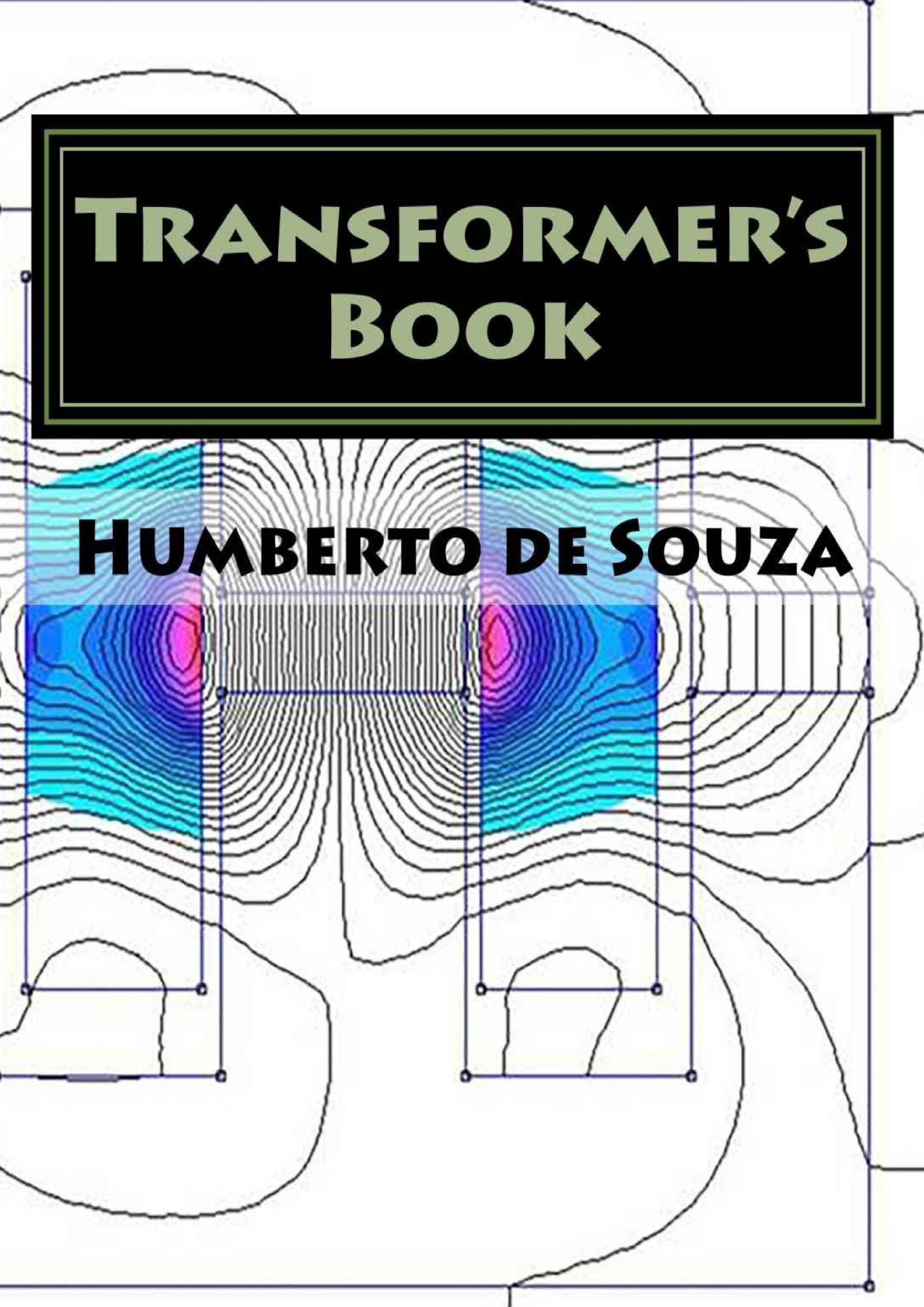
Transformer's Book

Humberto de Souza



Transformer’s Book

A travel over different aspects of transformers, inductors and transductors

Humberto de Souza

First Edition

2015

Transformers, Inductors and transductors Design with different aspects like skin depth, eddy currents, proximity, strange, final and gap effects. Optimization of size and discussion of special featured transformers, like Kfactor transformers design.

Rio de Janeiro

50729S Souza, Humberto de, 1948

Transformer’s Book - A travel over different aspects of transformers, inductors and transductors / Humberto de Souza. – Rio de Janeiro: 2015

306 P.: 25.4 Cm

ISBN 978-85-920396-0-8

1.Transformers 2.Additional Losses and Design

1 Title

CCD: 600

CDU: 537.37

First Edition

To my Father and Mother

Index:

Index of all volumes

Introduction

A Brief History

The transformer was primarily invented in 1831 by Michael Faraday and since then, scattered over the history in many industrial and residential applications.

Its great impulse took place after the win of Nicholas Tesla over Thomas Alva Edison in the “War of Currents” where both, defended opposed positions about AC or DC distribution of electrical Energy.

In that occasion, the selection of the Alternating Current demonstrated being more attractive then Direct Current distribution, due to its natural properties of manageability and easily conversion of voltage levels.

Exactly in the center of this discussion, was the transformer, the main responsible for the conversion of the levels of voltages in the electrical distribution systems.

What Is a Transformer?

A transformer is basically composed by two coils of metallic wires, insulated superficially by varnish, allowing being mounted together in a same iron core.

The existence of the Iron core, allows to couple magnetically the two coils in a better way, and due to its magnetic properties, reducing the currents developed in the coils. This particular is going to be seen later on.

When exists an AC voltage applied in the input coil called “Primary” will appear in the other coil, called “Secondary” a voltage that is proportional to the relation of turns between the two coils.

That property of transformation of the voltage from one level of voltage to another or several others is the main feature of it. The level of voltage is changed but the power is invariable, meaning that isn´t changed the energy level

The physics behind the scenes

We must understand that the main objective of this brief discussion is to give an aerial view about the mechanisms of the magnetism in magnetic materials. In addition a remembering of the magnetic terms and the definition of certain values involved in them.

We depart from the principle that everyone that opens this book understands entirely such subject.

This book is far from being a beginners book. It is intended to be a book for experienced engineers or technicians, that already design transformers and is concerned in advanced topics of optimization.

It's a book of few words . The subjects are treated in mathematics and visual basis in order to transmit concepts mainly through the experience of visual effects.

The mathematical is intentionally kept in a low level with the main aim to be understood by most of the people.

The book was designed in tree independent volumes that obeys a logical and a structural knowledge sequence oriented for better understanding.

Structure of the Matter and a brief view over atomic physics

Democritus, a philosopher of antiquity, proposed that the matter is formed of tiny invisible particles called “Atoms” that means “indivisible”. This idea was followed by His group called Atomists and during a lot of time; this theory was maintained as the main explanation of the matter.

For Democritus the huge variety of materials in nature came from the movements of the different types of the atoms

Others contributed with this theory, like Epicurus proposed a limit for the size of the atoms explaining its invisibility.

Aristotle changed the path, proposing that everything is composed by combinations of 4 elements, Fire, Air, Earth and Water.

John Dalton, in 1803, trying to explain the behavior of the various atmospheric gases and gas mixtures, resumed the atomic hypothesis. Just as Leucippus, Democritus and Epicurus, Dalton they believed that matter was composed of indivisible atoms and empty space. They imagined the atom as a small sphere with mass defined and characteristic properties. Thus, all chemical transformations could be explained by the arrangement of atoms. All matter is composed of atoms. These are the smallest particles that constitute it, are indivisible and indestructible, and cannot be transformed into other, even during chemical phenomena. The atoms of the same chemical element are identical in mass and behave equally in chemical transformations. Chemical transformations occur by separation and union of atoms. That is, atoms of a substance that are combined in a certain way, separate, joining together again in a different way.

The British Joseph John Thomson discovered the electron in 1897 through experiments involving cathode ray tubes. The cathode ray tube is in a vial containing only vacuum and an electrical device that makes the electrons jump from any conductive material and form bundles, which are themselves cathode. Thomson, studying cathode rays, discovered that these are affected by electric and magnetic fields and deduced that the deflection of cathode rays by these fields are deviations from trend of very small particles of negatively charged electrons.

Thomson proposed that the atom was therefore divisible, in positively and negatively charged particles, contrary to the indivisible atom model proposed by Dalton (and atomistic in Ancient Greece). The atom consists of several electrons embedded and incorporating in a large particle positive, like raisins in a pudding. The atomic model of the “pudding with raisins” remained in vogue until the discovery of the atomic nucleus by Ernest Rutherford.

In 1911, experimenting bombing sheets of gold with alpha particles (positively charged particles, released by radioactive elements), Rutherford made an important observation: the vast majority of particles passed through directly to the blade, and some others suffered minor deviations in very small number (one hundred thousand), suffered large deviations in the opposite direction.

From these observations, Rutherford came to the following conclusions:

In the atom there are empty spaces; most of the particles passed through the bypass without conceding any effect…

In the center of the atom has a nucleus very small and dense, some alpha particles collide with this core and returned without crossing the blade.

The nucleus has a positive charge; alpha particles passing near him were repelled and therefore suffered deviation in its trajectory.

By Rutherford atomic model, the atom consists of a central core, provided with positive electric charges (protons) surrounded by a cloud of negative charges (electrons).

Rutherford has also shown that virtually all mass is concentrated in the small atom core region.

Two years after Rutherford have created your model; the Danish scientist Niels Bohr completed it, creating what is now called planetary model. For Bohr, the electrons revolve in circular orbits around the nucleus. After these new studies were made and new atomic models were created. The model represents the atom as having a central part called core, containing protons and neutrons, serves to explain a large number of comments on materials.

Niels Bohr in the beginning of the XX century described the atom, departing from Quantum electromagnetic radiations, proposed by Albert Einstein and Max Plank, depicted more completely an atom as a planetary system like the model of Rutherford, where negative charges turns around positive and neutral charges. In reality the electrons are clouds of probability surrounding the nucleus.

There are many problems with this model, like there is with any model that is proposed, because the behavior of those mechanisms, are not completely understood, being all, mere parables about reality.

There are many constituents of universe and matter, space, time, particles and forces. The main forces of our interest are gravity and electromagnetic forces, being the last one, derived from the photons interactions.

Today there are so many theories about the fundamental structure of our universe and matter, but many of all are no totally understood, like string theory, super string theory, M theory and other that depicts partially the behavior of particles movements, time and forces. All in elegant theories and even opening ways for a most ambitious and vast theory of everything, where the relativity theory and Quantum mechanics could be unified, being the dream and the Holy Grail of the modern physicists.

But still today where such theories didn´t gain their structure to evolves the explanation of our physical and engineering models, explaining our daily life. The Bohr atomic model is still in the practice to explain them.

Magnetism and Magnetic Materials

Magnetism

To understand the cause of magnetism we must begin with the atoms, from which the materials are built, and consider first of all the structure of the individual atom. At the center there is the nucleus around which electrons orbit in a complicated manner.

We need concern ourselves here only with the electrons; we can ignore the nucleus because of its small magnetic contribution.

Each single electron possesses in the first place an electric charge; more important for us, however, is the fact that it also has its own magnetic (spin) moment. The single electron already behaves like a tiny bar magnet, and the ferromagnetism that we are trying to explain is ultimately only the effect, summed under prescribed conditions in a specific way, of a huge number of such electron magnets. Besides the spin each electron of the atom possesses a further magnetic moment (Orbital moment) caused by its rotation around the nucleus. A movement of charge of this kind produces a magnetic effect exactly as does an ordinary circulating current.

In a larger atom numerous electrons with spins and orbital moments, are present; in an iron atom, for example, there are 26 electrons arranged in shells. The abundance of magnetic moments in the individual atoms seems at first impossible to estimate. Fortunately, however there are very strict laws in the atom governing the magnitude and direction of the individual contributions. Their effect is for contributions either wholly or partially to cancel out, so that frequently there is no resultant magnetic moment in the single atom or in some cases only that of one or a few electrons.

For a single in isolation there is thus a definite magnetic moment which we may ascribe to a conceptual atomic magnet (Elementary magnet).

In a metal we are not therefore dealing with individual atoms existing completely independently of one another; on the contrary there is an interaction between the atoms which is of vital importance to the metal performance. We must therefore turn first to the structure of metals.

Metals are normally crystalline solid bodies consisting of a large number of grains (Crystallites) tightly united by grain boundaries (Fig 1.1)

Magnetism in Materials

In nature, magnetism is presented by some substances .Depending their individual natures; they present different amount and properties like ferromagnetism, ferrimagnetisms, anti-ferromagnetism paramagnetism and diamagnetism.

Ferromagnetism is a physical phenomenon where there is an existence of a magnetic ordainment of all the magnetic momentum of material that externally presents an observable force of attraction and retention by a magnet. At same form in the ferrimagnetism, is observed an ordainment of the magnetic momentum, but in this case, some of them in the opposite direction, but they are randomly distributed and is not enough to cancel the attraction effect.

In the Anti-ferromagnetism, is observed the same ordainment of all magnetic Momentum, but in both direction in a so ordered way, that cancels any observable attraction

Paramagnetism is the trend of all free individual momentum of the material to align parallel with an applied field. When do not exist any applied field it does not present any magnetic property.

The great majority the substances are diamagnetic. This property was first observed by Sebald Justinus Brugmans in 1778 and first nominated by Michael Faraday in September of 1945, when he had seen a piece of bismuth be repulsed by a magnetized bar. An interesting experience is a piece of a diamagnetic material being levitated by a magnetic bar. The experience of a superconductor being levitated by a magnetic field is due to this diamagnetic property.

At present we are interested only with the strongest of those five “magnetisms” Called ferromagnetism.

Ferromagnetic Materials

An iron atom presents numerous electrons, with spins and orbital moments.

In this example 26 electrons are arranged in shells. The contribution of the magnetic moments of individual atoms seems to be impossible to predict. Fortunately there are ways to describe the magnetism phenomena. We need to descent to the interior of the matter and begin with the atoms, from which the matter is built. In its simplified model in the center of the atoms are the protons and neutrons that form the nucleus around witch the electrons revolves in a complicated manner.

Our worry need be only with the electrons, because the amount of magnetism presented by the nucleus is negligible

An Electron presents an electrical charge and has its own magnetic (spin) moment. The single electron already behaves like tiny magnetic bar. The ferromagnetism we are beginning to explain is ultimately only the effect, summed under prescribed conditions in a specific way, of a huge number of such electron magnets. Besides the spin effect, each electron of the atom possesses an additional magnetic moment (Orbital Moment) caused by its rotation around the nucleus. A movement of charge of this kind produces a magnetic effect, exactly as done by awes of physics governing the magnitude and direction of the individual contributions.

For a single isolated atom there is a magnetic moment which we may relate to a conceptual atomic magnet. In a metal we are not therefore dealing with individual atoms. There are groups of atoms related. They are intrinsically coupled by their individual magnetic forces this groups we call “Domains”.

The ferromagnetic domains

In magnetic materials, there are groups of atoms linked together through their forces. We call them domains. Such domains works like them were a single magnet. The coupling forces between those domains are of big importance in such phenomena. In ferromagnetic materials they are of so technical importance, that at room temperature they are so aligned, that are all entirely paralleled working all like they were one only magnet.

If the temperature rises, atomic agitation contributes to misalignment of such domains. Reaching such point where this alignment is lost. This temperature is called the Curie temperature, being approximately 770 ºC for iron and 360 ºC for nickel

Metals are normally Crystalline solid bodies consisting of a large number of grains (Crystallites) tightly united by grain boundaries. In all crystallites the atoms are arranged regularly in a space structure; in individual crystallites however, the structure is generally oriented in different ways. There are quite a number of such structures; for example, at room temperature nickel has a cubic-face-centered structure, iron a cubic-space-centered structure (figs 1 and 2).

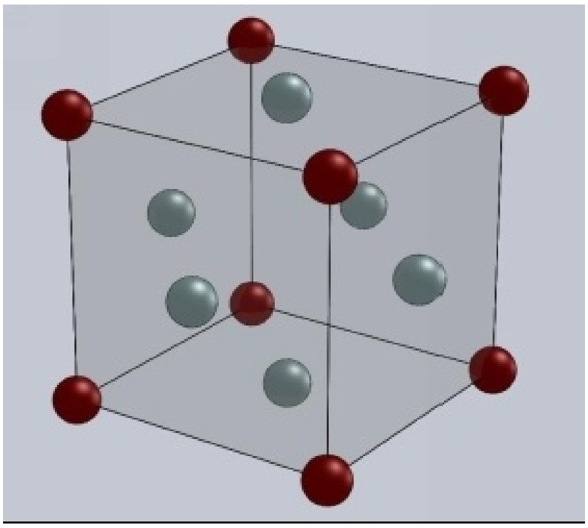


Fig. 1 – Space structure of the nickel

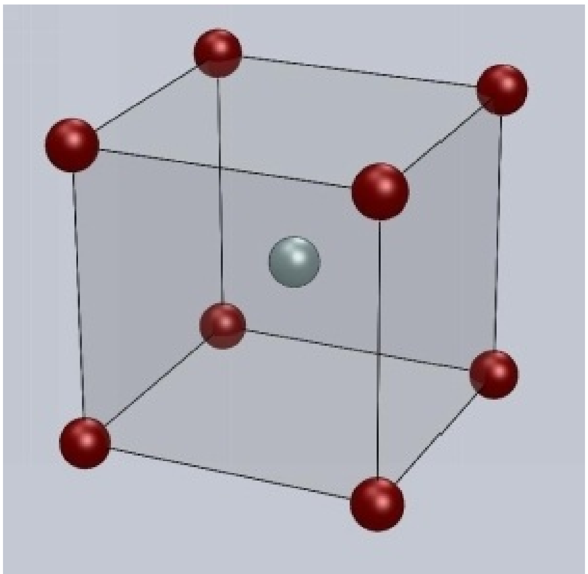


Fig. 2 Space structure of the iron

In a metal crystal of this type the electrons of the atoms are more weakly bound to the atomic nuclei than in a single free atom.

The outermost electrons specially make them independent and move around almost freely in the metal crystal. This leads to the high electrical conductivity of metals.

The electrons are then, however, in a state of mutual interaction and as a result their magnetic moments arrange themselves ant parallel almost exception. The free electrons are thus able to contribute virtually nothing to the magnetism of the metal, but the situation is different with the other electrons that have remained in the vicinity of their atomic nucleus. They do in fact to large extent cancel one another out of their magnetic effect. I a number of cases, however, as with the individual atoms, some magnetic moments remain uncompensated at each lattice site in the metal crystal there reposes, in a manner of speaking , a small magnet. These magnets are in fact prerequisite of ferromagnetism. If these tiny magnets were not intercooled in some way, they would possess all possible directions, as long as no external magnetic field is present. Just as the lattice atoms themselves execute thermal vibrations caused by temperature, the atomic magnets are continually changing their directions.

In most cases, however the atomic magnets in the solid are intercoupled. If the effect of coupling forces is to make the atomic magnets arrange themselves parallel at the lattice sites, we are concerned with ferromagnetism. The moments of the individual atoms then add up. These conditions apply, for example, for iron, nickel, and cobalt.

The coupling forces between atomic magnets at lattice sites thus decide the occurrence of ferromagnetism.

Ferromagnetic Domains and domain walls

In the ferromagnetic materials of technical importance the coupling forces inside are generally so powerful that atomic magnetic magnets at room temperature are almost entirely parallel-aligned. If, however, we raise the temperature, then thermal directional oscillations of the atomic magnets increase steadily and finally become so strong that the coupling forces are overcome. This occurs with various ferromagnetic metals and alloys at Cleary defined temperatures, known as the Curie temperatures. For iron, the Curie temperatures is 770 ºC; for nickel, 360 ºC.

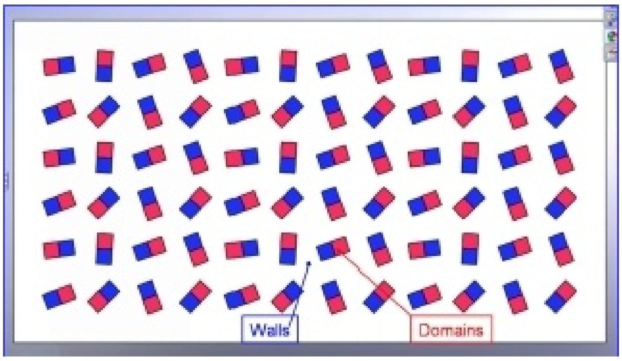


Fig. 3 Randomly arranged domains

Thus, it is only below the Curie temperature that the atomic magnets in a ferromagnetic material are aligned in parallel by the coupling forces.

The boundaries between the domains are called domain walls or Bloch walls (After F. Bloch). Such a wall is not only a boundary; it is more a transition layer with its own system of laws. Its thickness may range from several hundred of thousand atomic distances.

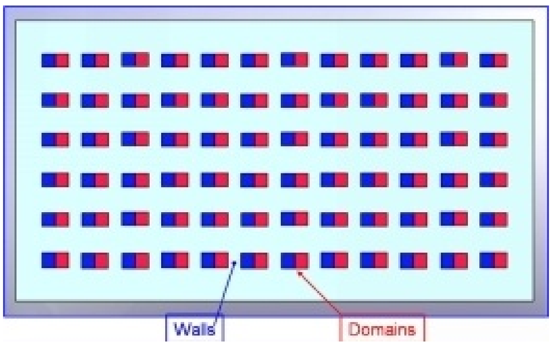


Fig. 4 Aligned arranged domains

Within the wall the orientation of the atomic magnets changes in screw like fashion from the direction of the one domain into that of the other, as fig. 5 shows diagrammatically. In so doing, torsions arise opposing the forces acting (for example the coupling and anisotropy forces); the wall has thereby certain energy content

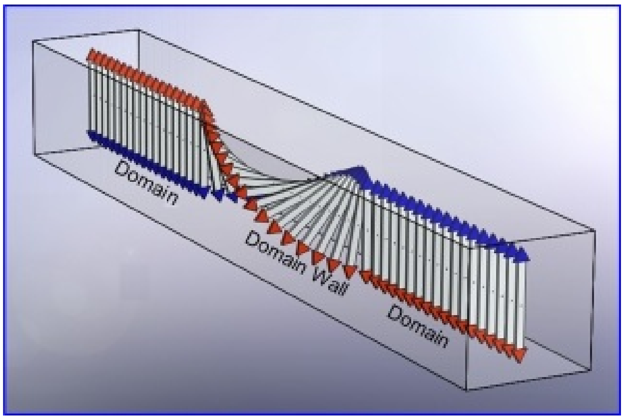


Fig. 5 – Domain and domain Wall

Initial Concepts

Ampere´s Law

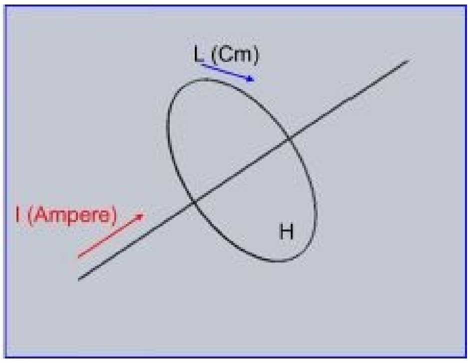


Fig. 6 A current produces a surrounding magnetic field

$$I := \oint H\,dl$$

$$l_{fe} = l$$

Considering that $I = N X I$

$$H = \frac{N \times I}{l_{fe}}$$

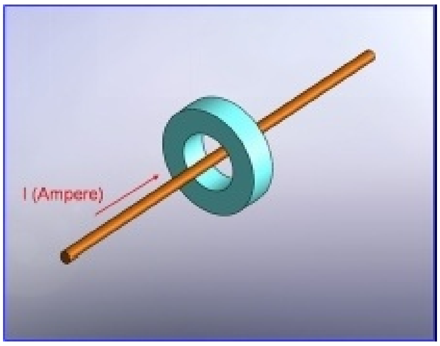


Fig. 7 – One turns crossing the center of the toroid

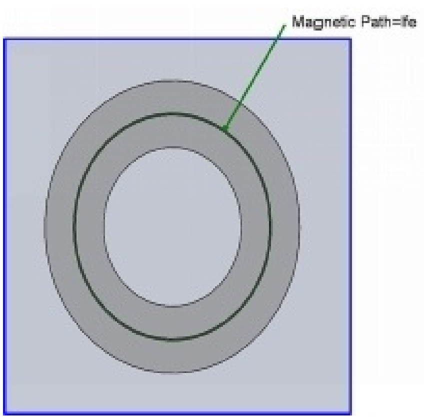


Fig. 8 – The magnetic path length in a toroidal core

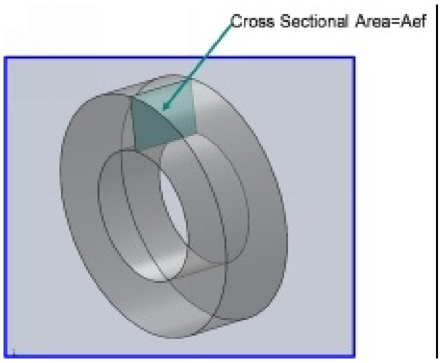


Fig. 8A The Cross Sectional area Aeff in green

The cross sectional area is that one perpendicular with the lines of magnetic force. The numer of line crossing it constitutes the flux and the number of lines per unit square is the flux density like gauss and Tesla units.

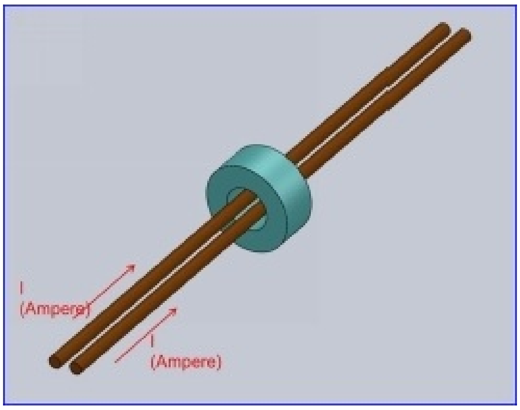


Fig. 9 - Two turns passing through the center of the toroid

The concept of one turn is like show in fig 7 and Fig. 9, where the turn is passing through the center of a Toroid. In this sense can be said in fig. 9 that there is one turn of 2 ampere or 2 turns of 1 ampere each.

