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Nathan Sivin

Granting the Seasons

The Chinese Astronomical Reform of 1280, With a Study of Its Many Dimensions and an Annotated Translation of Its Records

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Nathan Sivin

with the research collaboration of

the late Kiyosi Yabuuti 薮内清

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Granting the Seasons:

The Chinese Astronomical Reform of 1280, With a Study of its Many Dimensions and a Translation of its Records

授時暦叢考

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Dedicated to
Professor Chen Meidong 陈美东
and my other colleagues, past and present,
at the
Research Institute for the History of Natural Sciences,
Chinese Academy of Sciences

Contents

Introduction

This is a translation and study of the Season-granting system *(Shou-shih li* 授時曆, 1280). It is arguably the most innovative, and certainly the most sophisticated and influential, of China's many astronomical treatises. This inquiry looks at its many dimensions, not merely its computational techniques. The astronomical system that it contains was a method for generating annual almanacs. It was also an important component of the imperial charisma, a means of livelihood for officials and others, a bureaucratic project, an expenditure probably unequalled in the history of imperial Chinese astronomy, and a key tool for Mongol rule over a newly vanquished China. Understanding it will be indispensable for any history of astronomy that does not arbitrarily restrict itself to one civilization or another.

Granting the Seasons has several purposes. First, it is time that an important East Asian astronomical treatise be available in a Western language, and fitting that the first one fully translated be this one. The Season-granting system is a remarkable attempt to predict a wide range of important phenomena in the sky, using methods and assumptions that differ greatly from those of the European, Muslim, and Indian traditions. Unlike the last two, it developed without significant influence from the Greek tradition. On the other hand, the Season-granting system exerted great influence in East Asia.

The work of evaluating the strengths and weaknesses of Chinese predictive techniques has barely begun. The Evaluation that was included in the treatise is an important early contribution to this work. I have taken some additional approaches in notes to the Evaluation, but a systematic project would require a good many additional man-years on the parts of well-prepared scholars.

Second, given the technical complexity of this system and the social complexity of its creation and use, an introductory study can only begin to explain it. I have designed this book in a way that should facilitate solving more of the many problems of understanding. The astronomical terminology of my translation is not only internally consistent, but compatible with those of other Chinese systems. I have explained as clearly as present understanding permits what each step in the procedures is meant to accomplish. I have also pointed out where the gaps in our comprehension lie.

Third, because this system was the culmination of over a thousand uninterrupted years of mathematical astronomy in China, it is essential to understand what the norms of practice were with respect to such matters as computation, the recording of time and celestial locations, the use of instruments, the keeping of records and the social and political organization of astronomical work. Chapter 2 is an orientation on a wide range of such matters.

Finally, unlike previous studies of Western astronomical treatises, I have devoted equal effort—and almost equal space—to reconstructing the remarkable project that produced the reform, not only as a technical accomplishment but in its political, social, instrumental, intellectual, and other dimensions.

This book will also, I hope, prove itself useful to several kinds of reader. Its most obvious audience is people curious about an astronomical tradition that differs in many important ways from the other main traditions, those of Europe, the Muslim world, and India. Some will want to know about its techniques, and others about its technical institutions, thought, and practitioners. Other readers will want to understand an important but inadequately studied facet of Chinese culture, fundamental to imperial ritual and significant in daily life. Finally, there are astronomers who wonder about potentialities of their science that were not greatly developed in the West.

I have tried to make this book comprehensible to all those drawn to it. This has meant summarizing aspects of Mongol rulership in China with which specialists in society and politics are already familiar, and explaining technical usages that many astronomers already know. That is the price of writing a book that, although it strives for depth of explanation, is designed for anyone who wants to learn something from it, not mainly for specialists.

China in World Astronomy

All the astronomers of the ancient world studied the same sky, and used the same mathematical tools, to explore the same questions: Where will the celestial bodies be at a given time? When will their prominent phenomena—eclipses, changes in direction of planetary motion, and so on—take place? That is how many people who have read some astronomical history tend to put it; but that is too coarse a view. Each culture made its own sky. Each grouped the stars into its own constellations; each saw the sun, moon, and planets as moved by different movers or volitions. Ptolemy (circa A.D. 100–circa 175), the greatest computational astronomer of the Hellenistic world, explained that the planets move as they do because each was a living god, and that was its will—a view that Chinese would have considered nonsensical. The Greeks, on the other hand, would have laughed at the Chinese habit of seeing governmental bureaus in the heavens.

Out of the many techniques that mathematics offers, each astronomical tradition used a different subset, variously preferring numerical or geometric methods. Some, particularly those in the temperate zone, organized coordinate systems around the equatorial or ecliptic pole. Others, usually close to the equator, mainly measured events on the horizon, or with respect to celestial objects.

Once I had looked closely for a while at more than one tradition, I realized that any one of them used too narrow a range of possibilities to give an adequate picture of what the whole spectrum was. Only sheer parochialism has led Western historians to offer their histories of European thought and practice as "the history of astronomy."

It also became clear that narrow technical studies, as more and more of them appeared, would not spontaneously generate a picture large enough to be adequate. The charter myth of the research university, when the Prussians invented it in the early nineteenth century, was that, if specialists engaged in rigorous, tightly focused analytic research, less blinkered scholars would, from these technical results, build an overview. Others, atop their syntheses, would raise still more overarching levels of understanding. Eventually, out of this would come not only truly reliable, seamlessly integrated knowledge but, on its foundation, wisdom. This was a mirage. Except in a few areas of physical science and mathematics, that well-shaped tower of learning has failed to appear. Modern sub-sub-specialties, despite their bounteous harvest of fact, offer precious little connection to wisdom.

In no area is that more obvious than in the history of astronomy. Consider the longest uninterrupted, well-documented tradition in world history, that of China. As the list of sources cited in this book reminds us, a mass of primary documents has survived, partly because they were routinely printed centuries before Gutenberg. Monographs in Chinese, Japanese, and European languages based on their study have piled up to form a figurative mountain. Nevertheless, there is no usable overview of the tradition in any language but Chinese, and—until this one—no translation of any major source into any Western language. What we have learned over the past generation about the character of Chinese astronomy, its evolution, and its particularities lies outside the ken of the most learned historians of Europe. The situation is even worse for Islamic and Indian astronomy, for their sources are much less accessible. For that matter, only in the last generation have colleagues translated with high accuracy a handful of the most influential early European classics out of the Greek and Latin. There is still no comprehensive and intellectually substantial history of European astronomy as a whole.

This book is a modest effort to make a historically consequential non-European classic available in translation, accompanied as it must be by an explanation of how and why the astronomers did what they did. Any attempt, even a tentative one, to do that for one of the more sophisticated Chinese systems is bound to take up a good many pages. I merely hope I have made it easier for the tradition of China to take part in a dialogue with those of other cultures.

At the same time, I aim to portray the technical methods of astronomy as part of a continuum that enfolds every dimension of human activity, from algorithms to political maneuvering—what some scholars call a cultural manifold.[1] It is easy enough to recognize when we look about us today that first-rate science is a result not only of individual technical talent and effort, but of the ability to adopt certain institutional values, to establish productive relations with colleagues, to use the rhetoric of technical writing persuasively, and to attract money and other resources. Historians, when they write about the technical past, often ignore these matters, or set them to one side as mere context. Scientists of earlier times, like those today, knew that they were essential—at least those scientists who turned out to be productive.

Most readers will find it odd that the Season-granting system, which historians generally consider the high point of Chinese mathematical astronomy, came into being to inaugurate a regime that emphatically did not share China's culture, that had subdued the country after devastating it for generations. Its emperor, known to his own people as Great Khan, was by Chinese standards illiterate. How occupation politics and astronomical reform came to be coupled is one of the large questions I seek to shed light on. To what extent the Muslim and Chinese astronomers who served the same emperor influenced each other, and what set limits on the interchange, is another.

Contents of this Book

That is why this book begins by looking (to avoid even greater length) at only five representative dimensions of the manifold that created the Season-granting system: the cultural, political, bureaucratic, personal, and technical. The first chapter explains why the unlikely coincidence of astronomical reform and occupation politics led to new technical heights. It introduces many of the themes that later chapters explore in detail.

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[1] See Lloyd & Sivin 2002, Sivin 2005a and 2005b, and the forthcoming proceedings of the XXII International Congress of History of Science, Beijing, 2005.

The next five chapters provide the reader with the background needed to understand the translation in its several dimensions.

Chapter 2, a historical orientation, outlines aspects of the Chinese astronomical tradition pertinent to this study. That is all it does; a general history of the tradition would be out of place here. The Season-granting system, in the thirteenth century, was not the endpoint of this tradition, but those that succeeded it were either adaptations of it, or—from the Manchu conquest of 1644 on—hybrids of European astronomy with it. The chapter describes the computational systems used to generate Chinese ephemerides; the key problems as the tradition defined them; conventions for recording positions in space and time; the mathematical techniques on which astronomers relied; and the character of astronomical writings. In addition to describing these aspects, it gives a rough idea of how they changed and evolved.

Chapter 3 is a brief overview of the reform project, how it came about and what happened. Chapter 4 looks more closely than chapter 1 at the people who planned it and carried it out, what brought them together, why they were chosen, and what sort of staff they gathered. Chapter 5 reconstructs the observatory and its instruments. In order to make the historical role of the instruments clear, I also look into their predecessors over the preceding two centuries, as well as a few of the fourteenth century and later that throw light on those of circa 1280. I also take up the vexed questions of the extent to which the Season-granting system responded to foreign influence, and why.

Chapter 6 takes up the form in which the system was published, its relation to other astronomical treatises, and how it was transmitted before its publication. Because there were many studies of it in China, Japan, and Korea from the time of its publication on, I have tried to give a rough idea of the most important research in East Asia and the occident over the centuries to the present.

The translation follows. It attempts to be both literal and faithful, characteristics that are often at war. The treatise is also rich in tables. I reproduce them, replace lost ones that survive in later sources, and add new ones that aid in understanding and evaluating the techniques. There are also a few places where discursive text is so stereotyped and repetitive that I have put it—more readably, I believe—in tabular form.

Since the treatise is written concisely in technical language (sometimes with interlinear notes of its own), I have added a commentary that, for each step, clarifies the procedures being explained, and explains how they articulate with those that precede and follow.

I have added in appendices translations of two important contemporary documents, the description of instruments in the "Treatise on Astrology" of the Yuan dynasty's official history, and a biography of the renowned astronomer Kuo Shou-ching 郭守敬 written shortly after his death. The book ends with glossaries of Chinese technical terms and their English translations, designed as aids to anyone studying early astronomical documents.

It is easy to think of additions that would have enriched this book. One is a set of worked examples for the procedures in the Canon. Another is a spreadsheet that would have allowed readers to work their own examples. The first would have made this study a great deal longer, and it is long enough already. I do provide a few examples where they are needed to clarify the text; see, for instance, the commentaries to the Canon, 3.8 and 3.9. I experimented with a spreadsheet that necessarily would have been distributed separately, but it turned out not to be practicable. Even one that included only the first three sections of the Canon, to predict positions of the apparent sun, would have involved reiterated input from the user, so much that only one already quite familiar with Yuan astronomical practices could have used it successfully.

Principles of the Translation

The goal of faithfully transmuting thought from one language to another is never quite attainable, but nonetheless irresistible in the pursuit. Let me describe my approach.

A great deal of translation from classical Chinese is literal, preoccupied with accounting for every word in the original. This often leads to an English version that reads grotesquely. That would be defensible if the original were grotesque, but otherwise it is anything but faithful. Another kind of translation focuses on finding in English an equivalent in some sense of what the original meant to say. The problem is that what one translator considers equivalent someone else may well see as a failure to get the point. Such a translation often sacrifices too much of the source's language.

Like many colleagues, I simply try to balance translation that is as close as possible to the content *and* tone of the original—what the author would say if he were writing in modern English—with respect for its diction, structure, and rhetoric. There is no ready-made methodological formula. Translation is an art, not a science. One must often compromise when there is no happy medium—or at least none that one can find. I leave it to the reader to judge in each case whether a loose paraphrase of ideas or a word-by-word trot would have been more serviceable.

Since I am trying to reveal the document's original ideas and habits of thought, I have avoided translating directly into modern astronomical terminology. Still, readers encountering those ideas and methods for the first time will need some link to what they already know. Therefore, my commentary—unlike my translation—often explains concepts and approaches in minimally technical modern language. For the same reason, I begin each section of the Canon with a concise list, in accessible astronomical language, of what its steps aim to accomplish.

I also avoid translation into equations. I do not believe there is an ideal language of quantitative astronomy, independent of cultural accident, into which one can transpose ancient assertions to fully reveal their meaning. Such transposition imposes a complex of logic and value that the original was not pursuing, and thus does as much to hide the meaning as to reveal it. Equations may give an impression of generality that hides the limited scope of the verbal originals, or may imply limits that were not originally there.

One might defend translation into equations on the ground that if ancient Chinese had had modern algebra, or even computer programming languages, they would have used them. They were undeniably bright enough to have done so. But it is obviously not that simple. If they felt frustrated by having to write in classical Chinese, they were articulate enough to have complained. They did not. One does not have to read many of their scientific texts to realize that the authors were able to write in exactly as precise, as relaxed, or as ambiguous a way as each wanted. It is an elementary fallacy of reasoning to claim that, given the opportunity, they would have chosen to be modern astronomers.[2]

On the other hand, the readership for this book is likely to be more at home in mathematics than in the artful use of ambiguity. I therefore do not reject any means available to clarify every part of the treatise. That has led me from time to time, when commenting on passages about mathematical relationships, to use equations when they are the only way I can say clearly what the text is about.

There is finally the question of whether to translate. Some readers familiar with writing on Chinese history will be surprised that I transpose into English the names of reign periods (*nien-hao* 年號). That is hardly standard practice. Nevertheless, a date given as "year 4 of the Ta-yeh era (*Ta-yeh ssu nien* 大業四年)" conveys a great deal less information than when fully translated "year 4 of the Great Patrimony era." Translation makes it clear that this reign title—like all of them—is a political statement, a motto much like the American "New Deal." Imperial ritualists (who were, among other things, ideologues) composed names of eras that any contemporary reader would understand, mottoes meant to encourage confidence in the government. In this case, the great patrimony is unmistakably the state; it asserts that the ruler takes seriously his responsibility as its inheritor. There are, of course, many contexts in which a date does

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[2] For a final disproof of the frequent assertion that the Chinese written language was inherently unfitted for scientific writing, see Robinson 2004. The classical language translated in *Granting the Seasons* was no more limited in its expressivity than Ptolemy's Greek or Copernicus' Latin.

nothing more than record a date, in which case I translate *Ta-yeh ssu nien* "A.D. 608." Since the new year may fall between 19 January and 18 February, the overlap is not exact.

There is another reason never noticed in past debates on whether to translate names of reign periods.[3] Not only were reign periods supposed to transmit meaning, but so were the titles of astronomical systems. In fact one was often named for the other, as a glance at section 11 of the Evaluation, Englished below, makes clear. To translate neither is a good deal less trouble, but it deprives the reader of important information that every Chinese reader had.

Conventions

- The rich studies of scholars in China and Japan have provided much evaluative insight. Because their work is inadequately known outside East Asia, whenever possible I cite them, and refer to their critical studies of accuracy, precision, etc. Some readers may think it odd that I often cite computations from others instead of my own. I prefer to give credit to colleagues who have earned it.
- As for computations in this book that use the thirteenth-century procedures, many are by Takebe Katahiro 建部賢弘 (1664–1739), a great Japanese astronomer who worked within the Season-granting tradition as it was passed down in Japan. His corrections of the Evaluation are most valuable, because he was still using the complicated rounding-off and other computational habits of his Yuan predecessors. His calculations based on the Canon, I believe, are more likely to represent the result the reformers were striving for than my own could do. On the other hand, I have checked his calculations and have corrected a few. A full reconstruction of early computational practice remains badly needed; until colleagues have

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[3] The classical debate on the topic is Schafer 1952 and 1965 vs. Wright 1958. The historian Wright's position is that it is extremely difficult to find out what reign titles actually mean; the philologist Schafer's is that the research is worth while. I agree with both, but the difficulty that Wright pleads is characteristic of all classical Chinese.

done it, all results must remain tentative. Computations for which I do not cite a source are my own.

- Two systems for transliterating Chinese are in common use. Neither has much to be said for it from a linguistic point of view, although Sinologists are often vehement about their personal choice, and scold those who choose differently. I use the Wade-Giles system because almost all previous literature on the history of Chinese astronomy employs it. I make one exception. In order to make clear to non-Sinologist readers which are ancient place names and which modern, I write the former in Wade-Giles (which uses hyphens in multisyllabic words) and the latter in the Pinyin system (which runs the syllables together). Thus the Ming dynasty's northern capital was Pei-ching, which on the whole coincides with present Beijing.
- There are also two systems for writing Chinese characters, the simplified form *(chien-t'i tzu* 简体字) used in the People's Republic, and the traditional one *(fan-t'i tzu* 繁體字) that was the norm before 1950 and is still used outside the Chinese mainland. The former, when quoting ancient sources, is often problematic, since it sometimes collapses two or more traditional characters into one (e.g., it writes 里, 裏, and 裡 as 里). I simply reproduce the form that each source uses.
- I have striven to translate technical terms consistently, based on my study of all the surviving astronomical treatises and of astronomical problems in mathematical books. My primary sources were writing for educated readers of their time. Any faithful translation is bound to reflect what the nomenclature meant then, not earlier or later.
- In the translation there are two commentaries, one by the original authors or editors (printed in smaller type in the Chinese text) and one by myself. The first appears in the same type face but slightly smaller, and is set in angle brackets

  <like this.>

The second appears in a different type face, also slightly smaller than the text, and begins and ends in square brackets

  [like this.]

- In order to facilitate cross-reference and provide a tool that will be useful to other students of Chinese astronomy, I have numbered the sections and subsections of the Evaluation and the subsections of the Canon; its sections are already numbered. A reference to the Canon, subsection 4.0, denotes the introductory section of section 4, "Pacing the Travel of the Moon," which lists the constants used in the lunar theory. In the introductory list of constants for each section of the Canon, each number includes a "C" and takes the general form "4.C1."

For intermediate and final results of each computational step, the numbers take the form "4.6.1" in step 4.6, and so on. When a later step uses that result, I give the same number so that it is easy to trace the function of a result throughout the Canon. When the step gives no number for an earlier result, it comes from the previous step. When the result is clearly a technical term, I capitalize its translation to make that clear. It is impossible to rigorously distinguish technical terms from simple results of calculation; I have tried to make the numbering as useful as possible.

An excerpt from step 7.2 of the planetary theory will make this notation clear: "In each case set up the Intermediate Accumulation, add the Argument Interval Constant (7.C9) and Posterior Conjunction Parts for the desired [year], and cast out complete Argument Rates (7.C6). … The result is *tu* and parts of the Argument of Mean Conjunction (7.2.1) for the given star." Since Intermediate Accumulation and Posterior Conjunction Parts are not numbered, they are the output of the previous step, 7.1. They are numbered there. The Argument Interval Constant (7.C9) and Argument Rate (7.C6) are constants listed at the beginning of section 7. "Argument of Mean Conjunction" (7.2.1) is an intermediate term, the first outcome of step 7.2. The glossaries at the end of this book list all the numbered quantities and variables, and give their Chinese equivalents.

- In addition to the commentaries, I explicate the texts of the Canon and Evaluation with diagrams and illustrations. The diagrams are designed to be as clear as possible for readers, whether technically adept or not. For that reason, they are seldom to scale.

- For old-style Chinese books, I cite *chüan* 卷 and page numbers in the form *n*: *nn.*
- For the sake of readability, the translations in this book reduce all linear units to *ch'ih* and abbreviate that unit as "c." In measurements, 11.5c means 11½ *ch'ih* (or 11 *ch'ih* 尺 5 *ts'un* 寸*).* An astronomical *ch'ih* was equivalent to roughly 25cm (see p. 67).
- For the *tu* 度 or Chinese degree (see p. 89), I either write "*tu*" or use a superscript "t" after the integral part of a number, for instance, $365^{t}.25$. In discussions, I use "degree" only for the European degree. In the translation, however, for the sake of simplicity, I translate *tu* "Degree" in the capitalized names of subsections, constants and variables such as "Accumulated Degrees after the Standard Crossing."
- In dates, 272–79 means "from 272 to 279," and 272/79 means "at some unknown point between 272 and 279." I ordinarily translate sexagenary dates and years of reign eras into modern notation—e.g., "sexagenary year 16 of the Perfectly Great era *(chih-yuan chi-mao* 至元己卯)" becomes "1279." I translate them literally only to help the reader comprehend the text.
- The modern distinction between true and apparent motions was meaningless in China. Copernicus made a strong case that the apparent motions (those visible from the earth) are not the true ones as seen from the centers of rotation of the planets (which are not the sun, but are near it). Twentieth-century astronomers have accepted the relativity of all cosmological systems, but the distinction lingers on in some textbooks merely as a historical souvenir. I use "apparent" throughout this book when discussing motions from the modern point of view.
- In references to sources, "ch." refers to the Chinese unit *chüan* 卷. *Chüan* originally referred to rolls of silk used for manuscripts, but for the last two millennia the word has marked subdivisions of a text, long or short, rather like a European chapter. A reference to pages in a given *chüan* occurs in the conventional form 39: 42–54, i.e., *chüan* 39, pages 42–54. In a few cases where it is not feasible to write "ch.," I refer to *chüan* in ancient Chinese books as "chapter."

- In the translations of the Season-granting treatise, I note at the end of each subsection, in curly brackets, the pages it occupies, e.g., {1121–22}. In Appendix B, the biography of Kuo Shou-ching, these numbers mark the beginning of each original page.
- Books written before the twentieth century give ages in *sui.* A person is one *sui* at birth, and adds a *sui* at every new year. An age stated in *sui* is thus greater than in occidental years by one, or sometimes two. Since precision is seldom possible—we often have a year of birth, but seldom the actual date—in translation I normally subtract one from *sui* to give years. Anyone more interested in *sui* need only add one.
- In references to astronomical systems, "(#*n*)" refers to the number in table 2.1, which provides systematic information about each.
- Translations of book and article titles are my own except that those enclosed in quotation marks are those of the authors.
- For translation of official titles, I rely on the standard reference work, Hucker 1985 and, for titles he does not include, Farquhar 1990. The few not in either source are my own.

1 Astronomical Reform and Occupation Politics

This chapter has two aims: to furnish a bird's-eye view of the book's historical argument, and to explain its method. Let me deal with the method first.

This book is an experiment in doing away with the border between foreground and context, and studying a transition in astronomy as what some scholars call a cultural manifold. This term refers to all the dimensions of a given historical phenomenon or process. A cultural manifold includes not only the technical, cosmological, social, institutional, and other aspects of a complex set of events, but also the interactions that make all of these aspects add up to a single whole.

This approach implies that context is not an autonomous setting that may or may not be connected to inquiry. Technical work and its circumstances are parts of one thing. That one thing includes how people make a living, their relation to structures of authority, what bonds connect those who do the same work, how they communicate what they understand, what concepts and assumptions they use, and how they use them. I do not think of social factors determining thought, nor of ideas changing society. The point is to comprehend the interactions within a manifold as doers and thinkers respond to, and at the same time influence, institutions and prevalent values.

The concept of a cultural manifold is an aid to better understanding the technical high point of mathematical astronomy in China, the Season-granting system *(Shou-shih li* 授時曆) of 1280. That astronomical system was named for the classical idea that, because the emperor mediated between the celestial order and the state, one of his most basic rituals, at the new year, was to bestow a correct calendar on his people. Thus he granted them the seasons, and furthered the harmony between the two realms.

A large portion of the recent scholarship on Chinese astronomy has concentrated on explaining its computational methods. Such explication is essential, and this book is devoted to it among other things; but if done alone it sheds an inadequate light on astronomy as an enterprise on which people spend their lives. We can understand why this system's sophisticated techniques evolved when they did, as they did, only when we are as attentive to its political and bureaucratic circumstances as to those of persons and methods.

Astronomers in China compiled about two hundred systems for computing ephemerides between 104 B.C. and 1911, and sovereign governments officially adopted about fifty of them for short or long periods. The ephemeris that emperors granted to their people at the new year was not a calendar in the modern sense. European calendars have had nothing to do with the sky for more than 400 years. Rather than predicting celestial phenomena, they simply count off conventional cycles.

The Chinese calendar was an ephemeris, part of an almanac that combined it with extensive divinations of propitious days to do one thing or another. The ephemeris predicted the year's celestial events, including the winter solstice that determined the beginning of the year, the lunar conjunction that began each month, and eclipses and other phenomena of the planets such as their conjunctions or their passage through certain parts of the sky. By A.D. 100, astronomers were—mostly—getting the new year and the beginnings of months right, according to their own definitions. Still, predicting when an eclipse could be seen from a given place remained difficult in a tradition that preferred numerical to geometric methods. On top of that, a system that at first gave highly satisfactory predictions would, as time passed, lose accuracy due to minor sources of error adding up. Frequent astronomical reforms created new computational systems for generating annual ephemerides.[4]

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[4] Historians who mechanically translate *li* 曆 "calendar" have caused a great deal of confusion. I call the computational treatises "astronomical systems," and reserve "calendars" for the calendrical part of almanacs that astronomers used these systems to compute. I use "astronomical reform"

Every system included a canon: a set of step-by-step instructions, worked out so that a minor functionary with limited mathematical skills could calculate the annual ephemeris. When a step was mathematically complicated, all he had to do was look up the answer in a table that the canon provided.

The systems that survive are the ones whose treatises the official histories incorporate. The Season-Granting system, in addition to its sophistication, is remarkable for its documentation. It includes an evaluation that is just as long as the canon. The evaluation sets out in detail how astronomers used a remarkable archive of observations recorded over more than a thousand years to test the new astronomical system, proving that it would be more reliable than its predecessors. Scholars such as Ch'en Mei-tung 陈美东 and Yamada Keiji 山田慶児 have already used the evaluation to reconstruct the history of the reform.[5]

Drawing on the original documents and modern studies, I will examine five dimensions of the project—the cultural, the political, the bureaucratic, the personal, and the technical—and ask why they turn out to be complementary. These are only examples of pertinent dimensions, as my passing references to economic and intellectual matters indicate.

Cultural

It would be difficult to imagine a people in the first half of the thirteenth century more unlike the Chinese than the Mongols were. Even Europeans, to the extent that Chinese knew about them, seemed no more alien. It was not just that Mongols were nomadic and lived by raising animals. In fact they lived many kinds of lives, and the migratory herders were only the majority. Mongols were tribal, originally with no overarching government. Large political

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for the preparation of a new astronomical system. For a detailed explanation of four distinct meanings for *li,* see the orientation, p. 38.

Because this chapter is largely a conspectus of the book's detailed discussions, I provide references only for a few points where I do not cite sources later.

[5] Yamada 1980 and Chen 2003b.

forms came into being as individuals assembled federations to extort wealth from agrarian societies such as China. Exaction and division of spoils was the key to power that extended beyond one's tribe. The way most Mongols lived made them highly adaptable, always ready to move, and used to fighting. Succession to tribal leadership regularly involved violence; building wealth involved conquest or—more often—the credible threat of it; and everyone took part in internal and external war.

As the Mongols moved out of their heartland to conquer a great swath of the world, they learned to fashion a new "culture created for and bounded by the state," no longer ethnic but using the skills of many peoples. Originally their only specialists were "shamans, bards, and perhaps metalsmiths."[6] Its elite learned to need many advanced skills, from those of administration to those of cosmopolitan cooking. The Mongol rulers thought of talented people as a kind of booty, to be shared. They often sent experts across their domains as gifts.

For Mongols, technical skill implied spiritual force. As rulers, they strove "to mobilize and monopolize the spiritual forces of the realm" embodied in the natural world, ancestors, ritual specialists, artisans, and scholars, as well as priests and monks of every faith. They had long used divination intensively; now they could call on prognosticators from half of Eurasia and compare their findings. Because in China the same people tended to be experts in divination, astronomy, and astrology—which Mongols lumped together as "yin-yang"—the rulers particularly valued these elaborate traditions that had always served imperial courts.[7] They saw them not as competing with their own old methods, but complementing them.

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[6] This section largely draws on the penetrating analysis of Thomas Allsen (2001), especially pp. 198–211, from which the quotations come. As Christopher Beckwith has noted in comments on this manuscript, Chinese farmers did not necessarily experience Mongol exactions differently from taxation by a native government.

[7] I suppose that the Mongols' use of "yin-yang" as a single type of skilled practitioner was part of learning to think about sophisticated occupational categories.

In fact, "Muslim astronomers came to China because the Mongols wanted second opinions on the reading of heavenly signs and portents, not because they or their Chinese counterparts wanted scientific exchange."

When we study the uses of divination, it becomes obvious that the point was not which kind always came true. Competing forecasts could not dictate decisions to the Great Khan, but provided a diverse set of options to discuss, and a ritual for both broadening and focusing discussion. One might indeed say that prognosticators "divined the intentions of their masters, not future events." In the final analysis, the routine use of divination in the rituals of court and military campaign legitimated policy, added to the cosmic authority that backed the decisions of policymakers, and built morale.

Political

Chinggis Khan (r. 1206–27) launched the first Mongol empire. Some among the generation of his grandsons created hybrids of their traditional culture and that of the peoples they ruled within what became five empires. As part of that cyclonic conquest along the breadth of Eurasia, the Mongols vanquished the Chin regime in North China in 1234, and the Sung empire in the west and south by 1276.[8]

The new overlords of North China were poorly prepared to govern an agricultural and urban population. They saw their new subjects largely as providers of manpower and resources for further conquests. They did not see the point of farming, and turned vast areas into pasture land before their advisors convinced them that Chinese society could not survive without agriculture. Since the Mongol leaders were not used to reading and writing, they put together an administration from surrendered Uighurs, Jurchens, Khi-

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[8] I will examine these circumstances more closely in chapter 3. The most judicious account of the Mongol world conquest is that of Fletcher 1986; see also Thomas Allsen in Franke & Twitchett 1994, 321–413, and Barfield 1989. For a map of the Mongol empires at Khubilai's death in 1294, see Rossabi 1988, 111.

tan, Chinese, and others. It extracted wealth at high human cost. This new model of government as extortion terrified the Southern Sung Chinese, and made a negotiated surrender unthinkable.

It was Khubilai (Hu-pi-lieh, 忽必烈, (born 1215, reigned as Grand Forbear [T'ai-tsung 太宗], 1260–94), Chinggis' grandson, who came to understand Chinese culture and the benefits it offered its rulers. Although he could not read Chinese, and probably could not even speak much of it, his interest in the Chinese way of life attracted to him literati who nurtured it. As a young man, he gathered around him not only conventional scholars but members of Buddhist and Taoist movements. The example that concerns us is Liu Ping-chung 劉秉忠 (1216–74), who as a Ch'an monk joined Khubilai's entourage early and became his main political advisor. He proposed, and his Mongol patron accepted, basic Chinese structures of government.[9]

Liu was more than a persuasive courtier. He was celebrated as a philosopher, classicist, diviner, mathematician, astronomer, poet, calligrapher, and painter. He became the only Han Chinese in all of the Yuan period to serve as one of the Three Preceptors, the state's highest dignitaries. Liu was among the many Chinese literati whom Khubilai especially esteemed because of their skill at "yin-yang." Educated Chinese who learned astronomy usually knew astrology and divination as well.[10]

Khubilai began campaigning against the southern Sung shortly after 1250. His Chinese advisors convinced him that victory could be quicker, less bloody, and less ruinous if he installed in the north a style of government that southerners could understand and eventually accept. He and they invented a style of just that kind. In 1276,

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[9] On Khubilai's literacy, see Franke 1953 and Yoshikawa 1968–70. For the lives of Liu and others discussed below, see chapter 4.

[10] In this respect Khubilai was not atypical of the early Mongol rulers; Endicott-West 1999. Historians have tended to think of divination among the Chinese as a preoccupation of only the lower classes, but that was never true. See, for instance, re the Sung period, Liao Hsien-huei 2005 and Liu Hsiang-kuang 劉祥光 2005.

I use "Han" in this book not to refer exclusively to China's ethnic majority, but to distinguish the native population of China proper from the Mongols and their dependents from the Chinese periphery.

when the Mongols were pressing toward the Sung capital, the empress dowager actually surrendered the imperial seals of authority. The dynasty dragged on for three years longer only because refugee loyalists crowned two more baby emperors.

As early as 1251, Liu Ping-chung suggested an astronomical reform, as a way Chinese would recognize of asserting legitimate imperial authority. In 1273, he presented a concrete proposal, putting himself in charge, but nothing came of it and he died the next year. In 1276, Khubilai, certain that all of China would soon be in his hands, gave the order. Ritually marking the unification of China was an important matter, and a new system to generate the ephemeris was symbolically indispensable. Liu intended it to be more than a standard symbol of dynastic change. He wanted to take a step forward in technical practice as well. The Chin and then the Mongols had used the Revised Great Enlightenment system *(Ch'ung-hsiu Ta-ming li* 重修大明曆) of 1180 for a century. Whoever was in charge of pre-imperial Yuan astronomy in 1215 obviously chose this source for convenience, not because of its technical merits. By the late thirteenth century, it was showing its age in the many erroneous predictions it generated.

This was only one of a series of moves in the direction of imperial dominion. Khubilai had already taken a Chinese-style reign title *(nien-hao* 年號) in 1260, when he became Great Khan. In 1270 he had adopted the dynastic title Yuan 元, meaning "great," an outcome of Liu's studies in the *Book of Changes*. Liu then designed for him a new capital along classical lines, a bureaucracy, and a set of state rituals.

Another characteristic of Mongol politics is very much to the point. Mongols enjoyed the diversity of their trans-Asian order. Unlike Chinese scholar-officials, they did not think of other peoples as barbarians, or expect others to adopt their own culture. They drew avidly on the strengths of the many peoples they had brought under their sway. As Eugene Anderson and Paul Buell have recently pointed out, even the imperial palace's menus drew on the cooking of every part of their world from Persia to Russia to Korea.[11]

The Mongols, because of their widespread conquests, were exceptionally well informed about Islamic astronomy. Before their victory over the Southern Sung in 1276, they had two new, complete systems. Their authors were Chinggis Khan's companion the Khitan Yeh-lü Ch'u-ts'ai 耶律楚材 (1190–1244), and Jamāl al-Dīn (Cha-ma-la-ting 札馬剌丁, fl. 1267–91), a Muslim from the west, two generations later. Both became high officials.

But by 1276 Khubilai understood Liu Ping-chung's point that to mark the victory, and to begin the occupation productively, a recognizably Chinese system was essential.

Bureaucratic

Perhaps the most remarkable characteristic of Chinese astronomy is its bureaucratic character, which reflected its official status. There were always private practitioners. Laws against practice outside the civil service were occasionally decreed but seldom enforced. Even practitioners who had nothing to do with the imperial palace thought of astronomy as inherently connected with rulership and the state. Decisions about its technical priorities were consistently decided by bureaucratic criteria (a point to which I will recur below).[12]

It was not unusual for the palace to have more than one astronomical bureau or observatory. Their officials were generally entrenched, with overlapping responsibilities, but not necessarily communicating with each other. If their predictions failed, they were always prepared to deny responsibility. Astronomy was, in other words, part of a true bureaucracy, an organizational form that Chinese perfected early.

This time, however, the organizers of the reform (I will introduce them shortly) were able to avoid the organizational inertia. They

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[11] Buell & Anderson 2000.

[12] This side of the history of astronomy has been rather neglected, but see Arai 1990 and Sivin 1995d.

chose their own colleagues. They used their clout to situate the project in a new, independent organization, with its own new observatory, insulated from the bureaucratic stasis.

When the reform group began organizing in 1276, it became the third of three astronomical institutions. There were already Chinese and Muslim Directorates of Astronomy. Both were expected to observe, compute annual calendars, carry out divination for the remainder of the almanac, interpret astrological omens, and report their findings. The Islamic observers had a set of instruments that Jamāl al-Dīn built in 1267.

The newest instruments the Chinese had were about 150 years old, and not in good working order. The Chinese Directorate had just been established, and the Muslim one had recently moved from the old capital in Inner Mongolia. The two had been amalgamated as part of the new Palace Library, of which Jamāl al-Dīn was co-director, but there is no reason to believe that they worked together (see chapter 3, p. 145). Nor is there evidence that in the 1270's either organization was getting any consequential work done. Whether their staffs were incompetent is not the only pertinent question (some of them were quite able). Despite the change going on all about them, nothing indicates that their superiors were assigning them urgent work, or granting them the funds that would make them useful.

After 1276 that no longer mattered. The autonomous team planning the new system was barely visible—for some time it was simply called "the office *(chü* 局)"—but it was off to a running start. Although it already had acquired a group of experienced astronomers captured in the Northern Sung capital, it soon requisitioned from the Directorate 30 more specialists of its own choice, and began using the old instruments while designing new ones. When "the office" formally became the Astrological Commission (T'ai-shih yuan 太史院) in 1278, it was granted the sole responsibility for preparing and distributing the almanac. The same order converted the Directorate—which previously held that responsibility—into a training organization to supply the Commission with skilled manpower.

The new group, unlike their colleagues in the older offices, made many astronomical innovations in four politically turbulent years. Seven people planned, directed and supervised the project (eight if we include Liu Ping-chung, the most important planner, but he did not live to direct it). Another leader of the project had been brought from the south as a prisoner of war. All except he held non-astronomical posts in the middle and upper ranks of the regular civil service. The full staff of the commission included 77 officials, ranging in rank from 2a, a very high grade, to 9b, the lowest, 44 student observers, and sub-official employees whom no source took the trouble to count.

To comprehend how this diverse and highly competent group came together, we have to investigate the personal relations of its members. That is inevitably a long story, but as stories go, it has a simple plot.

Personal

This was not the first astronomical reform carried out by a number of generalist officials leading specialists assembled for the purpose, but it is the one we know most about. Those who took the largest part in planning and running the project were all Chinese. That made the Commission an anomaly in the Mongol administration. They were also an exceptionally motley group, ranging from an Assistant Supervisor of Waterways to two Vice Directors of the Secretariat, very high officials in the most powerful organ of the Chinese-style government.

Khubilai happened to know from experience that all seven of the planners and supervisors, especially Wang Hsun 王恂 (1235–81), whom he appointed to plan and carry out the project, were accomplished amateurs of "yin-yang." Kuo Shou-ching 郭守敬 (1231–1316), who had become the north's leading expert on water control, had been making astronomical instruments and observing with them since he was a teenager. Hsu Heng 許衡 (1209–81) was not only the leading representative of the Ch'eng-Chu Confucian tradition, but was trained as a diviner. The emperor added him as a consultant on astronomical principles, perhaps to make the new system ideologically attractive. Ch'en Ting 陳鼎 was once described as "a southern barbarian yin-yang" (*man-tzu yin-yang* 蠻子陰陽).[13] In more conventional terms, he was the official astronomer who had carried out the Southern Sung dynasty's last reform in 1271. When the Mongols took the Sung capital Lin-an 臨安 (present Hangzhou) in 1276, they captured him and some of his subordinates and shipped them north.

What brought these diverse figures together? All but Ch'en Ting, the only member already experienced in astronomical reform, were prominent in the network of the powerful Liu Ping-chung. Chang Wen-ch'ien 張文謙 (1217–83), as a child, had been a fellow student of Liu's. Kuo Shou-ching's grandfather, who raised and educated Kuo, was a close friend. He sent Kuo to become Liu's disciple. Wang Hsun was another disciple. Chang I 張易 (d. 1282) had been part of the same scholarly community as Liu. Liu or people close to him introduced all of them to Khubilai, who took them into his personal circle of advisors. Hsu Heng 許衡 and Yang Kung-i 楊恭懿 (1225–94), because of their fame as exemplary teachers, had joined that circle too; they had then stoutly supported Liu in building Chinese-style institutions in two successive capitals.

What attracted the future emperor to them? In addition to their talent, ambition and desire to please, it was the same thing that drew him to Liu Ping-chung, that is, their "yin-yang." They were working not for a faceless, alien state apparatus, but for a patron who knew them well and rewarded them generously.

The historical accounts stress the accuracy of these experts' prognostications, but of course to do that the authors had to overlook many that, without a doubt, failed. I doubt that Khubilai was so accommodating. It was not exceptional accuracy that made divination useful to him.

It is easier to appreciate his enthusiasm when we stop thinking about astrology as a pseudo-science, and become aware of its social

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[13] For the epithet, *Yuan Mi-shu chien chih* 元秘書監志, cited in Yamada 1980, 183.

dimension. It is not hard to understand that the Great Khan should have valued a court ritual that let him talk over vexatious problems with people he trusted outside the rigid structure of formal policy discussion. Once the discussions were over, he could make up his own mind.

Technical

Reforms of the century before 1280, such as the one that Ch'en Ting carried out for the Southern Sung government in 1270–71, were perfunctory. Few were based on original observations or new methods of computation. Some were a little better than their predecessors, and some were worse. If all a ruler and his highest officials demanded was something to symbolize a new cosmic order, they were likely to be satisfied with whatever their celestial functionaries could easily produce. But Khubilai and his personal advisors had devoted their lives to bringing China under Mongol dominion. With victory in sight, they wanted a political order understandable to the Chinese that would enable peace and stability without compromising the power of the Mongol nobles. Naturally, what mattered were the symbolic innovations that generated the imperial charisma. Among the most important of the customary innovations was an astronomical reform.

Part of Liu Ping-chung's vision that Khubilai had come to share was a recognizably Chinese astronomical system. The palace already had Middle Eastern and Central Asian systems and tables that could have been useful in the reform. It is likely that the Chinese astronomers knew they existed, but the evidence is not at all clear. Non-Han influence is remarkably unimportant in the Season-Granting system. The Great Khan obviously did not want in this system—designed for the Chinese subjects he was reuniting—the same diversity that Mongols expected in their banquets. In fact, as we will see, he discouraged his Han, Central Asian, and other astronomers from exchanging ideas or methods.[14]

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[14] For the evidence behind this conclusion, see p. 218.

One reason the new Chinese-style system was so impressive is that this was by all accounts the most elaborate and expensive reform ever carried out. It involved a new observatory, outfitted with a set of large bronze armillaries and other instruments, some of them unprecedented in design. Kuo Shou-ching also built two 40-foot high brick gnomons with a kind of pinhole camera that determined with remarkable exactitude the length of the tropical year. These instruments were so elaborate that they were not ready for use until after the reform was presented to the throne at the end of 1280; the nearly 100 initial observations over three years were made with a prototype armillary instrument and a wooden gnomon.[15]

Teams of observers also carried out a great latitude survey with portable equipment at 27 locations scattered from Siberia southward, covering roughly 3600 miles (6000 km) from north to south and 2000 miles (3300 km) from east to west. They recorded the latitude for each place, and for a number of them time differences of solar or lunar eclipse observations, variations in the ratio of day length to night length, and changes in the altitudes of the sun, the moon, and possibly the planets.

Khubilai was also ready to pay for a new level of detail in evaluating the accuracy of the new system. The Yuan astronomers not only drew on data in books, but also chose from an archive of observations passed down from one dynasty's observatory to the next. The evaluators included in their tests a series of early eclipse observations that specified not only the day but the time of the eclipse, often to the nearest quarter of an hour. This was not the norm. The records were, by and large, quite accurate. The Yuan group used them systematically and profusely, not only because of its high standard, but because its staffing and funds permitted it to do that.

At the same time, this massive support was double-edged. It enabled some kinds of innovation, and ruled out others. It is easy to appreciate the ingenuity that went into the system's eclipse technique. It combined third-order interpolation[16] that allowed a step

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[15] See below, p. 166.

[16] I.e., three-step linear interpolation.

ahead in accuracy with new developments in the direction of spherical trigonometry. It comes as something of a shock, then, to see how stale and unimaginative its planetary technique is. Most of it, surprisingly, turns out to be a copy of its counterpart a century earlier in the Chin regime, the system that the pre-dynastic Yuan had been using in North China. Although the Yuan astronomers did extend their improved interpolation method to the planets, on the whole their technique could not have reliably predicted many planetary phenomena.

It would be naive to conclude that the group was good at lunisolar astronomy but bad at planetary astronomy. It is striking that, although the Yuan astronomers used ancient records to test one feature of the new system after another, they did not evaluate the planetary theory at all. This is particularly surprising, since over the life of the project they amassed a very large body of planetary observations (see p. 589). The obvious hypothesis is that their imperial employer would not give priority to improving planetary prediction.

Their predecessors for many centuries were not notably more concerned than they about this problem. Systems rarely stood or fell on their planetary techniques alone. It is not hard to see why. China's earliest written documents, in the second millennium B.C., recorded eclipses of the sun and moon as baleful omens. From at least the second century B.C. on, as astronomers learned how to predict eclipses, the state came to be intensely concerned with unpredicted ones as highly visible omens—threats to the dynasty's mandate. That kept the solar and lunar techniques at the center of attention until, by A.D. 500, bureaucrats were predicting a large proportion of visible eclipses.

The political priority of eclipses kept planetary phenomena peripheral, no matter how technically interesting they were. Astronomers recognized that the motions of the five classical planets were very different from each other, but coming to grips with those differences would have required sustained, precise measurement over a number of years. That would have entailed new government priorities, additional time, and more money. Precisely because planetary prediction techniques were a weak point, challengers fairly often attacked a current system for predictions of planetary phenomena that had failed. By the end of the tenth century, one system was comparing its computations of planetary events with those of important predecessors, but even 300 years later that had not become the norm.[17]

Conclusions

Even money can't always buy talent. The remarkable concatenation of skills and ambition that drove this reform was possible because Khubilai—for his own reasons—took seriously his protégés who were skilled in "yin-yang." For reasons of state, he was willing to move them from their regular assignments for several years, and to give them exceptional autonomy. The history of the project fully makes sense only when we are aware of its many dimensions, public and private.

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[17] In a chronology that begins in the eighth century B.C., the concern with eclipses is not balanced by omens involving planets; Schaberg 2001: 100–1. For a polemical attack on predictions of planetary phenomena, see *Wei shu* 魏書, 107B: 2698–99.

2 Orientation

This chapter provides a briefing on the Chinese tradition of mathematical astronomy that will aid in understanding the rest of the book. It is not a general survey, but deals concisely with topics pertinent to the Yuan reform. It begins with what can be said in general about the social, political, and institutional matrices of astronomy. It then reviews the basic tools, observational and mathematical, that astronomers used. Finally, it introduces a few important points of comparison between the technical practices of China, Europe, and the Muslim world.

The approach to observation, prediction, and timekeeping that I sketch below existed in rudimentary form by the beginning of the first millennium A.D. Every aspect of astronomy evolved over the ensuing two thousand years. Here I will deal with only those aspects of change that are directly relevant to the reform of 1280.

The Social Organization of Astronomy

In imperial China, astrology (*t'ien-wen* 天文) and mathematical astronomy (*li* 曆), *li-fa* 曆法) were based on the observation and prediction of actual phenomena in the sky. Both were primarily governmental functions, and were interdependent. These complementary arts evolved from the responsibilities of early scribes who kept records and divined to determine whether courses of action were propitious or disastrous. By the mid first millennium B.C., scribes (then called *shih* 史) were prominent officials in local courts, for they were in charge of the rituals which lay at the heart of aristocratic life.[18]

Rituals had to be remembered in order to be performed correctly, and had to be carried out at times determined by divination. The position of ritualist-scribe, in other words, included both recording and divining. When warfare between many small states ended in

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[18] Cook 1995.

the formation of a single empire, in the late third century B.C., the title became Grand Scribe *(T'ai shih* 太史), and its holder became the head of a bureau of experts. This bureau quickly came to combine the state of the art in astronomy, divination, and record-keeping. More than a millennium before the Yuan reform, the primary connotation of its leader's title came to be "Grand Astrologer."[19]

This celestial orientation of the civil service came about because the imperial government's ideologists built its arguments for legitimacy on the idea of the Mandate of Heaven. This doctrine asserted that heaven, broadly understood as the natural order and sometimes personified, conferred the mandate to rule on a family with exceptional spiritual power and virtue, able to pass these qualities down a line of hereditary monarchs. Such a family's dedication to the responsibilities of emperors, which had mainly to do with the conscientious and correct performance of ritual, guaranteed that the social order would remain in harmony with that of the cosmos. If the ruler or those close to him neglected these responsibilities, heaven would send a warning in the form of unpredictable, baleful events in the sky or on the earth. Ignoring such warnings would lead sooner or later to the fall of the dynasty.

Since an emperor could never be quite sure that his virtue and his sincere dedication to duty measured up to those of his ancestors—that is, met the requirements of the cosmic order—this doctrine inevitably generated anxiety. The remedy was three kinds of activity designed to manage the danger.

Divination used a variety of methods to guide decision-making. Most did not require observation of the sky. Astrology was the special form that did depend on celestial events. (In China, omens appeared not only in the sky, but certain events on earth such as abnormal births, earthquakes and palace fires were also portents.) Astrology was what historians call judicial, devoted to judging the implications of unpredicted events for the emperor and those closest to him. Astronomers observed and compiled reports to discover when anomalies happened, and scoured records to determine what

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[19] See the fluctuations over time noted in Hucker 1985, s.v.

they meant. Specialists had to build and maintain instruments and timekeepers, observe, record, colligate, and compare the phenomena with those in the large archive that their predecessors had built up over the centuries. A main responsibility of the director was to report to the throne the new phenomenon and its interpretation. What happened then was not, as moderns tend to suppose, a reward for an accurate prediction and a penalty for a wrong one. What the record implies is a function rather like that of an economic advisor in present-day government, whose value depends, not on the accuracy of his forecasts, but of his ability to make economics play a proper role in his superiors' decisions, even when they are much more interested in political maneuvering. The Grand Astrologer introduced for discussion by courtiers issues that required resolution, but that, without his authority to stress their cosmic and dynastic implications, they might well ignore. It was his work to assert—by prediction and interpretation—the state's symbolic control over the cosmic order.

Symbolic control was a state monopoly, but its correlate in the lives of the privileged was symbolic harmony with, and an assured place in, the order of heaven, reflecting the place they had achieved in the social order. Archeologists have found a great many stellar planispheres painted on the ceilings of excavated tombs. Many of them portray the relations of individual stars and asterisms with good precision. That suggests that the noble and wealthy—not only officials—used them, and other symbolic means, to orient themselves after death within the harmony of the cosmos.[20]

Mathematical astronomy was the art of transforming the ominous into the predictable and therefore the no longer threatening. This function made it the complement of astrology.

Because modern cultures accord astronomy and astrology very different status, contemporary historians tend to treat these enterprises as though they always had been unrelated. That can badly

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[20] Ch'en Mei-tung 陈美东 1996 reproduces a good selection of tomb paintings as well as star charts. See the excellent orientation by Po Shu-jen 薄树人 in the same book and, on tomb iconography, Loewe 1979 and 1982.

distort our view of the past in every one of the literate civilizations. Ptolemy's *Almagest* provided the computations that made horoscopes based on the *Tetrabiblos* possible. Copernicus, Tycho Brahe, and Kepler were astrologers, refining methods to cast horoscopes for their employers or patrons. Analogously, computation, interpretation of portents, and determination of auspicious days were not distinct and unrelated duties, but equal parts of the Chinese astronomical bureau's responsibility. Most of those who received copies of the government almanacs—whether empresses or commoners—valued them at least as much for their daily prognostications as for their predictions of phenomena.

Astronomical Reforms

The consequential work of the astronomers was astronomical reforms, namely the proposal, preparation, testing, defense, and use of new computational systems. Improvement meant being able to generate more reliable calendars, that is, ephemerides that predicted more accurately the phenomena that defined time.

The word *li,* usually translated "calendar," has several distinct meanings, which can lead to much confusion if we do not distinguish them.

One is the art of computing the times or locations of certain future or past phenomena in the sky. It is that sense that corresponds to what historians have called "mathematical astronomy," and I so translate it.

Speaking of mathematical astronomy as "calendrical science" misses the point in two ways. It obscures the basic similarity between the Chinese technical literature, Islamic tractates—most historians of Muslim astronomy translate *zij* "table," although they are actually handbooks—and Western treatises from Claudius Ptolemy (circa A.D. 100–circa 175) to Georg von Peuerbach (1423–61). The Chinese handbooks were astronomical systems in the sense that I have defined it. All were linked to astrology. The political rationale of palace astronomy, what justified the state's control and the responsibility it took for the work, was the ritual of issuing the almanac: that is what "granting the seasons" refers to.

A second sense is a step-by-step sequence of computations that generates such forecasts and assembles them to make a complete ephemeris. That set of procedures I call an “astronomical system.” A project designed to produce a new computational system, meant (in principle, at least) to improve the content of almanacs, is an “astronomical reform (*kai li* 改曆)” in both ancient and modern Chinese.

Some historians have called it a “calendar reform,” but that is misleading. What *kai li* reforms is not a calendar but an astronomical system. The notion of a calendar reform in European history differs fundamentally from the Chinese practice. The term normally refers to the changes that Pope Gregory XIII made in 1582 to the Catholic church’s ecclesiastical calendar, to replace both the Julian calendar used since 46 B.C. and the various local lunisolar calendars of the sixteenth century. The goal of the Gregorian reform was not to improve predictions of solstices and lunations, but to get rid of them as components of dates. Its product was the purely solar calendar now used in most of the world for secular purposes. It is based simply on counting off months of 28 to 31 days and years of 365 or 366 days in a fixed order. This calendar reform, in other words, was a decisive rejection of astronomical reforms.[21]

A third *li* is the embodiment of the system, namely the computational treatise that results from a reform by a process that this chapter discusses. The Season-granting treatise, translated in this book, is an example.[22]

The fourth *li* that concerns us is the products of computational treatises, namely ephemerides published in almanacs. Historians usually refer to them simply as calendars, but this ignores the importance of their substantial content on divination. Almanacs predicted regular events such as the new moons that began each lunar month, and irregular ones such as eclipses. Their rich hemerological

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[21] For details see Anonymous 1961, chapter 14B–14C.

[22] See Chapter 6 for a conspectus of astronomical treatises.

data allowed users to anticipate propitious and unpropitious days for most of their activities—beginning a journey, planting crops, sponsoring a ritual, and so on. On the one hand, the almanac was a basic component of state ritual, for the ruler granted it annually to his subjects, and, on the other, its predictions and indications embodied governmental order and control. The government prepared versions meant only for official use, and others for public distribution. To avoid confusion, I use "calendar" (or "ephemeris") only when I mean the calendrical component of an almanac.[23]

The annual almanacs themselves, this fourth *li,* were the astronomers' least challenging work. Preparing a calendar was a low-level routine task. A computist was qualified well enough if he could add and subtract, multiply and divide. Handbooks such as the Canon of the Season-granting system laid out for him, step by step, what he was to do. When more than elementary operations were necessary, his handbooks provided him with tables in which he merely looked up a quantity as the instructions demanded. So long as he followed instructions and memorized the names of constants and variables, he had no need to understand the astronomy of what he was doing. Once he had worked his way through the Canon, he could set out in official form the whole ephemeris, beginning with a prediction for the winter solstice, for the civil New Year (defined as the second conjunction of the sun and moon after the solstice, and as the first day of the first civil month), for the first days of the remaining months, for solar and lunar eclipses, and for planetary phenomena. His colleagues, equally low-ranking diviners, would later add notes on lucky and unlucky days.

One of the most noticeable peculiarities of astronomical systems is the frequency with which the government changed the official one. This in turn motivated astronomers, inside and outside the civil service, to design new ones in the hope that they would be adopted. The result was roughly 200 systems through history. We have basic information about half of them; half of that half were of-

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[23] On the types, and the government's control of their circulation, see the important study in Tung Yü-yü 董煜宇 2006, and details in Tung 2004.

ficially adopted at one time or another. See table 2.1 for 98 of the best-known systems, and the Evaluation, section 11, for the Yuan astronomers' list of 43 systems they considered their predecessors.

This poses an obvious question. Since the quality of astronomical systems was generally high, why was it necessary to keep improving them? In the West, one system, the so-called Julian calendar of 46 B.C., served well enough until A.D. 1584. The reform enacted then, that of Pope Gregory, has been the basis of every European calendar since.[24] European ideas of calendars and their functions were quite different from those prevalent in China. That is not an answer, but a restatement of the problem.

There was no need for Chinese to be as demanding as they were. In one sense, by the reform of A.D. 85 (#4 in table 2.1), they had met the needs of agriculture and government for accurate predictions. In another, no single ephemeris could predict seasonal phenomena for the whole of an empire larger than Europe. Encompassing that climatic diversity was something no official almanac even tried to do. Doing so would have compromised the official picture of one government ruling over a single, cosmically unified empire.

Table 2.1 lists nearly a hundred astronomical systems from the Han period to the end of imperial China. It is far from a complete list. Of the roughly 200 systems recorded, I include only the 98 about which we have some basic information. Three sources cite almost all of them: the Season-granting system (SGS); Ch'en Mei-tung 陈美东 1995: 215–23, table 8-1 (CMT); and Ch'en Tsun-kuei 陈遵妫 1980–84: III, 1399–1407, table 50 (CTK, often inexact in its dates). Yabuuchi 1963 lists additional systems about which there is very little reliable information.

The table omits half a dozen early schemata that many historians before recent decades believed were those of dynasties or kingdoms long before the unification of the third century B.C. (items 1–6 in CTK). They generated only simple lunisolar calendars, and the data about them are negligible. Most specialists today see them as local

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[24] Actually the date of adoption varied widely from one country to another. See the table in Anonymous 1961: 414–16.

calendar schemata of the third century. Item 1 is a version of one of these, important because #2 was explicitly meant to reform it, and #4, a complete system, was based on it.

On these "old" Warring States calendars see Ch'en 2003a: 87–92.

The Title column contains only systems itemized in section 11 of the Evaluation. Titles in the form "*n*th-year epoch" refer to the sexagenary year number of the epoch. The Dynasty column uses the abbreviations F. (former, *ch'ien* 前), L. (later, *hou* 後), N. (northern, *pei* 北), E. (eastern, *tung* 東), S. (southern, *nan* 南), and W. (western, *hsi* 西). In the next to last column, inclusive dates are those of official use as given in section 11. Since a number of those dates are incorrect, the last column provides corrections when needed, as well as dates for systems not listed in the Evaluation. Single dates mark either systems that did not achieve official status or official systems for which the final dates remain unknown. I explain the corrections in the notes that follow the table, keyed by numbers in parentheses. Many dates require further study.

In the Dynasty column I add a plus sign when a system remained official past the end of a dynasty. Although I normally list only the person conventionally credited with authorship, most systems were the products of collaboration.

Table 2.1. Astronomical Systems in Imperial China

No.	SGS	CMT	CTK	Dynasty	Title	Title in CMT or CTK	Author	Dates, SGS	Dates
1	-	1	-	F. Han	-	Quarter-remainder (Ssu fen li 四分曆)	Anonymous	-	Unknown
2	-	2	7	F. Han	-	Grand Inception (T'ai ch'u li 太初曆)	Teng P'ing 鄧平	-	104 B.C-A.D.1/5
3	1	-	8	F. Han+	Triple Concordance (San t'ung li 三統曆)	-	[Liu Hsin 劉歆]	104 B.C–A.D. 85	A.D. 1/5–85 (1)
4	2	3	9	L. Han	Quarter-remainder (Ssu fen li 四分曆)	Later Han Quarter-remainder ([Hou Han] Ssu fen li [後漢] 四分曆)	Pien Hsin 編訢	85–206	-
5	3	4	10	L. Han+	Supernal Emblem (Ch'ien hsiang li 乾象曆)	-	Liu Hung 劉洪	206–37	-
6		5	11	Wei	-	Yellow Inception (Huang ch'u li 黄初曆)	Han I 韓翊	-	220
7	-	6	12	Wei	-	Grand Harmony (T'ai ho li 太和曆)	Kao T'ang-lung 高堂隆	-	227
8	4-	7	13	Wei+	Luminous Inception (Ching ch'u li 景初曆)	-	Yang Wei 楊偉	237–443	-
9	-	-	16	Tsin	-	Supernal Standard (Ch'ien tu li 乾度曆)	Li Hsiu 李修	-	c. 277
10	-	8	17	Tsin+	-	Perpetual Harmony (Yung ho li 永和曆)	Wang Shuo-chih 王朔之		

No.	SGS	CMT	CTK	Dynasty	Title	Title in CMT or CTK	Author	Dates, SGS	Dates
11	-	-	15	Tsin	-	Correct (Cheng li 正曆)	Liu Chih 劉智	-	c. 352
12	-	9	14, 20	Tsin	-	Grand Beginning (T'ai shih li 泰始曆)	Liu Chih	-	365
13	-	10	18	L. Ch'in	-	Triple Era First-year Epoch (San chi chia-tzu yuan li 三紀甲子元曆)	Chiang Chi 姜岌	-	384–417
14	-	11	19	N. Liang	-	Epochal Beginning (Hsuan shih li 玄始曆)	Chao Fei	-	c. 412–522 (2)
15	-	-	22	Sung	-	Seven Luminaries Past (Chi wang ch'i yao li 既往七曜曆)	Hsu Kuang 徐廣	-	c. 424
16	-	-	21	N. Wei	-	Triple Spreading Epoch (San hsuan yuan li 三宣元曆)	Ts'ui Hao 崔浩	-	c. 440
17	5	12	23, 24	Sung	Epochal Excellence (Yuan chia li 元嘉曆)	-	Ho Ch'eng-t'ien 何承天	443–63	(3)
18	6	13	25	Liang+	Great Enlightenment (Ta ming li 大明曆)	-	Tsu Ch'ung-chih 祖沖之	463–521	510–589 (4)
19	-	-	26	N. Wei	-	Luminous Enlightenment (Ching ming li 景明曆)	Kung-sun Ch'ung 公孫崇	-	507

No.	SGS	CMT	CTK	Dynasty	Title	Title in CMT or CTK	Author	Dates, SGS	Dates
20	-	-	27	N. Wei	-	Divine Tortoise (Shen kuei li 神龜曆)	Ts'ui Kuang 崔光	-	(5)
21	7	14	28	N. Wei+	Orthodox Glory (Cheng kuang li 正光曆)	-	Chang Lung-hsiang 張龍祥	521–40	518?-89 (5)
22	-	-	33	N. Ch'i	-	Numinous Pattern (Ling hsien li 靈憲曆)	Hsin-tu Fang 信都芳	-	by 539? (6)
23	8	15	29	E. Wei	Ascendant Harmony (Hsing ho li 興和曆)	-	Li Yeh-hsing 李業興	540–50	-
24	-	16	30	Liang	-	Great Unity (Ta t'ung li 大同曆)	Yü K'uo 虞劆	-	544
25	-	17	31	E. Wei	-	Nine-grid Chess (Chiu kung hsing ch'i li 九宮行棋曆)	Li Yeh-hsing	-	547 (7)
26	9	18	32	N. Ch'i	Celestial Preservation (T'ien pao li 天保曆)	-	Sung Ching-yeh 宋景業	550–66	550–77 (8)
27	-	-	-	N. Chou	-	Chou (Chou li 周曆?)	Ming K'o-jang 明克讓	-	559? (9)
28	10	19	34	N. Chou	Celestial Harmony (T'ien ho li 天和曆)	-	Chen Luan 甄鸞	566–79	-
29	-	20	35	N. Ch'i	-	Martial Tranquillity (Wu p'ing li 武平曆)	Liu Hsiao-sun 劉孝孫		

No.	SGS	CMT	CTK	Dynasty	Title	Title in CMT or CTK	Author	Dates, SGS	Dates
30	-	21	36	N. Ch'i	-	50th-year Epoch (Chia-yin yuan li 甲寅元曆)	Tung Chün 董峻	-	576
31	-	22	37	N. Ch'i	-	Meng-pin (Meng-pin li 孟賓曆)	Chang Meng-pin 張孟賓	-	576
32	11	23	38	N. Chou+	Great Emblem (Ta hsiang li 大象曆)	-	Ma Hsien 馬顯	579–84	-
33	12	24	39	Sui	Opening Sovereignty (K'ai huang li 開皇曆)	-	Chang Pin 張賓	584–608	584–609 (10)
34	42	25	40	Sui+	Sovereign Pole (Huang-chi li 皇極曆)	-	Liu Cho 劉焯	608	609
35	13	26	41	Sui	Great Patrimony (Ta yeh li 大業曆)	-	Chang Chou-hsuan 張胄玄	609–19	-
36	14	27	42	T'ang	Fifteenth-year Epoch (Wu-yin li 戊寅曆)	-	Fu Jen-chün 傅仁均	619–65	-
37	-	-	45	T'ang	-	Warp and Weft (Ching-wei li 經緯曆)	Ch'ü-t'an Lo 瞿曇羅 (Indian)	-	by 664 (11)
38	15	29	44	T'ang	Chimera Virtue (Lin-te li 麟德曆)	-	Li Ch'un-feng 李淳風	665–728	666–728 (12)

No.	SGS	CMT	CTK	Dynasty	Title	Title in CMT or CTK	Author	Dates, SGS	Dates
39	-	-	46	T'ang	-	Shining Residence (Kuang chai li 光宅曆)	Ch'ü-t'an Lo	-	698
40	-	30	47	T'ang	-	Divine Dragon (Shen lung li 神龍曆)	Nan-kung Yueh 南宮說	-	705
41	-	28	48	T'ang	-	Nine Upholders (Chiu-chih li 九執曆)	Anonymous [Indian]	-	718 (13)
42	16	31	49	T'ang	Great Expansion (Ta-yen li 大衍曆	-	I-hsing 一行	728–62	728–58 (14)
43	-	-	50	T'ang	-	Thousand Year (Ch'ien sui li 千歲曆)	Wang Po 王勃	-	8th cent.
44	-	-	51	T'ang	-	Seven Luminaries (Ch'i yao li 七曜曆)	Wu Po-shan 吳伯善	-	8th cent.
45	17	32	53	T'ang	Fivefold Era (Wu chi li 五紀曆)	-	Kuo Hsien-chih 郭獻之	762–85	763–85 (15)
46	-	-	52	T'ang	-	Perfect Virtue (Chih te li 至德曆)	Han Ying 韓穎	-	758–62
47	-	-	43	T'ang	-	Tallying with Heaven (Fu t'ien li 符天曆)	Ts'ao Shih-wei 曹士蒍		(16)

No.	SGS	CMT	CTK	Dynasty	Title	Title in CMT or CTK	Author	Dates, SGS	Dates
48	18	33	54	T'ang	Constant Epoch (Chen yuan li 貞元曆)	Constant Epoch (Cheng-yuan li 正元曆)	Hsu Ch'eng-ssu 徐承嗣	785–822	783–806 (17)
49	-	-	55	T'ang	-	Phenomenal Contemplation (Kuan hsiang li 觀象曆)	Hsu Ang 徐昂	-	807–22 (18)
50	19	34	56	T'ang	Extending Enlightenment (Hsuan ming li 宣明曆)	-	Hsu Ang	822–93	(19)
51	20	35	57	T'ang+	Reverence for the Arcana (Ch'ung hsuan li 崇玄曆)	-	Pien Kang 邊岡	893–956	(20)
52	-	-	59	F. Shu	-	Eternal Glory (Yung ch'ang li 永昌曆)	Hu Hsiu-lin 胡秀林	-	ca. 911?
53	-	-	60	F. Shu?	-	Correct Phenomena (Cheng hsiang li 正象曆)		-	after 911? (21)
54	-	-	58	5 Dyn.	-	Myriad Parts (Wan fen li 萬分曆)	Anonymous, popular	-	ca. 920?
55	-	-	61	L. Tsin	-	Adjusted Epoch (T'iao yuan li 調元曆)	Ma Ch'ung-chi 馬重績	-	938
56	-	-	62	S. T'ang	-	Centered and Upright (Zhong-zheng li 中正曆)	Ch'en Ch'eng-hsun 陳成勛	-	940

No.	SGS	CMT	CTK	Dynasty	Title	Title in CMT or CTK	Author	Dates, SGS	Dates
57	-	-	63	S. T’ang	-	Orderly Governance (Ch’i cheng li 齊政曆)	Unknown	-	940/975
58	-	-	64	L. Chou	-	Resplendent Arcana (Ming yuan li 明玄曆)	Wang Ch’u-ne 王處訥	-	956
59	21	36	65	L. Chou+	Veneration for Heaven (Ch’in t’ien li 欽天曆)	-	Wang P’u 王朴	956–60	956–63 (22)
60	22	37	66	Sung	Response to Heaven (Ying t’ien li 應天曆)	-	Wang Ch’u-ne	960–81	963–81
61	23	38	67	Sung	Supernal Epoch (Ch’ien yuan li 乾元曆)	-	Wu Chao-su 吳昭素	981–1001	
62	-	-	77	Liao	-	Great Enlightenment	Chia Chün 賈俊	-	994–1126 (23)
63	-	39	68	Sung	-	Perfect Way (Chih tao li 至道曆)	Wang Jui 王睿	-	995 (24)
64	24	40	69	Sung	Matching Heaven (I t’ien li 儀天曆)	-	Shih Hsu 史序	1001–24	-
65	-	41	70	Sung	-	Supernal Ascendance (Ch’ien hsing li 乾興曆)	Chang K’uei 張奎	-	1022
66	25	42	71	Sung	Reverence for Heaven (Ch’ung t’ien li 崇天曆)	-	Sung 宋行古		1070?-74 (25)

No.	SGS	CMT	CTK	Dynasty	Title	Title in CMT or CTK	Author	Dates, SGS	Dates
67	26	43	72	Sung	Resplendent Heaven (Ming t'ien li 明天曆)	-	Chou Ts'ung 周琮	1064–74	1065–70?
68	27	44	73	Sung	Oblatory Epoch (Feng yuan li 奉元曆)	-	Wei P'u 衛朴	1074–92	1075–94 (26)
69	28	45	74	Sung	Contemplation of Heaven (Kuan t'ien li 觀天曆)	-	Huang Chü-ch'ing 皇居卿	1092–1103	1094–1103 (27)
70	29	46	75	Sung	Augury of Heaven (Chan t'ien li 占天曆)	-	Yao Shun-fu 姚舜輔	1103–6	-
71	30	47	76	Sung	Era Epoch (Chi yuan li 紀元曆)	-	Yao Shun-fu	1106–27	1106–35
72	31	48	78	Chin	Great Enlightenment	-	Yang Chi 楊級	1127–80	-
73	33	49	79	Sung	Concordant Epoch (T'ung yuan li 統元曆)	-	Ch'en Te-i 陳得一	1135–67	-
74	34	50	80	Sung	Supernal Way (Ch'ien tao li 乾道曆)	-	Liu Hsiao-jung 劉孝榮	1167–76	1167–77 (28)
75	35	51	81	Sung	Splendor through Simplicity (Ch'un hsi li 淳熙曆)	-	Liu Hsiao-jung	1176–91	1177–91
76	32	53	82	Chin+	Revised Great Enlightenment (Ch'ung hsiu Ta ming li 重修大明曆)	-	Chao Chih-wei 趙知微	1180–1281	1182–1280 (29)

No.	SGS	CMT	CTK	Dynasty	Title	Title in CMT or CTK	Author	Dates, SGS	Dates
77	43	52	83	Chin	Thirty-second Year Epoch (I wei li 乙未曆)	-	Yeh-lü Lü 耶律履	-	c. 1180
78	-	54	84	Sung	-	Repeated Five-star Conjunction (Wu-hsing tsai chü li 五行再聚曆)	Shih Wan 石萬	-	1187
79	36	55	85	Sung	Coincident Epoch (Hui yuan li 會元曆)	-	Liu Hsiao-jung	1191–99	-
80	37	56	86	Sung	Concord with Heaven (T'ung t'ien li 統天曆)	-	Yang Chung-fu 楊忠輔	1199–1207	-
81	38	57	87	Sung	Spreading Joy (K'ai hsi li 開禧曆)	-	Pao Huan-chih 鮑澣之	1207–51	-
82	-	58	88	Yuan	-	Western Expedition Seventh-year Epoch (Hsi cheng keng-wu yuan li 西征庚午元曆)	Yeh-lü Ch'u-ts'ai 耶律楚材	-	c. 1221 (30)
83	39	59	89	Sung	Protection through Simplicity (Ch'un yu li 淳祐曆)	-	Li Te-ch'ing 李德卿	1250–52	1252–53 (31)
84	40	60	90	Sung	Coincidence with Heaven (Hui t'ien li 會天曆)	-	T'an Yü 譚玉	1253–71	-

No.	SGS	CMT	CTK	Dynasty	Title	Title in CMT or CTK	Author	Dates, SGS	Dates
85	-	-	91	Yuan	-	Myriad Year (Wan nien li 萬年曆)	Jamāl al-Dīn (Persian)	-	1267
86	41	61	92	Sung	Attainment of Heaven (Ch'eng t'ien li 成天曆	-	Ch'en Ting 陳鼎	1271–81	1271–76 (32)
87	-	-	93	Sung	-	Founded in Heaven (Pen t'ien li 本天曆)	Teng Kuang-chien 鄧廣薦	-	1276 (33)
88	44	62	94	Yuan+	Season-Granting system (Shou-shih li 授時曆)	-	Wang Hsun 王恂	1281	1281–1384
89	-	63	95	Ming	Muslim (Hui-hui li 回回曆)	-	Wu Po-tsung 吳伯宗	-	1382
90	-	64	96	Ming	-	Great Concordance 大統	Liu Chi 劉基	-	1384–1644 (34)
91	-	65	97	Ming	-	Sagely Longevity Myriad Year (Sheng shou wan nien li 聖壽萬年曆)	Chu Tsai-yü 朱載堉	-	c. 1590? (35)
92	-	66	98	Ming	-	Yellow Bell (Huang chung li 黃鍾曆)	Chu Tsai-yü	-	by 1581
93–94	-	67–68		Ming	-	Titles unknown	Hsing Yun-lu 邢雲路	-	c. 1610 (36)

No.	SGS	CMT	CTK	Dynasty	Title	Title in CMT or CTK	Author	Dates, SGS	Dates
95	-	70	99	Ming	-	New Method (Hsin fa li 新法曆)	Hsu Kuang-ch'i 徐光啟	-	1628
96	-	71	101	Ch'ing	-	Temporal Pattern (Shih hsien li 時憲曆)	J. A. Schall v. Bell	-	1644–1742
97	-	69	100	Ch'ing	-	Hsiao-an (Hsiao-an li 曉庵曆)	Wang Hsi-shan 王錫闡	-	c. 1656 (37)
98	-	72	102	Ch'ing	-	40th-year (Kuei-mao li 癸卯曆)	Ignatius Kögler	-	1742–1911

[Notes

1. So far as we now understand it, the Grand Inception system of Teng P'ing 鄧平 et al. was a set of basic lunisolar procedures that Liu Hsin extended to include what became the standard contents of astronomical systems. See Sivin 1995a , chap. 2 , p. 11.

2. CMT 255 gives the title as *Yuan shih li* 元始曆. Early sources differ about the correct title of the regnal era for which the system is named. The difference is almost certainly due to a historical taboo, but Ch'en Yuan 1928 does not list a pertinent one.

3. When continued in use under the Ch'i dynasty, this system was renamed Established Epoch *(Chien yuan* 建元).

4. This outstanding system, because it was attacked by a court favorite, was not adopted in 463, but was used under the Liang and Ch'en dynasties. See *Sung shu,* 13: 304–17. Later astronomers used the same title for systems #62, probably based on this one, and #72, which replaced the former in the north. Number 80, which in turn replaced #72, still drew on the same name although it was explicitly a revision.

5. Although Ts'ui proposed a system with the title as in #20, it was renamed Orthodox Glory when promulgated in 518 (apparently not 521; the source is slightly ambiguous). Li Yeh-hsing (see #23) was the astronomer most prominently involved. See *Wei shu,* 107A: 2662–63.

6. This date is quite uncertain; see the discussion in Ch'en Mei-tung 2003: 294.

7. CMT gives the title as *Chiu kung li* 九宮曆, and CTK as *Chiu kung hsing ta li* 九宮行荅曆. Ch'en Mei-tung corrects the error in 2003a, 288–89.

8. The terminal date of use depends on when one ends the N. Ch'i dynasty. Although conservative historians terminate it in 557 with the beginning of the N. Chou period, the last year of its last ruler was 577.

9. See Yabuuchi 1963, 455, item 12, and Ch'en Mei-tung 2003a, 304–5.

10. See *Sui shu,* 17: 435.

11. On the dates of this and #39, see Ch'en Mei-tung 2003a, 357.

12. Huang I-nung 1992 has made this correction, and has shown that the system was not used for certain years in this interval.

13. This is probably when this treatise was translated from Sanskrit or another Indian language. When it was compiled remains unknown (Ch'en 2003a, 359). Yabuuchi & Yano 1979 is a translation and study.

14. Item #46 replaced this system in 758; see *Hsin T'ang shu,* 27B: 635.

15. *Hsin T'ang shu,* 29: 695.

16. CTK lists only the epoch of this popular system (660), but it was compiled more than a century later; see Momo Hiroyuki 1964.

17. See Ch’en Mei-tung 2003a, 401. The difference between the titles given in the Yuan system and by the two modern sources is due to a taboo on the personal name of the third Sung emperor (Ch’en Yuan 1928, 154).

18. *Hsin T’ang shu,* 30A: 739.

19. This system was adopted in Korea from the early ninth century to 1392, and in Japan from 861 to 1684. In both countries the Season-granting system succeeded it; Ch’en Mei-tung 2003a, 407.

20. CTK gives the title as *Ch’ung yuan li* 崇元曆, another instance of taboo avoidance.

21. CTK gives the epoch of this item as “Hou Shu Yen-k’ang yuan-nien 後蜀延康元年,” and that of the last as “Hou Shu Yung-p’ing yuan-nien,” but basic reference works (e.g., Ku Ching 1995, 60, Li Ch’ung-chih 1981, 133) list the date of the latter as in Former Shu, and give no such reign title as the former in the Five Dynasties. See also Ch’en Mei-tung 2003a, 419.

22. *Sung shih,* 68: 1498.

23. For the dates, see Ch’en Mei-tung 2003a, 491. P’an Nai & Hsiang Ying (1980, 26) point out that, although many historians consider this system a minor revision of #18, there is no evidence to that effect. They believe that the Chin used it until #72 replaced it in 1137.

24. CTK unaccountably dates items 63–71 (its items 68–76) as Later Chou, but that dynasty ended in 960.

25. The final year for #67 may have been 1065 (or 1068; *Sung shih,* 82: 1929–30, is unclear). After its last year, #66 was reinstated. See also Ch’en Mei-tung 2003a, 463–64.

26. *Yü hai* 玉海*,* 10: 32a-33b.

27. Some early sources give the author’s name as Huang Chü-ch’ing 黄居卿; see Ch’en Mei-tung 2003a, 485.

28. *Yü hai,* 10: 35a-40b.

29. The Season-granting system lists this item out of chronological order, no doubt because it is a revision of #72. Chin used it until 1234, and Yuan from 1215 until it adopted #88 in 1281.

30. This pre-dynastic Yuan system is often cited (here by CMT) simply as *Keng-wu yuan li.* It survives in *Yuan shih,* ch. 57–58.

31. See my note to the Evaluation, s.v., for the correct dates.

32. This system was no longer official after the Sung government surrendered in 1276. The Yuan was using #76.

33. *Sung shih,* 82: 1952. This system was commissioned by a refugee court, and was used only locally.

34. Liu drafted this system in 1355, but it was not used until the Ming period; *Ming shih,* 31: 516–17.

35. On the dates of this and the next item, see the discussion in Ch'en Mei-tung 2003a, 612.

36. CTK devotes its items 67–68 to two lost treatises by Hsing Yun-lu. The excerpts in *Ku chin lü li k'ao* indicate that both are revisions of SGS. Of the remaining items in this table, 96 and the revision of it in 98 are treatises in the conventional mode. Whether that was true of 95 and 97 is uncertain.

37. On this item see Sivin & Fang 1976, 1379–80.]

The need for ever more accurate predictions arose from several principles. One was the claim that the emperor was the Son of Heaven, that his virtue and conscientious performance of ritual kept the state in harmony with the cosmic order. Another was the conviction, supported by precedents in early classics, that the emperor's astronomers were crucial to his success. A third was the inexhaustible need of a true bureaucracy—a form Chinese invented—for symbols of regularity and control that it could manipulate (especially when it was failing to maintain social order).[25]

Astronomy, astrology and divination were the responsibility of the Grand Astrologer (whose title in most periods remained *T'ai shih).* His staff included experts in all the skills required for all three kinds of tasks. This work gradually ramified into the scope of diverse institutions, sometimes including more than one observatory, more or less independent of each other. On the multiple organizations responsible for astronomy in the Yuan period, see pp. 143–146.

The Politics of Astronomical Reform

If astronomical reform were a straightforward technical task, new systems would have come into being only to resolve crises or to introduce major innovations, and each would have generated significantly more accurate ephemerides than its predecessor. The reality was quite different. Some systems were not significantly new; the Season-granting system's Ming successor (#90) was, as everyone knew, a trivial modification of it. Almost all the systems of the

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[25] The most frequently cited precedent for official astronomy was the "Canon of Yao" at the beginning of the *Documents of Antiquity (Shang shu* "Yao tien 尚書堯典"). On bureaucracy see Lloyd & Sivin 2002, 34–36.

Southern Sung period were minor revisions of a single one (see p. 140). Some systems were noticeably inferior in predictive accuracy to those they replaced (for instance, the immediate successors of the Great Expansion system of 728, #42). More than one excellent system was denied official status due to friction between factions or powerful individuals; the most famous example is the outstanding Great Enlightenment system of 463 (#18), not adopted until a later dynasty.

It is impossible to understand the historical distribution of astronomical treatises without attending to their political circumstances. A government compiles a set of treatises, on astronomy among other topics, for inclusion in the official history of a predecessor, often of the dynasty it has overthrown and replaced. By sponsoring the compilation, it demonstrates that it is a legitimate successor (see below, p. 228).

The tie of astronomical prediction to astrological analysis of phenomena, the bearing on the Mandate of Heaven of events that officials failed to predict, gave astronomy a secure place within the palace. Not all omens were baleful; officials occasionally manipulated good ones to bolster a faltering reign, as well as bad ones to encourage doubt about one.[26]

Astronomical reforms too were emblems of a new cosmic dispensation. Beginning with the Grand Inception era of 104–101 B.C., many reign periods were named after a new astronomical system (in this instance, #2). Renewing a reign was analogous to proclaiming American governmental transitions by such slogans as “the Great Society.” An astronomical reform further advertised a new or renewed mandate. An emperor often ordered up a reform as a gesture, even when he did not make any substantive changes in his policies (see below, p. 68). Over the centuries instituting a new astronomical system became the norm for each dynasty, and sometimes for each newly enthroned emperor. At such times of transi-

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[26] On the need to consider case by case the possibility of manipulation, see Bielenstein 1950 and Bielenstein & Sivin 1977. See Kern 2000 and Lippiello 2001 on the important issue of propitious omens.

tion, it became easy to introduce astronomical novelties in a reform; a very long reign often meant no reform, and no opportunity to innovate.[27]

Quarrels between political factions or interest groups could involve astronomical systems and the officials responsible for them. The most famous such donnybrook was the long campaign (1657–68) against the European-influenced Temporal Pattern system (#96), to protest the Manchu rulers' turning control over palace astronomy to Jesuit missionaries from 1644 on. The attacks on the Europeans resulted in imprisonment for them and execution for some of their Chinese supporters.[28]

The recruitment of astronomical talent was also subject to a political tension. Systems were sometimes repaired piecemeal, but this was usually just a stopgap. If the problem was significant, the normal solution was to replace the system. That is another reason why there were so many.

Who did the innovation come from? The majority of officials who staffed the various astronomical organizations in most periods of history were hereditary practitioners better at trivially adjusting step-by-step computational protocols than breaking through to new techniques. Grand Astrologers were generally an exception, but they were seldom chosen because of their desire for change or their propensity to innovate. Most were non-specialist appointees, expected to run a large organization engaged in routine work, and to minimize unsettling surprises.

Not surprisingly, much of the technical novelty came from outsiders who discovered serious weaknesses in the current system and proposed not minimal adjustments but a new system. Some who did so were officials; others were commoners. But to know as-

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[27] For an instructive example, see Sun Xiaochun 2007, 54–60.

[28] See Huang I-nung 1990a on the Jesuit takeover and 1990b, 1990c, and Hummel 1943–44, 2: 889–92, on the polemic.

tronomy in the first place they were necessarily well educated, and that usually implied a family history of civil service.[29]

The researches of private practitioners kept astronomy vital, but were also a potential threat. A better computational schema could be a priceless symbol of legitimacy in the hands of a potential rebel. It was tempting for rulers to forbid the private study of astronomy, as happened in a few periods of history.[30]

But doing so meant drying up the pool of talent that could rescue a failing official system. It was more profitable to co-opt those whose proposals proved worth pursuing, giving them appointments based partly on their skill and partly on their social standing. But such success too was dangerous, as any position was bound to be that gave one standing in an organization as a reward for proving its staff incompetent. An appointment as Grand Astrologer did not entitle one to replace fifty or a hundred subordinates. As we saw in chapter 1, the authorization of those who planned the Season-granting reform to set up an independent working group was extremely rare, and due to their exceptional access to power.

Mathematical astronomy as well as judicial astrology, as part of the apparatus of power, were politically charged from the start. That accounts for not only the frequency of reforms, but the high level of controversy surrounding them. Equally, the lack of open discord caused by the Season-granting reform was due to the emperor's strong backing of its organizers.

The realities of political organization also shaped the form and content of the astronomical treatises: their technical language; their rhetoric; their systematic arrangement, designed for bureaucrats in a large, highly articulated government; their step-by-step instructions, which presuppose that civil-service computists with limited skills would use them; and their emphasis on eclipses, the most potent early warning of a threatened Mandate of Heaven. Astronomi-

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[29] By "commoner" I mean a *pu-i* 布衣 (literally, one who wore cheap clothing), neither an official nor given special legal status due to membership in an official family.

[30] Such prohibitions were practically never enforced. The main exception was that of the Ming dynasty, discussed in chapter 5, p. 217.

cal systems were part of the technical infrastructure that shaped, reshaped, and maintained the special character of Chinese government.

Observatories

Observatories fitted with dedicated instruments go back over two thousand years. The government built all of those recorded, almost all of them in contemporary capitals. No intact observatory from imperial times survives, but we will see that some instruments from a fifteenth-century installation survive at the Purple Mountain Observatory, Nanjing.[31]

Table 2.2 Most important sites of observation[32]

Name, Ancient	Present	Abbr.	Lat., N	Long., E	UT to LT, hrs
Ch'ang-an 長安 (Han)	Xi'an	XH	34.35	108.88	7.26
Ch'ang-an (T'ang)	Xi'an	X	34.27	108.80	7.25
Ch'ü-fu 曲阜	Qufu	Q	35.53	117.02	7.80
Chien-k'ang 健康	Nanjing	N	32.03	118.78	7.92
Lin-an 臨安	Hangzhou	H	30.25	120.17	8.01
Lo-yang 洛陽	Luoyang	L	34.75	112.47	7.50
P'ing-ch'eng 平城	Datong	D	40.20	113.20	7.55
Pien-liang 汴梁	Kaifeng	K	34.78	114.33	7.62
Ta-tu 大都	Beijing	B	39.92	116.42	7.89
Yang-ch'eng 陽城	Gaocheng	G	34.26	113.02	7.53

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[31] For what archeologists believe is the site of an imperial observatory of A.D. 56, see Chung-kuo She-hui-k'o-hsueh Yuan K'ao-ku Yen-chiu-so 中国社会科学院考古研究所 1978.

[32] I use the abbreviations listed in the third column in tables throughout the book. The sixth column lists the interval between Universal Time and local time in modern hours and hundredths. The locations of the ancient capitals at which observers recorded celestial phenomena are taken from Stephenson & Houlden 1986, xii, Watanabe 1979, 285, and Steele 2000, 165. Like these sources, I give their geographical coordinates in degrees, not *tu*. The coordinates for Kao-ch'eng are from Tung Tso-pin 董作賓 et al. 1939, 6.

The sites that yielded the richest records of celestial events were dynastic capitals. Observations sometimes came from places far from the capital, as computation reveals, but in many cases local time was recorded under the assumption that it was the same as time at the capital (there were no time zones in ancient China). Ritualists asserted from time to time that, according to the ancient classics, the obligatory place for observations was Yang-ch'eng 陽城, the "center of the world" (present Gaocheng, Dengfeng county, Henan province 河南登封县告城), where the sun's shadow at the summer solstice was supposed to be 1.5 *ch'ih* (38cm) long. Various instruments were installed there from time to time, but it was not a consistent source of data.[33]

Mathematical Tools

Computation

Chinese in 1280 depended on computing devices, namely sets of small rods. They were prevalent for more than a millennium before the fourteenth century, when the abacus came into widespread use. One manipulated these computing rods in configurations that stood for numbers. One could arrange a small set of these rods in the squares of a grid—actual or imaginary—to stand for the coefficients of unknowns in any numerical equation, and rearrange them to stand for the results. The abacus that replaced it was a one-dimensional row of beads that could represent a single number, or (informally) a couple of them side by side. But counting rods, set out in a two-dimensional array, were able to represent numbers horizontally, vertically, or even diagonally. Rods arranged in a row of squares could stand for decimal numbers; a 3, a 2, and a 7 stood for 327. When the text directs the computist to "shift the result

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[33] For a detailed discussion see Needham et al. 1954–, 3: 284–91 and Tung Tso-pin et al. 1939. The claim that Yang-ch'eng was the center of the world *(ti chung* 地中) appears in the *Chou li* 周禮, 10: 5a. For additional information on the site see the Yuan source *Ho-shuo fang-ku chi* 河朔仿古記, 3: 5b–6a.

backward two columns," as in the Canon, step 6.6, the result is to move the horizontally arrayed number up two places in the powers of ten, effectively multiplying it by 100, so that the result in this case would be 32 700.[34]

The number system

The basis of Chinese numbers, as far back as they were recorded, was decimal. There were special characters for the numbers from 1 to 9, and others for powers of ten—tens, hundreds, thousands, and so forth.

In everyday writing it was possible to record any number unambiguously. Thus one could write 40070 as *"ssu wan ch'i shih* 四萬七十," literally "4 ten-thousands 7 tens." People grouped the digits of large numbers not in thousands but in *wan* 萬, ten thousands. Thus Chinese wrote not 210 423 624 but the equivalent of 2 1042 3624 (2 *wan wan* 1042 *wan* 3624).

Mathematicians could write numbers so large that they had no practical application, but there was not one standard system for doing so. Books on mathematics used much more compact forms of notation, with only digits, to represent the placement of rods on a grid. From the fifth century A.D. some texts leave blank spaces for zeroes. Since miscopying easily led to the wrong number of blanks, from about the sixth century on mathematicians used dots, circles, or squares to unambiguously mark such spaces. It was probably not until the thirteenth century that a circle became the standard symbol for the purpose in China (a circle as zero appears in Sumatra and Cambodia in Sanskrit inscriptions in A.D. 683).[35]

There was no standard practice through history for rounding off numbers. The Yuan astronomers rounded fractions down (see the Evaluation, pp. 261 and 275), but observed separate conventions for time expressions (e.g., p. 343).

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[34] On counting rods see Lam & Ang 1992, 20–27. Martzloff makes a convincing case that their use did not depend on a prepared board of any kind (1997, 209–11).

[35] Martzloff 1997, 204–8, for a detailed account with examples.

Fractions

Chinese ordinarily expressed fractions of the type 11/12 using the form "12 *fen chih* 11 (*shih-erh fen chih shih-i* 十二分之十一)," literally, "of 12 parts, 11." In this example, 11 would have been called the dividend or numerator (*shih* 實), and 12 the divisor or denominator (*fa* 法).

This form was simple and consistent, but in astronomical calculation it sometimes led to notation that can confuse modern readers. There were two peculiarities, the method adopted early for noting compound fractions, and, as an outcome of that, the consistent use of a single denominator for all of them.

When doing rod arithmetic, a quantity such as 29 43/81, days in the mean lunar month in the first complete astronomical system, is not at all convenient to manipulate. The simplest solution is to convert it to a compound fraction, in this instance 2392/81, by multiplying the whole number by the denominator of the fraction and adding the result to the numerator. Astronomical systems then usually recorded it as two separate quantities, the new numerator as the Month Factor (*yueh fa* 月法, 2392), and the denominator as the Day Denominator (*jih-fa* 日法, 81). They could then use both quantities independently in various computations. This usage easily confuses someone simply looking for a constant equal to 29 and a fraction. The same astronomers recorded the value adopted for day length as two other quantities, the Celestial Revolution Constant (*chou t'ien* 周天), 562 120, and the Concordance Cycle (*t'ung* or *t'ung-fa* 統法), 1539. Dividing the first by the second, as the system stipulates, yields about 365¼ days.[36]

Constants

Li Ch'un-feng 李淳風 (602–70), author of the Chimera Virtue system (#38, 665), considerably reduced the number of constants by adopting a single denominator for all fractions. "The old systems lacked uniformity, using Rule Cycles and Obscuration Cycles, Ep-

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[36] This example comes from the Triple Concordance system (#3), in *Han shu*, 21B: 991.

och Cycles and Era Cycles, Day Parts and Degree Parts; Li Ch'unfeng unified them all with his General Denominator *(tsung fa* 總法)." This factor, 1034, was not a power of 10. Its use as the denominator of all fractions greatly simplified their manipulation. Li set down his tropical year constant as 489 428, which is the equivalent of 489428/1034 = 365¼ days, and recorded his synodic month constant as 39 571, amounting to 39571/1034 = 29.5306 days. Some of his successors followed him in using a single divisor, but made neither its value nor its nomenclature uniform.[37]

It was inevitable that the universal denominator be a large number, since it had to be a common denominator for the diverse fractional values on which each system was based. Even very slight changes in the fractional parts of constants would require a different value for the universal denominator. Its remaining unchanged from one system to another usually signaled that the new one was borrowing from its predecessor rather than undertaking new measurements. Furthermore, as astronomers determined constants with greater precision, the common denominator was almost certain to become much greater—or else limiting the size of the General Denominator would compromise advances in precision.

The Yuan reformers went far beyond Li's modification, expressing all mixed numbers in decimal notation. This was not quite a radical step; it simply amounted to adopting a common denominator of 10 000 (called the Day Cycle in the Canon, 1.0). In practice, this change did away with much unnecessary complexity, ending the old fluctuation from system to system, and considerably simplifying calendrical calculation.[38]

Arithmetic

There are several different terms for the ordinary logistical operations, chosen depending on their circumstances or how their result was to be used.

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[37] *Hsin T'ang shu,* 26: 559–60; see also the discussion in Ang 1979, 79 and the discussion of epochs below, p. 73.

[38] Evaluation, section 11, and my comment on it; also p. 592 below.

"Addition" is simply *chia* 加.

"Subtract" is uniformly *chien* 減. The form "x *yü* y *hsiang chien (x* 與 *y* 相減)," can be confusing if one tries to apply the dictionary sense "each other" to *hsiang.* The phrase does not mean "subtract *x* and *y* from each other." As in the Canon, 7.14, it implies subtraction that may become scalar addition if the two quantities are of different signs. I maintain the ambiguity by translating the phrase "perform subtraction with."

For division, there are four common terms, complementary in their usage. "*Yueh* 約 (simplify)" is the ordinary term for the operation, but the Yuan treatise uses it for reducing a quantity to a larger metrological unit, as when one divides by 10 000 to convert day parts (ten-thousandths of a day) to days. *Shou* 收, literally "collect," refers to taking the fractional part expressed in one unit and dividing it by a factor that converts it to a larger—or simply a different—unit. An example in the Canon, section 2.4, divides day parts by 1200 to convert it to marks (hundredths of a day). A formula such as "Count a unit for each full *n (man* n *erh i* 滿 *n* 而一, or *ju* n *erh i* 如 *n* 而一)" designates a division by *n* from which one will use only the integral result. The form "cast out each full *n (man* n *ch'ü chih* 滿 *n* 去之)" refers to one in which one will use only the fractional remainder.

Multiplication also uses more than one term. The everyday word is *ch'eng* 乘, but in technical writing (e.g., in the Canon, 7.11) *yin* 因 appears with the same meaning.

Raising a quantity to a higher power is stated in the form "multiply by itself," if necessary more than once. Taking a square root has its own term, "*k'ai ping fang* 開平方," which appears inverted in the Canon, step 6.15, as "*p'ing fang k'ai chih* 平方開之, take [literally, 'open'] its square root."[39]

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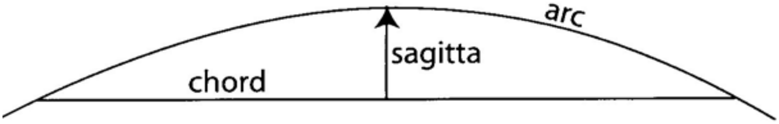
[39] The general discussion of computational terminology in Libbrecht 1973, 83–87 is based primarily on a source only a generation earlier than the Yuan reform.

Trigonometry

The familiar trigonometric functions—sine, cosine, tangent, and so on—did not exist in the ancient world. Sines and cosines emerged first in India in the sixth century, and the rest in Islam from the ninth century on. The first monograph on the subject appeared only in the thirteenth century, in Mongol Persia, probably not long before the Season-granting reform.[40]

Classical astronomers such as Hipparchus (fl. 147–127 B.C.) and Ptolemy, of course, had to solve numerical problems involving plane and spherical angles. Hipparchus discovered, for plane figures, that it was possible to do so by finding the relation between an arc, its chord, and its sagitta; that is, between the belly of an archer's bow, its stretched, straight bowstring, and an arrow just long enough to fill the space between string and belly at right angles to the string.



A table of chords such as the one extant in the *Almagest* can be used to solve a range of problems that involve right triangles. Formally speaking, this approach is related to trigonometric functions, a much later development, in a simple way, since the sine is half the chord of twice the arc, and the other functions are related to the sine. The simplicity of that relationship was not at all apparent until Indian and Islamic mathematicians worked out theories of trigonometric functions. Another predecessor of Ptolemy, Menelaus (fl. A.D. 98), clarified the corresponding arc-sagitta relations on a sphere.[41]

Early Chinese astronomy was a great deal less concerned with geometric relationships, and much more with temporal cycles, so

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[40] A book in draft by Glen Van Brummelen will give details of these developments.

[41] Pederson 1974, 56–78, gives a clear explanation of this development, and conveys the complicated problem-solving that went into Ptolemy's chord table.

exploring the relation of arcs and chords was less urgent. It was not until the late eleventh century that Shen Kua 沈括 (or K'uo, 1031–95) worked out a general formula for chords of two-dimensional arcs. The authors of the Season-granting system made the step to spherical solutions. The reform group, seeking an accurate conversion of ecliptic to equatorial coordinates, worked out a close approximation for a triangle with one spherical side. As J.–C. Martzloff has demonstrated, their seemingly eccentric use of 3 for π made possible considerably more accurate results than if they had chosen the precise value of the constant already available in China for eight centuries ($\pi = 355/113 = 3.141\,593$).[42]

Metrology

This section will outline the pertinent measures of space and time, including the elements of chronology.

Spatial Units

Early astronomical systems used various schemata of fractions, usually complicated, in metrology. We have seen that the Yuan system innovated by moving to decimal units. The basic unit of length was the *ch'ih* 尺. In the Yuan period the *ch'ih* was officially the equivalent of 30.72cm, although there was much variation among localities and trades. For several centuries there had been a special system of measures for ritual purposes, in which a *ch'ih* amounted to 24.525cm. Because astronomical institutions were regularly sub-

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[42] For a concise and clear explanation of Shen's work and that incorporated in the Season-granting reform, see Tu Shih-jan 杜石然 1993, 161–65. Martzloff's more detailed discussion (1997, 328–35) is the only more or less up-to-date account of Chinese arc-sagitta relations in a Western language. Unfortunately, the English translator of this book renders *shih* 矢 "arrow," unaware that the standard term in the history of mathematics is "sagitta." As Martzloff points out, the compilers of the *Yuan History* omitted the explanation, diagrams, and tables for the Difference between the Yellow and Red Ways (*huang ch'ih tao ch'a* 黄赤道差), but the editors of the *Ming History,* realizing their importance, incorporated them (*Ming shih,* 32, 551–82). The history of arc-sagitta studies between the late eleventh century and the Yuan period has yet to be adequately explored. I am grateful to Donald B. Wagner for discussions of the π problem.

ordinate to the Ministry of Rites, their work used this system. The Season-granting reform adopted it, even for the dimensions of the observatory.[43]

Although these measures were decimal, they were not part of a pure place-value system. Decimal fractions were expressed, not in *ch'ih,* but in mixed units. There was even a special measure, the *chang* 丈, for 10 *ch'ih.* Thus the first gnomon shadow length measurement in the Evaluation, section 1.1, amounts to 79.4855 *ch'ih.* This is formally recorded as *ch'i chang chiu ch'ih ssu ts'un pa fen wu li wu hao* 七丈九尺四寸八分五釐五毫, that is, 7 *chang* 9 *ch'ih* 4 *ts'un* 8 *fen* 5 *li* 5 *hao.* This nomenclature is entirely consistent, descending from the first power of 10 to the minus fourth power. The Canon sets down the value just given as 79.4855c.

Chronology

Documents before the unification of China were normally dated with the year in the reign of the local ruler, which made endless trouble for people investigating past records. When the empire began, annalists counted the years of a reign from the accession of each monarch. Thus 200 B.C. was the 7th year of the High Progenitor *(Kao-tsu* 高祖) of the Han dynasty.

Earlier, two local rulers had symbolically renewed their reigns by stopping the count midway and beginning a new one. In 163 B.C., a Han emperor did the same thing, but marked the new era with a new name, "Later Epoch *(hou yuan* 後元)." This was so impressive a way of asserting a renewed mandate that his successor did it twice. The annals of the next ruler, the Martial Emperor (Wu ti 武帝, r. 140–87), began with a propitiously named reign, "Established Epoch," and instituted new eras ten times in the 54 years he ruled. Some monarchs proclaimed new eras annually, or to celebrate a victory or a propitious portent, although many made no change at all. In fact, the rulers of the two dynasties that succeeded the Yuan did not change their initial era names.[44]

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[43] I Shih-t'ung 1978; Wang Chien-min 王建民 1982.

[44] Wilkinson 2000, 181–82, gives an excellent outline.

The formation of the empire meant a single ruler, and thus a single year count. There remained a great many uncertainties, particularly when one emperor replaced another, but an inquirer armed with tables of reigns and eras can—most of the time—be sure what a given date means.[45]

The earliest extant documents, from the mid second millennium B.C., used a cycle of 60 to count off days. Scribes labeled them by pairing off two series of characters employed for numbering things in order, the so-called ten "heavenly stems *(t'ien-kan* 天干)" and 12 "terrestrial branches (*ti-chih* 地支)." The short term for the two series together was "stems and branches *(kan-chih* 干支)." Ten or twelve days was too short a cycle for keeping track of extended periods of time, but the cycle of 60 pairs did nicely.

Table 2.3. Sexagenary Series

	0	10	20	30	40	50
1	I-A 甲子	I-K 甲戌	I-I 甲申	I-G 甲午	I-E 甲辰	I-C 甲寅
2	II-B 乙丑	II-L 乙亥	II-J 乙酉	II-H 乙未	II-F 乙巳	II-D 乙卯
3	III-C 丙寅	III-A 丙子	III-K 丙戌	III-I 丙申	III-G 丙午	III-E 丙辰
4	IV-D 丁卯	IV-B 丁丑	IV-L 丁亥	IV-J 丁酉	IV-H 丁未	IV-F 丁巳
5	V-E 戊辰	V-C 戊寅	V-A 戊子	V-K 戊戌	V-I 戊申	V-G 戊午
6	VI-F 己巳	VI-D 己卯	VI-B 己丑	VI-L 己亥	VI-J 己酉	VI-H 己未
7	VII-G 庚午	VII-E 庚辰	VII-C 庚寅	VII-A 庚子	VII-K 庚戌	VII-I 庚申
8	VIII-H 辛未	VIII-F 辛巳	VIII-D 辛卯	VIII-B 辛丑	VIII-L 辛亥	VIII-J 辛酉
9	IX-I 壬申	IX-G 壬午	IX-E 壬辰	IX-C 壬寅	IX-A 壬子	IX-K 壬戌
10	X-J 癸酉	X-H 癸未	X-F 癸巳	X-D 癸卯	X-B 癸丑	X-L癸亥

A good analogue would be pairing ten roman numerals (I to X) and twelve letters of the alphabet (A to L; see table 2.3). If we start reading downward with 01 as I-A, 02 as II-B, the tenth in the series is X-J. Because each series must eventually begin again, the eleventh through fourteenth are I-K, II-L, III-A, IV-B. The 60th in the series is

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[45] Chang P'ei-yü 张培瑜 1997 makes earlier chronologies obsolete. The most detailed reference work for reign and era titles, including obscure transitional ones, is Li Ch'ung-chih 李崇智 1981.

X-L. At that point the cycle ends and must begin again; the next member in sequence is I-A.

By the beginning of the empire in the third century B.C., there was a single chronology for all the imperial domains. Scribes began using the stem-branch series for years as well. At the same time, those who study imperial China do not need elaborate tables to compute the interval between two dates; one can count off sexagenary cycles in one's head or on one's fingers.

There is some uncertainty about whether the day count has remained uninterrupted since the oldest written records, circa 1450 B.C. What matters from our point of view is that since the earliest comprehensive system of astronomy, circa A.D. 1 (#3), both the year and day counts are unbroken.[46]

As the translation shows, the Season-granting treatise is typical in employing both the reign year and the sexagenary system: it uses one or the other when that does not cause ambiguity, or both to avoid it when necessary, and occasionally as a matter of prescribed form.

Date Reckoning

In the earliest known Chinese practice (as in that of Islam today), the month began at the first lunar crescent, and the day at sunrise or some point near it. The first crescent is easy to see—despite the inventiveness with which various peoples have complicated its definition—but it is hard to foretell. By the seventh century B.C., Chinese no longer based their conventions on visible events. For the royal government and the local courts that followed its lead in ritual matters, the year began on its shortest day, the month on a day

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[46] For mental computation of sexagenary dates, see Sivin 1966a. As Huang I-nung 1992 has shown, there are many discrepancies between datings prescribed in such official sources as the Standard Histories and those that appear in contemporary documents. This does not compromise the usefulness of court records for the study of government astronomical activities, but merely demonstrates, once again, the limited reach of China's centralized government.

when no moon is visible, and the day halfway between dusk and dawn.[47]

From at least the third century B.C. on, the evolving capabilities of astronomy made more sophisticated definitions possible:

1. the day began at midnight, usually determined as the midpoint between two noons. Noon was the moment when the sun cast the longest shadow of the day.

2. the astronomical month began with the conjunction of sun and moon, or lunation. Since this was not an observable event, it was necessarily calculated. In the Quarter Remainder system (#4, A.D. 85), the mean synodic month constant—the average length of a month—was 29 499/940 days, for reasons that will become clear below.

3. the astronomical year began with the winter solstice. The year in early systems was thus the average interval between two winter solstices, the mean tropical year. In the Quarter Remainder system, this quantity was 365¼ days.[48]

For the civil calendar on which administrative record-keeping depended, these cycles were unworkable. If we start this year at midnight, and count off 365¼ days, next year will have to begin at 6am; a month from midnight is not another midnight, but a little after noon—29 499/940 days later. For the purposes of everyday life, people have to count civil months and years as whole numbers of

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[47] Chang P'ei-yü 1997: "Conventions *(fan-li* 凡例*)*," p. 1.

[48] The European tradition began the year with the vernal equinox. This was to some extent a fundamental philosophical difference. For the Greeks, the equinox began the spring, and thus implied rebirth. The Chinese put the equinox in the middle of the spring, and thought of rebirth beginning at the solstice, when yin predominance had gone as far as it could, and yang began to reassert itself. The difference was also practical; the early ritual importance of the gnomon in China made the solstices, when the longest and shortest noon shadows of the year appear, natural choices. Furthermore, the large role of the planets in geometrically-oriented Western doctrine made their latitudes important; the zodiac, centered on the ecliptic (the solar orbit), was a band about 12° wide that included them. An equinox, one of its intersections with the equator, was an obvious place to begin the count. In China, where there was no concept of planetary latitude, that was not a natural choice at all.

days. Early peoples did this by alternate counts that averaged out in the long run. For example, counting three years of 365 days and then one of 366 days would yield an average of 365¼ days. But as later systems slightly altered their definitions of constants—in the Grand Inception system of circa A.D. 1 (#3), year length became 365 385/1539 days—the schemes of alternation became more complicated. The general strategy was to define

4. the civil month as 29 or 30 days, strictly alternating in the earliest calendars, although two of one or the other in sequence soon became acceptable, and

5. the civil year as twelve or thirteen months.

A further step was needed, however. Twelve such months would be 354 days long—well short of the astronomical year. A year of thirteen months, on the other hand, would be 383 or 384 days, far too long.

An intercalation rule was therefore necessary in order to add (that is, intercalate) just enough thirteen-month years to make the long-term average equal the length of the astronomical year. The rule used until A.D. 412 (as well as in Greece earlier, where it was called the cycle of Meton, or Metonic cycle) was to add seven extra "intercalary months *(jun-yueh* 閏月)" at more or less equal intervals every nineteen years. Thus the nineteen-year cycle contained (19 × 12) + 7, or 235 months. One astronomical month was thus 19/235 astronomical years, and, 365 ¼ days/year × 19/235 years/months = 29 499/940 days/month. In other words, the unkempt fraction was an artifact of the Metonic cycle itself. Once practitioners fixed the length of an astronomical year, the intercalation cycle determined the long-term mean month length.

Once astronomers widened their purview to include eclipses, planetary conjunctions, occultations, and so on, the cycles on which they depended became more and more complicated—and more likely to demand revision as time passed and small errors in their constants added up. That was, in fact, a fundamental pattern of the history of astronomy in China.[49]

This juggling of months and years to avoid fractions made life easier for bureaucrats and merchants, but impossible for astronomers. They therefore tended to rely on conventions of their own, discussed below.

Cycles

One can find every conception of time in every culture: cyclical, progressive, regressive. Each has its purpose and its applications. In astronomy, the character of time was unambiguously cyclical.[50]

An astronomical system assembled a large array of constants for the sun, moon, and planets. They represented diverse cycles—synodic, sidereal, and others—that constituted the basic rhythms of the universe. The Chinese cosmos was not created (leaving aside a few myths that did not affect astronomy). Nevertheless, since its main dimension was time, each cycle had to have a beginning. Computational systems counted off all cycles from a single initial point. But one needs to know more than this mathematical custom to understand how astronomers conceived their work.

The matrix of cosmic process was a cycle of very great length, which began and ended at what the Yuan treatise calls the Extended Era Superior Epoch *(yen chi shang yuan* 演積上元, one of its many names). From it astronomers counted off all of the computational cycles. The Evaluation writes of its inception, at which the sun, moon, and planets were conjoined "in the same degree" (a vaguer phrase then it seems; section 11, page 371).

The Superior Epoch approach was metaphysically attractive as a starting point for a great cosmic cycle, so that all celestial phenomena would have a common origin in the distant past. An unforeseen result was that the interval between this universal conjunction and the time of computation sometimes increased through history from

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[49] On early systems of cycles and their eventual breakdown see Sivin 1969.

[50] Needham 1965, Sivin 1966b. By regressive I mean time in which the general moral trend was downward, the opposite of progressive.

one system to the next, by up to several orders of magnitude. This is because the length of the period between two universal conjunctions was the least common denominator of all the subsidiary constant periods. As knowledge of solar, lunar, and planetary periods became more precise, the divisors of the fractional remainders tended to become larger. Their least common divisor, which defined the interval between two universal conjunctions and thus the great cycle, accordingly increased at a yet greater rate.

To take a simple example, the Triple Concordance system (#3, A.D. 1/5), in order to reconcile the denominators of all its constants, defined the length of the great cycle between Superior Epochs as 639,040 years. The Quarter-remainder system (#4, 85) required more than 2 x 10^{18} years for the same purpose. This quantity quickly became stupendous in length; for the sake of practicality, the focus shifted to an intermediate (that is, a recent) epoch.[51]

The event that began an intermediate epoch was not a universal conjunction, but was related to one in a simple way. One could then begin the new system's procedures with Accumulated Years, the interval from the intermediate epoch to the year that began the new system. Fiddling with the divisors of fractions in constants—"calculating back and forth," as the Evaluation circumspectly puts it—was another way to pare down the interval from the Superior Epoch (or intermediate epoch) to the period in which astronomers were applying the system.

Rejection of Superior Epochs

The author of the Evaluation considered the Season-granting system's rejection of this complicated apparatus a most desirable innovation. A step taken in the seventh century paved the way. In his Fifteenth-Year Epoch system (#14, 619), "Fu Jen-chün originally took 618 as the beginning point of his system, and for *ch'i*, lunations, and cycles of inequality and eclipse cycles, used correction factors to be added or subtracted. But from [626] on, calculations were based on accumulated years from a Superior Epoch." In other

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[51] See Sivin 1969, 17–22, for a full discussion.

words, this useful simplification caused such discomfort that Fu's opponents got rid of it after a mere eight years.[52]

The Season-Granting system made a final break with the tradition of endless adjustments, pointing out that the simple cyclical model that had justified them was irrelevant to computational practice. The Yuan system, like that of Fu, set its epoch in the period when the system was to be used. That remained the norm until the end of imperial China. The reformers ostentatiously rejected the universal denominator, unique to each system, that had been conventional for centuries. Section 11 of the Evaluation is devoted to this change.

Years

Determining the winter solstice, the point that the Chinese chose to begin the tropical year, poses special difficulties. It is an ideal choice from the points of view of philosophic and poetic cosmology, for it symbolizes—and not only in China—the low point of cosmic vitality, the seasonal moment at which nature dies and is reborn. But from the viewpoint of mathematical astronomy, this choice poses special difficulties. The noon shadow of the gnomon at the winter solstice is the longest of the year. At that season it changes very little from day to day, so little that when measuring the shadow's length, one cannot be certain even on which day the longest falls, much less the time of day. Nakayama has estimated that, relying on solstitial shadow readings of a gnomon two meters (roughly 8c) high, a mistake of 1cm in reading shadow length would result in an error of four or five days for the winter solstice and about eight days for the summer solstice, when the noon shadow is much shorter (1969, 242–44). Since the shadow of a gnomon the height of a man is markedly fuzzy, an accuracy of 1cm is not easy to attain.[53]

On top of this technical difficulty the state superimposed another one. Among the gestures early dynasties invented to signal a change of mandate—a decisive new deal—was a shift in the first

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[52] *Hsin T'ang shu* 新唐書, 25: 536.

[53] Nakayama 1969: 243; see also Bruin & Bruin 1977.

month of the civil year. To make a long story short, the final such shift, in 103 B.C., began the civil new year in the second lunar month after the one that contained the winter solstice. Even in the lunar calendar used informally today, the new year is still defined in this indirect way by the solstice. Astronomers, in order to avoid the complication that such shifts entail, used their own year. This "astronomical year" differed from the civil year in that its Astronomical First Month *(t'ien-cheng* 天正, literally "epochal month according to the sky") remained the one that contains the solstice. The Astronomical First Month occurs often in the Yuan treatise.

As the Evaluation, section 1.0, mentions, it was Tsu Ch'ung-chih 祖沖之 (ca. 430–510) (#18) who, circa 463, solved the problem of determining accurately the time of the winter solstice. His solution involves measuring the difference in the noon shadow length on two consecutive days well before the solstice, finding a day after the solstice in which the measurement falls between those two, and performing linear interpolation (of the kind demonstrated in the Evaluation), to determine a date and time. The Yuan astronomers further improved the results by using the Tall Gnomon with its Shadow Aligner to see a clearly defined shadow (p. 187).

Months

The lunar month, the interval between two conjunctions of the sun and moon, presented troublesome problems because the motions of both bodies varied in speed. That of the moon was particularly irregular. It took some time to move from a reliance on mean conjunctions—which considerably limited the accuracy of prediction—to adopting a reliable method for apparent ones.

One cannot verify the time of the conjunction, unlike that of most predictable events, by observation. The moon is normally invisible for a whole night, or frequently longer. It took until the late fourth century A.D. for astronomers in China to realize that one may use the maximal phase of a solar eclipse to specify closely the time of conjunction.[54]

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[54] See the Evaluation, section 4. Parallax can cause a small but noticeable discrepancy between maximal phase and conjunction. Benno van Dalen

As section 10.0 of the Evaluation explains in detail (p. 367), the move in calculation from the mean lunation to the apparent one was very gradual. The Supernal Emblem system of A.D. 206 (#5) was the first to seek the inequality of the moon's motion. Ho Ch'eng-t'ien 何承天 worked out a method for the apparent conjunction in the Epochal Excellence system (#17, mid fifth century), but it did not become the norm until Li Ch'un-feng's 李淳風 First-year Epoch system (#38) in 665. This change required a serious and unsettling reorientation.

The apparent lunation meant, as Fu put it, that even though "there may be three long or three short months [in sequence], solar eclipses will always fall on the first of the month and lunar eclipses on the 'full moon' [i.e., the 15th or 16th]," and that "once one determines the first of the month by applying the lunar slackening and hastening, the moon as it moves will no longer appear in the east on the last day of the month or in the west on the first."[55] In other words, once one determines the first day of the month by computing the apparent conjunction, the alternation of long and short months ceases to be even approximately regular. When astronomers who had built reputations on inventing schemes to keep it regular were assured that it no longer mattered, one could hardly expect them to be mollified.

On the other hand, from the viewpoint of practical chronology, there was something to be said for computing with the mean lunation. Using it meant that computations over a long time span from the epoch would be simple and reproducible. It would avoid the cumulative error inherent in the methods used to compute the apparent lunation, since they depended on approximations. This point became increasingly apparent as arguments about the apparent lunation continued for centuries, and as astronomers tested computational methods against records of past phenomena. As several

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has estimated that the time from the last crescent moon until apparent conjunction varies between 18.5 and 68.5 hours (email, 2006.12.14).

[55] *Hsin T'ang shu,* 25: 536, 27A: 597. "The lunar slackening and hastening" is the lunar anomaly.

schemes for calculating the apparent lunation succeeded each other over centuries, someone reconstructing past computations could seldom be certain when to shift from one to the other. It was also impossible to keep track of irregular time intervals that had been calculated by a variety of techniques, some inadequately known. The result was bound to be confusion.

Ho Ch'eng-t'ien incorporated the solar inequality in the theory of conjunctions (*Sung shu* 宋書, 13: 277). As the Evaluation's discussion makes clear, he failed to convince his fellow astronomers that there was a better way to minimize conjunctions on the last or second day of the month than the habitual alternation of long and short months. The old practice was not in fact a strict alternation, which would have yielded an average month length of exactly 29½ days. Even the earliest recorded calendrical schemas added extra 30-day months—i.e., extra thirtieth days—at intervals to make the long-term average approximate the mean lunar month (29.53059 days). This was essential in order for the calendar to predict conjunctions even approximately. But conventional astronomers drew the line at a succession of three or more long months (which had to be balanced sooner or later by additional short months). They found it esthetically unacceptable.

The methods of Liu Hsiao-sun 劉孝孫 (#29, 576) and Liu Cho 劉灼 (#34, 608) for predicting apparent lunations seemed so radical that experts who insisted on classical precedent refused to accept the latter's Sovereign Pole system. Their most clamorous critic was Chang Chou-hsuan 張冑玄 (circa 526–circa 612). He designed the Great Patrimony system (#35) that replaced Liu Cho's system in the same year. Fu Jen-chün's enterprising Fifteenth-year Epoch system (#36) became official in 619, but had to be successively revised to cut out techniques that seemed to be too advanced. The more or less regular alternation of mean lunations disappeared from the ephemeris only in 663. The change became permanent after the adoption of Li Ch'un-feng's system in 665. The pattern of month lengths beginning in the General Simplicity era, year 2, month 10 (7 Nov 671) was 30–30–29–30–30–29–29–29. Four long months in sequence first occur in the Penetration to Heaven era, year 2, months 8–12 (beginning 22 August 697). Using the apparent lunation was no longer audacious.[56]

Li was able to take the step to apparent conjunctions once and for all because astronomers had greatly improved their craft. They could cope by then with the chronological problems that the apparent lunation raised—and no doubt generational change had helped to lighten the old anxieties. For the short run, Li's technique of "advancing the new moon" was a compromise that minimized what classical-minded astronomers considered anomalous appearances of the first and last crescent (see the Evaluation, p. 368). "Advance," *chin* 進, refers to entering the conjunction in the ephemeris one day later, so that the month it begins is one day shorter. Li's instructions on this point are missing from the versions of his system in the two T'ang histories. A textual note merely remarks that maximal values of the lunar equation of center at conjunction "ought not to recur more than three times in a row; if they exceed this frequency, consider whether the Corrected [i.e., Apparent] Minor Surplus places [the conjunction] near midnight."[57]

Ch'i

The *ch'i* 氣 were twelve equal divisions of the tropical year, a solar counterpart of the month, each roughly 365.25/12 or 30.4 days long. Since the growing seasons are solar rather than lunar phenomena—folklore to the contrary notwithstanding—lunar months with their intercalations are useless for tracking seasonal change. Chinese further divided each *ch'i* into halves named nodal *ch'i (chieh-ch'i* 節氣, just after the transition or "node" that divided one *ch'i* from the next) and medial *ch'i (chung-ch'i* 中氣, between the nodes), so that the usual name for the system is the "24 *ch'i*." The traditional names (table 2.4) provide an accurate description of the yearly climatic cycle in rural north central China. That was no

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[56] Yabuuchi 1944a: 29–30.

[57] *Hsin T'ang shu,* 26: 560–83, esp. p. 567; Sung version in *Sung Shih* 宋史, 72: 1640. The equation of center is the difference between apparent and mean positions.

doubt adequate for the local court in which the cycle originated before the mid third century B.C., but the descriptions were irrelevant to most of the enormous Yuan empire.[58]

Table 2.4. The 24 *Ch'i* Divisions of the Tropical Year

Name	**Translation**	***Ch'i***	**Date**
Li ch'un立春	Enthronement of spring	1 N	28 Jan
Yü shui 雨水	Rainwater	1 M	-
Ching chih 驚蟄	Excited Insects	2 N	-
Ch'un fen 春分	Vernal Division	2 M	13 Mar
Ch'ing ming 清明	Pure and Bright	3 N	-
Ku yü 穀雨	Grain Rains	3 M	-
Li hsia 立夏	Enthronement of summer	4 N	28 Apr
Hsiao Man 小滿	Grain Small but Full	4 M	-
Mang chung 芒種	Bearded Grain	5 N	-
Hsia Chih 夏至	Summer Solstice	5 M	14 Jun
Hsiao Shu 小暑	Lesser Heat	6 N	-
Ta Shu 大暑	Greater Heat	6 M	-
Li ch'iu 立秋	Enthronement of Autumn	7 N	31 Jul
Ch'u shu 處暑	Abiding heat	7 M	-
Pai lu 白露	Hoarfrost	8 N	-
Ch'iu fen 秋分	Autumnal Division	8 M	15 Sep
Han lu 寒露	Cold Dew	9 N	-
Shuang chiang 霜降	Frost Settles	9 M	-
Li tung 立冬	Enthronement of Winter	10 N	30 Oct
Hsiao hsueh 小雪	Lesser Snow	10 M	-
Ta hsueh 大雪	Greater Snow	11 N	-
Tung chih 冬至	Winter Solstice	11 M	14 Dec
Hsiao han 小寒	Lesser cold	12 N	-
Ta han 大寒	Greater cold	12 M	-

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[58] This table lists the 24 *ch'i* in the conventional order, beginning at the civil new year. The third column gives the lunar month in which each most commonly occurs, with N to designate the nodal *ch'i* and M the medial *ch'i*. The fourth gives some of the corresponding Julian dates in 1280; dates vary considerably from year to year.

For intercalation, a subsidiary rule, usually mentioned in treatises because it was classical—they did not necessarily use it—stipulated that a month that did not contain a medial *ch'i* became an intercalary month (e.g., Canon, 4.6 and figure 29). This was another way to achieve the proper spacing.

Some post-classical sources further subdivided each *ch'i* into six equal smaller periods. The 72 phase *(hou* 候) subdivisions of the tropical year derive from seasonal indicators grouped informally through the "Monthly Ordinances" chapters of *Springs and autumns of Master Lü (Lü shih ch'un-ch'iu* 呂氏春秋, 239–235 B.C.) and later in the similar chapters of the *Records of Rites* classic *(Li chi,* "Yueh ling" 禮記月令, by A.D. 100). The *ch'i* and *hou* first appear together for astronomical purposes in the "Treatise on Mathematical Harmonics and Astronomy" in the *Standard History of the Wei,* which reflects astronomical practice in the mid third century A.D., and in the Canon, section 2.2.[59]

These phases represent a finely graded round of the seasons. The congruence of the phase system with yin-yang thought is clear, for instance, in the appearance and growth of yang functions immediately after the winter solstice (the maximum of yin) and in the many symmetries of the seventy-two categories. The correlation of *ch'i* and lunar months in table 2.4 and below is loose, since a "solar month" of nodal and medial *ch'i* is longer than a mean lunar month, and intercalation frequently upsets the pairing. The eleventh civil month can begin on the winter solstice or up to twenty-nine days earlier.

Days

The Canon of the Season-Granting system (4.6) mentions shifting the beginning of an apparent lunar month that falls before sunrise to the previous day. This does not imply that the day officially began at sunrise. The new day formally began at midnight (the mid-

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[59] Re the Monthly Ordinances, see the translations in Legge 1861-72, volume 4, and Knoblock & Riegel 2000, sections 1–12 , and the text in *Wei shu* 魏書, 107B: 2716–18. Compare the listing of *ch'i* and phase Intervals (with much minor variation) in the Canon, section 2.2.

dle, not the beginning, of the first duodenary double-hour), but even today one says in Chinese as in English "there will be an eclipse tonight" although it is expected at 0400 the next calendar day.

Units of less than a day

The vast extent of mainland China stretches across five time zones from 73° to 134°. The government of the People's Republic of China assigns all of it to a single time zone, Universal Time + 8 hours. As the list of observatories in table 2.2 shows, if the Chinese divisions were, as in most of the modern world, 15° wide, only one of the sites of observation in the table actually would fall within that time zone. As usual before modern times, all recorded times were local. Although most modern studies of early astronomical phenomena use the U.T. + 8 hours convention to express all times in China, that would make it impossible to compare the recorded times with computed ones. I therefore reduce all computed times to local times for the site of observation. Such comparisons involve some uncertainty, since the site of observation is not always known.

Most aspects of astronomical timekeeping were unproblematic in the thirteenth century. The oldest Chinese records subdivided the calendar day into twelve equal intervals *(shih* 時), which I translate "double-hour." This usage survived to the end of the empire.

By two thousand years ago, clepsydras—water clocks—were widely used for telling time at night. They divided the day into a hundred equal marks *(k'o* 刻), each equivalent to 14.4 minutes on a modern clock. These divisions were usual in computational astronomy—that is, events were predicted to hundredths of a day—from the seventh century A.D. on. Lunar eclipse times from 1052 on are so listed in the Evaluation. It also became common to divide the night into five equal watches, each subdivided into five calls (see below, p. 88).[60]

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[60] Hua T'ung-hsu 1991 is a fairly detailed history of Chinese water clocks. It draws on experimental studies of their precision. Hua does not analyze mechanical clocks in which water is used only to drive a gear system, but Li Chih-ch'ao 1997 discusses the astronomical clocks of that kind.

Double-hours

Each double-hour, from the seventh century on, was divided into a Beginning Half (ch'u 初) and a Standard Half (cheng 正). I write of these smaller units as "single-hours" to avoid confusing them with modern hours. Their length is the same as the modern hour, but they are part of a dissimilar system of time reckoning. The day began at midnight *(tzu cheng)* with the Standard Half of *tzu,* not with the Beginning Half. In other words, the first double-hour, *tzu,* began at what in China today is 2300h, not at midnight, and the day began half a double-hour later. Table 2.5 gives the system of double-hours and their modern equivalents.[61]

Table 2.5. Double-hour Reckoning

D.H.	1	2	3	4	5	6	7	8	9	10	11	12
Chinese	tzu 子	ch'ou 丑	yin 寅	mao 卯	ch'en 辰	ssu 巳	wu 午	wei 未	shen 申	yu 酉	hsu 戌	hai 亥
Modern, hr	23–01	01–03	03–05	05–07	07–09	09–11	11–13	13–15	15–17	17–19	19–21	21–23

In the period of disunion from the early third to late sixth centuries, many observational records used a complicated hybrid notation (table 2.6). It began as a notation for 24 directions, with eight of the ten stems and four of the eight trigrams of the *Book of Changes* inserted between the twelve branches to mark intermediate points. Thus a record of a lunar eclipse of 20 Aug 221 specifies "the sun was at denary 9, and the moon at denary 3"; in other words, since the sun and moon were at opposition, the sun was at circa 345° and the moon at circa 165°. This mixed notation still appears on three separate circles of the siting ("geomantic") compass. There is more

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[61] For a detailed general study of divisions of the day, with many tables, see Wang Li-hsing 王立兴 1986. Sōma et al. 2004 is a good deal less reliable, but summarizes research on Japanese time units as well.

than one circle because the orientation of each label shifted slightly over time.[62]

Table 2.6. Hybrid Notation for Single-hours

[A. Double-hour

B. Equivalent times and directions. Those given begin a period of two modern hours and an arc of 30° (30^{t}.44).

C. Initial half, name. Designations in this and the next column begin with D for denary stems, T for trigrams, B for branches. The numbers represent order in the original set, not here.

D. Standard half

E. Day Remainder (Canon, 2.4)]

A	B	C	D	E
1	2300h, 345°	*jen* 壬 (D9)	*tzu* 子 (B1)	0
2	0100h, 15°	*kuei* 癸 (D10)	*ch'ou* 丑 (B2)	833
3	0300h, 45°	*ken* 艮 (T4)	*yin* 寅 (B3)	1667
4	0500h, 75°	*chia* 甲 (D1)	*mao* 卯 (B4)	2500
5	0700h, 105°	*i* 乙 (D2)	*ch'en* 辰 (B5)	3333
6	0900h, 135°	*sun* 巽 (T6)	*ssu* 巳 (B6)	4167
7	1100h, 165°	*ping* 丙 (D3)	*wu* 午 (B7)	5000
8	1300h, 195°	*ting* 丁 (D4)	*wei* 未 (B8)	5833
9	1500h, 225°	*k'un* 坤 (T5)	*shen* 申 (B9)	6667
10	1700h, 255°	*keng* 庚 (D7)	*yu* 酉 (B10)	7500
11	1900h, 285°	*hsin* 辛 (D8)	*hsu* 戌 (B11)	8333
12	2100h, 315°	*ch'ien* 乾 (T1)	*hai* 亥 (B12)	9167

Astronomers extended the system of directions to serve as a time notation for 24 single-hours. The twelve celestial stems that already

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[62] On the hybrid system, Ch'en Chiu-chin 1983a, 121; for the eclipse record, *Chin shu* 晉書, 17: 500; re the compass, Needham et al. 1954– , 4 part 1: 293–98.

marked the double-hours were also used in this system to label the second or standard half of each. Thus, in the hybrid system, *tzu,* which ordinarily spanned 2300h to 0100h, stood for midnight to 0100h. The twelve added labels indicate the beginning half of each double-hour. The Canon, section 2.4, used this system when determining the time of day in hours and marks. But in timekeeping generally, the double-hour system remained predominant.

Subdivisions of the double-hour and day

Before the T'ang, astronomers divided the double-hour into more precise units of time. Timekeepers used alongside them a system of 100 equal marks *(k'o* 刻) per day, based on the divisions of the indicators in water clocks. The fact that there was not an integral number of them in a double-hour lessened the motivation to integrate the two systems.

The earliest set of double-hour subdivisions did not introduce new units, but used fractions. They did not employ the highly flexible notation already available to mathematicians for every possible fraction, but rather the special, limited set of terms *shao* 少 (literally, "smaller"), *pan* 半 ("half"), and *t'ai* 太 ("larger") to stand for ¼, ½, and ¾ respectively (table 2.7).

Authors in the Han period and afterward subdivided this notation by modifying these terms with two others, *jo* 弱 ("weaker") and *ch'iang* 強 ("stronger"). These meant respectively 1/12 weaker and stronger than what they modify, producing units of ten minutes each. This system has confused some historians, but the Luminous Inception system (#8, A.D. 237) clearly explains it.

The words for "weak" and "strong" qualified those for quarters.[63] The confusion arose because "weak" by itself meant 11 times as much as "strong" alone. That is because the former referred to 1/12 less than the *next* double-hour rather than the current one.

That precision was still useful for some purposes in the thirteenth century. Nevertheless, the authors of the Season-granting system use "weak" and "strong" generally as inexact terms, to mean "a little less than" and "a little more than."

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[63] *Chin shu* 晉書, 18: 548–49; see also Ch'en Chiu-chin 1983a, 121–22.

Table 2.7. Fractional Divisions of the Double-hour

Fraction	Translation	Twelfths	Fraction
(double-hour)	-	0	0
chiang 強	strong	1	1/12
shao jo 少弱	weak smaller	2	1/6
shao 少	smaller	3	1/4
shao ch'iang 少強	strong smaller	4	1/3
pan jo半弱	weak half	5	5/12
pan半	half	6	1/2
pan ch'iang半強	strong half	7	7/12
t'ai jo太弱	weak larger	8	2/3
t'ai 太	larger	9	3/4
t'ai ch'iang太強	strong larger	10	5/6
jo 弱	weak	11	11/12
(next double-hour)	-	12	1

Marks

The timekeeping system prevalent from the seventh century on subdivided not the hour but the day. The mark *(k'o* 刻) was tied to a specific method of timekeeping that relied on water clocks. It was named after the gradations on clepsydra floats.

According to three sources beginning in the late first century B.C., the system worked out for use in the capital before 200 divided the day into 45 marks and the night into 55 marks at the winter solstice. It stipulated changing the currently used clepsydra float every nine days (beginning after the winter solstice) for one engraved with one more day mark and one less night mark.[64]

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[64] It is likely but not certain that the winter solstice night marks and the summer solstice day marks were actually both 65. For the complex details

Long before the Yuan period, water clocks had attained high accuracy. Because there were 100 marks in a day, each double-hour contained 8⅓ of these intervals. Since the first was called the "initial mark *(ch'u k'o* 初刻)," , and the next mark 1, the one-third mark at the end was mark 8. When the double-hour was divided into an initial and standard half, each of these single-hours contained 4 1/6 marks, with the short one at the end of each. Table 2.8 shows equivalents in modern minutes for each mark in this schema. Since marks were intervals and not moments, when converting to modern equivalents I use the halfway point; thus mark 1 (the second mark in the double-hour) extends from 14 to 29 minutes past the hour, and 21 minutes past is the midpoint that represents it.

Table 2.8. Reckoning in Marks

Mark	Initial	1	2	3	4	5	6	7	8
Time, inclusive, minutes	0–14	14–29	29–43	43–58	58–72	72–86	86–101	101–115	115–120
Midpoint	7	21	36	51	65	79	94	108	118
Same, for each half of double-hour	0–14	14–29	29–43	43–58	58–60	-	-	-	-
Midpoint	7	21	36	51	59	-	-	-	-

The Season-Granting system integrated this time notation into what amounts to a consistent decimal system extending to 10 000 parts of the day. It was not, technically speaking, a purely decimal system, as noted earlier (p. 68), and in a few minor respects (discussed below, p. 89) it was not consistently a place-value system.

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see Ch'en Mei-tung 2003a, 134–35. I agree with Ch'en that the sources lack certain essential details and one or more is probably corrupt. For a general discussion of the system of marks, see Yen Lin-shan 阎林山 & Ch'üan Ho-chün 全和钧 1980.

Occasionally influential astronomers found it inelegant that a double-hour did not contain an integral number of marks, even though one needed only simple arithmetic to deal with it. The result was an occasional edict changing marks in a day to a number divisible by 12—until bureaucratic inertia again took command.[65] One hundred remained the persistent value until Jesuit missionaries took over palace astronomy in 1644. They changed the number to 96, making one mark equivalent to a European quarter-hour.

Subdivisions of the Night

Among the measures the government used to maintain public order in cities was a night watch. For this purpose, in addition to reading clepsydra marks at night, functionaries divided the interval from dusk to dawn (not from sunrise to sundown) into five equal night watches, and each watch into five equal points; see the Canon, 5.5. The watches were called *geng* 更 (formerly read *ching)* and the points *tien* 點, a word that in modern Chinese came to designate hours. In different times and places, points were also called "rods" *(ch'ou* 籌), "drums" *(ku* 鼓), or "calls" *(ch'ang* 唱). *Ch'ou* refers to counting-rods as well as to graduated clepsydra floats; drums and calls are those of the night watchmen who passed periodically through the lanes. At the equinoxes a watch would be about 2 hours and 10 minutes long, and a point or call about 26 minutes long. In winter, since the nights were longer, watches and points were longer, and in summer shorter.

In the early Quarter Remainder system (#4, A.D. 85), the system of clepsydra indicators fixed dusk at three *k'o* after sunset, and dawn at the same interval before sunrise. There were various attempts to improve on this crude interval, but in the Yuan system the interval was still a constant, 2.5 *k'o* (Canon, 5.C6). Before the Sung period, astronomers recorded celestial events that took place at night in variable watches and points. Thereafter, marks of fixed length replaced them. In the Evaluation, table 8.5, the first definite

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[65] Ch'en Chiu-chin 陈久金 1983a, 123. The numbers of marks used were 96, 108, and 120; the last such arrangement ended in A.D. 560.

record using the latter is from 1069, and there is only one isolated record in watches and points after 948.

The night watch system was not needed to generate the ephemeris. It remained indispensable because civil timekeeping still employed it, and because astronomers used it to test their procedures against records of eclipses observed before the eleventh century.

Celestial positions

The basic Chinese system for recording celestial positions, unlike that in Europe and the Muslim world for the past two millennia, was equatorial. Although it is likely that Hipparchus also gave equatorial locations, Ptolemy used the ecliptical coordinate system; it became standard in the Hellenistic world and its successors.[66]

Chinese practice was far from identical to the equatorial system used in Europe from about 1600 on, which measured right ascension parallel to the equator and declination perpendicular to it. Astronomers in East Asia expressed the counterpart of what we now call right ascension in *tu* 度 or Chinese degrees. The Babylonian sexagesimal tradition that has survived into modern astronomy divides circles into 360 degrees, degrees into 60 minutes, and minutes into 60 seconds. Chinese instead defined their basic unit of arc measurement as one day's mean solar travel among the stars. This makes the *tu* of the Yuan period equal to 360/365.2575, or 0.985 61, degrees. The Yuan treatise subdivides the *tu* into 100 parts (*fen* 分).

When necessary, astronomers added the terms *miao* 秒 and *wei* 微 to *fen,* each providing two additional decimal places. To indicate a fraction halfway between two regular places, they inserted "and a half (*pan* 半)." Thus the Count Factor (7.C3) in the planetary technique records the equivalent of 15$^{\mathrm{t}}$.219 062 5 as "15 *tu* 21 *fen* 90 *miao* 62 *wei* and a half". One could always press other fractions into use. For instance, the Canon, 7.3A, expresses the quantity 91$^{\mathrm{t}}$.314 375 as 91 *tu* 31 *fen* 43 *miao tai* 太, that is, 91$^{\mathrm{t}}$.3143¾.

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[66] The question of what coordinates Hipparchus used remains to be settled, but Duke 2002 provides strong arguments against the conventional wisdom that they were ecliptical.

Early Chinese astronomers measured distance perpendicular to the equator either as the arc from the north pole in *tu,* or linearly in *ch'ih* 尺 and *ts'un* 寸 (literally, "feet and decimal inches," of varying standard), rather than in *tu.* But most of the observational records from the second century A.D. on used *tu* measure for polar distance as well as right ascension. Because astronomers did not define an ecliptic pole, when they began to use subsidiary ecliptic measurements, they combined with longitude the arc from the phenomenon to the pole of the equator.[67]

Lunar Lodges

East Asians used the lunar lodges (*hsiu* 宿),[68] a system of 28 stars as well as of the arcs between them, to record the locations along the equator of celestial events and motions (what astronomers today call right ascensions). Unlike the Western zodiac, a system of twelve equal ecliptic arcs of 30° named after a dozen constellations, the lodges varied greatly in width.

The original purpose of the lunar lodges, as of the zodiac, was to allow observers to estimate fairly accurately the position of a star or celestial phenomenon without needing a graduated instrument. In the European system, one had only to remember the locations of a dozen stars spaced 30° apart.[69] Memorizing 28 was no great strain on the trained memories of Chinese astronomical functionaries. With a little practice it would not have been difficult to estimate right ascension to within about a degree. One could see that a posi-

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[67] For instance, with respect to the moon, see Ch'en 1995, 278–90.

[68] Although the 28 divisions are called "mansions" in the Muslim world and South Asia, that is not what Chinese meant by *hsiu.* From the *Book of Songs* on, it refers primarily to a temporary lodging, not to one's home.

[69] This does not imply that the Greeks placed the beginnings of the houses on determinative stars as the Chinese did. For instance, although the Greeks set the longitude of the spring equinox at the beginning of Aries, no prominent star of that constellation was located there in the time of Hipparchus or Ptolemy (email, Benno van Dalen, 2006.5.10).

tion slightly less than halfway between the lodge Ox, 7.2 *tu* wide, and the next lodge, Maid, was 3 *tu* in Ox.[70]

Table 2.9. Lunar Lodges

Lodge	Short name	Extension, tu	Determinative
Horn *(chiao* 角)	-	12.10	α Vir
Neck *(k'ang* 亢)	-	9.20	κ Vir
Root *(ti* 氐)	-	16.30	α^2 Lib
Chamber *(fang* 房)	-	5.60	π Sco
Heart *(hsin* 心)	-	6.50	σ Sco
Tail *(wei* 尾)	-	19.10	μ^1 Sco
Winnowing-basket *(chi* 箕)	Basket	10.40	γ Sgr
EAST, total	-	79.20	-
Southern Dipper *(nan tou* 南斗)	Dipper	25.20	φ Sgr
Led Ox *(ch'ien niu* 牽牛)	Ox	7.20	β Cap
Serving-maid *(hsu nü* 須女)	Maid	11.35	ε Aqr
Tumulus *(hsu* 虛)	-	8.9575	β Aqr
Rooftop *(wei* 危)	-	15.40	α Aqr
Hall *(ying-shih* 營室)	-	17.10	α Peg
Eastern Wall *(tung pi* 東壁)	Wall	8.60	γ Peg
NORTH, total	-	93.8075	-
Crotch *(k'uei* 奎)	-	16.60	ζ And
Pasture *(lou* 婁)	-	11.80	β Ari

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[70] This table reproduces for the convenience of the reader some of the information from the Evaluation, table 7.5. The names in the left column include long forms given in some texts, particularly early ones. The extension is that given in the Yuan treatise. The first identification of each determinative star is that proposed for 1280 by P'an Nai 潘鼐 (1989, table 71, p. 273). I have added alternatives, followed by a question mark, when a choice between two or three identifications is uncertain.

Lodge	**Short name**	**Extension, tu**	**Determinative**
Stomach *(wei* 胃)	-	15.60	35/41 Ari?
Mao *(mao* 昴)	-	11.30	17/η Tau?
Net *(pi* 畢)	-	17.40	ε Tau
Beak *(tzu* 觜)	-	0.05	$\phi^1/\lambda^1/\kappa$ Ori?
Triad *(shen* 參)	-	11.10	δ/ζ Ori?
WEST, total	-	83.85	-
Eastern Well *(tung ching* 東井)	Well	33.30	μ Gem
Cartborne Devils *(yü kuei* 輿鬼)	Devils	2.20	θ Cnc
Willow *(liu* 柳)	-	13.30	δ Hya
Seven Stars *(ch'i hsing* 七星)	Stars	6.30	α Hya
Strung Bow *(chang* 張)	Bow	17.25	$\upsilon^1/\nu/\mu$ Hya?
Wings *(i* 翼)	-	18.75	α Crt
Chariot Baseboard *(chen* 軫)	Baseboard	17.30	γ Crv
SOUTH, total	-	108.40	-

Each lodge (see table 2.9) began with a star, and was named after the constellation to which it belonged.[71] These determinative stars

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[71] Because the names of the lodges are archaic, their meanings are sometimes unclear. The best set of translations so far is that of Edward H. Schafer in *Pacing the Void,* his recreation of the sky as T'ang poets saw and imagined it (1977, 81). Mine differ only in small respects, mainly because I have tried to find single English words that reflect the senses implied in both the short and long versions of the names. There are only four less obvious differences. First, I do not translate *mao* 昴, but merely transcribe the Chinese word, because I believe that the graph originated as the name of this asterism, without at first any mundane meaning. Second, recent scholarship on the components of very early war chariots indicates that *chen* 軫 means, not an axletree, but the baseboards on the floor of a carriage atop which its sides were built (Barbieri-Low 2000 and his email to me of 2003.8.7; additional data, but not corrected translations, in Sun & Kistemaker 1997: 65–67). Third, because *ti* 氐 and its cognates have not been reliably identified in very early script, the word's original meaning is uncertain. Schafer and others equate it to *ti* 底, "base," because of an early

("determinatives" for short), were not necessarily the brightest stars in their constellations; in some cases their magnitude was as low as 4 or 5. They were far from equidistant. The width of a given lodge—the distance from its determinative star to that of the next lodge—varied from 0.05 *tu* to over 33 *tu*. They were not chosen for their proximity to either the equator or the ecliptic. All that did not matter, so long as one had learned to locate the pertinent stars of the 28 constellations and had memorized the distances between them. This unequal system proved quite satisfactory for a very long time across a wide range of cultures, including not only those of East Asia but those of India (where they were called "mansions," e.g., *nakshatra),* and later those encompassed in the realms of Islam (where the Arabic term was *manāzil)*. The characteristics of the system of mansions varied from time to time both in India and in China.[72]

The widths of the Chinese lodges changed gradually over the centuries. The cumulative change was great enough that the Evaluation includes a table of their varying equatorial extensions—that is, the distance in right ascension from their determinative star to that of the next lodge—between the Han and Yuan eras (Evaluation, table 7.5, columns A–F). The Canon (section 3.3) reminds the user, when investigating past phenomena, to use the lodge widths of that time rather than those of the Yuan survey. A record of an event located at 4 *tu* in Ox would have amounted to a R.A. of 18h36m in 104 B.C., 19h42m in A.D. 1030, and 19h56m in 1280.

