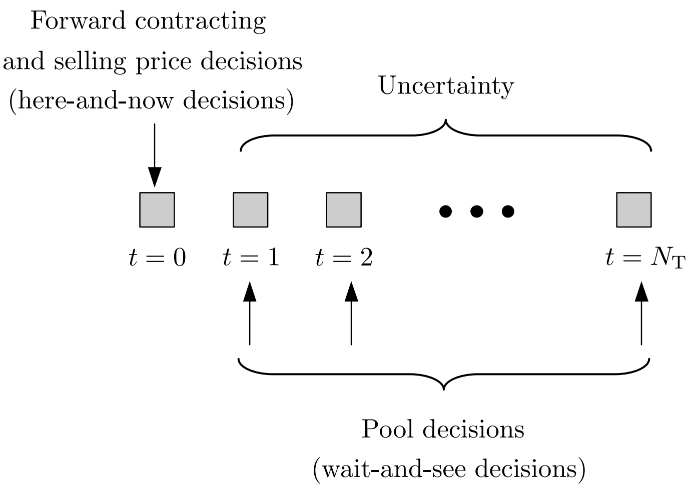

Fig. 8.2 Decision framework for a retailer

The example below illustrates the decision framework of a retailer.

Illustrative Example 8.2 (Two-stage decision framework).

Consider a single year planning horizon. The retailer has to determine the selling price for a set of clients. Four quarterly forward contracts, each spanning one of the four quarters of the year, and an annual contract are available for the retailer in the futures market. Additionally, the retailer trades in the pool either purchasing or selling electricity. The decision framework for the retailer is as follows:

1. At the beginning of the year, the retailer fixes the selling price offered to the clients and decides which quarterly and annual forward contracts to sign for the whole planning horizon.
2. For each realization of the pool prices of the year, the retailer decides the amount of energy to be traded each hour in the pool.

□

8.3 Uncertainty Characterization

There exist two main sources of uncertainty that a retailer must face in the medium-term. At the time when futures market purchasing and price setting decisions are made, pool prices and client demands throughout the planning horizon are unknown to the retailer. Observe that pool prices depend on the bids submitted by the market agents and that electricity demands of the clients are also unknown to the retailer when first-stage decisions are made at the beginning of the planning horizon.

In order to properly model both sources of uncertainty, we treat pool prices and client demands in future periods $t = 1, \ldots, N_{\mathrm{T}}$ as stochastic processes. For modeling purposes, it is usual to define several client clusters by grouping together those clients with similar characteristics. These characteristics are, for example, consumption patterns or response sensitivity to the price offered by the retailer.

We define the pool price in period t as λ_t^{P}, while E_{et}^{D} represents the electricity demand of client group e in period t.

As explained in Chapter 2, stochastic processes are represented by a finite set of scenarios. Let Ω denote the set of scenarios and N_Ω the number of scenarios in Ω. Each scenario ω comprises a vector of pool prices, a demand vector for each client group e, and a probability of occurrence π_ω. Note that the sum of the probabilities over all scenarios is equal to 1, i.e., $\sum_{\omega=1}^{N_\Omega} \pi_\omega = 1$.

This set of scenarios is arranged in a two-stage scenario tree as shown in Fig. 8.3. The futures market portfolio and the selling price offered to the clients are both decided at the first stage. In contrast, purchases and sales in the pool are decided at the second stage for each single scenario (realization) of pool prices and client demands.

The example below illustrates the scenarios characterizing the decision-making problem of a retailer.

Illustrative Example 8.3 (Client demand and pool price scenarios).

Table 8.1 provides an example of client demand and pool price scenarios. The market horizon spans two time periods and the number of client demand and pool price scenarios is three. Only one group of clients is considered. The probability of each scenario is 1/3.

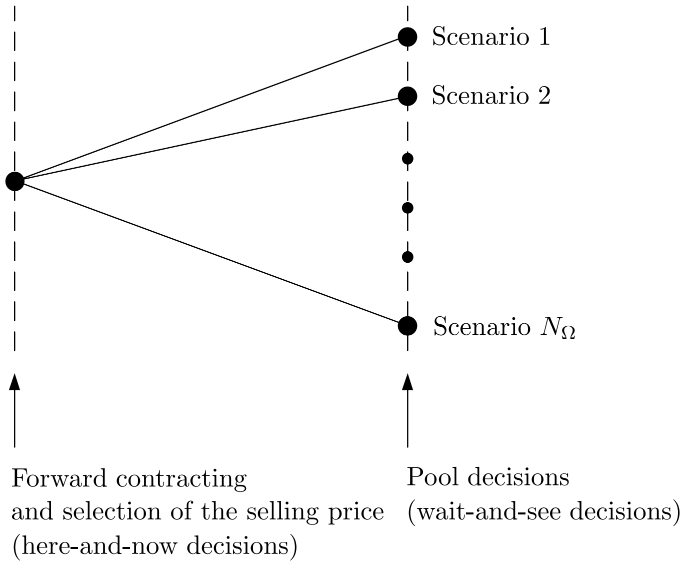

Fig. 8.3 Retailer scenario tree

Table 8.1 Illustrative Example 8.3: client demand and pool price scenarios

Scenario #	Probability	Client demand (MWh)		Pool price ($/MWh)	
		Period #		Period #	
		1	2	1	2
1	1/3	200	245	52	60
2	1/3	220	280	55	65
3	1/3	190	210	48	55

□

8.4 Market Structure

In this section we describe the two main trading floors used by retailers to procure the electricity to be supplied to their clients. Similarly to the case of a power producer, these trading floors are the futures market and the pool. For this reason, we refer the reader to Section 1.2 of Chapter 1 for a detailed discussion on these two markets. However, we include below some specific issues pertaining to the participation of a retailer in these two markets.

8.4.1 Futures Market

Typically, a retailer participates in the futures market to acquire part of the electricity that it sells to its clients. The main advantage of this market is that it allows the retailer to buy energy at a fixed price prior to its selling.

It is realistic to consider that a medium-sized retailer is a price-maker agent in the futures market, i.e., it has the capability of altering futures-market prices. In the proposed model, the influence of the retailer actions on forward contract prices is explicitly taken into account through the so-called forward contracting curves. Please, see Subsection 7.4.1 of Chapter 7 for further detail on forward contracting curves.

Fig. 8.4 shows an example of a forward contracting curve with three blocks. The contract price is represented in the y-axis, while the power purchased from the contract is indicated in the x-axis. We assume that the retailer participates in the futures market only purchasing electricity. Observe that the number of blocks in the forward contracting curve should be tailored to the considered problem.

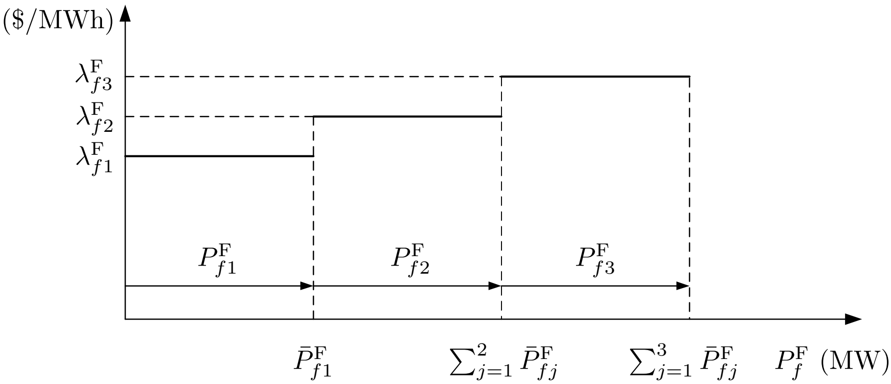

Fig. 8.4 Forward contract buying curve

The cost associated with the purchase of energy from forward contracts in period t is formulated as follows:

$$C_t^{\mathrm{F}} = \sum_{f \in F_t} \sum_{j=1}^{N_{\mathrm{J}}} \lambda_{fj}^{\mathrm{F}} P_{fj}^{\mathrm{F}} d_t, \; \forall t \tag{8.1}$$

$$0 \leq P_{fj}^{\mathrm{F}} \leq \bar{P}_{fj}^{\mathrm{F}}, \; \forall f, \forall j \tag{8.2}$$

$$P_f^{\mathrm{F}} = \sum_{j=1}^{N_{\mathrm{J}}} P_{fj}^{\mathrm{F}}, \; \forall f, \tag{8.3}$$

where F_t is the set of forward contracts available in period t.

Equations (8.1) express the cost of purchasing energy from forward contracts in each time period t. The cost in period t depends on the power contracted P_f^{F}, the energy price in each block of the forward contracting curve, and the duration of period t. Constraints (8.2) state that the power purchased from each block of the forward contracting curve is nonnegative and bounded by an upper limit. Finally, expressions (8.3) state for each contract that the power purchased is the sum of the power bought from each block.

The example below illustrates a forward contracting curve.

Illustrative Example 8.4 (Forward contracting curve).

Fig. 8.5 shows an example of a forward contracting curve with three blocks. In this case the retailer pays \$25/MWh for all the power contracted up to 10 MW. For the power lying between 10 and 20 MW the retailer pays \$35/MWh, and \$45/MWh for the power exceeding 20 MW. In this way, if the retailer signs a contract of 25 MW, it must pay 10 MW×\$25/MWh + 10 MW×\$35/MWh+5 MW×\$45/MWh = \$825 in each period in which the contract is available.

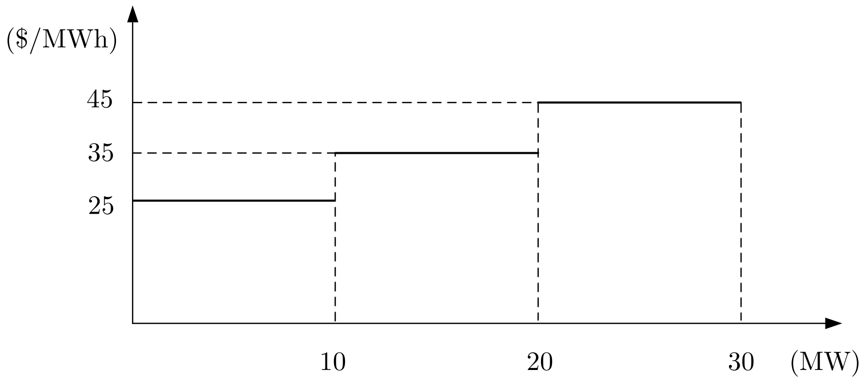

Fig. 8.5 Illustrative Example 8.4: forward contract buying curve for a retailer

□

8.4.2 Pool

A retailer participates in the pool to buy a part of the energy delivered to their clients. However, if profitable, the retailer can also sell energy in the pool.

Similarly to the producer case, pool prices are treated as a stochastic process, which is described using a set of scenarios, defined for $t = 1, \ldots, N_{\mathrm{T}}$ as $\{\lambda^{\mathrm{P}}_{1\omega}, \ldots, \lambda^{\mathrm{P}}_{N_{\mathrm{T}}\omega}\}$, $\forall \omega \in \Omega$.

Transactions of energy in the pool are wait-and-see decisions made by the retailer after fixing the futures market position. These transactions are modeled with the continuous variable $E^{\mathrm{P}}_{t\omega}$ that is positive if the retailer purchases energy in the pool in period t and scenario ω, being negative otherwise.

The net cost of trading in the pool is computed as:

$$C^{\mathrm{P}}_{t\omega} = \lambda^{\mathrm{P}}_{t\omega} E^{\mathrm{P}}_{t\omega}, \ \forall t, \forall \omega. \tag{8.4}$$

The example below illustrates the cost of trading in the pool.

Illustrative Example 8.5 (Pool trading cost).

We consider the pool prices previously presented in Table 8.1 and the transactions in the pool decided by the retailer provided in Table 8.2. Note that the retailer purchases electricity in all periods and scenarios except in period 1 of scenarios 1 and 3. In fact, the retailer sells 10 MWh in period 1 of scenario 3.

Table 8.2 Illustrative Example 8.5: pool trading cost

Scenario #	Purchased energy (MWh)	
	Period #	
	1	2
1	0	45
2	20	80
3	-10	10

The net cost of trading in the pool in each scenario and in each period is computed as follows:

1. Scenario 1
   - Period 1: $C^{\mathrm{P}}_{11} = 0 \times 52 = \$0$
   - Period 2: $C^{\mathrm{P}}_{21} = 45 \times 60 = \$2700$
2. Scenario 2

- Period 1: $C^{\mathrm{P}}_{12} = 20 \times 55 = \$1100$
- Period 2: $C^{\mathrm{P}}_{22} = 80 \times 65 = \$5200$

3. Scenario 3
   - Period 1: $C^{\mathrm{P}}_{13} = -10 \times 48 = -\$480$
   - Period 2: $C^{\mathrm{P}}_{23} = 10 \times 55 = \$550$

Observe that in period 1 of scenario 3 the retailer sells electricity in the pool, yielding a negative cost (revenue) in that period and scenario.

The expected cost in each period can be easily computed taking into account the probability of each scenario provided in Table 8.1:

- Period 1: $\sum_{\omega=1}^{3} \pi_\omega C^{\mathrm{P}}_{1\omega} = \frac{1}{3} \times (0 + 1100 - 480) = \$206.7$
- Period 2: $\sum_{\omega=1}^{3} \pi_\omega C^{\mathrm{P}}_{2\omega} = \frac{1}{3} \times (2700 + 5200 + 550) = \$2816.7$

□

8.5 Retailer Model

8.5.1 Client Modeling

A retailer must procure the electricity demand of its clients in each period of the planning horizon. Usually, retailers sell energy to their clients at a price that remains stable during a pre-specified planning horizon. Every retailer fixes a selling price, and the potential clients select their electricity supplier according to the prices offered by all available retailers. However, observe that a different price setting configuration can be considered. For instance, the selling price can be indexed to the pool price (pool price plus a fee). Another configuration consists in offering a time-of-use rate, considering several selling prices for different sets of hours within the day.

The set of all plausible clients is divided into a series of homogeneous groups containing clients with similar characteristics, e.g., consumption patterns, price-response, etc. Traditionally, consumers have been classified as residential, commercial, or industrial costumers. For the sake of generality, we consider that clients are divided into N_{E} groups.

We assume that clients behave elastically with respect to the selling price offered by the retailer. Here we denote by λ^{R}_e the price offered by the retailer to client group e. Roughly speaking, the elastic behavior of the clients means that if the selling price λ_e^{R} is too high, clients will choose a rival retailer for their electricity supply.

8.5.2 Price-Quota Curve

The relationship between the price offered and the actual demand supplied to a client (or group of clients) by the retailer is modeled through a stepwise price-quota curve. A price-quota curve determines the amount of electricity provided by the retailer that a group of clients is willing to buy at a given price [92, 103].

Price-quota curves may be as inelastic as needed to reflect client behavior. These curves are estimated by the retailer before solving the decision-making problem, and therefore, they are input data to the problem under consideration.

The total demand of client group e in period t is denoted by E_{et}^{D}. The client demand is unknown to the retailer and it is characterized as a stochastic process. Similarly to the pool price, the client demand is modeled using a set of scenarios. Thus, $E_{et\omega}^{\mathrm{D}}$ refers to the demand of client group e in period t and scenario ω.

The amount of demand of the total client group that the retailer supplies is contingent on the selling price that it offers. The variable E_{et}^{R} denotes the quantity of the demand of group e provided by the retailer in period t, where the relation $E_{et}^{\mathrm{R}} \leq E_{et}^{\mathrm{D}}$ is always satisfied. Since the energy provided by the retailer depends on the total demand of the client group, E_{et}^{R} is represented by a set of scenarios, $E_{et\omega}^{\mathrm{R}}$, $\forall \omega \in \Omega$. Note that $E_{et\omega}^{\mathrm{R}}$ is a continuous variable that depends on both the total demand $E_{et\omega}^{\mathrm{D}}$ and the selling price offered by the retailer λ_e^{R}. For each client group and each period the relationship between $E_{et\omega}^{\mathrm{R}}$ and λ_e^{R} is given by the price-quota curves. From a mathematical point of view, the price-quota curve for each client group in each period and scenario can be formulated as follows:

$$E_{et\omega}^{\mathrm{R}} = \sum_{i=1}^{N_{\mathrm{I}}} \bar{E}_{eti\omega}^{\mathrm{R}} \, v_{ei}, \; \forall e, \forall t, \forall \omega \tag{8.5}$$

$$\lambda_e^{\mathrm{R}} = \sum_{i=1}^{N_{\mathrm{I}}} \lambda_{ei}^{\mathrm{R}}, \; \forall e \tag{8.6}$$

$$\bar{\lambda}_{ei-1}^{\mathrm{R}} \, v_{ei} \leq \lambda_{ei}^{\mathrm{R}} \leq \bar{\lambda}_{ei}^{\mathrm{R}} \, v_{ei}, \; \forall e, \forall i \tag{8.7}$$

$$\sum_{i=1}^{N_{\mathrm{I}}} v_{ei} = 1, \; \forall e \tag{8.8}$$

$$v_{ei} \in \{0, 1\}, \; \forall e, \forall i. \tag{8.9}$$

Constraints (8.5)-(8.9) express the demand supplied by the retailer to each client group in each period and each scenario, $E^{\mathrm{R}}_{et\omega}$, as a function of the selling price, λ^{R}_{e}. Fig. 8.6 represents graphically a general price-quota curve with three blocks. The demand provided by the retailer is equal to the level of energy of the price-quota curve indicated by binary variables v_{ei} (8.5). As per (8.6)-(8.9), binary variables v_{ei} identify the interval of the price-quota curve corresponding to the selling price λ^{R}_{e}.

The formulation (8.5)-(8.9) allows defining a price-quota curve for each client group e, period t, and scenario ω. However, if the planning horizon is one year, it is reasonable to consider that clients do not switch retailers within the year, i.e., the percentage of demand supplied by the retailer to each group of clients is constant throughout the year. Therefore, if the vertical axis of the price-quota curve represents the percentage of demand supplied by the retailer, the no-client-switching assumption is equivalent to assuming identical price-quota curves for all periods.

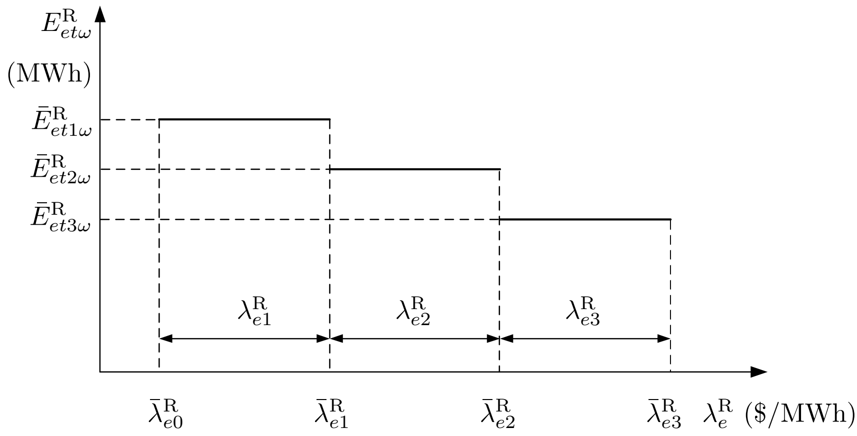

Fig. 8.6 Retailer: price-quota curve

The example below illustrates the concept of the price-quota curve.

Illustrative Example 8.6 (Price-quota curve).

Fig. 8.7 depicts an example of a price-quota curve for a given client group. We assume that the total demand of this client group is 100 MWh. As Fig. 8.7 shows, the higher the price offered by the retailer, the smaller the energy bought by the client group. If the selling price offered is less than \$50/MWh, the retailer provides 100 MWh, that is, the total demand of the client group. If the price is between 50 and \$80/MWh, the selling price is less competitive and the retailer provides 60 MWh of the total demand of the client group (100 MWh). For a selling price between 80 and \$100/MWh the retailer supplies only 20 MWh. Finally, if the selling price exceeds \$100/MWh, the retailer fails to provide any part of the demand of the client group.

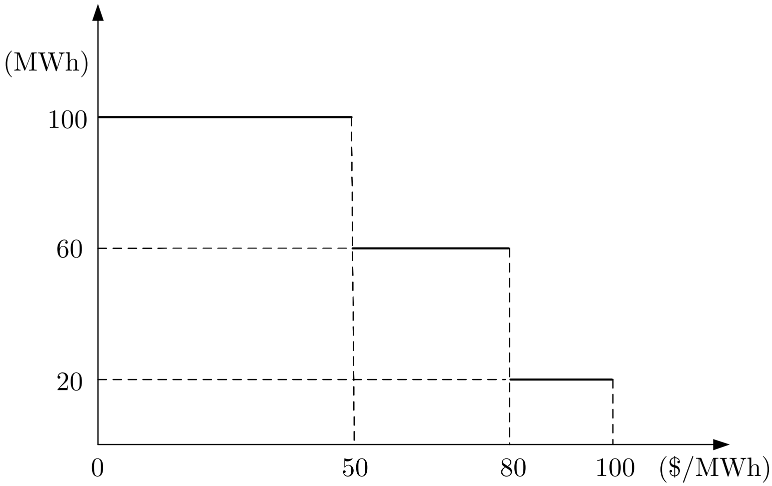

Fig. 8.7 Illustrative Example 8.6: retailer price-quota curve

□

8.5.3 Revenue from Selling to Clients

The main part of the revenue obtained by a retailer comes from the energy sold to the clients. This is not the only source of revenue, since the retailer can earn money from selling in the pool the exceeding energy bought from the futures market, i.e., exercising arbitrage between these markets. However, this revenue is generally much smaller than that obtained from selling to clients.

The revenue from selling to clients is equal to the product of the energy supplied by the retailer and the selling price. Since the energy supplied by the retailer is a stochastic process, the revenue obtained by the retailer in every period is a random variable. Mathematically, the revenue of the retailer from selling to client group e in period t and scenario ω is

$$R^{\mathrm{R}}_{et\omega} = \lambda^{\mathrm{R}}_{e} E^{\mathrm{R}}_{et\omega}, \ \forall e, \forall t, \forall \omega. \tag{8.10}$$

Observe that the revenue $R^{\mathrm{R}}_{et\omega}$ is equal to the product of two continuous variables, $E^{\mathrm{R}}_{et\omega}$ and λ^{R}_{e}. However, as the selling price and the demand supplied by the retailer are related by stepwise price-quota curves, the revenue can be equivalently recast into a linear expression. The mathematical formulation of the revenue obtained by the retailer from selling to client group e in period t and scenario ω is

$$R^{\mathrm{R}}_{et\omega} = \sum_{i=1}^{N_{\mathrm{I}}} \lambda^{\mathrm{R}}_{ei} \bar{E}^{\mathrm{R}}_{eti\omega}, \ \forall e, \forall t, \forall \omega. \tag{8.11}$$

The linear model of the revenue $R^{\mathrm{R}}_{et\omega}$ is depicted in Fig. 8.8. It should be emphasized that expressions (8.10) and (8.11) are equivalent if step-wise constant price-quota curves are used.

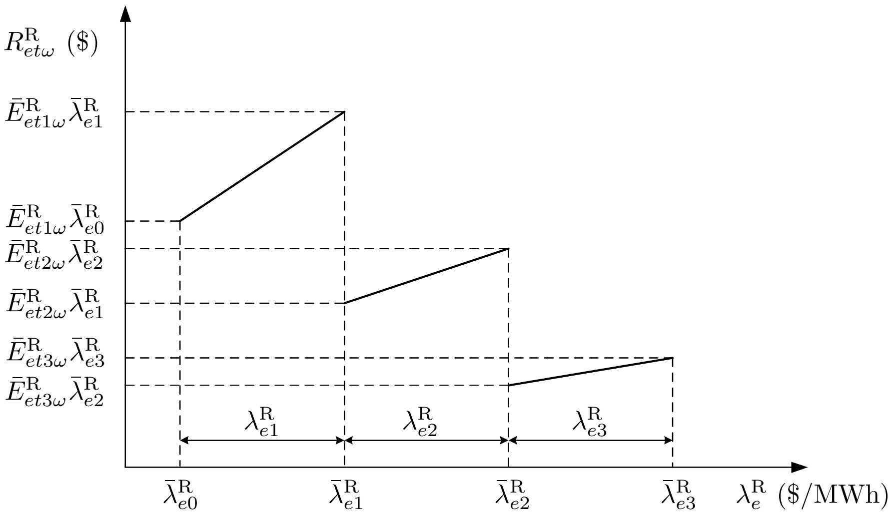

Fig. 8.8 Retailer: revenue from selling to clients

The example below illustrates the formulation of the revenues achieved from selling electricity to clients.

Illustrative Example 8.7 (Revenue).

The revenue associated with the price-quota curve illustrated in Fig. 8.7 is depicted in Fig. 8.9. This figure shows that the revenues associated with selling prices of 25 and \$90/MWh are equal to \$25/MWh×100 MWh = \$2500 and \$90/MWh×20 MWh = \$1800 respectively. Observe that the breakpoints (prices equal to 50, 80, and \$100/MWh) have associated two possible revenues. For instance, if the price is \$50/MWh the revenue can be either \$5000 (block of 100 MWh) or \$3000 (block of 60 MWh). If the objective is to maximize profit, the point with greatest revenue is always selected. However, the differences of revenues in the breakpoints can be neglected if enough blocks are used for the price-quota curve.

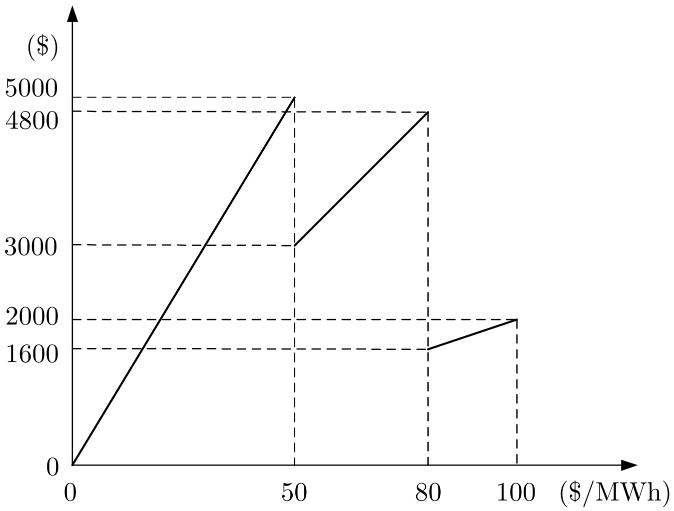

Fig. 8.9 Illustrative Example 8.7: retailer revenue from selling to clients

□

8.5.4 Energy Balance

The electric energy balance of the retailer in each period and scenario is expressed as

$$\sum_{e=1}^{N_{\mathrm{E}}} E_{et\omega}^{\mathrm{R}} = E_{t\omega}^{\mathrm{P}} + \sum_{f \in F_t} P_f^{\mathrm{F}} d_t + E_t^{\mathrm{PC}}, \ \forall t, \forall \omega, \tag{8.12}$$

where E_t^{PC} is a parameter that represents the quantity of energy previously contracted. For instance, E_t^{PC} may represent the energy corresponding to a forward contract signed several years ago that is in effect during period t. Observe that this constraint indicates that, for each period and scenario, the retailer must purchase in the pool the difference between the electricity demand of its clients and the energy purchased from forward contracts.

8.5.5 Expected Profit

The profit of the retailer is equal to the revenue obtained from selling electricity to clients minus the net cost of trading in the pool and minus the cost of forward contracting. The final profit attained by the retailer depends on the actual realizations of the stochastic processes (pool prices and client demands). The profit in scenario ω is mathematically expressed as

$$\sum_{t=1}^{N_{\mathrm{T}}}\left(\sum_{e=1}^{N_{\mathrm{E}}} R^{\mathrm{R}}_{et\omega} - C^{\mathrm{P}}_{t\omega} - C^{\mathrm{F}}_{t}\right) = \tag{8.13}$$

$$\sum_{t=1}^{N_{\mathrm{T}}}\left(\sum_{e=1}^{N_{\mathrm{E}}}\sum_{i=1}^{N_{\mathrm{I}}} \lambda^{\mathrm{R}}_{ei}\bar{E}^{\mathrm{R}}_{eti\omega} - \lambda^{\mathrm{P}}_{t\omega}E^{\mathrm{P}}_{t\omega} - \sum_{f\in F_t}\sum_{j=1}^{N_{\mathrm{J}}} \lambda^{\mathrm{F}}_{fj}P^{\mathrm{F}}_{fj}d_t\right), \ \forall\omega.$$

The expected profit expressed in (8.13) includes the revenue from selling to clients, the net cost of pool trading, and the cost of energy purchasing through forward contracts, respectively.

The profit of the retailer is a random variable whose volatility depends on the retailer actions and on the specific volatility of the stochastic processes involved in the problem. The expected value of the profit is computed as the sum over all scenarios of the profit in each scenario ω multiplied by its probability of occurrence, π_ω. The expected profit is expressed as follows:

$$\sum_{\omega=1}^{N_{\Omega}} \pi_\omega \sum_{t=1}^{N_{\mathrm{T}}}\left(\sum_{e=1}^{N_{\mathrm{E}}} R^{\mathrm{R}}_{et\omega} - C^{\mathrm{P}}_{t\omega} - C^{\mathrm{F}}_{t}\right). \tag{8.14}$$

8.5.6 Risk Modeling

The profit is a random variable subject to both the retailer actions and the stochastic processes involved in the problem. Usually the profit is characterized by the expected profit. However, a high expected profit does not always guarantee a high final profit. That is, there can exist some scenarios with low profit, or even losses.

The example below illustrates the concept of risk within a retailer context.

Illustrative Example 8.8 (Risk of losses).

The possibility of experiencing losses is depicted in Fig. 8.10. This figure shows both the probability density function (pdf) and the cumulative distribution function (cdf) of a continuous random variable representing the profit of the retailer. It can be observed in this figure that there exists a finite probability of experiencing a profit value less than 0 (loss). The probability of expecting losses is computed as the area below the pdf within the interval $(-\infty, 0)$. In this case, it can be observed in the cdf that this probability is equal to 0.08.

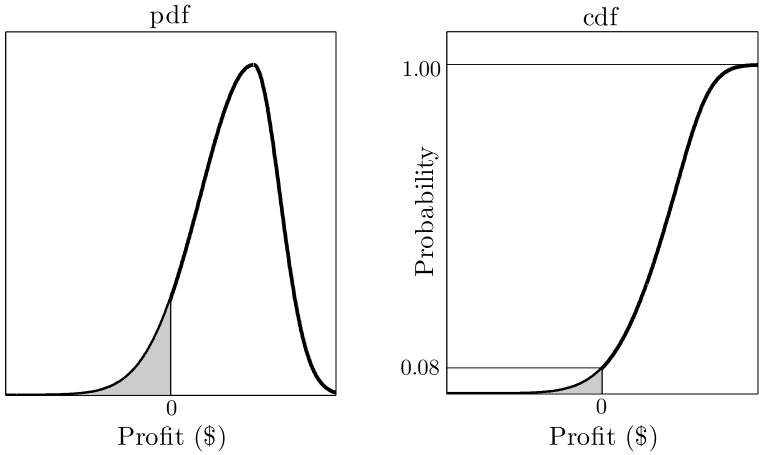

Fig. 8.10 Illustrative Example 8.8: profit distribution

□

It should be clear that the decisions made by the retailer have a significant influence on the resulting volatility of the profit. For instance, if the retailer only purchases electricity in the pool (risky source) the volatility of the profit is much higher than that if the retailer relies more on the futures market (unrisky source).

In order to consider the volatility of the profit in the decisions made by the retailer, it is appropriate to include a risk measure in the formulation of the problem. Including a risk measure limits the risk of experiencing a profit significantly smaller than the expected one. A well-known risk measure is the Conditional Value-at-Risk (CVaR) [124, 125]. For an α confidence level, the CVaR is approximately the expected profit of the $(1-\alpha) \times 100$ scenarios with the lowest profits. The CVaR is explained in detail in Subsection 4.3.5 of Chapter 4.

Using expression (8.13) for the profit, the CVaR for a confidence level α is computed as

$$\text{CVaR} = \text{Maximize}_{\zeta, \eta_\omega} \qquad \zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\Omega} \pi_\omega \, \eta_\omega \tag{8.15}$$

subject to

$$\zeta - \sum_{t=1}^{N_{\mathrm{T}}} \left(\sum_{e=1}^{N_{\mathrm{E}}} R^{\mathrm{R}}_{et\omega} - C^{\mathrm{P}}_{t\omega} - C^{\mathrm{F}}_{t} \right) \leq \eta_\omega, \ \forall \omega \tag{8.16}$$

$$\eta_\omega \geq 0, \ \forall \omega. \tag{8.17}$$

The optimal value of ζ represents the greatest value of the profit such that the probability of experiencing a profit less than or equal to ζ is less than or equal to $1-\alpha$. The auxiliary variable η_ω is equal to the excess of ζ over the profit in scenario ω, if this excess is positive.

As explained in Subsection 4.3.5 of Chapter 4, the CVaR can be equivalently included in the formulation of the problem as an additional constraint or in the objective function either alone or together with the expected profit.

8.6 Formulation

The formulation of the retailer problem is provided below

$$\begin{aligned}
&\mathrm{Maximize}_{P^{\mathrm{F}}_{fj}, \lambda^{\mathrm{R}}_{ei}, v_{ei}, E^{\mathrm{P}}_{t\omega}, \zeta, \eta_\omega} \\
&\sum_{\omega=1}^{N_\Omega} \pi_\omega \sum_{t=1}^{N_{\mathrm{T}}} \Big(\sum_{e=1}^{N_{\mathrm{E}}} \sum_{i=1}^{N_{\mathrm{I}}} \lambda^{\mathrm{R}}_{ei} \bar{E}^{\mathrm{R}}_{eti\omega} - \lambda^{\mathrm{P}}_{t\omega} E^{\mathrm{P}}_{t\omega} - \sum_{f \in F_t} \sum_{j=1}^{N_{\mathrm{J}}} \lambda^{\mathrm{F}}_{fj} P^{\mathrm{F}}_{fj} d_t \Big) \\
&\qquad + \beta \left(\zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\Omega} \pi_\omega \eta_\omega \right)
\end{aligned} \tag{8.18}$$

subject to

$$0 \leq P^{\mathrm{F}}_{fj} \leq \bar{P}^{\mathrm{F}}_{fj}, \ \forall f, \forall j \tag{8.19}$$

$$\bar{\lambda}^{\mathrm{R}}_{ei-1}\, v_{ei} \leq \lambda^{\mathrm{R}}_{ei} \leq \bar{\lambda}^{\mathrm{R}}_{ei}\, v_{ei}, \ \forall e, \forall i \tag{8.20}$$

$$\sum_{i=1}^{N_{\mathrm{I}}} v_{ei} = 1, \ \forall e \tag{8.21}$$

$$\sum_{e=1}^{N_{\mathrm{E}}} \sum_{i=1}^{N_{\mathrm{I}}} \bar{E}^{\mathrm{R}}_{eti\omega}\, v_{ei} = E^{\mathrm{P}}_{t\omega} + \sum_{f \in F_t} P^{\mathrm{F}}_{f}\, d_t + E^{\mathrm{PC}}_{t}, \ \forall t, \forall \omega, \tag{8.22}$$

$$\zeta - \sum_{t=1}^{N_{\mathrm{T}}} \left(\sum_{e=1}^{N_{\mathrm{E}}} \sum_{i=1}^{N_{\mathrm{I}}} \lambda^{\mathrm{R}}_{ei} \bar{E}^{\mathrm{R}}_{eti\omega} - \lambda^{\mathrm{P}}_{t\omega} E^{\mathrm{P}}_{t\omega} - \sum_{f \in F_t} \sum_{j=1}^{N_{\mathrm{J}}} \lambda^{\mathrm{F}}_{fj} P^{\mathrm{F}}_{fj} d_t \right) \leq \eta_\omega, \ \forall \omega \tag{8.23}$$

$$v_{ei} \in \{0, 1\}, \ \forall e, \forall i \tag{8.24}$$

$$\eta_\omega \geq 0, \ \forall \omega. \tag{8.25}$$

The objective function (8.18) comprises two terms: i) the expected profit and ii) the CVaR multiplied by the weighting factor β. The factor β models the tradeoff between expected profit and CVaR.

Constraints (8.19) bound the power contracted from each block of the forward contracting curve of each contract. Constraints (8.20)-(8.21) identify the block of the price-quota curve associated with each selling price. Constraints (8.22) impose the electric energy balance in each period and scenario. Constraints (8.23) are used to compute the CVaR. Finally, (8.24) and (8.25) constitute variable declarations.

8.7 Retailer Example

We present an example of reduced size to illustrate the formulation of the retailer problem described above.

We consider a planning horizon of two hourly periods. Two forward contracts spanning both periods are available. The forward contracting curves corresponding to each contract have two blocks. The parameters defining each contract are provided in Table 8.3.

Table 8.3 Retailer example: forward contracting curve data

Contract #	Usage period #	$\lambda^{\mathrm{F}}_{f1}$ (\$/MWh)	$\lambda^{\mathrm{F}}_{f2}$ (\$/MWh)	$\bar{P}^{\mathrm{F}}_{f1}$ (MW)	$\bar{P}^{\mathrm{F}}_{f2}$ (MW)
1	1-2	66.00	69.30	20	20
2	1-2	67.00	70.35	20	20

The client electricity demand and pool prices are modeled using a set of four equiprobable scenarios, which are shown in Table 8.4. In this example, a single group of clients is considered.

The response of the clients to the price offered by the retailer is represented through the price-quota curve shown in Fig. 8.11. A single price-quota curve is used for all periods and scenarios. As can be seen in Fig. 8.11, if the selling price is smaller than \$65/MWh, the retailer supplies 100% of the client demand. However, if this price is in between 65 and \$75/MWh, the retailer supplies 35% of the total demand of the clients. Likewise, if the selling price is in between 75 and \$85/MWh, the retailer just supplies 15% of the demand, and if the price is above \$85/MWh, the retailer supplies no demand.

Problem (8.18)-(8.25) is set up with a confidence level α equal to 0.95. The resulting problem, characterized by 23 constraints, 24 real variables, and 3 binary variables, is solved using CPLEX 10.2 [142] under GAMS [141].

Table 8.4 Retailer example: client demand and pool price scenarios

Scenario #	Client demand (MWh)		Pool price ($/MWh)	
	Period #		Period #	
	1	2	1	2
1	350	325	60	52
2	365	335	65	55
3	375	345	74	68
4	360	340	70	61

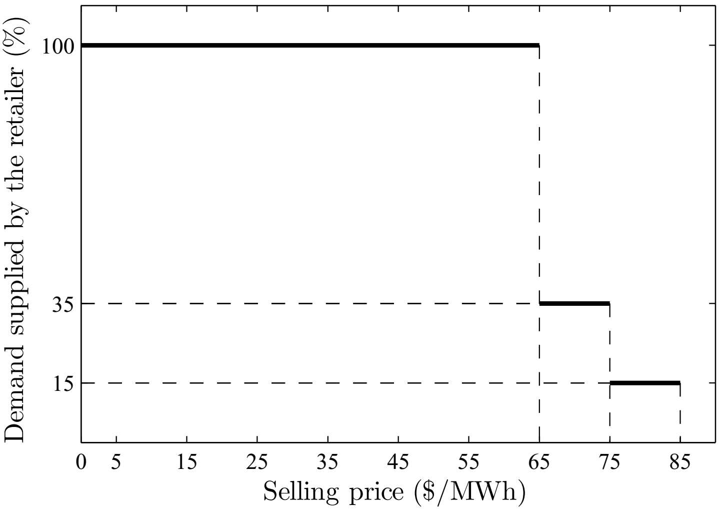

Fig. 8.11 Retailer example: price-quota curve

The expected profit, profit standard deviation, and CVaR for different values of β are presented in Table 8.5. As expected, the solution achieved for $\beta = 0$ attains the highest expected profit and the highest risk, measured as both the standard deviation and the CVaR of the profit distribution. For $\beta = 100$ the expected profit decreases 36.9% to attain a reduction of 98.1% in the profit standard deviation and an increase of 92.9% in the CVaR.

The efficient frontier, representing the expected profit versus the profit standard deviation and the CVaR for different values of β, is shown in Fig.

8.12. Observe that low-risk solutions (low standard deviation and high CVaR) are associated with a low expected profit.

Table 8.5 Retailer example: results

β	Expected profit ($)	Profit standard deviation ($)	CVaR ($)
0	3017.58	1482.62	976.50
1	2870.92	1229.07	1176.50
2	2007.42	105.30	1858.50
100	1903.96	28.59	1883.75

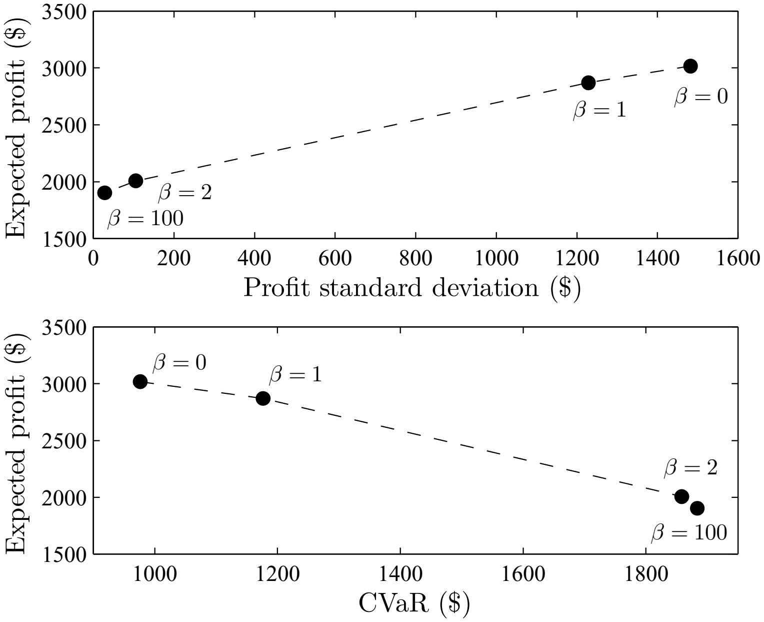

Fig. 8.12 Retailer example: efficient frontier

The amount of power purchased from forward contracts is given in Table 8.6. If $\beta = 0$, no contract is signed. This result is caused by the fact that expected pool prices are smaller than forward contract prices (Tables 8.3 and 8.4). Since forward contracts are used to hedge against the volatility of pool prices, the larger the parameter β, the greater the quantity of power contracted in the futures market through forward contracts.

Table 8.6 Retailer example: purchases from forward contracts (MW)

Contract #	β 0	1	2	100
1	0.0	20.0	20.0	27.4
2	0.0	0.0	20.0	20.0

Table 8.7 Retailer example: price offered to clients ($/MWh)

β 0	1	2	100
75	75	85	85

Table 8.7 provides the optimal selling price offered to clients for different values of β. As the value of β increases, so does the selling price offered to clients. The increase in the selling price with β results from the fact that the amount of energy purchased through forward contracts also increases. Since the energy purchased through forward contracts is more expensive on average than that obtained from the pool, the total procurement cost of the retailer is higher if risk is accounted for. Thus, this increase in the procurement cost leads to an increase in the selling price offered to clients.

Table 8.8 Retailer example: expected electricity procurement

β	Forward contract purchases (MWh)	Expected pool purchases (MWh)	Expected pool sales (MWh)
0	0.00	244.42	0.00
1	40.00	204.42	0.00
2	80.00	24.75	0.00
100	94.85	9.90	0.00

The electricity procurement using forward contracts and the pool is provided in Table 8.8. The quantity of electricity purchased through forward contracts grows as the parameter β increases. As previously explained, the more risk averse the retailer is, the greater the forward contract purchases. The opposite effect is observed in pool purchases, i.e., expected pool purchases decrease as the value of β increases. Finally, note that the expected pool sales are equal to zero for all values of β. This is so because the pool is used in all periods and scenarios for purchasing energy. This result is also observed in Tables 8.9 and 8.10 below.

Table 8.9 provides the energy purchases in each period and scenario for $\beta = 0$. In this case no contract is signed. Thus, all the electricity demand of the clients is supplied through the pool. Considering that for $\beta = 0$ the selling price offered to the clients is \$75/MWh and the price-quota curve shown in

Fig. 8.11, it can be concluded that the retailer provides 35% of the total demand of the clients. For instance, in period 1 and scenario 1 the retailer purchases 122.5 MWh in the pool, which represents 35% of the client demand in this period and scenario, which amounts to 350 MWh (see Table 8.4). A similar behavior can be observed in the remaining periods and scenarios.

Table 8.9 Retailer example: electricity purchases for $\beta = 0$ (MWh)

	Period 1		Period 2	
Scenario #	Futures market	Pool	Futures market	Pool
1	0.0	122.5	0.0	113.7
2	0.0	127.7	0.0	117.3
3	0.0	131.2	0.0	120.8
4	0.0	126.0	0.0	119.0

Table 8.10 provides the purchases of energy in each period and scenario for $\beta = 100$. In this case, forward contracts 1 and 2 are used. The increase in the value of β causes most of the energy purchased by the retailer to be obtained through forward contracts. For $\beta = 100$, the optimal selling price offered to the clients is \$85/MWh. According to the price-quota curve shown in Fig. 8.11, the retailer provides 15% of the total demand of the clients. As an example, in period 1 and scenario 1 the retailer purchases 52.5 MWh, 47.4 MWh in the futures market and 5.1 MWh in the pool, which represents 15% of the client demand in this period and scenario (350 MWh) (see Table 8.4).

Table 8.10 Retailer example: electricity purchases for $\beta = 100$ (MWh)

	Period 1		Period 2	
Scenario #	Futures market	Pool	Futures market	Pool
1	47.4	5.1	47.4	1.3
2	47.4	7.3	47.4	2.8
3	47.4	8.8	47.4	4.3
4	47.4	6.6	47.4	3.6

8.8 Retailer Case Study

A realistic case study based on the electricity market of the Iberian Peninsula, [106, 107] is examined below.

The planning horizon corresponds to the year 2004, which is divided into 72 periods, i.e., six periods per month. These six periods correspond to the following sets of hours: *Monday off-peak, Monday peak, weekday off-peak* (other than Monday), *weekday peak* (other than Monday), *weekend off-peak* and *weekend peak*, where peak hours are $\{11, 12, 13, 14, 19, 20, 21, 22\}$, and all other hours are considered off-peak. For example, the pool price in the first period of the planning horizon is equal to the average of pool prices during off-peak hours on all Mondays in January.

Three groups of clients are considered according to different consumption patterns, namely, residential, commercial, and industrial. The relationship between the selling price and the client demand supplied by the retailer to each group is modeled through a stepwise price-quota curve with 100 steps. Fig. 8.13 depicts the price-quota curves for each group of clients.

Three monthly and four quarterly contracts for peak and base hours are considered (14 contracts). Note that peak contracts are used in peak hours of weekdays, while base contracts are used in all periods over the lifespan of the contracts. Table 8.11 provides the price and the upper bound of the first block of the forward contracting curve of each contract. Nine additional equally-sized blocks are considered for each contract, with prices increasing 10% in each subsequent block.

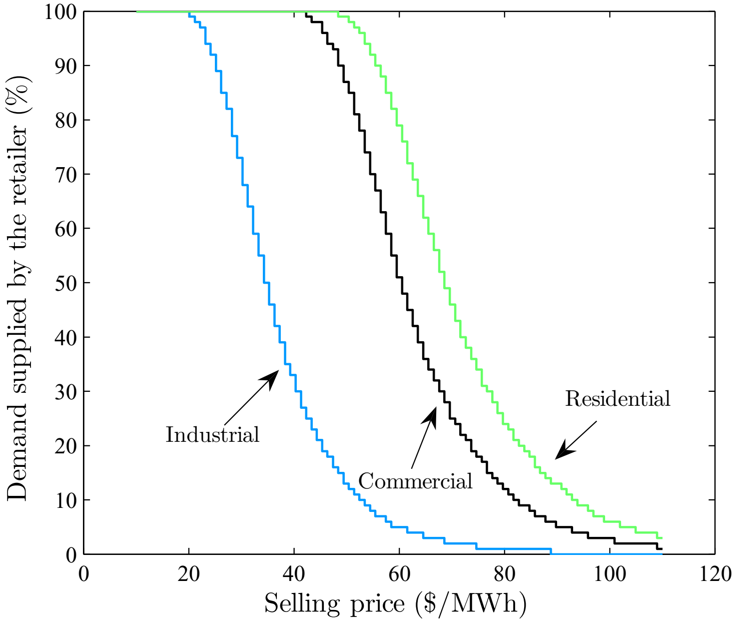

Fig. 8.13 Retailer case study: price-quota curves

For the sake of clarity, we consider that no previous forward contracts have been signed, i.e., $E_t^{\mathrm{PC}} = 0, \forall t = 1, \ldots, N_{\mathrm{T}}$.

Table 8.11 Retailer case study: forward contracting curve data

Contract #	Peak contracts $\lambda^{\mathrm{F}}_{f1}$ (\$/MWh)	$\bar{P}^{\mathrm{F}}_{f1}$ (MW)	Base contracts $\lambda^{\mathrm{F}}_{f1}$ (\$/MWh)	$\bar{P}^{\mathrm{F}}_{f1}$ (MW)
Monthly 1	37.60	500	34.29	500
Monthly 2	35.12	400	31.99	400
Monthly 3	31.43	200	29.42	200
Quarterly 1	34.72	500	31.90	500
Quarterly 2	33.89	200	32.21	200
Quarterly 3	37.29	100	34.66	100
Quarterly 4	36.12	100	33.34	100

By means of the scenario-generation procedure for non-contemporaneous stochastic processes presented in Subsection 3.4.3 of Chapter 3, a tree of 2000 scenarios of pool prices and client demands is built. After applying the scenario-reduction technique described in Subsection 3.3.3.3 of Chapter 3, a final tree of 200 scenarios is obtained. Figs. 8.14 and 8.15 show the resulting 200 scenarios of pool prices and client demands, respectively.

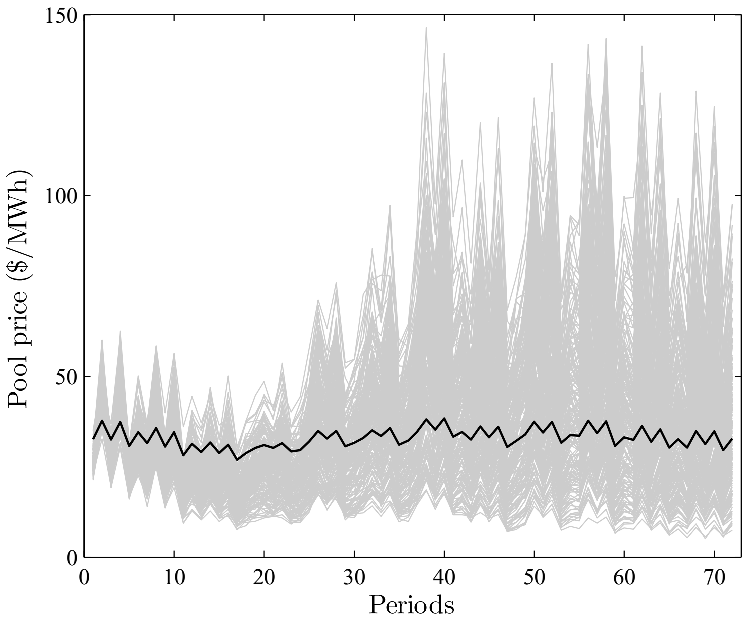

Fig. 8.14 Retailer case study: pool price scenarios

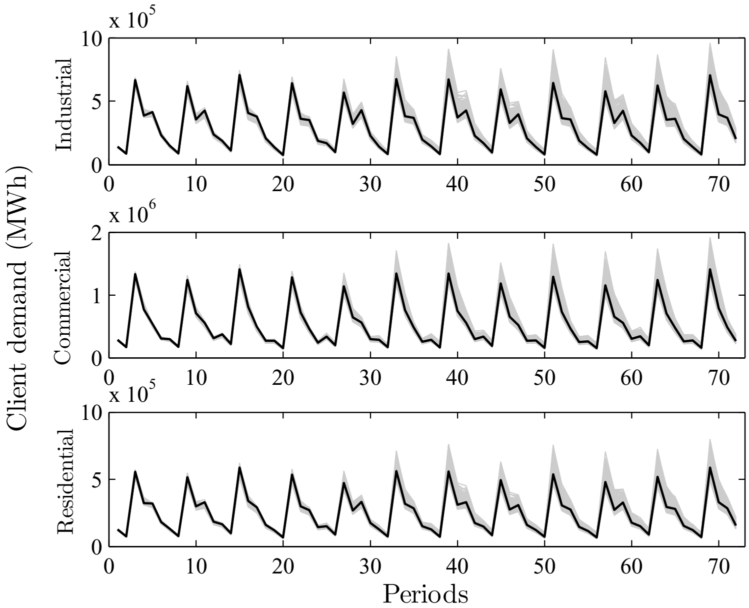

Fig. 8.15 Retailer case study: client demand scenarios

The case study presented above is solved using the model (8.18)-(8.25), where the goal is to maximize a multi-objective function formed by the expected profit plus the CVaR multiplied by the weighting factor β.

This problem, characterized by 15,341 constraints, 15,342 real variables, and 300 binary variables, is solved for different values of the risk parameter β at a confidence level $\alpha = 0.95$ using CPLEX 10.2 [142] under GAMS [141].

Fig. 8.16 shows two plots representing the efficient frontier of this problem. Specifically, Fig. 8.16 plots the expected profit versus both the profit standard deviation and the CVaR for different values of β. The expected profit ranges between \$1171.93 million ($\beta = 0$) and \$790.62 million ($\beta = 100$). The profit standard deviation ranges from \$367.31 million to \$97.24 million, and the CVaR between \$142.02 million and \$504.83 million. It can be observed that a 73.5% reduction in the profit standard deviation results in a 32.5% decrease in the expected profit, while the CVaR, which represents the expected value of the profit computed over the worst scenarios, increases 255.5%.

The power purchased from forward contracts for different values of β is provided in Table 8.12. The amount of power obtained through forward contracts increases as the parameter β grows. This result, together with Fig. 8.16, indicates that the futures market is a useful tool for hedging against profit volatility. It can be also observed that peak contracts are used more often than base contracts. This result is consistent since the highest volatility of pool prices occurs during peak periods.

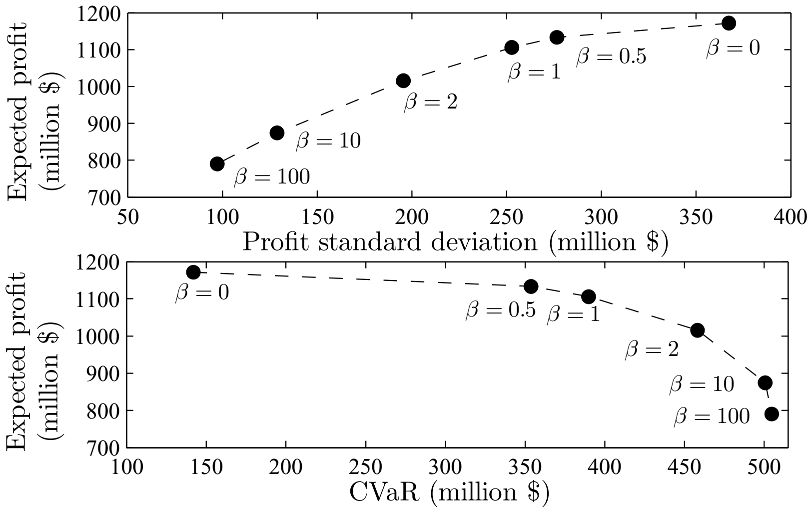

Fig. 8.16 Retailer case study: efficient frontier

Table 8.12 Retailer case study: power purchased from forward contracts (MW)

Contract #	β					
	0	0.5	1	2	10	100
Peak monthly 1	1000	1000	1500	1500	1500	1500
Peak monthly 2	0	400	400	400	800	800
Peak monthly 3	0	0	0	0	0	0
Peak quarterly 1	0	500	500	500	500	1,000
Peak quarterly 2	0	200	400	600	800	800
Peak quarterly 3	200	500	600	700	900	900
Peak quarterly 4	200	500	600	700	800	900
Base monthly 1	0	0	0	0	0	0
Base monthly 2	0	0	0	0	0	0
Base monthly 3	0	0	0	0	0	0
Base quarterly 1	0	0	0	0	0	0
Base quarterly 2	0	0	200	200	400	422
Base quarterly 3	0	300	400	500	700	700
Base quarterly 4	0	200	400	500	600	600

The optimal selling price obtained for different values of β is provided in Table 8.13. The selling price offered to each client group increases as the parameter β increases. This behavior is due to the use of a more expensive electricity source (forward contracts).

Fig. 8.17 depicts the selling price offered to each client group versus the profit standard deviation. Note that solutions with relatively smaller profit standard deviations involve higher selling prices. Thus, a retailer willing to

Table 8.13 Retailer case study including CVaR: price offered to clients ($/MWh)

Client group #	β					
	0	0.5	1	2	10	100
Industrial	40.30	48.38	48.38	56.46	60.51	63.54
Commercial	52.42	55.45	56.46	58.48	62.53	66.57
Residential	57.47	60.51	60.51	63.54	66.57	66.57

offer competitive selling prices has to assume a comparatively high level of risk.

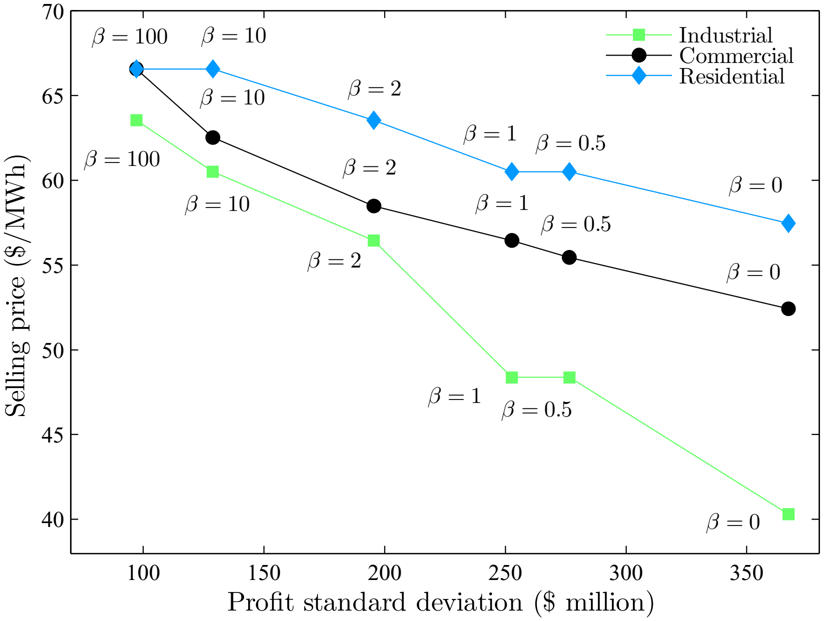

Fig. 8.17 Retailer case study: selling price versus profit standard deviation

The expected electricity procurements of the retailer for $\beta = 0$ and $\beta = 100$ are provided in Figs. 8.18 and 8.19, respectively. These figures illustrate the impact of risk aversion on the purchases of the retailer.

Fig. 8.18 shows the case of a risk-neutral retailer, i.e., $\beta = 0$. In this case, most of the purchases of electricity are made in the pool. As shown in Table 8.12, only peak monthly contract 1 and peak quarterly contracts 3 and 4 are used, whereas no base contracts are signed.

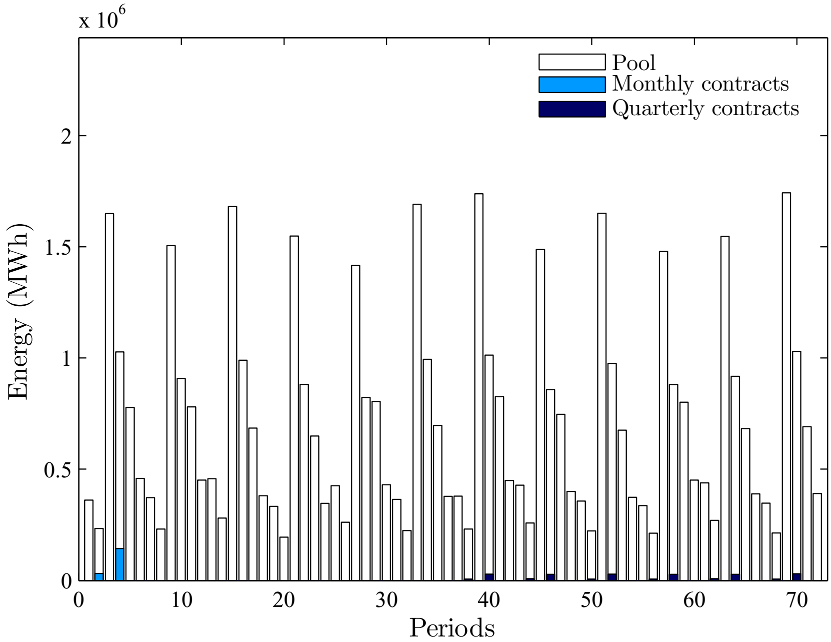

Fig. 8.18 Retailer case study: expected electricity procurement for $\beta = 0$

On the other hand, if risk is considered ($\beta = 100$), two relevant effects can be observed. First, the total amount of purchased electricity decreases relative to the risk-neutral case (Fig. 8.19 vs. Fig. 8.18). This result is a direct consequence of the increase in the selling prices due to the risk aversion of the retailer, which can also be observed in Table 8.13 and Fig. 8.17. The growth of the selling price along with the shapes of the price-quota curves shown in Fig. 8.13 results in a reduction in the total demand supplied by the retailer. Second, the purchases of energy from forward contracts increase if risk is accounted for.

Fig. 8.20 shows the adjusted pdfs of the profit for $\beta = 0$ and $\beta = 100$. The profit for $\beta = 0$ ranges from minus \$1057.1 million to plus \$1801.6 million and the expected profit is equal to \$1171.93 million. Thus, there exists a non-null probability of experiencing high losses in some scenarios. This explains why risk measures are incorporated in the modeling of the retailer problem. On the other hand, for $\beta = 100$, the profit varies from \$120.4 million to \$932.6 million, with an expected profit equal to \$790.62 million. In this case the expected profit is reduced by 32.5%, but the probability of experiencing losses is equal to zero.

Fig. 8.21 provides the percentages of the expected energy purchased from forward contracts and in the pool for different values of β. The energy in each

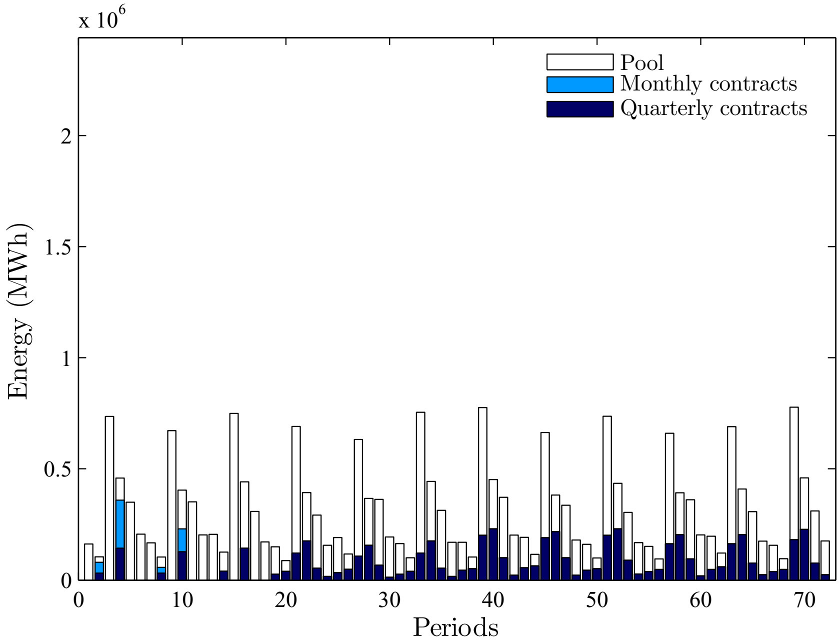

Fig. 8.19 Retailer case study: expected electricity procurement for $\beta = 100$

sector of the pie charts represents the expected value over all scenarios. For instance, we observe that for $\beta = 0$ more than 99% of the energy purchased by the retailer is obtained in the pool. As β increases, the percentage of energy purchased in the pool decreases. For $\beta = 2$, 88% of the expected electricity procurement of the retailer is obtained from the pool, and 7% by means of base contracts. Finally, note that from $\beta = 0$ to $\beta = 100$ the percentage of energy purchased in the pool decreases by 25%.

Solution times considering currently available computer technology is moderate and compatible within the decision-time framework of the problem under consideration.

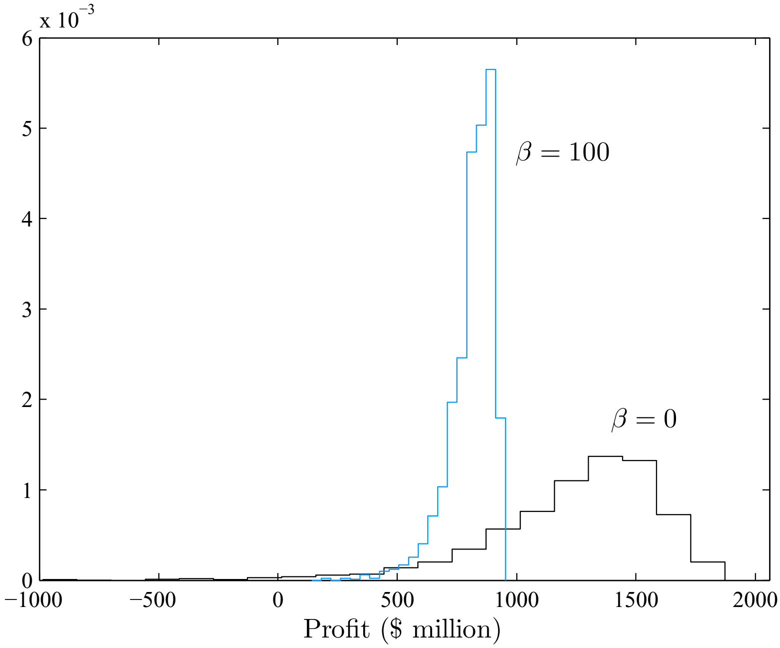

Fig. 8.20 Retailer case study: adjusted pdfs of the profit for $\beta = 0$ and $\beta = 100$

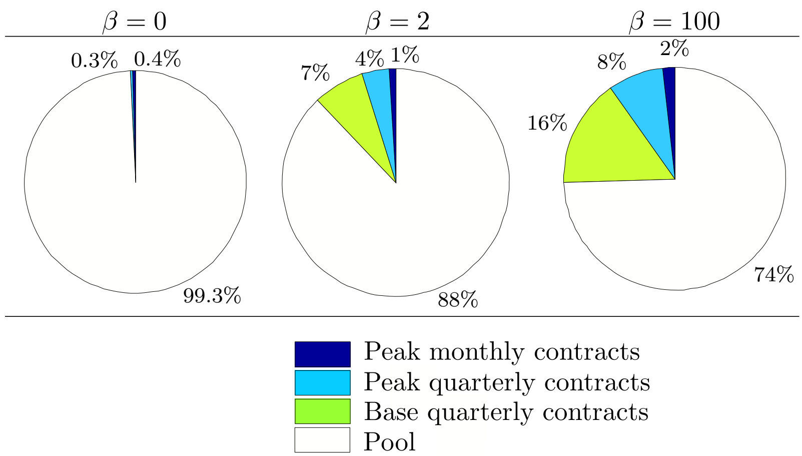

Fig. 8.21 Retailer case study: expected electricity procurement

8.9 Summary and Conclusions

This chapter presents a stochastic programming model that allows an electricity retailer to determine its medium-term forward contract portfolio and to offer optimal selling prices to clients.

To procure the electric energy to be sold to its clients, a retailer copes with two main challenges: while buying, it faces uncertain pool prices; while selling, it faces the uncertainty of client demand and the fact that clients may select a different retailer if selling prices are not competitive enough.

The pool price and the client demand are characterized by stochastic processes using time series models and a finite set of scenarios.

The influence of the selling price on the client demand supplied by the retailer is modeled through a stepwise price-quota curve.

The influence of the retailer on the futures market is explicitly taken into account through forward contracting curves.

The risk-aversion modeling is based on the CVaR. The objective function of the model comprises the expected profit plus the CVaR multiplied by a weighting factor. Solving the problem for different values of the weighting factor allows the representation of the so-called efficient frontiers.

This risk-constrained stochastic programming model is formulated as a mixed-integer linear programming problem that can be solved using commercial software. The proposed mixed-integer linear formulation proves both efficient and robust as demonstrated via both small and real-life case studies.

The numerical examples show that as the concern on risk increases (higher values of β), the reliance on pool purchases diminishes, yielding a higher participation in the futures market. In addition, as the retailer relies more and more on its purchases from the pool (small CVaR), the selling price offered to the clients decreases in order to attract as much consumption as possible.