Magnetization

We would rise the temperature of the core beyond the Curie temperature demagnetizing it completely. From this point, departing from zero current and rising gradually, measuring and plotting each point of the curve H-B, we would get the graph of Fig;10

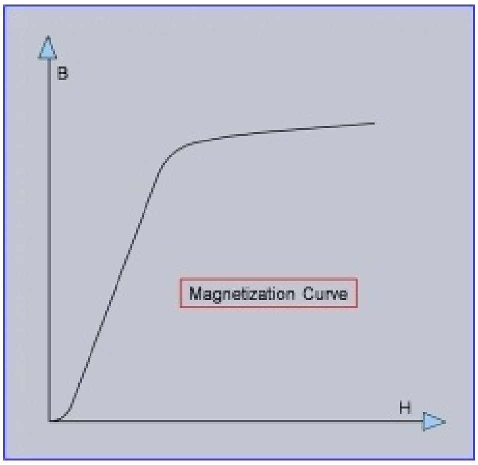


Fig. 10 – Magnetic material when magnetized first time produces a "Virgin" Curve.

When the current falls down diminishing H. Plotting again B–H curve, reaching the zero point at the H axis we can observe that the flux density is not zero. This is due to the fact that the interaction between domains, keep the magnetic material magnetized. This residual magnetization is called remanence.

In order to bring back the Flux Density B to zero, we need to apply the coercive force $–H_r$.

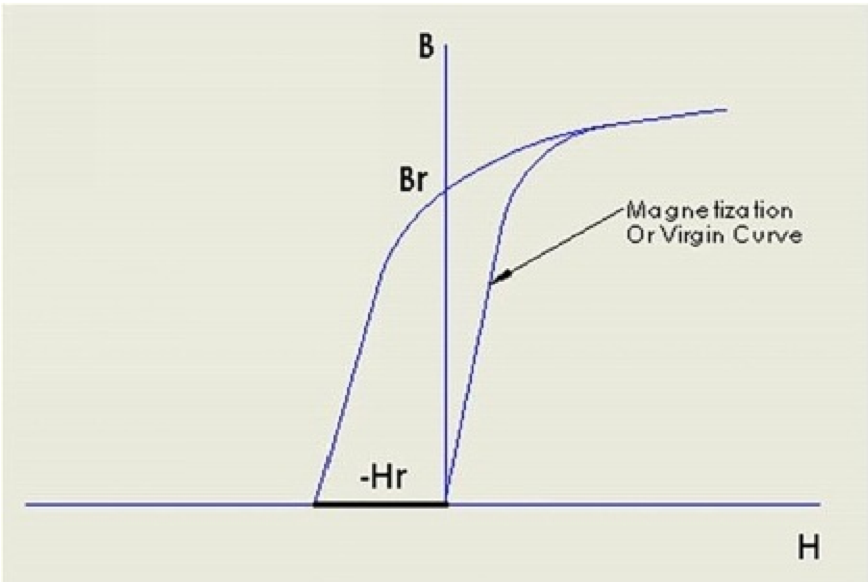


Fig. 11 The remanence maintain a magnetic field, even with the excitation current=0

Flux Density B:

We saw determining the flux density over the field H. The final imposition of flux density is dependent on the material characteristics such as permeability, but there is a way to directly impose flux density. Let consider de faraday´s law in its differential notation:

$$E = N \cdot \frac{d\phi}{dt}$$

$$Edt = n \cdot d\phi$$

$$\int Edt = \int nd\phi$$

Solving:

$$E \cdot t = n \cdot \phi$$

Making:

$$\phi = B \cdot A_{ef}$$

Solving both sides:

$$E \cdot t = N \cdot B \cdot A_{ef}$$

Rearranging:

$$B = \frac{E \cdot t}{N \cdot A_{ef}}$$

We can see in a rather simplified form that, with the arrangement of the coil and core was produced an integrator. Translating to the Simulink of the MatLab We can show now the dynamic System:

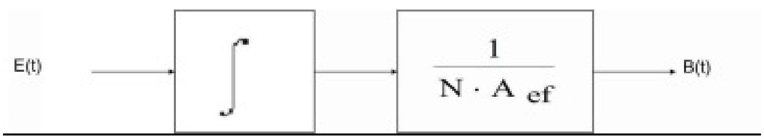


Fig. 12 The number of turns in a core with a cross sectional area produces an "Integrator"

This is produced by the growing field that induces an equal voltage in the opposite direction. The field is forced to grow slowly behaving like an integrator. The integrator constant is dependent solely of the number of turns and the cross sectional area of the core.

Matlab Model:

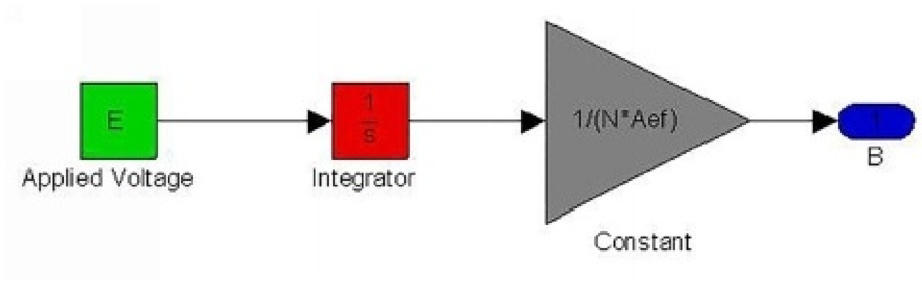


Fig. 13 - Simulink flux density calculator

In this model, we can see that the flux density is not dependent of the iron characteristics. It can be represented solely by a linear integrator and a constant term.

This can apparently be simple, but it is a very powerful model, since it is modeled in Laplace, where all models can be analyzed algebraically, without any difficulty. Besides the fact that structured on this basic model, we can build more complex structures.

We have experienced the construction of many complex systems involving different disciplines, like electrical circuits, thermal behavior and some mechanical structures like inertial movements of trains and more, all working together and giving dozens of information at same time, including animation and visualization, like in a simple videogame.

Let us consider 100 Volts applied to a coil of 100 Turns in a Core with cross sectional area of 100 Cm^2. At the end of 10 milliseconds:

$$E := 100$$

$$t := 10 \times 10^{-3} \qquad A_{ef} := \frac{100}{10000}$$

$$N := 100 \qquad B := \frac{E \cdot t}{N \cdot A_{ef}}$$

$$B = 1 \ \text{Tesla}$$

Let us see in the Matlab the behavior with the time:

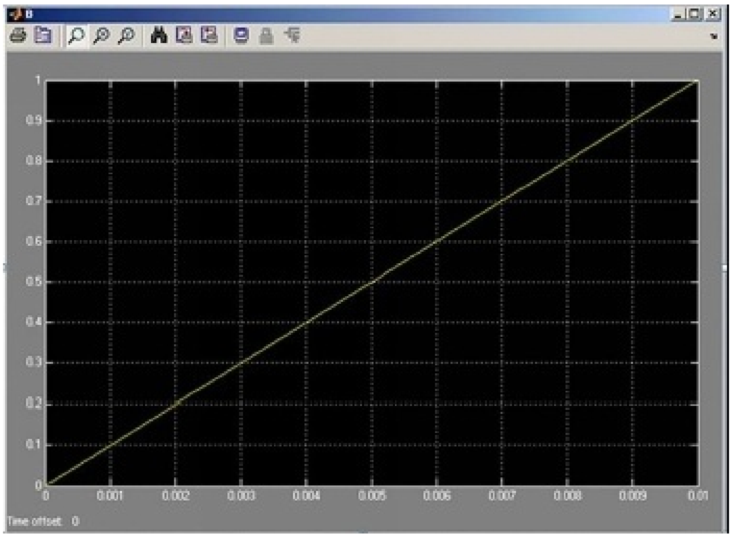


Fig. 14 – Behavior of the flux density in the core (Totally independent of the core material)

From this approach we can see now that the plotting of B-H curve can be done in a more simple way.

In a core at virgin state (Remanence=0), we can apply a constant voltage in the coil. By other side we can measure the H strength through the current, as shown in fig 15

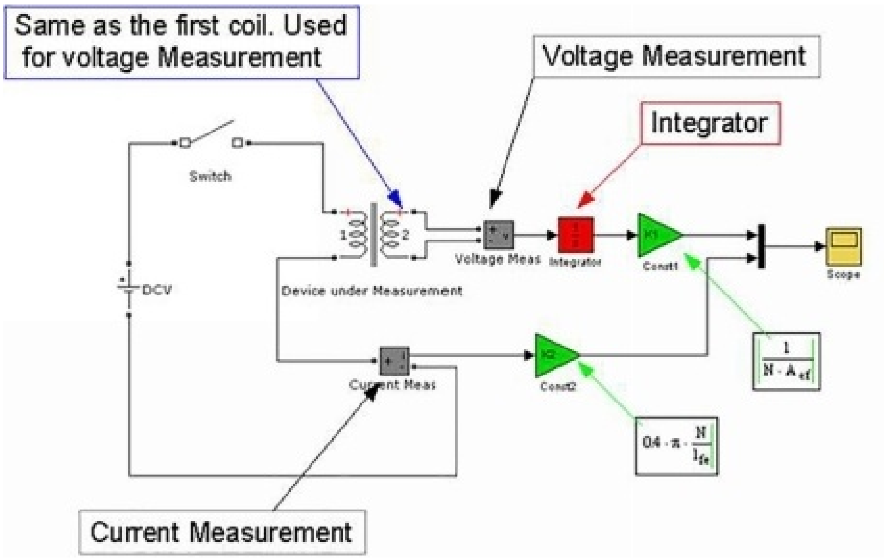


Fig. 15 – Simulink block diagram to plot the virgin curve of the core

Close the switch during the time required to the flux density reaching the maximum value of interest. Plot the results in a osciloscope of memory as illustrated in the fig.16

The voltage is applied to the vertical chanel input that will show the flux density axis. The Output of the const2 is applied to the Horizontal chanel, that will plot the field strenght H.

In the screen of the scope will be plotted the curve shown in fig.16 that represents the behaviour of the field strenght H as a function of the flux density stimulation of the core.

The analog to digital conversion is not represented in our circutry and they are represented soleily by the current and voltage “Meas” interfaces, stardards of the Matlab Symulink CAE system.

The addoption of any commercial interface with the real world for data acquisition is possisble and we have some special tools that can be used.

National Instruments also has a lot of DAQ (Data acquisition systems like Analog/Digital converters and many other appliances). They also have softwares for use for the implementation of such measurement systems.

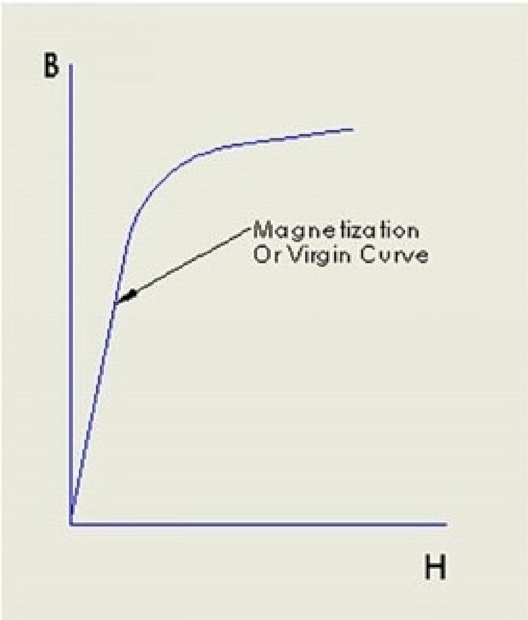


Fig. 16 Virgin B-H curve

Using the same system we can plot the entire hysteresis loop, changing only the switch and battery by a voltage generator. The constants are same.

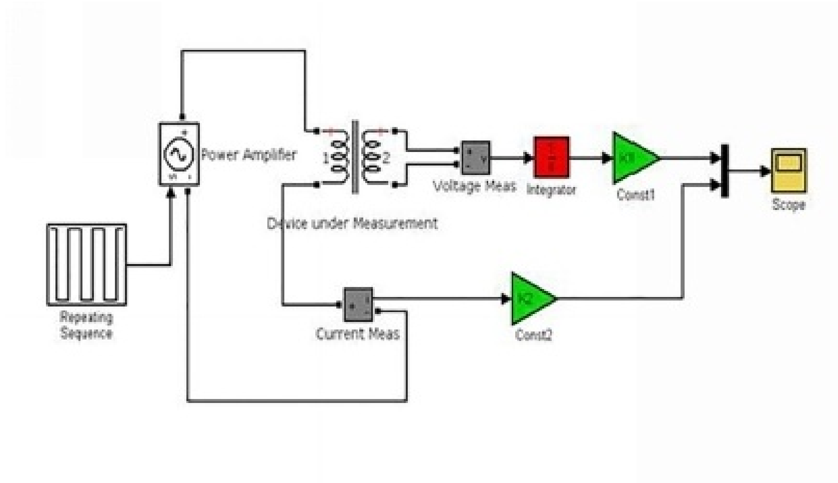


Fig. 17 - Simulink Block diagram for complete hysteresis loop plotting

Voltage applied

Fig 18

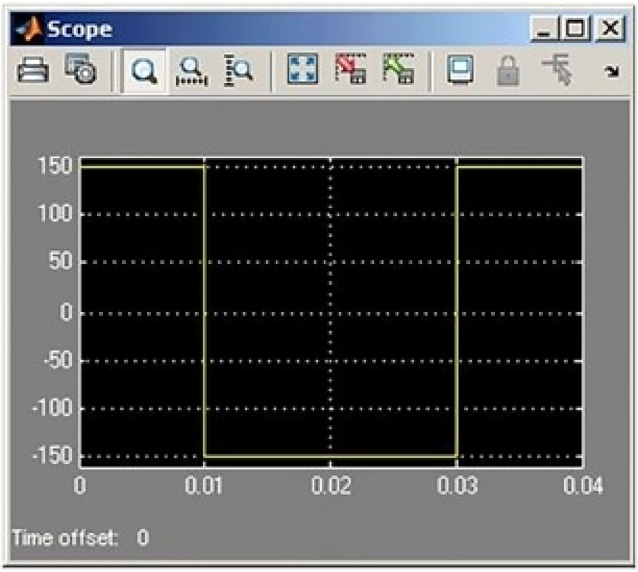


Fig. 18 – Voltage waveform of the oscillator

The Amplitude of voltage and frequency must be calculated for each case.

Flux Density

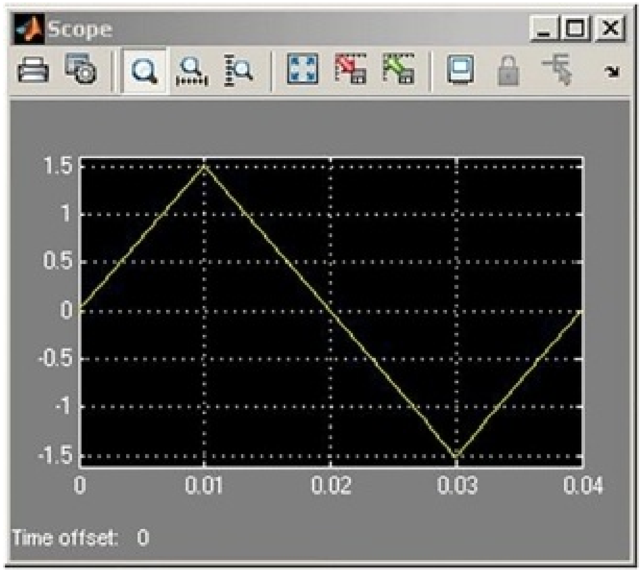


Fig. 19

Plotting using such system in which we connect the Vertical and horizontal channels of the oscilloscope to B and H signals respectively. In the oscilloscope will be formed a parametric Lissajous figure, we will observe the hysteresis loop like presented in fig. 20

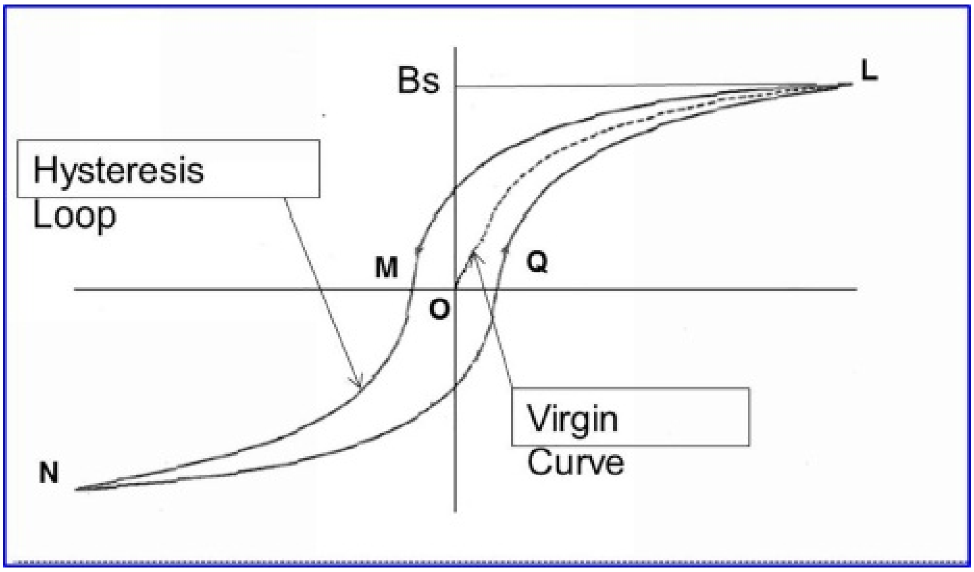


Fig. 20

Such figure, we can see the “Virgin curve” formed by the dashed line O-L. In this condition, the domains of the core will be aligned gradually without any anterior influences. Can be noted in the reverse path, when the curve L-M passes through the vertical axis (H=0). The core is not demagnetized, presenting a B, which is called remanence. This is due to the fact that there is a mutual influence between domains that keeps them aligned, maintaining the magnetization effect. To demagnetize, rearranging them to a rest state, where there isn't any mutual influence, we need to apply a reverse magnetic field called "Coercive force."

Permeability

Magnetic permeability is a physical quantity that defines the ability of a magnetic material to develop a magnetic flux density B, departing from a magnetic strength field H. Normally is represented by the Greek letter μ.

The relation between flux density B and the field strength H is:

$$\mu = B/H$$

The development of the flux density follows the alignment of the domains of the magnetic material contributing each one with its own magnetic field. Due to this fact is shown a (Multiplicative effect) of the flux density from the applied field strength.

Additionally, such multiplication factor is not constant, showing that the permeability presents a strong non linear behavior.

When all the domains were aligned the magnetic material reaches a condition that we call “Saturation” and the extra amount of flux density would be only that generated by the extra field strength.

Between the initial and final flux density (Saturation) there is a point where the permeability presents its maximum value.

There are many different permeability values:

Measurement of permeability:

Permeability of the virgin curve (Magnetization Curve):

With the same circuit with few modifications, we can plot the virgin curve directly in the oscilloscope or in a computer. In the circuit of fig 21, we are using two delayed steps (Step 1 and Step 2), Delayed one in relation to the other with a time window required to the measurement.

The comparator at the bottom of the diagram is responsible to open the window in the time required to the measurement.

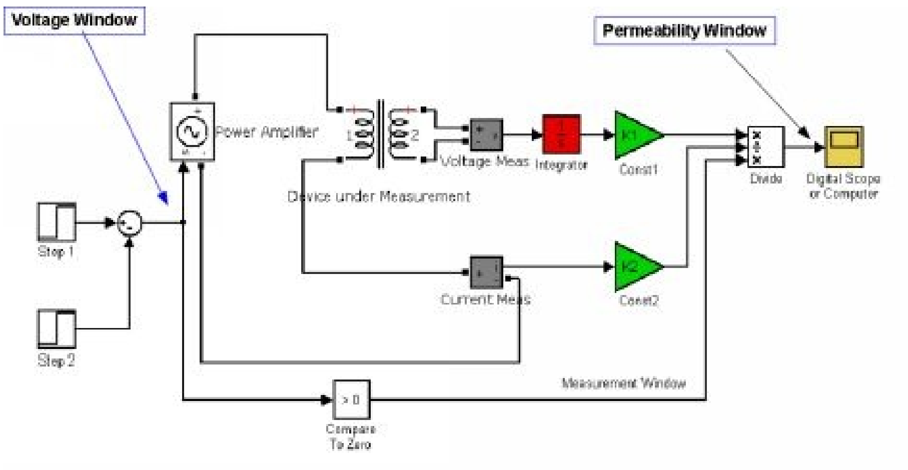


Fig. 21 Diagram in Simulink for the plotting the permeability of the "Virgin Curve"

Amplitude permeability

The relative permeability periodically alternating, following the flux density and field strength variation in an alternating fashion, forms a waveform of permeability.

The peak of such permeability waveform is called amplitude permeability.

Initial Permeability, Rayleigh Constant and Rayleigh law.

At low alternating periodic fields the magnetism behaves like a quadratic curve as described by Lord Rayleigh. This quadratic equation is known as Rayleigh law and occurs at Rayleigh´s region. The field, bellow certain level like shown in Fig. 22 the permeability is constant and the hysteresis loop obey the Rayleigh Law. The main parameters are the initial permeability μ_i and Rayleigh constant n.

$$B = \mu_i \times H + n \times H^2$$

The initial permeability is that incremental permeability found at very low ac fields of excitation. It is found in the data sheet of magnetic material, with this name: Initial permeability end have to be used only at very low excitation fields.

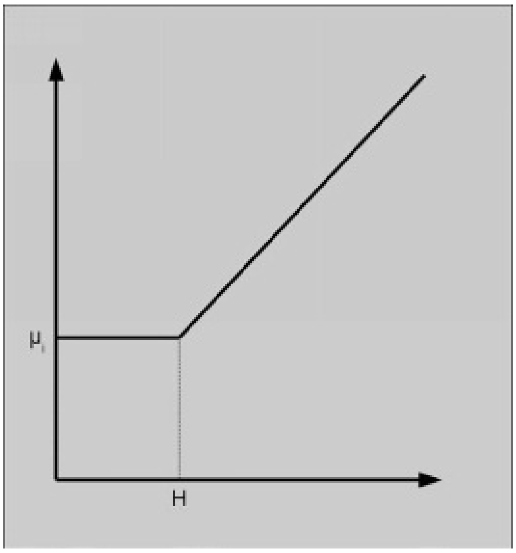


Fig. 22 Below certain point the permeability is constant

Absolute and relative permeability

This is manly referenced as the permeability of vacuum and can be found in many constant table of Physics as μ_o, however the absolute permeability of a certain material is the total permeability found in it:

$$m_a = m_0 \times m_r \qquad \text{wheremr is the Relative permeabilit}$$

Always in this book we will be dealing with relative permeability.

Complex permeability

Due to the phase shift between flux density and field strength in the alternating field their quotient in complex notation is likewise a complex quantity. In such a view the permeability comprises a real and imaginary part.

The customary notation applicable is in particular for a small excitation and under sinusoidal magnetization during linear behavior.

Apparent permeability

The relation is between the inductance of a coil with core and the inductance of the same coil without core.

$m_a = m_o \times m_r$ wheremr is the Relative permeat

Mapp=L/L’

Differential permeability

Is the permeability Mdiff at any point on the hysteresis loop for small values of B and H, that H, Thai is, db and dh tending to zero and gives the local gradients of the loop.

In the case of materials with a rectangular it is the quoted, for example, for the point of the coercive field strength end the point of remanence.

Magnetostriction

The deformation observed in a body of magnetic material when submitted to a magnetic field. It is observed a change in length or in volume. Both are important features of magnetic materials.

With alternating fields the behavior is rather different due to the fact that the skin depth of the eddy currents restricts the penetration and the material behaves differently in each surface layer in the direction towards the interior of the body.

The Magnetostriction is an important feature, mainly as a noise source in transformers

.

Inductance and Inductors

What is Inductance?

The inductance is the property of one component called inductor that presents a voltage drop proportional to the variation of current that flow through it.

There are two principles that govern the inductance:

1)A steady current creates a steady magnetic field as governed by Ampere’s Law.

2)A Variation of magnetic field produces a voltage in its terminals as governed by Faraday’s Law.

By the Faraday’s law, like defined in section “Flux Density B”, fig 12, we have:

$$B = \frac{1}{N \cdot A_{ef}} \cdot \int E\,dt$$

Represented in Block diagram:

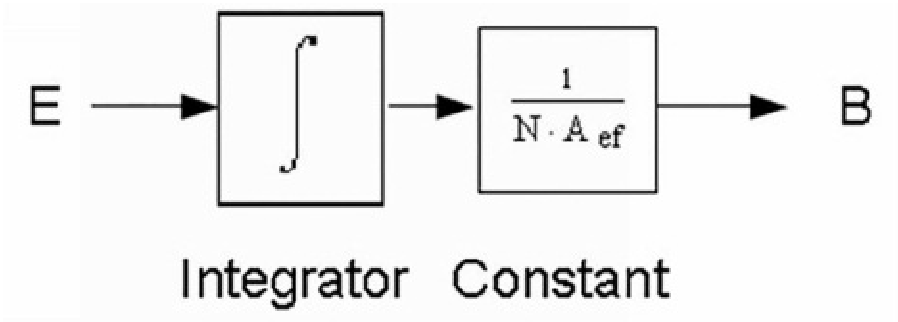


Fig. 23 – The flux density developed by the voltage

By the Ampere’s Law:

$$H = \frac{N \cdot I}{l_{fe}}$$

$$B = \mu_0 \cdot H = \frac{\mu_0 \cdot N \cdot I}{l_{fe}}$$

Handling:

$$I = \frac{B \cdot l_{fe}}{\mu_0 \cdot N}$$

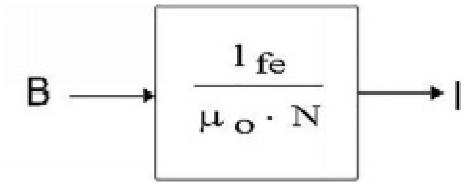


Fig. 24 – There isn´t any time delay between flux density and the current

Chaining the two blocks diagrams, we have:

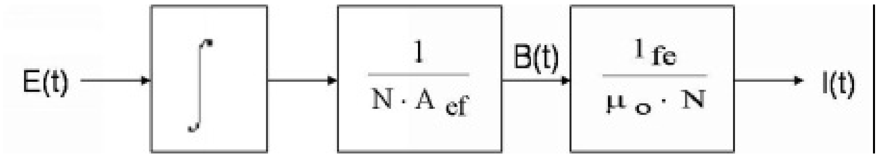


Fig. 25 I(t) as function of e(t)

Simplifying, we get:

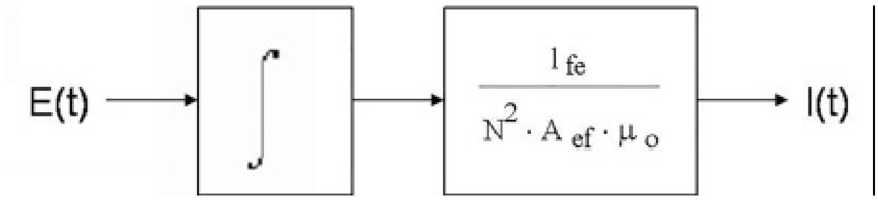


Fig. 26 I(t) as a function of E(t) in an inductor

Or in differential form:

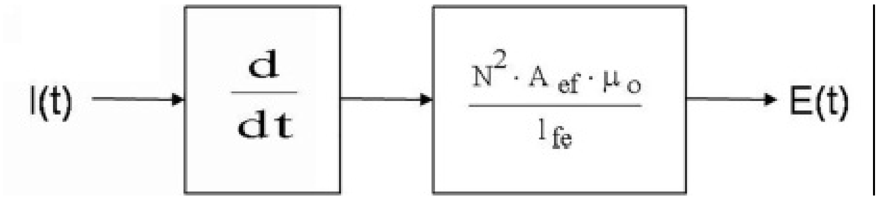


Fig. 27 Voltage as a function of the variation of current

In other words:

$$E = L \times \frac{di}{dt}$$

Where:

$$L = \frac{N^2 \times A_{ef} \times \mu_0}{l_{fe}}$$

We have simultaneously defined two things:

The first one is the inductance that is the ability of the inductor to inducing a voltage due to a variation of the current that flows through it.

