The Universe of Fluctuations

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The Universe of Fluctuations

The Architecture of Spacetime and the Universe

by

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Dedicated to the memory of my parents

"There was neither existence nor non existence"

- Rig Veda

Contents

Preface

In 1997, contrary to the ruling paradigm which was that of a dark matter filled, decelerating universe, my work pointed to a dark energy driven accelerating universe with a small cosmological constant. Moreover, the many supposedly accidental Large Number relations in cosmology, including the mysterious Weinberg formula were now deduced from the theory. Observational confirmation for this scenario came in 1998, while dark energy itself was finally reconfirmed in 2003, thanks to the Wilkinson Microwave Anisotropy Probe and the Sloan Digital Sky Survey.

The 1997, and subsequent work was the consequence of mainly three considerations: dark energy or the well known Zero Point Field, fuzzy spacetime and fluctuations. Indeed String Theory and Quantum Gravity approaches have had to discard the smooth spacetime of General Relativity and Quantum Field Theory, in a quest for a unified description of these two pillars of twentieth century physics.

This book is the result of some seventy five papers published in international journals, and partly an earlier book, "The Chaotic Universe: From the Planck to the Hubble Scale" (Nova Science, New York, 2001), as also several lectures delivered in Universities and institutes in the United States, Canada and Europe. It describes how, in a simple and somewhat conventional framework, an underpinning of Planck scale oscillators in the ubiquitous Zero Point Field or dark energy leads to a unified description of phenomena involving elementary particles and the cosmos. In particular, apart from the cosmology mentioned above, these considerations lead to a unified description of all interactions, including gravitation, though in an extended gauge field treatment. Furthermore, it brings out the character of gravitation as being quite different from other interactions. It is distributional in nature, over all elementary particles in the universe, rather than being a microphysical interaction in the sense of electromagnetism. This incidentally resolves a paradox pointed out years ago by Steven Weinberg, and which has been since overlooked. Pleasingly we recover conventional theory in a suitable limit - when the "quantum of area" is neglected, infact.

Chapter 1 gives a flavour of the considerations that lead to the above model. Chapters 2,3 and 4 summarize very briefly the standard models and other approaches like Quantum Superstring Theory and Loop Quantum Gravity.

Chapters 5 to 8 discuss the above scenario in detail while Chapter 9 deduces on the basis of dynamics, a simple formula which gives the masses of all known elementary particles with a maximum error of about three percent. Finally Chapter 10 describes some further experimental and observational consequences. These include a new short range force rather like the postulated B^3 force, the anomalous accelerations of the Pioneer spacecrafts and tests for detecting violations of Lorentz symmetry, planned for studies of Ultra High Energy Cosmic Rays. The treatment in the book makes it accessible to Graduate students in physics and Junior and Senior Researchers in High Energy Physics and Cosmology.

I am thankful to Professor G.'t Hooft and the Late Professor Ilya Prigogine for valuable discussions.

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DECEMBER 2004 *B.G. SIDHARTH*

1 THE UNIFICATION PARADIGM

"....the aim is to see complete nature as different aspects of one set of phenomena..."

R.P. Feynman

1.1 Introduction

If we look back at prehistory, we find bewildered man assigning to different natural phenomena, different controlling powers or deities. But gradually, we could discern underlying common denominators. Over the millennia man's quest for an understanding of the universe has been to perceive disparate phenomena in terms of a minimal set of simple principles. Today looking back we can see the logic of Occam's razor (literally, "A satisfactory proposition should contain no unnecessary complications"), or an economy of hypothesis– a far cry from prehistoric times.
In the words of F.J. Dyson[1], ".... the very greatest scientists in each discipline are unifiers. This is especially true in Physics. Newton and Einstein were supreme as unifiers. The great triumphs of Physics have been triumphs of unification. We almost take it for granted that the road of progress in Physics will be a wider and wider unification...".
Sir Isaac Newton was the first great unifier. He discovered the Universal Law of Gravitation: The force which kept the moon going round the earth, or the earth round the sun was also the force which kept binary stars going around each other and so on. All this was basically the same force of gravitation which brought apples down from a tree. This apart his Laws of Motion were also universal.
In the nineteenth century the work of Faraday, Ampere and others showed the close connection between the apparently totally dissimilar forces of electricity and magnetism. It was Maxwell who unified electricity not just with magnetism but with optics as well[2].
There was another great unification in the nineteenth century: Thermodynamics linked the study of heat to the kinetic theory of gases[3].
In the early part of the twentieth century Einstein fused space and time, giving them an inseparable identity, the Minkowski spacetime[4]. He went on

to unify space-time with gravitation in his General Theory of Relativity[5]. However the unification of electromagnetism and gravitation has eluded several generations of physicists, Einstein included [6].
Meanwhile, thanks to the work of De Broglie and others, the newly born Quantum Theory unified the two apparently irreconcilable concepts of Newton's "particles" and Huygen's waves[7].
Yet another unification in this century, which often is not recognised as such is the fusion of Quantum Mechanics and Special Relativity by Dirac, through his celebrated equation of the electron[7].
Another unification took place in the seventies due to the work of Salam, Weinberg, Glashow and others– the unification of electromagnetism with the weak forces. This has given a new impetus to attempts for unifying all interactions, gravitation included.
The weak force is one of two forces, the other being the strong force, discovered during the twentieth century itself. Earlier studies and work revealed that there seemed to be three basic particles in the Universe, the protons, the neutrons and the electrons. While the proton and the electron interact via the electromagnetic force, in the absence of this force the proton and the neutron appear to be a pair or a doublet. However the proton and the neutral neutron interact via "strong forces", forces which are about ten times stronger than the electromagnetic but have a much shorter range of just about $10^{-13} cms$. These are the forces which bind, for example, the protons in the nucleus.
The existence of the neutrino was postulated by Pauli in 1930 to explain the decay of the neutron, and it was discovered by Reines and Cowan in 1955. The weak force which is some 10^{-13} times the strength of the electromagnetic force is associated with neutrino type particles and has an even shorter range, $10^{-16} cms$. The neutrino itself has turned out to be one of the most enigmatic of particles, with peculiar characteristics, the most important of which is its handedness. This handedness property appears to be crucial for weak forces.
Later work revealed that while particles like the electron and neutrino, namely the leptons may be "truly" elementary, particles like the protons may be composite, infact made up of still smaller objects called quarks – six in all[8]. Today it is believed that the quarks interact via the strong forces.
All these "material" particles are Fermions, with half integral spin. Forces or interactions while originating in Fermions, are mediated by messengers like photons which are Bosons, with integral spin, spin 1 infact. This is crucial, for, now there is the formalism of gauge theory which can describe all these interactions.
In this sense gravitation is not a gauge force. It is supposedly mediated by particles of spin 2.
To picturize the above let us consider the interaction between a proton and an electron. A proton could be imagined to emit a photon which is then absorbed by the electron. These studies, in the late forties and fifties culminated

in the highly successful theory of Quantum Electro Dynamics or QED.
Instead of a single mediating particle we could think of multiplets, all having equal masses. With group theoretical inputs, one could shortlist, singlets with one particle like the photon, triplets, octets and so on as possible candidates[8].
Motivated by the analogy of electromagnetism mediated by the spin one photon, it was realized in the fifties that the W^+ and W^- Bosons could be possible candidates for the mediation of the weak force. However there had to be one more messenger so that there would be the allowable triplet. It was suggested by Ward and Salam that the third candidate could be the photon itself, which would then provide not only a description of the weak force but would also unify it with electromagnetism. However while the W particles were massive, the photon was massless so that they could not form a triplet. A heavy photon or Z^0 was then postulated to make up a triplet, while the photon was also used for the purpose of unification, and moreover a mixing of Z^0 and the photon was required for what has been called renormalization, that is the removal of infinities.
The question was how could the photon be massless while the W and Z particles would be massive? It was suggested that this could be achieved through the spontaneous breaking of symmetry[8]. For example a bar magnet when heated, looses its magnetism. In effect the North and South pole symmetry is broken. Conversely, when the magnet cools down, polarity or asymmetry is restored spontaneously. This infact is a phase transition from symmetry to asymmetry.
In our case, before the spontaneous breaking of symmetry or the phase transition, the Ws, Zs, and the photons would all be massless. After the phase transition, while the photons remain mass less, the others would acquire mass. This phase transition would occur at temperatures $\sim 10^{15^\circ}$ Centigrade. At higher temperatures there would be a single electroweak force. As the temperature falls to the above level electromagnetism and weak forces would separate out.
The next problem was, the inclusion of the strong forces. Clearly the direction to proceed appeared to be to identify the gauge character of the strong force– mediated by spin one particles, the gluons. (The approach differed from an earlier version of strong interaction in terms of Yukawa's pions.) This force binds the different quarks to produce the different elementary particles, other than the leptons. This is the standard model. It must be mentioned that in the standard model, the neutrino is a massless particle.
However we have not yet conclusively achieved a unification of the electroweak force and the strong force. We proceed by the analogy of the electroweak unification to obtain a new gauge force that has been called by Jogesh Pati and Abdus Salam as the electro nuclear force, or in a similar scheme the Grand Unified Force by Glashow and Georgi. It must be mentioned that one of the predictions is that the proton would decay with a life time of about 10^{32}

years, very much more than the age of the Universe itself. However some believe that we are near a situation where this should be observable[9]. This "unifying" theory still relies on eighteen arbitrary parameters, apart from being plagued by problems like the "hierarchy problem", which arises from the widely different energies and therefore masses associated with the various interactions, the as yet non-existent monopole, infinities or divergences (which have to be eliminated by renormalization), and so on[10]-[12].

The recent super Kamiokande determination of neutrino mass is the first evidence of what may be called, Physics beyond the standard model. Interestingly in this theory we would also require a right handed neutrino in this case.

Meanwhile extended particles had come into vogue from the seventies, with string theory[13]-[20]. Starting off with objects of the size of the Compton wavelength, the theory of superstrings now deals with the Planck length of about $10^{-33} cms$.

We have already noted that all interactions except gravitation which is mediated by spin 2 gravitons are generalizations of the electromagnetic gauge theory. String theory combines Special Relativity, and General Relativity - we need ten, $(9+1)$, dimensions for quantizing strings, and we also get a mass less particle of spin two which is the mediator of the gravitational force. This way there is the possibility of unifying all interactions including gravitation. Further, in the above ten dimensions there are no divergences. This is because the spatial extension of the string fudges the singularities (or vertices). However, we require, for verification of the string model, energies $\sim 10^{18} m_P$, as against the presently available $10^3 m_P$.

Another interesting feature of string theory is <u>duality</u>. There are five different solutions (compactifications) leading to the same physical picture. It is felt that these five theories are but different descriptions of a single, deeper, and may be more complicated theory.Over the past decade, the so called M-Theory has superseded Quantum Super String theory, though it is still not clear what the M stands for. M-Theory works in eleven dimensions.

The exotic dimensions, the Large Number of solutions and the non verifiable nature of the theory are some of the unsatisfactory features of this development[21], just as the 18 arbitrary parameters the hierarchy of energies, and the unseen monopoles are some of the unsatisfactory features of the standard model.

There have been other approaches to unifying gravitation and electromagnetism [22], including Loop Quantum Gravity: All these approaches differ from the standard model in that there is no longer a differentiable spacetime manifold, in which we could go down to arbitrarily small intervals. Rather there is a cut off at the Planck scale $\sim 10^{-33} cm$ and $10^{-42} sec$. We will be returning to these matters in Chapters 2 and 3 and thereafter.

1.2 Inertial Mass

In contrast to the Planck scale, ordinary Quantum Mechanics works at distances much greater than the Compton wavelength of elementary particles, roughly $10^{-12}cm$. In the domain of Quantum Field Theory, particles are points, spacetime is a continuum and Special Relativity holds. On the other hand in Quantum Gravity as mentioned we attempt to deal with phenomena at distances of the order of the Planck length or $10^{-33}cm$. As of now there has been no successful unification of Quantum Mechanics and General Relativity or gravitation.
At the Compton scale Quantum Mechanical phenomena like zitterbewegung, negative energy solutions and luminal velocities come in to play. Veneziano, one of the founding fathers of string theory, has termed the Compton scale as a "miracle" [23]. We will briefly indicate a scenario in which, we can trace the origin of inertial mass, gravitation, and even QCD type interactions. This would set the stage for considerations which we will encounter from Chapter 5 onwards.
We start from a Quantum Mechanical point of view on the lines elaborated in ref. [24]. Let us consider an equation deduced by Feynman[25] in a simple way,

$$\imath\hbar\frac{\partial C(x)}{\partial t} = \frac{-\hbar^2}{2m'}\frac{\partial^2 C(x)}{\partial x^2} \tag{1.1}$$

where $C(x) \equiv |\psi(x)>$ is the probability amplitude for the particle to be at the point x at some given moment of time.
To deduce equation (1.1) we follow the development of[25] and define a complete set of base states by the subscript $\imath$ and $U(t_2, t_1)$ the time elapse operator that denotes the passage of time between instants t_1 and t_2, t_2 greater than t_1. We denote by, $C_\imath(t) \equiv <\imath|\psi(t)>$, the amplitude for the state $|\psi(t)>$ to be in the state $|\imath>$ at time t, and

$$<\imath|U|j> \equiv U_{\imath j}, U_{\imath j}(t + \Delta t, t) \equiv \delta_{\imath j} - \frac{\imath}{\hbar}H_{\imath j}(t)\Delta t.$$

We can now deduce from the super position of states principle that,

$$C_\imath(t + \Delta t) = \sum_j [\delta_{\imath j} - \frac{\imath}{\hbar}H_{\imath j}(t)\Delta t]C_j(t)$$

and finally, in the limit,

$$\imath\hbar\frac{dC_\imath(t)}{dt} = \sum_j H_{\imath j}(t)C_j(t) \tag{1.2}$$

where the matrix $H_{\imath j}(t)$ is identified with the Hamiltonian operator. (To facilitate comparison we stick to the notation and development as given in[25]). Before proceeding to derive the Schrödinger equation, we apply equation (1.2) to the simple case of a two state system ($\imath, j = 1, 2$) (cf.ref.[25]). For a two state system we have

$$\imath\hbar \frac{dC_1}{dt} = H_{11}C_1 + H_{12}C_2$$

$$\imath\hbar \frac{dC_2}{dt} = H_{21}C_1 + H_{22}C_2$$

leading to two stationary states of energies $E - A$ and $E + A$, where $E \equiv H_{11} = H_{22}, A = H_{12} = H_{21}$. We can choose our zero of energy such that $E = 2A$. Indeed as has been pointed out by Feynman, when this consideration is applied to the hydrogen molecular ion, the fact that the electron has amplitudes C_1 and C_2 of being with either of the hydrogen atoms, manifests itself as an attractive force which binds the ion together, with an energy of the order of magnitude $A = H_{12}$.

To proceed, we consider in (1.2), the $\imath$ to be the space points $x_\imath$ and we denote $C(x_n) \equiv C_n$, the probability amplitude for the particle to be at this space point. Further let $x_{n+1} - x_n = b$. Then considering only the point x_n and its neighbours $x_{n\pm 1}$, the equation (1.2) goes over into

$$\imath\hbar \frac{\partial C(x_n)}{\partial t} = EC(x_n) - AC(x_n - b) - AC(x_n + b) \tag{1.3}$$

In the limit $b \to 0$, with our choice of the arbitrary zero of energy, (1.3) goes over into equation (1.1) where we have now dropped the subscript distinguishing the space point, and $m' = \hbar^2/2Ab^2$. (Shortly, we will see that infact, b does not $\to 0$, but rather $b \to \bar{b} > 0$.)

We now observe that while equation (1.1) resembles the free Schrödinger equation, it is infact not so, because as has been pointed out by Feynman, m' is not really the inertial mass, but an "effective mass" that emerges from the probability amplitude for the particle to be found at a neighbouring point.

The Schrödinger equation can be obtained from (1.1) if it can be shown that m' can somehow be replaced by m. This is what we propose to do.

To start with let us suppose that the particle has no mass other than the effective mass m', so that we can treat equation (1.1) as the Schrödinger type equation for such a particle which has only amplitudes to be at neighbouring points.

Let us now go one step further and suppose that the particle acquires non zero probability amplitudes to be present non locally at other than neighbouring points. We can then no longer work with equations (1.3) and (1.1). We will have to use the full equation (1.2) which explicitly exhibits this possibility. We rewrite equation (1.2) as

$$\imath\hbar \frac{dC_\imath(t)}{dt} = H_{\imath\imath}C_\imath(t) + H_{\imath,\imath-1}C_{\imath-1}(t) + H_{\imath,\imath+1}C_{\imath+1}(t)$$
$$+ \sum_j H_{\imath,\imath+j}(t)C_j(t), (j = \pm 2, \pm 3 \cdots,)$$

or as in the transition of equation (1.3) to equation (1.1),

$$\imath\hbar\frac{\partial C(x)}{\partial t} = \frac{-\hbar^2}{2m'}\frac{\partial^2 C(x)}{\partial x^2} + \int H(x,x')C(x')dx' \tag{1.4}$$

where we have replaced $H_{\imath j}$ by $H(x, x')$ and the points $x_\imath$ are in the limit taken for the time being to be a continuum. This is as in the well known case of the non-local Schrödinger equation for a non-local potential[26] but for a particle having only an effective mass.
The matrix $H(x, x')$ gives the probability amplitude for the particle at x to be found at x', that is,

$$H(x,x') = <\psi(x')|\psi(x)> \tag{1.5}$$

where as is usual we write $C(x) \equiv \psi(x)(\equiv |\psi(x)>$, the state of a particle at the point x).
Usually the amplitude $H(x, x')$ is non-zero only for neighbouring points x and x', that is, $H(x, x') = f(x)\delta(x - x')$. But if $H(x, x')$ is not of this form, then there is a non-zero amplitude for the particle to "jump" to an other than neighbouring point. In this case $H(x, x')$ may be described as a non local amplitude. Indeed such non-local amplitudes are implicit in the Dirac equation also and this will be commented on later.
We now give a quick derivation of how the inertial mass emerges from equation (1.4). The non local Schrödinger equation (1.4), given only the effective mass m', can be written, with the help of (1.5), as,

$$\imath\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m'}\frac{\partial^2\psi}{\partial x^2} + \int \psi^*(x')\psi(x)\psi(x')U(x')dx', \tag{1.6}$$

where,
i)$U(x) = 1$ for $|x| < R, R$ arbitrarily large and also $U(x)$ falls off rapidly as $|x| \to \infty; U(x)$ has been introduced merely to ensure the convergence of the integral; and
ii)$H(x, x') = <\psi(x')\psi(x) >= \psi^*(x')\psi(x)$.
(1.6) is an integro-differential equation of degree three.
The presence of the, what at first sight may seem troublesome, non-linear and non-local term, viz., the last term on the right side of (1.6) will be satisfactorily explained in the sequel.
In (1.6),in the first approximation $\psi(x)$ can be taken to be the solution of the Schrödinger like equation (1.1), viz.,

$$\imath\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m'}\frac{\partial^2\psi}{\partial x^2} \tag{1.7}$$

In effect, we linearize (1.6), so that we get,

$$\imath\hbar\frac{\partial\psi}{\partial t} = [-\frac{\hbar^2}{2m'}\frac{\partial^2}{\partial x^2} + m_0]\psi \tag{1.8}$$

where,

$$m_0 = \int \psi^*(x')\psi(x')U(x')dx' \tag{1.9}$$

This is the crux of the matter–the origin of the inertial mass in non local, what may be called, self-interaction amplitudes. To proceed we observe that in operator language, (1.8) becomes,

$$\bar{H} = \frac{p^2}{2m'} + m_0 \tag{1.10}$$

where $\bar{H}$ is the Hamiltonian operator, p the momentum operator and where, what can now be anticipated as a rest mass like term m_0, appears for a particle assumed not to have any rest mass in the absence of the non-local amplitude term in (1.6). Also we have replaced the Hamiltonian matrix H by $\bar{H}$ to stress that, to start with, in (1.4) and (1.6), the particle has no inertial mass. To facilitate comparison with the usual theory, we next multiply both sides of (1.10) by the constant $\frac{m'}{m}$, where,

$$m = (m_0 m')^{\frac{1}{2}}/c,$$

c being the velocity of light. (The reason for the appearance of the velocity of light, c can be seen below (cf.equation (1.12)) and the constant could be absorbed into the state vector, whose direction is all that matters. We then get,

$$\hat{H} = \frac{p^2}{2m} + mc^2 \tag{1.11}$$

The physical meaning of (1.11) is now clear. In an expansion of the classical relativistic expression for energy,

$$E = (p^2c^2 + m^2c^4)^{1/2}$$

as is well known, if we keep terms up to the order $(p/mc)^2$, we get,

$$E = \frac{p^2}{2m} + mc^2 \tag{1.12}$$

We can now easily identify m in (1.11) with the rest mass on comparing this equation with (1.12). (Interestingly it is not accidental that equation (1.11) corresponds to the approximation (1.12) as will be seen below). If further, we denote

$$H = \hat{H} - mc^2,$$

where H can be easily identified with the usual kinetic energy operator (or energy operator in non-relativistic theory, remembering that we are considering a free particle only), (1.11) becomes

$$H = \frac{p^2}{2m} \tag{1.13}$$

In a strictly non relativistic context, where the rest energy of the particle is not considered, the Hamiltonian is given by (1.13); otherwise, it is given approximately by (1.11).
We get from (1.6) or (1.11), the Schrödinger equation,

$$\imath\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \tag{1.14}$$

Interestingly, in the derivation of the Schrödinger equation (1.14), or the Hamiltonian (1.13), we have not used Newtonian mechanics! In our purely Quantum Mechanical approach based on probability amplitudes, if we use energy and momentum eigen states, then these operators become numbers in equations like (1.13). (This is the reverse of the usual procedure.) In effect we have deduced Newtonian mechanics.
All these considerations can be considered in a postulative development [27] and also generalized in a simple way to three dimensions, but there is no new physical insight.
The physical origin of the rest mass is clear from equation (1.6): in the two state hydrogen molecular ion case referred to earlier, it was the amplitude for the single electron to be with one hydrogen atom or the other which showed up as a binding energy. Similarly the amplitude of a particle to be at x or x' viz. the second term on the right side of equation (1.6) manifests itself as an (attractive) energy, which may be called the mass energy of the particle or the self energy or the energy of self interaction. This can be seen to be the particle's inertial mass.
We now come to this non local term in equation (1.6), the term which gives the inertial mass. Non locality implies superluminal velocities and the breakdown of causality which is not permissible in general. However without any contradiction to the theory it is well known that Quantum Mechanics allows such non locality, owing to the Uncertainty Principle [28], within the Compton wavelength of a particle. So there is no contradiction if the non local integral in (1.6) is taken within the region of the particle's Compton wavelength, that is, the inertial mass is a result of non local processes within the Compton wavelength of the particle.
Indeed the usual Dirac equation also has a non local character: The operator $c\boldsymbol{\alpha}.\boldsymbol{p}+\beta mc^2$ is equivalent to and replaces the non-local square-root operator, $(-\hbar^2\nabla^2 + m^2c^4)^{1/2}$. Here also, the non-local effects in the form of negative energies or zitterbewegung are encountered - again within the Compton wavelength region (cf.ref.[29, 7]). We will see in Chapters 5 and 6, in particular, that the Compton scale in any case is ill defined in the sense that spacetime points in it are ill defined - this arises out of a noncommutative geometry. We shall also see from an alternative point of view how the inertial mass or energy emerges from within this scale, as also characteristics like spin.
In the light of the preceding considerations, we can derive the Schrödinger equation from an alternative angle: It appears that the "point" particle is really spread over the non-locality region $\sim \bar{b} = \frac{\hbar}{mc}$, the Compton wavelength.

Further, the energy of the particle i.e., the energy tied up within this region viz., 2A is the inertial mass energy mc^2. We could now, speak of the amplitude for the particle at x to be found (locally) at a neighbouring point $x+b$, except that in the limit, $b \to \bar{b}$ (and not as earlier,0). The effective mass m' in equation (1.1) is then given by,

$$m' = \frac{\hbar^2}{2Ab^2} = m,$$

that is the mass itself!
So, equation (1.1) can be interpreted as the Schrödinger equation with this input.
It is worth re-emphasizing that it is the force of binding of non-local positions within the Compton wavelength, rather like the Hydrogen molecular ion binding, that manifests itself as inertial mass.
Finally we briefly comment on the appearance of the extra mass energy term in equations like (1.4), (1.6), (1.10), (1.11) or (1.12)[30, 31].
The Schrödinger equation is really the limiting case of the Dirac equation (which, as we will see in Chapter 5 is primary) in which process an inessential phase factor is dropped. Another way of looking at this is that the constant potential $m_o c^2$ does not affect the dynamics. That is the reason why the Schrödinger equation is not Galilean invariant, as a non relativistic theory should be, and infact exhibits the Sagnac effect, which a strictly Galilean invariant theory should not(Cf.ref.[32] for details).

1.3 Enter General Relativity

The fact that, as we saw in section 2, the mass generating non-local amplitudes are confined to a region of width $\sim \frac{\hbar}{mc}$ the Compton wavelength, suggests that the particle could be a Black Hole, because in this case also, there is a width, the horizon, inside which such unphysical phenomena appear. The simplest possibility that a particle could be a Schwarzchild Black Hole had been examined earlier by Markov, Motz and others[33]-[43]. This leads to a high particle mass, the Planck mass of $10^{-5}gm$, with the Planck size $\sim 10^{-33}cms$ [45]. Infact we can verify that in this case, the relation

$$R \sim \frac{2GM}{c^2}, \tag{1.15}$$

holds. Equation (1.15) gives, the Schwarzchild radius[46]. However, Quantum Mechanical properties, like spin, do not appear. Interestingly, Rosen[47] has shown that such a Planck particle, a mini Universe, can be deduced from the Schrödinger equation with the gravitational potential. Other authors have considered charged or spinning and even charged spinning Black Holes [48]-[58].

Let us approach the problem from a different angle. We consider a charged Dirac (spin half) particle. If we treat this as a spinning Black Hole, then a logical candidate would be the Kerr-Newman Black Hole, which is a charged and spinning version of the Schwarzchild black hole. But there is an immediate problem:The horizon of the Kerr-Newman Black Hole becomes in this case, complex[59, 46],

$$r_+ = \frac{GM}{c^2} + \imath b, b \equiv (\frac{GQ^2}{c^4} + a^2 - \frac{G^2M^2}{c^4})^{1/2} \tag{1.16}$$

where G is the gravitational constant, M the mass and $a \equiv L/Mc, L$ being the angular momentum. That is, we have a naked singularity apparently contradicting the cosmic censorship conjecture. However, in the Quantum Mechanical domain, (1.16) can be seen to be meaningful.

This is because the position coordinate for a Dirac particle is itself complex. The real part is the usual position while there is an imaginary part arising from zitterbewegung. Interestingly, in both cases, the imaginary part is of the order of $\frac{\hbar}{mc}$, the Compton wavelength, and leads to an immediate identification of these two equations. It is remarkable that (1.16) is a purely classical relation while the Dirac coordinate is purely Quantum Mechanical. We will see the significance of this later.

We must remember that our physical measurements are gross - they are really measurements averaged over a width of the order $\frac{\hbar}{mc}$. Similarly, time measurements are imprecise to the tune $\sim \frac{\hbar}{mc^2}$. This will be a recurrent theme in this book. Very precise measurements if possible, would imply that all Dirac particles would have the velocity of light, or in the Quantum Field Theory atleast of Fermions, would lead to divergences. (This is closely related to zitterbewegung and the non-Hermiticity of position operators in relativistic theory [60]). Physics begins after an averaging over the above unphysical space-time intervals. In the process as is known (cf.ref. [60]), the imaginary or non-Hermitian part of the Dirac position operator disappears. That is the naked singularity is shielded by the Quantum Mechanical zitterbewegung censor if, as Dirac originally implied, we renounce spacetime points [7].

It is relevant to mention here that one of the conceptual issues that has dogged Physicists is the question whether spacetime is an apriori background container, a purely geometrical structure in which matter and interaction have their play or whether it is the content that defines spacetime. The former was the Newtonian view while Leibnitz was more in tune with the latter picture. Einstein with his Special Relativity and then General Relativity made spacetime more physical - the geometry depended on the contents. In Quantum Theory, the concept of a spacetime point, which is at the heart of a continuum picture, loses its legitimacy due to Heisenberg's Uncertainty Principle - such points imply infinite momenta and energies. Even so, spacetime has continued to be considered a differentiable manifold, for much of twentieth century physics. Indeed Quantum Field Theory takes the Lorentzian underpinning for granted.

The picture becomes different, if we consider Planck scale phenomena whether in Quantum Gravity or String Theory ([37]-[45]). Here, we have to consider spacetime as resulting from fluctuating subconstituents at the Planck scale. As Wheeler[46] put it, "The Uncertainty Principle thus deprives one of any way whatsoever to predict, or even to give meaning to, 'the deterministic classical history of space evolving in time.' No prediction of spacetime, therefore no meaning for spacetime, is the verdict of the quantum principle. That object which is central to all of classical General Relativity, the four-dimensional spacetime geometry, simply does not exist, except in a classical approximation."
It is in this context that the above averages over minimum intervals or studies of a discrete space-time substructure become important. Already, the limitations of the concepts of rigid scales or space-time points as indicated, are coming to the force (Cf.ref.[24]).
We will encounter this aspect later in Chapter 5 where we will discuss "fuzzy" spacetime and complex coordinates more fully.
To continue, we treat for the moment a Dirac particle as a Kerr-Newman Black Hole of mass m, charge e and spin $\frac{\hbar}{2}$. The gravitational and electromagnetic fields at a distance are given by (cf.ref.[46],[61]-[63]),

$$\Phi(r) = -\frac{Gm}{r} + 0(\frac{1}{r^3}) E_{\hat{r}} = \frac{e}{r^2} + 0(\frac{1}{r^3}), E_{\hat{\theta}} = 0(\frac{1}{r^4}), E_{\hat{\phi}} = 0,$$
$$B_{\hat{r}} = \frac{2ea}{r^3} cos\theta + 0(\frac{1}{r^4}), B_{\hat{\theta}} = \frac{easin\theta}{r^3} + 0(\frac{1}{r^4}), B_{\hat{\phi}} = 0, \quad (1.17)$$

exactly as required. The remarkable fact has been well known that the Kerr-Newman metric (1.17) also exhibits the electron's purely Quantum Mechanical anomalous gyromagnetic ratio $g = 2$. Again (1.17) is purely classical and the g value purely Quantum Mechanical.
We now make some observations:
1. In the context of the Compton scale extension, the fact that we get the gravitational potential $\frac{m}{r}$ in equation (1.17) again confirms that mass comes from the Compton wavelength region.
2. In ordinary Quantum Mechanics, ψ being the wave function, $\psi\psi^*$ is proportional to the probability density. On the other hand, we saw that the mass density is produced by the non-linear, non-local amplitude $\psi\psi^*$ in the Compton wavelength region. More specifically it is known that it is χ, the negative energy part of the Dirac four spinor

$$\begin{pmatrix} \chi \\ \Theta \end{pmatrix}$$

(which dominates in this region), that is relevant [29]. That is, ρ being the material density,

$$\rho \alpha \chi\chi^* \quad (1.18)$$

For the two component neutrino in contrast, the divide between the positive and negative energy solutions does not exist, that is, $\chi = 0$, and the neutrinos

are massless.
It was shown in [64, 65], how gravitation can emerge from the Schrödinger equation self-consistently. Again, it is the identification of the material density in (1.18) which gives substance to that result. We will return to these aspects in Chapters 5, 6 and 8.
3. It is interesting to note that the above model of a particle could give a rationale for the left handedness of the neutrino. In the case of the neutrino, as the mass is vanishingly small, the Compton wavelength tends to infinity or turns out to be very large. On the other hand we encounter the negative energy solutions within this region. That is we encounter "negative energy" neutrinos only. The equation for a negative energy neutrino is (cf. ref.[60]).

$$(-p_o)v(p) = +\boldsymbol{\sigma}.\boldsymbol{p}v(p)$$

This is the equation for a left handed neutrino in the physical world of positive energy solutions. Infact it is known that[66] we need the Dirac 4 spinors only to preserve space reflection symmetry- working only with 2 spinors would violate this symmetry. This is the case at the Compton wavelength, where it is the negative energy 2 spinors of the Dirac 4 spinor which predominate [29]. So neutrinos show handedness. In the next Chapter, this will be seen to be true of quarks also.

1.4 Further Considerations

Ever since Einstein put forward his theory of gravitation in 1915, a problem that has vexed physicists including Einstein himself is the incorporation of electromagnetism into the theory of gravitation. Einstein himself said it all in his Stafford Little Lectures delivered in May 1921 at Princeton University[4], "... a theory in which the gravitational field and the electromagnetic field do not enter as logically distinct structures would be much preferable..."
The basic problem is that General Relativity belongs to the domain of classical physics whereas electromagnetism belongs to the domain of "elementary electrically charged particles", that is Quantum Theory, more specifically the theory of the electron. And, as J.A. Wheeler[46] put it, "the most evident shortcoming of the geometrodynamic model as it stands is this, that it fails to supply any completely natural place for spin $\frac{1}{2}$ in general and for the neutrino in particular", while "it is impossible to accept any description of elementary particles that does not have a place for spin half." This apart it should be remembered that the spacetime we speak of in General Relativity is not only deterministic, but we also speak in terms of definite points of spacetime. This is forbidden in Quantum Theory by the Uncertainty Principle. Infact four dimensional spacetime exists only as a classical approximation[46].
However, the characterization of an electron as a Kerr-Newman type Quantum Mechanical Black Hole or QMKNBH already gives a clue to a unified

picture of gravitation and electromagnetism: For example Equation (1.17) gives both for the electron (Cf.ref. [24] for a more detailed discussion).
It appears that the QMKNBH description applies to electrons and more generally leptons. Let us now approach the problem from a General Relativistic point of view. This will also reveal the origin of strong or, QCD type interactions. Taking the cue from the foregoing considerations, we now treat the particle as a relativistic fluid of "particlets" (or Ganeshas). Our starting point is the linearized theory [46]:

$$g_{\mu v} = \eta_{\mu v} + h_{\mu v}, h_{\mu v} = \int \frac{4T_{\mu v}(t - |\boldsymbol{x} - \boldsymbol{x}'|, \boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|} d^3 x' \tag{1.19}$$

The reason we consider the linearized theory is, that outside the Schwarzchild radius, as is well known the metric becomes asymptotically flat, and this is the region of our interest[5].
In (1.19), velocities comparable to the velocity of light c are allowed and also the stresses T^{jk} and momentum densities T^{0j} can be comparable to the energy momentum density T^{00}. As in ref.[46], we can easily deduce that, when $\frac{|\boldsymbol{x}'|}{r} << 1$, where $r \equiv |\boldsymbol{x}|$, and in a frame with origin at the centre of mass and at rest with respect to the particle, and in units in which the gravitational constant G is unity,

$$m = \int T^{00} d^3 x \tag{1.20}$$

$$S_k = \int \epsilon_{klm} x^l T^{m0} d^3 x \tag{1.21}$$

where m is the mass (or approximate mass because of the linear approximation), and S_k is the angular momentum. We next observe that,

$$T^{\mu v} = \rho u^\mu u^v \tag{1.22}$$

If we now work in the Compton wavelength region of the QMKNBH, we have, while $u^0 = 1$,

$$|u^l| = c \tag{1.23}$$

(This is the Quantum Mechanical input)
Substitution of (1.22) and (1.23) in (1.21) gives on using the Mean Value Theorem,

$$S_k = c < x^l > \int \rho d^3 x$$

As $< x^l > \sim \frac{\hbar}{2mc}$, using (1.20), we get, $S_k \approx \frac{\hbar}{2}$, as required for a spin half particle. Infact this relation becomes exact if we treat the QMKNBH as effectively a rotating shell distribution of radius $\hbar/2mc$ and, keeping in mind the fact that the interior region is in any case unphysical and is described by

complex spacetime coordinates.
The gravitational potential can similarly be obtained from (1.19) and (1.20),

$$\Phi = -\frac{1}{2}(g^{00} - \eta^{00}) = -\frac{m}{r} + 0(\frac{1}{r^3})$$

As we will see in detail in Chapter 5 the electromagnetic potential is given by, (Cf. also ref.[24, 67]),

$$A^\mu = \hbar \Gamma^{\mu\sigma}_\sigma \tag{1.24}$$

Using the expression for the Christoffel symbols, we have,

$$A_\sigma = \frac{1}{2}(\eta^{\mu v} \hbar_{\mu v}),_\sigma ,$$

so that, from (1.19),

$$A_0 = 2 \int \eta^{\mu v} \frac{\partial}{\partial t} [\frac{T_{\mu v}(t - |\boldsymbol{x} - \boldsymbol{x}'|, \boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|}] d^3 x'$$

Remembering that $|\boldsymbol{x} - \boldsymbol{x}'| \approx r$ for the distant region we are considering, we have,

$$A_0 \approx \frac{2}{r} \int \eta^{\mu v} [\frac{\partial}{\partial \tau} T_{\mu v}(\tau, \boldsymbol{x}'). \frac{d}{dt}(t - |\boldsymbol{x} - \boldsymbol{x}'|)] d^3 x' \approx \frac{2}{r} \int \eta^{\mu v} \frac{d}{d\tau} T_{\mu v}.(1+c) d^3 x',$$

or finally

$$A_0 \approx \frac{2c}{r} \int \eta^{\mu v} \frac{d}{d\tau} T_{\mu v} d^3 x' \tag{1.25}$$

as $c >> 1$, and where we have used the fact that in the Compton wavelength region, $|u_v| = c$.
It has already been observed that QMKNBH can be treated as a rotating shell distribution with radius $R \equiv \frac{\hbar}{2mc}$. So we have,

$$|\frac{du_v}{dt}| = |u_v| \omega \tag{1.26}$$

where ω, the angular velocity is given by,

$$\omega = \frac{|u_v|}{R} = \frac{2mc^2}{\hbar} \tag{1.27}$$

Interestingly we get the same relation (1.26) in the theory of the Dirac equation, remembering that in (1.20) and (1.21) the centre of mass is at rest:

$$\imath \hbar \frac{d}{dt}(c\alpha_\imath) = -2mc^2 (c\alpha_\imath),$$

where $c\alpha_\imath$ is the velocity operator (cf.ref.[7]). Finally, on using (1.24), (1.26) and (1.27) in (1.25), we get, displaying $G(= 1)$ explicitly,

$$\frac{e'e}{r} = A_0 \sim \frac{\hbar c^3}{r} \int \rho \omega d^3x' \sim (Gmc^3)\frac{mc^2}{r} \qquad (1.28)$$

where $e' = 1esu$ corresponds to the charge $n = 1$ and e is the test charge. Because of the approximations taken in deducing (1.28), and the changeover of units, a dimensional constant $(\frac{L}{T})^5$ has to be multiplied on the left side, which then becomes,(in units, $c = G = 1$),

$$e'e.(\text{dimensional constant}) \approx 1.6 \times 10^{-111} cm^2$$

The right side is,

$$Gm^2c^5 \approx 4.5 \times 10^{-111} cm^2,$$

in broad agreement with the left side.
Alternatively, using the values of G, m and c in (1.28), in usual units we get,

$$e \sim 10^{-10} esu,$$

which is correct.
Yet another way of looking at (1.28) is, that we get, as $e' = 1esu \sim 10^{10}$,

$$\frac{e^2}{Gm^2} \sim 10^{40}, \qquad (1.29)$$

Equation (1.29) is well known empirically and we will return to it repeatedly in later Chapters where it will be deduced alternatively. But equation (1.28) gives the reason for this "coincidental" relation.
In any case, equations (1.29) and (1.28) show the inter-relation between $e, m, c \text{and} G$.
So far we have been considering distances far from the particle: $|\boldsymbol{x}' - \boldsymbol{x}| >> |\boldsymbol{x}'|$. This is the approximation invoked in a transition from (1.19) to equations (1.20), (1.21) etc. Let us now see what happens when $|\boldsymbol{x}| \sim |\boldsymbol{x}'|$. In this case, we have from (1.19), expanding in a Taylor series about t,

$$h_{\mu v} = 4 \int \frac{T_{\mu v}(t, \boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|} d^3x' + (\text{terms independent of} \boldsymbol{x}) +$$
$$2 \int \frac{d^2}{dt^2} T_{\mu v}(t, \boldsymbol{x}').|\boldsymbol{x} - \boldsymbol{x}'| d^3x' + 0(|\boldsymbol{x} - \boldsymbol{x}'|^2) \qquad (1.30)$$

The first term gives a Coulombic $\frac{\alpha}{r}$ type interaction except that the coefficient α is of much greater magnitude as compared to the gravitational or electromagnetic case, because in this approximation, in an expansion of $(1/|\boldsymbol{x} - \boldsymbol{x}'|)$, all terms are of comparable order. To proceed further, using (1.26), we have,

$$\frac{d}{dt} T^{\mu v} = \rho u^v \frac{du^\mu}{dt} + \rho u^\mu \frac{du^v}{dt} = 2\rho u^\mu u^v \omega,$$

so that,

$$\frac{d^2}{dt^2}T^{\mu v} = 4\rho u^\mu u^v \omega^2 = 4\omega^2 T^{\mu v}$$

where ω is given by (1.27). Substitution in (1.30) gives,

$$h_{\mu v} \approx -\frac{\beta M}{r} + 8\beta M(\frac{Mc^2}{\hbar})^2 .r \tag{1.31}$$

β being a constant.
This resembles the QCD quark potential[68] to which we will return in the next Chapter, with both the Coulombic and confining parts. Taking for M the mass of a typical C quark $\sim 1.8Gev$, the ratio of the coefficients of the r term and the $\frac{1}{r}$ term as obtained from (1.31) is $\sim \frac{1}{\hbar^2}(Gev)^2$ as in the case of QCD. In any case these considerations suggest that we can get different interactions at different distances or scales in a unified picture based on linearized General Relativity and minimum spacetime scales, which can represent quarks also.
We can further refine this argument (Cf equations (1.25) and (1.28)). Outside the Compton wavelength, it was shown that

$$\frac{ee'}{r} = A_0 \approx \frac{2c\hbar}{r}\int \eta^{\mu\nu}\frac{d}{d\tau}T_{\mu\nu}d^3x' = \frac{2c\hbar}{r}\int \eta^{\imath\jmath}\frac{d}{d\tau}T_{\imath\jmath}d^3x',$$
$$= 2c\hbar(\frac{mc^2}{\hbar})\int \eta^{\imath\jmath}\frac{T_{\imath\jmath}}{r}d^3x' \tag{1.32}$$

where e' is the test charge.
As we approach the Compton wavelength however, we have to use equation (1.30), which after a division by m, the mass of the particle to be identified with the quark, and taking $\hbar = 1 = c$ to correspond to the usual theory, goes over to, as in equation (1.31),

$$-\frac{\alpha}{r} + \frac{\beta m_e}{l^2}r \tag{1.33}$$

where $\alpha \sim 1$ and $\beta \sim \frac{1}{m}$ and m_e is the electron mass. This is again the QCD potential with both the Coulombic and confining parts.
We now observe that the usual three dimensionality of space, as pointed out by Wheeler[46, 69] arises due to the double connectivity or spinorial behaviour of Fermions, which takes place outside the Compton wavelength due to the fact that as we have seen, while it is the negative energy components of the Dirac four-spinor which dominate inside, it is the positive energy components which predominate outside (cf.ref.[70, 71]) for details). Such a three dimensionality can also be similarly deduced using Penrose's spin network theory[72]. The interesting point here is that this three dimensionality is not apriori. Rather, it arises due to a holistic or Machian or environmental reason - the presence of other particles around. Interestingly, if we consider the Dirac equation in two (or one dimension) [73, 74], we encounter handedness and the absence of an invariant mass - features which in the light of our considerations are expected to arise at the Compton wavelength. As we approach the

Compton wavelength, we encounter mostly the negative energy components and the above double connectivity and therefore three dimensionality disappear: For the next simplest choice we have two or less dimensions. Infact, even in the purely classical case of a collection of relativistic particles, the various centres of mass form a two dimensional disk [75]. Indeed such a conclusion has been drawn alternatively at very small scales (cf.[76, 77]). Further recent experiments with nano tubes already reveal such low dimensional quantum behaviour[78, 79].
This leads to the following circumstance: We first have to consider two and one spatial dimensions. We now use the fact that as is well known[80] each of the $T_{ıj}$ in (1.32) is given by $(1/3)\epsilon$, where ϵ is the energy density. In this case it follows from (1.32) that the particle would have the charge $(2/3)e$ or $(1/3)e$, in two or one dimensions. Incidentally, this provides an explanation for the remarkable and well known fact that a third of charge appears to be concentrated in a core of the size of the order of the Compton wavelength[81]. This would also automatically imply that these fractionally charged particles cannot be observed individually, as they are by their very nature confined to dimensions of the order of their Compton wavelength. This is expressed by the confining part of the QCD potential (1.33). We now identify these confined particles with charge $(1/3)e$ and $(2/3)e$, with quarks and further justify this identification, below.
As in reference[70, 82], and as in the standard theory we consider the proton to be made up of two quarks of charge $(2/3)e$ with an intervening quark of charge $-(1/3)e$ all confined to a distance l which in the above light is of the order of the particle's Compton wavelength. For small displacements r of the central quark as in[70] we can easily see that the confining part of the potential is given by

$$V = \frac{e^2}{9l^2} \quad r \tag{1.34}$$

Comparing with (1.33) we get,

$$\frac{e^2}{9l^2} \quad \sim \frac{1}{m} \quad \frac{m_e}{l^2},$$

whence the quark's mass is given by

$$m \sim 10^3 m_e \tag{1.35}$$

as required.
Finally as we encounter predominantly the negative energy two spinor of the Dirac four spinor at the Compton wavelength, with negative helicity (cf.ref.[29]), the quarks display handedness which in conventional theory is due to the small Cabibo angle. So the puzzling, inexplicable features of quarks [8] turn out to be not so mysterious, after all.
It is interesting to note that just beyond the Compton wavelength, where we still do not encounter fractional charge or low dimensions, the mass of

the resulting particle would from (1.34) and (1.35) be given by $\sim 137m_e$ corresponding to the pion: In fact we have to consider two such Fermions which form the pion, as is well known, so that we recover the pion mass, $274m_e$.

