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Brick
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Lee

Advances in Quantitative Analysis of Finance and Accounting

Essays in Microstructure in Honor of David K. Whitcomb

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Editors

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Preface to Volume 3

Advances in Quantitative Analysis of Finance and Accounting is an annual publication designed to disseminate developments in the quantitative analysis of finance and accounting. The publication is a forum for statistical and quantitative analyses of issues in finance and accounting as well as applications of quantitative methods to problems in financial management, financial accounting, and business management. The objective is to promote interaction between academic research in finance and accounting and applied research in the financial community and the accounting profession.

This volume contains eleven papers in microstructure. These papers have been classified into three sections: i) Economics of Limit Orders, ii) Essays on Liquidity of Market, and iii) Market Rationality. The overall highlight of these papers can be found in the introduction written by Ivan Brick and Tavy Ronen.

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Introduction

Ivan E. Brick and Tavy Ronen
Rutgers University, USA

Once an obscure subfield of finance, Market Microstructure has emerged as a major stream of finance. In its narrowest sense, microstructure might be defined as the study of the level and the source of transactions costs associated with trading. It examines the organizational structure of exchanges and how the specific market structure enhances the efficiency, transparency and information dissemination of security trading. In a broader sense, this field has opened new methods and directions from which to examine pre-existing theories and puzzles in finance, in both the investments and corporate finance areas. It has seemingly created the most innovative and popular link between the two areas. In such, it can be viewed as way of thought, as opposed to a subfield.

A major contribution of microstructure can be seen in the advancement of our understanding of market efficiency. In particular, we can now use intraday data to examine the speed of information incorporation into security prices when major corporate announcements take place. Similarly, our understanding of asset pricing has been altered with the advent of high frequency data analysis. Traditional asset pricing models focus on the formation of equilibrium security prices based upon the moments of distribution of the underlying cash flows of the security and attribute changes in security prices to changes in information structure of the market. In contrast, market microstructure recognizes that the actual transaction prices and variances do not necessarily equal those determined by our financial models. Thus, the emphasis of market microstructure becomes the study of the deviations between the transaction price and the equilibrium price, with deviations attributed to such factors as liquidity, market structure, transaction costs, and inventory-based adjustments. Clearly, the growing body of research in this field has uncovered and revisited many of our traditional theories, shedding new light on the interpretation of our markets.

This book is a tribute to the field of microstructure and to David K. Whitcomb, Professor Emeritus at Rutgers University, who is one of its foremost pioneers. Like the field itself, David Whitcomb's contributions have had an impact both in their academic rigor and practical applications. His articles have appeared in *The American Economic Review, The International Journal of Finance, The Journal of Banking and Finance, The Journal of Finance, The Journal of Financial Economics, The Journal of Financial & Quantitative Analysis, The Journal of Industrial Economics, The Journal of Money, Credit & Banking, The Journal of Political Economy, Management Science, and The Review of Economics and Statistics.* He is author of one book, *Externalities and Welfare* (Columbia University Press, 1972), and co-author of two others, *The Microstructure of Securities Markets* (Prentice-Hall, February 1986), and *Transaction Costs and Institutional Investor Trading Strategy* (*Salomon Brothers Center for the Study of Financial Institutions Monograph Series*, 1988).

Besides his principal research interest in market microstructure, his other research interests include credit market theory, industrial organization, and economic theory. He is listed as one of the leading researchers in financial economics as measured by citations to his research in leading financial economics journals over the 25 years — 1974 to 1998 (see Chung, Cox, and Mitchell, "*Citation Patterns in the Finance Literature*," Financial Management, 2001).

Dave Whitcomb served as a faculty member in the Finance and Economics department at the Rutgers Business School for over 25 years, until he retired in 1999 as Professor Emeritus. Today, he devotes himself to Automated Trading Desk Inc. (ATD), the "microstructure" company he founded. ATD's brokerage subsidiary now trades over 65 million shares per day, mostly in the NASDAQ market and mostly via fully automated limit orders. Automated Trading Desk Inc. is the first expert system for fully automated limit order trading of common stocks. ATD is located in Mt. Pleasant, SC, has 50 full time employees and a subsidiary broker–dealer firm holding membership in the NASD, and trades about 65 million shares/day (over 2% of total NASDAQ volume). Whitcomb won the regional 2001 Entrepreneur of the Year award (sponsored by Ernst & Young, USA Toda, and NASDAQ) for financial services for the Carolinas.

In October 2002, we (Ivan Brick and Tavy Ronen) and Michael Long organized a conference at the Rutgers Business School of Rutgers University in honor of David K. Whitcomb. The conference was sponsored by the Whitcomb Center for Research in Financial Services. This conference showcased papers and research conducted by the leading luminaries in the field of microstructure and drew a broad and illustrious audience of academicians, practitioners and former students, all who came to pay tribute to David.

This book is a collection of 11 original studies in the field of microstructure, the first seven of which were presented at the conference in October 2002, across different subareas, and each reflecting the future directions of research. We have loosely divided the book into three sections: Economics of Limit Orders, Essays on Liquidity of Markets and Market Rationality.

The first section of the book addresses the important issue of optimal limit order book structure. This is a central focus of the microstructure literature today, in part because of the growing use of the limit order book in most major exchanges and markets, both domestically and internationally, in the trade of equities, derivatives, bonds, and foreign exchange. The chapters in this book that examine the optimality of the limit order book, as well as its characteristics and resulting efficiency all take a different perspective in analyzing this increasingly popular market mechanism. "Single Price Limit Order Books, Discriminatory Limit Order Books, and Optimality," by Lawrence Glosten establishes that the limit order book is not only inevitable, as suggested by his earlier paper, "Is the electronic limit order book inevitable?" (Glosten, *Journal of Finance*, September 1994), but also *optimal* in most instances. The analysis incorporates asymmetric information, inventory related costs and potential liquidity difficulties in the derivation and characterization of the equilibrium. The paper shows that a Centralized Limit order book is indeed optimal, implying that if a regulatory authority could choose and protect a single market mechanism, it would most probably choose the limit order book mechanism. Another interesting result of the paper is that a uniform price clearing mechanism can never be optimal in a setting where private information is present. The negative profits that Glosten shows to exist in such an environment are surprising in light of the fact that opening clearings on most exchanges use a uniform price procedure.

The second paper in this section, "Electronic Limit Order Books and Market Resiliency: Theory, Evidence, and Practice," by Mark Coppejans, Ian Domowitz, and Ananth Madhavan further addresses the question of market design by examining the liquidity provision of electronic limit order books. This is an important feature for market structure to consider, since despite the advantages of speed and simplicity attributed to automated auctions, a relevant concern is whether the lack of designated dealers compromise the consistency of liquidity levels. This paper develops a theoretical model to predict the impact of economic shocks on the resiliency of the limit order book system. Resiliency is defined as the speed with which the market absorbs economic shocks. The paper uses data from actual trade executions of an automated index futures market limit order book. While volatility shocks are found to reduce liquidity, the liquidity shocks dissipate quickly, implying that the electronic order limit book system is highly resilient. The policy implications of these findings are immediate: While trading halts following sharp market movements are desirable for efficient price discovery, they need not necessarily be long in duration to achieve their goal. Further, the results of this paper imply that informed traders take advantage of the depth reported by electronic limit order books to break up their trades and thereby minimize price impact of their trades.

The third paper in the limit order book section, "Notes on a Contingent Claims Theory of Limit Order Valuation" by Bruce Lehmann illustrates that limit order markets can create windows of opportunity for traders to pocket arbitrage profits if price priority rules govern order matching. These profits can be captured by simultaneously writing calls and placing a limit buy order, which in turn can be seen as a call option on a stock. The investor's profit is then the call option premium, assuming frictionless markets. Interestingly, the inclusion of time priority as a secondary execution rule does not completely eliminate potential arbitrage profit. This paper illustrates examples in which event time and calendar time differ but can coincide such as to precede continuous trading in most equity markets. The economics involve assuming that limit order traders (as suppliers of liquidity) span desired trading in event time.

In "The Option Value of the Limit Order Book," by Alex Frino, Elvis Jarnecic and Thomas H. McInish, the option value of the limit order book is calculated for a sample of ten actively traded stocks from the Australia Stock Exchange at 11 a.m. each day. The authors find that the option value of the limit order book is stable for the 11 a.m. snapshot over the sample period of September 3 to December 31, 2001. Interestingly, they also find that 33.1% of the option value of the limit order book is provided at the best ask and 34.7% at the best bid. Moreover, the paper concludes that the option value of the entire limit order book is more stable than both the value of an individual limit order option and the number of shares in the limit order book during that time period.

The second section of the book deals with the liquidity of capital markets. The first chapter of this section is "The Cross-Section of Daily Variation in Liquidity," by Tarun Chordia, Lakshmanan Shivakumar and Avanidhar Subrahmanyam. This paper analyzes cross-sectional heterogeneity in the time-series variation of liquidity in equity markets using a broad time series and cross-section of liquidity data. The authors find that average daily changes in liquidity exhibit significant heterogeneity in the cross-section; that is, the liquidity of small firms varies more on a daily basis than that of large firms.

A steady increase in aggregate market liquidity over the past decade is more strongly manifested in large firms than in small firms. The absolute stock return is an important determinant of liquidity. Cross-sectional differences in the resilience of a firm's liquidity to information shocks are analyzed. The sensitivity of stock liquidity to absolute stock returns is used as an inverse measure of this resilience, and the measure is found to exhibit considerable cross-sectional variation. Firm size, return volatility, institutional holdings, and volume are all found to be significant cross-sectional determinants of this measure.

In "Intraday Volatility on the NYSE and NASDAQ", Daniel Weaver examines differences in intraday volatility between stocks trading on the NYSE and NASDAQ under stable as well as stressful market conditions. Overall results as well as results broken down by industry group show that NYSE stocks exhibit lower volatility than those primarily traded on NASDAQ. Additional analysis that controls for firm specific factors known to be associated with volatility does not change the conclusion of the unrestricted results. In short — NYSE stocks are found to exhibit consistently lower intraday volatility than NASDAQ stocks. This finding is consistent with previous studies and suggests that a specialist market structure is associated with lower volatility.

The next paper, "The Intraday Probability of Informed Trading on the NYSE" by Michael Goldstein, Bonnie Van Ness and Robert Van Ness examines intraday trading patterns for a sample of NYSE stocks during the January through March 2002 time period. The authors use the Easley, Kiefer, O'Hara and Paperman (*Journal of Finance*, 1996) model to infer the probability of informed trading. The paper establishes that trading activity is positively related to the probability of informed trading which is most strongly apparent at both the beginning and the end of the trading period. The authors also document that the amount of regional trading activity is inversely related to the probability of informed trading.

Economic theory would suggest that the price of a NYSE seat should equal the present value of the benefits of being able to trade on the NYSE floor. Testing this proposition has been difficult, as NYSE seats have been relatively infrequently traded. However, in 1978, the NYSE has allowed the leasing of seats, which is the focus of the paper, "Leases, Seats, and Spreads: The Determinants of the Returns to Leasing a NYSE Seat," by Thomas Miller and Michael S. Pagano. These authors find that the lease rates for a sample of NYSE lease rates between 1995 and 2005 are a weighted average of past leasing returns and a set of fundamental factors, including NYSE quoted spreads, NYSE trading volume and market return. Interestingly, past leasing returns are shown to have a stronger impact upon current lease returns than do the fundamental factors.

The next chapter, "Decimalization and Market Quality," by Robin K. Chou and Wan-Chen Lee examines the impact of decimalization on the liquidity of stocks traded in the New York Stock Exchange. Economic theory would suggest that liquidity provided by market makers would be a function of the tick size. By January 29, 2001, all NYSE stocks traded in tick sizes of $0.01. The authors find that spreads decreased significantly after decimalization, but market depth and average volume per trade decreases as well. The authors argue that these results are due to front-runners, traders who offer marginally better prices to gain priority pushing market makers who are willing to provide greater depth to the market.

Section 3 of the book devotes itself to the rationality of the market. The first paper of this section, "The Importance of Being Conservative: An Illustration on Natural Selection in a Futures Market," Guo Ying Luo presents an evolutionary model of natural selection, with traders modeled as being pre-programmed with inherent behavioral rules. Two distinct types of traders are assumed. A conservative buyer has a lower probability of over-predicting the spot price than other traders. A conservative seller has a lower probability of under-predicting the spot price. Guo demonstrates that natural selection will redistribute wealth from less conservative traders to more conservative traders. As long as the conservative traders have some positive probability of making an accurate prediction of the spot price, the presence of these traders will ensure the convergence to an efficient market.

The final chapter of this section and book is "Speculative Non-Fundamental Components in Mature Stock Markets: Do They Exist and Are They Related?" by Ramaprasad Bhar and A. G. Malliaris. The authors assume that rational (or speculative) bubbles, when prices deviate from fundamental pricing factors may arise from asset price arbitrage conditions. The authors employ a new empirical methodology to test for the existence of these bubbles in four mature markets in the United States, Japan, England, and Germany. The methodology employed allows for the decomposition of stock prices into fundamental and non-fundamental factors. The paper finds support for the existence of rational bubbles and that bubbles in the US create bubbles in the other three markets. There is however no evidence for reverse causality.

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Section I

Economics of Limit Orders

Chapter 1

Discriminatory Limit Order Books, Uniform Price Clearing and Optimality

Lawrence R. Glosten
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The paper provides new results on the optimality of a centralized limit order book. In an environment in which traders optimally choose their trade quantity in response to the terms of trade they face, the analysis shows that a centralized limit order book is optimal in the following sense: the equilibrium in a limit order book corresponds to the welfare optimum for some set of welfare weights. The paper also provides a new analysis of a uniform price limit order book with endogenous trade.

Keywords: Market microstructure; market design; limit order markets.

1. Introduction

The answer to the question "Is the electronic limit order book inevitable?" in Glosten (1994) is a qualified "yes." Theoretically, the quote-based competition in a limit order book mimics the competition that occurs across exchanges. Thus, an efficient approach to market design is the development of the Centralized Limit Order Book (CLOB). In the past few years, the resilience of the electronic limit order book has become evident. Markets that have changed over to the electronic limit order book in Paris and Toronto have been quite successful. In the US, Nasdaq faces formidable competition from such trading venues as the ECN, Island. Thus, competition has indeed led to the electronic limit order book being a prominent trading venue. Neither the theoretical result nor the observed success of limit order markets says anything about the optimality of a CLOB. That is the focus of this paper, and the results generally support the inevitability of a CLOB — if a regulatory authority could choose and protect a single market mechanism it would quite likely choose a limit order book.

This paper takes the point of view that the market design question is most interesting for securities that face potential liquidity difficulties. Hence, problems with asymmetric information, inventory related costs and, potentially, a relatively few number of individuals willing to supply liquidity are all features of the analysis. Asymmetric information played an important part of the analysis in Glosten (1994) whereas, notably, a small number of strategic competing quoters did not. This feature recalls the analysis of Biais *et al.* (2000), which provides a characterization of equilibrium in a CLOB with strategic quoters. Like that paper, this paper focuses on some special cases of the environment in order to derive and characterize the equilibrium.

The question being asked in this paper places it in the relatively small literature that addresses the question of market design. It is most closely related to Viswanathan and Wang (VW) (2000), which examines the welfare properties of a discriminatory (each limit order pays of receives its limit price) CLOB with the equilibrium in a market with a finite number of strategic dealers all trading at the same price (or alternatively, a uniform price limit order book). The notable difference between this paper and VW is that while the distribution of trade sizes is specified exogenously in VW, this paper derives the equilibrium trade distribution based on the exogenously specified distribution of trader "types." That is, based on an individual's type and the terms of trade offered, the agent decides how large a trade to make. As the analysis of VW shows, and this paper confirms, the terms of trade determined by equilibrium in the discriminatory price CLOB are quite different from that in a uniform price clearing. Thus, one might expect the distributions of trade sizes to be different in the two settings. Consideration of elastic trade demand also allows a measure of welfare which includes the quoters. With inelastic trade, the cost to a trader is a benefit to the quoters and hence the total surplus is unaffected.

As with the papers cited above, the analysis is of the market at a point in time. Conceptually, the market is presumed to consist of a sequence of such equilibria. The paper does not analyze the trade-off between market orders and limit orders. This requires a dynamic model and is beyond the scope of this paper.

The outline of the paper is as follows. Section 2 lays out the economic environment and discusses the measure of welfare to be used. The subsequent section analyzes the optimum market design given this measure of welfare. This is followed by an analysis of equilibrium in a CLOB and a uniform price clearing with the major welfare result. The paper concludes with some observations on the relevance of the results for the regulation and design of markets.

2. The Economic Setting

The model to be analyzed considers the trade in a single security with a risky payoff, X. All of the analysis will be in terms of deviations from the current estimation of the value of the security. Hence, we can take $E[X] = 0$. The model considers a moment of time in which a single transaction takes place. Thus, the model is of the "Glosten–Milgrom" type rather than the "Kyle" type in which orders are aggregated. Following a trade, expectations will be updated and the market will continue on with another order.

The world is populated by two types of agents — a large number of potential "market order" users who observe the terms of trade and decide what quantity to buy or sell, and a relatively small number of agents who stand ready to take the other side of the market orders and hence supply liquidity by quoting. To conserve on verbiage, call the two market participant types "traders" and "quoters," respectively.

A trader observes the terms of trade and determines an optimal trade by setting his or her marginal valuation equal to the marginal price. More specifically, a typical trader of type t maximizes preferences which are a function of type, quantity and amount spent $U(t, Q, R(Q))$, where $R(Q)$ is the amount paid to buy Q shares ($Q > 0$), or the amount received to sell $-Q$ shares ($Q < 0$). Given the terms of trade, $R(.)$, the optimal amount to trade by a type t trader, $Q(t)$, is the solution to (if $Q(t)$ is not equal to zero)

$$\begin{aligned} &U_2(t, Q(t), R(Q(t)))/-U_3(t, Q(t), R(Q(t))) \\ &= V(t, Q(t), R(Q(t))) = R'(Q(t)), \end{aligned}$$

where $R'(.)$ is the first derivative of $R(.)$. We shall call $V(t, Q, R(Q))$ the marginal valuation of a trader of type t at the trade Q. For the analysis in this paper, it will be assumed that V does not depend upon $R(Q)$ and in that case we will write the condition that determines $Q(t)$ as $V(t, Q(t)) = R'(Q(t))$. In this case, V, with t fixed, is interpretable as individual t's demand curve for shares. To simplify the presentation, and provide for explicit derivations, the special case of a linear demand curve will be considered: $V(t, Q(t)) = t - Q(t)$. The coefficient of -1 on Q is without loss of generality since any other coefficient can be thought of as changing the units in which Q is measured.

In general, a trader's type would involve a specification of all the things that would matter in the portfolio and trading decision — information, existing position in the security, positions in securities with payoffs correlated with this specific security, etc. For tractability this paper assumes that the type is one dimensional. Thus, for example, and drawing from the ubiquitous normal exponential utility example, the type might be given by $t =$ constant $*$ $E[\text{payoff}|\text{information}]$ — endowment of shares. No one but this agent can know what his or her information is or endowment of shares, and hence the type of an arriving trader is a random variable Z, a particular realization of which is t. The random variable Z has a cumulative distribution $F(.)$ and density $f(.)$. As will be seen, distributions that satisfy the following will be particularly useful:

$$[1 - F(t)]/f(t) = a - bt, \quad \text{for } 0 < t < a/b, \quad a, b > 0;$$
$$F(t)/f(t) = a + bt, \quad \text{for } -a/b < t < 0.$$

For example, $b = 1$ corresponds to a uniform distribution on $(-a, a)$. Extending the domain of b to $b = 0$ corresponds to an exponential distribution. It should be noted that VW use a similar distribution restriction, but the distribution there is the exogenous distribution of trade quantities. Here it is the exogenous distribution of types.

There are N identical quoters, supplying liquidity to the market. Supplying liquidity is not costless, however. Specifically, if one of the quoter's participation in a trade is $q(t)$, then the cost to supplying liquidity is $C(t, q(t))$. Thus, in any symmetric equilibrium the total profit (to all quoters) from a trade from type t, $Q(t)$, will be $R(Q(t)) - NC(t, Q(t)/N)$. It is imagined that this cost arises from two sources. First, there may be trading on private information. Since this private information is included in the type, knowledge of the trader's type would lead the quoters to revise their expectations concerning the payoff, X, on the security. Of course, quoters do not directly observe type, but having observed a total trade, and knowing that a trader chooses a quantity optimally, the agent's type can be inferred from the trade. The second source of cost might be thought of as an inventory cost, and a convenient form for this cost is quadratic. Thus, a convenient specification for the cost function will be:

$$C(t, q) = e(t)q + \rho q^2/2,$$

where $e(t)$ is the revised expectation from seeing a trade from a type t trader:

$$e(t) = E[X|Z = t].$$

It is also useful to define the "upper tail" expectation $E(t)$:

$$E(t) = E[X|Z > t], \quad t > 0,$$

and to note that the derivative of $(1 - F(t))E(t)$ is $-f(t)e(t)$. It is assumed that $1 > e'(t) > 0$, and $e(0) = 0$.

There is, of course, a corresponding "lower tail" expectation but it will not be needed in this analysis since the model will analyze the market for types

$t > 0$ — i.e., the paper looks at the offer side of the market. The analysis of the bid side is symmetric.

The measure of welfare to be used in this paper is not uncontroversial. Specifically, the paper will consider a weighted sum of the profits to quoters and the "willingness to pay" (or "consumer surplus") of the trader averaged over all types t. Thus, if a trader of type t arrives, the quoters receive $R(Q(t)) - NC(t, Q(t)/N)$, while the surplus to the trader is the integral under his or her demand curve less the amount paid. The total surplus associated with this trader of type t is:

$$w_T(t) \int_0^{Q(t)} V(t, q)\mathrm{d}q - R(Q(t)) + w_Q[R(Q(t)) \\ - NC(t, Q(t)/N)] = \mathrm{SUR}(t).$$

The *ex ante* welfare is then $E[\mathrm{SUR}(Z)]$.

Given our assumption about the nature of the individual demand curve, the "willingness to pay" of a trader of type t is merely a monetizing of utility so that it can be compared with the profits of the quoters. What is more controversial is measuring *ex ante* welfare with the weighted average of the total surplus. In particular, the average willingness to pay is not the same thing as the *ex ante* willingness to pay. This measure is used, because it is quite tractable. Those who object, should mentally put quotation marks around the word optimal for the rest of the paper. It should also be noted that this formulation allows for a large number of welfare measures, depending upon the weights applied to individual types.

The measure allows for different weighting on the quoters and the traders, and for different weights for each type. To allow this seems reasonable. Furthermore, if the weight on quoter profits does not depend upon the type t, then maximization of $E[\mathrm{SUR}(Z)]$ can be thought of as maximizing trader surplus subject to the quoters earning at least some specified profit level (to cover fixed costs, for example). Choosing the profit level amounts to choosing the weight w_Q. With this setup, we can consider the optimal terms of trade in Section 3.

3. Optimum Terms of Trade

As previously mentioned, we will consider the simplest case of a linear demand curve, $V(t, q) = t - q$ and cost depending upon inventory and expectation revisions. In this environment trader surplus, at a trade $Q(t)$ is merely $tQ(t) - 0.5Q(t)^2 - R(Q(t))$, while total quoter surplus is $R(Q(t)) - e(t)Q(t) - 0.5\rho Q(t)^2/N$. Choosing the optimum terms of trade then consists of choosing the function $R(Q)$ and hence $Q(t)$ via the traders optimality condition to maximize the measure of welfare. It is easier, mathematically, however, to consider the problem of finding the optimal function $Q(t)$ which can then be used to find $R(Q)$. There are several constraints on the problem. First is the constraint that $R'(Q(t))$ be equal to the trader's marginal valuation $t - Q(t)$. Second, $Q(t)$ should be nonnegative for positive t. If this were not the case, then traders would be able to sell at the offer and buy at the bid. However, only quoters are allowed to do this. Thus, we will allow solutions of the form $Q(t) = 0$ for $-t_0 < t < t_0$. This, in effect, allows for the "zero quantity spread" as in Glosten (1994). Third, we will constrain $R(0)$ be zero. To allow this to be positive, for example, would require nontraders to pay for a trade they do not make. Finally, we must have $Q'(t)$ positive if $Q(t)$ is positive for t greater than t_0. This is to ensure that the second order condition holds for the trader's optimization problem. To see this, note that the second order condition for a trader of type t is $-1 - R''(Q(t)) < 0$, or $R''(Q(t)) > -1$. Differentiating the optimality condition $t - Q(t) - R'(Q(t)) = 0$, shows that $R''(Q(t)) = (1 - Q'(t))/Q'(t)$. The constraint above can only be satisfied if $Q'(t) > 0$.

Putting this all together, the welfare maximization problem is:

$$\text{Max} \int_{t_{0t}}^{\infty} \big\{ w_T(t)[tQ(t) - 0.5Q(t)^2 - R(Q(t)] \\ + w_Q[R(Q(t)) - e(t)Q(t) - 0.5\rho Q(t)^2/N]\big\} f(t)\mathrm{d}t$$

s.t.

$$R'(Q(t)) = t - Q(t),\ Q(t_0) = 0,\ t_0 \textit{free},\ Q(t) > 0 \Rightarrow Q'(t) > 0.$$

Define $g(t)$ to be $w_T(t)f(t)$. Furthermore, let $G(t)$ to be the upper tail integral of $g(t)$. Integrate by parts the integral in the maximization. The first term in square brackets will have the integrand:

$$G_T(t)[Q(t) + tQ'(t) - Q(t)Q'(t) - R'(Q(t))Q'(t)] = Q(t)G_T(t),$$

since $R'(Q(t)) = t - Q(t)$.

The second term will have the integrand (after substituting for $R'(Q(t))$):

$$Q'(t)[t - Q(t) - E(t) - \rho Q(t)/N]w_Q(1 - F(t)) \\ = w_Q\{[(1 - F(t))]Q'(t) - [(1 - F(t))E(t)]Q'(t) \\ - [1 - F(t)]Q(t)Q'(t)(1 + \rho/N)\}.$$

Integrate this expression again by parts (the square brackets surround the "u" term, the second term is "dv"). This, with the expression above for the trader welfare yields the integrand:

$$f(t)w_Q\{tQ(t) - (1+\rho/N)Q(t)^2/2 - e(t)Q(t) \\ - [(1-F(t))w_Q - G(t)]Q(t)/(w_Q f(t))\}.$$

Maximizing the integral of this merely requires calculus and yields the solution, $Q_o(t)$:

$$t - Q_o(t)(1+\rho/N) - e(t) - (w_Q(1-F(t)) - G(t))/f(t))w_Q.$$

In other words, at the optimum, the marginal value to the trader of an additional unit is set equal to the marginal cost of supplying that unit plus a term to ensure the minimum level profit. Solving:

$$Q_o(t)(1+\rho/N) = t - e(t) - [1-W(t)][(1-F(t))/(f(t))],$$

where $W(t)$ is the upper tail expectation of $w_T(t)$ relative to w_Q; i.e.:

$$W(t) = \int_t^\infty f(t)w_T(t)/w_Q \,\mathrm{d}t/(1-F(t)).$$

Notice that all that is important is the trader weight relative to the quoter weight.

The constraint on the derivative was not used. Since we have in mind a situation in which private information motivates only part of the trade, $e(t)$ should increase slower than t. For a wide class of distributions, $(1-F(t))/f(t)$ is nonincreasing and hence $Q_o(t)$ should be increasing at least for weights independent of t and less than one. The above also ignores the constraint that the optimum should be nonnegative and zero at t_0. Once the distribution function, weights and $e(.)$ are specified, t_0 can be found by setting the expression equal to zero. For example, for $1 - W(t) = 1 - w > 0$, $(1-F(t))/f(t) = a - bt$, and $e(t) = \alpha t$, the welfare optimum quantity for a trader of type t is given by:

$$Q_o(t) = B_o t - A_o \quad \text{for } t > t_o = A_o/B_o,$$
$$B_o = (1 - \alpha + b(1-w)/(1+\rho/N),$$
$$A_o = a(1-w)/[(1+\rho/N)].$$

The optimum involves a small trade spread since $t_o > 0$. To see this, note that $R'(0) = R'(Q(t_0)) = t_0 - Q(t_0) = t_0 > 0$. This is reminiscent of a CLOB when there is private information. In that case, the small trade spread arises out of quoters' realization that the first quote will be hit on not only small trades, but large trades as well. Thus, the small trade quote recognizes the informational consequences of all sized trades. The logic for the small trade spread in the optimum is different. Imagine reducing the small trade spread so that potential traders with small t traded a small quantity. The increase in trader welfare would be small since the surplus for small type traders is small. The effect on trader profits would be larger, however. By moving the marginal pricing schedule down, traders of other types would choose to make larger trades, and this would decrease the profits to the quoters. This latter effect is missing in the VW analysis, since in their model the quantity traded is specified exogenously.

Before going on to the analysis of the CLOB, it is useful to consider the aggregate profits to the quoters as a function of the relative weights, $W(t)$. Note that the integrand for the quoter profit term is (after integrating by parts):

$$\begin{aligned}&(1 - F(t))Q_o'(t)\{t - Q_o(t) - E(t) - \rho Q_o(t)/N\}\\&\quad = (1 - F(t))Q_o'(t)\{(1 - W(t))(1 - F(t))/f(t) + e(t) - E(t)\}.\end{aligned}$$

Since $E(t)$ exceeds $e(t)$, relative trader weights of one or larger would lead to the quoters getting negative profits. This suggests that realistic welfare optima should involve relative trader weights smaller than one, and hence a small trade spread seems likely for the optimum.

The above analysis is summarized in the following proposition.

Proposition 1

Let $V(t, Q) = t - Q$ be the demand curve for an individual of type t. Let $C(t, q) = e(t)q + 0.5\rho q^2$ be the cost to a single liquidity supplier of providing a quantity q. Then, the welfare optimum quantity purchased by a trader of type t is given by the following: $Q_o(t) = \{t - e(t) - (1 - W(t))(1 - F(t))/f(t))\}/(1 + \rho/N)$, $t > t_0$. Quoter profit is:

$$\int_{t_0}^{\infty} (1 - F(t))Q_o'(t)\left\{\frac{(1 - W(t))(1 - F(t))}{f(t)} + e(t) - E(t)\right\} \mathrm{d}t,$$

$$W(t) = \int_{t}^{\infty} f(t)w_T(t)/w_Q \,\mathrm{d}t/(1 - F(t)).$$

There are two robust features of the optimum. First, and as noted above, since $w_t(t)$ less than one is a reasonable restriction on the welfare weights, there will be a small trade spread. Second, the quantity chosen for the top "type" satisfies marginal value equals marginal cost, and this is independent of the weighting placed on quoter profits. As we will see, these are also features of the CLOB.

4. Discriminatory CLOB and Uniform Price Clearing

4.1. *CLOB*

In order to provide the analysis with the minimum complication, as above, I shall describe the equilibrium with the simplest specification — the marginal valuation of a trader is given by $V(t, Q) = t - Q$ and the cost function for the quoters is given by $C(t, q) = e(t) - \rho q^2/2$. The discriminatory limit order book with N competitors will be considered first.

Let $1 - F^*(p)$ be the probability that the next purchase arrival will lead to a stop-out price (highest price) greater than p, and let f^* be the associated density. The asterisk indicates that this distribution is derived from the exogenous type distribution, but needs to be derived as part of the equilibrium. Also, let $e^*(p)$ be the revised expectation of the payoff conditional on the stop-out price being p and $E^*(p)$ be the associated upper tail expectation. Consider the problem of quoter number 1. He or she will provide quantity $q'(p)\mathrm{d}p$ at the price p. Thus, the profit to quoter number 1 is:

$$\int_{p_0}^{\infty} f^*(p)\left\{\int_{p_0}^{p} q'(s)s\,\mathrm{d}s - e^*(p)q(p) - 0.5\rho q(p)^2\right\}\mathrm{d}p.$$

After integrating by parts, the profit can be expressed as:

$$\int_{p_0}^{\infty} (1 - F^*(p))(p - E^*(p) - \rho q(p))q'(p)\mathrm{d}p.$$

The probability that the stop-out price exceeds a price p is the probability that a trader's marginal valuation exceeds p at the trade $Q(p)$, the total number of shares offered at the price p or less. That is, $1 - F^*(p) = P\{Z - Q(p) > p\} = 1 - F(p + Q(p))$. Similarly, $E^*(p)$ is given by $E^*(p) = E(p + Q(p))$. The quoter under consideration considers the quantities supplied at each price by the other $N - 1$ quoters as given. Thus, $Q(p) = q(p) + (N - 1)q_L(p)$. Thus, from this quoters point of view, expected profits are given by:

$$\int_{p_0}^{\infty} (1 - F(p + Q(p)))(p - E(p + Q(p)) - \rho q(p))q'(p)\mathrm{d}p.$$

Maximizing this is a simple calculus in variations problem. The derivative of the integrand with respect to $q(p)$ is:

$$q'(p)f(p + Q(p))\{-p + e(p + Q(p)) + \rho q(p)\} - \rho(1 - F(p + Q(p)))q'(p).$$

The derivative with respect to $q'(p)$ is:

$$(1 - F(p + Q(p)))(p - E(p + Q(p)) - \rho q(p)).$$

After taking the derivative of this latter expression and setting it equal to the first expression and summing over all quoters one gets that the total amount supplied at a price p or less, $Q(p)$, is given as the solution to the differential equation:

$$f(p+Q(p))\left(1+\frac{N-1}{N}Q'(p)\right)(p-e(p+Q(p))-\rho Q(p)/N)$$
$$=1-F(p+Q(p)).$$

Recall that $Q(p)$ is the quantity offered at price p or less. Thus, p is the marginal price for a trade of size $Q(p)$. Now make two changes of variable. First, define the marginal price, by $p = R'(Q(p))$ and, define the function $p(Q)$ by $Q(p(Q)) = Q$. Evaluating at $p(Q)$ we have:

$$f(R'(Q)+Q)\left(1+\frac{N-1}{NR''(Q)}\right)(R'(Q)-e(R'(Q)+Q)-\rho Q/N)$$
$$=1-F(R'(Q)+Q).$$

Now evaluate the above at $Q_L(t)$ the traders optimum: $t-Q_L(t)=R'(Q_L(t))$ and note that the trader's first order condition implies that $1-Q'_L(t)=R''(Q_L(t))Q'_L(t)$. After substituting:

$$t-Q_L(t)-e(t)-\rho Q_L(t)/N-\frac{N(1-Q'_L(t))}{N-Q'_L(t)}\frac{(1-F(t))}{f(t)}=0.$$

Before examining this expression, which looks remarkably like the expression for the optimum, it is useful to get some intuition for how the competition between strategic quoters works in this market. Consider the effect of one quoter adding a small amount h, at the price p. If this quantity transacts at the price p, then the profit per unit is $p-E(p+Q(p))-\rho q(p)$. The upper tail expectation is used since this quantity will transact if the stop-out price is p or larger. The probability of this happening is $(1-F(p+Q(p)))$. Thus, the effect on expected profits at p is $(1-F(p+Q(p)))(p-E(p+Q(p))-\rho q(p))$. However, the addition of h shares at p shifts the whole schedule for prices larger than p. Now, in order to have a quantity at price s picked off, the type has to be $s+Q(s)+h$ or larger. At each price s, the marginal effect on expected profits is (since $q'(s)\mathrm{d}s$ is offered at s) $(1-F(s+Q(s)+h))\{s-E(s+Q(s)+h)-\rho(q(s)+h)\}q'(s)\mathrm{d}s$. For h small, the effect on profits is: $hq'(s)f(s+Q(s))\{-s+e(s+Q(s))+\rho q(s)\}-\rho(1-F(s+Q(s)))q'(s)\mathrm{d}s$. Integrating over all prices larger than p provides the total marginal effect of an increase in quantity at a price p on the expected profits at all larger prices:

$$(1 - F(p + Q(p)))(p - E(p + Q(p)) - \rho q(p))$$
$$+ \int_p^\infty q'(s)(f(s + q(s))(-s + e(s + Q(s)) + \rho q(s))$$
$$- \rho(1 - F(s + Q(s))).$$

At the optimum, the expected marginal effect at the price p and all higher prices should be zero. Taking the derivative of the above provides conditions identical to the ones analyzed above.

Consider the case of $(1 - F(t))/f(t) = a - bt$, and $e(t) = \alpha t$. There is a linear solution, given by $Q_L(t) = B_L t - A_L t > A_L/B_L, A_L = a(1 - B_L)/\{(1 + \rho/N)(1 - B_L/N)$, and B_L satisfies the quadratic equation $B^2 - BN\{1 + (1 - \alpha)/(\rho + N) + b/(1 + \rho/N)\} + N\{(1 - \alpha)/(1 + \rho/N) + b/(1 + \rho/N)\} = 0$.

Proposition 2 summarizes the above analysis.

Proposition 2

Suppose that a trader of type t has a demand curve given by $t - q$. Further suppose that the cost of supplying liquidity is $C(t, q) = e(t)q + 0.5\rho q^2$. If there are N competing quoters, then the equilibrium quantity traded by a trader of type t, $Q_L(t)$ satisfies the following differential equation:

$$t - Q_L(t)(1 + \rho/N) - e(t) - \frac{N(1 - Q'_L(t))}{N - Q'_L(t)} \frac{(1 - F(t))}{f(t)} = 0, \quad t > t_0.$$

As with Proposition 1, one can see that $(t, Q) = (0, 0)$ does not satisfy the equation and hence the equilibrium in the CLOB looks much like the optimum. It is also interesting to note that if there is a maximum type, T, then $T - E(T) - Q_L(T)(1 + \rho/N) = 0$, and, except for the risk sharing term, ρ/N, this is independent of the number of competitors. For the maximum type, marginal valuation is equal to marginal cost of taking the other side of the trade.

4.2. *Uniform price clearing*

Interestingly enough, the analysis of the uniform clearing price equilibrium is far more complicated than the discriminatory CLOB with endogenous trade. It is also far more complicated than the analysis of the uniform price clearing equilibrium with exogenous trade. With exogenous trade, the equilibrium is independent of the trade distribution. This is not the case with endogenous trade. The reason is that with endogenous trade, an agent adding quantity at a particular price has two effects. First, it increases his or her share of the order flow, but it also encourages greater order flow. How much greater order flow depends upon the distribution of the traders' types.

The analysis is carried out only for the special case of linear demand curves of traders and the special cost function arising out of private information and inventory costs. As before, we have that a typical agent, taking the actions of others as given maximizes ($f^*(p)$ is the endogenously determined distribution for the stop out price and $e^*(p)$ is the revision in expectations if the stop out price is p):

$$\int_0^\infty f^*(p)(pq(p) - e^*(p) - 0.5\rho q^2)\mathrm{d}p.$$

Notice that in this formulation, p is the average price rather than the marginal price in the CLOB analysis. Integrating by parts one obtains:

$$\int_0^\infty (1 - F^*(p))(pq'(p) + q(p) - q'E^*(p) - \rho q(p)q'(p))\mathrm{d}p.$$

Now, however, $1 - F^*(p)$ is given by the following (recall that F is the distribution of trader type):

$$1 - F^*(p) = 1 - F\left(p + Q(p) + \frac{Q(p)}{Q'(p)}\right)$$

and similarly for $E^*(p)$, where $Q(p)$ is the total quantity offered by all N competitors when the stop out price is p. Taking the other $N - 1$ quantities as given, a typical quoter maximizes the above. As before, this is a calculus of variations problem. After finding the first order condition, and then making the same change of variables as in the CLOB analysis, and manipulating the result one obtains that the equilibrium quantity purchases by a trader of type t is the solution to the differential equation:

$$\begin{aligned}&\frac{N-1}{N}(t - Q(t) - Q(t)P'(Q(t)) - e(t) - \rho Q(t)/N)f(t)\\&\quad = \frac{Q(t)P'(Q(t))}{N}\left(f(t) + \frac{\mathrm{d}}{\mathrm{d}t}f(t)(t - Q(t) - e(t) - \rho Q(t)/N\right)\end{aligned}$$

as well as $P'(Q(t))Q(t) + P(Q(t)) = t - Q(t)$. What makes this expression different from the exogenous trade case is the inclusion of the derivative term on the right-hand side of the equation. Without that term, the density of the type would disappear. The important thing to note about this expression is that it implies that in equilibrium there is no zero quantity spread. To see this, suppose that there is a t^* with $Q(t^*) = 0$ but $t^* > 0$, in which case $R'(0) = P(0) = t^*$. The right-hand side becomes zero, while the left-hand side is proportional to $(t^* - e(t^*)) f(t^*)$. Under the assumptions that we have made about $e(.)$, this expression is positive and hence the first-order condition is not satisfied. Thus, there are fundamental differences between the uniform price clearing and the CLOB. For the uniform price clearing competition, equilibrium is tied down by $P(0) = 0$, or price is equal to marginal cost. In the CLOB, the equilibrium is tied down by $V(T, Q(T)) = C_2(T, Q(T))$. Interestingly, the only way that the uniform price clearing and CLOB can both lead to quantities linear in the type, t, is if the distribution of types is uniform. These observations motivate the following welfare analysis.

4.3. *Welfare analysis*

The next proposition provides a comparison of the welfare optimum and equilibrium in the CLOB. The main result of this paper is that the equilibrium in a CLOB is the welfare optimum for some set of weights. In particular, if the equilibrium in the limit order book is linear, then the associated welfare weights are constant across types.

Proposition 3
Suppose that a trader of type t has marginal valuation $t - Q(t)$. Further suppose that the cost function for the quoters is of the form $e(t)q(t) + \rho q(t)^2/2$. Then, the optimum for some weight $w_N(t)$ and N quoters is implemented by the CLOB with N competing quoters.

Proof. Suppose that equilibrium in a CLOB with N quoters specifies that a trader of type t optimally chooses $Q_{LN}(t)$. Choose relative trader welfare weights $w(t)$ and number of quoters equal to N to satisfy the following equation:

$$1 - W(t) = N(1 - Q'_{LN}(t))/(N - Q'_{LN}(t)).$$

By inspection (refer to the expressions in Propositions 1 and 2), for these weights, $Q_o(t) = Q_{LN}(t)$.

If the CLOB equilibrium is linear, then $Q'_L(t)$ is a constant, in which case $W(t)$ and hence $w(t)$ are both constant. A related observation is that uniform price clearing can never be optimal as long as there is private information. If the optimum is of the form $Q(t) = bt$, then w must be equal to 1 and it was shown above that this leads to negative profits. This can certainly not be a feature of the equilibrium in the uniform price clearing.

In fairness, it should be stressed that the theorem does not say that any welfare function is maximized by the CLOB. Rather, the CLOB equilibrium corresponds to a particular welfare maximum.

5. Discussion

The limit order book form of market (though not necessarily centralized) is becoming the dominant form of trading throughout the world. This is happening on both a decentralized basis and by regulatory fiat. A prime example of the latter is the adoption by Nasdaq of new order handling rules, which requires Nasdaq dealers to give precedence to limit orders. This adoption was largely forced, and was the result of alleged non-competitive improprieties on the part of Nasdaq dealers. Nonetheless, the above analysis suggests that the move by the SEC was a good one — total welfare can be improved by such a move. It could be argued that Nasdaq is evolving to a hybrid market like the NYSE with an active limit order book and an active dealer. Perhaps, but at the same time, Nasdaq has been losing substantial market share to the ECN's. This points to the inevitability of the limit order book. The analysis in this paper suggests that this is not to be bemoaned.

This paper is not close to the last word on the subject. In particular, the model makes the unattractive assumption that there is a designated set of limit order submitters who have no motive for trade other than profit maximization. This ignores the important fact that traders can choose to use limit orders or market orders. Clearly, adding such a feature to the model would change the equilibrium conditions for the CLOB. However, it is not clear that the conclusion will change too much. As was noted, there are two robust features of the optimum. First, the quantity traded by the highest type is independent of the welfare weights. Second, the small trade spread is positive, and determined by the welfare weights. I find it likely that equilibrium in a CLOB with traders choosing to use market orders or limit orders would in fact exhibit both of these features. The terms of trade for the largest quantity is unlikely to be affected by individuals who have an active reason to trade but use limit orders. After all, trading at the highest price is a rare event. On the other hand, active traders may well reduce the small trade spread. However, they are unlikely to reduce it to zero as has been forcefully shown by Cohen *et al.* (1981). If the spread is reduced to zero, there is still execution uncertainty but no transaction cost advantage to using a limit order. Thus, as Cohen *et al.* argue, the small trade spread will persist.

It is interesting to note that the equilibrium in a uniform price book is quite difficult to analyze. Indeed, this paper provides no assurance that an equilibrium exists and it is known that there are cases in which an equilibrium does not exist, even with a large number of quoters. Perhaps this is related to the absence of such limit order books. Uniform price clearing is used at openings, when trade is aggregated, but not, to my knowledge, in continuous markets.

6. Conclusion

This paper provides a model in which to analyze the optimality of various market structures when trade is determined optimally rather than given exogenously. The main result is that the CLOB with discriminatory pricing (each limit order pays or receives its quote) implements an optimum. Thus, the CLOB is both "inevitable" and "optimal." The analysis shows that a uniform price limit order book (each limit order pays or receives the stop out price) will not implement an optimum. The analysis further shows how complex the analysis of a uniform price order book is with endogenous trade.

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Chapter 2

Electronic Limit Order Books and Market Resiliency: Theory, Evidence, and Practice

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The electronic limit order book has transformed securities markets. Advantages of speed, simplicity, scalability, and low costs drive the rapid adoption of this mechanism to trade equities, bonds, foreign exchange, and derivatives worldwide. But limit order book systems depend primarily on public limit orders to provide liquidity, raising natural questions regarding the resiliency of the mechanism under stress. This paper provides an analysis of the stochastic dynamics of liquidity and its relation to volatility shocks using data from a futures market. Aggregate market liquidity exhibits considerable variation, and is inversely related to volatility, as predicted by our model. However, liquidity shocks dissipate quickly, indicating a high degree of market resiliency. This fact has important practical implications, particularly as regards to institutional trading, and market protocols. We explore these practical issues in detail.

Keywords: Futures market; liquidity; automated auctions.

1. Introduction

The electronic limit order book has transformed securities markets. Advantages of speed, simplicity, and low costs drive the rapid adoption of electronic limit order books to trade equities, bonds, foreign exchange, and derivatives worldwide.[1] Unlike traditional markets, trading in an electronic limit order book does not require a physical exchange floor or intermediaries such as market

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[1] Outside the US and a handful of emerging markets, virtually all equity and derivative trading systems are automated. A partial list of major automated markets includes, for equities, the Toronto Stock Exchange, Euronext (Paris, Amsterdam, Brussels), Borsa Italiana, National Stock Exchange (India), London Stock Exchange, Tradepoint, SEATS (Australian Stock Exchange), Copenhagen Stock Exchange, Deutsche Borse, and Electronic Communication Networks such as Island. Fixed income examples include eSpeed, Euro MTS, BondLink, and BondNet. Foreign exchange examples are Reuters 2002 and EBS. Derivative examples include Eurex, Globex, Matif, and LIFFE. Domowitz (1993) provides a taxonomy of automated systems and updates are contained in Domowitz and Steil (1999).

makers, but depend primarily on public limit orders to provide liquidity. This feature of the electronic limit order book naturally raises questions regarding the mechanism's resiliency when subjected to stress. This paper provides an analysis, both theoretical and empirical, of the stochastic dynamics of liquidity and its relation to volatility. The results have important practical implications. We explore these practical issues in detail.

We use intraday order-level data obtained from the electronic market for Swedish stock index futures (henceforth OMX). We observe the instantaneous demand and supply curves at every point of time, yielding natural metrics for liquidity in terms of market depth, i.e., order flow necessary to move price by a given amount. We find that the variation in liquidity over time is economically and statistically significant, and goes beyond simple calendar time effects. The results suggest that traders can add value by strategic order placement behavior. We present evidence in favor of this hypothesis. In particular, the actual execution costs incurred by traders are significantly lower than the costs that would be incurred under a naïve strategy that ignores time-variation in liquidity. The cost differences are especially pronounced for larger trades, indicating the value of attempting to time trades to take advantage of periodic liquidity surpluses while avoiding liquidity deficits.

We then turn to an analysis of the dynamic relation between liquidity and volatility, as suggested by theory. The use of vector autoregressive models allows us to examine complicated liquidity dynamics and gain insights into the question posed here regarding the viability of systems that rely purely on public limit orders for liquidity. We find that volatility shocks reduce liquidity, as predicted by our model. Shocks to liquidity dissipate quickly, indicating a high degree of resiliency. This self-correcting ability turns out to be an attractive feature of the electronic limit order book, mitigating doubts with respect to the resilience of that form of market structure under pressure.

The paper proceeds as follows: We develop a theoretical model to examine liquidity dynamics in Section 2, empirical results are discussed in Sections 3 and 4, Section 5 discusses the practical implications of the results, and concluding remarks are offered in Section 6.

2. Theory

2.1. *Model framework*

To investigate questions concerning the resiliency of limit order book systems, we develop a simple model. Consider a market where trading takes place in a sequence of trading sessions, indexed by t. Consider a security whose fundamental value at time t is given by v_t. We assume — without loss of generality — that this value evolves as a martingale. For simplicity, the drift rate is normalized to zero. Let p_t denote the security's price at time t. Trading takes place in a series of public auction markets. Denote by N_t the number of price-sensitive traders at time t. The number of traders might vary from period to period as a function of other variables, such as information flows. These traders attempt to maximize their trading profits given a horizon; we focus on a trader with a horizon of k periods, but different traders might have different horizons. They observe a common (noisy) signal m_t regarding the current value of the security. The signal is an unbiased estimate of value. Purely for simplicity, we assume that a trader closes out their position in period $t + k$ with a market order. In period t, denote by Z_t the aggregate signed market order flow. This comes from discretionary traders, noise traders, and is assumed to be viewed by market participants as a stochastic shock with mean zero.

The expected profit of a price-sensitive trader is $(p_{t+k} - p_t)x_t$, where x_t is the position taken by the trader. Traders seek to maximize an objective function comprised of expected profit less a cost of carry. This cost is assumed to be proportional to the total risk of the position, i.e., $c(\sigma x_t)^2$, where σ is the fundamental volatility of the security over the horizon. In the case where future liquidity is stochastic, we include this variation too in interpreting σ. Each trader conjectures (this will be verified later) that the price in the future is

$$p_{t+k} = m_{t+k} - \Lambda_{t+k} x_t. \tag{1}$$

Since expectations follow a martingale $E_t[m_{t+k}] = m_t$, the optimal (profit maximizing) x_t is

$$x_t = \beta_t (m_t - p_t), \tag{2}$$

where $\beta_t = 1/2(\Lambda_{t+k} + c\sigma^2)$. Market clearing requires that in period t

$$\sum x_t(p_t) + Z_t = 0 \tag{3}$$

where the summation is taken over all N_t traders at time t. Substituting the demand schedule, this yields a price functional

$$p_t = m_t + \Lambda_t Z_t, \tag{4}$$

which is of the conjectured form with $\Lambda_t = (\sum \beta_t)^{-1}$. Note that the trader conjectures that other imbalances are, on average, zero when he or she liquidates their initial position. The parameter $D = \Lambda^{-1}$ is a measure of depth or liquidity;

it summarizes the expected price change in response to a unit of market order flow. From this equation, it can be seen that the volatility of price changes is due not only to the volatility of fundamentals, but also the volatility of liquidity interacting with the volatility of order imbalances.

2.2. *Liquidity dynamics*

Different assumptions regarding the process underlying trader arrivals generate different liquidity dynamics. We assume trader arrivals follow an autoregressive process. This assumption is reasonable and yields some interesting special cases. Specifically, we assume $N_t = \mu + \alpha N_{t-1} + \varphi$, where $\mu > 0$, φ is a shock, and $0 \leq \alpha \leq 1$.

An interesting special case of this model has $\alpha = 1$ and $\mu = 0$. (A constant number of traders is a subset of this case with no stochastic variation.) This assumption is reasonable if N itself is a function of primitives such as the cost of gathering information, maintaining a trading presence, etc. that themselves follow martingale-like processes. With no change expected in the number of traders, future liquidity is expected to equal current liquidity, so $E_t[\Lambda_{t+k}] = \Lambda_t$. Then, using the definition of $\Lambda_t = 2(\Lambda_{t+k} + c\sigma^2)/N_t$ (and assuming homogenous traders) this implies that current depth is $D_t = (N_t - 2)/2k\sigma^2$, i.e., depth is decreasing in σ and increasing in N. A decrease in N for any reason reduces depth *permanently*, so the market has *no resiliency* whatsoever.

An alternative special case occurs when $\alpha = 0$ and $\mu > 0$ so that N_t is drawn from a constant distribution. In this case, with homogenous traders, we get $E_t[\Lambda_{t+k}] = 2c\sigma^2 E[1/(N-2)] = \Lambda^*$, so that there exists some long-run or average liquidity level, D^*. Then, $D_t = N_t/2(\Lambda^* + c\sigma^2)$ and current depth is proportional to the number of current traders. All liquidity shocks (from change in N) are purely *transitory* in this special case. The market is *fully resilient*.

In the general case with $\mu > 0$ and $0 < \alpha < 1$, and k large depth is also autocorrelated and mean reverting to a value

$$D^* = \varphi^*/2(1-\alpha)(\Lambda^* + c\sigma^2). \tag{5}$$

Again, depth is inversely related to volatility. The term α captures market resiliency; higher values imply faster recovery of liquidity to its long-term value following a negative shock. The model shows that the extent to which volatility shocks reduce liquidity, and the resiliency of liquidity in response, are empirical questions.

3. Empirical Results

3.1. *Institutional details*

Trading in OMX contracts takes place via a consolidated automated trade execution system, including activity from Sweden, the UK, Denmark, and the Netherlands. It is worth noting that the cross-border automated limit order book system studied here is typical of many markets, including the Toronto Stock Exchange and Paris Bourse, allowing for some confidence that our results are not artifacts of special institutional arrangements.[2] We refer to the overall market as OMX, given the complete integration of trading across countries.

The electronic system functions as a continuous pure limit order book market. Trading on the order book is in round lots of 10 contracts. Orders are prioritized on the book in terms of price, then time. There are two ways in which a trade may be executed. Counterparty limit orders may match on the book in terms of price, in which case the maximum feasible size is filled. Alternatively, a trader may "hit the bid" or "lift the offer," taking up to as much quantity as advertised on the book. This is accomplished by executing a single keystroke and submitting desired volume. Once a trade is completed, unexecuted volume at the trade price remains on the order book, until cancelled. Cancellations of orders are possible at any time.

The trading day is six hours, beginning at 9:00 a.m and ending at 3:00 p.m, GMT. Unlike many automated markets, such as the Paris Bourse, there is no opening algorithm or batch auction at the beginning of the day. With that exception, the design and mechanics of the OMX market are quite similar to that described by Biais, Hillion, and Spatt (1995) for the CAC system, and by Domowitz (1993) for generic price/time priority continuous limit order systems.

Order and trade information are distributed directly from the trading system, making the OMX highly transparent. Specifically, market participants observe a transactions record (price and volume) and the five best bids and offers on

[2]The Paris Bourse data, for 40 stocks, is described and analyzed by Biais, Hillion, and Spatt (1995), and Gouriéroux, Le Fol, and Meyer (2000) provide a factor analysis of the order queue for a single stock. Hollifield, Miller, and Sandås (1999) and Sandås (2001) use OM data for a selection of 10 stocks traded on the equities order book. Some data also are available for trading on the Australian SEATS automated system, Toronto Stock Exchange, and Tel Aviv Stock Exchange.

the book, with aggregate volume at each price.[3] No "indicative" prices or other non-price expressions of trading interest are provided.

3.2. *Data*

Our database comprises the complete limit order book for OMX contracts from the period July 31, 1995 to February 23, 1996. The data are obtained from a trading house that chose the real-time feed, permitting the collection of some historical information for analysis.[4] Prices are denominated in Swedish currency (SEK), and volume is given in number of contracts. Information is time-stamped to the second. Transactions files and order information are matched. The order book is reconstructed from the raw data and completely consistent with transactions reported. An unusual, but valuable, feature of our database is that crosses are isolated, and matched in time with limit order book trading activity. Crosses arising from the so-called "upstairs" market (where large-block trades are negotiated and crossed) can bias any assessment of the real costs of trading and true underlying liquidity of the market if they are not isolated from the analysis.

3.3. *Liquidity metrics*

In what follows, we define market liquidity or depth as the number of contracts offered for sale at up to k ticks from the midquote. We distinguish between liquidity on the buy and sell sides, denoted by $D_b(k)$ and $D_a(k)$, respectively. These measures are natural in that they can be interpreted as the volume necessary to move the price by k ticks. More liquid markets are deeper in that they can accommodate larger trades for a given price impact. Casual inspection of depth by time of day suggests little time variation in liquidity, except for the open. This is incorrect. First, there is considerable variation in observed depth at different times of day even though on average they are roughly equivalent. Second, first-order autoregressive models of depth suggest a moderate degree of mean reversion in liquidity, and a large residual variance relative to mean

[3]There is some facility for the so-called "hidden orders" that are unobserved by traders. As in the analyses of Biais, Hillion, and Spatt (1995) and Hollifield, Miller, and Sandås (1999), we cannot ascertain the effects of such unobservable orders, but their importance in automated systems is generally very limited as discussed by Irvine, Benston, and Kandel (2000).

[4]We thank Lester Loops, who provided the raw numbers and some assistance with issues involved in merging the order and transactions records.

depth. Such results also suggest substantial time variation, but not necessarily that which would be captured by simple time-of-day analysis.

The expected trading costs facing a trader at any point of time, based on the prevailing demand and supply schedules, are also of interest. In particular, consider a market order of size Q (with the sign convention that $Q > 0$ represents a purchase and $Q < 0$ a sale) that, given the extant book, is executed at k different prices, with q_k shares executing at a price p_k, where $\sum q_k = Q$. The price impact of the trade is then defined in terms of the appropriately signed percentage difference between the weighted-average execution price and the pretrade midpoint:

$$p(Q) = \ln\left(\frac{\sum p_k q_k}{Q p_0}\right)\text{sign}(Q), \tag{6}$$

where p_0 is the midpoint of the bid-ask spread at the time of the trade. The price impacts thus defined are inversely related to the depth measures defined above. So, for example, if $D_b(k) = Q$, the total price movement associated with a buy order of size Q is k.[5]

Table 1 contains the expected price impact of trades, reported in percentage terms relative to the quote midpoint, by time of day. Calculations are done for hypothetical trades of 10–100 contracts in increments of 10, compared with the observed order book at a specific time of day, averaged over 105 trading days. Figures in the row marked "average" are computed based on the computations at 15-minute intervals over the trading day, averaged over intervals and trading days. Panel A contains data for transactions at the bid, and Panel B contains figures for transactions at the offer. The price impact of the trade is strictly increasing in order size, ranging from 7 to 15 basis points overall. Consistent with Table 1, the price impacts are much higher at the open, but do not vary by whether the order is a market buy or a market sell.

In equity market studies, it is increasingly common to model the price impact of a trade as a strictly concave function of size. Hasbrouck (1991), for example, advocates the use of square-root transformations for order size. Similar results are obtained by Madhavan and Smidt (1991), among others. By contrast, the price impacts here are nearly linear functions of size.

[5]The actual percentage price impact depends on the distribution of limit orders on the price grid. Suppose $p_0 = 100$, $Q = 50$, and at $p_1 = 110$ there are 20 contracts, and at $p_2 = 120$ there are 40 contracts. Then $p(50) = \ln[(110 \times 20 + 120 \times 30)/100 \times 50] = 0.148$. If there were 50 contracts at p_1, then $p(50) = 0.095$.

Table 1. Hypothetical price impacts by time of day.

Time	10	20	30	40	50	60	70	80	90	100
Panel A: Bid transactions										
9:15	0.08	0.09	0.10	0.12	0.13	0.14	0.15	0.16	0.17	0.19
10:15	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.12	0.13	0.14
12:15	0.07	0.08	0.09	0.10	0.11	0.12	0.12	0.13	0.14	0.15
14:15	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15
15:00	0.06	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14
Average	0.07	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15
Panel B: Offer transactions										
9:15	0.08	0.10	0.11	0.12	0.13	0.14	0.15	0.17	0.18	0.19
10:15	0.06	0.07	0.08	0.08	0.10	0.11	0.12	0.12	0.13	0.14
12:15	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.15
14:15	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15
15:00	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14
Average	0.07	0.07	0.09	0.10	0.10	0.11	0.12	0.13	0.14	0.15

This table contains the price impact of trades, reported in percentage terms relative to the quote midpoint, by time of day. Calculations are done for hypothetical trades of 10–100 contracts in increments of 10, compared with the observed order book at a specific time of day, averaged over 105 trading days. Figures in the row marked "average" are computed based on computations at 15 min intervals over the trading day, averaged over intervals and trading days. Panel A contains data for transactions at the bid, and Panel B contains figures for transactions at the offer. Trades at the bid are necessarily negative, and the absolute value is reported here.

The difference between our results and those based on NYSE or Nasdaq data might be the result of market structure. On the NYSE, for example, the trading crowd and specialist may step in to provide liquidity for large orders, while Nasdaq dealers may offer volume discounts to their customers. On an electronic limit order book like the OMX, however, traders are unwilling to offer large quantities at prices far away from the current price. Such limit orders constitute free options to the market, options that will be taken if the market moves by a large amount. The absence of depth at far prices implies that the price impact function is convex, because large trades incur proportionately greater costs.

It is also possible that the difference in the shape of the price impact function reflects upstairs trades. The data used to test models of the US equity markets do not identify large-block trades executed upstairs. These trades typically occur within the bid-ask spread, possibly biasing the estimated costs of execution for large orders downward. This is not an issue for us, since the computations in Table 1 use the current limit order book.