In modern terms, the changes over the centuries depended partly on the effect of the precession of the equinoxes on the coordinates of

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textual usage. A modern excavation has, however, yielded a manuscript of circa 200 B.C. that uses the graph for *ti* 柢, "root" (see, e.g., D. C. Lau 1982: 224). I therefore provisionally use that sense. See also Liu Ts'ao-nan 刘操南 1979.

[72] Several historians have tried to date the system of lodges by computing when their distances diverged minimally from the ecliptic or equator. The assumption that ancient Chinese chose them in that way is baseless, and those about the accuracy of observations in archaic times are equally arbitrary. See Needham et al. 1954–, 3: 248–50, for further discussion and an early example of fallacious dating.

the lodge determinative stars, since they varied in declination and their intervals were measured along the equator. For instance, the lodges Beak and Triad reversed their order as the width of Beak, over 1400 years, shrank from 2 *tu* until it effectively became negative.

Other changes resulted from different choices of determinative stars. Yabuuchi's computations indicate, among other interesting changes, that I-hsing 一行 (728, #42) changed the determinative of Crotch from ζ Andromedae to δ And. In the Yuan system the determinative was once again ζ And. In 1628 it shifted to η And.[73]

Another reason that distances between determinatives changed is that, when a star is barely visible to the naked eye and near other stars of more or less the same apparent size, it is hard for observers (and all the harder for sinecurists carrying out routine duties) consistently to pick out the same star. A good example is Mao. That narrow lodge incorporates the Pleiades, a confused agglomeration of dim stars, only one of them so much as third magnitude. This neglected uncertainty merits systematic study.

Finally, the coordinates of stars change due to their proper motions, but over the past 2000 years this effect has been negligible compared with the others.

The precision that the Season-Granting system brought to observational data served its coordinate system well. Previous reforms that carried out star surveys did not exceed a precision of ¼ *tu*. Kuo and his colleagues were the first to reach a consistent 1/20 *tu*.

The disuse of latitude measurements of determinative stars in the sophisticated Yuan system led to its fairly large inaccuracies in computing the width of the lunar lodges for epochs long before or after the epoch of 1280.[74]

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[73] Yabuuchi Kiyoshi 1969: 122–123; P'an Nai 1989, table 82, p. 347; Ch'en Tsun-kuei 1980–84: II, 324; T. Kiang 1972: 40; Pai Shang-shu 白尚恕 & Li Ti 李迪 1982.

[74] See my commentary on the Canon, section 3.9, p. 439.

Twelve Stations

Grand Year (*t'ai-sui* 太歲, *sui-yin* 歲陰, *t'ai-yin* 太陰, which some historians call the "year star" or "counter-Jupiter") is a fictitious (or factitious) body that revolves with the same period as Jupiter, but in the opposite sense. Its motion follows an order opposite to that of Jupiter's annual orbit. The Twelve Stations (*shih-erh tz'u* 十二次), which modern historians often call "Jupiter Stations," are equal divisions of the *equator* into twelve houses. In principle, they divide the sidereal revolution of Grand Year into twelve equal parts. When Jupiter began to play a role in cosmology, in the third century B.C., astronomers took its period as twelve years, so Grand Year traversed one station a year. Astronomers soon discovered that the period is roughly 11.86 years, and compensated for the gap between that figure and a dozen years.

There is no need to fully explicate this early system (table 9.5), which entered cosmological discourse over the course of a century before astronomers applied it in the reform of 104 B.C. By the Yuan it had no practical use in mathematical astronomy, despite the simplicity of its equal divisions compared with the unequal divisions of the lunar lodges. The Twelve Stations differ from the twelve houses of the zodiac found in Hellenistic and post-Hellenistic astronomy in crucial ways. The stations are equatorial, and the zodiacal houses ecliptic. The latter begin (or at least originally began) with the solstices or equinoxes, which the Greeks took as fixed points of ecliptic longitude. In the Chinese system, the four Standard Points (the equinoxes and solstices) fall not at the beginning of but midway in the pertinent stations: Star Marker (winter solstice), Downland (vernal equinox), Quail Head (summer solstice), and Longevity Star (autumnal equinox). Finally, the Stations are related to the double-hours. They therefore proceed—or rather Grand Year proceeds through them—in a sense opposed to that of the planetary revolutions.

The application of the Stations is not strictly astrological, for Grand Year is not exactly a planet. It is associated with yin and the earth, not the sky. What seem today to be the advantages of the system of stations over that of the lodges were negligible. On the other hand, the Stations turned out to be perfectly usable when, as happened a couple of times over history, astronomers employed them for cosmological reasons or for their putative associations with antiquity. The meanings of some of the names of the stations, like some of those of the lodges, are bound to be uncertain, since we know so little about what the often ambiguous words were intended in antiquity to evoke.[75]

Initial Points of Measurement

It was inevitable, in order to estimate the positions of celestial events without depending on instruments, that early Mediterranean astronomers tied fixed points to stars. As a result, although the cardinal direction points move slowly through the stars, it was natural to maintain the linkage of east and the vernal equinox with certain stars by convention, and shift them only at long intervals. Thus persons unknown around the time of Eudoxus (circa 370 B.C.) put the vernal point and due east at 15° in the constellation Aries, and Hipparchus two centuries later at the beginning of Aries (where a convention of occidental astrology has kept it).[76]

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[75] For an impression of the range of uncertainty, compare these translations with the always learned and thoughtful ones in Schafer 1977, 76–77. The main divergence between the two lists is that Schafer translates Chü-tzu 娵訾, the first name, "Loggerhead Turtle," based on the meaning of the graphic variant Tzu-hsi 觜蠵, noting that the meaning of Chü-tzu is very uncertain. I find it difficult to believe that this early group of names should have included that of a tropical sea turtle, and note that in his list of lodges Schafer translates the same Tzu 觜 "Beak." In his view as well as mine, Chü-tzu is recorded in early literature as a personal name and is therefore untranslatable. It is possible that both Chü-tzu and Shih-ch'en originated as names for the constellations (as did Shen 參 and, I think, Mao for lunar lodges), and were later attached to persons. In any case they have no ascertainable primary meaning to translate. I therefore transliterate them.

There is no satisfactory historical study of the Twelve Stations. The discussions in Saussure 1908 and 1930 are best used in conjunction with those in Shinjō 1933: 369–392, Nōda & Yabuuchi 1947: 113–137, and Liu T'an 1957. For fairly recent research see Ch'en 1995, 518–20 and table 19-3.

[76] Neugebauer 1975, 2: 595, 599.

For Chinese, the equivalence of *tu* and days of mean solar travel through the stars made the determination of the sun's equatorial position simple in principle. Chinese began the count at the winter solstice—unlike the Greeks and their successors, who began with the spring equinox. Yuan astronomers measured equatorial coordinates from 6 *tu* in Tumulus (6^t is the interval from the determinative to the beginning of the sixth *tu*). The initial point corresponds to the location of β Aquarii in December 1280 plus 6 *tu,* or to an R. A. of 21h17m = 319°.28. This was 36 minutes of arc backward from its position in 1950. Astronomers had to determine the position of the sun on the ecliptic, and of the moon on the White Way, by what amounted in modern terms to spherical projection of their computed equatorial locations.

Celestial Motions

The most challenging problems of prediction in ancient Chinese computational astronomy were those of solar eclipses and certain planetary phenomena. The earliest techniques assumed that the speeds of the sun, moon, and planets along their paths were constant. Predictive accuracy was thus inherently limited. Computation evolved as astronomers identified and numerically modeled actual orbits of the sun on the ecliptic and those of the moon and planets as they diverged from it.

Chinese recognized the considerable difference between the mean and apparent positions of the moon early in the third century A.D. By about 600 they were dealing satisfactorily with the solar equation of center, and by 666, with that of the moon. Their term for mean positions and periods was *p'ing* 平, literally "mean." *Ching,* as in *ching shuo* 經朔 ("regular conjunction," denoting the mean conjunction), serves the same purpose as *p'ing*. The Canon uses *ching shuo* regularly for the mean conjunction in the solar, lunar, and eclipse theories, and *p'ing shuo* for the same purpose in the planetary theory The Yuan system took over this unexplained distinction from the Revised Great Enlightenment system, from which it copied the planetary theory.[77] Since the basic meanings of these two terms are distinct, I avoid confusing them in translation.

Chinese astronomers' word for "apparent" was *ting* 定. It does not mean either "apparent" or "true." Its most pertinent literal senses in astronomical writing are, as an adjective, "determinate" and, as a verb, "to correct." Since the latter fits all its usages in the Yuan nomenclature of mathematical astronomy, I translate *ting* consistently as "corrected." That reflects its use in determining a position by adding to its mean value a correction (in early modern European terminology, the equation of center).

The imperial house and its astronomical bureaucracy, from the start, centered its attention on solar eclipses of large magnitude. Those phenomena particularly threatened the imperial mandate because the early classics depicted them that way, and everyone could see them. Computing them accurately depended on recognizing that the paths of the sun and the moon differ, and on mastering the inequalities of both solar and lunar motions. This task was further complicated by the need to determine what areas the narrow shadow of the moon darkened as it swept across the earth—that is, where people saw the sun's disc as eclipsed. This problem was much more difficult to surmount using Chinese numerical rather than Greek geometric methods.[78]

Planetary phenomena were less dramatic, and less compelling to laymen glancing at the sky as they occurred. None was as obviously ominous as the death and rebirth of the sun or moon in an eclipse. What was astrologically portentous—a conjunction of several planets, a planet's change of direction near a given star, or an occultation—was a matter of convention.

In principle, the computational difficulties posed by a single planet were much less serious than those involving the sun and moon in an eclipse. In practice, the aberrant movement of each planet through the stars with frequent reversals of direction, the di-

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[77] *Chin shih* 金史, 22: 514ff.

[78] I have discussed some of the difficulties of which early imperial astronomers became aware in Sivin 1969, 58–62.

vergence of each planetary orbit from the ecliptic, and the Chinese concern with occultations and other phenomena that involved more than one celestial body, made planetary computation at least as refractory as that of eclipses.

From an early time, Chinese traced celestial motions with respect to three "ways *(tao* 道)" in the sky. These were courses, not mere abstract loci of motion. The Red Way *(ch'ih tao* 赤道) corresponded to the celestial equator, parallel to the daily motions of the stars. The Yellow Way *(huang-tao* 黄道) was the path that the sun and planets followed, the ecliptic. The White Way *(pai-tao* 白道) was the path of the moon. Chinese astronomers had instruments with ecliptic as well as equatorial circles, but in practice they generally measured and recorded equatorial coordinates, and determined those along the ecliptic and the lunar orbit by conversion. The Season-Granting system systematically measured the intersection of the Red and Yellow Ways, and of the Yellow and White Ways. Like its predecessors, it wrote of positions on the latter two north of the equator as "inside," and south of it as "outside."

Solar

For modern astronomers, the precession of the equinoxes is an important factor in calculating past or future phenomena. Hipparchus evidently understood the precession as a shift of the solstitial and equinoctial points along the ecliptic at a rate of at least 1° per century. Ptolemy confirmed this rate, but explained the shift as due to a motion of the stellar background with respect to the same points, due to a revolution of the equatorial poles about the ecliptic poles in a cycle of about 25,000 years. He thought of the "fixed stars" as rotating slowly with the poles of the equator.[79]

Astronomers in China understood the shift quite differently, as a discrepancy in time as well as space, with the emphasis on time. They had no theory of precession that took the latitudes of stars into

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[79] Toomer 1994, 321–38, especially 321 note 2. Although Hipparchus' lower limit for precession was 1° per century, his upper limit was probably a good deal less than the modern value; Pedersen 1974, 239.

consideration. We have already seen that Chinese astronomers had no conception of the ecliptic pole, about which the pole of the equator precesses, or of ecliptic latitudes.

Unlike the Greeks, who counted the tropical year from the vernal equinox, for the Chinese the initial point of the year was the winter solstice. Their counterpart of the precession was the Annual Difference *(sui-ch'a* 歲差), the gap in *tu* between the position of the sun at the end of a sidereal year (Celestial Perimeter, Canon, 3.C1) and a tropical year (Year Numerator, 1.C2). In the Yuan system, the magnitude of the Annual Difference (3.C5) changed gradually, but for 1280 it was 0.0150 *tu,* measured along the equator. Modern computation for epoch 1280 puts it at 50″ = 0°.0139 = $0^{\mathrm{t}}.0141$.

In other words, this factor was an arc added to the circle that the sun describes between two winter solstices. Because the Season-granting system defined the *tu* as one day's share of the sun's motion in the mean tropical year, the Annual Difference was equivalent to 0.015 day or 1.5 marks. Although it is not intuitively obvious, this approach is operationally equivalent to the modern one.

The conception evolved in several steps. Beginning at the end of the first century B.C., astronomers became aware that in observations over long periods the winter solstice point appeared at different locations among the fixed stars. They were not certain, however, whether this was a gradual shift or the result of earlier observational errors. Second, Yü Hsi 虞喜 (281–356) accounted for the shift by a secular change. He compared the recorded positions over a span of 2700 years, from a legendary assertion in one of the classics—which he conventionally considered an observation—to the solstitial location in his own time. That enabled him to separate the sidereal from the tropical year. Since he took the first as unvarying, he concluded that the second, the period from one winter solstice to the next, diverged by 1/50 day per year. Given the definition of the *tu,* this amounted to $0^{\mathrm{t}}.02$, or a shift along the equator of 1 *tu* every 50 years. Circa 443, Ho Ch'eng-t'ien 何承天, beginning his time span with the same legendary source, corrected the value to 1 *tu* per century. A modern computation yields $0^{\mathrm{t}}.013$ per year for the period.

Two decades later, Tsu Ch'ung-chih 祖沖之 provided a firmer basis for the concept. He exploited a technique of Chiang Chi 姜岌 (#13) from circa 380 for determining the position of the sun with respect to the stars by its opposition to the moon at the maximal phase of a lunar eclipse. He got a precise result by using a number of historical records up to his own era. He also, for the first time, integrated the Annual Difference constant, and a length for the sidereal year, into his computational system. Although a series of errors in data made his value for the Annual Difference ($0^t.022$) very inaccurate, his method became standard, and soon led to extremely exact values. That of Liu Cho 劉灼 (#42, 604) was $0^t.013$.

The Season-granting system used a figure that was negligibly better ($0^t.015$ for its epoch, modern value $0^\circ.0141$; 3.C5). That is because its authors decided to make the tropical and sidereal year lengths symmetrical about 365¼ days. For the epoch, the sidereal year was 365.25 + 0.0075 = 365.2575 days, and the tropical year 365.25 – 0.0075 = 365.2425 days.[80]

The system had another peculiarity that affected the accuracy of the Annual Difference. The Yuan astronomers varied its magnitude linearly with time rather than making it constant. That secular variation shortened the mean tropical year by 0.0001 day per century, and lengthened the sidereal year by the same amount. Thus, although the tropical year for 1280 was 365.2425 days long, in the neighborhood of 1650 it became 365.2422 days long, briefly—and adventitiously—equal to the modern value.

Lunar

The Yuan authors treated the anomalistic month, the period of the moon's travel from perigee back to perigee, just as they did the sidereal year, the solar circuit among the stars. Since there was no explicit concept of perigee or apogee in their astronomical vocabulary, Chinese saw the anomalistic month—or, in their terms, the

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[80] On the early Annual Difference, see Ho Miao-fu 何妙福 1978, Ch'en Meidong 1995, 261–70 and Ch'en 2003a, 251, 265; for its variation in the Yuan, see the Canon, section 1, the Evaluation, section 2, and the detailed study in Nakayama 1969, 127–31.

revolution for which the Revolution Terminal Constant (Canon, 4.C2) is named—as corresponding to a circuit from the beginning of one phase of hastening (that is, of positive anomaly) to the next.[81]

Planetary

Astronomers conventionally referred to the planets as the "Five Stars *(wu-hsing* 五星)" or "Five Moving [Stars] *(wu-hsing* 五行)," that is, the five that have motions not shared with the fixed stars. Poets had many names for them, but those usual in astronomy came from a workaday set of designations based on the resonance of the planets with the Five Phases, a basic category of Chinese thought (see "planet" in the table), and an ancient set of literary terms (see "translation"). In the former set, Jupiter is literally "The Wood Star." This has nothing to do with the substance wood, but implies the associations of the functional Wood phase with east, spring, growth, and so on. The order in which astronomers customarily enumerated the planets, shown below, was the mutual generation order of the Five Phases, which governs normal transitions between phases of a process.[82]

Planet	**Equivalent**	**Poetic name**	**Translation**
Wood	Jupiter	Sui-hsing 歲星	Year star
Fire	Mars	Ying-huo 熒惑	Dazzling delusion
Earth	Saturn	Chen-hsing 鎮星	Commanding star
Metal	Venus	T'ai-po 太白	Great white
Water	Mercury	Ch'en-hsing 辰星	Hour star

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[81] The value of this constant in the Yuan system is 27.554 6 days, identical to the modern value. The perigee is the point in the moon's or a planet's orbit when it is closest to the earth, and its apparent speed is fastest; the farthest point is the apogee.

[82] On the associations of the phases, see, e.g., De Bary & Bloom 1999, 348, and on the concept, Sivin 1987, 70–80. Less common technical names for the planets were the Five Wefts *(wu wei* 五緯) or Five Walkers *(wu pu* 五步).

During the imperial period, astronomers concentrated on predicting the position of a planet at the times of conjunction with the sun, stations, appearance and disappearance—more or less the same phenomena that fascinated Europeans. As in other aspects of Chinese culture, what mattered most in astrology was relations, in this case the relations of planets to other celestial bodies. The record contains many instances in which a planet "trespassed" on a constellation (see below) or occulted a star. Astronomers never worked out a comprehensive and highly accurate approach to determining daily apparent sidereal positions of planets. A very few attempted to observe their position every day over a long period, but as in the West this never became the norm.

Anyone who studied the heavens was, of course, aware of the erratic course of the planets with respect to the fixed stars. Beginning when a planet is invisible (because it rises and sets with the sun), it reappears before sunrise, first gradually speeding up and then slowing down in forward (progressive) motion until it momentarily stands still (at its stations). Then it moves in reverse (retrogradation) against the starry backdrop until it is opposite the sun. That is its fastest backward speed. Then the sequence repeats in reverse order until the planet once again is invisible. This was the sequence for the first three planets listed above. How far it holds for the remaining two remained uncertain, because they never diverge far from the sun, so their locations near conjunction are very difficult to observe accurately. In both East and West, astronomers assumed that the sidereal periods of the last two were the same as that of the sun in its rotation about the earth.[83]

In the earliest period of astronomy, which depended on mean motions, astronomers divided the cycle of each planet into the phases of motion just discussed, or subdivisions of them. They treated each of these grades *(tuan* 段) as constant in speed. They treated the stations as periods with finite length, like those of pro-

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[83] On the issue of apparent planetary brightness, see Yabuuchi Kiyoshi 藪内清 1961.

gressive and retrograde motion, rather than as momentary points of transition between motion in two directions.

The realization that the planetary motions followed cycles of increasing and decreasing anomaly like those of sun and moon came about A.D. 560 (see below, p. 581). Systems from the Sovereign Pole (#34, 609) on record techniques for calculating the equation of center.

The grade divisions that were part of this effort differ greatly in size, with progressive motion taking several times as long as retrogradation, and the stations arbitrary in length but very short. Practitioners set up enough grades to offer steps of commensurable length—a different number for each planet—with the change in speed gradual within each. This table shows the number of phases in the cycle of each planet in three outstanding systems over more than 500 years:

[The three systems are:

A: Great Expansion system (#42, 728), from *Hsin Tang shu,* 28B: 683–88

B: Era Epoch system (#71, 1106), from *Sung shih,* 80: 1885–99

C: Season-granting system, Canon, tables 10.3a-10.3e]

Planet	Equivalent	A	B	C
Wood	Jupiter	8	14	14
Fire	Mars	10	18	18
Earth	Saturn	8	12	12
Metal	Venus	14	20	20
Water	Mercury	12	10	10

It is obvious that the division in the Era Epoch system was considerably more complex than that of its predecessor, far from a simple modification of it. It is also clear that the Era Epoch system set the standard from which its Yuan successor, and the best systems between the two, did not diverge.

The Great Expansion system, once again, was the first to offer an explicit technique for apparent planetary longitude. But a succession of linear phases is not an accurate solution, much less an elegant one, to such a complex problem. The departure of each planet's path from the Yellow Way was also not at all a trivial challenge, particularly for numerical rather than geometric computation. Even the best systems did not master it. Error in planetary techniques tended to be high throughout the Chinese tradition.

Ch'en Mei-tung has computed the implied planetary eccentricity and equation of center for twenty systems. The systems for which the errors are smallest, naturally enough, were those that undertook new observations. These include Reverence for Heaven (#66, 1024), Contemplation of Heaven (#69, 1092), and Era Epoch.[84] Ch'en further concluded that the error of these three systems in locating the perigee is asymmetrical. He argued that in practice, their authors empirically determined the equation of center. They first found the perigee, and then observed locations at other times, comparing them with mean locations. There remains considerable continuity of theoretical approach from one to another. This proposition merits further exploration.

The Season-granting system did not give the smallest errors for any planet; in fact, it bettered the results a decade earlier of Ch'en Ting (#86, 1271) in only three of ten results. This outcome is unexpected, since we know that the Yuan reformers collected massive planetary data (p. 589 below), and that Ch'en worked on both systems. Ch'en Mei-tung's research supports my contention that the astronomers did not use this data in the reform, and in fact copied with minor modifications the planetary technique of a predecessor (p. 32). Previous estimates of the accuracy of planetary techniques before modern times have concentrated on mean periods and constants. In order to achieve useful results, it will be necessary to compute carefully chosen phenomena using ancient methods and evaluate the extent of error.[85]

Keep in mind that the imperial sponsors of astronomy regularly expected a technique to yield accurate predictions over hundreds or

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[84] As we will see, the Season-granting system made new observations but did not incorporate them.

[85] Ch'en 1995, 455–464, especially tables 16-2 and 16-3. See also his table 14-1, pp. 393–97, which compares the planetary periods of 40 astronomical systems.

(if they were naive) thousands of years. Even in 1280, that would have required major innovations—among them, dealing with planetary latitudes as well as apparent longitudes.

Eclipses

The earliest methods of predicting eclipses using mean periods of the sun and moon are so crude that they can hardly have yielded useful results. It is more likely that, while officially employing such a method, unofficially astronomers used an eclipse cycle of 135 lunar months to predict new eclipses on the basis of ones observed earlier. This simple expedient remained accurate over long sequences of eclipses. It predicted when an eclipse would take place, but not where. One therefore could not be sure whether it would be visible at the observatory, or even in China. Like computational methods used later, this observation-based technique required a rule to the effect that only eclipses seen but not predicted counted as failures. Astrologers could not have counted as ominous eclipses predicted but not observed at a given place.[86]

The second stage, from A.D. 85 on, used various conceptions of eclipse limits. Early Chinese astronomers thought of these, not as arcs from the lunar node (the intersection of the sun's and moon's orbits), but as days of solar travel. Operationally speaking, since a *tu* amounted to a day of mean solar travel, the two were equivalent for mean motions.

The eclipse limit was the interval in which an eclipse could take place, measured between conjunction or opposition and the sun's transit of the lunar node. Let me first explain in modern terms. If sun and moon enter conjunction (or opposition) at or very near the node, the shadow of the moon (or earth) is bound to obscure the sun (or moon). If the two bodies move into conjunction at some distance from the node, so that sun, moon, and earth are not quite lined up vertically, the shadow will be thrown into empty space, and no eclipse will be visible from the earth. If they are somewhat

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[86] See Sivin 1969, 52–64, for a detailed argument, as well as Foley 1989 and Steele 2000, 176–77.

closer than that, part of the shadow will obscure the sun (or moon) so that an observer sees it only partially darkened. In the Canon, section 6.15, figure 44, S_5 is the sun at the lunar node; S_4 and S_6 show it at the partial eclipse limit. The limit for total eclipses remained less than 3° until the Great Expansion system (#42, 728).

Ch'en Mei-tung has studied the lunar eclipse limits in 34 systems from A.D. 206 to 1280, and the solar eclipse limits for 33 of these. He found that the closest approach to the theoretical limits for the latter came in the Concord with Heaven system (#80, 1199, CHS in the table), and that the Season-granting system's did not come close to matching it. This table compares, for solar eclipses, the theoretical limits for each condition with the average limits used in all the Chinese systems studied (under "Range"), with the Concord with Heaven system, and with the Season-granting system.[87]

Limit, *tu*, for	**Theoretical**	**Range**	**CHS**	**SGS**
No eclipse	12.55	18.06 > 12.38	12.42	12.98
Definite partial eclipse	< 9.75	15.95 > 9.60	-	-
Definite total eclipse	< 3.97	4.92 > 0	3.99	4.33

Eclipse prediction matured as astronomers incorporated the apparent motions of the sun and moon, and recognized the regular variation in the eclipse limit depending on whether the conjunction occurs south or north of the ecliptic (due, in modern terms, to geocentric parallax; see p. 499). The theory approached definitive form in the Sovereign Pole system of 608 (#34), fully incorporated the lunar anomaly in the Chimera Virtue system of 665 (#38), and redefined a *ch'i* as the sun's apparent motion in 1/24 of its orbit, rather than mean motion in 1/24 of the tropical year, in the Great Expansion system of 728 (#42, 728). That system also incorporated a first crude correction for the latitude of observation. John Steele has

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[87] Ch'en 1995, table 13-1, pp. 369–70; I have converted his results from degrees to *tu*. The "range" column gives highest and lowest values in Chinese sources. On the accuracy of the Great Expansion system (#42), see Hu T'ieh-chu 胡铁珠 2001.

suggested that an important factor in the Season-granting system's improved accuracy was an improvement in the accuracy of timekeeping. Near the end of the eleventh century, after average error had remained static at roughly 0.25 double-hour (30 minutes) since about 400, it was more than halved to 0.1 double-hour (12 minutes or a mark).[88]

Eclipse Records

The records of solar eclipses in section 9.3 of the Evaluation provide detail rarely recorded elsewhere: time to the nearest mark from the third century on as well as, from the late seventh century on, time for each phase. For lunar eclipses in section 9.4, the times of phases begin early in the fifth century.

The astrological treatises of the Standard Histories, and even their annals of individual emperors, regularly list eclipses. The treatises do not, however, regularly specify time and phase, and only occasionally say whether the eclipse was total and in which lunar lodge it occurred. The Evaluation's precursor, in I-hsing's Great Expansion system, includes an excerpt from the store of earlier observations used to calibrate the system. I-hsing tested his technique against records of 43 solar and 99 lunar eclipses.[89]

The compilation of observed eclipses in the astrological treatise of the *Old T'ang History* gives phases and times of day for only two of them. Both were recorded long after the Great Expansion system had been replaced for official use. The day of a lunar eclipse without the hour was probably too crude a test to be useful to I-hsing, the best astronomer of his time. A canon of detailed observational records must have existed then, since it survived until the thirteenth century, and was incorporated—partly or fully—in the Evaluation,

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[88] Steele 2000, 232. He compares this to an accuracy of observation in the Islamic world of between six and ten minutes (pp. 122–23).

[89] *Hsin T'ang shu,* 27B: 625–27. The still earlier polemic against the Opening Sovereignty system (#33, 584) mentioned 181 recorded solar eclipses from the mid third century B.C. to the early fifth century A.D.; *Sui shu,* 17: 425. This list does not survive.

section 9. Reconstructing the form and extent of this source will not be an easy matter.[90]

One may well ask why computation of a solar eclipse a thousand years earlier is a useful test of a system's ability to predict phenomena. The reasoning was similar to that applied today when experimenters use laboratory mice to test drugs. Very large doses produce conspicuous effects. These effects sometimes magnify very subtle outcomes of normal human dosage, outcomes that in clinical use—at a much later stage in testing—show up only in large statistical samples. Analogously, comparing a computation with an observational record from centuries earlier may result in a discrepancy large enough to presage eventual small failures over a shorter official life. The gradually growing frequency of small failures in predictions of solar eclipses, after all, was the most frequent reason for replacing a system.

Because a lunar eclipse is visible from half of the earth at one time, it is much easier to compute a verifiable lunar than solar eclipse. Lunar eclipses after the third century A.D., when the major problems of prediction were solved, became inherently less ominous, and of correspondingly less interest to astrologers. For a couple of centuries, treatises did not record eclipses of the moon. Records of them regained importance from A.D. 384, when the Triple Era First-year Epoch system (#13) began using such records to determine the location of the sun among the stars. That approach gave greatly improved results, since in a lunar eclipse the sun is in opposition to the moon. Certain of the later histories also incorporated many lunar eclipse reports, valuing them less for purposes of political and moral interpretation than for utility in perfecting astronomical prediction. The collection of data in section 9.4 of the Yuan treatise is far from exhaustive (for instance, it omits four lunar eclipses recorded in the summary of I-hsing's Evaluation), but it is unique

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[90] The only T'ang reports that note time in marks and magnitude are for 761 and 768, See *Chiu T'ang shu* 舊唐書, 36: 1324, 1326; cf. reports of the same events without these details, p. 1318. Steele 2000, 273, partially translates the reports. They do not appear in *Hsin T'ang shu.*

among extant printed records for consistently including details of phases and times of day.[91]

Eclipse magnitudes are a neglected topic of historical study. It is clear, however, that the prediction of magnitudes began as a function of the moon's distance from the lunar node during an eclipse, and was therefore related to the conception of eclipse limit. Before the Sung period, astronomers used an assortment of fractions to record the magnitudes of partial solar eclipses, that is, the part of the Sun's diameter that the shadow covered: quarters, thirds, tenths, and even, occasionally in the fifth and sixth centuries, fifteenths. The idiosyncratic latter fraction seems to have been an artifact of techniques used to predict eclipses.[92]

The Canon, 6.16, lists the Season-Granting System's Five Limits (i.e., phases) of total lunar eclipses: Beginning of Loss *(k'uei ch'u* 虧初), Eclipse Totality *(shih chi* 食既), Eclipse Maximum *(shih shen* 食甚), Rebirth of Light *(sheng kuang* 生光), and Disk Restoration *(fu man* 復滿). Early systems, however, record only three phases. They call totality *either* Eclipse Totality *or* Eclipse Maximum, use a variety of terms for first and last contact, and do not note the second or fourth phase. This is true of the Sung systems whose techniques survive. Astronomers began dividing eclipses into five phases only in the immediate predecessor of the Season-Granting system, the Revised Great Enlightenment system.[93]

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[91] Another rich—but frequently inaccurate—source of eclipse records is the thirteenth-century encyclopedia *Wen-hsien t'ung k'ao* 文憲通考, ch. 282–83 for solar eclipses and ch. 285 for lunar. It occasionally notes visibility and, even less frequently, magnitude. Its coverage does not overlap that of the Evaluation except for the period 1063–74. On the Triple Era First-year Epoch system's use of lunar eclipses see *Hsin T'ang shu,* 27A: 616.

[92] E.g., *Wei shu,* 105A: 2337, 2341; on the connection with eclipse prediction, see *Sui shu,* 17: 432–33. Ch'en Mei-tung 1995, 347–84, devotes a chapter to lunar eclipse magnitudes.

[93] *Sung shih*, 69: 1546, 73: 1658, 75: 1725, 78: 1826, 80: 1884; *Chin shih*, 22: 494–95. Note that the Canon, 6.16, still speaks of the Three Limits as well as the Five Limits of lunar eclipses. In the Evaluation, table 8.6, items 28–33 and 39–45, from the years 1270–80, report a maximum of only three phases. For details on the terminology of eclipse phases, see Steele 2000, 192.

Observed and Calculated Eclipses

A rule of thumb among historians of early astronomy is that unaided observers are unlikely to notice an unexpected solar eclipse unless its magnitude is at least 0.9 on a scale of 1. The eye compensates so well for dimming, and the sun is so bright, that the moon's shadow must cover it almost completely before its light seems to lessen. Nevertheless, both massive records and modern experiments make it clear that trained observers can do much better. The records in section 9.2 indicate that as early as the eighth century B.C., observers—who were not necessarily trained astronomers—could see large partial eclipses.

It is obvious from the data in the Evaluation that official astronomers were regularly observing solar eclipses of small magnitude. The most usual way to scrutinize the sun's disk for an impinging shadow was to look at its reflection in a bowl of water. Some sources mention using oil for the same purpose, although it is unlikely that this would serve better than water. Another way is to examine the shade under leafy trees, since small gaps between the leaves act like pinholes and form small images of the sun. Observation of either kind—even using the actual pinholes that Yuan astronomers had at their disposal—was obviously tedious work, but manageable if guided by predictions of the day on which the event might be visible.[94]

A matter that comes up from time to time through the early history of astronomy in China is reporting calculated eclipses as observed. My comments on pertinent sections of the Evaluation offer several examples. This expedient arises from the contrast between the relative ease of predicting a solar eclipse and the extreme difficulty in early systems of determining whether it will be visible from a given site. The earliest effective means of prediction evidently involved counting off 135-month cycles from an observed eclipse (see p. 106). As computations of eclipse limits and then apparent solar and lunar motions became more accurate, there were still enough

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[94] For a general discussion see Po Shu-jen 1983; for an example of the use of oil, *Yü hu ch'ing hua* 玉壺清話, 1: 6.

failures that some practitioners chose to cover them up. Misrepresentations occasionally occur even in the records that the Evaluation of the Season-Granting system used. See, for instance, the discussion at the end of section 9.3 of the Evaluation.

Accuracy

Most attempts to gauge the accuracy of old eclipse techniques have not gone beyond assessing the accuracy of their constants. A recent monograph by John Steele has taken an important further step. It has surveyed all known early Chinese eclipse observations and predictions that specify time as well as day. These records fall into scattered groups, but Steele's analysis, summarized in table 2.10, indicates regular (but not at all linear) improvement in accuracy of both recording and forecasting.

One may compare with the mean error of the Yuan computations—about 18 minutes—that of calculations for solar eclipses after 1644 by Jesuit missionaries and their successors in the imperial palace. A recent study indicates that mean error was (in modern terms) 14–16 minutes until 1742, 8 minutes until 1859, and 2 minutes thereafter. In this respect, by the end of the empire (1911) European innovations yielded flamboyantly slow progress.[95]

Table 2.10. Accuracy of Observed and Computed Eclipses Fifth to Thirteenth Century

[**A:** Dates of timed eclipses detailed in the source

B: Title of the source. All are Standard Histories except the *Hsuan ming li,* the astronomical system used in Japan to compute eclipses, and the *Ch'ung hsiu ta ming li,* the predecessor of the Yuan system

C: Number of timed solar eclipses in the source

D: Number of timed lunar eclipses in the source

E: Average error of time in report of observation, hours

F: Average error of prediction, hours]

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[95] Lü Ling-feng 吕凌峰 & Shih Yun-li 石云里 2003; cf. Stephenson & Fatoohi 1995.

A	B	C	D	E	F
434–40	Sung shu 宋書	-	5	2.9	2.7
585–96	Sui shu 隋書	2	9	1.8	2.3
761–68	Chiu T'ang shu 舊唐書[96]	2	-	0.52	-
937–1526	Hsuan ming li 宣明曆 (#50)[97]	-	73	-	0.98
1168–1245	Sung shih 宋史	2	3	0.7	0.5
434–1280	Ch'ung hsiu ta ming li (#76)[98]	28	22	0.4	0.5
Same	Yuan shih[99]	28	22	0.34	0.27
Average, solar	-	-	-	0.41	-
Same, lunar	-	-	-	0.52	-
Average, solar, 1040–99	-	-	-	0.16	-
Same, lunar	-	-	-	0.21	-

[The data in this table come from Steele 2000, Chapter 6.

Although the accuracy of eclipse computations with the *Hsuan ming li* degrades quickly for events far from its epoch, the Season-granting system remains accurate over several centuries.

There are also timed records for the period 1040–99 in *Wen-hsien t'ung k'ao*, eight solar and twenty lunar. They include numerous scribal errors. Equally important, Steele's comparison of its records with those that also appear in the Evaluation indicates that time-recording practices of the former are discrepant and apparently idiosyncratic (Steele 2000, 201–208). I therefore exclude this source from the evaluation.]

Stars

We have seen that, for recording celestial events, the 28 lunar lodges provided locations in what amounts to right ascension, and astronomers used *tu* and various other measures to specify distance

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[96] Of the two eclipses reported, the timing of the second is so far off (1.33 hours) that it implies a badly calibrated clepsydra. The three times for the first eclipse show an average error of only 0.25 hour.

[97] These Japanese timings purport to be observations, but were actually calculated using the Chinese Extending Enlightenment system (#50, 822, official in Japan 861–1684).

[98] These data regarding the Season-granting system's predecessor (#76, 1080) come from the Evaluation.

[99] These are the corresponding data for the Season-granting system.

at right angles to the equator and the other main orbital circles. The fixed stars played no regular part in astronomical reforms. Nevertheless astronomers carried out stellar surveys fairly often, especially over the last millennium, preparing tables that often became the basis of star maps. This work, like many other kinds of astronomical enterprise, garnered support as part of the state's assertion of symbolic control over the cosmic order.

Early historians ascribed the first census of asterisms and stars to two obscure astrologers in local courts of the fourth or third century B.C., Shih Shen-fu 石申夫 and Kan Te 甘德. The surviving documents give lunar lodge location and polar distance (crudely speaking, right ascension and declination) for the determinative stars of 120 asterisms. There has been much debate about when these observations took place, but independent studies using diverse quantitative tests indicate a date of about 70 B.C. The texts ascribed to Shih, Shen, and a third pre-Han astrologer have been greatly altered in transmission. One can only agree with Sun Xiaochun 孫小淳 that the extant compendium entitled *Shih shih hsing ching* 石氏星經 (Astrological treatise of Master Shih) is "not a book, but a title used by historians for all astronomical source material from [the time of Shih] up to the Tang"—that is, until it was included in a collection of 718. P'an Nai 潘鼐's reconstruction of the three treatises indicates that, in their final form, they identified 1464 or 1465 stars, a number that no later Chinese survey notably exceeded.[100]

The "Book of Celestial Offices," in *Records of the Grand Scribe* (circa 100 B.C.), records the oldest large-scale star survey of which we have the results. It is a comprehensive handbook of court astrology that concentrates on the interpretation of every kind of celestial event. The chapter on stars lists 530 of them, without coordinates or other astronomical details.[101]

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[100] Sun & Kistemaker 1997, chapter 3, especially p. 39; Sun 1994; P'an 1989, table 30, pp. 102–9 and, for a comparison of various surveys, table 99, pp. 414–24.

[101] *Shih chi* 史記, ch. 27, "T'ien kuan shu 天官書"; annotated edition in Kao P'ing-tzu 高平子 1965. *Shih chi* is the first of the Standard Histories, and this is the first such astronomical treatise.

Gradual shifts in the equatorial locations of the lunar lodge determinative stars (due to the Annual Difference) motivated fairly frequent observational studies. I-hsing (early eighth century), armed with a new armillary sphere with circles for the ecliptic and lunar orbit as well as the equator, set a new standard in data-gathering. He observed not only the lodge determinatives but other stars whose positions did not accord with those in old records. He measured their ecliptic as well as their equatorial coordinates.

I-hsing inspired a spate of new instrument-building between circa 976 and circa 1106, including at least eight new armillary spheres (see p. 176), as well as attempts to make stellar surveys more comprehensive while improving the data on locations. In the same period, there were three surveys of the whole visible sky, one of part of the sky, and three of the lodges. The Yuan reform also included a survey of stars and constellations, which did not become part of the Season-granting treatise.[102]

Precision and Accuracy

A fundamental distinction in the exact sciences is that between precision and accuracy. The first expresses the fineness with which one performs an operation or expresses a measurement; the second measures how well the result of a measure conforms to a true or standard value. It is easy to read the time on a run-of-the-mill watch to the nearest second, its precision; but if it is carelessly set, the time it tells may be inaccurate by hours.

What limits *precision* are measuring tools and practices and computational procedures. The Season-Granting System recorded time, for instance, to the nearest mark, an interval generally equivalent to 14.4 minutes. The precision is 1 (that is, ±½) mark. But it would be a mistake to conclude from that figure (which is simply 1/100 day) that all times are correct to the nearest four tenths of a minute. Expressing a mark as 15 minutes would be less accurate, but would

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[102] P'an Nai 1989, 140–46 and chapter 6; Po Shu-jen 1996, 19–20. For surviving celestial maps see Harley & Woodward 1994 and Ch'en Mei-tung 1996. On the Yuan survey, see chapter 6 (p. 235) below.

give a clearer idea of its precision. The *accuracy* of a given computation may be a great deal less than ±½ mark, depending on how it was derived and with what standard one compares it. The accuracy is unlikely to exceed the precision except by accident. The only standard available to ancient astronomers was observation, but empirical records too differed considerably in precision and accuracy. For instance, a very precise armillary sphere, if carelessly mounted and not calibrated, might yield consistently inaccurate readings. We will encounter this problem in the historical account below.

Historians today draw on the outcomes of modern computation, which is extremely precise. Its accuracy, although in most respects high, is limited. For events in the distant past, as I will show in the next subsection, some components of inaccuracy are irreducible.

Chang P'ei-yü's 张培瑜 reference book of 1997 is by far the best for ancient dates and certain basic astronomical phenomena. It lists such results as the greatest magnitude and time of an eclipse at eight ancient capitals to a precision of roughly a minute in time and 0.01 (0.08 inch) in magnitude. Chang finds that the absolute accuracy of the moon's position in the first century B.C. is only within 10 minutes (see also Stephenson 1997). With all this in mind, I have sought to record quantitative results in forms that avoid a spurious impression of accuracy or, for that matter, of precision.

The Enigma of Delta-T

Today's methods of computing ancient astronomical phenomena give much more accurate results than any ancient astronomer could dream of. But one limitation of the modern computations furnished here calls for comment.

Astronomers can compute the phases of a solar eclipse that took place 2000 years ago to a precision of a few seconds. Accuracy, however, is another matter. The main source of uncertainty is the variable ΔT. Since it was defined, specialists have been debating—or, to be more precise, talking past each other about—how to determine it. There is still no generally accepted standard.

Because of fluctuations in the rate of the earth's rotation, the length of a day based on the sun's travel changes gradually and not quite uniformly.[103] For that reason, mean solar time measured from one point on earth—Universal Time (UT), based on Greenwich Mean Time—is not satisfactory as a basis for computing celestial phenomena far in the past or future. Astronomers have come to use a measure independent of such variations, called since 1952 Ephemeris Time (ET), since 1984 Terrestrial Dynamic Time (TD), and now based on atomic timekeeping. Great effort and ingenuity for half a century have gone into defining the quantity ΔT, which for a given moment defines the difference between the invariant measure and the varying Universal Time (TD – UT). Records of observed eclipses make it possible to trace the value of ΔT in the pre-telescopic past.

The International Astronomical Union defined the term in 1952 as

$$\Delta T = 24.349 + 72.3165T + 29.949T^2 + 1.821\mathrm{B} \text{ seconds},$$

where the B term is negligible and T is the interval in Julian centuries from A.D. 1900. Standard reference works of the next generation such as the *Canon of Solar Eclipses* by Hermann Mucke and Jean Meeus (1983) used this definition, but astronomers today tend to prefer proprietary formulas.[104]

F. Richard Stephenson has devoted much of his career to tracing changes in astronomical constants. He has repeatedly sought formulas for ΔT that closely fit records of ancient eclipses set down in various parts of the world. The formulas of other well-qualified investigators yield results that differ appreciably from his for ancient eclipses. Still other values, for instance, those in the last release of the outstanding planetarium program, Starry Night Pro 6.2, do not agree with any of these formulas.[105]

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[103] The causes are, among others, tidal friction and changes in sea level due to variation in the polar ice caps. See Espenak & Meeus 2007, 17.

[104] See Mucke 2003 for a good discussion of the problem.

[105] See, for instance, Stephenson & Yau 1992 and Stephenson 1997. For different results, see Mucke & Meeus 1983 for the International Astronomical Union, and Pang et al. 1998. The uncertainties vary between dif-

In 2004 Stephenson abandoned the quest for a single formula. He published a table of ΔT uncertainties century by century, based on a comprehensive collection of recorded observations. The authoritative eclipse canon of Espenak and Meeus reproduces his table with negligible modifications.[106]

On the other hand, even if a consensus eventually develops, considerable uncertainty on another level will remain. As we know from studies of recent phenomena, the change in ΔT includes, as Meeus puts it, "unpredictable irregularities … the UT is not a uniform time." Astronomers treat it as regular simply "because they need a uniform time scale for their accurate calculations." The short-term fluctuations which their formulas must ignore when exploring the pre-modern past sensibly affect the timing of eclipses. As Stephenson has shown, with respect to most historical eclipses we have too little evidence to cope with these changes consistently in computation.[107] For instance, Morrison & Stephenson's graph for solar eclipses shows that values for ΔT derived from solar eclipses at the beginning of the first century A.D. vary over about four hours.[108]

With these uncertainties in mind, for eclipses *anno domini* in sections 9.3 and 9.4, I have computed times and magnitudes using the I.A.U. formula for ΔT and the Besselian elements in Mucke & Meeus 1983 for the solar eclipses. For the lunar eclipses I have used the data for TD of maximal phase and semi-duration in the same source. In view of the considerable uncertainty, the evaluations in section 9 can be only general and tentative.

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ferent formulas by over 40% for a well-documented solar eclipse of A.D. 689, and nearly 30% for one of 1107.

[106] Morrison & Stephenson 2004, 332; Espenak & Meeus 2007, 13. Rather than the usual parabolic curve-fitting, Morrison and Stephenson fit a more flexible curve by cubic splines. Still, the curve is fitted to highly varying data.

[107] Stephenson 1997, Pang et al. 1998, and Mucke 2003. The quotations are from Meeus 1998, 77.

[108] P. 330, figure 2; see also figure 1, p. 229, for large divergences in values based on telescopic observations.

Comparisons with Europe and Islam

Before it is feasible to broadly compare the astronomy of China with that of other cultures, studying the circumstances of such work in each culture, as well as the thought behind the technical procedures, is essential. Most comparisons have been restricted to mathematical accuracy. Such narrow conclusions are misleading since, as we have already seen, the many dimensions of astronomical enterprise are parts of an intimately intertwined whole. Let me encourage adequately compendious comparisons by listing a few salient points of similarity and difference, proceeding from the political to the mathematical. They concern the whole period over which computational astronomy evolved in China up to the Season-granting reform; that is, from roughly 200 B.C. to A.D. 1280. This survey will not do justice to Indian astronomy, about which there is too little work that meets modern standards (particularly, but not only, of chronology and dating of sources) to allow a broad range of juxtapositions.

Politics

Over the millennium and a half up to 1280, neither what we now call the European nations nor the Islamic domains were a single political unit. As people saw it circa A.D. 200 from the cultural sphere of Greek astronomy around the Mediterranean sea, most of Europe was barbarian wilderness. In the aftermath of Alexander's conquests, which spread Greek technical culture, parts of North Africa and the Middle East were centers of it while there was none in Europe north of the Alps or West of Italy. By 600, Europe, with few exceptions, had rejected classical culture and driven it eastward. By the thirteenth century, its most advanced kingdoms were still recovering from that loss. The astronomical centers of the Muslim world were politically independent of each other until the Mongol conquests. Western astronomers worked under a great variety of governments and with very diverse relationships to them.

On the other hand, at the end of this period China remained the same political unit as in its beginning. Over the long period we are considering, there was some expansion of borders, and some brief periods of division into two or three polities. Still, the dynastic system maintained its authority. Every ruler claimed sovereignty over All under Heaven, even when more than one regime coexisted. Even the Mongol rulers took on the persona of an orthodox emperor—at least that was the persona Chinese saw. Astronomers did their work mostly as officials in the centralized bureaucracy of a regime that did not permit independent local governments. The astronomers who did not belong to the civil service were, in their writings, usually making a case to be given a post, or criticizing official practice in order to urge adoption of an alternative.

Culture

Each of the three areas had the advantage of a common language of learning. In the Islamic world this was Arabic, although many Iranian scholars wrote in Persian. In Europe the shift from Greek to Latin was complicated, but much of classical culture remained more or less intact until circa 500. The enormous alterations in local cultures depended on much more complex causes than linguistic change. The incorporation of vernaculars greatly affected patterns of thought, but this was not a large factor in scientific change or other aspects of formal learning by 1280.

In China, at any time up to the early twentieth century, anyone who could read a freshly written astronomical document could read one written in 100 B.C. In belles lettres one can find shifts comparable to that between classical and scholastic medieval Latin, but writing on technical subjects, despite the evolution of its content, changed little in style or syntax.[109]

Among other things, these common linguistic bases nurtured something approaching homogeneity of learned culture within each

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[109] This generalization needs to be qualified for manuscripts before the second century A.D., of which many have been excavated since the mid twentieth century. Orthography (or its Chinese equivalent) was gradually standardized after that era. Most authoritative early books were printed soon after A.D. 1000. Shaughnessy 1997 and Galambos 2006 discuss the problems from different viewpoints.

of the three areas, more so in China than in Europe, and more so in Europe than in Islam. The transformation under way in the countries of Europe by 1280 was largely due to borrowings from Islamic Spain. Its level of secular scholarly culture was still much lower than in China or the Muslim world.

Livelihood

In all three cultures, the work that kept most astronomers employed was astrology. In all three, astrologer and astronomer was a single occupation, a specialized kind of diviner. But the character of livelihoods differed greatly. In the Greek world, those who wrote on astronomy and astrology were private practitioners or teachers. In medieval Europe they were teachers or clients of local rulers. In the Islamic world they taught, depended on patronage, or served mosques in some capacity that involved conventional astronomical problem-solving. In China, ruled as a centralized empire by the time mathematical astronomy was well under way, practitioners were usually technical functionaries. Most of the well-known figures mentioned in this book were at the middle level of officialdom, and the rest were employed in lowly posts. Some, particularly in the Yuan period, were high officials for whom study of the heavens was an avocation. Whatever their status, they had secure and stable civil-service careers of a kind that were out of the question for their counterparts elsewhere. There was much risk in such careers, for there were no limits on imperial power, and factional battles were endemic in the palace. Still, the political casualties were mainly at the top of the ladder, higher than astronomers generally reached. Nevertheless, most technical functionaries shunned breaking with established practices.

Another important difference has to do with conventional roles. In early Europe, most of those who taught astrology presented themselves as philosophers or mathematicians. Aristotle divided the responsibility: philosophers defined physical reality, and astronomers fitted their data into metaphysical frameworks that they were not entitled to disturb. With the coming of the universities, as the subject found an institutional place in the faculty of philosophy, philosophers continued to set out the framework of the heavens that astronomers filled in with observations and computational techniques. This division of European labor, which allowed mathematical astronomy no role in defining the cosmos, continued until long after 1280. In the Muslim world, astronomers began to challenge this bar from about 1250 on. In China, however, there was never a dichotomy of natural philosophy and technical astronomy. Practitioners who looked beyond solutions to technical problems worked out their own cosmologies—a wide range of them in various periods, which historians have mostly neglected.[110]

These differences affect what people wrote and how they wrote it. Chinese writing was on the whole oriented toward the civil service. Technical officials spoke with the voice of the government, but they were not heard outside the court. They practically never wrote for the public. What we know about their practices comes largely from the official histories, written after the end of the dynasty in which they worked. Personal literary collections and compendia of jottings—most of them published posthumously—supplement this data to some extent, but are of little value by themselves for reconstructing astronomical practice. [111]

By 1280 printing on paper had become the usual mode of publication in China, but had not yet begun in western Eurasia.[112] In the Islamic world, after Ptolemy's *Almagest* became available circa 800,

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[110] Again Ch'en Mei-tung is an exception. See his survey covering the late thirteenth to late fourteenth centuries in 2003a, 545–54.

[111] Conventional literary collections, such as *Su wei kung wen chi* 蘇魏公文集, by the technical polymath Su Sung 蘇頌 (1020–1101), contain much less data of technical interest than one would expect. On the other hand, *Meng ch'i pi-t'an* 夢溪筆談, the jottings of his contemporary Shen Kua 沈括 (1031–95), is an exception—a main source of qualitative information about technical enterprise of the time.

[112] Paper reached the Muslim world by the seventh century, but was not manufactured there until the twelfth, and printing did not follow before modern times; it reached Europe in the tenth century, was made there in the twelfth century, and was used for printing from the fifteenth century on. See Tsuen-hsuin Tsien (1985) in Needham et al. 1954– , 5 part 1: 293–319.

teachers of astronomy compiled many introductory works, and from circa 900 on, attempted to improve upon the classical heritage. Regardless of whether the authors were clients of the powerful, employees of institutions, or private practitioners, they wrote as individuals, and their readership depended on their personal networks of support. In Europe, after the general collapse of high culture between 200 and 600, no advanced work was available until Ptolemy was translated from Arabic into Latin in the twelfth century. The treatises, handbooks, and textbooks written from antiquity to the 13th century were to some extent intended for students or patrons and, beyond that, available to the few individuals who could read and afford manuscripts.[113]

Social organization

We actually know very little about the organization of astronomy as a vocation before 1280 in Europe and the Islamic world. It is clear that practitioners valued their relationships with colleagues, but most worked alone or with their masters or disciples. The emphasis on texts by most historians has resulted in very little attention to personal interactions. For instance, how many Alexandrian astronomers worked at that city's Museum, often described as a great library and research institute, and in what sorts of relationships, remain a mystery.

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[113] On the transmission of astronomy between cultures, see Montgomery 2000. A number of scholars, on the basis of careless reading, have argued that Ptolemy's *Almagest* and Euclid's *Elements* were available in China at the time of the reform; see, for instance, Yen Tun-chieh 严敦杰 1943. The evidence for this claim comes from a list of books in Arabic or Persian in the Islamic observatory in 1273, which gives transliterated titles or authors' names, and Chinese descriptions (*Yuan Mi-shu-chien chih* 元秘書監志, 7: 13b–14b). In this list, "Wu-hu-lieh-ti 兀忽列的" sounds like "Euclid," but the book is devoted to "computational methods (*suan fa* 算法)". "Mai-che-ssu-ti 麥者思的" also resembles "al-Majisti," Latinized as *Almagest,* but the description, "Standards for construction of astronomical instruments (*Tsao ssu-t'ien i shih* 造司天儀式)," rules out that identification. Only about one per cent of the Almagest is devoted to the construction of an armillary sphere and a parallactic instrument. These impossible identifications are among many in the wishful study of Tasaka (1942), carried out in the unpropitious circumstances of wartime Japan.

Chinese astronomers, as I have already made clear, belonged to technical branches of a very large and complex civil service. As Chapter 4 (p. 169) shows, the Astrological Commission that carried out the Yuan reform had a staff of at least 121—and it was only one of three astronomical bureaus in the capital. All of its members were salaried, even the students.

Practitioners of "yin-yang" who were not officials worked as individuals or with their masters or disciples. Those in such relationships saw teaching as the transmission of technical classics or other established teachings. That, like civil service status, tended to discourage iconoclastic behavior. On the other hand, occasionally someone challenged the official astronomical system, a risky step but one that sometimes led to a high post among those available to technical personnel. In some cases challengers presented themselves as innovators, but normally, following convention, claimed to represent the superior methods of a teacher in their lineage. In either case, whether their aim was to improve government practice or leap into the civil service—more likely, both—they were attempting to join a very large-scale bureaucracy with distinct patterns of activity, communication, ascription, and reward.

Instruments and observation

In early times, as I have mentioned, Western instruments were mounted in the plane of the ecliptic, and those of East Asia in the equatorial plane. An equatorial mounting makes it easy to follow the diurnal revolution of the stars; an ecliptic one simplifies tracking not only the sun but the moon and planets, the orbits of which lie near that of the sun. As the European shift to equatorial coordinates in the sixteenth century implies, this is not a greatly significant difference, depending primarily on the design of instruments and habits of computation. Long before the thirteenth century, armillary spheres across Eurasia carried circles of both kinds.

There was a great deal more to the armamentarium of the observatory than armillary devices, as the discussion of Islamic and Chinese instruments in chapter 5 implies. One of the more obvious differences was the habit in western Eurasia of telling time at night by the stars, using astrolabes and other instruments, and the preference for water clocks in China.[114]

Another salient difference was the Chinese use of gnomons for determining the winter solstice. Despite the difficulties that that poses (p. 256), the links of gnomons to antiquity gave them "ceremonial significance for the Chinese court," preventing their replacement by armillary instruments.[115] Kuo Shou-ching's Tall Gnomon, nearly 10 meters high, with a pinhole device to create a sharply defined image, overcame the limitations of conventional versions a fifth as high (see below, pp. 183–189).

In the thirteenth century, observation everywhere in the world depended on the naked eye. The Chinese standard of precision documented in the remainder of this book had no competition in Western Europe at the time—in all likelihood, not for another three centuries. Our ignorance of Islamic and Indian practice, and the poorly investigated utility of the very large instruments built in their domains, makes it impossible to draw broader comparisons.

The discussion of metrology earlier in this chapter has revealed that there were no universal measures, and that spatial and temporal units changed considerably over time. Even the Chinese divisions of the day differed from those of Western Eurasia. Copernicus escaped the utter confusion of European lunisolar chronology by counting Egyptian years of exactly 365 days, as his predecessors had done.

Detailed written chronologies in every era made ancient lunisolar dates usable in astronomy. This chronological continuity was possible because of China's centrally imposed order, as opposed to the socio-political chaos of European history.

In general, Europeans until the time of Tycho Brahe (1546–1601) were primarily interested in recording and computing the con-

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[114] I suggest in chapter 5, p. 206, that Kuo Shou-ching designed a nocturlabe under Islamic influence. There is no evidence that this instrument was widely used in China, or that it played a role in the astronomical reform.

[115] Nakayama 1969, 244.

spicuous phenomena of planets (stations, apogee, perigee). By the time of the Yuan reform, Chinese astronomical systems lacked powerful techniques for calculating planetary phases and positions at any desired time. Shen Kua proposed to record coordinates of the planets three times a night for five years. His proposal was not carried out. A proposal by two of the reform group argued for a program of nightly observations over twenty or 30 years in order to perfect the system, but it too was not carried out.[116]

Not until Tycho Brahe and his associates (including Kepler) did any European reap the full benefit of prolonged observations enhanced in both quantity and quality. Bureaucratic obstacles often impeded observational innovations in China, but there were none to stop a Danish nobleman with a few assistants and patronage—fitfully paid though it was—from a Bohemian king.

In sum, there were many differences, large and small, between practice in the chief centers at both ends of Eurasia. By 1280, European astronomers had not yet overcome the damage from the cultural debacle of a millennium earlier, but their backwardness by Chinese standards tells us nothing about their prospects for the long run. Although incorporating the Muslim and Indian centers of astronomy into such a comparison would be highly desirable, too much of the research on technical practice remains to be done.

Astronomical computation

In mathematical matters the differences were prominent and important. Greek astronomy drew on a tradition in which geometry predominated. Ptolemy lacked trigonometric functions, but used arc-sagitta ratios and chords to deal with a variety of tasks in plane and spherical measurement within a geometric perspective.[117]

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[116] Sivin 1995c, p. 18; for the proposal, see below, p. 166. Kuo's disciple and successor Ch'i Lü-ch'ien responded to the spirit of this proposal when, in 1323/1325, after the system had been used for nearly half a century, he carried out a daily series of sun shadow and planetary observations in order to test its accuracy. He did slightly adjust a solar constant. *Yuan shih,* 172: 4031.

[117] See the example in Toomer 1994, 7–9.

This geometric approach strongly influenced the techniques of Indians in the Hellenistic period, and of Muslims from the eighth century on. True trigonometry, algebra, and other tools entered Europe via the Islamic world centuries later, and became even more sophisticated.

Chinese astronomy and its counterparts in Korea, Japan, and Vietnam are often called algebraic in approach but, in view of their diverse techniques, "numerical" is more apt. The first computational systems, up to circa A.D. 85, largely relied on simple numerical cycles and linear interpolation. As the Season-granting system shows, astronomy evolved over the centuries to include such methods as equations of higher order and third-order interpolation. But problems such as predicting a solar eclipse, which amounts to studying the intersection of a conical shadow with a spherical earth, are considerably easier to deal with using tools of the Greek type. In the eleventh century Shen Kua began using arc-sagitta methods to deal with problems, particularly in eclipse prediction, that today would fall into the province of spherical geometry or trigonometry. The Yuan system elaborated such methods. There is no evidence of influence from abroad, and indeed the Yuan approach remained geometrically less sophisticated than Ptolemy's a millennium earlier.[118]

All of the civilizations I have discussed relied on computing devices, whether counting rods and matrices of some kind or reckoning in dust or sand. The abacus was used regularly in the Roman world, and vague descriptions of what may be some form of abacus occur in a Chinese mathematical treatise variously dated in the very late second, early third, or late sixth century. The indubitable abacus appears in the fourteenth century, and came into common use beginning in the second half of the sixteenth century. It is convenient and fast for simple logistical manipulations. It quickly became the norm in mercantile culture. When it replaced counting rods, the ease with which mathematicians had dealt with problems in multi-

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[118] On Shen, see Sivin 1995c, 19; on proto-trigonometry see the Canon, section 4.17.

ple unknowns was lost until the rise of computing with writing instruments and paper. At the time of the Yuan reform, use of rod arithmetic was still the norm.[119]

Records

Chinese governmental organization made it feasible, despite the enormous loss of documents to social turmoil and natural catastrophe, to compile and maintain records of observations and computational methods spanning many centuries (discussed in chapter 6). In Greek antiquity and the Middle Ages there was no such continuity in society and government. Hipparchus, early in the Greek astronomical tradition, was sensible enough to draw on Babylonian constants. Ptolemy of Alexandria, roughly 300 years later, used Mesopotamian observations from 747 B.C. on—that is, over 900 years before his own time.[120] That continuity was soon lost in Europe until the recovery of Ptolemy's *Almagest* from the Islamic world in the twelfth century.

The physical form and medium of the earliest records are uncertain except for Mesopotamia and China, for both of which actual examples survive. From Babylonia we have mathematical tables as well as omen and other texts in cuneiform script (that is, in combinations of slender wedge shapes) impressed on clay tablets. Many Chinese records of both observed and computed events from before the first century B.C. have been excavated in the last four decades. Some are written on silk, and some on narrow wooden strips that had to be strung together to make documents of any length. Silk was too costly for routine use. Archeologists have unearthed a fair proportion of silk documents, but that is because the writings come chiefly from the graves of aristocrats and officials. Paper, although invented in the third century B.C., remained too expensive to replace wood until the third century A.D. or shortly before.[121] From Greek and Roman antiquity no original astronomical documents

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[119] Martzloff 1988, 210–16.

[120] Neugebauer 1962, 157; Toomer 1994, 9.

[121] Tsien 1985, 86.

survive. The difficulty of dating early Indian manuscripts leaves this question open.

Ptolemy, author of the first extant Greek handbooks of astronomical calculation, avoided the inconvenience of writing numbers with letters of the alphabet—the Greek norm—by borrowing Babylonian sexagesimal notation for fractions. He retained, in combination with it, the unit fractions (made by combining fractions with 1 as numerator) conventional in the Hellenistic world. For instance, he expressed ¾ as ½ + ¼ rather than, in the Middle Eastern mode, as sexagesimal ;45 (that is, 45/60). The latter was a rather hermetic notation, but one that made elaborate computation easily feasible for those who learned it. Muslim astronomers later found it worth while to adapt this notation to Arabic and Persian. Chinese astronomers, as soon as they began recording the coordinates of celestial events, used the decimal system described earlier for whole numbers, with simple fractions for less than a *tu*. The Season-granting system used decimal numbers for fractions as well as integers.[122]

There were great libraries that collected technical books in the ancient Mediterranean region as well as in China, but comparisons are not easy to draw. The best known of the west, those of Alexandria in Egypt and Pergamon in what is now western Turkey, were funded less to enable research than to earn for their peripheral cities renown as cultural centers. Their rulers built reputations as patrons, and their collections were rich, but there is remarkably little evidence that their most famous intellectuals worked in their libraries. We have no idea whether Ptolemy was among those who did. These institutions did not survive (at least as cultural institutions) into the Middle Ages. Astronomical collections did, in Islamic centers such as Marāgha in Iran and Samarkand in Central Asia.

The Chinese libraries catalogued in six of the 25 Standard Histories documented nothing more or less than the reference collections of the central administration, comprising books of every sort, old and new, that made their way into the court's repositories, as well

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[122] Toomer 1994, 6–7, and, p. 68 above.

as the compilations and archival records of its agencies. The variation in quality of the catalogues shows that their curatorial standard varied greatly from one period to another. Like their western counterparts, the palace libraries were in no sense public. On the other hand, they were available to officials in the capital on what today would be called a need-to-know basis. The histories indicate that most of those in charge liberally interpreted that criterion.[123]

Comparing the catalogues of different eras makes it clear that a large proportion of the books listed in one did not survive into the next. This was the norm when dynasties changed as a result of war but, even when the transition was pacific, the wooden construction of palace buildings invited fires, natural and manmade. As Kuo Shou-ching's biography indicates (p. 588 below), the extant Season-granting treatise in four chapters is all that survives of the fifteen books in 105 chapters that made up the official archive of the reform.

Despite this attrition, however, it is obvious from the history of the reform project in chapters 3–5 below that its staff began it by gathering and studying the rich and detailed records of their predecessors' observations and computational techniques. There was nothing remotely comparable to this millennial trove of data in contemporary astronomical centers outside China.

Finally, the great volume of scholarship devoted to reconstituting and studying the rich and uninterrupted Chinese astronomical record up to 1280 is unequalled in Europe, where the primary literature is much smaller, and less of it is available in forms that facilitate study. The excavated literature of ancient Mesopotamia has been ingeniously studied by the few scholars able to read it, but they believe that its quantity is miniscule by comparison with what remains buried. For Islam and India, the few historians of astronomy have made significant but limited headway on the large, still

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[123] For which library catalogues (that is, bibliographic treatises) have survived, see Wilkinson 2000, table 31, p. 512. There is a fairly large literature of supplements to those that have survived and reconstructions of those that have not. E.g., for the Yuan, four reconstructions have been generally available, of which Lo Chu-yun 雒竹筠 1999 is the most useful.

mostly unprinted literature, and there is no immediate prospect of their number increasing.

Conclusion

The Season-granting system culminates a long, thickly textured, minutely documented tradition. That, in fact, is how it presents itself. Almost every section of the Evaluation begins with a review of how predecessors dealt with the problem and how the Yuan reformers went beyond them.[124]

Kuo Shou-ching's biography quotes his extensive summary of his predecessors' main innovations over more than a millennium, the basic tasks of research that underlay the new system, and its most important accomplishments (see below, pp. 579–588). The excerpt takes up about a third of his biography. The author of the biography, like the historians who compiled the *Yuan History's* Season-granting treatise, found it essential to poise the reform atop a tradition. That was a conventional way of introducing any reform.

These accounts of the past resemble the first document in the history of European science, Aristotle's account of his predecessors in Book I of the *Metaphysics.* There the philosopher, as he describes earlier work, consistently portrays his own as the endpoint of it all. In order to do so, he reads into the inquiries of previous thinkers issues that were important to him but that what we know of their own words does not reflect. He leaves his reader with an impression of missed opportunities up to his own time, an impression quite missing in Kuo's own writing.[125]

Neither Aristotle nor Kuo was motivated by a belief in progress. The doctrine that the long-term direction of history was upward, inevitably headed toward betterment, did not exist anywhere in the world before the European Enlightenment. Neither of the two authors affirmed that understanding was bound to improve in later

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[124] The exceptions are section 3, on lunar lodge extensions, and (more surprisingly) section 9, on eclipse predictions. The Evaluation does not take up planetary computation.

[125] Aristotle, *Metaphysics,* 983a–993a.

generations. They were claiming something quite distinct, unpredictable in ancient thought and practice, namely occasional innovation.

Aristotle was writing (or rather lecturing) for his pupils, in some of whom he awakened the aspiration to outdo him by launching new inquiries. Kuo was writing for the emperor. In principle, all Chinese officials could address their sovereign, but most could expect that only their bureaucratic superiors would read their reports. Kuo was one of the few astronomical functionaries in all of history whose intimacy with the monarch let him to be confident that the latter would want to know (even if he could not read) what he wrote.[126]

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[126] See above, p. 24.

3 The Project: Origins and Process

This chapter takes up the circumstances in which the astronomical reform began, and recounts the complicated sequence of events that accomplished it. In order to understand those events, it is essential to reflect on what was remarkable about the times.

Origins

The Yuan dynasty

Historians of China over the centuries thought of the Yuan dynasty, to use the words of the *Cambridge History of China* (1994), as a brief "alien regime" of no great significance. There is, however, a broader point of view. A modern student of the period began a study with a salutary reminder that "at its apogee in the mid-thirteenth century, the 'Great Mongol State' *(yeke mongghol ulus)* was the largest contiguous land empire in the history of mankind," and China was just one badly integrated part of its "sedentary sector."[127]

According to the conventional enumeration, the Yuan period succeeded the Sung as the legitimate dynasty in 1279, and the Ming took its place in 1368. Both of these transitions concern us, since the Season-granting system came into being to ritually mark the first change, and maintained its authority after the second.

The Mongols' power in China began much earlier. They differed from earlier and later "alien" dynasties from the north that ruled China. The latter were from the forest lands of Manchuria, not the central steppe where the Mongols lived. Such regimes came into being to extort wealth from Chinese empires by "pillage, tribute payments, border trade, and international re-export of luxury goods." They resorted to warfare—meant to devastate—only with governments that refused to pay up, or who broke their agreements.

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[127] Franke & Twitchett 1994, subtitle; Allsen 1983, 243.

The relationships Mongols preferred were mutually beneficial. Large gifts—a share of tax income—guaranteed the Chinese peace and quiet; warfare, which was cheap for tribal nomads, was expensive for them. The Mongol leaders shared their largesse in the form of luxuries and other marks of status with chiefs of thinly spread tribes with rudimentary economies, who could challenge the Chinese state only as part of a large confederation. The northern "shadow empires" that rose and fell alongside strong Chinese dynasties sought a profitable but distant relationship, for governing would overstrain their manpower and skills. It was only when the polities to the south fell apart, in turn threatening the survival of the nomadic leaders, that some such confederations, willy-nilly, took over and declared themselves dynasties.[128]

The Jurchen people—originally from Manchuria—had occupied most of China's territory north of the Yangzi river from 1126 on. Over nearly a century, they built a dynasty on the Chinese pattern. Their leaders called themselves the Chin dynasty. Mongols from the Inner Asian grasslands, led by Chinggis Khan (reigned 1206–27), began substantial military campaigns against this regime about 1211.[129]

Chinggis launched his world conquest almost accidentally. The Chin dynasty had refused to yield to his threats, confident in its own aggressive army. At the same time, his marginal status among Mongol khans prompted him to take great risks in battle, the surest way to raise his prestige. He unified his allies, building a centrally run order with a highly disciplined cavalry.

By 1215 Mongol offensives forced the Chin to abandon their capital, near the present Beijing, and to reconsolidate in more defensible territory. Chinggis himself returned to the steppe, leaving others to govern his new territory. His sons continued his many campaigns, convinced by their successes that they could dominate the whole world. In 1234 Chinggis' son and successor as Great Khan, Ögödei

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[128] This understanding of shadow empires is based on the anthropological study of Thomas J. Barfield (1989 and 2001).

[129] Franke & Twitchett 1994 is an up-to-date account of this period.

(reigned 1229–41), annihilated the Chin, taking over almost all of the north. The occupation was "ironically, simply a consequence of … having completely destroyed the Jurchen Chin regime which they had planned to extort," and coping with the void that resulted.

From 1242 on, as Chinggis' immediate heirs died, the effort lost cohesion and momentum, but not for long. His grandsons surged forth from their grasslands after 1252 with remarkable and fearsome suddenness. They, like the previous generation, did enormous destruction to those who did not accept their demand to surrender. For instance, a fairly reliable estimate of the population of North China makes it only a third as large in 1290 as in 1207.[130]

Khubilai (1215–94) was only one of Chinggis' grandsons who were directing the Mongol conquest of the world. The khans were not at all united. The others contested his elevation to Great Khan in 1260, so that he effectively ruled only North China and the domains to the north of the Great Wall. With his sway thus limited, in that year he took the advice of his Han counselors and instituted a Chinese reign title and a number of imperial institutions. In other words, he represented himself as emperor of China. His capital remained in what is now Inner Mongolia. In 1266 he ordered the building of a new capital in the Chinese style, which became Ta-tu 大都, on the site of present-day Beijing. At the end of the lunisolar year that corresponds to 1271—actually on 18 January 1272—a document in classical Chinese formally announced a new dynasty, the Yuan 元. His paramount Chinese advisor, Liu Ping-chung 劉秉忠, took this archaic term for "great" from the first word of the *Book of Changes.* Khubilai gradually adopted more of the rituals and other usages of Chinese dynasties, claiming, like them, dominion over all of civilization. As we saw in chapter 1, the Southern Sung surrendered in 1276 and the Yuan took over its governmental functions.

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[130] Barfield 1989, 197. The estimate is that of Frederick W. Mote in Franke & Twitchett 1994, 618–22.

Only a refugee court's claims for the legitimacy of two baby emperors postponed the conventional transition until 1279.[131]

This was acculturation at dizzying speed. Before their conquest of Eurasia began, the Mongols had no use for money; "the barter system and seizure by force were the chief means of obtaining goods." They did not settle on written language of their own until 1269. This was nearly a decade after Khubilai solemnly took on the dignity of an emperor. Only seven years more passed until, with all of China in his hands, he ordered the astronomical reform that is the topic of this book.[132]

The special circumstances of education for Han Chinese in this period are also worth pausing over. The Northern Sung period (960–1127) had set a precedent for polymathy. A number of important generalist officials mastered, and used innovatively, a broad knowledge of what would now be called science, medicine, and engineering. Su Sung 蘇頌 (1020–1101) and Shen Kua 沈括 (1031–95) are merely the best known. Such breadth ceased to be frequent in South China in the twelfth and early thirteenth centuries. But in the northern domains of the Mongols, a range of technical skills, including medicine, divination, astrology, and astronomy, became not only prominent but something like the norm among scholarly Chinese.[133] The high regard of Mongols for artisans, diviners, and curers—much higher than for scholars—no doubt had something to do with this trend.[134]

These abilities became prevalent in part because most schools that taught the classics disappeared when the Mongols ruled the

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[131] For the text of Khubilai's edict see Langlois 1981, 3–4. For detailed accounts of his empire-building, see Rossabi 1988 and Franke & Twitchett 1994.

[132] On currency, Farquhar 1990, 445, and on the language, p. 127. See also Endicott-West 1989.

[133] On Su, see Chuang T'ien-ch'üan 庄添全 et al. 1993; on Shen, consult Hang-chou Ta-hsueh Sung Shih Yen-chiu Shih 杭州大学宋史研究室1985, Chang Chia-chü 张家驹 1978, and Sivin 1995c.

[134] The social status of craftsmen, unlike the others, has been badly neglected in Western studies of the Yuan, but see Chü Ch'ing-yuan 菊清遠 1935.

north. Khubilai became enthusiastic about officially supported schools, but founded only a few of them, none large. The common patterns of education were elementary tuition within the family and learned individuals teaching disciples. Of the astronomical experts introduced in chapter 1, Wang Hsun was mainly taught by his mother, and Kuo Shou-ching by his grandfather; Hsu Heng was among the many scholars who supported himself by taking on private pupils. The disappearance of the civil service examinations as a standard route to appointment did away with the main reason for a single orthodox curriculum. What teachers taught their older pupils depended largely on the peculiarities of their own experience and on their opinions about what would enable a livelihood.[135]

The cliché about "Confucians" knowing nothing but the classics and belles lettres is usually misinformed, but with respect to the Yuan it is risible. To see why, it is necessary to understand how Chinese lived under Mongol occupation.[136]

The Status of Chinese

The Mongols legally constituted their regime as a four-tier system. They ruled. Those highly placed could demand Han slaves, or agricultural lands with peasant households attached. Because theirs was not a literate culture, they relied on a second tier of Uighurs, Khitans, Jurchens, other West Asians, Persians and even Europeans such as Marco Polo—all of whom they called *se-mu jen* 色目人—for record-keeping and administrative expertise. This class, neither Han nor Mongol, shouldered the practical burdens of management and implementation, including direct oversight of Chinese officials.

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[135] There is a good discussion in Han Chih-yuan 韩志远 2003, 243–247. After the examinations resumed (in 1313/15), they were based once again on an orthodox curriculum (Bol 1997), although they did not notably increase access to office for Han Chinese (Elman 2000, 30–37).

[136] Most modern Sinologists use "Confucian" vaguely to mean any conventionally educated man, and various other undefined persons. I use the term here exclusively for initiates into formal (although sometimes putative) master-disciple ritual lineages. I have discussed the reasons for Northern Sung polymathy in Sivin 1995c. See also Needham et al. 1960, 5–9.

Han Chinese and others the Mongols had conquered in North China were the third class. Those of the south who did not become their subjects until 1276 were the lowest of the low.[137]

Administrative positions often paired a Mongol with a fully qualified official of another group, preferably the second. The Chinese-style bureaucracy had no power over the first two tiers. It governed only Chinese, primarily those in the Metropolitan Province ("the belly," *fuli* 腹裹), the large area surrounding the capital. The decisions of Chinese officials could survive Mongol opposition only when the will of the emperor backed them, one by one.[138]

Many powerful Mongols resolutely opposed giving Chinese executive power. Khubilai's reunification and reign succeeded largely because he realized that Chinese institutions were indispensable in that vast realm. He found Han scholars who were able to adapt them to a limited but important role in the discriminatory system. Just as important, he overcame the resistance of other Mongol nobles to that role. From the Mongol point of view, the purpose of government was to extract the wealth of the subjugated in return for allowing them to live normal lives. The architects of the Chinese system argued successfully to Khubilai at one stage after another that indulgence would encourage a stable order and in the long run would mean greater wealth for all. In this they were often at odds with a group of financial experts from Central Asia (Ahmad, the most famous, appears below), who had no sympathy for the Han people and exploited them without favor. The success of the Central

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[137] A portion of the south Chinese educated elite, despite their abysmal political status, were able to maintain their wealth and local social standing. The Mongols were not tempted toward hands-on administration of what they considered the highly exotic south. As a share of its great wealth poured into their coffers, they saw no reason to disrupt its social and economic structure. See Smith 1998 for an example.

Scholars over the centuries have criticized the notion of four classes as far from rigorous, and shot through with exceptions. Their improved analyses have also been vague and shot through with exceptions, and have not accommodated the juridical status of the original. For a review see Yip 1980.

[138] On the Metropolitan Province see Farquhar 1981, especially pp. 51–52.

Asians led to a couple of Chinese uprisings—which Mongols were not temperamentally inclined to countenance from surrendered peoples. After that, even Khubilai was of two minds in his support for what we might call limited Han self-rule. It is not surprising that in the long run the Central Asians predominated.

The four-tier hierarchy affected astronomy, as it did every other social activity. The role of economic inequality in encouraging mathematical study is obvious. After the career path that led via state examinations to the civil service disappeared, studies of numbers and of the stars remained attainable sources of livelihood. Gentlemen had studied them all along as useful accomplishments, even as a form of self-cultivation. Now, in Mongol-ruled North China, these skills offered pathways to respectable, adequate livelihoods. For instance, Hsu Heng, who eventually became the leading Confucian master of his time, apprenticed himself as a young man to a diviner when his prospects for a conventional career faded. The prognosticator's ability to think rationally about the future was not only a desirable skill, but was inherently fascinating. In China, from beginning to end, astronomy and astrology were closely linked to divination in a continuum of technical and interpretive skills.

In the south, until 1276, classical studies and belles-lettres remained the way to official careers and high repute. In the north under non-Han occupation, with those goals unattainable, a number of promising young men found "yin-yang" attractive. It yielded the astronomers who interest us access to the Mongol court, political eminence, and a secure place in history.

Between the Mongols' conquest of North China and their victory in the south, these circumstances favored outstanding mathematicians and astronomers. As conditions changed—particularly after Chinese officials lost the prestige that their influence on Khubilai had given them—the Yuan period ceased to be exceptional from the viewpoint of mathematics, astronomy, astrology, and divination.[139]

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[139] In addition to the experts on astronomy discussed here, the outstanding mathematician Li Chih 李治 (or Li Yeh 李冶), equally renowned as a litterateur, owed his official career to Khubilai. See Chan Hok-lam &

The Mongol social hierarchy also goes far toward explaining the remarkable fact that traditional Chinese and Islamic astronomy, despite their bureaucratic proximity, influenced each other so little. I will return to this topic in chapter 5 (p. 218).

Astronomy before the reform

Khubilai's enemy, the Southern Sung dynasty (1127–1278), was not distinguished for its astronomy. In its century and a half, it adopted eleven systems for official use. Only the first, the Era Epoch system (#71, 1106), which originated in the Northern Sung period and which the southern court continued to use until 1135, was outstanding for innovation. There was no important new instrumentation or observational survey in the Southern Sung, and later systems were derivatives of the first. The final reform of any substance came with the Attainment of Heaven system (#86, 1271), credited to Ch'en Ting 陳鼎, who after the Mongol victory worked on the Season-granting system.[140]

The Mongols who first invaded North China were not avid for exact astronomical prediction. They did not share the passion of Chinese rulers for ritual control over time and the cosmos. Nevertheless, the Mongol leaders learned that for any group which sought imperial dominion, calculating and issuing annual almanacs was a ritual *sine qua non.* Among the usages of their predecessor the Chin that the Mongol regime in North China adopted was the Revised Great Enlightenment system (#76, 1180). "Granting the seasons" was one of the many ways in which they asserted legitimate succession. They used that system in the north for nearly half a century until Khubilai replaced it with the Season-granting system.

Ho Peng Yoke in Rachewiltz 1993, pp. 316–35. On the dearth of first-rate mathematicians and astronomers later in the Yuan and in the Ming and Ch'ing periods see Kuo Yung-fang 郭永芳 1987. This needs to be qualified for the Qing; see Sivin & Fang 1976.

[140] For further details on the content of this section, a well-documented source is Ch'en Mei-tung 陈美东 2003a, 497–524. The very last Sung reform (#87, 1276), was unofficial and inconsequential.

Two other predecessors of the latter were unofficial systems of non-Han origin.[141]

One came from the time of Chinggis. Yeh-lü Ch'u-ts'ai 耶律楚材 (1190–1244) was an ethnic Khitan, descended in the ninth generation from the founder of the Liao dynasty. Yeh-lü's father, a high official in the Chin government, created the 32nd Year Epoch system (#77), which the Chin did not officially adopt. Yeh-lü Ch'u-ts'ai was a polymath, deeply learned in Chinese culture. Chinggis added him to his entourage because of his skill at "yin-yang." In 1220–21 Yeh-lü, accompanying Chinggis on his expeditions, found himself in Samarkand, where he competed in lunar eclipse prediction with a local expert. He incorporated some of what he learned in the Western Expedition Seventh-year Epoch system (#82, based on his father's treatise), but his khan saw no need for a new official system. Yeh-lü later became convinced that the "astronomy of the Western Regions *(hsi yü li* 西域曆)"—which could refer to anywhere west of Sichuan—had a better technique for planets than that of China. Presumably on this basis, he designed a second system that he gave a Uighur name, but we know nothing about its character. Under Ögödei, the next Great Khan, Yeh-lü became the equivalent of prime minister.[142] He strove to make the Mongol overlords of North China aware of Chinese needs, with occasional successes.

Yeh-lü's first system, the Western Expedition Seventh-year Epoch system, although affected by his stay in Central Asia, did not differ fundamentally from the Revised Great Enlightenment system. It improved some constants and tables. Its main innovation was the League Distance Correction *(li-ch'a* 里差), which its author applied to adjust local time for differences in longitude. The method behind this correction was sound but, due to error in estimating the distance from Samarkand, it was about 40% too low.[143]

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[141] The historical sketch at the beginning of the Evaluation discusses both (pp. 251ff).

[142] Among the Mongols before Khubilai, such Chinese titles labeled not an officeholder but the crony of a leader.

[143] More precisely, it adjusted for differences in right ascension. Igor de Rachewiltz provided an extensive biography of Yeh-lü in Rachewiltz 1993,

Khubilai became intrigued with astrology early, and not only that of China. Before becoming Great Khan, he had gotten to know a Syrian polyglot named Ai-hsueh 愛薛. Khubilai “in 1263 commanded Ai-hsueh to take charge of a bureau for the astrology and astronomy, as well as medicine, of the Western regions. Afterward, when he was reassigned to the Muslim Medical Office, he was ordered to remain in charge of the bureau.” That is as much as we know about this astronomical institution.[144]

Khubilai next expressed interest in a second forerunner of the Season-granting system, imported from the Muslim world. Historians usually assert that Khubilai’s brother Hülegü sent Jamāl al-Dīn to China from the newly built observatory at Marāgha in northwest Persia, but there is no solid evidence for such a connection. Jamāl is mainly known for the seven typical instruments that he built *(tsao* 造) for Khubilai in 1267.[145]

In 1267 Jamāl submitted to Khubilai a Myriad Year system (#85), which used the twelve zodiacal signs instead of the lunar lodges, and a system of 360 degrees. Khubilai did not adopt Jamāl’s system. The Evaluation remarks that the monarch “to a small extent *(shao* 稍) promulgated it,” intimating that he had a few almanacs

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135–75. It lists the main Chinese sources. Li Kuo-ch’ing et al. 1977 have studied Yeh-lü’s work, and Sun Hsiao-ch’un 1998 examines the origins of the League Distance Correction.

[144] *Yuan shih,* 134: 3249; on Ai-hsueh, see Allsen 2001, 27. The Office was an organization for therapy. The single term for astrology and astronomy in this quotation was *hsing-li* 星曆.

[145] Hartner 1950. The record is in *Yuan shih*, 48: 998–99. For the identification of the instruments, see below, p. 178, and for an assessment of their influence in China, p. 219. "Jamāl al-Dīn” is a name inferred from the Chinese version, Cha-ma-la-ting 札馬剌丁, or Cha-ma-lu-ting 札馬魯丁 in some documents. Miyajima 1982, 407–408, and van Dalen 2000, 147, give his name as Jamāl al-Dīn Muh<underdot>ammad ibn Tāh<underdot>ir ibn Muh<underdot>ammad al-Zaydī al-Bukhārī. Aydin Sayili 1960, 191, mentions *as doubtful* the identification of Khubilai’s guest with this native of Bokhara. See also Tasaka 1942 and 1957, Yamada 1980, 48–58, and, for the date of the observatory, Allsen 2001, 162–163. For an analysis earlier than Hartner’s but less satisfactory, see Johnson 1940.

distributed to his Muslim subjects. The system was soon lost, and we are ignorant of its contents.[146]

As soon as Khubilai dreamt of ruling China, he had astronomical instruments at his disposal. An organizational extension of his will was a more complicated matter. We first hear of one in Khubilai's domains in the year he became Great Khan: "In 1260, in accord with the previous Chin institution, a Bureau of Astronomy (Ssu-t'ien t'ai 司天臺) was established, and officials appointed for it." Khubilai used the years 1260–64 to set up a basic governmental structure on the Chin model. The Bureau was part of this first cluster of institutions, many of them pro forma. It was located in Shang-tu 上都 (built shortly after 1256), Khubilai's capital in Inner Mongolia. The appointments of officials, so far as we know, were perfunctory and nominal, below the normal ladder of ranks. The "Treatise on Astrology" of the *Yuan History* lists some omens observed in this period. Since it also lists earlier ones, however, these do not imply the beginning of an astronomical research program.[147]

When Khubilai formally declared his dynastic ambitions in 1271, he understandably appointed a regular official—but one barely high enough in grade to belong at the bottom of the regular hierarchy. "To the duties of the official in charge of the Offering Response

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[146] The "Treatise on Commodities" of the *Yuan shih* gives data on "non-quota tax income" for 1328. It notes that, of 3,123,195 almanacs subject to tax, only 5257 were Muslim (94: 2404). One can easily believe that 61 years earlier they sold to "a small extent." But whether these almanacs were based on Jamāl's system is unknown. At the end of 1313 one K'o-li-ma-ting 可里馬丁 submitted a new Myriad Year system to the throne *(Yuan shih,* 24: 559). It is likely but not certain that its author was a Muslim. Although the system was important enough to note in the annals, there is no evidence that it became official in any sense.

[147] Although *ssu-t'ien t'ai* (literally, "Platform for Administration of Heaven") sometimes refers to an observatory and the word has been so translated, I agree with Hucker 1985, item 5783, that this is incorrect. In the Yuan, for instance, this complex institution was prepared to do all the work of palace astronomy, with specialists designated in astrology, divination, computation of the ephemeris, observation, and time reckoning. An observatory was only part of its equipment. See *Yuan Mi-shu-chien chih* 元秘書監志, 7: 2a–8a, and the discussion below, p. 169. The normal word for an observatory in Yuan documents is *ling-t'ai* 靈臺.

Gate-tower of Shang-tu was added that of acting Director of Astronomy." What archeologists believe are the remains of three walls of Shang-tu survive. Above the north gate of the inner wall is a platform of a size and shape to accommodate observational instruments. As for its level of activity, no astronomical system of the Mongol period originated in the Shang-tu Bureau. There is no evidence that any of its specialists were assigned to the Season-granting project.[148]

A third predecessor of Wang Hsun's astronomical reform group was what eventually became "the Muslim Directorate of Astronomy (Hui-hui ssu-t'ien chien 回回司天監), … in charge of observing phenomena and carrying out the procedures for generating the ephemeris (*yen li* 衍曆). … Before Khubilai became emperor, he gave an order to recruit Muslims who were engaged in study of the stars [i.e., astrologers]. Jamāl al-Dīn and others, because of their skill, were presented to him, but there was no astronomical organization [to which they could be assigned]. In 1271 a Bureau of Astronomy was first established." Another source says more specifically that the government "appointed officials for a Muslim Bureau of Astronomy, with Jamāl al-Dīn as Superintendent." Since no record mentions any instrument of Sung origin in Shangtu, its instruments may well have been those Jamāl built based on designs from outside China.[149]

These data are more than a little enigmatic. In 1260 in Shang-tu there was some sort of Bureau of Astronomy, with officials of some sort; in 1271 a gatekeeper became its Acting Director; and in the same year a Muslim Bureau appeared. Jamāl is on the scene as not Director but Superintendent, a post of low and ambiguous status. It is quite possible that there was no observatory until Jamāl provided his exotic instruments, and that in 1271 the platform above the north gate became the headquarters of the Muslim Bureau. All that

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[148] *Yuan shih,* 90: 2296–97. On the remains of the Shang-tu observatory see Lu Ssu-hsien & Li Ti 1981. They believe, but offer no evidence to prove, that its instruments were those Jamāl al-Dīn built.

[149] *Yuan shih,* 7: 136, 90: 2296–97. "Superintendent" *(t'i-tien* 提點) is a term often used for posts below the permanent administration.

remains uncertain. What the Bureau's personnel were doing is even more uncertain. Jamāl had submitted his astronomical system several years earlier, and there is no trace of any new project for them.

By 1273 Jamāl had been promoted to Acting Director of the Palace Library, organized only a few months earlier for installation in the new capital, Ta-tu, still under construction. Within a year he had become the second of its two regular co-directors. Also being organized for the new capital in 1273 was a Han Chinese Directorate of Astronomy (Han-erh ssu-t'ien chien 漢兒司天監) with its own observatory *(ling-t'ai* 靈臺). In 1274 (when key buildings were ready for governmental use), both directorates were subordinated to the Palace Library, and in 1275 were amalgamated to form a single directorate. But amalgamation does not mean unification. In 1276 the Directorate was ordered to submit separate reports on Chinese and Muslim activities.

What was the point of these elaborate maneuvers? An obvious guess is that they were meant to give Islamic astronomy authority over the Chinese version, or to encourage the pooling of skills. But that guess does not withstand scrutiny. There is no evidence of such authority, and Muslim influence on either instrumentation or predictive techniques, as we will see, remained negligible.

That suggests a more down-to-earth cause for bureaucratic realignments, namely Jamāl al-Dīn's advancement in the Mongol civil service. At the time, imperial esteem mattered a great deal more than managerial competence (about the latter, in Jamāl's case, we know nothing). His personal standing greatly increased in the early 1270's, as signaled by such honorific titles as Grand Academician of the Academy of Scholarly Worthies and Grand Master for Palace Attendance. But his astrological star ended its ascent in 1276. With the Sung vanquished, the astronomical reform, never subject to Jamāl's authority, became the focus of Khubilai's support. The next steps made the Muslim Directorate decidedly peripheral.

Jamāl's movement out of the limelight culminated when "in 1278, an Astrological Commission (T'ai-shih yuan 太史院) was separately established, accompanying the Directorate. The administration of the almanac's [preparation and] distribution belonged to the Commission; arrangements for training fell to the Directorate."[150] Since it is hardly likely that an observatory in Mongolia would take over educating future astronomers for the new Commission, I understand this ambiguous statement to be saying that both the Commission—the organization in charge of the reform—and the quasi-amalgamated Directorate in the new capital, with a technical training mission, formally came into being as part of this transition.

The Process

The next three chapters will discuss the astronomical reform's instruments, staff, and records. Here I will concentrate on clarifying the sequence of events—summarized in table 3.1—which turns out to be not at all simple.

The government inaugurated the Season-granting project in July 1276 by appointing two officials to plan it. A collection of the Yuan Palace Library's reports indicates that the Directorate was already metamorphosing into a support organization in 1276, when it had to send 30 qualified personnel to the headquarters of the reform.[151]

Between 1276 and 1278, the growing staff of the reform was busy with planning, and with surveying previous astronomical systems to determine what the main problems were and what constants and techniques needed improvement. During this period, those in charge simply referred to the organization as "the office *(chü* 局)." It formally became the Commission in 1279, and the construction of its building and instruments began. The empire-wide survey of the altitude of the north pole (equivalent to geographic latitude) and other quantities at many sites went under way in the same year (p. 577). But this was only a first step; the survey could not have been

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[150] *Yuan shih,* 90: 2297.

[151] *Yuan Mi-shu-chien chih,* ch. 7. The chapter also provides detailed records of the textbooks used for each specialization in the Bureau's school, and actual examination questions used to test applicants, students in the midst of their studies, and candidates for astronomical positions.

completed in time to be included in the first version of the Season-granting system.[152]

Table 3.1. Astronomy in the Mongol Court

Year	Events
1251	Liu Ping-chung suggests astronomical reform
1256	Khubilai begins building capital in Inner Mongolia
1260	Khubilai becomes Great Khan; capital completed; establishes astronomical bureau as part of Chinese-style institutions
1267	Jamāl al-Dīn presents instruments, computational system; building of capital at Beijing begun
1271	Muslim Directorate of Astronomy established
1272	Yuan dynasty proclaimed
1273	Jamāl becomes Associate Director of Imperial Library; Liu Ping-chung formally proposes astronomical reform
1274	Mongols begin moving government to new capital; Jamāl becomes co-director of library, with Muslim and Chinese Directorates of Astronomy subordinate; Liu Ping-chung dies
1275	Directorates of Astronomy combined
1276	Southern Song surrenders; move of capital to Ta-tu completed; Season-granting reform ordered; responsibility of Directorates downgraded
1279	Construction of new instruments and observatory; latitude survey begun
1280	Season-granting system submitted to throne in December

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[152] *Yuan shih,* 9: 183; 10: 198, 209; 88: 2219–20; 90: 2297. See Kuo's account of conduct (below, p. 577), and *Yuan Mi-shu-chien chih,* 7: 12b–13b, 9: 1a, 2a–2b, 4b.

On 15 April 1279, Kuo was sent an order to set out from the two Yuan capitals and "pass through Ho-nan-fu 河南府 to the Southern Sea to observe gnomon shadow lengths" (*Yuan shih,* 10: 210). This was evidently a first trial to be conducted by Kuo himself. Orders dated 19 July 1284 and 9 April 1285 directed others to observe in multiple locations (13: 267, 275).

Year	Events
1281	Season-granting system becomes official
1286	First documents of the reform submitted; observatory completed
1290	Complete documentation of system submitted?; instruments completed?

The edict promulgating the new system was drafted by an official of the Commission named Li Ch'ien 李謙 between 29 June and 27 July 1280. The date signals pressure that the reformers could hardly ignore. Wang Hsun composed and presented the new system to the throne in December of 1280. The edict was promulgated on 18 December 1280, making the system the basis of the almanac effective the beginning of the next civil year, 22 January 1281.[153]

That suggests remarkably quick completion of the project. Actually it was far from complete. A finished version of the Evaluation was ordered in 1283, a clear indication that the project was taking longer than expected. Its compilation was in the hands of the Li Ch'ien just mentioned.[154] Kuo's biography informs us that in 1282 he took responsibility for putting the project's documents in final form. He submitted the archive to the throne in two or more batches, one shortly after he became Grand Astrologer in 1286, and the remainder, according to Ch'en Mei-tung's 陈美东 estimate, by about 1290.[155] We learn from his list of its contents that the *Yuan History's* version of the astronomical treatise is only half the length of Wang Hsun's draft, and a small fraction as large as the final documentation.[156]

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[153] The draft edict is preserved in *Pan Shou-shih li chao* 頒授時曆詔. For the date of promulgation, see Khubilai's annals in *Yuan shih,* 11: 227.

[154] On Li Ch'ien's authorship, see the Evaluation, p. 253.

[155] Ch'en 2003b, 90–91.

[156] See below, p. 588. For the documents, see Yang Huan 楊桓, *Chin Shou-shih li ching li i piao* 進授時曆經曆議表, and on their disposition, *Yuan shih,* 14: 287.

We will see in chapter 5 that the new observatory and its instruments were also not completed until *after* the government promulgated the Season-granting system in 1281 (p. 181). As chapter 1 noted, the observers, in the first phase, did not use Kuo Shou-ching's celebrated Tall Gnomon, but a wooden prototype, and they made their measurements of lunar-lodge coordinates with an old and extensively repaired armillary sphere. The measurements begun earliest, those of noon sun shadow lengths, occupied from late 1277 to late 1279. In the same interval, the observers began determining equatorial and ecliptic positions of the sun and moon. Some of the basic work, for instance measurement of planetary positions, and of lunar latitude to determine the nodes of the moon's path, continued into 1280 and probably beyond.[157]

In other words, what was submitted to the throne by December 1280 and soon promulgated was a tentative and incomplete version that the reform group only gradually replaced. Materials of great importance worked out at a later stage were preserved in the court, but did not find their way into the *Yuan History*. The "Treatise on Mathematical Astronomy" of the *Ming History* concludes tentatively that what received official status was a draft of the Canon:

> According to the *Yuan History*, in 1280 the Season-Granting system was completed. In 1282 Wang Hsun 王恂 died. By that time the new system had been promulgated, but the procedures for computation and the numerical ready reckoners[158] were not yet in definitive drafts. Kuo Shou-ching set them in order and edited them, worked out the exact values, and compiled them into two chapters. The exemplar now in the Directorate of Astronomy records that it was "composed by Wang Hsun, Great Officer of Fine Counsel and Grand Astrologer, by imperial order." Might it not be that Wang prepared a draft and Kuo brought it to fruition?

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[157] Ch'en 2003a, 526–39, gives the most reliable detailed chronology.

[158] A ready reckoner is a collection of tables and other aids to simplify computation.

> The *Yuan History* did no selective compilation at all, but simply reproduced Li Ch'ien's record of evaluation and the first draft of the Canon.[159]

The Ming authors had access to the original documents, and incorporated in their own treatise some helpful ready-reckoning tables that Kuo had submitted in the Yuan period. I have added the most important of them to the translation below.

The terminology of the Ming tables corresponds well enough to that of the Yuan treatise that the editors of the *Ming History* must have used the Yuan tables largely as they found them. Their mention of two chapters implies that Kuo edited the Canon for the *Yuan History* as well as the final three-chapter version. That is not necessarily true, however, which may account for their final hesitancy. What they wrote up to that phrase was paraphrasing Kuo's biography, and it makes no such assertion (cf. below, p. 588).

The project's mission was a system that could generate accurate ephemerides indefinitely. The high quality of its predictions for nearly four centuries implies that the authors came closer to their goal than most—but only after much more extensive work than that presented in 1280. The abridged treatise in the *Yuan History* is presumably based on the revised version, but we cannot be certain.

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[159] *Ming shih,* 34: 623, 32: 547.

4 The Astronomers

A wealth of data has survived about the participants in the astronomical reform. The concern of this chapter is what that information reveals about basic patterns. I will clarify the intellectual, social, and political relations of those who directed the project, and work out an idea of its human scale. I will outline for each of the main participants how he became involved in the project, what responsibility he accepted, and what he accomplished. It will be possible at the end of this chapter to clarify the common threads in their involvement, what brought them together.

Leading statesman normally merit full-scale biographies in the dynastic histories. The official history of the Yuan is exceptional in grouping together brief accounts for everyone but emperors— first empresses and imperial secondary wives, then male Mongols, Han Chinese last.[160]

The information available about each participant varies greatly. The official histories often build a biography around an important piece of the subject's writing. For instance, that of Kuo Shou-ching 郭守敬 summarizes the memorial on water-control problems that he presented to Khubilai when they first met, and incorporates in its entirety the document that he and his colleagues submitted when they completed the new astronomical system. Together the two occupy two thirds of the biography. Wang Hsun's 王恂 is half as long as Kuo's astronomical report alone—too short to include a document. On the other hand, documents aside, the factual information about Wang is roughly as extensive as for Kuo.

Who planned and took charge of the reform? Different accounts name different people. To resolve some of the inconsistencies, one has to piece together data excavated from the complex organization

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[160] Ch. 157 combines the biographies of Liu, Chang Wen-ch'ien, and another important early advisor of Khubilai; ch. 164, also untitled, gives those of Yang Kung-i, Kuo Shou-ching, Wang Hsun, and five other scholars who served the early Yuan dynasty and became academicians.

of the official histories, which customarily sorted different kinds of information into annals, treatises, and biographies. Some assertions conflict because they are part of disparate documents from different sources. The historiographers, who often worked under severe pressure of time just after the end of a dynasty, could not always reconcile these differences. One can hardly expect more; at least their assertions were consistently grounded in the record.

The immediate task is to reconstruct the cast of characters mentioned in various parts of the *Yuan History* and other sources. It will be best to begin with Liu Ping-chung, who envisioned the reform. My account of him will be a bit longer than those of the others, partly to clarify the early origins of the system, and partly to throw light on the involvement of the one protagonist who was not skilled in astronomy, namely the emperor Khubilai. Each step of the reform depended on his personal approval and support.

Because modern historians of astronomy have tended to give Kuo Shou-ching more, and Wang Hsun less, than his share of attention, it is not simple to balance the accounts of the two. But a conspectus of the group responsible for the reform is feasible. That is what I essay in the conclusion.

The entry in Khubilai's annals gives more or less the official view:[161]

> On 23 July 1276, because of cumulative errors in the [Revised] Great Enlightenment system, the Admonisher Wang Hsun was commanded to work with astrologers (*jih-kuan* 日官) from southern China to set up a bureau and make a new astronomical system, with Vice Commissioner for Military Affairs Chang I 張易 to supervise the work. Chang and Wang memorialized: 'These days mathematical astronomers rarely comprehend the principles underlying astronomy. It would be best to have an old and respected scholar such as Hsu Heng 許衡 with whom to discuss and decide on such matters.' A decree ordered Hsu to proceed to the capital.

The Evaluation, when enumerating those ordered "to reform the ephemerides and make a new astronomical system," adds "Assistant Supervisor of Waterways Kuo Shou-ching" after those of Hsu

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[161] *Yuan shih,* 9: 183.

and Wang. Kuo's biography in the *Yuan History* gives his name first: he and Wang were, "as subordinates, to lead the astronomical officials of the north and south, dividing up responsibility for observation and computation. Chang Wen-ch'ien 張文謙 and Chang I were ordered as superiors to manage, deliberate, and report, and Hsu Heng to consult on the work."[162]

The contemporary account of the observatory, written to celebrate its completion, first names Wang, Kuo, Hsu and Chang Wen-ch'ien. It adds three names that do not recur in later sources: first it praises the mathematician Yang Kung-i 楊恭懿 for "guidance with respect to fundamentals, and support from start to finish." It finally specifies that the government assigned Tuan Chen 段貞 (fl. ca. 1279), the Acting Director of the Ministry of Works and concurrent head of the Directorate for Imperial Manufactories, to provide craftsmen in earth, wood, metal, and stone, and charged Aniga 阿你哥 of the Ministry of Education with the artisanry and decoration of the astronomical and timekeeping instruments.[163]

To sum up, although the annals of the *Yuan History* omitted Kuo from the list of early participants, earlier sources closer to the events make him a participant in—although not the leader—of the reform. Other data substantiate his involvement. We can proceed to consider the seven named in these various sources, as well as Ch'en Ting, the ranking southern astronomer.

Liu Ping-chung

The Season-granting reform was the brainchild of Liu Ping-chung 劉秉忠 (1216–74), the most influential and powerful Han Chinese in the early Yuan period. Although Liu died two years before Khubilai authorized the reform, it was he who motivated the

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[162] See the Evaluation below, p. 253; for Kuo's biography, p. 573 below and *Yuan shih,* 164: 3847.

[163] *T'ai-shih-yuan ming* 太史院銘, p. 217. Aniga (1244–1306; the Chinese form of his name varies greatly) was a Nepalese who received high rank and honors during a career in Khubilai's court, due to his artistry in metal-casting and other media. See his biography in *Yuan shih,* 203: 4545–46, and Rossabi 1988, 171.

ruler to do so. He also collected the talent and the massive support that made the project possible. Those who carried out the Season-granting reform, as the discussions below make clear, were all his protégés or political allies.[164]

Liu (original name Liu K'an 劉侃) came of a family of Hsing-t'ai 邢臺, Hsing prefecture (present Hebei province), who had been administrators under the Liao and Chin dynasties. His father submitted to the Mongols when they invaded his prefecture in 1220, and served under them. Ping-chung studied not only the orthodox *Book of Changes* but Buddhism, divination, astrology, and astronomy. To support his family, he took a clerical job in the prefectural administration at the age of 16, but left at 22 to become a Ch'an monk. Khubilai had learned from his astute mother (a Nestorian Christian) the usefulness of Chinese advisors. When he began to wield influence in the Mongol court, he summoned a few eminent Chinese to counsel him. Hai-yun 海雲 (1203–57), the most influential Ch'an master of his time, was the first to accept. On his way to Karakorum (the ancient Mongol capital) in 1242 to meet with Khubilai, Hai-yun recognized Liu's talent and took him along. During the meetings, he often called on Liu's erudition. Khubilai appreciated what Liu, at 26 a year younger than he, might contribute to his own education, and kept him on as an advisor. Liu remained a monk until finally, in 1264, Khubilai gave him several high appointments, commanded him to live as a layman, and conferred on him the personal name Ping-chung ("steadfast loyalty").[165]

When Khubilai first became responsible for governing Chinese, Liu suggested how to proceed. He helped him with the details of administration and—equally important to his Han subjects—ritual. Because of Liu's learning, his mastery of the classics, and his skill at prose, poetry, divination, calligraphy, painting, and most of the

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[164] For information about Liu, see Rachewiltz 1993, 245–69, and sources cited there, especially *Yuan [ch'ao] ming ch'en shih lueh,* 7: 1a–5a, and *Yuan shih,* 157: 3687–94.

[165] For an account of Hai-yun and his relation to Liu, see Y. H. Jan in Rachewiltz 1993, 224–42, and Huang Ch'un-ho 黄春和 1988. Chinese Ch'an 禪 is the original of the more generally known Japanese Zen.

other gentlemanly arts, he was able to attract Han scholars and officials to carry out the plans on which he and Khubilai agreed. At the same time, as an ascetic Buddhist who unobtrusively advised Khubilai without seeking personal power, he avoided offending Mongol nobles who distrusted Chinese literati.

A period of mourning when his father died allowed Liu to return to Hsing-t'ai (or rather to the nearby intellectual center at Purple Gold Mountain) from 1246 to 1249 to study, teach, and cultivate his relations with fellow scholars.[166] He introduced a number of them into Khubilai's circle of advisors, including several who became the main participants in the astronomical reform. In 1251 Liu, aided by Chang Wen-ch'ien and others he had recruited, reformed the administration of his native prefecture, then under Khubilai's direct rule. The changes revived an area so devastated by war that it had nearly been abandoned. The success of this early project increased the ruler's confidence in the Han scholars, and prompted him to give them more responsibility for troubleshooting. When Khubilai became Great Khan in 1260, Liu and his associates proposed a Chinese-style administration parallel to the Mongol one. Although Liu remained behind the scenes, several of his colleagues held important posts in the Secretariat, the most important of the new state organs.[167]

Liu planned and built two capital cities, Shang-tu in Inner Mongolia ("upper capital," built 1256–60) and Ta-tu ("great capital," built 1267–76). The latter was more or less the present Beijing. "It can be argued that formally [Ta-tu] was more a part of the classical tradition of Chinese urban planning than any city that had come before it."[168] Liu and his colleagues, who had already organized an administration, also designed a complete set of court ceremonials

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[166] This center, a little over 60 km northeast of Hsing-t'ai, began as a refuge from Mongol raids. See the informative discussion in Fan Yü-ch'i 范玉琪 1987 and, for the dates, Pai Kang 白钢 1996, 410–411.

[167] *Yuan shih,* 157: 3695.

[168] Steinhardt 1999, 158. Her account of Mongol capitals, pp. 147–60, makes it clear that many aspects of Khubilai's rule remained unsurpassed in the Yuan.

based on a study of precedents, appointing and training people to perform them.

Among the many political, managerial, and ritual measures to reconcile Mongol rulership with Chinese patterns, as early as 1251 Liu proposed an astronomical reform. He concluded "it will be desirable, on the accession of a new ruler [this subtly assumes that Khubilai will become Great Khan], to promulgate an astronomical system and a new epoch." There is an intriguing afterthought: "Let water clocks be set up in the capital and in administrative cities, so that the people become aware of time."

He fleshed out this proposal in 1273, designating the project his own responsibility. We do not know how much work he did on it before his death in the next year. He cited errors that had accumulated in the official Revised Great Enlightenment system (#76) over 70 years of use. More to the point, however, was the value of a new system for marking a decisive change of imperial mandate, which implied a new cosmic dispensation. This would, of course, have to be a Chinese system, so Jamāl al-Dīn's Islamic system of 1267 (#85) was not in the running. By 1276, with the collapse of the Sung, the Mongol emperor was ready to order up a new astronomical system.[169]

Wang Hsun

Wang Hsun's (1235–81) father was a local official when the Chin ended; he did not offer his services to the Yuan, but devoted himself to philosophical teachings, astrology, astronomy and mathematical harmonics. Wang, an extraordinary student, became expert in mathematics while in his early teens. He impressed Liu Ping-chung in the late 1240's when the latter passed through the Wangs' native place, about twenty miles north of Hsing-t'ai. Liu accepted

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[169] Although historians usually date Liu's astronomical reform proposal to 1273, in the famous memorial of 1251 outlining the basic elements of Chinese-style government he stresses the need for a new system; *Yuan shih,* 157: 3691.

him as a disciple and took him to the community at Purple Gold Mountain.[170]

In 1253, when Wang was 18, Liu presented him to Khubilai, who appointed him to guide the studies of the heir apparent. Wang quickly advanced in status as he indoctrinated his charge in the doctrines of conscientious rulership into which Wang's father and Liu had initiated him. In teaching, he emphasized the importance of mathematics—as he put it, "one of the six arts taught to aristocrats in antiquity. For establishing the state or making the family secure it is a large matter."

By the time he was 30, he had played an important role in establishing the National University, where members of the Mongol ruling clan and other nobles learned Chinese statecraft and cultivation. He was prominent in taking the steps that led up to a well-built imperial government, among them a review in 1269 of the administrative structures that had grown up ad hoc, and the design of court rituals.

Khubilai in 1276

> was aware that Wang had mastered mathematical techniques, and … commanded him to lead the astronomical reform project. … In 1279 he assumed the posts of Grand Master of Excellent Counsel and Grand Astrologer. In 1280 the system was complete. [The emperor] bestowed the name "Season-granting system" and, in the winter of that year, promulgated it throughout the realm. In 1281, shortly after mourning his father … Wang died at the age of 46.

Although Wang was the youngest of the working group, he was clearly in charge of the project, and responsible for at least some, and possibly most, of its innovations in mathematical astronomy. On the other hand, modern historians have been practically unanimous in assuming that Kuo Shou-ching was the project's chief mover. The result has been a notable disparity. For example, between 1990 and 1997, historians of science devoted nine books—three comprising papers from conferences in Hsing-t'ai—and thir-

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[170] *Yuan shih,* 164: 3843–45. On Wang's mathematical ability see *Kuo-ch'ao wen lei* 國朝文類, 39: 9a. Ch'i Lü-ch'ien 齊履謙 asserted that Wang was a fellow student of Liu's, but that is an error; see Pai Kang 1996, 404–5.

teen separate articles to the "scientific giant *(k'o-hsueh chü-jen* 科學巨人)" Kuo. Over the same period there was not a single essay, much less a book, on Wang, although the information about him is not scanty.

Kuo Shou-ching

Kuo (1231–1316), like Liu Ping-chung, was a native of Hsing-t'ai. Because his parents died early, his grandfather, a good friend of Liu's, raised and educated him. Kuo Jung 郭榮, a polymath, taught his grandson many skills, among them astronomy, astrology, divination, geography, and water control. As a child, Shou-ching began making astronomical instruments he had read about, and used them for observation.[171]

In 1247–48, when Kuo Shou-ching was about fifteen, his grandfather sent him to study with Liu. There he met the associates of Liu with whom he later worked on the astronomical reform. By the time Kuo was twenty, his work for Liu's reconstruction of his native prefecture gave him experience in water control projects. He not only gained a local reputation for skill at planning and supervising projects, but his work inspired the celebrated poet Yuan Hao-wen 元好問 (1190–1257) to eulogize the bridge-building talent of "the student Kuo." When Chang Wen-ch'ien recommended him to Khubilai in 1262, Kuo—by then an accomplished hydraulic engineer—presented the Great Khan with proposals for waterway and irrigation projects in the region south of Peking. He was duly appointed to the Directorate of Waterways. In 1271, the year Khubilai asserted his dynastic ambitions, Kuo became Supervisor (grade 3B, a high rank for a technical post). When his organization was incorporated in the Ministry of Works (1276), he became Director of one of its four bureaus. That was the year in which organization of the astro-

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[171] The earliest biography of Kuo is translated in appendix B. On Kuo's native village, see Shen Chen 沈震 & Hua P'eng-hui 华朋慧 2003.

nomical reform began, but Kuo was busy with his new responsibilities.[172]

What led to his incorporation in the project early the next year was, no doubt, Hsu Heng's observation, given prominence in the Evaluation (p. 253), that the old system failed because it had merely modified a predecessor rather than "actually testing its techniques against observations of the celestial phenomena." Chang I nominated Kuo to do that testing. He was ordered early in 1277 to repair an old armillary sphere and other instruments and to carry out observations for the reform. When he finished those tasks, the order concluded, he was to return to his previous position.[173]

The assignment was not highly ambitious. It implies that the organizers of the project originally planned to reuse battered instruments well over a century old and designed for a different latitude. But once Kuo joined the project, he made a compelling case for building new ones, and proceeded with alacrity. He made the earliest measurements of noon shadows with a wooden prototype of the unprecedented 40c gnomon only a couple of months afterward.[174]

An inscription written in 1286 for the new observatory implicitly contrasted Kuo's strengths with those of Wang Hsun. It describes the main figures whom the emperor appointed to create the new system, beginning with Wang and Kuo:[175]

> His Majesty's servitor the Admonisher of the Heir Apparent, Wang Hsun, had mastered the arts of mathematical calculation. The expansion and contraction, slackening and hastening, of the solar and lunar motion, the progressive and retrograde movement of the Five Stars, visibility, invisibility, dusk and dawn phenomena, and the Centered Star, as they respond to the seasonal cycles: he could com-

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[172] "Hsing-chou hsin shih ch'iao chi 邢州新石橋記 (Memoir of the new stone bridge in Hsing prefecture)," *Yuan Hao-wen ch'üan chi* 元好問全集, 33: 756–58. The dates when Kuo was at Purple Gold Mountain are the estimate of Fan Yü-ch'i (1987, 51).

[173] *Yuan Mi-shu-chien chih,* 2: 2a–2b.

[174] See below, p. 166, and Ch'en Mei-tung 2003b, 61–66.

[175] *T'ai-shih-yuan ming*, p. 217. Both were given these astronomical appointments in 1279. The "Five Stars" are the planets, and "Centered Star" is a star's upper transit of the meridian (i.e., culmination).

> pute them all, so that he was soon appointed Director of the Astrological Commission. His Majesty's servitor the Director of Waterways, Kuo Shou-ching, was ingenious in his understanding of the cosmic cycles and remarkable at design. Observational and demonstrational instruments, gnomons and water clocks for determining the time of day and pacing the motions of the heavenly bodies: he could set out the dimensions of them all, so that he was soon given the post of Associate Director.

When Wang died in 1281, he left the documents of the project unfinished. Kuo took up this work, sorted them categorically, edited them, and had them copied into separate manuscripts.[176] Despite this massive labor of editing—or perhaps because it preoccupied him—Kuo was not appointed immediately to be Wang's successor as Grand Astrologer. This happened in 1286, but completing the documentation of the project, as we have seen (p. 148), took some time longer.

In 1291 Kuo once more took up water control. When Khubilai died in 1294, he was appointed Administrator, a less demanding position than before, in the Astrological Commission, and given a high honorific post. Still he continued as an expert in hydraulic engineering. He died at 85, still on active duty in the court, in 1316.

Chang I

Chang, a very eminent civil servant, died in disgrace. For that reason, he was not only denied a biography in the *Yuan History,* but no one even wrote an account of conduct or funerary inscription for him. We therefore know little about him except for official responsibilities mentioned here and there. There is some disagreement about his native place, but he evidently came from the vicinity of T'ai-yuan 太原, in present Shanxi. He was a contemporary, more or less, of Liu Ping-chung, and died in 1282.[177] Although Chang did

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[176] See below, p. 588, which also lists the books that resulted.

[177] Yuan Chi 袁冀 (1974) has gathered a wide variety of scattered references. Fan Yü-ch'i 1987, 51, makes a case that Chang was born about the same time as Liu, not as Wang and Kuo. Pai Kang 1996, 404–407, argues that Chang was born circa 1215, but his reasoning, based on the relation of

not come from the vicinity of Hsing-t'ai, in the late 1240's he was among Liu's associates at Purple Gold Mountain. Liu almost certainly introduced him to Khubilai circa 1247.[178]

Divination was one of the services that Khubilai expected of Chang. By 1259 the latter esteemed him to the point that Chang was able to recommend others. He was among the group that helped the Great Khan to set up Chinese-style institutions when he inaugurated the Yuan dynasty in 1260, and became one of the first high functionaries in the Secretariat. By 1275 he had attained rank 2b, by 1280, 2a, and the next year, 1b, next to the top of the civil service. As we have seen, in 1276 he became one of the supervisors of the astronomical reform.

In April of 1282, two shady figures assassinated Ahmad 阿合馬, the Central Asian financial expert of mixed Iranian and Turkic extraction who was pushing relentlessly for total control over finances, often seeking to neutralize the influence of the Secretariat. The murder does not seem to have been a direct result of this mutual enmity. Nevertheless, Chang, apparently duped by a forged document, was implicated, and was executed with the conspirators.[179]

Hsu Heng

Hsu Heng 許衡 (1209–81) came of a farming family of Ho-nei 河內 (present-day Henan). As W. T. De Bary once put it, he became "the leading intellectual figure of the age, an activist at court and the most influential teacher of the time, whose followers were to dominate the Yüan educational system."[180]

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his sobriquet with those of Liu and Chang Wen-ch'ien, is not persuasive. I estimate his year of birth as circa 1216, and thus his age at death as circa 66.

[178] Pai Kang 1996, 411.

[179] See the biography of Ahmad by Herbert Franke in Rachewiltz 1993, 539–57, especially pp. 550–51, and for background, Rossabi 1988, 178-184. Pai Kang 1996, 423–28, argues that Chang was involved in the plot.

[180] Chan & De Bary 1982, 10; *Yuan shih,* 158: 3716–30; *Lu chai i shu* 魯齋遺書, ch. 12–14; Yao Ta-li 1983.

Growing up in wartorn North China, he could hardly have aspired to such a career. Instead, he was apprenticed to a diviner, and became expert in astronomy and astrology. When in 1238, under the Mongols, Yeh-lü Ch'u-ts'ai organized a special civil service examination, Hsu passed. Thus formally registered as a scholar, he was exempt from taxes and periodic labor service. Beginning at age 33, when he encountered and began to study the writings of Ch'eng-Chu Confucianism, he became its first great teacher in the north. Hsu was the second in direct transmission from Chu Hsi 朱熹 (1130–1200). A famous seventeenth-century treatise that defined the orthodox lineages of Confucian studies gave Hsu his own.[181]

Khubilai understood Hsu's emphasis on moral cultivation, ritual, and practical learning. The officials who introduced Hsu to the ruler in 1260 were allies of Liu Ping-chung, but Hsu was not one of Liu's protégés. From about 1266 on, as the Great Khan's imperial ambitions materialized, he sought Hsu's counsel regularly. Hsu served with Wang Hsun in reviewing the government's administrative structure and in preparing court rituals, and soon was appointed to a high executive post in the Secretariat. Ahmad, the bane of its Chinese officials, inevitably tried to undercut him. This was not a mere personal disagreement, but rather a maneuver in a long battle over how best to exploit the Mongols' Chinese subjects.

Hsu was a modest man, a teacher above all, willing to act on principle but with no appetite for palace infighting. He left the front lines when Liu's associates nominated him to establish the National University and serve as its first Chancellor. Nevertheless the pressure from Ahmad continued, and Hsu's health deteriorated. He retired in 1273 at the age of 60.

We have already seen that in 1276, when Wang Hsun and Chang I needed an eminent scholar learned in astronomical principles to take part in their project, Hsu returned to officialdom. He is said to have done so gladly, perhaps because his responsibility as a consultant took him to the new capital in Beijing without exposing him

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[181] *Sung-Yuan hsueh an* 宋元學案, 90: 1b–6b. The same chapter, pp. 7b–8a, includes Chang Wen-ch'ien in Hsu's line.

to Ahmad's line of fire. Hsu's poor health forced him to resign again shortly after he and his colleagues submitted the system to the throne late in 1280, and he died in March of the next year.

The obvious question is whether in the project Hsu was a mere figurehead, meant to dignify it with his age and eminence, or made an important contribution. But such an either/or question is beside the point. His biography indicates that, prestige aside, he did have something important to say about principles: "Hsu Heng believed that the winter solstice is the foundation of an astronomical system, and that finding a foundation for a system depends on the observational determination of *ch'i*"—that is, of the divisions of the year.[182]

The point, so stated, will not be obvious to modern readers, but at the time it was an important one: the success of the system in generating ephemerides over a long period depends indeed on an highly accurate figure for the length of the tropical year. Hsu's colleagues understood his point. His conviction inspired Kuo to greatly improve the gnomon; the result was indeed enhanced accuracy. It may also have motivated the team to incorporate the secular variation in tropical year length mentioned above (p. 101).

Chang Wen-ch'ien

Chang 張文謙 (1217–83) was a native of Sha-ho 沙河 in Hsing prefecture, about 35 miles south of Hsing-t'ai. He and Liu Ping-chung, a year older, studied in the same local school. Chang's father, the first official in his family, wanted him to become a scholar, but the fall of the Chin ended the examinations as a route to office. Chang was "thoroughly conversant with the disciplines based on the study of regularities *(tung chiu shu-shu* 洞究術數)," that is, divination, astrology, astronomy, and other branches of numerology and mathematics.[183]

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[182] *Yuan shih,* 158: 3728.

[183] *Yuan shih,* 157: 3695–98; Rachewiltz 1993: 270–81, and the list of sources cited there. The quotation is from *Yuan shih,* p. 3697. On *shu-shu* see Ho Peng Yoke 1991 and 2003.

Chang, like Hsu Heng, passed the special civil service examination of 1238, but how that affected his livelihood we do not know. In 1247, among those Liu had recommended to Khubilai, Chang was the first called to Karakorum. Khubilai found him useful in secretarial matters. In 1251 Chang joined Liu, who had returned to Hsing prefecture, to plan the program of reforms that revitalized it. By 1260, when Khubilai became Great Khan, he was so impressed by Chang's energy and efficiency that he appointed him, at the age of 43, second in command of the Secretariat in Shang-tu. There Chang befriended Hsu Heng, and often asked for his advice. In another reform project in present Hebei province, among the capable officials in Chang's entourage was the young hydraulic engineer Kuo Shou-ching.[184] In 1262 Chang introduced Kuo to Khubilai. By 1271 Chang was Minister of Revenue, at the apex of the Chinese bureaucracy.

His various responsibilities brought him repeatedly into collisions with Ahmad. This is no doubt why in 1276 Chang welcomed (or conceivably even requested) his assignment to supervise the astronomical reform group. He had a good knowledge of astronomy, but there is no concrete record of his involvement in either the technical work or reporting on it.

The next year brought him high honors as a Senior Academician of the Institute for the Glorification of Literature. In 1283 he was appointed Vice Commissioner of Military Affairs, a post in which he made a number of proposals for reform before dying in March of that year.

Yang Kung-i

Yang 楊恭懿 (1225–94) was a native of Feng-Yuan Route Command 奉元路 (present Xi'an 西安 prefecture, Shaanxi province). Ho Peng Yoke has described him as "an expert in calendar science. He

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[184] Chang's mission was as Pacification Commissioner in 1260. Kuo was 29. See Rachewiltz et al. 1993, 273.

made a contribution to the new calendar no less significant than that of Kuo Shou-ching and his associates."[185]

Like Hsu Heng, Yang was an early devotee of Chu Hsi's teachings; unlike Hsu, Yang refused a summons to meet Khubilai. He changed his mind in 1274, when a high official was deputed to invite him in person, and, once he reached Shang-tu, a Prince of the State personally interviewed him—unimaginable honors for a commoner. Divining at the Great Khan's request, among other things, demonstrated Yang's qualifications as an advisor. While the campaigns against the Sung were being fought, he returned home, but in 1279 he was ceremoniously invited back. He was assigned to the astronomical reform commission that the other participants had joined in 1276. When the Season-granting system was presented to the throne in 1280, Khubilai appointed Yang an Academician of the Academy of Scholarly Worthies, with concurrent duty in the Astrological Commission. He retired instead and, over the next dozen years, refused even higher honors.

As for Yang's skill in astronomy, a memorial that he and Hsu Heng submitted, along with an essay of Yang's on lunisolar conjunctions, shows exceptional acumen. The memorial presented the draft of the new system to the throne in December 1280. It plays an important role in the story of the project:[186]

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[185] See Ho's life of Kuo in Rachewiltz 1993, 291. See also *Yuan ch'ao ming ch'en shih lueh,* 13: 12a–17b, and *Yuan shih,* 164: 3841–43, preceding the biographies of Wang Hsun and Kuo Shou-ching. Some sources give Yang's native place as Kao-ling 高陵, apparently an alternative designation for the same place. His notice in *Yuan ch'ao ming ch'en shih lueh* miscopies Feng-Yuan as Feng-hsien 奉先.

[186] This translation is based on the text in the biography of Yang Kung-i in *Yuan shih,* 164: 3842. Three excerpts from a longer version reveal that this one, although more complete than any other single quotation, is abridged (*Yuan ch'ao ming ch'en shih lueh,* 9: 3b–4a, 13: 15a, 17a–17b). For the date of the memorial as the twelfth lunar month, year 17 of the era, see 13: 17a. The words in parentheses are my additions from the other sources.

Shu Wang Tsan-shan chia chuan hou 書王贊善家傳後 (Afterword to the family biography of Wang Hsun), an appreciation of Wang Hsun by an eminent literary figure two generations later, attributes the proposal to Wang Hsun alone. But the two excerpts in ch. 13 of *Yuan ch'ao ming ch'en*

We have brought together astronomical officials of north and south to study thoroughly more than 40 astronomical treatises from the Han period on. We have given serious reflection to methods of computation. [We have spent our days and nights in making and testing observations.]

Our old instruments were of little use, but the new ones were not yet ready, so we could not make detailed and authoritative studies of the corrected [i.e., apparent] motions of the sun and moon, and the periods of the planets. [With respect to the survey of the four directions (i.e., at many sites,) we had not yet carried out the observations or ancillary studies.]

Now we have provisionally used a new armillary instrument and a wooden gnomon *(hsin i mu piao* 新儀木表), and compared [the data] with those using the old ones. We have derived measurements for the noon shadow length and the corrected location of the sun at the winter solstice of this year, the varying intervals between the lunar lodge determinative stars, the height of the pole at Ta-tu, and the length of day and night marks [at various locations north and south]. Comparing these with results from the old systems, we have created new procedures and computed a new "Eighteenth-year Epoch system *(hsin-ssu li* 辛巳曆)."

Although some aspects of this system may not be perfectly exact[, we estimate that the errors are minute]. If we compare it with the work of previous reformers who adhered consistently to the old habits, arbitrarily locating epochs [in remote antiquity] and altering the Day Divisor [i.e., trivially modifying the constant for tropical year length], at least we have nothing to be ashamed of.

[Because Your Majesty respects the seasons of the sky, and wishes to promulgate a correct ephemeris to bestow on his people, the utmost exactitude is indispensable to setting a standard for future dynasties]. It is essential that every year we make [new] observations and revisions, so that, as [improvements] accumulate over twenty or 30 years, we will have perfected our methods. We can thus emulate [the work of] the astronomical officials of the Three Periods [of legendary high antiquity] who, carrying out their duties generation af-

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shih lueh come from the spirit way inscription *(shen tao pei* 神道碑) for Yang's tomb, and thus are more likely to be quoted from his own writing.

ter generation, observing for long periods, had no need to be concerned with adjusting year lengths.

Several aspects of this document are historically consequential. First, it says clearly that no major new instruments were available until shortly before the end of 1280. Since it does not say clearly which "new armillary instrument" it used at that time, the phrase is ambiguous, and may well refer to the Simplified Instrument. We cannot be sure that it was the instrument finally installed in the observatory. In view of the timing, it was probably a prototype. Second, the authors proposed for the first time to make testing and improvement of a new astronomical system a permanent enterprise. That did not, however, become normal practice.[187]

Third, I will argue in chapter 5 that, with the one exception mentioned here, the observatory's large instruments were not ready for use until after the Season-granting system was submitted and became official. If that is the case, one can hardly claim that the observatory was essential to the astronomical reform of 1276–80. What, then, justified the large expenditure of effort and money?[188]

To some extent, building it symbolized the new government's commitment to the Chinese cosmic rituals of monarchy. Beyond that, from the astronomers' point of view, the observatory enabled the long-term program that Yang proposed—the transformation of astronomical reform from an act into a process. They had no way of

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[187] Two centuries earlier, Shen Kua 沈括, when Grand Astrologer, tried to carry out a five-year daily program of planetary observations, but recalcitrant sinecurists in his observatory stymied him; *Meng ch'i pi-t'an* 夢溪筆談, item 148.

[188] This is by necessity a guess based on the elaborateness of the project. There are no data on actual spending, and no comparable data on other reforms to allow comparison. One derives an impression of financial scale from a notation in Khubilai's annals of an extraordinary grant to the Commission on 25 Nov 1279 of 1078 *liang* 兩 (40.2 kg) of silver; *Yuan shih,* 10: 210. This is more probably to pay for construction of the observatory than for salaries, which would be budgeted. It is impossible to convert this amount reliably into today's currency (in which the same amount of bullion would sell for only US$20,707), but its recording as a separate item indicates its importance. On the complexities of early Yuan money see Yang 1952, 62–64.

knowing that this transition would not take place. In any case, the government stood ready to pay the bills.

Ch'en Ting

Ch'en Ting appeared in chapter 1 as a "southern barbarian yin-yang." We know nothing about him except that the *Yuan History* credits him with the Attainment of Heaven system (#86, 1271), the last official system of the Southern Sung period, and that after the capital surrendered in 1276 he was sent north to take part in the Yuan astronomical reform. Whatever Ch'en Ting's official position during and after the project, his work merited his promotion in 1288 to Vice Director of the Palace Library. He was replaced in the position after a year, and does not appear in the record again.[189]

As we have seen (p. 152), the order that launched the reform assigned Wang Hsun to work with "astrologers from southern China." This obviously refers to Ch'en and the subordinates sent north with him, and indicates that they were in Ta-tu at the very beginning of the project.

In the judgment of Ch'en Mei-tung, Ch'en Ting's system of 1271 was a bit better than the mediocre average for the Southern Sung. What he contributed to the Yuan project was an experience of astronomical reform, its planning and execution, that the northerners lacked. He brought along several other astronomical officials, and very likely the records of the observatory that (along with any that the Mongols had captured earlier from the Chin) made it possible to test the Season-granting system against the past in such detail. The

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[189] Ch'en Ting's name also appears in the Evaluation as Ch'en Ting-ch'en 陳鼎臣. Oddly enough, the *Sung History's* account of the Attainment of Heaven system does not mention him under either name (*Sung shih* 宋史, 82: 1950–52), nor does either name appear elsewhere in the *Sung shih.* The Ch'en Ting mentioned twice in surviving portions of *Sung hui yao* 宋會要 lived over a century earlier. I suspect that when the failure of #86 led to the demotion of a number of astronomical officials, Ch'en was put in charge by default, and that his rank was too low to mandate giving him credit. The authors of the Evaluation understandably would have accepted his own view of who was responsible for that system. On the promotion in 1288, see *Yuan Mi-shu chien chih,* 9: 8a, which lists him as Ch'en Ting.

memorial of 1280 quoted above mentions that "the southern astronomical officials thoroughly studied more than 40 astronomical treatises …" None of the extant reports specify what else they did.[190]

Specialists

We have already seen that the astronomical reform was ordered and its organization began in mid 1276, and the Astrological Commission was formally established early in 1278 (p. 146). At that point its complement was seven officials, led by Wang Hsun. It expanded to ten in 1308, and fifteen in 1316, although the reform was over well before 1300. These numbers evidently referred only to those in the middling and higher ranks. The "Treatise on Officials" of the *Yuan History* lists a full complement of 66 officials in all ranks belonging to the Commission, comprising sixteen administrators, thirteen observers, fourteen computists, four timekeepers, two instructors, two editors, and fifteen printers (the Commission had its own printing office). The appointees below civil service rank included eleven clerks and 44 students. This count does not include consultants such as Hsu Heng and Chang I, or the artisanal officials seconded to the Commission to build instruments.

The older Directorate of Astronomy, which as part of the same transition became mainly a teaching institution, had a full complement of 39 ranked officials—all mid– or low-ranking administrators or teachers, none observers or computists—16 clerks, and 75 students.[191]

Conclusion

It is possible and useful at this point to essay some generalizations about the leading members of the astronomical reform group. We must leave aside Ch'en Ting, a target of opportunity about

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[190] For their names see below, p. 253.

[191] *Yuan shih,* 88: 2219–20, 90: 2296–97. These enumerations may or may not be for the same undisclosed date. A list of salaries for civil servants in the "Treatise on Food and Commodities" includes the Directorate, but not the Commission. It is not translatable into modern values (96: 2462–63).

whom we know practically nothing. Let me review the names of those whose careers I have summarized. Table 4.1 lists them in order of age in 1280, and notes their year of birth and age at death (Liu Ping-chung died before 1280):

Table 4.1. North Chinese Leaders of the Astronomical Reform

Name	Age in 1280	Born	Age at Death
Hsu Heng	67	1209	68
Liu Ping-chung	[64]	1216	58
Chang Wen-ch'ien	63	1217	66
Chang I	ca. 64	ca. 1216	ca. 66
Yang Kung-i	55	1225	69
Kuo Shou-ching	49	1231	85
Wang Hsun	45	1235	46

Given the era of devastation and reconstruction that we are considering, it is only to be expected that all the protagonists were self-made men.

Consider the important careers that became possible for people close to Liu Ping-chung. He grew up with Chang Wen-ch'ien and Kuo Shou-ching's grandfather; he met Wang Hsun and other statesmen-to-be early on Purple Gold Mountain. Chang I, Hsu, and Yang became his allies in the capital. Together they built as much of a traditional Han political order as Khubilai was willing to countenance and protect. Their mutual support was crucial in this enterprise. Hsu Heng and Chang Wen-ch'ien, as we have seen, were important figures in the propagation of Ch'eng-Chu Confucian studies in the north.

We can conclude, then, that the people who carried out the most elaborate of China's many astronomical reforms, although highly skilled, were not specialists—Kuo aside—but civil-service generalists, and that among them were important figures in forming the governmental institutions and even traditions of self-cultivation of late imperial China.

5 The Observatory and its Instruments

The Observatory

Construction of the Astrological Commission's headquarters and observatory began in 1279. The compound measured roughly 123m × 92m, with a main building 17m tall. It occupied, more or less, the site of today's Chinese Academy of Social Sciences on the north side of Jianguomennei Avenue (Jianguomennei dajie 建国门内大街), within a long walk of the Forbidden Palace. The southeast corner of its walled compound fell close to the present-day Jianguomen subway entrance. Not a trace of the observatory remains.

A dedicatory text written in 1286 by Yang Huan 楊桓 evidently marked its completion.[192] Carved on a stone stele in the Commission's compound, the inscription described its buildings and the activities that took place in them. Because no drawing survives, the sizes and alignments of the parts remain a matter for conjecture.

I Shih-t'ung 伊世同 has plausibly reconstructed the headquarters.[193] Figure 1 is an aerial view of the compound, and figure 2 is a ground plan with the buildings numbered. One entered the compound through a gateway at the center of the south side, proceeding northward on an axial path through a passageway in the building containing the stele (5) and an opening in the long building where the almanacs were printed (4), finally arriving at the main building with part of the observatory on its roof (1). On the way one passed two long buildings used for teaching and for preparing offerings in the shrine (6) and subsidiary towers that held, on one's left, the Tall Gnomon (3), and on the right, what other sources call

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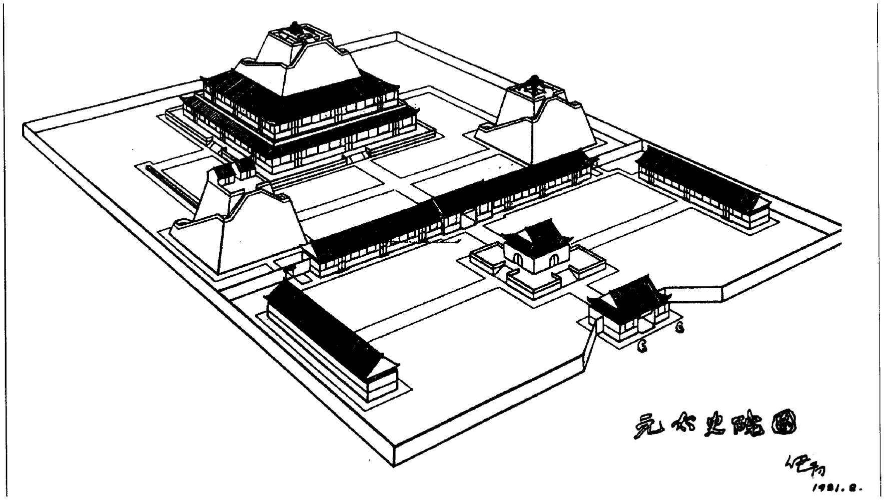
[192] *T'ai-shih-yuan ming* 太史院銘. Yang (1234–99) spent his career in the Palace Library.

[193] For the location of the compound, see I Shih-t'ung 1981. Guesswork is unavoidable, as comparing I's drawings with the very different ones of Yamada Keiji 1980, 218–19, makes clear. I's are in many respects superior; Yamada's bird's-eye view reverses right and left, and is not as close to the architectural practice of the time. See also Li Qibin 1997.

the Ingenious Instrument *(ling-lung i 玲瓏儀)* but this one calls the Ingenious Armillary Sphere *(ling-lung hun i 玲瓏渾儀)* (2). Figure 3 is a plan projection of the main building's three stories, in which letters designate the suites of rooms.[194]

Fig. 1. The Yuan observatory, aerial view. Adapted from I Shih-t'ung 1981.



Here is the pertinent section of the rather literary stele text:[195]

In the spring of 1279, a fine piece of land below the eastern wall of the capital was chosen and acquired, and the work began. The walls are 200½ paces from north to south, and one quarter less from east to west [i.e., 123 × 92m].

> From the center [of the compound] rises an observatory *(ling-t'ai 靈臺)*, more than 70c [22 m.] high, with three stories. The lower and middle stories are surrounded by a walkway lined with outer rooms. In the lower story, the structure that first meets the eye holds the offices (A) where the management of the Commission is carried out. Its head is called the Director; next [in command] is the Associate Administrator; and next are the Assistant Administrators. With the prestige of Steward-bulwarks of the State, they provide leadership

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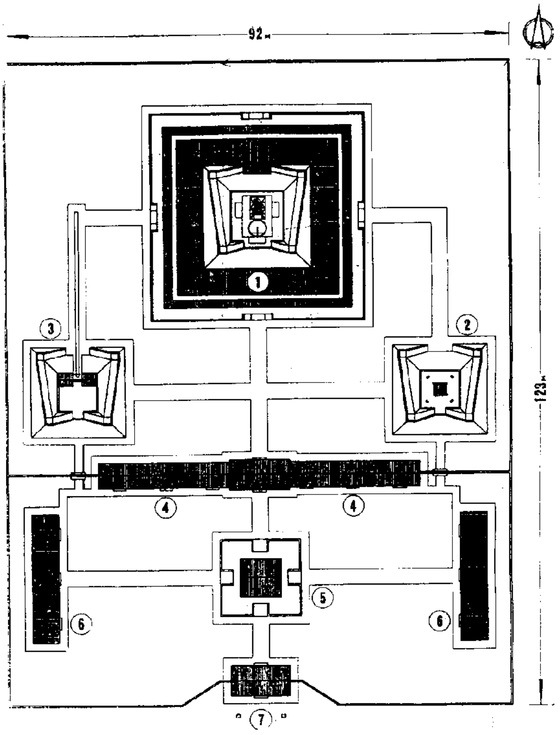
[194] Since there were eight suites, their numbering conventionally used the names of the *Book of Changes'* Eight Trigrams *(pa kua 八卦)*.

[195] *T'ai-shih-yuan ming*, pp. 217–18. For a translation of the entire text, which needs to be used critically, see Mercier 2003, 218–26.

from above, without a set quota of personnel. Their [direct] subordinates [housed in this building] include those in charge [of sections, *chu shih* 主事], clerks and interpreter-translators *(ling i shih* 令譯史), administrators, and those in charge of stores. The rooms on either side are for meetings of the leaders, to deliberate or issue orders.[196]

Fig. 2. Yuan observatory, plan view. From Ch'en, p. 533, figure 6-30.



As for astrological and astronomical computation, the 70 scholars are organized into three bureaus. The first is Computation. Its officers include the Five Seasonal Officials, the Royal Astrologers and their assistants, and the Calendar Clerks, distributed through the

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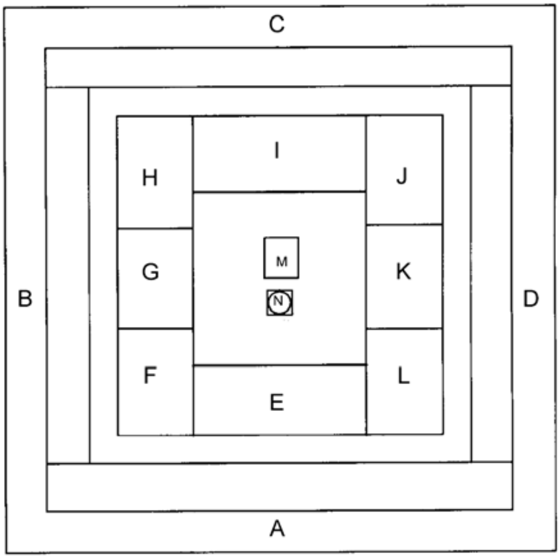
[196] *Ling i shih* is not an official title, but a combination of *ling shih* 令史 and *I shih* 譯史. The interpreter-translators did not have the task of translating foreign astronomical writings. Since the Mongols "systematically placed peoples of different ethnic, communal and linguistic backgrounds side by side in the Yuan bureaucracy," interpreting and translating was a normal function in government offices (Allsen 2001, 7).

Morning Building (D).[197] The second is Observation. Its officers include an Observatory Director and Astronomical Observers and their assistants. The third is Water Clocks. Its officers include a Supervisor of Water Clocks as well as Timekeepers. [The personnel of these two bureaus] are distributed through the Evening Building (B). The acquisition and provision of instruments and supplies take place in the Yin Building (C).

Fig. 3. Yuan observatory, rooms of the main building. Adapted from Ch'en, p. 533, figure 6-31.



In the middle level, room E exhibits the luminaries [in star maps?]. Room L holds a water-driven armillary clepsydra. Room F holds a "spherical sky" globe and a depiction of the "sky as a carriage-cover" cosmographic model. Rooms G and K show what [features of the sky] are visible from different sites, north and south, according to the "spherical sky" and "sky as a carriage-cover" models. Room I con-

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[197] Seventy is either a round number or the figure at a time different from that of the record in the *Yuan History;* see p. 169. The Five Seasonal Officials divided up the administration of computing; they were named after the four seasons and the transitions between them, which together corresponded to the Five Phases. See the Canon, section 2.1.

tains the place of Counter-Jupiter[; that is, a shrine to its god?]. Room H holds books on astrology and celestial observations. Room J holds books on ancient and modern mathematics and computational astronomy.[198]

Atop the observatory are installed the Simplified Instrument and the Upward-facing Instrument, and the Direction-determining Table set within (? *fu* 敷) the former.

Below the observatory, to its left, there is a separate smaller platform, with a walkway lined with peripheral rooms to beautify it on all sides. Atop it is the Ingenious Armillary Sphere *(ling-lung hun i* 玲瓏渾儀). To the right of the observatory is set up the Tall Gnomon, with a hall in front [i.e., to the south] of it. Emplaced northward from the gnomon is the stone shadow template, with a scale engraved on it in *ch'ih,* tenths, and hundredths, to measure the shadow. To the sides of the shadow template are joined a construction for use with it (?).[199] The space above it is open to the sky so that [the observers] can measure the shadow.