In other words at scales greater than the Compton wavelength the above description will correspond to that of an electron, while at scales $\geq_\sim$ the Compton wavelength, it corresponds to a pion and at scales $\leq_\sim$ the Compton wavelength, it corresponds to a quark, which is physically meaningful.

In any case we recover the usual structure of the proton in terms of the three quarks. So quarks are electrons (or positrons) at a smaller scale!

It has been discussed in detail in ref.[24], how we can accommodate antiparticles, for example positrons.

Thus, it appears that the treatment of leptons and quarks as QMKNBH leads to meaningful results in a unified description. On the other hand, these are the most fundamental constituents of matter, according to current thinking.

1.5 Prospect

In any case, we will explore in this book, a model originating in the above consideration, in which spacetime is fuzzy at the Compton scale. As Weinberg puts it [83], "The infinities in ordinary quantum field theories can be traced to the fact that the fields describe point particles..."

We also investigate how the Compton scale can emerge from an even more fundamental scale, the Planck scale. As we will see in Chapters 5 and 6 and subsequent Chapters, the picture that emerges is: From a dark energy or Quantum Vacuum or Zero Point Field background, oscillators at the Planck scale are formed. Such oscillators "condense" into elementary particles at the Compton scale, which are also the lowest energy and therefore stable states. At the same time these Planck oscillators provide an underpinning for the entire Universe, which on the one hand is a collection of elementary particles, and on the other is an excited state of a collection of Planck oscillators. While a unified picture of gravitation with other interactions is obtained, more fundamentally gravitation emerges as a distributional and residual interaction, being far from a fundamental interaction between particles. This characterization, infact resolves a paradox as we will see in Chapter 8, first pointed out by Weinberg decades ago, and since overlooked.

In this model, spacetime is not only not smooth but is chaotic, as described in an earlier book, "The Chaotic Universe" (Cf.ref.[24]). At the same time the cosmology that emerges, as we will see in Chapter 6, is one of fluctuations, that of a dark energy driven accelerating Universe with a small cosmological constant - as indeed was subsequently confirmed by observation. These ideas suggest in Chapter 9 a mass spectrum formula - one which gives the masses of all known elementary particles with an error of three percent or less.

Finally in Chapter 10 we discuss some further experimental or observational consequences. These include the violation of Lorentz symmetry at ultra high energies, a new "Gravitomagnetic" type interaction and the already observed anomalous acceleration of the Pioneer spacecrafts.

2 STANDARD MODELS OF PHYSICS AND COSMOLOGY

"I am inclined to suspect that the renormalization theory is something that will not survive in the future, and that the remarkable agreement between its results and experiments should be looked on as a fluke..."

P.A.M. Dirac

2.1 The Strong and Weak Interactions

A major achievement of the twentieth century has been the incorporation of three of the four fundamental interactions, viz., electromagnetism, weak interactions and the strong interaction within a unified mathematical framework. This framework is the non Abelian gauge field theory which we will see a little later and again in Chapter 7 ([11, 12, 84, 85, 68, 86]). Though the three forces remain different, the underlying mechanism is the same. From this point of view they could be thought to be different aspects of a single underlying process.

Thus there are leptons and there are quarks. The difference between these sets of particles which are perceived today arise because the Universe has become cold. At sufficiently high energies $\sim 10^{15} GeV$, leptons and quarks would be interchangeable and so also all the three forces would have the same strength. It must be mentioned that the above energy is still beyond the reach of foreseeable accelerators.

Apart from leptons and quarks, which are Fermions, or "material" particles, the fields are mediated by Bosons. These are the photons for electromagnetism, the W and Z Bosons for weak interactions and the gluons for the strong interactions.

Quarks were conceived following the work of Gellmann, Ne'eman and Zweig in the sixties. The motivation had been the overabundance of resonances observed in hadron or strong interaction collisions. These resonances could be classified on the one hand according to the Regge trajectories that plot the angular momentum J versus the mass squared, M^2 [87]. We will touch upon this briefly in the next Chapter. On the other hand, there was the SU(3) classification scheme which related particles of the same spin but different quantum numbers by introducing elemental entities - the quarks - whose

combinations could account for all observed hadrons.
It is now believed that there are six kinds of quarks: The down (d), the up (u), the strange (s), the charmed (c), the bottom (b) and the top (t). We attribute to the quarks three colours, red, green and blue which are generalizations of the positive and negative charges. It is these colours which characterize strong interaction and hence this field has come to be known as Quantum Chromo Dynamics (QCD). It may be observed that the leptons do not have any colour and so they do not participate in the strong interactions.
A peculiarity of quarks is their fractional charge - they have either the charge $\frac{1}{3}$ or the charge $\frac{2}{3}$ with their corresponding anti particles having opposite charges. So quarks can combine in two different ways to form hadrons, that is particles like protons and neutrons: Either as quark, anti-quark pairs or as a triplet of quarks, such that the total charge is either one or zero.
In electromagnetism, or Quantum Electro Dynamics (QED), two charged particles interact by the exchange of a photon, more correctly a virtual photon [88]. This exchange takes place within the Heisenberg Uncertainty time. There is a conservation of electric charge in the process. This combined with the masslessens of the photon is characteristic of the $U(1)$ Group which characterizes QED.
QCD is modelled on QED. However QCD which is described by the SU(3) group is more complicated because it describes interactions of three different colours, unlike QED which deals with just one charge. In QCD the interaction between different colours is expressed in terms of eight massless particles, the gluons, unlike the single photon of QED. Another profound difference is that the gluons do carry colour unlike the photon which is chargeless. The nett result of all this is that there is an effect opposite to that encountered in the charge screening of QED. In this latter case, an electron is surrounded by virtual electron-positron pairs. The electron attracts the positrons and repels the electrons of these pairs with the result that at large enough distances, the electron charge is shielded by the positrons and so appears reduced. In QCD on the other hand, virtual gluon pairs, themselves carrying colour are formed around a quark, no doubt. But there is now an anti screening effect as if the red component of a gluon is attracted to the red of a quark, for example. So at relatively larger distances, the colour charge of a quark increases and again contrary to the QED scenario, decreases as we approach the quark. The QCD force can therefore be compared to rubber bands - as we stretch, the elastic force manifests itself, but if the bands slacken at close range, the force decreases and even disappears. It is as if there is confinement at large distances and freedom at shorter, asymptotic distances.
The QCD potential can be written as [68, 89]

$$V(r) = -\frac{\alpha(r)}{r} + \frac{r}{\beta^2}$$

This consists of the Coulombic part $\propto -\frac{1}{r}$ and a confining part $\propto r$. Because of this latter, which dominates for large r, free quarks cannot be observed in

nature. On the other hand, the Coulombic part ensures that for small r, the inter quark force vanishes, a circumstance which is called asymptotic freedom. Professors Wilczek, Politzer and Gross were awarded the 2004 Nobel Prize in Physics for this work, done thirty years earlier.
The neutrinos are closely associated with the weak interactions. Though the neutrinos are leptons, they differ from their counterparts in that they are massless (or more precisely, they have a very tiny mass). A massless Fermion exhibits handedness, that is, its spin is either aligned in the direction of its motion (righthanded) or it is aligned anti parallel to its motion (lefthanded). This extra property of handedness characterizes the weak force which violates parity, unlike the other forces (though even the quarks exhibit handedness!). Only lefthanded particles and righthanded anti particles bear a weak charge while the righthanded particles and the lefthanded anti particles are neutral from the point of view of the weak interaction. This interaction acts on doublets of particles, which latter are described by the SU(2) Group, in which particles of a doublet pair can be transformed into one another. The weak interactions are mediated by the W Bosons. However a suitable mixture yields both the photon of electromagnetism and the Z° characterizing weak interactions. This theory therefore combines electromagnetic and weak interactions and is incorporated in the SU(2) XU(1) group [8].
An important difference between the weak forces on the one hand and QED and QCD on the other is that the intermediate particles of the weak interactions, the W and Z Bosons are not massless, but rather have large masses $\sim 100GeV$. This is characteristic of the fact that the weak charge is not invariably conserved and moreover has an extremely short range $\sim 10^{-15}cms$. We will return to this point later.
One of the problems that has plagued modern field theories is that of infinities. Indeed this problem was encountered early in the twentieth century itself when an attempt was made to model the electron as a tiny sphere. If the radius of the sphere was then made to shrink indefinitely, the energy of the electron increased without limit [201]. In QED for instance, if we approach the electron through the shield of screening positrons, the bare charge of the electron would be infinite. It is only the physically observable charge, at a distance, screened by the positron charges, which is finite. It is as if the infinite bare negative charge has been cancelled or neutralized by the infinite screening positive charge, the nett result being the observed finite physical charge. Loosely speaking this procedure is called "renormalization".
Mathematically, we encounter divergent integrals [90]. The infinities are eliminated in two steps. In the first step, called regularization, we introduce constraints, for example a cut off (or a lattice structure), to get a finite result dependent on the regularization parameter like the cut off. Counter terms (dependent on these parameters) are then added to the Lagrangian, such that they cancel the parameter dependent integrals. This generally leads to a rescaling of the mass, charge etc. This is the process that is called Renor-

malization.
The concept of Renormalization is unsatisfactory from the logical point of view as well as from the point of view of internal consistency. It has provoked unease among Physicists such as Dirac quoted above [91] or as we will see later in Chapter 5,'t Hooft and several others. Its merit however, has been that phenomenologically speaking, it works.

2.2 Gauge Fields

It has now come to be recognized that the physical principle governing the fundamental interactions between the elementary particles is gauge invariance. This principle, as we shall see in detail in Chapters 5 and 7, was originally introduced by Hermann Weyl, though in a different form and with a different motivation viz., the attempt to give a unified General Relativistic description of electromagnetism and gravitation [92]. At that time these were the only two known interactions and electrons and protons were the only known elementary particles. Weyl's original theory was soon dismissed as adhoc. But nevertheless it was recognized that gauge invariance was a symmetry of Maxwell's equations with useful implications.
Then in the 1950s Yang and Mills (and Shaw) tried to extend gauge symmetry to other interactions. It must be emphasized that both in Special Relativity and General Relativity there are no absolute frames of reference in the Universe. The physics within a system is independent of the choice of the reference frame. However in Special Relativity this freedom of choice of reference frame is a global symmetry- the Lorentz symmetry. In General Relativity on the other hand, the reference frame is to be defined locally, that is at each and every point in the gravitational field. There are the connections - the affine connections or Christofell symbols which relate nearby frames in General Relativity, something which is not required in Special Relativity [6]. Weyl attempted to investigate if there were similar connections associated with electromagnetism [92]. Just as in General Relativity, all physical measurements are relative, so also could the norm of a physical vector depend on its location? If so, a new connection would be required to relate the lengths of the vectors at different positions. This clearly would be a local property. It was called Gauge Invariance. Let us see how this can be expressed mathematically [84, 93]. In essence we have to multiply the norm of a vector $f^\mu(x^\mu) \equiv f(x)$ at $x \equiv x^\mu$ by a scale factor $S(x^\mu) \equiv S(x)$, which latter would represent the change in scale from point to point. So we have for a small displacement to the point $x + dx$, the equations

$$S(x+dx) = 1 + \partial_\mu S dx^\mu$$

$$Sf = f + (\partial_\mu S) f dx^\mu + \partial_\mu f dx^\mu$$

If f is a constant vector, then we have

$$(\partial_\mu + \partial_\mu S) f dx^\mu$$

As can be seen from the above, the derivative $\partial_\mu S$ is the new mathematical connection associated with the gauge transformation. Weyl identified this connection with the electromagnetic potential A_μ. This is motivated by the fact that a second gauge change with a scale factor Λ leads to

$$\partial_\mu S \to \partial_\mu S + \partial_\mu \Lambda$$

which mimics the behavior under a gauge transformation of the electromagnetic potential in classical theory,

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

With the advent of Quantum Theory, Weyl himself realized that his old idea could be given a new interpretation. Rather than being a change of scale, a gauge transformation could be interpreted as a phase transformation. This is because if

$$\psi \to \psi e^{-\imath \lambda} \tag{2.1}$$

then for the electromagnetic potential we would have

$$A_\mu \to A_\mu - \partial_\mu \lambda \tag{2.2}$$

Equation (2.1) together with equation (2.2) is a symmetry transformation of the Schrödinger equation. All this is nothing but the well known minimum coupling algorithm,

$$p_\mu \to p_\mu - e A_\mu$$

The reason that this reinterpretation of gauge transformations is acceptable is that the Quantum Mechanical phase is not a directly measurable quantity. It is now clear that electromagnetism can be interpreted as a Quantum Mechanical local gauge theory. This time it is the local phase of the wave function which is the physical degree of freedom that depends on its spacetime position.

The modern rebirth of gauge theory stemmed from a study of the strong forces mediated by the Yukawa Meson, and Heisenberg's iso spin interpretation of the identity of neutrons and protons when electromagnetic interactions are switched off. That is the strong force was invariant in the SU(2) isotopic spin group.

The difficulty was that iso spin is not a local gauge symmetry, because it is an internal Quantum number independent of spacetime location. So there was no question of an iso spin potential connection whose Quantum would be the Yukawa Meson.

Nevertheless in 1954 Yang and Mills went ahead to treat strong interactions as a gauge invariant field theory by postulating that the local gauge group was the SU(2) iso spin group, in analogy with the electromagnetic case. This time the proposed connection was a linear combination of the angular momentum operators,

$$A_\mu = \sum_\imath A^\imath_\mu(x) L_\imath \tag{2.3}$$

This is a generalization of the electromagnetic case. In the latter, the operators $L_\imath$ are replaced by the unit matrix and the coefficients $A_\mu(x)$ are proportional to the phase change $\delta_\mu \lambda$. As can be seen from (2.3) the Yang-Mills potential is both a field in spacetime and an operator in iso spin space. It must be observed that like the electromagnetic field the Yang-Mills field is mediated by zero mass Bosons. This is because a massive intermediary would imply a term of the form $m^2 A_\mu A^\mu$, which is clearly not gauge invariant.
Let us now see how a symmetry group transformation leads us to a connection which can be identified with the gauge potential field. Indeed, for an arbitrary non-Abelian group, the symmetry transformation is given by

$$U\Psi = \exp\left(\imath q \sum_k \Theta^k(x) F_k\right)\Psi \tag{2.4}$$

In (2.4), the fact that $\Theta^k(x)$ are continuous functions of x defines the local transformation. q is the coupling constant for the gauge group in question. F_k are the generators of the internal symmetry group, satisfying the commutation relations

$$[F_\imath, F_j] = \imath \epsilon_{\imath jk} F_k \, ,$$

In (2.4) if an infinitesimal transformation of the spacetime coordinate is carried out, we get instead of the usual derivative, the gauge covariant derivative describing the changes in both the external and internal components of $\Psi(x)$ viz.,

$$D_\mu \Psi_\beta = \sum_\alpha \left[\delta_{\beta\alpha}\partial_\mu - \imath q (A_\mu)_{\beta\alpha}\right]\Psi_\alpha \tag{2.5}$$

where A_μ are given by

$$(A_\mu)_{\alpha\beta} = \sum_k (\partial_\mu \Theta^k)(F_k)_{\alpha\beta}$$

A special case of (2.5) is the U(1) electromagnetic gauge group, for which this reduces to the usual form with the minimal coupling

$$D_\mu \Psi = (\partial_\mu - \imath q A_\mu)\Psi$$

Thus for the electromagnetic gauge group the gauge covariant derivative is the familiar canonical momentum. It must be noted that the potential A_μ is both an external field and as well, an internal space operator. Furthermore in the non-Abelian gauge group, an internal operator part of the potential would contain a linear combination of the group generators, F_k which do not in general commute. However as we saw above, the problem has been that we cannot incorporate a mass for the gauge field in an invariant manner. This is achieved by considering an additional field. In the case of weak interaction this is the Higgs field, which breaks the symmetry and leads to a mass generating mechanism. We will be returning to this point later in Chapter 7.

2.3 Standard Cosmology

In the sixties, it was not suspected that Elementary Particle Physics would be intimately connected with cosmology, which was at the other end of the spectrum in terms of sizes! But it was subsequently realized that further experimentation on theoretical particle models would require energies that could not be available in foreseeable particle accelerators. Fortunately the Big Bang model of cosmology provides a scenario in the early Universe where such high energies were accessible and consequently particle physics predictions become testable. The very interesting development that has emerged is that Particle Physics and cosmology have got linked by this high energy bridge.

The so called Big Bang model arose from three main observations. The first was the discovery in the 1920s that the Universe is expanding, in the sense that the basic constituents, the galaxies (as then believed) showed red shifts. Furthermore as Hubble discovered, the farther the galaxy, the greater its speed of recession. This is Hubble's Law: $v = Hr$, where H is the Hubble constant.

Another important observation was about light element abundance - or over-abundance - in the Universe. In the 1940s Gamow and coworkers provided an explanation for this. The early Universe must have been very hot and dense. The synthesis of light elements took place when the Universe was at a temperature of $10^9 K$. However heavier elements were formed later, inside the stars, and were strewn about by supernova explosions.

Finally there was a cosmic footprint of an explosion from a very early hot and dense state. This was the residual background radiation from that early event. In the present epoch however the earlier intense radiation would have cooled, and it was calculated that it would be in the form of microwaves. Exactly such a cosmic background microwave radiation footprint was accidentally discovered in 1965 by Penzias and Wilson. This effectively overthrew a competing model of that time - the Steady State Model, which has now become history [80].

So the picture to emerge [80, 59, 46] was that the Universe was born in a titanic explosion or Big Bang, as Gamow had christened it. Exactly at the time of the Big Bang some fourteen billion years ago, it is reckoned, all the matter and energy of the Universe was concentrated at a single point, where the density and curvature would be infinite. This is the Big Bang singularity. Following the Big Bang, matter and energy has been flung all round and even today the galaxies (or clusters of galaxies) are rushing outward due to that initial impact.

The question that arises is, will the expansion of the Universe continue for ever, or would it slow down to a halt and then collapse? The answer to this would depend on the mass/energy density of the Universe. If this value is greater than a critical value, then the gravitational attraction will ultimately prevail over the expansion and the Universe would collapse. But if the density is less than the critical value, the Universe would go on expanding for ever. This critical density is given by,

$$\rho_{crit} = \frac{3H^2}{8\pi G} = 2 \times 10^{-29} h^2 g/cm^3$$

Observations seem to indicate that the density of the Universe was close to the critical value. Further an observation of the speeds of rotation along the radii of the galaxies indicated that the galaxies themselves contained more matter than met the eye. This lead to "Dark Matter" being invoked. Dark matter has not been directly detected, nor can it be precisely characterized, even though there have been a number of possible candidates. For example invisible Black Holes or even difficult to detect brown dwarf stars. Exotic massive particles have also been proposed as also massive neutrinos or monopoles. With dark matter thrown in, it was believed that the Universe had the critical density to reverse the expansion.

Though the Big Bang model could explain several observations, there were subtler questions which came to haunt. These were: How come the density of the Universe, which could have been anything, is infact so close to the critical density in a process spread over billions of years? More precisely such a close critical density today would imply that even after about a billionth of a second after the Big Bang the density was equal to the critical density accurate to some twenty five decimal places. Alternatively this means that the Universe or space is very flat. This need not have been so.

And then the Universe appears uniform on large scales. For instance the cosmic microwave background radiation is uniform in temperature to a high degree of accuracy. How can this be so for regions separated by such vast distances, that since the Big Bang light itself has not had enough time to connect them. This is called the horizon problem.

Finally how do we account for the small scale inequalities or lumps in the Universe which we see as galaxies?

In 1981 Alan Guth proposed his inflation Theory ([73, 94]). According to this there was a super fast or super rapid expansion in the early stages of the Universe, so that the size of the Universe exploded to several times its original size within a small fraction of a second.

To put it simply this super fast or exponential expansion flattens out the Universe, thus explaining the first problem. The horizon problem is also accounted for: Due to the super fast expansion or inflation, distant regions were much closer together than with an usual expansion. So they would be at the

same temperature. Furthermore Quantum fluctuations in the inflation field would cause fluctuations in density, that is they would seed the formation of galaxies. Finally it may be added that given the inflationary scenario, the fact that exotic particles like magnetic monopoles are not detected is also explained. The rapid inflation would have diluted such particles and made them unobservable.
A time line of the Universe would be [95]

$$1 \qquad t = 10^{-43} secs, \quad T = 10^{32} K$$

The Planck era of Quantum Gravity would have just ended and the Universe would be described by a Grand Unified Theory

$$2 \qquad t = 10^{-35} secs, \quad T = 10^{28} K$$

The Grand Unified symmetry is broken. The size of the Universe would still be only a millimeter across

$$3 \qquad t = 10^{-10} secs, \quad T = 10^{15} K$$

At this stage electroweak symmetry is broken. Already the Universe has swelled to a size of $10^{14} cms$.

$$4 \qquad t = 10^{-5} secs, \quad T \sim 10^{12} K$$

QCD is switched off and quarks combine to form hadrons

$$5 \qquad t \sim 3 min, \quad T \sim 10^{9} K$$

Nucleosynthesis begins and nuclei of lighter elements like Helium and Lithium begin to form

$$6 \qquad t = 10^{-5} yrs, \quad T \sim 4000 K$$

Electrons and nuclei combine to form neutral atoms as charged particles are no longer present. So there is no scattering of photons and radiation in general including the Cosmic Microwave Background Radiation.
Interestingly Optical and Radio Astronomy cannot probe beyond this time

$$7 \qquad t \sim 10^{9} yrs, \quad T \sim 10 K$$

Galaxy formation begins

$$8 \qquad t \sim 10^{10} yrs, \quad T \sim 2.7 K$$

This is the Universe of today.
The above was the model till 1997. That year, the author put forward an alternative model which infact went against the then existing belief. On the contrary, this model predicted a dark energy driven accelerating ever expanding Universe. In 1998 dramatic confirmation for the new model came from the observations of Perlmutter, Schmidt, Kirshner and others. We will come back to this in Chapter 6.

3 DIFFERENT APPROACHES: QUANTUM SUPERSTRINGS AND QUANTUM GRAVITY

"Bodies are formed by ultimate atoms (sub constituents) in constant vibration"

Kanada, Ancient Indian Thinker, C.700 B.C.

3.1 String Theory

We saw in the last two Chapters that inspite of some success, the standard theory has failed to quantize gravitation. It was also seen that one of the obstacles was the point spacetime concept ingrained in these theories. For the past few decades Quantum Gravity schemes as also string theory have broken out of this limitation. Let us first consider string theory.
We begin with the important work of T. Regge in the fifties [87, 96, 97], in which he mathematically analysed using techniques like analytically continuing the angular momentum into the complex plane, particle resonances. These resonances seem to fall along a straight line plot, with the angular momentum being proportional to the square of the mass.

$$J \propto M^2, \tag{3.1}$$

All this suggested that resonances had angular momentum, on the one hand and resembled extended objects, that is particles smeared out in space.
This went contrary to the belief that truly elementary particles were points in space. Infact at the turn of the twentieth century, Poincare, Lorentz, Abraham and others had toyed with the idea that the electron had a finite extension, but they had to abandon this approach as noted earlier, because of a conflict with Special Relativity. The problem is that if there is a finite extension for the electron then forces on different parts of the electron would exhibit a time lag, requiring the so called Poincare stresses for stability [201, 327, 98].
In this context, it may be mentioned that in the early 1960s, Dirac came up with an imaginative picture of the electron, not so much as a point particle, but rather a tiny closed membrane or bubble. (We mentioned that the electron could be thought of as a shell, in Chapter 1). Further, the higher energy level oscillations of this membrane would represent the "heavier electrons" like muons [99].

Then, in 1968, G. Veneziano came up with a unified description of the Regge resonances (3.1) and other scattering processes. Veneziano considered the collision and scattering process as a black box and pointed out that there were in essence, two scattering channels, s and t channels. These, he argued gave a dual description of the same process [23, 100].

In an s channel, particles A and B collide, form a resonance which quickly disintegrates into particles C and D. On the other hand we have in a t channel scattering particles A and B approach each other, and interact via the exchange of a particle q. The result of the interaction is that particles C and D emerge. If we now enclose the resonance and the exchange particle q in an imaginary black box, it will be seen that the s and t channels describe the same input and the same output: They are essentially the same.

There is another interesting hint which we get from Quantum Chromo Dynamics. Let us come back to the inter-quark potential [68, 89]. As we saw in the last Chapter, there are two interesting features of this potential. The first is that of confinement, which is given by a potential term like

$$V(r) \approx \sigma r, \quad r \to \infty,$$

where σ is a constant. This describes the large distance behavior between two quarks. The confining potential ensures that quarks do not break out of their bound state, which means that effectively free quarks cannot be observed.

The second interesting feature is asymptotic freedom. This is realized by a Coulombic potential

$$V_c(r) \approx -\frac{\propto (r)}{r} (\text{small}\, r)$$

$$\text{where } \propto (r) \sim \frac{1}{ln(1/\lambda^2 r^2)}$$

The constant σ is called the string tension, because there are string models which yield $V(r)$. This is because, at large distances the inter-quark field is string like with the energy content per unit length becoming constant. Use of the angular momentum - mass relation indicates that $\sigma \sim (400MeV)^2$. Such considerations lead to strings which are governed by the equation [101, 102, 14]

$$\rho\ddot{y} - Ty'' = 0, \tag{3.2}$$

$$\omega = \frac{\pi}{2l}\sqrt{\frac{T}{\rho}}, \tag{3.3}$$

$$T = \frac{mc^2}{l}; \quad \rho = \frac{m}{l}, \tag{3.4}$$

$$\sqrt{T/\rho} = c, \tag{3.5}$$

T being the tension of the string, l its length and ρ the line density and ω in (3.3) the frequency. The identification (3.4) gives (3.5), where c is the velocity

of light, and (3.2) then goes over to the usual d'Alembertian or massless Klein-Gordan equation. (It is worth noting that as $l \to 0$ the potential energy which is $\sim \int_0^l T(\partial y/\partial x)^2 dx$ rapidly oscillates.)
Further, if the above string is quantized canonically, we get

$$\langle \Delta x^2 \rangle \sim l^2. \tag{3.6}$$

The string effectively shows up as an infinite collection of Harmonic oscillators [14]. It must be mentioned that (3.6) and (3.4) both show that l is of the order of the Compton wavelength. This has been called one of the miracles of string theory by Veneziano [23]. In fact the minimum length l turns out to be given by $T/\hbar^2 = c/l^2$, which from (3.4) and (3.5) is seen to give the Compton wavelength.
This is a description of what may be called a "Bosonic String". These theories have certain technical problems, for example they allow the existence of tachyons. Further they do not easily meet the requirements of Quantum Theory, as for example the commutation relations. The difficulties are resolved only in twenty six dimensions.
If the relativistic quantized string is given rotation [104], then we get back the equation for the Regge trajectories given in (3.1) above. Here we are dealing with objects of finite extension rotating with the velocity of light rather like spinning Black Holes. It must be pointed out that, in superstring theory, there is an additional term a_0

$$J \leq (2\pi T)^{-1} M^2 + a_0 \hbar, \text{ with } a_0 = +1(+2) \text{ for the open (closed) string.} \tag{3.7}$$

In Equation (3.7) a_0 comes from a zero-point energy effect. When $a_0 = 1$ we have the usual gauge Bosons and when $a_0 = 2$ we have the gravitons.
The theory of Quantum Superstrings in contrast requires only ten dimensions. Here, Quantum operators describing anti-commuting variables satisfy anti-commutation relations. Indeed this bivalence is a hallmark of supersymmetry itself.
The extra dimensions that appear in String theories reduce to the four dimensions of the physical spacetime by virtue of the fact that the redundant dimensions are treated as curled up into a negligible extension, in the manner suggested by Kaluza and later Klein in the early twentieth century. Kaluza's original motivation had been to unify electromagneitsm and gravitation by introducing a fifth negligible coordinate. The curling up takes place at the Planck scale [105].
A finite extension for an elementary particle, as in String theories can be shown to lead to new commutation relations, as was done by Snyder in the forties. In this case two space coordinates like x and y do not commute. Snyder's original motivation had been to fudge and eliminate singularities and divergences in Quantum fields. We will return to all this later particularly in Chapter 5, but remark that what this implies is that space coordinates in

some sense take on the character of momenta in addition, though this happens at very small scales or high energies. Effectively there is a modification of the Uncertainty Principle

$$\Delta x \geq \frac{\hbar}{\Delta_P} + l^2 \frac{\Delta_P}{\hbar} \tag{3.8}$$

What all this means is we cannot go down to lower and lower scales arbitrarily. As we approach the minimum length we return to the larger Universe [17]. We will return to this point several times.

The interesting thing about Quantum Superstring theory is the natural emergence of the spin 2 graviton as can be seen from (3.7), or as Witten puts it, the theory "predicts" gravitation.

Meanwhile supersymmetry or SUSY developed in parallel. This theory requires that each particle with integral spin has a counterpart with the same mass but having half integral spin. That is Bosons have their supersymmetric counterparts in Fermions. SUSY is then broken so that the counterparts would have a much greater mass, which would then account for the fact that these latter have not been observed. Nevertheless the fact that in this theory gravitation can be unified with the other forces makes it attractive.

Infact this had lead to Supergravity in which the spin 2 graviton has the spin 3/2 counterpart, the gravitino. Supergravity requires eleven spacetime dimensions, one more than Superstring theory.

Unfortunately Supergravity began to fade from the mid eighties because of the fact that, as shown by Witten and others, handedness cannot easily emerge on reduction to the four physical spacetime dimensions from eleven. On the other hand the Quantum Super String theory was in comparison altogether more satisfactory. We could say that the earlier bosonic String theory worked in a spacetime that was Bosonic, there being no place for spin. QSS works in a Fermionic spacetime where we have the modification (3.8), to which we will return in Chapter 5 and following Chapters.

So in the mid eighties ten dimensional QSS displaced Supergravity. There were five QSS theories - $E_8 \times E_8$ heterotic, $SO(32)$ heterotic, the Type I, the Type IIA and Type IIB. Of these the Type I is an open string while the others form closed loops. The $E_8 \times E_8$ appeared to explain many features of elementary particles and their forces.

However there were some disturbing questions. Why were there five different theories? After all we need a unique theory. And then why ten dimensions, while supersymmetry allows eleven dimensions? Another not very convincing factor was the fact that particles were being represented as one dimensional strings. Surely a more general formulation as noted above would have two dimensional surfaces or membranes or even p-dimensional entities which we may call p-branes. This generalization resembles the earlier attempt of Dirac's representing particles as a shell or membrane. Infact if the radius of the circle shrinks, the membrane begins to resemble a rolled up object in ten dimensions. It reduces to a Type IIA Superstring.

In such deformations certain topological properties can remain conserved. A good example is a knot in a set of field lines. Such knots or solitons remain as such and exhibit a particle type behaviour. A magnetic monopole can be characterized in this way, that is as a twisted knot of magnetic lines. It can be said to carry a topological charge. This is to be contrasted with the charges carried by particles like electrons and quarks which can be put within the framework of the Noether Conservation Theorem. In this context an interesting conjecture is that of Montonen and Olive [106]: There could be a dual formulation in which the roles of the usual charges and topological charges are reversed. In such a formulation for example a particle with charge e would show up as a soliton with charge $\frac{1}{e}$.
Over the past few years, a variant called M Theory arising from these generalizations has attracted much attention. This theory also uses supersymmetry, which is broken so that the postulated particles do not have the same mass as the known particles. Further these new masses must be much too heavy to be detected by current accelerators. The advantage of supersymmetry is that a framework is now available for the unification of all the interactions including gravitation. It may be mentioned that under a SUSY transformation, the laws of physics are the same for all observers, which is the case in General Relativity (gravitation) also. Under SUSY there can be a maximum of eleven dimensions, the extra dimensions being curled up as in Kaluza-Klein theories. In this case there can only be an integral number of waves around the circle, giving rise to particles with quantized energy. However for observers in the other four dimensions, it would be quantized charges, not energies. The unit of charge would depend on the radius of the circle, the Planck radius yielding the value e. This is the root of the unification of electromagnetism and gravitation in these theories.
The relevance of all this is that p-branes can be characterized as solitons. For example a ten dimensional string can show up as a p-brane with $p = 5$. In this case a strongly interacting string would be the dual of a weakly interacting 5-brane. In 1990 the Montonen-Olive duality which was between electricity and magnetism in ordinary four dimensional space, was generalized to four dimensional Superstrings.
This duality was called S-duality, to distinguish it from the well known T-duality which relates two kinds of particles that arise when the string loops around by a compact dimension: There would be vibrations on the one hand and multiple windings on the other. Winding particles over a circle of radius r correspond to vibrating particles in a circle of radius $1/r$ and conversely on the lines of (3.8). We will be returning to this aspect in Chapter 5 and subsequently, as such a behaviour is characteristic of minimum spacetime intervals. In this picture the solitonic interaction is given by the reciprocal of the string interaction, in conformity with the Montonen-Olive conjecture.
A further interesting development was the realization that in the reduction of the dimensions of spacetime to four dimensions the string and the cor-

responding soliton each acquire a T-duality. Moreover the T-duality of the solitonic string is the S-duality of the fundamental string and conversely. We have here a duality of dualities. It also implies that the interaction charges in one Universe show up as sizes in the dual.
Further the eleventh and extra dimension of the M-Theory could be shrunk, so that there would be two ten dimensional Universes connected by the eleven dimensional spacetime. Now particles and strings would exist in the parallel Universes which can interact through gravitation. The interesting aspect of the above scenario is that it is possible to conceive of all the four interactions converging at an energy far less than the Planck energy ($10^{19} GeV$). Infact the Planck energy is so high that it is beyond foreseeable experiments. Thus this would bring the eleven dimensional M-Theory closer to experiment. There have been further developments involving what are called Dirichlet surfaces. It is now suspected that Black Holes can be treated as intersecting black branes wrapped around seven curled up dimensions. There is here, an interesting interface between M-Theory and Black Hole physics [107]. In M-Theory, the position coordinates become matrices and this leads to, as we will see in detail in Chapter 5, though from a different perspective, a non-commutative geometry or fuzzy spacetime in which spacetime points are no longer well defined [108]

$$[x, y] \neq 0$$

From this point of view the mysterious M in M-Theory could stand for Matrix, rather than membrane. In any case, as we will argue in Chapter 5, fuzzy spacetime may well hold the key for the unification of all interactions.
So M-Theory is the new avatar of QSS. Nevertheless it is still far from being the last word. There are still any number of routes for compressing ten dimensions to our four dimensions. There is still no contact with experiment. It also appears that these theories lead to an unacceptably high cosmological constant and so on.

3.2 Loop Quantum Gravity

An alternative approach was developed in the mid eighties by Abhay Ashtekar, Ted Jacobson, Lee Smolin, Carlo Rovelli and others [110]. This has come to be known as Loop Quantum Gravity (LQG). In this approach too, spacetime is no longer a differentiable manifold. However there is an ingenious use of Quantum Theory with two important concepts of General Relativity viz., background independence and diffeomorphism invariance.
To put it roughly, according to the former principle the geometry of spacetime is an evolving dynamical quantity, which can be obtained from suitable equations. The latter principle which is closely connected with the former means that any arbitrary set of coordinates can be used to describe spacetime phenomenon. Surprisingly the above considerations lead to the conclusion that

space is quantized [110]. Indeed, we will argue in Chapter 5 that such a prescription is built into a Quantum theoretic description of the Universe.
So in LQG, a volume in space or the surface of this volume cannot be arbitrarily small - there are fundamental minimum units, viz., at the Planck scale l_P. So the minimum area would be l_P^2 and the minimum volume would be l_P^3. The minimum area turns out to be fundamental [111].
A polyhedral volume is considered to be a node and is depicted by a dot while the enclosing flat surfaces are depicted by lines sticking out of the dot. So any arbitrary volume would be, what may be called a network of these dots and lines. Infact the important idea is that this network of dots and lines *is* space rather than being a structure embedded in space. Such a depiction pleasingly concurs with Roger Penrose's spin network proposal of the 1970s [72]. Every Quantum state corresponds to one of the possible networks formed by nodes and lines. The nodes and lines with further characterization would also represent respectively particles and fields. Motion is now a result of discrete changes in the networks. Any process as in Quantum Theory is described by probabilities which have been worked out for the changes in the spin networks. When we introduce time the spin networks become spin foams. Clearly dots become lines and lines become surfaces due to the extra dimension. However the flow of time is no longer smooth as is the case in standard theories. Rather time progresses in discrete steps, each of duration of the Planck time. In other words the progress of time can be pictured by a discrete sequence of spin networks.
Though Loop Quantum Gravity has made some progress over the years, as Lee Smolin, one of the founders [112] puts it, "Many open questions remain to be answered in Loop Quantum Gravity. Some are technical matters that need to be clarified. We would also like to understand how, if at all, Special Relativity must be modified at extremely high energies. So far our speculations on this topic are not solidly linked to Loop Quantum Gravity calculations. In addition, we would like to know that classical General Relativity is a good approximate description of the theory for distances much larger than the Planck length, in all circumstances. (At present we know only that the approximation is good for certain states that describe rather weak gravitational waves propagating on an otherwise flat spacetime.) Finally, we would like to understand whether or not Loop Quantum Gravity has anything to say about unification: Are the different forces, including gravity, all aspects of a single, fundamental force? String theory is based on a particular idea about unification, but we also have ideas for achieving unification with Loop Quantum Gravity."

4 OTHER APPROACHES

"... it would be meaningless to speak about the "division" of the original particles. Experimentally, the concept of "dividing" had lost its meaning"

W. Heisenberg

4.1 Introduction

We saw that inspite of the success of the gauge theoretic formulation of the fundamental interactions there has been unease particularly about the infinities and their cancellations, apart from the Big Bang singularity itself which John Wheeler has termed the greatest crisis confronting physics [46]. From the 1930s itself Bohr, Heisenberg and Dirac were already thinking about the minimum fundamental spacetime interval. Infact Bohr wrote to Dirac [113] "I believe firmly the solution of the present troubles will not be reached without a revision of our general physical ideas still deeper than that contemplated in the present Quantum Mechanics."

Heisenberg on the other hand attempted a version of QED based on a lattice structure for spacetime. Heisenberg's papers lead Born in the late thirties and forties to develop a theory of reciprocity between spacetime and energy-momentum, in which a fundamental length was incorporated.

Also in the forties with the same motivation, Snyder as already noted [114, 115] worked out a covariant scheme of quantized spacetime. Several other scholars including Yang and Schild also worked on a similar structure in the forties. This was pursued subsequently by Hill in 1950, Das in 1960, Gol'fand in 1963, Kadyshevski, also in 1963 [24, 116, 117],(Cf.ref.[24] for a detailed bibliography). However the concept of a fundamental length lapsed into oblivion thanks to the success of the renormalization program and the gauge formulation.

Nevertheless Dirac was a critic of these successful programmes, right from the very beginning. Thus he wrote "Recent work by Lamb, Schwinger and Feynman and others has been very successful... but the resulting theory is an ugly and incomplete one." According to his recent biographer Kragh [282], throughout the remainder of his life he never wavered in the verdict that

"these [normalization] rules, even though they may lead to results in agreement with observations, are artificial rules, and I just cannot accept that the present foundations [of relativistic Quantum Field Theory] are correct." In fact, in his very last paper, published posthumously under the title "The Inadequacies of Quantum Field Theory", Dirac reiterated the following: "Just because the results [of conventional renormalization theory] happen to be in agreement with observation does not prove that one's theory is correct. After all, the Bohr theory was correct in simple cases. It gave very good answers, but still the Bohr theory had the wrong concepts. Correspondingly, the renormalized kind of Quantum Theory with which physicists are working nowadays is not justifiable by agreement with experiments." [118].
Scholars like Caldirola [119] and others continued to write about the Chronon, or a basic unit of time, though. These ideas were revived in the late eighties through the work of Bombelli [120], Finkelstein [121] and others.
Much of the motivation for studying spacetime with a fundamental length has come from as mentioned a realization that gravitation and Quantum Mechanics may not be unified within the context of a differentiable spacetime manifold. It has undoubtedly been recognized that a major problem in introducing quantized spacetime would be that Lorentz symmetry and General Relativistic covariance would both be violated [122]. However it is interesting that the work on Loop Quantum Gravity, as mentioned in the previous Chapter preserves General Relativistic principles within the context of a quantized spacetime. It may also be mentioned that in his last paper, in 1976, Heisenberg discussed his general dissatisfaction with the quark model and also pointed out that iterative sub divisions of spacetime might loose their meaning as we approach immeasurably small intervals [123]. We could give some logical backing to Heisenberg's intuition: the process of subdivision to a single point would require an infinite series of steps and would therefore be meaningless from the point of view of physical measurement.
The next and subsequent Chapters will investigate these aspects. But let us briefly survey some interesting approaches.

4.2 Quantum General Relativity

In the above context Prugoveski has done considerable work on Quantum Geometries incorporating into their structure a fundamental length. In his own words [113] a summary of some of his results is:

"1. The central concept of this framework is that of quantum frame and superframe bundle. The frames and superframes in such bundles take over the role played by the local Lorentz frames of classical General Relativity. Due to their informational completeness, these local quantum frames and superlocal quantum superframes are capable of taking over the role played by complete sets of (compatible) observables in orthodox Quantum Mechanics.