The proposed modeling framework is flexible enough to accommodate a variety of features characterizing both retailers and trading floors.

8.10 Notation

The notation used throughout this chapter is stated below for quick reference.

Indices and Numbers:

e Index of client groups, running from 1 to N_{E}.

f Index of forward contracts, running from 1 to N_{F}.

j	Index of blocks in the forward contracting curves, running from 1 to N_{J}.
i	Index of blocks in the price-quota curves, running from 1 to N_{I}.
t	Index of time periods, running from 1 to N_{T}.
ω	Index of scenarios, running from 1 N_{Ω}.

Real Variables:

C_t^{F}	Cost of purchasing from forward contracts in period t (\$).
$C_{t\omega}^{\mathrm{P}}$	Cost of purchasing from the pool in period t and scenario ω (\$).
$E_{t\omega}^{\mathrm{P}}$	Energy traded in the pool in period t and scenario ω (MWh).
E_{et}^{R}	Energy supplied by the retailer to client group e in period t (MWh). $E_{et\omega}^{\mathrm{R}}$ represents its value for scenario ω.
P_f^{F}	Power contracted from forward contract f (MW).
P_{fj}^{F}	Power contracted from block j of forward contracting curve of forward contract f (MW).
$R_{et\omega}^{\mathrm{R}}$	Revenue obtained by the retailer from selling to client group e in period t and scenario ω (\$).
λ_e^{R}	Selling price offered by the retailer to client group e (\$/MWh).
$\lambda_{ei}^{\mathrm{R}}$	Selling price associated with block i of the price-quota curve of client group e (\$/MWh). Limited to $\bar{\lambda}_{ei}^{\mathrm{R}}$.
ζ	Auxiliary variable used to calculate the CVaR (\$).
η_ω	Auxiliary variable related to scenario ω used to calculate the CVaR (\$).

Binary Variables:

v_{ei}	0/1 variable that is equal to 1 if the selling price offered by the retailer to client group e belongs to block i of the price-quota curve, being 0 otherwise.

Random Variables:

E_{et}^{D}	Random variable modeling the electricity demand of client group e in period t (MWh). $E_{et\omega}^{\mathrm{D}}$ represents the realization of this random variable in scenario ω.

λ_t^{P} Random variable modeling the price of energy in the pool in period t (\$/MWh). $\lambda_{t\omega}^{\mathrm{P}}$ represents the realization of this random variable in scenario ω.

Constants:

d_t Duration of period t (h).
E_t^{PC} Energy contracted prior to the beginning of the planning horizon that is used in period t (MWh).
$\bar{E}_{eti\omega}^{\mathrm{R}}$ Energy associated with block i of the price-quota curve of client group e in period t and scenario ω (MWh).
$\bar{P}_{fj}^{\mathrm{F}}$ Upper limit of the power contracted from block j of the forward contracting curve of forward contract f (MW).
$\lambda_{fj}^{\mathrm{F}}$ Price of block j of the forward contracting curve of forward contract f (\$/MWh).
α Confidence level used in the calculation of the CVaR.
β Weighting factor used to materialize the tradeoff between expected profit and CVaR.
π_ω Probability of occurrence of scenario ω.

Sets:

F_t Set of forward contracts available in period t.
Ω Set of scenarios.

8.11 Exercises

Exercise 8.1. Extend formulation (8.18)-(8.25) so that the retailer can buy and sell in the futures market.

Exercise 8.2. Replace CVaR in (8.18)-(8.25) by an alternative risk measure (see Chapter 4).

Exercise 8.3. Reformulate (8.18)-(8.25) avoiding parameter β while still modeling the tradeoff of expected profit versus CVaR.

Exercise 8.4. Indicate ways to make formulation (8.18)-(8.25) more realistic.

Exercise 8.5. Solve the example in Section 8.7 (whose GAMS code is provided in Section A.6 of Appendix A) considering just two of the scenarios provided in Table 8.4. Compare all solutions involving just two arbitrarily selected scenarios of those in Table 8.4.

Exercise 8.6. Reduce the number of available forward contracts in the example of Section 8.7 and compare the results obtained as the number of available forward contracts decreases.

Exercise 8.7. Include a new block in the price-quota curve in Fig. 8.11 and compare the results obtained with those of the original example.

Exercise 8.8. Shift \$20/MWh to the right the price-quota curve in Fig. 8.11 and solve again the example of Section 8.7. Compare the results obtained with those of the original case.

Exercise 8.9. Include a new group of clients in the example of Section 8.7 and compare the results obtained with those of the original instance.

Exercise 8.10. Calculate VSS and EVPI ($\beta = 0$) for the example in Section 8.7.

Chapter 9
Energy Procurement by Consumers

9.1 Introduction

We study in this chapter the electricity procurement problem faced by a large consumer. Such a consumer refers to an entity with a significant electricity consumption that participates in the electricity market, and has the capability of signing bilateral contracts with suppliers. Petrochemical industries, aluminum production complexes, or vehicle-assembling facilities are examples of large consumers. Supply sources available for the consumer include forward contracts, bilateral contracts, the pool, and self-production.

Since forward contracts are considered in the chapters devoted to producers and retailers (Chapters 7 and 8, respectively), they are intentionally overlooked in this chapter. The modeling of these contracts in the consumer problem is similar to their modeling in the producer or retailer problem.

A bilateral contract is a private arrangement between a supplier (e.g., a producer or a retailer) and a consumer, which allows the latter to buy electrical energy prior to its physical delivery. The consumer may participate in the pool to buy electricity at a low price during specific time periods. However, if the consumer only purchases in the pool, it has to deal with the uncertainty inherent to pool prices. In addition to buying from the pool and signing bilateral contracts, we assume that the consumer owns a production unit to satisfy part of its electricity demand when pool prices are comparatively high. The consumer can also use this unit to sell energy in the pool.

A large variety of contracts are generally at the consumer disposal, including those that penalize deviations over the quantities agreed upon. Signing these contracts makes sense for a consumer with capability of answering to these penalties by changing its consumption pattern.

When planning for a medium-term horizon, the consumer has to decide which bilateral contracts to sign before making the short-term decisions related to pool trading and self-production. For larger time horizons, the consumer may also consider building self-production facilities. In this chapter,

A. J. Conejo et al., *Decision Making Under Uncertainty in Electricity Markets*, International Series in Operations Research & Management Science 153,
DOI 10.1007/978-1-4419-7421-1_9,

we focus on a medium-term planning horizon, where the objective of the consumer is to determine its bilateral contract portfolio.

This problem is mathematically formulated as a multi-stage stochastic programming problem where the pool price is modeled as a stochastic process using a set of scenarios, [14, 84]. In order to attain tractability, scenario-reduction techniques are generally needed to reduce the number of scenarios representing the pool price. Risk aversion is explicitly considered and mathematically formulated by means of the Conditional Value-at-Risk, CVaR, [124, 125]. Further details on scenario reduction and risk modeling are found in Chapters 3 and 4.

Relevant references addressing energy procurement problems for consumers include [21, 23, 25, 67, 71, 77, 143, 115].

The rest of this chapter is organized as follows. Section 9.2 describes the decision framework for the energy procurement problem faced by a consumer in a medium-term horizon, i.e., from one to several months. Section 9.3 provides a detailed model for the considered procurement problem involving bilateral contracting and the pool, as well as a detailed mathematical formulation based on multi-stage stochastic programming. Section 9.5 includes a detailed example to illustrate all the technical aspects of the proposed model, as well as the characteristics of the decisions derived from the use of such a model. Section 9.6 reports and discusses results from a realistic case study. Finally, Section 9.7 concludes providing a summary for the chapter and some relevant conclusions.

9.2 Decision Framework and Uncertainty Model

9.2.1 Decision Framework

We consider a medium-term planning horizon that may comprise from one month to several years. Periodically, the consumer makes bilateral contract decisions for future supply, and throughout the planning horizon the consumer decides its pool involvement and operates its existing production unit.

Without loss of generality, hereinafter we focus on a planning horizon of four weeks, and, in order to attain tractability, each day of the planning horizon is divided into three eight-hour periods, namely *valley*, *shoulder* and *peak*. This constitutes an appropriate tradeoff between solution accuracy and computational burden. The resulting time span comprises 84 periods, where the pool price in each eight-hour period is obtained as the average of the hourly pool prices during those 8 hours, while the electricity demand of the consumer is computed as the sum of the hourly demand over all hours included in each period. Needless to say, the time discretization above should be tailored to the needs of the consumer under consideration.

The procurement alternatives that we consider available for the consumer are:

1. Monthly contracts, which are selected at the beginning of the month.
2. Weekly contracts, which are decided at the beginning of each week.
3. Pool purchases or sales, which are decided one day ahead.
4. Self-production, which is also a day-ahead decision.

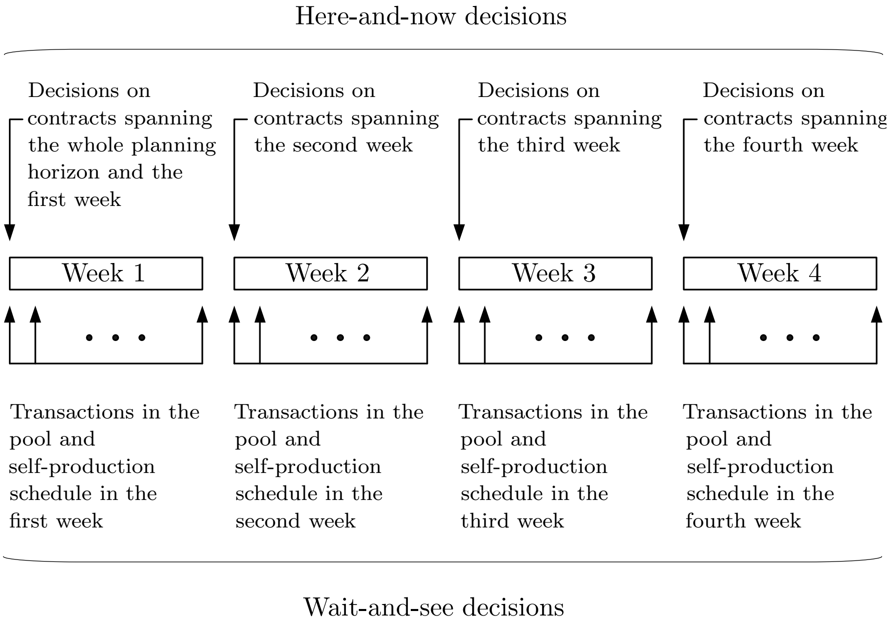

Fig. 9.1 Decision framework of the consumer model

Fig. 9.1 shows the considered decision framework. Decisions on contracts are made with uncertain pool prices, i.e., with *imperfect information* on them, and thus, they are here-and-now decisions (Fig. 9.1). In contrast, decisions on pool trading and self-production are deferred in time with respect to the bilateral contract decisions, i.e., they are made close to the time when the actual pool prices are known. Thus, in the medium-term, it is an appropriate approximation to consider that they are made with *perfect information*, i.e., prices are assumed to be known when the consumer purchases and sells in the pool and schedules its self-production unit. These decisions are thus wait-and-see decisions (Fig. 9.1).

In order to account for the impact of pool price uncertainty on contracting decisions, we propose a multi-stage stochastic programming model, and define five stages, each corresponding to the beginning or the end of a single week.

Under this framework, the pool price is modeled as a stochastic process that is represented through a set of scenarios.

It should be emphasized that the proposed tool is designed to yield the optimal contracting decisions for the first week. In this respect, by the time the decisions for the second week must be made, pool prices and demands of the first week are known and pool price scenarios are generated for the following weeks. Note that the price scenarios for the remaining weeks may be different from the original estimates. A similar situation arises at the end of the second and third weeks. Therefore, in order to make optimal decisions for weekly contracts used in the second, third, and fourth weeks, the proposed approach should also be run at the beginning of these weeks with an updated set of price scenarios and with the selection of monthly contracts fixed at the beginning of the month.

9.2.2 Pool Price and Demand

Since decisions on bilateral contracts are dependent on future pool prices, an adequate modeling of these prices is required. To this end, we define the collection $\{\lambda_1^{\mathrm{P}}, \dots, \lambda_{N_{\mathrm{T}}}^{\mathrm{P}}\}$ of pool prices in future periods $t = 1, \dots, N_{\mathrm{T}}$ as a stochastic process, λ_t^{P}, and model the stochastic process pertaining to pool prices using a set of scenarios. Each scenario represents a realization of the pool prices for all periods of the planning horizon. In this way, the set of pool price scenarios $\{\lambda_{1\omega}^{\mathrm{P}}, \dots, \lambda_{N_{\mathrm{T}}\omega}^{\mathrm{P}}\}$, $\forall \omega \in \Omega$, represents the collection of random variables $\{\lambda_1^{\mathrm{P}}, \dots, \lambda_{N_{\mathrm{T}}}^{\mathrm{P}}\}$, where ω is the index of scenarios, Ω is the set of scenario indices, and N_{T} is the number of time periods in the planning horizon. Each scenario has a probability of occurrence, π_ω, such that the sum of the probabilities over all scenarios is equal to 1, i.e., $\sum_{\omega \in \Omega} \pi_\omega = 1$.

The electricity demand of the consumer can also be modeled as a stochastic process. However, we consider that a large consumer has a sufficiently precise knowledge of its own demand so that this demand can be accurately forecast. Note that this may not be the case in developing countries where electricity demand variations for a particular consumer may be large even within a medium-term horizon. Note that the level of uncertainty of the demand is much smaller than that pertaining to the electricity price faced by the large consumer while buying from or selling to the pool. This drastic difference on the uncertainty level leads us to consider the consumer demand deterministic.

The illustrative example below clarifies the decision framework and decision tree for a consumer.

Illustrative Example 9.1 (Consumer decision framework and scenario tree).

Fig. 9.2 shows an example of a five-stage scenario tree for a four-week electricity procurement problem. In this tree, two branches leave each node, resulting in $2^4 = 16$ scenarios. Since each week is divided into seven days, each comprising three periods of eight hours, the resulting vector of random variables is made up of 84 components.

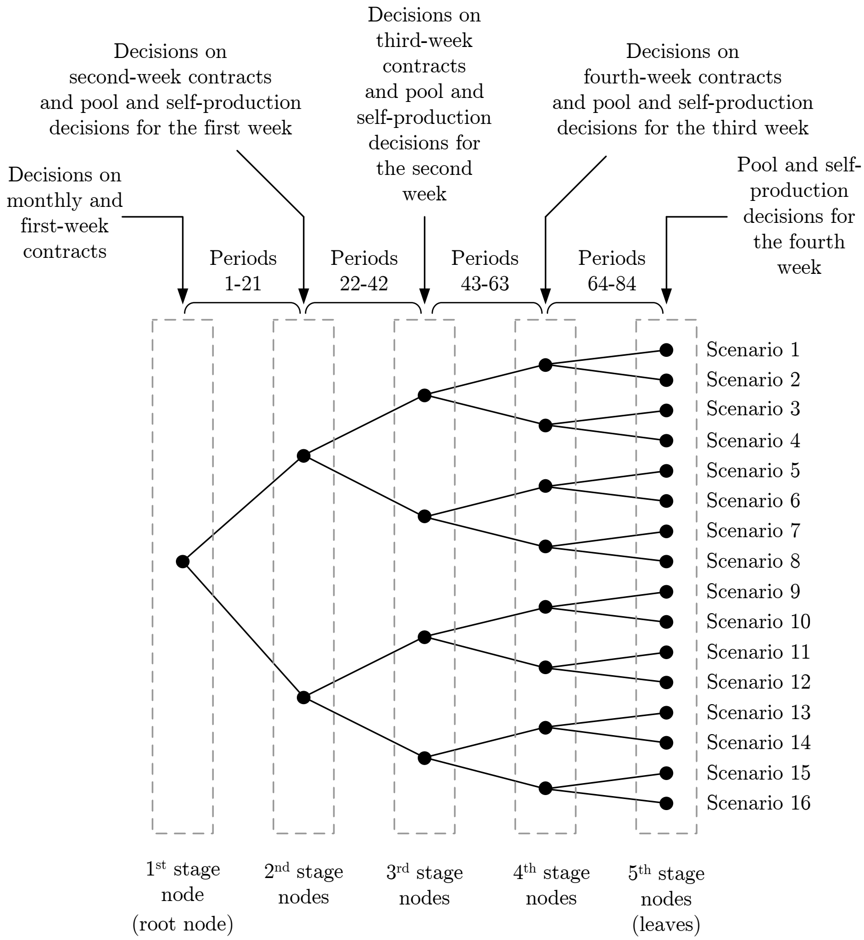

Fig. 9.2 Consumer scenario tree

At the root node, decisions on the monthly and first-week contracts are made. Since first-stage decisions are made before knowing the future pool prices, they have to be the same for each single scenario. This means that these decisions are made with non-anticipativity with respect to the considered 16 pool price scenarios.

The two branches leaving the root node represent two possible pool price realizations for the first week (periods 1 to 21). Throughout this week, pool and self-production decisions are made. We consider that these decisions are made with perfect information. In other words, these decisions are assumed to be made after the realization of the first-week pool prices, i.e., at the beginning of the second week. Observe in Fig. 9.2 that since nodes represent decision points, two different pool and self-production decisions can be made for the first week at the second stage.

Decisions on contracts spanning the second week are made at the second-stage nodes. Again, two different decisions on second-week contracts can be made. Similarly, at the third stage, bilateral contracts for the third week and decisions about pool purchasing and self-production for the second week are made, and so on.

It should be noted that all scenarios coinciding in the same branch have the same values of the corresponding random variables, i.e., $\lambda^{\mathrm{P}}_{t\omega} = \lambda^{\mathrm{P}}_{t\omega'}$ if scenarios ω and ω' are coincident in period t. In other words, they are identical 21-component vectors. □

9.3 Model

We model and formulate in this section the electricity procurement problem faced by a large consumer.

9.3.1 Bilateral Contracts

The consumer can sign bilateral contracts with producers to supply a part of its electricity demand. We consider that bilateral contracts are private arrangements between two parties outside an organized market.

Although multiple types of bilateral contracts exist in practice, we analyze in this chapter a particular one. We define contract b as a physical delivery contract stipulating that the consumer receives a quantity of energy from an electricity supplier at a price equal to the average value of a reference price associated with the contract, λ^{B}_b, and the pool price, λ^{P}_t, in each period t spanned by the contract. Since pool prices are represented by a set of scenarios, the final price of the energy purchased from bilateral contract b also depends on pool price scenarios. Thus, the price of electricity for bilateral contract b in period t and scenario ω, $\lambda^{\mathrm{B}}_{bt\omega}$, is expressed as

$$\lambda^{\mathrm{B}}_{bt\omega} = \frac{\lambda^{\mathrm{B}}_b + \lambda^{\mathrm{P}}_{t\omega}}{2}, \ \forall b, \forall t \in T_b, \forall \omega, \tag{9.1}$$

where T_b represents the set of time periods in which contract b is defined.

Although final contract prices depend on future pool prices, the uncertainty associated with bilateral contract purchasing is smaller than that related to trading in the pool. This is clarified below.

Consider a bilateral contract b. According to (9.1), if the price in the pool λ_t^{P} is equal to price λ_b^{B}, the consumer pays for the energy supplied through contract b a final price equal to the pool price. However, if the pool price is x \$/MWh greater than λ_b^{B}, the consumer pays $\left(\lambda_b^{\mathrm{B}} + \frac{x}{2}\right)$ \$/MWh for the electricity obtained from contract b. Note that this value is smaller than the price in the pool, which is equal to $\left(\lambda_b^{\mathrm{B}} + x\right)$ \$/MWh. On the other hand, if the pool price is x \$/MWh smaller than λ_b^{B}, the consumer pays a price equal to $\left(\lambda_b^{\mathrm{B}} - \frac{x}{2}\right)$ \$/MWh, which is greater than that in the pool, $\left(\lambda_b^{\mathrm{B}} - x\right)$ \$/MWh. Thus, we conclude that bilateral contracts allow hedging against high pool prices, but at the cost of paying a higher price when pool prices are relatively low.

The illustrative example below clarifies the type of contracts considered throughout this chapter.

Illustrative Example 9.2 (Contract).

Fig. 9.3 depicts a contract involving seven time periods, where the reference contract price λ_b^{B} is equal to \$50/MWh. Observe that the volatility of the final contract prices is smaller than that associated with pool prices.

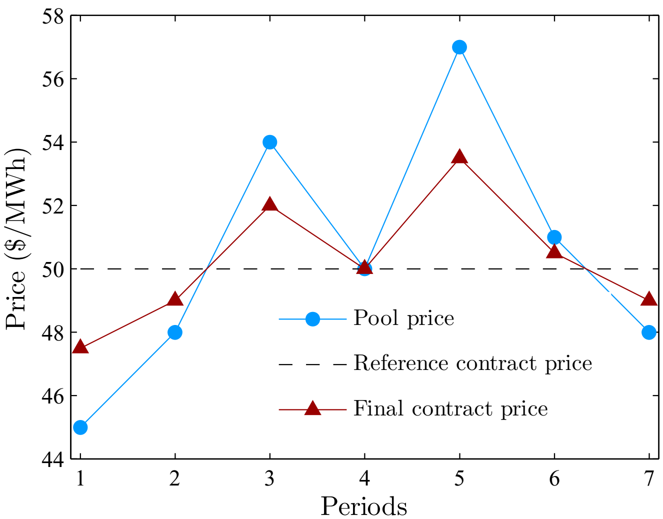

Fig. 9.3 Illustrative Example 9.2: example of contract

□

Variables defining contract decisions are introduced below. The selection of bilateral contract b is modeled through binary variable $s_{b\omega}$, which is equal to 1 if contract b is selected in scenario ω and 0 otherwise. Note that, in general, $s_{b\omega}$ depends on the pool price scenario ω. This is a consequence of the fact that decisions on weekly contracts for the second, third, and fourth weeks depend on the pool prices in previous weeks. For the sake of clarity, we consider that all contract decisions are scenario dependent, including monthly and first-week contracts. Consequently, we must impose non-anticipativity conditions on the contract decisions. Thus, if the decision on the selection of contract b is made at stage K_b, then $s_{b\omega} = s_{b\omega'}$ must be satisfied in all scenarios $\omega,\ \omega' \in \Omega$ which are coincident up to stage K_b. Specifically, if contract b is decided at the beginning of the planning horizon (root node), the relation $s_{b\omega} = s_{b\omega'}$ is imposed on all pairs of scenarios $\omega,\ \omega' \in \Omega$.

Non-anticipativity constraints are explained in Subsection 9.3.5 below and illustrated through Illustrative Examples 9.7 and 9.8.

Decision variable $P^{\mathrm{B}}_{b\omega}$ models the power purchased from contract b in scenario ω. This power is delivered in every period of the time span of the contract. Similarly to binary variable $s_{b\omega}$, $P^{\mathrm{B}}_{b\omega}$ depends on the pool price scenario and is also subject to the non-anticipativity conditions.

If contract b is selected, the power purchased $P^{\mathrm{B}}_{b\omega}$ is bounded by upper and lower limits $P^{\mathrm{B,max}}_{b}$ and $P^{\mathrm{B,min}}_{b}$ as follows:

$$P^{\mathrm{B,min}}_{b}\ s_{b\omega} \leq P^{\mathrm{B}}_{b\omega} \leq P^{\mathrm{B,max}}_{b}\ s_{b\omega},\ \forall b, \forall \omega. \tag{9.2}$$

For each contract b with parameters $\{\lambda^{\mathrm{B}}_{b}, P^{\mathrm{B,max}}_{b}, P^{\mathrm{B,min}}_{b}, T_b\}$, the consumer has to decide both whether or not that contract is signed and the level of contracted power.

The cost of purchasing from bilateral contract b in scenario ω, $C^{\mathrm{B}}_{b\omega}$, is expressed as

$$C^{\mathrm{B}}_{b\omega} = \sum_{t \in T_b} \lambda^{\mathrm{B}}_{bt\omega} P^{\mathrm{B}}_{b\omega} d_t,\ \forall b, \forall \omega. \tag{9.3}$$

Since time periods comprise several hours, the energy purchased in period t from bilateral contract b is equal to the product of the power associated with contract b, $P^{\mathrm{B}}_{b\omega}$, and the duration of period t in hours, d_t. It should be emphasized that if contract b is not selected in scenario ω, binary variable $s_{b\omega}$ is equal to 0, and constraints (9.2) and (9.3) set the cost of purchasing through contract b to 0 in scenario ω.

The illustrative example below clarifies the data required to define a contract.

Illustrative Example 9.3 (Contract example).

Consider a bilateral contract b involving 3 hours and minimum and maximum powers equal to 50 and 100 MW, respectively. The reference price for this contract is \$40/MWh, while pool prices during the three considered hours for a given scenario ω are 39, 45 and \$38/MWh, respectively.

The power to be purchased through this contract is bounded as follows (constraints (9.2) above):

$$50\ s_{b\omega} \leq P^{\mathrm{B}}_{b\omega} \leq 100\ s_{b\omega}\ .$$

The cost of buying from this contract is (equation (9.3) above)

$$C^{\mathrm{B}}_{b\omega} = \left(\frac{40+39}{2} + \frac{40+45}{2} + \frac{40+38}{2}\right) P^{\mathrm{B}}_{b\omega}\ .$$

□

9.3.2 Pool

The consumer participates in the pool by both buying and selling energy. For instance, the consumer may sell in the pool part of the electricity generated by its self-production unit.

Transactions in the pool are wait-and-see decisions since they depend on pool price scenarios (Fig. 9.1). The energy traded by the consumer in the pool in period t and scenario ω is modeled by the continuous variable $E^{\mathrm{P}}_{t\omega}$, which is positive if the consumer buys energy in the pool in period t and scenario ω, and negative if it sells.

The net cost of trading in the pool is then computed as

$$C^{\mathrm{P}}_{t\omega} = \lambda^{\mathrm{P}}_{t\omega} E^{\mathrm{P}}_{t\omega},\ \forall t, \forall \omega. \tag{9.4}$$

If a consumer purchases in the pool in period t and scenario ω, $E^{\mathrm{P}}_{t\omega}$ and $C^{\mathrm{P}}_{t\omega}$ are both positive. On the other hand, if a consumer sells in the pool, $E^{\mathrm{P}}_{t\omega}$ is negative, yielding a negative cost (revenue), i.e., $C^{\mathrm{P}}_{t\omega} < 0$.

The illustrative example below provides an example of the cost of pool purchases by a consumer.

Illustrative Example 9.4 (Pool Trading Cost).
Considering the data in Illustrative Example 9.3 above, pool costs per hour are as follows:

$$\begin{aligned} C^{\mathrm{P}}_{1\omega} &= 39 E^{\mathrm{P}}_{1\omega} \\ C^{\mathrm{P}}_{2\omega} &= 45 E^{\mathrm{P}}_{2\omega} \\ C^{\mathrm{P}}_{3\omega} &= 38 E^{\mathrm{P}}_{3\omega}\ . \end{aligned}$$

□

9.3.3 Self-Production

It is common for a large electricity consumer to own a self-production facility to supply a part of its electricity demand. This self-production unit can be, for example, a co-generation facility [67].

The energy produced by the consumer in period t and scenario ω is modeled by variable $E^{\mathrm{S}}_{t\omega}$. As Fig. 9.2 shows, pool prices are considered known when self-production decisions are made. Hence, they are wait-and-see decisions.

The production cost of the self-production unit is modeled using a convex piecewise linear function. If the auxiliary variable $E^{\mathrm{S}}_{nt\omega}$ is defined as the energy produced within block n of the piecewise linear production cost in period t and scenario ω, $E^{\mathrm{S}}_{t\omega}$ can be expressed as

$$E^{\mathrm{S}}_{t\omega} = \sum_{n=1}^{N_{\mathrm{N}}} E^{\mathrm{S}}_{nt\omega}, \ \forall t, \forall \omega. \tag{9.5}$$

It is relevant to note that no binary variables are required to model the production cost above.

Thus, the production cost $C^{\mathrm{S}}_{t\omega}$ is computed using the auxiliary variables $E^{\mathrm{S}}_{nt\omega}$ as

$$C^{\mathrm{S}}_{t\omega} = \sum_{n=1}^{N_{\mathrm{N}}} S^{\mathrm{S}}_{n} E^{\mathrm{S}}_{nt\omega}, \ \forall t, \forall \omega \tag{9.6}$$

$$0 \leq E^{\mathrm{S}}_{1t\omega} \leq \bar{E}^{\mathrm{S}}_{1}, \ \forall t, \forall \omega \tag{9.7}$$

$$0 \leq E^{\mathrm{S}}_{nt\omega} \leq \bar{E}^{\mathrm{S}}_{n} - \bar{E}^{\mathrm{S}}_{n-1}, \ n = 2, \ldots, N_{\mathrm{N}}, \forall t, \forall \omega \ , \tag{9.8}$$

where S^{S}_{n} represents cost slopes and overlining indicates upper bound.

Since we consider a medium-term problem, the minimum output and start-up cost of this unit are disregarded. As a result, binary variables modeling the commitment status of the self-production unit are not required. Note that this simplification is typical in medium-term studies [21].

The maximum quantity of energy that the consumer can produce in a period is equal to the capacity of the unit multiplied by the duration of the period, and is denoted by $\bar{E}^{\mathrm{S}}_{N_{\mathrm{N}}}$. Therefore, the output of the self-production unit is bounded as follows:

$$0 \leq E^{\mathrm{S}}_{t\omega} \leq \bar{E}^{\mathrm{S}}_{N_{\mathrm{N}}}, \ \forall t, \forall \omega. \tag{9.9}$$

Note that constraints (9.9) are implicitly satisfied by enforcing constraints (9.5) and (9.7)-(9.8). This is so because constraints (9.7)-(9.8) bound above all continuing blocks in (9.5) of $E^{\mathrm{S}}_{t\omega}$.

The illustrative example below clarifies the self-production and the cost of self-production by a consumer.

Illustrative Example 9.5 (Self-production energy and cost).

Consider a self-production unit with cost slopes of 30 and $35/MWh, and power production blocks of sizes 20 and 40 MW, respectively.

The energy production function of this unit is defined by the constraints below:

$$E^{\mathrm{S}}_{t\omega} = E^{\mathrm{S}}_{1t\omega} + E^{\mathrm{S}}_{2t\omega},\ \forall t, \forall \omega$$
$$0 \leq E^{\mathrm{S}}_{1t\omega} \leq 20,\ \forall t, \forall \omega$$
$$0 \leq E^{\mathrm{S}}_{2t\omega} \leq 40,\ \forall t, \forall \omega\ .$$

The cost function is

$$C^{\mathrm{S}}_{t\omega} = 30\ E^{\mathrm{S}}_{1t\omega} + 35\ E^{\mathrm{S}}_{2t\omega},\ \forall t, \forall \omega\ .$$

□

9.3.4 Energy Balance

The electricity demand of the consumer needs to be satisfied in each period for all pool price scenarios using the electricity from the bilateral contracts, the pool, and the self-production unit. The consumer has an accurate knowledge of its future electricity demand, which in period t is denoted by E^{D}_t.

Mathematically, the electric energy balance of the consumer is formulated as

$$E^{\mathrm{S}}_{t\omega} + E^{\mathrm{P}}_{t\omega} + \sum_{b \in B_t} P^{\mathrm{B}}_{b\omega} d_t = E^{\mathrm{D}}_t - E^{\mathrm{PC}}_t\ , \forall t, \forall \omega, \tag{9.10}$$

where B_t is the set of bilateral contracts available in period t, and E^{PC}_t is a constant that represents the quantity of energy previously contracted. For instance, the term E^{PC}_t can represent the energy stipulated in a yearly bilateral contract signed at the beginning of the year and currently in force.

Since we consider that the objective of the consumer is to minimize the electricity procurement cost, the arbitrage between bilateral contracts and the pool is not allowed. In other words, buying from bilateral contracts to sell in the pool is not permitted. Thus, only self-produced energy can be sold in the pool, which is formulated as:

$$E^{\mathrm{P}}_{t\omega} \geq -E^{\mathrm{S}}_{t\omega},\ \forall t, \forall \omega. \tag{9.11}$$

Note that the assumption above (made for the sake of simplicity) should be relaxed if the demand of the consumer is clearly uncertain. Relaxing that assumptions allows higher flexibility to the consumer that generally translates into a lower procurement cost.

The illustrative example below clarifies the energy balance to be performed by a consumer.

Illustrative Example 9.6 (Energy balance).

Consider a given scenario ω and a given hourly time period t where only 1 bilateral contract b is available. The demand in this time period is 500 MW, and 200 MW are available from an annual bilateral contract. The energy balance and non-arbitrage constraints are

$$E_{t\omega}^{\mathrm{S}} + E_{t\omega}^{\mathrm{P}} + P_{b\omega} \times 1 = 500 - 200$$
$$E_{t\omega}^{\mathrm{P}} \geq -E_{t\omega}^{\mathrm{S}} \; .$$

Note that the "1" in the first equation above refers to 1 hour. □

9.3.5 Non-anticipativity

Non-anticipativity constraints are used in stochastic programming problems where decision variables are defined as dependent on the scenario index. In this case it is necessary to require that a decision made in period t is identical for all possible realizations of the random variables in time periods following t. For example, the selection of monthly contracts at the beginning of the month has to be identical for all possible pool price scenarios.

As explained in Chapter 2, the use of these constraints is particularly relevant in multi-stage problems. In these problems, decisions are made at the nodes of the scenario tree. Hence, the decision made at a specific node of the tree only affects the branches leaving that node. In general, as shown in Fig. 9.2, several scenarios are coincident at each node of the tree. Therefore, without enforcing non-anticipativity constraints, a decision variable associated with a decision node can have different values in different scenarios. The uniqueness of the decisions for coincident scenarios is stated by the non-anticipativity constraints.

In the electricity procurement problem of a large consumer, only variables modeling bilateral contract decisions, $s_{b\omega}$ and $P_{b\omega}^{\mathrm{B}}$, are subject to the non-anticipativity constraints. In fact, variables modeling pool trading and self-production are wait-and-see decisions, and they are considered to be made with perfect information. Wait-and-see variables are not constrained by non-anticipativity conditions and thus can take different values in different scenarios.

Note also that non-anticipativity constraints are the only equations linking variables belonging to different scenarios.

The illustrative example below clarifies non-anticipativity constraints for a consumer.

Illustrative Example 9.7 (Non-anticipativity constraints).

Fig. 9.4 shows an example where scenarios ω and ω' are coincident at node n. Non-anticipativity constraints force the values of variables $s_{b\omega}$ and $s_{b\omega'}$, and $P^{\mathrm{B}}_{b\omega}$ and $P^{\mathrm{B}}_{b\omega'}$ to be respectively equal.

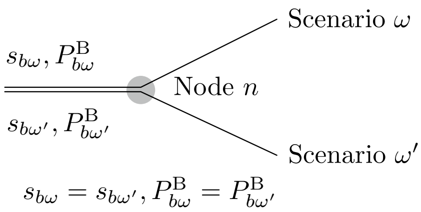

Fig. 9.4 Illustrative Example 9.7: non-anticipativity constraints

□

Non-anticipativity constraints are highly dependent on the structure of the scenario tree. For the consumer problem, we propose storing the structure of the scenario tree in a matrix of 0s and 1s denoted by $\boldsymbol{A}$. Element $A(\omega, k)$ is equal to 1 if scenarios ω and $\omega + 1$ are equal up to stage k, and 0 otherwise. The size of matrix $\boldsymbol{A}$ is $(N_\Omega - 1) \times (N_{\mathrm{K}} - 1)$, where N_Ω and N_{K} are the numbers of scenarios and stages, respectively.

The illustrative example below clarifies non-anticipativity constraint matrix for a consumer.

Illustrative Example 9.8 (Non-anticipativity constraint matrix).

The 15×4 matrix $\boldsymbol{A}$ for the scenario tree shown in Fig. 9.2 is the following:

$$\boldsymbol{A} = \begin{pmatrix} 1&1&1&1 \\ 1&1&1&0 \\ 1&1&1&1 \\ 1&1&0&0 \\ 1&1&1&1 \\ 1&1&1&0 \\ 1&1&1&1 \\ 1&0&0&0 \\ 1&1&1&1 \\ 1&1&1&0 \\ 1&1&1&1 \\ 1&1&0&0 \\ 1&1&1&1 \\ 1&1&1&0 \\ 1&1&1&1 \end{pmatrix}. \tag{9.12}$$

Note that $A(\omega, 1) = 1$, $\omega = 1, \ldots, 15$, because all scenarios are coincident up to the first stage. Analogously, element $A(2, 4)$ in (9.12) is equal to 0 because scenarios 2 and 3 become different at stage 4, as can be seen in

Fig. 9.2. In fact, in the third week (periods 43-63), scenarios 2 and 3 contain different pool prices.

□

In the procurement problem faced by a large consumer, non-anticipativity constraints affect only here-and-now decisions, which are those modeling the decisions on bilateral contracts, i.e., $s_{b\omega}$ and $P^{\mathrm{B}}_{b\omega}$. As a result, non-anticipativity constraints can be formulated as

$$\begin{aligned} s_{b\omega} = s_{b\omega+1},\ &\forall b, \omega = 1, \ldots, N_{\Omega} - 1 \\ &\text{if } A(\omega, K_b) = 1\ , \end{aligned} \tag{9.13}$$

$$\begin{aligned} P^{\mathrm{B}}_{b\omega} = P^{\mathrm{B}}_{b\omega+1},\ &\forall b, \omega = 1, \ldots, N_{\Omega} - 1 \\ &\text{if } A(\omega, K_b) = 1, \end{aligned} \tag{9.14}$$

where K_b is the stage at which decisions on contract b are made.

Note that if constraints (9.2) and (9.14) are enforced, constraints (9.13) are implicitly satisfied.

9.3.6 Expected Cost

The procurement cost faced by the consumer consists of the cost of buying from bilateral contracts, the cost of trading in the pool, and the production cost of the self-production facility. Note that the procurement cost depends on the pool price scenarios. In scenario ω, it is mathematically expressed as

$$\begin{aligned} &\sum_{t=1}^{N_{\mathrm{T}}} \left(\sum_{b \in B_t} C^{\mathrm{B}}_{b\omega} + C^{\mathrm{P}}_{t\omega} + C^{\mathrm{S}}_{t\omega} \right) = \\ &\sum_{t=1}^{N_{\mathrm{T}}} \left(\sum_{b \in B_t} \lambda^{\mathrm{B}}_{bt\omega} P^{\mathrm{B}}_{b\omega} d_t + \lambda^{\mathrm{P}}_{t\omega} E^{\mathrm{P}}_{t\omega} + \sum_{n=1}^{N_{\mathrm{N}}} S^{\mathrm{S}}_{n} E^{\mathrm{S}}_{nt\omega} \right), \forall \omega. \end{aligned} \tag{9.15}$$

The three terms in (9.15) correspond to the cost of purchasing from bilateral contracts, the cost of pool trading, and the cost of operating the self-production facility, respectively.

Based on (9.15), the expected procurement cost is computed as the sum over all scenarios of the cost in each scenario times its probability of occurrence. Mathematically, the expected cost is formulated as

$$\sum_{\omega=1}^{N_{\Omega}} \pi_{\omega} \sum_{t=1}^{N_{\mathrm{T}}} \left(\sum_{b \in B_t} C^{\mathrm{B}}_{b\omega} + C^{\mathrm{P}}_{t\omega} + C^{\mathrm{S}}_{t\omega} \right). \tag{9.16}$$

9.3.7 Risk

We model the risk of cost variability in this problem using the Conditional Value-at-Risk for a given confidence level α. The interested reader is referred to Chapter 4 for a detailed discussion on risk measures and risk management. The CVaR is approximately the expected cost of the $(1-\alpha)\times 100\%$ scenarios with greatest cost. Taking into account the expression for the procurement cost (9.15), the CVaR is computed by solving the following linear optimization problem, [124, 125],

$$\text{CVaR} = \text{minimum}_{\zeta,\eta_\omega} \qquad \zeta + \frac{1}{1-\alpha}\sum_{\omega=1}^{N_\Omega} \pi_\omega \eta_\omega \tag{9.17}$$

subject to

$$\sum_{t=1}^{N_\mathrm{T}} \left(\sum_{b \in B_t} C_b^\mathrm{B}(\omega) + C_{t\omega}^\mathrm{P} + C_{t\omega}^\mathrm{S} \right) - \zeta \le \eta_\omega, \ \forall \omega \tag{9.18}$$

$$\eta_\omega \ge 0, \ \forall \omega. \tag{9.19}$$

The optimal value of ζ is the value-at-risk (VaR) and is the smallest value of the cost such that the probability that the actual cost exceeds or equals ζ is less than or equal to $1-\alpha$. In addition, η_ω provides the excess of the cost in scenario ω over ζ, provided that this excess is positive.

9.4 Formulation

The mathematical formulation of the multi-stage stochastic problem faced by the large consumer is

$$\begin{aligned} &\text{Minimize}_{s_{b\omega}, P_{b\omega}^\mathrm{B}, E_{t\omega}^\mathrm{P}, E_{nt\omega}^\mathrm{S}, \zeta, \eta_\omega} \\ &\sum_{\omega=1}^{N_\Omega} \pi_\omega \sum_{t=1}^{N_\mathrm{T}} \Bigg(\sum_{b \in B_t} \lambda_{bt\omega}^\mathrm{B} P_{b\omega}^\mathrm{B} d_t + \lambda_{t\omega}^\mathrm{P} E_{t\omega}^\mathrm{P} \\ &\qquad\qquad + \sum_{n=1}^{N_\mathrm{N}} S_n^\mathrm{S} E_{nt\omega}^\mathrm{S} \Bigg) \\ &\qquad\qquad + \beta \left(\zeta + \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\Omega} \pi_\omega \eta_\omega \right) \end{aligned} \tag{9.20}$$

subject to

$$P_b^{\mathrm{B,min}} s_{b\omega} \leq P_{b\omega}^{\mathrm{B}} \leq P_b^{\mathrm{B,max}} s_{b\omega}, \ \forall b, \forall \omega \tag{9.21}$$

$$0 \leq E_{1t\omega}^{\mathrm{S}} \leq \bar{E}_1^{\mathrm{S}}, \ \forall t, \forall \omega \tag{9.22}$$

$$0 \leq E_{nt\omega}^{\mathrm{S}} \leq \bar{E}_n^{\mathrm{S}} - \bar{E}_{n-1}^{\mathrm{S}}, \ n = 2, \ldots, N_{\mathrm{N}}, \forall t, \forall \omega \tag{9.23}$$

$$\sum_{n=1}^{N_{\mathrm{N}}} E_{nt\omega}^{\mathrm{S}} + E_{t\omega}^{\mathrm{P}} + \sum_{b \in B_t} P_{b\omega}^{\mathrm{B}} d_t = E_t^{\mathrm{D}} - E_t^{\mathrm{PC}}, \forall t, \forall \omega \tag{9.24}$$

$$E_{t\omega}^{\mathrm{P}} \geq -\sum_{n=1}^{N_{\mathrm{N}}} E_{nt\omega}^{\mathrm{S}}, \ \forall t, \forall \omega \tag{9.25}$$

$$\begin{aligned} P_{b\omega}^{\mathrm{B}} = P_{b\omega+1}^{\mathrm{B}}, \ \forall b, \omega = 1, \ldots, N_{\Omega} - 1, \\ \text{if } A(\omega, K_b) = 1 \end{aligned} \tag{9.26}$$

$$\begin{aligned} \sum_{t=1}^{N_{\mathrm{T}}} \Bigg(\sum_{b \in B_t} \lambda_{bt\omega}^{\mathrm{B}} P_{b\omega}^{\mathrm{B}} d_t + \lambda_{t\omega}^{\mathrm{P}} E_{t\omega}^{\mathrm{P}} \\ + \sum_{n=1}^{N_{\mathrm{N}}} S_n^{\mathrm{S}} E_{nt\omega}^{\mathrm{S}} \Bigg) - \zeta \leq \eta_{\omega}, \ \forall \omega \end{aligned} \tag{9.27}$$

$$\eta_{\omega} \geq 0, \ \forall \omega \tag{9.28}$$

$$s_{b\omega} \in \{0, 1\}, \ \forall b \in B, \forall \omega. \tag{9.29}$$

The objective function (9.20) consists of two parts. The first term corresponds to the expected procurement cost formulated in (9.16). The second term is equal to the CVaR multiplied by a weighting factor β. The factor β is used to model the tradeoff between the expected cost and the risk of cost variability, which is measured by the CVaR. Typically, a solution with low expected cost involves a high risk of experiencing large costs in some scenarios. Solving the problem (9.20)-(9.29) for different values of β yields a set of solutions with different values of expected cost and CVaR. Consequently, it is possible to represent the expected cost versus the CVaR or the cost standard deviation for different values of β, thus obtaining the so-called efficient frontier.

Constraints (9.21) bound the power purchased from bilateral contracts. Constraints (9.22)-(9.23) model the production block bounds of the self-production facility owned by the consumer. Constraints (9.24) impose the energy balance for each period and each scenario. Constraints (9.25) avoid the arbitrage between bilateral contracts and the pool. Constraints (9.26) set the non-anticipativity conditions. Constraints (9.27) are used to compute the CVaR. Finally, constraints (9.28) and (9.29) constitute variable declarations.

9.5 Consumer Example

In this section, we test and illustrate the formulation previously presented using a detailed example. We consider a planning horizon of four hourly periods, in which contract decisions are made at the beginning of each period. We consider six contracts: two contracts spanning the four periods, and one contract for each single period. The parameters defining each contract are provided in Table 9.1.

Table 9.1 Consumer example: bilateral contract data

Contract #	Usage Period	λ_b^{B} (\$/MWh)	$P_b^{\mathrm{B,max}}$ (MW)	$P_b^{\mathrm{B,min}}$ (MW)
1	1-4	48.5	75	20
2	1-4	50.0	75	20
3	1	49.5	50	20
4	2	51.0	50	20
5	3	49.0	50	20
6	4	50.0	50	20

The electricity demand of the consumer is considered to be deterministic and therefore, defined as independent of pool price scenarios. Table 9.2 shows the electricity demand of the consumer for each period.

Table 9.2 Consumer example: electricity demand (MWh)

Period #			
1	2	3	4
200	250	225	275

The pool price is modeled using a set of 16 equiprobable scenarios. Table 9.3 provides the pool price for each period and scenario. The corresponding scenario-tree structure is depicted in Fig. 9.5. Note that each node is a decision point and each branch represents the realization of a pool price value. For instance, the two branches leaving the root node represent the two possible realizations of the pool price in period 1, namely, 44 and \$50/MWh.

Table 9.3 Consumer example: pool price scenarios ($/MWh)

	Period #					Period #			
Scenario #	1	2	3	4	Scenario #	1	2	3	4
1	44	47	45	45	9	50	52	50	51
2	44	47	45	47	10	50	52	50	55
3	44	47	46	47	11	50	52	51	52
4	44	47	46	49	12	50	52	51	53
5	44	44	42	42	13	50	49	47	49
6	44	44	42	45	14	50	49	47	50
7	44	44	44	45	15	50	49	48	50
8	44	44	44	46	16	50	49	48	53

The consumer owns a 100-MW self-production unit with a linear production cost equal to $45/MWh.

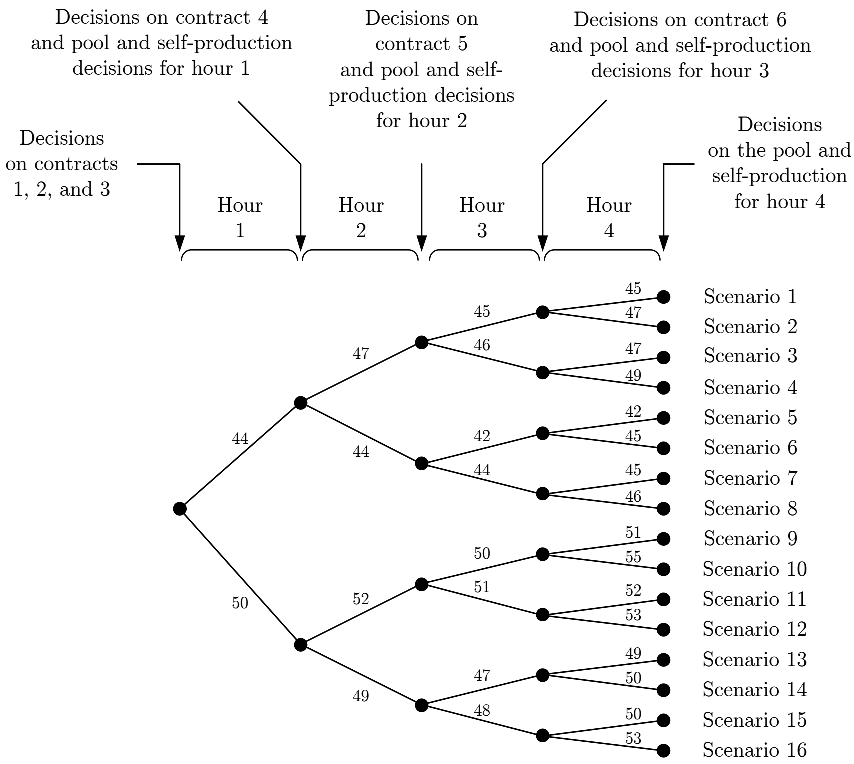

Fig. 9.5 Consumer example: scenario tree

Problem (9.20)-(9.29) is set up considering a confidence level α equal to 0.95. The resulting problem includes 480 constraints, 338 real variables, and 96 binary variables. A GAMS code to solve this problem is provided in Section A.7 of Appendix A.

The optimal solutions in terms of expected cost, cost standard deviation and CVaR are provided in Table 9.4. The solution obtained for $\beta = 0$ attains the lowest expected cost and the highest risk, measured in terms of the standard deviation and the CVaR of the procurement cost. For $\beta = 10$ the expected cost increases 1.3% to attain a reduction of 52.5% in the standard deviation of the cost and 1.7% in the CVaR.

Table 9.4 Consumer example: results

β	Expected cost (\$)	Cost standard deviation (\$)	CVaR (\$)
0	44,073.44	1776.70	46,525.00
1	44,217.97	1340.87	46,012.50
2	44,581.25	908.23	45,750.00
10	44,643.75	843.47	45,737.50

The efficient frontier in terms of the expected cost and the cost standard deviation is depicted in Fig. 9.6. Low-risk solutions (low standard deviation) are related to high expected costs. Observe that the solution obtained for $\beta = 1$ is particularly relevant, because it reduces the cost standard deviation by 24.5%, leading to a small increase of 0.3% in the expected cost.

The amounts of power purchased from bilateral contracts 1 to 3, which are decided at the beginning of the planning horizon, are listed in Table 9.5. For $\beta = 0$ no contracts are signed because expected pool prices are smaller than the reference contract prices (check Tables 9.1 and 9.3). It should be noted that the larger the parameter β, the lower the variability of the cost and the higher the quantity of the power contracted. These results together with those presented in Table 9.4 indicate that the risk associated with the volatility of the procurement cost is efficiently hedged by the use of bilateral contracts.

Table 9.5 Consumer example: purchases from contracts 1, 2, and 3 (MW)

Contract #	β			
	0	1	2	10
1	0	75	75	75
2	0	0	75	75
3	0	0	0	50

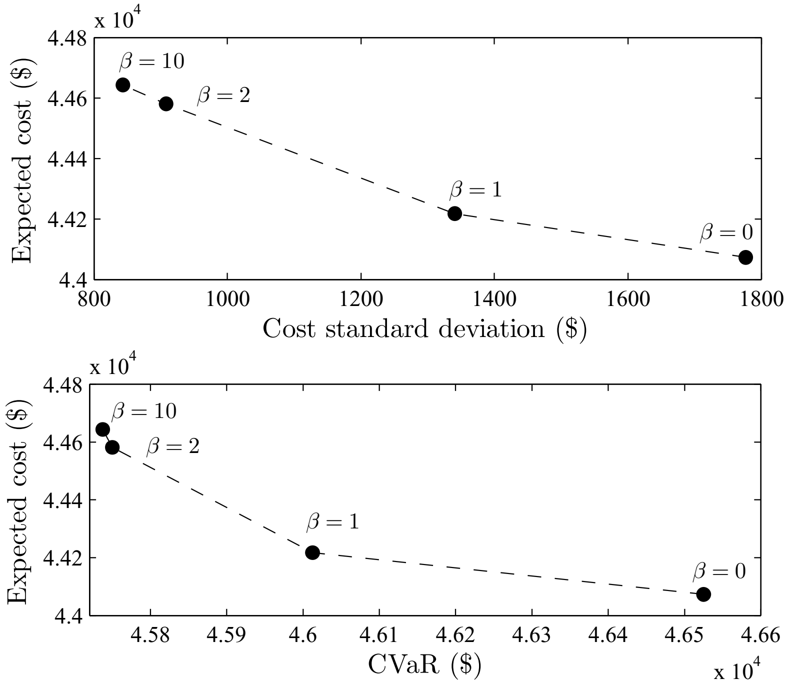

Fig. 9.6 Consumer example: efficient frontier

Table 9.6 Consumer example: expected electricity procurement

β	Expected contract purchases (MWh)	Expected pool purchases (MWh)	Expected pool sales (MWh)	Expected self-production (MWh)
0	31.25	656.25	0.00	262.50
1	356.25	331.25	0.00	262.50
2	656.25	118.75	87.50	262.50
10	706.25	93.75	112.50	262.50

The expected electricity procurement is provided in Table 9.6. Note that the purchases from bilateral contracts grow as the parameter β increases and the risk aversion of the consumer becomes more significant in consequence. The opposite effect is observed for the pool purchases, i.e., the expected pool purchases decrease as the value of β increases and as a result, the expected pool sales increase with the value of β. For large values of this parameter, the demand is mostly supplied by contracts, and the energy generated through the self-production unit is sold in the pool if the pool price is higher than the production cost.

Finally, it is worth mentioning that the self-production remains constant for different values of β. The self-production unit operates in those periods and scenarios where pool prices are higher than the self-production cost. If the electricity demand of the consumer is not supplied in these periods and scenarios by bilateral contracts, the self-production unit is used to supply part of the electricity demand of the consumer. On the other hand, if the electricity demand is entirely satisfied by bilateral contracts, the electricity obtained from the self-production unit is sold in the pool. This behavior can be observed in Tables 9.7 and 9.8.

Table 9.7 Consumer example: electricity procurement for $\beta = 0$

	Period #											
	1			2			3			4		
Scenario #	C	P	SP	C	P	SP	C	P	SP	C	P	SP
1	0	200	0	0	150	100	0	225	0	0	275	0
2	0	200	0	0	150	100	0	225	0	0	175	100
3	0	200	0	0	150	100	0	125	100	0	175	100
4	0	200	0	0	150	100	0	125	100	0	175	100
5	0	200	0	0	250	0	0	225	0	0	275	0
6	0	200	0	0	250	0	0	225	0	0	275	0
7	0	200	0	0	250	0	0	225	0	0	275	0
8	0	200	0	0	250	0	0	225	0	0	175	100
9	0	100	100	0	150	100	50	75	100	50	125	100
10	0	100	100	0	150	100	50	75	100	50	125	100
11	0	100	100	0	150	100	50	75	100	50	125	100
12	0	100	100	0	150	100	50	75	100	50	125	100
13	0	100	100	0	150	100	0	125	100	0	175	100
14	0	100	100	0	150	100	0	125	100	0	175	100
15	0	100	100	0	150	100	0	125	100	50	125	100
16	0	100	100	0	150	100	0	125	100	50	125	100

C: Energy purchased from bilateral contracts (MWh), P: Energy traded in the pool (MWh) and SP: Self-produced energy (MWh).

Table 9.7 provides the energy traded through bilateral contracts and the pool, as well as the self-production in each period and scenario for $\beta = 0$. In this case, neither the first-period nor the second-period contract is signed. The self-production unit operates in all periods and scenarios where pool prices are higher than the production cost. During period 1, the expected demand (200 MWh) is completely supplied by purchases from the pool in scenarios 1-8. The consumer uses solely the pool because the pool price is smaller than the self-production cost (Table 9.3). However, in scenarios 9-16, the self-production cost is smaller than the pool price, and the unit is operated then at its full capacity. A similar behavior can be observed in the remaining periods. In periods 3 and 4, one-period contracts are signed in those scenarios where pool prices are higher than the reference contract prices. This occurs in scenarios 9-12 of period 3, and in scenarios 9-12 and 15-16 of period 4.

Table 9.8 Consumer example: electricity procurement for $\beta = 10$

	Period #											
	1			2			3			4		
Scenario #	C	P	SP	C	P	SP	C	P	SP	C	P	SP
1	200	0	0	150	0	100	150	75	0	150	125	0
2	200	0	0	150	0	100	150	75	0	150	25	100
3	200	0	0	150	0	100	150	-25	100	150	25	100
4	200	0	0	150	0	100	150	-25	100	150	25	100
5	200	0	0	150	100	0	150	75	0	150	125	0
6	200	0	0	150	100	0	150	75	0	150	125	0
7	200	0	0	150	100	0	150	75	0	150	125	0
8	200	0	0	150	100	0	150	75	0	150	25	100
9	200	-100	100	200	-50	100	200	-75	100	200	-25	100
10	200	-100	100	200	-50	100	200	-75	100	200	-25	100
11	200	-100	100	200	-50	100	200	-75	100	200	-25	100
12	200	-100	100	200	-50	100	200	-75	100	200	-25	100
13	200	-100	100	200	-50	100	150	-25	100	150	25	100
14	200	-100	100	200	-50	100	150	-25	100	150	25	100
15	200	-100	100	200	-50	100	150	-25	100	200	-25	100
16	200	-100	100	200	-50	100	150	-25	100	200	-25	100

C: Energy purchased from bilateral contracts (MWh), P: Energy traded in the pool (MWh) and SP: Self-produced energy (MWh).

Complementarily, Table 9.8 provides the purchases of energy in each period and scenario for $\beta = 10$. In this case, contracts 1-3 are signed. For instance, observe that in period 1 the entire consumer demand is supplied through bilateral contracts. Note that in scenarios 9-16 of period 1 the energy produced by the self-production unit is sold in the pool, which is due to the fact that pool prices are higher than the production cost in these scenarios. A similar behavior is observed in the remaining periods.

9.6 Consumer Case Study

The performance of the proposed decision-making model is further illustrated through a realistic case study. The planning horizon considered corresponds to four weeks (28 days). Each day is divided into 3 periods, namely *valley*, *shoulder*, and *peak*, and each period comprises eight hours. Thus, the planning horizon includes 84 time periods. The sets of hours in each time period are

$$\begin{aligned} valley &= \{1, 2, 3, 4, 5, 6, 7, 8\} \\ shoulder &= \{9, 10, 15, 16, 17, 18, 23, 24\} \\ peak &= \{11, 12, 13, 14, 19, 20, 21, 22\}. \end{aligned}$$

The following ARIMA model is used to generate pool prices during the planning horizon,

$$\begin{aligned}&\left(1-\phi_1 B^1-\phi_2 B^2\right)\left(1-\phi_3 B^3-\phi_{24} B^{24}-\phi_{27} B^{27}\right)\\&\left(1-\phi_{21} B^{21}-\phi_{42} B^{42}\right)\left(1-B^3\right)\left(1-B^{21}\right)\ln\left(\lambda_t^{\mathrm{P}}\right)=\\&\qquad\left(1-\theta_3 B^3\right)\left(1-\theta_{21} B^{21}\right)\epsilon_t,\ \forall t.\end{aligned} \tag{9.30}$$

Model (9.30) is obtained using historical price data of the Iberian Electricity Market spanning one year. The error term ϵ_t is assumed to be a white noise process. The standard deviation of ϵ_t is considered constant and equal to 2.42 ln($/MWh). The parameters of (9.30) are provided in Table 9.9.

Table 9.9 Consumer case study: parameters of the ARIMA model (9.30)

ϕ_1	=	0.7257	ϕ_{21}	=	-0.1134	ϕ_{42}	=	-0.0976
ϕ_2	=	-0.1603	ϕ_{24}	=	0.0661	θ_3	=	0.7491
ϕ_3	=	0.1206	ϕ_{27}	=	-0.0852	θ_{21}	=	0.8304

In order to represent the pool prices, 2500 equiprobable scenarios are generated. This set of scenarios corresponds to a tree with a $25\times10\times5\times2$ scenario structure, i.e., from the root node 25 branches leave, from each second-stage node 10 branches leave, and so on.

We assume that the consumer has a precise knowledge of its future demand, whose forecast is plotted in Fig. 9.7.

This consumer has the possibility of signing four monthly bilateral contracts of the type described in Subsection 9.3.1. Two of these monthly contracts can be used in all periods (base contracts) and the other two can only be used in peak periods of weekdays (peak contracts), spanning from 8am to 8pm (12 hours). In addition, one base and one peak weekly contracts are considered for each of the four weeks. The characteristics of these contracts are provided in Table 9.10.

The consumer owns a 100-MW self-production unit. Since each time period comprises 8 hours, the maximum energy that can be produced in each period is equal to 800 MWh. Table 9.11 lists the data for the piecewise linear cost function of this unit that involves three blocks.

After applying a suitable scenario-reduction technique (the Variant 2 of the algorithm explained in Subsection 3.3.3 of Chapter 3), the resulting tree contains just 200 scenarios. Fig. 9.8 shows the 200 scenarios of pool prices for the 84 periods considered. The bold line in Fig. 9.8 indicates the expected pool prices obtained by the ARIMA model (9.30).

The resulting problem, which includes 90,691 constraints, 72,202 real variables, and 2400 binary variables, is solved for different values of the weighting factor β. Fig. 9.9 provides two plots depicting the expected cost versus both

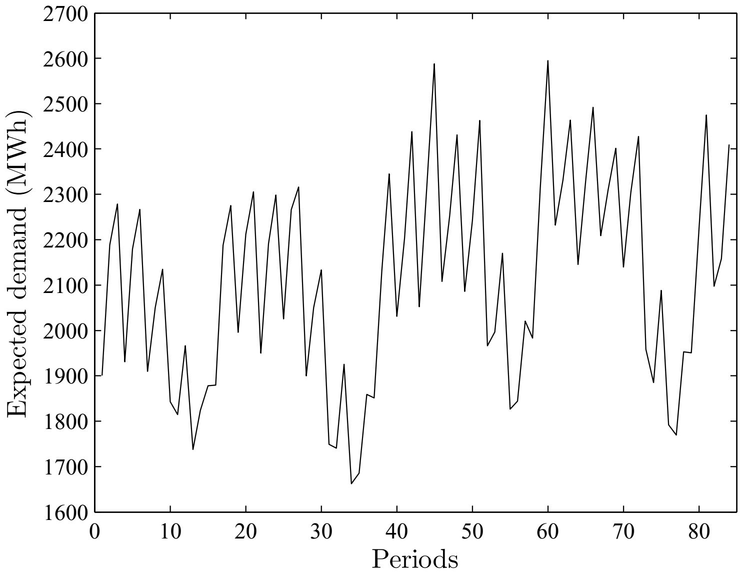

Fig. 9.7 Consumer case study: expected demand

Table 9.10 Consumer case study: bilateral contract data

Contract #	Usage period	Contract type	λ_b^{B} (\$/MWh)	$P_b^{\mathrm{B,max}}$ (MW)	$P_b^{\mathrm{B,min}}$ (MW)
1	4 weeks	Base	54.4	30	15
2	4 weeks	Peak	68.1	20	10
3	4 weeks	Base	55.4	30	15
4	4 weeks	Peak	64.1	20	10
5	1st week	Base	45.9	50	25
6	1st week	Peak	56.0	40	20
7	2nd week	Base	56.7	50	25
8	2nd week	Peak	63.2	40	20
9	3rd week	Base	57.4	50	25
10	3rd week	Peak	64.1	40	20
11	4th week	Base	57.8	50	25
12	4th week	Peak	64.4	40	20

the cost standard deviation and the CVaR for different values of β (efficient frontier). The expected cost ranges from \$7.19 million if risk is ignored ($\beta = 0$), to \$7.48 million if risk is accounted for ($\beta = 5$). The cost standard deviation and the CVaR vary from \$1.03 million to \$0.81 million, and from \$9.56 million to \$9.27 million, respectively. Thus, a 3% reduction in the CVaR results in a 4% increase in the expected cost. This result is important since it provides relevant information to the decision maker. Note that if the risk taken is high (β close to 0), the increase in expected cost resulting from

Table 9.11 Consumer case study: production cost data of the self-production unit

Block 1 (MWh)	Block 2 (MWh)	Block 3 (MWh)	Cost 1 ($/MWh)	Cost 2 ($/MWh)	Cost 3 ($/MWh)
320	600	800	33	36	39

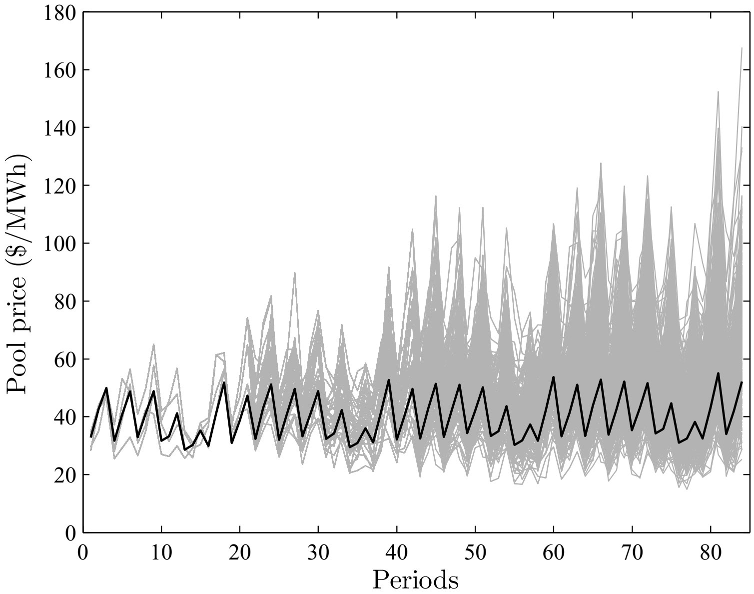

Fig. 9.8 Consumer case study: pool price scenarios

reducing risk is small. However, if the risk taken is low (high value of β), any additional decrease in risk causes a high increase in the expected cost.

Table 9.12 provides the power contracted at the beginning of the planning horizon through monthly and first-week contracts. As expected, the contracted power increases as the value of β increases. This fact indicates that bilateral contracts are used by the consumer as a risk-hedging tool. Note that for $\beta = 5$ all monthly and first-week contracts are signed.

Fig. 9.10 shows the expected electricity procurement for each period in the first week for $\beta = 5$. Note that representing both the energy procurement and the bilateral contract purchases during the first week is appropriate because first-week decisions are actually implemented by the consumer. In contrast to first-week decisions, the energy procurement and the bilateral contracts signed for the second and following weeks can be modified once actual values of pool prices and demands within the first week are known and the decision model is run again.

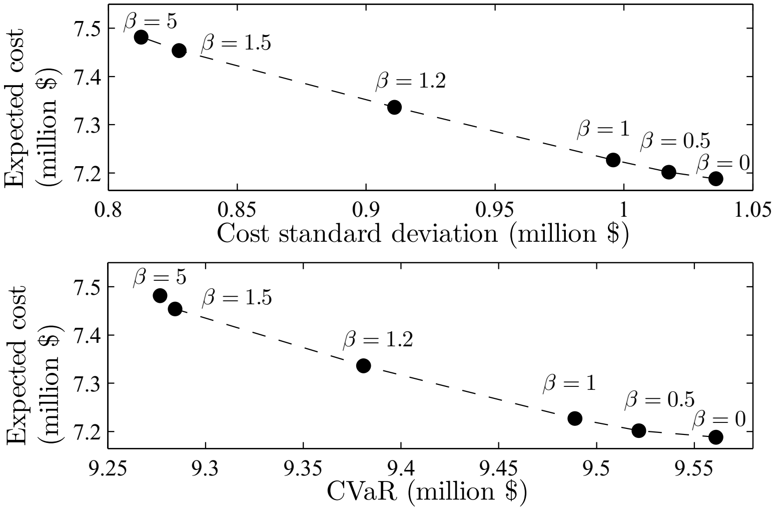

Fig. 9.9 Consumer case study: efficient frontier

Table 9.12 Consumer case study: power purchased from monthly and first-week contracts (MW)

	β					
Contract #	0	0.5	1	1.2	1.5	5
1	0	0	0	30	30	30
2	0	0	20	20	20	20
3	0	0	0	0	30	30
4	0	20	20	20	20	20
5	0	0	0	0	0	50
6	0	0	40	40	40	40

Each bar in Fig. 9.10 represents the expected demand in each period, and shows the fraction of the expected demand that is supplied by each available source (i.e., bilateral contracts, the pool, and the self-production unit). Monthly contracts 1-4 and weekly contracts 5 and 6 are used during the first week. However, the energy contracted is insufficient to supply the total demand of the consumer. Therefore, the pool and the self-production unit are also used. The self-produced energy corresponds mostly to peak periods. For example, in periods 3, 9, and 18 the entire consumer demand is supplied using bilateral contracts and the self-production unit. Plots similar to that in Fig. 9.10 can be obtained for other values of β.