In front of the observatory, in the eastern and western corners, is an office in which to carry out the printing of the almanac. Next southward are set up, as above (?),[200] a kitchen for ritual offerings *(shen ch'u* 神廚) and a school of mathematics.

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[198] Needham et al. 1960, chapters 6 and 7, gives a history of medieval water-powered astronomical clocks. For the "spherical sky" and "sky as a carriage-cover" cosmographies, see Cullen 1996, 35–39. The second of these schemata, closely related to the invention of the armillary sphere, may be what "'spherical sky' globe" refers to. The "armillary clepsydra" would be a water-driven astronomical clock.

[199] *Kuei p'ang chia i lien kou k'o kuei* 圭旁夾以連冓可圭 probably refers to the shadow aligner, but the sentence is too unclear—or garbled—to permit certainty. Note that "left," as in other early sources, is defined by a south-facing standard, and is therefore to the east. Left in this usage is hierarchically superior to right.

[200] There is no obvious referent for "as above *(ju shang* 如上)." The kitchen would not be unusual in a government compound equipped with a shrine that required food for offerings. It may or may not have been used also to prepare ordinary food for the staff. Liu Ping-chung and a colleague had proposed in 1273 that the Mongol government set up its first printing office; see Allsen 2001, 179–80. By this time some agencies had printing offices of their own. The inscription says nothing about accommodations for the students.

Predecessors of the Yuan Project

Chinese Predecessors

So far as we know, officials manufactured eight bronze armillary spheres in the Northern Sung period between 979 and 1124—the first one nineteen years after the dynasty began, and the last one three years before the loss of North China. They made three more in the Southern Sung up to 1193.[201]

As the Yuan "Treatise on Astrology" puts it,

> after the debacle of the [Northern] Sung, all of its observational instruments became the property of the Chin dynasty. When the Yuan arose, establishing its ritual vessels [i.e., its capital] at Yen-ching 燕京 [i.e., Ta-tu], at first it continued to use the old Chin [instruments]. But their tracks and rings [were damaged and] no longer worked together, so [the instruments] could no longer be used. That being so, the [future] Grand Astrologer, Kuo Shou-ching, contributed the Simplified Instrument, Upward-facing Instrument, and the other instruments he invented, which had attained unprecedented precision.[202]

In other words, some of the instruments of the northern Sung court survived the fall of that dynasty in 1127 as well as the defeat of its successor the Chin in 1234. The Yuan poet Wu Shih-tao 吳師道 (1283–1344) wrote about coming across a deserted observatory in the fields south of the capital, and discerning an inscription on an instrument—almost certainly an armillary sphere—dating it in the reign era 1049–53.[203]

When the Chin took the Sung capital, Pien 汴 (present Kaifeng, Honan), they moved another major observational instrument to their own capital, Yen 燕 (or Yen-ching). This was the bronze armillary sphere made in 1089 as part of a great water-driven celestial

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[201] P'an Nai 潘鼐 1989, 165–66. See also Li Chih-ch'ao 李志超 1987.

[202] *Yuan shih,* 48: 989–90. It is not certain whether this means precision or accuracy, but Chinese understood the difference by this time. See Sivin 1989.

[203] "Chiu yueh nien-san-jih ch'eng wai chi yu 九月廿三日城外紀遊 (Record of an excursion outside the city gates on the 23rd of the ninth month)," an undated poem in *Li-pu chi* 理部集, 5: 19a.

clock, based on a separate instrument cast by Shen Kua 沈括 in 1074.[204]

The Chin history relates what happened to the remainder of the clock in the aftermath of that victory: "When the Chin had taken Pien, they carted off [the parts] to Yen. The celestial gear-wheel … [and other components of the clock], over a long time, had been thrown away or wrecked; only the bronze armillary sphere had been set up in the observatory of the Astrological Service. The distance from Pien to Yen was over 600 km, and the up-and-down configuration of the places differed. [After the move,] one could no longer quite center the sighting tube on the pole star. Nor could one [even] observe it without lowering the tube 4 *tu.*" The discrepancy in "up-and-down configuration"—that is, the difference between the altitude of the pole at the two sites, equivalent to the discrepancy in latitude, was just over 5°.[205]

This passage goes on to detail later damage to the armillary sphere. When in 1215 the Chin government moved southward to Pien to avoid Mongol attacks, "because the armillary sphere had been cast in [large] units, [the technicians] were unwilling to break it up. Since it would have been impossible to transport the whole thing in a cart, they abandoned it when they left."[206]

Shen's armillary sphere of 1089 enters the Yuan record in 1234 when, in the aftermath of the Mongol victory in North China, Chinggis's successor ordered that "the Temple of Confucius and the armillary sphere be repaired." In 1237 there was another order that "the Temple of Confucius and the observatory again be repaired." Why did he take this trouble—twice—so long before the Mongols

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[204] There has been some controversy over whether this instrument, which Kuo later used, was Shen's original sphere, but the sources cited in P'an Nai 1983, 236–37, resolve this question.

[205] *Chin shih,* 22: 523–24. It was not unusual for new dynasties to change the names of bureaus.

[206] For a more extensive but not consistently reliable translation see pp. 131–33 in Needham et al. 1960, a history of the celestial clock. I Shih-t'ung 1994 discusses the history of Shen's instrument to the present. Yamada 1980, 26–30, looks into which Northern Sung armillaries survived until 1127.

contemplated an astronomical reform? That was almost certainly because of another Mongol acquisition, a living one. Yeh-lü Ch'u-ts'ai 耶律楚材, a Chin official, stayed behind in Yen when in 1215 the Mongols drove his government southward. In 1218, although less than 30 years old, he became an advisor of Chinggis. His responsibilities, as we have already seen (p. 141), included astrology and astronomy. Eager to mediate between Mongol and Chinese culture, he was prepared to appreciate the potential value of the old, patched armillary sphere.[207]

In the first phase of the Yuan reform, Kuo Shou-ching and his colleagues did a good deal of the basic observational work with whatever usable instruments were on hand. A document by Kuo tells us (p. 575 below) that in the Directorate of Astronomy there was an armillary sphere built in Pien between 1049 and 1054. This would not have been the one that Wu Shih-tao saw in an abandoned observatory; Wu was not yet born in 1279. We have no idea how or when it found its way to Ta-tu. Its installation implies that the instrument of 1089 was past repair, although Kuo found it useful as a basis for the design of his own instrument.[208]

Islamic Predecessors

We know that Jamāl al-Dīn constructed seven instruments of Persian type in Khubilai's court in 1267. The *Yuan History* records their names in a hybrid of Persian and Arabic transliterated into Chinese, with very brief descriptions. So far as we can identify them, they comprised an armillary sphere, a parallactic ruler, equinoctial and solstitial plane sundials, celestial and terrestrial globes, and an astrolabe or nocturlabe. Astronomers in China already had armillary spheres and celestial globes, both of which often appeared in large astronomical clocks. The celestial globe may have motivated Kuo to build a free-standing one for his observatory. It is pos-

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[207] *Yuan shih,* 2: 33, 34. Yamada believes that Yeh-lü had the armillary set up in the Temple of Confucius (1980, 31), but the wording does not imply any connection between the two.

[208] See also *Yuan shih,* 164: 3847.

sible that he made a nocturlabe as well (p. 205). The other Islamic instruments that had no counterpart in China did not establish themselves there. Once we have looked closely at the project's own instruments, we will be prepared to estimate the extent of Islamic influence (p. 219).[209]

The Instruments of the Reform Project

Kuo Shou-ching's account of conduct lists more than a dozen instruments he designed for the Commission's observatory, and four intended for astronomers involved in the empire-wide survey. Copies of three of Kuo's observational instruments, reproductions cast between 1438 and 1446, survive at the Purple Mountain Observatory, Nanjing: the armillary sphere ultimately based on that of Shen Kua, the Simplified Instrument, and a cast bronze gnomon 8c high. Archeologists have studied and restored one of his two 40c gnomons, namely the one at present-day Gaocheng, Henan. About a few more of the instruments we have enough details to permit their reconstruction. Of the rest, we know very little, and we cannot be sure what several were.[210]

Keeping these uncertainties in mind, I will enumerate the important instruments that were built for use in the project, and assess what was special about each. In order to make clear the kinds of evidence we have, I have translated in appendices A and B the main

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[209] For these transcriptions see *Yuan shih*, 48: 998–99. Although some historians write of Jamāl's "bringing" these instruments from the West, *Yuan shih* clearly states that he constructed (*tsao* 造) them. Several able Japanese and European scholars have worked on their identification. My list is based on Miyajima 1982. For a report in English, see Yabuuchi 1997, 14–16.

[210] For the account of conduct, see below, p. 573, and Kuo's *Yuan shih* biography, 164: 3847. More or less detailed accounts of eight instruments appear in the history's "Treatise on Astrology," 48: 990–97, translated in Appendix A. Some of them are discussed in this chapter. There are also contemporary poetic inscriptions for the Ingenious Instrument and the Tall Gnomon by Yang Huan, and for the Simplified Instrument and the Upward-facing Instrument by Yao Sui 姚燧. These lack prosaic, descriptive introductions like the one translated above. See also I Shih-t'ung 1983.

sources that describe them. Here I will first take up the question of how far the astronomical reform depended on them.

We have already seen that the construction of the new instruments did not get under way until 1279. Only two—one a vaguely described armillary instrument and one a wooden gnomon—were ready before 1280. On the other hand, the dated measurements of solstitial sun shadow lengths using the gnomon, recorded in section 1 of the Evaluation, began in July of 1277 and concluded in January of 1280. The memorial of Yang Kung-i that submitted the new system (p. 166) confirms that the observations on which the new system was based—in the form promulgated at the end of 1280—could not have used most of the instruments made for the new observatory after 1279. As for when the fundamental data were gathered, we have a perplexing but valuable clue.

The annals of Khubilai include, in the midst of unrelated entries, nearly identical items for 17 June 1288 and 7 April 1289: the first says that an "armillary sphere was completed," and the second that "the casting of an armillary sphere was completed."[211] Although casting is not the final task in producing a bronze instrument, the annals of an emperor do not pause over intermediate steps; the sense of the two announcements is the same. Neither sphere of these dates is mentioned by any source closer to the primary documents, nor by any later source. It is difficult to imagine that, three years after Kuo submitted several final documents of the reform, he had reason to make two identical instruments that contemporary accounts of his own work do not mention.

P'an Nai has suggested a plausible and consequential answer to this puzzle. He notes, first, that the largest instruments on which work began in 1279 would, in the best circumstances, have taken considerable time to cast, finish the surfaces, assemble, calibrate, test, and do final preparations for use. Given the many tasks that

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[211] *Yuan shih,* 15: 312, 321. The second says *"chu hun-t'ien i ch'eng* 鑄渾天儀成," and the first lacks the initial character. To add to the ambiguity, section 3 of the Evaluation refers to the Simplified Instrument as "the new armillary sphere" (p. 292).

the reform entailed, and the heavy burden on the shoulders of Kuo after the death of his colleagues, completing the instruments could easily have taken a decade. The two reports, nine and ten years after manufacture of the instruments began, simply announce the completion of two of them. In explaining why both accounts name what seems to be the same instrument, P'an reminds us that at the time terminology was loose—an essential point for resolving several ambiguities in the record. As many examples show, writers on astronomy generally use *hun-t'ien* 渾天 or *hun-t'ien i* 渾天儀 for an armillary sphere, but sometimes they mean by these terms a solid celestial sphere. In this instance we cannot say which of the two is which; but the most likely conclusion is that one instrument of each kind was completed with nearly a year between.[212] I will argue below (p. 194) that the Ingenious Instrument was an armillary sphere.

The corollary is striking. The wooden model of the Tall Gnomon and the Simplified Instrument seem to have been used first; according to Kuo's biography (below, p. 583), he planned prototypes of the Tall Gnomon and the Simplified Instrument, rather than an armillary sphere, to serve as his most basic instruments. But, if it took a decade to complete the Ingenious Instrument, can we assume that any of the large permanent instruments were in place long before 1286? In that year, with Kuo's documents of the reform partly compiled, and at least one inscription that described the observatory written, the reform had drawn to an end. If that is the case, it is quite possible that the new large instruments—whatever their long-term utility—played no part at all in it. There is no clear evidence that they played any part in the system as submitted at the end of 1280. The ongoing testing and refining of the system no doubt took advantage of them. In fact, the solar theory of the Canon states expressly that the equatorial extensions of the 28 lunar lodges were measured "with the new armillary sphere"—although it does not say when (p. 426). But that is an exception; it appears that old

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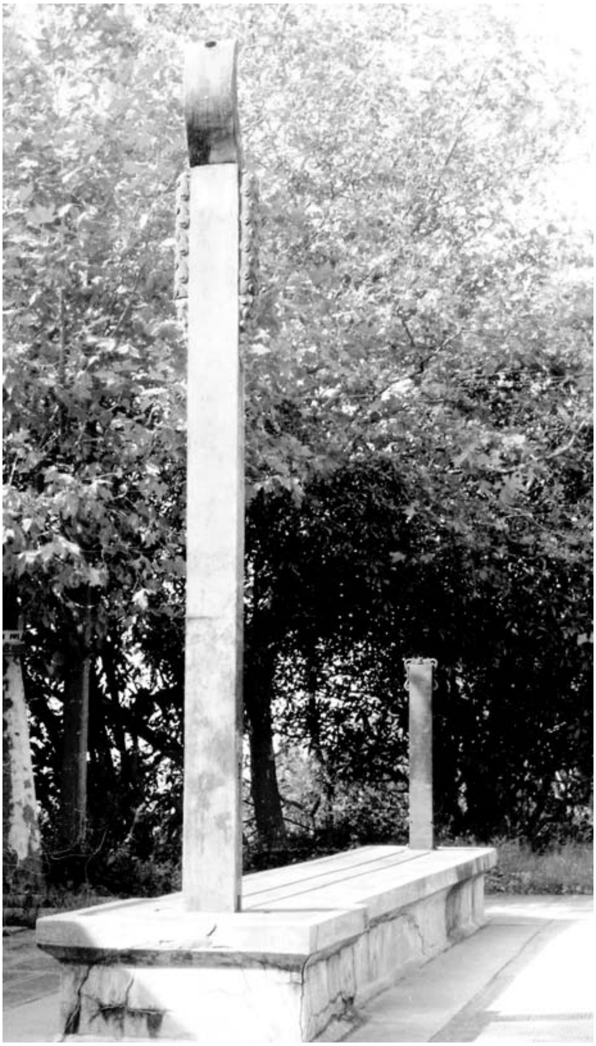
[212] P'an 1983, 242–243.

equipment and prototypes yielded most of the data for the Season-granting system that the *Yuan History* records.

Let us now pass on to the project's individual instruments.

Fig. 4. A conventional 8-*ch'ih* gnomon of traditional design, the restored 15th-century copy of the Yuan original, at Purple Mountain Observatory, Nanjing. The shorter upright holds an inscription. This and other photographs were taken by the author at Purple Mountain Observatory in October 1977.



The Tall Gnomon

The Tall Gnomon *(kao piao* 高表) is an instrument for measuring the length of the sun's shadow at noon and tracing its variation through the year (figure 4). At 40c (10m), it is five times the height of the standard gnomon long used for that purpose.

Fig. 5. The Tall Gnomon at Gaocheng, after reconstruction. Photo courtesy of the Institute for the History of Natural Sciences, Chinese Academy of Sciences, Beijing.

The detailed description of it (pp. 569–571), summarized in the Evaluation, section 1, is the only surviving account of its dimensions and parts. The plan of 1279, as approved, envisaged building five large gnomons in important cities. Two were actually built, one at the Commission and one, not in a city, but in the ancient observatory at what is now Gaocheng, Henan. Archeologists located the remains of the latter early in the twentieth century, and subsequently restored it.[213]

Contemporary Chinese sources say unanimously that the new system's observations came from the capital, not from Gaocheng.[214] Why, then, was the second gnomon built in such an obscure place? We do not know why the collaborators did not carry out the approved proposal to build multiple urban observatories. It would have been natural to argue, once the plan for an empire-wide sky survey using portable 8c gnomons was approved, that enhanced precision at more than one location would not be worth the enormous cost and allocation of skilled manpower—if indeed enough experienced astronomers were available.

Gaocheng was a special case. Conventionally educated Yuan Chinese believed the classical assertion that the sages who founded the Chou dynasty circa 1050 B.C. inaugurated a state ritual when they used a gnomon to determine the site of its capital:[215]

> [The Grand Minister of Education] used the method of the earth template *(t'u-kuei* 土圭) to measure the position (?) on the earth. He set the standards for the sun's shadow in order to determine the center of the world *(ti chung* 地中). … The place where at the extreme of the sun [i.e., the summer solstice, *jih chih* 日至] the shadow is 1.5c long is called "the center of the world." That is where heaven and earth join, where the four seasons meet, where wind and rain are confluent, where yin and yang are harmonious, and therefore where all things are prosperous and secure. There [the founders] established the king's domain …

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[213] On the initial plan for five gnomons, see *Yuan shih,* 10: 209. Re the Gaocheng gnomon, Tung Tso-pin 董作賓 et al. 1939, especially pp. 55–65, discuss the texts. Chang Chia-t'ai 张家泰 1978 describes the modern restoration of the gnomon and experimentation with it.

[214] Mercier 2003 has confirmed this finding by analyzing the latitude implied in the gnomon shadow records, and has added a number of useful comments.

[215] *Chou li* 周禮, 10: 4b–5b. This orthodox classic purports to outline the structure and functions of the early Chou government, but was actually written at some time between the early second and late first century B.C. "Position" in the first sentence is *shen* 深, literally "depth."

The Evaluation of the Yuan treatise unmistakably echoes this passage when it says that "At the center of the world, the shadow of an 8c gnomon is 13c and a fraction long at the winter solstice and 1.5c long at the summer solstice" (p. 256). Wherever the "center of the world" originally may have been, and whenever its location may first have been determined, by at least the first century B.C. astronomical authors were identifying it with Yang-ch'eng 陽城—that is, today's Gaocheng. Kuo and his colleagues found there, among other relics, one of the gnomons eight *ch'ih* high used for the sky survey of A.D. 725, the model for the one they themselves carried out. Even more important than this historical link was the ritual and aspirational link between the founding of the Chou, the golden era of high antiquity, and that of the Yuan. That is, the Tall Gnomon at Gaocheng marked Yang-ch'eng as the center of the world. We have no record that anyone used it for observation.[216]

There is a most significant discrepancy between the remains of the Gaocheng gnomon and the description of the Ta-tu instrument. The former includes a template and a building, but no trace of a bronze gnomon. The text goes into detail about the gnomon and template, but does not even hint that a building was included.[217]

Leading historians have argued persuasively that the text (and, oddly enough, the preamble to the Evaluation, p. 255) described the original design, not the instrument actually built. Serious problems with the design led to a fundamental revision visible in the Gaocheng instrument. Li Ti has estimated from the dimensions given for the bronze gnomon that, if solid, it would have weighed close to twenty metric tons; a sturdy hollow one would still have been very heavy. In either case, the fabricators would have had to mold an extremely long and heavy post and to attach at its upper end two bronze uprights to hold the crosspiece horizontal, as set

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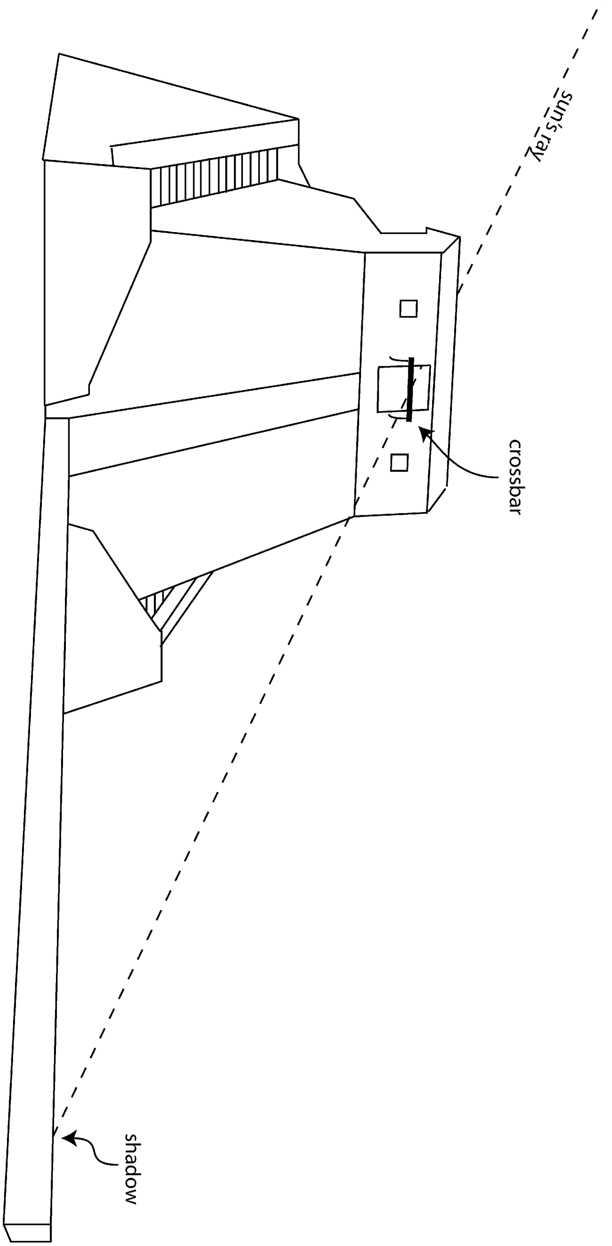
[216] On the meridian survey of 725, see Tung Tso-pin et al. 1939 and Beer et al. 1960. For a general discussion of gnomons, see Needham et al. 1954– , 3: 286–300, and Bruin & Bruin 1977.

[217] On the discrepancy and its meaning see Li Ti 2001 and Ch'en Mei-tung 2003a, 525–26.

Fig. 6. The Tall Gnomon, with Li Ti's conjectural reconstruction of the crossbar and support, showing the shadow cast by the center of the crossbar on the horizontal shadow template.



using a water-level channel atop it. It would have been very difficult to install the gnomon precisely perpendicular, and the crosspiece perfectly parallel, to the ground; if later on either tilted slightly, adjusting the gnomon would have been practically impossible. There is the additional problem of inspecting—necessarily from above—the water-level indicator ten meters off the ground. It is obvious from the surviving instrument that simplifying the design easily avoided these serious problems.

The fundamental engineering task was straightforward: to hold a thin horizontal rod in place at exactly 40c above the shadow template in such a way that a technician could verify that it was level. All one needed to meet this specification was a building that solidly supported the crosspiece, provided access to it, and allowed a workman to easily adjust it when necessary. The building that has survived at Gaocheng accomplished precisely that, as Li Ti's diagram (figure 6) shows, with its unproblematic two uprights.

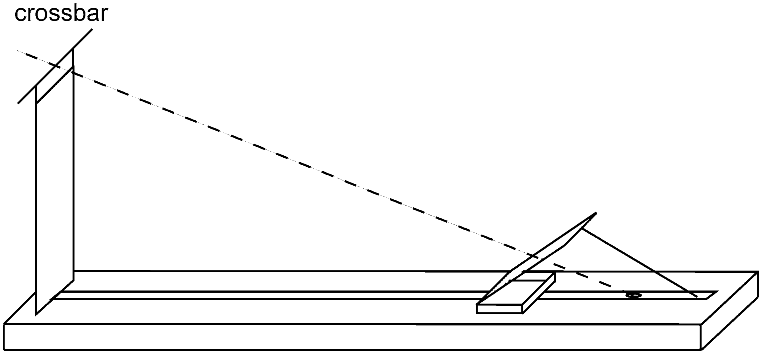
The gnomon was invented more than a millennium earlier to cast its own shadow. It became a ritual fixture in imperial astronomy, and standard equipment for every observatory. When Kuo realized that an enormous pillar was a most unwieldy mount for a small object designed to form an image rather than cast a shadow, he had the understanding and the authority to discard it.

In order to estimate the usefulness of the instrument, it is necessary to consider two others used with it, the Shadow Aligner and the Observing Table.

The Shadow Aligner

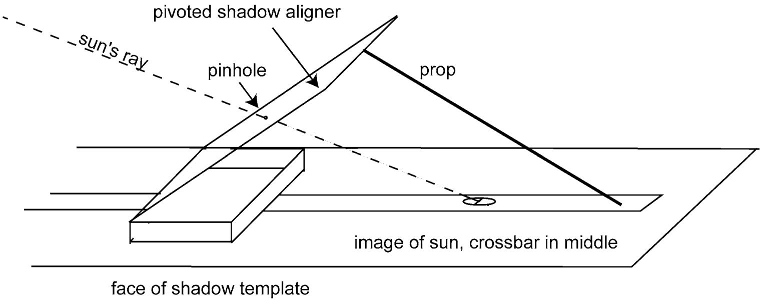
The description of the Shadow Aligner (*ying-fu* 景符) in the "Treatise on Astrology" (appendix A) is clear about its rationale and its design. The problem with multiplying the height of a gnomon by five is that the shadow becomes even less distinct. The solution was a sort of pinhole camera that casts on the shadow template an image of the crosspiece silhouetted against the sun (figure 7).

Fig. 7. Movement of sun's ray across the gnomon crossbar, pinhole of the shadow aligner, and the image cast on the horizontal template. This and the next figure are adapted from Ch'en, p. 527, figure 6-25.



As the observer moves this device, it rides along the scale of the template until an image is perceptible. Since the plate carrying the pinhole and the scale on which it casts the image are not parallel, the simple mechanism for tilting the plate allows the image to be adjusted for optimal sharpness (see the detail in figure 8). There is a tradeoff. A very small pinhole casts a sharp but dim image, and that from a larger one is easier to see but is, as the description puts it, "indistinctly visible." Even so, the result is a considerable improvement, as a modern experiment has shown.[218]

Fig. 8. Shadow Aligner, shadow cast by pinhole on template (detail).



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[218] Bruin & Bruin 1977, especially pp. 24–25. I Shih-t'ung 1996, 212, has estimated that a workable size for the pinhole is 2 or 3mm. See also Kuo Sheng-ch'ih 郭盛炽 et al. 1983.

The Observing Table

The second ancillary instrument, the Observing Table *(k'uei chi* 窺几), uses the gnomon to determine the shadow of the moon or a star at night. This makes it possible, for instance, to determine a star's polar distance at culmination. Since one can use an armillary instrument with altazimuth rings for the same purpose, the main purpose of the Observing Table was perhaps to associate such an observation with the gnomon. The astronomers and their patrons would have considered such an association to be ritually significant, even though this device cannot yield the high precision of sun shadow measurement. Figure 9 illustrates a modern reconstruction of the instrument.[219]

The problem with this use of the gnomon at night is that the luminaries—even, sometimes, the moon—do not form a pinhole image clear enough to read on the template scale. To use the Observing Table, the recumbent observer, instead of lining up the celestial object, the crosspiece, and the scale, rests his head on the shadow template. Looking upward, he moves the sliding table above his head down the scale to visually line up the top of the crosspiece and the object. He moves one of the two observation delimiters *(k'uei-hsien* 窺限)—crosswise strips with beveled edges—to mark that alignment on the table's scale, which is graduated like that of the shadow template. He then does the same thing for the bottom of the crosspiece, using the other delimiter. The opening between them corresponds to the space between the "shadows" of the top and bottom. Half the difference between the two positions corresponds to the point on the scale where the celestial object would be lined up with the center of the crosspiece. Projecting that point perpendicularly downward to the template's scale yields, not where the shadow of the object would fall if it cast one, but one point on a right triangle 40c high; solving for the horizontal leg of the triangle, a trivial problem in similar triangles, yields the virtual shadow reading on the template.

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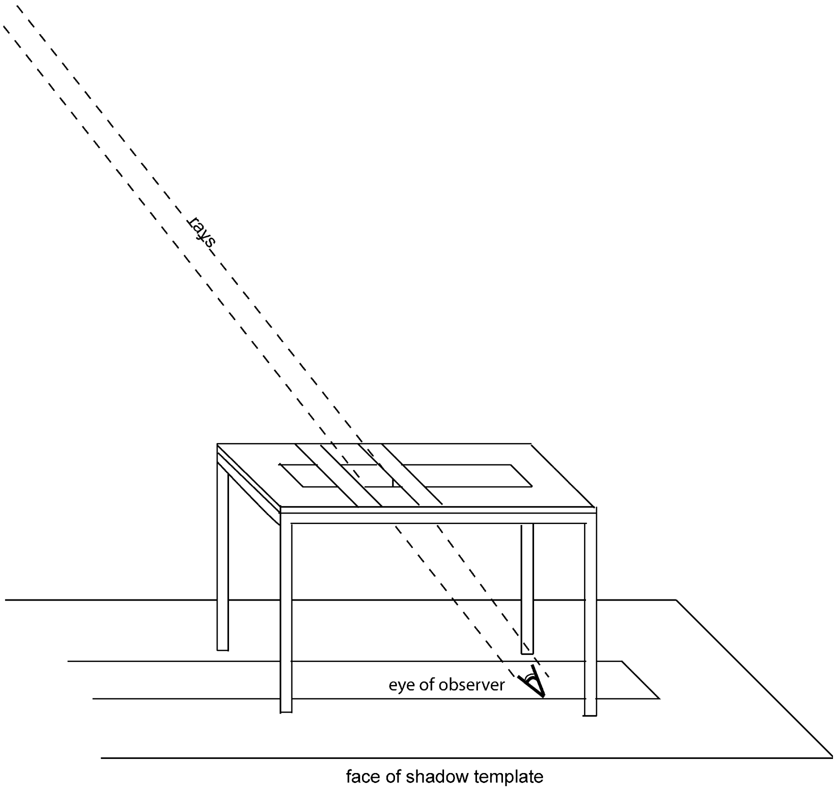
[219] P'an Nai and Hsiang Ying 1980, 60; figure 9 is based on the redrawn version in Ch'en Mei-tung 2003a, 528.

Fig. 9. Conjectural reconstruction of the Observing Table. An observer under the table visually lines up the two movable delimiter strips with the gnomon crossbar at the moment that it divides the moon or obscures a star, and then reads from above the positions of the strips on the graduated scale alongside the slit. Adapted from Ch'en, p. 528, figure 6-26.



The Ingenious Instrument

No straightforward description of the Ingenious Instrument *(ling-lung i* 玲瓏儀) survives. Conjectures about what it was—and even about what the poetic *ling-lung* in its name means—vary wildly. We have already seen in the description of the observatory that this instrument occupied its own tower (p. 172). That suggests that it was large and important.

All the interpretations are based mainly on the one substantial source, the contemporary *Inscription on the Ingenious Instrument*

(Ling-lung i ming 玲瓏儀銘). Yang Huan wrote this inscription for display with the instrument at the Commission's observatory. He composed it with four characters to the line and rhymes. It is flowery and laden with clichés, a good example of conventional inscriptions for bureaucratic apparatus. Much of it has the flavor of a metrical introduction to astronomy; I will translate only the part pertinent to the instrument's identification. What follows sacrifices meter and rhyme in the interest of what scant clarity the inscription offers. I also avoid false clarity, maintaining the ambiguities of the original, always desirable when translating poetry:

> The Grand Astrologer, in charge of the sky,
> Uses fully his encompassing knowledge,
> Designs the implements of his art,
> Each to be used in its own way.
> Surpassing in its uses,
> This instrument is ingenious.
> More than ten myriad eyes,
> Ascension, declination both set out.
> Its body the same as heaven,
> Responsive to compass and try square [i.e., meticulously built].
> Its whole body empty, bright,
> Inside and outside revealed.
> Celestial images thickly arrayed;
> No one can count their number.
> Each located sequentially in its lodge,
> Its distance from the pole measured in degrees.
> A man who observes from within,
> When he sights them he knows.
> The ancient sages were truly fertile
> But never achieved this design;
> Only in our sovereign's great era
> Was this work [of astronomy] fully accomplished.

This source is highly ambiguous, and other scattered bits of evidence do not make its meaning clear. For instance, "eyes" *(mu* 目) could be a metaphor for the luminaries, although several commentators read the word in diverse literal senses (observers' eyes, small openings, etc.). "Celestial images" *(hsiang* 象, which often means "counterparts") might be the luminaries themselves or representations of them. It is impossible to be sure what sort of instrument this inscription describes. I will summarize the main interpretations and the most pertinent evidence for each.

The most obvious hypothesis is that the Ingenious Instrument was an armillary sphere. We have seen that a memorial of Yang Kung-i and Hsu Heng, late in 1280, mentioned the existence of a "new armillary sphere" (above, p. 166). That could refer only to this device or the Simplified Instrument. In fact the stele text written for the observatory's completion refers not to an "Ingenious Instrument" but to the "Ingenious Armillary Sphere *(ling-lung hun i* 玲瓏渾儀)" (above, p. 172). In an inscription expressly for the installation, this is strong evidence indeed.

Fig. 10. Armillary Sphere, a 15th-century copy of one designed by Kuo Shou-ching, now at Purple Mountain Observatory.

In the mid fifteenth century, as I have mentioned, the Ming built its own complement of instruments, including an armillary sphere. Historians disagree on whether the surviving one at the Purple Mountain Observatory (Figure 10) was based on Kuo's design or an earlier one, but the evidence strongly suggests that it is a copy of a Yuan instrument (see below, p. 216). The only likely candidate is this one.

Modern historians who deny that the Ingenious Instrument was an armillary sphere disagree about what it was. Their counterproposals include a demonstrational planetarium used to entertain official visitors; a kind of planetarium based on a water-driven celestial globe, covered in an unspecified translucent material; a globe similar in design but primarily an observational instrument; and a kind of open basketwork sphere rigid enough to support an observer inside.[220] All except the armillary-sphere proposal depend on a literal but not very close reading of a very ambiguous poetic text, and pay too little attention to engineering to be persuasive. None offers evidence that the technology for making a large globe of rigid basketwork or of material one can see through was available in the 1280's.

No source mentions clockwork or any other mechanism in connection with the Ingenious Instrument. The inscription's reference to "its whole body empty, bright," conflicts with the Sung and Yuan practice of installing clockwork-driven celestial globes in cabinets that showed only the hemisphere corresponding to the visible sky (see below, p. 567). None of this, or anything else, rules out identifying the instrument as an armillary sphere.

Finally, Kuo's biography, listing his motive for designing each instrument, says that "because, although the shape of the celestial globe was correct, it was not adapted to his uses *(mo shih so yung* 莫

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[220] These are the hypotheses of Li Ti 1977, reprinted with trivial modifications in Li 1996 and Yamada 1978, 324–27, and 1980, 207–9, elaborated and criticized in Po Shu-jen 薄树人 1982, 323–25, Ch'en Mei-tung 2003b, 156–60, Tu Sheng-yun 杜升云 2003, and Li Chih-ch'ao 1987, 106–8. For a detailed critique of the early proposals see P'an Nai 1983, 243–45.

適所用), he made an Ingenious Instrument" (p. 576). Vague though that is, it suggests a useful observational device. In short, although the debates over the identity of the *ling-lung i* will no doubt continue indefinitely, we need not hesitate to conclude that the Ingenious Instrument was the thirteenth-century prototype—the only likely prototype—of the surviving fifteenth-century armillary sphere.

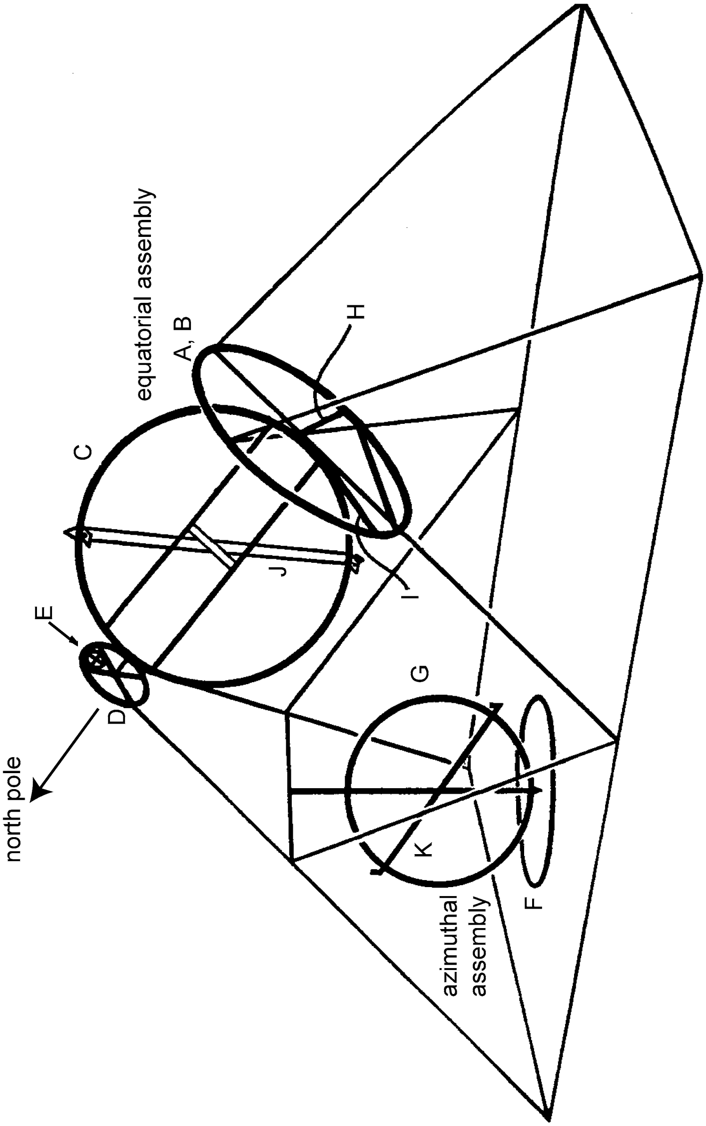
The Simplified Instrument

Fig. 11. The Simplified Instrument, cast bronze, a fifteenth-century copy of one designed by Kuo Shou-ching. Now at Purple Mountain Observatory.

Elaborate armillary spheres like the one in figure 10 contain concentric circles for equatorial, ecliptic, and azimuthal measurements, or some combination of them. They are most versatile instruments, but the multiplication of circles makes them more difficult to build and use than three separate devices would be.

Kuo rethought the conventional design by unpacking the circles. He discarded the ecliptic components, for observations in ecliptic coordinates were not very important to Chinese astronomers. When they wanted them, they could use the armillary sphere. In any case, the Yuan treatise does not record direct ecliptic observations.

Fig. 12. The Simplified Instrument, schematic diagram showing equatorial and azimuthal assemblies. Modified from Needham et al. 1954-, 3: 371, figure 166.



As figure 12 shows schematically, Kuo mounted the equatorial and azimuthal components in separate groups on a rectangular base 18c by 12c (4.6m × 3m). In the upper part of the diagram is an assembly of equatorial circles mounted on an axis pointing toward the north pole. The "hundred-mark circle *(pai k'o huan* 百刻環)" (A), is a diurnal circle fixed in place and divided into 100 marks. The "Red Way circle *(ch'ih tao huan* 赤道環)" (B), is a mobile circle of the same size for reading right ascension. It sits on A, and four roller bearings permit the observer to rotate it. The "four excursions double circle *(ssu yu [shuang] huan* 四游雙環)" (C), is a ring with a movable sight for reading declination; D is the "orbital circle *(kuei huan* 規環)." Parallel to A, it carries the small "pole-determining circle *(ting chi huan* 定極環)," E, 6 *tu* in diameter. Astronomers used the latter to measure the distance from the north pole to certain nearby stars at their culminations (see below, p. 214). The azimuthal assembly, in the lower left, comprises the "yin azimuth circle *(yin wei huan* 陰緯環)" (F), a circle fixed in the plane of the horizon for reading azimuth, and the "standing movable circle *(li yun huan* 立運環)" (G), for observing altitude. Mounted on the various mobile circles are four diametral sights. H and I, on B, make it possible to determine the equatorial arc between two stars. The two others are J on C and K on G.[221]

The segregation of functions into two isolated groups had several obvious advantages. Building a smoothly functioning and sturdy machine became much simpler when one did not have to incorporate a large nest of movable circles, all perfectly concentric. Those irrelevant to the task at hand no longer blocked the observer's view of the sky. One needed to manipulate only the circles needed for the task at hand. Kuo worked into the instrument some incidental but valuable improvements. Ch'en Mei-tung points out that, although the Simplified Instrument was smaller than the Northern Sung in-

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[221] As P'an & Hsiang 1980, 36, points out, the roller bearings, which enable one ring to rotate upon another, seem to be Kuo's invention.

struments, it was more finely made, and Kuo increased its precision from the Sung's 1/4 or 1/12 *tu* to 1/20 *tu*.[222]

Joseph Needham has described the instrument as an "equatorial torquetum," related to the Middle Eastern (and later European) device that included an equatorial disk, separate ecliptic circles, and a kind of protractor and plumb bob for reading altitude.[223] It did not separate its components in the same way as the Chinese device segregated its equatorial and azimuthal parts. Its use was primarily converting between ecliptic and equatorial coordinates, a task for which the Chinese one was not equipped. If there is a relationship, it is an attenuated one. In any case, no torquetum was included among the instruments from Marāgha (pp. 142 and 178), nor is there evidence that any—or even knowledge of any—reached China. The insight that led to the Simplified Instrument is straightforward, at least in the rare instrumentalist such as Kuo who had the imagination, power, and funds not only to imagine a large new instrument but to build it.

Kuo's account of conduct mentions two additional instruments, a Standing Revolving Instrument *(li yun i* 立運儀) and a Pole-Observing Instrument *(hou chi i* 候極儀), that are not itemized in the "Treatise on Astrology." Although he may have invented these as discrete devices, comparing their names with those of the Simplified Instrument's parts make it clear that in the Yuan observatory they were sub-assemblies: the first, the vertical circle in the altazimuth assembly, and the second, the parts that allow observation of the pole star's distance from the celestial pole (the circle of 3 *tu* radius) and the apertures through which one sights it (figure 13).

Further detail is unnecessary. The description of the Simplified Instrument in the "Treatise on Astrology" gives detailed measurements of all the main parts, but says nothing about how and how much the astronomers used it. There is no reason to believe that

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[222] The Yuan instrument divided each *tu* into ten parts, and the observers routinely estimated positions halfway between the parts; Ch'en 2003b, 136.

[223] Needham et al. 1954– , 3: 370–75.

they employed it in the first phase of the Season-granting system that ended in 1280. It should not be difficult, through study of these dimensions and the extant instrument, to form an idea of its utility and accuracy.[224]

Fig. 13. The Simplified Instrument, north polar assembly (circle D in figure 12).

The Upward-facing Instrument *(yang i* 仰儀), the fourth of the large instruments in the Yuan observatory, was a bowl-shaped scaphe sundial. What little we know about it comes mostly from a poetic inscription, copied into the Yuan astrological treatise with this perfunctory prose introduction:

> The design of the Upward-facing Instrument: it is made of bronze in the shape of a cooking pan, and set into a tiled *(or* brick, *chuan* 甎)

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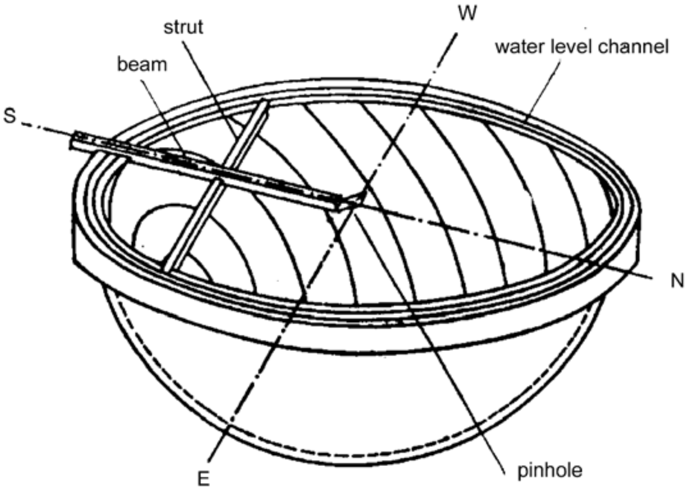
[224] I Shih-t'ung 1987 has made a most interesting proposal. He argues that the basic components of the Simplified Instrument were originally separate devices constructed in wood for the hurried first phase of observation, and not combined until the casting of the bronze Simplified Instrument for the permanent observatory. This is quite possible, but the fact remains that the primary documents mention only one wooden instrument, the early version of the Tall Gnomon. The general estimate of historians that the observations completed before 1281 used a repaired Sung armillary sphere better fits the evidence.

> platform. On its inside are painted the degrees of the sky's circumference, and on its lip the locations corresponding to the twelve double-hours. It would seem that it is for determining [phenomena up in] the sky by looking downward.[225]

From the inscription we glean that the instrument was a hemisphere 12c across and 6c deep (2.9m × 1.5m), recessed into a platform and open to the sky (Figure 14). It was inscribed with a series of concentric circles and arcs of circles radiating from a point that corresponded to the south pole, and with a series of lines (not shown in the figure) perpendicular to the arcs.

Fig. 14. The Upward-facing Instrument, a scaphe sundial that incorporates a pinhole for focusing the solar image. Adapted from Ch'en, p. 530, figure 6-27.



More than its large size distinguished this device from the usual flat sundial. Jutting over the bowl from the southern side was a thin beam, supported by a crosswise strut. As the inscription tells us,

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[225] *Yang i ming* 仰儀銘, quoted in *Yuan shih*, 48: 993–94. *Yuan wen lei* 元文類, 17: 215, quotes only the metrical inscription. Circa 1700, the great historian of astronomy Mei Wen-ting 梅文鼎 wrote a useful commentary on the poetic inscriptions for the Upward-facing Instrument and the Simplified Instrument in *Erh i ming pu chu* 二儀銘補注. For modern discussions of the instrument see I Shih-t'ung 1996, Ch'en Mei-tung 2003a, 529, and Needham et al. 1954– , 3: 301–302, 369.

At its end turns a sighting plate (*chi pan* 璣板),
Pierced to admit a mustard-seed.[226]

In other words, its northern end supported a small bronze plate carrying a pinhole. The hole, aligned vertically with the exact center of the hemisphere, served much the same purpose as the Shadow Aligner, casting an actual image of the sun that moved gradually across the bowl. Observing this image, one could read the time of day on one of the arcs and, on the perpendicular lines, the beginnings of the 24 *ch'i*, the equal divisions of the tropical year. As another couplet indicates,

And from whence an eclipse comes
One can inspect its birth and death.

That is, one can read from the solar image in the bowl the time and direction of the eclipse's beginning and end. This astronomical application went considerably beyond the customary use of the scaphe sundial outside China.

Obviously the large size of the instrument was meant to enhance the results. So much guesswork would be required to reconstitute it that it is impossible to estimate its precision or accuracy.

The scaphe sundial was well-known in the Islamic world. Although none was included in the instruments from Marāgha, it may well have originated in the West. Kuo was the first in China to build one. By adding a pinhole he improved the method for reading the shadow, making this sundial a considerably more versatile device. Smaller scaphe sundials without pinholes, usable only for timekeeping, were manufactured in Korea from the late seventeenth century on. Those who have studied them trace their origin to Kuo.[227]

The Direction-determining Table

The Direction-determining Table (*cheng-fang-an* 正方案, figure 15) was primarily a device for setting up mobile instruments. The

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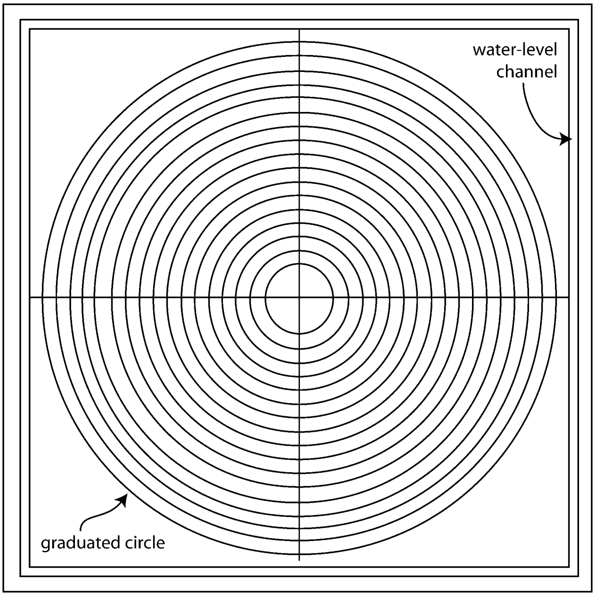
[226] So reads the *Yuan History*. In the original, *chi pan* is instead *chi chang* 機杖, an "operating (*or* moving) strut." The physical meaning of this term is not clear. Since *pan* and *chang* are easily confused, I suggest that the former is correct.

[227] I Shih-t'ung 1986a, pp. 204–208; Jeon 1974, 46–49, and 1998, 72–73.

straightforward description in the *Yuan History,* translated below (p. 567), indicates that it was shaped like a tabletop, a meter square and 0.1c (2.5cm) thick. Its center supported a small gnomon. From the center of the table radiated a series of nineteen circles, the first with a radius of 0.1c, each of the others with a radius 0.1c greater than that of the preceding one. The periphery was graduated in the 365.2575 *tu* of the sidereal circle, and around the edge was a water-level channel.[228]

Fig. 15. The Direction-determining Table, a device for orienting instruments. The outermost circle is a degree scale. Adapted from Ch'en, p. 531, figure 6-28.



To prepare the bed for an instrument, one set the table in place, poured water into the channel, and leveled the table as a basis for leveling the bed. The table was supplied with three gnomons. There was one 1.5c (0.36cm) high for use near the equinoxes, and one each 1c and 3c high for use in the vicinity of the solstices. The three compensated for differences in the shadow's length and in the way it changed from day to day through the seasons. One set the appro-

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[228] This description and the quotation below come from the "Treatise on Astrology" of *Yuan shih,* 48: 995–96.

priate gnomon upright and observed the shadow of its top from sunrise to sunset, marking the two points where it crossed each circle. Measuring the morning and afternoon intersections on the same circle and halving the distance between them yielded a point on the line running north and south from the center. Several such points derived from more than one circle enabled the observer to draw an accurate north-south line. The astronomers could have reached the same goal by determining the longest shadow of the day, which would fall at noon due north of the post (or due south in low latitudes), but the table did the job with short, easily portable gnomons, and no accurate timekeeper was needed.

The table also let the mobile teams verify the local altitude of the celestial pole, another main goal of their survey. They first approximated this datum by sighting. The procedure for verification is not entirely clear, but it seems to involve turning the table on its side. Thus one uses it as a quadrant bounded by the line from the pole (as previously determined) to the center and the one from the center to the equator, and sights along the first of these lines. The plumb line attached to this proto-quadrant corrects the observed altitude.[229]

Although Kuo's biography lists the Direction-determining Table among four instruments designed "for the use of the observers who traveled to various locations" (below, p. 577), the contemporary description of the observatory notes that one was installed beneath the Simplified Instrument. The Ming copy of that instrument incorporated in its base a device of the same kind, although shortly before or shortly after the Manchu conquest of 1644 its face was ground down and replaced by a sundial.[230]

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[229] On a predecessor 550 years earlier, I-hsing's 一行 "inverted try square (*fu chü* 覆矩)," which used a graduated quadrant with a plumb bob to observe the altitude of any celestial body, see Li Ti 1964.

[230] P'an & Hsiang 1980, 39. I Shih-t'ung 1982, after examining the Ming Simplified Instrument, documented the alteration of the Direction-determining Table (some traces remain).

The Celestial Globe

The mechanically driven celestial globe (*hun hsiang* 渾象) was far from new in China. Su Sung's astronomical clock of 1092 incorporated a water-powered clockwork globe, and its predecessors stretch backward in time at least to the third century A.D.[231]

The astrological treatise of the *Yuan History* devotes a passage to the globe designed for the Astrological Commission (translated below, p. 566). The description mentions the moon, but says nothing about stars and constellations. It refers to a gear-driven mechanism, but does not describe it. Su Sung's globe, inside his clock-tower, was meant—in principle, at least—to verify what the observer atop it sighted. There is no indication that the Yuan device served any astronomical purpose other than showing some portion of what would be visible in the night sky.[232]

The old armillary sphere that Kuo used initially for the Season-granting system's observations came, as we have seen, from Su's clock (p. 177). The Yuan celestial globe may or may not have included such additional features of the earlier one as an array of stars, the lunar lodges, the Celestial River (the Milky Way), and colored balls representing the sun, moon, and planets, which one could use to follow their apparent motions.

Other Instruments

The uncertainty about details of the instruments just discussed pales to insignificance beside our ignorance about several others. For them we have no data except a few words in Kuo's biography about his motivation for creating each. Po Shu-jen 薄树人 has applied his considerable ingenuity and learning to speculate about their identity. Let me summarize what can be said on the basis of his research and that of others.[233]

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[231] Needham et al. 1960, chapter 6; Combridge 1975 is important.

[232] *Yuan shih,* 48: 993. Kuo's biography (below, p. 575) mentions the globe. Other sources add nothing substantial.

[233] Po 1982. For the context see below, pp. 575ff. Yamada 1980, 209–10, is largely devoted to correcting errors in Needham et al. 1954– , 3: 369–70.

- "The sun has its central path, and the moon its nine motions; Kuo united them and made an Adjusting Instrument *(cheng-li i* 證理儀)." The lunar orbit diverges about 6° from the ecliptic, and the moon's latitude varies according to its distance along its path from the lunar node, the intersection of the two. Very early Chinese astronomers treated this variation not as a continuous one, but as though the moon divided its time between a stepwise succession of four paths to the north of the ecliptic, one coincident with it, and four to its south. But by a millennium before the Yuan reform, it was normal to compute the moon's longitude and latitude simply as continuous functions of its distance from the lunar node. The Adjusting Instrument may have been an observational instrument that measured the latitude of the moon, although the Ingenious Instrument (unlike the Simplified Instrument) could have done the same thing. On the other hand, the instrument could have been a device to demonstrate the new approach. Since it was not installed on one of the observatory's towers, it was presumably either demonstrational, small, or seldom needed.[234]
- "Because the verification of an astronomical system rests on Crossing Coincidences, he made an Instrument for Solar and Lunar Eclipses *(jih-yueh-shih i* 日月食儀)." Crossing Coincidences were passages of the sun or moon through the lunar nodes (Canon, section 6). If this description can be taken literally, it implies that the instrument was used for observation, perhaps for locating the nodes. More than that it is impossible to say.
- "The sky has its Red Way [i.e., the celestial equator]; a circle serves in its place. The two poles rise and sink below it, with gradations marking [positions]; thus he made a Star-dial Time-determining Instrument *(hsing-kuei ting shih i* 星晷定時儀)."

This description is not at all clear, and does not fit any Chinese instrument before or after. The name of the putative time-determining Instrument does not appear in any earlier or later astronomical treatise. "Star Dial" does appear earlier in poetry and

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[234] Ch'en Mei-tung 1995, 278–90, takes up approaches to computing lunar latitudes. See also Ch'ü An-ching 曲安京 2003.

belles lettres. It originated in the late fifth century as a metaphor for the Northern Dipper constellation. In poetry the dipper's handle chronicled the season by the direction toward which its handle pointed at a given time of night. This, by extension, alluded to the passage of time. But no one applied this term to an instrument that early astronomers used.

The description is problematic in still another respect. Immediately following it in Kuo's account of conduct is the sentence "Those listed above comprise thirteen kinds", but there are only twelve items in the list.[235] It is likely that the count of thirteen instruments was an error; copyists often confused 十二 and 十三. That does not dispose of the problem. A Star-dial Time-determining Instrument also has no precedent in the astronomical literature.

Po Shu-jen (1982) has proposed a bold solution to this puzzle. In the 1620's, he noticed, a book named *Standards For Sun, Moon, and Star Dials (Jih yueh hsing kuei shih* 日月星晷式) used *hsing kuei* for "astrolabe." We already know that an astrolabe or nocturlabe was among the instruments that came to the Yuan court from Iran (p. 178 above).

Po's proposal is problematic, for he neglected to give any information about the book he cited, and I can find no record of it. On the other hand, "star dial" occurs in easily accessible seventeenth-century sources. It always refers to devices of Western origin. The most useful is a long passage in the Ming history from a memorial on timekeeping, written in 1629 by Hsu Kuang-ch'i 徐光啟 (1562–1633), the eminent statesman, Christian convert, and champion of the Jesuit missionaries. Among the European instruments that he

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[235] Ch'en Tsun-kuei (1955, 133) managed this problem with the hypothesis that the "Star-dial Time-determining Instrument" is actually two, a Star Dial and a Time-determining Instrument. This could easily be the case in an unpunctuated text. If we credit this hypothesis, the text becomes stylistically inconsistent with the others in providing a single description that, although it is meant for two instruments, reads as though it applies to only one. Ch'en also did not suggest what these instruments might have been. The editors of the *Yuan History*, perhaps aware of the discrepancy, omitted the questionable sentence.

planned to manufacture, he speaks of "star dials with revolving plates (*chuan p'an hsing kuei* 轉盤星晷)" and "star dials with multiple plates (*chung p'an hsing kuei* 重盤星晷)." He goes on to describe a star dial:

> One makes a pin (*chu* 柱) of bronze, and installs multiple plates on it. On the inner one are engraved the degrees in the round of the sky, and the twelve houses as divisions for the nodal *ch'i*. On the outer one are engraved the hours and marks. A slit cut in the crosspiece is used for sighting stars. The method of using [the device] is to rotate the outer plate so that its gradation corresponding to the moment of midnight is lined up with the appropriate nodal *ch'i* on the inner plate, and then to turn [the crosspiece] northward to sight [the two stars closest to the celestial pole]. When one sees the two stars together in the slit, one notes the gradation on the plate at which the indicator points; that is the exact time.[236]

This is not a description of an astrolabe, but a tolerably accurate one of a nocturlabe or nocturnal, a much simpler instrument for telling the time at night. The dozen houses of the zodiac on the base plate of the originals had nothing to do with the twelve Chinese divisions of the tropical year (see above, p. 79). The zodiacal houses are unambiguously European, but the twelve *ch'i* were an obvious way for a Chinese to think of them. The instrument's crosspiece is the alidade, and the pin on which it pivots corresponds to the pole. Nocturnals were used mainly near the *western* end of the Islamic world; more accurate instruments were common further east.

Of course this description from 1620 does not clear up the question of what instrument Kuo made 350 years earlier. It implies that it may well have been a nocturlabe.

As Po reminds us, "It is unlikely that [Kuo's instrument] is simply a copy, for in that case he hardly would have claimed it was an innovation. But what modifications or developments he incorporated remains, for the time being, an enigma." We cannot conclude on the basis of current knowledge that the Star-dial Time-determining Instrument was a nocturlabe or (less likely) an astro-

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[236] *Ming shih* 明史, 25: 359–62.

labe, but without this plausible hypothesis its identity is a still deeper enigma.[237]

Finally, Kuo designed four instruments for the use of the traveling survey teams. In addition to the Direction-determining Table, discussed above because it was also installed in the observatory, they included a Ball Gnomon (*wan-piao* 丸表), a Suspended [vertical] Standard Instrument *(hsuan cheng-i* 懸正儀), and a Mounted [horizontal] Standard Instrument *(tso cheng-i* 座正儀)." For these we have only names, and not a word of description.

- What the Ball Gnomon might be is doubly confused by a variant reading in some editions of the *Yuan History.* Omitting one stroke from wan piao 丸表 makes the term chiu piao 九表, "nine gnomons." This makes no sense as the name of a single device, but has confused historians not aware that this form is absent from better recensions of Kuo's biography.

Wan implies sphericity; its earliest use was for a crossbow pellet or some such small ball-shaped object. *Piao,* a word with a broad range of meanings, was the normal term for a gnomon. Combining the two, Yamada Keiji has speculated hesitantly that it might have been a kind of portable hemispherical scaphe sundial. He did not explain how an accurate scaphe sundial could be portable. Po Shujen has suggested more literally, with becoming diffidence, that it might be a gnomon mounted on the surface of a sphere, an idea he did not work out. I have found no further evidence on which to base speculation of any kind.[238]

- As one of a set of basic devices for traveling observers, the Suspended Standard Instrument was most likely a plumb line for ensuring that an instrument was perpendicular to the ground. The

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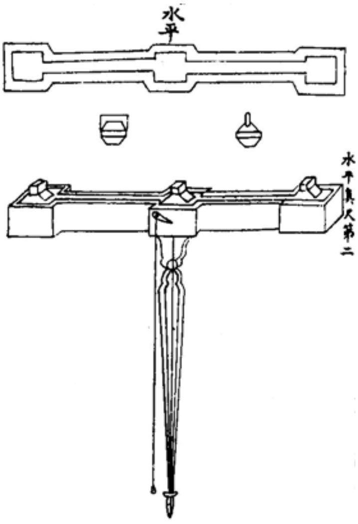
[237] Po 1982, 320–21, 326. I am grateful to David King and Benno van Dalen for their identification and comments (email to me from the latter, 2005.4.17). Oestmann 2001 writes on the history of the nocturlabe in Europe alone. Liu Lu 劉潞 1998, an illustrated catalogue of instruments in the former Ch'ing Palace collection, does not include astrolabes or nocturlabes.

[238] Yamada 1980, 209; Po 1982, 323–24; Ch'en 2003b, 150–51. Needham et al. 1954–, 3: 370, misread the name of the device as *chiu piao hsuan* 九表懸.

instructions for the Direction-determining Table, for instance, stipulate that when turning it on its side to verify the altitude of the celestial pole, one "suspend a line *(hsuan sheng* 懸繩)" to ascertain its alignment (p. 202). The verb corresponds to that in the name of this device, a clue to its use.

- The most likely identity for the Mounted Standard Instrument is a horizontal level. Portable equipment may not always have incorporated water-level channels. The channels built into the Direction-determining Table would have served to install large mobile instruments. It is likely that Kuo built a separate level for small ones, and that it was in some way, as his list implies, innovative.

Fig. 16. A water level of 1103. The top view shows the shape of the water channel. Between it and the more-or-less three-quarter view are side (left) and front (right) views of one of the three floats.



Po Shu-jen has suggested a simple, compact design derived from one that appears in a military manual of 1040; a handbook of building standards of 1103 includes a similar but more coherent illustration. It is in the form of an elongated bar with three fairly deep water reservoirs, connected by a trough and equipped with a plumb line. Visually lining up the floats that fit in each reservoir would make it easy to check leveling. This is just one possibility, and presumably Kuo would have improved upon its design.[239]

- Finally, the mobile survey teams used, not the fixed Tall Gnomon, but portable ones of the classic size, 8c high. The one still installed at the Purple Mountain Observatory (figure 4) may have been one of these. I Shih-t'ung surmises that its upright was made in the Yuan period.[240]
- The instruments just discussed were not the sum of Kuo's inventions. He also built incense clocks and other devices. Under Khubilai's successor, Kuo made two even more elaborate water-powered clocks, a largely ornamental "clepsydra" for the imperial hall of audience and, in 1298 for the observatory, one that generally fits the description of Su's entire astronomical clock (see p. 593).[241]

Table 5.1 summarizes what we know, and what we can conjecture, about the main instruments made for the Commission from 1279 on.

[In the "source" column of Table 5.1, A is Kuo's autobiography (Appendix B), and T is the "Treatise on Astronomy" of the *Yuan History* (cited in Appendix A); (T) indicates that the instrument occurs as a sub-assembly of another.]

Table 5.1. Instruments Made for the Reform

Translation	Name	Identification	Source
Adjusting Instrument	*cheng-li i* 證理儀	uncertain	A
Ball Gnomon	*wan-piao* 丸表	unknown	A
Celestial Globe	*hun-hsiang* 渾象	celestial globe	A

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[239] *Wu ching tsung yao* 武經總要 (1040), 11: 2a–3a; see also the description in *Ying tsao fa shih* 營造法式 (1103), 3: 3b–4a, s.v. *ting p'ing* 定平 (leveling). Figure 16 is based on the latter source, 29: 3b. See the discussion in Ch'iao Hsun-hsiang 乔迅翔 2006.

[240] I Shih-t'ung 1984, 225.

[241] On the first of the later clocks, see *Yuan shih,* 48: 994–95, translated in Needham et al. 1960, 135–36, and on that of Su, *Hsin i-hsiang fa yao* 新儀象法要, 1: 12, translation in Needham et al. 1960, 24–25; on predecessors see pp. 17–18. Li Chih-ch'ao 1997, 102–106, has conjecturally restored the ornamental clock.

Translation	Name	Identification	Source
Direction-determining Table	*cheng fang an* 正方案	device for leveling and orienting instrument	A,T
Ingenious Instrument	*ling-lung i* 玲瓏儀	armillary sphere	A
Mounted Standard Instrument	*tso cheng-i* 座正儀	horizontal level?	A
Observing Table	*k'uei chi* 窺几	sliding table for sighting moon or star using gnomon	A,T
Pole-observing Instrument	*hou chi i* 候極儀	polestar sighting circle	A,(T)
Shadow Aligner	*ying-fu* 景符	pinhole focuser	A,T
Simplified Instrument	*chien-i* 簡儀	armillary with separate equatorial and azimuthal circles	A,T
Solar and Lunar Eclipse Instrument	*jih-yueh-shih i* 日月食儀	lunar node locator?	A
Star-dial Time-determining Instrument	*hsing-kuei ting shih i* 星規定時儀	nocturlabe?	A
Suspended Standard Instrument	*hsuan cheng-i* 懸正儀	plumb line?	A
Tall Gnomon	*kao piao* 高表	gnomon, 40c	A,T
[Portable Gnomon]	*piao* 表	gnomon, 8c	artifact
Upward-facing Instrument	*yang-i* 仰儀	scaphe sundial with pinhole focuser	A,T
Vertical Revolving Instrument	*li yun i* 立運儀	vertical circle	A,(T)

Some of the Yuan instruments that survived the dynasty's fall were abandoned by its successor. They were left at the foot of its new observatory at the southeast corner of the Beijing city wall. People who knew enough about astronomy to identify them, up to the London Missionary Society's Alexander Wylie in the late nineteenth century, saw them there. The best account was by Mei Ku-ch'eng 梅瑴成 (or Chueh-ch'eng, 1681–1763), an astronomer exceptionally well informed on historical matters:

> In 1713 and 1714, when serving provisionally as an editorial official in the Study for Enlightenment of the Young [Meng yang chai 蒙養齋, an institute that taught European mathematics and astronomy to Manchu princes], I frequently went to the observatory to make observations. I saw that below it were a good many derelict ancient in-

> struments. There were a Simplified Instrument, an Upward-facing Instrument, and other instruments made in the Yuan period. All carried the inscription "made under the supervision of Wang Hsun and Kuo Shou-ching." Although they were not undamaged, one could examine their design and imagine the great effort that had gone into their manufacture. Unawares, one was overcome by respect [for their makers]. … [Most were destroyed by the orders of the Catholic missionaries who replaced them with European instruments], leaving only three. But in the winter of 1744, an order came down to set up the three in front of the Purple Tenuity Hall [of the imperial palace]. These paradigmatic relics of the ancients may survive for a thousand ages!"

They did not, in fact, survive into the twentieth century.[242]

Ming Instruments

The "Treatise on Astrology" of the Ming history, like that of the Yuan, chronicles the dynasty's instruments. What the treatise has to say about instrument-building up to 1500 casts some light on the Yuan instruments, and allows us to compare the two dynasties in this respect. Its very first report, of an event shortly after the Ming forces triumphantly entered Beijing late in 1368, seems so unpropitious that it calls for examination:[243]

> When the emperor T'ai-tsu had pacified the Yuan, his Directorate of Astronomy submitted a crystal water clock. It contained two wooden puppets which, on the hour, automatically struck a gong and drum. T'ai-tsu, on the ground that it offered no benefit, had it smashed.

Historians have understood this episode as an anti-technological tantrum by the new ruler. A slightly different account in a less prominent early source prompts a different interpretation: "In November 1368 the Directorate of Astronomy submitted the water

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[242] *Ts'ao-man chih-yen* 操縵卮言, 29b–30a. Wylie 1876 cites Matteo Ricci's notice. I Shih-t'ung 1994 gives an excellent overview of what scholars knew at that time about the instruments connected with the Yuan reform, and about their present whereabouts.

[243] *Ming shih,* 25: 357–59. I have added to my several quotations from this source, in square brackets, a few details from other reliable sources.

clock from the Crystal Palace, which the Yuan ruler had had made."[244]

It happens that the last ruler of the Yuan did have an elaborate water-driven clock made—the first since that of Kuo two generations earlier, and the last such device in imperial China. His annals in the *Yuan History* describe its appearance, although they give no details about its mechanism. Li Chih-ch'ao has suggested a reasonable motivation for the Ming ruler: that he saw this elaborate water clock as mere detritus of the Yuan's defeat, and had no use for it.[245] We can conclude that, since this report had nothing to do with the manufacture of new astronomical instruments, it set no precedent for their later production. Let us, then, continue the chronology of instruments where the first quotation ends.

> In 1384, the Directorate made a star-sighting dial *(kuan hsing p'an* 觀星盤, i.e., a nocturlabe or astrolabe?). In 1385, an observatory was set up on Cockcrow Hill *(Chi ming shan* 雞鳴山, in the north of Nan-ching, the southern capital). In 1391, [an edict ordered that] an armillary sphere be cast in bronze.[246]
>
> In 1437, the Director of the Provisional Directorate of Astronomy, Huang-fu Chung-ho 皇甫仲和, reported in a memorial: "The observatory in Nan-ching is outfitted with [a celestial sphere,] an armillary sphere, a Simplified Instrument, and a gnomon and template, with which to observe the angular motions of the Seven Governors [i.e., the sun, moon, and planets]. In Pei-ching, we are merely observing atop the city wall at the Uniform Transformation Gate *(Ch'i hua men* 齊化門), without any [stationary] instruments. I beg that you command a member of this office to proceed to Nan-ching, [superintend

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[244] *Ming t'ung chi* 明通紀. This source does not seem still to exist, but Li Chih-ch'ao 1997, 110, cites a quotation in *Ku chin t'u-shu chi ch'eng* 古今圖書集成. There is an obvious discrepancy in wording between this source and the official history, and others between this and the one discussed just below. Li suggests that they result from the chaos in the immediate aftermath of the Ming victory.

[245] *Yuan shih,* 43: 918. This account does not say which palace building it was in.

[246] *Ming shih* says merely that the sphere "was cast," but compilations closer to the primary sources specify in their records of the edict that it was completed; *Ming shih lu* 明實錄, 208: 4b, *Huang Ming ta cheng chi* 皇明大政紀, 5: 21b.

workmen who will] copy in wood the instruments there, and transport [the copies] to Pei-ching. There we will verify by comparison the height of the pole above the earth, and then reproduce [the models] by casting each in bronze. In this way we anticipate that we will be able to make reliable observations." This was approved. In the winter of the next year [i.e., late 1438], an armillary sphere and a Simplified Instrument were cast in bronze at Pei-ching. [The text next quotes a conventional, uninformative inscription that the emperor wrote for the observatory].[247]

In 1446, an official of the [Pei-ching] Directorate said "No gradations are inscribed on the Simplified Instrument. It is installed in a low place, so that when we try to observe the sun or other luminaries, platforms and buildings on all sides block our view. The gnomon and template are set up on an exposed platform, so that the light is dispersed in all directions and the image lacks definition (? *ying wu ting tse* 影無定則). The building that contains the clepsydra is a low one. At night, when the reservoir [in the highest part of the building] quickly empties, it is difficult to supply it with water to keep [the display of] hours and marks correctly adjusted. We request permission to correct these problems according to the specifications." A reply gave permission.

In the winter of the next year [late 1447], P'eng Te-ch'ing 彭德清, the Director, said "In Pei-ching, the distance of the north pole above the horizon in *tu,* and the times of sunrise and sunset, differ from those in Nan-ching, as do the lengths of day and night in summer and winter. Now the clepsydra floats in the palaces and government offices, all of which follow the old Nan-ching standard, are unusable." An instruction ordered the Directorate of Palace Eunuchs to make corrected [floats].[248]

In 1455, a Simplified Instrument and bronze clepsydra were again made for the palace observatory. Between 1465 and 1477, the Minister Chou Hung-mo 周洪謨 asked for permission to make a *hsuan-chi*

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[247] Pei-ching 北京, the Ming's second capital, was the former Ta-tu, minus its northern part but extended to the south—the Beijing of 1980, more or less. The version of the memorial in the *Ming shih lu,* 27: 5b–6a, includes the celestial sphere, which it calls *hun-t'ien i;* it calls the armillary sphere *hsuan-chi yü-heng* 璿璣玉衡. I discuss this term just below.

[248] On the graduated clepsydra floats used to tell the time, see p. 86 above and the Canon, section 5.

yü-heng 璿璣玉衡. Emperor Hsien-tsung commanded him to have it made and submit it to the throne.[249]

In 1478, the Director of the Observatory asked for permission to repair the enclosure of the gnomon (? *kuei ying t'ang* 晷影堂). This was approved.[250]

In 1489 the Director, Wu Hao 吳昊, said: "When we verify the locations of the sun at the solstices and equinoxes, the Yellow and Red Ways [i.e., the ecliptic and equator] ought to intersect in the lunar lodges Wall and Baseboard. According to the armillary sphere of ancient design at the Observatory, the two intersect in Crotch and Baseboard, which does not conform to the phenomena.[251] Its north and south axes do not correspond [in placement] to the altitudes of the two poles. Furthermore, its sighting tube does not give results that correspond to the rising and setting of the sun. That is why, although it is installed, we do not use it.

The Simplified Instrument, which we do use, is based on the old design by Kuo Shou-ching . The north pole assembly's supports with the cloud decor *(yun chu* 雲柱 are too short,[252] so that, when using the instrument to measure the distance of fixed stars *(ching-hsing* 經星) from the pole, errors are unavoidable. We request permission to repair these instruments or to build new ones that will embody designs of our own time."

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[249] *Hsuan-chi yü-heng* is a term of splendidly indefinite meaning. It derives from the spurious "Shun tien 舜典 (Canon of Shun)" chapter of the Documents classic *(Shang shu* 尚書), which speaks of using the *hsuan-chi yü-heng* to regulate or regularize the Seven Governors. Over the centuries scholars have variously interpreted this term as a ritual implement, or some such instrument as an early armillary sphere. Cullen & Farrer 1983 have argued conclusively that we have no idea what this term meant in the earliest sources. In the early Ming, astronomers often used it simply to mean "armillary sphere." On the other hand, by the late fifteenth century there was no shortage of such instruments in the capital. Since this proposal was approved, it is likely that Chou, who was not an astronomical official, had an archaistic instrument of some sort in mind. Such a request would not be exceptional as a gesture, or part of a program, to restore symbolically the golden age of antiquity.

[250] This building was evidently meant to remedy the problem noted in 1446.

[251] For the lunar lodges, see table 2.9.

[252] These wishbone-shaped supports hold up the small circle (D) in Figure 12.

> The matter was referred to the Ministry of Rites. After repeated discussions, the Vice Director [of the Directorate of Astronomy], Chang Shen 張紳, was ordered to make wooden models. [Replacing the bronze instruments was] to await trials [with them]. Permission was given to correct the ecliptic gradations [of the armillary sphere].[253]

There is no record of further action on this request.

In order to read these documents profitably, we need to pay attention to their circumstances.

First, there was more than one observatory. The Ming, after its victory in 1368, maintained its predecessor's observatory in what had been Ta-tu, now renamed Pei-ching ("northern capital"). In 1385 it set one up in its new capital, Nan-ching ("southern capital," present-day Nanjing). After the government moved back to Pei-ching (over the period 1409–21), it built still another observatory there in 1438 or a little later.

Second, the new Chinese government, unlike its Mongol predecessor, was in no hurry to create an astronomical system of its own. Until 1384 it simply relied on the Yuan system to generate its annual almanacs. In that year it adopted the Great Concordance system (#90), drafted in the late Yuan period, which made some trivial improvements in the Season-granting system but was otherwise identical to it. The Nan-ching observatory was set up the year after that shift.[254]

If we bear those facts in mind, the passage from the Ming history permits several conclusions that cast light on the Yuan instruments and their posterity:

- The Yuan instruments that survived the dynastic transition were moved to Nan-ching, but it took nearly twenty years to build an observatory for them. Its construction followed the nominal astronomical reform of 1384 rather than contributing to it. The Great

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[253] For further details see *Ming shih lu,* 182: 1b–2a, memorial dated 1501.

[254] For an overview of the Ming system see Li Kuo-ch'ing 历国青 et al. 1977. For the chronology see Li Cheng-fu 黎正輔 1963. I note in comments on the translation several respects in which the Great Concordance system edited the Yuan treatise to improve clarity.

Concordance system did not demand extensive new observations for the two and a half centuries that it remained in official use.

In other words, the Season-granting system, in its original and minimally modified forms, remained official for more than 350 years, a record. This does not mean that for all of that time it continued to yield unsurpassable predictions. What did matter was that the government did not take seriously any rival proposal until, in 1644, the new Manchu regime granted authority over almanac-making to Jesuit missionaries.

- Huang-fu Chung-ho's replicas of the Nan-ching instruments, made between 1438 and 1446 for Pei-ching, are the ones that after many vicissitudes have survived to the present. Much evidence indicates that they are reconstructions of Kuo Shou-ching's equipment. An announcement of 1528 that the equipping of the Ming observatory was complete asserts that its instruments "are limited to the Yuan style *(i Yuan fa wei tuan* 以元法為斷)." Mei Ku-ch'eng wrote from experience in Pei-ching that "the Ming on its observatory platform duplicated the designs of the Yuan armillary sphere, Simplified Instrument, and celestial globe"; he used these instruments himself in 1713–14. P'an Nai's fastidious comparison of the features and dimensions of the Ming armillary sphere with four spheres of the eleventh and twelfth centuries shows no affinity to any of the Sung models. The extant Simplified Instrument is also based on that of Kuo Shou-ching, with some minor modifications.[255]
- The memorial of 1446 suggests that the installation of these instruments in the Pei-ching observatory, a quarter century after the move of the capital, was carelessly done. The document of 1447 reveals that Huang-fu's instruments, despite his intentions, were not corrected for the latitude of Pei-ching. The use there of clepsydra floats calibrated for Nan-ching would lead to such obviously unsat-

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[255] See the detailed arguments in P'an 1975, 86, P'an & Hsiang 1980, 50–53, and P'an 1983. Needham et al. 1954–, 3: 368–69, translates a description of several Yuan instruments circa 1600 by the Jesuit missionary Matteo Ricci, but what Ricci saw were actually the Ming reproductions. For Mei's memoir see *Ts'ao-man chih-yen*, pp. 27a, 29b–30a.

isfactory results that competent astronomers would have caught the error quickly. The fact that the improved instruments were still not satisfactory a generation after their manufacture indicates that the observatory—as happened recurrently through history—was firmly in the hands of sinecurists. That is not surprising, since there is no evidence of pressure from above for substantial improvement.

I will conclude with some comparisons. The astronomical situation in the early Ming did not at all resemble the urgency of the Yuan reform. It was closer to the historical norm: nothing happened quickly, and the innovation was minimal. Even when the observatory was fully outfitted in 1528, a century and a half after the inauguration of the Ming period, there is no evidence that the instruments were capable of refined observation.

How can we explain this disparity between the priorities of the Yuan and Ming governments? The lack of a great astronomer in the late fourteenth century will not do. Nothing in the record suggests that the government sought out technical talent. Ming experts could at least have improved on earlier computational refinements. As we saw earlier (p. 67), the Yuan astronomers, for instance, had considerably elaborated the arc-sagitta methods that originated two centuries before their time, in ways analogous to spherical trigonometry. That was a natural direction for further evolution, but none occurred. Given the lavish expenditures of the Ming palace on other things, one can hardly argue that a full-scale reform was unaffordable.

The issue that overrode the others, I believe, was political priority. In the thirteenth century, Khubilai and his advisors saw an astronomical reform as essential to begin the new era of Chinese reunification. In the fourteenth, T'ai-tsu obviously did not. The expulsion of the Mongols by a Han leader gave him a mandate to rule that needed no justification. Furthermore, one of his earliest statutory enactments, in the first year of the new dynasty, was to forbid the study of both astrology and mathematical astronomy outside the court, with severe penalties. The predictable outcome was that no one applied for positions in the astronomical bureau;

that would have amounted to confessing criminal activity. A mere five years later, in 1373, a further edict dealt with the problem in equally Draconian fashion: "Personnel of the Directorate of Astronomy are never to be allowed to transfer [outside the Directorate]. Their male descendants are to study only astrology and mathematical astronomy *(t'ien hsueh li suan* 天學曆算), and may not be permitted to study for other occupations. Those who fail to study [astrology and astronomy] are to be transported to the Southern Sea [generally, Southeast Asia] to fill vacancies in the armed forces."[256] In the 31 years of T'ai-tsu's rule, no emergency occurred to change his mind about astronomical priorities.

One need not affirm the Great Man theory of history to appreciate these developments. A *leitmotif* of this book is that strong-willed and engaged emperors (not at all the norm) could shape an empire to their liking to a greater extent than was the case in other large states. Both Khubilai and Ming T'ai-tsu were among the most prominent of those rulers whose administrations became instruments of their wills. And in China, as this study makes abundantly clear, astronomy, astrology, and divination were affairs of state.

Influence on the System from the West

It may seem that the clearly perceptible influence of Islamic culture on the Mongols should have greatly affected their new Chinese-style astronomical system. But it is not that simple. Although the domains of the Mongols stretched from Sakhalin to Bagdad, they were never under a single administration. The vibrancy of Islam in the western part of those domains—ruled though they were by Khubilai's brother—produced imperceptible reverberations in China.

Still, there are two obvious points where transmission might have taken place.[257]

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[256] *Ta Ming hui tien,* ch. 223.

[257] Van Dalen 2002, 333–36, has studied transmission from China to the Muslim world. The main result was hybrid Islamic-Chinese astronomical systems found in Persian writings from the Mongol period on.

One is in the person of Jamāl al-Dīn, who built Muslim instruments (see p. 142), and who later, as noted in chapter 1, was the hierarchic superior of the old Chinese Directorate of Astronomy as well as the Muslim Directorate. The other is in the relation of the Season-granting reform group to the Muslim Directorate.

Fig. 17. A sight, part of the Simplified Instrument.

What evidence is there that Jamāl's instruments influenced those built in China, or altered Chinese cosmological ideas? There are nearly as many opinions as there are historians who have raised the question. Let me sum up what the sources document or believably suggest.

As for instruments, the two plausible influences are embodied in the scaphe sundial (p. 200) and—merely likely—the nocturlabe (p. 206). Only the latter was included in the list of Jamāl's instruments. The nocturnal is not usual observatory equipment, but a simple timekeeping device. It was not as accurate as a good water clock, but more easily portable for the teams the reformers sent out to survey the sky in many parts of the empire. The scaphe sundial was well-known in the Islamic world. There is no reason to doubt that it originated in the West. So far as we know, Kuo was the first in China to build one, so the influence, although obscure, would have been direct.

Po Shu-jen noticed two constituents of the Simplified Instrument that he thought derived from Islamic practice. First, instead of the conventional sighting tubes it used diametral bars with sighting vanes, which the Chinese called "ears *(erh* 耳)." The vane on each end of a bar had a small opening with a vertical wire centered in it, so that one lined up the two wires to sight an object with maximal accuracy. Figure 17 depicts one of the sights (the Ming copy in the photograph lacks the wires), and figure 13 shows the relation between a sight and a larger part of the instrument. This is a plausible influence, although a distinctly minor one. This design did not influence the Seaston-Granting project's instruments of traditional design.

Second, each of the 100 marks of the Simplified Instrument's fixed diurnal circle is subdivided into 36 equal parts, each the equivalent of 24 seconds of time (see appendix A, p. 563). Po has suggested that the division of circles on Jamāl al-Dīn's instruments into 360 degrees prompted this division. But the connection of 360 degrees with 3600 time minima is not obvious, and Po did not explain it.

Was there influence on less concrete aspects of astronomy? Po has pointed out that Jamāl's armillary sphere would have indicated the ecliptic pole for first time, but that pole does not figure in the Season-granting system or in cosmology prior to European influence in the seventeenth century. The terrestrial globe may have inspired Yuan and early Ming speculation on the sphericity of the earth, but none of those who wrote on this topic betray inspiration from Muslim sources. One might add to Po's proposals the zodiac and the 360-degree circle used in the Muslim Myriad Year system (see above, p. 142). But neither appears in the Season-granting system, or for that matter in other non-Islamic writings on astronomy in the Yuan period.[258]

Other scholars have speculated about still other influences. An interesting example is Sun Xiaochun & Jacob Kistemaker's discovery that in 1099, nearly two centuries before the Yuan reform, "the Steward *(feng-chih* 奉職) Ch'ou He-shang 醜和尚 presented [to the court] illustrations of [four devices, including] a simplified instrument. An order was given to the responsible officials to manufacture them." This passage gives no clue as to what it means by "simplified instrument *(chien-i* 簡儀)," a term that does not speak for itself. What part of the world this "simplified instrument" came from, whether it was similar to that of Kuo Shou-ching, and whether it was actually constructed, we do not know.[259]

Why, then, was influence from Western Eurasia in Hellenistic, Indian, or Islamic form either missing or negligible? Historians have tended to chalk up the lack of effect to Chinese xenophobia or lack of interest in the arts of the peoples around them, whom dull-

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[258] Po Shu-jen 1982, 325–26; Ch'en Mei-tung 2003a, 528–29.

[259] Sun & Kistemaker 2001, 64. The primary source is *Chin shih,* 11: 251. The authors translate Ch'ou He-shang's name as "shabby monk," and reason that such a term might describe a foreigner, "most probably an Arab or a Persian." In fact, *ch'ou* (ugly) was never used to mean "shabby." The authors ignored the incompatible official title, because the surname Ch'ou is not listed in standard biographical reference works. They did not realize that non-Han nobles with this family name figure in four of the Standard Histories. The *Yuan History* has some biographical information on four people with this surname, all Khitan aristocrats, and *Chin shih* mentions Ch'ou He-shang again at 133: 2847.

The record of the discussion that followed delivery of this conference paper by Sun & Kistemaker (pp. 72–73) includes a number of additional proposals, such as one that the Tall Gnomon "was stimulated by the trend towards large instruments which had already started among the Arabic scholars." This trend did not prompt, in China before 1279, a gnomon of unusual height, or any other exceptionally large instrument.

minded people considered culturally inferior. Like all such stereotypes, those have no explanatory value.

Much more to the point is what the primary sources *do not tell us* about cooperation between Muslim and Han specialists. They are rich in detail about bureaucratic propinquity, but they do not mention Islamic astronomers on the astronomical reform team, or cooperating with Chinese. We have seen that although Jamāl, as co-Director of the Palace Library in the period of the reform, had some authority over the old Directorates of Astronomy, he was instructed to report on the Islamic and Han components separately (p. 145). He had no authority over the Directorate. It is overwhelmingly likely that, had he participated in the reform, his name would have figured in its copious documentation—but it does not appear once.

An obvious source of material evidence would be Chinese translations of Islamic astronomical manuals or handbooks. There is not a single one from the Mongol period. I do not mean that none survive; there is no trace of them even in early catalogues of Yuan sources. On the other hand, there were many such works in Arabic or Persian that could have been translated. The records of the Palace Library in the Yuan period, compiled circa 1350, list 195 of them in the collection of the northern [i.e. Muslim] observatory in 1273, and an additional 47 in the Superintendent's—Jamāl's—household.[260]

What is more curious still, as soon as the Ming had replaced the Yuan, translations and other evidence of communication between Muslim and Han astronomers became prominent. In the year of victory, 1368, the Ming captured more than 200 books on astronomy, astrology, and presumably divination, in foreign languages—which implies Arabic and Persian. The Grand Progenitor, the first Ming emperor, had heard that the Muslim astronomers were skillful, and insisted on finding out what their books contained. He sought out

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[260] Compare the lists of Yuan astronomical and astrological books in Huang Yü-chi 黃虞稷 et al. 1958, and Lo Chu-yun 雒竹筠 1999, 229–34, with the list of untranslated ones in *Yuan Mi-shu-chien chih* 元秘書監志, 7: 13b–14b (see also Ma Chien 馬堅 1955). Muslim works on other topics were translated; for instance Huang, p. 263, lists a *Hui-hui k'o shu* 回回課書 (Islamic divination manual).

Muslims to read them. Although he did not order an astronomical reform, he cared about the importance of astronomy in imperial ideology.

Between 1368 and 1370 he reorganized his astronomical bureaucracy twice. The resulting Directorate of Astronomy integrated a bureau for Islamic astronomy. In 1382 he personally commanded two Han officials to cooperate with the foreign astronomers in translating some of the books. The Muslims were to translate orally, with the former turning their dictation into readable Chinese. Some of the resulting books in Chinese survive, particularly Kūshyār ibn Labbān's *Introduction to Astrology,* an encyclopedia of divination, and multiple versions of an Islamic system of computational astronomy. The emperor further ordered the astronomers, once the translations were finished, to peruse them "from time to time" and make observations based on the Muslim methods.[261]

The express goal of these commands was cosmic monarchy. The Grand Progenitor aimed to cultivate himself, "forefend calamities, conform to the heart of heaven, and establish the destinies of the people." He saw Islamic astronomy as a means to those ideological goals. This was not xenophobia.[262]

All of this suggests that, instead of relying on clichés about Chinese attitudes, we would do better to explore Mongol policy. The explicit imperial call in 1382 for cooperation in sharing Muslim astronomical knowledge had no precedent at all under the Mongols. It is a mistake to assume that Khubilai's enthusiasm for things Chinese grew from a vision of a new technical culture that integrated

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[261] For details see Ch'en Mei-tung 2003a, 562–80, and Ho 1969 on early changes in the organization of the astronomical bureau. For a close study of the Islamic system's sources, see van Dalen 2000. Kūshyār's book was translated in 1383 into *Ming i t'ien-wen shu* 明譯天文書. Yano Michio (1997) has prepared an English version from Arabic and Chinese sources. The best-known Muslim system is *Hui-hui li-fa* 回回曆法 (Computational methods of Muslim astronomy). Van Dalen speculates that some work on the *Hui-hui li-fa* 回回曆法 was done in the period of the Yuan reform (2002, 330, 336–38, and personal communication, 2006.8.31).

[262] *Ming shih,* 31: 516–17, and Wu Po-tsung's 吳伯宗 preface to *Ming i t'ien-wen shu*.

talent from throughout the Mongol domains. I argued in Chapter 1 that what the Mongol elite sought was not a single best technique of prognostication, but a number of predictions from which they themselves could select when forming decisions (p. 23). Khubilai was exceptionally catholic in his interests, but, like other Mongol nobles, he preferred to keep those who gratified them separate. The record indicates that he wanted all the options to flow to him, and offer him the widest possible range of choice. In keeping secret what his various astrologer-astronomers produced, he did no more than his brethren. The Persian Marāgha observatory, for instance, was in a designated "prohibited area."[263]

Throughout the history of imperial China, all policy decisions were in principle those of the ruler. Emperors usually depended on high officials and advisors to make most of them. In the early Yuan period, Khubilai's policy with respect to the Chinese people remained, by and large, literally personal. He settled all but trivial disagreements, and did not hesitate to humiliate any official, Chinese, Mongol or other, who pushed too hard. In Khubilai's long reign, he did not revive the examination system, the traditional arrangement for selecting officials; he insisted on picking talent himself. His personal quirks conditioned the use as well as the provision of talent. In fact, in the fifteen years after the astronomical reform, as he aged and began to drink heavily, his decisions became more and more erratic.

There was, after all, a larger scheme of governance than the one in the minds of Han officials. In this four-tier society (p. 137), Chinese belonged to the two lowest classes. The traditional bureaucracy had no power at all over the first two tiers. Whatever else they did, officials' most concrete responsibility was to extract wealth from those they administered and to deliver it to the Mongols' agents. Khubilai and his successors saw no advantage in encouraging cooperation between his strictly delimited classes of subjects. The Mongols designed their society to keep their diverse subjects

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[263] Allsen 2001, 206.

apart, unable to join forces against their greatly outnumbered masters.

To sum up, in the period of the Season-granting system's creation, how much knowledge of non-Chinese astronomy could affect a reform was a matter for the ruler himself to decide, just as he personally selected, one by one, who would take charge of the project. He consistently chose to restrict the flow of information that might affect his prerogatives.

Once the Yuan period was over, the first Ming emperor, with a different set of priorities, promptly reversed this policy of segregation. This was not merely a shift in personal taste, but a contrast between fundamental values of the Mongol and Chinese rulers. And there is a deeper irony. Although Khubilai avoided interchange, the result was still singularly innovative; but the open door between Muslim and Chinese astronomy under the Ming did not tempt its sinecurists toward astronomical originality.

Conclusion

Despite the great disparity in what we know about individual instruments, it is clear that the observatory established for the reform was—in time—outstandingly equipped. We have little information about the astronomical teams dispersed through the empire, but their results indicate that, with unprecedented support, they too carried out a considerably more ambitious survey than their predecessors in the early eighth century were able to do.

Although we must admit that Kuo Shou-ching's role in the astronomical reform was far from predominant, his instruments were essential to its high standard, and his editorial role was indispensable for the system's survival. He was involved in the project longer than anyone else. Nevertheless, though he was recognized in his time as a peerless instrument-maker, and his biography credits the devices discussed above to him, other members of the reform group, in addition to leading the computational innovation, may well have played some part in equipping the observatory.

6 The Records

The astronomical treatises from the second century B.C. to the early twentieth century are based on the documentation for certain astronomical reforms, but the surviving versions are heavily revised and highly stylized. The relationship between what astronomical officials did and wrote, and what compilers of the Standard Histories (*cheng-shih* 正史) eventually published, bears considerably on our understanding of astronomy. This chapter will therefore explore that relationship. Since the many studies—early and recent, in Asia and elsewhere—of the Season-granting system were written by people in different circumstances with different aims, values, and biases, I will also review some of the most important secondary researches and remark on the motivations and assumptions that formed them. What we read in the astronomical treatises[264] of the Standard Histories is only indirectly related to the content and organization of the documents from which its editors compiled them. About the original records we know practically nothing except what a few of the treatises happen to tell us. Their stylization is another consequence of the state's sponsorship, for two thousand years, of astronomy and astrology. Historians take their sources where they find them. But the Season-granting system offers an exceptional opportunity to explore how technical bureaucracy formed the original record, and historiographic bureaucracy shaped the final one. Once the latter was published, its many scholarly uses become a topic of interest to this inquiry.

The Standard Histories and their Treatises

Each imperial dynasty kept records of everything that governance made important. Into this prodigious archive went every word

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[264] Usually entitled "Treatise on Mathematical Astronomy *(li chih* 曆志)," but sometimes part of a "Treatise on Mathematical Harmonics and Astronomy *(lü li chih* 律曆志)," and in one case entitled with bureaucratic panache "Treatise on the Administration of Heaven *(ssu t'ien chih* 司天志)."

and act of the emperor (recorded by specialist officials assigned to witness both), every memorial submitted to the palace, every edict that issued from it, reports on the operations of every department of government, and accounts of the lives and careers of people (and peoples) who merited attention for one reason or another.

The eventual distillation of this record, compiled after the end of the dynasty by its successor, was an official Standard History. The histories contained, among other items, annals of each reign, genealogies of aristocrats, biographies of eminent individuals, and treatises on matters that concerned the state, ranging through rituals, the civil service, food and commodities, and watercourse management, to court music, and of course astronomy and astrology.

Despite their wealth of vivid detail and quotation, the intent of the histories was didactic and political. Each provided models of good and bad in the conduct of governments and individuals. At the same time, each documented the legitimacy of the dynasty that it memorialized. The compilation also implied that the later dynasty which ordered it was the legitimate successor, with the authority to assess its predecessor's policies, decisions, and conduct, and to form the definitive view of it. In other words, officially compiling a history of the last dynasty was one of the ritual acts that bolstered the legitimacy of both regimes.

The Republic of China began in 1912 as a final rejection of the imperial tradition, but republics too, if they hope to last, must meet certain expectations their people already have. It is not surprising, then, that one of the early projects of the new government was a draft history of the Ch'ing dynasty that it had destroyed.[265] Eventually, the Nationalist government, after moving to Taiwan, issued a full-fledged Standard History in 1961. In that way, twelve years after the victory of the People's Republic and the Nationalist government's own retreat, the latter asserted that it still was entitled to rule all of China.

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[265] *Ch'ing shih kao* 清史稿 (Draft history of the Ch'ing), ordered 1914, completed 1927.

This same pattern of dual legitimation was responsible for the *Standard History of the Yuan (Yuan shih* 元史). The first Ming emperor ordered its compilation in 1368, the very year of his victory, and accepted its delivery in 1370. The result signaled that the Ming was the Yuan's true successor. At the same time, it placed the Mongol period—although it resulted from an alien invasion—in the legitimate sequence of dynasties. Thus the astronomical treatises we know most about are those sponsored by governments that historians in later dynasties placed in the orthodox line.[266]

Among the treatises *(chih* 志) in each history is often one on mathematical astronomy *(li chih* 曆志) or mathematical harmonics and astronomy *(lü li chih* 律曆志), one on astrology *(t'ien-wen chih* 天文志), and one on portentous phenomena in the sky and on earth correlated with the Five Phases *(wu-hsing chih* 五行志). The distribution of information varies. In most histories, the first category sets out one or more systems of computation. We have seen that each is a series of procedures for forecasting phenomena in time and space. The second and third record observations of unpredictable events. Because these were omens, the treatises also usually set out their significance with respect to the emperor and those around him. If there is historical information about the circumstances of an astronomical reform, the instruments used, and so on, it is likely to appear in the astrological treatise. The complementarity of astrology and astronomy, in other words, keeps the two kinds of treatise closely linked. The intricate classification of omens ties together the treatises on astrology and Five-Phases phenomena, both of which record them.

Comparing the treatises in different histories makes it clear that there was no fixed pattern. For instance, of the three histories of predecessors compiled in the Yuan period—those of the Sung, Liao, and Chin periods—the first gives detailed interpretations of omens

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[266] For lists of the 24 histories generally recognized ca. 1900 and of the treatises in each, see Wilkinson 2000, 501–15. Han Yu-shan 1955 is still informative on general aspects of Chinese historiography. Both authors confusingly translate *li* 曆 as "calendar" and *t'ien-wen* 天文 as "astronomy."

in its "Treatise on Astrology"; the second has no such treatise; and the third lists only phenomena without interpretations. Such structural differences had to do partly with what records were available. For instance, interpretations were those of the former dynasty's astrologers, and were not always available to the new one's historians. The choice of records reflected the historians' judgments about what was technically correct, what made historical sense, and what was morally or politically desirable—or risky. Astrology was a sensitive topic but, fortunately for our exploration, its handmaiden mathematical astronomy generally avoided shaky ground.

Astronomical Treatises

An astronomical system is the basic unit of Chinese mathematical astronomy, and a treatise in the Standard Histories is the document that sets it out. Remarkably few books on the subject were published in any other form.

Historic sources mention roughly 200 systems; enough information is extant for about 100 of them that we have some idea—occasionally a detailed view—of their technical character. As we have seen, less than 50 were officially adopted by imperial governments at one time or another. Of the latter group, Section 11 of the Evaluation lists and gives basic information about 41, as well as three important unofficial systems. In order to provide a broader view, table 2.1 comprises the names, authors, and dates of 98 systems for which some technical data are available.

As I have pointed out, the published astronomical treatises are not the original documents that set out new computational systems. Like that of the Season-granting system, they are concise summaries of the archival sources, compiled by successor dynasties as part of a history. They give some parts in detail, summarize some, and omit others. The Season-Granting system, despite its condensation (see p. 588), is among the fullest treatises that survive.

Canons

Most astronomical treatises, in their surviving forms, contain a canon (see above, p. 21). That of the *Yuan History,* translated below in chapters 9 and 10, is a good specimen, although more sophisticated than most. The more or less complete ones roughly resemble various European handbooks written for the same purpose, from Ptolemy's *Handy Tables*[267] to seventeenth-century productions.

All that survives of many Chinese treatises is their list of constants. But the Standard Histories include enough fairly complete canons to give us an idea of how their techniques evolved. Historians must infer these developments. Canons, sets of instructions for minimally skilled computists, neither explain nor justify their procedures.

Evaluations

Once a system became the basis for the annual ephemeris and therefore part of the imperial charisma, its potential or actual shortcomings became a source of anxiety in the palace. Narratives of the first systems in the Han period, from the end of the second century B.C. on, already cite contemporary criticisms of both details and general principles. By the end of the second century A.D., those who had studied calendrical problems historically knew that, no matter how satisfactory a system was when adopted, with the passage of time errors would accumulate and become obvious.[268]

As one system replaced another, critical discussions recurred. Some pointed out shortcomings of systems long in use, or proposed piecemeal improvements. Some attempted to forestall adoption of new systems that the throne had approved. Eventually, not only to make such interventions unnecessary but to minimize bickering, some governments appointed experts to test a new system thoroughly before or just after adoption, to ensure that it was superior to the one currently in use.

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[267] Halma 1822–25.

[268] See the example in Sivin 1969, 59–62.

When such systematic assessments began is not clear. The so-called evaluation of Tsu Ch'ung-chih's 祖沖之 exceptional Great Enlightenment system (#18, A.D. 464) is actually an attack on it by Tai Fa-hsing 戴法興—a literatus with limited astronomical skill but firmly fixed views—and Tsu's replies. The earliest internal critique about which we have adequate information is the Evaluation of the Great Expansion system (#42, 728). Only a summary survives. The key feature of the Yuan evaluation, computing events in the past and testing them against contemporary reports of observation, is already there. But that was not the main purpose of the eighth-century evaluation. Written by I-hsing 一行 (683–727), the author of the new system, its chief concern was validating the metaphysical basis of its constants, based on the numerology of the *Book of Changes*.[269]

In the published Supernal Epoch system (#61, 981), a fragment of an evaluation compares its predictions of planetary phenomena with those of two important predecessors. The method resembles that of the Yuan Evaluation, which, however, ignores the planets.[270]

The Season-granting system's evaluation drew on a large and detailed archive of observational records, stretching over a great span of time. The resulting document is as long and as detailed as the system's canon—in that sense possibly unprecedented, and never emulated. Still, the original version of the evaluation was almost certainly considerably larger than the published one. As table 6.1 shows, the four chapters of the whole surviving astronomical treatise—evaluation and canon—are insignificant compared with the 105 chapters of documentation that Kuo Shou-ching presented to the throne.[271]

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[269] For the Great Enlightenment system, see Sung shu 宋書, 13: 304–17, Nan shih 南史, 72: 1773–74, and Yen Tun-chieh 2000; for the Great Expansion system, consult Hsin t'ang shu 新唐書, ch. 27B; partial translation in Ang 1979: 419–63.

[270] *Sung shih* 宋史, 70: 1592–94.

[271] The data in this table come from Kuo Shou-ching's biography (Appendix B). All the titles listed in Huang Yü-chi 黄虞稷 et al. 1958 and Lo Chu-yun 雒竹筠 1999 come from the Yuan treatise and from this source.

Table 6.1. Writings of the Project

Translation	**Title**	**Size, *chüan***
Wang Hsun's drafts:	-	-
Pacing the Motions	*T'ui pu* 推步	7
Ready Reckoner	*Li-ch'eng* 立成	2
Draft of the Evaluation	*Li i ni-kao* 曆議擬藁	3
Hemerology based on the Rotations of the Gods	*Chuan-shen hsuan-tse* 轉神選擇	2
Annotated Divination with the three calendars	*Shang chung hsia san li chu shih* 上中下三曆註式	12
Kuo's compilations:	-	-
History of the Astronomical Reform	*Hsiu-kai yuan-liu* 修改源流	1
Critical Annotations on Times	*Shih-hou chien chu* 時候箋注	2
Designs of the Instruments	*I-hsiang fa-shih* 儀象法式	2
Study of Eclipses, Ancient and Modern	*Ku chin chiao-shih k'ao* 古今交食考	1
Study of Gnomon Shadow [Measurements] for the Solstices	*Erh chih kuei ying k'ao* 二至晷景考	20
Study of the Moon's Travel	*Yueh-li k'ao* 月離考	1
Study of the Detailed Movements of the Five Stars	*Wu-hsing hsi hsing k'ao* 五星細行考	50
Newly Observed Degrees of Lodge Entry and Polar Distance of Stars in Miscellaneous Asterisms among the 28 Lunar Lodges	*Hsin ts'e erh-shih-pa she tsa tso chu hsing ju hsiu ch'ü chi* 新測二十八舍雜座諸星入宿去極	1
Newly Observed Unnamed Stars	*Hsin ts'e wu ming chu hsing* 新測無名諸星	1
Total	-	105

The canon of the next dynasty, the Ming (1368–1644), is considerably more elaborate than its immediate predecessor, but no evaluation accompanies it. Since its authors produced it by slightly modifying the Yuan canon, there was not enough novelty to justify one.[272]

Transmission and Publication

As we have just seen, the astronomical treatise in a Standard History is a new compilation that reproduces some parts of the documentation in detail, summarizes some, and omits others. The Season-Granting system is among the fullest astronomical treatises that survive, exceeded in size only by the two that came after it. These are the treatise of the Great Concordance system of 1384, a superficial revision of the Season-Granting system that incorporates some valuable materials that the editors of the *Yuan History* omitted, and that of the Temporal Pattern system (*Shih-hsien li* 時憲曆, #96) of 1644, the curious hybrid of Chinese and obsolescent European content that the Jesuits used.[273]

Both the Yuan and the Ming treatises omit the star tables of the Season-granting system. Until recently it has seemed that they are lost, but that may not be true. Since the 1980's, the data on fixed stars in the "Astronomical Anthology (*T'ien-wen hui ch'ao* 天文匯鈔)," a manuscript in the Beijing Library, have attracted attention because they generally correspond to information in the Yuan treatise. One opuscule in this anthology includes 75 diagrams of one or more constellations. They label each star, often with its equatorial position within a lunar lodge and its distance from the north pole—the equivalents of right ascension and declination. This is new, since normally coordinates appear in a text, not a star chart. The dia-

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[272] *Ming shih,* ch. 31–39. For the complex history of its writing and revisions, and the considerable self-censorship involved, see Han Ch'i 韩琦 1997.

[273] The revised Ming system of 1384 was based on the *Ta-t'ung li-fa t'ung kuei* 大統立法通軌 (Standards of the Great Concordance system) of Yuan T'ung 元統. According to Huang Tsung-hsi 黄宗羲, this was the last work that had access to the Yuan system's collection of worked computational examples *(Li ts'ao* 曆草; *Shou-shih li ku,* preface, p. 1a). See Liu Tun 刘钝 1982 on the worked examples, and Ch'en Mei-tung 2003a, 555–58, on the differences between the Yuan and Ming systems.

grams include a total of 283 asterisms containing 3175 stars, and give data for 739 of them.

P’an Nai 潘鼐 has noticed that the name of this little compilation somewhat resembles that of Kuo Shou-ching’s edition of star observations (table 6.1).[274] The similarity of purport and wording is suggestive. The astronomical record for a century or so after the Yuan reform includes no record that officials undertook large-scale programs of stellar observation. On the other hand, the scale of the survey in the manuscript leaves no room for doubt that it was a government project.

Sun Hsiao-ch’un 孙小淳 has done an error analysis of the data, and has concluded that the errors in several parameters are consistently minimal circa 1380. That is a persuasive argument, but not a definitive one; a systematic error of construction or calibration in the instrument used could have biased the results. We can conclude, first, that the observations behind the star treatise were probably made close to 1380, but could have been as early as circa 1280; and, second, that in the century between the two dates, the lack of evidence for innovation in astronomy suggests that the compilation, even if it did not copy Kuo’s model, drew upon it.[275]

All of the original writings that came out of the 1280 astronomical reform remained unpublished. They were archived for the use of future astronomers, just as past archival documents had played an indispensable part in that reform. With the possible exception that I have just discussed, none survived to the present, but they were still available to historians when the Ming compiled the *Yuan History.*

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[274] The MS is “San-yuan lieh she ju hsiu ch’ü chi tu 三垣列舍入宿去極度 (Degrees of lodge entry and polar distance [of asterisms] within the Three Walls [i.e., the circumpolar region] and the lunar lodges),” and Kuo’s compilation is “Hsin ts’e erh-shih-pa she tsa tso chu hsing ju hsiu ch’ü chi 新測二十八舍雜座諸星入宿去極 (Newly observed degrees of lodge entry and polar distance of stars in miscellaneous constellations among the 28 lunar lodges).”

[275] P’an 1989, chapter 7, especially tables 74 and 75; Ch’en Ying 陈鹰 1986; Sun 1996.

Considering the chaos and mass destruction during most transitions between dynasties, which led to a great attrition of books, the long-term survival of palace astronomical archives—such as those that the Evaluation repeatedly cites—is remarkable but undeniable. But those archives, as part of the imperial library, were in no sense public, or even accessible to officials in general.[276]

We have no idea how the documents eventually fell into the hands of the Ming government. The *Yuan History* does not include the usual bibliographic treatise based on a catalogue of the palace collection.[277] The bibliographic treatise of the *Ming History* and various unofficial supplements do not list any documents of the reform as extant. On the other hand, the authors of the Ming astronomical treatise incorporated the ready reckoners, and had access to tables not included in the Yuan treatise. We do not know whether they used the copies of the collection in the imperial library or those in the office of the Astrological Commission.[278]

The distillation of astronomical records into treatises in the Standard Histories made them public in a limited sense. Once the government began printing the Histories, it distributed them to local offices.[279] Civil servants on business, as well as anyone to whom the magistrate granted access, could thus read them. Local officials eventually sponsored reprints, often having them published in the printing shops connected to regional offices. Their most usual aim was to spread historical knowledge as a spur to orthodoxy. As private printing expanded from the eleventh century on, the histories became available to any member of the literate minority who could

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[276] Ts'ui Chen-hua 崔振华 & Chang Shu-ts'ai 张书才 1997, a catalogue of the surviving astronomical archives of the Ch'ing period, lists about 1500 items.

[277] There have been many attempts to reconstitute it, but their listings of the pertinent items obviously come from Kuo's biography (e.g., Lo Chuyun 1999: 230–31).

[278] Li Yin-chi 李银姬 & Ching Ping 景冰 1998, 74, find that the table of times for dawn and dusk in the Ming History were revised for the latitude of Nanjing, the early Ming capital.

[279] Official printing of the classics and histories began in the 980's, according to Tsien Tsuen-hsuin in Needham 1954–, 5 part 1: 162.

afford the massive set, or who could read someone else's copy. Eventually the government set questions on the civil service examinations that required every potential examinee to memorize astronomical and mathematical treatises. This was an irregular practice, but once there was a precedent, students were likely to prepare themselves.[280]

The end of the empire in 1911 did not end the reproduction of the Histories. The 21 histories that governments recognized by 1400 expanded to 24 by 1911, and to 25 in 1921. A command of the classical language spread beyond the offspring of the old scholar-official class, and highly educated people expected each other to have some acquaintance with the histories. They remained the main sources for thinking about the historic past. In both the Republic before 1949 and the People's Republic after, large groups of scholars produced corrected editions of the whole set. The one edited over a period of twenty years at the behest of Mao Tse-tung established texts of exceptionally high quality. The participation of historians of science resulted in reliable tables and other technical content. The decision to print the series in old-style script *(fan-t'i* 繁體) avoided the loss of information inevitable with the use of simplified script *(chien-t'i* 簡體). That is the version of the histories I have used in this study.[281]

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[280] Elman 2000, chapter 9, refutes the misconception that the imperial examinations never mandated study of the quantitative sciences.

[281] *Erh-shih-ssu shih* 二十四史 (The twenty-four histories), published in 241 volumes between 1962 and 1975 by the Chung-hua Shu-chü (Beijing). Since 1988, a searchable digital version of that version as well as the *Draft History of the Ch'ing Period* (*Ch'ing shih kao* 清史稿) has been available (Anonymous 1988). For details on various versions of the Histories see Wilkinson 2000, 501–8.

Literacy in the restricted sense of being able to read and write enough characters for mundane purposes had been spreading for centuries, as Rawski 1979 and others have shown. To read the histories with comprehension required not this scant literacy but a good command of ancient Chinese and of traditional literature.

Studies

This section enumerates a number of important publications on the Yuan reform. I will look first at early studies from East Asia, written inside the tradition that produced the Season-granting system to facilitate its teaching and use. I will also survey some landmarks of modern scholarship, Eastern and Western, meant to incorporate an understanding of that tradition into historical and other learning. I will summarize for each group their significance and the discrepant assumptions that underlie them.

Early Studies in East Asia

Many Chinese, Japanese, and Korean scholars before 1900 wrote commentaries and other aids to learning the Season-Granting system's techniques of computation. When neighbors to the east of China adapted the Yuan system under local titles for use in their courts (Korea between 1441 and 1444, and Japan in 1684), local experts took up the challenges it posed. Its study increased mathematical proficiency in the two countries to the point that astronomers eventually mastered its complexities.[282]

What inspired a good many of the annotators in all three countries was the Evidential Studies *(k'ao-cheng* 考證) movement. It began among Chinese intellectuals in the seventeenth century, spread to some extent through East Asia, and predominated from the mid eighteenth century to the present day. Its aim was to restore the original meaning of the ancient classics using philological methods. Its tools included mathematical, astronomical, medical, and other techniques useful for dating and for explicating ancient terminology.[283] Some commentaries were thus textbooks for training official and unofficial astronomers, and some were works of scholarship. Four authors stand out among those whose work I have drawn on.

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[282] Nakayama 1969, 120; Jeon 1974, 79–82; Lee Eun-Hee 1997; Shih Yun-li 石云里 1998. Li Yin-chi & Ching Ping 1998 point out that the Korean court began using the calendrical part of the Canon in 1309, but its procedures for planetary motions and eclipses continued to be those of the Extending Enlightenment system (#50) for more than a century longer.

[283] Elman 1984 is the standard source for Evidential Studies.

Shou-shih li ku 授時曆故 (The Season-granting system restored), written in the third quarter of the seventeenth century by the great generalist Huang Tsung-hsi 黄宗羲 (1610–95), is a masterly guide to using the system, full of valuable critical comments. The title reflects Huang's dedication to restoring the classics, and the importance of astronomy to this quest.

Mei Wen-ting 梅文鼎 (1633–1721), the preeminent astronomer of his time, left several short technical monographs, as always acute on aspects of the Yuan system.[284]

Seki Takakazu (or Kōwa, 関孝和, circa 1640–1708), Japan's most innovative mathematician and a famed teacher of mathematicians, responded circa 1680 to the Yuan system's computational challenges with three manuscripts, including a ready reckoner and an explication.[285]

Seki's disciple Takebe Katahiro 建部賢弘 (1664–1739), mathematician to two Shoguns, wrote a systematic and detailed introduction to calculating ephemerides with the system. His manuscript "Jujireki kaigi 授時曆解議 (Explication of the Season-Granting System)" circa 1690, provides worked-out examples of every step in calculation. It is the most sophisticated collection of worked examples available to me. My commentary frequently cites this book.

Seki, Takebe, and other Japanese authors of textbooks greatly admired the Yuan system's secular variation in the length of the tropical year (see above, p. 101), so much so that they tended to apply it not only to year length and the Annual Difference but to all solar constants; Nakayama Shigeru calls this "the only original idea in Japanese astronomy.") These "improvements" introduced sys-

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[284] I have found particularly useful his *Erh i ming pu chu* 二儀銘補注, *Li-hsueh p'ien-ch'i* 曆學駢枝, *Shou-shih li p'ing li ting san ch'a hsiang shuo* 授時曆平立定三差詳說, *Chien-tu ts'e-liang* 塹堵測量 (On the mensuration of moat walls), and *Ta-t'ung li chih* 大統曆志.

[285] I have consulted *Juji hatsumei* 授時発明, *Jujireki kyō rissei* 授時曆経立成, and *Jujireki kyō rissei no hō* 授時曆経立成の法. See Horiuchi 1994, Martzloff 1998, Feng Li-sheng 冯立升 2001, and Wang Jung-pin 王荣彬 2001.

tematic but negligible errors into their results. Seki's ingenuity and clarity outweigh such miniscule divergences.[286]

A Chinese author earlier than any of the four, but lacking their skills, deserves notice for his quirky insights. Hsing Yun-lu 邢雲路 (fl. 1580) was involved in more than one proposal to reform the Ming computational system, in one instance cooperating with early Jesuit missionaries. Because in his time (the mid Ming period) the private study of astronomy was illegal, his initiatives provoked threats by court astronomers. His status as an official barely protected him from judicial punishment. Hsing's *Studies of Mathematical Harmonics and Astronomy, Ancient and Modern (Ku chin lü li k'ao* 古今律曆考, 1607) is a large survey and evaluation of systems up to his time, with special attention to the Yuan system. He tended to be more occupied with cosmological questions and problems of spatial visualization than later authors. His book shows an awareness of what might be called the spherical trigonometry of the Yuan system, although he expressed it without the European concepts that the missionaries began to introduce a little later.

Hsing's reputation among historians has been low, because he was not very competent in astronomy. Those limits were a result, after two centuries, of the imperial ban on private study. His book, like his other technical activities, was an attempt to make some headway against the damage.[287]

Teachers in various parts of East Asia compiled many useful tables and other aids to make the system easier to use for students (and conceivably for holders of computational sinecures). Especially early is the Korean *Susiryŏk ch'ŏppŏp ipsŏng* 授時曆捷法立成 (Expeditious ready reckoner for the Season-Granting system) of 1343. Kang Po 姜保, trained by an astronomer who had studied in Yuan China, provided in this book nearly twenty tables meant to simplify tasks that without them would not be extremely demanding.[288]

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[286] Horiuchi 1994 is an excellent study; see also Nakayama 1969, 229.

[287] For a perceptive and well-balanced discussion see Ch'en Mei-tung 2003a, 616–18. On Hsing's giant gnomon see Shih Yun-li 石云里 & Wang Miao 王淼 2003.

[288] See Jeon 1974, 109.

A good example of an elementary text is the anonymous Japanese *Genshi Jujirekikei zukai* 元史授時暦經圖解 (Illustrated explication of the *Yuan History's* Canon of the Season-Granting system, 1697), which combines elementary astronomical principles with worked examples and a few illustrations.[289]

Whether written to deepen understanding or to train beginners, all of the commentaries I have enumerated, early and late, in all three countries, studied the Yuan treatise as an authoritative, officially sponsored document, the state of the art in computational astronomy. Hsing apparently hoped to improve upon it. Mei and Huang were the foremost custodian of astronomical standards and a leading historian and political thinker. Although born in the Ming period, they did their writing after 1644, when there was no longer an effective ban on private study. In Japan and Korea, where private study of the stars was not prohibited, textbooks opened the prospect of official careers. Both countries strongly discouraged occupational mobility—even from one generation to the next—but technical skills sometimes opened irregular opportunities.

Modern Studies

Since our concern is the study of the Season-granting system, and I cite throughout this book the most important publications about it, here I will merely sketch how and why scholarship evolved in the way it did. To conclude, I will summarize some of the assumptions and motivations behind these extremely diverse studies.

Substantial studies from outside the imperial Chinese perspective actually began more than 250 years ago. The Jesuit missionary Antoine Gaubil was born in 1689 and lived in China from 1722 to his death in 1759. His formidable command of astronomy and of the classical Chinese language, combined with his resolve to introduce the empire's early knowledge of celestial computation to Europeans, gave his writings greater depth than any others before the

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[289] See also *Jujireki kyō enkai rakki* 授時暦経諺解略記. A few examples of Chinese elementary texts survive, e.g., *Shou-shih li yao fa* 授時暦要法 (Essential methods of the Season-Granting system), compiled after the Yuan period.

twentieth century, and of most before 1950. He was in China to further Roman Catholicism and to work as an astronomer, but his understanding of early technical issues was outstanding. His histories of Chinese astronomy in French naturally paid ample attention to the Yuan system. Joseph Needham had reason to write in 1959 that "for any thorough study of Chinese astronomy Gaubil is needed even today." Only the rapid strides that research in China has taken since 1980 have made his work optional—but it remains useful.[290]

The next landmark in studies of the Season-granting system was the writings of the indefatigable Alexander Wylie (1815–87), a Scotsman who spent 30 years in China on behalf of the London Missionary Society. Wylie was well read in Chinese sources, and often wrote on early mathematics. He set down in 1876 a compendious account of the Yuan observatory and its instruments, the Islamic instruments of the same period, and the astronomical reform. In it he narrates his discovery of the surviving instruments (see above, p. 210). What makes this essay stand out is Wylie's discovery that Kuo Shou-ching's original Simplified Instrument and Ingenious Instrument still existed. He came upon them abandoned in a shed below the Jesuit observatory on the Pei-ching city wall (they eventually disappeared).[291]

Mikami Yoshio 三上義夫, an excellent historian of Japanese mathematics, provided in 1913 a conspectus in English of what mathematicians had accomplished in both China and Japan. It contains an intelligent chapter on the mathematics of the Yuan reform (which Mikami, like most historians after him, treated as the creation of Kuo Shou-ching alone). L. Gauchet, a French missionary who nearly a century ago wrote several of the earliest non-Asian studies of Taoism, published in 1917 a first discussion in modern terms of the spherical proto-trigonometry in the Yuan system.

General histories of astronomy by Chinese began to appear in the twentieth century. They placed the Yuan system in the long sweep of time, but their focus remained almost entirely on computation

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[290] Needham et al. 1954– , 3: 182–83.

[291] Wylie 1876.

and institutions (this was also true of scientific history in the Occident until the 1970's). The survey by Chu Wen-hsin 朱文鑫, published in 1934, was reliable and handy enough that it was not superseded until the massive (but still traditional) history by Ch'en Tsun-kuei 陈遵妫 (1980–84). Yen Tun-chieh 严敦杰., a pioneer of the history of mathematics, published his reading notes on the Season-granting system in 1985. The modern works of Ch'en Mei-tung 陈美东 (1995, 2003a) have carried synoptic, critical research beyond all his twentieth-century predecessors.

Until late in the twentieth century, occidental historians of astronomy remained sublimely oblivious of non-European work, and of Gaubil's writings. Most were inclined to take seriously the publications by missionaries that claimed, among the many weaknesses that required foreigners to save the Chinese, the latter had had no talent whatever for science, they had a language unfitted for its pursuit, and, by an obvious deduction, they had never done any technical work could be taken seriously.

It remained for Joseph Needham's *Science and Civilisation in China,* from 1954 on, to demonstrate the ignorance and prejudice that underlay such deductive disdain. The outcome is an incipient consciousness that technical change is best understood by studying it as a world enterprise—but it has been slow in coming.

Needham's volume on mathematics and astronomy (1959) is still the frequent first resort of curious Westerners. It is part of a tightly integrated and intellectually iconoclastic historical survey. Needham and his collaborators are informative on many neglected aspects such as instrumentation; the book is exceptionally clear, and well illustrated. Nevertheless, although no new survey in any European language has superseded it, it remains the least successful tome in the series. Needham's curiosity about quantitative matters was less acute than about other topics, so that he relied more than elsewhere on old secondary sources. He failed to understand the centrality of the almanac in the astronomical tradition, dismissing the reiterated astronomical reforms as "of minor scientific interest," and discussing them only cursorily.[292]

Several studies broader than monographs but narrower than surveys are useful for both the background and certain aspects of the Yuan reform. The Chinese pioneers of the history of mathematics—Li Yen 李儼 (1954–55, 1957), and Li & Tu Shih-jan 杜石然 1963), Ch'ien Pao-ts'ung 錢寶琮 (1956), and Yen Tun-chieh (1966)—made many contributions of this kind.

In Japan the emphasis of scientific history tended to be squarely on that of astronomy. Yabuuchi Kiyoshi 藪内清, who signed his writings in English as Kiyosi Yabuuti, studied each great period of the Chinese art; his survey of the Sung and Yuan (1967) was embedded in a volume on science, medicine, and technology in those two dynasties. Nakayama Shigeru's *A History of Japanese Astronomy* (1969) was the first book in English to provide detailed and reliable technical information on the Season-granting system. Because of the system's important role in Japan, Nakayama gave it considerable attention. The work of both scholars on the Season-granting system has finally appeared in a monographic Japanese translation of the treatise and a very concise commentary (2006).

There have been many modern East Asian monographs and collections, long and short, on the *Shou-shih li.* Ch'en Mei-tung 2003a cites and gives references to the most valuable from China. Of Japanese studies, Hirose Hideo 広瀬秀雄 1968–70, although brief, is particularly worth reading. The establishment of the Kuo Shou-ching Memorial Hall (Kuo Shou-ching Chi-nien-kuan 郭守敬纪念馆, circa 1985) in Xingtai (Hsing-t'ai 邢臺), Kuo's native place, prompted conferences, books, and even an irregular journal. Kuo was of course their main focus, but a good many of these publications dealt with other aspects of the reform and with its other participants. Some are laudatory and conventional, but some take up previously unexplored topics or use local materials for the first time.[293]

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[292] Needham et al. 1954– , 3: 390. See, for instance, Nakayama 1969, 3.

[293] The three books available to me are Kuo Shou-ching Chi-nien-kuan 1987 and 1996, and Ch'en Mei-tung & Hu K'ao-shang 胡考尚 2003. The

There are a couple of book-length biographies of Kuo, but none of his colleagues in the reform group. The lives of Kuo by Li Ti 李迪 (1966) and P'an Nai 潘鼐 & Hsiang Ying 向英 (1980) are well-informed, concise accounts for popular audiences; the latter includes no documentation. Excellent, substantial chapters on Liu Ping-chung 劉秉忠, Kuo, Hsu Heng 許衡, and Chang Wen-ch'ien 張文謙—four of the leading figures in the project—are among those on "eminent personalities" in Rachewiltz et al. 1993. Rossabi 1988 is most helpful for understanding the motivations of Khubilai.

For the studies reviewed in this subsection, the range of aims and assumptions over two and a half centuries is enormous. Gaubil typified eighteenth-century Jesuit missionaries in his desire to convince Europeans that China had an advanced culture and merited a respectful approach to religious conversion from the top down (the other Catholic orders mainly proselytized among the poor and powerless). Wylie typified the nineteenth-century Protestants doing the work of European empire-building, for whom the Chinese were fallen and urgently in need of saving. His familiarity with and high regard for the culture, technical as well as literary, made him exceptional among his peers. Most of the work by Mikami and other leading Japanese scholars of astronomy document an assumption of continuity between Chinese and Japanese mathematical culture; they are exploring their own history.

Chinese historians of astronomy from the 1930's on were academic scientists tracing their own heritage. This was a politically momentous pursuit. They were not just documenting past intellectual glories, they were demonstrating that science was not inherently foreign. In the first half of the twentieth century their country was chaotic, and it remained poor in the second half. Nevertheless, it had played an important role in creating astronomy, and was regularly producing distinguished practitioners.

Memorial Hall's journal is entitled *Kuo Shou-ching yen-chiu* 郭守敬研究; I have been able to read only nos. 2 (1986) and 5–8 (1988–99). I am grateful to Ch'en Mei-tung for his help in getting these important sources.

The activities of the Kuo Shou-ching Memorial Hall have not greatly differed from those of similar institutions in known or putative birthplaces all over China, dedicated to legendary sage-kings, celebrated philosophers, and technical paragons. They were important in building local pride, and from the 1980's on the government encouraged them as a means to create a tourist industry. The state's attempts to canonize a few heroes of learning—planetoids and eminences on the dark side of the moon named after them, postage stamps portraying them (figures 18, 19), books for children about them—have supported the abiding tendency to see the Season-granting system as the one-man creation of Kuo Shou-ching.

Figs. 18, 19. Chinese postage stamps depicting Kuo Shou-ching (from a series of imaginary depictions of ancient scientists, 1962), and the 15th-century copy of the Yuan armillary sphere, 1953.

Conclusion

We have seen a succession of aims and assumptions as the writings of the Season-granting project moved out of the astronomical archives to become, in print, relics of a dead dynasty. For the small educated public, from the time the *Yuan History* appeared, the abridged treatise in it the became the canonical document of the reform. As a result, people studied the treatise from many points of view, encompassing many motivations. Both the viewpoints and the assumptions changed greatly over the centuries I have surveyed. For Hsing Yun-lu, the Season-granting treatise was a document to be improved upon and used for making better almanacs. Commentators in China, Japan, and Korea taught it as a means to a career or an avocation, or a tool useful in restoring the pristine classics of antiquity. Gaubil's intellectual aims and the Protestant missionaries' activism inspired very diverse interpretations abroad. As imperial China expired, and afterward—in the decades of tumultuous change throughout East Asia before 1950—the circumstances of the many first-rate Chinese and Japanese, and the few non-missionary Europeans, who studied the Yuan reform shaped other frames of understanding and other goals.

Since the inauguration of the People's Republic in 1949, the government's intention to widen and prescribe the content of education has meant a massive investment in historical study of China. Since 1980, as a result of comparatively settled circumstances for academic study, of a more open intellectual ambiance, and of economic growth, Chinese researchers have begun exploring in unprecedented directions. Their interest in new methodologies and regular give and take with historians elsewhere have been major forces for historiographic innovation in the West as well as in East Asia.

7 Evaluation of the Season-Granting System

[The evaluation consists of eleven sections in two chapters (7 and 8 in this translation). Here is a list of the topics covered, with indications in modern terminology of what each section assesses:

Chapter 7

1. Determination of *ch'i* from observation: solstices
2. Year surplus and annual difference: tropical and sidereal year and their relation
3. Angular extensions of the lunar lodges: system for recording the equivalent of right ascension
4. Tread of the sun: mean solar motion
5. Expansion and contraction of the solar motion: apparent motion of the sun
6. Slackening and hastening of the lunar motion: apparent motion of the moon
7. Crossing cycle of the White Way: nodical month

Chapter 8

8. Day and night marks: length of day and night
9. Eclipses: accuracy of eclipse predictions at times distant from the epoch
10. Corrected conjunctions: apparent conjunctions
11. Disuse of Accumulated Years and Day Divisor: adoption of an epoch close to the date of promulgation]

Introduction by the Compilers of the Yuan History

The seasons clearly understood, the ephemerides in order: since the times of the Yellow Emperor, Yao, Shun, and the glorious monarchs of the Three Dynasties, none has failed to give these aims due weight.

[The first phrase of the chapter alludes to the *Chou i* 周易, the *Book of Changes*, where the Commentary on the Images *(Hsiang chuan* 象傳) to hexagram 49 asserts that “the lordly man, by ordering the ephemerides, clearly understands the seasons.” This short introduction implies that Khubilai, who ordered the Season-granting reform, was one of these paragons.]

Their writings appear adequately in the record that has come down to us. We are far removed from antiquity, and no longer know the old techniques in detail. Nevertheless, if we go back to the essentials, they amount simply to putting predictions to the test of time in order to reach accord with the celestial phenomena. In the Han period, Liu Hsin 劉歆, in making the Triple Concordance system (#3), first instituted the method of Accumulated Years and Day Divisor to serve as the standard for pacing the celestial motions *(t'ui-pu* 推步).

> [All references to astronomical systems used officially prior to the Yuan system, and to their authors, are numbered in the form "(#3)." The numbers correspond with the list of systems given in table 2.1, which provides cross-references to section 11 below.
>
> The authors are giving Liu Hsin (46 B.C.–A.D. 23) credit for the classical approach to calculating the ephemeris, in which one counted off constant periods from an epoch to give the desired date of inception of cyclic phenomena. Accumulated Years and Day Divisor are among the constants Liu created for that purpose. "Pacing," in the sense of "pacing off," is a common word—borrowed from surveying—for tracing successive positions of heavenly bodies by observation or calculation. By extension it has come to mean astronomical prediction in general.]

Later generations carried on this tradition, through T'ang and Sung, until dozens of experts had appeared who improved the epoch and the techniques. But surely it was not that [the revisions were so frequent merely] because the reformers wanted to differ [from their predecessors]. It would seem, rather, that some irregularities are inherent in the celestial motions, but an astronomical system must use set methods. Thus with the passing of time discrepancies are inevitable. Once they appear, correcting them is unavoidable.

> [Chinese thinkers did not believe that nature was bound by law and thus had to be quantitatively regular in its activities (nor was that notion prevalent in Greek antiquity, or the Christian West before the Enlightenment). Astronomers in China often expressed the view that prediction is inherently limited because the celestial motions are to some extent unpredictable (Sivin 1989, Henderson 1984). That did not prevent the best astronomers from continuing to apply number and measure to forecast previously intractable phenomena.]

The Yuan house, when it began, continued to use the Chin dynasty's [Revised] Great Enlightenment system (#76, 1180). In sexagenary year 17 (1220; hereafter s.y. 17), the Grand Progenitor (i.e., Chinggis) undertook a military campaign in the west. A lunar eclipse predicted for the full moon of the fifth month did not appear, and on the first of the second and fifth months a sliver of moon was visible in the southwest. The Secretariat-Director Yeh-lü Ch'u-tsai 耶律楚材, on the ground that [predictions using] the Great Enlightenment system were retarded with respect to the sky, reduced the day parts in a nodal *ch'i,* lessened the ten-thousandth parts of the Celestial Perimeter, reduced the rate for the Crossing Terminal Constant, and set in order the remainder for the moon's period of revolution. He tested by observation the varying speeds of the Two Luminaries (i.e., the inequalities of the sun and moon) and adjusted the techniques for predicting the risings and settings of the Five Stars, in order to correct the shortcomings of the Great Enlightenment system.

[The author's point is that Yeh-lü made changes in the values of synodic and sidereal constants (for Celestial Perimeter, *chou ti'en* 周天, see Canon, 3C.1; for nodal *ch'i, chieh-ch'i* 節氣, 1C.8; for the nodical month, *chiao chung* 交終, 6C.2). In other words, Yeh-lü adjusted the system in breadth. The last sentence analogously has him testing the whole range of feasible predictions against the phenomena, including the planets.

The modern editors of the *Yuan shih* correct "T'ai-tsung 太宗, Khubilai" to read "T'ai-tsu 太祖, Chinggis"; see p. 1150, n. 1.]

Then, beginning with the Intermediate Epoch as sexagenary year 7 (1210), when the army of our state made a punitive expedition southward [i.e., against the Chin] and largely pacified the realm, he computed the Superior Epoch to be the winter solstice which fell in the standard half of double-hour 1, s.d. 59, the first day of month 11, the Astronomical First Month of s.y. 7. At that moment the sun and moon were aligned [concentrically] like the edges of an annular jade disc, and the Five Planets [were lined up like] strung pearls, in general conjunction at 6 *tu* in the lunar lodge Tumulus—a propitious sign vouchsafed in response to the Grand Progenitor's assumption of the Mandate.

[A grand universal conjunction of the sun, moon, and planets, phrased in this stereotypical formula, is the most splendid sign of a new cosmic and political order. In systems prior to the Yuan, it was normal to compute the time of such a conjunction in the remote past and to consider it the initial point of all astronomical cycles. The Intermediate Epoch is a date close to the actual period in which the system is used. For convenience, astronomers treated it as the epoch for most calculations. In this instance the Superior Epoch falls 20,275,270 years or 337,921 sexagenary cycles and 10 years before 1210. That is to say, it was the winter solstice which began the year 20,274,061 BC. The greatest Japanese scholar of the Season-Granting System, Takebe Katahiro 建部賢弘, gave the constant a value of 20,275,260, which made the interval ten years less. That is because he depended on a bad text that also put the Superior Epoch in s.y. 7.

The word I translate "solstice" is *chih* 至. It literally means "extreme," also an alternative sense of "solstice" in English. "The army of our state" refers to an expeditionary force under the command of Chinggis. Backdating statehood was frequent in official historiography.]

Furthermore, because of the great distance between the Western Regions and the Central Plains, he created the League Distance Correction *(li-ch'a* 里差) with which to augment or diminish [the time correction for longitude]. Even though east and west are a myriad leagues apart, no longer [was this distance responsible for] errors. He subsequently named his astronomical system the Western Expedition Seventh-year Epoch system *(Hsi cheng keng-wu yuan li* 西征庚午元曆, #82). He submitted it to the throne in a memorial, but it was not promulgated and used officially. Nor did the Season-granting system include the League Distance Constant.

[Although the Mongols never officially adopted Yeh-lü's system, the *Yuan History* outlines it in chapters 56–57, directly after the account of the Season-Granting System translated here. It was natural that Yeh-lü's Intermediate Epoch be a significant year for the establishment of the dynasty. The one chosen fell in the early period of Chinggis' campaigns against the Khitan Tartars, who were ruling North China after he became Khan in 1206. The linkage of this year with the Superior Epoch made it cosmically significant, tying it to the legitimacy of later Mongol rule of all China. The Mongol victory in North China did not actually come until 1215; 1210 was the year that Chinggis ended his status as a tributary of the Chin.

This computational system was not the only contribution of the great Khitan statesman Yeh-lü Ch'u-ts'ai (1190–1244) to the Mongols. Yeh-lü's biography occupies ch. 146 of the *Yuan History.* It contains nothing germane except for a couple of stories about his astrological prescience. The excellent biography in Rachewiltz 1993: 136–75 is detailed except with respect to his technical interests. On his astronomy, see Ch'en Mei-tung 2003a: 518–20.]

In year 4 of the Perfectly Great era (1267), Jāmal al-Dīn (Cha-ma-lu-ting 扎馬魯丁) of the Western Region compiled and submitted to the Throne the Myriad Years System (Wan nien li 萬年曆, #85). Emperor Shih-tsu [i.e., Khubilai] promulgated it to a limited extent.

In year 13 (1276), [the Yuan] pacified [i.e., conquered] the Sung state. It was subsequently decreed that the former Left Vice Director of the Secretariat Hsu Heng 許衡, the Admonisher to the Heir Apparent Wang Hsun 王恂, and the Assistant Supervisor of Waterways Kuo Shou-ching 郭守敬 were to reform the ephemerides and make a new astronomical system.

[Compare the account in chapter 5, p. 152.]

Hsu and the others realized that, although the Chin had carried out an astronomical reform, all they had done was to slightly modify the [century-old] Era Epoch system of the Sung (#71), without actually testing its techniques against observations of the celestial phenomena. Thereupon, in collaboration with astronomical officials of the north and south [i.e., those of the Yuan and the Southern Sung] such as Ch'en Ting-ch'en 陳鼎臣, Teng Yuan-lin 鄧元麟, Mao P'eng-i 毛鵬翼, Liu Chü-yuan 劉巨淵, Wang Su 王素, Yueh Hsuan 岳鉉, and Kao Ching 高敬, they carried out research on methods of mathematical astronomy through the ages, and observed the alternating phenomena and cyclical motions of the sun, moon, and planets in order to determine their variations, distinguishing what had remained the same from what differed. They deliberated and chose values between their extremes *(chung shu* 中數) to serve as the foundations of a system.

[Ch'en Mei-tung 2003b, 65, gives reason to believe that, in this list, the first three names and the last name are those of southerners.]

The new system was completed on the winter solstice of year 17 (14 December 1280). A decree bestowed upon it the name "Season-Granting system." In year 18 (1281) it was promulgated throughout the realm. In year 20 (1283) a decree ordered the Advisor to the Heir Apparent Li Ch'ien 李謙 to prepare an evaluation of the system. It was to reveal the subtle but profound means by which the new system sought [exact] conformity with the sky, and to demonstrate by comparative study the errors that had led men of earlier times into misunderstanding. [He concluded that] "truly the Season-Granting System deserves to become a perpetual legacy. From antiquity to the present, in the refinement of its approach to computation and verification there seems to be none that surpasses it."

[This appears to be a quotation, possibly from the original preface of Li Ch'ien. Although Li would have been expected to draw a conclusion of this kind, none appears in the final Evaluation as the *Yuan History* reproduces it. That is not surprising. Although Li was in charge of writing the evaluation—based on the rough draft originally submitted by the reformers—Kuo Shou-ching edited the final version, and the editors of the *Yuan history* further revised it.]

The canon of astronomy compiled by Hsu, Wang, and Kuo and the evaluation by Li still exist, and are reliable. Therefore we publish them here in their entirety. The Myriad Years system has not been passed down to our time. Although the Seventh-year Epoch system was never used as the official system, the documents that embody it survive. We have appended it so that scholars of the future may refer to it.

[As Chapter 6 has already made clear, the history's version of the Canon and Evaluation are greatly condensed. The wording in the second sentence *(chü chu yü p'ien* 具著于篇) does not mean what it seems to say, namely that the history reproduces the original texts in their entirety. Its summary version, the historians imply, adequately incorporates them in the historical record.]

Thus have we composed the "Treatise on Mathematical Astronomy." {52: 1119–20}

Evaluation of the Season-Granting System, Part 1

1. Determination of Ch'i from Observation

The Celestial Way in its cyclical motion is endless as a ring. As a starting point from which to establish their techniques, those who order the ephemerides attend to the transition between the waning and waxing of yin and yang [i.e., the solstices]. The motive agencies of yin and yang, of waning and waxing: from what may we perceive them? If only we observe the advance and recession of the gnomon's shadow, these motive agencies can no longer elude us. The means of observation is simply to erect a gnomon, measure its shadow, and from it determine the inceptions of the *ch'i* and solstices (section 1). The methods worked out by men of former ages—the wise who created them and the able who transmitted them—are, generally speaking, all we need. If only we can ponder with concentration and search with rigor, there comes a meeting of our minds with the inherent pattern of the phenomena (*hsin yü li hui* 心與理會). In this way, it is not impossible that we may go beyond what our predecessors have transmitted and created, and improve on them in some way.

The old method was to choose a place both level and broad, construct a water-level channel, and, using an inkline, erect in the center [of the place] a gnomon (*piao* 表) with which to measure the noonday shadow. But because the gnomon was short, measures finer than *ch'ih* and parts—decimal or fractional—were not easy to distinguish. If the gnomon is made longer, the divisions of the *ch'ih* too can be lengthened, but what then causes difficulty is that the shadow becomes empty [i.e., diffuse, *tan* 淡) and pale, so that it is impossible to read the actual [length of the] shadow. Our predecessors wanted to locate the center of the empty shadow in order to determine the real one. They sometimes set up a sighting-tube, sometimes erected a smaller gnomon [to supplement the large one], and sometimes made a compass of wood.

[I do not know what the last phrase refers to.]

In each case, they read the sunlight shining across the tip of the gnomon down onto the face of the template scale (*kuei-mien* 圭面).

Now the gnomon is bronze, 36 *ch'ih* [= 9m] in height, with [the top] divided [to support] two dragons at the [upper] end. They hold between them a horizontal beam [6c wide and 0.3c thick]; its distance to the face of the template is 40c. This is five times the length of the [classical] eight-*ch'ih* gnomon. The scale of the template is engraved with *ch'ih* and tenths. One of the old tenths is now enlarged to five of them, so that differences in small fractions of an inch are easier to distinguish.

[I write dimensions in *ch'ih* in the form "36c."]

[The astronomers] separately created a shadow aligner (*ying-fu* 景符) in order to obtain the real shadow. They made it of bronze *[or* copper] leaf, 0.2c broad, and twice longer than its width. In its center is pierced a hole the size of a needle or a mustard seed. A rectangular frame serves as a base. At the [lower] end there is a hinge *(chi-chou* 機 軸) that allows it to be opened and closed. One may [thus] prop up the shadow aligner so that it slants with the end toward the north higher than that toward the south [i.e., perpendicular to the sun's rays]. One moves the device back and forth within the diffuse shadow until the pinhole catches the sun's light, [forming an image] the size of a rice-grain or so, with the horizontal beam indistinctly visible in the middle of it.

The old methods involved observing the shadow of the end of the gnomon, so that what one obtained was the shadow cast by the upper edge of the body [i.e., the upper limb] of the sun. Now one takes it from the horizontal beam, so that one actually obtains the shadow of the sun's center [i.e. the silhouette of the beam]. There cannot be an iota of inaccuracy.

[See the discussion of this and other instruments in chapter 5 above, and consult figures 7 and 8. A more detailed description of the forty-*ch'ih* gnomon at the capital, incorporating a nearly identical account of the shadow aligner, appears in the "Treatise on Astrology." I translate it in appendix A. The two accounts differ in two trivial respects; in both, this version makes better sense.

Several authors have followed Joseph Needham's transcription of *ying-fu* as *ching-fu,* and his translation of it as "shadow definer" (1954–, 3: 299). That phrase describes the device's function, but does not translate its name.]

At the center of the world, the shadow of an eight-*ch'ih* gnomon is 13c and a fraction long at the winter solstice and 1.5c long at the summer solstice.

[On the center of the world, see p. 61.]

Now at the capital [i.e., Ta-tu, present Beijing], using the long gnomon, the winter solstice shadow is 79.8c and a fraction long, corresponding to 15.96c with the eight-*ch'ih* gnomon. The summer solstice shadow is 11.7c and a fraction long, corresponding to 2.34c with the eight-*ch'ih* gnomon. Although the [noon] shadow length varies with location, it is always true that when the shadow is longest it is the winter solstice, and when shortest it is the summer solstice. Still it is not an easy matter to determine the double-hour and mark [i.e. exact time of day] of the solstices and other *ch'i* transitions *(ch'i chih* 氣至). But it would seem that if the *ch'i* is correct for the solstices, the [24] *ch'i* transitions for the whole year will be correct.

[In the Canon, computing the exact time of the winter solstice (Subsection 1.1) is the basis for determining those of the remaining 23 *ch'i* (1.2, and section 3 for apparent motions).]

In the Liu Sung period, Tsu Ch'ung-chih 祖沖之 (#18, 463) measured shadows [of the same length] over the 23 or 24 days before and after the [approximate day of the] solstice, halved [the time interval], and took the midpoint as the winter solstice. He went on to compute the double-hour and mark of the solstice by comparing measurements of the Daily Difference *(jih-ch'a* 日差).

[See the commentary at the end of 1.1 below.]

In the Sovereign Protection era (1049–54) of the Sung period, it was Chou Ts'ung 周琮 (#67, 1064) who read the shadows at Enthronement of Winter and Enthronement of Spring [45 or 46 days before and after the solstice; Canon, 2.2]. He understood that because of their greater distance from the solstice and the somewhat increased Daily Difference, the computation became easier. Later astronomical systems from the Era Epoch system (#71) on were more elaborate in their techniques, but on the whole they did not depart from Tsu's method.

The new system is based on actual observations of noon shadows of the sun's center as days and months accumulate before and after [the winter solstice], pairing days both close to the solstice and far away for which the Daily Rate [i.e., daily change in shadow length] is similar, and studying both the coincidences and differences in length [by use of linear interpolation]. As a starting point, rather than depending on shadow lengths for a mere couple of days, the compilers took the preponderant numerical result [from many measurements] as the corrected one.