2. In the quantum geometric regime all the counterparts of the constraints that emerge from classical field theories and from classical General Relativity are geometrized by means of gauge groups, which inject them into Maurer-Cartan structural equations that govern the construction of connection forms in principal quantum frame and superframe bundles.
3. The elements of all massive quantum frame bundles possess an operational interpretation at the microlevel, obtained by replacing the test particles of classical relativity with quantum test bodies (i.e., geometro-stochastic excitons).
4. The quantum-geometric evolution of fields in mutual local interactions within a quantum spacetime supermanifold is described by a perpetually ongoing process of creation and annihilation of the geometro-stochastic excitons associated with them.
5. In the resulting framework for geometric Quantum Gravity, matter and quantum fields in free fall propagate by parallel transport along stochastic paths; those paths are the limits of broken paths corresponding to time-ordered segmentations of a quantum gravitational spacetime supermanifold.
6. The causal time-ordering in a quantum gravitational supermanifold is intrinsic, since it is implicit in the (local) proper time marked by the massive constituents of the quantum frames. This proper time emerges from an adaptation to geometro-stochastic excitons of De Broglie's (1923, 1924) original idea that a natural time is inscribed in all matter in existence, since each elementary quantum object of rest-mass m can be viewed as a natural clock with mean period $T = 2\pi/m$ in Plank natural units.
7. The quantum general relativistic covariance principle is embedded in a quantum gravitational supergroup, which incorporates the semidirect product of two types of sub-groups of gauge transformations: one type pertains to the metric equivalence classes of quantum superframes that are interrelated by supergauge transformations originating from superoperator representations of the diffeomorphism group, and gives rise to equivalence classes of mean metrics; the other type describes changes of quantum superframes within the equivalence class for each of these mean metrics, and is provided by superunitary representation of the Poincare group.
8. The strong equivalence principle is embedded in the above type of Poincare gauge invariance, as well as in the mode of the quantum-geometric propagation of quantum fields, which takes place by parallel transport along the arcs of the broken paths that are the horizontal lifts of geodesics of the Levi-Civita connection in each of the metric equivalence classes. These geodesics lie in the base manifold resulting from the natural fibration of the quantum spacetime supermanifold into superfibres of quantum superframes lying above the various points in that base manifold - which can be viewed as a "classical" spacetime manifold that labels the mean stochastic locations of geometro-stochastic excitons and of the fields producing or annihilating them.
9. The quantum superposition principle is embedded in the path integrals that

describe the outcome of the quantum-geometric propagation of all quantum superfields along all possible stochastic causal paths. The causality of these paths is embodied in the above described features of quantum propagation, which reflect a microcausal time-ordering and an adaptation of the geodesic postulate to quantum general relativistic propagation. Their stochasticity is not due to the presence of probability measures over paths, as in classical stochastic processes; rather, it is due to the fact that, in constructing the limits leading to the quantum-geometric propagators of quantum fields and superfields, the superpositions of propagators for parallel transport are taken, with purely geometric weighting factors, over all possible broken causal paths consisting of geodesic arcs".

4.3 Scale Relativity

Another interesting approach is that of Laurent Nottale who has introduced the idea of Scaled Relativity [124]-[133]. The main idea is a generalization of Einstein's Principle of Relativity to scale transformations. It emerges that spacetime is scale dependent, that is fractal and so is no longer a differentiable continuum. These new scale relativistic transformations lead to the appearance of a minimal and a maximal length scale in nature, which are invariant under dilatations. The minimal length scale is the Planck length and the maximal scale is $\Lambda^{-\frac{1}{2}}$ where Λ is the cosmological constant.
Nottale has related the fractal and renormalization group approaches to develop a new version of stochastic Quantum Mechanics. In this approach, the correspondence principle and the Schrödinger equation are obtained by replacing the classical time derivative by a Quantum covariant derivative. There are many applications of this approach, ranging from the mass spectrum of elementary particles through cosmology. One interesting consequence is that the flat rotation curves of galaxies which lead to the invoking of Dark Matter in conventional theories are explained in terms of the fractality of space. Similarly density fluctuations are explained without the necessity of invoking inflation. So also the horizon problem referred to in Chapter 2 can be overcome without inflation. There are also interesting manifestations of "quantized" systems in the macro Universe, e.g., quantized orbits in the solar system and so on.

4.4 Cantorian Spacetime

A very interesting idea introduced by El Naschie is that of an infinite dimensional transfinite Cantorian spacetime [134]-[143]. This reductionist approach is then linked to the global thermodynamic interpretation of Quantum Mechanics. Quantum Mechanics would now appear to be the result of a turbulent but homogenous diffusion process in a transfinite non smooth micro spacetime with an area like Quantum path. Interestingly the four dimensionality of micro spacetime is now the consequence of the discrete Maxwell Boltzmann distribution of the elementary Cantor sets forming this space. It turns out that the effective Hausdorff dimension is given by $4+\phi^3$ where ϕ is the golden mean. It is only at the Planck energy scale, that the infinite dimensions of the underlying space appears. This apart many of the paradoxes of Quantum Mechanics are traced back to the underlying unstable and non smooth Cantorian geometry. Several other results are proposed by El Naschie with this geometry - the fine structure constant, for example. A transfinite heterotic String Theory is also postulated. However, El Naschie tries to demonstrate that the strings themselves emerge from the underlying sizzling set of Cantor spacetime points. There is also an interesting treatment of Quantum Gravity and also, a mass spectrum for the elementary particles is exhibited.

5 FUZZY SPACETIME AND THE PLANCK SCALE

"It is somewhat puzzling to the present author why the lattice structure of space and time had escaped attention from other investigators up till now..."

G. 't Hooft

5.1 The Origins of Fuzzy Spacetime

The Theory of Relativity (Special and General) and Quantum Theory have been often described as the two pillars of twentieth century physics. Each in its own right explained aspects of the universe to a certain extent. But there are still many unanswered questions. For example spacetime singularities (like the Big Bang), termed by John Wheeler as the Greatest Crisis of Physics, the many divergences encountered in particle physics, some eighteen arbitrary parameters in the standard model, elusive monopoles (and Higgs bosons), gravitational waves and Dark Matter and so on.
To quote t' Hooft (drawing a comparison with planetary orbits) [144], "What we do know is that the standard model, as it stands today, cannot be entirely correct, in spite of the fact that the interactions stay weak at ultrashort distance scales. Weakness of the interactions at short distances is not enough; we also insist that there be a certain amount of stability. Let us use the metaphor of the planets in their orbits once again. We insisted that, during extremely short time intervals, the effects of the forces acting on the planets have hardly any effect on their velocities, so that they move approximately in straight lines. In our present theories, it is as if at short time intervals several extremely strong forces act on the planets, but, for some reason, they all but balance out. The net force is so weak that only after long time intervals, days, weeks, months, the velocity change of the planets become apparent. In such a situation, however, a reason must be found as to why the forces at short time scales balance out. The way things are for the elementary particles, at present, is that the forces balance out just by accident. It would be an inexplicable accident, and as no other examples of such accidents are known in Nature, at least not of this magnitude, it is reasonable to suspect that the true short distance structure is not exactly as described in the standard model, but that there are more particles and forces involved, whose nature

is as yet unclear."
Yet it was almost as if Rudyard Kipling's "The twain shall never meet" was true for these two intellectual achievements - General Relativity and Quantum Theory, a view endorsed by Pauli, who went as far as to say that we should not try to put together what God had intended to be separate. For decades there have been fruitless attempts to unify electromagnetism and gravitation, or Quantum Theory and General Relativity: As Wheeler put it [46], the problem has been, how to incorporate curvature into Quantum Theory or spin half into General Relativity:
"It is impossible to accept any description of elementary particles that does not have a place for spin $\frac{1}{2}$. What, then, has any purely geometric description to offer in explanation of spin $\frac{1}{2}$ in general? More particularly and more importantly, what place is there in quantum geometrodynamics for the neutrino–the only entity of half-integral spin that is a pure field in its own right, in the sense that it has zero rest mass and moves with the speed of light? No clear or satisfactory answer is known to this question today. Unless and until an answer is forthcoming, pure geometrodynamics must be judged deficient as a basis for elementary particle physics."
At the same time it is also remarkable that both these disparate theories share one common platform: An underlying differentiable spacetime manifold, be it the Reimannian spacetime of General Relativity or the Minkowski spacetime of Relativistic Quantum Theory (including Quantum Field Theory).
However this underlying common feature has been questioned by Quantum Gravity theories including the author's own model on the one hand and Quantum Superstrings on the other amongst more recent approaches, which try to provide a unified description as we saw earlier (Cf.ref.[145] and several references therein). We will now argue from a perspective which shares this spirit, that unification and a geometrical structure for Quantum Theory are possible if differentiable spacetime is discarded in favour of fuzzy spacetime Indeed Einstein himself had anticipated this. As he observed around 1930 itself [146]
"... It has been pointed out that the introduction of a space-time continuum may be considered as contrary to nature in view of the molecular structure of everything which happens on a small scale. It is maintained that perhaps the success of the Heisenberg method points to a purely algebraic method of description of nature that is to the elimination of continuous functions from physics. Then however, we must also give up, by principle the space-time continuum. It is not unimaginable that human ingenuity will some day find methods which will make it possible to proceed along such a path. At present however, such a program looks like an attempt to breathe in empty space."
Infact Clifford had also anticipated such ideas much earlier [147]: "I hold in fact (1) That small portions of space are in fact of a nature analogous to little hills on a surface which is on the average flat; namely, that the ordinary laws of geometry are not valid in them. (2) That this property of being curved or distorted is continually being passed on from one portion of space to another

after the manner of a wave..."
Perhaps smooth spacetime is an approximation? Infact Mandelbroit's work on fractals has clearly brought out that the smooth curves of Classical Mathematics are to be replaced in real life, by fractal structures, previously dismissed as pathological cases [148]. At the same time this new description has many ramifications and leads, in our formulation, to a cosmology which correctly predicted the latest iconoclastic observations, for example that the Universe is accelerating and expanding for ever with a small cosmological constant while supposedly sacrosanct constants like the fine structure constant seem to be changing with time.
We start with a physical model so as to clarify ideas as in Chapter 1. The Kerr-Newman Black Hole of Classical Physics and General Relativity describes the electron's purely Quantum Mechanical g=2 factor. But the price one has to pay is the naked singularity, or equivalently, the complex space coordinate. Curiously enough, the space coordinate of the Dirac electron has precisely the same non Hermitian or complex character. In Quantum Theory, this is due to zitterbewegung effects, which are eliminated, as Dirac pointed out, by averaging over the Compton scale: spacetime points have no physical meaning [7]. Compton scale intervals, complex coordinates, spin and noncommutative geometry, are all symptomatic or indicative of the underlying fuzzy spacetime.
From Galilean-Newtonian Mechanics to Quantum Field Theory, the concept is Newtonian, in that spacetime is a container or stage within which the actors of matter, energy and interactions play their parts, even modifying the stage. However, the new concept of spacetime is Liebnitzian, in that, the actors create or define the stage itself [149]. It is now possible to circumvent spacetime singularities and even the famous divergences.

5.2 Further Considerations

To see all this in greater detail, we observe that if we treat an electron as a Kerr-Newman Black Hole, then even though we get the correct Quantum Mechanical $g = 2$ factor, the horizon of the Black Hole becomes complex [24, 46].

$$r_+ = \frac{GM}{c^2} + \imath b, b \equiv (\frac{GQ^2}{c^4} + a^2 - \frac{G^2M^2}{c^4})^{1/2} \qquad (5.1)$$

G being the gravitational constant, M the mass and $a \equiv L/Mc, L$ being the angular momentum. While (5.1) exhibits a naked singularity, and as such has no physical meaning, we note that the position coordinate for a Dirac particle in conventional theory is given by

$$x = (c^2 p_1 H^{-1} t) + \frac{\imath}{2} c\hbar (\alpha_1 - cp_1 H^{-1}) H^{-1} \qquad (5.2)$$

an expression that is very similar to (5.1). Infact the imaginary parts of both (5.1) and (5.2) are the same, being of the order of the Compton wavelength. It is at this stage that a proper physical interpretation begins to emerge. Dirac himself observed as noted, that to interpret (5.2) meaningfully, it must be remembered that Quantum Mechanical measurements are really averaged over the Compton scale: Within the scale there are the unphysical zitterbewegung effects: for a point electron the velocity equals that of light.
Once such a minimum spacetime scale is invoked, then we have a non commutative geometry as shown by Snyder more than fifty years ago [114]:

$$[x,y] = (\imath a^2/\hbar)L_z, [t,x] = (\imath a^2/\hbar c)M_x, etc.$$

$$[x,p_x] = \imath\hbar[1+(a/\hbar)^2p_x^2]; \tag{5.3}$$

The relations (5.3) are compatible with Special Relativity. Indeed such minimum spacetime models were studied for several decades, precisely to overcome the divergences encountered in Quantum Field Theory [24],[115]-[121],[150, 152].
Before proceeding further, it may be remarked that when the square of a, which we will take to be the Compton wavelength (including the Planck scale, which is a special case of the Compton scale for a Planck mass viz., $10^{-5}gm$), in view of the above comments can be neglected, then we return to point Quantum Theory.
It is interesting that starting from the Dirac coordinate in (5.2), we can deduce the non commutative geometry (5.3), independently. For this we note that the α's in (5.2) are given by

$$\boldsymbol{\alpha} = \begin{bmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{bmatrix} \quad ,$$

the σ's being the Pauli matrices. We next observe that the first term on the right hand side is the usual Hermitian position. For the second term which contains α, we can easily verify from the commutation relations of the σ's that

$$[x_\imath, x_j] = \beta_{\imath j} \cdot l^2 \tag{5.4}$$

where l is the Compton scale.
There is another way of looking at this. Let us consider the coordinate in (5.2) or (5.1) to be complex, reminiscent of Newman's original complexification of the coordinate. We now try to generalize this complex coordinate to three dimensions. Then we encounter a surprise - we end up with not three, but four dimensions,

$$(1,\imath) \to (I,\sigma),$$

where I is the unit 2×2 matrix. We get the special relativistic Lorentz invariant metric at the same time. (In this sense, as noted by Sachs [153], Hamilton who made this generalization would have also hit upon Special Relativity, if

he had identified the fourth coordinate with time).
That is,

$$x + \imath y \to I x_1 + \imath x_2 + j x_3 + k x_4,$$

where $(\imath, j, k)$ now represent the Pauli matrices; and, further,

$$x_1^2 + x_2^2 + x_3^2 - x_4^2$$

is invariant.
While the usual Minkowski four vector transforms as the basis of the four dimensional representation of the Poincare group, the two dimensional representation of the same group, given by the right hand side in terms of Pauli matrices, obeys the quaternionic algebra of the second rank spinors (Cf.Ref.[154, 155, 153] for details).
To put it briefly, the quarternion number field obeys the group property and this leads to a number system of quadruplets as a minimum extension. In fact one representation of the two dimensional form of the quarternion basis elements is the set of Pauli matrices. Thus a quarternion may be expressed in the form

$$Q = -\imath \sigma_\mu x^\mu = \sigma_0 x^4 - \imath \sigma_1 x^1 - \imath \sigma_2 x^2 - \imath \sigma_3 x^3 = (\sigma_0 x^4 + \imath \boldsymbol{\sigma} \cdot \boldsymbol{r})$$

This can also be written as

$$Q = -\imath \begin{pmatrix} \imath x^4 + x^3 & x^1 - \imath x^2 \\ x^1 + \imath x^2 & \imath x^4 - x^3 \end{pmatrix}.$$

As can be seen from the above, there is a one to one correspondence between a Minkowski four-vector and Q. The invariant is now given by $Q\bar{Q}$, where $\bar{Q}$ is the complex conjugate of Q.
However, as is well known, there is a lack of spacetime reflection symmetry in this latter formulation. If we require reflection symmetry also, we have to consider the four dimensional representation,

$$(I, \boldsymbol{\sigma}) \to \left[\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix} \right] \equiv (\Gamma^\mu)$$

(Cf.also.ref. [66] for a detailed discussion). The motivation for such a reflection symmetry is that usual laws of physics, like electromagnetism do indeed show the symmetry.
We at once deduce spin and Special Relativity and the geometry (5.3). This is a transition that has been long overlooked [187]. Conversely it must be mentioned that spin half itself is relational and refers to three dimensions, to a spin network infact [72]. That is, spin half is not meaningful in a single particle Universe.
Equally interesting is the fact that starting from the geometry (5.3) we can

deduce the Dirac equation itself as we will see in the next section.
While a relation like (5.4) above has been in use recently, in non commutative models, and as noted, was an independent starting point due to the work of Snyder, we would like to stress that it has been overlooked that the origin of this non commutativity lies in the original Dirac coordinates.
The above relation shows on comparison with the position-momentum commutator that the coordinate $\boldsymbol{x}$ also behaves like a "momentum". This can be seen directly from the Dirac theory itself where we have [7]

$$c\boldsymbol{\alpha} = \frac{c^2 \boldsymbol{p}}{H} - \frac{2\imath}{\hbar} \hat{x} H \tag{5.5}$$

In (5.5), the first term is the usual momentum. The second term is the extra "momentum" $\boldsymbol{p}$ due to zitterbewegung.
Infact we can easily verify from (5.5) that

$$\boldsymbol{p} = \frac{H^2}{\hbar c^2} \hat{x} \tag{5.6}$$

where $\hat{x}$ has been defined in (5.5).
We finally investigate what the angular momentum $\sim \boldsymbol{x} \times \boldsymbol{p}$ gives - that is, the angular momentum at the Compton scale. We can easily show that

$$(\boldsymbol{x} \times \boldsymbol{p})_z = \frac{c}{E} (\boldsymbol{\alpha} \times \boldsymbol{p})_z = \frac{c}{E} (p_2 \alpha_1 - p_1 \alpha_2) \tag{5.7}$$

where E is the eigen value of the Hamiltonian operator H. Equation (5.7) shows that the usual angular momentum but in the context of the Compton scale, leads to the "mysterious" Quantum Mechanical spin.
In the above considerations, we started with the Dirac equation and deduced the underlying non commutative geometry of spacetime. Interestingly, starting with Snyder's non commutative geometry, based solely on Lorentz invariance and a minimum spacetime length, which we have taken to be the Compton scale, (5.3), it is possible to deduce the relations (5.7), (5.6) and the Dirac equation itself as noted [101] and as we will see shortly.
We have thus established the correspondence between considerations starting from the Dirac theory of the electron and Snyder's (and subsequent) approaches based on a minimum spacetime interval and Lorentz covariance. It can be argued from an alternative point of view that Special Relativity operates outside the Compton wavelength (Cf.ref.[24]).
We started with the Kerr-Newman Black Hole. Infact the derivation of the Kerr-Newman Black Hole itself begins with a Quantum Mechanical spin yielding complex shift, which Newman has found inexplicable even after several decades [61]-[63]. As he observed, "...one does not understand why it works. After many years of study I have come to the conclusion that it works simply by accident". And again, "Notice that the magnetic moment $\mu = ea$ can be thought of as the imaginary part of the charge times the displacement of

the charge into the complex region... We can think of the source as having a complex center of charge and that the magnetic moment is the moment of charge about the center of charge... In other words the total complex angular momentum vanishes around any point z^a on the complex world-line. From this complex point of view the spin angular momentum is identical to orbital, arising from an imaginary shift of origin rather than a real one... If one again considers the particle to be "localized" in the sense that the complex center of charge coincides with the complex center of mass, one again obtains the Dirac gyromagnetic ratio..."

The unanswered question has been, why does a complex shift somehow represent spin about that axis? The answer to this question lies in the above considerations. Complexified spacetime is symptomatic of fuzzy spacetime and a non commutative geometry and Quantum Mechanical spin [154]. Indeed Zakrzewski has shown in a classical context that non commutativity implies spin [156, 157].

The above considerations used the Quantum Mechanical spin together with classical relativity, though the price to pay for this was minimum spacetime intervals and noncommutative geometry. Is this the path towards a reconciliation of electromagnetism and gravitation?

5.3 Quantum Geometry

One of the earliest attempts to unify electromagnetism and gravitation, was Weyl's gauge invariant geometry. The basic idea was [92] that while

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{5.8}$$

was invariant under arbitrary transformations in General Relativity, a further invariant, namely,

$$\Phi_\mu dx^\mu \tag{5.9}$$

which is a linear form should be introduced. $g_{\mu\nu}$ in (5.8) would represent the gravitational potential, and Φ_μ of (5.9) would represent the electromagnetic field potential. As Weyl observed, "The world is a $3+1$ dimensional metrical manifold; all physical field - phenomena are expressions of the metrics of the world. (Whereas the old view was that the four-dimensional metrical continuum is the scene of physical phenomena; the physical essentialities themselves are, however, things that exist "in" this world, and we must accept them in type and number in the form in which experience gives us cognition of them: nothing further is to be "comprehended" of them.)$\cdots$"

This was a bold step, because it implied the relativity of magnitude multiplied effectively on all components of the metric tensor $g_{\mu\nu}$ by an arbitrary function of the coordinates. However, the unification was illusive because the $g_{\mu\nu}$ and Φ_μ were really independent elements. As Einstein noted, in Stafford Little Lectures delivered in May 1921 at Princeton University [4], "...if we introduce

the energy tensor of the electromagnetic field into the right hand side of (the gravitational field equation) we obtain (the first of Maxwell's systems of equations in tensor density form), for the special case $(\sqrt{-g}\rho\frac{dx_\nu}{ds} =)r^\mu = 0, \cdots$ This inclusion of the theory of electricity in the scheme of General Relativity has been considered arbitrary and unsatisfactory... a theory in which the gravitational field and the electromagnetic field do not enter as logically distinct structures would be much preferable..."
A more modern treatment is recapitulated below [6].
The above arbitrary multiplying factor is normalized and we require that,

$$|g_{\mu\nu}| = -1, \tag{5.10}$$

For the invariance of (5.10), $g_{\mu\nu}$ transforms now as a tensor density of weight minus half, rather than as a tensor in the usual theory. The covariant derivative now needs to be redefined as

$$T^{\iota\cdots}_{\kappa\cdots,\sigma} = T^{\iota\cdots}_{\kappa\cdots,\sigma} + \Gamma^{\iota}_{\rho\sigma}T^{\rho\cdots}_{\kappa\cdots} - \Gamma^{\rho}_{\kappa\sigma}T^{\iota\cdots}_{\rho\cdots} - nT^{\iota\cdots}_{\kappa\cdots}\Phi_\sigma, \tag{5.11}$$

In (5.11) we have introduced the Φ_μ, and n is the weight of the tensor density. This finally leads to (Cf.ref.[6] for details).

$$\Phi_\sigma = \Gamma^\rho_{\rho\sigma}, \tag{5.12}$$

Φ_μ in (5.12) is identified with the electromagnetic potential, while $g_{\mu\nu}$ gives the gravitational potential as in the usual theory. The affine connection is now given by

$$\Gamma^\lambda_{\iota\kappa} = \frac{1}{2}g^{\lambda\sigma}(g_{\iota\sigma,\kappa} + g_{\kappa\sigma,\iota} - g_{\iota\kappa,\sigma}) + \frac{1}{4}g^{\lambda\sigma}(g_{\iota\sigma}\Phi_\kappa + g_{\kappa\sigma}\Phi_\iota - g_{\iota\kappa}\Phi_\sigma) \equiv \begin{pmatrix} \lambda \\ \iota\kappa \end{pmatrix} \tag{5.13}$$

The essential point, and this was the original criticism of Einstein and others, is that in (5.13), $g_{\mu\nu}$ and Φ_μ are independent entities.
Let us now analyze the above from a different perspective. Let us write the product $dx^\mu dx^\nu$ of (5.8) as a sum of half its anti-symmetric part and half the symmetric part. The invariant line element in (5.8) now becomes $(h_{\mu\nu} + \hbar_{\mu\nu})dx^\mu dx^\nu$ where h and $\hbar$ denote the anti-symmetric and symmetric parts respectively of g. h would vanish unless the commutator

$$[dx^\mu, dx^\nu] \approx l^2 \neq 0 \tag{5.14}$$

l being some fundamental minimum length. Infact h can be characterized as

$$h_{\mu\nu} = \eta^{\rho\sigma}\epsilon_{\rho\sigma\mu\nu},$$

where η is an antisymmetric tensor and ϵ is the Levi-Civita tensor density.
As pointed out a little earlier the noncommutative geometry given in (5.14) was studied by Snyder and others though from a different perspective. In this

case it has been shown in detail by the author [158, 71] that under a time elapse transformation of the wave function, (or, alternatively, as a small scale transformation),

$$|\psi' >= U(R)|\psi > \tag{5.15}$$

we get

$$\psi'(x_j) = [1 + \imath\epsilon(\imath x_j \frac{\partial}{\partial x_j}) + 0(\epsilon^2)]\psi(x_j) \tag{5.16}$$

Equation (5.16) has been shown to lead to the Dirac equation when ϵ is the Compton time. A quick way to see this is as follows: At the Compton scale we have,

$$|\boldsymbol{L}| = |\boldsymbol{r} \times \boldsymbol{p}| = |\frac{\hbar}{2mc} \cdot mc| = \frac{\hbar}{2},$$

that is, we get the Quantum Mechanical spin. Next, we can easily verify, that the choice,

$$t = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \boldsymbol{x} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}$$

provides a representation for the coordinates in (5.3), apart from scalar factors. As can be seen, this is also a representation of the Dirac matrices. Substitution of the above in (5.16) leads to the Dirac equation

$$(\gamma^\mu p_\mu - mc^2)\psi = 0$$

because

$$E\psi = \frac{1}{\epsilon}\{\psi'(x_j) - \psi(x_j)\}, \quad E = mc^2,$$

where $\epsilon = \tau$ (Cf.ref.[150]).
Indeed, as noted, Dirac himself had realized that his electron equation needed an average over spacetime intervals of the order of the Compton scale to remove zitterbewegung effects and give meaningful physics. This again is symptomatic of an underlying fuzzy spacetime described by a noncommutative spacetime geometry (5.14) or (5.4) [154].
The point here is that under equation (5.14), the coordinates $x^\mu \to \gamma^{(\mu)}x^{(\mu)}$ where the brackets with the superscript denote the fact that there is no summation over the indices. Infact, in the theory of the Dirac equation it is well known [159]that,

$$\gamma^k\gamma^l + \gamma^l\gamma^k = -2g^{kl}I \tag{5.17}$$

where γ's satisfy the usual Clifford algebra of the Dirac matrices, and can be represented by

$$\gamma^k = \sqrt{2}\begin{pmatrix} 0 & \sigma^k \\ \sigma^{k*} & 0 \end{pmatrix} \tag{5.18}$$

where σ's are the Pauli matrices. As noted by Bade and Jehle (Cf.ref.[159]), we could take the σ's or γ's in (5.17) and (5.18) as the components of a contravariant world vector, or equivalently we could take them to be fixed

matrices, and to maintain covariance, to attribute new transformation properties to the wave function, which now becomes a spinor (or bi-spinor). This latter has been the traditional route, because of which the Dirac wave function has its bi-spinorial character. In this latter case, the coordinates retain their usual commutative or point character. It is only when we consider the equivalent former alternative, that we return to the noncommutative geometry (5.14).
That is, in the usual commutative spacetime the Dirac spinorial wave functions conceal the noncommutative character (5.14).
Indeed we can verify all these considerations in a simple way as follows:
First let us consider the usual spacetime. This time the Dirac wave function is given by

$$\psi = \begin{pmatrix} \chi \\ \Theta \end{pmatrix},$$

where χ and Θ are spinors. It is well known that under reflection while the so called positive energy spinor Θ behaves normally, $\chi \to -\chi, \chi$ being the so called negative energy spinor which comes into play at the Compton scale [29]. That is, the space is doubly connected. Because of this property as shown in detail [71], there is now a covariant derivative given by, in units, $\hbar = c = 1$,

$$\frac{\partial \chi}{\partial x^\mu} \to [\frac{\partial}{\partial x^\mu} - nA^\mu]\chi \tag{5.19}$$

where

$$A^\mu = \Gamma^{\mu\sigma}_\sigma = \frac{\partial}{\partial x^\mu} log(\sqrt{|g|}) \tag{5.20}$$

Γ denoting the Christofell symbols.
A^μ in (5.20)is now identified with the electromagnetic potential, exactly as in Weyl's theory except that now, A^μ arises from the bi spinorial character of the Dirac wave function or the double connectivity of spacetime. Further, as shown already [160], the mass density of the particle is given by,

$$\rho = \chi\chi^*$$

Indeed ρ vanishes outside the Compton scale for any particle.
What all this means is that the so called ad hoc feature in Weyl's unification theory is really symptomatic of the underlying noncommutative spacetime geometry (5.14). Given (5.14) we get both gravitation and electromagnetism in a unified picture, because both are now the consequence of spacetime geometry. We could think that gravitation arises from the symmetric part of the metric tensor (which indeed is the only term if $0(l^2)$ is neglected) and electromagnetism from the antisymmetric part (which manifests itself as an $0(l^2)$ effect). It is also to be stressed that in this formulation, we are working with noncommutative effects at the Compton scale, this being true for the Weyl like formulation also. We will see this in a little greater detail later.

That is, once we abandon smooth spacetime manifolds and consider non-commutative geometries defined by, for example (5.3) or (5.4) or (5.14), then we are lead to multiply connected manifolds which conceal the Quantum Mechanical spin half and a unified description of Quantum Mechanics and Geometrodynamics becomes possible. We will return to this point in Chapter 7. Finally it may be mentioned that the fact that n in (5.19) is integral, explains the discreteness of electric charge.

5.4 The Unification of Gravitation and Electromagnetism

The identification of the Kerr-Newman Black Hole of classical physics with the Quantum Mechanical electron already points to a unified description of gravitation and electromagnetism. This can be seen directly from the non commutative geometry (5.4) or (5.14) [161]. Indeed let us start with the expression for the metric As before, rewriting the product of the two coordinate differentials in (5.8) in terms of the symmetric and non symmetric combinations, we get for the right side $\frac{1}{2}g_{\mu\nu}[(dx^\mu dx^\nu + dx^\nu dx^\mu) + (dx^\mu dx^\nu - dx^\nu dx^\mu)]$, so that, we can write

$$g_{\mu\nu} = \eta_{\mu\nu} + kh_{\mu\nu} \tag{5.21}$$

where the first term on the right side of (5.21) denotes the usual flat spacetime and the second term denotes the effect of the non commutativity, k being a suitable constant.

It must be noted that if $l, \tau \to 0$ then equation (5.21) reduces to the usual formulation. From a physical point of view, if we are dealing with time and length scales much greater than the Compton wavelength, so that the order $0(l^2)$ terms can be neglected, then the usual commutative geometry works, with the usual derivatives and more generally differential geometry. In that sense, and at such scales we can attribute the same meaning to coordinate differentials like dx^μ. However this formulation breaks down at and inside the scale (l, τ) as discussed earlier. In what follows, in order to see the effect of the non commutative geometry, we will consider scales, near the minimum (l, τ) scale, and continue to use the concept of derivatives and differentials as a good approximation, while incorporating the effects of departure from the commutative geometry at the same time.

The effect of the non commutative geometry is therefore to introduce a departure from flat spacetime, as can be seen from (5.21). Indeed, as is well known (Cf.ref.[5]), this is exactly as in the case of General Relativity and the second term on the right of (5.21) plays the role of the usual energy momentum tensor. However it must be borne in mind that we are now dealing with elementary particles. For an elementary particle, the material density vanishes outside its Compton wavelength and therefore also the minimum scale. On the other hand it should be borne in mind that at and near the minimum

scale itself we have the departure from the usual commutative geometry, as we will see below.
Infact remembering that the second term of the right side of (5.21) is small, this can straightaway be seen to lead to a linearized theory of General Relativity [5]. Exactly as in standard literature we could now deduce the General Relativistic relation

$$\partial_\lambda \partial^\lambda h^{\mu\nu} - (\partial_\lambda \partial^\nu h^{\mu\lambda} + \partial_\lambda \partial^\mu h^{\nu\lambda})$$
$$-\eta^{\mu\nu} \partial_\lambda \partial^\lambda h + \eta^{\mu\nu} \partial_\lambda \partial_\sigma h^{\lambda\sigma} = -k\bar{T}^{\mu\nu} \tag{5.22}$$

It must be mentioned that the energy momentum type term on the right side of (5.22) arises due to the fact that the derivatives ∂^λ and ∂^μ no longer commute and this leads to an additional contribution as can be verified from the left side of (5.22). To show this special origin of the right side term, we have used $\bar{T}$ instead of the usual T. More explicitly, it follows from the foregoing that (Cf.ref.[162])

$$\frac{\partial}{\partial x^\lambda}\frac{\partial}{\partial x^\mu} - \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\lambda} \text{goes over to} \frac{\partial}{\partial x^\lambda}\Gamma^\nu_{\mu\nu} - \frac{\partial}{\partial x^\mu}\Gamma^\nu_{\lambda\nu} \tag{5.23}$$

Normally in conventional theory the right side of (5.23) would vanish. Let us designate this non vanishing part on the right by

$$\frac{e}{c\hbar}F^{\mu\lambda} \tag{5.24}$$

We have shown here that the non commutativity in momentum components leads to an effect that can be identified with electromagnetism and infact from expression (5.24) we have

$$A^\mu = \hbar \Gamma^{\mu\nu}_\nu \tag{5.25}$$

where A_μ can be identified with the electromagnetic four potential (Cf.also ref.[24]). To see this in the light of the usual gauge invariant minimum coupling (Cf.ref.[24]), we start with the effect of an infinitesimal parallel displacement of a vector in this non commutative geometry,

$$\delta a^\sigma = -\Gamma^\sigma_{\mu\nu} a^\mu dx^\nu \tag{5.26}$$

As is well known, (5.26) represents the effect due to the curvature and non integrable nature of space - in a flat space, the right side would vanish. Considering the partial derivatives with respect to the μ^{th} coordinate, this would mean that, due to (5.26)

$$\frac{\partial a^\sigma}{\partial x^\mu} \to \frac{\partial a^\sigma}{\partial x^\mu} - \Gamma^\sigma_{\mu\nu} a^\nu \tag{5.27}$$

Letting $a^\mu = \partial^\mu \phi$,, we have, from (5.27)

$$D_{\mu\nu} \equiv \partial_\nu \partial^\mu \to D'_{\mu\nu} \equiv \partial_\nu \partial^\mu - \Gamma^\mu_{\lambda\nu} \partial^\lambda$$
$$= D_{\mu\nu} - \Gamma^\mu_{\lambda\nu} \partial^\lambda \quad (5.28)$$

Now we can also write

$$D_{\mu\nu} = (\partial^\mu - \Gamma^\mu_{\lambda\lambda})(\partial_\nu - \Gamma^\lambda_{\lambda\nu}) + \partial^\mu \Gamma^\lambda_{\lambda\nu} + \Gamma^\mu_{\lambda\lambda} \partial_\nu$$

So we get

$$D_{\mu\nu} - \Gamma^\mu_{\lambda\lambda} \partial_\nu = (p^\mu)(p_\nu)$$

where

$$p^\mu \equiv \partial^\mu - \Gamma^\mu_{\lambda\lambda}$$

Or,

$$D_{\mu\mu} - \Gamma^\mu_{\lambda\lambda} \partial_\mu = (p^\mu)(p_\mu)$$

Further we have

$$D'_{\mu\mu} = D_{\mu\mu} - \Gamma^\mu_{\lambda\mu} \partial^\lambda$$

Thus, (28) gives, finally,

$$D'_{\mu\nu} = (p^\mu)(p_\nu)$$

That is we have

$$\frac{\partial}{\partial x^\mu} \to \frac{\partial}{\partial x^\mu} - \Gamma^\nu_{\mu\nu}$$

Comparison with (5.25) establishes the required identification.

It is quite remarkable that equation (5.25) is once again mathematically identical to Weyl's unified formulation, which we saw in the previous section, though this was not originally acceptable because of the adhoc insertion of the electromagnetic potential. Here in our case it is a consequence of the noncommutative geometry (Cf.refs.[24] and [163] for a detailed discussion).

We can see this in even greater detail as follows. The gravitational field equations can be written as [5]

$$D\phi^{\mu\nu} = -k\bar{T}^{\mu\nu} \quad (5.29)$$

where

$$\phi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h \quad (5.30)$$

and D is the D'Alembertian.

It also follows, if we use the usual gauge and equation (5.25) that

$$\partial_\mu h^{\mu\nu} = A^\nu \quad (5.31)$$

in this linearised theory.

Whence, remembering that we have (5.21), operating on both sides of equation (5.29) with ∂_μ we get Maxwell's equations of electromagnetism on using (5.30) and (5.31).

This is not surprising because as is well known if equation (5.25) holds as in the Weyl formulation, then in the absence of matter the general relativistic field equations (5.22) reduce to Maxwell equations [6]. In any case, all this provides a rationale for the fact that from (5.29) we get the equation for spin 2 gravitons (Cf.ref.[5]) while from the Maxwell equations, we have spin 1 (vector) photons. We will return to this point in Chapter 7.

5.5 The Planck Scale

As we have seen, and this as noted being true in Quantum Gravity as well as in Quantum Super String Theory, we encounter phenomena at a minimum scale. It is well known, and this was realized by Planck himself, that there is an absolute minimum scale in the Universe, and this is,

$$l_P = \left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}} \sim 10^{-33} cm$$

$$t_P = \left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} \sim 10^{-42} sec \tag{5.32}$$

Yet what we encounter in the real world is, not the Planck scale, but the elementary particle Compton scale. The explanation given for this is that the very high energy Planck scale is moderated by the Uncertainty Principle. The question which arises is, exactly how does this happen? We will now present an argument to show how the Planck scale leads to the real world Compton scale, via fluctuations and the modification of the Uncertainty Principle.
We note that (5.32) defines the absolute minimum physical scale [163, 164, 23, 165]. Associated with (5.32) is the Planck mass

$$m_P \sim 10^{-5} gm \tag{5.33}$$

There are certain interesting properties associated with (5.32) and (5.33). l_P is the Schwarzchild radius of a Black Hole of mass m_P while t_P is the evaporation time for such a Black Hole via the Beckenstein Radiation [166]. Interestingly t_P is also the Compton time for the Planck mass, a circumstance that is symptomatic of the fact that at this scale, electromagnetism and gravitation become of the same order [24]. Indeed all this fits in very well with Rosen's analysis that such a Planck scale particle would be a mini Universe [47, 65]. We will now invoke a time varying gravitational constant (to be discussed in detail in Chapters 6 and 8.)

$$G \approx \frac{lc^2}{m\sqrt{N}} \alpha(\sqrt{N} t)^{-1} = T^{-1} \tag{5.34}$$

which resembles the Dirac cosmology and features in another scheme to be discussed in the next Chapter, in which (5.34) arises due to the fluctuation in

the particle number [167, 168, 169, 170, 24]. In (5.34) m and l are the mass and Compton wavelength of a typical elementary particle like the pion while $N \sim 10^{80}$ is the number of elementary particles in the Universe, and T is the age of the Universe.
In this scheme wherein as we shall see in Chapter 6, (5.34) follows from the theory, we use the fact that given N particles, the fluctuation in the particle number is of the order $\sqrt{N}$, while a typical time interval for the fluctuations is $\sim \hbar/mc^2$, the Compton time. We will come back to this point later. So anticipating later work we have

$$\frac{dN}{dt} = \frac{\sqrt{N}}{\tau}$$

whence on integration we get,

$$T = \frac{\hbar}{mc^2}\sqrt{N}$$

and we can also deduce its spatial counterpart, $R = \sqrt{N}l$, which is the well known empirical Eddington formula. We will return to this later.
Equation (5.34) which is an order of magnitude relation is consistent with observation [171, 172] while it may be remarked that the Dirac cosmology itself has inconsistencies.
Substitution of (5.34) in (5.32) yields

$$l = N^{\frac{1}{4}} l_P,$$
$$t = N^{\frac{1}{4}} t_P \quad (5.35)$$

where t as noted is the typical Compton time of an elementary particle. We can easily verify that (5.35) is correct. It must be stressed that (5.35) is not a fortuitous empirical coincidence, but rather is a result of using (5.34), which again as noted, can be deduced from theory.
(5.35) can be rewritten as

$$l = \sqrt{n} l_P$$
$$t = \sqrt{n} t_P \quad (5.36)$$

wherein we have used (5.32) and (5.34) and $n = \sqrt{N}$.
We will now compare (5.36) with the well known relations, referred to earlier,

$$R = \sqrt{N} l \quad T = \sqrt{N} t \quad (5.37)$$

The first relation of (5.37) is the well known Weyl-Eddington formula referred to while the second relation of (5.37) is given also on the right side of (5.34). We now observe that (5.37) can be seen to be the result of a Brownian Walk process, l, t being typical intervals between "steps" (Cf.[24, 173, 174]). We demonstrate this below after equation (5.39). On the other hand, the typical intervals l, t can be seen to result from a diffusion process themselves. Let us consider the well known diffusion relation,

$$(\Delta x)^2 \equiv l^2 = \frac{\hbar}{m} t \equiv \frac{\hbar}{m} \Delta t \tag{5.38}$$

(Cf.[173],[175]-[177]). What is being done here is that we are modelling fuzzy spacetime by a double Wiener process to be touched upon later, which leads to (5.38). This will be seen in more detail, below.
Indeed as l is the Compton wavelength, (5.38) can be rewritten as the Quantum Mechanical Uncertainty Principle

$$l \cdot p \sim \hbar$$

at the Compton scale (Cf. also [178]) (or even at the De Broglie scale).
What (5.38) shows is that a Brownian process defines the Compton scale while (5.37) shows that a Random Walk process with the Compton scale as the interval defines the length and time scales of the Universe itself (Cf.[174]).
Returning now to (5.36), on using (5.33), we observe that in complete analogy with (5.38) we have the relation

$$(\Delta x)^2 \equiv l_P^2 = \frac{\hbar}{m_P} t_P \equiv \frac{\hbar}{m_P} \Delta t \tag{5.39}$$

We can now argue that the Brownian process (5.39) defines the Planck length while a Brownian Random Walk process with the Planck scale as the interval leads to (5.36), that is the Compton scale.
To see all this in greater detail, it may be observed that equation (5.39) (without subscripts)

$$(\Delta x)^2 = \frac{\hbar}{m} \Delta t \tag{5.40}$$

is the same as the equation (5.38), indicative of a double Wiener process. Indeed as noted by several scholars, this defines the fractal Quantum path of dimension 2 (rather than dimension 1) (Cf.e.g. ref.[176]).
Firstly it must be pointed out that equation (5.40) defines a minimum spacetime unit - the Compton scale (l, t). This follows from (5.40) if we substitute into it $\langle \frac{\Delta x}{\Delta t} \rangle_{max} = c$. If the mass of the particle is the Planck mass, then this Compton scale becomes the Planck scale.
Let us now consider the distance traversed by a particle with the speed of light through the time interval T. The distance R covered would be

$$\int dx = R = c \int dt = cT \tag{5.41}$$

by conventional reasoning. In view of the equation (5.40), however we would have to consider firstly, the minimum time interval t (Compton or Planck time), so that we have

$$\int dt \to nt \tag{5.42}$$

Secondly, because the square of the space interval Δx (rather than the interval Δx itself as in conventional theory) appears in (5.40), the left side of (5.41) becomes, on using (5.42)

$$\int dx^2 \to \int (\sqrt{n}dx)(\sqrt{n}dy) \tag{5.43}$$

Whence for the linear dimension R we would have

$$\sqrt{n}R = nct \quad \text{or} \quad R = \sqrt{n}l \tag{5.44}$$

Equation (5.43) brings out precisely the fractal dimension $D = 2$ of the Brownian path while (5.44) is identical to (5.35) or (5.37) (depending on whether we are dealing with minimum intervals of the Planck scale or Compton scale of elementary particles). Apart from showing the Brownian character linking equations (5.35) and (5.40), incidentally, this also provides the justification for what has so far been considered to be a mysterious Large Number coincidence viz. the Eddington formula (5.37).
There is another way of looking at this. It is well known that in approaches like that of the author or Quantum Super String Theory, at the Planck scale we have a non commutative geometry encountered earlier [179, 162] Indeed as noted, (5.3) follows without recourse to Quantum Superstrings, merely by the fact that l_P, t_P are the absolute minimum space time intervals as we saw earlier.
The non commutative geometry (5.3), as is known, is symptomatic of a Modified Uncertainty Principle at this scale [180]-[184]

$$\Delta x \approx \frac{\hbar}{\Delta p} + l_P^2 \frac{\Delta p}{\hbar} \tag{5.45}$$

The relation (5.45) would be true even in Quantum Gravity. The extra or second term on the right side of (5.45) as noted in Chapter 3 expresses the well known duality effect - as we attempt to go down to the Planck scale, infact we are lead to the larger scale. The question is, what is this larger scale? If we now use the fact that $\sqrt{n}$ is the fluctuation in the number of Planck particles (exactly as $\sqrt{N}$ was the fluctuation in the particle number as in (5.34)) so that $\sqrt{n}mpc = \Delta p$ is the fluctuation or uncertainty in the momentum for the second term on the right side of (5.45), we obtain for the uncertainty in length,

$$\Delta x = l_P^2 \frac{\sqrt{n}m_P c}{\hbar} = l_P \sqrt{n}, \tag{5.46}$$

We can easily see that (5.46) is the same as the first relation of (5.36). The second relation of (5.36) follows from an application of the time analogue of

(5.45).
Thus the impossibility of going down to the Planck scale because of (5.3) or (5.45), manifests itself in the fact that as we attempt to go down to the Planck scale, we infact end up at the Compton scale. In the next section we will give another demonstration of this result. This is how the Compton scale is encountered in real life.
Interestingly while at the Planck length, we have a life time of the order of the Planck time, as noted above it is possible to argue on the other hand that with the mass and length of a typical elementary particle like the pion, at the Compton scale, we have a life time which is the age of the Universe itself as shown by Sivaram [166, 185].
Interestingly also Ng and Van Dam deduce the relations like [186]

$$\delta L \leq (L l_P^2)^{1/3}, \delta T \leq (T t_P^2)^{1/3} \tag{5.47}$$

where the left side of (5.47) represents the uncertainty in the measurement of length and time for an interval L, T. We would like to point out that if in the above we use for L, T, the size and age of the Universe, then ΔL and ΔT reduce to the Compton scale l, t.
In conclusion, Brownian double Wiener processes and the modification of the Uncertainty Principle at the Planck scale lead to the physical Compton scale.