3.4. *Realized price impact costs*

We view the costs in Table 1 as a benchmark, being produced from a completely naïve trading strategy. Table 2 contains the actual price impact of trades, reported in percentage terms relative to the quote midpoint, by time of day. We compute these impacts except that we use the *realized* executions from an incoming market order in computing the trade price. Calculations are done for actual trades of 10–100 contracts in increments of 10, compared with the observed order book at the time of trade, over 105 trading days.

In contrast to Table 1, the realized impacts in Table 2 are virtually constant across order sizes. This pattern is true for both trades on the bid and offer sides. It also is true for off-exchange crosses. These results can be explained by discretionary timing of trades. In Admati and Pfleiderer (1988), for example, it is optimal for discretionary uninformed traders to trade at the same time. This, in turn, implies liquidity clustering in an environment in which informed trading further exaggerates the clustering effect. In Scharfstein and Stein (1990), large order flows, observable here through the book, encourage entry by traders, suggesting that greater liquidity should be correlated with more and larger trades. A similar herding effect in the case of discretionary timing is predicted by Spiegel and Subrahmanyam (1995). An even sharper result is obtained by Mendelson and Tunca (2000). In their model, discretionary liquidity traders adjust order sizes along with changing market depth, equalizing trading costs across size of transaction.

Overall, our results are strongly supportive of the Mendelson-Tunca (2000) model of discretionary trading equilibrium. It also is evident that traders obtain substantially lower costs than they would through a naïve order submission

Table 2. Actual price impacts by time of day.

	10	20	30	40	50	60	70	80	90	100
Bid side	0.04	0.04	0.04	0.04	0.05	0.05	0.04	0.04	0.06	0.04
Offer side	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.07
Cross bid	–	–	0.05	0.04	0.05	0.05	0.05	0.07	0.03	0.05
Cross offer	–	–	0.04	0.05	0.05	0.04	0.06	0.06	0.02	0.05

This table contains the price impact of trades, reported in percentage terms relative to the quote midpoint, broken down by time of day, by side (bid or offer), and for regular trades and crosses. Calculations are done for actual trades of 10–100 contracts in increments of 10, compared with the observed order book at the time of trade, over 105 trading days. Trades at the bid are necessarily negative, and the absolute value is reported here.

strategy, especially for large orders, even ignoring crosses. For example, the hypothetical price impact of a trade on the bid side for 100 contracts is 275% larger than realized price impact costs. These findings support the predictions of the Admati-Pfleiderer (1988) model.

4. Dynamics of Liquidity

We now turn to an investigation of the dynamics of market liquidity and volatility. As suggested by our theoretical model, a general vector autoregression of liquidity and volatility metrics is the approach we use here. Our primary interest, beyond a characterization of the dynamics of liquidity, is in the dynamic relationship of returns with depth. We therefore specify the vector $Y_t = (D_{bt}, D_{at}, |\Delta m|_t)'$, where $|\Delta m|_t$ is the absolute value of the change in the quote midpoint and depth on the bid and sell side, D_{bt}, D_{at}, are six ticks away.

4.1. *Identification*

We begin with the following complete dynamic system or structural model,

$$RY_t = \sum_{s=1}^{q} B_s Y_{t-s} + v_t, \tag{7}$$

where Y_t and v_t are the vectors and R and B_s, $s = 1, \ldots, q$, are the matrices. This is closely related to a reduced form model,

$$Y_t = \sum_{s=1}^{q} A_s Y_{t-s} + \eta_t, \tag{8}$$

where $A_s = R^{-1}B_s$ and $R^{-1}v_t$.

Use of the complete dynamic system, as opposed to simply the reduced form, has two main advantages. First, estimates of the complete model also include *contemporaneous* influences, permitting description of current period effects on market liquidity itself. Second, it permits explicit delineation of the identification conditions required to isolate shocks to market liquidity. These conditions often are hidden in the estimation of the reduced form alone, confusing inference with respect to the shocks of interest.[6]

In terms of estimation, the difficulty often encountered in structural estimation is that there are more parameters than moments. Therefore, we have

[6]There is a large literature devoted to this point, starting with Sims (1986) and explicated in more detail in Hamilton (1994).

to make meaningful restrictions in order to identify R and B_s. The identification conditions chosen here are expressed in terms of the variance–covariance matrix of υ_t and the elements of the matrices R and B_s. Identification is similar to that of a Wold causal chain.[7] In our case, the covariance matrix of the structural error is block diagonal, restrictions are imposed on R such that the matrix is block triangular, and returns follow a unit root process by a restriction imposed on $B_1 \equiv B$. We make the latter assumptions explicit below, once the elements of Y have been specified.

4.2. *Specification and estimation of market liquidity dynamics*

Theoretical treatments of the relationship between liquidity and returns are essentially static in nature. Our approach to identification is therefore empirical, using elements of the techniques in Swanson and Granger (1997) and Sims (1986). The combination of techniques involves the use of different identification schemes, each allowing the assessment of the strength of various correlations among the variables. The scheme below represents a choice based on this procedure, but also is intuitively plausible in nature.

The variance–covariance matrix of the structural error vector is taken to be block diagonal. In particular, it is assumed that shocks to liquidity on the bid and offer sides of the market are contemporaneously correlated. The correlation of current and lagged absolute returns is left unrestricted, however, following the large literature on volatility clustering. Lag lengths are truncated at $s = 1$. The matrix of contemporaneous effects, R, is specified as

$$R = \begin{bmatrix} 1 & 0 & -\rho_{13} \\ 0 & 1 & -\rho_{23} \\ 0 & 0 & 1 \end{bmatrix}. \tag{9}$$

Based on the above identification conditions, Equation 8 is estimated by method of moments, and the standard errors are computed using the usual GMM form.

4.3. *Impulse response functions*

The dynamic responses of returns to market liquidity shocks, and those of depth on one side of the market to shocks on the other side, are computed based on the estimated version of Equation 8 specified by full simultaneous equations

[7] See, for example, Sims (1986).

model,

$$Y_t = \sum_{s=1}^{q} \hat{R}^{-1}\hat{B}_s Y_{t-s} + \hat{R}^{-1}\hat{v}_t. \tag{10}$$

This autoregression is transformed into its infinite order vector moving average representation, through the device of matching moments.[8] The moving average representation is then used to generate the impulse response functions.

Results are reported in Table 3 for liquidity measured in terms of number of contracts available at six ticks away from the quote midpoint. Results for shocks to liquidity and absolute returns are presented for shocks to liquidity on the bid side (Panel A), on the offer side (Panel B), and for shocks to absolute returns (Panel C). Dynamic responses are given for the first five minutes, as well as average responses over time periods following the initial shock, up to 60 min. Shocks to market liquidity consist of an increase in depth of 30 contracts. Shocks to volatility are in units of 5 ticks.[9] Responses for liquidity are measured in terms of number of contracts; those for spreads and returns are given in terms of ticks.

Volatility has a contemporaneous, statistically significant negative effect on liquidity, regardless of side of market. The result stands in sharp contrast to the typically trading volume/volatility relationship, in which the positive correlation between variables typically is attributable to information effects (e.g., Blume, Easley, and O'Hara, 1994). Note that trading volume and the absolute value of price changes are positively correlated, and there is some evidence that the volatility/volume correlation extends to common factors in prices and volumes.[10] In an open limit order book system, higher volatility increases the value of the free option stemming from liquidity provision to the order book. Periods of higher information intensity and concomitant higher volatility increase the likelihood of adverse selection, and adverse selection effects have been found to be large in electronic markets.[11] In both cases, the incentive to provide liquidity to the book in the form of limit orders decreases,

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[8] See Hamilton (1994, Chapter 11).

[9] The precise scaling is immaterial, given the linearity of the system. A shock of 90 contracts to depth, for example, results in a response that is three times what is given in the table. The size of the shocks illustrated here was chosen to be approximately one standard deviation.

[10] See, for example, Karpoff (1987), Gallant, Rossi, and Tauchen (1992), and Hasbrouck and Seppi (2001).

[11] See Kofman and Moser (1997) and Coppejans and Domowitz (1999).

Table 3. Coefficient estimates for the model of depth and volatility.

	Bid depth	Offer depth	$\lvert\Delta\text{midquote}\rvert$
Constant	2.730	2.626	3.247
	(0.084)	(0.087)	(0.114)
$\lvert\Delta\text{midquote}\rvert_t$	−0.085	−0.070	–
	(0.010)	(0.010)	
Bid depth$_{t-1}$	0.373	0.032	−0.054
	(0.017)	(0.012)	(0.015)
Offer depth$_{t-1}$	0.146	0.321	−0.021
	(0.012)	(0.017)	(0.015)
$\lvert\Delta\text{midquote}\rvert_{t-1}$	−0.006	0.001	0.193
	(0.008)	(0.008)	(0.019)

This table contains estimates of the dynamic simultaneous equations model,

$$RY_t = BY_{t-1} + v_t,$$

in which $Y_t = (D_{bt}, D_{at}, |\Delta m_t|)'$, where $|\Delta m_t|$ is volatility, measured as the absolute value of the change in the quote midpoint, D_{bt} is depth of market, measured in lots of 10 contracts on the bid side of the order book at 6 ticks away from the quote midpoint, and D_{at} is the same measure, computed for the offer side of the book. The matrix, R, is given by Equation 9. Figures in the table are coefficient estimates (GMM robust standard errors in parentheses) for the regression of each of the elements of Y_t (column headings) on the variables in the left-hand column. Estimation is based on 5-minute intervals.

and market liquidity falls. The good news is that the effects on liquidity are relatively short-lived, so that the market self-corrects.

Conversely, increases in market liquidity lower future price volatility. The result is intuitively plausible, and consistent with the findings of Bollerslev and Domowitz (1991) in their investigation of the relationship between volatility dynamics and generic order book systems. The effects are economically larger, and statistically significant, on the bid side of the market, relative to the offer side. The difference might be thought to represent variability in this particular sample, since there is no obvious reason for a disparity. On the other hand, the literature on trading costs suggests that costs are substantially higher for sells than for buys in both traditional market structure (Keim and Madhavan, 1998) and electronic venues (Domowitz and Steil, 1999). Evidence from these cost studies is consistent with the fact that volatility does not respond significantly to offer-side depth, remaining relatively high even when the market is relatively deep on the sell side.

4.4. *The dynamic relationship between liquidity and volatility*

The dynamic responses of shocks to liquidity and volatility are summarized in Table 4 for liquidity defined in terms of number of contracts 6 ticks away from the midquote. As in the previous analysis, we report the initial 5-minute effect, as well as averages over subperiods within the hour following the shocks. The magnitude of the shocks to liquidity is as discussed previously. Shocks to volatility represent an increase of five ticks, or about 0.1% of contract value.[12]

Table 4. Dynamic responses to shocks in depth and volatility.

	Bid depth	Offer depth	$\|\Delta \text{midquote}\|$
Panel A: 30 contract shock to depth on bid side			
5 min	11.33	1.068	−1.619
5–10 min	7.833	0.915	−1.283
15–25 min	0.855	0.234	−0.230
30–60 min	0.023	0.009	−0.008
Panel B: 30 contract shock to depth on offer side			
5 min	0.492	9.678	−0.639
5–10 min	0.426	6.411	−0.497
15–25 min	0.111	0.495	−0.081
30–60 min	0.005	0.009	−0.002
Panel C: 10 tick shock to volatility			
5 min	−0.280	−0.191	0.996
5–10 min	−0.206	−0.184	0.604
15–25 min	−0.029	−0.018	0.023
30–60 min	−0.001	−0.000	0.000

This table contains the dynamic responses (impulse response function estimates) of bid-side depth, offer-side depth, and volatility, measured as the absolute value of midquote returns, to shocks to market depth on the buy side (Panel A), market depth on the sell side (Panel B), and volatility (Panel C). Calculations are based on 5-minute intervals, and use coefficient estimates of a complete dynamic simultaneous equations model, also estimated over 5-minute periods. Figures in the first row, labeled "5 min" are responses to the initial shock. The remainder of the rows give figures for average effects over the interval indicated (e.g., 15–25 min is the response calculated for 5-minute periods, starting at 15 min and ending at 25 min, averaged over the period). Depth responses are given in number of contracts. Volatility responses are given in number of ticks.

[12] Average 5-minute volatility over the estimation period is 3.67 ticks, with a standard deviation of 3.6 ticks. A move of two standard deviations is approximately the size of the average bid-ask spread.

Increases in market liquidity lower volatility. The volatility impacts of the liquidity shocks die away quickly, with the responses over the 15–25 minute interval being only 16–18% of the average impacts over the first 10 minutes. The standard deviation of volatility is about 3.5, and the 5-minute impact is -1.619, so a shock of $(3.5/1.619) \times 30$ or 65 contracts to depth is required to move volatility by one standard deviation. Shocks to liquidity on one side of the market move the other side of the market in the same direction as the initial shock. These results are unsurprisingly similar to those obtained using the structural VAR system incorporating midquote returns.

Shocks to volatility not only have a contemporaneous effect on liquidity, but also a more persistent effect over time. Higher volatility clearly decreases liquidity over the hour following the shock. The effects are especially strong only in the first 10 minutes following the volatility event, consistent with our overall findings of high natural market resiliency. Further, the magnitude of the effect of a volatility shock is relatively small. An increase in volatility of 5% of value decreases bid depth by only 14 contracts, for example, less than the average trade size.

5. Practical Issues

The results have practical implications for institutional traders and market protocols. This section discusses some of these issues, focusing on resiliency issues.

5.1. *Institutional trading*

Limit order book markets function well, particularly for small trades originating from a retail clientele. But large trades leave large footprints, creating problems for institutional traders. Large block matching through upstairs markets is not always feasible because of concerns regarding anonymity and the ability to discover "naturals," i.e., traders of similar size on the other side. Moreover, upstairs trades are not suited to long lists or lists with portfolio constraints.

Consequently, many institutional traders adopt strategies that breakup trade over extended horizons. These strategies can be manual, but often, taking advantage of automated venues, the strategy itself can be automated. For example, many institutional managers use the value-weighted average price (VWAP) as their benchmark price in evaluating trade performance. Consistent with this, some traders attempt to realize VWAP by using a simple break-up strategy.

Typically, the order is broken up for execution over the day to participate proportionately in the day's volume. Our findings suggest that this strategy is suboptimal; efforts to take advantage of time-varying liquidity may result in substantially better executions.

5.2. *Optimal trading strategies*

The model can also be used to probe the optimal trading strategies of a discretionary uninformed trader who submits market orders. Let Q denote the desired order, and suppose the trader breaks up his or her order for execution over h periods. This type of logical participation strategy is attractive because it obtains a better average price for the trader by trading Q/h for h periods instead of Q in a single period. But if the trader can observe D_t a better strategy is to trade larger amounts when $D_t > D^*$ (and vice versa) to strategically time trades during periods of large liquidity. The optimal strategy can be solved using a dynamic programming method; this yields, for example, an optimal trade size as a function of current depth, estimated future depth, time remaining, and shares remaining. In practical situations, the dynamics of liquidity might be quite complex and this strategy might be estimated using Monte Carlo simulations.

5.3. *Market structure, trading protocols, and resiliency*

As noted above, the electronic limit order book system typically does not feature a designated dealer. In the system analyzed here, with a single index contract, market participants can readily step in to provide liquidity if depth is low, explaining the high degree of resiliency. However, with large numbers of individual securities, this might be more difficult and require automation.

In terms of market structure, the fact that volatility shocks reduce liquidity supports arguments for trading halts following sharp market movements. This would give market participants time to recover, it is argued. On the other hand, impulse response functions show that shocks to liquidity dissipate quickly, indicating a high degree of resiliency. The results on resiliency are encouraging in this regard because they indicate that a halt, if necessary, need not be very long in duration. Natural liquidity providers will step in following shocks quickly. This "self-correcting" ability of the electronic limit order book is an important element of this mechanism's success, and belies arguments that the "free-option" problem is potentially fatal with respect to automated market viability.

6. Conclusion

The rapid adoption of electronic limit order book systems (or automated auctions) for equities, derivatives, and bonds worldwide has generated considerable practitioner and academic interest in the operation of such markets. In particular, many questions concern the nature and characteristics of liquidity in automated systems because of their reliance on public limit orders. This paper provides an analysis of the stochastic dynamics of liquidity and its relation to volatility shocks using data from a futures market. Aggregate market liquidity exhibits considerable variation, and is inversely related to volatility, as predicted by our model. However, liquidity shocks dissipate quickly, indicating a high degree of market resiliency. This fact has important practical implications, particularly as regards to institutional trading and market protocols.

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Chapter 3

Notes for a Contingent Claims Theory of Limit Order Markets

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This paper provides a roadmap for building a contingent claims theory of limit order markets grounded in a simple observation: limit orders are equivalent to a portfolio of cash-or-nothing and asset-or-nothing digital options on market order flow. However, limit orders are not conventional derivative securities: order flow is an endogenous, nonprice state variable; the underlying asset value is a construct, the value of the security in different order flow states; and arbitrage trading or hedging of limit orders is not feasible. Fortunately, none of these problems is fatal since options on order flow can be conceptualized as bets implicit in limit orders, arbitrage trading can be replaced by limit order substitution, and plausible assumptions can be made about the endogeneity of order flow states and their associated asset values. The analysis yields two main results: Arrow–Debreu prices for order flow "states" are proportional to the slope of the limit order book and the limit order book at one time proves to be identical to that at an earlier time adjusted for the intervening net order flow when all information arrives via trades.

Keywords: Arbitrage; contingent claim; digital option; information regime; limit order; limit order book; limit order market dynamics; market order; state price.

1. Introduction

A dozen or so years ago, I visited David Whitcomb in his New York apartment to view his limit order trading system. He invited me to do so under the misapprehension that I might be able to help him find a buyer, a notion of which he was rapidly disabused.[1] I came away remembering one detail of the system: limit orders were always canceled if they were not executed within two minutes because the specialist's commission — a number on the order of a mil at the time as I recall — was only paid on orders that resided on the book more than two minutes. The idea that stuck with me is that one could learn a great deal about optimal trading strategies by paying close attention to the minutiae of the microstructure of a market. Anyone who feels the need to verify the claim that

[1]As I recall, the asking price was $10 million and the proposed commission was 10%.

I see value in paying attention to such minutiae should examine the description of the Tokyo Stock Exchange in Lehmann and Modest (1995).

The research reported here marries this observation with a simple idea: the apparatus of state pricing can be applied profitably in market microstructure. Just as Hicks (1939) noted that conventional microeconomic theory could be applied to economic dynamics by treating the same goods on different dates as different commodities and Arrow (1964) and Debreu (1959) showed that dynamics under uncertainty could be analyzed similarly by treating the same good on different dates in different states of nature as different commodities, it has long seemed to me that one could exploit this strategy in microstructure by defining states in terms of trade prices, quotes, and quantities. Limit order markets are natural environments within which to apply this idea precisely because the mechanical nature of the order execution process makes for a clear definition of order flow states.

The idea of using contingent claims analysis in a microstructure setting is hardly new: since at least Copeland and Galai (1983), it has been commonplace to view quotes or limit orders as free options given by limit order traders or market makers to market order traders who may possess superior information regarding asset values. On this view, a limit buy (sell) order is a call (put) option with a strike price equal to the limit price with an implicit option premium — actually a contingent premium since it is not paid unless the option is exercised — given by the spread between the limit price and the midpoint of the bid-ask spread, albeit one with an uncertain expiration date since limit orders are typically good until cancelled. This spread reflects the by now classic balance between losses to informed investors and profits from uninformed liquidity traders along with traders who falsely believe they possess value-relevant private information.

Examining this implicit option from the perspective of limit order traders sheds quite different light on the sources of value in limit orders. A limit buy (sell) order for X_t shares involves the receipt (expenditure) of $P_t X_t$ dollars in exchange for (delivery of) X_t shares. In the language of exotic options markets, the first payoff is that of X_t cash-or-nothing digital call options struck at P_t and the second is the same as that on X_t asset-or-nothing digital call options struck at P_t as well. Since limit order markets only result in transactions at a given price $P_t(q)$ when a market order of size Q_t is large enough to hit the limit orders posted at the price (i.e., when $Q_t \geq q$), one can view these implicit derivative securities as options on order flow.

The chapter is laid out as follows. Section 2 describes both my assumptions about limit order markets and the digital options implicit in limit orders. Section 3 is devoted primarily to the identification of the Arrow–Debreu prices for order flow "states" based on the implicit options embedded in the limit order book. Care is needed since these implicit digital options differ in important ways from their analogues in conventional derivative asset markets since the nature of the underlying asset, the definition of states of nature, and the notion of arbitrage are not entirely straightforward in this context. The penultimate section derives some perhaps surprising implications for limit order book dynamics and the final section provides brief concluding remarks.

2. Limit Orders as Order Flow Derivatives

Consider a limit order book market for a single security with a marginal price schedule $P_t(q)$ where q is positive for buy orders and negative for sell orders. Limit orders are placed prior to time t and so $P_t(q) \in \mathcal{F}_{t-1}$, where $\mathcal{F}_{t-1}$ is public information available before time t including, at minimum, past quotes along with transactions prices, quantities, and times. Assume that $P_t(q)$ is continuous; that is, there is no minimum price variation or minimum tick.

Since incoming market orders are executed against standing limit orders, the overall cost of a market order for Q_t shares is $\int_0^{Q_t} P_t(q)\mathrm{d}q$, where Q_t and q are positive for buy orders and negative for sell orders — that is, the market order walks up or down the book until it is filled. Note that all orders at the same limit price are treated symmetrically in this limit order book — that is, price priority is strictly maintained while both size and time priority are ignored. Neither omission should be of great importance in this market since one can *always* step ahead of an existing order by posting one at an infinitesimally better price.

This market should be thought of as one that operates without frictions, save for enough slippage to permit limit order traders to cancel old orders and/or submit new ones before the next market order arrives. There are no dealers like NYSE specialists who can see the book before taking a position or stepping in front of an existing limit order in the book. There are no hidden limit orders that market order traders might "ping" to discover how much immediacy resides in an undisclosed portion of the book. There are no limits on how many tiers of the book a single market order can clear out as there are on the Tokyo Stock Exchange. Some of these frictions can be handled by careful definition of the "state" of the book and some cannot. I will not deal with them in what follows.

No special assumptions are made about order flows at this point. There can be limit or market orders that follow arbitrary stochastic processes that depend in arbitrary ways on the slope, depth, and history of the limit order book as well as other public information. To be sure, most of the heavy lifting in microstructure models involves the determination of the optimal order placements of informed investors and/or optimizing liquidity traders, all of which requires numerous assumptions about the economic setting. The present exercise is much easier since it only involves the *mechanics* of order execution in a stylized limit order market. Some modest assumptions about order flows and execution will be made in the next two sections.

The analysis of contingent claims written on order flows begins with the three possible events that might transpire at time t: the arrival of a market buy order (i.e., $Q_t > 0$), of a market sell order (i.e., $Q_t < 0$), or of information without an intervening trade including, perhaps, that the clock has ticked without the arrival of a trade (i.e., $Q_t = 0$). This last prospect will often lead to a revision of the limit order book, an instance of the good until cancelled feature of limit orders, and is discussed in Section 3. In this context, my assumption that there is a tatônnement-like process permitting limit order traders to freely cancel and submit orders prior to the next market order arrival means that limit order traders, by assumption, have placed the limit orders they think appropriate given all of the information available to them at time t.

Now suppose a market buy order arrives for $Q_t > q$ shares at time t — the sell side being symmetric — and consider a limit order that offered one share at price $P_t(q)$. Since this limit order is executed for sure (i.e., $Q_t > q$), the limit order trader exchanges one share of stock for $P_t(q)$ dollars. In the language of exotic options, a contingent claim that pays a fixed cash flow in a given eventuality is called a cash-or-nothing digital call option and so the cash flow from a limit sell order is the payoff on a cash-or-nothing digital call option struck at $P_t(q)$. Similarly, a contingent claim that pays a share in a given state is called an asset-or-nothing digital call option and so the share transfer in this transaction is the payoff of an asset-or-nothing digital call option struck at $P_t(q)$ as well. I will follow Ingersoll's (2000) nomenclature and refer to cash-or-nothing digital calls and puts as digital options and to asset-or-nothing digital calls and puts as digital shares.

Hence, limit orders implicitly bundle digital option and share positions. A limit buy order is equivalent to a long position in a digital option and a short position in a digital share, each struck at $P_t(q)$. Each implicit digital option pays $P_t(q)$ dollars if $Q_t \geq q$ and expires worthless if $Q_t < q$. Similarly, each implicit digital share converts into a share if $Q_t \geq q$ and expires worthless otherwise. By the same token, a limit sell order is equivalent to a short position in a digital option and a long position in a digital share. The former requires the payment of $P_t(q)$ dollars when $Q_t \leq q$ and the latter involves the receipt of a share in these circumstances. Both expire worthless if $Q_t > q$.

Since a limit order involves no up front cash flows, the value of these two implicit options must be the same. Accordingly, any analysis that delivers the value of one option position implicitly determines that of the other position as well. In addition, any such valuation must take account of the fact that these derivatives are implicitly written on order flow — a nonprice state variable.

This is a weaker restriction than that derived in Ingersoll (2000) for conventional digital options or shares. After all, the underlying asset price, a quantity measured in dollars, is always available to compute the relative value of digital options and shares in any state of nature. However, market order size is a nonprice state variable and there is no observable that corresponds to the value of a share in different order flow states. This inability to mechanically assign a value to the digital share on expiration or to a claim to a dollar in the same state of nature is the main obstacle to the straightforward computation of the value of the contingent claims implicit in limit order books.

A digital call (put) can be replicated by a bull (bear) spread in conventional call (put) options with an infinitesimal spread between the two strike prices. This observation is of limited utility for limit order valuation purposes since it is hard to imagine a market in which it is easy to form a bull spread in conventional options at the appropriate strikes just before a market order arrives that expires on the arrival of the market order. It may be of some use in reverse: inferences regarding the prices of order flow states implicit in the limit order book can provide insight into microstructural effects on the value of very short expected maturity options.

However, this fact does yield one modest insight concerning the riskiness of the derivatives implicit in limit orders. As is well known, at-the-money digital options with short time to expiration are highly sensitive to changes in the underlying asset value because the absolute values of their deltas and gammas — that is, the sensitivity of the bull spread to changes in the underlying asset price and its square, respectively — can both be large in absolute value and fluctuate a great deal. Moreover, the gamma of such a digital call can change signs and its delta is unbounded. Accordingly, the prices of limit orders should be quite sensitive to changes in both order flow dynamics and the elasticity of limit order prices with respect to order flow. Perhaps this sensitivity explains why changes in the market for liquidity in a stock are associated with volatility in limit order cancellations and submissions.

More revealing for valuation purposes, however, is a spread in digital calls with an infinitesimal spread between the two strike prices. Since a digital call (put) is equivalent to a spread in conventional calls (puts), a spread in digital call (put) options can be replicated by a spread in bull (bear) spreads in conventional options or, in more common parlance, by a butterfly spread in three conventional calls (puts). As shown in Breeden and Litzenberger (1978), a butterfly spread in conventional options with infinitesimal differences across the three strike prices has a payoff proportional to that of a pure Arrow–Debreu claim that pays off only when the stock price on expiration equals the intermediate strike price. Similarly, the payoff of a bull spread in digital calls (puts) on order flow pays off only in the intermediate order flow state when there is an infinitesimal spread in the order flow states. Hence, the analysis of the contingent claims implicit in limit orders yields a new interpretation of the *slope* of the limit order book.

3. Limit Order Valuation and Order Flow Bets

Hence, the contingent claims implicit in limit orders can be viewed as the payoffs on two American, deferred premium, one-touch, binary or digital options. They are American options because limit orders are generally good until cancelled and they are one-touch options because they pay a fixed quantity of shares or dollars if the market order exceeds the tier of the book on which the limit order resides. They are also very short maturity options since they last, at most, one trading day. We need only value the digital options or digital shares implicit in limit orders to provide a contingent claims interpretation of the sources of value in limit order books.

There are nontrivial challenges associated with exploiting this observation. The "underlying asset" is order flow — a non-price state variable — and there is no observable or risk premium for order flow risk. Moreover, order flow is not an exogenous set of outcomes but rather is the endogenous product of the trading decisions of informed and uninformed investors. In addition, the application of arbitrage reasoning — the natural starting point for the valuation of contingent claims — is hampered by the presence of short sales restrictions and the absence of natural hedging instruments. Finally, any such analysis must account for the American nature of these implicit order flow derivatives.

There is a simple but useful device for thinking through these issues: imagining a parallel but separate market for wagering on order flow in which participants bet by buying and selling order flow contingent claims. In such a market, claim q pays \$1 if the next order is for exactly q shares — i.e., when $Q_t = q$ — and zero otherwise, making it an Arrow–Debreu claim on order flow. As is readily apparent, the cash flows associated with any trading strategy in the actual limit order market can be perfectly replicated in the betting market. When this fictitious betting market is isomorphic to general arbitrage-free financial markets, we can freely import analysis from that setting to this one when it is convenient to do so. Additional insights into the contingent claims interpretation can be gleaned by being precise about when the betting market produces exactly the same state prices as those implicit in the limit order market.

Before proceeding, it is worthwhile to consider the role of interest rates in the analysis. As is commonplace in the analysis of generic derivatives, the time value of money drops out of the relevant pricing relations if the numeraire is a savings account that accrues at the relevant risk-free interest rate. There is an even better reason to do so in the present setting: interest rates are a second order consideration over (most) market microstructure time scales and so most researchers are content to simply set the riskless rate to zero. On this hypothesis, state prices are also risk neutral probabilities for order flow states.

One nettlesome problem associated with the betting market analogy concerns the endogeneity of order flow. Since order flow states represent the transactions of market order traders, such traders have an incentive to place bets on market orders of given sizes and then to submit the corresponding market orders in the limit order market. Accordingly, there can, in general, be no trade in the betting market in equilibrium since trades in the actual limit order market can be used to perfectly manipulate the betting market. The simplest solution is to assume that market order traders cannot wager in the betting market prior to the placement of market orders. That is, the natural solution to the problem of endogeneity of order flow is to assume there is no intermarket front-running.[2]

[2] See the bluffing subsection of Lehmann (2005) for a more detailed analysis. I do not mean to suggest that this issue is easy to deal with in substance but rather that it is easy to find assumptions under which order flow endogeneity is not a problem.

The limit order market analogue is Assumption 3 of Glosten (1994), which requires traders to use only public information when submitting limit orders. That is, informed and uninformed traders interpret the information content of posted limit orders in the same way under this assumption. This circumstance will arise when informed investors cannot use limit orders by assumption, which is the usual interpretation of Glosten (1994). However, this assumption will also hold when informed traders have no incentive to post limit orders at prices different from those that would be submitted by uninformed traders. If the supply schedule of limit orders that would be submitted by uninformed traders is common knowledge, informed traders would have no incentive to post orders off the supply schedule.[3] They will have the incentive to post orders on the schedule if it lies above their valuations for limit sell orders and below their valuations for limit buy orders.

Accordingly, suppose the set of possible market order sizes is given by $\mathbb{Q}_t \subseteq \mathbb{R}$ and that market participants agree that all order sizes in $\mathbb{Q}_t$ are possible.[4] Note that $\mathbb{Q}_t$ is likely to be bounded since the market will not provide bets for all order sizes if the adverse selection problem is severe enough. In the absence of frictions in the betting market, the usual arbitrage argument insures the existence of a set of (not necessarily unique) strictly positive state prices $\{\psi_t(q), q \in \mathbb{Q}_t\}$, where $\psi_t(q) = E_\psi[1_{Q_t=q}|\mathcal{F}_{t-1}]$ with $E_\psi[\bullet|\bullet]$ denoting the associated risk neutral expectation.

While nothing in what follow depends on it, it is natural to ask whether spanning would naturally arise in the betting market.[5] That is, would bettors naturally write claims on all of $\mathbb{Q}_t$? The answer to this question is yes because the size of the subsequent market order is observable and its endogeneity has been assumed away. If there were at least two investors with different shadow prices for some non-traded claim for q^* shares, some investor could earn an arbitrage profit by buying a claim that pays \$1 when the next market order is for q^* shares from the low valuation investor and selling it to the high valuation investor. Such riskless profit opportunities will not arise if this betting market is

[3]By informed trader, I mean those who actually possess value-relevant private information as well as noise traders who think they are informed but who are not.

[4]Agreement on the possible is necessary for equilibrium to exist in frictionless markets because any market participant who believed a given state is impossible will be happy to sell an infinite quantity of bets that pay off in that state.

[5]Spanning is not necessary since order flow is a non-price state variable and $\mathbb{Q}_t$ can simply be defined to be the set of trade sizes offered, which is trivially spanned by construction.

arbitrage-free. Accordingly, it is natural to assume that state prices $\{\psi_t(q),\ q \in \mathbb{Q}_t\}$ are unique because the menu of claims that will be offered in arbitrage-free betting markets spans $\mathbb{Q}_t$. The analogue in the actual limit order market is that its book should have orders posted at all feasible trade sizes, a prediction clearly at variance with the evidence from actual limit order markets.

In any event, the question at hand is whether these state prices can be used to value limit orders. What is missing is $V_t(q)$, the value of the asset if a market order of size q arrives at time t. If $V_t(q)$ is common knowledge among market participants and the state prices in the betting and limit order markets are identical, the value of a claim to a share in order flow state q is simply $\psi_t(q)V_t(q)$. The existence of such values crops up in the analysis of generic financial markets in the definition of states of nature: the typical assumption is that there is a deterministic mapping between cash flows and asset values on the one hand and their associated states of nature on the other. The problem in the present setting, of course, is that there is no observable mapping between the value of the asset being traded and order flow states. The absence of such a mapping motivates Assumption 2 of Glosten (1994): the existence of a function $V_t(q)$ that is both nondecreasing in q and common knowledge among market participants, although I will assume it is strictly increasing for simplicity.

Limit order valuation is straightforward when state prices from the betting market are identical to those implicit in the book. Consider a market buy order since the sell side is symmetric. The digital option implicit in the corresponding limit sell order pays $P_t(q)$ when $Q_t > q$. The implicit digital share value is given by the value of a claim to a share in each order flow state integrated over order flow states. Since the values of the implicit digital option and digital share are equal:

$$E_\psi[1_{Q_t \geq q} P_t(q) | \mathcal{F}_{t-1}] = E_\psi\left[\int_q^\infty 1_{Q_t = u} V_t(u)\mathrm{d}u | \mathcal{F}_{t-1}\right]$$
$$\Rightarrow P_t(q)\int_q^\infty \psi_t(u)\mathrm{d}u = \int_q^\infty \psi_t(u)V_t(u)\mathrm{d}u \quad (1)$$

and, hence, the limit price is given by:

$$P_t(q) = \frac{\int_{u \geq q} \psi_t(u)V_t(u)\mathrm{d}u}{\int_{u\geq q}\psi_t(u)\mathrm{d}u}$$
$$\equiv E_\psi[V_t(Q_t)|Q_t \geq q, \mathcal{F}_{t-1}] \equiv \frac{E_\psi[V_t(Q_t)1_{Q_t\geq q}|\mathcal{F}_{t-1}]}{E_\psi[1_{Q_t \geq q}|\mathcal{F}_{t-1}]}, \quad (2)$$

which is the risk neutral analogue of the upper tail expectation formula in Glosten (1994).[6]

The only remaining question is whether state prices from the betting market are identical to those implicit in the book. This brings us to the third basic problem with the betting market analogy: the absence of short sales restrictions. In a betting market, all gambles are in zero net supply and, thus, there are obviously no short sales restrictions. However, the limit order market permits no short selling of limit orders and so the frictionless markets assumption cannot apply. Since state prices implicit in the book are given by bull spreads in the relevant limit orders, this absence of short sales in the limit order book would appear to be an insuperable barrier to the application of the betting market analogy.

The way out of this conundrum is to recall that a zero net investment portfolio always has two interpretations. The standard one is as an arbitrage portfolio in which any long positions are financed by short positions. However, a zero net investment portfolio also represents a feasible change in an existing portfolio with no short positions in which security purchases are financed by security sales. A feasible change is one in which none of the asset sales is so large so as to create a short position in any asset and an arbitrage portfolio can always be scaled so that no long position is exhausted in the absence of indivisibilities.

It is this second interpretation that is the right one to apply in this context. A limit order trader can always contemplate a swap of all or part of an existing limit order for another at a different price point or, for that matter, for none at all. If one limit order trader with no private information perceives that a limit order is mispriced, all such traders will want to cancel their orders if there

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[6]As Glosten emphasizes, the fact that it is an upper tail expectation reflects the discriminatory nature of the limit order book. As Equation (2) clearly shows, this observation is independent of the probabilities used in the calculation. In contrast, the conditional mean $P_t(q) = E_{\psi}[V_t(Q_t)|Q_t = q, \mathcal{F}_{t-1}]$ is the limit order schedule in a nondiscriminatory book. Most limit order markets open with a single price auction in which all market orders are consolidated into an aggregate net order and executed against the book at the same price. As noted above, this price can be treated as the "liquidating" asset value embedded in the expectations $V_t(q)$. Note also that these single price auctions are isomorphic to the batch auctions studied by Huberman and Stanzl (2004). It may be possible use their results to place restrictions on the "terminal asset values" determined in opening call auctions which might, in turn, deliver additional intertemporal restrictions on order flow state prices. See the end of Section 4.

is too much depth at this price point and they will submit new ones at said price if it there is too little depth. Limit order substitution by sufficiently many patient or value traders is a perfect substitute for explicit arbitrage in these circumstances.[7]

The mathematics of risk neutral upper tail expectations naturally dovetails with the observation that bull spreads in limit orders reveal the associated implicit state prices. The slope of the limit order book at q is proportional to the risk neutral probability $\psi_t(q)$ since:

$$P_t'(q) = \lambda_t(q)[P_t(q) - V_t(q)]; \quad \lambda_t(q) = \frac{\psi_t(q)}{\int_q^\infty \psi_t(u)\mathrm{d}u} \tag{3}$$

by Leibniz's rule where $\lambda_t(q)$ is the hazard function familiar from survival analysis. Accordingly, the risk neutral probabilities implicit in $P_t(q)$ can be extracted from $P_t'(q)$ given $V_t(q)$ via the recursion:[8]

$$\psi_t(q) = \lambda_t(q) \exp\left\{-\int_0^q \lambda_t(u)\mathrm{d}u\right\}. \tag{4}$$

Of course, $V_t(q)$ can be uncovered from $P_t'(q)$ given $\lambda_t(q)$ and, thus, $\psi_t(q)$ as well.

There is one state that cannot be valued by limit order substitution alone: the null "trade" $Q_t = 0$ that corresponds to changes in the state of the book that arise when nontrade-related information arrives in the market. Recall that limit orders were predetermined with respect to market orders conditional on public information by assumption through the tatônnement-like process by which traders could freely cancel and replace their limit orders before the next

[7]This version of the *Fundamental Theorem of Asset Pricing* corresponds to one in which the portfolio choice problem of an investor with (possibly state dependent) von Neumann–Morgenstern preferences will not have an interior maximum if arbitrage opportunities are present. Alternatively, optimizing behavior on the part of market order traders along the lines of Assumption 1 of Glosten (1994) along with uninformed risk neutral limit order traders is sufficient as well.

[8]After making the relevant substitutions, the discrete version of this formula:

$$\psi_t(q_k) = \frac{\sum_{j=k+1}^{\infty} \psi_t(q_j)[V_t(q_j) - P_t(q_k)]}{P_t(q_k) - V_t(q_k)}; \quad \sum_{j=1}^{\infty} \psi_t(q_j) = 1.$$

is identical to Equation (5) of Banz and Miller (1978) with the option price set to zero, $P_t(q)$ equal to the underlying asset price, and $V_t(q)$ equal to the strike price. The formal similarity ends there as Banz and Miller are not concerned with bull and bear spreads in digital options as opposed to Breeden and Litzenberger (1978), who (implicitly) construct state prices from such spreads.

market order arrived. In this model, limit order traders can revise their limit orders in light of any nontrade-related information as well before exposing them to a market order when $0 \in \mathbb{Q}_t$.

The analogue in the betting market is a provision for the cancellation of all existing bets when the null "trade" arrives. The wagers on this revised claim are refunded to all market participants and new bets are placed, which results in the discovery of new order flow state prices during the tatônnement-like process. That is, these bets are not pure Arrow–Debreu securities because they pay a dollar if state $q \in \mathbb{Q}_t$ occurs, zero if a market order for any other size $Q_t \neq 0$ arrives, and $-\varphi_t(q)$ if $Q_t = 0$, where $1 > \varphi_t(q) > 0$ is the initial price paid for this claim.[9]

It is a straightforward matter to construct pure contingent claims by augmenting the asset menu — that is, the set of trade size bets — with one additional claim: a savings account that pays a dollar irrespective of order flow outcomes since the interest rate has been normalized to be zero. For each $q \neq 0$, a portfolio that is long

$$1 - \frac{\varphi_t(q)}{1 + \int_{\substack{u \in \mathbb{Q}_t \\ u \neq 0}} \varphi_t(u)\mathrm{d}u}$$

units of bet q, short

$$\frac{\varphi_t(q)}{1 + \int_{\substack{u \in \mathbb{Q}_t \\ u \neq 0}} \varphi_t(u)\mathrm{d}u}$$

units of each bet $u \neq q$, and long

$$\frac{\varphi_t(q)}{1 + \int_{\substack{u \in \mathbb{Q}_t \\ u \neq 0}} \varphi_t(u)\mathrm{d}u}$$

[9] Of course, the bets could have been designed so that there was no need to refund the wager. That is, bets that pay $1 - \psi_t(q)$ if state $q \in \mathbb{Q}_t$ occurs, $-\psi_t(q)$ if a market order for any other size $Q_t \neq 0$ arrives, and nothing if $Q_t = 0$ have no up-front costs and, hence, require no refunds if the expected risk neutral payoff is zero as in $\psi_t(q)[1 - \psi_t(q)] + [1 - \psi_t(q)][-\psi_t(q)] = 0$. The formulation in the text has the virtue of showing how the valuation of Arrow–Debreu securities for all of $\mathbb{Q}_t$ save for 0 — that is, $q \in \mathbb{Q}_t \backslash 0$ — is affected by the presence of the null trade.

units of the riskless savings account pays one dollar in state $q \neq 0$ and zero otherwise.[10] Similarly, a portfolio that is short

$$\frac{1}{1 + \int_{\substack{u \in \mathbb{Q}_t \\ u \neq 0}} \varphi_t(u)}$$

units of each bet q and long

$$\frac{1}{1 + \int_{\substack{u \in \mathbb{Q}_t \\ u \neq 0}} \varphi_t(u)}$$

units of the riskless savings account will yield a dollar when $q = 0$ and zero otherwise. Hence, state prices are given by the prices of these portfolios:

$$\psi_t(q) = \varphi_t(q) \frac{2}{1 + \int_{\substack{u \in \mathbb{Q}_t \\ u \neq 0}} \varphi_t(u)\mathrm{d}u}, \qquad \psi_t(0) = \frac{1 - \int_{\substack{u \in \mathbb{Q}_t \\ u \neq 0}} \varphi_t(u)\mathrm{d}u}{1 + \int_{\substack{u \in \mathbb{Q}_t \\ u \neq 0}} \varphi_t(u)\mathrm{d}u}, \qquad (5)$$

which sum to one because these are risk neutral probabilities due to the normalization of the interest rate to zero.

Hence, a betting market coupled with access to riskless savings is isomorphic to the actual limit order market and so state prices calculated in one setting can be freely transferred to the other. All that was necessary was care in constructing the claims in the betting market so that they possessed the relevant features of the contingent claims implicit in limit orders. These characteristics include arbitrage via limit order substitution, the existence of asset values for each order flow state, and enough time for the limit order book to be refreshed — after either a market order has executed or the arrival of information via the "null trade" — before the next market order arrives so that market order traders cannot "front run" the book. Informed investors will have no incentive to post limit orders off of the supply schedule because the schedule that would be posted by uninformed traders is common knowledge in these circumstances.

[10] This portfolio can be formed by any limit order trader who is long more than

$$\frac{\varphi_t(q)}{1 + \int_{\substack{u \in \mathbb{Q}_t \\ u \neq 0}} \varphi_t(u)}$$

units of each bet. Since $\varphi_t(q) < 1$, one unit of the qth bet suffices in the absence of indivisibilities.

4. Limit Order Book Dynamics

What happens to state prices if information arrives in the market *only* via a market order at time $t-1$? The answer lies in the change in the information available in the market between time $t-1$ and time t. Since the information state of the market was $\mathcal{F}_{t-2}$ at time $t-1$ and no nontrade-related information hit the market, the new information state is $\mathcal{F}_{t-1} = \{Q_{t-1}, \mathcal{F}_{t-2}\}$. As a result, the state of the world in which the trades at times t and $t-1$ are q and Q_{t-1}, respectively, is identical to that in which the trade at time $t-1$ is $\bar{Q}_{t-1} = q + Q_{t-1}$ due to the structure of information flows when $Q_{t-1} \neq 0$. Hence, state prices at times t and $t-1$ must be linked by:

$$\begin{aligned}\psi_t(q) &= E_\psi[1_{Q_t=q}|\mathcal{F}_{t-1}] = E_\psi[1_{Q_t=q}|Q_{t-1}, \mathcal{F}_{t-2}] \\ &= E_\psi[1_{\bar{Q}_{t-1}=q+Q_{t-1}}|\mathcal{F}_{t-2}] = \psi_{t-1}(q + Q_{t-1}),\end{aligned} \tag{6}$$

when information arrives only via market orders.

This restriction generalizes in an obvious way for longer intervals during which all information arrival is trade-related. If the null trade does not occur between times 1 and T, the recursion suggested by Equation (6) reveals that:

$$\begin{aligned}\psi_T(q) &= E_\psi[1_{Q_T=q}|\mathcal{F}_{T-1}] = E_\psi[1_{\bar{Q}_{T-1}=q+Q_{T-1}}|\mathcal{F}_{T-2}] \\ &= E_\psi[1_{\bar{Q}_{T-2}=q+Q_{T-1}+Q_{T-2}}|\mathcal{F}_{T-3}] \\ &= E_\psi[1_{\bar{Q}_{T-3}=q+Q_{T-1}+Q_{T-2}+Q_{T-3}}|\mathcal{F}_{T-4}] = \cdots \\ &= E_\psi\left[1_{\bar{Q}_1=q+\sum_{t=1}^{T-1} Q_t}|\mathcal{F}_0\right] \\ &= \psi_1\left(q + \sum_{t=1}^{T-1} Q_t\right) \equiv \psi_1\left(q + Q_1^{T-1}\right),\end{aligned} \tag{7}$$

where Q_1^{T-1} is cumulative signed volume between times 1 and $T-1$. It is as though all bets are placed at time zero and the bet on the value of cumulative net order flow given by Q_1^{T-1} is the one that paid off.

What is the analogue of these relations in the actual limit order market? The answer is concealed in the equivalence between trades of q and Q_{t-1} at times t and $t-1$, respectively, and one of $\bar{Q}_{t-1} = q + Q_{t-1}$ at time $t-1$ when $\{q, Q_{t-1}, \bar{Q}_{t-1}\} \neq 0$. The limit order book is discriminatory and so limit order traders place orders as though they might confront a single order of $\bar{Q}_{t-1}$ at time $t-1$ or an order of size Q_{t-1} at time $t-1$ and another of size q at time t before limit order traders have a chance to cancel and replace their limit orders. When limit orders do so, they need not fear any such order splitting on the demand side. This is the essence of part ii of Proposition 3 of Glosten (1994). Continuation of the recursion in Equation (6) yields the required relation $\psi_T(q) = \psi_1\left(q + Q_1^{T-1}\right)$ between the time t and time 0 limit order books when all information arrives via trades.

The discriminatory nature of the book coupled with information arrivals that are entirely driven by order flow has a surprising implication for limit order book dynamics: moving up or down the limit order book at a point in time is isomorphic to moving across limit order books over time in these circumstances. It is as though there is a single market order of size Q_1^T arrives at time one and walks up or down the book to the marginal price $P_1\left(Q_1^T\right)$. Put differently, the marginal price at time T is path independent and Markovian, depending only on Q_1^T if the "null trade" representing value-relevant, nontrade-related information arrives at time $T + 1$ or later.

Note that I am not assuming that the actual time zero limit order book has orders posted at all of the possible values of cumulative signed order flow Q_1^T that can occur given the stochastic process generating market order flow. As noted earlier, we would expect the actual book to have finite depth at time zero such that sufficiently large values of Q_1^T are not bid or offered. Put differently, actual markets can force informed market order traders to break up their orders over time within the implicit constraints associated with free entry and competition in limit order placement.

However, limit order markets that routinely remain in information epochs — that is, periods in which all information arrives via market orders — are more likely to satisfy the assumption that limit order traders can freely cancel and replace limit orders before the next market order arrives. Limit order traders will post orders at time $t - 1$ on the price schedule $P_1\left(Q_1^{T-1} + q\right)$ if no value-relevant, nontrade-related information arrives between times one and $t - 1$. If a market order arrives at time t, limit order traders will post limit orders on the marginal price schedule $P_1\left(Q_1^T + q\right)$. If the null trade arrives at time t, limit order traders can cancel all of their orders and repost them when they have assimilated the new information. In fact, this process can be automated if limit order traders know how the set of feasible trades evolve over time as well as the hypothetical marginal price at time zero for each feasible future level of cumulative net order flow. It is certainly common for microstructure theorists to assume that markets experience long information regimes of this sort.

Most markets open (sometimes more than once per day) and some markets close with a single price auction. While the marginal price schedule is given by the upper tail expectation $P_t(q) = E_\psi[V_t(Q_t)|Q_t \geq q, \mathcal{F}_{t-1}]$ due to the discriminatory nature of the book, the uniform nature of single price auctions produces prices that are simple expectations $P_T^{\mathrm{A}}\big(Q_T^{\mathrm{A}}\big) = E_\psi\big[V_T\big(Q_T^{\mathrm{A}}\big)|\mathcal{F}_{T-1}\big]$, where T is the time of the single price auction and A denotes auction. This completes the picture of dynamics in such markets: the upper tail expectation is also given by $P_t(q) = E_\psi\big[P_t^{\mathrm{A}}\big(Q_T^{\mathrm{A}}\big)|Q_t \geq q, \mathcal{F}_{t-1}\big]$ due to the law of iterated expectations and the expected midquote of the postauction limit order book is given by $P_T^{\mathrm{A}}\big(Q_T^{\mathrm{A}}\big)$. It may prove useful to connect this analysis with Huberman and Stanzl's (2004) study of sequences of batch auction markets that are isomorphic to single price auctions.

5. Conclusion

This paper provides a roadmap for building a contingent claims theory of limit order markets. It is grounded in a simple observation: limit orders are equivalent to a portfolio of cash-or-nothing and asset-or-nothing digital options on order flow. However, limit orders are not conventional derivative securities: order flow is an endogenous, nonprice state variable; the underlying asset value is a construct, the value of the security in different order flow states; and arbitrage trading or hedging of limit orders is not feasible. Fortunately, none of these problems is fatal since options on order flow can be conceptualized as bets implicit in limit orders, arbitrage trading can be replaced by limit order substitution, and plausible assumptions can be made about the endogeneity of order flow states and their associated asset values.

Perhaps surprisingly, limit order books and the state prices implicit in them have a simple dynamic stochastic structure under plausible conditions. A special feature of limit order books is their discriminatory nature and a market order that walks up or down the book as it executes is identical to an order that executes one share at a time *over time* when the only new information is the arrival of yet another market order. Hence, prices at time t can be read off of the time one limit order book if the only new information arriving between those two times is trade-related.

While I have said nothing about it in this paper, the most interesting aspect of this analysis is its implications for the empirical analysis of limit order markets. Implicit state prices can shed light on basic microstructure questions such as the role of risk aversion, if any, in the provision of immediacy in limit order markets. The successful identification of episodes in which information arrives only via trade can serve to better measure information flows and to provide new insight into the process of price formation. The expectational linkages between periodic single price auctions and continuous trading in limit order markets and even between these markets and options markets should shed considerable light on the economics of the associated intertemporal and cross-market comparisons.

I have called the paper "Notes…" because the theory is as yet incomplete. It seems to me to be worthwhile to present it in this somewhat embryonic state since both the broad outline of the theory and the work needed to complete it seem reasonably clear. And I thought that its link to my glimpse of David Whitcomb's limit order trading system, the progenitor of his extremely successful company Automated Trading Desk, made it fitting as well.

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Chapter 4

The Option Value of the Limit Order Book

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Previous studies of the limit order book report that low depths accompany wide spreads and that spreads widen and depths fall in response to higher volume, but some postulate a positive relationship between spreads and depth during normal trading periods. We calculate the option value of the limit order book at 11:00 a.m. for 10 actively traded firms listed on the Australian Stock Exchange. Simultaneously this approach enables us to consider the spread and depth of the limit order book. We find that 33.1% of the option value of the limit order book is provided at the best ask and 34.7% at the best bid. We find that the option value of the limit order book is greatest at the best bid price and the best ask price and is more stable through time than the option value of individual shares or share quantities in the book. Also, consistent with the arguments of Cohen *et al.* (1981), we find evidence of equilibrium in the supply and demand of liquidity.

Keywords: Limit orders; options.

1. Introduction

Cohen, Maier, Schwartz, and Whitcomb (1981) define an equilibrium market spread, where the forces that tend to widen and narrow the spread are in balance. We investigate whether this equilibrium is stable over time.

The limit order book is an important source of liquidity for most exchanges.[1] Some exchanges such as the New York Stock Exchange (NYSE) and the Tokyo Stock Exchange rely partially on limit orders. But limit orders are the only source of liquidity on screen-based, order-driven exchanges such as the Paris Bourse, the Stock Exchange of Singapore, and the Australian Stock Exchange (ASX). Previous work has focused on individual characteristics of the limit order book. Examining NYSE data, Lee, Mucklow, and

[1] A limit order is an order for a specified quantity to be executed at a specified price or better. Limit orders to buy are called bids and limit orders to sell are called asks or offers. The difference between the best bid and the best ask is called the spread. The schedule of limit orders is called the limit order book.

Ready (1993) report that low depths accompany wide spreads, and that spreads widen and depths fall in response to higher volume. Harris (1991, 1994) and Bacidore (1997) show that reductions in minimum price variations are associated with narrower spreads and smaller quantities at the best quotes, suggesting a positive relationship between spread and depth. Biais, Hillion, and Spatt (1995) examine the interaction between the order book and order flow on the Paris Bourse.[2]

In path-breaking work, Copeland and Galai (1983) model limit orders as free options offered to the market. According to Option Pricing Theory, the value of an individual limit order is a function of the stock price, the exercise or limit order price, the time until execution, the rate of interest, and the volatility of the stock. The option pricing approach allows us to consider the spread and depth of the limit order book simultaneously. Liquidity suppliers, in their efforts to optimize gains from liquidity traders and losses to informed traders, focus on the prices at which limit orders are placed and the quantity of these orders.[3] Volatility, of course, is outside the control of liquidity suppliers. Suppose an increase in volatility increases the option value of the limit order book. Liquidity suppliers respond by placing limit orders that are further from the mean of the bid and ask (i.e., that are more out of the money) and/or by reducing their quantity, both of which decrease the option value of the limit order book. But we know nothing about whether the behavior of liquidity suppliers and the relationship between spread and depth leaves the option value of the book increased, decreased or unchanged.

We estimate the option value of limit orders standing in the book at 11:00 a.m. each day for 10 actively traded stocks traded on the ASX. We also examine the stability of the option value of the limit order book. We report evidence that the option value of the limit order book is more stable over our sample period than either the value of an individual limit order option or the number of shares in the limit order book.

[2]There have been numerous studies examining the behavior and determinants of the spread (McInish and Wood, 1992), as well as studies relating the spread to liquidity (Amihud and Mendelson, 1991) and transactions costs (Chan and Lakonishok, 1995). Concentrating on the bid-ask spread, Copeland and Galai conclude that the spread is a positive function of the price level and return variance and a negative function of market activity and depth.

[3]In the short-lived options examined here we can reasonably assume that interest rates are zero. Further, as explained below, initially we assume that the time until expiration of each option is fixed. But we intend to explore the efficacy of this assumption in subsequent analysis.

Table 1. Differences between exchange traded options and limit orders.

	Exchange-traded options	Limit orders
Writers' preferred expiration price	At the money	Early execution by uninformed traders
Cancelable?	Only after repurchase	Yes
Cost to counterparty	Yes	No
Expiration date	Fixed by exchange	Fixed by writer
Can trade in the money?	Yes	No

The first column indicates five characteristics of an option. These characteristics are described for exchange traded options in column two and for limit orders in column three.

Our goal is to enhance our knowledge concerning the option value of the limit order book. But to guard against extending the limit order analogy too far, it may be useful to explore differences between exchange-traded options and limit order options. Table 1 summarizes these differences. First, exchange-traded option writers generally hope that the options they have written will expire at the money because the consequent failure of the option to be exercised will reduce transaction costs. But liquidity suppliers placing limit orders hope that their orders will be exercised by noise traders. Second, limit order suppliers can cancel the option and are likely to do so if the likelihood of execution by an informed trader increases, but exchange-traded option contracts must be repurchased before they can be cancelled. Third, options have a premium, which is a cost to the buyer and compensates the writer whereas limit orders are provided freely to the market. Fourth, option contracts have an expiration fixed by the exchange whereas limit orders have their expiration determined by the liquidity supplier. Finally, exchange-traded options can trade in the money whereas limit orders cannot.

2. The ASX Market Structure

The ASX trading environment consists of a network of interconnected terminals, which form part of the SEATS. SEATS was based on a similar system known as Computer Automated Trading System (CATS) developed by the Toronto Stock Exchange. The SEATS displays the complete limit order book of all unexecuted orders to participating brokers. Details of unexecuted limit orders displayed include the associated price, broker, and quantity. In order to execute a trade on the ASX, brokers place orders relating to clients and their own principal demands through SEATS terminals in their offices.[4] The SEATS then automatically executes orders which cross according to price and then time precedence rules. Orders that are not crossed remain displayed in the limit order book.

Trading normally takes place on a continuous basis between 10:00 a.m. and 16:00 p.m.[5] However, immediately prior to commencement of trading a call market operates on the ASX where stocks are opened in groups, following a two and half period referred to by the ASX as the pre-open phase during which limit orders may be submitted and amended, but no trade takes place.

In order to identify limit orders as being at a specific price tick away from the best bid and best ask, it is necessary to recognize different minimum price ticks applicable to various securities. Minimum ticks stipulate the minimum distance in cents at which an order can be placed next to another of a different price. They dictate the minimum price movement and minimum bid ask spread faced by investors. Minimum price variations at various stock price levels are shown in Table 2. From November 1995, the minimum price ticks have been as follows: up to $0.10, 0.1 cent; over $0.10 to $0.50, 0.5 cent; over $0.50, 1 cent.

Table 2. Mean number of shares.

Stock	Bid5	Bid4	Bid3	Bid2	Bid1	Ask1	Ask2	Ask3	Ask4	Ask5
AMP	2,539	3,222	3,190	5,458	4,889	5,887	2,996	3,706	2,162	1,763
ANZ	6,315	9,115	5,677	7,772	6,527	10,423	7,108	11,197	4,781	6,726
BHP	20,764	21,349	17,416	27,481	20,126	22,741	24,801	28,538	23,566	29,267
CBA	2,076	2,237	2,853	2,283	3,581	4,433	3,118	2,623	2,907	1,434
NAB	2,858	2,950	2,486	5,364	9,946	4,734	3,701	3,370	4,988	6,443
NCP	5,392	5,790	8,767	6,259	9,660	10,250	5,775	7,459	3,776	3,789
RIO	595	1,489	1,838	1,628	3,548	4,295	2,683	1,431	961	1,338
TLS	106,686	119,191	151,157	158,790	116,582	117,524	152,826	153,353	117,723	103,370
WBC	4,100	9,208	6,240	9,761	8,643	10,704	8,302	11,874	6,042	9,346
WOW	3,625	8,295	5,295	5,671	6,224	8,248	7,615	8,177	4,438	4,062
Average	15,495	18,285	20,492	23,047	18,973	19,924	21,892	23,173	17,134	16,754

For each firm in our sample and for the full sample, we report the mean number of shares at the five best bid prices and five best ask prices at 11:00 a.m.

[4]A SEATS terminal allows the operator to enter limit orders in their order book without transferring them to the active market. Hence, the contents of the order book are private to the operator until such time as the operator transfers the order to the active market. This unrevealed demand is not captured by the exchange and is not available for study.

[5]Trading halts occur when the ASX receives information, which is deemed to be price-sensitive. The market reopens with a call after trading halts.

3. Data and Methodology

3.1. *Databases and sample selection*

We begin with a data set comprising all limit orders and on-market trades on the ASX for the ten largest firms in terms of market capitalization.[6] The ten firms included in the sample comprise more than one-half of the market capitalization of the ASX. The ticker symbols for these firms are: AMP, ANZ, BHP, CBA, NAB, NCP, RIO, TLS, WBC, and WOW. We collect data daily for the period 3 September 2001 to 31 December 2001.

These data are obtained from the SEATS database of the Securities Industry Research Centre of Asia-Pacific (SIRCA) located at the University of Sydney. The SEATS database includes records describing all trades and orders placed on the ASX. The data are captured on-line and in real time. Very detailed information is included in each order and trade including the date, time to the nearest hundredth of a second, stock code, price, volume, broker, and various flags relating to order type (e.g., short, fill-or-kill). SEATS record details relating to all limit orders, including undisclosed limit orders.

3.2. *Reconstruction of the limit order schedule*

Relatively well-traded stocks are used because they typically exhibit depth during most of the trading day. Each limit order and all amendments to or cancellations of limit orders are captured and used to reconstruct the entire limit order book as it existed during the period examined. Reconstructing the schedule required the incorporation of particular institutional features of the market such as the strict enforcement of priority rules, and the treatment of market orders. A snapshot of the recreated schedule was taken at 11:00 a.m. each day. Each snapshot includes the best bid and ask prices and associated quantities and the four possible price steps on each side of the best bid and ask prices and the quantities at each of these price steps. We report the schedule for only four price steps away from the best bid and ask prices because we find that the option value of a limit order placed further away from the schedule is negligible. Our sample comprises a total of 840 snapshots.