[Here I follow the interpretation, based on analyzing results, of Ch'en Meitung 2003b, 182, and his table 3-1.]

They thus were able to diminish by 19 marks and 20 parts [0.1920 day = 4h36m] the Great Enlightenment system['s prediction of the epochal winter solstice of 1280]. The new system goes on to determine double-hours and marks of the winter and summer solstice as given [in the Canon], based on precise Daily Difference measurements of the midday shadow of the sun's center accumulated over years of observation. {1120–22}

[Here and later this Evaluation tends to use technical terminology loosely. The Daily Difference (3.13.3) is the daily change in midday gnomon shadow length; "Daily Rate" is a term not found in the solar theory of the Canon. The author is using both terms to refer to the same procedure, interpolation of the kind exemplified in the next section—a procedure for which neither term is apposite. The improvement in the solstitial prediction over that of the Great Enlightenment system is also noted in the conclusion of subsection 2.1 below. On interpolation see Ch'ü An-ching et al. 2001a and 2001b.]

1.1. Computation of Winter Solstice, Year 14 of the Perfectly Great Period, S.Y. 14 (14 December 1277)

On the 14th of month 11 of that year, s.d. 36, the shadow was 79.4855c long. By the 21st, s.d. 43, the shadow was 79.541c long. On the 22d, s.d. 44, the shadow was 79.455c long. Comparing the shadows for s.d. 36 and 44, their difference was 0.0305c. This became the Shadow Difference *(kuei ch'a* 晷 差), and was shifted forward two columns [in the computing-rod columns]. Comparing the shadows for days 43 and 44, the remainder, 0.086c, became the divisor and was divided into the Difference, yielding 35 marks [i.e. 0.35 day]. This was subtracted from the Day Interval of 800 marks [i.e., 8 days' interval corresponding to the Shadow Difference], leaving a remainder of 765 marks. This was halved, and to the result was added the number of marks in half a day, making altogether 432½ marks. Simplifying [i.e. dividing] by 100 to give days, the result was four days. The remainder was multiplied by 12 and simplified by 100 to give double-hours, the result being 3 double-hours. Because a full 50 in the remainder is calculated as another double-hour, the sum was 4 double-hours. The remainder was gathered in [i.e. divided] by 12, yielding 3 marks. Counting off exclusively from s.d. 36, which began the interval, the result was that 3 marks in the beginning half of the 5th double-hour, s.d. 40, was the winter solstice of s.y. 14. This is a case of taking the shadows 4 days before and after the solstice.

[Christopher Cullen has remarked in private correspondence (email, 2002.4.2) that the most accurate translation of *suan wai* 算外 is “outside the count.” I agree with him, but use the less literal “count off exclusively” to make the term’s usage here more readily understandable.]

On the 9th of month 11, s.d. 31, the shadow was 78.6355c long. By the 26th, s.d. 48, the shadow was 78.7935c long. On the 27th, s.d. 49, the shadow was 78.55c long. Performing subtraction on the shadows for days 31 and 49, and again on the shadows for days 48 and 49, and solving by the method given above also yielded 3 marks in the beginning half of the first double-hour, s.d. 40. By the 28th, s.d. 50, the shadow was 78.3045c. Using the shadows for days 49 and 50 and that for day 31, and solving by the method given above, the result also agreed. This is a case of taking the shadows 8 and 9 days before and after the solstice.

On the new moon of month 11, s.d. 23, the shadow was 75.9865c. On day 2, s.d. 24, the shadow was 76.377c. By the 6th of month 12, s.d. 57, the shadow was 75.851c. Solving by the method given above, it was also at 3 marks in the beginning half of the first double-hour. This is a case of taking the shadows 17 days before and after the solstice.

On the 21st of month 10, s.d. 13, the shadow was 70.971c. By the 16th of month 12, s.d. 7, the shadow was 70.76c. On the 17th, s.d. 8, the shadow was 70.1565c. Solving by the method given above, the result was also 3 marks in the beginning half of the first double-hour. This is a case of taking the shadows 27 days before and after the solstice.

On the 5th of month 6, s.d. 60, the shadow was 13.08c. Passing on to the new moon of month 5, year 15, s.d. 20, the shadow was 13.0385c. On day 2, s.d. 21, the shadow was 12.9205c. Solving by the method given above, the result also agreed. This is a case of taking the shadows 160 days before and after the solstice. {1122–23}

[This set of five examples for the winter solstice of 1277 illustrates the use of measurements at varying intervals before and after the solstice for interpolation. See table 7.1 below for a series of gnomon shadow length measurements used for a year’s solstices, arranged in chronological order.

In each of the five examples, the problem and technique are the same. The problem is finding the exact time of the solstice from shadows that can be measured only at noon. Direct readings of noon shadow length cannot accurately determine the time, for the change from day to day very close to the solstice is practically imperceptible.

Kuo uses Tsu Ch'ung-chih's linear interpolation method, already mentioned in the text. The Shadow Difference is the difference in length between the first and last measurement. The Day Interval is the number of whole days between the two measurements, expressed in marks, units of 0.01 day. Shifting the counting rods forward converts to days, in preparation for division by the difference in shadow length for the two consecutive days (the Daily Difference). The result, after the subtraction and halving of the interval, yields days from the first measurement to the actual solstice. The rest of the procedure simply reduces the remainder first to double-hours and then, within the relevant hour, to marks.

Fig. 20. Determination of winter solstice by linear interpolation, adapted from Nakayama 1969, 124, figure 12.

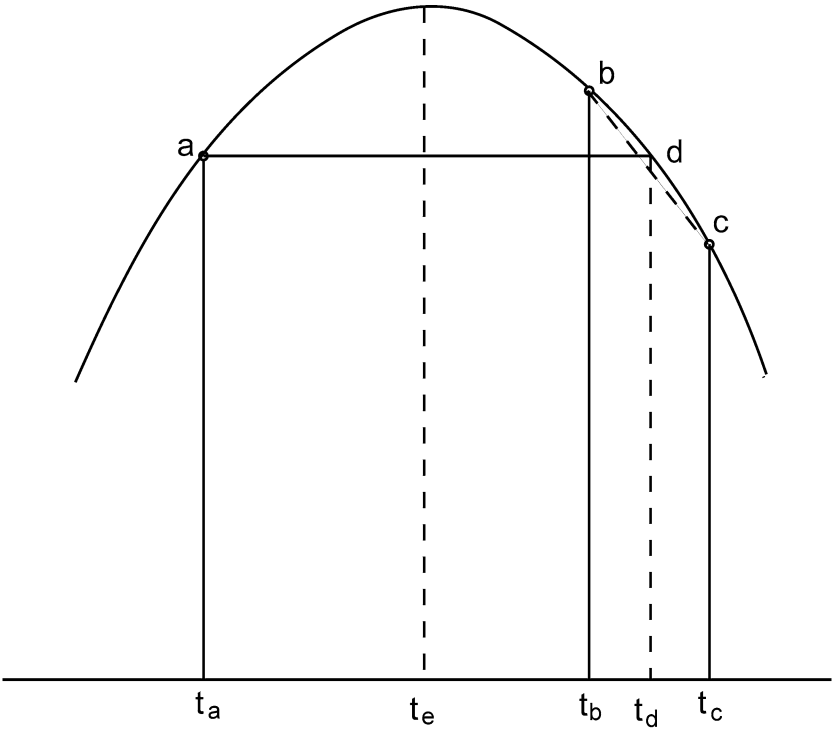


Figure 20 illustrates (in a graphic form that Yuan astronomers did not use) the reasoning behind the procedure. It plots shadow length *(a* to *d)* against date. For each determination, the astronomers used one noon shadow length before the solstice, *a,* and two after the solstice, or vice versa. Of the pair, one, *b,* would be longer than *a,* and one, *c,* shorter. The dotted lines show graphic equivalents of the interpolation. The astronomers first calculated t_d, when the shadow length after the solstice was the same as that at time *a.* This could be done with reasonable accuracy by interpolating along the line *bc* rather than the corresponding curve. Halfway between t_a and t_d lies the solstice at t_e.

The technique relies on two assumptions. Stated in modern terms, for periods as short as a day, at some distance from the solstice, the change in shadow length is in simple proportion to the change in time. This is a feasible approximation given the limited accuracy of the method. It also assumes, like all its predecessors, that the rate of change of shadow length is symmetrical about the solstice. Actually it is symmetrical about the solar perigee. Unknown to the astronomers, solstice and perigee nearly coincided in the period when observations for the Season-Granting system were gathered (the perigee was at 270°.55; see Nakayama 1969, 131, 133). This circumstance, coincident with the precision of the greatly enlarged Yuan gnomon, led to results of unprecedented accuracy.

The astronomers were reading shadow length to three decimal places, and were estimating a fourth decimal place as 0 or 5; that is, the precision is an optimistic 0.0005c (in modern terms, 0.1mm). This translation conveys the form in which they set down their observations. It was not customary in China to write zeros at the end of a number with decimal parts, so a shadow length recorded in this section as 79.1 would imply 79.1000c. Note that this precision differs from that of observations of other kinds, as well as from that of calculations. The latter were carried out to a precision of 0.0001c, ordinarily without regard to the precision of observation.

The text's claim that one can determine the solstice precisely from shadows measured as much as 160 days earlier or later is correct, within limits, only because accumulating error is offset by the rounding-off procedure. The astronomers, as usual, dropped the decimal parts and took the integer when they rounded off the quotient of Shadow Difference and Daily Difference. This is sufficient to offset an error which tends to make the answer high. For example, the result calculated from shadow lengths 27 days before and after the solstice, had 3.83 been rounded up to 4, would be 4 marks rather than 3.

But this technique is much more precise than it is accurate; a difference of several marks in various time determinations would be negligible. The use of interpolation and of a pinhole to define the precise point of shadow are difficult to evaluate mathematically, but data like those in table 7.2 at the end of section 1, which compares the given determinations of solstices with dates based on recent computations, suggest that the Yuan innovations improved accuracy at the summer solstice to roughly ±0.05 day, and at the winter solstice to an even smaller uncertainty.

The conversion of day remainder to double-hours and marks is as given in the Canon, section 2.4. The rule about counting an hour remainder of 50 or greater in the next hour comes about because, although the day begins at midnight, the first double-hour runs from 11 P.M. to 1 A.M. Thus an hour remainder of less than 50 (corresponding to a day remainder of less than 0.0416) falls in the second or "standard" *(cheng* 正) half of double-hour 1, and one between 50 and 100 falls in the first or "initial" *(ch'u* 初) half of the second double-hour.]

1.2. Computation of Summer Solstice, Year 15 of the Perfectly Great Period, S.Y. 15 (15 June 1278)

On the 19th of month 5, s.d. 38, the shadow was 11.7775c long. Passing on to the 28th, s.d. 47, the shadow was 11.78c long. On the 29th, s.d. 48, the shadow was 11.8055c long. Performing subtraction on the shadows for days 38 and 47, the remainder was 0.0025. Shifted forward two columns, it became the dividend. Performing subtraction on the shadows for days 47 and 48, the remainder, 0.0255, became the divisor. Upon dividing, 9 marks was the result. This was subtracted from the Day Interval of 900 marks, leaving a remainder of 891 marks. After halving, adding the marks in half a day, and simplifying by 100, the result was four days. Multiplying the remainder by 12 and simplifying by 100, the result was 11 double-hours. Gathering in the remainder by 12 to give marks, the result was 3 marks. Counting off exclusively from s.d. 38, which began the interval, the result was that s.d. 42, standard half of the 12th double-hour, 3 marks, was the summer solstice. This is a case of taking the shadows 4 days before and after the solstice.

On the 15th of month 12, year 14, s.d. 6, the shadow was 71.343c long. Passing on to day 2, month 11, year 15, s.d. 18, the shadow was 70.7595c long. On day 3, s.d. 19, the shadow was 71.406c long. The shadows for days 6 and 19 were subtracted and those for days 18 and 19 were subtracted, and the results were divided. This also agreed. This is a case of taking the sun's shadows 156 days before and after the solstice.

On the 12th of month 12, year 14, s.d. 3, the shadow was 72.9725c long. On the 13th, s.d. 4, the shadow was 72.4545c long. On the 14th, s.d. 5, the shadow was 71.909c long. Passing on to the 4th of month 11, year 15, s.d. 20, the shadow was 71.9575c long. On the 5th, s.d. 21, the shadow was 72.505c long. On the 6th, s.d. 22, the shadow was 73.0335c long. Taking [various combinations of] readings before and after the solstice, the times of day obtained all agreed. This is a case of taking the sun's shadows 158 and 159 days before and after the solstice.

On the 7th of month 12, year 14, s.d. 58, the shadow was 75.417c long. On the 8th, s.d. 59, the shadow was 74.9595c long. On the 9th, s.d. 60, the shadow was 74.486c long. Passing on to the 9th of month 11, year 15, s.d. 25, the shadow was 74.5205c long. On the 10th, s.d. 26, the shadow was 75.0035c long. On the 11th, s.d. 27, the shadow was 75.4495c long. The shadows for days 59 and 26 were subtracted to give the dividend, and the shadows for days 58 and 59 were subtracted to give the divisor, and the results were divided. Alternatively the shadows for days 59 and 60 were subtracted, and the shadows for days 25 and 26 or for days 26 and 27 were subtracted [to give the divisor]. Solving by the method given above, in every instance the result agreed. This is a case of taking the sun's shadows 163 and 164 days before and after the solstice. {1123–24}

[As in the previous case, there is some variation in the times computed using various readings, but the text does not note it. For instance, the figures for 159 days before and after the solstice, read on s.d. 3, 4, and 22, give 2 marks instead of 3 if rounded off as usual. In the first example, the single value before the solstice does not quite fall between the pair of values read after the solstice.

Why did that not matter to the author? He may have recognized that the difference between 11.7775 and 11.78c is negligible. On the other hand, he may have been aware that consecutive Daily Differences in shadow length are sufficiently close that the error introduced is actually insignificant. Even sizable rounding-off errors are ordinarily masked. For instance, in the first example, the quotient of 0.25 and 0.0255 was 9.8039. This was rounded off to 9, when the difference in the computed time of the solstice was only 0.4 mark.]

1.3. Computation of Winter Solstice, Year 15, S.Y. 15 (14 Dec 1278)

On the 19th of month 11 of that year, s.d. 35, the shadow was 78.3185c long. Passing on to the 9th of intercalary month 11, s.d. 55, the shadow was 78.3635c long. On the 10th, s.d. 56, the shadow was 78.0825c long. Taking the shadows for days 35 and 55 and subtracting, the remainder was 0.045c. This became the Shadow Difference, and was shifted forward two columns. Taking the shadows for days 55 and 56 and subtracting, the remainder, 0.281c, became the divisor. Division yielded 16 marks. This was added to the Day Interval of 2000 marks, the sum halved, the marks in half a day added, and the result simplified by 100. The result was 10 days. The remainder was multiplied by 12 and simplified by 100 to give double-hours. The full 50 in the remainder was advanced into the next double-hour, so that the sum was 7 double-hours. The remainder was gathered in by 12 to give [3] marks. Counting off exclusively from s.d. 35, which began the interval, the result was that s.d. 45, beginning half of double-hour 8, 3 marks, was the winter solstice of s.y. 15. This is a case of taking the shadows 10 days before and after the solstice.

On the 12th of month 11, s.d. 28, the shadow was 75.8815c long. On the 13th, s.d. 29, the shadow was 76.3015c long. On the 15th of intercalary month 11, s.d. 1, the shadow was 76.3665c long. On the 16th, s.d. 2, the shadow was 75.953c long. On the 17th, s.d. 3, the shadow was 75.5045c long. The shadows for days 29 and 1 were subtracted to give the dividend, and the shadows for days 28 and 29 were subtracted to give the divisor. They were divided, also yielding s.d. 45, beginning half of double-hour 8, 3 marks. Alternatively the shadows for days 1 and 2 were subtracted [to give the divisor], and this computation also agreed. When the shadows for days 28 and 2 were subtracted to give the dividend and the shadows for days 2 and 3 were subtracted to give the divisor, division gave the same result. This is a case of taking the shadows 16 and 17 days before and after the solstice.

On the 8th of month 11, s.d. 24, the shadow was 74.0375c long. On the 20th of intercalary month 11, s.d. 6, the shadow was 74.12c long. On the 21st, s.d. 7, the shadow was 73.6145c long. The shadows for days 24 and 6 were subtracted to give the dividend, and the shadows for days 6 and 7 were subtracted and the remainders divided, also giving the same result. This is a case of taking the shadows 21 days before and after the solstice.

On the 26th of month 6, s.d. 15, the shadow was 14.4525c long. On the 27th, s.d. 16, the shadow was 14.638c long. By the 2d of month 4, year 16, s.d. 15, the shadow was 14.481c long. Subtracting the shadows for the two days 15, and subtracting the shadows for the later day 15 and day 16, computation gave the same result. This is a case of taking the shadows 150 days before and after the solstice.

On the 28th of month 5, s.d. 47, the shadow was 11.78c long. By the 29th of month 4, year 16, s.d. 42, the shadow was 11.863c long. On the 30th, s.d. 43, the shadow was 11.783. Subtracting the shadows for days 47 and 43, and subtracting the shadows for days 42 and 43, computation also gave the same result. This is a case of taking the shadows 178 days before and after the solstice. {1125–26}

[The next-to-last sentence of the first paragraph of this section gives the sexagenary day erroneously as 36 *(chi-hai* 己 亥) instead of 35 *(wu-hsu* 戊 戌); I have corrected it in the translation. Since this cannot be due to confusing either graphic forms nor sounds, it is likely that the compiler, while copying the stereotyped form of this report from the first record in the series (Sec. 1.1), inadvertently copied the day given there as well.]

1.4. Computation of Summer Solstice, Year 16, S.Y. 16 (15 Jun 1279)

On the 19th of month 4, s.d. 32, the shadow was 12.3695c long. On the 20th, s.d. 33, the shadow was 12.2935c long. By the 19th of month 5, s.d. 2, the shadow was 12.264c long. Taking the shadows for days 33 and 2 and subtracting, the remainder was 0.0295c. This became the Shadow Difference, and was shifted forward two columns. Taking the shadows for days 32 and 33 and subtracting, the remainder, 0.076c, became the divisor. Division yielded 38 marks. This was added to the Day Interval of 2900 marks, the sum halved, the marks in half a day added, and the result simplified by 100. The result was 15 days. The remainder was multiplied by 12 and simplified by 100 to give 2 double-hours. The remainder was gathered in by 12 to give 2 marks. Counting off exclusively from s.d. 33 which began the interval, the result was that s.d. 48, standard half of double-hour 3, 2 marks was the summer solstice. This is a case of taking the shadows 15 days before and after the solstice.

On the 21st of month 3, s.d. 5, the shadow was 16.3905c long. On the 16th of month 6, s.d. 29, the shadow was 16.0995c long. On the 17th, s.d. 30, the shadow was 16.311c long. The shadows for days 5 and 30 were subtracted, and the shadows for days 29 and 30 were subtracted. Computing by the method given above gave the same result. This is a case of taking the shadows 42 days before and after the solstice.

On the 2d of month 3, s.d. 46, the shadow was 21.305c long. By the 7th of month 7, s.d. 49, the shadow was 21.1955c long. On the 8th, s.d. 50, the shadow was 21.4865c long. The shadows for days 46 and 49 were subtracted, and the shadows for days 49 and 50 were subtracted. Computing by the method given above gave the same result. This is a case of taking the shadows 61 and 62 days before and after the solstice.

On the new moon of month 3, s.d. 45, the shadow was 21.611c long. By the 8th of month 7, s.d. 50, the shadow was 21.4865c long. On the 9th, s.d. 51, the shadow was 21.9155c long. The shadows for days 45 and 50 were subtracted, and the shadows for days 50 and 51 were subtracted. Computing by the method given above gave the same result. This is a case of taking the shadows 62 and 63 days before and after the solstice.

On the 18th of month 2, s.d. 32, the shadow was 26.0345c long. By the 21st of month 7, s.d. 3, the shadow was 25.899c long. On the 22d, s.d. 4, the shadow was 26.259c long. The shadows for days 32 and 3 were subtracted, and the shadows for days 3 and 4 were subtracted. Computing by the method given above gave the same result. This is a case of taking the shadows 75 and 76 days before and after the solstice.

On the 3d of month 2, s.d. 17, the shadow was 32.1955c long. By the fifth of month 8, s.d. 17, the shadow was 31.5965c long. On the sixth, s.d. 18, the shadow was 32.0265c long. The shadows for the earlier day 17 and for day 18 were subtracted, and the shadows for the later day 17 and for day 18 were subtracted. Computing by the method given above gave the same result. This is a case of taking the shadows 90 days before and after the solstice.

On the 19th of the standard [i.e. 1st] month, s.d. 4, the shadow was 38.5015c long. By the 18th of month 8, s.d. 30, the shadow was 37.823c long. On the 19th, s.d. 31, the shadow was 38.3105c long. The shadows for days 4 and 31 were subtracted, and the shadows for days 30 and 31 were compared [i.e., subtracted]. Computing by the above method gave the same result. This is a case of taking the shadows 103 and 104 days before and after the solstice. {1126–27}

1.5. Computation of Winter Solstice, Year 16, S.Y. 16 (14 Dec 1279)

On the 24th of month 10, s.d. 35, the shadow was 76.74c long. By the 25th of month 11, s.d. 6, the shadow was 76.58c long. On the 26th, s.d. 7, the shadow was 76.1425c long. Taking the shadows for days 35 and 6 and subtracting, the remainder was 0.16c. This became the Shadow Difference, and was shifted forward two columns. Taking the shadows for days 6 and 7 and subtracting, the remainder, 0.4375c, became the divisor. Division yielded 36 marks. This was subtracted from the Day Interval of 3100 marks, the remainder, 3064 marks, halved, 50 marks added, and [the result] simplified by 100. The result was 15 days. The remainder was multiplied by 12 and simplified by 100 to give double-hours. The full 50 in the remainder was advanced into the next double-hour, and the sum was 10 double-hours. The remainder was gathered in by 12 to give marks, yielding 2 marks. Counting off exclusively from s.d. 35 which began the interval, the result was that s.d. 50, beginning half of double-hour 11, 2 marks, was the winter solstice. This is a case of taking the shadows 15 and 16 days before and after the solstice.

On the 18th of month 10, s.d. 29, the shadow was 74.0525c long. On the 19th, s.d. 30, the shadow was 74.545c long. On the 20th, s.d. 31, the shadow was 75.025c long. By the 28th of month 11, s.d. 9, the shadow was 75.32c long. On the 29th, s.d. 10, the shadow was 74.8525c long. On the new moon of month 12, s.d. 11, the shadow was 74.365c long. On the second, s.d. 12, the shadow was 73.8715c long. The shadows for days 31 and 10 were subtracted, and the shadows for days 30 and 31 were subtracted. Computing as above gave the same result. If the shadows for days 9 and 10 are subtracted to give the divisor, computing gives the same result. This is a case of taking the shadows 18 and 19 days before and after the solstice.

If the shadows for days 30 and 11 are subtracted, and the shadows for days 29 and 30 are subtracted, and the computation carried out; or if the shadows for days 30 and 31 are subtracted and the computation carried out; or if the shadows for days 11 and 10 are subtracted and the computation carried out; or if the shadows for days 11 and 12 are subtracted and the computation carried out; or if the shadows for days 29 and 12 are subtracted, and the shadows for days 29 and 30 are subtracted, and the computation carried out, the result is the same. This is a case of taking the shadows 20 days before and after the solstice.

On the 16th of month 10, s.d. 27, the shadow was 73.015c long. On the 3d of month 12, s.d. 13, the shadow was 73.32c long. On the 4th, s.d. 14, the shadow was 72.8425c long. The shadows for days 27 and 14 were subtracted, and the shadows for days 13 and 14 were subtracted. Computing gave the same result. This is a case of taking the shadows 23 days before and after the solstice.

On the 14th of month 10, s.d. 25, the shadow was 71.9225c long. On the 15th, s.d. 26, the shadow was 72.469c long. On the 5th of month 12, s.d. 15, the shadow was 72.2725c long. When the shadows for days 26 and 15 were subtracted, and the shadows for days 25 and 26 were subtracted, and the computation carried out; or when the shadows for days 26 and 27 were subtracted and the computation carried out, the result was the same. This is a case of taking the shadows 24 days before and after the solstice.

On the 7th of month 10, s.d. 18, the shadow was 67.745c long. On the 8th, s.d. 19, the shadow was 68.3725c long. On the 9th, s.d. 20, the shadow was 68.9775c long. On the 12th of month 12, s.d. 22, the shadow was 68.145c long. When the shadows for days 19 and 22 were subtracted, and the shadows for days 18 and 19 were subtracted, and the computation carried out; or when the shadows for days 19 and 20 were subtracted, and the computation carried out, the result was the same. This is a case of taking the shadows 31 and 32 days before and after the solstice.

On the new moon of month 10, s.d. 12, the shadow was 63.87c long. On the 18th of month 12, s.d. 28, the shadow was 64.2975c long. On the 19th, s.d. 29, the shadow was 63.625c long. The shadows for days 12 and 29 were subtracted, and the shadows for days 28 and 29 were subtracted. Computing gave the same result. This is a case of taking the sun's shadows 38 days before and after the solstice.

On the 22d of month 9, s.d. 3, the shadow was 57.825c long. On the 28th of month 12, s.d. 38, the shadow was 57.58c long. On the 29th, s.d. 39, the shadow was 56.915c long. The shadows for days 3 and 38 were subtracted, and the shadows for days 38 and 39 were subtracted. Computing gave the same result. This is a case of taking the sun's shadows 47 and 48 days before and after the solstice.

On the 20th of month 9, s.d. 1, the shadow was 56.4925c long. By the 29th of month 12, s.d. 39, the shadow was 56.915c long. By the new moon of the standard month of year 17, s.d. 40 [i.e., the next day], the shadow was 56.25c long. [The shadows for] days 1 and 40 were subtracted, and the shadows for days 39 and 40 were subtracted. Computing gave the same result. This is a case of taking the sun's shadows 50 days before and after the solstice.

Based on the above computations of the double-hours and marks of the winter and summer solstices for successive years, the astronomers concluded that the winter solstice preceding year 18 of the Perfectly Great period, s.y. 18, must have fallen 6 marks after midnight on s.d. 56; that is, in the beginning half of the 2d double-hour, 1 mark. (0129/0143h 14 Dec 1280). {1128–30}

[The last paragraph of this section registers, without data, a computation of the Season-Granting System's epochal winter solstice on the basis of the observations in this section. Accurate prediction of solstices was not a matter of difficulty, even though the precision of observed shadow lengths fell well below the recorded 0.0001 *ch'ih*. Chang Pei-yü's modern computation of the solstice (1997) puts the solstice at 0149h, roughly 8 marks after midnight.

Table 7.1 gathers all of the gnomon shadow measurements for a month before and after the winter solstice of December 1277 (s.y. 14), and arranges them in chronological order, to serve as the basis of figure 21. I have added to the table for comparison a series of measurements shortly before and after the summer solstice of June 1278 (s.y. 15). I also give the average daily increment of shadow length for each pair of measurements. It is obvious that the daily change is minimal in the vicinity of both solstices, so that interpolation is necessary. Table 7.2 compares solstitial dates given in this section with those computed by the method of the Canon and those computed by modern methods in Chang 1997.]

Table 7.1. Shadow Lengths

Year	Month	Day	S. D.	Shadow, ch'ih	Increment/day	Event
14	10	21	13	70.971	-	-
14	11	1	23	75.9865	+.502	-
14	11	2	24	76.377	+.391	-
14	11	9	31	78.6355	+.323	-
14	11	14	36	79.4855	+.170	-
14	11	18	40	-	-	Winter solstice
14	11	21	43	79.541	-	-
14	11	22	44	79.455	−.086	-
14	11	26	48	78.7935	−.165	-
14	11	27	49	78.55	−.243	-
14	11	28	50	78.3045	−.246	-
14	12	6	57	75.851	−.351	-
14	12	7	58	75.417	−.434	-
14	12	8	59	74.9595	−.458	-
14	12	9	60	74.486	−.474	-
14	12	12	3	72.9725	−.505	-
14	12	13	4	72.4545	−.518	-
14	12	14	5	71.909	−.546	-
14	12	15	6	71.343	−.566	-
14	12	16	7	70.76	−.583	-
14	12	17	8	70.1565	−.604	-
15	5	1	20	13.0385	-	-
15	5	2	21	12.9205	−.118	-

15	5	19	38	11.7775	−.067	-
15	5	23	42	-	-	Summer solstice
15	5	28	47	11.78	-	-
15	5	29	48	11.8055	+.026	-

Table 7.2. Comparison of Solstice Determinations

Solstice	Given in Sec. 1.1 (month, day, marks)	Computed using Canon	Computed, modern (Chang 1993)
Winter, s.y. 14 (1277)	11.18.32	11.18.33	11.18.35
Summer, s.y. 15 (1278)	5.23.95	5.23.95	5.23.93
Winter, s.y. 15	11.29.57	11.29.575	11.29.59
Summer, s.y. 16 (1279)	5.5.19	5.5.19	5.5.18
Winter, s.y. 16	11.9.81	11.9.81	11.9.83

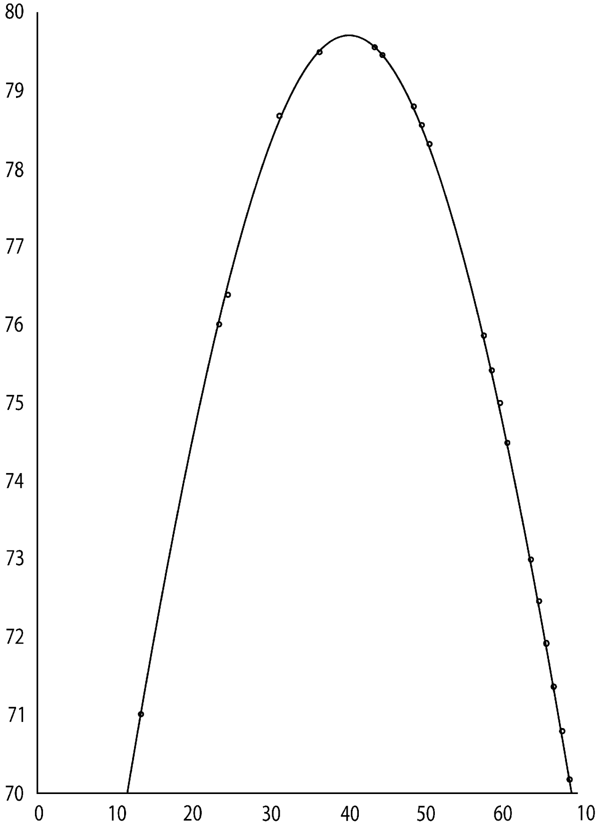
2. Year Surplus and Annual Difference

The *tu* in the round of the sky and the days in the round of the year both number 365. In addition to this integral factor *(ch'üan ts'e* 全 策) there is an odd fraction, in both instances roughly one quarter. Passing from this year's winter solstice to that of next year, over 365 days the sun moves through one circuit. Four circuits take 1460 days and accumulate a surplus of 1 day; splitting this into four gives ¼. But the sky fraction is always in excess of this, and the year fraction never amounts to this. There is something in these quantities which cannot be reconciled, but the discrepancy is extremely subtle.

[The "sky fraction" is the fraction of a day above 365 that it takes the sun to return to a given position among the stars. It is thus the fractional part of the Celestial Perimeter *(chou-t'ien,* Canon, 3C.1). In Occidental terms, it is the sidereal year fraction. The "year fraction" (an informal term for the Year Surplus) is the corresponding difference between two solstices, the fractional part of the Year Cycle (1C.6), or the tropical year fraction. The Canon uses not the Year Surplus but the Series Surplus (1.C3) of 52 425 parts, amounting to 5.2425 days. The Yuan astronomers expressly made the two remainders symmetrical about the fraction ¼, a matter the author explains below.

Although in the commentary below I speak of tropical and sidereal years, that terminology is merely a help to the comprehension of modern readers. The discussion here is about fractional parts of constants, not lengths of years.]

Fig. 21. Observed gnomon shadow lengths before and after the winter solstice of December 1277, used for linear interpolation in 1.1.



Our predecessors were at first not aware of this. At the end of the Han, Liu Hung 劉洪 was the first to realize that [predicted] winter solstices were retarded with respect to the celestial phenomena. Declaring that the Year Surplus fraction was too large, he composed the Supernal Emblem system (#5, A.D. 206), in which he reduced the Year Surplus fraction [i.e. the fractional part of the tropical year constant], 2500, to 2462. In the Tsin, Yü Hsi 虞喜 (early fourth century), and in the Sung, Ho Ch'eng-t'ien 何承天 (#17, 443) and Tsu Ch'ung-chih (#18, 463), asserted that there must be a factor for the difference between the [fractional parts for the sidereal and tropical] years. They created the Annual Difference technique.

This method involved diminishing the annual remainder and adding to the round of the sky, so that as the Year Surplus gradually becomes less, the Celestial Perimeter [i.e., the sidereal cycle] gradually becomes greater [than a fixed average value by equal amounts]. By subtracting greater and less one obtains the difference which corresponds to the annual recession [with respect to the stars] of the sun's [solstitial] position. The Year Surplus and Celestial Perimeter were thus truly coordinated in function. From then on the Annual Difference was established and the method for corrected solar motion successful. But once [the value of the Annual Difference] becomes greater or less than the correct value, how can one hope to obtain agreement with the celestial phenomena?

[On the Annual Difference, see the orientation, p. 99.

I have added table 7.3 to indicate variations in tropical year length based on records of solstices.]

Table 7.3. Determinations of Tropical Year Length from Historical Records of Solstices

[Labels:

A	sexagenary day and marks of the winter solstice
B	elapsed days since the preceding entry
C	elapsed calendar years
D	tropical year length = B/C]

Observer	Year	System	A	B	C	D
Tsu Ch'ung-chih	462	18	27.55	-	-	-
Liu Cho	603	34	46.54	51498.99	141	365.2410
I-hsing	727	42	36.72	45290.18	124	365.2433
Chou Tsung	1049	67	45.02	117608.30	322	365.2431
Yao Shun-fu	1099	70	7.16	18262.14	50	365.2428
Kuo Shou-ching?	1278	88	45.58	65378.42	179	365.2425

[Takebe Katahiro, "Jujireki kaigi 授時曆解議," 1*:* 32b-33a, has proposed the determinations listed in this table as the six mentioned but not specified in the text. That is not likely—the text mentions the interval between the first and last items as a seventh instance, not the sixth—but these at least illustrate the trend. He has confused the sexagenary ordinal for the year with that for Column A in the first case, but I have corrected the error above. The values in Column D reflect the practice of dropping remainders rather than rounding them off to the nearest number; or, to put it operationally, of carrying the division out only to the number of decimal places wanted. All except the fourth tropical year value would be one digit higher in the last significant figure if rounded off according to modern practice. The last value, for instance, becomes 365.242 569 8 if the division is continued. The last four items in Column D give a rough idea of the tendency from the T'ang period on to decrease the tropical year length.]

Since s.y. 39 of the Great Enlightenment period of the Liu Sung era (462) there have been six instances of "measuring the shadow and verifying the *ch'i*" in order to derive true values *(chen shu* 真數) for the hours and marks of winter solstices. By taking days, double-hours, and marks elapsed in the interval [between each such determination and the epochal Yuan solstice] and dividing by years in the interval, [the authors of the Season-Granting system] obtained in each case the Year Surplus in use at the time [of the first solstice in each pair]. They further divided days, double-hours, and marks elapsed between [the winter solstices of] s.y. 39 of the Great Enlightenment period (462) and s.y. 15 of the Perfectly Great period (1278) by the interval in years. The result was that each year contains 365.2425 days. This was a reduction of .0011 day from the value used in the Great Enlightenment system (#76). They designated for current use [the fractional part] the Year Surplus. They added the remainder of 0.0075 to the nominal ¼, designating the total, $365^{t}.2575$, the Celestial Perimeter. Subtracting the positive and negative fractions [of the Celestial Perimeter and Year Surplus], they divided the result, 0.0150 day, into an integral *tu*, giving 66 and a fraction years in which the sun moves backward one *tu*. If this 66 years is divided into an integral *tu*, one obtains just 0.0150 day, which they designated the Annual Difference.

[Several scholars over the past 400 years have identified a problem in this passage. The interval between the solstice of 1278, in the 12th month, 14.58 days, and that of 452, in the 12th month, 20.56 days, when divided by the interval of 816 years, does not yield the value given for the tropical year, but rather 365.2427 days. Ch'en Mei-tung 陈美东 1995, 229–31, has given the best solution that the ambiguous wording and the scanty evidence permit. He argues that the astronomers used the intervals between the six unspecified solstice determinations and the 1280 solstice, and further used the interval from the 462 solstice to that of 1280, yielding seven values. They obtained the final value either by averaging or, perhaps more likely, by taking the most frequent value. I follow this analysis in resolving the ambiguity of the passage.

In the Great Enlightenment system, the Year Surplus amounted to 0.2536 day; see *Chin shih,* 21: 442–43. Note that the period of backward motion, 66.67 years per *tu,* is rounded downward.]

They next checked the Annual Difference against the meridian transit recorded in the "Institutes of King Yao." At the winter solstice the sun was then at the junction of the lunar lodges Servingmaid and Tumulus. They also checked [the value] against the early histories. At the winter solstice of year 2 of the Epochal Concord era of the Han (A.D. 85), the sun was 21^{t} in Southern Dipper. In year 9 of the Grand Epoch era of the Tsin (384), it had receded to 17^{t} in Southern Dipper. In year 10 of the Epochal Excellence era of the [Liu] Sung (433), it was at the end of the 14th *tu* in Southern Dipper. In year 10 of the Great Unity era of the Liang (544) it was at 12^{t} in Southern Dipper. In year 18 of the Opening Sovereignty era of the Sui (598) it was still at 12^{t} in Southern Dipper. In year 12 of the Opening Epoch era of the T'ang (724) it was at 9½^{t} in Southern Dipper. At present it has receded to 10^{t} in Winnowing-basket. Taking the number of years in the interval to the present and the number of *tu* in the interval to the current location and comparing them, there is a maximum of 70+ years and a minimum of not less than fifty years in which the difference amounts to one *tu*.

[The "Institutes of King Yao" is a chapter of *Documents of Antiquity (Shang shu* "Yao tien" 尚書 堯典). Before modern times classicists believed it to be archaic, but the most critical now consider it an addition of the mid third century B.C. (e.g., Loewe 1993, 377–78). Imprecise observations recorded in the early histories are responsible for the variation, but the evaluator would not have understood it that way. On this meridian transit see the Canon, 3.3.

A modern value of the Annual Difference Rate for the year 1280 would be 70.94 tropical years per *tu* (71.93 years per degree), corresponding to 50".1188 or 0.0141 *tu* per year (1°23' or 1^{t}.41 per century).]

In the Felicitous Epoch period of the Sung, in the Concord with Heaven astronomical reform (#80, 1199), they took the Annual Difference Rate of the Great Expansion system (#42), 82 years, and the rate [computed] for the interval [from the solstitial point for the] Opening Epoch period, 55 years, and halved [their sum], taking the mean value. They obtained 67 years as the difference constant for a solar retrogression of 1 *tu*. Applied today, tested against the Way of Heaven [i.e., the phenomena], it is truly precise.

[The text is asserting that the compilers of the Concord with Heaven system attained a correct figure for what is now called the precessional constant by an arbitrary procedure, averaging the value given in an early eighth-century system with a value computed for the interval from then to the time of compilation. The actual result, 68.5 years per *tu,* is not very close to the Season-granting system's value of 66⅔. Rationalizations of this sort are far from rare in the history of Chinese astronomy. The Canon's approach was more rigorous, but the evaluator tended more than his colleagues to appreciate adventitious but useful results.]

Even so, when we consider astronomical systems past and present, we see that what agrees with [the phenomena of] the present cannot correspond perfectly to those of the past; what is precise with respect to antiquity we cannot confirm by observing today. Now with the Season-Granting system, when investigating past events one adds to the Year Surplus and diminishes the Annual Difference; when computing future events, one adds to the Annual Difference and diminishes the Year Surplus. When computing backward to determine winter solstices since the Springs and Autumns period, again and again [cross-checks] agree; for computing forward to determine future [solstices], this solar theory can remain faultless forever. This is not merely accuracy for the present.

[This paragraph refers to the secular variation in the sidereal and tropical year lengths discussed in an author's note at the beginning of the Canon.]

The exactitude (*shu-mi* 疏密) of 49 determinations of the winter solstice from the Springs and Autumns period on, according to six astronomical systems—the Great Expansion system and others—is set out below. {1130–32}

2.1. Winter Solstice Marks

[Because the author sets out what follows in a table, I present it as table 7.4, adding the results of modern computation. The original table compares computations of day and mark to the nearest mark (0.01 day) given by six important systems for 48 winter solstices. I have numbered each prediction. Since the Evaluation states before and after table 7.4 that there are 49 solstices, one record must have been lost or removed before the treatise was incorporated in the *Yuan History*.

The headings are:

Event. I give each event as recorded, abbreviated as follows: longest shadow, l.s.; sun furthest south, s.f.s.; winter solstice, w.s. Despite this varying terminology for events, the text that follows the table calls all of them winter solstices.

The six systems are recorded by the number assigned each one in table 2.1:

#42. Great Expansion (A.D. 728)

#50. Extending Enlightenment (822)

#71. Era Epoch (1106)

#80. Concord with Heaven (1199)

#76. Great Enlightenment (Revised, 1180)

#88. Season-Granting (1280)

These are not the same systems as those given in table 7.3, which begins in 462. It is based on independent shadow length observations.

The additional information in table 7.4 is:

A. The corresponding date in the Julian calendar.

B. Sexagenary day and marks, by modern computation. The day is from Chang 1997, and marks is converted from the time in item C.

C. Date and time of the solstice, by modern computation of Chinese standard time, from Chang. The numbers of the notes (in parentheses) are the last item in this column.

The table records the epochal year *(yuan-nien* 元 年) simply as “year 1” and marks, where *k’ung* 空 occurs for them, as “00.”]

Table 7.4. Winter Solstice Marks

Event							A
#42	**#50**	**#71**	**#80**	**#76**	**#88**	**B**	**C**
1. W.s., Duke Hsien, year 15, s.y. 15, month 1, new moon, s.d. 51							27 Dec 884 B.C.
53.22	52.88	54.33	52.02	54.35	51.99	55.50	29 Dec 1154h
2. W.s., Duke Hsi, year 5, s.y. 3, month 1, new moon, s.d. 48							26 Dec 656
48.94	48.66	49.74	48.27	49.89	48.14	50.91	27 Dec 2145h
3. W.s., Duke Chao, year 20, s.y. 16, month 1, new moon, s.d. 26							26 Dec 523
26.45	26.20	27.25	25.92	27.29	25.83	28.23	27 Dec 0533h
4. L.s., Liu Sung pd., Epochal Excellence era, year 12, s.y. 12, month 11, day 15, s.d. 5							20 Dec A.D. 435
5.35	5.32	5.39	5.51	5.41	5.47	5.81	20 Dec 1933h
5. L.s., Epochal Excellence era, year 13, s.y. 13, month 11, day 26, s.d. 11							20 Dec 436
10.59	10.57	10.63	10.75	10.65	10.71	11.06	20 Dec 0127h
6. L.s., Epochal Excellence era, year 15, s.y. 15, month 11, day 18, s.d. 21							20 Dec 438
21.08	21.06	21.12	21.24	21.14	21.19	21.54	20 Dec 1304h
7. L.s., Epochal Excellence era, year 16, s.y. 16, month 10, day 29, s.d. 26							20 Dec 439
26.33	26.30	26.37	26.48	26.37	26.44	26.78	20 Dec 1850h
8. L.s., Epochal Excellence era, year 17, s.y. 17, month 11, day 10, s.d. 31							19 Dec 440
31.57	31.55	31.61	31.72	31.63	31.68	32.03	20 Dec 0038h
9. L.s., Epochal Excellence era, year 18, s.y. 18, month 11, day 21, s.d. 36							19 Dec 441
36.82	36.79	36.85	36.97	36.87	36.93	37.27	20 Dec 0631h
10. L.s., Epochal Excellence era, year 19, s.y. 19, month 11, day 3, s.d. 42							20 Dec 442
42.06	42.04	42.10	42.21	42.11	42.17	42.51	20 Dec 1216h
11. W.s., Great Enlightenment era, year 5, s.y. 38, month 11, s.d. 22							20 Dec 461
21.70	21.68	21.73	21.89	21.74	21.79	22.13	20 Dec 0301h

Event							**A**
#42	**#50**	**#71**	**#80**	**#76**	**#88**	**B**	**C**
12. L.s., Ch’en pd., Celestial Excellence era, year 6, s.y. 22, month 11, s.d. 27							19 Dec 565
27.12	27.13	27.05	27.24	27.08	27.17	27.40	19 Dec 0935h
13. L.s., Radiant Greatness era, year 2, s.y. 25, month 11, s.d. 42							18 Dec 568
42.80	42.86	42.79	42.97	42.81	42.90	43.13	19 Dec 0308h
14. L.s., Grand Establishment era, year 4, s.y. 29, month 11, day 29, s.d. 4							19 Dec 572
3.83	3.78	3.77	3.95	3.98	3.87	4.10	19 Dec 0223h
15. L.s., Grand Establishment era, year 6, s.y. 31, month 11, day 20, s.d. 14							18 Dec 574
14.32	14.33	14.25	14.43	14.27	14.36	14.59	19 Dec 1404h (1)
16. L.s., Grand Establishment era, year 9, s.y. 34, month 11, day 23, s.d. 29							18 Dec 577
30.04	30.06	29.99	30.16	30.00	30.08	30.31	19 Dec 0729h
17. L.s., Grand Establishment era, year 10, s.y. 35, month 11, day 5, s.d. 35							19 Dec 578
35.30	35.30	35.23	35.40	35.24	35.33	35.55	19 Dec 1312h
18. L.s., Sui pd., Opening Sovereignty era, year 4, s.y. 41, month 11, day 11, s.d. 6							18 Dec 584
6.77	6.78	6.69	6.86	6.71	6.86	7.01	19 Dec 0020h (2)
19. L.s., Opening Sovereignty era, year 5, s.y. 42, month 11, day 22, s.d. 12							18 Dec 585
12.01	12.02	11.92	12.11	11.55	12.10	12.26	19 Dec 0620h (3)
20. L.s., Opening Sovereignty era, year 6, s.y. 43, month 11, day 3, s.d. 17							18 Dec 586
17.25	17.26	17.18	17.34	17.19	17.34	17.50	19 Dec 1207h
21. L.s., Opening Sovereignty era, year 7, s.y. 44, month 11, day 14, s.d. 22							19 Dec 587
22.50	22.51	22.42	22.59	22.44	22.59	22.75	19 Dec 1758h
22. L.s., Opening Sovereignty era, year 11, s.y. 48, month 11, day 28, s.d. 43							19 Dec 591
43.48	43.49	43.43	43.57	43.41	43.56	43.72	19 Dec 1711h
23. W.s., Opening Sovereignty era, year 14, s.y. 51, month 11, new moon, s.d. 58							18 Dec 594
59.21	59.22	59.13	59.30	59.14	59.29	59.43	19 Dec 1025h

Event							A
#42	#50	#71	#80	#76	#88	B	C
24. L.s., T'ang pd., Constant Contemplation era, year 18, s.y. 41, month 11, s.d. 22							19 Dec 644
21.43	21.45	21.31	21.50	21.32	21.44	21.60	18 Dec 1420h
25. L.s., Constant Contemplation era, year 23, s.y. 46 , month 11, s.d. 48							19 Dec 649
47.65	47.68	47.53	47.72	47.54	47.66	47.80	18 Dec 1913h
26. L.s., Dragon Conjunction era, year 2, s.y. 59, month 11, day 4, s.d. 55/56							19 Dec 662
55.83	55.86	55.69	55.88	55.71	55.82	55.97	18 Dec 2313h
27. L.s., Paradigmatic Phoenix era, year 1, s.y. 13, month 11, s.d. 9							18 Dec 676
9.25	9.28	9.10	9.28	9.12	9.22	9.36	18 Dec 0845h
28. L.s., Perpetual Simplicity era, year 1, s.y. 19, month 11, s.d. 40							18 Dec 682
40.72	40.75	40.57	40.76	40.58	40.68	40.82	18 Dec 1947h
29. L.s., Opening Epoch era, year 10, s.y. 59, month 11, s.d. 10							18 Dec 722
10.49	10.54	10.31	10.50	10.32	10.46	10.54	18 Dec 1252h
30. L.s., Opening Epoch era, year 11, s.y. 60, month 11, s.d. 15							18 Dec 723
15.74	15.77	15.55	15.74	15.56	15.70	15.78	18 Dec 1841h
31. W.s., Opening Epoch era, year 12, s.y. 1, month 11, s.d. 20							17 Dec 724
20.98	21.03	20.80	20.99	20.81	20.95	21.02	18 Dec 0031h
32. S.f.s., Sung pd., Luminous Virtue era, year 4, s.y. 44, month 11, s.d. 5							17 Dec 1007
5.15	5.26	4.74	4.82	4.74	4.80	4.77	16 Dec 1825h
33. L.s., Sovereign Protection era, year 2, s.y. 27, month 11, day 30, s.d. 50							16 Dec 1050
50.65	50.79	50.22	50.25	50.22	50.23	50.22	16 Dec 0519h
34. L.s., Epochal Plenty era, year 6, s.y. 60, month 11, s.d. 43							16 Dec 1083
43.73	43.85	43.26	43.27	43.26	43.26	43.23	16 Dec 0528h
35. L.s., Epochal Plenty era, year 7, s.y. 1, month 11, s.d. 48							15 Dec 1084
48.97	49.10	48.50	48.51	48.50	48.51	48.48	15 Dec 1125h

Event							A
#42	#50	#71	#80	#76	#88	B	C
36. L.s., Epochal Protection era, year 3, s.y. 5, month 11, s.d. 9							15 Dec 1088
9.94	10.08	9.48	9.48	9.48	9.48	9.45	15 Dec 1046h
37. L.s., Epochal Protection era, year 4, s.y. 6, month 11, s.d. 14							15 Dec 1089
15.19	15.32	14.72	14.72	14.72	14.72	14.69	15 Dec 1635h
38. W.s., Epochal Protection era, year 5, s.y. 7, month 11, s.d. 19							15 Dec 1090
20.44	20.56	19.96	19.97	19.96	19.96	19.94	15 Dec 2230h
39. W.s., Epochal Protection era, year 7, s.y. 9, month 11, s.d. 30							15 Dec 1092
30.92	31.05	30.45	30.45	30.45	30.45	30.41	15 Dec 0957h
40. W.s., Epochal Tally era, year 1, s.y. 15, month 11, s.d. 1							15 Dec 1098
2.39	2.52	1.91	1.91	1.91	1.91	1.87	15 Dec 2058h
41. W.s., Reverent Tranquillity era, year 3, s.y. 21, month 11, s.d. 33							15 Dec 1104
33.86	33.99	33.37	33.36	33.37	33.37	33.33	15 Dec 0755h
42. W.s., Sustained Prosperity era, year 2, s.y. 48, month 11, s.d. 9							15 Dec 1191
10.12	10.27	9.57	9.47	9.57	9.46	9.46	15 Dec 1101h
43. S.f.s., Felicitous Epoch era, year 3, s.y. 54, month 11, s.d. 40							14 Dec 1197
41.59	41.74	41.03	40.92	41.03	40.92	40.92	14 Dec 2204h
44. S.f.s., Excellent Serenity era, year 3, s.y. 60, month 11, s.d. 11							14 Dec 1203
13.05	13.21	12.49	12.37	12.49	12.37	12.38	15 Dec 0904h
45. S.f.s., Excellent Order era, year 5, s.y. 9, month 11, s.d. 59							14 Dec 1212
60.25	60.41	59.69	59.56	59.68	59.56	59.56	14 Dec 1323h
46. S.f.s., Sustained Order era, year 3, s.y. 27, month 11, s.d. 33							14 Dec 1230
34.65	34.83	34.07	33.63	34.07	33.92	33.93	14 Dec 2216h
47. S.f.s., Simplicity Protection era, year 10, s.y. 47, month 11, s.d. 18							14 Dec 1250
19.94	19.71	18.96	18.77	18.94	18.78	18.79	14 Dec 1901h

Event #42	#50	#71	#80	#76	#88	B	A C
48. W.s., present dynasty, Perfect Epoch era, year 17, s.y. 17, month 11, s.d. 56.06							14 Dec 1280
56.87	57.05	56.25	56.04	56.24	56.06	56.08	14 Dec 0149h

[Note 1. The measurements are for day 21, s.d. 14, not for day 20.

Note 2. This year is predynastic Sui, normally recorded under the Ch'en period.

Note 3. The measurements are for day 23, s.d. 12, not for day 22.

Note 4. The measurements are for day 4, s.d. 17, not for day 3.

Note 5. An alternative but less likely reading is "Day 4, s.d. 56, solstice; s.d. 57, longest shadow." *Chih* 至 may mean not only "solstice" but "to," as in *"x* to *y."]*

For the period of more than 2160 years spanned above since the time of Duke Hsien in the Spring and Autumn period, we have used six systems—Great Expansion, Extending Enlightenment, Era Epoch, Concord with Heaven, Great Enlightenment, and Season-Granting—to compute a total of 49 winter solstices. [Computations using] the Great Expansion system agree [with the observational record] 32 times and disagree 17 times. The Spreading Enlightenment system agrees 26 times and disagrees 23 times. The Era Epoch system agrees 35 times and disagrees 14 times. The Concord with Heaven system agrees 38 times and disagrees 11 times. The Great Enlightenment system agrees 34 times and disagrees 15 times. The Season-Granting system agrees 39 times and disagrees in 10 instances.

[Agreement evidently means falling in the same sexagenary day. Since the observers measured only the length of the shadow at noon, the Evaluation accepts results within half a day. A count of the results for the forty-eight items in the table gives 18, 22, 14, 11, 15, and 10 disagreements respectively, and implies that in the lost forty-ninth item, the first two items disagreed with the observational record and the remainder agreed.

In the discussion that follows, the evaluator analyzes the contents of the table to determine which system's computations are correct. In the first three comments, he assumes that the very early observational records are accurate, and that the discrepancies are due to non-uniform ("erratic") solar motion. This is not surprising in a culture that tends to affirm the perfection of survivals from antiquity.]

We note that:

1. For the winter solstice, Duke Hsien, year 15, s.y. 15, new moon of the standard month, s.d. 51, the Season-Granting system gives the s.d. as 51 and the Concord with Heaven system, giving it as 52, is a day behind the sky (item 1). By the winter solstice, Duke Hsi, year 5, s.y. 3, new moon of the standard month, s.d. 48, both the Season-Granting and Concord with Heaven systems give the s.d. as 48, agreeing with the sky (item 2). Going on to the winter solstice, Duke Chao, year 20, s.y. 16, new moon of the standard month, s.d. 26, both the Season-Granting and Concord with Heaven systems give the s.d. as 25, and both are a day early (item 3). If one were to adjust the procedure to make [the Yuan system's prediction agree

with the phenomenon], the Duke Hsien and Duke Hsi observations would no longer agree. Thus we know that the winter solstice in the time of Duke Chao recorded in the Spring and Autumn annals is evidence that the sun's angular motion was erratic (*jih tu shih hsing* 日度失行).

2. According to the Great Expansion system's investigation of past winter solstices, the southernmost position of the sun in year 13 of the Epochal Excellence era of the Liu Sung period, s.y. 13, month 11, fell on s.d. 11 (item 5), but the Great Expansion, Sovereign Pole (#34), and Chimera Virtue (#38) systems agree in predicting the solstice for s.d. 10. All are a day early. This is because the sun's angular motion was erratic, and is not an error of the three systems, for checking with the Season-Granting system today also yields day 10.

3. For the winter solstice of year 5 of the Great Enlightenment era, s.y. 38, month 11, s.d. 22, all the systems yield s.d. 21 (item 11); this is evidently also due to a discrepancy in the sun's angular motion.

4. For the longest shadow, year 4 of the Grand Establishment era of the Ch'en period, s.y. 29, month 11, s.d. 4, the Great Expansion and Season-Granting systems both give the s.d. as 3, which is a day early (item 14).

5. For the longest shadow, year 9 of the Grand Establishment era, s.y. 34, month 11, s.d. 29, the Great Expansion and Season-Granting systems both yield s.d. 30, which is a day late (item 16).

In one instance both systems are too early, and in the other too late. If they are made to agree for s.y. 29 they will err in s.y. 34, and vice versa. This is also evidence that the sun's angular motion is erratic.

6. For the longest shadow, year 11 of the Opening Sovereignty era, s.y. 48, month 11, s.d. 43, the Great Expansion, Concord with Heaven, and Season-Granting systems all yield s.d. 43, agreeing with the sky (item 22). By the winter solstice, year 14 of the Opening Sovereignty era, s.y. 51, month 11, s.d. 58, the Great Expansion, Concord with Heaven, and Season-Granting systems all yield s.d. 59 (item 23). If they are made to agree for year 48 they will err in

year 51, and vice versa. The winter solstice of year 14 of the Opening Sovereignty era, s.y. 51, is another instance of the sun's erratic angular motion.

7. For the longest shadow, year 18 of the Constant Contemplation era of the T'ang period, s.y. 41, month 11, s.d. 22, all the systems yield s.d. 21 (item 24). For the longest shadow, 23d year of the Constant Contemplation era, s.y. 46, month 11, s.d. 48, all the systems yield s.d. 47 (item 25). According to the *Evaluation of the Great Expansion System (Ta yen li i* 大衍曆議), computing winter solstices of the Perpetual Simplicity and Opening Epoch eras (items 28–31), we know that the two winter solstices [of the Constant Contemplation era] were recorded by officials of the Astronomical Bureau from current ephemerides, and were certainly not obtained from shadow measurements. Thus they do not agree [with computations by other systems]. Now, checking with the Season-Granting system, one also finds this to be the case.

[The *Evaluation* no longer survives, but there is a long summary in the *New T'ang History's* account of the system. The evaluator refers to *Hsin t'ang shu* 新唐書, 27A: 594: "Up to the Unicorn Virtue era (664–65), what was set down in the *Veritable Records* was in fact dependent on current ephemerides, not obtained from shadow measurements." This would include not only Constant Contemplation (627–49, items 24–25) but Dragon Conjunction (661–63, item 26). The evaluator may be overlooking the latter, or it may be that he had access to the original *Evaluation* and that its account differed in some respect. The *Veritable Records* was a detailed compilation, an important resource for the *History*.]

[8]. Since the, Sung [i.e. Liu Sung] period, there have been 17 determinations of the *ch'i* from shadow measurements. For the southernmost position of the sun in the Luminous Virtue era, s.y. 44, s.d. 5, the Concord with Heaven and Season-Granting systems, both yielding s.d. 4, are a day early (item 32).

[The seventeen determinations probably comprise the sixteen items which note "winter solstice," and the missing 49th item.]

For the southernmost position of the sun in the Excellent Serenity era, s.y. 60, s.d. 11, the Concord with Heaven and Season-Granting systems, both yielding s.d. 12, are a day late (item 44). For one event they err on the early side, and for the other on the late side. If one were to adjust the procedure to make [the prediction in item 9] agree [with the phenomenon], most of the other sixteen predictions would then be later than the phenomena. If one were to do the same for the Excellent Serenity [prediction], most of the other sixteen predictions would then be ahead of the phenomena. This is also evidence that the sun's angular motion is erratic.

[The text below mentions ten instances, but the numbering above is confused. Either 7 or 8, and either 9 or 10, are missing.]

The preceding ten instances are those in which the Season-Granting system did not agree [with observation], but according to this line of reasoning it does not actually disagree. It would seem that "if you group likes together you discover the successes; if you sort out differences you discover the abnormalities." Now with respect to winter solstices, if we omit the ten cases in which the sun's angular motion was erratic or in which astronomical officials depended on the contemporary computational system, the Season-Granting system was on target 39 times. There is only one instance, that of Duke Hsien (item 1), in which the Concord with Heaven system and the current system do not agree. Computing the Duke Hsien winter solstice, the Great Expansion system falls 2 days behind the phenomena, the Great Enlightenment system falls 3 days behind the phenomena, and the Season-Granting system agrees with the phenomena. Computing forward, for the winter solstice of the Perfectly Great era, s.y. 17 (item 48), the Great Expansion system is 81 marks behind the phenomena, the Great Enlightenment system is 19 marks behind the phenomena, the Concord with Heaven system is 1 mark *[should be* 2 marks] ahead of the phenomena, and the Season-Granting system agrees with the phenomena. Compared with the various systems of former eras, the Season-Granting system is the most precise. Perhaps "as we sit there, we are able to call before us the solstices of a thousand years." {1132–40}

[The purpose of this lengthy argument is to prove that, although the Season-Granting system appears to be merely better on the whole than the other five systems considered, its superiority is absolute. The evaluator explains away all ten cases in which the date computed by the Yuan system does not coincide with the date on which the longest noon shadow was observed, making its score perfect.

Two observations, he learns from the Evaluation of the Great Expansion system, were faked (item 7). The other eight were subject to what he considers irregular fluctuations in the tropical year cycle. For each he notes that all or several systems, including the Season-Granting system, agree on a prediction which nevertheless is not supported by the observations, or that the Season-Granting and one or more other systems are early for one solstice and late for another. He concludes that the discrepancy is not linear in character. His assumption that the solar motion "is erratic" is vague in formulation, but has ample precedents; see Henderson 1984 and Sivin 1989. His stance here is perhaps best understood in the light of what alternatives he was rejecting.

Most of the discrepancies the evaluator cites arise (as we would say) out of either the inaccuracy of the shadow reading or the shift from one day to the next caused by a discrepancy in the date or by rounding off. He underestimates the former source of error, and pays little attention to these latter shifts; but his unwillingness to reject observations from very early classics is typical of imperial astronomy. It is partly due, no doubt, to excessive respect for the record transmitted from antiquity, but also to a reluctance to reduce the small store of early observations by subjecting them to scrutiny as critical as for later ones. Although in this Evaluation the author exhibits more talent for neatly turned phrases than for astronomical insight, the same reluctance to question early, even archaic, records lies at the foundation of Kuo Shou-ching's otiose secular variation in tropical year length, and the reform group's trust in unreliable records of eclipses in some of the very early classics (subsections 9.1–9.2).

Comparison of the day and time in column B computed using modern techniques with those for the various systems shows an increase in accuracy over the five centuries, as well as the generally superior results of the Season-granting system.

The quotation at the end of the passage is from Mencius *(Meng-tzu,* 4B/26). Originally, of course, it was not about computation but about imagination; see the discussion in Sivin 2000, 128–29. The phrase in quotation marks near the beginning is probably proverbial rather than a classical allusion; I have not found a source for it.]

2.2. Comparisons of the Exactitude of Ancient and Modern Astronomical Systems

Comparing the Season-Granting system with the old systems, its greater accuracy becomes apparent. It seems that "if working backward [a system] can agree with the phenomena of many centuries before, working forward it can be used perpetually." This was a conviction of our predecessors. Among those the ancients considered masters at regulating the ephemerides, Ho Ch'eng-t'ien of the Sung, Liu Cho 劉焯 of the Sui, and Fu Jen-chün 傅仁均 and the monk I-hsing 一行 of the T'ang are most outstanding. Now when we check [computations based on] their systems for the *Ch'i* Interval Constant corresponding to the winter solstice, Perfectly Great era, s.y. 17, not one fails to err; but when we use the new system to compute phenomena far in the past, it never fails to correspond perfectly. From this we can realize its exactitude.

[The ten tests that follow compare another system with the new one by using the former to compute the epochal winter solstice of the Season-Granting system, and then using the latter to compute the winter solstice preceding the first year in which the older system was used. Earlier systems normally adopted epochs set long before the time in which they were compiled. The Yuan astronomers knew that this was a source of error, and that calculating the actual epochs of earlier systems in the far distant past would not be a fair trial.

The *Ch'i* Interval Constant is the interval between the beginning of the sexagenary cycle preceding the epoch and the epoch itself, or 550 600 day parts. In the Yuan system that corresponds to a sexagenary date of day 56, 6 marks, for the epochal solstice (see Canon, 1.0 and 1.1; the day count is exclusive). Item 10 of table 7.4 lists the winter solstice of 442, but ignores the time of day of the observation. The table does not include any of the other solstices taken up in this section.]

1. The interval from the winter solstice, Epochal Excellence era of Emperor Wen of the Sung period, year 19, s.y. 19, month 11, s.d. 42, 11 marks (20 December A.D. 442; table 7.4, item 10), to the Perfectly Great era of the current dynasty, year 17, s.y. 17, is 838 years. The *Ch'i* Interval for month 11 of that year puts the winter solstice at s.d. 56, 6 marks (14 December 1280; table, item 48). Computing it by the Epochal Excellence system (#17), one obtains s.d. 58, 2 days later than by the Season-Granting system. Computing backward by the Season-Granting system to determine the winter solstice of the Epochal Excellence period, s.y. 19, yields s.d. 42, which agrees with the Epochal Excellence system.

2. The interval from the winter solstice, Great Patrimony era of the Sui period, year 3, s.y. 4, month 11, s.d. 7, 52 marks (19 December 607), to the Perfectly Great era, year 17, s.y. 17, is 673 years. Computing by the Sovereign Pole system (#34), one obtains s.d. 57

for the winter solstice [of 1280], one day later than by the Season-Granting system. Computing backward by the Season-Granting system to determine the winter solstice of the Great Patrimony era, s.y. 4, yields s.d. 7, which agrees with the Sovereign Pole system.

3. The interval from the winter solstice, Martial Virtue era of the T'ang period, epochal year, s.y. 15, month 11, s.d. 5, 64 marks (19 Dec 618), to the Perfectly Great era, year 17, s.y. 17, is 662 years. Computing by the Fifteenth-year Epoch system (#36), one obtains s.d. 57 for the winter solstice [of 1280], one day later than by the Season-Granting system. Computing backward by the Season-Granting system to determine the winter solstice of the Martial Virtue era, s.y. 15, yields s.d. 5, which agrees with the Fifteenth-Year Epoch system.

4. The interval from the winter solstice, Opening Epoch era, year 15, s.y. 4, month 11, s.d. 36, 72 marks (18 Dec 727), to the Perfectly Great era, year 17, s.y. 17, is 553 years. Computing by the Great Expansion system (#42), one obtains s.d. 56[, 87 marks,] for the winter solstice [of 1280], 81 marks later than by the Season-Granting system. Computing backward by the Season-Granting system to determine the winter solstice of the Opening Epoch era, s.y. 4, yields s.d. 36, which agrees with the Great Expansion system but is 4 marks earlier.

5. The interval from the winter solstice, Eternal Felicitation era, epochal year, s.y. 38, month 11, s.d. 49, 76 marks (17 Dec 821), to the Perfectly Great era, year 17, s.y. 17, is 459 years. Computing by the Extending Enlightenment System (#50), one obtains s.d. 57 for the winter solstice [of 1280], one day later than by the Season-Granting system. Computing backward by the Season-Granting system to determine the winter solstice of the Eternal Felicitation era, s.y. 38, yields s.d. 49, which agrees with the Spreading Enlightenment system.

6. The interval from the winter solstice, Grand Peace and National Prosperity era of the Sung period, year 5, s.y. 17, month 11, s.d. 43, 63 marks (16 December 980), to the Perfectly Great era, year 17, s.y. 17, is 300 years. Computing by the Supernal Epoch system

(#61), one obtains s.d. 57 for the winter solstice [of 1280], one day later than by the Season-Granting system. Computing backward by the Season-Granting system to determine the winter solstice of the Grand Peace and National Prosperity era, s.y. 17, yields s.d. 43, which agrees with the Supernal Epoch system.

7. The interval from the winter solstice, General Peace era, year 3, s.y. 37, month 11, s.d. 28, 53 marks (16 December 1000), to the Perfectly Great era, year 17, s.y. 17, is 280 years. Computing by the Matching Heaven system (#64), one obtains s.d. 57 for the winter solstice [of 1280], one day later than by the Season-Granting system. Computing backward by the Season-Granting system to determine the winter solstice of the General Peace era, s.y. 37, yields s.d. 28, which agrees with the Matching Heaven system.

8. The interval from the winter solstice, Reverent Tranquillity era, year 4, s.y. 22, month 11, s.d. 38, 62 marks (15 December 1105), to the Perfectly Great era, year 17, s.y. 17, is 175 years. Computing by the Era Epoch system (#71), one obtains s.d. 56[, 25 marks,] for the winter solstice [of 1280], 19 marks later than by the Season-Granting system. Computing backward by the Season-Granting system to determine the winter solstice of the Reverent Tranquillity era, s.y. 22, yields s.d. 38, which agrees with the Era Epoch system but is 2 marks earlier.

9. The interval from the winter solstice, Great Discipline era of the Chin period, year 19, s.y. 36, month 11, s.d. 6, 64 marks (15 December 1179), to the Perfectly Great era, year 17, s.y. 17, is 101 years. Computing by the [Revised] Great Enlightenment system (#76), one obtains s.d. 56[, 25 marks,] for the winter solstice [of 1280], 19 marks later than by the Season-Granting system. Computing backward by the Season-Granting system to determine the winter solstice of the Great Order era, s.y. 36, yields s.d. 6, which agrees with the Great Enlightenment system but is 9 marks earlier.

<This is apparently because the Great Enlightenment winter solstice was imprecisely measured.>

10. The interval from the winter solstice, Felicitous Epoch era, year 4, s.y. 55, month 11, s.d. 46, 17 marks (15 December 1198), to the Perfectly Great era, year 17, s.y. 17, is 82 years. Computing by

the Concord with Heaven system (#80), one obtains s.d. 56[, 5 marks,] for the winter solstice [of 1280], 1 mark earlier than by the Season-Granting system. Computing backward by the Season-Granting system to determine the winter solstice of the Felicitous Epoch era, s.y. 55, yields s.d. 46, which agrees with the Concord with Heaven system. {1140–42}

3. Angular Extensions of the Lunar Lodges along the Celestial Perimeter

The sequence of lodging-places [of the moon, *hsiu* 宿] manifest themselves in the sky as 28 hostels (*she* 舍). In angular measure they comprise 365-odd *tu*. Were it not for the solar motion, there would be no standard for the angular extensions [of the lodges]; and were it not for the sequential hostels, there would be no way record their angular extensions. By depending on these two, we derive the angular extent of the celestial perimeter.

[This gives the impression that the lodges *(hsiu)* are abstract divisions of the sky, and hostels *(she)* are the visible stars that mark them. Actually the evaluator is not so fastidious; in fact he does not use *she* again after this passage. Neither the Yuan astronomers nor their predecessors made such a distinction. Reasons of compositional style led astronomical writers to substitute *she* for *hsiu,* regardless whether they were referring to the star that marked the beginning of a lodge (the determinative star), or to the section of the sky defined by an arc of the equator between two lodges. This is the case in such important early writings as the *"Book of Celestial Offices (T'ien kuan shu* 天 官 書)," ch. 27 of the *Shih chi* 史 記 (circa 100 B.C.). Shen Kua 沈括, a fastidious stylist of two centuries earlier, uses *she* in a precise context to mean "lodge determinatives" in item 129 of his *Meng Ch'i pi-t'an* 夢溪筆談, and *hsiu* for the same purpose, in a similar argument, in item 147.

Although *hsiu* used to be translated "lunar mansion," that betrays inattention to what the word means. *Hsiu* is not where one lives but where one spends the night when away from home. *She* is a word of similar coloration. When Hellenistic astrology entered China, translators used *she* for its zodiacal houses.]

The physical sky is spherical, centered between its north and south poles, and bound transversely by the Red Way *[ch'ih-tao* 赤道, the equator], in and out [i.e., north and south] of which in a regular way move the sun, moon, and five planets. The sky rotates leftward and the sun, moon, and planets revolve in the contrary sense toward the right. When the ancients represented the sun, moon, planets and stars in their ephemerides, they were referring to this [orderly moving array].

The angular intervals between the hostels as measured in successive eras, however, have not been constant. Those variations that were not a matter of subtle shifts would have been due to inaccuracy in our predecessors' observations. In olden times they used a sighting-tube; now observations employ the two wires *(erh hsien* 二線) of the new armillary sphere.

> [This actually refers to the diametral sights of the Simplified Instrument; see p. 196. Kuo's summary of major innovations in the new system (appendix B, p. 585) calls them "interval wires *(chü hsien* 矩線)."]

Below we set out degrees and decimal parts of the angular extensions so measured, along with variant observations in earlier eras.

> [The table that follows in the text shows successive changes in the extents of lodges through six sets of observations over nearly fourteen hundred years. Table 7.5 reproduces these data in columns A–F. I have added useful supplementary information, based partly on Yabuuchi 1936, 57–65. The information given in each column is as follows (for details, see the notes that follow the table): For additional information, see the orientation, table 2.9 (p. 91).
>
> **Lodge:** Translation of the name (short form) of the lunar lodge.
>
> **A:** Observations by Lo-hsia Hung 落下閎 of the Han (epoch of computation circa 104 B.C.; note 2)
>
> **B:** Observations by I-hsing of the T'ang (circa 730)
>
> **C:** Observations in the Sovereign Protection period of the Sung (1050)
>
> **D:** Observations in the Epochal Plenty period (1080)
>
> **E:** Observations in the Reverent Tranquillity period (1100)
>
> **F:** Observations in the Perfectly Great period (1280)
>
> **G:** The right ascension of each determinant for 1280, modern computations, expressed in *tu*]

Table 7.5. Extensions of the Lunar Lodges in Various Periods

Lodge	A	B	C	D	E	F	G
Horn	12 (12)	12 (12)	12 (12)	12	12 (12)	12.1 (12.0)	194.72
Neck	9 (9)	9 (9)	9 (9)	9	9¼ (9¼)	9.2 (9.3)	206.70
Root	15 (15)	15 (16)	16 (15)	16	16 (16–16¼)	16.3 (16.3)	216.01

Lodge	**A**	**B**	**C**	**D**	**E**	**F**	**G**
Chamber	5 (5)	5 (5/6)	5 (6)	6	5¾ (5½)	5.6 (5.6)	232.34
Heart	5 (4/5)	5 (6)	6 (6)	6	6¼ (6¼)	6.5 (6.5)	237.93
Tail	18 (19)	18 (19)	19 (19)	19	19¼ (19¼)	19.1 (19.3)	244.48
Basket	11 (10)	11 (10/11)	10 (10/11)	11	10½ (10¼)	10.4 (10.35)	263.72
Eastern Quarter	75	75	77	79	79	79.2	-
Dipper	26+ (27)	26 (26)	25 (26)	25	25 (25½)	25.2 (25.3)	274.09
Ox	8 (8)	8 (7/8)	7 (7)	7	7¼ (7¼)	7.2 (7.15)	299.36
Maid	12 (12)	12 (12)	11 (11/12)	11	11¼ (11½)	11.35 (11.4)	306.51
Tumulus	10 (9/10)	10⅓ (9)	10⅓ (9)	9⅓	9⅓ (9)	8.95 (8.9)	317.92
Rooftop	17 (16/17)	17 (16)	16 (15/16)	16	15½ (15½)	15.4 (15.3)	326.86
Hall	16 (17)	16 (17)	17 (17)	17	17 (17)	17.1 (17.1)	342.19
Wall	9 (8/9)	9 (8)	9 (8)	9	8¾ (8½)	8.6 (8.45)	359.287
Northern Quarter	98+	98¼	95¼	94¼	94	93.8	-
Crotch	16 (16)	16 (16)	16 (16/17)	16	16½ (16½)	16.6 (16.65)	2.47
Pasture	12 (11)	12 (11/12)	12 (12)	12	12 (11¾)	11.8 (11.8)	19.14
Stomach	14 (15)	14 (15)	15 (15)	15	15 (15¼)	15.6 (15.4)	30.95
Mao	11 (11)	11 (11)	11 (10)	11	11¼ (11¼)	11.3 (11.2)	46.35
Net	16 (18)	17 (16)	18 (17/18)	17	17¼ (17½)	17.4 (17.4)	57.58
Beak	2 (1)	1 (0/1)	1 (0)	1	½ (0)	0.05 (-0.1)	74.93
Triad	9 (8)	10 (10)	10 (11)	10	10½ (10¾)	11.1 (11.2)	74.93
Western Quarter	80	81	83	82	83	83.85	-
Well	33 (33)	33 (33)	34 (33)	34	33¼ (33¼)	33.3 (33.1)	86.10
Devils	4 (4)	3 (3)	2 (3)	2	2½ (2½)	2.2 (2.3)	119.25
Willow	15 (15)	15 (14)	14 (14)	14	13¾ (13½)	13.3 (13.4)	121.58

Lodge	A	B	C	D	E	F	G
Stars	7 (7)	7 (6/7)	7 (6)	7	6¾ (6¼)	6.3 (6.3)	134.98
Bow	18 (17)	18 (17)	18 (17)	17	17¼ (17¼)	17.25 (17.25)	141.25
Wings	18 (18)	18 (18/19)	18 (19)	19	18¾ (18¾)	18.75 (18.85)	158.51
Baseboard	17 (17)	17 (17)	17 (17)	17	17 (17¼)	17.3 (17.4)	177.34
Southern Quarter	112	111	110	110	109.25	108.40	-
Total	365+	365¼	365¼	365¼	365¼	365.25	-

{1142–45}

[**Note 1.** In columns A-F, values calculated by Yabuuchi, in parentheses, follow the observed value. The presentation here differs from that in Yabuuchi 1936 in that the quantities in columns A - F are given in *tu* to the precision reflected in the *Yuan shih* passage: to the nearest 1 *tu* for columns A - D, ¼ *tu* for column E, and as usual, 0.05 *tu* for the Season-Granting system figures in column F. This allows accuracy to be estimated by inspection. The average deviation of readings in column F, ignoring signs, is 0.13 *tu*.

Note 2. When Yabuuchi's calculated value falls (within 1/10 of the level of precision) on either side of the midpoint between two recording steps (e.g., between 0.4^t and 0.6^t for a precision of 1^{t}), I record both the upper and lower rounded values. Thus I set down a calculated value of $9^t.45$ as "(9/10)," meaning "on the borderline between 9 and 10." This rule does not apply to the miniscule lodge Beak.

Note 3. In the original table, values unchanged from the nearest entry in a column to the left are indicated by a blank. To avoid misreading, I have filled in the blanks.

Note 4. The last row in the table gives the total extension of all 28 lodges.

A few textual matters are worth mentioning:

- In Dipper, column A, the plus sign (+) stands for the words "and a fraction *(chi fen* 及 分)."
- In Tumulus, columns B–E, the *Yuan History* text includes the words *shao ch'iang* 少 強. Historians have usually interpreted them as "3/16." There was more than one system of fractional terminology, which gave them different values, in the period covered by the table (see the orientation, p. 85, and table 2.7). Since here they are reflected in the total for the Northern Quarter as ¼, the evaluator evidently understood them in that way.
- If we examine the sources of columns B through E, it is clear that the compilers took some latitude with their data. Column B comes from the *Old History of the T'ang (Chiu T'ang shu),* 34: 1240–41, in which Tumulus has a fractional

remainder recorded as *hsu fen ch'i-pai-ch'i-shih-chiu t'ai* 虛分七百七十九太, or 779¾. Dividing it by the system's standard divisor, 3040, gives 0.2565. The record of the new instruments of the Sovereign Protection period and their observational results, on which column C is based, records the extent of Tumulus as simply 10^{t} *(Sung shih,* 76: 1746). It seems therefore that the compilers of the table interpolated the fraction passively, by leaving that space blank so that the preceding value would be read. Column D is from the star chart of *Essentials of the Method for the New Instrument (Hsin i-hsiang fa yao* 新儀象法要), 2: 7b-9a, on which see Needham et al. 1960. The data are also scattered through *Wen hsien t'ung k'ao* 文獻通考, ch. 279. The fractional part there is *shao ch'iang* 少強, as in the comment above. Column E comes from a source cognate to but earlier than the *Sung History,* 79: 1856–57. The latter records the fraction as *shao miao ch'i-shih-erh* 少秒七十二, which can only mean 0.2572, and gives the total for the Northern Quarter as 94^{t}.0072. The total given in *Yuan shih,* 94¾, does not tally with the individual entries; I follow the modern text's emendation to 94 (p. 1144, note 10).

The evaluator understandably ignores the appended .0072, and indeed one wonders why he included the other miniscule quantities. Yabuuchi suggests that he added them, obviously not as a result of observation, but to make the total extent of the lodges equal the Celestial Perimeter, the number of *tu* that the mean sun passes through in a sidereal year. The author gives the Celestial Perimeter in each system with a fractional remainder exactly equal to that of Tumulus. The T'ang figure is 365^{t}.2565 (p. 1237), and that in the *Sung History* corresponding to column D is 365^{t}.2572 (p. 1851). The tradition of assigning fractional parts arbitrarily to one lodge (instead of distributing them among the lodges) began in the Han with Dipper. It became stabilized by the T'ang with Tumulus. Shen Kua, in the late eleventh century, spoke of this as established practice: " … thus it was imperative that stars located directly on a *tu* division be designated the lodge determinatives. Only at the end of Tumulus [*wei* 未 → *mo* 末] was there a fraction. This was simply the fractional day parts [in a year], for it is Tumulus to which astronomers assigned [what were called in the Han] the 'Dipper Parts'" *(Meng-ch'i pi-t'an* 夢溪筆談, item 147). Shen's hypothesis that the lodges and their determinatives were selected to make all extensions but one integral is incorrect. The Yuan astronomers maintained the tradition of assigning fractional parts to Tumulus. The Canon, 3.3, unlike column F of this table, gives the extent of Tumulus as 8^{t}.9575.

The fractional parts given for Tumulus in columns B and D represent a level of precision that there is no reason to believe had been attained at the time of observation. Comparison with calculated values reveals that this spurious precision was gained at some cost in accuracy.

- In Eastern Well, column C, we follow the Editors of the *Yuan History* (p. 1145, note 11) in inserting “34” in the empty space to conform with *Sung shih,* 76: 1746, and with computation. One would otherwise read the value as 33 *tu*. The *Sung History* gives the extent of Heart as 4^{t}. This is evidently an error, since it would result in a Celestial Perimeter of a fraction over 363^{t}.
- The total originally given for the Northern Quarter in column E above, 94¾, does not tally with the extents of the individual lodges. The modern editors of the *Yuan History* have therefore emended it to 94 (52: 1144, note 10).]

4. Tread of the Sun

The sun, appendage of heaven, most prominent of the emblems hanging there! As soon as the Great Brightness comes to life, all the [stars of the] lunar lodges go out. When the ancients wished to measure what degree the sun was treading [i.e., its position], they had to begin with a meridian transit at dusk, dawn, or midnight, and from it determine crosswise the distance [between transit and sun] and then lengthwise the place [that the sun] occupied.

[On the “tread” of the sun—its apparent position and motion—see Canon, section 3, Introduction. The author uses “crosswise” and “lengthwise” rhetorically to emphasize the indirection of the procedure.

One cannot directly observe the sun’s place among the stars. Early astronomers crudely estimated it by observing stars that rose just after the sun set, or set just before the sun rose. How close to the sun such stars are visible varies considerably with the angle of the ecliptic to the horizon, which changes with the seasons, and other variables. A star that crosses the meridian at midnight is approximately half a circle away from one that accompanies the sun across the meridian at noon, so observing it can provide a more accurate estimate.]

But the exact times of dusk, dawn, and midnight are not easy to determine truly. Once there is a discrepancy in the double-hour and marks, it is impossible to avoid error in the interval and position.

In the Tsin era, Chiang Chi 姜岌 (A.D. 384, #13) was the first to determine the degree in which the sun was located from observations of opposition in lunar eclipses. The Era Epoch system (#71; in use 1106–27) went on to note the elongation of Great White [i.e. Venus], and to determine by observation after dusk or before dawn the position of the planet among the stars, thus deriving the tread [i.e., the position] of the sun.

[These two methods were relatively accurate ways of measuring the position of the sun among the stars. See the orientation above, p. 101. The elongations of Jupiter and Venus from the Sun were germane because a planet's rising just after sunset or setting just before sunrise provided another method of locating the sun.]

Now the astronomers have used the total phase of the lunar eclipse at full moon in the Perfectly Great period, s.y. 14, month 4, s.d. 10 (18 May 1277; see below, 9.4, item 36) to compute the position of the sun on the Red Way at the winter solstice as 10^t in Winnowing-basket, and its position on the Yellow Way [i.e., the ecliptic] as 9-odd *tu*. Over the three years from the standard month of that year to the end of s.y. 16, they have measured nightly the position of the moon with respect to the lunar lodges, and the degrees of elongation of the Year Star and Great White [i.e. Jupiter and Venus]. Upon colligating these observations, they obtained a total of 134 such determinations, all of which placed the sun in Winnowing-basket [at the winter solstice of December 1278], in agreement with the location of opposition during the lunar eclipse. Computed by the method given in the Great Enlightenment system as revised by Chao Chih-wei 趙知微 of the Chin era (#76, in use 1180–1281), the winter solstice would still be located $0^t.3664$ in Dipper, an actual discrepancy of $0^t.7664$ compared with the measurement according to the new system. {1145–46}

[See the different phrasing of this point in Kuo's biography, p. 584. The solstice was "still" in Northern Dipper ca, 1180 because by 1280 its location had gradually shifted backward from the beginning of that lodge to the end of Winnowing-basket.]

5. Expansion and Contraction of the Solar Motion

"In the motion of sun and moon there is a winter and a summer" is a way of saying that the angular motions of the sun and moon differ in winter and summer. Laymen know only that the sun travels a *tu* each day, making a round of the sky once a year, but are unaware that there are seasonal differences—expansion and contraction (*ying-so* 盈縮), augmentation and diminution (*sun-i* 損益). Chang Tzu-hsin 張子信 (circa 560) of the Northern Ch'i era accumulated observations of the exact times of conjunctions and eclipses *[or* conjunction eclipses], and became aware that the solar motion incorporated [an inequality that he called the] *Ch'i* Entry Difference *(ju ch'i ch'a* 入氣差). He did not, however, derive a correct [formula for] augmentation and diminution. Chao Tao-yen 趙道嚴 (roughly contemporary) went on to determine the advance and recession in the solar motion [i.e. its inequality] on the basis of gnomon shadow length, and then to originate an Expansion/Contraction factor for predicting eclipses. Liu Cho (#34, 608) instituted degrees of corrected [solar] motion which rise and fall in the fourfold sequence of the seasons. Later generations, although their [numerical values for the] diminution and augmentation factors did not remain the same, transmitted and employed [Liu's theory] as the ancestral standard [for corrected motion].

> [We know little about the astronomical work of Chang and Chao. No outlines of their astronomical systems have survived. The history of the Sui period, not that of their own, records the two *(Sui shu,* 17: 418, 20: 561). Expansion and the other terms are discussed later in this section. "Corrected (*ting* 定)" as a component of astronomical terms is the Chinese equivalent of "apparent" with respect to motions. On apparent vs. true motions, see the orientation, p. 98.
>
> The best analysis of developing knowledge of the solar inequality is Ch'en Mei-tung 1995, chapters 11–12; for the period up to the eighth century see also Yabuuchi 1944a, 61–74. Li Yen 1957, Yen Tun-chieh 1966, and Ch'ü An-ching in Ch'ü et al. 1994, discuss early interpolation schemes for modeling it.]

In the comings and goings of yin and yang, variation comes about through gradual cumulation. At the winter solstice, the sun's daily travel is a bit over one *tu,* and it has emerged a bit less than 24^{t} outside [i.e., to the south of] the Red Way. From then on, the solar track proceeds gradually northward for 88.91 days, until three days before Vernal Division[, i.e., the equinox, the sun] crosses and is located [momentarily] on the Red Way. Its actual motion has been $91^{\mathrm{t}}.31$ (90°), and [its speed] is now just at its mean value. From then on, its motion in the phase of expansion *(ying* 盈) diminishes daily. After it travels another 93.71 days, on the day of the summer solstice it has entered a bit less than 24 *tu* inside [i.e. north of] the Red Way. Its actual travel has been $91^{\mathrm{t}}.31$, and its daily motion is a bit less than one *tu*. The previous expansion parts [i.e. the equation of center, the difference between apparent and mean position on the ecliptic] have diminished to the point that there is no remainder.

From then on the track of the sun proceeds gradually southward for 93.71 days, until three days after Autumnal Division[, i.e., the equinox, the sun] crosses and is located on the Red Way. Its actual motion has been $91^t.31$, and [its speed] is now just at its mean value. From then on, its motion in the phase of contraction *(so* 縮) diminishes daily as it travels on for 88.91 days. It emerges a bit less than 24^t outside the Red Way. Its actual motion has been $91^t.31$, and it is once again at the winter solstice. The previous contraction parts have been completely diminished, so that there is no remainder.

> [Vernal Division and Autumnal Division are the *ch'i* periods of 15+ days halfway between the solstices, centered on the equinoxes; see Canon, 2.2. The mean value of the sun's speed is of course 1^t per day.]

There are both diminution and augmentation in expansion and contraction, augmentation at the beginning and diminution at the end [of each phase]. From the winter solstice to Vernal Division, and from Vernal Division to the summer solstice, the tread of the sun turns from its northern road to its western one, and from its western road to its southern one. Thus there is augmentation in the phase of expansion. When augmentation reaches its maximum, diminution sets in. When diminution has proceeded to the point where there is no remainder, contraction sets in. From the summer solstice to Autumnal Division, from Autumnal Division to the winter solstice, the tread of the sun turns from its southern road to its eastern one, and from its eastern road to its northern one. Thus there is augmentation in the phase of contraction. When augmentation reaches its maximum, diminution sets in. When diminution has proceeded to the point where there is no remainder, expansion again sets in. The beginning of expansion and the end of contraction both take 88.91 days for the sun to travel through one aspect. The beginning of contraction and the end of expansion both take 93.71 days for the sun to travel through one aspect. The maximal difference in the phases of expansion and contraction *(ying-so chi ch'a* 盈縮極差) is $2^t.40$. This was derived from solar shadow observations, and has also been determined by mathematical inference *(suan-shu t'ui-k'ao* 算術推考), which agrees with measurement. {1146–47}

> [This passage is concerned with the inequality of the sun's motion. It rephrases in rather literary language the division of the tropical year into phases as given in the Canon, 3.1, and the gradual changes tabulated in 3.7 and visually represented in figure 30. This notion of four "roads" *(lu* 陸) goes back to *Tso chuan,* Chao 4/352/2; Legge 1861-72, 5: 595, for 538 B.C. In the Canon, the phase of expansion is the half-year in which the daily apparent solar motion is greater than the mean (i.e., when the equation of center, the "remainder" spoken of above, is positive). The phase of contraction is that in which the apparent motion is less than the mean. An aspect, related to the Aspectual Limit discussed in the Canon, 3.0, is a quadrant of the sun's circuit through the stars. Each of the two phases is subdivided into two Extents, or quadrants, according to whether the equation of center is increasing or decreasing (Beginning and End Extent respectively). "Diminution" and "augmentation" are terms for decrease and increase in what would now be called the solar or lunar equation of center. One reads the arguments of apparent daily motion within each quadrant, forward or backward from the solstices, depending on the Extent.
>
> The figure of $2^t.40$ for maximal equation of center is not, despite the Evaluation's assertion to that effect, the product of both observation and mathematical inference. It is merely half the difference between the days required for the sun to cover a quarter-cycle in the two halves of a phase ([91.31 – 88.91]/2). This difference is in effect rounded off from $2^t.4014$, the difference between the Expansion Beginning/Expansion End Extent in the Canon, 3. As for mathematical inference, an independent check is provided by table 9.3 in the Canon, which was built up by accumulating finite differences. There the maximal equation of center (column C - column A), which occurs between 44^t and 45^t, is between $2^t.3085$ and $2^t.31$.]

6. Slackening and Hastening of the Lunar Motion

The ancient astronomical systems asserted that the mean [daily sidereal] motion of the moon is 13 and 7/19 *tu*. In the Han, Keng Shou-ch'ang 耿壽昌 (fl. 57–52 B.C.) believed that when the sun and moon arrive at Led Ox and Eastern Well, the sun exceeds its degree [i.e., its mean daily motion] and the moon travels 15^t [per day]. Only when they reach Pasture and Horn do they attain the mean motion. What makes this true is the equator. Chia K'uei 賈逵 (fl. A.D. 92) considered that the reason for inaccurate contemporary predictions of the times of day of lunar conjunctions, crescent moons, oppositions, and lunar eclipses was apparently failure to apprehend the notion of a slackening and hastening [i.e., inequality] of the lunar motion. Li Fan 李梵 and Su T'ung 蘇統 (both fl. A.D. 85) held that the due slackening and hastening of the lunar motion do not necessarily take place at Led Ox, Eastern Well, Pasture, and Horn, but actually are consequences of distance and separation along the lunar path. Liu Hung 劉洪, in composing his Supernal Emblem system (#5, 206), devoted more than twenty years of serious thought to this problem before he comprehended the pattern. He set forth his Difference Rate (*ch'a lü* 差 率) to encompass the numerical basis of advance and recession, diminution and augmentation. Those who made new systems after his time all followed him until, in the T'ang, I-hsing (#42, 728) worked out the numerical basis of the serpentine coils of the Nine Ways, thus mastering the pattern of quick and slow in the lunar motion.

[The four lunar lodges mentioned are roughly a quarter-circle apart (see table 7.5). The irregularity that Keng pointed out is, in modern language, due to using equatorial coordinates to measure motion along the inclined solar and lunar orbits. By "distance and separation," Li and Su evidently meant what we would consider the angle in right ascension between the moon's location on its orbit and the nodes, and the distance in declination to the equator. The Difference Rate was basic to Liu's treatment of the lunar variation in longitude and latitude. On the Nine Ways, see the next comment.

For a graphic representation of the next paragraph, see figure 35 in the Canon, 4.0.]

Scholars of earlier times said that the moon and the planets hastened when near the sun and slackened when far from it. Astronomers instituted a method by which the Revolution Entry [*ju chuan* 入轉, i.e. anomalistic] cycle of days was divided into two sequences of slackening and hastening, within both of which they established Beginning and End Extents. There is augmentation in the Beginning Extents and diminution in the End Extents. At the beginning of hastening and at the end of slackening, the Degree Rate (*tu-lü* 度率, Canon, 3.7) of the moon's travel is greater than the mean motion. At the beginning of slackening and at the end of hastening, the Degree Rate falls short of the mean motion. On the first day of Revolution

Entry, the moon travels a bit over 14½ *tu*, and from then on its motion gradually abates. After seven days' travel it becomes just equal to the mean angular motion. [This quarter-cycle] is called the Beginning Extent of the hastening sequence. Its Accumulated Degrees is $5^{t}.42$ in excess when compared with the mean motion. From then on its hastening diminishes daily. After another seven days' travel the moon's motion is very slightly over 12 *tu*. The previous augmentation has been completely diminished, so that there is no remainder. [This quarter-cycle] is called the End Extent of the hastening sequence. From then on the moon's angular motion is slack [i.e., less than the mean velocity]. After another seven days' travel the moon's motion is just equal to the mean angular motion. [This quarter-cycle] is called the Beginning Extent of the slackening sequence. Its Accumulated Degrees falls short of the mean motion by $5^{t}.42$. From then on its slackening diminishes daily as its angular motion gradually increases. After another seven days' travel it is again travelling a bit over 14½ *tu*. The previous augmentation has also been completely diminished, so that there is no remainder. [This quarter- cycle] is called the End Extent of the slackening sequence. The cycle of Revolution Entry is actually 27.5546 days (Canon, 4.C2). The maximal difference factor for either slackening or hastening is $5^{t}.42$.

In the old systems each day was one Extent [i.e. one step in the argument of anomaly], so that they all used 28 extents. But now it has been determined by experience *(ting yen* 定驗) that the times of advance and recession of Revolution [Entry] Parts are not uniform. [The Canon] now divides the day into 12 Extents, for a total of 336 Extents [in the anomalistic month cycle; 4.C6]. Halving this total gives Extents for half the cycle, and breaking it into quarters gives Extents in an Aspectual Limit. {1147–48}

[The first paragraph of section 6 sketches the gradual definition of a concept of lunar equation of center from the early systems which considered only mean motion to the Nine Ways of I-hsing. The mean value given for the moon's motion among the stars is that of the early Quarter Remainder system (#4). Equivalent to $13^{t}.3684$ (corresponding to a sidereal month of 27.3219 days), it nicely approximates the modern value of $13^{t}.3687$.

Much of the discussion is about problems due to Chinese systems' dependence on equatorial coordinates. Early techniques superimposed the projection problem upon that of the moon's proper motion, making the latter difficult to treat directly. (European astronomy of the past three centuries has also been equatorial, but the earlier ecliptic-oriented tradition and the simplicity of conversion using spherical geometry made it easy to think in both systems.) To paraphrase what the Evaluation tells us, Keng Shou-ch'ang saw that the moon's projection on the equator moves more quickly near the beginnings of the northern and southern quadrants (i.e., the solstices) than near the beginnings of the eastern and western quadrants (the equinoxes), which is true. Chia K'uei correctly pointed out that projecting motion on or near the ecliptic onto the equator is an entirely different matter from the lunar equation of center. Li Fan and Su T'ung almost understood the lunar anomaly as an independent notion, but they had not yet quite distinguished the apogee and perigee from the lunar nodes. This Liu Hung did, leaving it to I-hsing five hundred years later to work out a numerical treatment of anomaly in nine increments.

Unlike the predecessors whom the Evaluation names, I-hsing dealt with variation in both the solar and lunar motions. He grounded his discussion of the moon in the archaic notion of nine sequential orbits through which the sun and moon step, a notion that spoke of paths east and west as well as north and south of the central path. But I-hsing made "Nine Ways" a mere figure of speech for the stepwise divergence of the moon's orbit from the ecliptic. He visualized the Nine Ways heuristically as nine connected, intertwined orbits including the ecliptic, about which the system was symmetrical, with the moon moving from one to another at regular intervals; see Needham 1954– , 3: 392–393, and more detailed discussions in Yabuuchi 1969: 298–304, and Ch'en Chiu-chin 1982. In principle this visualization blurred the distinction between the moon's orbit and that of the sun. In practice, I-hsing did not encourage such confusions, and so far as I know they did not ensue. It was the nine-step table of anomaly that his predecessors considered the fundamental part of his legacy. The evaluator makes it clear at the beginning of the next section that he takes neither the old conceit's symmetry about the ecliptic nor its directional orientation of the other eight ways literally.

The mature conception of lunar anomaly sketched out in the paragraphs that follow the historical outline is the same as that used in the Canon, 4.3 and 4.4. The principles on which it is based are identical to those of the solar inequality theory. Revolution Entry is days elapsed since perigee, the beginning of the anomalistic month. The cycle of anomaly is divided into two sequences—hastening, from perigee to apogee, in which the apparent moon is ahead of the mean moon (i.e., in which the equation of center is positive), and slackening, in which the apparent moon lags behind the mean moon. The tabulated maximal and minimal values of daily lunar travel in the Canon, 4.4, are $14^{t}.6764$ and $12^{t}.0462$. The maximum tabulated equation of center is $5^{t}.4281$ for hastening and $5^{t}.4248$ for slackening. Both are, as customary, rounded to $5^{t}.42$.

Although the author phrased his discussion of the apparent solar motion consistently in terms of paths distributed in space (above, section 5), here the discussion is concerned with the separation in *tu* between the right ascensions of the sun, the mean moon, and the apparent moon. One is reminded that, although spatial discourse is often found in Chinese astronomical treatises, their authors usually—not always—meant it heuristically rather than as a description of a physical reality on which theory depended.

The last paragraph of the text refers to the Cycle Extent Constant and associated constants set out at the beginning of Section 4 of the Canon. Because the moon travels so rapidly, the day is too long an interval at which to tabulate successive values for the equation of center. The Extent, defined as 0.082 day (8.2 marks, with 12.2 Extents per day, 336 per month), provides a more satisfactory increment. It is easily confused, however, with a quadrant of the anomalistic month cycle, called by the same name. Its length (1h 58m 4s in modern reckoning), was obviously chosen to approximate the Chinese double-hour.]

7. Crossing Cycle of the White Way

Set midway between the two poles, north and south, winding transversely around the body of the sky to mark the degrees of the lunar lodges: that is the Red Way. Entering inside and emerging outside [i.e., extending north and south of] the equator, serving as the track of the solar motion: that is the Yellow Way. What is called the White Way crosses and penetrates the Yellow Way; the moon's motion gives rise to it.

The ancients divided [the moon's orbit] into eight motions, naming them after the directions. With the one on the Yellow Way they made nine. In the light of deeper knowledge, they are actually one. Because [this single orbit] shifts with its intersection, changing position without stasis, the ancients took the far-fetched expedient of [designating Nine Ways and] naming them after the colors that correspond to the directions.

The path of the moon (*yueh tao* 月道) moves in and out of the sun's path (*jih tao* 日道); they cross twice. When they intersect at conjunction the sun is covered by the moon and, when at opposition, the moon is confronted by the sun; thus there may be eclipses in both cases. But when [the moon] passes the intersection it may be nearer or farther [in latitude from the sun], and the immersion of the eclipse can be deep or shallow. All of this can be determined by calculation.

[See figure 44 in the Canon, step 6.15.]

What is called the crossing cycle is the round of days in which the lunar path moves in and out of the solar path [i.e., the nodical month]. The [angular] distance of the solar path from the Red Way [i.e., the obliquity of the ecliptic] is 24 *tu*. The entry of the moon's path inside or its emergence outside the path of the sun does not exceed 6 *tu* [in latitude]. Its distance from the Red Way is no more than 30^{t} at its furthest, and no less than 18^{t} at its nearest. When it emerges outside the Yellow Way it is yang; when it enters inside it is yin. The cycle of yin and yang is divided into four aspects (see p. 300). When the moon is situated on the Yellow Way it is at the Standard Crossing. When it emerges 6^{t} outside the Yellow Way it is at the Crossing Halfway Point. When it is again situated on the Yellow Way it is at the Intermediate Crossing. When it enters 6^{t} inside the Yellow Way it is at the Crossing Halfway Point. These are the four aspects. One aspect occupies seven days of its own, and in each the moon travels 91 *tu*. Travel through the cycle of four aspects is called "completion of one crossing [cycle, *i chiao chih chung* 一交之終]." Reckoning it in days, one obtains 27.212 224 days. Each crossing recedes 1 93/200 *tu* [or $1^{t}.465$] with respect to the sky. A total of 249 crossings recedes through the sky slightly more than one cycle, which begins once again as it ends.

When the Standard Crossing is located at the Vernal Standard Point, the Intersection Halfway Point, emerging 6^{t} outside the Yellow Way, is 18^{t} inside the Red Way. When the Standard Crossing is located at the Autumnal Standard Point, the Intersection Halfway Point, emerging 6^{t} from the Yellow Way, is 30^{t} outside the Red Way. When the Intermediate Crossing is located at the Vernal Standard Point, the Intersection Halfway Point, entering 6^{t} inside the

Yellow Way, is 30^t inside the Red Way. When the Intermediate Crossing is located at the Autumn Standard Point, the Intersection Halfway Point, entering 6^t inside the Yellow Way, is 18^t outside the Red Way. At the Vernal and Autumnal Standard Points, the distance between the lunar lodge position of the Standard Crossing of the moon's path and the Red Way, and that of the Standard Crossing of the Yellow Way and the Red Way, falls short of $14\frac{2}{3}^t$ from east to west [i.e., along the equator] .

When at the summer solstice, in the yin sequence, the moon's path falls on the inner side [i.e., to the north of the ecliptic], and when, at the winter solstice, in the yang sequence, it falls on the outer side [i.e., to the south], the gap between [the ecliptic intersections of] the moon's path and the Red Way is greater. When at the summer solstice, in the yang sequence, the moon's path falls on the outer side, and when, at the winter solstice, in the yin sequence, it falls on the inner side, the gap between the moon's path and the Red Way is less.

It would thus appear that [the line between] the two intersections of the moon's path may be more or less inclined [to the equator] as [the moon's path] falls inside or outside [the ecliptic], according to whether the sequence is yin or yang. When the inclination is less, [the triangle formed by the intersections of these three great circles] is tighter and narrower; when greater, it is looser and wider. The difference [between the arc connecting the two intersections as measured along the moon's path and the projection of this arc on the equator] varies accordingly.

Now the astronomers have worked out a representation *(hsiang* 象) and instituted a technique to compute this difference. At its greatest it does not exceed $3^t.50$, and at its least it is not less than $1^t.30$. Such is the difference, greater and lesser, between the moon's path and the Red Way. {1148–49}

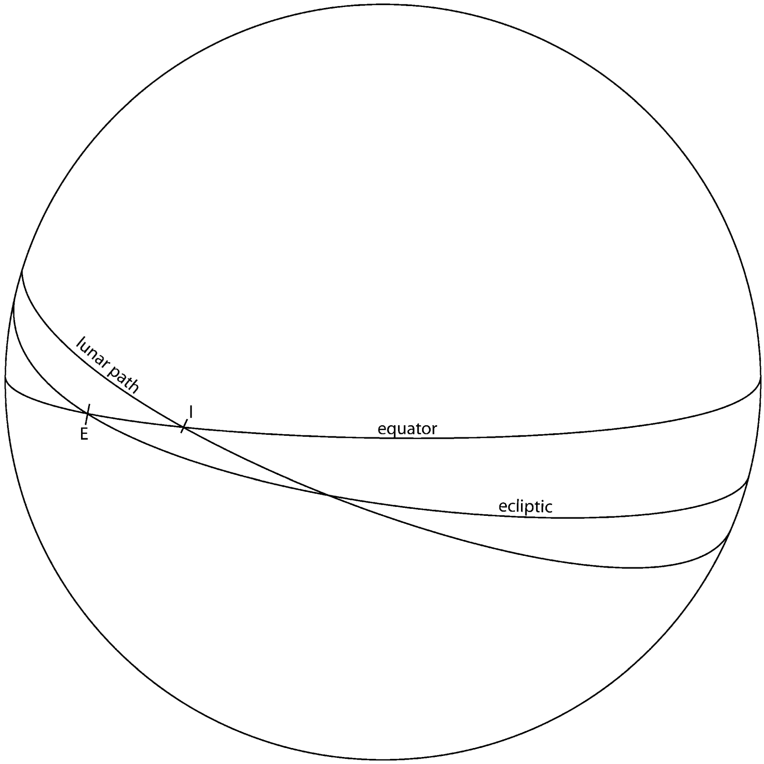
[The subject of this section is the "crossing cycle *(chiao chou* 交周)" or nodical month, the time it takes the moon, beginning at an intersection of its orbit with the sun's path, the ecliptic, to return to the same intersection. This problem was complicated by the Chinese use of equatorial coordinates to record observations. That made it necessary to consider the projection of both the lunar orbit and the ecliptic on the equator. The Yuan astronomers mastered the conversion between ecliptic and equatorial coordinates (Canon, 3.7–14), which had been troublesome up to their time. The lunar nodes are central to eclipse prediction, but the Evaluation, like the Canon (section 4), considers them part of the lunar theory rather than primarily of eclipse theory.

The conventional language for the various orbits used colors. The equator was the Red Way, the ecliptic the Yellow Way, and the lunar orbit the White Way. As the authors note, early astronomers used an even more complicated scheme of colors for the eight or nine paths (at various inclinations from the equator) into which they divided the moon's orbit—an expedient that the Season-Granting system firmly rejected.

The Yuan astronomers' more sophisticated understanding based the occurrence of an eclipse on the proximity of sun and moon to these intersections. In this section, the Standard Crossing is the moon's descending node, and the rest of the schema is as described in the Canon, 4.9 – 4.12 (see also figure 43, step 6.11). The evaluator gives the inclination of the moon's path approximately here as 6^t. This is added to or subtracted from the obliquity of the ecliptic (for this purpose, rounded to $24^t = 23^{\circ}.4'$) to yield maximum and minimum distance to the equator. He puts the retrogradation of the node with respect to the stars at $1^t.465$ (= 1°.27’) per sidereal month. This implies a nodical year length of 345.6797 days (= sidereal year length x [1 - (monthly retrogradation / sidereal month length)]). A modern value for the nodical or eclipse year is 346.6200 days (epoch 1900), implying a monthly retrogradation of 1°.24’. The cycle of 249 sidereal months = 250 nodical months (= 18.63 tropical years) gives a total nodical retrogradation of $364^t.785$. This is indeed slightly greater than one nodical cycle, although 236 sidereal months would give $345^t.74$, a closer approximation. For a table of nodical periods in various systems, see Ch'en Mei-tung 1993, 254–60.

Historically the most problematic aspect of this section has been the meaning of the difference between the two Standard Crossings, which, the author tells us laconically, falls short of 14⅔ *tu.* This, it turns out, refers to the maximum value of the Elongation Difference, $14^t.66$ (= 14°27’), described with equal brevity in the Canon, 4.13. No one was able to explain it until Huang Tsung-hsi's 黃宗羲 *Shou-shih li ku* 授時曆故, of the mid seventeenth century. Actually, the evaluator is not at all obscure. The Elongation Difference is, as he says, the difference in right ascension between the equinoctial point (Standard Crossing of ecliptic and equator) and the Standard Crossing of the moon's path and the equator (see figure 22). The author does not say that this disparity is maximal when the descending node, or Standard Crossing, is located at the preceding solstitial point.

Fig. 22. The Elongation Difference, the equatorial arc between the intersection of the ecliptic and the equator (E, the equinoctial point) and the intersection of the lunar orbit and the equator (I).



The confusion of later scholars about the meaning of the Elongation Difference is understandable. This arc was of no interest in earlier Chinese astronomy (nor in the European tradition), and came to be understood only by a greater effort of geometric visualization—not necessarily of spherical geometry, but of some approximation to it—than pre-Yuan astronomers, and most post-Yuan astronomers, were called upon to make. There is no doubt, however, that the authors of the Season-granting system learned to visualize angles in space, and to compute them with their arc-sagitta proto-trigonometry.

The last paragraph is concerned with the difference between the length of the arc along the White Way from the moon to the nearest lunar node, and that of the equatorial projection of this arc. The difference between the two varies between $3^{t}.50$ (3°27’) and $1^{t}.30$ (1°17’), it says, according to whether the moon’s path is outside or inside the area between the ecliptic and the equator.

That determines whether its latitudinal arc adds to or subtracts from the obliquity of the ecliptic.]

8. Day and Night Marks

When the sun has risen it is day; when the sun has set it is night. One cycle of day and night together is one hundred marks. Dividing this by the twelve double-hours, in each double-hour there are 8⅓ marks. This is the same everywhere, regardless of whether one is in the north or south. If the day is shorter the night is longer; if the night is shorter the day is longer. This is the pattern, and it is spontaneously so.

At the Vernal and Autumnal Divisions the sun is on the Red Way as it rises and sets.

[That is, it is at the intersection of the ecliptic and the equator.]

Day and night are precisely equal, each 50 marks long. From Vernal Division to the summer solstice, the sun enters within [i.e. moves to the north of] the Red Way, gradually moving nearer to the pole; the nights grow shorter and the days longer. From Autumnal Division to the winter solstice, the sun emerges outside the Red Way, gradually moving further from the pole; the days grow shorter and the nights longer. If we measure at the center of the world, the longest day does not exceed 60 marks and the shortest does not exceed 40 marks.

To the south of the center of the world, at the summer solstice [the north pole] is further from where the sun rises and sets, so there can be longest days of less than 60 marks. At the winter solstice it is closer to where the sun rises and sets, so there can be shortest days not limited to 40 marks. To the north of the center of the world at the summer solstice [the pole] is closer to where the sun rises and sets, so there can be longest days not limited to 60 marks. At the winter solstice it is further from where the sun rises and sets, so there can be shortest days of less than 40 marks.

Now at the capital, at the winter solstice the sun rises in the 5th double-hour, beginning half, 2 marks, and sets in the 9th double-hour, standard half, 2 marks. Thus day marks is 38 and night marks is 62. At the summer solstice the sun rises in the 3rd double-hour, standard half, 2 marks, and sets in the 11th double-hour, beginning half, 2 marks. Thus day marks is 62 and night marks is 38. It would seem that [changes in the site of observation] from north to south and [consequent differences] in the altitude of the pole are responsible for variations in the time of sunrise and sunset. In the Season-Granting system day and night marks are based uniformly on the capital as the standard. Actual measurements of the height of the north pole at various places are to be found in the "Treatise on Astrology."

{1149–50; END OF *YUAN SHIH,* CHAPTER 52}

[This section reflects section 5 of the Canon, especially the determination of day length in 5.4 and the calculation of night length for places outside the capital by extrapolation from solstitial night length in 5.9. For the list of polar altitudes and solstitial shadow lengths, see the *Yuan History*, 48: 1000–1, and appendix B, table B.1. The risings and settings of the sun at Ta-tu correspond to 4:29 A.M. and 7:29 P.M. on the summer solstice, for a day length of fifteen hours by modern reckoning. Those given for the winter solstice correspond to 7:29 A.M. and 4:29 P.M. for a day length of nine hours.]

8 Evaluation of the Season-Granting System, Part 2

9. Eclipses

The test of an astronomical system's exactitude is its treatment of eclipses. In this art of pacing the celestial motions, exactitude is hard to come by. There is always [uncertainty about] whether the [predicted] time of day is early or late, and whether the immersion is too shallow or too deep.

[In other words, uncertainty about the time and magnitude of the eclipse.]

If exact agreement [with the phenomena] be the goal, there can be no room for happenstance *(ou-jan* 偶然). Calculation of the exact time of eclipses must be based on the corrected motions of the sun and moon *(ch'an li t'iao-nü* 躔離朓朒). Determination of Eclipse Parts must be based on distance to the crossings [i.e. the lunar nodes]. When expansion or contraction in *Ch'i* Entry, or slackening or hastening in Revolution entry, is not correct, the predicted conjunction is bound to be either early or late. And if the conjunction errs, early or late, how can the double-hour and mark [predicted] for the eclipse be exact?

Sun and moon both travel eastward, but the sun is slower, the moon quicker. When the moon overtakes the sun, that is one meeting *[hui* 會, i.e., conjunction]. On the way to the crossing [i.e., a lunar node] there are a yang argument and a yin argument. The meeting [of sun and moon] at a crossing may take place before or after its midpoint. Added to this are variations in the locale of sighting—north-south, east-west—and differences in observers' eyes—higher or lower, perpendicular or aslant. That is why, with respect to the magnitude of Eclipse Parts, astronomers have not attained unanimity with respect to principles.

But now, with conjunctions correct, there are no longer discrepancies of earlier or later in the time of day. With *ch'i* and marks right on center, there is no error on the strong or weak side in Eclipse Parts. Calculating backward, the *Songs,* the *Documents,* the *Spring and Autumn Annals,* or [writings] from the Three States on [i.e., up to the beginning of the empire], do not record an eclipse with which [the Season-Granting system] fails to agree. Since it agrees with what is past, when used indefinitely into the future it can without further ado remain free of defect. {53: 1153–54}

[The argument of this introduction, which draws on the concepts of the Canon, section 6, is best explained by paraphrase: Eclipse prediction is the primary test of any astronomical system. Accurate prediction of time (double-hour and mark) and magnitude (Eclipse Parts) depend on successfully incorporating a number of factors into an eclipse theory. One is the position of apparent conjunction. This is in turn based on knowledge of the apparent lunar and solar motions: expansion/contraction (solar equation of center) in the tropical year cycle (*Ch'i* Entry), and slackening/hastening (lunar equation of center) in the anomalistic month cycle (Revolution Entry). Another important variable is proximity to the lunar nodes. The nodes, which the Season-Granting system calls the Standard Crossing and the Intermediate Crossing, are the intersections of the ecliptic and the lunar orbit, where the right ascensions and declinations of the sun and moon are equal. The Yuan astronomers measured the distance from the nodes with an argument in yin and yang sequences (Canon, 6.11). Other factors to be considered are the site for which the prediction is made and variations in parallax that depend on the angle of observation. The eclipse theory of the Season-Granting system, the Evaluation tells us, has overcome all these difficulties. The success of tests comparing computations of past eclipses with observational records implies accuracy equally far into the future. This assumes, of course, that past and future are, astronomically speaking, complementary.]

9.1. Two Solar Eclipses Recorded in the *Songs* and the *Documents*

[The two eclipses cited here are recorded in canonic ancient writings, the *Documents of Antiquity (Shang shu* 尚書, later also called *Shu ching* 書經) and the Mao recension of the *Book of Songs* (*Mao shih* 毛詩, also called *Shih ching* 詩經). Neither record was set down for astronomical purposes. Classicists have understood both of these records as reflecting consternation due to unanticipated solar eclipses.]

1. The *Book of Documents,* "The Expedition to Yin" (Yin cheng 殷征): "When Chung-k'ang established his dominion over all within

the Four Seas . . . On the first day of the third month of autumn, the stars *(ch'en* 辰) did not come together in House."

<We note that the Great Expansion system [#42, 728] places the eclipse in year 5 of Chung-k'ang's accession to the throne, s.y. 30. The interval to s.y. 18 [1281] is 3408 years. For the first day of month 9, s.d. 47, the Supernumerary Crossing is 26.5421 days, and falls within the Eclipse Limit.>

[The interval cited places this first eclipse in 2128 BC. The exact date is beside the point. Critical scholars now believe that only a few of the documents in the book are as early as circa 1040 B.C. Critical scholars now consider this particular chapter of the "old text" classic (Legge 1861-72, 3: 162, 165), despite its archaic language, a forgery of the early fourth century A.D. (see, for instance, Loewe 1993, 376–77). The understanding of "the stars did not come together" as meaning that a predicted eclipse failed to occur would be unlikely even were this quotation authentic.

The "Comprehensive Crossing *(fan chiao* 泛交)" is Crossing Entry Comprehensive Days *(ju chiao fan jih* 入交汎日), or days elapsed in the current nodical month (Canon, 6.1.1; *fan* is written differently there). The value given here is only 0.6791 days from the Standard Crossing, the descending node.]

2. The *Book of Songs*, the Lesser Odes, "Concourse of the Tenth Month" (Hsiao ya, "Ssu yueh chih chiao 小雅 四月之交") in which the great officers lampooned King Yu: "At the concourse *(chiao* 交) of the tenth month, on the first day, the 28th sexagenary day, the sun was eclipsed, most ominous."

<We note that Yü K'uo 虞𠠎, the Grand Astrologer of the Liang period, asserted that the first day of month 10, s.d. 28, was a conjunction in year 6 of King Yu, s.y. 2. The Great Expansion system also took this to be true. Computing on the basis of the Season-Granting system, on the new moon of month 10, s.d. 28, the Supernumerary Crossing is 14.5709 days, and falls within the Eclipse Limit.>{1154}

[A *Book of Songs* collection was evidently written down at some unknown time between 1000 and 600 B.C. (Loewe 1993, 415). This passage occurs in a ritual ode, no. 193 (Karlgren 1950, 138). The date given in the Evaluation's note corresponds to 6 September 776 B.C. Its computation puts the "concourse," which it understands as the conjunction, only 0.1944 day from the Intermediate Crossing, or ascending node. Yü K'uo (fl. 543–44) originated a system that was not officially adopted. The text from the Evaluation of the Great Expansion system occurs in *Hsin T'ang shu*, 27A: 625.

The political motivation of this ode is important, for it provides the only context from which to attempt dating an eclipse for which the text gives so little information. Hopeful scholars have often taken positions on whether this date is correct; for references see Hartner 1934, Nōda Chūryō 1943, 365–406, and Ch'en Tsun-kuei 1980–84, 859–64.

In any case the likelihood is vanishingly small that Chou observers recorded the eclipse of 6 September 776. Its region of centrality fell to the north of Asia, passing far above the Chou kingdom at approximately 77° north latitude. Chang P'ei-yü's computation shows that that eclipse would have been completely invisible from Ch'ang-an, and would have had a magnitude of only 0.02 at T'ai-yuan (1997, s.v.). It is impossible to take seriously the proposition that in the early eighth century astronomers could have seen anything less than a nearly total eclipse. It is quite unlikely that they could have seen one with a magnitude of 0.02 a thousand years later than that. Hartner makes a more technically persuasive argument that "the solar eclipse of 30 November 735 B.C., which attained a magnitude at Ch'ang-an of 11 inches (11/12 of the sun's diameter), is the only one that can claim to be identical with the eclipse in the *Book of Songs* (p. 234)." Chang's calculation bears out Hartner's. The debate will no doubt go on.]

9.2. Thirty-seven Solar Eclipses from the Spring and Autumn Period

[This group of eclipse records comes from the *Spring and Autumn Annals (Ch'un ch'iu* 春 秋), compiled (according to the traditional account) from annals of the state of Lu 魯, in present-day Shandong province. The period covered by this chronological list of historic events, from 722 to 481 B.C., is usually called the Spring and Autumn Period, after the book. Early followers of Confucius attributed this dated list of events to him, so that it eventually entered the state-sponsored canon. Alongside its covenants, battles, betrayals, enthronements, and state rituals, it includes stereotyped notations of solar eclipses. Even that early it is probable that they were politically significant as astrological omens. The "royal month 2" is the 2nd month in the civil calendar. At the time, the year began with the lunar month that contained the winter solstice. In 256 B.C., the state of Ch'in abolished the Chou kingship to form the first Chinese empire. Two months later, it shifted the civil first month to its permanent location, which put the solstice thereafter in month 11.

There are actually 36 records in this subsection, not 37. It omits without explanation the large partial eclipse of 19 Aug 655, listed in the annals under year 5 of Duke Hsi, In view of the number specified, it is likely that an editor or copyist inadvertently omitted this one after the Evaluation was completed.

The author's notes in this section cite the three main traditions of interpretation of the Spring and Autumn Annals, namely the Tso 左, Kung-yang 公羊, and Ku-liang 穀梁 traditions (all three named for the scholars who originated or at least transmitted them). The first primarily supplements the laconic Annals with more detailed anecdotal and historical records. It repeats eleven of the eclipse reports, and comments on others, but does not add any reports. The Kung-yang and Ku-liang traditions are more concerned (as we can see here) with working out arcane moral and political implications of the Annals—by interpreting not only precisely what the text says but also what it does not say.

Both assume that every variation in wording and every omission were clues to Confucius' subtle understanding of the events he describes. The aim of the two is therefore hidden political and moral meanings. Scholars are still debating the origins and evidentiary value of these documents. In my view, all three compilations draw indirectly on documents of the dates indicated, but were set down in their current form, a very different one, in the late fourth century B.C. or later (for discussions see Loewe 1993, 67–76, and Brooks & Brooks 1998: 8).

The Evaluation also cites commentators on the *Annals* such as Tu Yü 杜預 (A.D. 222–85), and astronomers such as Chiang Chi 姜岌 (fl. late fourth century) and I-hsing 一行 (683–727). These and other astronomers cited attempted to use early solar eclipse records as a basis for a universal chronology.

The authors accurately extracted these records from the *Ch'un-ch'iu.* As Stephenson & Yau 1992 show, the observational records are generally reliable. What proves problematic, as I will show below, is the days on which months began. Lunisolar conjunctions are not observable, and could not be predicted accurately in the Springs and Autumns period.

Following this section, table 8.1 gathers basic data on each eclipse and compares it with modern findings, for evaluation in my commentary that follows it. The locale for modern computations is Ch'ü-fu 曲阜, capital of the ancient state of Lu.]

1. In the third year of Duke Yin 隱公, s.y. 58, spring, royal month 2, s.d. 6 (23 Jan 720 BC), the sun was eclipsed.

<Tu Yü asserts that the failure to record whether it was the first day of the month was an omission of the recording officials. The *Kung-yang Tradition* says that in cases of solar eclipses, the *Annals* sometimes mentions the first day of the month and sometimes does not; sometimes it records the sexagenary day and sometimes does not; [the eclipse] sometimes errs on the early side and sometimes on the late side. "Errs on the early side" means that the first day of the month precedes [the eclipse]; "errs on the late side" means that the first day of the month follows the eclipse. The *Ku-liang Tradition* says "when the *Annals* mentions the

sexagenary day but does not mention the first day of the month, the eclipse took place at the dark of the moon [i.e. the last day of the month]."

Chiang Chi, in his critical study of solar eclipses in the *Annals,* asserts that "in this year, in month 2, the first day was s.d. 36, and it did not contain a s.d. 6; apparently there was an error in intercalation. On s.d. 6 of month 3, Nodal Elongation Parts *(ch'ü chiao fen* 去交分) fell within the Eclipse Limit." The Great Expansion system agrees with Chiang Chi. Computing according to the current Season-Granting system, in that year, first day of month 3, s.d. 6 (22 Feb 720), the time of the eclipse fell during daylight, and Nodal Elongation Parts was 26.6631 days, falling within the Eclipse Limit *(shih hsien* 食限).>

[The Evaluation paraphrases the *Kung-yang Tradition*, but quotes the *Ku-liang Tradition* (6/Yin 3/1; see also the *Han History,* 27B2: 1479). Tu (A.D. 222–84) was a major political figure and a devoted student of the *Annals* and the *Tso Tradition*. The note refers to his *Long chronology of the Spring and Autumn Annals (Ch'un-ch'iu ch'ang li* 春秋長曆).

Chiang was the author of the Triple Era System *(San chi li* 三紀曆, #13). It was official 384–417 under the Later Ch'in dynasty, but is not listed in section 11 below, since the *Yuan History* did not recognize this regime. He notes that the s.d. 6 closest to month 2 was the new moon of month 3. It was natural from the viewpoint of Chiang's time to explain this discrepancy by a misplaced intercalary month. If what should have been month 2 was designated intercalary month 1 instead, what would otherwise have been month 3 would have been recorded as the second. There may well have been an intercalary month in that year according to Chiang's system, although this was not the case according to the Season-granting and later systems.

Present knowledge of the archaic stage of Chinese astronomy suggests, however, that in the eighth century B.C. the scribe-astrologers of the various states still prepared their calendars by empirical methods—trial-and-error prediction using rough constants—so that there was as much variation from one to the other as in the later Greek city-states. Errors in conjunctions of as much as two days were by no means rare. That is the point of the note from the Kung-yang tradition, concerned with discrepancies between solar eclipses and first days of calendar months, not those between predicted and observed eclipses.

Astronomers up to the late third century A.D.—Tu Yü's time—did not think of these discrepancies as errors. They relied on the mean speeds of the sun and moon to compute conjunctions and eclipses, so they could not attain high accuracy. In order to work within the limitations of their art, they presumed that solar eclipses need not fall on the first day of the month (e.g., see the author's note to item 12 below). The “Treatise on the Five Phases” of the *History of the Former Han* records a number of eclipses as observed on the last day of the month *(hui* 晦) rather than the first *(shuo* 朔; Sivin 1969, 6). From the modern viewpoint, this signals errors in calculating the ephemeris. Since the treatise does not record eclipses on the second of the month, we can infer an accuracy of roughly ±1 day. Early astronomers, because they did not always see eclipses on the first of the month, did not associate eclipses of the sun with conjunctions. As Section 10 of the Evaluation shows, even systems considerably after the Han are far from reliable in predicting apparent conjunctions. For instance, the evaluation of the Great Expansion system of 727 (#42; *Hsin T'ang shu,* 32: 832) notes that of 93 solar eclipses recorded in the seventh through tenth centuries, 90 took place on the first day of the month, two on the last day, and one on the second.

In the Yuan notes to the remaining records, Nodal Elongation Parts is usually abbreviated *chiao fen.* Neither form is part of the Season-granting system's eclipse technique. The term belongs, rather, to an old, crude method that forecasts an eclipse if Nodal Elongation Parts falls within the eclipse limit; that is, if conjunction takes place within a certain number of days and thus of *tu* from the lunar node. See the Luminous Inception system (#8, A.D. 237) in the *Tsin History (Chin shu* 晉書), 28: 545–46. It uses the equivalent term Nodal Elongation Degrees *(ch'ü chiao tu* 去交度). On the limit inherent in the Yuan system, see p. 107 above.]

2. In year 3 of Duke Huan 桓公, s.y. 9, first day of month 7, s.d. 29 (18 Jun 709), the sun was eclipsed.

<Chiang Chi believed that since in this year, month 7, the first day was s.d. 60, and it did not contain a s.d. 29, there was also an error in intercalation. For the new moon of month 8, which was s.d. 29, Nodal Elongation Parts fell within the Eclipse Limit. The Great Expansion system agrees with Chiang Chi. Computing with the current system, in this year, new moon of month 8, s.d. 29 (17 Jul 709), the time of the eclipse fell during daylight, and the eclipse was 6.14 parts.>

[This note gives Eclipse Parts, a measure of magnitude and (by implication) duration on a scale of 10. The outcome of calculation was that the eclipse was partial. As table 8.1 indicates, this eclipse would have been total in northeast China, and was no doubt recorded for that reason.]

3. In year 17 of Duke Huan, s.y. 23, winter, first day of month 10 (10 Sep 695), the sun was eclipsed.

<The *Tso Tradition* asserts that the failure to record the sexagenary day was an omission of the recording officials. Calculating according to

the Great Expansion system, in month 11, Nodal Elongation Parts fell within the Eclipse Limit, so there must indeed have been an error in intercalation. Computing according to the current system, in this year, month 11 (10 Oct 695), the time of the eclipse fell during daylight, and Nodal Elongation Parts was 26.8560 days, falling within the Eclipse Limit.>

4. In year 18 of Duke Chuang 莊公, s.y. 42, spring, royal month 3 (15 Feb 676), the sun was eclipsed.

<The Ku-liang Tradition says that the Annals mentions neither the sexagenary day nor the first day of the month because the eclipse took place at night. Calculating according to the Great Expansion system, in this year, first day of month 5 (15 Apr 676), Nodal Elongation Parts fell within the Eclipse Limit; in month 3 no eclipse was to be expected. Computing according to the current system, in this year, first day of month 3, it was not within the Eclipse Limit. On the first day of month 5, s.d. 49, the time of the eclipse fell during daylight, and Nodal Elongation Parts fell within the Eclipse Limit. It would seem that "3 (*san* 三)" is an error for "5 (*wu* 五)."

[The quotation from the *Ku-liang Tradition* is from 64/Chuang 18/1; see also the *Han History*, 27B2: 1483. Brush-written forms of the two characters are easily confused.]

5. In year 25 of Duke Chuang, s.y. 49, first day of month 6, s.d. 8 (27 May 669), the sun was eclipsed.

< Calculating according to the Great Expansion system, on the first day of month 7, s.d. 8, Nodal Elongation Parts fell within the Eclipse Limit. Computing according to the current system, in this year, first day of month 7, s.d. 8, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 27.0489 days, falling within the Eclipse Limit. There was an error in intercalation.>

[In view of Chang's computation in table 8.1, the original record in the Annals, rather than the Season-granting System, is correct, and there is no such error.]

6. In year 26 of Duke Chuang, s.y. 50, winter, first day of month 12, s.d. 60 (10 Nov 668), the sun was eclipsed.

<Computing according to the current system, in this year, first day of month 12, s.d. 60, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 14.3551 days, falling within the Eclipse Limit.>

7. In year 30 of Duke Chuang, s.y. 54, first day of month 9, s.d. 7 (28 Aug 664), the moon was eclipsed.

<Computing according to the current system, in this year, first day of month 10, s.d. 7 (27 Sep 664), the time of the eclipse fell during daylight, and Nodal Elongation Parts was 14.4696 days, falling within the Eclipse Limit. There was an error in intercalation. The Great Expansion system agrees.>

[Here, as in item 5, the "error in intercalation" is specious; the original record, not the Season-granting System, is correct. The missing observation of the eclipse of 19 Aug 655, noted above (p. 314), fits between this item and the next.]

8. In year 12 of Duke Hsi 僖公, s.y. 10, spring, first day of royal month 3, s.d. 7 (5 Feb 648), the sun was eclipsed.

<Mr. Chiang says that on the first day of month 3 there was a transit of the node, but there should not have been an eclipse; this item must be misclassified. On the first day of the 5th month, s.d. 7 (6 Apr 648), Nodal Elongation Parts fell within the Eclipse Limit. The Great Expansion system agrees. Computing according to the current system, in this year, first day of the 5th month, s.d. 7, the time of the eclipse fell during daylight, and Nodal Elongation parts was 26.5192 days, falling within the Eclipse Limit. It would seem that "3" was an error for "5.">

9. In year 15 of Duke Hsi, s.y. 13, summer, month 5 (2 Apr 645), the sun was eclipsed.

<The *Tso Tradition* says "the failure to record the first day of the month and the sexagenary day was an omission of the recording officials." Calculating according to the Great Expansion system, on the first day of month 4, s.d. 50 (4 Mar 645), Nodal Elongation Parts fell within the Eclipse Limit; thus there was a discrepancy of one [month in] intercalation. Computing according to the current system, in this year, first day of month 4, s.d. 50, Nodal Elongation Parts was 1.1316 days, falling within the Eclipse Limit.>

[Modern calculation does not yield any eclipse corresponding to this record or any simple correction of it. For the quotation from the *Tso Tradition*, see 108/Hsi 15/6.]

10. In the epochal [i.e., first] year of Duke Wen 文公, s.y. 32, first day of month 2, s.d. 60 (5 Jan 626), the sun was eclipsed.

<Mr. Chiang says that month 2, day 1, was s.d. 31, and the month did not contain a day 60. On the first day of month 3, s.d. 60 (3 Feb 626),

[Nodal Elongation Parts] fell within the Eclipse Limit. According to the Great Expansion system this was also the case. Computing according to the current system, in this year, first day of month 3, s.d. 60, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 26.5917 days, falling within the Eclipse Limit. There was an error in intercalation.>

11. In year 15 of Duke Wen, s.y. 46, first day of month 6, s.d. 38 (27 May 612), the sun was eclipsed.

<Computing according to the current system, in this year, first day of month 6, s.d. 38, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 26.4473 days, falling within the Eclipse Limit.>

12. In year 8 of Duke Hsuan 宣公, s.y. 57, autumn, month 7, s.d. 1 (24 Jun 601), the sun was eclipsed.

<According to Tu Yü, in month 7, s.d. 1, there was an eclipse on the last day of the month. Mr. Chiang notes that on the first day of month 10, s.d. 1 (20 Sep 601), there was an eclipse. The Great Expansion system agrees. Computing according to the current system, in this year, first day of month 10, s.d. 1, the time of the eclipse fell during daylight, and the [magnitude of the] eclipse was 9.81 parts. It would seem that "7 *(ch'i* 七)" was an error for "10 *(shih* 十)."">

[This is possibly due to an error in copying the number of the month. Early forms of the characters for 7 and 10 are similar (cf. item 4).]

13. In year 10 of Duke Hsuan, s.y. 59, summer, month 4, s.d. 53 (6 Mar 599), the sun was eclipsed.

<Computing according to the current system, on the first day of this month, s.d. 53, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 14.0968 days, falling within the Eclipse Limit.>

14. In year 17 of Duke Hsuan, s.y. 6, month 6, s.d. 40 (16 May 592), the sun was eclipsed.

<Mr. Chiang says that there should not have been an eclipse on the first day of month 6, which was s.d. 41. The Great Expansion system says that in this year [the first day of] month 5 fell within the Nodal Limit 交限 [=Eclipse Limit] , whereas on the first day of month 6, s.d. 41, Nodal Elongation Parts had already exceeded the Eclipse Limit; there must be a mistake. Computing according to the current system, in this year, the first day of month 5, s.d. 12 (17 Apr 592), fell within the Eclipse Limit. On the first day of month 6, s.d. 41, the Supernumerary Crossing

was 2 days, exceeding the Eclipse Limit. The Great Expansion system is correct.>

15. In year 16 of Duke Ch'eng 成公, s.y. 23, first day of month 6, s.d. 3 (9 May 575), the sun was eclipsed.

<Computing according to the current system, in this year, first day of month 6, s.d. 3, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 26.9835 days, falling within the Eclipse Limit.>

16. In year 17 of Duke Ch'eng, s.y. 24, first day of month 12, s.d. 54 (21 Nov 574), the sun was eclipsed.

<Mr. Chiang says that in month 12, the first day was s.d. 25, and the month did not contain a day 54; there appears to have been an error in intercalation. Calculating according to the Great Expansion system, on the first day of month 11, s.d. 54 (22 Oct 574), Nodal Elongation Parts fell within the Eclipse Limit. Computing according to the current system, in this year, first day of month 11, s.d. 54, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 14.2897 days, falling within the Eclipse Limit. This agrees with the Great Expansion system.>

[According to modern computation, the first day of month 12 was indeed s.d. 54, but the eclipse, a partial (M = 0.66 at Ch'ü-fu), took place at the beginning of month 11, 22 Oct 574 (Chang P'ei-yü 1997, 17, 985).]

17. In year 14 of Duke Hsiang 襄公, s.y. 39, first day of month 2, s.d. 32 (14 Jan 559), the sun was eclipsed.

<Computing according to the current system, in this year, first day of month 2, s.d. 32, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 14.1393 days, falling within the Eclipse Limit.>

18. In year 15 of Duke Hsiang, s.y. 40, autumn, first day of month 8, s.d. 54 (29 Jun 558), the sun was eclipsed.

<Mr. Chiang says that on the first day of month 7, s.d. 54 (31 May 558), there was an eclipse; this is an error in intercalation. The Great Expansion system agrees. Computing according to the current system, in this year, first day of month 7, s.d. 54, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 26.3394 days, falling within the Eclipse Limit.>

19. In year 20 of Duke Hsiang, s.y. 45, winter, first day of month 10, s.d. 53 (31 Aug 553), the sun was eclipsed.

<Computing according to the current system, in this year, first day of month 10, s.d. 53, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 13.7600 days, falling within the Eclipse Limit.>

20. In year 21 of Duke Hsiang, s.y. 46, autumn, first day of month 7, s.d. 47 (21 Jun 552), the sun was eclipsed.

<Computing according to the current system, on the first day of this month, s.d. 47, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 14.3682 days, falling within the Eclipse Limit.>

[As table 8.1 indicates, this item actually fell on 20 August, which was a s.d. 47. It is likely that the next item, only one month later, is the result of a scribal error based on the same eclipse.]

21. In winter [of the same year], first day of month 10, s.d. 17 (19 Sep 552), the sun was eclipsed.

<Mr. Chiang says that this is an instance of eclipses in closely succeeding months *(pi yueh* 比 月), and must be a misplaced item. The Great Expansion system also considered that to be the case. Computing according to the current system, [the first day of] month 10 had already exceeded the Eclipse Limit, so that there could not have been successive eclipses. Chiang's explanation must be correct.>

22. In year 23 of Duke Hsiang, s.y. 48, spring, first day of royal month 2, s.d. 10 (5 Jan 550), the sun was eclipsed.

<Computing according to the current system, on the first day of this month, s.d. 10, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 26.5703 days, falling within the Eclipse Limit.>

23. In year 24 of Duke Hsiang, s.y. 49, autumn, first day of month 7, s.d. 1 (19 Jun 549), the sun was eclipsed totally *(chi* 既).

<Computing according to the current system, on the first day of this month, s.d. 1, the time of the eclipse fell during daylight, and the eclipse was 9.06 parts.>

[The magnitude given corresponds to 11 inches. Among the 36 eclipses this was the one of largest magnitude. According to modern computation, it, like item 2, was a total eclipse as seen from Chü-fu.]

24. On the first day of month 8, s.d. 30 (18 July 549), the sun was eclipsed.

<According to the "Treatise on Astrology" of the *History of the Former Han,* Tung Chung-shu 董仲舒 (circa 179–104? B.C.) believed that "the eclipses fell in successive months, both total *(yu chi* 又既)." The Great Expansion system says that there could not have been eclipses in successive months, and that this is among the misplaced items. Computing according to the current system, Nodal Elongation Parts do not corre-

spond, so there should not have been an eclipse. The explanation of the Great Expansion system is correct.>

[For Tung's interpretation of these two items, both of which he accepted, see *Han shu*, 27B2: 1491. In *li fen pu hsieh* 立分不叶, I emend the first word to the visually similar *chiao* 交, so that the phrase reads "Nodal Elongation Parts" as in other items.]

25. In year 27 of Duke Hsiang, s.y. 52, winter, first day of month 12, s.d. 12 (11 Nov 546), the sun was eclipsed.

<Mr. Chiang says that on the first day of month 11, s.d. 12 (13 Oct 546), Nodal Elongation Parts fell within the Eclipse Limit, so there should have been an eclipse. The Great Expansion system agrees. Computing according to the current system, in this year, on the first day of month 11, s.d. 12, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 0.0825 days, falling within the Eclipse Limit.>

26. In year 7 of Duke Chao 昭公, s.y. 3, summer, first day of month 4, s.d. 41 (18 Mar 535), the sun was eclipsed.

<Computing according to the current system, on the first day of this month, s.d. 41, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 27.0298 days, falling within the Eclipse Limit.>

27. In year 15 of Duke Chao, s.y. 11, first day of month 6, s.d. 54 (18 May 527), the sun was eclipsed.

<Computing according to the Great Expansion system, on the first day of month 5, s.d. 54 (18 Apr 527), there was an eclipse, so that there was an error of one [month in] intercalation. Computed according to the current system, in this year, first day of month 5, s.d. 54, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 13.9567 days, falling within the Eclipse Limit.>

28. In year 17 of Duke Chao, s.y. 13, summer, first day of month 6, s.d. 11 (26 April 525), the sun was eclipsed.

<Mr. Chiang says that on the first day of month 6, sexagenary day 42, Nodal Elongation Parts do not correspond, so there could not have been an eclipse. There must have been an error. The Great Expansion system says that it should have fallen on the first day of month 9 (22 Aug 525); there could not have been an eclipse in month 6, so Mr. Chiang is correct. Computing according to the current system, in this year, first day of month 9, s.d. 11, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 26.7650 days, falling within the Eclipse Limit.>

[Modern computation puts the maximum phase of this eclipse on 21 August, 1734h.]

29. In year 21 of Duke Chao, s.y. 17, first day of month 7, s.d. 19 (10 Jun 521), the sun was eclipsed.

<Computing according to the current system, on the first day of this month, s.d. 19, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 26.8794 days, falling within the Eclipse Limit.>

30. In year 22 of Duke Chao, s.y. 18, winter, first day of month 12, s.d. 10 (23 Nov 520), the sun was eclipsed.

<Computing according to the current system, on the first day of this month, s.d. 10, Nodal Elongation Parts was 14.1800 days, falling within the Eclipse Limit. Tu Yü computed [the first day] in his *Long Chronology* as s.d. 40, which is incorrect.>

[This report is correctly quoted from the *Ch'un-ch'iu,* but none of the three traditions of interpretation remark on it.]

31. In year 24 of Duke Chao, s.y. 20, summer, first day of month 5, s.d. 32 (9 Apr 518), the sun was eclipsed.

<Computing according to the current system, on the first day of this month, s.d. 32, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 26.3839 days, falling within the Eclipse Limit.>

32. In year 31 of Duke Chao, s.y. 27, first day of month 12, s.d. 48 (14 Nov 511), the sun was eclipsed.

<Computing according to the current system, on the first day of this month, s.d. 48, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 26.6128 days, falling within the Eclipse Limit.>

33. In year 5 of Duke Ting 定公, s.y. 33, spring, first day of month 3, s.d. 48 (16 Feb 505), the sun was eclipsed.

<Computing according to the current system, on the first day of month 3, s.d. 28, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 14.0334 days, falling within the Eclipse Limit.>

[The sexagenary date in the text, not that in the note, is correct.]

34. In year 12 of Duke Ting, s.y. 40, first day of month 11, s.d. 3 (22 Sep 498), the sun was eclipsed.

<Computing according to the current system, in this year, first day of month 10, s.d. 3 (24 Aug 498), the time of the eclipse fell during daylight, and Nodal Elongation Parts was 14.2622 days, falling within the Eclipse Limit. It would seem that there was an error of one [month in] intercalation.>

35. In year 15 of Duke Ting, s.y. 43, first day of month 8, s.d. 17 (22 Jul 495), the sun was eclipsed.

<Computing according to the current system, on the first day of this month, s.d. 17, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 13.7685 days, falling within the Eclipse Limit.>

36. In year 14 of Duke Ai, s.y. 57, summer, first day of month 5, s.d. 57 (19 Apr 481), the sun was eclipsed.

<Computing according to the current system, on the first day of this month, s.d. 57, the time of the eclipse fell during daylight, and Nodal Elongation Parts was 26.9201 days, falling within the Eclipse Limit.>

Computing the above solar eclipses—two recorded in the *Songs* and *Documents* and 37 from the 242 years of the Spring and Autumn period—according to the Season-Granting system, we find that only those for Duke Hsiang, year 21, first day of month 10, s.d. 17 [item 21], and year 24, first day of month 8, s.d. 30 [item 24], do not fall within the Eclipse Limit. It seems that since the beginning of astronomy there has been no such thing as eclipses in proximate months. The other 35 eclipses all took place on the first day of the month. When in some cases the Annals does not record the sexagenary day, or does not state that the eclipse fell on the first day of the month, the *Kung-yang Tradition* and *Ku-liang Tradition* take this to mean that the eclipse took place on the last or second day of the month, but they are in error. The *Tso Tradition's* understanding that the lacunae are omissions by the recording officials is correct.

As for the discrepancies of one or two months (一日二日 → 一月二月) in certain records, this is apparently due to the inaccuracy of the old systems, with their tendency toward inappropriate intercalation. Chiang Chi and I-hsing have already said what needs to be said on this subject. When Confucius wrote his books, he simply relied on contemporary astronomical records. This was of no consequence to the great principles of his work, so he did not feel obliged to go into detail as far as he might have. {1154–61}

[After a great many adjustments of date, only two records disagree significantly with computations according to the new system. Both events, the Evaluation implies, are peculiar in that they closely follow other solar eclipses (item 21 after three months, and item 24 after one). Neither, as table 8.1 shows, was a visible eclipse. The author is aware of how crude the early Chou calendar was.

Still, as I remarked at the beginning of this section, there were problems even more basic than misplaced intercalations to reckon with.

There is no evidence that Confucius was involved in the making of what, hundreds of years after his death, became classics generally accepted by scholars. Nevertheless, Yuan astronomers shared the traditional faith that he compiled the *Spring and Autumn Annals.* The evaluator felt the need to absolve Confucius of blame for perpetuating the eclipse-recording errors. He implies that the moral import of the *Annals* outweighed considerations of accuracy, and that the Sage could have corrected the details had he wished.