5.6 The Universe as Planck Oscillators

In the previous section, we had argued that a typical elementary particle like a pion could be considered to be the result of $n \sim 10^{40}$ evanescent Planck scale particles. We will return to this line of thinking again particularly in Chapter 8. The argument was based on random motions and also on the modification to the Uncertainty Principle. We will now consider the problem from a different point of view, which not only reconfirms the above result, but also enables an elegant extension to the case of the entire Universe itself.
Let us consider an array of N particles, spaced a distance Δx apart, which behave like oscillators, that is as if they were connected by springs. We then have [187, 188]

$$r = \sqrt{N \Delta x^2} \tag{5.48}$$

$$ka^2 \equiv k\Delta x^2 = \frac{1}{2} k_B T \tag{5.49}$$

where k_B is the Boltzmann constant, T the temperature, r the extent and k is the spring constant given by

$$\omega_0^2 = \frac{k}{m} \tag{5.50}$$

$$\omega = \left(\frac{k}{m}a^2\right)^{\frac{1}{2}} \frac{1}{r} = \omega_0 \frac{a}{r} \tag{5.51}$$

We now identify the particles with Planck masses, set $\Delta x \equiv a = l_P$, the Planck length. It may be immediately observed that use of (5.50) and (5.49) gives $k_B T \sim m_P c^2$, which ofcourse agrees with the temperature of a Black Hole of Planck mass. Indeed, as noted, Rosen had shown that a Planck mass particle at the Planck scale can be considered to be a Universe in itself. We also use the fact alluded to that a typical elementary particle like the pion can be considered to be the result of $n \sim 10^{40}$ Planck masses. Using this in (5.48), we get $r \sim l$, the pion Compton wavelength as required. Further, in this latter case, using (5.48) and the fact that $N = n \sim 10^{40}$, and (5.49),i.e. $k_B T = kl^2/N$ and (5.50) and (5.51), we get for a pion, remembering that $m_P^2/n = m^2$,

$$k_B T = \frac{m^3 c^4 l^2}{\hbar^2} = mc^2,$$

which of course is the well known formula for the Hagedorn temperature for elementary particles like pions. In other words, this confirms the conclusions in the previous section, that we can treat an elementary particle as a series of some 10^{40} Planck mass oscillators. However it must be observed from (5.49) and (5.50), that while the Planck mass gives the highest energy state, an elementary particle like the pion is in the lowest energy state. This explains why we encounter elementary particles, rather than Planck mass particles in nature. Infact as already noted [24], a Planck mass particle decays via the Beckenstein Radiation within a Planck time $\sim 10^{-42} secs$. On the other hand, the lifetime of an elementary particle would be very much higher.

In any case the efficacy of our above oscillator model can be seen by the fact that we recover correctly the masses and Compton scales in the order of magnitude sense and also get the correct Beckenstein and Hagedorn formulas as seen above, and get the correct estimate of the mass of the Universe itself, as will be seen below.

Using the fact that the Universe consists of $N \sim 10^{80}$ elementary particles like the pions, the question is, can we think of the Universe as a collection of nN or 10^{120} Planck mass oscillators? This is what we will now show. Infact if we use equation (5.48) with

$$\bar{N} \sim 10^{120},$$

we can see that the extent $r \sim 10^{28} cms$ which is of the order of the diameter of the Universe itself. Next using (5.51) we get

$$\hbar\omega_0^{(min)} \langle \frac{l_P}{10^{28}} \rangle^{-1} \approx m_P c^2 \times 10^{60} \approx Mc^2 \tag{5.52}$$

which gives the correct mass M, of the Universe which in contrast to the earlier pion case, is the highest energy state while the Planck oscillators individually are this time the lowest in this description. In other words the

Universe itself can be considered to be described in terms of normal modes of Planck scale oscillators.
We will return to these considerations later: this and the preceding considerations merely set the stage.

5.7 Modelling Fuzzy Spacetime as a Double Wiener Process

As noted earlier, fuzzy spacetime or complexification of coordinates could be modelled by a double Wiener process. Here, we have to consider the forward and backward time derivatives [176] which are unequal,

$$\frac{d}{dt^+}, \frac{d}{dt^-}$$

For simplicity we consider the problem in one space dimension to start with. So we have

$$\frac{d}{dt^+}x(t) = b_+, \quad \frac{d}{dt^-}x(t) = b_-, \tag{5.53}$$

From (5.53) we define two new velocities

$$V = \frac{b_+ + b_-}{2}, \quad U = \frac{b_+ - b_-}{2} \tag{5.54}$$

It may be pointed out that in the absence of the double Wiener process, U given in (5.54) vanishes while V gives the usual velocity. It is now possible to introduce a complex velocity

$$W = V - \imath U \tag{5.55}$$

Once the complex velocity W is introduced, we can then add and subtract the two Fokker-Planck equations

$$\partial\rho/\partial t + div(\rho b_1) = D\Delta\rho,$$

$$\partial\rho/\partial t + div(\rho b_-) = -D\Delta\rho,$$

to get

$$\partial\rho/\partial t + div(\rho V) = 0,$$

the equation of continuity, and

$$div(\rho U) - D\Delta\rho = 0,$$

where it can be shown that

$$U = D\nabla ln\rho,$$

In the above ρ is the probability density and D is the diffusion constant. Thence it is possible to lead up to the Schrödinger equation, where $\psi = \sqrt{\rho}e^{\imath S}$, S being related to the velocity V by

$$V = 2D\nabla S$$

(cf.ref.[176] for details), which means that, from the above classical considerations of the diffusion equation, we arrive at the Quantum Mechanical equation.
From (5.55) it appears that the consequence of the above theory or equations (5.53) to (5.55) is that the coordinate x goes over into a complex coordinate

$$x \to x + \imath x' \tag{5.56}$$

To see this in detail, let us rewrite (5.54) as

$$\frac{dX_r}{dt} = V, \quad \frac{dX_\imath}{dt} = U, \tag{5.57}$$

where we have introduced a complex coordinate X with real and imaginary parts X_r and $X_\imath$, while at the same time using derivatives with respect to time as in conventional theory.
We can now see from (5.55) and (5.57) that

$$W = \frac{d}{dt}(X_r - \imath X_\imath) \tag{5.58}$$

That is, in this non relativistic development either we use forward and backward time derivatives and the usual space coordinate as in (5.53), or we use the derivative with respect to the usual time coordinate but introduce complex space coordinates as in (5.56).
However, unlike in the Nelsonian theory, in which the above considerations were shown to lead to the Schrödinger equation, we have taken (5.56) as a starting point for a generalization to three, but as it turns out, actually four dimensions and non commutative spacetime as seen earlier.
Finally, it may be remarked that the original Nelsonian theory itself has been criticized by different scholars [189]-[193].

5.8 Other Issues

1. In Dirac's theory of displacement operators [7] the operator $d_x \equiv \frac{d}{dx}$ is a purely imaginary operator, and is given by

$$\delta x(d_x + \bar{d}_x) = \delta x^2 d_x \bar{d}_x = 0$$

if

$$0(\delta x^2) = 0$$

as is tacitly assumed. However if

$$0(\delta x^2) \neq 0 \tag{5.59}$$

then the operator d_x becomes complex, and therefore, also the momentum operator, $p_x \equiv \imath\hbar d_x$ and the position operator. In other words if (5.59) holds good then we have to deal with complex or non-Hermitian coordinates. The implication of this is that (Cf.[161] for details) spacetime becomes non- commutative as seen in earlier sections.

In any case here is the mysterious origin of the complex coordinates and spin. The complex coordinates lead to the Kerr-Newman metric and the electron's field including the anomalous gyro magnetic ratio which are symptomatic of the electron's spin. It also means that the naked singularity is shielded by the fuzzy spacetime (Dirac's original averages over the zitterbewegung interval) or equivalently the noncommutative geometry (5.4) (Cf. also [108]).

2. Ever since Dirac deduced theoretically the existence of the monopole in 1931, it has eluded physicists [194, 99]. At the same time the possibility of realising huge amounts of energy using monopoles has been an exciting prospect. In 1980 when the fiftieth Anniversary of the monopole was being commemorated, Dirac himself expressed his belief that the monopole did not exist [195]. Some scholars have indeed dismissed the monopole [196, 197], while in a model based on quantized vortices in the hydrodynamical formulation, the monopole field can be mathematically identified with the momentum vector [24]. Monopoles had also been identified with solitons [106].

In any case, it has been noted that the existence of free monopoles would lead to an unacceptably high density of the Universe [196], which in the light of latest observations of eternal expansion [206, 207] would be difficult to reconcile.

We will now show that monopoles arise due to the non commutative structure of spacetime being ignored, and this would also provide an explanation for their being undetected.

Let us start by reviewing Dirac's original derivation of the monopole (Cf.ref.[99]). He started with the wave function

$$\psi = Ae^{\imath\gamma}, \tag{5.60}$$

He then considered the case where the phase γ in (5.60) is non integrable. In this case (5.60) can be rewritten as

$$\psi = \psi_1 e^{\imath S}, \tag{5.61}$$

where ψ_1 is an ordinary wave function with integrable phase, and further, while the phase S does not have a definite value at each point, its four gradient viz.,

$$K^\mu = \partial^\mu S \tag{5.62}$$

is well defined. We use temporarily natural units, $\hbar = c = 1$. Dirac then goes on to identify K in (5.62) (except for the numerical factor hc/e) with the electromagnetic field potential, as in the Weyl gauge invariant theory.
Next Dirac considered the case of a nodal singularity, which is closely related to what was later called a quantized vortex (Cf. for example ref.[198]). In this case a circuit integral of a vector as in (5.62) gives, in addition to the electromagnetic term, a term like $2\pi n$, so that we have for a change in phase for a small closed curve around this nodal singularity,

$$2\pi n + e \int \boldsymbol{B} \cdot d\boldsymbol{S} \tag{5.63}$$

In (5.63) $\boldsymbol{B}$ is the magnetic flux across a surface element $d\boldsymbol{S}$ and n is the number of nodes within the circuit. The expression (5.63) directly leads to the monopole.
Let us now reconsider the above arguments in terms of recent developments. The Dirac equation for a spin half particle throws up as we saw a complex or non Hermitian position coordinate. Dirac as noted identified the imaginary part with zitterbewegung effects and argued that this would be eliminated once it is realized that in Quantum Mechanics, spacetime points are not meaningful and that on the contrary averages over intervals of the order of the Compton scale have to be taken to recover meaningful physics [7]. Over the decades the significance of such cut off spacetime intervals has been stressed by T.D. Lee and several other scholars as noted earlier [24, 152, 120, 116]. Indeed with a minimum cut off length l, it was shown that there would be a non commutative spacetime structure, and infact at the Compton scale we would have, as in (5.4),

$$[x, y] = 0(l^2) \tag{5.64}$$

and similar relations.
In fact starting from the Dirac equation itself, we can deduce directly the non commutativity (5.64) as we saw in Section 2.
Let us now return to Dirac's formulation of the monopole in the light of the above comments. As noted above, the non integrability of the phase S in (5.61) gives rise to the electromagnetic field, while the nodal singularity gives rise to a term which is an integral multiple of 2π. As is well known [173] we have

$$\boldsymbol{\nabla} S = \boldsymbol{p} \tag{5.65}$$

where $\boldsymbol{p}$ is the momentum vector. When there is a nodal singularity, as noted above the integral over a closed circuit of $\boldsymbol{p}$ does not vanish. In fact in this case we have a circulation given by

$$\Gamma = \oint \boldsymbol{\nabla} S \cdot d\boldsymbol{r} = \hbar \oint dS = 2\pi n \tag{5.66}$$

It is because of the nodal singularity that though the $\boldsymbol{p}$ field is irrotational, there is a vortex - the singularity at the central point associated with the vortex makes the region multiply connected, or alternatively, in this region we cannot shrink a closed smooth curve about the point to that point. In fact if we use the fact as seen above that the Compton wavelength is a minimum cut off, then we get from (5.66) using (5.65), and on taking $n = 1$,

$$\oint \nabla S \cdot d\boldsymbol{r} = \int \boldsymbol{p} \cdot d\boldsymbol{r} = 2\pi mc \frac{l}{2mc} = \frac{h}{2} \tag{5.67}$$

($l = \frac{\hbar}{2mc}$ is the radius of the circuit and $h = 2\pi$ in the above natural units). In other words the nodal singularity or quantized vortex gives us the mysterious Quantum Mechanical spin half (and other higher spins for other values of n). In the case of the Quantum Mechanical spin, there are $2 \times n/2 + 1 = n+1$ multiply connected regions, exactly as in the case of nodal singularities. Indeed in the case of the Dirac wave function, which is a bi-spinor $\begin{pmatrix} \Theta \\ \phi \end{pmatrix}$, as we saw, far outside the Compton wavelength, it is the usual spinor Θ, preserving parity under reflections that predominates, whereas at and near the Compton scale it is the spinor ϕ which predominates, where under a reflection ϕ goes over to $-\phi$. This double connectivity of the Dirac spinor was shown to lead immediately to the same electromagnetic potential we had obtained from the nonintegrability of the phase above, which again was identical to that from Weyl's gauge invariant theory.
As we saw in Section 4, given nonintegrability, we have

$$\frac{\partial}{\partial x^\mu} \to \frac{\partial}{\partial x^\mu} - \Gamma^\nu_{\mu\nu} \tag{5.68}$$

We can identify

$$A_\mu = \Gamma^\nu_{\mu\nu} \tag{5.69}$$

from the above using minimum electromagnetic coupling exactly as in Dirac's monopole theory.
If we use (5.68), we will get, the commutator relation, as seen in Section 4,

$$\frac{\partial}{\partial x^\lambda}\frac{\partial}{\partial x^\mu} - \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\lambda} \to \frac{\partial}{\partial x^\lambda}\Gamma^\nu_{\mu\nu} - \frac{\partial}{\partial x^\mu}\Gamma^\nu_{\lambda\nu} \tag{5.70}$$

Let us now use (5.69) in (5.70): The right side does not vanish due to the electromagnetic field (5.69) and we have a non-commutativity of the momentum components of Quantum Theory. Indeed the left side of (5.70) can be written as

$$[p_\lambda, p_\mu] \approx \frac{0(1)}{l^2}, \tag{5.71}$$

l being the Compton wavelength. In (5.71) we have utilized the fact that at the extreme scale of the Compton wavelength, the Planck scale being a special case, the momentum is mc.
From (5.69), (5.70) and (5.71), we have,

$$Bl^2 \sim \frac{1}{e} = \left(\frac{\hbar c}{e}\right), \qquad (5.72)$$

where B is the magnetic field.
Equation (5.72) is the well-known equation for the magnetic monopole. Indeed it has been shown by Saito and the author [194, 199] that a non commutative spacetime at the extreme scale shows up as a powerful magnetic field.
To recapitulate, the monopole was shown by Dirac to arise because of two separate issues. The first was the non integrability of the phase S given in (5.61), which gave rise to the electromagnetic potential on the lines of the Weyl potential (5.69) (which latter was dismissed because it was adhoc). The other issue was that of nodal singularities or alternatively the multiply connected nature of space which gave rise to a term like $2\pi n$ as in (5.63). In effect there would be free monopoles. However all this was considered in the context of the usual commutative Minkowski spacetime. Effectively this means that terms $\sim 0(l^2)$ as in (5.64) are neglected.
However once such terms are included, in other words once the non commutative structure of spacetime to this order is recognized, firstly the previously supposedly adhoc Weyl electromagnetic formulation automatically follow as in (5.69) and furthermore the first term in the monopole expression (5.63) immediately gives the Quantum Mechanical spin, and the elusive monopole appears as the spin and the magnetic effect at the Compton (or Planck scale). Indeed in recent times the fact that non commutative spacetime gives rise to spin has, as noted, been recognized.
3. One could argue that the non commutative relations are an expression of Quantum Mechanical spin. To put it briefly, for a spinning particle the non commutativity arises when we go from canonical to covariant position variables. As mentioned Zakrzewski [156] has shown that we have the Poisson bracket relation

$$\{x^j, x^k\} = \frac{1}{m^2} R^{jk}, (c = 1),$$

where R^{jk} is the spin.
4. The characterization of the metric in terms of symmetric and non symmetric components as seen in Section 4 is similar to the torsional formulation of General Relativity [200]. However in this latter case, there is no contribution to the differential interval from the torsional (that is non commutative) effects. The non commutative contribution is there, however, and herein comes the extended, rather than point like particle.
In any case the indicated attempt at unification of electromagnetism and gravitation had made part headway, but unless the underpinning of a non commutative geometry is recognized, the full significance does not manifest itself. We will return to this in Chapter 8.
5. We reiterate that the minimum spacetime intervals are at or below the Compton scale where the momentum p equals mc. For a Planck mass

$\sim 10^{-5} gms$, this is also the Planck scale, as in Quantum Superstring theory.
In Snyder's original work, the commutation relations hold good outside the minimum spacetime intervals, and are Lorentz invariant. This is quite pleasing because in any case, even in Quantum Field Theory, we use Minkowski spacetime.
6. The above non commutative geometry also holds the key to the mysterious extra dimensions of Quantum Superstrings. This has been discussed in detail in references [24, 101]. But to see in a simple way, we note that the coordinates y and z show up as some sort of a momenta, though with a different dimensional multiplying constant as the analogue of the Planck constant. That is instead of the single x momentum, p_x, we have two extra "momenta", this being the same for the y and z momenta also. This leads to the well known $9+1$ dimensions of Quantum Superstrings, though because for all these extra "momenta", the multiplying factor, the analogue of the Planck constant is different, so these extra dimensions are suppressed or curled up in the Kaluza-Klein sense.
7. A concept which one encounters in Quantum Super String theory and more generally in the presence of the non commutative geometry is that of duality. We will briefly examine this now and see its significance in relation to electrodynamic theory. In fact as we saw in (5.45) we have [17, 23],

$$\Delta x \sim \frac{\hbar}{\Delta p} + \alpha' \frac{\Delta p}{\hbar} \tag{5.73}$$

where $\alpha' = l^2$, which in Quantum Superstrings Theory $\sim 10^{-66} cm^2$. This is an expression of the duality relation,

$$R \to \alpha' / R$$

This is symptomatic of the fact that we cannot go down to arbitrarily small spacetime intervals, below the Planck scale in this case but that the macro Universe is connected with the micro Universe or in Witten's words, "when one accelerates past the string scale - instead of probing short distances one just watches the propagation of large strings." (Cf.ref.[17]).
In this light, an interesting meaning to the duality relation arising from (5.73) has been discussed in [102, 173].
We will now see a curious connection between the forgoing micro-macro link with the apparently disparate concept of the Feynman-Wheeler action at a distance theory, which had been quite successful.
Our starting point is the so called Lorentz-Dirac equation [201]:

$$ma^\mu = F^\mu_{in} + F^\mu_{ext} + \Gamma^\mu \tag{5.74}$$

where

$$F^\mu_{in} = \frac{e}{c} F^{\mu\nu}_{in} v_\nu$$

and similarly

$$F^{\mu}_{ext} = \frac{e}{c} F^{\mu\nu}_{ext} v_{\nu}$$

and $\varGamma^{\mu}$ is the Abraham radiation reaction four vector related to the self force and, given by

$$\varGamma^{\mu} = \frac{2}{3} \frac{e^2}{c^3} (\dot{a}^{\mu} - \frac{1}{c^2} a^{\lambda} x_{\lambda} v^{\mu}) \tag{5.75}$$

Equation (5.74) is the relativistic generalization for a point electron of an earlier equation proposed by Lorentz, while equation (5.75) is the relativistic generalization of the original radiation reaction term due to energy loss by radiation. It must be mentioned that the mass m in equation (5.74) consists of a neutral mass and the original electromagnetic mass of Lorentz, which latter does tend to infinity as the electron radius $\rightarrow 0$. We thus have here the forerunner of renormalisation in Quantum Theory.
There are three unsatisfactory features of the Lorentz-Dirac equation (5.74). Firstly the third derivative of the position coordinate in (5.74) through $\varGamma^{\mu}$ gives a whole family of solutions. Except one, the rest of the solutions are run away - that is the velocity of the electron increases with time to the velocity of light, even in the absence of any forces. This energy can be thought to come from the infinite self energy we get when the size of the electron shrinks to zero. If we assume adhoc an asymptotically vanishing acceleration then we get a physically meaningful solution, though this leads to a second difficulty, that of violation of causality of even the physically meaningful solutions.
It has been shown in detail elsewhere (Cf. [202] that these acausal, non local effects take place within the Compton time. In any case, the Quantum Mechanical Compton scale is a region of non local acausal effects, as noted.
We now come to the Feynman-Wheeler action at a distance theory [203, 204]. They showed that the apparent acausality of the theory would disappear if the interaction of a charge with all other charges in the Universe, such that the remaining charges would absorb all local electromagnetic influences was considered. The rationale behind this was that in an action at a distance context, the motion of a charge would instantaneously affect other charges, whose motion in turn would instantaneously affect the original charge. Thus considering a small interval in the neighbourhood of the point charge, they deduced,

$$F^{\mu}_{ret} = \frac{1}{2} \{F^{\mu}_{ret} + F^{\mu}_{adv}\} + \frac{1}{2} \{F^{\mu}_{ret} - F^{\mu}_{adv}\} \tag{5.76}$$

The left side of (5.76) is the usual causal field, while the right side has two terms. The first of these is the time symmetric field while the second can easily be identified with the Dirac field above and represents the sum of the responses of the remaining charges calculated in the vicinity of the said charge. Also here it can be shown that we encounter effects within the Compton scale (Cf.ref.[202]). We thus return to the concept, from a theory based on relations of extended particles and duality, of a manifestation of holism: The original instantaneous action at a distance theory has been shown to lead to the minimum Compton scale concept and this will be shown to be a result of cosmic fluctuations in Chapter 8.

6 THE UNIVERSE OF FLUCTUATIONS

"Our physical world ... is a world of instabilities and fluctuations ..."

I. Prigogine

6.1 The "Old" Cosmology

The Newtonian Universe was one in which there was an absolute background space in which the basic building blocks of the Universe were strewn about– these were stars. This view was a quantum jump from the earlier view, based on the Greek model in which stars and other celestial objects were attached to transparent material spheres, which prevented them from falling down.

When Einstein proposed his General Theory of Relativity early in the last century, the accepted picture of the Universe was one where all major constituents were stationary. This had puzzled Einstein, because the gravitational pull of these constituents should make the Universe collapse. So he invented his famous cosmological constant, essentially a repulsive force that would counterbalance the attractive gravitational force.

Shortly thereafter there were two dramatic discoveries which completely altered that picture. The first was due to astronomer Edwin Hubble, who discovered that the basic constituents or building blocks of the Universe were not stars, but rather, huge conglomerations of stars called galaxies. The second discovery was the fact that these galaxies are rushing away from each other – far from being static, the Universe was exploding. There was no need for the counterbalancing cosmic repulsion any more and Einstein dismissed this as his greatest blunder.

Over the next forty odd years, these observations evolved into the Big Bang theory, according to which all the matter in the Universe, possibly some 13.7 billion years ago, was concentrated in a speck, at the birth of the Universe, which was characterized by an inconceivable explosion or bang. This lead to the matter being flung outwards, and that is what keeps the galaxies rushing outwards even today. In the mid sixties confirmation for the Big Bang model of the Universe came from the detection of a cosmic footprint. The energy of the initial Big Bang would today still be available in the form of cosmic microwaves, which accidentally were discovered as we saw in Chapter

2 [46, 59, 5].

Over the next three decades and more, the Big Bang theory was refined further and further. An important question was, would the Universe continue to expand for ever, though slowing down, or would the expansion halt one day and the Universe collapse back again? Much depended on the material content or density of the Universe. If there was enough matter, and therefore gravitation, then the expansion would halt and reverse. If not the Universe would expand for ever. However the observed material content, more correctly density, of the Universe appeared to be insufficient to halt the expansion.

At the same time there were a few other intriguing observations. For example the velocities within galaxies, instead of sharply falling off with distance from their centres, flattened out. All this led astronomers to invoke Dark Matter, that is as yet undetected and possibly exotic matter. This matter could be in the form of Black Holes within galaxies, or brown dwarf stars which were too faint to be detected, or even massive neutrinos which were otherwise thought to be massless or who knows what. With Dark Matter thrown in, it appeared that the Universe had sufficient material content to halt, and even reverse the expansion. That is, the Universe would expand up to a point and then collapse.

There still were several subtler problems to be addressed. One was the famous horizon problem. To put it simply the Big Bang was an uncontrolled or random event and so, different parts of the Universe in different directions were disconnected at the very earliest stage and even today, light would not have had enough time to connect them. So today they need not be the same, just as people in different parts of the world need not share the same habits or dress. Observation however shows that the Universe is by and large uniform, rather like people in different countries showing the same habits or dress. That would not be possible without some form of intercommunication which would violate Einstein's Special Theory of Relativity, according to which no signal can travel faster than light.

The next problem was, that according to Einstein, due to the material content in the Universe, space should be curved whereas the Universe appears to be flat. There were other problems as well. For example astronomers predicted that there should be the monopoles encountered earlier, that is, simply put, either only North magnetic poles or only South magnetic poles, unlike the North South combined magnetic poles we encounter. Such monopoles have failed to show up.

Some of these problems as we briefly saw were sought to be explained by what has been called inflationary cosmology whereby, early on, just after the Big Bang the explosion was super fast [73, 205].

What would happen in this case is, that different parts of the Universe, which could not be accessible by light, would now get connected. At the same time, the super fast expansion in the initial stages would smoothen out any distortion or curvature effects in space, leading to a flat Universe and in the

process also eliminate the monopoles.
One other feature that has been studied in detail over the past few decades is that of structure formation in the Universe. To put it simply, why is the Universe not a uniform spread of matter and radiation? On the contrary it is very lumpy with planets, stars, galaxies and so on, with a lot of space separating these objects. This has been explained in terms of fluctuations in density, that is, accidentally more matter being present in a given region. Gravitation would then draw in even more matter and so on. These fluctuations would also cause the cosmic background radiation to be non uniform or anisotropic. Such anisotropies are in fact being observed.
From early 1998, the conventional wisdom of cosmology that had concretized from the mid sixties onwards, began to be challenged. It had been believed that the density of the Universe is near its critical value, separating eternal expansion and ultimate contraction, while the nuances of the Dark Matter theories were being fine tuned. However the work of Perlmutter and others [206, 207] began appearing in 1998 and told a different story. These observations of distant supernovae indicated that contrary to widely held belief, the Universe was not only not decelerating, it was actually accelerating.
This paradigm shift permeated to the popular press too. For example an article in the Scientific American [208] observed, ”In recent years the field of cosmology has gone through a radical upheaval. New discoveries have challenged long held theories about the evolution of the Universe... Now that observers have made a strong case for cosmic acceleration, theorists must explain it.... If the recent turmoil is anything to go by, we had better keep our options open.”
On the other hand, the Physics World observed [209], ”A revolution is taking place in cosmology. New ideas are usurping traditional notions about the composition of the Universe, the relationship between geometry and destiny, and Einstein’s greatest blunder.”
The infamous cosmological constant was resurrected and now it was ”dark energy” that was in the air, rather than Dark Matter.
Shortly before these dramatic discoveries, the author had presented a cosmological model based on fluctuations in an all permeating Zero Point Field - or dark energy [210, 211, 168, 169]. This model while consistent with astrophysical observations, predicted an ever expanding and accelerating Universe with a small cosmological constant. It deduces from theory the so called Large Number coincidences including the purely empirical Weinberg formula that connects the pion mass to the Hubble constant [80, 28] – ”coincidences” that have troubled and mystified scientists from time to time. Let us now examine this cosmology and some of its implications. We will first go over the essentials and then examine the nuances.

6.2 Dark Energy and Fluctuations

We first observe that the concept of a Zero Point Field (ZPF) or Quantum Vacuum (or Aether) is an idea whose origin can be traced back to Max Planck himself. Quantum Field Theory attributes the ZPF to the virtual Quantum effects of an already present electromagnetic field [29]. What is the mysterious energy of supposedly empty vacuum?

It may sound contradictory to attribute energy or density to the vacuum. After all vacuum is a total void. However, over the past four hundred years, it has been realized that it may be necessary to replace the vacuum by a medium with some specific physical properties. For instance Descartes the seventeenth century French philosopher mathematician proclaimed that the so called empty space above the mercury column in a Torricelli tube, that is, what is called the Torricelli vacuum, is not a vacuum at all. Rather, he said, it was something which was neither mercury nor air, something he called aether.

The seventeenth century Dutch Physicist, Christian Huygens required such a non intrusive medium like aether, so that light waves could propagate through it, rather like the ripple waves on the surface of a pond. Hence the word luminiferous aether. In the nineteenth century the aether was reinvoked. Firstly in a very intuitive way Faraday could conceive of magnetic effects in vacuum in connection with his experiments on induction. Based on this, the aether was used for the propagation of electromagnetic waves in Maxwell's Theory of electromagnetism, which infact laid the stage for Special Relativity. This aether was a homogenous, invariable, non-intrusive, material medium which could be used as an absolute frame of reference atleast for certain chosen observers. However the experiments of Michelson and Morley towards the end of the nineteenth century, lead to its downfall, and thus was born Einstein's Special Theory of Relativity in which there is no such absolute frame of reference. The aether lay shattered once again.

Very shortly thereafter the advent of Quantum Mechanics lead to its rebirth in a new and unexpected avatar. Essentially there were two new ingredients in what is today called the Quantum vacuum. The first was a realization that Classical Physics had allowed an assumption to slip in unnoticed: In a source or charge free "vacuum", one solution of Maxwell's Equations of electromagnetic radiation is no doubt the zero solution. But there is also a more realistic non zero solution. That is, the electromagnetic radiation does not necessarily vanish in empty space.

The second ingredient was the mysterious prescription of Quantum Mechanics, the Heisenberg Uncertainty Principle, according to which it would be impossible to precisely assign momentum and energy on the one hand and spacetime location on the other. Clearly the location of a vacuum with no energy or momentum cannot be specified in spacetime.

This leads to what is called a Zero Point Field. For instance a Harmonic oscillator, a swinging pendulum for example, according to classical ideas has

zero energy and momentum in its lowest position. But the Heisenberg Uncertainty endows it with a fluctuating energy. This fact was recognized by Einstein himself way back in 1913 who contrary to popular belief, retained the concept of aether though from a different perspective [212]. It also provides an understanding of the fluctuating electromagnetic field in vacuum.
From another point of view, according to classical ideas, at the absolute zero of temperature, there should not be any motion. After all the zero is when all thermodynamic motion ceases. But as Nernst, father of the third law of Thermodynamics himself noted, experimentally this is not so. There is the well known superfluidity due to Quantum Mechanical – and not thermodynamic – effects. This is the situation where supercooled Helium moves in a spooky fashion.
This mysterious Zero Point Field or Quantum vacuum energy has since been experimentally confirmed in effects like the Casimir effect which demonstrates a force between uncharged parallel plates separated by a charge free medium, the Lamb shift which demonstrates a minute oscillation of an electron orbiting the nucleus in an atom-as if it was being buffetted by the Zero Point Field-, the anomalous Quantum Mechanical gyromagnetic ratio $g = 2$ and so on [68][213]-[217],[46].
The Quantum Vacuum is a far cry however, from the passive aether of olden days. It is a violent medium in which charged particles like electrons and positrons are constantly being created and destroyed, almost instantly, infact within the limits permitted by the Heisenberg Uncertainty Principle for the violation of energy conservation. One might call the Quantum Vacuum as a new state of matter, a compromise between something and nothingness. Something which corresponds to what the Rig Veda described thousands of years ago: "Neither existence, nor non existence."
Quantum Vacuum can be considered to be the lowest state of any Quantum field, having zero momentum and zero energy. The properties of the Quantum Vacuum can under certain conditions be altered, which was not the case with the erstwhile aether. In modern Particle Physics, the Quantum Vacuum is responsible for phenomena like quark confinement, a property whereby it would be impossible to observe an independent or free quark, the spontaneous breaking of symmetry of the electroweak theory, vacuum polarization wherein charges like electrons are surrounded by a cloud of other opposite charges tending to mask the main charge and so on. There could be regions of vacuum fluctuations comparable to the domain structures of ferromagnets. In a ferromagnet, all elementary electron-magnets are aligned with their spins in a certain direction. However there could be special regions wherein the spins are aligned differently.
Such a Quantum Vacuum can be a source of cosmic repulsion, as pointed by Zeldovich and others [218, 24]. However a difficulty in this approach has been that the value of the cosmological constant turns out to be huge, far beyond what is observed. This has been called the cosmological constant problem

[219].

There is another approach, sometimes called Stochastic Electrodynamics which treats the ZPF as primary and attributes to it Quantum Mechanical effects [220, 221]. It may be re-emphasized that the ZPF results in the well known experimentally verified Casimir effect [222, 223]. We would also like to point out that contrary to popular belief, the concept of aether has survived over the decades through the works of Dirac, Vigier, Prigogine, String Theorists like Wilczek and others [224]-[229]. As pointed out it appears that even Einstein himself continued to believe in this concept [230].

We would first like to observe that the energy of the fluctuations in the background electromagnetic field could lead to the formation of elementary particles.

Indeed this was Einstein's belief. As Wilczek put it, "Einstein was not satisfied with the dualism. He wanted to regard the fields, or ethers, as primary. In his later work, he tried to find a unified field theory, in which electrons(and of course protons, and all other particles) would emerge as solutions in which energy was especially concentrated, perhaps as singularities. But his efforts in this direction did not lead to any tangible success."

Let us see how this can happen. In the words of Wheeler [46], "From the zero-point fluctuations of a single oscillator to the fluctuations of the electromagnetic field to geometrodynamic fluctuations is a natural order of progression..."

Let us consider, following Wheeler a Harmonic oscillator in its ground state. The probability amplitude is

$$\psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-(m\omega/2\hbar)x^2}$$

for displacement by the distance x from its position of classical equilibrium. So the oscillator fluctuates over an interval

$$\Delta x \sim (\hbar/m\omega)^{1/2}$$

The electromagnetic field is an infinite collection of independent oscillators, with amplitudes X_1, X_2 etc. The probability for the various oscillators to have amplitudes X_1, X_2 and so on is the product of individual oscillator amplitudes:

$$\psi(X_1, X_2, \cdots) = exp[-(X_1^2 + X_2^2 + \cdots)]$$

wherein there would be a suitable normalization factor. This expression gives the probability amplitude ψ for a configuration $B(x, y, z)$ of the magnetic field that is described by the Fourier coefficients $X_1, X_2, \cdots$ or directly in terms of the magnetic field configuration itself by

$$\psi(B(x, y, z)) = Pexp\left(-\int\int \frac{\mathbf{B(x_1)} \cdot \mathbf{B(x_2)}}{16\pi^3\hbar c r_{12}^2} d^3x_1 d^3x_2\right).$$

P being a normalization factor. Let us consider a configuration where the magnetic field is everywhere zero except in a region of dimension l, where it is of the order of $\sim \Delta B$. The probability amplitude for this configuration would be proportional to

$$\exp[-(\Delta B)^2 l^4/\hbar c)$$

So the energy of fluctuation in a region of length l is given by finally [46, 231, 145]

$$B^2 \sim \frac{\hbar c}{l^4}$$

In the above if l is taken to be the Compton wavelength of a typical elementary particle, then we recover its energy mc^2, as can be easily verified. In Chapter 1, we had seen how inertial mass and energy can be deduced on the basis of Quantum Mechanical effects within the Compton scales. The above gives us back this result in the context of the ZPF.

It may be mentioned that Einstein himself had believed that the electron was a result of such a condensation from the background electromagnetic field (Cf.[232, 24] for details). We will return to this point again. We also take the pion to represent a typical elementary particle, as in the literature. To proceed, as there are $N \sim 10^{80}$ such particles in the Universe, we get

$$Nm = M \tag{6.1}$$

where M is the mass of the Universe. A justification for (6.1), which is consistent, is that as the Universe at large is electrically neutral, the particles interact via the gravitational force, which is very weak in any case.

In the following we will use N as the sole cosmological parameter.

Equating the gravitational potential energy of the pion in a three dimensional isotropic sphere of pions of radius R, the radius of the Universe, with the rest energy of the pion, we can deduce the well known relation [233, 176, 234]

$$R \approx \frac{GM}{c^2} \tag{6.2}$$

where M can be obtained from (6.1).

We now use the fact that given N particles, the fluctuation in the particle number is of the order $\sqrt{N}$[234, 235, 168, 169, 210, 211], while a typical time interval for the fluctuations is $\sim \hbar/mc^2$, the Compton time, the fuzzy interval we encountered in the previous Chapter. We will come back to this point later in this Chapter, in the context of the minimum Planck scale: Particles are created and destroyed - but the ultimate result is that $\sqrt{N}$ particles are created. So we have, as we saw briefly earlier,

$$\frac{dN}{dt} = \frac{\sqrt{N}}{\tau} \tag{6.3}$$

whence on integration we get, (remembering that we are almost in the continuum region),

$$T = \frac{\hbar}{mc^2}\sqrt{N} \tag{6.4}$$

Later in this Chapter, and also in Chapter 8, we will analyze in greater detail, the above and subsequent relations and continue with this preliminary treatment. We can easily verify that the equation is indeed satisfied where T is the age of the Universe. Next by differentiating (6.2) with respect to t we get

$$\frac{dR}{dt} \approx HR \tag{6.5}$$

where H in (6.5) can be identified with the Hubble constant, and using (6.2) is given by,

$$H = \frac{Gm^3c}{\hbar^2} \tag{6.6}$$

Equation (6.1), (6.2) and (6.4) show that in this formulation, the correct mass, radius, Hubble constant and age of the Universe can be deduced given N as the sole cosmological or large scale parameter. Equation (6.6) can be written as

$$m \approx \left(\frac{H\hbar^2}{Gc}\right)^{\frac{1}{3}} \tag{6.7}$$

Equation (6.7) has been empirically known as an "accidental" or "mysterious" relation. As observed by Weinberg[28], this is unexplained: it relates a single cosmological parameter H to constants from microphysics. We will touch upon this micro-macro nexus again. In our formulation, equation (6.7) is no longer a mysterious coincidence but rather a consequence.

As (6.6) and (6.5) are not exact equations but rather, order of magnitude relations, it follows, on differentiating (6.5) that a small cosmological constant $\wedge$ is allowed such that

$$\wedge < 0(H^2)$$

This is consistent with observation and shows that $\wedge$ is very small – this has been a puzzle, the so called cosmological constant problem alluded to, because in conventional theory, it turns out to be huge [219]. But it poses no problem in this formulation. We shall further characterize $\wedge$ later in this Chapter.

To proceed we observe that because of the fluctuation of $\sim \sqrt{N}$ (due to the ZPF), there is an excess electrical potential energy of the electron, which infact we have identified as its inertial energy. That is [168, 234],

$$\sqrt{N}e^2/R \approx mc^2.$$

On using (6.2) in the above, we recover the well known Gravitation-electromagnetism ratio viz.,

$$e^2/Gm^2 \sim \sqrt{N} \approx 10^{40} \tag{6.8}$$

or without using (6.2), we get, instead, the well known so called Weyl-Eddington formula,

$$R = \sqrt{N}l \tag{6.9}$$

(It appears that this was first noticed by H. Weyl [236]). Infact (6.9) is the spatial counterpart of (6.4). If we combine (6.9) and (6.2), we get,

$$\frac{Gm}{lc^2} = \frac{1}{\sqrt{N}} \propto T^{-1} \tag{6.10}$$

where in (6.10), we have used (6.4). This was the relation we encountered in the previous chapter. Following Dirac (cf.also [172]) we treat G as the variable, rather than the quantities m, l, c and $\hbar$ (which we will call micro physical constants) because of their central role in atomic (and sub atomic) physics.
Next if we use G from (6.10) in (6.6), we can see that

$$H = \frac{c}{l} \quad \frac{1}{\sqrt{N}} \tag{6.11}$$

Thus apart from the fact that H has the same inverse time dependance on T as G, (6.11) shows that given the microphysical constants, and N, we can deduce the Hubble constant also, as from (6.11) or (6.6).
Using (6.1) and (6.2), we can now deduce that

$$\rho \approx \frac{m}{l^3} \quad \frac{1}{\sqrt{N}} \tag{6.12}$$

Next (6.9) and (6.4) give,

$$R = cT \tag{6.13}$$

(6.12) and (6.13) are consistent with observation.
Finally, we observe that using $M, G$and$H$ from the above, we get

$$M = \frac{c^3}{GH}$$

This relation is required in the Friedman model of the expanding Universe (and the Steady State model too).
The above model predicts a dark energy driven ever expanding and accelerating Universe with a small cosmological constant whose density keeps decreasing. This seemed to go against the accepted idea that the density of the Universe equalled the critical density required for closure and that aided by Dark Matter, the Universe was decelerating. However, as noted, from 1998 onwards, following the work of Perlmutter, Schmidt and co-workers, these otherwise apparently heretic conclusions have been vindicated.

It may be mentioned that the observational evidence for an accelerating Universe was the American Association for Advancement of Science's (Science Magazine) Breakthrough of the Year, 1998 while the evidence for nearly seventy five percent of the Universe being dark energy, based on the Wilkinson Microwave Anisotropy Probe (WMAP) and the Sloan Sky Digital Survey was the Breakthrough of the Year, 2003 [Cf.ref.Science, December 1998 and Science, December, 2003].