Fig. 9.11 illustrates the expected electricity procurement during both the first week and the whole planning horizon for different values of β. The energy represented in each sector of the pie charts is the expected value over all

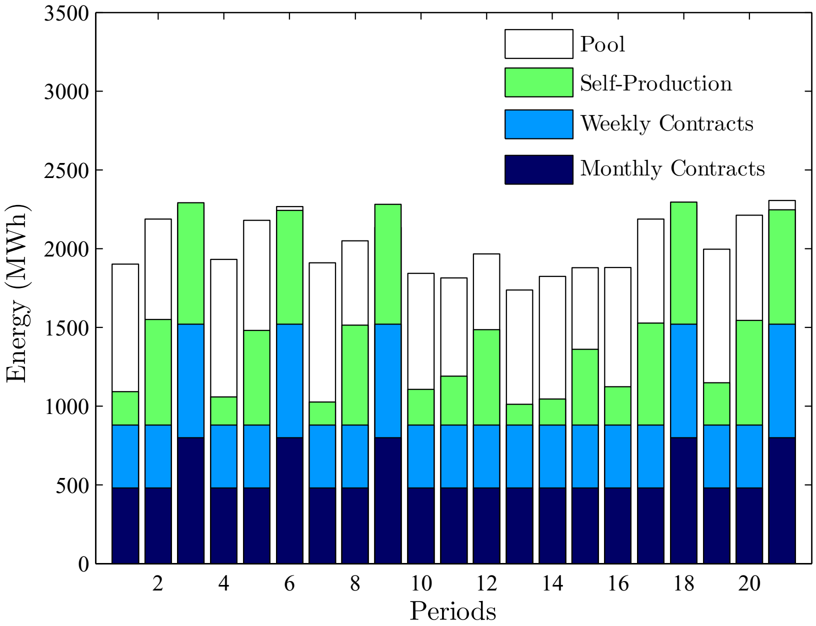

Fig. 9.10 Consumer case study: expected electricity procurement for the first week and $\beta = 5$

scenarios. Again, the share of bilateral contracts increases considerably with β in order to hedge against exposure to high pool prices. For $\beta = 0$ the consumer mostly relies on the pool and no monthly contract is signed. The consumer procures only 2% of its expected demand through weekly contracts. Note that self-production supplies approximately 26% of the expected electricity procurement of the consumer.

For $\beta = 1.2$, one base monthly contract and two peak monthly contracts are signed, representing 16% of the expected demand during the first week. The purchases from weekly contracts over the whole planning horizon increase from 2% to 4%. Note that the purchases from the pool are reduced by 17%.

For $\beta = 5$, 51% of the energy consumed in the first week is procured exclusively through bilateral contracts. In contrast, the pool satisfies only 26% of the energy consumed in the first week.

It is worth noting that the proportion of energy self-produced over the four weeks by the consumer remains steady at 26% for different values of β.

Fig. 9.12 depicts the cost distributions for $\beta = 0$ and $\beta = 5$ using adjusted probability density functions. It can be observed that the interval spanned

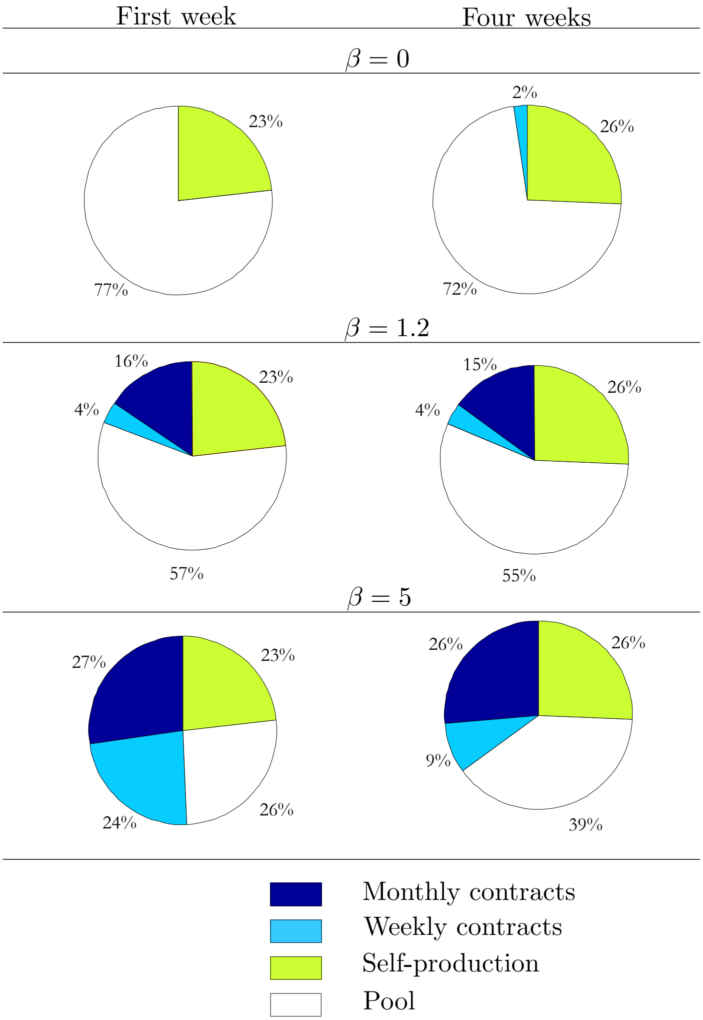

Fig. 9.11 Consumer case study: expected electricity procurement

by the cost for $\beta = 0$ is wider than that resulting for $\beta = 5$. This indicates that the volatility of the procurement cost is higher in the risk-neutral case ($\beta = 0$). The cost in the worst scenario is reduced from \$11.4 million ($\beta = 0$) to \$10.2 million ($\beta = 5$), which means a 10.5% reduction.

Finally, it is relevant to note that using currently available optimization software and current personal computer technology, the required computer time to solve problems similar to the one discussed above is compatible with the considered decision-time framework.

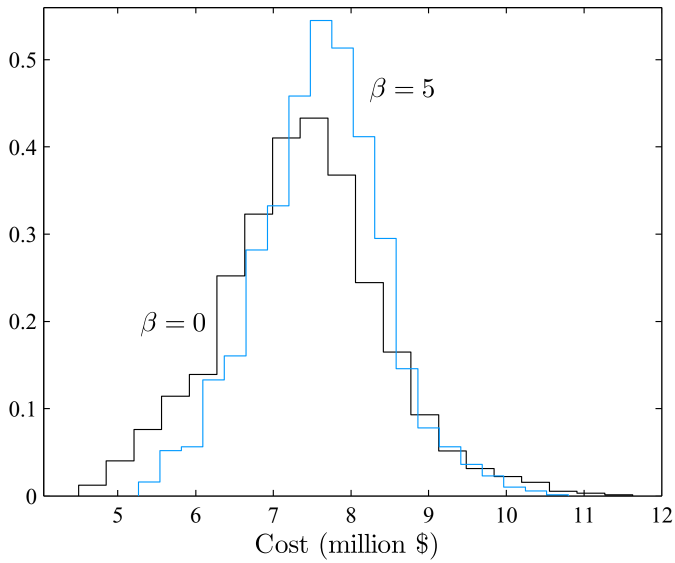

Fig. 9.12 Consumer case study: adjusted pdfs of the cost for $\beta = 0$ and $\beta = 5$

9.7 Summary and Conclusions

This chapter provides a methodology for modeling the energy procurement problem of a large consumer within a medium-term planning horizon. This methodology is based on a multi-stage stochastic programming problem, where risk aversion is modeled through the CVaR. This stochastic programming problem is converted into an equivalent deterministic mathematical programming problem that can be efficiently solved.

The pool price is modeled as a stochastic process that is represented using a set of scenarios. The number of scenarios is trimmed down through an appropriate scenario-reduction algorithm so that the resulting mixed-integer linear programming problem is solvable using a commercial solver.

The proposed methodology is relevant for a consumer to make informed decisions on electricity procurement. Moreover, it allows resolving the tradeoff between minimum expected cost and minimum risk of cost variability.

The conclusions below are worth mentioning:

1. Bilateral contracting and the availability of a self-production unit allow a consumer to efficiently hedge the risk of cost volatility.
2. Procuring energy through bilateral contracts results generally in a higher average cost but a lower risk of cost variability.

3. The higher average cost is the result of not being able to buy in the pool when prices are significantly low because the required energy has been already bought through a bilateral contract.
4. Multi-stage stochastic programming constitutes an appropriate modeling framework to make decisions on electricity procurement under uncertainty.
5. The considered multi-stage stochastic model translates into a tractable mixed-integer linear programming problem, which can be solved using available optimization software.
6. Appropriate scenario-generation and scenario-reduction techniques allow constructing tractable models that can be solved for realistic problems in reasonable computing times.
7. Electricity procurement decisions in futures market can be derived using models similar to the ones developed in this chapter for bilateral contracting.

Finally, it should be noted that the proposed model can be easily extended to take into account additional features of the considered markets (other types of contracts, different decision frameworks, different pool structures, etc.), and other constraints of the consumers (minimum load requirements, procurement differentiation by season, etc.).

9.8 Notation

The notation used throughout this chapter is stated below for quick reference.

Indices and Numbers:

b	Index of bilateral contracts.
k	Index of stages, running from 1 to N_{K}.
n	Index of blocks in the piecewise linear production cost function of the self-production unit, running from 1 to N_{N}.
t	Index of time periods, running from 1 to N_{T}.
ω	Index of scenarios, running from 1 to N_{Ω}.

Real Variables:

$C^{\mathrm{B}}_{b\omega}$	Cost of purchasing from bilateral contract b in scenario ω (\$).

$C_{t\omega}^{\mathrm{P}}$ Cost of purchasing from the pool in period t and scenario ω (\$).
$C_{t\omega}^{\mathrm{S}}$ Production cost incurred by the self-production unit in period t and scenario ω (\$).
$E_{t\omega}^{\mathrm{P}}$ Energy traded in the pool in period t and scenario ω (MWh).
$E_{nt\omega}^{\mathrm{S}}$ Energy produced in block n of the piecewise linear production cost function of the self-production unit in period t and scenario ω (MWh).
$E_{t\omega}^{\mathrm{S}}$ Energy produced by the self-production unit in period t and scenario ω (MWh).
$P_{b\omega}^{\mathrm{B}}$ Power purchased from bilateral contract b in scenario ω (MW). Bounded by $P_b^{\mathrm{B,min}}$ and $P_b^{\mathrm{B,max}}$.
ζ Auxiliary variable used to calculate the CVaR (\$).
η_ω Auxiliary variable related to scenario ω used to calculate the CVaR (\$).

Binary Variables:

$s_{b\omega}$ 0/1 variable that is equal to 1 if bilateral contract b is selected in scenario ω, and 0 otherwise.

Random Variables:

ϵ_t Error term in the ARIMA model of pool prices (ln(\$/MWh)).
λ_t^{P} Random variable modeling the price of energy in the pool in period t (\$/MWh). $\lambda_{t\omega}^{\mathrm{P}}$ represents the realization of this random variable in scenario ω.

Constants:

$\boldsymbol{A}$ Non-anticipativity matrix of the consumer problem. Element $A(\omega, k)$ is equal to 1 if scenarios ω and $\omega+1$ are equal up to stage k, being 0 otherwise.
d_t Duration of period t (h).
$\bar{E}_t^{\mathrm{D}}$ Expected demand of the consumer (MWh).
E_t^{PC} Energy contracted prior to the beginning of the planning horizon that is used in period t (MWh).

$\bar{E}_n^{\mathrm{S}}$	Upper bound of the energy associated with block n of the piecewise linear production cost of the self-production unit (MWh).
K_b	Stage at which decisions on bilateral contract b are made.
S_n^{S}	Slope of block n of the piecewise linear production cost of the self-production unit (\$/MWh).
α	Confidence level used in the calculation of the CVaR.
β	Weighting factor.
γ_e	Parameter representing the relationship between the pool price and the demand of client group e.
θ_u	Parameter related to delay u used in the ARIMA model of pool prices.
λ_b^{B}	Reference price of electricity for bilateral contract b (\$/MWh).
$\lambda_{bt\omega}^{\mathrm{B}}$	Price of electricity for bilateral contract b in period t and scenario ω (\$/MWh).
π_ω	Probability of occurrence of scenario ω.
ϕ_u	Parameter related to delay u used in the ARIMA model of pool prices.

Sets:

B	Set of bilateral contracts.
B_t	Set of bilateral contracts available in period t.
T_b	Set of time periods over which contract b is defined.
Ω	Set of scenarios.

Others:

B^u	Backshift operator related to delay u.

9.9 Exercises

Exercise 9.1. Draw and describe a decision framework (similar to that in Fig. 9.1) for a consumer that makes purchasing decisions on a weekly basis and that, in addition to the pool, has bilateral contracts available for the whole week, for weekdays, and for the weekend.

Exercise 9.2. Draw and describe a scenario tree (similar to that in Fig. 9.2) for the consumer in Exercise 9.1 above. Consider that each stage of this problem is adequately described by means of 3 scenarios.

Exercise 9.3. Describe and illustrate graphically (with a figure similar to 9.3) a contract spanning the 168 hours of a week.

Exercise 9.4. Write non-anticipativity conditions for the consumer problem in Exercises 9.1 and 9.2 above.

Exercise 9.5. Reformulate problem (9.20)-(9.29) to include forward contracts of the type considered in Chapter 7.

Exercise 9.6. Modify the example in Section 9.5 to include forward contracts. Using the GAMS code in Section A.7 of Appendix A, report results involving a variety of forward contracts.

Exercise 9.7. Modify the example in Section 9.5 by considering a reduced number of bilateral contracts. Compare the results obtained with those reported in Section 9.5.

Exercise 9.8. Modify the example in Section 9.5 by changing the number of considered scenarios. How do the obtained results compare with those reported in Section 9.5?

Exercise 9.9. Discuss the advantages of using the CVaR in problem (9.20)-(9.29) over other risk measures and particularly over the profit variance.

Exercise 9.10. Modify problem (9.20)-(9.29) to include demand scenarios as in the retailer problem studied in Chapter 8.

Chapter 10
Market Clearing Considering Equipment Failures

10.1 Introduction

Electricity markets generally comprise a great variety of mechanisms allowing market agents to carry out energy transactions in accordance with their market strategy. For instance, in futures markets, agents can engage in long-term energy transactions seeking to reduce the risk level on profit/utility variability by hedging against the financial losses derived from pool price uncertainty.

Most of the current electricity markets, if not all, include a day-ahead market, as well as adjustment markets and balancing markets, through which the real-time balance between generation and demand is ensured. Of the three, the *day-ahead market* is generally the most important one in terms of trading volume, and as such, is often seen as a reference by the rest of trading structures. The set of these sequentially arranged short-term trading floors is usually referred to as the *pool*. Further details are provided in Chapter 1.

In the pool, the Market Operator receives energy offers from producers and energy bids from consumers, and determines, for every hour, the power production of every producer, the consumption level of every consumer, and the price at which every producer/consumer is paid/charged for its energy production/consumption, [130]. The aim is to maximize the net social welfare. This process is known as *market clearing* and constitutes the leitmotif of this chapter.

A general overview of an electricity market, including a description of its structure, agents and sequential organization is provided in Chapter 1. The technical literature is rich in references on methods to clear electricity markets. Consider those presented in [4,16,17,53,57,58,121,130,134,137,138] as a representative sample of them.

The rest of this chapter is organized as follows. Section 10.2 starts from the fundamentals of a market-clearing procedure and culminates in the concept of *stochastic security-constrained market clearing*. It also includes several def-

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initions concerning power system security, which are of special importance to understand the development carried out in this chapter. Section 10.3 introduces the notion of *expected load not served* as a stochastic metric to quantify the damage caused to the system by a set of credible disturbances. In Section 10.4, the mathematical formulation of the stochastic security-constrained market clearing is presented. An illustrative example and a case study are analyzed in depth in Sections 10.6 and 10.7, respectively. Finally, Section 10.8 summarizes the chapter and provides some relevant conclusions.

10.2 Stochastic Security-Constrained Market Clearing

10.2.1 Main Features

The market-clearing procedure applied by the Market Operator should produce a feasible dispatch, be transparent and fair, and maximize the net social welfare. The data required by the market-clearing procedure is the offering and bidding information provided by producers and consumers, respectively. The economic offering information presented by any producer for every hour generally consists of a set of energy blocks and their corresponding prices. Likewise, a generator can complement this simple economic data by declaring a start-up price and by providing some information on its inter-temporal technical limitations, i.e., its minimum up and down times, and ramping capabilities.

Market-clearing procedures currently implemented in electricity markets are significantly different from one another. As an example, the market clearing algorithm used in the market of the Iberian Peninsula, [106], consists of a sequence of single-period auctions, where inter-temporal constraints are enforced ex post using simple heuristic rules. The independent consideration of every hour may result in inefficient solutions. Therefore, in this market, the simplicity and transparency of the clearing algorithm prevails over the optimality of the resulting energy dispatch. In general, this criterion is predominant in European electricity markets. For the sake of optimality and maximum economic efficiency, a multi-period market-clearing procedure is required in order to take into consideration inter-temporal constraints within a global optimization problem, [4]. The electricity markets established in the East Coast of the US (e.g., [100, 111]) go beyond this idea, and their functioning is presently based on a much more sophisticated market-clearing procedure in which even network constraints are taken into account through a *dc load flow model*.

10.2.2 Introducing Security Constraints

Day-ahead markets are cleared well in advance of the scheduling horizon, e.g., 14-38 hours ahead. As a result, the outcomes of the market-clearing procedures are subject to uncertainty as for their final implementation due to unforseen disturbances occurring in real-time such as generator and line outages or sudden variations of load. The ability of a power system to survive a credible set of these disturbances (also named as *contingencies*) without extensive load disconnections is usually understood as *system security*, [108].

In terms of security, two types of actions can be distinguished. Preventive security actions are designed to put the system in a state such that in the event of a credible disturbance enough resources are available to fast perform the corrective security actions that guarantee the normal operation of the system once the disturbance has taken place.

The reserves are the services traded in the market to physically materialize the preventive and corrective security actions by means of their scheduling and their subsequent deployment, respectively. In practice, scheduling reserves means operating the system at less than its full capacity, while the deployment of reserves usually translates into the redispatch of units previously committed in the day-ahead market, the voluntary curtailment of specific loads, and/or even the quick start-up of extra generating units to palliate unexpected shortages of energy supply.

Nowadays, there exist mainly two paradigms concerning how reserves should be traded and scheduled in electricity markets, [58]. The first one starts from the premise that energy and reserve are weakly coupled services, and consequently, reserves are scheduled in a series of auctions run once energy schedules have been determined in a separate market, [134, 137]. This is the current practice in the wholesale electricity market of the Iberian Peninsula, for example. As the aforementioned premise is generally false, sequential market-clearing procedures may provide solutions too far away from the optimal operating point, or even infeasible. On the contrary, the other paradigm is based on simultaneous market-clearing procedures, which assume that energy and reserves are in fact strongly interrelated and allow avoiding the need for *ad hoc* operator intervention, uneconomical out-of-merit operation, the start-up of extra units, as well as unnecessary load shedding, [121, 138]. This approach is usually associated with the US East Coast electricity markets and is very often called *security-constrained market clearing*.

10.2.3 Setting Reserve Requirements: Deterministic and Probabilistic Approaches

Another key issue is how to establish the amount of reserves required in a power system. In order to deal with this problem, two different approaches, deterministic or probabilistic, can be applied. The deterministic approach, [108], is normally based on rule-of-thumb type criteria such as scheduling enough reserve to cover the loss of the largest production unit (known as the N-1 criterion), or to supply a portion of the hourly demand, or even a combination of both. Nevertheless, note that these preventive security measures do not account for the probability of occurrence of the contingencies they are supposed to cover. Therefore, if these probabilities are high enough, the amount of scheduled reserves may be insufficient. Conversely, if the probabilities of occurrence are sufficiently low, reserve requirements may be overestimated. In the probabilistic approach, [12, 41, 53, 68, 70], however, reserve needs are set by considering both the probability of occurrence of every possible failure event and the damage caused to the system by each of them. This normally requires, on the one hand, the evaluation of a huge number of possible disturbances, which may result in computational intractability, and on the other, the acquisition of the statistical failure data necessary to compute outage probabilities; data usually exposed to important uncertainties. In order to circumvent these drawbacks, some authors advocate the use of a *hybrid* deterministic/probabilistic approach [16, 17, 57] that basically consists in considering just a reduced set of the most credible contingencies to estimate the probabilities of failures and expected outcomes.

A market-clearing procedure combining a deterministic/probabilistic hybrid approach to set the reserve needs and the implementation of preventive security actions constitutes a so-called *stochastic security-constrained market-clearing procedure*, which is the subject of study in this chapter.

10.2.4 Solution Algorithm

From a mathematical point of view, numerous techniques have been proposed in the technical literature to solve market-clearing procedures, namely, heuristics, dynamic programming, Lagrangian relaxation, mixed-integer linear programming, etc. Among these methodologies, Lagrangian relaxation used to be one of the most employed approaches because of its capability of solving large-scale problems. Its main flaw is that, as a consequence of the non-convex nature of the market-clearing problems we are dealing with, heuristic algorithms are required to find feasible solutions, which may be suboptimal. On the contrary, mixed-integer linear programming, [99], guarantees convergence to the optimal solution in a finite number of steps while providing a flexible and accurate modeling framework. Furthermore, efficient mixed-integer algorithms such as the branch-and-cut technique are currently available within comercial solvers for optimization. For all these reasons, mixed-integer linear programming is the solution methodology selected to solve the stochastic security constrained market-clearing procedure presented in this chapter. Needless to say, the price to pay is the need to formulate a *mixed-integer linear* model.

10.2.5 Security-Related Definitions

Some relevant security-related concepts are explicitly defined below for clarity.

Reserve is the capability of a power system above and below its current operating point to act in response of a contingency within a specified time interval. A contingency is the unexpected failure or outage of a system component, such as a generator or transmission line, as well as significant, unexpected load variations.

Reserve consists of spinning and non-spinning reserves. Spinning reserve is the portion of reserve comprising the generation synchronized to the system and the load voluntarily interruptible. Non-spinning reserve is the generating reserve not synchronized to the system but capable of serving demand within a given time frame. Therefore, its use entails the start-up of the production unit providing this service.

Spinning reserve can be up- or down-going. A generating unit providing up-/down-spinning reserve must be ready to increase/decrease its production if required. In the case of a consumer, the criterion is the opposite. A load committed to provide up-/down-spinning reserve can be ordered to decrease/increase its consumption if necessary.

Finally, reserve deployment is the collection of security corrective actions through which the reserve capacity is transformed into actual energy production or consumption in the aftermath of a contingency.

10.3 Stochastic Security Metrics

In this section, the keystone characterizing the electricity market-clearing with stochastic security is introduced.

A *security metric* is a measure quantifying the level of security of a power system, usually expressed in terms of the load involuntarily shed as a result of a disturbance. A *stochastic* security metric should take into account the probability of occurrence of the contingencies capable of threatening the system security.

In the context of a stochastic security-constrained market clearing, stochastic security metrics are intended to appraise the risk that load shedding incidents take place following the occurrence of contingencies.

10.3.1 Probabilistic Metrics

In the technical literature, [12, 57], various stochastic metrics have been proposed, namely, the *expected load not served* (ELNS), the *loss-of-load probability* (LOLP), and the *loss-of-load expectation* (LOLE).

The ELNS measures the average amount of energy not supplied to loads as a result of load shedding events. As its own name indicates, the expected load not served is a weighted average energy value accounting for both the probability of contingencies and the damage that these contingencies cause to the system in terms of lost load.

The LOLP is computed as the probability that failure events lead to load shedding. As opposed to the ELNS, however, the loss-of-load probability is a dimensionless number that does not provide any information on the *severity* of the disturbance, i.e., on the energy not supplied. This lack of a clear physical meaning makes the LOLP a less intuitive metric to work with by system operators.

The LOLE assesses the expected number of hours during which loss-of-load events could happen. As the LOLP, the loss-of-load expectation fails to provide an estimation of the damage done to the system by contingencies.

From a mathematical viewpoint, both the LOLE and the LOLP require the use of binary variables to be considered within a mixed-integer linear programming problem, [16, 57]. On the contrary, the ELNS can be expressed linearly, without binary variables, as follows:

$$\mathrm{ELNS}_j = \sum_{\omega=1}^{N_\Omega} \sum_{t=1}^{N_\mathrm{T}} \pi_\omega L^{\mathrm{shed}}_{jt\omega} d_t, \tag{10.1}$$

where ELNS_j is the expected load not served to consumer j during the scheduling horizon, π_ω is the probability of occurrence of scenario ω, d_t is the duration of time period t, and $L^{\mathrm{shed}}_{jt\omega}$ is the load shedding imposed on consumer j in period t and scenario w, given by:

$$L^{\mathrm{shed}}_{jt\omega} = \sum_{j:(j,n)\in M_L} L^{\mathrm{C}}_{jt\omega} - \sum_{i:(i,n)\in M_G(t,\omega)} P^{\mathrm{G}}_{it\omega} + \sum_{r:(n,r)\in \Lambda(t,\omega)} f_{t\omega}(n,r), \quad \forall n, \forall t, \forall \omega. \tag{10.2}$$

In equation (10.2), $P^{\mathrm{G}}_{it\omega}$, $L^{\mathrm{C}}_{jt\omega}$, and $f_{t\omega}(n,r)$ are, respectively, the power output of unit i, the power consumed by load j, and the power flow through line (n,r), in period t and scenario ω.

Logically, the involuntary load shedding cannot be negative, nor greater than the actual load:

$$0 \leq L^{\text{shed}}_{jt\omega} \leq L^{\text{C}}_{jt\omega}, \quad \forall j, \forall t, \forall \omega. \tag{10.3}$$

A *scenario* ω describes a plausible realization of the stochastic processes involved in an optimization problem (see Chapter 2 for further details). In the case we are dealing with in this chapter, each possible scenario ω is determined by a contingency k occurring at a specific time period τ. The scenario in which no contingency happens during the whole scheduling horizon defines the pre-contingency state of the system and, hereafter, is denoted as scenario $\omega = 0$.

For all the reasons previously indicated, the expected load not served is usually deemed as a superior security assessment index, and as such, it is the stochastic security metric considered in the subsequent market-clearing formulation.

The following illustrative example is aimed to clarify the probabilistic metrics presented above.

Illustrative Example 10.1 (Scenario, LOLP, LOLE, ELNS). Consider the small system shown in Fig. 10.1. This system comprises three buses, three lines and three generating units. Line resistances are null and thus, active power losses are disregarded. Line reactances are all 0.13 p.u. The capacities of all three lines are the same and equal to 55 MW.

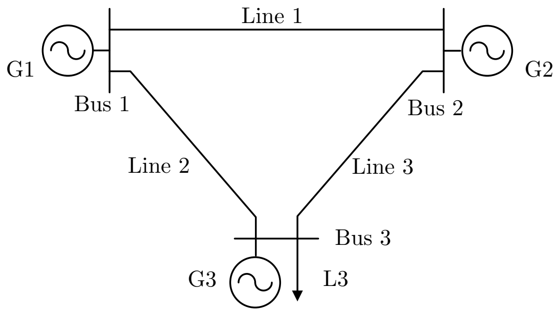


Fig. 10.1 Three-node system

Table 10.1 Hourly demand at node 3

Period #	1	2	3	4
L^{S}_{jt} (MW)	30	80	110	40

A load, with the hourly demand profile shown in Table 10.1, is located at node 3. As inferred from this table, the scheduling horizon comprises four hourly periods.

Assume that generating units 1 and 2, as well as transmission lines 1 and 3, are 100% reliable, i.e., they never fail. On the contrary, unit 3 and line 2 are known to fail with given probabilities.

By indexing the failures of generator 3 and line 2 with $k = 1, 2$, respectively, and considering that any one of these single contingencies can occur during any one of the four hours of the scheduling horizon, $\tau = 1, \ldots, 4$, and that the joint failure of unit 3 and line 2 is not possible, the following nine scenarios ω can be defined:

Scenario $\omega = 0$, in which no contingency happens during the entire scheduling horizon.
Scenarios $\omega = 1, \ldots, 4$, defined by pairs $(k, \tau) = (1, \tau)$ with $\tau = \omega$, where the failure of unit 3 takes place in time period τ.
Scenarios $\omega = 5, \ldots, 8$, defined by pairs $(k, \tau) = (2, \tau)$ with $\tau = \omega - 4$, where the outage of line 2 occurs in time period τ.

Let be the probabilities π_ω associated with the scenarios above the following:

$\pi_\omega = 0.05; \omega = 1, \ldots, 4$
$\pi_\omega = 0.01; \omega = 5, \ldots, 8$
$\pi_0 = 1 - \sum_{\omega=1}^{8} \pi_\omega = 1 - 4 \times 0.05 - 4 \times 0.01 = 0.76.$

Suppose next that the load shedding actions reported in Table 10.2 are expected from the application of a certain scheduling program settled in the electricity market under each one of these scenarios. Then, the LOLP, LOLE

Table 10.2 Involuntary load shedding (MWh)

Period #	Scenario #								
	0	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0	0	0
2	0	1	1	0	0	0	3	0	0
3	0	0	5	5	0	0	0	5	0
4	0	0	0	0	0	0	0	0	0

and ELNS allied to that scheduling program are:

LOLP $= \pi_1 + \pi_2 + \pi_3 + \pi_6 + \pi_7 = 3 \times 0.05 + 2 \times 0.01 = 0.17$
LOLE $= 1\text{h}\ \pi_1 + 2\text{h}\ \pi_2 + 1\text{h}\ \pi_3 + 1\text{h}\ \pi_6 + 1\text{h}\ \pi_7 = 4\text{h} \times 0.05 + 2\text{h} \times 0.01 = 0.22$ h
ELNS $= 1$ MWh $\pi_1 + (1 + 5)$ MWh $\pi_2 + 5$ MWh $\pi_3 + 3$ MWh $\pi_6 + 5$ MWh $\pi_7 = 12$ MWh $\times 0.05 + 8$ MWh $\times 0.01 = 0.68$ MWh.

□

10.3.2 Security Criteria Based on the ELNS

There exist three manners to enforce a stochastic security criterion based on the ELNS, [16, 17]. The first one consists in including a set of constrains limiting the magnitude of the ELNS either node-by-node, area-by-area or over the whole system. Likewise, these constrains can be imposed over the entire scheduling horizon, or just over a subset of time periods (e.g., hour-by-hour, peak hours, off-peak hours, etc.). The second way is to add a penalty function, increasing monotonically with ELNS, to the objective function of the market-clearing problem through the so-called *value of lost load* [85, 138, 139] provided by each consumer. The third criterion is just a combination of the two previous ones.

Imposing bounds on the ELNS has three drawbacks, namely:

1. The upper bound limits would have to be specified by a regulatory agency by means of an arduous and non-discriminatory process calling for transparency and equity.
2. In cases where there is no enough reserve or sufficient transmission capacity, the specified upper bounds on ELNS may not be satisfied, leading to an infeasible market-clearing problem.
3. In the absence of an economic penalty of the ELNS, the upper bound would be reached as a way to minimize the social cost of scheduling and deploying reserves albeit there may be sufficient reserve resources and transmission capacity available to avoid load shedding events.

For simplicity and in order to circumvent these snags, in the market-clearing formulation presented in the following section, we just use a stochastic security criterion solely based on an economic penalty of the ELNS within the objective function. Therefore, according to this criterion, the system operator is willing to allow some involuntary load shedding if the contingencies responsible for the loss-of-load events occur with sufficiently low probabilities and the corresponding increase in expected social cost is small.

10.4 Market-Clearing Formulation

In order to account for the equipment-failure uncertainty affecting the operation of a power system, the market clearing is formulated as a two-stage stochastic programming problem, [14], where the first stage represents the electricity market, its constraints and rules, and the second stage represents the power system, its operation and physical limitations. Thus, the variables pertaining to the first stage correspond to the market decisions (preventive actions), which consider the plausible realizations of the uncertain events and their possible impact on the real-time operation of the power system (corrective actions).

The structuring of the stochastic programming problem into two stages is widely used in electricity markets where decisions must be usually made before the uncertain events materialize (e.g., one day in advance in a day-ahead market).

10.4.1 Assumptions

For the market-clearing formulation considered and for the sake of tractability, the following assumptions are made:

1. Only single contingencies can occur over the length of the scheduling horizon, where such a contingency may be a simultaneous compounded failure. That is, non-simultaneous, sequential contingencies are disregarded. Therefore, a *hybrid* deterministic/probabilistic approach for setting reserve requirements is considered.
2. The proposed market-clearing formulation is intended to determine the reserve services required to economically face every possible, but single, contingency occurring at a specific time period τ of the scheduling horizon. This means that once a contingency has taken place, the system operator should employ a model analogous to the one presented here so as to reassess reserve needs.
3. The costs of the energy corresponding to the deployed reserves by units or loads are assumed to be equal, respectively, to their energy offer costs $(\lambda_{\mathrm{G}it}(m))$ or to their utilities $(\lambda_{\mathrm{L}jt})$. Nevertheless, any other criterion about the cost of the deployment of reserve could be straightforwardly implemented within the presented formulation with a minimum of changes.
4. A linear representation of the network is considered through a dc load flow model.

10.4.2 Variables

Variables that do not depend on any particular scenario realization in each time period are:

1. Start-up and shut-down plan of each generating unit $\left(u_{it}, C_{it}^{\mathrm{SU}}\right)$.
2. Scheduled power output for each generating unit $\left(P_{it}^{\mathrm{S}}\right)$.
3. Scheduled power consumption for each load $\left(L_{jt}^{\mathrm{S}}\right)$.
4. The down/up spinning reserve scheduled for each generating unit $\left(R_{it}^{\mathrm{D}}, R_{it}^{\mathrm{U}}\right)$.
5. The non-spinning reserve scheduled for each off-line generating unit $\left(R_{it}^{\mathrm{NS}}\right)$.
6. The reserve scheduled for each load $\left(R_{jt}^{\mathrm{D}}, R_{jt}^{\mathrm{U}}\right)$.

These variables are *first-stage variables* and constitute *here-and-now* decisions, i.e., decisions pertaining to the market clearing that are made before the realization of any one of the scenarios (contingencies).

Variables pertaining to each particular scenario in each time period are:

1. The changes and adjustments in the start-up and shut-down plan of each generating unit required for making feasible the deployment of non-spinning reserves $\left(v_{it\omega}, C^{\mathrm{A}}_{it\omega}\right)$.
2. The deployment of down/up spinning reserve by each generating unit with scheduled spinning reserve $\left(r^{\mathrm{D}}_{it\omega}, r^{\mathrm{U}}_{it\omega}\right)$.
3. The deployment of non-spinning reserve by each generating unit with scheduled non-spinning reserve $\left(r^{\mathrm{NS}}_{it\omega}\right)$.
4. The deployment of reserve by each load with scheduled reserve $\left(r^{\mathrm{D}}_{jt\omega}, r^{\mathrm{U}}_{jt\omega}\right)$.
5. The involuntarily load shed by each consumer $\left(L^{\mathrm{shed}}_{jt\omega}\right)$.
6. The network related variables, namely, angle of every node $(\delta_{nt\omega})$, flow through each line $(f_{t\omega}(n,r))$ and power injection at every node (actual power productions, $P^{\mathrm{G}}_{it\omega}$, and consumptions, $L^{\mathrm{C}}_{jt\omega}$).

These variables, which do depend on the particular scenarios, are related to the actual operation of the power system, are *second-stage variables* and constitute *wait-and-see* decisions.

10.4.3 Structure

Within the two-stage stochastic programming model described in this chapter, four parts can be distinguished. The objective function to be minimized, included in Subsection 10.4.4, groups separately those terms representing the costs pertaining to the electricity market and those representing the costs incurred during the real-time operation of the power system. Three sets of constraints are included: the constraints modeling the market and its rules (in Subsection 10.4.5), those modeling the actual operation of the power system once a particular scenario ω is realized (in Subsection 10.4.6), and finally, the linking constraints (in Subsection 10.4.7), which bind the market decisions to the actual operation of the power system through the deployment of reserves provided by units and loads.

The resulting model is formulated as a large-scale mixed-integer linear programming problem that can be solved using commercially available software, [141].

10.4.4 Objective function

The objective function below to be minimized is the expected cost (EC):

$$
\begin{aligned}
\mathrm{EC} = \sum_{t=1}^{N_\mathrm{T}} \mathrm{EC}_t = & \\
= & \sum_{t=1}^{N_\mathrm{T}} \sum_{i=1}^{N_\mathrm{G}} C_{it}^{\mathrm{SU}} \\
& + \sum_{t=1}^{N_\mathrm{T}} d_t \left[\sum_{i=1}^{N_\mathrm{G}} \left(C_{it}^{R^\mathrm{U}} R_{it}^\mathrm{U} + C_{it}^{R^\mathrm{D}} R_{it}^\mathrm{D} + C_{it}^{R^\mathrm{NS}} R_{it}^\mathrm{NS} \right) \right. \\
& \qquad \left. + \sum_{j=1}^{N_\mathrm{L}} \left(C_{jt}^{R^\mathrm{U}} R_{jt}^\mathrm{U} + C_{jt}^{R^\mathrm{D}} R_{jt}^\mathrm{D} \right) \right] \\
& + \sum_{\omega=0}^{N_\Omega} \pi_\omega \left\{ \sum_{t=1}^{N_\mathrm{T}} \sum_{i=1}^{N_\mathrm{G}} C_{it\omega}^{\mathrm{A}} \right. \\
& \qquad + \sum_{t=1}^{N_\mathrm{T}} d_t \left[\sum_{i=1}^{N_\mathrm{G}} \sum_{m=1}^{N_{\mathrm{O}it}} \lambda_{\mathrm{G}it}(m) p_{\mathrm{G}it\omega}(m) - \sum_{j=1}^{N_\mathrm{L}} \lambda_{\mathrm{L}jt} L_{jt\omega}^{\mathrm{C}} \right. \\
& \qquad\qquad \left. \left. + \sum_{j=1}^{N_\mathrm{L}} V_{jt}^{\mathrm{LOL}} L_{jt\omega}^{\mathrm{shed}} \right] \right\}, \qquad (10.4)
\end{aligned}
$$

where EC_t is the expected cost of the system in period t and $C_{it}^{R^\mathrm{U}}$, $C_{it}^{R^\mathrm{D}}$, and $C_{it}^{R^\mathrm{NS}}$ are, respectively, the offer costs of the up-, down-, and non-spinning reserves of unit i in period t. Likewise, $C_{jt}^{R^\mathrm{U}}$ and $C_{jt}^{R^\mathrm{D}}$ represent, respectively, the offer costs of the up- and down-spinning reserves of load j in period t.

The objective function above includes six terms (from line 2 to line 7 in equation (10.4)):

1. The start-up offer cost of the generating units,
2. the offer cost of contracting up/down spinning and non-spinning reserves from generating units,
3. the offer cost of contracting up/down spinning reserves from loads,
4. the cost resulting from the adjustments of the start-up and shut-down plan of generating units previously established by the market clearing,
5. the energy offer cost of the generating units minus the demand utility,
6. the cost of the load shedding.

The energy offer cost of the generating units is introduced as a second-stage cost since its computation requires the knowledge of the availability of production units throughout the scheduling horizon, and therefore, this cost is dependent on the scenario realization.

10.4.5 Electricity Market Constraints

These constraints model the functioning of the market, and consequently, involve first-stage variables, i.e., variables that do not depend on scenario ω. The functioning of the market specifically relies on the equilibrium between generation offers and demand bids. This equilibrium must be attained in each time period and is mathematically expressed as

$$\sum_{i=1}^{N_{\mathrm{G}}} P_{it}^{\mathrm{S}} = \sum_{j=1}^{N_{\mathrm{L}}} L_{jt}^{\mathrm{S}}, \ \ \forall t. \tag{10.5}$$

The participation of producers and consumers in the electricity market is restricted to their production and consumption limits. This is stated by means of constraints (10.6) and (10.7), respectively,

$$P_i^{\min}\, u_{it} \le P_{it}^{\mathrm{S}} \le P_i^{\max}\, u_{it}, \quad \forall i, \forall t \tag{10.6}$$

$$L_{jt}^{\mathrm{S,\,min}} \le L_{jt}^{\mathrm{S}} \le L_{jt}^{\mathrm{S,\,max}}, \ \ \forall j, \forall t. \tag{10.7}$$

$L_{jt}^{\mathrm{S,\,min}}$ and $L_{jt}^{\mathrm{S,\,max}}$ are parameters submitted as part of the demand-side bids. In the case of inelastic demand, the two limits are equal, that is, $L_{jt}^{\mathrm{S,\,min}} = L_{jt}^{\mathrm{S,\,max}} = L_{jt}^{\mathrm{S}}$.

Next, we introduce a small example to illustrate the constraints modeling the energy market equilibrium.

Illustrative Example 10.2 (Market equilibrium). Consider the three-node system described in Illustrative Example 10.1. Data for the generators are given in Table 10.3. For simplicity, the energy and reserve offers of the

Table 10.3 Illustrative Example 10.2: generator data

Generator i	1	2	3
$P_i^{\min}$ (MW)	10	10	10
$P_i^{\max}$ (MW)	100	100	50
$\lambda_{it}^{\mathrm{SU}}$ ($)	100	100	100
λ_{Git} ($/MWh)	30	40	20
$C_{it}^{R^{\mathrm{U}}}$ ($/MWh)	5.0	7.0	8.0
$C_{it}^{R^{\mathrm{D}}}$ ($/MWh)	5.0	7.0	8.0
$C_{it}^{R^{\mathrm{NS}}}$ ($/MWh)	4.5	5.5	7.0

units are assumed to be the same throughout the scheduling horizon consisting of four periods. Each generator offers a single block of energy at a marginal cost of λ_{Git} and a start-up cost offer of $\lambda_{it}^{\mathrm{SU}}$. The maximum amounts of reserve services offered by generators are the largest possible, i.e., $P_i^{\max} - P_i^{\min}$

for up and down-spinning reserves and $P_i^{\max}$ for non-spinning reserves. On the other hand, the load offers to sell up- and down-spinning reserves at \$70/MWh. For this, the load accepts to be partially curtailed in each period up to 10% of its demand (see Table 10.1 in Illustrative Example 10.1) in order to provide additional up-spinning reserve. Likewise, it commits itself to increase its consumption by the same quantity so as to contribute to down-spinning reserve when required. Beyond these limits, the value of lost load is assumed to be \$1000/MWh.

As the only load in this system, located at node 3, is inelastic (i.e., the consumer is willing to pay anything for the energy), the market equilibrium boils down to

$$\begin{aligned}
&P_{11}^{\mathrm{S}} + P_{21}^{\mathrm{S}} + P_{31}^{\mathrm{S}} = 30 \quad && (t = 1) \\
&P_{12}^{\mathrm{S}} + P_{22}^{\mathrm{S}} + P_{32}^{\mathrm{S}} = 80 \quad && (t = 2) \\
&P_{13}^{\mathrm{S}} + P_{23}^{\mathrm{S}} + P_{33}^{\mathrm{S}} = 110 \quad && (t = 3) \\
&P_{14}^{\mathrm{S}} + P_{24}^{\mathrm{S}} + P_{34}^{\mathrm{S}} = 40 \quad && (t = 4).
\end{aligned}$$

The amount of energy placed by generating units in the electricity market is bounded by their production limits,

$$\begin{aligned}
10\ u_{1t} \le P_{1t}^{\mathrm{S}} \le 100\ u_{1t}, \forall t \\
10\ u_{2t} \le P_{2t}^{\mathrm{S}} \le 100\ u_{2t}, \forall t \\
10\ u_{3t} \le P_{3t}^{\mathrm{S}} \le 50\ u_{3t}, \forall t.
\end{aligned}$$

□

According to the rationale behind a security-constrained market-clearing procedure, in which energy and reserve are simultaneously dispatched, the scheduling of reserves proceeds as follows: if generating unit i has been committed to the electricity supply in period t (i.e., $u_{it} = 1$), then it can also provide spinning reserve (upward or downward) for that period as stated through equations (10.8) and (10.9):

$$0 \le R_{it}^{\mathrm{U}} \le R_{it}^{\mathrm{U,\,max}}\ u_{it}, \quad \forall i, \forall t \tag{10.8}$$

$$0 \le R_{it}^{\mathrm{D}} \le R_{it}^{\mathrm{D,\,max}}\ u_{it}, \quad \forall i, \forall t. \tag{10.9}$$

Otherwise ($u_{it} = 0$), generating unit i can still contribute to cover the need for non-spinning reserves in the system as expressed by equation (10.10):

$$0 \le R_{it}^{\mathrm{NS}} \le R_{it}^{\mathrm{NS,\,max}}\ (1 - u_{it}), \quad \forall i, \forall t. \tag{10.10}$$

On the other hand, since loads are assumed to be always connected to the grid, they can only provide spinning reserve:

$$0 \leq R^{\mathrm{U}}_{jt} \leq R^{\mathrm{U,\ max}}_{jt}, \quad \forall j, \forall t \tag{10.11}$$

$$0 \leq R^{\mathrm{D}}_{jt} \leq R^{\mathrm{D,\ max}}_{jt}, \quad \forall j, \forall t. \tag{10.12}$$

We exemplify below the constraints modeling the scheduling of reserves.

Illustrative Example 10.3 (Reserve scheduling constraints). Considering the reserve offers indicated in Illustrative Example 10.2 and submitted to the electricity market by the three generators of the three-node system detailed in Illustrative Example 10.1, equations (10.8)–(10.12) become as stated below:

1. Generation-side reserves.

   a. Spinning:

$$\begin{aligned} \text{Unit 1:} \qquad & 0 \leq R^{\mathrm{U}}_{1t} \leq 90\ u_{1t}, \quad \forall t \\ & 0 \leq R^{\mathrm{D}}_{1t} \leq 90\ u_{1t}, \quad \forall t. \end{aligned}$$

$$\begin{aligned} \text{Unit 2:} \qquad & 0 \leq R^{\mathrm{U}}_{2t} \leq 90\ u_{2t}, \quad \forall t \\ & 0 \leq R^{\mathrm{D}}_{2t} \leq 90\ u_{2t}, \quad \forall t. \end{aligned}$$

$$\begin{aligned} \text{Unit 3:} \qquad & 0 \leq R^{\mathrm{U}}_{3t} \leq 40\ u_{3t}, \quad \forall t \\ & 0 \leq R^{\mathrm{D}}_{3t} \leq 40\ u_{3t}, \quad \forall t. \end{aligned}$$

   b. Non-spinning:

$$\begin{aligned} \text{Unit 1:} \qquad & 0 \leq R^{\mathrm{NS}}_{1t} \leq 100\ (1 - u_{1t}), \quad \forall t \\ \text{Unit 2:} \qquad & 0 \leq R^{\mathrm{NS}}_{2t} \leq 100\ (1 - u_{2t}), \quad \forall t \\ \text{Unit 3:} \qquad & 0 \leq R^{\mathrm{NS}}_{3t} \leq 50\ (1 - u_{3t}), \quad \forall t. \end{aligned}$$

2. Demand-side reserves.

$$\text{Period 1:} \qquad 0 \le R^{\mathrm{U}}_{31} \le 3$$
$$0 \le R^{\mathrm{D}}_{31} \le 3.$$

$$\text{Period 2:} \qquad 0 \le R^{\mathrm{U}}_{32} \le 8$$
$$0 \le R^{\mathrm{D}}_{32} \le 8.$$

$$\text{Period 3:} \qquad 0 \le R^{\mathrm{U}}_{33} \le 11$$
$$0 \le R^{\mathrm{D}}_{33} \le 11.$$

$$\text{Period 4:} \qquad 0 \le R^{\mathrm{U}}_{34} \le 4$$
$$0 \le R^{\mathrm{D}}_{34} \le 4.$$

□

Finally, the scheduling program must also account for the cost associated with the expected start-ups required for its practical implementation. This cost is modeled through the constraints below

$$C^{\mathrm{SU}}_{it} \ge \lambda^{\mathrm{SU}}_{it}(u_{it} - u_{i,t-1}), \quad \forall i, \forall t \tag{10.13}$$
$$C^{\mathrm{SU}}_{it} \ge 0, \quad \forall i, \forall t. \tag{10.14}$$

Specifically, equation (10.14) states that the start-up cost of a generating unit must be either zero or positive. The nature of the optimization problem seeking to minimize the expected cost incurred by the market participants guarantees that variable C^{SU}_{it}, which represents the start-up cost of generating unit i in period t, takes the minimum value that is possible. In particular, this value is equal to $\lambda^{\mathrm{SU}}_{it}$ if $u_{it} = 1$ and $u_{i,t-1} = 0$, i.e., if unit i is started up in period t, and 0 otherwise. Further details on the modeling of the start-up cost of a generating unit are provided in [3, 20].

The following example is intended to illustrate equations (10.13) and (10.14).

Illustrative Example 10.4 (First-stage start-up costs). Consider generating unit 1 of the three-node system in Illustrative Example 10.1 and the first two periods of the scheduling horizon. Its start-up offer cost is equal to \$100 in both periods (see Table 10.3 in Illustrative Example 10.2), and therefore, constraints (10.13) and (10.14) render:

$$\text{Period 1:} \qquad C^{\mathrm{SU}}_{11} \ge 100(u_{11} - u_{10})$$
$$C^{\mathrm{SU}}_{11} \ge 0,$$

where u_{10} is a binary variable indicating the initial status (on/off) of unit 1.

$$\text{Period 2:} \qquad C_{12}^{\mathrm{SU}} \geq 100(u_{12} - u_{11})$$
$$C_{12}^{\mathrm{SU}} \geq 0.$$

□

Note that the rules governing the functioning of the market, setting the balance point, and consequently, determining the scheduling program, can be much more complex than those presented above. For instance, the market balance can be enforced per node by incorporating network considerations into the market formulation. However, there is no need for this, and neither advantage nor improvement is expected to result from a market representation with a higher degree of sophistication. This is so because the stochastic programming framework, on which this market-clearing paradigm is built, guarantees the enforcement of all relevant constraints. This means that market decisions are made accounting for the physical limitations imposed by the subsequent real-time operation of the power system, mathematically modeled via the second-stage of the stochastic programming problem. In other words, the scheduling program satisfying the market balance (10.5) envisions all *plausible paths* to be followed by system variables during the real-time operation, and thus, constitutes an optimal pre-positioning of generators and loads to respond to transmission capacity problems, operation limits, and/or failure events, which is the issue of interest in this chapter.

10.4.6 Real-Time Operating Constraints

The constraints considered in this subsection model the real-time operation of the power system and therefore, involve second-stage variables, i.e., variables that do depend on scenarios ω.

The actual operation of a power system revolves around the continuous power balance between production and consumption, which is formulated as

$$\sum_{i:(i,n)\in M_{\mathrm{G}}(t,\omega)} P_{it\omega}^{\mathrm{G}} - \sum_{j:(j,n)\in M_{\mathrm{L}}} \left(L_{jt\omega}^{\mathrm{C}} - L_{jt\omega}^{\mathrm{shed}}\right) - \sum_{r:(n,r)\in \Lambda(t,\omega)} f_{t\omega}(n,r) = 0, \quad \forall n, \forall t, \forall \omega, \tag{10.15}$$

where $f_{t\omega}(n,r)$ is the power flow through line (n,r) from n to r, which, in accordance with a dc load flow model, is given by

$$f_{t\omega}(n,r) = B(n,r)\big(\delta_{nt\omega} - \delta_{rt\omega}\big), \quad \forall (n,r) \in \Lambda(t,\omega), \forall t, \forall \omega. \tag{10.16}$$

Note that the set of transmission lines, Λ, and the mapping of the set of generating units into the set of nodes, M_{G}, are contingent on scenario ω, since contingencies often translate into modifications of the system topology and network configuration.

Power balance constraints are illustrated below using a small example.

Illustrative Example 10.5 (Power balance constraints). Consider the three-node system and the nine scenarios ω introduced in Illustrative Example 10.1. The power balance at each node of the power system must be constantly satisfied on a real-time basis, no matter which the outcomes of the uncertain variables are. This implies specifically that equations (10.15) must be fulfilled for every possible state of the system, each determined by a plausible mode of failure.

In this illustrative example, three states of the three-node power system indicated below can be distinguished.

1. The pre-contingency state:

   This is the state free of failures, represented in Fig. 10.2.

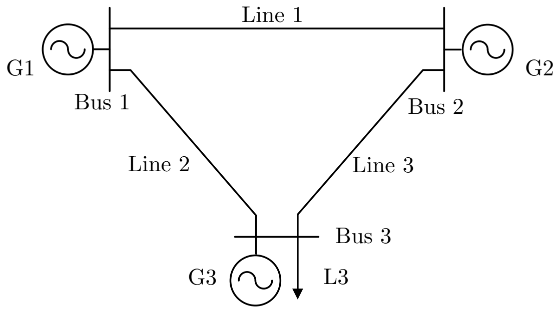


Fig. 10.2 Illustrative Example 10.5: state of the system free of failures

The power balance equations (10.15) for this system state are

$$\begin{aligned} &\text{Node 1:} \quad P^{\mathrm{G}}_{1t} = f_t(1,2) + f_t(1,3) \\ &\text{Node 2:} \quad P^{\mathrm{G}}_{2t} = f_t(2,1) + f_t(2,3) \\ &\text{Node 3:} \quad P^{\mathrm{G}}_{3t} + L^{\mathrm{shed}}_{3t} - L^{\mathrm{C}}_{3t} = f_t(3,1) + f_t(3,2). \end{aligned}$$

These equality constraints should be enforced for every scenario $\omega = 0, \ldots, 8$, in every time period $t < \tau$, with τ being the time interval where the contingency characterizing the scenario takes place.

2. System state determined by the outage of unit 3:

This state is defined by the loss of unit 3, as illustrated in Fig. 10.3.

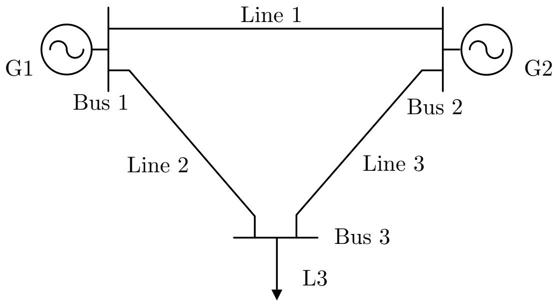


Fig. 10.3 Illustrative Example 10.5: state of the system without unit 3

The power balance equations (10.15) in this case are

$$\begin{aligned}
&\text{Node 1:} \quad P^{\mathrm{G}}_{1t} = f_t(1,2) + f_t(1,3)\\
&\text{Node 2:} \quad P^{\mathrm{G}}_{2t} = f_t(2,1) + f_t(2,3)\\
&\text{Node 3:} \quad L^{\mathrm{shed}}_{3t} - L^{\mathrm{C}}_{3t} = f_t(3,1) + f_t(3,2).
\end{aligned}$$

These equalities are applied to scenarios $\omega = 1, \ldots, 4$, for time periods $t \geq \tau$.

3. System state defined by the outage of line 2:

   The network configuration resulting from the outage of line 2 is depicted in Fig. 10.4.

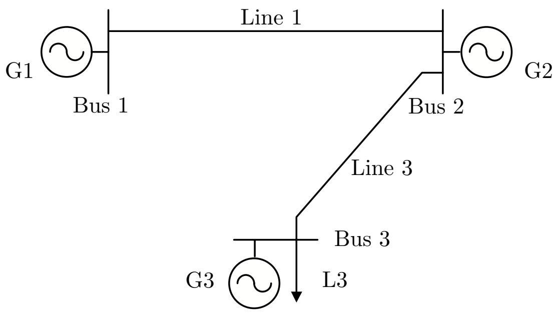


Fig. 10.4 Illustrative Example 10.5: state of the system without line 2

The power balance constraints (10.15) for this state are

$$\begin{aligned}
&\text{Node 1:} \quad P_{1t}^{\mathrm{G}} = f_t(1,2)\\
&\text{Node 2:} \quad P_{2t}^{\mathrm{G}} = f_t(2,1) + f_t(2,3)\\
&\text{Node 3:} \quad P_{3t}^{\mathrm{G}} + L_{3t}^{\mathrm{shed}} - L_{3t}^{\mathrm{C}} = f_t(3,2).
\end{aligned}$$

These constraints are enforced for scenarios $\omega = 5, \ldots, 8$, in time periods $t \geq \tau$.

Needless to say, this state is characterized by a reduced transmission capacity of the system that surely entails difficulties to evacuate the energy produced by the generating units.

□

Constraints (10.17) and (10.18) below set the generation limits of each unit for each period and scenario. Note that these inequalities are analogous to the first-stage constraints (10.6), except that the ones below are enforced per scenario through the on/off variables $v_{it\omega}$.

$$P_{it\omega}^{\mathrm{G}} \geq P_i^{\min}\, v_{it\omega}, \quad \forall i, \forall t, \forall \omega \tag{10.17}$$

$$P_{it\omega}^{\mathrm{G}} \leq P_i^{\max}\, v_{it\omega}, \quad \forall i, \forall t, \forall \omega. \tag{10.18}$$

The set of constraints (10.19) expresses the power output as the sum of the accepted energy blocks, whose lower and upper limits are imposed by (10.20). That is, equations (10.19) and (10.20) allow us to approximate the energy offer cost function of generating units by blocks. This formulation of the energy offers implies for every unit that the costs of the energy blocks are monotonically increasing. To model non-convex energy offer cost functions, additional binary variables are required, as stated for instance in [3].

$$P_{it\omega}^{\mathrm{G}} = \sum_{m=1}^{N_{\mathrm{O}it}} p_{\mathrm{G}it\omega}(m), \quad \forall i, \forall t, \forall \omega \tag{10.19}$$

$$0 \leq \; p_{\mathrm{G}it\omega}(m) \leq p_{\mathrm{G}it}^{\max}(m), \quad \forall m, \forall i, \forall t, \forall \omega. \tag{10.20}$$

Equations (10.19) and (10.20) are exemplified below.

Illustrative Example 10.6 (Generator power output by blocks). Let us consider the three-node system in Illustrative Example 10.1. Suppose that generating unit 1 submits to the electricity pool the energy offer cost function represented in Fig. 10.5 for the first period of the scheduling horizon. As can be observed, this function is comprised of three energy blocks. Table 10.4 constitutes the numerical translation of Fig. 10.5. Hence, constraints (10.19) and (10.20) become

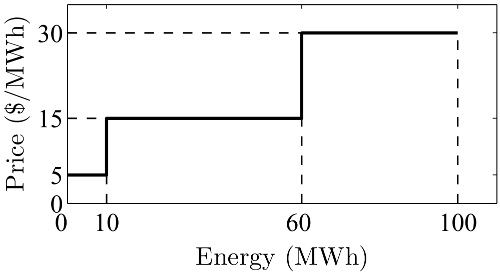


Fig. 10.5 Illustrative Example 10.6: energy offer cost function submitted by unit 1 for the first period of the scheduling horizon

Table 10.4 Illustrative Example 10.6: energy blocks and their cost

Block #	1	2	3
Size (MWh)	10	50	40
Cost ($/MWh)	5	15	30

$$\begin{aligned} P^{\mathrm{G}}_{11\omega} = p_{\mathrm{G}11\omega}(1) + p_{\mathrm{G}11\omega}(2) + p_{\mathrm{G}11\omega}(3), \quad \forall \omega \\ 0 \leq \; p_{\mathrm{G}11\omega}(1) \leq 10, \quad \forall \omega \\ 0 \leq \; p_{\mathrm{G}11\omega}(2) \leq 50, \quad \forall \omega \\ 0 \leq \; p_{\mathrm{G}11\omega}(3) \leq 40, \quad \forall \omega. \end{aligned}$$

It should be noted that, for consistency, the minimum power output of unit 1 is equal to its first offered power block, i.e.,

$$P_1^{\min} = p_{\mathrm{G}it}^{\max}(1) = 10 \text{ MW}. \tag{10.21}$$

As we assume that the market-clearing procedure is performed on a hourly basis, note that capacity (MW) is numerically the same figure as energy (MWh).

□

The line flow limits in each period and scenario are enforced by the group of constraints (10.22), hereinafter referred to as *transmission capacity constraints*.

$$-f^{\max}(n,r) \leq f_{t\omega}(n,r) \leq f^{\max}(n,r), \quad \forall (n,r) \in \Lambda(t,\omega), \;\; \forall t, \forall \omega. \tag{10.22}$$

The following example is aimed to illustrate this set of constraints.

Illustrative Example 10.7 (Transmission capacity constraints). Consider again the three-node system presented in Illustrative Example 10.1 and the scenarios ω listed in it. Constraints (10.22) become

$$
\begin{aligned}
&\text{Line 1:} \quad && -55 \leq f_{t\omega}(1,2) \leq 55, \quad \forall t, \forall \omega \\
&\text{Line 2:} \quad && -55 \leq f_{t\omega}(1,3) \leq 55, \quad \forall t; \forall \omega = 0,1,\ldots,4 \\
& && \qquad\qquad\qquad\qquad \text{and } \forall t < \omega - 4; \forall \omega = 5,\ldots,8 \\
&\text{Line 3:} \quad && -55 \leq f_{t\omega}(2,3) \leq 55, \quad \forall t, \forall \omega,
\end{aligned}
$$

where

$$
\begin{aligned}
f_{t\omega}(1,2) &= B(1,2)\big(\delta_{1t\omega} - \delta_{2t\omega}\big) = \frac{\delta_{1t\omega} - \delta_{2t\omega}}{0.13}, \quad \forall t, \forall \omega \\
f_{t\omega}(2,3) &= B(2,3)\big(\delta_{2t\omega} - \delta_{3t\omega}\big) = \frac{\delta_{2t\omega} - \delta_{3t\omega}}{0.13}, \quad \forall t, \forall \omega,
\end{aligned}
$$

and

$$
f_{t\omega}(1,3) = \begin{cases}
B(1,3)\big(\delta_{1t\omega} - \delta_{3t\omega}\big) = \dfrac{\delta_{1t\omega} - \delta_{3t\omega}}{0.13}, & \forall t; \omega = 0,1,\ldots,4 \\[2ex]
B(1,3)\big(\delta_{1t\omega} - \delta_{3t\omega}\big) = \dfrac{\delta_{1t\omega} - \delta_{3t\omega}}{0.13}, & \forall t < \omega - 4; \omega = 5,\ldots,8 \\[2ex]
0, & \forall t \geq \omega - 4; \omega = 5,\ldots,8.
\end{cases}
$$

□

Lastly, note that the amount of involuntary load shedding imposed on each consumer has to be positive and smaller or equal than its actual demand. This is mathematically expressed as

$$
0 \leq L^{\text{shed}}_{jt\omega} \leq L^{\text{C}}_{jt\omega}, \quad \forall j, \forall t, \forall \omega. \tag{10.23}
$$

According to equations (10.15) and (10.23), load shedding variables can be mathematically seen as fictitious generators, located at load nodes, with a maximum capacity equal to the magnitude of the corresponding load.

Finally, this part of the formulation, which involves the actual operation of the system (second-stage variables), can also include ramping constraints and minimum up/down time of generating units as stated in [3,20]. Ramping constraints are imposed on variables $P^{\text{G}}_{it\omega}$, which represent the power outputs of generating units during the actual operation of the power system, by using binary variables $v_{it\omega}$ representing their on/off status. Minimum up/down time constraints are, in turn, imposed through these 0/1 variables. For the sake of conciseness, the mathematical formulation of these constraints is omitted here. The interested reader may consult [20].

10.4.7 Linking constraints

These constraints relate market decisions to the real-time operation of the power system through the deployment of reserves provided by units and loads. Hence, from a stochastic programming perspective, these coupling constraints encompass both first- and second-stage variables.

The real-time power outputs of generating units result from the power productions settled in the electricity market plus the different types of reserves deployed to accommodate system failures. This is stated as follows:

$$P^{\mathrm{G}}_{it\omega} = P^{\mathrm{S}}_{it} + r^{\mathrm{U}}_{it\omega} + r^{\mathrm{NS}}_{it\omega} - r^{\mathrm{D}}_{it\omega}, \ \ \forall t, \forall \omega, \forall i \in G(t,\omega), \tag{10.24}$$

where $G(t,\omega)$ is the set of available generating units in period t and scenario ω.

The group of equations (10.24) is further clarified by means of the example below.

Illustrative Example 10.8 (Composition of generator power outputs). Considering the three generators of the three-node system in Illustrative Example 10.1 and the nine scenarios ω enumerated in it, equation (10.24) renders:

$$\begin{aligned}
&\text{Unit 1:} \quad P^{\mathrm{G}}_{1t\omega} = P^{\mathrm{S}}_{1t} + r^{\mathrm{U}}_{1t\omega} + r^{\mathrm{NS}}_{1t\omega} - r^{\mathrm{D}}_{1t\omega}, \ \ \forall t, \forall \omega \\
&\text{Unit 2:} \quad P^{\mathrm{G}}_{2t\omega} = P^{\mathrm{S}}_{2t} + r^{\mathrm{U}}_{2t\omega} + r^{\mathrm{NS}}_{2t\omega} - r^{\mathrm{D}}_{2t\omega}, \ \ \forall t, \forall \omega \\
&\text{Unit 3:} \quad P^{\mathrm{G}}_{3t\omega} = P^{\mathrm{S}}_{3t} + r^{\mathrm{U}}_{3t\omega} + r^{\mathrm{NS}}_{3t\omega} - r^{\mathrm{D}}_{3t\omega}, \\
&\qquad\qquad\qquad \forall t < \omega; \forall \omega = 1, \ldots, 4 \text{ and } \forall t; \forall \omega = 0, 5, \ldots, 8.
\end{aligned}$$

□

An analogous decomposition can be carried out for the power consumed by each load:

$$L^{\mathrm{C}}_{jt\omega} = L^{\mathrm{S}}_{jt} - r^{\mathrm{U}}_{jt\omega} + r^{\mathrm{D}}_{jt\omega}, \ \ \forall j, \forall t, \forall \omega. \tag{10.25}$$

The amount of reserve that a generating unit i is willing to deploy in time period t as a response to a failure event is limited to the quantity established in the electricity market for each type of reserve. Constraints (10.26)–(10.28) below constitute the mathematical implementation of these limits for up-, down-, and non-spinning reserves, respectively:

$$0 \leq r^{\mathrm{U}}_{it\omega} \leq R^{\mathrm{U}}_{it}, \ \ \forall i, \forall t, \forall \omega \tag{10.26}$$

$$0 \leq r^{\mathrm{D}}_{it\omega} \leq R^{\mathrm{D}}_{it}, \ \ \forall i, \forall t, \forall \omega \tag{10.27}$$

$$0 \leq r^{\mathrm{NS}}_{it\omega} \leq R^{\mathrm{NS}}_{it}, \ \ \forall i, \forall t, \forall \omega. \tag{10.28}$$

Similar constraints are to be enforced for each load j with scheduled reserve:

$$0 \le r^{\mathrm{U}}_{jt\omega} \le R^{\mathrm{U}}_{jt}, \ \ \forall j, \forall t, \forall \omega \tag{10.29}$$

$$0 \le r^{\mathrm{D}}_{jt\omega} \le R^{\mathrm{D}}_{jt}, \ \ \forall j, \forall t, \forall \omega. \tag{10.30}$$

The deployment of reserves during the real-time operation of the power system can entail changes in the start-up and shut-down planning settled in the electricity market. This changes require the adjustment of the start-up costs computed in the market (first-stage), adjustment that is mathematically modeled through the following set of equations:

$$C^{\mathrm{A}}_{it\omega} = C^{\mathrm{SU}}_{it\omega} - C^{\mathrm{SU}}_{it}, \quad \forall i, \forall t, \forall \omega \tag{10.31}$$

$$C^{\mathrm{SU}}_{it\omega} \ge \lambda^{\mathrm{SU}}_{it}(v_{it\omega} - v_{i,t-1,\omega}), \quad \forall i, \forall t, \forall \omega \tag{10.32}$$

$$C^{\mathrm{SU}}_{it\omega} \ge 0, \quad \forall i, \forall t, \forall \omega. \tag{10.33}$$

Note that variable $C^{\mathrm{SU}}_{it\omega}$ accounts for the start-up cost incurred by generating unit i during the actual operation of the power system in period t and scenario ω. Thus, $C^{\mathrm{A}}_{it\omega}$ is zero if the commitment status of unit i in period t is as scheduled in the market. Its value is positive if unit i has to be started up in period t against what was set in the market and it is negative if a scheduled start-up for unit i in period t is finally cancelled due to the realization of scenario ω.

On the other hand, it should be noted that, in objective function (10.4), the start-up costs are separated into two terms: the first one, C^{SU}_{it}, accounts for the costs resulting from the scheduled start-ups of each generating unit (first-stage costs). The second term, $C^{\mathrm{A}}_{it\omega}$, covers the costs caused by the adjustments required in the unit commitment plan in order to respond to a particular contingency (second-stage costs). However, from a computational point of view, this splitting of the start-up costs is not necessary, since the ultimate start-up costs incurred by generators, including both scheduling and the subsequent adjustments, can be merged into the single second-stage term, $C^{\mathrm{SU}}_{it\omega}$, through (10.31). This way, constraints (10.13), (10.14) and (10.31), together with variables C^{SU}_{it} and $C^{\mathrm{A}}_{it\omega}$, can be eliminated.