The sense requires the emendation in the last paragraph of the note (following Takebe, "Jujireki kaigi 授時暦解議," 2: 33b). The error is not "one or two days" (as the text would read without emendation) in any record. As explained in connection with item 1, faulty intercalation usually shifts the month number by one. A shift of two (items 4 and 8), three (item 12), or four months (item 28) may or may not be so simply explainable. We know that in the Spring and Autumn period scribes sometimes inserted two or three intercalary months in a row to make the ephemeris correspond with seasonal changes (below, section 10).

Table 8.1 summarizes the data given in this subsection, and supplements it with modern computations. The information given in each column is as follows (for details, see the notes that follow the table):

Number: As given in the translation.

Year: As listed in the Annals, Duke and regnal year.

Date, *Annals:* Corresponding B.C. date in the Julian calendar with s.d. in parentheses.

Date, SGS: Date computed using the Season-Granting system. S.d. is noted here and in the next column only if not previously provided.

Date, Modern: Closest date yielded by modern computation, from Chang 1997, pp. 959ff.

M: Decimal magnitude at Ch'ü-fu for the eclipse, from Chang (divergences in Stephenson & Yau 1992: 41 are within 0.05). M ≥ 1 is a total eclipse.

References to the notes that follow the table are in parentheses.]

Table 8.1. Solar Eclipses, 720–481 B.C.

No.	Year	Date, Annals	Date, SGS	Date, Modern	M
1	Yin 3	23 Jan 720 (6)	22 Feb 720	same as SGS	0.44
2	Huan 3	18 Jun 709 (29)	17 Jul 709	same as SGS	1.06
3	Huan 17	10 Sep 695 (7)	10 Oct 695	same as SGS	0.55
4	Chuang 18	15 Feb 676 (49)	15 Apr 676	same as SGS	0.70

No.	Year	Date, Annals	Date, SGS	Date, Modern	M
5	Chuang 25	27 May 669 (8)	25 Jun 669	same as Annals	0.90
6	Chuang 26	10 Nov 668 (60)	same	same	0.74
7	Chuang 30	28 Aug 664 (7)	27 Sep 664	same as Annals	0.85
8	Hsi 12	5 Feb 648 (7)	6 Apr 648	same as SGS	0.27
9	Hsi 15	2 Apr 645 (50)	4 Mar 645	no eclipse	(1)
10	Wen 1	5 Jan 626 (60)	3 Feb 626	same as SGS	0.79
11	Wen 15	27 May 612 (38)	same	28 Apr 612	0.86
12	Hsuan 8	24 Jun 601 (1)	20 Sep 601	same as SGS	0.88 (2)
13	Hsuan 10	6 Mar 599 (53)	same	same	0.69
14	Hsuan 17	15 May 592 (40)	17 Apr 592 (12)	same as SGS	0 (3)
15	Ch’eng 16	9 May 575 (3)	same	same	0.95
16	Ch’eng 17	21 Nov 574 (54)	22 Oct 574	same as SGS	0.66
17	Hsiang 14	14 Jan 559 (32)	same	same	0.60
18	Hsiang 15	29 Jun 558 (54)	31 May 558	same as SGS	0.37
19	Hsiang 20	31 Aug 553 (53)	same	same	0 (4)
20	Hsiang 21	21 Jun 552 (47)	same	20 Aug 552	0.70
21	same	19 Sep 552 (17)	rejects	same as above?	(5)
22	Hsiang 23	5 Jan 550 (10)	same	same	0.88
23	Hsiang 24	19 Jun 549 (1)	same	same	1.08
24	same	18 Jul 549 (30)	rejects	same as above?	(5)
25	Hsiang 27	11 Nov 546 (12)	13 Oct 546	same as SGS	0.92
26	Chao 7	18 Mar 535 (41)	same	same	0.36
27	Chao 15	18 May 527 (54)	18 Apr 527	same as SGS	0.88
28	Chao 17	24 Apr 525 (11)	22 Aug 525	21 Aug 525 (10)	0.84
29	Chao 21	10 Jun 521 (19)	same	same	0.61
30	Chao 22	23 Nov 520 (10)	same	same	0.60
31	Chao 24	9 Apr 518 (32)	same	same	0.58
32	Chao 31	14 Nov 511 (48)	same	same	0.58

No.	Year	Date, Annals	Date, SGS	Date, Modern	M
33	Ting 5	16 Feb 505 (48)	same	same	0.39
34	Ting 12	22 Sep 498 (3)	24 Aug 498	same as Annals	0.88
35	Ting 15	22 Jul 495 (17)	same	same	0.52
36	Ai 14	19 Apr 481 (57)	same	same	0.84

[**Note 1.** There was no solar eclipse visible in China between 19 Sep 647 and 11 Nov 641 B.C.

Note 2. Stephenson 1997: 226 gives M = 1.04, and treats this item as one of the three total eclipses in this series. The magnitude at Ch'ü-fu was actually 0.91.

Note 3. The eclipse of 17 Apr, an annular eclipse at between 20° and 30° south latitude, would have been invisible at Ch'ü-fu. It was apparently calculated rather than observed.

Note 4. Item 19 was invisible at Ch'ü-fu; at Hangzhou it would have been a partial of negligible magnitude, M = 0.09.

Note 5. Items 21 and 24 are evidently errors (either scribal or computational) for the eclipses recorded in items 20 and 23.

The thrust of the Evaluation's critique in subsection 9.2 has been to confirm the Season-Granting system by showing that computation using it tallies with *and corrects* the archaic records. The author cites chronologists and the Great Expansion system of five centuries earlier because of their close study of records in the early classics. He assumes that the reports and their dates in the *Annals* are based on observation, at worst affected by minor errors of intercalation or copyists' mistakes. But once we compare the Yuan data with the results of modern computation, both given in the table, we learn that neither the original records nor the Yuan computations are consistently trustworthy evidence that solar eclipses were visible in China on the dates indicated.

The original record and the Season-Granting system agree on the month of the eclipse in only seventeen of the 36 cases; in seventeen cases (mostly in the earliest records) they disagree, and in two other cases the evaluator rejects records as beyond correction (items 21 and 24). Of the seventeen disagreements, thirteen differ by one month, two by two months, and two by more than that. Modern computation agrees with the *Annals* eighteen times, agrees with the Season-granting system 26 times, once differs from it by one day, and once finds no eclipse corresponding to either source. In the seventeen cases of agreement as to month, modern computation shows that both the record and the Yuan computation are off in one case by one month (item 11), and in one case by two months (item 20).

For such early records, the month given is a less valuable datum than the sexagenary day. The numbering of months depended on schemes of intercalation that differed from one state to the next, that were not always consistently applied, and that astronomer-scribes did not yet know how to test against the conjunction. The sexagenary day, on the other hand, is based on an uninterrupted day count since at least the fifteenth century B.C. Redoubtable scholars have looked for breaks in it, but no one has found one. The author often corrects eclipse dates, but only once is able to correct a sexagenary day (item 14).

Of the 36 records, two were total eclipses, and fourteen would have been large partials (M of 0.79 or greater) at Ch'ü-fu. (It is conceivable that some or all the eclipses in this source were recorded elsewhere in the state of Lu, but the difference in M would not have been great.) Of these fourteen easily visible eclipses, in two cases only the original record is correct; in seven, the Season-Granting system is correct; in five others both are correct. Of all the 31 potentially observable at Ch'ü-fu, for two the original record is correct; for thirteen only the Season-Granting system is correct; for fifteen others both are correct; and for one neither is correct.

We can draw two conclusions, pending our scrutiny of other data. First, some records in the *Annals* were not based on observation. Second, the Season-Granting system, although capable of predicting with good accuracy solar eclipses that occur *somewhere in the world* on the first day of a given month, has not proven in this demanding subsection to be greatly reliable for determining which eclipses will be visible at a given place.]

9.3. Solar Eclipses from the Three States on

[In this and the next subsection, each item includes an observational record, calculations of the double-hour and mark of the phenomenon according to the Season-Granting system and its immediate predecessor, the Great Enlightenment system, and an evaluation of the computational results. The older system is the focus of comparison because, official when the Season-granting system was submitted, it was the obvious competitor.

These records, more recent than in 9.2, are a great deal more sophisticated. The phenomenon, unless otherwise identified, is the middle of the eclipse, "Eclipse Maximum" *(shen* 甚, *shih shen* 食甚*, shih chi* 食既*).* When the record does not specify the phase, this is also generally true. Other items report "First Loss" *(k'uei ch'u* 虧初), i.e., first contact, which begins the eclipse, or Restoration of Fullness *(fu man* 復滿), fourth contact, which ends it. The comments to this subsection again will make it clear that not all of these putative observational records are the results of observation.

The evaluation employs a scale of five grades. As outlined below (p. 340), it is verbal, but the words have quantitative meanings:

Name	Chinese	Accuracy, marks
Exact	*mi-ho* 密 合	less than 1
Close	*ch'in* 親	1–2
Fairly close	*tz'u ch'in* 次親	2–3
Off	*shu* 疏	3–4
Far off	*shu-yuan* 疏遠	4 or more

This system is not quite consistent, for when scoring computations of lunar eclipses expressed in the watch-point system (see the orientation), the evaluator uses it without modification for watches, which vary in duration up to 26 minutes.

I cite records in official histories and other sources that cast light on the entries. For additional references, see Chuang Wei-feng & Wang Li-hsing 1988, s.v. Their data make it obvious that the Evaluation depended on archival records, not on the previous Standard Histories. For instance, in eclipses during the period of division, it often cites observations from a northern dynasty when the only published records are southern, or vice versa.

Table 8.3 at the end of this subsection recapitulates important data and compares it with modern computations.]

1. Epochal year of the Manifest Might era of the Shu kingdom (Shu Chang-wu 蜀 章 武), s.y. 38, eclipse on the last day of month 6, s.d. 5, 8th double-hour (1300/1500h 5 Aug 221). Season-Granting system: Eclipse Maximum, 8th double-hour, 5 marks. Great Enlightenment system: Eclipse Maximum, 8th double-hour, 5 marks. Both are close. According to computation by both systems, s.d. 5 is the first day of month 7.

[In the final collapse of the Han dynasty, the ruler of the Shu kingdom in the far west was one of three warlords who declared themselves its successor. This eclipse took place only four months after the beginning of his reign. He used the official system of A.D. 206 (#5), still based on mean motions of the sun and moon. It thus remained possible to observe an eclipse a day earlier than the date listed in the ephemeris as the conjunction. That explains this contemporary record of an eclipse seen on the last day of the month. The two late systems are astronomically correct in making s.d. 5 the first day of month 7.

In the official histories, this eclipse first appears, not under Shu, but under the contemporary Wei kingdom, which later historians came to recognize as the legitimate dynasty *(San kuo chih* 三國志, 2: 78). The time of day is recorded much later in *Chin shu* 晉書, 17: 500.

For this and other early records that do not specify the phase, the Evaluation's assumption that the reported time corresponds to Eclipse Maximum is a guess. Since the precision of the two late systems evaluated was much greater than that available in the third century, rigid use of the test protocol undermines the evaluator's results where the five grades cannot be rigorously applied. Here, as in a number of instances below, Takebe corrects the evaluator's computations (see table 8.3).]

2. Year 3 of the Yellow Inception (Huang-ch'u 黄初) era of the Wei period, s.y. 39, eclipse on the last day of month 11, s.d. 57, eclipse in the double-hour of the southwest corner (1500/1600h 19 Jan 223). Season-Granting system: Eclipse Maximum, 9th double-hour, 2 marks. Great Enlightenment System: Eclipse Maximum, 9th double-hour, 3 marks. The Season-Granting system is close, the Great Enlightenment system fairly close. According to computation by both systems, s.d. 57 is the new moon of month 12.

[This record and the next express the time in the hybrid system discussed in the orientation (p. 83). The southwest corresponds to the trigram *k'un* 坤 of the *Book of Changes (Chou i* 周易*)*, which in turn corresponds to the second half of the eighth hour, or 1500–1600h. The word translated "corner *(wei* 維)" in this context denotes a point midway between the cardinal directions.]

3. Year 5 of the Renewed Great Penetration (Chung-ta-t'ung 中大通) era of the Liang period, s.y. 50, eclipse on the first day of month 4, s.d. 56, denary hour 3 (*ping* 丙, 1100/1200h 10 May 533). Season-Granting system: Beginning of Loss, 7th double-hour, 4 marks. Great Enlightenment system: Beginning of Loss, 7th double-hour, 4 marks. Both are close.

[This eclipse appears in *Wei shu* 魏書, 105: 2343, with the additional specification that "loss began at due south [on the sun's disk]."]

4. Epochal year of the Supreme Purity (T'ai-ch'ing 太清) era, s.y. 4, eclipse on the first day of the standard month, s.d. 36, eclipse in the 9th double-hour (1500/1700h 6 Feb 547). Season-Granting system: Eclipse Maximum, 9th double-hour, 1 mark. Great Enlightenment system: Eclipse Maximum, 9th double-hour, 3 marks. The Season-Granting system is fairly close, the Great Enlightenment system close.

[The notation of the date is idiosyncratic, since the Liang dynasty's Martial Emperor did not adopt the era name Supreme Purity until month 4 of that year (May 547). The hour specification might mean 1500/1600h if the hybrid system were in use, but that is uncertain for this era. There is evidently a misprint in one of the numbers, since the difference between "close" and "fairly close" should be only 1 mark. *Wei shu,* 105: 2344, specifies that the eclipse "began at the southwest corner [of the sun's disk]." Although the word for "corner" here *(chiao* 角) is not the same as in item 2, the concept appears to be the same.]

5. Year 8 of the Grand Establishment (T'ai-chien 太 建) era of the Ch'en period, s.y. 33, eclipse on the first day of month 6, s.d. 45, eclipse between the 4th double-hour and the 1st denary hour (circa 0600h 12 Jul 576). Season-Granting system: Eclipse Maximum, 4th double-hour, 2 marks. Great Enlightenment system: Eclipse Maximum, 4th double-hour, 4 marks. The Season-Granting system is fairly close, the Great Enlightenment system far off.

6. Epochal year of the Perpetual Thriving (Yung-lung 永 隆) era of the T'ang period, s.y. 17, eclipse on the first day of month 11, s.d. 9, Eclipse Maximum in the 6th double-hour, 4 marks (0958/1011h 27 Nov 680). Season-Granting system: Eclipse Maximum, 6th double-hour, 7 marks. Great Enlightenment system: Eclipse Maximum, 6th double-hour, 5 marks. The Season-Granting system is off, the Great Enlightenment system close.

[*Hsin T'ang shu* 新 唐 書, 32: 828, places the eclipse at 16^t in the lodge Tail (roughly 257°).]

7. Epochal year of the Opening Brilliance (K'ai-yao 開耀) era, s.y. 18, eclipse on the first day of month 10, s.d. 3, Eclipse Maximum in the 6th double-hour, beginning half (0900/1000h 16 Nov 681). Season-Granting system: Eclipse Maximum, 5th double-hour, standard half, 3 marks. Great Enlightenment system: Eclipse Maximum, 5th double-hour, standard half, 1 mark. The Season-Granting system is close, the Great Enlightenment system off.

[The same source, p. 829, puts the eclipse at 4^t in the lodge Tail (roughly 245°).]

8. Year 8 of the Successive Sage (Ssu-sheng 嗣聖) era, s.y. 28, eclipse on the first day of month 4, s.d. 39, Eclipse Maximum in the 4th double-hour, 2 marks (0529/0543h 4 May 691). Season-Granting system: Eclipse Maximum, 3d double-hour, 8 marks. Great Enlight-

enment system: Eclipse Maximum, 4th double-hour, initial mark [i.e. 0 marks]. Both are fairly close.

[Since there are only 8.33 marks in a double-hour, it is impossible that both systems err by 2 to 3 marks. This record makes sense only if the evaluator assumed that the eclipse fell at the end of the short 8th mark (1:55–1:59), the outset of the initial mark of the next double-hour.

Actually Emperor Chung-tsung's Successive Sage era ("successive" in OED sense 1b, "following another [sage] of the same kind in a regular sequence or series") was one of two ephemeral reigns. It lasted only from 23 January to 27 February 684. The official history of the T'ang puts this eclipse in year 2 of the Celestial Grant era (T'ien shou 天授) of the female usurper Wu Tse-t'ien 武則天, who took the throne and the title of emperor later in 684 *(Chiu T'ang shu* 舊唐書, 36: 1317; *Hsin T'ang shu,* 4: 91, 32: 829; the latter notice gives the position of the eclipse as 7^t in the lodge Mao, or about 53°).

In this and the next two items, the Evaluation continues the use of "Successive Sage" for nearly two decades after the era actually ended, ignoring no fewer than sixteen rapid-paced eras in the reign of China's only female emperor. The point was, perhaps, to deny her legitimacy. Chung-tsung actually reigned again after Wu's death, from 705 to 710, so that item 12 pertains to his rule as well.]

9. Year 17, s.y. 37, eclipse on the first day of month 5, s.d. 46, Eclipse Maximum in the 9th double-hour, beginning half (1500/1600h 23 May 700). Season-Granting system: Eclipse Maximum, 9th double-hour, beginning half, 2 marks. Great Enlightenment system: Eclipse Maximum, 9th double-hour, standard half, initial mark. The Season-Granting system is fairly close, the Great Enlightenment system far off.

[For this and the next two items, see *Hsin T'ang shu,* 4: 101 and 32: 829. These notices record the positions of the two eclipses as 15^t in Net (72°) and 1^t in Horn (193°) respectively.]

10. Year 19, s.y. 39, eclipse on the first day of month 9, s.d. 2, Eclipse Maximum in the 9th double-hour, 3 marks (1543/1558h 26 Sep 702). Season-Granting system: Eclipse Maximum, 9th double-hour, 1 mark. Great Enlightenment system: Eclipse Maximum, 9th double-hour, 4 marks. The Season-Granting system is fairly close, the Great Enlightenment system close.

11. Epochal year of the Luminous Dragon (Ching-lung 景 龍) era, s.y. 44, eclipse on the first day of month 6, s.d. 4, Eclipse Maximum

in the 7th double-hour, standard half (1200/1300h 4 Jul 707). Season-Granting system: Eclipse Maximum, 7th double-hour, standard half, 2 marks. Great Enlightenment system: Eclipse Maximum, 8th double-hour, beginning half, initial mark. The Season-Granting system is fairly close, the Great Enlightenment system far off.

[The era name is again historically off. The Divine Dragon era (Shen-lung 神龍) was not changed to the Luminous Dragon era until month 9 (5 Oct 707). This peccadillo seems due to carelessness rather than to a concern for dynastic legitimacy, since the interregnum of Wu Tse-t'ien ended with the beginning of the Divine Dragon era at the beginning of 705. *Hsin T'ang shu*, 32: 829, notes that the eclipse was nearly total. This would have been true only in the extreme south of China.]

12. Year 9 of the Opening Epoch (K'ai-yuan 開 元) era, s.y. 58, eclipse on the first day of month 9, s.d. 42, Eclipse Maximum in the 7th double-hour, standard half, 3 marks (1243/1258h 26 Sep 721). Season-Granting system: Eclipse Maximum, 7th double-hour, standard half, 1 mark. Great Enlightenment system: Eclipse Maximum, 7th double-hour, standard half, 2 marks. The Season-Granting system is fairly close, the Great Enlightenment system close.

[I accept the emendation of the *Yuan shih* editors (元 → 九; 53: 1164n4), based on the sexagenary year (which is incorrect for 713), and records of the same eclipse in the *Hsin T'ang shu*, 5: 128 and 32: 830. The latter do not specify double-hour or phase, but the second says that the eclipse took place at 18^t in Chariot Platform (193°).]

13. Year 6 of the Felicitous Ephemeris (Ch'ing-li 慶 曆) era of the Sung period, s.y. 23, eclipse on the first day of month 3, s.d. 18, Restoration of Fullness in the 9th double-hour, standard half, 3 marks (1643/1658h 9 Apr 1046). Season-Granting system: Restoration of Fullness, 9th double-hour, standard half, 3 marks. Great Enlightenment system: Restoration of Fullness, 9th double-hour, standard half, 1 mark. The Season-Granting system is in exact agreement, the Great Enlightenment system fairly close.

14. Epochal year of the Sovereign Protection (Huang-yu 皇祐) era, s.y. 26, eclipse on the first day of the standard month, s.d. 31, Eclipse Maximum in the standard half of the 7th double-hour (1200/1300h 5 Feb 1049). Season-Granting system: Eclipse Maximum, 7th double-hour, beginning half, 3 marks. Great Enlighten-

ment system: Eclipse Maximum, 7th double-hour, standard half, initial mark. The Season-Granting system is close, the Great Enlightenment system in exact agreement.

> [Here the Evaluation ignores (as it normally does when rounding) the 4th mark in the beginning half of a double-hour, which only subsumes two minutes. Cf. the comment to item 8. The encyclopedia *Wen-hsien t'ung k'ao* 文獻通考 (1224), 282: 2250, gives the magnitude of this eclipse as 0.1. Modern reckoning makes it 0.18 at Kai-feng, the Northern Sung capital.]

15. Year 5, s.y. 30, eclipse on the first day of month 10, s.d. 33, Eclipse Maximum in the 8th double-hour, 1 mark (1314/1329h 13 Nov 1053). Season-Granting system: Eclipse Maximum, 8th double-hour, 3 marks. Great Enlightenment system: Eclipse Maximum, 8th double-hour, initial mark. The Season-Granting system is fairly close, the Great Enlightenment system close.

16. Epochal year of the Perfect Harmony (Chih-ho 至和) era, s.y. 31, eclipse on the first day of month 4, s.d. 31, Eclipse Maximum in the 9th double-hour, standard half, 1 mark (1614/1629h 10 May 1054). Season-Granting system: Eclipse Maximum, 9th double-hour, standard half, 1 mark. Great Enlightenment system: Eclipse Maximum, 9th double-hour, standard half, 2 marks. The Season-Granting system is in exact agreement, the Great Enlightenment system close.

17. Year 4 of the Excellent Protection (Chia-yu 嘉祐) era, s.y. 36, eclipse on the first day of the standard month, s.d. 33, Restoration of Fullness in the 8th double-hour, 3 marks (1343/1358h 15 Feb 1059). Season-Granting system: Restoration of Fullness, 8th double-hour, beginning half, 2 marks. Great Enlightenment system: Restoration of Fullness, 8th double-hour, beginning half, 2 marks. Both are close.

18. Year 6, s.y. 38, eclipse on the first day of month 6, s.d. 49, Beginning of Loss in the beginning of the 8th double-hour *(wei ch'u k'o* 未初刻; 1300/1314h? 20 Jun 1061). Season-Granting system: Beginning of Loss, 8th double-hour, initial mark. Great Enlightenment system: Beginning of Loss, 8th double-hour, 1 mark. The Season-Granting system is close, the Great Enlightenment system fairly close.

[The official history notes that "the weather was cloudy and it was invisible," but a source closer to the original documents says "the Directorate of Astronomy reported 'this will be an eclipse of magnitude 0.65.' At the initial mark of the 8th double-hour, the eclipse began at the west [side of the sun's disk], but by the time its magnitude was 0.4, the sky became cloudy and darkened. There was thunder and lightning, followed shortly by the maximum phase. This is an instance of the saying about the armillary sphere: 'if obscured by clouds, an unobservable solar eclipse does not portend a calamity.'" This question of meaning was an important one, because omens reflected on the emperor's virtue; it is a matter of central concern in the compilation of eclipse records. *(Sung shih* 宋史, 53: 1083; *Sung hui yao,* "Rui i" 宋會要，瑞異, 2: 2082b). Chuang and Wang 1988, 189, obscure the meaning of the last sentence by incompletely quoting it.]

19. Year 3 of the Peace through Order (Chih-p'ing 治平) era, s.y. 43, eclipse on day 1 of month 9, s.d. 49, Eclipse Maximum in the 8th double-hour, 2 marks (1329/1343h 22 Sep 1066). Season-Granting system: Eclipse Maximum, 8th double-hour, 3 marks. Great Enlightenment system: Eclipse Maximum, 8th double-hour, 4 marks. The Season-Granting system is close, the Great Enlightenment system fairly close.

20. Year 2 of the Splendid Tranquillity (Hsi-ning 熙寧) era, s.y. 46, eclipse on the first day of month 7, s.d. 2, Eclipse Maximum in the 5th double-hour, 3 marks (0743/0758h 21 Jul 1069). Season-Granting system: Eclipse Maximum, 5th double-hour, 5 marks. Great Enlightenment system: Eclipse Maximum, 5th double-hour, 4 marks. The Season-Granting system is fairly close, the Great Enlightenment system close.

[The *Sung History,* in both the annals and the "Treatise on Astrology," records that because of cloudy weather the eclipse was unobservable *(Sung shih,* 14: 271, 52: 1083).]

21. Year 3 of the Epochal Plenty (Yuan-feng 元豐) era, s.y. 57, eclipse on day 1 of month 11, s.d. 26, Eclipse Maximum in the 6th double-hour, 6 marks (1026/1041h 14 Dec 1080). Season-Granting system: Eclipse Maximum, 6th double-hour, 5 marks. Great Enlightenment system: Eclipse Maximum, 6th double-hour, 2 marks. The Season-Granting system is close, the Great Enlightenment system far off.

[*Sung shih,* 16: 303, notes that cloudy weather also made this eclipse unobservable.]

22. Epochal year of the Sustained Sagehood (Shao-sheng 紹聖) era, s.y. 11, eclipse on the first day of month 3, s.d. 9, Eclipse Maximum in the 8th double-hour, 6 marks (1426/1441h 19 Mar 1094). Season-Granting system: Eclipse Maximum, 8th double-hour, 5 marks. Great Enlightenment system: Eclipse Maximum, 8th double-hour, 5 marks. Both are close.

[This is another error in the reign era, shared by *Sung shih* (18: 339, 52: 1083). Sustained Sagehood did not begin until 29 Apr 1094. The confusion about changes of reign is further compounded in two sources from the Liao kingdom in the north, which give different reign eras of the same Khitan monarch for this same eclipse (*Ch'i-tan kuo chih* 契丹國志, 9: 4; *Liao shih* 遼史, 25: 303).]

23–25. Epochal year of the Great Prospect (Ta-kuan 大觀) era, s.y. 24, eclipse on the first day of month 11, s.d. 49, Beginning of Loss in the 8th double-hour, 2 marks (1329/1343h 16 Dec 1107); Eclipse Maximum in the 8th double-hour, 8 marks (1455/1459h); Restoration of Fullness in the 9th double-hour, 6 marks (1626/1641h).

Season-Granting system: Beginning of Loss, 8th double-hour, 3 marks; Eclipse Maximum, 9th double-hour, initial mark; Restoration of Fullness, 9th double-hour, 6 marks.

Great Enlightenment system: Beginning of Loss, 8th double-hour, initial mark; Eclipse Maximum, 8th double-hour, 7 marks; Restoration of Fullness, 9th double-hour, 5 marks.

The Season-Granting system is close with respect to Beginning of Loss and Eclipse Maximum and in exact agreement with respect to Restoration of Fullness; the Great Enlightenment system is fairly close with respect to Beginning of Loss, and close with respect to Eclipse Maximum and Restoration of Fullness.

[Here, unlike the situation in item 8, the author distinguishes 7 marks, 8 marks, and the initial mark of the next double-hour. According to the Annals, after this eclipse, "because it did not reach the magnitude that had been forecast, Ts'ai Ching 蔡 京 and other [high ministers of state] led the officials of the court in congratulating [the emperor]" *(Sung shih,* 20: 379). Actually its magnitude at the capital was 0.79.]

26. Year 32 of the Sustained Ascendancy (Shao-hsing 紹興) era, s.y. 19, eclipse on the first day of the standard month, s.d. 5, Beginning of Loss in the 9th double-hour, beginning half (1500/1600h 17 Jan 1162). Season-Granting system: Beginning of Loss, 9th double-hour, 1 mark. Great Enlightenment system: Beginning of Loss, 8th double-hour, 7 marks. Both are close.

[The Evaluation is reading *shen ch'u* 申初 as if it were *shen ch'u k'o* 申初刻. The entries in the *Sung History* do not specify the time; one (52: 1084) says that the eclipse took place in the lodge Serving-maid (302°/313°).]

27. Year 10 of the Splendor through Simplicity (Ch'un-hsi 淳熙) era, s.y. 40, eclipse on the first day of month 11, s.d. 59, Eclipse Maximum in the 6th double-hour, standard half, 2 marks (1029/1043h 17 Nov 1183). Season-Granting system: Eclipse Maximum, 6th double-hour, standard half, 2 marks. Great Enlightenment system: Eclipse Maximum, 6th double-hour, standard half, 1 mark. The Season-Granting system is in exact agreement, the Great Enlightenment system close.

[According to a biography in the *Sung History*, the Grand Astrologer predicted that this eclipse would occur at 8^t in Heart (242°; *Sung shih*, 388: 11919).]

28. Epochal year of the Felicitous Epoch (Ch'ing-yuan 慶元) era, s.y. 52, eclipse on the first day of month 3, s.d. 23, Beginning of Loss in the 7th double-hour, beginning half, 2 marks (1129/1143h 12 Apr 1195). Season-Granting system: Beginning of Loss, 7th double-hour, beginning half, 1 mark. Great Enlightenment system: Beginning of Loss, 7th double-hour, beginning half, 2 marks. The Season-Granting system is close, the Great Enlightenment system in exact agreement.

[*Sung shih* 52: 1085 puts the eclipse in the lodge Pasture (19°/31°).]

29. Year 2 of the Excellent Serenity (Chia-t'ai 嘉泰 period, s.y. 59, eclipse on the first day of month 5, s.d. 41, Beginning of Loss in the 7th double-hour, beginning half, 1 mark (1114/1129h 23 May 1202). Season-Granting system: Beginning of Loss, 6th double-hour, standard half, 3 marks. Great Enlightenment system: Beginning of Loss, 7th double-hour, beginning half, 3 marks. Both are fairly close.

[The last sentence originally reads “close,” but I emend to restore a word that has evidently dropped out of the text (親 → 次親). Without it the scoring is obviously in error.

Feng ch’uang hsiao tu 楓窗小牘, a collection of jottings of the early thirteenth century, contains an account of an embarrassing competition related to this eclipse: “The Grand Astrologer believed that the time of the eclipse would fall in the 7th double-hour, standard half. The non-official Chao Ta-yu 趙大猷 asserted that it would occur in the 7th double-hour, beginning half, 3 marks, and its magnitude would be 0.3. An edict ordered the editorial official Chang Ssu-tsai 張嗣在 to supervise observations with the armillary sphere, and Assistant Director of the Palace Library Chu Ch’in-tse 朱欽則 and others to recheck them. The eclipse turned out as Chao said” (2: 28). As table 8.3 reveals, Chao’s prediction—evidently of Eclipse Maximum, not Beginning of Loss—was indeed the most accurate of those recorded for this eclipse. *Sung shih,* 52: 1085, situates the eclipse in the lodge Net (57/74°).]

30. Year 9 of the Excellent Order (Chia-ting 嘉定) era, s.y. 13, eclipse on the first day of month 2, s.d. 21, Eclipse Maximum in the 9th double-hour, standard half, 4 marks (1658/1700h 19 Feb 1216). Season-Granting system: Eclipse Maximum, 9th double-hour, standard half, 3 marks. Great Enlightenment system: Eclipse Maximum, 9th double-hour, standard half, 2 marks. The Season-Granting system is close, the Great Enlightenment system fairly close.

[According to *Sung shih,* 542: 1085, the eclipse took place in Hall (337/354°).]

31. Year 3 of the Protection through Simplicity (Ch’un-yu 淳祐) era, s.y. 40, eclipse on day 1 of month 3, s.d. 14, Eclipse Maximum in the 6th double-hour, beginning half, 2 marks (0929/0943h 22 Mar 1243). Season-Granting system: Eclipse Maximum, 6th double-hour, beginning half, 1 mark. Great Enlightenment system: Eclipse Maximum, 6th double-hour, beginning half, initial mark. The Season-Granting system is close, the Great Enlightenment system fairly close.

[The text does not designate the observed phase Eclipse Maximum; the modern editors of the *Yuan shih* insert it from one edition (53: 1188n5).]

32. Epochal year of the Renewed Succession (Chung-t’ung 中統) era of the present dynasty, s.y. 57, eclipse on the first day of month 3, s.d. 5, Eclipse Maximum in the 9th double-hour, standard half, 2 marks (1629/1643h 12 Apr 1260). Season-Granting system: Eclipse

Maximum, 9th double-hour, standard half, 1 mark. Great Enlightenment system: Eclipse Maximum, 9th double-hour, beginning half, 3 marks. The Season-Granting system is close, the Great Enlightenment system off.

[This record too gives an idiosyncratic date. The Renewed Succession era was not promulgated until month 5, s.d. 23, or 29 Jun 1260. Since the Yuan did not yet rule all of China, later historians refer to this as the epochal year of the Luminous Order (Ching-ting 景 定) era of the Sung (e.g., *Sung shih,* 52: 1086). The ephemerides of the rival polities were identical at that point.]

33–35. Year 14 of the Perfectly Great era, s.y. 14, eclipse on the first day of month 10, s.d. 53, Beginning of Loss in the 7th double-hour, standard half, initial mark (1200/1213h 28 Oct 1277); Eclipse Maximum in the 8th double-hour, beginning half, 1 mark (1314/1329h); Restoration of Fullness in the 8th double-hour, standard half, 2 marks (1414/1429h). Season-Granting system: Beginning of Loss, 7th double-hour, standard half, initial mark; Eclipse Maximum, 8th double-hour, beginning half, 1 mark; Restoration of Fullness, 8th double-hour, standard half, 1 mark. Great Enlightenment system: Beginning of Loss, 7th double-hour, standard half, 3 marks; Eclipse Maximum, 8th double-hour, standard half, 1 mark; Restoration of Fullness, 9th double-hour, beginning half, 2 marks.

The Season-Granting system is in exact agreement with respect to Beginning of Loss and Eclipse Maximum, and close with respect to Restoration of Fullness; the Great Enlightenment system is off with respect to Beginning of Loss, and far off with respect to Eclipse Maximum and Restoration of Fullness.

◆ ◆ ◆

In earlier times, when astronomers computed past eclipses, results that fell in the same mark as the observed eclipse were termed "in exact agreement"; those which on comparison [differed by] one mark were called "close"; those [which differed] by two marks were called "fairly close"; those [which differed] by three marks were called "off"; and those [which differed] by four marks were called "far off";

[See the table on p. 330. Because extant documents do not generally record methods of scoring predictions, we do not know when the Yuan conventions originated. The best-known previous system, the Resplendent Heaven system of Chou Ts'ung 周琮 circa 1064 (#67, 1064), uses only three grades of error. These are close *(ch'in* 親; see below, the amount shown or less), near *(chin* 近, the amount shown or less), and far *(yuan* 遠, the amount shown or more). That system employs them in a more comprehensive way than in the Yuan reform *(Sung shih, 75*: 1740).

Discrepancy in Parameter	**Close**	**Near**	**Far**
Time of eclipse, marks	2	4	5
Magnitude of eclipse, marks	1	2	3
Position of event, tu	2	3	4
Gnomon shadow length, *ch'ih*	0.2	0.3	0.4

The Evaluation refers to one or more lost systems that used five grades, evidently the predecessor of its own five.]

We have now applied the Season-Granting and Great Enlightenment systems to study critically ancient solar eclipses ranging from the epochal year of the Manifest Might era of the Later Han period down to the present dynasty, a total of 35 items. {1161–69}

[In the last paragraph, the author refers to the Manifest Might era of the Shu kingdom as if it belonged to the Later Han dynasty, which he did not do in item 1 (p. 330). Readers of history were sensitive to seeming slips of this kind; his phrasing quietly—though inconsistently—endorses the claim of Shu's ruler to be the legitimate successor of the Han. I do not know what he had in mind, but such an assertion is not likely to be inadvertent.

♦ ♦ ♦

The Evaluation, under each item above, has compared the scoring for computations according to the new system and its official predecessor. A final paragraph which sums up the results is best set out in table 8.2.

To it I have added mean scores according to the rather arbitrary equivalents shown with each rating. Unlike tables 8.3 and 8.4, this one does not correct the computations, since at this point we are concerned only with the original evaluation's rankings. I obtained mean scores by multiplying the number of discrepancies by the magnitude of each in marks, adding, and dividing by the number of scores. The lower score for the Yuan system indicates its overall superiority.

Table 8.2. Scores for Accuracy of Solar Eclipse Predictions Compared with Observations in the Evaluation

Accuracy	**A**	**B**
Exact agreement (0)	2	7
Close (1)	16	17
Fairly close (2)	8	10
Off (3)	3	1
Far Off	6	0
Mean score	1.9	1.1

A: Great Enlightenment system

B: Season-Granting system

By the third century A.D., when this eclipse series begins, there were no longer discrepancies between the month or sexagenary day of the record and the time computed using the Yuan system. Nor does the date given for each item differ, according to modern calculations, from that of the actual eclipse.

The records in subsection 9.3 are concerned not merely with the day of the eclipse, as in 9.2, but with the double-hour. They record it in diverse contemporary forms. For eclipses from the late seventh century on, they specify one or more phases of the eclipse, and more or less consistently record the time to the nearest mark.

Table 8.3 compares data from this section with the results of modern computations. Although the information it provides is in several respects comparable to that of table 8.1, changed goals of prediction and improvements in recording technique make different information pertinent.]

Table 8.3. Solar Eclipses, A.D. 221–1277

[**Number:** As given above.

Date: Julian day, month, and year

A: Phenomenon. BL=Beginning of Loss, EM=Eclipse Maximum, RF=Restoration of Fullness; enclosed in parentheses if it is the author's conjecture rather than a specification in the record.

B: Time as given in the record of observation, double-hour and mark; A = beginning half, B = standard half, H=set out in a hybrid system; see the orientation, table 2.6, for details.

C: Time as reconstructed by the evaluator, if different from B. Numbers in parentheses in this and other columns are references to notes that follow the table.

D: Time according to modern computation for the pertinent phase observable at the capital where recorded (see column J). The point of observation, if not at the capital, is unknown, and the time is not necessarily reliable. The date is given first as local time in hours and minutes, and then as converted to a time in double-hours and marks commensurate with the times in the other columns.

E: Interval from time according to modern calculation to time of observation, that is, B or C minus D, in marks. Not calculated in cases where the author had to estimate the time to the nearest mark or the phase of the eclipse (see below). Rounding of fractional marks follows the Evaluation's practice of rounding a 5-minute mark that ends a double-hour to a full one, but dropping a 2-minute mark that ends a single-hour.

F: Time calculated by the evaluator using the Season-Granting system, double-hour and marks.

G: Time calculated by Takebe when he corrects the Evaluation's computation, with page reference to "Jujireki kaigi."

H: Time calculated by the evaluator using the Great Enlightenment system, double-hour and marks.

I: Time calculated by Takebe when he corrects that given in H.

J: Greatest magnitude of the eclipse at the capital where it was recorded (see table 2.2), according to Chang P'ei-yü's modern computation. The modern name of the capital is abbreviated as follows:

H=Hangzhou 杭州

K=Kaifeng 开封
L=Luoyang 洛阳
N=Nanjing 南京
X=Xi’an 西安

Although the Han and T’ang capitals at Xi’an (then called Ch’ang-an 长安) differed slightly in location, the discrepancy is negligible in this connection.

A lower-case “x” in column J before the number indicates that the eclipse is not mapped in Stephenson & Houlden 1986 because of its low magnitude as seen from China.]

No	Date	A	B	C	D	E	F	G	H	I	J
1	5 Aug 221	[EM]	8	8,4? (1)	1411h=8,4	-	8,5		8,5	8,6 (63a)	x 0.15 L
2	19 Jan 223	[EM]	H	9,4	1514h=9,1	-	9,2		9,3	9,0 (37a)	0.95 L
3	10 May 533	[BL]	H	7,3? (2)	1045h=6,7	-	7,4		7,4	-	0.42 N
4	6 Feb 547	[EM]	9	9,0? (3)	1706h=10,0	-	9,1		9,3	-	0.69 N
5	12 Jul 576	[EM]	H	4,0	0610h=4,4	-	4,2	-	4,4	4,5 (39b)	0.57 N
6	27 Nov 680	EM	6,4	-	0917h=6,1	+3	6,7	-	6,5	-	0.49 X
7	16 Nov 681	EM	6A	6A,0	0742h=4A,2	-	5B,3	-	5B,1	-	0.63 X
8	4 May 691	EM	4,2	? (4)	(4)	-	3,8	-	4,0	4,1 (42a)	0.48 X
9	23 May 700	EM	9A	9A,0	1503h=9A,0	-	9A,2	-	9B,0	-	0.67 X
10	26 Sep 702	EM	9,3	-	1506h=9,0	+3	9,1	-	9,4	-	0.95 X
11	4 Jul 707	EM	7B,0	-	1152h=7A,3	+1	7B,2	-	8A,0	-	0.56 X

No	Date	A	B	C	D	E	F	G	H	I	J
12	26 Sep 721	EM	7B,3	-	1110h=7A,0	+7	7B,1	-	7B,2	-	0.50 X
13	9 Apr 1046	RF	9B,3	-	1554h=9A,3	+4	9B,3	-	9B,1	-	x 0.46 K
14	5 Feb 1049	EM	7B	7B,0	1211h=7B,0	-	7A,3	7B,2 (46b)	7B,0	-	x 0.18 K
15	13 Nov 1053	EM	8,1	-	1325h=8,1	0	8,3	-	8,0	-	0.68 K
16	10 May 1054	EM	9B,1	-	1617h=9B,1	0	9B,1	-	9B,2	9B,3 (47b)	0.72 K
17	15 Feb 1059	RF	8,3	8A,3 (5)	1416h=8,5	-2	8A,2	8A,1 (48a)	8A,2	8B,0 (48a)	x 0.40 K
18	20 Jun 1061	BL	8,0?	7B,4?	1259h=7,8	-	8A,0	-	8A,1	-	0.94 K
19	22 Sep 1066	EM	8,2	-	1323h=8,1	+1	8,3	-	8,4	-	0.72 K
20	21 Jul 1069	EM	5,3	-	0729h=5,2	+1	5,5	-	5,4	5,5 (50a)	0.82 K
21	14 Dec 1080	EM	6,6	-	0947h=6,3	+3	6,5	-	6,2	-	0.95 K
22	19 Mar 1094	EM	8,6	-	1501h=9,0	–3	8,5	-	8,5	8,7 (51a)	0.79 K
23	16 Dec 1107	BL	8,2	-	1321h=8,1	+1	8,3	-	8,0	-	-
24	same	EM	8,8	-	1500h=9,0	–1	9,0	-	8,7	-	0.79 K
25	same	RF	9,6	-	1624h=9,5	+1	9,6	-	9,5	-	
26	17 Jan 1162	BL	9A	9,0	1534h=9A,2		9,1	-	8,7	8,8 (52b)	0.53 H
27	17 Nov 1183	EM	6B,2	-	1013h=6B,0	+2	6B,2	-	6B,1	6B,2 (53a)	0.71 H
28	12 Apr 1195	BL	7A,2	-	1126h=7A,1	+1	7A,1	-	7A,2	7A,0 (54a)	0.59 H
29	23 May 1202	BL	7A,1	-	1109h=7A,0	+1	6B,3	-	7A,3	-	x 0.21 H
30	19 Feb 1216	EM	9B,4	-	1700h=10A,0	-1	9B,3	-	9B,2	9B,1 (55a)	x 0.13 H
31	22 Mar 1243	EM	6A,2	-	0927h=6A,1	0	6A,1	-	6A,0	-	0.87 H

No	Date	A	B	C	D	E	F	G	H	I	J
32	12 Apr 1260	EM	9B,2	-	1623h=9B,1	+1	9B,1	-	9A,2	9A,3 (56a)	0.60 H
33	28 Oct 1277	BL	7B,0	-	1209h=7B,0	0	7B,0	-	7B,3	-	-
34	same	EM	8A,1	-	1331h=8A,2 (6)	-1	8A,1	-	8B,1	-	0.90 H
35	same	RF	8B,2	-	1449h=8B,3	-1	8B,1	-	9A,2	-	-

[Notes

1. In view of the author’s evaluation, he may be interpreting B as 8,4 or 8,6.

2. The author may be reading the quantity in column B as 7,3 or 7,5.

3. As noted in the text, an error in the author’s evaluation makes his interpretation uncertain.

4. The error in the author’s evaluation makes it impossible to reconstruct his interpretation. In column D, the maximum phase of the eclipse would not have been visible at Xi’an.

5. the author may be reading B as 8,1 or 8,3; I assume he reads it as 8,3.

6. This assumes that the record came from Hangzhou, the Southern Sung capital.]

[Table 8.3 is designed to facilitate comparing early eclipse records, Sung-Yuan computations, and modern counterparts of the latter. The evaluator is sometimes careless in his computations using the Season-Granting system and its predecessor (columns F and H); in such cases, columns G and I give Takebe's recalculations. In column D, I have converted modern computations of local time for the pertinent phase at the contemporary capital to double-hours and marks to make them commensurate with the reports of observation as well as with the outcomes of the Yuan computations.

This set of records is remarkable because they give time of day as well as date, but the choice of eclipses is odd. The text does not explain why the astronomers evaluated only these events.

On the one hand, they are by no means the most prominent. Nine are below 0.5 in magnitude, so that Stephenson & Houlden 1986 does not list six of them as visible in East Asia, and Watanabe 1979 does not list seven. Some were not even observable, texts in the histories tell us, because of bad weather, and the recorded phase of item 8 would not have been visible at the capital in any circumstances (note 4).

On the other hand, the Evaluation ignores a number of exceptionally prominent events. For example, between those of large magnitude in 1243 and 1277 is one (that of 12 April 1260, M = 0.60) easily seen with the techniques official astronomers routinely used. The evaluator also ignores the eclipse of 14 May 1249, which anyone at the capital could see as nearly total (M = 0.94); only three eclipses in this list were equally large. The omitted eclipse of 25 June 1275 was total, unlike every one of the items in this subsection. It elicited a dramatic report: "… at the maximum phase, everything was dark; one could not recognize people even at close range, and birds retired to their nests. It took from the 6th to the 7th double-hour for the sun's brightness to return" *(Sung shih,* 67: 1474). Other important historiographic sources such as *Sung hui yao* cite a number of eclipses not included here, sometimes with extensive comments.

The most likely reason for the selection is that the various astronomical domains of the bureaucracy carried out their responsibilities with fluctuating vigor, and did not regularly share their data. That was the case from the early empire to the twentieth century. Shen Kua 沈括, who headed the Astronomical Bureau from 1072 to circa 1075, found his quest for new observational data stymied by obstruction from the careerists in the Bureau *(Meng ch'i pi-t'an* 夢溪筆談, item 148). The records in section 9 are simply a selection from those available in the Directorate of Astronomy when it undertook its work.

There are two notable gaps in the *Yuan History's* series. The first, between 223 and 533, is not surprising, for this was a period of division and short-lived regimes, when the transmission of information was likely to falter. The second, from 721 to 1046, is not so easy to explain, because the Standard Histories, fairly widely available, recorded many eclipses for the interim. But 721 is only shortly before I-hsing completed the Great Expansion system (#42) in 727. That system included the Evaluation's celebrated precursor. As the many references to the Great Expansion system in subsection 9.2 attest, the Yuan astronomers drew on the records prior to 727 used to test it. The Directorate's own archive lacked timed observations from that point until the mid eleventh century. In this subsection, because comparison with the Revised Great Enlightenment system is so important, the series of seven eclipses from 1180 (when it became official) to 1277 (when it was still in use) was essential to the case the Evaluation is making.

In an ingenious study of solar eclipse observations from the Ming period (1368–1644), Liu Tz'u-yuan 刘次沅 & Chuang Wei-feng 庄威凤 (1998) compare the hundred or so official records with roughly 700 in local and private writings. They find a striking difference between the two groups. The government records attempt an exhaustive account of events visible from the capital, without concern for their magnitude. Unofficial ones, which largely depend on naked-eye observation, are mainly concerned with total or easily visible near-total eclipses. Given the elaborate equipment at the imperial observatory, partial eclipses are no greater challenge than total ones, and there is no preference for the latter. Still, Liu and Chuang find errors of one kind or another in about a tenth of the official records, and about seven tenths of the rest.

The eclipse records in this subsection are much more reliably timed than the early series in subsection 9.2. There is nothing problematic in the dates of the eclipses aside from item 1, where the eclipse is listed for the correct sexagenary day but the last day of the preceding month. Techniques for alternating long and short months, that is, for calculating the mean conjunction and relating it to the 60-day cycle, were good enough by the early sixth century (item 3 on) that such errors were rare. This is not to say that a system of that period could have predicted events many centuries after or before, as the evaluator calls upon the Season-Granting system to do. Small inaccuracies in constants add up to large errors over a long stretch of time, as we have seen in subsection 2.2 of the Evaluation, but that problem does not arise here. By the late seventh century (item 6), the records more or less consistently give marks as well as double-hours.

In every case the computation according to the Season-Granting system agrees with the record of observation to within a single-hour. That makes it possible for the author, at the end of this subsection, to form an assessment crucial to the Season-Granting system's official status. He compares overall the series of computations using it with those based on its immediate predecessor. This was a conventional way of demonstrating the superiority of a new system, in which solar eclipse prediction was customarily the crucial test. The outcome is clear-cut.

It is odd that the reign eras in the Evaluation's dates are so often not the standard ones, as my notes reveal. This is not a shortcoming peculiar to this evaluation. Especially in periods of disunion or of dynastic instability, it could take several centuries for historians to settle on a politically orthodox choice. Confusion often lingers indefinitely, as the note on item 22 indicates.

♦ ♦ ♦

We can draw some interesting conclusions from this table, from both what is in it and what is not. Let me assess the value of the author's comparisons from three points of view: the value of the observations, the accuracy of the evaluator's judgments, and the implications of both for the superiority of the Season-granting system over its predecessor.

The value of the observations The reported times of observation (column B) are in some cases given only to the nearest double-hour or single-hour. In other cases, the hybrid system of notation makes the time uncertain. In these twelve instances (items 1–5, 7–9, 14, 17–18, and 26), the evaluator arbitrarily estimates a time to the nearest mark (column C), and if necessary a phase as well. But the uncertainty is irreducible. In item 4, for example, the original record specifies only that the eclipse took place in the 9th double-hour, and does not specify the phase for which it records the time. The modern calculated time for the greatest phase is 1706 hours, in the initial mark of the 10th double-hour (10.0). The evaluator also assumes, when the phase is not recorded, that the report was based on observation of the maximum, which is far from certain. Because the eclipse could have taken place at the beginning or end of the double-hour, the uncertainty of the observation could be (in Western terms) anywhere between just over six minutes and two hours. Column E therefore omits for eleven of these twelve items comparisons between the times of observation and times calculated by modern procedures (one, item 17, can be used on the basis of a reasonable assumption; see note 5 to the table).

Still, the results show that the reported times are quite accurate. For the 24 items compared, the mean divergence is about 1¼ marks, or a little over a quarter-hour. This is nearly at the limit of accuracy of a water-clock. Since the early computational systems could hardly reach this level of accuracy, we can conclude that these are in general actual records of observation. That is not necessarily true in particular cases. The accuracy improves over the six hundred years, but it is excellent for the earliest eclipses in the series with the exception of item 12—better than predictions by contemporary methods could have yielded.

The accuracy of the Evaluation's judgments Now let us move on to the Evaluation's computations using the methods of the new system. The author undercuts the value of his comparison by frequent carelessness in computation, which leads to errors of as much as 3 marks; see the corrections by Takebe in columns G and I. Only two of these affect the Season-Granting system's computations (items 14 and 17). Because both apply to approximate times that the evaluator has arbitrarily made more exact, I ignore them as well as his report. Takebe corrects thirteen of the 35 Great Enlightenment calculations, six of which I am not obliged to ignore. Whether the greater frequency of the evaluator's errors is due to his unfamiliarity with the earlier system, I cannot say.

Table 8.4 uses the same form and scoring system as the evaluator's own comparisons (table 8.2), but the sixth column, which compares original records of observation with modern computations, includes only the 24 items for which evaluations are meaningful. I follow the author's rounding practice (see p. 343), summarizing and averaging scores for the Season-Granting and Great Enlightenment systems (SGS, GES) as well as those based on Takebe's corrections of the latter. Again, lower scores indicate higher accuracy. The outcome indicates that the author's errors improved the score of the Season-Granting system, and worsened that of the Great Enlightenment system, without his knowing that that would happen. In both cases, however, the net difference is close to negligible.]

Table 8.4. Scores for Accuracy of Solar Eclipse Predictions Compared with Modern Computations

Score	SGS	SGS, corrected	GES	GES, corrected	Obs. vs. modern
Exact (0)	6	6	3	3	4
Close (1)	9	8	8	7	12
Fairly close (2)	9	10	5	7	2
Off (3)	3	2	7	8	4
Far off (4 or more)	7	8	11	9	2
Number	34	34	34	34	24
Mean score	2.3	2.4	2.9	2.8	1.1

Implications. Although the evaluator claims absolute superiority for the Season-Granting system's prediction of solstices (subsection 2.1 above), here the argument is based on preponderance (the weighting of the results is not sufficiently explicit to let us call the claim statistical). That the evaluator considered an hour of modern time reckoning "far off" is impressive. In fact, when we check the Season-Granting system's computations against modern ones, the times agree within a single-hour, and are usually much better. The later records in the series show a general improvement as the interval between the eclipse and the epoch of the Yuan system decreases.

Unlike the records of very early eclipses in the last subsection, these, we can be confident, are more generally based on actual observations, recorded with the best accuracy attainable in each period. An astrologer at the capital, guided by calculation and trained in spotting phenomena of small magnitude, could have observed every eclipse in this group (although whether this was feasible for those of small magnitude such as no. 1 remains arguable).

There remain enigmas for further investigation to untangle. In two eleventh-century cases (items 18 and 20) the records in the Standard Histories candidly note that, although the eclipses were predicted, it was impossible to observe them. It is of course possible that they were seen elsewhere. It is more likely that the observers were recording, as astrologically significant phenomena, eclipses that they had predicted but could not see. We know that both items were calculated in advance; the historians believed that they took place even though they were not observable.]

9.4 Lunar Eclipses of Former Eras

1–2. Year 11 of the Epochal Excellence era of the Sung period, s.y. 11, eclipse on the full moon of month 7, s.d. 13 (4 Sep A.D. 434), Beginning of Loss in the 4th watch, 2d call; Eclipse Totality in the 4th watch, 4th call. Season-Granting system: Beginning of Loss, 4th watch, 3d point; Eclipse Totality at the 4th watch, 4th point. Great Enlightenment system: Beginning of Loss, 4th watch, 2d point; Eclipse Totality, 4th watch, 5th point. The Season-Granting system is close with respect to Beginning of Loss and in exact agreement with respect to Eclipse Totality; the Great Enlightenment system is in exact agreement with respect to Beginning of Loss and close with respect to Eclipse Totality.

[As the orientation notes (p. 110), according to the Season-Granting system, Eclipse Totality *(shih chi* 食既) is the second phase of a lunar eclipse, the beginning of totality, rather than the third or maximal phase, Eclipse Maximum, in which the shadow reaches its greatest magnitude. Until a century earlier, however, astrologers used it for the latter. The Evaluation evidently treats Eclipse Totality in records from the fifth century on as if the term already had the Yuan meaning. The distinction between Eclipse Totality and Eclipse Maximum became standard only in 1180. The Canon, 6.16, still speaks of the Three Limits as well as the Five Limits of lunar eclipses. Items 28–30 and 39–45 below, from the years 1270–80, report a maximum of only three phases, with totality as Eclipse Maximum.

No records prior to 200 B.C. relate the distinct phases of observed lunar eclipses and their times. The astronomical systems recorded in the *History of the Liu Sung era (Sung shu* 宋書, ch. 12–13), which give some indication of practice during the period of items 1–5, predict nothing but totality, which they call Eclipse Maximum. Takebe notes that neither the Great Enlightenment nor the Season-Granting system predicts item 3, which calls its only event "Eclipse Totality," as a total eclipse ("Jujireki kaigi," 2: 58b-59b). That is the case even though (according to modern calculations) all the eclipses for which the term are used in this list, including item 3, were actually total.

Observers use "Eclipse Maximum" for the maximal phase of partial eclipses. As for item 15, which uses the ambiguous "maximum *(shen* 甚)," the record in *Sung shih*, 52: 1095, uses the unambiguous word "totality *(chi* 既)" to indicate that the eclipse was total. The Sung predictive systems, however, refer in general to the maximal phase of total eclipses as Eclipse Maximum.

In order to evaluate the accuracy of retrospective predictions in this subsection, I therefore treat listings of Eclipse Totality in the observational record as referring to the maximal phase, and convert the results of calculating Eclipse Totality back to Eclipse Maximum to make them commensurable with the record (reference to Takebe confirms the conversions; see the references in table 8.6 below). Items 34–38 are exceptions, for they record all five phases of one eclipse in the Perfectly Great era, the period of the Yuan reform.

Another point germane to evaluating these records is the usage that dates astronomical events between midnight and dawn to the preceding night. For that reason, it is necessary to correct the dates of all such events, as I do in table 8.6.

According to *Sung shu,* 12: 262, re item 2, the moon when eclipsed was at the end of 15^t in the lodge Hall.]

3. Year 13, s.y. 13, eclipse on the full moon of month 12, s.d. 30 (8 Jan 437), Eclipse Totality in the 1st watch, 3d call. Season-Granting system: Eclipse Totality, 1st watch, 3d point. Great Enlightenment system: Eclipse Totality, 1st watch, 4th point. The Season-Granting system is in exact agreement, the Great Enlightenment system close.

[The modern editors of the History have emended the day of the record (*53*: 1170n7). It reads "sexagenary day 6 己巳," but the full moon of this month, the fifteenth, fell on s.d. 30 癸巳. According to Takebe's checks, neither system predicts this eclipse as total. Neither of the two records in the history of the Liu Sung period *(Sung shu,* 12: 262–263, 13: 309) gives a sexagenary day. The first of the two is most detailed: "The [predicted] time of onset *(chia-shih* 加時)

was double-hour 10 (1700/1900h), but the eclipse began only in the beginning half of double-hour 12, and Eclipse Totality [i.e., the maximal phase] was in the 1st watch, 3d call, at 4t in Devils [RA = 7h16m]."]

4–5. Year 14, s.y. 14, eclipse on the full moon of month 11, s.d. 24 (28 Dec 437), Beginning of Loss in the 2d watch, 4th call; Eclipse Totality in the 3d watch, 1st call. Season-Granting system: Beginning of Loss, 2d watch, 5th point; Eclipse Totality, 3d watch, 2d point. Great Enlightenment system: Beginning of Loss, 2d watch, 4th point; Eclipse Totality, 3d watch, 2d point. The Season-Granting system is close with respect to both Beginning of Loss and Eclipse Totality; the Great Enlightenment system is in exact agreement with respect to Beginning of Loss and close with respect to Eclipse Totality.

[The *Sung History (Sung shu,* 12: 263; cf. 269n61) mistakenly lists this phenomenon for month 12; it offers the additional information that at totality the moon was at 38t in Eastern Well [RA = 7h19m] and the sun was at 22t in Dipper [18h35m].]

6. Year 2 of the Renewed Great Penetration (Ta-chung-t'ung 中大通) era of the Liang period, s.y. 47, lunar eclipse on the full moon of month 5, s.d. 27, 1st double-hour (2300h 25 Jun/0100h 26 Jun 530). Season-Granting system: Eclipse Maximum in the 1st double-hour, standard half, initial mark. Great Enlightenment system: Eclipse Maximum in the 1st double-hour, standard half, initial mark. Both are in exact agreement.

[Although in this instance the record gives only the double-hour, the Evaluation assumes that it refers to the outset of the standard half; in other words, it considers only the interval of 14.4 minutes following the midpoint. The assumption that this is Eclipse Maximum is also gratuitous.]

7. Year 9 of the Great Unity (Ta-t'ung 大同) era, s.y. 60, eclipse on the full moon [15th] of month 3, s.d. 42 (4 May 543), Beginning of Loss in the 3d watch, 3d call.

Season-Granting system: Beginning of Loss, 3d watch, 1st point. Great Enlightenment system: Beginning of Loss, 3d watch, 3d point. The Season-Granting system is fairly close, the Great Enlightenment system in exact agreement.

8. Year 12 of the Opening Sovereignty (K'ai-huang 開皇) era of the Sui period, s.y. 49, eclipse on the full moon [15th] of month 7,

s.d. 56 (28 August 592), Beginning of Loss in the 1st watch, 3d call. Season-Granting system: Beginning of Loss, 1st watch, 4th point. Great Enlightenment system: Beginning of Loss, 1st watch, 5th point. The Season-Granting system is close, the Great Enlightenment system fairly close.

[The Sui history *(Sui shu* 隋書, 17: 433), notes that "according to [the Great Patrimony system, #35], the motion of the moon put it at 7^t in Hall. The time predicted was the 11th double-hour (1900/2100h), when the moon would be at *ch'en t'ai-ch'iang shang* 辰太強上, with 12 5/12 of 15 parts eclipsed [M = 0.83], and obscuration beginning in the northwest. Now [the eclipse] has been observed; the shadow began, rising from the northwest, in the 1st watch, 3d rod, with exactly 2 1/12 of 3 parts eclipsed [M = 0.69]. This agrees with the notation in the ephemeris." The shadow "rose" because the Chinese convention put north at the bottom and south at the top.

The notation of the unclear passage in the last paragraph uses some adaptation of the hybrid system in table 2.6.]

9–11. Year 15, s.y. 52, eclipse on the full moon [16th] of month 11, s.d. 7 (22 Dec 595), Beginning of Loss in the 1st watch, 4th point; Eclipse Maximum in the 2d watch, 3d point; Restoration of Fullness in the 3d watch, 1st point. Season-Granting system: Beginning of Loss, 1st watch, 3d point; Eclipse Maximum, 2d watch, 2d point; Restoration of Fullness, 2d watch, 5th point. Great Enlightenment system: Beginning of Loss, 1st watch, 5th point; Eclipse Maximum, 2d watch, 3d point; Restoration of Fullness, 2d watch, 5th point. The Season-Granting system is close with respect to Beginning of Loss, Eclipse Maximum, and Restoration of Fullness; the Great Enlightenment system is close with respect to both Beginning of Loss and Restoration of Fullness, and in exact agreement with respect to Eclipse Maximum.

[According to the same source (p. 434), "the motion of the moon put it at 17^t in Well [RA = 6h5m]. The time predicted was the 12th double-hour (2100/2300h), when the moon would be at *i pan shang* 乙半上, with 9 7/12 of 15 parts eclipsed [M = 0.64], and obscuration in the northwest. That night, after the 1st watch, 4th rod, the moon began to be eclipsed at *ch'en shang* 辰上, with obscuration in the southeast. By the 2d watch, 3d rod, with the moon at at *ssu shang* 巳上, eclipsed a bit more than 2 parts of 3 [M = 0.67+], it began its rebirth, until at the 3d watch, 1st rod, the moon was at *ping shang* 丙上 and fullness was restored."]

12. Year 16, s.y. 53, eclipse on the full moon [15th] of month 11, s.d. 1 (10 Dec 596), Restoration of Fullness in the 4th watch, 3d rod. Season-Granting system: Restoration of Fullness, 4th watch, 4th point. Great Enlightenment system: Restoration of Fullness, 4th watch, 5th point. The Season-Granting system is close, the Great Enlightenment system fairly close.

[The Sui history (17: 434) gives more data, but does not date this early morning eclipse to the previous day: "Year 16, month 11, day 16, s.d. 2, the motion of the moon put it at 17^{t} in Well [R.A. as in the last item]. The time predicted was the 2d double-hour (0100/0300h), when the moon would be at *wei t'ai jo shang* 未太弱上 with 12 5/12 of 15 parts eclipsed [M = 0.83], and obscuration beginning in the southeast. On the night of the 15th [the eclipse] was observed. By the 3d watch, rod 1, the moon was at *ping shang* 丙上. When seen amongst clouds, it was eclipsed a bit more than 3 parts of 15 [M = 0.20+], with obscuration beginning due east. [When the moon] reached *ting shang* 丁上 the eclipse was total. Thereafter rebirth was from the southeast. At the 4th watch, 3d rod, the moon was at wei mo 未末, and fullness was restored. Thus [Chang] Chou-hsuan was not entirely on target."]

13. Year 12 of the Celestial Fortune (T'ien-fu 天福) era of the Later Han period [of the Five Dynasties, Feb 947 - Feb 951], s.y. 44, eclipse on the full moon [15th] of month 12, s.d. 32 (28 Jan 948), Beginning of Loss in the 4th watch, 4th point. Season-Granting system: Beginning of Loss, 4th watch, 5th point. Great Enlightenment system: Beginning of Loss, 4th watch, 1st point. The Season-Granting system is close, the Great Enlightenment system fairly close.

14. Year 4 of the Sovereign Protection era of the Sung period, s.y. 29, eclipse on the full moon [15th] of month 11, s.d. 53 (8 Dec 1052), Beginning of Loss in the 3d double-hour, 4 marks. Season-Granting system: Beginning of Loss, 3d double-hour, 2 marks. Great Enlightenment system: Beginning of Loss, 3d double-hour, 1 mark. The Season-Granting system is fairly close, the Great Enlightenment system off.

15. Year 8 of the Excellent Protection era, s.y. 40, eclipse on the full moon [16th] of month 10, s.d. 20 (8 Nov 1063), Maximum (*shen* 甚) in the 4th double-hour, 7 marks. Season-Granting system: Eclipse Maximum, 5th double-hour, initial mark. Great Enlighten-

ment system: Eclipse Maximum, 5th double-hour, initial mark. Both are close.

16–18. Year 2 of the Splendid Peace era, s.y. 46, eclipse on the full moon [14th] of intercalary month 11, s.d. 44 (30 Dec 1069), Beginning of Loss in the 12th double-hour, 6 marks; Eclipse Maximum in the 1st double-hour, 5 marks; Restoration of Fullness in the 2d double-hour, 4 marks. Season-Granting system: Beginning of Loss, 12th double-hour, 6 marks; Eclipse Maximum, 1st double-hour, 5 marks; Restoration of Fullness, 2d double-hour, 3 marks. Great Enlightenment system: Beginning of Loss, 1st double-hour, initial mark; Eclipse Maximum, 1st double-hour, 6 marks; Restoration of Fullness, 2d double-hour, 4th mark. The Season-Granting system is in exact agreement with respect to Beginning of Loss and Eclipse Maximum, and close with respect to Restoration of Fullness; the Great Enlightenment system is fairly close with respect to Beginning of Loss, close with respect to Eclipse Maximum, and in exact agreement with respect to Restoration of fullness.

19–20. Year 4, s.y. 48, eclipse on the full moon [14th] of month 11, s.d. 33 (9 Dec 1071), Beginning of Loss in the 4th double-hour, 2 marks; Maximum in the 4th double-hour, 6 marks. Season-Granting system: Beginning of Loss, 4th double-hour, initial mark; Eclipse Maximum, 4th double-hour, 5 marks. Great Enlightenment system: Beginning of Loss, 4th double-hour, 4 marks; Eclipse Maximum, 4th double-hour, 7 marks. Both are fairly close with respect to Beginning of Loss and close with respect to Eclipse Maximum.

21–23. Year 6, s.y. 50, eclipse on the full moon [15th] of month 3, s.d. 55 (24 Apr 1073), Beginning of Loss in the 12th double-hour, 1 mark; Maximum in the 12th double-hour, 6 marks; Restoration of Fullness in the 1st double-hour, 4 marks. Season-Granting system: Beginning of Loss, 11th double-hour, 7 marks; Eclipse Maximum, 12th double-hour, 5 marks; Restoration of Fullness, 1st double-hour, 3 marks. Great Enlightenment system: Beginning of Loss, 12th double-hour, 2 marks; Eclipse Maximum, 12th double-hour, 7 marks; Restoration of Fullness, 1st double-hour, 4 marks. The Season-Granting system is fairly close with respect to Beginning of Loss, and close with respect to both Eclipse Maximum and Restoration of Fullness; the Great Enlightenment system is close with respect to both Beginning of Loss and Eclipse Maximum, and in exact agreement with respect to Restoration of Fullness.

[*Wen-hsien t'ung k'ao* (285: 2261) adds: "… In the 12th double-hour, 1 mark, obscuration began in the southeast."]

24–25. Year 7, s.y. 51, eclipse on the full moon [14th] of month 9, s.d. 46 (7 Oct 1074), Beginning of Loss in the 4th watch, 5th point (circa 0288h); Eclipse Totality in the 5th watch, 3d point (circa 0412h). Season-Granting system: Beginning of Loss, 4th watch, 5th point; Eclipse Totality, 5th watch, 3d point. Great Enlightenment system: Beginning of Loss, 4th watch, 3d point; Eclipse Totality, 5th watch, 2d point. The Season-Granting system is in exact agreement with respect to both Beginning of Loss and Eclipse Totality; the Great Enlightenment system is fairly close with respect to Beginning of Loss and close with respect to Eclipse Totality.

[*Wen-hsien t'ung k'ao,* immediately after the citation in the last note, has "In the 2d double-hour, 1 mark (0114/0129h), obscuration began due east. By mark 6 (0226/0241h), it was total at 1 11/12^{t} in Mound. By the time of sunrise *(? chih ming k'o* 志明刻), restoration was not visible."

Steele 2000, 207–8, points out that Restoration of Fullness should have been visible on a day when moonset came at 0527 and sunrise at 0617. Modern computations for the two phases at 0329h and 0524h put the *Yuan shih* much closer, although far off, rather than in agreement, by the standard of the Evaluation.]

26–27. Year 4 of the Reverent Tranquillity (Ch'ung-ning 崇寧) era, s.y. 22, eclipse on the full moon [15th] of month 12, s.d. 15 (21 Jan 1106), Maximum in the 10th double-hour, 3 marks; Restoration of Fullness in the 11th double-hour, initial mark. Season-Granting system: Eclipse Maximum, 10th double-hour, 1 mark; Restoration of Fullness, 10th double-hour, 7 marks. Great Enlightenment system: Eclipse Maximum, 10th double-hour, 3 marks; Restoration of Fullness, 11th double-hour, 2 marks. The Season-Granting system is fairly close with respect to Eclipse Maximum and Restoration of Fullness; the Great Enlightenment system is in exact agreement with respect to Eclipse Maximum and fairly close with respect to Restoration of Fullness.

[According to modern computation, the sun set circa 1830h on 21 Jan, and the moon was already partly eclipsed when it rose. The maximum phase was at 1741 hrs.]

28–30. Year 7 of the Perfectly Great era of the present dynasty, s.y. 7, eclipse on the full moon [16th] of month 3, s.d. 52 (7 Apr 1270), Beginning of Loss in the 2d double-hour, 3 marks; Eclipse Maximum in the 3d double-hour, initial mark; Restoration of Fullness in the 3d double-hour, 6 marks. Season-Granting system: Beginning of Loss, 2d double-hour, 2 marks; Eclipse Maximum, 3d double-hour, initial mark; Restoration of Fullness, 3d double-hour, 6 marks. Great Enlightenment system: Beginning of Loss, 2d double-hour, 4 marks; Eclipse Maximum, 3d double-hour, 1 mark; Restoration of Fullness, 3d double-hour, 7 marks. The Season-Granting system is close with respect to Beginning of Loss, and in exact agreement with respect to Eclipse Maximum and Restoration of Fullness; the Great Enlightenment system is close with respect to Beginning of Loss, Eclipse Maximum, and Restoration of Fullness.

31–33. Year 9, s.y. 9, eclipse on the full moon [15th] of month 7, s.d. 8 (10 Aug 1272), Beginning of Loss in the 2d double-hour, initial mark; Eclipse Maximum in the 2d double-hour, 6 marks; Restoration of Fullness in the 3d double-hour, 3 marks. Season-Granting system: Beginning of Loss, 1st double-hour, 7 marks; Eclipse Maximum, 2d double-hour, 4 marks; Restoration of Fullness, 3d double-hour, 1 mark. Great Enlightenment system: Beginning of Loss, 2 double-hours, 2 marks; Eclipse Maximum, 2d double-hour, 6 marks; Restoration of Fullness, 3d double-hour, 2 marks. The Season-Granting system is close with respect to Beginning of Loss, and fairly close with respect to both Eclipse Maximum and Restoration of Fullness; the Great Enlightenment system is fairly close with respect to Beginning of Loss, in exact agreement with respect to Eclipse Maximum, and close with respect to Restoration of Fullness.

34–38. Year 14, s.y. 14, eclipse on the full moon [14th] of month 4, s.d. 10 (18 May 1277), Beginning of Loss in the 1st double-hour, 6 marks; Eclipse Totality in the 2d double-hour, 3 marks; Maximum in the 2d double-hour, 5 marks; Rebirth of Light in the 2d double-hour, 7 marks; Restoration of Fullness in the 3d double-hour, 4 marks. Season-Granting system: Beginning of Loss, 1st double-hour, 6 marks; Eclipse Totality, 2d double-hour, 4 marks; Eclipse Maximum, 2d double-hour, 5 marks; Rebirth of Light, 2d double-hour, 6 marks; Restoration of Fullness, 3d double-hour, 4 marks. Great Enlightenment system: Beginning of Loss, 2d double-hour, initial mark; Eclipse Totality, 2d double-hour, 7 marks; Eclipse Maximum, 2d double-hour, 7 marks; Rebirth of Light, 2d double-hour, 8 marks; Restoration of Fullness, 3d double-hour, 6 marks. The Season-Granting system is in exact agreement with respect to Beginning of Loss, Eclipse Maximum, and Restoration of Fullness, and close with respect to Eclipse Totality and Rebirth of Light; the Great Enlightenment system is fairly close with respect to Beginning of Loss, Eclipse Maximum, and Restoration of Fullness, far off with respect to Eclipse Totality, and close with respect to Rebirth of Light.

[The Great Enlightenment system's Eclipse Totality and Eclipse Maximum fall in the same mark. The scores suggest that the error is in the former, and occurred in the original Evaluation. It is odd that Takebe corrects only the computation for Rebirth of Light, making the time identical with those of Eclipse Totality and Eclipse Maximum (table 8.6, column H.]

39–41. Year 16, s.y. 16, eclipse on the full moon [16th] of month 2, s.d. 30 (29 Mar 1279), Beginning of Loss in the 1st double-hour, 5 marks; Maximum in the 2d double-hour, 2 marks; Restoration of Fullness in the 2d double-hour, 7 marks. Season-Granting system: Beginning of Loss, 1st double-hour, 5 marks; Eclipse Maximum, 2d double-hour, 2 marks; Restoration of Fullness, 2d double-hour, 7 marks. Great Enlightenment system: Beginning of Loss, 1st double-hour, 7 marks; Eclipse Maximum, 2d double-hour, 3 marks; Restoration of Fullness, 2d double-hour, 7 marks. The Season-Granting system is in exact agreement with respect to Beginning of Loss, Eclipse Maximum, and Restoration of Fullness; the Great Enlightenment system is fairly close with respect to Beginning of Loss, close with respect to Eclipse Maximum, and in exact agreement with respect to Restoration of Fullness.

[The text reads "s.d. 10 (*kuei-yu* 癸酉)," but such a day does not occur in month 2. The Editors of the *Yuan shih* (53: 1175n8) note that the eclipse actually took place shortly after midnight on s.d. 31, and that a time after midnight would be dated the previous day. The Editors' point is that correcting from s.d. 10 to s.d. 30 (kuei-ssu 癸巳), unlike emending to day 31 (chia-wu 甲 午), requires a correction of only one character. It is possible that the mistake was due to copying the s.d. number from the entry for items 34–38.]

42–44. Eclipse on the full moon [14th] of month 8, s.d. 26 (21 Sep 1279), Beginning of Loss in the 2d double-hour, 5 marks; Maximum in the 3d double-hour, initial mark; Restoration of Fullness in the 3d double-hour, 4 marks. Season-Granting system: Beginning of Loss, 2d double-hour, 3 marks; Eclipse Maximum, 3d double-hour, initial mark; Restoration of Fullness, 3d double-hour, 4 marks. Great Enlightenment system: Beginning of Loss, 2d double-hour, 7 marks; Eclipse Maximum, 3d double-hour, 2 marks; Restoration of Fullness, 3d double-hour, 4 marks. The Season-Granting system is fairly close with respect to Beginning of Loss, and in exact agreement with respect to both Eclipse Maximum and Restoration of Fullness; the Great Enlightenment system is fairly close with respect to both Beginning of Loss and Eclipse Maximum, and in exact agreement with respect to Restoration of Fullness.

45. Year 17, s.y. 17, eclipse on the full moon [15th] of month 8, s.d. 21 (10 Sep 1280), in daylight, Restoration of Fullness in the 11th double-hour, 1 mark. Season-Granting system: Restoration of Fullness, 11th double-hour, 1 mark. Great Enlightenment system: Restoration of Fullness, 11th double-hour, 4 marks. The Season-Granting system is in exact agreement, the Great Enlightenment system off. {1169–76}

[The sun set about 1710 hrs, and the moon rose eclipsed.

The remainder of this subsection compares the scores of computation. They fall naturally into table 8.5, to which, like 8.2, I have added mean scores.]

Table 8.5. Scores for Accuracy of Lunar Eclipse Predictions Compared with Observations in the Evaluation

Accuracy (score)	SGS	GES
Exact agreement (0)	18	11
Close (1)	18	17

Accuracy (score)	**SGS**	**GES**
Fairly close (2)	9	14
Off (3)	0	2
Far off (4)	0	1
Mean Score	0.8	1.2

[Table 8.6, analogous to table 8.3, compares the data in this subsection with the outcomes of modern computation.

No.: As given above

Date: Julian day, month, and year. The record assigns eclipse phenomena after midnight to the preceding day

A: Phenomenon: BL=Beginning of Loss, ET=Eclipse Totality, EM=Eclipse Maximum, RL=Rebirth of Light, RF=Restoration of Fullness. In this table the evaluator infers only once a phase not given in the record (item 6, which also infers the time)

B: Time as given in the record, double-hour and mark in the form “4,3” or watch and point (or a synonym of point) in the form “4w3”

C: Time according to modern computation for the phase, observable at the capital where recorded (see column I). The date is given first as local time in hours, and then as converted to the same time units as in column B. I have used Ch’en Chiu-chin 1983a, tables 7–11, for conversion to night watches

D: Discrepancy between B and D; that is, time of observation minus time according to modern calculation, expressed in the units originally recorded

E: Time calculated using the Season-Granting system, units as in column B

F: Time calculated by Takebe when he corrects that given in E, with page reference to “Jujireki kaigi.” All of Takebe’s corrected times are in double-hours and marks

G: Time calculated using the Great Enlightenment system, units as in column B

H: Time calculated by Takebe when he corrects that given in G

I: Greatest umbral magnitude of the eclipse at the capital where it was recorded, according to modern computation; 1 or greater is total. M is followed by the name of the capital, abbreviated as in table 8.3. Numbers in parentheses are references to notes that follow the table.]

Table 8.6. Lunar Eclipses, A.D. 434–1280

No.	Date	A	B	C	D	E	F	G	H	I
1	4 Sep 434	BL	4w,2	0021h=3w,4	+3w	4w,3	4,1 (58a)	4w,2	3,5 (58b)	N
2	same	ET	4w,4	0214h=4w,4		4w,4	-	4w,5	4,4 (58b)	1.5 N
3	8 Jan 437	ET	1w,3	1944h=2w,1	-3w	1w,3	-	1w,4	-	1.0 N
4	28 Dec 437	BL	2w,4	2201h=2w,5	-1w	2w,5	-	2w,4	-	N
5	same	ET	3w,1	2353h=3w,3	-2w	3w,2	-	3w,2	-	1.3 N
6	26 Jun 530	[EM]	[1,0	2249h=12,7	+2	1B,0	-	1B,0	-	0.9 N
7	4 May 543	BL	3w,3	2345h=3w,2	+1w	3w,1	-	3w,3	-	N
8	28 Aug 592	BL	1w,3	2142h=2w,2	-4w	1w,4	2,4 (61a)?	1w,5	-	0.6 X
9	22 Dec 595	BL	1w,4	1950h=2w,1	-2w	1w,3	1,5 (62a)	1w,5	-	X
10	same	EM	2w,3	2114h=2w,4	-1w	2w,2	-	2w,3	-	0.6 X
11	same	RF	3w,1	2238h=3w,1	-	2w,5	-	2w,5	-	X
12	10 Dec 596	RF	4w,3	0232h=4w,3	-	4w,4	-	4w,5	4,4 (63a)	1.8 X
13	28 Jan 948	BL	4w,4	0346h=4w,5	-1w	4w,5	-	4w,1	5,1 (64a)	1.3 K

14	8 Dec 1052	BL	3,4	0407h=3,4	-	3,2	-	3,1	3,4 (64b)	1.7 K
15	8 Nov 1063	EM	4,7	0657h=4,8	-1	5,0	-	5,0	5,1 (65a)	1.6 K
16	30 Dec 1069	BL	12,6	2257h=12,8	-2	12,6	-	1,0	-	K
17	same	EM	1,5	0026h=1,5	-	1,5	-	1,6	-	0.8K
18	same	RF	2,4	0155h=2,3	+1	2,3	-	2,4	-	K
19	9 Dec 1071	BL	4,2	0510h=4,0	+2	4,0	-	4,4	-	K
20	same	EM	4,6	0617h=4,5	+1	4,5	-	4,7	-	0.4 K
21	24 Apr 1073	BL	12,1	2104h=12,0	+1	11,7	11,8 (66b)	12,2	-	K
22	same	EM	12,6	2224h=12,5	+1	12,5	-	12,7	-	0.6 K
23	same	RF	1,4	2344h=1,3	+1	1,3	-	1,4	-	K
24	7 Oct 1074	BL	4w,5	0257h=4w,5	-	4w,5	-	4w,3	-	K
25	same	ET	5w,3	0401h=5w,3	-1w	5w,3	-	5w,2	-	1.8 K
26	21 Jan 1106	EM	10,3	1741h=10,2	+1	10,1	-	10,3	10,5 (68a)	0.8 K
27	same	RF	11,0	1909h=11,0	-	10,7	10,8 (68a)	11,2	-	K
28	7 Apr 1270	BL	2,3	0130h=2,2	+1	2,2	-	2,4	-	

29	same	EM	3,0	0300h=3,0	-	3,0	-	3,1	-	0.8 K
30	same	RF	3,6	0430h=3,6	-	3,6	-	3,7	-	K
31	10 Aug 1272	BL	2,0	0107h=2,0	-	1,7	-	2,2	-	K
32	same	EM	2,6	0223h=2,5	+1	2,4	-	2,6	-	0.6 K
33	same	RF	3,3	0339h=3,2	+1	3,1	-	3,2	-	K
34	18 May 1277	BL	1,6	0041h=1,6	-	1,6	-	2,0	-	K
35	same	ET	2,3	0146h=2,3	-	2,4	-	2,7	-	K
36	same	EM	2,5	0222h=2,5	-	2,5	-	2,7	-	1.3 K
37	same	RL	2,7	0258h=2,8	-1	2,6	-	2,8	2,7 (70a)	K
38	same	RF	3,4	0403h=3,4	-	3,4	-	3,6	-	K
39	29 Mar 1279	BL	1,5	0030h=1,6	-1	1,5	-	1,7	-	K
40	same	EM	2,2	0137h=2,2	-	2,2	-	2,3	-	0.4 K
41	same	RF	2,7	0244h=2,7	-	2,7	2,8 (70b)	2,7	-	K
42	21 Sep 1279	BL	2,5	0222h=2,5	-	2,3	-	2,7	-	(1) K
43	same	EM	3,0	0312h=3,0	-	3,0	-	3,2	-	0.2 K

44	same	RF	3,4	0402h=3,4	-	3,4	-	3,4	-	K
45	10 Sep 1280	RF	11,1	1932h=11,2	-1	11,1	10,7 (71b)	11,4	-	1.6 K

[Note 1. Due to a scribal ellipsis, Takebe's check of these three items is missing from 2: 71b, but they are included in his total score on 2: 72a.

Since in this subsection the lunar eclipses cover roughly the same period of time as the solar eclipses in the last, one would expect the recorded and computed dates to agree here too. That turns out to be the case, keeping in mind that it was normal practice to record eclipses after midnight and before dawn under the date of the previous day. Again the authors of the Evaluation have frequently erred in their computations of time, as Takebe's corrected results in columns F and H show.]

Table 8.7. Scores for Accuracy of Lunar Eclipse Predictions

[A: Observed vs. modern computation
B: SGS vs. observed
C: SGS vs. modern
D: GES vs. observed
E: GES vs. modern

Score	**A**	**B**	**C**	**D**	**E**
Exact (0)	19	17	21	10	4
Close (1)	18	16	11	16	19
Fairly close (2)	5	10	8	12	14
Off (3)	1	1	3	5	4
Far off (4 or more)	1	0	1	1	3
Number	44	44	44	44	44
Mean score	0.7	0.9	0.9	1.3	1.6

[Table 8.7 summarizes scores for accuracy of computations using the Season-Granting and Great Enlightenment systems, and compares the results with the observational records and with modern computations. It is similar to table 8.4 for solar eclipses, except that lacunae in Takebe's recomputations (see note 1 to table 8.6) make his results useless. Of the 45 lunar eclipses, we can evaluate 44; the evaluator's guess about phase and time rules out item 6. I have not attempted to weight the scores to reflect the fact that some times are given in double-hours and others in night watches that can be nearly twice as long. Given the considerable uncertainty of modern computations, and thus the limited value of this table, there is no point in doing so.

It is much easier to predict for a lunar eclipse than for a solar eclipse that it will be visible from a given location, because an eclipse of the moon is visible from anywhere in the hemisphere of the earth facing it. It is not surprising, therefore, that the scores in this table are consistently much lower, and therefore better, than in the tables for the solar eclipses. Again the computational errors that Takebe corrects do not skew the results consistently in favor of either system.

The only clear error in prediction is a minor one; item 6 was not umbral in Nanjing or elsewhere; it was a penumbral eclipse.]

10. Corrected Conjunctions

[“Corrected” is the equivalent of “apparent” or “true” as used for celestial events in early Europe; see the orientation, p. 98. This section discusses how the conception evolved, and how the Season-Granting system employed it.]

The mean motion of the sun is one *tu*, and that of the moon 13 7/19 *tu*[, per day]. In the space of a day and a night, the moon moves ahead of the sun by 12-odd *tu*. After 29 days and 53 marks [i.e., 29.53 days] of travel, it catches up with the sun once more and they share the same *tu*. This is what one calls the regular conjunction. Calling it that indicates that the actual coincidence of sun and moon (*ho-shuo* 合朔) does not greatly differ from it.

[On ”regular conjunction,” see the orientation, p. 97.]

The sun has its expansion and contraction, and the moon its slackening and hastening [i.e., their inequalities]. Only by adding the values of expansion or contraction and of slackening or hastening to, or subtracting them from, [the location of regular conjunction] does one obtain the corrected conjunction (*ting shuo* 定朔).

The ancients, in instituting their techniques, cleaved to the simple, for they could not yet be exact. At first they made use of the regular conjunction, [alternating] one large [i.e., 30-day] with one small [29-day] month. Thus [they predicted] solar eclipses on the last(?) or second day of the month,

[The text says “on the first or second day,” which does not fit the context. I translate on the tentative hypothesis that *tsai shuo* 在朔 is a textual error for *tsai hui* 在晦.]

and lunar eclipses before or after the full moon. In the Han period Chang Heng 張衡 (A.D. 78–139) treated the slackening and hastening of the moon’s motion by dividing it into the Nine Ways [see section 7 above]. In the Liu Sung period, Ho Ch’eng-t’ien 何承天 (circa A.D. 443), on the basis of the expansion and contraction of the solar motion, determined a Corrected Minor Surplus (*ting hsiao yü* 定小餘 [i.e., a correction factor applied to the time of regular conjunction or lunar eclipse], with the result that there could be [a sequence of] three large or two small months. In the Sui, Liu Hsiao-sun 劉孝孫 and Liu Cho 劉焯 wished to follow this technique, but fashionable critics rejected [Liu Cho’s Sovereign Pole system (#42), influenced by Liu Hsiao-sun,] as eccentric, and in the end [the two] could not get it adopted for official use.

[See the discussion of this topic in the orientation, p. 77.]

In the T'ang, Fu Jen-chün 傅仁均 finally employed [the corrected conjunction in an official system, #36, 619], until, beginning in month 9 of year 19 of the Constant Contemplation era (September 645), four months in succession were large. [Because of the dismay that this caused, unofficial court] practice reverted to reckoning by the mean conjunction. In the epochal year of the Chimera Virtue era (664) Li Ch'un-feng's 李淳風 First-year Epoch system (#15) became official. From that time on, the use of the corrected conjunction became prevalent. Because the moon was repeatedly visible on the last day of the month, Li also instituted the technique of "advancing the new moon." This meant that when the Minor Surplus of the first day of the month is greater than 3/4 of the Day Divisor, [the conjunction] was to be hypothetically *(hsu* 虛) advanced one day. Later generations followed this practice.

Nevertheless, as Yü K'uo 虞劇 once said, "a month begins when [sun and moon] meet at conjunction. If their positions actually coincide, what reason is there to be doubtful about successive large months? If sun and moon remain separated, what reason is there to permit only alternate small months?" I-hsing (683–727) too said "if what happens in the sky is exactly [as predicted], what harm can there be in four large months or three small months?"

Now we simply take the day in which the exact time of the luminaries' meeting falls as that of the corrected conjunction. We do not "advance the new moon" even though the Minor Surplus falls within the Advance Limit [i.e., the conjunction falls within the last quarter of the day]. How apt men are to be content in their old usages!

[The historical issue was whether to maintain the mean lunation, with its more-or-less alternation of 29-day (small) and 30-day (large) months, or to adopt the corrected lunation, so that each month would begin with the actual day of conjunction and vary in length without any simple pattern. See the discussion in chapter 2, p. 77.

The Evaluation is incorrect when it asserts that appearances of the last crescent within 24 hours or so of the first day of the month were the main issue. More important, the apparent conjunction spaced *first and last* appearances of the crescent moon around the first of the month in a more orderly way than the mean conjunction did.

In the above discussion and its continuation below, the Evaluation is partly paraphrasing its predecessor, the Evaluation of the Great Expansion system (#42; *Hsin T'ang shu,* 27A: 596–597). Yabuuchi 1944a: 25–28, 30, 244, and 258 explicates the primary sources.]

At first, astronomical techniques depended on the mean conjunction, recognizing only the alternation of one large and one small month. This astronomers considered an unalterable aspect of their method. When they first heard the arguments [for permitting] three large months and two small months in succession, none believed them to be correct. From the beginnings of mathematical astronomy, it was not until the Chimera Virtue era (#38, 665) that the corrected conjunction officially came into use. [But the idea of allowing a succession of] four large months or three small ones is an obvious one, whether arrived at by reasoning on principles or on numerical relations. The men of T'ang found themselves no longer able to "accord with heaven *(jo t'ien* 若天)" while employing only the mean conjunction.

Coming now to the Perfectly Great era of our own dynasty, sustained discussions preceded the changes [that the new system made]. As for "advancing the new moon," the aim of that technique was merely to avoid visibility of the moon on the last day of the month. Now it is obvious that when the conjunction falls in the 10th, 11th, or 12th double-hour [1700/2300h, the last hours of the day], the interval from the 4th double-hour [0500/0700, or dawn] of the previous day is 7 to 9 double-hours (i.e., up to 19 hours). It is certainly true that advancing the new moon one day will keep the moon from being visible on the last day of the month. Nevertheless, if the conjunction should fall between the 5th and 9th double-hours, according to this technique the date should not be advanced. But the interval to the 4th double-hour of the previous day can exceed

14 or 15 *tu* [of lunar motion]; in such a case, why should one want to avoid the moon being visible on the last day of the month?

After all, the alternating visibility and invisibility of the moon is grounded in the spontaneous order of the Celestial Way *(t'ien tao chih tzu-jan* 本天道之自然), but advancing or retarding the date of conjunction arises from human arbitrariness. Is it not best to forsake the artificial and make use of the natural—that is, no longer meaninglessly to advance the conjunction—and attain the truth of the matter? Where [one can know] the perfect pattern, why be concerned about what people say? Let this serve as a principle for the cognoscenti. {1176–77}

[The last crescent may be visible as little as 9 double-hours (18.5 hours) before the apparent conjunction (see chapter 2, note 54). The point is thus that "advancing the new moon" makes less frequent, but cannot prevent, the old moon rising shortly before the sun on the last day of the month.

It is clear from the Canon, 4.6, that the Season-Granting system has rejected the compromise of six centuries earlier. The Yuan astronomers entered a conjunction falling between midnight and dawn in the ephemeris in accordance with the date of the previous day. This increases the frequency with which the moon is seen on the last day of the month (it amounts to beginning the month one day earlier). It is obvious that in the Yuan this was no longer a matter for concern.

The metaphysics of this paragraph is significant, and its language reappears later. Its theme is the contrast between the spontaneous order of the cosmos ("the perfect pattern") and human arbitrariness. *Tzu-jan* 自然, "spontaneity, spontaneous, happening without need for an external cause," means "nature" in modern Chinese, but took on that sense only in the late nineteenth century (Lloyd & Sivin 2002, 200). In early writing it may be translated "natural" only in the restricted sense of an obvious outcome.]

11. Disuse of Accumulated Years and Day Divisor

The Calendrical Art was created as a means to pace the tread and travel of the sun and moon, to determine the excess or depletion *(ying-hsu* 盈虛) of *ch'i* and lunation.

["Accumulated Years and Day Divisor *(chi-nien jih-fa* 積年日法)" refers to two characteristics of early systems that the Yuan astronomers rejected. The first is the habit of counting all cycles from a single epoch very far in the past. The second is the use of elaborate fractional parts (e.g., in a synodic month length of 29 499/940 days). The Yuan system treats the two as a unit because the interval from the grand epoch to the present is usually a common multiple of the denominators of such fractions. See, for instance, Sivin 1969: 17–19.

The "tread of the sun" and the "travel of the moon" are terms that occur in the headings of the Canon, sections 3 and 4. *Ying-hsu* is about determining whether solar and lunar intervals incorporate a fractional remainder (excess) or not (depletion) when compared with the pertinent constants, and if so, what these remainders are.]

Without considering the starting points of [all these cycles], one has no means to observe and come to know the celestial paths, and reach accord with them [through systems of prediction].

[*T'ien tao* 天道, which I translate as "the celestial paths," is ambiguous; it also means "the Way of Heaven," which implies yet deeper cosmological significance for epochs.]

But the motions of the sun and moon are not constant in speed, and the cycles of *ch'i* and lunation are irregular. The ancients, instituting their techniques, made a point of finding a [single] initial point long ago when all the constants begin. This they called the Extended Accumulation Superior Epoch (*yen chi shang yuan* 演纪上元). At that moment the sun, moon, and planets were in the same degree, adjoined [concentrically like a jade] ring and [lined up like] strung pearls (*ho pi lien chu* 合璧連珠). But this [epoch] lay far in the past, and [astronomers, from one system to the next,] increased [the interval] until it reached stupendous size. Those who came later found the [complexity of the] computations irksome. By calculating back and forth they cut down the size of these numbers, at the same time augmenting or diminishing the Day Divisor. They believed that in doing so they were producing reformed techniques. But the result was that the Accumulated Years and Day Divisors used in successive periods of history have not been commensurable.

[This discussion justifies two linked simplifications introduced in the Season-Granting system. The second, discussed below, adopted a single denominator for all fractions. The flat annular jade ring and string of pearls are conventional similes applied to the legendary great conjunction, the first for the sun and moon and the second for the planets. "Cut down the size" refers to the use of an intermediate epoch (see above, p. 74).]

This being the case, before [each system using such an epoch] had been long in use, errors gradually reappeared. That, it would seem, is because the Celestial Way is spontaneous, not to be reached hit or miss through artificial and far-fetched reasoning. In the cyclical motions of the Seven Governors through the sky, their advances and recessions have their own regular measures.

[In other words, the inequalities of the sun, moon, and planets are not ineffable, but are the result of knowable regularities.]

If one investigate from beginning to end, determining and verifying the complete cycles, then the numerical regularities of the emblematic phenomena *(hsiang-shu* 象數) become clear, with no leeway for obscurity. And then what need is there to reject simple techniques based on what is in front of one's eyes, and pursue grandiose methods involving millions and myriads of years?

Now the Season-Granting system has taken s.y. 18 of the Perfectly Great era as its epoch. The constants it employs are uniformly based in the sky [i.e., on observation]. Ten-thousandth to hundredth, hundredth to mark, mark to day, all of these divisions take 100 as their increment. By comparison with the Accumulated Years and Day Divisors of other systems, in which iteration based on forced reasoning is the product of artifice, it has succeeded in spontaneous.

["Spontaneous *(tzu-jan*)" implies oneness with the spontaneity of the cosmic order.]

Someone asked: "The ancients used to say that, as a foundation on which to erect a system of astronomy, it is necessary first to establish the epoch; after the epoch is correct one sets the Day Divisor; and when the divisor is set one measures the celestial perimeter and determines the equinoxes and solstices. This being so, the inclusion of Accumulated Years and Day Divisor in astronomical systems merits respect because of its long usage *(shang* 尚). Since the time of the Yellow Emperor, astronomical systems have transmitted [this practice as part of] their inheritance from one to the next, seventy or eighty in all. That a system could discard it and yet succeed is unheard of. Now is your hacking all this away with one slash not simply because, ignoring the foundations, [your investigations] have failed to yield the right method?"

[I read *she* 舍 in the third tone to mean "discard."]

This, we believe, is not the case. In the Tsin period Tu Yü [222–84] had a saying that "those who regulate the ephemerides ought to conform to the sky [i.e. its observable phenomena] and seek agreement with it, rather than forcing agreement to obtain validation from it." The extended accumulation *[yen chi* 演積, i.e. Superior Epoch] technique used in former eras amounts to nothing more than forcing agreement to obtain validation.

Now, because the old astronomical system has become rather imprecise, there has come a command to correct it. Techniques that lack accuracy are what must be changed; for how have we leisure to follow in the steps of old usages?

With all this in mind, we have taken the Accumulated Years and Day Divisors of the various astronomical systems since the Han, along with the number of years each was in official use, and set them out below. We have also appended [the equivalent of] Extended Epochs and Divisors [for the Season-Granting system] in order to set the doubts of certain critics at rest.

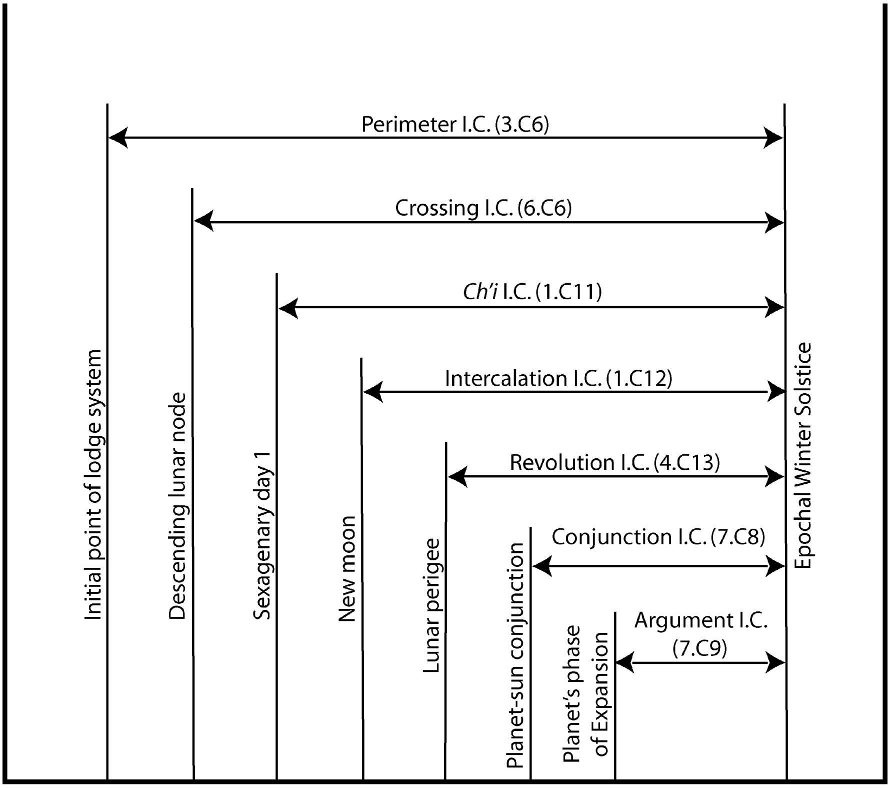
[For discussions of earlier usages involving Extended Epochs and Divisors see pp. 73 and 63 respectively. This section slightly exaggerates the originality of the Season-Granting system. There are T'ang precursors for both of the simplifications described here, but no one until the Yuan combined the two and made them standard practice.

The sole epoch in the Yuan system was the winter solstice of 14 December 1280. The astronomers determined, by observation and by calculation from empirical data, a series of constants for intervals between all of the important events that began astronomical cycles, such as sexagenary day 1 and the lunar perigee, just before the epochal solstice, and the solstice itself. They uniformly named them "interval cycles *(ying* 應)." Figure 23 shows their relationships; figures 26, 32, and 44 relate some of them to other quantities (for a detailed discussion of an analogous table, see Ch'en Mei-tung 陈美东 2003a, 221–26).

The Yuan astronomers' rejection of the Day Divisor improved on early practice in two respects. One was that their common divisor for all fractions did away with a plethora of miscellaneous divisors that added to the irksomeness of computation. For instance, in the Grand Inception system (#2), the lunation value was 29 43/81 days; the Quarter Remainder system (#4) changed it to 29

499/940 days. Second, by taking 10 000 as their common divisor, the Yuan authors made it possible to use what amounted to decimal notation.]

Fig. 23. Interval constants, which embody the time interval in day parts from beginnings of important cycles (vertical lines) to the epochal winter solstice of 14 December 1280 (vertical line at right). The constants are roughly in order of their distance from the solstice, but the last two differ for each planet.



[The list of astronomical systems that follows is far from comprehensive. The astronomers were actually constructing, to the extent that was possible, a sequence of official systems adopted by legitimate dynasties. In periods when China was divided into two or more rival regimes, historians had to choose between them. For a more comprehensive enumeration, see table 2.1 in the orientation. I have added the numbers in that table to the names below, as elsewhere, in the form #*n*.

In the list, Accumulated Years is for every system the interval from the Superior Epoch or the intermediate epoch, not to the inception of that system (which would have been more informative), but to the Yuan system's epoch. The Day Divisor is generally the divisor of the fractional part of the lunation constant (e.g., in #2, 81, from 29 43/81). This was not always the meaning of Day Divisor, since the names of constants varied considerably from one system to another. In some instances the Day Divisor is the divisor of the fractional part of the tropical year constant. For example, in #13, fractional days in the year constant is expressed as Hour Divisor *[ch'en-fa* 辰法 = 286] divided by Day Divisor *[jih-fa* 日法 = 1144], or 1/4 *(Sui shu*, 17: 435). In other cases neither divisor is so named, and the Evaluation gives the lunation divisor, whatever its creators called it. The translator's notes to each item specify the meaning where it is pertinent. The "Extended Epochs and Day Divisors" for the Season-Granting system referred to in the text above are the three sets of figures that item 44 of the list compares with observation (p. 386).

"Ahead of [or behind] the sky" in each item is the error in winter solstice prediction, expressed in marks, at the end of the period when the system was in official use. Whether the evaluator was comparing the calculated solstices with observational records is uncertain. Takebe has shown that the listed discrepancies are, with good consistency, those between the solstice as calculated by the original system and that of the Yuan. For reasons discussed in chapter 7, section 1, the precision of this computation increased markedly during the fifth century.

Where an account of the canon, including its constants and methods, survives, I cite it after the translation of the item, noting accounts that survive only in summaries.

The dates given below for the various systems do not always coincide with those that result from modern historical studies. Here I merely translate, but I have incorporated in table 2.1 several corrections based on recent research.]

1. Triple Concordance system *(San t'ung li* 三統曆, #3)

<[Adopted in the] epochal year of the Grand Inception *(T'ai-ch'u* 太初) era of the Western Han period, s.y. 14 (104 B.C.). Composed by Teng P'ing 鄧平 [and Liu Hsin 劉歆]. In official use for 188 years, until s.y. 22 of the Epochal Concord *(Yuan-ho* 元和) period of the Eastern Han (A.D. 85). Behind the sky 78 marks [i.e., 0.78 day].>

Accumulated Years: 144,511

Day Divisor: 81.

[*Han shu,* 21B: 991–1011.]

2. Quarter Remainder system *([Hou Han] ssu fen li* [後漢] 四分曆, #4)

<Year 2 of the Epochal Concord period of the Eastern Han, s.y. 22 (85). Composed by Pien Hsin 編訢. In official use 121 years, until s.y. 23

of the Established Security *(Chien-an* 建安) era (206). Behind the sky 7 marks.>

Accumulated Years: 10,561

Day Divisor: 4

[*Hou Han shu*, *chih* 3: 3058–81 .]

3. Supernal Emblem system *(Ch'ien hsiang li* 乾象曆, #5)

<Year 11 of the Established Security period, s.y. 23 (206). Composed by Liu Hung 劉洪. In official use 31 years, until s.y. 54 of the Luminous Inception *(Ching-ch'u* 景初曆) era of the Wei period (237). Behind the sky 7 marks.>

Accumulated Years: 8452

Day Divisor: 1457

[*Chin shu*, 17: 504–31.]

4. Luminous Inception system *(Ching ch'u li* 景初曆, #8)

<Epochal year of the Luminous Inception era of the Wei period, s.y. 54 (237). Composed by Yang Wei 楊偉. In official use 206 years, until s.y. 20 of the Epochal Excellence *(Yuan-chia* 元嘉) era of the Liu Sung period, s.y. 20 (443). Ahead of the sky 50 marks.>

Accumulated Years: 5089

Day Divisor: 4559

[*Chin shu*, 18: 536–62; *Sung shu*, 12: 233–58.]

5. Epochal Excellence system *(Yuan chia li* 元嘉曆, #17)

<Year 20 of the Epochal Excellence era of the Liu Sung period, s.y. 20 (443). Composed by Ho Ch'eng-t'ien 何承天. In official use twenty years, until year 7 of the Great Enlightenment *(Ta-ming* 大明) era, s.y. 40 (463). Ahead of the sky 50 marks.>

Accumulated Years: 6541

Day Divisor: 752

[*Sung shu*, 13: 271–88.]

6. Great Enlightenment system *(Ta ming li* 大明曆, #18)

<Year 7 of the Great Enlightenment era of the Liu Sung period, s.y. 40 (463). Composed by Tsu Ch'ung-chih 祖沖之. In official use 58 years, until the Orthodox Glory *(Cheng-kuang* 正光) era of the Northern Wei period, s.y. 38 (521). Behind the sky 29 marks.>

Accumulated Years: 52,757

Day Divisor: 3939

[*Sung shu*, 13: 291–304.]

7. Orthodox Glory system *(Cheng kuang li* 正光曆, #21)

<Year 2 of the Orthodox Glory era of the Later [i.e., Northern] Wei period, s.y. 38 (521). Composed by Li Yeh-hsing 李業興. In official use nineteen years, until the Ascendant Harmony *(Hsing-ho* 興和) era [of the Eastern Wei period], s.y. 57 (540). Ahead of the sky 13 marks.>

Accumulated Years: 168,509

Day Divisor: 74,952

[*Wei shu*, 107A: 2663–86.]

8. Ascendant Harmony system *(Hsing ho li* 興和曆, #23)

<Year 2 of the Ascendant Harmony era, s.y. 57 (540). Composed by Li Yeh-hsing. In official use ten years, until the Celestial Preservation *(T'ien-pao* 天保) era of the Northern Ch'i period, s.y. 7 (550). Ahead of the sky 99 marks.>

Accumulated Years: 204,737

Day Divisor: 208,530

[*Wei shu*, 107B: 2699–2723.]

9. Celestial Preservation system *(T'ien pao li* 天保曆, #26)

<Epochal year of the Celestial Preservation era of the Northern Ch'i period, s.y. 7 (550). Composed by Sung Ching-yeh 宋景業. In official use seventeen years, until the Celestial Harmony *(T'ien-ho* 天和) era of the [Northern] Chou period, s.y. 23 (566). Behind the sky 1 day and 87 marks.>

Accumulated Years: 111,257

Day Divisor: 23,660

[The canons of this and the next three systems have all disappeared, and only a few details remain. Here the Day Divisor is derived not from the length of the mean synodic month but from that of the mean tropical year.]

10. Celestial Harmony system *(T'ien ho li* 天和曆, #28)

<Epochal year of the Celestial Harmony era of the Later [i.e., Northern] Chou period, s.y. 23 (566). Composed by Chen Luan 甄鸞. In official use thirteen years, until the Great Emblem *(Ta-hsiang* 大象) era, s.y. 36 (579). Ahead of the sky 40 marks.>

Accumulated Years: 876,507

Day Divisor: 23,460

11. Great Emblem system *(Ta hsiang li* 大象曆, #32)

<Epochal year of the Great Emblem era, s.y. 36 (579). Composed by Ma Hsien 馬顯. In official use five years, until the Opening Sovereignty *(K’ai-huang* 開皇) era of the Sui period, s.y. 41 (584). Behind the sky 10 marks.>

Accumulated Years: 42,355

Day Divisor: 12,992

[I have emended the author’s name from Feng 馮 to Ma 馬 following the modern editors, *Yuan shih,* 53: 1189n9, and *Sui shu,* 17: 419.]

12. Opening Sovereignty system *(K’ai huang li* 開皇曆, #33)

<Year 4 of the Opening Sovereignty era of the Sui period, s.y. 41 (584). Composed by Chang Pin 張賓. In official use 24 years, until the Great Patrimony *(Ta-yeh* 大業) era, s.y. 5 (608). Behind the sky 7 marks.>

Accumulated Years: 4,129,697

Day Divisor: 102,960

[Summary account in *Sui shu,* 17: 421–23. There the Day Divisor *(jih-fa)* is 181,920, and the Obscuration Divisor *(pu-fa* 蔀法) is 102,960.]

13. Great Patrimony system *(Ta yeh li* 大業曆, #35)

<Year 4 of the Great Patrimony era, s.y. 5 (608). Composed by Chang Chou-hsuan 張胄玄. In official use eleven years, until the Martial Virtue *(Wu-te* 武德) era of the T’ang period, s.y. 16 (619). Behind the sky 7 marks.>

Accumulated Years: 1,428,317

Day Divisor: 1144

[Full account in *Sui shu,* 17: 435–56.]

14. Fifteenth-year Epoch system *(Wu-yin li* 戊寅曆, #36

<Year 2 of the Martial Virtue era of the T’ang period, s.y. 16 (619). Composed by the Man of the Way Fu Jen-chün 道士傅仁均. In official use 46 years, until the Chimera Virtue *(Lin-te* 麟德) era, s.y. 2 (665). Behind the sky 47 marks.>

Accumulated Years: 165,003

Day Divisor: 13,006

[*Chiu T’ang shu,* 32: 1153–70; *Hsin T’ang shu,* 25: 537–54. I follow *Yuan shih,* 53: 1189n10, in deleting one character from this passage. In the seventh century, “man of the Way” could designate a Buddhist or Taoist monk or a layman devoted to immortality practices.]

15. Chimera Virtue system *(Lin-te li* 麟德曆, #38

<Year 2 of the Chimera Virtue era, s.y. 2 (665). Composed by Li Ch'un-feng 李淳風. In official use 63 years, until the Opening Epoch *(K'ai-yuan* 開元) era, s.y. 5 (728). Behind the sky 12 marks.>

Accumulated years: 270,497

Day Divisor: 1340

[*Chiu T'ang shu,* 33: 1175–1219; *Hsin T'ang shu,* 26: 560–84. The title given here is a short form of Chimera Virtue Sexagenary First-Year Epoch system *(Lin-te chia-tzu yuan li* 麟德甲子元曆). Note that the proximate epoch of this system, like that of #14, is the year preceding official use, actually the year in which it was first used to compute the almanac in advance for use in the year of official adoption. On this system's new departure with respect to the Day Divisor, see p. 63.]

16. Great Expansion system *(Ta-yen li* 大衍曆, #42)

<Year 16 of the Opening Epoch era, s.y. 5 (728). Composed by the Buddhist monk I-hsing 僧一行. In official use 34 years, until the Precious Response *(Pao-ying* 寶應) era, s.y. 39 (762). Ahead of the sky 13 marks.>

Accumulated Years: 96,962,297

Day Divisor: 3040

[Canon, *Chiu T'ang shu,* 34: 1231–82; *Hsin T'ang shu,* 28A: 637–28B: 694. Summary of evaluation, *Hsin T'ang shu,* 27A: 588–27B: 634. *Hsin T'ang shu,* 28A: 637, gives Accumulated Years as 96,661,740 to A.D. 724, but but *Chiu T'ang shu,* p. 1231, reads 96,961,740 and *Yuan shih* agrees.]

17. Fivefold Era system (*Wu chi li* 五紀曆, #45)

<Epochal year of the Precious Response era, s.y. 39 (762). Composed by Kuo Hsien-chih 郭獻之. In official use 23 years, until the Constant Epoch *(Zhen-yuan* 貞元) era, s.y. 2 (785). Behind the sky 24 marks.>

Accumulated Years: 270,497

Day Divisor: 1340

[The formal title is Precious Response Fivefold Era system *(Pao ying wu chi li* 寶應五紀曆). *Hsin T'ang shu,* 29: 697–736. The account of the genesis of this system and its main differences from its predecessor on pp. 695–97 do not reveal the connotation of *wu chi*. Compare the constants in item 15.]

18. Constant Epoch system *(Chen yuan li*貞元曆, #48)

<Epochal year of the Constant Epoch era, s.y. 2 (785). Composed by Hsu Ch'eng-ssu 徐承嗣. In official use 37 years, until the Eternal Felicitation *(Ch'ang-ch'ing* 長慶) era, s.y. 39 (822). Ahead of the sky 15 marks.>

Accumulated Years: 403,397

Day Divisor: 1095

[This system is largely based on items 15 and 16.]

19. Extending Enlightenment system *(Hsuan ming li* 宣明曆, #50)

<Year 2 of the Eternal Felicitation era, s.y. 39 (822). Composed by Hsu Ang 徐昂. In official use 71 years, until the Luminous Fortune *(Ching-fu* 景福) era, s.y. 50 (893). Ahead of the sky 4 marks.>

Accumulated Years: 7,070,597

Day Divisor: 8400

[*Hsin T'ang shu,* 30A: 745–69. According to Takebe's calculations (3: 35a), this system was actually ahead of the sky 6 marks in 893.]

20. Reverence for the Arcana system *(Ch'ung hsuan li* 崇玄曆, #51

<Year 2 of the Luminous Fortune era, s.y. 50 (893). Composed by Pien Kang 邊岡. In official use fourteen years, or [during and] after, 63 years, until the Manifest Virtue *(Hsien-te* 顯德) era of the [Later] Chou period, s.y. 53 (956). Ahead of the sky 4 marks.>

Accumulated Years: 53,947,697

Day Divisor: 13,500

[*Hsin T'ang shu,* 30B: 779–804. This system was used during the T'ang period for fourteen years, and from start to finish for a total of 63 years.]

21. Veneration for Heaven system *(Ch'in t'ien li* 欽天曆, #59

<Year 3 of the Manifest Virtue era of the Chou period of the Five Dynasties, s.y. 53 (956). Composed by Wang P'u 王朴. In official use five years, until the Established Prosperity *(Chien-lung* 建隆) era of the Sung period, s.y. 57 (960). Ahead of the sky 2 marks.>

Accumulated Years: 72,698,777

Day Divisor: 7200

[Incomplete account in *Chiu Wu tai shih* 舊五代史, 140: 1867–77, complete in *Hsin Wu tai shih* 新五代史, 58: 674–703. The second constant given here was originally called Concordance Divisor *(t'ung fa* 統法).]

22. Response to Heaven system *(Ying t'ien li* 應天曆, #60)

<Epochal year of the Established Prosperity era of the Sung period, s.y. 57 (960). Composed by Wang Ch'u-ne 王處訥. In official use 21 years, until the Grand Peace and Strengthened State *(T'ai-p'ing hsing-kuo* 太平興國) era, s.y. 18 (981). Behind the sky 2 marks.>

Accumulated Years: 4,825,877

Day Divisor: 10,002

[Full account in *Sung shih*, 68: 1491–70: 1591. That account comprises and compares this and the next two items, which are modifications of it. This item originally calls the second constant Epoch Divisor *(yuan fa* 元法); item 23 calls it Epoch Rate *(yuan lü* 元率), and item 24 Ancestral Divisor *(tsung fa* 宗法).]

23. Supernal Epoch system *(Ch'ien yuan li* 乾元曆, #61

<Year 6 of the Grand Peace and Strengthened State era, s.y. 18 (981). Composed by Wu Chao-su 吳昭素. In official use twenty years, until the General Peace *(Hsien-p'ing* 咸平) era, s.y. 38 (1001). In agreement.>

Accumulated Years: 30,544,277

Day Divisor: 2940

24. Matching Heaven system *(I t'ien li* 儀天曆, #64

<Year 4 of the General Peace era, s.y. 38 (1001). Composed by Shih Hsu 史序. In official use 23 years, until the Celestial Sage *(T'ien-sheng* 天聖) era, s.y. 1 (1024). In agreement.>

Accumulated Years: 716,777

Day Divisor: 10,100

25. Reverence for Heaven system *(Ch'ung t'ien li* 崇天曆, #66

<Year 2 of the Celestial Sage era, s.y. 1 (1024). Composed by Sung Hsing-ku 宋行古. In official use 40 years, until the Peace through Order *(Chih-p'ing* 治平) era, s.y. 41 (1064). Behind the sky 54 marks.>

Accumulated Years: 97,556,597

Day Divisor: 10,590

[Full account in *Sung shih*, 71: 1603–73: 1682. According to Takebe's calculations (3: 44b), this system was behind the sky 51 marks in 1064.]

26. Resplendent Heaven system *(Ming t'ien li* 明天曆, #67

<Epochal year of the Peace through Order era, s.y. 41 (1064). Composed by Chou Ts'ung 周琮. In official use ten years, until the Splendid Tranquillity *(Hsi-ning* 熙寧) era, s.y. 51 (1074). In agreement.>

Accumulated Years: 711,977

Day Divisor: 39,000

[I follow the editors of the *Yuan History* in correcting a misprint (十 → 千). Full account in *Sung shih*, 74: 1685–75: 1738.]

27. Oblatory Epoch system *(Feng yuan li* 奉元曆, #68

<Year 7 of the Splendid Peace era, s.y. 51 (1074). Composed by Wei P'u 衛朴. In official use eighteen years, until the Epochal Protection *(Yuan-yu* 元祐) era, s.y. 9 (1092). Behind the sky 7 marks.>

Accumulated Years: 83,185,277

Day Divisor: 23,700

[The Sung history does not give an account of this system, on which the polymath Shen Kua collaborated; see Sivin 1995a: chap. 3, pp. 18–20. Li Jui 李銳 (1765–1814) has ably reconstructed the canon in *Pu-hsiu Sung Feng yuan shu* 補修宋奉元術.]

28. Contemplation of Heaven system *(Kuan t'ien li* 觀天曆, #69

<Year 7 of the Epochal Protection era, s.y. 9 (1092). Composed by Huang Chü-ch'ing 皇居卿. In official use eleven years, until the Reverent Tranquility *(Ch'ung-ning* 崇寧) era, s.y. 20 (1103). Ahead of the sky 6 marks.>

Accumulated Years: 5,944,997

Day Divisor: 12,030

[*Sung shih*, 77: 1797–78: 1845. The original calls the second constant the Concordance Divisor.]

29. Augury of Heaven system *(Chan t'ien li* 占天曆, #70

<Year 2 of the Reverent Tranquility era, s.y. 20 (1103). Composed by Yao Shun-fu 姚舜輔. In official use three years, until s.y. 23 (1106). Behind the sky 4 marks.>

Accumulated Years: 25,501,937

Day Divisor: 28,080

[See Li Jui's reconstruction in *Pu-hsiu Sung Chan t'ien shu* 補修宋占天術.]

30. Era Epoch system *(Chi yuan li* 紀元曆, #71

<Year 5 of the Reverent Tranquility era, s.y. 23 (1106). Composed by Yao Shun-fu. In official use 21 years, until the Celestial Coincidence *(T'ien-hui* 天會) era of the Chin period, s.y. 44 (1127). In agreement.>

Accumulated Years: 28,613,467

Day Divisor: 7290

[*Sung shih,* 79: 1847–80, 1905.The Accumulated Years value apparently should be 28,613,641, since in the Sung history, Accumulated Years to 1106 is 28,613,466 (p. 1847).]

31. Great Enlightenment system *(Ta ming li* 大明曆, #72

<Year 5 of the Celestial Coincidence era of the Chin period, s.y. 44 (1127). Composed by Yang Chi 楊級. In official use 53 years, until the Great Discipline *(Ta-ting* 大定) era, s.y. 37 (1180). In agreement.>

Accumulated Years: 383,768,657

Day Divisor: 5230

[The authors of *Chin shih* (21: 441) believe that this lost system was based on item 30, with minor modifications.]

32. Revised Great Enlightenment system *(Ch'ung hsiu Ta ming li* 重修大明曆, #76

<Year 20 of the Great Discipline era, s.y. 37 (1180). Revised by Chao Chih-wei 趙知微. In official use [by the Jin and the pre-dynastic and early Yuan] 101 years, until the Perfectly Great era of the Yuan dynasty, s.y. 18 (1281). Behind the sky 19 marks.>

Accumulated Years: 88,639,757

Day Divisor: 5230

[*Chin shih,* 21: 441–22: 519. Some historians cite this system as Chih-wei li 知微曆 after its reviser.]

33. Concordant Epoch system *(T'ung yuan li* 統元曆, #73

<Year 5 of the Sustained Ascendancy *(Shao-hsing* 紹興) era of the Later [i.e., Southern] Sung period, s.y. 52 (1135). Composed by Ch'en Te-i 陳得一. In official use 32 years, until the Supernal Way era, s.y. 24 (1167). In agreement.>

Accumulated Years: 94,251,737

Day Divisor: 6930

[*Sung shih,* 83: 1955–2021, gives a full combined account of this and the next three items. Items 33–35 call the latter constant the Epoch Divisor, and item 36, the Concordance Rate *(t'ung lü* 統率). Their numerical values vary. The text gives the author's name as 德一, but the editors correct it; *Yuan shih,* 53: 1189n12.]

34. Supernal Way system *(Ch'ien tao li* 乾道曆, #74

<Year 3 of the Supernal Way era, s.y. 24 (1167). Composed by Liu Hsiao-jung 劉孝榮. In official use nine years, until the Splendor through Simplicity *(Ch'un-hsi* 淳熙) era, s.y. 33 (1176). Behind the sky 1 mark.>

Accumulated Years: 91,645,937

Day Divisor: 30,000

35. Splendor through Simplicity system *(Ch'un hsi li* 淳熙曆, #75

<Year 3 of the Splendor through Simplicity era, s.y. 33 (1176). Composed by Liu Hsiao-jung. In official use fifteen years, until the Sustained Splendor *(Shao-hsi* 紹熙) era, s.y. 48 (1191). In agreement.>

Accumulated Years: 52,422,077

Day Divisor: 5640

36. Coincident Epoch system *(Hui yuan li* 會元曆, #79

<Year 2 of the Sustained Splendor era, s.y. 48 (1191). Composed by Liu Hsiao-jung. In official use eight years, until the Felicitous Epoch *(Ch’ing-yuan* 慶元) era, s.y. 56 (1199). Behind the sky 10 marks.>

Accumulated Years: 25,494,857

Day Divisor: 38,700

37. Concord with Heaven system *(T’ung t’ien li* 統天曆, #80

<Year 5 of the Felicitous Epoch era, s.y. 56 (1199). Composed by Yang Chung-fu 楊忠輔. In official use eight years, until the Spreading Joy *(K’ai-hsi* 開禧) era, s.y. 4 (1207). Ahead of the sky 6 marks.>

Accumulated Years: 3917

Day Divisor: 12,000

[For a combined account of this and the next two items see *Sung shih*, 84: 2023–90. This system calls the latter constant the Factor Divisor *(ts’e fa* 策法).]

38. Spreading Joy system *(K’ai hsi li*開禧曆, #81

<Year 3 of the Spreading Joy era, s.y. 4 (1207). Composed by Pao Huan-chih 鮑澣之. In official use 44 years, until the Protection through Simplicity *(Ch’un-yu* 淳祐) era, s.y. 48 (1251). Behind the sky 7 marks.>

Accumulated Years: 7,848,257

Day Divisor: 16,900

39. Protection through Simplicity system *(Ch’un yu li* 淳祐曆, #83

<Year 10 of the Protection through Simplicity system, s.y. 47 (1250). Composed by Li Te-ch’ing 李德卿. In official use one year, until s.y. 49 (1252). In agreement.>

Accumulated Years: 120,267,677

Day Divisor: 3530

40. Coincidence with Heaven system *(Hui t’ien li* 會天曆, #84

<Epochal year of the Protective Treasure *(Pao-yu* 寶祐) era, s.y. 50 (1253). Composed by T’an Yü 譚玉. In official use eighteen years, until the General Simplicity *(Hsien-ch’un* 咸淳) era, s.y. 8 (1271). Behind the sky 1 mark.>

Accumulated Years: 11,356,157

Day Divisor: 9740

[This and the next item were apparently trivial modifications of their predecessors.]

41. Attainment of Heaven system *(Ch’eng t’ien li* 成天曆, #86)

<Year 7 of the General Simplicity era, s.y. 8 (1271). Composed by Ch’en Ting 陳鼎. In official use ten years, until the Perfectly Great *(Chih-*

yuan 至元) era [of the Yuan period], s.y. 18 (1281). Behind the sky 1 mark.>

Accumulated Years: 71,758,157

Day Divisor: 7420

[The original discussion in *Sung shih,* 84: 2023, gives the first constant as 71,758,147. The Yuan text has this system in official use "4 years *(ssu nien* 四年)," but that is an obvious copying error.]

What follows are two systems that were never officially used, but are outstanding among those documented.

42. Sovereign Pole system *(Huang-chi li* 皇極曆, #34)

<Great Patrimony era (Sui, 605–617). Composed by Liu Cho 劉焯. Because of faultfinding, not officially used. By year 2 of the Martial Virtue era of the T′ang period, s.y. 16 (619), ahead of the sky 43 marks.>

Accumulated Years: 1,009,517

Day Divisor: 1242

[In 583 Liu was ordered to prepare a new system; it was about to be adopted in 608 when hostile criticism blocked it. He died the next year. See *Sui shu,* 18: 460–61, and the canon of Liu's system, pp. 461–98.]

43. Thirty-Second Year Epoch system *(I-wei li* 乙未曆, #77)

<Year 20 of the Great Discipline era, s.y. 37 (1180). Composed by Yeh-lü Lü 耶律履. Never officially used. By s.y. 18 (1281), behind the sky 19 marks.>

Accumulated Years: 40,453,126

Day Divisor: 20,690

[This system competed unsuccessfully with the Revised Great Enlightenment system for adoption *(Chin shih,* 21: 442). According to Takebe's computation, by 1281 #77 was behind the sky only 2 marks.]

44. Season-Granting system (#88)

<Year 18 of the Perfectly Great era of the Yuan period, s.y. 18 (1281), as epoch.>

Accumulated Years, Day Divisor: not used

A. From observation, year 18 of the Perfectly Great era, s.y. 18:

Ch′i Interval Constant *(ch′i-ying* 氣應): 55.0600 days

Intercalation Interval Constant *(jun-ying* 閏應): 20.1850 days

Regular conjunction: day 34.8750

B. Taking the Day Divisor as 2190, and the interval between s.y. 36 of the Extended Era Superior Epoch and s.y. 18 of the Perfectly Great era as 98,251,422, and computing:

Ch'i Interval Constant: 55.0602 days

Intercalation Interval Constant: 20.1853 days

Regular conjunction: day 34.8749

C. Taking the Day Divisor as 8270, and the interval between s.y. 1 of the Extended Era Superior Epoch and s.y. 18 [of the Perfectly Great era] as 5,670,557, computing, and counting off from s.d. 1:

Ch'i Interval Constant: 55.0533 days

Intercalation Interval Constant: 20.1808 days

Regular conjunction: day 34.8725

D. Taking the Day Divisor as 6570, and the interval between the Extended Era Superior Epoch, s.y. 1, and s.y. 18 [of the Perfectly Great era] as 39,752,537, and computing:

Ch'i Interval Constant: 55.0631 days

Intercalation Interval Constant: 20.1919 days

Regular conjunction: day 34.8712

{1177–88; END OF SECTION 11, of *YUAN SHIH,* CHAPTER 53, AND OF THE EVALUATION.}

[The evaluation does not specify the point of these four calculations; Takebe treats them as examples without explaining their origin (3: 88b-91a). The Day Divisor and interval for items B–D do not, as one might suppose, come from an earlier system.

In each instance, the *Ch'i* Interval Constant is the interval from the beginning of the last sexagenary day cycle to the epochal winter solstice—in other words, the sexagenary date of the solstice (1.C11). The Intercalation Interval Constant is the interval from the last new moon to the solstice (1.C12). The Regular Conjunction (i.e., its sexagenary date, 1.3.3) is simply the difference between the two (see Canon, 1.0, and, for variant terminology, Libbrecht 1973: 469–70). Since the three values in each case tally to the fourth decimal place, we can be sure that the evaluator obtained the Regular Conjunction by subtraction rather than independent calculation.

The evaluator's larger point, perhaps, is that these fundamental tropical-year and synodic-month intervals calculated traditionally by counting off from a Superior Epoch are at best good approximations to values derived from observation. He evidently means to demonstrate that any fractional divisor for the lunation constant is a matter of convention, and that the notion of a Superior Epoch is equally arbitrary. The range of variation in the four results is within roughly a mark, at the limit of observational accuracy. There were certainly many astronomers in 1280 who did not agree with the Yuan system's rejection of the old fixtures. I suspect these entries were meant to quiet them. We have, in fact, no record of a protest.

Now let us consider the list as a whole. It prompts several observations:

1. As the examples of errors given in the notes make clear, the Evaluation's information and computations, here as elsewhere, are not always reliable.

2. This is far from a complete list of systems, even of official ones, as table 2.1 shows. The number is arbitrary, depending on how many competing dynasties in periods of division an author acknowledges (see Chu Wen-hsin 1934, 31–34 for a list of overlaps). The aim of this list in the *Yuan History* is political, namely to present a seamless succession of systems used officially in what the dynasty recognized as its legitimate predecessors. That act of recognition means that the Mongol dynasty is the equally legitimate endpoint. The authors compromised this ideal for the three centuries before their own time, when China was divided north and south, again for political reasons. Their list accepts both the Sung and its contemporary northern enemies—the Liao, Chin, and the pre-dynastic phase of their own Yuan period. They also listed two systems that were never officially adopted, out of appreciation for their technical quality. In the second case, the fact that its author was the father of the Yuan dignitary Yeh-lü Ch'u-ts'ai probably played some part in its recognition here.

3. The systems that attained official recognition were by and large (but far from always) the most innovative, accurate, and influential. Not all substantial innovations appeared in computational systems. For instance, in a treatise on the cosmos *(An t'ien lun* 案天論, 320/336), Yü Hsi 虞喜 first identified the Annual Difference (see the orientation, p. 99).

4. How long a system was officially used says nothing about its originality or technical quality. When conditions for research were stable, access to the literature relatively easy, and trained talent abundant, heightened competition might cause a system to be replaced soon by a technically superior one, or one with more powerful backing, In a period of low standards, or one in which the government effectively suppressed the private practice of computational astronomy—or during a long reign—a mediocre system might retain official status for decades. Many an adequate system was replaced for reasons of state by one that was only superficially different, or less accurate. The Southern Sung prohibition against astronomical study by private individuals, although not rigorously enforced, may have shrunk the pool of talent as that of the Ming later did.

5. From the third or second century B.C. on, the almanac and its ephemeris were part of the regalia of monarchy. It gradually became fairly usual to promul-

gate a new astronomical system, symbolically reattuning the political and cosmic orders, as soon as possible after the beginning of a dynasty. Well-known examples are #33 for the Sui, #36 for the T'ang, #59 for the Later Chou, #60 for the Sung, and of course the Season-Granting system for the Yuan). The Grand Inception system (#2) set the precedent for new systems associated with a new regnal era within a dynasty, especially an era that justified itself by claiming renewal or revival. This emblematic aim accounts for the frequency with which new systems were adopted in the first or second years of reign eras (ten each in the above list).

6. Nor is the influence of a system on its successors a reliable sign of its astronomical importance. The officials of an Astronomical Bureau generally immersed in routine operations were not likely to keep rethinking their art, but at the same time the state machinery periodically demanded tokens of renewal. The systems of the T'ang after the Great Expansion system (#42), the best of its time, are modifications of it. But several of these late T'ang systems were also considerably influenced by the Chimera Virtue system (#38), which was notably less distinguished. On the other hand, the twenty systems of the Sung and Chin, with a few exceptions, represent minor alterations of #60, #73, and #80. These three systems are not technically enterprising enough to merit such authority.

To sum up, the 44 astronomical systems listed here, the 98 listed in table 2.1, and the hundred or so others about which we know little or nothing, saw use or neglect in part because of their technical quality and in part because of the court politics that pervaded astronomical practice. If we wish to understand their content and originality, comprehending the political and personal as well as the astronomical dimension is indispensable.]

9 Canon of the Season-Granting System, Part 1

[The Canon consists of seven sections, divided into two chapters. I list them here with the purpose of each stated in familiar terms:

Chapter 9

1. Pacing the *Ch'i* and Lunations: basic lunisolar divisions of the civil calendar
2. Pacing the Putting Forth and Gathering In: other customary divisions
3. Pacing the Tread of the Sun: theory of the apparent sun
4. Pacing the Travel of the Moon: theory of the apparent moon

Chapter 10

5. Pacing the Centered Star: elements of time reckoning
6. Pacing Crossing Coincidences: eclipse theory
7. Pacing the Five Stars: theory of the planets

In these two chapters I give only technical terms in translation only. For the Chinese originals, see the glossary in Appendix C.]

1. Pacing the Ch'i and Lunations

[The purpose of each step is to compute

1.1. date (and day parts) of winter solstice for given year.

1.2. date of mean seasonal *ch'i* (each 1/24 year) after the solstice.

1.3. date of mean conjunction preceding winter solstice.

1.4. dates of mean quarter moons, full moon, and next conjunction.

1.5. date at which to insert 16-day rather than 15-day *ch'i.*

1.6. date at which to insert 29-day rather than 30-day month.]

The 18th year of the Perfectly Great (Chih-yuan 至元) period, sexagenary year 18 (24 Nov 1280 - 12 Dec 1281), is the epoch.

[This section gives the procedures for generating the civil calendar, and therefore largely depends on mean periods. For this purpose, s.y. 18 is the epochal year. For the epochal day, see my comment on 1.C11.

The *ch'i* are intermediate divisions of the tropical year analogous to lunar months. Unlike lunar phenomena, because they are based on the year, they are directly related to the seasonal agricultural cycle. Originally (before the first astronomical system) there were probably twelve equal divisions of 30.44 days each, slightly longer than a lunar month. These were halved to give twelve each of equal, alternating "nodal ch'i (chieh-ch'i 節氣)" and "medial ch'i (chung-ch'i 中氣)," 15.22 days long.]

<Whether computing upward [i.e., backward] for past events or downward for the future, one calculates from an epoch. The secular variation in both the celestial perimeter and the year *(chou sui hsiao-chang* 周歲消長) is one digit per century. Interval constants *(ying* 應) between the beginnings of various cycles and the epoch must be recomputed for a given time; they cannot be used to fix an epoch.>

[According to the theory of secular variation adopted in the Yuan system, the Year Numerator (tropical year length, 1.C2) diminishes and the Celestial Perimeter (sidereal year length, 3.C1) increases at the rate of one digit (corresponding to 1 part or 0.0001 day) per century; see below, step 1.1. Instead of all cycles beginning at a single epoch as in early systems, this one counts each type from its initial point immediately prior to the epoch. The interval constants perform that task.

In each section of the Canon, a list of the pertinent constants precedes the series of instructions for computation. I number each constant for later reference in the form “1.C1.” For Chinese nomenclature see the technical glossary.]

Day Cycle	10 000	1.C1
Year Numerator	3 652 425 parts	1.C2

[These two quantities constitute a typical expression of an astronomical constant which in modern practice would be written as an integer with fractional remainder, or which in ancient Western practice were written in sexagesimal numbers. Their meaning here becomes clear only when they are combined to make the fraction 36 52 425 / 10 000 or 365.2425, the length of the tropical year. This constant, when expressed straightforwardly for other purposes, is called the [Tropical] Year Cycle (1.C6). Many of the constants below are used as numerator with the Day Cycle—the number of parts into which the system divided a day—as denominator. Their dimension is thus “parts *(fen* 分).”]

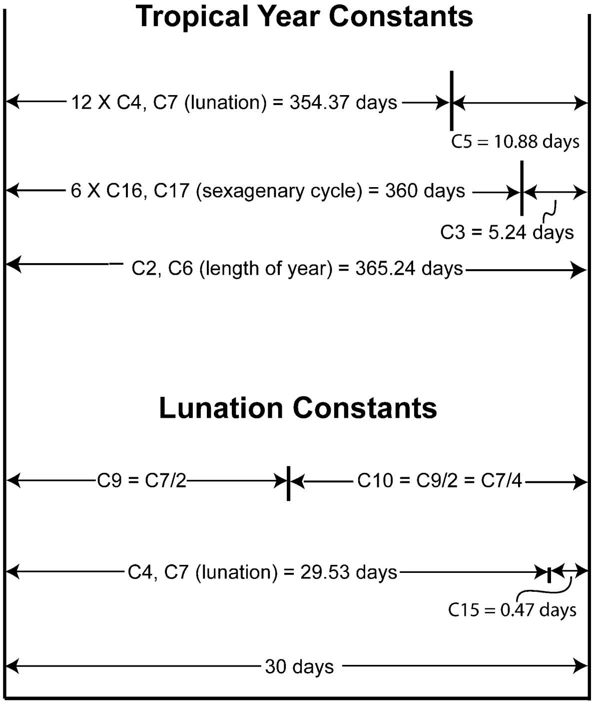
Series Surplus	52 425 parts	1.C3

[This constant, when divided by the Day Cycle, amounts to 5.2425 days, the difference between the Tropical Year Cycle and six sexagenary cycles. Since astronomers counted up days in cycles of 60, for *n* integral years from a given event, *n* times the Series Surplus gives the number of the sexagenary day. Adding the result to the sexagenary date of the earlier event, and casting out cycles of 60 if the sum is greater than that, yields the sexagenary date of the new one.

For instance, the solstice of December 1282 falls exactly two solar years after the epochal day, during which time the Series Surplus accumulates to become 104 850 parts, or roughly 10½ days. When this is added to the Ch’i Interval Constant (the number of days from s.d. 1 to the epochal solstice, 1.C11 below), the result is 655 450 parts, or 65.450 days. Casting out cycles of 60 days (1.C16, 1.C17), the remainder represents 5.45 days elapsed in the current sexagenary cycle, and corresponds to a date of s.d. 6 plus .45 day (1048 hours local time by modern reckoning), for the mean solstice.

Figure 24 portrays the relation between some of the constants in this section.]

Fig. 24. Relationships between fundamental constants based on the tropical year and the synodic month.



Lunation Numerator 295 305.93 parts 1.C4

[This is the length of the lunation expressed in parts; cf. the Lunation Factor below. The character that marks hundredths of a part or 10^{-6} is *miao* 秒.]

Series Intercalation Constant 108 753.84 parts 1.C5

[This is the difference between the length of a tropical year and the length of 12 lunations (354.37 days). This excess accumulates at the rate of over 10 days per year. When it becomes equal to the number of days in a synodic month (i.e. the Lunation Factor below), an intercalary month is added to the calendar. This happens a little more often than once every three years.]

Year Cycle 365.2425 days 1.C6

Lunation Factor 29 days 530 5.93 parts 1.C7

Ch'i Factor 15 days 2184.375 parts 1.C8

[This is the length of one *ch’i,* or 1/24 of a tropical year. For the method used to express the fractional part, see p. 89.]

Full Moon Factor 14 days 7652.965 parts 1.C9

[This is half of the Lunation Factor.]

Quarter Moon Factor 7 days 3826.4825 parts 1.C10

[This is one quarter of the Lunation Factor, and represents the mean interval between the new moon and first quarter. See also 6.C5.]

Ch’i Interval Constant 550 600 parts 1.C11

[This is the first of the interval constants that record the time between the beginning of a given cycle—in this instance the sexagenary cycle—and the epoch of the system. The sexagenary cycle that contained the epoch, in other words, began 55.0600 days before it. That amounted to the sexagenary date of the epoch|. The latter was as customary at the moment of winter solstice, which fell at 1:26 A.M. on s.d. 56. This was year 17, month 11, day 21, of the civil calendar, day 21 of the Astronomical First Month of year 18, or 14 December 1280 in the Julian calendar. An interesting study of how the reformers empirically determined the various interval constants is Ch’en Mei-tung 2003b, 193–201.]

Intercalation Interval Constant 201 850 parts 1.C12

[20.1850 days is the interval from the last new moon to the epoch.]

Disappearance Limit 7815.625 parts 1.C13

[This amounts to 16 days minus the Ch’i Factor (1.C8). This and the next two constants are used in the procedures for determining whether in the calendar a *ch’i* period will be 15 or 16 days long (step 1.5) and whether a month will be 29 or 30 days long (1.6).]

Ch’i Repletion Constant 2184.375 parts 1.C14

[This amounts to the *Ch’i* Factor (1C.8) minus 15 days; thus the sum of this constant and the last is 10 000 parts, or one day.]

Lunation Deficiency Constant 4694.07 parts 1.C15

[This amounts to 30 days minus the Lunation Factor (1.C7).]

Decad Cycle 600 000 1.C16

[This absolute number corresponds to the number of day parts in one sexagenary cycle. The next constant expresses the same quantity in days.]

Era Divisor 60 1.C17.

[In the sequence of steps that follows. I number results in the form “1.1.1.”]

1.1. To Compute the Winter Solstice in the Astronomical First Month

Set up [in counting rods] the Interval Count (the number of years from epoch, 1.1.1) and multiply by the Year Numerator (1.C2)

<In computing backward for past events, add one digit per century; in calculating forward for future events, subtract one digit per century.>

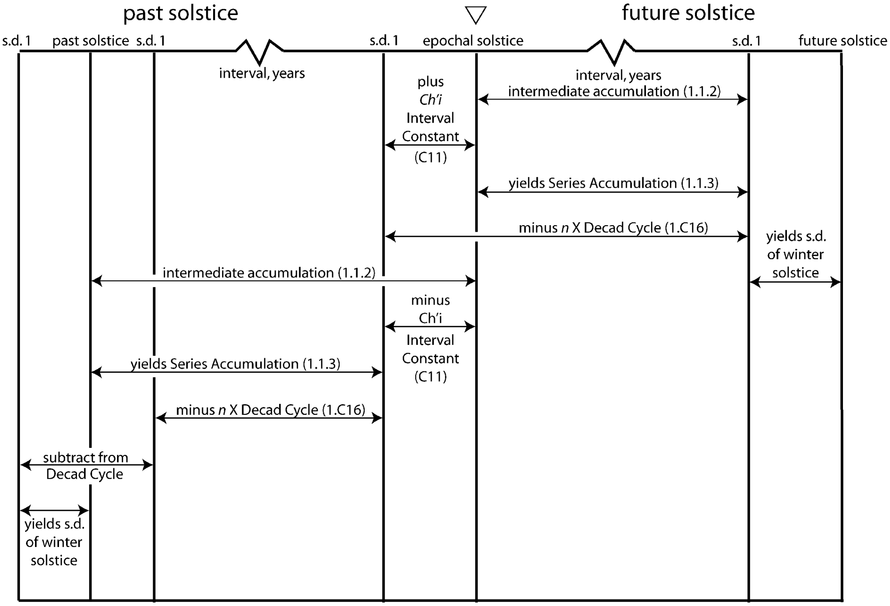
to yield the Intermediate Accumulation (1.1.2). Add the Ch′i Interval Constant (1.C11) to yield the Series Accumulation (1.1.3). Cast out full Decad Cycles (1.C16); simplify (i.e. divide) the remainder by the Day Cycle (1.C1) to yield days. What is not a full cycle [i.e., the remainder] is expressed as parts. The day, counted exclusively from s.d. 1, yields the desired date, double-hour, and hundredths of a day of the winter solstice in the Astronomical First Month (1.1.4).

<If investigating the past, subtract the Ch′i Interval Constant (1.C11) from the Intermediate Accumulation, subtract full Decad Cycles (1.C16) as many times as possible, and subtract the remainder from the Decad Cycle. The rest is as above.> {1192–93}

[The proliferation of named terms tends to hide the simplicity of this counting-off operation, which figure 25 makes clear. In computational astronomy, dates were always expressed within the current sexagenary cycle. Thus the figure represents the Ch'i Interval Constant by a line segment, the interval from s.d. 1 at the far left to the epochal solstice that falls in that day cycle.

The purpose of this procedure is to find the date (i.e. ordinal number of the day within the sexagenary cycle) of the winter solstice of the desired year. The Astronomical First Month is by definition the month containing the solstice. When the integral number of years from the epochal winter solstice to the desired year is multiplied by the Year Numerator (day parts per year), the result (the Intermediate Accumulation) is day parts, or days x 10^4. This intermediate quantity corresponds to the number of whole days in an integral number of tropical years. If the count begins at a winter solstice it will also end at a winter solstice if the count is exclusive (or the day before if the count is inclusive). Adding the *Ch'i* Interval Constant—the interval between the first day of the sexagenary cycle containing the epochal solstice and the solstice itself—extends the count backward in time to s.d. 1. The Series Accumulation is thus the number of day parts elapsed between s.d. 1 before the epoch and the winter solstice of the desired year. Subtracting $60n$ x 10^4 day parts—that is, canceling out full sexagenary cycles—leaves the number of day parts from day 1 of the *current* cycle to the desired winter solstice. Division by 10^4 converts to days; the result is the sexagenary date of the solstice. When parts are converted to days, any remainder will be ten-thousandths of a day. On the conversion of the remainder to double-hours and marks, see step 2.4.]