6.3 Issues and Ramifications

i) The above cosmology exhibits a time variation of the gravitational constant of the form

$$G = \frac{\beta}{T} \tag{6.14}$$

Indeed this is true in a few other schemes also, including the so called Brans-Dicke and Dirac cosmologies (Cf. [237, 238, 24]). Interestingly it can be shown that such a time variation can explain the precession of the perihelion of Mercury (Cf.[170]). It can also provide an alternative explanation for Dark Matter and the bending of light (while the Cosmic Microwave Background Radiation is also explained (Cf.[24])).
It is also possible to deduce the existence of gravitational waves given (6.14). To see this quickly let us consider the Poisson equation for the metric $g_{\mu\nu}$

$$\nabla^2 g_{\mu\nu} = G\rho u_\mu u_\nu \tag{6.15}$$

The solution of (6.15) is given by

$$g_{\mu\nu} = G \int \frac{\rho u_\mu u_\nu}{|\boldsymbol{r} - \boldsymbol{r}'|} d^3\boldsymbol{r} \tag{6.16}$$

Indeed equations similar to (6.15) and (6.16) hold for the Newtonian gravitational potential also. If we use the second time derivative of G from (6.14) in (6.16), along with (6.15), we can immediately obtain the D'alembertian wave equation for gravitational waves, instead of the Poisson equation:

$$Dg_{\mu\nu} \approx 0$$

ii) Recently a small variation with time of the fine structure constant has been detected and reconfirmed by Webb and coworkers [239, 240]. This observation is consistent with the above cosmology. We can see this as follows. We use an equation due to Kuhne [241]

$$\frac{\dot{\alpha}_z}{\alpha_z} = \alpha_z \frac{\dot{H}_z}{H_z}, \tag{6.17}$$

If we now use the fact that the cosmological constant Λ is given by

$$\Lambda < 0(H^2) \tag{6.18}$$

as can be seen from (6.5), in (6.17), we get using (6.18),

$$\frac{\dot{\alpha}_z}{\alpha_z} \leq \beta H_z \tag{6.19}$$

where $\beta < -\alpha_z < -10^{-2}$.
Equation (6.19) can be shown to be the same as

$$\frac{\dot{\alpha}_z}{\alpha_z} \leq -1 \times 10^{-5} H_z. \tag{6.20}$$

which is the same as Webb's result.
We give another derivation of (6.20) in the above context wherein, as the number of particles in the Universe increases with time, we go from the Planck scale to the Compton scale.
This can be seen as follows: In equation (6.8), if the number of particles in the Universe, $N = 1$, then the mass m would be the Planck mass. In this case the classical Schwarzchild radius of the Planck mass would equal its Quantum Mechanical Compton wavelength. To put it another way, all the energy would be gravitational (Cf.[24] for details). However as the number of particles N increases with time, according to (6.4), gravitation and electromagnetism get differentiated and we get (6.8) and the Compton scale.
It is known that the Compton length, due to zitterbewegung causes a correction to the electrostatic potential which an orbiting electron experiences, rather like the Darwin term [29].
Infact we have

$$\begin{aligned} \langle \delta V \rangle &= \langle V(\boldsymbol{r} + \delta \boldsymbol{r}) \rangle - V \langle (\boldsymbol{r}) \rangle \\ &= \langle \delta r \frac{\partial V}{\partial r} + \frac{1}{2} \sum_{\imath j} \delta r_\imath \delta r_j \frac{\partial^2 V}{\partial r_\imath \partial r_j} \rangle \\ &\approx 0(1) \delta r^2 \nabla^2 V \end{aligned} \tag{6.21}$$

Remembering that $V = e^2/r$ where $r \sim 10^{-8} cm$, from (6.21) it follows that if $\delta r \sim l$, the Compton wavelength then

$$\frac{\Delta \alpha}{\alpha} \sim 10^{-5} \tag{6.22}$$

where $\Delta\alpha$ is the change in the fine structure constant from the early Universe. (6.22) is an equivalent form of (6.20) (Cf.ref.[241]), and is the result originally obtained by Webb et al (Cf.refs.[239, 240]).
iii) The latest observations of distant supernovae referred to above indicate that the closure parameter $\Omega \leq 1$.
Remembering that Ω is given by [5]

$$\Omega = \frac{8\pi G}{3H^2}\rho$$

we get therefrom on using (6.1)

$$\frac{H^2}{2G}R^3 = mN$$

which immediately leads to the mysterious Weinberg formula (6.7). Thus this is the balance between the cosmos at large and the micro cosmos. We will return to this point in Chapter 8.

iv) In General Relativity as well as in the Newtonian Theory, we have, without a cosmological constant

$$\ddot{R} = -\frac{4}{3}\pi G\rho R \tag{6.23}$$

We remember that there is an uncertainty in time to the extent of the Compton time τ, and also if we now use the fact that G varies with time, (6.23) becomes on using (6.14),

$$\ddot{R} = -\frac{4}{3}\pi G(T-\tau)\rho R$$

$$= -\frac{4}{3}\pi G\rho R + \frac{4}{3}\pi\rho R\left(\frac{\tau}{T}\right)G \tag{6.24}$$

Remembering that at any point of time, the age of the Universe, that is T itself is given by (6.4), we can see from (6.24) that this effect of time variation of G, which again is due to the background Zero Point Field is the same as an additional density, the vacuum density given by

$$\rho_{vac} = \frac{\rho}{\sqrt{N}} \tag{6.25}$$

This term in (6.24) is also equivalent to the presence of a cosmological constant Λ as discussed above. On the other hand, we know independently that the presence of a vacuum field leads to a cosmological constant given by (Cf.ref.[24] and references therein)

$$\Lambda = G\rho_{vac} \tag{6.26}$$

Equation (6.26) is pleasingly in agreement with (6.24) and (6.25) that is, the preceding considerations of fluctuational creation: Infact, due to fluctuational creation ρ_{vac} should be given by

$$\rho_{vac} = \sqrt{N}m/R^3,$$

as $\sqrt{N}$ particles are created. This gives, on using (6.9),

$$\rho_{vac} = \frac{m}{l^3 N} = \rho/\sqrt{N},$$

which is (6.25).

In other words quantitatively we have reconfirmed that it is the background Zero Point Field that manifests itself as the cosmological constant described in Section 2. This also gives as pointed out an explanation for the so called cosmological constant problem [219] viz., why is the cosmological constant so small?

v) In the above cosmology of fluctuations, our starting point was the creation of $\sqrt{N}$ particles within the minimum time interval, a typical elementary particle Compton time τ. A rationale for this, very much in the spirit of the condensation of particles from a background Zero Point Field as discussed at the beginning of Section 2, can also be obtained in terms of a phase transition from the Zero Point Field or Quantum Vacuum as we will see in the sequel. In this case, particles are like the Benard cells which form in fluids, as a result of a phase transition. While some of the particles or cells may revert to the Zero Point Field, on the whole there is a creation of $\sqrt{N}$ of these particles. If the average time for the creation of the $\sqrt{N}$ particles or cells is τ, then at any point of time where there are N such particles, the time elapsed, in our case the age of the Universe, would be given by (6.4) (Cf. [242, 243]). While this is not exactly the Big Bang scenario, there is nevertheless a rapid creation of matter from the background Quantum Vacuum or Zero Point Field. Thus over 10^{40} particles would have been created within a fraction of a second.

In any case when $\tau \to 0$, we recover the Big Bang scenario with a singular creation of matter, while when $\tau \to$ Planck time we recover the Prigogine Cosmology (Cf.[24] for details). However in neither of these two limits we can deduce all the above consistent with observation Large Number relations which therefore have to be branded as accidents.

One of the puzzles has been, why is there an excess of matter over anti matter? After all, if particles are created from the Quantum Vacuum, the probability for the creation of a particle equals the probability for the creation of an anti particle. At this stage what may be called a probability symmetry breaking appears. Let us see a simple example. If a couple has n children, then the probability that there would be n/2 boys is actually less than half, even though the probability for the birth of a boy equals the probability for the birth of a girl. For instance if n is six, the probability for three boys is 5/16, which is less than one third! Ofcourse for large n this figure would be much closer to half, but even a minute fractional difference would lead to large scale particle - anti particle asymmetry.

vi) The above cosmological model is related to the fact that there are minimum spacetime intervals l, τ. Indeed in this case as we saw in the previous Chapter, there is an underlying non commutative geometry of spacetime [244, 162, 101] given by

$$[x, y] \approx 0(l^2), [x, p_x] = \imath\hbar[1 + \beta l^2], [t, E] = \imath\hbar[1 + \gamma\tau^2] \tag{6.27}$$

Interestingly (6.27) implies as we saw, modification to the usual Uncertainty Principle. (This in turn has also been interpreted in terms of a variable speed

of light cosmology [245, 246, 247]).
The relations (6.27), lead to the modified Uncertainty relation

$$\Delta x \sim \frac{\hbar}{\Delta p} + \alpha' \frac{\Delta p}{\hbar} \tag{6.28}$$

(6.28) appears also in Quantum Super String Theory and is related to the well known duality relation

$$R \to \alpha'/R$$

(Cf.[17, 23]). In any case (6.28) is symptomatic of the fact that we cannot go down to arbitrarily small spacetime intervals. We now observe that the first term of (6.28) gives the usual Uncertainty relation. In the second term, we write $\Delta p = \Delta N mc$, where ΔN is the Uncertainty in the number of particles, in the Universe. Also $\Delta x = R$, the radius of the Universe where

$$R \sim \sqrt{N} l,$$

the famous Eddington relationship (6.9). It should be stressed that the otherwise empirical Eddington formula, arises quite naturally in a Brownian characterisation of the Universe as has been pointed out in the previous Chapter (Cf. for example ref.[187]). Put simply (6.9) is the Random Walk equation.
We now get back,

$$\Delta N = \sqrt{N}$$

This is the uncertainty in the particle number, we used earlier. Substituting this in the time analogue of the second term of (6.28), we immediately get, T being the age of the Universe,

$$T = \sqrt{N} \tau$$

which is equation (6.4). So, our cosmology is self consistent with the modified relation (6.28). The fluctuational effects are really couched in the modification of the Heisenberg Principle, as given in (6.28).
Interestingly these minimum spacetime considerations can be related to the Feynman-Wheeler Instantaneous Action At a Distance formulation (Cf.[248, 202, 161]), a point which we shall elaborate further in the sequel.
We finally remark that relations like (6.27) and (6.28), which can also be expressed in the form, a being the minimum length,

$$[x, p_x] = \imath\hbar[1 + \left(\frac{a}{\hbar}\right)^2 p^2]$$

(and can be considered to be truncated from a full series on the right hand side (Cf.[179])), could be deduced from the rather simple model of a lattice - a one dimensional lattice for simplicity. In this case we will have (Cf.[24])

$$[x, p_x] = \imath\hbar cos \left(\frac{p}{\hbar} a\right),$$

where a is the lattice length, l the Compton length in our case. The energy time relation now leads to a correction to the mass energy formula, viz

$$E = mc^2 cos(kl), k \equiv p/\hbar$$

This is the contribution of the extra term in the Uncertainty Principle and we will return to it in Chapter 10, in the context of observational tests.
vii) As noted the Planck scale is an absolute minimum scale in the Universe. In this section, ii) we argued that with the passage of time the Planck scale would evolve to the present day elementary particle Compton scale. To recapitulate: We have by definition

$$\hbar G/c^3 = l_P^2$$

where l_P is the Planck length $\sim 10^{-33} cms$. If we use G from (6.10) in the above we will get

$$l = N^{1/4} l_P \quad (6.29)$$

Similarly we have

$$\tau = N^{1/4} \tau_P \quad (6.30)$$

In (6.29) and (6.30) l and τ denote the typical elementary particle Compton length and time scale, and N is the number of such elementary particles in the Universe.
We could explain these equations in terms of the Benard cell like elementary particles referred to above. This time there are a total of $n = \sqrt{N}$ Planck particles and (6.29) and (6.30) are the analogues of equations (6.4) and (6.9) in the context of the formation of such particles. Indeed as we saw a Planck mass, $m_P \sim 10^{-5} gms$, has a Compton life time and also a Beckenstein Radiation life time of the order of the Planck time. These spacetime scales are much too small and we encounter much too large energies from the point of view of our experimental constraints. As noted in the previous chapter our observed scale is the Compton scale, in which Planck scale phenomena are moderated. In any case it can be seen from the above that as the number of particles N increases, the scale evolves from the Planck to the Compton scale.
So, the scenario which emerges is, that as the Universe evolves, Planck particles form the underpinning for elementary particles, which in turn form the underpinning for the Universe by being formed continuously.
This can be confirmed by the following argument: We can rewrite (6.29) as

$$l = \nu' \sqrt{T} \quad (6.31)$$

$$\nu' = l_P/\sqrt{\tau} \approx \hbar/m_P$$

wherein we have used (6.4). Equation (6.31) as we saw earlier, is identical to the Brownian diffusion process which is infact the underpinning for equations like (6.4) or (6.9), except that this time we have the same Brownian Theory operating from the Planck scale to the Compton scale, instead of from the Compton scale to the edge of the Universe as seen above (Cf. also [187, 24]). Interestingly, let us apply the above scenario of $\sqrt{n}$ Planck particles forming an elementary particle, to the extra term of the modified Uncertainty Principle (6.28), as we did earlier in this section in (iv). Remembering that $\alpha' = l_P^2$ in the theory, and $\Delta p = N^{1/4} m_P c$, in this case, we get, as $\Delta x = l$,

$$l = N^{1/4} l_P,$$

which will be recognized as (6.29) itself! Thus once again we see how the above cosmology is consistently tied up with the non commutative spacetime expressed by equations (6.27) or (6.28).
It may be mentioned that, as indeed can be seen from (6.29) and (6.30), in this model, the velocity of light remains constant.
viii) We would now like to comment further upon the Compton scale and the fluctuational creation of particles alluded to above. In this case particles are being produced out of a background Quantum Vacuum or Zero Point Field which is pre spacetime. First a Brownian process alluded to above defines the Planck length while a Brownian random process with the Planck scale as the fundamental interval leads to the Compton scale (Cf. also ref.[249]).
This process is a phase transition, a critical phenomenon. To see this briefly, let us start with the Landau-Ginzburg equation [188]

$$-\frac{\hbar^2}{2m}\nabla^2\psi + \beta|\psi|^2\psi = -\alpha\psi \tag{6.32}$$

Here $\hbar$ and m have the same meaning as in usual Quantum Theory. It is remarkable that the above equation (6.32) is identical with a similar Schrödinger like equation based on amplitudes which we encountered in Chapter 1, where moreover $|\psi|^2$ is proportional to the mass (or density) of the particle (Cf. ref.[24] for details). The equation in question is,

$$\imath\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m'}\frac{\partial^2\psi}{\partial x^2} + \int \psi^*(x')\psi(x)\psi(x')U(x')dx', \tag{6.33}$$

In (6.33), $\psi(x)$ is the probability of a particle being at the point x and the integral is over a region of the order of the Compton wavelength. From this point of view, the similarity of (6.33) with (6.32) need not be surprising considering also that near Critical Points, due to universality diverse phenomena like magnetism or fluids share similar mathematical equations. Equation (6.33) was shown to lead to the Schrödinger equation with the particle acquiring a mass (Cf.also ref.[250]).
Infact in the Landau-Ginzburg case the coherence length is given by

$$\xi = \left(\frac{\gamma}{\alpha}\right)^{\frac{1}{2}} = \frac{h\nu_F}{\Delta} \tag{6.34}$$

which can be easily shown to reduce to the Compton wavelength (Cf. also ref.[74]).
Thus the emergence of Benard cell like elementary particles from the Quantum Vacuum mimics the Landau-Ginzburg phase transition. In this case we have a non local growth of correlations reminiscent of the standard inflation theory.
As is known, the interesting aspects of the Critical Point theory (Cf.ref.[251]) are universality and scale. Broadly, this means that diverse physical phenomena follow the same route at the Critical Point, on the one hand, and on the other this can happen at different scales, as exemplified for example, by the course graining techniques of the Renormalization Group [252]. To highlight this point we note that in Critical Point phenomena we have the reduced order parameter $\bar{Q}$ (which gives the fraction of the excess of new states) and the reduced correlation length $\bar{\xi}$ (which follows from (6.34)). Near the Critical Point we have relations [253] like

$$(\bar{Q}) = |t|^{\beta}, (\bar{\xi}) = |t|^{-\nu}$$

Whence

$$\bar{Q}^{\nu} = \bar{\xi}^{\beta} \qquad (6.35)$$

In (6.35) typically $\nu \approx 2\beta$. As $\bar{Q} \sim \frac{1}{\sqrt{N}}$ because $\sqrt{N}$ particles are created fluctuationally, given N particles, and in view of the fractal two dimensionality of the path

$$\bar{Q} \sim \frac{1}{\sqrt{N}}, \bar{\xi} = (l/R)^2 \qquad (6.36)$$

This gives the Eddington formula,

$$R = \sqrt{N} l$$

which is nothing but (6.9).
There is another way of looking at this. The noncommutative geometry (6.27) brings out the primacy of the Quantum of Area. Indeed this has been noted from the different perspective of Black Hole Thermodynamics too [111]. We would also like to point out that a similar treatment leads from the Planck scale to the Compton scale.
In other words the creation of particles is the result of a Critical Point phase transition and subsequent coarse graining (Cf. also ref.[111]).
The above model apart from mimicking inflation also explains as we saw, the so called miraculous Large Number coincidences.
The peculiarity of these relations as we saw is that they tie up large scale parameters like the radius or age of the Universe or the Hubble constant with microphysical parameters like the mass, charge and the Compton scales of an elementary particle and the gravitational constant. That is, the Universe appears to have a Machian or holistic feature. One way to understand why the large and the small are tied up is to remember, as we saw in the earlier

chapter, that there is an underpinning of normal mode Planck oscillators, that is, collective phenomena all across the Universe.

We will return to this point in Chapter 8, but to re-emphasize: It has been known that there is a deep connection between a stochastic and Brownian behaviour on the one hand and Critical Point phenomena and the Renormalization Group on the other hand. Fractality itself is a manifestation of resolution dependent measurements, while Renormalization Group considerations arise due to coarse graining at different resolutions. A good example of the fractal behaviour is Quantum Mechanics itself which as noted earlier has been shown to have the fractal dimension 2.

In the above context, we will now argue that there is a manifestation of what may be called "scaled" Quantum Mechanics, at different scales in the Universe, and not just at the usual Quantum scale.

It has already been argued that in the Universe at large, there appear to be the analogues of the Planck constant at different scales [254, 255]. Infact we have

$$h_1 \sim 10^{93} \tag{6.37}$$

for super clusters;

$$h_2 \sim 10^{74} \tag{6.38}$$

for galaxies and

$$h_3 \sim 10^{54} \tag{6.39}$$

for stars. And

$$h_4 \sim 10^{34} \tag{6.40}$$

for Kuiper Belt objects. In equations (6.37) - (6.40), the $h_\imath$ play the role of the Planck constant, in a sense to be described below. The origin of these equations is related to the following empirical relations

$$R \approx l_1 \sqrt{N_1} \tag{6.41}$$

$$R \approx l_2 \sqrt{N_2} \tag{6.42}$$

$$l_2 \approx l_3 \sqrt{N_3} \tag{6.43}$$

$$R \sim l\sqrt{N} \tag{6.44}$$

and a similar relation for the KBO (Kuiper Belt objects)

$$L \sim l_4 \sqrt{N_4} \tag{6.45}$$

where $N_1 \sim 10^6$ is the number of superclusters in the Universe, $l_1 \sim 10^{25} cms$ is a typical supercluster size $N_2 \sim 10^{11}$ is the number of galaxies in the Universe and $l_2 \sim 10^{23} cms$ is the typical size of a galaxy, $l_3 \sim 1$ light years is a typical distance between stars and $N_3 \sim 10^{11}$ is the number of stars in a galaxy, R being the radius of the Universe $\sim 10^{28} cms, N \sim 10^{80}$ is the number of elementary particles, typically pions in the Universe and l is

the pion Compton wavelength and $N_4 \sim 10^{10}, l_4 \sim 10^5 cm$, is the dimension of a typical KBO (with mass $10^{19} gm$ and L the width of the Kuiper Belt $\sim 10^{10} cm$ cf.ref.[24]).
The size of the Universe, the size of a supercluster etc. from equations like (6.41)-(6.45), as described in the references turn up as the analogues of the Compton wavelength. For example we have

$$R = \frac{h_1}{Mc} \tag{6.46}$$

One can see that equations (6.37) to (6.46) are a consequence of gravitational orbits (or the Virial Theorem) and the conservation of angular momentum viz.,

$$\frac{GM}{L} \sim v^2, MvL = H \tag{6.47}$$

(Cf.refs.[254, 255]), where L, M, v represent typical length (or dispersion in length), mass and velocities at that scale and H denotes the scaled Planck constant.
It also appears that equations (6.41) to (6.45) resemble a typical Random Walk relation (Cf.[256]) of Brownian motion.
All this is suggestive but empirical. The question arises whether there is any theoretical justification. To investigate this further we observe that if we use (6.47) along with the relation,

$$L = vT$$

where T is a typical time scale, for example the time period for an orbit, we get the relations

$$L^2 = \frac{H}{M} T \quad \left(H = \frac{GM^2}{v} \right) \tag{6.48}$$

(48) is nothing but the well known diffusion equation of Nelson viz.,

$$\Delta x^2 = \nu \Delta t, \quad \nu = \frac{h}{m} \tag{6.49}$$

where ν is the diffusion constant, h the Planck constant and m the mass of a typical particle.
We now observe that as is well known, the relations (6.48) or (6.49) lead to an equation identical to the Quantum Mechanical Schrödinger equation (Cf.ref.[176] for a detailed derivation)

$$h_\imath \frac{\partial \psi}{\partial t} + \frac{h_\imath^2}{2m} \nabla^2 \psi = 0 \tag{6.50}$$

(for different $h_\imath$). Indeed this is not surprising because one can rewrite equation (6.49) as

$$m\Delta x \frac{\Delta x}{\Delta t} = h = \Delta x \cdot \Delta p \tag{6.51}$$

which gives the well known Uncertainty relation. Conversely, from the Uncertainty Principle we could get back (6.48) or (6.49).

Interestingly it has been shown that this is true, not just for the special form of the diffusion constant, but also for any other form of the diffusion constant [257]. Another interesting point is that starting from (6.48) or (6.49), we can deduce equations like (6.41), which describe a Brownian path [187].

In any case the steps leading to equation (6.50) and (6.50) itself provide the rationale for the scaled De Broglie or Compton lengths, for example equation (6.46), which follow from (6.51).

All this can be linked to Critical Point Theory and the Renormalization Group exactly as above. Relations like (6.41) to (6.45) would then be the result of equations (6.35) and (6.36) at different scales, just as (6.9) resulted from them.

We also observe that a Schrödinger equation like procedure has been used though in an empirical way by Agnese and Festa [258] to derive a Titius-Bode type relation for planetary distances which now appear as quantized levels. This consideration has been extended in an empirical way to also account for quantized cosmic distances [259].

Interestingly if we consider a wave packet of the generalized Schrödinger equation (6.50) with h_1 given by (6.1) for the Universe itself, we have for a Gaussian wave packet

$$R \approx \frac{\sigma}{\sqrt{2}} \left(1 + \frac{h_1^2 T^2}{\sigma^4 M^2}\right)^{1/2} \left(\approx \frac{1}{\sqrt{2}} \frac{h_1 T}{\sigma M}\right) \tag{6.52}$$

where R and T denote the radius and age of the Universe, M its mass and $\sigma \sim R$ is the spread of the wave packet. As $R \approx cT$ (6.52) gives us back (6.46), that is the “Compton wavelength” of the Universe treated as a wave packet.

Interestingly also we can pursue the reasoning of equations like (6.37) to the case of terrestrial phenomena. Let us consider a gas at standard temperature and pressure. In this case, the number of molecules $n \sim 10^{23}$ per cubic centimeter, so that $r \sim 1cm$ and with the same l, we can get a "scaled" Planck constant $\tilde{h} \sim 10^{-44} << h$, the Planck constant.

In this case, a simple application of the WKB approximation, leads immediately from the Schrödinger equation at the new scale to the classical Hamilton-Jacobi theory, that is to classical mechanics.

Equations like (6.41) are the analogue of the well known Eddington formula. Similarly we can have the analogue of the mysterious Weinberg relation linking the pion mass to the Hubble constant, from $H^2 = M^3 LG$. For this we need to define the analogue of the Hubble constant H

$$\hat{H} = \frac{v}{L}$$

to get

$$M = \left(\frac{\hat{H}H^2}{Gv}\right)^{\frac{1}{3}}$$

which is the required relation.
We can now argue that just as matter in the form of elementary particles, forms or condenses within the Compton wavelength from a background Quantum Vacuum in a phase transition, matter at other scales, for example stars and galaxies also could be considered to condense or cluster by a similar mechanism. This would give a rationale for the observed lumpiness of the Universe. Similar considerations apply for the other scales referred to.
ix) We will now argue, following the above reasoning, that the difference between electromagnetism at the micro scale and gravitation at the macro scale is merely a matter of the difference in the time and length scales.
Infact the operative equations are (6.35) and (6.36):

$$\bar{Q}^{\nu} = \bar{\xi}^{\beta}$$

where $\bar{Q}$ and $\bar{\xi}$ are the reduced order parameter and correlation length.
We now have

$$\bar{Q} \sim \frac{1}{\sqrt{N}}, \bar{\xi} = (l/R)^2$$

which gives, Eddington like relations.
Now if we consider the representation of the Hamiltonian as the differential time operator we will get

$$H(T) = \frac{d}{dT} = \frac{d}{\sqrt{N}d\tau} = \frac{H(\tau)}{\sqrt{N}} \tag{6.53}$$

$H(T)$ in (6.53) denotes gravitation represented by the coupling constant Gm^2 and $H(\tau)$ in (6.53) denotes electromagnetism represented by the coupling constant e^2 and m referring to the same elementary particle. Whence if (6.53) is consistent, we should have,

$$\frac{e^2}{Gm^2} \sim \sqrt{N} \tag{6.54}$$

Infact this is (6.8) - the well known empirical and supposedly accidental relation - the ratio of the coupling constants encountered earlier.
Let us now consider the analogue of the microscopic relation,

$$m\frac{l^2}{\tau} = h$$

for the macro or cosmic scale. We then get

$$h \to ML^2/T = h_1 \sim 10^{93} \tag{6.55}$$

This equation which is the same as (6.37), is infact perfectly meaningful because h_1 in (6.55) is the Godel spin of the Universe [254, 259]. Infact (6.55) immediately leads to

$$R = \frac{h_1}{Mc} \tag{6.56}$$

which is (6.46). Equations (6.55) and (6.56) show that the Universe itself seems to follow a Quantum Mechanical behaviour with a scaled up Planck constant h_1 as argued previously.

The above considerations in the context of universality and scaling effects of Critical Point Phenomena and the Renormalization Group mean: The Universe is a coarse grained scaled up version of the micro world, gravitation being the counterpart of electromagnetism should be given by their mutual scaled ratio. Let us see if this model is correct.

In such a coarse graining, we know that at a Critical Point we have for the coupling constants,

$$J^{(1)}/kT_c^{(1)} = 1 \, J^{(2)}/kT_c^{(2)} = 1$$

where from the theory, in our case,

$$T_c^{(1)}/T_c^{(2)} = l/R$$

Whence we get

$$J^{(1)}/J^{(2)} = l/R \tag{6.57}$$

As $J^{(1)} = Gm^2$ and $J^{(2)} = e^2$ are the coupling constants at the two scales, does (6.57) give the correct ratio? Infact it gives us back (6.8). In other words, as can be seen from (6.53) or (6.57), the "weak" gravitational interaction is a manifestation of the much longer time periods involved on the macro or cosmic scale, while the much stronger electromagnetic interaction is a manifestation of the much smaller scale of time at the micro level. This can be elaborated upon in the following way.

The electromagnetic energy of a typical elementary particle, for example the pion is given by

$$\text{Energy} = \frac{e^2}{l} = \frac{\hbar}{\tau}$$

On the other hand its gravitational energy is given by

$$\text{Gravitational Energy} = \frac{Gm^2}{l} = \frac{\hbar}{T} \tag{6.58}$$

Whence,

$$\frac{Gm^2}{e^2} = \frac{l}{R} = \frac{1}{\sqrt{N}} \tag{6.59}$$

In both these cases, as we have been dealing with a microscopic particle, the Heisenberg Uncertainty Principle holds. So while the electromagnetic energy plays out in the Compton time τ, the gravitational energy plays out during the life time of the Universe. Sivaram [260] uses in (6.58) the relation $T = \frac{1}{H}$, where H is the Hubble constant, to get, as a curiosity, the mysterious Weinberg formula again.

Let us now consider the gravitational energy of all the N particles in the Universe. This is given by

$$E = \frac{NGm^2}{l}$$

The energy E has a low Beckenstein temperature and as can be easily calculated from the Beckenstein Radiation decay formula viz.,

$$T = 8.4 \times 10^{-24} (E/c^2)^3$$

the life time is T, the age of the Universe itself.

Interestingly if the above considerations are carried over to the Planck scale versus the Compton scale, we can easily verify that there is no new scaled down Planck constant, as for example in (6.55)– that is the considerations remain the same as those at the Compton scale. However let us see what we get if in analogy to (6.35) and (6.36)we compare the Planck and Compton scales. This time, the Critical Point relations lead to the known relation,

$$l = \sqrt{n} l_P, \quad \tau = \sqrt{n} \tau_P.$$

Furthermore, (6.53), with a similar notation leads to,

$$H(\tau) = \frac{H(\tau_P)}{\sqrt{n}}$$

which also we have encountered earlier in this Chapter. It is just,

$$m = m_P / \sqrt{n}$$

Further, the Beckenstein Radiation life time of a Planck mass, gives this time - the Planck Compton time.

This can be illustrated by the following amusing description in Indian Mythology. Brahma, the creator of the Universe has a very very long day– while he takes a bath, many time consuming and momentous events take place on the earth. By Brahma's reckoning, however, the time elapsed is still miniscule. Interestingly the ratio of the time scales would be the same as above, because of the fact that the estimate for the age of the Universe or Brahma's day is exactly of the same order of magnitude as modern estimates.

6.4 The Nature of Space Time

As we noted earlier all of Classical Physics and Quantum Theory, is based on the Minkowski spacetime, as for example in the case of Quantum Field

Theory, or Reimannian spacetime as in the case of General Relativity. In the non relativistic theories, Newtonian spacetime, is used, which is a special case of Minkowskian spacetime. But in all these cases the common denominator is that we are dealing with a differentiable manifold.

As we saw in the previous Chapter, this breaks down however in Quantum Gravity including the author's approach, String Theory and other approaches, be it at the Planck scale, or at the Compton scale [180, 164, 261, 262]. The underlying reason for this breakdown of a differentiable spacetime manifold is the Uncertainty Principle–as we go down to arbitrarily small spacetime intervals, we encounter arbitrarily large energy momenta. As Wheeler put it [46], "no prediction of spacetime, therefore no meaning for spacetime is the verdict of the Quantum Principle. That object which is central to all of Classical General Relativity, the four dimensional spacetime geometry, simply does not exist, except in a classical approximation." Before proceeding to analyze the nature of spacetime beyond the classical approximation, let us first analyze briefly the nature of classical spacetime itself.

We can get an insight into the nature of the usual spacetime by considering the well known formulation of Quantum Theory in terms of stochastic processes more precisely, a double Wiener process which, as we saw, models fuzzy spacetime [175, 178, 176, 24].

In the stochastic approach, we deal with a double Wiener process which leads to a complex velocity $V - \imath U$. It is this complex velocity that leads to Quantum Theory from the usual diffusion theory as seen in the previous Chapter. To see this in a simple way, let us write the usual diffusion equation as

$$\Delta x \cdot \Delta x = \frac{h}{m} \Delta t \equiv \nu \Delta t \tag{6.60}$$

We saw that equation (6.60) can be rewritten as the usual Quantum Mechanical relation,

$$m \frac{\Delta x}{\Delta t} \cdot \Delta x = h = \Delta p \cdot \Delta x \tag{6.61}$$

We are dealing here, with phenomena within the Compton or De Broglie wavelength.

We now treat the diffusion constant ν to be very small, but non vanishing. That is, we consider the semi classical case. This is because, a purely classical description, does not provide any insight.

It is well known that in this situation we can use the WKB approximation [263]. Whence the right hand side of the wave function,

$$\psi = \sqrt{\rho} e^{\imath / \hbar S}$$

goes over to, in the one dimensional case, for simplicity,

$$(p_x)^{-\frac{1}{2}} e^{\frac{1}{\hbar}} \int p(x) dx$$

so that we have, on comparison,

$$\rho = \frac{1}{p_x} \tag{6.62}$$

ρ being the probability density. In this case the condition $U \approx 0$, that is, the velocity potential becoming real, implies

$$\nu \cdot \nabla ln(\sqrt{\rho}) \approx 0 \tag{6.63}$$

This semi classical analysis suggests that $\sqrt{\rho}$ is a slowly varying function of x, infact each of the factors on the left side of (6.63) would be $\sim 0(h)$, so that the left side is $\sim 0(h^2)$ (which is being neglected). Then from (6.62) we conclude that p_x is independent of x, or is a slowly varying function of x. The equation of continuity now gives

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}(\rho \boldsymbol{v}) = \frac{\partial \rho}{\partial t} = 0$$

That is the probability density ρ is independent or nearly so, not only of x, but also of t. We are thus in a stationary and homogenous scenario. This is strictly speaking, possible only in a single particle Universe, or for a completely isolated particle, without any effect of the environment. Under these circumstances we have the various conservation laws and the time reversible theory, all this taken over into Quantum Mechanics as well. This is an approximation valid for small, incremental changes, as indeed is implicit in the concept of a differentiable spacetime manifold.
Infact the well known displacement operators of Quantum Theory which define the energy momentum operators are legitimate and further the energy and momenta are on the same footing only under this approximation[264].
We would now like to point out the well known close similarity between the formulation mentioned above and the hydrodynamical formulation for Quantum Mechanics, which also leads to identical equations on writing the wave function as above. These two approaches were reconciled by considering quantized vortices at the Compton scale (Cf.[24, 265]).
To proceed further, we start with the Schrödinger equation

$$\imath\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \psi + V\psi \tag{6.64}$$

Remembering that for momentum eigen states we have, for simplicity, for one dimension

$$\frac{\hbar}{\imath}\frac{\partial}{\partial x}\psi = p\psi \tag{6.65}$$

where p is the momentum or p/m is the velocity v, we take the derivative with respect to x (or $\boldsymbol{x}$) of both sides of (6.64) to obtain, on using (6.65),

$$\imath\hbar \frac{\partial (v\psi)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 (v\psi) + \frac{\partial V}{\partial x}\psi + Vv\psi \tag{6.66}$$

We would like to compare (6.66) with the well known equation for the velocity in hydrodynamics[266, 267], following from the Navier-Stokes equation,

$$\rho \frac{\partial v}{\partial t} = -\nabla p - \rho \alpha T g + \mu \nabla^2 v \tag{6.67}$$

In (6.67) v is a small perturbational velocity in otherwise stationary flow between parallel plates separated by a distance d, p is a small pressure, ρ is the density of the fluid, T is the temperature proportional to $Q(z)v, \mu$ is the Navier-Stokes coefficient and α is the coefficient of volume expansion with temperature. Also required would be

$$\beta \equiv \frac{\Delta T}{d}.$$

v itself is given by

$$v_z = W(z) exp(\sigma t + \imath k_x x + \imath k_y y), \tag{6.68}$$

z being the coordinate perpendicular to the fluid flow.
We can now see the parallel between equations (6.66) and (6.67). To verify the identification we would require that the dimensionless Rayleigh Number

$$R = \frac{\alpha \beta g d^4}{\kappa \nu}$$

should have an analogue in (6.66) which is dimensionless, κ, ν being the thermometric conductivity and viscosity.
Remembering that

$$\nu \sim \frac{h}{m}$$

and

$$d \sim \lambda$$

where λ is the Compton wavelength in the above theory (Cf.[24, 160] for details) and further we have

$$\rho \propto f(z) g = V \tag{6.69}$$

for the identification between the hydrostatic energy and the energy V of Quantum Mechanics, it is easy using (6.69) and earlier relations to show that the analogue of R is

$$(c^2/\lambda^2) \cdot \lambda^4 \cdot (m/h)^2 \tag{6.70}$$

The expression (6.70) indeed is dimensionless and of order 1. Thus the mathematical identification is complete.
Before proceeding, let us look at the physical significance of the above considerations (Cf.[268] for a graphic description.) Under conditions of stationery

flow, when the difference in the temperature between the two plates is negligible there is total translational symmetry, as in the case of the displacement operators of Quantum Theory. But when there is a small perturbation in the velocity (or equivalently the temperature difference), then beyond a critical value the stationarity and homogeneity of the fluid is disrupted, or the symmetry is broken and we have the phenomena of the formation of Benard cells, which are convective vortices and can be counted. This infact is the "birth" of space It must be stressed that before the formation of the Benard cells, there is no "space", that is, no point to distinguish from or relate to another point. Only with the formation of the cells are we able to label space points. In the context of the above identification, the Benard cells would correspond to the formation of "quantized vortices" at the Compton (Planck) scale from the ZPF, as we saw, which latter had been discussed in detail in the literature (Cf.[24] and [198]) from the ZPF. This phase transition would correspond to the "formation" of spacetime. As discussed in detail in [24] these "quantized vortices" can be identified with elementary particles. Interestingly, as noted Einstein himself considered electrons as condensates from a background electromagnetic field[232]. All this ties up with the discussion in the previous section.
However in order to demonstrate that the above formulation is not a mere mathematical analogy, we have to show that the critical value of the wave number k in the expression for the velocity in the hydrodynamical flow (6.68) is the same as the critical length, the Compton length. In terms of the dimensionless wave number $k' = k/d$, this critical value is given by[266]

$$k'_c \sim 1$$

In the case of the "quantized vortices" at the Compton scale l, remembering that d is identified with l itself we have,

$$l'_c(\equiv)k'_c \sim 1,$$

exactly as required.
In this connection it may be mentioned that due to fluctuations in the Zero Point Field or the Quantum Vacuum, there would be fluctuations in the metric given by[46]

$$\Delta g \sim l_P/l$$

where l_P is the Planck length and l is a small interval under consideration. At the same time the fluctuation in the curvature of space would be given by

$$\Delta R \sim l_P/l^3$$

Normally these fluctuations are extremely small but as discussed in detail elsewhere[168], this would imply that at the Compton scale of a typical elementary particle $l \sim 10^{-11} cms$, the fluctuation in the curvature would be ~ 1. This is symptomatic of the formation of what we have termed above as

elementary particle “quantized vortices”.
Further if a typical time interval between the formation of such “quantized vortices” which are the analogues of the Benard cells is τ, in this case the Compton time, then as in the theory of the Brownian Random Walk[256], the mean time extent would be given by

$$T \sim \sqrt{N}\tau \qquad (6.71)$$

where N is the number of such quantized vortices or elementary particles (Cf.also [24, 265]). This is equation (6.4) - that is, the equation (6.71) holds good for the Universe itself because T the age of the Universe $\sim 10^{17} secs$ and N the number of elementary particles $\sim 10^{80}, \tau$ being the Compton time $\sim 10^{-23} secs$. Interestingly, this “phase transition” nature of time would automatically make it irreversible, unlike the conventional model in which time is reversible. We will return to these considerations later in this section. It may be mentioned that an equation similar to (6.71) can be deduced by the same arguments for space extension also as indeed we did, and this time we get back the well known Eddington formula viz.,

$$R \sim \sqrt{N}l \qquad (6.72)$$

where R is the extent or radius of the Universe and l is the cell size or Compton wavelength. We can similarly characterize the formation of elementary particles themselves from cells at the Planck scale.
Once we recognize the minimum spacetime extensions, then we immediately are lead to the underlying non commutative geometry encountered in the earlier chapter and given by equation (6.27):

$$[x, y] = 0(l^2), [x, p_x] = \imath\hbar[1 + 0(l^2)], [t, E] = \imath\hbar[1 + 0(\tau^2) \qquad (6.73)$$

As we saw relations like (6.73) are Lorentz invariant. At this stage we recognise the nature of spacetime as given by (6.27) in contrast to the stationary and homogeneous spacetime discussed earlier. Witten [17, 269] has called this Fermionic spacetime as contrasted with the usual Bosonic spacetime. Indeed we traced the origins of the Dirac equation of the electron to (6.27). We also argued that (6.27) provides the long sought after reconciliation between electromagnetism and gravitation[161, 162].
The usual differentiable spacetime geometry can be obtained from (6.27) if l^2 is neglected, and this is the approximation that has been implicit.
Thus spacetime is a collection of such cells or elementary particles. As pointed out earlier, this spacetime emerges from a homogeneous stationary non or pre spacetime when the symmetry is broken, through random processes. The question that comes up then is, what is the metric which we use? This has been touched upon earlier, and we will examine it again.
We first make a few preliminary remarks. When we talk of a metric or the distance between two ”points” or ”particles”, a concept that is implicit is that

of topological "nearness" - we require an underpinning of a suitably Large Number of "open" sets[270]. Let us now abandon the absolute or background spacetime and consider, for simplicity, a Universe (or set) that consists solely of two particles. The question of the distance between these particles (quite apart from the question of the observer) becomes meaningless. Indeed, this is so for a Universe consisting of a finite number of particles. For, we could isolate any two of them, and the distance between them would have no meaning. We can intuitively appreciate that we would infact need distances of intermediate or more generally, other points.

In earlier work[271, 231], motivated by physical considerations we had considered a series of nested sets or neighbourhoods which were countable and also whose union was a complete Hausdorff space. The Urysohn Theorem was then invoked and it was shown that the space of the subsets was metrizable. Let us examine this in more detail.

Firstly we observe that in the light of the above remarks, the concepts of open sets, connectedness and the like reenter in which case such an isolation of two points would not be possible.

More formally let us define a neighbourhood of a particle (or point or element) A of a set of particles as a subset which contains A and atleast one other distinct element. Now, given two particles (or points) or sets of points A and B, let us consider a neighbourhood containing both of them, $n(A, B)$ say. We require a non empty set containing atleast one of A and B and atleast one other particle C, such that $n(A, B) \subset n(A, C)$, and so on. Strictly, this "nested" sequence should not terminate. For, if it does, then we end up with a set $n(A, P)$ consisting of two isolated "particles" or points, and the "distance" $d(A, P)$ is meaningless.

We now assume the following property[271, 231]: Given two distinct elements (or even subsets) A and B, there is a neighbourhood N_{A_1} such that A belongs to N_{A_1}, B does not belong to N_{A_1} and also given any N_{A_1}, there exists a neighbourhood $N_{A_{\frac{1}{2}}}$ such that $A \subset N_{A_{\frac{1}{2}}} \subset N_{A_1}$, that is there exists an infinite topological closeness.

From here, as in the derivation of Urysohn's Lemma[272], we could define a mapping f such that $f(A) = 0$ and $f(B) = 1$ and which takes on all intermediate values. We could now define a metric, $d(A, B) = |f(A) - f(B)|$. We could easily verify that this satisfies the properties of a metric.

With the same motivation we will now deduce a similar result, but with different conditions. In the sequel, by a subset we will mean a proper subset, which is also non null, unless specifically mentioned to be so. We will also consider Borel sets, that is the set itself (and its subsets) has a countable covering with subsets. We then follow a pattern similar to that of a Cantor ternary set [270, 273]. So starting with the set N we consider a subset N_1 which is one of the members of the covering of N and iterate this process so that N_{12} denotes a subset belonging to the covering of N_1 and so on.

We note that each element of N would be contained in one of the series of subsets of a sub cover. For, if we consider the case where the element p belongs to some $N_{12\cdots m}$ but not to $N_{1,2,3\cdots m+1}$, this would be impossible because the latter form a cover of the former. In any case as in the derivation of the Cantor set, we can put the above countable series of sub sets of sub covers in a one to one correspondence with suitable sub intervals of a real interval (a, b).