The set of constraints (10.31)–(10.33) is exemplified below.

Illustrative Example 10.9 (Second-stage start-up cost adjustments). Consider the generating unit 1 of the three-node system in Illustrative Example 10.1 and just the first period of the scheduling horizon. Its start-up offer cost is equal to \$100 in such a period (see Table 10.3 in Illustrative Example 10.2), and thus, constraints (10.31)–(10.33) become

$$\begin{aligned}
&C^{\mathrm{A}}_{11\omega} = C^{\mathrm{SU}}_{11\omega} - C^{\mathrm{SU}}_{11}, \ \ \forall \omega \\
&C^{\mathrm{SU}}_{11\omega} \ge 100(v_{11\omega} - v_{10\omega}), \ \ \forall \omega \\
&C^{\mathrm{SU}}_{11\omega} \ge 0, \ \ \forall \omega,
\end{aligned}$$

where $v_{10\omega}$ is the initial commitment status of unit 1 in scenario ω. It holds that $v_{10\omega} = u_{10}, \forall \omega$. □

The formulation of the proposed market-clearing procedure ends up with the constraints modeling the non-anticipativity character of the information. Specifically, these constraints state that the system operator cannot anticipate the outcomes of the random events, or equivalently, that no corrective action can be executed before the occurrence of a contingency, that is,

$$L^{\text{shed}}_{jt\omega} = 0, \qquad \forall j, \forall t < \tau, \forall \omega = (k, \tau) \tag{10.34}$$

$$r^{\text{U}}_{it\omega} = r^{\text{D}}_{it\omega} = r^{\text{NS}}_{it\omega} = 0, \qquad \forall i, \forall t < \tau, \forall \omega = (k, \tau) \tag{10.35}$$

$$r^{\text{U}}_{jt\omega} = r^{\text{D}}_{jt\omega} = 0, \qquad \forall j, \forall t < \tau, \forall \omega = (k, \tau) \tag{10.36}$$

$$v_{it\omega} = u_{it}, \qquad \forall i, \forall t < \tau, \forall \omega = (k, \tau). \tag{10.37}$$

Condition (10.34) assumes that involuntary load shedding is a corrective action that the system operator can perform just in response to a failure event, but not in advance. Being so, the market-clearing procedure presented in this chapter relies on the premise that the electric energy system to be *dispatched* counts on sufficient available resources to supply the demand under normal operating conditions. Otherwise, a feasible scheduling program cannot be found unless constraints (10.34) are removed from the formulation. Note that current electricity systems are designed to serve energy demand within its typical range of variation without shedding load, and in this sense, a demand value out of this range is treated as a contingency.

Likewise, observe that equations (10.34)–(10.37) fix the decisions variables pertaining to scenario $\omega = 0$, in which no contingency occurs during the entire scheduling horizon, to the values set in the market.

The following illustrative example is aimed to further clarify the group of non-anticipativity constraints (10.34)–(10.37).

Illustrative Example 10.10 (Non-anticipativity constraints). Consider the pre-contingency state of the three-node system in Illustrative Example 10.1. In Illustrative Example 10.5, we introduce the power balance equations for this system state as follows:

$$\begin{aligned}
&\text{Node 1:} \quad P^{\text{G}}_{1t} = f_t(1,2) + f_t(1,3)\\
&\text{Node 2:} \quad P^{\text{G}}_{2t} = f_t(2,1) + f_t(2,3)\\
&\text{Node 3:} \quad P^{\text{G}}_{3t} + L^{\text{shed}}_{3t} - L^{\text{C}}_{3t} = f_t(3,1) + f_t(3,2).
\end{aligned}$$

We can incorporate into these balance equations the non-anticipativity condition (10.34) expressing that $L^{\text{shed}}_{3t} = 0$ in the absence of system failures. Thus, the power balance at node 3 becomes

$$P^{\text{G}}_{3t} - L^{\text{C}}_{3t} = f_t(3,1) + f_t(3,2).$$

Moreover, during the pre-contingency state, it holds

$$P_{1t}^{\mathrm{G}} = P_{1t}^{\mathrm{S}}$$
$$P_{2t}^{\mathrm{G}} = P_{2t}^{\mathrm{S}}$$
$$P_{3t}^{\mathrm{G}} = P_{3t}^{\mathrm{S}}.$$

In other words, as long as no failure event occurs, the real-time operation of the power system should follow the scheduling program settled in the electricity market. □

10.4.8 Formulation

Lastly, the whole formulation of the proposed market-clearing procedure to cope with equipment failures is presented below for completeness.

$$\begin{aligned}
&\text{Minimize}_{C_{it}^{\mathrm{SU}}, R_{it}^{\mathrm{U}}, R_{it}^{\mathrm{D}}, R_{it}^{\mathrm{NS}}, R_{jt}^{\mathrm{U}}, R_{jt}^{\mathrm{D}}, C_{it\omega}^{\mathrm{A}}, p_{\mathrm{G}it\omega}(m), L_{jt\omega}^{\mathrm{C}}, L_{jt\omega}^{\mathrm{shed}}} \\
&\mathrm{EC} = \sum_{t=1}^{N_{\mathrm{T}}} \mathrm{EC}_t = \\
&= \sum_{t=1}^{N_{\mathrm{T}}} \sum_{i=1}^{N_{\mathrm{G}}} C_{it}^{\mathrm{SU}} \\
&+ \sum_{t=1}^{N_{\mathrm{T}}} d_t \left[\sum_{i=1}^{N_{\mathrm{G}}} \left(C_{it}^{R^{\mathrm{U}}} R_{it}^{\mathrm{U}} + C_{it}^{R^{\mathrm{D}}} R_{it}^{\mathrm{D}} + C_{it}^{R^{\mathrm{NS}}} R_{it}^{\mathrm{NS}} \right) \right. \\
&\qquad \left. + \sum_{j=1}^{N_{\mathrm{L}}} \left(C_{jt}^{R^{\mathrm{U}}} R_{jt}^{\mathrm{U}} + C_{jt}^{R^{\mathrm{D}}} R_{jt}^{\mathrm{D}} \right) \right] \\
&+ \sum_{\omega=0}^{N_{\Omega}} \pi_{\omega} \left\{ \sum_{t=1}^{N_{\mathrm{T}}} \sum_{i=1}^{N_{\mathrm{G}}} C_{it\omega}^{\mathrm{A}} \right. \\
&\qquad + \sum_{t=1}^{N_{\mathrm{T}}} d_t \left[\sum_{i=1}^{N_{\mathrm{G}}} \sum_{m=1}^{N_{\mathrm{O}it}} \lambda_{\mathrm{G}it}(m) p_{\mathrm{G}it\omega}(m) - \sum_{j=1}^{N_{\mathrm{L}}} \lambda_{\mathrm{L}jt} L_{jt\omega}^{\mathrm{C}} \right. \\
&\qquad\qquad \left. \left. + \sum_{j=1}^{N_{\mathrm{L}}} V_{jt}^{\mathrm{LOL}} L_{jt\omega}^{\mathrm{shed}} \right] \right\}
\end{aligned} \tag{10.38}$$

subject to

$$\sum_{i=1}^{N_{\mathrm{G}}} P_{it}^{\mathrm{S}} = \sum_{j=1}^{N_{\mathrm{L}}} L_{jt}^{\mathrm{S}}, \quad \forall t. \tag{10.39}$$

$$P_i^{\min}\ u_{it} \le P_{it}^{\mathrm{S}} \le P_i^{\max}\ u_{it}, \quad \forall i, \forall t \tag{10.40}$$

$$L_{jt}^{\mathrm{S,\ min}} \le L_{jt}^{\mathrm{S}} \le L_{jt}^{\mathrm{S,\ max}}, \ \ \forall j, \forall t. \tag{10.41}$$

$$0 \le R_{it}^{\mathrm{U}} \le R_{it}^{\mathrm{U,\,max}}\ u_{it}, \ \ \forall i, \forall t \tag{10.42}$$

$$0 \le R_{it}^{\mathrm{D}} \le R_{it}^{\mathrm{D,\,max}}\ u_{it}, \ \ \forall i, \forall t \tag{10.43}$$

$$0 \le R_{it}^{\mathrm{NS}} \le R_{it}^{\mathrm{NS,\ max}}\ (1 - u_{it}), \ \ \forall i, \forall t. \tag{10.44}$$

$$0 \le R_{jt}^{\mathrm{U}} \le R_{jt}^{\mathrm{U,\ max}}, \ \ \forall j, \forall t \tag{10.45}$$

$$0 \le R_{jt}^{\mathrm{D}} \le R_{jt}^{\mathrm{D,\ max}}, \ \ \forall j, \forall t. \tag{10.46}$$

$$C_{it}^{\mathrm{SU}} \ge \lambda_{it}^{\mathrm{SU}}(u_{it} - u_{i,t-1}), \quad \forall i, \forall t \tag{10.47}$$

$$C_{it}^{\mathrm{SU}} \ge 0, \quad \forall i, \forall t. \tag{10.48}$$

$$\sum_{i:(i,n)\in M_{\mathrm{G}}(t,\omega)} P_{it\omega}^{\mathrm{G}} - \sum_{j:(j,n)\in M_{\mathrm{L}}} \left(L_{jt\omega}^{\mathrm{C}} - L_{jt\omega}^{\mathrm{shed}}\right) - \sum_{r:(n,r)\in \Lambda(t,\omega)} f_{t\omega}(n,r) = 0, \ \ \forall n, \forall t, \forall \omega. \tag{10.49}$$

$$f_{t\omega}(n,r) = B(n,r)\big(\delta_{nt\omega} - \delta_{rt\omega}\big), \ \ \forall (n,r) \in \Lambda(t,\omega), \forall t, \forall \omega. \tag{10.50}$$

$$P_{it\omega}^{\mathrm{G}} \ge P_i^{\min}\ v_{it\omega}, \quad \forall i, \forall t, \forall \omega \tag{10.51}$$

$$P_{it\omega}^{\mathrm{G}} \le P_i^{\max}\ v_{it\omega}, \quad \forall i, \forall t, \forall \omega \tag{10.52}$$

$$P_{it\omega}^{\mathrm{G}} = \sum_{m=1}^{N_{\mathrm{O}it}} p_{\mathrm{G}it\omega}(m), \quad \forall i, \forall t, \forall \omega \tag{10.53}$$

$$0 \le \ p_{\mathrm{G}it\omega}(m) \le p_{\mathrm{G}it}^{\max}(m), \quad \forall m, \forall i, \forall t, \forall \omega. \tag{10.54}$$

$$-f^{\max}(n,r) \le f_{t\omega}(n,r) \le f^{\max}(n,r), \quad \forall (n,r) \in \Lambda(t,\omega), \ \ \forall t, \forall \omega. \tag{10.55}$$

$$0 \le L_{jt\omega}^{\mathrm{shed}} \le L_{jt\omega}^{\mathrm{C}}, \ \ \forall j, \forall t, \forall \omega. \tag{10.56}$$

$$P_{it\omega}^{\mathrm{G}} = P_{it}^{\mathrm{S}} + r_{it\omega}^{\mathrm{U}} + r_{it\omega}^{\mathrm{NS}} - r_{it\omega}^{\mathrm{D}}, \ \ \forall t, \forall \omega, \forall i \in G(t,\omega) \tag{10.57}$$

$$L_{jt\omega}^{\mathrm{C}} = L_{jt}^{\mathrm{S}} - r_{jt\omega}^{\mathrm{U}} + r_{jt\omega}^{\mathrm{D}}, \ \ \forall j, \forall t, \forall \omega. \tag{10.58}$$

$$0 \leq r_{it\omega}^{\mathrm{U}} \leq R_{it}^{\mathrm{U}}, \quad \forall i, \forall t, \forall \omega \tag{10.59}$$

$$0 \leq r_{it\omega}^{\mathrm{D}} \leq R_{it}^{\mathrm{D}}, \quad \forall i, \forall t, \forall \omega \tag{10.60}$$

$$0 \leq r_{it\omega}^{\mathrm{NS}} \leq R_{it}^{\mathrm{NS}}, \quad \forall i, \forall t, \forall \omega. \tag{10.61}$$

$$0 \leq r_{jt\omega}^{\mathrm{U}} \leq R_{jt}^{\mathrm{U}}, \quad \forall j, \forall t, \forall \omega \tag{10.62}$$

$$0 \leq r_{jt\omega}^{\mathrm{D}} \leq R_{jt}^{\mathrm{D}}, \quad \forall j, \forall t, \forall \omega. \tag{10.63}$$

$$C_{it\omega}^{\mathrm{A}} = C_{it\omega}^{\mathrm{SU}} - C_{it}^{\mathrm{SU}}, \quad \forall i, \forall t, \forall \omega \tag{10.64}$$

$$C_{it\omega}^{\mathrm{SU}} \geq \lambda_{it}^{\mathrm{SU}}(v_{it\omega} - v_{i,t-1,\omega}), \quad \forall i, \forall t, \forall \omega \tag{10.65}$$

$$C_{it\omega}^{\mathrm{SU}} \geq 0, \quad \forall i, \forall t, \forall \omega. \tag{10.66}$$

$$L_{jt\omega}^{\mathrm{shed}} = 0, \qquad \forall j, \forall t < \tau, \forall \omega = (k, \tau) \tag{10.67}$$

$$r_{it\omega}^{\mathrm{U}} = r_{it\omega}^{\mathrm{D}} = r_{it\omega}^{\mathrm{NS}} = 0, \qquad \forall i, \forall t < \tau, \forall \omega = (k, \tau) \tag{10.68}$$

$$r_{jt\omega}^{\mathrm{U}} = r_{jt\omega}^{\mathrm{D}} = 0, \qquad \forall j, \forall t < \tau, \forall \omega = (k, \tau) \tag{10.69}$$

$$v_{it\omega} = u_{it}, \qquad \forall i, \forall t < \tau, \forall \omega = (k, \tau). \tag{10.70}$$

$$u_{it}, v_{it\omega} \in \{0, 1\}, \forall i, \forall t, \forall \omega. \tag{10.71}$$

The objective function (10.38) is the expected cost of operating the power system, which consists of the costs related to the electricity-market stage (start-up and shut-down planning of generating units, and reserve scheduling), and the costs incurred during the real-time operation of the power system (reserve deployment, demand supply and load shedding events).

Equations (10.39)–(10.48) form the group of constraints directly linked to the electricity-market stage: market balance (10.39)–(10.41), reserve scheduling (10.42)–(10.46), and first-stage start-up costs (10.47)–(10.48).

Constraints (10.49)–(10.56) model the real-time operation of the power system through the dc load flow model of the power balance equations (10.49)–(10.50), the decomposition of generator power outputs by blocks (10.51)–(10.54), the transmission capacity limits (10.55), and the involuntary load shedding bounds (10.56).

The group of equations (10.57)–(10.69) constitute the so-called *linking constraints*, which couple market decisions with operating decisions made in real time through the deployment of reserves by generating units and loads (10.57)–(10.63), the changes in the start-up and shut-down planning of units to make such a deployment feasible (10.64)–(10.66), and the constraints enforcing the non-anticipatory character of information in the decision-making process (10.67)–(10.70).

Finally, constraints (10.71) restrict variables u_{it} and $v_{it\omega}$ to be binary.

10.5 Computing Scenario Probabilities

In this section, the basic concepts required to calculate the probability π_ω associated with scenario ω are briefly introduced. Recall that each scenario ω corresponds to a pair (k, τ) expressing the occurrence of a contingency k within a time period τ.

A contingency is related to an equipment failure (generator or transmission line outages). It is widely accepted that the time between two consecutive equipment failures, as well as the time to repair a failure, both follow exponential distributions. These distributions are determined, respectively, by the mean time to failure (MTTF) and the mean time to repair (MTTR), which are obtained from historical data, [12, 13].

In the market-clearing formulation above, the MTTR is assumed to be much greater than the 24 hourly periods that usually comprise the scheduling horizon of day-ahead electricity markets. This way, repairs are ignored so that once some equipment fails it is assumed to be unavailable for the rest of the scheduling horizon.

Probabilities π_ω are derived from the probabilities associated with the following two random events:
Random event $A(k, \tau)$: *contingency k happens within the time period τ,*

$$P[A(k,\tau)] = \int_{\tau-1}^{\tau} \lambda_k e^{-\lambda_k t} dt = e^{-\lambda_k \tau}(e^{\lambda_k} - 1), \tag{10.72}$$

where λ_k is the inverse of the mean time to the occurrence of contingency k.
Random event $B(k)$: *contingency k does not occur during the entire scheduling horizon,*

$$P[B(k)] = 1 - \int_{0}^{T} \lambda_k e^{-\lambda_k t} dt = e^{-\lambda_k T}, \tag{10.73}$$

where T is the time length of the scheduling horizon (e.g., 24 hours).

Therefore, under the assumption of statistically independent contingencies, the probability π_0 of the pre-contingency scenario $\omega = 0$, in which no contingency occurs during the whole scheduling horizon, can be computed as the product of the probabilities $P[B(k)]$ for all k, i.e.,

$$\pi_0 = \prod_{k=1}^{K} P[B(k)] = \prod_{k=1}^{K} e^{-\lambda_k T}, \tag{10.74}$$

where K is the cardinality of the set of pre-selected contingencies.

On the other hand, the probability π_ω of the scenario ω in which contingency k occurs during the time period τ given that the remainder of system components are available for the whole scheduling horizon can be calculated as

$$\pi_\omega = \pi(k,\tau) = P[A(k,\tau)] \prod_{\substack{y=1 \\ y\neq k}}^{K} P[B(y)] = e^{-\lambda_k \tau}(e^{\lambda_k} - 1) \prod_{\substack{y=1 \\ y\neq k}}^{K} e^{-\lambda_y T}. \quad (10.75)$$

It should be noted that probabilities π_0 and π_ω do not add up to 1 because sequential contingencies have been disregarded.

The calculation of probabilities π_0 and π_ω is illustrated below.

Illustrative Example 10.11 (Scenario probabilities). Let us consider the three-node system in Illustrative Example 10.1. Unlike in that example, in this one the equipment failures probabilities are not known beforehand, but they have to be estimated from the available historical data.

Suppose that a detailed statistical analysis of these data reveals the following information:

1. Outages of units 1 and 2, and transmissions lines 1 and 3, have been never reported. As a result, these system devices are assumed to be 100% reliable, which translates into a mean time to failure tending to infinite.
2. Time between two consecutive failures of unit 3 follows an exponential distribution characterized by a MTTF equal to 500 h.
3. Analogously, time between two consecutive failures of line 2 follows an exponential distribution with a MTTF of 1000 h.

Therefore, by indexing the failures of generator 3 and line 2 with $k = 1, 2$, respectively, and considering a scheduling horizon spanning four hourly periods, the scenario enumeration provided in Example 10.1 can be used here.

First, we compute the probabilities of random events $A(k,\tau)$ and $B(k)$:

$$\begin{aligned}
P[A(1,\tau)] &= \exp(-\lambda_1 \tau)(\exp(\lambda_1) - 1) = \exp\left(-\tfrac{1\text{ h}}{500\text{ h}}\right)\left(\exp\left(\tfrac{1\text{ h}}{500\text{ h}}\right) - 1\right) \\
&= 1.9980 \times 10^{-3} \\
P[A(2,\tau)] &= \exp(-\lambda_2 \tau)(\exp(\lambda_2) - 1) = \exp\left(-\tfrac{1\text{ h}}{1000\text{ h}}\right)\left(\exp\left(\tfrac{1\text{ h}}{1000\text{ h}}\right) - 1\right) \\
&= 0.9995 \times 10^{-3} \\
P[B(1)] &= \exp(-\lambda_1 T) = \exp\left(-\tfrac{4\text{ h}}{500\text{ h}}\right) = 0.9920 \\
P[B(2)] &= \exp(-\lambda_2 T) = \exp\left(-\tfrac{4\text{ h}}{1000\text{ h}}\right) = 0.9960.
\end{aligned}$$

Then, the probability π_0 that none of contingencies k occurs during the scheduling horizon is given by

$$\pi_0 = \prod_{k=1}^{K} P[B(k)] = P[B(1)]P[B(2)] = 0.9920 \times 0.9960 = 0.9880.$$

Likewise, the probability $\pi_\omega, \omega = 1, \dots, 4$ that the failure of unit 3 occurs during the interval $\tau = \omega$, given that all other system components are available for the entire scheduling horizon, is

$$\pi_\omega = \pi(1,\omega) = P[A(1,\omega)] \prod_{\substack{y=1 \\ y\neq 1}}^{2} P[B(y)] = P[A(1,\omega)]P[B(2)]$$
$$= 1.9980 \times 10^{-3} \times 0.9960 = 1.9900 \times 10^{-3};\ \ \forall\omega = 1,\ldots,4.$$

Finally, the probability $\pi_\omega, \omega = 5,\ldots,8$ that only the outage of line 2 in time period $\tau = \omega - 4$ happens during the whole scheduling horizon is

$$\pi_\omega = \pi(2,\omega-4) = P[A(2,\omega-4)] \prod_{\substack{y=1 \\ y\neq 2}}^{2} P[B(y)] = P[A(2,\omega-4)]P[B(1)]$$
$$= 0.9995 \times 10^{-3} \times 0.9920 = 0.9915 \times 10^{-3};\ \ \forall\omega = 5,\ldots,8.$$

Note that $\sum_{\omega=0}^{9} \pi_\omega \lesssim 1$, as the missing probability corresponds to the simultaneous and sequential outages of unit 3 and line 2, which are significantly more unlikely and are not considered here.

□

10.6 Market-Clearing Example

In this example, the market-clearing procedure formulated in Section 10.4 is applied to the three-node system described in Illustrative Example 10.1. This procedure is intended to produce a scheduling program pre-positioning generating units and loads in the best possible manner to face a set of preselected contingencies.

As in Illustrative Example 10.1, the failure events considered here consist of the outages of unit 3 and transmission line 2 ($k = 1$ and $k = 2$, respectively). The occurrence of any one of these outages in any one of the four time periods comprising the scheduling horizon ($\tau = 1,\ldots,4$) makes up a set of eight contingencies (k,τ). As a result, the following nine scenarios are considered:

Scenario $\omega = 0$, in which no contingency happens during the entire scheduling horizon.

Scenarios $\omega = 1,\ldots,4$, defined by pairs $(k,\tau) = (1,\tau)$ with $\tau = \omega$, where the outage of unit 3 takes place in time period τ.

Scenarios $\omega = 5,\ldots,8$, defined by pairs $(k,\tau) = (2,\tau)$ with $\tau = \omega - 4$, where the failure of line 2 occurs in time period τ.

The mathematical model detailed in Section 10.4 is used next to solve the market-clearing problem according to the economic and technical data provided in Illustrative Example 10.2. Minimum up- and down-times of the units are considered to be zero, and therefore, the corresponding minimum up- and down-time constraints are not binding. The ramping capabilities of generating units are the largest possible, i.e, $P_i^{\max}$ MW/h. The three units are assumed to be off-line at the beginning of the study horizon ($t = 0$).

The probabilities of failure scenarios computed in Illustrative Example 10.11 are considered. Specifically, these probabilities are:

$$\pi_0 = 0.9880$$
$$\pi_\omega = 1.9900 \times 10^{-3}; \ \forall \omega = 1, \ldots, 4$$
$$\pi_\omega = 0.9915 \times 10^{-3}; \ \forall \omega = 5, \ldots, 8.$$

As previously pointed out, probabilities π_ω do not sum up to one because compounded and sequential failures have been discarded from the analysis. Their sum is indeed approximately equal to 0.999926. A simple criterion to force these probabilities to add up to one consists in just distributing proportionally the missing probability amongst them. By rounding off to five decimal places the new probabilities π'_ω become

$$\pi'_0 = \frac{\pi_0}{\sum_{\omega=1}^{9} \pi_\omega} = 0.98807$$
$$\pi'_\omega = \frac{\pi_\omega}{\sum_{\omega=1}^{9} \pi_\omega} = 1.99015 \times 10^{-3}; \ \forall \omega = 1, \ldots, 4$$
$$\pi'_\omega = \frac{\pi_\omega}{\sum_{\omega=1}^{9} \pi_\omega} = 0.99157 \times 10^{-3}; \ \forall \omega = 5, \ldots, 8,$$

which add up to 1.00000.

Table 10.5 provides the scheduling program resulting from the market-clearing procedure (solution to problem (10.38)–(10.71)).

Table 10.5 Market-clearing example: scheduling program. Scenario probabilities from Illustrative Example 10.11. Powers in MW

Period #	P_t^{S}			R_t^{U}				R_t^{D}				R_t^{NS}		
	G1	G2	G3	G1	G2	G3	L3	G1	G2	G3	L3	G1	G2	G3
1	0	0	30	0	0	0	0	0	0	0	0	0	0	0
2	30	0	50	0	0	0	0	0	0	0	0	0	0	0
3	60	0	50	22.5	0	0	0	5	0	0	0	0	0	0
4	0	0	40	0	0	0	0	0	0	0	0	40	0	0

The expected social cost is \$7050.7. The total amounts of scheduled reserves per period are 0, 0, 27.5, and 40 MW, which represent, respectively, the 0, 0, 25, and 100% of the hourly demands. Table 10.6 provides the amount of load that is involuntarily shed per scenario and period.

Therefore, the expected load not served (ELNS) associated with the proposed scheduling program is

Table 10.6 Market-clearing example: involuntary load shedding (MWh)

Period #	Scenario #								
	0	1	2	3	4	5	6	7	8
1	0	30	0	0	0	0	0	0	0
2	0	50	50	0	0	0	0	0	0
3	0	27.5	27.5	27.5	0	5	5	5	0
4	0	0	0	0	0	0	0	0	0

$$\text{ELNS} = (30 + 2 \times 50 + 3 \times 27.5) \times 1.99015 \times 10^{-3} + \\ + 3 \times 5 \times 0.99157 \times 10^{-3} = 0.4378 \text{ MWh}.$$

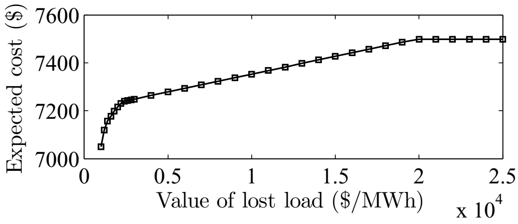
The load shedding events numerically detailed in Table 10.6 are justified solely by economic reasons. That is, there is no any technical limitation preventing the system from supplying the complete demand. The low probabilities characterizing the occurrence of outages of unit 3 and line 2 *encourage* the system operator to *relax* the preventive measures in order to reduce expected costs, barely undermining the reliability of the system. In fact, the expected load not served represents just 0.17% of the total energy consumption over the entire duration of the scheduling horizon.

As those loss-of-load incidents respond solely to economic criteria, a simple manner to avoid them is increasing the value of lost load. As an example, if this value is increased up to \$25,000/MWh, no load shedding occurs during the entire scheduling horizon under any scenario. Table 10.7 details the scheduling programm derived from the market-clearing procedure for this new value of lost load.

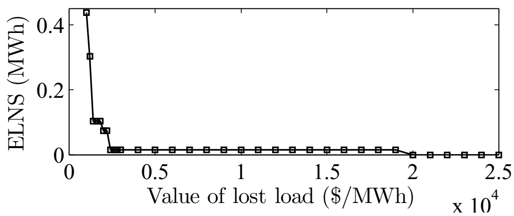
Table 10.7 Market-clearing example: scheduling program for V^{LOL} = \$25,000/MWh. Powers in MW

Period #	P_t^{S}			R_t^{U}				R_t^{D}				R_t^{NS}		
	G1	G2	G3	G1	G2	G3	L3	G1	G2	G3	L3	G1	G2	G3
1	0	0	30	0	0	0	0	0	0	0	0	30	0	0
2	30	0	50	50	0	0	0	0	0	0	0	0	0	0
3	60	0	50	0	0	0	5	5	0	0	0	0	45	0
4	0	0	40	0	0	0	0	0	0	0	0	40	0	0

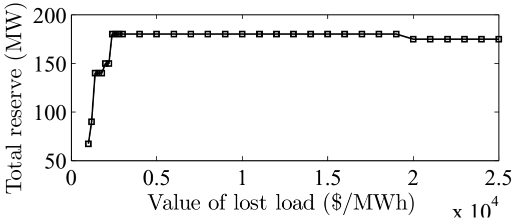
In this case, note that the quantity of scheduled reserves represents 67% of the total demand. Likewise, the expected social cost associated with this new program amounts to \$7498, which translates into an increase of 6.34% with respect to the previous one.



(a)



(b)



(c)

Fig. 10.6 Market-clearing example: impact of the magnitude of the value of lost load on: (a) the expected social cost; (b) the expected amount of load not served (ELNS); (c) the total amount of reserve scheduled in the market

Fig. 10.6 shows the evolution of the expected social cost, the expected load not served (ELNS) and the total amount of reserves scheduled in the market as a function of the value of lost load. It can be inferred from this figure that the changes undergone by these three magnitudes are clearly interrelated. The linear stretch characterizing most part of the evolution of the expected cost corresponds to a range of the value of lost load in which both the ELNS and the total scheduled reserve remain constant. The scheduling programs generated by the market-clearing procedure for a value of lost load within this range are all identical. As a result, the increase in the rate of the referred linear evolution is precisely proportional to the value of lost load, with the amount of load shed being the proportionality constant.

On the other hand, in light of the results provided in this figure, there is no doubt that the reduction of the frequency and magnitude of load shedding events leads to an increase in the amount of reserve scheduled in the electricity market, and consequently, to a rise in the expected cost of system operation. In plain words, prevention implies cost. As an example, observe that the initial stretch of the ELNS evolution is characterized by a sharp drop, which coincides with a brisk increase in expected cost and total scheduled reserve.

Nonetheless, there is an exception in this general behavior. If we examine in detail Fig. 10.6(c), we can appreciate a small decrease in the total reserve occurring exactly for the value of lost load from which the ELNS becomes zero and the expected cost becomes flat. That is, this step change in the amount of reserve scheduled in the market is intended to eliminate load shedding events once and for all. This exception can be easily understood by means of Tables 10.8 and 10.9. The first one provides a comparison between the scheduling programs obtained for two different values of lost load, \$15,000/MWh and \$25,000/MWh. Note that these values are, respectively, smaller and greater than the value of lost load for which the referred decrease in reserve takes place. The numbers presented in square brackets correspond to those results in which the two schedules differ, with the value enclosed corresponding to V^{LOL}= \$25,000/MWh. Table 10.9 illustrates the magnitude of load shedding events associated with both scheduling programs. The format used for this second table is analogous to that of Table 10.8.

Table 10.8 Market-clearing example: reserve schedule for V^{LOL} = \$15,000/MWh [\$25,000/MWh]. Powers in MW

Period #	P_t^{S}			R_t^{U}				R_t^{D}				R_t^{NS}		
	G1	G2	G3	G1	G2	G3	L3	G1	G2	G3	L3	G1	G2	G3
1	0	0	30	0	0	0	0	0	0	0	0	30	0	0
2	30	0	50	50	0	0	0	0	0	0	0	0	0	0
3	60	0	50	0	0	0	0 [5]	5	0	0	0	0	55 [45]	0
4	0	0	40	0	0	0	0	0	0	0	0	40	0	0

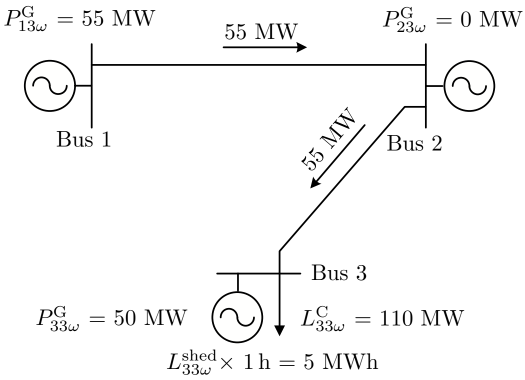
Table 10.9 Market-clearing example: involuntary load shedding (MWh) for V^{LOL} = \$15,000/MWh [\$25,000/MWh]

Period #	Scenario #								
	0	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	5 [0]	5 [0]	5 [0]	0
4	0	0	0	0	0	0	0	0	0

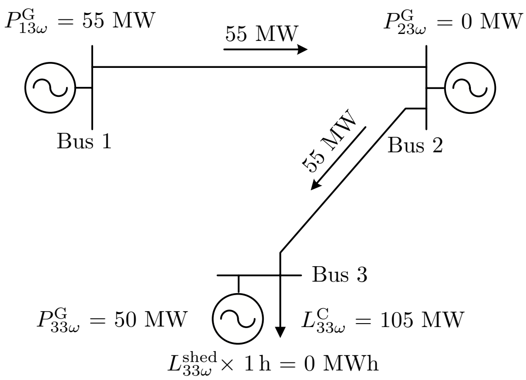
The only differences that can be detected between both scheduling programs concern period 3, a period characterized by the highest demand level (110 MW). The loss of line 2 (connecting nodes 1 and 3) brings about a substantial reduction of the network transmission capability, which, in particular, poses great difficulties for the system to meet the demand during period 3. Specifically, in the absence of this line, the maximum load that can be supplied at node 3 is 105 MW, 50 of which are generated by unit 3 located at that node. The remainder 55 MW come from unit 1 through lines 1 and 3, which become both saturated as a consequence. Against this situation, the system operator has two alternatives. The first one basically consists in doing nothing, and thereby, if the outage of line 2 finally materializes, those 5 MW missing to satisfy the complete demand are irremediably shed without the consent of the corresponding consumer. This corrective action would entail a very high cost, but the probability of having to put it into practice is also very low. Therefore, this operation strategy turns out to be the most economic option if the value of lost load is sufficiently low (e.g., \$15,000/MWh). In contrast, the system operator can *ask for the consent* of the consumer at node 3 to reduce its consumption in 5 MW in the case that the outage of line 2 comes into effect. Needless to say, this is speaking in a figurative sense. In fact, this *request* is negotiated in the electricity market through the scheduling and contracting of reserves with the load. This market operation also entails a cost. However, unlike the cost associated with the involuntary load shedding, this cost is usually much lower (the system operator has the permission from the load!), but *certain*, that is, it constitutes a cost to incur irrespective of the scenario that eventually materializes. Consequently, this option becomes profitable for values of lost load high enough (e.g., \$25,000/MWh). These two possible situations are depicted in Fig. 10.7.

On the other hand, the system operator *contracts* non-spinning reserve with unit 2 in order to cover the likely loss of the production of unit 3 without the need for resorting to involuntary load shedding actions. For this purpose, 55 MW of non-spinning reserve from unit 2 are scheduled if the value of lost load is equal to \$15,000/MWh. On the contrary, if this value is \$25,000/MWh, just 45 MW of this reserve are required since the system operator can also make use of the additional 5 MW of up-spinning reserve that, in this case, are contracted from the demand to eliminate the loss-of-load events pertaining to the outage of line 2. This issue explains the total reserve reduction in Fig. 10.6(c) and is further clarified in Fig. 10.8.

Lastly, Table 10.10 displays a detailed breakdown of the expected social costs both in absolute terms and in percentages of the total value (number indicated in parentheses). System operation costs include the cost of the energy required to materialize the deployment of reserves plus the cost of the energy transactions carried out in the electricity market. It can be observed that the *cost* associated with the scheduling of reserves is an increasing function of the value of lost load, as opposed to the decreasing trend exhibited by the cost of the energy not served. In short, this table highlights the intuitive fact



(a) $V^{\mathrm{LOL}} = \$15{,}000$/MWh

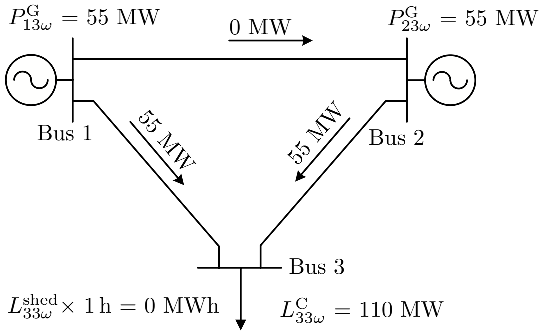


(b) $V^{\mathrm{LOL}} = \$25{,}000$/MWh

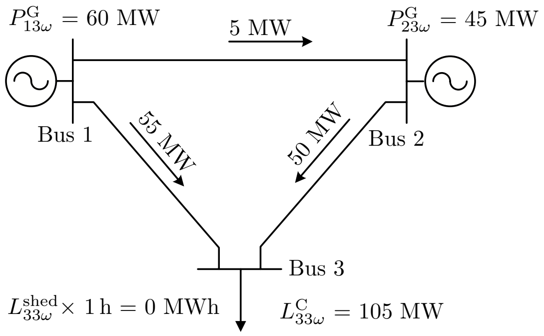
Fig. 10.7 Market-clearing example: effect of the outage of line 2 during period $t = 3$ in scenarios $\omega = 5$, 6 and 7

Table 10.10 Market-clearing example: breakdown of expected social costs ($)

V^{LOL} ($/MWh)	Total	Start-up	System operation	Reserve scheduling	Energy not served
1000	7051 (100)	200 (3)	6096 (86)	318 (5)	438 (6)
15,000	7428 (100)	200 (3)	6112 (82)	893 (12)	223 (3)
25,000	7498 (100)	200 (3)	6110 (81)	1188 (16)	0 (0)



(a) $V^{LOL} = \$15{,}000$/MWh



(b) $V^{LOL} = \$25{,}000$/MWh

Fig. 10.8 Market-clearing example: effect of the outage of unit 3 during period $t = 3$ in scenarios $\omega = 1$, 2 and 3

that, in order to eliminate likely load shedding events, and thus making the system security prevail over economic objectives, the system operator has to reinforce the preventive actions, i.e., to schedule a greater amount of reserves and/or to schedule more expensive reserves. Whichever the option chosen, this task entails additional costs.

10.7 Market-Clearing Case Study

In this section, the proposed electricity market-clearing model with stochastic security is further tested over a 24-h scheduling horizon on the 24-node power system described in Appendix B. This system comprises 17 loads, 12 generating units and 34 transmission lines.

In addition to further illustrating the model described in Section 10.4, this case study is aimed to appraise the impact of unit failures on both the magnitude and the composition of the expected cost of system operation. To this end, we consider a set of credible contingencies made up of all the single outages of the six generating units located at buses 13, 18 and 21–23. The capacities of these units sum up to 2351 MW, which represents almost 71% of the total capacity installed in the power system (3325 MW). The remaining six units are assumed to be 100% reliable, and consequently, they never fail. Likewise, neither sequential, nor compounded failures are taken into account in the analysis. Being so, the number of scenarios to be considered in the market-clearing formulation is 145, which results from the outage of every generating unit susceptible to failure occurring within any time period of the scheduling horizon (6×24), plus the scenario free of contingencies ($6 \times 24 + 1 = 145$).

As stated in Section 10.5, the time between two consecutive equipment failures is modeled using an exponential distribution univocally specified by the mean time to failure (MTTF). For simplicity and without loss of generality, in this case study we assume that the six generating units subject to a possible failure have all the same MTTF. Thus, we analyze three cases, namely, A, B and C, each characterized by a different value of the mean time to failure: 3000, 1000 and 200 hours, respectively, as indicated in Table 10.11. Note that, in relative terms, case A corresponds to a robust power system, while case C yields an electric system with a notably reduced level of reliability. Case B constitutes a situation halfway between cases A and C. The probability π_0 of the pre-contingency scenario, in which no failure event happens during the entire scheduling horizon, is indeed equal to 0.95, 0.87, and 0.55 for cases A, B, and C, respectively.

Table 10.11 Market-clearing case study: mean time to failure (MTTF) of generating units

Case	MTTF (h)
A	3000
B	1000
C	200

In order to make the proposed market-clearing problem tractable and solvable in a reasonable time using an ordinary computer, non-spinning reserves are not modeled. The mathematical treatment of such a type of reserve within a mixed-integer programming formulation requires binary variables $v_{it\omega}$ to model the commitment status of every generating unit in each period and scenario. Therefore, the number of these variables drastically increases with the number of scenarios considered. Hence, if non-spinning reserves are excluded from the market-clearing problem, the on/off state of every generating unit different from the one failing remains equal to the commitment schedule settled in the electricity market throughout the whole scheduling horizon. Logically, in the aftermath of a failure event, the binary variable representing the on/off status of the affected generating unit becomes necessarily zero. Disruptions caused by unit outages are then handled by means of spinning reserves and/or load shedding. From a mathematical viewpoint, non-spinning reserves are ruled out of the market-clearing problem by enforcing that $v_{it\omega} = u_{it}$, for all i, t and ω, with the consequent reduction in binary variables. However, in spite of this simplifying measure, the resulting market clearing is still hard to solve and includes 1,471,318 constraints and 1,004,197 variables, out of which 216 are binary.

Table 10.12 shows the breakdown of the expected cost for the three previously referred cases, A, B and C. The numbers in parentheses represent the proportion in percentages of each expected cost component with respect to the total. A value of lost load equal to \$2000/MWh is considered. By simple

Table 10.12 Market-clearing case study: breakdown of expected social cost (\$) for V^{LOL} = \$2000/MWh

Case	Total	Start-up	System operation	Reserve scheduling	Energy not served
A	470,094 (100)	5521 (1)	420,328 (89)	38,675 (8)	5570 (1)
B	479,744 (100)	5503 (1)	423,348 (88)	42,523 (9)	8370 (2)
C	511,423 (100)	5436 (1)	434,734 (85)	42,719 (8)	28,534 (6)

inspection of this table, we come to the conclusion that the reliability level of a power system has a relevant impact on how costly its operation is. Just observe the rise experienced by the expected total cost as the system becomes less reliable (from case A to case C). Particularly, note the remarkable increase of the cost associated with the expected load not served as the system reliability worsens. In an attempt to curb likely load shedding events, the amount of reserve scheduled in the electricity market increases and so does its associated cost, at least in absolute terms. The cost of system operation (that related to the energy production and consumption) is impacted by the system reliability as well, albeit to a lesser extent relatively speaking. Its rise has to do with the fact that more expensive, but also more flexible generating units (faster and wider ramping units) are committed to the energy dispatch program resulting from the market clearing so as to enhance the availability of spinning reserves.

As seen in the example of Section 10.6, a straightforward manner to *force* the scheduling of additional reserve consists in increasing the value of lost load. In this respect, Table 10.13 displays both the expected load not served and the amount of programmed reserve for different values of this parameter, specifically 2000, 4000 and \$10,000/MWh. Observe that, in cases A and B, as the value of lost load becomes higher, the megawatts of demand that are expected to be involuntarily shed decrease as a result of planning more reserve. This statement does not hold, however, for case C, in which both the amount of expected load not served and the quantity of scheduled reserve remain unchanged and equal to 14.27 and 9450 MW, respectively. The technical limitations and physical restrictions of the power system and its agents (generating units and loads) impose an upper bound on the amount of reserve that can be provided. This bound is precisely 9450 MW. Besides, with this quantity of reserve, the expected load not served throughout the scheduling horizon can be reduced, in case C, up to 14.27 MW at the most.

Table 10.13 Market-clearing case study: expected load not served (ELNS) and amount of scheduled reserve for different values of lost load (V^{LOL})

(a) ELNS (MW)

Case	V^{LOL} (\$/MWh)		
	2000	4000	10,000
A	2.79	1.53	1.47
B	4.19	4.10	4.10
C	14.27	14.27	14.27

(b) Total reserve (MW)

Case	V^{LOL} (\$/MWh)		
	2000	4000	10,000
A	8913	9360	9450
B	9400	9450	9450
C	9450	9450	9450

Finally, the costs of energy and security per megawatt and hour are shown in Table 10.14 for different values of lost load and cases A, B and C. The cost of energy is defined as the sum of the start-up and system operation costs. In order to express this cost in per megawatt and hour, the above sum is divided by the time length of the scheduling horizon (24 h) and the total capacity installed in the power system (3325 MW). Similarly, the cost of security is computed as the sum of the cost derived from load shedding events and the cost pertaining to the scheduling of reserves. Since the need for reserves and the occurrence of load shedding incidents are directly linked to unit failures, the result of this sum is divided by the time span of the scheduling horizon (24 h) and the aggregated capacity of all the units that are susceptible to fail (2351 MW). The cost of energy represents the cost stemming from the energy supply, while the cost of security expresses the cost of balancing supply and demand at all buses, in all time periods and for all scenarios. Observe in Table 10.14 that the cost of energy per megawatt and hour is not affected, at least appreciably, by the value of lost load or the reliability level of the power system. On the contrary, the cost of security is considerably influenced by both factors. In this sense, as the value of lost load increases and/or the reliability of system components deteriorates, the cost of security rises. This is a direct consequence of the scheduling of additional reserves and/or the upsurge of load shedding phenomena and their associated cost.

Table 10.14 Market-clearing case study: costs of energy and security in \$ per MWh

(a) Energy

Case	V^{LOL} (\$/MWh)		
	2000	4000	10,000
A	5.34	5.34	5.34
B	5.37	5.37	5.37
C	5.52	5.52	5.52

(b) Security

Case	V^{LOL} (\$/MWh)		
	2000	4000	10,000
A	0.78	0.86	1.02
B	0.90	1.05	1.48
C	1.26	1.77	3.29

On the other hand, note also that the cost of security keeps well below the cost of energy in all cases. In Section 11.6 of Chapter 11, we witness how the marked uncertain nature of wind power production can push the cost of system security up, much more than unit failures, at the same time that its zero energy production cost can lead to a significant reduction in the cost of energy.

Lastly, given that the market-clearing tool described in this chapter is computationally demanding, we conclude this case study pointing out that the CPU time required to solve the market-clearing problem with equipment failures keep below 40 minutes in all instances using CPLEX 11.2.1 under GAMS 23.0 [141] on a Linux-based server with two processors Quad Core AMD Opteron 2356 clocking at 2.3 GHz and 8 GB of RAM.

10.8 Summary and Conclusions

The functioning of electricity markets is strongly conditioned by the particularities of the product *electricity*, which concern its production, consumption, and especially, its transmission. The time gap existing between market *agreements* on energy transactions and their realization during the real-time operation of the power system leads to uncertainties as for the availability of system resources at operation time. Reserves are the ancillary services intended to accommodate these uncertainties, enabling the actual implementation of the market outcomes in real time while maintaining the normal functioning of the physical system.

In this challenging context, engineers and economists are called to design market-clearing procedures following the principles of economic efficiency without threatening system security.

In this chapter, a stochastic security-constrained market-clearing procedure is presented as a promising tool to attain this goal. According to the rationale behind the proposed procedure, energy and reserves are simultaneously traded within a common optimization framework. The security level linked to the resulting scheduling programs is appraised through a security measure, the *expected load not served*, that accounts for the probability of occurrence and magnitude of failure events.

Mathematically, this market-clearing procedure is formulated as a large-scale mixed-integer linear programming problem that can be solved using commercially available software.

The most relevant aspects of the proposed market-clearing formulation are pointed up using a small example and a larger-scale case study based on a 3-node and a 24-node electric systems, respectively. The main conclusions drawn from the analysis are:

1. Market-clearing procedures able to cope with equipment failures are imperative in order to resolve the tradeoff between system security and economic efficiency.
2. Network conditions play a fundamental role in the design of the preventive and corrective actions aimed to manage possible equipment failures.
3. The value of lost load has a great impact on the dispatch program resulting from the market clearing, especially on the reserve schedule. In fact, the value assigned to the involuntary load shedding can be seen as a parameter useful to strengthen system security by programming more reserves.
4. Comparatively, the cost related to the energy supply is barely affected by the reliability level of system components or the value of lost load. In contrast, the cost of security, which includes the cost of the reserve scheduling and the cost associated with the expected load not served, is strongly dependent on both factors.

10.9 Notation

The notation used throughout this chapter is stated below for quick reference.

Indices and Numbers:

i Index of generating units, running from 1 to N_{G}.
j Index of loads, running from 1 to N_{L}.

k Index of contingencies.
m Index of energy blocks offered by generating units, running from 1 to $N_{\mathrm{O}it}$ (number of blocks of energy offered by unit i in period t).
n, r Indices of system buses.
t Index of time periods, running from 1 to N_{T}.
τ Index of contingency occurrence intervals, running from 1 to N_{T}.
ω Index of failure scenarios, running from 1 to N_{Ω}.

Real Variables:

C_{it}^{SU} Cost due to the scheduled start-up of unit i in period t (\$). $C_{it\omega}^{\mathrm{SU}}$ is the actual start-up cost incurred by unit i in period t and scenario ω.
L_{jt}^{S} Power scheduled for load j in period t (MW). Bounded by $L_{jt}^{\mathrm{S,min}}$ and $L_{jt}^{\mathrm{S,max}}$.
P_{it}^{S} Power output scheduled for unit i in period t (MW).
R_{it}^{U} Spinning reserve up scheduled for unit i in period t (MW). Limited to $R_{it}^{\mathrm{U,\,max}}$.
R_{it}^{D} Spinning reserve down scheduled for unit i in period t (MW). Limited to $R_{it}^{\mathrm{D,\,max}}$.
R_{it}^{NS} Non-spinning reserve scheduled for unit i in period t (MW). Limited to $R_{it}^{\mathrm{NS,\,max}}$.
R_{jt}^{U} Spinning reserve up scheduled for load j in period t (MW). Limited to $R_{jt}^{\mathrm{U,\,max}}$.
R_{jt}^{D} Spinning reserve down scheduled for load j in period t (MW). Limited to $R_{jt}^{\mathrm{D,\,max}}$.
$C_{it\omega}^{\mathrm{A}}$ Cost due to the change in the start-up plan of unit i in period t and scenario ω (\$).
$P_{it\omega}^{\mathrm{G}}$ Power output of unit i in period t and scenario ω (MW). Bounded by P_i^{min} and P_i^{max}.
$p_{\mathrm{G}it\omega}(m)$ Power output corresponding to the m-th block of energy offered by unit i in period t and scenario ω (MW). Limited to $p_{\mathrm{G}it}^{\mathrm{max}}(m)$.
$L_{jt\omega}^{\mathrm{C}}$ Power consumed by load j in period t and scenario ω (MW).
$r_{it\omega}^{\mathrm{U}}$ Spinning reserve up deployed by unit i in period t and scenario ω (MW).
$r_{it\omega}^{\mathrm{D}}$ Spinning reserve down deployed by unit i in period t and scenario ω (MW).
$r_{it\omega}^{\mathrm{NS}}$ Non-spinning reserve deployed by unit i in period t and scenario ω (MW).

$r^{\mathrm{U}}_{jt\omega}$	Reserve up deployed by load j in period t and scenario ω (MW).
$r^{\mathrm{D}}_{jt\omega}$	Reserve down deployed by load j in period t and scenario ω (MW).
$L^{\mathrm{shed}}_{jt\omega}$	Load shedding imposed on consumer j in period t and scenario ω (MW).
$f_{t\omega}(n,r)$	Power flow through line (n,r) in period t and scenario ω (MW). Limited to $f^{\max}(n,r)$.
$\delta_{nt\omega}$	Voltage angle at node n in period t and scenario ω (rad).

Binary Variables:

u_{it}	0/1 variable equal to 1 if unit i is scheduled to be committed in period t.
$v_{it\omega}$	0/1 variable equal to 1 if unit i is online in period t and scenario ω.

Constants:

$C^{R^{\mathrm{U}}}_{it}$	Offer cost of the up-spinning reserve submitted by unit i in period t (\$/MWh).
$C^{R^{\mathrm{D}}}_{it}$	Offer cost of the down-spinning reserve submitted by unit i in period t (\$/MWh).
$C^{R^{\mathrm{NS}}}_{it}$	Offer cost of the non-spinning reserve submitted by unit i in period t (\$/MWh).
$C^{R^{\mathrm{U}}}_{jt}$	Offer cost of the up-spinning reserve submitted by load j in period t (\$/MWh).
$C^{R^{\mathrm{D}}}_{jt}$	Offer cost of the down-spinning reserve submitted by load j in period t (\$/MWh).
d_t	Duration of time period t (h).
$\lambda^{\mathrm{SU}}_{it}$	Start-up offer cost of unit i in period t (\$).
$\lambda_{\mathrm{G}it}(m)$	Marginal cost of the m-th block of energy offered by unit i in period t (\$/MWh).
$\lambda_{\mathrm{L}jt}$	Utility of consumer j in period t (\$/MWh).
V^{LOL}_{jt}	Value of load shed for consumer j in period t (\$/MWh).
π_{ω}	Probability of scenario ω.
$B(n,r)$	Absolute value of the imaginary part of the admittance of line (n,r) (per unit).

Sets:

$G(t,\omega)$	Set of available generating units in period t and scenario ω.
$\Lambda(t,\omega)$	Set of transmission lines that are functional in period t and scenario ω.
M_{L}	Mapping of the set of loads (consumers) into the set of buses.
$M_{\mathrm{G}}(t,\omega)$	Mapping of the sets of available generating units into the set of buses in period t and scenario ω.

10.10 Exercises

Exercise 10.1. Considering the three-node system described in Illustrative Example 10.1 compute the number of failure scenarios to be taken into account in the following cases:

a. The probabilities of occurrence of sequential and compound failures are known to be nil, i.e., only the failure of a single piece of equipment at a certain time period can happen during the whole scheduling horizon.
b. The probabilities of occurrence of sequential failures are equal to zero, i.e., only the failure of one or several pieces of equipment at a certain time period can occur during the entire scheduling horizon.
c. Every piece of equipment in the system may fail at any time period of the scheduling horizon with a non-zero probability.

Exercise 10.2. Obtain generalized formulae for the calculation of the number of possible failure scenarios in the cases enumerated in the previous exercise.

Exercise 10.3. A certain scheduling program, derived from the stochastic security-constrained market-clearing procedure detailed in this chapter, is associated with the load shedding events reported in Table 10.15.

Table 10.15 Exercise 10.3: involuntary load shedding (MWh)

Period #	Scenario #								
	0	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0	0	0
2	0	1	1	0	0	0	2	0	0
3	0	7	5	0	0	0	13	5	2
4	0	7	2	0	0	0	10	4	0

Knowing that the scenario probabilities are

$$
\begin{aligned}
\pi_\omega &= 0.001; \omega = 1, \dots, 4 \\
\pi_\omega &= 0.005; \omega = 5, \dots, 8 \\
\pi_0 &= 1 - \textstyle\sum_{\omega=1}^{8} \pi_\omega = 1 - 4 \times 0.001 - 4 \times 0.005 = 0.976,
\end{aligned}
$$

calculate the LOLP, LOLE and ELNS values associated with such a scheduling program.

Exercise 10.4. Repeat Illustrative Example 10.11, but considering the following updated statistical information:

a. Outages of unit 1 and of transmission lines 1 and 3 have been never reported.
b. Time between two consecutive failures of unit 1 follows an exponential distribution characterized by a MTTF equal to 500 hours. This same probability distribution applies also to unit 3.
c. Time between two consecutive failures of line 2 follows an exponential distribution with a MTTF of 1000 hours.

Exercise 10.5. Extend the market-clearing formulation described in Section 10.4 to include ramping limits of generating units and minimum-up and -down time constraints. References [3, 20] may be of significant help to accomplish such a task.

Exercise 10.6. Solve the example in Section 10.6 with a transmission capacity for line 3 reduced to 35 MVA. Is it possible to eliminate in this case the load shedding events for a sufficiently high value of lost load? Why?

Exercise 10.7. Solve the example in Section 10.6 for the scenario probabilities provided in Illustrative Example 10.1. Can it be stated that the three-node system is less reliable under these probabilities? Why? Provide numerical results to support your answer.

Exercise 10.8. Decompose the expected social cost obtained in Exercise 10.7 above by scenario. Is there any connection between the probability of each scenario and its expected cost proportion?

Exercise 10.9. Investigate ways to price energy and reserve transactions in the context of an electricity market-clearing with stochastic security. Start your inquiry consulting references [6, 16, 17, 58]. How the *price of energy* and the *price of security* are defined?

Exercise 10.10. With the aid of references such as [134] and [137], describe briefly the functioning of sequential electricity markets in which energy and reserve are separately scheduled and priced.

Chapter 11
Market Clearing under Uncertainty: Wind Energy

11.1 Introduction

Power systems are subject to a great variety of uncertainties. Restructuring and competition in electricity systems are definitely contingent on the available means to overcome the difficulties brought by these uncertainties. In the case of electricity trade, holding a competitive framework constitutes a task tougher than for other commodities due to the particularities of the electricity transactions. As a key example, the transmission of electrical energy is such that the production and consumption at a given bus of the system do affect the entire system. Therefore, the actions to be carried out in order to accommodate these uncertainties *without compromising the consistency of the trade* call for a global management able to involve, if required, the participation of every agent in the system.

In Chapter 10, the reserve was introduced as the instrument to face power system uncertainties, and thus, to support energy transactions in electricity markets while guaranteeing a secure use of the electrical infrastructure. In pursuit of a comprehensive restructuring, the reserve, like energy, is managed, scheduled and traded through electricity markets. In Chapter 10, we concluded that market-clearing procedures with stochastic security are appropriate tools to solve the tradeoff between economic efficiency and system security. In practice, reserve scheduling translates into pre-positioning generating units and loads in the best possible manner to respond effectively to the realization of uncertainties in accordance with the available resources and their cost. The response usually consists in altering the production and consumption levels of generators and loads.

The magnitude and characteristics of the uncertainties present in power systems are crucial to the planning of the reserve. In Chapter 10, we study the uncertainties related to equipment failures. One of the main features of this type of uncertainty is its discrete nature, i.e., there exists a finite number

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International Series in Operations Research & Management Science 153,
DOI 10.1007/978-1-4419-7421-1_11,

of plausible realizations of the uncertain variables leading to a finite number of system states.

In this chapter, the market-clearing methodology proposed in the previous one is revised in order to account for the uncertainty associated with the continuous stochastic phenomenon representing the *wind power production*. Relevant references that highlight and address the new challenges arising from the large-scale penetration of wind generation into power systems include [1, 15, 34, 35, 37, 38, 42, 63, 64, 94, 109].

The rest of this chapter is organized as follows. Section 11.2 provides a general view of some of the most burning issues surrounding the large-scale usage of wind energy in current power systems. Section 11.3 introduces the market-clearing model to manage wind power uncertainty emphasizing the differences with respect to the treatment of equipment failures. In Section 11.4, three performance metrics providing condensed information on the benefits and costs of wind integration are presented. An illustrative example is analyzed in detail in Section 11.5. In Section 11.6, the computational problems associated with the proposed market-clearing procedure are posed and discussed with the aid of a 24-node case study. Lastly, Section 11.7 summarizes and concludes the chapter.

11.2 Wind Power Production

11.2.1 A Look to the Near Future: Wind Generation

The issue of climate change has sparked discussion on the benefits of limiting industrial emissions of greenhouse gases in comparison with the costs that such alterations would entail. Reaching an agreement on this controversial subject seems to be difficult due to the time scales and uncertainties involved. For the time being, climate change can only be strictly appraised in terms of the projected impacts for the coming decades, centuries, or even millennia of increasing temperatures, rising sea levels, heat waves, droughts, reduced crop yields and species extinction. However, beyond this controversy, one thing seems clear: if we desire to avoid the worst and irreversible damages of a plausible climate change, then global greenhouse gas emissions must begin to decline in the near future.

Governments in industrialized nations are currently subject to public opinion demanding actions to head off the worst impacts of climate change. These actions require addressing fundamental, long term structural adjustments to the global economy. A significant part of these structural changes is to be carried out in the electricity generation sector, where curbing emissions of greenhouse gases causing global warming is nowadays one of the most pressing issues. In this sector, the measures undertaken to meet emission reduction targets have basically consisted in increasing the level of penetration of renewable and low carbon electricity generation resources, with wind power generation being the resource par excellence. In fact, at this time, wind energy is the only power generation technology that can enable the cuts in CO_2 emissions from the power sector necessary for fulfilling the emission reduction targets agreed in some industrialized countries [37, 63].

According to the data provided by the Global Wind Energy Council (GWEC) in [64], the world's total installed wind capacity amounted to 158.5 GW at the end of 2009, more than 38 GW of which came online in 2009 alone, representing a 41.5% growth rate with respect to the wind capacity that was installed just in 2008 and a year-on-year increase of 31.7%. These 158.5 GW of global wind capacity are expected to save around 200 million tons of CO_2 every year. Furthermore, wind energy is firmly penetrating the world's energy markets, with a market for turbine installations valued at about $63 billion during 2009. This emerging economic activity is responsible for the creation of many new jobs. The GWEC estimates that over half a million people were working in the wind industry in 2009, a number that is expected to increase up to millions in the near future. The GWEC also forecasts that in 2014 the global wind generating capacity will reach 409 GW, 62.5 GW of which will be added precisely during 2014, compared to 38.3 GW of wind capacity introduced in 2009. The annual growth rates during the period 2010-2014 will average 20.9% in terms of total installed capacity.

Wind generation is free of emissions and promotes sustainable development. The energy source, the force of the wind, is indigenous and, as such, geopolitically generous, encouraging the self-sufficiency in energy of nations. Moreover, it uses no water. Technically, wind power is fast to deploy, economical and competitive, and generates employment. As wind farms do not consume fuel, they can reduce fuel costs and offer a hedge against fuel price volatility. Wind power plants have low forced outage rates and can contribute to lessen the need for polluting generation sources.

11.2.2 All That Glitters Is Not Gold: Wind Impact on System Security

Wind power cannot be dispatched in a traditional sense due to the inherent randomness of the natural phenomenon involved: the wind. Therefore, wind generation is variable and uncertain, and consequently, its large-scale integration into a power system constitutes a unique challenge for system operators and planners [1, 34, 35].

Actually, the management of uncertainties in a power system is not new for practitioners. The energy demand is also variable and uncertain, just like wind generation, and system agents have been coping with the natural variability and stochasticity of demand since the dawn of the power industry. Then, what is the problem? Where is the challenge?

The answers to these questions are manifold. First, the integration of wind generation into a power system entails the consideration of additional amounts of uncertainty and variability in the operation of the system. This basic fact may be of the utmost importance taking into account the exigent wind penetration marks that several countries have set out to achieve. Part of this variability can be predicted some hours or days ahead. The uncertain part of the variability is managed with reserves in the power system. As the level of wind power generation increases, the need for spinning and non-spinning reserves increases to maintain system security, and this increase translates into higher reserve costs. In either case, the variability of wind power production, uncertain or predictable, requires operating the power system with a higher degree of flexibility in order to coordinate the following of the fluctuating load and the variable output of wind generation. This greater flexibility usually leads to operate conventional generation at lower/higher production levels in an attempt to accommodate the inherent variability of wind generation by ramping up or down. These ramping excursions may often end up with the start-up or shut-down of conventional units.

On the other hand, the variability of power output has an impact on the capacity credit of wind generation. The capacity credit is a measure of the amount of conventional generation that could be displaced by the renewable production without making the system any less reliable. Hence, capacity credits for wind plants do not approach nameplate ratings due to the non-controllability of the underlying energy source. In plain words, this means that in practice 1 MW of installed wind capacity is not adequate to cover 1 MW of demand in a reliable and secure manner.

For all these reasons, new system operation planning methods are required to deal with the intricate nature of wind generation while preserving or even enhancing the current reliability and economic performance of power systems.

11.2.3 Accommodating Wind Uncertainty in Electricity Markets

The economic performance of power systems is contingent on the correct functioning of the electricity markets. According to the most basic economic principles, electricity markets driven by the *invisible hand* of competition should guarantee the economically efficient operation of electrical systems. However, when wind power *comes on stage* in high levels, the competitive functioning can be altered as the energy transactions settled in these markets may not be implemented in real-time, exactly as agreed, for security reasons. Therefore, the proper integration of wind generation into power systems calls for markets decisions relying on economic criteria while maintaining or even improving the reliability of the electrical infrastructure and its operation.

The reliability and security of power systems is largely dependent on the reserve management, which, in turn, is conditional on the amount of wind power planned [1]. In this respect, there are several issues related to reserves that need to be assessed:

1. Scheduling and allocation of reserves: how much capacity has to be allocated for each day. The system operator must ensure that an adequate amount of reserve is kept in power plants in order to face the unpredictable variability of wind generation.
2. Deployment or use of reserves: how the scheduled reserves are utilized in real time as the wind uncertainty is revealed.

All these issues can be tackled suitably by means of a revised version of the stochastic security-constrained market-clearing procedure introduced in Chapter 10. As stated in this chapter, this procedure constitutes a flexible methodology able to reconcile economic efficiency and system security for clearing electricity markets with high levels of uncertainty, as in the case of markets working with high penetration of wind power. In this methodology, reserve requirements are driven by the tradeoff between the value of lost load and those of the energy and reserve offers. The procedure embeds the advantages of the simultaneous energy and reserve market-clearing paradigm [58] and makes use of the modeling capability of the stochastic programming [14].

The market-clearing model is formulated as a two-stage stochastic programming problem, where the first and second stages represent, respectively, the electricity market and the real-time operation of the power system. In consequence, the variables belonging to the first stage are the market decisions that envisage the impact of the plausible realizations of wind power uncertainty on system variables, i.e., on second-stage variables.

11.2.4 The Handicap: The Computational Burden

The main drawback of this type of market-clearing model with stochastic security is, by far, the computational burden involved. In this sense, it should be noted that the methodology presented in Chapter 10, and tailored here to integrate wind generation, is built on a global optimization framework accounting for every state of the system under every possible realization of the uncertain events (equipment failures and/or wind power). For large-scale systems, this translates into a two-stage stochastic programming problem with a computationally heavy second stage including many variables and constraints. Although, currently, the technical literature offers various techniques to alleviate this burden (some of which have been studied in Chapter 3), they may be insufficient.

Hopefully, the steady progress in computing technologies could well overcome those difficulties in the future. In this sense, it is comforting to know that the complex algorithms used at present to run the large areas covered by some of the ISOs in the US would have been deemed impractical 25 years ago due to their computational requirements. Moreover, in the meantime, the market-clearing models presented can be used by the system operator as a tool to evaluate the reserve needs from a market viewpoint in the short, mid and long term [94]. This analysis would be based on historical records and forecasts of market agents' behavior and wind power production.

11.3 Market-Clearing Model

In order to cope with the uncertainty of wind generation, some adjustments are to be implemented within the market-clearing model presented in Chapter 10. This section examines the required changes in detail. In spite of the necessary revision to be performed, the main features characterizing the previous market-clearing procedure remain intact. Thus, the management of the stochastic nature of wind power generation is accomplished through a two-stage stochastic programming problem where the first stage models the functioning of the electricity market and the second one, the real-time operation of the power system.

Mathematically, we build an optimization problem in which the objective function, representing the expected social cost, is subject to three sets of constraints: the first one modeling the energy and reserve transactions in the electricity market; the second one modeling the actual operation of the power system once the wind power production is certainly known; and lastly, the linking constraints that couple market decisions with real-time operating actions executed via the deployment of reserves.

From a mathematical programming viewpoint, the optimization model is formulated as a large-scale mixed-integer linear problem [99] that can be solved using commercially available software [141].

11.3.1 Assumptions

Apart from the considerations concerning the linear approximation of the network and the cost of the reserve deployment (we refer the reader to Subsection 10.4.1 in Chapter 10), the market-clearing formulation designed to handle wind uncertainty is based on the following additional assumptions:

1. Without loss of generality, equipment failures are not considered. This assumption allows us to underline the differences between the treatment of wind uncertainty and equipment contingencies.
2. For the sake of simplicity, wind generation is located at a single node of the power network. If wind power is produced and then injected at different nodes within the grid, the rigorous management of wind uncertainty calls for the statistical analysis and modeling of the likely spatial interrelations among wind sites. The technique developed in Chapter 3 to deal with spatially dependent wind stochastic processes can be used for this purpose.
3. Wind generation is assumed to be a regulated activity and thus wind producers do not compete in the market. Consequently, wind generation is either considered a negative demand or spilled, and thus, paid a regulated tariff. Note that this is the case in which wind power is treated in most energy systems throughout the world. Additionally, observe that this assumption is equivalent to considering that wind generators offer in the pool their actual productions at zero prices.

It should be stressed that these three assumptions are made just in order to free the proposed market-clearing procedure of complexities that can complicate its understanding unnecessarily.

11.3.2 Wind Uncertainty Characterization

We model wind power generation in time period t as a random variable denoted by P_t^{WP}. The collection $\boldsymbol{P}^{\mathrm{WP}}$ of dependent random variables P_t^{WP} throughout a scheduling horizon of N_{T} periods, i.e., $P^{\mathrm{WP}} = \{P_t^{\mathrm{WP}}, t = 1, 2, \ldots, N_{\mathrm{T}}\}$, constitutes a stochastic process [110]. In stochastic programming (see Chapters 2 and 3), the treatment of a stochastic process within an optimization problem usually requires its discretization into a set of scenarios w describing its plausible realizations.