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[6]While the ASX has an "upstairs" market that can be used for executing very large trades, the focus of this study is on-market trading since it is on market orders which make up the limit order book. Off-market trading may however be related to the limit order book insofar as traders executing off-market trades would withdraw limit orders, which they had placed earlier.

3.3. *Calculation of variables and the option value of a limit order*

The option value of one share is calculated at each price step using the Black and Scholes Option Pricing Model and an iterative technique as in Latane and Rendleman (1976).

Black and Scholes (1973) originally developed a model to price European call options. Their model states that the European call option price, C^E, is a function of five determinants: the asset price S, the exercise price X, the option's maturity T, the riskless rate of interest r, and the asset's volatility as captured by s^2, the asset's annualized return variance. The price of a European call option is directly related to S, R, s^2, and T, and is inversely related to X. A European put option's value is directly related to X, R, s^2, and T, and is inversely related to S.

As inputs we need exercise price, stock price, the risk-free rate of interest, the time period that the limit order remains outstanding, and the volatility of the underlying stock. These inputs are calculated as follows:

Exercise price: The exercise price for calls (puts) is the ask (bid) quote.

Stock price: We use the midpoint of the best bid and ask.

Risk-free rate: The risk-free rate of interest is assumed to be 5% over the sample period. Given that limit orders remain in the schedule for a short time, the value of the limit order is not sensitive to the interest rate assumed.

Option life: It is necessary to make an assumption regarding the life of the limit order. Limit orders on the ASX remain in the schedule until they are matched with market orders, amended or cancelled since the limit order book is not cleared overnight. Berkman (1996) argues that because "on some markets it might take several minutes before an instruction to cancel a limit order is executed" options in the limit order book have maturity, which gives them option value. This maturity depends not only on the mechanics of the trading system, but also on the vigilance of traders. Obviously, if limit orders are supplied for a longer length of time they will have a greater option value. But because the value of each option in the limit order book is affected by a change in assumed life, a change in assumed life does not affect the ranking of option values in the limit order book. Initially, we assume that all limit order options have a maturity of five hours. We replicated our analysis assuming various option lives and the conclusions reported below are not changed. Our approach is in contrast to Harris and Panchapagesan (2005) who estimate the life of limit order options.

Volatility: A measure of the volatility of the underlying stock is also necessary in order to calculate the option value of a limit order. In order to avoid endogeneity, stock prices are not used to calculate stock volatility.[7] Instead, we use the implied volatility based on the nearest-to-maturity, at-the-money call option trading on the ASX. Limiting our sample to large capitalization firms insures that both the stock and the associated option are actively traded. The option quotes are sampled at the same time as the stock quotes.[8]

3.4. *The limit order schedule and its option value*

Once the option value of an individual limit order is calculated, the total option value of the limit order book at that price is calculated by multiplying by the quantity at that price step. The option value of the limit order book for each snapshot is the sum of these values at the best bid and ask prices and the first four permissible ticks on each side of the best bid and ask. We also calculated the option value of the bids and asks separately in a similar manner.

4. Empirical Results

4.1. *An intraday examination of the limit order schedule*

Figure 1 and Table 2 show the mean number of shares at the best bid and ask and at the first four price ticks on each side of these quotes. Only schedules with positive quantities at each of the ten possible ticks are reported. The stocks in our sample have full schedules each day. There are more full schedules at 11:00 a.m. than at other times of the day.[9]

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[7]Endogeneity may arise because stock price movements would influence both the difference between limit order prices and equilibrium price and stock volatility measures calculated directly from stock prices, thereby causing significant correlation between such problems.

[8]In a previous version of this paper, we used the formula for implied standard deviation from Brenner and Subrahmanyam (1988). We switched to the current method because it is used by the ASX to calculate implied volatilities that are distributed to the public.

[9]In an earlier version of this paper, we considered two alternate ways of presenting the results for thinner stock that might not always have full schedules. One treats zero quantities as missing observations and the other includes them as zero in the calculation of the mean. The alternate approaches did not affect our conclusions and hence are not presented.

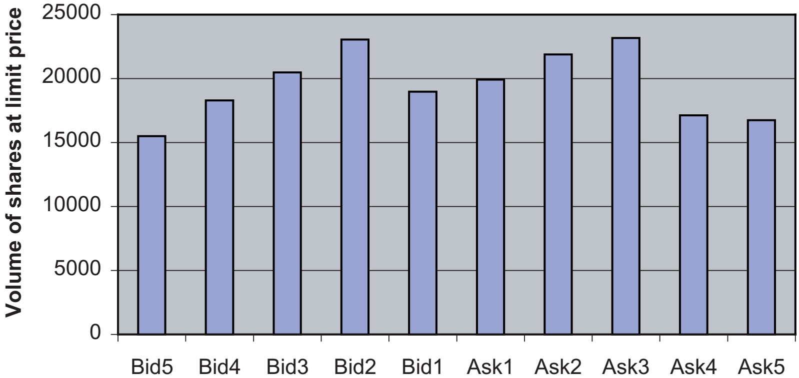

Figure 1. Mean number of shares at the bid and ask. Considering all limit orders for all of the firms in our sample, we report the mean aggregate number of shares and the BBO and at each of the first four ticks on each side of the BBO at 11:00 a.m.

For convenience, we label the best bid as BID1 and each tick away from the best bid as BID2, BID3, and so forth. We use comparable notation for the ask side. On average, there are about 18,000 shares at the best bid and about 20,000 shares at the best ask. For each of the firms the number of shares at the best ask is greater than at the best bid. In fact, at each price step away from the best bid and ask the number of shares at the bid is greater than the number of shares at the ask.[10] Interestingly, there are more shares at BID2 than at BID1 for five of our ten firms. On the other hand, ASK1 has more shares than ASK2 for eight of the ten firms.

The mean implied volatility for each of the ten firms is presented in Table 3. We do not believe that there is anything unusual about these implied volatilites that would affect our results.

Table 4 and Figure 2 present the mean of the value of a single option at each tick for each firm. Of course, the option value declines as the ticks move away from the best bid or ask since the options become more and more out of the money. An option for a single share at the either BID1 or ASK1 is worth

[10]In this regard it should be noted that there are other alternatives for trade. As stated above approximately 33% of trade by value is executed off-market. It is possible that sellers are more likely to seek off-market execution thereby resulting in a balance in demand and supply when both on-market and off-market trading are considered together. However, we are unable to measure the level of buyer versus seller initiated trading off-market.

Table 3. Implied volatility.

Stock	Implied volatility
AMP	0.2519
ANZ	0.2404
BHP	0.3031
CBA	0.2385
NAB	0.2501
NCP	0.4197
RIO	0.3161
TLS	0.2440
WBC	0.2094
WOW	0.2550
Average	0.2728

For each of the firms in our sample and for the entire sample, we present the mean implied volatility. The volatility is calculated each day for each option series. The volatilities measure the stock's implied standard deviation and are annualized.

Table 4. Option value of a single option.

Stock	Bid5	Bid4	Bid3	Bid2	Bid1	Ask1	Ask2	Ask3	Ask4	Ask5
AMP	0.0223	0.0256	0.0292	0.0332	0.0375	0.0380	0.0337	0.0297	0.0260	0.0227
ANZ	0.0192	0.0224	0.0260	0.0299	0.0343	0.0348	0.0304	0.0264	0.0228	0.0196
BHP	0.0103	0.0129	0.0161	0.0199	0.0242	0.0245	0.0201	0.0164	0.0132	0.0105
CBA	0.0583	0.0621	0.0663	0.0706	0.0752	0.0760	0.0714	0.0670	0.0629	0.0590
NAB	0.0468	0.0507	0.0548	0.0591	0.0637	0.0646	0.0600	0.0556	0.0515	0.0476
NCP	0.0343	0.0380	0.0420	0.0463	0.0508	0.0513	0.0467	0.0424	0.0385	0.0348
RIO	0.0719	0.0759	0.0802	0.0845	0.0891	0.0900	0.0855	0.0811	0.0769	0.0728
TLS	0.0010	0.0019	0.0034	0.0058	0.0094	0.0096	0.0060	0.0035	0.0020	0.0011
WBC	0.0105	0.0130	0.0160	0.0194	0.0235	0.0238	0.0198	0.0163	0.0133	0.0108
WOW	0.0098	0.0124	0.0154	0.0190	0.0232	0.0235	0.0193	0.0157	0.0126	0.0100
Average	0.0284	0.0315	0.0349	0.0388	0.0431	0.0436	0.0393	0.0354	0.0320	0.0289

For each of the firms in our sample and overall, we present the value of a single option at each of the five best bid prices and ask prices. The values are in AUD and are calculated using the Black-Scholes option pricing formula and the implied volatilities presented in Table 3.

about $0.04 (recall that all values are in AUD) and an option at BID5 or ASK5 is worth about $0.03.[11]

[11] Further analysis has indicated that values further away from the schedule are relatively greater when it is assumed that limit orders remain for longer time periods. However, irrespective of the time period assumed, limit orders submitted at the best bid and ask continue to have much greater per unit value than limit orders submitted further away.

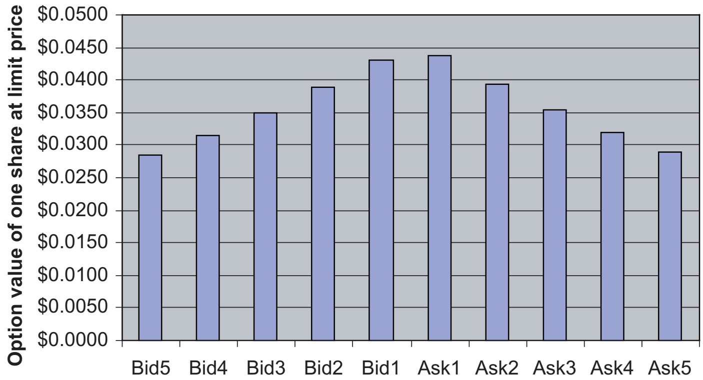

Figure 2. Mean option value (AUD) of one share. For the best bid and the best ask and the four ticks nearest to the best bid and best ask, respectively, we report the mean option value of a limit order for one share across the stocks in our sample at 11:00 a.m.

The option value at each price step is calculated by multiplying the option value per share at a given price step times the number of shares offered or sought at that price step. Table 5 and Figure 3 report option values at the best bid and ask and the first four steps on each side of these quotes considering only full schedules. The mean value of the limit order book at the first five ticks is $1262.97 on the ask side and $1199.01 on the bid side. On average 34.7% ($418.51/1262.97) of the liquidity is provided at the best ask and 33.1% ($416.41/$1199.01) at the best bid.

Table 5. Mean option value of the limit order book.

	Bid5	Bid4	Bid3	Bid2	Bid1	Ask1	Ask2	Ask3	Ask4	Ask5
AMP	54.34	81.15	91.95	187.97	187.29	212.35	100.15	114.87	54.09	41.78
ANZ	115.12	253.78	144.18	222.90	212.27	370.48	218.97	291.53	121.01	129.89
BHP	211.91	263.26	276.20	565.75	472.46	559.99	506.22	449.28	307.08	306.26
CBA	79.50	104.86	237.23	176.03	413.54	263.82	329.19	126.09	128.29	63.97
NAB	139.00	163.55	140.30	324.28	665.11	303.97	242.78	172.67	258.52	317.06
NCP	187.31	209.95	366.98	290.47	480.51	531.62	270.69	319.83	139.24	124.62
RIO	44.61	111.28	149.69	136.12	314.72	374.17	243.59	106.35	70.41	103.33
TLS	108.55	207.58	475.89	917.87	1,066.78	1,115.89	886.97	601.85	219.06	99.62
WBC	45.89	139.46	89.93	178.64	206.78	261.07	156.88	191.91	88.68	142.94
WOW	41.24	104.38	79.49	107.51	144.61	191.73	169.84	133.81	56.36	38.87
Average	102.75	163.92	205.18	310.75	416.41	418.51	312.53	250.82	144.28	136.83

For each of the firms in our sample and overall, we present the mean value of the limit orders standing in the book at 11:00 a.m. For a given stock for a given day, the option value of the limit order book at a particular step in the book is calculated by multiplying the value of a single option at that step times the number of shares at that step. The process is repeated for each stock for each day, giving 840 observations in total.

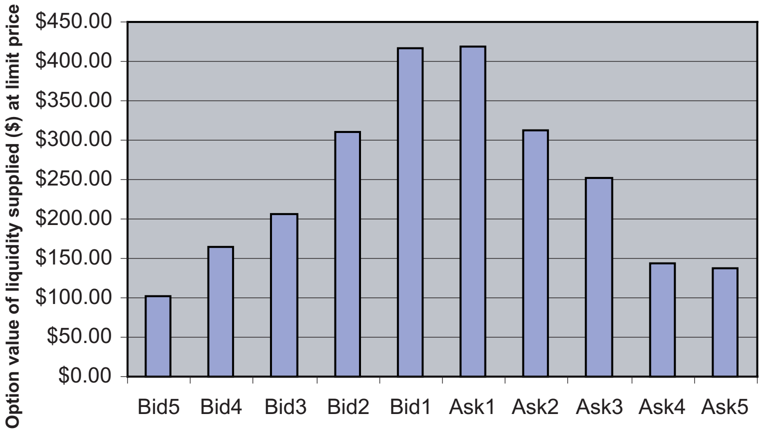

Figure 3. Mean option value (AUD) of the limit order book. For the best bid and best ask and each of the four ticks closest to the best bid and ask, we provide the mean option value of the limit order book across the firms in our sample at 11:00 a.m.

4.2. *Robustness of results across size of stocks and time periods*

In order to investigate whether the documented option values are consistent across time periods, we segment the sample into two periods — 3 September 2001 to 30 October, 2002 (period 1) and 31 October 2001 to 31 December 2001 (period 2).

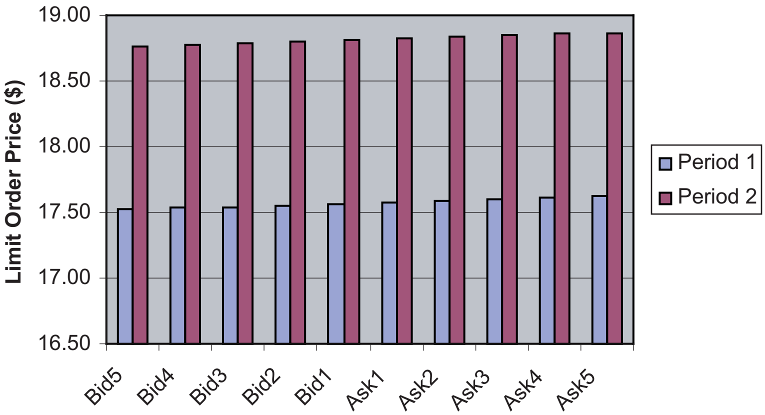

Figure 4. Mean bid and ask of limit orders for all firms, by period. We report the mean bid and ask at the five best ask prices and the five best bid prices at 11:00 a.m. for two periods — period 1 (3 September 2001) and period 2 (30 October 2001).

The mean bid price and ask price for the five best bid prices and ask prices are presented in Figure 4 and Table 6. Figures 5–7 repeat Figures 1–3, respectively, but showing the results by period. Numerical data are provided in Table 6. Prices declined from period 1 to period 2. The option value of a single shares declined 20% for the best bid and best ask. In response to the lower option cost of placing limit orders, the quantities in the limit order book increased 45% for the best bid and 38% for the best ask. As a result of the offsetting movements of quantity and option value, the option value of the limit order book was little changed between periods 1 and 2.

Table 6. Option value of the limit order book, by period.

	Period	Bid5	Bid4	Bid3	Bid2	Bid1	Ask1	Ask2	Ask3	Ask4	Ask5
Limit	1	17.52	17.53	17.54	17.55	17.56	17.58	17.59	17.60	17.61	17.62
price	2	18.77	18.78	18.79	18.80	18.81	18.83	18.84	18.85	18.86	18.87
Quantity	1	12,683	16,647	17,144	19,143	15,478	16,725	17,460	19,787	13,062	9,656
	2	18,307	19,922	23,840	26,950	22,467	23,123	26,325	26,559	21,207	23,851
Option value	1	0.0331	0.0363	0.0398	0.0436	0.0480	0.0485	0.0441	0.0402	0.0367	0.0336
of one share	2	0.0237	0.0267	0.0301	0.0339	0.0382	0.0388	0.0344	0.0306	0.0272	0.0242
Option value	1	102.61	188.07	218.67	321.83	407.52	417.82	334.54	245.73	146.57	96.30
of book	2	102.89	139.78	191.70	299.68	425.29	419.20	290.51	255.91	141.98	177.37

For the five best bid prices and five best ask prices at 11:00 a.m., we report means of the limit price, number of shares, option value of one share, and option value of the book. All prices are in AUD. We report these results for two periods — 3 September 2001 to October 30, 2001 (period 1) and (31 October 2001 to 31 December 2001 (period 2).

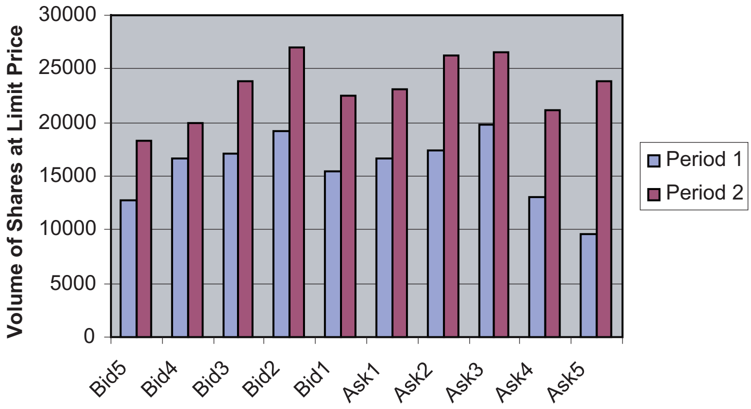

Figure 5. Mean number of shares standing at each limit order price. We report the depth at the five best ask prices and the five best bid prices at 11:00 a.m. for two periods — period 1 (3 September 2001) and period 2 (30 October 2001).

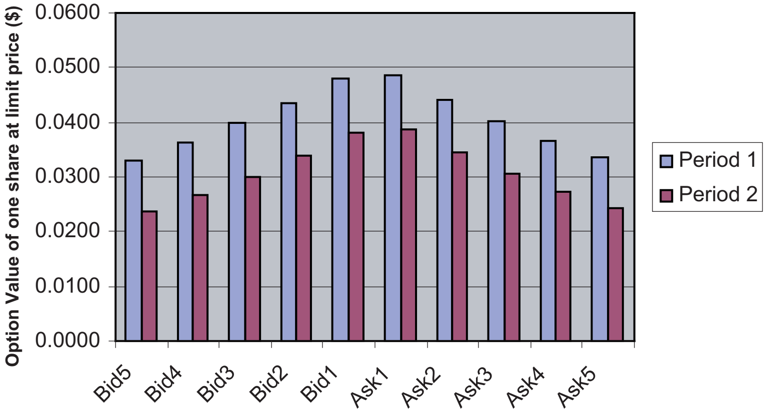

Figure 6. Mean option value (AUD) of one share standing at each limit price. We report the option value of a single share at the five best ask prices and the five best bid prices at 11:00 a.m. for two periods — period 1 (3 September 2001) and period 2 (30 October 2001).

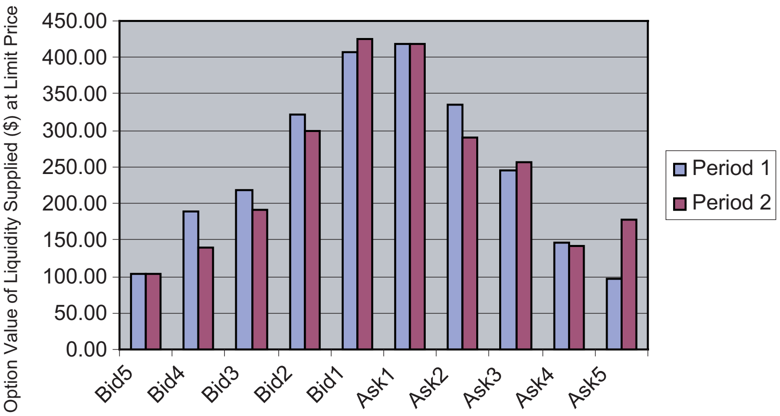

Figure 7. Mean option value (AUD) of liquidity supplied at each limit price. We report the option value of liquidity supplied at the five best ask prices and the five best bid prices at 11:00 a.m. for two periods — period 1 (3 September 2001) and period 2 (30 October 2001).

5. Summary and Conclusions

We use the option pricing approach of Copeland and Galai (1983) to calculate the option value of the limit order book at 11:00 a.m. for the 10 most highly capitalized firms listed on the ASX during the last four months of 2001. As inputs into the option value calculation, we use implied volatilities from each firm's associated option prices. The option value of the limit order book provides a measure of liquidity that (1) is based on established finance theory, (2) is a monetary amount, and (3) combines the two main intraday proxies for liquidity, spreads, and quantities in the limit order schedule. Spreads and order sizes have previously typically been considered separately.

We find that the option value of the limit order book is greatest at the best bid price and the best ask price, and is more stable through time than the option value of individual shares or share quantities in the book.

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Section II

Essays on Liquidity of Markets

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Chapter 5

The Cross-Section of Daily Variation in Liquidity

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In this paper, we analyze cross-sectional heterogeneity in the time-series variation of liquidity in equity markets. Our analysis uses a broad time-series and cross-section of liquidity data. We find that average daily changes in liquidity exhibit significant heterogeneity in the cross-section; the liquidity of small firms varies more on a daily basis than that of large firms. A steady increase in aggregate market liquidity over the past decade is more strongly manifest in large firms than in small firms. Absolute stock returns are an important determinant of liquidity. We investigate cross-sectional differences in the resilience of a firm's liquidity to information shocks. We use the sensitivity of stock liquidity to absolute stock returns as an inverse measure of this resilience, and find that the measure exhibits considerable cross-sectional variation. Firm size, return volatility, institutional holdings, and volume are all significant cross-sectional determinants of this measure.

Keywords: Liquidity; friction; spreads; depth; information shocks.

Liquidity is the grease that facilitates the smooth functioning of financial markets. A lack of liquidity is a form of friction (Stoll, 2000) that can have adverse effects on asset values, as demonstrated by Amihud and Mendelson (1986). Recent events such as the 1998 bond market crisis have heightened regulatory concerns about liquidity crises.[1] The study of liquidity is important, from a scientific as well as a practical standpoint.

Many studies of liquidity have documented that liquidity varies in the cross-section. Papers that focus on the cross-sectional determinants of liquidity include Benston and Hagerman (1974), Branch and Freed (1977), Stoll (1978),

[1] See the *Wall Street Journal*, "Illiquidity is Crippling the Bond World," (October 19, 1998) p. C1, "Illiquidity means it has become more difficult to buy or sell a given amount of any bond but the most popular Treasury issue. The spread between prices at which investors will buy and sell has widened, and the amounts in which Wall Street firms deal have shrunk across the board for investment grade, high-yield (or junk), emerging market and asset-backed bonds…The sharp reduction in liquidity has preoccupied the Fed because it is the *lifeblood* of markets." (emphasis added).

and Easley, Kiefer, O'Hara, and Paperman (1996). Of late, there has been interest in examining the time-series variation in market-wide liquidity (see Chordia, Roll, and Subrahmanyam (CRS), 2001).

While cross-sectional and time-series variations in liquidity have been analyzed in separate strands of literature, not much is known about how the time-series behavior of liquidity varies in the cross-section. There are sound reasons to study this issue. An immediate question is whether any trends in liquidity over the recent past are discernible uniformly in the cross-section. Another issue is whether the extent of day-to-day variation in liquidity differs across firms. A third question is whether there are cross-sectional differences in the ability of equity markets to provide liquidity when information shocks buffet the value of the security. That is, how resilient is liquidity to information flows that affect the value of the company?

The latter question raises the issue of how to measure the sensitivity of liquidity to information flows. Stock returns move both because of information as well as temporary price pressures; the second type of movement is reversible. Since daily stock returns exhibit extremely low serial correlation in our sample, we use the daily absolute return as a proxy for daily information flow, and use the sensitivity of liquidity to absolute returns as an inverse measure of the resilience of liquidity to information shocks. Inventory and asymmetric information arguments suggest that this resilience could be very different across firms with differing market capitalization and differing levels of trading activity. However, since there is no extant evidence on this issue, an empirical question of interest is whether the time-series sensitivity of liquidity to information varies significantly in the cross-section, and if so, what cross-sectional attributes capture the heterogeneity in this relationship.

Motivated by the above observations, we seek to document cross-sectional heterogeneity in the time-series variation of liquidity, and in the sensitivity of liquidity to daily stock price fluctuations.[2] Specifically, we ask the following questions: (i) Are any trends in liquidity over the recent past discernible uniformly in both small and large stocks? (ii) Is the extent of day-to-day

[2]In this paper, we do not attempt to shed explicit light on the inventory vs. asymmetric information hypotheses. That is an exercise which can be better conducted using transaction-by-transaction data. Our goal here is to present stylized facts on cross-sectional heterogeneity in daily liquidity variations; however, studies such as Glosten and Harris (1988) suggest that the inventory component is small at daily horizons. Further, while some important studies have analyzed cross-sectional differentials in liquidity around specific events (see Goldstein and Kavajecz, 2000; Corwin and Lipson, 2000) our focus here is on long-term variations in liquidity across a multitude of events.

variation in liquidity uniform across all firms in the cross-section? (iii) How does the relation between liquidity and absolute stock returns vary in the cross-section? (iv) What firm-specific characteristics explain cross-sectional variation in this co-movement?

Apart from the straightforward goal of understanding more about the general topic of liquidity, our study has asset pricing implications. For instance, larger liquidity improvements for some firms relative to others imply a greater reduction in their costs of capital. In addition, knowing the determinants of the relation between liquidity and stock price movements can aid in the development of trading strategies; for example, stocks whose resilience to stock price movements is small imply higher trading costs during periods of important news announcements. From an academic standpoint, understanding the time-series relation as well as the cross-sectional relation between liquidity and stock price movements can help us to gain a better understanding of why stock liquidity moves over time.

In our empirical analysis, we depart from the existing cross-sectional studies of liquidity by using a broad time-series and cross-section of liquidity data. Specifically, we use daily liquidity data on more than 1200 NYSE stocks over more than 2500 trading days; whereas most existing cross-sectional studies of liquidity (e.g., Benston and Hagerman, 1974; Branch and Freed, 1977; Stoll, 1978) use data over an year or less for a relatively small sample of stocks. Our comprehensive sample allows us to enhance the reliability of our results, and, unlike existing studies, we study both the time-series and cross-section of liquidity.

We find that the increase in liquidity over the past decade, while manifest across the cross-section, is more pronounced for the larger stocks. Further, the daily liquidity of small firms is far more volatile than that of large firms. We also find that daily absolute returns are an important determinant of day-to-day variations in liquidity; in particular, spreads vary strongly and positively with absolute returns. This result obtains for returns computed using closing prices as well as the midpoint of the last bid and ask quotes during a day, so it is not an artefact of bid-ask bounce.[3] In addition, individual stock liquidity

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[3]We interpret the relation between liquidity and absolute returns as representing the resilience of liquidity to information flows, i.e., we take daily absolute returns as a measure of daily information flow. This interpretation is supported by the finding that daily returns exhibit virtually zero serial correlation, so that noise does not appear to be significant factor in daily returns; in addition, return variations due to changes in liquidity premia are related to signed, not absolute returns. See footnote 6 for a more detailed explanation.

is also strongly and positively related to a five-day moving average of lagged absolute returns (where the latter variable, given volatility persistence, proxies for expected future volatility). After controlling for concurrent absolute stock returns and recent stock volatility, concurrent and recent market movements do not appear to be important in determining stock liquidity.

The co-movement between liquidity and absolute stock returns, an inverse measure of the resilience of a firm's liquidity to information shocks, exhibits considerable cross-sectional heterogeneity. We explore the cross-sectional determinants of this co-movement. Return volatility, stock market volume, and firm size strongly and negatively affect this relation. Variability of volume and the level of the stock price are positively related to this relation. Institutional holdings influence the relation negatively in large firms. In sum, the cross-sectional results demonstrate that the resilience of equity market liquidity to stock price movements is (*ceteris paribus*) greatest for large firms, firms with high trading volume, firms with high return volatility, and firms with low variability in trading activity. Greater institutional holdings are positively associated with this capacity in large firms.

The rest of the paper is organized as follows. Section 1 describes the data. Section 2 documents the time-series response of liquidity to absolute returns, and analyzes the cross-sectional determinants of the response coefficient. Section 3 concludes the chapter.

1. Data

The data sources are the Institute for the Study of Securities Markets (ISSM) and the New York Stock Exchange (NYSE) TAQ (trades and automated quotations). The ISSM data cover 1988–1992 inclusive while the TAQ data are for 1993–1998. We use only NYSE stocks to avoid any possibility of the results being influenced by differences in trading protocols.

1.1. *Inclusion requirements*

Stocks are included or excluded during a calendar year depending on the following criteria:

- To be included, a stock had to be present at the beginning and at the end of the year in both the CRSP and the intraday databases.

- If the firm changed exchanges from NASDAQ to NYSE during the year (no firms switched from the NYSE to the NASDAQ during our sample period), it was dropped from the sample for that year.
- Because their trading characteristics might differ from ordinary equities, assets in the following categories were also expunged: certificates, ADRs, shares of beneficial interest, units, companies incorporated outside the US, Americus Trust components, closed-end funds, preferred stocks and REITs.
- To avoid the influence of unduly high-priced stocks, if the price at any month-end during the year was greater than $999, the stock was deleted from the sample for the year.

Intraday data were purged for one of the following reasons: trades out of sequence, trades recorded before the opening or after the closing time, and trades with special settlement conditions (because they might be subject to distinct liquidity considerations). Our preliminary investigation revealed that autoquotes (passive quotes by secondary market dealers) were eliminated in the ISSM database but not in TAQ. This caused the quoted spread to be artificially inflated in TAQ. Since there is no reliable way to filter out autoquotes in TAQ, only BBO (best bid or offer)-eligible primary market (NYSE) quotes are used. Quotes established before the opening of the market or after the close were discarded. Negative bid-ask spread quotations, transaction prices, and quoted depths were discarded. Following Lee and Ready (1991), any quote less than five seconds prior to the trade is ignored and the first one at least five seconds prior to the trade is retained.

For each stock we define the following variables:

QSPR: The quoted bid-ask spread associated with the transaction.
RQSPR: The quoted bid-ask spread divided by the midpoint of the quote (%).
ESPR: The effective spread, i.e., the difference between the execution price and the midpoint of the prevailing bid-ask quote.
RESPR: The effective spread divided by the midpoint of the prevailing bid-ask quote (%).
DEPTH: The average of the quoted bid and ask depths.
$DEPTH: The average of the ask depth times ask price and bid depth times bid price.

COMP = RQSPR/\$DEPTH: Spread and depth combined in a single measure. COMP is intended to measure the average slope of the liquidity function in percent per dollar traded.

Our initial scanning of the intraday data revealed a number of anomalous records that appeared to be keypunching errors. We thus applied filters to the transaction data by deleting records that satisfied the following conditions:

1. QSPR > \$5.
2. ESPR/QSPR > 4.0.
3. RESPR/RQSPR > 4.0.
4. QSPR/PRICE > 0.4.

These filters removed fewer than 0.02% of all transaction records. In addition, because we later document the relation between liquidity and absolute returns, days for which stock return data was not available from CRSP were dropped from the sample.

1.2. *Summary statistics*

Panel A of Table 1 presents the cross-sectional averages of the liquidity measures in each year of our sample period, as well as for the entire sample. The variables are first averaged for each firm for each year, and then averaged cross-sectionally. As can be seen, the effective spread is lower than the quoted spread, because a large proportion of transactions take place within the spread. The table also indicates that the quoted and effective spreads have generally decreased over time during our sample period.[4] However, focusing on the cross-sectional standard deviation for the variables, we notice that the averages hide significant cross-sectional variation in liquidity, particularly in the depth and relative spread variables.

Panels B–E of Table 1 show the trend in the liquidity variables across size quartiles. As can be seen, both quoted and effective spreads have shown a steady decline across both small and large firms. For instance, the quoted (effective) spread for the smallest firms has declined from \$0.21 (\$0.16) in 1988 to \$0.18 (\$0.12) in 1998 and for the largest firms it has decreased from \$0.23 (\$0.17) to \$0.14 (\$0.09). The relative quoted and effective spreads have

[4]This is also pointed out in Chordia, Roll, and Subrahmanyam (2001), who look at time-series variation in *aggregate market* liquidity.

Table 1. Average liquidity measures by year, 1988–1998.

Year		QSPR	RQSPR(%)	ESPR	RESPR(%)	DEPTH^{+}	$_DEPTH^{++}	COMP
Panel A: All firms								
1988	Mean	0.238	1.705	0.174	1.275	5.730	0.110	1.534
($n = 1193$)	Std. Dev.	0.079	1.895	0.062	1.523	8.456	0.116	7.088
1989	Mean	0.226	1.618	0.149	1.121	6.738	0.144	2.099
($n = 1185$)	Std. Dev.	0.080	2.126	0.053	1.650	9.413	0.156	15.970
1990	Mean	0.229	2.141	0.139	1.416	6.128	0.113	5.750
($n = 1208$)	Std. Dev.	0.085	2.926	0.046	2.232	8.733	0.124	38.211
1991	Mean	0.226	2.044	0.145	1.397	6.088	0.119	5.315
($n = 1255$)	Std. Dev.	0.088	2.789	0.052	2.075	8.030	0.136	42.497
1992	Mean	0.221	1.775	0.143	1.212	6.565	0.139	2.691
($n = 1311$)	Std. Dev.	0.086	2.449	0.053	1.799	8.850	0.157	16.948
1993	Mean	0.221	1.492	0.139	0.988	6.703	0.154	1.709
($n = 1392$)	Std. Dev.	0.080	1.974	0.052	1.419	8.650	0.177	15.478
1994	Mean	0.210	1.361	0.136	0.919	6.586	0.148	1.077
($n = 1466$)	Std. Dev.	0.067	1.599	0.044	1.170	8.653	0.182	9.349
1995	Mean	0.196	1.302	0.130	0.904	7.539	0.175	1.056
($n = 1495$)	Std. Dev.	0.054	1.687	0.034	1.267	9.654	0.217	9.796
1996	Mean	0.193	1.173	0.130	0.823	7.140	0.174	0.749
($n = 1545$)	Std. Dev.	0.050	1.454	0.032	1.106	9.065	0.202	4.884
1997	Mean	0.171	0.948	0.120	0.688	5.450	0.146	0.718
($n = 1548$)	Std. Dev.	0.069	1.270	0.045	0.997	6.537	0.155	5.686
1998	Mean	0.159	0.914	0.109	0.637	3.662	0.098	0.672
($n = 1444$)	Std. Dev.	0.061	1.026	0.044	0.754	4.102	0.092	2.089
All years	Mean	0.207	1.469	0.136	1.015	6.216	0.140	2.020
($n = 15042$)	Std. Dev.	0.077	2.010	0.050	1.508	8.364	0.164	19.200
Panel B: Quartile 1								
1988	Mean	0.212	3.623	0.158	2.775	6.114	0.028	5.308
($n = 298$)	Std. Dev.	0.063	2.884	0.048	2.351	10.724	0.020	13.561
1989	Mean	0.204	3.700	0.133	2.619	6.531	0.033	7.846
($n = 296$)	Std. Dev.	0.061	3.344	0.036	2.688	11.132	0.025	31.378
1990	Mean	0.202	5.115	0.122	3.462	5.895	0.022	22.115
($n = 302$)	Std. Dev.	0.066	4.505	0.036	3.604	10.375	0.017	74.337
1991	Mean	0.197	5.059	0.129	3.559	6.225	0.023	20.633
($n = 314$)	Std. Dev.	0.066	4.205	0.042	3.205	9.529	0.016	83.872
1992	Mean	0.202	4.317	0.134	3.022	6.512	0.027	10.131
($n = 328$)	Std. Dev.	0.061	3.788	0.040	2.831	10.953	0.023	32.844
1993	Mean	0.210	3.343	0.138	2.271	5.740	0.036	6.291
($n = 348$)	Std. Dev.	0.058	3.181	0.038	2.324	8.723	0.031	30.425

Table 1. (*Continued*)

Year		QSPR	RQSPR(%)	ESPR	RESPR(%)	DEPTH^{+}	\$_DEPTH^{++}	COMP
1994	Mean	0.210	2.886	0.139	2.007	5.400	0.035	3.789
($n = 366$)	Std. Dev.	0.053	2.566	0.034	1.901	7.874	0.025	18.541
1995	Mean	0.198	2.799	0.133	1.984	5.623	0.038	3.769
($n = 374$)	Std. Dev.	0.047	2.756	0.027	2.095	7.777	0.024	19.336
1996	Mean	0.195	2.571	0.133	1.844	6.555	0.043	2.620
($n = 386$)	Std. Dev.	0.047	2.323	0.031	1.801	10.359	0.030	9.524
1997	Mean	0.182	2.088	0.128	1.540	4.684	0.036	2.555
($n = 387$)	Std. Dev.	0.053	2.088	0.036	1.668	7.343	0.021	11.192
1998	Mean	0.177	1.940	0.123	1.370	3.198	0.028	2.215
($n = 361$)	Std. Dev.	0.063	1.548	0.044	1.163	6.011	0.014	3.722
All years	Mean	0.198	3.324	0.134	2.348	5.638	0.032	7.501
($n = 3760$)	Std. Dev.	0.059	3.241	0.039	2.475	9.246	0.024	37.902
Panel C: Quartile 2								
1988	Mean	0.257	1.552	0.188	1.143	4.502	0.055	0.627
($n = 299$)	Std. Dev.	0.089	0.704	0.069	0.581	8.208	0.033	0.495
1989	Mean	0.253	1.410	0.164	0.949	5.561	0.072	0.446
($n = 297$)	Std. Dev.	0.107	0.841	0.068	0.705	10.495	0.049	0.393
1990	Mean	0.247	1.824	0.149	1.180	5.709	0.058	0.743
($n = 302$)	Std. Dev.	0.087	1.135	0.042	0.962	10.340	0.038	0.651
1991	Mean	0.244	1.657	0.155	1.094	4.994	0.061	0.708
($n = 314$)	Std. Dev.	0.073	0.991	0.040	0.781	8.274	0.040	1.150
1992	Mean	0.242	1.383	0.157	0.915	4.495	0.066	0.458
($n = 328$)	Std. Dev.	0.073	0.671	0.046	0.528	6.362	0.038	0.413
1993	Mean	0.240	1.285	0.153	0.834	5.060	0.075	0.370
($n = 348$)	Std. Dev.	0.060	0.608	0.038	0.457	8.358	0.046	0.399
1994	Mean	0.225	1.243	0.145	0.812	4.683	0.072	0.396
($n = 366$)	Std. Dev.	0.048	0.506	0.031	0.377	7.482	0.052	0.344
1995	Mean	0.208	1.196	0.137	0.814	6.058	0.081	0.328
($n = 374$)	Std. Dev.	0.045	0.630	0.026	0.524	10.451	0.055	0.323
1996	Mean	0.207	0.985	0.138	0.672	4.525	0.081	0.267
($n = 387$)	Std. Dev.	0.041	0.411	0.026	0.331	6.629	0.060	0.211
1997	Mean	0.180	0.823	0.126	0.585	3.914	0.072	0.233
($n = 386$)	Std. Dev.	0.047	0.393	0.032	0.318	5.359	0.037	0.237
1998	Mean	0.171	0.814	0.118	0.561	2.810	0.056	0.300
($n = 361$)	Std. Dev.	0.074	0.367	0.055	0.259	2.647	0.025	0.262
All years	Mean	0.223	1.266	0.147	0.854	4.732	0.069	0.431
($n = 3762$)	Std. Dev.	0.075	0.750	0.048	0.588	7.974	0.045	0.523

Table 1. (*Continued*)

Year		QSPR	RQSPR(%)	ESPR	RESPR(%)	DEPTH$^+$	$_DEPTH^{++}	COMP
Panel D: Quartile 3								
1988	Mean	0.252	1.051	0.181	0.759	5.528	0.112	0.198
($n = 298$)	Std. Dev.	0.079	0.573	0.062	0.485	8.614	0.065	0.189
1989	Mean	0.228	0.871	0.151	0.587	6.799	0.151	0.120
($n = 296$)	Std. Dev.	0.047	0.328	0.031	0.264	8.641	0.089	0.109
1990	Mean	0.242	1.049	0.146	0.659	5.439	0.106	0.207
($n = 302$)	Std. Dev.	0.058	0.527	0.034	0.447	7.324	0.051	0.235
1991	Mean	0.234	0.978	0.148	0.634	5.781	0.115	0.177
($n = 313$)	Std. Dev.	0.062	0.419	0.040	0.331	7.214	0.060	0.215
1992	Mean	0.224	0.903	0.143	0.589	6.422	0.135	0.166
($n = 328$)	Std. Dev.	0.077	0.483	0.054	0.376	8.470	0.073	0.630
1993	Mean	0.225	0.833	0.140	0.525	6.378	0.146	0.110
($n = 348$)	Std. Dev.	0.090	0.312	0.063	0.221	7.525	0.096	0.094
1994	Mean	0.214	0.836	0.136	0.542	6.015	0.132	0.133
($n = 367$)	Std. Dev.	0.083	0.306	0.061	0.241	7.324	0.081	0.137
1995	Mean	0.199	0.763	0.130	0.511	6.918	0.158	0.101
($n = 373$)	Std. Dev.	0.073	0.312	0.051	0.245	7.792	0.119	0.106
1996	Mean	0.192	0.724	0.127	0.493	7.362	0.168	0.090
($n = 386$)	Std. Dev.	0.067	0.316	0.043	0.249	8.487	0.101	0.109
1997	Mean	0.170	0.561	0.118	0.400	5.525	0.142	0.075
($n = 388$)	Std. Dev.	0.109	0.311	0.070	0.269	6.405	0.073	0.076
1998	Mean	0.151	0.566	0.103	0.386	3.636	0.097	0.115
($n = 361$)	Std. Dev.	0.048	0.268	0.032	0.185	2.535	0.045	0.133
All years	Mean	0.210	0.818	0.137	0.545	5.990	0.134	0.133
($n = 3760$)	Std. Dev.	0.081	0.416	0.055	0.325	7.501	0.084	0.236
Panel E: Quartile 4								
1988	Mean	0.233	0.621	0.168	0.441	6.778	0.244	0.050
($n = 298$)	Std. Dev.	0.073	0.220	0.063	0.152	5.286	0.141	0.053
1989	Mean	0.219	0.512	0.148	0.345	8.053	0.319	0.030
($n = 296$)	Std. Dev.	0.085	0.172	0.064	0.119	6.626	0.195	0.026
1990	Mean	0.225	0.601	0.140	0.382	7.466	0.264	0.049
($n = 302$)	Std. Dev.	0.111	0.248	0.064	0.174	5.977	0.151	0.096
1991	Mean	0.230	0.549	0.145	0.352	7.336	0.274	0.043
($n = 314$)	Std. Dev.	0.128	0.210	0.073	0.140	6.713	0.177	0.082
1992	Mean	0.217	0.509	0.138	0.332	8.794	0.324	0.029
($n = 327$)	Std. Dev.	0.118	0.181	0.066	0.137	8.500	0.198	0.022

Table 1. (*Continued*)

Year		QSPR	RQSPR(%)	ESPR	RESPR(%)	DEPTH^{+}	$_DEPTH^{++}	COMP
1993	Mean	0.208	0.497	0.128	0.316	9.603	0.358	0.025
($n = 348$)	Std. Dev.	0.101	0.202	0.059	0.163	9.220	0.227	0.022
1994	Mean	0.192	0.505	0.122	0.333	10.182	0.350	0.029
($n = 367$)	Std. Dev.	0.072	0.201	0.039	0.162	10.462	0.248	0.028
1995	Mean	0.178	0.453	0.119	0.311	11.501	0.420	0.021
($n = 374$)	Std. Dev.	0.042	0.170	0.018	0.138	11.003	0.286	0.020
1996	Mean	0.179	0.418	0.120	0.285	10.054	0.403	0.019
($n = 386$)	Std. Dev.	0.037	0.161	0.021	0.126	9.469	0.263	0.018
1997	Mean	0.153	0.327	0.107	0.232	7.655	0.333	0.017
($n = 387$)	Std. Dev.	0.042	0.117	0.028	0.090	6.300	0.191	0.015
1998	Mean	0.138	0.319	0.094	0.218	4.992	0.214	0.025
($n = 361$)	Std. Dev.	0.050	0.136	0.037	0.103	3.831	0.106	0.021
All years	Mean	0.195	0.476	0.128	0.318	8.475	0.322	0.030
($n = 37604$)	Std. Dev.	0.088	0.207	0.054	0.150	8.180	0.217	0.045

$^{+}$Thousands of shares.
$^{++}$$ millions.
For each firm, liquidity measures are averaged within each year cross-sectionally. This table reports the cross-sectional mean and standard deviation for liquidity measures for each year as well as for all years combined. QSPR: the quoted bid-ask spread, RQSPR: the quoted bid-ask spread divided by the midpoint of the quote (%), ESPR: the effective spread, i.e., the difference between the execution price and the midpoint of the prevailing bid-ask quote, RESPR: the effective spread divided by the mid-point of the prevailing bid-ask quote (%), DEPTH: the average of the quoted bid and ask depths, $DEPTH: the average of the ask depth times ask price and bid depth times bid price, COMP = RQSPR/$DEPTH.

also declined significantly for all size quartiles but this could be the result of the dramatic increase in prices in the 1990s. For the smallest quartile of firms, depth has increased from an average of 6,114 shares in 1988 to 6,555 shares in 1996 and for the largest quartile of firms, depth has increased from an average of 6,778 shares in 1988 to 10,054 shares in 1996. The increasing trend in aggregate market liquidity, while manifest throughout the cross-section, is more pronounced for the largest stocks. This suggests that it is the largest stocks that have benefited more from technological innovations that have led to an increase in liquidity over time.

We next examine how day-to-day changes in liquidity vary in the cross-section. Panel A of Table 2 presents the summary statistics for the absolute daily changes in liquidity measures (%). Changes in liquidity exhibit significant cross-sectional variation. For example, the average absolute daily change in the

Table 2. Summary statistics for percentage daily changes in liquidity measures, 1988–1998.

Year		DQSPR	DRQSPR	DESPR	DRESPR	DDEP	D_$DEP	DCOMP
Panel A: Average for absolute percentage change in liquidity measures								
All firms	Mean	18.15	18.31	30.10	30.33	54.76	54.79	65.97
	Std. Dev.	6.65	6.63	128.87	131.68	18.08	18.05	22.49
Quartile 1	Mean	23.17	23.39	55.65	56.21	65.25	65.27	76.97
(small)	Std. Dev.	6.10	6.06	255.47	261.11	20.87	20.72	25.85
Quartile 2	Mean	20.64	20.78	30.04	30.18	60.86	60.90	74.63
	Std. Dev.	5.68	5.62	12.37	12.31	17.17	17.21	21.62
Quartile 3	Mean	16.77	16.89	21.29	21.40	52.54	52.57	63.93
	Std. Dev.	4.77	4.72	9.07	9.03	12.19	12.17	16.01
Quartile 4	Mean	12.11	12.24	13.59	13.72	40.58	40.60	48.57
(large)	Std. Dev.	3.84	3.80	5.99	5.95	8.82	8.81	11.44

	DQSPR	DRQSPR	DESPR	DRESPR	DDEPTH	D$DEPTH	DCOMP
Panel B: Cross-correlation of percentage change in liquidity measures							
DQSPR	1.00	0.99	0.19	0.19	−0.20	−0.20	0.48
DRQSPR(%)	0.99	1.00	0.19	0.21	−0.20	−0.20	0.48
DESPR	0.19	0.19	1.00	0.99	−0.12	−0.12	0.16
DRESPR(%)	0.19	0.21	0.99	1.00	−0.11	−0.12	0.17
DDEPTH	−0.20	−0.20	−0.12	−0.11	1.00	1.00	−0.59
D$DEPTH	−0.20	−0.20	−0.12	−0.12	1.00	1.00	−0.59
DCOMP	0.48	0.48	0.16	0.17	−0.59	−0.59	1.00

For each firm, percentage daily change in liquidity measures are averaged within each year. Panel A presents the firm-year averages and standard deviations for absolute percentage changes in liqudity measures. Summary statistics are also presented for firms sorted into quartiles, where the sorting is done each year based on the market capitalization at end of prior year. Panel B presents the time-series averages for cross-correlation in liquidity changes. For each firm and year, cross-correlation across liquidity measures are computed. These are then averaged across firm-years and the below tables present these averages. The prefix "D" denotes daily percentage change. QSPR: the quoted bid-ask spread, RQSPR: the quoted bid-ask spread divided by the midpoint of the quote (%), ESPR: the effective spread, i.e., the difference between the execution price and the midpoint of the prevailing bid-ask quote, RESPR: the effective spread divided by the mid-point of the prevailing bid-ask quote (%), DEPTH: the average of the quoted bid and ask depths, $DEPTH: the average of the ask depth times ask price and bid depth times bid price, COMP = RQSPR/$DEPTH.

quoted spread is about 12% for quartile 4, which consists of the largest firms, but as much as 23% for quartile 1, which consists of the smallest firms. The variability of absolute changes in the spread measures are also largest for small firms.

Panel B of this table presents the time-series averages of the cross-correlation in daily liquidity changes. Variations in the liquidity measures are highly correlated with each other; and changes in spread are negatively correlated with changes in depth. In addition, changes in quoted spread are positively correlated with changes in effective spread, viz., the correlation between DQSPR and DESPR is 0.19.

2. The Relation Between Liquidity and Stock Volatility

To this point, we have described cross-sectional heterogeneity in the daily level and day-to-day variation in liquidity. Motivated partially by the evidence in Chordia, Roll, and Subrahmanyam (2001) that stock market returns are the most important determinant of aggregate market liquidity, we now turn to the issue of whether there is cross-sectional heterogeneity in the relation between liquidity and stock price movements. In order to build up to the empirical analysis in Section 2.2, we provide a simple theoretical setting in the following section.

2.1. *Theoretical background*

Consider the following framework. A standard Kyle (1985)-type setting (e.g., Subrahmanyam, 1991) indicates that the slope of the pricing schedule λ when the market maker is risk averse is given by

$$\lambda = \frac{Rv_\delta}{4} + \sqrt{\frac{(Rv_\delta)^2}{4} + \frac{nv_\delta}{(n+1)^2 v_z}}, \tag{1}$$

where v_δ is the volatility of the asset value (δ being the asset's terminal payoff), R the risk aversion of the market maker, v_z the volatility of noise trading, and n is the number of informed traders. Henceforth, we use λ as a theoretical proxy for the empirical liquidity measures we describe in the next section.

Define $K = R/4$ and $A \equiv n/[(n+1)^2 v_z]$. Then, the derivative of λ with respect to v_δ is given by

$$\frac{d\lambda}{dv_\delta} = K + \frac{8K^2 v_\delta + A}{\sqrt{4K^2 v_\delta^2 + Av_\delta}}, \tag{2}$$

which is positive. From the above it follows that

$$\frac{d^2\lambda}{dv_\delta^2} = -\frac{A^2}{4(4K^2 v_\delta^2 + A^2)^{3/2}} < 0, \tag{3}$$

and

$$\frac{d^2\lambda}{dv_\delta\, dv_z} = -\frac{A^2 v_\delta v_z}{4(4K^2 v_\partial^2 + A^2)^{3/2}} < 0. \tag{4}$$

Thus, the response of λ to v_δ is positive but decreasing in v_δ and v_z. In other words, λ is concave in v_δ, and the response of λ to v_δ is decreasing in v_z. An increase in v_δ implies an increase in profit potential for informed traders as well as greater inventory risk and thus results in greater illiquidity. However, for progressively larger values of asset volatility, unit increases in asset volatility have increasingly smaller impacts on lambda. Further, for large values of the variance of noise trading, the adverse selection problem is small, so a marginal increase in v_δ does not have much of an effect on illiquidity, but for small values of v_z the opposite is true. We use the above framework as a guide to our analysis.[5] Note that with normally distributed asset values, we have $v_\delta = (\pi/2)[E|\delta|]^2$, so that $E[|\delta|]$ is monotonically related to v_δ. In order to keep the measures comparable across stocks, we use a scaled (dimensionless) estimate for $E[|\delta|]$. Thus, in Section 2.2.1 to follow, we use the absolute return over a trading day as a proxy for the information flow v_δ. Of course, a potential concern that the absolute return could be a temporary (reversible) price pressure unrelated to information flow. However, the average daily serial correlation in the cross-section of stocks during our sample period is close to zero (−0.021). This suggests that the bulk of daily return movements is due to information flows; which justifies the use of the absolute return as a proxy for v_δ. In addition to the contemporaneous absolute return, we also use a moving average of lagged absolute returns over the past week as a proxy for the market maker's estimate of v_δ. To control for market-wide changes in volatility, we use the absolute market return as well as the moving average of past five-day absolute market returns.

In our cross-sectional work in Section 2.2.2, our goal is to identify the sources of cross-sectional variation in the sensitivity of liquidity to absolute returns. In this context, the second derivatives in Equations (3) and (4) above can be interpreted as capturing the cross-sectional relation between the response of liquidity to new information (the first derivative) and cross-sectional proxies for v_δ and v_z. This interpretation allows us to test hypotheses regarding the sign

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[5]The theory is meant to guide the interpretation of our results. We do not intend our empirical analysis to be interpreted as a test of the theory. A richer theoretical analysis would consider a dynamic setting with multiple securities, an exercise that is beyond the scope of this work. The empirical analysis can simply be viewed as the answer to the following questions of applied interest: Suppose the price of stock X is flat on a given day but falls by 5% on the next day. By how much can its liquidity be expected to change on the second day relative to the first? Further, what attributes of stock X allow one to characterize how its relation between liquidity and price movements differs from that for another stock Y?

of the second derivatives in Equations (3) and (4). The specific proxies for v_δ and v_z that we use are described in Section 2.2.2.

2.2. *Empirical analysis*

2.2.1. *Time-series regressions*

We now document cross-sectional differences in the ability of a stock's liquidity to withstand information flows. As pointed out in the previous subsection, an inverse measure of this ability is the extent of co-movement between a stock's liquidity and contemporaneous absolute returns. To estimate this quantity, we run the following regression for each stock i:

$$\begin{aligned} X_{it} = {} & \alpha_{0i} + \alpha_{1i}\text{DABSRET}_{it} + \alpha_{2i}\text{DABL5RET}_{it} + \alpha_{3i}\text{DABSMRET}_t \\ & + \alpha_{4i}\text{DABL5MRET}_t + \alpha_{5i}\text{MX}_t + \alpha_{6i}\text{MX}_{t-1} + \alpha_{7i}\text{MX}_{t+1} \\ & + \sum_{j=1}^{5} \alpha_{i7+j}\text{X}_{it-j} + \varepsilon_{it}, \end{aligned} \tag{5}$$

where

X_{it}: Proportional change in stock i's liquidity measure at date t. We consider percentage changes in quoted spread (DQSPR), relative quoted spread (DRQSPR), effective spread (DESPR), relative effective spread (DRESPR), depth (DDEPTH), dollar depth (D$_DEPTH) and the composite liquidity measure (DCOMP),

ABSRET_{it}: Absolute value of return for stock i on date t,

DABSRET_{it}: Change in the absolute value of the contemporaneous return for stock i across dates $t-1$ and t,

DABL5RET_{it}: Change in the cumulative absolute return for stock i over the past five days across dates $t-1$ and t, i.e., $\text{ABSRET}_{it-1} - \text{ABSRET}_{it-6}$,

DABSMRET_t: Change in the absolute value of the contemporaneous market return across dates $t-1$ and t,

DABL5MRET_t: Change in the cumulative absolute market return over the past five days across dates $t-1$ and t, i.e., $\text{ABSMRET}_{t-1} - \text{ABSMRET}_{t-6}$,

MX_t: Equally-weighted market liquidity measure that corresponds to the dependent variable X,

ε_{it}: Error term which is assumed to follow an AR(1) process.

As argued earlier, the contemporaneous change in the daily absolute stock return, DABSRET_{it}, results from information shocks, and thus, reflects new information about a company. The regression coefficient associated with this variable captures the sensitivity of liquidity to stock price fluctuations, and is an inverse estimate of the resilience of stock's liquidity to information shocks.[6]

Since volatility is persistent,[7] the lagged change in absolute stock returns, DABL5RET_{it}, partially captures the market's assessment of stock return volatility this week as compared to that of last week, and thus proxies for changes in the market maker's estimate of inventory risk. Our theory suggests that the coefficient on DABL5RET_{it} should also have a positive sign. This coefficient is an additional measure of the resilience of liquidity to stock price fluctuations. Since information shocks may be either stock-specific or economy-wide, we include DABSMRET_t and DABL5MRET_t as the market counterparts of the individual stock measures.

Our model also includes market-wide liquidity and five lagged values of liquidity changes; further, we assume an auto-regressive error structure. The five lags of liquidity changes capture autocorrelation, and the market liquidity variables capture commonality as well as weekly seasonalities in aggregate market liquidity. The above equation is estimated separately for each firm in each year using a maximum likelihood estimation procedure.[8] The average adjusted R^2's in these regressions vary from 34% for regressions based on DQSPR to 15% for regressions of DCOMP.

Table 3 presents the annual cross-sectional averages for the coefficients α_1 through α_4,[9] as well as the averages sorted into size quartiles. Panels A and B report the averages for the coefficient on DABSRET (α_1). With the exception of regressions based on DDEPTH and D$_DEPTH, the average coefficient on DABSRET, α_1, is positive and statistically significant in the regressions.

[6]The reader may wonder whether day-to-day returns contain a liquidity premium, which could result in a reverse causality whereby liquidity causes return fluctuations. However, we look at absolute returns, whereas the liquidity premium theory is one involving signed returns. Further, while infrequent liquidity crises could lead to stock market crashes, we find it implausible that day-to-day liquidity variations are an important factor in day-to-day return variation. The notion that returns mainly reflect information is supported by our earlier statement that day-to-day returns appear to follow a random walk in our sample; the serial correlation in these returns is virtually zero. Also, our results are robust to using closing quote-midpoints to calculate returns (see Section 2.2.3 to follow), thus alleviating concerns about bid-ask bounce.

[7]The persistence of volatility is well-known (see, e.g., Bollerslev, Chou, and Kroner, 1992).

[8]The results are qualitatively the same when OLS is used.

[9]For convenience, we drop the *i* subscripts on the coefficients in the discussion that follows.

Table 3. Average coefficients from time-series regression using daily data, 1988–1998.