Fig. 25. The basic procedures for computing sexagenary dates of winter solstices, future and past, beginning with the one that falls on the Season-granting system's epoch.



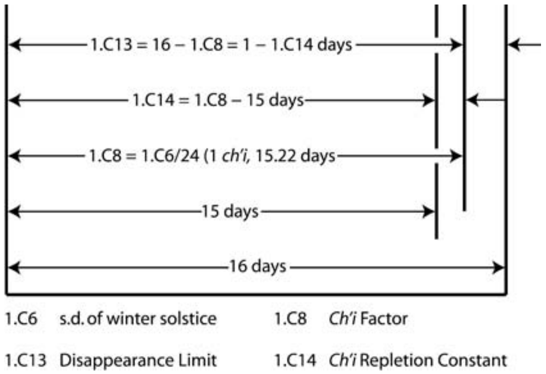
The first textual note refers back to the first such note of section 1, and gives instructions for dealing with secular variation in tropical year length when counting either forward or backward. The second textual note explains what moderns call the reversal of signs necessary when using this technique to calculate winter solstices in the past.]

1.2. To Determine Subsequent *Ch'i*

Set up the day and parts for the winter solstice in the Astronomical First Month (1.1.6–1.1.7), and add the *Ch'i* Factor (1.C8) repeatedly. When the day fills [i.e., becomes equal to or greater than] the Era Divisor (1.C17), subtract the latter. Count off exclusively as before. Each application of this procedure yields the sexagenary date and hundredth and ten-thousandth day parts for one subsequent *ch'i* (1.2.1). {1193}

[This procedure adds to the interval between s.d. 1 and the solstice (i.e., the sexagenary date of the solstice) a factor which amounts to 1/24 of the number of days in a tropical year, or the length of one mean *ch'i*. Since the solstice itself is one of the twenty-four *ch'i,* the result is the date (to the nearest 10^{-4} day) on which the next mean *ch'i* begins. Reiteration yields successive *ch'i* beginnings. Figure 26 shows the relationship between the values of the pertinent constants (their relations are further developed in step 1.5). Each sixth *ch'i* is centered on, and named for, one of the Standard Points (an equinox or solstice, 3.5). For the names of the twenty-four *ch'i*, see below, subsection 2.2.

Fig. 26. Relations between constants used to compute the length of a given *ch'i*.



The date of the regular (i.e., mean) *ch'i* is adequate for generating the civil calendar, but step 3.12 introduces the corrected *ch'i* as part of the solar theory. It refers to its starting point, mean *ch'i,* as "fixed *ch'i (heng-ch'i* 恆氣)," a term that does not appear elsewhere in the treatise.]

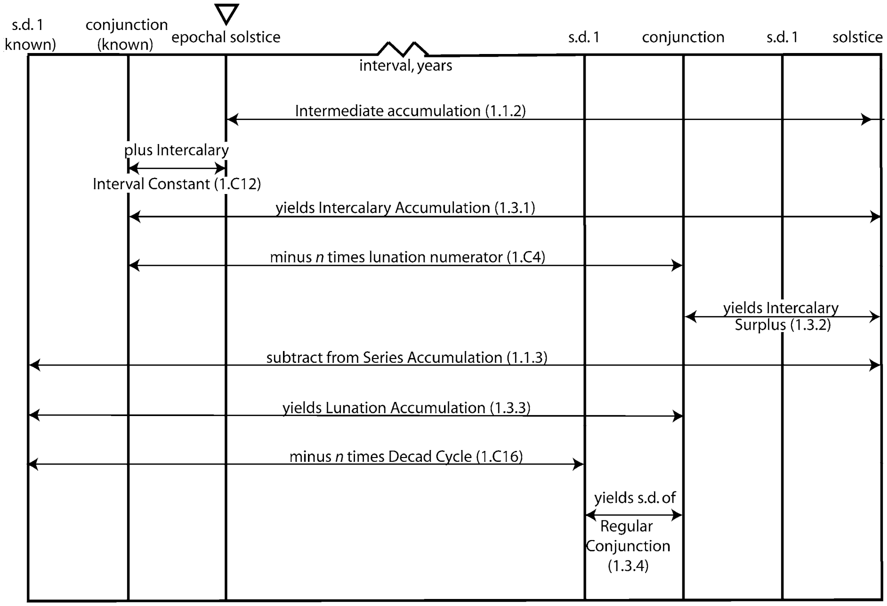
1.3. To Compute the Regular Conjunction [preceding] the Astronomical New Year

Set up the Intermediate Accumulation (1.1.2) and add the Intercalation Interval Constant (1.C12) to yield the Intercalary Accumulation (1.3.1). Cast out full Lunation Numerators (1.C4). The remainder is the Intercalary Surplus (1.3.2). Subtract it from the Series Accumulation (1.1.3) to yield the Lunation Accumulation (1.3.3). Subtract full Decad Cycles (1.C16). Simplify the remainder by the Day Cycle (1.C1) to yield days. The remainder is expressed as parts. The result is the desired days, hundredths, and ten-thousandth day parts for the Regular Conjunction of the Astronomical New Year (1.3.4).

<In investigating the past, subtract the Intercalary Interval Constant (1.C12) from the Intermediate Accumulation (1.1.2). Cast out full Luna-

tion Numerators. Subtract the remainder from the Lunation Numerator to yield the Intercalary Surplus. Simplify the resulting quantity by the Day Cycle (1.C1) to yield days. The remainder is expressed as parts. Subtract the result from winter solstice days and parts. If there is not enough to subtract, add the Era Divisor (1.C17) and then subtract. Count off as above.>{1193}

Fig. 27. Steps in computing future mean conjunctions of the sun and moon. The procedure for past conjunctions is analogous (cf. figure 24).



[The first sentence of the text instructs the calculator to add day parts from new moon to epoch to day parts from epoch to the winter solstice of the year desired; the sum is day parts from pre-epoch new moon to solstice. Repeated subtraction of the Lunation Numerator casts out all full lunations, so that the count now extends from the new moon immediately preceding winter solstice in the year desired to the solstice itself. Needless to say, one does not actually go through these endlessly repeated subtractions, but merely divides by the Lunation Numerator and drops the integer in the result. The text is phrased in this conventional way to indicate that the rest of the procedure will use only the remainder. Subtracting this remainder, the Intercalary Surplus, from the interval between s.d. 1 and the desired winter solstice, gives [(solstice – day 1) – (solstice – new moon)] = (day 1 – new moon), that is, the number of days between pre-epoch day 1 and desired new moon. As in the winter solstice procedure, the calculator casts out sexagenary cycles, converts the remaining day parts to whole days, and counts off from day 1 to yield the date. This date is based on the mean lunation, not on the actual position of the moon, and thus cannot predict its conjunction with the sun. For that purpose one must consider variation in the length of the lunation (see below, 4.6). Takebe, in his example of a past event (for A.D. 724, 4: 4a), makes a simple computational error that affects his later results for past lunar phenomena, including those in section 4.

Fig. 28. The computation of past conjunctions preceding the winter solstice of a given year. This figure is for the new moon and solstice in different day cycles.

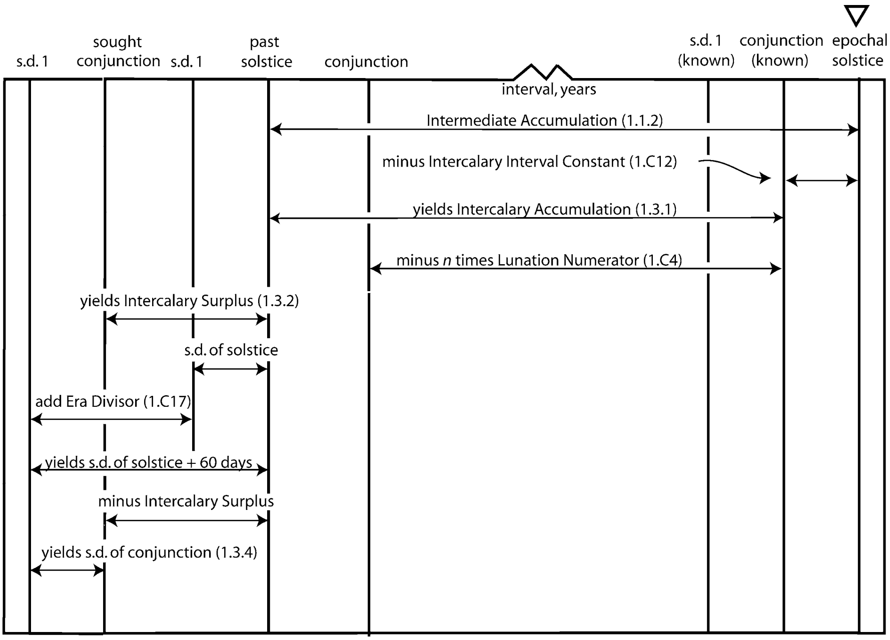


Figure 27 shows the steps for calculating future conjunctions. As the text note makes clear, the manipulation of cycles in this procedure differs somewhat depending on whether the desired mean conjunction is before or after the epoch, and, in the case of retrospective predictions, on whether the desired new moon falls in the same sexagenary day cycle. Since the regular procedure for past conjunctions simply reverses the one for future lunations (cf. the two in figure 25), figure 28 diagrams the special case in which a new sexagenary cycle begins between the past conjunction and the solstice.]

1.4. To Find Subsequent Quarters, Full Moon, and Next Lunation

Set up the days and ten-thousandth day parts of the regular [i.e., mean] new moon of the Astronomical First Month and repeatedly add the Quarter Moon Factor (1.C10). When the number of days equals or exceeds the Era Divisor (1.C17), cast the latter out. Each application of the procedure yields the days, hundredths, and ten-thousandth day parts for quarters, full moon, and next lunation (1.4.1). {1193}

[This simple reiterative procedure for mean lunar phenomena resembles that for successive mean *ch'i* following the winter solstice (1.2). When computing the date of a mean lunation, full moon, or quarter, one minimizes iteration by using the Lunation Factor (1.C7) and the Full Moon Factor (1.C9) in the same way.]

1.5. To Compute the Disappearance Date

Set up the hundredths and ten-thousandth day parts of [the sexagenary date that begins] the *ch'i* period (1.2) containing a Disappearance,

<If [the day parts] exceed the Disappearance Limit (1.C13), it is a *ch'i* period containing a Disappearance.>

and multiply by 15. Subtract the product from the *Ch'i* Factor (1.C8). Count a unit for each *Ch'i* Repletion Constant (1.C14) contained in the remainder. This gives the number of days to be added to sexagenary days of the regular *ch'i* period (1.2) to count off the Disappearance Date (1.5.1).{1193}

[The word *ch'i* can refer either to the whole *ch'i* period of 15+ days or to the moment at which one of these periods begins. In the calendar a *ch'i* period must be 15 or 16 days long, but the average must be 1/24 year (the *Ch'i* Factor; see figure 26). The simple Disappearance Date procedure determines when to enter in the ephemeris a 16-day *ch'i* rather than the normal 15-day one. As with month length in the next step, the issue is the date on which the accumulated fractional remainder of the *ch'i* adds up to a day. The accumulated remainder disappears as the extra day is recorded. "Disappearance" refers to the dropping of the remainder at that point; "elimination" in the next step is analogous.

The Disappearance Date serves no technical predictive function, since at what point within the *ch'i* period one inserts the extra day is astronomically immaterial. The procedure is typical of a tendency toward over-specification from a purely astronomical point of view—a point of view not to be found in the early astronomy of any civilization. To call such over-specification meaningless would be myopic. The Disappearance Date had *astrological* significance based on that of the *ch'i.* It even appears as a hemerological indication in some civil almanacs.]

1.6. To Compute the Elimination Date

Set up the hundredths and ten-thousandth day parts of [the sexagenary date that begins] the month containing an Elimination,

<If the parts are less than the Lunation Deficiency Constant, it is a month that contains an Elimination.>

and multiply by 30. Count a unit for each Lunation Deficiency Constant (1.C15) contained in the product. This gives the number of days to be added to sexagenary days of the regular conjunction (1.3.3) to count off the Elimination Date (1.6.1). {1193–94}

[This procedure does with respect to calendar month length what the Disappearance Date procedure does for the length of a *ch'i* period. Although the Season-Granting system considers the length of the mean synodic month to be 29.530 593 days, civil calendar months must be either 29 or 30 days long. Since the mean lunation value is slightly more than 29½ days, the astronomers take the normal calendar month as 30 days rather than 29. The procedure begins at the inception of a month containing an Elimination, that is, one 29 rather than 30 days long. It uses the Lunation Deficiency Constant, which is the difference between the mean lunation value and 30 days, and thus is analogous to the Disappearance Limit. Each time, the procedure yields a number of days from the beginning of the month up to 30. Like the Disappearance Date, the Elimination Date serves no astronomical function.

The terminology of Disappearance and Elimination was originally integral, and the latter had nothing to do with lunar periods. For details, see *Hou Han shu* 後漢書, Treatises, 3: 3445.]

2. Pacing the Putting Forth and Gathering In

["Putting Forth and Gathering In *(fa-lien* 發斂*)"* is a general term for alternating modes of activity in a cyclical process. Here it refers to the active and inactive modalities in the year, and also, in step 2.4, to day and night in the diurnal cycle. Its purpose is to determine the placement of intercalary months and *ch'i* divisions of the tropical year, and to enable certain basic time conversions.

The procedures in this section divide the cycle of the seasons into parts. Some commentators explain the vague old phrase *fa-lien* as a term for the solar inequality. The authors of the Season-Granting system do not look at it that way, for they treat the latter in the next section rather than here.

The purpose of each step is to

2.1. Compute dates of governance by each of the Five Phases.

2.2. Tabulate *ch'i* (named intervals of 1/24 year) and phases (same, 1/72 year).

2.3. Determine the mean month that does not contain a mean medial *ch'i* and thus is intercalary

2.4. Convert fractional day of phenomenon to double-hour and mark.]

Earth Kingship Factor	3.043 687 5 days	2.C1

[This is one-fifth of the *Ch'i* Factor (1.C8), or 1/120 of a tropical year. Since the phase Earth governs one-fifth of each *ch'i* period, the *Ch'i* Factor is used in 2.1 to determine this part.]

Lunation Intercalation Constant	9062.82 parts	2.C2

[This is one-twelfth of the Series Intercalation Constant (1.C5), the difference between mean tropical year length (divided into twelve equal *ch'i*) and one calendar year of twelve mean lunations, or (365.2425 – [12 x 29.530 593])/12. It thus represents the average monthly increase in the interval between the beginning of a *ch'i* and the beginning of a lunation—or, for that matter, between the beginnings of any pair of solar and lunar phenomena. If, for instance, the winter solstice (the medial *ch'i* of the 11th month) falls at the moment of lunar conjunction today, the next medial *ch'i*, called Greater Cold, will begin 9062.82 day parts = 0.91 day after the new moon of the twelfth month.

Although this constant is expressed as an integral number of day parts, step 2.3 uses it as a decimal fraction. The dimension of the result of its application there is days, not parts.]

Double-hour Divisor	10 000	2.C3
Single-hour Divisor	5000	2.C4
Mark Divisor	1200	2.C5

{1194}

[These three constants are used in the general procedure for reducing remainders expressed in day parts to time of day (2.4). The double-hour is 1/12 day; the single-hour, half of that, corresponds to a modern hour; and the mark is 1/100 day. The three constants listed above are 12 times the day parts in a double-hour, a single-hour, and a mark.]

2.1. To Compute the Periods of Governance of the Five Phases

The starts of the four nodal *[ch'i]* called Four Enthronements *(ssu-li* 四立) begin the governance *[yung-shih* 用事 of four of the Five Phases, as follows]: [Enthronement] of Spring [3], Wood; of Summer [9], Fire; of Autumn [15], Metal; and of Winter [21], Water. For the beginning date of the governance of Earth (2.1.1), subtract the Earth Kingship Factor (2.C1) from the date of the medial *ch'i* at each of the four seasonal transitions. {1194}

[This step apportions the year among the Five Phases, key categories for cosmological processes. Four phases are associated with the seasons. Earth, which represents the neutral point of dynamic yin-yang balance, is distributed equally among the transitions between the four seasons. The primacy of Wood, the immature yang phase of incipience and growth, begins at Enthronement of Spring (in the Yuan period, circa 27 January; now circa 5 February), and similarly with the other three phases that represent mature yang and immature and mature yin. Earth occupies 1 1/5 *ch'i* (rounded to the nearest full day) at the end of each season. Thus the dominion of each phase covers a total period of (6 – 1.2) = 4.8 of the 24 *ch'i,* with slight adjustment for rounding. The *ch'i* given over in their entirety to the phase Earth are Grain Rains, Greater Heat, Frost Settles, and Greater Cold. I have added the sequential numbers counting from winter solstice as in the list below. Each season begins with a nodal *ch'i,* and an Earth *ch'i* ends it. The 1/5 of a nodal *ch'i* preceding each of these four medial *ch'i*, plus the medial *ch'i* itself, make the required 1 1/5 *ch'i*.]

2.2. *Ch'i* and Phase Intervals

[*Ch'i* 氣 and Phase *(hou* 候) are divisions of the tropical year into twelve and 72 equal intervals, based on popular usages. Here, as usual, the *ch'i* are subdivided into alternating nodal and medial halves (see the orientation, p. 79). Since each of the 24 *ch'i* is 15.22 days long, the three phases into which each is divided is a period of 5.07 days. Obviously two *ch'i* amount to more than one lunar month; the occasional intercalary month absorbs the surplus.]

Civil First Month [Spring]

Enthronement of Spring, nodal *ch'i* of the first month [3]

[I have added to the name of each *ch'i* its ordinal number after the winter solstice. This is the number required to compute the dates of additional *ch'i* in step 1.2.]

East Wind Breaks the Freeze
Dormant Insects First Stir
Fish Ascend to the Ice

Rainwater, medial *ch'i* of the first month [4]

Otter Offers Fish

[This refers to the old notion that animals, not only humans, order their lives by ritual; the point is that the otter makes an offering of its prey before it eats it; see also the behavior of the hawk mentioned under the seventh month and the wild dog under the ninth.]

Wild Goose *(hou-yen* 候鴈) Flies North

Plants and Trees Begin to Sprout

Second Month

Excited Insects, nodal *ch'i* of the second month [5]

Peach First Blossoms

Oriole *(ts'ang-keng* 倉鶊) Sings

Hawk Transforms into Dove

Vernal Division [i.e., equinox], medial *ch'i* of the second month [6]

Black Bird *(hsuan-niao* 玄鳥, i.e. swallow) Arrives

Thunder Sounds [cf. eighth month]

First Lightning

Third Month

Pure and Bright, nodal *ch'i* of the third month [7]

Paulownia Buds First Blossom

Mole Transforms into Button-quail

Rainbow First Appears

Grain Rains, medial *ch'i* of the third month [8]

Duckweed First Grows

Calling-dove Preens

Hoopoe Alights in the Mulberry Tree

Fourth Month [Summer]

Enthronement of Summer, nodal *ch'i* of the fourth month [9]

Green Frogs Peep

Earthworms Emerge

Royal Gourd *(wang-kua* 王瓜) Grows

[Thiadiantha dubia Bge. or Tricosanthes cucumeroides?]

Grain Small but Full, medial *ch'i* of the fourth month [10]

Bitter Vegetable in Seed
Fragile Greens Die
"Wheat-autumn" Arrives [i.e. Wheat yellows]

Fifth Month
Bearded Grain, nodal *ch'i* of the fifth month [11]
Praying Mantis Born
Shrike First Calls
Tongue-Twister Silent
[*Fan-she* 反舌, mockingbird or bush warbler?]
Summer Extreme [i.e., solstice], medial *ch'i* of the fifth month [12]
Deer Shed Antlers
Cicadas First Sing
Midsummer Plant Grows
[This may be Pinellia tuberifera Ten.]

Sixth Month
Lesser Heat, nodal *ch'i* of the sixth month [13]
Warm Wind Arrives
Crickets Settle in the Walls
Hawks First Clutch
Greater Heat, medial *ch'i* of the sixth month [14]
Decayed Grass Becomes Fireflies
Soaked Earth Makes Summer Muggy
Heavy Rains from Time to Time

Seventh Month [Autumn]
Enthronement of Autumn, nodal *ch'i* of the seventh month [15]
Cooling Wind Arrives
Hoarfrost Settles
Cold Cicada Chirps
[Cosmopsaltria opalifera?]
Abiding heat, medial *ch'i* of the seventh month [16]
Hawks Offer Birds

Sky and Earth First Severe
Grain Presented

Eighth Month

Hoarfrost, nodal *ch'i* of the eighth month [17]

Wild Goose Comes
Black Bird Goes Back [cf. second month]
Flocking Birds Store up Food

Autumnal Division [i.e., equinox], medial *ch'i* of the eighth month [18]

Thunder First Restrains its Sound
Hibernating Insects Close the Entrances [to their burrows]
Water First Dries Up

Ninth Month

Cold Dew, nodal *ch'i* of the ninth month [19]

Wild Goose Comes and Sojourns
Small Birds Enter the Great Water and Become Bivalves *(ko* 蛤)
Chrysanthemum Bears Yellow Blossoms

Frost Settles, medial *ch'i* of the ninth month [20]

Wild Dog Offers Prey
Plants and Trees Turn Yellow and Drop Their Leaves
Hibernating Insects All Burrow in

Tenth Month [Winter]

Enthronement of Winter, nodal *ch'i* of the tenth month [21]

Water First Turns to Ice
Ground First Freezes
Pheasants Enter the Great Water and Become Large Bivalves *(shen* 蜃)

Lesser Snow, medial *ch'i* of the tenth month [22]

Rainbow Hides and is Invisible
Sky *Ch'i* Rises; Earth *Ch'i* Settles
Everything Closed and Stopped up; Winter Sets in

Eleventh Month

Greater Snow, nodal *ch'i* of the eleventh month [23]

Flying Fox Silent
Tiger First Mates
North China Iris Emerges

Winter Extreme [i.e., solstice], medial *ch'i* of the eleventh month [24]

Earthworms Curl Up
Elaphure Sheds Antlers
Streams and Springs Stir

Twelfth Month

Lesser Cold, nodal *ch'i* of the twelfth month [1]

Wild Goose Heads Northward
Magpie First Nests
Pheasant Crows

Greater Cold, medial *ch'i* of the twelfth month [2]

Hen Broods
Birds of Prey Fierce and Quick
Ice Thick and Solid on the Streams and Marshes {1194–97}

[It is impossible to be certain of the plant and animal species mentioned in this section by their archaic names. A few of my tentative identifications differ from those in Legge 1861-75 and Knoblock & Riegel 2000; the sources they translate vary textually from the Yuan version. I mostly base my translations on the overlap of glosses in early commentaries with listings in the materia medica literature.]

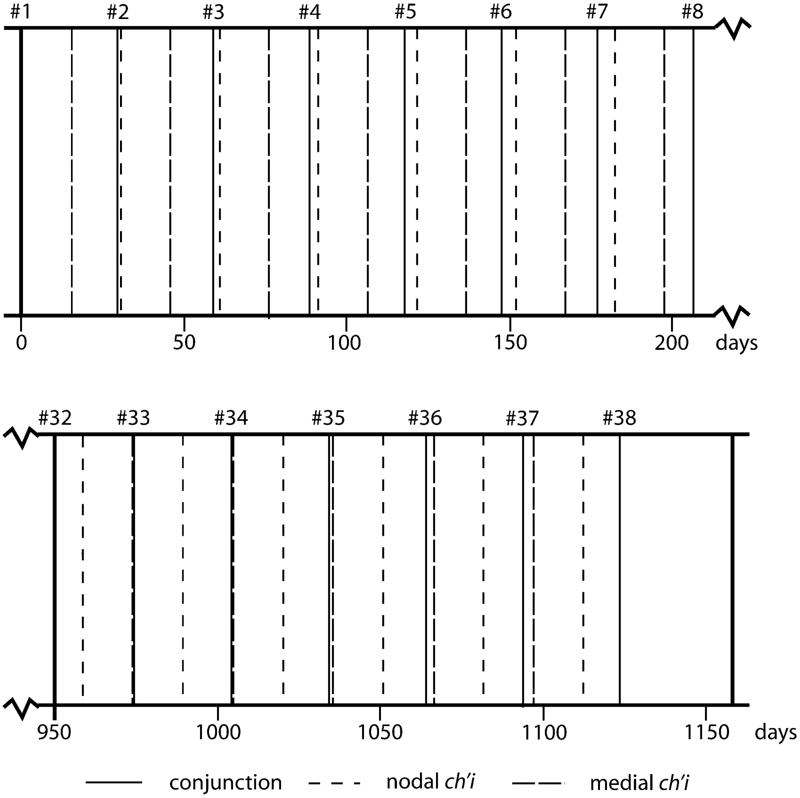
2.3. To Compute the Distance of Medial *Ch'i* from Regular Conjunction

Set up the Intercalary Surplus at the Astronomical New Year (1.3.2) and simplify by the Day Cycle (1.C1) to give days. Counting off gives the distance of the winter solstice from the regular conjunction. By repeatedly adding the Lunation Intercalation Constant (2.C2), each time one obtains the count in days for the distance between a regular conjunction and a medial *ch'i (*2.3.1).

<By subtracting the Lunation Factor (1.C7) when this distance becomes equal to it, one derives full intercalary months; but to insert them

[in the ephemeris] one must wait for an apparent lunation which does not contain [the first day of] a medial *ch'i*.> {1197}

Fig. 29. Relation of two sequences of nodal and medial *ch'i* and conjunctions.



[The Intercalary Surplus (from the procedure for computing regular conjunction of the Astronomical New Year) is the number of day parts from the new moon preceding the winter solstice to the solstice itself, so that the division by 10 000 gives the interval between conjunction and solstice in days. When counted off exclusively from the beginning of the month, it yields what one might call the lunar date of the solstice. The Lunar Intercalation Constant expresses the monthly increase in the interval between conjunctions and solar phenomena. Thus its reiterated addition generates a series of gradually increasing intervals between successive regular conjunctions and beginnings of medial *ch'i* periods. This growing interval will eventually exceed the Lunation Factor, the number of days in the mean lunation, an event which mandates inserting an intercalary month. As the note indicates, the month after which it is inserted depends on the apparent rather than the mean lunation (see 4.6).

Figure 29 shows the relationship between lunations and *ch'i*. I have folded it to make it easier to read; bottom left follows from top right, with a gap of 24 lunar months between. The numbers of conjunctions in the sequence are at the top. This sequence begins as conjunction #1 coincides with a nodal *ch'i*. Conjunction #2 follows after 29.53 days, and a second nodal *ch'i* after 30.44 days. A medial *ch'i* is visible between each pair of conjunctions, 15.22 days after each nodal *ch'i*. Skipping ahead, we find that a medial *ch'i* falls just before the end of conjunction #32, and just after the beginning of conjunction #34. Thus the lunar month that follows #33, because it does not contain a medial *ch'i,* requires intercalation.]

2.4. To Compute Time of Day of Putting Forth and Gathering In

Set up the desired hundredths and ten-thousandth day parts, multiply by 12, and count 1 for each full Double-hour Divisor (2.C3) in the result to yield the number of double-hours elapsed. The remainder is collected *(shou* 收, i.e., divided) by the Mark Divisor (2.C5) to yield marks. Count off [double-hours and marks] exclusively from the standard half of hour 1 (i.e. from midnight) to yield the double-hour and mark of [the phenomenon] (2.4.1).

<If the (parts) equal or exceed the Single-hour Divisor (2.C4), treat them as a full double-hour but count from the beginning half of double-hour 1 [i.e., from 11 P.M. of the previous day].> {1197}

[As the heading indicates, this is a general method for converting remainders in day parts to time of day expressed in double-hours and marks. On hour reckoning, see the orientation, p. 82.

The numerical value of the Double-hour Divisor is the same as that of the Day Cycle. The difference lies in its use. To reduce day parts to days, one divides by 10 000; but to convert to double-hours one multiplies by 12 x (day parts / 10 000). One must divide the remainder by (12 x 100) to reduce it to marks. Note the distinction between hundredths of a day counted off consecutively from midnight, a great convenience for astronomical computation, and hundredths of a day counted off from the beginning half of a double-hour when that is where the remainder falls. A double-hour contains 8 1/3 marks.

The fact that the first daily double-hour begins at 2300h but the day begins at midnight complicates the process of counting off double-hours and marks, at least for semi-competent computists. A day remainder of 4 marks (.04 day) falls near the end of *the first half of a double-hour in that day,* but this is the standard, not beginning, half of the first double-hour (I designate it 1B). A day remainder of 7 marks corresponds to an elapsed time of less than a double-hour but more than half of one. It therefore falls in the beginning half of the day’s 2nd double-hour (hour 2A, corresponding to between 0100/0200h).

This second example is of the kind mentioned in the note, where the parts equal to the Single-hour Divisor (= 0.0417 day) are treated as a double-hour but counted off exclusively from the beginning half of the first hour (2300h) rather than from midnight. Huang Tsung-hsi’s 黃宗羲 paraphrase is a little more straightforward: “If [12 x day parts] amounts to a full Double-hour Divisor, the time falls in the standard half of an hour; if it amounts to a full Single-hour Divisor, the time falls in the beginning half of an hour” (*Shou-shih li ku* 授時曆故, 1: 10a).

The tables “Double-hour Reckoning” (2.4) and “Reckoning in Marks” (2.7) in the orientation convert directly from the day and double-hour remainders, or from the results of computations as directed in this section, to yield double-hours and marks.]

3. Pacing the Tread of the Sun

[The literary term *jih-ch’an* 日躔 (also written 日纏) used here for solar motion refers both to the sun’s travel and, more often, to its location among the stars. It implies apparent rather than mean motion. *Ch’an* itself is often a verb that describes a planet’s “treading” a certain lunar lodge (e.g. Evaluation, Section 4). The next section uses the analogous term *yueh-li* 月離, which I translate “travel of the moon.”

The purpose of each step is to

3.1. Compute days from solstices to mean lunar syzygies.

3.2. Compute (with or without table) equation of center for sun.

3.3. Tabulate angular equatorial extensions of lunar lodges (to record coordinates).

3.4. Compute apparent equatorial position of sun at winter solstice.

3.5. Same for equinoxes and summer solstice.

3.6. Compute angular equatorial arcs from solstice or equinox to endpoints of lunar lodges.

3.7. Tabulate corresponding equatorial and ecliptic arcs from solstice or equinox for conversions.

3.8. Compute angular ecliptic extensions of lunar lodges.

3.9. Tabulate angular ecliptic extensions of lunar lodges (to obviate computation for periods near epoch).

3.10. Compute apparent ecliptic position of sun at winter solstice.

3.11. Same for equinoxes and summer solstice.

3.12. Same for midnight preceding solstices and equinoxes.

3.13. Same for midnight positions for every day.

3.14. Same for noon positions.

3.15. Compute ecliptic arc from noon position of sun to previous solstice, for next step.

3.16. Convert to equatorial arc.

3.17. Tabulate ecliptic positions of the Twelve Stations with respect to lunar lodges.

3.18. Compute time that apparent sun enters one of the Twelve Stations.]

Celestial Perimeter Parts	3 652 575 parts	3.C1
Celestial Perimeter	$365^{t}.2575$	3.C2
Celestial Semi-perimeter	$182^{t}.628\ 75$	3.C3
Aspectual Limit	$91^{t}.314\ 375$	3.C4
Annual Difference	$0^{t}.0150$	3.C5

[The last four of these constants are expressed in *tu,* the Chinese version of degrees. For the Annual Difference, see the orientation, p. 99, and the Evaluation, section 2.

The Celestial Semi-perimeter and the Aspectual Limit are simply one-half and one-quarter of the Celestial Perimeter. "Aspect (*hsiang* 象)" is a literary word for celestial phenomena that correspond to some aspect of the microcosm (in other contexts *hsiang* means "counterpart"). The Yuan system uses it occasionally in the technical sense of a quadrant of a cycle of motion. It is not exactly a geometric notion, but a division of a sequence of computations into four distinct phases.]

Perimeter Interval Constant	3 151 075 parts	3.C6

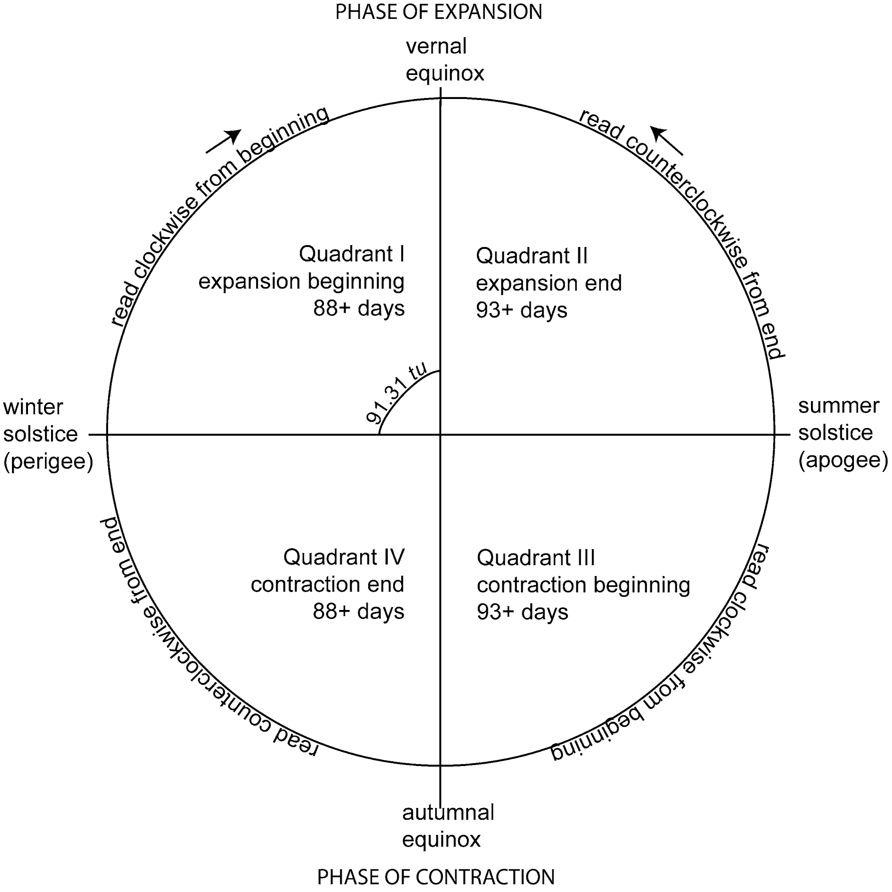
[This constant, which amounts to $315^{t}.1075$, is the interval between 6^{t} in the lunar lodge Tumulus, the initial point of the Chinese system that corresponds to right ascension, and the winter solstice at the epoch of the Season-Granting system. In other words, it is analogous to the r.a. of the sun at the epochal solstice. Because a *tu* is also one day's solar motion, it is also the number of days elapsed since the mean sun was last at 6^{t} in Tumulus.

The Yuan astronomers set the zero point by entirely traditional means. Beginning with the configuration of the sun, moon and planets at the epoch, they calculated backward to find a Superior Epoch at which all were in conjunction. The outcome depends, of course, on how one defines conjunction. Their unspecified definition put this grand universal meeting at 6^{t} in Tumulus (Evaluation, p. 251). Although they had expressly abandoned the old practice of basing the whole computational system on a general conjunction located in the remote past, it still had enough rhetorical authority to determine for the purpose of step 3.4 what is, from the viewpoint of modern astronomy, an arbitrary point.]

Year Semicycle 182 days 6212.5 parts 3.C7

[This is half the length of the Year Cycle or tropical year, and thus the sum of the next two constants.]

Fig. 30. Solar inequality: relation of phase of expansion or contraction and seasonal cycle.



Expansion Beginning/Contraction end Extent
88 days 9092.25 parts 3.C8

Contraction Beginning/Expansion End Extent
93 days 7120.25 parts 3.C9

{1157–58}

[The solar inequality theory divided the sun's path into four quadrants of $91^t.31$ (= 90°), symmetrical about the solstices but not the equinoxes. Their boundaries are called the Four Standard Points *(ssu-cheng* 四正), named after the four seasons (see below, 3.5). The Winter and Summer Standard Points coincide with the beginnings of the *ch'i* periods Summer Extreme and Winter Extreme, which include the solstices. The Vernal and Autumnal Standard Points precede the beginning of the *ch'i* Vernal Division and follow the beginning of Autumnal Division, the equinoxes, by an amount equal to the maximum equation of center, 2.4 *tu*.

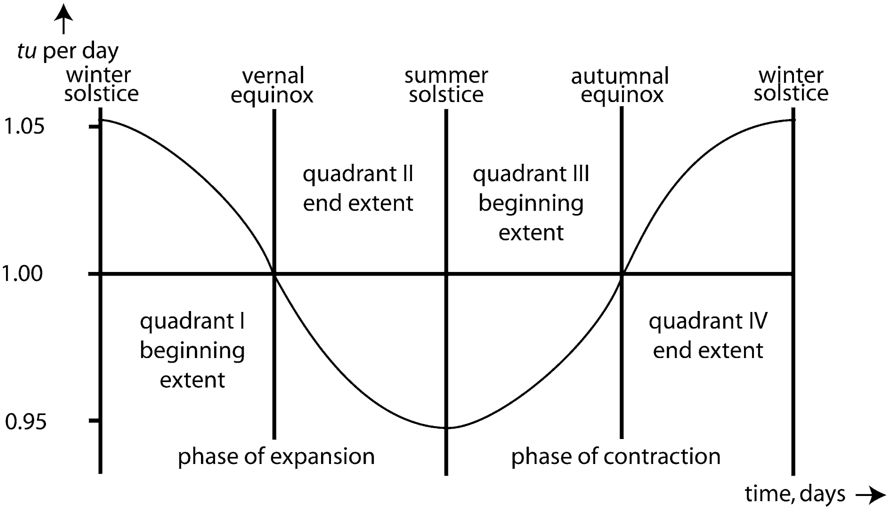
Thus 88.9 days after the winter solstice, the apparent sun crosses the equator, halfway in angular measure between the solstices at the Vernal Standard Point. At the moment halfway in time between the solstices, 2.4 days later, it enters the *ch'i* Vernal Division. At the same time, by definition, the mean sun is halfway between the solstices and thus is crossing the Vernal Standard Point. This relationship is simpler than it looks. The system of *ch'i* employed in the Season-Granting system is one of mean *ch'i*, equal divisions of the tropical year (above, 2.2). The Standard Points are the beginnings of corrected *ch'i*, and correspond to the apparent equinoxes and solstices.

The inequality is zero at the solstices and symmetrical about them. Chinese astronomers did not distinguish the perigee and apogee of the sun's orbit from the winter and summer solstices respectively. According to the constants given here, the sun takes 88+ days to traverse each of the two quadrants before and after the winter solstice, and 93+ days to traverse each of the two quadrants before and after the summer solstice. The period during which the sun moves between the winter and summer solstices, in which the equation of center (3.2) is positive, is called the "phase of expansion *(ying* 盈)." The half-year in which the equation of center is negative is the "phase of contraction *(so* 縮)." Figures 30 and 31, two ways of looking at the same cycle, show their relationship.

[In principle, the constants for section 3, with the exceptions of the Annual Difference (3.C5) and the Perimeter Interval Constant (3.C6), are subject to the secular variation in tropical and sidereal year length introduced in 1.0. Nevertheless, we are told to apply it only in Section 3.4 below. Commentators (i.e., Huang Tsung-hsi, 2: 1a, 3a) suggest that it was applied more consistently, and Japanese users of the system did precisely that; see p. 239. The question remains open. Since the effect of the variation is negligible, I deal with it only where it comes up in the text.]

[Figure 31 is a rough graph of solar velocity in *tu* per day. The horizontal line at 1^t per day is mean speed; the curve is apparent speed. Inspection reveals that in Quadrants I and IV the latter value is positive (greater than the mean speed of the sun), and in Quadrants II and III negative (less than the mean). The equation of center is the summation of differences between apparent and mean velocity, and thus would correspond to the area between the curve and the line of mean speed. The equation of center is 0 at the starting point, the winter solstice; it accumulates until the end of Quadrant I (2.4 days before the vernal equinox, the inception of Vernal Division), when it reaches its maximum. From there on it diminishes, remaining positive, until the areas between the curves of apparent and mean motion become equal at the summer solstice. The process reverses in Quadrants III and IV. The slope of the curve represents the change in velocity of the apparent solar motion. From it we see that the phase of Expansion is marked by deceleration, and that of Contraction by acceleration.

Fig. 31. Solar velocity: change in speed of the apparent sun in relation to the phases of expansion and contraction.



From the modern point of view, the change in observed velocity is due overwhelmingly to variation in the distance between the sun and the earth as the latter moves on its elliptical orbit. The idea of varying distance, however, played no part in the design of Chinese techniques. Astronomers neither stated nor questioned the assumption that the earth was the center of the sun's path. One might argue that it is implicit in their computational techniques, but it is irrelevant to the numerical—not geometrical—method used here.

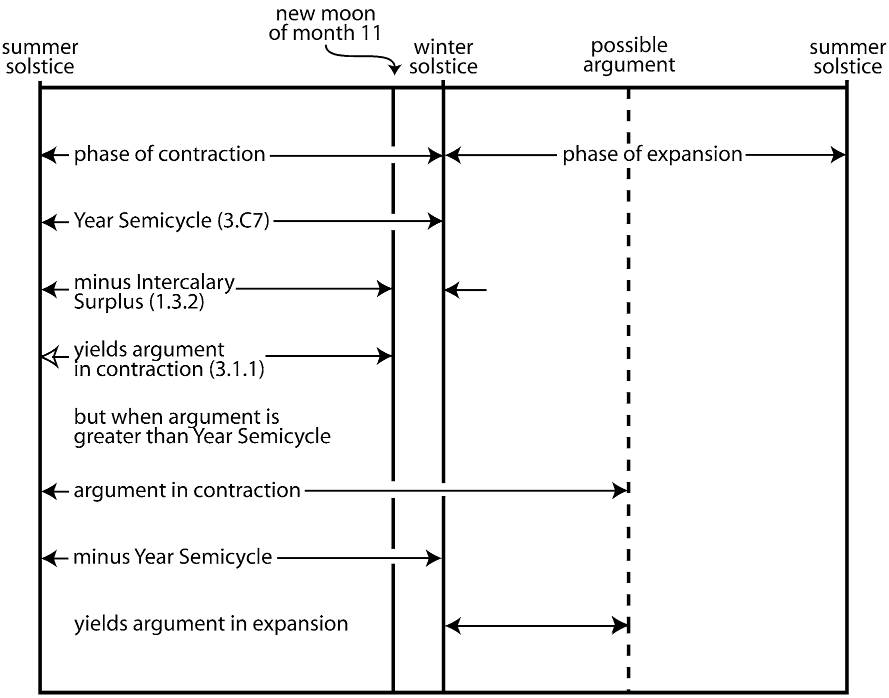
The lunar inequality theory in Section 4 is based on a numerical model analogous to that of the solar theory (compare figures 30 and 35). The terminology differs, but in both instances process-oriented yin-yang metaphysics underlies the language of symmetric, opposed, and complementary phases.]

3.1. To Compute the Argument in the Phases of Expansion and Contraction for the Regular New Moon, Quarters, and Full Moon of the Astronomical First Month

Set up the Year Semicycle (3.C7) and subtract the days and day parts of the Intercalary Surplus (1.3.2), yielding the argument in the phase of contraction for the regular conjunction of the Astronomical First Month.

<After the winter solstice it is the expansion phase; after the summer solstice it is the contraction phase.>

Fig. 32. Argument for lunar phases: method of computing the argument in the phases of expansion and contraction for the regular new moon, quarters, and full moon of the Astronomical First Month.



By repeatedly adding the Quarter Moon Factor (1.C10), each time one obtains the argument in the expansion or contraction phase (3.1.1) in days and day parts for quarter moons, full moons, and succeeding new moons.

<After the argument amounts to a Year Semicycle (3.C7), cast out the latter, for this is the transition between the expansion and contraction phases.> {1198}

[Since the Intercalary Surplus is the interval between the Astronomical New Year and winter solstice, subtracting it from half the length of the tropical year leaves the interval from the summer solstice to the new moon of the Astronomical First Month, which falls in the contraction phase (figure 32). The rule in the second note begins the count again at each solstice. The arguments *(ju* 入 … *li* 曆) are thus days elapsed from the solstices to mean syzygies and quarters. Subsequent steps will use them to compute the apparent position of the sun at the time of these synodic phenomena. Generally, “arguments” are days elapsed for looking up in a table or, less frequently, for computation.

Because the Intercalary Surplus is expressed in parts, but this step specifies days and parts, it is necessary to divide it by the Day Cycle (1.C1).]

3.2A. To Find the Expansion/Contraction Difference

[The Expansion/Contraction Difference corresponds to the equation of center.]

Inspect the argument. When in the phase of expansion, if it is less than the Expansion Beginning/Contraction End Extent (3.C8), it becomes the Beginning Extent [in Quadrant I]. If greater, subtract the Year Semicycle (3.C7) from it; the remainder is the End Extent [in Quadrant II]. When in the phase of contraction, if it is less than the Contraction Beginning/Expansion End Extent (3.C9) it becomes the Beginning Extent [in Quadrant III]]. If greater, subtract the Year Semicycle from it; the remainder is the End Extent [in Quadrant IV].

In the case of an argument at the beginning of expansion or the end of contraction [i.e. in Quadrant I or IV], set up the Cubic [i.e. third-order] Difference (3.2.1) 31, multiply it by the [argument in the] Beginning/End Extent, and add the Square [i.e. second-order] Difference (3.2.2) 24 600, again multiplying by the [argument in the] Beginning/End Extent. Subtract the result from the Corrected [i.e. first-order] Difference (3.2.3) 5 133 200. Again multiply the remainder by the [argument in the] Beginning/End Extent. In the result, count each full 100 000 000 *(i* 億= *wan-wan* 萬萬) as one *tu*, and shift anything less backward [in the computing-rod columns] to yield parts.

[Shifting a result backward by *n* columns in the computational array amounts to multiplying the divisor by 10^n; see the orientation, p. 61.]

In the case of an argument at the beginning of contraction or the end of expansion [i.e. in Quadrant II or III], set up the Cubic Difference 27, multiply it by the [argument in the] Beginning/End Extent, and add the Square Difference 22 100, again multiplying it by the [argument in the] Beginning/End Extent. Subtract the result from the Corrected Difference 4 870 600. Again multiply the remainder by the [argument in the] Beginning/End Extent. In the result, count each full 100 000 000 as one *tu*, and shift anything less backward [in the computing-rod columns] to give parts.

The result [in either case] is the desired Expansion/Contraction Difference (3.2.4). {1198}

[The outcome of this step, the Expansion/Contraction Difference, is the equation of center. Adding it corrects a mean location to give the apparent position (see the orientation, p. 97, on mean and apparent motions).

The first paragraph begins with the argument of expansion or contraction, that is to say, with the number of days elapsed since the preceding solstice, and defines a new argument called the Beginning Extent or the End Extent, which is the number of days from the synodic phenomenon being considered to the solstitial point nearest it in space (not necessarily in time). Thus the Beginning/End Extent in Quadrants I and IV is read from the winter solstice, and in Quadrants II and III from the summer solstice. Subtraction of the Year Semicycle (half a tropical year) in Quadrants II and IV counts days backward from the end of the quadrant.

The sun's speed and the daily increment to the equation of center are identical in Quadrants I and IV, and also in II and III. The remainder of the procedure provides the equivalent of two third-order formulas for the equation of center in *tu*,

$$y = 0.051\,332x - 0.000\,246x^2 - .000\,000\,31x^3$$

for Quadrants I and IV and

$$y = 0.048\,706x - 0.000\,221x^2 - 0.000\,000\,27x^3$$

for Quadrants II and III, where x is the Beginning or End Extent. The dimension of y is *tu*. The commentary at the end of step 3.2B explains the terminology of the coefficients.

In principle, the extents of the quadrants should be determined so that where they meet, the equation of center yielded by either formula is identical. This is true only to within three decimal places. The equation of center in Quadrants I and IV when x = 88.909 225 days (C8) is 2.401 423 *tu*, and that in Quadrants II and III when x = 93.712 025 days (C9) is 2.401 325 *tu*. The two extents are not independent, as we have seen; they add up to half the tropical year. A more satisfactory division would be 88.675 50 days for Quadrants I and IV, which implies 93.945 75 days for Quadrants II and III. The maximum equation of center (at the point in each quadrant furthest from the solstice) would be 2.401 349 in both cases.

Be that as it may, the discrepancy of 0.001 *tu* is negligible. By the thirteenth century, the problem of predicting conjunctions had long been solved (for earlier attempts, see p. 77). Still, given the limits of observational accuracy in establishing the constants, as well as the nature of the problem, an element of arbitrariness remained in the equations. The authors evidently decided to impose the condition that the difference between the two extents be 4.802 800 days, or twice the "official" round figure of 2.4014 for the maximum equation of center. This was for their purpose a significant theoretical gain at the cost of a negligible loss of fit where the quadrants overlap.]

3.2B. Another Method

Set up day parts in the argument for the current quadrant and multiply by the Expansion/Contraction Parts [from table 9.1] for the day being studied. Reduce by 100 000 to get parts, and add to the listed Expansion/Contraction Accumulation. Reduce by 10 000 to yield *tu*; the remainder is parts. This also yields the Expansion/Contraction Difference (3.2.4). {1199}

[This ready-reckoning method avoids the relatively onerous procedure just given, substituting simple linear interpolation from tables provided in the text. The extant Canon lacks them but, like other important materials omitted from it, they appear among the ready-reckoning tables of the Great Concordance system of the Ming (which is based on the Season-Granting system, and does not differ fundamentally from it). No doubt this and other tables (9.8, 10.4) were omitted from the *Yuan History's* treatise because of their optional character and because even without them it was larger than its predecessors. They were available separately (even outside China) in the fifteenth century.

In order to demonstrate the simple construction of the Ming table (34: 624–40), I reproduce it as table 9.1. It is in two parts, first for Quadrants I and IV and then for Quadrants II and III. Because the Ming version nicely represents other tables included in the system but omitted from the *Yuan History*, it is well worth examining closely. Its columns are labeled as follows:

A. Accumulated Days *(chi-jih* 積日)

B. Square and Cubic Combined Difference *(p'ing-li ho-ch'a* 平立合差; solar acceleration, 10^{-8} *tu*/day/day)

C. Expansion/Contraction Addition Parts *(ying* [or *so] chia fen* 盈縮加分, solar inequality), 10^{-8} *tu*/day)

D. Expansion/Contraction Accumulated Degrees *(ying* [or *so] chi tu* 盈縮積度, equation of center, 10^{-8} *tu)*

E. Expansion/Contraction Angular Motion *(ying* [or *so] hsing tu* 盈縮行度, true solar velocity, *tu*/day)]

Table 9.1. Solar Equation of Center, Ready Reckoner

A. For Beginning of Expansion and End of Contraction (Quadrants I and IV)

A	B	C	D	E
0	49,386	5,108,569	-	1.051 085
1	49,572	5,059,183	5,108,569	1.050 591
2	49,758	5,009,611	10,167,752	1.050 096
3	49,944	4,959,853	15,177,363	1.049 598
4	50,130	4,909,909	20,137,216	1.049 099
5	50,316	4,859,779	25,047,125	1.048 597
6	50,502	4,809,463	29,906,904	1.048 094
7	50,688	4,758,961	34,716,367	1.047 589
8	50,874	4,708,273	39,475,328	1.047 082
9	51,060	4,657,399	44,183,601	1.046 573
10	51,246	4,606,339	48,841,000	1.046 063
11	51,432	4,555,093	53,447,339	1.045 550
12	51,618	4,503,661	58,002,432	1.045 036
13	51,804	4,452,043	62,506,093	1.044 520
14	51,990	4,400,239	66,958,136	1.044 002
15	52,176	4,348,249	71,358,375	1.043 482
16	52,362	4,296,073	75,706,624	1.042 960
17	52,548	4,243,711	80,002,697	1.042 437
18	52,734	4,191,163	84,246,408	1.041 911
19	52,920	4,138,429	88,437,571	1.041 384
20	53,106	4,085,509	92,576,000	1.040 855
21	53,292	4,032,403	96,661,509	1.040 324
22	53,478	3,979,111	100,693,912	1.039 791

A	B	C	D	E
23	53,664	3,925,633	104,673,023	1.039 256
24	53,850	3,871,969	108,598,656	1.038 719
25	54,036	3,818,119	122,470,625	1.038 181
26	54,222	3,764,083	116,288,744	1.037 640
27	54,408	3,709,861	120,052,827	1.037 098
28	54,594	3,655,453	123,762,688	1.036 554
29	54,780	3,600,859	127,418,141	1.036 008
30	54,966	3,546,079	131,019,000	1.035 460
31	55,152	3,491,113	134,565,079	1.034 911
32	55,338	3,435,961	128,056,192	1.034 359
33	55,524	3,380,623	141,492,153	1.033 806
34	55,710	3,325,099	144,872,776	1.033 250
35	55,896	3,269,389	148,197,875	1.032 693
36	56,082	3,213,493	151,467,264	1.032 134
37	56,268	3,157,411	154,680,757	1.031 574
38	56,454	3,101,143	157,838,168	1.031 011
39	56,640	3,044,689	160,939,311	1.030 446
40	56,826	2,988,049	163,984,000	1.029 880
41	57,012	2,931,223	166,972,049	1.029 312
42	57,198	2,874,211	169,903,272	1.028 742
43	57,384	2,817,013	172,777,483	1.028 170
44	57,570	2,759,629	175,594,496	1.027 596
45	57,756	2,702,059	178,354,125	1.027 020
46	57,942	2,644,303	181,056,184	1.026 443
47	58,128	2,586,361	183,700,487	1.025 863
48	58,314	2,528,233	186,286,848	1.025 282
49	58,500	2,469,919	188,815,081	1.024 699
50	58,686	2,411,419	191,285,000	1.024 114
51	58,872	2,352,733	193,696,419	1.023 527
52	59,058	2,293,861	196,049,152	1.022 938
53	59,244	2,234,803	198,343,013	1.022 348
54	59,430	2,175,559	200,577,816	1.021 755
55	59,616	2,116,129	202,753,375	1.021 161
56	59,802	2,056,513	204,869,504	1.020 565
57	59,988	1,996,711	206,926,017	1.019 967

A	B	C	D	E
58	60,174	1,936,723	208,922,728	1.019 367
59	60,360	1,876,549	210,859,451	1.018 765
60	60,546	1,816,189	212,736,000	1.018 161
61	60,732	1,755,643	214,552,189	1.017 556
62	60,918	1,694,911	216,307,832	1.016 949
63	61,104	1,633,993	218,002,743	1.016 339
64	61,290	1,572,889	219,636,736	1.015 728
65	61,476	1,511,599	221,209,625	1.015 115
66	61,662	1,450,123	222,721,224	1.014 501
67	61,848	1,388,461	224,171,347	1.013 884
68	62,034	1,326,613	225,559,808	1.013 266
69	62,220	1,264,579	226,886,421	1.012 645
70	62,406	1,202,359	228,151,000	1.012 023
71	62,592	1,139,953	229,353,359	1.011 399
72	62,778	1,077,361	230,493,312	1.010 773
73	62,964	1,014,583	231,570,673	1.010 145
74	63,150	951,619	232,585,256	1.009 516
75	63,336	888,469	233,536,875	1.008 884
76	63,522	825,133	234,425,344	1.008 251
77	63,708	761,611	235,250,477	1.007 616
78	63,894	697,903	236,012,088	1.006 979
79	64,080	634,009	236,709,991	1.006 340
80	64,266	569,929	237,344,000	1.005 699
81	64,452	505,663	237,913,929	1.005 056
82	64,638	441,211	238,419,592	1.004 412
83	64,824	376,573	238,860,803	1.003 765
84	65,010	311,749	239,237,376	1.003 117
85	65,196	246,739	239,549,125	1.002 467
86	65,382	181,543	239,795,864	1.001 815
87	65,568	116,161	239,977,407	1.001 161
88	65,754	50,593	240,093,568	1.000 505
89	0	0	240,144,161	1.000 000

B. For Beginning of Contraction and End of Expansion (Quadrants II and III)

A	B	C	D	E
0	44,362	4,848,473	-	0.951 516
1	44,524	4,804,111	4,848,473	0.951 959
2	44,686	4,759,587	9,652,584	0.952 405
3	44,848	4,714,901	14,412,171	0.952 851
4	45,010	4,670,053	19,127,072	0.953 300
5	45,172	4,625,043	23,797,125	0.953 750
6	45,334	4,579,871	28,422,168	0.954 202
7	45,496	4,534,537	33,002,039	0.954 655
8	45,658	4,489,041	37,536,576	0.955 110
9	45,820	4,443,383	42,025,617	0.955 567
10	45,982	4,397,563	46,469,000	0.956 025
11	46,144	4,351,581	50,866,563	0.956 485
12	46,306	4,305,437	55,218,144	0.956 946
13	46,468	4,259,131	59,523,581	0.957 409
14	46,630	4,212,663	63,782,712	0.957 874
15	46,792	4,166,033	67,995,375	0.958 340
16	46,954	4,119,241	72,161,408	0.958 808
17	47,116	4,072,287	76,280,649	0.959 278
18	47,278	4,025,171	80,352,936	0.959 749
19	47,440	3,977,893	84,378,107	0.960 221
20	47,602	3,930,453	88,356,000	0.960 696
21	47,764	3,882,851	92,286,453	0.961 172
22	47,926	3,835,087	96,169,304	0.961 650
23	48,088	3,787,161	100,004,391	0.962 129
24	48,250	3,739,073	103,791,552	0.962 610
25	48,412	3,690,823	107,530,625	0.963 092
26	48,574	3,642,411	111,221,448	0.963 576
27	48,736	3,593,837	114,863,859	0.964 062
28	48,898	3,545,101	118,457,696	0.964 549
29	49,060	3,496,203	122,002,797	0.965 038
30	49,222	3,447,143	125,499,000	0.965 529
31	49,384	3,397,921	128,946,143	0.966 021
32	49,546	3,348,537	132,344,064	0.966 515

A	B	C	D	E
33	49,708	3,298,991	135,692,601	0.967 011
34	49,870	3,249,283	138,991,592	0.967 508
35	50,032	3,199,413	142,240,875	0.968 006
36	50,194	3,149,381	145,440,288	0.968 507
37	50,356	3,099,187	148,589,669	0.969 009
38	50,518	3,048,831	151,688,856	0.969 512
39	50,680	2,998,313	154,737,687	0.970 017
40	50,842	2,947,633	157,736,000	0.970 524
41	51,004	2,896,791	160,683,633	0.971 033
42	51,166	2,845,787	163,580,424	0.971 543
43	51,328	2,794,621	166,426,211	0.972 054
44	51,490	2,743,293	169,220,832	0.972 568
45	51,652	2,691,803	171,964,125	0.973 082
46	51,814	2,640,151	174,655,928	0.973 599
47	51,976	2,588,337	177,296,079	0.974 117
48	52,138	2,536,361	179,884,416	0.974 637
49	52,300	2,484,223	182,420,777	0.975 158
50	52,462	2,431,923	184,905,000	0.975 681
51	52,624	2,379,461	187,336,923	0.976 206
52	52,786	2,326,837	189,716,384	0.976 732
53	52,948	2,274,051	192,043,221	0.977 260
54	53,110	2,221,103	194,317,272	0.977 789
55	53,272	2,167,993	196,538,375	0.978 321
56	53,434	2,114,721	198,706,368	0.978 853
57	53,596	2,061,287	200,821,089	0.979 388
58	53,758	2,007,691	202,882,376	0.979 924
59	53,920	1,953,933	204,890,067	0.980 461
60	54,082	1,900,013	206,844,000	0.981 000
61	54,244	1,845,931	208,744,013	0.981 541
62	54,406	1,791,687	210,589,944	0.982 084
63	54,568	1,737,281	212,381,631	0.982 628
64	54,730	1,682,713	214,118,912	0.983 173
65	54,892	1,627,983	215,801,625	0.983 721
66	55,054	1,573,091	217,429,608	0.984 270
67	55,216	1,518,037	219,002,699	0.984 820

A	B	C	D	E
68	55,378	1,462,821	220,520,736	0.985 372
69	55,540	1,407,443	221,983,557	0.985 926
70	55,702	1,351,903	223,391,000	0.986 481
71	55,864	1,296,201	224,742,903	0.987 038
72	56,026	1,240,337	226,039,104	0.987 597
73	56,188	1,184,311	227,279,441	0.988 157
74	56,350	1,128,123	228,463,752	0.988 719
75	56,512	1,071,773	229,591,875	0.989 283
76	56,674	1,015,261	230,663,648	0.989 848
77	56,836	958,587	231,678,909	0.990 415
78	56,998	901,751	232,637,496	0.990 983
79	57,160	844,753	233,539,247	0.991 553
80	57,322	787,593	234,384,000	0.992 125
81	57,484	730,271	235,171,593	0.992 698
82	57,646	672,787	235,901,864	0.993 273
83	57,808	615,141	236,574,651	0.993 849
84	57,970	557,333	237,189,792	0.994 427
85	58,132	499,363	237,747,125	0.995 007
86	58,294	441,231	238,246,488	0.995 588
87	58,456	382,937	238,687,719	0.996 171
88	58,618	324,481	239,070,656	0.996 756
89	58,780	265,863	239,395,137	0.997 342
90	58,942	207,083	239,661,000	0.997 930
91	59,104	148,141	239,868,083	0.998 519
92	59,266	89,037	240,016,224	0.999 110
93	59,428	29,771	240,105,261	0.999 703
94	0	0	240,135,032	1.000 000

[The relationship of columns B, C, and D is most easily understood by comparison with figures 30 and 31 above and their explanations. In figure 31, apparent solar velocity is the first derivative, and solar acceleration the second derivative, of the equation of center, with respect to either time or angular measure. Column B of table 9.1 is the solar acceleration, a series of differentials that correspond to the daily change in column C, the solar velocity. Column D is the equation of center, the primary yield of the table. It is the summation of values in column C. The differentials in column C, called Expansion/Contraction

Parts in the text above, are the daily increments of inequality, the absolute values of apparent minus mean solar velocity. Since the mean velocity by definition is 1 *tu* per day, the sun's daily motion (or Expansion/Contraction Angular Motion) in column E is simply (1 + Parts) in Quadrants I and IV, and (1 - Parts) in Quadrants II and III. Thus, in step 3.13, one reads the daily ecliptic apparent motion from this table as follows:

Quadrant	**Beginning Standard Point**	**Argument**
I	winter	x
II	vernal	$93 - x$
III	summer	x
IV	autumnal	$88 - x$

The lost Yuan ancestor of the *Ming History's* table may not have been identical in form, but the procedure in the two histories is the same.

Although the tabulated values for the equation of center in column D of figure 9.1 correspond within 10^{-8} *tu* to the cubic formulas given in 3.2A, the table is constructed by the simplest possible numerical manipulation. There is a constant difference of 186 (i.e., 0.000 001 86 *tu)* between successive values in column B for Quadrants I and IV; the constant difference is 162 for Quadrants II and III. The table unfolds by successive addition and subtraction beginning with three numbers: the initial values in columns B and C, and the constant interval between values in column B. Thus a given value in column C is obtained by subtracting the previous line's values for C and B; D is the sum of the previous line's values for C and D; and E is the difference between the previous line's values for E and 0.01 B.

The "Origins of Techniques *(fa yuan* 法原)" section of the *Ming History's* astronomical treatise (33: 591–92), explains clearly and concretely the derivation of the three values. It summarizes the very detailed, illustrated treatment in Mei Wen-ting's monograph *Shou-shih p'ing li ting san ch'a hsiang shuo* 授時平立定三差詳說 (Detailed explanation of the square, cubic, and corrected differences in the Season-granting system).

Some of the initial values are artifacts of curve-fitting using the technique that Mei Wen-ting calls "building up differences *(to-chi [li] chao-ch'a* 垛積立招差)," or what historians of mathematics call the method of finite differences. Yen Tun-chieh (1966: 217–220) elegantly demonstrated that the Yuan astronomers' derivation of a third-order constant difference table was directly adapted from a procedure used in the revised Great Enlightenment system (#32, 1180) to generate the table of sunrise times found in *Chin shih,* 21: 463–465.

Further details in the Ming treatise make it clear that, although the reformers had neither formally defined concepts nor a highly developed symbolic language for what much later became the differential and integral calculus, they employed constant differences to the same effect in making their tables. The table is accurate to the eighth decimal place. The only exception is column E, in which values are rounded off to six places. Negligible inaccuracies enter the last decimal place in table 9.1 because for Quadrants I and IV, it consistently drops remainders less than 0.9, and for Quadrants II and II, rounds up those less than 0.9 to the next digit when subtracting C(*x*) from 1.

Tables similar to those in the Ming treatise appear in the Korean *Expeditious Ready Reckoner for the Season-Granting system* (*Susiryŏk ch'ŏppŏp ipsŏng* 授時曆捷法立成, 1343). They provide only the equation of center, rounded off inconsistently to four decimal places. They give values for every 10 marks (0.1 day). Since these values come from linear interpolation between integral values of *x* calculated according to step 3.2A, they represent a saving of effort only if values for divisions of the day finer than 10 marks are not wanted. If, for instance, one wished to compute the equation of center after 14.37 days, after looking up the tabulated value for 14.30 days in the table, one would still have to interpolate for the additional 0.07 days. This might yield marginally greater accuracy than a single interpolation for 0.37 days, but only if unequal intervals were being used. In other words, fifteenth-century Korean practice, although based on the Season-Granting system, was in this respect a great deal more approximate than that of China. Ch'en Mei-tung 1995: 320–21 corrects the version in the *Koryŏsa* 高麗史 of 1451.

According to Mei's explanation, the empirical foundation of the Yuan formula for equation of center is observations of the Accumulated Difference for intervals of 1 to 6 *ch'i*, which yield a constant second differential. This is scarcely a plausible claim. The Accumulated Difference must be based on measurement of celestial longitude in *tu.* The table records it to better than 10^{-4} *tu*, although we have reason to believe that the effective limit of observational accuracy was on the order of 0.05 *tu.* The Second Difference is constant to within at least 10^{-6} *tu*. That implies the same precision in Regular Daily Difference, and with an increment of 15 days per step, makes a precision of 10^{-4} *tu* imperative. It thus appears that the formula for equation of center is based on observations that were tidied up in some way that only further investigation will disclose.]

3.3. Angular Extension of the Lunar Lodges on the Red Way

[I incorporate the list of extensions in table 9.2, along with additional useful data:

A = Accumulated *tu,* 6^{t} in Tumulus to beginning of lodge

B = Accumulated *tu*, 0^t in Dipper to end of lodge
C= Location of Standard Point in lodge, *tu*
D = Endpoint Elongation Arc (see 3.6), Standard Point to end of lodge]

Table 9.2. Equatorial Extensions of Lunar Lodges

Lodge	Ext., *tu*	A	B	C	D
Horn	12.10	236.3075	298.1575	-	24.6144
Neck	9.20	248.4075	307.3575	-	33.8144
Root	16.30	257.6075	323.6575	-	50.1144
Chamber	5.60	273.9075	329.2575	-	55.7144
Heart	6.50	279.5075	335.7575	-	62.2144
Tail	19.10	286.0075	354.8575	-	81.3144
Basket	10.40	305.1075	365.2575	10^t, Winter	0.4000
EAST, total	79.20	-	-	-	-
Dipper	25.20	315.5075	25.20	-	25.60
Ox	7.20	340.7075	32.40	-	32.80
Maid	11.35	347.9075	43.75	-	44.15
Tumulus	8.9575	359.2575	52.7075	-	53.1075
Rooftop	15.40	2.9575	68.1075	-	68.5075
Hall	17.10	18.3575	85.2075	-	85.6075
Wall	8.60	35.4575	93.8075	$5^t.7069$, Spring	2.8931
NORTH, total	93.8075	-	-	-	-
Crotch	16.60	44.0575	110.4075	-	19.4931
Pasture	11.80	60.6575	122.2075	-	31.2931
Stomach	15.60	72.4575	137.8075	-	46.8931
Mao	11.30	88.0575	149.1075	-	58.1931
Net	17.40	99.3575	166.5075	-	75.5931
Beak	0.05	116.7575	166.5575	-	75.6431
Triad	11.10	116.8075	177.6575	-	86.7431

Lodge	Ext., *tu*	A	B	C	D
WEST, total	83.85	-	-	-	-
Well	33.30	127.9075	210.9575	4^t.5713, Summer	28.7287
Devils	2.20	161.2075	213.1575	-	30.9287
Willow	13.30	163.4075	226.4575	-	44.2287
Stars	6.30	176.7075	232.7575	-	50.5287
Bow	17.25	183.0075	250.0075	-	67.7787
Wings	18.75	200.2575	268.7575	-	86.5287
Baseboard	17.30	219.0075	286.0575	4^t.7856, Autumn	12.5144
SOUTH, total	108.40	-	-	-	-

The extensions of the lodges on the Red Way given above have been measured with the new armillary sphere. When used as constants they give results corresponding exactly to observation. When calculating past events, however, use lodge extensions of the time as the standard. {1199–1200}

[The authors provide these lodge extensions for use in subsequent computations of equatorial position. The original part, the first two columns, duplicates the information in column F of the Evaluation's table 7.5. I have added other data for later reference. Columns A and C are computed. Columns B and D are derived from Huang Tsung- hsi's *Shou-shih li ku,* 2: 13b-14b. Huang's quantities corresponding to those in column D assume unequal and unsymmetrical quadrants, and have therefore required correction. The values in columns C and D hold only for the years close to the epochal year of the system, since the Standard Points (solstices and equinoxes) on the equator shift gradually with the precession of the equinoxes.

The last paragraph of text cautions against applying the new and improved values for lodge extensions to past events, but does not explain why. Successive stellar surveys had long since shown that the extensions changed, but Kuo and his associates did not seek a way to calculate secular changes in equatorial extension (cf. the discussion of ecliptic extensions in 3.9). They had no reason to believe that the changes were gradual and continuous. As we would explain it today, precession, and to a much smaller extent proper motion, change the right ascensions of determinative stars at rates that vary from one star to another. Not only have the breadths of the lodges—that is, the intervals between the right ascensions of two determinatives—varied, but a pair of lodges that differ considerably in declination have changed their order (see the orientation, p. 94).

The discussion in the Evaluation (section 3) reflects uncertainty on its author's part about whether there actually was a secular shift. The authors of the Canon, who were better astronomers, assume one, but are not specific. There is no point in using early observations if only inaccuracy distinguishes them from later ones. The authors do not suggest any method for coping with future shifts, but instead treat their unprecedentedly precise values as "constants," in the sense of "regular values *(ch'ang shu* 常數)"—fixed for a time.

It was not feasible to predict the changing sidereal extents of the lodges. Extrapolating from ancient tables of lodge extensions did not solve the problem, since early observations were much less precise than Kuo's, and were fitfully recorded. There are isolated surveys of 104 B.C. and A.D. 724, and reiterated surveys only after 1049. Table 7.5 of the Evaluation, collected over a period of 1400 years, must represent all of the sets of observations available to Kuo.

For a given astronomical system, it is not always prudent to assume that the authors based their lodge extensions on contemporary stellar observations. The tables of extensions given as a routine part of astronomical systems were often inaccurate, based on old surveys. The Season-Granting system, however, based its extensions on fresh observations.

Computation of events in the past, although restricted in accuracy by the limitations of old lodge extensions, was not, after all, the main purpose of the Yuan system. Astronomers made such predictions primarily for testing a system against its predecessors. Employing the lodge extensions of the latter did not put the new techniques at a disadvantage.]

3.4. To Compute the Position of the Sun on the Red Way at Winter Solstice

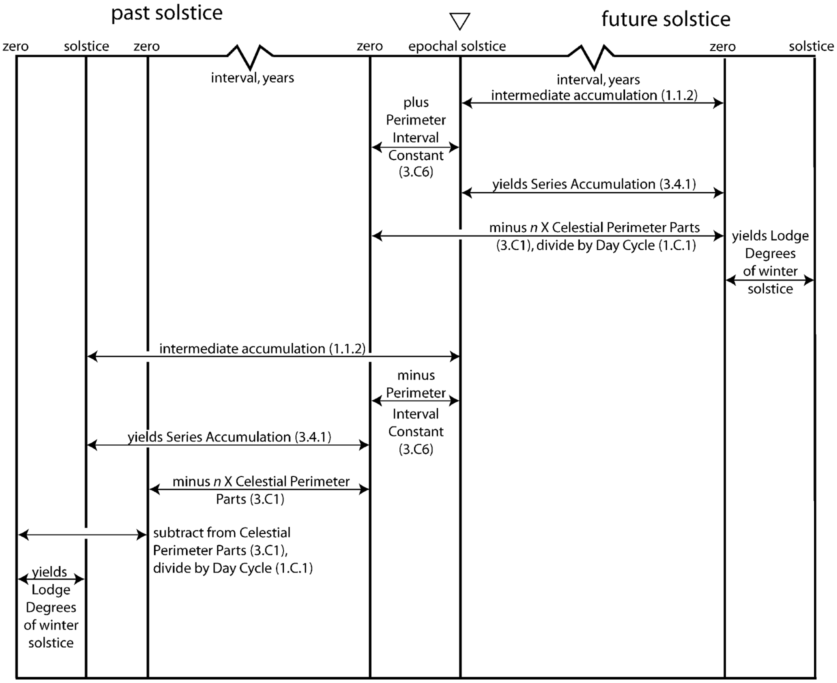
Set up the Intermediate Accumulation (1.1.2) and add it to the Perimeter Interval Constant (3.C6) to yield the Series Accumulation (3.4.1). Cast out full Celestial Perimeter Parts (3.C1)

<When computing backward for past events decrease by one digit per century; when calculating forward for future events increase by one digit per century.>

as often as possible. Reduce the residue by the Day Cycle (1.C1) to yield *tu*. If there is a remainder from this operation, shift it backward [in the computing-rod columns] to reduce it *(t'ui yueh* 退約) to decimal and ten-thousandth parts of a *tu*. Count off exclusively along the Red Way beginning at 6^{t} of the lodge Tumulus, casting out complete lodges until less than a lodge remains. The result is the desired Red Way lodge degrees for the exact time of the sun's location on the day of winter solstice in the Astronomical First Month (3.4.2) in *tu* and parts.

<When calculating backward, subtract the Perimeter Interval Constant from the Intermediate Accumulation, and cast out Celestial Perimeter [Parts]. Any remainder is subtracted from Celestial Perimeter [Parts] and the remainder reduced by the Day Cycle to give *tu*. The rest of the procedure is as above. In cases where the contemporary extensions of the lodges are known, count off the angular distance according to them.> {1200}

Fig. 33. Equatorial position of sun at winter solstice: relation of values when computing past and future solstices.



[Having set out various preliminary operations in 3.1–3.3, the authors now begin the sequence of computations that yields daily position of the apparent sun at midnight and noon (3.4–3.16). The procedure in this step for the equatorial location of the mean sun depends on the definition of Chinese angular measure, the *tu*, as one day's travel of the mean sun through the stars; thus quantities expressed in days and *tu* are interchangeable. See figure 33, and compare the analogous procedure for time of conjunction in figure 25. The Intermediate Accumulation is the number of day parts from winter solstice at the calendrical epoch to winter solstice of the current year. The Perimeter Interval Constant is the angular distance of $315^t.1075$ from 6^t in the lodge Tumulus—the canonic zero point of right ascension—to the sun's location at the epochal winter solstice, 10^t in Winnowing-basket. For more on the significance of this zero point—marked "zero" in the figure—see the commentary to step 3.17.

Because of the day-degree equivalence, the Series Accumulation is the number of *tu* traversed by the sun from its passage through the zero point preceding epoch until the current winter solstice. This Series Accumulation has nothing to do with the identically titled working term given earlier (1.1.3). "Accumulation" terms (Intermediate Accumulation is another) are often used for intermediate variables. Celestial Perimeter Parts are simply the number of degree parts in a full sidereal circle. Casting out full circles gives the distance from the zero point to winter solstice expressed as less than a circle—in other words, the equivalent of right ascension in *tu*. Division by the Day Cycle (10 000) converts degree parts to *tu*. The remainder appears in the computing-rods as degree parts.

The right ascension of the sun is counted off along the lodges one by one. For instance, if the distance from zero point were derived as 50^t, we would count the remaining $2^t.8575$ of Tumulus, the $15^t.40$ of Rooftop, the $17^t.10$ of Hall, and the $8^t.60$ of Wall, for a total of $44^t.0575$ to be cast out. Thus the sun's location would be at $50 - 44.0575 = 5^t.9425$ in the lodge Crotch.

One can make this counting off less tedious by referring to column A of table 9.2, in which I have inserted accumulated *tu* from 6^t in Tumulus to the beginning of each lodge. One need merely find the last entry in the table that is less than the equatorial location of the solstice in *tu*, thus identifying the lodge, and subtract the tabulated value, yielding *tu* within the lodge. In this case the accumulation to Crotch is given directly in column A as $44^t.0575$.

This step on the whole is parallel to 1.1. That is natural, since step 3.4 determines the sun's position on the date computed in 1.1. It is likely that the authors reflected similarity of purpose in similarity of structure. That would explain the elaboration of what, in the simplest possible terms, amounts to moving the position of the solstice backward or forward from that of the epoch by an amount equal to the product of the Annual Difference and the number of years elapsed. The effect of the secular variation in tropical year length, already incorporated in the Intermediate Accumulation 1.1.2 and mentioned in the first text note, must modify Celestial Perimeter Parts (C1) as well.]

3.5. To Find the Position of the Sun on the Red Way at the Four Standard Points

Set up the position of the sun on the Red Way at the exact time of day of winter solstice in the Astronomical First Month and repeatedly add the Aspectual Limit. Cast out full extensions on the Red Way of successive lodges. Each application of the procedure yields *tu* and hundredths and ten-thousandth parts for the position of the sun at the Vernal, Summer, and Autumnal Standard Points (3.5.1). {1200}

[This procedure adds one quarter of a tropical year at each application, and thus yields the right ascension of the mean sun at the summer solstice and the apparent equinoxes (i.e., at the points $2^t.4$ before the mean vernal equinox and after the mean autumnal equinox, figure 30). Trivial though the step may be, it serves an important function in the sequence of solar computations. The basic coordinate system of the Season-Granting system is equatorial, but the sun's motion can be computed in a simple way only along its own orbit, the ecliptic, and then converted to equatorial coordinates. The Four Standard Points (3.0) are the four points at which the equatorial and ecliptic arcs of the apparent sun are equal. This is true of the solstices because at those points in space and time, by Chinese definition, the transition from the phase of expansion to that of contraction takes place. The corresponding modern explanation would be that the solstices are the points at which motion along the great circles of the ecliptic and equator is parallel. It is true of the equinoxes because there the Red and Yellow Ways cross and thus momentarily coincide (3.0, commentary on C7). Once one has determined the mean equatorial arc of the sun at these four points, one knows the mean ecliptic arc, and can use it to compute intermediate positions.]

3.6. To Find Accumulated Degrees on the Red Way to the Lunar Lodges from the Four Standard Points

Set up the total extensions on the Red Way of the lunar lodges in which the Four Standard Points are located and subtract from them the position of the sun on the Red Way in *tu* and parts at the Four Standard Points. The remainder is the Endpoint Elongation Degrees (3.6.1). Each time the extension of a lunar lodge is added, the successive result is Accumulated Red Way Degrees from [one of] the

Four Standard Points to [and including] the lodge (3.6.2) in *tu* and parts. {1200}

[Here one calculates the complement of the sun's mean solstitial or equinoctial position within a lodge by subtracting it from the total extension of the lodge. Adding the extension of subsequent lodges to Endpoint Elongation Degrees yields the angular distance between the solstice or equinox and the farthest point of each lodge. This farthest point, of course, coincides with the beginning of the next lodge.

In table 9.2, column C gives the position of each Standard Point in its lunar lodge for the epoch, and column D gives Endpoint Elongation Degrees for each.]

3.7. Yellow Way–Red Way Rates

[Table 9.3 provides the data for this step. It serves the dual purpose of providing the ecliptic arc corresponding to points along the equator, and the equatorial arc for points on the ecliptic.

A. Accumulated Degrees (ecliptic after solstices, equatorial after equinoxes)
B. Degree Rate (3.7.1, same)
C. Accumulated Degrees (equatorial after solstices, ecliptic after equinoxes)
D. Degree Rate (same)
E. Accumulated Difference (3.7.2)
F. Difference Rate (3.7.3)]

Table 9.3. Yellow Way–Red Way Rates

A	B	C	D	E	F
0	1	-	1.0865	-	0.0082
1	1	1.0865	1.0863	0.0082	0.0246
2	1	2.1728	1.0860	0.0328	0.0411
3	1	3.2588	1.0857	0.0739	0.0576
4	1	4.3445	1.0849	0.1315	0.0741
5	1	5.4294	1.0843	0.2056	0.0907
6	1	6.5137	1.0833	*0.2963	*0.1073
7	1	7.5970	1.0823	0.4036	0.1240
8	1	8.6793	1.0812	0.5276	0.1408
9	1	9.7605	1.0801	0.6684	0.1576
10	1	10.8406	1.0786	0.8260	0.1745
11	1	11.9192	1.0772	1.0005	0.1916
12	1	12.9964	1.0755	1.1921	0.2087

A	B	C	D	E	F
13	1	14.0719	1.0740	1.4008	0.2258
14	1	15.1459	1.0720	1.6266	0.2430
15	1	16.2179	1.0704	*1.8696	0.2605
16	1	17.2883	1.0684	*2.1301	0.2779
17	1	18.3567	1.0663	2.4080	0.2955
18	1	19.4230	1.0642	2.7035	*0.3130
19	1	20.4872	1.0622	3.0165	0.3307
20	1	21.5494	1.0599	3.3472	0.3485
21	1	22.6093	1.0575	3.6957	0.3663
22	1	23.6668	1.0554	4.0620	0.3842
23	1	24.7222	1.0530	4.4462	0.4020
24	1	25.7752	1.0506	4.8482	0.4200
25	1	26.8258	1.0482	5.2682	0.4379
26	1	27.8740	1.0456	5.7061	0.4559
27	1	28.9196	1.0432	6.1620	0.4738
28	1	29.9628	1.0408	6.6358	0.4917
29	1	*31.0036	1.0382	7.1275	0.5095
30	1	32.0418	1.0355	7.6370	*0.5273
31	1	33.0773	*1.0332	8.1643	0.5450
32	1	34.1105	1.0306	8.7093	0.5626
33	1	35.1411	1.0280	9.2719	0.5801
34	1	36.1691	1.0254	9.8520	0.5974
35	1	37.1945	1.0229	10.4494	0.6145
36	1	38.2174	1.0203	*11.0639	0.6314
37	1	39.2377	1.0177	11.6953	0.6481
38	1	40.2554	1.0152	12.3434	0.6647
39	1	41.2706	1.0126	13.0081	0.6808
40	1	42.2832	*1.0102	13.6889	0.6967
41	1	43.2934	1.0075	14.3856	0.7124
42	1	44.3009	1.0049	15.0980	0.7276
43	1	45.3058	1.0027	15.8256	0.7426
44	1	46.3085	1.0000	*16.5682	*0.7571
45	1	47.3085	0.9974	17.3253	*0.7712
46	1	48.3059	0.9951	18.0965	0.7850
47	1	49.3010	0.9925	18.8815	0.7984

A	B	C	D	E	F
48	1	50.2935	0.9901	19.6799	0.8112
49	1	51.2836	0.9876	20.4911	0.8237
50	1	52.2712	0.9851	21.3148	0.8357
51	1	53.2563	0.9827	22.1505	0.8472
52	1	54.2390	0.9803	22.9977	0.8583
53	1	55.2193	0.9780	23.8560	0.8688
54	1	56.1973	0.9755	24.7248	0.8789
55	1	57.1728	0.9731	25.6037	0.8885
56	1	58.1459	0.9708	26.4922	0.8977
57	1	59.1167	0.9685	27.3899	0.9063
58	1	60.0852	0.9661	28.2962	0.9144
59	1	61.0513	0.9639	29.2106	0.9222
60	1	62.0152	0.9616	30.1328	0.9294
61	1	62.9768	0.9594	31.0622	0.9361
62	1	63.9362	*0.9572	31.9983	0.9426
63	1	*64.8934	0.9551	32.9409	*0.9485
64	1	65.8485	0.9529	33.8894	0.9538
65	1	66.8014	0.9509	34.8432	0.9590
66	1	67.7523	0.9487	35.8022	0.9638
67	1	68.7010	0.9470	36.7660	0.9681
68	1	69.6480	0.9450	37.7341	0.9719
69	1	70.5930	0.9427	38.7060	0.9756
70	1	71.5357	0.9412	39.6816	0.9789
71	1	72.4769	0.9392	40.6605	0.9818
72	1	73.4161	0.9385	41.6423	0.9845
73	1	74.3546	0.9353	42.6268	0.9868
74	1	75.2899	0.9343	43.6136	* 0.9891
75	1	76.2242	0.9329	44.6027	0.9910
76	1	77.1571	0.9315	45.5937	0.9925
77	1	78.0886	0.9304	46.5862	0.9940
78	1	79.0190	0.9286	47.5802	0.9952
79	1	79.9476	0.9275	48.5754	0.9962
80	1	80.8751	0.9265	49.5716	0.9972
81	1	81.8016	0.9255	50.5688	0.9979
82	1	82.7271	0.9244	51.5667	0.9984

A	B	C	D	E	F
83	1	83.6515	0.9238	52.5651	0.9989
84	1	84.5753	0.9228	53.5640	0.9993
85	1	85.4981	0.9222	54.5633	0.9996
86	1	86.4203	0.9215	55.5629	0.9997
87	1	87.3418	0.9212	56.5626	0.9999
88	1	88.2630	0.9210	57.5625	1
89	1	89.1840	0.9204	58.5625	1
90	1	90.1044	0.9204	59.5625	1
91	0.3125	91.0248	0.2877	60.5625	0.3125
91.3125	91.3125	91.3125	-	60.8750	-

{1200–9}

[The edition on which this translation is based corrects many copyists' errors in previous ones, especially rife in the numerical tables. Textual corruptions in table 9.3 are not difficult to identify, since each value in column A, C, or E, when added to the difference value to its right, generates the next value in the same column. Values corrected in the modern edition are marked with an asterisk.

One or more of the data in column D, Degree Rates, for 72, 73, and 74 *tu,* is almost certainly still in error. The differences between successive rows in this column decrease in a fairly regular way; for instance, the differences for 68–71 *tu* are 0.0023, 0.0015, and 0.0020 and those for 74–77 *tu* are 0.0014, 0.0014, and 0.0011. But those for 71–74 *tu* are 0.0007, 0.0032, and 0.0010. The first of these three differences is considerably smaller, the second much larger, and the third slightly smaller than the differences between data in the rest of the column leads one to expect. On the other hand, cross-checking with other columns does not reveal which of the values are incorrect.

I have corrected two additional errors overlooked by the Chung-hua Shu-chü editors. There are two columns (rows in the table above) for 91 *tu.* In the first, column B is given as 0.31, but should be 0.3125. In the second, column B is blank, but should be 91.3125, (p. 1224, n. 15; *Ming shih,* pp. 561 and 569, gives the second value incorrectly as 91.31).

We have seen that, in principle, one reads locations after the equinoxes (Quadrants II and IV, figure 30) backward from the following solstice. That is the point of subtracting the equinox–sun arc from a quarter-circle or Aspectual Limit (rounded for the purpose of this table to 91^{t}.3125, as the last entry in column B should show). Instead of carrying out the subtraction, however, one accomplishes the same end by reading column C instead of column A, and vice versa. (The commentary to 3.10 discusses an exception to these rules.)

Thus one reads table 9.3 in this way:

Known Quantity	Quadrant	Enter Column	Read Column
Red Way (equatorial) arc	I, III	C	A
Red Way	II, IV	A	C
Yellow Way (ecliptic) arc	I, III	A	C
Yellow Way arc	II, IV	C	A

Degree Rates (3.7.1) are first differentials of Accumulated Degrees, used for linear interpolation as in 3.8. The values in column E, despite its title, are not accumulated differences. The corresponding table of the *Ming History* uses more apposite terms *(Ming shih,* 32: 553–61, 569). It labels the data in column E above (its second row) Yellow Way Sagitta Degrees *(huang tao shih tu* 黃道矢度 = 3.7.2), and those in column F above (its third row) Yellow Way Sagitta Differences *(huang-tao shih ch'a* 黃道矢差 = 3.7.3). It also points out the Yuan system's error. Yellow Way Sagitta Degrees are used in the solar theory to compute columns C and D, and in the lunar theory to compute polar distances of points on the moon's path (4.18). Yellow Way Sagitta Differences are successive differences used for linear interpolation.]

3.8. To Compute the Extensions on the Yellow Way of the Lunar Lodges

Set up Accumulated Degrees on the Red Way from [one of] the Four Standard Points through [one of] the lunar lodges and subtract Accumulated Degrees on the Red Way. The remainder is multiplied by the Yellow Way Rate and divided by the Red Way Rate. Add the result to Accumulated Degrees on the Yellow Way to give Yellow Way Accumulated Degrees in [i.e. read from the beginning of one of] the 28 lodges. By subtracting Yellow Way Accumulated Degrees for the preceding lodge one obtains the extension on the Yellow Way of the desired lodge (3.8.1) in *tu* and parts.

<Round off *(chiu-chin* 就近) ten-thousandth parts to the nearest hundredth.> {1209}

[This is a linear interpolation procedure for ecliptic extension based on tables 9.3 and 9.4. The ecliptic extensions of the lodges are reckoned as the ecliptic projections of equatorial arcs, counted from a Standard Point, between successive determinative stars. This is convenient for observers using the equatorial ring of an armillary sphere or the Simplified Instrument. For those using an ecliptic ring, the beginning point of an ecliptic lodge would coincide with an observation of the determinative only when the latter lies close to the equator.

The first Accumulated Degrees parameter is the number of *tu* and parts from the preceding Standard Point to the end of the lodge as listed in table 9.2, column D (the Endpoint Elongation Arcs of step 3.6). The second is *tu* listed in table 9.3. One looks up the value following the rules set out in the commentary to step 3.7. One then converts the difference between the tabulated quantity and the exact arc by the simple proportion

$$\Delta\alpha : \Delta\lambda :: \text{Equatorial Rate} : \text{Ecliptic Rate}.$$

Since the value in column B corresponding to any arc of 90^t or less is 1, this proportion is equal to the value in column D after the solstices, and its reciprocal after the equinoxes. If we call the value in column D δ (since it is the difference between successive values in column C), after the solstices $\Delta\lambda = \Delta\alpha / \delta$, and after the equinoxes $\Delta\lambda = \delta \bullet \Delta\alpha$. One adds the interpolated result to the value read from the table.

The difference between two successive calculated values of ecliptic Accumulated Degrees, analogous to the equatorial Endpoint Elongation Degrees, 3.6.1, gives a lodge extension. When the second Accumulated Degrees is smaller than the first—which happens when there is a new Standard Point between the two lodge endpoints (that is, within the second lodge)—one adds an Aspectual Limit (3.C4), $91^t.3144$, to the second before subtracting the first. Column D of table 9.4 shows where the standard points fall.

An example will make this typical procedure clear. Let us determine the ecliptic extension of the lodge Ox, which the sun enters soon after the winter solstice. We begin by consulting table 9.2, Column D, to determine equatorial Accumulated Degrees to the end of the previous lodge, Dipper; the result is 25.60 *tu.* Since we are concerned with the quadrant between winter solstice and the vernal equinox, and know the equatorial extension, we enter column C in table 9.3, and read column A. In column C, the value next lower than the one from table 9.2 is 24.7222 *tu.* Subtracting the two leaves 0.8778 *tu,* and the ecliptic arc that corresponds to 24.7222 is 23. To interpolate, we multiply by the rate in column B that corresponds to the ecliptic arc, 1, and divide by the ecliptic rate in column D, 1.0530. That yields 0.8336 *tu.* Adding it to the ecliptic arc already looked up, the result is 23.8336 *tu,* the ecliptic arc from the winter Standard Point to the end of Dipper. Repeating the procedure to give the corresponding arc for Ox yields 32.80. The difference between the two arcs, that to the end of Dipper and that to the end of Ox (32.80 – 25.60) gives the ecliptic extension of Ox as 6.8986 *tu,* as listed in table 9.4. The commentary to 3.11 will cast further light on this procedure.]

3.9. Angular Extensions of the Lunar Lodges on the Yellow Way

[Table 9.4 transcribes the tabulated ecliptic extensions of the lunar lodges in column A. I have added information parallel to that in table 9.2.

A. Extension given, *tu*

B. Extension, computed by method of 3.8, *tu*

C. Accumulated Degrees, 6^{t} in Tumulus to beginning of lodge, *tu* (based on values in column A)

D. Standard Point to end of lodge, *tu*]

Table 9.4. Extensions on the Yellow Way of Lunar Lodges at the Epoch

Lodge	A	B	C	D
Horn	12.87	12.8711	239.0838	26.4207
Neck	9.56	9.5576	251.9538	35.9783
Root	16.40	16.4056	261.5138	52.3839
Chamber	5.48	5.4841	277.9138	57.8680
Heart	6.27	6.2734	283.3938	64.1414
Tail	17.95	17.9512	289.6638	82.0926
Basket	9.59	9.5900 Winter	307.6138	0.3682
Above are the seven lodges of the East, totaling $78^{t}.12$ [comp. $78^{t}.1330$].				
Dipper	23.47	23.4654	317.2038	23.8336
Ox	6.90	6.8986	340.6738	30.7322
Maid	11.12	11.1180	347.5738	41.8502
Tumulus	9.0075	8.9987	358.6938	50.8489
Rooftop	15.95	15.9471	2.4438	66.7960
Hall	18.32	18.3226	18.3938	85.1186
Wall	9.34	9.3385 Vernal	36.7138	3.1427
Above are the seven lodges of the North, totaling $94^{t}.1075$ [$94^{t}.0889$].				
Crotch	17.87	17.8683	46.0538	21.0110
Pasture	12.36	12.3692	63.9238	33.3801
Stomach	15.81	15.8145	76.2838	49.1946
Mao	11.08	11.0771	92.0938	60.2718
Net	16.50	16.5057	103.1738	76.7775
Beak	0.05	0.0466	119.6738	76.8241
Triad	10.28	10.2909	119.7238	87.1151

Above are the seven lodges of the West, totaling 83^{t}.95 [83^{t}.9723].				
Well	31.03	31.0167 Summer	130.0038	26.8174
Devils	2.11	2.1106	161.0338	28.9280
Willow	13	13.0003	163.1438	41.9283
Seven Stars	6.31	6.3092	176.1438	48.2376
Strung Bow	17.79	17.7903	182.4538	66.0278
Wings)	20.09	20.0790	200.2438	86.1068
Baseboard	18.75	18.7572 Autumnal	220.3338	13.5496
Above are the seven lodges of the South, totaling 109^{t}.08 [109t.0633].				
[Total	365.2575	365.2575]		

[In Table 9.4, I supplement the ecliptic extensions in the text (column A) with computed extensions (column B) for comparison. The added column D gives the computed ecliptic intervals at the end of 1280 from the preceding Standard Point to the end of the listed lodge. The values in the column C are the successive differences in these intervals (although the third and fourth decimal figures repeated for all entries indicate that they were based on values precise to only 0.01 *tu).*

The discrepancies between columns A and B are mostly within the limit of precision, and only three differ by slightly more than 0.01 *tu*. The total extents of three of the quadrants (which are not actually quarter-circles) are also very close to the computed values. The northern quadrant is .099^{t} too large because of the convention that, after the lodge extensions are rounded to two places, the computist adjusts the extent of the lodge Tumulus (which contains the zero point) to make the total of ecliptic extensions 365.2575 *tu.* Note the analogous adjustment in the equatorial extension of Tumulus (table 9.2, column B).]

The above extensions of the lunar lodges on the Yellow Way are computed from observations on the Red Way according to the current system, taking into consideration the Annual Difference of the winter solstice, as a basis for pacing [i.e., calculation]. Whether investigating the past or future, each time [the solstice] shifts one *tu* according to the Annual Difference, make adjustments as directed to yield the lodge extensions. {1209–10}

[This explains a gradual change in ecliptic lodge extensions due to the Annual Difference (3.0), the secular shift in the winter solstice point. These long-term changes in the ecliptic extensions of the lunar lodges occur because Chinese measured their beginnings and endpoints by great circles from the equatorial pole, rather than from the ecliptic pole as in the European zodiac. These shifts imply computing a new counterpart of this table using the method of step 3.8 every time precession amounts to 1 *tu.* Thus table 9.4 is usable for only about 67 years (see also the Evaluation, section 2).

To demonstrate the utility of this secular correction, let us repeat the example just given, but this time for 1750. In the 470 years since the epoch of the previous example, the lodges, shifting at the rate of 0.015 *tu* per year, have moved +7 *tu.* The values of the Endpoint Elongation Arc for Basket, Dipper, and Ox, 0.40^{t}, 25.60^{t} and 32.80^{t} in 1280, have become $7^{t}.40$, $32^{t}.60$, and $39^{t}.80$. Using the quantities in table 9.3, the Yellow Way Accumulated Degrees become $6^{t}.8181$, $30^{t}.5391$ and $37^{t}.5525$. The difference between the last two values makes the extension of Ox in 1750 $7^{t}.0134$. Despite the shift of 7 *tu* in locations, its extension is less than 2 per cent more than $6^{t}.8986$, the width in 1280.

From the modern point of view, there is also a slow change in the equatorial extension of the lodges. It is due mainly to the declinations of the determinative stars, which make the rate of drift vary from one star to another. The compilers of the Yuan treatise were aware that there was such a shift, as we have seen in step 3.3, but they did not see the need to express the change mathematically as part of the system of computation. In thirteenth-century China there was no theory of precession that considered the effect of declination. The lodge determinatives were treated mathematically as though they were all located on the equator. The much smaller effect due to the proper motions of the determinative stars is negligible within the precision of ancient systems. It could not be measured until after the astronomical telescope was perfected.

My preliminary investigations, too complex to merit discussion here, indicate that the technique for determining the ecliptic coordinates of lodge determinatives led to considerable errors for as little as a century before or after 1280. These inaccuracies were largely due to ignoring the declinations of the stars. The astronomers might have avoided such errors if they had measured stellar positions in the ecliptic plane as well and compared them with equatorial coordinates. Their instruments could do that, but the Yuan astronomers made a number of technical compromises (which deserve separate study) to avoid integrating direct ecliptic measures in their procedures. I suspect that was because comparing two types of empirical data would have forced them to confront difficulties in design and application of their technique for the equatorial-ecliptic transform. In this respect it appears once again that, when a well-crafted theory came into conflict with the soundest attainable empirical basis its originators, like pre-modern astronomers elsewhere, tended to favor the former.

This inaccuracy of computed ecliptic lodge extensions is consequential only from the modern point of view. The fundamental stellar datum was not ecliptic position in a given lunar lodge, but a position with respect to a Standard Point, that is, a solstice or an equinox. An erroneous width assigned to a particular lodge, so long as it was used consistently for a given epoch, would not have affected the more fundamental arc between the Standard Point and the star.]

3.10. To Compute the Position of the Sun on the Yellow Way at the Exact Time of Winter Solstice

Set up the position of the sun on the Red Way at the exact time of winter solstice in the Astronomical First Month (3.4). Subtract Red Way Accumulated Degrees (table 9.3). Multiply the remainder by the Yellow Way Rate and count 1 for each Red Way Rate. To the result add Yellow Way Accumulated Degrees. The sum is Yellow Way Solar Degrees at the exact time of the winter solstice in the Astronomical First Month (3.10.1) of that year, in *tu* and parts. {1210}

[This is another linear interpolation technique analogous to that for the ecliptic extensions of the lodges (3.8). It parallels that of step 3.4 for mean equatorial position.

After using the Accumulated Degrees table to convert tabulated equatorial *tu* to ecliptic *tu*, one transforms the remainder of the equatorial arc by the proportion

$$\Delta\alpha : \Delta\lambda :: \text{Equatorial Rate} : \text{Ecliptic Rate}.$$

The result of 3.4 is here converted to an ecliptic arc in preparation for adding the equation of center to give the corrected position of the sun. The next step converts it to position in a lunar lodge.]

3.11. To Compute the Position of the Sun on the Yellow Way at the Exact Time of the Four Standard Points

Set up the Yellow Way–Red Way Difference (3.11.1) for the winter solstices of the desired year and the next year and subtract. In the remainder, count 1 for each 4 [i.e., divide by 4] and add [repeatedly] to the Aspectual Limit to give Corrected Aspect Degrees (3.11.2) for [each of] the Four Standard Points.

Set up the position of the sun on the Yellow Way at the exact time of winter solstice [from 3.10]. Successively add Corrected Aspect Degrees to obtain the [positions at the others of the] Four Standard Points, and cast out complete lunar lodges on the Yellow Way. Each [step] yields the Yellow Way lodge position for the exact time of the corrected *ch'i* of one of the Four Standard Points (3.11.3) in *tu* and hundredths. {1211}

[This procedure, parallel to step 3.5, derives an ecliptic counterpart to the Aspectual Limit (3C.4), the quarter-circle that one adds repeatedly to the sun's position at winter solstice to yield its location at the other three Standard Points.

The Yellow Way–Red Way Difference is, in the words of a seventeenth-century expert, "the remainder when *tu* on the Yellow Way is subtracted from *tu* on the Red Way for the time of winter solstice" *(Shou-shih li ku,* 2: 18b). In other words, it is the difference between the magnitudes of the ecliptic and equatorial arcs from the beginning of the lodge in which the winter solstice is located to the solstitial point itself. For the epoch, 1281, this difference between the equatorial and ecliptic arcs of the solstice amounts to 0^{t}.2758. To find the same quantity for the next solstice, one first subtracts the Annual Difference from this year's equatorial arc to give next year's equatorial counterpart. One then finds the corresponding ecliptic arc with the help of table 9.3, and subtracts the two arcs; the outcome for 1282 is 0^{t}.2747. The difference between the arcs for the two consecutive solstices (0^{t}.0011), added to the equatorial circle, gives a reasonable approximation to the ecliptic round. A quarter of the difference (0^{t}.0003) does the same for the desired ecliptic quarter-circle.

The method given here, as Takebe points out, adapts that given in the Era Epoch system (#71, 1106), using a simple, linear equatorial-ecliptic transform ("Jujireki kaigi 授時暦解議," 5: 38b, 4: 16b-17b; *Sung shih,* 79: 1858–61). The reformers borrowed the term "Yellow Way–Red Way Difference" from that system. In order to move from one Standard Point on the sun's equatorial path to the next, one can add to the first a quarter-circle, an Aspectual Limit. But that quantity is a quarter of a sidereal, not of a tropical, round. At this point, the Yuan astronomers—or their eventual editor—introduced a source of confusion. They took over the Aspectual Limit *(hsiang-hsien)* of the Era Epoch system (originally 91^{t}.3109 for a tropical year round of 365^{t}.2436), without explaining that the quantity was not the one already defined, or even stating it (it became 91^{t}.3105 for their year value, as the Yuan astronomers rounded it). Later sources overcame this confusion by giving distinct names to the tropical year quadrant. The *Ming History,* followed by Takebe, calls it the *Ch'i* Aspectual Limit *(ch'i hsiang-hsien* 氣象限). Huang Tsung-hsi, although he uses the quantity correctly, compounds the confusion by calling it the Celestial Perimeter Aspectual Limit *(sui-chou hsiang-hsien* 歲周象限), which implies that it is sidereal (*Sung shih*, 79: 1851; Takebe, 4: 17a; *Ming shih,* 35: 686, 695; *Shou-shih li ku,* 2: 20b). Adding the tropical quadrant, 91^{t}.3105, to the correction factor, 0^{t}·0003, gives the ecliptic quarter-circle, Corrected Aspect Degrees, as 91^{t}.3108 for 1281.

The second paragraph of this step gives instructions for locating the transition points between the four ecliptic quadrants. One adds Corrected Aspect Degrees (the equatorial quadrant plus one quarter of the annual correction) to the lodge position of the winter solstice point to give *tu* from the beginning of that lodge to the vernal equinoctial point. One then looks up the ecliptic extensions of lodges for the desired year in table 9.4 or its equivalent for the year in question, and subtracts them one at a time until the remainder is less than the extension of the next lodge. What remains is the arc from the beginning of that lodge to the equinox.

Corrected Aspect Degrees is, despite the name, obviously a mean rather than an apparent value. “Corrected *(ting* 定*)*” is used carelessly to distinguish it from the sidereal Aspectual Limit. As Takebe has remarked, this method calls for needless computation; the crucial parameter, the second-order difference, remains constant for centuries on end. The procedure also lacks elegance in that there is a discontinuity once a century when the secular variation adds a digit to or subtracts one from tropical year length. For example, between 1179 and 1180 the second-order difference is a normal $0^{t}.0013$. Between 1180 and 1181 (one century before the epoch), however, it becomes $-0^{t}.0004$, leading to anomalous results for Corrected Aspect Degrees. There is another discontinuity when the winter solstice point moves from one lodge into the next.

Takebe provides a simple substitute for the Corrected Aspect Degrees procedure, at the same time avoiding these discontinuities:

C.A.D. = 1/4 x (365.2575 – [23 / 25 (365.2575 – tropical year length)])

For 1280 this gives an ecliptic round of $365^{t}.2437$ and Corrected Aspect Degrees of $91^{t}.3109$ (Takebe rounds down to $91^{t}.3108$). For several centuries Takebe’s formula remains within $0^{t}.0001$ of the non-anomalistic results of Kuo’s procedure, while entirely avoiding the anomalies.

As for accuracy, step 3.11 appears to be a source of error, although of small magnitude when used for dates near the epoch. Its rationale within the Season-Granting system is the assignment of different circumferences in *tu* to the equatorial and ecliptic arcs between two winter solstices. Such a technique, which accepts geometrical implausibilities for practical purposes within basically numerical reasoning, is typical of the Era Epoch system. But how does it work in the Yuan system?

The second-order difference becomes more understandable when we keep in mind that the shift in the equatorial arc of the winter solstice from one year to the next is the Annual Difference (3.C5). It implies, in fact, that the tropical and sidereal years are not only units of time but circles with different circumferences (although the authors of the treatise did not state this point). The second-order difference is most simply explained as the difference between the equatorial shift of the winter solstice point and its slightly larger precession along the ecliptic. Numerically, it makes no difference whether one visualizes this as a pure increment of precession, or an enlarged ecliptic circle.

From a geometrical point of view it does make a difference. In the Season-Granting system, which made the breakthrough into geometrical visualization in astronomy, one expects assumptions about the relation between the equator and the ecliptic to be well integrated and used consistently. To the contrary, we have found that the authors substituted without notice or explanation an obsolete Aspectual Limit constant for that given earlier. There is also a discrepancy between what is treated in this section as an ecliptic round of $365^{t}.2437$ and the sum of ecliptic extensions given in table 9.4 as $365^{t}.2575$.

The latter quantity is in fact the Celestial Perimeter (3.C2). It is involved because in table 9.4 the authors use the contemporary Aspectual Limit (without specifying it in step 3.8) to compute the equatorial extensions of the lodges that contain the Standard Points. The extensions listed in column C are the differences in successive ecliptic Accumulated Degrees in column D. For instance, in 1281 the extension of Wall is the difference between Accumulated Degrees of $85^{t}.1186$ for Hall and $3^{t}.1427$ for Wall; thus $(3.1427 + 91.3144) - 85.1186 = 9^{t}.3385$. If the Aspectual Limit of the Era Epoch system had been used consistently instead, the extension would have become $9^{t}.3350$. Only the extensions of the four transitional lodges would be affected, each diminished by $0^{t}.0035$ (a quantity below the limit of observational precision). The sum of the lodges would decrease by a total of $0^{t}.0138$ to $365^{t}.2437$. We are left with discrepancies between, first, the values for ecliptic location of the Four Standard Points calculated as this section directs and, second, those implied in table 9.4 (as the differences between the extension of the lodge in column C and the arc from the Standard Point to the end of the lodge in column D). Trial calculations indicate that neither value for the Aspectual Limit perfectly reconciles the two sets of values.

These discrepancies are unnoticeable when extensions are rounded to two decimal places. Nevertheless, they fall short of the customary internal consistency of the Season-Granting system. They indicate that the borrowed computational step was not fully worked into the new system. They also remind us that the thirteenth-century breakthrough into geometric thinking went only so far.]

3.12. To Find the Position of the Sun at Midnight preceding the Four Standard Points

Set up the days and day parts of the fixed [i.e., mean] *ch'i (heng-ch'i* 恆氣) for [one of] the Four Standard Points (1.2.1).

<The winter and summer solstices are the endpoints of the expansion and contraction phases, so [at those two points] fixed [*ch'i*] values may be taken as corrected values.>

Count the Expansion/Contraction Difference (3.2.4) as day parts, and add [to position on the Yellow Way of the regular sun at the equinoxes] if in the phase of expansion, or subtract if in the phase of contraction. The result is the time of the corrected *ch'i* of [one of] the Four Standard Points in days and day parts (3.12.1).

Set up the day parts of the corrected *ch'i* which amount to less than a day (Incomplete Day Parts, 3.12.2), multiply by the sun's travel for the day (table 9.1) and count 1 for each full Day Cycle (1.C1). Subtract the result from the position of the sun on the Yellow Way at the exact time of the Four Standard Points, yielding in each case the position of the sun at midnight preceding the corrected *ch'i* of [one of] the Four Standard Points in *tu* and parts (3.12.3). {1211}

[Here one computes the apparent location of the sun on the ecliptic at the midnights that precede the four Standard Points, the apparent solstices and equinoxes. On that basis, one proceeds to find the apparent location at any midnight (3.13) and then the aim in section 6, the location at the moment of an eclipse.

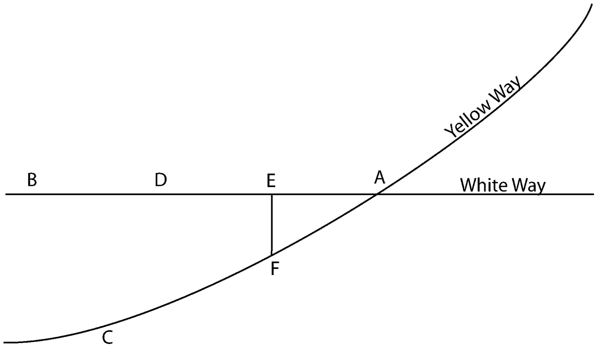
One begins with the mean *ch'i* of the solstices and equinoxes expressed in sexagenary days and parts, from 1.1 and 1.2, and the ecliptic lodge positions of the Standard Points. This first step converts the mean equinoxes to their corrected counterparts by looking up the equation of center (Expansion/Contraction Difference, 3.2.4; see 3.2B) and subtracting it for the vernal equinox or adding it for the autumnal equinox. As the note in the text reminds us, at the solstices it is zero, so no conversion is needed. The result in each case is the interval from the midnight that began the current sexagenary cycle to a Standard Point. Casting out complete days, we are left with Incomplete Day Parts from a Standard Point back to the midnight that preceded it. Since we are dealing with the apparent solar motion, days and *tu* are not interchangeable. One must determine degree parts by the proportion

degree parts : daily solar travel :: day parts : Day Cycle.

Since day parts is already a fraction of the Day Cycle, dividing it by daily solar travel yields degree parts.

The sun's apparent daily motion for the days that contain the Standard Points appears in the tables that once accompanied step 3.2, and that survive in the *Ming History* (table 9.1). Actually, only two values are needed. From column E, the values corresponding to 0 Accumulated Days are $1^t.051\ 085$ for the winter solstice and $0^t.951\ 516$ for the summer solstice. At the equinoxes the sun's daily travel is 1^t by definition. One then subtracts the result of solving the proportion, *tu* along the ecliptic, from the arc between the Standard Point's location in a lodge back to the preceding midnight, giving the location of the preceding midnight.]

Fig. 34. Arc from midnight to Standard Point: relation of values when computing position of the sun at midnight preceding the four Standard Points.



[Figure 34 depicts the relations involved in this step. In it,

AB	is the equator
AC	ecliptic
A	Autumnal Standard Point
B	equatorial position of the sun at the beginning of the current sexagenary cycle
C	beginning of the current lodge on the ecliptic
D	mean equinox
E	equatorial position of the sun at midnight
F	ecliptic counterpart of E
AE	equatorial arc corresponding to day parts from the apparent equinox to the preceding midnight
AF	corresponding ecliptic arc to midnight
CF	ecliptic arc from the beginning of the lodge to midnight.]

3.13. To Find the Position of the Sun on the Yellow Way at Midnight at the Beginning of each Day after the Four Standard Points

Take the interval from the day of the corrected *ch'i* of one of the Four Standard Points to the day of the corrected *ch'i* of the next Standard Point as the Day Interval (3.13.1), and the angular distance from the sun's position at midnight preceding the corrected *ch'i* of the first Standard Point to the sun's position at midnight preceding the corrected *ch'i* of the next Standard Point as the Degree Interval (3.13.2). Reckon successively the corrected motions [on the Yellow Way] in the Day Interval and subtract [their sum] from the Degree Interval, counting 1 for each Day Interval in the remainder, to yield the Daily Difference *(jih-ch'a* 日差, 3.13.3).

<If the Degree Interval is greater [than the sum], add [the Daily Difference]; if smaller, subtract it.>

Add to or subtract from the Daily Angular Motion Rate measured from the Four Standard Points (3.13.4), giving the daily motion in corrected *tu (*3.13.5). Add successively to positions of the sun on the Yellow Way at midnight preceding the Four Standard Points, casting out completed lunar lodges in sequence, to give the Daily Position on the Yellow Way of the Sun at Midnight (3.13.6) in *tu* and parts. {1211}

[This is the basic technique for computing the apparent ecliptic position of the sun among the stars at the beginning of each day. The procedure amounts, in principle, to determining successive daily corrected motions of the sun from tables such as table 9.1 that list the equation of center. The object is to generate a new table of apparent ecliptic positions that can be used to prepare the ephemeris of the coming year.

Although the interval between positions of the sun at midnight preceding two consecutive Standard Points should be identical to the sum of daily tabulated motions over the quadrant, that is not quite the case, for reasons discussed below. One adds one at a time the daily motions, corrected to compensate for the small discrepancy, to the lunar lodge position of the sun at midnight on the day in which the winter solstice occurs. The result is solar positions at sequential midnights up to the one before the vernal equinox, and so on through the four seasons.

One first determines the Day Interval, the number of whole days (that is, intervals from one midnight to another) between two quadrants. The simplest way to do so, as Huang points out, is to take the *ch'i* of midnight before the first Standard Point (3.12), add Corrected Aspect Degrees (3.11.1) corresponding to a quadrant, and add or subtract the equation of center (0 at solstices, ±2.4014 *tu* at equinoxes) to give the apparent rather than the mean position of the sun (the interchangeability of days and *tu* is in play).

At the Winter Solstice of 1281, adding the tropical-year quadrant of 91.3147 days, subtracting 2.4014 *tu,* the equinoctial equation of center, and rounding down, gives 88 days for the Day Interval. As for the Degree Interval, one can avoid lodge-by-lodge counting by using the tables of the equation of center and related functions preserved in the *Ming History* (table 9.1 above). Using successive days in the Day Interval to enter column A, one looks up in column E the corrected daily motion of the sun within each quadrant. Adding the first quantity in column E to the initial position of the sun at midnight of the day containing the winter solstice (3.12) yields the position of the sun at midnight of the next day, and so on daily.

The daily motions in table 9.1 are not, however, rigorously usable without modification, for several reasons:

1. The daily quantities in the tables of equation of center are not computed from midnight to midnight, but at one-day intervals forward and backward from the moments of the solstices.

2. The tables ignore the difference between the Day Interval and the actual time elapsed between two Standard Points. The total of Day Intervals in Astronomical Year 1281, for instance, is only 364 days.

3. Part of the discrepancy between the Degree Interval for a quadrant and the sum of tabulated daily motions over the quadrant is due to imprecision and accumulated error.

The Yuan astronomers could have chosen to deal explicitly with the first two of these sources of discrepancy, at the cost of some additional complication. They chose instead to cope with all three at once by a short cut, empirically adjusting the sum of tabulated motions to make it equal the Degree Interval.

It is not actually necessary to add all the successive corrected angular motions from column E of table 9.1 to obtain the desired position. Column D, the equation of center, is the sum of apparent motions minus (in table 9.1, column A) or plus (in 9.1, B) the number of days in column A. For Quadrant I, for instance, one reads in part A, under 88 *tu,* an equation of center of 240,093,568 x 10^{-8}, or 2.4009, *tu*. Adding this result to the argument, 88 days or *tu* of mean solar motion, yields $90^{t}.4009$ as the sum of the corrected motions. The correction for the whole period is thus 90.4009 - 88 = $2^{t}.4009$, the equation of center listed in column D. Actually, since in Quadrants I and IV the Day Interval can be only 88 or 89 days, and in Quadrants II and II only 93 or 94 days, one needs only four values (see Takebe, 4: 18b-22b, 5: 40a-42b).

The computist applies this adjustment not at the end of the quadrant, but to each tabulated Daily Angular Motion Rate. For that purpose he divides the discrepancy by the Day Interval to give a much smaller factor, the Daily Difference, and adds this fixed increment to or subtracts it from each daily motion. For

1281, for instance, in the quadrant between the Winter and Vernal Standard Points, the Daily Difference is 2.400 935 68 / 88 = 0^t.0273. He finally adds this quantity one at a time to the tabulated daily motions, to build up a table of corrected ecliptic positions for each day of the year. The next steps (3.14–3.16) will convert these to a table of equatorial locations at noon.

A more detailed investigation of this procedure would be worth while. Perhaps it was just as well that the authors, instead of attempting more sophisticated ameliorations at the limit of precision of the system, cut the Gordian knot with this crude but small linear correction.]

3.14. To Find the Daily Position on the Yellow Way of the Sun at Noon

Set up the daily corrected solar motion [for the given day], halve it, and add to the position of the sun on the Yellow Way at the preceding midnight to yield the Position of the Sun on the Yellow Way at Noon (3.14.1) in *tu* and parts. {1211}

[This step determines noon positions by the simplest means, linear interpolation between midnight locations.]

3.15. To Find Daily Accumulated Degrees on the Yellow Way at Noon

Take the interval from the position of the sun on the Yellow Way at the exact time of [one of] the two solstices to the position of the sun on the Yellow Way at noon of the given day as Yellow Way Accumulated Degrees from the solstice (3.15.1) in *tu* and parts. {1212}

[This step sets up the arc from the noon position of the sun to the previous solstice. It will serve in the next step as the argument for conversion to equatorial position at noon of the given day.]

3.16. To Find Daily Degrees on the Red Way at Noon

Set up Yellow Way Accumulated Degrees for noon of the desired day and cast out complete Aspectual Limits. Count the remainder from the Standard Point divisions. Subtract Yellow Way Accumulated Degrees (table 9.3), multiply [the remainder] by the Red Way Rate, and count 1 for each complete Yellow Way Rate in the dividend. Add the result to Red Way Accumulated Degrees (table 9.3) and the [number of] Aspectual Limits previously cast out to give the desired Equatorial Accumulated Degrees (3.16.1) in *tu* and parts. Add the position of the sun on the Red Way at a solstice and count off [by lunar lodges]. The result is the Daily Position of the Sun on the Red Way at Noon (3.16.2) in *tu* and parts. {1212}

[This step uses the table and a simple proportion to convert the daily apparent position from ecliptic to equatorial coordinates. It is the basis for a main product of this section, namely a table of daily positions of the noon sun, measured conventionally along the equator. It is the obverse of the equatorial-ecliptic transform given in 3.8 for ecliptic Accumulated Degrees. For which column to enter, see the instructions in the commentary to 3.7 (p. 435).

The Aspectual Limit subtracted from the ecliptic arc is not the same as that subsequently added to the equatorial arc. The former must be the *Ch'i* Aspectual Limit—or rather its adjusted form, Corrected Aspect Degrees, one-quarter of the ecliptic circle (3.11.1). The latter is equivalent to the quarter-circle arc incorporated in the system of equatorial lodge extensions, namely the sidereal Aspectual Limit (3.C4; Takebe, 4: 23a-24a, 5: 43a-44a).]

3.17. Positions of the Twelve Stations on the Yellow Way with respect to the Lunar Lodges

Table 9.5. Ecliptic Locations of the Twelve Stations

Entry in Station	Markpoint	Location in Lodge, *tu*
Chü-tzu 娵訾	12	Rooftop, 12.6491
Downland (*hsiang-lü* 降婁)	11	Crotch, 1.7363
Big Weir (*ta-liang* 大梁)	10	Stomach, 3.7456
Shih-ch'en (*shih-ch'en* 實沉)	9	Net, 6.8805
Quail Head (*ch'un-shou* 鶉首)	8	Well, 8.3494
Quail Fire (*ch'un-huo* 鶉火)	7	Willow, 3.8680
Quail Tail (*ch'un-wei* 鶉尾)	6	Strung Bow, 15.2606
Longevity Star (*shou-hsing* 壽星)	5	Chariot Crosspiece, 10.0797
Big Fire (*ta-huo* 大火)	4	Base, 1.1452
Split Wood (*hsi-mu* 析木)	3	Tail, 3.0115
Star Marker (*hsing-chi* 星紀)	2	Dipper, 3.7685
Black Hollow (*hsuan-hsiao* 玄枵)	1	Serving-maid, 2.0638

{1212–13}

[Table 9.5 sets out the initial points of the Twelve Stations, equal divisions of the equator that follow an order opposite to that of the annual motions (see the orientation, p. 95). For heuristic purposes, I have changed the order of the data in each column.

The list of entry points appears here in preparation for calculating the time of the sun’s entry into each Station in 3.18. That the Season-Granting system failed to include a procedure to compute tables for years other than 1281 indicates the technical unimportance of the Twelve Stations. They are included because custom gave them a place in annual almanacs among the data needed for divination.

Neither the Yuan nor the Ming history gives a method for calculating this table. Huang Tsung-hsi *(Ming shih,* 2: 22a-25b) and Takebe (4: 24a-26b; 5: 44a-45b) have reconstructed the procedure. It places the reference point for counting off lodges, 6^{t} in Tumulus (3.4), at the center of Black Hollow, which explains why it is Station 1. Since the equatorial arc from the reference point to the Winter Standard Point is known, and the equatorial width of each station is 1/12 of the Celestial Perimeter, it is a simple matter to derive the arc from the Winter Standard Point to the border of the next station. One then converts it to an ecliptic arc (by the method of step 3.8), and finally counts it off among the lodges. For subsequent Stations, one repeats the computation for each 1/12 of the equatorial circle.

As an aid in evaluating the accuracy of 3.17, I have added table 9.6. It provides computations for Astronomical Year 1281.

Table 9.6. Ecliptic Locations of the Twelve Stations by the Method of Takebe
A. Station
B. Equatorial Arc from Standard Point, *tu*
C. Ecliptic Lodge Location
D. Ecliptic Arc from Standard Point, *tu*
E. Ecliptic Extension, *tu*
F. Equatorial Lodge Location
G. Implied error (column F - Table 9.5, col. C), *tu]*

A	B	C	D	E	F	G
Chü-tzu	Winter, 65.3691	Rooftop, 12.2616	63.4981	32.70	Rooftop, 12.6492	+0.0001
Downland	Vernal, 4.4929	Crotch, 1.5998	4.8792	32.24	Crotch, 1.7365	+0.0002
Big Weir	34.9310	Stomach, 3.6379	37.1237	30.03	Stomach, 3.7436	-0.0020
Shih-ch’en	65.3691	Net, 7.1760	67.1524	28.30	Net, 6.8806	+0.0001
Quail Head	Summer, 4.4929	Well, 9.0642	4.1368	28.66	Well, 8.3361	-0.0133
Quail Fire	34.9310	Willow, 4.0023	32.7961	30.70	Willow, 3.8681	+0.0001

A	B	C	D	E	F	G
Quail Tail	65.3691	Bow, 14.8404	63.4981	32.70	Bow, 15.2605	-0.0001
Longevity Star	Autumnal, 4.4929	Baseboard, 9.2785	4.8792	32.24	Baseboard, 10.0868	+0.0071
Big Fire	34.9310	Root, 1.1166	37.1237	30.03	Root, 1.1454	+0.0002
Split Wood	65.3691	Tail, 3.1547	67.1524	28.30	Tail, 3.0110	+0.0005
Star Marker	Winter, 4.4929	Dipper, 0.4929	4.1368	28.66	Dipper, 3.7686	+0.0001
Black Hollow	34.9310	Maid, 2.1310	32.7961	30.70	Maid, 2.0639	+0.0001
TOTAL	-	-	-	365.26	-	-0.0079

[The table gives computed initial points (column F) and intermediate results for all Twelve Stations to provide a check on the initial points given in the text. I have based it on Takebe's method, which is superior to that of Huang. The small error in the *Yuan History*'s computational results (table 9.6, column G, duplicated in the *Ming History*, 35: 696–697, with one insignificant variant) does not show an obvious pattern. If the Yuan authors' approach and Takebe's used different values of the aspectual limit, and if Wang Hsun and his colleagues were in other respects systematic, large errors not merely due to rounding would be concentrated in the Stations following the Standard Points. Instead, the two largest errors fall at the only two points where in the course of the computation it is necessary to add an Aspectual Limit to column D, table 9.4, in order to determine the lodge position on the Yellow Way. Since the two errors are opposite in sign, however, they do not seem due to straightforward application of an inaccurate constant. Since the algebraic sum of the errors is $0^{t}.0079$ for the full circle or an average of $0^{t}.0020$ for an Aspectual Limit, it is possible that the compilers of the Yuan treatise, unconcerned for accuracy to more than two decimal places, used in some less than obvious way the compromise Aspectual Limit of $91^{t}.3125$ already encountered in step 3.7.]

3.18. To Find Time of Entry into the Twelve Stations

In each case set up the initial point of the station in the lunar lodge in *tu* and parts and subtract the position of the sun at midnight preceding the given day. Multiply the remainder by the Day Cycle (1.C1) to give the numerator. The corrected angular motion for the day (3.13.5) is the divisor. Count 1 for each complete divisor in the numerator. The result, found according to the "time of day of putting forth and gathering in" [technique], is the double-hour and marks of entry into a station (3.18.1). {1213}

[Given an entry point from step 3.17, one determines, by inspecting the values generated by 3.13, the ecliptic position of the sun at the midnight immediately preceding. Since the sun follows the order of the lodges rather than the stations, and enters each station from its end, not its beginning, this midnight location will be in the next station, not the preceding one, in table 9.2. The difference between the two points is the arc in *tu* between midnight and the sun's entry of the station in the same day. Division by the corrected daily motion (step 3.2)—solving a proportion involving the apparent daily motion and the mean daily motion of 1 *tu,* both expressed in parts—yields the corresponding interval of solar travel in day parts. The computist then converts day parts to double-hours and marks by the method of step 2.4.]

4. Pacing the Travel of the Moon

[The purpose of each step is to

4.1. Compute elapsed time in anomalistic month for mean conjunction preceding winter solstice.

4.2. Same for mean quarter moons, full moon, and next conjunction.

4.3. Provide instructions for using table (next item).

4.4. Tabulate moon's equation of center, apparent daily motion, and arc from perigee.

4.5. Compute lunar equation of center for time of day.

4.6. Compute date of apparent lunar syzygy.

4.7. Compute ecliptic positions of sun and moon at apparent lunar syzygy.

4.8. Convert to equatorial position of moon at apparent lunar syzygy.

4.9. Compute time interval from moon's perigee to transit (on its own orbit) of equator.

4.10. Convert to date of apparent lunar transit of equator.

4.11. Compute position of apparent lunar transit of equator.

4.12. Compute equatorial arc from winter solstice to transit of apparent moon.

4.13. Compute corrections for arc from equinox to transit and for polar distance of lunar orbit.

4.14. Compute equatorial positions of apparent moon for solstices and equinoxes.

4.15. Compute equatorial position of apparent moon at transit.

4.16. Compute equatorial positions of apparent moon for second transit and points halfway between transits.

4.17. Compute declination for points halfway between transits, and angle between equator and lunar orbit.

4.18. Compute distance from moon's position on its orbit to equatorial pole.

4.19. Compute positions of apparent moon on its orbit for second transit and points halfway between transits.

4.20. Compute position of moon on its orbit at apparent lunar syzygy.

4.21. Compute times of midnights, dawns, and dusks for apparent moon at syzygies.

4.22. Compute daily positions of apparent moon on its orbit at midnight.

4.23. Compute daily positions of apparent moon on its orbit at dusk and dawn.

4.24. Compute daily lunar lodge positions of apparent moon on its orbit at dusk and dawn.]

Revolution Terminal Parts	275 546 parts	4.C1
Revolution Terminal Constant	27 d. 554 6 parts	4.C2
Revolution Midway Constant	13 d. 777 3 parts	4.C3

[This section is devoted to the "travel of the moon," its apparent motion. The first two constants are different expressions for the length of the anomalistic month (modern value 27.554 55 days; see the orientation, p. 101), and the third is half the second. The names of the second and third constants are pronounced identically, since *chung* 終 in 4.C2 meaning "terminal," and *chung* 中 in 4.C3 meaning "middle, midway," are homophones.]

Beginning Extent Constant	84	4.C4
Intermediate Extent Constant	168	4.C5
Cycle Extent Constant	336	4.C6

[Because the moon's daily motion is so large, it is necessary to calculate its equation of center more frequently than once a day. The interval defined for this purpose is the Extent, rounded off from Revolution Terminal Parts/Cycle Extent Constant = 820 day parts or 8.2 marks; one day is 12.2 Extents. An anomalistic month contains *approximately* 336 Extents, equivalent to one Cycle Extent Constant, two Intermediate Extent Constants, or four Beginning Extent Constants (see the commentary to step 4.5B).]

Mean Lunar Motion	$13^{t}.368\ 75$	4.C7

[This constant represents mean daily travel of the moon among the stars. Divided into the Year Cycle (1.C6), it implies a mean sidereal month value of 27.320 62 days for the epoch (modern value 27.321 66 days). It is very close to the approximation 13 7/19° or $13^{t}.3684$, widely used in the West as well as in China (see the orientation, p. 72).]

Revolution Difference	1 d. 9759.93 parts	4.C8

[This is the difference between the lengths of the synodic and anomalistic months, and thus by implication the increase per lunation in the elongation of mean conjunction from the lunar perigee.]

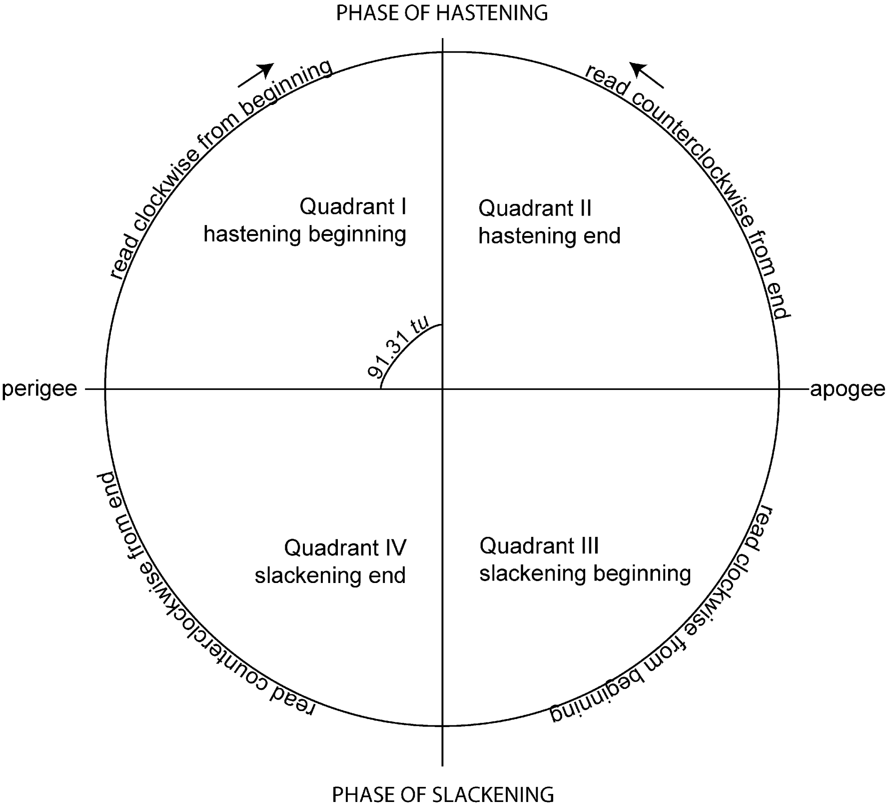
Quarter Moon Factor	7 d. 3826.4825 parts	4.C9

[This value for one-quarter of the synodic month is also given as 1.C10.]

Upper Quarter Moon Constant	$91^{t}.314\,375$	4.C10
Full Moon Constant	$182^{t}.628\,75$	4.C11
Lower Quarter Moon Constant	$273^{t}.943\,125$	4.C12

[These are quadrants of the Celestial Perimeter (3.C2), the ecliptic circle. The “upper” quarter moon follows the sun in its orbit and the “lower” quarter precedes it.]

Fig. 35. Lunar inequality



Revolution Interval Constant 131 904 parts 4.C13 {1213–14}

[Analogous to other interval constants, this gives the time, equivalent to 13.1904 days, between the lunar perigee and the epochal winter solstice of 1280.]

[Standard Crossing Constant $14^{t}.66$ 4C.14.]

[For this constant, unnamed in the text, see the commentary to step 4.13 (p. 475). Kuo's autobiography calls it the Standard Crossing Constant (p. 588).

Figure 35 is an overview of the lunar inequality theory, analogous to figure 30. In this instance the phase of hastening runs between perigee and apogee, and that of slackening, between apogee and perigee.]

4.1. To Compute the Revolution Entry of the Regular Conjunction of the Astronomical First Month

Set up the Intermediate Accumulation (1.1.2), add the Revolution Interval Constant (4C.13), and subtract the Intercalary Surplus (1.3.2). Cast out complete Revolution Terminal Parts (4.C1). The residue is simplified [i.e. divided] by the Day Cycle (1.C1) to give days, and the remainder is parts. This is Revolution Entry of the Regular Conjunction of the Astronomical First Month (4.1.1) in days and day parts.

<When computing backward, to the Intermediate Accumulation add the desired Intercalation Surplus and subtract the Revolution Interval Constant, casting out complete Revolution Terminal [Parts]. Subtract the remainder from Revolution Terminal [Parts]. The rest of the procedure is the same.> {1214}

[The Revolution Entry is time elapsed in the current anomalistic month. When calculating future events, one begins with the number of day parts from the epoch to winter solstice of the desired year, the Intermediate Accumulation. Adding the Revolution Interval Constant, the interval between the beginning of an anomalistic month (the pre-epoch lunar perigee) and the epoch, and subtracting the Intercalary Surplus, the interval between the conjunction and the solstice of the desired year, yields the number of day parts from the perigee to the mean conjunction of the Astronomical First Month. Casting out Revolution Terminal Parts (full anomalistic months) leaves day parts between the beginning of the current anomalistic month and the conjunction. Dividing this remainder by the Day Cycle (day parts per day) converts it to days.]

4.2. To Find the Revolution Entry of Quarter Moons, Full Moon, and the Next Conjunction

Set up days and day parts of the Revolution Entry for the Regular Conjunction of the Astronomical First Month. Add to it repeatedly the Quarter Moon Factor (4.C9), casting out complete Revolution Terminal Constants (4C.2). What results are the Revolution Entry of the quarter moon, full moon, and next conjunction (4.2.1) in days and day parts.

<If directly seeking the next conjunction, add the Revolution Difference (4.C8).> {1214}

[This is simple addition, parallel to 1.4. Since the Revolution Difference is the difference between the synodic and anomalistic periods, each addition increments the date in the anomalistic month from one conjunction to the next.]

4.3. To Find the Argument in the Phases of Slackening and Hastening of Regular Conjunctions, Quarter Moons, and Full Moons

In each case inspect Revolution Entry Days and Day Parts. If less than the Revolution Midway Constant (4.C3), read it as the argument in the phase of hastening (4.3.1); if greater, subtract the Revolution Midway Constant and read it as the argument in the phase of slackening (4.3.2). {1214}

[This step tells the computist how to read column A of table 9.7 (see step 4.4).

The astronomers divided the anomalistic month here, like the sidereal year in the solar theory, into two phases of inequality. The lunar hastening and slackening correspond to the sun's expansion and contraction respectively, and are similarly defined and used (see figure 35). The phase of hastening is the beginning half of the anomalistic month. It covers quadrants I and II of the moon's anomalistic motion between perigee and apogee. Apparent position is in advance of mean position, and the equation of center is positive. In the phase of Slackening, the apparent moon is retarded, and the equation of center negative. That phase includes quadrants III and IV. Quadrants I and III are Beginning Extents; quadrants II and IV are End Extents. In the latter, the values in column B diminish. Since the Revolution Midway Constant is simply half the length of the anomalistic month, subtracting it means that one counts the argument from the beginning of the second half, the phase of slackening.

The word order “slackening and hastening *(ch’ih-chi* 遲疾),” which runs the two together to make a technical term for the lunar inequality, is a habit of everyday language, like the preference in English for “hot and cold” over “cold and hot,” and has no other significance.]

4.4. Slackening/Hastening Degrees, Revolution Degrees, and Accumulated Degrees

[Table 9.7 tabulates the lunar equation of center and other quantities:

A. Revolution Entry Days

B. Beginning or End Extents (B marks Beginning, E End)

C. Slackening or Hastening Degrees (+ marks Hastening; – marks Slackening)

D. Revolution Corrected Degrees, *tu*

E. Revolution Accumulated Degrees, *tu]*

Table 9.7. Slackening/Hastening Degrees, Revolution Corrected Degrees, and Accumulated Degrees

A	B	C	D	E
0	B 0	+0	14.6764	0
1	12.20	+1.3077	14.5573	14.6764
2	24.40	+2.4963	14.4029	29.2337
3	36.60	+3.5305	14.2130	43.6366
4	48.80	+4.3748	13.9877	57.8496
5	61	+4.9938	13.7271	71.8373
6	73.20	+5.3522	13.4446	85.5644
7	E 82.60	+5.4281	13.2353	99.0090
8	70.40	+5.2947	12.9475	112.2443
9	58.20	+4.8735	12.6948	125.1918
10	46	+4.1996	12.4777	137.8866
11	33.80	+3.3086	12.2960	150.3643
12	21.60	+2.2359	12.1496	162.6603
13	9.40	+1.0168	12.0462	174.8099
14	B 2.80	*-0.3088	12.0852	186.8561
15	15	-1.5923	12.2122	198.9413
16	27.20	-2.7488	12.3752	211.1535
17	39.40	-3.7423	12.5730	223.5287
18	51.60	-4.5380	*12.8063	236.1017

19	63.80	-5.1004	13.0753	248.9080
20	76	-5.3938	13.3377	261.9833
21	E 79.80	-5.4248	13.5712	275.3210
22	67.60	-5.2223	13.8511	288.8922
23	55.40	-4.7399	14.0955	302.7433
24	43.20	-4.0131	14.3046	316.8388
25	31	-3.0772	14.4782	331.1434
26	18.80	-1.9677	14.6163	345.6216
27	6.60	-0.7201	14.7154	360.2379

{1215–17}

[As in table 9.3, values corrected in the modern edition are marked with an asterisk. The entry in column C for day 17 incorporates a correction from Ch’en Mei-tung 1995, 304.

Revolution Entry Days (column A), as we have seen in step 4.3, is days in the anomalistic month, counted from the lunar perigee. Beginning/End Extents *(ch’u mo hsien* 初末限, 4.4.1, column B) is a simple cumulation of Extents at the constant rate of 12.20 per day (see the Commentary to constant 4.C6 and steps 3.2A and 4.5A). At the point where adding 12.20 exceeds the Beginning Extent Constant (84), one subtracts the total from the Intermediate Extent Constant (168). The result is End Extents, and one subtracts 12.20 instead of adding it. Slackening/Hastening Degrees (4.4.2, column C) is the daily lunar equation of center (see the commentary to 4.3). Revolution Corrected Degrees (4.4.3, column D) is corrected (i.e., apparent) daily lunar motion. Revolution Accumulated Degrees (4.4.4, column E), the summation of the latter, amounts to the elongation of the moon from perigee.

The maximal equation of center (column C, row 7) falls far short of the modern value, 6°.29 (= 6^{t}.38; Nakayama 1969, 140).]

4.5A. To Find the Slackening/Hastening Difference

Set up the argument in the phase of slackening or hastening in days and day parts and multiply by 12.20 Extents. If the result is less than the Beginning Extent Constant (4C.4), it becomes the Beginning Extent (4.5.1, in Quadrants I and III). If greater, subtract from the Intermediate Extent Constant (4C.5), and read the remainder as the End Extent (4.5.2, in Quadrants II and IV).

Set up the Cubic Difference 325, multiply by the Beginning/End Extent and add the Square Difference 28 100, again multiplying by the Beginning/End Extent. Subtract the result from the Corrected Difference 11 110 000. Multiply the remainder again by the Beginning/End Extent. In the result, count each full 100 000 000 (*i*) as 1 *tu,* shifting backward anything less to give parts. The result is the Slackening/Hastening Difference (4.5.3). {1217}

[The aim of this procedure is to find the lunar equation of center for day parts. The method is parallel to that of 3.2A but simpler. Here the equivalent single formula for the equation of center is $y = 0.111x - 0.000\ 281x^2 - 0.000\ 003\ 25x^3$. Before computing by this formula, one converts the argument from days to Extents. The remainder is negative in the End Extents (Quadrants II and IV), but the procedure uses its absolute value.]

4.5B.Another Method

Set up the argument in the phases of slackening and hastening in days and day parts and subtract the Slackening/Hastening Argument Day Rate. Multiply the remainder by Diminution/Augmentation Parts below [i.e. in the same row of the table as the argument]. For each 820 in the result, count 1. If Augmentation, add it to, or if Diminution, subtract it from, Slackening/Hastening Degrees listed under the argument. The outcome here too is the Slackening/Hastening Difference. {1218}

Table 9.8. Lunar Equation of Center, Ready Reckoner

[This is a ready-reckoning method similar in principle to 3.2B. The *Yuan History* omitted the table that gives the required values in steps of 1 Extent, but it survives in *Ming shih,* 34: 657–72. I reproduce it here as table 9.8. It lists values for 168 Extents, or half of an anomalistic month.