The second one is the calculation of an inductor, departing from their dimensions and Physical parameters.

The term Inductance was named by the English Physicist and mathematician Olivier Heaviside. The unity for inductance is the Henry with the symbol H in honor of Joseph Henry.

Is attributed the value of 1 Henry to an inductor that submitted to a current variation of 1 Amp per second, display a voltage drop of 1 volt. Inversely if an inductor of 1 Henry is submitted to a voltage of 1 volt, it will present a variation of current of 1 amp per second.

By the same origin, other consequent relations must be defined:

$$f = \frac{L \times I}{N}$$

$$B = \frac{L \times I}{A_{ef} \times N}$$

The complete formula for inductance:

$$L = \frac{N^2 \times A_{ef} \times m_o}{l_{fe}}$$

Including the relative Permeability, we have:

$$L = \frac{N^2 \times A_{ef} \times m_o \times m_r}{l_{fe}}$$

$$L = \frac{N^2 \times \mu_0}{\dfrac{l_{fe}}{A_{ef} \times \mu_r}}$$

$$R = \frac{l_{fe}}{A_{ef} \times \mu_r}$$

Reluctance

The reluctance of the magnetic circuit is the ratio of the MMF to the magnetic flux. It is analogous to a resistance in an electrical circuit.

Thus:

$$R = \frac{MMF}{F}$$

Where:

R = Reluctance

MMF =Magnetomotive force (MMF) in ampere

F is the magnetic flux in Webers

The reluctance can be used exactly as resistances in an electric circuit that can be put in series or parallel or even combination of them in a more complex array.

Let us see some possibilities:

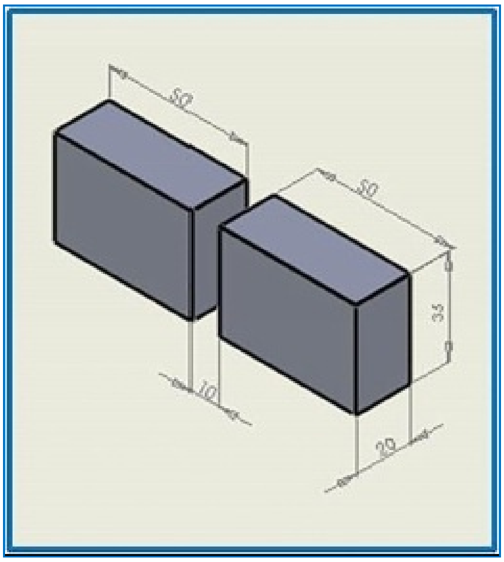


Fig. 28 – Core geometry for the calculation of reluctance

There are 2 parts of an iron core; the length of each piece is 50 cm. The effective area is 29 * 35 = 1015Cm^2. There is an air gap of 10 cm. The permeability of the core is 2000. What is the total reluctance?

$l_{fe} := 50$

$\mu_r := 2000$

$A_{ef} := 1015$

$l_g := 10$

$$R = \frac{l_{fe}}{\mu_r \cdot A_{ef}} + \frac{l_g}{A_{ef}} + \frac{l_{fe}}{\mu_r \cdot A_{ef}}$$

$$R := \frac{50}{2000 \cdot 1015} + \frac{10}{1015} + \frac{50}{2000 \cdot 1015}$$

$$R = 9.901 \cdot 10^{-3}$$

There is an silicon iron core with lfe=158cm, lg=2cm, Aef=100Cm^2

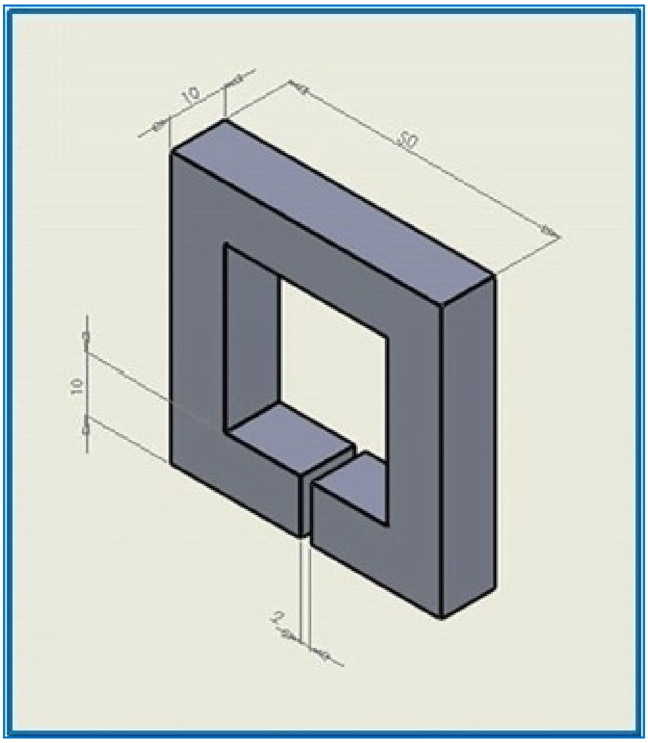


Fig. 29 – Core geometry for the calculation of reluctance

$$l_{fe} := 158$$

$$\mu_r := 2000$$

$$A_{ef} := 100$$

$$l_g := 2$$

$$R = \frac{l_{fe}}{\mu_r \times A_{ef}} + \frac{l_g}{A_{ef}}$$

$$R := \frac{158}{2000 \times 100} + \frac{2}{100}$$

$$R = 0.02$$

Now let us arrange the formula to calculate the inductance.

$$L = \frac{N^2 \times \mu_0}{R}$$

Adjusting the unit for centimeter, we get:

$$L = \frac{0.4 \cdot \pi \cdot N^2}{10^8 \cdot R}$$

$$N := 100$$

$$L := \frac{0.4 \cdot \pi \cdot N^2}{10^8 \cdot R}$$

Having 100 turns, we get: $L = 6.044 \cdot 10^{-3}$

If we neglect the spreading of the flux in the gap, we can consider that we have reluctance with a resultant permeability:

$$\frac{l_{fe}}{\mu_{res} \cdot A_{ef}} = \frac{L_{fe}}{\mu_r \cdot A_{ef}} + \frac{l_g}{A_{ef}}$$

$$\frac{1}{\mu_{res}} = \frac{1}{\mu_r} + \frac{l_g}{l_{fe}}$$

$$\mu_{res} = \frac{1}{\frac{1}{\mu_r} + \frac{l_g}{l_{fe}} \cdot \frac{\mu_r}{\mu_r}}$$

$$\mu_{res} = \frac{\mu_r}{1 + \mu_r \cdot \frac{l_g}{l_{fe}}}$$

Calculating:

$$\mu_{res} := \frac{2000}{1 + \frac{2}{158} \cdot 2000}$$

$$\mu_{res} = 75.99$$

This is the resultant permeability of the total magnetic circuit:

And the total reluctance is:

$$R := \frac{158}{\mu_{res} \cdot 100}$$

$$R = 0.021$$

Calculating the flux density of the Inductor:

$$B = \frac{L \cdot I \cdot 10^{8}}{N \cdot A_{ef}} = \frac{0.4 \cdot \pi \cdot N \cdot \mu_{res} \cdot I}{l_{fe}}$$

Considering the current in the inductor equal to100A, we have:

For:

$I := 100$

$L = 6.044 \cdot 10^{-3}$

$N = 100$

$A_{ef} = 100$

$$\frac{L \times 100 \times 10^{8}}{N \times A_{ef}} = 6.044\ 10^{3}$$

6044 Gausses

Losses in Magnetics Materials

Losses

Among the most important features of the cores used in magnetic components are the losses.

They are responsible for heating and efficiency in most applications. Mostly the losses are surveyed by their own manufactures that presents their behavior in the specifications sheets, through graphs or tables as a function of frequency and flux density.

In the last times is customary that they present the losses in the format of a modified Steinmetz equation for their manufactured materials.

The measurement of losses:

The classical method of measurement of losses is through the observation of the behavior of the current and voltage as response for an excitation applied to a sample.

The application of the excitation can be done through a curled coil in the core sample with has a convenient shape. There is another coil that is used to read the voltage, measuring indirectly the flux density by the application of a numerical external constant over the value measured. This numerical constant depends on the core geometry and the number of turns of the coil.

A sinusoidal voltage is applied, the voltage and current are measured and computed the instantaneous product of both. The average value over each cycle is also computed and the result is the average loss in watts of that material. Plotting all the values, changing the frequency and the flux density (By the voltage), the total figure of the material can be surveyed.

In many cases the measurement can be done through an digital oscilloscope that can perform the measurements, multiplication and integration, giving at the end, the average calculation of the product over the entire cycle.

Another solution is the employment of special data acquisition modules like that one's of National Instruments, which can acquire data in a fast fashion. All the other calculation done, inside a computer through, MatLab or even in NI software Labview or on an specially designed software.

In certain materials like ferrites is important that all the measurements must be made under rigorous temperature control.

Mostly there are two ways to control the temperature in the samples being tested: one is the employment of a environmental chamber, but the problem is that a massive body with a wide external surface, must be attached, to get good coupling between internal environment temperature controlled and the material under test.

One other method is the "Bath method" that the material is steeped in a bath of vegetal oil with controlled temperature.

The two methods must take in consideration of the influence of the observer in the measurement and consequently care must be taken to not influence the measures with the excitations that must be done in a short period of time to no lead power to the DUT (Device under test). Is interesting to give some calculated interval, between test in accordance with the total mass and the power delivered in each measurement.

Representation of losses

The modeling of losses in the cores is complex and many attempts have been made toward this, direction in order to we get the means to calculate losses in different formulations of problems, in the broad spectrum of applications that we try to cover.

The historical model is the famous equation proposed by Steinmetz in 1892, known as Steinmetz equation.

$$P_i = K \times B^b$$

In which Pi was the average loss per unit volume or Weight, K a constant, B the flux density, and b a coefficient.

Currently, most manufacturers provide data through a “Modified Steinmetz equation”, in which has been introduced the dependence of frequency f.

$$P_i = K \times f^a \times B^b$$

Unfortunately, these data relate only to sinusoidal excitation, and for the applications of oriented grain silicon steel, used in distribution applications, is useful, but for others, with rich harmonic waveforms or in cases of ferrite to switching power supplies or even inductors with rich harmonic currents, it is impossible to use them.

General Losses

Due to the complexity of the matter, one looks for the phenomenological laws summarizing in simple terms the regularities observed in experiments.

The natural path is the separation of the phenomena involved, to study in detail each one of them.

We can then classify 3 different aspects contributing to the total losses. P_h, P_{cl} and P_{exc}, termed hysteresis loss, classical loss, and excess loss.

$$P = P_h + P_{cl} + P_{exc}$$

Such separation of terms reflects the existence of three scales in the magnetization process; the scale set by the specimen geometry (Classical loss), the scale of domain wall-pinning mechanisms (Hysteresis loss) and the scale of magnetic domains.

As can be seen the strongly nonlinear character of the magnetization process, there is no obvious reason why the superposition law expressed should hold true under broad conditions.

Hysteresis losses:

Hysteresis losses are the consequence of the fact that on microscopic scale the magnetization process proceeds through sudden jumps of the magnetic domains walls that are pressed by the external field. The local eddy currents induced by induction, change accompanying the wall jump, dissipating a finite amount of energy through the Joule effect. The sum over the jumps gives the hysteresis loss associated with the jump sequence.

The jumps are so short (Of the order of 10^{-8} ~10^{-9} S) that the external field is unable to influence the internal jump dynamics.

The only effect of the field is to compress or expand the time interval between subsequent jumps in inverse proportion to the field rate of change which yield a number of jumps per unit time and an amount of energy dissipated per unit time, proportional to the magnetization frequency. Therefore for the hysteresis loss in a loop of peak induction B, one obtains the typical expression:

$$P_h = 4 \times K_{hist} \times B^b \times f \qquad \frac{w}{m^3}$$

The loss per cycle Wh=Ph/f is thus independent of frequency.

The parameters Khyst and b, include the structural aspects affecting domain wall pinning and magnetization reversal.

Their value will differ from material to material. The rule b =1.6 has claimed to have some general validity and is known as Stainmetz Law.

Classical losses:

The classical loss is that one calculated directly from Maxwell Equations for a perfect homogeneous conducting medium, that is, with non structural inhomogenities and magnetic domains.

The scale relevant to the classical loss is the scale set by the system geometry. For example in a lamination, the slab thickness d. This scale controls the boundary condition for Maxwell equations, which in turn determine the distribution of eddy currents in the specimen cross-section and ensuring joule dissipation.

$$P_{cl} = \frac{p^2 \times s \times d^2}{6} \times (B \times f)^2 \qquad \frac{w}{m^3}$$

It is calculated for a lamination of thickness d, and electrical conductivity s , frequency f and peak induction B.

Excess Losses

The excess loss results from the smooth, large scale motion of domain walls across the specimen cross section, when the fine scale jumps responsible for the hysteresis loss are disregarded.

Eddy currents tends to concentrate around the moving domain walls, as a fact that gives rise to loss higher than the classical ones because of the quadratic dependence of the local loss (J^2*s)/Dn) on the intensity of the eddy current density.

According to the picture just described, excess losses are expected to be ubiquitous as the presence of magnetic domains. However the importance of excess losses in comparison with classical and hysteresis losses will substantially depend on the size and arrangement of magnetic domains.

As general rule, the smaller the domain structure the smaller the excess loss contribution. This conclusion can be given a precise quantitative measure in the case of lamination of thickness d containing longitudinal bar like magnetic domains of random width.

In this case Maxwell Equations can be exactly solved, giving the result:

$$P_{Exc} = \frac{48}{\pi^3} \cdot \left(\sum_{odd\_n} \frac{1}{n^3} \right) \cdot \frac{2 \cdot L}{d} \cdot P_{cl} = 1.63 \cdot 2 \cdot \frac{L}{d} \cdot P_{cl} \quad \left(\frac{W}{m^3} \right)$$

Where 2*L Represents the average domain width and P_{cl} given by the formula for classical losses:

$$P_{cl} = \frac{\pi^2 \cdot \sigma \cdot d^2}{6} \cdot (B \cdot f)^2 \quad \frac{W}{m^3}$$

f=frequency

B=peak induction

Identifies the ratio 2*L/d

The prediction is helpful in the interpretation of losses in grain oriented silicon steels, where domain structures are often not far from the one assumed in the model. However one should not forget the regular arrangement of longitudinal domains assumed in the model can be far from the conditions encountered in ordinary materials. In many cases the domain size is no longer itself the important parameter, because micro structural features introduce additional characteristic lengths that tend to control the loss problem.

In these more complex cases the excess loss approximately follows a law of type:

$$P_{exc} = K_{exc} \cdot \sqrt{\sigma}\,(B \cdot f)^{\frac{3}{2}} \qquad \left(\frac{W}{m^3}\right)$$

Where the parameter K_{exc} includes the effect of various relevant of micro structural factors. The equation above applies to grain oriented silicon steel also, a fact that should not be interpreted as a contradiction with respect to the immediate anterior equation. The point is that the ratio 2*L/d in that equation is unknown. In particular, there is no reason why the ratio should be a constant of the material.

Facts affecting the total losses

According to the previous explanations an approximate expression for the total power loss, valid at frequencies low enough to avoid the onset off important eddy current shielding is:

$$P = 4 \cdot K_{hyst} \cdot B^2 \cdot f + \frac{\pi^2 \cdot \sigma \cdot d^2}{6} \cdot (B \cdot f)^2 + K_{exc} \cdot \sqrt{\sigma} \cdot (B \cdot f)^{\frac{3}{2}} \qquad \frac{W}{m^3}$$

This equation applies to the particular case of a material magnetized under controlled sinusoidal induction rate. However there are several situations where magnetic material operates under distorted flux conditions, for which a generalization of above is needed. This generalization is obtained by calculating the instantaneous power loss at individual points of the magnetization cycle and then by carrying out the time average over the particular wave shape involved. This more general expression reads:

$$P = P_h + \frac{\sigma \cdot d^2}{12} \cdot \overrightarrow{\left(\frac{dB}{dt}\right)^2} + K_{exc} \cdot \sqrt{\sigma} \cdot \overrightarrow{\left(\frac{dB}{dt}\right)^{\frac{3}{2}}} \qquad \frac{W}{m^3}$$

Where the arrows are not meaning “vectorize”, but instead time averaging over the magnetization cycle.

The hysteresis loss is independent of the induction wave-shape , provided the induction is monotone in each magnetization half-cycle.

Loss behavior in different magnetic materials.

The dependence of losses on frequency and induction from the combination of the three terms of the equation above, the final behavior can be rather different from material to material. Ferrites, Silicon Iron, Nickel Iron and others, present different amounts of each of them. Unfortunately the manufactures do not separate their losses, but this is not a problem because generally they present curves and even coefficients of the Steinmetz equation.

Soft ferrites for high frequency applications:

Most applications at high frequencies up to 100Mhz, leads to the use of soft ferrites due to their non metallic nature partially avoid eddy current dissipation. In most applications, mixed Mn-Zn or Ni-Zn are employed, in which it is possible to tailor material properties to an specific application by trimming the concentration of the component atoms.

The electrical resistivity is in the order of 10^7 Wm for Ni-Zn and 10^{-2} up to 10 Wm for Zn-Mn ferrites. The initial susceptibility is constant and the loss angle is negligible up to frequencies of some Mhz to Mn-Zn and 100Mhz for Ni-Zn ferrites.

Searching for a general loss model which could include harmonics and DC currents:

The MSE Approach - (Modified Steinmetz Equation)

Based on the physical understanding as mentioned above that the loss depends of the average of derivative of the flux density over time [16] we can obtain

$$\left(\frac{dB}{dt}\right)_B = \frac{1}{\Delta B} \cdot \int_0^T \left(\frac{dB}{dt}\right)^2 dt$$

Where ΔB is the peak-to-peak flux amplitude and T is the period of the flux waveform. From this, an “equivalent frequency” is defined as:

$$f_{eq} = \frac{2}{\Delta B^2 \cdot \pi^2} \cdot \int_0^T \left(\frac{dB}{dt}\right)^2 dt$$

The loss is then estimated with the modified Steinmetz equation (MSE)

$$\overline{Pv} = k \cdot \left(f_{eq}\right)^{(\alpha - 1)} \cdot B^{\beta} \cdot f_r$$

Where B is the peak flux density amplitude, PV with the bar on the top meaning time average of power loss per unit volume and:

$$f_r = \frac{1}{T}$$

Is the repetition frequency.

The GSE Approach (General Steinmetz Equation)

The work presented in [17] by Jieli Li, Tarek Abdallah, and Sullivan presents the approach of the GSE formulation that can be used to calculate loss for any given waveform:

$$Pv = \frac{1}{T} \cdot \int_0^T K1 \cdot \left(\left| \frac{dB}{dt} \right| \right)^{\alpha} \cdot \left(|B(t)| \right)^{\beta - \alpha} dt$$

As presented by the Authors the equation above is the Generalized Steinmetz equation.

They presented the case where the waveform is sinusoidal, the equation must behave like the Steinmetz equation for sinusoidal case. For that, they replaced B(t) by a sinusoidal wave, resulting in:

$$P_V = k1 \cdot \omega^{\alpha} B^{\beta} \cdot \int_0^t \frac{1}{T} \cdot \left(|\cos \omega t| \right)^{\alpha} \cdot \left(|\sin \omega t| \right)^{\beta - \alpha} dt$$

With:

$$T = \frac{2 \cdot \pi}{\omega}$$

The integral here is independent of ώ, and so the equation agrees with Steinmetz equation. But before, we need to get the value of k1 in the equation.

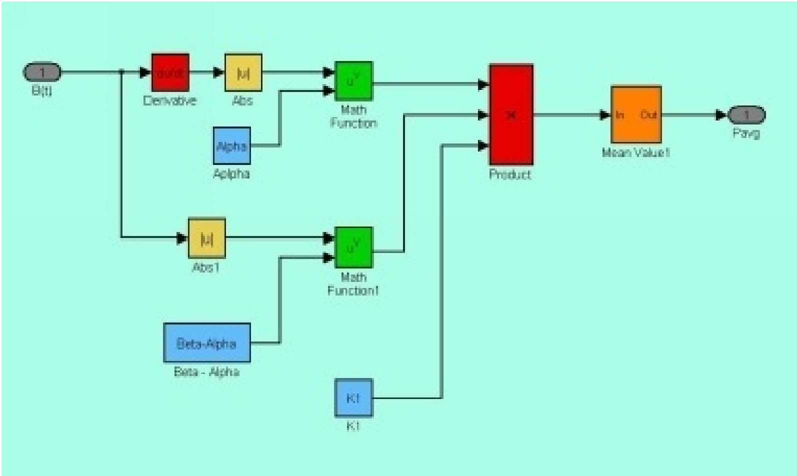
$$K_1 = \frac{k}{(2 \cdot \pi)^{\alpha - 1} \cdot \int_0^{2\pi} \left(|\cos\theta| \right)^{\alpha} \cdot \left(|\sin\theta| \right)^{\beta - \alpha} d\theta}$$

As mentioned and presented by the Authors in their work, the GSE has good results within determined limits.

Now, we can give a Simulink system that can calculate the GSE presented above. Firstly we must calculate the value of K1, a and b ,

The values a and b can be taken from the values given by the manufacturer in the specifications for the ordinary (Sinusoidal) Steinmetz equation. We can calculate K1 from a given sinusoidal B signal in the input and adjusting K1 in the system fo a same result of the Steinmetz equation. That is all!

An advantage is that the GSE has a DC-bias sensitivity. A disadvantage of the GSE is the accuracy limitation if the third or another relatively low-ordered harmonic of the flux density becomes significant, i.e. if multiple peaks are occurring in the flux density waveform.



To overcome this problem, the GSE was upgraded to the improved Generalized Steinmetz Equation The AGES splits the waveform in one major and one or more minor

loops and thus takes sub loops of the full hysteresis loop into account. This is achieved by a recursive algorithm which calculates the iron losses for each loop separately by

$$P_{av}=\frac{1}{T}\cdot\int_0^T K_1\cdot\left(\left|\frac{Db}{dt}\right|\right)^{a}\cdot\left(|DB|\right)^{(b-a)}dt$$

where B is the peak-to-peak flux density of the current major or minor loop of the waveform.

It should be pointed out that all modifications of the Steinmetz equation have the well known problem that the Steinmetz coefficients vary with frequency. Thus, for waveforms with a high harmonic content, it can be difficult to find applicable coefficients which give good result sover the full frequency range of the applied waveform.

Combination of fields

Ampere´s Law in depth.

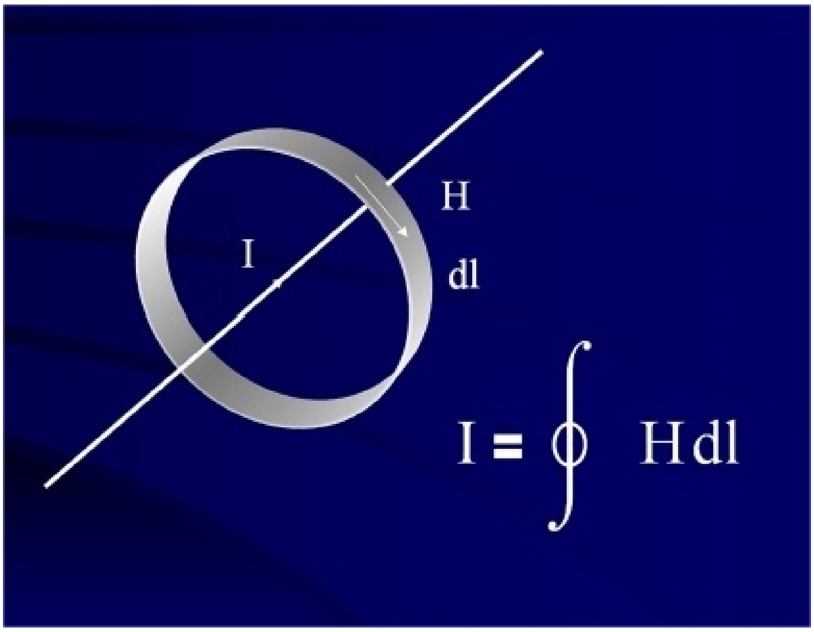


Fig. 30 The representation of the Apere´s Law

The current in a conductor always creates a magnetic field around it with the intensity of 1.25 Oersted for each Amp/cm.

A tangential field to a conductor generates current in it. They are fundamentally interrelated, generating one, intrinsically creates another (Inter-causality).

Practical Example:

In a plier’s ammeter, the current is measured by the intensity of the magnetic field, no matter the size of the pickup ring instrument which is around the conductor.

The current is measured, only in the conductor that is going through inside of the pickup ring.

For the Sake of simplicity

To simplify the analysis, we will establish the hypothesis of an infinite height Planar Conductor with a flat field on their faces. This assumption of course, is a simplification of the effect, do not existing in practice, however, serving for the present purposes and not introducing any misconception. In reality the field decays while going away from the conductor.

Magnetic field produced by currents in endless sheets

As will be shown in the following figure, an uniform current in an infinite plate, produces a uniform field on each side of the sheet, but with opposite polarities. A current density of 1A / Cm generates a field of 0.628 Oersted on each face. With m =1, B= (Gauss), H (Oersted)

In a real conductor, the field quickly decays as it leaves the conductor but in our hypothetical infinite sheet, remains constant.