	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	All years
Panel A: Average coefficient for DABSRET												
DQSPR	192.01	244.63	200.32	201.19	231.44	226.50	206.94	188.57	182.52	216.58	231.62	210.71
Mean coeff.	(24.96)	(35.29)	(33.27)	(35.85)	(39.17)	(44.78)	(44.85)	(44.11)	(39.84)	(46.74)	(37.36)	(125.33)
t-statistic	90.4*	94.0*	94.4*	95.5*	95.9*	96.2*	96.3*	95.3*	95.6*	95.6*	96.9*	95.19*
% coeff. >0												
DRQSPR	186.33	245.57	206.07	199.62	233.57	230.41	208.49	185.90	182.41	212.10	233.53	210.81
Mean coeff.	(31.30)	(35.04)	(34.56)	(37.84)	(38.58)	(45.87)	(45.58)	(43.63)	(39.76)	(45.69)	(37.70)	(129.24)
t-statistic	89.7*	93.5*	94.2*	94.5*	95.3*	96.7*	96.2*	94.7*	95.2*	94.9*	96.6*	94.77*
% coeff. >0												
DESPR	430.60	334.09	215.78	243.82	263.01	200.03	194.32	179.10	205.75	238.62	270.75	248.38
Mean coeff.	(29.34)	(18.41)	(13.58)	(20.63)	(19.51)	(21.18)	(19.04)	(19.13)	(16.35)	(25.36)	(30.48)	(67.09)
t-statistic	91.3*	85.6*	78.0*	83.6*	81.3*	77.6*	75.1*	71.6*	76.6*	89.7*	97.7*	82.41*
% coeff. >0												
DRESPR	426.04	324.75	221.56	244.63	264.31	202.18	192.99	174.90	197.95	242.56	277.38	247.81
Mean coeff.	(28.06)	(14.89)	(14.72)	(19.44)	(20.30)	(20.98)	(19.74)	(18.95)	(18.15)	(21.18)	(32.61)	(65.09)
t-statistic	91.0*	85.3*	79.9*	83.5*	80.8*	77.7*	75.4*	71.8*	75.2*	89.3*	97.0*	82.27*
% coeff. >0												
DDEPTH	17.38	−88.27	−87.68	−66.92	−29.41	−28.53	2.37	20.62	46.25	66.53	62.70	−3.79
Mean coeff.	(0.85)	(−3.78)	(−4.30)	(−3.53)	(−1.75)	(−1.81)	(0.14)	(1.14)	(2.71)	(4.23)	(3.93)	(−0.70)
t-statistic	52.3	44.1*	42.1*	45.0*	50.2	48.5	49.0	50.6	50.3	54.3*	55.3*	49.45
% coeff. >0												

Table 3. (*Continued*)

D$_DEPTH	21.70	−67.97	−107.05	−63.39	−35.14	−32.46	3.76	22.52	45.28	64.48	65.86	−3.65
Mean coeff.	(1.07)	(−2.83)	(−5.84)	(−3.31)	(−2.09)	(−2.04)	(0.21)	(1.19)	(2.53)	(3.94)	(3.75)	(−0.65)
t-statistic	53.0*	45.4*	39.3*	46.0*	49.4	48.5	48.9	50.3	50.5	53.4*	54.8*	49.25
% coeff. >0												
DCOMP	293.16	520.88	470.77	396.96	416.48	438.91	404.00	365.18	325.28	332.73	344.16	388.68
Mean coeff.	(9.84)	(16.64)	(18.83)	(16.52)	(18.32)	(20.80)	(19.43)	(18.31)	(15.72)	(16.33)	(16.58)	(55.66)
t-statistic	66.5*	75.9*	78.7*	74.8*	74.5*	77.2*	75.0*	71.8*	71.0*	70.5*	73.3*	73.48*
% coeff. >0												

	Quartile 1 (small)	Quartile 2	Quartile 3	Quartile 4 (high)
Panel B: Average coefficient for DABSRET across size quartiles				
DQSPR	157.72	242.32	238.75	203.75
Mean coeff.	(43.44)	(61.21)	(78.80)	(82.01)
t-statistic	89.4*	96.5*	97.3*	97.5*
% coeff. >0				
DRQSPR	156.46	241.03	239.51	205.89
Mean coeff.	(44.06)	(65.19)	(80.59)	(81.51)
t-statistic	88.3*	96.2*	97.2*	97.4*
% coeff. >0				
DESPR	262.44	326.42	243.94	162.45
Mean coeff.	(24.10)	(44.05)	(43.87)	(43.01)
t-statistic	77.9*	85.9*	85.4*	80.5*
% coeff. >0				
DRESPR	254.29	326.64	244.84	167.02
Mean coeff.	(22.35)	(43.37)	(43.94)	(43.76)
t-statistic	77.6*	86.1*	85.1*	80.2*
% coeff. >0				

Table 3. (*Continued*)

	Quartile 1 (small)	Quartile 2	Quartile 3	Quartile 4 (high)
DDEPTH	126.66	33.54	−59.05	−113.54
Mean coeff.	(10.80)	(2.63)	(−5.80)	(−14.04)
t-statistic	60.3*	52.1*	44.2*	41.4*
% coeff. >0				
D$_DEPTH	136.95	35.06	−59.41	−124.23
Mean coeff.	(11.33)	(2.74)	(−5.60)	(−15.33)
t-statistic	61.2*	52.0*	43.7*	40.4*
% coeff. >0				
DCOMP	219.98	393.94	500.17	438.10
Mean coeff.	(14.00)	(25.57)	(38.27)	(40.55)
t-statistic	63.0*	73.0*	79.7*	78.0*
% coeff. >0				

	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	All years
Panel C: Average coefficient for DABL5RET												
DQSPR	304.80	394.62	299.11	320.27	332.71	305.18	289.57	263.77	254.30	279.13	289.07	300.48
Mean coeff.	(14.65)	(16.29)	(16.58)	(16.44)	(17.86)	(16.80)	(17.24)	(16.30)	(20.12)	(18.73)	(16.99)	(56.10)
t-statistic	70.0*	74.4*	77.9*	76.4*	75.6*	76.9*	75.7*	75.3*	77.3*	74.6*	76.7*	75.6*
% coeff. >0												
DRQSPR	238.06	315.61	298.98	254.82	292.31	293.20	261.00	205.52	207.93	223.35	271.38	258.02
Mean coeff.	(10.93)	(12.78)	(16.50)	(14.31)	(15.36)	(16.44)	(15.68)	(12.64)	(14.26)	(14.83)	(15.54)	(47.57)
t-statistic	64.1*	68.1*	75.5*	70.7*	71.9*	73.2*	71.6*	68.5*	68.8*	68.7*	74.8*	70.5*
% coeff. >0												
DESPR	397.03	441.76	256.88	272.76	226.25	158.99	97.44	124.29	214.68	256.76	326.33	246.31
Mean coeff.	(10.09)	(10.15)	(6.68)	(7.77)	(6.98)	(5.55)	(3.32)	(4.58)	(8.05)	(11.43)	(15.40)	(26.25)
t-statistic	69.9*	68.5*	62.7*	65.4*	62.6*	57.8*	54.6*	56.0*	61.6*	70.9*	76.7*	64.1*
% coeff. >0												

Table 3. (*Continued*)

	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	All years
DRESPR	335.79	363.53	238.39	231.61	181.55	141.56	71.05	70.22	170.63	207.93	300.74	204.84
Mean coeff.	(8.32)	(8.87)	(6.06)	(7.24)	(5.83)	(4.92)	(2.42)	(2.62)	(6.12)	(9.11)	(13.00)	(21.95)
t-statistic	66.2*	64.6*	62.2*	61.3*	59.9*	57.1*	53.5*	52.2	56.6*	65.0*	74.3*	61.0*
% coeff. >0												
DDEPTH	127.41	−122.45	−248.37	−87.46	10.02	−79.94	77.11	190.75	155.86	117.69	34.86	24.12
Mean coeff.	(2.05)	(−1.74)	(−3.90)	(−1.43)	(0.18)	(−1.46)	(1.32)	(3.19)	(2.90)	(2.30)	(0.77)	(1.39)
t-statistic	54.9*	49.8	46.9*	51.0	52.7	51.3	54.2*	54.8*	54.0*	56.3*	56.3*	53.1*
% coeff. >0												
D$_DEPTH	227.73	−35.23	−260.85	−4.14	33.93	−77.42	106.57	260.89	221.60	155.06	64.04	70.55
Mean coeff.	(3.58)	(−0.49)	(−4.18)	(−0.07)	(0.61)	(−1.40)	(1.82)	(4.38)	(4.09)	(3.03)	(1.43)	(4.04)
t-statistic	56.6*	52.0	46.2*	53.9*	54.5*	52.3	54.9*	56.2*	55.4*	57.4*	56.3*	54.3*
% coeff. >0												
DCOMP	21.12	326.59	439.25	184.65	82.07	170.03	79.24	−22.41	−69.11	−148.73	6.60	85.48
Mean coeff.	(0.28)	(3.88)	(5.81)	(2.61)	(1.17)	(2.52)	(1.18)	(−0.33)	(−1.10)	(−2.46)	(0.12)	(4.13)
t-statistic	47.5	54.5*	57.1*	53.0*	49.9	51.0	50.3	49.3	47.1*	46.1*	47.1*	50.1
% coeff. >0												

	Quartile 1 (small)	Quartile 2	Quartile 3	Quartile 4 (high)
Panel D: Average coefficient for DABL5RET across size quartiles				
DQSPR				
Mean coeff.	220.76	291.40	331.16	357.13
t-statistic	(17.36)	(23.84)	(35.02)	(46.71)
% coeff. > 0	68.5*	72.7*	76.9*	84.0*
DRQSPR				
Mean coeff.	163.07	254.86	292.13	320.42
t-statistic	(12.89)	(20.35)	(30.61)	(40.76)
% coeff. > 0	60.2*	68.7*	73.1*	79.9*

Table 3. (*Continued*)

	Quartile 1 (small)	Quartile 2	Quartile 3	Quartile 4 (high)
DESPR				
Mean coeff.	213.73	231.68	257.65	281.15
t-statistic	(8.69)	(10.48)	(16.73)	(29.01)
% coeff. > 0	58.6*	61.1*	65.7*	70.9*
DRESPR				
Mean coeff.	143.38	204.76	218.01	251.71
t-statistic	(5.95)	(9.24)	(14.07)	(25.83)
% coeff. > 0	53.9*	58.9*	62.8*	68.2*
DDEPTH				
Mean coeff.	−0.29	−31.44	73.14	53.82
t-statistic	(−0.01)	(−0.83)	(2.01)	(1.79)
% coeff. > 0	51.8*	51.7*	54.1*	54.7*
D$_DEPTH				
Mean coeff.	87.19	1.58	121.60	71.04
t-statistic	(2.50)	(0.04)	(3.30)	(2.39)
% coeff. > 0	53.9*	53.0*	55.1*	55.3*
DCOMP				
Mean coeff.	183.36	128.84	8.96	23.00
t-statistic	(4.56)	(2.84)	(0.21)	(0.63)
% coeff. > 0	53.3*	50.8	48.1*	48.1
	53.3*	50.8	48.1*	48.1*

Table 3. (*Continued*)

*Significant at the 5% level based on two-sided binomial test.
For each firm and year, the following regression is estimated using daily data:

$$X_t = \alpha_1 + \alpha_2 \text{DABSRET}_t + \alpha_3 \text{DABL5RET} + \alpha_4 \text{DABSMRET} + \alpha_5 \text{DABL5MRET}_t + \alpha_6 \text{MX}_t + \alpha_7 \text{MX}_{t-1} + \alpha_8 \text{MX}_{t+1} + \sum_{j=1}^{5} \alpha_{9j} \text{X}_{t-j} + \varepsilon_{it},$$

where, $X_t = \%$ Change in liquidity measure on day t (either % change in quoted spread or relative quoted spread or effective spread or relative effective spread or depth or dollar depth or volume or dollar volume or composite liquidity measure). "D" denotes daily percentage change. QSPR: the quoted bid-ask spread, RQSPR: the quoted bid-ask spread divided by the midpoint of the quote (%), ESPR: the effective spread, i.e., the difference between the execution price and the midpoint of the prevailing bid-ask quote, RESPR: the effective spread divided by the midpoint of the prevailing bid-ask quote (%), DEPTH: the average of the quoted bid and ask depths, $DEPTH: the average of the ask depth times ask price and bid depth times bid price, COMP = RQSPR/$DEPTH.

$\text{MX}_t =$ % change in equally weighted market-liquidity measure from day $t-1$ to day t

$\text{DABSRET}_t =$ Absolute return in day t – Absolute return in day $t-1$

$\text{DABL5RET}_t =$ ABL5RET(t) – ABL5RET($t-1$)

ABL5RET(t) = Cumulative absolute return over days $t-1$ to $t-5$

$\text{DABSMRET}_t =$ Absolute equally weighted market return in day t – Absolute market return in day $t-1$

$\text{DABL5MRET}_t =$ ABL5MRET(t) – ABL5MRET($t-1$)

ABL5MRET(t) = Cumulative absolute market-return over days $t-1$ to $t-5$

Panel A of this table presents the cross-sectional averages and t-statistics for α_1 and the percentage of firms with positive coefficients. Panel B presents this information averaged across all years for firms sorted into size quartiles. These panels also present the cross-sectional average adjusted R-squares from the above regression. Panels C and D present the same information for α_3, Panels E and F for α_4 and Panels G and H for α_5.

In addition, the vast majority of the coefficients on DABSRET are positive for all measures of liquidity except depth. The positive value suggests that spreads increase with the magnitude of absolute returns, i.e., liquidity is lower when the contemporaneous stock volatility is large. This is consistent with the theory in Section 1, suggesting that new information, as proxied by stock price volatility, increases adverse selection risks and inventory risks faced by liquidity providers, and thus impacts spreads. The estimates are economically significant, suggesting, for example, that a change in ABSRET of 0.1% changes spreads by about 20%.

In the regressions of DEPTH and $_DEPTH, α_1 tends to be insignificant in many years. The coefficient is negative during the years 1989–1993 and positive during 1994–1998. This suggests that stock price movements may impact liquidity mainly through spreads rather than depth. Alternatively, with an increase in return volatility and higher spreads, depth may actually increase for some stocks as the inside depth is wiped out or as investors submit more limit orders instead of market orders in order to capture the bid-ask spread. Finally, $\alpha_1$is significantly positive in regressions of DCOMP (which combines depth and spread), but is positive for fewer firms in this regression than in the regressions based on spreads.

From Panel B, which reports the average coefficients sorted by size quartiles (firm size is computed as of the end of the previous year), we observe that the average coefficients in the spread regressions decrease monotonically across quartiles 2–4.[10] However, these averages are smaller for quartile 1 than for quartile 2. In addition, the percentage of firms with positive coefficients is lowest for quartile 1 and the standard errors in the spread regressions are the highest, suggesting that the coefficients for the smallest firms are estimated less precisely. Doubtless, these findings could be influenced by the low transaction frequency in small firms.

The coefficients for the depth measures are negative for the two largest quartiles and positive for the two smallest quartiles. This implies that for the smaller stocks, spreads and depth are both positively related to absolute returns. Again, limit order submissions may account for this finding. In particular, since smaller stocks have higher spreads, with an increase in volatility, it may become advantageous for investors to place limit orders instead of market orders in order to capture the bid-ask spread. The probability of executing a limit order

[10]Quartile 1 represents the smallest firms and quartile 4 the largest.

is greater when the magnitude of stock price fluctuations is higher. Thus, for the small stocks, an increase in limit order submissions in response to greater stock price fluctuations may explain the positive coefficient in the regression of depth on absolute stock returns.

An alternative explanation for results for the depth regressions is as follows. For small stocks, depth at the inside quotes is likely to be small, so that as return volatility increases, the inside depth is more likely to be eliminated in response to incoming orders. Outside the inside quotes, depth may be higher so that we may see an increase in depth for the small stocks as volatility increases. For larger stocks, the inside depth may be larger and thus may not be completely eliminated in response to an increase in volatility. This may lead to a decrease in depth for the largest stocks as return volatility increases.

Overall, there is significant interquartile variation in the relation between liquidity and contemporaneous stock price movements. However, it remains an open question as to which firm characteristics (such as price, size, trading volume, etc.) drive this time-series relationship. This issue is addressed in the next subsection.

Panels C and D present the average coefficients from the regression of changes in liquidity on changes in the past moving average of absolute returns (α_2). Similar to Panel A, the coefficients are positive and highly significant for spread measures, but not for depth measures of liquidity. This indicates that increases in recent stock volatility also lead to higher spreads and thus lower liquidity. In Panel D, the coefficients for spread measures increase monotonically across quartiles. Further, as in Panel B, the coefficients appear to be noisiest for the smallest quartile of firms where the percentage of coefficients with positive values is the least.[11]

The above results establish a strong relationship between the spread measures of liquidity and contemporaneous stock price movements as well as stock volatility in the recent past. However, the coefficients from the depth regressions do not show a consistent pattern, and the proportion of positive coefficients is

[11]The finding that the results for the contemporaneous absolute return (DABSRET) are similar to those for the past moving average of returns (DABL5RET) indicates that our results on the relation between liquidity and stock price movements are not an artefact of a spurious relation between spread and contemporaneous absolute returns caused by bid-ask bounce effects. The relation between liquidity and past absolute returns is unlikely to be caused by bid-ask bounce. Nevertheless, we examine the robustness of our results to the bid-ask bounce effect in Section 2.4

in the range of 40–60%; for this reason we do not focus on these coefficients in our cross-sectional analysis.

The results for the absolute market return (measured by the absolute value of the CRSP equally weighted return), while not reported for brevity, indicate the following pattern. In contrast to the results for α_1, the vast majority of the α_3 (the coefficient on DABSMRET) estimates tend to be only weakly significant. The sign for α_3 is not consistent across the years. Further, in all regressions, the proportion of firms with coefficients that are positive (or negative) is very close to 50%. Even when firms are sorted into quartiles, the proportion of firms with positive (or negative) coefficients (not reported for brevity), although often statistically significant, tend to be around 50%. The results for α_4 are similar. For all liquidity measures other than quoted spreads and relative quoted spreads, the coefficient on DABSMRET tend to be insignificant. This suggests that it is individual stock return volatility and not market return volatility that impacts the liquidity of a given stock. Hence, we do not consider DABSMRET in the cross-sectional analysis to follow.

2.2.2. *Cross-sectional determinants of the response of liquidity to absolute returns*

The previous subsection documented cross-sectional heterogeneity in the extent of co-movement between liquidity and daily absolute stock returns, alternatively in the resilience of a liquidity to information shocks (for brevity we will henceforth use the term "response coefficient" for this co-movement). In this section, we explore whether firm-specific characteristics explain the cross-sectional variation in these response coefficients. That is, we try to identify variables which help explain why the ability of liquidity to withstand information shocks varies across firms. In particular, we attempt to isolate variables that are associated with the inventory and/or asymmetric information problems faced by market makers on the trading floor. First, we hypothesize that the smaller a firm, the larger the increase in adverse selection risks and inventory risks following information shocks, proxied by stock price movements. In addition, market makers of small firms with a low supply of outstanding shares may have difficulty turning around their inventory. This suggests that, *ceteris paribus*, the liquidity of smaller firms should be more strongly associated with fluctuations in stock prices than those of larger firms. Hence, we include firm size as an explanatory variable in our cross-sectional analysis.

To obtain further guidelines for the choice of cross-sectional variables, we proceed as follows. As we pointed out earlier, we interpret the second derivative in the theoretical analysis of Section 2.1 as measuring how the relation between liquidity and stock price movements varies in the cross-section. The theory indicates that this response is decreasing in volatility of the asset value and the volume of uninformed trade (recall the sign of the second derivatives in Equations (3) and (4)). In our cross-sectional analysis, we use proxies for these variables as well as other variables indicated by a priori intuition. Thus, we use a measure of return volatility measured over the prior calendar-year as a proxy for the asset variance v_δ. We also include share turnover because we expect that more volume would cause market makers to be less concerned about reversing their inventory. This variable also proxies for liquidity trading, v_z, as per Section 2.1. However, the more volatile the trading activity, the more difficulty the market maker will have in predicting the arrival of reversing transactions and, hence, greater will be the inventory risk; based on this intuition we also include the volatility of share turnover.

The next two variables we consider are the price per share and the percentage of a firm's stock held by institutions. We include price per share to account for inadequate scaling of the response coefficients across low-price and high-priced firms. In addition, we conjecture that market makers have more reasons to be concerned about inventory if a greater proportion of stock is held by institutions as institutional orders tend to be larger. Thus, we include the institutional holdings variable.

Based on the above reasoning, the explanatory variables used in the cross-sectional analysis are as follows:

SIZE: Market capitalization as of end of the previous year.

INSTPC: The percentage of the outstanding shares of a company held by institutions as of the end of the previous year.

PRICE: The closing stock price level as of the previous year.

STDRET: The volatility (standard deviation) of daily returns as of the previous year.

AVETURN: The average daily stock turnover (trading volume/number of shares outstanding) in the previous year.

STDTURN: The standard deviation of daily turnover in the previous year.

Table 4 presents the summary statistics for these variables across all firms as well as across firms sorted into size quartiles. On average, institutions

Table 4. Summary statistics for determinants of the response of liquidity to absolute stock returns.

	All firms	Quartile 1 (small)	Quartile 2	Quartile 3	Quartile 4 (large)
INSTPC					
Mean	46.32	32.09	44.40	52.01	56.73
Median	47.51	30.14	44.68	54.59	58.86
Std. dev	22.25	19.13	21.88	21.99	17.47
SIZE					
Mean	2.60	0.09	0.38	1.18	8.77
Median	0.62	0.08	0.36	1.08	4.49
Std. dev	7.59	0.07	0.16	0.49	13.37
PRICE					
Mean	27.84	10.61	22.27	30.01	48.48
Median	23.63	8.63	20.00	27.75	42.38
Std. dev	23.89	8.03	12.85	14.71	32.85
STDRET (*100)					
Mean	2.27	3.24	2.21	1.94	1.71
Median	1.97	2.74	2.08	1.83	1.59
Std. dev	1.36	2.07	0.91	0.76	0.59
AVETURN (*1000)					
Mean	3.05	2.77	3.20	3.26	2.98
Median	2.35	1.97	2.31	2.61	2.45
Std. dev	3.02	3.68	3.24	2.78	2.15
STDTURN (*1000)					
Mean	3.76	4.38	4.55	3.65	2.47
Median	2.41	2.79	2.87	2.53	2.53
Std. dev	5.60	7.37	6.68	4.00	2.80
NTRANS					
Mean	112.32	19.29	41.05	87.51	299.48
Median	46.26	12.58	27.57	62.47	176.72
Std. dev	237.56	22.08	47.21	97.42	403.62

This table presents the averages across firm-years of explanatory variables used to explain cross-sectional variation of response of liquidity to information. The variable definitions are as follows:
SIZE: market capitalization (in $billions) as of 31st December of each year.
INSTPC: the percentage of the company held by institutions as of 31 December of each year.
PRICE: the closing stock price level as of 31 December of each year.
STDRET: the volatility (standard deviation) of daily returns estimated separately for each firm and each calendar-year.
AVETURN: the average daily stock turnover (trading volume/number of shares outstanding) estimated separately for each firm and each calendar-year.
STDTURN: the standard deviation of daily turnover estimated separately for each firm and each calendar-year.
NTRANS: average daily number of transactions, estimated separately for each firm and each calendar-year.

hold about 46% of the shares in our sample firms. This percentage, however, increases monotonically from the smallest quartile of firms, where the average institutional holding is 32%, to the largest quartile where the corresponding figure is 57%. The increase in institutional holdings across size quartiles is consistent with the preference of institutions to hold more stock in the larger firms. The average price of the shares varies from \$10.61 for the smallest quartile to \$48.48 for the largest quartile. Not surprisingly, the volatility of returns decreases across the size quartiles, while turnover increases across these groups. On average, 0.31% of the stocks for all firms are "turned-over" on each day of trading. To provide more perspective on cross-sectional variation in trading activity, we also provide statistics on the daily number of transactions across the size quartiles and for the entire sample (though we do not use this variable in our cross-sectional regressions). The pattern in the numbers for the average daily number of transactions, an alternative measure of trading activity, are similar to those for turnover. Specifically, for the median small firm, there are about 13 trades in a day. However, this number increases to about 177 for the median firm in the largest quartile. These indicate the existence of significant variation in the explanatory variables and trading activity across the size quartiles.

We now turn our attention to the cross-sectional regressions of the response coefficients estimated from our time-series regressions. Table 5 presents the Fama–Macbeth averages and t-statistics based on year by year cross-sectional regression of the time-series coefficient α_1 from the previous section on the above variables. For brevity, we only report results for the proportional spread measures and the composite measures; the results for the unscaled measures are similar. The averages are presented for regressions estimated across all firms and across firms in each of the size quartiles. Our variables explain between 5 and 26% of the cross-sectional variation in spread response to absolute returns; the explanatory power is lower for the COMP variable.

Many of our variables tend to be important in explaining the cross-sectional variation of the response coefficients. Consistent with Equations (3) and (4) of Section 1, STDRET, SIZE, and AVETURN are all negatively and significantly related to the response coefficient for spreads and composite liquidity measure.[12] The coefficients are also economically significant; as an example, a one standard deviation change in STDRET changes the response coefficient by

[12]It is well known that volatility is *positively* related to the *level* of the bid-ask spread in the cross-section (see, e.g., Benston and Hagerman, 1974). However, as documented here, there is a *negative* relation between volatility and the *time-series response of spreads to absolute returns*.

Table 5. Cross-sectional regression estimates for DABSRET. Average coefficients from cross-sectional regression and Fama–Macbeth *t*-statistics, where the dependent variable is the coefficient on DABSRET (change in absolute value of concurrent stock return) in the time-series regressions.

	DRQSPR					DRESPR					DCOMP				
	All firms	Quartile 1 (small)	Quartile 2	Quartile 3	Quartile 4 (large)	All firms	Quartile 1 (small)	Quartile 2	Quartile 3	Quartile 4 (large)	All firms	Quartile 1 (small)	Quartile 2	Quartile 3	Quartile 4 (large)
INTERCEP	298.26	96.69	251.78	320.68	427.83	261.77	−156.06	163.48	25.86	189.48	681.33	167.46	686.32	1118.74	1397.20
	(16.89)	(3.27)	(4.76)	(9.24)	(7.28)	(5.69)	(−0.89)	(1.70)	(0.58)	(4.70)	(21.17)	(1.93)	(4.18)	(6.91)	(6.50)
SIZE	−19.51	2.61	−49.68	−43.30	−49.53	−84.92	−79.68	−144.16	−166.73	−74.27	3.11	39.71	64.21	5.75	−183.46
	(−4.03)	(0.48)	(−2.51)	(−6.40)	(−7.13)	(−11.07)	(−3.23)	(−3.81)	(−10.23)	(−7.76)	(0.15)	(1.59)	(1.05)	(0.13)	(−8.24)
INSTPC	0.00	−0.28	0.09	−0.45	−0.60	0.74	0.37	0.11	0.34	0.27	−0.13	0.54	0.35	−1.33	−2.14
	(−0.03)	(−0.81)	(0.38)	(−3.75)	(−4.19)	(2.34)	(0.54)	(0.19)	(0.99)	(0.83)	(−0.27)	(0.35)	(0.42)	(−1.95)	(−2.66)
PRICE	1.74	12.04	4.36	3.53	0.99	4.79	27.83	11.75	10.49	3.10	1.52	21.79	3.72	2.15	0.66
	(9.19)	(12.48)	(3.73)	(8.61)	(4.96)	(9.00)	(4.66)	(7.38)	(9.06)	(6.18)	(2.86)	(5.51)	(0.62)	(0.68)	(0.97)
STDRET*	−54.42	−10.68	−56.15	−74.04	−83.41	−75.77	−17.04	−68.15	−9.96	−12.42	−110.20	−8.61	−96.50	−30.95	−30.28
100	(−15.24)	(−2.44)	(−5.61)	(−6.56)	(−4.14)	(−5.78)	(−0.88)	(−5.57)	(−0.92)	(−0.80)	(−9.07)	(−0.55)	(−4.76)	(−6.27)	(−4.86)
AVETURN*	−13.53	−12.15	−14.62	−7.95	−11.51	−23.82	−39.65	−33.35	−31.81	−12.38	−32.03	−19.85	−15.05	−22.70	−42.02
1000	(−5.37)	(−4.49)	(−3.44)	(−3.25)	(−4.01)	(−6.67)	(−5.06)	(−5.20)	(−6.83)	(−2.80)	(−3.70)	(−1.90)	(−1.15)	(−0.23)	(−4.07)
STDTURN*	63.22	33.88	51.56	41.77	86.35	91.79	14.36	70.53	92.89	14.75	91.82	−21.54	−58.61	31.91	28.38
100	(5.69)	(3.37)	(2.53)	(3.45)	(3.69)	(4.66)	(2.84)	(2.93)	(4.78)	(0.74)	(2.17)	(−0.38)	(−0.68)	(0.42)	(3.37)
Adj. *R*-sq. (%)*	17.83	24.31	20.44	24.83	21.66	11.65	13.68	19.47	25.69	23.74	5.55	4.62	5.60	9.19	12.86

*Average adjusted r-square across years.

First, yearly time-series regressions are run for each stock to estimate the response of its liquidity to absolute returns (Table 3). Then, the coefficients from these regressions are regressed annually on the explanatory variables in Table 4. This table reports the Fama–Macbeth averages of the coefficients from these yearly cross-sectional regressions. *T*-statistics are in parentheses. The prefix "D" denotes daily perctentage change, RQSPR: the quoted bid-ask spread divided by the midpoint of the quote (%), RESPR: the effective spread divided by the midpoint of the prevailing bid-ask quote (%), COMP = RQSPR/$DEPTH.

about 27%. Given the magnitudes of the response coefficients documented in the previous section, this is a substantial effect.

PRICE and STDTURN are positively related to the response coefficient. The response coefficient is decreasing in size and trading volume as measured by the turnover, suggesting that for larger firms and firms that have a higher trading volume, the impact of contemporaneous stock volatility on liquidity is smaller. Thus, the liquidity of larger stocks and stocks that have higher trading volumes, is more resilient to stock price movements than the liquidity of smaller stocks and stocks with lower trading volumes. This is consistent with the notion that it is the former category of stocks that are widely held and extensively followed. The response coefficient is also decreasing in return volatility. Thus, the liquidity of stocks that have higher return volatility exhibits a lower response to information shocks.

The impact of contemporaneous absolute returns on spreads increases with price and this impact decreases monotonically across size quartiles. A possible explanation for this result is as follows. Table 4 documents that smaller firms have lower stock prices. The tick size,[13] which represents the minimum institutionally mandated change in stock prices, is more likely to be binding for lower priced stocks. The response coefficient is then constrained by the tick size, especially for lower priced stocks. Thus, as price increases, the constraint will be less binding, and the impact of the stock price on the response coefficient should be higher. Furthermore, this response coefficient should decrease across size quartiles because as price increases across the size quartiles, the tick size becomes less binding.

The variability of turnover is positively related to the response coefficient for the spread. This result is consistent with the intuition that market maker inventory is riskier for stocks with more variable turnover, so that liquidity responds strongly to stock price movements for stocks with higher variability in trading activity.

The relation between the spread and absolute returns is insignificantly related to institutional holdings for the quoted spread, but positively for the proportional effective spread, for the entire sample of firms. Focusing on the within quartile regressions, the largest two quartiles have a significantly negative relationship between the quoted spread's response coefficient and institutional

[13] See Chordia and Subrahmanyam (1995) and Ball and Chordia (2000) for a detailed analysis of the tick size.

holdings. This suggests that as the impact of stock price movements on quoted spreads increases in the largest quartiles, institutions can step up to supply liquidity because they hold more of the largest stocks and being well diversified they are in a position to profit from information shocks to a given stock.

The regression results for the composite liquidity measure tend to be qualitatively similar to those based on quoted spreads. This is not surprising, since the composite measure is based on quoted spreads and depth.

In sum, the cross-sectional results demonstrate that the resilience of a firm's equity market to information shocks is (*ceteris paribus*) greatest for firms with high market capitalization, trading volume, and return volatility, and low variability in trading activity. Larger institutional holdings are positively associated with this capacity in large firms.

2.2.3. *Robustness checks*

It is worth emphasizing that the Fama–Macbeth statistics reported for the regressions in Table 5 are based on only 11 years of data. Naturally, power in the tests is a concern with such a short sample period. The significance observed in the regressions in spite of this shortcoming reinforces our confidence in the results, and provides evidence of stability in the cross-sectional estimates across the years. In this section, we address two other concerns regarding our estimation procedure.

The first concern relates to the fact that we compute absolute returns using actual transaction prices, so that there is the possibility that the co-movement between spreads and absolute returns could be an artifact of bid-ask bounce, given that the component of return due to bid-ask bounce depends on the size of the bid-ask spread. To address this issue, we create a data-set of returns that is free of bid-ask bounce. In particular, we use a time series of the midpoint of the quoted bid-ask spread prevailing at the time of the last trade of the day to calculate the daily return for each stock.[14] We redid the regressions using this return series and found that the results in Tables 3–5 were qualitatively unaltered. These results are not reported for brevity but are available upon request.

A second concern regarding our results is that in the two-step procedure we have used thus far, the response coefficients are measured with error in the

[14]We do not use the closing bid-ask spreads because without any transaction, these spreads may be economically suspect. However, for the largest stocks, the closing bid-ask spreads are often also the spreads at which the last transaction of the day took place.

second stage cross-sectional regressions. But these response coefficients are the dependent variables in the cross-sectional regressions and as long as the estimation errors are not related to the firm characteristics in any systematic manner, the cross-sectional regression coefficients will not be biased. Nevertheless, to address this issue, we estimate a single panel regression using both time-series as well as cross-sectional data. The panel regression allows us to estimate, the response coefficient and the impact of the characteristic on the response coefficient in a single step.

The following regression is estimated across all firms and all days in the sample (i.e., 1988–1998), using the bid-ask bounce free return data:

$$\begin{aligned} X_{it} = {} & a_0 + a_1 \text{DABSRET}_{it} + a_2 \text{DABSRET}^*_{it}\text{SIZE}_{it} \\ & + a_3 \text{DABSRET}^*_{it}\text{INSTPC}_{it} + a_4 \text{DABSRET}^*_{it}\text{PRICE}_{it} \\ & + a_5 \text{DABSRET}_{it} * \text{STDRET}_{it} + a_6 \text{DABSRET}^*_{it}\text{AVETURN}_{it} \\ & + a_7 \text{DABSRET}^*_{it}\text{STDTURN}_{it} + b_1 \text{DABL5RET}_{it} \\ & + b_2 \text{DABL5RET}^*_{it}\text{SIZE}_{it} + b_3 \text{DABL5RET}^*_{it}\text{INSTPC}_{it} \\ & + b_4 \text{DABL5RET}^*_{it}\text{PRICE}_{it} + b_5 \text{DABL5RET}^*_{it}\text{STDRET}_{it} \\ & + b_6 \text{DABL5RET}^*_{it}\text{AVETURN}_{it} + b_7 \text{DABL5RET}^*_{it}\text{STDTURN}_{it} \\ & + c_1 \text{DABSMRET}_t + c_2 \text{DABL5MRET}_t + c_3 \text{MX}_t + c_4 \text{MX}_{t-1} \\ & + c_5 \text{MX}_{t+1} + \sum_{j=1}^{5} c_{5+j} X_{it-j} + \varepsilon_{it}, \end{aligned}$$

Table 6. Estimates from panel regressions run across all years and all firms, 1988–1998.

Dependent variable (X_t)	DRQSPR		DRESPR		DCOMP	
	Coeff.	t-statistic	Coeff.	t-statistic	Coeff.	t-statistic
Intercept	5.88	438.35	6.16	33.28	33.49	394.77
DABSRET_t	135.06	73.14	88.83	3.11	344.20	31.93
DABL5RET_t	205.43	23.98	719.82	5.47	415.34	8.31
DABSMRET	−0.42	−0.15	74.99	1.74	87.04	5.29
DABL5MRET	223.00	16.31	187.37	0.92	1120.95	14.27
DABSRET_t*SIZE	−1.56	−20.90	−2.34	−2.10	−4.91	−11.32
DABSRET_t*INSTPC	0.02	0.60	1.04	2.46	0.36	2.24
DABSRET_t*PRICE	1.09	30.27	2.26	4.17	0.90	4.28
DABSRET_t*STDRET*100	−10.63	−38.53	−15.68	−3.59	−31.75	−19.64
DABSRET_t*AVETURN*1000	−4.51	−17.60	−10.96	−2.85	−15.15	−10.15

Table 6. (*Continued*)

Dependent variable (X_t)	DRQSPR		DRESPR		DCOMP	
	Coeff.	t-statistic	Coeff.	t-statistic	Coeff.	t-statistic
DABSRET$_t$*STDTURN*100	22.19	16.14	61.10	2.97	54.48	6.80
DABL5RET$_t$*SIZE	1.88	5.10	5.09	0.92	5.30	2.46
DABL5RET$_t$*INSTPC	0.94	7.15	−1.01	−0.51	−1.41	−1.84
DABL5RET$_t$*PRICE	2.23	12.99	−5.92	−2.30	−6.18	−6.18
DABL5RET$_t$*STDRET*100	−20.55	−16.68	−221.68	−11.49	−30.38	−4.21
DABL5RET$_t$*AVETURN*1000	−51.51	−4.16	18.99	1.02	−12.36	−1.71
DABL5RET$_t$*STDTURN*100	27.84	4.25	24.49	0.25	45.23	1.19
Adj. R-sq. (%)	22.35		0.14		2.89	

The following regression is estimated across all firms and all days in the sample (i.e., 1988–1998):

$$
\begin{aligned}
X_t = {} & \alpha_0 + \alpha_1 \text{ DABSRET}_t + a1 \text{ DABSRET} * \text{ SIZE} + a2 \text{ DABSRET} * \text{ INSTPC} \\
& + a3 \text{ DABSRET} * \text{ PRICE} + a4 \text{ DABSRET} * \text{ STDRET} \\
& + a5 \text{ DABSRET} * \text{ AVETURN} + a6 \text{ DABSRET} * \text{ STDTURN} + \alpha_2 \text{ DABL5RET}_t \\
& + b1 \text{ DABL5RET} * \text{ SIZE} + b2 \text{ DABL5RET} * \text{ INSTPC} + b3 \text{ DABL5RET} * \text{ PRICE} \\
& + b4 \text{ DABL5RET} * \text{ STDRET} + b5 \text{ DABL5RET} * \text{ AVETURN} \\
& + b6 \text{ DABL5RET} * \text{ STDTURN} + \alpha_3 \text{ DABSMRET}_t + \alpha_4 \text{ DABL5MRET}_t \\
& + \alpha_5 MX_t + \alpha_6 MX_{t-1} + \alpha_7 MX_{t+1} + \sum_{j=1}^{5} \alpha_{8j} X_{t-j} + \varepsilon_{it}
\end{aligned}
$$

where X_t = % change in liquidity measure from day $t-1$ to day t, "D" denotes daily percentage change, RQSPR: the quoted bid-ask spread divided by the midpoint of the quote (%), RESPR: the effective spread divided by the midpoint of the prevailing bid-ask quote (%), COMP = RQSPR/$DEPTH.
MX_t = % change in equally weighted market-liquidity measure from day $t-1$ to day t
DABSRET$_t$ = absolute return in day t − absolute return in day $t-1$ (calculated from midpoints of closing bid-ask quotes)
SIZE = market-capitalization as of December 31 of previous year
INSTPC = percentage institutional holding as of December 31 of previous year
PRICE = stock price as of December 31 of previous year
STDRET = standard-deviation of returns measured during the previous calendar year
AVETURN = average turnover measured during the previous calendar year
STDTURN = standard-deviation of turnover measured during the previous calendar year
DABL5RET$_t$ = ABL5RET(t) − ABL5RET($t-1$)
ABL5RET(t) = cumulative absolute return over days $t-1$ to $t-5$ (calculated from midpoints of closing bid-ask quotes)
DABSMRET$_t$ = absolute equally-weighted market return in day t − absolute market return in day $t-1$
DABL5MRET$_t$ = ABL5MRET(t) − ABL5MRET($t-1$)
ABL5MRET(t) = cumulative absolute market-return over days $t-1$ to $t-5$

where the variables are as defined earlier, except that individual stock returns are computed using the return dataset that is free of bid-ask bounce.

In the above regression, the coefficients on the interaction terms capture the sensitivity of the response coefficient to the relevant cross-sectional variable.[15] For example, the coefficient a_2 captures the cross-sectional relation between firm size and the time-series liquidity response to DABSRET. An advantage of this panel regression relative to the two-stage regressions used for Table 5 is that the panel regression can increase the power of the model by avoiding estimation errors which arise in the first step of the two-stage regressions.

The panel regression results are presented in Table 6.[16] Size, turnover, and return volatility are all negatively related to the response of liquidity to absolute returns, and the standard deviation of turnover is positively related to this response. Notice that the explanatory power is considerably higher for the quoted spread regressions than for the effective spread regressions; perhaps because effective spreads, especially for infrequently-traded stocks, are estimated noisily.[17] In general, however, the panel regressions support the central findings of Table 5.

3. Conclusion

A voluminous literature has explored the cross-sectional determinants of the spread. This line of literature treats liquidity essentially as a fixed property of a given stock. Yet, recent research indicates that market-wide liquidity exhibits substantial intertemporal variation. This paper connects the cross-sectional and time-series studies of liquidity by taking a first step toward documenting cross-sectional heterogeneity in time-series variation in liquidity. We first examine whether there are differences across firms in recent liquidity trends as well as in daily fluctuations in liquidity. Next, we explore cross-sectional differences in the capacity of a firm's equity market to provide liquidity when information shocks affect the value of the stock. Low serial correlation in daily stock returns

[15]The intercept a_0 corresponds to the intercept from the first-stage time-series regression, while the coefficients a_1 and b_1 corresponds to the intercept from the second stage cross-sectional regression.

[16]We also estimated the panel regression using returns computed from transaction prices. The results were qualitatively similar to those reported in Table 7.

[17]Panel regressions by size quartile, while not reported here for brevity, yielded results that were largely similar to those in Tables 5 and 6.

indicates that much of daily return movements are information-related, which justifies our use of daily absolute stock returns as a proxy for volatility.

We depart from existing cross-sectional studies of liquidity by using a comprehensive sample of more than 1200 stocks over more than 2800 days, and examining the cross-section and time-series of liquidity simultaneously. Our main results are as follows:

- Daily average liquidity changes exhibit considerable cross-sectional variation. Small firms tend to have greater proportional liquidity changes on average than large firms.
- The increase in aggregate market liquidity over the past decade has been more pronounced for large firms than for small firms.
- Daily absolute returns are an important determinant of daily variations in liquidity.
- We take the degree of co-movement between liquidity and absolute returns as an inverse measure of the resilience of a firm's liquidity to information shocks. Our cross-sectional analysis indicates that size, volume, and volatility are all negatively related to this co-movement coefficient; however, the volatility of volume is positively related to the co-movement.
- Institutional holdings are negatively related to the co-movement between liquidity and absolute stock returns. Our rationale for this result is that liquidity trades are more likely to emanate from institutions in large companies, so that the liquidity of large stocks is better able to withstand large stock price fluctuations.

The results in the last two items above indicate that the resilience of a firm's equity market to stock price fluctuations is largest for firms with large market capitalization, trading volume, and return volatility, but small variability of trading volume. Larger institutional holdings are positively related to this resilience. These results help shed light on which types of firms are likely to be most costly to trade during periods of information arrival. They also shed light on the cross-sectional determinants of the heterogeneity in the time-series movements of liquidity across different types of stocks.

Our work suggests some interesting topics for future research. While we provide some theoretical analysis, a further exploration of the issues we address (e.g., in the context of a multisecurity dynamic model) may be worthwhile. In addition, it may also be worthwhile to analyze the relation between liquidity and returns, and how this relation varies in cross-section. For instance, stock returns have been high during the 1990s, while liquidity has increased. Further, large firms have performed better than small firms, and their liquidity has increased more than that of small firms. It would be interesting to document how much of the greater price appreciation for large firms can be attributed to increases in their liquidity.

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Chapter 6

Intraday Volatility on the NYSE and NASDAQ

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This paper compares intraday volatility on two different market structures: specialist and multiple dealer. Return volatility based on 15-minute returns is compared for stocks trading on the NYSE and NASDAQ. It is hypothesized in the paper that the price continuity obligation of specialists will lead to lower volatility for stocks traded in that market structure. The results support the hypothesis. Regressions are performed to control for firm-specific variables. The results of the regressions do not alter the conclusions of the paper that a specialist-based market is associated with a lower level of volatility than a multiple dealer market.

Keywords: Affirmative obligations; market structure; NASDAQ; NYSE; volatility.

1. Introduction

Is market structure related to volatility? Previous studies have found that volatilities for securities listed on the New York Stock Exchange (NYSE) are lower than comparable securities on NASDAQ.[1] The relationship is found to be robust to the introduction of the Order Handling Rules (OHR) and a reduction in tick size from \$1/8 to \$1/16. However, previous studies examine volatility based on daily returns, which may be information rather than market structure related. This study extends previous studies by examining intraday volatility for a large sample of NYSE and Nasdaq stocks that trade on penny ticks.

The NYSE is an order-driven market run by a single market maker called a specialist. Supply and demand meet on the floor of the exchange. There is a single limit order book where public limit orders are stored until they can be executed or are canceled by the submitter. The limit order book is maintained by the specialist. NYSE specialists have a number of affirmative obligations. Not the least among them is the requirement to maintain a continuous market. That is, price movements should occur smoothly rather than in large jumps. Additionally, NYSE specialists are required to be supply liquidity when no one

[1] See Huang and Stoll (1996), Bessembinder and Kaufman (1997), and Bessembinder (1998).

else wishes to. Both of these features may serve as "shock absorbers" when public supplied liquidity is thin.

In contrast to the NYSE, Nasdaq is largely a quote-driven market without any priority rules.[2] Competing market makers, or dealers, establish quotes for the stocks they make markets in. In contrast to NYSE specialists, Nasdaq dealers have no affirmative obligations. In addition to dealers, there are a number of alternative trading systems (such as Instinet and Island) that also compete with dealer quotes by exposing public limit orders to the market. Again, these public limit orders have no affirmative obligations to supply liquidity or maintain an orderly market.

Large market orders can be motivated by information or liquidity needs. In the former case, prices will permanently change following the execution of the order. In the latter case, large orders use up available liquidity in a market and temporarily move prices away from current valuations. This hurts investors who are unlucky enough to execute orders on the wrong side of the market during these temporary price fluctuations. For example, assume that a very large liquidity-motivated sell order comes in and uses up all liquidity at the current bid price of $10, and at subsequent lower prices down to $9.25. Before additional liquidity can each the market, a small investor's order to sell 1,000 shares arrives. The investor sells her stock for $9.25. Shortly thereafter additional liquidity arrives at the market and the inside (highest) bid price rebounds to $10. The investor that sold 1,000 shares received $75 less than she would have if she had known the price decline was temporary and waited.

The institutional features of the NYSE should theoretically make the type of temporary price fluctuations just illustrated, smaller and less frequent — resulting in lower volatility for stocks traded on the NYSE in comparison to Nasdaq. The impact of large orders on price will be greatest when the market for a stock is thinnest — during the middle of the trading day. However, most previous studies examine volatility using end of day prices — not intraday. For example, Huang and Stoll (1996) and Bessembinder and Kaufman (1997) examine a matched sample of NYSE and NASDAQ stocks. These two studies examine volatility before the OHR went into effect in 1997. Both studies found that on average firms listed on the NYSE exhibited lower volatility than comparable Nasdaq issues. Bessembinder (1998) examines daily return volatility

[2]While some ECNs have internal priority rules, there are no priority rules which dictate how orders are directed to dealer quotes which account for over 60% of trading on Nasdaq.

before and after the OHR for a sample of firms that switch from NASDAQ to the NYSE during either 1996 or 1997.[3] Bessembinder finds that the lower volatility of NYSE stocks found by the earlier studies persists after the OHR. For 23 firms, Bessembinder is also able to determine that the observed difference in volatilities between the two market structures is still present after tick sizes are reduced from $1/8 to $1/16. Therefore, the observed lower volatility of NYSE stocks was not impacted by the adoption of new trading rules on NASDAQ or a lowering of the tick size.

In this paper, I compare intraday volatility levels on the two markets. The results show that NYSE stocks have lower intraday volatility than NASDAQ stocks. This result still holds even after controlling for factors known to be associated with volatility, and during stable as well as stressful market conditions. Results are reported overall as well as by industry group. The lower level of volatility on the NYSE is found both overall and in almost all of the industry groups examined. This finding provides further evidence that the structure of trading at the NYSE lends itself to lower volatility for its listed stocks.

The remainder of this study is organized as follows. Section 2 describes the methodology used and how the sample is chosen. Section 3 discusses the results. Section 4 provides some concluding remarks.

2. Sample and Methodology

The initial sample consists of all common stocks listed on the NYSE as of August 1, 2001. Preferred stocks, warrants, when-issued stocks, units, trusts, and ADRs are excluded. Using ticker symbols as the identification criteria, three-digit SIC codes are obtained from the Center for Research in Security Prices (CRSP) files dated December 2000. Due to listing changes (ticker symbol changes, listing after December 2000, missing SIC codes on CRSP, or occasionally ticker misidentification) 173 of the 1,700 common stocks do not have SIC codes and are dropped.

Calculating intraday volatility requires historical trade records. The Trade and Quote files (TAQ) produced by the NYSE are used to obtain trade records for the stocks in the sample. TAQ contains detailed trade records including the price, quantity, and time of the trade along with any trade condition codes

[3]The study does look at intraday returns but defines them as the return from 10 a.m. to 4 p.m. This definition is closer to a daily return than a true intraday return with multiple holding periods during the day.

and error correction codes. Only trades occurring between 9:30 a.m. and 4 p.m. are used in this study. Included trades are further limited to those with a sale condition code of regular way or NYSE Direct+. Trades that are identified as later being corrected or canceled are excluded.

Stable as well as stressful market conditions are examined in this study. To choose the stable period, closing values for the DJIA and NASDAQ Composite Index are examined over the period July to August 2001. A period of time when both indexes appeared to be fairly stable is chosen. This period is the first 10 trading days of August 2001. This period contains over 20 million trades that are analyzed for this study.

Consistent with previous market microstructure studies, inactive stocks are then excluded. For the purposes, of this study an inactive stock is one that has less than 20 trades per day on average during the month of July 2001. I find that 153 NYSE stocks have less than 20 trades per day on average and exclude them from the sample leaving 1,374 stocks. All NASDAQ stocks with a daily average of at least 20 trades during July 2001 are then identified as potential matches for the NYSE sample.

Next, I identify all SIC codes for the NYSE sample and then extract from the NASDAQ sample all stocks that have matching SIC codes. I find that 254 stocks from the NYSE sample representing 83 industry groups did not have stocks with the same SIC code on NASDAQ. There are then a total of 1,120 NYSE stocks from 158 different industry groups in the final NYSE sample, while there are 2,110 SIC-matching stocks in the NASDAQ sample.

To determine whether any volatility differences found for the stable period described above also exist during stressful market conditions, I examine three days during 2001 that are characterized by large daily changes in index values. Those three days are May 16, October 3, and October 25. The Standard and Poor's 500 index rose by 1.2, 1.9, and 1.4%, respectively on those days.

The number of stocks in each industry group is contained in Table 1. Note that while some groups have equal numbers of stocks in the NYSE and NASDAQ subsamples (see for example SIC codes 736 and 799), most do not. For example there are 43 NYSE stocks in SIC code 131 (crude petroleum and natural gas), but only 14 NASDAQ stocks. Similarly, there are only 26 NYSE stocks in SIC code 737 (computer and data processing services), but 429 NASDAQ stocks. This mismatch creates a dilemma in matching.

Since the goal is to examine intraday volatility on the NYSE and corresponding NASDAQ stocks, other factors known to affect volatility should be

Table 1. Number of stocks in each industry group.

SIC	Industry group	Number of stocks	
		NYSE	NASDAQ
109	Miscellaneous metal ores	1	1
131	Crude petroleum and natural gas	43	14
138	Oil and gas field services	27	4
152	Residential building construction	6	5
162	Heavy construction, except highway	4	5
173	Electrical work	4	2
201	Meat products	5	1
202	Dairy products	2	5
203	Preserved fruits and vegetables	7	3
204	Grain mill products	7	1
205	Bakery products	3	2
206	Sugar and confectionery products	3	2
208	Beverages	11	1
209	Miscellaneous food and kindred products	3	4
222	Broadwoven fabric mills, manmade	3	1
227	Carpets and rugs	1	1
232	Men's and boys' furnishings	5	3
239	Miscellaneous fabricated textile products	1	1
243	Millwork, plywood and structural members	2	1
251	Household furniture	5	3
252	Office furniture	3	1
263	Paperboard mills	6	2
265	Paperboard containers and boxes	2	3
272	Periodicals	7	3
273	Books	3	3
274	Miscellaneous publishing	1	1
275	Commercial printing	8	3
282	Plastics materials and synthetics	6	3
283	Drugs	23	202
286	Industrial organic chemicals	7	1
287	Agricultural chemicals	8	2
289	Miscellaneous chemical products	7	3
302	Rubber and plastics footwear	1	2
308	Miscellaneous plastics products	8	7
314	Footwear, except rubber	4	4
322	Glass and glassware, pressed or blown	5	2
326	Pottery and related products	2	1
327	Concrete, gypsum, and plaster products	4	1
329	Misc. nonmetallic mineral products	1	1
331	Blast furnace and basic steel products	14	6
332	Iron and steel foundries	1	1
335	Nonferrous rolling and drawing	8	5

Table 1. (*Continued*)

SIC	Industry group	Number of stocks	
		NYSE	NASDAQ
336	Nonferrous foundries (castings)	1	1
339	Miscellaneous primary metal products	1	1
344	Fabricated structural metal products	8	3
346	Metal forgings and stampings	1	1
349	Miscellaneous fabricated metal products	5	2
351	Engines and turbines	4	1
352	Farm and garden machinery	8	1
353	Construction and related machinery	14	4
354	Metalworking machinery	3	1
355	Special industry machinery	2	25
356	General industrial machinery	14	6
357	Computer and office equipment	23	69
358	Refrigeration and service machinery	6	7
361	Electric distribution equipment	3	1
362	Electrical industrial apparatus	7	13
364	Electric lighting and wiring equipment	4	4
365	Household audio and video equipment	1	11
366	Communications equipment	14	101
367	Electronic components and accessories	26	150
369	Misc. electrical equipment and supplies	6	9
371	Motor vehicles and equipment	27	9
372	Aircraft and parts	6	2
374	Railroad equipment	4	1
381	Search and navigation equipment	2	5
382	Measuring and controlling devices	17	54
384	Medical instruments and supplies	20	84
385	Ophthalmic goods	3	2
386	Photographic equipment and supplies	4	4
393	Musical instruments	1	1
394	Toys and sporting goods	4	2
399	Miscellaneous manufactures	8	6
401	Railroads	6	3
421	Trucking and courier services, except air	1	14
441	Deep sea foreign trans. of freight	3	1
451	Air transportation, scheduled	12	7
452	Air transportation, nonscheduled	1	3
481	Telephone communication	19	56
483	Radio and television broadcasting	11	16
484	Cable and other pay TV services	4	18
489	Communication (not elsewhere classified)	2	12
491	Electric services	39	2
493	Combination utility services	32	3

Table 1. (*Continued*)

SIC	Industry group	Number of stocks	
		NYSE	NASDAQ
494	Water supply	4	3
495	Sanitary services	4	7
501	Motor vehicles, parts, and supplies	4	5
503	Lumber and construction materials	3	1
504	Professional and commercial equipment	3	15
505	Metals and minerals, except petroleum	3	1
506	Electrical goods	7	15
508	Machinery, equipment, and supplies	2	3
509	Miscellaneous durable goods	1	5
511	Paper and paper products	1	2
512	Drugs, proprietaries, and sundries	3	9
513	Apparel, piece goods, and notions	5	4
514	Groceries and related products	5	4
519	Miscellaneous nondurable goods	4	4
521	Lumber and other building materials	1	1
531	Department stores	11	3
533	Variety stores	7	5
541	Grocery stores	7	4
551	New and used car dealers	4	1
553	Auto and home supply stores	4	1
561	Men's and boys' clothing stores	1	1
562	Women's clothing stores	1	7
565	Family clothing stores	5	8
566	Shoe stores	3	1
571	Furniture and homefurnishings stores	4	2
573	Radio, television, and computer stores	3	13
581	Eating and drinking places	14	26
591	Drug stores and proprietary stores	5	1
593	Used merchandise stores	1	1
594	Miscellaneous shopping goods stores	10	8
596	Nonstore retailers	2	16
602	Commercial banks	34	113
603	Savings institutions	14	56
614	Personal credit institutions	6	11
615	Business credit institutions	9	5
616	Mortgage bankers and brokers	1	8
621	Security brokers and dealers	18	9
628	Security and commodity services	9	4
631	Life insurance	18	5
632	Medical service and health insurance	16	2
633	Fire, marine, and casualty insurance	18	12
635	Surety insurance	6	2

Table 1. (*Continued*)

SIC	Industry group	Number of stocks	
		NYSE	NASDAQ
636	Title insurance	5	1
641	Insurance agents, brokers, and service	4	3
653	Real estate agents and managers	3	1
655	Subdividers and developers	4	2
671	Holding offices	35	15
679	Miscellaneous investing	5	4
701	Hotels and motels	17	1
721	Laundry, cleaning, and garment services	2	2
729	Miscellaneous personal services	2	1
731	Advertising	4	14
732	Credit reporting and collection	4	2
733	Mailing, reproduction, stenographic	1	2
735	Miscellaneous equipment rental and leasing	7	3
736	Personnel supply services	7	8
737	Computer and data processing services	26	429
738	Miscellaneous business services	10	49
751	Automotive rentals, no drivers	3	1
781	Motion picture production and services	2	3
784	Video tape rental	1	3
794	Commercial sports	4	4
799	Misc. amusement, recreation services	8	10
801	Offices and clinics of medical doctors	1	5
805	Nursing and personal care facilities	2	1
806	Hospitals	6	3
807	Medical and dental laboratories	5	12
808	Home health care services	1	4
809	Misc. health and allied services	6	12
824	Vocational schools	1	2
871	Engineering and architectural services	4	4
873	Research and testing services	3	54
874	Management and public relations	7	21
	Total	1,120	2,110

This table lists the number of NYSE and NASDAQ common stocks for each SIC industry group in the sample. The sample excludes units, trusts, closed end funds, ADRs, and when-issued securities. Listed is the three-digit SIC code, the industry group name, and the number of NYSE and NASDAQ stocks that are included in the industry group.

considered. In particular, trading activity and spread width have been shown in other studies to be associated with volatility. These factors can be controlled for by first matching on criteria in addition to SIC code (i.e., trading activity and spread width) and then comparing volatilities between matched pairs of stocks. Given the unequal subsample sizes, this methodology will exclude many stocks.[4] An alternative methodology which allows for a larger sample is to control for the other factors in regressions which include all stocks in an SIC code. To maximize the number of observations, I use this methodology.

The next step is to calculate intraday volatility. Consistent with Schwert (1990) among others, I measure the volatility of 15-minute returns during the trading day. Using returns controls for any differences in price levels across the samples. As a first step, the trading day is divided into 26 15-minute intervals. Then the price of the last trade at or before the end of the 15-minute interval is determined.[5] If there was no trade in the interval, the last price of the previous trading interval is used. In other words a price is used until a new trade occurs. For the stable market period, the last trade's price from July is used to start the series in the event there is no trading in the first 15-minute interval of August. For the stressful market periods, the last trade from the previous day is used. Then the return for each 15-minute interval is calculated (i.e., 9:45–10:00; 10:00–10:15; …; 3:45–4:00). Three different measures of volatility are then calculated for each stock for each of the ten trading days in the sample: the simple standard deviation of return; the standard deviation of continuously compounded returns; and the average of squared returns. All three measures yield qualitatively similar results, so I only report the simple standard deviation of return.

3. Results

I first compare average volatility on the NYSE and NASDAQ for the stable market period. For each stock I find the average intraday volatility over the ten-day period. I then compute the average over all stocks in an industry group. NYSE and NASDAQ averages are calculated separately. Differences between

[4]Mayhew (2001) gives a good discussion of the problems faced in finding matches for securities. He matches option contracts according to date, price, and volume. He finds that by also including volatility as a matching criterion he loses 98% of his observations.

[5]Returns based on observed prices are known to be upward biased due to what is known in the academic literature as bid-ask bounce. A correction for this is to use the midpoint of the bid-ask spread instead of the trade price to calculate returns. Bessembinder (1998) calculates returns on both prices and spread midpoints. The different methods yield quantitatively similar results. I do control for differences in spread width across market centers later in regressions. I find that the results are not driven solely by differing spread widths between the two market centers.

market centers for each industry group are then calculated. I also calculate the average across all stocks for a market center. The results are reported in Table 2. Overall the average intraday volatility of NYSE stocks is 0.41%. NASDAQ stocks exhibit average intraday volatility more than twice as large as NYSE stocks or 0.98%. This finding is consistent with previous studies.

Examining Table 2 by industry group reveals that NYSE stocks exhibit higher average intraday volatility than their NASDAQ counterparts in only 6 of the 158 industry groups examined (204 — grain mill products, 239 — misc. fabricated textile products, 252 — office furniture, 326 — pottery, 511 — paper and paper products, and 593 — used merchandise stores). For stocks in each of the 152 remaining industry groups, NYSE-listed stocks consistently exhibit lower intraday volatility than their NASDAQ counterparts.

For the stressful market condition sample, I find the average intraday volatility for each stock for each of the three trading days examined. The results are reported in Table 3. Overall the average intraday volatility of NYSE stocks ranges from a low of 0.43% on May 16 to a high of 0.65% on October 3, 2001. NASDAQ stocks exhibit average intraday volatility more than twice as large as NYSE stocks or a minimum of 1.03% on May 16 to 1.72% on October 3, 2001. On each day, the differences between NYSE and NASDAQ stocks is larger than that reported for the stable market period (Table 2).

Examining Table 3 by industry group reveals that NYSE stocks exhibit higher average intraday volatility than their NASDAQ counterparts in less than 15% of the 158 industry groups examined each trading day. Specifically NYSE stocks exhibit lower volatility than their NASDAQ industry counterparts for 144, 145, and 137 of the 158 industry groups on May 16, October 3, and October 25, respectively. The number of cases where the difference is negative is lower than reported for the stable market condition period, however the number of cases of a positive difference is very small suggesting that on average NYSE listed stocks have lower volatility than their NASDAQ counterparts.

Overall, the magnitude of the differences between NYSE and NASDAQ stocks in this study is much larger than those reported in Bessembinder (1998) for his sample of stocks that left NASDAQ and listed on the NYSE. Therefore, the differences may be due to the characteristics of the firms in the NASDAQ and NYSE samples used in this study. To control for firm-specific characteristics, I regress each stock's average volatility against variables known to be associated with volatility.

Table 2. Comparison of NYSE and NASDAQ intraday volatility: stable market conditions.