Case I

If $N_{1,2,3\cdots m} \to$ an element of the set N as $m \to \infty$, that is if the set is closed, we would be establishing a one to one relationship with points on the interval (a, b) and hence could use the metric of this latter interval, as seen earlier.

Case II

It is interesting to consider the case where in the above iterative countable process, the limit does not tend to an element of the set N, that is set N is not closed and has what we may call singular points. We could still truncate the process at $N_{1,2,3\cdots m}$ for some $m > L$ arbitrary and establish a one to one relationship between such truncated subsets and arbitrarily small intervals in a, b. We could still speak of a metric or distance between two such arbitrarily small intervals.

This case is of interest because we described elementary particles as, what we have called Quantum Mechanical Kerr-Newman black holeblack holes or vortices, where we have a length of the order of the Compton wavelength as seen in the previous sections, within which spacetime as we know it breaks down. Such cut offs as seen lead to a non commutative geometry (6.27) and what may be called fuzzy spaces[67, 46].(We note that the centre of the vortex is a singular point). In any case, the number of particles in the Universe is of the order 10^{80}, which approximates infinity from a physicist's point of view.

Interestingly, we usually consider two types of infinite sets - those with cardinal number n corresponding to countable infinities, and those with cardinal number c corresponding to a continuum, there being nothing in between [273]. This is the well known but unproven Continuum hypothesis.

What we have shown with the above process is that it is possible to conceive an intermediate possibility with a cardinal number $n^p, p > 1$.

In the above considerations three properties are important: the set must be closed i.e. it must contain all its limit points, perfect i.e. in addition each of its points must be a limit point and disconnected i.e. it contains no non null open intervals. Only the first was invoked in Case I.

Finally we notice again the holistic feature. A metric emerges by considering large encompassing sets. It may be remarked that much of Quantum Theory, like much of Classical Theory was couched in the concepts of Newtonian two body mechanics and determinism. The moment we consider even a three body problem, as was realized by Poincare more than a century ago, the picture gets altered. As he noted [274], "If we knew exactly the laws of nature and the situation of the Universe at the initial moment, we could predict exactly the situation of that same Universe at a succeeding moment. But even if it were the case that the natural laws had no longer any secret for us, we could still know the situation approximately. If that enabled us to predict the succeeding situation with the same approximation, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by the laws. But it is not always so; it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible." In a similar vein, Prigogine observes [268], "Our physical world is no longer symbolized by the stable and periodic planetary motions that are at the heart of classical mechanics. It is a world of instabilities and fluctuations..."

Indeed, the departure from the two body formulation began with electromagnetism itself, which has to invoke the environment.

We now return to the current view of Planck scale oscillators in the background dark energy or Quantum Vacuum. In this context we saw in the last Chapter that elementary particles can be considered to be normal modes of $n \sim 10^{40}$ Planck oscillators in the ground state, while the entire Universe itself has an underpinning of $\bar{N} \sim 10^{120}$ such oscillators, there being $N \sim 10^{80}$ elementary particles in the Universe [187, 249]. These Planck oscillators are formed out of the Quantum Vacuum (or dark energy). Thus we have, $m_P c^2$ being the energy of each Planck oscillator, m_P being the Planck mass, $\sim 10^{-5} gms$,

$$m = \frac{m_P}{\sqrt{n}} \tag{6.74}$$

$$l = \sqrt{n} l_P, \tau = \sqrt{n} \tau_P, n = \sqrt{N} \tag{6.75}$$

where m is the mass of a typical elementary particle, taken to be a pion in the literature. The ground state of $\bar{N}$ such Planck oscillators would be, in analogy to (6.74),

$$\bar{m} = \frac{m_P}{\sqrt{\bar{N}}} \sim 10^{-65} gms \tag{6.76}$$

The Universe is an excited state and consists of N such ground state levels and so we have, from (6.76)

$$M = \bar{m} N = \sqrt{N} m_P \sim 10^{55} gms,$$

as required, M being the mass of the Universe.

Due to the fluctuation $\sim \sqrt{n}$ in the levels of the n oscillators making up an elementary particle, the energy is, remembering that mc^2 is the ground state,

$$\Delta E \sim \sqrt{n} mc^2 = m_P c^2,$$

using (6.75), and so the indeterminacy time is

$$\frac{\hbar}{\Delta E} = \frac{\hbar}{m_P c^2} = \tau_P,$$

as indeed we would expect.
The corresponding minimum indeterminacy length would therefore be l_P. One of the consequences of the minimum spacetime cut off as we saw is that the Heisenberg Uncertainty Principle takes an extra term as we saw in the previous Chapter [145]. Thus as we saw

$$\Delta x \approx \frac{\hbar}{\Delta p} + \alpha \frac{\Delta p}{\hbar}, \; \alpha = l^2 (\text{or}\, l_P^2) \tag{6.77}$$

where l (or l_P) is the minimum interval under consideration. This is just (6.28). The first term gives the usual Heisenberg Uncertainty Principle.
Application of the time analogue of (6.77) for the indeterminacy time Δt for the fluctuation in energy $\Delta \bar{E} = \sqrt{N}mc^2$ in the N particle states of the Universe gives exactly as above,

$$\Delta t = \frac{\Delta E}{\hbar}\tau_P^2 = \frac{\sqrt{N}mc^2}{\hbar}\tau_P^2 = \frac{\sqrt{N}m_P c^2}{\sqrt{n}\hbar}\tau_P^2 = \sqrt{n}\tau_P = \tau,$$

wherein we have used (6.75). In other words, for the fluctuation $\sqrt{N}$, the time is τ. It must be re-emphasized that the Compton time τ of an elementary particle, is an interval within which there are unphysical effects like zitterbewegung - as pointed out by Dirac, it is only on averaging over this interval, that we return to meaningful Physics. This gives us,

$$dN/dt = \sqrt{N}/\tau \tag{6.78}$$

Equation (6.78) is identical to (6.3), the starting point for the cosmology discussed. Here we have derived this relation from a consideration of the underlying Planck oscillators. On the other hand due to the fluctuation in the $\sqrt{\bar{N}}$ oscillators constituting the Universe, the fluctuational energy is similarly given by

$$\sqrt{\bar{N}}\bar{m}c^2,$$

which is the same as (6.76) above. Another way of deriving (6.78) is to observe that as $\sqrt{n}$ particles appear fluctuationally in time τ_P which is, in the elementary particle time scales, $\sqrt{n}\sqrt{n} = \sqrt{N}$ particles in $\sqrt{n}\tau_P = \tau$. That is, the rate of the fluctuational appearance of particles is

$$\left(\frac{\sqrt{n}}{\tau_P}\right) = \frac{\sqrt{N}}{\tau} = dN/dt$$

which is (6.78). From here by integration,

$$T = \sqrt{N}\tau$$

T is the time elapsed from $N = 1$ and τ is the Compton time. This gives T its origin in the fluctuations - there is no smooth "background" (or "being") time - the root of time is in "becoming". It is the time of a Brownian double Wiener process: A step l gives a step in time $l/c \equiv \tau$ and therefore $\Delta x = \sqrt{N} l$ gives $T = \sqrt{N}\tau$. Time is born out of acausal fluctuations which are random and therefore irreversible. Indeed, there is no background time. Time is proportional to $\sqrt{N}$, N being the number of particles which are being created spontaneously from the ZPF.

The time we use is what may be called Stationary time and it is an approximation as we saw [178]. Further, Quantum Mechanics, gravitation etc. follow from here. In Quantum Mechanics, the measurement of the observer triggers the acausal collapse of the wave function - an irreversible event - but the wave function itself satisfies a deterministic and reversible equation paradoxically. Yet the Universe is "irreversible". It appears spontaneous irreversibility or indeterministic time [275] is the real time. This can be contrasted to the usual time reversible Quantum Theory.

We observe that [276]

$$\psi(r,t) = \frac{1}{(2\pi\hbar)^{3/2}} \int a(p) \exp\left[\frac{\imath}{\hbar}\left(p \cdot r - \frac{p^2}{2m}t\right)\right] dp,$$

$a(p)$ being independent of time. So we have at any other time t':

$$a(p) = \frac{1}{(2\pi\hbar)^{3/2}} \int \psi(r',t') \exp\left[-\frac{\imath}{\hbar}\left(p \cdot r' - \frac{p^2}{2m}t'\right)\right] dr'$$

Substitution yields the result

$$\psi(r,t) = \int K(r,t';r',t')\psi(r',t')dr', \tag{6.79}$$

the Kernel function K being given by

$$K(r,t;r',t') = \frac{1}{(2\pi\hbar)^3} \int \exp\left\{\frac{\imath}{\hbar}\left[p \cdot (r - r') - \frac{p^2}{2m}(t - t')\right]\right\} dp$$

or after some manipulation, in the form

$$K(r,t;r',t') = \left[\frac{2\pi\imath\hbar}{m}(t - t')\right]^{-3/2} \exp\left[\imath\frac{m}{2\hbar}\frac{|r - r'|^2}{(t - t')}\right]$$

The point is that in (6.79) $\psi(r,t)$ at t is given in terms of a linear expansion of $\psi(r,t')$ at earlier times t'. But what is to be noted is, the symmetry between t and t'. This is not surprising as the original Schrödinger equation remains unchanged under $t \to -t$.

Thus it is possible to understand the fluctuations encountered in Section 2, that is, the equation (6.78) which was the starting point for fluctuational energy in terms of the underpinning of Planck scale oscillators in the Quantum Vacuum.

We would now like to make some remarks. Starting from a completely different point of view namely Black Hole Thermodynamics, Landsberg [277]

deduced that the smallest observable mass in the Universe is $\sim 10^{-65} gms$, which is exactly the minimum mass given in (6.76).
Further due to the fluctuational appearance of $\sqrt{N}$ particles, the fluctuational mass associated with each of the N particles in the Universe is

$$\frac{\sqrt{N}m}{N} = \frac{m}{\sqrt{N}} \sim 10^{-65} gms,$$

that is once again the smallest observable mass or ground state mass in the Universe.

6.5 Further Considerations

1. We will now provide yet another rationale for (6.3) or (6.78). Let us start with equations encountered earlier, viz., (6.9), (6.8) and (6.7) respectively

$$R = \sqrt{N} l$$

$$\frac{Gm^2}{e^2} = \frac{1}{\sqrt{N}} \sim 10^{-40}$$

or the Weinberg formula

$$m = \left(\frac{H\hbar^2}{Gc}\right)^{\frac{1}{3}}$$

where $N \sim 10^{80}$ is the number of elementary particles, typically pions, in the Universe. On the other hand (6.8) which is the ratio of the electromagnetic and gravitational coupling constants, is deducible from (6.7). The very mysterious feature of (6.7) was stressed by Weinberg as we saw "...it should be noted that the particular combination of $\hbar, H, G$, and c appearing (in the formula) is very much closer to a typical elementary particle mass than other random combinations of these quantities....
In contrast, (the formula) relates a single cosmological parameter, H, to the fundamental constants $\hbar, G, c$ and m, and is so far unexplained..."
Relations like (6.9) and (6.8) inspired the Dirac Large Number Cosmology. All these relations are to be taken in the order of magnitude sense.
We will now take a different route and provide an alternative theoretical rationale for equations (6.71), (6.8) and (6.9), and in the process light will be shed on the new cosmological model and the nature of gravitation.
Following Sivaram [33] we consider the gravitational self energy of the pion. This is given by

$$\frac{Gm^2}{l} = Gm^2/(\hbar/mc)$$

If this energy were to have a life time of the order of the age of the Universe, T, then we have by the Uncertainty relation

$$\left(\frac{Gm^3c}{\hbar}\right)(T) \approx \hbar \tag{6.80}$$

As $T = \frac{1}{H}$, this immediately gives us the Weinberg formula (6.7). It must be observed again that (6.80) gives a time dependent gravitational constant G. We could also derive (6.7) by using a relation given by Landsberg [277]. We use the fact that the mass of a particle is given by

$$m(b) \sim \left(\frac{\hbar^3 H}{G^2}\right)^{1/5} \left(\frac{c^5}{\hbar H^2 G}\right)^{b/15} \tag{6.81}$$

where b is an unidentified constant. Whence we have

$$m(b) \sim G^{-3/5} G^{-3b/15} = G^{-(b+1)/5}$$

The mass that would be time independent, if G were time dependent would be given by the value

$$b = -1$$

With this value of b (6.81) gives back (6.7).

Let us now proceed along a different track. We rewrite (6.80) as

$$G = \frac{\hbar^2}{m^3 c} \cdot \frac{1}{T} \tag{6.82}$$

If we use the fact that $R = cT$, then (6.82) can be written as

$$G = \frac{\hbar^2}{m^3 R} \tag{6.83}$$

Let us now use the well known relation encountered earlier [59]

$$R = \frac{GM}{c^2}, \tag{6.84}$$

There are several derivations of (6.84). For example in a uniformly expanding Friedman Universe, we have

$$\dot{R}^2 = \frac{8\pi G \rho R^2}{3}$$

If we substitute the value $\dot{R} = c$ at the radius of the Universe, then we recover (6.84). If we use (6.84) in (6.83) we will get

$$G^2 = \frac{\hbar^2 c^2}{m^3 M} \tag{6.85}$$

Let $M/m = N$ be called the number of elementary particles in the Universe. Infact this is just (1). Then (6.85) can be written as (6.10),

$$G = \frac{\hbar c}{m^2 \sqrt{N}}$$

which can also be written as (6.8)

$$Gm^2/e^2 \sim \frac{1}{\sqrt{N}}$$

Whence we get (6.9)

$$\sqrt{N} l = R$$

We now remark that (6.82) shows an inverse dependence on time of the gravitation constant, while (6.10) shows an inverse dependence on $\sqrt{N}$. Equating the two, we get back,

$$T = \sqrt{N} \tau$$

the relation (6.4) which we have encountered several times. If we now take the time derivative of (6.10) and use (6.4), we get (6.3)

$$\dot{N} = \frac{\sqrt{N}}{\tau}$$

This equation is the same as (6.78) or (6.3). To put it briefly in a phase transition from the Quantum Vacuum $\sqrt{N}$ particles appear within the Compton time τ. In terms of our unidirectional concept of time, we could say that particles appear and disappear, but the nett result is the appearance of $\sqrt{N}$ particles.

We now make a few remarks. Firstly it is interesting to note that $\sqrt{N}m$ will be the mass added to the Universe. Let us now apply the well known Beckenstein formula for the life time of a mass M viz., [59],

$$t \approx G^2 M^3 / \hbar c^4$$

to the above mass. The life time as can be easily verified turns out to be exactly the age of the Universe!

A final remark. To appreciate the role of fluctuations in the otherwise mysterious Large Number relations, let us follow Hayakawa [234] and consider the excess of electric energy due to the fluctuation $\sim \sqrt{N}$ of the elementary particles in the Universe and equate it to the inertial energy of an elementary particle. We get

$$\frac{\sqrt{N} e^2}{R} = mc^2$$

This gives us back (6.8) if we use (6.84). If we use (6.9) on the other hand, we get

$$e^2/mc^2 = l,$$

another well known relation from micro physics.

2. We note that as is well known, a background ZPF of the kind we have

been considering can explain the Quantum Mechanical spin half as also the anomalous $g = 2$ factor for an otherwise purely classical electron [278, 279, 226]. The key point here is (Cf.ref.[278]) that the classical angular momentum $\boldsymbol{r} \times m\boldsymbol{v}$ does not satisfy the Quantum Mechanical commutation rule for the angular momentum $\boldsymbol{J}$. However when we introduce the background Zero Point Field, the momentum now becomes

$$\boldsymbol{J} = \boldsymbol{r} \times m = \boldsymbol{v} + (e/2c)\boldsymbol{r} \times (\boldsymbol{B} \times \boldsymbol{r}) + (e/c)\boldsymbol{r} \times \boldsymbol{A}^0, \tag{6.86}$$

where $\boldsymbol{A}^0$ is the vector potential associated with the ZPF and $\boldsymbol{B}$ is an external magnetic field introduced merely for convenience, and which can be made vanishingly small.
It can be shown that $\boldsymbol{J}$ in (6.86) satisfies the Quantum Mechanical commutation relation for $\boldsymbol{J} \times \boldsymbol{J}$. At the same time we can deduce from (6.86)

$$\langle J_z \rangle = -\frac{1}{2}\hbar\omega_0/|\omega_0| \tag{6.87}$$

Relation (6.87) gives the correct Quantum Mechanical results referred to above.
From (6.86) we can also deduce that

$$l = \langle r^2 \rangle^{\frac{1}{2}} = \left(\frac{\hbar}{mc}\right) \tag{6.88}$$

(6.88) shows that the mean dimension of the region in which the fluctuation contributes is of the order of the Compton wavelength of the electron. By relativistic covariance (Cf.ref.[279]), the corresponding time scale is at the Compton scale. Thus once again we return to the Compton scale, as at the beginning of this Chapter.
3. In the light of the preceding considerations, let us now investigate the neutrino and weak interactions. We start by following Hayakawa [234] to balance the gravitational force and the Fermi energy of the "cold" background neutrinos and further identify it with the intrinsic energy of the neutrinos to get

$$\frac{GN_\nu m_\nu^2}{R} = \frac{N_\nu^{2/3}\hbar^2}{m_\nu R^2} = m_\nu c^2 \tag{6.89}$$

(All this is in the Large Number sense) m_ν is the neutrino mass. From (6.89) we can immediately deduce that

$$m_\nu = 10^{-8} m_e, N_\nu \sim 10^{90} \tag{6.90}$$

Both the relations in (6.90) are known to be correct.
We then use the fact that due to the fluctuation in the number of neutrinos, we have an energy which is the inertial energy again:

$$\frac{\bar{g}^2\sqrt{N_\nu}}{R} \approx m_\nu c^2 \tag{6.91}$$

where $\bar{g}^2$ gives the weak interaction coupling constant.
Interestingly we saw a similar relation for the electrons

$$\frac{e^2\sqrt{N}}{R} = mc^2 \tag{6.92}$$

From (6.91) and (6.92) on using (6.90) we get

$$\bar{g}^2/e^2 \sim 10^{-13} \tag{6.93}$$

which ofcourse is again known to be correct.
We have thus recovered from theory the well known values of the weak coupling constant and the neutrino mass. We would next like to show that there is a complete parallel between the Large Number Relations for elementary particles with similar relations for the neutrino. We start with the simplest relation, which can be easily verified

$$N_\nu m_\nu = Nm = M = 10^{55} gm,$$

M being the mass of the Universe. We next return to the fact used above in (6.89) and consider the equality of the gravitational mass of a particle due to the remaining n particles with the inertial mass of the particle

$$\frac{Gnm^2}{r} = mc^2 \tag{6.94}$$

In (6.94), if n is replaced by N and r is replaced by the radius of the Universe, we get the mass of an elementary particle like the pion. On the other hand if in (6.94) we replace n by the number of neutrinos N_ν instead of N then we recover the mass of the neutrino. Finally if we take $n = 1$ and $r = l_P$, the Planck scale we recover the Planck mass m_P, which indeed is to be expected because as Rosen had shown and we saw earlier, the Planck mass Black Hole is a Universe in itself [47].
Similarly we see the complete parallel between (6.91) and (6.92). To proceed further we consider (6.10) in an alternative form viz.,

$$\hbar = \frac{Gm^2\sqrt{N}}{c} \tag{6.95}$$

For the neutrino number and neutrino mass given in (6.90), (6.95) gives

$$\hbar' = \frac{Gm_\nu^2\sqrt{N_\nu}}{c} = 10^{-12}\hbar \tag{6.96}$$

(6.96) shows that the magnetic moment of the neutrino is given by

$$\mu_\nu \sim 10^{-11}\,\text{Bohr} \quad \text{magnetons} \tag{6.97}$$

Indeed (6.97) is consistent with observation [280]. That is for the neutrino we have effectively $\hbar'$ given by (6.96), instead of $\hbar$. It is then simple to verify that the analogue of the Eddington formula (6.9) applies for the neutrinos viz.,

$$R = \sqrt{N_\nu} l_\nu,$$

where $l_\nu = \frac{\hbar'}{m_\nu c}$, the neutrino analogue of the Compton length.
It has been shown on the basis of Black Hole radiation life times that we have

$$\frac{Gm^2}{l} = \frac{\hbar}{T}, T = 10^{17} sec \tag{6.98}$$

where T is the life time of the Universe (Cf. also [33]). Indeed as we saw (6.98) is just a variant of the Weinberg formula, and can now be interpreted as the fact that the gravitational self energy of the elementary particle, viz., $\frac{Gm^2}{l}$ has a life time of the order of the age of the Universe, due to the Uncertainty Principle. It can immediately be verified that for the neutrino we have the equation

$$\frac{Gm_\nu^2}{l_\nu} = \frac{\hbar'}{T} \tag{6.99}$$

In the author's model, it has been shown that [24] the pion can be considered to be an electron positron bound state so that we have

$$l = \frac{e^2}{m_e c^2} \tag{6.100}$$

where l is the pion Compton wavelength. Similarly one could consider the pion to also be the bound state of a quark anti-quark in QCD so that we have

$$\frac{g^2}{m_q c^2} = l \tag{6.101}$$

where m_q is the quark mass and g^2 is the strong interaction coupling constant. There is an immediate analogue of (6.100) and (6.101) for the neutrino viz.,

$$l_\nu = \frac{\bar{g}^2}{m_\nu c^2} \tag{6.102}$$

Finally it may be pointed out that there is an immediate analogue of the Weinberg formula (6.7) viz.,

$$m_\nu = \left(\frac{H\hbar'^2}{Gc} \right)^{1/3} \tag{6.103}$$

It must be mentioned that these analogues like (6.91), (6.96), (6.99), (6.102) and (6.103) between the neutrino and an elementary particle are not mere numerical coincidences. This is because the various relations for the elementary particles are the result of a theoretical structure, and are not mere accidents. What the foregoing means is that the neutrino has a similar theoretical structure.

To see this in greater detail, we note that in the case of the Planck scale underpinning for the Universe of elementary particles, as was discussed earlier in the previous Chapter, we have,

$$r = \sqrt{N \Delta x^2}$$

$$k l_P^2 \equiv k \Delta x^2 = \frac{1}{2} k_B T \tag{6.104}$$

where k_B is the Boltzmann constant, T the temperature, r the extent and k which resembles the spring constant is given by

$$\omega_0^2 = \frac{k}{m}$$

where ω_0 is the frequency of a Planck mass viz.,

$$\frac{m_P c^2}{\hbar}$$

In the case of elementary particles it was shown that with $r \sim l$ the pion Compton wavelength we get

$$k_B T = \frac{m^3 c^4 l^2}{\hbar^2} = mc^2, \tag{6.105}$$

This as noted agrees with the Hagedorn temperature for elementary particles. For the neutrino a similar argument using the above equations including (6.104) gives, with the neutrino parameters m_ν, l_ν and $\hbar'$ substituted in (6.105),

$$k_B T = m_\nu c^2 \tag{6.106}$$

Equation (6.106) gives for the neutrino mass

$$T \sim 1^\circ K \tag{6.107}$$

which corresponds to the "cold" cosmic background temperature. This is completely consistent with our starting point in (6.89), where we consider the Fermi energy of the "cold" cosmic neutrinos. Infact the Fermi energy term in (6.89) (or the temperature (6.107)) is the only difference between elementary particles and neutrinos - this is what leads to different values for m_ν, N_ν etc. as compared to m, N etc.

We have seen that the weak interactions given by the coupling constant in (6.93) is a parallel of the electromagnetic interaction. Ofcourse in the standard electroweak theory [85] the neutrino mass is taken to be zero. However after the SuperKamiokande experiments, it has been realised that some modification in the standard model is required. We have seen above that it is the "cold" cosmic background or equivalently the Fermi energy which gives the neutrino its mass on the one hand and the weak interaction on the other.

Further as can be seen from (6.91) and (6.92) the origin of the weak and the electromagnetic interaction is the same viz., the fluctuation in particle number in the Universe.
If we observe the parallel in the equations (6.100), (6.101) and (6.102), we can interpret (6.102) as describing a bound state of two neutrinos. The result is a particle of Compton wavelength l_ν, that is a heavy particle of mass $10^4 m$. Such a particle would ofcourse be very shortlived. Indeed particles of this order of mass, for example the $*\gamma$ resonances are known [281].
Finally it must be noted that the much smaller mass of the neutrino - approximately a vanishing mass - causes the four component Dirac electron equation to split into two component neutrino equations, as in standard theory, and thus gives the neutrino its handedness.
In summary we have shown that the neutrino can be described as a "cold" (old) electron.

6.6 A Final Comment

We have alluded to relations like (6.4), (6.7),(6.8), (6.9) and (6.10), the so called Large Number relations. In all these cases it turns out that T, the age of the Universe is proportional to a suitable power of N the number of particles in the Universe. Rather than dismiss these relations as mere coincidences, Dirac suspected that these pointed to a relationship with time. In his words [282]: "I call this principle the

Large Numbers Hypothesis

According to it, all the very large dimensionless numbers, which turn up in Nature, are related to one another, just like $t = 7 \times 10^{39}$ and e^2/Gm_em_p.
There is one further very large dimensionless number which we have to take into consideration. That is the total mass of the Universe when expressed in units of, say, the proton mass. That will be, if you like, the total number of protons and neutrons in the Universe. It may be, of course, that the Universe is infinite and that, therefore, this total number is infinite. In that case we should not be able to talk about it. Yet we can use another number to replace it. We need only consider that portion of the Universe which is sufficiently close to us for the velocity of recession to be less than, let us say, half the velocity of light. We are then considering just a certain chunk of this infinite Universe, for which recession velocities are less than half the velocity of light. We then ask, what is the total mass of this chunk of the Universe? That again will be a very Large Number and will replace the total mass of the Universe, to give us a definite number when the Universe is infinite.
We may try to estimate this total mass using the mass of those stellar objects which we can observe, and making an allowance for unobservable matter. We do not know very well how big that allowance should be: there may be quite

a lot of unobservable matter in the form of intergalactic gas or Black Holes or things like that. Still, it is probable that the amount of Dark Matter is not very much greater than the amount of visible matter. If you make an assumption of that kind, you find that the total mass, in terms of the proton mass, is

$$\frac{\text{total mass}}{\text{proton mass}} = 10^{78},$$

with a suitable factor allowed for the invisible matter. We, therefore get a number which is, roughly, the square of t (in atomic units).

Now, according to the Large Number Hypothesis, all these very large dimensionless numbers should be connected together. We should then expect that

$$\frac{\text{total mass}}{\text{proton mass}} = 10^{78} :: t^2,$$

Using the same argument again, we are therefore led to think that the total number of protons in the Universe is increasing proportionally to t^2. Thus, there must be creation of matter in the Universe, a continuous creation of matter.

There have been quite a number of cosmological theories working with continuous creation of matter. A theory like that was very much developed by Hoyle and others. The continuous creation which I am proposing here is entirely different from that. Their continuous creation theory was introduced as a rival to the Big Bang theory, and it is not in favor at the present time. The continuous creation which I have here is essentially different from Hoyle's continuous creation, because Hoyle was proposing a steady state of the Universe, with continuous creation to make up for the matter which is moving beyond our region of vision by the expansion. In his steady-state theory, he had G constant. Now, in the present theory, G is varying with time, and that makes an essential difference.

I propose a theory where there is continuous creation of matter, together with this variation of G. Both the assumption of continuous creation and the variation of G follow from the Large Number Hypothesis.

This continuous creation of matter must be looked upon as something quite independent of known physical processes. According to the ordinary physical processes, which we study in the laboratory, matter is conserved. Here we have direct nonconservation of matter. It is, if you like, a new kind of radioactive process for which there is nonconservation of matter and by which particles are created where they did not previously exist. The effect is very small, because the number of particles created will be appreciable only when we wait for a very long time interval compared with the age of the Universe."

There was an inconsistency in this Dirac cosmology, namely the relation,

$$R \propto T^{1/3}.$$

He vacillated over the decades between versions using the conservation of energy and also violating it.

In our cosmology, using fluctuations, all these apparently disparate relations are derived from underlying principles, not to mention the prediction of a dark energy driven accelerating Universe with a permissible cosmological constant. That is what science is all about - finding a minimum set of principles to explain a maximal set of observations.

7 QUANTUM GEOMETRY

"The world is a $3+1$ dimensional metrical manifold; all physical field-phenomena are expressions of the metrics of the world"

H. Weyl

7.1 Introduction

We have already seen in Chapter 5 that, based on a discrete spacetime non-commutative geometrical approach, it was possible to reconcile electromagnetism and gravitation [161]. It is of course well known that nearly ninety years of effort has gone in to get a unified description of electromagnetism and gravitation starting with Hermann Weyl's original Gauge Theory. It is only in the recent years as noted that approaches in Quantum Gravity and Quantum Superstrings, amongst a few other theories are pointing the way to a reconciliation of these two forces. These latest theories discard the differentiable spacetime of earlier approaches and rely on a lattice like approach to spacetime, wherein there is a minimum fundamental interval which replaces the point spacetime of earlier theories. Indeed to again quote 't Hooft, "It is some what puzzling to the present author why the lattice structure of space and time had escaped attention from other investigators up till now...." [24, 151, 152] Infact we will see that within this approach, it is possible to get a rationale for the De Broglie wavelength and the mysterious Bohr-Sommerfeld quantization relations as well [163]. Nevertheless, the link with the gauge theories of other interactions, based as they are, on spin 1 particles, is not clear, because the graviton is a spin 2 particle (or alternatively, the gravitational metric is a tensor).

7.2 A Gauge like Formulation

In this latter context, we will now argue that it is possible for both electromagnetism and gravitation to emerge from a gauge like formulation [283]. In Gauge Theory, which as we saw in Chapter 2, is a Quantum Mechanical generalization of Weyl's original geometry, we generalize, as is well known, the original phase transformations, which are global with the phase λ being a constant, to local phase transformations with λ being a function of the coordinates [284]. As is well known this leads to a covariant gauge derivative. For example, the transformation arising from $(x^\mu) \to (x^\mu + dx^\mu)$,

$$\psi \to \psi e^{-\imath \lambda} \tag{7.1}$$

leads to the familiar electromagnetic potential gauge,

$$A_\mu \to A_\mu - \partial_\mu \lambda \tag{7.2}$$

The above transformation, ofcourse, is a symmetry transformation. In the transition from (7.1) to (7.2), we expand the exponential, retaining terms only to the first order in coordinate differentials.
Let us now consider the case where there is a minimum cut off in the spactime intervals. As we saw this leads to a noncommutative geometry (Cf.ref.[161])

$$[dx_\mu, dx_\nu] = O(l^2) \tag{7.3}$$

where l is the minimum scale. From (7.3) it can be seen that if $O(l^2)$ is neglected, we are back with the familiar commutative spacetime. The new effects of fuzzy spacetime arise when the right side of (7.3) is not neglected. Based on this we had argued in Chapter 5 that it is possible to reconcile electromagnetism and gravitation [162, 71, 285, 187]. If in the transition from (7.1) to (7.2) we retain, in view of (7.3), squares of differentials, in the expansion of the function λ we will get terms like

$$\{\partial_\mu \lambda\} dx^\mu + (\partial_\mu \partial_\nu + \partial_\nu \partial_\mu) \lambda \cdot dx^\mu dx^\nu \tag{7.4}$$

where we should remember that in view of (7.3), the derivatives (or the product of coordinate differentials) do not commute as indeed we saw in Chapter 5. As in the usual theory the coefficient of dx^μ in the first term of (7.4) represents now, not the gauge term but the electromagnetic potential itself: Infact, in this noncommutative geometry, it can be shown that this electromagnetic potential reduces to the potential in Weyl's original gauge theory [283, 162]. Without the noncommutativity, the potential $\partial_\mu \lambda$ would lead to a vanishing electromagnetic field. However as we saw Dirac pointed out in his famous monopole paper in 1930 that a non integrable phase $\lambda(x, y, z)$ leads as above directly to the electromagnetic potential, and moreover this was an alternative formulation of the original Weyl theory [195, 194].
Returning to (7.4) we identify the next coefficient with the metric tensor giving the gravitational field:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (\partial_\mu \partial_\nu + \partial_\nu \partial_\mu) \lambda dx^\mu dx^\nu \tag{7.5}$$

Infact one can easily verify that ds^2 of (7.5) is an invariant. We now specialize to the case of the linear theory in which squares and higher powers of the deviation from the Minkowski metric, $h^{\alpha\beta}$ can be neglected. In this case it can easily be shown that

$$2\Gamma^{\beta}_{\mu\nu} = h_{\beta\mu,\nu} + h_{\nu\beta,\mu} - h_{\mu\nu,\beta} \tag{7.6}$$

where in (7.6), the Γs denote Christofell symbols. From (7.6) by a contraction we have

$$2\Gamma^{\mu}_{\mu\nu} = h_{\mu\nu,\mu} = h_{\mu\mu,\nu} \tag{7.7}$$

If we use the well known gauge condition [5]

$$\partial_{\mu}\left(h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h_{\mu\nu}\right) = 0, \text{ where } h = h^{\mu}_{\mu}$$

then we get

$$\partial_{\mu}h_{\mu\nu} = \partial_{\nu}h^{\mu}_{\mu} \equiv \partial_{\nu}h \tag{7.8}$$

(7.8) shows that we can take the λ in (7.4) as $\lambda = h$, both for the electromagnetic potential A_{μ} and the metric tensor $h_{\mu\nu}$. (7.7) further shows that the A_{μ} so defined becomes identical to Weyl's gauge invariant potential [6]. However it is worth reiterating that in the present formulation, we have a noncommutative geometry, that is the derivatives do not commute and moreover we are working to the order where l^2 cannot be neglected. Given this condition both the electromagnetic potential and the gravitational potential are seen to follow from the gauge like theory. By retaining coordinate differential squares, we are even able to accommodate apart from the usual spin 1 gauge particles, also the spin 2 graviton which otherwise cannot be accommodated in the usual gauge theory. If however $O(l^2) = 0$, then we are back with commutative spacetime, that is a usual point spacetime and the usual gauge theory describing spin 1 particles.

We had reached this conclusion in Chapter 5 (Cf. ref. [161]), though from a different, non gauge point of view. The advantage of the present formulation is that it provides a transparent link with conventional theory on the one hand, and shows how the other interactions described by non Abelian gauge theories smoothly fit into the picture.

Finally it may be pointed out that we had already argued that a fuzzy spacetime input explains why the purely classical Kerr-Newman metric gives the purely Quantum Mechanical anomalous gyromagnetic ratio of the electron [67, 154], thus providing a link between General Relativity and electromagnetism. This provides further support to the above considerations.

7.3 Gauge Fields

Let us now return to the gauge field itself. As is well known, this could be obtained as a generalization of the above phase function λ to include fields with internal degrees of freedom. For example λ could be replaced by A_μ given by [93]

$$A_\mu = \sum_\imath A^\imath_\mu (x) L_\imath , \tag{7.9}$$

The gauge field itself would be obtained by using Stoke's Theorem and (7.9). This is a very well known procedure: considering a circuit, which for simplicity we can take to be a parallelogram of side dx and dy in two dimensions, we can easily deduce the equation for the field, viz.,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \imath q[A_\mu , A_\nu], \tag{7.10}$$

q being the gauge field coupling constant.
In (7.10), the second term on the right side is typical of a non Abelian gauge field. In the case of the(U(1) electromagnetic field, this latter term vanishes. Further as is well known, in a typical Lagrangian like

$$L = \imath\bar{\psi}\gamma^\mu D_\mu\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - m\bar{\psi}\psi \tag{7.11}$$

D denoting the Gauge covariant derivative, there is no mass term for the field Bosons. Such a mass term in (7.11) must have the form $m^2 A^\mu A_\mu$ which unfortunately is not Gauge invariant.
This as we saw in Chapter 2, was the shortcoming of the original Yang-Mills Gauge Theory: The Gauge Bosons would be massless and hence the need for a symmetry breaking, mass generating mechanism.
The well known remedy for the above situation has been to consider, in analogy with superconductivity theory, an extra phase of a self coherent system (Cf.ref.[93] for a simple and elegant treatment and also refs. [284] and [85]). Thus instead of the gauge field A_μ, we consider a new phase adjusted gauge field after the symmetry is broken

$$W_\mu = A_\mu - \frac{1}{q}\partial_\mu\phi \tag{7.12}$$

The field W_μ now generates the mass in a self consistent manner via a Higgs mechanism. Infact the kinetic energy term

$$\frac{1}{2}|D_\mu\phi|^2 \quad , \tag{7.13}$$

where D_μ in (7.13) denotes the Gauge , now becomes

$$|D_\mu\phi_0|^2 = q^2|W_\mu|^2|\phi_0|^2 \, , \tag{7.14}$$

Equation (7.14) gives the mass in terms of the ground state ϕ_0.
The whole point is as follows: The symmetry breaking of the gauge field manifests itself only at short length scales signifying the fact that the field is mediated by particles with large mass. Further the internal symmetry space

of the gauge field is broken by an external constraint: the wave function has an intrinsic relative phase factor which is a different function of spacetime coordinates compared to the phase change necessitated by the minimum coupling requirement for a free particle with the gauge potential. This cannot be achieved for an ordinary point like particle, but a new type of a physical system, like the self coherent system of superconductivity theory now interacts with the gauge field. The second or extra term in (7.12) is effectively an external field, though (7.14) manifests itself only in a relatively small spatial interval. The ϕ of the Higgs field in (7.12), in analogy with the phase function of Cooper pairs of superconductivity theory comes with a Landau-Ginzburg potential $V(\phi)$.

Let us now consider in the gauge field transformation, an additional phase term, $f(x)$, this being a scalar. In the usual theory such a term can always be gauged away in the U(1) electromagnetic group. However we now consider the new situation of a noncommutative geometry referred to above,

$$[dx^\mu, dx^\nu] = \Theta^{\mu\nu}\beta, \beta \sim 0(l^2) \tag{7.15}$$

where l denotes the minimum spacetime cut off. Equation (7.15) is infact Lorentz covariant. Then the f phase factor gives a contribution to the second order in coordinate differentials,

$$\frac{1}{2}\left[\partial_\mu B_\nu - \partial_\nu B_\mu\right]\left[dx^\mu, dx^\nu\right]$$
$$+\frac{1}{2}\left[\partial_\mu B_\nu + \partial_\nu B_\mu\right]\left[dx^\mu dx^\nu + dx^\nu dx^\mu\right] \tag{7.16}$$

where $B_\mu \equiv \partial_\mu f$.

As can be seen from (7.16) and (7.15), the new contribution is in the term which contains the commutator of the coordinate differentials, and not in the symmetric second term. Effectively, remembering that B_μ arises from the scalar phase factor, and not from the non-Abelian gauge field, in equation (7.10) A_μ is replaced by

$$A_\mu \to A_\mu + B_\mu = A_\mu + \partial_\mu f \tag{7.17}$$

Comparing (7.17) with (7.12) we can immediately see that the effect of noncommutativity is precisely that of providing a new symmetry breaking term to the gauge field, instead of the ϕ term, (Cf.refs. [242, 286]) a term not belonging to the gauge field itself.

On the other hand if we neglect in (7.15) terms $\sim l^2$, then there is no extra contribution coming from (7.16) or (7.17), so that we are in the usual non-Abelian gauge field theory, requiring a broken symmetry to obtain an equation like (7.17). This is not surprising because as noted several times if we neglect the term $\sim l^2$ in (7.15) then we are back with the usual commutative theory and the usual Quantum Mechanics.

7.4 Quantum Geometry

As we saw in Chapter 5, one of the earliest attempts to unify electromagnetism and gravitation, was Weyl's gauge invariant geometry. The basic idea was that while

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{7.18}$$

was invariant under arbitrary transformations in General Relativity, a further invariant, namely,

$$\Phi_\mu dx^\mu \tag{7.19}$$

which is a linear form should be introduced. $g_{\mu\nu}$ in (7.18) would represent the gravitational potential, and Φ_μ of (7.19) would represent the electromagnetic field potential.
A more modern treatment was considered (Cf. Chapter 5): the above arbitrary multiplying factor is normalized and we require that,

$$|g_{\mu\nu}| = -1, \tag{7.20}$$

For the invariance of (7.20), $g_{\mu\nu}$ transforms now as a tensor density of weight minus half, rather than as a tensor in the usual theory. The covariant derivative then had to be redefined. This finally lead to

$$\Phi_\sigma = \Gamma^\rho_{\rho\sigma}, \tag{7.21}$$

Φ_μ in (7.21) is identified with the electromagnetic potential, while $g_{\mu\nu}$ gives the gravitational potential as in the usual theory.
Unfortunately, as we saw $g_{\mu\nu}$ and Φ_μ are independent entities.
We then analyzed the above from a different perspective considering the noncommutative geometry (7.3) or (7.15),

$$[dx^\mu, dx^\nu] \approx l^2 \neq 0$$

l being some fundamental minimum length. In this case as we saw, the antisymmetric part of $g_{\mu\nu}$ that is $h_{\mu\nu}$ under reflection, behaved as $h_{\mu\nu} \to -h_{\mu\nu}$ as in the case of the tensor density metric tensor above.
We then argued that (7.15) lead to the Dirac equation - that is, the noncommutative geometry manifests itself as, in the usual commutative formation, the Dirac spinor:

$$\psi = \begin{pmatrix} \chi \\ \Theta \end{pmatrix}, \tag{7.22}$$

where χ and Θ are two spinors. Under reflection while the so called positive energy spinor Θ in (7.22) behaves normally, $\chi \to -\chi, \chi$ being the so called negative energy spinor which we encountered at the Compton scale. So there is now a covariant derivative near the Compton scale (in natural units),

$$\frac{\partial \chi}{\partial x^\mu} \to \left[\frac{\partial}{\partial x^\mu} - nA^\mu\right] \chi \tag{7.23}$$

where, in (7.23),

$$A^\mu = \Gamma^{\mu\sigma}_\sigma = \frac{\partial}{\partial x^\mu} \log(\sqrt{|g|}) \tag{7.24}$$

Γ denoting the Christofell symbols.
A^μ in (7.24) is now identified with the electromagnetic potential and we recover (7.21).
That is the so called ad hoc feature in Weyl's unification theory is really symptomatic of the underlying noncommutative spacetime geometry (7.15) or the double connectivity of space implied in (7.22). Given (7.15) we get both gravitation and electromagnetism in a unified picture.
Let us now consider the above ideas in the context of the De Broglie-Bohm formulation [163]. We start with the Schrödinger equation

$$\imath\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \tag{7.25}$$

In (7.25), the substitution

$$\psi = Re^{\imath S/\hbar} \tag{7.26}$$

where R and S are real functions of $\boldsymbol{r}$ and t, leads to,

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0 \tag{7.27}$$

$$\frac{1}{\hbar}\frac{\partial S}{\partial t} + \frac{1}{2m}(\boldsymbol{\nabla} S)^2 + \frac{V}{\hbar^2} - \frac{1}{2m}\frac{\nabla^2 R}{R} = 0 \tag{7.28}$$

where

$$\rho = R^2, \boldsymbol{v} = \frac{\hbar}{m}\boldsymbol{\nabla} S$$

and

$$Q \equiv -\frac{\hbar^2}{2m}(\nabla^2 R/R) \tag{7.29}$$

Using the theory of fluid flow, it is well known that (7.27) and (7.28) lead to the Bohm alternative formulation of Quantum Mechanics (Cf.refs.[287, 198] for a simple treatment). In this theory there is a hidden variable namely the definite value of position while the so called Bohm potential Q in (7.29) can be non local, two features which do not find favour with physicists. (In our formulation however, the definite value of the position coordinate is fudged by the fuzzyness of spacetime.)
It must be noted that in Weyl's geometry, even in a Euclidean space there is a covariant derivative and a non vanishing curvature R.
Santamato (Cf.refs. [288, 289, 290, 291]) exploits this latter fact, within the context of the De Broglie-Bohm theory and postulates a Lagrangian given by

$$L(q, \dot{q}, t) = L_c(q, \dot{q}, t) + \gamma(\hbar^2/m)R(q, t),$$

He then goes on to obtain the equations of motion like (7.25),(7.26), etc. by invoking an Averaged Least Action Principle

$$I(t_0,t_1) = E\left\{\int_{t_0}^{t} L^*(q(t,\omega),\dot{q}(t,\omega),t)dt\right\}$$
$$= \text{minimum} \tag{7.30}$$

with respect to the class of all Weyl geometries of space with fixed metric tensor. Equation (7.30) now leads to the Hamilton-Jacobi equation

$$\partial_t S + H_c(q,\nabla S,t) - \gamma(\hbar^2/m)R = 0, \tag{7.31}$$

Equation (7.31) leads to the Schrödinger equation (in curvilinear coordinates)

$$\imath\hbar\partial_l\psi = (1/2m)\left\{\left[(\imath\hbar/\sqrt{g})\,\partial_\imath\sqrt{g}A_\imath\right]g^{\imath k}\left(\imath\hbar\partial_k + A_k\right)\right\}\psi$$
$$+\left[V - \gamma\left(\hbar^2/m\right)\dot{R}\right]\psi = 0, \tag{7.32}$$

As can be seen from (7.32), the Quantum potential Q is now given in terms of the scalar curvature R.
We have already related the arbitrary functions Φ of Weyl's formulation with a noncommutative spacetime geometry (7.15).
This throws further light on Santamato's postulative approach of extending the De Broglie-Bohm formulation.
At an even more fundamental level, our formalism gives us the rationale for the De Broglie wave length itself. Because of the noncommutative geometry in (7.15) space becomes multiply connected, in the sense that a closed circuit cannot be shrunk to a point within the interval. Let us consider the simplest case of double connectivity. In this case, if the interval is of length L, we will have,

$$\Gamma = \int_c m\boldsymbol{V}\cdot d\boldsymbol{r} = h\int_c \boldsymbol{\nabla}S\cdot d\boldsymbol{r} = h\oint dS = mV\pi L = \pi h \tag{7.33}$$

whence

$$L = \frac{h}{mV} \tag{7.34}$$

We had encountered equations like (7.33) earlier in Chapter 5, but in the narrower context of monopoles. In (7.33), the circuit integral was over a circle of diameter L. Equation (7.34) shows the emergence of the De Broglie wavelength. This follows from the noncommutative geometry of spacetime, rather than the physical Heisenberg Uncertainty Principle. Remembering that Γ in (7.33) stands for the angular momentum, this is also the origin of the Wilson-Sommerfeld quantization rule, an otherwise mysterious Quantum Mechanical prescription.
What we have done is to develop a Quantum Geometrical picture, based on

(7.3) or (7.15).
We finally remark that as seen above and also in Chapter 5 the double connectivity of space gives the Quantum Mechanical spin, while the non integrability of the phase gives the electromagnetic field of the particle (Cf. also [195]). Lastly the energy within this region with radius given by the Compton wavelength, viz.

$$\int \rho c^2 d\Omega = mc^2,$$

that is we get the mass, as well (Cf. also ref.[24]).
In other words, the considerations of fuzzy spacetime or equivalently the Kerr-Newman metric seen briefly in Chapter 1, yield at the Compton scale , the mass, spin and electromagnetic field of the elementary particle.