The creation of this model, aims to simplify the understanding of the combinations of the fields in different configurations, avoiding the detailed description of its variations.

The main objective is to be able to show the different interactions between the magnetic fields that result from combinations of windings in various structures.

Minor differences in decay in the spaces for these representations show no significant deviations when compared to real models, and its real conceptual goal is the representation of all the interactions between fields in a more simple way.

Besides, this book is not intended to be a matter for beginners, but a source of information for experienced designers. The subjects contained in this book, are far of being information found in any common book of transformers. Due to this fact, we feel ourselves very free to make such suppositions, considering that the reader would not make absurd considerations about the subjects being approached.

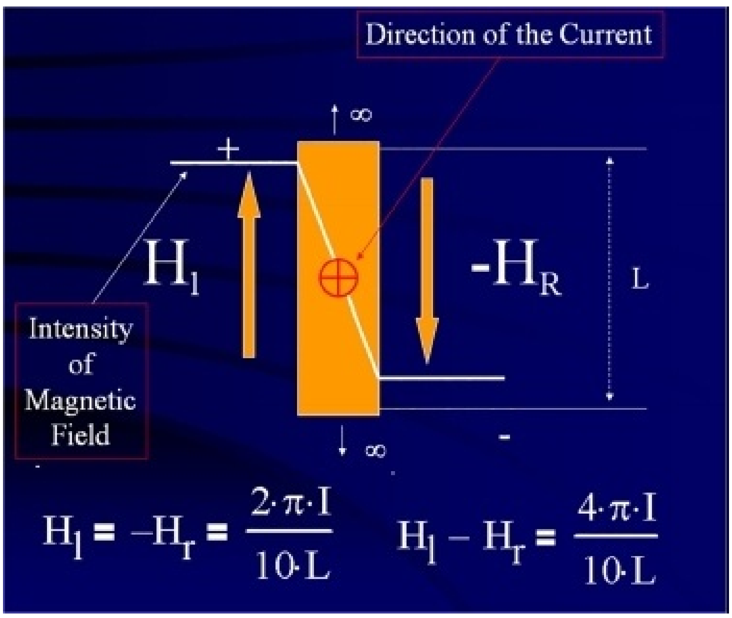


Fig. 31 Suposition of an infinite bar surrounded by a magnetic field

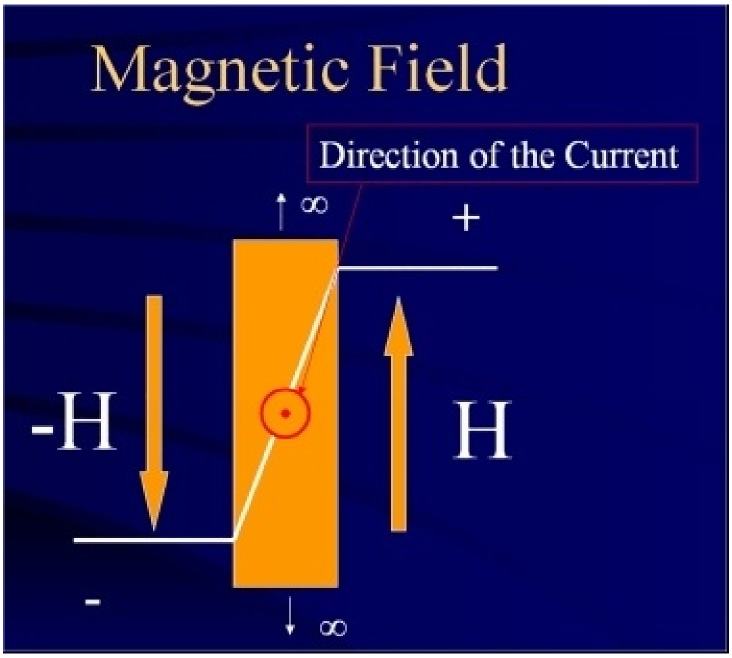


Fig. 32 The direction of the fiel obeys the right hand rule

Addition of fields in two parallel plates

With two endless parallel sheets with equal currents but different directions, the fields are added in the space between them and cancelled in the part external to them.

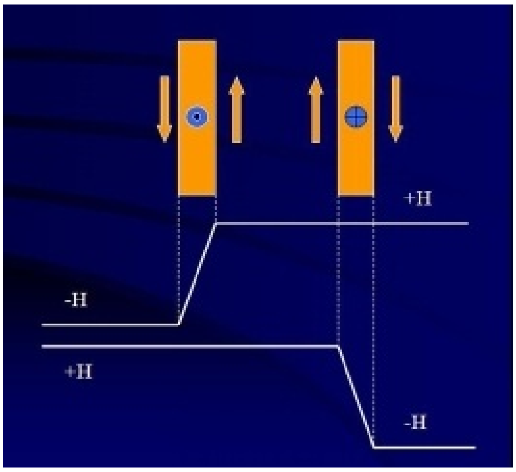


Fig. 33 Adding the field of the two paralleled bars (Neglecting the decay of the fields)

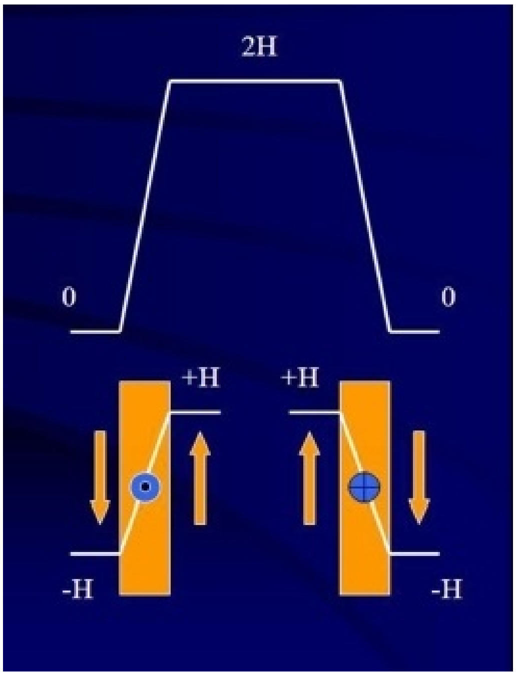


Fig. 34 – The resultant field (Neglecting the decay)

Infinite Solenoid

This distribution field, does not change when we join the sheets to allow current to flow in a continuous loop.

This forms the classical non finite length solenoid in which the flow is uniform inside and on the outside is zero

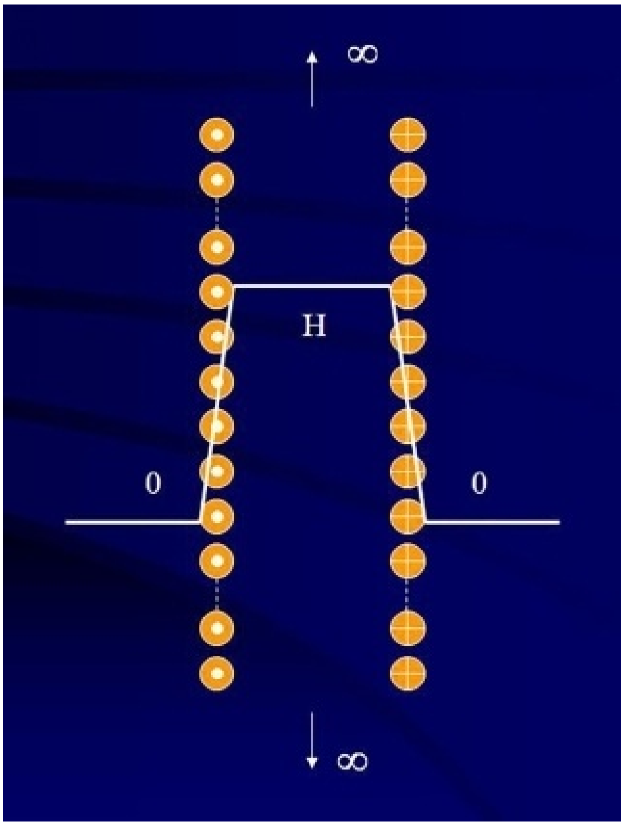


Fig. 35

If the solenoid consisting of a coil rather than a sheet, then:

$$H = \frac{4 \times \pi}{10} \times \frac{N \times I}{l} = 1.257 \times \frac{N \times I}{l}$$

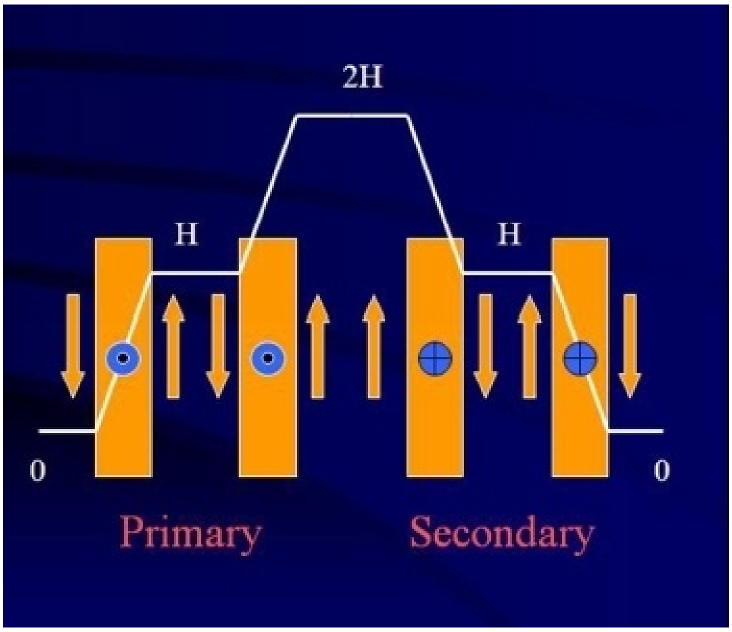


Fig. 36 – If the coil has two layers the interaction of the fields can be seen.

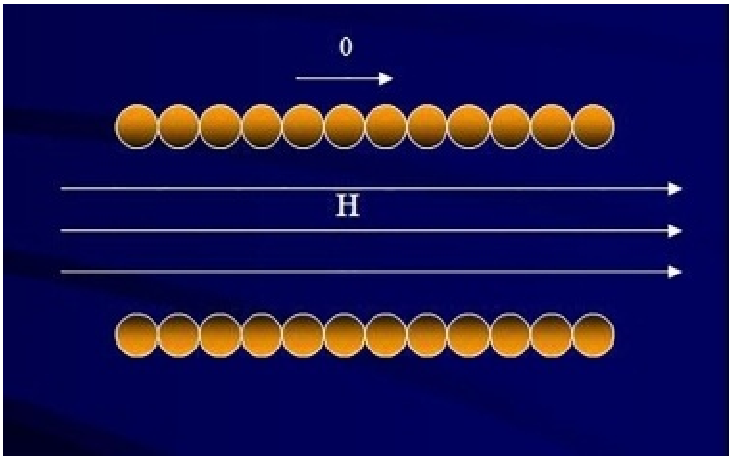


Fig. 37 – Flux = 0 outside and H inside

Addition of fields in several layers:

Two overlapping solenoids

As a solenoid is on top of another, the field of each solenoid is added to the field formed by the previous. Thus we have over the first solenoid H = 0, on top of the second solenoid H = 1 on the third solenoid H = 2 and beneath the third solenoid H = 3.

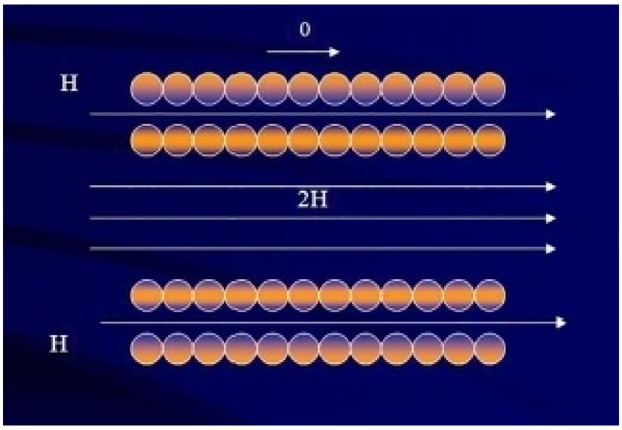


Fig. 38

We can see that inside the solenoid H=2;

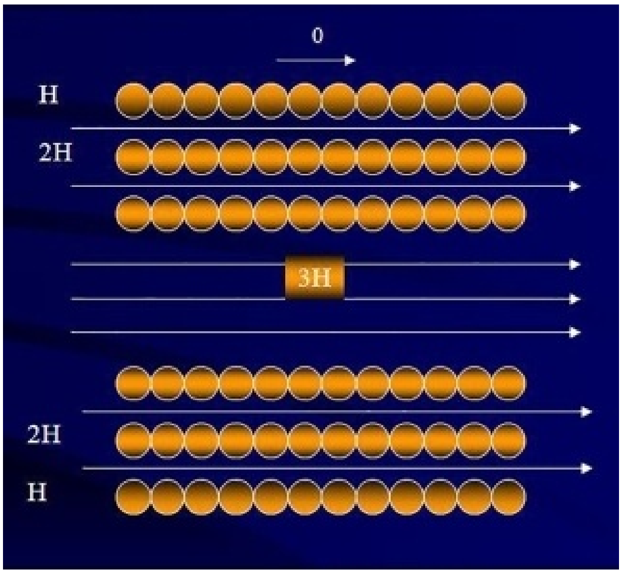


Fig. 39

We can see that inside the solenoid H=3

Permeability:

$$m = \frac{B}{H}$$

Relutance:

$$R = \frac{l}{m \times A}$$

In reality, the magnetic core may be of ferrite, iron, or metal composition, their internal fields depends solely of the reluctance, and H Field manipulated by the solenoid.

In transformers, the secondary current produces an “H field comparison that induces a counter current in the primary winding (as defined by Lenz’s law), and this additional field, does not contribute to the core flux.

We can call this phenomenon “countercurrent confrontation “and the fields produced by the primary and secondary are the same, the only difference being the H field necessary for the magnetization of the core.

$$Np \cdot Ip = Ns \cdot Is$$

Np=Number of Turns

Ip=Current of Primary

Ns=Number of turns of Secondary

Is=current of secondary

The above relationship provides relation of currents in a transformer:

$$\frac{Np}{Ns} = \frac{Is}{Ip}$$

In transformers with adjacent primary and secondary, the MMF distribution depends only on the relative arrangements between them. The overall curvatures are irrelevant, as well as their position in the core.

In inductors, MMF produced by the current is absorbed or “confronted” by the reluctance of core instead of “countercurrent confrontation ” of another winding. Thus, (portions of high reluctance) of the core relative to the winding diagram determine the FMM profile.

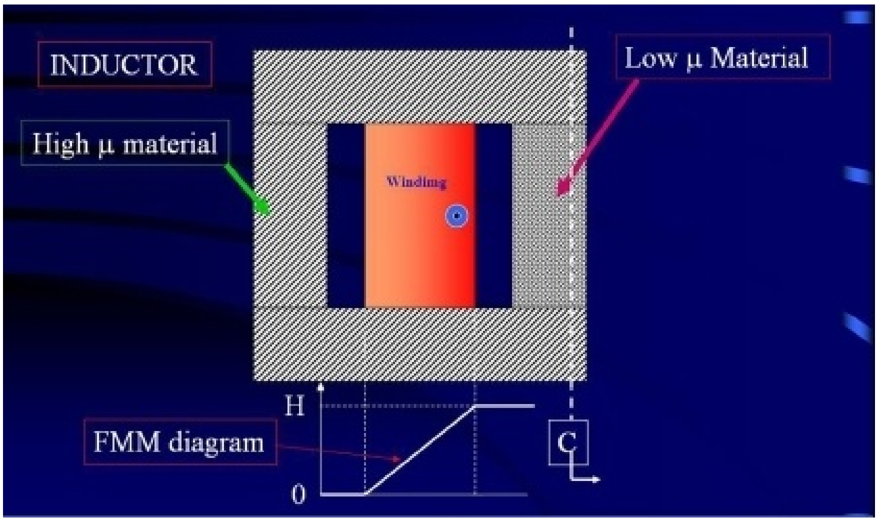


Fig. 40

Note that the core is shaped like a pot, and the axis the dotted line ending below in the "C" inside the square. The same pattern will be repeated to the right from the center.

The more reddish shows the increase in the H field toward the high reluctance element (low permeability).

Note that we are advancing very slowly with the intention to fix very well the concepts involved in the combination of fields. This is a part of my course of transformers

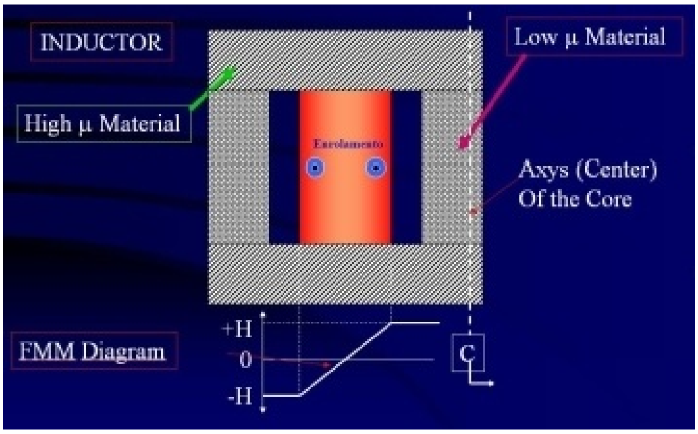


Fig. 41 Evolution of MMF (H Field) diagram due to low mu material in both sides

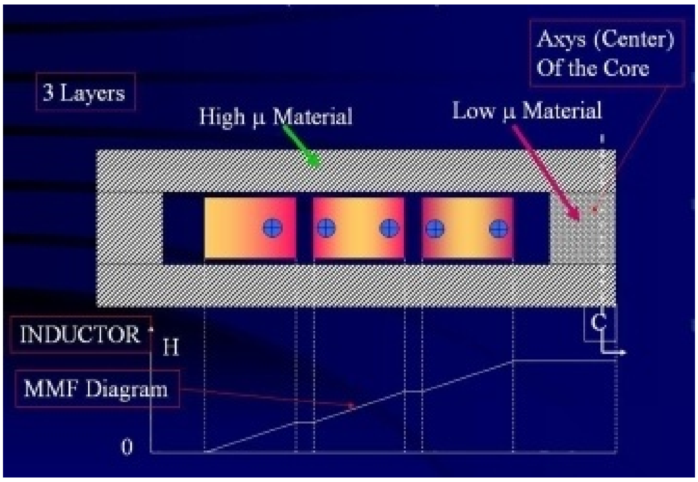


Fig. 42 Evolution of MMF diagram due to low mu material at right

Low m material (High reluctance) in the top side of the core.

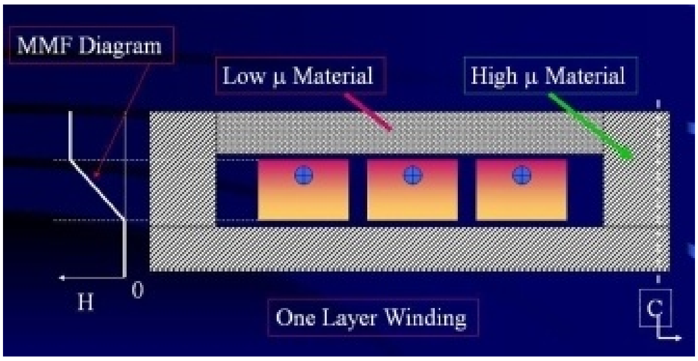


Fig. 43Evolution of H Field due to low mu material at the top

Low mu material (High reluctance) in the top and in the bottom side of the core.

We can see very clear that the mmf (H) diagrams grows toward the low magnetic material (Higher reluctance).

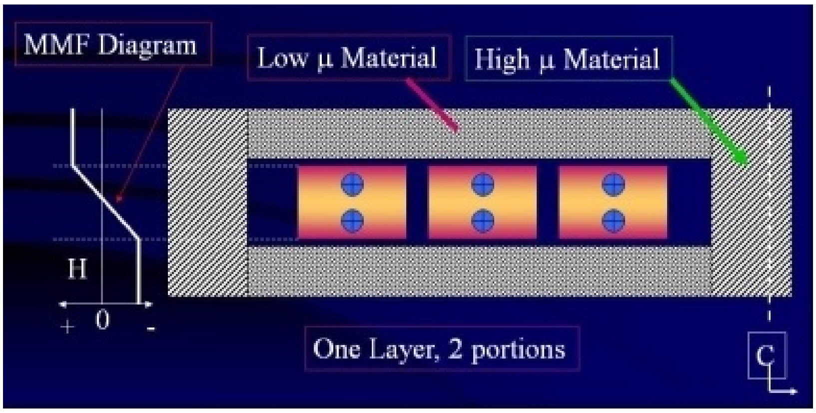


Fig. 44

Simulation by FEA (Finite Element Analysis)

On a core 'E' "curled up a primary and a secondary winding, passing cross currents" I "in each. Each coil has 3 layers and currents are equal. The core is of silicon-Iron sheet. There is no concern for the flux in the core.

This research is going to be done in FEMM.

Simulation by FEA (Finite Elements Analysis by FEMM

We use the FEMM because it is very powerfull and can analise skin depth problems and in addition can be found in Tnternet totally free.

The software was developed by David Meeker, Ph.D and is in the words of it´s own author: A Windows finite element solver for 2D and axisymmetric magnetic, electrostatic, heat flow, and current flow problems with graphical pre- and post-processors.

Is a very powerfull tool and can be foun at Internet in:

http://www.femm.info/wiki/HomePage

Other parts of the software like the Triangle was developed in Computer Science division at Uinversity of California at Berkeley and the software LUA, developed in Pontifícia Universidade Católica do Rio de Janeiro – Brasil.

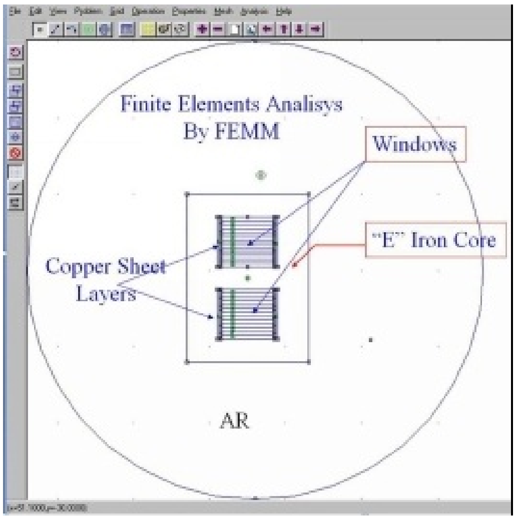


Fig. 45 Can be seen the regions and the setting of materials and the definition of air spaces.

Although the differential equations that describe magnetic phenomena in transformers (And other magnetic components), are relatively compact, it is very difficult to get closed solutions, except for extremely simple geometries. Here is where the idea of finite elements breaks the problem of complex geometries in a large number of simple structural elements, for example triangles. The process solution is reduced through simple geometries.

The advantage of breaking the complex geometry in a large number of simple elements, it is that turned a small problem, but difficult to solve in a big problem, but of relatively simple solution. Specifically the triangulation serves this purpose, as the case that reduces to a simple linear algebra solution with perhaps tens of thousands of unknowns

The FEMM uses triangular elements. About each element, the solution is approximated by linear interpolation of the values of the three vertices of the triangle. The linear algebra problem is formed by the choice of A based on the minimization of the total energy.

Triangulating …

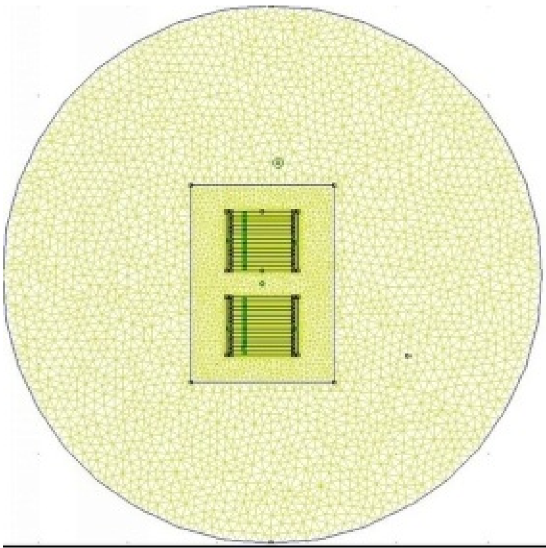


Fig. 46 – The mesh of triangles is generated by the software

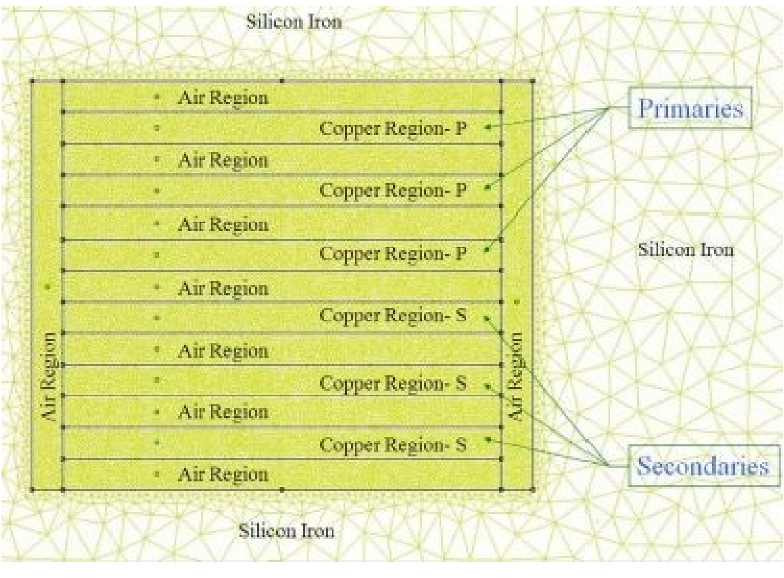


Fig. 47 – Now we can see a closed view of the definitions of material and the mesh of triangles

Solving the model:

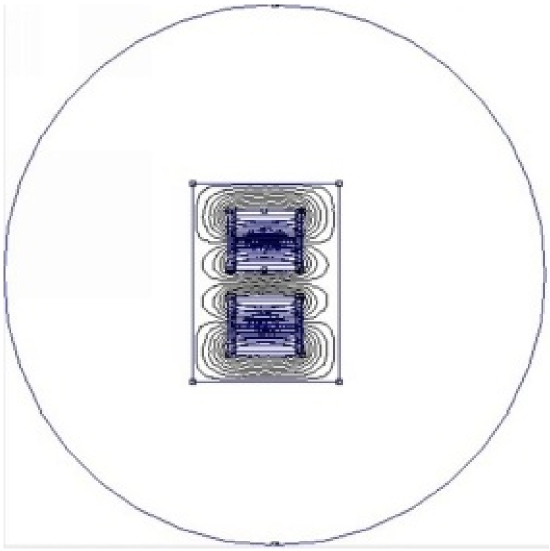


Fig. 48 – The flux seen at distance

The figure below, show the contours of the magnetic strength H in the window of the transformer.