In the context of this chapter, each scenario $\omega \in \Omega$ is a vector of wind power productions with N_{T} components, i.e.:

$$\text{Scenario } \omega : \{P_{1\omega}^{\mathrm{WP}}, \ldots, P_{N_{\mathrm{T}}\omega}^{\mathrm{WP}}\}.$$

In other words, $P_{t\omega}^{\mathrm{WP}}$ represents the realization of random variable P_t^{WP} in scenario ω.

Each scenario ω has a probability of occurrence π_ω, such that the summation of the probabilities over all scenarios is equal to 1, i.e., $\sum_{\omega=1}^{N_\Omega} \pi_\omega = 1$.

Illustrative Example 11.1 (Wind power scenarios). Table 11.1 provides an example of wind power scenarios. The scheduling horizon spans four time periods. The wind power uncertainty is modeled via three scenarios: *as forecast*, *high* and *low*, with probabilities 0.6, 0.2 and 0.2, respectively.

Table 11.1 Illustrative Example 11.1: wind power scenarios

Period #	$P^{\mathrm{WP}}_{t\omega}$ (MW)		
	As forecast	High	Low
1	6	9	2
2	20	30	13
3	35	50	25
4	8	12	6

□

The treatment of the uncertainty associated with wind power in a stochastic programming framework should generally comprise two phases. The first one consists in the accurate modeling of the wind power stochastic behavior by generating a sufficient number of scenarios representing the most plausible realizations of wind power throughout the scheduling horizon (see Chapter 3, Section 3.2). The second phase is required because the size of this initial scenario set is usually too large, resulting in an optimization model that is intractable. Hence, to achieve tractability, statistical techniques [44,74,75,97] are applied to reduce the number of scenarios while retaining the essential features of the original scenario set (see Chapter 3, Section 3.3).

Finally, the set of scenarios together with the decision timing of the problem can be arranged in a two-stage scenario tree as illustrated in Fig. 11.1. The first stage (root node) represents the electricity market, where decisions pertaining to the scheduling of energy and reserve are made. The second stage (leaf nodes) constitutes the real-time operation of the power system which revolves around the deployment of reserves in order to accommodate the specific realization (scenario) of the wind power production. From an intuitive point of view, the aim of any stochastic programming model in the market-clearing procedure is to *locate* the root node at a stochastically equidistant position with respect to the leaf nodes.

11.3.3 Wind Uncertainty vs. Equipment Failures

The market-clearing formulation introduced in Chapter 10 to deal with equipment failures can be easily revised in order to handle the uncertainty of wind power generation. The changes to be undertaken are relatively minor and the aim of this section is precisely to highlight the resulting differences.

The decisions made in the electricity market should account for the contribution of wind generation to the energy supply. Needless to say, this contribution is uncertain by the time the market is cleared, and therefore, the scheduling of production units and loads should be carried out with the aim

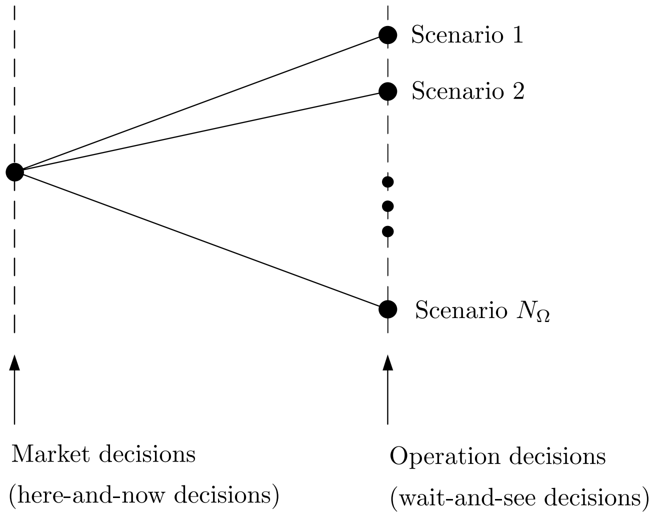


Fig. 11.1 Scenario tree example for the market-clearing problem with wind generation

of leaving enough *room* in the power network to accommodate the eventual outcome of the wind power production at minimum cost. Mathematically, the *room* left in the power system for the wind generation is modeled through a continuous variable $P_t^{\text{WP, S}}$ that is added to the market equilibria:

$$\sum_{i=1}^{N_\text{G}} P_{it}^\text{S} + P_t^{\text{WP, S}} = \sum_{j=1}^{N_\text{L}} L_{jt}^\text{S}, \quad \forall t. \tag{11.1}$$

Variable $P_t^{\text{WP, S}}$ can be interpreted as the amount of wind power *scheduled* in the electricity market for period t and is logically bounded by the wind generation limits, i.e.:

$$P_t^{\text{WP, min}} \leq P_t^{\text{WP, S}} \leq P_t^{\text{WP, max}}, \quad \forall t, \tag{11.2}$$

where $P_t^{\text{WP, min}}$ and $P_t^{\text{WP, max}}$ are generally considered zero and equal to the wind power installed capacity, respectively.

Illustrative Example 11.2 (Market equilibria). Let us consider the three-node system presented in Illustrative Example 10.1. We place a 60-MW wind generator at node 2; see Fig. 11.2.

For this system, the market equilibria (11.1) become

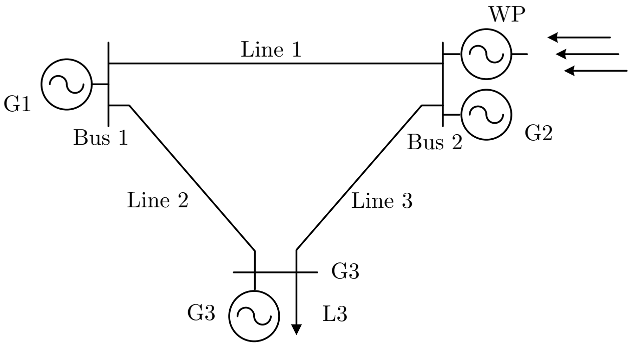


Fig. 11.2 Illustrative Example 11.2: three-bus system including a wind generator at bus 2

$$\begin{aligned}
&P_{11}^{\mathrm{S}} + P_{21}^{\mathrm{S}} + P_{31}^{\mathrm{S}} + P_{1}^{\mathrm{WP,\,S}} = 30 \quad && (t=1)\\
&P_{12}^{\mathrm{S}} + P_{22}^{\mathrm{S}} + P_{32}^{\mathrm{S}} + P_{2}^{\mathrm{WP,\,S}} = 80 \quad && (t=2)\\
&P_{13}^{\mathrm{S}} + P_{23}^{\mathrm{S}} + P_{33}^{\mathrm{S}} + P_{3}^{\mathrm{WP,\,S}} = 110 \quad && (t=3)\\
&P_{14}^{\mathrm{S}} + P_{24}^{\mathrm{S}} + P_{34}^{\mathrm{S}} + P_{4}^{\mathrm{WP,\,S}} = 40 \quad && (t=4).
\end{aligned}$$

The contribution of wind generation to the energy supply is constrained by the wind production limits, i.e.,

$$0 \le P_t^{\mathrm{WP,\,S}} \le 60, \quad \forall t.$$

□

On the other hand, the modeling of the real-time operation of the power system should include the actual power $P_{t\omega}^{\mathrm{WP}}$ injected into the grid from wind resources. Equation (11.3) states the power balance at node n' at which the wind power generation is injected.

$$\sum_{i:(i,n)\in M_{\mathrm{G}}} P_{it\omega}^{\mathrm{G}} - \sum_{j:(j,n)\in M_{\mathrm{L}}} \left(L_{jt\omega}^{\mathrm{C}} - L_{jt\omega}^{\mathrm{shed}}\right) + P_{t\omega}^{\mathrm{WP}} - S_{t\omega} - \sum_{r:(n,r)\in \Lambda} f_{t\omega}(n,r) = 0, \;\; n = n', \forall t, \forall \omega. \quad (11.3)$$

The continuous variable $S_{t\omega}$ represents the wind power generation *spilled* (and therefore not utilized) in period t and scenario ω. The amount of wind power spilled has to be greater than or equal to zero, and smaller than or equal to the actual wind power production, i.e.,

$$0 \le S_{t\omega} \le P_{t\omega}^{\mathrm{WP}}, \;\; \forall t, \forall \omega. \quad (11.4)$$

The occurrence of wind power spillage can be due to both economic and technical reasons. As we already know, the uncertain nature of wind generation requires scheduling reserves to guarantee and preserve system security. This operation entails a cost. Consequently, if the benefit inherent to the cost-free character of wind energy is smaller than the cost associated with the management of its unpredictability, then wind spillage turns out to be profitable. Likewise, the variability of wind generation calls for a flexible operation of the power system. Hence, the physical limitations of the electrical infrastructure impose a cap on the amount of wind power that can be injected into the network and the remaining wind power production has to be spilled as a consequence.

For completeness, equation (11.5) establishes the power balance at every node n different from node n' at which the wind power generation is injected.

$$\sum_{i:(i,n)\in M_{\mathrm{G}}} P^{\mathrm{G}}_{it\omega} - \sum_{j:(j,n)\in M_{\mathrm{L}}} \left(L^{\mathrm{C}}_{jt\omega} - L^{\mathrm{shed}}_{jt\omega}\right) - \sum_{r:(n,r)\in \Lambda} f_{t\omega}(n,r) = 0, \quad \forall n \neq n', \forall t, \forall \omega. \quad (11.5)$$

Note that both the set Λ of transmission lines and the mapping M_{G} of the set of loads into set of nodes are independent of the scenario ω in (11.3) and (11.5), because we do not take into account equipment failures in this chapter.

Illustrative Example 11.3 (Power balance constraints). Consider the three-node system depicted in Illustrative Example 11.2 and the three scenarios ω modeling wind power uncertainty in Illustrative Example 11.1.

The power balance constraints (11.3) and (11.5) for the scenario named *as forecast* ($\omega = 1$) are

$$\begin{aligned}
&\text{Node 1:} && P^{\mathrm{G}}_{1t1} = f_{t1}(1,2) + f_{t1}(1,3), \quad \forall t. \\
&\text{Node 2:} && P^{\mathrm{G}}_{211} + 6 - S_{11} = f_{11}(2,1) + f_{11}(2,3) \quad (t=1) \\
& && P^{\mathrm{G}}_{221} + 20 - S_{21} = f_{21}(2,1) + f_{21}(2,3) \quad (t=2) \\
& && P^{\mathrm{G}}_{231} + 35 - S_{31} = f_{31}(2,1) + f_{31}(2,3) \quad (t=3) \\
& && P^{\mathrm{G}}_{241} + 8 - S_{41} = f_{41}(2,1) + f_{41}(2,3) \quad (t=4). \\
&\text{Node 3:} && P^{\mathrm{G}}_{3t1} + L^{\mathrm{shed}}_{3t1} - L^{\mathrm{C}}_{3t1} = f_{t1}(3,1) + f_{t1}(3,2), \quad \forall t,
\end{aligned}$$

where

$$\begin{aligned}
0 \le S_{11} \le 6 \\
0 \le S_{21} \le 20 \\
0 \le S_{31} \le 35 \\
0 \le S_{41} \le 8.
\end{aligned}$$

Similarly, Equations (11.3) and (11.5) for the scenario referred to as *high* ($\omega = 2$) become

$$\begin{aligned}
\text{Node 1:}\quad & P^{\mathrm{G}}_{1t2} = f_{t2}(1,2) + f_{t2}(1,3), \quad \forall t. \\
\text{Node 2:}\quad & P^{\mathrm{G}}_{212} + 9 - S_{12} = f_{12}(2,1) + f_{12}(2,3) \qquad (t=1) \\
& P^{\mathrm{G}}_{222} + 30 - S_{22} = f_{22}(2,1) + f_{22}(2,3) \quad (t=2) \\
& P^{\mathrm{G}}_{232} + 50 - S_{32} = f_{32}(2,1) + f_{32}(2,3) \quad (t=3) \\
& P^{\mathrm{G}}_{242} + 12 - S_{42} = f_{42}(2,1) + f_{42}(2,3) \quad (t=4). \\
\text{Node 3:}\quad & P^{\mathrm{G}}_{3t2} + L^{\mathrm{shed}}_{3t2} - L^{\mathrm{C}}_{3t2} = f_{t2}(3,1) + f_{t2}(3,2), \quad \forall t,
\end{aligned}$$

with

$$\begin{aligned}
0 \le S_{12} \le 9 \\
0 \le S_{22} \le 30 \\
0 \le S_{32} \le 50 \\
0 \le S_{42} \le 12.
\end{aligned}$$

Finally, the power balance constraints (11.3) and (11.5) for the scenario labelled as *low* ($\omega = 3$) are

$$\begin{aligned}
\text{Node 1:}\quad & P^{\mathrm{G}}_{1t3} = f_{t3}(1,2) + f_{t3}(1,3), \quad \forall t. \\
\text{Node 2:}\quad & P^{\mathrm{G}}_{213} + 2 - S_{13} = f_{13}(2,1) + f_{13}(2,3) \qquad (t=1) \\
& P^{\mathrm{G}}_{223} + 13 - S_{23} = f_{23}(2,1) + f_{23}(2,3) \quad (t=2) \\
& P^{\mathrm{G}}_{233} + 25 - S_{33} = f_{33}(2,1) + f_{33}(2,3) \quad (t=3) \\
& P^{\mathrm{G}}_{243} + 6 - S_{43} = f_{43}(2,1) + f_{43}(2,3) \qquad (t=4). \\
\text{Node 3:}\quad & P^{\mathrm{G}}_{3t3} + L^{\mathrm{shed}}_{3t3} - L^{\mathrm{C}}_{3t3} = f_{t3}(3,1) + f_{t3}(3,2), \quad \forall t,
\end{aligned}$$

with

$$\begin{aligned}
0 \le S_{13} \le 2 \\
0 \le S_{23} \le 13 \\
0 \le S_{33} \le 25 \\
0 \le S_{43} \le 6.
\end{aligned}$$

□

The last change to be introduced into the market-clearing formulation has to do with the non-anticipativity constraints (10.34)–(10.36) stated in Chapter 10 (page 381). Recall that the aim of these constraints is to guarantee that no corrective action is executed before the occurrence of a failure event. As a consequence, the real-time operation of the power system follows the scheduling program settled in the electricity market as long as no contingency takes place. The insertion of such constraints into the market-clearing model is closely linked to the discrete nature of equipment contingencies, which manifests itself as a system failure occurring at a certain instant τ in time within the scheduling horizon. Prior to that instant, there is no sign of the impending failure, there is no way to preempt it, and therefore, there is nothing to react against. In contrast, wind power constitutes a stochastic process whose uncertainty is revealed on a continuous basis, which compels the system operator to undertake corrective measures from the very beginning of the scheduling horizon. For this reason, the non-anticipativity constraints (10.34)–(10.36) become inconsistent when dealing with wind uncertainty, and consequently, they are removed from the revised market-clearing formulation presented in this chapter.

11.3.4 Breaking Down the Expected Cost

On the assumption that generating units are 100% reliable, and as a result, there is no need to cope with the uncertainty of unit unavailability, the operation costs associated with the energy production can be easily split up into two terms. The first one constitutes a first-stage cost related to the energy and reserve scheduling in the electricity market, and the second one is a second-stage cost derived from the reserves deployed in real time to accommodate the uncertain variability of wind generation. This breakdown is possible thanks to the *decomposable* nature of the generator power outputs,

$$P^{\mathrm{G}}_{it\omega} = P^{\mathrm{S}}_{it} + r^{\mathrm{U}}_{it\omega} + r^{\mathrm{NS}}_{it\omega} - r^{\mathrm{D}}_{it\omega}, \quad \forall i, \forall t, \forall \omega. \tag{11.6}$$

According to equation (11.6), the power produced by unit i in time period t and scenario ω can be seen as the summation of the power production agreed in the electricity market, P^{S}_{it}, plus the amount of power generated in the form of reserves during the real-time operation of the power system, $r^{\mathrm{U}}_{it\omega} + r^{\mathrm{NS}}_{it\omega} - r^{\mathrm{D}}_{it\omega}$. In order to translate equation (11.6) into costs via the energy cost function of the generating unit i under consideration, we need to decompose both additive terms into blocks. With this aim in mind, we define variables $p_{\mathrm{G}it}(m)$ and $r_{\mathrm{G}itw}(m)$ representing, respectively, the power produced and the reserve deployed from the m-th block of energy offered by unit i in period t and scenario ω. This way, the block decomposition of P^{S}_{it} is accomplished by equations (11.7) and (11.8).

$$0 \leq \; p_{\mathrm{G}it}(m) \leq p^{\max}_{\mathrm{G}it}(m), \quad \forall m, \forall i, \forall t \tag{11.7}$$

$$P^{\mathrm{S}}_{it} = \sum_{m=1}^{N_{\mathrm{O}it}} p_{\mathrm{G}it}(m), \quad \forall i, \forall t. \tag{11.8}$$

Equation (11.9) below is analogous to equation (11.8) and states the decomposition of the reserve deployment by blocks through variables $r_{\mathrm{G}itw}(m)$. Equations (11.10) and (11.11) enforce that the blocks of reserve are added (or subtracted in case of down-spinning reserve) to the blocks of energy. Thus, the cost of the deployment of reserve per unit, period and scenario is equal to $\sum_{m=1}^{N_{\mathrm{O}it}} \lambda_{\mathrm{G}it}(m) r_{\mathrm{G}it\omega}(m)$.

$$r_{it\omega}^{\mathrm{U}} + r_{it\omega}^{\mathrm{NS}} - r_{it\omega}^{\mathrm{D}} = \sum_{m=1}^{N_{\mathrm{O}it}} r_{\mathrm{G}it\omega}(m), \quad \forall i, \forall t, \forall \omega \tag{11.9}$$

$$r_{\mathrm{G}it\omega}(m) \leq p_{\mathrm{G}it}^{\max}(m) - p_{\mathrm{G}it}(m), \quad \forall m, \forall i, \forall t, \forall \omega \tag{11.10}$$

$$r_{\mathrm{G}it\omega}(m) \geq -p_{\mathrm{G}it}(m), \quad \forall m, \forall i, \forall t, \forall \omega. \tag{11.11}$$

Illustrative Example 11.4 (Block decomposition). Assume that, for the first period of the market horizon, generating unit 1 of the three-node system in Fig. 11.2 submits to the electricity market the three-block energy offer cost function depicted in Fig. 11.3 and numerically described in Table 11.2.

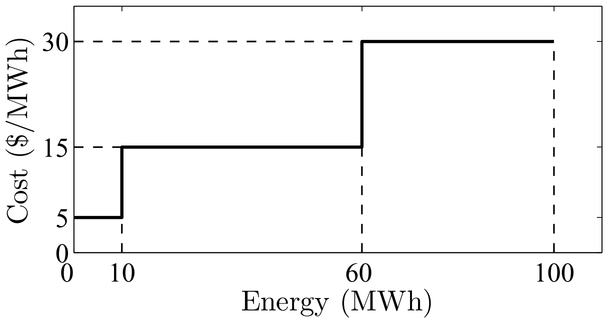


Fig. 11.3 Illustrative Example 11.4: energy offer cost function submitted by unit 1 for the first period of the scheduling horizon

Table 11.2 Illustrative Example 11.4: energy blocks and their cost

Block #	1	2	3
Size (MWh)	10	50	40
Cost ($/MWh)	5	15	30

In this case, constraints (11.7) and (11.8) become

$$
\begin{aligned}
0 \leq\ & p_{\mathrm{G}11}(1) \leq 10 \\
0 \leq\ & p_{\mathrm{G}11}(2) \leq 50 \\
0 \leq\ & p_{\mathrm{G}11}(3) \leq 40 \\
P_{11}^{\mathrm{G}} = p_{\mathrm{G}11}&(1) + p_{\mathrm{G}11}(2) + p_{\mathrm{G}11}(3).
\end{aligned}
$$

Likewise, the decomposition of the deployed reserves into blocks is carried out by means of equations (11.9)–(11.11) as follows:

$$
\begin{aligned}
r_{11\omega}^{\mathrm{U}} + r_{11\omega}^{\mathrm{NS}} - r_{11\omega}^{\mathrm{D}} &= r_{\mathrm{G}11\omega}(1) + r_{\mathrm{G}11\omega}(2) + r_{\mathrm{G}11\omega}(3), \ \ \forall\omega \\
r_{\mathrm{G}11\omega}(1) &\leq 10 - p_{\mathrm{G}11}(1), \ \ \forall\omega \\
r_{\mathrm{G}11\omega}(2) &\leq 50 - p_{\mathrm{G}11}(2), \ \ \forall\omega \\
r_{\mathrm{G}11\omega}(3) &\leq 40 - p_{\mathrm{G}11}(3), \ \ \forall\omega \\
r_{\mathrm{G}11\omega}(1) &\geq -p_{\mathrm{G}11}(1), \ \ \forall\omega \\
r_{\mathrm{G}11\omega}(2) &\geq -p_{\mathrm{G}11}(2), \ \ \forall\omega \\
r_{\mathrm{G}11\omega}(3) &\geq -p_{\mathrm{G}11}(3), \ \ \forall\omega.
\end{aligned}
$$

□

Finally, the introduction of the generation decomposition above into the objective function of the market-clearing model yields the following revised expression of the expected cost:

$$
\begin{aligned}
\mathrm{EC} = \sum_{t=1}^{N_\mathrm{T}} \mathrm{EC}_t = \\
= & \sum_{t=1}^{N_\mathrm{T}} \sum_{i=1}^{N_\mathrm{G}} C_{it}^{\mathrm{SU}} \\
& + \sum_{t=1}^{N_\mathrm{T}} d_t \Bigg[\sum_{i=1}^{N_\mathrm{G}} \sum_{m=1}^{N_{\mathrm{O}it}} \lambda_{\mathrm{G}it}(m) p_{\mathrm{G}it}(m) - \sum_{j=1}^{N_\mathrm{L}} \lambda_{\mathrm{L}jt} L_{jt}^{\mathrm{S}} \\
& \qquad + \sum_{i=1}^{N_\mathrm{G}} \left(C_{it}^{R^\mathrm{U}} R_{it}^{\mathrm{U}} + C_{it}^{R^\mathrm{D}} R_{it}^{\mathrm{D}} + C_{it}^{R^\mathrm{NS}} R_{it}^{\mathrm{NS}} \right) \\
& \qquad + \sum_{j=1}^{N_\mathrm{L}} \left(C_{jt}^{R^\mathrm{U}} R_{jt}^{\mathrm{U}} + C_{jt}^{R^\mathrm{D}} R_{jt}^{\mathrm{D}} \right) \Bigg] \\
& + \sum_{\omega=1}^{N_\Omega} \pi_\omega \Bigg\{ \sum_{t=1}^{N_\mathrm{T}} \sum_{i=1}^{N_\mathrm{G}} C_{it\omega}^{\mathrm{A}} \\
& \qquad + \sum_{t=1}^{N_\mathrm{T}} d_t \Bigg[\sum_{i=1}^{N_\mathrm{G}} \sum_{m=1}^{N_{\mathrm{O}it}} \lambda_{\mathrm{G}it}(m) r_{\mathrm{G}it\omega}(m) \\
& \qquad\qquad + \sum_{j=1}^{N_\mathrm{L}} \lambda_{\mathrm{L}jt} \left(r_{jt\omega}^{\mathrm{U}} - r_{jt\omega}^{\mathrm{D}} \right) \\
& \qquad\qquad + \sum_{j=1}^{N_\mathrm{L}} V_{jt}^{\mathrm{LOL}} L_{jt\omega}^{\mathrm{shed}} \Bigg] \Bigg\} .
\end{aligned}
\tag{11.12}
$$

Specifically, two changes can be identified in (11.12) with respect to the definition of the expected cost (10.4) provided in Chapter 10 (page 368), namely:

1. The second-stage energy cost $\lambda_{\mathrm{G}it}(m) p_{\mathrm{G}it\omega}(m)$ in (10.4) (page 368) is broken down into the first- and second-stage energy costs $\lambda_{\mathrm{G}it}(m) p_{\mathrm{G}it}(m)$ and $\lambda_{\mathrm{G}it}(m) r_{\mathrm{G}it\omega}(m)$, respectively, in line with the power decomposition proposed in this subsection. Note that the use of subscript ω is the key element to make the difference in terms of notation.
2. Following the same idea as in point 1 above, the second-stage utilities $\lambda_{\mathrm{L}jt} L_{jt\omega}^{\mathrm{C}}$ in (10.4) (page 368) is split up into the first- and second-stage utility terms $\lambda_{\mathrm{L}jt} L_{jt}^{\mathrm{S}}$ and $\lambda_{\mathrm{L}jt}(r_{jt\omega}^{\mathrm{U}} - r_{jt\omega}^{\mathrm{D}})$, respectively.

From the point of view of stochastic programming and the nature of the problem, this alternative expression for the expected cost is conceptually clearer as it distinguishes systematically the costs pertaining to market decisions (first-stage) and the costs associated with the corrective actions to be implemented during the real-time operation of the power system (second stage). Nevertheless, from a computational viewpoint, this reformulation of the expected cost is more inefficient, since it introduces unnecessary variables and constraints as, for example, variables $r_{\mathrm{G}it\omega}(m)$.

Illustrative Example 11.5 (Expected cost). According to the economic data provided in Illustrative Example 10.2 for the three-node system illustrated in Fig. 11.2, the expected cost resulting from the operation of this small-scale system in time period $t = 1$ is expressed as:

$$
\begin{aligned}
\mathrm{EC}_1 = {} & C_{11}^{\mathrm{SU}} + C_{21}^{\mathrm{SU}} + C_{31}^{\mathrm{SU}} \\
& + 30\, p_{\mathrm{G}11}(1) + 40\, p_{\mathrm{G}21}(1) + 20\, p_{\mathrm{G}31}(1) \\
& + 5\, R_{\mathrm{G}11}^{\mathrm{U}} + 5\, R_{\mathrm{G}11}^{\mathrm{D}} + 4.5\, R_{\mathrm{G}11}^{\mathrm{NS}} \\
& + 7\, R_{\mathrm{G}21}^{\mathrm{U}} + 7\, R_{\mathrm{G}21}^{\mathrm{D}} + 5.5\, R_{\mathrm{G}21}^{\mathrm{NS}} \\
& + 8\, R_{\mathrm{G}31}^{\mathrm{U}} + 8\, R_{\mathrm{G}31}^{\mathrm{D}} + 7\, R_{\mathrm{G}31}^{\mathrm{NS}} \\
& + 70\, R_{\mathrm{L}31}^{\mathrm{U}} + 70\, R_{\mathrm{L}31}^{\mathrm{D}} \\
& + 0.6\, \big[C_{111}^{\mathrm{A}} + C_{211}^{\mathrm{A}} + C_{311}^{\mathrm{A}} \\
& \qquad + \; 30\, r_{\mathrm{G}111}(1) + 40\, r_{\mathrm{G}211}(1) + 20\, r_{\mathrm{G}311}(1) \\
& \qquad + 1000\, L_{311}^{\mathrm{shed}}\big] \\
& + 0.2\, \big[C_{112}^{\mathrm{A}} + C_{212}^{\mathrm{A}} + C_{312}^{\mathrm{A}} \\
& \qquad + \; 30\, r_{\mathrm{G}112}(1) + 40\, r_{\mathrm{G}212}(1) + 20\, r_{\mathrm{G}312}(1) \\
& \qquad + 1000\, L_{312}^{\mathrm{shed}}\big] \\
& + 0.2\, \big[C_{113}^{\mathrm{A}} + C_{213}^{\mathrm{A}} + C_{313}^{\mathrm{A}} \\
& \qquad + \; 30\, r_{\mathrm{G}113}(1) + 40\, r_{\mathrm{G}213}(1) + 20\, r_{\mathrm{G}313}(1) \\
& \qquad + 1000\, L_{313}^{\mathrm{shed}}\big].
\end{aligned}
$$

Note that the utility of the only load in the three-node system is not included in the expected cost for being inelastic. Also, no cost of wind power spillage is considered. □

11.3.5 Wind Spillage Cost

The expected cost defined in (11.12) fails to account for other relevant aspects of wind power production, as those linked to the amount of system carbon emissions avoided by wind sources and to the pressure imposed by some market policies promoting wind power integration. In order to fill this gap, we redefine the objective function z of the proposed market-clearing model as the summation of the expected cost (11.12) and the so-called (expected) wind spillage *cost*:

$$
z = \mathrm{EC} + \sum_{\omega=1}^{N_\Omega} \sum_{t=1}^{N_\mathrm{T}} \pi_\omega V_t^{\mathrm{S}} S_{t\omega}, \tag{11.13}
$$

where V_t^{S} is the cost associated with the spillage of wind power generated in period t .

Note that the wind spillage term is not included into the expected cost as it is just an *fictitious* cost difficult to determine and whose only purpose is to serve the system operator as a mechanism to alter the energy and reserve dispatch program in favor of wind generation.

11.3.6 Formulation

In short, the final formulation of the market-clearing model to deal with wind power production uncertainty remains as follows:

$$
\begin{aligned}
&\mathrm{Minimize}_{C_{it}^{\mathrm{SU}},p_{\mathrm{G}it}(m),L_{jt}^{\mathrm{S}},R_{it}^{\mathrm{U}},R_{it}^{\mathrm{D}},R_{it}^{\mathrm{NS}},R_{jt}^{\mathrm{U}},R_{jt}^{\mathrm{D}},C_{it\omega}^{\mathrm{A}},r_{\mathrm{G}it\omega}(m),r_{jt\omega}^{\mathrm{U}},r_{jt\omega}^{\mathrm{D}},L_{jt\omega}^{\mathrm{shed}}} \\
&\quad \mathrm{EC} = \sum_{t=1}^{N_{\mathrm{T}}} \mathrm{EC}_t = \\
&\quad = \sum_{t=1}^{N_{\mathrm{T}}} \sum_{i=1}^{N_{\mathrm{G}}} C_{it}^{\mathrm{SU}} \\
&\quad + \sum_{t=1}^{N_{\mathrm{T}}} d_t \Bigg[\sum_{i=1}^{N_{\mathrm{G}}} \sum_{m=1}^{N_{\mathrm{O}it}} \lambda_{\mathrm{G}it}(m) p_{\mathrm{G}it}(m) - \sum_{j=1}^{N_{\mathrm{L}}} \lambda_{\mathrm{L}jt} L_{jt}^{\mathrm{S}} \\
&\qquad + \sum_{i=1}^{N_{\mathrm{G}}} \left(C_{it}^{R^{\mathrm{U}}} R_{it}^{\mathrm{U}} + C_{it}^{R^{\mathrm{D}}} R_{it}^{\mathrm{D}} + C_{it}^{R^{\mathrm{NS}}} R_{it}^{\mathrm{NS}} \right) \\
&\qquad + \sum_{j=1}^{N_{\mathrm{L}}} \left(C_{jt}^{R^{\mathrm{U}}} R_{jt}^{\mathrm{U}} + C_{jt}^{R^{\mathrm{D}}} R_{jt}^{\mathrm{D}} \right) \Bigg] \\
&\quad + \sum_{\omega=1}^{N_{\Omega}} \pi_\omega \Bigg\{ \sum_{t=1}^{N_{\mathrm{T}}} \sum_{i=1}^{N_{\mathrm{G}}} C_{it\omega}^{\mathrm{A}} \\
&\qquad + \sum_{t=1}^{N_{\mathrm{T}}} d_t \Bigg[\sum_{i=1}^{N_{\mathrm{G}}} \sum_{m=1}^{N_{\mathrm{O}it}} \lambda_{\mathrm{G}it}(m) r_{\mathrm{G}it\omega}(m) \\
&\qquad\quad + \sum_{j=1}^{N_{\mathrm{L}}} \lambda_{\mathrm{L}jt} \left(r_{jt\omega}^{\mathrm{U}} - r_{jt\omega}^{\mathrm{D}} \right) \\
&\qquad\quad + \sum_{j=1}^{N_{\mathrm{L}}} V_{jt}^{\mathrm{LOL}} L_{jt\omega}^{\mathrm{shed}} \Bigg] \Bigg\}
\end{aligned} \tag{11.14}
$$

subject to

$$\sum_{i=1}^{N_G} P_{it}^{\mathrm{S}} + P_t^{\mathrm{WP,\ S}} = \sum_{j=1}^{N_L} L_{jt}^{\mathrm{S}}, \ \ \forall t. \tag{11.15}$$

$$P_i^{\min}\ u_{it} \le P_{it}^{\mathrm{S}} \le P_i^{\max}\ u_{it}, \quad \forall i, \forall t \tag{11.16}$$

$$L_{jt}^{\mathrm{S,\ min}} \le L_{jt}^{\mathrm{S}} \le L_{jt}^{\mathrm{S,\ max}}, \ \ \forall j, \forall t \tag{11.17}$$

$$P_t^{\mathrm{WP,\ min}} \le P_t^{\mathrm{WP,\ S}} \le P_t^{\mathrm{WP,\ max}}, \ \ \forall t. \tag{11.18}$$

$$0 \le \ p_{\mathrm{G}it}(m) \le p_{\mathrm{G}it}^{\max}(m), \quad \forall m, \forall i, \forall t \tag{11.19}$$

$$P_{it}^{\mathrm{S}} = \sum_{m=1}^{N_{\mathrm{O}it}} p_{\mathrm{G}it}(m), \quad \forall i, \forall t. \tag{11.20}$$

$$0 \le R_{it}^{\mathrm{U}} \le R_{it}^{\mathrm{U,\,max}}\ u_{it}, \ \ \forall i, \forall t \tag{11.21}$$

$$0 \le R_{it}^{\mathrm{D}} \le R_{it}^{\mathrm{D,\,max}}\ u_{it}, \ \ \forall i, \forall t \tag{11.22}$$

$$0 \le R_{it}^{\mathrm{NS}} \le R_{it}^{\mathrm{NS,\ max}}\ (1 - u_{it}), \ \ \forall i, \forall t. \tag{11.23}$$

$$0 \le R_{jt}^{\mathrm{U}} \le R_{jt}^{\mathrm{U,\ max}}, \ \ \forall j, \forall t \tag{11.24}$$

$$0 \le R_{jt}^{\mathrm{D}} \le R_{jt}^{\mathrm{D,\ max}}, \ \ \forall j, \forall t. \tag{11.25}$$

$$C_{it}^{\mathrm{SU}} \ge \lambda_{it}^{\mathrm{SU}}(u_{it} - u_{i,t-1}), \quad \forall i, \forall t \tag{11.26}$$

$$C_{it}^{\mathrm{SU}} \ge 0, \quad \forall i, \forall t. \tag{11.27}$$

$$\begin{aligned}\sum_{i:(i,n)\in M_G} P_{it\omega}^{\mathrm{G}} - \sum_{j:(j,n)\in M_L} \left(L_{jt\omega}^{\mathrm{C}} - L_{jt\omega}^{\mathrm{shed}}\right) + P_{t\omega}^{\mathrm{WP}} - S_{t\omega} \\ - \sum_{r:(n,r)\in \Lambda} f_{t\omega}(n,r) = 0, \ \ n = n', \forall t, \forall \omega.\end{aligned} \tag{11.28}$$

$$\begin{aligned}\sum_{i:(i,n)\in M_G} P_{it\omega}^{\mathrm{G}} - \sum_{j:(j,n)\in M_L} \left(L_{jt\omega}^{\mathrm{C}} - L_{jt\omega}^{\mathrm{shed}}\right) \\ - \sum_{r:(n,r)\in \Lambda} f_{t\omega}(n,r) = 0, \ \ \forall n \ne n', \forall t, \forall \omega.\end{aligned} \tag{11.29}$$

$$f_{t\omega}(n,r) = B(n,r)\big(\delta_{nt\omega} - \delta_{rt\omega}\big), \ \ \forall (n,r) \in \Lambda, \forall t, \forall \omega. \tag{11.30}$$

$$P^{\mathrm{G}}_{it\omega} \geq P^{\min}_{i}\ v_{it\omega}, \quad \forall i, \forall t, \forall \omega \tag{11.31}$$

$$P^{\mathrm{G}}_{it\omega} \leq P^{\max}_{i}\ v_{it\omega}, \quad \forall i, \forall t, \forall \omega. \tag{11.32}$$

$$r^{\mathrm{U}}_{it\omega} + r^{\mathrm{NS}}_{it\omega} - r^{\mathrm{D}}_{it\omega} = \sum_{m=1}^{N_{\mathrm{O}it}} r_{\mathrm{G}it\omega}(m), \ \ \forall i, \forall t, \forall \omega \tag{11.33}$$

$$r_{\mathrm{G}it\omega}(m) \leq p^{\max}_{\mathrm{G}it}(m) - p_{\mathrm{G}it}(m), \ \ \forall m, \forall i, \forall t, \forall \omega \tag{11.34}$$

$$r_{\mathrm{G}it\omega}(m) \geq -p_{\mathrm{G}it}(m), \ \ \forall m, \forall i, \forall t, \forall \omega. \tag{11.35}$$

$$-f^{\max}(n,r) \leq f_{t\omega}(n,r) \leq f^{\max}(n,r), \quad \forall (n,r) \in \Lambda, \ \ \forall t, \forall \omega. \tag{11.36}$$

$$0 \leq L^{\mathrm{shed}}_{jt\omega} \leq L^{\mathrm{C}}_{jt\omega}, \ \ \forall j, \forall t, \forall \omega \tag{11.37}$$

$$0 \leq \ \ S_{t\omega} \ \ \leq P^{\mathrm{WP}}_{t\omega}, \ \ \forall t, \forall \omega. \tag{11.38}$$

$$P^{\mathrm{G}}_{it\omega} = P^{\mathrm{S}}_{it} + r^{\mathrm{U}}_{it\omega} + r^{\mathrm{NS}}_{it\omega} - r^{\mathrm{D}}_{it\omega}, \ \ \forall i, \forall t, \forall \omega \tag{11.39}$$

$$L^{\mathrm{C}}_{jt\omega} = L^{\mathrm{S}}_{jt} - r^{\mathrm{U}}_{jt\omega} + r^{\mathrm{D}}_{jt\omega}, \ \ \forall j, \forall t, \forall \omega. \tag{11.40}$$

$$0 \leq r^{\mathrm{U}}_{it\omega} \leq R^{\mathrm{U}}_{it}, \ \ \forall i, \forall t, \forall \omega \tag{11.41}$$

$$0 \leq r^{\mathrm{D}}_{it\omega} \leq R^{\mathrm{D}}_{it}, \ \ \forall i, \forall t, \forall \omega \tag{11.42}$$

$$0 \leq r^{\mathrm{NS}}_{it\omega} \leq R^{\mathrm{NS}}_{it}, \ \ \forall i, \forall t, \forall \omega. \tag{11.43}$$

$$0 \leq r^{\mathrm{U}}_{jt\omega} \leq R^{\mathrm{U}}_{jt}, \ \ \forall j, \forall t, \forall \omega \tag{11.44}$$

$$0 \leq r^{\mathrm{D}}_{jt\omega} \leq R^{\mathrm{D}}_{jt}, \ \ \forall j, \forall t, \forall \omega. \tag{11.45}$$

$$C^{\mathrm{A}}_{it\omega} = C^{\mathrm{SU}}_{it\omega} - C^{\mathrm{SU}}_{it}, \quad \forall i, \forall t, \forall \omega \tag{11.46}$$

$$C^{\mathrm{SU}}_{it\omega} \geq \lambda^{\mathrm{SU}}_{it}(v_{it\omega} - v_{i,t-1,\omega}), \quad \forall i, \forall t, \forall \omega \tag{11.47}$$

$$C^{\mathrm{SU}}_{it\omega} \geq 0, \quad \forall i, \forall t, \forall \omega. \tag{11.48}$$

$$u_{it}, v_{it\omega} \in \{0,1\}, \forall i, \forall t, \forall \omega. \tag{11.49}$$

The objective function (11.14) is the expected cost of power system operation, which includes the costs pertaining to the electricity-market stage (start-up planning of generating units, and energy and reserve scheduling) and the costs derived from the real-time operation of the power system (*ad hoc* adjustments to the start-up and shut-down planning of generating units, reserve deployment, and load shedding actions).

The group of equations (11.15)–(11.27) is related to the electricity-market stage, which basically boils down to the balance between energy offers by producers and energy bids by consumers (11.15), generation limits and demand bounds (11.16)–(11.18), the decomposition of generator power outputs by blocks (11.19)–(11.20), the reserve scheduling constraints (11.21)–(11.25), and the modeling of the costs pertaining to the start-up planning of generating units (11.26)–(11.27).

The set of constraints (11.28)–(11.38) models the real-time operation of the power system, which revolves around the real-time power balance between generation and demand (11.28)–(11.29). In particular, equation (11.28) constitutes the mathematical expression of the power balance at the node into which the wind power production is injected. The power balance equations are complemented with a dc model of the transmission network (11.30), the generations limits of production units (11.31)–(11.32), the decomposition of the deployed reserves by blocks (11.33)–(11.35), the transmission capacity constraints (11.36), and the load shedding and wind spillage bounds (11.37)–(11.38).

The set of equations (11.39)–(11.48), referred to as *linking constraints*, couple market decisions with operating decisions made in real time through the deployment of reserves by generating units and loads (11.39)–(11.45), and the changes in the start-up and shut-down planning of units that make such a deployment possible (11.46)–(11.48).

Finally, constraints (11.49) constitute binary variable declarations.

11.4 Wind Benefits and Costs at a Glance: Performance Metrics

In the economic appraisal of wind generation, two conflicting and opposite effects come into play. On the one hand, the integration of wind power into an electric system brings about a reduction in the system operation cost provided that wind generation, with zero fuel cost, replaces conventional units with significant fuel costs (thermal units). On the other hand, since wind generation is variable and difficult to predict accurately, it contributes to system uncertainty and demands the scheduling of additional reserve capacity to sustain system security. This increase in the required reserve leads to an increase in the operation cost.

Therefore, these two opposing effects should be taken into account while assessing the integration of wind generation, its cost and its profitability. For this purpose, three performance metrics are defined below. Specifically, these metrics are used to clarify the two conflicting economic effects of wind generation: the first effect being the fuel cost reduction and the second one, the reserve cost increase due to wind uncertainty. In addition, these metrics succeed in providing condensed and valuable information on the role of wind generation in the economic performance of the electricity market.

11.4.1 Average Benefit (AB)

The *Average Benefit* in \$/MWh is a measure of the decrease in expected cost as a result of injecting an additional MWh of wind power into the network.

$$\mathrm{AB} = \frac{\sum_{t=1}^{N_\mathrm{T}} \frac{\mathrm{DC}_t^0 - \mathrm{EC}_t^*}{d_t \hat{P}_t^{\mathrm{WP}}}}{N_\mathrm{T}}, \tag{11.50}$$

where EC_t^* is the objective function optimal value in period t (see equation (11.12)), $\hat{P}_t^{\mathrm{WP}}$ represents the wind power generation forecast in period t and DC_t^0 is the *deterministic* cost of the system in period t without wind power generation. Note that DC_t^0 is obtained by solving the proposed model for a deterministic case in which no wind is considered.

11.4.2 Average Uncertainty Cost (AUC)

The *Average Uncertainty Cost* in \$/MWh is a measure of the equivalent deterministic cost of a wind generator producing exactly the forecast power.

$$\mathrm{AUC} = \frac{\sum_{t=1}^{N_\mathrm{T}} \frac{\mathrm{EC}_t^* - \mathrm{DC}_t^\mathrm{F}}{d_t \hat{P}_t^{\mathrm{WP}}}}{N_\mathrm{T}}, \tag{11.51}$$

where DC_t^F is the *deterministic* cost in period t assuming that the wind power is perfectly known and coincides with its forecast. DC_t^F is obtained by solving the proposed model for a deterministic case in which only the wind power forecast scenario is considered.

11.4.3 Net Average Benefit (NAB)

The *Net Average Benefit* in \$/MWh is a measure of the profitability of the wind power generation injected at a given bus of the system.

$$\mathrm{NAB} = \mathrm{AB} - \mathrm{AUC}. \tag{11.52}$$

Illustrative Example 11.6 (Computing performance metrics). Let us consider the three-node system represented in Illustrative Example 11.2, with a wind generator located at node 2. Suppose that the market-clearing procedure introduced in this chapter provides a scheduling program with the associated expected cost per period shown in Table 11.3. This table also provides the *deterministic* costs resulting from the problems in which either wind generation is not included (DC_t^0) or it is considered certain and equal to the forecast ($\mathrm{DC}_t^{\mathrm{F}}$). Note that the scheduling horizon is comprised of four hourly periods.

Table 11.3 Illustrative example 11.6: costs per period

Period #	1	2	3	4
EC_t^* (\$)	628	1438.5	1865	664
DC_t^0 (\$)	700	2000	2800	800
$\mathrm{DC}_t^{\mathrm{F}}$ (\$)	580	1400	1750	640

Assuming the wind power production forecast indicated in Table 11.4, the average benefit of wind generation is computed as

Table 11.4 Illustrative example 11.6: wind power production forecast

Period #	1	2	3	4
$\hat{P}_t^{\mathrm{WP}}$ (MW)	6	20	35	8

$$\mathrm{AB} = \frac{\frac{\$700-\$628}{1\mathrm{h}\times 6\mathrm{MW}} + \frac{\$2000-\$1438.5}{1\mathrm{h}\times 20\mathrm{MW}} + \frac{\$2800-\$1865}{1\mathrm{h}\times 35\mathrm{MW}} + \frac{\$800-\$664}{1\mathrm{h}\times 8\mathrm{MW}}}{4} = \$20.9473/\mathrm{MWh}.$$

Analogously, its average uncertainty cost is given by

$$\mathrm{AUC} = \frac{\frac{\$628-\$580}{1\mathrm{h}\times 6\mathrm{MW}} + \frac{\$1438.5-\$1400}{1\mathrm{h}\times 20\mathrm{MW}} + \frac{\$1865-\$1750}{1\mathrm{h}\times 35\mathrm{MW}} + \frac{\$664-\$640}{1\mathrm{h}\times 8\mathrm{MW}}}{4} = \$4.0527/\mathrm{MWh}.$$

Therefore, the net average benefit associated with the wind power generation is

$$\mathrm{NAB} = \$20.9473/\mathrm{MWh} - \$4.0527/\mathrm{MWh} = \$16.8946/\mathrm{MWh}.$$

□

11.5 Market-Clearing Example with Wind Generation

In this section, we test and illustrate the market-clearing formulation previously presented using the three-node system with a wind generator located at node 2 depicted in Illustrative Example 11.2. The technical and economic data defining this small example are provided in Illustrative Examples 10.1 and 10.2. Throughout this section, we examine the influence of several basic aspects of wind generation on the scheduling program derived from the market-clearing model and on its associated expected cost composition. In particular, we appraise the impact of: i) wind generator location; ii) network congestion; iii) wind spillage cost, and iv) wind penetration and uncertainty levels.

To start with, Fig. 11.4 offers a breakdown of the expected cost that results from three different situations, namely:

a) The wind power production is perfectly known and coincides with the forecast. Therefore reserves are not needed.
b) The three-node system is dispatched excluding wind generation.
c) The power produced by the wind generator is uncertain. Its stochastic behavior is modeled via the three wind power scenarios provided in Table 11.1 of Illustrative Example 11.1.

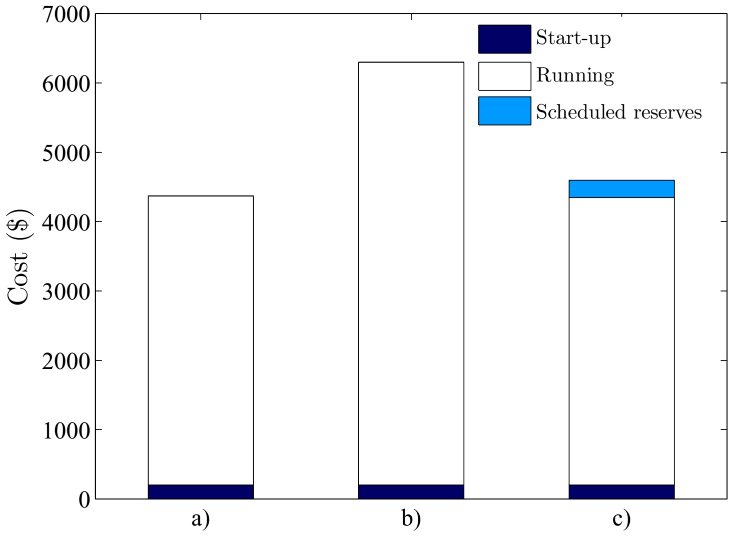


Fig. 11.4 Market-clearing example with wind generation. a) No wind generation uncertainty; b) no wind generation; c) uncertain wind generation

By comparing case a) or c) with case b), it becomes clear that the inclusion of wind energy sources into the generation mix of this small-scale power system leads to a substantial reduction in the running costs, that is, in the costs of energy production. In instances a) and c), the energy supply to the load at node 3 can be partially covered by wind sources. As a result, a part of the energy that otherwise would be produced by the other three units, with significant fuel costs, is displaced by renewable and free energy. On the other hand, case c) shows an expected cost somewhat greater than that of case a). As can be observed, the difference is mainly due to the cost of the reserves required to accommodate the uncertain variability of wind generation, cost that cannot be disregarded, albeit it is significantly smaller than the cost associated with the energy production.

11.5.1 Impact of wind generator location and network congestion

Next, we explore how the location of the wind generator impacts on the expected cost. The magnitude of this impact is dependent on both the wind potential at the specific site and the network conditions. More precisely, the wind site characteristics shape the stochastic behavior of the power produced by the wind generator installed at it, and the capacity of the network determines the capability of this wind generator to evacuate its uncertain and variable power production toward the consumption nodes. Being so, let us assume that the wind characteristics are the same all over the three-node system. Consequently, the three wind power scenarios listed in Table 11.1 serve us to model the wind power production irrespective of the node at which the wind generator is placed. Then, the network condition remains as the only cause responsible for the impact of wind generator location on the expected cost.

Fig. 11.5 illustrates the breakdown of the expected cost as a function of the wind generator position within the power system. The node location “0” stands for the case in which no wind generation is considered. Note that the expected cost and its components are identical regardless of the node at which the wind generator is situated. The reason for this is that the grid has enough room to accommodate and take advantage of the whole wind power production. According to the network data provided in Illustrative Example 10.1 for this small-scale system, the transmission capacities of all the lines is equal to 55 MW. For the sake of clarity, hereinafter, the three-node system with these capacity values will be referred to as the *base case*. We also define the *network-congested* case, which is equal to the former except for the fact that the capacity of line 3 is reduced to 25 MW ($f^{\max}(2,3) = 25$) to highlight the effect of network bottlenecks on the expected cost. Fig. 11.6 is analogous to Fig. 11.5, but for the network-congested case. Observe that if the wind generator is located at node 2, the expected cost suffers from an important rise, which is the result of a substantial increase in running costs and a small reduction in reserve costs. This rise is justified by a significant

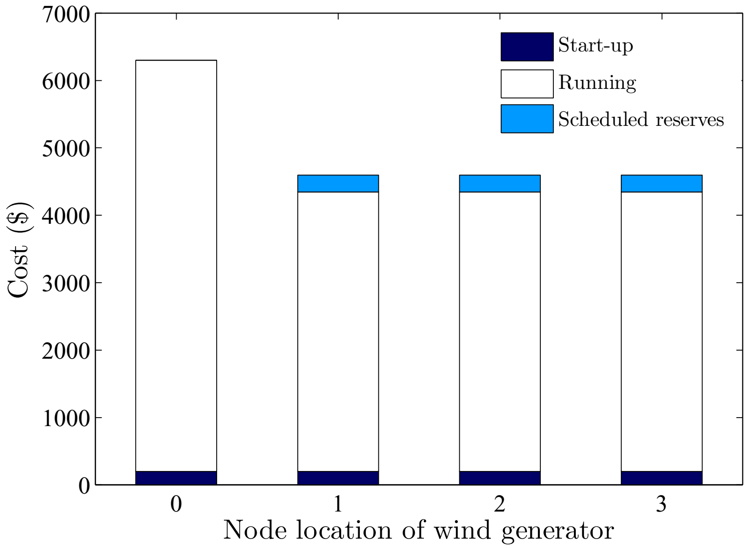


Fig. 11.5 Market-clearing example with wind generation. Base case ($f^{\max}(2,3) = 55$ MW). Expected cost breakdown

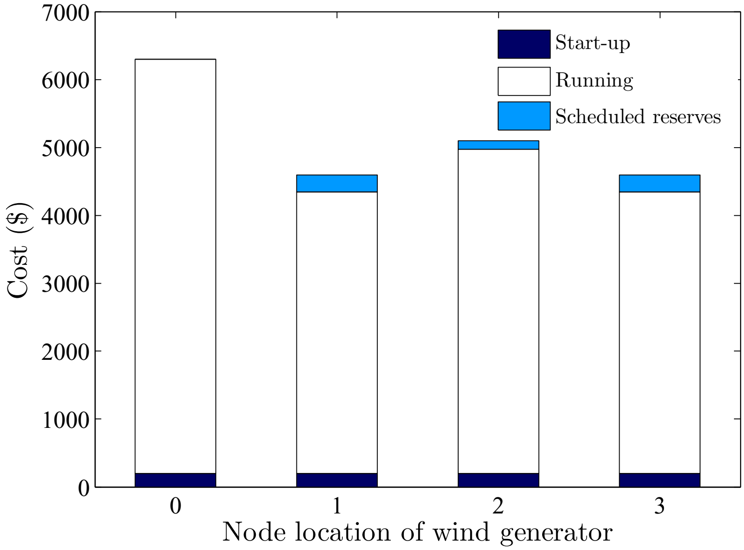


Fig. 11.6 Market-clearing example with wind generation. Network-congested case ($f^{\max}(2,3) = 25$ MW). Expected cost breakdown

upsurge of wind spillage phenomena. If wind is spilled, the power system can be operated under a diminished level of uncertainty and less reserves need to be scheduled as a consequence. On the contrary, the share of renewable and free energy in the electricity supply decreases and costly energy resources have to be used instead.

Table 11.5 Market-clearing example with wind generation. Impact of network congestion on performance metrics

Performance metrics	$f^{\max}(2,3) = 55$ MW	$f^{\max}(2,3) = 25$ MW	
	WP at buses 1, 2 or 3	WP at buses 1 or 3	WP at bus 2
AB	20.95	20.95	17.34
AUC	4.05	4.05	3.37
NAB	16.90	16.90	13.97

The performance metrics defined in Section 11.4 can be used to summarize the information regarding the impact of network congestion on the profitability of wind generation. Table 11.5 provides the value of these metrics computed for both the base and the network-congested cases. Additionally, the metrics are calculated for each possible node location of the wind generator. It should be noted in Table 11.5 that, under no congestion ($f^{\max}(2,3) = 55$ MW), the three metrics take on the same values regardless of where the wind generator is located. However, under network congestion ($f^{\max}(2,3) = 25$ MW), the metrics do change depending upon the node at which the wind power unit is placed. If it is located at nodes 1 or 3, the metric values are the same and equal to those obtained without congestion, but if it is located at node 2 the three metric values are comparatively smaller. In the latter case, the *average benefit* (AB) decreases since the system cannot benefit fully from the wind potential at that node due to the physical restrictions imposed by the network. The *average uncertainty cost* (AUC) also decreases as the network acts as a filter cutting off, and thus reducing, the uncertain variability of wind power production. From a numerical perspective, the AUC decreases because the associated deterministic cost $\mathrm{DC}_t^{\mathrm{F}}$, computed from the wind power forecast, is also a congested case and therefore the deterministic and stochastic problems yield similar results. As the reduction in AB is greater than the decrease of AUC, the *net average benefit* finally diminishes. This means that, under economic criteria, nodes 1 or 3 are more suitable sites than bus 2 to locate the wind farm if the capacity of line 3 is limited to 25 MW.

On the other hand, network bottlenecks can have an important effect on the scheduling program provided by the market-clearing model introduced in this chapter. As an illustration of this, in Table 11.6, we compare the scheduling and dispatching programs obtained for both the base and network-congested cases with the wind generator located at node 2. The results shown in square brackets for period 3 correspond to the network-congested case. For conciseness, the results of periods 1, 2 and 4 for this case are omitted because they are identical to the corresponding ones in the base case. Observe that

Table 11.6 Market-clearing example with wind generation. Comparison of scheduling programs (congested and uncongested network). Powers in MW

Period #	$P_t^{\mathrm{WP,\,S}}$	P_t^{S}			R_t^{U}				R_t^{D}				R_t^{NS}		
		G1	G2	G3	G1	G2	G3	L3	G1	G2	G3	L3	G1	G2	G3
1	6	0	0	24	0	0	4	0	0	0	0	0	0	0	0
2	30	0	0	50	0	0	0	0	0	0	0	0	17	0	0
3	35	25	0	50	10	0	0	0	15	0	0	0	0	0	0
	[15]	[45]	[0]	[50]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]
4	8	0	0	32	0	0	2	0	0	0	0	0	0	0	0

the hourly wind production *scheduled* in period 2 is equal to the *high* wind power scenario for both the base and the network-congested cases. This result, which may be surprising a priori, poses the following relevant question: would it not be more sensible to program wind power production at the level of the value forecasted for that period? Intuitively, we may feel tempted to respond *yes* because, first, this is precisely the solution schedule for the other time periods (except for period 3 in the network-congested case), and second, the *as forecast* wind power scenario is by far the one with the highest probability (0.6). However, our tireless search for the economic optimum leads us to answer with an emphatic *no* for the following reason: to guarantee supplying the load at bus 3 for any realization of wind power production in period 2, the market operator contracts 17 MW of reserve with unit 1 for that period. Clearing the market with a wind power schedule in period 2 of 30 MW results in unit 1 being out of the energy market in that period and thus allows the market operator to contract those 17 MW of reserve from the non-spinning reserve offer of this unit. In contrast, if we fix the wind power scheduled in period 2 to its forecast value (20 MW), then unit 1 is committed in the energy market. Consequently, the required 17 MW of reserve are obtained from the spinning reserve offer of unit 1, which is more expensive, and as a result, a higher expected cost is obtained.

On the other hand, in the network-congested case, the optimal-market clearing solution is achieved by scheduling only 15 MW of wind power in period 3. This is the maximum amount of wind power that can be injected at bus 2 without making use of any type of reserve. Consequently, the excess of wind generation is spilled, reserves are not required and unit 1 is committed (with a scheduled power of 45 MW) in order to meet the period load of 110 MW by transferring power through line 2.

11.5.2 Impact of wind spillage cost

To gain a greater insight into the economic effects of wind spillage, we consider below a wind spillage cost equal to \$100/MWh. This value is arbitrary, which is immaterial for the purposes of this example. Yet, this cost should be in practice sufficiently high to account for other benefits associated with wind power that are not included in the model, e.g., avoided emissions, and to constitute a numerical translation of the policies intended to encourage the integration of wind energy. Ultimately, this cost translates into a dispatching priority for the wind farm regardless of the operation cost. Fig. 11.7 shows the breakdown of the expected cost taking into account such a spillage cost. Once more, if the wind generator is placed at node 2, line 3 becomes congested and the wind potential cannot be completely exploited. However, unlike in a free cost wind spillage situation (see Fig. 11.6), in this case, the system operator tries its best to curb the waste of wind through a more flexible operation of the power system based on the scheduling of additional and more expensive reserves (e.g., reserve from the load). This enhanced flexibility enables a greater use of the wind sources, and therefore, the running costs remain almost unchanged, or even decrease. Fig. 11.8 clarifies this by means of two pie charts illustrating the breakdown in percentages of the expected cost for the two different values of wind spillage cost that we have considered, i.e., 0 and \$100/MWh. Note that the portion corresponding to the cost of reserves increases from 2% to 18% when the wind spillage becomes costly. This increase is aimed at getting the power system ready to *absorb* the largest possible amount of wind energy. Nonetheless, in spite of this additional expense devoted to increase system reserves, some of the wind power production still has to be irremediably spilled due to network congestion. This stresses the fundamental role that the network plays in the integration of wind generation: managing system uncertainties requires not only generating units and loads willing to alter their energy production and consumption if needed, but also an electrical infrastructure robust enough to allow for such alterations.

11.5.3 Impact of wind penetration and uncertainty levels

Lastly, we analyze the impact of the wind integration level on the three-node system. For this purpose, the base and the network-congested cases are solved next for a large set of conditions characterized by different wind power penetration and uncertainty levels. The wind power penetration level is defined by setting the wind power forecast scenario to a percentage of the hourly demand (x-axis in Figs. 11.9(a) and 11.9(b)), while the other two scenarios (*low* and *high*) are built as given percentages of the wind power

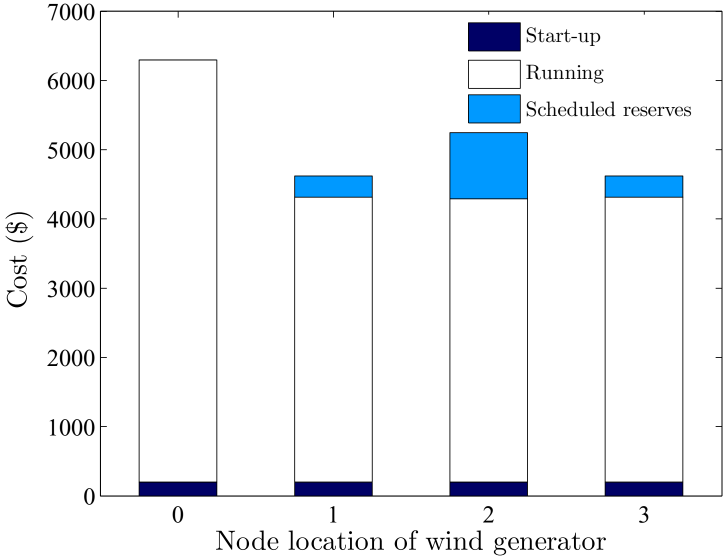
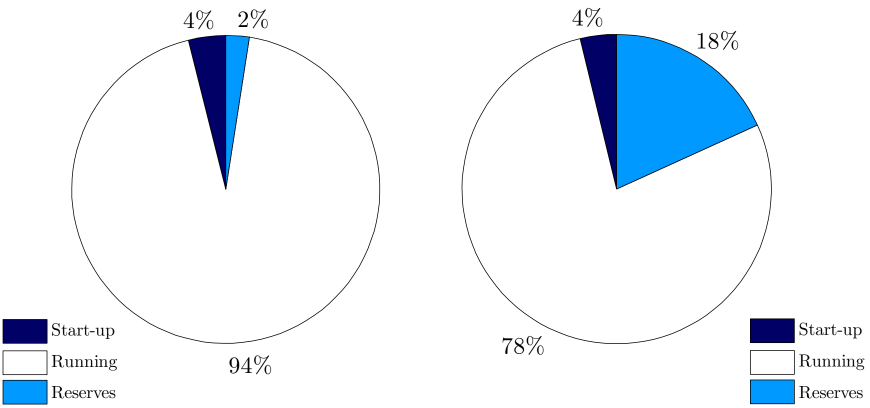


Fig. 11.7 Market-clearing example with wind generation. Network-congested case with a cost of wind power spillage equal to $100/MWh. Expected cost breakdown



(a) V^{S} = \$0/MWh (b) V^{S} = \$100/MWh

Fig. 11.8 Market-clearing example with wind generation. Wind generator at node 2. Effect of wind spillage cost on the expected cost breakdown

forecast, below and above respectively. The uncertainty level is defined as a measure of how far from the wind power forecast scenario the *low* and *high* scenarios are. The uncertainty level considered ranges from 0% to 100% with

10% increments in such a way that $x\%$ uncertainty means that the *high* and *low* scenarios are equal to the wind power forecast multiplied by $(1 + x/100)$ and $(1 - x/100)$, respectively.

The eleven curves depicted in Fig. 11.9(a) show the evolution of the expected social cost in the base case as the wind power penetration increases, each curve representing a different uncertainty level. A decreasing trend in the expected cost can be observed as the wind power penetration increases. However, it should be noted that this trend is less marked as the uncertainty associated to wind production grows.

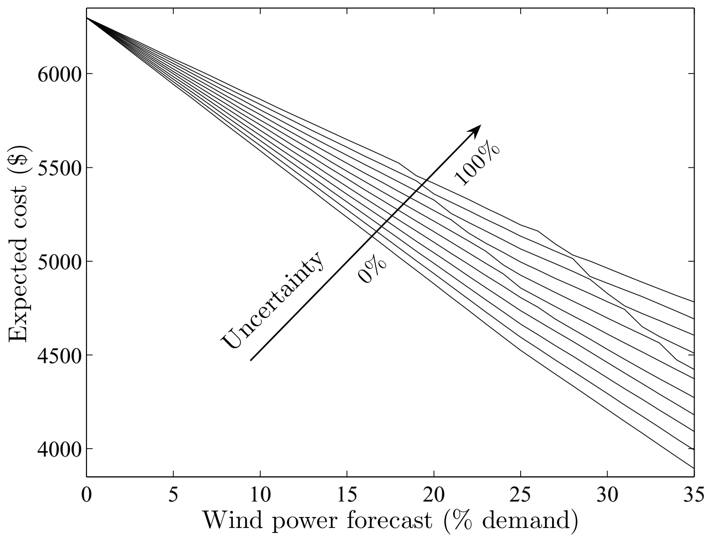
Fig. 11.9(b) illustrates the evolution of the expected cost for the network-congested case. The same wind power penetration and uncertainty levels as in Fig. 11.9(a) are considered. By comparing both figures, the constraining effect of the network is made evident. Qualitatively, it can be observed that the effect of the network is twofold. On the one hand, the slopes of the eleven curves decrease for increasing wind power penetration levels, which translates into a decrease of the reduction rate of the expected cost. The reason for this is that the network imposes a *cap* on the amount of wind power that can be injected into the system. Thus, once the network becomes congested, the wind is spilled and, consequently, the system cannot benefit from high wind scenarios. On the other hand, the curves corresponding to low uncertainty levels stay closer to one another in comparison with those of Fig. 11.9(a). This is due to the fact that for those conditions in which the wind power penetration level is high ($> 20\%$ approximately) and the three wind power scenarios are close enough (uncertainty level below 50% approximately), not only the *high* realization of wind may cause network congestion, but also the lowest ones, and then the differences among the values of the expected cost corresponding to these conditions are reduced.

11.6 Market-Clearing Case Study with Wind Generation

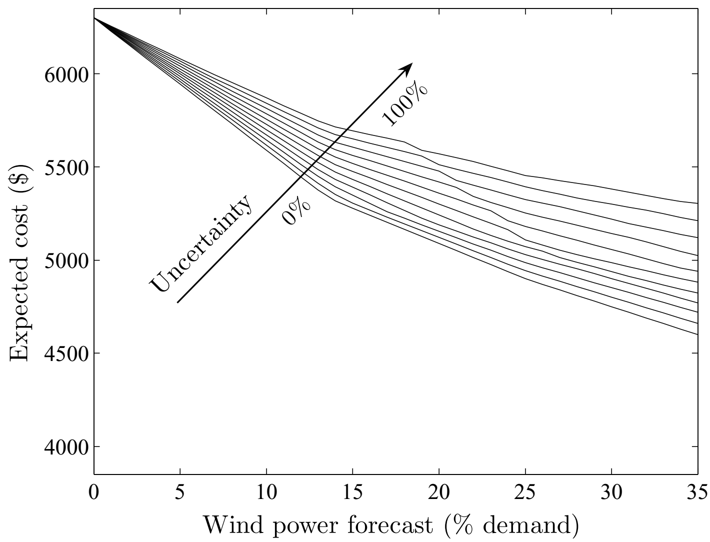
The features of the market-clearing model proposed to deal with wind generation uncertainty are further illustrated through a more realistic case study based on the 24-node system described in Appendix B. The scheduling horizon considered corresponds to one day (24 hours).

Specifically, the objective of this case study is threefold:

1. To provide additional insight into the impact of wind generation on the cost of scheduling reserves by using a more realistic data set.
2. To identify non-spinning reserves as the main origin of the high computational burden associated with stochastic security-constrained market-clearing models.



(a) Base Case



(b) Network-congested case

Fig. 11.9 Market-clearing example with wind generation. Wind generator at node 2. Expected cost as a function of wind penetration and wind uncertainty

3. To list and evaluate some straightforward strategies aimed at alleviating such computational burden.

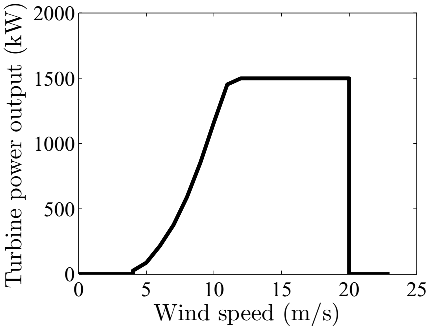
As explained in Subsection 11.3.2, the accurate modeling of the wind power stochastic behavior usually requires generating a sufficient number of scenarios representing the most plausible realizations of wind power throughout the scheduling horizon. For this purpose, we can use one of the scenario-generation techniques described in Chapter 3. However, there also exists a simple alternative to these more elaborate methodologies: we can just build the scenario set directly from historical records, that is, we can just employ *historical scenarios*. Needless to say, the use of this alternative is contingent on the availability of a good and extensive data collection.