SIC	Industry group	NYSE	NASDAQ	Difference
	All stocks in sample	0.41%	0.98%	0.57
109	Miscellaneous metal ores	0.30	1.28	0.98
131	Crude petroleum and natural gas	0.48	0.87	0.40
138	Oil and gas field services	0.48	0.70	0.22
152	Residential building construction	0.50	1.57	1.07
162	Heavy construction, except highway	0.49	1.58	1.09
173	Electrical work	0.63	2.14	1.51
201	Meat products	0.43	0.50	0.07
202	Dairy products	0.25	0.95	0.70
203	Preserved fruits and vegetables	0.30	0.53	0.22
204	Grain mill products	0.22	0.05	−0.17
205	Bakery products	0.40	0.72	0.33
206	Sugar and confectionery products	0.19	0.59	0.40
208	Beverages	0.26	0.44	0.18
209	Miscellaneous food and kindred products	0.23	0.84	0.61
222	Broadwoven fabric mills, manmade	0.39	0.71	0.32
227	Carpets and rugs	0.27	1.07	0.80
232	Men's and boys' furnishings	0.40	0.53	0.13
239	Miscellaneous fabricated textile products	1.96	1.59	−0.37
243	Millwork, plywood and structural members	0.39	0.77	0.38
251	Household furniture	0.39	1.00	0.61
252	Office furniture	0.60	0.50	−0.09
263	Paperboard mills	0.29	0.64	0.34
265	Paperboard containers and boxes	0.27	0.60	0.34
272	Periodicals	0.38	0.75	0.37
273	Books	0.32	0.81	0.49
274	Miscellaneous publishing	0.47	1.91	1.44
275	Commercial printing	0.29	1.63	1.34
282	Plastics materials and synthetics	0.33	0.75	0.42
283	Drugs	0.37	1.01	0.65
286	Industrial organic chemicals	0.34	1.07	0.73
287	Agricultural chemicals	0.40	1.57	1.16
289	Miscellaneous chemical products	0.41	0.66	0.26
302	Rubber and plastics footwear	0.36	0.79	0.43
308	Miscellaneous plastics products	0.43	1.14	0.71
314	Footwear, except rubber	0.43	0.72	0.29
322	Glass and glassware, pressed or blown	0.51	0.92	0.40
326	Pottery and related products	0.47	0.41	−0.06
327	Concrete, gypsum, and plaster products	0.48	0.59	0.11
329	Miscellaneous nonmetallic mineral products	0.25	0.51	0.26
331	Blast furnace and basic steel products	0.49	0.76	0.27
332	Iron and steel foundries	0.44	0.79	0.35
335	Nonferrous rolling and drawing	0.57	0.83	0.26
336	Nonferrous foundries (castings)	0.24	0.43	0.19

Table 2. (*Continued*)

SIC	Industry group All stocks in sample	NYSE 0.41%	NASDAQ 0.98%	Difference 0.57
339	Miscellaneous primary metal products	0.42	1.07	0.64
344	Fabricated structural metal products	0.41	1.44	1.02
346	Metal forgings and stampings	0.69	0.82	0.13
349	Miscellaneous fabricated metal products	0.34	1.88	1.54
351	Engines and turbines	0.43	0.50	0.07
352	Farm and garden machinery	0.36	0.71	0.35
353	Construction and related machinery	0.38	0.64	0.26
354	Metalworking machinery	0.34	0.87	0.54
355	Special industry machinery	0.33	0.84	0.51
356	General industrial machinery	0.29	0.72	0.43
357	Computer and office equipment	0.59	1.39	0.80
358	Refrigeration and service machinery	0.36	0.89	0.53
361	Electric distribution equipment	0.43	0.75	0.32
362	Electrical industrial apparatus	0.38	0.96	0.59
364	Electric lighting and wiring equipment	0.43	0.76	0.33
365	Household audio and video equipment	0.28	1.35	1.07
366	Communications equipment	0.50	1.35	0.85
367	Electronic components and accessories	0.59	0.92	0.33
369	Miscellaneous electrical equipment and supplies	0.51	2.05	1.54
371	Motor vehicles and equipment	0.35	0.82	0.47
372	Aircraft and parts	0.21	0.61	0.41
374	Railroad equipment	0.63	3.16	2.53
381	Search and navigation equipment	0.26	0.96	0.70
382	Measuring and controlling devices	0.44	1.01	0.57
384	Medical instruments and supplies	0.37	1.10	0.74
385	Ophthalmic goods	0.75	0.92	0.18
386	Photographic equipment and supplies	0.86	1.40	0.55
393	Musical instruments	0.21	0.94	0.73
394	Toys and sporting goods	0.62	1.31	0.69
399	Miscellaneous manufactures	0.30	1.34	1.04
401	Railroads	0.34	0.43	0.09
421	Trucking and courier services, except air	0.30	0.58	0.28
441	Deep sea foreign trans. of freight	0.41	1.09	0.68
451	Air transportation, scheduled	0.32	0.64	0.32
452	Air transportation, nonscheduled	0.44	0.96	0.51
481	Telephone communication	0.56	1.62	1.06
483	Radio and television broadcasting	0.31	0.99	0.68
484	Cable and other pay TV services	0.31	1.40	1.09
489	Communication services (not elsewhere classified)	0.50	1.24	0.74
491	Electric services	0.31	1.19	0.89
493	Combination utility services	0.34	1.02	0.68
494	Water supply	0.24	0.68	0.44
495	Sanitary services	0.40	0.80	0.40

Table 2. (*Continued*)

SIC	Industry group All stocks in sample	NYSE 0.41%	NASDAQ 0.98%	Difference 0.57
501	Motor vehicles, parts, and supplies	0.44	0.92	0.49
503	Lumber and construction materials	0.28	0.69	0.40
504	Professional and commercial equipment	0.40	1.08	0.68
505	Metals and minerals, except petroleum	0.72	2.41	1.68
506	Electrical goods	0.36	1.23	0.87
508	Machinery, equipment, and supplies	0.35	1.54	1.19
509	Miscellaneous durable goods	0.59	0.75	0.16
511	Paper and paper products	0.66	0.54	−0.12
512	Drugs, proprietaries, and sundries	0.46	0.87	0.41
513	Apparel, piece goods, and notions	0.54	0.83	0.28
514	Groceries and related products	0.38	0.62	0.25
519	Miscellaneous nondurable goods	0.51	1.36	0.85
521	Lumber and other building materials	0.29	0.70	0.42
531	Department stores	0.44	1.29	0.85
533	Variety stores	0.43	0.57	0.15
541	Grocery stores	0.32	0.81	0.48
551	New and used car dealers	0.43	0.90	0.47
553	Auto and home supply stores	0.50	0.52	0.02
561	Men’s and boys’ clothing stores	0.36	0.91	0.54
562	Women’s clothing stores	0.40	0.81	0.41
565	Family clothing stores	0.45	0.76	0.31
566	Shoe stores	0.51	0.73	0.22
571	Furniture and homefurnishings stores	0.38	0.89	0.51
573	Radio, television, and computer stores	0.47	1.29	0.83
581	Eating and drinking places	0.38	0.69	0.30
591	Drug stores and proprietary stores	0.31	1.79	1.48
593	Used merchandise stores	0.54	0.49	−0.06
594	Miscellaneous shopping goods stores	0.58	1.24	0.67
596	Nonstore retailers	0.43	1.25	0.82
602	Commercial banks	0.21	0.51	0.30
603	Savings institutions	0.33	0.45	0.12
614	Personal credit institutions	0.30	0.84	0.54
615	Business credit institutions	0.32	0.71	0.39
616	Mortgage bankers and brokers	0.23	1.05	0.82
621	Security brokers and dealers	0.31	0.96	0.65
628	Security and commodity services	0.25	0.63	0.38
631	Life insurance	0.27	0.43	0.16
632	Medical service and health insurance	0.42	1.09	0.67
633	Fire, marine, and casualty insurance	0.35	0.58	0.23
635	Surety insurance	0.24	0.63	0.39
636	Title insurance	0.33	1.71	1.38
641	Insurance agents, brokers, and service	0.25	0.73	0.48
653	Real estate agents and managers	0.35	1.90	1.55

Table 2. (*Continued*)

SIC	Industry group All stocks in sample	NYSE 0.41%	NASDAQ 0.98%	Difference 0.57
655	Subdividers and developers	0.26	0.89	0.63
671	Holding offices	0.34	0.47	0.13
679	Miscellaneous investing	0.49	0.77	0.28
701	Hotels and motels	0.35	0.77	0.41
721	Laundry, cleaning, and garment services	0.28	0.68	0.40
729	Miscellaneous personal services	0.36	0.65	0.29
731	Advertising	0.28	1.10	0.82
732	Credit reporting and collection	0.26	0.81	0.54
733	Mailing, reproduction, stenographic	0.21	0.78	0.57
735	Miscellaneous equipment rental and leasing	0.56	0.69	0.12
736	Personnel supply services	0.48	0.87	0.39
737	Computer and data processing services	0.44	1.42	0.97
738	Miscellaneous business services	0.50	1.52	1.01
751	Automotive rentals, no drivers	0.60	2.01	1.41
781	Motion picture production and services	0.34	1.51	1.17
784	Video tape rental	0.35	0.83	0.48
794	Commercial sports	0.41	0.60	0.19
799	Miscellaneous amusement, recreation services	0.37	1.14	0.77
801	Offices and clinics of medical doctors	0.39	1.37	0.98
805	Nursing and personal care facilities	0.54	0.76	0.22
806	Hospitals	0.39	0.74	0.35
807	Medical and dental laboratories	0.43	1.04	0.61
808	Home health care services	0.40	0.83	0.43
809	Health and allied services (not elsewhere classified)	0.46	0.70	0.24
824	Vocational schools	0.31	1.51	1.20
871	Engineering and architectural services	0.44	1.24	0.80
873	Research and testing services	0.40	1.00	0.60
874	Management and public relations	0.48	1.49	1.01

This table reports the average intraday volatility for NYSE and NASDAQ common stocks for the first 10 trading days in August 2001. Returns are calculated each day for the 25 15-minute intervals from 9:45 a.m until 4 p.m. Volatility is defined as the standard deviation of return. The results are broken down by SIC industry group. The columns labeled NYSE and NASDAQ report the average volatility over the 10-day period for firms in that industry group (in percentage terms). The last column reports the difference between NYSE and NASDAQ average volatility for each industry group.

First, the activity level of trading in a stock has been shown to be directly related to a stock's volatility. All things being equal, a higher activity level is thought to be evidence of a difference of opinion regarding a stock's value. Jones, Kaul, and Lipson (1994) have shown the number of trades to have more explanatory power than volume of trading — as far as volatility is concerned.

Table 3. Comparison of NYSE and NASDAQ intraday volatility: stressful market conditions.

SIC	Industry group	May 16, 2001			October 3, 2001			October 25, 2001		
	All stocks in sample	NYSE 0.43	NASDAQ 1.03	Diff. 0.60	NYSE 0.65	NASDAQ 1.72	Diff. 1.06	NYSE 0.56	NASDAQ 1.35	Diff. 0.79
109	Miscellaneous metal ores	0.24	1.11	0.88	0.14	2.03	1.89	0.14	1.72	1.58
131	Crude petroleum and natural gas	0.51	0.67	0.16	0.64	0.99	0.36	0.64	1.19	0.55
138	Oil and gas field services	0.48	0.49	0.02	0.71	0.92	0.21	0.71	1.05	0.33
152	Residential building construction	0.81	1.71	0.90	0.91	1.48	0.56	0.91	1.28	0.37
162	Heavy construction, except highway	0.51	0.71	0.20	0.86	1.57	0.71	0.86	0.79	−0.08
173	Electrical work	0.66	1.36	0.69	1.14	7.40	6.27	1.14	3.82	2.69
201	Meat products	0.46	0.19	−0.27	0.54	0.83	0.29	0.54	0.51	−0.03
202	Dairy products	0.51	1.10	0.59	0.53	2.31	1.78	0.53	1.36	0.82
203	Preserved fruits and vegetables	0.33	0.79	0.46	0.42	1.10	0.68	0.42	0.98	0.56
204	Grain mill products	0.26	0.37	0.11	0.29	0.35	0.07	0.29	–	–
205	Bakery products	0.39	0.43	0.04	0.75	1.20	0.45	0.75	1.18	0.43
206	Sugar and confectionery products	0.18	1.01	0.82	0.27	1.07	0.80	0.27	0.72	0.45
208	Beverages	0.32	0.41	0.10	0.27	0.96	0.69	0.27	0.40	0.12
209	Misc. food and kindred products	0.27	1.35	1.08	0.36	0.78	0.41	0.36	0.71	0.34

Table 3. (*Continued*)

SIC	Industry group	May 16, 2001			October 3, 2001			October 25, 2001		
		NYSE	NASDAQ	Diff.	NYSE	NASDAQ	Diff.	NYSE	NASDAQ	Diff.
	All stocks in sample	0.43	1.03	0.60	0.65	1.72	1.06	0.56	1.35	0.79
222	Broadwoven fabric mills, manmade	0.18	0.66	0.48	0.44	1.01	0.57	0.44	0.82	0.39
227	Carpets and rugs	0.19	1.05	0.86	0.58	1.81	1.23	0.58	1.96	1.38
232	Men's and boys' furnishings	0.43	0.59	0.16	0.93	0.75	−0.17	0.93	0.58	−0.34
239	Misc. fabricated textile products	1.87	1.11	−0.76	0.91	1.14	0.22	0.91	0.00	−0.91
243	Millwork, plywood and structural	0.58	0.10	−0.48	0.47	1.61	1.14	0.47	0.88	0.42
251	Household furniture	0.53	0.61	0.08	0.80	1.30	0.50	0.80	1.45	0.66
252	Office furniture	0.33	0.41	0.08	0.87	0.90	0.03	0.87	0.43	−0.45
263	Paperboard mills	0.40	0.63	0.23	0.37	1.02	0.65	0.37	0.99	0.61
265	Paperboard containers and boxes	1.19	0.71	−0.48	0.61	1.23	0.62	0.61	0.76	0.15
272	Periodicals	0.33	1.19	0.85	0.59	1.38	0.79	0.59	1.14	0.54
273	Books	0.42	0.88	0.46	0.48	1.38	0.90	0.48	1.54	1.06
274	Miscellaneous publishing	0.50	1.56	1.05	0.92	2.11	1.19	0.92	1.59	0.67
275	Commercial printing	0.32	1.76	1.44	0.36	3.02	2.65	0.36	1.49	1.13
282	Plastics materials and synthetics	0.42	0.77	0.35	0.44	1.03	0.59	0.44	1.18	0.75
283	Drugs	0.38	1.02	0.64	0.46	1.43	0.97	0.46	1.19	0.73
286	Industrial organic chemicals	0.43	1.99	1.56	0.51	2.22	1.71	0.51	1.76	1.25

Table 3. (*Continued*)

SIC	Industry group All stocks in sample	May 16, 2001			October 3, 2001			October 25, 2001		
		NYSE 0.43	NASDAQ 1.03	Diff. 0.60	NYSE 0.65	NASDAQ 1.72	Diff. 1.06	NYSE 0.56	NASDAQ 1.35	Diff. 0.79
287	Agricultural chemicals	0.43	1.84	1.41	0.68	3.00	2.32	0.68	0.63	−0.05
289	Miscellaneous chemical products	0.41	0.79	0.38	0.56	0.82	0.26	0.56	0.87	0.31
302	Rubber and plastics footwear	0.34	1.13	0.79	0.59	1.03	0.44	0.59	0.98	0.39
308	Miscellaneous plastics products	0.40	1.19	0.79	0.64	0.99	0.35	0.64	1.40	0.76
314	Footwear, except rubber	0.47	0.96	0.49	0.74	1.40	0.67	0.74	1.13	0.40
322	Glass and glassware	0.53	0.71	0.17	0.87	1.79	0.92	0.87	1.35	0.48
326	Pottery and related products	0.94	0.85	−0.09	0.84	0.79	−0.05	0.84	2.35	1.51
327	Concrete, gypsum, and plaster prod.	0.33	0.70	0.38	0.70	0.97	0.27	0.70	0.76	0.06
329	Misc. nonmetallic mineral products	0.43	0.72	0.28	0.50	1.73	1.23	0.50	1.00	0.50
331	Blast furnace and basic steel prod.	0.82	0.90	0.08	0.99	0.72	−0.28	0.99	1.22	0.23
332	Iron and steel foundries	0.30	0.86	0.55	0.58	2.38	1.81	0.58	0.32	−0.26
335	Nonferrous rolling and drawing	0.69	0.51	−0.18	0.86	2.06	1.20	0.86	1.20	0.33
336	Nonferrous foundries (castings)	0.21	0.66	0.44	0.32	0.63	0.31	0.32	0.96	0.64
339	Miscellaneous primary metal prod.	0.42	0.67	0.25	0.40	2.95	2.55	0.40	1.99	1.59
344	Fabricated structural metal products	0.32	0.99	0.67	0.55	1.58	1.03	0.55	1.49	0.94
346	Metal forgings and stampings	0.41	1.06	0.65	1.13	2.06	0.94	1.13	1.93	0.81
349	Misc. fabricated metal products	0.31	1.38	1.07	0.53	2.71	2.18	0.53	1.81	1.28
351	Engines and turbines	0.60	0.50	−0.10	0.52	1.04	0.52	0.52	0.52	0.01
352	Farm and garden machinery	0.50	0.77	0.27	0.67	1.34	0.67	0.67	0.67	0.00

Table 3. (*Continued*)

SIC	Industry group All stocks in sample	May 16, 2001			October 3, 2001			October 25, 2001		
		NYSE	NASDAQ	Diff.	NYSE	NASDAQ	Diff.	NYSE	NASDAQ	Diff.
		0.43	1.03	0.60	0.65	1.72	1.06	0.56	1.35	0.79
353	Construction and related machinery	0.47	0.95	0.48	0.70	1.14	0.44	0.70	0.83	0.13
354	Metalworking machinery	0.34	0.38	0.04	0.61	0.91	0.30	0.61	1.66	1.05
355	Special industry machinery	0.72	0.90	0.18	0.45	1.58	1.13	0.45	1.39	0.94
356	General industrial machinery	0.36	0.83	0.47	0.43	1.13	0.70	0.43	0.94	0.51
357	Computer and office equipment	0.65	1.25	0.59	1.00	2.31	1.31	1.00	1.70	0.70
358	Refrigeration and service machinery	0.35	0.97	0.62	0.54	0.95	0.41	0.54	1.82	1.28
361	Electric distribution equipment	0.42	0.65	0.23	0.63	1.47	0.83	0.63	0.81	0.18
362	Electrical industrial apparatus	0.33	0.72	0.39	0.60	1.81	1.21	0.60	1.38	0.78
364	Electric lighting and wiring equip.	0.51	1.00	0.49	0.89	2.17	1.28	0.89	1.41	0.52
365	Household audio and video equip.	0.18	1.07	0.89	0.87	2.15	1.28	0.87	1.67	0.80
366	Communications equipment	0.45	1.26	0.81	0.90	2.30	1.39	0.90	1.85	0.95
367	Electronic components and access.	0.65	0.94	0.29	1.11	1.75	0.64	1.11	1.25	0.15
369	Misc. electrical equipment and supplies	0.32	1.17	0.85	0.61	2.26	1.65	0.61	1.62	1.01
371	Motor vehicles and equipment	0.39	0.70	0.31	0.87	1.02	0.16	0.87	0.92	0.05
372	Aircraft and parts	0.23	0.46	0.23	0.53	1.08	0.55	0.53	0.70	0.17
374	Railroad equipment	0.37	2.11	1.74	1.16	5.60	4.45	1.16	–	–
381	Search and navigation equipment	0.23	0.72	0.49	0.42	1.48	1.06	0.42	1.17	0.75
382	Measuring and controlling devices	0.49	0.98	0.49	0.68	1.61	0.93	0.68	1.25	0.56
384	Medical instruments and supplies	0.48	1.00	0.52	0.46	1.54	1.08	0.46	1.26	0.80
385	Ophthalmic goods	0.57	0.85	0.29	0.38	1.08	0.70	0.38	0.94	0.56
386	Photographic equipment and supp.	0.45	1.04	0.58	1.08	3.17	2.09	1.08	1.17	0.09
393	Musical instruments	0.11	0.75	0.64	0.34	1.19	0.85	0.34	1.22	0.88
394	Toys and sporting goods	0.51	0.63	0.12	0.89	2.45	1.56	0.89	1.40	0.50
399	Miscellaneous manufactures	0.56	1.34	0.79	0.53	2.04	1.51	0.53	1.27	0.75
401	Railroads	0.38	0.86	0.48	0.48	0.64	0.16	0.48	0.45	−0.03

Table 3. (*Continued*)

SIC	Industry group	May 16, 2001			October 3, 2001			October 25, 2001		
	All stocks in sample	NYSE	NASDAQ	Diff.	NYSE	NASDAQ	Diff.	NYSE	NASDAQ	Diff.
		0.43	1.03	0.60	0.65	1.72	1.06	0.56	1.35	0.79
421	Trucking and courier serv., except air	0.55	0.56	0.01	0.41	1.10	0.70	0.41	0.84	0.43
441	Deep sea foreign trans. of freight	0.38	0.88	0.50	0.62	1.72	1.10	0.62	0.91	0.29
451	Air transportation, scheduled	0.28	0.77	0.49	0.91	1.17	0.25	0.91	1.44	0.53
452	Air transportation, nonscheduled	0.51	0.30	−0.20	0.58	0.80	0.22	0.58	0.73	0.15
481	Telephone communication	0.65	1.29	0.64	0.89	2.48	1.59	0.89	1.86	0.97
483	Radio and television broadcasting	0.35	1.00	0.65	0.67	2.07	1.40	0.67	1.24	0.57
484	Cable and other pay TV services	0.35	1.18	0.83	0.64	2.41	1.77	0.64	1.48	0.85
489	Communication services	0.37	1.09	0.72	0.85	1.63	0.78	0.85	1.44	0.60
491	Electric services	0.29	1.49	1.20	0.41	1.34	0.93	0.41	1.78	1.37
493	Combination utility services	0.30	1.95	1.65	0.50	1.61	1.11	0.50	1.75	1.25
494	Water supply	0.31	0.68	0.36	0.33	0.65	0.31	0.33	1.40	1.07
495	Sanitary services	0.41	0.75	0.34	0.43	0.54	0.11	0.43	1.05	0.61
501	Motor vehicles, parts, and supplies	0.38	0.66	0.28	0.50	1.96	1.46	0.50	1.39	0.89
503	Lumber and construction materials	0.25	0.94	0.69	0.48	0.28	−0.21	0.48	0.77	0.28
504	Professional and commercial equip.	0.41	0.99	0.58	0.55	1.99	1.45	0.55	1.19	0.65
505	Metals and minerals, except petrol	0.49	0.49	0.00	0.58	2.90	2.31	0.58	2.76	2.18
506	Electrical goods	0.55	0.96	0.40	0.71	1.84	1.13	0.71	1.42	0.71
508	Machinery, equipment, and supplies	0.39	0.94	0.55	0.49	3.04	2.55	0.49	2.03	1.55
509	Miscellaneous durable goods	0.47	0.75	0.28	0.42	1.21	0.79	0.42	0.90	0.47
511	Paper and paper products	0.59	0.32	−0.27	0.80	0.61	−0.19	0.80	0.81	0.00
512	Drugs, proprietaries, and sundries	0.33	1.28	0.95	0.26	0.97	0.71	0.26	1.05	0.79
513	Apparel, piece goods, and notions	0.51	0.73	0.22	1.09	1.54	0.44	1.09	1.07	−0.02
514	Groceries and related products	0.43	0.58	0.15	0.50	0.70	0.20	0.50	0.89	0.39

Table 3. (*Continued*)

SIC	Industry group All stocks in sample	May 16, 2001			October 3, 2001			October 25, 2001		
		NYSE 0.43	NASDAQ 1.03	Diff. 0.60	NYSE 0.65	NASDAQ 1.72	Diff. 1.06	NYSE 0.56	NASDAQ 1.35	Diff. 0.79
519	Miscellaneous nondurable goods	0.43	0.71	0.28	0.66	1.50	0.84	0.66	1.88	1.22
521	Lumber and other building materials	0.31	1.36	1.05	0.45	1.91	1.46	0.45	1.63	1.18
531	Department stores	0.37	0.67	0.30	0.85	1.37	0.51	0.85	1.04	0.19
533	Variety stores	0.38	0.72	0.34	0.87	1.36	0.48	0.87	0.61	−0.26
541	Grocery stores	0.33	0.79	0.45	0.47	0.79	0.33	0.47	0.51	0.04
551	New and used car dealers	0.63	0.55	−0.08	0.94	1.14	0.19	0.94	1.32	0.38
553	Auto and home supply stores	0.45	0.62	0.17	0.67	1.15	0.48	0.67	0.75	0.08
561	Men’s and boys’ clothing stores	0.45	0.58	0.13	1.54	1.46	−0.08	1.54	1.27	−0.27
562	Women’s clothing stores	0.51	0.75	0.24	0.94	1.10	0.16	0.94	0.78	−0.16
565	Family clothing stores	0.28	0.74	0.46	0.74	1.48	0.74	0.74	1.06	0.32
566	Shoe stores	0.46	0.79	0.33	0.96	1.06	0.10	0.96	0.93	−0.04
571	Furniture and homefurnishing stores	0.41	1.08	0.67	1.09	2.28	1.19	1.09	0.50	−0.59
573	Radio, television, and computer stores	0.42	1.26	0.84	0.80	2.17	1.37	0.80	1.33	0.53
581	Eating and drinking places	0.45	0.74	0.29	0.67	1.07	0.40	0.67	0.76	0.09
591	Drug stores and proprietary stores	0.39	1.29	0.90	0.56	2.53	1.97	0.56	2.91	2.35
593	Used merchandise stores	0.58	0.46	−0.12	2.55	0.35	−2.20	2.55	0.79	−1.77
594	Misc. shopping goods stores	0.52	1.09	0.57	0.87	1.40	0.53	0.87	1.24	0.38
596	Nonstore retailers	0.62	1.47	0.84	0.71	2.03	1.32	0.71	1.25	0.54
602	Commercial banks	0.25	0.58	0.32	0.38	0.71	0.33	0.38	0.66	0.28
603	Savings institutions	0.36	0.43	0.07	0.42	0.66	0.24	0.42	0.58	0.16
614	Personal credit institutions	0.39	0.58	0.20	0.56	1.56	0.99	0.56	1.41	0.85
615	Business credit institutions	0.44	0.85	0.41	0.70	1.94	1.23	0.70	0.75	0.05
616	Mortgage bankers and brokers	0.19	1.10	0.92	0.41	1.16	0.74	0.41	1.07	0.66

Table 3. (*Continued*)

SIC	Industry group All stocks in sample	May 16, 2001			October 3, 2001			October 25, 2001		
		NYSE	NASDAQ	Diff.	NYSE	NASDAQ	Diff.	NYSE	NASDAQ	Diff.
		0.43	1.03	0.60	0.65	1.72	1.06	0.56	1.35	0.79
621	Security brokers and dealers	0.40	0.98	0.58	0.56	2.52	1.97	0.56	1.02	0.46
628	Security and commodity services	0.28	0.77	0.49	0.48	1.39	0.91	0.48	1.17	0.69
631	Life insurance	0.28	0.55	0.27	0.42	0.77	0.35	0.42	0.79	0.37
632	Medical service and health ins.	0.49	2.20	1.70	0.77	2.13	1.35	0.77	0.62	−0.15
633	Fire, marine, and casualty insurance	0.36	0.60	0.25	0.59	0.79	0.21	0.59	1.06	0.47
635	Surety insurance	0.34	0.98	0.63	0.53	1.04	0.51	0.53	0.89	0.36
636	Title insurance	0.36	1.68	1.32	0.55	1.57	1.02	0.55	1.63	1.09
641	Insurance agents, brokers, and service	0.40	0.54	0.14	0.66	0.34	−0.32	0.66	0.83	0.16
653	Real estate agents and managers	0.51	1.46	0.95	0.36	8.83	8.47	0.36	5.78	5.41
655	Subdividers and developers	0.30	0.64	0.34	0.35	0.81	0.46	0.35	1.34	0.99
671	Holding offices	0.35	0.44	0.09	0.59	0.59	−0.01	0.59	0.56	−0.03
679	Miscellaneous investing	0.43	0.48	0.05	0.94	2.06	1.12	0.94	0.50	−0.44
701	Hotels and motels	0.35	0.62	0.28	0.67	0.17	−0.50	0.67	1.18	0.51
721	Laundry, cleaning, and garment serv.	0.35	0.90	0.55	0.20	0.98	0.78	0.20	1.02	0.82
729	Miscellaneous personal services	0.27	0.41	0.14	0.54	0.84	0.30	0.54	0.68	0.14
731	Advertising	0.36	1.23	0.86	0.48	2.19	1.71	0.48	1.66	1.18
732	Credit reporting and collection	0.33	0.75	0.43	0.37	0.87	0.50	0.37	0.58	0.21
733	Mailing, reproduction, stenographic	0.33	1.37	1.04	0.34	1.26	0.92	0.34	1.42	1.08
735	Misc. equipment rental and leasing	0.74	0.64	−0.10	1.09	1.78	0.70	1.09	0.58	−0.51
736	Personnel supply services	0.62	0.94	0.32	0.90	1.30	0.40	0.90	0.93	0.02
737	Computer and data processing serv.	0.44	1.29	0.85	0.74	2.30	1.56	0.74	1.75	1.02
738	Miscellaneous business services	0.53	1.25	0.71	1.01	1.78	0.77	1.01	1.99	0.98
751	Automotive rentals, no drivers	0.63	1.64	1.01	1.54	4.01	2.47	1.54	3.83	2.29

Table 3. (*Continued*)

SIC	Industry group	May 16, 2001			October 3, 2001			October 25, 2001		
		NYSE	NASDAQ	Diff.	NYSE	NASDAQ	Diff.	NYSE	NASDAQ	Diff.
	All stocks in sample	0.43	1.03	0.60	0.65	1.72	1.06	0.56	1.35	0.79
781	Motion picture production and services	0.47	1.37	0.90	0.73	1.20	0.47	0.73	1.34	0.61
784	Video tape rental	0.27	1.31	1.04	0.90	1.21	0.30	0.90	0.80	−0.11
794	Commercial sports	0.52	0.37	−0.15	0.64	0.77	0.14	0.64	0.75	0.11
799	Misc. amusement, recreation serv.	0.42	1.89	1.47	0.64	1.54	0.89	0.64	0.95	0.31
801	Offices and clinics of medical doctors	0.50	1.33	0.83	0.44	2.44	2.00	0.44	2.64	2.20
805	Nursing and personal care facilities	0.45	0.83	0.38	0.72	0.28	−0.43	0.72	0.62	−0.10
806	Hospitals	0.47	1.60	1.14	0.66	0.48	−0.18	0.66	1.32	0.66
807	Medical and dental laboratories	0.57	0.92	0.35	0.50	1.06	0.56	0.50	0.96	0.46
808	Home health care services	0.31	1.00	0.69	0.80	0.75	−0.06	0.80	0.83	0.03
809	Health and allied services	0.46	0.68	0.22	0.43	0.72	0.29	0.43	0.76	0.33
824	Vocational schools	0.75	1.81	1.06	0.40	2.00	1.60	0.40	2.58	2.18
871	Engineering and architectural services	0.69	0.94	0.25	0.55	2.69	2.14	0.55	1.74	1.19
873	Research and testing services	0.36	0.94	0.58	0.62	1.60	0.98	0.62	1.32	0.71
874	Management and public relations	0.41	1.28	0.87	0.97	2.46	1.49	0.97	2.06	1.09

This table reports the average intraday volatility for NYSE and NASDAQ common stocks for three days during 2001–May 16, October 3 and October 25. Returns are calculated each day for the 25 15-minute intervals from 9:45 a.m until 4 p.m. Volatility is defined as the standard deviation of return. The results are broken down by SIC industry group. The columns labeled NYSE and NASDAQ report the average volatility for firms in that industry group (in percentage terms). The last column for each day reports the difference between NYSE and NASDAQ average volatility for each industry group. A period (.) in a cell indicates no stock from that SIC group traded on that day.

Accordingly, I use the number of trades as the activity measure in the regressions. For the stable market condition period, the variable *Activity* is defined as the average daily number of trades for a stock over the ten trading days from August 1–14, 2001. For the stressful market condition period, the number of trades for each stock on the day considered is used. In both cases, the scale is in hundreds of trades.[6]

The second variable of interest is spread width. As mentioned earlier, using observed prices to calculate volatility may impart an upward bias in returns, that could in turn cause higher observed volatility. If this is the case, then there should be a positive relationship between spread and volatility. Since I am using return volatility to control for price, I use percentage-spread width as the measure of spread in the regressions. For the stable market period, the variable *Spread* is defined as the time-weighted average percentage spread for each stock over the period August 1–3, 2001.[7] For the stressful market periods, daily spreads are calculated. The scale is in percentage points. Spreads are based on calculated NBBOs excluding crossed quotes. Spreads from individual market centers are filtered to exclude errors (e.g., a bid or ask equal to zero).

To capture any remaining differences between stocks from the two market centers, I include a dummy variable called *Listed*, which is given the value of 1 if the stock is an NYSE issue. Otherwise, the *Listed* variable is given a value of zero. Finding that the *Listed* parameter value is negative indicates that the smaller volatility for NYSE issues documented in Table 2 is not driven by differences in activity level and spread width between the two market centers, but is more likely associated with where the issue is traded (NYSE vs. NASDAQ).

The results of the regressions, for the stable market period, are contained in Table 4. Panel A reports the parameter estimates including all stocks. The parameter estimates for in Panel A are of the expected sign (positive) and statistically significant, indicating that these two variables explain at least part

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[6]There is a potential problem with including the number of trades in the regressions in that there are structural differences between the NYSE and NASDAQ that cause differences in the number of trades reported. On an intermediated market like NASDAQ a public order to buy and a public order to sell will be recorded as two trades since they both trade with a dealer. On the NYSE the same occurrence would be recorded as one trade. The advent of alternative trading systems on NASDAQ has increased the number of trades where public orders meet each other, so the problem is not as severe as it may have been using data from earlier periods.

[7]A three-day period was used due to the large amount of data involved. Over 14 million quotes were analyzed over the three-day period. Time weighting gives less weight to spreads that exist only briefly and therefore is thought to be a better indicator of normal spread width for a stock.

Table 4. Control regressions: stable market conditions.

SIC	Industry group	Intercept	Activity	Spread	Listed	Adj. R^2
A. All stocks in sample		0.538*	0.001*	0.334*	−0.286*	0.68
B. SIC groups with a sufficient number of observations						
131	Crude petroleum and natural gas	0.361*	0.013*	0.340*	−0.117*	0.83
138	Oil and gas field services	0.556*	0.003	0.150	−0.160	0.13
283	Drugs	0.579*	0.000	0.317*	−0.256*	0.64
331	Blast furnace and basic steel products	0.637*	0.008	0.116	−0.273*	0.19
355	Special industry machinery	0.550*	0.000	0.349*	−0.388	0.39
356	General industrial machinery	0.657*	−0.001	0.067	−0.387*	0.83
357	Computer and office equipment	0.841*	−0.001	0.247*	−0.387*	0.49
362	Electrical industrial apparatus	0.637*	0.008	0.238*	−0.374*	0.66
366	Communications equipment	0.779*	−0.001	0.281*	−0.370*	0.63
367	Electronic components and accessories	0.659*	0.000	0.279*	−0.171*	0.49
371	Motor vehicles and equipment	0.375*	0.003	0.402*	−0.211*	0.88
382	Measuring and controlling devices	0.618*	0.000	0.299*	−0.269*	0.59
384	Medical instruments and supplies	0.621*	0.004	0.270*	−0.368*	0.64
481	Telephone communication	0.924*	−0.001	0.303*	−0.451*	0.60
483	Radio and television broadcasting	0.830*	−0.007	0.115*	−0.541*	0.44
484	Cable and other pay TV services	0.283	0.005	0.420*	−0.074	0.73
491	Electric services	0.690*	0.009*	0.357*	−0.521*	0.79
493	Combination utility services	−0.095	0.023*	0.536*	0.185	0.76
506	Electrical goods	0.441*	0.080	0.316*	−0.432*	0.86
581	Eating and drinking places	0.440*	0.000	0.224*	−0.153*	0.64
602	Commercial banks	0.301*	0.003	0.202*	−0.145*	0.61
603	Savings institutions	0.251*	−0.001	0.210*	0.012	0.73
621	Security brokers and dealers	0.201*	0.006*	0.546*	−0.101	0.94
631	Life insurance	0.202*	0.011	0.363*	−0.075	0.48
633	Fire, marine, and casualty insurance	0.319*	0.000	0.198*	−0.039	0.42
671	Holding offices	0.159*	0.001	0.329*	0.035	0.78
737	Computer and data processing services	0.695*	0.000	0.345*	−0.387*	0.66
738	Miscellaneous business services	0.362*	0.005	0.495*	−0.447*	0.73
873	Research and testing services	0.554*	−0.001	0.370*	−0.290	0.56
874	Management and public relations	0.715*	0.008	0.241*	−0.426*	0.74

This table reports the results of control regressions where the dependent variable is the average intraday volatility for NYSE and NASDAQ common stocks for the first ten trading days in August 2001. Average intraday volatility for each stock is regressed against Activity (the average daily number of trades in 100s), Spread (the time-weighted percentage spread), and Listed (an indicator variable with the value of 1 if the stock is listed on the NYSE; otherwise it is assigned the value of zero). A negative value for Listed indicates that intraday volatility is lower on the NYSE than on NASDAQ. Panel A reports the overall results which includes all stocks in the sample. Panel B reports results of regressions performed on industry groups with at least 20 stocks. An asterisk next to the parameter estimate indicates that the estimate is statistically significant at the 10% level. Test of significance are t-tests. The last column lists the adjusted R^2 — a measure of how well the model works.

of the observed differences reported in Table 2. The *Listed* parameter estimate is negative and statistically significant. This indicates that after controlling for differences between the characteristics of the stocks in the two samples, those stocks listing on the NYSE still exhibit a statistically significant lower level of volatility.

Panel B of Table 4 gives parameter estimates for industry groups that have at least 20 stocks between the two market center subgroups. There are 30 such industry groups. Of the 30 *Listed* parameter estimates, 27 are negative and 20 of those 27 are statistically significant. None of the three positive *Listed* parameter estimates are statistically significant.

For the stressful market periods, the parameter estimates for *Activity* and *Spread* are similar to those reported in Table 4. Therefore, only estimates of the variable of interest, *Listed*, are reported. Examining Panel A of Table 5, reveals

Table 5. Control regressions: stressful market conditions.

SIC	Industry group	Listed parameter value		
		May 16, 2001	October 3, 2001	October 25, 2001
A. All stocks in sample		−0.339*	−0.462*	−0.375*
B. SIC groups with a sufficient number of observations				
131	Crude petroleum and natural gas	0.155	−0.467*	−0.500*
138	Oil and gas field services	0.004	−0.198	−0.589*
283	Drugs	−0.379*	−0.277*	−0.487*
331	Blast furnace and basic steel products	−0.077	0.692*	.
355	Special industry machinery	0.013	−1.038*	−0.466
356	General industrial machinery	−0.327	−0.360	−0.128
357	Computer and office equipment	−0.202	−0.679*	−0.751*
362	Electrical industrial apparatus	−0.470*	−0.622	−0.528
366	Communications equipment	−0.667*	−0.273	−0.512
367	Electronic components and accessories	−0.171*	−0.500*	−0.146
371	Motor vehicles and equipment	−0.186	0.308	−0.193*
382	Measuring and controlling devices	−0.184*	−0.240	−0.246*

Table 5. (*Continued*)

SIC	Industry group	Listed parameter value		
		May 16, 2001	October 3, 2001	October 25, 2001
384	Medical instruments and supplies	−0.254*	−0.255	−0.477*
481	Telephone communication	−0.200	−1.171*	−0.771*
483	Radio and television broadcasting	−0.271	−0.041	−0.294
484	Cable and other pay TV services	−0.093	−0.676	−0.433
491	Electric services	−1.609*	−0.492	0.447
493	Combination utility services	−0.864*	0.500	−0.967*
506	Electrical goods	−0.079	−0.524	−0.162
581	Eating and drinking places	−0.056	0.058	−0.247*
602	Commercial banks	−0.048	0.046	−0.053
603	Savings institutions	0.045	−0.120	−0.084
621	Security brokers and dealers	−0.438*	−0.738*	−0.093
631	Life insurance	−0.252*	−0.307	−0.229
633	Fire, marine, and casualty insurance	0.008	0.215	−0.115
671	Holding offices	0.060	0.350*	0.053
737	Computer and data processing services	−0.486*	−0.809*	−0.549*
738	Miscellaneous business services	−0.534*	−0.706*	−0.504
873	Research and testing services	−0.466*	−0.371	−0.564
874	Management and public relations	−0.545*	0.649	−0.928

This table reports the results of regressions performed to control for firm variables known to be associated with volatility for NYSE and NASDAQ common stocks for three days during 2001–May 16, October 3, and October 25. Intraday volatility for each stock is regressed against Activity (the number of trades in 100s), Spread (the time-weighted percentage spread), and Listed (an indicator variable with the value of 1 if the stock is listed on the NYSE; otherwise it is assigned the value of zero). A negative value for Listed indicates that intraday volatility is lower on the NYSE than on NASDAQ for that SIC code on that day. Reported are the parameter values for the variable of interest — the Listed indicator variable. Panel A reports the overall results that include all stocks in the sample. Panel B reports results of regressions performed on industry groups with at least 20 stocks. An asterisk next to the parameter estimate indicates that the estimate is statistically significant at the 10% level. Test of significance are t-tests. A period (.) in a cell indicates an insufficient number of observations for that SIC code for that day.

that for each of the three stressful market condition days, the *Listed* parameter estimates are larger than that reported for the stable market period (Table 4) for the samples including all stocks. This is consistent with the observed larger differences for unconditional volatility under stressful market conditions reported in Table 3 versus those for stable market conditions reported in Table 2.

Examining Panel B of Table 5 and comparing the results to those reported in Table 4 reveals a relationship similar to that reported for Tables 2 and 3. The results in Panel B of Table 5 are not as strong as those reported in Table 4, yet reach the same general conclusion. The results of the overall and industry group regressions, for both stable and stressful market conditions, suggests that the observed lower level of volatility for NYSE-listed stocks is driven by the trading location of the stock and not differences in firm specific characteristics between the two market center groups.

4. Conclusion

This study examines differences in intraday volatility between stocks trading on the NYSE and NASDAQ under stable as well as stressful market conditions. Overall results as well as results broken down by industry group, show that NYSE stocks exhibit lower volatility than those primarily traded on NASDAQ. Additional analysis that controls for firm specific factors known to be associated with volatility does not change the conclusion of the unrestricted results. In short — NYSE stocks exhibit consistently lower intraday volatility than NASDAQ stocks. This finding is consistent with previous studies and suggests that a specialist market structure is associated with lower volatility.

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Chapter 7

The Intraday Probability of Informed Trading on the NYSE

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Trading and quoting exhibit distinct intraday patterns. Using transaction data for a sample of NYSE stocks, we analyze the intraday probability of informed trading using a model developed by Easley, Kiefer, O'Hara, and Paperman (1996). We find a crude inverted U-shaped pattern in the probability of informed trading on an intraday basis. We find that trading activity, measured by the number of trades, is positively related to the probability of informed trading, and the amount of regional activity is inversely related to the probability of informed trading.

Keywords: Intraday; NYSE; probability of informed trading.

JEL: G10; G14

1. Introduction

Informed traders hold an important role in finance. Through their actions, informed traders transmit private information into prices, increasing the overall informational efficiency of the market. The question of how informed traders actually transmit their information to the market through trading strategies has been the focus of considerable debate. As Glosten and Milgrom (1985) and others have shown, the presence of informed traders cause other market participants to alter their trading and quoting behavior. As a result, informed traders have an incentive to hide their trading among other, uninformed traders so as to reduce the impact of their informed trading, as noted in Kyle (1985) and Admati and Pfleiderer (1988).

Theory indicates that informed traders trade throughout the day so as to profit from their information while attempting to blend in with the trades of uninformed traders. However, informed traders' trading patterns may vary over time as there is considerable evidence that trading and the trading environment varies considerably throughout the trading day. For example, Wood, McInish, and Ord (1985) document a U-shaped pattern in volume and returns on the New York Stock Exchange (NYSE). In addition, McInish and Wood (1992), Brock and Kleidon (1992), Lee, Mucklow, and Ready (1993), and Chan, Chung, and Johnson (1995) find that spreads are widest at the beginning of the trading day, narrow during the day, then widen near the close.[1]

In spite of the distinct patterns of spreads and trading volume, it is not certain when, exactly, informed traders trade relative to the trading of uninformed traders. For example, in contradiction to the theory, it could be that, in practice, informed traders follow a uniform trading pattern throughout the day while uninformed trading varies widely. Alternatively, it could be that information arrives at different times over the course of the day. Even when trying to hide among uninformed traders, informed traders will attempt to trade at different intensities at different times depending on the arrival rate of their information. As a result, the probability of an informed trade can differ over the course of the day depending on the trading pattern of the informed trader and the arrival rate of information over the day.

We therefore investigate whether the probability of informed trading is different during different times of the day.[2] To do so, we use the methodology in Easley, Kiefer, O'Hara, and Paperman (1996). This technique allows us to examine the probability of an informed trader trading over different intervals while allowing for different probabilities for the existence of private information for each interval. Overall, we find that the probability of an informed trade is highest at the beginning, the end, and — surprisingly — the middle of the day. Although volume and spreads drop in the middle of the day, the probability that a trade is informed is highest at this time. Note that this does not mean that the informed trader trades more often during the middle of the day; in fact, the trading of informed traders drops in the middle of the day, resulting in a U-shaped pattern similar to the overall trading pattern. However, the uninformed traders also drop off in the middle of the day, and in fact, drop off *more* than the informed traders. As a result, although there are fewer informed trades, they comprise a higher percentage of the overall trading volume in the middle of the

[1] The empirical findings of Chung, Van Ness, and Van Ness (1999) suggest that this pattern may be a result of competition from the limit order book.

[2] Chan, Christie, and Schultz (1995) examine the intraday pattern of spreads for Nasdaq-traded stocks. They find that, unlike NYSE spreads, Nasdaq spreads decline throughout the day and the magnitude of the decline is largest during the last 30 minutes of trading.

day. In addition, we find that while the pattern is similar across trading volume quartiles, we find that the most often traded stocks have the lowest probability of informed trading.

2. Probability of Informed Trading Model

To examine the probability of informed trading during different time periods during the day, we use the Easley, Kiefer, O'Hara, and Paperman (1996) model (hereafter referred to as EKOP) to estimate the probability of informed trading (PI). EKOP develop a trade flow model using order imbalances of buys and sells to generate the probability that the market maker will face an informed trader. The inputs for the model are the total buys (B) and sells (S) for each interval during the estimation period. The model parameters, $\theta = (\alpha, \mu, \varepsilon, \delta)$, are estimated by maximizing the following likelihood function:

$$L(\theta|M) = \prod_{i=1}^{I} L(\theta|B_i, S_i),$$

where the likelihood for an interval is given by:

$$L(\theta|B, S) = (1-\alpha)e^{-\varepsilon}\frac{\varepsilon^B}{B!}e^{-\varepsilon}\frac{\varepsilon}{S!} + \alpha\delta e^{-\varepsilon}\frac{\varepsilon^B}{B!}e^{-(\mu+\varepsilon)}\frac{(\mu+\varepsilon)^S}{S!}$$
$$+ \alpha(1-\delta)e^{-(\mu+\varepsilon)}\frac{(\mu+\varepsilon)^B}{B!}e^{-\varepsilon}\frac{\varepsilon^S}{S!}.$$

In this model, α is the probability of an information event occurring during that interval, δ the probability that a given signal is low, μ the arrival rate of informed traders given a signal, and ε is the arrival rate of uninformed traders. The probability of informed trading (PI) is calculated as:

$$PI = \frac{\alpha\mu}{\alpha\mu + 2\varepsilon}$$

where $\alpha\mu$ is the amount of informed trading. (For an informed trader to trade, he/she has to have information on which to trade. Thus, $\alpha\mu$ is the amount of informed trading as it is the probability of an information event times the arrival rate of informed traders.)

To evaluate the changing nature of the probability of an informed trade over the course of the day, we estimate the EKOP model on an intraday basis, dividing the day into 13 half-hour intervals. The EKOP model is then estimated over each one of these periods individually, producing one estimate for each stock during each of the 13 half-hour intervals. We therefore treat each interval as an EKOP "trading day".[3] For each of the 13 intervals, a different estimate of each of model parameters (α, μ, ε, δ) are developed based on the total buys and sells for each period. As a result, the estimation will allow for a differing probability of information arrival during each half-hour period, i.e., each of the thirteen intervals may have a different α. (Note also that the EKOP model allows for no new information to arrive.) In this way, we can estimate the probability of an informed trade as the result of both the likelihood of new information arrival and the arrival rate of informed traders.

3. Data

The data for the study is obtained from the Trades and Quotes (TAQ) file provided by the NYSE. We use NYSE-listed stocks for the months of January, February, and March 2002. We use several filters to screen the data. First, to remove unusually low-priced stocks that might have unusually high percentage bid-ask spreads, we only include stocks with an average price over the time period of more than $5. Additionally, we eliminate firms with fewer than 7,800 trades over our three-month time period to eliminate stocks with low levels of trading activity that might unfairly bias our results econometrically.

These filters result in a sample of 625 NYSE-listed firms. When we divide our sample into quartiles based on the number of trades, we have 156 firms per quartile, with the exception of the last quartile (the smallest in terms of the number of trades) which has 157 firms. To employ the EKOP model, the number of buy and sell trades must be estimated for each stock for each time period. As we are later interested in whether trading on the regional exchanges affect the proportion of informed trading, we examine only NYSE trades. Similar to EKOP, we classify trades as buys or sells based on the classification method described in Lee and Ready (1990), where trades above the midpoint of the bid and ask are considered buy trades, and those below sells. As we are examining NYSE trades in NYSE-listed securities, we use the NYSE quotes for classifying trades.[4]

We further describe our sample in Table 1. The average price per share is about $38. Most of the sample has a price lower than $50. The average number of trades per firm over the entire period is 55,334; the average trade size for

[3] Functionally, the methodology is exactly equivalent to EKOP, except while their trading period is the entire day, our trading period is one of the 13 intraday trading intervals.

[4] Blume and Goldstein (1997) show that the NYSE provides the best intermarket quote 90–95% of the time.

Table 1. Descriptive statistics.

Variable	Mean	Standard deviation	Percentile				
			Min	25	50	75	100
Price ($)	37.91	20.12	5.77	23.79	35.20	48.22	151.30
Trade size	1457.68	1049.77	350.43	813.13	1133.95	1695.43	8865.94
Number of trades	55334.37	35655.40	8503	30396	44316	68068	254568
Spread	0.0507	0.0190	0.0169	0.0390	0.0469	0.0582	0.2197
% Spread	0.0016	0.0009	0.0005	0.0010	0.0014	0.0020	0.0075

This table presents average NYSE statistics for our sample. Price is the price of each security in dollars. Trade size is the average number of shares per trade. The number of trades shows the activity of each firm. Spread is the difference in the offer and bid prices, and % spread is the difference in the spread divided by the midpoint of the spread.

each firm is about 1,458 shares. The average dollar bid-ask spread for these firms is about 5 cents a share, resulting in an average percentage spread of 0.16%.

To observe intraday variation, we divide the trading day, which runs from 9:30 a.m. to 4:00 p.m., into 13 intervals. The first interval is from 9:30 a.m. to 10:00 a.m., which comprises the first 31 min of trading. The next 12 intervals divide the remainder of the day into successive, consecutive segments of 30 min each. The EKOP model is then estimated over each one of these periods individually, producing one estimate for each stock during each of the 13 half-hour intervals from 9:30 a.m. to 4:00 p.m.[5]

4. Intraday Results

Table 2 presents the intraday data on the probability of informed trading for our sample. We find that the probability of informed trading exhibits a crude inverted U-shaped intraday pattern, shown visually in Figure 1. The probability of an informed trade during the first half-hour of trading is 20.6%, but then drops to its lowest overall value during the day of 19.6% during the 10:00 a.m. to 10:30 a.m. half-hour. After that point, the probability increases throughout the day, reaching a peak during the 12:30 p.m. to 1:00 p.m. half-hour at 22.8%, after which it declines throughout the rest of the day, ending the day with a

[5]Interval 10 is omitted in the regression analysis.

Table 2. The probability of informed trading by time interval.

	Interval	Mean	Standard deviation	Percentile				
				Min	25	50	75	100
1	9:30–10:00	0.2060	0.0768	0.0000	0.1609	0.1974	0.2360	0.7167
2	10:00–10:30	0.1961	0.0755	0.0000	0.1523	0.1963	0.2281	0.8970
3	10:30–11:00	0.2063	0.0788	0.0000	0.1617	0.1991	0.2418	0.8939
4	11:00–11:30	0.2091	0.0735	0.0000	0.1665	0.2057	0.2435	0.7565
5	11:30–12:00	0.2062	0.0710	0.0000	0.1640	0.2009	0.2414	0.7056
6	12:00–12:30	0.2121	0.0699	0.0000	0.1654	0.2095	0.2523	0.5065
7	12:30–1:00	0.2279	0.0717	0.0000	0.1771	0.2205	0.2707	0.6970
8	1:00–1:30	0.2225	0.0684	0.0592	0.1755	0.2161	0.2635	0.5715
9	1:30–2:00	0.2140	0.0759	0.0000	0.1666	0.2061	0.2491	0.5952
10	2:00–2:30	0.2080	0.0761	0.0000	0.1641	0.1992	0.2452	0.7906
11	2:30–3:00	0.2111	0.0823	0.0000	0.1633	0.2025	0.2490	0.8182
12	3:00–3:30	0.1998	0.0798	0.0000	0.1553	0.1931	0.2341	0.9141
13	3:30–4:00	0.1974	0.0768	0.0000	0.1518	0.1866	0.2309	0.7599

This table shows the intraday levels of the probability of informed trading. The probability of informed trading metric is calculated using the model of Easley, Kiefer, O'Hara, and Paperman (1996). We calculate the mean, standard deviation and quartiles for each interval of the trading day. The trading day is segregated into 13 30-minute intervals.

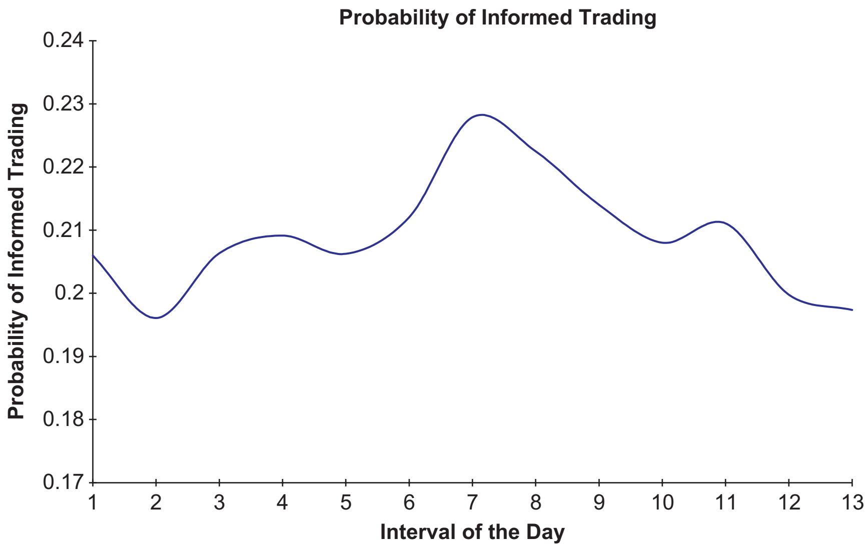

Figure 1. Intraday probability of informed trading.

probability of 19.8%. Thus, unlike the results for both volume and spread, the probability of an informed trade is highest in the middle of the day.

Table 2 also indicates the relative level of dispersion across stocks over each of the half-hour intervals. As Table 2 indicates, the pattern of the probability of informed trading over the course of the day is similar whether measured by the mean or the median for each interval, providing evidence that the overall result is not generated by outliers. The 25th and 75th percentiles also indicate a similar trend over the course of the day, further supporting the results.[6] Reassuringly, an examination of the stocks with the largest probability of an informed trade in each interval indicates that the middle of the day has a lower maximum probability than the beginning and end of the day, contrary to the overall trend and thus further reducing the likelihood that the midday results are the result of a number of outliers.

An examination of the magnitude of the probability of informed trading over the course of the day indicates that the overall probability varies between 19.5 and 23% (19 and 22% for the median). These numbers are slightly higher than those found by EKOP using 1990 data.[7] On one hand, it is reassuring that the overall magnitude of these numbers is similar to EKOP's results. On the other hand, these results indicate a slight increase in the probability of an informed trade over the intervening years. As the minimum tick size has dropped from $0.125 to $0.01 over this time period, it is possible that the lower tick size has enabled informed traders to be able to trade more frequently.

As Table 2 indicates some dispersion in the data, it is possible that this result is an artifact of aggregation of the data and not a general trend. To investigate this possibility, we segregate our sample into quartiles based on the number of trades for each firm (the first quartile has the most active stocks; the fourth quartile has the least active).[8] Table 3 and Figure 2 provide data on each quartile over the course of the day, indicating that this overall result is consistent over each quartile. In addition, Table 3 and Figure 2 indicate that as trading activity

[6]Interestingly, the probability of an informed trade for the middle 50% of the stocks (between the 25th and 75th percentiles) varies only from about 15% to about 27%. Thus, although different, there is not a great deal of variation across most stocks.

[7]EKOP examined 30 NYSE-listed stocks each from the first, fifth, and eighth NYSE volume deciles. They found the probability of an informed trade to be 16% for the most active stocks, 21% for the middle decile, and 22% for the eighth decile stocks.

[8]We also reanalyzed the data using market value of equity to segregate the firms into quartiles instead of number of trades. All results, including the intraday plots, are substantively similar.

Table 3. Intraday probability of informed trading by trading activity quartile.

	Interval	Whole sample	*Q*1	*Q*2	*Q*3	*Q*4
1	9:30–10:00	0.2060	0.1855	0.1929	0.2145	0.2310
		(0.91)	(0.99)	(0.56)	(0.39)	(2.24)*
2	10:00–10:30	0.1961	0.1769	0.1808	0.1980	0.2286
		(4.33)*	(0.77)	(3.45)*	(4.73)*	(3.41)*
3	10:30–11:00	0.2063	0.1753	0.1947	0.2134	0.2420
		(0.76)	(0.25)	(0.62)	(1.31)	(0.34)
4	11:00–11:30	0.2091	0.1742	0.2070	0.2186	0.2368
		(0.17)	(0.27)	(0.99)	(0.39)	(0.54)
5	11:30–12:00	0.2063	0.1761	0.2030	0.2158	0.2301
		(0.87)	(0.28)	(0.26)	(0.21)	(2.40)*
6	12:00–12:30	0.2121	0.1710	0.2079	0.2267	0.2427
		(1.26)	(0.58)	(0.09)	(3.32)*	(0.38)
7	12:30–1:00	0.2279	0.1919	0.2195	0.2413	0.2589
		(6.94)*	(1.39)	(3.41)*	(5.82)*	(4.27)*
8	1:00–1:30	0.2225	0.1841	0.2177	0.2349	0.2533
		(5.20)*	(0.41)	(3.20)*	(3.16)*	(3.89)*
9	1:30–2:00	0.2140	0.1732	0.2172	0.2204	0.2452
		(1.84)**	(0.81)	(2.13)*	(3.13)*	(0.35)
10	2:00–2:30	0.2080	0.1739	0.1950	0.2185	0.2446
		(0.22)	(0.47)	(0.83)	(0.09)	(0.76)
11	2:30–3:00	0.2111	0.1723	0.2006	0.2135	0.2579
		(0.77)	(0.62)	(0.32)	(0.37)	(2.98)*
12	3:00–3:30	0.1998	0.1660	0.1893	0.2060	0.2379
		(2.90)*	(0.78)	(2.10)*	(2.39)*	(1.55)
13	3:30–4:00	0.1974	0.1758	0.1768	0.2005	0.2363
		(3.86)*	(0.26)	(2.46)*	(6.03)*	(1.32)

*Statistically significant at the 5% level.
**Statistically significant at the 10% level.
This table contains the differences in means for each intervals of the trading day. This table tests to see if the mean probability of informed trading for interval 1 is different from the mean probability of informed trading for the rest of the day, etc. Quartiles are based on the number of trades (*Q*1 is the most active; *Q*4 is the least; T-statistics are in parantheses).

decreases, the probability of an informed trade increases. These results support and augment the results found in EKOP using volume.

Table 3 and Figure 2 also indicate that while the general pattern of the probability of informed trading over the course of the day is similar for the lowest three trading activity quartiles, the pattern is noticeably different for

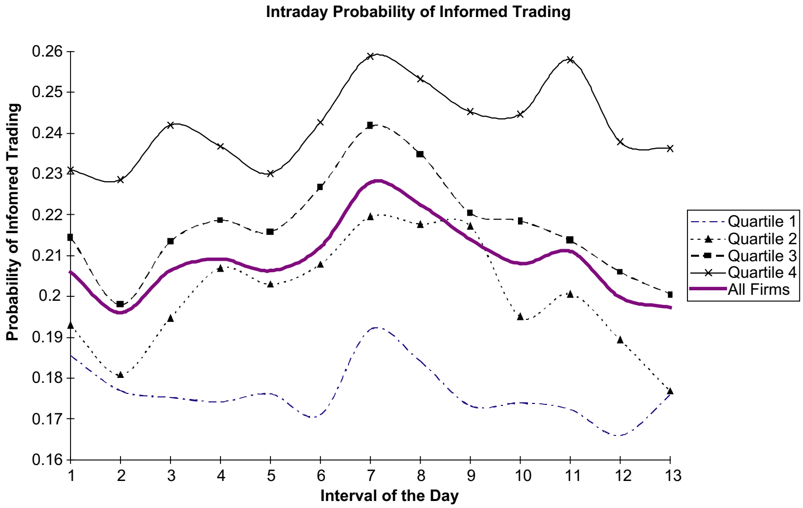

Figure 2. Probability of informed trading for all firms and quartiles. Quartiles are sorted by number of trades (quartile 1 is the most active, quartile 4 is the least active).

the most actively traded stocks. While all quartiles have the highest probability of informed trading during the 12:30 p.m. to 1:00 p.m. half-hour, the most actively traded stocks have an otherwise relatively constant probability of informed trades, with slight increases at the beginning and end of the day. Thus, similar to EKOP, we find that the less active stocks are more similar to each other than they are to the most active stocks.

Table 4 shows the correlations between the probability of informed trading in each interval. Overall, the probabilities of informed trading in one interval are moderately correlated with any other interval, with the correlations between intervals hovering around 0.45–0.50. These results indicate that if informed traders are active during an interval, there is a positive, but not guaranteed, probability that they will be active during other intervals during the same day.

An unresolved question is why the probability of informed trading increases so dramatically in the middle of the day. One possibility is that informed traders maintain a constant trading pattern throughout the day. Figure 3 indicates that this is not the case. In fact, informed traders arrive at about the same rate as uninformed traders overall. However, while both groups exhibit a U-shaped pattern, the U-shaped pattern is more pronounced for the uninformed traders than the informed traders. Thus, while the informed traders alter their trading patterns to match the arrival rates of uninformed traders, they do so incompletely.

Table 4. Intraday correlations.