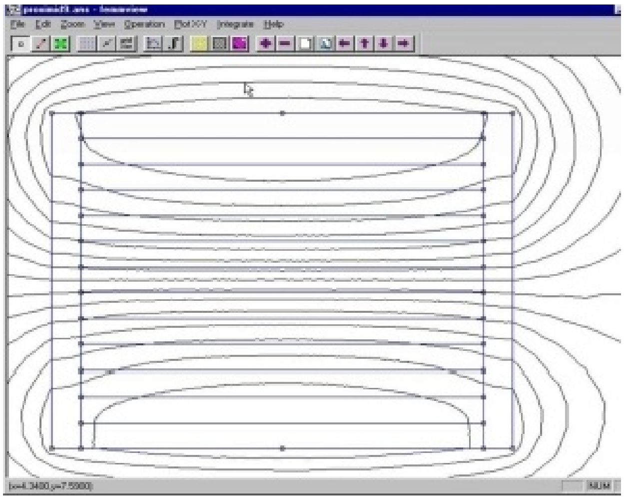


Fig. 49 The field strength lines

The red line at left define the path where we will measure the H field.

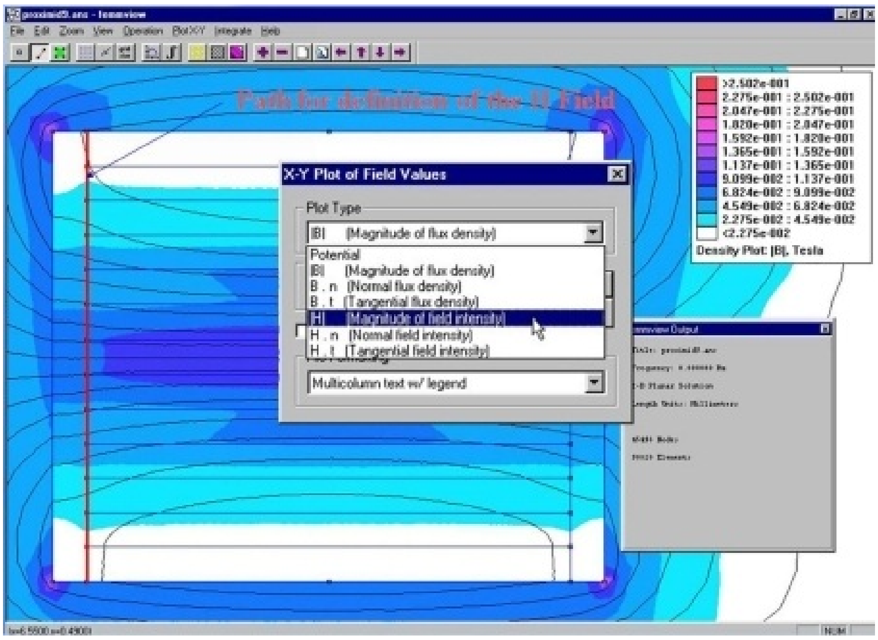


Fig. 50 – Path for definition of the H field

Calculating the field H over the path:

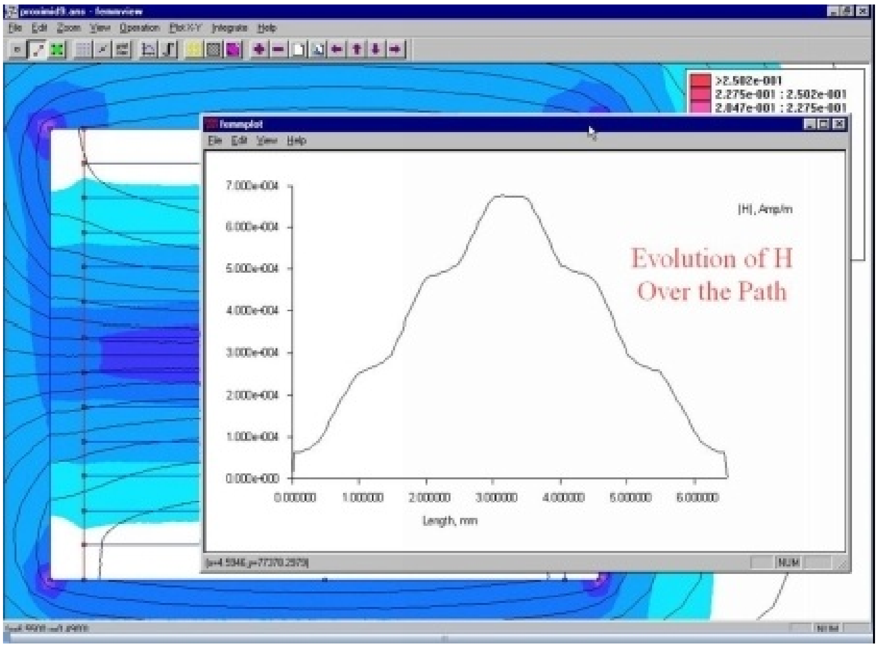


Fig. 51 – H field over the path

This is behaving exactly as we have seen in fig 36.

We can see the evolution of H, rising to as it approaches the secondary, reaching the maximum amplitude and reducing from there. Note clearly, the effect of the “counter current”.

The energy in the windings is proportional to the square of the magnetic field (H^2).

Note that the H2 is directly related to the energy of the magnetic field and is a valuable measure of the energy stored in the volume of the entire coil.

We can see the total energy per unit area by measuring the magnetic energy contained in the window of the transformer. The Finite elements software is a valuable tool in order to get this value.

Defining an area where we will measure the energy of the magnetic field.

This will be done within the primary winding region (Marked in green)

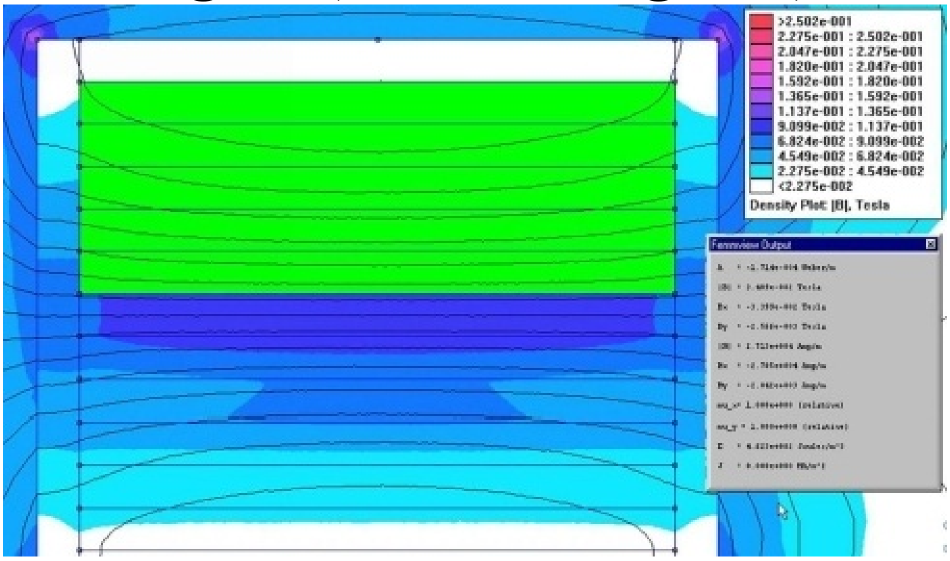


Fig. 52 The area of measurement was defined by the color green

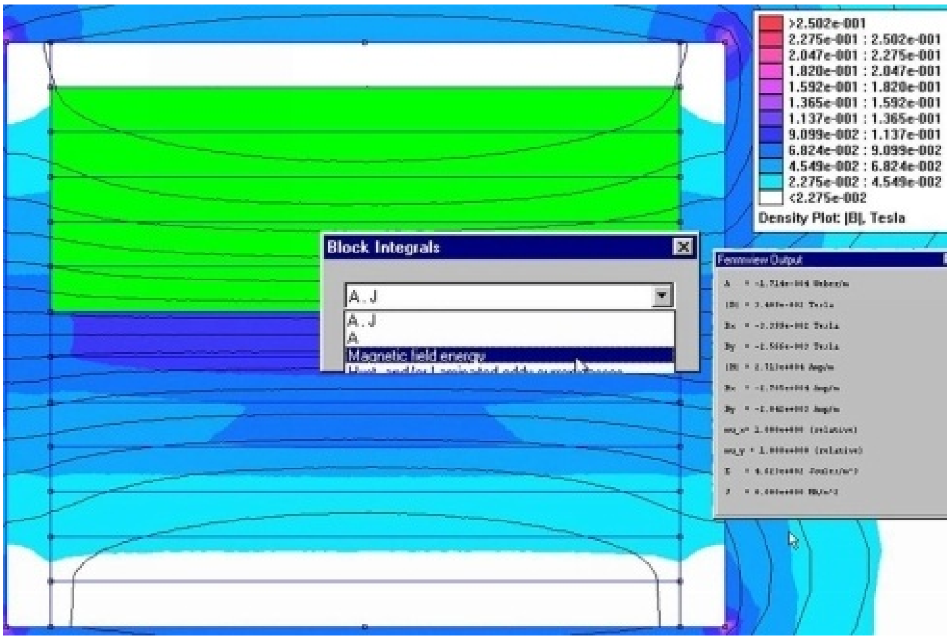


Fig. 53 – The kind of measurement is selected (Magnetic field Energy

After solution…

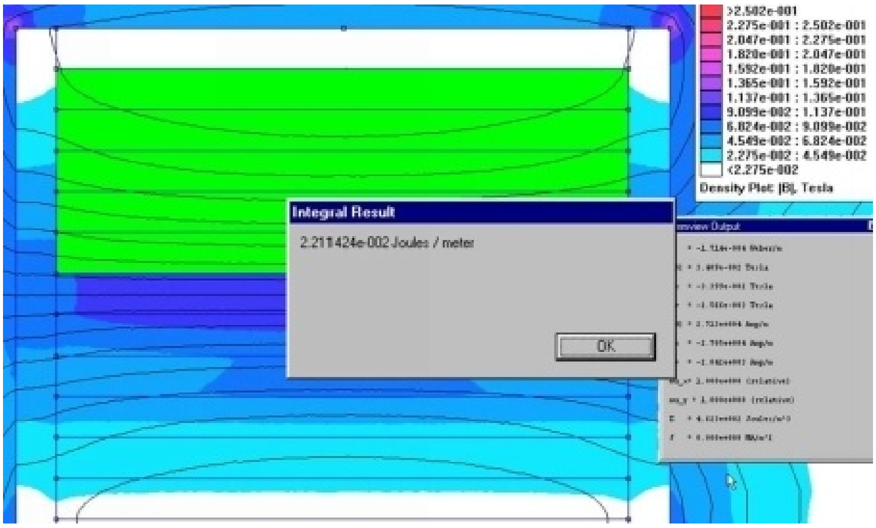


Fig. 54

We got the value of the energy in Joules of the area 2.211424*10^{-2} Joules/meter

We can see that if we have the amount of energy per meter, and the mean length of the coil, we can get the inductance, because:

$$W = \frac{L \cdot I^2}{2}$$

Handling, we can obtain the Inductance L

$$L = \frac{2 \cdot W \cdot Lenght}{I^2}$$

This inductance is commonly referred to as “leakage inductance” being relative to the individual field windings which do not embrace the others and thus characterized as “dispersed.”

Equivalent Circuit of a transformer:

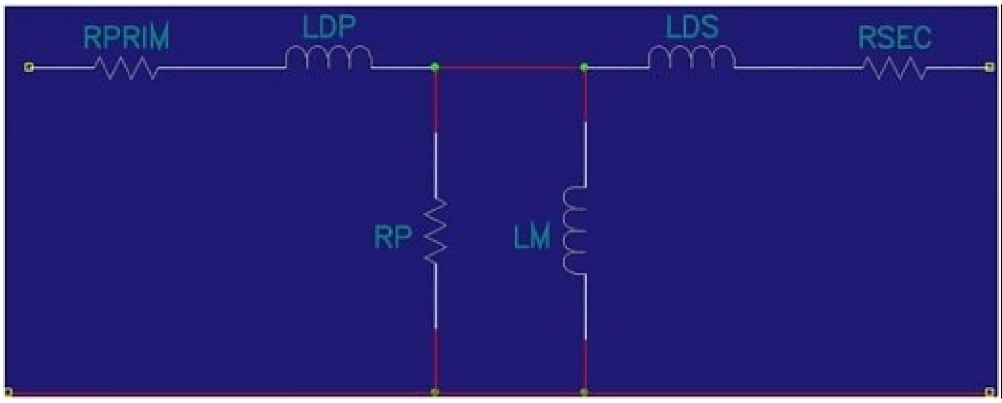


Fig. 55 – The equivalent diagram considering the turns relation n between prim and sec = 1

Rprim = Resistanceofthewindingdoftheprimary

Ldp=Leakage inductance related to the primary

Rp=Core loss related to the primary

Lm=Mutual Inductance Related to the primary

Rsec=Resistance of the winding of the secondary

Lds=Leakage inductance related to the secondary

Leakage Inductance

What is leakage Inductance

The definition of a fundamental inductance equation departing from the energy stored in magnetic field:

For considerations of leakage inductance due to a particular layer, it is only necessary to consider the other winding layers which influence in it.

The distribution of leakage flux through any layer depends only on its current and summation of the currents of the layers that lay between it and the layer which is adjacent to MMF zero position. For leakage inductance analysis purposes, we should consider the spaces of winding layers as parts that lie between zero and the maximum field. To these parts we will call "Portion".

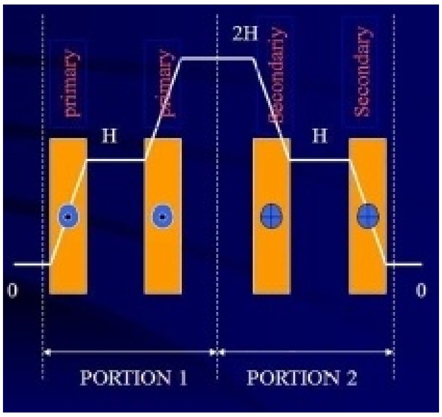


Fig. 56 – Definition of "Portion"

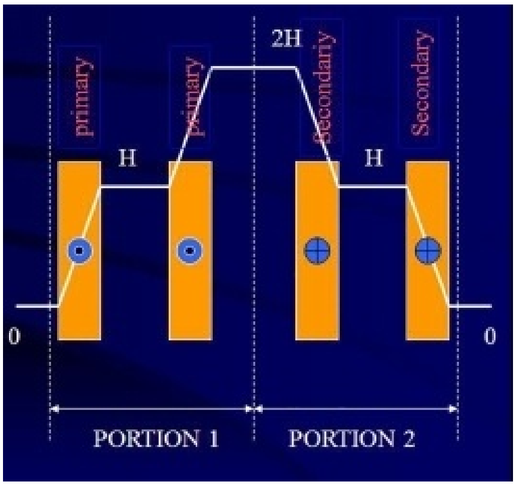


Fig. 57 Portion in interleaved coils

As a result of interleaving windings, we have a reduction of amplitudes of the magnetic fields in the transformer. Consequently we can drastically reduce the leakage inductance.

The density of the energy in the magnetic field is:

$$w = \frac{B \cdot H}{2}$$

And the total energy field with constant density in a given volume is:

$$W = \frac{B \cdot H \cdot V}{2}$$

At this point the flux density B is merely the current in the windings multiplied by the permeability of vacuum, number of Turns of coils, etc … and is not the flux density present in the core (determined by the effective cross-sectional area, voltage, frequency, etc.)

If the field is not homogeneous, this relationship can be applied to all units of infinitesimal volume and the total energy can be determined by integration:

$$W = \frac{1}{2} \cdot \int B \cdot H dv$$

If we combine with the relations:

$$W = \frac{L \cdot I^2}{2}$$

$$B = \mu_0 \cdot H$$

We get:

$$L = \frac{\mu_0}{I^2} \cdot \int H^2 dv$$

That we can call "Inductance general equation"

Inductance formula definition (Toroidal Case):

Initially we will determine the formula of inductance of an inductor with toroidal core. Additionally we will determine the formula for the leakage inductance of a transformer

As in a toroidal Core the field is homogeneous we can calculate de volume of the Core simply by:

$$V = l_{fe} \times A_{ef}$$

$$L = \frac{\mu_0}{I^2} \times \int H^2 dv = \frac{\mu_0 \times H^2 \times V}{I^2}$$

Replacing V:

$$L = \frac{\mu_0 \times H^2 \times l_{fe} \times A_{ef}}{I^2}$$

$$\mu_0 = 4 \times \pi \times 10^{-7} \times \frac{newton}{amp^2}$$

At present case de measures are in Centimeters.

We have:

$$L = \frac{0.4 \times \pi \times H^2 \times L_{fe} \times A_{ef}}{10^8 \times I^2}$$

Replacing H^2, We have:

$$H^2 = \frac{N^2 \times I^2}{l_{fe}^2}$$

$$L = \frac{0.4 \cdot \pi}{10^8} \cdot \left(\frac{N^2 \cdot I^2}{l_{fe}^2} \right) \cdot \frac{l_{fe} \cdot A_{ef}}{I^2}$$

Simplifying:

$$L = \frac{0.4 \cdot \pi \cdot N^2 \cdot A_{ef}}{10^8 \cdot l_{fe}}$$

~

Introducing the relative permeability, we get:

$$L = \frac{0.4 \cdot \pi \cdot N^2 \cdot A_{ef} \cdot \mu_r}{10^8 \cdot l_{fe}}$$

Inductance Formula definition (Leakage inductance Case):

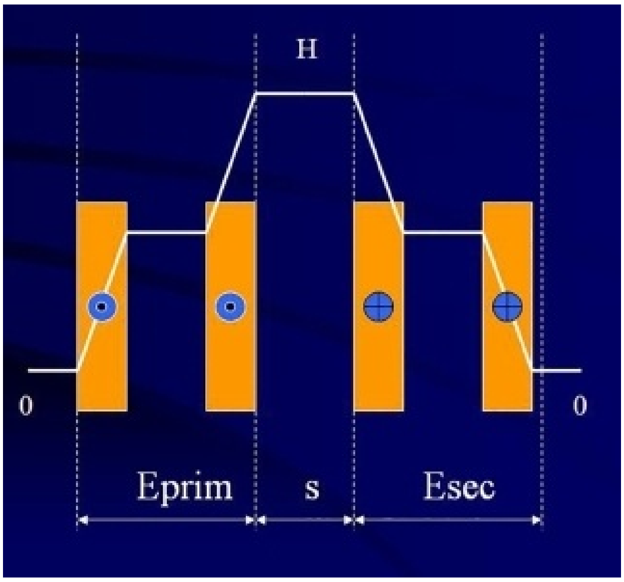


Fig. 58

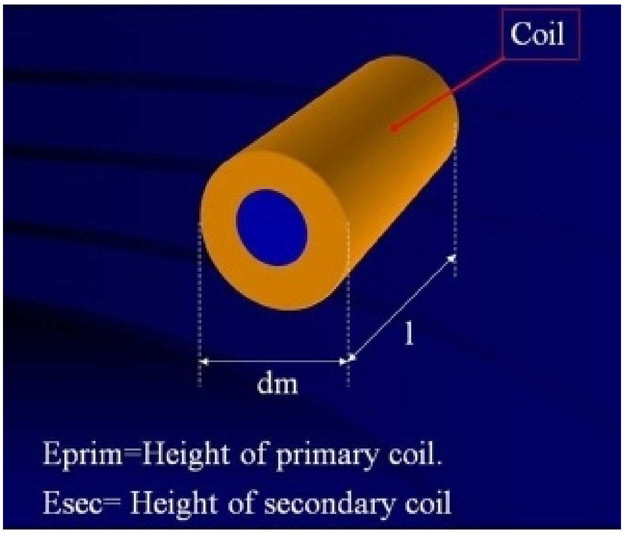


Fig. 59 - Circular coil for the determination of the leakage inductance equation

Equation of general inductance arranged for circular winding. In this way we are changing dv by dr, then rearranging:

$$L = \frac{\mu_0}{I^2} \cdot l \cdot \pi \cdot d_m \cdot \int_{r1}^{r2} H^2 dr$$

Soon:

$$\int_{r1}^{r2} H^2 dr = \left(\frac{N \cdot I}{l}\right)^2 \cdot \left(\frac{E_{prim} + E_{sec}}{3} + s\right)$$

Finally:

$$L = \frac{\mu_0 \cdot \pi \cdot d_m}{l} \cdot N^2 \cdot \left(\frac{E_{prim} + E_{sec}}{3} + s\right)$$

Decomposition of the Equation illustrating the elements of the leakage inductance:

Primary Leakage Inductance:

$$L_{prim} = \frac{\mu_0 \cdot \pi \cdot d_m \cdot N^2}{l} \cdot \left(\frac{E_{prim}}{3}\right)$$

Secondary Leakage Inductance:

$$L_{sec} = \frac{\mu_0 \cdot \pi \cdot d_m \cdot N^2}{l} \cdot \left(\frac{\sum se}{3} \right)$$

Contribution of the Intersection gap:

$$L_{inters} = \frac{\mu_0 \cdot \pi \cdot d_m \cdot N^2}{l} \cdot (s)$$

It is common practice to be considered the contribution of the intersection gaps as part of the primary leakage inductance, however, this depends on the settings, primaries and secondaries, number of layers and etc

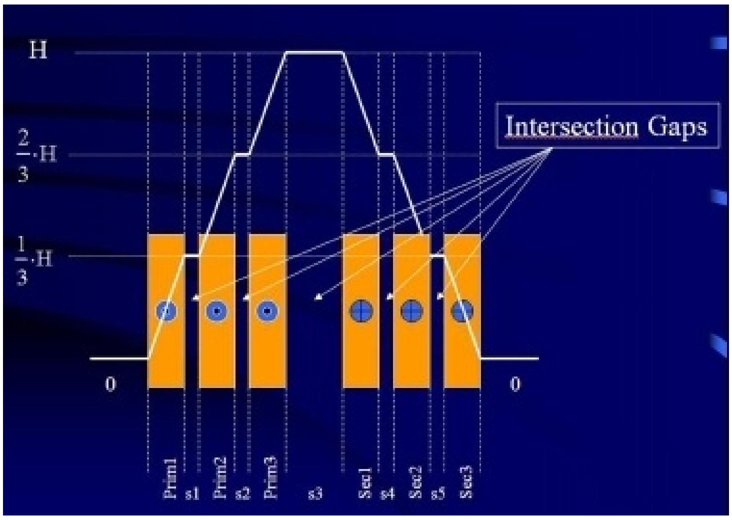


Fig. 60 – Account of the intersection gaps

The inductance of the gap is due to the local field. ASimple way to treat the problem, is to check the proportion of the field that acts on that layer are relative to the total field, creating the proportionality constant k For each intersection gap.

$$Ls_n = \frac{\mu_0 \cdot \pi \cdot d_m \cdot N^2}{l} \cdot \left(K_n^2 \cdot s_n \right)$$

The total leakage inductance is the contribution given by inductances of the layers plus the contribution given by spaces (intersection Gaps).

$$L_{tot} = L_{cam(1)} + \ldots L_{cam(n)} + L_{inter(1)} + \ldots L_{inter(n)}$$

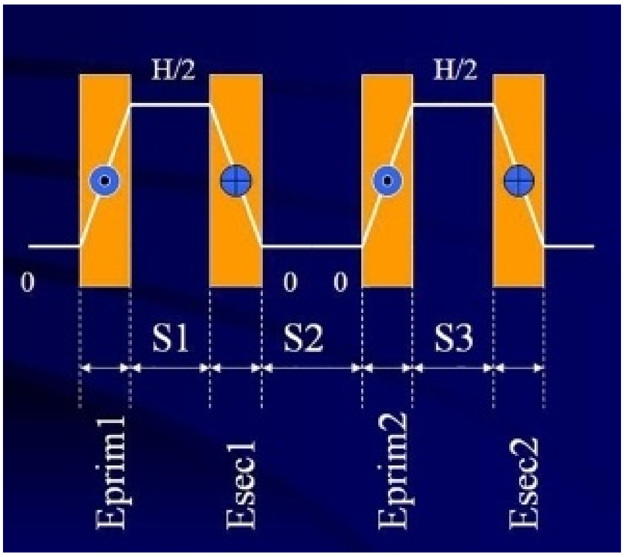


Fig. 61 The null field spaces

We should note that the spaces that are experiencing null fields should not be taken into consideration because it does not contribute to the total energy.

This is the case of the previous slide, where we have the field H = 0 in the intersection of gap between secondary 1 and primary 2.

Applying the same principle:

$$L = \frac{\mu_0 \cdot \pi \cdot d_m}{l^2} \cdot l \cdot \left[\int_0^{E_{prim1}} \left(\frac{\frac{H}{2}}{E_{prim1}} \cdot r \right)^2 dr + \int_0^{E_{prim2}} \left(\frac{\frac{H}{2}}{E_{prim2}} \cdot r \right)^2 dr + \int_0^{E_{sec1}} \left(\frac{\frac{H}{2}}{E_{sec1}} \cdot r \right)^2 dr + \int_0^{E_{sec2}} \left(\frac{\frac{H}{2}}{E_{sec2}} \cdot r \right)^2 dr + \int_0^{(s_1+s_3)} H^2 dr \right]$$

Solving:

$$L = \mu_0 \cdot \pi \cdot \frac{d_m}{l^2} \cdot l \cdot \left[\frac{1}{12} \cdot E_{prim1} \cdot H^2 + \frac{1}{12} \cdot E_{prim2} \cdot H^2 + \frac{1}{12} \cdot E_{sec1} \cdot H^2 + \frac{1}{12} \cdot E_{sec2} \cdot H^2 + (s_1 + s_3) \cdot H^2 \right]$$

Doing Eprim1=Eprim2=Eprim e Esec1=Esec2=Esec, We Have:

$$L = \mu_0 \cdot \pi \cdot \frac{d_m}{l^2} \cdot l \cdot \left(\frac{1}{6} \cdot E_{prim} + \frac{1}{6} \cdot E_{sec} + s_1 + s_3 \right) \cdot H^2$$

$$L = \mu_0 \cdot \pi \cdot \frac{d_m}{l^2} \cdot l \cdot \left(\frac{1}{6} \cdot E_{prim} + \frac{1}{6} \cdot E_{sec} + s_1 + s_3 \right) \cdot \frac{N^2 \cdot I^2}{l^2}$$

That simplifying finally crops:

$$L = \frac{\mu_0 \cdot \pi \cdot d_m \cdot N^2}{l} \cdot \left(\frac{E_{prim} + E_{sec}}{6} + s_1 + s_3 \right)$$

Final Formula:

$$L = \frac{\mu_0 \cdot \pi \cdot d_m \cdot N^2}{l} \cdot \left(\frac{E_{prim} + E_{sec}}{6} + s_1 + s_3 \right)$$

Where:

Dm=Average diameter of the coil

L=Length of the coil

N = Number of the turns of that coil.