A. Extents

B. Day Rate

C. Diminution/Augmentation Parts

D. Accumulated Degrees, *tu*

E. Angular Motion per Extent, Phase of Hastening, *tu*

F. Angular Motion per Extent, Phase of Slackening, *tu*]

A	B	C	D	E	F
0	-	0.110 815 75	-	1.2071	0.9855
1	0.0820	0.110 234 25	0.110 815 75	1.2065	0.9861
2	0.1640	0.109 633 25	0.221 050 00	1.2059	0.9867
3	0.2460	0.109 012 75	0.330 683 25	1.2053	0.9873
4	0.3280	0.108 372 75	0.439 696 00	1.2047	0.9879
5	0.4100	0.107 713 25	0.548 068 75	1.2040	0.9886
6	0.4920	0.107 034 25	0.655 782 00	1.2033	0.9893
7	0.5740	0.106 335 75	0.762 816 25	1.2026	0.9900
8	0.6560	0.105 617 75	0.869 152 00	1.2019	0.9907
9	0.7380	0.104 880 25	0.974 769 75	1.2012	0.9914
10	0.8200	0.104 123 25	1.079 650 00	1.2004	0.9922
11	0.9020	0.103 346 75	1.183 773 25	1.1996	0.9929
12	0.9840	0.102 550 75	1.287 120 00	1.1988	0.9937
13	1.0661	0.101 735 25	1.389 670 75	1.1980	0.9946
14	1.1481	0.100 900 25	1.491 406 00	1.1972	0.9954
15	1.2301	0.100 045 75	1.592 306 25	1.1963	0.9962
16	1.3121	0.099 171 75	1.692 352 00	1.1955	0.9971
17	1.3941	0.098 278 25	1.791 523 75	1.1946	0.9980
18	1.4761	0.097 365 25	1.889 802 00	1.1937	0.9989
19	1.5581	0.096 432 75	1.987 167 25	1.1927	0.9999
20	1.6401	0.095 480 75	2.083 600 00	1.1918	1.0008
21	1.7221	0.094 509 25	2.179 080 75	1.1908	1.0018
22	1.8041	0.093 518 25	2.273 590 00	1.1898	1.0028

A	B	C	D	E	F
23	1.8861	0.092 507 75	2.367 108 25	1.1888	1.0038
24	1.9681	0.091 477 75	2.459 616 00	1.1878	1.0048
25	2.0502	0.090 428 25	2.551 093 75	1.1867	1.0059
26	2.1322	0.089 359 25	2.641 522 00	1.1856	1.0069
27	2.2142	0.088 270 75	2.730 881 52	1.1846	1.0080
28	2.2962	0.087 162 75	2.819 152 00	1.1835	1.0091
29	2.3782	0.086 035 25	2.906 314 75	1.1823	1.0103
30	2.4602	0.084 888 25	2.992 350 00	1.1812	1.0114
31	2.5422	0.083 721 75	3.077 238 25	1.1800	1.0126
32	2.6242	0.082 535 75	3.160 960 00	1.1788	1.0138
33	2.7062	0.081 330 25	3.243 495 75	1.1776	1.0150
34	2.7882	0.080 105 25	3.324 826 00	1.1764	1.0162
35	2.8702	0.078 860 75	3.404 931 25	1.1751	1.0174
36	2.9522	0.077 596 75	3.483 792 00	1.1739	1.0187
37	3.0343	0.076 313 25	3.561 388 75	1.1726	1.0200
38	3.1163	0.075 010 25	3.637 702 00	1.1713	1.0213
39	3.1983	0.073 687 75	3.712 712 25	1.1700	1.0226
40	3.2803	0.072 345 75	3.786 400 00	1.1686	1.0239
41	3.3623	0.070 984 25	3.858 745 75	1.1673	1.0253
42	3.4443	0.069 603 25	3.929 730 00	1.1659	1.0267
43	3.5263	0.068 202 75	3.999 333 25	1.1645	1.0281
44	3.6083	0.066 782 75	4.067 536 00	1.1631	1.0295
45	3.6903	0.065 343 25	4.134 318 75	1.1616	1.0309

A	B	C	D	E	F
46	3.7723	0.063 884 25	4.199 662 00	1.1602	1.0324
47	3.8543	0.062 405 75	4.263 546 25	1.1587	1.0339
48	3.9363	0.060 907 75	4.325 952 00	1.1572	1.0354
49	4.0183	0.059 390 25	4.386 859 75	1.1557	1.0369
50	4.1004	0.057 853 25	4.446 250 00	1.1541	1.0384
51	4.1824	0.056 296 75	4.504 103 25	1.1526	1.0400
52	4.2644	0.054 720 75	4.560 400 00	1.1510	1.0416
53	4.3464	0.053 125 25	4.615 120 75	1.1494	1.0432
54	4.4284	0.051 510 25	4.668 246 00	1.1478	1.0448
55	4.5104	0.049 875 75	4.719 756 25	1.1462	1.0464
56	4.5924	0.048 221 75	4.769 632 00	1.1445	1.0481
57	4.6744	0.046 548 25	4.817 853 75	1.1428	1.0497
58	4.7564	0.044 855 25	4.864 402 00	1.1411	1.0514
59	4.8384	0.043 142 75	4.909 257 25	1.1394	1.0531
60	4.9204	0.041 410 75	4.952 400 00	1.1377	1.0549
61	5.0024	0.039 659 25	4.993 810 75	1.1359	1.0566
62	5.0845	0.037 888 25	5.033 470 00	1.1342	1.0584
63	5.1665	0.036 097 75	5.071 358 25	1.1324	1.0602
64	5.2485	0.034 287 75	5.107 456 00	1.1306	1.0620
65	5.3305	0.032 458 25	5.141 743 75	1.1287	1.0638
66	5.4125	0.030 609 25	5.174 202 00	1.1269	1.0657
67	5.4945	0.028 740 75	5.204 811 25	1.1250	1.0676
68	5.5765	0.026 852 75	5.233 552 00	1.1231	1.0694

A	B	C	D	E	F
69	5.6585	0.024 945 25	5.260 404 75	1.1212	1.0713
70	5.7405	0.023 018 25	5.285 350 00	1.1193	1.0733
71	5.8225	0.021 071 75	5.308 368 25	1.1174	1.0752
72	5.9045	0.019 105 75	5.329 440 00	1.1154	1.0772
73	5.9865	0.017 120 25	5.348 545 75	1.1134	1.0792
74	6.0685	0.015 115 25	5.365 666 00	1.1114	1.0812
75	6.1506	0.013 090 75	5.380 781 25	1.1094	1.0832
76	6.2326	0.011 046 75	5.393 872 00	1.1073	1.0852
77	6.3146	0.008 983 25	5.404 918 75	1.1053	1.0873
78	6.3966	0.006 900 25	5.413 902 00	1.1032	1.0894
79	6.4786	0.004 797 75	5.420 802 25	1.1011	1.0915
80	6.5606	0.002 675 75	5.425 600 00	1.0990	1.0936
81	6.6426	0.000 534 25	5.428 275 75	1.0968	1.0958
82	6.7246	0.000 356 16	5.428 810 00	1.0966	1.0959
83	6.8066	0.000 178 08	5.429 166 16	1.0965	1.0961
84	6.8886	0.000 178 08	5.429 344 24	1.0961	1.0965
85	6.9706	0.000 356 16	5.429 166 16	1.0959	1.0966
86	7.0526	0.000 534 25	5.428 810 00	1.0958	1.0968
87	7.1346	0.002 675 75	5.428 275 75	1.0936	1.0990
88	7.2167	0.004 797 75	5.425 600 00	1.0915	1.1011
89	7.2987	0.006 900 25	5.420 802 25	1.0894	1.1032
90	7.3807	0.008 983 25	5.413 902 00	1.0873	1.1053
91	7.4627	0.011 046 75	5.404 918 75	1.0852	1.1073

A	B	C	D	E	F
92	7.5447	0.013 090 75	5.393 872 00	1.0832	1.1094
93	7.6267	0.015 115 25	5.380 781 25	1.0812	1.1114
94	7.7087	0.017 120 25	5.365 666 00	1.0792	1.1134
95	7.7907	0.019 105 75	5.348 545 75	1.0772	1.1154
96	7.8727	0.021 071 75	5.329 440 00	1.0752	1.1174
97	7.9547	0.023 018 25	5.308 368 25	1.0733	1.1193
98	8.0367	0.024 945 25	5.285 350 00	1.0713	1.1212
99	8.1187	0.026 852 75	5.260 404 75	1.0694	1.1231
100	8.2008	0.028 740 75	5.233 552 00	1.0676	1.1250
101	8.2828	0.030 609 25	5.204 811 25	1.0657	1.1269
102	8.3648	0.032 458 25	5.174 202 00	1.0638	1.1287
103	8.4468	0.034 287 75	5.141 743 75	1.0620	1.1306
104	8.5288	0.036 097 75	5.107 456 00	1.0602	1.1324
105	8.6108	0.037 888 25	5.071 358 25	1.0584	1.1342
106	8.6928	0.039 659 25	5.033 470 00	1.0566	1.1359
107	8.7748	0.041 410 75	4.993 810 75	1.0549	1.1377
108	8.8568	0.043 142 75	4.952 400 00	1.0531	1.1394
109	8.9388	0.044 855 25	4.909 257 25	1.0514	1.1411
110	9.0208	0.046 548 25	4.864 402 00	1.0497	1.1428
111	9.1028	0.048 221 75	4.817 853 75	1.0481	1.1445
112	9.1848	0.049 875 75	4.769 632 00	1.0464	1.1462
113	9.2669	0.051 510 25	4.719 756 25	1.0448	1.1478
114	9.3489	0.053 125 25	4.668 246 00	1.0432	1.1494

A	B	C	D	E	F
115	9.4309	0.054 720 75	4.615 120 75	1.0416	1.1510
116	9.5129	0.056 296 75	4.560 400 00	1.0400	1.1526
117	9.5949	0.057 853 25	4.504 103 25	1.0384	1.1541
118	9.6769	0.059 390 25	4.446 250 00	1.0369	1.1557
119	9.7589	0.060 907 75	4.386 859 75	1.0354	1.1572
120	9.8409	0.062 405 75	4.325 952 00	1.0339	1.1587
121	9.9229	0.063 884 25	4.263 546 25	1.0324	1.1602
122	10.0049	0.065 343 25	4.199 662 00	1.0309	1.1616
123	10.0869	0.066 782 75	4.134 318 75	1.0295	1.1631
124	10.1689	0.068 202 75	4.067 536 00	1.0281	1.1645
125	10.2510	0.069 603 25	3.999 333 25	1.0267	1.1659
126	10.3330	0.070 984 25	3.929 730 00	1.0253	1.1673
127	10.4150	0.072 345 75	3.858 745 75	1.0239	1.1686
128	10.4970	0.073 687 75	3.786 400 00	1.0226	1.1700
129	10.5790	0.075 010 25	3.712 712 25	1.0213	1.1713
130	10.6610	0.076 313 25	3.637 702 00	1.0200	1.1726
131	10.7430	0.077 596 75	3.561 388 75	1.0187	1.1739
132	10.8250	0.078 860 75	3.483 792 00	1.0174	1.1752
133	10.9070	0.080 105 25	3.404 931 25	1.0162	1.1764
134	10.9890	0.081 330 25	3.324 826 00	1.0150	1.1776
135	11.0710	0.082 535 75	3.243 495 75	1.0138	1.1788
136	11.1530	0.083 721 75	3.160 960 00	1.0126	1.1800
137	11.2350	0.084 888 25	3.077 238 25	1.0114	1.1812

A	B	C	D	E	F
138	11.3171	0.086 035 25	2.992 350 00	1.0103	1.1823
139	11.3991	0.087 162 75	2.906 314 75	1.0091	1.1835
140	11.4811	0.088 270 75	2.819 152 00	1.0080	1.1846
141	11.5631	0.089 359 25	2.730 881 25	1.0069	1.1856
142	11.6451	0.090 428 25	2.641 522 00	1.0059	1.1867
143	11.7271	0.091 477 75	2.551 093 75	1.0048	1.1878
144	11.8091	0.092 507 75	2.459 616 00	1.0038	1.1888
145	11.8911	0.093 518 25	2.367 108 25	1.0028	1.1898
146	11.9731	0.094 509 25	2.273 590 00	1.0018	1.1908
147	12.0551	0.095 480 75	2.179 080 75	1.0008	1.1918
148	12.1371	0.096 432 75	2.083 600 00	0.9999	1.1927
149	12.2191	0.097 365 25	1.987 167 25	0.9985	1.1937
150	12.3012	0.098 278 25	1.889 802 00	0.9980	1.1946
151	12.3832	0.099 171 75	1.791 523 75	0.9971	1.1955
152	12.4652	0.100 045 75	1.692 352 00	0.9962	1.1963
153	12.5472	0.100 900 25	1.592 306 25	0.9954	1.1972
154	12.6292	0.101 735 25	1.491 406 00	0.9946	1.1980
155	12.7112	0.102 550 75	1.389 670 75	0.9937	1.1988
156	12.7932	0.103 346 75	1.287 120 00	0.9929	1.1996
157	12.8752	0.104 123 25	1.183 773 25	0.9922	1.2004
158	12.9572	0.104 880 25	1.079 650 00	0.9914	1.2012
159	13.0392	0.105 617 75	0.974 769 75	0.9907	1.2019
160	13.1212	0.106 335 75	0.869 152 00	0.9900	1.2026

A	B	C	D	E	F
161	13.2032	0.107 034 25	0.762 816 25	0.9893	1.2033
162	13.2853	0.107 713 25	0.655 782 00	0.9886	1.2040
163	13.3673	0.108 372 75	0.548 068 75	0.9879	1.2047
164	13.4493	0.109 012 75	0.439 696 00	0.9873	1.2053
165	13.5313	0.109 633 25	0.330 683 25	0.9867	1.2059
166	13.6133	0.110 234 25	0.221 050 00	0.9861	1.2065
167	13.6953	0.110 815 75	0.110 815 75	0.9855	1.2071
168	13.7773	-	-	-	-

[Since the Day Rate is elapsed time tabulated in constant steps of 0.082 days per Extent— the listing of 8.2008 under 100 Extents implies the precise value, 0.082 008—subtracting it leaves fractional parts of an Extent. Diminution/Augmentation Parts, like Expansion/Contraction Parts in 3.2B, are differences between successive values for the equation of center. Slackening/Hastening Degrees, as in 4.4, are tabulated equation of center values for integral Extents. The term is ambiguous, but the *Ming History* version (35: 691) calls it Slackening/Hastening Accumulated [Degrees], thus identifying it with column D. That column indeed gives the summation of column C for the beginning of each extent. Accumulated Degrees is symmetrical about 84 extents.

To summarize the procedure, one multiplies the increment in the equation of center for a given Extent by Extent parts expressed as a fraction of 820 (the Day Rate expressed in Day Parts), and adds or subtracts the result from the momentary equation of center for the beginning of the Extent. The use of the Day Rate of 0.082 day per Extent is not only approximate, but depends solely on the moon's mean daily motion rather than on the difference between solar and lunar motions.]

4.6. To Find the Date of Corrected Conjunction, Quarter Moons, and Full Moon

Take the Expansion/Contraction Differences and Slackening/Hastening Differences for the regular conjunction, quarters, and full moon, and add if their sign is the same or subtract if their sign is different.

<Expansion and slackening, contraction and hastening, are of the same sign; expansion and hastening, contraction and slackening, are of different signs.>

Multiply by 820 and divide by the listed angular motion per Extent in the phase of slackening or hastening, yielding the Addition/Subtraction Difference (4.6.1).

<In the case of expansion or slackening, add; in the case of contraction or hastening, subtract.>

Add this to or subtract it from days and day parts of regular conjunction, quarters, and full moon, yielding corrected conjunction, quarters, and full moon (4.6.2) in days and day parts. In case the day parts of a corrected quarter or conjunction are less than day parts of sunrise, reckon backward one day.

Count off exclusively days thus obtained in the sexagenary cycle, each time giving the date and time of corrected conjunction, quarter, or full moon (4.6.3). When the denary stem (*kan* 干) of the corrected conjunction is the same as that of the succeeding conjunction, the month is large [i.e. 30 days in the calendar]. When they are different, the month is small [i.e. 29 calendar days]. When there is no medial *ch'i* within the month, it is an intercalary month. {1218}

[First one combines the solar and lunar equations of center algebraically. The text is confusing at first sight. We expect the phases of expansion and hastening to carry the same sign, since both are defined by apparent position ahead of mean position. Actually, the convention defined in the note is straightforward. If both sun and moon are ahead of their mean positions, one must in fact subtract their equations of center to yield the difference between their mean and apparent elongation (Addition/Subtraction Difference). The tabulated angular motion is corrected lunar motion per Extent (0.082 day), tabulated separately for the two phases in table 9.8 (columns E and F). Dividing by the tabular value after multiplying by 820 day parts (0.082 day) per Extent converts the dimension of the correction from Extents to day parts.

The remark about sunrise refers to the civil practice that refers a nighttime event to the date preceding midnight. The *Yuan History* does not give the procedure for determining sunrise parts until step 5.3. Since the first, denary part of the sexagenary day number (the so-called “celestial stem”) repeats itself every 10 days, if the stems agree, the length of the apparent lunar month is 30 days.]

4.7. To Compute the Lunar Lodge Positions of the Sun and Moon at the Exact Times of Corrected Conjunctions, Quarter Moons, and Full Moons

Set up days and day parts of the arguments of regular conjunction, quarters, and full moon in the phases of expansion and contraction, and add or subtract the Addition/Subtraction Differences. The results are the arguments of corrected conjunction, quarters, and full moon. If within the phase of expansion, they become the Intermediate Accumulation (4.7.1). If within the phase of contraction, add the Year Semicycle (3.C7) and the result is the Intermediate Accumulation. Count days as *tu*. If the Expansion/Contraction Difference is in the phase of expansion, add it; if in the phase of contraction, subtract it. The result is Corrected Accumulated Degrees (4.7.2) for the exact time of day. Take the lunar lodge position of the sun on the Yellow Way at the exact time of the winter solstice, add and count off, obtaining in each case the position of the sun at the exact time of corrected conjunction, quarter moons, and full moon (4.7.3).

At the exact time of conjunctions the sun and moon are in the same degree, which is thus the position of the moon at the exact time of corrected conjunction. For quarter moons and full moon, in each case take the position of quarter or full moon and add Corrected Accumulated [Degrees]. The result is Corrected Accumulated Degrees of the moon's motion for the corrected quarter or full moon (4.7.4). Add and count off as above, obtaining in each case the position of the moon on the Yellow Way at the exact time of corrected quarter or full moon (4.7.5). {1218}

[The Addition/Subtraction Differences are computed as in step 4.6, and added to mean intervals measured from winter solstice to yield the position of the mean sun at the time of the apparent phenomena. Because it is the mean sun, elapsed days are elapsed *tu*. The common working term "Intermediate Accumulation" designates the result. Algebraic addition of the solar equation of center yields *tu* from the solstitial point to the apparent sun at the moment of the phenomenon. At apparent conjunction this is also the location of the moon. For the other phenomena, one successively adds Aspectual Limits to the sun's apparent position to give that of the moon. This computation uses the moon's projection on the ecliptic rather than its position on its own orbit, which comes into play only for restricted purposes in the eclipse technique.]

4.8. To Compute the Positions on the Red Way of the Moon at the Exact Times of Corrected Conjunctions, Quarter Moons, and Full Moons

In each case set up corrected Yellow Way Accumulated Degrees of the moon's motion for corrected conjunction, quarter, or full moon. Cast out full Aspectual Limits (3.C4). Subtract Yellow Way Accumulated Degrees from the result. Multiply the remainder by the Red Way Rate (3.7) and count 1 for each full Yellow Way Rate in the answer. To the result add the Accumulated Degrees on the Red Way below [i.e. in the same column of table 9.3] and the Aspectual Limits previously cast out. The outcome in each case is Red Way Accumulated Degrees for the exact time of day (4.8.1). Take the position of the sun on the Red Way at the exact time of winter solstice (3.4), add, and count off. In each case the result is the position of the moon on the Red Way at the exact time of corrected conjunction, quarter, or full moon (4.8.2) in *tu* and parts.

<If [the initial Accumulated Degrees] is less than an Aspectual Limit or if a [Celestial] Semi-perimeter has been cast out, the result falls after a solstice. If a full Aspectual Limit or three of them have been cast out, it falls after an equinox.> {1218–19}

[This method parallels that for transforming the sun's apparent daily position at noon from ecliptic to equatorial coordinates (3.16). The note is a corollary of the definition of quadrants.]

4.9. To Compute from the Mean Crossing after a [Given] Conjunction the Argument in the Phase of Slackening or Hastening for the Revolution Entry

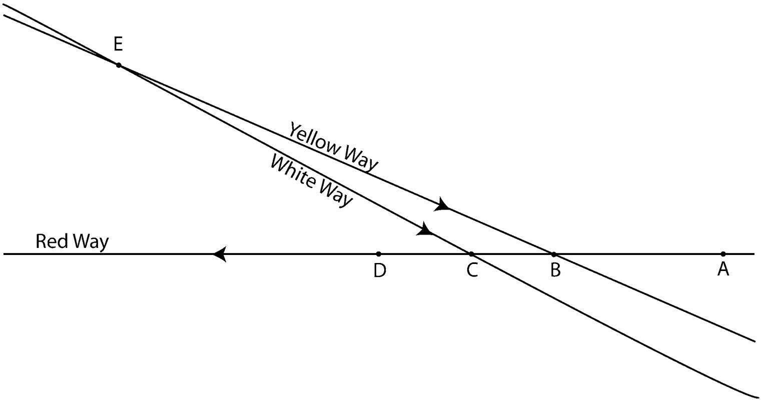
Set up days and day parts of the Crossing Terminal Constant (6.C2) and deduct from it Crossing Entry Days and parts for the regular conjunction; the result is days after conjunction to Mean Crossing (4.9.1). Add this to the Revolution Entry of the regular conjunction (4.1.1) to yield the Revolution Entry of the Mean Crossing following the conjunction (4.9.2). If less than the Revolution Midway Constant (4.C3), it becomes the argument in the phase of hastening; if greater, subtract the constant and the result becomes the argument in the phase of slackening. {1219}

[This step begins computation involving the lunar orbit or White Way. The terminology relates that path to the Red Way (the equator) and the Yellow Way (the ecliptic).

Because the word *chiao* 交 can refer in different contexts either to an intersection or to the passage of the sun or moon through one, I translate it below, alone or in combination, by the comparably ambiguous word "crossing." In varying circumstances, it can refer to any intersection of two orbits: the equinoctial points (where the equator and ecliptic cross), the lunar nodes (junctions of the ecliptic and the moon's path), and the intersections of the lunar orbit and equator. See figure 36 for the spatial relations of these circles. It shows

A	beginning of lodge
B	equinoctial point
C	Standard Crossing
D	end of lodge
E	descending lunar node at solstice (distance not to scale), and
BC	Elongation Difference (4.13.2)
CD	Accumulated Degrees after the Standard Crossing (4.16.1)]

Fig. 36. Lunar intersections.



[In the lunar theory of the Yuan system, *chiao* when used as a noun without specification usually (but not always) refers to the equatorial intersections of the lunar orbit rather than to the lunar nodes. Logically, in this step it could refer to either without violating the sense. In the next (4.10), again only days are involved. Step 4.11 calculates the ecliptic arc from solstice to corrected transit, and 4.12 uses that arc. The intersection involved must thus be either the lunar node or the ecliptic projection of the equatorial intersection. Step 4.13 clearly has to do with the latter. By reading back along this chain, I identify the Mean Crossing in the present section as the equatorial intersection (C in figure 36).

Takebe rejected the four steps 4.9–4.12 on the ground that they were not used later in the lunar theory, and suggested that they had been thoughtlessly injected from some earlier system. This view is difficult to accept. The result of each procedure from here on is used in the next. The Beginning/End Extent and position with respect to the solstice computed in 4.12 are the starting point of 4.13.

But something is wrong *before* step 4.9. The Crossing Terminal Constant is not listed at the beginning of Section 4; nor is Crossing Entry Days the product of any previous step. It is possible that the authors, or more likely a later editor, moved these steps from the Canon's eclipse theory (section 6) without revising them to fit them into this sequence.

The Crossing Terminal Constant is very close to the Western nodical month (modern value 27.212 220 days), the mean period between successive transits of the same node. That constant is defined later, in the eclipse theory, as 27.212 224 days (6.C2). There it is clearly the interval between two transits of the same lunar node. Here, if I am correct, this constant is used for the interval between two equatorial intersections.

As for Crossing Entry Days, there is no such quantity anywhere in the Canon. But Huang Tsung-hsi, in his version of this step, calls it instead Crossing Entry Comprehensive Days, a term that serves the same function in step 6.1 *(Shou-shih li ku,* 4: 12b, renumbered 4.7). I therefore recommend for further study the hypothesis that at least steps 4.9 – 4.12 were moved from section 6.

This interpretation implies that the Yuan astronomers incorporated a greater innovation in their lunar theory than commentators have given them credit for. Their predecessors counted the moon's position on its orbit from the ecliptic intersection (the node). By moving the reference point to the equatorial intersection, the authors of the Season-granting system took advantage of the relatively fixed character of the latter. The lunar node makes a complete circuit of the ecliptic in slightly over 18 years; the equatorial intersection librates on the equator to a maximum of only $14^{t}.66$ from the equinoctial point in half that time (see 4.13). Whether this change led to increased computational accuracy remains to be determined. The authors perhaps chose it because it facilitates visualizing spatial relations in a system primarily oriented to the equator.

The two equatorial intersections of the lunar orbit are called the Standard Crossing *(cheng-chiao* 正交; see 4.10) and Intermediate Crossing *(chung-chiao* 中交; see 4.16). The former is the reference point for calculating transits (passages of the moon through an intersection), hence its name. The moon crosses it after its transit of what modern astronomers call the descending node. The

Standard Crossing differs from the Mean Crossing in being an apparent phenomenon. The term “Mean Crossing” can in principle be applied to either intersection (since either can follow a conjunction), but since its date is used in step 4.10 to calculate the Standard Crossing, it must refer to the mean transit following the descending node.

Nodal Crossing Entry Days is thus the date of the conjunction in what one might call the month defined by transits of the Standard Crossing. The subtraction gives the mean period from conjunction to the next transit of the same intersection. Adding Revolution Entry yields the mean period from the previous lunar perigee to the same transit. The outcome of this computational step is the date of the transit in the current anomalistic month. If it falls in the second half of the mean anomalistic month, one subtracts half the length of the month in order to read the argument from the beginning of the phase of slackening.]

4.10. To Find the Date of the Standard Crossing

Set up the regular conjunction and add days after conjunction to Mean Crossing. Using the argument in the phase of slackening/hastening, proceed as before until you have derived the Slackening/Hastening Difference (4.5.3). If in the phase of slackening, add it [to the previous result]; if in hastening, subtract it. The result is days and day parts of the Standard Crossing. Count the days off exclusively in the current sexagenary cycle, yielding the Standard Crossing Date (4.10.1). {1219}

[In this procedure one first obtains from the results of the last step a value corresponding to the lunar date of the Mean Crossing, and adds to it the lunar equation of center computed by either procedure of step 4.5.]

4.11. To Compute the Position of the Moon on the Yellow Way at the Exact Time of Standard Crossing

Set up days after conjunction to Mean Crossing, and multiply by the Mean Lunar Motion in *tu* (4.C7) to give Endpoint Elongation Degrees (4.11.1). Add this to the Intermediate Accumulation for the regular conjunction (4.7.1) to give corrected Accumulated Degrees for the interval from the winter solstice to the Standard Crossing. Add to this interval the lunar lodge position of the sun on the Yellow Way at winter solstice and count off. The result is the lunar lodge position of the moon on the Yellow Way at the exact time of the Standard Crossing (4.11.2) in *tu* and parts. {1219}

[This procedure begins its count from the mean conjunction, when the positions of mean sun and moon coincide. Multiplying the time between conjunction and Mean Crossing by the moon's daily mean motion converts the time to *tu*. The working term Endpoint Elongation Degrees marks this intermediate value, but as usual does not imply any connection with the identically named intermediate value in 3.6. One then adds Endpoint Elongation Degrees to the interval from winter solstice to the mean sun at the moment of conjunction, read as *tu* rather than as days. The result is the interval from the solstitial point to the moon's transit of the node. Added to the ecliptic position of the solstice, it yields the ecliptic lunar lodge position of the lunar orbit-equator intersection (i.e., of its projection on the ecliptic). This is by definition the position of the apparent moon at its transit of this intersection (the Standard Crossing).]

4.12. To Find the Solstice after which, and the Beginning/End Extent in which, the Standard Crossing is located

Set up Accumulated Degrees and parts for the interval between winter solstice and the Standard Crossing. If less than a Year Semicycle (3C.7), read it after the winter solstice. If greater, cast out the Year Semicycle and read the remainder after the summer solstice. Following the solstice, if [the quantity just derived] is less than an Aspectual Limit (3C.4), read it in the Beginning Extent (4.12.1); if greater, subtract a Year Semicycle and read the result in the End Extent (4.12.2). {1219}

[Unlike the simpler argument in the anomalistic month derived in step 4.3, this argument is in principle identical with that of step 3.2A and figure 30 (see figure 35). The *Ming History* adds to its corresponding version (35: 701, step 4.9) rules for succeeding months.]

4.13. To Find Corrected Difference, Elongation Difference, and Corrected Limit Degrees

Set up *tu* in the Beginning or End Extent and multiply by $14^{t}.66$; count 1 for each full Aspectual Limit in the answer. The result is the Corrected Difference (4.13.1). Subtract this from $14^{t}.66$; the remainder is the Elongation Difference (4.13.2). Multiply the Corrected Difference by 24 and take 1 for each $14^{t}.66$ in the answer. If the [Standard] Crossing falls after the winter solstice, the sign of the result makes it subtract; if after the summer solstice, its sign makes it add.

In either case add to or subtract from 98^t to give Corrected Limit Degrees (4.13.3) in *tu,* hundredths and ten-thousandth parts. {1220}

[The quantity $14^t.66$—although untitled and not among the constants at the head of section 4—is one of the five innovations of which the authors of the Season-Granting system were proudest (see appendix B, p. 588). Fundamental to the theory of the lunar node, it represents the maximum value of the Elongation Difference, the distance along the equator from the nearest equinoctial point to the Standard Crossing (the equator's intersection with the lunar orbit, arc BC in figure 36). Because of the importance of this value, I have added it to the list at the beginning of section 4 as the Standard Crossing Constant (4C.14).

The Elongation Difference reaches its maximum when the descending node coincides with the summer solstice. The difficulty lay in determining the Elongation Difference when the lunar node and the solstice do not coincide, given the rudimentary computational tools for spherical problems available to the Yuan astronomers. They drastically simplified this task by simple linear interpolation between 0^t and the maximum. This way of finding intermediate values of complex functions was a common source of inaccuracy in earlier astronomy. The reformers would have been aware of its uses and limitations. The outcome in this case is more accurate than one might expect, as the maximum values in figure 37 indicate. Nevertheless, the astronomers applied the results in a way that led to substantial errors. This approach may signal a conviction that the exact value was not crucial.

Fig. 37. Comparison of the Elongation Difference given by modern computation and the results of linear interpolation used in the Yuan system.

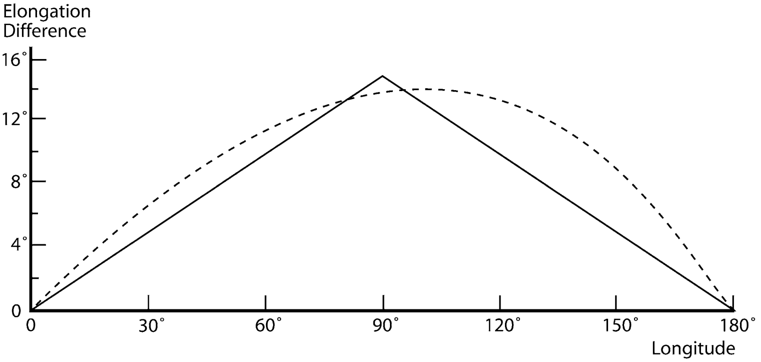


Figure 37 is a graph of the zigzag function resulting from this step, plotted together with modern values in degrees (redrawn from Nakayama's detailed discussion in 1969: 259–62). The Elongation Difference thus takes on the range of values linearly interpolated over a quarter-cycle. The Corrected Difference is the difference between this value and the maximum, $14^{t}.66$. It should not be confused with the constants of the same name in steps 3.2 and 4.5.

Corrected Limit Degrees is another crude correction for variation in the polar distance of the lunar orbit (see figure 38 below). Since it is also based on the Corrected Difference, it is accurate only for the greatest value. At the solstice the polar distance correction is maximal, equal to the obliquity of the ecliptic (given for this purpose the obsolete value of 24^{t} rather than the $23^{t}.9$ used elsewhere in the system). At other times the correction factor, to be added to or subtracted from 98^{t}, is also a linear function, the maximum value multiplied by the argument within a given quadrant and divided by the number of *tu* in a quadrant (cf. *Ming shih,* 35: 9a). The mean value for polar distance is thus the round sum of an Aspectual Limit and the inclination of the lunar orbit, a total of 98^{t}. The arc varies between 74^{t} and 122^{t} (Nakayama 1969, p. 261, diagram 7).

The word I have translated above as “the sign [of the result]” is *ming* 名, a special mathematical usage; see Libbrecht 1973, 484.]

4.14. To Find Lunar Lodge Position on the Red Way at the Four Standard Points

Set up degrees on the Red Way at the exact time of the winter solstice; count them as Standard Degrees for the winter solstice (4.14.1). Repeatedly add Aspectual Limits, obtaining Standard Accumulated Degrees (4.14.2) for the vernal equinox, summer solstice, and autumnal equinox respectively. Count off with respect to the lunar lodges, casting out completed lodges. The result is Red Way Lunar Lodge Degrees for the Four Standard Points (4.14.3) in *tu* and hundredths and ten-thousandth parts. {1220}

[This step determines the equatorial position of the apparent moon in a lunar lodge for the beginnings of the four quadrants, to be used in the next section. It differs from step 3.5 only in phrasing.]

4.15. To Find the Moon’s Lunar Lodge Position on the Red Way at Standard Crossing

Add the Elongation Difference to or subtract it from the lunar lodge position on the Red Way of the Vernal or Autumnal Standard Point to give the moon’s lunar lodge position on the Red Way at Standard Crossing (4.15.1) in *tu* and ten-thousandth parts.

<Following the winter solstice, add the Elongation Difference in the Beginning Extent or subtract it in the End Extent, and read from the

Vernal Standard Point. Following the summer solstice, subtract it in the Beginning Extent or add it in the End Extent, and read from the Autumnal Standard Point.> {1220}

[See the commentary to 4.13 on the Elongation Difference. The note in the text defines the equinoctial point closest to the intersection.]

4.16. To Find Accumulated Lunar Lodge Degrees on the Red Way after the Standard Crossing, and Beginning/End Extent Entry

Set up *tu* and parts on the Red Way for the whole breadth of the lunar lodges occupied by the Vernal and Autumnal Standard Points. Subtract the moon's lunar lodge position on the Red Way in *tu* and parts at the Standard Crossing. The remainder is Accumulated Degrees after the Standard Crossing (4.16.1). Repeatedly add extensions on the Red Way of the lunar lodges. After casting out a complete Aspectual Limit, read the result after the Crossing Halfway Point (4.16.2). After again casting one out, read the result after the Intermediate Crossing (4.16.3). After again casting one out, read the result after the Crossing Halfway Point (4.16.4). Inspect Accumulated Degrees for each crossing. If less than half an Aspect[ual Limit], read it in the Beginning Extent; if greater, subtract it from an Aspectual Limit and read the remainder in the End Extent. {1220}

[Accumulated Degrees is the equatorial interval from the Standard Crossing to the end of the lunar lodge in which it is located (arc B in figure 36). This term is used in counting off the positions in lunar lodges corresponding to Beginning/End Extent Entry (4.16.2). These are three points at one-quadrant intervals from the Standard Crossing: the intersection following the ascending node (Intermediate Crossing) and the two points on the lunar orbit equidistant from the Standard and Intermediate Crossings (Crossing Halfway Points). The latter are thus points of maximal declination, but what is calculated here are the equatorial positions of their projections. In the next-to-last sentence of this section, the word "crossing" applies to Crossing Halfway Points as well as to the Standard and Intermediate Crossings.

The phrasing of this section does not hold when a new lunar lodge begins between the equinoctial point and the crossing. To avoid confusion, the corresponding section of the Great Concordance system recasts the first sentence *(Ming shih,* 35: 702, 4.14): "Determine by inspection the lodge in which the White Way's Standard Crossing of the Red Way is located. Then set up the entire extent on the Red Way of that lunar lodge and deduct the position within the lodge of the White Way's Standard Crossing with the Red Way …"]

4.17. To Find Degrees inside or outside the Red Way and Corrected Difference for the White Way when the Moon's Position is at the Crossing Halfway Point after the Standard Crossing

<[Following "White Way":] Formerly called "the Nine Paths.">

Set up *tu* and parts for the Corrected Difference (4.13.1) at each crossing, multiply by 25, and for each 61 in the result count 1. Determine by inspection whether the position in the lunar lodges of the moon's transit of the Yellow Way falls after the winter solstice—in which case subtract the result thus obtained—or after the summer solstice—in which case add it. In each instance add it to or subtract it from $23^{t}.90$, yielding Degrees inside or outside the Red Way for the White Way at the Crossing Halfway Point after the moon's motion [through the Standard Crossing] (4.17.1) in *tu* and parts. Divide by 1/6 of the Celestial Perimeter, $60^{t}.876\ 25$, to give the Corrected Difference (4.17.2).

<If after the moon's motion through the equatorial Standard Crossing, [the Halfway Point] is outside; if after the Intermediate Crossing, it is inside.> {1220–21}

[In astronomical usage "inside *(nei* 内)" is south of the equator, and "outside *(wai* 外)" north.

The result of this procedure is the angle of the White Way with the Red Way. The ratio 25/61 (= 0.4098) approximates 6/14.66 (= 0.4093) and is substituted in order to avoid operations with decimal fractions. We have already encountered $14^{t}.66$ as the maximal value of the Corrected Difference; 6^{t} is a round value for the inclination of the lunar orbit to the ecliptic. Here the Season-Granting system's value for the angle between equator and ecliptic is $23^{t}.90$. Thus the method for calculating 4.17.1 is equivalent to

$$\theta = \varepsilon \pm \psi_{max} ([E_{\max} - E] / E_{\max})$$

where θ is the obliquity of the lunar orbit, ε the obliquity of the ecliptic, ψ the angle between the lunar orbit and the ecliptic, and E the Elongation Difference ($14^{t}.66$ - Corrected Difference) from 4.13 (see figure 36). In other words, this step linearly interpolates the Elongation Difference to crudely approximate the desired angle at a given time.

The Corrected Difference computed at the end of this step is not the same as the one that begins it; it is

$$D = \theta / r$$

where r is the Celestial Perimeter divided by 2π. Because the Shen Kua tradition of spherical proto-trigonometry uses 3 for π, 2π = 6 (orientation, p. 67).

Note that the word translated "Crossing" at the beginning of 4.17 refers to four points, the Crossing Halfway Points as well as the Crossings. The second textual note implies that the sign in the first equation depends on which Crossing Halfway Point the computist is considering.]

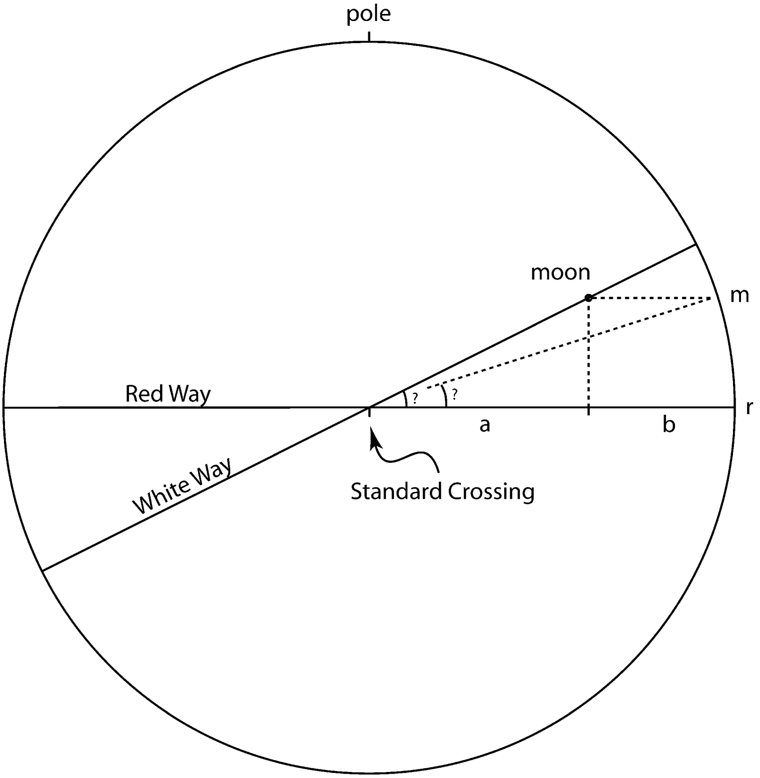
4.18. To Find the Polar Distance of the Moon's Position on the White Way inside or outside the Red Way

Set up the Beginning or End Extent of the moon's daily position on the Red Way after [Standard] Crossing and subtract it from an Aspectual Limit. The remainder is White Way Accumulation (4.18.1). Subtract from this the corresponding Accumulated Degrees and multiply the remainder by its Difference Rate (3.7.3). The result thus obtained is simplified by 100 and added to the corresponding Accumulated Difference (3.7.2) to give the Daily Accumulated Difference (4.18.2). Subtract this from 1/6 the Celestial Perimeter. Multiply the remainder by the Corrected Difference to give the daily angle of the moon's position inside or outside the Red Way. If inside subtract it from, or if outside add it to, an Aspectual Limit to give the daily polar distance of the moon's position on the White Way (4.18.3) in *tu* and hundredths and ten-thousandth parts. {1221}

[Figure 38 is a projection centered on the Standard Crossing. Its radius is $a + b$, where a is the distance from the Standard Crossing to the moon's equatorial position, and b is the sagitta. The polar distance is the arc from the equatorial pole to m. The inclination of the moon's orbit to the equator is θ, and δ is the angle between the equator and the moon.

When the arc from Standard Crossing to the moon's position (see 4.12) is subtracted from a quarter-cycle, the result is the interval from the moon to the next Crossing Halfway Point, that is, the point of maximum declination of the lunar orbit. There are two steps in using table 9.3 to determine Daily Accumulated Difference (the sagitta, b, in the figure). The first is to look up integral elapsed *tu* (Accumulated Degrees) in column C to yield Accumulated Difference. The second is to linearly interpolate fractional parts by finding the Difference Rate in column F and adding the result. As in step 4.17, 1/6 of the Celestial Perimeter is r, the radius of the celestial circle. Subtracting the sagitta from it leaves a.

Fig. 38. Variation in north polar distance of the moon with declination.



From the simple proportion

$$\delta / a = \theta / r$$

declination is

$$\delta = a \cdot \theta / r$$

or *a* times the Corrected Difference from 4.17. Because Chinese astronomers normally spoke of the polar arc rather than of the declination, they determined the former from the latter by adding to or subtracting from a quarter-cycle.]

4.19. To Find the White Way Accumulation and Lunar Lodge Position for the Moon at Each Crossing

Set up Corrected Limit Degrees, subtract Beginning/End Extents, and multiply [the former by the remainder]. Shift backward to read as hundredths of a *tu*; this is the Corrected Difference (4.19.1).

<After Standard or Intermediate Crossings add it; after Crossing Halfway Points subtract it.>

Add the Difference to, or subtract it from, Red Way Accumulated Degrees after the Standard Crossing (4.16.1). The result is Corrected Accumulated Degrees for the moon's position on the White Way (4.19.2). From it subtract Accumulated Degrees on the White Way for the preceding lunar lodge. In each case one obtains the moon's lunar lodge position on the White Way (4.19.3), to a hundredth of a *tu*. {1221}

[This is the first of two steps that give the apparent position of the moon on the lunar orbit itself. The Standard Crossing is an obvious starting point, since there equatorial and White Way positions coincide (although their lunar lodge coordinates do not necessarily coincide).

The sequence of subtraction and multiplication at the beginning is a customary way of dealing with symmetrical second-degree functions. The procedure amounts to

$$f(y) = kx\,(B - x),$$

where $f(y)$ is maximal at $x = B / 2$, and 0 at $x = 0$ and $x = B$. The constant, in this case $k = 0.01$, reflects the shift in decimal value. B is normally the last value of x in an extent. Thus if x is the Beginning/End Extent, B would be the Aspectual Limit constant.

The authors, instead of using the Aspectual Limit, substitute Corrected Limit Degrees, the crude polar distance correction value from 4.13. Evidently they wanted to use a value that depended on declination, but this was an unfortunate choice. Corrected Limit Degrees, as we have seen (4.13, commentary) varies linearly between 74^t and 122^t. The result is a function that does not reach 0 at the end of a Beginning/End Extent, but see the next step.

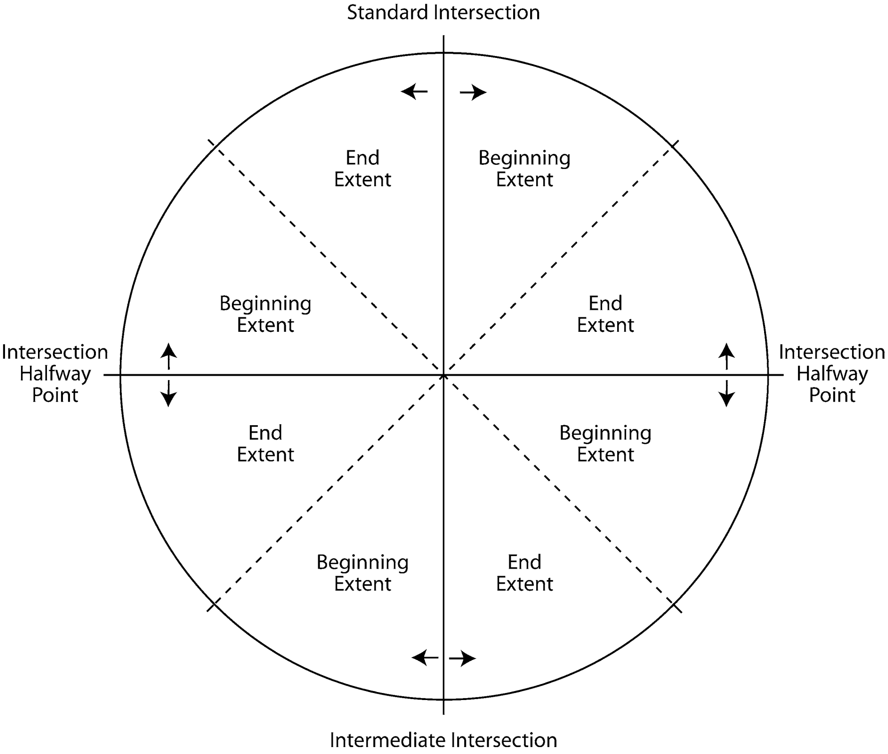
The computist applies the Corrected Difference so derived to the moon's equatorial position measured from the Standard Crossing. Subtracting the result from the White Way extension of the lodge gives *tu* within the lodge.

Although the text specifies locating the moon on the White Way with respect to a lunar lodge, the system does not include a procedure for computing the White Way positions of the lodges. Whether the computist was expected to approximate them with ecliptic positions or derive them by some empirical conversion from the latter, one can only guess. Yabuuchi & Nakayama 2006, 28, suggest that nothing is omitted and this step simply uses equatorial lodge positions, an even less accurate approximation. The *Ming History* recasts this part of the lunar theory, unambiguously determining the position of each lodge from the intersection of the equator with that of the moon's orbit *(Ming shih,* 35: 702–3).]

4.20. To Compute the Moon's Lunar Lodge Position on the White Way at the Exact Time of Corrected Conjunction, Quarter Moons, and Full Moon

In each case take the interval between the moon's lunar lodge position on the Red Way at the Standard Crossing and its position at the exact time of the desired corrected conjunction, quarter, or full moon as Accumulated Degrees after the Standard Crossing (4.20.1). After casting out a complete Aspectual Limit, read the result after the Crossing Halfway Point. After casting it out again, read the result after the Intermediate Crossing. After casting it out again, read the result after the Crossing halfway Point.

Fig. 39. Arguments of White Way positions. The arrows show the directions in which positions are read.



Inspect Accumulated Degrees after each crossing. If less than half an Aspect[ual Limit], read it in the Beginning Extent; if greater, subtract it from an Aspectual Limit and read the remainder in the End

Extent. Subtract the Beginning/End Extent from Corrected Limit Degrees and multiply [the former by the remainder]. Shift backward to read as hundredths of a *tu*. Each 100 of these is counted as a *tu* and read as the Corrected Difference (4.20.2).

<After Standard or Intermediate Crossings add it; after Crossing Halfway Points subtract it.>

Add the Difference to, or subtract it from, Red Way Accumulated Degrees for the moon's position after the Standard Crossing. The result is Corrected Accumulated Degrees (4.20.3). Add the lunar lodge position of the Standard Crossing and cast out the White Way position of the lunar lodge in which the moon is located. In each case one obtains the moon's lunar lodge position on the White Way at the exact time of corrected conjunction, quarter, or full moon (4.20.4) in *tu* and hundredths and ten-thousandth parts. {1221–22}

4.21. To Find the Revolution Entry for the Exact Times of, and Midnights, Dawns, and Dusks Associated with, Corrected Conjunctions, Quarter Moons, and Full Moons

Set up days and day parts of the Revolution Entry for regular conjunction, quarters, and full moon (4.1.1, 4.1.2). Add or subtract the Addition/Subtraction Difference (4.6.1) for corrected conjunction, quarters, and full moon. The result is the Revolution Entry for the exact time of corrected conjunction, quarters, and full moon (4.21.1). Subtract Incomplete Day Parts (3.12.2) for corrected conjunction, quarters, and full moon; the result is the Revolution Entry for midnight (4.21.2). Add Dawn Parts (5.3.4)[to the latter] and the result is the Revolution [Entry] for dawn (4.21.3). When Dusk Parts (5.3.5) are added, the result is the Revolution [Entry] for dusk (4.21.4). {1222}

[This step converts what I have called mean dates in the anomalistic month (computed in step 4.2) to apparent synodic phenomena by adding the correction factor from 4.6, which incorporates the solar and lunar equations of center. The computist then determines dates and times of sunset, midnight, and sunrise for the nights of the lunar phenomena as in 5.3 (q.v.).

Incomplete Day Parts here is the interval between midnight and the time of apparent conjunction, quarter, or full moon. A later step calls it Sunset Parts, which means the same thing (5.3.3); the brief section on Pacing the Centered

Star (i.e., timekeeping) in the *Ming History* does not use either term *(Ming shih,* 35: 712). The compilers of the history do not explain how to derive and use Dawn Parts and Dusk Parts until the eclipse theory (5.3); whether their employment here is due to rearrangement of the original canon is not clear.]

4.22. To Find the Position of the Moon at Midnight

Set up Incomplete Day Parts for corrected conjunction, quarters, and full moon, and multiply by Revolution Corrected Degrees (4.4.3) for the current Revolution Entry Day. Simplify by 10 000; the result is Revolution Degrees for the time of day of the phenomenon (4.22.1). This is subtracted from Corrected Accumulated Degrees for the exact time of day (4.20.4). The remainder is Corrected Accumulated Degrees for midnight (4.22.2). Add and count off as specified earlier, obtaining in each case the moon's lunar lodge position at midnight (4.22.3) in *tu* and hundredths and ten-thousandth parts. {1222}

[Table 9.7 above tabulates Revolution Corrected Degrees, or apparent daily lunar motion, as a function of anomaly (i.e., Revolution Entry Days). Since the dimension of Incomplete Day Parts is parts, the result after division by 10 000 is *tu* of lunar motion between midnight and the phenomenon. Subtracting it algebraically from the moon's White Way position at the time of the phenomenon gives the moon's position at midnight.]

4.23. To Find the Position of the Moon at Dawn and Dusk

Set up Dawn and Dusk Parts (5.3.4, 5.3.5) for the desired day, multiply by Revolution Corrected Degrees for the current Revolution Entry Day, and simplify by 10 000. The result is Revolution Degrees for dawn and dusk (4.23.1). In each case add Corrected Accumulated Degrees for midnight; the result is Corrected Accumulated Degrees for dawn and dusk (4.23.2). Add and count off as before, obtaining in each case the moon's lunar lodge position at dawn and dusk (4.23.3) in *tu* and hundredths and ten-thousandth parts. {1222}

[This procedure converts the day parts for the interval between dawn or dusk and midnight—which changes daily—to *tu*, and algebraically adds the arc thus obtained to the moon's position at midnight.]

4.24. To Find the Daily Dawn and Dusk Lunar Lodge Positions of the Moon on the White Way

Reckon successively Revolution Corrected Degrees corresponding to Day Intervals to determine Revolution Accumulated Degrees (4.24.1). Perform subtraction with the Degree Interval corresponding to the respective dawn and dusk lunar lodge positions [on the day of] corrected conjunction, quarters, and full moon. The remainder is divided by the Day Interval, and the result is the Daily Difference (4.24.2).

<If the Degree Interval is greater, add it [below]; if smaller, subtract it.>

Add it to or subtract it from daily Revolution Corrected Degrees, giving [the moon's] corrected [daily] motion in *tu* (4.24.3). Repeatedly add it to the moon's dusk and dawn positions for corrected conjunction, quarters, and full moon. Add and count off as before. The results are the moon's daily dawn and dusk lunar lodge positions on the White Way (4.24.4).

<After conjunction use the dusk [position]; after full moon use the dawn [position]; at conjunction and full moon, use both dusk and dawn [positions].>

{1222; END OF *YUAN SHIH,* CHAPTER 54}

[This method is parallel in structure to 3.13, analogously correcting tabulated daily apparent lunar motion (Revolution Corrected Degrees, table 9.7). The latter value is based only on distance from the line of apsides (the line between lunar apogee and perigee), but the correction applied here involves the moon's phases. The correction is calculated afresh here for each quarter of the lunation, since the Degree Interval is counted between synodic phenomena.]

10 Canon of the Season-Granting System, Part 2

5. Pacing the Centered Star

[This section sets out the elements of timekeeping; see the orientation, p. 82. The Centered Star *(chung-hsing* 中星) refers to whatever star is culminating (crossing the meridian) at a given moment. Most often it designates a star in one of the lunar lodges, since astronomers knew its distance from the determinative, and thus its equatorial location. Determining culminations is not part of generating the ephemeris, but in the Yuan laymen still widely used rules of thumb based on sighting a star crossing the meridian to tell the time of night.

The Centered Star in the planetary technique, step 7.1, is an arc, not directly related to a planet's culmination or to the timekeeping techniques in this section. The purpose of each step is to

5.1. Tabulate declination and polar distance of ecliptic, and day and night lengths

5.2. Compute daily polar distance of ecliptic, using table

5.3. Compute length of day and night

5.4 Compute times of sunrise and sunset

5.5. Compute lengths of night watches and subdivisions

5.6. Compute times of night watches and subdivisions

5.7. Compute the nightly arc of stellar motion

5.8. Compute positions of stars that culminate at dusk, dawn, and each night watch

5.9. Compute day and night marks for any latitude]

At Ta-tu, the north pole rises above the earth 40⅚ *tu* 5.C1

[The latitude of the Yuan capital Ta-tu 大都, modern Beijing, is now reckoned as 39°55' = 39°.92 = $40^{t}.50$, so the error is roughly 1%. The contrast is obvious between the reporting of this value with an imprecise fraction (5/6 = 0.83) and the long decimal appendages of the other constants. I am unable to account for the poor precision of the measurement. In this case the error of 0.3 *tu* is probably due to a slight elevation of the horizon, or the observatory's position in the southeast part of the capital, or both (Li Qibin 1997: 63).

The altitude of the pole is an observed value, recorded as it was measured, while the next two constants (differently named) are interpolated or extrapolated from data. What superficially appears to be the excessive and misleading precision of other constants may or may not have aimed to minimize loss of accuracy in further calculations using them. On the fraction *(t'ai ch'iang* 太強) used here in the figure for the polar altitude, see the orientation (p. 85).]

Polar Distance at Winter Solstice	$115^{t}.2173$	5.C2
Polar Distance At Summer Solstice	$67^{t}.4113$	5.C3

[These two polar distances are the arcs from the horizon to the equatorial pole; 5.C4 and 5.C5 are day and night lengths at the solstices in day parts. The last constant, 0.025 day, 2.5 marks (36 minutes by modern reckoning), is the more or less arbitrary interval between dawn and sunrise, and between sunset and dusk.]

Winter Solstice Day/Summer Solstice Night	3815.92 parts	5.C4
Summer Solstice Day/Winter Solstice Night	6184.08 parts	5.C5
Dusk/Daybreak [Parts]	250 parts	5.C6

{1225}

5.1. Polar Degrees of the Yellow Way inside or outside the Red Way, Polar Distance, and Midday and Midnight Parts

[The meanings of the quantities tabulated in table 10.1 are as follows:

A: Ecliptic Accumulated Degrees (5.1.1) = ecliptic *tu* from the solstice

B: Inside/Outside Degrees (5.1.2) = declination, counted from the equator to the ecliptic

C: Inside/Outside Difference (5.1.3) = successive differences in B, expressed in degree parts (.01 *tu)*; thus $B_{n+1} = B_n - .01C_n$. It is also successive differences in the values of D and of E, but E increases as D decreases

D: Polar Distance before/after Winter Solstice (5.1.4) = a quarter-cycle ($91^{t}.3143$) plus B

E: Polar Distance before/after Summer Solstice (5.1.5) = a quarter-cycle minus B

F: Winter Day/Summer Night Parts (5.1.6), given in the text below as Midday/Midnight Parts, is half the length of day or night, expressed in day parts. The original table gives F and G in a confusing form, with the thousands and hundreds in large type and the rest in small type. Thus the first entry under F is "*i-ch'ien chiu-pai ling ch'i chiu liu* 一千九百 O七九六," or 1900**0796.** It is easily misread as 1900.0796, but is actually 1907.96.

G: Summer Day/Winter Night Parts (5.1.7) is the complement of F, i.e. G = 5000 - F

H: Day-Night Difference (5.1.8) = successive differences in F and G.]

Table 10.1. Polar Arc of the Ecliptic

A	B	C	D	E	F	G	H
0	23.9030	0.33	115.2173	67.4113	1907.96	3092.04	0.09
1	23.8997	0.99	115.2140	67.4146	1908.05	3091.95	0.29
2	23.8898	1.66	115.2041	67.4245	1908.34	3091.66	0.47
3	23.8732	2.31	115.1875	67.4411	1908.81	3091.19	0.66
4	23.8501	2.99	115.1644	67.4642	1909.47	3090.53	0.85
5	23.8202	3.65	115.1345	67.4941	1910.32	3089.68	1.04
6	23.7837	4.32	115.0980	67.5306	1911.36	3088.64	1.22
7	23.7405	4.98	115.0548	67.5738	1912.58	3087.42	1.42
8	23.6907	5.65	115.0050	67.6236	1914.00	3086.00	1.61
9	23.6342	6.36	114.9485	67.6801	1915.61	3084.39	1.79
10	23.5706	7.02	114.8849	67.7437	1917.40	3082.60	1.99
11	23.5004	7.69	114.8147	67.8139	1919.39	3080.61	2.18
12	23.4235	8.39	114.7378	67.8908	1921.57	3078.43	2.37
13	23.3396	9.08	114.6539	67.9747	1923.94	3076.06	2.56
14	23.2488	9.75	114.5631	68.0655	1926.50	3073.50	2.74
15	23.1513	10.47	114.4656	68.1630	1929.24	3070.76	2.94
16	23.0466	11.14	114.3609	68.2677	1932.18	3067.82	3.14
17	22.9352	11.85	114.2495	68.3791	1935.32	3064.68	3.30
18	22.8167	12.54	114.1310	68.4976	1938.62	3061.38	3.51
19	22.6913	13.25	114.0056	68.6230	1942.13	3057.87	3.69
20	22.5588	13.95	113.8731	68.7555	1945.82	3054.18	3.88
21	22.4193	14.66	113.7336	68.8950	1949.70	3050.30	4.07
22	22.2727	15.37	113.5870	69.0416	1953.77	3046.23	4.26
23	22.1190	16.06	113.4333	69.1953	1958.03	3041.97	4.43
24	21.9584	16.78	113.2727	69.3559	1962.46	3037.54	4.62
25	21.7906	17.47	113.1049	69.5237	1967.08	3032.92	4.80
26	21.6159	18.20	112.9302	69.6984	1971.88	3028.12	4.98
27	21.4339	18.90	112.7482	69.8804	1976.86	3023.14	5.16
28	21.2449	19.60	112.5592	70.0694	1982.02	3017.98	5.35
29	21.0489	20.27	112.3632	70.2654	1987.37	3012.63	5.49
30	20.8462	20.99	112.1605	70.4681	1992.86	3007.14	5.67
31	20.6363	21.68	111.9506	70.6780	1998.53	3001.47	5.85
32	20.4195	22.35	111.7338	70.8948	2004.38	2995.62	6.01
33	20.1960	23.03	111.5103	71.1183	2010.39	2989.61	6.16

A	B	C	D	E	F	G	H
34	19.9657	23.71	111.2800	71.3486	2016.55	2983.45	6.33
35	19.7286	24.37	111.0429	71.5857	2022.88	2977.12	6.48
36	19.4849	25.03	110.7992	71.8294	2029.36	2970.64	6.63
37	19.2346	25.66	110.5489	72.0797	2035.99	2964.01	6.78
38	18.9780	26.31	110.2923	72.3363	2042.77	2957.23	6.92
39	18.7149	26.93	110.0292	72.5994	2049.69	2950.31	7.05
40	18.4456	27.52	109.7599	72.8687	2056.74	2943.26	7.19
41	18.1704	28.14	109.4847	73.1439	2063.93	2936.07	7.32
42	17.8890	28.72	109.2033	73.4253	2071.25	2928.75	7.44
43	17.6018	29.29	108.9161	73.7125	2078.69	2921.31	7.56
44	17.3089	29.84	108.6232	74.0058	2086.25	2913.75	7.68
45	17.0105	30.38	108.3248	74.3038	2093.93	2906.07	7.78
46	16.7067	30.90	108.0210	74.6076	2101.71	2898.29	7.89
47	16.3977	31.41	107.7120	74.9166	2109.60	2890.40	7.98
48	16.0836	31.91	107.3979	75.2307	2117.58	2882.42	8.08
49	15.7645	32.36	107.0788	75.5498	2125.66	2874.34	8.17
50	15.4409	32.85	106.7552	75.8734	2133.83	2866.17	8.26
51	15.1124	33.26	106.4267	76.2019	2142.09	2857.91	8.32
52	14.7798	33.64	106.0941	76.5345	2150.41	2849.59	8.40
53	14.4434	34.07	105.7577	76.8709	2158.81	2841.19	8.46
54	14.1027	34.45	105.4170	77.2116	2167.27	2832.73	8.54
55	13.7582	34.81	105.0725	77.5561	2175.81	2824.19	8.59
56	13.4101	35.15	104.7244	77.9042	2184.40	2815.60	8.64
57	13.0586	35.47	104.3729	78.2557	2193.04	2806.96	8.69
58	12.7039	35.78	104.0182	78.6104	2201.73	2798.27	8.75
59	12.3461	36.07	103.6604	78.9682	2210.48	2789.52	8.78
60	11.9854	36.33	103.2997	79.3289	2219.26	2780.74	8.81
61	11.6221	36.59	102.9364	79.6922	2228.07	2771.93	8.84
62	11.2562	36.83	102.5705	80.0581	2236.91	2763.09	8.89
63	10.8879	37.05	102.2022	80.4264	2245.80	2754.20	8.90
64	10.5174	37.24	101.8317	80.7969	2254.70	2745.30	8.92
65	10.1450	37.44	101.4593	81.1693	2263.62	2736.38	8.94
66	9.7706	37.61	101.0849	81.5437	2272.56	2727.44	8.97
67	9.3945	37.76	100.7088	81.9198	2281.53	2718.47	8.97
68	9.0169	37.91	100.3312	82.2974	2290.50	2709.50	8.98

A	B	C	D	E	F	G	H
69	8.6378	38.07	99.9521	82.6765	2299.48	2700.52	9.00
70	8.2571	38.17	99.5714	83.0572	2308.48	2691.52	9.00
71	7.8754	38.28	99.1897	83.4389	2317.48	2682.52	9.01
72	7.4926	38.38	98.8069	83.8217	2326.49	2673.51	9.01
73	7.1088	38.47	98.4231	84.2055	2335.50	2664.50	9.01
74	6.7241	38.54	98.0384	84.5902	2344.51	2655.49	9.01
75	6.3387	38.62	97.6530	84.9756	2353.52	2646.48	9.01
76	5.9525	38.67	97.2668	85.3618	2362.53	2637.47	9.01
77	5.5658	38.73	96.8801	85.7485	2371.54	2628.46	9.00
78	5.1785	38.77	96.4928	86.1358	2380.54	2619.46	9.00
79	4.7908	38.81	96.1051	86.5235	2389.54	2610.46	9.00
80	4.4027	38.85	95.7170	86.9116	2398.54	2601.46	9.00
81	4.0142	38.88	95.3285	87.3001	2407.54	2592.46	9.00
82	3.6254	38.89	94.9397	87.6889	2416.54	2583.46	8.97
83	3.2365	38.90	94.5508	88.0778	2425.51	2574.49	8.97
84	2.8475	38.92	94.1618	88.4668	2434.48	2565.52	8.97
85	2.4583	38.93	93.7726	88.8560	2443.45	2556.55	8.97
86	2.0690	38.94	93.3833	89.2453	2452.42	2547.58	8.96
87	1.6796	38.94	92.9939	89.6347	2461.38	2538.62	8.96
88	1.2902	38.95	92.6045	90.0241	2470.34	2529.66	8.96
89	0.9007	38.95	92.2150	90.4136	2479.30	2520.70	8.96
90	0.5112	38.95	91.8255	90.8031	2488.26	2511.74	8.95
91	0.1217	12.17	91.4360	91.1926	2497.21	2502.79	2.79
91.31	0	0	91.3143	91.3143	2500	2500	0

{1226–34}

[Note: In column E, entry for 46^{t}, I have emended 74.6067 to 74.6076, as required by the relationship between columns D and E discussed above.

As Ch’en Mei-tung & Li Tung-sheng point out (1990), columns F and G amount to a table of daily day and night lengths of a kind that served to calibrate clepsydra floats. One would simply choose values at the appropriate interval. They also point out that the average error in day and night length (0.7 minutes) for the 24 *ch’i* intervals in the Yuan system is a considerable improvement over the smallest previous error (3.8 minutes).]

5.2. To Find Daily Polar Distance Degrees of the Yellow Way inside or outside the Red Way

Set up Accumulated Degrees on the Yellow Way for midnight preceding the desired dawn. Cast out complete Year Semicycles (3.C7). If [the remainder is] less than an Aspectual Limit it becomes the Beginning Extent. If greater, subtract from a Year Semicycle, and the remainder becomes the argument in the End Extent. Cast out full Accumulated Degrees. Multiply the remainder by the Inside/Outside Difference for the corresponding increment and simplify [i.e., divide] the result by 100. Subtract the result thus obtained from Inside/Outside Degrees to give Degrees inside or outside the Red Way (5.2.1) in *tu*. If inside subtract from, or if outside add to, an Aspectual Limit. The result is the desired Polar Distance Degrees (5.2.2) in *tu* and ten-thousandth parts. {1234}

[The data come from table 10.1. The rules for reading the argument are of the familiar kind first seen in 3.2a. The computist looks up integral *tu*, and interpolates parts using successive differences in declination. The result is declination; adding to or subtracting from a quadrant yields the polar distance of the ecliptic.]

5.3. To Find the Daily Midday, Midnight, Sunrise, Sunset, Dawn and Dusk Parts

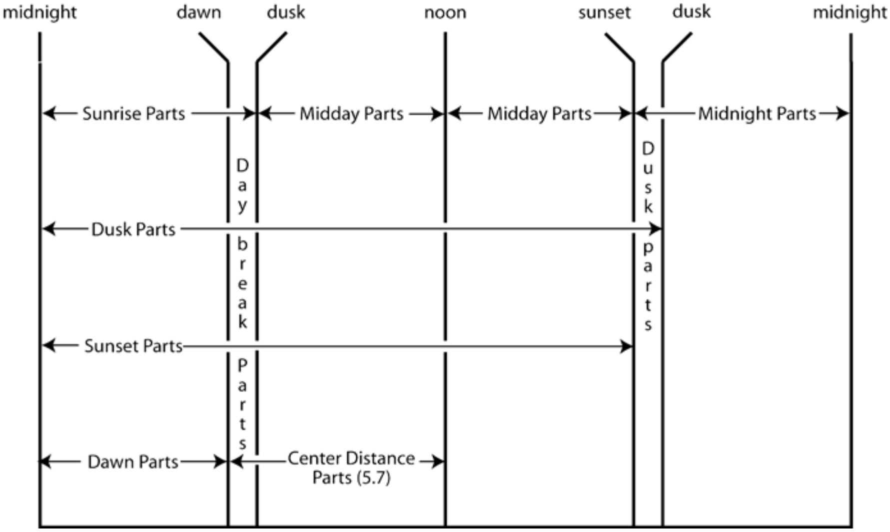
Set up the desired argument in the Beginning/End Extent and cast out full Accumulated Degrees. Multiply the remainder by the Day/Night Difference (5.1.8) and simplify by 100. The result, when added to or subtracted from Midday and Midnight Parts (5.1.6) for the corresponding increment, becomes Midday/Midnight Parts for the day (5.3.1).

<If it is greater before than after, subtract; if smaller before than after, add.>

Then take Midnight Parts as Sunrise Parts (5.3.2). Subtract from the Day Cycle (1.C.1) and the remainder is Sunset Parts (5.3.3). Subtract Dusk/Daybreak Parts from Sunrise Parts and the remainder is Dawn Parts (5.3.4). Add [Dusk/Daybreak Parts] to Sunset Parts to give Dusk Parts (5.3.5). {1234–35}

[This procedure begins with table lookup and interpolation for day and night length. The note directs the computist to subtract the fractional part if the value of the variable is decreasing and add if it is increasing. Sunrise Parts is the interval from midnight to sunrise. Because the division is symmetrical about midnight, it is equal in length to Midnight Parts, between sunset and midnight. Subtracting Midnight Parts from day length (expressed as 10 000) leaves Sunset Parts, the interval from midnight to sunset. Dusk/Daybreak parts (5.C6) is 250 day parts before sunrise in the first case and after sunset in the second. Figure 40 shows all these relationships.]

Fig. 40. Relations between parts of the day in computing day and night length.



5.4. To Find Day/Night Marks and the Time of Sunrise and Sunset in Double-hours and Marks

Set up Midnight Parts, double it, and simplify by 100 to give Night Marks (5.4.1). Subtract it from 100 marks to leave Day Marks (5.4.2). Computing with Sunrise or Sunset Parts according to "Putting forth and Gathering in," the result is the desired time of day (5.4.3) in double-hours and marks. {1235}

[This procedure gives the relative lengths of night and day on a given date in double-hours and marks. These lengths change every day; one might think of them as what the number of marks on night and day clepsydra indicators would be if they were changed daily—which was not the case (orientation, p. 86). "Putting forth and Gathering in" is the general term in section 2 for subdividing the cycles of the year and day. This procedure converts Sunrise and Sunset Parts for a given day length to double-hours and marks by the standard method of step 2.4.]

5.5. To Find Watch-Point Rates

Set up Dawn Parts, double it, and simplify by 5 to give the Watch Rate (5.5.1). Again, simplify the Watch Rate by 5 to give the Point Rate (5.5.2). {1235}

[For purposes of civil timekeeping, the night was the interval between dusk and dawn. Astronomers divided it into five equal watches *(keng* 更), which they subdivided into points *(tien* 點; see the orientation, p. 88). Over the year, since the number of watches and points is constant, the length of each varies from day to day. Doubling Dawn Parts (the time from dusk to midnight) for a given day yields the interval from dusk to dawn. Dividing by 5 and 25 gives the length of a watch and a point respectively.]

5.6. To Find the Times of Watches and Points in Double-hours and Marks

Set up the desired number of [i.e., time in] watches and points and multiply them by the Watch and Point Rates. Add Dusk Parts (5.3.5) for the current day. Compute according to "Putting forth and Gathering in," yielding the desired time of day (5.6.1) in double-hours and marks. {1235}

[The first operation converts elapsed time in watches and points to elapsed time in day parts from dusk. Adding Dusk Parts extends the count backward to the preceding midnight. One is then ready to apply the standard conversion of day parts to double-hours and marks from step 2.4. Like double-hours and marks, watches and points were used to record both elapsed time (as in this instance) and a point in time (as in 6.17). If 2 watches and 3 points have elapsed, the time is point 4 in the 3rd night watch.]

5.7. To Find Center Distance Degrees and Watch Difference Degrees

Set up half the Day Cycle (1.C.1) and subtract from it Dawn Parts (5.3.4) for the desired day. The remainder is Center Distance Parts (5.7.1). Multiply by $366^t.2575$ and count 1 for each Day Cycle in the product. The result thus obtained is Center Distance Degrees (5.7.2). Subtract it from $183^t.128\ 75$. Double the remainder and divide by 5 to give Watch Difference Degrees (5.7.3) in *tu* and parts. {1235}

[Subtracting the midnight-dawn interval from 5000 gives Center Distance Parts, that is, day parts from dawn to noon (seen as the center of the day). Multiplying this quantity by the combined daily motion of the fixed stars and the sun (Celestial Perimeter + 1^t) and dividing by 10 000 (day parts per day) converts it to Center Distance Degrees, the arc from dawn to noon, or noon to dusk, in *tu*. Subtracting from half the same daily motion yields the arc from midnight to dawn. Doubling gives stellar motion for dusk to dawn. Division by 5 gives stellar motion per watch, or Watch Difference Degrees. This and the next step are useful for telling time at night by observing meridian transits of stars.]

5.8. To Find the Centered Star for Dusk, Daybreak, and the Five Watches

Set up Center Distance Degrees. Add the sun's position on the Red Way for noon of the day being studied and count off [lunar lodges]. The result is the lunar lodge position of the Centered Star at dusk (5.8.1). Count this as the Centered Star for the first watch. Repeatedly add Watch Difference Degrees and cast out complete Red Way lunar lodges, yielding the lunar lodge position of the Centered Star for each watch and for break of day (5.8.2) in *tu* and ten-thousandth parts.

As for day and night marks and parts, the Centered Star, and the rates for the Nine Domains, determine these on the basis of how many *tu* the north pole rises above the earth at each place.

<All of the rates mentioned above will coincide with those computed from gnomon and clepsydra measurements.> {1235–36}

[The arc from dawn to noon is the same as that from noon to dusk. Adding it to the sun's noon position gives the right ascension of a star crossing the meridian at dusk. One converts it to lunar lodge position and repeatedly adds the arc that corresponds to stellar motion in the interval of one watch as given in the last step.

The value of this procedure is limited by the use of a constant interval of 250 parts between sunset and dusk. Actually the first visibility of a star depends on the season and on the star's altitude. On the other hand, since the purpose of this step is to provide for timekeeping functionaries a simple way to determine each night watch, no great accuracy is essential.

This step uses two terms for dawn, *ming* 明 and *hsiao* 曉. Although their meaning is the same, *ming* is ordinarily paired with *hun* 昏, "dusk." I translate the two as "daybreak" and "break of day" respectively. Step 5.3 uses the synonym *ch'en* 辰, which I translate "dawn."

The second paragraph, referring to the Nine Domains, is clearly an introduction to the next step. See the commentary below. "The rates" would refer to Watch/Point Rates.]

5.9. To Find Clepsydra Marks at [Any] Location within the Nine Domains

At each location make instrumental observations or measure time with the clepsydra to determine local night length in clepsydra marks for winter or summer solstice. Perform subtraction with 50 marks; the remainder is Solstice Difference Marks (5.9.1). Set up, for the desired day, *tu* and parts on the Yellow Way inside or outside the Red Way (5.1.2). Multiply by Solstice Difference Marks, advance one column, and count 1 for each 239 in the result. The outcome is, if inside, subtracted from, or if outside, added to, 50 marks to give the desired night marks (5.9.2). When this is subtracted from 100, the remainder is day marks (5.9.3).

<Find rates for the double-hours and watches of sunrise and sunset according to the methods given.> {1236}

[Solstice Difference Marks is an empirical local measure of the difference between solstitial and equinoctial day length (the latter is constant at 50 marks). The remainder of the procedure is a simple linear interpolation from this single datum, based on a value of $23^{t}.90$ for the greatest value of ecliptic declination (equal to the obliquity of the ecliptic at the solstices and 0 at the equinoxes). A shift of one column in the computing rods enables using $23^{t}.90$ as the divisor without needing to keep track of decimals.

It is difficult to understand the use of so indirect and inaccurate a procedure. The authors of this system must have known that Day/Night Parts is not simply proportional to solar declination. Columns F and G of table 10.1 would have provided a sounder basis for interpolation.

The Nine Domains *(chiu fu* 九服) is a literary allusion to an early theory of monarchy that envisioned the state in terms of concentric squares, with the king's domain in the center and the abodes of the outermost barbarians on the periphery *(Chou li* 周禮, 33: 6b, second or first century B.C.). Here the term designates latitudinal zones in which day lengths differ. This step was evidently added because the observational survey made the equivalent of latitude data available (but still with no overt conception of latitude). On the Nine Domains in astronomy see Ch'ü An-ching 曲安京, Yuan Min 袁敏, & Wang Hui 王辉 2001.]

6. Pacing Crossing Coincidences

[”Crossing Coincidences *(chiao-hui* 交會)” refers to the method for predicting eclipses. As we have seen (step 4.9), the word “crossing *(chiao)*” refers to the intersection of two Ways, or to the transit of sun or moon across such an orbital intersection. this section (unlike 5), refers throughout to their passage through the lunar nodes. Earlier eclipse theory generally used the word “coincidence *(hui* 會)” to refer to conjunctions. The combination of *chiao* and *hui* in the title of this section suggests conjunctions at or near the lunar node, that is, the condition under which eclipses of the sun take place. The section includes not only solar but lunar eclipses, which take place near opposite nodes.

The purpose of each step is to

6.1. Compute time elapsed in current mean nodical month to conjunction

6.2. Same for mean lunisolar opposition, and next conjunction

6.3. Same for midnight preceding apparent conjunction and opposition and each day

6.4. Compute exact times of corresponding apparent phenomena

6.5. Compute arc from lunar node to apparent conjunction

6.6. Compute correction for effect of parallax on time of maximal phase of eclipse

6.7. Compute position of apparent sun at maximal phase of eclipse

6.8. Compute second correction for parallax

6.9. Compute third correction for parallax

6.10. Combine parallax corrections

6.11. Compute interval from node to syzygy, corrected for parallax, for solar eclipse.

6.12. Same for lunar eclipse

6.13. Compute solar eclipse magnitude and duration

6.14. Same for lunar eclipse

6.15. Compute times of three phases for solar eclipse

6.16. Compute times of three or five phases for lunar eclipse

6.17. Compute time of lunar eclipse in night watches and subdivisions

6.18. Determine direction of occultation for solar eclipse

6.19. Same for lunar eclipse

6.20. Compute magnitude of solar or lunar eclipse at sunrise or sunset

6.21. Compute ecliptic location of sun at maximum phase of solar or lunar eclipse]

Crossing Terminal Parts	272 122.24 parts	6.C1
Crossing Terminal Constant	27 days 2122.24 parts	6.C2
Crossing Midway Constant	13 days 6061.12 parts	6.C3

[The first two are different forms of the nodical month value, the interval between two passages of the moon through the same node (modern value 27.212 19 days), and the third is half a nodical month. The Chinese originals of the last two terms are pronounced identically (cf. 4.C3).]

Crossing Difference	2 days 3183.69 parts	6.C4

[This is the difference between synodic and nodical month length.]

Crossing Opposition Constant	14 days 7652.965 parts	6.C5

[This is simply half the synodic month value, identical to the Full Moon Factor (1.C9) in Section 1.]

Crossing Interval Constant	260 187.86 parts	6.C6

[This, analogous to other interval constants *(ying),* is the number of day parts between the moon's last transit of the lunar node and the calendrical epoch. It is equivalent to 26.018 786 days.]

Crossing Terminal Constant	$363^t.7934$	6.C7
Crossing Midway Constant	$181^t.8967$	6.C8

[The two constants already given (6.C2–6.C3) are expressed here as *tu* of lunar travel per nodical month. Each is the period times the daily mean sidereal motion of the moon (4.C7).]

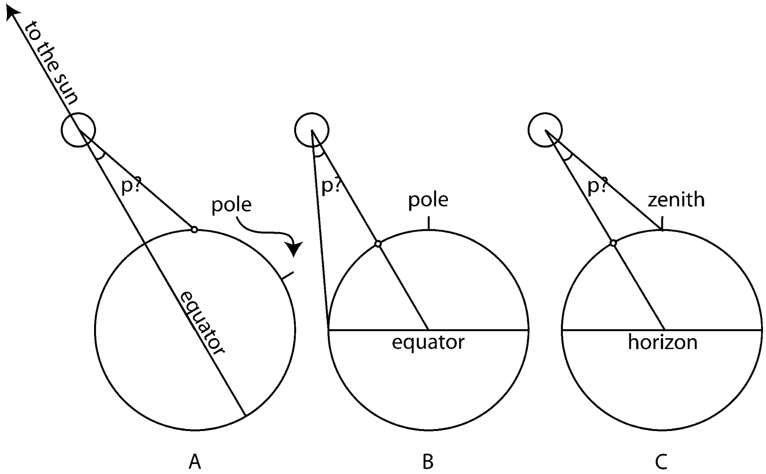
Standard Crossing Constant	$357^t.64$	6.C9
Intermediate Crossing Constant	$188^t.05$	6.C10
Solar Eclipse Yang Argument Limit	6^t	6.C11
Corrected Divisor	60	6.C12
Yin Argument Limit	8^t	6.C13
Corrected Divisor	80	6.C14
Lunar Eclipse Limit	$13^t.05$	6.C15
Corrected Divisor	87	6.C16
[Posterior Standard	15½ *tu*	6.C17]
[Anterior Standard	$166^t.3968$	6.C18]

{1236–37}

[In order to comprehend the two constants 6.C9 and 6.C10, we have to understand the three limits 6.C11, 6.C13, and 6C.15, the bases of eclipse prediction. They set the condition of a solar eclipse as a conjunction of the sun and moon within 6^t of a lunar node when north of the ecliptic, or 8^t when south of it, and that of a lunar eclipse as opposition within $13^t.05$ of a node in either case. A main limitation on accuracy was that the Season-Granting system, for this special purpose, based conjunction on the mean rather than the apparent longitude of the moon.

The solar eclipse limit is inherently more problematic than the lunar because of the moon's parallax. We observe the positions of celestial objects against the starry background, not from the center of the earth, but from its surface. If we are looking at a very distant body—say, the sun—the difference that our location on the surface makes when measuring its position—the parallax—is negligible. But when an observer is far from the line between the earth's center and the nearby moon, the discrepancy in where the object appears to be among the stars is large enough that accurately measuring its location requires a correction that shifts the observer to that line. The Yuan astronomers had no explicit conception of parallax, a quintessentially geometric idea. Among other compromises, they were unprepared to take into account the varying inclination of the moon with respect to the equator. Nevertheless, they applied three empirical corrections that amounted to correcting for parallax. Figure 41 shows the three. In each diagram the moon is the small circle and the earth the larger one; the observer is the very small circle on the surface of the earth, and the parallax component is the angle *p*.]

Fig. 41. Three-part empirical correction for parallax.



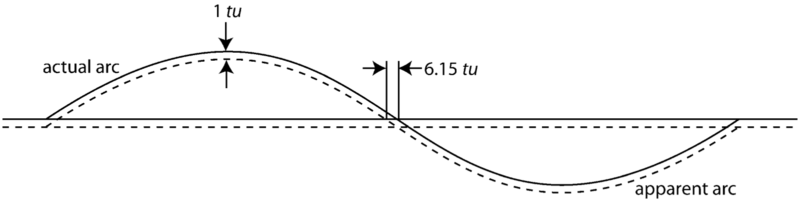
First, in step 6.6, as Nakayama's detailed analysis (1969: 143–49) shows, the astronomers worked out an approximate technique that compensated for the position of an observer in the northern hemisphere (diagram A). The South-North Difference of step 6.8 corrected for the moon's declination as measured by an observer on the earth's equator (B). The East-West Difference of step 6.9 addressed horizontal parallax (C), so-called because it is maximal when the moon is on the observer's horizon (for the Ptolemaic approach, see Evans

1998, 253–54). As Section 9 of the Evaluation shows, this combined technique enabled an improvement in predictions of solar as well as lunar eclipses.

The two solar eclipse limits imply, practically speaking, a 7^t limit on either side of an ecliptic that is in effect 1^t south of where it appears to be. That 1^t latitudinal displacement is equivalent to a displacement along the ecliptic of roughly $6^t.153$, as figure 42 shows.

The Standard Crossing Constant and the Intermediate Crossing Constant tacitly apply this correction to displacement along the ecliptic. The first, $6^t.153$ less than the Crossing Terminal Constant, yields the corrected arc to the second node after it. If we begin at a descending node, this is the arc to the next descending node (Intermediate Crossing Constant). The second, $6^t.153$ more than the Crossing Midway Constant, gives the arc to the next lunar node (Standard Crossing Constant). The Corrected Divisors that correspond to each limit are used to compute measures of eclipse magnitude (see 6.13 and 6.14).]

Fig. 42. Limits of the constant parallax term and their displacement. The relations of the two arcs are exaggerated for clarity.



6.1. To Compute the Crossing Entry of the Regular Conjunction of the Astronomical First Month

Set up the Intermediate Accumulation (1.1.2), add the Crossing Interval Constant (6.C6), and subtract the Intercalary Surplus (1.3.2). Cast out complete Crossing Terminal Parts (6.C1). Simplify the remainder by the Day Cycle (1.C1) to give days, and the remainder is ten-thousandth parts; this is Crossing Entry Comprehensive Days (6.1.1) and ten-thousandth day parts for the regular conjunction of the Astronomical First Month.

<In computing backward, to the Intermediate Accumulation itself add the Intercalary Surplus for the desired [month], subtract the Crossing Interval Constant, and cast out complete Crossing Terminal Parts. Subtract the remainder from Crossing Terminal Parts. The rest of the procedure is as above.> {1237}

[This procedure is precisely parallel to 4.1, but yields time elapsed in the current nodical month to conjunction. The next steps find the interval from the apparent lunar nodes to apparent syzygy by applying corrections to this result. In the Yuan system generally, “comprehensive *(fan* 汎)” designates intermediate terms in procedures for corrected (i.e., apparent) quantities, and does not, so far as I can tell, have a specifically physical meaning.]

6.2. To Find the Crossing Entry of the Next Conjunction and Opposition

Set up Crossing Entry Comprehensive Days and ten-thousandth parts for the regular conjunction of the Astronomical First Month and repeatedly add to it the Crossing Opposition Constant (6.C5), casting out complete Crossing Terminal Days (6.C2). What results are Crossing Entry Comprehensive Days (6.2.1) and ten-thousandth parts for the next conjunction and full moon. {1237}

[This procedure corresponds to 4.2, and yields mean opposition and next conjunction. “Crossing Terminal Days” refers to the Crossing Terminal Constant, which as 6C.2 is expressed in days.]

6.3. To Find Crossing Entry for [Midnight preceding] Corrected Conjunctions and Oppositions and for Midnight of Each Day

In each case set up Crossing Entry Comprehensive Days (6.1.1, 6.2.1) and ten-thousandth parts and deduct the Minor Surplus for regular conjunction or opposition. The result is Crossing Entry for midnight (6.3.1) [preceding] the corrected conjunction or opposition. If the corrected day involves augmentation or diminution, proceed with it. If not, employ the mean [day] as corrected. For large months add two days, and for small months one day. As remainder, in either case add 7877.76 parts. The result is Crossing Entry for midnight of the next conjunction (6.3.2). Add increments of one day, casting out complete Crossing Terminal Constants (6.C2). The results are Crossing Entry Comprehensive Days (6.3.3) and ten-thousandth day parts for midnight of each day. {1237–38}

[The Minor Surplus *(hsiao yü* 小餘) is fractional day parts of conjunction or opposition, that is to say the interval between midnight and the exact time of the apparent syzygy. Normally the Season-Granting system would call this Incomplete Day Parts. When it copied this procedure from the Revised Great Enlightenment System (#76, *Chin shih* 金史, 22: 490), it retained the term Minor Surplus, characteristic of an earlier style of nomenclature. Neither this source nor the Season-Granting system specifies where or how to find the Minor Surplus. That is unfortunate, since it is responsible for the conversion from mean (regular) to apparent (corrected) syzygy. If the term refers to the corresponding remainder in step 4.1 of the Yuan system, i.e., "day parts of the Revolution Entry (4.1.1)" of the mean conjunction of the Astronomical First Month, it cannot accomplish that conversion. See also Takebe, 6: 17b.

Deducting the Minor Surplus may or may not change the calendar day of the conjunction. One determines by inspection whether the integral day counts to mean and corrected conjunctions are the same; that is the point of determining "augmentation or diminution." The difference between a large calendar month of 30 days and a mean nodical month is 2.787 776 days, and that for a small month one day less. Hence the constants added to yield Crossing Entry.]

6.4. To Find Crossing Entry for the Exact Times of Conjunctions and Oppositions

Set up Crossing Entry Comprehensive Days and ten-thousandth parts for regular conjunction or opposition and add or subtract the Addition/Subtraction Difference (4.6.1) for the exact time of corrected conjunction or full moon. [The result] is Crossing Entry for the date and exact time of conjunction or opposition (6.4.1). {1238}

[This is the usual method for converting the time of mean synodic phenomena to that of corresponding apparent phenomena, as detailed in step 4.6.]

6.5. To Find Crossing Regular Degrees and Crossing Corrected Degrees

Set up Crossing Entry Comprehensive Days and ten-thousandth day parts for regular conjunction or opposition (6.1.1, 6.2.1) and multiply by the Mean Lunar Motion in *tu* (4.C7) to yield Crossing Regular Degrees (6.5.1). Add the Expansion/Contraction Difference (3.2.4) if in the phase of expansion, or subtract if in the phase of contraction, to yield Crossing Corrected Degrees (6.5.2). {1238}

[Multiplying by the Mean Lunar Motion converts days between a nodal transit and the mean conjunction or opposition to yield the arc from the node to the event. Adding algebraically the solar equation of center yields the arc to the apparent sun. Because this approach does not correct for the difference between the mean and apparent syzygy, it can lead to a maximal error of $0^{t}.68$. For a theoretical discussion from the point of view of modern astronomy, see Nakayama 1969: 143–44.]

6.6. To Find Eclipse Maximum Corrected Parts for Solar and Lunar Eclipses

Solar Eclipses. Inspect day parts of the corrected conjunction. If less than half a Day Cycle (1.C1), subtract it from half a Cycle and read the result [as time] before midday *(chung ch'ien* 中前). If greater, subtract half a Cycle and read it after midday. Perform subtraction and multiplication with half a Cycle, and shift the result backward two columns. For each 96 in the result take 1, yielding the Time Difference (6.6.1). If before midday subtract it from, and if after, add it to, day parts of corrected conjunction (4.6.2), yielding Eclipse Maximum Corrected Parts (6.6.2). To day parts before or after midday add the Time Difference to yield Noon Interval Corrected Parts (6.6.3).

Lunar Eclipses. Inspect day parts of corrected opposition. If less than ¼ of a Day Cycle, read before double-hour 4. If greater [but less than ½ cycle], again subtract from half a Cycle and read after double-hour 4. If [greater than ½ but] less than ¾ [cycle], subtract half a Cycle and read before double-hour 10. If greater [than ¾ but less than a cycle], subtract from a Day Cycle and read after double-hour 10. Multiply parts before or after double-hour 4 or 10 by itself and shift the result backward two columns. For each 478 in the result take 1, yielding the Time Difference (6.6.1). If before midnight *(tzu ch'ien* 子前) subtract [the Time Difference] from, and if after midnight, add it to, day parts of corrected opposition, yielding Eclipse Maximum Corrected Parts (6.6.2).

In either case computing by the "Putting forth and Gathering in" method results in the Eclipse Maximum (6.6.4) in double-hours and marks. {1238}

[The Time Difference is a correction to the time of apparent syzygy which yields the time of the middle of the eclipse (Eclipse Maximum Corrected Parts; both times are expressed as intervals after midnight). The Time Difference, *d,* is an empirical measure of the effect of parallax on the time of the Eclipse Maximum, formulated for solar eclipses as

$$d = (5000t - t^2) / 9600$$

and for lunar eclipses as

$$d = (5000t - t^2) / 47800$$

where t is in day parts. Times are measured as usual from the beginning of the standard half, not the beginning half, of the double-hour. For lunar eclipses, t is measured from 6 A.M. or 6 P.M. (standard half of the double-hour 4 or 10), whichever is closer to the time of apparent syzygy. This approach to time correction goes back to the Phenomenal Contemplation system (#49, of 807); the Great Enlightenment system made it symmetrical about noon.

For "Putting forth and Gathering in," see step 2.4).]

6.7. To Find the Arguments in the Phases of Expansion and Contraction of Solar and Lunar Eclipse Maxima, and Solar Motion Corrected Degrees

Set up the argument of regular conjunction or opposition in the phase of expansion or contraction, in days and day parts (3.1.1). Add to it days and corrected parts for the Eclipse Maximum. Subtract from it days and day parts for regular conjunction or opposition. The result is the argument in the phase of expansion or contraction for the Eclipse Maximum (6.7.1). Find the Expansion/Contraction Difference (3.2.4) according to the solar motion technique. If in the phase of expansion add it, and if in the phase of contraction subtract it, yielding Corrected Degrees of the argument in the phase of expansion or contraction of the Eclipse Maximum (6.7.2). {1239}

[This step algebraically adds the time interval between mean conjunction or opposition and the maximal phase of the eclipse to the argument from which step 3.2 derives the equation of center. This correction yields the position of the apparent sun at eclipse maximum.]

6.8. To Find the South-North Difference

Inspect corrected *tu* of the argument in the phase of expansion or contraction for the solar Eclipse Maximum. If less than an Aspectual Limit, it becomes the Beginning Extent; if greater, subtract it from the Celestial Semi-perimeter (3.C3) and read the remainder as the End Extent. Multiply *tu* in the Beginning or End Extent by itself and count 1 for each 1870 in the result to give *tu*. Shift anything less [than a *tu]* backward and read as ten-thousandths of a *tu*. Subtract [*tu* and parts] from $4^{t}.46$. The remainder is the South-North Comprehensive Difference (6.8.1). Multiply by Noon Interval Corrected Parts (6.6.3) and divide by Midday Parts (5.1). Subtract the result from the Comprehensive Difference to yield the [South-North] Corrected Difference (6.8.2).

> <If the Comprehensive Difference is not large enough to allow this subtraction, subtract it instead to yield the Corrected Difference. [Subsequently] subtract what would be added and add what would be subtracted.>

In the case of [an argument] at the beginning of expansion or the end of contraction [i.e. in quadrant I or IV], if it precedes the crossing, subtract yin arguments and add yang arguments; if it follows the crossing, add yin arguments and subtract yang arguments. In the case of [an argument] at the beginning of contraction or the end of expansion [i.e. in Quadrant II or III], if it precedes the crossing, add yin arguments and subtract yang arguments; if it follows the crossing, subtract yin arguments and add yang arguments. {1239}

[As my commentary at the beginning of section 6 explains, the Yuan astronomers dealt empirically with the parallax of the moon as the sum of three distinct values. The first was a constant term of $6^{t}.15$ which corrects for the apparent southward displacement of the lunar orbit when seen from Ta-tu in the northern hemisphere. The second, used here, is the South-North Difference. When measured at midday, it is zero at the equinoxes and $4^{t}.46$ (maximum value of the parallax) at the solstices. The third, the East-West Difference, is treated in the next step.

The South-North Difference is a correction for the moon's declination and altitude when seen by an observer on the equator. In this procedure, the intermediate South-North Comprehensive Difference is equivalent to

$$S = P_{\max} - \lambda^2 / 1870,$$

where P is the parallax. The South-North Difference, D, amounts to

$$D = (MS - NS) / M,$$

where M is Midday Parts, the interval between noon and sunset, and N is Noon Interval Corrected Parts, the interval between noon and syzygy after the empirical correction for Eclipse Maximum has been applied (6.6). Yin arguments are north of the lunar orbit, and yang arguments south of it. The arrangement of quadrants makes the sense of the argument the same as that of the lunar node's motion in Quadrants I and IV. The sense is contrary (and thus subtractive) in Quadrants II and III.]

6.9. To Find the East-West Difference

Inspect corrected *tu* of the argument in the phase of expansion or contraction of the solar Eclipse Maximum. Perform subtraction and multiplication with the Annual Semi-perimeter and count 1 for each 1870 in the result to give *tu*. Shift anything less backward and read it as ten-thousandth parts of a *tu*. [The result in *tu* and parts] is the East-West Comprehensive Difference (6.9.1). Multiply by Noon Interval Corrected Parts (6.6.3) and divide by 1/4 of a Day Cycle (1.C1) to yield the [East-West] Corrected Difference (6.9.2).

> <If the result is greater than the Comprehensive Difference, double the Comprehensive Difference and subtract [the Corrected Difference derived above]; the remainder becomes the Corrected Difference. Addition and subtraction are as given.>

In the case of an argument before the middle of expansion [Quadrant I], if it precedes the crossing, subtract yin arguments and add yang arguments; if it follows the crossing, add yin arguments and subtract yang arguments. If after the middle [Quadrant II], if it precedes the crossing, add yin arguments and subtract yang arguments; if it follows the crossing, subtract yin arguments and add yang arguments. In the case of an argument before the middle of contraction [Quadrant III], if it precedes the crossing, add yin arguments and subtract yang arguments; if it follows the crossing, subtract yin arguments and add yang arguments. If after the middle [Quadrant IV], if it precedes the crossing, subtract yin arguments and add yang arguments; if it follows the crossing, add yin arguments and subtract yang arguments. {1239}

[This procedure gives what would now be considered the horizontal parallax, which increases with the interval from noon. It is maximal ($4^{t}.46$) at equinoctial sunrise or sunset, and 0 at solstitial sunrise or sunset. The intermediate East-West Comprehensive Difference may be expressed as

$$S' = \lambda\,(A - \lambda) / 1870,$$

where A is a semicircle. The Corrected East-West Difference is

$$E = S'N / 2500.$$

The note in text gives a second formula for the case $E > S'$, according to which

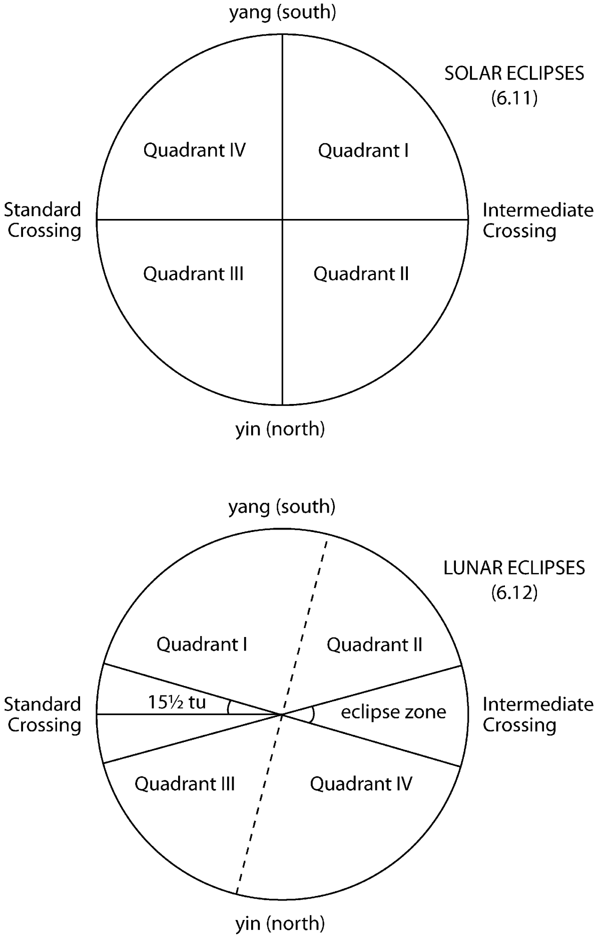
$$E = S'\,(5000 - N) / 2500.$$

The arrangement of quadrants has the same effect here as for the East-West Difference in step 6.8.]

6.10. To Find Standard Crossing and Intermediate Crossing Limit Degrees for Solar Eclipses

Set up *tu* of the Standard Crossing Constant (6.C9) and Intermediate Crossing Constant (6C.10) and add or subtract the South-North Difference (6.8.2) and East-West Difference (6.9.2). The remainders are Standard Crossing Limit Degrees (6.10.1) and Intermediate Crossing Limit Degrees (6.10.2) and ten-thousandth parts. {1240}

Fig. 43. Arguments of yin and yang phases for solar and lunar eclipses.



[This step assembles the components of parallax. The $6^t.15$ term, as noted at the beginning of Section 6, is tacitly incorporated in the two constants used here. The results thus represent the total parallax correction to be applied in 6.11, in the form of the corrected mean interval between two nodes. This procedure is fixed at the latitude of Beijing, and does not accommodate variations in the site of observation.]

6.11. To Find Degrees in the Yin or Yang Phase preceding or following the Crossing for Solar Eclipses

Inspect Crossing Corrected Degrees (6.5.2). If less than Intermediate Crossing Limit [Degrees], subtract it from the latter and read [the result] as Degrees in the Yang Phase preceding the Crossing (6.11.1). If greater, subtract the Intermediate Crossing Limit and read [the result] as Degrees in the Yin Phase following the Crossing (6.11.2). If less than the Standard Crossing Limit [Degrees], subtract it from the latter and read [the result] as Degrees in the Yin Phase preceding the Crossing (6.11.3). If greater, subtract the Standard Crossing Limit [Degrees] and read [the result] as Degrees in the Yang Phase following the Crossing (6.11.4). {1240}

[This step applies the parallax correction to the node-syzygy interval (6.5). Step 6.15 refers to the result of this step as Corrected Limit Entry Degrees. The assignment of arguments to quadrants is conventional; compare figure 43 to figure 30.]

6.12. To Find Degrees in the Yin or Yang Phase preceding or following the Crossing for Lunar Eclipses

Inspect Crossing Corrected Degrees (6.5.2). If less than the Crossing Midway [Constant] in *tu* (6.C8), read in the yang phase. If greater, subtract the Crossing Midway Constant and read in the yin phase. Inspect the Yin/Yang Argument. If less than the Posterior Standard, 15½ *tu* (6.C17), read as Degrees following the Crossing (6.12.1). If greater than the Anterior Standard (6.C18), $166^t.3968$, again subtract the Crossing Midway Constant. Read the remainder as Degrees preceding the Crossing (6.12.2) and parts. {1240}

[Although the authors employ a time correction for parallax in the lunar eclipse theory (6.6), they do not correct the node-syzygy arc; indeed the undesirability of such a correction is obvious. The quadrants are thus delimited by the Crossing Midway Constant arc ($181^t.8967$, 6.C8). The Anterior Standard is simply the Crossing Midway Constant minus the Posterior Standard. The Posterior Standard follows the node, hence “posterior.” I have added both to the list of constants at the beginning of section 6.

The extreme limit of 15½ *tu* on either side of the nodes amounts approximately to the sum of the lunar eclipse limit ($13^{t}.05$) and the greatest value of the solar equation of motion ($2^{t}.40$). See figure 43.]

6.13. To Find Solar Eclipse Parts

Inspect *tu* preceding or following the crossing. In each case subtract it from the Yin or Yang Argument Eclipse Limit (6.C11, 6.C13).

<If subtraction is impossible, there will be no eclipse.>

In the remainder count 1 for each Corrected Divisor (6.C12, 6.C14), yielding Solar Eclipse Parts (6.13.1). {1240}

[Solar Eclipse Parts is a measure of eclipse magnitude and duration. The computist derives it from the corrected distance obtained in 6.11. Dividing by the appropriate Corrected Divisor provides an arbitrary factor varying between 0 and 10 parts, where 10 parts signals totality. Court astronomers considered accurate prediction of Eclipse Parts in solar eclipses at least as important as prediction of time. A small error in time will not render the eclipse unobservable, but an error of several tenths in magnitude may in effect do so (only eclipses that are nearly total are visible to the naked eye).]

6.14. To Find Lunar Eclipse Parts

Inspect *tu* before or after the crossing.

<to which the South-North and East-West Differences have not been applied.>

Subtract it from the Eclipse Limit.

<If subtraction is impossible, there will be no eclipse.>

In the remainder count 1 for each Corrected Divisor (6.C16), yielding Lunar Eclipse Parts (6.14.1). {1240}

[No lunar parallax constant complicates the lunar eclipse limit. The Corrected Divisor in this case makes Lunar Eclipse Parts vary between 0 and 15 *tu*, of which the astronomers arbitrarily counted the region between 10 and 15 *tu* as the range of totality. The difference between the solar and lunar eclipse limits is physically due to the large diameter of the earth’s shadow cone when it falls on the moon. The time required for the moon to pass through it makes for a long phase of totality in lunar eclipses. The Season-Granting system envisages totality as the time it takes for the shadow to move from 10 to 15 parts and back, or a third the duration of the eclipse. In practice, the duration varies in much more complicated ways. As a measure of magnitude, Lunar Eclipse Parts is quite different from, but commensurate with, the 15^{t} lunar eclipse limit of early astronomical systems. Use of the latter constant began with the Luminous Inception system (#8, A.D. 237), which used *tu* from the moon to the node to indirectly signify magnitude.]

6.15. To Find Corrected Use [Parts] and Time of Day of the Three Limits for Solar Eclipses

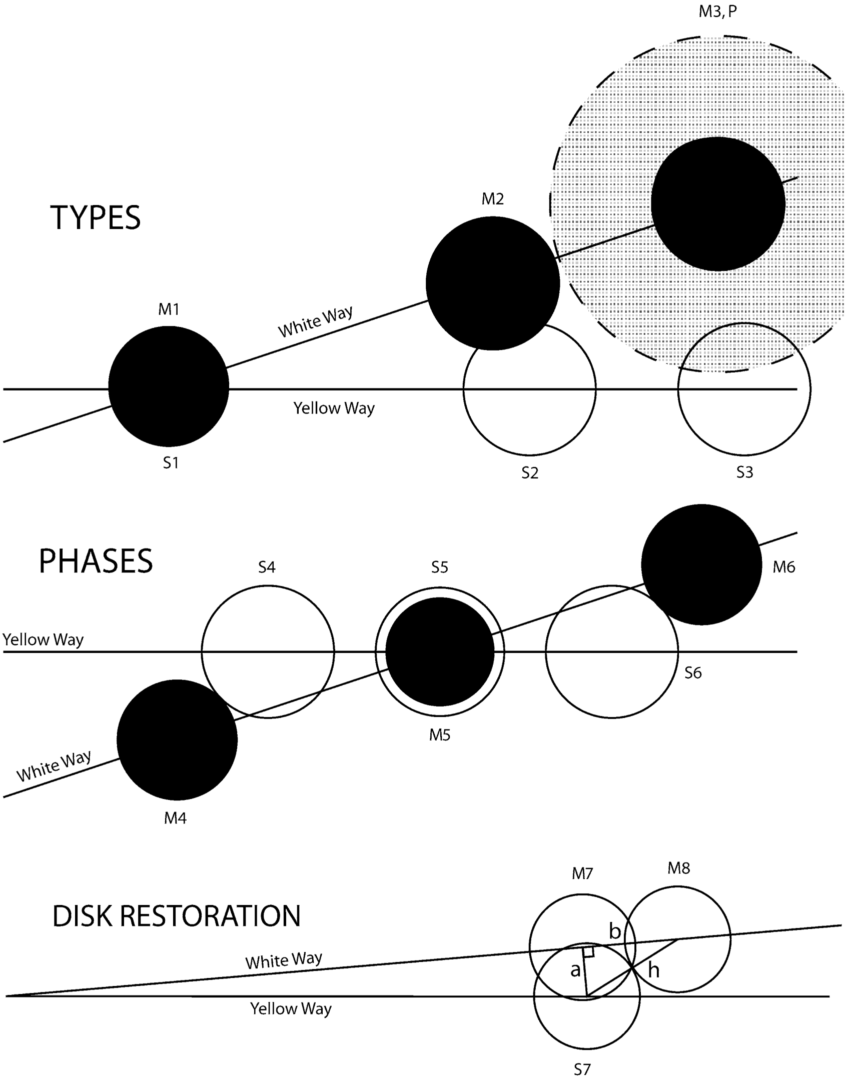
Set up Solar Eclipse Parts. Subtract and multiply with 20 parts and take the square root. Multiply the result by 5740 and take 1 for each complete Corrected Limit Entry Degrees value (6.11.1–6.11.4), yielding Corrected Use Parts (6.15.1). Subtract the result from Eclipse Maximum Corrected Parts (6.6.2) to give First Loss [Parts] (6.15.2), and add to Eclipse Maximum Corrected Parts to give Disk Restoration [Parts] (6.15.3). Compute according to "Putting forth and Gathering in," yielding double-hours and marks of the Three Limits for Solar Eclipses (6.15.4). {1240–41}

[Figure 44 shows the spatial relationships responsible for the various types of eclipse and their phases.

In all three parts of the figure, the horizontal line is part of the Yellow Way or ecliptic, as seen from the central earth. The White Way, the moon's path, is at an angle of about 5° to it (exaggerated here for clarity). At their intersection, the sun (S_1) and the moon's shadow (M_1) completely overlap, and the resulting Eclipse Maximum is a total solar eclipse. Since the moon's shadow may be slightly smaller than the apparent sun, depending on the lunar distance from the earth, the result may also be an annular eclipse (see S_5, M_5 in the "phases" diagram, size of ring exaggerated). When sun and moon move into conjunction at a small distance from the intersection of the two orbits, so that the lunar shadow (M_2) does not completely obscure the sun (S_2), the result is a partial eclipse. For lunar eclipses only, Chinese astronomers also computed the times of what we call penumbral eclipses, in which the fully dark shadow, the umbra (M_3), does not overlap the sun (S_3), but the larger, diffuse shadow, the penumbra (P), does.

The second diagram shows the phases of a single solar eclipse. Eclipse Maximum for this annular eclipse has taken place at or very near the node, when the moon's shadow (M_5), moving much more quickly than the sun (S_5), overtakes and coincides with it. A little earlier, the shadow (M_4) has just contacted the sun (S_4), beginning First Loss as the apparent circle of the sun begins to diminish. Past the node, as the shadow (M_6) moves beyond the sun (S_6) and no longer overlaps it, the phase is Disk Restoration. The five limits for lunar eclipses (6.16) incorporate phases for the beginnings and ends of the penumbral (S3, P) phase as well.

Fig. 44. Types and phases of eclipses.



We can now analyze the current computational step. It obtains a measure of duration, symmetrical about the middle of the eclipse, from Solar Eclipse Parts. The maximum value of the latter is half the 20 parts used as the constant factor here; the underlying assumption is that, as astronomers today (and Mei Wenting in the mid seventeenth century) would phrase it, the apparent diameter of the shadow is twice that of the sun's disk. That is not a very accurate estimate; what it tells us is that the procedure did not depend on these relative sizes.

Since the Yuan astronomers were not visualizing a conical lunar shadow, it is easier to understand their approach if one looks at the problem as a simple one of distances. This procedure treats the two bodies as if the sun were standing still and the moon moving past it toward the node. The bottom diagram of disk restoration—which, unlike the first two, is drawn very roughly to scale—does not show the moon's shadow as black, in order to reveal the geometric relationships (which played no part in the Chinese procedure).

The greatest phase of this partial eclipse takes place when the moon is at M_7, so that *a,* the line between the moon's center and that of the sun, is perpendicular to the White Way. When the moon's shadow has moved to M_8, it no longer impinges on the sun (which for readability, I have shown as still at S_7). The result is Disk Restoration, the end of the eclipse. In the right triangle *abh, h,* its hypotenuse, is 10 parts, and side *a* is (10 - *S),* where *S is* Eclipse Limit Parts. To find the arc between the two positions of the moon is a simple matter of applying the Pythagorean theorem: $\sqrt{10^2 - (10 - S)^2} = \sqrt{S(20 - S)}$. Corrected Use Parts is the interval from first or last contact to the maximal phase, equivalent to

$$T = 5740 \sqrt{S(20 - S)} / L$$

where *L* is Corrected Limit Entry (the argument from 6.11). Why the authors did not choose a more descriptive name than "use parts," I have no idea.

The diameter of the sun's disk amounts very roughly to 70 parts, but this procedure sets it at 10 parts, half of the constant 20; it is therefore necessary to multiply by 7, and again by the duration of 1 extent (see the commentary to 4.C6). The latter is 0.082 day, or 820 when expressed in ten-thousandths of a day. The product of the two constants is 5740. There is a detailed analysis of Corrected Use Parts in Mei Wen-ting's *Li-hsueh p'ien-ch'i* 曆學駢枝, ch. 1, appendix; see also Takebe, 6: 40b-42b.]

6.16. To Find Corrected Use [Parts] and Time of Day of the Three Limits and Five Limits for Lunar Eclipses

Set up Lunar Eclipse Parts. Subtract and multiply with 30 parts and take the square root. Multiply the result by 5740 and take 1 for each complete Corrected Limit Entry Degrees value (6.12), yielding Corrected Use Parts (6.16.1). Subtract it from Eclipse Maximum Corrected Parts to give First Loss [Parts] (6.16.2). Add it to Eclipse Maximum Corrected Parts to give Disk Restoration [Parts] (6.16.3). Compute according to "Putting forth and Gathering in," yielding double-hours and marks of the Three Limits for Lunar Eclipses (6.16.4).

In the case of total lunar eclipses, subtract and multiply parts within totality *[should be* Lunar Eclipse Parts, 6.14.1] with 10 parts and take the square root. Multiply the result by 5740 and take 1 for each complete Corrected Limit Entry Degrees value, yielding Parts inside Totality (6.16.5). Subtract this from Corrected Use Parts to give Parts outside Totality (6.16.6). Subtract Corrected Use Parts from Eclipse Maximum Corrected Parts to yield First Loss [Parts] (6.16.7). Add [Parts] outside Totality to yield Eclipse Totality [Parts] (6.16.8). Once again add [Parts] inside Totality to yield Eclipse Maximum [Parts] (6.16.9). Again add [Parts] inside Totality to yield Rebirth of Light [Parts] (6.16.10). Once again add [Parts] outside Totality to yield Disk Restoration [Parts] (6.16.11). Compute by the "Putting forth and Gathering in" method, and the results are double-hours and marks of the Five Limits for Lunar Eclipses (6.16.12). {1241}

[The first procedure, for partial eclipses, is parallel to 6.15; the constant factor, twice the maximum value of Lunar Eclipse Parts, is 30. The second method sets a variable limit on the duration of totality (designated as Parts inside Totality) within the constant Lunar Eclipse Limit angle of 13^{t}.05. The sequence of contacts and the intervals between them is thus First Loss or first contact—(Parts outside Totality)—Eclipse Totality or second contact—(Parts inside Totality)—Eclipse Maximum or middle of the eclipse—(Parts inside Totality)—Rebirth of Light or third contact—(Parts outside Totality)—Disk Restoration or fourth contact. Note that "parts within totality" in the first sentence of the second paragraph is not the working term that appears later in the paragraph, but is an error for "Lunar Eclipse Parts." No such confusion appears in the *Ming History* version (36: 719).]

6.17. To Find Entry of Watches and Points for Lunar Eclipses

Set up Dawn Parts for the day in which the Eclipse Maximum falls, double and simplify by 5 to yield the Watch Divisor (6.17.1). Again simplify the Watch Divisor by 5 to yield the Point Divisor (6.17.2). Then set up the various parts in Beginning and End Extents. If greater than Dusk Parts (5.3.5), deduct Dusk Parts. If less than Dawn Parts (5.3.4), add Dawn Parts. Divide by the Watch Divisor to give the number of watches (6.17.3). Collect (2.4) incomplete parts with the Point Divisor to give the number of points (6.17.4). Count off the number of watches and points exclusively from the first point of the first watch, to give Entry of [i. e., time in] Watches and Points (6.17.5). {1241}

[The "various parts" are day parts of the Three or Five Limits, as appropriate, from the last step (cf. *Ming shih*, 36: 720). "Collect" refers to division that converts a quantity from one unit to another (see the orientation, p. 65).

The manipulations with Dawn and Dusk Parts convert the normal time from midnight to time from dusk, and extend it to dawn by the doubling in the definition of the Watch Divisor. The two divisors partition the dusk-to-dawn interval into five watches and 25 points. The phrasing of 6.17.5, *so ju keng tien* 所入更點, is a reminder that the beginning of the count is not zero but the 1st point of the 1st watch.]

6.18. To Find the Direction from which a Solar Eclipse Begins

If the eclipse takes place in the yang phase, it begins in the southwest [limb of the sun], reaches maximum due south, and restoration is in the southeast. If the eclipse takes place in the yin phase, it begins in the northwest, reaches maximum due north, and restoration is in the northeast. If the eclipse is of greater than 8 parts, it begins due west and restoration is due east.

<This discussion applies to an eclipse observed on the meridian.> {1241–42}

[These are long-established empirical rules for the direction of solar occultation (6.18.1). In the yang phase, the sun is to the north and the moon to the south. As the moon proceeds eastward, its shadow darkens the sun from the west. If the occultation is deep, the moon's direction of travel is nearly due east-west. The approximate demarcation value of 8 parts applies to Solar Eclipse Parts (6.13.1, maximum 10).]

6.19. To Find the Direction from which a Lunar Eclipse Begins

If the eclipse takes place in the yang phase, it begins in the northeast [limb of the moon], reaches maximum due north, and restoration is in the northwest. If the eclipse takes place in the yin phase, it begins in the southeast, reaches maximum due south, and restoration is in the southwest. If the eclipse is greater than 8 parts, it begins due east and restoration is due west.

<This discussion also applies to an eclipse observed on the meridian.> {1242}

[When determining the direction of lunar occultation (6.19.1), at 8 parts out of a maximum of 15 the astronomers consider the direction of occultation horizontal.]

6.20. To Find Visibility Parts of an Eclipse Carried with the Rising or Setting Sun or Moon

Inspect Sunrise Parts and Sunset Parts (5.3.2, 5.3.3) for the sun on the day [of the eclipse]. If greater than that for First Loss and less than for Eclipse Maximum, it will be a carried eclipse *(tai-shih* 帶食). In each case, perform subtraction with Eclipse Maximum Parts (6.6.2) and Sunrise or Sunset Parts. The remainder is Carried Eclipse Difference (6.20.1). With it multiply parts eclipsed (i.e. Eclipse Parts, 6.13.1, 6.14.1) and take 1 for each Corrected Use Parts value (6.15.1) in the product.

<In the case of a total lunar eclipse, subtract Parts inside Totality (6.16.5) from the Carried Eclipse Difference. Advance the remainder one column. Take 1 for each value of Parts outside Totality (6.16.6) and subtract the result from Totality Parts [i.e., Lunar Eclipse Parts, 6.14.1]. The result is Visibility Parts of an eclipse carried with the rising or setting moon. If subtraction is impossible, a total eclipse is carried at rising or setting.>

Subtract the result from parts eclipsed. The result is Visibility Parts (6.20.2) of an eclipse carried with the rising or setting sun or moon.

<When the Eclipse Maximum falls within Day [Parts], at dawn it is gradually advancing and at dusk it has already receded. When it falls within Night [Parts], at dawn it has already receded and at dusk it is gradually advancing.> {1242}

[A “carried eclipse” is an eclipse which takes place at sunrise or sunset, that is, in which sunrise or sunset falls between first and last contact. Visibility Parts, or magnitude, is determined by linear interpolation within Eclipse Parts. It bears the same relation to Eclipse Parts as the Carried Eclipse Difference does to Corrected Use Parts (the divisor here). In the case of total lunar eclipses, the first note in the text restricts this interpolation to the zone of partiality.]

6.21. To Find the Lunar Lodge Position of Solar or Lunar Eclipse Maximum

Set up corrected degrees of the argument in the phase of expansion or contraction for the solar or lunar eclipse maximum (6.7.2). If in the phase of expansion, take it directly as Corrected Accumulation (6.21.1). If in the phase of contraction, add a Year Semicycle (3.C7) to yield the Corrected Accumulation.

<In the case of opposition, further add the *tu* in a Celestial Semiperimeter (3.C3).>

Add *tu* of the sun's position on the Yellow Way at the exact time of winter solstice in the Astronomical First Month and count off. The result in each case is lunar lodge degrees of the solar or lunar Eclipse Maximum (6.22.2) and ten-thousandth *tu* parts. {1242}

[This step refers the argument of the middle of the eclipse to the winter solstice (Corrected Accumulation), adds it to the sun's ecliptic winter solstice position (3.10.1), and counts off with respect to the lunar lodges (table 9.4) to give the location of the eclipse maximum in the lodge system.]

7. Pacing the Five Stars

[This section is copied with little substantive modification from its predecessor the Revised Great Enlightenment system of 1180 (#76; *Chin shih,* 22: 496–519). A large proportion of the constants, even when their units differ, are the same. Of the eighteen procedures, only three subsections are substantively different (7.1, 7.13, and 7.16). Five, very minor changes of wording aside, are identical (7.8–7.12). The other ten are paraphrases that do not differ in meaning from their predecessors.

The planetary theory in this section begins with three constants used for all five planets, and then gives, separately for each of the five, constants and tabular models of the phases of motion.

The purpose of each step is to

7.1. Compute arc from winter solstice to first mean planet-sun conjunction and beginning of each grade (portion of orbit in which the system treats planetary speed as constant).

7.2. Same for arc from planetary perigee, where equation of center is zero.

7.3. Compute (with or without table) equation of center for planet.

7.4. Compute cyclical dates of apparent conjunction and beginning of each grade.

7.5. Same for dates in civil calendar.

7.6. Compute apparent position of planet from time of mean conjunction and beginning of each grade.

7.7. Compute apparent position of planet at midnight preceding beginning of each grade.

7.8. Compute days and *tu* elapsed in the apparent planet's movement through each grade.

7.9. Compute mean planetary velocity for each grade.

7.10. Compute daily increase in velocity for each grade.

7.11. Same for grades in which previous step is inapplicable.

7.12. Compute apparent position of planet at midnight for each day.

7.13. Compute days from solstice to mean conjunction, first appearance, and last appearance.

7.14. Compute difference between daily velocities of sun and planet at mean conjunction, first appearance, and last appearance.

7.15. Compute intermediate quantity for next step.

7.16. Compute date and position of apparent planetary conjunction.

7.17. Compute dates of apparent last and first appearance for Mars, Jupiter, and Saturn.

7.18. Same for Venus and Mercury.]

Argument Degrees	$365^{t}.2575$	7.C1
Argument Midway Constant	$182^{t}.628\,75$	7.C2
Argument Factor	$15^{t}.219\,062\,5$	7.C3

{1242–43}

[The first two constants are the same as the Celestial Perimeter (the sidereal year cycle in *tu*, 3.C2) and the Celestial Semi-perimeter (half the sidereal year cycle, 3.C3) in the solar theory. The Argument Factor is 1/24 of Argument Degrees, and is thus the sidereal arc in *tu* which corresponds to a mean *ch'i,* nodal or medial. Its units differ from those of the *Ch'i* Factor (1.C8), 1/24 of the tropical year in days.

Below, "cycle" marks synodic constants, and "argument," sidereal constants.]

a. The Wood Star (Jupiter)

Cycle Rate	3 988 800 parts	7.C4a
Cycle Day Constant	398.8800 days	7.C5a

[These are two expressions of the synodic period. For a comparison of these and other constants with Han and modern values and with those of Ptolemy (which were authoritative in Europe until after 1600), see table 10.2.

Section 7 gives a separate, parallel list of constants and table of grades of motion for each planet. I designate the planets and their Five-Phases designations by a small letter after the number of the constant. Thus the treatise calls Jupiter “the Wood Star *(mu-hsing* 木星)” and I label its Cycle Rate “7.C4a.” My comments on each constant for Jupiter also apply to those for the other planets.

Letter	Planet	Phase
a	Jupiter	Wood
b	Mars	Fire
c	Saturn	Earth
d	Venus	Metal
e	Mercury	Water

The Yuan astronomers subdivide particular constants; hence I subdivide their numbers. For instance, the differing dawn and evening values for Mercury’s Invisibility/Appearance Constant become 7.C13e1 and 7.C13e2.

I have added table 10.2, which tabulates the planetary constants and compares those of the Han period, 1200 years earlier, and those of Ptolemy in the mid second century A.D.]

Table 10.2. Constants of the Planets

Synodic Period in Days	**Jupiter**	**Mars**	**Saturn**	**Venus**	**Mercury**
Yuan, Cycle Day Constant	398.88	779.93	378.09	583.90	115.88
Han value (1)	398.85	779.44	378.03	584.03	115.78
Modern value (2)	398.88	779.94	378.09	583.92	115.88
Sidereal Period in Days (3)	-	-	-	-	-
Yuan, Count Rate/Day Cycle	4331.30	686.96	10747.88	365.26	365.26
Modern value	4332.59	686.98	10759.20	365.26	365.26
Sidereal Period in Years	-	-	-	-	-
Yuan, Degree Rate/Day Cycle	11.86	1.88	29.43	1.00	1.00
Ptolemy’s value	11.87	1.88	29.45	-	-
Modern value	11.86	1.88	29.46	-	-
Equivalent, Years and Days	-	-	-	-	-
Yuan	11y 313d	1y 322d	29y 155d	1y	1y

[Note 1. For the Quarter Remainder system (#4. A.D. 85), based on Sivin 1969: 19, table III, converted to days using that system’s tropical year length, 365.25 days.

Note 2. Based on Smart 1963: 422.

Note 3. The Quarter Remainder system did not incorporate a distinct sidereal year.

The accuracy of the fundamental planetary constants in table 10.2 indicates how early and how strong the concern with accurate mean constants was in Chinese astronomy. The early Quarter Remainder system's values, tabulated for comparison, already approximate the corresponding modern values. In the Yuan values, all divergences are on the low side, implying that the rounding-off procedures of the time were among the systematic limits to accuracy.

For planetary constants through Chinese history see Ch'en Mei-tung 1995, table 14-1, pp. 393–97.]

Argument Rate 43 312 964.865 parts 7.C6a

[The Argument Rate is the mean sidereal period of revolution expressed in day parts, corresponding to slightly over 4331 days, that is, to 11.86 years, as in the next item.]

Degree Rate 118 582 parts 7.C7a

[This is the mean sidereal period in years, expressed in ten-thousandths of a year. In this form step 7.2 uses it to convert day parts since conjunction to *tu* of mean planetary motion. Since the mean sidereal periods of Venus (7.C7d) and Mercury (7.C7e) are the same as that of the sun, their degree rates are both 10 000. Thus for these planets, as well as for the sun, 10 000 day parts since conjunction amount to 1 *tu*.]

Conjunction Interval Constant 1 179 726 parts 7.C8a

[The Conjunction Interval Constant (just under 118 days) is the interval between the previous conjunction of planet and sun and the calendrical epoch.]

Argument Interval Constant 18 999 481 parts 7.C9a

[The Argument Interval Constant (nearly 1900 days, or 5 years 74 days) is the time elapsed between the planet's perigee (the beginning of its phase of Expansion), and the epoch. The perigee—more formally, the start of the Beginning Extent in Expansion—is the point where the planet's momentary equation of center is zero, as it changes from negative to positive. Cf. the commentary on the Expansion Beginning/Contraction End Extent for the sun, 3.C8, and for the corresponding Revolution Interval Constant for the moon, 4.C13.]

Expansion/Contraction Cubic Difference 236, add 7.C10a

Square Difference 25 912, subtract 7.C11a

Corrected Difference 10 897 000 7.C12a

[These coefficients, analogous to those in 3.2a, are used below to calculate inequalities (7.3A). This set of values for Jupiter corresponds to an inequality of

$$\Delta\alpha = (1089.7000x - [259\ 12x^2 + 236x^3]) / 10000000,$$

where x is the arc from perigee, the initial point for the equation of center. The maximum value of $\Delta\alpha$ is $5^{t}.994$. The *Ming History* reconstructs the derivation of these formulas for the five planets (33: 595–610). Like the sun (3.2A), Mars and Venus are equipped with two sets of constants each.]

Invisibility/Appearance Constant 13^{t} 7.C13a

[This constant represents how far from the sun the planet must be to be visible. From the modern viewpoint, planetary visibility is a complicated matter. It depends not only on the geometrical configuration, but on the planet's apparent brightness as affected by its distance from the earth, phase with relation to the sun, and albedo (that is, reflectivity). For instance, in the case of Venus, its phase appears to be smaller when it is closest to the earth than when farthest away, so that its brightness varies by only one magnitude; but Mercury, because it is more reflective at outer transit, is about four magnitudes brighter when farthest from the earth (Yabuuchi 1961, 26).

Each of the planets is provided with a table of grades *(tuan* 段), intervals between its major phenomena within each of which the authors treat the basic parameters as constant, allowing linear interpolation. As the text implies (e.g., step 7.5), a grade can be either a defined interval or the moment that begins it. For instance, Conjunction Invisibility begins at the moment of conjunction and extends to the next characteristic phenomenon.

Tables 10.3a–10.3e list constants for from ten to twenty steps in each planet's synodic period. I have added totals. The division into grades differs for each planet, but each table supplies the same four constants for every phenomenon. The values in each are symmetrical about retrogradation. Values of zero are blank in the original tables.]

Original column labels for all five:

A. Grade Days *(tuan-jih* 段日, 7.C14)
B. Mean Degrees *(p'ing-tu* 平度, 7.C15)
C. Limit Degrees *(hsien-tu* 限度, 7.C16)
D. Initial Motion Rate *(ch'u hsing lü* 初行率, 7.C17, in *tu* and parts).]

Table 10.3a. Grades of Motion for the Wood Star (Jupiter)

Grade	A, days	B, *tu*	C, *tu*	D, *tu*
Conjunction Invisibility	16.86	3.86	2.93	0.23
Dawn Hastening, Beginning	28	6.11	4.64	0.22
Dawn Hastening, End	28	5.51	4.19	0.21
Dawn Slackening, Beginning	28	4.31	3.28	0.18
Dawn Slackening, End	28	1.91	1.45	0.12
Dawn Station	24	-	-	-

Grade	**A, days**	**B, *tu***	**C, *tu***	**D, *tu***
Dawn Retrogradation	46.58	4.88125	0.32875	-
Evening Retrogradation	46.58	4.88125	0.32875	0.16
Evening Station	24	-	-	-
Evening Slackening, Beginning	28	1.91	1.45	-
Evening Slackening, End	28	4.31	3.28	0.12
Evening Hastening, Beginning	28	5.51	4.19	0.18
Evening Hastening, End	28	6.11	4.64	0.21
Evening Invisibility	16.86	3.86	2.93	0.22
Total (added)	398.88	53.1625	33.6375	-

{1244–45}

[The columns require some explanation:

A. Grade Days is the mean length of each grade in days. The total of this column for each planet is the Cycle Day Constant (7.C5), the synodic period. Earlier systems called Grade Days "Regular [i.e., mean] Days, *ch'ang-tu* 常日."

B. Mean Degrees is the corresponding arc in *tu,* and thus is zero for stationary phases. Earlier called *ch'ang-tu* 常度.

C. Limit Degrees is the mean increase in the planet's distance from the sun over the period of the grade, in *tu*. It too is zero for stations. The total of Limit Degrees for the outer planets is the synodic period of the planet minus (only approximately) that of the sun, the mean tropical year. For Mars, the total is 414.6865 = 779.93 - 365.2435. For the inner planets, the total of Limit Degrees is the planet's synodic period. For a valuable but narrowly positivistic analysis of Limit Degrees and its antecedents, see Ch'ü An-ching 曲安京 2006.

D. The Initial Motion Rate is, in modern terms, a mean figure for the velocity of the planet at the beginning of each grade. The authors express it in *tu* and parts. Yuan Chinese, like Europeans before the early modern period, did not have a clear mathematical conception of velocity that would have led them to speak explicitly of *tu* per day. Unlike the other tabulated quantities, which are averaged over the extent of the grade, the Initial Motion Rate applies to the event that defines the grade and begins it. Step 7.7 uses this constant for interpolation. Only for Mercury and Venus does this parameter exceed 1 *tu.*

The values are almost symmetrical about conjunction as well, but that is not the case for the Initial Motion Rate. Mars and Saturn employ two different equations of center, presumably to deal with the large eccentricity of their orbits. Mercury, which also has a markedly eccentric orbit, does not, perhaps because its closeness to the sun makes the lack of precision less obvious.]

b. The Fire Star (Mars)

Cycle Rate	7 799 290 parts	7.C4b
Cycle Day Constant	779.9290 days	7.C5b
Argument Rate	6 869 580.43 parts	7.C6b
Degree Rate	18 807.5 parts	7.C7b
Conjunction Interval Constant	567 545 parts	7.C8b
Argument Interval Constant	5 472 938 parts	7.C9b
Expansion Beginning/Contraction End Cubic Difference	1135, subtract	7.C10b1

[This item, unlike the other cubic differences, stipulates subtracting rather than adding. Item 7.C11b2 below adds *fu* 負, which in mathematics designates a negative number. The commentators do not explain these peculiarities, and I cannot account for them.]

Square Difference	831 189, subtract	7.C11b1
Corrected Difference	88 478 400	7.C12b1
Contraction Beginning/Expansion End Cubic Difference	851, add	7.C10b2
Square Difference	30 235 *fu* 負, subtract	7.C11b2
Corrected Difference	29 976 300	7.C12b2
Invisibility/Appearance Constant	19^{t}	7.C13b

Table 10.3b. Grades of Motion for the Fire Star (Mars)

Grade	A, days	B, *tu*	C, *tu*	D, *tu*
Conjunction Invisibility	69	50	46.50	0.73
Dawn Hastening, Beginning	59	41.80	38.87	0.72
Dawn Hastening, End	57	39.08	36.34	0.70
Dawn Secondary Hastening, Beginning	53	34.16	31.77	0.67 {1247}
Dawn Secondary Hastening, End	47	27.04	25.15	0.62
Dawn Slackening, Beginning	39	17.72	16.48	0.53
Dawn Slackening, End	29	6.20	5.77	0.38
Dawn Station	8	-	-	-
Dawn Retrogradation	28.9645	8.65675	6.46325	-
Evening Retrogradation	28.9645	8.65675	6.46325	0.44
Evening Station	8	-	-	-
Evening Slackening, Beginning	29	6.20	5.77	-
Evening Slackening, End	39	17.72	16.48	0.38

Grade	A, days	B, *tu*	C, *tu*	D, *tu*
Evening Secondary Hastening, Beginning	47	27.04	25.15	0.53
Evening Secondary Hastening, End	53	34.16	31.77	0.62 {1248}
Evening Hastening, Beginning	57	39.08	36.34	0.67
Evening Hastening, End	59	41.80	38.87	0.70
Evening Invisibility	69	50	46.50	0.72
Total	779.93	449.3135	414.6865	-

{1245–48}

c. The Earth Star (Saturn)

Cycle Rate	3 780 916 parts	7.C4c
Cycle Day Constant	378.0916 days	7.C5c
Argument Rate	17 478 845.66 parts	7.C6c
Degree Rate	294 255 parts	7.C7c
Conjunction Interval Constant	175 643 parts	7.C8c
Argument Interval Constant	52 240 561 parts	7.C9c
Expansion Cubic Difference	283, add	7.C10c1
Square Difference	41 022, subtract	7.C11c1
Corrected Difference	15 146 100	7.C12c1
Contraction Cubic Difference	331, add	7.C10c2
Square Difference	15 126, subtract	7.C11c2
Corrected Difference	1 117 500	7.C12c2
Invisibility/Appearance Constant	18^{t}	7.C13

Table 10.3c. Grades of Motion for the Earth Star (Saturn)

Grade	A, days	B, *tu*	C, *tu*	D, *tu*
Conjunction Invisibility	20.40	2.40	1.49	0.12
Dawn Hastening	30	3.40	2.11	0.11
Dawn Secondary Hastening	29	2.75	1.71	0.10
Dawn Slackening	26	1.50	0.83	0.08
Dawn Station	30	-	-	-
Dawn Retrogradation	52.6458	3.62545	0.28455	-

Grade	A, days	B, *tu*	C, *tu*	D, *tu*
Evening Retrogradation	52.6458	3.62545	0.28455	0.10 {1250}
Evening Station	30	-	-	-
Evening Slackening	26	1.50	0.83	-
Evening Secondary Hastening	29	2.75	1.71	0.08
Evening Hastening	30	3.40	2.11	0.10
Evening Invisibility	20.40	2.40	1.49	0.11
Total	378.09	27.3509	12.8491	-

{1248–50}

d. The Metal Star (Venus)

Cycle Rate	5 839 026 parts	7.C4d
Cycle Day Constant	583.9026 days	7.C5d
Argument Rate	36 525.75 parts	7.C6d
Degree Rate	10 000	7.C7d
Conjunction Interval Constant	5 716 330 parts	7.C8d
Argument Interval Constant	119 639 parts	7.C9d
Expansion/Contraction Cubic Difference	141, add	7.C10d
Square Difference	3, subtract	7.C11d
Corrected Difference	3 515 500	7.C12d
Invisibility/Appearance Constant	10½^{t}	7.C13d

Table 10.3d. Grades of Motion for the Metal Star (Venus)

Grade	A, days	B, *tu*	C, *tu*	D, *tu*
Conjunction Invisibility (1)	39	49.50	47.64	1.275
Evening Hastening, Beginning	52	65.50	63.04	1.265
Evening Hastening, End	49	61	58.71	1.255
Evening Secondary Hastening, Beginning	42	50.25	48.36	1.235
Evening Secondary Hastening, End	39	42.50	40.90	1.16
Evening Slackening, Beginning	33	27	25.99	1.02
Evening Slackening, End	16	4.25	4.09	0.62
Evening Station	5	-	-	-
Evening Retrogradation	10.9513	3.6987	1.5913	-
Evening Retrogradation Invisibility	6	4.35	1.63	0.61

Grade	**A, days**	**B, *tu***	**C, *tu***	**D, *tu***
Conjunction Retrogradation Invisibility	6	4.35	1.63	0.82
Dawn Retrogradation	10.9513	3.6987	1.5913	0.61
Dawn Station	5	-	-	-
Dawn Slackening, Beginning	16	4.25	4.09	-
Dawn Slackening, End	33	27	25.99	0.62
Dawn Secondary Hastening, Beginning	39	42.50	40.90	1.02
Dawn Secondary Hastening, End	42	50.25	48.36	1.16
Dawn Hastening, Beginning	49	61	58.71	1.235
Dawn Hastening, End	52	65.50	63.04	1.255
Dawn Invisibility	39	49.50	47.64	1.265
Total	583.9026	616.0974	583.9026	-

{1250–53}

e. The Water Star (Mercury)

Cycle Rate	1 158 760 parts	7.C4e
Cycle Day Constant	115.8760 days	7.C5e
Argument Rate	36 525.75 parts	7.C6e
Degree Rate	10 000	7.C7e
Conjunction Interval Constant	700 437 parts	7.C8e
Argument Interval Constant	2 055 161 parts	7.C9e
Expansion/Contraction Cubic Difference	141, add	7.C10e
Square Difference	2165, subtract	7.C11e
Corrected Difference	3 877 000	7.C12e
Dawn Invisibility/Appearance Constant	16½^{t}	7.C13e1
Evening Invisibility/Appearance Constant	19^{t}	7.C13e2

Table 10.3e. Grades of Motion for the Water Star (Mercury)

Grade	**A, days**	**B, *tu***	**C, *tu***	**D, *tu***
Conjunction Invisibility	17.75	34.25	29.08	2.1558
Evening Hastening	15	21.38	18.16	1.7034
Evening Slackening	12	10.12	8.59	1.1472

Grade	A, days	B, *tu*	C, *tu*	D, *tu*
Evening Station	2	-	-	-
Evening Retrogradation Invisibility	11.1880	7.8120	2.1080	-
Conjunction Retrogradation Invisibility	11.1880	7.8120	2.1080	1.0346
Dawn Station	2	-	-	-
Dawn Slackening	12	10.12	8.59	-
Dawn Hastening	15	21.38	18.16	1.1472
Dawn Invisibility	17.75	34.25	29.08	1.7034
Total	115.8760	147.1240	115.8760	-

{1253–54}

[As the commentary to table 10.3a explains, the tables just given break down the mean synodic motion of each planet into grades. These steps correspond to a sequence of straight lines that crudely trace the contour of a curve. The procedures that follow use them to determine the main apparent planetary phenomena: arcs of direct and retrograde motion, the stations between them, and periods of invisibility due to rising and setting close to the sun. The technique goes on to predict a planet's apparent positions and times for conjunction with the sun and for disappearance and reappearance of the planet, but not for any desired time.]

7.1. To Compute Mean Conjunction, and Intermediate Accumulation and Centered Star, for each Grade, following Winter Solstice in the Astronomical First Month for the Five Stars

Set up the Intermediate Accumulation (1.1.2), add the Conjunction Interval Constant (7.C8), and cast out the Cycle Rate (7.C4) for the star. The remainder is Anterior Conjunction [Parts] (7.1.1). Then subtract this from the Cycle Rate; the remainder is Posterior Conjunction [Parts] (7.1.2). Simplify [i.e., divide] by the Day Cycle (1.C1) to obtain the Mean Conjunction (7.1.3), Intermediate Accumulation (7.1.4) and Centered Star (7.1.5) for the star following winter solstice in the Astronomical First Month.

<If counted off as days, it is Intermediate Accumulation for that day; if counted off as *tu*, it is the Centered Star for that day.>

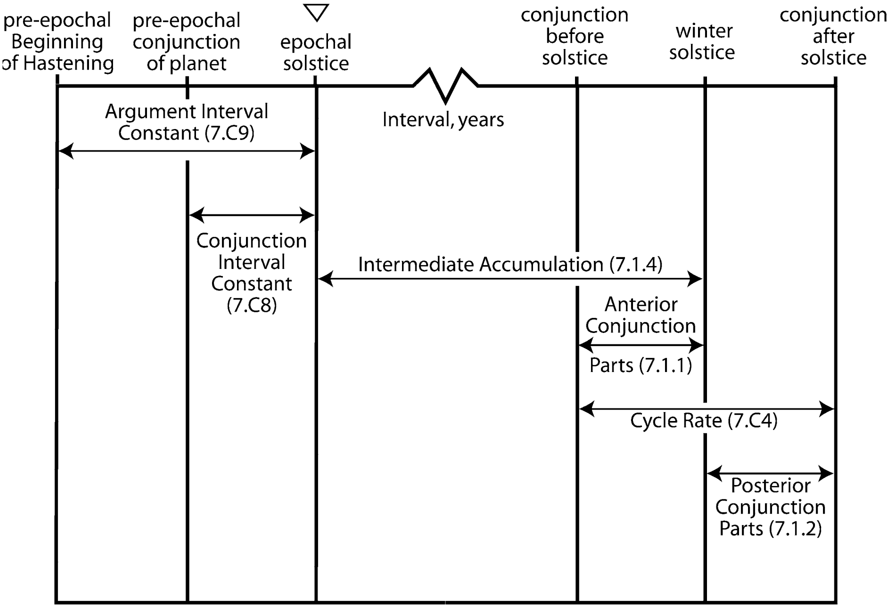
Repeatedly add Grade Days (7.C14 in table 10.3) to the Intermediate Accumulation. The result [of each iteration] is the Intermediate Accumulation for each grade. Repeatedly add [Mean] Degrees

(7.C15) to the Centered Star—subtract when passing through retrogradation—and the result is the Centered Star for each grade (7.1.6).

<In calculating backward, from the Intermediate Accumulation itself subtract the Conjunction Interval Constant, casting out complete Cycle Rates. The remainder is the desired Posterior Conjunction Parts.> {1255}

[The Intermediate Accumulation with which this step begins is the number of day parts from the epochal winter solstice to the solstice in the current Astronomical First Month, corresponding to elapsed tropical years. Adding the interval between pre-epoch conjunction and epoch (Conjunction Interval Constant) and casting out completed synodic revolutions of the planet leaves the interval between the last mean planet-sun conjunction and the current winter solstice. This is Anterior Conjunction Parts. Subtracting this from the mean interval between conjunctions, the Cycle Rate, leaves Posterior Conjunction Parts, or the interval between the current winter solstice and the next Mean Conjunction. For the relations between these constants, see figure 45.]

Fig. 45. Relation between constants in steps 7.1 and 7.2.



[In the planetary theory, the Centered Star is simply the arc (not days) from winter solstice to the mean beginning of each grade. It has nothing to do with transits of the meridian. Adding tabulated Mean Degrees to the Centered Star yields the next grade's Centered Star—the arc from the solstice to its mean beginning. This anomalous usage, like others in the planetary theory, was simply copied from the Revised Great Enlightenment system in *Chin shih*, 22: 514.]

7.2. To Compute Mean Conjunction and the Argument of Each Grade for the Five Stars

In each case set up the Intermediate Accumulation, add the Argument Interval Constant (7.C9) and Posterior Conjunction Parts for the desired [year], and cast out complete Argument Rates (7.C6). In the remainder, take 1 for each Degree Rate (7.C7) and read as *tu*. Shift anything that remains backward and read as ten-thousandth parts. The result is the Argument of Mean Conjunction (7.2.1) in *tu* and parts for the star. Repeatedly add Limit Degrees (7.C16) for each grade to obtain the argument for each grade (7.2.2).

<In calculating backward, from the Intermediate Accumulation itself subtract the Argument Interval Constant and cast out complete Argument Rates. Subtract the remainder from the Argument Rate, and add the remainder to Posterior Conjunction Parts for that year. The rest of the procedure is as above.> {1255}

[This step begins with the interval between the epoch and the winter solstice of the year being studied, extending it backward to the zero point of the equation of center and forward to the mean conjunction of planet and sun after the solstice (see figure 45). That conjunction begins the initial sequence of grades.

Casting out the mean sidereal period of the planet's revolution in days leaves the interval in days from the zero point to mean conjunction in the current revolution of the planet. Dividing what remains by the same period expressed in ten-thousandth parts of a year (and thus a full cycle of *tu)* and shifting the counting rods backward yields the same interval in *tu.* One then computes additional grades by adding successively the arcs that correspond to each.]

7.3A. To Find the Expansion/Contraction Difference

Set up *tu* and ten-thousandth parts of the Argument [of Mean Conjunction]. If less than the Argument Midway Constant (7.C2), read it in the phase of expansion. If greater, cast out an Argument Midway Constant and read the remainder in the phase of contraction. Inspect the argument in expansion or contraction. If less than 91^{t}.314 375, it is read in the Beginning Extent; if greater, subtract it from the Argument Midway Constant and read the remainder in the End Extent.

As for the Fire Star [Mars], if its argument in the phase of expansion is less than 60^{t}.876 25, read it in the Beginning Extent. If greater, subtract it from the Argument Midway Constant and read the remainder in the End Extent. If the argument in the phase of contraction is less than 121^{t}.7525, read it in the Beginning Extent. If greater, subtract it from the Argument Midway Constant and read the remainder in the End Extent.

Set up the Cubic Difference for the star being studied (7.C10) and multiply it by the [argument in the] Beginning or End Extent. Cast out the additive or subtractive Square Difference (7.C11). Again multiply the result by the [argument in the] Beginning or End Extent. Cast out the additive or subtractive Corrected Difference (7.C12), and once more multiply the result by the [argument in the] Beginning/End Extent. Complete hundred millions (*i* 億) become *tu*. Shift anything that remains backward to yield ten-thousandth parts. The result is the desired Expansion/Contraction Difference (7.3.1). {1255–56}

[This procedure is parallel to those for the equation of center of the sun (Expansion/Contraction Difference, 3.2A) and moon (Slackening/Hastening Difference, 4.5A), using the tabulated constants for each planet. The asymmetric constants for Mars arise from an empirical attempt to overcome the problem posed by what to the historian of astronomy is the planet's considerable eccentricity. The two are complementary, adding up to half the sidereal year value. Benno van Dalen suggests that the equations for Mars have two negative coefficients because the function has to provide positive values for a range of arguments different from those of the other planets (email, 3 Mar 2007).]

7.3B. Another Method

Set up the argument in the phase of expansion or contraction and divide by the Argument Factor (7.C3) to yield the Factor Number (7.3.2). Anything left over becomes the Factor Surplus (7.3.3). Multiply [the latter] by the Diminution/Augmentation Rate (7.3.4) below it [in the table] and divide by the Argument Factor. Add the result thus obtained to, if Augmentation, or subtract from, if Diminution, the Expansion/Contraction Accumulation (7.3.5) below it [in the table]. This also yields the Expansion/Contraction Difference. {1256}

[This ready-reckoning method is parallel to those in steps 3.2B and 4.5B. It uses a set of five tables, all omitted from the published system. Those I have added survive in *Ming shih* 34: 672–84. There are also simpler versions in the Korean *Ch'iljŏngsan Naepiŏn* 七政算内篇 (1444), 158: 397–401. The latter includes a version with only columns A-C rounded to the equivalent of two decimal places, with many copyist's errors and arbitrarily rounded figures. Here are the tables 10.4a–10.4e from the *Ming History,* with their original labels:

A. Factor Number (7.3.2)
B. Diminution/Augmentation Rate (7.3.4), *tu*
C. Expansion/Contraction Accumulation (7.3.5), *tu*
D. Corrected Motion (7.3.6), *tu*
E. Accumulated Motion (7.3.7), *tu*
Column C sums the values in column B, as does E for those in D.]