Interval	Interval												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1.0000												
2	0.5122	1.0000											
3	0.5389	0.3795	1.0000										
4	0.5075	0.5090	0.4976	1.0000									
5	0.4825	0.4794	0.4840	0.6386	1.0000								
6	0.4085	0.3680	0.4606	0.5107	0.4359	1.0000							
7	0.4302	0.4275	0.4720	0.5598	0.5561	0.5175	1.0000						
8	0.3694	0.4008	0.4192	0.4954	0.4983	0.4621	0.5651	1.0000					
9	0.4167	0.4369	0.3992	0.5490	0.5236	0.4633	0.4937	0.4915	1.0000				
10	0.4016	0.4879	0.4151	0.4489	0.5172	0.4491	0.4666	0.4958	0.4713	1.0000			
11	0.4471	0.4331	0.3707	0.5219	0.5350	0.4101	0.4653	0.4714	0.4769	0.4947	1.0000		
12	0.3873	0.4375	0.4760	0.4957	0.5052	0.4327	0.4712	0.4650	0.4535	0.5349	0.5000	1.0000	
13	0.4125	0.3429	0.4393	0.4452	0.4529	0.4247	0.4415	0.4432	0.4689	0.4175	0.4545	0.4476	1.0000

All numbers are statistically different at the 1% level.
This table examines the correlations of the probability of informed metric of Easley, Kiefer, O'Hara, and Paperman (1996) for each of the 13 30-minute intervals of the trading day.

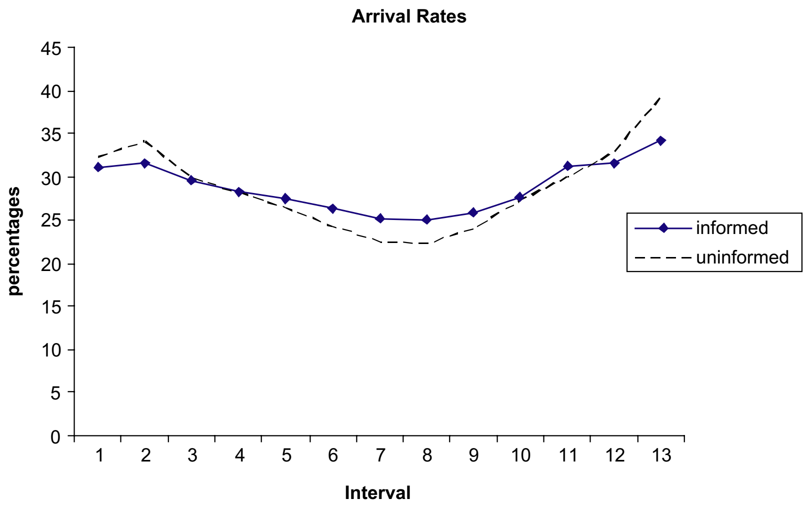

Figure 3. Arrival rates of informed and uninformed traders over the course of the day.

Although the probability of informed trading may increase in the middle of the day, it does not imply that costs immediately increase at this time. To estimate the costs of the increase in informed trading, we estimated the asymmetric information component of the bid-ask spread using the GKN (1991) model as modified for transactions data by Neal and Wheatley (1998). GKN's (1991) model allows expected returns to be serially dependent. This dependence is assumed to have the same impact on both transaction returns and quote midpoint returns. Under these conditions, the difference between the two returns filters out the serial dependence. The transaction return is

$$\mathrm{TR}_t = E_t + \pi(s_q/2)(Q_t - Q_{t-1}) + (1-\pi)(s_q/2)Q_t + U_t,$$

where E_t is the expected return from time $t-1$ to t, π and $(1-\pi)$ are the fractions of the spread due to order processing costs and adverse selection costs, respectively. s_q is the percentage bid-ask spread, assumed to be constant through time. Q_t is a $+1/-1$ buy–sell indicator and U_t captures public information innovations.

GKN assume the quote midpoint is measured immediately following the transaction at time t. As in Neal and Wheatley (1998), we will use an upper case T subscript to preserve the timing distinction for the quote midpoint. The midpoint return is

$$\mathrm{MR}_T = E_T + (1-\pi)(s_q/2)Q_T + U_T.$$

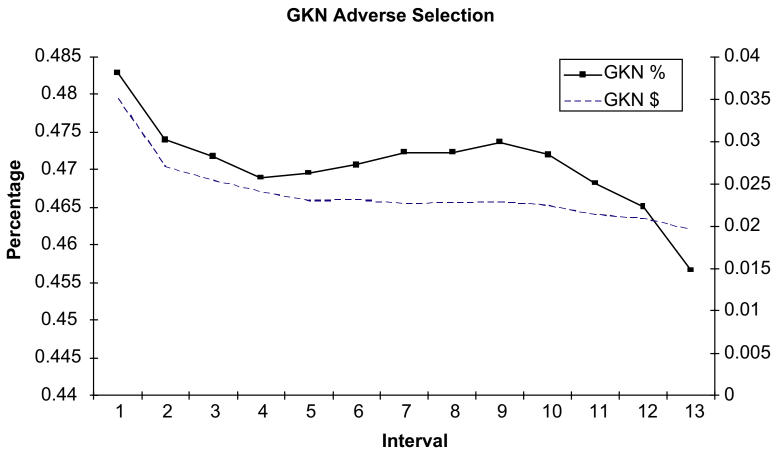

Figure 4. GKN adverse selection component throughout the day.

Subtracting the midpoint return from the transaction return and multiplying by two yields:

$$2\text{RD}_t = \pi s_q(Q_t - Q_{t-1}) + V_t,$$

where $V_t = 2(E_t - E_T) + 2(U_t - U_T)$.

Relaxing the assumption that s_q is constant and including an intercept yields:

$$2\text{RD}_t = \pi_0 + \pi_1 s_q(Q_t - Q_{t-1}) + V_t.$$

Figure 4 indicates that while the percentage of the bid-ask spread due to the asymmetric information increases in the middle of the day, the overall costs do not as the mean bid-ask spread decreases over the course of the day. Thus, although there is an increased probability of an informed trade in the middle of the day *conditional on the existence of a trade*, the overall costs do not increase as the probability of a trade is reduced.

5. Factors that Might Affect the Overall Probability Informed Trading

We can see from the previous section that the overall probability of informed trading is correlated both by stock specific factors such as the number of trades and by intertemporal factors, such as the time of day. Indeed, informed trading is likely to be affected by a variety of factors, such as the size of the bid-ask spread, trading activity, trade size, price, and risk. Other factors that might affect the probability of informed trading include intermarket order flow considerations that affect the types of orders sent to the NYSE. We look at each of these in turn:

5.1. *Spread*

Informed traders would like to trade more frequently as spreads decline as they need low trading costs in order to maximize profits.[9] On the other hand, an increase in the activities of informed traders might increase adverse selection faced by other market participants, potentially causing them to increase the bid-ask spread. Thus, the relation between bid-ask spread and the probability of informed trading is uncertain. Many researchers (see McInish and Wood, 1992; Brock and Kleidon, 1992; Lee, Mucklow, and Ready, 1993; Chan, Chung, and Johnson, 1995) have shown that NYSE spreads are smallest during the middle of the trading day. Thus, it is possible that more informed trading takes place during this time.

5.2. *Price*

Researchers show that there is an inverse relation between price and the bid-ask spread (see, for example, Benston and Hagerman, 1974). Given the aforementioned relation between spread and informed trading, it is possible that informed trading is related to price through the bid-ask spread.

5.3. *Trading activity, order flow, and regional exchanges*

As informed traders would like to hide among uninformed trades, more informed trading should result from higher levels of overall trading activity. However, trading activity in NYSE-listed stocks may also increase due to activity on the regional exchanges. One possibility is that trading on regional exchanges is associated with uninformed, retail trades. Another possibility, however, is that informed traders use the regional exchanges to hide some of their trading activity. Thus, it is uncertain whether the probability of informed trading is affected by an increase in regional trading activity. Most researchers agree that, relative to the regional exchanges, the NYSE captures

[9] For example, the previous section suggested that the probability of informed trading has increased slightly as bid-ask spreads have decreased over the past decade.

more informed order flow (see, Bessembinder and Kaufman, 1997; Chordia and Subrahmanyam, 1995; Easley *et al.*, 1996; Lin, Sanger, and Booth, 1995). Consequently, we believe that the probability of informed trading on the NYSE may be influenced by the level of activity on the regional exchanges.

5.4. *Trade size*

Informed traders face a dilemma when confronted with the question of optimal trade size. On one hand, informed traders are more likely to trade in larger trade sizes in order to maximize the value of their information and at the same time reduce transactions costs. On the other hand, informed traders make their trades easier to identify by choosing larger trade sizes than those chosen by uninformed traders.

5.5. *Risk*

The riskiness of a stock should affect risk-reward relationship for informed trading. We distinguish between two different types of risk: (1) the risk of dealing in one stock versus another and (2) differing risk experienced for a stock over time. Since we are investigating the probability of informed trading and speculate that informed traders are concerned about trading costs, we incorporate the standard deviation of spread as our measure of risk.

To examine how each of these variables affects the probability of an informed trade, we run the following regression:[10]

$$\begin{aligned} PI = \alpha &+ \beta_1 * \text{Spread} + \beta_2 * \ln(\text{NumTrade}) + \beta_3 * \ln(\text{TradeSize}) \\ &+ \beta_4 * \text{Nsize} + \beta_5 * \ln(\text{Price}) + \beta_6 * \text{Risk1} + \beta_7 * \text{Risk2} \\ &+ \beta * \text{Interval} + \varepsilon, \end{aligned}$$

where Spread is the average proportional spread for stock i during interval t, NumTrade is the number of trades for stock i during interval t, TradeSize is the average trade size for stock i during interval t, Nsize is the relative (normalized) trade size for stock i during interval t, Price is the average price for stock i during interval t, Risk1 is stock i's mean value of the SPREAD for over the 13 trading intervals, Risk2 is stock i's normalized mean value of the SPREAD for over the 13 trading intervals, and Interval is Dummies for each time interval.[11]

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[10] We replicate our analysis using average daily trading volume and find quantitatively similar results.

[11] Interval 10 is omitted in the regression analysis.

Note that, for robustness, we include two measures of trade size. The second measure is relative, or normalized, trade size, which also proxies information arrival (see Karpoff, 1987). We measure normalized trade size for stock i during interval t, $\text{NSIZE}_{i,t}$, by:

$$\text{NSIZE}_{i,t} = \frac{\text{Size}_{i,t} - X_i}{D_i},$$

where X_i is the mean trade size for security i across intervals and D_i is the standard deviation of $\text{SIZE}_{i,t}$.

In addition, we use more than one risk measure for completeness. We compute the standard deviation of the proportional spread for each interval. The first risk proxy for each security in the sample, RISK1_i, is stock i's mean value of the SPREAD over the 13 trading intervals. To obtain the second measure of risk, $\text{RISK2}_{i,t}$ we normalize each interval's standard deviation within a particular stock issue by subtracting the mean value for that issue and dividing by the standard deviation of the $\text{SPREAD}_{i,t}$.

Table 5 presents the regression results for the whole sample and for each of the quartiles. In this analysis, we only use NYSE data as we compute the

Table 5. Probability of informed trading regressions.

	Whole sample	Trading activity quartile			
		*Q*1	*Q*2	*Q*3	*Q*4
Intercept	0.5698	−0.0819	0.5620	0.7999	0.5177
	(26.35)*	(−0.65)*	(6.61)*	(10.36)*	(9.54)*
Spread	0.0740	−0.7211	0.1021	0.2523	−0.2068
	(1.05)	(−1.36)	(0.80)	(1.79)	(−2.02)*
Log (number of trades)	−0.0363	0.0290	−0.0241	−0.0554	−0.0410
	(−17.59)*	(2.03)	(−2.40)*	(−6.12)*	(−7.47)*
Log (trade size)	−0.0016	0.0050	−0.0146	−0.0085	0.0020
	(−0.70)	(0.50)	(−3.21)*	(−2.29)*	(0.61)
Nsize	−0.0015	−0.0219	0.0065	−0.0012	−0.0017
	(−0.93)	(−2.57)*	(1.85)	(−0.50)	(−0.79)
Log (price)	−0.0171	−0.0110	−0.0180	−0.0275	0.0045
	(−4.74)*	(−0.75)	(−2.58)*	(−3.63)*	(0.71)
Risk 1	−0.4208	2.9211	−5.5450	−2.3715	6.6763
	(−0.19)	(0.21)	(−0.98)	(−0.39)	(2.26)*
Risk 2	−0.0007	−0.0006	0.0007	−0.0034	0.0005
	(−0.53)	(−0.11)	(0.27)	(−1.69)	(0.28)

Table 5. (*Continued*)

	Whole sample	Trading activity quartile			
		$Q1$	$Q2$	$Q3$	$Q4$
Interval 1	0.0074 (1.20)	0.0744 (2.51)*	−0.0053 (−0.41)	0.0152 (1.62)	0.0012 (0.14)
Interval 2	−0.0015 (−0.34)	0.0339 (1.76)	−0.0089 (−0.99)	−0.0029 (−0.42)	−0.0040 (−0.60)
Interval 3	0.0033 (0.80)	0.0176 (1.06)	0.0021 (0.26)	0.0024 (0.39)	0.0017 (0.28)
Interval 4	0.0030 (0.75)	0.0072 (0.45)	0.0095 (1.22)	0.0044 (0.73)	−0.0027 (−0.44)
Interval 5	−0.0028 (−0.70)	0.0117 (0.74)	0.0044 (0.57)	−0.0010 (−0.16)	−0.0141 (−2.35)*
Interval 6	0.0002 (0.04)	0.0044 (0.27)	0.0015 (0.19)	0.0075 (1.24)	−0.0054 (−0.90)
Interval 7	0.0139 (3.45)*	0.0234 (1.46)	0.0184 (2.35)*	0.0150 (2.45)*	0.0082 (1.35)
Interval 8	0.0077 (1.91)	0.0113 (0.71)	0.0169 (2.14)*	0.0021 (0.35)	0.0055 (0.90)
Interval 9	0.0011 (0.28)	−0.0014 (−0.09)	0.0122 (1.57)	0.0048 (0.79)	−0.0073 (−1.21)
Interval 11	0.0071 (1.76)	−0.0029 (−0.18)	0.0052 (0.67)	0.0046 (0.76)	0.0139 (2.31)*
Interval 12	−0.0001 (−0.02)	−0.0036 (−0.22)	−0.0027 (−0.34)	0.0029 (0.46)	−0.0011 (−0.18)
Interval 13	0.0076 (1.67)	0.0197 (1.04)	−0.0023 (−0.24)	0.0037 (0.49)	0.0118 (1.71)
Adj. R^2	0.1164	0.0266	0.0272	0.0996	0.0491

*Statistically significant at the 5% level.
The dependent variable is the probability of informed (PIN) trading computed using the model of Easley, Kiefer, O'Hara, and Paperman (1996). We regress PIN against spread, activity (number of trades), trade size, price, our two measures of risk, and dummy variables for the intervals of the trading day (omitting interval 10). Quartiles are based on the number of trades ($Q1$ is the most active; $Q4$ is the least; T-statistics are in parantheses).

probability of informed trading using NYSE trades. We find that trading activity is the primary determinant of the probability of informed trading on an intraday basis. Interestingly, the risk variables are not significant in any regression, perhaps because the informed trader (being informed) is not subject to much risk. The regression results also confirm that interval 7 (the 12:30 to 1:00 half-hour) has an increased probability of informed trading, even after controlling for spread and trading activity. Results for the quartiles generally support the overall sample results, although the results for both the most and least active stocks indicate that interval 7 is not a significant determinant of trading.

Another possibility is that the type of order flow received by the NYSE will affect the probability of an informed trade. As noted above, trading on the regional exchanges might affect the uninformed order flow received by the NYSE. To investigate this hypothesis, we compute a measure of the regional exchanges' relative volume, $\text{REGIONAL}_{I,t}$, as the ratio of the average number of shares traded on the regional exchanges to the average number of shares traded on the NYSE for each stock i during interval t.

Table 6 presents the regression results including regional exchange trading activity as an explanatory factor. We find evidence of an inverse relationship between the probability of informed trading and the number of trades on the regional exchanges, implying that trading on the regional exchanges does, in fact, affect the probability of informed order flow on the NYSE.

Table 6. Probability of informed trading regressions with regional activity.

	Whole sample	Trading activity quartile			
		$Q1$	$Q2$	$Q3$	$Q4$
Intercept	0.4486	−0.2274	0.4097	0.6725	0.4854
	(18.62)*	(−1.37)	(4.45)*	(8.86)*	(9.89)*
Regional	−0.0046	−0.0069	−0.0084	−0.0040	−0.0075
	(−3.22)*	(−0.91)	(−2.54)*	(−1.68)	(−4.07)*
Spread	0.0539	−0.3255	0.0015	−0.0030	−0.1084
	(0.77)	(−0.56)	(0.01)	(−0.02)	(−1.16)
Log (number of trades)	−0.0291	0.0400	−0.0155	−0.0526	−0.0311
	(−9.35)*	(2.08)*	(−1.41)	(−5.50)*	(−5.18)*
Log (trade size)	0.0039	0.0152	−0.0042	−0.0043	0.0037
	(1.67)	(1.25)	(−0.77)	(−1.11)	(1.24)
Nsize	−0.0022	−0.0223	0.0053	−0.0019	−0.0017
	(−1.40)	(−2.62)*	(1.50)	(−0.79)	(−0.84)
Log (price)	−0.0003	−0.0006	−0.0002	−0.0002	−0.0001
	(−4.33)*	(−1.64)	(−1.14)	(−1.44)	(−0.71)
Risk 1	4.3488	−3.0021	2.1984	14.6359	6.009
	(2.40)*	(−0.24)	(0.42)	(3.46)*	(2.84)
Risk 2	−0.0006	−0.0013	0.0012	−0.0026	0.0004
	(−0.42)	(−0.23)	(0.43)	(−1.30)	(0.22)

Table 6. (*Continued*)

	Whole sample	Trading activity quartile			
		$Q1$	$Q2$	$Q3$	$Q4$
Interval 1	0.0102	0.0683	0.0003	0.0232	0.0048
	(1.64)	(2.32)*	(0.03)	(2.44)*	(0.56)
Interval 2	−0.0017	0.0298	−0.0082	−0.0004	−0.0039
	(−0.37)	(1.55)	(−0.91)	(−0.05)	(−0.60)
Interval 3	0.0036	0.0155	0.0029	0.0045	0.0022
	(0.87)	(0.93)	(0.36)	(0.73)	(0.35)
Interval 4	0.0035	0.0064	0.0104	0.0059	−0.0020
	(0.84)	(0.40)	(1.34)	(0.97)	(−0.32)
Interval 5	−0.0024	0.0121	0.0049	−0.0003	−0.0137
	(−0.60)	(0.76)	(0.64)	(−0.06)	(−2.28)*
Interval 6	0.0010	0.0050	0.0023	0.0083	−0.0045
	(0.24)	(0.32)	(0.30)	(1.38)	(−0.74)
Interval 7	0.0145	0.0246	0.0186	0.0151	0.0086
	(3.59)*	(1.53)	(2.37)*	(2.47)*	(1.43)
Interval 8	0.0086	0.0126	0.0175	0.0026	0.0064
	(2.12)*	(0.79)	(2.22)*	(0.42)	(1.06)
Interval 9	0.0015	−0.0008	0.0122	0.020	−0.0071
	(0.37)	(−0.05)	(1.57)	(0.82)	(−1.18)
Interval 11	0.0066	−0.0031	0.0046	0.0040	0.0132
	(1.63)	(−0.19)	(0.59)	(0.67)	(2.21)*
Interval 12	−0.0010	−0.0043	−0.0039	0.0023	−0.0018
	(−0.25)	(−0.27)	(−0.48)	(0.36)	(−0.29)
Interval 13	0.0053	0.0170	−0.0053	0.0023	0.0098
	(1.15)	(0.87)	(−0.53)	(0.29)	(1.42)
Adj. R^2	0.1167	0.0287	0.0271	0.0955	0.0543

*Statistically significant at the 5% level.
The dependent variable is the probability of informed (PIN) trading computed using the model of Easley, Kiefer, O'Hara, and Paperman (1996). PIN is regressed against regional trading activity (regional), spread, activity (number of trades), trade size, price, two measures of risk, and dummy variables for the intervals of the trading day. Quartiles are based on the number of trades (Q1 is the most active; Q4 is the least; T-statistics are in parantheses).

6. Conclusion

Using the methodology in EKOP, we analyze the probability of informed trading on an intraday basis. Unlike previous studies investigating intraday spreads and volume, we find a crude inverted U-shaped pattern. These results are robust to levels of trading activity, although more actively traded stocks show less of this pattern than more infrequently traded stocks. While our results support those found by EKOP, more recent data indicates that the probability of an informed trade has increased slightly over time, perhaps because of the significant decrease in the bid-ask spread during the past decade.

Overall, we find that the time of day remains a significant determinant of the probability of informed trades even after controlling for trading activity and bid-ask spread. Interestingly, trading on the regional exchanges changes the probability of informed trading on the NYSE.

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Chapter 8

Leases, Seats, and Spreads: The Determinants of the Returns to Leasing a NYSE Seat

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We study the returns to leasing a New York Stock Exchange (NYSE) seat during 1995–2005 and find that these returns are a weighted average of past leasing returns and a set of fundamental factors such as average NYSE quoted spreads, the dollar value of NYSE trading volume, and the return on the overall stock market. Our partial adjustment model explains 70–80% of the variation in leasing returns and 80–85% of this explanatory power is attributable to a simple AR(1) process. Quoted spreads, trading volume, and stock market returns are all significant factors that positively affect leasing returns, albeit to a lesser extent than past returns to leasing. In addition, NYSE seat lessors rely more heavily on past values of these fundamental factors rather than coincident or forward-looking values of spreads, volume, returns, etc. This is in contrast to previous results on exchange seat prices which find that only unexpected changes in fundamental factors such as those noted above have a significant impact on exchange seat prices. In addition, unlike previous research on seat prices, which report that these prices follow a random walk, we do not find that leasing returns behave in this manner.

Keywords: Empirical; leasing; market microstructure; stock exchanges.

1. Introduction

Given recent headlines in the popular press, the economic value of a seat on a securities exchange, along with what determines this value, have become important issues here in the US, as well as in Europe and Asia where stock markets are rapidly consolidating.[1] The need to derive an accurate market value of an exchange seat is becoming increasingly critical to exchange members and regulators as more exchanges merge and/or become for-profit, publicly traded entities via demutualization.

Prior research by Schwert (1977), Chiang, Gay, and Kolb (1987), and Keim and Madhavan (2000) has focused on the value of seat prices on stock and commodities exchanges. However, as these and other studies have noted, exchange

[1] For example, one of the debates surrounding the proposed reverse merger of Archipelago with the New York Stock Exchange (NYSE) is whether or not the implied value of $1.76 million for a NYSE seat is a fair one for exchange members (for more details, see the April 21, 2005 article in *The Wall Street Journal* titled, "NYSE to Acquire Electronic Trader and Go Public", p. A1).

seats trade fairly infrequently (e.g., 1–2 times a month at most) and thus suffer from nonsynchronous trading effects first noted in Fisher (1966).[2] Our contribution to the literature on exchange seat valuation incorporates data on lease rates of stock exchange seats and the returns they generate rather than focusing on the prices of infrequently traded seats. Using a unique data set of annual lease payments for NYSE seats, we are the first to examine the time series behavior of the returns associated with leasing a NYSE seat. This is an important innovation because, since 1978 when leasing became permissible, NYSE seats are typically leased far more frequently (e.g., once a week, on average) than they are sold.[3] Thus, our data are less likely to suffer from the nonsynchronous trading effects noted above.[4]

In addition, by focusing on the returns to leasing a NYSE seat, we can gain another perspective on what determines the value of an exchange seat. For example, the returns to leasing a seat should be, in equilibrium, equal to the returns to owning a seat (less any capital gain or loss from selling the seat). We therefore build upon the valuation model for an exchange seat first proposed by Schwert (1977) in order to identify which factors determine the returns to leasing a NYSE seat.

We find that a partial adjustment model similar to the one first explored by Lintner (1956) explains 70–80% of the variation in leasing returns during 1995–2005.[5] Our model indicates that leasing returns are a weighted average of past leasing returns and a set of fundamental factors such as average quoted spreads, the dollar value of trading volume, and the return on the overall stock market. Interestingly, we find that NYSE leasing returns are highly

[2]The nonsynchronous trading patterns of common stocks were also initially explored in detail from a market microstructure perspective in Schwartz and Whitcomb (1977a, b), thus spawning a new branch of financial research.

[3]As noted in Keim and Madhavan (2000), leasing of NYSE seats was not permitted prior to 1978 but is now a common practice at the exchange.

[4]As a robustness check, we use Schwert's (1990) time series adjustment for the return data of infrequently traded securities and obtain results that are similar to those reported here based on unadjusted leasing returns. Thus, we can infer from this result that nonsynchronous trading effects are less problematic for our leasing data. To conserve space, we focus on the results based on unadjusted leasing returns.

[5]Although seat leasing on the NYSE began in 1978, we were able to hand-collect data from the *NYSE Weekly Bulletins* starting in 1995 because, prior to 1995, the data were recorded only on paper cards and are no longer maintained by the NYSE. Inquiries to NYSE officials and visits to the NYSE archives confirmed that data for 1978–1994 are not available. Consequently, we focus our analysis on the monthly data we were able to obtain during 1995–2005.

autoregressive with 80–85% of the model's explanatory power attributable to a simple AR(1) process.

Quoted spreads, trading volume, and stock market returns are all significant factors that positively affect leasing returns, albeit to a lesser extent than past leasing returns. In addition, NYSE seat lessors rely more on past values of these fundamental factors rather than coincident or forward-looking values of spreads, volume, returns, etc. This is in contrast to previous results on seat prices by Schwert (1977) and Chiang *et al.* (1987) which both find that only unexpected changes in fundamental factors such as those noted above have a significant impact on exchange seat prices. In addition, unlike Schwert (1977) and Chiang *et al.* (1987), who both report seat prices follow a random walk, we do not find that leasing returns behave in this manner. Our results are consistent with the notion that NYSE seat lessors form their expectations of leasing returns in a rational, adaptive fashion in response to past movements in key fundamental factors and place relatively little weight on current or near-future fluctuations in these factors.

Our results are robust to several possible alternative specifications including different autoregressive processes, possible nonlinear and interactive relations between the independent variables, the omission of outliers, alternative transformations of the fundamental factors, and two different forms of the dependent variable (based on new leases and renewals of prior leases). Thus, the partial adjustment model of NYSE leasing returns provides reliable estimates of the factors determining lease rates and sheds new light on the economic value of a seat on a securities exchange.

2. Relevant Literature

As noted in Section 1, prior research on the seats of a securities exchange has focused on the pricing, trading, and returns of the seats rather than returns on leasing exchange seats. Given that the returns related to owning a seat will, in equilibrium, be equal to the returns on leasing a seat, less any capital gain or loss, a brief review of the relevant literature on seat prices is warranted. For example, Schwert (1977) posits a valuation model for NYSE and American Stock Exchange (ASE) seats based on trading volume and stock return data during 1926–1972. Schwert (1977) finds that returns on these seats follow a random walk and that only unexpected changes in the prices of listed stocks and the volume of shares traded affect these seat returns. The author also observes a significant nonsynchronous trading, or "Fisher (1966) effect," in terms of the infrequent trading of exchange seats relative to the trading of the securities listed on an exchange.

Keim and Madhavan (2000) build on Schwert's early valuation model by examining how the trading of NYSE seats might affect the returns of the broader stock market. The authors find no significant relation between the returns on NYSE seats and the returns on the S&P 500 index. However, they find that the number of NYSE seat trades during a month (particularly if the number is unexpectedly high) can have a significantly negative effect on monthly returns to the S&P 500. Keim and Madhavan suggest that this inverse relation between trading in NYSE seats and stock returns is due to the seat trading activity's role as a proxy for liquidity at the NYSE and a manifestation of overall market sentiment.

Chiang *et al.* (1987) extend Schwert's initial empirical analysis to study the pricing behavior of the seats of three commodity exchanges. They report results similar to Schwert's finding that commodity seats follow a random walk and that unexpected changes in trading volume can have a significant effect on commodity seat returns. In addition, they find that, due to the possible diversification benefits of seat ownership, commodity seats are "less risky than the commodity market as a whole but more risky than a typical individual commodity."

Overall, the empirical results noted above suggest seat prices are affected by exchange-specific factors such as trading volume and market-wide conditions such as the overall price level of the stock market, as well as the broad market's returns and riskiness. Keim and Madhavan's analysis also suggests that seat trading can serve as a proxy for an exchange's liquidity as well as an indicator of stock market sentiment. In contrast, our focus on the returns to leasing a securities exchange seat provides an alternative way of studying the value of a seat on the NYSE.

3. An Empirical Model of Returns on Leasing NYSE Seats

As noted in the previous sections, the return on leasing a seat on a securities exchange has not been studied (primarily due to the lack of lease rate data). Schwert (1977) and Keim and Madhavan (2000) both present valuation models for exchange seats that are relevant to our task because, in equilibrium, the return on leasing a seat, lr_t, should be equal to the return on owning a seat, less any expected capital gain or loss on the future sale of the seat. Both

Schwert (1977) and Keim and Madhavan (2000) set up a simple valuation model for seat ownership that is based on discounting expected profits from trading securities on the exchange at an appropriate discount rate.

Our model draws most closely from Schwert's (1977) derivation and Lintner's (1956) partial adjustment model to form an empirically testable model of NYSE seat leasing returns. That is, we specify below a Schwert-type model of the profits from leasing a seat and then use Lintner's notion that seat lessors might have a "target" leasing return which they strive to obtain in an adaptive fashion.[6] The net result of this approach is that returns on leasing a NYSE seat can be described as a weighted average of an autoregressive process and a set of fundamental factors that have been shown in McInish and Wood (1992), among others, to affect spreads and trading volume.

Drawing on Schwert's (1977) profit function for an exchange seat owner, we start with a definition of the single-period return on leasing a NYSE seat as follows:

$$lr_t^* = \%\ \text{Spread}_t \cdot \text{Volume}_t - TC(\text{Volume}_t), \tag{1}$$

where lr_t^* is the "target" net return on leasing a NYSE seat during month-t, % Spread_t is the volume-weighted quoted spread expressed as a percentage of the average stock price traded during month-t, Volume_t is the dollar value of trading volume during month-t, and $TC(\text{Volume}_t)$ is the total cost function of leasing a seat and trading on the exchange during month-t.

The first term on the right-hand side of Equation (1) represents the total revenue earned by a seat owner with total dollar trading volume, Volume_t, defined as the product of the volume-weighted average stock price and the total number of shares traded during the period. In turn, the total number of shares traded can be influenced by factors such as the return and volatility of the overall market (denoted as R_{mt} and SD_{mt}, respectively) as well as the average level of stock prices, Average P_t (e.g., higher stock prices can signify a bull market and attract a larger set of investors). Thus, we specify Volume_t as a function of these factors:

$$\begin{aligned} \text{Volume}_t &= \text{Average } P_t \cdot \text{Total Shares Traded}_t \\ &= \text{Average } P_t \cdot f(R_{mt}, SD_{mt}, \text{Average } P_t). \end{aligned} \tag{2}$$

[6]Our model allows for lessees to form rational expectations with no adaptive adjustment because the rational expectation version of the model is simply a more restricted form of the general model presented here.

Substituting Equation (2) into Equation (1) and using the fact that the total cost function is itself a function of total dollar volume, we obtain our leasing return equation as a function of five fundamental variables:

$$lr_t^* = g(\$\text{ Spread}_t, R_{mt}, SD_{mt}, \text{Average } P_t, \text{Volume}_t), \tag{3}$$

where \$ Spread_t is the quoted spread (in dollars) = % $\text{Spread}_t \cdot \text{Average } P_t$ during month-t.

Equation (3) indicates that the returns to leasing a NYSE seat should be a function of five fundamental variables that are related to NYSE-specific trading activity (\$ Spread_t and Volume_t) and general market conditions (R_{mt}, SD_{mt}, and Average P_t) at the time the lease contract is signed.

However, as Lintner (1956) first noted in the context of dividend policy, financial decision-makers typically have a "target" rate in mind which they strive to attain in an adaptive manner when making financial projections. This concept can be used here to examine whether or not NYSE seat lessors form their expectations adaptively or immediately upon receiving new information. In addition, the model presented below enables us to test if seat lessors base their lease rates on past, future, and/or coincident realizations of the fundamental factors identified above in Equation (3).

A simple Lintner-type of model of leasing returns can be expressed as follows:

$$\Delta lr_t = lr_t - lr_{t-1} = a + c(lr_t^* - lr_{t-1}) + u_t, \tag{4}$$

where lr_t^* is the targeted level of return on leasing a NYSE seat, a and c are the parameter estimates measuring the drift and speed of adjustment of the model, respectively, and u_t is an error term with zero mean.

Equation (4) posits that changes in leasing returns adjust toward the target leasing return, lr_t^*, at a rate equal to the parameter, c.

Substituting Equation (3) into Equation (4) yields our full empirical specification of leasing NYSE seat returns:

$$\begin{aligned} lr_t = a + c\{&g(\$\text{ Spread}_t, R_{mt}, SD_{mt}, \text{Average } P_t, \text{Volume}_t)\} \\ &+ (1-c)lr_{t-1} + u_t. \end{aligned} \tag{5}$$

Note that c is a parameter that identifies the weight of the fundamental factors in determining the NYSE seat lessor's return whereas $(1 - c)$ represents the relative weight of past lease returns. As noted earlier, Equation (5) demonstrates that leasing returns are expected to be a weighted average of fundamental factors and past lease returns.[7] Also, note that if leasing returns follow a pure random walk (without drift), then we expect the parameter, c, and the intercept, a, to both equal 0. This empirical model provides us with a convenient way of testing how NYSE seat lessors actually determine the lease rates they charge seat lessees.

We can test Equation (5) using a time series of monthly observations for leasing returns and the relevant fundamental factors. All of the fundamental factors, except for the market riskiness variable (SD_{mt}), are expected to be positively related to the observed monthly leasing return, lr_t, because they are expected to raise the cash flows associated with controlling a NYSE seat (via higher spreads and/or more trading volume). Further, we can determine whether leading, lagging, or coincident fundamental factors best explain the variation in leasing returns.

4. Sample

We use monthly lease rate data on NYSE seats (expressed on an annual basis in dollars) for the period of August, 1995 to March, 2005. These data were hand-collected from the month-end issues of the *NYSE Weekly Bulletin*, thus yielding 116 observations. To compute the returns from these annual lease payments, we divide the lease payment by the NYSE seat price from the prior month (to avoid the possible timing problem of having a lease payment matched with a seat price that occurred after the lease agreement). We then de-annualize these data into monthly return estimates in order to reflect the reality that an entire year's worth of lease payments are not earned in the first month of the lease. This transformation also ensures that the leasing return data are consistent with the monthly data for the fundamental factors used in our analysis.

[7]Equation (5) also enables us to test whether or not the unexpected components of these five fundamental factors solely affect leasing returns. For example, we obtained the unexpected components of these factors by using the residuals from a set of AR(1) models estimated for each of the five variables. These five sets of residuals were then used to re-estimate Equation (5). The results of this test (not reported here to conserve space) indicate that these unexpected components are all insignificant determinants of leasing returns. Thus, in contrast to prior work by Schwert (1977) and Chiang *et al.* (1987) on seat prices, the returns to leasing a NYSE seat are driven mostly by readily observed fundamental factors and not by the unexpected component of these factors.

The five fundamental factors in Equation (5) were obtained from the NYSE's *Factbook* and *Statistics Archive*.[8] Quoted spreads ($ Spread) are monthly averages based on the share volume-weighted spreads, in dollars, of all NYSE common stocks that traded during a particular month. The dollar volume of trading (Volume) represents the total dollar volume of shares traded on the NYSE during a month (expressed in billions of dollars). The average stock price level (Average P) indicates the average price per share (in dollars) of all NYSE common stocks that traded during the month. The monthly return on the overall stock market (R_m) is computed using the NYSE Composite Index (a float-adjusted, market capitalization-weighted index of all NYSE common stocks). The standard deviation of the daily returns on the NYSE Composite Index for a given month is used as a proxy for the overall volatility of the stock market (SD_m).

5. Empirical Results

5.1. *Summary statistics*

Table 1 reports summary statistics for the variables used in our analysis. The first two rows of Table 1 report the mean annualized returns on new leases and renewed leases at 13.5 and 12.9%, respectively. Returns to leases that were renewed are lower and less volatile than new leases because renewal lease rates typically reflect lower adverse selection costs (as the lessee has now established a reliable payment record on the prior lease). As expected, the dollar volume of monthly trading is highly volatile with a standard deviation of $237.6 billion. The average volume-weighted spread was $0.112 and also varied considerably during 1995–2005 with the minimum of $0.03 per share occurring in 2003–2005 after the 2000–2001 decimalization initiative and the maximum of $0.19 occurring early in the sample period (e.g., during July 1996).[9] The median monthly return on the NYSE Composite Index of 1.04% is essentially the same level as the median monthly return of 1.05% on new NYSE seat leases. However, the volatilities of R_m and lr are substantially different with the NYSE Composite Index exhibiting much greater monthly fluctuations (with a standard deviation of 4.10% for the NYSE Composite versus 0.22% for new leases).

[8]The assistance of NYSE personnel, Bill Tschirhart, Steve Wheeler, Steve Fuller, and Francoise Baron, in directing us to the relevant NYSE data sources is greatly appreciated.

[9]Decimalization refers to the process during 2000–2001 where the NYSE gradually phased in the trading of stocks in penny increments (rather than in increments of eighths or sixteenths of a dollar). This initiative led to immediate reductions in bid-ask spreads for most NYSE stocks.

Table 1. Summary statistics.

Description	Variable	*N*	Mean	Median	SD	Min	Max
Yearly return on new leases	*annual lr-new*	116	0.1347	0.1338	0.0293	0.0500	0.2121
Yearly return on renewal leases	*annual lr-renewal*	116	0.1289	0.1274	0.0268	0.0500	0.2036
NYSE dollar trading volume ($ bil.)	*Volume*	116	716.24	773.85	237.58	267.00	1172.50
NYSE share trading volume (mil.)	*shvol*	116	20,443.1	20,602.6	8776.0	7055.7	41,498.6
Average price on NYSE ($/Share)	*Average P*	116	37.0042	38.1500	6.4596	24.5000	47.5000
Market return	R_m	116	0.0084	0.0104	0.0410	−0.1498	0.0929
SD of market return	SD_m	116	0.0093	0.0087	0.0039	0.0026	0.0211
Volume weighted bid/ask spread	$ *Spread*	116	0.1117	0.1400	0.0585	0.0300	0.1900
Sale price of NYSE seat ($)	*nyseseat*	116	1,692,638	1,587,500	468,424	950,000	2,650,000
Monthly return on new leases	*lr-new*	116	0.0106	0.0105	0.0022	0.0041	0.0162
Monthly return on renewal leases	*lr-renewal*	116	0.0101	0.0100	0.0020	0.0041	0.0156

Overall, the summary statistics indicate there is substantial variability over time in terms of spreads, trading volume, lease rates, and market returns.

5.2. *Historical trends in leasing returns and seat prices*

Figures 1 and 2 illustrate the time series behavior of the returns to leasing and owning NYSE seats during 1995–2005. Figure 1 shows NYSE seat prices on the left axis and annual lease rates on the right axis (both in dollars). As can be seen in this figure, seat prices and lease rates track each other quite closely with a +0.83 correlation between these two time series. However, the lack of a perfect correlation between seat prices and lease rates implies that the return on leasing a NYSE seat varies over time. In fact, Figure 2 displays this variation in leasing returns by plotting the time series of monthly returns associated with new and renewed leases. Given the similarity in the summary statistics for new and renewal leasing returns reported in Table 1, it is not surprising to see that these two sets of returns move in tandem (with a +0.95 correlation between them). Taken together, Figures 1 and 2 show substantial variation in seat prices,

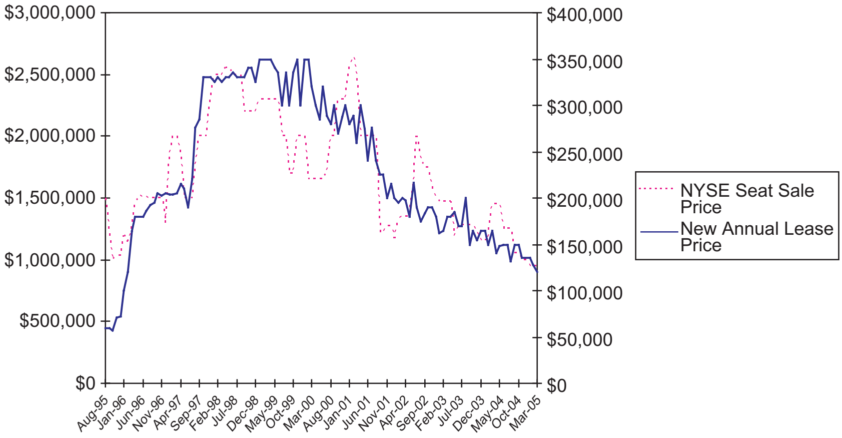

Figure 1. NYSE seat prices and lease rates (1995–2005).

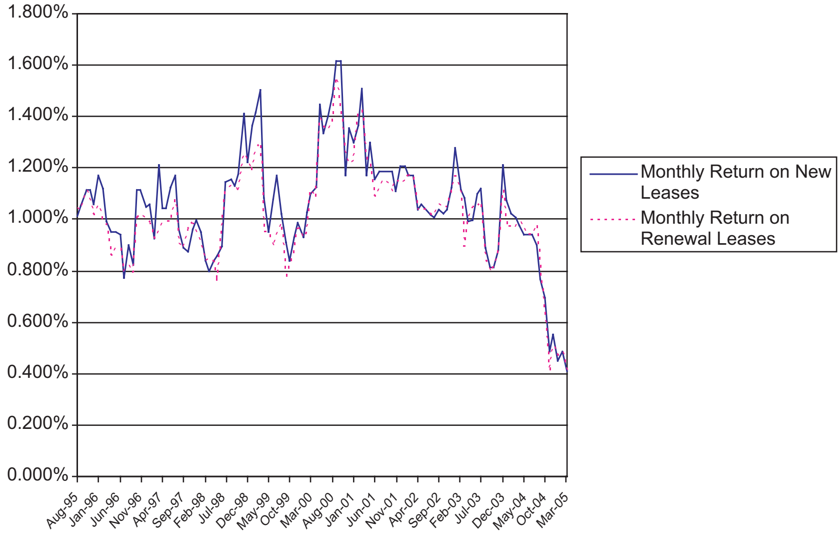

Figure 2. Monthly NYSE lease returns.

lease rates, and leasing returns over the sample period with leasing returns peaking in September 2000 and ultimately reaching their nadir in March 2005.[10]

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[10]Due to the unusually low leasing returns observed during the first three months of 2005, we checked the robustness of our results by omitting these returns and re-estimating our model. Our results are not materially different when the 2005 returns are dropped from the sample and thus we focus our discussion here on the results based on the full sample.

5.3. *Empirical tests of the partial adjustment model of NYSE seat leasing returns*

Table 2 presents some multivariate results based on our "full model" as described by Equation (5). The first column of results are based on the return on new leases and fundamental factors that are lagged one month while the second column reports the results of our empirical model based on the return on renewed leases and lagged fundamental variables. The third and fourth sets of columns report our results based on these same two dependent variables but include fundamental variables that *lead* the leasing returns by one month. For all four sets of results, an AR(1) model, along with the five fundamental factors shown in Equation (5), are estimated via the maximum likelihood method.[11]

Comparing the first two columns of Table 2 with the last two columns, we can see that the models based on lagged fundamental factors display greater statistical significance in terms of parameter estimates and explanatory power. In fact, although the AR(1) parameter is highly significant in all four specifications, only the results in the first two columns show any statistically significant parameter estimates for the fundamental factors. The lagged values of market return, R_m, and volume weighted average dollar spread, \$ Spread, are significant for both new and renewal leases (and the lagged dollar value of trading volume is also significant for new leases). In contrast, the leading values of these same fundamental factors exhibit no statistically significant relations with either new or renewal leases. In addition, the average NYSE stock price, Average P, and the volatility of the NYSE Composite Index, SD_m, are not significant determinants of either type of leasing return for any of the four sets of results.

Our results suggest NYSE seat lessors base their lease pricing decisions on past values of three fundamental factors (spreads, market returns, and trading volume) as well as past leasing returns themselves. A forward-looking model does not perform as well but this does not necessarily mean the market for seat leases is inefficient. As can be seen by the near-ideal levels of the Durbin–Watson statistic for the lagged versions of the full model (i.e., 2.10 and 1.93),

[11]Equation (5) was also estimated without an AR(1) component and exhibited substantial first-order serial correlation. By including the AR(1) component, the effects of first- and all higher-order serial correlations are essentially removed from the leasing returns time series. Higher-order autoregressive models such as AR(2) and AR(3) do not improve upon our AR(1) results. Thus, we focus on the AR(1) model's results.

Table 2. Full model of monthly return on leases.

	Variable	Lagged explanatory variables		Leading explanatory variables	
		New leases	Renewal leases	New leases	Renewal leases
	Intercept	0.006116**	0.006548***	0.009062***	0.0108***
		2.30	2.72	3.23	4.28
NYSE dollar trading volume	*Volume*	2.157E−6**	8.89E−7	9.195E−7	−3.904E−9
		2.01	1.01	0.78	0.00
Average price on NYSE	*Average P*	0.0000246	0.0000192	0.0000327	−0.000010
		0.33	0.31	0.41	−0.16
Market return	R_m	0.006857***	0.004371***	−0.000831	0.000326
		3.28	2.62	−0.36	0.18
SD of market return	SD_m	−0.0176	0.0107	−0.008237	−0.000276
		−0.50	0.38	−0.21	−0.01
Volume weighted spread	$ *Spread*	0.0164*	0.0156*	−0.004141	−0.005077
		1.68	1.79	−0.40	−0.55
Autoregressive parameter	AR*(1)*	0.854***	0.899***	0.8389***	0.8950***
		14.92	18.11	13.81	17.74
	R^2	0.7377	0.7935	0.6420	0.7319
	Durbin–Watson	2.10	1.93	2.12	1.92
	No. observ.	115	115	113	113

***Significant at the 0.01 level.
**Significant at the 0.05 level.
*Significant at the 0.10 level.

there is no first-order serial correlation evident in our model and this suggests that decision-makers are not making systematic errors when determining lease rates based on past data. Thus, this finding indicates lessors form their decisions rationally using relevant, recent market- and exchange-related factors.

The relatively high AR(1) parameter estimates (e.g., +0.854 and +0.899 in the first two columns) indicate that most of the model's weight is placed on past leasing returns and the fundamental factors carry a weight of only 0.10–0.15. In other words, our results suggest that leasing returns are highly autoregressive with 80–85% of the model's explanatory power attributable to a simple AR(1) process. However, since the AR(1) parameter is significantly different than 1.0, we cannot state that leasing returns follow a random walk model (with or without drift). The relatively large weight on the AR(1) parameter, the high explanatory power of the lagged model, and the lack of serial correlation suggests that the partial adjustment model specified by Equation (5) can provide rationally formed, unbiased NYSE seat leasing return estimates.

Table 3 reports the results for the "reduced form" of Equation (5) where Average P and SD_m have been omitted from the specification due to their overall lack of significance. The model's results are based on the lagged fundamental factors that were shown to be significant determinants of leasing returns in the

Table 3. Reduced form model of monthly return on leases.

	Variable	Lagged explanatory variables	
		New leases	Renewal leases
	Intercept	0.00694***	0.007119***
		5.00	5.43
NYSE dollar trading volume	*Volume*	2.0319E−6**	1.02E−6
		2.03	1.24
Market return	R_m	0.006969***	0.004244**
		3.40	2.60
Volume weighted bid/ask spread	$ *Spread*	0.0166*	0.0169**
		1.96	2.08
Autoregressive factor	AR*(1)*	0.8468***	0.8978***
		14.99	18.67
	R^2	0.7365	0.7931
	Durbin–Watson	2.11	1.94
	No. observ.	115	115

***Significant at the 0.01 level.
**Significant at the 0.05 level.
*Significant at the 0.10 level.

full model (i.e., R_m, \$ Spread, and Volume). Similar to the results of Table 2, the reduced form model shows parameter estimates that are theoretically correct (all three fundamental variables are positively related to leasing returns) and statistically significant (with one exception being the relatively low t-statistic for Volume when renewal leasing returns are used as the dependent variable).

Table 3 provides a more parsimonious version of our model that retains essentially the same levels of statistical significance and explanatory power as the full model described by Equation (5). In sum, NYSE seat leasing returns are positively related to past realizations of the leasing returns themselves, as well as past values of dollar trading volume, the return on the overall market, and the volume-weighted average spread. These findings are also consistent with the notion that NYSE seat lessors establish their lease rates according to a Lintner-type partial adjustment model rather than a forward-looking expectational model.[12]

6. Conclusion

This paper is the first to examine the time series behavior of the returns associated with leasing NYSE seats. Using monthly leasing data from 1995–2005, we develop and test a partial adjustment model in the spirit of Lintner's (1956) empirical specification of dividend policy. We find that leasing returns are a weighted average of past leasing returns and a set of fundamental factors such as the average quoted spreads on the NYSE, the dollar value of NYSE trading volume, and the return on the overall stock market. We also find that leasing returns are highly autoregressive with 80–85% of the model's explanatory power attributable to a basic AR(1) process.

Although past leasing returns are a key determinant of future leasing returns, quoted spreads, trading volume, and stock market returns are all significant

[12]For example, as noted earlier, Schwert-type (1977) unexpected components of Equation (5)'s fundamental factors do not have a statistically significant relationship with our dependent variables. In addition, we do not find that adding quadratic or interaction terms based on the fundamental factors provide any significant improvement over our basic model. Thus, there does not appear to be a nonlinear relation between the fundamental factors and NYSE seat leasing returns. Last, we do not find that leasing returns are significant determinants of excess stock returns and thus we can conclude that leasing returns do not act as meaningful proxies for "market sentiment" or some other asset pricing "factor." Thus, it seems leasing returns react to, rather than anticipate, trends in spreads, market returns, trading volume, and the leasing returns themselves.

factors that positively affect leasing returns. NYSE seat lessors also rely more on past values of these fundamental factors rather than coincident or forward-looking values of spreads, volume, and returns. In addition, unlike previous research which reports that seat prices follow a random walk, we do not find that leasing returns behave in this manner. Our results are consistent with the notion that NYSE lessors form their expectations of leasing returns in a rational, adaptive fashion in response to past movements in key fundamental factors and place relatively little weight on current or near-future fluctuations in these factors.

Acknowledgments

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Chapter 9

Decimalization and Market Quality

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We study the impact of decimalization on the New York Stock Exchange (NYSE). Empirical results indicate that spreads decrease significantly after decimalization, but the market depth and average volume per trade also decline significantly. It is likely that more front-runners are entering the market. We test the degree of front-running surrounding decimalization directly and confirm the conjecture. Furthermore, stocks with different characteristics have different reactions to decimalization. Our results lend support to multiple optimal tick sizes for stocks with different characteristics, instead of a uniform minimum tick size for all stocks.

Keywords: Decimalization; front-running; market quality; tick size.

JEL Classification: G10; G18; G20

1. Introduction

Tick size represents the minimum allowable variation in stock price and it is also the implicit cost born by investors and the revenues to market makers. Researchers have sought to determine whether there exists an optimal tick size, both theoretically and empirically. The impact of changes in tick size has been extensively examined in the literature and it is directly related to the existence of an optimal tick size.

The recent move to a decimal system in quoting bid and ask prices at $0.01 increments on the New York Stock Exchange (NYSE) has generated a great amount of interest. Traditionally, stock prices in the US equity markets were quoted in fractions such as $1/2, $1/4, $1/8, $1/16, etc. On August 28, 2000, the NYSE started the process of decimalization in four stages. The decimal pricing program was completed on January 29, 2001, two months earlier than the US Securities and Exchange Commission's (SEC) required conversion date.

The usual argument for a smaller tick size is that a reduced tick size will lower the spread and enhance the market liquidity. Proponents of decimalization argue that a smaller tick size will induce price competition, narrow spreads, and reduce trading costs (Hart, 1993; Peake, 1995; O'Connell, 1997; and others). Hence, a smaller tick size will increase the liquidity and the overall market quality. A decimal pricing system is also easier to use and understand than a fractional one. Furthermore, a smaller tick size after decimalization would make the NYSE more competitive in world equity markets.

Opponents of a small tick size argue that a reduced tick size provides less market-making revenues and decreases the number of market makers willing to provide liquidity. Harris (1996) examines the influences of tick size on order exposures, and shows that traders display more size when the minimum tick is large. With a smaller tick size, some investors may offer marginally better prices to gain priority and this tends to discourage other investors from submitting limit orders. This is known as the front-running problem (Fishman and Longstaff, 1992; Harris, 1996, 1999; Danthine and Moresi, 1998; and others).[1]

Harris (1996, 1999) argues that traders who offer liquidity risk higher transaction costs if other traders employ front-running trading strategies so as to profit from the information conveyed by their displayed orders. Tick size determines the profitability of front-running. It is conjectured that there will be more front-runners in a decimal pricing system, because the costs of front-running are greatly reduced.

As Huang and Stoll (2001) argue, traders of high-priced stocks have incentives to reduce their negotiation costs by trading only at a certain number of available prices, which results in price clustering. Decimal quoting will increase the execution time and trading costs.[2] Hence, large traders tend to quote and trade on a larger tick size in order to reduce bargaining costs. Besides, both Angel (1997) and Seppi (1997) also point out an extremely small tick size set by the regulatory agency may not necessarily be optimal for all kinds of stocks and traders.

The American Stock Exchange (AMEX) adopted \$1/16 ticks for all stocks priced over \$1 on May 7, 1997. Following that, on June 24, 1997, the NYSE

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[1]Fishman and Longstaff (1992) note that front-running refers to the situation where brokers trade ahead of their customers. However, front-running by the broker, with whom the order has been placed, is illegal. Order jumping, where floor traders make an offer slightly better than the existing limit order or the order being shopped by another floor broker is legal (Gibson, Singh, and Yerramilli, 2003). The front-running we are referring to is really scooping (or penning). This is also the definition used by Harris (1996, 1999).

[2]For example, if a buyer wants to buy a stock at \$20 and a seller wants to sell the same stock at \$22, then there could be rounds of bargaining between traders in a decimal pricing system where the tick size is \$0.01.

changed its minimum tick size of eighths and started quoting sixteenths as an intermediate step towards full decimalization.[3] Several studies (Ahn, Cao, and Choe, 1996; Bacidore, 1997; Porter and Weaver, 1997; Ahn, Cao, and Choe, 1998; Bollen and Whaley, 1998; MacKinnon and Nemiroff, 1999; Goldstein and Kavajecz, 2000; Ronen and Weaver, 2001; and others) have examined the impact of tick size changes on market quality. All of them find decreases in quoted spreads and effective spreads but do not find definite conclusions about quoted depth and trading volume.[4] Porter and Weaver (1997) and Bollen and Whaley (1998) document that the effects of a reduced tick size are not uniform across stocks with different characteristics. In addition, Goldstein and Kavajecz (2000), Jones and Lipson (2001), and Bollen and Busse (2003) point out that not all traders benefit from tick size reductions.[5]

Studies on the NYSE decimalization include Bacidore, Battalio, Jennings, and Farkas (2001), Chakravarty, Wood, and Van Ness (2004), and Gibson, Singh, and Yerramilli (2003). They all find significantly lower quoted and effective spreads. Gibson, Singh, and Yerramilli (2003) further show that almost all the reduction of spreads occurs in the order-processing component. Chung, Van Ness, and Van Ness (2004), and Bessembinder (2003) focus on comparing trading costs between NASDAQ and the NYSE, after NASDAQ adopted the decimal pricing system. They generally find lower spreads and depth on both markets after decimalization.

Our study focuses on the market quality of the NYSE during the decimalization process. The samples include the first three pilots as well as all the NYSE stocks decimalized in the final stage. A broad set of market quality measures are examined, which include relative quoted spread, relative effective spread, quoted market depth, trading volume, number of trades, and volume per trade.

Previous studies show that the change in tick size may have different influences on stocks with different characteristics (Harris, 1994; Porter and Weaver,

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[3]The Toronto Stock Exchange (TSE) switched to a decimal trading system on April 15, 1996.

[4]For example, MacKinnon and Nemiroff (1999) show that investors gain from the reduction of tick size on the TSE, because effective spreads decrease and trading activities increase. Ronen and Weaver (2001) discover that while spreads decline, depth does not on the AMEX.

[5]Goldstein and Kavajecz (2000) conclude that a reduced tick size is beneficial for liquidity demanders who trade small orders, but not for those who submit large orders. Jones and Lipson (2001) find an increase in trading costs for institutional trades following the switch to sixteenths on the NYSE. Bollen and Busse (2003) analyze changes in equity mutual fund trading costs following two reductions in tick size in the US equity markets. Consistent with Jones and Lipson's (2001) results, they find a significant increase in trading costs following the move to decimal pricing for actively managed funds.

1997; Ahn, Cao, and Choe, 1998; Bollen and Whaley, 1998; and others). Thus, we repeat the tests by dividing the full samples into quintiles by their pre-event average price levels, average number of trades, and average volume per trade.

Other than tests commonly found in the literature, this study provides several valuable contributions to the existing literature. First, we propose a direct measure of front-running. Most existing studies test the front-running hypothesis by inferences from market quality measures such as liquidity and trading activities, which can be viewed as indirect evidence. In contrast, we provide direct evidence of front-running by proposing a new measure that can be used to estimate changes in the degree of front-running surrounding decimalization.

Second, we study the entire decimalization process. We conduct tests on the first three pilot stocks, as well as on the final-stage samples, which include all NYSE stocks. This paper offers more comprehensive results on the effects of decimalization than previous studies, which generally use limited samples. Third, the study provides further insights on the existence of multiple optimal tick sizes for stocks with different attributes. Angel (1997) argues that there are multiple optimal tick sizes for stocks with different price levels and Seppi (1997) demonstrates that large traders have a larger optimal tick size. Based on these, we test the influences of decimalization on stocks with a broad category of characteristics such as price levels, trading frequencies, and volume per trade.

Our empirical results indicate that, consistent with the literature, both spreads and depth decline significantly after decimalization. As Bacidore (1997) points out, both decreases in spreads and depth suggest that traders whose trade size does not exceed the reduced depth will enjoy lower transaction costs after decimalization. However, the impact on large traders is unclear. This makes it difficult to determine whether the market quality is improved for all traders after decimalization.

Our results show that volume per trade decreases significantly after decimalization, especially for stocks that are actively traded or traded with a larger order size. Such results demonstrate that, after decimalization, either small traders participate more frequently or there are more front-runners entering the market. To examine these conjectures, we examine the changes in degree of front-running surrounding decimalization, and show that the degree of front-running does increase, especially for higher-priced stocks.

The results also show that quotes of stocks with higher prices cluster more intensively after decimalization. The clustering indices decrease for lower-priced stocks, but not for higher-priced stocks. This indicates that the pre-decimal sixteenth tick size is binding for lower-priced stocks, but not for higher-priced stocks. After decimalization, traders of higher-priced stocks use a subset of available prices, trying to reduce negotiation costs. Overall, our empirical results show that decimal pricing may not be optimal for all stocks.

The rest of this paper is organized as follows. Section 2 describes the data. Section 3 explains the research methodology. Section 4 presents the empirical results and Section 5 concludes the paper.

2. Data

Decimalization on the NYSE began on August 28, 2000, when seven listed issues (the first pilot) started trading in dollars and cents instead of fractions. The pilot was extended to an additional 57 stocks (the second pilot) on September 25, 2000 and to another 94 issues (the third pilot) on December 4, 2000. The program was completed on January 29, 2001, when all the NYSE-listed stocks (the final stage) were converted to decimal pricing.

We retrieve the tick-by-tick data from the Trade and Quote (TAQ) database published by the NYSE. Only issues that are identified as common stocks are included. Preferred and convertible preferred stocks, closed-end funds, and American Depository Receipts (ADRs) are excluded. For the first three pilots, the sample periods cover six months before and after decimalization (a total of 12 months). Due to the large amount of tick data when all stocks became decimalized during the final stage, we only include one month of data surrounding decimalization for the final-stage samples.[6]

A set of trade and quote prices are filtered, because they are likely to be erroneous or do not reflect the true trading cost.[7] We also exclude the trades and quotes that are time-stamped outside the regular NYSE trading hours, from

[6]Our sample period is comparable to other studies of the NYSE tick size change. For example, Bollen and Whaley (1998) use all stocks (2,709 issues) on the NYSE with a sample period of 20 trading days before and after the tick size changed from $1/8 to $1/16. Goldstein and Kavajecz (2000) apply a sample period of four weeks before and after the same event, but they only examine 100 stocks.

[7]Trades are excluded if they are coded in the TAQ database as being out of sequence, involve an error or a correction, represent exchange distribution or exchange acquisition, involve non-standard settlement, or involve price changes of 25% (since prior trade) or more if the prior trade price is more than two dollars. Trades that are not preceded by valid same day quotes are also omitted. Quotes are excluded if either bid or ask is non-positive, the difference between ask and bid is non-positive, or if the quotes are associated with trading halts or designated order imbalances or are non-firm.

9:30 a.m. to 4:00 p.m. The total numbers of stocks in our final samples for each of the four stages are 7, 49, 82, and 2,088, respectively.

3. Research Methodology

We employ an event-study type approach. The average market quality measures surrounding decimalization are tested for differences. We first calculate the market quality measures for the sample period before and after decimalization for each stock, and then compute the cross-sectional mean, median, and standard deviation. Statistics of the standard t-test (for mean) and Wilcoxon signed rank test (for median), which test differences in market quality, are reported. Furthermore, in order to test the impact of decimalization on stocks with different characteristics, we use multivariate regressions with dummy variables representing stocks in different characteristic groups.