Eprim and Esec = Height of primary and secondary coils respectively.
S1 e S3 width of gap space between coils

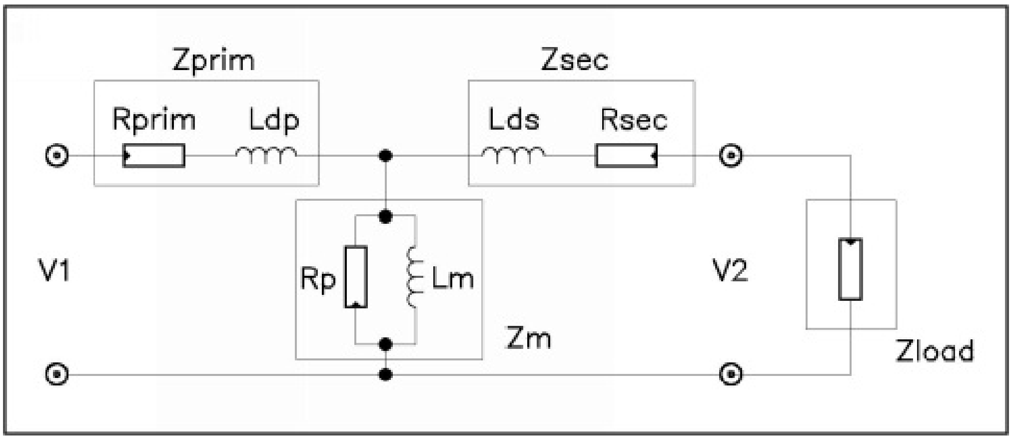
We can note Ldp and Lds as the dispersion inductances on the primary and secondary. These inductances are in series with the resistance of the primary and secondary (Rprim and rsec). The mutual inductance and loss resistance (Lm and Rp) are part of the diagram, but they are out of our present interest.

With the inductance general equation, we are free to define (deduce) any formula for any arrangement of coils, to calculate the stray (Dispersion) inductances. We need only arrange the problem strategically to solve the particular problem.

The equivalent circuit of a transformer and measurements of AC Resistance:

The equivalent circuit:

As defined in the anterior chapter a transformer has a general equivalent circuit like presented Below in which we now add a load Z.



The general transfer function to it, using chain matrix is:

$$\begin{pmatrix}A & B\\ C & D\end{pmatrix} = \begin{pmatrix}1 & Z_{prim}\\ 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ \frac{1}{Z_m} & 1\end{pmatrix}\cdot\begin{pmatrix}1 & Z_{sec}\\ 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ \frac{1}{Z_{load}} & 1\end{pmatrix}$$

Solving:

$$\begin{pmatrix}A & B\\ C & D\end{pmatrix} = \begin{bmatrix} 1+\frac{Z_{prim}}{Z_m}+\frac{\left[\left(1+\frac{Z_{prim}}{Z_m}\right)\cdot Z_{sec}+Z_{prim}\right]}{Z_{load}} & \left(1+\frac{Z_{prim}}{Z_m}\right)\cdot Z_{sec}+Z_{prim} \\ \frac{1}{Z_m}+\frac{\left(\frac{1}{Z_m}\cdot Z_{sec}+1\right)}{Z_{load}} & \frac{1}{Z_m}\cdot Z_{sec}+1 \end{bmatrix}$$

AC resistance Measurement

For the first work we must to measure the input impedance with output short circuited. The effect of Z_m will be neglected and we can do it infinite impedance. Thus considering this, our expression change to:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1+\frac{Z_{prim}}{Z_{sec}} & Z_{prim} \\ \frac{1}{Z_{sec}} & 1 \end{pmatrix}$$

Z_{in} is:

$$\frac{A}{C} = \frac{1+\frac{Z_{prim}}{Z_{sec}}}{\frac{1}{Z_{sec}}}$$

$$Z_{in} = \frac{A}{C} = \left(1+\frac{Z_{prim}}{Z_{sec}}\right) \times Z_{sec} = Z_{sec} + Z_{prim}$$

Consider that the turn's relation n, must be taken into consideration to the proper reflection of Z_{sec} to the primary.

For the measurement of the AC resistance, we must connect the secondary in short circuit, injecting in the primary ac voltage or current, measuring the active power.

The measurement instrument must have a wide band to assure precision . A digital oscilloscope would be a good choice.

With the power and the current measured, we can calculate the value of the total AC resistance for the given frequency (That of measurement).

A precise impedance bridge can be used for this measurement and also we can directly measure the dispersion inductance of primary and secondary. There are impedance bridges with various measurement frequencies that would be useful in our testing's.

Measurement of DC resistance

For the measurement of the DC resistance a Thomson bridge that works in Direct Current is the correct choice. There are other instruments with four terminals for measurement of DC resistance that guarantee good precision.

This same chosen instrument will be used in the transformer temperature tests, elsewhere in this book.

High frequencies response:

For High frequency, is customary introduce the stray capacitance in the output of the transformer, but the Zm in this case also, is considered virtually infinite. Then, our complete matrix change to:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{bmatrix} 1 + \dfrac{(Z_{sec}+Z_{prim})}{Z_{load}} & Z_{sec}+Z_{prim} \\ \dfrac{1}{Z_{load}} & 1 \end{bmatrix}$$

The transfer function:

$$\frac{V_2}{V_1} = \frac{1}{A} = \frac{1}{\left[1 + \dfrac{(Z_{sec}+Z_{prim})}{Z_{load}}\right]}$$

Considering that:

$$Z_{load} = \frac{1}{\dfrac{1}{R_{load}} + s \cdot C_{stray}}$$

$$Z_{sec} = R_{sec} + s \cdot L_{sec}$$

$$Z_{prim} = R_{prim} + s \cdot L_{prin}$$

$$\frac{V_2}{V_1} = \frac{1}{\left[1 + (R_{sec} + s \cdot L_{sec} + R_{prim} + s \cdot L_{prim}) \cdot \left(\dfrac{1}{R_{load}} + s \cdot C_{stray}\right)\right]}$$

Turns relation 1:n

$L_d = L_{prim} + n^2 \cdot L_{sec}$

$R_{tot} = R_{prim} + n^2 \cdot R_{se}$

$$\frac{V_2}{V_1} = \frac{1}{\left[1 + (R_{tot} + s \cdot L_d) \cdot \left(\dfrac{1}{R_{load}} + s \cdot C_{stray}\right)\right]}$$

$$\frac{V_2}{V_1} = \frac{1}{L_d \cdot C_{stray} \cdot \left[s^2 + s \cdot \left(\dfrac{1}{C_{stray} \cdot R_{load}} + \dfrac{R_{tot}}{L_d}\right) + \left(\dfrac{1}{L_d \cdot C_{stray}} + \dfrac{R_{tot}}{R_{load}} \cdot \dfrac{1}{L_d \cdot C_{stray}}\right)\right]}$$

$$\omega_o^2 = \frac{1}{L_d \cdot C_{stray}}$$

$$\tau_1 = C_{stray} \cdot R_{load}$$

$$\tau_2 = \frac{L_d}{R_{tot}}$$

$$R_{rel} = \frac{R_{tot}}{R_{load}}$$

$$\frac{V_2}{V_1} = \frac{\omega_o^2}{\left[s^2 + s \cdot \left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right) + \omega_o^2 \cdot (1 + R_{rel})\right]}$$

$$\zeta = \left[\frac{\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)}{2 \cdot \omega_o \cdot \sqrt{1 + R_{rel}}}\right]$$

And the normalized frequency Ω:

Doing $s = i\omega$

And

$$i\Omega = i \cdot \frac{\omega}{\omega_o}$$

Then there are 3 conditions:

$\zeta > 1$

$$\frac{V_2}{V_1} = \frac{1}{\left[\frac{i \cdot \Omega}{\left(\sqrt{1 + R_{rel}}\right)} + \zeta + \sqrt{\zeta^2 - 1}\right] \cdot \left[\frac{i \cdot \Omega}{\left(\sqrt{1 + R_{rel}}\right)} + \zeta - \sqrt{\zeta^2 - 1}\right] \cdot (1 + R_{rel})}$$

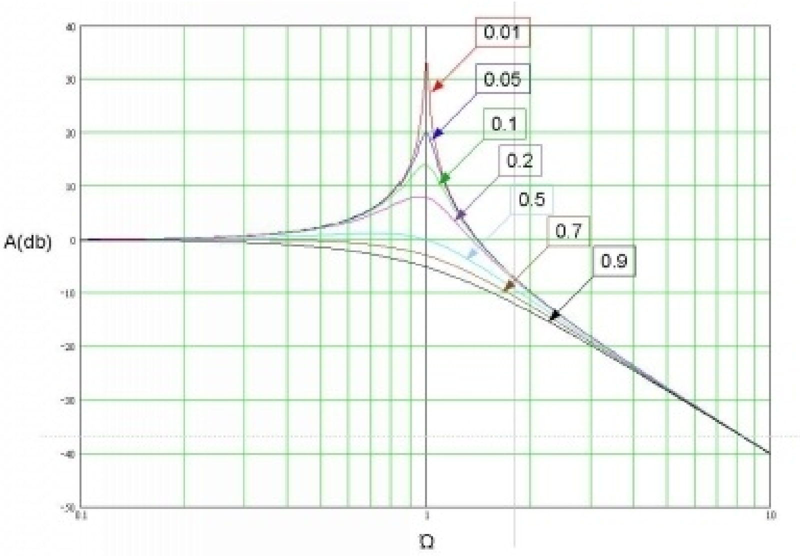
$\zeta = 1$

$$\frac{V_2}{V_1} = \frac{1}{\left[\frac{i \cdot \Omega}{\left(\sqrt{1 + R_{rel}}\right)} + \zeta\right]^2 \cdot (1 + R_{rel})}$$

$\zeta < 1$

$$\frac{V_2}{V_1} = \frac{1}{\left[\frac{i \cdot \Omega}{\left(\sqrt{1 + R_{rel}}\right)} + \zeta + i \cdot \sqrt{\left|\zeta^2 - 1\right|}\right] \cdot \left[\frac{i \cdot \Omega}{\left(\sqrt{1 + R_{rel}}\right)} + \zeta - i \cdot \sqrt{\left|\zeta^2 - 1\right|}\right] \cdot (1 + R_{rel})}$$

$$A_{x,k} := 20 \cdot \log\left[\left|\frac{1}{\left[\frac{i \cdot \Omega_x}{\left(\sqrt{1 + R_{rel}}\right)} + \zeta_k + i \cdot \sqrt{1 - \zeta_k^2}\right] \cdot \left[\frac{i \cdot \Omega_x}{\left(\sqrt{1 + R_{rel}}\right)} + \zeta_k - i \cdot \sqrt{1 - \zeta_k^2}\right] \cdot (1 + R_{rel})}\right|\right]$$



In the graph can be seen the various curves for the value of V = Damping factor. For the maximum gain flatness, the damping factor must be set between 0.5-0.7.

The minimization of the leakage inductance and stray capacitances can be obtained through dividing the winding in many parts, besides trying to reduce the stray capacitance by avoiding faces between same coils. That is an exhaustive work, which needs to be planned carefully.

In the slide below we can see the distribution of windings of primary and secondary in order to minimize the wanted property.

In 1 is the normal distribution of primary and secondary.

In 2 we have the second option, reducing the leakage inductance.

In 3 we have additional reduction of the leakage by the division of the secondary and the superposition of more one primary.

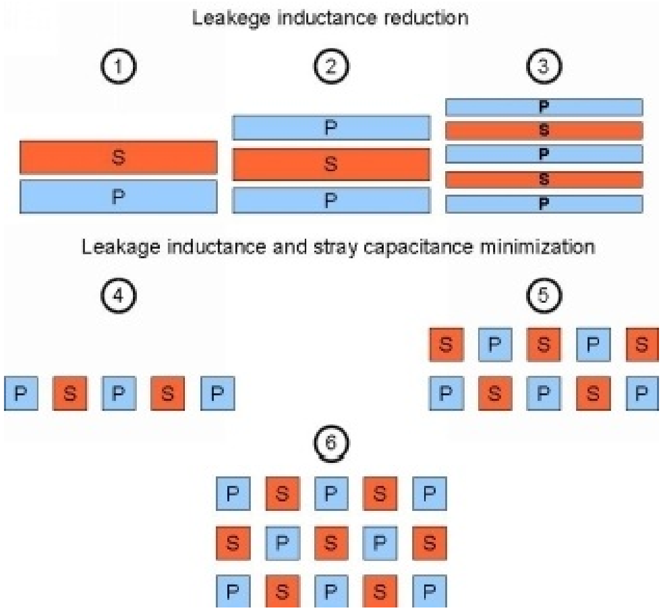
In 4, there is an equivalent solution like 3, but laterally arranged with extra bonus in the reduction of the stray capacitance.

In 5 extra reduction of leakage inductance and stray capacitance

In 6 there are still more reduction of the leakage inductance and stray capacitance.

We can increase indefinitely the number of coils, reducing additionally the leakage inductance and stray capacitance. Care must be taken with the order of the coils.

For the deduction of the formulas to be employed in each case, we must define the H profile of the coil and use the “Inductance general equation” rearranged for that special situation Integrating as a function of dv, dr or other.



Leakege inductance reduction
1
2
3
S
P
P
S
P
P
S
P
S
P
Leakage inductance and stray capacitance minimization
4
5
P S P S P
S P S P S
P S P S P
6
P S P S P
S P S P S
P S P S P

Medium frequencies Response

For this condition, the complete equation will be used:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{bmatrix} 1 + \frac{Z_{prim}}{Z_m} + \frac{\left(1 + \frac{Z_{prim}}{Z_m}\right) \cdot Z_{sec} + Z_{prim}}{Z_{load}} & \left(1 + \frac{Z_{prim}}{Z_m}\right) \cdot Z_{sec} + Z_{prim} \\ \frac{1}{Z_m} + \frac{\left(\frac{1}{Z_m}\right) \cdot Z_{sec} + 1}{Z_{load}} & \frac{1}{Z_m} \cdot Z_{sec} + 1 \end{bmatrix}$$

The transfer function would be:

$$\frac{V_2}{V_1} = \frac{1}{A} = \frac{1}{\left[1 + \frac{Z_{prim}}{Z_m} + \frac{\left(1 + \frac{Z_{prim}}{Z_m}\right) \cdot Z_{sec} + Z_{prim}}{Z_{load}}\right]}$$

Where:

$$Z_m = \frac{1}{\frac{1}{R_p} + \frac{1}{s \cdot L_m}}$$

$$Z_{sec} = R_{sec} + s \cdot L_{sec}$$

$$Z_{prim} = R_{prim} + s \cdot L_{prim}$$

Due to other considerations the power factor of the Z_{load} can be added for a more correct response of the system.

Low frequencies response

Evidently the transformer is not intended to work below its operating frequency, but in certain cases, certain classes of transformers need transiently to work in very low frequencies, like instrumentation transformers and so forth.

With the decreasing of frequency, the transformer is lead to the short circuit or something near that, when frequency approaches 0.

We must consider the impedance of the generator to compute the real frequency response, manly in applications of audio amplifiers or instrumentation. Someone can smile or even To laugh when hearing about transformers to audio amplifiers, but still today the best of the best of audio amplifier use Vacuum Tubes and transformers. Vacuum tubes and transformers working besides last generation DSP, wireless and so forth. Is incredible but it's true. The transistors with their non linear natures are very strange in audio environment. The feedback is improper to correct inter modulation products and other transients generated in the nonlinearities and dynamic properties of such amplifiers.

We can modify our equation to work with generator impedance, considering that the Zprim and Zsec are Zero, reducing our equation, and we get:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 + \frac{Z_{gen}}{Z_m} + \frac{Z_{gen}}{Z_{load}} & Z_{gen} \\ \frac{1}{Z_m} + \frac{1}{Z_{load}} & 1 \end{pmatrix}$$

$$\frac{V_2}{V_1} = \frac{1}{\left(1 + \frac{Z_{gen}}{Z_m} + \frac{Z_{gen}}{Z_{load}}\right)}$$

Where:

$$Z_m = \frac{1}{\frac{1}{R_p} + \frac{1}{s \cdot L_m}}$$

$$\frac{V_2}{V_1} = \frac{1}{\left[1 + Z_{gen}\left(\frac{1}{R_p} + \frac{1}{s \cdot L_m}\right) + \frac{Z_{gen}}{Z_{load}}\right]}$$

Skin Depth and Proximity Effect

Skin Depth

Ampere Law in more Detail:

The surface current in a conductor always creates a magnetic field around it

It can be said that a tangential field to a conductor generates this surface current.

They are fundamentally related, generating one inherently creates another (Inter causal)

This may seem counterintuitive therefore a magnet placed on a conductive plate do not creates a perpetual flow of current (except in a superconductor).

To understand the principle on which a field always has a corresponding surface current, it requires that we analyze how surface currents evolve over time.

We are assuming that this matter in most cases is not of full reader's knowledge and thus we are going to present it step by step in order to make it the most clearly possible.

Diffusion of magnetic field in a Conductor

If a magnetic field "H" is suddenly generated at = 0 in the neighborhood of a conductive plate. A corresponding surface current will flow in the conductor perpendicular to the field (right hand rule). Resistive effects quickly try to reduce this surface current, allowing the magnetic field penetrating the short distance in the conductor. By the fact, in the ampere law the current and field are inter causal, if a conductor is subjected to a sudden current, the diffusion chain will also be gradual, starting from the outer surface toward the inside

Fig. 62

The current now flows at a finite depth in the conductor. The IxR resistive drops are smaller but still present, and the magnetic field continues to spread toward the inside of the conductor. But at a lower rate. As the surface current will gradually gaining a larger flow area, the magnetic field diffusion rate reduces its speed to the square root of the time (t = 1,4,16, etc.).

Eventually the field reaches the opposite surface where the field now develops.

Since then the total current begins to decrease as comes into being an opposite surface current on the other side of the conductor, who now has the same magnetic field, until t = infinity when the current will be zero.

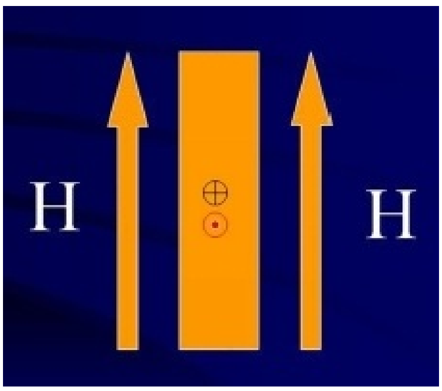


Fig. 63

Simple Rules in order to handle fields and currents in a single face.

(Follows the right hand rule)

Direction of the current

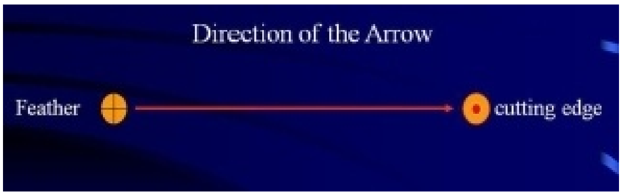


Fig. 64

Simple Rules

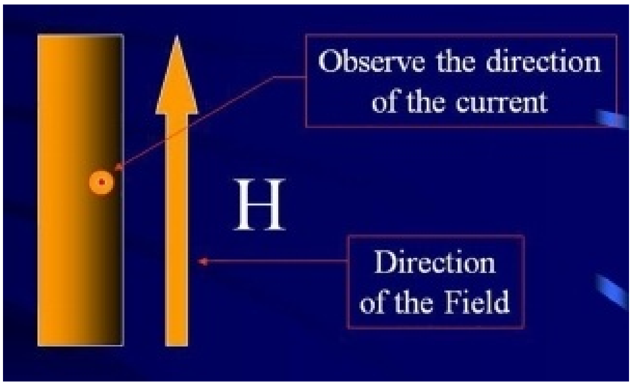


Fig. 65 Field towards up in the right side

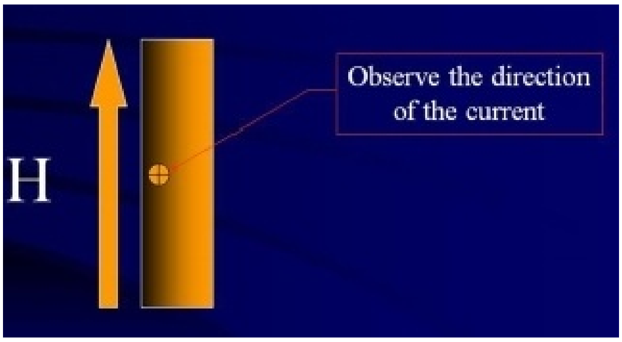


Fig. 66 Field Towards up in the left side

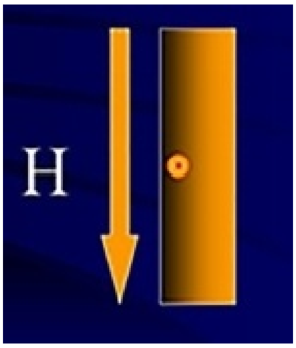


Fig. 67 Field towards down in the left side

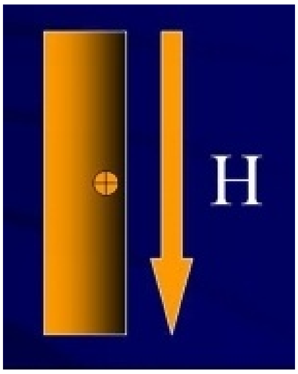


Fig. 68 Field towards down in the right side

If a conductor is suddenly subjected to a homogeneous field

will be formed two films that will gradually grow toward the inside of the conductor as shown in the following figure.

Fig. 69 Field suddenly subjected by an homogeneous field

Note that the current gradual diffusion is contrary, according to the directions of the fields on both sides of the bar.

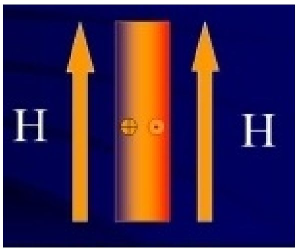


Fig. 70 The difusing in both directions

For t = Infinite the currents will be diffusing across the bar equally in both directions, canceling each other. Therefore the final resulting current is zero.

Looking from another angle, bar dipped in a homogeneous field shows no electrical potential, as described by the Faraday Law, where the time variation of the field is required to produce voltage potential.

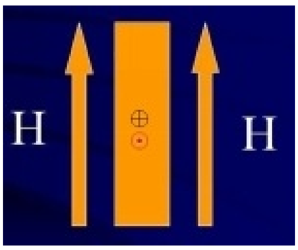


Fig. 71 At t=infinity

Diffusion interrupted by change of field or current direction

We can see that being the current diffusion gradual and if there is a sudden variation in the direction, there is no time for the complete diffusion of the current in the conductor. Then, by reversing the magnetic field, there will be the extinction of current distribution in the original direction, recreating the process for the other direction of the current.

We can see also that the higher the frequency, the shorter the time given to the diffusion, decreasing with this penetration of the current into the conductor. It is clear that there will be an external film of the current distribution and its thickness is inversely proportional to frequency. In reality it is inversely proportional to the square root.

This diffusion has an exponential decay towards the interior of the conductor being considered the foil thickness penetration where the diffusion displays a magnitude equal to 1/e.

The exponential area is:

$$S_{exp} = \int_0^{\infty} e^{-x} dx$$

$$S_{exp} = 1$$

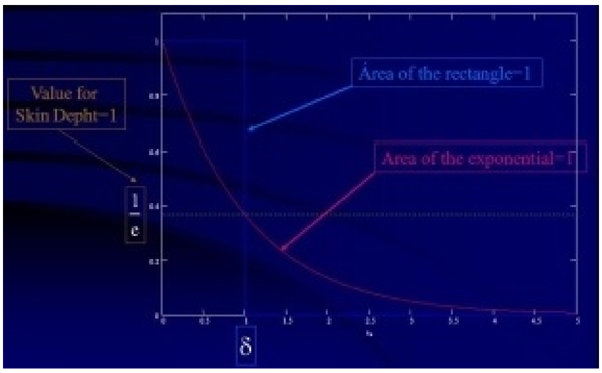


Fig. 72 The definition of the “thicknes” of the skin depth from the exponential

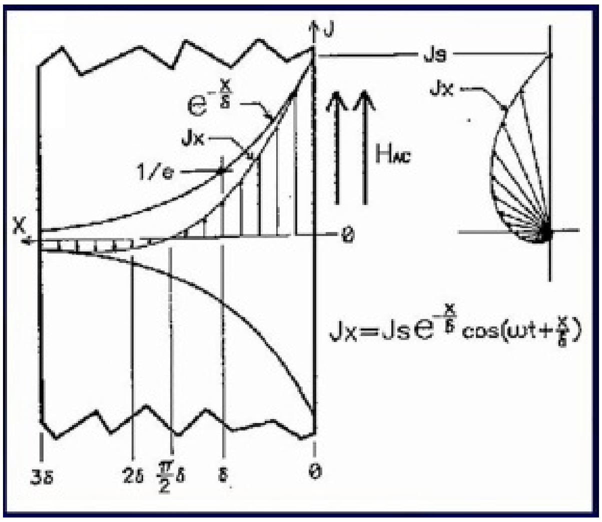


Fig. 73 The logaritimic exponential

The instantaneous current is also delayed of one radian per deepening of the skin, thus the vector of AC current forms a logarithmic spiral, as was shown in the previous figure.