In this case study, 3018 equiprobable wind speed scenarios are constructed using the wind speed data publicly available in [45] and collected from a location in the state of Kansas (U.S.) by Dr. Gary Johnson. This wind data set comprises average hourly speed measurements between May 1995 and February 2005. We consider a wind farm having 100 commercial wind generators, all of which correspond to the same type of turbine model, model A or model B, with nameplate capacities of 1.5 and 2.5 MW, respectively. Therefore, the total installed wind power capacity of the farm is 150 or 250 MW depending on which of the two turbine models is used.

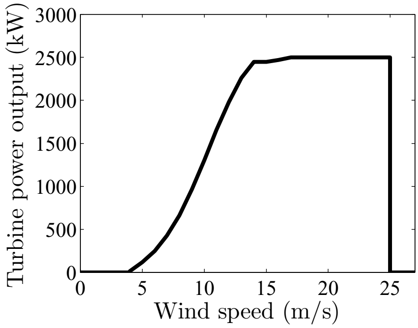
An additional complex issue is how to transform those 3018 wind speed scenarios into wind power scenarios. For the sake of simplicity, we resolve to just aggregate the power output of each individual wind turbine within the wind farm, which, in turn, is estimated from its characteristic power curve. Fig. 11.10 depicts the power curves of turbine models A and B, which express wind turbine power output as a function of the wind speed input. This constitutes an approximate approach to the speed-power conversion problem, which relies on the unlikely assumption that the characteristics of the wind are the same all over the plant at each instant.

On the other hand, the size of the scenario set obtained above (3018 scenarios) is too large, resulting in an optimization model that is intractable (with 290,000 binary variables approximately). Hence, to achieve tractability, statistical techniques, as those described in Chapter 3, are applied in order to reduce the number of scenarios while retaining the essential features of the original scenario set. The reduced scenario set resulting from this process includes 20 scenarios. Solution results are discussed below.

Table 11.7 provides a breakdown of the expected cost of the system into the cost pertaining to the energy supply and the cost of security for two different locations of the wind plant (WP), node 7 and node 20, and two different installed wind power capacities (150 and 250 MW). It also provides the costs of energy and security per installed MW and hour (as defined in Section 10.7 of Chapter 10). Given that the expected amount of load not served is zero for all the variants examined in this case study, the cost of security becomes equivalent to the cost associated with the scheduling of reserves. Wind potential is assumed to be the same at nodes 7 and 20, that is,



(a) Turbine model A



(b) Turbine model B

Fig. 11.10 Market-clearing case study with wind generation. Turbine speed-power curve

the same wind power scenarios can be used to model the stochastic behavior of wind generation at both sites. Since the wind generation is assumed to be free, the cost of the energy is obtained by dividing the total cost of the energy by the length of the scheduling horizon (24 h) and the total power capacity of the electric system excluding the wind power capacity (3325 MW). On the contrary, as wind generation is responsible for the need for reserves, the cost of security is obtained by dividing the total cost of the scheduled reserve by the installed wind power capacity in the system (150 or 250 MW) and the time span of the scheduling horizon (24 h). Note that the cost of security is comparatively relevant. In this sense, it should be taken into account that, on the one hand, wind constitutes an important source of cheap renewable energy but, on the other hand, reserves are required in order to accommodate its unpredictable variability and, thus, to maintain system security and reliability. Therefore, the integration of wind generation into a power system reduces the cost of energy at the expense of increasing the cost of security, i.e., the cost of being able to balance supply and demand at all buses, at all times and for all conditions.

Table 11.7 Market-clearing case study with wind generation. Costs of energy and security

WP at bus #	Installed wind capacity (MW)	Total Cost ($)		Cost ($) per MWh	
		Energy	Security	Energy	Security
7	150	384,233	13,107	4.81	3.64
7	250	376,701	16,897	4.72	2.82
20	150	384,245	13,093	4.82	3.64
20	250	376,644	16,835	4.72	2.81

In the same vein, we can compare the results provided in Table 11.7 with those presented in Table 10.14 of Chapter 10 (Section 10.7, page 398) about the effect of unit failures on the costs of energy and security. From the comparison, it can be concluded that wind production makes the maintenance of system security economically more costly than equipment failures. In general, one megawatt of wind generation needs to be backed up with a greater amount of reserves than one megawatt produced by a conventional unit subject to a possible failure. In contrast, a megawatt obtained from the force of wind is free, while a megawatt resulting from the combustion of fossil fuels is not. Therefore, wind generation contributes to reduce the cost of energy.

Regarding computational issues, the mathematical programming problem associated with the proposed market-clearing model is large-scale, as it includes 228,562 constraints, 153,409 continuous variables, and 4536 binary variables. The model is hard to solve and the computational burden is mostly due to the binary variables required to determine the on/off status of generating units. Note that in order to model the deployment of non-spinning reserves, one binary variable, $v_{it\omega}$, is required per unit, period and scenario. Hence, it seems reasonable to question whether it is worthwhile to include non-spinning reserves in the model. Tables 11.8 and 11.9 show, respectively, a comparison between the expected costs and CPU times with and without non-spinning reserve. Computation times have been obtained using CPLEX 11.2.1 under GAMS 23.0 [141] on a Linux-based server with two processors Quad Core AMD Opteron 2356 clocking at 2.3 GHz and 8 GB of RAM.

In all cases, excluding non-spinning reserve implies an increase of the expected cost lower than 0.21% with respect to the solution including non-spinning reserve. However, the decrease in computation time is dramatic (above 98.6%), which is mostly due to the fact that the number of binary variables without non-spinning reserve is 216 vs. 4536 with non-spinning reserve.

All the results presented in Table 11.8 pertain to optimal solutions, i.e., to solutions that are obtained considering a *mip gap* of 0%. The mip gap is defined for mixed-integer programming (mip) minimization problems as the difference in percent between the objective function value associated with the best integer solution found and the objective function lower bound. Hence, an easy manner of mitigating the computational demand of the proposed

Table 11.8 Market-clearing case study with wind generation. Valuation of non-spinning reserves (NSR). Expected costs

WP at bus #	Installed wind capacity (MW)	Expected cost ($)		Cost increase (%)
		With NSR	Without NSR	
7	150	397,340	397,610	0.07
7	250	393,598	394,426	0.21
20	150	397,338	397,607	0.07
20	250	393,479	394,217	0.19

Table 11.9 Market-clearing case study with wind generation. Valuation of non-spinning reserves (NSR). CPU times

WP at bus #	Installed wind capacity (MW)	CPU time (s)		Time reduction (%)
		With NSR	Without NSR	
7	150	1555	21	98.6
7	250	23,448	21	99.9
20	150	1536	21	98.6
20	250	18,355	22	99.9

market-clearing model is to work with a mip gap greater than zero at the risk of getting a suboptimal solution. Table 11.10 shows the expected cost increase stemming from the consideration of mip gaps equal to 1 and 0.5%. Likewise, Table 11.11 includes the corresponding reduction in calculation time. Observe that a very remarkable decrease in cpu time can be achieved at the expense of a small increase in the expected cost. These tables also highlight the computational advantages obtained from the application of a *warm-start strategy*. This strategy consists in using the market-clearing solution without non-spinning reserve as the initial first integer feasible solution of the mip algorithm to solve the market-clearing model with non-spinning reserves. Logically, this tactic does not entail any increase in the expected cost, as it just affects the search, but not the final solution. In contrast, it brings about a substantial reduction in computational time (see Table 11.11), because the search for the optimal solution is usually shortened. Needless to say, the warm-start strategy can be combined with a positive mip gap.

Finally, we conclude this section presenting several guidelines to further relieve the computational burden of the market-clearing problem with stochastic security. These guidelines have to do with modeling simplifications and are intended to reduce the number of binary variables v_{itw} used to model non-spinning reserves. These simplifications are:

Table 11.10 Market-clearing case study with wind generation. Expected cost increase (%) resulting from the application of several strategies to relieve the computational burden

WP at bus #	Installed wind capacity (MW)	Mip gap (%)	
		1	0.5
7	150	0.54	0.19
7	250	0.66	0.13
20	150	0.64	0.20
20	250	0.47	0.09

Table 11.11 Market-clearing case study with wind generation. Time reduction (%) achieved through the application of several strategies to relieve the computational burden

WP at bus #	Installed wind capacity (MW)	Mip gap (%)		Warm start (%)
		1	0.5	
7	150	81.2	75.2	24.7
7	250	94.6	91.1	81.3
20	150	85.9	76.0	29.2
20	250	90.9	88.4	57.7

1. To limit the number of generating units providing non-spinning reserve. This is a realistic assumption since not all the generating units meet the required conditions to offer this ancillary service.
2. To bundle the hourly wind power production in intervals greater than 1 hour. This way of proceeding may be appropriate and necessary in long- and medium-term analysis, in which, as indicated in Subsection 11.2.4, the proposed model is used as a planning tool for reserve valuation based on a market-clearing procedure.
3. To reduce the number of scenarios representing the stochastic behavior of wind power. This reduction has been applied in this case study as a mandatory measure to make the problem tractable, but it can also be employed to enhance computational efficiency thanks to the availability of accurate and efficient scenario-reduction techniques (see Chapter 3).

11.7 Summary and Conclusions

The increase of wind power generation throughout the world has been remarkable in recent years, and its growth prospects in the decades to come are simply staggering. Wind is a form of renewable energy increasingly appealing from an economic viewpoint. It is gradually shaping up as a good bet for governments in industrialized and developing countries to curb emissions of greenhouse gases in response to public pressure on climate change.

However, wind power production is also uncertain and variable. As a result, it cannot be dispatched at the will of the producer. New methodologies are therefore required to incorporate wind generation into electricity markets enhancing their economic performance without making the power system any less reliable.

In this chapter, the market-clearing model with stochastic security introduced in Chapter 10 is revisited to help meet that goal. The necessary modifications are mostly intended to adapt the market-clearing formulation to the intricacies of wind generation. The wind power production is treated as a continuous stochastic process, and as such, requires a discretization based on scenarios to be embedded into a stochastic programming optimization framework. The cardinality of the scenario set needs to be reduced using appropriate statistical tools with the aim of making the resulting mixed-integer linear programming problem tractable. Even after this reduction, the optimization problem is highly demanding computationally, and thereby, additional simplifications may have to be implemented to achieve the optimal or an approximate solution in reasonable time for realistic applications.

We illustrate the market-clearing formulation with a small example and a more realistic case study, based on a 3-node and a 24-node electric systems, respectively. Extensive simulations allow us to conclude that:

1. Wind generation decreases expected operation costs, but increases the costs pertaining to scheduling of reserves.
2. The reserve cost due to wind power uncertainty is relevant in comparison to the costs related to energy production.
3. Network congestion may seriously hinder the cost reduction achievable by wind generation.
4. The consideration of a wind spillage cost may have a significant impact on scheduled reserves and on generation/demand scheduling.

11.8 Notation

The notation used throughout this chapter is stated below for quick reference.

Indices and Numbers:

i	Index of generating units, running from 1 to N_{G}.
j	Index of loads, running from 1 to N_{L}.

m	Index of energy blocks offered by generating units, running from 1 to $N_{\mathrm{O}it}$ (number of blocks of energy offered by unit i in period t).
n, r	Indices of system buses.
t	Index of time periods, running from 1 to N_{T}.
ω	Index of wind power scenarios, running from 1 to N_{Ω}.

Real Variables:

C_{it}^{SU}	Cost due to the scheduled start-up of unit i in period t (\$). $C_{it\omega}^{\mathrm{SU}}$ is the actual start-up cost incurred by unit i in period t and scenario ω.
L_{jt}^{S}	Power scheduled for load j in period t (MW). Bounded by $L_{jt}^{\mathrm{S,min}}$ and $L_{jt}^{\mathrm{S,max}}$.
P_{it}^{S}	Power output scheduled for unit i in period t (MW).
$p_{\mathrm{G}it}(m)$	Power output scheduled from the m-th block of energy offered by unit i in period t (MW). Limited to $p_{\mathrm{G}it}^{\max}(m)$.
R_{it}^{U}	Spinning reserve up scheduled for unit i in period t (MW). Limited to $R_{it}^{\mathrm{U,\,max}}$.
R_{it}^{D}	Spinning reserve down scheduled for unit i in period t (MW). Limited to $R_{it}^{\mathrm{D,\,max}}$.
R_{it}^{NS}	Non-spinning reserve scheduled for unit i in period t (MW). Limited to $R_{it}^{\mathrm{NS,\,max}}$.
R_{jt}^{U}	Spinning reserve up scheduled for load j in period t (MW). Limited to $R_{jt}^{\mathrm{U,\,max}}$.
R_{jt}^{D}	Spinning reserve down scheduled for load j in period t (MW). Limited to $R_{jt}^{\mathrm{D,\,max}}$.
$P_{t}^{\mathrm{WP,S}}$	Scheduled wind power in period t (MW).
$C_{it\omega}^{\mathrm{A}}$	Cost due to the change in the start-up plan of unit i in period t and scenario ω (\$).
$P_{it\omega}^{\mathrm{G}}$	Power output of unit i in period t and scenario ω (MW). Bounded by $P_{i}^{\min}$ and $P_{i}^{\max}$.
$L_{jt\omega}^{\mathrm{C}}$	Power consumed by load j in period t and scenario ω (MW).
$r_{it\omega}^{\mathrm{U}}$	Spinning reserve up deployed by unit i in period t and scenario ω (MW).
$r_{it\omega}^{\mathrm{D}}$	Spinning reserve down deployed by unit i in period t and scenario ω (MW).
$r_{it\omega}^{\mathrm{NS}}$	Non-spinning reserve deployed by unit i in period t and scenario ω (MW).
$r_{jt\omega}^{\mathrm{U}}$	Reserve up deployed by load j in period t and scenario ω (MW).

$r^{\mathrm{D}}_{jt\omega}$ Reserve down deployed by load j in period t and scenario ω (MW).

$r_{\mathrm{G}it\omega}(m)$ Reserve deployed from the m-th block of energy offered by unit i in period t and scenario ω (MW).

$L^{\mathrm{shed}}_{jt\omega}$ Load shedding imposed on consumer j in period t and scenario ω (MW).

$S_{t\omega}$ Wind power generation spillage in period t and scenario ω (MW).

$f_{t\omega}(n,r)$ Power flow through line (n,r) in period t and scenario ω (MW). Limited to $f^{\max}(n,r)$.

$\delta_{nt\omega}$ Voltage angle at node n in period t and scenario ω (rad).

Binary Variables:

u_{it} 0/1 variable equal to 1 if unit i is scheduled to be committed in period t.

$v_{it\omega}$ 0/1 variable equal to 1 if unit i is online in period t and scenario ω.

Random Variables:

P^{WP}_{t} Random variable modeling the wind power generation in period t (MW). P^{WP}_{tw} represents the realization of this random variable in scenario ω.

Constants:

$C^{R^{\mathrm{U}}}_{it}$ Offer cost of the up-spinning reserve submitted by unit i in period t (\$/MWh).

$C^{R^{\mathrm{D}}}_{it}$ Offer cost of the down-spinning reserve submitted by unit i in period t (\$/MWh).

$C^{R^{\mathrm{NS}}}_{it}$ Offer cost of the non-spinning reserve submitted by unit i in period t (\$/MWh).

$C^{R^{\mathrm{U}}}_{jt}$ Offer cost of the up-spinning reserve submitted by load j in period t (\$/MWh).

$C^{R^{\mathrm{D}}}_{jt}$ Offer cost of the down-spinning reserve submitted by load j in period t (\$/MWh).

d_t Duration of time period t (h).

$\lambda_{it}^{\mathrm{SU}}$	Start-up offer cost of unit i in period t (\$).
$\lambda_{\mathrm{G}it}(m)$	Marginal cost of the m-th block of energy offered by unit i in period t (\$/MWh).
$\lambda_{\mathrm{L}jt}$	Utility of consumer j in period t (\$/MWh).
V_{jt}^{LOL}	Value of load shed for consumer j in period t (\$/MWh).
V_t^{S}	Cost of wind power spillage in period t (\$/MWh).
π_ω	Probability of wind power scenario ω.
$B(n,r)$	Absolute value of the imaginary part of the admittance of line (n,r) (per unit).

Sets:

Λ	Set of transmission lines.
M_{L}	Mapping of the set of loads (consumers) into the set of buses.
M_{G}	Mapping of the sets of generating units into the set of buses.
Ω	Set of scenarios.

11.9 Exercises

Exercise 11.1. Revise the market-clearing model presented in Section 11.3 to consider wind producers as competitive agents in the electricity market. List and justify the required changes.

Exercise 11.2. Extend the market-clearing formulation described in Section 11.3 to include ramping limits of generating units and minimum up and down time constraints. Then, work out to enforce these constraints in the base case of the example in Section 11.5 and observe their effects on the expected cost.

Exercise 11.3. Include a risk measure (e.g., the CVaR) in the objective function (11.12). Use the example in Section 11.5 to analyze how the risk-aversion level of the system operator affects the resulting scheduling programs.

Exercise 11.4. Represent the required amount of reserves as a function of the wind power penetration and uncertainty levels for the base case of the example in Section 11.5. That is, build a figure analogous to Fig. 11.9(a) for the total quantity of scheduled reserves.

Exercise 11.5. Solve the base case of the comprehensive example in Section 11.5 fixing the scheduled wind power production in period 3 to 50 MW (i.e., $P_3^{\mathrm{WP,\,S}} = 50$ MW). What can be said about the solution schedule? Does the expected cost change? Why? Use the GAMS code provided in Section A.9 of Appendix A.

Exercise 11.6. Solve the base case of the comprehensive example in Section 11.5 fixing the scheduled wind power production in period 2 to its forecast value (i.e., $P_2^{\mathrm{WP,\,S}} = 20$ MW). Break down the expected cost into components and justify why this solution is economically worse than the one provided in that example.

Exercise 11.7. For the network-congested case of the example in Section 11.5, find the value of lost load below which load shedding events begins to occur.

Exercise 11.8. For the network-congested case of the example in Section 11.5, find the value of wind spillage cost above which involuntary load shedding becomes more profitable than spilling wind.

Exercise 11.9. Solve the comprehensive example in Section 11.5 considering that the wind power scenarios in Table 11.1 are equiprobable and the load offers to sell both up- and down-spinning reserve at $20/MWh. Elaborate on the obtained results.

Exercise 11.10. Compute the average uncertainty cost (AUC), the average benefit (AB) and the net average benefit (NAB) from the results obtained in Exercise 11.9.

Appendix A
GAMS codes

A.1 Introduction

In Chapters 5–11 different decision-making models pertaining to producers, consumers, retailers, and market/system operators are proposed and formulated. Model performances are assessed by solving and discussing examples and realistic case studies.

The objective of this Appendix is to help the reader to implement the models presented in this book. For this reason, the GAMS codes used to formulate the examples in the aforementioned chapters are included in this Appendix. We refer the interested reader to [19, 141] for further details on GAMS formulation.

Specifically, the list of examples whose GAMS codes are included in this Appendix is

1. Producer Pool Example (Section 5.7).
2. Wind Producer Example (Section 6.6).
3. Producer Futures Market Example. No Unit Unavailability (Section 7.7).
4. Producer Futures Market Example. Unit Unavailability (Section 7.8).
5. Retailer Example (Section 8.7).
6. Consumer Example (Section 9.5).
7. Market-Clearing Example (Section 10.6).
8. Market-Clearing Example with Wind Generation (Section 11.5).

A.2 GAMS code for the Producer Pool Example (Section 5.7)

```
$title POOL PRODUCER
```

```
***************************************************************************
*       DATA
***************************************************************************

SETS
G       Units                   /g1 * g2/
T       Periods                 /t1 * t3/
W       Scenarios               /w1 * w12/;

alias(W,WW);

SCALARS
beta    Weighting parameter     /0/
alpha   Confidence level        /0.95/;

PARAMETERS
CP(G)   Production cost
/ g1    12
  g2    16 /

PGmax(G) Capacity
/ g1    100
  g2    50 /

PGmin(G) Minimum power ouput
/ g1    10
  g2    10 /


Rup(G)  Ramp-up ramp
/ g1    25
  g2    15 /

Rdw(G)  Ramp-down ramp
/ g1    25
  g2    15 /

PG0(G)  Initial power status
/ g1    50
  g2    25 /

PAmax(G) Maximum range power in the AGC service
/ g1    15
  g2    10 /

AD(W)   Day-ahead non-anticipativity vector
/ w1    1
  w2    1
  w3    1
  w4    0
  w5    1
  w6    1
  w7    1
  w8    0
  w9    1
  w10   1
  w11   1
  w12   0 /

AR(W)   Regulation non-anticipativity vector
/ w1    1
  w2    1
  w3    1
  w4    0
  w5    1
  w6    1
  w7    1
```

```
  w8     0
  w9     1
  w10    1
  w11    1
  w12    0 /

AA(W)    Adjustment nonanticipativity vector
/ w1     1
  w2     0
  w3     1
  w4     0
  w5     1
  w6     0
  w7     1
  w8     0
  w9     1
  w10    0
  w11    1
  w12    0 /

prob(W);


TABLE lambdaD(W,T) Day-ahead price scenarios
         t1      t2      t3
w1       11.0    16.5    12.1
w2       11.0    16.5    12.1
w3       11.0    16.5    12.1
w4       11.0    16.5    12.1
w5       13.2    18.7    16.5
w6       13.2    18.7    16.5
w7       13.2    18.7    16.5
w8       13.2    18.7    16.5
w9       15.4    19.8    17.6
w10      15.4    19.8    17.6
w11      15.4    19.8    17.6
w12      15.4    19.8    17.6;

TABLE lambdaR(W,T) Regulation price scenarios
         t1      t2      t3
w1       5.5     8.5     6.5
w2       5.5     8.5     6.5
w3       5.0     7.0     5.5
w4       5.0     7.0     5.5
w5       6.5     9.5     8.0
w6       6.5     9.5     8.0
w7       6.0     8.5     7.5
w8       6.0     8.5     7.5
w9       7.0     10.5    10.5
w10      7.0     10.5    10.5
w11      6.5     9       9.5
w12      6.5     9       9.5;

TABLE lambdaA0(W,T) Adjustment price scenarios (inverse demand curve intercept)
         t1      t2      t3
w1       17      28      15
w2       10      17      13
w3       12      17      15
w4        9      14      11
w5       17      22      19
w6       14      19      15
w7       16      25      21
w8       13      17      14
w9       17      28      24
w10      14      23      20
w11      16      23      21
w12      13      19      18;
```

```
TABLE gammaA(W,T) Adjustment price scenarios  (inverse demand curve slope)
         t1      t2      t3
w1       -0.01   -0.03   -0.02
w2       -0.01   -0.03   -0.02
w3       -0.01   -0.03   -0.02
w4       -0.01   -0.03   -0.02
w5       -0.01   -0.03   -0.02
w6       -0.01   -0.03   -0.02
w7       -0.01   -0.03   -0.02
w8       -0.01   -0.03   -0.02
w9       -0.01   -0.03   -0.02
w10      -0.01   -0.03   -0.02
w11      -0.01   -0.03   -0.02
w12      -0.01   -0.03   -0.02;

TABLE OD(W,T) Day-ahead price order
         t1      t2      t3
w1       1       1       1
w2       1       1       1
w3       1       1       1
w4       1       1       1
w5       2       2       2
w6       2       2       2
w7       2       2       2
w8       2       2       2
w9       3       3       3
w10      3       2       3
w11      3       2       3
w12      3       3       3;

prob(W)=1/card(W);

***************************************************************************
*        DECLARATION OF VARIABLES
***************************************************************************

VARIABLES
z                Objective function

** FIRST-STAGE VARIABLES

PD(G,T,W)        Power sold in the day-ahead market

** SECOND-STAGE VARIABLES

PR(G,T,W)        Power sold in the regulation market
wReg(G,T,W)      01 variable modeling if regulation service is provided

** THIRD-STAGE VARIABLES

PA(G,T,W)        Power sold in the adjustment market
PG(G,T,W)        Power output of the unit

** VARIABLES ASSOCIATED WITH RISK MODELING

xi               Auxiliary variable used to calculate the CVaR
eta(W)           Auxiliary variable used to calculate the CVaR;

POSITIVE VARIABLES PR(G,T,W), PD(G,T,W), eta(W), PA(G,T,W), EA(T,W);
BINARY VARIABLE wReg(G,T,W);


EQUATIONS
ObjectiveFunction

** DAY-AHEAD MARKET CONSTRAINTS
```

```
NonDecreasingDayAhead
NonAnticipativityDayAhead

** REGULATION MARKET CONSTRAINTS

MaximumReg
NonAnticipativityReg

** ADJUSTMENT MARKET CONSTRAINTS

NonAnticipativityAdjustment
PowerAdjustment

** UNIT CONSTRAINTS

Capacity
MinimumOutput
RampUp
RampDown
RampUp0
RampDown0
Power
Power0

** CVAR CONSTRAINTS

CVaR;

ObjectiveFunction..  z =e= sum(W, prob(W)*sum(T, sum(G,lambdaD(W,T)*PD(G,T,W))
+sum(G,lambdaR(W,T)*PR(G,T,W))+(lambdaA0(W,T)*EA(T,W)+
gammaA(W,T)*EA(T,W)*EA(T,W))-sum(g,CP(G)*(PD(G,T,W)+PA(G,T,W))) ))
+beta*(xi-1/(1-alpha)*sum(w,prob(W)*eta(W)));

NonDecreasingDayAhead(G,T,W,WW)$((OD(W,T)+1=OD(WW,T)) and (AD(W)=0)
and (AD(WW)=0))..   PD(G,T,W)-PD(G,T,WW) =l= 0;

NonAnticipativityDayAhead(G,T,W)$(AD(W)=1)..     PD(G,T,W)-PD(G,T,W+1) =e= 0;

MaximumReg(G,T,W)..      PR(G,T,W) =l= PAmax(G)*wReg(G,T,W);

NonAnticipativityReg(G,T,W)$(AR(W)=1)..  PR(G,T,W)-PR(G,T,W+1) =e= 0;

NonAnticipativityAdjustment(G,T,W)$(AA(W)=1)..   PA(G,T,W)-PA(G,T,W+1) =e= 0;

PowerAdjustment(T,W)..   sum(G,PA(G,T,W)) =e= EA(T,W);

Capacity(G,T,W)..        PD(G,T,W)+0.5*PR(G,T,W)+PA(G,T,W) =l= PGmax(G);

MinimumOutput(G,T,W)..   PD(G,T,W)-0.5*PR(G,T,W)+PA(G,T,W) =g= PGmin(G);

RampUp(G,T,W)$(ord(T) gt 1)..    PG(G,T,W)-PG(G,T-1,W) =l=
(1-wReg(G,T,W))*Rup(G);

RampDown(G,T,W)$(ord(T) gt 1)..  PG(G,T-1,W)-PG(G,T,W) =l=
(1-wReg(G,T,W))*Rdw(G);

RampUp0(G,T,W)$(ord(T) eq 1)..   PG(G,T,W)-PG0(G) =l= (1-wReg(G,T,W))*Rup(G);

RampDown0(G,T,W)$(ord(T) eq 1).. PG0(G)-PG(G,T,W) =l= (1-wReg(G,T,W))*Rdw(G);

Power(G,T,W)$(ord(T) gt 1)..     PD(G,T,W)+PA(G,T,W) =e=
0.5*(PG(G,T,W)+PG(G,T-1,W));

Power0(G,T,W)$(ord(T) eq 1)..    PD(G,T,W)+PA(G,T,W) =e=
0.5*(PG(G,T,W)+PG0(G));
```

```
CVaR(W)..                         xi-sum(T, sum(G,lambdaD(W,T)*PD(G,T,W))+
sum(G,lambdaR(W,T)*PR(G,T,W))+(lambdaA0(W,T)*EA(T,W)+
gammaA(W,T)*EA(T,W)*EA(T,W))-sum(G,CP(G)*(PD(G,T,W)+PA(G,T,W))) ) =l= eta(W);

***************************************************************************
*        MODEL
***************************************************************************

MODEL PoolProducer /ALL/;
PoolProducer.optcr=0;
Option iterlim = 1e8;
Option reslim = 1e10;
Option miqcp=cplex;
SOLVE PoolProducer USING miqcp MAXIMIZING z;
```

A.3 GAMS code for the Wind Producer Example (Section 6.6)

```
$title SHORT-TERM TRADING FOR A WIND POWER PRODUCER

* DA: Day-ahead market
* AM: Adjustment market
* BM: Balancing market

***************************************************************************
*        DATA
***************************************************************************

SET
D  Index for DA price scenarios /d1, d2/
A  Index for price differences between DA and AM /a1, a2/
L  Index for scenarios modeling wind production in between DA and AM /l1, l2/
W  Index for scenarios modeling wind production after AM /w1, w2/
K  Index for scenarios representing imbalance price ratios /k1, k2/;

ALIAS(D,Daux);

SCALARS
dt       Time span between two consecutive time periods (h) /1/
Pmax     Wind farm capacity (MW) /100/
alfa     Confidence level /0.95/
beta     Risk-aversion parameter /0/;

TABLE P(L,W)   Wind power production
         w1      w2
l1       100     50
l2       0       40;

PARAMETERS
lambdaD(D)       Day-ahead market prices
/ d1      50
  d2      20 /

MAvsMD(A)       Price difference between day-ahead and adjustment markets
/ a1      -10
  a2      3 /

r_pos(K)        Imbalance price ratio for positive energy deviations
/ k1      1
```

```
  k2      0.9 /

r_neg(K)       Imbalance price ratio for negative energy deviations

/ k1      1.5
  k2      1 /

lambdaI(D,A)    Adjustment market prices

pi(D,L,A,W,K)  Scenario probabilities
/ d1.l1.a1.w1.k1        0.0098
  d1.l1.a1.w1.k2        0.0042
  d1.l1.a1.w2.k1        0.0294
  d1.l1.a1.w2.k2        0.0126
  d1.l1.a2.w1.k1        0.0182
  d1.l1.a2.w1.k2        0.0078
  d1.l1.a2.w2.k1        0.0546
  d1.l1.a2.w2.k2        0.0234
  d1.l2.a1.w1.k1        0.0343
  d1.l2.a1.w1.k2        0.0147
  d1.l2.a1.w2.k1        0.0245
  d1.l2.a1.w2.k2        0.0105
  d1.l2.a2.w1.k1        0.0637
  d1.l2.a2.w1.k2        0.0273
  d1.l2.a2.w2.k1        0.0455
  d1.l2.a2.w2.k2        0.0195
  d2.l1.a1.w1.k1        0.0147
  d2.l1.a1.w1.k2        0.0063
  d2.l1.a1.w2.k1        0.0441
  d2.l1.a1.w2.k2        0.0189
  d2.l1.a2.w1.k1        0.0273
  d2.l1.a2.w1.k2        0.0117
  d2.l1.a2.w2.k1        0.0819
  d2.l1.a2.w2.k2        0.0351
  d2.l2.a1.w1.k1        0.05145
  d2.l2.a1.w1.k2        0.02205
  d2.l2.a1.w2.k1        0.03675
  d2.l2.a1.w2.k2        0.01575
  d2.l2.a2.w1.k1        0.09555
  d2.l2.a2.w1.k2        0.04095
  d2.l2.a2.w2.k1        0.06825
  d2.l2.a2.w2.k2        0.02925 /;

lambdaI(D,A) = lambdaD(D) + MAvsMD(A);

***************************************************************************
*        DECLARATION OF VARIABLES
***************************************************************************

VARIABLES

z                        Objective function value
profit(D,L,A,W,K)        Profit per scenario

** FIRST-STAGE VARIABLES

Pd(D)                    Power sold in the day-ahead market

** SECOND-STAGE VARIABLES

Pa(D,L)                  Power traded in the adjustment market
Ps(D,L)                  Final power schedule

** THIRD-STAGE VARIABLES

desvP(D,L,W)             Positive energy deviation
desvN(D,L,W)             Negative energy deviation
```

```
** VARIABLES ASSOCIATED WITH RISK MODELING

CVaR                    Conditional value-at-risk
var                     Value-at-risk
eta(D,L,A,W,K)          Auxiliary variable

***************************************************************************
*        MATHEMATICAL CHARACTERIZATION OF VARIABLES
***************************************************************************

POSITIVE VARIABLES Pd(D), Ps(D,L), desvP(D,L,W), desvN(D,L,W), eta(D,L,A,W,K);

Pd.up(D) = Pmax;
Ps.up(D,L) = Pmax;

***************************************************************************
*        EQUATIONS
***************************************************************************

EQUATIONS

OF                         Objective function
CVaRfunction               CVaR definition
PROFITfunction(D,L,A,W,K)  Profit definition
DesvDEF(D,L,W)             Definition of energy imbalances
MAXdesvPOS(D,L,W)          Maximum positive energy deviation
MAXdesvNEG(D,L,W)          Maximum negative energy deviation
PsDEF(D,L)                 Computation of final power schedule
curve(D,Daux)              Non-decreasing condition of offering curves
noANTICIPd(D,Daux)         Non-anticipativity constraints for the DA market
CVaRrest(D,L,A,W,K)        Constraints associated with risk modeling;

OF.. z =e=  sum((D,L,A,W,K), pi(D,L,A,W,K)*profit(D,L,A,W,K)) + beta*CVaR;

CVaRfunction..

CVaR =e= var - (1/(1-alfa))*sum((D,L,A,W,K), pi(D,L,A,W,K)*eta(D,L,A,W,K));

PROFITfunction(D,L,A,W,K)..

profit(D,L,A,W,K) =e= dt*lambdaD(D)*Pd(D)
                        + dt*lambdaI(D,A)*Pa(D,L)
                        + lambdaD(D)*r_pos(K)*desvP(D,L,W)
                        - lambdaD(D)*r_neg(K)*desvN(D,L,W);

DesvDEF(D,L,W).. desvP(D,L,W)-desvN(D,L,W) =e= dt*(P(L,W) - Ps(D,L));

MAXdesvPOS(D,L,W).. desvP(D,L,W) =l= P(L,W)*dt;

MAXdesvNEG(D,L,W).. desvN(D,L,W) =l= Pmax*dt;

PsDEF(D,L).. Ps(D,L) =e= Pd(D) + Pa(D,L);

curve(D,Daux)$(lambdaD(Daux) GT lambdaD(D)).. Pd(D)-Pd(Daux) =l= 0;

noANTICIPd(D,Daux)$(lambdaD(Daux) EQ lambdaD(D)).. Pd(D) =e= Pd(Daux);

CVaRrest(D,L,A,W,K).. var - profit(D,L,A,W,K) - eta(D,L,A,W,K) =l= 0;

***************************************************************************
*        MODEL
***************************************************************************

MODEL WindProdProblem /ALL/;
OPTION iterlim = 1e8;
OPTION reslim = 1e10;
```

```
Option lp=cplex;
WindProdProblem.optcr=0;
SOLVE WindProdProblem USING lp MAXIMIZING z;
```

A.4 GAMS code for the Producer Futures Market Example. No Unit Unavailability (Section 7.7)

```
$title  PRODUCER (NO UNIT UNAVAILABILITY)

**************************************************************************
*        DATA
**************************************************************************

SETS
G        Generators                                /g1 * g2/
W        Scenarios                                 /w1 * w4/
T        Periods                                   /t1 * t4/
F        Forward contracts                         /f1 * f2/
J        Blocks in the forward contracting curve  /j1 * j2/
SPeak(T) peak hours;

SPeak(T)=no; SPeak('t2')=yes; SPeak('t4')=yes;

SCALARS
beta     Weighting factor /0/
alpha    Confidence level /0.95/;

PARAMETERS
CG(G)    Production cost
/ g1     31
  g2     32 /

Pmax(G)  Capacity of the units
/ g1     300
  g2     250 /

Pmin(G)  Minimum output of the units
/ g1     0
  g2     0 /

H(G)     Relationship between the production in peak and off-peak hours
/ g1     0.33
  g2     0.33 /

d(T)    Duration in hours of every period t
EPC(T)  Energy previously contracted
prob(W) Scenarios probability;

d(T)=1;
EPC(T)=0;
prob(W)=0.25;


TABLE PFmax(F,J) Upper limit of the forward contracting blocks
         j1      j2
f1       100     100
f2       100     100;

TABLE lambdaF(F,J) Price of the forward contracting blocks
```

```
        j1      j2
f1      35      34
f2      34.5    32;

TABLE lambdaP(W,T) Pool price
        t1      t2      t3      t4
w1      33      43      29      36
w2      32      39      26      33
w3      34      48      28      37
w4      33      45      26      39;

***************************************************************************
*       DECLARATION OF VARIABLES
***************************************************************************

VARIABLES
        z               Objective function

** FIRST-STAGE VARIABLES

        PF(F,J)         Power sold through forward contracts

** SECOND-STAGE VARIABLES

        EP(T,W)         Energy traded in the pool
        EG(G,T,W)       Energy generated by the units

** VARIABLES ASSOCIATED WITH RISK MODELING

        xi              Auxiliary variable used to calculate the CVaR
        eta(W)          Auxiliary variable used to calculate the CVaR

***************************************************************************
*       MATHEMATICAL CHARACTERIZATION OF VARIABLES
***************************************************************************

POSITIVE VARIABLES   PF(F,J),EP(T,W),EG(G,T,W),eta(W);

***************************************************************************
*       EQUATIONS
***************************************************************************

EQUATIONS
ObjetiveFunction

** FORWARD CONTRACT CONSTRAINTS

LimitContractMax

** UNIT CONSTRAINTS

LimitGenMin
LimitGenMax
RelPeakBase

** ENERGY BALANCE

EnergyBalance

** CVAR CONSTRAINTS

CVaRConstraint;

ObjetiveFunction..              z =e= sum(W,prob(W)*sum(T,sum(F,sum(J,PF(F,J)*
lambdaF(F,J)))*d(T)+lambdaP(W,T)*EP(T,W)-sum(G,CG(G)*EG(G,T,W))))+
beta*(xi-1/(1-alpha)*sum(W,prob(W)*eta(W)));
```

```
LimitContractMax(F,J)..         PF(F,J) =l= PFmax(F,J);

LimitGenMin(G,T,W)..            EG(G,T,W) =l= Pmax(G)*d(T);

LimitGenMax(G,T,W)..            EG(G,T,W) =g= Pmin(G)*d(T);

RelPeakBase(G,T,W)$SPeak(T)..   EG(G,T-1,W) =g= H(G)*EG(G,T,W);

EnergyBalance(T,W)..            sum(G,EG(G,T,W)) =e= EP(T,W)+
sum(F,sum(J,PF(F,J)*d(T)))+EPC(T);

CVaRConstraint(W)..        xi-(sum(T,sum(F,sum(J,PF(F,J)*lambdaF(F,J)))*d(T)
+lambdaP(W,T)*EP(T,W)-sum(G,CG(G)*EG(G,T,W)))) =l= eta(W);

***************************************************************************
*        MODEL
***************************************************************************

MODEL Producer /ALL/;
Producer.optcr=0;
Option iterlim = 1e8;
Option reslim = 1e10;
Option lp=cplex;
SOLVE Producer USING lp MAXIMIZING z;
```

A.5 GAMS code for the Producer Futures Market Example. Unit Unavailability (Section 7.8)

```
$title PRODUCER UNIT AVAILABILITY

***************************************************************************
*        DATA
***************************************************************************

SETS
G        Generators                             /g1 * g2/
W        Scenarios                              /w1 * w20/
T        Periods                                /t1 * t4/
F        Forward contracts                      /f1 * f2/
J        Blocks in the forward contracting curve  /j1 * j2/
SPeak(T) peak hours;

SPeak(T)=no; SPeak('t2')=yes; SPeak('t4')=yes;

SCALARS
beta     Weighting factor /0/
alpha    Confidence level /0.95/;

PARAMETERS
CG(G)    Production cost
/ g1     31
  g2     32 /

Pmax(G)  Capacity of the units
/ g1     300
  g2     250 /

Pmin(G)  Minimum output of the units
/ g1     0
```

```
  g2     0 /

H(G)     Relationship between the production in peak and off-peak hours
/ g1     0.33
  g2     0.33 /

prob(W) Scenarios probability
/ w1     0.24
  w2     0.24
  w3     0.24
  w4     0.24
  w5     0.0025
  w6     0.0025
  w7     0.0025
  w8     0.0025
  w9     0.0025
  w10    0.0025
  w11    0.0025
  w12    0.0025
  w13    0.0025
  w14    0.0025
  w15    0.0025
  w16    0.0025
  w17    0.0025
  w18    0.0025
  w19    0.0025
  w20    0.0025/

d(T)    Duration in hours of every period t
EPC(T)  Energy previously contracted;

d(T)=1;
EPC(T)=0;

TABLE PFmax(F,J) Upper limit of the forward contracting blocks
         j1      j2
f1       100     100
f2       100     100;

TABLE lambdaF(F,J) Price of the forward contracting blocks
         j1      j2
f1       35      34
f2       34.5    32;

TABLE lambdaP(W,T) pool price
        t1      t2      t3      t4
w1      33      43      29      36
w2      32      39      26      33
w3      34      48      28      37
w4      33      45      26      39
w5      33      43      29      36
w6      32      39      26      33
w7      34      48      28      37
w8      33      45      26      39
w9      33      43      29      36
w10     32      39      26      33
w11     34      48      28      37
w12     33      45      26      39
w13     33      43      29      36
w14     32      39      26      33
w15     34      48      28      37
w16     33      45      26      39
w17     33      43      29      36
w18     32      39      26      33
w19     34      48      28      37
w20     33      45      26      39
;
```

```
TABLE a_aux(W,T) availability of unit 2
         t1      t2      t3      t4
w1       1       1       1       1
w2       1       1       1       1
w3       1       1       1       1
w4       1       1       1       1
w5       0       1       1       1
w6       0       1       1       1
w7       0       1       1       1
w8       0       1       1       1
w9       1       0       1       1
w10      1       0       1       1
w11      1       0       1       1
w12      1       0       1       1
w13      1       1       0       1
w14      1       1       0       1
w15      1       1       0       1
w16      1       1       0       1
w17      1       1       1       0
w18      1       1       1       0
w19      1       1       1       0
w20      1       1       1       0
;
PARAMETER a(G,W,T);
a('g1',w,t)=1;
a('g2',w,t)=a_aux(w,t);

***************************************************************************
*        DECLARATION OF VARIABLES
***************************************************************************

VARIABLES
         z               Objective function

** FIRST-STAGE VARIABLES

         PF(F,J)         Power sold through forward contracts

** SECOND-STAGE VARIABLES

         EP(T,W)         Energy traded in the pool
         EG(G,T,W)       Energy generated by the units

** VARIABLES ASSOCIATED WITH RISK MODELING

         xi              Auxiliary variable used to calculate the CVaR
         eta(W)          Auxiliary variable used to calculate the CVaR

***************************************************************************
*        MATHEMATICAL CHARACTERIZATION OF VARIABLES
***************************************************************************

POSITIVE VARIABLES   PF(F,J),EP(T,W),EG(G,T,W),eta(W);

***************************************************************************
*        EQUATIONS
***************************************************************************

EQUATIONS

ObjetiveFunction

** FORWARD CONTRACT CONSTRAINTS

LimitContractMax

** UNIT CONSTRAINTS
```

```
LimitGenMin
LimitGenMax
RelPeakBase

** ARBITRAGE CONSTRAINTS

LimitBuyPool

** ENERGY BALANCE

EnergyBalance

** CVAR CONSTRAINTS

CVaRConstraint;

ObjetiveFunction..               z=e=sum(W,prob(W)*sum(T,lambdaP(W,T)*EP(T,W)+
sum(F,sum(J,PF(F,J)*lambdaF(F,J)))*d(T)-sum(G,CG(G)*EG(G,T,W))))+
beta*(xi-1/(1-alpha)*sum(W,prob(W)*eta(W)));

LimitContractMax(F,J)..          PF(F,J)=l=PFmax(F,J);

LimitGenMin(G,T,W)..             EG(G,T,W)=l= PMAX(G)*d(T)*a(G,W,T);

LimitGenMax(G,T,W)..             EG(G,T,W)=g= PMIN(G)*d(T)*a(G,W,T);

LimitBuyPool(T,W)..              EP(T,W)=g=(a('g2',W,T)-1)*
sum(F,sum(J,PF(F,J)*d(T)));

RelPeakBase(G,T,W)$SPeak(T)..    EG(G,T-1,W)=g=H(G)*EG(G,T,W);

EnergyBalance(T,W)..             sum(g,EG(G,T,W))=e=EP(T,W)+
sum(F,sum(J,PF(F,J)*d(T)));

CVaRConstraint(W)..              -(sum(T,lambdaP(W,T)*EP(T,W)+
sum(F,sum(J,PF(F,J)*lambdaF(F,J)))*d(T)-sum(G,CG(G)*EG(G,T,W))))+xi-eta(W)=l=0;

***************************************************************************
*        MODEL
***************************************************************************

MODEL ProducerAvailability /ALL/;
ProducerAvailability.optcr=0;
Option iterlim = 1e8;
Option reslim = 1e10;
Option lp=cplex;
SOLVE ProducerAvailability USING lp MAXIMIZING z;
```

A.6 GAMS code for the Retailer Example (Section 8.7)

```
$title RETAILER

***************************************************************************
*        DATA
***************************************************************************

SETS
```

```
W       Scenarios                               /w1 * w4/
T       Periods                                 /t1 * t2/
F       Forward contracts                       /f1 * f2/
E       Client groups                           /e1 * e1/
J       Blocks in the forward contracting curve  /j1 * j2/
I       Blocks in the piece-quota curve         /i1 * i3/
Ft(F,T) Forward contracts available in period t;

Ft(F,T)=yes;

SCALARS

lambdaRmax       Maximum selling price   /120/
lambdaRmin       Minimum selling price   /20/
beta             Weighting factor        /0/
alpha            Confidence level        /0.95/;

PARAMETERS

EPC(T)           Energy previously contracted
ED(T,E,W)        Stochastic demand of the clients
ER(T,E,I,W)      Stochastic demand supplied by the retailer
d(T)             Duration in hours of every period t
prob(W)          Scenario probability;

TABLE lambdaF(F,J) Price of the forward contracting blocks
         j1      j2
f1       66.00   69.30
f2       67.00   70.35;

TABLE PFmax(F,J) Upper limit of the forward contracting blocks
         j1      j2
f1       20      20
f2       20      20;

TABLE k(I,E) Demand supplied in each price-quota curve block (per unit)
         e1
i1       1.00
i2       0.35
i3       0.15;

TABLE lambdaRI(I,E) Price of each price-quota curve block
         e1
i1       65.0
i2       75.0
i3       85.0;

TABLE lambdaP(W,T) Pool prices
         t1      t2
w1       60      52
w2       65      55
w3       74      68
w4       70      61;

ED('t1',E,'w1')=350; ED('t2',E,'w1')=325; ED('t1',E,'w2')=365;
ED('t2',E,'w2')=335; ED('t1',E,'w3')=375; ED('t2',E,'w3')=345;
ED('t1',E,'w4')=360; ED('t2',E,'w4')=340;
d(T)=1;
prob(W)=1/3;
ER(T,E,I,W)=ED(T,E,W)*k(I,E);
EPC(T)=0;

***************************************************************************
*        DECLARATION OF VARIABLES
***************************************************************************

VARIABLES
```

```
        z               Objective function

** FIRST-STAGE VARIABLES

        PF(F,J)         Power bought from forward contracts
        lambdaR(E,I)    Auxilary variable for computing the selling price
        v(E,I)          Selection of block I in price-quota curve of group E

** SECOND-STAGE VARIABLES

        EP(T,W)         Energy traded in the pool

** VARIABLES ASSOCIATED WITH RISK MODELING

        xi              Auxiliary variable used to calculate the CVaR
        eta(W)          Auxiliary variable used to compute the CVaR;

***********************************************************************
*       MATHEMATICAL CHARACTERIZATION OF VARIABLES
***********************************************************************

POSITIVE VARIABLES PF(F,J),eta(W);
BINARY VARIABLES v(E,I);


***********************************************************************
*       EQUATIONS
***********************************************************************

EQUATIONS

ObjetiveFunction

** FORWARD CONTRACT CONSTRAINTS

LimitContractMax

** CLIENT CONSTRAINTS

LimitPriceMin
LimitPriceMax
PriceQuota

** ENERGY BALANCE CONSTRAINTS

EnergyBalance

** CVAR CONSTRAINTS

CVaRConstraint;

ObjetiveFunction..      z =e= sum(W,prob(w)*sum(T,sum((E,I),ER(T,E,I,W)*
lambdaR(E,I))-lambdaP(W,T)*EP(T,W)-sum(F$Ft(F,T),sum(J,PF(F,J)*
lambdaF(F,J)))*d(T)))+beta*(xi-1/(1-alpha)*sum(W,prob(W)*eta(W)));

LimitContractMax(F,J).. PF(F,J) =l= PFmax(F,J);

LimitPriceMin(E,I)$(ord(I) gt 1)..      lambdaR(E,I) =g=
lambdaRI(I-1,E)*v(E,I);

LimitPriceMax(E,I).. lambdaR(E,I) =l= lambdaRI(I,E)*v(E,I);

PriceQuota(E)..         sum(I,v(E,I)) =e= 1;

EnergyBalance(T,W)..    sum((E,I),ER(T,E,I,W)*v(E,I)) =e= EP(T,W)+
sum(F$Ft(F,T),sum(J,PF(F,J)*d(T)))+EPC(T);
```

```
CVaRConstraint(W)..  xi-(sum(T,sum((E,I),ER(T,E,I,W)*lambdaR(E,I))
-lambdaP(W,T)*EP(T,W)-sum(F$Ft(F,T),sum(J,PF(F,J)*lambdaF(F,J)))*d(T)))
=l= eta(W);

***************************************************************************
*        MODEL
***************************************************************************

MODEL Retailer /ALL/;
Retailer.optcr=0;
Option iterlim = 1e8;
Option reslim = 1e10;
Option mip=cplex;
SOLVE Retailer USING mip MAXIMIZING z;
```

A.7 GAMS code for the Consumer Example (Section 9.5)

```
$title  CONSUMER

***************************************************************************
*        DATA
***************************************************************************

SETS
B        Bilateral contracts                              /b1 * b6/
K        Stages                                           /k1 * k4/
W        Scenarios                                        /w1 * w16/
T        Periods                                          /t1 * t4/
N        Blocks in the piecewise linear production cost   /n1/
Bt(B,T)  Mapping of bilateral contracts active in period T
Kb(B,K)  Mapping of decisions on bilateral contracts made in stage K;

Bt(B,T)=no; Bt('b1',t)=yes; Bt('b2',t)=yes; Bt('b3','t1')=yes;
Bt('b4','t2')=yes; Bt('b5','t3')=yes; Bt('b6','t4')=yes;

Kb(B,K)=no; Kb('b1','k1')=yes; Kb('b2','k1')=yes; Kb('b3','k1')=yes;
Kb('b4','k2')=yes; Kb('b5','k3')=yes; Kb('b6','k4')=yes;

SCALARS

beta    Weighting factor /0/
alpha   Confidence level /0.95/;

PARAMETERS

ED(T) Demand
/ t1     200
  t2     250
  t3     225
  t4     275 /

lambdaBo(B) Reference price of electricity for bilateral contracts
/ b1     48.5
  b2     50.0
  b3     49.5
  b4     51.0
  b5     49.0
  b6     50.0 /
```

```
PBmax(B) Upper bound of bilateral contract purchases
/ b1     75
  b2     75
  b3     50
  b4     50
  b5     50
  b6     50 /

PBmin(B) Lower bound of bilateral contract purchases
/ b1     20
  b2     20
  b3     20
  b4     20
  b5     20
  b6     20 /

EGT(N) Blocks of the lineal approximation of the variable costs
/ n1     100 /

SG(N) Slope of the approximation of the variable costs
/ n1     45 /

lambdaB(B,T,W)   Price of electricity for bilateral contracts
d(T)             Duration in hours of every period t
EPC(T)           Energy previously contracted
prob(W)          Scenario probability;

TABLE lambdaP(W,T) Pool price
         t1      t2      t3      t4
w1       44      47      45      45
w2       44      47      45      47
w3       44      47      46      47
w4       44      47      46      49
w5       44      44      42      42
w6       44      44      42      45
w7       44      44      44      45
w8       44      44      44      46
w9       50      52      50      51
w10      50      52      50      55
w11      50      52      51      52
w12      50      52      51      53
w13      50      49      47      49
w14      50      49      47      50
w15      50      49      48      50
w16      50      49      48      53;

TABLE A(W,K) Non-anticipativity matrix
         k1      k2      k3      k4
w1       1       1       1       1
w2       1       1       1       0
w3       1       1       1       1
w4       1       1       0       0
w5       1       1       1       1
w6       1       1       1       0
w7       1       1       1       1
w8       1       0       0       0
w9       1       1       1       1
w10      1       1       1       0
w11      1       1       1       1
w12      1       1       0       0
w13      1       1       1       1
w14      1       1       1       0
w15      1       1       1       1;

d(T)=1;
EPC(T)=0;
```

```
lambdaB(B,T,W)=(lambdaBo(B)+lambdaP(W,T))/2;
prob(W)=0.0625;

***************************************************************************
*        DECLARATION OF VARIABLES
***************************************************************************

VARIABLES

   z             Objective function value
   PB(B,W)       Power purchased from bilateral contracts
   s(B,W)        Selection of bilateral contracts
   EG(N,T,W)     Energy produced by the self-production unit
   EP(T,W)       Energy traded in the pool

** VARIABLES ASSOCIATED WITH RISK MODELING

   xi            Auxiliary variable used to calculate the CVaR
   eta(W)        Auxiliary variable used to calculate the CVaR;

***************************************************************************
*        MATHEMATICAL CHARACTERIZATION OF VARIABLES
***************************************************************************

POSITIVE VARIABLES EG(N,T,W),eta(W);
BINARY VARIABLES s(B,W);

***************************************************************************
*        EQUATIONS
***************************************************************************

EQUATIONS

ObjetiveFunction

** BILATERAL CONTRACT CONSTRAINTS

LimitContractMax
LimitContractMin

** SELF-PRODUCTION CONSTRAINTS

SelfProduction1
SelfProduction

** ENERGY BALANCE

EnergyBalance

** ARBITRAGE CONSTRAINTS

Arbitrage

** NON-ANTICIPATIVITY CONSTRAINTS

Non_Anticipativity

** CVAR CONSTRAINTS

CVaR;

ObjetiveFunction..                z =e= sum(W,prob(W)*sum(T,sum(B$Bt(B,T),
lambdaB(B,T,W)*PB(B,W)*d(T))+lambdaP(W,T)*EP(T,W)+sum(N,SG(N)*EG(N,T,W))))+
beta*(xi+1/(1-alpha)*sum(W,prob(W)*eta(W)));

LimitContractMax(B,W)..           PB(B,W) =l= PBmax(B)*s(B,W);
```

```
LimitContractMin(B,W)..          PB(B,W) =g= PBmin(B)*s(B,W);

SelfProduction1(N,T,W)$(ord(N) eq 1)..          EG(N,T,W) =l= EGT(N);

SelfProduction(N,T,W)$(ord(N) gt 1)..          EG(N,T,W) =l= EGT(N)-EGT(N-1);

EnergyBalance(T,W)..             sum(N,EG(N,T,W))+EP(T,W)+sum(B$Bt(B,T),
PB(B,W)*d(T)) =e= ED(T)-EPC(T);

Arbitrage(T,W)..                 sum(N,EG(N,T,W)) =g= -EP(T,W);

Non_Anticipativity(B,W,K)$(KB(B,K) and (A(W,K)=1) and (ord(W) lt card(W)))..
PB(B,W) =e= PB(B,W+1);

CVaR(W)..                    sum(T,sum(B$Bt(B,T),lambdaB(B,T,W)*PB(B,W)*d(T))
+lambdaP(W,T)*EP(T,W)+sum(N,SG(N)*EG(N,T,W)))-xi-eta(W) =l= 0;

***************************************************************************
*        MODEL
***************************************************************************

MODEL Consumer /ALL/;
Consumer.optcr=0;
Option iterlim = 1e8;
Option reslim = 1e10;
Option mip=cplex;
SOLVE Consumer USING mip MINIMIZING z;
```

A.8 GAMS code for the Market-Clearing Example (Section 10.6)

```
$title MARKET CLEARING UNDER EQUIPMENT FAILURES

***************************************************************************
*        DATA
***************************************************************************

SETS
N   Nodes           /n1 * n3/
I   Units           /i1 * i3/
J   Loads           /j1 * j1/
T   Periods         /t1 * t4/
W   Scenarios       /w0 * w8/
M   Offer blocks    /m1 * m1/
S   System states   /s0*s2/;

ALIAS(N,R);

SCALARS
NT         Number of time periods /4/,
dt         Duration of time periods /1/
x          Reactance of transmission lines /0.13/
Pbase      Power base /41/;

TABLE GDATA(I,*) Generation technical data
         PMIN   PMAX    Istatus    Pini
i1       10     100     0          0
i2       10     100     0          0
i3       10     50      0          0;
```

```
TABLE RDATA_G(I,T,*) Generation-side reserve data
                 Rspin_max_up    Rspin_max_dn    Rnspin_max
i1.t1*t4         90              90              100
i2.t1*t4         90              90              100
i3.t1*t4         40              40              50;


TABLE CostBlock(I,T,M) Offer cost of energy blocks
                 m1
i1.t1*t4         30
i2.t1*t4         40
i3.t1*t4         20;

TABLE WidthBlock(I,T,M) Width of energy blocks
                 m1
i1.t1*t4         100
i2.t1*t4         100
i3.t1*t4         50;

TABLE Cr_schG(I,T,*) Generation-side reserve offer cost
                 up      dn        NS
i1.t1*t4         5       5         4.5
i2.t1*t4         7       7         5.5
i3.t1*t4         8       8         7;

TABLE ML(J,N) Mapping of the set of loads into the set of nodes
         n1      n2      n3
j1       0       0       1;

TABLE NetState(N,N,S)   Network states
         s0      s1      s2
n1.n1    0       0       0
n1.n2    1       1       1
n1.n3    1       1       0
n2.n1    1       1       1
n2.n2    0       0       0
n2.n3    1       1       1
n3.n1    1       1       0
n3.n2    1       1       1
n3.n3    0       0       0;

TABLE GenState(I,N,S)   States of unit availability
         s0      s1      s2
i1.n1    1       1       1
i1.n2    0       0       0
i1.n3    0       0       0
i2.n1    0       0       0
i2.n2    1       1       1
i2.n3    0       0       0
i3.n1    0       0       0
i3.n2    0       0       0
i3.n3    1       0       1;

PARAMETERS

elast_limits(J,T,*)   Elasticity limits of loads
/  j1.t1.sup     30
   j1.t2.sup     80
   j1.t3.sup     110
   j1.t4.sup     40
   j1.t1.inf     30
   j1.t2.inf     80
   j1.t3.inf     110
   j1.t4.inf     40 /

RDATA_D(J,T,*) Demand-side reserve data
```

```
/  j1.t1.Rspin_max_up     3
   j1.t2.Rspin_max_up     8
   j1.t3.Rspin_max_up     11
   j1.t4.Rspin_max_up     4
   j1.t1.Rspin_max_dn     3
   j1.t2.Rspin_max_dn     8
   j1.t3.Rspin_max_dn     11
   j1.t4.Rspin_max_dn     4 /

prob(W) Scenario w probability
/  w0    0.98807312
   w1    0.00199015
   w2    0.00199015
   w3    0.00199015
   w4    0.00199015
   w5    0.00099157
   w6    0.00099157
   w7    0.00099157
   w8    0.00099157 /

TAU(W) Period in which the contingency defining scenario w occurs
/  w0    5
   w1    1
   w2    2
   w3    3
   w4    4
   w5    1
   w6    2
   w7    3
   w8    4 /

NumO(I,T)        Number of energy blocks offered by each unit
lambdaSU(I,T)    Start-up offer cost
lambdaL(J,T)     Demand utility
Cr_schD(J,T,*)   Demand-side reserve offer cost
Vlol(J,T)        Value of lost load
b(N,N)           Imaginary parts of the admittance of lines
fmax(N,N)        Transmission capacity limits
conec(N,N,T,W)   Network state in time period t and scenario w
MG(I,N,T,W)      Mapping of the set of units into the set of nodes
G(I,T,W)         Set of available units per scenario and period;

NumO(I,T) = 1;
lambdaSU(I,T) = 100;
b(N,R)$(ord(N) ne ord(R)) = -1/x;
G(I,T,W) = 1;
lambdaL(J,T) = 0;
Cr_schD(J,T,'up') = 70;
Cr_schD(J,T,'dn') = 70;
Vlol(J,T) = 1000;
fmax(N,R)$(ord(N) ne ord(R)) = 55;

loop(T,
    loop(W,
         if(ord(T) < TAU(W),
           conec(N,R,T,W) =  NetState(N,R,'s0');
           MG(I,N,T,W) = GenState(I,N,'s0');
         else
           if(ord(W) <= 5,
                conec(N,R,T,W) = NetState(N,R,'s1');
                MG(I,N,T,W) = GenState(I,N,'s1');
                G('i3',T,W) = 0;
           else
                conec(N,R,T,W) = NetState(N,R,'s2');
                MG(I,N,T,W) = GenState(I,N,'s2');
           );
        );
```

```
    );
);

**************************************************************************
*       DECLARATION OF VARIABLES
**************************************************************************

VARIABLES
   EC                   Expected cost (objective function value)

** FIRST-STAGE VARIABLES
   Ps(I,T)              Power output schedule
   Ls(J,T)              Demand schedule
   R_spin_schG_d(I,T)   Generation-side spinning reserve schedule (downward)
   R_spin_schG_u(I,T)   Generation-side spinning reserve schedule (upward)
   R_spin_schD_d(J,T)   Demand-side spinning reserve schedule (downward)
   R_spin_schD_u(J,T)   Demand-side spinning reserve schedule (upward)
   R_nspin_schG(I,T)    Generation-side non-spinning reserve schedule
   u(I,T)               Scheduled commitment status

** SECOND-STAGE VARIABLES
   Lc(J,T,W)            Power consumption in real time
   Lshed(J,T,W)         Amount of involuntarily load shed
   Pg(I,T,W)            Generator power output in real time
   pgblock(I,T,W,M)     Power produced from energy blocks
   r_spin_depG_d(I,T,W) Generation-side spinning reserve deployment (downward)
   r_spin_depG_u(I,T,W) Generation-side spinning reserve deployment (upward)
   r_spin_depD_d(J,T,W) Demand-side spinning reserve deployment (downward)
   r_spin_depD_u(J,T,W) Demand-side spinning reserve deployment (upward)
   r_nspin_depG(I,T,W)  Generation-side non-spinning reserve deployment
   angle(N,T,W)         Voltage angle
   Pinj(N,T,W)          Power injection
   f(N,R,T,W)           Power flow
   v(I,T,W)             Real-time commitment status
   Csu(I,T,W)           Start-up cost;

**************************************************************************
*       MATHEMATICAL CHARACTERIZATION OF VARIABLES
**************************************************************************

POSITIVE VARIABLES
Csu(I,T,W), Lshed(J,T,W), r_spin_depG_d(I,T,W), r_spin_depG_u(I,T,W),
r_nspin_depG(I,T,W), r_spin_depD_d(J,T,W), r_spin_depD_u(J,T,W),
R_spin_schG_d(I,T), R_spin_schG_u(I,T), R_spin_schD_d(J,T), R_spin_schD_u(J,T),
R_nspin_schG(I,T), pgblock(I,T,W,M);


BINARY VARIABLES u(I,T), v(I,T,W);

** Elasticity limits of demand
Ls.up(J,T)= elast_limits(J,T,'sup');
Ls.lo(J,T)= elast_limits(J,T,'inf');

** Reference node
angle.fx('N1',T,W)=0;

** Non-anticipativity constraints
Lshed.fx(J,T,W)$(ord(T) LT TAU(W))= 0;
r_spin_depG_d.fx(I,T,W)$(ord(T) LT TAU(W))= 0;
r_spin_depG_u.fx(I,T,W)$(ord(T) LT TAU(W))= 0;
r_spin_depD_d.fx(J,T,W)$(ord(T) LT TAU(W))= 0;
r_spin_depD_u.fx(J,T,W)$(ord(T) LT TAU(W))= 0;
r_nspin_depG.fx(I,T,W)$(ord(T) LT TAU(W))= 0;

**************************************************************************
*       EQUATIONS
**************************************************************************
```

```
EQUATIONS

ECFunction      Objective function

** ELECTRICITY MARKET CONSTRAINTS

** Production limits
MAX_PROD(I,T)
MIN_PROD(I,T)

** Market equilibria
MARKET_EQU(T)

** Scheduled reserve determination constraints

** Generation side
RESERVE_SCH_SPIN_G_u(I,T) Spinning up
RESERVE_SCH_SPIN_G_d(I,T) Spinning down
RESERVE_SCH_NSPIN_G(I,T)  Non-spinning

** Demand side
RESERVE_SCH_SPIN_D_u(J,T) Spinning up
RESERVE_SCH_SPIN_D_d(J,T) Spinning down

** REAL-TIME OPERATING CONSTRAINTS

** Start-up costs
SUo(I,W)          T = 1
SU(I,T,W)         T > 1

** Generation limits
GL1(I,T,W)
GL2(I,T,W)

** Decomposition of generator power outputs into blocks
GL3(I,T,W,M)
GL4(I,T,W)

** Involuntary load shedding constraints
LIMIT_ENS(J,T,W)

** Network constraints
PB(N,T,W)           Power balance
DEF_Pinj(N,T,W)     Definition of power injection
DEF_FLOW(N,R,T,W)   Defintion of power flows
MAX_CAP(N,R,T,W)    Transmission capacity

** LINKING CONSTRAINTS
Pgenerated(I,T,W) Composition of generator power outputs
Pconsumed(J,T,W)  Composition of the power consumption

** Deployed reserve determination constraints

** Generation side
RESERVE_DPL_SPIN_G_u(I,T,W) Spinning up
RESERVE_DPL_SPIN_G_d(I,T,W) Spinning down
RESERVE_DPL_NSPIN_G(I,T,W)  Non-spinning

** Demand side
RESERVE_DPL_SPIN_D_u(J,T,W) Spinning up
RESERVE_DPL_SPIN_D_d(J,T,W) Spinning down;


ECFunction..

EC =e=  dt*sum((I,T),
```

```
Cr_schG(I,T,'up')* R_spin_schG_u(I,T)+ Cr_schG(I,T,'dn')* R_spin_schG_d(I,T)
+ Cr_schG(I,T,'NS')* R_nspin_schG(I,T)) + dt*sum((J,T),
Cr_schD(J,T,'up')* R_spin_schD_u(J,T)+ Cr_schD(J,T,'dn')* R_spin_schD_d(J,T))
+ dt*sum((T,W),
prob(W)*(sum((I,M)$(ord(M) LE NumO(I,T)), CostBlock(I,T,M)* pgblock(I,T,W,M))
- sum(J, lambdaL(J,T)* Lc(J,T,W))
+ sum(J, Vlol(J,T)*Lshed(J,T,W))))
+ sum((I,T,W),prob(W)*Csu(I,T,W));

MIN_PROD(I,T).. Ps(I,T) =g= GDATA(I,'PMIN')* u(I,T);

MAX_PROD(I,T).. Ps(I,T) =l= GDATA(I,'PMAX')*u(I,T);

MARKET_EQU(T).. sum(I, Ps(I,T)) =e= sum(J, Ls(J,T));

RESERVE_SCH_SPIN_G_u(I,T)..

                R_spin_schG_u(I,T) =l= RDATA_G(I,T,'Rspin_max_up')*u(I,T);

RESERVE_SCH_SPIN_G_d(I,T)..

                R_spin_schG_d(I,T) =l= RDATA_G(I,T,'Rspin_max_dn')*u(I,T);

RESERVE_SCH_NSPIN_G(I,T)..

                R_nspin_schG(I,T) =l= RDATA_G(I,T,'Rnspin_max')*(1-u(I,T));


RESERVE_SCH_SPIN_D_u(J,T).. R_spin_schD_u(J,T) =l= RDATA_D(J,T,'Rspin_max_up');

RESERVE_SCH_SPIN_D_d(J,T).. R_spin_schD_d(J,T) =l= RDATA_D(J,T,'Rspin_max_dn');

SUo(I,W).. Csu(I,'t1',W) =g= lambdaSU(I,'t1')*(v(I,'t1',W)-GDATA(I,'Istatus'));

SU(I,T,W)$(ord(T) GT 1).. Csu(I,T,W) =g= lambdaSU(I,T)*(v(I,T,W)-v(I,T-1,W));

GL1(I,T,W).. Pg(I,T,W) =g= GDATA(I,'PMIN')* v(I,T,W);

GL2(I,T,W).. Pg(I,T,W) =l= GDATA(I,'PMAX')*v(I,T,W);

GL3(I,T,W,M)$(ord(M) LE NumO(I,T)).. pgblock(I,T,W,M) =l= WidthBlock(I,T,M);

GL4(I,T,W).. Pg(I,T,W) =e= sum(M$(ord(M) LE NumO(I,T)), pgblock(I,T,W,M));

LIMIT_ENS(J,T,W).. Lshed(J,T,W)=l= Lc(J,T,W);

PB(N,T,W).. sum(I$(MG(I,N,T,W) EQ 1),Pg(I,T,W))- sum(J$(ML(J,N) EQ 1),
                                Lc(J,T,W)-Lshed(J,T,W))-Pinj(N,T,W) =e= 0;

DEF_Pinj(N,T,W).. Pinj(N,T,W) =e= sum(R$(conec(N,R,T,W) EQ 1),f(N,R,T,W));

DEF_FLOW(N,R,T,W)$(conec(N,R,T,W) EQ 1)..

         f(N,R,T,W)=e= -b(N,R)*(angle(N,T,W)-angle(R,T,W));

MAX_CAP(N,R,T,W)$(conec(N,R,T,W) EQ 1).. f(N,R,T,W) =l= fmax(N,R);

Pgenerated(I,T,W)$(G(I,T,W) EQ 1)..

     Pg(I,T,W) =e= Ps(I,T)+ r_spin_depG_u(I,T,W)+r_nspin_depG(I,T,W)
                 -r_spin_depG_d(I,T,W);

Pconsumed(J,T,W)..

     Lc(J,T,W) =e= Ls(J,T)- r_spin_depD_u(J,T,W) + r_spin_depD_d(J,T,W);

RESERVE_DPL_SPIN_G_u(I,T,W).. r_spin_depG_u(I,T,W) =l= R_spin_schG_u(I,T);
```

```
RESERVE_DPL_SPIN_G_d(I,T,W).. r_spin_depG_d(I,T,W) =l= R_spin_schG_d(I,T);

RESERVE_DPL_NSPIN_G(I,T,W).. r_nspin_depG(I,T,W) =l= R_nspin_schG(I,T);

RESERVE_DPL_SPIN_D_u(J,T,W).. r_spin_depD_u(J,T,W)=l= R_spin_schD_u(J,T);

RESERVE_DPL_SPIN_D_d(J,T,W).. r_spin_depD_d(J,T,W)=l= R_spin_schD_d(J,T);

***************************************************************************
*        MODEL
***************************************************************************

MODEL MCU /ALL/;
MCU.optcr=0;
Option iterlim = 1e8;
Option reslim = 1e10;
Option mip=cplex;
SOLVE MCU USING mip MINIMIZING EC;
```