The event-study type approach has an inherit shortcoming. Decimalization is a rare event and all stocks experience it at the same time. During the event period, important events other than decimalization may interfere with the market conditions. However, since it is difficult to observe many different tick size change events during a short period of time on the same market, most previous studies are confined to a single event and apply an event-study approach.[8] In this study we have four sets of samples and if the results are consistent throughout, then they can be viewed as being more powerful in detecting the impact of decimalization. The measures used to gauge the market quality and the regression models are described in the following sections.

3.1. *Spreads*

A common measure of trading costs is the quoted spread, which measures the difference between the quoted bid and ask prices. The relative quoted spread (QS) is calculated as:

$$QS_{it} = \frac{A_{it} - B_{it}}{M_{it}}, \tag{1}$$

where A_{it} is the quoted ask price for stock i at time t, B_{it} is the quoted bid price for stock i at time t, and M_{it} is the mid-point of quoted ask and bid prices.

[8]For example, see Ahn, Cao, and Choe (1996, 1998), Porter and Weaver (1997), Bacidore (1997), Bollen and Whaley (1998), and Goldstein and Kavajecz (2000). An exception of this approach is Bessembinder (2000).

Relative quoted spreads are likely to be biased estimators of trading costs, because trades do not always occur at the quoted prices. To overcome this bias, a second measure of trading costs, the effective spread, is computed. It measures the difference between the actual traded price and the mid-point of the quoted bid and ask prices and provides a better measure of the actual trading costs. The relative effective spread (ES) is calculated as:

$$ES_{it} = 2\left[\frac{|P_{it} - M_{it}|}{M_{it}}\right], \tag{2}$$

where P_{it} is the transaction price for stock i at time t and M_{it} is the mid-point of the bid and ask prices of the quotes applicable to the transaction. We use the Lee and Ready (1991) algorithm to determine the quotes that correspond to the current trades.[9]

3.2. *Depth*

It is inappropriate to conclude that all investors benefit from decimalization based on drops in spread size alone, because the quoted depth may decrease at the same time. We examine changes in market depth by comparing the cross-sectional averages of three depth measures, i.e., bid size, ask size, and total order size surrounding decimalization.

The ideal measure of market depth is the cumulative depth of different quote prices in the limit order book. However, because of database constraints, we are only able to measure depth at the inside quotes.[10] Nonetheless, the depth at the inside quotes could still be an informative liquidity measure, because the orders behind the inside quotes may be subject to frequent cancellations and thus do not always reliably represent the market depth.[11]

3.3. *Trading activities*

Another important factor determining market liquidity is trading volume. Harris (1994) shows that when execution costs decline, there is an increased propensity

[9]Lee and Ready (1991) note that trades are often reported with a delay. They recommend using the quotes that are time-stamped at least 5 seconds preceding the current trades, which we follow.

[10]The depth beyond inside quotes is generally unavailable from the publicly accessible databases. Most existing studies apply the same approach. Exceptions include Goldstein and Kavajecz (2000) and Bacidore, Battalio, Jennings, and Farkas (2001), who are able to observe the cumulative depth in the limit order book.

[11]Bacidore, Battalio, Jennings, and Farkas (2001) find that, on the NYSE, the limit order cancellation rate is 43% before decimalization and increases to 53% afterwards.

to trade and it is expected that the total volume will increase after the tick size decreases. On the other hand, with a smaller tick size, investors may offer marginally better prices to gain priority and more small traders may participate after decimalization, such that the average volume per trade will decrease.

To examine these conjectures, we calculate changes in average daily volume, average daily number of trades, and average volume per trade. The average volume per trade (*VPT*) is defined as:

$$VPT_i = \frac{Vol_{it}}{N_{it}}, \tag{3}$$

where Vol_{it} is the daily volume and N_{it} is the daily number of trades for stock i at day t, respectively.

3.4. *Clustering*

Clustering is the tendency for prices to occur on a subset of available prices, which is defined with respect to a certain price grid. Clustering exists if not all price positions are used equally. We use the clustering index proposed by Huang and Stoll (2001), which is similar to the Herfindahl Index, to compare clustering surrounding decimalization. The clustering index is defined as:

$$C_i = \sum_k (O_{ik} - O^*_{ik})^2, \tag{4}$$

where O_{ik} is stock *i's* observed trade or quote frequency at price k and O^*_{ik} is the theoretical frequency under the assumption of a uniform distribution, which is 1/16 before decimalization and 0.01 afterwards.

3.5. *Front-running*

To maximize their profits, front-runners tend to undercut the current quotes by the minimum allowable tick size. To measure the degree of front-running before decimalization, we first calculate the percentage of successive positive bid changes and negative ask changes with only one tick difference ($1/16 or $0.0625) over the total number of successive positive bid and negative ask changes. If, for some investors, front-running is profitable before decimalization, then it would also be profitable, after decimalization, by undercutting exiting quotes by a size that is no larger than $0.0625, or 6.25 post-decimal ticks.

Those who do not find front-running profitable before decimalization may start to front run after decimalization, because of reduced tick size. In other words, as more investors find it profitable to front run the existing orders, we would expect the probability of successive positive bid changes and negative ask changes that are less than six post-decimal ticks[12] to be greater than the probability of those that are less than one pre-decimal tick.[13]

Define FR_{pre} as the cumulative percentage of positive bid changes and negative ask changes that are less than or equal to one tick ($0.0625) before decimalization, i.e.,

$$FR_{\text{pre},i} = \frac{\sum_t I(0 < B_{i,t+1} - B_{i,t} \leq 0.0625) + \sum_t I(-0.0625 \leq A_{i,t+1} - A_{i,t} < 0)}{\sum_t I(0 < B_{i,t+1} - B_{i,t}) + \sum_t I(A_{i,t+1} - A_{i,t} < 0)}, \tag{5}$$

where $I(\cdot)$ denotes an indicator function which takes the value of one if the statement is true and zero otherwise. $B_{i,t}$ is the bid quote and $A_{i,t}$ is the ask quote of stock i at time t before decimalization.

After decimalization, define FR_{post} as the cumulative percentage of positive bid changes and negative ask changes that are less than or equal to six post-decimal ticks ($0.06). The equation for FR_{post} can be defined similarly to Equation (5) by replacing 0.0625 (-0.0625) with 0.06 (-0.06). If the degree of front-running is higher after decimalization, then the cross-sectional average of $FR_{\text{post},i}$ will be larger than that of $FR_{\text{pre},i}$.

3.6. *Multivariate regression test*

Based on prior studies and results in the preceding sections, stocks with different characteristics seem to react differently to decimalization. To further test this hypothesis, we partition stocks in the final-stage sample into quintiles by their pre-event average price levels, average number of trades, and average volume

[12] Since a quote change of 6.25 post-decimal ticks is not possible, we would rather perform our tests based on six post-decimal ticks, which is biased against the chance of finding a higher degree of front-running after decimalization.

[13] Certainly, not all tick size changes within one pre-decimal tick or within six post-decimal ticks are motivated by front-running. The assumption is that the proportion of quote changes less than $0.0625, which are not motivated by front-running, is stable surrounding decimalization. The differences in the probabilities of quote changes that are less than $0.0625 surrounding decimalization can then be reasonably attributed to front-running.

per trade, and regress the market quality measures of spreads and depth on various dummy variables that represent stocks' characteristics. The model is defined as follows:

$$PPS(\text{PDEPTH}) = \text{Intercept} + \sum_{i=2}^{5} \alpha_i DP_i + \sum_{j=2}^{5} \beta_j DNT_j + \sum_{k=2}^{5} \gamma_k DVPT_k + \varepsilon, \quad (6)$$

where *PPS* is percentage changes in relative quoted spread before and after decimalization; PDEPTH is percentage changes in market depth; DP_i, DNT_j, and $DVPT_k$ denote the stock belonging to the ith quintile of the pre-event average stock price, the jth quintile of the pre-event average number of trades and the kth quintile of the pre-event average share volume per trade, respectively. Thus the coefficients of dummy variables will denote the difference between the change in relative quoted spread and in market depth between stocks in the first quintile to those in the other quintiles.

4. Empirical Results

4.1. *Spreads and depth*

Table 1 shows the raw and percentage changes[14] of the relative quoted spread and relative effective spread after switching to decimal pricing for the last stage of decimalization.[15] Consistent with Bacidore, Battalio, Jennings, and Farkas (2001), Bessembinder (2003), Chakravarty, Wood, and Van Ness (2004), and Gibson, Singh, and Yerramilli (2003), all spread measures drop significantly after decimalization. For example, the relative quoted spread (relative effective spread) declines by 22.65% (24.72%), also significant at the 1% level. As previously discussed, the spread reduction only tells one side of the story. If the quote size and trade size also decrease after decimalization, then it would be erroneous to conclude that all market participants benefit from decimalization.

[14]The relative spread is known to be affected by various stock characteristics such as stock prices and trading activities. For example, see Tinic and West (1972) and Stoll (1978). To control for differences due to these factors, percentage changes in relative spreads are also reported. Similarly, percentage changes are reported for other market quality measures.

[15]To conserve space, we omit the results of the first three pilots, which are qualitatively similar to those of the final stage and available upon request.

Table 1. Changes in spreads and market depth surrounding decimalization.

	Before	After	Change	Change (%)
Relative quoted spread				
Mean	0.0134	0.0107	−0.0027**	−0.2265**
Median	0.0079	0.0059	−0.0015**	−0.2479**
Percentage of negatives			89.38%	
Relative effective spread				
Mean	0.0090	0.0069	−0.0021**	−0.2472**
Median	0.0054	0.0038	−0.0011**	−0.2740**
Percentage of negatives			90.35%	
Market depth				
Mean	96.25	47.02	−49.24**	−0.3871**
Median	63.41	37.74	−24.75**	−0.4174**
Percentage of negatives			93.83%	

For the final-stage samples, the sample period is one month surrounding decimalization. The relative quoted spread (QS) is calculated as: $QS_{it} = (A_{it} - B_{it})/M_{it}$, where A_{it} is the quoted ask price for stock i at time t, B_{it} is the quoted bid price for stock i at time t, and M_{it} is the mid-point of quoted ask and bid prices.

The relative effective spread (ES) is calculated as:

$$ES_{it} = 2\left[\frac{|P_{it} - M_{it}|}{M_{it}}\right],$$

where P_{it} is the transaction price for stock i at time t and M_{it} is the mid-point of bid and ask prices of the quotes applicable to the current transaction. Market depth is defined as the sum of order size behind the best bid and the best ask prices. Percentage of negatives is the percentage of stocks with negative changes in mean. Superscripts ** and * denote the significance levels of 1 and 5%, respectively, for the t-test (for mean) and Wilcoxon signed rank test (for median) of differences.

Table 1 also reports significant declines in market depth after decimalization.[16] The reduction is 38.71%. Also consistent with the literature, the market liquidity for the inside quotes is deteriorating after decimalization.

From Table 1, we can see that both spreads and depth decrease significantly after decimalization. For small investors whose order size does not exceed the quoted depth, the move to decimal pricing is beneficial, because they enjoy lower transaction costs without sacrificing liquidity. For large investors, the result is unclear, because the benefits from smaller spreads may be more than offset by the potential losses from drops in depth.

[16] We also perform separate tests on bid size and ask size. The results are similar and only those for the total market depth are presented.

4.2. *Trading activities*

Table 2 presents the change in trading activities, as measured by daily trading volume, number of trades, and volume per trade surrounding decimalization. The daily trading volume for the final-stage samples decreases by 14.07%, significant at the 1% level, while the trading volume for the third pilot increases significantly by 17.38%.[17] These lead to inconsistent results. However, trading volume is likely to be affected by general market conditions. Indeed, the total market trading volume increases by 17.41, 33.15, and 15.03% during the sample periods of the first three pilots, respectively, and the increases in trading volume for the three pilots are either smaller than or comparable to those for the entire market.

We cannot therefore rule out the possibility that the increases in trading volume for the first three pilots are partially induced by general market increases. However, for the final-stage samples, the decrease in trading volume fairly represents the general market conditions, because it includes most of the listed stocks except for those in the first three pilots. Although no definite conclusions can be made from them, these results at least show that trading volume does not necessarily increase after decimalization. Such results are consistent with Ahn, Cao, and Choe (1996, 1998), Bacidore (1997), Porter and Weaver (1997), and Chakravarty, Wood, and Van Ness (2004). All of them find no significant increases in volume after the tick size reduction.

We calculate another two measures of trading activity — number of trades and average volume per trade — which are also relevant in measuring changes in market quality due to decimalization.

The raw and percentage changes in the number of trades for the final stage are 7.42 and 0.66%, respectively. The mean increase in the number of trades is significant at the 1% level. Table 2 shows that there is significant drop in average volume per trade. For example, the average volume per trade decreases by 14.34% in the final stage, which is significant at the 1% level. Trade sizes may have fallen in large part because of the greater prevalence of large traders slicing and dicing their orders.[18] The higher trading frequency and smaller size per trade after decimalization show that postdecimal front-running problems are likely to be more severe. We will directly test this hypothesis in Section 4.4.

[17]Unreported results show that the trading volume for the first two pilots also increases significantly by 10.39 and 16.81%, respectively.

[18]We thank the referee for pointing out the insight.

Table 2. Changes in trading activity surrounding decimalization.

	Before	After	Change	Change (%)
Trading volume				
Mean	516859	431922	−84938**	−0.1407**
Median	86345	70987	−7201**	−0.1827**
Percentage of negatives			71.68%	
Number of trades				
Mean	184.14	191.55	7.42**	0.0066
Median	61.05	61.42	−0.15	−0.0061
Percentage of negatives			51.36%	
Average volume per trade				
Mean	1705.61	1362.66	−342.95**	−0.1434**
Median	1380.50	1133.12	−207.67**	−0.1716**
Percentage of negatives			76.05%	

For the final-stage samples, the sample period is one month surrounding decimalization. The average volume per trade (VPT) is defined as: $VPT = Vol_{it}/N_{it}$, where Vol_{it} is the daily trading volume and N_{it} is the daily number of trades for stock i at day t. Percentage of negatives is the percentage of stocks with negative changes in mean. Superscripts ** and * denote the significance levels of 1 and 5%, respectively, for the t-test (for mean) and Wilcoxon signed rank test (for median) of differences.

Overall, it seems that decimalization decreases spread size at the expense of market liquidity.

4.3. *Clustering*

Table 3 reports the clustering indices for both quote and trade prices surrounding the final-stage decimalization. From Panel A, it seems that clustering is less severe after decimalization.[19] Nonetheless, Harris (1991), Hameed and Terry (1998), and Huang and Stoll (2001) document that price clustering increases with stock price levels. We thus divide the samples by their pre-event stock prices and repeat the test. From Panel B, we can see that the decrease in the clustering index is mostly due to the decreases in low-priced stocks.[20] For high-priced stocks, the clustering index is actually increasing.

It appears that investors who trade high-priced stocks on the NYSE seek to reduce their negotiation costs by trading only at a certain number of available

[19] In contrast, Hameed and Terry (1998) find that price clustering increases as tick size decreases, and the negative relationship is pervasive across stocks of different price levels.

[20] We only report the clustering index of the trade price for stocks grouped by price levels. The results of bid quotes and ask quotes are similar and thus omitted.

Table 3. Degree of price clustering surrounding decimalization.

	Bid price	Ask price	Trade price
Panel A: Clustering index of quote and trade prices			
Before	0.0419	0.0426	0.0396
After	0.0309	0.0307	0.0272
Change	−0.0110**	−0.0119**	−0.0124**
Change (%)	−0.0317**	−0.0983**	−0.0472**

	Low priced	2	3	4	High priced
Panel B: Changes in clustering index for trade price by stock price levels					
Before	0.0700	0.0516	0.0325	0.0190	0.0249
After	0.0402	0.0335	0.0224	0.0149	0.0253
Change	−0.0298**	−0.0181**	−0.0101**	−0.0040*	0.0004
Change (%)	−0.2269**	−0.1831**	−0.1128**	−0.0688*	0.2181**

The degree of clustering is calculated for the final-stage samples. The cross-sectional averages are calculated and compared surrounding decimalization. The clustering index is defined as: $C_i = \sum_k (O_{ik} - O^*_{ik})^2$, where O_{ik} is stock $i's$ observed trading frequency at price k and O^*_{ik} is the theoretical frequency under the assumption of a uniform distribution, which is 1/16 before decimalization and 0.01 after decimalization. Superscripts ** and * denote the significance levels of 1 and 5%, respectively, for the t-test (for mean) and Wilcoxon signed rank test (for median) of differences.

prices after decimalization, which results in additional price clustering. After decimalization, only low-priced stocks experience decreases in clustering and this provides evidence supporting that a uniform minimum tick size may not be optimal for high-priced stocks.

4.4. *Front-running*

Table 4 reports the degree of front-running surrounding the final stage of decimalization. From Panel A, it is interesting to see that the degree of front-running increases significantly by 7.89% after decimalization. This result provides direct evidence showing that there are more front-runners in a decimal pricing system, because of decreased costs of front-running.[21]

[21] Orders will sometimes come in from one participant and execute at multiple prices, which could somehow bias the front-running tests. Our estimator of front running can only capture the first order of a series of orders submitted by the front-runners. Thus, it is downward biased. However, even with this downward biased estimator, we are still able to detect an increase in the front-running activity after decimalization, thus we believe our results on front-running tests are quite robust.

Table 4. Changes in degree of front-running surrounding decimalization.

Panel A: Changes in degree of front-running					
Before			0.7546		
After			0.7850		
Change			0.0305**		
Change (%)			0.0789**		
	Low priced	2	3	4	High priced
Panel B: Changes in degree of front-running by average stock prices					
Before	0.7873	0.7782	0.7420	0.7672	0.7277
After	0.7763	0.8024	0.7836	0.8053	0.7731
Change	−0.0110*	0.0242**	0.0416**	0.0381**	0.0454**
Change (%)	−0.0046	0.0489**	0.0799**	0.0649**	0.0792**

The change in degree of front-running is defined as the difference between the cumulative percentage of successive positive bid changes and negative ask changes that are less than one predecimal tick ($0.0625) and the cumulative percentage of those that are less than six postdecimal ticks ($0.06).

Define FR_{pre} as the cumulative percentage of positive bid changes and negative ask changes that are less than or equal to one tick ($0.0625) before decimalization, i.e.,

$$FR_{\text{pre},i} = \frac{\sum_t I(0 < B_{i,t+1} - B_{i,t} \leq 0.0625) + \sum_t I(-0.0625 \leq A_{i,t+1} - A_{i,t} < 0)}{\sum_t I(0 < B_{i,t+1} - B_{i,t}) + \sum_t I(A_{i,t+1} - A_{i,t} < 0)},$$

where $I(.)$ denotes an indicator function which takes the value of one if the statement is true and zero otherwise. $B_{i,t}$ is the bid quote and $A_{i,t}$ is the ask quote of stock i at time t before decimalization. Define FR_{post}, the cumulative percentage after decimalization, similarly by replacing 0.0625 (−0.0625) with 0.06 (−0.06) in the equation above. Superscripts ** and * denote the significance levels of 1 and 5%, respectively, for the t-test (for mean) and Wilcoxon signed rank test (for median) of differences.

Since the stepping-ahead strategies may be relatively more profitable for higher-priced stocks, we repeat the front-running test by grouping stocks into quintiles by their average pre-event prices. The results are reported in Panel B of Table 4. The degree of front-running after decimalization increases for all quintiles, except for the lowest-priced stocks. After decimalization, higher-priced stocks also tend to have higher increases in the degree of front-running. That is, except for limit orders of the lowest-priced stocks, more limit orders are undercut by front-runners, especially for those of the higher-priced stocks. Investors of high-priced stocks may be less willing to provide liquidity. These results provide direct evidence for the front-running hypothesis of Harris (1996, 1999).

4.5. *A closer look at decimalization*

Tables 5 and 6 report the results of regressing percentage changes in relative quoted spreads and in market depth on various stock characteristics, respectively.[22]

The larger the pre-event average number of trades is, the larger the reductions are in both spreads and depth after decimalization. Similar results are obtained for the pre-event average share volume per trade. For example, from Model (2) in Table 5, the coefficient of –0.1890 indicates that the percentage

Table 5. Regression results of percentage changes in relative quoted spreads on stock characteristic variables.

	Model (1)	Model (2)	Model (3)	Model (4)
Intercept	−0.2382**	−0.1330**	−0.1337**	−0.1281**
DP_2	0.0270*			0.0343**
DP_3	0.0165			0.0460**
DP_4	0.0053			0.0858**
DP_5	0.0085			0.1266**
DNT_2		−0.0500**		−0.0509**
DNT_3		−0.1001**		−0.1142**
DNT_4		−0.1300**		−0.1671**
DNT_5		−0.1890**		−0.2360**
$DVPT_2$			−0.0639**	−0.0292*
$DVPT_3$			−0.0925**	−0.0406**
$DVPT_4$			−0.1272**	−0.0619**
$DVPT_5$			−0.1816**	−0.0882**
Adjusted R^2	0.0007	0.1224	0.1070	0.2062
F-statistic	1.36	73.74	63.53	46.17
N	2088	2088	2088	2088

We regress changes in relative quoted spreads on various dummy variables that represent stocks' characteristics. The model is defined as follows:

$$PPS = \text{Intercept} + \sum_{i=2}^{5} \alpha_i DP_i + \sum_{j=2}^{5} \beta_j DNT_j + \sum_{k=2}^{5} \gamma_k DVPT_k + \varepsilon,$$

where PPS is percentage changes in relative quoted spread before and after decimalization. DP_i, DNT_j, and $DVPT_k$ denote the stock belonging to the ith quintile of the pre-event average stock price, the jth quintile of the pre-event average number of trades and the kth quintile of the pre-event average share volume per trade, respectively. Superscripts ** and * denote the significance levels of 1 and 5%, respectively.

[22]The regressions results for percentage changes in relative effective spreads are similar to those of relative quoted spreads. To save space, we report the results for percentage changes in relative quoted spreads only.

Table 6. Regression results of percentage changes in market depth on stock characteristic variables.

	Model (1)	Model (2)	Model (3)	Model (4)
Intercept	−0.3973**	−0.2744**	−0.2224**	−0.1827**
DP_2	−0.0117			−0.0180
DP_3	0.0358*			0.0360*
DP_4	−0.0053			0.0397*
DP_5	0.0312*			0.1021**
DNT_2		−0.0617**		−0.0510**
DNT_3		−0.1219**		−0.1217**
DNT_4		−0.1725**		−0.1861*
DNT_5		−0.2084**		−0.1975**
$DVPT_2$			−0.1085**	−0.0791*
$DVPT_3$			−0.1692**	−0.1268**
$DVPT_4$			−0.2276**	−0.1756**
$DVPT_5$			−0.3194**	−0.2453**
Adjusted R^2	0.0056	0.1081	0.2258	0.2939
F-statistic	3.94	64.24	153.19	73.40
N	2088	2088	2088	2088

We regress changes in market depth on various dummy variables that represent stocks' characteristics. The model is defined as follows:

$$\text{PDEPTH} = \text{Intercept} + \sum_{i=2}^{5} \alpha_i DP_i + \sum_{j=2}^{5} \beta_j DNT_j + \sum_{k=2}^{5} \gamma_k DVPT_k + \varepsilon,$$

where PDEPTH is percentage changes in market depth before and after decimalization. DP_i, DNT_j, and $DVPT_k$ denote the stock belonging to the ith quintile of the pre-event average stock price, the jth quintile of the pre-event average number of trades and the kth quintile of the pre-event average share volume per trade, respectively. Superscripts ** and * denote the significance levels of 1 and 5%, respectively.

drop in relative quoted spreads are 18.90% larger for stocks in the highest quintile of average number of trade. Nevertheless, from Model (2) in Table 6, it is also found that stocks that are most actively traded experience a drop of 20.84% in market depth. This shows that stocks with different characteristics react differently to decimalization. Although the most actively traded stocks experience the largest drop in spreads, they are also the ones that experience the largest decrease in market depth. Such result shows that actively traded stocks seem to be the ones that have lost out after decimalization, which is consistent with Bollen and Busse (2003).

The evidence shows that spreads and market depth fall monotonically with average number of trades and average share volume per trade. A possibility is that front-running (or penning) is more prevalent when trades are more frequent, and that the act of front-running tightens spreads. Front-running also leads big participants to split their orders more often and to use limit orders less, which decreases depth.

5. Conclusions

We have studied the impact of decimalization on the market quality of NYSE. Consistent with the literature, the results show significant reductions in spreads and in market depth. Since spreads decrease, it is expected that more investors are willing to trade and trading volume should increase. However, our findings show that this is not necessarily the case. Trading volume increases for the first three pilots, which may be partially induced by the general increases in market trading volume during those sample periods. However, in the final stage, trading volume decreases.

With a smaller tick size, investors may offer marginally better prices to gain priority. The average size per trade is expected to decrease and we find that this is indeed the case. To directly test the front-running hypothesis, we examine the degree of front-running by a proposed new measure. There appear to be more front runners after decimalization, especially for stocks that are actively traded or are higher priced.

We also find that high-priced stocks cluster more intensively after decimalization. This indicates that investors who trade high-priced stocks wish to reduce their negotiation costs by using only a certain number of available prices, which results in additional price clustering.

We further group our final-stage samples by average stock prices, number of trades, and volume per trade and perform dummy variable regressions to test if stocks with different characteristics have different reactions to decimalization. The results present the evidence that high-priced stocks, actively traded stocks, and large traders seem to be the ones that have lost out after decimalization. These empirical results are in line with Angel (1997) and Seppi (1997), who support the existence of multiple optimal tick sizes for stocks with different characteristics, rather than a uniform minimum tick size for all stocks.

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Section III

Market Rationality

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Chapter 10

The Importance of Being Conservative: An Illustration of Natural Selection in a Futures Market

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With an aversion to losses, there are some traders in financial markets who are not overly aggressive in bidding too high and some traders who are not overly aggressive in selling at too low of a price. This paper shows in an evolutionary model of natural selection within the context of a futures market that as long as these conservative traders have some positive probability (however small) of making an accurate prediction of the spot price, their presence ensures the convergence to an efficient market. This result is distinct from previous related research. Furthermore, although such conservative traders are not aggressive nor do they always participate in the market, in the long run they not only survive but eventually increase their share of aggregate wealth.

Keywords: Conservative traders; futures market; market efficiency; market rationality; natural selection; survivorship.

1. Introduction

We often hear recommendations from financial advisers such as "don't buy on margin"; "sell early, buy late"; "refrain from being overly aggressive in bidding" (e.g., see Seiver, 1991), etc. This advice reflects the importance of being conservative in financial markets. For some traders there is the preoccupation of avoiding losses, and this may result in some of these conservative traders not being overly aggressive in bidding too high or for some of these conservative traders not being too low in their sales offers. The avoidance of losses is paramount in their behavior. Can conservative traders survive in financial markets? How would their presence impact the market? This paper addresses these questions. So far the current literature has not yet discussed these questions.

The current literature has two main streams of frameworks that are used to examine traders' behavior. One is the rational expectation approach (e.g., Blume and Easley, 1992; Bray, 1981; Feiger, 1978; Figlewski, 1978, 1982; Grossman, 1976, 1978; Hellwig, 1980; Jordan, 1983; Radner, 1979). The other is the evolutionary approach of natural selection (e.g., Alchian, 1950; Biais and Shadur, 1993; Fischer and Verrecchia, 1997; Friedman, 1953; Luo, 1995, 1998, 2001, 2003). The former approach requires rationality on the part of market participants. The latter one abandons rationality on all traders' part. All traders are modeled as being preprogrammed with some inherent behavioral rules. To examine the survival issue, the evolutionary approach of natural selection is a robust framework to use since it addresses the short run dynamics and the long run outcome of the markets.

This paper takes the framework of futures markets in Luo (1998). The framework is briefly described as follows. Consider a commodity futures market, where the commodity is assumed to be nonstorable and must be sold in the spot market at the end of each time period. Traders enter the market sequentially over time at the beginning of each time period. Traders are engaged in buying or selling contracts in order to make trading profits. The spot price is determined in the beginning of each time period before the futures market opens, but is unknown to all market participants. In each time period, each trader's prediction about the spot price together with trader's wealth provides this trader's demand function for contracts. Each trader's wealth in each time period is defined to be the accumulated profits up to that time period. The futures market is a Walrasian market structure. Each time period, the futures price is the futures market clearing price which equates the aggregate net demand for contracts with the supply of contracts from producers in that time period.

Traders participate in the markets either as a buyer or a seller but not as a buyer in one time period and a seller in another time period. Each trader's trading type (buyer or seller) is randomly determined upon each trader's entry and fixed in all future time periods. A fraction of wealth allocated by each trader in his or her trading activities is also randomly determined upon each trader's entry period and fixed in all future time periods. In addition, upon entry of each trader, each trader is endowed with an initial amount of wealth and a probability distribution of the prediction error with respect to the spot price. Each trader's prediction error in a time period represents the amount by which each trader overpredicts or underpredicts the spot price at that time period. The endowment of the probability distribution is trader specific and it is fixed in all subsequent time periods. These probability distributions describe differing views of traders in predicting the spot price. A trader's prediction of the spot price is generated through this trader's prediction error distribution. In this framework, the traders are modeled as unsophisticated and they merely act upon their predetermined trading type, and a predetermined fraction of wealth that they allocate on trading activities, and their predetermined prediction error distribution.

In this model, a conservative buyer's behavioral rule is modeled as the low probability of overprediction of the spot price provided that the buyer has a strictly positive probability of predicting correctly the spot price. Similarly, a conservative seller's behavioral rule is modeled as the low probability of underprediction of the spot price provided that the seller has a strictly positive probability of predicting correctly the spot price. The traders with such behavioral rules resemble the real world conservative traders who try to avoid losses by not overbidding very much as a buyer and not underbidding very much as a seller. Suppose that in each time period, with positive probability the entering buyer (seller) has an arbitrarily low probability of overpredicting (underpredicting) the spot price and has a strictly positive probability (however small) of predicting correctly the spot price. Under this assumption, this paper illustrates through simulations the convergence of the futures price to the spot price and the cause of this convergence. Before discussing the intuition behind this numerical result, a conservative trader will be defined, for the purpose of this paper. A buyer (seller) is said to be more conservative than other buyers if this buyer (seller) has a lower probability of overpredicting (underpredicting) the spot price than other buyers (other sellers), assuming he or she has a strictly positive probability of predicting correctly the spot price. A more (less) conservative trader is could be either a more (less) conservative buyer or a more (less) conservative seller. A buyer (seller) would make a loss to this buyer's (seller's) trading counterpart if this buyer (seller) trades at the time period in which this buyer (seller) ovepredicts (underpredicts) the spot price. Hence, a conservative buyer (seller) with a low probability of overpredicting (underpredicting) the spot price will not make a loss very often. Given the fact that a conservative buyer (seller) has a strictly positive probability of predicting correctly the spot price, this conservative buyer (seller) potentially would make gains in some time periods. A more conservative buyer (seller) with even lower probability of overpredicting (underpredicting) the spot price would make losses even less often. Over time, as more and more conservative traders enter the markets, they will gradually take wealth away from less conservative traders. Those more and more conservative traders will eventually come to dominate the market and drive the futures price to the spot price.

The natural selection force in the markets takes the form of the constant wealth redistribution over times among traders. This constant wealth redistribution process gradually takes wealth away from less conservative traders and rewards wealth to the more conservative traders. This is the critical factor accounting for this convergence.

Section 2 describes briefly the model. The numerical simulations are discussed in Section 3. Section 4 concludes the paper.

2. The Model

Consider a dynamic model of a commodity futures market. Time is discrete and indexed by t, where $t = 1, 2, \ldots$. The commodity is nonstorable and it must be sold at the end of each time period. Hence, futures contracts are one-period in length. The futures market opens at the beginning of each time period. The futures market closes after all transactions in the futures market are completed. Traders participate in the futures market by buying or selling contracts. The aggregate supply of futures contracts from producers at time t, $t = 1, 2, \ldots,$ denoted as S_t, is randomly determined each time period from an interval $[\underline{S}, \overline{S}]$ according to a given probability distribution. After the futures market closes, the spot market opens at the end of each time period. Denote the spot price at time t as p_t. The spot price at time t (p_t) is determined at the beginning of time period t before traders purchase or sell their contracts, but p_t is unknown to all the market participants. It is revealed to all market participants at the end of time period in the spot market after the futures market at time t closes. The price of a futures market at time t (where $t = 1, 2, \ldots$) is denoted as p_t^{f}. The price of the commodity in the spot market p_t and the price of a futures contract p_t^{f} at time t (where $t = 1, 2, \ldots$) are always quoted as nonnegative multiples (m) of a unit size d (> 0). The spot price p_t is randomly drawn at the beginning of time period t (before the futures market opens) from the set $\{d, d + 1, \ldots, (\bar{m} - 1)d, \bar{m}d\}$, where $\bar{m}$ is a positive integer, according to a distribution function. The futures price p_t^{f} is merely the market clearing price in the commodity futures market and for any t, p_t^{f} is constrained to be greater than or equal to d.

Traders are assumed to enter the market sequentially over time and participate with previously entered traders for the purpose of making profits. At the beginning of time period t, where $t = 1, 2, \ldots,$ a single trader (called trader t) is allowed to enter the market and participate either as a buyer or a seller in all future time periods. But no trader is allowed to be a buyer (or seller) in some time periods and to be a seller (or buyer) in other time periods. Whether trader t is a buyer or a seller is randomly determined upon entry. Once the trading type for trader t is determined, it is fixed for all future time periods. Specifically, trader $t's$ trading type, denoted as z_t, is randomly taken from a set $\{-1, 1\}$ in the beginning of time period t upon entry according to a given discrete distribution function with its support being $\{-1, 1\}$. If $z_t = 1$, it indicates that trader t is a buyer. If $z_t = -1$, it indicates that trader t is a seller. Once trader $t's$ trading type is determined at time period t, it is fixed in all future time periods. Furthermore, it is assumed that z_t is independent of p_t for $t = 1, 2, \ldots$ The timing of events for the model is illustrated in Fig. 1.

Furthermore, each trader spends a fraction of his or her total wealth on trading activity. This fraction could be viewed as a reflection of a trader's inherent attitude toward risk. A smaller fraction indicates more risk aversion. The remaining fraction of total wealth can be viewed as risk-free and earns no return. This fraction is randomly determined in the beginning of his or her entry period and fixed thereafter. Denote f_t as trader t's fraction of wealth allocated for trading activities. f_t is randomly taken from an interval $(0, 1]$ according to a distribution with its support being $(0, 1]$ in the beginning of time period t. And furthermore, f_t is independent of p_t for $t = 1, 2, \ldots$

Each trader is assumed to be endowed with initial wealth V_0, where $0 < V_0 < \infty$, in the entry time period. The futures market is assumed to have

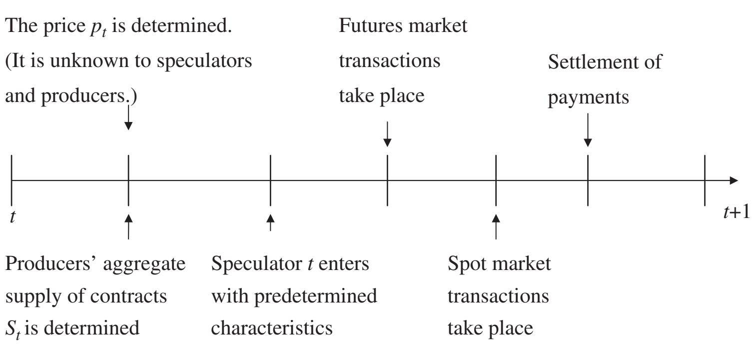

Figure 1. The timing of events.

no traders at time 0. After time 0 traders enter sequentially. Denote trader t's wealth at the end of time period s (where $s \geq t$) as V_s^t and $V_{t-1}^t = V_0$. The futures market will not open if there are only sellers in the futures market.

Trader t's prediction about the spot price (where $t = 1, 2, \ldots$) at time s (where $s \geq t$) is characterized by b_s^t. If trader t is a buyer, trader t is willing to buy contracts up to what his or her trading wealth permits at a price no higher than his or her prediction b_s^t, at time s, where $s \geq t$. Therefore, for $z_t = 1$, trader t's demand for futures contracts in time period s, denoted as $q_s^t(p_s^{\mathrm{f}})$, is

$$q_s^t(p_s^{\mathrm{f}}) = \begin{cases} \left(\dfrac{f_t V_{s-1}^t}{p_s^{\mathrm{f}} + d}, \dfrac{f_t V_{s-1}^t}{p_s^{\mathrm{f}}}\right] & \text{if } p_s^{\mathrm{f}} = b_s^t - rd, \quad \text{where } r = 1, 2, \ldots, \hat{R}, \\ \left[0, \dfrac{f_t V_{s-1}^t}{p_s^{\mathrm{f}}}\right] & \text{if } p_s^{\mathrm{f}} = b_s^t, \\ 0 & \text{if } p_s^{\mathrm{f}} > b_s^t. \end{cases}$$

where $\hat{R}$ is characterized by $\begin{cases} b_s^t - (\hat{R}+1)d < 0, \\ b_s^t - \hat{R}d \geq 0. \end{cases}$

If trader t is a seller, at time s, where $s \geq t$, speculator t is willing to sell contracts up to what his or her trading wealth permits at a price no lower than his or her prediction b_s^t. To ensure that sellers honor their contracts, sellers are constrained to invest $f_t/(\bar{m} - 1)$ of their total wealth.[1] Therefore, for $z_t = -1$ speculator t's demand for futures contracts in time period s, denoted as $q_s^t(p_s^{\mathrm{f}})$, is

$$q_s^t(p_s^{\mathrm{f}}) = \begin{cases} 0 & \text{if } p_s^{\mathrm{f}} < b_s^t, \\ \frac{1}{\bar{m}-1}\left[-\dfrac{f_t V_{s-1}^t}{p_s^{\mathrm{f}}}, 0\right] & \text{if } p_s^{\mathrm{f}} = b_s^t, \\ \frac{1}{\bar{m}-1}\left[-\dfrac{f_t V_{s-1}^t}{p_s^{\mathrm{f}}}, -\dfrac{f_t V_{s-1}^t}{p_s^{\mathrm{f}} + d}\right) & \text{if } p_s^{\mathrm{f}} = b_s^t + rd, \quad \text{where } r = 1, 2, \ldots. \end{cases}$$

A negative number of futures contracts indicates the quantity of sales. Figure 2 provides examples of this type of demand curves for a buyer and for a seller.

In addition, trader t's prediction about the spot price at time period s, where $s \geq t$, is modeled as $b_s^t = p_s + \nu_s^t$, where ν_s^t is trader t's prediction error with respect to the spot price at time s. The ν_s^t may be correlated across traders.

[1] Since the maximum loss that a seller would incur is when the spot price takes the highest value $\bar{m}d$ and the futures price takes the lowest value d, the constraints for investing $f_t/(\bar{m} - 1)$ of the total wealth by the seller ensures no defaults on any contract.

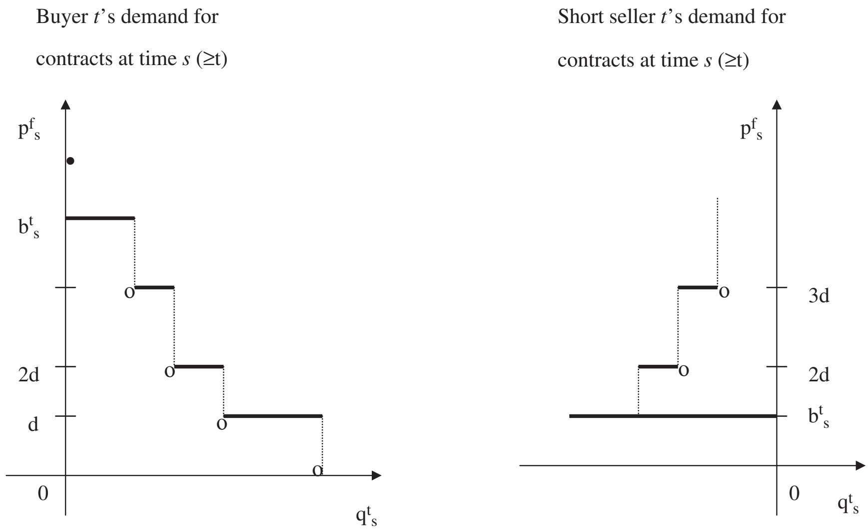

Figure 2. Traders' demands for contracts.

It is assumed that $\nu_s^t \in \{-\underline{N}d, -(\underline{N}-1)d, \ldots -2d, -d, 0, d, 2d, \ldots (\overline{N}-1)d, \overline{N}d\}$,[2] where $\underline{N}$ and $\overline{N}$ are both positive whole numbers. The smaller a prediction error is, the more accurate is the information reflected in the prediction. It is assumed that ν_s^t is independent of p_s at time s. For a fixed t, the ν_s^t are assumed to be independently and identically distributed across time $s = t, t+1, \ldots$[3] Therefore, denote

$$\theta_1^t = \Pr\left(\nu_s^t > 0\right), \quad \theta_2^t = \Pr\left(\nu_s^t = 0\right) \quad \text{and} \quad \theta_3^t = \Pr\left(\nu_s^t < 0\right),$$

where superscript t indicates trader t. The vector $\left(\theta_1^t, \theta_2^t, \theta_3^t\right)$ describes the probability distribution of trader $t's$ overprediction, exact prediction and underprediction. This probability distribution characterizes trader $t's$ predictive ability. Once trader $t's$ predictive ability $\left(\theta_1^t, \theta_2^t, \theta_3^t\right)$ is determined in the beginning of his or her entry time period t, it is fixed thereafter. Specifically, it is assumed that the vector $\left(\theta_1^t, \theta_2^t, \theta_3^t\right)$ is randomly taken in the beginning of time period t after trader $t's$ trading type is realized from a set $\Theta = \{(\theta_1, \theta_2, \theta_3) \in (0, 1)^3 : \theta_1 + \theta_2 + \theta_3 = 1\}$ according to an given distribution $F_z(\theta_1, \theta_2, \theta_3)$.

[2] To prevent unrealistic negative predictions, the range of predictions sometimes may have to be truncated if $p_s + \nu_s^t < d$. More precisely, $b_s^t = \max[p_s + \nu_s^t, d]$.

[3] This reflects the impossibility of speculators learning from other traders and learning from their past experiences.

The distribution $F_z(\theta_1, \theta_2, \theta_3)$ has the following two properties: (i) for any given $\epsilon > 0$, $\int_{\substack{0<\theta_1<\epsilon \\ \theta_2>0}} dF_{z=1}(\theta_1, \theta_2, \theta_3) > 0$ and (ii) for any given $\epsilon > 0$, $\int_{\substack{0<\theta_3<\epsilon \\ \theta_2>0}} dF_{z=-1}(\theta_1, \theta_2, \theta_3) > 0$. Property (i) suggests that conditional on trader t is a buyer, there is a positive probability at time period t that the entering buyer has an arbitrarily low probability of overpredicting the spot price and has a strictly positive probability of predicting correctly the spot price. Similarly, property (ii) means that conditional on trader t is a seller, there is a positive probability at time period t that the entering seller has an arbitrarily low probability of underpredicting the spot price and has a strictly positive probability of predicting correctly the spot price. In other words, the properties indicate that with a positive probability (however small) each time period the entering buyer (or seller) has a lower probability of overpredicting (or underpredicting) the spot price than the previously entered buyers (or sellers).

Furthermore, the vector $\left(\theta_1^t, \theta_2^t, \theta_3^t\right)$ is independent of p_t and f_t for a fixed t. *It is assumed that the random vector* $\left(\theta_1^t, \theta_2^t, \theta_3^t, z_t, f_t\right)$ *is independently and identically distributed across traders* $t = 1, 2, \ldots$.

All payments among all participants in the futures market are settled at the end of each time period. Hence, trader t's profit at the end of time period s is $\left(p_s - p_s^{\mathrm{f}}\right)q_s^t$ and consequently, trader t's wealth at the end of time period s is

$$V_s^t = V_{s-1}^t + \left(p_s - p_s^{\mathrm{f}}\right)q_s^t.$$

The futures price at time s, p_s^{f}, is the futures market clearing price, determined by the following equality of the aggregate net demand from traders with the aggregate supply of the producers,

$$\sum_{t=1}^{s} q_s^t\left(p_s^{\mathrm{f}}\right) = S_s. \tag{1}$$

The solution, p_s^{f}, to the above equation is not unique due to the shape of the stepwise demand function and the vertical supply from producers. If there are multiple solutions to Equation (1), then the highest number is chosen as a futures price. Furthermore, since the spot price is constrained to be at least d, to be consistent, the futures price is set to equal to d if the highest number among the solutions to Equation (1) is below d. All transactions are executed at the futures market clearing price. If the maximum number of contracts demanded by traders at the futures market clearing price exceeds the number of contracts supplied by producers and if there are traders whose predictions coincide with the futures market clearing price, the remaining supply could be allocated among those traders proportionately to their wealth. Nevertheless, how the remaining supply is allocated does not affect the paper's results.

In fact, the above futures market mechanism is a Walrasian market. From the above, it is evident that the more wealth a trader has, the more contracts that this trader demands. Consequently, the more influence this trader's prediction has over the futures price.

The futures price at time s, p_s^{f}, is the futures market clearing price at time s, which is determined by Equation (1). As can be seen, the futures price at time period s, p_s^{f}, is a function of $S_1, S_2, \ldots, S_s$; $p_1, p_2, \ldots, p_s$; $f_1, f_2, \ldots f_s$; $z_1, z_2, \ldots z_s$; $v_t^t, v_{t+1}^t, \ldots, v_s^t$, for all $t \leq s$; and V_0.

The above outlines the model in Luo (1998) except that the distribution function from which the vector $(\theta_1^t, \theta_2^t, \theta_3^t)$ is drawn differs. Luo (1998) assumes that with a positive probability each time period the entering trader has an arbitrarily high probability of predicting correctly the spot price. This means that with a positive probability each time period the entering trader has a higher probability of predicting the spot price than the previously entered traders. This is the crucial assumption used in Luo (1998) to show that with probability 1, the proportion of times that the futures price equals to the spot price converges to one as time goes to infinity. The assumption used this paper is less restrictive in the sense that the allowance for the buyers (sellers) who do not overpredict (underpredict) the spot price very much and who have a strictly positive probability of predicting the spot price would drive the convergence of the futures price to the spot price. The intuition behind this convergence of the futures price to the spot price is as follows. Notice that if a buyer (or a seller) overpredicts (or underpredicts) the spot price, then potentially they could make a loss. Hence, the buyers (or sellers) who happen to have a low probability of overpredicting (or underpredicting) the spot price would not make losses very much. Those traders resemble the real world conservative traders who try to avoid losses by not overbidding the spot price as a buyer nor underbidding the spot price as a seller. For the sake of discussion, a buyer is called more conservative if this buyer has a lower probability of overpredicting the spot price than other buyers provided that this buyer has a strictly positive probability (however small) of predicting correctly the spot price; similarly, a seller is called more conservative than other sellers if this seller has a lower probability of underpredicting the spot price than other sellers provided that this seller has a strictly positive probability (however small) of predicting correctly the spot price. If there is a positive probability each time period that the entering buyer (or seller) has a lower probability of overpredicting (or underpredicting) the spot price than previous entered buyers (or sellers), then more conservative buyers (or sellers) will eventually enter the markets. Assuming that those conservative traders have a strictly positive probability (however small) of predicting correctly the spot price, then those traders will gradually take wealth away from the previously entered traders. Over times, this constant wealth redistribution process rewards more and more conservative buyers (or sellers) who eventually come to dominate the markets and set the futures price at the spot price.

The intention of this paper is to illustrate through simulations the intuition behind this result. In other words, this paper examines by way of simulations the convergence and the cause of the convergence of the futures price to the spot price, in the presence of conservative traders.

Section 3 provides the details of the simulation model and its results.

3. The Simulation Model

The following simulates the model in Section 2. It numerically illustrates how the conservative traders impact the markets and whether conservative traders could survive in the long run.To highlight the importance of conservative traders, the simulation model will restrict participants' predictions to be accurate with a small probability. The following numerical illustrations characterize a futures and spot market in the presence of conservative traders. The characteristics are as follows.

(1) The spot price is randomly drawn from $\{2, 4, 6, 8, 10\} = \{d, 2d, 3d, 4d, 5d\}$, where $d = 2$.
(2) The aggregate quantity of futures contracts supplied by producers at time t (S_t) is a random draw from [0.25,1} each time period.
(3) All traders are initially endowed with wealth equal to 0.2 ; at each time period the entering trader has an equal chance of being a buyer and a seller; and the fraction of the trader's wealth allocated to trading activities is randomly drawn from a uniform distribution over (0, 1].
(4) Trader t's prediction error distribution, assigned in his or her entry time period is $(\theta_1^t, \theta_2^t, \theta_3^t)$ where θ_2^t is randomly drawn from a uniform distribution over $(0, 0.1)$. This constraint departs from the model of Luo (1998)

where an entering trader could have an arbitrarily high probability of predicting the spot price. θ_1^t is randomly drawn from a uniform distribution over $(0, 1 - \theta_2^t)$ and $\theta_3^t = 1 - \theta_1^t - \theta_2^t$. In the period t and in subsequent time period s $(s > t)$ the probability of drawing $v_s^t > 0$, $v_s^t = 0$ and $v_s^t < 0$ is determined by θ_1^t, θ_2^t and θ_3^t, respectively. If in time period s, for trader t, an underprediction is realized, v_s^t is assigned to $-4d$, $-3d$, $-2d$, or $-d$ according to a uniform distribution. Similarly, if an overprediction is realized, v_s^t is assigned to $4d$, $3d$, $2d$, or d according to a uniform distribution. The prediction $b_s^t = \max[p_s + v_s^t, d]$.

One thousand simulations were conducted with this model and the markets are followed from time period 1 to 7500. The percentage of times that the futures price equals to the spot price between time period 50 and 7500 are shown in Fig. 3. While at time period 50 for 31.5% of the simulations the futures price equals the spot price, at time periods 5000 and 7500 the futures price equals the spot price for 96.2% and 98.3% of the simulations, respectively. It is clear that the futures price converges to the spot price. Even though traders are restricted to an arbitrarily small probability of accurately predicting the spot price, convergence still occurs. (Of course, if this restriction were lifted, convergence would occur faster.)

To provide some further insight into the causes of the convergence, Tables 1–3 describe the distribution of traders by wealth at time period 100, 1000, and 7500, respectively. For assembling these table all traders are initially ranked by their wealth from lowest to highest. The total wealth of all traders at

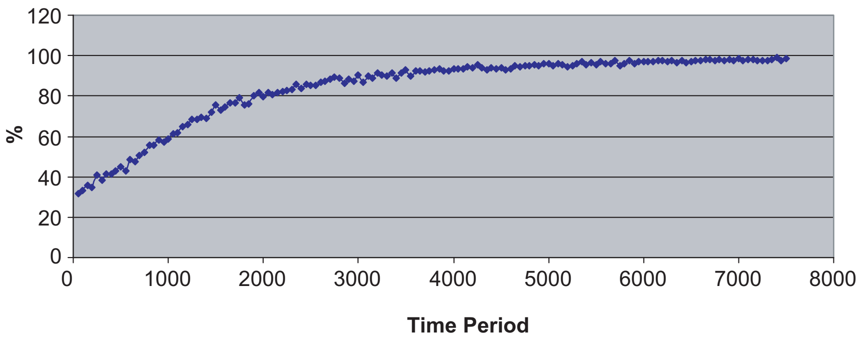

Figure 3. The percentage of times that futures price equals to the spot price.

time period 100, 1000, and 7500 was calculated and divided into eight groups (listed in column 1 of Tables 1–3, respectively). The cumulative percentage of wealth at time period 100, 1000, and 7500 is shown in column 2 of Tables 1–3. The complete list of cumulative percentage of traders accounting for cumulative percentages of wealth shown in column 2 are listed in the corresponding column 3 of Tables 1–3. The percentage of traders whose wealth belongs to the corresponding wealth groups is displayed in the corresponding column 4 of those tables. For example, in column 4 of Table 1, at time period 100, there are 31.2% of the traders whose wealth fall into the first wealth group. Column 5 lists traders' conservativeness, which refers to the weighted average of the probability of overpredicting by the buyers and the probability of underpredicting by the sellers (the weights being the proportion of the buyers and the

Table 1. Distribution of traders by wealth at time period 100.

Wealth group	Cumulative percentage of wealth	Cumulative percentage of traders	Percentage of traders	Traders' conservativeness	Probability of correct prediction
1 (lowest)	5.0	31.20	31.20	55.11	4.85
2	10.0	52.70	21.50	43.67	4.99
3	25.0	85.80	33.10	43.21	5.08
4	50.0	96.12	10.32	46.54	5.16
5	75.0	98.67	2.55	45.38	5.20
6	90.0	99.51	0.84	44.97	5.34
7	95.0	99.75	0.24	44.43	5.37
8 (highest)	100.0	100.0	0.25	44.44	5.36

Table 2. Distribution of traders by wealth at time period 1000.

Wealth group	Cumulative percentage of wealth	Cumulative percentage of traders	Percentage of traders	Traders' conservativeness	Probability of correct prediction
1 (lowest)	5.0	32.53	32.53	60.36	4.57
2	10.0	46.05	13.52	54.19	4.76
3	25.0	76.40	30.35	41.93	5.18
4	50.0	94.78	18.38	32.67	5.58
5	75.0	98.96	4.18	21.95	5.94
6	90.0	99.76	0.80	11.28	6.14
7	95.0	99.90	0.14	5.73	6.17
8 (highest)	100.0	100.00	0.10	4.48	6.13

Table 3. Distribution of traders by wealth at time period 7500.

Wealth group	Cumulative percentage of wealth	Cumulative percentage of traders	Percentage of traders	Traders' conservativeness	Probability of correct prediction
1 (lowest)	5.0	34.16	34.16	57.07	4.70
2	10.0	45.01	10.85	55.09	4.77
3	25.0	71.26	26.25	47.56	5.00
4	50.0	96.37	25.11	32.07	5.51
5	75.0	99.58	3.21	17.87	5.99
6	90.0	99.91	0.33	9.55	5.94
7	95.0	99.97	0.06	4.89	6.03
8 (highest)	100.0	100.00	0.03	2.41	6.11

proportion of the sellers). The lower is the weighted average of the probability of overpredicting by the buyers and probability of underpredicting by the sellers, the more conservative the traders are as a group. For example, in column 5 of Table 1, there is 55.11% weighted average of probability of overpredicting by the buyers and probability of underpredicting by the sellers in the lowest wealth group at time period 100. The last column lists the average probability of correct prediction for the traders in each of the wealth groups. For example, the last column of Table 1 indicates that in the lowest wealth group, where there are 31.2% of the traders, the average probability of traders' correct prediction of the spot price is 4.85%. (Recall that the simulations constrain the probability of correct predictions to be less than 10%.)

First of all, from Tables 1–3, it can be seen that the average probabilities of traders' correct prediction of the spot price across the wealth groups are similar at time periods 100, 1000, and 7500. However, traders' conservativeness across wealth groups shifts through time periods 100, 1000, and 7500. For example, traders' conservativeness in the highest wealth group decreases from 44.44% at time period 100 to 4.48% at time period 1000 and further decreases to 2.41% at time period 7500. This suggests that it is the more conservative traders who eventually own the top 5% of wealth. Furthermore, the total wealth distribution across traders shifts as traders' conservativeness across the wealth groups shifts although traders' correct prediction probability distributions are similar at time periods 100, 1000, and 7500. At time periods 100, 1000, and 7500, in the highest wealth group (group 8), the percentage of traders owning the top 5% of total wealth decreases from 0.25% at time period 100 to 0.1%

at time period 1000 and further decreases to 0.03% at time period 7500 as the traders' conservativeness in the highest wealth group decreases from 44.44% at time period 100 to 4.48% at time period 1000 and further decreases to 2.41% at time period 7500. This trend indicates that the wealth is redistributing over times towards more conservative traders away from less conservative traders.

Table 4 shows the distribution of the traders according to conservativeness at time periods 100, 1000, and 7500. Notice that this distribution remains stable through time. For example, the percentage of traders over- (under-) predicting in the case of buyers (sellers) less than 5% of the time is between 2.76% and 2.79%. Table 5 describes the distribution of the probabilities of traders' correct prediction across traders' conservativeness groups, which is also very stable across time periods 100, 1000, and 7500. However, the wealth distribution across traders' conservativeness does shift across time periods 100, 1000, and 7500. This can be seen from Table 6. In Table 6, the total wealth distribution across traders shifts from the less conservative traders group to the more conservative traders group. For example, the group of traders with traders' conservativeness between 0.5 and 1.00 has 42.38% of total wealth at time period 100. By the time period 1000 and 7500 their wealth is reduced to 16.17% and 16%, respectively. While the most conservative group of traders with traders' conservativeness between 0.00 and 0.05 owns 3.82% of total wealth at time period 100, their wealth increases to 17.66% of total wealth at time period 1000 and further increases to 19.99% of total wealth at time period 7500.

One can also organize a similar sets of tables by separating the buyers from the sellers. In other words, all traders are separated into a buyer group and a seller group. In the buyer group (seller group) all buyers (sellers) are ranked by their wealth from the lowest to the highest. Total wealth of all buyers (sellers) at time periods 100, 1000, and 7500 was calculated and divided into eight groups. A set of tables similar to Tables 1–6 are put together in

Table 4. Distribution of traders by traders' conservativeness.

Traders' conservativeness	Period 100	Period 1000	Period 7500
0.5–1.0	44.94	44.93	45.00
0.25–0.50	37.28	37.19	37.13
0.10–0.25	11.83	11.99	11.98
0.05–0.10	3.20	3.09	3.10
0.00–0.05	2.76	2.79	2.79

Table 5. Distribution of traders' probability of correct prediction.

Traders' conservativeness	Period 100	Period 1000	Period 7500
0.5–1.0	4.80	4.82	4.81
0.25–0.50	5.16	5.17	5.17
0.10–0.25	5.11	5.13	5.13
0.05–0.10	5.04	5.12	5.10
0.00–0.05	5.30	5.11	5.09

Table 6. Distribution of wealth by traders' conservativeness.

Traders' conservativeness	Period 100	Period 1000	Period 7500
0.5–1.0	42.38	16.17	16.00
0.25–0.50	35.73	30.50	27.01
0.10–0.25	13.18	24.18	24.50
0.05–0.10	4.88	11.49	12.49
0.00–0.05	3.82	17.66	19.99

Tables 7–12 for the buyer group and in Tables 13–18 for the seller group. In those tables, the buyers' conservativeness refers to the average probability of buyers' overprediction of the spot price. The sellers' conservativeness refers to the average probability of sellers' underprediction of the spot price. Again, as the probability of overpredicting (underpredicting) for buyers (sellers) decreases the buyers' (sellers') conservativeness increases. Both sets of tables reveal the same numerical results as the above.

Table 7. Distribution of buyers by wealth at time period 100.

Wealth group	Cumulative percentage of wealth	Cumulative percentage of buyers	Percentage of buyers	Buyers' conservativeness	Probability of correct prediction
1 (lowest)	5.0	39.15	39.15	47.69	4.89
2	10.0	60.15	20.99	46.82	5.01
3	25.0	83.87	23.72	47.02	5.07
4	50.0	93.87	10.00	47.24	5.23
5	75.0	97.71	3.84	45.38	5.20
6	90.0	99.13	1.42	44.97	5.34
7	95.0	99.56	0.44	44.44	5.37
8 (highest)	100.0	100.00	0.44	44.44	5.36

Table 8. Distribution of buyers by wealth at time period 1000.

Wealth group	Cumulative percentage of wealth	Cumulative percentage of buyers	Percentage of buyers	Buyers' conservativeness	Probability of correct prediction
1 (lowest)	5.0	41.39	41.39	54.04	4.61
2	10.0	54.94	13.55	48.98	4.78
3	25.0	80.91	25.97	41.86	5.30
4	50.0	94.94	14.03	33.10	5.82
5	75.0	98.82	3.87	22.50	6.23
6	90.0	99.69	0.88	11.66	6.36
7	95.0	99.86	0.17	5.41	6.22
8 (highest)	100.0	100.00	0.14	4.33	6.16

Table 9. Distribution of buyers by wealth at time period 7500.

Wealth group	Cumulative percentage of wealth	Cumulative percentage of buyers	Percentage of buyers	Buyers' conservativeness	Probability of correct prediction
1 (lowest)	5.0	41.79	41.79	53.36	4.78
2	10.0	51.42	9.63	50.97	4.83
3	25.0	71.35	19.94	48.44	4.91
4	50.0	93.58	22.23	34.23	5.49
5	75.0	99.17	5.59	21.05	6.24
6	90.0	99.85	0.68	8.74	6.57
7	95.0	99.95	0.09	2.86	6.53
8 (highest)	100.0	100.00	0.05	1.32	6.42

Table 10. Distribution of buyers by buyers' conservativeness.

Buyers' conservativeness	Period 100	Period 1000	Period 7500
0.5–1.0	44.70	44.83	45.01
0.25–0.50	37.48	37.24	37.12
0.10–0.25	11.90	12.03	11.98
0.05–0.10	3.23	3.11	3.10
0.00–0.05	2.69	2.78	2.79

Evidently, across time periods 100, 1000, and 7500, the wealth has been shifted from less conservative traders to the more conservative traders. This constant wealth redistribution over time from the less conservative traders to the more conservative traders serves as a natural selection in the markets.

Table 11. Distribution of buyers' probability of correct prediction.

Buyers' conservativeness	Period 100	Period 1000	Period 7500
0.5–1.0	4.81	4.87	4.81
0.25–0.50	5.12	5.16	5.16
0.10–0.25	5.12	5.14	5.13
0.05–0.10	5.19	5.13	5.10
0.00–0.05	5.25	5.11	5.09

Table 12. Distribution of wealth by buyers' conservativeness.