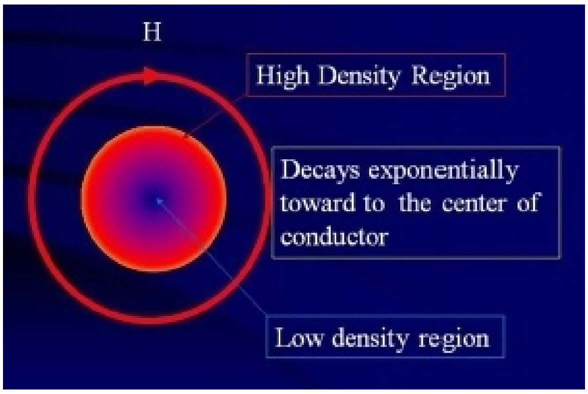


Fig. 74 - Current diffusion in a circular conductor

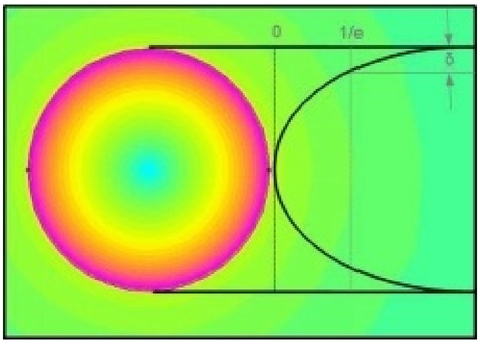


Fig. 75 The exponential diffusion

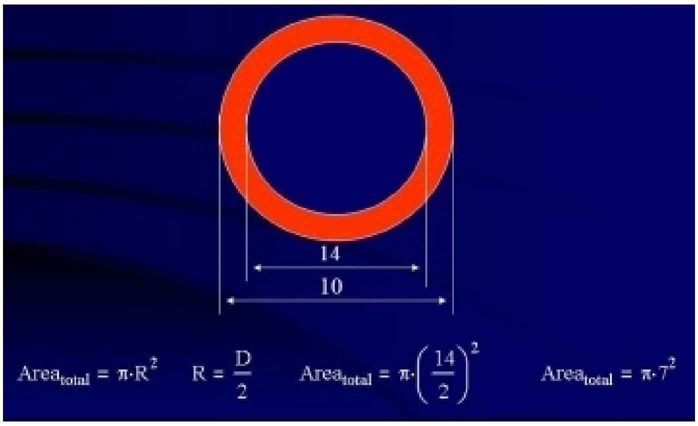


Fig. 76 Considering the skin depth of the circular wire, working like a circular pipe

It can be seen in the figure above the distribution of field in a conductor for an alternating current with a given frequency.

$$\frac{R_{ac}}{R_{dc}} = \frac{S_{tot}}{S_{ring}} = \frac{7^2}{7^2 - 5^2} = \frac{49}{49 - 25} = \frac{49}{24}$$

$$\frac{R_{ac}}{R_{dc}} = \frac{49}{24} = 2.04$$

Skin Depth

$$\delta = \sqrt{\frac{\rho \times \lambda}{\pi \times \mu_0 \times \mu_r \times C}} = \sqrt{\frac{\rho}{\pi \times \mu_0 \times \mu_r \times f}}$$

Where:

□ = Skin Depth

□ = Conductor Resistivity

μ_o= *Vacuum permeability*.

μ_r=*Conductorpermeability*.

f = *Frequency in Hertz.*

λ =*Wavelength of the frequency.*

C= *Speed of light in Vacuum.*

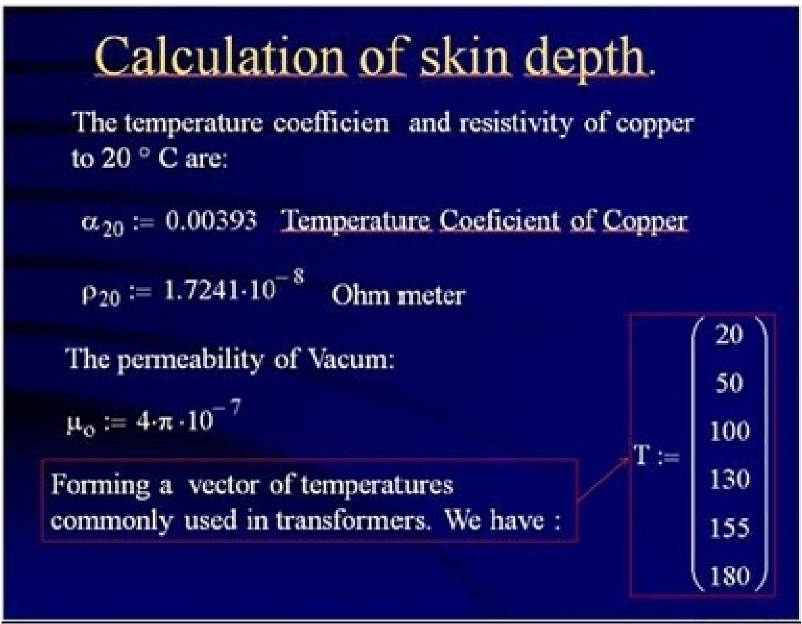


Fig. 77 – Main constants and values of the temperatures

Calculation of resistivity of copper for each temperature

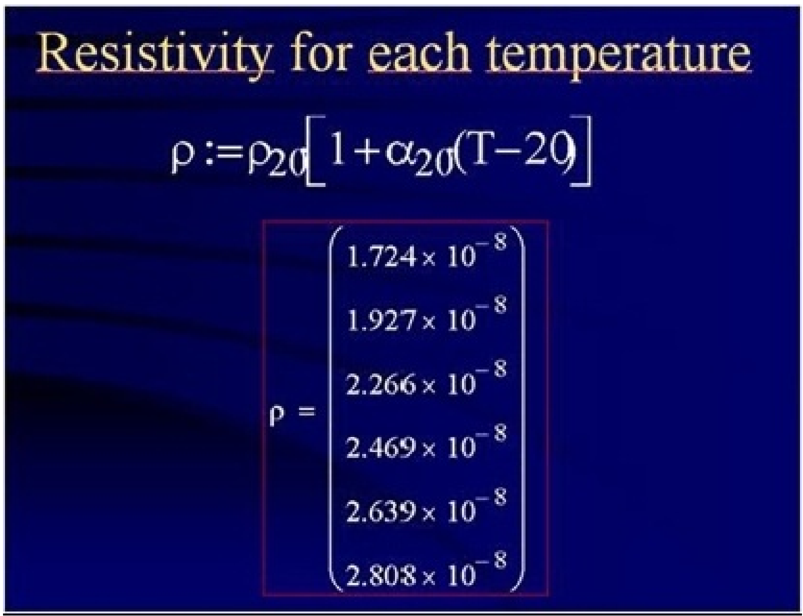


Fig. 78 – Resistivity as a function of temperature and resistivity at each temperature

$$d = \left(\frac{\rho}{\mu_o \cdot \pi \cdot f}\right)^{\frac{1}{2}} = \left(\frac{\rho}{\mu_o \cdot \pi}\right)^{\frac{1}{2}} \cdot \left(\frac{1}{f}\right)^{\frac{1}{2}} = \frac{Constate_d}{\sqrt{f}}$$

Values of “Constante$_d$” for Copper and Aluminum in function of temperatures

Temp	Cu	Al
20	66,1	81,5
50	69,9	86,1
100	75,8	93,4
130	79,1	97,5
155	81,8	100,8
180	84,4	104

Fig. 79 Constants of Cu and Al at each temperature

Proximity Effect:

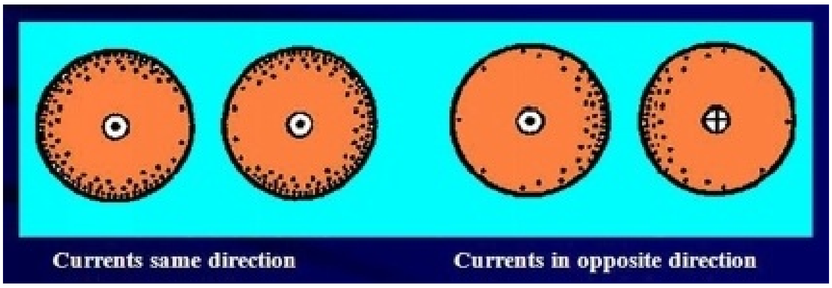


Fig. 80 the diffusion of current depending the relative direction of conductors

In both conductors of the next slide, we can see the spacing and alignment of greater diffusion regions, depending evidently the relative direction of the currents.

This effect is known as “proximity effect” because it involves effects due to proximity of conductors and fields.

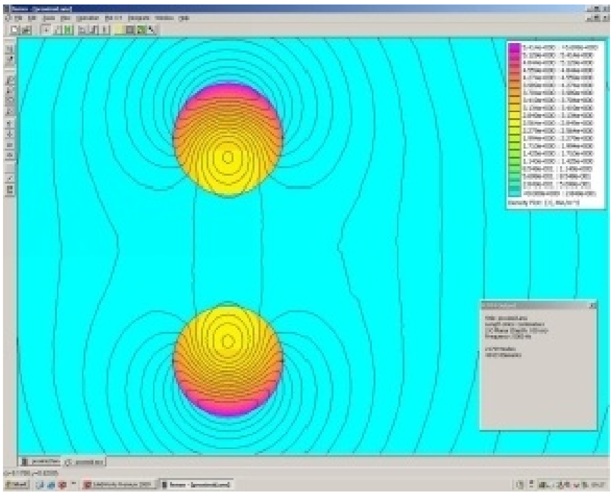


Fig. 81 currents in same direction

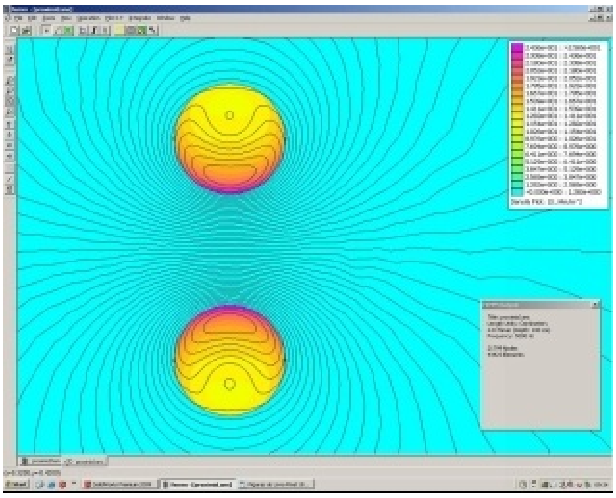


Fig. 82 Currents in opposite directions

Proximity & Skin Effect

The proximity effect in parallel conductors are combined with the skin effect, generating a rather much more complex effect what is known as “Skin proximity effect”, most wrongly mentioned in the literature as “proximity effect“.

The proximity effect is dominant over the classic skin effects in windings of high frequency or high current in magnetic components (Transformers, Inductors or transductors).

Ordinary proximity effect is combined with the skin properties of the current, creating a combined effect notoriously difficult to be calculated, especially when it involves multiple conductors.

The proximity effect develops considerable losses.

In extreme cases proximity effect losses may involve tens, hundreds and sometimes thousands of times greater losses than the direct current.

Proximity Effect three-dimensional configurations.

Three-dimensional configurations are effectively one-dimensional when they are contained in two of three dimensions. An isolated round wire is a case: Due to symmetry, the currents and fields vary radially in cylindrical coordinates, with no axial or circumferential variations.

This symmetry is destroyed with two wires, and the complexity grows with the number of conductors and their consequent interactions.

There were no practical formulas available to calculate these phenomena in windings of transformers and conventional inductors, until the advent of Dowell work.

Simplifying assumptions

- The layers of windings are assumed to homogeneous conductive plates.
- The layers of windings are considered infinitely long portions of a coaxial solenoid (The flow is uniform anywhere and axially between the layers).
- The thickness of the layers is much smaller than the radius of curvature (may be analyzed locally as a flat plate portion).
- All the coils have the same distance (No partial layers).
- All layers on a winding portion have the same thickness.

Consequences:

- These assumptions reduce the problem to one dimension; Currents and fields vary only radially. Not varying axially or circumferentially. Similarly to the case of a circular isolated conductor.

Transformation from a round conductor winding to metal plate or sheet

Dowell transformed circular wire windings into an "Equivalent Sheet" as follows.

- First replaces round wires by squares of equal areas.
- After join the squares by moving them.
- After stretch it to cover the previous distance.
- The new thickness "h" is equal to the plate thickness of the square of the conductive plate and the resistivity is increased by a factor s /h in order to maintain the same resistance.

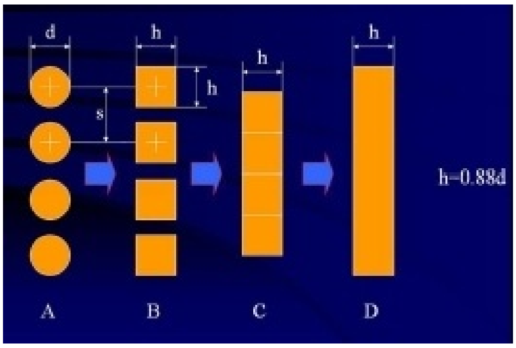


Fig. 83 Sequence of transformation from round conductors to sheet - Dowell

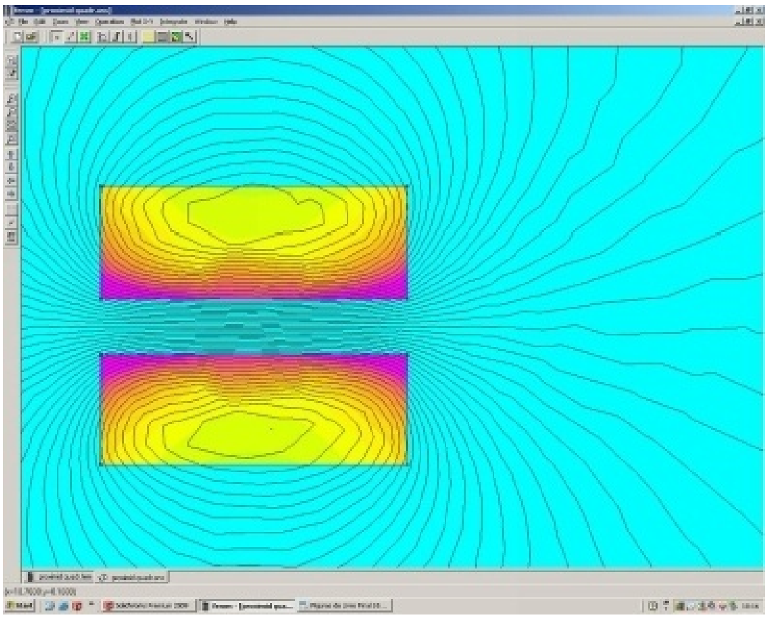


Fig. 84

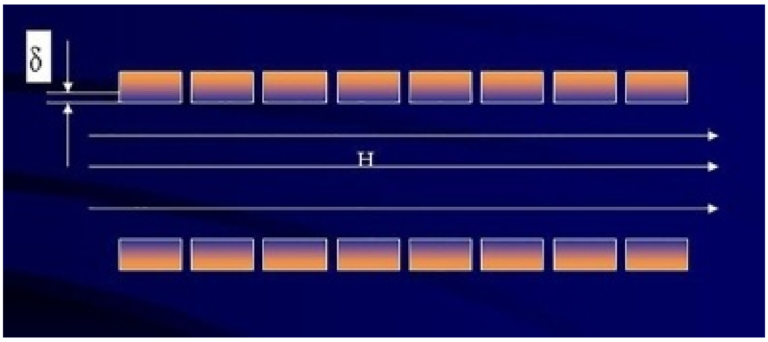


Fig. 85 Bar conductors

We can see right the highlight showing the exponential decay of the current distribution in the conductor bar

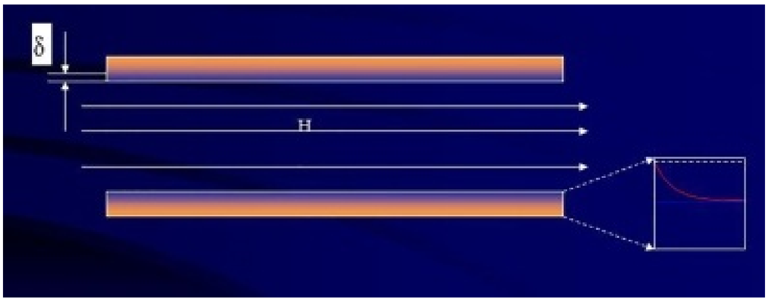


Fig. 86 - Sheet conductor

Eddy Currents

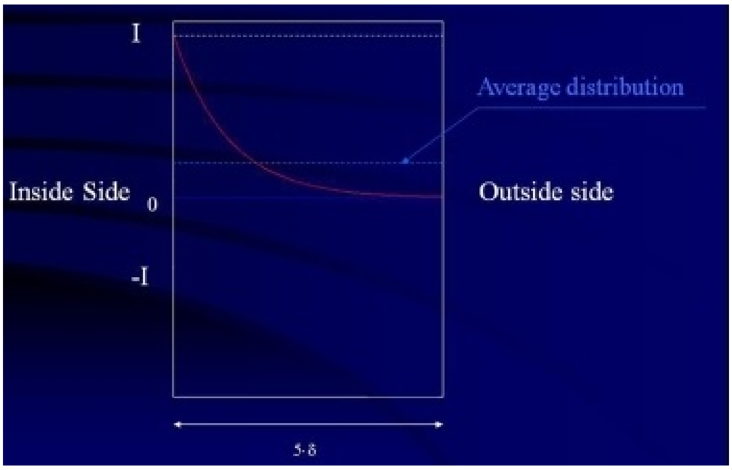


Fig. 87 the diffusion in one layer

In the second layer due to the external field (inside field of the first layer), a contrary current is formed due to the fact of the field be on the outside (as seen by the second layer). See right hand rule.

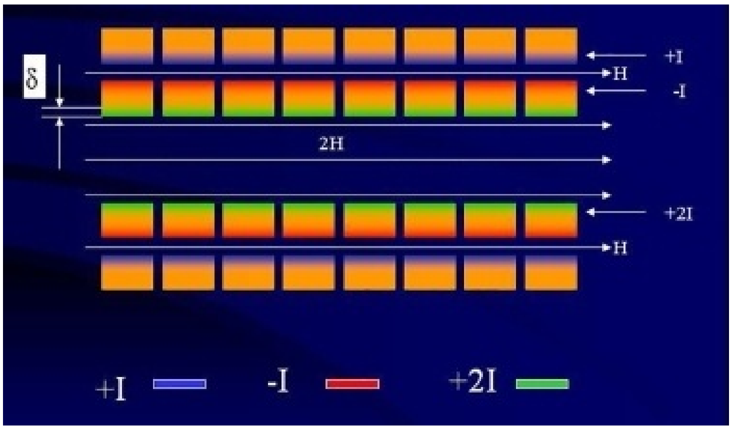


Fig. 88 Fields and diffusion in 2 layers

Can be seen that with increasing field toward the center of the solenoid, there is a gradual increase of currents flowing through each conductor layer.

Through highlights we can see those effects.

Forming a unique sheet:

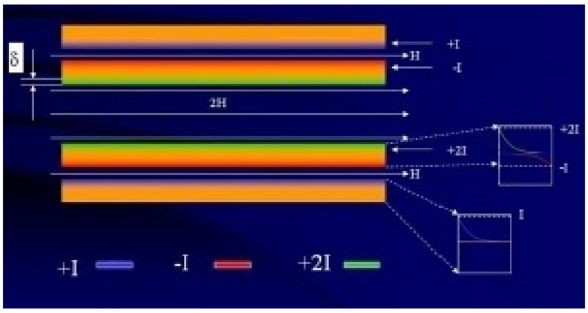


Fig. 89 The profile of the diffusion of the current in different layers

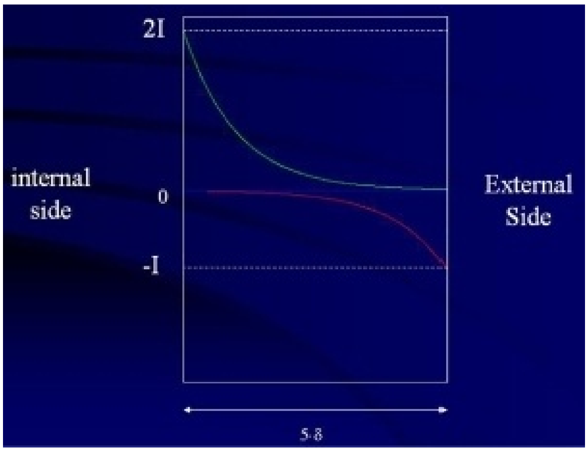


Fig. 90 Diffusion of the current in the second layer

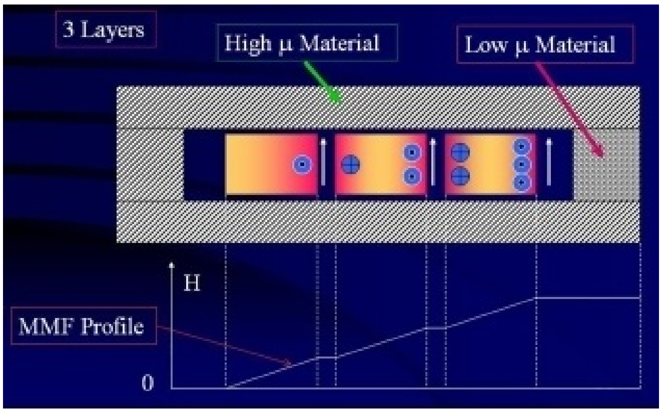


Fig. 91 The diffusion of the different layers and their effect on the H profile

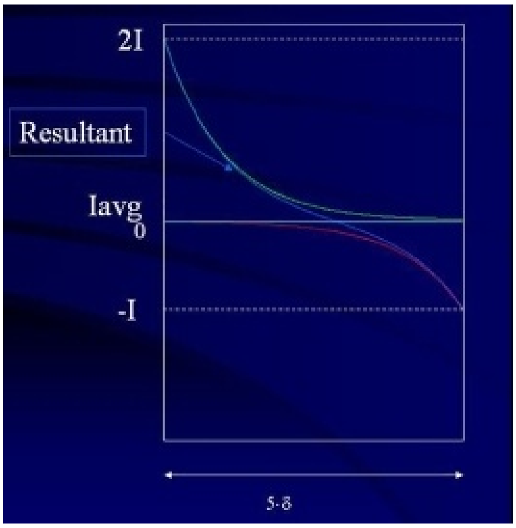


Fig. 92 The resultant current for the second layer for 5 skin depth

We can observe a resultant by a combination of the two contrary currents.

The net average of this distribution is the total current flowing in the conductor, as shown as Iavg in the next slide.

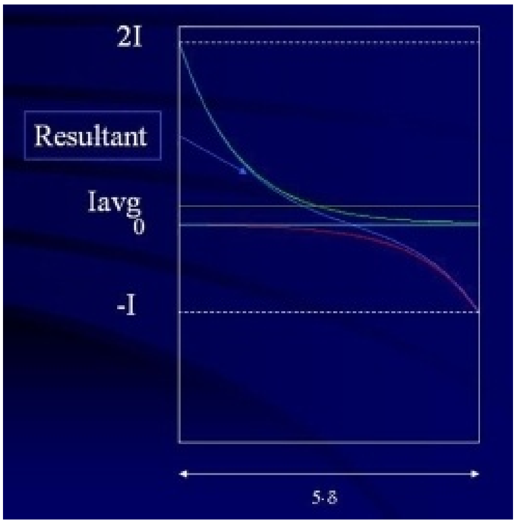


Fig. 93

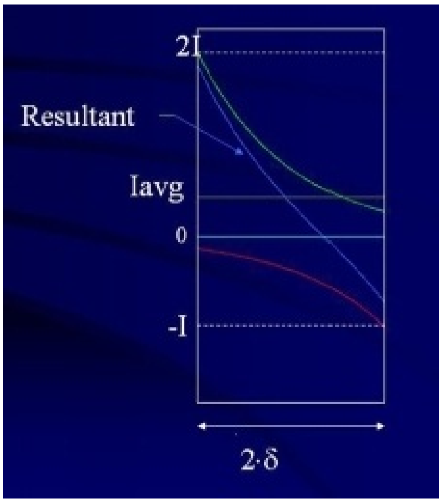


Fig. 94 the resultant for 2 skin depths

We can see in the previous and next slides that due to the conductor thickness there is a variation of the resulting profile, thus changing the average value.

This property as will be seen later, sets the optimal value of the thickness of the conductor for each application.

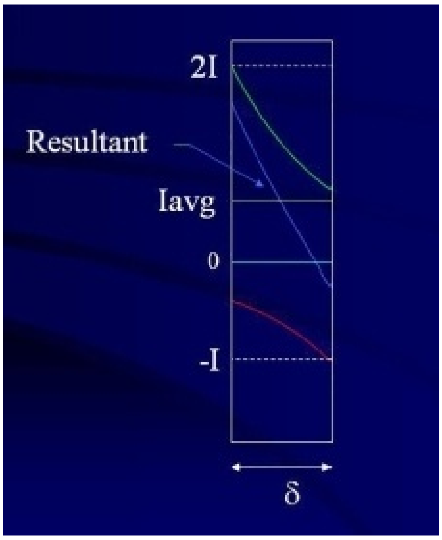


Fig. 95 The resultant for 1 skin depth

Square of the current

If we imagine a conductor, such as sliced into infinitesimal slices, losses will be proportional to the square of the current multiplied by the resistance of each slice. Thus if we set up a system to calculate the total loss of the conductor due to the resulting quadratic profile, we can define a kind of the conductor AC resistance.

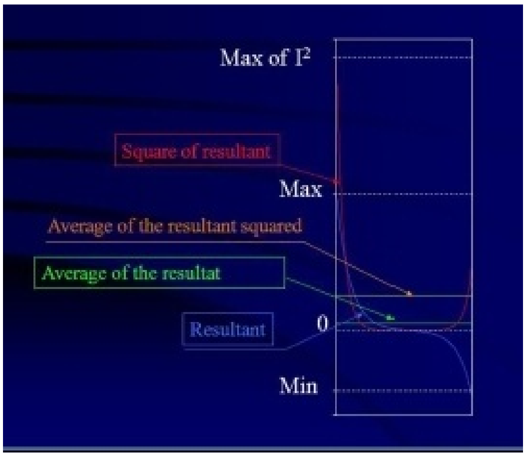


Fig. 96 The resultants

Evolution of the Average of the resultant squared for the average value.