A.9 GAMS code for the Market-Clearing Example with Wind Generation (Section 11.5)

```
$title MARKET CLEARING UNDER WIND GENERATION UNCERTAINTY

***************************************************************************
*        DATA
***************************************************************************
SETS
N        Nodes                   /n1 * n3/
I        Units                   /i1 * i3/
J        Loads                   /j1 * j1/
T        Periods                 /t1 * t4/
W        Wind power scenarios    /w1 * w3/
M        Offer blocks            /m1 * m1/;

ALIAS(N,R);

SCALARS
NT          Number of time periods /4/
dt          Duration of time periods /1/
x           Reactance of transmission lines /0.13/
Pbase       Power base /41/
wind_bus    System node at which wind generation is injected /2/;

TABLE GDATA(I,*) Generation technical data
         PMIN   PMAX    Istatus    Pini
i1       10     100     0          0
i2       10     100     0          0
i3       10     50      0          0;

TABLE RDATA_G(I,T,*) Generation-side reserve data
                 Rspin_max_up    Rspin_max_dn    Rnspin_max
i1.t1*t4         90              90              100
i2.t1*t4         90              90              100
i3.t1*t4         40              40              50;

TABLE CostBlock(I,T,M) Offer cost of energy blocks
```

```
                m1
i1.t1*t4        30
i2.t1*t4        40
i3.t1*t4        20;

TABLE WidthBlock(I,T,M) Width of energy blocks
                m1
i1.t1*t4        100
i2.t1*t4        100
i3.t1*t4        50;

TABLE Cr_schG(I,T,*) Generation-side reserve offer cost
                up      dn        NS
i1.t1*t4        5       5         4.5
i2.t1*t4        7       7         5.5
i3.t1*t4        8       8         7;

TABLE ML(J,N) Mapping of the set of loads into the set of nodes
         n1      n2      n3
j1       0       0       1;

TABLE MG(I,N) Mapping of the set of units into the set of nodes
         n1      n2      n3
i1       1       0       0
i2       0       1       0
i3       0       0       1;

TABLE WP(T,W) Scenarios of wind power production
         w1      w2      w3
t1       6       9       2
t2       20      30      13
t3       35      50      25
t4       8       12      6;

PARAMETERS
elast_limits(J,T,*)   Elasticity limits of loads
/  j1.t1.sup     30
   j1.t2.sup     80
   j1.t3.sup     110
   j1.t4.sup     40
   j1.t1.inf     30
   j1.t2.inf     80
   j1.t3.inf     110
   j1.t4.inf     40 /

RDATA_D(J,T,*) Demand-side reserve data
/  j1.t1.Rspin_max_up     3
   j1.t2.Rspin_max_up     8
   j1.t3.Rspin_max_up     11
   j1.t4.Rspin_max_up     4
   j1.t1.Rspin_max_dn     3
   j1.t2.Rspin_max_dn     8
   j1.t3.Rspin_max_dn     11
   j1.t4.Rspin_max_dn     4 /

prob(W) Scenario w probability
/  w1    0.6
   w2    0.2
   w3    0.2 /

NumO(I,T)         Number of energy blocks offered by each unit
lambdaSU(I,T)     Start-up offer cost
lambdaL(J,T)      Demand utility
Cr_schD(J,T,*)    Demand-side reserve offer cost
Vlol(J,T)         Value of lost load
Vspill(T)         Cost of wind power spillage
conec(N,N)        Network topology
```

```
b(N,N)           Imaginary parts of the admittance of lines
fmax(N,N)        Transmission capacity limits;


NumO(I,T) = 1;
lambdaSU(I,T) = 100;
lambdaL(J,T) = 0;
Cr_schD(J,T,'up') = 70;
Cr_schD(J,T,'dn') = 70;
Vlol(J,T) = 1000;
Vspill(T) = 0;
conec(N,R)$(ord(N) ne ord(R)) = 1;
b(N,R)$(ord(N) ne ord(R)) = -1/x;
fmax(N,R)$(ord(N) ne ord(R)) = 55;

***************************************************************************
*       DECLARATION OF VARIABLES
***************************************************************************

VARIABLES

   EC                     Expected cost (objective function value)

** FIRST-STAGE VARIABLES

   Ps(I,T)                Power output schedule
   Ls(J,T)                Demand schedule
   WPs(T)                 Wind power schedule
   R_spin_schG_d(I,T)     Generation-side spinning reserve schedule (downward)
   R_spin_schG_u(I,T)     Generation-side spinning reserve schedule (upward)
   R_spin_schD_d(J,T)     Demand-side spinning reserve schedule (downward)
   R_spin_schD_u(J,T)     Demand-side spinning reserve schedule (upward)
   R_nspin_schG(I,T)      Generation-side non-spinning reserve schedule
   u(I,T)                 Scheduled commitment status

** SECOND-STAGE VARIABLES

   Lc(J,T,W)              Power consumption in real time
   Lshed(J,T,W)           Amount of involuntarily load shed
   s(T,W)                 Amount of spilled wind power production
   Pg(I,T,W)              Generator power output in real time
   pgblock(I,T,W,M)       Power produced from energy blocks
   r_spin_depG_d(I,T,W)   Generation-side spinning reserve deployment (downward)
   r_spin_depG_u(I,T,W)   Generation-side spinning reserve deployment (upward)
   r_spin_depD_d(J,T,W)   Demand-side spinning reserve deployment (downward)
   r_spin_depD_u(J,T,W)   Demand-side spinning reserve deployment (upward)
   r_nspin_depG(I,T,W)    Generation-side non-spinning reserve deployment
   angle(N,T,W)           Voltage angle
   Pinj(N,T,W)            Power injection
   f(N,R,T,W)             Power flow
   v(I,T,W)               Real-time commitment status
   Csu(I,T,W)             Start-up cost;

***************************************************************************
*       MATHEMATICAL CHARACTERIZATION OF VARIABLES
***************************************************************************

POSITIVE VARIABLES

Csu(I,T,W), Lshed(J,T,W), r_spin_depG_d(I,T,W), r_spin_depG_u(I,T,W),
r_nspin_depG(I,T,W), r_spin_depD_d(J,T,W), r_spin_depD_u(J,T,W),
R_spin_schG_d(I,T), R_spin_schG_u(I,T), R_spin_schD_d(J,T), R_spin_schD_u(J,T),
R_nspin_schG(I,T), s(T,W), pgblock(I,T,W,M), WPs(T);

BINARY VARIABLES u(I,T), v(I,T,W);

** Elasticity limits of demand
```

```
Ls.up(J,T)= elast_limits(J,T,'sup');
Ls.lo(J,T)= elast_limits(J,T,'inf');

** Reference node

angle.fx('N1',T,W)=0;

** Wind generation limits

WPs.up(T)= 60;

**************************************************************************
*        EQUATIONS
**************************************************************************

EQUATIONS

ECFunction       Objective function

** ELECTRICITY MARKET CONSTRAINTS

** Production limits

MAX_PROD(I,T)
MIN_PROD(I,T)

** Market equilibria

MARKET_EQU(T)

** Scheduled reserve determination constraints

** Generation side

RESERVE_SCH_SPIN_G_u(I,T) Spinning up
RESERVE_SCH_SPIN_G_d(I,T) Spinning down
RESERVE_SCH_NSPIN_G(I,T)  Non-spinning

** Demand side

RESERVE_SCH_SPIN_D_u(J,T) Spinning up
RESERVE_SCH_SPIN_D_d(J,T) Spinning down

** REAL-TIME OPERATING CONSTRAINTS

** Start-up costs

SUo(I,W)          T = 1
SU(I,T,W)         T > 1

** Generation limits

GL1(I,T,W)
GL2(I,T,W)

** Decomposition of generator power outputs into blocks

GL3(I,T,W,M)
GL4(I,T,W)

** Involuntary load shedding constraints

LIMIT_ENS(J,T,W)

** Limit of wind power generation spillage
```

```
LIMIT_SPILLAGE(T,W)

** Network constraints

PB(N,T,W)           Power balance
PB_wind_bus(N,T,W)  Power balance at wind bus
DEF_Pinj(N,T,W)     Definition of power injection
DEF_FLOW(N,R,T,W)   Definition of power flows
MAX_CAP(N,R,T,W)    Transmission capacity

** LINKING CONSTRAINTS

Pgenerated(I,T,W)   Composition of generator power outputs
Pconsumed(J,T,W)    Composition of the power consumption

** Deployed reserve determination constraints

** Generation side

RESERVE_DPL_SPIN_G_u(I,T,W) Spinning up
RESERVE_DPL_SPIN_G_d(I,T,W) Spinning down
RESERVE_DPL_NSPIN_G(I,T,W)  Non-spinning

** Demand side

RESERVE_DPL_SPIN_D_u(J,T,W) Spinning up
RESERVE_DPL_SPIN_D_d(J,T,W) Spinning down
;

ECFunction..
EC =e=  dt*sum((I,T),
Cr_schG(I,T,'up')* R_spin_schG_u(I,T)+ Cr_schG(I,T,'dn')* R_spin_schG_d(I,T)
+ Cr_schG(I,T,'NS')* R_nspin_schG(I,T)) + dt*sum((J,T),
Cr_schD(J,T,'up')* R_spin_schD_u(J,T)+ Cr_schD(J,T,'dn')* R_spin_schD_d(J,T))
+ dt*sum((T,W),
prob(W)*(sum((I,M)$(ord(M) LE NumO(I,T)), CostBlock(I,T,M)* pgblock(I,T,W,M))
- sum(J, lambdaL(J,T)* Lc(J,T,W))
+ sum(J, Vlol(J,T)*Lshed(J,T,W)) + Vspill(T)*s(T,W)))
+ sum((I,T,W),prob(W)*Csu(I,T,W));

MIN_PROD(I,T).. Ps(I,T) =g= GDATA(I,'PMIN')* u(I,T);

MAX_PROD(I,T).. Ps(I,T) =l= GDATA(I,'PMAX')*u(I,T);

MARKET_EQU(T).. sum(I, Ps(I,T)) + WPs(T) =e= sum(J, Ls(J,T));

RESERVE_SCH_SPIN_G_u(I,T)..
                 R_spin_schG_u(I,T) =l= RDATA_G(I,T,'Rspin_max_up')*u(I,T);

RESERVE_SCH_SPIN_G_d(I,T)..

                 R_spin_schG_d(I,T) =l= RDATA_G(I,T,'Rspin_max_dn')*u(I,T);

RESERVE_SCH_NSPIN_G(I,T)..
                 R_nspin_schG(I,T) =l= RDATA_G(I,T,'Rnspin_max')*(1-u(I,T));

RESERVE_SCH_SPIN_D_u(J,T)..
                 R_spin_schD_u(J,T) =l= RDATA_D(J,T,'Rspin_max_up');

RESERVE_SCH_SPIN_D_d(J,T)..
                 R_spin_schD_d(J,T) =l= RDATA_D(J,T,'Rspin_max_dn');

SUo(I,W).. Csu(I,'t1',W) =g= lambdaSU(I,'t1')*(v(I,'t1',W)-GDATA(I,'Istatus'));

SU(I,T,W)$(ord(T) GT 1).. Csu(I,T,W) =g= lambdaSU(I,T)*(v(I,T,W)-v(I,T-1,W));

GL1(I,T,W).. Pg(I,T,W) =g= GDATA(I,'PMIN')* v(I,T,W);
```

```
GL2(I,T,W).. Pg(I,T,W) =l= GDATA(I,'PMAX')* v(I,T,W);

GL3(I,T,W,M)$(ord(M) LE NumO(I,T)).. pgblock(I,T,W,M) =l= WidthBlock(I,T,M);

GL4(I,T,W).. Pg(I,T,W) =e= sum(M$(ord(M) LE NumO(I,T)), pgblock(I,T,W,M));

LIMIT_ENS(J,T,W).. Lshed(J,T,W) =l= Lc(J,T,W);

LIMIT_SPILLAGE(T,W).. s(T,W) =l= WP(T,W);

PB(N,T,W)$(ord(N) NE wind_bus)..

sum(I$(MG(I,N) EQ 1),Pg(I,T,W))- sum(J$(ML(J,N) EQ 1),
                                 Lc(J,T,W)-Lshed(J,T,W))-Pinj(N,T,W) =e= 0;

PB_wind_bus(N,T,W)$(ord(N) EQ wind_bus)..
sum(I$(MG(I,N) EQ 1),Pg(I,T,W))- sum(J$(ML(J,N) EQ 1), Lc(J,T,W)-Lshed(J,T,W))
                                 + WP(T,W)-s(T,W)-Pinj(N,T,W) =e= 0;

DEF_Pinj(N,T,W)..
      Pinj(N,T,W) =e= sum(R$(conec(N,R) EQ 1),f(N,R,T,W));

DEF_FLOW(N,R,T,W)$(conec(N,R) EQ 1)..
         f(N,R,T,W) =e= -b(N,R)*(angle(N,T,W)-angle(R,T,W));

MAX_CAP(N,R,T,W)$(conec(N,R) EQ 1)..
         f(N,R,T,W) =l= fmax(N,R);

Pgenerated(I,T,W)..
     Pg(I,T,W) =e= Ps(I,T)+ r_spin_depG_u(I,T,W)+r_nspin_depG(I,T,W)
                  -r_spin_depG_d(I,T,W);

Pconsumed(J,T,W)..
     Lc(J,T,W) =e= Ls(J,T)- r_spin_depD_u(J,T,W) + r_spin_depD_d(J,T,W);

RESERVE_DPL_SPIN_G_u(I,T,W)..
                 r_spin_depG_u(I,T,W) =l= R_spin_schG_u(I,T);

RESERVE_DPL_SPIN_G_d(I,T,W)..
                 r_spin_depG_d(I,T,W) =l= R_spin_schG_d(I,T);

RESERVE_DPL_NSPIN_G(I,T,W)..
                 r_nspin_depG(I,T,W) =l= R_nspin_schG(I,T);

RESERVE_DPL_SPIN_D_u(J,T,W)..
                 r_spin_depD_u(J,T,W)=l= R_spin_schD_u(J,T);

RESERVE_DPL_SPIN_D_d(J,T,W)..
                 r_spin_depD_d(J,T,W)=l= R_spin_schD_d(J,T);

***************************************************************************
*        MODEL
***************************************************************************

MODEL MCU_WP /ALL/;
MCU_WP.optcr=0;
Option iterlim = 1e8;
Option reslim = 1e10;
Option mip=cplex;
SOLVE MCU_WP USING mip MINIMIZING EC;
```

Appendix B
24-Node System Data

This appendix lists the characteristics of the 24-node system used to examine the market-clearing models described in Chapters 10 and 11. Unless stated otherwise in the main text of these chapters, the characteristics itemized next apply integrally.

B.1 Network data

The 24-node system considered in this book is based on the single-area version of the IEEE Reliability Test System–1996 [54]. The system comprises 34 transmission lines as illustrated in Fig. B.1. Line resistances are null and thus, active power losses are disregarded. The values of reactance and capacity of transmission lines are listed in Table B.1. Line reactances are given in per unit on a 100-MVA base.

B.2 Generator data

Technical data for the generators are given in Table B.2. The power system includes 12 generating units. Unit 10 is a hydro generator whereas the rest of them are all thermal. In particular, units 8 and 9 are nuclear power plants. Table B.3 indicates the node location of each unit. The nuclear and hydro generators are assumed to be must-run units.

For simplicity, the energy and reserve offers of the units are assumed to be the same throughout the scheduling horizon consisting of 24 hourly periods. Tables B.4 and B.5 provide, respectively, the blocks of energy submitted by each generating unit and their corresponding offer costs. Table B.6 lists the start-up cost *declared* by each unit. Each generator offers the maximum possible amount of up and down-spinning reserve ($P^{\max} - P^{\min}$), both at

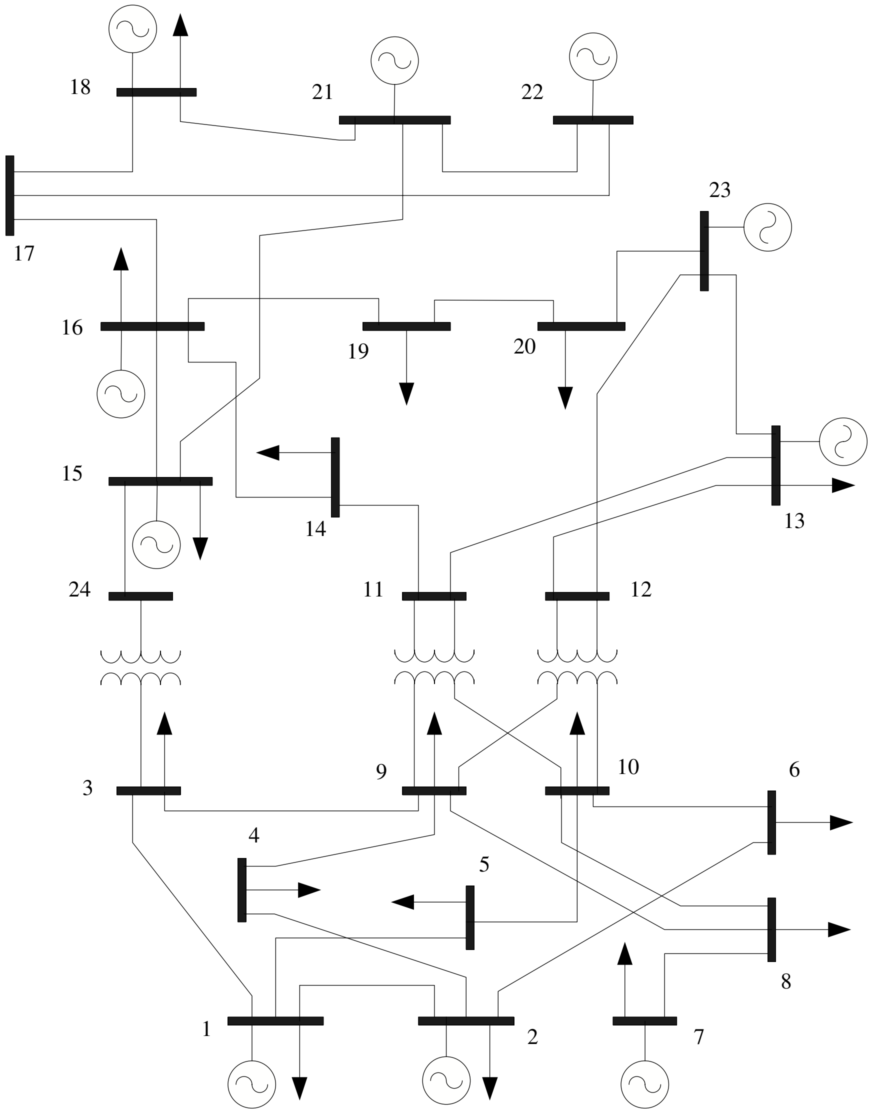


Fig. B.1 24-node system

Table B.1 24-node system: reactance and capacity of transmission lines

From node	To node	Reactance (p.u.)	Capacity (MVA)
1	2	0.0146	175
1	3	0.2253	175
1	5	0.0907	175
2	4	0.1356	175
2	6	0.2050	175
3	9	0.1271	175
3	24	0.0840	400
4	9	0.1110	175
5	10	0.0940	175
6	10	0.0642	175
7	8	0.0652	175
8	9	0.1762	175
8	10	0.1762	175
9	11	0.0840	400
9	12	0.0840	400
10	11	0.0840	400
10	12	0.0840	400
11	13	0.0488	500
11	14	0.0426	500
12	13	0.0488	500
12	23	0.0985	500
13	23	0.0884	500
14	16	0.0594	500
15	16	0.0172	500
15	21	0.0249	1000
15	24	0.0529	500
16	17	0.0263	500
16	19	0.0234	500
17	18	0.0143	500
17	22	0.1069	500
18	21	0.0132	1000
19	20	0.0203	1000
20	23	0.0112	1000
21	22	0.0692	500

Table B.2 24-node system: technical data of generating units

Unit #	1	2	3	4	5	6	7	8	9	10	11	12
$P^{\min}$ (MW)	30.4	30.4	75	206.85	12	54.25	54.25	100	100	300	108.5	140
$P^{\max}$ (MW)	152	152	300	591	60	155	155	400	400	300	310	350
RD (MW/h)	152	152	300	540	60	155	155	400	400	300	310	240
RU (MW/h)	152	152	300	540	60	155	155	400	400	300	310	240
SD (MW/h)	152	152	300	540	60	155	155	400	400	300	310	240
SU (MW/h)	152	152	300	540	60	155	155	400	400	300	310	240
D^{T} (h)	4	4	8	10	2	8	8	1	1	0	8	48
U^{T} (h)	8	8	8	12	4	8	8	1	1	0	8	24
S^0 (h)	0	0	2	1	1	2	0	0	0	0	0	0
V^0 (h)	22	22	0	0	0	0	10	769	16	24	10	300
u^0 (h)	1	1	0	0	0	0	1	1	1	1	1	1

$P^{\min}$: minimum power output; $P^{\max}$: maximum power output; RD: ramp-down limit; RU: ramp-up limit; SD: shut-down ramp limit; SU: start-up ramp limit; D^{T}: minimum down time; U^{T}: minimum up time; S^0: time periods the generating unit has been offline at the beginning of the scheduling horizon; V^0: time periods the generating unit has been online at the beginning of the scheduling horizon; u^0: initial commitment status (1 online, 0 otherwise)

Table B.3 24-node system: node location of generating units

Unit #	1	2	3	4	5	6	7	8	9	10	11	12
Node location	1	2	7	13	15	15	16	18	21	22	23	23

a cost equal to 25% of its highest offer cost of energy production. However, only generators located at buses 7, 15 and 16 offer non-spinning reserve. We consider that these units constitute the set of generators with the required features to provide this reserve service. Then, each of these generating units offers the maximum possible amount of non-spinning reserve ($P^{\max}$) at a cost equal to 20% of its highest offer cost of energy production.

B.3 Demand data

The load profile for the considered 24-hour scheduling horizon is provided in Table B.7. The 24-node system sketched in Fig. B.1 includes 17 loads. Both the nodal location of these loads and their contribution in percentage to the total system demand are listed in Table B.8. All these loads offer identical rates of \$50/MWh for up and down-spinning reserves. Loads can be curtailed from their normal levels up to 2% of their scheduled consumption for all hours to provide up-spinning reserves. Likewise, the demands can increase the same

Table B.4 24-node system: size (MWh) of the energy blocks offered by generating units

Unit #	Block #			
	1	2	3	4
1	30.40	45.60	45.60	30.40
2	30.40	45.60	45.60	30.40
3	75.00	75.00	90.00	60.00
4	206.85	147.75	118.20	118.20
5	12.00	18.00	18.00	12.00
6	54.25	38.75	31.00	31.00
7	54.25	38.75	31.00	31.00
8	100.00	100.00	120.00	80.00
9	100.00	100.00	120.00	80.00
10	300.00	0.00	0.00	0.00
11	108.50	77.50	62.00	62.00
12	140.00	87.50	52.50	70.00

Table B.5 24-node system: offer costs ($/MWh) submitted by generating units

Unit #	Block #			
	1	2	3	4
1	11.46	11.96	13.89	15.97
2	11.46	11.96	13.89	15.97
3	18.60	20.03	21.67	22.72
4	19.20	20.32	21.22	22.13
5	23.41	23.78	26.84	30.40
6	9.92	10.25	10.68	11.26
7	9.92	10.25	10.68	11.26
8	5.31	5.38	5.53	5.66
9	5.31	5.38	5.53	5.66
10	0.00	0.00	0.00	0.00
11	9.92	10.25	10.68	11.26
12	10.08	10.66	11.09	11.72

quantity to contribute to down-spinning reserve. Beyond these limits, the value of lost load is assumed to be $2000/MWh.

Table B.6 24-node system: start-up costs of generating units

Unit #	Start-up cost ($)	Unit #	Start-up cost ($)
1	1430.4	7	312.0
2	1430.4	8	0.0
3	1725.0	9	0.0
4	3056.7	10	0.0
5	437.0	11	624.0
6	312.0	12	2298.0

Table B.7 24-node system: load profile

Hour	System demand (MW)	Hour	System demand (MW)
12-1 am	1775.835	noon-1 pm	2517.975
1-2	1669.815	1-2	2517.975
2-3	1590.300	2-3	2464.965
3-4	1563.795	3-4	2464.965
4-5	1563.795	4-5	2623.995
5-6	1590.300	5-6	2650.500
6-7	1961.370	6-7	2650.500
7-8	2279.430	7-8	2544.480
8-9	2517.975	8-9	2411.955
9-10	2544.480	9-10	2199.915
10-11	2544.480	10-11	1934.865
11-noon	2517.975	11-12	1669.815

Table B.8 24-node system: node location and distribution of the total system demand

Load #	Node #	% of system load	Load #	Node #	% of system load
1	1	3.8	10	10	6.8
2	2	3.4	11	13	9.3
3	3	6.3	12	14	6.8
4	4	2.6	13	15	11.1
5	5	2.5	14	16	3.5
6	6	4.8	15	18	11.7
7	7	4.4	16	19	6.4
8	8	6.0	17	20	4.5
9	9	6.1			

Appendix C
Exercise solutions

This chapter includes the solutions to some of the exercises proposed in Chapters 2 to 11. These exercises are computationally oriented and serve as a complement to the illustrative examples solved throughout these chapters. Exercises from Chapter 1 are not solved due to their conceptual nature. The purpose of this appendix is to help the reader to further comprehend the nature of the models and solutions techniques described in this book, as well as to develop appropriate skills to formulate and solve similar models.

C.1 Exercises from Chapter 2

Solution to Exercise 2.3

An electricity producer owns a 100-MW unit with a production cost of \$25/MWh, and faces an uncertain electricity selling price for the next week. For simplicity, we consider that the price is uncertain but constant throughout the week. Scenario price data are provided in Table C.1.

Table C.1 Solution to Exercise 2.3: scenario data for the producer

Scenario #	Probability (per unit)	Price (\$/MWh)
1	0.2	50
2	0.6	46
3	0.2	44

Additionally, this producer has the possibility of selling up to 60 MW at \$45/MWh over the next week, by signing a bilateral contract before that week, i.e., prior to knowing the actual pool price it has to face.

The two-stage stochastic programming problem that model the decision-making process of the producer is as follows:

$$\begin{aligned}
&\text{Maximize}_{P_1^{\text{C}},P_2^{\text{C}},P_3^{\text{C}},P_1,P_2,P_3}\\
&\Pi^{\text{S}} =\\
&168(0.2 \times 45P_1^{\text{C}} + 0.6 \times 45P_2^{\text{C}} + 0.2 \times 45P_3^{\text{C}})+\\
&168(0.2 \times 50P_1 + 0.6 \times 46P_2 + 0.2 \times 44P_3)-\\
&168 \times 25 \left[0.2 \times (P_1^{\text{C}} + P_1) + 0.6 \times (P_2^{\text{C}} + P_2) + 0.2 \times (P_3^{\text{C}} + P_3)\right]\\
&\text{subject to}\\
&P_1^{\text{C}} + P_1 \leq 100\\
&P_2^{\text{C}} + P_2 \leq 100\\
&P_3^{\text{C}} + P_3 \leq 100\\
&0 \leq P_1^{\text{C}}, P_2^{\text{C}}, P_3^{\text{C}} \leq 60\\
&0 \leq P_1, P_2, P_3\\
&P_1^{\text{C}} = P_2^{\text{C}} = P_3^{\text{C}}\ .
\end{aligned}$$

Variables P_1^{C}, P_2^{C} and P_3^{C} represent the power sold through the bilateral contract in scenarios 1, 2 and 3, respectively. They should be equal so that no information is anticipated (as enforced by the last constraint). Variables P_1, P_2, and P_3 represent the power sold in the pool for scenarios 1, 2, and 3, respectively.

The objective function is the expected profit of the producer: revenue from selling through the bilateral contract (first row of the objective function) plus revenue from selling in the pool (second row) minus production cost (third row). Powers are multiplied by 168 to obtain the energy produced throughout the week.

The first three constraints enforce the capacity limit of the production unit for the three scenarios, while the next two constraints are the contract bound and nonnegativity declarations for all the variables. The last constraint enforces the non-anticipativity of the information.

The solution to this problem is $P_1^{\text{C}*} = P_2^{\text{C}*} = P_3^{\text{C}*} = 0$, $P_1^* = P_2^* = P_3^* = 100$ MW, which means that the power producer decides to sell its full production capacity to the pool regardless of the eventual pool price realization. This is so because the expected value of the pool price during the week, \$46.4/MWh, is greater than the price of the available bilateral contract, \$45/MWh.

The optimal expected profit is $\Pi^{\text{S}*}$ =\$359,520.

Solution to Exercise 2.4

This exercise is built upon the solution to Exercise 2.3 above. The decision-making process of the producer of Exercise 2.3 is as follows:

a. At the first stage (root of the tree in Fig. C.1), the producer has to decide how much to sell through the contract. This is modeled through variables $P_1^{\mathrm{C}} = P_2^{\mathrm{C}} = P_3^{\mathrm{C}} = P^{\mathrm{C}}$.
b. The second stage reproduces pool selling for each of the three considered price scenarios (leaves of the tree in Fig. C.1). This is modeled through variables P_1, P_2 and P_3.

The corresponding scenario tree of the producer decision-making process is provided in Fig. C.1.

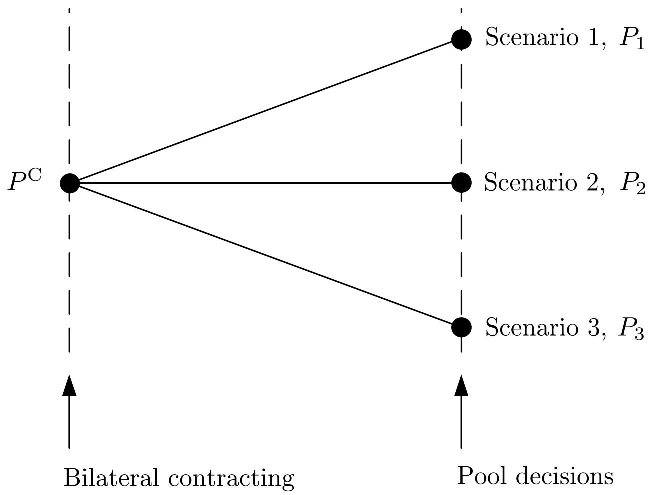


Fig. C.1 Solution to Exercise 2.4: scenario tree for the producer

Solution to Exercise 2.5

This exercise is build upon the solution to Exercises 2.3 and 2.4 above.
EVPI is computed as follows. First, the problem below is solved,

$$
\begin{aligned}
&\text{Maximize}_{P_1^{\mathrm{C}},P_2^{\mathrm{C}},P_3^{\mathrm{C}},P_1,P_2,P_3}\\
&\Pi^{\mathrm{P}} =\\
&168(0.2\times 45P_1^{\mathrm{C}}+0.6\times 45P_2^{\mathrm{C}}+0.2\times 45P_3^{\mathrm{C}})+\\
&168(0.2\times 50P_1+0.6\times 46P_2+0.2\times 44P_3)-\\
&168\times 25\left[0.2\times(P_1^{\mathrm{C}}+P_1)+0.6\times(P_2^{\mathrm{C}}+P_2)+0.2\times(P_3^{\mathrm{C}}+P_3)\right]\\
&\text{subject to}\\
&P_1^{\mathrm{C}}+P_1\le 100\\
&P_2^{\mathrm{C}}+P_2\le 100\\
&P_3^{\mathrm{C}}+P_3\le 100\\
&0\le P_1^{\mathrm{C}},P_2^{\mathrm{C}},P_3^{\mathrm{C}}\le 60\\
&0\le P_1,P_2,P_3\ .
\end{aligned}
$$

Note that in this problem a contracting variable is defined per scenario, which implies perfect information. In other words, the contracting decision is made dependent on the pool price scenario. This problem is similar to that of Exercise 2.3 but eliminating the last constraint (non-anticipativity constraint). Its optimal objective function value is $\Pi^{\mathrm{P}*}$ =\$361,536.

Then, EPVI is calculated as

$$\text{EPVI}=\Pi^{\mathrm{P}*}-\Pi^{\mathrm{S}*}=\$361{,}536-\$359{,}520=\$2016,$$

where $\Pi^{\mathrm{S}*}$ is the objective function optimal value of the problem in Exercise 2.3 above. Therefore, the error associated with the pool price forecast is expected to entail a cost of \$2016 to the power producer.

Next, VSS is computed as follows. Since the average price value is $(0.2\times 50+0.6\times 46+0.2\times 44)$ =\$46.4/MWh, the deterministic version of the problem in Exercise 2.3 has the form

$$
\begin{aligned}
&\text{Maximize}_{P^{\mathrm{C}},P}\\
&168\left[45P^{\mathrm{C}}+46.4P-25(P+P^{\mathrm{C}})\right]\\
&\text{subject to}\\
&P^{\mathrm{C}}+P\le 100\\
&0\le P^{\mathrm{C}}\le 60\\
&0\le P\ ,
\end{aligned}
$$

where P^{C} is the power sold through the bilateral contract and P the power sold in the pool. The solution to this problem is $P^{\mathrm{C}*}=0$ MW and $P^*=100$ MW, implying a profit of \$359,520.

Then, we solve the stochastic problem of Exercise 2.3 but fixing P^{C} to its optimal value obtained from solving the deterministic problem above, i.e., to 0,

$$
\begin{aligned}
&\text{Maximize}_{P_1^{\mathrm{C}}, P_2^{\mathrm{C}}, P_3^{\mathrm{C}}, P_1, P_2, P_3} \\
&\Pi^{\mathrm{D}} = \\
&168(0.2 \times 45 P_1^{\mathrm{C}} + 0.6 \times 45 P_2^{\mathrm{C}} + 0.2 \times 45 P_3^{\mathrm{C}}) + \\
&168(0.2 \times 50 P_1 + 0.6 \times 46 P_2 + 0.2 \times 44 P_3) - \\
&168 \times 25 \left[0.2 \times (P_1^{\mathrm{C}} + P_1) + 0.6 \times (P_2^{\mathrm{C}} + P_2) + 0.2 \times (P_3^{\mathrm{C}} + P_3)\right] \\
&\text{subject to} \\
&P_1^{\mathrm{C}} + P_1 \leq 100 \\
&P_2^{\mathrm{C}} + P_2 \leq 100 \\
&P_3^{\mathrm{C}} + P_3 \leq 100 \\
&0 \leq P_1^{\mathrm{C}}, P_2^{\mathrm{C}}, P_3^{\mathrm{C}} \leq 60 \\
&0 \leq P_1, P_2, P_3 \\
&P_1^{\mathrm{C}} = P_2^{\mathrm{C}} = P_3^{\mathrm{C}} = 0 \, ,
\end{aligned}
$$

whose solution is $P_1^* = P_2^* = P_3^* = 100$ MW and $\Pi^{\mathrm{D}*} = \$359{,}520$,

VSS is then computed as

$$\mathrm{VSS} = \Pi^{\mathrm{S}*} - \Pi^{\mathrm{D}*} = \$359{,}520 - \$359{,}520 = \$0,$$

where Π^{S} is the objective function optimal value resulting from the stochastic version of the producer problem described in Exercise 2.3. Consequently, no economic advantage is obtained from using a stochastic approach to solve this problem over a deterministic one. In the solution to Exercise 6.7, the reader can find an example of a problem in which the VSS is different from zero.

C.2 Exercises from Chapter 3

Solution to Exercise 3.1

We denote by $y_{t\omega}$ the realization of process y_t in scenario ω. For this process, the ARMA(2,0,1) model is

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} - \theta_1 \epsilon_{t-1} + \epsilon_t, \tag{C.1}$$

that particularized for scenario ω results in

$$y_{t\omega} = \phi_1 y_{t-1\omega} + \phi_2 y_{t-2\omega} - \theta_1 \epsilon_{t-1\omega} + \epsilon_{t\omega}. \tag{C.2}$$

For each period and scenario, the scenario-generation procedure consists in randomly generating a value of the white noise term $\epsilon_{t\omega}$ and evaluating expression (C.2) to obtain $y_{t\omega}$.

Being $y_{0\omega} = 125.2134$, $y_{1\omega} = 120.1054$, and $\epsilon_{0\omega} = 0.0124$ (for all ω), the procedure used to obtain three four-period scenarios is the following:

Scenario 1.

Period 1: $\epsilon_{11} = 1.5861$.
$y_{11} = 0.8124 \times 125.2134 + 0.2514 \times 120.1054 - 0.8714 \times 0.0124 + 1.5861 = 131.3429$.
Period 2: $\epsilon_{21} = 2.0336$.
$y_{21} = 0.8124 \times 131.3429 + 0.2514 \times 125.2134 - 0.8714 \times 1.5861 + 2.0336 = 138.8331$.
Period 3: $\epsilon_{31} = 2.3946$.
$y_{31} = 0.8124 \times 138.8331 + 0.2514 \times 131.3429 - 0.8714 \times 2.0336 + 2.3946 = 146.4301$.
Period 4: $\epsilon_{41} = -2.5249$.
$y_{41} = 0.8124 \times 146.4301 + 0.2514 \times 138.8331 - 0.8714 \times 2.3946 - 2.5249 = 149.2509$.

Scenario 2.

Period 1: $\epsilon_{12} = 0.5396$.
$y_{12} = 0.8124 \times 125.2134 + 0.2514 \times 120.1054 - 0.8714 \times 0.0124 + 0.5396 = 130.2964$.
Period 2: $\epsilon_{22} = 0.6054$.
$y_{22} = 0.8124 \times 130.2964 + 0.2514 \times 125.2134 - 0.8714 \times 0.5396 + 0.6054 = 137.4666$.
Period 3: $\epsilon_{32} = -2.5648$.
$y_{32} = 0.8124 \times 137.4666 + 0.2514 \times 130.2964 - 0.8714 \times 0.6054 - 2.5648 = 141.3421$.
Period 4: $\epsilon_{42} = -1.8885$.
$y_{42} = 0.8124 \times 141.3421 + 0.2514 \times 137.4666 + 0.8714 \times 2.5648 - 1.8885 = 149.7319$.

Scenario 3.

Period 1: $\epsilon_{13} = 2.7544$.
$y_{13} = 0.8124 \times 125.2134 + 0.2514 \times 120.1054 - 0.8714 \times 0.0124 + 2.7544 = 132.5112$.
Period 2: $\epsilon_{23} = -0.3347$.
$y_{23} = 0.8124 \times 132.5112 + 0.2514 \times 125.2134 - 0.8714 \times 2.7544 - 0.3347 = 136.3959$.
Period 3: $\epsilon_{33} = 0.9922$.
$y_{33} = 0.8124 \times 136.3959 + 0.2514 \times 132.5112 + 0.8714 \times 0.3347 + 0.9922 = 145.4052$.

Period 4: $\epsilon_{43} = 0.2239$.
$y_{43} = 0.8124 \times 145.4052 + 0.2514 \times 136.3959 - 0.8714 \times 0.9922 + 0.2239 = 151.7764$.

The resulting set of scenarios of process y_t is depicted in Fig. C.2.

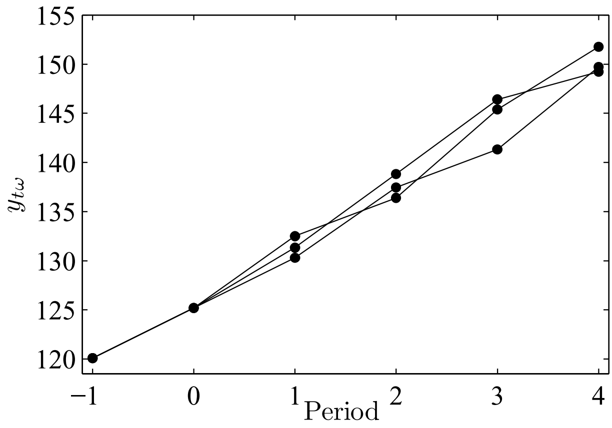


Fig. C.2 Solution to Exercise 3.1: scenarios of y_t

Solution to Exercise 3.4

We follow the algorithm provided in Subsection 3.2.3 of Chapter 3.

Scenario 1.

Iteration 1: We randomly generate $u_1, u_2 \sim \mathrm{U}(0,1)$, where $u_1 = 0.9388$ and $u_2 = 0.9961$.
The time to failure (t_f) and the time to repair (t_r) are obtained using the randomly generated values of u_1 and u_2.
$t_f = -2500 \times \ln(0.5383) = 157.9$, $t_r = -150 \times \ln(0.9961) = 0.6$.
The unit remains available throughout the first 158 hours, and it is off in hour 159.
For each scenario new values of t_f and t_r are successively obtained in order to fulfill the planning horizon (8760 hours).

Iteration 2: $u_1 = 0.0782$ and $u_2 = 0.4427$.
$t_f = -2500 \times \ln(0.0782) = 6372$, $t_r = -150 \times \ln(0.4427) = 122.2$.
The unit remains available from hour 160 to hour 6531, and it is off from hour 6532 to hour 6653.

Iteration 3: $u_1 = 0.1067$ and $u_2 = 0.9619$.
$t_f = -2500 \times \ln(0.1067) = 5595.4$, $t_r = -150 \times \ln(0.9619) = 5.8$.
The unit remains available from hour 6654 up to the end of the planning horizon (hour 8760).

Scenario 2.

Iteration 1: $u_1 = 0.0046$ and $u_2 = 0.7749$.
$t_f = -2500 \times \ln(0.0046) = 13,454.2$, $t_r = -150 \times \ln(0.7749) = 38.2$.
The unit remains available the whole planning horizon (8760 hours).

Scenario 3.

Iteration 1: $u_1 = 0.8173$ and $u_2 = 0.8687$.
$t_f = -2500 \times \ln(0.8173) = 504.3$, $t_r = -150 \times \ln(0.8687) = 21.1$.
The unit remains available the first 504 hours, and it is off from hour 505 to hour 525.

Iteration 2: $u_1 = 0.0844$ and $u_2 = 0.3998$.
$t_f = -2500 \times \ln(0.0844) = 6179.4$, $t_r = -150 \times \ln(0.3998) = 137.5$.
The unit remains available from hour 526 to hour 6704, and it is off from hour 6705 to hour 6842.

Iteration 3: $u_1 = 0.2599$ and $u_2 = 0.8001$.
$t_f = -2500 \times \ln(0.2599) = 3368.9$, $t_r = -150 \times \ln(0.8001) = 33.4$.
The unit remains available from hour 6843 up to the end of the planning horizon (hour 8760).

The resulting set of scenarios is depicted in Fig. C.3.

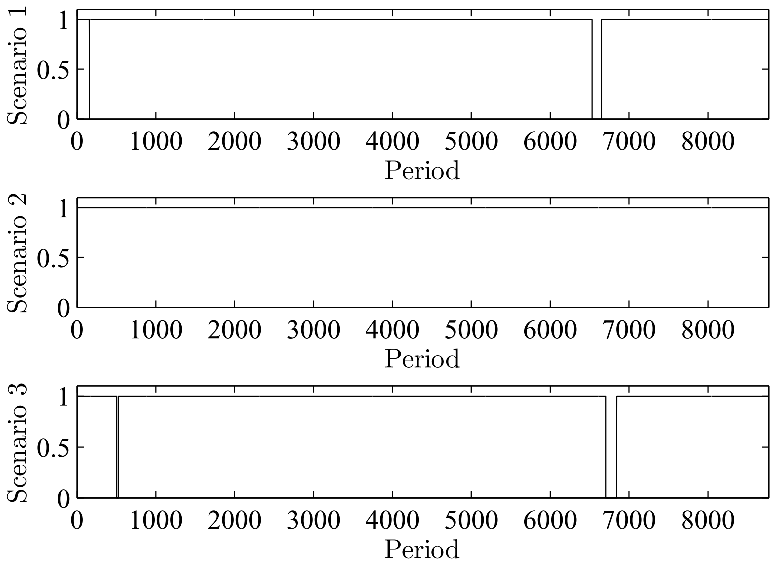


Fig. C.3 Solution to Exercise 3.4: availability scenarios

Solution to Exercise 3.6

The variant 1 of the fast forward selection algorithm described in Subsection 3.3.3 of Chapter 3 works as follows:

Step 0. We evaluate the cost function $\nu(\omega, \omega') = \|P_\omega - P_{\omega'}\|, \forall \omega, \omega' \in \Omega$, where $\Omega = \{1, 2, 3, 4, 5\}$. The values of function ν are arranged into a symmetric matrix with zero diagonal elements,

$$\nu = \begin{pmatrix} 0 & 5 & 22 & 30 & 35 \\ 5 & 0 & 17 & 25 & 30 \\ 22 & 17 & 0 & 8 & 13 \\ 30 & 25 & 8 & 0 & 5 \\ 35 & 30 & 13 & 5 & 0 \end{pmatrix} \text{MW}.$$

Step 1. For each scenario ω that is candidate to be selected from Ω, we choose the one that minimizes the resulting Kantorovich distance between the reduced and original sets, Ω_S and Ω, respectively. Distance values are

$$\begin{aligned}
d_1 &= \pi_2\ \nu(1,2) + \pi_3\ \nu(1,3) + \pi_4\ \nu(1,4) + \pi_5\ \nu(1,5) = 16.40 \text{ MW} \\
d_2 &= \pi_1\ \nu(2,1) + \pi_3\ \nu(2,3) + \pi_4\ \nu(2,4) + \pi_5\ \nu(2,5) = 13.90 \text{ MW} \\
d_3 &= \pi_1\ \nu(3,1) + \pi_2\ \nu(3,2) + \pi_4\ \nu(3,4) + \pi_5\ \nu(3,5) = 12.20 \text{ MW} \\
d_4 &= \pi_1\ \nu(4,1) + \pi_2\ \nu(4,2) + \pi_3\ \nu(4,3) + \pi_5\ \nu(4,5) = 14.60 \text{ MW} \\
d_5 &= \pi_1\ \nu(5,1) + \pi_2\ \nu(5,2) + \pi_3\ \nu(5,3) + \pi_4\ \nu(5,4) = 18.60 \text{ MW}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\Omega_S^{[1]} &= \{3\} \\
\Omega_J^{[1]} &= \{1, 2, 4, 5\}.
\end{aligned}$$

Step 2. We update the cost matrix,

$$\begin{aligned}
\nu^{[2]}(1,2) &= \min\{\nu(1,2), \nu(1,3)\} = 5 \text{ MW} \\
\nu^{[2]}(1,4) &= \min\{\nu(1,4), \nu(1,3)\} = 22 \text{ MW} \\
\nu^{[2]}(1,5) &= \min\{\nu(1,5), \nu(1,3)\} = 22 \text{ MW} \\
\nu^{[2]}(2,1) &= \min\{\nu(2,1), \nu(2,3)\} = 5 \text{ MW} \\
\nu^{[2]}(2,4) &= \min\{\nu(2,4), \nu(2,3)\} = 17 \text{ MW} \\
\nu^{[2]}(2,5) &= \min\{\nu(2,5), \nu(2,3)\} = 17 \text{ MW} \\
\nu^{[2]}(4,1) &= \min\{\nu(4,1), \nu(4,3)\} = 8 \text{ MW} \\
\nu^{[2]}(4,2) &= \min\{\nu(4,2), \nu(4,3)\} = 8 \text{ MW} \\
\nu^{[2]}(4,5) &= \min\{\nu(4,5), \nu(4,3)\} = 5 \text{ MW} \\
\nu^{[2]}(5,1) &= \min\{\nu(5,1), \nu(5,3)\} = 13 \text{ MW} \\
\nu^{[2]}(5,2) &= \min\{\nu(5,2), \nu(5,3)\} = 13 \text{ MW} \\
\nu^{[2]}(5,4) &= \min\{\nu(5,4), \nu(5,3)\} = 5 \text{ MW}.
\end{aligned}$$

Thus,

$$\nu^{[2]} = \begin{pmatrix} 0 & 5 & 22 & 22 & 22 \\ 5 & 0 & 17 & 17 & 17 \\ 22 & 17 & 0 & 8 & 13 \\ 8 & 8 & 8 & 0 & 5 \\ 13 & 13 & 13 & 5 & 0 \end{pmatrix} \text{MW}.$$

In accordance with the updated cost matrix $\nu^{[2]}$, we take from $\Omega_J^{[1]}$ the scenario ω that, if included in the subsequent reduced set $\Omega_S^{[2]}$, minimizes the Kantorovich distance between this set and the original one, Ω. Distances are

$$\begin{aligned}
d_1^{[2]} &= \pi_2\ \nu^{[2]}(2,1) + \pi_4\ \nu^{[2]}(4,1) + \pi_5\ \nu^{[2]}(5,1) = 4.30 \text{ MW} \\
d_2^{[2]} &= \pi_1\ \nu^{[2]}(1,2) + \pi_4\ \nu^{[2]}(4,2) + \pi_5\ \nu^{[2]}(5,2) = 4.55 \text{ MW} \\
d_4^{[2]} &= \pi_1\ \nu^{[2]}(1,4) + \pi_2\ \nu^{[2]}(2,4) + \pi_5\ \nu^{[2]}(5,4) = 9.40 \text{ MW} \\
d_5^{[2]} &= \pi_1\ \nu^{[2]}(1,5) + \pi_2\ \nu^{[2]}(2,5) + \pi_4\ \nu^{[2]}(4,5) = 10.15 \text{ MW}.
\end{aligned}$$

Hence,

$$\begin{aligned}
\Omega_S^{[2]} &= \Omega_S^* = \{1,3\}, \\
\Omega_J^{[2]} &= \Omega_J^* = \{2,4,5\}.
\end{aligned}$$

Step 3. Lastly, we add the probabilities of the non-selected scenarios in Ω_J^* to the probabilities of the selected ones in Ω_S^*.
Given that

1. Scenario 1 in Ω_S^* is the *closest* one to scenario 2 in Ω_J^* ($\nu(2,1) = 5$, while $\nu(2,3) = 17$),
2. Scenario 3 in Ω_S^* is the *closest* one to scenario 4 in Ω_J^* ($\nu(4,3) = 8$, while $\nu(4,1) = 30$), and
3. Scenario 3 in Ω_S^* is the *closest* one to scenario 5 in Ω_J^* ($\nu(5,3) = 13$, while $\nu(5,1) = 35$),

it follows that

$$\begin{aligned}
\pi_1^* &= \pi_1 + \pi_2 = 0.45, \\
\pi_3^* &= \pi_3 + \pi_4 + \pi_5 = 0.55.
\end{aligned}$$

In conclusion, the variant 1 of the forward selection algorithm yields a reduced scenario set $\Omega_S^* = \{1,3\}$ with associated probabilities $\pi_1^* = 0.45$ and $\pi_3^* = 0.55$.

Solution to Exercise 3.9

We follow the algorithm described step by step in Subsection 3.4.2 of Chapter 3.

Step 1. Fit univariate ARMA models to stochastic processes $\boldsymbol{Y}^a$ and $\boldsymbol{Y}^b$. The fitted models are provided in the statement of the exercise,

$$\begin{pmatrix} y_t^a \\ y_t^b \end{pmatrix} = \begin{pmatrix} 0.908 & 0 \\ 0 & 0.850 \end{pmatrix} \begin{pmatrix} y_{t-1}^a \\ y_{t-1}^b \end{pmatrix} + \begin{pmatrix} \varepsilon_t^a \\ \varepsilon_t^b \end{pmatrix} + \begin{pmatrix} 0.182 & 0 \\ 0 & 0.203 \end{pmatrix} \begin{pmatrix} \varepsilon_{t-1}^a \\ \varepsilon_{t-1}^b \end{pmatrix}.$$

Step 2. Compute variance-covariance matrix $\boldsymbol{G}$ ($N_{\mathrm{T}} = 2$ and $K = 1$):

$$\boldsymbol{G}_{11} = 0.50^2 \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.25 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{pmatrix}$$

$$\boldsymbol{G}_{22} = 0.20^2 \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.04 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.04 \end{pmatrix}$$

$$\boldsymbol{G}_{12} = \boldsymbol{G}_{21}^T = 0.50 \times 0.20 \times \begin{pmatrix} 0.7 & 0.1 & 0 \\ 0.3 & 0.7 & 0.1 \\ 0 & 0.3 & 0.7 \end{pmatrix} = \begin{pmatrix} 0.07 & 0.01 & 0 \\ 0.03 & 0.07 & 0.01 \\ 0 & 0.03 & 0.07 \end{pmatrix}.$$

Therefore,

$$\boldsymbol{G} = \left(\begin{array}{ccc|ccc} 0.25 & 0 & 0 & 0.07 & 0.01 & 0 \\ 0 & 0.25 & 0 & 0.03 & 0.07 & 0.01 \\ 0 & 0 & 0.25 & 0 & 0.03 & 0.07 \\ \hline 0.07 & 0.03 & 0 & 0.04 & 0 & 0 \\ 0.01 & 0.07 & 0.03 & 0 & 0.04 & 0 \\ 0 & 0.01 & 0.07 & 0 & 0 & 0.04 \end{array}\right).$$

Step 3. Determine orthogonal matrix $\boldsymbol{L}$ by applying the Cholesky decomposition to matrix $\boldsymbol{G}$,

$$\boldsymbol{L} = \left(\begin{array}{ccc|ccc} 0.5000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5000 & 0 & 0 & 0 \\ \hline 0.1400 & 0.0600 & 0 & 0.1296 & 0 & 0 \\ 0.0200 & 0.1400 & 0.0600 & -0.0864 & 0.0945 & 0 \\ 0 & 0.0200 & 0.1400 & -0.0093 & -0.1270 & 0.0616 \end{array}\right)$$

Step 4. Simulate $N_{\mathrm{T}} \times N_{\mathrm{S}} = 2 \times 2 = 4$ independent standard normal errors, $\boldsymbol{\xi}$, e.g.,

$$\boldsymbol{\xi} = \left(\begin{array}{c} \boldsymbol{\xi}^a \\ \hline \boldsymbol{\xi}^b \end{array}\right) = \left(\begin{array}{c} \xi_1^a \\ \xi_2^a \\ \hline \xi_1^b \\ \xi_2^b \end{array}\right) = \left(\begin{array}{c} -0.4326 \\ -1.6656 \\ \hline 0.1253 \\ 0.2877 \end{array}\right).$$

Step 5. Cross-correlate the standard normal errors generated in the previous step by using the orthogonal transformation (3.31) described in Subsection 3.4.2 of Chapter 3,

$$\left(\begin{array}{c}\hat{\varepsilon}_0^a\\ \varepsilon_1^a\\ \varepsilon_2^a\\ \hline \hat{\varepsilon}_0^b\\ \varepsilon_1^b\\ \varepsilon_2^b\end{array}\right) = \left(\begin{array}{ccc|ccc} 0.5000 & 0 & 0 & 0 & 0 & 0\\ 0 & 0.5000 & 0 & 0 & 0 & 0\\ 0 & 0 & 0.5000 & 0 & 0 & 0\\ \hline 0.1400 & 0.0600 & 0 & 0.1296 & 0 & 0\\ 0.0200 & 0.1400 & 0.0600 & -0.0864 & 0.0945 & 0\\ 0 & 0.0200 & 0.1400 & -0.0093 & -0.1270 & 0.0616 \end{array}\right) \left(\begin{array}{c}-0.3000\\ -0.4326\\ -1.6656\\ \hline -0.1200\\ 0.1253\\ 0.2877\end{array}\right)$$

$$= \left(\begin{array}{c}-0.1500\\ -0.2163\\ -0.8328\\ \hline -0.0835\\ -0.1443\\ -0.2389\end{array}\right).$$

Step 6. Lastly, introduce the cross-correlated normal errors generated in Step 5 above into the ARMA models fitted in Step 1 so as to simulate two values of stochastic processes $\boldsymbol{Y}^a$ and $\boldsymbol{Y}^b$, i.e.,

$$\begin{aligned}\begin{pmatrix} y_1^a \\ y_1^b \end{pmatrix} &= \begin{pmatrix} 0.908 & 0 \\ 0 & 0.850 \end{pmatrix} \begin{pmatrix} y_0^a \\ y_0^b \end{pmatrix} + \begin{pmatrix} \varepsilon_1^a \\ \varepsilon_1^b \end{pmatrix} + \begin{pmatrix} 0.182 & 0 \\ 0 & 0.203 \end{pmatrix} \begin{pmatrix} \varepsilon_0^a \\ \varepsilon_0^b \end{pmatrix} \\ &= \begin{pmatrix} 0.908 & 0 \\ 0 & 0.850 \end{pmatrix} \begin{pmatrix} 9.25 \\ 5.60 \end{pmatrix} + \begin{pmatrix} -0.2163 \\ -0.1443 \end{pmatrix} + \begin{pmatrix} 0.182 & 0 \\ 0 & 0.203 \end{pmatrix} \begin{pmatrix} -0.30 \\ -0.12 \end{pmatrix} \\ &= \begin{pmatrix} 8.1281 \\ 4.5913 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\begin{pmatrix} y_2^a \\ y_2^b \end{pmatrix} &= \begin{pmatrix} 0.908 & 0 \\ 0 & 0.850 \end{pmatrix} \begin{pmatrix} y_1^a \\ y_1^b \end{pmatrix} + \begin{pmatrix} \varepsilon_2^a \\ \varepsilon_2^b \end{pmatrix} + \begin{pmatrix} 0.182 & 0 \\ 0 & 0.203 \end{pmatrix} \begin{pmatrix} \varepsilon_1^a \\ \varepsilon_1^b \end{pmatrix} \\ &= \begin{pmatrix} 0.908 & 0 \\ 0 & 0.850 \end{pmatrix} \begin{pmatrix} 8.1281 \\ 4.5913 \end{pmatrix} + \begin{pmatrix} -0.8328 \\ -0.2389 \end{pmatrix} \\ &+ \begin{pmatrix} 0.182 & 0 \\ 0 & 0.203 \end{pmatrix} \begin{pmatrix} -0.2163 \\ -0.1443 \end{pmatrix} = \begin{pmatrix} 6.5081 \\ 3.6344 \end{pmatrix}.\end{aligned}$$

In short, the pair of correlated scenarios $\left(\hat{\boldsymbol{Y}}^a, \hat{\boldsymbol{Y}}^b\right)^T$ spanning two periods are

$$\begin{pmatrix} \hat{\boldsymbol{Y}}^a \\ \hat{\boldsymbol{Y}}^b \end{pmatrix} = \begin{pmatrix} 8.1281 & 6.5081 \\ 4.5913 & 3.6344 \end{pmatrix}.$$

C.3 Exercises from Chapter 4

Solution to Exercise 4.1

The formulation of the problem is the following:

$$\text{Maximize}_{P_f^{\mathrm{F}}, v_t, P_{t\omega}^{\mathrm{P}}, P_{t\omega}^{\mathrm{G}}}$$

$$\sum_{f=1}^{3}\sum_{t=1}^{4}\lambda_f^{\mathrm{F}}P_f^{\mathrm{F}} + \sum_{\omega=1}^{5}\pi_\omega\left[\sum_{t=1}^{4}\left(\lambda_{t\omega}^{\mathrm{P}}P_{t\omega}^{\mathrm{P}} - C^{\mathrm{G}}P_{t\omega}^{\mathrm{G}}\right)\right] \tag{C.3}$$

subject to

$$0 \le P_f^{\mathrm{F}} \le 25, \quad f = 1, 2, 3 \tag{C.4}$$

$$\sum_{f=1}^{3} P_f^{\mathrm{F}} + P_{t\omega}^{\mathrm{P}} = P_{t\omega}^{\mathrm{G}}, \quad t = 1, \ldots, 4;\ \omega = 1, \ldots, 5 \tag{C.5}$$

$$20v_t \le P_{t\omega}^{\mathrm{G}} \le 100v_t, \quad t = 1, \ldots, 4;\ \omega = 1, \ldots, 5 \tag{C.6}$$

$$P_{t\omega}^{\mathrm{P}} \ge 0, \quad t = 1, \ldots, 4;\ \omega = 1, \ldots, 5 \tag{C.7}$$

$$v_t \in \{0, 1\}, \quad t = 1, \ldots, 4, \tag{C.8}$$

where P_f^{F} is the power purchased from forward contract f, $P_{t\omega}^{\mathrm{P}}$ and $P_{t\omega}^{\mathrm{G}}$ are the power sold in the pool and the power generated in period t and scenario ω, respectively, v_t is a binary variable that is equal to 1 if the unit is online in period t, being 0 otherwise, λ_f^{F} is the selling price of contract f, $\lambda_{t\omega}^{\mathrm{P}}$ is the pool price in period t and scenario ω, C^{G} is the linear production cost of the unit, and π_ω is the probability of scenario ω.

The optimal solution to this problem is obtained for $\{P_1^{\mathrm{F}*}, P_2^{\mathrm{F}*}, P_3^{\mathrm{F}*}\} = \{0, 0, 0\}$ and $\{v_1^*, v_2^*, v_3^*, v_4^*\} = \{0, 1, 1, 1\}$. The expected profit is equal to \$734.4.

The profit per scenario is provided in Table C.2.

Table C.2 Solution to Exercise 4.1: profit per scenario

Scenario #	Profit (\$)
1	534.0
2	1280.0
3	258.0
4	420.0
5	1180.0

Solution to Exercise 4.5

The formulation of the problem described in Exercise 4.1 including the VaR is the following:

$$\text{Maximize}_{P^{\mathrm{F}}_f, v_t, P^{\mathrm{P}}_{t\omega}, P^{\mathrm{G}}_{t\omega}, \eta, \theta_\omega}$$

$$(1-\beta)\left[\sum_{f=1}^{3}\sum_{t=1}^{4}\lambda^{\mathrm{F}}_f P^{\mathrm{F}}_f + \sum_{\omega=1}^{5}\pi_\omega \sum_{t=1}^{4}\left(\lambda^{\mathrm{P}}_{t\omega}P^{\mathrm{P}}_{t\omega} - C^{\mathrm{G}}P^{\mathrm{G}}_{t\omega}\right)\right] + \beta\eta \qquad \text{(C.9)}$$

subject to

$$0 \le P^{\mathrm{F}}_f \le 25, \quad f = 1,2,3 \qquad \text{(C.10)}$$

$$\sum_{f=1}^{3} P^{\mathrm{F}}_f + P^{\mathrm{P}}_{t\omega} = P^{\mathrm{G}}_{t\omega}, \quad t = 1,\dots,4;\ \omega = 1,\dots,5 \qquad \text{(C.11)}$$

$$20v_t \le P^{\mathrm{G}}_{t\omega} \le 100v_t, \quad t = 1,\dots,4;\ \omega = 1,\dots,5 \qquad \text{(C.12)}$$

$$\sum_{\omega=1}^{5}\pi_\omega\theta_\omega \le 1-\alpha \qquad \text{(C.13)}$$

$$\eta - \left[\sum_{f=1}^{3}\sum_{t=1}^{4}\lambda^{\mathrm{F}}_f P^{\mathrm{F}}_f + \sum_{t=1}^{4}\left(\lambda^{\mathrm{P}}_{t\omega}P^{\mathrm{P}}_{t\omega} - C^{\mathrm{G}}P^{\mathrm{G}}_{t\omega}\right)\right] \le M\theta_\omega, \quad \omega = 1,\dots,5 \qquad \text{(C.14)}$$

$$P^{\mathrm{P}}_{t\omega} \ge 0, \quad t = 1,\dots,4;\ \omega = 1,\dots,5 \qquad \text{(C.15)}$$

$$v_t \in \{0,1\}, \quad t = 1,\dots,4 \qquad \text{(C.16)}$$

$$\theta_\omega \in \{0,1\}, \quad \omega = 1,\dots,5. \qquad \text{(C.17)}$$

The results in terms of the expected profit, the VaR, and the first-stage variables for different values of β are provided in Table C.3. Observe that the expected profit decreases as the VaR increases and that the risk is reduced by using the forward contracts. Note that selling energy through a forward contract requires that the unit of the producer must be online in all periods within the time span of the contract (4 periods).

Table C.3 Exercise 4.3: results

β	Expected profit (\$)	VaR (\$)	P^{F}_1 (MW)	P^{F}_2 (MW)	P^{F}_3 (MW)	v_1	v_2	v_3	v_4
0	734.4	420.0	0	0	0	0	1	1	1
0.12	726.4	488.0	0	0	20	1	1	1	1
1	718.5	495.0	0	0	25	1	1	1	1

Solution to Exercise 4.8

The formulation of the problem described in Exercise 4.1 including second-order stochastic dominance constraints is

$$\text{Maximize}_{P_f^{\text{F}}, v_t, P_{t\omega}^{\text{P}}, P_{t\omega}^{\text{G}}, s_{\omega\upsilon}}$$
$$\sum_{f=1}^{3}\sum_{t=1}^{4}\lambda_f^{\text{F}}P_f^{\text{F}} + \sum_{\omega=1}^{5}\pi_\omega\sum_{t=1}^{4}\left(\lambda_{t\omega}^{\text{P}}P_{t\omega}^{\text{P}} - C^{\text{G}}P_{t\omega}^{\text{G}}\right) \tag{C.18}$$

subject to

$$0 \le P_f^{\text{F}} \le 25, \quad f = 1, 2, 3 \tag{C.19}$$

$$\sum_{f=1}^{3} P_f^{\text{F}} + P_{t\omega}^{\text{P}} = P_{t\omega}^{\text{G}}, \quad t = 1, \ldots, 4;\ \omega = 1, \ldots, 5 \tag{C.20}$$

$$20v_t \le P_{t\omega}^{\text{G}} \le 100v_t, \quad t = 1, \ldots, 4;\ \omega = 1, \ldots, 5 \tag{C.21}$$

$$\eta - \left[\sum_{f=1}^{3}\sum_{t=1}^{4}\lambda_f^{\text{F}}P_f^{\text{F}} + \sum_{t=1}^{4}\left(\lambda_{t\omega}^{\text{P}}P_{t\omega}^{\text{P}} - C^{\text{G}}P_{t\omega}^{\text{G}}\right)\right] \le s_{\omega\upsilon},$$
$$\omega = 1, \ldots, 5;\ \upsilon = 1, \ldots, 4 \tag{C.22}$$

$$\sum_{\omega=1}^{5}\pi_\omega s_{\omega\upsilon} \le \sum_{\upsilon'=1}^{4}\tau_\upsilon' \max\big(k_\upsilon - k_{\upsilon'}, 0\big), \quad \upsilon = 1, \ldots, 5 \tag{C.23}$$

$$P_{t\omega}^{\text{P}} \ge 0, \quad t = 1, \ldots, 4;\ \omega = 1, \ldots, 5 \tag{C.24}$$

$$v_t \in \{0, 1\}, \quad t = 1, \ldots, 4 \tag{C.25}$$

$$s_{\omega,\upsilon} \ge 0, \quad \omega = 1, \ldots, 5;\ \upsilon = 1, \ldots, 4. \tag{C.26}$$

In this exercise two benchmarks with four scenarios are used. The probability and the value of the benchmark for each scenario, $\{\tau_\upsilon, k_\upsilon\}$, $\upsilon = 1, \ldots, 4$, are provided in Table C.4. Observe that the second benchmark is identical to the first one but displaced \$50 *to the right*. Therefore, we can say that benchmark 2 is more restrictive than benchmark 1.