8 HOW FUNDAMENTAL IS GRAVITATION?

"The existence of gravity clashes with our description of the rest of physics by quantum fields"

Edward Witten

8.1 Introduction

More than five thousand years ago, the Rig Veda repeatedly raised the question: "How is it that *though unbound* the sun does not fall down?"
This was a question that puzzled thinking man over the millennia. Indian scholars right up to Bhaskaracharya who lived about a thousand years ago believed in some attractive force which was responsible for keeping the celestial bodies from falling down.
The same problem was addressed by Greek thinkers about two thousand five hundred years ago. They devised transparent material spheres to which each of the celestial objects was attached - the material spheres prevented them from falling down. Further, all motions were circular, for, the Greeks believed that circles and spheres were perfect figures.
Unfortunately it was this answer to the age old question, which held up further scientific progress till the time of Kepler, for even Copernicus accepted the transparent material spheres.
Kepler had a powerful tool in the form of the accurate observations of Tycho Brahe. He also had the advantage of the Indian numeral system, which via the Arabs reached Europe just a few centuries earlier. These lead him to his famous laws of elliptical orbits with definite periods correlated to distances from the Sun.
This couching of natural phenomena in the terse language of mathematical symbols that could be manipulated, was the beginning of modern science.
The important point was that the Greek answer to the problem of why heavenly objects do not fall down - the transparent material spheres - was now demolished. The age old question of why celestial bodies do not fall down came back to haunt again. Kepler himself speculated about some type of a magnetic force between the Sun and the Planets, rather on the lines of earlier speculations in India.

It was Newton who provided the breakthrough.
To quote Hawking [300],"*The Philosophiae Naturalis Principia Mathematica* by Isaac Newton, first published in Latin in 1687, is probably the most important single work ever published in the physical sciences. Its significance is equalled in the biological sciences only by *The Origin of Species*. The original impulse which caused Newton to write the *Principia* was a question from Edmund Halley as to whether the elliptical orbits of the planets could be accounted for on the hypothesis of an inverse square force directed towards the Sun. This was something that Newton had worked out some years earlier but had not published, like most of his work on mathematics and physics. However, Halley's challenge, and the desire to refute the suggestions of others such as Hooke and Descartes, spurred Newton to try to write a proper account of this result."
Newton using Galileo's ideas of Mechanics, thus stumbled upon the Universal Law of Gravitation.
This held sway for nearly two hundred and twenty five years, before Einstein came out with his own theory of gravitation. There was no force in the mechanical sense that Newton and preceding scholars had envisaged it to be. Rather it was due to the curvature of spacetime itself. Einstein's bizarre ideas have had some experimental verification while there are some other experimental consequences, such as gravitational waves, which need to be confirmed.
After Einstein's formulation of gravitation a problem that has challenged and defied solution has been that of providing a unified description of gravitation along with other fundamental interactions. Infact Einstein spent the last decades of his life in this fruitless quest. As he said [107] "I have become a lonely chap who is mainly known because he doesn't wear socks and who is exhibited as a curiosity on special occasions."
One of the earliest attempts was as seen earlier that of Hermann Weyl, which though elegant was rejected on the grounds that in the final analysis, it was not really a unification of gravitation with electromagnetism but rather an adhoc prescription.
Modern approaches to this problem have as discussed, finally lead to the abandonment of a smooth spacetime manifold. Instead, the Planck scale is now taken to be a minimum fundamental scale. This has been discussed in Chapters 5 and 6.
We had argued from different points of view to arrive at the otherwise empirically known equations [187, 249]

$$R = \sqrt{\bar{N}} l_P = \sqrt{N} l$$
$$l = \sqrt{n} l_P \tag{8.1}$$

where l_P, l and R are the Planck length, the pion Compton wavelength and the radius of the Universe and $\bar{N}, N$ and n are certain Large Numbers. Some of these are well known empirically for example $N \sim 10^{80}$ being the number

of elementary particles, which typically are taken to be pions in the literature, in the Universe.
One way of arriving at the above relations while not assuming any values for n, N and $\bar{N}$ a priori is by considering a series of N Planck mass oscillators which are created out of the Quantum Vacuum. In this case (Cf. also ref.[301]) we have

$$r = \sqrt{Na^2} \tag{8.2}$$

In (8.2) a is the distance between the oscillators and r is the extension and, N is as yet unspecified. Equations (8.1) follow from equation (8.2).
There is another way of arriving at equations (8.1) (Cf.ref.[168]). For this, we observe that the position operator for the Klein-Gordan equation is given by [60],

$$\boldsymbol{X}_{op} = \boldsymbol{x}_{op} - \frac{\imath\hbar c^2}{2} \frac{\boldsymbol{p}}{E^2}$$

Whence we get

$$\hat{X}^2_{op} \equiv \frac{2m^3c^4}{\hbar^2} X^2_{op} = \frac{2m^3c^6}{\hbar^2} x^2 + \frac{p^2}{2m} \tag{8.3}$$

It can be seen that purely mathematically (8.3) for $\hat{X}^2_{op}$ defines the Harmonic oscillator equation, this time with quantized, what may be called space levels. It turns out that these levels are all multiples of $(\frac{\hbar}{mc})^2$. This Compton length is the Planck length for a Planck mass. Accordingly we have for any system of extension r,

$$r^2 \sim Nl^2$$

which gives back equation (8.1). It is also known that the Planck length is the Schwarzchild radius of a Planck mass, that is we have

$$l_P = Gm_P/c^2 \tag{8.4}$$

Using equations (8.1) and (8.4), we will now deduce afresh a few new and valid and a number of otherwise empirically known relations involving the various microphysical parameters and large scale parameters, relations which we encounter particularly in Chapters 5 and 6. Some of these relations are deducible from the others. Many of these relations featured (empirically) in Dirac's Large Number Cosmology. We follow Dirac and Melnikov in considering $l, m, \hbar, l_P, m_P$ and e as microphysical parameters [80, 172] but in a departure leaving out G from the list. Large scale parameters include the radius and the mass of the Universe, the number of elementary particles in the Universe, the Hubble consent and so on.
All this will enable us to reexamine the nature of gravitation. It must also be observed again that the Large Number relations below are to be considered in the Dirac sense, wherein for example the difference between the electron and pion (or proton) masses is irrelevant [28].

8.2 Interrelationships

We will use the following well known equation which has been obtained through several routes:

$$\frac{GM}{c^2} = R \tag{8.5}$$

For example in an uniformly expanding flat Friedman spacetime, we have [28]

$$\dot{R}^2 = \frac{8\pi G\rho R^2}{3}$$

If we substitute $\dot{R} = c$ at the radius of the Universe in the above we recover (8.5).
We now observe that from the first two relations of (8.1), using the Compton wavelength expression we get

$$m = m_P/\sqrt{n} \tag{8.6}$$

This was equation (6.74) of Chapter 6. Using also the second relation in (8.1) we can easily deduce

$$\bar{N} = Nn \tag{8.7}$$

Using (8.1) and (8.5) we have

$$M = \sqrt{\bar{N}}m_P \tag{8.8}$$

Interestingly (8.8) can be obtained directly, without recourse to gravitation or (8.5), from the energy of the Planck oscillators as we saw in Chapter 5 (Cf.ref.[294]). Combining (8.8) and (8.6) we get

$$M = \left(\sqrt{\bar{N}n}\right) m \tag{8.9}$$

Further if we use in the last of equation (8.1) the fact that l_P is the Schwarzchild radius that is equation (8.4), we get,

$$G = \frac{lc^2}{nm} \tag{8.10}$$

We now observe that if we consider the gravitational energy of the $\bar{N}$ Planck oscillators (which do not have any other interactions) we get,

$$\text{Gravitational Energy} = \frac{G\bar{N}m_P^2}{R} \tag{8.11}$$

If this is equated to the inertial energy in the Universe, Mc^2, as can be easily verified we get back (8.5). In other words the energy of the $\bar{N}$ Planck oscillators underlying the Universe, that is, its inertial energy content equals

the gravitational energy of all the $\bar{N}$ Planck oscillators.
Similarly if we equate the gravitational energy of the n Planck oscillators constituting the pion we get

$$\frac{Gm_P^2 n}{R} = mc^2 \tag{8.12}$$

Using in (8.12) equation (8.4) we get

$$\frac{l_P m_P n}{R} = m$$

Whence it follows on using (8.7), (8.6) and (8.1),

$$n^{3/2} = \sqrt{N}, \, n = \sqrt{\bar{N}} \tag{8.13}$$

Substituting the value for n from (8.13) into (8.10) we will get

$$G = \frac{lc^2}{\sqrt{N}m} \tag{8.14}$$

If we use (8.13) in (8.9) we will get

$$M = Nm, \tag{8.15}$$

a relation we encountered in Chapter 6. Alternatively we could use (8.15) which expresses the fact that the mass of the Universe is given by the mass of the N elementary particles in it and deduce equations (8.12), (8.13) and (8.14). Using the expressions for the Planck length as a Compton wavelength and equating it to (8.4) we can easily deduce

$$Gm^2 = \frac{e^2}{n} = \frac{e^2}{\sqrt{\bar{N}}} \tag{8.16}$$

wherein we have also used $\hbar c \sim e^2$ and (8.6). Equation (8.16) is the empirically well known equation which was used by Dirac in his Cosmology and which we deduced in Chapter 6. Interestingly, as we have deduced (8.16), rather than use it empirically, this points to a unified description of electromagnetism and gravitation as noted earlier.
Before proceeding further we make the following observations: Equation (8.9) gives the inertial mass (or energy) of the Universe purely in terms of an underpinning of Planck oscillators, without any reference to gravitation. In essence, we have equated this to the energy of gravitation of the constituent elementary particles in the Universe. Explicitly, we have,

$$\sqrt{\bar{N}} nmc^2 = \frac{G\bar{N}m_P^2}{R}$$

Using some of the preceding relations, this gives

$$G = \frac{lc^2}{nm},$$

which is (8.10) (or (8.14)). Further, the universal underpinning of $\bar{N}$ oscillators manifests itself as N elementary particles, each with an underpinning of n oscillators, interacting amongst themselves via the gravitational interaction.

Interestingly also rewriting (8.14) as

$$G = \frac{l^2 c^2}{Rm}$$

wherein we have used (8.1) and further using the fact that $H = c/R$, where H is the Hubble constant we can deduce

$$m \approx \left(\frac{H\hbar^2}{Gc}\right)^{\frac{1}{3}} \tag{8.17}$$

Equation (8.17) is the so called mysterious Weinberg formula, known empirically [28]. To quote Weinberg again, "...it should be noted that the particular combination of $\hbar, H, G$, and c appearing (in the formula) is very much closer to a typical elementary particle mass than other random combinations of these quantities; for instance, from $\hbar, G$, and c alone one can form a single quantity $(\hbar c/G)^{1/2}$ with the dimensions of a mass, but this has the value $1.22 \times 10^{22} MeV/c^2$, more than a typical particle mass by about 20 orders of magnitude!

"In considering the possible interpretations (of the formula), one should be careful to distinguish it from other numerical "coincidences"... In contrast, (the formula) relates a single cosmological parameter, H, to the fundamental constants $\hbar, G, c$ and m, and is so far unexplained."

We will come back to this point but remark that (8.14) brings out gravitation in a different light- it appears as a distributional effect over the N elementary particles of the Universe, rather than as a microphysical constant. It is a manifestation of $\bar{N}$ underlying Planck oscillators, at the level of N elementary particles, as remarked. In fact all this shows up gravitation as the excess or residual energy in the Universe.

Finally it may be observed that (8.14) can also be rewritten as

$$N = \left(\frac{c^2 l}{mG}\right)^2 \sim 10^{80} \tag{8.18}$$

and so also (8.10) can be rewritten as

$$n = \left(\frac{lc^2}{Gm}\right) \sim 10^{40}$$

It now immediately follows that

$$\bar{N} \sim 10^{120}$$

Looking at it this way, given G and the microphysical parameters we can deduce from equations like (8.18) the numbers $N, \bar{N}$ and n. So if gravitation were indeed a microphysical fundamental constant, then we would be back again with Weinberg's long overlooked paradox: The number of particles in the Universe would no longer be a free parameter, but rather would depend on the fundamental constants.

To throw further light on this matter, we observe that the gravitational energy of the N elementary particles if equated to its inertial energy, gives,

$$\frac{GN^2 m}{R} = mc^2$$

which immediately gives (8.5), or, if we use the Eddington formula

$$R = \sqrt{N} l,$$

and the expression for the Hubble constant derived in Chapter 6 (equation (6.11)),

$$H = \frac{c}{l} \cdot \frac{1}{\sqrt{N}}$$

to the Weinberg formula (8.17).

Or, we could directly consider the gravitational self interaction of a particle (Cf.ref.[65] for details). Our starting point is the action functional

$$S = -(8\pi G)^{-1} \int d^4 x \phi \Delta^2 \phi + \int d^4 x \Psi^* \left(\imath\hbar \frac{\partial \Psi}{\partial t} + \frac{\hbar^2}{2m} \Delta^2 \Psi - m\phi\Psi \right)$$

where ϕ is some potential whose nature is not as yet specified, G being some coupling constant. The extremum conditions of action with respect to Ψ^* and Ψ lead to the Schrödinger equation with the interaction potential ϕ:

$$\imath\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta^2 \Psi + m\phi\Psi \tag{8.19}$$

and to the Poisson equation for the potential itself

$$\Delta^2 \phi = 4\pi G m \Psi^* \Psi \tag{8.20}$$

Thus, the equations (8.19) and (8.20) describe a self-interacting particle. It is well known that an exact solution to (8.20) is given by

$$\phi(\boldsymbol{r}, t) = -G \int_{\Omega} d\Omega(\boldsymbol{r}) \frac{\rho(\boldsymbol{r}, t)}{|\boldsymbol{r} - \boldsymbol{r}'|}, \tag{8.21}$$

where Ω is the three dimensional region which confines the particle, and we have defined

$$\rho(\boldsymbol{r},t) = m\Psi^*(\boldsymbol{r},t)\Psi(\boldsymbol{r},t) \tag{8.22}$$

From (8.21), we can immediately see that for distances far outside the region Ω, that is $|\boldsymbol{r}| << |\boldsymbol{r}'|$, the potential ϕ has the form

$$\phi \approx \frac{GM}{r}, \tag{8.23}$$

where $r = |\boldsymbol{r}|$, and we have defined M as,

$$M = \int_\Omega d\Omega(\boldsymbol{r})\rho(\boldsymbol{r},t) = m\int_\Omega d\Omega(\boldsymbol{r})\Psi^*(\boldsymbol{r},t)\Psi(\boldsymbol{r},t) \tag{8.24}$$

The integral on the right hand side of (8.24) is conserved in time due to (8.19):

$$\frac{\partial}{\partial t}\int_\Omega d\Omega(\boldsymbol{r})\Psi^*(\boldsymbol{r},t)\Psi(\boldsymbol{r},t) = 0 \tag{8.25}$$

Thus the quantity M is constant, and we can interpret (8.23) and (8.24) as follows. The attractive potential (8.23) is now the classical gravitational potential, M is the gravitational mass, G being the gravitational constant. If we prescribe the unit value to the above conserved functional and interpret it as the norm square, I^2, or the full probability

$$I^2 = \int_\Omega d\Omega(\boldsymbol{r})\Psi^*(\boldsymbol{r},t)\Psi(\boldsymbol{r},t) = 1, \tag{8.26}$$

then the gravitational mass coincides with the inertial mass,

$$m = M, \tag{8.27}$$

and the quantity (8.22) now can be interpreted as the mass probability density. The source term on the right side of (8.20) is equal to the particle probability density itself.

Now, let us consider the self-consistent problem - the particle in its own potential well. We cannot obtain an exact solution. However, we can approximately describe some features of such a solution. The first assumption will be that we deal only with a spherically symmetric wave function: $\Psi = \Psi(r,t)$ where r is a radial coordinate. Then the mass probability density has the same dependence: $\rho = \rho(r,t)$. It can be easily shown that for any spherical mass distribution, the potential (8.21) is reducible to a simple form

$$\phi(r,t) = G\int_0^r dr'\frac{m(r',t)}{r'^2} - \int_0^\infty dr'\frac{m(r',t)}{r'^2}, \tag{8.28}$$

where we denote

$$m(r,t) = 4\pi\int_0^r dr' r'^2\rho(r',t), \tag{8.29}$$

and $m(r,t)$ is just the mass inside a ball of radius r. Certainly, the solution (8.28) gives an exact formula (8.23) with the mass (8.27) for the point mass distribution. Further, we shall use the value Φ instead of the potential ϕ:

$$\phi(r,t) = mG\Phi(r,t)$$

This allows us to rewrite (8.19) in the form

$$\imath \frac{2m}{\hbar}\frac{\partial}{\partial t}\Psi + \Delta^2\Psi - \frac{2m^3G}{\hbar^2}\Phi\Psi = 0 \tag{8.30}$$

The coefficient of Φ in (8.30) has the dimensionality of inverse length. Thus, we denote

$$l_G = \frac{\hbar^2}{2m^3G}, \tag{8.31}$$

Equation (8.31) is nothing but (8.17) the Weinberg formula again if we identify l_G with the radius of the Universe.
All this shows that the mass m of an elementary particle is very Machian, rather than being microphysical, if G is microphysical. But in our model of underlying Planck oscillators, it is m which is microphysical and G that is distributional.
Finally we remark that if in (8.31) we take m to be the Planck mass then l_G becomes the Schwarzchild radius, in conformity with our earlier observation that a Planck mass Black Hole is a mini Universe in itself.

8.3 Comments

Thus the many so called Large Number coincidences and the mysterious Weinberg formula can be deduced on the basis of a Planck scale underpinning for the elementary particles and the whole Universe. This was done from a completely different point of view, namely using fuzzy spacetime and fluctuations in the author's 1997 model discussed in Chapter 6 a model that successfully predicted as we saw a dark energy driven accelerating Universe with a small cosmological constant [168, 24]. However the above treatment brings out the primary role of the Planck scale oscillators in the Quantum Vacuum.
We now briefly comment on an alternative treatment of gravitation, the Sakharov-Zeldovich metric elasticity of space approach [295]. Essentially Sakharov argues that the renormalization process in Quantum Field Theory which removes the Zero Point energies is altered in General Relativity due to the curvature of spacetime, that is the renormalization or subtraction no longer gives zero but rather there is a residual energy similar to the modification in the molecular bonding energy due to deformation of the solids. We see this in a little more detail following Wheeler [46]. The contribution

to the Lagrangian of the Zero Point energies can be given in a power series as follows

$$L(r) = A\hbar \int k^3 dk + B\hbar^{(4)}r \int kdk$$
$$+\hbar[C(^{(4)}r)^2 + Dr^{\alpha\beta}r_{\alpha\beta}] \int k^{-1}dk$$
$$+(\text{higher-order terms}). \quad (8.32)$$

where A, B, C etc. are of the order of unity and r denotes the curvature. By renormalization the first term in (8.32) is eliminated. According to Sakharov, the second term is the action principle term, with the exception of some multiplicative factors. (The higher terms in (8.32) lead to corrections in Einstein's equations). Finally Sakharov gets

$$G = \frac{c^3}{16\pi B\hbar \int kdk} \quad (8.33)$$

Sakharov then takes a Planck scale cut off for the divergent integral in the denominator of (8.33). This immediately yields

$$G \approx \frac{c^3 l_P^2}{\hbar} \quad (8.34)$$

Infact using relations like (8.1), (8.6) and (8.13), it is easy to verify that (8.34) gives us back (8.10).

According to Sakharov (and (8.34)), the value of G is governed by the Physics of Fields and Particles and is a measure of the metrical elasticity at small spacetime intervals. It must be emphasized that in this approach G is a microphysical constant.

However in our interpretation of (8.14), G appears as the expression of a residual energy over the entire Universe: To reiterate, the entire Universe has an underpinning of the $\bar{N}$ Planck oscillators and is made up of N elementary particles, which again each have an underpinning of n Planck oscillators. The n oscillator elementary particle is a stable ground state, the $\bar{N}$ oscillator Universe is not. It must be reiterated that (8.34) obtained from Sakharov's analysis shows up G as a microphysical parameter because it is expressed in their terms. This is also the case in Dirac's cosmology. This is also true of (8.10) because n relates to the micro particles exclusively.

However when we use the relation (8.13), which gives n in terms of N, that is links up the microphysical domain to the large scale domain, then we get (8.14). With Sakharov's equation (8.34), the mysterious nature of the Weinberg formula remains. But once we use (8.14), we are effectively using the large scale character of G – it is not a microphysical parameter. This is brought out by (8.18), which is another form of (8.14). If G were a microphysical parameter, then the number of elementary particles in the Universe would depend solely on the microphysical parameters and would not be a large scale parameter as noted. The important point is that G relates to elementary particles and the whole Universe [294]: To reiterate, the energy of the Universe can be specified solely in terms of the underpinning of Planck oscillators without any recourse to gravitation, as in (8.8), or it can be exhibited equivalently as arising due to the gravitational interactions amongst the different particles of the Universe, as in (8.5). That is why (8.14) or equivalently the Weinberg formula (8.17) relate supposedly microphysical parameters to a cosmological parameter. Once the character of G as brought out by (8.14) is recognized, the mystery disappears. This explains Weinberg's paradox.

8.4 Gravitation as Weak Electromagnetism

The question that arises is, can we similarly consider the electromagnetic interaction between elementary particles to be the residual energy of the underpinning Planck oscillators, between elementary particles. In other words in (8.14) if we replace N by a number P which is $\sim 0(1)$ then we should get instead of Gm^2 the gravitational coupling, e^2 the electromagnetic coupling. Infact we get

$$e^2 \approx lmc^2 \tag{8.35}$$

which is indeed true! This also brings out a fundamental difference between the two interactions: electromagnetism deals with a few particles, that is, $P = N \sim 1$; whereas gravitation deals with the $N \sim 10^{80}$ particles of the entire Universe.

We can further support the above characterization in equation (8.16) of gravitation as a form of "weak electromagnetism" (or "weak electric force") or electromagnetism as a form of strong gravitation as follows: (It must be borne in mind that the terms weak electromagnetism and strong gravitation were used several years ago in different contexts). Firstly we observe that an equation like (8.5) with a numerical factor 2 on the right side (which in the Large Number context is not important) gives the Schwarschild radius of a Black Hole of mass M. If Gm^2, M for the moment being replaced by m, is substituted by e^2 in (8.5), then we should get the corresponding "Schwarschild radius" for electromagnetism treated as strong gravitation. Indeed we then get (8.35) giving the Compton wavelength for the mass m. In other words the Compton wavelength shows up as a non gravitational but rather "electromagnetic Schwarschild radius" on the scale of elementary particles.

Let us now consider the temperature and life time of a Black Hole in the context of the Hawking-Beckenstein Radiation. In the usual theory we have [59] in standard notation

$$T = \frac{\hbar c^3}{8\pi Gkm} \tag{8.36}$$

$$\frac{dm}{dt} = -\frac{\beta}{m^2}, \tag{8.37}$$

where β is given by

$$\beta = \frac{\hbar c^4}{(30.8)^3 \pi G^2}$$

This leads to the usual Black Hole life time given by

$$t = \frac{1}{3\beta} m^3 = 8.4 \times 10^{-24} m^3 secs, \quad (8.38)$$

If now we carry out the substitution $Gm^2 \to e^2$ in the above we have instead of (8.36), the relation

$$kT \sim mc^2 \quad (8.39)$$

Equation (8.39) is the well known relation encountered in Chapter 5 expressing the Hagedorn temperature of elementary particles [260]. Similarly instead of (8.37) we will get

$$\frac{dm}{dt} = -\frac{\hbar c^4}{\Theta^3 e^4} m^2, \; \Theta^3 = (30.8)^3 \pi$$

Whence we get for the life time

$$\frac{\hbar c^4}{\Theta^3 e^4} t = \frac{1}{m} \quad (8.40)$$

From (8.40) we get, for the pion, a life time

$$t \sim 10^{-23} secs,$$

which is the pion Compton time. So the Compton time shows up as an "electromagnetic Beckenstein Radiation life time."

Thus for elementary particles, working within the context of gravitational theory, but with a scaled up coupling constant, we get the meaningful relations (8.35), (8.40) and (8.39) giving the Compton length and Compton time as also the Hagedorn temperature as the analogues of the Schwarzchild radius, radiation life time and Black Hole temperature obtained with the usual gravitational coupling constant.

We now make the following remarks:

1. The role of the Planck scale in Quantum Gravity considerations as noted in Chapter 5 is well known. We reiterate that what has been done is that the same reasoning used in the theory of Black Holes within a purely gravitational framework can be extended to electromagnetic considerations, and then this leads to the Compton scale of elementary particles. In this sense, there is just a rescaling.

2. The Planck scale considerations, as has been noted lead to a modification of the Uncertainty Principle (Cf. [165, 145] and several references therein). There is now, as we saw, in addition to the usual Heisenberg Uncertainty term, an additional term given by

$$\Delta x = l_P^2 \frac{\Delta p}{\hbar} \tag{8.41}$$

As the Uncertainty in the momentum $\Delta p \sim \sqrt{n} m_P$, given the fact that as pointed out in the beginning there are n Planck oscillators defining a typical elementary particle, we have from (8.41), as noted in Chapters 5 and 6,

$$l = l_P^2 \sqrt{n} \frac{m_P c}{\hbar} = \sqrt{n} l_P$$

which is just (8.1). So the modification of the Uncertainty relation due to Planck scale considerations leads to the Compton scale.
3. Already we have referred to Sakharov's formulation of gravitation in terms of the background Zero Point Field (or Quantum Vacuum). In this context let us recapitulate the following well known fact encountered in Chapter 6. Due to the Zero Point oscillators, there is an electromagnetic field density ΔB over an interval L given by

$$(\Delta B)^2 \sim \frac{e^2}{L^4} \tag{8.42}$$

So the energy over an extension $L = l$ is given from (8.42) by $\frac{e^2}{l}$ which is the energy mc^2 of the elementary particle itself,

$$\frac{e^2}{l} = mc^2 \tag{8.43}$$

If on the other hand we replace in (8.43) e^2 by Gm^2, we get, reverting to the length L

$$\frac{Gm^2}{L} \approx mc^2$$

whence

$$L \approx \frac{Gm}{c^2} \tag{8.44}$$

(8.44) shows that we can similarly obtain from the fluctuating background Zero Point Field a Black Hole, infact a Planck scale Black Hole, it being well known as we saw, that a Planck mass is a Schwarzchild Black Hole at the Planck scale (Cf. also ref.[47]). From this point of view, Planck mass particles (or oscillators) are created from the fluctuation of the Zero Point Field and then lead up to elementary particles as indicated above. In any case, this again brings out the interchangability, $e^2 \to Gm^2$.
It is interesting to note that the substitution of $Gm^2 \to e^2 \to g_w^2$ for the neutrino, gives us relations similar to (8.39) and (8.43) (the latter was noted in Chapter 6). That is we get, this time,

$$T = \frac{m\hbar c^3}{8\pi g_w^2} = \frac{m\hbar c^3 \cdot 10^{13}}{8\pi \cdot e^2} = \frac{m_e c^2 \cdot \hbar c \cdot 10^5}{8\pi e^2} \sim 1^\circ,$$

corresponding to the Cosmic Background temperature as we saw, and,

$$l_v = \frac{g_\omega^2}{m_\nu c^2},$$

as already encountered. (Conversely, if we use $T \sim 1^\circ K$, then we recover $g_\omega^2 \sim 10^{-13} e^2$). The whole point is that as we also saw in Chapter 6, there is a complete parallel between the neutrino and an elementary particle which is particularly meaningful in the context of a Planck oscillator underpinning. This can be expressed by,

$$h, m, N, n, T, \bar{N} \to h', m', N', n', 1^\circ K, \bar{N}'$$

where, as we saw in Chapter 6, $h' \sim 10^{-12} h, N'$, the number of neutrinos $\sim 10^{90}, m'$, the neutrino mass is $10^{-10} m, T$ is the Hagedorn temperature and $1^\circ K$ is the corresponding temperature for the neutrino which is the Cosmic Background temperature (and which lead to the Fermi temperature considerations in Chapter 6 (cf. equation (6.88) $n' \sim 10^{60}$ is the number of underlying Planck oscillators for a neutrino and $\bar{N}' \sim 10^{125}$ is the number of Planck oscillators providing the underpinning for all the neutrinos.
Further, the considerations in Section 5 of Chapter 6 showed that gravitation and electromagnetism could be thought to be different due to the different "rates" at which these interactions played themselves out. This can be extended to the weak interactions also. The rates are different because of the difference in the number of subconstituents.
4. We have seen above how from the background Zero Point Field Planck scale oscillators can "condense". Let us suppose that n such particles are formed. We can then use the well known fact that [75] for a collection of ultra relativistic particles, in this case the Planck oscillators, the various centres of mass form a two dimensional disk of radius l given by

$$l \approx \frac{\beta}{m_e c} \tag{8.45}$$

where in (8.45) m_e($\approx m$ in the Large Number sense) is the electron mass and β is the angular momentum of the system. Further l is such that for distances $r < l$, we encounter negative energies (exactly as for the Compton length). It will at once be apparent that for an electron, for which $\beta = \frac{\hbar}{2}$, (8.45) gives the Compton wavelength. We can further characterize (8.45) as follows: By the definition of the angular momentum of the system of Planck particles moving with relativistic speeds, we have

$$\frac{\hbar}{2} = m_P c \int_0^l r^2 dr d\Theta \sim m_P c \sigma l^3 = m_e c l \tag{8.46}$$

In (8.46) we have used the fact that the disk of mass centres is two dimensional, and σ has been inserted to stress the fact that we are dealing with a two dimensional density, so that σ while being unity has the dimension

$$\left[\frac{1}{L^2}\right]$$

The right side of (8.46) gives the angular momentum for the electron. From (8.46) we get

$$\sigma l^2 m_P = m_e \tag{8.47}$$

which ofcourse is correct.
Alternatively from (8.47) we can recover $n \sim 10^{40}$, in the Large Number sense.
5. In Chapter 5 we argued that, based on the small scale structure of spacetime, it is possible to effect a reconciliation of electromagnetism and gravitation. The argument was based on the effects of a minimum spacetime interval and the resulting noncommutative geometry. In Chapter 6 we could see a reconciliation of both these interactions, based on the fluctuation in particle number. In Chapter 7 we saw how taking the minimum spacetime intervals (or the noncommutative geometry) into account, it is possible to even have an extended gauge treatment of gravitation. Then in this Chapter we have shown that gravitation can infact be considered to be a distributional effect at the scale of the Universe rather than a fundamental interaction. Even electromagnetism can be thought of as such an effect though at the scale of elementary particles. Undoubtedly, it is the underpinning of Planck oscillators that is fundamental. How can we reconcile the large scale "fluctuational" view with the Quantum view?
We first observe that it is not surprising that Quantum Theory itself should be the result of fluctuations in the Universe as a whole. In fact as pointed out [234] the fluctuation in the mass of a typical elementary particle, for example the pion, due to the fluctuation $\sim \sqrt{N}$ of the particle number $N \sim 10^{80}$ is given by

$$\Delta m \approx \frac{G\sqrt{N}m^2}{c^2 R}$$

Whence

$$(\Delta mc^2)T = \frac{G\sqrt{N}m^2}{R}T = \frac{G\sqrt{N}m^2}{c} \approx \hbar \tag{8.48}$$

where T is the age of the Universe and R its radius, which equals cT.
The right side of (8.48) gives the reduced Planck constant $\hbar$, in the order of magnitude sense in which (8.48) itself is an expression of the uncertainty relation

$$\Delta E \Delta t \approx \hbar$$

Equation (8.48) again suggests the origin of the Quantum Theory in cosmic fluctuations. However, it is just a restatement of (8.14) or (8.16).
Recently in the above context, Inaba [302] has considered a simple model in the Robertson-Walker geometry and argued that a fluctuation in this case yields a random motion which reduces to the usual Quantum Theory.

He deduces for a nearly flat Robertson-Walker Universe from a minimum average curvature principle, the Hamilton-Jacobi equation for a single particle.

$$\partial S + \frac{1}{2m} g^{\imath j} (\nabla S) + V - \alpha R = 0, \tag{8.49}$$

where the curvature R is given by

$$R = R^{(b)} + R'; \quad R^{(b)} = 6 \left(\frac{\dot{a}}{a} + \frac{\ddot{a}^2}{a^2} \right),$$

R' being the fluctuation effect and $R^{(b)}$ being the curvature in the standard Robertson-Walker geometry.
Equation (8.49) leads by the standard Madelung- Bohm or Nelson theory to the Schrödinger equation

$$\imath \hbar \partial_\imath \psi = \frac{\hbar^2}{2m} \Delta \psi + V \psi - \frac{\hbar^2}{4m} R^{(b)} \tag{8.50}$$

Inaba then argues that (8.50) is indeed the Quantum Mechanical equation in the classical Robertson-Walker geometry- it is the perturbation R' in the Robertson-Walker geometry that leads to (8.50).
We can reexamine the above conclusion as follows: We first observe that in the random motion of N particles over an extent R, l the fluctuation in the length is given by

$$l \approx \frac{R}{\sqrt{N}} \tag{8.51}$$

What is very interesting is that using for R the actual radius of the Universe $\sim 10^{28} cm$, and for N the actual number of particles in the Universe, (8.51) reduces to the well-known and oft encountered Eddington formula.
Further the diffusion equation describing the motion of a particle with position given by $x(t)$ subject to random corrections is given by the well-known equation

$$|\Delta x| = \sqrt{< \Delta x^2 >} \approx \nu \sqrt{\Delta t}$$

where the diffusion constant ν is related to the mean free path l and the mean velocity v

$$\nu \approx lv \tag{8.52}$$

Identifying l of (8.52) with that in (8.51), we, as in the case of Nelson's derivation, arrive at the Hamilton-Jacobi equation (8.49) and thence the Schrödinger equation (8.50) [24, 275, 303, 176, 304]. Incidentally, this also provides a rationale to the otherwise adhoc identification in Nelsonian theory viz.,

$$v = \hbar / m$$

Thus using the equations of Brownian motion in the context of all the particles in the Universe, we arrive at the same equations (8.49) and (8.50) of

Inaba based on a minimum curvature principle and Santamato's geometric Quantum Mechanics which we saw in the previous Chapter.
In fact one can look upon the above results in terms of the fluctuation of the metric itself. In Santamato's original formulation [288, 289, 290, 291], the geometry is Weyl's gauge invariant geometry, where there is no invariant length and in fact we have

$$\delta l^2 \sim l^2 \delta g_{\imath k} \tag{8.53}$$

It must be stressed that (8.53) is valid for arbitrary vectors, in which case l would be their length.
Using the usual geometrodynamic formula for the fluctuation of the metric [46], we have

$$l^2 \delta g_{\imath k} \approx \Delta g_{\imath k} \approx \frac{l_P}{l}, \tag{8.54}$$

where l_P is the Planck length.
Whence we get

$$\delta g_{\imath k} \sim 1 \tag{8.55}$$

if l is of the order $10^{-11} cm$ or the electron Compton wavelength a result we encountered earlier.
Similarly using (8.51) in (8.53), we recover (8.55), as in the Weyl geometry. On the other hand, we have already seen in Chapter 6 that the fluctuation $\sim \sqrt{N}$ can itself be deduced from the point of view of the energy levels in the Universe, considered as a collection of Planck oscillators. All this establishes the equivalence of the two approaches and reconfirms the Machian feature, from a more general viewpoint.
Finally it may be remarked that Feynman himself observed a long time ago that gravitation must be the result of a fluctuation in the metric [299].

9 THE ELUSIVE MASS SPECTRUM

"How can reality be predicated of that which is subject to change, and reassumes no more its original character? Earth is fabricated into a jar ; the jar is divided into two halves ; the halves are broken to pieces; the pieces become dust ; the dust becomes atoms."

Vishnu Purana

One of the problems that has eluded a solution is that of a mass spectrum for elementary particles. In other words, why should there be such a plethora of particles, and why should they have such and such masses? Is there any formula, based on dynamics, which would generate all these known masses? Such a formula would be intimately tied up with inter quark interactions. We will now use the QCD potential to deduce such a formula, which as will be seen, surprisingly covers all known elementary particles. The well known QCD potential as we saw is given by [68, 24]

$$U(r) = -\frac{\alpha}{r} + \beta r \tag{9.1}$$

where in units $\hbar = c = 1, \alpha \sim 1$. The first term in (9.1) represents the Coulombic part while the second term represents the confining part of the potential.

Let us consider the pion made up of two quarks along with a third quark, one at the centre and two at the ends of an interval of the order of the Compton wavelength, r. Then the central particle experiences the force

$$\frac{\alpha}{(\frac{x}{2}+r)^2} - \frac{\alpha}{(\frac{x}{2}-r)^2} \approx \frac{-2\alpha x}{r^3} \tag{9.2}$$

where x is the small displacement from the mean position. Equation (9.2) gives rise to the Harmonic oscillator potential, and the whole configuration resembles the tri-atomic molecule. This is pleasingly similar to the oscillator scenario we encountered in Chapters 5 and 8 wherein the pions themselves were made up of Planck scale oscillators.

Before proceeding we can make a quick check on (9.2). We use the fact that the frequency is given by

$$\omega = \left(\frac{\alpha^2}{m_\pi r^3}\right)^{\frac{1}{2}} = \frac{\alpha}{(m_\pi r^3)^{\frac{1}{2}}}$$

whence the mass of the pion m_π is given by

$$(h\omega \equiv)\omega = m_\pi \tag{9.3}$$

Remembering that $r = 1/m_\pi$, use of (9.3) gives $\alpha = 1$, which ofcourse is correct.
To proceed the energy levels of the Harmonic oscillator are now given by,

$$\left(n + \frac{1}{2}\right) m_\pi,$$

or if there are small m such oscillators, we have

$$E = m_P = m\left(n + \frac{1}{2}\right) m_\pi \tag{9.4}$$

where m_P is the mass of the corresponding elementary particle. The formula (9.4) gives the mass of all known elementary particles with an error of less than one percent for sixty three percent of the particles, less than two percent for ninety three percent of the particles, and less than three percent for all particles with the lone exception of $\omega(782)$, in which case the error is 3.6%. The known elementary particles for which the formula (9.4) is valid include the recently discovered $Ds(2317)$ and the $1.5GeV$ Pentaquark, discovered after the above formula was deduced.

Remarks

Firstly it is surprising that there is such a good fit for all the particles [281] considering that only bare details of the interaction have been taken into consideration. Once other details are included, the agreement could be even better. Secondly, it may be mentioned that a similar approach, but using the proton as the base particle had lead to interesting, but not such comprehensive results [70, 82, 305, 242]. Furthermore, instead of starting with quarks in (9.1) or (9.2), we could have very well started with electrons and positrons. Indeed, this would be in the spirit of Chapter 1 namely that quarks are electrons at a smaller scale.