Table 10.4a. Expansion/Contraction Difference for the Wood Star (Jupiter)

A	B	C	D	E
0	**Augmentation**	**Expansion**		
	1.59008481	0.00000000	16.80914731	16.80914731
1	1.42013561	1.59008481	16.63919811	33.44834542
2	1.20027188	3.01022042	16.41933438	49.86767980
3	0.93049362	4.21049230	16.14955612	66.01723590
4	0.61080083	5.14098592	15.82986333	81.84709925
5	0.24119352	5.75178676	15.46025602	97.30735527
6	**Diminution**	**[Expansion]**		
	0.24119352	5.99298028	14.97786898	112.28522425
7	0.61080083	5.75178676	14.60826167	126.89348592
8	0.93049362	5.14098592	14.28856888	141.18205480
9	1.20027188	4.21049230	14.01879062	155.20084542
10	1.42013561	3.01022042	13.79892689	168.99977231
11	1.59008481	1.59008481	13.62897769	182.62875000
0	**Augmentation**	**Contraction**		
	1.59008481	0.00000000	13.62897769	196.25772769
1	1.42013561	1.59008481	13.79892689	210.05665458
2	1.20027188	3.01022042	14.01879062	224.07544520
3	0.93049362	4.21049230	14.28856888	238.36401408
4	0.61080083	5.14098592	14.60826167	252.97227575

A	B	C	D	E
5	0.24119352	5.75178676	14.97786898	267.95014473
6	0.24119352	5.99298028	15.46025602	283.41040075
7	0.61080083	5.75178676	15.82986333	299.24026408
8	0.93049362	5.14098592	16.14955612	315.38982020
9	1.20027188	4.21049230	16.41933438	331.80915458
10	1.42013561	3.01022042	16.63919811	348.44835269
11	1.59008481	1.59008481	16.80914731	365.25750000

Table 10.4b. Expansion/Contraction Difference for the Fire Star (Mars)

A	B	C	D	E
0	**Augmentation**	**Expansion**		
	11.58039334	0.00000000	26.79945584	26.79945584
1	7.97005072	11.58039334	23.18911322	49.98856906
2	4.59976313	19.55044406	19.81882563	*69.70939469
3	1.46957252	24.15020719	16.68863502	86.49602971
4	**Diminution**	**[Expansion]**		
	0.54285064	25.61977971	14.67621186	101.17224157
5	1.66275085	25.07692907	13.55631165	114.72855322
6	2.60262072	23.41417822	12.61644178	127.34499500
7	3.36250217	20.81155750	11.85656033	139.20155533
8	3.94239524	17.44905533	11.27666726	150.47822259
9	4.42257296	13.50666009	10.79648954	161.27471213
10	4.56200750	9.08408713	10.65705500	171.93176713
11	4.52207963	4.52207963	10.69698287	182.62875000
0	**Augmentation**	**Contraction**		
	4.52207963	0.00000000	10.69698287	193.32573287
1	4.56200750	4.52207963	10.65705500	203.98278787
2	4.42257296	9.08408713	10.79648954	214.77927741
3	3.94239524	13.50666009	11.27666726	226.05594467
4	3.36250217	17.44905533	11.85656033	237.91250500
5	2.60262072	20.81155750	12.61644178	250.52894678
6	1.66275085	23.41417822	13.55631165	264.08525843
7	0.54285064	25.07692907	14.67621186	278.76147029

A	B	C	D	E
8	Diminution	[Contraction]		
	1.46957252	25.61977971	16.68863502	295.45010531
9	4.59976313	24.15020719	19.81882563	315.26893094
10	7.97005072	19.55044406	23.18911322	338.45804416
11	11.58039334	11.58039334	26.79945584	365.25750000

Table 10.4c. Expansion/Contraction Difference for the Earth Star (Saturn)

A	B	C	D	E
0	**Augmentation**	**Expansion**		
	2.20010346	0.00000000	17.41916596	17.41916596
1	1.95021814	2.20010346	17.16928064	34.58844660
2	1.64047765	4.15032160	16.85954015	51.44798675
3	1.27088211	5.79079925	16.48994461	67.93793136
4	0.84143135	7.06168136	16.06049385	83.99842521
5	0.35212550	7.90311271	15.57118800	99.56961321
6	**Diminution**	**[Expansion]**		
	0.35212550	8.25523821	14.86693700	114.43655021
7	0.84143135	7.90311271	14.37763115	128.81418136
8	1.27088211	7.06168136	13.94818039	142.76236175
9	1.64047765	5.79079925	13.57858485	156.34094660
10	1.95021814	4.15032160	13.26884436	169.60979096
11	2.20010346	2.20010346	13.01895904	182.62875000
0	**Augmentation**	**Contraction**		
	1.63005751	0.00000000	13.58900499	196.21775499
1	1.48998064	1.63005751	13.72908186	209.94683685
2	1.27989652	3.12003815	13.93916598	223.88600283
3	0.99980516	4.39993467	14.21925734	238.10526017
4	0.64970658	5.39973983	14.56935592	252.67461609
5	0.22960073	6.04944641	14.98946177	267.66407786
6	**Diminution**	**[Contraction]**		
	0.22960073	6.27904714	15.44866323	283.11274109
7	0.64970658	6.04944641	15.86876908	298.98151017
8	0.99980516	5.39973983	16.21886766	315.20037783
9	1.27989652	4.39993467	16.49895902	331.69933685

A	B	C	D	E
10	1.48998064	3.12003815	16.70904314	*348.40837999
11	1.63005751	1.63005751	16.84912001	365.25750000
0	**Augmentation**	**Expansion**		
	0.53004889	0.00000000	15.74911139	15.74911139
1	0.50021318	0.53004889	15.71927568	31.46838707
2	0.44055565	1.03026207	15.65961815	47.12800522
3	0.35107631	1.47081772	15.57013881	62.69814403
4	0.23177516	1.82189403	15.45083766	78.14898169
5	0.08265219	2.05366919	15.30171469	93.45069638
6	**Diminution**	**[Expansion]**		
	0.08265219	2.13632138	15.13641031	108.58710669
7	0.23177516	2.05366919	14.98728734	123.57439403
8	0.35107631	1.82189403	14.86798619	138.44238022
9	0.44055565	1.47081772	14.77850685	153.22088707
10	0.50021318	1.03026207	14.71884932	167.93973639
11	0.53004889	0.53004889	14.68901361	182.62875000
0	**Augmentation**	**Contraction**		
	0.53004889	0.00000000	14.68901361	197.31776361
1	0.50021318	0.53004889	14.71884932	212.03661293
2	0.44055565	1.03026207	14.77850685	226.81511978
3	0.35107631	1.47081772	14.86798619	241.68310597
4	0.23177516	1.82189403	14.98728734	256.67039331
5	0.08265219	2.05366919	15.13641031	271.80680362
6	**Diminution**	**[Contraction]**		
	0.08265219	2.13632138	15.30171469	287.10851831
7	0.23177516	2.05366919	15.45083766	302.55935597
8	0.35107631	1.82189403	15.57013881	318.12949478
9	0.44055565	1.47081772	15.65961815	333.78911293
10	0.50021318	1.03026207	15.71927568	349.50838861
11	0.53004889	0.53004889	15.74911139	365.25750000

Table 10.4d. Expansion/Contraction Difference for the Metal Star (Venus)

A	B	C	D	E

A	B	C	D	E
0	**Augmentation**	**Expansion**		
	0.53004889	0.00000000	15.74911139	15.74911139
1	0.50021318	0.53004889	15.71927568	31.46838707
2	0.44055565	1.03026207	15.65961815	47.12800522
3	0.35107631	1.47081772	15.57013881	62.69814403
4	0.23177516	1.82189403	15.45083766	78.14898169
5	0.08265219	2.05366919	15.30171469	93.45069638
6	**Diminution**	**[Expansion]**		
	0.08265219	2.13632138	15.13641031	108.58710669
7	0.23177516	2.05366919	14.98728734	123.57439403
8	0.35107631	1.82189403	14.86798619	138.44238022
9	0.44055565	1.47081772	14.77850685	153.22088707
10	0.50021318	1.03026207	14.71884932	167.93973639
11	0.53004889	0.53004889	14.68901361	182.62875000
0	**Augmentation**	**Contraction**		
	0.53004889	0.00000000	14.68901361	197.31776361
1	0.50021318	0.53004889	14.71884932	212.03661293
2	0.44055565	1.03026207	14.77850685	226.81511978
3	0.35107631	1.47081772	14.86798619	241.68310597
4	0.23177516	1.82189403	14.98728734	256.67039331
5	0.08265219	2.05366919	15.13641031	271.80680362
6	**Diminution**	**[Contraction]**		
	0.08265219	2.13632138	15.30171469	287.10851831
7	0.23177516	2.05366919	15.45083766	302.55935597
8	0.35107631	1.82189403	15.57013881	318.12949478
9	0.44055565	1.47081772	15.65961815	333.78911293
10	0.50021318	1.03026207	15.71927568	349.50838861
11	0.53004889	0.53004889	15.74911139	365.25750000

Table 10.4e. Expansion/Contraction Difference for the Water Star (Mercury)

A	B	C	D	E
0	**Augmentation**	**Expansion**		
	0.58005818	0.00000000	15.79912068	15.79912068

A	B	C	D	E
1	0.54020722	0.58005818	15.75926972	31.55839040
2	0.47053446	1.12026540	15.68959696	47.24798736
3	0.37103987	1.59079986	15.59010237	62.83808973
4	0.24172348	1.96183973	15.46078598	78.29887571
5	0.08258526	2.20356321	15.30164776	93.60052347
6	**Diminution**	**[Expansion]**		
	0.08258526	2.28614847	15.13647724	108.73700071
7	0.24172348	2.20356321	14.97733902	123.71433973
8	0.37103987	1.96183973	14.84802263	138.56236236
9	0.47053446	1.59079986	14.74852804	153.31089040
10	0.54020722	1.12026540	14.67885528	167.98974568
11	0.58005818	0.58005818	14.63900432	182.62875000
0	**Augmentation**	**Contraction**		
	0.58005818	0.00000000	14.63900432	197.26775432
1	0.54020722	0.58005818	14.67885528	211.94660960
2	0.47053446	1.12026540	14.74852804	226.69513764
3	0.37103987	1.59079986	14.84802263	241.54316027
4	0.24172348	1.96183973	14.97733902	256.52049929
5	0.08258526	2.20356321	15.13647724	271.65697653
6	**Diminution**	**[Contraction]**		
	0.08258526	2.28614847	15.30164776	286.95862429
7	0.24172348	2.20356321	15.46078598	302.41941027
8	0.37103987	1.96183973	15.59010237	318.00951264
9	0.47053446	1.59079986	15.68959696	333.69910960
10	0.54020722	1.12026540	15.75926972	349.45837932
11	0.58005818	0.58005818	15.79912068	365.25750000

[The modern editors of the *Ming History* do not provide critical notes. Asterisks in these tables indicate errors that I have corrected by cross-checking.]

7.4. To Find the Corrected Accumulation for Each Grade from Mean Conjunction

For each grade, set up the Intermediate Accumulation for the star. Add the Expansion/Contraction Difference if in the phase of expansion, or subtract if contraction. The result is Corrected Accumulation Days (7.4.1) and ten-thousandth parts for the given grade.

Add days and day parts of the winter solstice in the Astronomical First Month, and cast out complete Era Divisors (1.C17). Count off what remains exclusively from s.d. 1 to yield the date [of the grade, 7.4.2]. {1256}

[This corrects the mean arc of the planet's position with respect to the zero point of the equation of center, the conjunction. The computist adds the equation of center to it, and then converts it to a sexagenary date by the procedure of step 1.2.]

7.5. To Find the Month and Day of Mean Conjunction and Each Grade

For each grade, set up the Corrected Accumulation and add Intercalary Days and day parts for the Astronomical First Month. Divide by the Lunation Factor, yielding the number of months. What remains is days and day parts since entry of the lunation. Count off the number of months exclusively from the Astronomical First Month, taken as the eleventh [civil] month. The result is Lunation Entry Mean Conjunction Days (7.5.1) and day parts for the grade. Intervals between sexagenary dates give the month and day [of the grade, 7.5.2], counted from the corrected [conjunction]. {1256}

[Intercalary Days is days in the Intercalary Surplus (1.3.2), the interval from the lunisolar conjunction that begins the Astronomical First Month to the solstice. Adding the Intercalary Surplus to the Corrected Accumulation gives the interval from that first mean lunisolar conjunction of the year to the apparent phenomenon being studied. Dividing by the length of the synodic month, the Lunation Factor (1.C7), gives elapsed mean synodic months. Counting off gives the number of the month in the civil calendar, which counted the Astronomical First Month as the eleventh month. The last sentence asserts that, after the computist determines one civil calendar date, he can arrive at those for other grades simply by comparing the sexagenary dates of the Corrected Accumulations from 7.4. Here "grade" refers not to the interval but to the moment at which each begins.

The phrase in the last sentence that I have translated "counted from corrected [conjunction]" is ambiguous. Parallels in earlier treatises, cited in note 24 to the *Yuan shih* text, 55: 1263, make its meaning clear. They do not, however, mandate altering the text, as the modern editors chose to do.]

7.6. To Find the Corrected Star for the Exact Times of Mean Conjunction and of each Grade

For each grade, set up the Centered Star. Add the Expansion/Contraction Difference if in the phase of expansion, or subtract if in the phase of contraction.

<In the case of Venus double it, and for Mercury triple it.>

The result is the Corrected Star (7.6.1) for each grade. Add the sun's position on the Yellow Way at the exact time of winter solstice in the Astronomical First Month and count off. The result is lunar lodge position in *tu* and ten-thousandth parts for the exact time of the grade and star (7.6.2). {1256}

[The Corrected Star is the arc from winter solstice to the apparent beginning of each grade. The computist starts with the arc to the mean beginning of each grade from step 7.1, and applies the equation of center. Adding the sun's solstitial lunar lodge position along the ecliptic (3.11.2) gives the lunar lodge position on the ecliptic of the apparent beginning of the grade.

As for the note in text, its point is unclear, and it is part of a generally inadequate procedure for the inner planets. The Ming history, which seldom diverges substantially from the Season-Granting System, omits this note *(Ming shih,* 36: 730).]

7.7. To Find the Corrected Star for Midnight at the Beginning of the First Day of Each Grade

For each grade, multiply the Initial Motion Rate (7.C17) by day parts for the exact time of that grade, and simplify by 100. If in progressive motion subtract, or if in retrograde motion add, the Corrected Star [position] for the exact time [that the grade begins] on that day. The result is the Corrected Star for midnight at the beginning of the grade (7.7.1). Add and count off as before to obtain the desired value. {1256–57}

[This step converts the time interval between midnight and the apparent beginning of each grade—that is, when the apparent sun reaches that grade—to *tu* by linear interpolation. The Initial Motion Rate is a tabulated interpolation factor that represents the initial speed of the planet in each grade, expressed in degree parts (0.01 *tu)*. Division by 100 converts it to *tu.* Algebraically subtracting the arc thus derived from the lunar lodge position of the phenomenon leaves the position of the planet at midnight. On the interpolation method, see Ch'e I-hsiung 車一雄 1982.]

7.8. To Find the Day Rate and Degree Rate for Each Grade

For each grade, take the interval from its sexagenary date to that of the next grade as the Day Rate (7.8.1). Perform subtraction with the lunar lodge position of the star for that grade and that of the next grade. The remainder is the Degree Rate (7.8.2). {1257}

[See the commentary to 7.9.]

7.9. To Find Mean Motion Parts for Each Grade

For each grade, set up the Degree Rate and divide by the Day Rate for that grade. The result is Mean Motion Degrees (7.9.1) and ten-thousandth parts for that grade. {1257}

[Steps 7.8 and 7.9 together derive the mean planetary velocity over the extent of each grade, since the Degree Rate is *tu* covered, and the Day Rate is days elapsed. The Day Rate is based on sexagenary dates of the apparent phenomena from 7.4; the Degree Rate must accordingly be based on the arc for an integral number of days. It thus uses the values to midnight computed in 7.7.]

7.10. To Find the Increase/Decrease Difference and Daily Difference for Each Grade

For each grade, perform subtraction with Mean Motion Parts preceding and following to yield the Comprehensive Difference (7.10.1) for that grade. Double and shift backward one column to yield the Increase/Decrease Difference (7.10.2). Add to or subtract from Mean Motion Parts for the grade to yield First/Last Day Motion Parts (7.10.3).

<When the preceding [Mean Motion Parts] is greater than the following, add for the first day and subtract for the last. When the preceding is smaller than the following, subtract for the first day and add for the last.>

Double the Increase/Decrease Difference to give the General Difference (7.10.4). Decrease the Day Rate (7.8.1) by 1 and divide [the General Difference by the result] to yield the Daily Difference (7.10.5). {1257}

[Beginning with Mean Motion Parts—in modern terms, the mean planetary velocity over the extent of each grade—here one derives Motion Parts for the first and last day of each grade. The Comprehensive Difference is that between the mean velocity of the next and the previous grade. The Increase/Decrease Difference, since the computist obtains it by doubling and shifting backward a column (that is, a power of ten), is one-fifth of that. It is the difference between first-day and average velocity for the grade, and between average and last-day velocity. The aim of this step seems to amount to smoothing the curve.

Thus First/Last Day Motion Parts amounts to

$$v = v_b \pm 1/5\,(v_c - v_a),$$

where *a, b,* and *c* mark the velocities of the previous, current, and next grade. The reason for the factor of 5 is not obvious. Takebe does not provide a convincing explanation, nor can I. It is no doubt pertinent that this step was copied verbatim from the Revised Great Enlightenment system.

Doubling the Increase/Decrease Difference yields the General Difference

$$v_a - v_c = (v_a - v_b) + (v_b - v_c),$$

the total increment in velocity over the extent of the current grade. Since the Day Rate is the interval from the first day of one grade to the first day of the next, 1 less is that from the first to the last day of the same grade. Dividing the increment of velocity by the interval gives the daily increment in velocity, the Daily Difference. In principle this amounts to the mean daily acceleration of the planet over the grade. In 1280, of course, the language of velocity and acceleration would have seemed as outlandish in China as in Italy.]

7.11. To Find the Increase/Decrease Difference for Grades Preceded or Followed by Invisibility, Slackening, or Retrogradation

When the grade is preceded by [Conjunction] Invisibility, set up First Day Motion Parts for the following grade and add half the Daily Difference to give Last Day Motion Parts [for the current grade, 7.11.1]. When it is followed by invisibility, set up Last Day Motion Parts for the preceding grade and add half the Daily Difference to give First Day Motion Parts [for the current grade, 7.11.2]. In either case] subtract the result from Mean Motion Parts for the grade of invisibility. The remainder is the Increase/Decrease Difference (7.11.3).

When the grade is preceded by slackening, set up Last Day Motion Parts for the preceding grade, double the corresponding Daily

Difference and subtract, yielding First Day Motion Parts. When it is followed by slackening, set up First Day Motion Parts for the following grade, double the corresponding Daily Difference and subtract, yielding Last Day Motion Parts. [From either result] subtract Mean Motion Parts for the grade of slackening, and the remainder is the Increase/Decrease Difference.

<This refers to preceding or following slackening grades in the vicinity of a station.>

For the Wood, Fire, or Earth Star (Jupiter, Mars, or Saturn) in retrograde motion, multiply *(yin* 因) Mean Motion Parts by 6 and shift backward one column to yield the Increase/Decrease Difference.

For the Metal Star (Venus) when preceded or followed by retrogradation or invisibility, multiply Mean Motion Parts by 3, halve the result, and shift backward a column to yield the Increase/Decrease Difference.

When the current grade is preceded by retrogradation, set up First Day Motion Parts for the following grade and subtract the corresponding Day Difference to yield Last Day Motion Parts [for the current grade]. When it is followed by retrogradation, set up Last Day Motion Parts for the preceding grade and subtract the corresponding Day Difference to yield First Day Motion Parts. [From either result] subtract Mean Motion Parts for the current grade. The remainder is the Increase/Decrease Difference.

For the Water Star (Mercury) in retrograde motion, halve Mean Motion Parts to yield the Increase/Decrease Difference.

In every case add the Increase/Decrease Difference to or subtract it from Mean Motion Parts to yield First/Last Day Motion Parts.

<When the preceding [Mean Motion Parts] is greater than the following, add for first day [Motion Parts] and subtract for last. When the preceding is smaller than the following, subtract for first day [Motion Parts] and add for last.>

Again double the Increase/Decrease Difference to give the General Difference (7.11.4). Decrease the Day Rate by 1 and divide the [General Difference by the result] to yield the Daily Difference (7.11.5). {1257–58}

[Step 7.10 gives the general procedure for the Increase/Decrease Difference; this one gives the many exceptions. The general procedure is a matter of interpolation between the preceding and following grade. The special cases are those grades for which such interpolation is not feasible, namely those bordering on:

1. Retrogradation, preceded by a station and succeeded by another grade of retrogradation, or vice versa. In retrogradation there is no Mean Motion Parts (that is, no meaningful mean velocity);

2. Conjunction Invisibility, which as the maximum of velocity in direct motion cannot itself be the difference between the speeds of the previous and next grades. It also has no tabulated preceding grade, since the end and beginning of the table are not—formally speaking—continuous; and

3. Slackening before or after a station. This case presents the same problems as retrogradation, since either the preceding or following grade offers no Mean Motion Parts. In all the versions of table 10.3, a grade of slackening precedes the first station, which is then followed by retrogradation (centered on the perigee, with minimal velocity), then the second station, and then a second grade of slackening.

Since interpolation is ruled out, this step offers instead a variety of extrapolations from the usable previous or subsequent grade. The reasoning behind these special cases is not clear, and surviving evidence does not enable its reconstruction. The redoubtable Takebe makes a valiant effort to do just that (6: 70b-71b), but his results are inconclusive. It is impossible to say with confidence to what extent these adjustments are empirical and to what extent based on deductive reasoning. The Yuan authors took this step, like the previous one, more or less verbatim from the Revised Great Enlightenment system. I say “more or less” because there are some very minor discrepancies. Only one affects the meaning: The phrase I have translated “for the Water Star (Mercury) in retrograde motion *(shui-hsing t’ui hsing che* 水星退行者)” reads, in the earlier version, “for the Water Star *(shui-hsing che* 水星者).”

Let me restate the above adjustments in a modern form that will help some readers to understand their language.

The quantity that each instance aims at is the Increase/Decrease Difference, the difference between the velocity on the first or last day and the average velocity for the grade; I will designate it Δv. One derives Motion Parts for the first or last day in the current grade, v, from that for the previous or next grade, which I will further designate $v_{n\pm 1}$, and the Daily Difference, which we have seen amounts to the acceleration, and which I will therefore call a. One then derives Δv from v and Mean Motion Parts for the excluded grade, which I will call v_g. The procedure for particular planets is much simpler, involving only v_g.

The handling of each exception is as follows:

Conjunction Invisibility:

$$v = v_{n\pm1} + a/2;\ \Delta v = v_g - v$$

Slackening:

$$v = v_{n\pm1} - 2a;\ \Delta v = v - v_g$$

Jupiter, Mars, Saturn:

$$\Delta v = 3/5\ v_g$$

Venus:

$$\Delta v = 3/20\ v_g$$

This technique applies only to Conjunction Retrogradation Invisibility; Evening Retrogradation Invisibility uses the technique of 7.10.

Retrogradation:

$$v = v_{n\pm1} - a;\ \Delta v = v - v_g$$

Mercury:

$$\Delta v = ½\ v_g$$

The two final steps in every instance that lead from the Increase/Decrease Difference to the Daily Difference for the grade being studied are identical to those of step 7.10.]

7.12. To Find the Star's Lunar Lodge Position for Midnight at the Beginning of each Day

For each grade, set up First Day Motion Parts. Repeatedly augment or diminish it by the Daily Difference. If it is becoming smaller diminish it, or if it is becoming larger augment it, to yield Daily Motion Degrees (7.12.1) in *tu* and ten-thousandth parts. Then add [to First Day Motion Parts] if the motion is progressive or subtract if retrograde, casting out complete lunar lodges. The result is the lunar lodge position at midnight at the beginning of each day of the star's motion (7.12.2). {1258}

[Velocity for the second day is in modern terms the first day's velocity plus or minus acceleration; that for the third day is the second day's velocity plus or minus acceleration, and so on. The planet's position on a given day—that, and not velocity, is the result of this step—is its position at midnight preceding the beginning of the grade (from 7.7) plus the sum of all these daily arcs up to the desired midnight. Although the term for the result says literally "motion *(hsing* 行)," it clearly is a matter of position, that is, where the planet has moved to among the lodges.]

7.13. To Find the Arguments in the Phases of Expansion and Contraction for Mean Conjunction, Appearance, and Invisibility for the Five Stars

For the grade, set up days and ten-thousandth parts of the Corrected Accumulation (7.4) for the star.

<If it contains full days and day parts of the Year Cycle (1.C6), cast out the latter. The remainder will fall after the winter solstice in the Astronomical First Month of the following year.>

If it is less than a Year Semicycle (3.C7), read it as the argument in the phase of expansion. If it is a full Year Semicycle [or greater,] cast out the Year Semicycle and read it as the argument in the phase of contraction. In either case, if less than the Beginning Extent Constant (4.C4), it becomes the Beginning Extent [Quadrants I and III]; if greater, subtract the Year Semicycle from it and the remainder becomes the End Extent [Quadrants II and IV]. The result thus obtained is the argument in the phase of expansion or contraction for the mean conjunction, appearance or invisibility for [one of] the Five Stars (7.13.1) in days and ten-thousandth parts. {1258}

[The arguments are set out within quarters of the tropical year as in steps 3.1–3.2a and figure 30. They are days elapsed from the solstice to the phenomenon in question. In the case of a planet it is, of course, possible for its Corrected Accumulation to exceed the length of a tropical year, in which case the Note in Text directs that it be shifted to the next year.]

7.14. To Find the Motion Difference for Mean Conjunction, Appearance, and Invisibility for the Five Stars

For the first day of each grade, perform subtraction with Motion Parts for the star and for the sun. The remainder is the Motion Difference (7.14.1). In the case of the Metal Star in retrograde motion or the Water Star at retrograde conjunction, take the star's Motion Parts for the first day of the grade and combine it with that of the sun for the first day of the grade to give the Motion Difference. If within the grade of Evening Invisibility or Dawn Appearance for the Water Star, directly take Motion Parts for the sun on the first day of the grade as the Motion Difference. {1258–59}

[This step, paraphrased from the Yuan system's predecessor, computes the difference between daily velocities of the sun and the planet—the speed of elongation, so to speak. Motion Parts for the planet is First Day Motion Parts from 7.10.3 or 7.11.2; that for the sun is the difference between two successive midnight positions from 3.13.6. The grade of retrograde conjunction for Mercury is called Conjunction Retrogradation Invisibility.

The phrase "perform subtraction with *(hsiang chien* 相減)" signals that the two quantities may have different signs and that the computist is to adjust the operation (see p. 65). The first two sentences give the general procedure. It applies to the conjunctions of the outer planets, and to those of the inner planets near apogee, when the planet and the sun are moving in the same direction, with the planet's speed larger than that of the sun. One straightforwardly subtracts them.

When an inner planet is moving in retrograde, at or near solar conjunction, its direction is opposite to that of the sun, so one determines the difference in their velocities by scalar addition. For Mercury, Evening Invisibility and Dawn Appearance belong to the grade of Evening Station. Since during those two days, according to the schema, the planet does not move, there is nothing to subtract from the sun's Motion Parts.

A literal translation of the third sentence would begin "In the case of the retrograde motions of the Metal and Water Stars at retrograde conjunction (*jo chin, shui erh-hsing t'ui-hsing tsai t'ui-ho che* 若金, 水二星退行在退合者)...," which does not make sense. The editors of the modern edition point out that the corresponding passage in two of the Yuan system's predecessors reads *"jo chin tsai t'ui-hsing, shui tsai t'ui-ho che* 若金在退行, 水在退合者." The Season-Granting system's reading is patently inferior. I follow *Chin shih,* 22: 517, and the canon of the Western Expedition Seventh-year Epoch system (#82, circa 1221; *Yuan shih,* 57: 1339), which also closely followed the Chin system.

The phrase "Evening Invisibility or Dawn Appearance" in the last sentence is not at all clear, for neither grade is so named in either the Yuan system or its predecessors.

7.15. To Find the Comprehensive Accumulation for Corrected Conjunction, Corrected Appearance, and Corrected Invisibility for the Five Stars

For the Wood, Fire, and Earth Stars, take the Corrected Accumulation (7.4.1) for mean conjunction, dawn appearance, or evening invisibility in days as the Comprehensive Accumulation (7.15.1) in days and ten-thousandth parts for corrected conjunction, invisibility, or appearance.

For the Metal and Water Stars, set up *tu* and ten-thousandth parts of the Expansion/Contraction Difference for the grade being studied.

<Double it for the Water Star.>

In each case divide by the Motion Difference for that grade to yield days. Shift any remainder backward to divide it, and read as ten-thousandth parts. If in mean conjunction, evening appearance, or dawn invisibility, subtract if expansion or add if contraction; if in retrograde conjunction, evening invisibility, or dawn appearance, add if expansion or subtract if contraction. In each case add to or subtract from the Corrected Accumulation to yield the Comprehensive Accumulation for corrected conjunction, appearance, or invisibility (7.15.1).

[The Yuan astronomers took over this step without significant change from their Chin predecessor. The Comprehensive Accumulation is an intermediate quantity used in the next step to derive a corrected value.

For the outer planets and the conjunctions near apogee of Venus and Mercury, the Corrected Accumulation (the apparent position of the planet) serves directly as the intermediate. For the conjunctions near perigee of the two planets, they and the sun are moving in opposite directions at nearly equal speeds. That is why the Expansion/Contraction Difference comes into play. Takebe suggests that its doubling for Mercury is due to the greater daily speed of that planet than of Venus (6: 73a-74a), but there is no simple relationship between velocity and Expansion/Contraction Difference.]

7.16. To Find Corrected Conjunction Corrected Accumulation and [Corrected Conjunction] Corrected Star for the Five Stars

For the Wood, Fire, and Earth Stars, in each case divide the Motion Difference for mean conjunction (7.14.1) into the solar Expansion/Contraction Accumulation (7.3.5) for the first day of the grade. The result is Interval Conjunction Difference Days (7.16.1). Shift any remainder backward to divide it, and read as ten-thousandth parts. Subtract from [the combined result] solar Expansion/Contraction Accumulation, yielding Interval Conjunction Difference Degrees (7.16.2). In each case set up Comprehensive Accumulation for corrected conjunction for the star, and subtract Interval Conjunction

Difference Days if in the phase of expansion, or add it if in the phase of contraction, to yield Corrected Conjunction Corrected Accumulation Days (7.16.3) and ten-thousandth parts for the star. Subtract Interval Conjunction Difference Degrees if in the phase of expansion, or add if in the phase of contraction, to yield Corrected Conjunction Corrected Star Degrees (7.16.4) and ten-thousandth parts.

For the Metal and Water Stars, for progressive and retrograde conjunctions, in each instance divide the Motion Difference for mean conjunction or retrograde conjunction into the solar Expansion/Contraction Accumulation for that day. The result is Interval Conjunction Difference Days (7.16.1). Shift any remainder backward to divide it, and read as ten-thousandth parts. For progressive motion add to, or if in retrograde subtract from, solar Expansion/Contraction Accumulation to yield Conjunction Interval Difference Degrees (7.16.2). For progressive conjunctions, if in the phase of expansion add, or if in the phase of contraction subtract, the Comprehensive Accumulation for corrected conjunction for the star to yield Corrected Conjunction Corrected Accumulation Days (7.16.3) and ten-thousandth parts for the star. For retrograde conjunctions, in the phase of expansion subtract Interval Conjunction Difference Days from, or in the phase of contraction add to, Interval Conjunction Difference Degrees. If in the phase of expansion add [the result] to, or if in the phase of contraction subtract it from, the Corrected Conjunction Comprehensive Accumulation for the star in retrograde to yield Corrected Conjunction Corrected Accumulation Days and ten-thousandth parts for the star in retrogradation. Count it off; the result is Corrected Conjunction Corrected Star Degrees (7.16.4) for retrogradation in *tu* and ten-thousandth parts.

To days and ten-thousandth parts of the winter solstice in the Astronomical First Month (1.1.4) add Corrected Conjunction Corrected Accumulation Days and ten-thousandth parts for the star, casting out complete Decad Cycles (1.C16) and counting off exclusively from s.d. 1. The result thus obtained is the Corrected Conjunction Date (7.16.5) and ten-thousandth day parts.

To *tu* and ten-thousandth parts of the lunar lodge position of the sun on the Yellow Way at the exact time of winter solstice in the Astronomical First Month (3.11.2) add *tu* and ten-thousandth parts of Corrected Conjunction Corrected Star Degrees for the star, casting out complete lunar lodges on the Yellow Way. The result thus obtained is the lunar lodge position on the Yellow Way of Corrected Conjunction (7.16.6) in *tu* and ten-thousandth parts.

<To find Conjunction Invisibility Corrected Days for the Five Stars directly: For the Wood, Fire, and Earth Stars, from the position of the sun on the Yellow Way at midnight in *tu* subtract the lunar lodge position of the star on the Yellow Way at midnight. If the remainder is less than solar Motion Parts for that day, it corresponds to Conjunction Invisibility on that day.

For the Metal and Water Stars, from the lunar lodge position of the star on the Yellow Way at midnight subtract the position of the sun on the Yellow Way at midnight. If the remainder is less than Motion Parts of the Metal or Water Star for that day, it corresponds to conjunction invisibility on that day. In the case of Conjunction Retrogradation Invisibility for the Metal or Water Star, determine by inspection when the lunar lodge position of the sun on the Yellow Way at midnight of the day being studied has not yet reached the lunar lodge position of the Metal or Water Star, but on the next day will have moved past the their lunar lodge position, [or] that the Metal or Water Star in retrograde motion will have passed the lunar lodge position of the sun. The result corresponds to Corrected Conjunction Retrogradation Invisibility Corrected Days (7.16.7) for that day.> {1259–60}

[The title of this step does not convey its purpose, namely to determine the date and location of the apparent planetary conjunction as preparation for computing the dates and locations of invisibility and appearance. In that sense it is analogous to the procedures for apparent lunar conjunction in steps 4.6–4.8. The Corrected Star (7.6.1) is the apparent beginning of each grade.

In view of the great complexity of this step, I see no point in explicating each of the procedures that it includes. The authors revised it considerably, unlike the other steps in this part of the planetary procedure, from the version in the *Chin shih* (22: 518), but it is by and large no clearer. The order of the words "conjunction, retrogradation, invisibility" varies, with no discernible change in purpose; I have translated uniformly to avoid adding to the confusion.]

7.17. To Find Corrected Appearance/Invisibility Corrected Accumulation Days for the Wood, Fire, and Earth Stars

In each case set up days and ten-thousandth parts of the Comprehensive Accumulation (7.15.1) for corrected appearance or corrected invisibility. If dawn add, or if evening subtract, 91 days 3106 parts. If [the result is] less than a Year Semicycle (3.C7), square [the Comprehensive Accumulation]. If greater, subtract it from a Year Cycle instead, and square the remainder. Divide by 75 to yield parts. Read full hundreds as *tu*, and shift any remainder backward to divide it, yielding ten-thousandth parts. Multiply by *tu* of appearance or of invisibility for the star and divide by 15. Divide the result thus obtained by the Motion Difference (7.14.1) for the grade to yield days. Shift any remainder backward to divide it, and read as ten-thousandth parts. If appearance add, or if invisibility subtract, the Comprehensive Accumulation to yield Corrected Appearance/Invisibility Corrected Accumulation Days (7.17.1) and ten-thousandth parts for the star. Add and count off as before. The result is the date of Corrected Appearance or Corrected Invisibility (7.17.2) and ten-thousandth day parts. {1260}

[The first number is half of a Year Semicycle, and a quarter of a tropical year. The purpose of this step is to compensate for an unspecified effect that lengthens the period of invisibility. A historian of science would think of it as geocentric parallax. The constants used imply a maximal value of 4.5 *tu;* actually, the maximum varies from one planet to the next as a function of distance from the earth.

Although this and the next step divide what astronomers in the Greek tradition called the outer and inner planets, there was no such conception in Yuan China. Mercury and Venus were marked off from the other planets primarily because their sidereal period was the same as that of the sun, and in this context because predicting invisibility and reappearance required a different method.]

7.18. To Find Corrected Appearance/Invisibility Corrected Accumulation Days for the Metal and Water Stars

In each case divide the Motion Difference for the day of invisibility or appearance into the solar Expansion/Contraction Accumulation for the first day of the grade to yield days. Shift any remainder backward to divide it, yielding ten-thousandth parts. In the case of evening appearance or dawn invisibility, add if in the phase of expansion or subtract if in the phase of contraction. In the case of dawn appearance or evening invisibility, subtract if in the phase of expansion or add if in the phase of contraction. Add [days and day parts] to or subtract from days and day parts of the Comprehensive Accumulation for corrected appearance or corrected invisibility for the star to yield the Regular Accumulation (7.18.1). If less than a Year Semicycle, read it after the winter solstice. If greater, subtract [the Year Semicycle] and read the remainder after the summer solstice. In either case, if less than 91 days 3106 parts, square the result, or if greater, instead subtract from the Year Semicycle and then square [the result]. If dawn after the winter solstice or evening after the summer solstice, take 1 for each 18 to yield parts. If evening after the winter solstice or dawn after the summer solstice, take 1 for each 75 to yield parts. Then multiply by *tu* of appearance or invisibility for the star and divide by 15. Divide the result thus obtained by full Motion Parts to yield days, and shift any remainder backward to divide it, yielding ten-thousandth day parts. Add to or subtract from the Regular Accumulation to yield the Corrected Accumulation (7.18.2). In the case of dawn appearance or evening invisibility, add if after the winter solstice or subtract if after the summer solstice. In the case of evening appearance or dawn invisibility, subtract if after the winter solstice or add if after the summer solstice. The result is Corrected Appearance/Corrected Invisibility Corrected Accumulation Days (7.18.3) and ten-thousandth day parts for the star. Add and count off as before. The result is the date of Corrected Appearance or Corrected Invisibility (7.18.4) and ten-thousandth day parts.

{1260–61; END OF *YUAN SHIH,* CHAPTER 55, AND OF THE TREATISE}

[A detailed evaluation of this section on the planets will be worth the effort of some future scholar, but its lack of clarity and originality do not justify one in this introductory study. Let me close with a few general observations.

Despite some minor adjustments when the Yuan astronomers adopted the Chin planetary procedures, they left unchanged a great many compromises. These greatly undercut the value of this section. The reliance on mean values, the recourse to linear interpolation, the inattention to any counterpart of planetary latitude, made this section inferior to the rest of the Canon. It became part of the system only because its authors did not give high priority to accurate planetary predictions.

The Expansion/Contraction cycle is in every case one year, in effect based on the sun's period rather than those of the planets. That implies, among other things, that the perigee, the point at which the planet approaches the earth most closely and is therefore midway through its retrograde motion, ought to coincide with the winter solstice. This, as we have seen (p. 261), happens not to be the case, although in the late thirteenth century the two points happened to be very close. But over 350 years of use, as they moved apart, the problem grew.

Finally, for an overview of this section's results, consider the maximum equation of center for each planet:

A = Maximum Equation of Center, *tu (Ming shih,* 34: 640–84)
B = Modern Value (converted to *tu)*
C = Error in A, *tu*
D = Error in A, per cent
E = Eccentricity (Smart 1962: 422)

Planet	**A**	**B**	**C**	**D**	**E**
Jupiter	5.992	5.48	+0.51	+9	0.048 4
Mars	25.620	10.75	+14.87	+138	0.093 4
Saturn	8.255	6.80	+1.46	+21	0.055 7
Venus	2.136	0.81	+1.33	+164	0.006 8
Mercury	2.286	24.05	-21.76	-90	0.205 6
Sun [Earth]	2.401	1.93	+0.47	+24	0.016 7

We can conclude that in every case the error in maximum equation of center is so high that the predictions of apparent phenomena are inferior to those of previous sections of the Canon. They are also far from the best in previous systems.]

11 Conclusion

I will draw a few conclusions about the Season-granting system that the current state of knowledge permits, and end with a handful of thoughts on the place of the reform project in the history of science.

The Problem of Evaluation

In many respects the history of astronomy was cumulative, but its changes in predictive ability were not at all uniformly for the better. Only by overlooking the fluctuations can we see it as a cavalcade of progress.

In China, some changes that contemporaries widely acclaimed as positive turned out to be astronomically retrograde. Others rejected because of savage criticism were first-rate. Pitched battles were not only due to technical disagreements; the importance of the almanac to state ritual made computational systems a natural target for attacks by political factions. Evaluations after the fact by historians carry their own difficulties. This section will consider those difficulties and apply its conclusions to the Season-granting system.

Fifty years ago evaluation was a simple matter. The paramount question for most historians of astronomy was how well ancient knowledge approximated that of today, how much of ancient work gave the right answers, and how much was error. As Joseph Needham phrased his central assumption in 1974, "there is only one unitary science of Nature, approached more or less closely, built up more or less successfully and continuously, by various groups of mankind from time to time." The teleological force of objective modern knowledge, like an immense magnetic field, pulled all the ancient sciences hesitatingly, against the drag of the past, toward that goal.[294] Early actors in this pageant of progress could only blindly further or obstruct a plot they had no power to shape.

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[294] Needham 1954– , 5 part 2: xxi.

Positivism, odd though its suppositions are, has been the dominant mode of assessment within the exact sciences. Ptolemy and the Yuan evaluator agreed with the authors of modern astronomy textbooks that accuracy and precision are all that counts in results. The fallacy lies in letting these criteria crowd out all the others in reconstructing the multidimensional past.

Over the past half century, students of history have moved far beyond such assumptions. They have affirmed that the perceptions and values of past scientists were those of their time and place, in many respects not at all like those that animate exact science today. They have opened to inquiry the questions of how and why change takes place. That reorientation has depended on close attention to what those who did astronomy said or implied they were trying to accomplish, and how their aims were rooted in the particularity of their culture. No explanation in this book has depended on technical inspiration springing spontaneously into the minds of Great Men, or on scientists blundering toward modern techniques. There is no doubt that Wang Hsun and Kuo Shou-ching were remarkable thinkers, but they lived in Yuan China and constantly depended upon others. The concern with cultural manifolds that this book exemplifies is simply an aid to understanding how the Yuan astronomers found their way from their own starting point to the completed Season-granting system.

On the other hand, since ancient astronomers valued and sought to improve both accuracy and precision, no historical analysis that ignores them can be adequate. What I am trying to reconstruct is the complex of social, political, intellectual, and technical relations and meanings within which individual astronomers did their own work.

In order to avoid being deceived by my quite fallible preferences and presuppositions, I regularly evaluate books, projects, and so on in three steps. They are no guarantee against self-deception, but they at least make it harder to ignore.

1. What were the authors or planners trying to accomplish? Putting the question this way avoids the temptation to begin with what I would have tried to do if I were in their place.

2. To what extent did they accomplish this goal?

3. To what extent is it worth accomplishing?

Let me now apply these questions to the Season-granting system.

First, we are fortunate enough to have a general statement of aims and two early lists of what the reform accomplished.

The highly rhetorical introduction to the Evaluation (chapter 7) puts it in general terms: the reformers, aware that no computational system can provide perfect results forever, simply wished to improve on the faltering techniques of the Yuan system's predecessor, the Revised Great Enlightenment system (#76, 1180).

The account of conduct of Kuo Shou-ching (the biography based in part on his own writings, Appendix B) enumerates the new instruments built for the project; the empire-wide observational survey of polar altitudes and other parameters; the seven topics of research to improve fundamental quantities and functions (winter solstice, tropical year length, apparent solar and lunar motions, motion of the lunar nodes, coordinate system, and day and night length); and what Kuo presented as five new methods (for computing apparent solar and lunar motions, for conversion between equatorial right ascension and ecliptic longitude, for determining the obliquity of the ecliptic, and for converting the moon's position on its orbit to ecliptic longitude).[295]

The section headings of the Evaluation (chapters 7–8) amount to a list of the claims for improvement in technique that the evaluators tested. To those just listed, it adds methods for sidereal as well as tropical year length, the inclination of the lunar path to that of the ecliptic, eclipse prediction, and apparent lunisolar conjunctions. It also points out the use of a recent rather than an extremely remote epoch for calculation, and the use of 10 000 as the numerator of all

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[295] To minimize the need for explanation, in this section I list modern equivalents of the original terms.

fractions (as close as possible in the Chinese number system to using decimals throughout).

The editors of the *Yuan History* revised the Evaluation to some extent, but it is still the only critique of any system to survive in close to complete form. Backed up by Kuo's biography, these headings give us—for the first and last time in the history of Chinese astronomy—a comprehensive list of aims. There is good reason to believe that the reformers aimed for more innovation than they attained. Most prominently, the two lists do not even mention the planets, although the astronomers accumulated many observations of them.

Nevertheless, to ignore the other aims of the reformers is to be satisfied with an inadequate account. They meant their work not only "to serve as the foundations of a system" of official astronomy (as the Evaluation put it), but to create a much more encompassing system, namely the dynasty that began with (again in their own words), "the Grand Progenitor's assumption of the Mandate."[296] They were dedicated functionaries of the Mongol regime in North China, and in all of their very diverse careers worked to spread its power to the south, to reunite China. For traditionally educated north Chinese this was a social and moral as well a political goal.

Second, by and large, with the help of organizational support and despite the organizational drag, the astronomers achieved the aims with which the reform began. Their employers were fully satisfied, and they themselves were clear about what they had accomplished. The result was on one level a system that no one significantly improved until the Manchus replaced it with European astronomy—in an obsolescent Renaissance version—at the outset of a new dynasty in 1644. It was the Manchu invasion from the north, not dissatisfaction with the system's accuracy, that ended the official use of the Season-granting system in that year. In Japan and Korea, where versions of the Season-granting system became official later than 1281 (p. 238), its use continued longer.

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[296] See above, pp. 253 and 251.

On another level, the outcome was a regime that fulfilled traditional expectations about its cosmological and ritual basis, to which a state ritual for granting a correct almanac at the new year was essential. In this sense their success was measured by later dynasties' acceptance of the Yuan, despite its non-Han character and its un-Chinese social structure, as the legitimate successor to the Sung and predecessor of the Ming. One reason this happened was because the reform group made possible, through their very diverse careers, a government that remained to some recognizable extent based on the millennial Chinese pattern.

Third, did the astronomers pursue worthwhile aims? Well, what aims were worth while? As for those that affected society, politics, cosmology, and the other dimensions of their project, they were as worth while as any that enabled the continuity of China's brilliant civilization.

As for their scientific aims, two generations ago, in the era of a naive positivist history of science, one hardly needed to ask that question; the only right aims were those advertised for science ca. 1950. But in the astronomy of the early twenty-first century, the prediction of solar, lunar, and planetary phenomena is no longer problematic. One past generation of astronomers after another devoted their working lives to these tasks, but they have long since become trivial: no longer taught in introductory undergraduate courses, solved in a few seconds without calculation by anyone with access to an online database or a free computer application.

The few remaining positivists have simply shifted their stances: the right aims are those of immaculate European astronomy that connect Hipparchus and Ptolemy (via a short but boring detour into the Middle East) with Copernicus, Kepler, and Newton. Ignorance of non-European astronomy is not in their eyes mere ignorance, but a sign of good taste.[297]

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[297] General histories of the field do not put it so crassly, but so far—despite the positive example of its institutional founder, George Sarton (e.g., 1927–48)—they have made no effort to achieve the balance of attention that their titles advertise. Most, from Neugebauer's *The Exact Sciences in Antiquity* (1962) to Evans' *The History and Practice of Ancient Astronomy*

On the other hand, scholars who have investigated all the great world traditions are aware that there is nothing lily white about the origins of modern science. Ptolemy was a resident of Alexandrian Egypt (circa A.D. 100–circa 175) who wrote in Greek; no historian has produced concrete evidence that he was Caucasian. As Europe sedulously rid itself of mathematical learning between A.D. 200 and 600, its advanced sciences found a home first in Nestorian Syria and then in the Islamic world (which before long occupied Spain). There, far from remaining pure-bred Greek, they promiscuously cohabited with the learning of the ancient Middle East, Edessa, India, Central Asia, and China. Beginning circa 1000, a few Christians from northern Europe began carrying home from Muslim Spain Latin translations of ancient learning. To comprehend it, scholars of the far west were entirely dependent for some time on Muslim commentaries and compendia that synthesized all Eurasia's knowledge. By the time Greek manuscripts (in no case intact ones of classical times) reached Europe from Byzantium, the working methods of their recipients had been transformed once and for all by such non-European novelties as Arabic numerals and algebra, Indian trigonometric functions, and Chinese paper and printing.[298]

In short, there is nothing pure about the "Greek impulse" in early modern science. Science—between the earliest Hellenic borrowings of Mesopotamian observational records to the increasing eminence of East Asian mathematicians and computational specialists today—is the outcome of an endless series of territorial, ethnic, and racial mixings.

In assessing the value of the Yuan astronomers' ambitions, then, if we were to ask what they contributed to modern astronomy, the

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(1998), ignore China. Neugebauer feels no need to qualify generalizations such as "Up to Newton all astronomy consists in modifications, however ingenious, of Hellenistic astronomy" (p. 4). Hoskin's recent *The Cambridge Concise History of Astronomy* (1999), at least, gives China one and a half pages out of 362.

[298] Montgomery 2000 is a well informed and detailed account of these peregrinations. Actually Gutenberg's printing enterprise came at almost the same time as the large-scale retrieval of Greek texts from Constantinople.

answer is that we have no idea. The outworn "toward modern science" model confirmed scholars before circa 1980 in their ignorance of the roles of Indian and Chinese knowledge in the gestation of early modern science. Studies across these cultural boundaries are among our most urgent needs. Van Dalen and his colleagues have traced Chinese influence in the Islamic world, and it may well be traceable in Europe, but only research can decide.[299]

For the moment, we need not leave the third question entirely hanging. Since the authors of the Yuan system set out to design a system of unprecedented accuracy, we can at least apply modern mathematics to determine whether we can take seriously their own evaluation. That cannot answer the question of its historical value, but it provides a starting point.

Accuracy of the Season-granting system

Calendrical functions

In the Yuan system, the prediction of solstices, lunations, and other phenomena on which the basic calendar depends are far more accurate than any practical activity requires. For instance, for solstices in the vicinity of the epoch, the system reduced average computational error to between 0.1 and 0.5 marks (1 to 7 minutes) from a previous average of between 1 and 2 marks (10 to 30 minutes). At the same time, the error in observation of the solstices averaged 1 to 4 marks. Previous systems had been generating reliable calendars for roughly a thousand years before the Yuan era, but this was a more than trivial incremental improvement.[300]

Eclipses

As can be seen in the commentaries, the Season-Granting system's eclipse theory was in the aggregate more innovative than

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[299] A promising first step is the study of Chinese astronomical parameters in Persian manuscripts in van Dalen et al. 1997.

[300] The quantitative conclusions come from Ch'en Mei-tung 1995, pp. 75–77. Ch'en discusses in detail the theoretical reasons for limited observational accuracy in this case. Table 7.2 above shows that error in the epochal winter solstice of 1277 was considerably below the average.

most, but some of its predecessors were quite original in one respect or another. The excellence of the thirteenth-century system came from choosing among and combining the best methods available with new techniques where they were needed.

In the accuracy of eclipse predictions, a great many variables converge: the quality of apparent solar and lunar motions, of the obliquity of the lunar orbit with respect to the ecliptic, of timekeeping, and so on. A detailed analysis of how these interact to determine the prediction has yet to be done, but there is no need for such elaboration here. I have compared predictions of solar and lunar eclipses by the Yuan system, by its immediate predecessor, and by modern computation, in the tables and commentaries of the Evaluation, sections 9.3 and 9.4. Again they demonstrate that the Season-granting system indeed attained the clear superiority in accuracy that the Evaluation claimed.

Planets

There is no important innovation in the planetary technique of the Season-Granting system, leaving aside some revised constants, and the use of third-order interpolation that characterized the whole system. We have seen that the planetary procedures, even their words, were largely copied from the Revised Great Enlightenment system of 1180 (#76). The most satisfactory explanation for the scanty innovation is that the astronomers could not give the planetary technique high enough priority to merit more than perfunctory effort (see chapter 1, p. 32). My commentary on the planetary section of the Canon summarizes its failings. It is no surprise that the Yuan system's planetary predictions do not rank high in accuracy among those from A.D. 900 on. On the other hand, the Evaluation made no claim to the contrary.

On the question of accuracy in general, we can conclude that the Evaluation gave a fair picture of the Season-granting system's strengths. One may differ with one detail or another, but the document makes a good case for the value of the reform.

A Final Word

The precision and accuracy of the Season-granting system are important topics, but they do not merit the last word. Given our ignorance, it will be some time before the results of investigations can make possible a textured estimate of the Yuan astronomers' accomplishment. We can, at least, reach some conclusions about the system's place in the history of astronomy, Chinese and worldwide.

The Yuan astronomical reform took place at the end of a long and destructive war, which ironically made unprecedented support available to an exceptional team of astronomers for a project limited only by the time available. It drew on a thorough study of its predecessors, on new instruments of the first quality, and on important computational novelties. Its most telling limitation was in planetary prediction, which had relatively low priority. Another was the unwieldiness of its numerical approach, even with a proto-trigonometry, for solving what are a great deal easier to formulate as problems in solid geometry. Nevertheless, in China this was the most mature system of its tradition, not noticeably bettered for 350 years.

As for the system's place in world astronomy, the sophistication of the treatise makes studying it essential if we are to understand the place of computational science in the great civilizations. It is comparable to documents elsewhere in what we can learn from it about the possibilities of astronomy in its time and place. Its story of inquiry driven by a bureaucratic project of larger dimensions than was conceivable elsewhere is one example. The strengths and limits of numerical vs. geometric methods that I have just mentioned is another. China's rich records can throw light on the intermarriage of various technical traditions in the Islamic world that eventually made possible the emergence of modern astronomy in Europe. Chinese astronomy, in any open-minded history of astronomy, will reward attention commensurate with its importance. The Season-granting system has allowed us a close look at the high point of that tradition, which spanned without interruption nearly two thousand years.

Appendix A The Instruments of Kuo Shou-ching

These fairly detailed descriptions of eight important instruments discussed in chapter 5 come from the "Treatise on Astrology" of the *Yuan History.* I have offered evidence in chapter 5 that these accounts are based on the specifications originally proposed, and that the instruments finally built differ from them in important respects. Nevertheless, since none of the instruments made for the Ta-tu observatory has survived, the dimensions and other features given below are essential to understanding how they worked and why Kuo designed them as he did.[301]

The Simplified Instrument

In this instance we have a detailed description:

> The design of the Simplified Instrument: At the corners are pedestals (*fu* 趺). The instrument is 18c long and a third less [i.e., 12c] in width. The faces of the pedestals are 0.6c wide at the top and 0.8c wide at the bottom. Their thickness is the same as the upper width. Within the instrument are three longitudinal and three transverse bars. To the south there are two [longitudinal bars], which extend northward to rest against the southernmost transverse bar. To the north there is one, which extends southward to the middle transverse bar. Around the face of the pedestal is a water channel 0.1c deep {991} and 0.05c wider [i.e., 0.15c wide, for leveling the instrument]. At the four corners there are bases that extend 0.2c inside and outside the pedestal. There are also water channels around the bases, 0.1c in both depth and width, connected to the surrounding channels. There are bases too at 3 o'clock and 9 o'clock [i.e., at the midpoints of the long sides]. Their width is 4 *wei* greater, and their length 2/3 greater than the width, with similar water channels.
>
> [*Kuang chia ssu wei* 廣加四維. *Wei* is not a standard measure, and the usual meanings of *ssu wei*, the four cardinal points or their midpoints, are inapplicable

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[301] *Yuan shih,* 48: 990–98. See page numbers in curly brackets. Kuo's biography (Appendix B, p. 574) also briefly characterizes the set.

here. Wylie, who saw the copy of the instrument and may have seen the original, takes the width to be 1.4c and the length, consequently, 2.3c. I can offer no better conjecture than his.]

There are two standards for the cloud support at the north pole, 0.4c in diameter and 12.8c long. Their lower parts are [pedestals with] tortoise-and-cloud ornamentation, resting on the bases at the northern corners. From left and right the standards face inward [that is, converge toward each other]. Their inclination conforms to that of the Red Way [i.e., the celestial equator], and they meet at the upper orbit[al circle, which marks the north pole].

The orbital circle *(kuei huan* 規環, D) is 2.4c in diameter, 0.15c wide, and twice as thick. Inside it are stretchers *(chü* 距) which intersect in a slanting cross. Its width and thickness are the same as that of the orbital circle. In its center is an opening 0.05c wide and 0.15c square at the top, and at the bottom 0.25c wide and 0.1c square, so that it holds the axial pivot of the north pole. [The support] ascends obliquely from the pedestal with cloud ornamentation. At a distance of 7.2c from the pedestal is a transverse bar (L). From its center, it is 6.8c to the center of the opening.

[The letters I have added are keyed to figure 12, p. 251.]

Two more dragon pillars rest on the bases at 3 o'clock and 9 o'clock, to the north of the centers of the bases. They are decorated with dragons above and the shapes of hills below. They rise obliquely northward to support the northern framework [that is, they intersect the standards at the transverse bar].

There are two support pillars with cloud ornamentation for the south polar [opening]. They rest to the south of the centers of the bases at 3 o'clock and 9 o'clock. Their widths, thicknesses, and forms are identical to those of the northern supports [on the same bases]. Slanting toward the southeast and southwest corners, they intersect to form a cross, the upper ends of which are flush with the southeastern and southwestern edges of the hundred-mark circle *(pai k'o huan* 百刻環, A). Their oblique southward situation conforms to that of the Red Way. Each is 11.5c long.

[The crossed pillars directly support the fixed diurnal circle (A), on which the mobile one (B) revolves.]

At 3.8c above the surface of the pedestal is a transverse bar that holds up the hundred-mark circle. Below [the circle] there are two more dragon pillars, which rest on the southeast and southwest bases. They slant northward [to support the ends of the crossed] pillars. The configurations of their ends are identical to those of the northern supports.

[The last sentence is ambiguous. It may also mean “their ends and configurations are identical … “]

The four excursions double circle *[ssu yu shuang huan* 四游雙環, C, a double meridian circle bearing a sighting bar across its diameter] is 6c in diameter, 0.2c wide, and 0.1c thick. There is a space of 0.1c between the circles, and [spacers] join them at north, south, east, and west. Along the north-south line there are round openings to accommodate the axial pivots of the north and south poles. The degrees of the Celestial Perimeter are set out on both faces of the circles. They begin at the south pole and proceed to the north pole, where the excess parts are added on.

[The “excess parts” are the fractional parts of Celestial Perimeter or sidereal year circle of 365^{t}.257 5. Wylie’s “old minutes” is probably a typographical error for “odd minutes.”]

At a distance of 0.4c on both sides of the axial pivots are straight stretchers of the same width and thickness as the rings. At the centers of the stretchers are transverse crosspieces *(heng kuan* 橫關), joined at their eastern and western ends to the stretchers, and of the same width and thickness. The centers of these crosspieces are joined to form a thickness of 0.3c, in which there is an opening 0.08c square to accommodate the axial pivot of the sighting bar *(k’uei heng* 窺橫, J).

The sighting bar is 5.94c long; its width and thickness are the same as those of the circles. At the waist [i.e., center, of its length] is a round opening of 0.05c to accommodate the axis on which it pivots. The two ends of the bar are pointed like a ritual scepter *(kuei shou* 圭首), enabling one to read the intervals between gradations [on the circle]. At a distance of 0.05c from the pointed ends are perpendicular transverse ears *[ts’e li heng erh* 側立橫耳, sighting vanes] 0.22c high, as wide as the face of the circle, and 0.03c thick. At the center of each is a round opening with a diameter of 0.06c. The bar is divided lengthwise by a wire [*hsien* 線, aligned with the sighting apertures] along which one reads *tu* and parts.

[The word I have translated “reads” is *chih* 知, literally “knows.”]

The hundred-mark circle is 6.4c in diameter, and its face is 0.2c wide. Along its circumference are set out the twelve double-hours and the 100 marks, each of the latter divided into 36 parts. It is 0.2c thick, and from halfway up it is 0.3c wide. There are also the crossed stretchers [mentioned above], provided to support the Red Way circle [*ch'ih tao huan* 赤道環, the mobile equatorial circle, B]. Along the breadth of the hundred-mark circle's face are placed four recumbent rollers *[yuan-chou* 圓軸, i.e., roller bearings]. This allows the Red Way circle to revolve without being impeded by roughness. The rollers are recessed 0.1c into the south polar framework and transfixed in place.

["Roughness *(se-chih* 澀滯)" evidently refers to friction.]

The Red Way circle is identical to the hundred-mark circle in diameter, width, and thickness. On its face are delicately engraved the lunar lodge demarcations *(lieh she* 列舍) and the *tu* and parts of the celestial perimeter. Within it are the crossed stretchers, 0.3c wide, with an empty space of 0.1c, and 0.1c thickness. At its center, the opening 0.1c in diameter is to accommodate the axial pivot for the south pole.

There are two bounding crosspieces *[chieh-heng* 界衡, pivoted sighting bars, H and I], each 5.94c long and 0.3c wide. The ends [of the sighting bars] taper to 0.5c [wide], and *tu* and parts are engraved to correspond with those on the face of the circle. In their waists are openings by which to attach them to the Red Way circle and the south polar pivot.

The two ends of the sighting bars on this circle, from the outside of the sighting apertures to the end of the sights' [pointed] tips, are twice as thick as the rest. Thus the two sights rotate on the face of the circle without wobbling up and down, making it easier to read *tu* and parts.

[The photograph of the corresponding sight on the fifteenth-century reproduction (figure 17) does not closely correspond to this description. For a necessarily detailed attempt to work out the original form, see Ch'en Mei-tung 2003b, 131–33.]

The axial pivots of the poles are made of steel, and are 0.6c long. Half of this is the shaft, and half the pivot. The dimension of the shaft is identical to that of the [openings at the] center of the orbital ring's stretchers, leaving just enough space for pivots of 0.1c diameter. There is a hole *(k'ung* 空) in the center of the north polar axis, pierced transversely at its bottom through both sides. From the center extends a wire that winds round the shaft [? *ch'ü ch'i pen* 曲其本] and emerges from the ends of the transverse hole, where it is fastened. Of the wire, three tenths remain in the hole, and are also fastened there. These wires pierce from top to bottom, passing through the ends of the bounding crosspieces. In the centers of the crosspieces are holes that extend to their bottoms. Channels are prepared along them to accommodate the wires. The latter directly enter the long openings in the inner boundaries. When they reach the waist, there are other holes. There [the wires] emerge from the underside of the crosspiece and are fastened in place.

[This translation is highly uncertain, since the text is ambiguous and the wires were either not incorporated in the Ming reproduction of the instrument or have not survived.]

The Pole-determining Circle *(ting chi huan* 定極環, E) is 0.05c wide and twice as thick, configured to correspond to the celestial vault.

[This apparently refers not to shape—which does not correspond—but to the centering of the circle on the true celestial pole.]

Its center subtends a diameter of 6 *tu,* with 1 *tu* corresponding to 0.1c or so. The pole star is 3 *tu* distant from the unmoving point of the sky, so the circle encloses just enough space for the star's rotation. Inside it are slanting stretchers in the form of a cross, their width and thickness the same as that of the circle. [The north-south stretcher is] connected to the orbital [circle] above. In the center of the stretchers' intersection is a hole, 0.005c in diameter. From this to the center of the north polar pivot is 0.65c.

A bronze plate is also installed and connected to the crossing point of the south polar cloudy framework, 0.2c square and 0.05c thick. Its northern face is beveled toward its center, where a thickness of only 0.001c remains, and in it there is a round hole of 0.01c diameter. From this to the center of the south polar pivot is also 0.65c.

There are two more circles. One, the [fixed] yin azimuth circle *(yin wei huan* 陰緯環, F), has the compass directions engraved on its face. It is horizontally emplaced, centered on the horizontal crosspieces in the northern part of the pedestal. The other, the standing revolving circle *(li yun huan* 立運環, G), has *tu* and parts engraved on its face, and is situated below the cloud-ornamented pillar [i.e., the transverse bar] of the north pole. It [pivots] on the center of the horizontal circle. At the top it connects to the horizontal bar of the framework, and at the bottom it reaches the pedestal's crosspieces. There are axial pivots at top and bottom so that it can rotate.

Within it is a vertical stretcher with an opening in its center in order to emplace the sighting bar (K) so that it can move upward and downward for observing the altitude of the sun, moon, and asterisms above the horizon.

The four excursions circle mentioned above rotates between east and west. [Its sights] can move upward and downward in order to measure *tu* and parts from the pole to the sun, moon, planets, the lunar lodge determinatives *(lieh she* 列舍), and the inner and outer celestial offices *[chung wai kuan* 中外官, constellations within and outside the circumpolar region]. The Red Way circle revolves to line up the lunar lodge determinative stars; then one lines up the two wires of the bounding crosspieces to measure the [locations in] *tu* and parts of the sun, moon, planets, and celestial offices among the lunar lodges.

For the hundred-marks circle, when one turns the bounding crosspieces so that the two wires line up with the sun, they point out the time in double-hours and marks for daylight marks. One determines night marks by use of the stars. Thus, comparing this to older instruments, [this one] does away with the interference of equatorial circles and cloud pillars in measuring the risings and settings of the sun, moon, and planets.{990–93}

The Celestial Globe

The design of the Celestial Globe (literally, "armillary counterpart, *hun hsiang* 渾象"): it is round as a crossbow pellet, 6c (1.5m) in diameter, and the *tu* and parts of the Celestial Perimeter are marked on it longitudinally and latitudinally. The Red Way [i.e., the equator] occupies the middle, distant from the two poles by a quarter of the Celestial Perimeter. The Yellow Way goes in and out of the Red Way, in each case by 23 11/12 *tu* [=23°.57]. The moon travels on the White Way, its distance [from the Red Way] irregular.

[The Red Way is the celestial equator, the Yellow Way the ecliptic, and the White Way the lunar orbit. "Going in and out" refers to the obliquity of two orbits. Although the angle of the moon's path to the ecliptic is constant, determining that of the lunar orbit with the equator is complicated. See the Canon, 4.13 and 4.14.]

One uses a bamboo strip divided regularly into celestial degrees to work out its intersection with the Yellow Way, moving it about as time passes. First one uses the Simplified Instrument to observe the number of *tu* of [the moon's] location in the pertinent lunar lodge and of its polar distance. One sets [these values] with the strip and then determines the [moon's] location with respect to the Yellow and Red Ways. Ascertaining both distance and precision is then quite easy. Checking [the result] by computation is standard.

The sphere is installed in a rectangular cabinet, with the north and south poles 40 5/6 *tu* (40°.25) above and below its top surface, so that half of the globe is visible and half invisible. The gears responsible for the mechanism's motion are hidden within the cabinet.{993}

[The sphere was installed to show visibility at the latitude of Ta-tu, present Beijing. There is no further information about the mechanism that drove the globe. Many precedents suggest that it was water-powered; see especially Needham et al. 1960, 133–42.]

The Upward-facing Instrument

The discussion is almost entirely devoted to reproducing (not altogether accurately) the contemporary metrical inscription for the instrument by Yao Sui 姚燧 (1201–78). Yao aimed for literary elegance—in this instance, at the expense of clear and useful information. Therefore, rather than translate the inscription at length, I summarize it and translate its brief prose introduction in chapter 5 (p. 198).

The Lamp Water Clock for the Palace of Supernal Brightness

I have not translated this description. This clepsydra was not part of the reform project, or directly connected with the observatory. There is a good English version in Needham, Wang & Price 1960, 135–36.

The Direction-determining Table

The Direction-Determining Table is 4c square and 0.1c thick. On all sides, 0.05c from the edges, runs a water channel. [To prepare the table], one first determines the center and then draws a cross, its arms extending as far as the water channels. One draws an orbital ring 0.1c from the center. From the outside inscribe rings *(kuei* 規)

0.1c apart, for a total of nineteen rings. Then, 0.03c inside the outer ring, draw a double ring and in it set out the *tu* of the Celestial Perimeter. Inside [the nest of circles] make a disk *(yuan* 圓) 0.2c in diameter and the same height. At the center, attached to the bottom [fitted into a hole?], install a post [i.e., a gnomon] 1.5c high. At the summer solstice use one 0.5c shorter, and at the winter solstice one twice as long.

[Whether twice as long as the 1.5c post or the 1c post is unclear. The longest post could thus be 2c or 3c. This ambiguity is best resolved by an experiment.]

To correctly orient [an instrument to the four directions], place the table on a flat site, pour water into the channels, and level it visually. Then set up a post at its center. When the shadow of the post shifts westward inside the outer ring, mark with ink [where] the shadow [of its top intersects the ring]. Each time it shifts a little, again mark [its intersection with a ring]. Do this for all the circles until [the shadow] has shifted eastward beyond the outermost one. Measure the lines connecting the [two] intersections with each ring with a cord and fold it in half to find the midpoints. When the marked [midpoints] line up with the post, and the [day's] shadow is shortest, the instrument is correctly oriented north and south. When one has reviewed the markings for all the rings and worked out the correct north-south orientation, the east-west orientation, as a consequence, will be correct.

[Shortly] before and after the solstices, as the sun moves from east to west [above] the rings, the variation in north-south extension [of the shadow] is minimal. Nevertheless, one can use [the points] where the shadow moves in and out of the outer ring to establish an east-west line, which makes a correct orientation possible. [Shortly] before and after the equinoxes, as the sun moves from east to west, the variation in the north-south length is maximal, so that in some instances the morning and evening intersections with the outer ring are unreliable. Then one must take [the intersections with] the rings near the central one to depend on for the determination, and collate the results from a series of days to get a truer result.

Another method useful for observation: First, determine by sighting how far the northern pole is above the earth, that is, the number of *tu* upward from the leveled table. If the number of *tu* runs downward, [it corresponds to] how far the southern pole is below the earth. Use the inkline to mark a line [from the corresponding gradation on the outer circle] obliquely through the center, as well as a horizontal line crossing it, which marks the obliquity of the equator in the belly of the sky *(t'ien fu ch'ih-tao hsieh shih* 天腹赤道斜勢). Then turn the table up on its side and suspend [from the center] a line perpendicular [to the horizon line, i.e., a plumb line], to determine the correct orientation.

This is the standard procedure for emplacing instruments.{995–96}

The Template and Gnomon

[This section is easily misread because it uses the term *kuei-piao* in two distinct senses. The problem is that *piao* can mean a gnomon, a graduated scale, or something equipped with a scale. The most common sense of *kuei-piao* is "gnomon," as used in the title of this section and elsewhere. But that cannot be the meaning in the first sentence, for the gnomon is not made of stone.]

The template scale *(kuei-piao* 圭表) is made of stone. It is 128c long, 4.5c wide, and 1.4c thick. Its platform is 2.6c high. At the northern and southern ends are water reservoirs with a diameter of 1.5c and 0.2c deep. [The northern one is] one foot north of the gnomon, and is vertically in line with the center of the gnomon's crosspiece. For the outer 120c [of the template], the center part is 0.4c wide, with sections on either side of it 0.1c wide, graduated in *ch'ih,* tenths and hundredths, up to the northern end. On each side, 0.1c [from the edges,] are water channels, 0.1c wide and deep. The water in the northern and southern reservoirs fills them to permit keeping [the template] horizontal.

[An alternate understanding of "*ch'ih,* tenths and hundredths" is "*ch'ih* and tenths."]

The gnomon *(piao* 表) is 50c long, 2.4c wide, and half as thick. It stands upright in the stone platform at the southern end of the template *(kuei).* It extends 14c into the platform and the earth [below it], so that the part above is 36c high. On both sides of its [upper] end are dragons. [The lower] halves of their bodies are attached to the gnomon, and they hold aloft a horizontal crosspiece *(heng liang* 横梁). From the center of the crosspiece to the head of the gnomon is 4c, so that down to the face of the template is a total of 40c. The crosspiece is 6c long and 0.3c in diameter. Atop it is a water channel for keeping it horizontal. At the two ends and in the waist at the middle [of the crosspiece] there are horizontal holes 0.02c in diameter with iron [rods] 0.5c long piercing them. From cords tied to these which meet in the middle hangs a metal weight for keeping [the gnomon] perpendicular and for preventing tilting due to subsidence.

["Cords tied" may also mean "wires fastened."]

Note that when a gnomon is shorter, the gradations [of the scale] are more crowded, so that the finest divisions between them are not easy to discriminate. When it is longer, the gradations are somewhat further apart. What makes [the latter arrangement] difficult is that the shadow becomes empty and indefinite, so that it is hard to catch a solid shadow. Our predecessors, wishing to find something truly solid within the empty shadow, have sometimes included a sighting tube, sometimes set up a small gnomon [on the template scale], sometimes made wooden dividers (? *kuei* 規) for the purpose. The reason for all of them is to catch the sunlight at the end [of the gnomon] as it falls on the surface of the scale *(piao mien* 表面).

[The problem is that a taller gnomon casts a fuzzier and lighter shadow than a shorter one; the text uses "empty" as an antonym of "solid." The expedients of earlier astronomers measured the actual shadow that the top of the gnomon cast.]

Now we will make a gnomon of bronze, 36c high, its [upper] end fitted with two dragons that hold up a horizontal crosspiece. From [the crosspiece] down to the surface of the scale is a total of {997} 40c, so that it is five times [the height of] an 8-*ch'ih* gnomon. The template scale is engraved with *ch'ih* and tenths, but what was formerly 0.1c has now been expanded fivefold, so that its fine subdivisions are easy to distinguish.{996–97}

The Shadow Aligner

The design of the Shadow Aligner (*ying-fu* 景符): it uses bronze leaf, 0.2c broad, and twice longer than its width.

[The Chinese wording of this sentence is not at all clear. It seems to mean that the plate is 0.6c long; but "twice as long as," meaning 0.4c long, is another possible understanding. *T'ung* 銅 can refer to copper or any of its common alloys; bronze is merely the most commonly used in China.

This description almost coincides with that at the beginning of the Evaluation (p. 255).]

In its center is pierced a hole the size of a needle['s point] or a mustard seed. A rectangular frame serves as a base. At the [lower] end there is a hinge that allows it to be opened and closed. One may [thus] prop up the shadow aligner so that it slants with the end toward the north higher than that toward the south [i.e., set it perpendicular to the sun's rays]. One moves the device back and forth within the empty [i.e., diffuse] shadow until the pinhole catches the sun's light, [forming an image] the size of a rice-grain or so, with the crosspiece indistinctly visible in the middle of it.

[There may be something wrong with the word *yin-jan* 隱燃, which I have translated "indistinctly," since the image of the crosspiece is more distinct than its shadow. The version of the last sentence in the Evaluation is superior to this one, and I follow it here.]

The old methods involved observing the shadow of the end of the gnomon, so that what one obtained was the shadow cast by the upper edge of the body [i.e., the upper limb] of the sun. Now one takes it from the horizontal beam, so that one actually obtains the shadow of the sun's center [i.e. the silhouette of the crosspiece]. There cannot be an iota of inaccuracy.

A shadow length near the summer solstice of year 16 of the Perfectly Great period, s. y. 16 (15 Jun 1279), on the 19th of month 4, s.d. 32 (30 May), was 12.3695 feet long. One near the winter solstice (14 Dec 1279), on the 24th of month 10, s.d. 35 (29 Nov), was 76.7400 feet long.{997}

[The author took these two measurements, examples of the precision the device permits, from the Evaluation, sections 1.4 and 1.5, where each is one of a series.]

The Observing Table

The design of the Observing Table *(k'uei chi* 闚几): It is 6c long, 2c wide, and twice as high [as it is wide, i.e., 4c]. Below [the top surface] is a pedestal, 0.3c wide and 0.2c thick. The framework above them is 0.4c wide and as thick as the pedestal. The surface is made of wooden boards a full 0.1c thick [? *hou chi ts'un* 厚及寸; fastened to the framework], with legs at the four corners. [They are] supported by diagonal wooden [braces], for it is essential that [the structure] remain exactly rectangular. In the center of the surface is an opening for the light [? *ming ch'iao* 明竅], 4c long and 0.2c wide. [Strips] within 0.1c [of the two edges] are divided into *ch'ih,* and 0.3c inside, they are inscribed with fine divisions to correspond to those on the shadow template. From the surface of the table to the top of the crosspiece vertically is 36c; this is taken as the standard.

[The text reads *"erh-shh-liu ch'ih* 二十六尺," i.e., 26c, but that is an obvious copying error for *"san-shih-liu ch'ih* 三十六尺."]

There are [two] observation delimiters *(k'uei-hsien* 闚限), each 2.4c long and 0.2c wide. Their spines are 0.05c thick, and both edges are beveled.

Choose a place on the surface of the table that corresponds [to the sight line past the crosspiece to the celestial object], and place the delimiters at its two extremities.

Leave 0.2c to correspond to the thickness and width, {998} and constrain the [sight line's] access to the framework. Wait until the celestial object is [vertically] centered [i.e., at its upper transit of the meridian], and sight upward from beneath the table. Observe and note the northern and southern [sight lines] across the gnomon's crosspiece. Take the measurement halfway between the two as the direct shadow *(chih ying* 直景). Then determine the shadow magnitude (? *ying shu* 景數) by observing the body at its greatest distance on the same day [i.e., at the lower transit]. [Use the result] to compute the altitude of the celestial object.{997–98}

[For a discussion of this instrument and its uses, see p. 189. "Celestial object" is literally "star or moon, *hsing yueh* 星月." Astronomers commonly used *hsing* for the planets as well as the fixed stars.]

Appendix B The Account of Conduct of Kuo Shou-ching

The "account of conduct" *(hsing-chuang* 行狀) is a document that the family or disciples of a deceased person of high status submitted to the palace historiographical office for possible use in an official biography. It normally combines a biographical narrative with a sample of the subject's writing—often literary or official documents. The largest component of this one is a long memorial.

The author of Kuo's account, Ch'i Lü-ch'ien 齊履謙 (1263–1329), was an outstanding astronomer, a polymath, and a learned devotee of Chu Hsi's 朱熹 philosophy. He was one of the first students in the Astrological Commission, and assisted Kuo in compiling the records of the reform. Like him (and undoubtedly with his support), Ch'i earned distinction in the Astrological Commission and eventually high honors in the general civil service.[302]

This translation is based not only on the account of conduct but on the life in the *Yuan History.*[303] The latter, Kuo's official biography, follows the account of conduct fairly closely, but with cuts and a good deal of minor stylistic revision. I have added section headings.

This is not a complete translation. I have omitted a number of long technical passages on Kuo's activities in hydraulic engineering, and a description of the Simplified Instrument that goes over the same ground as the more detailed account in appendix A.

Biography

> The name of the Associate Director of the Astrological Commission was Kuo Shou-ching. His style-name was Jo-ssu 若思 ("as if in thought"). He was a native of Hsing-t'ai in Shun-te Circuit 順德邢臺

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[302] Biography in *Yuan shih,* 172: 4028–32.

[303] *Yuan wen lei* 元文類, 50: 715–23 and *Yuan shih,* 164: 3845–52. The biography by Mei Wen-ting 梅文鼎 in *Ta-t'ung li chih* 大統曆志, 8: 35a–41b adds nothing new. See also the excellent modern biography by Ho Peng Yoke in Rachewiltz 1993, 282–99.

(in modern Hebei). From the beginning his personal integrity was exceptional, and [as a child] he did not engage in play. His grandfather Kuo Jung 郭榮, whose sobriquet was The Old Man of Mandarin-Duck Stream *(Yuan-shui weng* 鴛水翁), had mastered the five orthodox classics, and was greatly accomplished in mathematics and water control.

At the time, the Grand Guardian Liu Ping-chung (T'ai-pao Liu Wen-chen-kung 太保劉文貞公 = 劉秉忠), the Assistant Director of the Left Chang Wen-ch'ien (Tso-ch'eng Chang Chung-hsuan-kung 左丞張忠宣公 = 張文謙), the member of the Bureau of Military Affairs Chang I (Shu-mi Chang kung I 樞密張公易), and the Admonisher Wang Hsun (Tsan-shan Wang kung Hsun 贊善王公恂), were studying together at Purple Gold Mountain, west of the prefectural capital. Liu was also a comrade of Kuo Jung, so Kuo Jung sent Shou-ching to study with him.{715}

[It is conventional in such a biography to refer to important characters, even when children, by high titles that they later attained. The title given for Kuo was not his highest, but the last that he held.]

Water Control

[Here, a lengthy passage that I omit discusses the skill in water control that Kuo developed early in life. This led to a local reputation by the time he was capped (normally at the age of roughly nineteen years). Before long, Liu presented him to Khubilai, then young and just developing an interest in the conquest of South China. This section details Kuo's proposals for hydraulic projects to the future emperor.]

… In 1265 he was appointed Vice Director in the Directorate of Waterways. … {715–16}

New Astronomical Instruments

In 1276[, after the victory over the Sung,] the Directorate of Waterways was incorporated into the Ministry of Works. Kuo was appointed director of a bureau in the Ministry. In this year the government established the office [for the astronomical reform]. Kuo's responsibilities shifted to work on the new astronomical system.

Earlier, Liu Ping-chung became aware that the Great Enlightenment system, used successively by the Liao and Chin dynasties for more than two centuries, was gradually running behind the sky [that is, its predictions were consistently late]. He proposed correcting it, but then he died [in 1274. In 1276, with] South China pacified, the emperor gave thought to adopting Liu's proposal. A memorial [sent to the ruler for approval] had Kuo and Wang as subordinates, leading astronomical officials of the north and south [i.e., of the Sung and Yuan governments], dividing up responsibility for observation and computation, all subordinate to Chang Wen-ch'ien and Chang I, who were in charge. It also proposed that the former Assistant Director of the Left in the Department of State Affairs, Hsu Heng 許衡, who was capable of elucidating the underlying principles of the ephemeris, be assigned to take part as a consultant.

[The "more than two centuries" actually included three Great Enlightenment systems (#62, used by Liao 994–1126, #72, used by Chin 1127–80, and its revised version, #76, used by Chin 1180–1234 and by pre-dynastic and dynastic Yuan 1234–81). The biographer refers to the latter.]

Kuo began [this memorial] by saying that the foundation of mathematical astronomy is observation. Among observational instruments, none is more important than armillary spheres and gnomons (*i piao* 儀表). The armillary sphere now in the Directorate of Astronomy was made in the Sung period, between 1049 and 1054, in Pien [the Sung capital Pien-liang 汴梁, modern Kaifeng, Henan]. Its gradations did not correspond to those of the present location [Ta-tu 大都, the capital, modern Beijing]. Comparing the altitude of the pole for the two locations, south and north, it differed by about 4 *tu.* The stone [base] of the gnomon, with age, also had tilted. He studied its flaws thoroughly and moved it [back] into place. He also planned an elevated, dry location on which to erect a double wooden fence, and within it to build a Simplified Instrument (*chien i* 簡儀) and a Tall Gnomon (*kao piao* 高表) to serve as a standard for testing (? *yung hsiang bi fu* 用相比覆).

[The last phrase might instead mean "to compare [their results]"; but the Yuan astronomers, so far as we know, did not compare them. On the instruments, see appendix A. The point of this paragraph is that Kuo made the old instruments usable again, and planned superior new ones.]

Because, he was convinced, the pivot of the sky rotates [literally, "moves"] with the pole [star] attached [but not coincident], and the ancients who set out sighting-tubes to observe the latter could not aim them correctly, Kuo made a Pole-Observing Instrument (*hou chi i* 候極儀). Because once the polestar and the pole were in their proper places, he had a standard for the body of the sky, he made a Celestial Globe *(hun-t'ien hsiang* 渾天象). Because, although the shape of the celestial globe was correct, it was not adapted to his uses, he made an Ingenious Instrument *(ling-lung i* 玲瓏儀). Because using the gnomon's right angles to observe the perfect sphere of the sky is not as good as using a sphere to find a sphere, he made an Upward-facing Instrument *(yang i* 仰儀).

[Note the shift in this paragraph from instruments planned to those later made. The version of *Yuan wen lei* cited mispunctuates at the beginning of the last sentence.]

In antiquity there were horizontal and vertical meridians which formed an unvarying network, but Kuo changed this and made a Vertical Revolving Instrument *(li yun i* 立運儀). The sun has its central path, and the moon its nine motions; Kuo united them and made an Adjusting Instrument *(cheng-li i* 證理儀). Because the higher the gnomon, the vaguer and less exact its shadow, he made a Shadow Aligner *(ying-fu* 景符). Because, although the moon has its light, observing its shadow is difficult, he made an Observing Table *(k'uei-chi* 闚几). Because the verification of an astronomical system rests on Crossing Coincidences [i.e., simultaneous passages of the sun and moon through the lunar nodes], he made an Instrument for Solar and Lunar Eclipses *(jih-yueh-shih i* 日月食儀). The sky has its Red Way; a circle serves in its place. The two poles rise and sink below it, with gradations marking [positions]; thus he made a Star-dial Time-determining Instrument *(hsing-kuei ting shih i* 星晷定時儀). The instruments listed above comprise thirteen kinds.

[This is a list of the instruments that Kuo designed to carry out the observations for the Yuan reform. Some were actually components of others, such as the shadow definer of the great gnomon.

I have generally followed the translations of their titles in Needham et al. 1954– , 3: 369–70 and its figures 163–66. That discussion did not use the account of conduct, but is based on the derivative account in the *Yuan shih,* 164: 3847). The authors also consulted the detailed descriptions of five of the instruments that I have translated in appendix A. But their "verification instrument" mistranslates *cheng-li i,* overlooking the point that the instrument included rings for the ecliptic and the lunar orbit—which is all we can conclude about its design. As for the statement that "the verification of an astronomical system rests on Crossing Coincidences" (that is, on solar eclipses), see the Canon, section 6. Needham et al. treat the star-dial time-determining instrument as two mechanisms, but see p. 205 above. How Ch'i arrived at a count of thirteen here is unclear, so the count does not resolve the question.]

In addition, he made a Direction-determining Table *(cheng-fang-an* 正方案), a Ball Gnomon (*wan-piao* 丸表), a Suspended [vertical] Standard Instrument *(hsuan cheng-i* 懸正儀), and a Mounted [horizontal] Standard Instrument *(tso cheng-i* 座正儀), four kinds in all, for the use of the observers who traveled to various locations. He also made a diagram with a compass facing upward and a trysquare facing downward (? *yang kuei fu chü t'u* 仰規覆矩圖), a diagram showing various places according to different models of the cosmos (? *i fang hun kai t'u* 異方渾蓋圖), a diagram showing times of sunrise, sunset, and day length *(jih ch'u ju yung-tuan t'u* 日出入永短圖), five in all, usable for reference with the instruments listed above.

[How the second group of diagrams adds up to five—that is, which one includes more than one illustration—is also not clear. It is possible that these are books rather than single diagrams. Since the names are not accompanied by descriptions, the translations of the titles are tentative.]

The Polar Altitude Survey

In 1279, the office was reorganized to form the Astrological Commission, with Wang Hsun as Director and Kuo with additional duty *(t'ung* 同) as Associate Director. It was given its own official seal, and a headquarters was set up for it. In the same year [Kuo] submitted memorials containing designs for the instruments. In the presence of the emperor, he elucidated in a comprehensive and systematic way what was attractive about the project. Although this took from the time of morning audience until evening, the monarch did not tire of it. As a result, Kuo sent up a memorial:

"In the Opening Epoch era of the T'ang period [actually in 721–725], I-hsing 一行 ordered Nan-kung Yueh 南宮說 to observe gnomon shadows throughout the realm. [The records that] appear in books include those of thirteen places. Today the realm is bigger than that of the T'ang. If we do not carry out observations in far-separated places, there will be discrepancies [for which we cannot compensate] in the magnitudes and times of solar and lunar eclipses, in the lengths of day and night, and in the heights of sun, moon, and stars [i.e., planets] in the sky. At the moment there are few qualified observers but, to begin with, we can set up gnomons north to south and make direct measurements of their shadows."

The ruler approved his memorial, and provided Kuo with fourteen [specialist] observers from the Directorate. He sent them out in succession by separate routes.

They first determined that at Nan-hai the north pole emerged from the earth 15 *tu*; at the summer solstice the shadow would fall to the south of the gnomon and was 1.16c long; day length was 54 marks, and night length 46 marks. {717–18}

[I have transcribed the data that follow in discursive but stereotyped form into table B.1, where they are easier to read and more useful than in textual form.

A: Altitude of the North Pole, *tu*

B: Gnomon Shadow Length on the Summer Solstice, *ch'ih* (a negative number indicates a southward shadow)

C: Day Length, marks

D: Night Length, marks

E: Latitude of modern site, if available, degrees converted to *tu]*

Table B.1. Sites of Gnomon Observation Survey

Place	Modern Location	A	B	C	D	E
Nan-hai 南海	Central Vietnam	15	–1.16	54	46	
Mt. Heng (Heng-yueh 衡岳	Mt. Heng, Henan	25	0	56	44	27.6
Yueh-t'ai 岳臺	Kaifeng 开封, Henan vicinity	35	1.48	60	40	-
Ho-lin 和林	Outer Mongolia	45	3.24	64	36	-
T'ieh-le 鐵勒	Outer Mongolia	55	5.10	70	30	57
Northern Sea (Pei-hai 北海)	Siberia?	65	6.78	82	18	-
Shang-tu 上都	Dolon Nor, Inner Mongolia	43¼	-	-	-	-
Pei-ching 北京	Xilinguo 锡林郭, Inner Mongolia	42¾	-	-	-	42
I-tu 益都	Yidu, Shandong	37¼	-	-	-	37.2
Teng-chou 登州	Penglai 蓬莱, Shandong	38¼	-	-	-	38.4
Kao-li 高麗	Korea	38¼	-	-	-	
Hsi-ching 西京	Datong 大同, Shanxi	40¼	-	-	-	40.7
T'ai-yuan 太原	Taiyuan, Shaanxi	38¼	-	-	-	38.5
An-hsi-fu 安西府	Guanzhong 关中, Shaanxi	34 7/12		-	-	35
Hsing-yuan 興元	Nanzheng 南郑, Shaanxi	33 7/12	-	-	-	34
Ch'eng-tu 成都	Chengdu, Sichuan	31 7/12	-	-	-	30.1

Place	Modern Location	A	B	C	D	E
Hsi-liang-chou 西涼州	Wuwei 武威, Gansu	40¾	-	-	-	39
Tung-p'ing 東平	Dongping, Shandong	35¾	-	-	-	36.4
Ta-ming 大名	Daming, Hebei	36	-	-	-	36.8
Nan-ching 南京	Kaifeng, Henan	34 5/6	-	-	-	35.3
Yang-ch'eng 陽城	Dengfeng 登封, Henan	34 2/3	-	-	-	35.0
Yang-chou 揚州	Jiangdu 江都, Jiangsu	33	-	-	-	32.9
O-chou 鄂州	Wuchang 武昌, Hubei	31½	-	-	-	31.0
Chi-chou 吉州	Ji'an 吉安, Jiangxi	26½	-	-	-	27.5
Lei-chou 雷州	Haikang 海康, Guangdong	20¾	-	-	-	21.2
Ch'iung-chou 瓊州	Qiongshan 瓊山, Guangdong	19¾	-	-	-	-

[Note: Yuan place names, unless otherwise specified, are those of routes *(lu* 路); modern names, those of counties.

The locations fall into two groups. The first includes six sites plainly chosen for their positions 10 *tu* apart. The listings vaguely identify those at the extremes. For the second and larger group, chosen for the importance of the places, the astronomers recorded only polar altitude. The observations were obviously made with a conventional 8c gnomon to a precision of 1/12 *tu.* See also chapter 1, p. 31. Nan-hai, literally "Southern Sea," may not be the name of a single location.

A few of Ch'en Mei-tung's identifications of places (2003b, 201–4) differ slightly from mine. Uncertainties due to the names used for places make it impossible to resolve the differences. Ch'en's analysis indicates that errors in the latitudes vary from +0.6 to –0.8 degrees, and that errors in day lengths for the first five locations are consistently between 0.2 and 0.4 marks (3 and 6 minutes) low. Neither is bad for accuracy, but the error for day length suggests a slight systematic error in the manufacture or operation of the instruments. The most detailed modern study of this survey estimates a mean accuracy of ±0°.5. The accuracy decreases, and uncertainty increases, with distance from the capital (Li Kuo-ch'ing 历国青 et al. 1977). See also Chang Chia-t'ai 张家泰 1978 and Po Shu-jen 薄树人 1963.]

Predecessors of the Reform

In 1280, upon completion of the [new] astronomical system, Kuo took up the Directorship of the Astrological Commission. He and the other commissioners sent up a memorial:

"We have heard that, of the tasks of rulers, none is more weighty than the ephemeris. From when the Yellow Emperor 'met the sun and derived the constants,' the emperor Yao used intercalary months to determine the seasons and complete the year, and Shun "attended to the *hsuan-chi* 璿璣 and *yü-heng* 玉衡 to set in order the seven governors," through the Three Dynasties [Hsia, Shang, and Chou], there was no set method for computational astronomy."

[*Shih chi* 史記, 1:6; *Shang shu* 尚書, 020100.

This last sentence, as is customary in such memorials, presents classical allusions to remind its readers that from the legendary beginnings of monarchy in China, astronomy has remained a central concern—or so went the conventional wisdom. The platitude about the Yellow Emperor, from *Memoirs of the Grand Astrologer,* refers to his putative invention of the imperial new year ritual of welcoming the sun—reborn after its death at the winter solstice—and his creation of astronomical forecasting (the meaning of the phrase is contentious); that of Shun is from the canon that bears his name.

As for *hsuan-chi* and *yü-heng* in the latter passage, historians ancient and modern have tried to identify these terms with astronomical instruments or with something in the sky. From the Han period on, astronomers came to understand *ch'i cheng* 七政, which earlier meant "the seven [concerns] of government," as "the seven governors," denoting the sun, moon, and planets. Kuo understands the term in that way. As for what he meant by *hsuan-chi* and *yü-heng,* that is impossible to say (see p. 214, note).

Below, as elsewhere, I have inserted the numbers and dates of adoption of the cited astronomical systems from table 2.1. In this appendix, I have appended to them the numbers given in the Evaluation, section 11, to enable following the Yuan astronomers' own sequence of references. I use the form "(#3/1)," where 3 is the number in the comprehensive table and 1 that in the Yuan list.]

By the [late] Chou and Ch'in periods, "the Intercalary Surplus would fall out of step," but in the Western Han period the Triple Concordance system (#3/1, A.D. 1/5) was made.

[The quotation is from *Shih chi*, 26: 1257. The author is adapting for his own rhetorical purpose a phrase that marked an entirely legendary collapse of astronomical competence in a time much earlier than the late Chou. On the Intercalary Surplus, see the Canon, 1.3.2.]

One hundred thirty years later, the disagreements [about the best approach to astronomical prediction] were finally settled. In the Eastern Han period, the Quarter Remainder system (#4/2, A.D. 85) was made. After more than 70 years, the instrumentation [for accurate observation] was finally complete. After another 121 years, Liu Hong 劉洪 made the Supernal Emblem system (#5/3, 206). He was the first to realize that the motion of the moon varies in speed. After another 180 years, Chiang Chi 姜岌 made the Triple Era First-year Epoch system (#13/10, 384). He was the first to realize that [one can determine exactly] the position of the sun in the lunar lodges by its opposition [to the moon] at the time of a lunar eclipse.

[This capsule history of innovations in mathematical astronomy links its landmarks tightly by assertions about the intervals between them. On the many arithmetical errors, see my comments at the end of this sequence.]

After another 57 years, Ho Ch'eng-t'ien 何承天 made the Epochal Excellence system (#17/5, 443). He was the first to realize that one could determine [Corrected] Major and Minor Surpluses for lunations, full moons, and crescents [and thus incorporate the expansion and contraction of the solar motion in the lunar technique]. After another 65 years, Tsu Ch'ung-chih 祖沖之 made the Great Enlightenment system (#18/6, 463). He was the first to realize that in the sun's motion there is a constant for the Annual Difference, and that the pole star was more than 1 *tu* away from the unmoving place [i.e., the pole of the sky]. After another 52 years, Chang Tzu-hsin 張子信 [circa 560] was the first to realize that the paths of the sun and moon diverged laterally and intersected, and that the [speeds of the] five stars [i.e., the planets] varied, and [he incorporated] stationary points and retrograde motions.

[Chang's system had no official status; it took some time for official systems to incorporate his innovations.]

After another 33 years, Liu Cho 劉焯 made the Sovereign Pole system (#34/42, 608). He was the first to realize that there were an expansion and a contraction [i.e., an inequality] in the sun's motion. After another 35 years, Fu Jen-chün 傅仁均 made the Fifteenth-year Epoch system (#36/14, 619). He chose from among ancient observations (*chiu i* 舊儀),

[This compound would normally mean "old instruments," but that does not fit the context. This translation is decidedly tentative.]

but was the first to use Corrected [i.e., apparent] Lunations. After another 46 years, Li Ch'un-feng 李淳風 made the Chimera Virtue system (#38/15, 665). Because in the old systems [the fractional parts of] various cycles (*chang pu yuan-shou* 章蔀元首) did not have commensurable denominators, he was the first to make one general denominator [for all fractional parts of quantities]. He also [had a technique to] advance the new moon to avoid the moon's first appearance falling at dawn of the last day of the month.

[See above, p. 63, and the Evaluation, p. 368. On the "various cycles" see Sivin 1969, especially pp. 14 and 20.]

After another 63 years, the Buddhist priest I-hsing 僧一行 made the Great Expansion system (#42/16, 728). He dealt with the possibility of four [consecutive] long months [followed by] three short ones, and worked out differences in eclipse [predictions] depending on nine regions (*chiu-fu* 九服). After another 94 years, Hsu Ang 徐昂 made the Extending Enlightenment system (#50/19, 822). He was the first to realize that in solar eclipses there were three [systematic] differences, those of the *ch'i*, the mark, and the double-hour. After another 236 years, Yao Shun-fu 姚舜輔 made the Era Epoch system (#71/30, 1106). He was the first to realize that there is a constant difference in the Comprehensive Remainder for the maximal phase of eclipses.

[On "nine regions," see the Evaluation, p. 368. Re "three differences," *Chiu T'ang shu,* 30A: 742. Hsu actually proposed four corrections, but the fourth was negligible in magnitude. The Time Difference corrected the time of the apparent lunation to determine the maximum phase of the eclipse; the other three were corrections to eclipse magnitude. Ch'en Mei-tung 2003a, 404–6, finds Hsu's methods flawed, but his influence in pointing out the need for corrections important. For the "constant difference" cf. the Canon, step 6.8.]

What we have set out above covers 1182 years, during which the astronomical system was reformed 70 times, and thirteen experts created innovative methods.{718}

[The intervals apply to the year when each system originated, rather than, as in Table 2.1, that when it was adopted. This listing, on either understanding, is rife with arithmetical errors. The numbers given (if we count "more than 70" as 70) do add up to the total of 1182 years, but that is far from the actual interval between A.D. 1 and 1106. On the other hand, the assertions about the character of innovations are correct.]

Topics of Research

[What Kuo summarizes here is investigations by the Yuan astronomers themselves that served as foundations for the reform.]

After another 174 years, we venture to believe, this sagely dynasty has unified all within every direction and created this realm. [Its ruler] has specifically commanded us to carry out a reform and set in order a new astronomical system. For this purpose, we have newly made a Simplified Instrument and a Tall Gnomon, and have arrived at constants based on observations using them. We have carried out research *(k'ao-cheng* 考正) on seven items:

1. Winter solstice

From Enthronement of Winter (30 October) of 1276 on, based on daily observations of the gnomon shadow, we matched shadow lengths before and after the winter solstice to serve as standards. We determined that the winter solstice of [the beginning of] 1277 would fall on sexagenary day 35, 8½ marks after midnight; that the summer solstice of 1277 would fall on s.d. 37 {719}, 70 marks after midnight; that the winter solstice of [the beginning of] 1278 would fall on s.d. 40, 33 marks after midnight; that the winter solstice of [the beginning of] 1279 would fall on s.d. 45, 57½ marks after midnight; and that the winter solstice of [the beginning of] 1280 would fall on s.d. 50, 81½ marks after midnight. In each instance this was 18 marks earlier than the time predicted according to the Great Enlightenment system. [The results from] dates near [the solstices] tallied with those far away; those earlier and later corresponded to the standard.

[Compare the Evaluation, sections 1.1–1.5, and see the Evaluation, subsection 1.1, for a discussion of solstice determination.]

2. Year Surplus

From the Great Enlightenment system of the Liu Sung dynasty on, there have been a total of six projects that, by observing gnomon shadow lengths to verify the *ch'i* periods, have determined the true times of winter solstices. We have used [their results] to determine the intervals [i.e., tropical year length], in every case taking their times and using them along with the Year Surplus. Now we have tested [the outcome] for four years. Our results tally, with no discrepancy. The result is that (? *jeng* 仍), in the 810 years since sexagenary year 39 of the Great Enlightenment era (462), each year comprises 365 days, 24 marks, and 25 parts [i.e., 365.2425 days]. The 25 parts is the appropriate quantity to serve as the Year Surplus of the present system.

[For the Year Surplus, see the Evaluation, section 2 and table 7.3. This mean tropical year length actually holds only for circa 1280. Kuo's secular variation of the Annual Difference gradually changes the lengths of both the tropical and the sidereal year. See the Introduction, p. 101. The last sentence is defective; it should begin "The 24 marks and 25 parts … "]

3. Tread of the sun

Using the maximum phase of the lunar eclipse on the full moon of the 4th month, sexagenary year 14 (0205 hrs 19 May 1277), we computed that the tread of the sun [i.e., its apparent motion] for the winter solstice put it 10 *tu* in the lodge Basket on the Red Way, and 9 *tu* and a fraction in Basket on the Yellow Way. We proceeded to measure the tread of the sun for every day, whether depending on the stars to determine the position of the moon, depending on the moon to determine the position of the sun, or directly depending on the positions of the stars to determine that of the sun. After the method was established, we calculated [positions] beginning with month 1 of s.y. 14 (5 Feb 1277) and ending with month 12 of s.y. 16 (last day 1 Feb 1280), a total of three years. Altogether we obtained 134 instances, every one of which tallied with the location in Basket and the lunar eclipse.

4. Travel of the moon

From sexagenary year 14 to the present, based on daily observations of the moon's successive positions as it moves, we have converted these to accord with Revolution Entry (*ju chuan* 入轉) on the Yellow Way for locations of [the moon's] fastest and slowest motion as well as of mean motion. We attained such results for 51 instances in 13 revolutions. Discarding invalid results (*pu-chen ti* 不真的), we were left with 30 events. We concluded from these that results for Revolution Entry in the Great Enlightenment system fell behind the phenomena. Further, having tested [this hypothesis] on eclipses, [we found that] adding 30 marks to [predictions using] the Great Enlightenment system brought them into correspondence with the way of heaven.

[Revolution Entry (Canon, 4.1.1) is the time elapsed in the current anomalistic month. This indicates that the basic method of the Yuan system's predecessor was sound, so that a constant correction could be used—at least for a range of months within a short span of years. The mention of discarding invalid results, although "invalid" remains undefined, indicates careful testing.]

5. Crossing entry

From the 5th month of sexagenary year 14 on, based on daily observations of the moon's angular distance from the pole, we have compared them with the polar distance of the Yellow Way to determine the intersection of the moon's path with the latter. We derived a total of eight instances. We proceeded to use the solar eclipse [prediction] method. In every case where we determined the [greatest] magnitude, we found that, for the exact time of Crossing Entry, the discrepancy [between computation and prediction] did not exceed that of the Great Enlightenment system.

[The text that I have translated "for the exact time of Crossing Entry" reads *te ju shih-k'o* 得入時刻. Its counterpart in the *Yuan History* (164: 3850), *te ju chiao shih-k'o* 得入交時刻, is clearer. Crossing Entry *(ju chiao* 入交, Canon, 6.1), analogous to Revolution Entry, corresponds to the time elapsed in the current nodical month, and is thus directly related to the moon's distance from the intersection of its path with the ecliptic. This paragraph is a little too concise to be entirely clear, but evidently the astronomers were using the times of the maximal phase of observed partial eclipses to test their computations of the moon's position with respect to the node.]

6. Angular extensions of the lunar lodges

Since the Grand Inception system (#2, 104 B.C.), the angular distances [between the lodge determinatives] have varied, some increasing and some decreasing. As for the [Revised] Great Enlightenment system (#76), it specified fractional parts as ¼, ½, or ¾ of a *tu,* but these were [the result of] arbitrary fudging. In no instance did [the astronomers] determine the extension by observation.

Our new instrument is finely engraved with the degrees and fractional degrees of the celestial round, with each *tu* divided into 36 parts. Interval wires *(chü-hsien* 距線) take the place of a sighting-tube. The fractional parts of lodge extensions are based on observations rather than on arbitrary fudging.

[See the discussion of the lodge extensions in the Evaluation, section 3. For those of #76, see *Chin shih* 金史, 21: 454–55. "Interval wires" (on the Simplified Instrument) are perpendicular wires mounted at opposite ends of an alidade. When lined up on an object, they allow the observer to read its location precisely.]

7. Sunrise, sunset, day and night marks

The Great Enlightenment system took Pien-ching 汴京 [the Southern Sung capital, modern Kaifeng, Henan] as the standard for time of sunrise and sunset and for day and night marks [for a given date]. The number of marks is not the same as for Ta-tu. Now we have proceeded, using measurements of the altitude of the north pole at our location and of the angle—whether inside or outside—of the Yellow Way [with the horizon], to establish a method for determining daily sunrise and sunset and day and night marks.

Our result is that at the summer solstice, when the day is longest, the sun rises in the 3rd double-hour, standard half, 2 marks [0436 hrs], and sets in the 11th double-hour, beginning half, 2 marks [1936 hrs]. Thus day marks is 62 and night marks is 38. At the winter solstice, when the day is shortest, the sun rises in the 5th double-hour, beginning half, 2 marks [0736 hrs], and sets in the 9th double-hour, standard half, 2 marks [1636 hrs]. Thus day marks is 38 and night marks is 62. This standard can serve as an eternal one.{718–19}

[The equivalents in the last paragraph are approximate, since the modern equivalent of a mark is 14.4 minutes. On the variation in day length through the year, see the Canon, 5.4, and the Evaluation, section 8. Here Kuo does not criticize the method of his predecessor, but points out the need to correct for what we would now call a difference in latitude.]

New Methods

We have created five new methods:

1. Expansion and contraction of the sun's motion

We used the corrected *ch'i* of the Four Standard Points (3.11.3), establishing {720} rising and descending limits on this basis. We set up [a scheme of] interpolation (*chao-ch'a* 招差) to yield motion parts for each day, beginning and end [extents], maximal differences, and accumulated degrees. These were more accurate than in ancient times.

[This is a laconic but accurate account of the solar theory, dividing the sun's apparent motion through the tropical year into quadrants, using a third-order formula to determine the solar equation of center, and calculating the arc of apparent motion from one end of a quadrant. For details, see the Canon, steps 3.2A and 3.2B, commentary.]

2. Slackening and hastening of the moon’s motion

The old astronomical systems all used 28 extents. Now we have applied [the new system’s standard] 10 000 divisions to 820 to give one extent, producing 336 of them. We then used reiterated interpolation to determine the advance and recession of Revolution Parts, the degree of slackening or hastening varying with time. It would seem that earlier systems did not incorporate this [method].

[This states the gist of the Yuan system’s lunar theory, which divided the anomalistic month cycle into 336 extents. See the commentary to the Canon, 4.C4–4C.6 and, for details on the lunar inequality, the commentary to section 4 and subsections 4.5A and 4.5B. Martzloff 1997: 339 (briefly) and Li Yen 1957 discuss predecessors of the interpolation method.]

3. Difference between [positions on] the Yellow and Red Way

The old method used 101 *tu,* subtracting and multiplying. Now we have used computational methods that encompass arcs and sagittae, squares and circles, and oblique and perpendicular lines, to find the [Daily] Angular Motion Rate (3.13) and [Daily] Accumulated Difference (4.18). This rate and difference truly correspond closely to the way of heaven.

[This is a highly condensed way of referring to the proto-trigonometric method used to convert positions of the sun and moon on their orbits to equatorial locations. See the Orientation on this method.]

4. Inside and outside angles of the Yellow and Red Ways

According to numerous years of observations, the maximal inside and outside angles are $23^{t}.90$. We have used circular capacity (? *yuan-jung* 圓容), rectilinear lines, sagittae (? *shih-chieh* 矢接), and sides of triangles to form a method to find daily polar distance [of the Yellow Way]. [The results of calculation] tally with those of observation.

[This paragraph refers to the method for computing what the next paragraph calls a “perpendicular arc” between a position on the equator and the corresponding position on the ecliptic. ”Inside and outside” means to the north and south of the equator. The Yuan system’s constant for what modern astronomy calls the obliquity of the ecliptic was $23^{t}.90$, or 23°.56. The translation of this paragraph is tentative; the text is possibly corrupt.]

5. Nodical cycle of the White Way

According to the old method, converting [position on] the Yellow Way into that on the White Way was a matter of deriving one oblique line from another. Now we use a perpendicular arc (*li hun* 立渾) to make a comparative measurement and from it determine the Standard Crossing (4.10.1) for the moon['s orbit] and the Red Way according to the distance from the vernal or autumnal Standard Point. With the Standard Crossing Constant for the Yellow and Red Ways as $14^t.66$, we have established this as our method. When we compute each position of the [White Way – Red Way] intersections among the lunar lodges for successive months, they are entirely in accordance with the inherent principles.{719–20}

[For the Standard Crossing, see the Evaluation, section 7, and the commentaries to the Canon, 4.9, 4.13, and 4.17. What this passage calls the "Standard Crossing Constant" is unnamed there (see the commentary, p. 475). The term *li hun* is unclear, but probably refers to the meridional ring of an armillary sphere, and by implication, to the arc perpendicular to the equator between the two circles.].

Documents of the Reform

[See the summary of these two lists in Table 6.1.]

In 1282 the Grand Astrologer, Master Wang [Hsun], died. By that time the new system had been promulgated, but the procedures for computation and the numerical ready reckoners were not yet in definitive drafts. Kuo then set the materials in order, sorted them categorically, edited them, and copied them into separate manuscripts. He divided them into *Pacing the Motions (T'ui pu* 推步) in seven chapters, *Ready Reckoner (Li-ch'eng* 立成) in two chapters, *Draft of the Evaluation (Li i ni-kao* 曆議擬藁) in three chapters, *Hemerology based on the Rotations of the Gods (Chuan-shen hsuan-tse* 轉神選擇) in two chapters, and *Forms for Annotation from the Three Almanacs (Shang chung hsia san li chu shih* 上中下三曆註式) in twelve chapters.

[Yang Huan's 楊桓 contemporary preface to the last two works survives (*Shou-shih li chuan shen chu shih hsu* 授時曆轉神注式序). He explains that the first treatise (in only one chapter according to him) traces the rotating dominance of various gods, which one must learn in order to choose auspicious days; the second revises the full range of conventional forms for annotating the almanac. Its techniques are to a large extent those that diviners still use. On a Korean printed edition of the ready reckoner that has survived only in Korea, see Li Yin-chi 李银姬 & Ching Ping 景冰 1998. Ho Peng Yoke 2003 discusses ancient divination practices still in use.]

In 1286 Kuo succeeded to the post of Grand Astrologer. He submitted a document presenting [the records of the system] to the throne. He added *Critical Annotations on Times (Shih-hou chien chu* 時候箋注) in two chapters and *History of the Astronomical Reform (Hsiu-kai yuan-liu* 修改源流) in one chapter. His writings on observations included *Designs of the Instruments (I-hsiang fa-shih* 儀象法式) in two chapters, *Study of Gnomon Shadow Measurements for the Solstices (Erh chih kuei ying k'ao* 二至晷景考) in twenty chapters, *Study of the Detailed Movements of the Five Stars (Wu-hsing hsi hsing k'ao* 五星細行考) in 50 chapters, *Study of Eclipses, Ancient and Modern (Ku chin chiao-shih k'ao* 古今交食考) in one chapter, *Newly Observed Degrees of Lodge Entry and Polar Distance of Stars in Various Asterisms among the 28 Lunar Lodges (Hsin ts'e erh-shih-pa she tsa tso chu hsing ju hsiu ch'ü chi* 新測二十八舍雜座諸星入宿去極) in one chapter, *Newly Observed Unnamed Stars (Hsin ts'e wu ming chu hsing* 新測無名諸星) in one chapter, and *Study of the Moon's Motion (Yueh-li k'ao* 月離考) in one chapter. These were also stored in the office [of the Commission]. {720}

[These lists comprise fifteen books in 105 chapters. They reveal how extensive the records of the Season-Granting reform were, and how small a portion of its documents was incorporated into the *Yuan History's* astronomical treatise. The four chapters of the latter are probably based on *Pacing the Motions* and *Draft of the Evaluation,* which when finished comprised ten chapters.

Other interesting implications of these titles come readily to mind. First, it is obvious that divination was an integral part of the astronomers' work. The last two works in the first paragraph of this translation, and probably the first in its second paragraph, have to do with divination. They total sixteen chapters, perhaps four times the length of the published Season-granting treatise.

Second, although Kuo's book on historical eclipse observations occupies only one chapter (presumably with some overlap in the Evaluation's draft), that on contemporary planetary observations, in 50 chapters, is by far the largest document that he compiled. This suggests that the perfunctory and unoriginal theory of the planets in the Canon, and the silence with respect to planetary motions in the surviving Evaluation and in this biography, are due not to lack of interest but to lack of opportunity to innovate. I have already suggested (p. 32) that the limitation was inseparable from the support that made the reform possible.

Third, another outcome of the reform was a star survey that significantly increased the number of known stars and provided data on their coordinates.]

Water Control, Continued

[Not translated. This section summarizes the hydraulic projects that took up most of Kuo's efforts from 1291 on. The anecdote in the next paragraph gives the flavor.] {721}

Biography, Continued

In 1294 Kuo took up an appointment as Grand Academician of the Institute for the Glorification of Literature and Associate Director of the Astrological Commission. In 1298 [Khubilai's successor] had Kuo summoned to Shang-tu to consult on building an aqueduct [to there] from T'ieh-fan-kan 鐵幡竿.

[Kuo's first title was purely honorary, and the second amounted to letting him retire from the active directorship of the Commission. T'ieh-fan-kan is a mountain range near Shang-tu, the Yuan's northern capital in what is now Inner Mongolia, on the border between Chinese agricultural and Mongolian pastoral lands. A mobile palace *(lu-chang* 廬帳) was an ensemble of tents that the emperor or a member of his family used when traveling or hunting.]

Kuo said in a memorial that where mountain streams pour tumultuously downward year after year, one can succeed in building aqueducts and weirs only on a large scale, at least 50 to 70 paces wide. The executive officials, who wanted to minimize the expenditure, considered what he said extravagant, and reduced the width by a third. The year after [the channel was built], it rained heavily, the mountain streams poured downward, and the drainage channels could not accommodate them. [The flood] inundated people and their animals and tents, nearly impinging on a mobile palace. The day after, when the emperor was hunting in the north, he remarked to the high ministers with him "The Grand Astrologer Kuo is a godlike person. Too bad they did not accept what he said!"

In 1303, an edict went out to metropolitan and provincial officials to the effect that officials who had reached the age of 69 would be permitted to retire. Because Kuo was an old servitor of the emperor *(chiu ch'en* 舊臣), and remained active on behalf of the court, his was the only request not approved. The custom to this day of not [permitting] Hanlin Academicians, Grand Astrologers, and Directors of the Bureau of Astronomy to retire follows the precedent set by Kuo. He died in 1316 at the age of 85. {721}

[The edict of 1303 was significant because refusal of permission to retire implied indispensability. Onerous though retention may have been for someone of Kuo's age, it was a great personal compliment from the emperor.]

Character

[The remainder of this biography differs fundamentally from what comes before. The next two sections are not based on Kuo's own documents, but give Ch'i Lü-ch'ien's personal view of him. They do not appear in the *Yuan History's* biography.

There is no doubt about Ch'i's technical skill, but this section and the next suggest pedestrian, conventional judgment of people. His own choice of Kuo's most important accomplishments in computation and instrumentation seem arbitrary, and the remarks on Kuo's character at the end of this biography are there not for their penetration but as proof that important people praised him. Since the celebrities cited were family friends and mentors, their testimony does not carry as much weight as that of less partial witnesses would have done. Analogously, the "pure virtue" and the echo of Confucius as "model teacher for all generations" in the first sentence of this section do not reflect anything in Kuo's biography. They are mere effusions of piety on the part of a protégé. More to the point is the mention of "practical studies," an expertise that the Mongols valued.]

Kuo, because of his pure virtue and his practical studies, is a model teacher for all generations. There are three respects in which he was peerless:

1. Studies of water control

[Not translated.] "… Because what he said was reliable and backed by evidence, Kuo's studies of water control are peerless."

2. Studies of computational astronomy

In the ancient astronomical systems, for both the circumference of the sky and the cycle of the year, the Minor Surplus was the same as ¼ of a degree of the sun['s travel].

[On the Minor Surplus, see the Canon, p. 501. "Subdivided" is literally "broken, *p'o* 破."]

From the Han and Wei periods on, astronomers gradually became aware that this quantity was not uniform. They came to speak of subdivided parts. But the methods they instituted were not uniform, and they made [their quantities] larger or smaller to suit themselves.

Kuo based his rate on periods of 100 years. Under the Minor Surplus, he added 1 [to tropical year length] and subtracted it [from sidereal year length]. {722} By doing this, whether he was calculating backward to antiquity or forward to the future, [his results] unfailingly corresponded [to observation].

[Kuo's "rate" is the secular variation in year length; see p. 101.]

Furthermore, from Grand Inception (#2, 104 B.C.) to [Revised] Great Enlightenment (#76, A.D. 1180), there were more than 70 famous astronomical systems, of which 43 were put to use in their time. Their originators generally adjusted the numerators and denominators of the fractional parts [of important constants], impressing people for a while. In this period some excelled, no more than three or so: the Epochal Excellence system (#17) of the [Liu] Sung period, the Great Expansion system of the T'ang (#42), and the recent Era Epoch system (#71). Even these failed to attain perfection. When [we] test [their computations against] the celestial phenomena, although at first they were highly accurate, by the end of their period of official use they were no longer effective.

The system that Kuo created was not only based on rigorous observation, but its techniques were attentive to detail and comprehensive. It has been in use for nearly 50 years, but there has not been a single error in which it has run ahead of, or fallen behind, the sky. Because Kuo rejected the limitations of Accumulated Years and Day Divisor, as well as the inferior practice of freely adjusting the numerators and denominators of the fractional parts, his studies of computational astronomy are peerless.

[See the Evaluation, section 11.]

3. Studies of the design of astronomical instruments

The old instruments generally put obstacles in the way of their users, and the graduations of their rings were engraved only in *tu,* lacking finer markings. When seeking a star in the sighting-tube, as [the eye] gradually moves toward the circumference, what one sees gradually opens out [i.e., the field of view shifts outward], so that it becomes difficult to find the target. …

[At this point the text surveys some of the components of the Simplified Instrument, emphasizing the diametral bars with sighting vanes that solved this problem. I have not translated that passage, since it merely reiterates details of the already detailed description translated in appendix A (p. 561).]

When using the old 8c [gnomon], it was said that the [noon] shadow at the summer solstice was 1.5c, and that for 10 000 *li* [of southward travel by an observer] it[s length] would change by 0.1c *(i ts'un* 一寸). These assertions are found in such books as the *Officials of Chou* and the *Chou Gnomon*. As to the notion that for 10 000 *li* [of southward travel the noon shadow] would change by 0.1c, I-hsing of the T'ang period already refuted it. [When using] the 8c gnomon, because the upright is low, the shadow moves quickly. Astronomers passed this instrument down and used it from antiquity to the present, without anyone improving on it.

[The *Officials of Chou (Chou kuan* 周官) was an antiquarian name for *Chou li* 周禮 (mid second century B.C.), one of the orthodox classics. Scholars in the Yuan believed that it and the *Chou Gnomon (Chou pi* 周髀, between 50 B.C. and A.D. 100), were written in high antiquity. On I-hsing's survey and the conclusions he drew from it, see Ang 1979: 395–407.]

In the gnomon he made, Kuo quintupled [the height] of the old one. He suspended [at the top] a horizontal bar so that, each day at noon, the aperture of the shadow definer made the shadow of the horizontal bar converge [i.e., focus]. As he measured it, he took the intermediate quantity [that corresponds to its center, *che ch'ü chung shu* 折取中數]. Compared with the old gnomon, which could only catch the shadow, his approach was indeed judicious.

[The Astrological Treatise (Appendix A) puts it more clearly.]

During the reign of Khubilai, Kuo submitted a seven-jewel lamp water clock. Today in the Palace of Supernal Brightness *(T'ien ming tien* 天明殿), it is set up and used for every imperial audience. The bells and drums in it respond to the time, sounding by themselves. He also once presented a "wooden ox and gliding horse *(mu-niu liu-ma* 木牛流馬)." Although they did not quite reduplicate the old design by Chu-ko [Liang] 諸葛亮, they were marvelously ingenious.

[The first machine seems to be identical with the "lamp clepsydra of the Palace of Great Brightness," which the "Treatise on Astrology" describes in fair detail as a large water-driven celestial clock. The character of the "wooden ox and gliding horse," credited with no details to the ingenious and resourceful strategist Chu-ko Liang 諸葛亮 (181–234) in his biography, is entirely shrouded in the mists of time. Needham's extensive study of wheelbarrows (which does not cite this source) plausibly identifies the "wooden ox" as a wheelbarrow designed for military use. He argues cursorily that the "gliding horse" is another type of wheelbarrow, but some Chinese scholars believe that it was a four-wheeled cart pushed by hand. Here Ch'i implies that Kuo's versions were automatic in some sense. On the “lamp clepsydra” see *Yuan shih.* 48: 994–95, tr. Needham et al. 1960: 135–36; for Chu-ko’s biography, *San-kuo chih* 三國志, 35: 927; on the wheelbarrow, Needham et al. 1954– , 4 part 2: 258–81.]

During the reign of Temür 鐵木兒 (r. 1295–1307), Kuo submitted a cabinet-mounted incense clock (? *kuei hsiang lou* 櫃香漏). He also made a screen-mounted incense clock (? *p'ing-feng hsiang lou* 屏風香漏), and a traveling clepsydra for use when the emperor journeyed for the suburban and ancestral sacrifices.

[On incense clocks see Bedini 1994.]

In 1298, Kuo set up the Observatory's water-driven celestial armillary clock. It contained a total of 25 mechanical wheels of various sizes, all of wood with carved gear teeth *(ch'ung-ya* 衝牙), which turned and engaged each other. On the upper part [of its globe] were painted spots [to represent] the stars and their degrees. Rings [to represent] the paths of the sun and moon crossed it obliquely. The globe turned leftward following the sky, and the rings [of the armillary sphere bearing] the sun and moon turned rightward according to their motions.

[This is an elaborate instrument in the tradition of the enormous astronomical clock of 1092 and its many predecessors, studied in Needham et al. 1960, especially pp. 133–42. The text calls this globe *shui hun yun hun-t'ien-lou* 水渾運渾天漏, which does not make sense; in translating, I disregard the first *hun*. The instrument’s name does not prove that it was used in conjunction with observation. Such mechanisms were designed primarily to impress dignitaries.]

Kuo once wished to reconstruct the seismoscope that Chang Heng made, and the sealed room used to observe the *ch'i*. Although he never carried out these projects, he did not fail to exhaustively study their principles. In all these respects, Kuo's studies of the design of astronomical instruments are peerless. {721–22}

[On the seismoscope of A.D. 132, see Sleeswyk & Sivin 1983, and, on the ancient technique of *hou ch’i* 候氣, Bodde 1959, Hulsewé 1979, Chang Chih-ch’eng 張志誠 1991, and Huang Yilong & Chang Chih-ch’eng 1996.]

Kuo’s Uniqueness

Early in Kuo's life, when he was fourteen or fifteen years old, he got a rubbing of a stone engraving of a lotus-blossom clepsydra. He was already able fully to understand its underlying principles. When he accompanied Chang Wen-ch'ien on the latter's mission in Taming 大名, he had occasion to do metal casting on a large scale. [The eventual result] was the bronze clepsydra vessels now in the Observatory.

He also obtained an illustration of the *hsuan-chi* mentioned in the *Book of Documents (Shang-shu hsuan-chi t'u* 尚書璿璣圖). [To reconstruct it,] he bent strips of bamboo into circles *(kuei* 規) to serve as his instrument. He piled up earth to make a platform, and on it he observed the determinative stars of the 28 lunar lodges and the other large stars.

[The author of Kuo's illustration obviously understood the *hsuan-chi,* discussed in my commentary above (p. 580), as an astronomical instrument. This is apparently another juvenile accomplishment of Kuo's. The last paragraph may refer to an apocryphal book, *Shang-shu hsuan-chi ch'ien* 尚書璿璣鈐, of which only fragments survive.]

When Kuo came to be useful [to the court], in the simplicity and ease with which he drew charts and the precision of his astronomical observations, he was so inventive that he was never able to keep his consultations private (? *pu-neng ssu ch'i i* 不能私其議). Ordinary officials were unable to share in the merit he earned.

[This translation is tentative, because at least one character is missing from the text.]

Wang Hsun, Director of the Astrological Commission, was a strong-willed and self-centered man. But every time he visited Kuo and saw the technical quality of his work, he had to express his admiration.

Hsu Heng's judgments were the model for his time. When he spoke of Kuo, he would put his hands up to his temples and say "Heaven has protected our Yuan dynasty! Surely in any generation a man like this is hard to find. You might say that he surpasses everyone in a thousand ages." {722–23}

Appendix C Technical Terms

This glossary makes it possible to look up all constants and named results (intermediate and final) in the Canon by their English or Chinese names. Each item gives the number that identifies it in the translation of the Canon. In order to avoid wasting space, I have slightly condensed the forms of some terms given in the translation. “Lunar lodge,” for instance, is given here as “Lodge.”

Named Results, English

Accumulated Degrees after the Crossing Halfway Point (*pan-chiao hou chi-tu* 半交後積度, 4.16.2, 4.16.4)

Accumulated Degrees after the Intermediate Crossing (*chung-chiao hou chi-tu* 中交後積度, 4.16.3)

Accumulated Degrees after the Standard Crossing (*cheng-chiao hou chi-tu* 正交後積度, 4.16.1, 4.20.1)

Accumulated Difference (*chi-ch'a* 積差, 3.7.2)

Accumulated Motion (*hsing ch- tu* 行積度, 7.3.7)

Accumulated Red Way Degrees from Standard Point to lodge (*ssu-cheng hou ch'ih-tao hsiu chi-tu* 四正後赤道宿積度, 3.6.2)

Addition/Subtraction Difference (*chia-chien-ch'a* 加減差, 4.6.1)

Anterior Conjunction [Parts] (*ch'ien ho* 前合, 7.1.1)

Argument for grade (*chu tuan ju-li* 諸段入曆, 7.2.2)

Argument in Expansion/Contraction for Eclipse Maximum (*shih-shen ju ying-so li* 食甚入盈縮曆, 6.7.1)

Argument in Expansion/Contraction for Mean Conjunction/Appearance/ Invisibility for Star (*wu-hsing p'ing ho hsien fu ju ying-so li* 五星平合見伏入盈縮曆, 7.13.1)

Argument in Expansion/Contraction for moon phases (*ju ying-so li* 入盈縮曆, 3.1.1)

Argument of Mean Conjunction (*p'ing-ho ju-li* 平合入曆, 7.2.1)

Argument of moon phase in the Phase of Hastening (*chi-li* 疾曆, 4.3.1);

Argument of moon phase in the Phase of Slackening (*ch'ih-li* 遲曆, 4.3.2)

Beginning Extent (*ch'u hsien* 初限, 4.5.1)

Beginning/End Extents (*ch'u mo hsien* 初末限, 4.4.1)

Carried Eclipse Difference (*tai-shih ch'a* 帶食差, 6.20.1)

Center Distance Degrees (*chü-chung tu* 距中度, 5.7.2)

Center Distance Parts (*chü-chung fen* 距中分, 5.7.1)

Centered Star (*chung-hsing* 中星, 7.1.5)

Centered Star for each grade (*chu tuan chung-hsing* 諸段中星, 7.1.6)

Comprehensive Accumulation parts for corrected conjunction/invisibility/ appearance (*fan-chi* 汎積, 7.15.1)

Comprehensive Difference *(fan ch'a* 汎差, 7.10.1)

Corrected Accumulated Degrees *(ting chi-tu* 定積度, 4.7.2, 4.20.3)

Corrected Accumulated Degrees for Dawn and Dusk *(ch'en hun ting chi-tu* 晨昏定積度, 4.23.2)

Corrected Accumulated Degrees for Midnight *(yeh-pan ting chi-tu* 夜半定積度, 4.22.2)

Corrected Accumulated Degrees for the Moon's Position on the White Way *(yueh-li pai-tao ting chi-tu* 月離白道定積度, 4.19.2)

Corrected Accumulated Degrees of the Moon's Motion for Corrected Quarter/Full Moon *(ting hsien, wang yueh hsing ting chi-tu* 定弦望月行定積度, 4.7.4)

Corrected Accumulation *(ting chi* 定積, 6.21.1, 7.18.2)

Corrected Accumulation Date (*ting chi jih-ch'en* 定積日辰, 7.4.2)

Corrected Accumulation Days *(ting chi* 定積日, 7.4.1)

Corrected Appearance/Invisibility, Date *(ting hsien ting fu jih ch'en* 定見定伏日辰, 7.17.2, 7.18.4)

Corrected Appearance/Corrected Invisibility Corrected Accumulation Days *(ting hsien ting fu ting chi jih* 定見定伏定積日, 7.18.3)

Corrected Appearance/Invisibility Corrected Accumulation Days *(ting hsien fu ting chi jih* 定見伏定積日, 7.17.1)

Corrected Aspect Degrees *(ting hsiang tu* 定象度, 3.11.2)

Corrected Conjunction Corrected Accumulation Days *(ting-ho ting-chi jih* 定合定積日, 7.16.3)

Corrected Conjunction Corrected Star Degrees *(ting-ho ting-hsing tu* 定合定星度, 7.16.4)

Corrected Conjunction Date *(ting ho jih-ch'en* 定合日辰, 7.16.5)

Corrected Conjunction/Opposition,Crossing Entry, Date and time *(ting shuo wang chia-shih ju chiao jih* 定朔望加時入交日, 6.4.1)

Corrected Conjunction Retrogradation Invisibility Corrected Days *(ting ho fu t'ui ting jih* 定合伏退定日, 7.16.7)

Corrected Conjunction, Lodge Position on Yellow Way *(ting ho so ch'an huang-tao hsiu tu* 定合所躔黄道宿度, 7.16.6)

Corrected Conjunction/Quarter/Full Moon, Date *(ting shuo, hsien, wang jih* 定朔弦望日, 4.6.2)

Corrected Conjunction/Quarter/Full Moon, Date and Time *(ting shuo, hsien, wang jih ch'en* 定朔, 弦, 望日辰, 4.6.3)

Corrected Degrees of the Argument in Expansion/Contraction of the Eclipse Maximum *(shih-shen ju ying-so li ting tu* 食甚入盈縮曆定度, 6.7.2)

Corrected Difference *(ting ch'a* 定差, 3.2.3, 4.13.1, 4.17.2, 4.19.1, 4.20.2)

Corrected Limit Degrees *(ting hsien-tu* 定限度, 4.13.3)

Corrected Limit Entry Degrees (*ju ting-hsien hsing tu* 入定限行度, 6.11.4, note)

Corrected Motion *(hsing ting tu* 行定度, 7.3.6)

Corrected Star for Grade *(chu tuan ting-hsing* 諸段定星, 7.6.1)

Corrected Star for Midnight at Beginning of Grade *(chu tuan ch'u jih ch'en ch'ien yeh-pan ting hsing* 諸段初日晨前夜半定星, 7.7.1)

Corrected Use Parts (*ting yung fen* 定用分, 6.15.1, 6.16.1)

Crossing Corrected Degrees *(chiao ting tu* 交定度, 6.5.2)

Crossing Entry Comprehensive Days, Midnight (*yeh-pan ju chiao fan jih* 夜半入交汎日, 6.3.3)

Crossing Entry Comprehensive Days, Next Conjunction and Full Moon (*tz'u shuo wang ju chiao fan jih* 次朔望入交汎日, 6.2.1)

Crossing Entry Comprehensive Days, Regular Conjunction of Astronomical First Month *(t'ien-cheng ching shuo ju chiao fan jih* 天正經朔入交汎日, 6.1.1)

Crossing Entry, Midnight [preceding] Corrected Conjunction/Opposition *(ting shuo wang yeh-pan ju chiao* 定朔望夜半入交, 6.3.1)

Crossing Entry, Midnight of Next Conjunction (*tz'u shuo yeh-pan ju chiao* 次朔夜半入交, 6.3.2)

Crossing Regular Degrees *(chiao ch'ang-tu* 交常度, 6.5.1)

Cubic Difference *(li-ch'a* 立差, 3.2.1)

Daily Accumulated Difference *(mei jih chi-ch'a* 每日積差, 4.18.2)

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Daily Motion in Corrected *Tu (mei jih hsing ting tu* 每日行定度, 3.13.5)

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Daily Position of the Sun on the Red Way at Noon *(mei jih wu chung ch'ih-tao jih tu* 每日午中赤道日度, 3.16.2)

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Dawn Parts *(ch'en fen*, 晨分, 5.3.4)

Day Interval *(hsiang chü jih* 相距日, 3.13.1)

Day Marks for given date *(chou k'o* 晝刻, 5.4.2); for given location *(chou k'o* 晝刻, 5.9.3)

Day Rate for each grade *(chu tuan jih-lü* 諸段日率, 7.8.1)

Day-Night Difference *(chou yeh ch'a* 晝夜差, 5.1.8, table)

Days after Conjunction to Mean Crossing *(shuo hou p'ing-chiao* 朔後平交, 4.9.1)

Degree Interval *(hsiang chü tu* 相距度, 3.13.2)

Degree Rate *(tu-lü* 度率, 3.7.1)

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Degrees in the Yang Phase preceding the Crossing *(yang li chiao-ch'ien tu* 陽曆交前度, 6.11.1)

Degrees in the Yin Phase following the Crossing *(yin li chiao-hou tu* 陰曆交後度, 6.11.2)

Degrees in the Yin Phase preceding the Crossing *(yin li chiao-ch'ien tu* 陰曆交前度, 6.11.3)

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Direction of Solar Occultation *(jih-shih so ch'i* 日食所起, 6.18.1)

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Factor Surplus *(ts'e-yü* 策餘, 7.3.3)

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Inside/Outside Difference (*nei wai ch’a* 内外差, 5.1.3, table)
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Lunation Entry Mean Conjunction Days (*ju yueh ching shuo jih* 入月經朔日, 7.5.1)
Mean Conjunction (*p’ing-ho* 平合, 7.1.3)
Mean Motion Degrees (*p’ing-hsing tu* 平行度, 7.9.1)
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Revolution Entry of the Quarter Moon, Full Moon, and Next Conjunction *(hsien, wang, tz'u shuo ju chuan jih* 弦, 望, 次朔入轉日, 4.2.1)
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Winter Day/Summer Night Parts *(tung chou hsia yeh* 東晝夏夜, 5.1.6, table)
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Constants, English

Annual Difference: *sui-ch'a* 歲差 (0^t.0150), 3.C5
Anterior Standard: *ch'ien-chun* 前準 (166^t.3968), 6.C18
Argument Degrees: *li-tu* 曆度 (365^t.2575), 7.C1
Argument Factor: *li-ts'e* 曆策 (15^t.219 062 5), 7.C3
Argument Interval Constant: *li-ying* 曆應 (varies with planet), 7.C9
Argument Midway Constant: *li-chung* 曆中 (182^t.628 75), 7.C2
Argument Rate: *li-lü* 曆率 (varies with planet), 7.C6
Aspectual Limit: *hsiang-hsien* 象限 (91^t.314 375), 3.C4
Beginning Extent Constant: *ch'u-hsien* 初限 (84), 4.C4
Celestial Perimeter Parts: *chou-t'ien fen* 周天分 (3 652 575 parts), 3.C1
Celestial Perimeter: *chou-t'ien* 周天 (365^t.2575), 3.C2
Celestial Semi-perimeter: *pan-chou-t'ien* 半周天 (182^t.628 75), 3.C3
Ch'i Factor: *ch'i-ts'e* 氣策 (15 days 2184.37 5 parts), 1.C8
Ch'i Interval Constant: *ch'i-ying* 氣應 (550 600 parts), 1.C11
Ch'i Repletion Constant: *ch'i-ying* 氣盈 (2184.375 parts), 1.C14
Conjunction Interval Constant: *ho-ying* 合應 (varies with planet), 7.C8
Contraction Beginning/Expansion End Extent: *so ch'u ying mo hsien* 縮初盈末限 (93 days 7120.25 parts), 3.C9
Corrected Divisor: *ting-fa* 定法 (60), 6C12; (80), 6.C14; (87), 6.C16
Crossing Difference: *chiao-ch'a* 交差 (2 days 3183.69 parts), 6.C4
Crossing Interval Constant: *chiao-ying* 交應 (260 187.86 parts), 6.C6
Crossing Midway Constant: *chiao-chung* 交中 (13 days 6061.12 parts), 6.C3; (181^t.8967), 6.C8
Crossing Opposition Constant: *chiao-wang* 交望 (14 days 7652.965 parts), 6.C5
Crossing Terminal Constant: *chiao-chung* 交終 (27 days 2122.24 parts), 6.C2; (363^t.7934), 6.C7
Crossing Terminal Parts: *chiao-chung fen* 交終分 (272 122.24 parts), 6.C1
Cycle Day Constant: *chou-jih* 周日 (varies with planet), 7.C5
Cycle Extent Constant: *chou-hsien* 周限 (336), 4.C6
Cycle Rate: *chou-lü* 周率 (varies with planet), 7.C4

Day Cycle: *jih-chou* 日周 (10 000), 1.C1
Decad Cycle: *hsun-chou* 旬周 (600 000), 1.C16
Degree Rate: *tu-lü* 度率 (varies with planet), 7.C7
Disappearance Limit: *mo-hsien* 沒限 (7815.625 parts), 1.C13
Double-hour Divisor: *ch'en-fa* 辰法 (10 000), 2.C3
Dusk/Daybreak [Parts]: *hun-ming* 昏明 (250 parts), 5.C6
Earth Kingship Factor: *t'u-wang-ts'e* 土王策 (3.043 6875 days), 2.C1
Era Divisor: *chi-fa* 紀法 (60), 1.C17
Expansion Beginning/Contraction End Extent: *ying ch'u so mo hsien* 盈初縮末限 (88 days 9092.25 parts), 3.C8
Expansion/Contraction Corrected Difference: *ting-ch'a* 定差 (varies with planet), 7.C12
Expansion/Contraction Cubic Difference: *ying-so li-ch'a* 盈縮立差 (varies with planet), 7.C10
Expansion/Contraction Square Difference: *p'ing-ch'a* 平差 (varies with planet), 7.C11
Full Moon Constant: *wang* 望 (182^{t}.628 75), 4.C11
Full Moon Factor: *wang-ts'e* 望策 (14 days 7652.965 parts), 1.C9
Grade Days: *tuan-jih* 段日 (varies with planet), 7.C14
Initial Motion Rate: *ch'u-hsing-lü* 初行率 (varies with planet), 7.C17
Intercalation Interval Constant: *jun-ying* 閏應 (201 850 parts), 1.C12
Intermediate Crossing Constant: *chung-chiao* 中交 (188^{t}.05), 6.C10
Intermediate Extent Constant: *chung-hsien* 中限 (168), 4.C5
Invisibility/Appearance Constant: *fu-hsien* 伏見 (varies with planet), 7.C13
Limit Degrees: *hsien-tu* 限度 (varies with planet), 7.C16
Lower Quarter Moon Constant: *hsia-hsien* 下弦 (273^{t}.943 125), 4.C12
Lunar Eclipse Limit: *yueh-shih hsien* 月食限 (13^{t}.05), 6.C15
Lunation Deficiency Constant: *shuo-hsu* 朔虛 (4694.07 parts), 1.C15
Lunation Factor: *shuo-ts'e* 朔策 (29 days 530 5.93 parts), 1.C7
Lunation Intercalation Constant: *yueh-jun* 月閏 (9062.82 parts), 2.C2
Lunation Numerator: *shuo-shih* 朔實 (295 305.93 parts), 1.C4
Mark Divisor: *k'o-fa* 刻法 (1200), 2.C5
Maximum Elongation Difference, *tu*: [unnamed] (14^{t}.66), 4C.14]
Mean Degrees: *p'ing-tu* 平度 (varies with planet), 7.C15
Mean Lunar Motion: *yueh p'ing-hsing* 月平行 (13^{t}.368 75), 4.C7
Perimeter Interval Constant: *chou-ying* 周應 (3 151 075 parts), 3.C6
Polar Distance At Summer Solstice: *hsia-chih ch'ü chi* 夏至去極 (67^{t}.4113), 5.C3
Polar Distance at Winter Solstice: *tung-chih ch'ü chi* 冬至去極 (115^{t}.2173), 5.C2
Polar Distance, Ta-tu: Ta-tu *pei-chi ch'u ti* 大都北極出地 (40⅚ *tu)*, 5.C1
Posterior Standard: *hou-chun* 後準 (15½ tu), 6.C17
Quarter Moon Factor: *hsien-ts'e* 弦策 (7 d. 3826.4825 parts), 1.C10, 4.C9
Revolution Difference: *chuan-ch'a* 轉差 (1 d. 9759.93 parts), 4.C8
Revolution Interval Constant: *chuan-ying* 轉應 (131 904 parts), 4.C13

Revolution Midway Constant: *chuan-chung* 轉中 (13 d. 7773 parts), 4.C3
Revolution Terminal Constant: *chuan-chung* 轉終 (27 d. 5546 parts), 4.C2
Revolution Terminal Parts: *chuan-chung fen* 轉終分 (275 546 parts), 4.C1
Series Intercalation Constant: *t'ung-jun* 通閏 (108 753.84 parts), 1.C5
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Single-hour Divisor: *pan-ch'en-fa* 半辰法 (5000), 2.C4
Solar Eclipse Yang Argument Limit: *jih-shih yang-li hsien* 日食陽曆限 (6^{t}), 6.C11
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Summer Solstice Day/Winter Solstice Night parts: *hsia-chih chou tung-chih yeh* 夏至晝冬至夜 (6184.08 parts), 5.C5
Upper Quarter Moon Constant: *shang-hsien* 上弦 (91^{t}.314 375), 4.C10
Winter Solstice Day/Summer Solstice Night parts: *tung-chih chou, hsia-chih yeh* 冬至晝夏至夜 (3815.92 parts), 5.C4
Year Cycle: *sui-chou* 歲周 (365.2425 days), 1.C6
Year Numerator: *sui-shih* 歲實 (3 652 425 parts), 1.C2
Year Semicycle: *pan-sui-chou* 半歲周 (182 days 6212.5 parts), 3.C7

Named Results, Chinese

ch'a-lü 差率: Difference Rate, 3.7.3
ch'ang-chi 常積: Regular Accumulation, 7.18.1
cheng chi-tu 正積度: Standard Accumulated Degrees, 4.14.2
cheng-chiao hsien-tu 正交限度: Standard Crossing Limit Degrees, 6.10.1
cheng-chiao tsai erh chih hou ch'u hsien 正交在二至後初限: Standard Crossing after Solstice in Beginning Extent, 4.12.1
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ting-ch'a 定差: Expansion/Contraction Corrected Difference (varies with planet), 7.C12
ting-fa 定法: Corrected Divisor (60), 6.C12; (80), 6.C14; (87), 6.C16
tu-lü 度率: Degree Rate (varies with planet), 7.C7
t'u-wang-ts'e 土王策: Earth Kingship Factor (3.043 687 5 days), 2.C1
tuan-jih 段日: Grade Days (varies with planet), 7.C14
tung-chih chou, hsia-chih yeh 冬至晝夏至夜: Winter Solstice Day/Summer Solstice Night parts (3815.92 parts), 5.C4
tung-chih ch'ü chi 冬至去極: Polar Distance at Winter Solstice (115^{t}.2173), 5.C2
t'ung-jun 通閏: Series Intercalation Constant (108 753.84 parts), 1.C5
t'ung-yü 通餘: Series Surplus (52 425 parts), 1.C3
wang 望: Full Moon Constant (182^{t}.628 75), 4.C11
wang-ts'e 望策: Full Moon Factor (14 days 7652.965 parts), 1.C9
wu ming 無名 [unnamed]: [Maximum Elongation Difference, *tu* (14^{t}.66), 4C.14]
ying ch'u so mo hsien 盈初縮末限: Expansion Beginning/Contraction End Extent (88 days 9092.25 parts), 3.C8
ying-so li-ch'a 盈縮立差: Expansion/Contraction Cubic Difference (varies with planet), 7.C10
yueh p'ing-hsing 月平行: Mean Lunar Motion (13^{t}.368 75), 4.C7
yueh-jun 月閏: Lunation Intercalation Constant (9062.82 parts), 2.C2
yueh-shih hsien 月食限: Lunar Eclipse Limit (13^{t}.05), 6.C15

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I cite the Standard Histories in the text by their English translations in italics, e.g., *Yuan History;* in footnotes, I use their original titles. The edition used is that of Chung-hua Shu-chü, 1962–75. Classics are cited from texts in standard concordances unless otherwise noted. Abbreviations:
PP: Pai pu ts'ung-shu chi-ch'eng 百部叢書集成 series
SK: Ssu k'u ch'üan shu 四庫全書 series
TG: *Tōhō gakuhō* 東方學報 (Kyoto)
TKY: *Tzu-jan k'o-hsueh-shih yen-chiu* 自然科学史研究
YWL: *Yuan wen lei* 元文類

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Index-Glossary

This index includes names of people, book titles, methods and ideas, as well as Chinese technical terms discussed in all except the Canon (chapters 9–10). Of the 98 astronomical systems listed in Table 2.1, it lists only those discussed in the body of the book. The names of terms, constants, intermediate quantities, and results in the Canon are given in Appendix C. "SGS" below is an abbreviation for "Season-granting system."