Table C.4 Solution to Exercise 4.8: benchmark data

Scenario #	Probability	Benchmark 1 (\$)	Benchmark 2 (\$)
1	0.25	300	350
2	0.25	450	500
3	0.25	600	650
4	0.25	700	750

Imposing each benchmark, the resulting profits per scenario are provided in Table C.5. The expected profit and the optimal first-stage variables are provided in Table C.6. Note that imposing benchmark 2 yields a smaller expected profit. This result is coherent since benchmark 2 is more restrictive than benchmark 1. Observe also that the profit associated with the worst scenarios (scenarios {1,3}) is higher in the solution to benchmark 2, at the cost of a smaller profit in the best scenarios (scenarios {2,4,5}).

Table C.5 Solution to Exercise 4.8: profit per scenario for benchmarks 1 and 2

Scenario #	Profit ($)	
	Benchmark 1	Benchmark 2
1	560.00	562.12
2	1144.00	1115.15
3	336.00	350.00
4	488.00	493.94
5	1104.00	1077.27

Table C.6 Solution to Exercise 4.8: expected profit and first-stage decisions for benchmarks 1 and 2

Benchmark #	Expected profit ($)	P_1^{F} (MW)	P_2^{F} (MW)	P_3^{F} (MW)	v_1	v_2	v_3	v_4
1	726.4	0	0	20.0	1	1	1	1
2	719.7	0	0	24.2	1	1	1	1

C.4 Exercises from Chapter 5

Solution to Exercise 5.1

First, observe that considering the producer as a price-taker agent in the adjustment market implies that $\lambda_{t\omega}^{\mathrm{A}} = \lambda_{t\omega}^{\mathrm{A0}}$ and $\gamma_{t\omega}^{\mathrm{A}} = 0$. In addition, this assumption causes that variable $P_{t\omega}^{\mathrm{A}}$ is no longer needed. As a result, the objective function and the set of constraints modeling the CVaR in problem (5.48)-(5.63) must be modified. This way, the objective function (5.48) results in

$$\text{Maximize}_{P^{\text{A}}_{gt\omega}, P^{\text{D}}_{gt\omega}, P^{\text{G}}_{gt\omega}, P^{\text{R}}_{gt\omega}, w_{gt\omega}, \zeta, \eta_\omega}$$
$$\sum_{\omega=1}^{N_\Omega} \pi_\omega \sum_{t=1}^{N_\text{T}} \left[\sum_{g=1}^{N_\text{G}} \lambda^{\text{D}}_{t\omega} P^{\text{D}}_{gt\omega} + \sum_{g=1}^{N_\text{G}} \lambda^{\text{R}}_{t\omega} P^{\text{R}}_{gt\omega} + \sum_{g=1}^{N_\text{G}} \lambda^{\text{A}}_{t\omega} P^{\text{A}}_{gt\omega} \right.$$
$$\left. - \sum_{g=1}^{N_\text{G}} C^{\text{P}}_g (P^{\text{D}}_{gt\omega} + P^{\text{R}}_{gt\omega}) \right] + \beta \left(\zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\Omega} \pi_\omega \eta_\omega \right), \tag{C.27}$$

whereas the CVaR constraints (5.60) yield

$$\zeta - \sum_{t=1}^{N_\text{T}} \left[\sum_{g=1}^{N_\text{G}} \lambda^{\text{D}}_{t\omega} P^{\text{D}}_{gt\omega} + \sum_{g=1}^{N_\text{G}} \lambda^{\text{R}}_{t\omega} P^{\text{R}}_{gt\omega} + \sum_{g=1}^{N_\text{G}} \lambda^{\text{A}}_{t\omega} P^{\text{A}}_{gt\omega} - \right.$$
$$\left. \sum_{g=1}^{N_\text{G}} C^{\text{P}}_g (P^{\text{D}}_{gt\omega} + P^{\text{A}}_{gt\omega}) \right] \leq \eta_\omega, \ \forall \omega. \tag{C.28}$$

Observe also that constraint (5.54), which computes the value of variable $P^{\text{A}}_{t\omega}$, is not needed and can be removed from the final formulation of the problem.

In order to extend formulation (5.48)-(5.63) for considering offering curves instead of single power quantities in both the regulation and the adjustment markets, it is necessary to relax the non-anticipativity constraints (5.52) and (5.53). Thus, we replace vectors $\boldsymbol{A}^{\mathbf{R}}$ and $\boldsymbol{A}^{\mathbf{A}}$ in (5.52) and (5.53) by the new vectors $\boldsymbol{A}^{\mathbf{R}'}$ and $\boldsymbol{A}^{\mathbf{A}'}$, respectively. This way, $\boldsymbol{A}^{\mathbf{R}'}$ is a vector whose element $A^{\text{R}'}(\omega)$ is equal to 1 if scenarios ω and $\omega+1$ have identical regulation prices. In the same way $\boldsymbol{A}^{\mathbf{A}'}$ is a vector whose element $A^{\text{A}'}(\omega)$ is equal to 1 if scenarios ω and $\omega+1$ have identical adjustment prices. Then, the resulting non-anticipativity constraints are

$$P^{\text{R}}_{gt\omega} - P^{\text{R}}_{gt\omega+1} = 0, \ \forall g, \forall t, \ \forall \omega; \text{ if } A^{\text{R}'}(\omega) = 1 \tag{C.29}$$
$$P^{\text{A}}_{gt\omega} - P^{\text{A}}_{gt\omega+1} = 0, \ \forall g, \forall t, \ \forall \omega; \text{ if } A^{\text{A}'}(\omega) = 1. \tag{C.30}$$

Additionally, it is necessary to enforce that the resulting offering curves in the regulation and the adjustment markets must be non-decreasing. This can be stated using an analogous formulation to that used for modeling the offering curves in the day-ahead market (Subsection 5.4.1 of Chapter 5). The resulting formulation for the non-decreasing offering curves in the regulation and the adjustment markets is

$$P^{\text{R}}_{gt\omega} - P^{\text{R}}_{gt\omega'} \leq 0, \ \forall g, \forall t, \forall \omega; \text{ if } O^{\text{R}}(t,\omega) + 1 = O^{\text{R}}(t,\omega'), \text{ if } Q^{\text{D}}(\omega,\omega') = 1 \tag{C.31}$$

$$P^{\text{A}}_{gt\omega} - P^{\text{A}}_{gt\omega'} \leq 0, \ \forall g, \forall t, \forall \omega; \text{ if } O^{\text{A}}(t,\omega) + 1 = O^{\text{A}}(t,\omega'), \text{ if } Q^{\text{R}}(\omega,\omega') = 1, \tag{C.32}$$

where $\boldsymbol{O}^{\mathbf{R}}$ and $\boldsymbol{O}^{\mathbf{A}}$ are matrices containing the ranking for the regulation and the adjustment prices, respectively, (analogous to matrix $\boldsymbol{O}^{\mathbf{D}}$ defined for the day-ahead price ranking in Subsection 5.4.1 of Chapter 5), $\boldsymbol{Q}^{\mathbf{D}}$ is a matrix whose element $Q^{\mathrm{D}}(\omega, \omega')$ is equal to 1 if the day-ahead prices in scenarios ω and ω' are identical; and, in the same manner, $\boldsymbol{Q}^{\mathbf{R}}$ is a matrix whose element $Q^{\mathrm{R}}(\omega, \omega')$ is equal to 1 if the regulation prices in scenarios ω and ω' are the same. This formulation allows obtaining different offering curves in the regulation market for different realizations of the day-ahead prices. Likewise, for each realization of the regulation prices a different offering curve in the adjustment market is derived.

Solution to Exercise 5.2

The on/off status of each producer unit is modeled by means of the binary variable v_{gt}, which is equal to 1 if unit g is online in period t, being 0 otherwise. Here, we define this variable as a here-and-now decision, being its value equal for all possible scenarios.

In order to account for the binary variable v_{gt}, constraints (5.55) and (5.56) should be replaced by the following ones:

$$P^{\mathrm{D}}_{gt\omega} + \frac{1}{2} P^{\mathrm{R}}_{gt\omega} + P^{\mathrm{A}}_{gt\omega} \leq P^{\mathrm{G,max}}_{g} v_{gt}, \ \forall g, \forall t, \forall \omega \tag{C.33}$$

$$P^{\mathrm{D}}_{gt\omega} - \frac{1}{2} P^{\mathrm{R}}_{gt\omega} + P^{\mathrm{A}}_{gt\omega} \geq P^{\mathrm{G,min}}_{g} v_{gt}, \ \forall g, \forall t, \forall \omega. \tag{C.34}$$

If the on/off status of the units is accounted for, the start-up and shutdown costs should be also considered. The start-up cost is formulated as

$$C^{\mathrm{U}}_{gt} \geq C^{\mathrm{U0}}_{g} \left(v_{gt} - v_{gt-1} \right), \ \forall g, \forall t \tag{C.35}$$

$$C^{\mathrm{U}}_{gt} \geq 0, \ \forall g, \forall t, \tag{C.36}$$

where C^{U}_{gt} is a variable modeling the start-up cost incurred by unit g in period t, and C^{U0}_{g} is a parameter equal to the start-up cost of unit g.

The shutdown cost is

$$C^{\mathrm{D}}_{gt} \geq C^{\mathrm{D0}}_{g} \left(v_{gt-1} - v_{gt} \right), \ \forall g, \forall t \tag{C.37}$$

$$C^{\mathrm{D}}_{gt} \geq 0, \ \forall g, \forall t, \tag{C.38}$$

where C^{D}_{gt} is a variable modeling the shutdown cost incurred by unit g in period t, and C^{D0}_{g} is a parameter equal to the shutdown cost of unit g.

The start-up and shutdown costs have to be included in the objective function and the CVaR constraints of problem (5.48)-(5.63). The objective function considering start-up and shutdown costs is

$$\begin{aligned}
&\text{Maximize}_{C^{\mathrm{D}}_{gt}, C^{\mathrm{U}}_{gt}, P^{\mathrm{A}}_{t\omega}, P^{\mathrm{A}}_{gt\omega}, P^{\mathrm{D}}_{gt\omega}, P^{\mathrm{G}}_{gt\omega}, P^{\mathrm{R}}_{gt\omega}, u_{gt}, w_{gt\omega}, \zeta, \eta_\omega} \\
&\sum_{\omega=1}^{N_\Omega} \pi_\omega \sum_{t=1}^{N_\mathrm{T}} \left[\sum_{g=1}^{N_\mathrm{G}} \lambda^{\mathrm{D}}_{t\omega} P^{\mathrm{D}}_{gt\omega} + \sum_{g=1}^{N_\mathrm{G}} \lambda^{\mathrm{R}}_{t\omega} P^{\mathrm{R}}_{gt\omega} + \lambda^{\mathrm{A0}}_{t\omega} P^{\mathrm{A}}_{t\omega} + \gamma^{\mathrm{A}}_{t\omega} \left(P^{\mathrm{A}}_{t\omega}\right)^2 \right. \\
&\left. - \sum_{g=1}^{N_\mathrm{G}} \left(C^{\mathrm{P}}_g (P^{\mathrm{D}}_{gt\omega} + P^{\mathrm{R}}_{gt\omega}) + C^{\mathrm{U}}_{gt} + C^{\mathrm{D}}_{gt} \right) \right] + \beta \left(\zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\Omega} \pi_\omega \eta_\omega \right).
\end{aligned} \tag{C.39}$$

The resulting CVaR constraints are

$$\begin{aligned}
&\zeta - \sum_{t=1}^{N_\mathrm{T}} \left[\sum_{g=1}^{N_\mathrm{G}} \lambda^{\mathrm{D}}_{t\omega} P^{\mathrm{D}}_{gt\omega} + \sum_{g=1}^{N_\mathrm{G}} \lambda^{\mathrm{R}}_{t\omega} P^{\mathrm{R}}_{gt\omega} + \lambda^{\mathrm{A0}}_{t\omega} P^{\mathrm{A}}_{t\omega} + \gamma^{\mathrm{A}}_{t\omega} \left(P^{\mathrm{A}}_{t\omega}\right)^2 - \right. \\
&\qquad \left. \sum_{g=1}^{N_\mathrm{G}} \left(C^{\mathrm{P}}_g (P^{\mathrm{D}}_{gt\omega} + P^{\mathrm{R}}_{gt\omega}) + C^{\mathrm{U}}_{gt} + C^{\mathrm{D}}_{gt} \right) \right] \le \eta_\omega, \ \forall \omega.
\end{aligned} \tag{C.40}$$

Solution to Exercise 5.3

The second adjustment market is an additional trading floor in which producers can submit offering curves after the first adjustment market is cleared. Traditionally, the volume of energy associated to the successive adjustment markets is smaller as the time between the market clearing and the energy delivery decreases. The decision-making framework for the producer pool problem including a second adjustment market is provided in Fig. C.4. Observe that different adjustment markets may have associated different planning horizon. For instance, in this exercise the first adjustment market comprises all hours of the next day (24), whereas the second one only covers the last 12 hours.

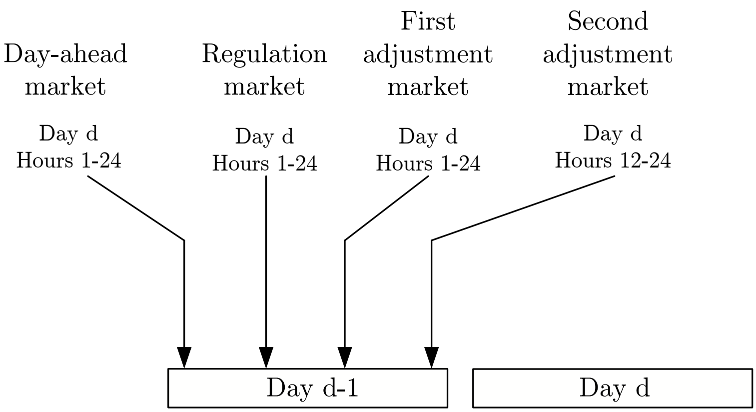


Fig. C.4 Solution to Exercise 5.3: decision framework

If we include a second adjustment market, the decision-making process changes with respect to the case with a single adjustment market. If fact, an additional stage has to be included in the stochastic programming problem in order to account for the decisions associated with the second adjustment market. Fig. C.5 provides the scenario tree for the decision-making process including day-ahead, regulation, first adjustment, and second adjustment markets.

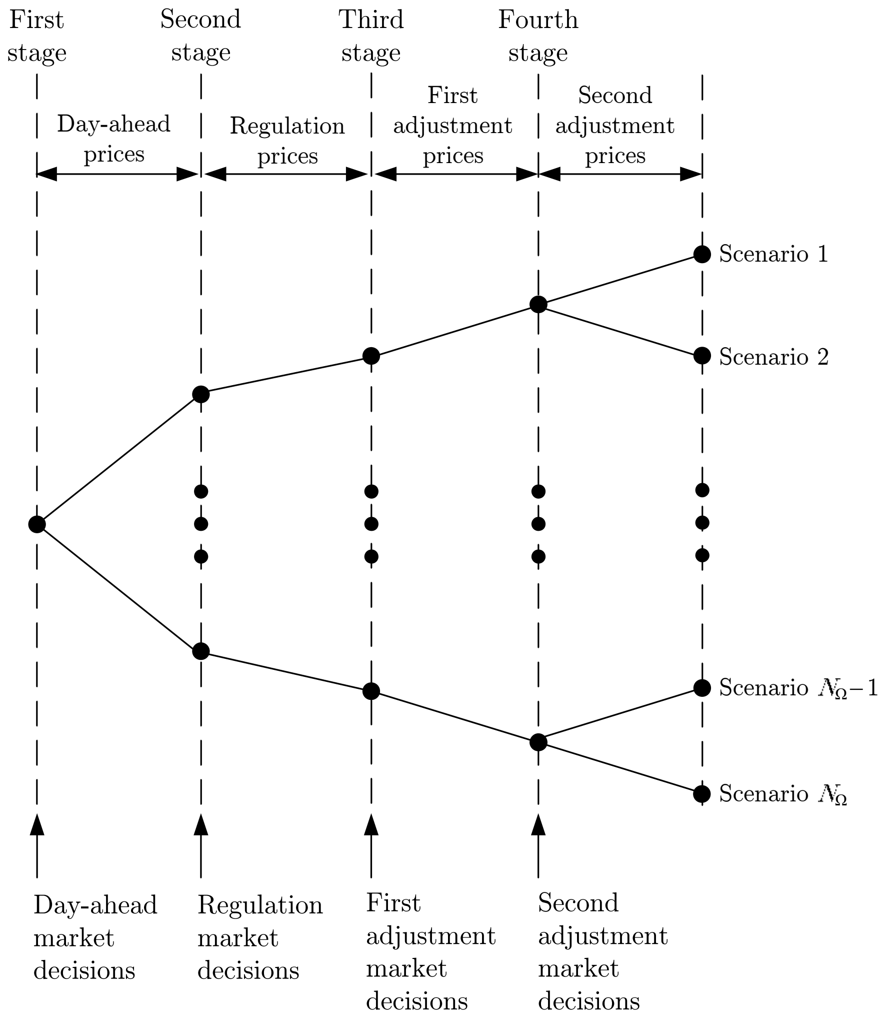


Fig. C.5 Solution to Exercise 5.3: scenario tree

C.5 Exercises from Chapter 6

Solution to Exercise 6.3

According to equation (6.15) in Subsection 6.3.2 of Chapter 6, the imbalance cost C_t incurred by the wind power producer in period t is given by

$$C_t = \lambda_t^{\mathrm{D}}(1 - r_t^{+})\Delta_t = \$24/\mathrm{MWh} \times (1 - 0.85) \times (100 - 75)\mathrm{MWh} = \$90,$$

since $\Delta_t = 100 - 75 = 25$ MWh > 0. As a result, the profit obtained from the energy trading in this period is

$$R_t = \lambda_t^{\mathrm{D}} E_t - C_t = \$24/\mathrm{MWh} \times 100 \text{ MWh} - \$90 = \$2310.$$

Solution to Exercise 6.5

The scenario tree built from the statistical information on the day-ahead market price, the price difference between day-ahead and adjustment markets, the imbalance price ratios and the wind power production provided in the statement of the exercise is depicted in Fig. C.6. Prices are in \$/MWh, powers in MW and ratios are non-dimensional numbers. For illustrative purposes, we detail below the computation of some of the probabilities π_ω:

$$\begin{aligned}
\pi_1 &= 0.30 \times 0.60 \times 0.50 \times 0.50 \times 0.45 = 0.02025\\
\pi_2 &= 0.30 \times 0.60 \times 0.50 \times 0.50 \times 0.55 = 0.02475\\
&\vdots\\
\pi_9 &= 0.30 \times 0.40 \times 0.50 \times 0.35 \times 0.45 = 0.00945\\
\pi_{10} &= 0.30 \times 0.40 \times 0.50 \times 0.35 \times 0.55 = 0.01155\\
\pi_{11} &= 0.30 \times 0.40 \times 0.50 \times 0.65 \times 0.45 = 0.01755\\
\pi_{12} &= 0.30 \times 0.40 \times 0.50 \times 0.65 \times 0.55 = 0.02145\\
&\vdots\\
\pi_{17} &= 0.70 \times 0.60 \times 0.50 \times 0.50 \times 0.45 = 0.04725\\
\pi_{18} &= 0.70 \times 0.60 \times 0.50 \times 0.50 \times 0.55 = 0.05775\\
&\vdots\\
\pi_{29} &= 0.70 \times 0.40 \times 0.50 \times 0.35 \times 0.45 = 0.02205\\
\pi_{30} &= 0.70 \times 0.40 \times 0.50 \times 0.35 \times 0.55 = 0.02695\\
\pi_{31} &= 0.70 \times 0.40 \times 0.50 \times 0.65 \times 0.45 = 0.04095\\
\pi_{32} &= 0.70 \times 0.40 \times 0.50 \times 0.65 \times 0.55 = 0.05005.
\end{aligned}$$

Solution to Exercise 6.7

The offering strategy model (6.36)–(6.41) in Chapter 6 does not take into account the existence of an adjustment market and as a result, the 32 scenarios in the tree represented in Fig. C.6 can be reduced to 16 as shown in Fig. C.7.

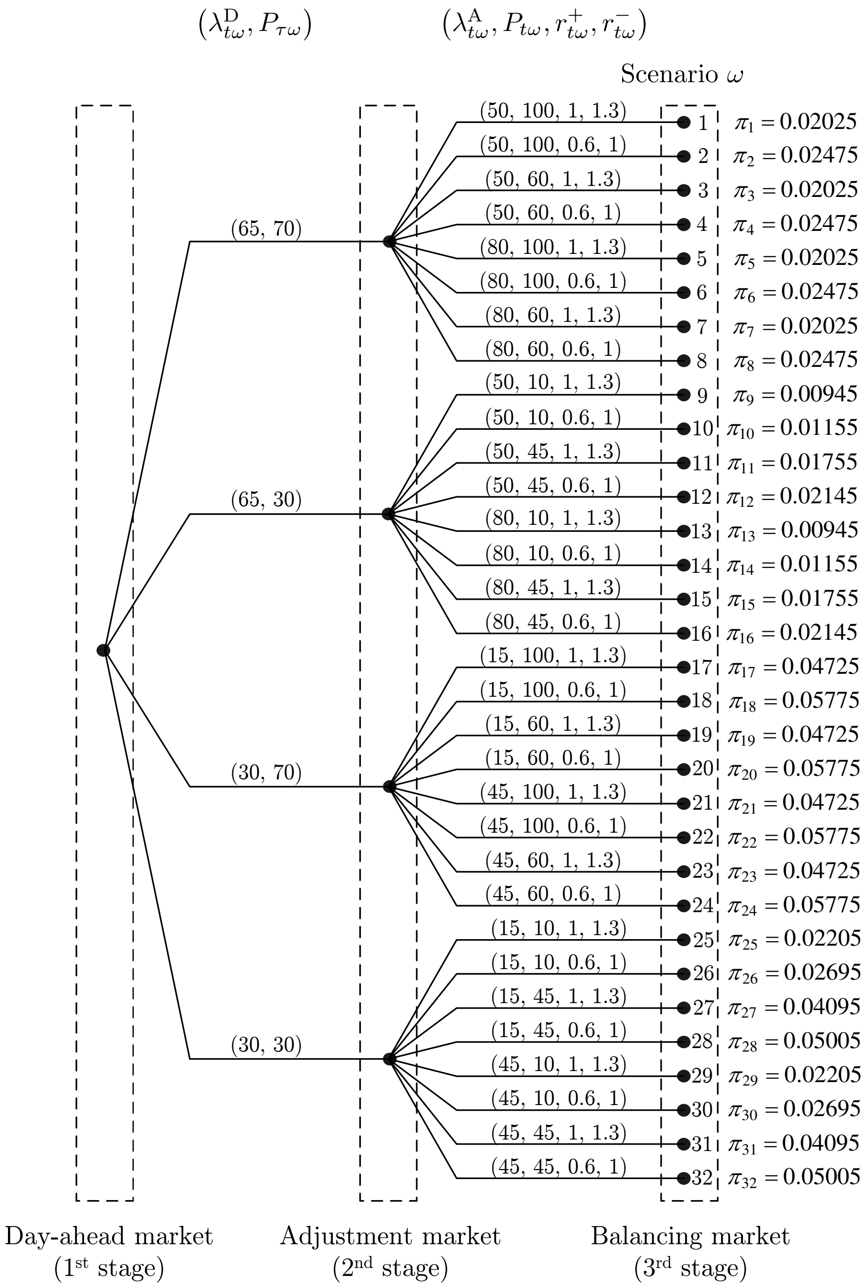


Fig. C.6 Solution to Exercise 6.5: scenario tree

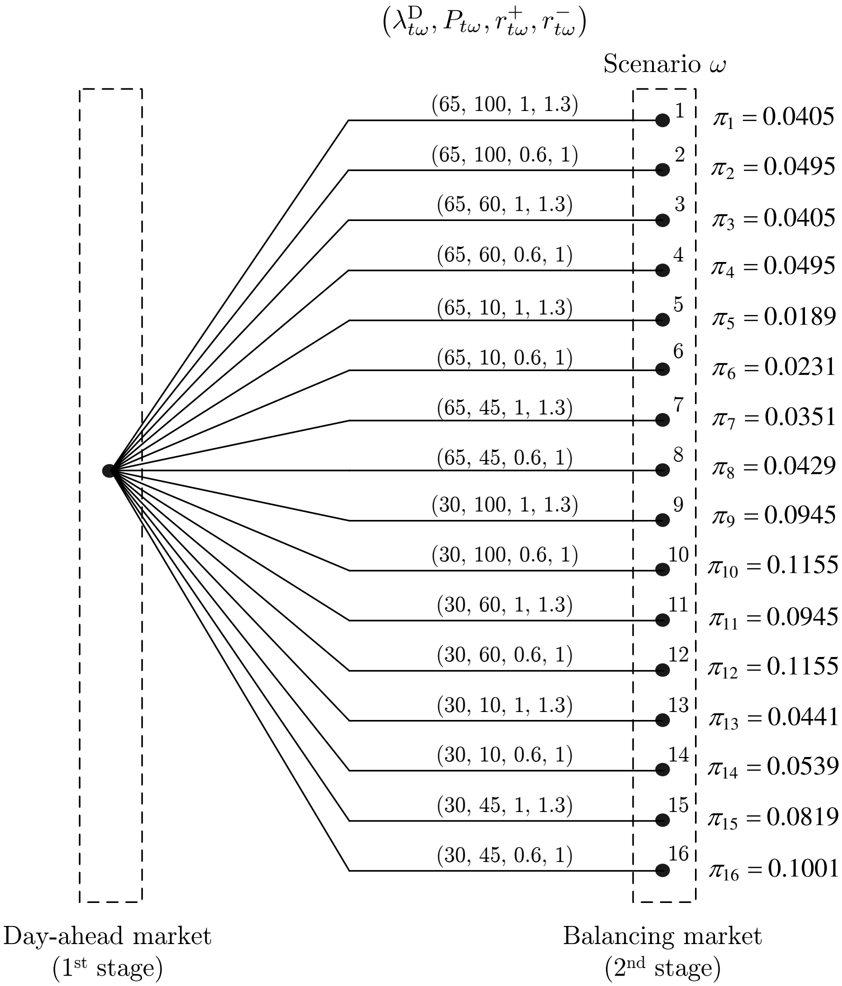


Fig. C.7 Solution to Exercise 6.7: two-stage scenario tree (excluding the adjustment market)

The deterministic approach to solve the wind producer problem simply comes down to selling the expected wind power production ($\hat{P}$) in the day-ahead market, which is computed as follows:

$$\hat{P} = \xi\{P\} = \sum_{\omega=1}^{16} \pi_\omega P_\omega = (0.0405 + 0.0495 + 0.0945 + 0.1155) \times 100$$
$$+ (0.0405 + 0.0495 + 0.0945 + 0.1155) \times 60$$
$$+ (0.0189 + 0.0231 + 0.0441 + 0.0539) \times 10$$
$$+ (0.0351 + 0.0429 + 0.0819 + 0.1001) \times 45 = 61.10 \text{ MW}.$$

If we fix decision variable P^{D} in model (6.36)–(6.41) to 61.10 MW, the resulting expected profit ($z^{\mathrm{D}*}$) is \$2306.8. Therefore, the Value of Stochastic Solution is

$$\mathrm{VSS} = z^{\mathrm{S}*} - z^{\mathrm{D}*} = 2308 - 2306.8 = \$1.2$$

where $z^{\mathrm{S}*}$ is the expected profit associated with the stochastic solution calculated in Exercise 6.6. The VSS represents a percentage of 0.052% with respect to $z^{\mathrm{S}*}$.

Finally, Table C.7 provides the expected values of perfect information as improvements in percentage with respect to the expected profit resulting from model (6.36)–(6.41).

Table C.7 Solution to Exercise 6.7: expected values of perfect information

Perfect information on	λ^{D}	P	(r^+, r^-)
EVPI (%)	0	7.22	7.22

C.6 Exercises from Chapter 7

Solution to Exercise 7.1

Constraints (7.11) of Chapter 7 are reformulated as

$$u_{gt\omega} P_g^{\mathrm{G,min}} a_{gt\omega} d_t \leq E_{gt\omega}^{\mathrm{G}} \leq u_{gt\omega} P_g^{\mathrm{G,max}} a_{gt\omega} d_t, \ \forall g, \forall t, \forall \omega, \tag{C.41}$$

where $u_{gt\omega}$ is a binary (0/1) variable modeling the off/on status of unit g during period t and scenario ω, and $P_g^{\mathrm{G,min}}$ is the minimum power output of unit g.

It should be noted that expressions (C.41) are linear but involve binary variables.

Additionally, note that if the start-up costs of the units are incorporated in the objective function of the forward contracting problem of a producer, problem (7.18)-(7.25), constraints (C.41) may render obsolete constraints (7.21).

Solution to Exercise 7.4

The formulation of the forward contracting problem for a producer avoiding parameter β is provided below,

$$\text{Maximize}_{P^{\text{F}}_{fj}, E^{\text{P}}_{t\omega}, E^{\text{G}}_{gt\omega}, \zeta, \eta_\omega}$$

$$\zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\Omega} \pi_\omega \eta_\omega \tag{C.42}$$

subject to

$$0 \leq P^{\text{F}}_{fj} \leq \bar{P}^{\text{F}}_{fj}, \ \forall f, \forall j \tag{C.43}$$

$$0 \leq E^{\text{G}}_{gt\omega} \leq P^{\text{G,max}}_g d_t, \ \forall g, \forall t, \forall \omega \tag{C.44}$$

$$E^{\text{G}}_{gt\omega} \geq H_g E^{\text{G}}_{gt'\omega}, \ \forall g, \forall \omega, \forall t, t' \mid W(t,t') = 1 \tag{C.45}$$

$$\sum_{g=1}^{N_{\text{G}}} E^{\text{G}}_{gt\omega} = E^{\text{P}}_{t\omega} + \sum_{f \in F_t} \sum_{j=1}^{N_{\text{J}}} P^{\text{F}}_{fj} d_t + E^{\text{PC}}_t, \ \forall t, \forall \omega \tag{C.46}$$

$$\zeta - \sum_{t=1}^{N_{\text{T}}} \left(\sum_{f \in F_t} \sum_{j=1}^{N_{\text{J}}} \lambda^{\text{F}}_{fj} P^{\text{F}}_{fj} d_t + \lambda^{\text{P}}_{t\omega} E^{\text{P}}_{t\omega} - \sum_{g=1}^{N_{\text{G}}} C^{\text{G}}_g E^{\text{G}}_{gt\omega} \right) \leq \eta_\omega, \ \forall \omega \tag{C.47}$$

$$\eta_\omega \geq 0, \ \forall \omega \tag{C.48}$$

$$E^{\text{P}}_{t\omega} \geq 0, \ \forall t, \forall \omega. \tag{C.49}$$

Observe that the objective function of the problem above is the CVaR of the profit distribution. The interested reader should check the definition of the CVaR in Chapter 3.

To manage risk using the above formulation, the parameter α is used. A value of $\alpha = 0$ provides the risk-neutral decision (i.e., the expected value of the profit is maximized), while an α value close to 1 involves a risk-averse decision, which avoids scenarios of low profit at the cost of reducing the expected profit.

Solution to Exercise 7.7

Table C.8 provides the results for different values of β and all possible pairs of scenarios. In Table 7.8 of Section 7.7 of Chapter 7, it can be observed that the average values of the four pool price scenarios are $\{35.25, 32.54, 36.75, 35.75\}$, measured in \$/MWh. Therefore, scenarios $\{3, 4\}$ contain the highest pool prices, whereas the lowest ones are associated with scenarios $\{1, 2\}$. For this reason, in the case considering scenarios $\{3, 4\}$ the forward contracts are not signed (forward contracts are not competitive with respect to the pool). The same fact occurs for pairs of scenarios $\{1, 3\}$ and $\{1, 4\}$. On the contrary, in cases with scenarios $\{1, 2\}$, $\{2, 3\}$, and $\{2, 4\}$, the producer sells power through forward contracts for all values of β.

Solution to Exercise 7.8

Table C.8 Solution to Exercise 7.7: expected profit and CVaR for different values of β and pairs of scenarios

Scenarios	β	Expected profit ($)	CVaR ($)	Power sold through contract 1 (MW)	Power sold through contract 2 (MW)
1-2	0	7436.00	5962.00	100	82
	10	7117.50	6117.50	200	100
1-3	0	11,088.75	9254.50	0	0
	10	11,088.75	9254.50	0	0
1-4	0	10,357.25	9254.50	0	0
	10	10,357.25	9254.50	0	0
2-3	0	8887.52	5552.03	100	0
	10	7992.50	6117.50	200	100
2-4	0	8357.25	5962.00	100	82
	10	7742.50	6117.50	200	100
3-4	0	12,191.50	11,460.00	0	0
	10	12,191.50	11,460.00	0	0

Table C.9 provides the results for different values of β and for three cases considering (i) all contracts, (ii) only contract 1, and (iii) only contract 2. Observe that for $\beta = 0$ the contracts are not used in any case. For this reason, the results in terms of the expected profit and the CVaR are the same in the three cases. For the rest of cases, observe that the best solution (highest expected profit and highest CVaR) is always obtained if the two contracts are considered. Notice also that the results obtained only considering the first contract outperforms those obtained with only the second contract. This result is a consequence of the higher selling price of the first contract (see Table 7.7 in Section 7.7 of Chapter 7).

Solution to Exercise 7.9

First of all, note that computing EVPI or VSS requires a risk-neutral formulation, which leads to $\beta = 0$.

Considering $\beta = 0$ and solving the particular instance of problem (7.18)-(7.25) that corresponds to the example in Section 7.7 of Chapter 7 renders an objective function optimal value $\Pi^{\mathrm{S}*} = \$9548.13$

To compute EVPI, non-anticipativity constraints need to be relaxed in the problem above. Doing so results in an objective function optimal value $\Pi^{\mathrm{P}*} = \$9938.75$.

EVPI is then computed as $\Pi^{\mathrm{P}*} - \Pi^{\mathrm{S}*} = \$390.62$.

To compute VSS two problems need to be solved. The first one involves solving the considered instance of problem (7.18)-(7.25) for the average scenario (just that scenario). The solution to this deterministic problem allows deriving the optimal value of the first-stage variables, i.e., the forward con-

Table C.9 Solution to Exercise 7.8: expected profit and CVaR for different values of β and numbers of available contracts

Available contracts	β	Expected profit (\$)	CVaR (\$)	Power sold through contract 1 (MW)	Power sold through contract 2 (MW)
1-2	0	9548.13	4555.00	0	0
	1	9278.50	5962.00	100	82
	10	8367.50	6117.50	200	100
1	0	9548.13	4555.00	0	0
	1	9522.38	5552.03	100	0
	10	8967.50	5817.50	200	0
2	0	9548.13	4555.00	0	0
	1	9322.38	5352.03	0	100
	10	9322.38	5352.03	0	100

tracting decisions. In this case, the power sold through forward contracts is equal to 0.

Then, the considered instance of problem (7.18)-(7.25) is solved fixing the first-stage variables to the optimal values obtained from solving the deterministic problem above. The objective function optimal value of this problem is $\Pi^{D*} = \$9548.13$, which is the same value obtained from solving the stochastic problem (7.18)-(7.25) for $\beta = 0$. Therefore, the VSS is computed as $\Pi^{S*} - \Pi^{D*} = 0$. This result is a consequence of the fact that the optimal quantities of power sold through forward contracts resulting from solving both the deterministic and the risk-neutral stochastic problems are equal to zero. This result is coherent since the forward contracts are tools used by the market agents in order to reduce the risk of profit variability, which is not accounted for in neither the deterministic nor the risk-neutral stochastic problems.

C.7 Exercises from Chapter 8

Solution to Exercise 8.3

Problem (8.18)-(8.25) can be equivalently formulated by replacing the objective function (8.18) by a new objective function including the CVaR of the profit. The resulting formulation is

$$\text{Maximize}_{P^{\text{F}}_{fj}, \lambda^{\text{R}}_{ei}, v_{ei}, E^{\text{P}}_{t\omega}, \zeta, \eta_\omega}$$
$$\zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\Omega} \pi_\omega \eta_\omega \tag{C.50}$$

subject to

$$0 \leq P^{\text{F}}_{fj} \leq \bar{P}^{\text{F}}_{fj}, \ \forall f, \forall j \tag{C.51}$$
$$\bar{\lambda}^{\text{R}}_{ei-1} v_{ei} \leq \lambda^{\text{R}}_{ei} \leq \bar{\lambda}^{\text{R}}_{ei} v_{ei}, \ \forall e, \forall i \tag{C.52}$$
$$\sum_{i=1}^{N_{\text{I}}} v_{ei} = 1, \ \forall e \tag{C.53}$$
$$\sum_{e=1}^{N_{\text{E}}} \sum_{i=1}^{N_{\text{I}}} \bar{E}^{\text{R}}_{eti\omega} v_{ei} = E^{\text{P}}_{t\omega} + \sum_{f \in F_t} P^{\text{F}}_f d_t + E^{\text{PC}}_t, \ \forall t, \forall \omega, \tag{C.54}$$
$$\zeta - \sum_{t=1}^{N_{\text{T}}} \left(\sum_{e=1}^{N_{\text{E}}} \sum_{i=1}^{N_{\text{I}}} \lambda^{\text{R}}_{ei} \bar{E}^{\text{R}}_{eti\omega} - \lambda^{\text{P}}_{t\omega} E^{\text{P}}_{t\omega} - \sum_{f \in F_t} \sum_{j=1}^{N_{\text{J}}} \lambda^{\text{F}}_{fj} P^{\text{F}}_{fj} d_t \right) \leq \eta_\omega, \ \forall \omega \tag{C.55}$$
$$v_{ei} \in \{0, 1\}, \ \forall e, \forall i \tag{C.56}$$
$$\eta_\omega \geq 0, \ \forall \omega. \tag{C.57}$$

Observe that the objective function (8.18) presented in Section 8.6 of Chapter 8 comprises the expected profit plus the CVaR multiplied by β. However, in (C.50), there is a single term modeling the CVaR. In this situation the tradeoff between expected profit and CVaR is not materialized by means of the parameter β, but by varying the confidence level α. This way, if $\alpha = 0$ the objective function is equivalent to maximizing the expected profit. This results from the definition of the CVaR, which can be approximately defined as the expected profit for the $(1-\alpha) \times 100\%$ "worst" scenarios. Therefore, if $\alpha = 0$, the $(1-\alpha) \times 100\%$=100% "worst" scenarios correspond to the whole set of scenarios. The value $\alpha = 0$ corresponds to the case $\beta = 0$ in problem (8.18)-(8.25). On the other hand, for values of α higher than 0, the $\alpha \times 100\%$ "best" profit scenarios of the profit distribution are disregarded from the objective function, and the optimal decision vector is only determined considering the "worst" zone of the profit distribution. In summary, values of α close to 0 yield solutions involving low risk aversion, whereas values of α close to 1 result in risk-averse solutions.

Solution to Exercise 8.8

If the price-quota curve in Fig. 8.11 of Chapter 8 is *shifted to the right* \$20/MWh, the retailer supplies the 100% of the client demand for selling prices up to \$85/MWh. For prices between \$85/MWh and \$95/MWh, it supplies the 35% of the demand, and the 15% for prices between \$95/MWh and

\$105/MWh. In this new case, the solutions attained in terms of the expected profit and the CVaR for different values of β are provided in Table C.10. Observe that the expected profit and the CVaR are significantly higher than those reported in Table 8.5 of Section 8.7 of Chapter 8. This a consequence of the fact that the clients are willing to pay an additional amount of \$20/MWh with respect to the original case. Since the pool and the forward prices remain equal to those in the original case, the profit obtained by the retailer is significantly higher.

Table C.10 Solution to Exercise 8.8: expected profit and CVaR for different values of β

β	Expected profit (\$)	CVaR (\$)
0	15,605.0	9990.0
1	15,458.3	10,190.0
2	15,271.7	10,350.0
5	14,993.0	10,418.0
15	14,672.3	10,444.0

The power purchased through forward contracts and the selling price offered to the clients is provided in Table C.11. Observe that the reduction in the risk faced by the retailer is achieved by increasing only the forward contract purchases whereas the selling price remains stable. This is due to the high price that the clients are willing to pay, which is higher than the purchase prices in the pool and the futures market.

Table C.11 Solution to Exercise 8.8: forward contract purchases and selling price

β	Contract 1 (MW)	Contract 2 (MW)	Selling price (\$/MWh)
0	0	0	85
1	20	0	85
2	20	20	85
5	40	20	85
15	40	40	85

Solution to Exercise 8.9

In this exercise we consider an additional group of clients whose demand scenarios are provided in Table C.12. The price-quota curve of this group of clients includes 3 blocks. In the first block the 100% of the demand is supplied and it comprises selling prices smaller than \$60/MWh. In the second block the 75% of the demand is provided and the selling prices lie between \$60/MWh and \$70/MWh. Finally, the third block contains the selling prices between \$70/MWh and \$80/MWh, and it has associated the 25% of the demand.

Table C.12 Solution to Exercise 8.9: demand scenarios for the second client group (MWh)

Scenario #	Period	
	1	2
1	225	175
2	240	215
3	200	235
4	240	215

Considering the formulation of problem (8.18)-(8.25) and the data characterizing the second group of clients, the results in terms of the expected profit and the CVaR for different values of β are provided in Table C.13. Observe that considering an additional group of clients causes the expected profit and the CVaR to increase with respect to their values in the example of Section 8.7 of Chapter 8. In Table C.14, the forward contract purchases and the selling prices for the two client groups are provided.

Table C.13 Solution to Exercise 8.9: expected profit and CVaR for different values of β

β	Expected profit (\$)	CVaR (\$)
0	5391.3	729.0
1	4737.2	2181.5
2	3873.7	2863.5
5	3595.0	2931.5
15	3274.3	2957.5

C.8 Exercises from Chapter 9

Solution to Exercise 9.1

Fig. C.8 illustrates the decision framework of the consumer, which is as follows:

1. Before the beginning of the week (e.g., at 10pm of the previous week Sunday), the consumer decides (a) which contracts to sign spanning the

Table C.14 Solution to Exercise 8.9: forward contract purchases and selling prices

β	Contract 1 purchases (MW)	Contract 2 purchases (MW)	Selling price for client group 1 ($/MWh)	Selling price for client group 2 ($/MWh)
0	0	0	75	70
1	20	0	75	80
2	20	20	85	80
5	40	20	85	80
15	40	40	85	80

whole week and (b) which contracts to sign spanning the working days of the week. These are here-and-now decisions.

2. Before the beginning of Saturday (e.g., at 10 pm of Friday), the consumer decides which contracts to sign spanning the weekend. These are here-and-now decisions for weekend prices but wait-and-see decisions for prices pertaining to the working days.
3. Throughout the week the consumer trades in the pool. These are wait-and-see decisions.

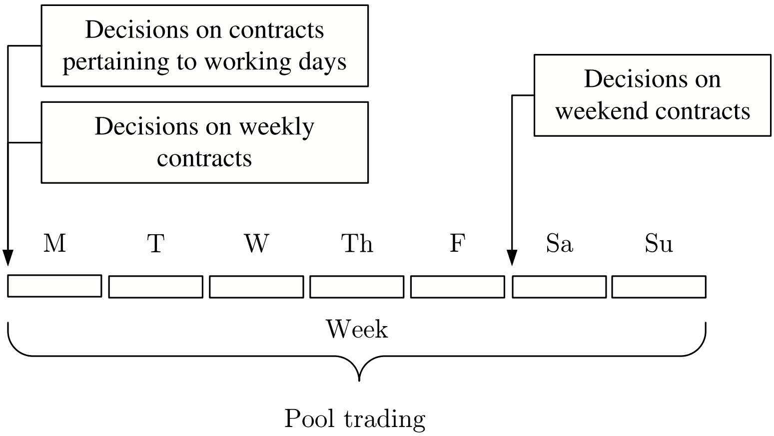


Fig. C.8 Solution to Exercise 9.1: decision framework for the consumer

Solution to Exercise 9.6

The two forward contracts described in Table C.15 are included as buying options for the consumer of the example in Section 9.5 of Chapter 9.

Table C.16 provides the expected cost and the CVaR for different values of the risk-aversion parameter β.

Table C.15 Solution to Exercise 9.6: forward contract data

Contract #	Usage period	Price ($/MWh)	Upper bound (MW)
1	1-4	48.5	30
2	1-4	50	30

Table C.16 Solution to Exercise 9.6: cost results

β	Expected cost ($)	CVaR ($)
0	44,073.44	46,525.00
1	44,328.59	45,622.50
2	44,938.75	45,197.50
10	44,938.75	45,197.50

Table C.17 provides the power purchased from bilateral and forward contracts. Observe that only those bilateral contracts decided at the beginning of the planning horizon and constituting therefore first-stage decision variables are reported (bilateral contracts 1, 2 and 3).

Table C.17 Solution to Exercise 9.6: power purchased from bilateral and forward contracts (MW)

β	Bilateral contracts			Forward contracts	
	1	2	3	1	2
0	0	0	0	0	0
1	75	0	0	30	0
2	75	65	0	30	30
10	75	65	0	30	30

By comparing, respectively, Tables C.16 and C.17 with Tables 9.4 and 9.5 in Chapter 9, the observations below are in order:

1. The incorporation of forward contracts reduces the risk faced by the consumer (CVaR) at the cost of increasing the expected cost.
2. The power purchased from forward contracts increases as the parameter β grows.
3. The power purchased from bilateral contracts 2 and 3 diminishes if forward contracts are considered.

Solution to Exercise 9.8

The considered reduced set of scenarios pertaining to the example in Section 9.5 of Chapter 9 are provided in Table C.18.

Table C.18 Solution to Exercise 9.8: pool price scenarios ($/MWh)

	Period #					Period #			
Scenario #	1	2	3	4	Scenario #	1	2	3	4
1	44	44	42	42	5	50	52	50	51
2	44	44	42	45	6	50	52	50	55
3	44	44	44	45	7	50	52	51	52
4	44	44	44	46	8	50	52	51	53

Considering the scenarios in Table C.18, the results obtained are provided in Tables C.19 and C.20.

Table C.19 provides cost results, including expected cost, cost standard deviation and CVaR, while Table C.20 provides electricity procurement results, including expected contract purchases, expected pool purchases and sales, and expected production by the self-production unit.

Table C.19 Solution to Exercise 9.6: cost results

β	Expected cost ($)	Cost standard deviation ($)	CVaR ($)
0	43,950.00	2302.21	46,500.00
1	44,085.94	1741.36	46,012.50
2	44,446.88	1182.06	45,750.00
10	44,509.38	1109.44	45,737.50

Table C.20 Solution to Exercise 9.6: expected electricity procurement

β	Expected contract purchases (MWh)	Expected pool purchases (MWh)	Expected pool sales (MWh)	Expected self-production (MWh)
0	75.00	662.50	0.00	212.50
1	375.00	362.50	0.00	212.50
2	675.00	162.50	100.00	212.50
10	725.00	137.50	125.00	212.50

By comparing, respectively, Tables C.19 and C.20 with Tables 9.4 and 9.6 in Chapter 9, the observations below are in order:

1. The results in terms of expected cost, cost standard deviation and CVaR for the case with 8 scenarios are very similar to those obtained with 16 scenarios.
2. In the case with 8 scenarios the purchases from bilateral contracts and the absolute value of the energy traded in the pool increase with respect to the original case.
3. The expected production of the self-production unit resulting from the case with 8 scenarios decreases with respect to the value obtained in the original case.

C.9 Exercises from Chapter 10

Solution to Exercises 10.1 and 10.2

a) If sequential and compound failures are ignored, the number of failure scenarios N_Ω is given by

$$N_\Omega = N_\mathrm{T} \times N_\mathrm{E} + 1 = 4 \times 6 = 25,$$

where N_T is the number of periods spanning the scheduling horizon and N_E is the number of pieces of equipment (lines and generators) in the power system.

b) If sequential failures are disregarded, but compound failures (defined as the simultaneous outage of several pieces of equipment) are taken into account, the number of failure scenarios is computed as

$$N_\Omega = N_\mathrm{T} \times \sum_{i=1}^{N_\mathrm{E}} \binom{N_\mathrm{E}}{i} + 1 = N_\mathrm{T} \times \left(2^{N_\mathrm{E}} - 1\right) + 1 = 4 \times (2^6 - 1) + 1 = 253.$$

c) If every piece of equipment in the system may fail at any time period of the scheduling horizon, the number of failure scenarios is

$$N_\Omega = \sum_{i=0}^{N_\mathrm{E}} N_\mathrm{T}^i \binom{N_\mathrm{E}}{i} = \sum_{i=0}^{6} 4^i \binom{6}{i} = 15{,}625.$$

Solution to Exercise 10.3

According to the load shedding events provided in Table 10.15 of Chapter 10, the LOLP, LOLE, and ELNS associated with the scheduling program are

$$\mathrm{LOLP} \quad = \pi_1 + \pi_2 + \pi_6 + \pi_7 + \pi_8 = 2 \times 0.001 + 3 \times 0.005 = 0.017$$

LOLE $= 3$ h $\pi_1 + 3$ h $\pi_2 + 3$ h $\pi_6 + 2$ h $\pi_7 + 1$ h $\pi_8 = 6$ h $\times\ 0.001 + 6$ h $\times\ 0.005 = 0.036$ h

ELNS $= (1 + 7 + 7)$ MWh $\pi_1 + (1 + 5 + 2)$ MWh $\pi_2 + (2{+}13{+}10)$ MWh $\pi_6 + (5{+}4)$ MWh $\pi_7 + 2$ MWh $\pi_8 = 23$ MWh $\times\ 0.001 + 36$ MWh $\times\ 0.005 = 0.203$ MWh.

Solution to Exercise 10.6

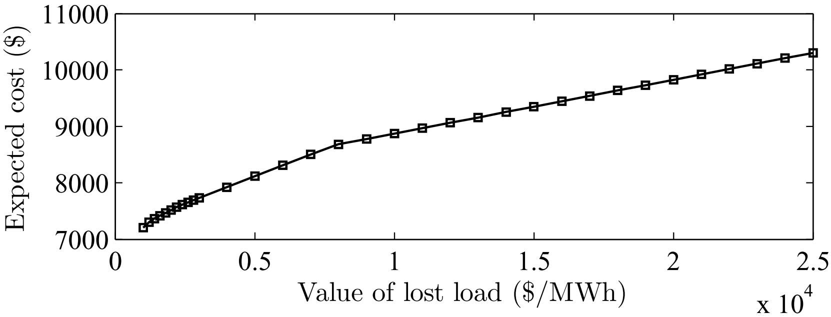
Fig. C.9 shows the evolution of the expected cost, the expected load not served (ELNS), and the total amount of reserves scheduled in the market as the value of lost load increases. Observe that the expected load not served remains different from zero for values of lost load as high as \$25,000/MWh. In particular, from a value of lost load higher than \$7825/MWh, the expected load not served remains constant and equal to its lowest possible value, which turns out to be 0.0954 MWh.

Table C.21 provides the power and reserve schedules resulting from the market-clearing procedure for a value of lost load equal to \$25,000/MWh. Note that the 11 MW of up-spinning reserve offered by the only load in the system in period 3 are accepted in the electricity market and consequently scheduled in that period. This way, the consumption level at node 3 can be voluntarily reduced to 99 MW in period 3. Even so, a part of the load still needs to be shed for technical reasons (see Table C.22 below) under every scenario ω such that the failure of unit 3 or the outage of line 2 takes place before period 4, i.e., under scenarios $\omega = 1, 2, 3, 5, 6,$ and 7. If unit

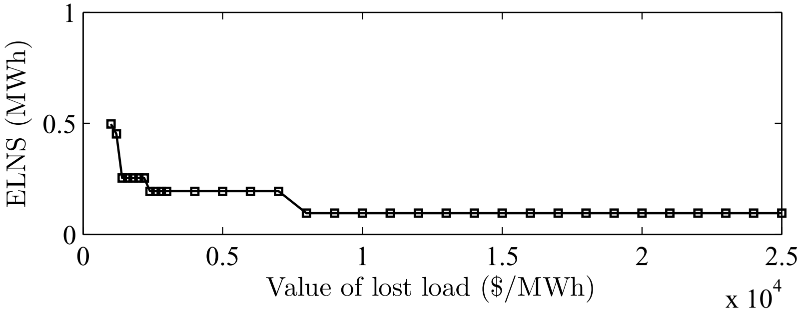
Table C.21 Solution to Exercise 10.6: scheduling program for $V^{\mathrm{LOL}} = \$25{,}000$/MWh. Powers in MW

Period #	P_t^{S}			R_t^{U}				R_t^{D}				R_t^{NS}		
	G1	G2	G3	G1	G2	G3	L3	G1	G2	G3	L3	G1	G2	G3
1	0	0	30	0	0	0	0	0	0	0	0	30	0	0
2	30	0	50	50	0	0	0	0	0	0	0	0	0	0
3	60	0	50	15	0	0	11	25	0	0	0	0	15	0
4	0	0	40	0	0	0	0	0	0	0	0	40	0	0

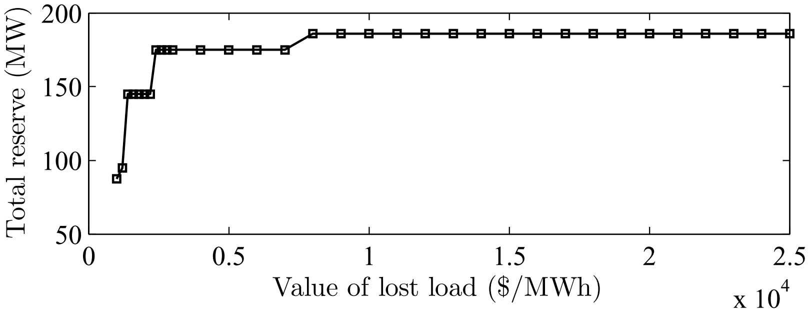
3 is forced out, the maximum load that can be supplied at node 3 is 90 MW. This power comes from generating units 1 and 2 through lines 2 and 3, whose transmission capacities are 55 and 35 MW, respectively. Therefore, if unit 3 becomes unavailable before period 4, 9 MW of demand need to be involuntarily curtailed in period 3. This situation is graphically represented in Fig. C.10. Likewise, if line 2 is forced out (but not unit 3), the maximum demand that can be satisfied at node 3 is 85 MW, 50 of which are produced by unit 3 located at that node. The remaining 35 MW come from unit 1 through lines 1 and 3. As a result, if the outage of line 2 occurs before period 4, 14 MW of load have to be necessarily shed in period 3. This situation is illustrated in Fig. C.11.



(a)



(b)



(c)

Fig. C.9 Solution to Exercise 10.6: impact of the value of lost load on: (a) the expected social cost; (b) the expected amount of load not served (ELNS); (c) the total amount of reserve scheduled in the market

Table C.22 Solution to Exercise 10.6: involuntary load shedding (MWh) for $V^{\mathrm{LOL}} =$ \$25,000/MWh

Period #	Scenario #								
	0	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	9	9	9	0	14	14	14	0
4	0	0	0	0	0	0	0	0	0

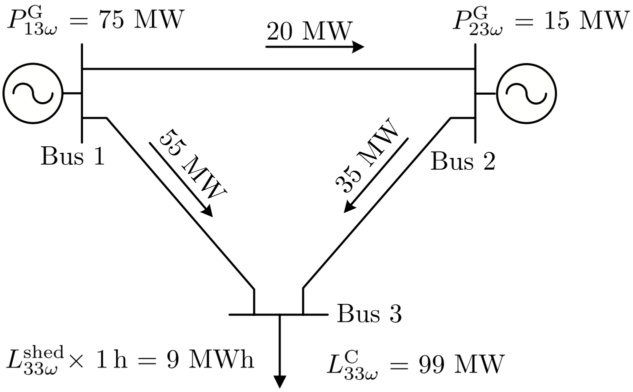


Fig. C.10 Solution to Exercise 10.6: effect of the outage of unit 3 during period $t = 3$ in scenarios $\omega = 1$, 2 and 3

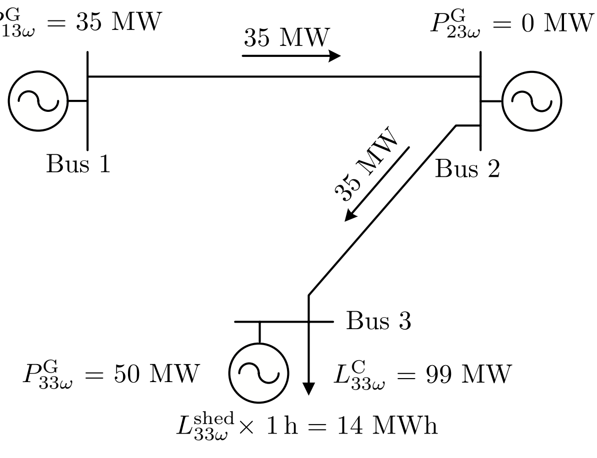


Fig. C.11 Solution to Exercise 10.6: effect of the outage of line 2 during period $t = 3$ in scenarios $\omega = 5$, 6 and 7

C.10 Exercises from Chapter 11

Solution to Exercise 11.2

Consider the following additional notation:

RD_i	Ramp-down limit of unit i.
RU_i	Ramp-up limit of unit i.
SD_i	Shut-down ramp limit of unit i.
SU_i	Start-up ramp limit of unit i.
D_i^{T}	Minimum down time of unit i.
U_i^{T}	Minimum up time of unit i.
S_i^0	Time periods unit i has been off-line at the beginning of the scheduling horizon.
V_i^0	Time periods unit i has been online at the beginning of the scheduling horizon.
u_i^0	Initial commitment status of unit i (1 online, 0 otherwise).

Following [20], the ramp-up and start-up ramp rates of each generating unit i are enforced by equations (C.58) below. Analogously, the shut-down ramp and ramp-down limits of each unit i are imposed through equations (C.59) and (C.60), respectively.

$$
\begin{aligned}
P_{it\omega}^{\mathrm{G}} \leq{} & P_{i(t-1)\omega}^{\mathrm{G}} + \mathrm{RU}_i v_{i(t-1)\omega} \\
& + \mathrm{SU}_i (v_{it\omega} - v_{i(t-1)\omega}) \\
& + P_i^{\max}(1 - v_{it\omega}), \quad \forall i, \forall t, \forall \omega \qquad \text{(C.58)} \\
P_{it\omega}^{\mathrm{G}} \leq{} & P_i^{\max} v_{i(t+1)\omega} \\
& + \mathrm{SD}_i (v_{it\omega} - v_{i(t+1)\omega}), \\
& \forall i, \forall t = 1, 2, \ldots, N_{\mathrm{T}} - 1, \forall \omega \qquad \text{(C.59)} \\
P_{i(t-1)\omega}^{\mathrm{G}} - P_{it\omega}^{\mathrm{G}} \leq{} & \mathrm{RD}_i v_{it\omega} \\
& + \mathrm{SD}_i (v_{i(t-1)\omega} - v_{it\omega}) \\
& + P_i^{\max}(1 - v_{i(t-1)\omega}), \ \forall i, \forall \omega, \forall t. \qquad \text{(C.60)}
\end{aligned}
$$

The minimum up-time constraints are written as follows:

$$\sum_{t=1}^{T_i^{\mathrm{ON}}} (1 - v_{it\omega}) = 0, \ \ \forall i, \ \ \forall \omega \tag{C.61}$$

$$\sum_{t_1=t}^{t+U_i^{\mathrm{T}}-1} v_{it_1\omega} \geq U_i^{\mathrm{T}} (v_{it\omega} - v_{i(t-1)\omega})$$
$$\forall i, \forall t = T_i^{\mathrm{ON}} + 1, \ldots, N_{\mathrm{T}} - U_i^{\mathrm{T}} + 1, \ \ \forall \omega \tag{C.62}$$

$$\sum_{t_1=t}^{N_{\mathrm{T}}} \left[v_{it_1\omega} - (v_{it\omega} - v_{i(t-1)\omega}) \right] \geq 0$$
$$\forall i, \forall t = N_{\mathrm{T}} - U_i^{\mathrm{T}} + 2, \ldots, N_{\mathrm{T}}, \ \ \forall \omega, \tag{C.63}$$

where T_i^{ON} is the number of initial periods during which unit i must be online. T_i^{ON} is mathematically expressed as

$$T_i^{\mathrm{ON}} = \mathrm{Min}\{N_{\mathrm{T}}, \left(U_i^{\mathrm{T}} - V_i^0\right) u_i^0\} \ .$$

Equations (C.61) constrain the initial status of the units according to T_i^{ON}. Equations (C.62) are enforced throughout the subsequent periods to comply with the minimum up time constraint during all the possible sets of consecutive periods of size U_i^{T}. Lastly, constraints (C.63) model the final $U_i^{\mathrm{T}} - 1$ periods in which if unit i is started up, it remains online until the end of the time horizon.

Minimum down-time constraints are obtained from the minimum up-time constraints (C.61)–(C.63) if $v_{it\omega}$, U_i^{T}, and V_i^0 are replaced by $1 - v_{it\omega}$, D_i^{T}, and S_i^0, respectively. That is,

$$\sum_{t=1}^{T_i^{\mathrm{OFF}}} v_{it\omega} = 0, \ \ \forall i, \ \ \forall \omega \tag{C.64}$$

$$\sum_{t_1=t}^{t+D_i^{\mathrm{T}}-1} (1 - v_{it_1\omega}) \geq D_i^{\mathrm{T}} (v_{i(t-1)\omega} - v_{it\omega})$$
$$\forall i, \forall t = T_i^{\mathrm{OFF}} + 1, \ldots, N_{\mathrm{T}} - D_i^{\mathrm{T}} + 1, \ \ \forall \omega \tag{C.65}$$

$$\sum_{t_1=t}^{N_{\mathrm{T}}} \left[1 - v_{it_1\omega} - (v_{i(t-1)\omega} - v_{it\omega}) \right] \geq 0$$
$$\forall i, \forall t = N_{\mathrm{T}} - D_i^{\mathrm{T}} + 2, \ldots, N_{\mathrm{T}}, \ \ \forall \omega, \tag{C.66}$$

where T_i^{OFF} is the number of initial periods during which unit i must be off-line. T_i^{OFF} is mathematically expressed as

$$T_i^{\mathrm{OFF}} = \mathrm{Min}\{N_{\mathrm{T}}, \left(D_i^{\mathrm{T}} - S_i^0\right) \left(1 - u_i^0\right)\} \ .$$

Fig. C.12 illustrates the evolution of the expected cost as the amount of wind generation increases. A 50% level of uncertainty, as defined in Section 11.5 of Chapter 11, is considered. Two instances are compared, namely, the base case and a new case study in which the unit ramping capabilities are reduced so that ramping constraints become binding. Observe that ramp limitations and network bottlenecks have a similar detrimental effect on the expected cost reduction achieved by wind integration.

In order to make ramping constraints binding, we can set, for example, the ramping limits of generating units to the values listed in Table. C.23.

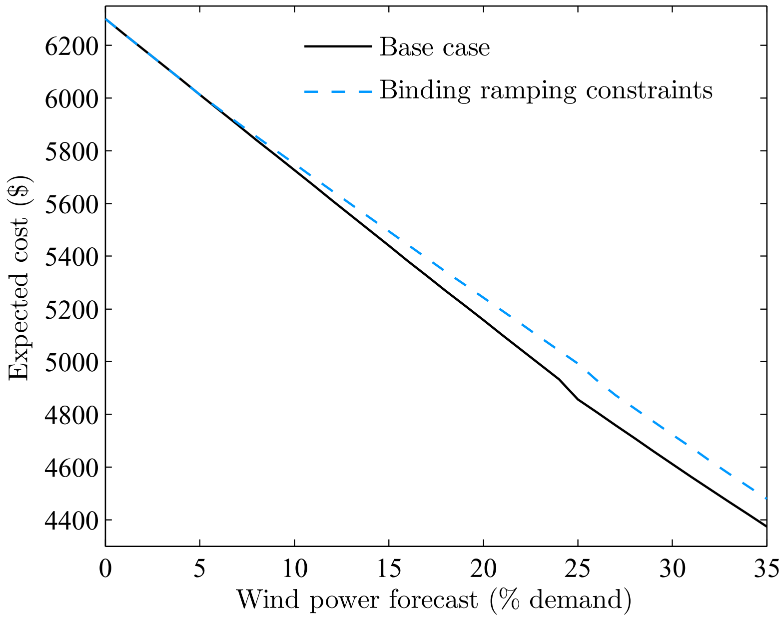


Fig. C.12 Solution to Exercise 11.2: effect of binding ramping constraints on the expected cost

Table C.23 Solution to Exercise 11.2: ramp limits in MW/h

Generator #	1	2	3
RD_i	20	90	12
RU_i	90	90	22
SD_i	100	100	50
SU_i	30	100	50

Solution to Exercise 11.6

Table C.24 provides the optimal production schedule and the optimal contracting of reserves with units and the load if decision variable $P_2^{\mathrm{WP,\,S}}$ is fixed to 20 MW. The results shown in square brackets for period 2 correspond to the base case described in Section 11.5 of Chapter 11, in which $P_2^{\mathrm{WP,\,S}} = 30$ MW. The results for the rest of periods are omitted because they are identical in both cases.

Table C.24 Solution to Exercise 11.6: scheduling program if $P_2^{\mathrm{WP,\,S}} = 20$ MW [30 MW]. Powers in MW

Period #	$P_t^{\mathrm{WP,\,S}}$	P_t^{S}			R_t^{U}				R_t^{D}				R_t^{NS}		
		G1	G2	G3	G1	G2	G3	L3	G1	G2	G3	L3	G1	G2	G3
1	6	0	0	24	0	0	4	0	0	0	0	0	0	0	0
2	20	10	0	50	7	0	0	0	10	0	0	0	0	0	0
	[30]	[0]	[0]	[50]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[17]	[0]	[0]
3	35	25	0	50	10	0	0	0	15	0	0	0	0	0	0
4	8	0	0	32	0	0	2	0	0	0	0	0	0	0	0

In order to satisfy the demand at bus 3 for any realization of the wind power production in period 2, the market operator contracts 17 MW of reserve with unit 1 for that period. If the market is cleared with a wind power schedule in period 2 of 20 MW, unit 1 is committed to take part in the energy dispatch in that period with 10 MW and consequently, the market operator has to contract those 17 MW of reserve from the spinning reserve offer of this unit. In contrast, if the market is cleared with a wind power schedule in period 2 of 30 MW, then unit 1 is out of the energy market. Consequently, the required 17 MW of reserve are obtained from the non-spinning reserve offer of this unit, which is cheaper. Thus, as can be observed in Table C.25, a lower expected cost is achieved by reducing the cost associated with the reserve scheduling.

Table C.25 Solution to Exercise 11.6: breakdown of expected social costs ($)

$P_2^{\mathrm{WP,\,S}}$ (MW)	Total	Start-up	System operation	Reserve scheduling	Energy not served
20	4604.0	200	4146.0	258.0	0
30	4595.5	200	4146.0	249.5	0

Solution to Exercise 11.9

Table C.26 provides the scheduling program yielded by the market-clearing procedure described in Chapter 11 if the three wind power scenarios in Table 11.1 are assumed to be equiprobable and the load offers to sell both up- and down-spinning reserve at \$20/MWh. The results shown in square brackets for period 3 correspond to the network-congested case, in which the capacity of line 3 is reduced to 25 MW. The results of periods 1, 2 and 4 in the network-congested case are omitted because they are identical to the corresponding ones in the base case.

Table C.26 Solution to Exercise 11.9: scheduling program. Powers in MW

Period #	$P_t^{\mathrm{WP,S}}$	P_t^{S}			R_t^{U}				R_t^{D}				R_t^{NS}		
		G1	G2	G3	G1	G2	G3	L3	G1	G2	G3	L3	G1	G2	G3
1	6	0	0	24	0	0	4	0	0	0	0	0	0	0	0
2	30	0	0	50	0	0	0	0	0	0	0	0	17	0	0
3	60	0	0	50	0	0	0	11	0	0	0	0	24	0	0
	[36]	[24]	[0]	[50]	[0]	[0]	[0]	[11]	[1]	[0]	[0]	[0]	[0]	[0]	[0]
4	8	0	0	32	0	0	2	0	0	0	0	0	0	0	0

Note that in the periods of highest demand (2 and 3), the hourly wind production scheduled in the market exceeds the wind power forecast in those periods, especially in the base case, for which the scheduled wind production in period 3 (60 MW) is even higher than the wind power value corresponding to the *high* scenario (50 MW). The reason for this is that clearing the market with a wind power value of 60 MW in period 3 allows the market operator to solely commit unit 3 (with a scheduled power of 50 MW) to meet the period load of 110 MW, provided that 24 MW of non-spinning reserve are contracted with unit 1 to guarantee the supply if the lowest values of wind power materialize. On the contrary, in the network-congested case (see results in square brackets) and for any realization of wind, unit 1 needs to be started up in order to supply the load at bus 3 by transferring power through line 2. Consequently, the market schedule includes a power generation of 24 MW for unit 1, while non-spinning reserves are not required.

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