Table 9.1. Baryons

Particle and Mass	Mass from Formula	Error %	(m,n)
$p(938)$	959	$-2.23881,$	$(2,3)$
$n(939)$	959	$-2.12993,$	$(2,3)$
$P_{11} ****N(1440)$	1438.5	$(0.138889,)0$	$(1,10)$
$D_{13} ****N(1520)$	1507	$(0.855263,)$	$(2,5)$
$S_{11} ****N(1535)$	1507	1.9442	$(2,5)$
$S_{11} ****N(1650)$	1644	$(0.363636,)0$	$(8,1)$
$D_{15} ****N(1675)$	1644	$1.85075,$	$(8,1)$
$F_{15} ****N(1680)$	1644	$2.14286,$	$(8,1)$
$D_{13} ***N(1700)$	1712.5	$(-0.705882,)0$	$(1,12)$
$P_{11} ***N(1710)$	1712.5	$(-0.116959,)0$	$(1,12)$
$P_{13} ****N(1720)$	1712.5	$(0.465116,)0$	$(1,12)$
$P_{13} ****N(1900)$	1918	$-0.947368,$	$(4,3)$
$F_{17} **N(1990)$	1986.5	$0.201005,$	$(1,14)$
$F-15 **N(2000)$	1986.5	$0.7,$	$(1,14)$
$D_{13} **N(2080)$	2055	$1.20192,$	$(2,7)$
$S_{11} *N(2090)$	2123.5	$-1.57895,$	$(1,15)$
$P_{11} *N(2100)$	2123.5	$(-1.09524,)$	$(1,15)$
$G_{17} ****N(2190)$	2123.5	$(3.05936,)0$	$(1,15)$
$D_{15} **N(2200)$	2260.5	$-2.72727,$	$(3,5)$
$H_{19} ****N(2220)$	2260.5	$(-1.8018,)0$	$(3,5)$
$G_{19} ****N(2250)$	2260.5	$(-0.444444,)0$	$(3,5)$
$I_{1;11} ***N(2600)$	2603	$(-0.115385,)0$	$(2,9)$
$K_{1;13} **N(2700)$	2671.5	1.05556	$(1,19)$
$P_{33} ****\varDelta(1232)$	1233	$(-0.0811688,)0$	$(2,4)$
$P_{33} ***\varDelta(1600)$	1575.5	$(1.5625,)0$	$(1,11)$
$S_{31} ****\varDelta(1620)$	1644	$(-1.46148,)0$	$(8,1)$
$D33 ****\varDelta(1700)$	1712	$(-0.705882,)0$	$(1,12)$
$P31 *\varDelta(1750)$	1781	$-1.77143,$	$(2,6)$
$S_{31} **\varDelta(1900)$	1918	$-0.947368,$	$(4,3)$
$F35 ****\varDelta(1905)$	1918	$(-0.682415,)0$	$(4,3)$
$P31 ****\varDelta(1910)$	1918	$(-0.418848,)0$	$(4,3)$
$P33 ***\varDelta(1920)$	1918	$(0.104167,)0$	$(4,3)$
$D35 ***\varDelta(1930)$	1918	$(0.621762,)0$	$(4,3)$
$D33 *\varDelta(1940)$	1918	$1.13402,$	$(4,3)$

Particle and Mass	Mass from Formula	Error %	(m,n)
$F37 * * * * \Delta(1950)$	1918	1.64103,	$(4,3)$
$F35 * * \Delta(2000)$	1986	0.7,	$(1,14)$
$S_{31} * \Delta(2150)$	2123.5	1.25581,	$(1,15)$
$G_{37} * \Delta(2200)$	2260	$-2.72727,$	$(1,16)$
$H_{39} * * \Delta(2300)$	2329	$-1.26087,$	$(2,8)$
$D_{35} * \Delta(2350)$	2329	0.893617,	$(2,8)$
$F_{37} * \Delta(2390)$	2397.5	$-0.292887,$	$(1,17)$
$G_{39} * * \Delta(2400)$	2397.5	0.125,	$(1,17)$
$H_{3;11} * * * * \Delta(2420)$	2397.5	$(0.950413,)0$	$(1,17)$
$I_{3;13} * * \Delta(2750)$	2740	0.363636,	$(8,2)$
$K_{3;15} * * \Delta(2950)$	2945.5	0.152542,	$(1,21)$
$\Lambda(1115)$	1096	1.7000,	$(16,0)$
$P_{01} * * * * \Lambda(1600)$	1575.5	1.53125,	$(1,11)$
$S_{01} * * * * \Lambda(1405)$	1438.5	$-2.3130,$	$(1,10)$
$D_{03} * * * * \Lambda(1520)$	1507	0.855263,	$(2,5)$
$P01 * * * \Lambda(1600)$	1575.5	$(1.5625,)0$	$(1,12)$
$S01 * * * * \Lambda(1670)$	1644	1.55689,	$(8,1)$
$D03 * * * * \Lambda(1690)$	1712.5	$-1.30178,$	(1.12)
$S01 * * * \Lambda(1800)$	1781	$(1,05556,)0$	$(2,6)$
$P01 * * * \Lambda(1810)$	1781	$(1.60221,)0$	$(2,6)$
$\Lambda(1820)$	1849.5	$(2.14286,)$	$(1,13)$
$D05 * * * * \Lambda(1830)$	1849.5	$-1.03825,$	$(1,13)$
$P03 * * * * \Lambda(1890)$	1918	$-1.48148,$	$(4,3)$
$* \Lambda(2000)$	1986.5	0.7,	$(1,14)$
$F07 * \Lambda(2020)$	2055	$-1.73267,$	$(2,7)$
$G07 * * * * \Lambda(2100)$	2123.5	$-1.09524,$	$(1,15)$
$F05 * * * \Lambda(2110)$	2123.5	$(-0.616114,)0$	$(1,15)$
$D03 * \Lambda(2325)$	2329	$-0.172043,$	$(2,8)$
$H09 * * * \Lambda(2350)$	2329	0.893617,	$(2,8)$
$* * \Lambda(2585)$	2603	0.309478,	$(2,9)$
$P11 * * * * \Sigma + (118)$	1164.5	2.10261,	$(1,8)$
$P11 * * * * \Sigma 0(119)$	1164.5	2.34899,	$(1,8)$
$* * * * \Sigma - (119)$	1164.5	2.75689,	$(1,8)$
$P13 * * * * \Sigma(1385)$	1370	$(0.108),$	$(4,2)$
$* \Sigma(1480)$	1438.5	2.83784,	$(1,10)$
$* * \Sigma(1560)$	1575.5	$-0.961538,$	$(1,11)$

Particle and Mass	Mass from Formula	Error %	(m, n)
$D13 ** \Sigma(1580)$	1575.5	0.316456,	$(1, 11)$
$S11 ** \Sigma(1620)$	1644	-1.48148,	$(8, 1)$
$P11 *** \Sigma(1660)$	1644	(0.963855,)0	$(8, 1)$
$D13 **** \Sigma(1670)$	1644	1.55689,	$(8, 1)$
$** \Sigma(1690)$	1712.5	-1.30178,	$(1, 12)$
$S11 *** \Sigma(1750)$	1781	(-1.77143,)0	$(2, 6)$
$P11 * \Sigma(1770)$	1781	-0.621469,	$(2, 6)$
$D15 **** \Sigma(1775)$	1781	(-0.338028,)0	$(2, 6)$
$P13 * \Sigma(1840)$	1849.5	-0.48913,	$(1, 13)$
$P11 ** \Sigma(1880)$	1849.5	1.64894,	$(1, 13)$
$F15 **** \Sigma(1915)$	1918	(-0.156658,)0	$(4, 3)$
$D13 *** \Sigma(1940)$	1918	(1.13402,)0	$(4, 3)$
$S11 * \Sigma(2000)$	1986.5	0.7,	$(1, 14)$
$F17 **** \Sigma(2030)$	2055	-1.23153,	$(2, 7)$
$F15 * \Sigma(2070)$	2055	0.724638,	$(2, 7)$
$P13 ** \Sigma(2080)$	2055	1.20192,	$(2, 7)$
$G17 * \Sigma(2100)$	2123	-1.09524,	$(1, 15)$
$*** \Sigma(2250)$	2260	(-0.444444,)0	$(3, 5)$
$** \Sigma(2455)$	2466	-0.448065,	$(4, 4)$
$** \Sigma(2620)$	2603	0.648855,	$(2, 9)$
$* \Sigma(3000)$	3014	-0.466667,	$(4, 5)$
$* \Sigma(3170)$	3151	0.599369,	$(2, 11)$
$P11 **** \Xi 0, \Xi - (13)$	1301.5	1.01156,	$(1, 9)$
$**** \Xi(1321)$	1301.5	1.47615,	$(1, 9)$
$P13 **** \Xi(1530)$	1507	1.50327,	$(2, 5)$
$* \Xi(1620)$	1644	-1.48148,	$(8, 1)$
$*** \Xi(1690)$	1712.5	-1.30178,	$(1, 12)$
$D13 *** \Xi(1820)$	1849.5	-1.59341,	$(1, 13)$
$*** \Xi(1950)$	1918	1.64103,	$(4, 3)$
$*** \Xi(2030)$	2055	-1.23153,	$(2, 7)$
$* \Xi(2120)$	2123.5	-0.141509,	$(1, 15)$
$** \Xi(2250)$	2260.5	-0.444444,	$(1, 16)$
$** \Xi(2370)$	2397.5	-1.13924,	$(1, 17)$
$* \Xi(2500)$	2534.5	-1.36,	$(1, 18)$

Particle and Mass	Mass from Formula	Error %	(m, n)
$****\,\Omega-(1672)$	1644	1.67464,	$(8, 1)$
$***\Omega-(2250)$	2260.5	$(-0.444444,)0$	$(1, 16)$
$**\,\Omega-(2380)$	2397.5	$-0.714286,$	$(1, 17)$
$**\,\Omega-(2470)$	2466	0.161943,	$(4, 4)$
$****\,\Lambda c+2285)$	2260.5	1.09409,	(1.16)
$+***\,\Lambda c+(2593)$	2603	$-0.385654,$	$(2, 9)$
$+***\,\Lambda c+(2625)$	2603	0.838095,	$(2, 9)$
$+*\,\Lambda c+(2765)$	2740	0.904159,	$(8, 2)$
$+**\Lambda c+(2880)$	2877	0.104167,	$(2, 10)$
$****\,\Sigma c(2455)$	2466	$-0.448065,$	$(4, 4)$
$***\Sigma c(2520)$	2534.5	$-0.555556,$	$(1, 18)$
$\Xi c+(2466)$	2466	0,	$(4, 4)$
$***\Xi c0(2471)$	2466	0.202347,	$(4, 4)$
$***\Xi c+(2574)$	2603	$(1.12665,)0$	$(2, 9)$
$***\Xi c0(2578)$	2603	$(0.96974,)0$	$(2, 9)$
$\Xi c(2645)$	2671.5	$-0.982987,$	$(1, 19)$
$***\Xi c(2790)$	2808.5	$-0.645161,$	$(1, 20)$
$***\Xi c(2815)$	2808.5	$0, 248668,$	$(1, 20)$
$***\Omega c0(2697)$	2671.5	0.964034,	$(1, 19)$
$***\Lambda b0(5624)$	5617	$(0.124467,)0$	$(2, 20)$

Table 9.2. Mesons

Particle and mass	Mass From Formula	Error %	(m, n)
$*\pi^{\pm}(139)$	137	−1.43885	
$*\pi^{0}(135)$	137	1.481481	
$K^{\pm}$	496	1.9	$(1, 3)$
$*\eta(547)$	548	0.182815	$(8, 0)$
$*f_0(600)$	616.5	(2.75)0	$(1, 4)$
$*\rho(770)$	753.5	−2.14286	(1.5)
$*\omega(782)$	753.5	−3.6445	$(1, 5)$
$*\eta'(958)$	959	0.104384	$(2, 3)$
$*f_0(980)$	959	−2.14286	$(2, 3)$
$*a_0(980)$	959	−2.14286	$(2, 3)$
$*\phi(1020)$	1027.5	0.735294	$(1, 7)$
$*h_1(1170)$	1164.5	(−0.47009)0	$(1, 8)$
$*b_1(1235)$	1233	(−0.16194)0	$(2, 4)$
$a_1(1260)$	1233	(−2.14286)0	$(2, 4)$
$f_2(1270)$	1233	−2.91339	$(2, 4)$
$f_1(1285)$	1301.5	1.284047	$(1, 9)$
$*\eta(1295)$	1301.5	0.501931	$(1, 9)$
$\pi(1300)$	1301.5	0.115385	$(1, 9)$
$a_2(1320)$	1301.5	−1.40152	$(1, 9)$
$*f_0(1370)$	1370	0	$(4, 2)$
$h_1(1380)$	1370	0.72464	$(4, 2)$
$\pi_1(1400)$	1370	−2.14286	$(4, 2)$
$f_1(1420)$	1438.5	1.302817	$(1, 10)$
$*\omega(1420)$	1438.5	(1.302817)0	$(1, 10)$
$f_2(1430)$	1438.5	0.594406	$(1, 10)$
$*\eta(1440)$	1438.5	−0.10417	$(1, 10)$
$*a_0(1450)$	1438.5	−0.7931	$(1, 10)$
$*\rho(1450)$	1438.5	−0.7931	$(1, 10)$
$*f_0(1500)$	1507	(0.466667)0	$(2, 5)$
$f_1(1510)$	1507	−0.19868	$(2, 5)$
$*f_2'(1525)$	1507	−1.18033	$(2, 5)$
$f_2(1565)$	1575.5	0.670927	$(1, 11)$

Particle and mass	Mass From Formula	Error %	(m, n)
$h_1(1595)$	1575.5	−1.22257	$(1, 11)$
$\pi_1(1600)$	1575.5	−1.53125	$(1, 11)$
$\chi(1600)$	1575.5	−1.53125	$(1, 11)$
$a_1(1640)$	1644	0.243902	$(8, 1)$
$f_2(1640)$	1644	0.243902	$(8, 1)$
$\eta_2(1645)$	1644	(0.06079)0	$(8, 1)$
$\omega(1670)$	1644	(1.55688)0	$(8, 1)$
$*\omega_3(1670)$	1644	−1.55689	$(8, 1)$
$*\pi_2(1670)$	1644	−1.55689	$(8, 1)$
$*\phi(1680)$	1712.5	1.934524	$(1, 12)$
$*\rho_3(1690)$	1712.5	1.331361	$(1, 12)$
$*\rho(1700)$	1712.5	(0.735294)0	$(1, 12)$
$a_2(1700)$	1712.5	0.735294	$(1, 12)$
$f_0(1710)$	1712.5	(0.146199)0	$(1, 12)$
$\eta(1760)$	1781	1.193182	$(2, 6)$
$*\pi(1800)$	1781	−1.05556	$(2, 6)$
$f_2(1810)$	1781	−1.60221	$(2, 6)$
$*\phi_3(1850)$	1849.5	(−0.02703)0	$(1, 13)$
$\eta_2(1870)$	1849.5	−1.09626	$(1, 13)$
$\rho(1900)$	1918	0.947368	$(4, 3)$
$f_2(1910)$	1918	0.418848	$(4, 3)$
$f_2(1950)$	1918	−1.64103	$(4, 3)$
$\rho_3(1990)$	1986.5	−0.17588	$(1, 14)$
$X(2000)$	1986.5	−0.675	$(1, 14)$
$f_2(2010)$	1986.5	(−1.16915)0	$(1, 14)$
$f_0(2020)$	1986.5	1.65842	$(1, 14)$
$*a_4(2040)$	2055	0.735294	$(2, 7)$
$f_4(2050)$	2055	0.243902	$(2, 7)$
$\pi_2(2100)$	2123.5	1.119048	$(1, 15)$
$f_0(2100)$	2123.5	1.119048	$(1, 15)$
$f_2(2150)$	2123.5	−1.23256	$(1, 15)$

Particle and mass	Mass From Formula	Error %	(m,n)
$\rho_2(2150)$	2123.5	-1.23256	$(1,15)$
$f_0(2200)$	2260.5	2.75	$(1,16)$
$f_J(2220)$	2260.5	1.824324	$(1,16)$
$\eta(2225)$	2360	1.595506	$(1,16)$
$\rho_3(2250)$	2260	0.466667	$(1,16)$
$*f_2(2300)$	2329	1.26087	$(2,8)$
$f_4(2300)$	2329	1.26087	$(2,8)$
$D_s(2317)$	2329	0.5	$(2,8)$
$f_0(2330)$	2329	-0.04292	$(2,8)$
$*f_2(2340)$	2329	-0.47009	$(2,8)$
$\rho_5(2350)$	2329	-0.89362	$(2,8)$
$a_6(2450)$	2466	-0.89362	$(4,4)$
$f_6(2510)$	2534.5	0.976096	$(1,18)$
$*K^*(892)$	890.5	-0.16816	$(1,6)$
$*K_1(1270)$	1233	2.91338	$(2,4)$
$*K_1(1400)$	1370	-2.14286	$(4,2)$
$*K^*(1410)$	1438.5	2.021277	$(1,10)$
$*K_0^*(1430)$	1438.5	0.594406	$(1,10)$
$*K_2^*(1430)$	1438.5	0.594406	$(1,10)$
$K(1460)$	1438.5	-1.4726	$(1,10)$
$Pentaquark(1.5GeV)$	1.5	0	$(2,5)$
$K_2(1580)$	1575.5	-0.28481	$(1,11)$
$K(1630)$	1644	0.858896	$(8,1)$
$K_1(1650)$	1644	-0.36364	$(8,1)$
$*K^*(1680)$	1712.5	(1.934524)0	$(1,12)$
$*K_2(1770)$	1781	(0.621469)0	$(2,6)$

Particle and mass	Mass From Formula	Error %	(m,n)
$*K_3^*(1780)$	1781	(0.05618)0	$(2,6)$
$*K_2(1820)$	1849.5	1.620879	$(1,13)$
$K(1830)$	1849.5	1.065574	$(1,13)$
$K_0^*(1950)$	1918	-1.64103	$(4,2)$
$K_2^*(1980)$	1986.5	0.328283	$(1,14)$
$*K_4^*(2045)$	2055	(0.488998)0	$(2,7)$
$K_2(2250)$	2260.5	0.466667	$(1,16)$
$K_3(2320)$	2329	0.387931	$(2,8)$
$K_5^*(2380)$	2397.5	0.735294	$(1,17)$
$K_4(2500)$	2466	-1.36	$(4,4)$
$K(3100)$	3082.5	-0.56452	$(1,22)$
$*D^{\pm}(1869.3)$	1849.5	-1.05922	$(1,13)$
$*D_0^{\pm}(1968.5)$	1986.5	0.914402	$(1,14)$
$*D_0^*(2007)$	1986.5	-1.02143	$(1,14)$
$D_{\pm}^*(2010)$	1986.5	-1.16915	$(1,14)$
$D_S(2317)$	2329	0.51791	$(2,8)$
$*D_1(2420)$	2397.5	-0.92975	$(1,17)$
$D_1^{\pm}(2420)$	2397.5	-0.97067	$(1,17)$
$D_2^*(2460)$	2466	0.243902	$(4,4)$
$D_{\pm}^*(2460)$	2466	0.243902	$(4,4)$
$D_{S1}^{\pm}(2536)$	2534.5	-0.07885	$(1,18)$
$D_{SJ}(2573)$	2534.5	-1.49631	$(1,18)$
$*B^{\pm}(5278)$	5274.5	-0.08524	$(1,38)$
$*B^0(5279.4)$	5274.5	-0.09281	$(1,38)$
$B_j(5732)$	5754	-0.47009	$(4,10)$
$*B_S^0(5369.6)$	5343	-0.49538	$(2,19)$
$B_{SJ}^*(5850)$	5822.5	-0.47009	$(1,42)$
$*B_c^{\pm}(6400)$	6370.5	0.4609	$(3,15)$
$*\eta c(1S)(2979)$	2945.5	-1.12454	$(1,21)$
$*J/\psi(1S)(30968)$	3082.5	-0.46402	$(1,22)$
$*\chi c_0(1P)(3415.1)$	3425	0.289889	$(2,12)$
$*\chi c_1(1P)(3510.5)$	3493.5	-0.48426	$(1,25)$
$*\chi c_2(1P)(3556)$	3562	0.168729	$(4,6)$

Particle and mass	Mass From Formula	Error %	(m, n)
$*\psi(2S)(3685.9)$	3699	0.355408	$(2, 13)$
$*\psi(3770)$	3767.5	$(-0.06631)0$	$(1, 27)$
$*\psi(3836)$	3836	0	$(8, 3)$
$*\psi(4040)$	4041.5	$(0.037129)0$	$(1, 29)$
$*\psi(4160)$	4178.5	$(0.444712)0$	$(1, 30)$
$*\psi(4415)$	4452.5	0.84937	$(1, 32)$
$*\gamma(1S)(9460.3)$	9453	-0.07716	$(2, 34)$
$\chi b_0(1P)(9859.9)$	9864	0.041583	$(16, 4)$
$*\chi b_1(1P)(9892.7)$	9864	-0.29011	$(16, 4)$
$*\chi b_2(1P)(9912.6)$	9864	-0.49029	$(16, 4)$
$*\gamma(2S)(10023)$	10001	0.21949	$(2, 36)$
$*\chi b_0(2P)(10232)$	10275	0.42026	$(2, 37)$
$*\chi b_1(2P)(10255)$	10275	0.1945027	$(2, 37)$
$*\chi b_2(2P)(10268)$	10275	0.068173	$(2, 37)$
$*\gamma(3S)(10355)$	10343.5	0.11105	$(1, 75)$
$*\gamma(4S)(10580)$	10549	-0.29301	$(2, 38)$
$*\gamma(10860)$	10891.5	0.290055	$(3, 26)$
$*\gamma(11020)$	11028.5	0.077132	$(1, 80)$

10 EXPERIMENTAL EFFECTS

"I want to know how God created this world. I am not interested in this or that phenomenon, in the spectrum of this or that element. I want to know His thoughts, the rest are details."

A. Einstein

10.1 Introduction

We will now consider the experimental consequences of fuzzy spacetime considerations we encountered, particularly in Chapters 5 and 6. Indeed the paradigmatic shift to an accelerating Universe is already a major confirmation of the theory developed in Chapter 6. Let us consider some other effects. We start with the possible correction to Einstein's formula, which we encountered earlier in Chapter 6. Recent observations of Ultra High Energy Cosmic Rays suggest that there could be a small violation of Lorentz symmetry at energies $\sim 10^{20}eV$, as we will see below. This is because there is the so called Lorentz compatible GZK cut off, beyond which there should not be any particles reaching the earth from cosmological distances. This has prompted several authors including Glashow and Coleman, Mestres, Jacobson and others in recent years to speculate on the form of Lorentz violation. Actually as pointed out by 't Hooft, the author himself and others, a high energy violation of Lorentz symmetry is expected in schemes where spacetime is discrete. Such schemes as we saw in Chapter 5 have been studied for a long time - from the work of Snyder, Finkelstein, Kardyshevskii, Wolf, the author himself and others [114, 115, 121, 24, 116, 150]. This is encountered in Lattice Gauge Theory too [252], though more as a computational tool. More recently, 't Hooft and others have re-examined lattice theories. This time the motivation has been more on the lines of minimum spacetime intervals [22]. What happens in this case is there is a departure from Lorentz symmetry[24, 307]. Typically we have an energy momentum relation (with units such that $c = 1 = \hbar$)

$$E^2 = m^2 + p^2 - l^2p^4 \tag{10.1}$$

where l is a minimum length interval, which could be typically the Planck length and more generally the Compton length, (which reduces to the Planck length for a Planck mass). Interestingly we could arrive at (10.1) from an alternative point of view, starting directly from the noncommutativity of spacetime, or the Modified Uncertainty Principle which results from these considerations as we saw in Chapters 5 and 6 (Cf. also [145]):

$$[x,p] = \hbar' = \hbar[1 + \left(\frac{l}{\hbar}\right)^2 p^2]\, etc \tag{10.2}$$

where we have temporarily re-introduced $\hbar$. (10.2) shows that effectively $\hbar$ is replaced by $\hbar'$. So,

$$E = (m^2 + p^2)^{\frac{1}{2}}(1 + l^2 p^2)^{-1}$$

or

$$E^2 = m^2 + p^2 - 2l^2 p^2, \tag{10.3}$$

neglecting higher order terms. (10.3) is of the same form as (10.1). We now examine a few implications of (10.1).

10.2 Modified Dispersion

Let us consider an effect similar to the Compton effect [276], but with (10.1) replacing the usual energy momentum formula. Here if $\boldsymbol{k}_0$ is the incident radiation and $\boldsymbol{k}$ is the scattered radiation at an angle Θ, as in the usual theory we get from the energy and momentum conservation laws,

$$k_0 - k = E - m \tag{10.4}$$

and

$$\boldsymbol{k}_0 - \boldsymbol{k} = \boldsymbol{p} \tag{10.5}$$

Further algebraic manipulation of (10.4) and (10.5) gives

$$kk_0(1 - cos\Theta) = m(k_0 - k) + \frac{l^2}{2}[Q^2 + 2mQ]^2 = mQ + \frac{l^2}{2}[Q^2 + 2mQ]^2$$

where

$$E - m = Q = k_0 - k$$

Whence, we get the frequency k as, (in natural units),

$$k = \frac{mk_0 + \frac{l^2}{2}[Q^2 + 2mQ]^2}{[m + k_0(1 - cos\Theta)]} \tag{10.6}$$

Alternatively, let us denote the additional change in frequency due to the noncommutativity of spacetime or the presence of minimum spacetime intervals by ϵ, so that

$$k + \epsilon = \bar{k}$$

$\bar{k}$ being the usual compton frequency. With this, we get, instead of (10.6),

$$\epsilon = \frac{l^2[Q^2 + 2mQ]^2}{2\{m + k_0(1 - cos\Theta)\}} \tag{10.7}$$

The relations (10.6) or (10.7) enable us to observe the effect of the violation of Lorentz symmetry as embodied in (10.1).
It must be reiterated that in the usual formulation there is a restriction on the energy of the cosmic rays which we receive due to the presence of the GZK cut off (Cf.refs.[308] to [311]): The Microwave Cosmic Background Radiation limits the propagation of Ultra High Energy Cosmic Rays due to inelastic collisions with the background photons. Particles with energy greater than a few$10^{19}eV$ cannot propagate further than about $50Mpc$. On the other hand the very small departure from Lorentz symmetry as in (10.6) or (10.7) would lead to significant effects at higher and higher energies and could explain the observed Ultra High Energy Cosmic Ray events of energy greater than $10^{20}eV$.

10.3 Particle Behaviour

Owing to (10.1) we have a modified Klein-Gordan equation

$$(D + l^2\nabla^4 - m^2)\psi = 0 \tag{10.8}$$

where D denotes the usual D'Alembertian.
Just to get a feel, it would be interesting to consider the extra effect in (10.8). For simplicity we take the one dimensional case. As in conventional theory if we separate the space and time parts of the wave function we get

$$l^2u^{(4)} + u^{(2)} + \lambda u = 0, \quad \lambda = E^2 - m^2 > 0 \tag{10.9}$$

where $u^{(n)}$ denotes the nth space derivative.
Whence if in (10.9) we take,

$$u = e^{\alpha x}$$

and $\alpha^2 = \beta$ we get,

$$l^2\beta^2 + \beta + \lambda = 0$$

whence

$$\beta = \frac{-1 \pm \sqrt{1 - 4l^2\lambda}}{2l^2}$$

So

$$\beta \approx \frac{-1 \pm \{1 - 2l^2\lambda\}}{2l^2} \tag{10.10}$$

From (10.10) it is easy to deduce that there are two extra solutions, as can be anticipated by the fact that (10.8) is a fourth order equation, unlike the usual second order Klein-Gordan equation. Thus we have

$$\beta = -\lambda (< 0)$$

giving the usual solutions, but additionally we have

$$\beta = -\left(\frac{1 - \lambda l^2}{l^2}\right) (< 0) \tag{10.11}$$

What do the two extra solutions in (10.11) indicate? To see this we observe that α is given by, from (10.11)

$$|\alpha| \approx \pm \frac{1}{l} \tag{10.12}$$

In other words (10.12) corresponds to waves with wavelength of the order l, which is intuitively quite reasonable.

What is interesting is that if l is an absolute length then the extra effect is independent of the mass of the particle. In any case the solutions from (10.12) are GZK violating solutions, arising as they do, from the modified energy momentum formula (10.1).

We now make some remarks. Departures from Lorentz symmetry of the type given in (10.1) have as noted, been studied, though from a phenomenological point of view [308, 309, 310, 311, 312, 313]. These arise mostly from an observation of Ultra High Energy Cosmic Rays. Given Lorentz Symmetry, there is the GZK cut off already alluded to, such that particles above this cut off would not be able to travel cosmological distances and reach the earth. However as mentioned, there are indications of a violation of the GZK cut off (Cf.references [308]-[313]).

In any case some of the effects following, for example from (10.1), like (10.6) or (10.7) can be detected, it is hoped by the GLAST Satellite to be launched by NASA in 2006 or shortly thereafter [314].

Interestingly, if in (10.1) or (10.8) we take, purely on an ad hoc basis, $-l^2$ rather than $+l^2$, we get two real exponential solution of (10.8). One of them is an increasing exponential leading to very high probabilities for finding these particles.

10.4 A New Short Range Force

As we saw in Chapter 5, in some ways the General Relativistic gravitational field resembles the electromagnetic field, particularly in certain approximations, as for example when the field is stationary or nearly so and the velocities are small. In this case the equations of General Relativity can be put into a form resembling those of Maxwell's Theory, and then the fields have been called Gravitoelectric and Gravitomagnetic [315]. Experiments have also been suggested for measuring the Gravitomagnetic force components for the earth [316].

We can ask whether such a consideration can be applied to elementary particles, if in fact they can be considered in the context of General Relativity. As already mentioned in Chapters 1 and 4, apart from Quantum Gravity, there have been different approaches for studying elementary particles via General Relativity [317, 24, 113] and references therein. We will now show that it is possible to extend the Gravitomagnetic and Gravitoelectric formulations to elementary particles within the framework of the theory developed in [24].
We saw that the linearized General Relativistic equations could describe the properties of elementary particles, such as spin, mass, charge and even the very Quantum Mechanical anomalous gyromagnetic ratio $g = 2$, apart from several other characteristics [161, 318, 168, 82].
We merely report that the linearized equations of General Relativity, viz.,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, h_{\mu\nu} = \int \frac{4T_{\mu\nu}(t - |\boldsymbol{x} - \boldsymbol{x}'|, \boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|} d^3x' \tag{10.13}$$

where as usual,

$$T^{\mu\nu} = \rho u^u u^v \tag{10.14}$$

lead, on using (10.14) in (10.13), to the mass, spin, gravitational potential and charge of an electron, if we work at the Compton scale (Cf. Chapters 1, 5 and ref.[24] for details). Let us now apply the macro Gravitoelectric and Gravitomagnetic equations to the above case. Infact these equations are (Cf.ref.[315]).

$$\nabla \cdot \boldsymbol{E}_g \approx -4\pi\rho, \nabla \times \boldsymbol{E}_g \approx -\partial \boldsymbol{H}_g/\partial t, etc. \tag{10.15}$$

$$\boldsymbol{E}_g = -\nabla\phi - \partial \boldsymbol{A}/\partial t, \quad \boldsymbol{H}_g = \nabla \times \boldsymbol{A} \tag{10.16}$$

$$\phi \approx -\frac{1}{2}(g_{00} + 1), \boldsymbol{A}_\imath \approx g_{0\imath}, \tag{10.17}$$

The subscripts g in the equations (10.15) and (10.16) are to indicate that the fields E and H in the macro case do not really represent the electromagnetic field, but rather resemble them. Let us apply equation (10.16) to equation (10.13), keeping in mind equation (10.17). We then get, considering only the order of magnitude, which is what interests us here, after some manipulation

$$|\boldsymbol{H}| \approx \int \frac{\rho V}{r^2} \bar{r} \approx \frac{mV}{r^2} \tag{10.18}$$

and

$$|\boldsymbol{E}| = \frac{mV^2}{r^2} \tag{10.19}$$

V being the speed.
In (10.18) and (10.19) the distance r is much greater than a typical Compton wavelength, to make the approximations considered in deriving the Gravitomagnetic and Gravitoelectric equations meaningful.
Remembering that we have, by the Uncertainty Principle,

$$mVr \approx h,$$

the electric and magnetic fields in (10.18) and (10.19) now become

$$|\boldsymbol{H}| \sim \frac{h}{r^3}, |\boldsymbol{E}| \sim \frac{hV}{r^3} \tag{10.20}$$

We now observe that (10.20) does not really contain the mass of the elementary particle. Could we get a further insight into this new force?

Indeed in the above linearized General Relativistic characterization of the electron, it turns out as indicated that the electron can be represented by the Kerr-Newman metric which incidentally also gives the anomalous gyromagnetic ratio $g = 2$. (This result has recently been reconfirmed by Nottale [319] from a totally different point of view, using scaled relativity). It is well known that the Kerr-Newman field has extra electric and magnetic terms (Cf.[62]), both of the order $\frac{1}{r^3}$, exactly as indicated in (10.20).

It may be asked if there is any candidate as yet for the above mass independent, spin dependent (through h) short range force. There is already one such experimental candidate - the inexplicable $B_{(3)}$ [320] short range force, first detected in 1992 at Cornell and since it is claimed, confirmed by subsequent experiments. It differs from the usual $B_{(1)}$ and $B_{(2)}$ long range fields of Special Relativity.

Interestingly, if we think of the above force as being mediated by a "massive" particle, that is, work with a massive vector field we can recover (10.19) and (10.20) [321]. In this case there is an upper limit on the mass of the photon $\sim 10^{-48}g$, that is, less than a trillionth the mass of a neutrino.

A final comment: It is quite remarkable that equations like (10.15), (10.16) and (10.17) which resemble the equations of electromagnetism, have in the usual macro considerations no connection whatsoever with electromagnetism except in appearance. This would seem to be a rather miraculous coincidence. In fact the above considerations of section 2 and linearized General Relativistic theory of the electron as also the Kerr-Newman metric formulation, demonstrate that the resemblance to electromagnetism is not an accident, because in this latter formulation, both electromagnetism and gravitation arise from the metric (Cf.also refs.[67, 161, 318, 24]).

10.5 Gravitational Effects

We may next point out the following. Let us introduce the minimum cut off l into the Schwarzchild metric. This gives

$$d\tau^2 = d\tau_0^2 - \frac{2MG}{r}(\frac{l}{r})(dt^2 - dr^2)$$

where $d\tau_0^2$ is the unmodified metric. The above shows that G is replaced by

$$G(1 + \frac{l}{r}).$$

Apart from the fact that this is equivalent to an extra force,

$$\text{Force} \quad \propto \frac{GMl}{r^3},$$

it is also equivalent to the time varying G encountered earlier in Chapter 6 and given by

$$\dot{G} = -G/t$$

The above follows because

$$\frac{r}{t} = \frac{l}{\tau}$$

where r and t are the radius and age of the Universe and l and τ are a typical Compton length and time. This variation is within the observed limits.
It is interesting to note that in the above analysis, if we take l to be the radius of the Universe and M to be its mass, then the extra force gives the observed cosmological constant. (Interestingly, the Universe itself shows up as a Schwarzchild Black Hole, as shown elsewhere [24]).
Further, apart from known results, the above variation of G with time explains the otherwise inexplicable anomalous accelerations of the Pioneer spacecrafts observed over the past several years by J.D. Anderson of JPL, Pasadena, and co-workers [322, 323, 324].
Infact from the usual orbital equations we have [325]

$$v\dot{v} \approx -\frac{GM}{2tr}(1 + ecos\Theta) - \frac{GM}{r^2}\dot{r}(1 + ecos\Theta)$$

v being the velocity of the spacecraft. It must be observed that the first term on the right side is the new effect. There is now an anomalous acceleration given by

$$a_r = \langle \dot{v} \rangle \text{anom} = \frac{-GM}{2trv}(1 + ecos\Theta)$$
$$\approx -\frac{GM}{2t\lambda}(1 + e)^3$$

where

$$\lambda = r^4 \dot{\Theta}^2$$

If we insert the values for the Pioneer spacecrafts we get

$$a_r \sim -10^{-7} cm/sec^2$$

This is the observed as yet unexplained anomalous acceleration even though Anderson and collaborators have tried several other explanations for over a decade.

10.6 Bosons as Bound States of Fermions

In our formulation, Fermions are primary - Bosons are bound states of Fermions. This has been discussed in detail in (ref. [24]). The question is, does the photon fit into this scheme? Indeed a long time ago, Darwin showed that the massless, force free Dirac theory was formally identical to source free electrodynamics in a vacuum [326]. In the absence of a suitable physical interpretation this mathematical identity has for long been considered to be a mere mathematical coincidence (Cf.ref.[326]). After all, photons are spin one particles, while the Dirac equation represents spin half particles. At the same time, it has also been recognized for a long time - Einstein and Meyer were one of the first to point this out - that the spinorial representation of the Lorentz group is more fundamental than the vectorial representation [317].
In the light of the above observations we would now like to point out that the above circumstance is not a mere coincidence, but has a definite physical interpretation.
We firstly make some preliminary remarks: Both in electromagnetic theory and in the Dirac theory, the D'Alembertian equation

$$D\psi_\mu = 0 \tag{10.21}$$

where D is the D'Alembertian operator, is satisfied by the respective components. This is merely an expression of Lorentz invariance. At this point the two theories diverge. This is because an equation like (10.21) requires the value of ψ at say $t = 0$ and so also the value of $\frac{\partial \psi}{\partial t}$ for specifying the solution. This does not pose any problem in electromagnetic theory, but is not acceptable in Quantum Theory, because the Quantum Mechanical wave function ψ contains as complete a description of the state as is possible and there is no room for derivatives as initial conditions. This is also the reason why (10.21), or the Quantum Mechanical Klein-Gordan equation gives negative probability densities. So the order of (10.21) needs to be depressed to make it a first order equation, which infact is the starting point of the Dirac theory and leads to the Dirac equation,

$$(\gamma^\mu p_\mu - m)\,\psi = 0 \tag{10.22}$$

It may be mentioned that two component spinors belonging to the representation

$$D^{(\frac{1}{2}0)} \mathrm{or} D^{(0\frac{1}{2})}$$

of the Lorentz group are solutions of the Dirac equation (10.22). But these are no longer invariant under reflections [66]. It is to preserve this invariance that we have to consider the 4×4 representation

$$D^{(\frac{1}{2}0)} \oplus D^{(0\frac{1}{2})}$$

Under reflections, the two spinors transform into each other thus maintaining the overall invariance [60]. We also note that, as is known [327], the Maxwell equations can also be written in the form of neutrino equations. Defining a four vector such that

$$\chi_j = E_j + \imath B_j, \chi_0 = 0 \tag{10.23}$$

we can rewrite the Maxwell equations in the form

$$\beta_\mu \frac{\partial \chi_\nu}{\partial x_\mu} = -\frac{1}{c} j_\nu \tag{10.24}$$

where in a particular representation, for example,

$$\beta_0 = IXI, \quad \beta_1 = -\sigma_3 \otimes \sigma_2,$$

$$\beta_2 = \sigma_2 \otimes I, \quad \beta_3 = \sigma_1 \otimes \sigma_2,$$

the σ's being the Pauli matrices and wherein for our source free vacuum case, the current four vector on the right hand side of equation (10.24) vanishes. It is easy to show that the four component equation (10.24) breaks down into two component neutrino like equations, except that both these equations are coupled owing to the additional condition $\chi_0 = 0$ in (10.23). This has been the problem in identifying (10.24) with the Dirac theory.
In the above context let us now approach the above considerations from the opposite point of view, that of the Dirac equation. It is well known that the four linearly independent four spinor Dirac wave functions are given by [29], apart from multiplicative factors,

$$\begin{bmatrix} 1 \\ 0 \\ \frac{p_z c}{E+mc^2} \\ \frac{p+c}{E+mc^2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ \frac{p-c}{E+mc^2} \\ \frac{-p_z c}{E+mc^2} \end{bmatrix} \begin{bmatrix} \frac{p_z c}{E+mc^2} \\ \frac{p+c}{E+mc^2} \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} \frac{p+c}{E+mc^2} \\ \frac{-p_z c}{E+mc^2} \\ 0 \\ 1 \end{bmatrix} \tag{10.25}$$

where p_z is the z component of the momentum and

$$p_\pm = p_x \pm \imath p_y,$$

in a representation given by,

$$\gamma_\imath = \gamma_0 \begin{bmatrix} 0 & \sigma_\imath \\ \sigma_\imath & 0 \end{bmatrix}, \gamma_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

the σ's being the Pauli matrices.
If we consider the z axis to be in the direction of motion, for simplicity and take the limit $m \to 0$, the spinors in (10.25) become,

$$\psi_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \psi_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \psi_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \psi_4 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \tag{10.26}$$

It should be noticed that in (10.26) $\psi_1 = \psi_3$, and $\psi_2 = \psi_4$ so that effectively, two of the spinors vanishes exactly and we are using with two solutions as in the case of the solutions χ of (10.24).(The mass zero four component Dirac spinor does not represent a neutrino unless an auxiliary condition, which effectively destroys the lower two or upper two components is imposed [60]). It can now be seen from the above considerations that the source free vacuum electromagnetic field can be considered to be a composite of a neutrino and an anti neutrino. It may be mentioned that the possibility of Bosons being bound states of Fermions, rather than being primary has been discussed by the author and other scholars [328, 24].

10.7 Can we Harness the Zero Point Field?

Let us start with the well known Casimir effect, encountered in Chapter 6. The essential idea of the Casimir effect is that the interaction between the ZPF and matter leads to macroscopic consequences. For example if we consider two parallel metallic plates in a conducting box, then we should have a Casimir force given by [213]

$$F = \frac{-\pi^2}{240} \frac{\hbar c A}{l^4}$$

where A is the area of the plates and l is the distance between them. More generally, the Casimir force is a result of the boundedness or deviation from a Euclidean topology of or in the Quantum Vacuum. These Casimir forces have been experimentally demonstrated [222, 329, 330, 331].
Returning to the ZPF as the ubiquitous dark energy, we observe that [46], a fluctuating electromagnetic field can be modelled as an infinite collection of independent Harmonic oscillators as noted in Chapter 6. Quantum Mechanically, the ground state of the Harmonic oscillators is described by, as we saw in Chapter 6,

$$\psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-(m\omega/2\hbar)x^2}$$

which exhibits the probability for the oscillator to fluctuate, mostly in the region given by

$$\Delta x \sim (\hbar/m\omega)^{1/2}$$

An infinite collection of such oscillators can be modelled by

$$\psi(\xi_1, \xi_2, \cdots) = const. \exp[-(\xi_1^2 + \xi_2^2 + \cdots)],$$

which gives the probability amplitude for an electromagnetic field configuration $B(x, y, z), \xi_1$, etc. being the Fourier coefficients. Finally, as a consequence there is a fluctuating magnetic field given by

$$B = \frac{\sqrt{\hbar c}}{l^2} \tag{10.27}$$

where l is the extent over which the fluctuation is measured. Further these fluctuations typically take place within the time τ, a typical elementary particle Compton time (Cf.ref.[187]) as seen in Chapter 6. This begs the question whether such ubiquitous fields could be tapped for terrestrial applications or otherwise.
We now invoke the well known result from macroscopic physics that the current in a coil is given by

$$\imath = \frac{NBA}{R\Delta t} \tag{10.28}$$

where N is the number of turns of the coil, A is its area and R the resistance. Introducing (10.27) into (10.28) we deduce that a coil in the ZPF would have a fluctuating electric current given by

$$\imath \approx \frac{NA}{R} \cdot \frac{e}{l^2 \tau} \tag{10.29}$$

In principle it should be possible to harness the current (10.29). While this current is small, if we have a superconductor, then R would be very small and the current would be much larger. The question is, whether such an application is possible, on the earth or in an orbiting space craft, for example.

10.8 Retrospect

We have seen in Chapter 6 that a major confirmation for the ideas encountered in this book has come from the observation of a dark energy driven accelerating Universe, with a small cosmological constant. Moreover inexplicable, empirical "coincidental" relations are deduced from the theory.
Apart from the effects seen in this Chapter, it may also be mentioned that the mass spectrum formula encountered in Chapter 9, not just gives the masses of all known elementary particles, but also, the subsequently discovered $Ds(2317)$ and the pentaquark agree with the formula, which infact gives any number of other particle masses.
It may also be pointed out that the violation of time reversal in the Kaon decay also has an explanation within the Compton time minimum interval. This has been discussed in detail (Cf.refs.[24, 332]).
Finally, there is also an Aharonov-Bohm type of an extra effect as described in ref.[24]. Briefly, given that an electron can be described by a Kerr-Newman metric, it can be approximated by a solenoid and we could expect an Aharonov-Bohm type of effect, due to the vector potential $\boldsymbol{A}$ which would give rise to shift in the phase in a two slit experiment for example [333]. This shift is given by

$$\Delta\delta_{\hat{B}} = \frac{e}{\hbar} \oint \boldsymbol{A} \cdot \boldsymbol{ds} \tag{10.30}$$

while the shift due to the electric charge would be

$$\Delta\delta_{\hat{E}} = \frac{e}{\hbar} \oint A_0 dt \tag{10.31}$$

where A_0 is the electrostatic potential. In the above formulation we would have

$$\boldsymbol{A} \sim \frac{1}{c} A_0 \tag{10.32}$$

Substitution of (10.32) in (10.30) and (10.31) shows that the magnetic effect is $\sim \frac{v}{c}$ times the electric effect.

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