Buyers' conservativeness	Period 100	Period 1000	Period 7500
0.5–1.0	43.07	16.49	17.98
0.25–0.50	35.43	29.98	27.82
0.10–0.25	12.93	22.33	20.93
0.05–0.10	4.87	11.06	10.89
0.00–0.05	3.70	2.01	2.24

Table 13. Distribution of sellers by wealth at time period 100.

Wealth group	Cumulative percentage of wealth	Cumulative percentage of sellers	Percentage of sellers	Sellers' conservativeness	Probability of correct prediction
1 (lowest)	5.0	11.92	11.92	63.27	4.74
2	10.0	18.90	6.98	59.67	4.80
3	25.0	36.00	17.10	56.31	4.85
4	50.0	60.79	24.79	49.38	4.98
5	75.0	83.13	22.34	39.69	5.11
6	90.0	94.38	11.25	31.77	5.16
7	95.0	97.44	3.06	25.24	5.31
8 (highest)	100.0	100.00	2.56	20.73	5.39

Eventually, the buyers with lower and lower probability of overprediction or the sellers with lower and lower probability of underprediction are gradually rewarded with wealth and come to dominate the markets and set the futures price to the spot price. This constant wealth redistribution over times towards the more conservative traders is the critical factor accounting for the convergence.

Table 14. Distribution of sellers by wealth at time period 1000.

Wealth group	Cumulative percentage of wealth	Cumulative percentage of sellers	Percentage of sellers	Sellers' conservativeness	Probability of correct prediction
1 (lowest)	5.0	23.48	23.48	66.82	4.51
2	10.0	36.75	13.26	58.89	4.74
3	25.0	69.35	32.60	43.58	5.12
4	50.0	93.80	24.45	32.28	5.39
5	75.0	98.91	5.10	20.37	5.58
6	90.0	99.73	0.82	11.64	5.89
7	95.0	99.87	0.15	8.03	5.96
8 (highest)	100.0	100.00	0.13	7.56	5.99

Table 15. Distribution of sellers by wealth at time period 7500.

Wealth group	Cumulative percentage of wealth	Cumulative percentage of sellers	Percentage of sellers	Sellers' conservativeness	Probability of correct prediction
1 (lowest)	5.0	24.58	24.58	63.51	4.56
2	10.0	37.27	12.69	57.94	4.75
3	25.0	70.55	33.29	45.40	5.08
4	50.0	97.25	26.69	30.68	5.44
5	75.0	99.57	2.32	17.60	5.69
6	90.0	99.89	0.32	8.85	5.88
7	95.0	99.95	0.06	4.38	6.01
8 (highest)	100.0	100.00	0.05	2.46	6.12

Table 16. Distribution of sellers by sellers' conservativeness.

Sellers' conservativeness	Period 100	Period 1000	Period 7500
0.5–1.0	45.17	45.02	44.99
0.25–0.50	37.10	37.14	37.13
0.10–0.25	11.77	11.95	11.99
0.05–0.10	3.15	3.08	3.10
0.00–0.05	2.82	2.80	2.79

Table 17. Distribution of sellers' probability of correct prediction.

Sellers' conservativeness	Period 100	Period 1000	Period 7500
0.5–1.0	4.80	4.81	4.82
0.25–0.50	5.17	5.17	5.17
0.10–0.25	5.06	5.12	5.13
0.05–0.10	4.98	5.11	5.10
0.00–0.05	5.21	5.09	5.08

Table 18. Distribution of wealth by sellers' conservativeness.

Sellers' conservativeness	Period 100	Period 1000	Period 7500
0.5–1.0	38.81	16.01	15.04
0.25–0.50	38.47	31.47	26.68
0.10–0.25	14.52	26.41	26.37
0.05–0.10	4.24	11.96	13.32
0.00–0.05	3.96	14.15	18.60

4. Conclusion

While one might initially feel that conservative traders may not be aggressive enough in their bidding for survival nor aggressive enough to always participate in the market, the presence of a large number of conservative traders means the occurrence of market efficiency. Because of traders' conservativeness, these traders seldom make mistakes to sufficiently drive their wealth to zero. Furthermore, because conservative traders seldom make very large losses, in the long run their market share of aggregate wealth steadily grows, and it is their aggregate participation which causes the market to converge to market efficiency. Unlike Luo (1998), it is illustrated that it is not necessary for some traders to be perfectly informed price predictors in order for market efficiency to occur or in order for the traders to survive; rather, what is sufficient is that there is a sufficiently large number of conservative traders who seldom make significant losses and who make correct predictions of the spot prices with some probability (however small). The natural selection in the markets, taking the form of the constant wealth redistribution among traders, gradually rewards conservative traders with wealth by taking wealth away from aggressive traders. This constant wealth redistribution process eventually ensures the existence of long-run efficient markets.

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Chapter 11

Speculative Nonfundamental Components in Mature Stock Markets: Do they Exist and are they Related?

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Economists have long conjectured that movements in stock prices may involve speculative components, called bubbles. A bubble is defined as the difference between the market value of a security and its fundamental value. The topic of asset bubbles remains controversial because the existence of a bubble is inherently an empirical issue and no satisfactory test has yet been devised to estimate the magnitude of a bubble. This paper proposes a new methodology for testing for the existence of rational bubbles. Unlike previous authors, we treat both the dividend that drives the fundamental part and the nonfundamental process as part of the state vector. This new methodology is applied to the four mature markets of the US, Japan, England, and Germany to test whether a speculative component was present during the period of January 1951 to December 1998 in these markets. The paper also examines whether there are linkages between these national speculative components. We find evidence that the nonfundamental component in the US market causes the other three markets but we find no evidence for reverse causality.

Keywords: Kalman filter; mature stock markets; speculative bubbles.

1. Introduction

Campbell (2000) concludes that many economists accept market efficiency as the well-established paradigm of finance but also acknowledge that asset prices are too volatile. For example, NASDAQ, from its peak on March 10, 2000 when it stood at 5,048.62 to its low of 1,454.04 on September 21, 2001, declined by 71.25%. Is this significant decline caused only because of substantial revisions of the market fundamentals or can it be viewed as the bursting of a speculative bubble?

The literature on bubbles or speculative components follows two broad methodologies: speculative components are assumed to be rational or irrational.

The rational bubble approach is presented in Blanchard and Fisher (1989) who argue that in solving the difference equation that arises from asset price arbitrage conditions, the efficient market solution is not unique. There is also a rational bubble solution with asset prices deviating from fundamentals and expected to grow at the rate of return of the riskless asset. In contrast to the rational bubble approach, Keynes (1936) postulated the "irrational bubble" approach where asset prices are driven by "animal spirits". In this paper, we only consider "rational bubbles" and use the terms "rational bubble" and "speculative nonfundamental component" interchangeably.

This paper modifies certain empirical methodologies to test for the presence of rational bubbles and then explores the relationship among these bubbles in mature stock markets. Section 2 introduces general ideas about asset bubbles. In Section 3, we motivate our empirical methodology by reviewing the important contributions of Wu (1995, 1997) and other econometricians. Section 4 examines global stock market integration and Section 5 highlights our new methodology for testing for the existence of asset bubbles and then applies it to the stock markets of the US, Japan, England, and Germany. We find evidence of asset bubbles in all four stock markets. We then proceed to examine whether these bubbles travel across mature economies. Our main findings and conclusions are given in the last section.

2. Rational Asset Bubbles

The standard definition of the fundamental value of an asset is the summed discounted value of all future cash flows generated by such an asset. The difference, if any, between the market value of the security and its fundamental value is termed a speculative bubble. The existence of bubbles is inherently an empirical issue that has not been settled. A number of studies such as Blanchard and Watson (1982) and West (1988) have argued that dividend and stock price data are not consistent with the "market fundamentals" hypothesis, in which prices are given by the present discounted values of expected dividends. These results have often been construed as evidence for the existence of bubbles or fads.

In addition, Shiller (1981) and LeRoy and Porter (1981) have argued that the variability of stock price movements is too large to be explained by the discounted present value of future dividends. Over the past century US stock prices are 5–13 times more volatile than can be justified by new information about future dividends. Campbell and Shiller (1988a, b) and West (1987, 1988) remove the assumption of a constant discount rate. However, a variable discount rate provides only marginal support in explaining stock price volatility. These authors reject the null hypothesis of no bubbles.

Wu (1997) examines a rational bubble, able to burst and restart continuously. The specification is parsimonious and allows easy estimation. The model fits the data reasonably well, especially during several bull and bear markets in this century. Such rational bubbles can explain much of the deviation of US stock prices from the simple present-value model. Wu's work is reviewed in more detail in the next section. Beyond the existence or not of bubbles, economists have also studied in detail the implications of a stock market bubble to the economy at large. Binswanger (1999) offers a comprehensive review of these issues and Chirinko and Schaller (1996) argue that bubbles have existed over certain periods in the US stock market but real investment decisions have been determined by fundamentals. Hayford and Malliaris (2001) argue that easy monetary policy may have contributed to the US stock market bubble during 1995–2000.

3. Review of Key Empirical Papers

To motivate our methodological contribution to testing for asset bubbles, we examine few influential papers in this area.

3.1. *Flood and Garber (1980)*

Flood and Garber (1980) test the hypothesis that price-level bubbles did not exist in a particular historical period. The existence of a price-level bubble places such extraordinary restrictions on the data that such bubbles are not an interesting research problem during normal times. Since hyperinflations generated series of data extraordinary enough to admit the existence of a price-level bubble, the German episode is an appropriate and interesting period to search for bubbles. The authors build a theoretical model of hyperinflation in which they allow price-level bubbles. Then, they translate the theoretical model into data restrictions and use these restrictions to test the hypothesis that price-level bubbles were not partly responsible for Germany's massive inflation during the early 1920s.

Cagan (1956) used the following monetary model in his study of seven hyperinflations:

$$m_t - p_t = y + \alpha\pi_t + \varepsilon_t, \quad \alpha < 0, \quad t = 1, 2, 3, \ldots.. \tag{1}$$

The variables m and p are the natural logarithms of money and price at time t. The anticipated rate of inflation between t and $t+1$ is π and ε is a stochastic disturbance term. The rational-expectations assumption requires $\pi_t = E(\pi_t|I_t)$, where $\pi_t = p_{t+1} - p_t$ is the mathematical expectations operator, and I is the information set available for use at time t.

The solution of Equation (1) is

$$p_t = -\alpha A_0 \psi^t + \left[m_t - y + \psi^{-1} \sum E(\psi_{t+1} - w_{t+i} II_t)\psi^{-1} - \varepsilon_t\right],$$

where $\psi \equiv (\alpha - 1)/\alpha > 1$, $\mu_{t+i} = m_{t+i+1} - m_{t+i}$, $w_{t+i} = \varepsilon_{t+i+1} - \varepsilon_{t+i}$ and A is an arbitrary constant.

For this model, market fundamentals are defined as

$$\left[m_t - y + \psi^{-1} \sum E(\psi_{t+1} - w_{t+i} II_t)\psi^{-1} - \varepsilon_t\right],$$

price-level bubbles are then captured by the term $-\alpha A_0 \psi^t$.

Rational-expectations models normally contain the assumption $A = 0$, which prevents bubbles. Notice that if $A \neq 0$, then the price will change with t even if market fundamentals are constant. The definition of a price-level bubble as a situation in which $A \neq 0$ is appropriate for two reasons. First, A is an arbitrary and self-fulfilling element in expectations. Second, if $A \neq 0$, then agents expect prices to change through time at an ever-accelerating rate, even if market fundamentals do not change. Since economists usually consider price bubbles to be episodes of explosive price movement that are unexplained by the normal determinants of market price, $A \neq 0$ will produce a price-level bubble. The results of the empirical analysis presented by Flood and Garber support the hypothesis of no price-level bubbles.

3.2. *West (1987)*

The test compares two sets of estimates of the parameters needed to calculate the expected present discounted value (PDV) of a given stock's dividend stream, with expectations conditional on current and all past dividends. In a constant discount rate model the two sets are obtained as follows. One set is obtained by regressing the stock price on a suitable set of lagged dividends. The other set is obtained indirectly from a pair of equations. One is an arbitrage equation yielding the discount rate, and the other is the ARIMA equation of the dividend process.

Under the null hypothesis that the stock price is set in accord with a standard efficient markets model, the regression coefficients in all equations may be estimated consistently. When the two sets of estimates of the expected PDV parameters are compared, then, they should be the same, apart from sampling error. This equality will not hold under the alternative hypothesis that the stock price equals the sum of two components: the price implied by the efficient markets model and a speculative bubble.

A stock price is determined by the arbitrage condition:

$$p_t = bE(p_{t+1} + d_{t+1})|I_t, \tag{2}$$

where p is the real stock price in period t, b the constant *ex ante* real discount rate, $0 < b = 1/(1+r) < 1$, r the constant expected return, E is mathematical expectation, d is the real dividend paid in period $t+1$, and I denotes information common to traders at period t.

As long as the transversality condition $\lim_{n\to\infty} b^n E[p_{t+n}|I_t] = 0$ holds, the unique forward solution to Equation (1) is $p_t^* \sum b^i E[d_{t+i}|I_t]$. If this condition fails, there is a family of solutions to Equation (2). Any p that satisfies $p_t = p_t^* + c_t$, $E[c_t|I_{t-1}] = b^{-1}c_{t-1}$ is also a solution. c is by definition a speculative bubble. The aim of West is to test $p_t = p_t^*$, versus $p_t = p_t^* + c$.

Checking for the equality of the two sets in long-term annual data on the Standard and Poor's 500 Index (1871–1980) and the Dow Jones Index (1928–1978), the author finds that the null hypothesis of no bubbles is rejected and the coefficients in the regression of price on dividends are biased upwards.

3.3. *Ikeda and Shibata (1992)*

Using a stochastic dividend-growth model, the paper provides a general analysis of fundamental-dependent bubbles in stock prices. Given that dividends follow a continuous Markov process, a stock price is specified as a function of dividends as well as of time. The authors derive a partial differential equation with respect to this price function from an arbitrage equation. Provided that a free-disposal condition is satisfied, a fundamental price process is defined as the forward-looking particular solution of this equation and a price bubble as the general solution of the corresponding homogeneous equation.

Consider a stock share that yields dividends $D(t)$ at time t. These dividends follow a geometric Brownian motion with positive drift:

$$\begin{aligned} \mathrm{d}D(t) &= gD(t)\,\mathrm{d}t + \sigma D(t)\mathrm{d}z(t), \\ D(0) &= D_0, \quad g - \sigma^2/2 > 0, \quad \sigma > 0. \end{aligned} \tag{3}$$

The constants g and σ are, respectively, the expected value and the standard deviation of the instantaneous rate of dividend growth. dz is an independent increment of a standard Wiener process, z, with the initial condition $z(0) = 0$. Since ln D follows a normal distribution, the time series of dividend payments have a positive trend.

The stochastic dividend-payment process described in Equation (3) is the only source of randomness. Assume risk neutrality of investors, free disposability of the stock and also that the cum-dividend stock price is determined by the following two conditions:

$$E[\mathrm{d}P(t)|\Omega_t]/\mathrm{d}t + D(t) = rP(t), \tag{4}$$

$$P(t) \geq 0, \quad \text{with } r > 0 \ \ \forall t \in [0, \infty]\text{w.p.1}, \tag{5}$$

where, $E[\cdot|\Omega_t]$ represents mathematical expectations conditional on Ω, and parameter r denotes the constant riskless interest rate.

The rational expectations stochastic process of the stock price is obtained then by solving the nonhomogeneous partial differential Equation (4), subject to the dividend payment process (3) and the price positivity condition (5). The authors find that the fundamentals dependency stabilizes bubble dynamics and that stock prices with fundamentals-dependent bubbles can be less volatile than fundamentals. Furthermore, fundamentals-dependent bubbles exhibit various transition patterns, such as nonmonotonic movements and monotonic shrinkage in magnitude and volatility.

3.4. *Wu (1997)*

The paper estimates a rational stochastic bubble using the Kalman filtering technique. The bubble grows at the discount rate in expectation and it can collapse and restart continuously, allowing for the possibility of a negative bubble. The log of dividends follows a general ARIMA (p, 1, q) process. The model for stock prices with the bubble component, the dividend process and the bubble process are expressed in the state-space form with the bubble being treated as an unobserved state vector. The model parameters are estimated by the method of maximum likelihood and obtain optimal estimates of stochastic bubbles through the Kalman filter.

Consider the standard linear rational expectations model of stock price determination:

$$[E_t(P_{t+1} + D_t) - P_t]/P_t = r, \tag{6}$$

where p is the real stock price at time t, D is the real dividend at time t, E is the mathematical expectation conditional on information available at time t and r is the required real rate of return, $r > 0$. The log-linear approximation of

Equation (6) can be written as follows:

$$q = k + \psi E_t p_{t+1} + (1 - \psi) d_t - p_t, \tag{7}$$

where q is the required log gross return rate, Ψ is average ratio of the stock price to the sum of the stock price and the dividend, $k = -\ln(\Psi) - (1 - \Psi)\ln(1/\Psi - 1)$, $p = \ln(P)$, and $d = \ln(D)$.

The general solution to Equation (7) is given by

$$p_t = (k - q)/(1 - \psi) + (1 - \psi) \sum_{1=0}^{\infty} \psi^i E_t(d_{t+i}) + b_t = p_t^{\mathrm{f}} + b_t, \tag{8}$$

where b_t satisfies the following homogeneous difference equation:

$$E_t(b_{t+i}) = (1/\psi)^i b_t. \tag{9}$$

In Equation (7), the no-bubble solution p is exclusively determined by dividends, while b can be driven by events extraneous to the market and is referred to as a rational speculative bubble. After defining the stock price equation, the parametric bubble process and the dividend process in a state-space form, the bubble is treated as an unobserved state vector, which can be estimated by the Kalman filtering technique.

Wu finds statistically significant estimate of the innovation variance for the bubble process. During the bull market of the 1960s, the size of the bubble is 40–50% of the stock price. Negative bubbles are found during the 1919–1921 bear market, in which case the bubble explains 20–30% of the decline in stock prices.

3.5. *Wu (1995)*

The model reviewed in the previous Section 3.4 has also been used by the same author to estimate the unobserved nonfundamental component of the exchange rate and to test whether it is significantly different from zero. Using the monetary model of exchange rate determination, the solution for the exchange rate is the sum of two components. The first component, called the fundamental solution, is a function of the observed market fundamental variables. The second component is an unobserved process, which satisfies the monetary model and is called the stochastic bubble. The monetary model, the market fundamental process and the bubble process are expressed in the state-space form, with the bubble being treated as a state variable. The Kalman filter can then be used to estimate the state variable.

The author finds no significant estimate of a bubble component during the period 1974–1988. Similar results were obtained for the subsample, 1981–1985, during which the US dollar appreciated most drastically and a bubble might have occurred.

Sections 5–7 elaborate our extensions of the five key papers reviewed in this section. The added advantage of our methodological innovation is that it allows us to test for possible linkages between national bubbles. Section 4 describes the rationale for searching for linkages between national bubbles.

4. Global Stock Market Integration

Once bubbles are confirmed empirically in the four mature stock markets, we proceed to test linkages between these markets in terms of these nonfundamental components. In this context, we adopt a subset VAR methodology presented in Lutkepohl (1993, p. 179). The approach builds into it the causal relations between the series and this gives us the opportunity to analyze the potential global interaction among these national equity markets through the speculative component of the prices. The potential existence of global linkages among equity markets has attracted great interest among scholars because of its impact on global diversification.

During the past 30 years, world stock markets have become more integrated, primarily because of financial deregulation and advances in computer technology. Financial researchers have examined various aspects of the evolution of this particular aspect of world integration. In analyzing the results of such studies, one could deduce that greater global integration implies lesser benefits from international portfolio diversification. If this is true, how can one explain the ever-increasing flow of big sums of money invested in international markets? To put differently, while Tesar and Werner (1992) confirm the home bias in the globalization of stock markets, why are increasing amounts of funds invested in nonhome equity markets?

The analysis of the October 19, 1987 stock market crash may offer some insight in answering this question. Roll (1988, 1989), King and Wadhwani (1990), Hamao, Musulis, and Ng (1990), and Malliaris and Urrutia (1992) confirm that almost all stock markets fell together during the October 1987 crash despite the existing differences of the national economies while no significant interrelationships seem to exist for periods prior and post the crash. Malliaris and Urrutia (1997) also confirm the simultaneous fall of national stock market returns because of the Iraqi invasion of Kuwait in July 1990. This evidence supports the hypothesis that certain global events, such as the crash of October 1987 or the invasion of Kuwait in July 1990, tend to move world equity markets in the same direction, thus reducing the effectiveness of international diversification. On the other hand, in the absence of global events, national markets are dominated by domestic fundamentals, and international investing increases the benefits of diversification. Exceptions exist, as in the case of regional markets, such as the European stock markets reported in Malliaris and Urrutia (1996).

A review of the literature on linkages among international stock markets can be found in McCarthy and Najand (1995). These authors adopt the state space methodology to infer the linkage relationships between the stock markets in Canada, Germany, Japan, UK, and the US. The authors claim that this approach not only determines the causal relationship, in the Granger sense, but it delivers the result with minimum number of parameters necessary. They report that the US market exerts the most influence on other markets. Since these authors use daily data, there is some overlap in the market trading time and they attempt to take care of that in the interpretation of their results. The main finding is consistent with similar findings by other researchers, such as, Eun and Shim (1989), who examine nine stock markets in the North America and Europe over period 1980–1985 in a VAR framework.

From this rapid review of global stock market integration, it becomes apparent that the topic of linkages between bubbles has not been addressed. Our methodology for testing the existence of bubbles in national markets has the additional advantage that it renders itself for also testing for possible linkages between bubbles in these mature stock markets. We augment our contribution to the literature by exploring this issue also.

5. Our Methodological Contribution

The purpose of our study is to search empirically for fundamental and nonfundamental components in the national stock markets of the US, Japan, Germany, and the UK, using a state-of-the-art econometric methodology. The word nonfundamental or bubble in this context implies the deviation of the observed stock price from the fundamental part driven by the dividend process. Once this nonfundamental part is estimated we investigate how this might be traveling between these four markets.

We focus on the postwar period in these four countries as opposed to the authors reviewed in the previous section who concentrate on only the US.

All data are monthly returns of the S&P 500, Nikkei 225, Dax-30, and FT-100 indexes ranging from January 1951 to December 1998, that is, 576 observations. All data are converted to real values using the corresponding CPI measures and Global Financial Data provided the data. In order to establish the soundness of our methodology we have reproduced the results from Wu (1997) using annual US data (also obtained from Global Financial Data) covering the period 1871–1998. These results are available in Bhar and Malliaris (2001).

Since the nonfundamental part is not observed, the modeling problem is necessarily that of a partially observed system. Wu (1997) employs a similar concept but our implementation is quite different as described in Bhar and Malliaris (2001). We follow Shumway and Stoffer (2000, p. 306) to develop a Dynamic Linear Model, DLM, to treat both the dividend process and the nonfundamental process as part of the unobserved components, the state vector. These states are filtered out of the observations that include the observed dividend and the price, which form the measurement vector.

We also establish the superior performance of our stock price model with a nonfundamental component compared to the simple stock price model with a GARCH error. Our modeling approach makes this comparison straightforward within the same maximum likelihood framework. Wu (1997) does not report any model adequacy tests and the precise moment conditions needed in the GMM estimation are not reported either. On the other hand, our models are subjected to a battery of diagnostic tests applicable to partially observed state space systems. Since Bhar and Malliaris (2001) describe the details of the models we adopt, in Sections 6 and 7 we only outline briefly the essential elements of our approach. In Section 8, we describe the procedure for testing the propagation of the nonfundamental parts between the four countries.

6. Dynamic Linear Model with Nonfundamental Component

Our starting point is Equations (8) and (9) described earlier. As our preliminary investigations reveal that both the log real price and log real dividend series are nonstationary, we choose to work with the first differenced series. Thus, Equation (8) becomes,

$$\Delta p_t = \Delta p_t^{\mathrm{f}} + \Delta b_t, \tag{10}$$

where $\Delta p_t^{\mathrm{f}} \equiv (1-\psi)\sum_{i=0}^{\infty}\psi^i E_t(d_{t+i}) - (1-\psi)\sum_{i=0}^{\infty}\psi^i E_{t-1}(d_{t-1+i})$. Assume the parametric representation of Equation (9) is

$$b_{t+1} = \frac{1}{\psi}b_t + \varepsilon_\eta, \quad \varepsilon_\eta : N\big(0, \sigma_\eta^2\big), \tag{11}$$

$$\Delta b_t = \frac{1}{\psi}(b_t - b_{t-1}). \tag{12}$$

In order to express the fundamental component of the price, Δp_t^{f}, in terms of the dividend process, we fit an appropriate AR model of sufficient order so that the Akaike information criterion, AIC, is minimized. We find that for the Japanese data a AR(1) model is sufficient whereas for the other three countries we need AR(3) models. The infinite sums in the expression for Δp_t^{f} may be expressed in terms of the parameters of the dividend process once we note the following conditions. First, the differenced log real dividend series is stationary, therefore the infinite sum converges. Second, any finite order AR process can be expressed in companion form (VAR of order 1) by using extended state variables, i.e., suitable lags of the original variables (Campbell, Lo, and MacKinlay, 1997, p. 280). And third, using demeaned variables the VAR(1) process can be easily used for multiperiod ahead forecast (Campbell, Lo, and MacKinlay, 1997, p. 280).

Assuming the demeaned log real dividend process has the following AR(3) representation,

$$\Delta d_t = \phi_1 \Delta d_{t-1} + \phi_2 \Delta d_{t-2} + \phi_3 \Delta d_{t-3} + \varepsilon_\delta, \quad \varepsilon_\delta : N\left(0, \sigma_\delta^2\right) \tag{13}$$

the companion form may be written as,

$$\begin{bmatrix} \Delta d_t \\ \Delta d_{t-1} \\ \Delta d_{t-2} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta d_{t-1} \\ \Delta d_{t-2} \\ \Delta d_{t-3} \end{bmatrix} + \begin{bmatrix} \varepsilon_\delta \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

$$\text{or} \quad X_t = \Phi X_{t-1} + \Xi_t, \tag{15}$$

where the definitions of X_t, Φ, and Ξ_t are obvious from comparison of Equations (9) and (10). Following Campbell, Lo, and MacKinlay (1997, p. 280), Δp_t^{f} may be expressed as (with I being the identity matrix of the same dimension as Φ)

$$\Delta p_t^{\mathrm{f}} = \Delta d_t + \psi\Phi(I - \psi\Phi)^{-1}\Delta X_t. \tag{16}$$

We can now express Equation (5) in terms of the fundamental component and the bubble component,

$$\Delta p_t = \Delta d_t + e'\psi\Phi\,(I - \psi\Phi)^{-1}\,\Delta X_t + \Delta b_t, \tag{17}$$

where $e' \equiv [1 \quad 0 \quad 0]$.

Equations (12), (14), and (17) can now be set up as a DLM and this is shown in Appendix A. The details of the estimation procedure of such a DLM are described in Appendix C.

7. Dynamic Linear Model with Garch Error

In order to compare the performance of the model discussed in the previous section, we develop the DLM for a model without the nonfundamental component. We maintain the same framework so that a comparison can be more meaningful. This is in contrast to the approach taken by Wu (1997), where the nobubble solution was estimated in the GMM framework. We also note that the model should account for the stylized fact of correlations in the variance of the stock return series. This is done by incorporating the GARCH(1,1) effect in the price equation (17) without the bubble component. In this context we adopt the methodology of Harvey, Ruiz, and Sentana (1992) and follow Kim and Nelson (1999, p. 144) to suitably augment the state vector of the DLM so that the time varying conditional variance could be accounted for.

In essence, the price difference equation (17) should now become,

$$\Delta p_t = \Delta d_t + e'\psi\Phi(I - \psi\Phi)^{-1}\Delta X_t + \varepsilon_{p,t}, \tag{18}$$

$$\varepsilon_{p,t} \sim N(0, h_t), \quad h_t = \alpha_0 + \alpha_1\varepsilon^2_{p,t-1} + \beta_1 h_{t-1}. \tag{19}$$

In order to set up the DLM in this case of Garch error together with dynamic of the dividend process, we include the details in Appendix B. Section 8 takes up the issues in modeling the linkages between the markets in the subset VAR framework.

8. Subset VAR Framework for Establishing Linkages Between Markets

The methodology developed in this paper allows us to decompose the stock prices in their fundamental and the nonfundamental components. We, analyze the linkage relationship both through the fundamental as well as through the speculative component. This helps us to understand whether the market linkages are through the fundamental or through the speculative components of the price series. Also, since we are dealing with monthly data, the time overlap problem between markets is largely nonexistent.

The econometric procedure we adopt is referred to as the subset VAR. Use of standard VAR approach to study causal relations between variables is frequently employed. A typical VAR model involves a large number of coefficients to be estimated and thus estimation uncertainty remains. Some of the coefficients may in fact be zero. When we impose zero constraints on the coefficients in full VAR estimation problem what results is the subset VAR. But, since most often no priori knowledge is available that will guide us to constrain certain coefficients, we base the modeling strategy on information provided by the AIC (Akaike Information Criterion) and the HQ (Hannan-Quinn) model selection criteria. Actual mathematical definitions and the details of this approach can be found in Lutkepohl (1993, Ch. 5). Below, we describe this procedure very briefly.

We first obtain the order of the VAR process for the four variables using the information criterion mentioned above. The top-down strategy starts from this full VAR model and the coefficients are deleted one at a time (from the highest lag term) from the four equations separately. Each time a coefficient is deleted, the model is estimated using least-squares algorithm and the information criterion is compared with the previous minimum one. If the current value of the criterion is greater than the previous minimum value, the coefficient is maintained otherwise it is deleted. The process is repeated for each of the four equations in the system. Once all the zero restrictions are determined the final set of equations are estimated again which gives the most parsimonious model. We also check for the adequacy of this model by examining the multivariate version of the portmanteau test for whiteness of the residuals as suggested by Lutkepohl (1993, p. 188). Once the subset VAR model is estimated there is no further need for testing causal relations and/or linkages between the variables. The causality testing is built into the model development process. Therefore, we examine linkages between the four markets in our study using this subset VAR model.

As mentioned earlier, we explore linkages between these markets in two stages. In the first stage, the fundamental price series are all found to be stationary, and hence in this case the modeling is done using the levels of the variables. We find evidence of one unit root in the speculative components of the price series for all the four markets. As we suspect existence of a cointegrating relation between these speculative components, we explore this using Johansen's cointegration test and find evidence of one cointegrating vector. It is, therefore, natural to estimate a vector error correction model, which is essentially a restricted VAR model with the cointegrating relation designed into it. As suggested in Lutkepohl (1993, p. 378), we examine the causal relation between these variables in the same way as for a stable system. In other words, we explore the linkages as for the fundamental price component but in this case we use first differenced form and use the lagged values of the cointegrating vector as well.

9. Discussion of Results

We analyze the monthly data, covering the postwar period, for the four mature stock markets of Germany, Japan, UK, and the US. In Table 1, we present the estimation results of the nonfundamental component solutions. It is clear that most of the parameters are statistically significant. The discount parameter, ψ, is close to the respective sample values while the significant σ_η for all the four countries imply highly variable speculative components. The estimated parameters of the dividend processes are close to their respective univariate estimation (not reported here) results. As is evident from Table 2, the significant ARCH and the Garch parameters indicate appropriateness of the error specification for the log price difference series for the models with no speculative components. There is substantial persistence in the variance process.

Next, we analyze the residual diagnostics in order to ascertain the appropriateness of the model (Table 3) for the monthly data series for all four countries. We find evidence of whiteness on residuals from the portmanteau test and the

Table 1. Parameter estimates from the state space model using Kalman filter nonfundamental solution for monthly data.

	ψ	σ_η	ϕ_1	ϕ_2	ϕ_3	σ_δ
Germany	0.9980*	0.0470*	−0.0009	0.0611*	0.0947*	0.0475*
	(0.0011)	(0.0010)	(0.0400)	(0.0210)	(0.0271)	(0.0002)
Japan	0.9989*	0.0570*	−0.0879*			0.0511*
	(0.0010)	(0.0013)	(0.0370)			(0.0007)
UK	0.9983*	0.0535*	−0.5210*	−0.3669*	−0.1324*	0.0407*
	(0.0047)	(0.0009)	(0.0144)	(0.0214)	(0.0225)	(0.0003)
USA	0.9964*	0.0416*	−0.7218*	−0.3553*	−0.0969*	0.0287*
	(0.0020)	(0.0009)	(0.0350)	(0.0453)	(0.0387)	(0.0007)

Estimates reported here are obtained from maximizing the innovation form of the likelihood function. Numerical optimization procedure in GAUSS is used without any parameter restriction. The standard errors (reported below the parameters in parentheses) are obtained from the Hessian matrix at the point of convergence. These estimates are robust to different starting values including different specification of the prior covariance matrix. Significance at 5% level is indicated by *.

Table 2. Parameter estimates from the state space model using Kalman filter GARCH (1,1) error for price equation: monthly data.

	ψ	ϕ_1	ϕ_2	ϕ_3	σ_δ	α_0	α_1	β_1
Germany	0.8526*	0.0047	0.0631	0.0848*	0.0475*	0.0001*	0.1108*	0.8633*
	(0.0391)	(0.0407)	(0.0409)	(0.0415)	(0.0014)	(5.14e-5)	(0.0299)	(0.0341)
Japan	0.5437*	−0.0906*			0.0511*	0.0000	0.0988*	0.8869*
	(0.0372)	(0.0407)			(0.0015)	(0.0000)	(0.0232)	(0.0301)
UK	0.2830*	−0.5331*	−0.3425*	−0.1148*	0.0407*	0.0004*	0.2307*	0.6107*
	(0.0380)	(0.0411)	(0.0440)	(0.0399)	(0.0012)	(0.0001)	(0.0541)	(0.0910)
USA	0.3189*	−0.7213*	−0.3271*	−0.0901*	0.0288*	0.0001*	0.0657*	0.8365*
	(0.0344)	(0.0413)	(0.0484)	(0.0400)	(0.0008)	(4.62e-5)	(0.0274)	(0.0533)

Estimates reported here are obtained from maximizing the innovation form of the likelihood function. Numerical optimization procedure in GAUSS is used without any parameter restriction. The standard errors (reported below the parameters in parentheses) are obtained from the Hessian matrix at the point of convergence. These estimates are robust to different starting values including different specification of the prior covariance matrix. GARCH (1,1) error for state space system implemented following Harvey, Ruiz, and Sentana (1992). Significance at 5% level is indicated by *.

Table 3. Residual diagnostics and model adequacy tests: monthly data.

	Portmanteau	ARCH	KS test	MNR	Recursive *T*
Nonfundamental solution					
Germany	0.253	0.158	0.176	0.586	0.903
Japan	0.061	0.206	0.093	0.379	0.972
UK	0.366	0.199	0.136	0.467	0.931
USA	0.377	0.327	0.048	0.425	0.894
With GARCH (1,1) error					
Germany	0.254	0.195	0.175	0.466	0.806
Japan	0.017	0.194	0.089	0.186	0.771
UK	0.307	0.179	0.139	0.571	0.907
USA	0.353	0.283	0.047	0.418	0.846

Entries are *p*-values for the respective statistics except for the KS statistic. These diagnostics are computed from the recursive residual of the measurement equation, which corresponds to the real dividend process. The null hypothesis in portmanteau test is that the residuals are serially uncorrelated. The ARCH test checks for no serial correlations in the squared residual up to lag 26. Both these test are applicable to recursive residuals as explained in Wells (1996, p. 27). MNR is the modified Von Neumann ratio test using recursive residual for model adequacy (see Harvey, 1990, Ch. 5). Similarly, if the model is correctly specified then Recursive *T* has a Student's *t*-distribution (see Harvey, 1990, p. 157). KS statistic represents the Kolmogorov–Smirnov test statistic for normality. 95 and 99% significance levels in this test are 0.057 and 0.068, respectively. When KS statistic is less than 0.057 or 0.068 the null hypothesis of normality cannot be rejected at the indicated level of significance.

lack of ARCH effect in the residuals from ARCH test results. The US data also support the normality of the residuals. More importantly, however, the tests for model adequacy are captured by the von-Neumann ratio and the recursive t-test. As pointed out in Harvey (1990, p. 157), the von-Neumann test provides the most appropriate basis for a general test of misspecification with recursive residuals. In this context, the dynamic linear models for both the approaches perform extremely well.

Figure 1 plots the nonfundamental price ratio for the sample period and the substantial variation of the speculative component is visible for all the countries. Except for the US, there is evidence of negative component for the other three countries in the initial part of the sample period. Each country was affected differently by the oil price shock of the 1970s. The most severe impact appears to have occurred in the UK. The fall in the speculative percentage during the October 1987 stock market crash is evident for all countries. It is also worth

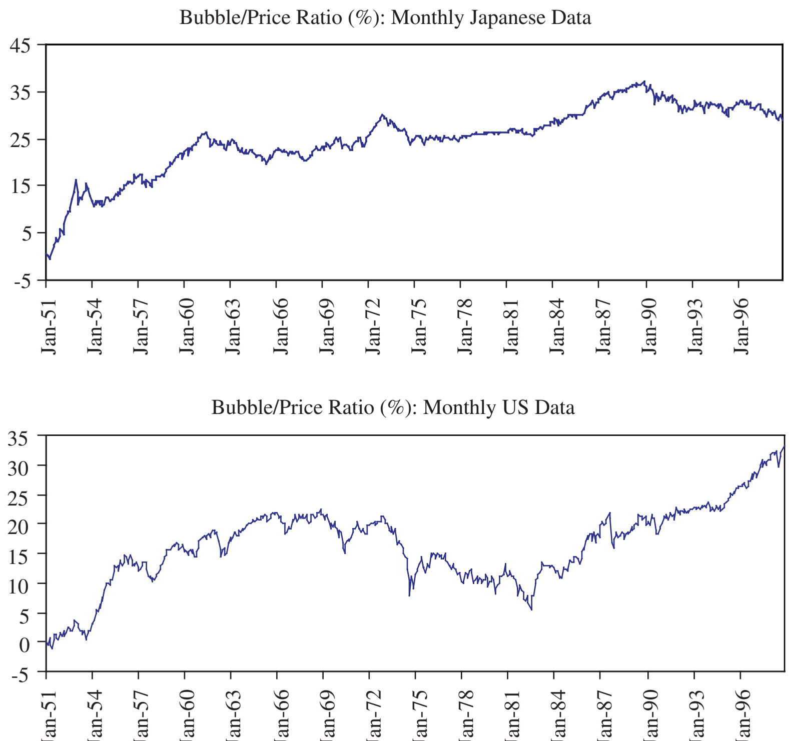

Figure 1. Plots using the smoothed estimates of the nonfundamental component from the state space model monthly data for Japan and USA.

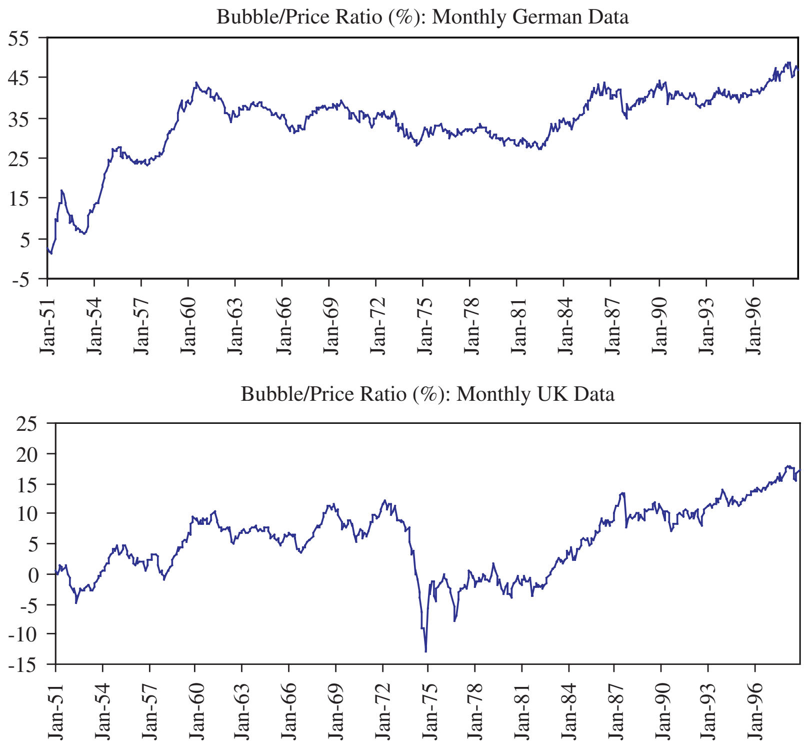

Figure 2. Plots using the smoothed estimates of the nonfundamental component from the state space model monthly data for Germany and UK.

observing that there is a general upward trend for the nonfundamental price ratio toward the later part of sample period for Germany, UK, and the US but not for Japan. The figures also provide the visual evidence of the collapsing and self-starting nature of the stochastic nonfundamental component we have attempted to capture in this study.

In order to quantify the performance improvement of the nonfundamental solution compared to the case with GARCH(1,1) errors, we present in Table 4 the in sample fitting statistics, RMSE and MAE.

These criteria are defined as,

$$\text{RMSE} = \frac{1}{T}\sum_{t=1}^{T}(\hat{p}_t - p_t)^2, \quad \text{MAE} = \frac{1}{T}\sum_{t=1}^{T}|\hat{p}_t - p_t|,$$

where $\hat{p}_t$ is the fitted price and T is the number of observations. The entries in Table 4 confirm that the nonfundamental component solution does a credible

Table 4. Nonfundamental solution versus solution with GARCH error compared monthly data.

	RMSE	MAE
Nonfundamental solution		
Germany	0.796	0.795
Japan	1.730	1.730
UK	0.247	0.366
USA	0.117	0.895
With GARCH (1,1) error		
Germany	2.945	2.945
Japan	4.394	4.395
UK	0.719	0.838
USA	1.734	1.735

RMSE and MAE stand for "root mean squared error" and "mean absolute error", respectively. These are computed from the differences between the actual log prices and the fitted log prices from the corresponding estimated model. Additional details are in the text.

job in terms of both metrics. For example, in the case of US the metric RMSE is reduced to 7% and the metric MAE to 52% of the solution with Garch error, respectively.

We indicated earlier the importance and the extent of investigation into the study of market linkages by various researchers. In this paper, we are able to focus on this aspect in two different levels. The study of stochastic bubbles through the dynamic linear models enables us to decompose the price into a fundamental and a bubble component. It is, therefore, natural to examine whether the market linkages exist via both these components. McCarthy and Najand (1995) demonstrated the influence of the US market on several other OECD countries using daily data which might have unintended consequences of trading time overlap in these markets. Using monthly data over a period of 48 years, we are in a better position to analyze the market interrelationships.

VAR methodology is often employed to study causal relationships. If some variables are not Granger-causal for the others, then zero coefficients are obtained. Besides, the information in the data may not be sufficient to provide precise estimates of the coefficients. In this context the top-down strategy of the subset VAR approach described in the earlier section is most suitable. For the fundamental price series we adopt this approach in the levels of the variables since these are all found to be stationary. Using the Hannan–Quinn criterion, we start our VAR model with a lag of one and follow the subset analysis process described before. This gives us the model presented in Table 5. As with McCarthy and Najand (1995) we find strong evidence of the US dominance on all the other three countries, but no reverse causality. This is a particularly important finding in the sense that this causality exists in the fundamental components of the prices. Intuitively, this evidence suggests that the US economy, as represented by the stock market data, acts as the engine of global growth. For Germany and Japan, the causality from the US is significant at the 5% level whereas for the UK it is significant at the 1% level only. The overall significance of this modeling approach is also established by testing the multivariate version of the portmanteau test to detect whiteness of the residuals.

We also apply the top-down strategy for the subset VAR approach to the nonfundamental components to examine the causality between the four markets. Since the nonfundamental components are found to be nonstationary (results for the unit root tests not included), we model this using the first difference of the log prices. With the nonstationary speculative price series it is natural to expect some long-term equilibrium relationship between these variables. We detected one cointegrating vector using Johansen's procedure and this has been described in Table 6. We follow the same procedure (as for the fundamental prices) to obtain the subset VAR model, including the cointegrating vector that

Table 5. Subset VAR estimation results for linkages between markets in fundamental prices.

	GR (−1)	JP (−1)	UK (−1)	US (−1)	Constant
Germany	0.2074*			0.1904*	1.7063*
	(3.40)			(3.89)	(8.23)
Japan	−0.1029			0.1878*	6.1837*
	(−1.91)			(3.08)	(23.95)
UK			0.0939*	0.1078**	5.0729*
			(1.97)	(1.76)	(18.02)
US					4.4358*
					(25.50)

Details of the methodology for determining the subset VAR relations are given in the text. This has been done in the level variables since the fundamental price series are stationary. The numbers in parentheses are t-statistics for the corresponding coefficient. Significance at 5 and 10% level are indicated by * and **, respectively. The p-value for the multivariate portmanteau statistic for residual white noise is 0.017. This is described in Lutkepohl (1993, p. 188). This indicates that the model adequately represents the relationship documented here.

Table 6. Subset VAR estimation results for linkages between markets in nonfundamental prices.

	ΔGR (−1)	ΔJP (−1)	ΔUK (−1)	ΔUS (−1)	Coint (−1)	Constant
ΔGermany	0.1289* (2.94)			0.1904* (3.91)	0.0071* (2.47)	0.0033 (1.74)
ΔJapan	−0.1436* (−2.67)			0.1915* (3.20)	0.0167* (4.76)	0.0048* (2.09)
ΔUK			0.0956* (1.99)	0.1064** (1.73)		0.0016 (0.74)
ΔUS					0.0009* (3.57)	0.0038* (2.21)

The nonfundamental prices are found nonstationary and Johansen's procedure identified existence of one cointegrating vector. The lagged value of this cointegrating vector (COINT) has been used in estimating the subset VAR relations for the linkages between the markets. The details of the unit root and the cointegration tests are not reported here but can be obtained from the authors. The estimated cointegrating vector (normalized on GR) including TREND and constant terms is given below. The numbers in parentheses are *t*-statistics for the corresponding coefficient. Significance at 5 and 10% levels are indicated by * and **, respectively.

$$\text{GR}(-1) - 1.5826\,\text{JP}(-1) + 2.7303\,\text{UK}(-1) - 3.2545\,\text{US}(-1) + 0.0054\,\text{TREND} + 2.3772$$

The *p*-value for the multivariate portmanteau statistic for residual white noise is 0.068. This is described in Lutkepohl (1993, p. 188). This indicates that the model adequately represents the relationship documented here.

describes the causal relationship between these markets. Table 6 shows that causality exists from the US to the other three markets. Also, these linkages are significant at the 5% level for Germany and Japan and only at the 1% level for the UK. Similar to the fundamental prices, there is no reverse causality in the speculative price components as well. It is also observed that the strength of this causality from the US to Japan is slightly stronger for the speculative price process, 0.1915 as opposed to 0.1878 for the fundamental prices.

It is also noted from Table 6 that the coefficients of the error correction term i.e. "Coint (−1)" are statistically significant. This implies that the modeled variables i.e. the changes in log prices, adjust to departures from the equilibrium relationship. The magnitude of the coefficient "Coint (−1)" for the Japanese log price difference is much higher than the others, capturing, first the upward and later, the downward trend in the Japanese market. Although, the existence of an error correction model implies some form of forecasting ability, we do not pursue this in this paper. Finally, we note the multivariate portmanteau test for whiteness of residuals in Table 6. This again supports the model adequacy and hence the inferences drawn are statistically meaningful.

10. Conclusions

Economists have long conjectured that movements in stock prices may involve speculative bubbles because trading often generates over-priced or under-priced markets. A speculative bubble is usually defined as the difference between the market value of a security and its fundamental value. Although there are several important theoretical issues surrounding the topic of asset bubbles, the existence of bubbles is inherently an empirical issue that has not yet been settled.

This paper reviews several important tests and offers a new methodology that improves upon the existing ones. In particular, we implement the state space form in such a way that it treats both the dividend process and the nonfundamental process as part of the state vector in a dynamic linear model that allows for a straightforward comparison with the model that only allows GARCH errors. The new methodology is applied to the four mature markets of the US, Japan, England, and Germany to test whether a nonfundamental component was present during the period of January 1951 to December 1998. To establish the soundness of our methodology, we have also applied a battery of diagnostic tests. Our methodology establishes that asset prices in the US, Japan, UK, and Germany have deviated from fundamentals during our sample period. These deviations we call "rational bubbles" or "speculative nonfundamental components".

Once we find evidence of nonfundamental components in these four mature stock markets, we next ask the question whether these are interrelated. We avoid using the technical term of contagion because it has a very specific meaning. Several authors use contagion to mean a significant increase in cross-market linkages, usually after a major shock. For example, when the Thai economy experienced a major devaluation of its currency during the summer of 1997, the spreading of the crisis across several Asian countries has been viewed as a contagion. Unlike the short-term cross-market linkages that emerge as a result of a major, often regional economic shock, we are interested in this paper in long-run linkages. Speculative effects often take long time, that is several years to develop and one is interested in knowing if such processes travel from one mature economy to another. Our statistical tests of the long-term linkages between the four mature stock markets provide evidence that the US stock market nonfundamental component or bubble causes a bubble in the other three markets but we find no evidence for reverse causality. Thus, in contrast to numerous studies showing that these four mature stock markets are cointegrated, our decomposition of the national markets returns into fundamental and nonfundamental components offers the additional insight that it is the US nonfundamental component that statistically causes the emergence of bubbles in Japan, Germany and the UK. Such evidence suggests that global diversification can be more effective if the US stock market becomes more successful in reducing the emergence of bubbles at home.

Appendix A: Setting up the DLM with Nonfundamental Component

Equation (17) in the main text represents the measurement equation of the DLM and we need to suitably define the state equation for the model. An examination of Equations (12) and (14) suggests that the following state equation represent the dynamics of the dividend and the nonfundamental process:

$$\begin{bmatrix} \Delta d_t \\ \Delta d_{t-1} \\ \Delta d_{t-2} \\ \Delta d_{t-3} \\ b_t \\ b_{t-1} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\psi} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta d_{t-1} \\ \Delta d_{t-2} \\ \Delta d_{t-3} \\ \Delta d_{t-4} \\ b_{t-1} \\ b_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_\delta \\ 0 \\ 0 \\ 0 \\ \varepsilon_\eta \\ 0 \end{bmatrix}, \tag{A.1}$$

$$\begin{pmatrix} \varepsilon_\delta \\ \varepsilon_\eta \end{pmatrix} : N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\delta^2 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix} \right). \tag{A.2}$$

We are in a position now to define the measurement equation of the DLM in terms of the state vector in Equation (A.1). This is achieved by examining Equation (17) and defining a row vector, $M \equiv e'\psi\Phi(I - \psi\Phi)^{-1} = [m_1, m_2, m_3]$, as follows:

$$\Delta p_t = \Delta d_t + [m_1,\ m_2,\ m_3] \begin{bmatrix} \Delta d_t - \Delta d_{t-1} \\ \Delta d_{t-1} - \Delta d_{t-2} \\ \Delta d_{t-2} - \Delta d_{t-3} \end{bmatrix} + \Delta b_t,$$

or

$$\begin{bmatrix} \Delta p_t \\ \Delta d_t \end{bmatrix} = \begin{bmatrix} (1+m_1) & (m_2-m_1) & (m_3-m_2) & -m_3 & 1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \Delta d_t \\ \Delta d_{t-1} \\ \Delta d_{t-2} \\ \Delta d_{t-3} \\ b_t \\ b_{t-1} \end{bmatrix}. \tag{A.3}$$

Equation (A.3) determines the measurement equation of the DLM without any measurement error. In other words, the evolution of the state vector in Equation (A.1) results in the measurement of the measurement vector through Equation (A.3). Equations (A.1) and (A.3) represent the DLM for the model with nonfundamental component when the dividend process is described by the AR(3) system in Equation (14). In our sample this is the case for Germany, UK, and the US. Since the data for Japan required only an AR(1) process for the dividend in Equation (14), the DLM, in this case, may be written directly as:

$$\begin{bmatrix} \Delta d_t \\ \Delta d_{t-1} \\ b_t \\ b_{t-1} \end{bmatrix} = \begin{bmatrix} \phi_1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\psi} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta d_{t-1} \\ \Delta d_{t-2} \\ b_{t-1} \\ b_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_\delta \\ 0 \\ \varepsilon_\eta \\ 0 \end{bmatrix}, \tag{A.4}$$

$$\begin{pmatrix} \varepsilon_\delta \\ \varepsilon_\eta \end{pmatrix} : N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\delta^2 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix} \right). \tag{A.5}$$

Similarly, the measurement equation for the DLM of the solution with nonfundamental component for the Japanese data becomes,

$$\begin{bmatrix} \Delta p_t \\ \Delta d_t \end{bmatrix} = \begin{bmatrix} (1+m_1) & -m_1 & 1 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta d_t \\ \Delta d_{t-1} \\ b_t \\ b_{t-1} \end{bmatrix}, \tag{A.6}$$

where $M \equiv e'\psi\Phi(I-\psi\Phi)^{-1} = [m_1]$, since $e' = [1]$, $\Phi = [\phi_1]$.

We have now completed the DLM for the solutions with nonfundamental component for all the four markets in our sample. The parameters of the models, embedded in these equations, may be estimated by maximum likelihood method as described in Appendix C. At the same time both the filtered and the smoothed estimates of the nonfundamental component series are inferred from the observed price and the dividend series.

Appendix B: Setting up the DLM with Garch Error

For Germany, UK and the USA with AR(3) representation of the dividend change process, the state equation with GARCH(1,1) error becomes,

$$\begin{bmatrix} \Delta d_t \\ \Delta d_{t-1} \\ \Delta d_{t-2} \\ \Delta d_{t-3} \\ \varepsilon_{p,t} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta d_{t-1} \\ \Delta d_{t-2} \\ \Delta d_{t-3} \\ \Delta d_{t-4} \\ \varepsilon_{p,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_\delta \\ 0 \\ 0 \\ 0 \\ \varepsilon_{p,t} \end{bmatrix}, \tag{B.1}$$

$$\begin{pmatrix} \varepsilon_\delta \\ \varepsilon_{p,t} \,|\omega_{t-1} \end{pmatrix} : N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\delta^2 & 0 \\ 0 & h_t \end{bmatrix} \right), \quad h_t = \alpha_0 + \alpha_1 \varepsilon_{p,t-1}^2 + \beta_1 h_{t-1}, \tag{B.2}$$

and ω_{t-1} is the information set at time $t-1$. This is equivalent to Equation (A.1) in this context. The corresponding measurement equation becomes,

$$\begin{bmatrix} \Delta p_t \\ \Delta d_t \end{bmatrix} = \begin{bmatrix} (1+m_1) & (m_2 - m_1) & (m_3 - m_2) & -m_3 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta d_t \\ \Delta d_{t-1} \\ \Delta d_{t-2} \\ \Delta d_{t-3} \\ \varepsilon_{p,t} \end{bmatrix}. \tag{B.3}$$

For the Japanese data with an AR(1) dividend change process, the DLM may be written following the approach above. The state Equation (B.1) becomes,

$$\begin{bmatrix} \Delta d_t \\ \Delta d_{t-1} \\ \varepsilon_{p,t} \end{bmatrix} = \begin{bmatrix} \phi_1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta d_t \\ \Delta d_{t-1} \\ \varepsilon_{p,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_\delta \\ 0 \\ \varepsilon_{p,t} \end{bmatrix}, \qquad \text{(B.4)}$$

$$\begin{pmatrix} \varepsilon_\delta \\ \varepsilon_{p,t} \,|\omega_{t-1} \end{pmatrix} : N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\delta^2 & 0 \\ 0 & h_t \end{bmatrix}\right), \quad h_t = \alpha_0 + \alpha_1 \varepsilon_{p,t-1}^2 + \beta_1 h_{t-1}. \qquad \text{(B.5)}$$

The corresponding measurement equation becomes,

$$\begin{bmatrix} \Delta p_t \\ \Delta d_t \end{bmatrix} = \begin{bmatrix} (1+m_1) & -m_1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta d_t \\ \Delta d_{t-1} \\ \varepsilon_{p,t} \end{bmatrix}. \qquad \text{(B.6)}$$

In the case of stock price solutions with GARCH error, the parameters to be estimated are those of the dividend process and the GARCH(1,1) coefficients. The procedure for this is the same as that for the case with nonfundamental component and is described in detail in Appendix C.

Appendix C: Estimating the Parameters of the DLM

In this appendix, we describe briefly how the unknown parameters in the DLM may be estimated. Our aim is to present an overview of the filtering and smoothing algorithm (known as Kalman filter and smoother) and the optimization of the likelihood function. Before proceeding, however, it is advantageous to express the DLM in term of suitable notations. Since the discussion here is applicable to both the bubble solution and the nobubble solution described earlier, we will not make any distinction between the two once the DLM have been defined.

We consider the DLM with reference to the following state and measurement equations:

$$y_t = \Gamma y_{t-1} + w_t \quad \text{(state equation)}, \qquad \text{(C.1)}$$

$$z_t = A_t y_t + v_t \quad \text{(measurement equation)}. \qquad \text{(C.2)}$$

In this DLM, y_t is a $p \times 1$ vector of unobserved state variables, Γ is the $p \times p$ state transition matrix governing the evolution of the state vector. w_t is the $p \times 1$ vector of independently and identically distributed, zero-mean normal vector with covariance matrix Q. The state process is assumed to have started with the initial value given by the vector, y_0, taken from normally distributed variables with mean vector μ_0 and the $p \times p$ covariance matrix, Σ_0.

The state vector itself is not observed but some transformation of these is observed but in a linearly added noisy environment. In this sense, the $q \times 1$ vector z_t is observed through the $q \times p$ measurement matrix A_t together with the $q \times 1$ Gaussian white noise v_t, with the covariance matrix, R. We also assume that the two noise sources in the state and the measurement equations are uncorrelated.

The next step is to make use of the Gaussian assumptions and produce estimates of the underlying unobserved state vector given the measurements up to a particular point in time. In other words, we would like to find out,

$$E(y_t|\{z_{t-1}, z_{t-2}, \ldots, z_1\})$$

and the covariance matrix,

$$P_{t|t-1} = E[(y_t - y_{t|t-1})(y_t - y_{t|t-1})'].$$

This is achieved by using Kalman filter and the basic system of equations is described below.

Given the initial conditions $y_{0|0} = \mu_0$, and $P_{0|0} = \Sigma_0$, for observations made at time $1, 2, 3, \ldots, T$,

$$y_{t|t-1} = \Gamma y_{t-1|t-1}, \tag{C.3}$$

$$P_{t|t-1} = \Gamma P_{t-1|t-1}\Gamma' + Q, \tag{C.4}$$

$$y_{t|t} = y_{t|t-1} + K_t(z_t - A_t z_{t|t-1}), \tag{C.5}$$

where the Kalman gain matrix

$$K_t = P_{t|t-1}A_t'[A_t P_{t|t-1}A' + R]^{-1}, \tag{C.6}$$

and the covariance matrix $P_{t|t}$ after the tth measurement has been made is,

$$P_{t|t} = [I - K_t A_t]P_{t|t-1}. \tag{C.7}$$

Equation (C.3) forecasts the state vector for the next period given the current state vector. Using this one step ahead forecast of the state vector it is possible to define the innovation vector as,

$$\nu_t = z_t - A_t y_{t|t-1} \tag{C.8}$$

and its covariance as,

$$\Sigma_t = A_t P_{t|t-1} A_t' + R. \tag{C.9}$$

Since in finance and economic applications all the observations are available, it is possible to improve the estimates of state vector based upon the whole sample. This is referred to as Kalman smoother and it starts with initial conditions at the last measurement point i.e., $y_{T|T}$ and $P_{T|T}$. The following set of equations describes the smoother algorithm:

$$y_{t-1|T} = y_{t-1|t-1} + J_{t-1}(y_{t|T} - y_{t|t-1}), \tag{C.10}$$

$$P_{t-1|T} = P_{t-1|t-1} + J_{t-1}(P_{t|T} - P_{t|t-1})J_{t-1}', \tag{C.11}$$

where

$$J_{t-1} = P_{t-1|t-1}\Gamma'[P_{t|t-1}]^{-1}. \tag{C.12}$$

It should be clear from the above that to implement the smoothing algorithm the quantities $y_{t|t}$ and $P_{t|t}$ generated during the filter pass must be stored.

With reference to the DLM for the bubble and the nobubble solutions it is obvious that the parameters of interest are embedded in the matrices Γ and Q, since by construction of our models $R \equiv 0$. The description of the above filtering and the smoothing algorithms assumes that these parameters are known. In fact, we want to determine these parameters and this achieved by maximizing the innovation form of the likelihood function. The one step ahead innovation and its covariance matrix are defined by Equations (C.8) and (C.9) and since these are assumed to be independent and conditionally Gaussian, the log likelihood function (without the constant term) is given by

$$\log(L) = -\sum_{t=1}^{T} \log|\Sigma_t(\Theta)| - \sum_{t=1}^{T} \nu_t'(\Theta)\Sigma_t^{-1}(\Theta)\nu_t(\Theta). \tag{C.13}$$

In this expression, Θ is specifically used to emphasize the dependence of the log likelihood function on the parameters of the model. Once the function is maximized with respect to the parameters of the model, the next step of smoothing can start using those estimated parameters.

Maximization of the function in Equation (C.13) may be achieved using one of two approaches. The first one depends on algorithm like Newton–Raphson and the second one is known as the EM (Expectation Maximization) algorithm. In this paper we employ the Newton–Raphson technique to achieve our objective and since the likelihood function is reasonably well behaved, maximization is achieved quite quickly. In some modeling situations it may not be so straightforward. EM algorithm has been reported to be quite stable in the presence of bad starting values, although it may take longer to converge. Some researchers report that when good starting values are hard to obtain, a combination of the two approaches may be useful. In that situation it is preferable to employ EM algorithm first in order to obtain an intermediate estimates and then switch to the Newton–Raphson method. Interested readers may refer to Shumway and Stoffer (2000, p. 323).

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