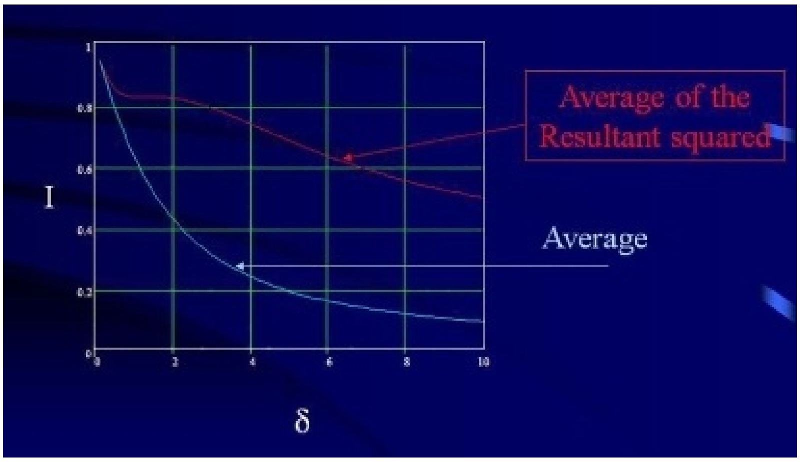


Fig. 97 – The average of the resultant squared

Second Layer

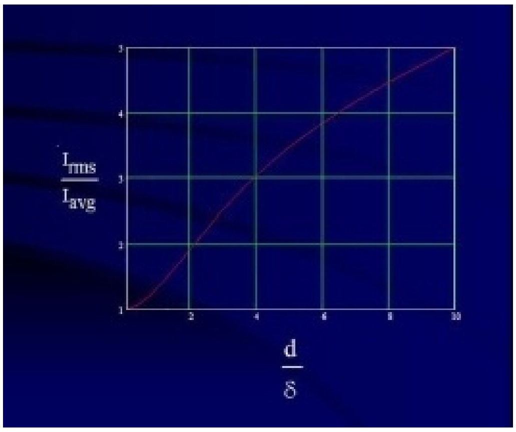


Fig. 98 Second Layer

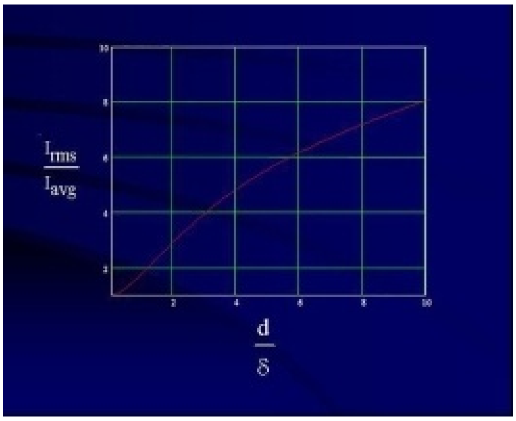


Fig. 99 Third Layer

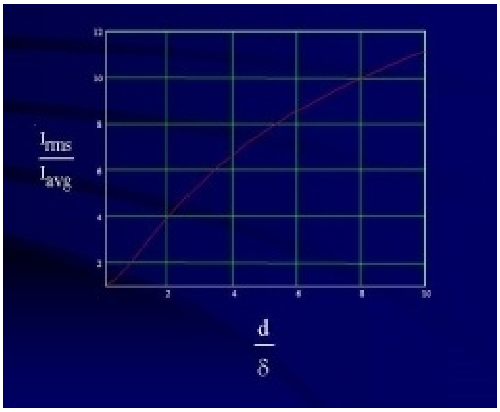


Fig. 100 Fourth Layer

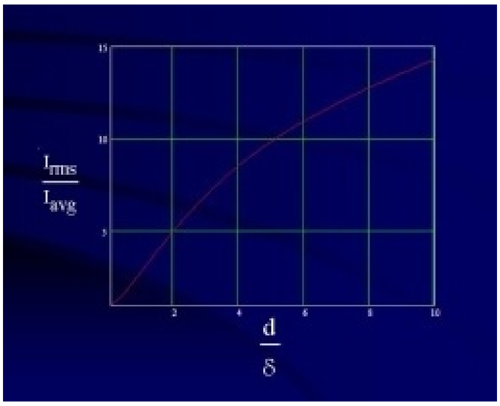


Fig. 101 Fifth Layer

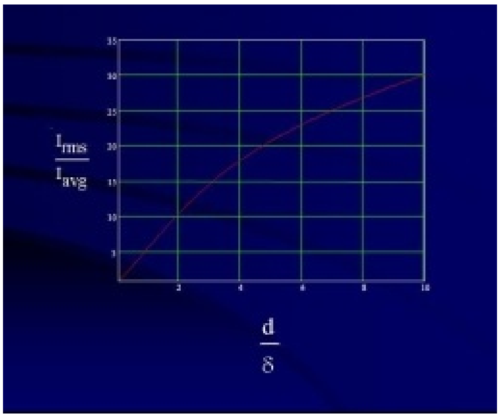


Fig. 102 Tenth Layer

Resistance relation.

$$F_r = \frac{R_{ac}}{R_{dc}} = \frac{P_{tot}}{Iav^2 \times R}$$

Note: I_{AV} does not match the average of a waveform of current, but the average profile of evolution of the current in the conductor.

$$Fr = \frac{R_w}{R_{wo}} = \frac{Resist_{ac}}{Resist_{dc}}$$

Relation for a particular layer:

$$Fr = M + D \times \left(p^2 - p\right.$$

Where p is the number of the layer from the place where

H = 0.

$$M = X \times \frac{\sinh(2 \times X) + \sin(2 \times X)}{\cosh(2 \times X) - \cos(2 \times X)}$$

Or:

$$M = X \times \frac{e^{2 \times X} - e^{-2 \times X} + 2 \times \sin(2 \times X)}{e^{2 \times X} + e^{-2 \times X} - 2 \times \cos(2 \times X)}$$

$$D = 2 \times X \times \frac{\sinh(X) - \sin(X)}{\cosh(X) + \cos(X)}$$

Or:

$$D = 2 \times X \times \frac{e^{X} - e^{-X} - 2 \times \sin(X)}{e^{X} + e^{-X} + 2 \times \cos(X)}$$

Where:

$$X=\frac{d}{d}$$

Being d = conductor diameter.

and d = skin depth.

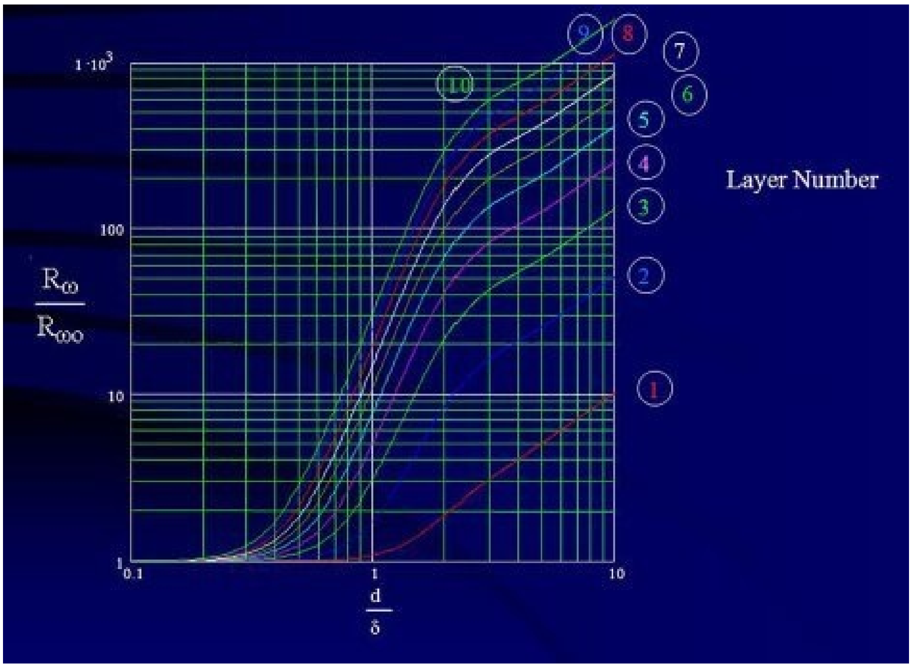


Fig. 103

Determination of Fr for the entire coil:

For practical purposes, most often, it is necessary to establish the resistance not for a particular layer, but for all the winding. To achieve that, we must calculate the average resistance of all layers.

Relation Fr for the entire winding Portion":

$$Fr=\frac{\sum_{P=1}^{m}\left[M+D\cdot\left(P^2-P\right)\right]}{m}$$

m being the number of layers for that **“Portion”**.

$$F_r = M + \frac{(m^2 - 1)}{3} \cdot D$$

And the skin depth:

$$\delta = \left(\frac{\rho}{\mu_0 \cdot \pi \cdot f}\right)^{\frac{1}{2}} = \left(\frac{\rho}{\mu_0 \cdot \pi}\right)^{\frac{1}{2}} \cdot \left(\frac{1}{f}\right)^{\frac{1}{2}} = \frac{Constant}{\sqrt{f}}$$

As presented before

Temp	Cu	Al
20	66,1	81,5
50	69,9	86,1
100	75,8	93,4
130	79,1	97,5
155	81,8	100,8
180	84,4	104

Fig. 104 – Constants for Cu and Al for each temperature

Sample Calculation of AC resistance for primary and secondary windings due to skin and proximity effect

To illustrate, we are going calculate the resistance of two windings (primary and secondary) of a transformer curled with a copper sheet.

- Core E42 / 20.
- Primary 20 turns of 0.05mm copper sheet. Width of 25.39mm - 20 layers.
- Secondary windings 5 turns of 0.1 mm copper sheet. Width of 25.39mm - 5

layers.

- The average length of the coil is 9.9 cm.
- The frequency of use is 100kHz of sinusoidal waveform.
- The resistance is calculated for a temperature of 100° C.

Specific resistance of the copper:

$$r_c := 2.226 \times 10^{-6}$$

DC resistance of the Primary:

Cross sectional area of the primary in Cm^2:

$$Acu_{prim} := 0.005 \times 2.53$$

$$Acu_{prim} = 0.013$$

$$lcu_{media} := 9.9$$

$$N_p := 20$$

$$Rdc_{prim} = r_c \times \frac{lcu_{media} \times N_p}{Acu_{prim}} = 2.266 \times 10^{-6} \times \frac{198}{0.013}$$

$$Rdc_{prim} = 0.035$$

DC resistance of the Secondary

Cross sectional area of the secondary in Cm^2:

$$Acu_{sec} := 0.01 \times 2.53$$

$$Acu_{sec} = 0.025$$

$$Icu_{media} := 9.9$$

$$N_s := 5$$

$$Rdc_{sec} := r_c \times \frac{Icu_{media} \times N_s}{Acu_{sec}}$$

$$Rdc_{sec} = 4.34 \cdot 10^{-3}$$

For 100^0 Celsius and f=100.000 Hz the skin depth is:

$$d := \frac{76}{\sqrt{f}}$$

$$d = 0.24$$

$$d_{prim} := 0.05$$

$$X_{prim} := \frac{d_{prim}}{d}$$

$$M_{prim} := X_{prim} \times \frac{(\sinh(2 \times X_{prim}) + \sin(2 \times X_{prim}))}{(\cosh(2 \times X_{prim}) - \cos(2 \times X_{prim}))}$$

$$M_{prim} = 1.001$$

$$D_{prim} := 2 \times X_{prim} \times \frac{(\sinh(X_{prim}) - \sin(X_{prim}))}{(\cosh(X_{prim}) + \cos(X_{prim}))}$$

$$D_{prim} = 2.497 \cdot 10^{-3}$$

Number of turns of primary:

$$m_{prim} := 20$$

$$Fr_{prim} := M_{prim} + D_{prim} \cdot \left(\frac{m_{prim}^{2} - 1}{3} \right)$$

$$Fr_{prim} = 1.333$$

$$Rac_{prim} := Rdc_{prim} \cdot Fr_{prim}$$

$$Rac_{prim} = 0.047$$

$$d_{sec} := 0.01$$

$$X_{sec} := \frac{d_{sec}}{d}$$

$$X_{sec} = 0.059$$

$$M_{sec} := X_{sec} \cdot \frac{(\sinh(2 \cdot X_{sec}) + \sin(2 \cdot X_{sec}))}{(\cosh(2 \cdot X_{sec}) - \cos(2 \cdot X_{sec}))}$$

$$M_{sec} = 1$$

$$D_{sec} := 2 \cdot X_{sec} \cdot \frac{(\sinh(X_{sec}) - \sin(X_{sec}))}{(\cosh(X_{sec}) + \cos(X_{sec}))}$$

$$D_{sec} = 3.997 \cdot 10^{-6}$$

Number of layers of secondary:

$$m_{sec} := 5$$

$$Fr_{sec} := M_{sec} + D_{sec} \cdot \left(\frac{m_{sec}^{2} - 1}{3} \right)$$

$$Rac_{sec} := Rdc_{sec} \cdot Fr_{sec}$$

$$Rac_{sec} = 4.34 \cdot 10^{-3}$$

Dowell Relations – Simplifying the figure:

Real and imaginary parts for the determination of the AC Resistance and Ac inductance of the wires:

$$\alpha = \sqrt{\frac{i \cdot \omega \cdot \mu_0 \cdot h}{\rho}}$$

$$M = \alpha \cdot h \cdot \coth(\alpha \cdot h)$$

$$D = 2 \cdot \alpha \cdot h \cdot \tanh\left(\frac{\alpha \cdot h}{2}\right)$$

Where:

$$M_{re} = Re(M)$$

$$M_{im} = Im(M)$$

$$D_{re} = Re(D)$$

$$D_{im} = Im(D)$$

$$\alpha = \sqrt{\frac{i \cdot \omega \cdot \mu_0 \cdot h}{\rho}}$$

Doing $h = 1$:

$$\alpha = \sqrt{\frac{i \cdot \omega \cdot \mu_0}{\rho}}$$

Doingώ=2 × π × f;

$$\alpha = \sqrt{\frac{i \cdot 2 \cdot \pi \cdot f \cdot \mu_0}{\rho}}$$

Expanding in the complex plane, we have:

$$a = \left[\left[\pi \cdot f^2 \cdot \left(\frac{\mu_0^2}{\rho^2}\right)\right]^{\frac{1}{2}}\right]^{\frac{1}{2}} + i \cdot \mathrm{signum}(f \cdot \mu_0 \cdot \rho) \cdot \left[\left[\pi \cdot f^2 \cdot \left(\frac{\mu_0^2}{\rho^2}\right)\right]^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

As $f \cdot \mu_0/\rho$ is always >0, we have

$$a = \left[\left[\left(\pi^2 \cdot f^2 \cdot \frac{\mu_0^2}{\rho^2}\right)\right]^{\frac{1}{2}}\right]^{\frac{1}{2}} + i \cdot \left[\left[\left(\pi^2 \cdot f\right)^2 \cdot \frac{\mu_0^2}{\rho^2}\right]^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

Simplifying:

$$a = \left[\left(\pi \cdot f \cdot \frac{\mu_0}{\rho}\right)\right]^{\frac{1}{2}} + i \cdot \left[\left(\pi \cdot f \cdot \frac{\mu_0}{\rho}\right)\right]^{\frac{1}{2}}$$

We may note that the real and imaginary parts are exactly same and that in reality they are the inverse of the skin depth. Then, doing:

$$\frac{1}{\delta} = \left(\pi \cdot f \cdot \frac{\mu_0}{\rho}\right)^{\frac{1}{2}} \quad \textbf{Or} \quad \delta = \left(\frac{\rho}{\pi \cdot f \cdot \mu_0}\right)$$

We are replacing h by d and by1/ , then:

$$ah = \frac{d}{\delta} + i \cdot \frac{d}{\delta}$$

Now replacing d/δ per x, we have:

$$a \cdot h = \frac{d}{\delta} + i \cdot \frac{d}{\delta} = x + i \cdot x$$

In the equations M and D. We have:

$$M = (x + i \cdot x) \cdot \coth(x + i \cdot x)$$

As we are interested in the real and imaginary parts:

$$M_{re} = \mathrm{Re}[(x + i \cdot x) \cdot \coth(x + i \cdot x)]$$

The solution over the complex plane, results:

$$M_{re} = \left(x \cdot \sinh(x) \cdot \frac{\cosh(x)}{\sinh(x)^2 + \sin(x)^2}\right) + x \cdot \sin(x) \cdot \frac{\cos(x)}{\sinh(x)^2 + \sin(x)^2}$$

That simplifies to:

$$M_{re} = -x\cdot\frac{(\sin(x)\cdot\cosh(x)+\sin(x)\cdot\cos(x))}{-\cosh(x)^2+\cos(x)^2}$$

The solution for the imaginary M:

$$M_{im} = \mathrm{Im}[(x+i\cdot x)\cdot\coth(x+i\cdot x)]$$

The solution over the complex plane, results:

$$M_{im} = x\cdot\sinh(x)\cdot\frac{\cosh(x)}{\sinh(x)^2+\sin(x)^2} - x\cdot\sin(x)\cdot\frac{\cos(x)}{\sinh(x)^2+\sin(x)^2}$$

That simplifies to:

$$M_{im} = x\cdot\frac{(\sinh(x)\cdot\cosh(x)-\sin(x)\cdot\cos(x))}{\cosh(x)^2-\cos(x)^2}$$

Now for D part:

As we are interested in the real and imaginary parts:

$$D_{re} = \mathrm{Re}\left[2\cdot(x+i\cdot x)\cdot\tanh\left(\frac{x+i\cdot x}{2}\right)\right]$$

The solution over the complex plane, results:

$$D_{re} = 2\cdot x\cdot\sinh\left(\frac{x}{2}\right)\cdot\frac{\cosh\left(\frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right)^2+\cos\left(\frac{x}{2}\right)^2} - 2\cdot x\cdot\sin\left(\frac{x}{2}\right)\cdot\frac{\cos\left(\frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right)^2+\cos\left(\frac{x}{2}\right)^2}$$

That simplifies to:

$$D_{re} = -2\cdot x\cdot\frac{\left(\sinh\left(\frac{x}{2}\right)\cdot\cosh\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)\right)}{\left(\cos\left(\frac{x}{2}\right)^2-1+\cosh\left(\frac{x}{2}\right)^2\right)}$$

The solution for the imaginary D:

$$D_{im} = \mathrm{Im}\left[2\cdot(x+i\cdot x)\cdot\frac{\tanh(x+i\cdot x)}{2}\right]$$

The solution over the complex plane, results:

$$D_{im} = 2\cdot x\cdot\sinh\left(\frac{x}{2}\right)\cdot\frac{\cosh\left(\frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right)^2+\cos\left(\frac{x}{2}\right)^2} + 2\cdot x\cdot\sin\left(\frac{x}{2}\right)\cdot\frac{\cos\left(\frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right)^2+\cos\left(\frac{x}{2}\right)^2}$$

That simplifies to:

$$D_{im} = 2 \cdot \Delta \cdot \frac{\left(\sinh\left(\frac{\Delta}{2}\right) \cdot \cosh\left(\frac{\Delta}{2}\right) + \sin\left(\frac{\Delta}{2}\right) \cdot \cos\left(\frac{\Delta}{2}\right)\right)}{\left(\cos\left(\frac{\Delta}{2}\right)^2 - 1 + \cosh\left(\frac{\Delta}{2}\right)^2\right)}$$

Dowel complete relations:

The total voltage drop across the portion may be obtained by adding the resistive drops and voltages induced in the tops of the conductors.

These resistive drops and induced voltages are calculated from the real and imaginary parts of the equations, which, by itself are parts of the overall impedance of the portion.

$$R_W = R_{wo} \cdot \left[M_{re} + \frac{\left(m^2 - 1\right) \cdot D_{re}}{3}\right]$$

$$R_W = F_r \cdot R_{wo}$$

Where:

$$F_r = \left[M_{re} + \frac{\left(m^2 - 1\right) \cdot D_{re}}{3}\right]$$

The inductance of Lwo dispersion due to the flow cutting the conductors can be calculated by FMM diagram in the classic way.

The energy in the inductances and consequent contribution provided by the gaps between the layers do not enter the correction calculations.

$$L_W = L_{wo} \cdot \left[\frac{3 \cdot M_{im} + \left(m^2 - 1\right) \cdot D_{im}}{m^2 \cdot \left|m^2 \cdot a^2\right|}\right]$$

or

$$L_W = F_L \cdot L_{wo}$$

Where:

$$F_L = \frac{3 \cdot M_{im} + \left(m^2 - 1\right) \cdot D_{im}}{m^2 \cdot \left|a^2 \cdot h^2\right|}$$

Now Simplifying:

We Take the relationships of Dowell in its two real terms (related to the resistive voltage drops).

$$D=-2\cdot\lambda r\cdot\frac{\left(\sinh\left(\frac{1}{2}\cdot\lambda r\right)\cdot\cosh\left(\frac{1}{2}\cdot\lambda r\right)+\sin\left(\frac{1}{2}\cdot\lambda r\right)\cdot\cos\left(\frac{1}{2}\cdot\lambda r\right)\right)}{\left(\cos\left(\frac{1}{2}\cdot\lambda r\right)^2-1+\cosh\left(\frac{1}{2}\cdot\lambda r\right)^2\right)^2}$$

$$M=-\lambda r\cdot\frac{\left(\sinh(\lambda r)\cdot\cosh(\lambda r)+\sin(\lambda r)\cdot\cos(\lambda r)\right)}{\left(-\cosh(\lambda r)^2+\cos(\lambda r)^2\right)}$$

Be D:

$$D=-2\cdot\lambda r\cdot\frac{\left(\sinh\left(\frac{1}{2}\cdot\lambda r\right)\cdot\cosh\left(\frac{1}{2}\cdot\lambda r\right)+\sin\left(\frac{1}{2}\cdot\lambda r\right)\cdot\cos\left(\frac{1}{2}\cdot\lambda r\right)\right)}{\left(\cos\left(\frac{1}{2}\cdot\lambda r\right)^2-1+\cosh\left(\frac{1}{2}\cdot\lambda r\right)^2\right)^2}$$

Expanding in series, we have:

$$D=\frac{1}{3}\cdot\lambda r^4-\frac{17}{1260}\cdot\lambda r^8+O(\lambda r^{12})$$

Truncating the series:

$$D=\frac{1}{3}\cdot\lambda r^4$$

Be M:

$$M=-\lambda r\cdot\frac{\left(\sinh(\lambda r)\cdot\cosh(\lambda r)+\sin(\lambda r)\cdot\cos(\lambda r)\right)}{\left(-\cosh(\lambda r)^2+\cos(\lambda r)^2\right)}$$

Expanding in series for simplification:

Expanding in series, we have:

$$M=1+\frac{4}{45}\cdot\lambda r^4-\frac{16}{4725}\cdot\lambda r^8+o(\lambda r^9)$$

Truncating the series:

$$M=1+\frac{4}{45}\cdot\lambda r^4$$

Like the relation between AC and DC is:

$$F_r=M+D\cdot\frac{\left(p^2-1\right)}{3}$$

$$F_r = 1 + \frac{4}{45} \cdot \lambda r^4 + \frac{1}{3} \cdot \lambda r^4 \cdot \frac{(p^2 - 1)}{3}$$

Collecting terms, result in:

$$F_r = \left(\frac{1}{45} + \frac{1}{9} \cdot p^2\right) \cdot \lambda r^4 + 1$$

Rearranging:

$$Fr = 1 + \frac{5 \cdot p^2 - 1}{15} \cdot \frac{\lambda r^2}{3}$$

Or Finally:

$$Fr = 1 + Y \cdot \frac{\lambda r^4}{3}$$

Where:

$$Y = \frac{5 \cdot p^2 - 1}{15}$$

Originally we were using “p” to the layer number “m” to the number of layers, we are now using “p” for the number of layers to avoid confusion with “M”

Applicability

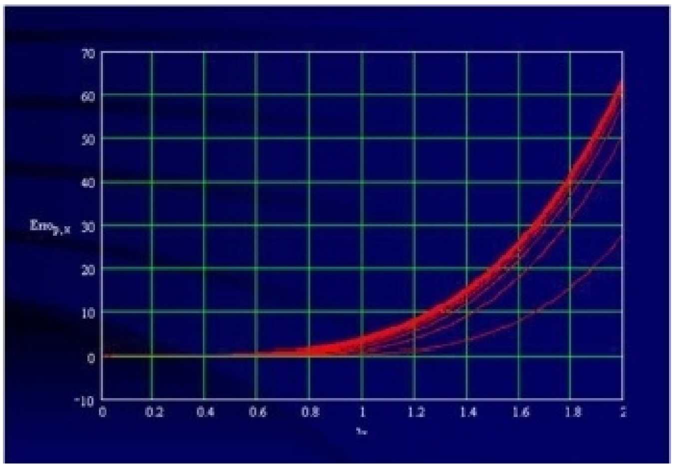
Let us study the error that can be introduced by the application of this methodology of simplification:

Number of layer from 1 to 100;

λ changing from 0 up to 2.

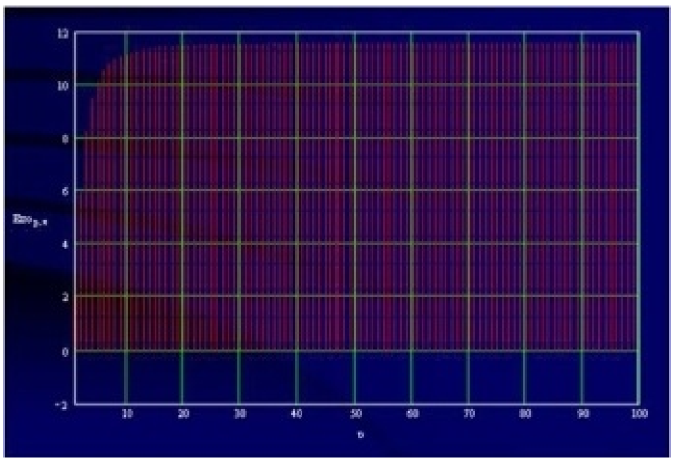
Doing a comparison between the two systems, we obtained:

$$Erro_{p,x} = \frac{Frsimplif_{p,x} - Frnorm_{p,x}}{Frnorm_{p,x}} \times 10$$



We can see in the above chart that for d/δ, the maximum error is 4.041% of which is perfectly acceptable for this type of application.

For thicknesses of up to 1.2 * δ is the maximum error of 8.3791%, which is still acceptable.



For thicknesses of up to 1.3 * d the maximum error is 11:54%, which stars to be incompatible.

It is observed that in the previous figure from a number of layers the maximum error is practically constant.

For a fast evaluation of the conditions of a determined application this may be seen as a good system.

Skin and Proximity effects facing complex waveforms

Behavior for complex waveforms

In the majority of applications, the power transformers are used to supply industrial power equipment, computers and a huge spectrum of modern apparatus that involves nonlinearities. Such equipments generating harmonics and a lot of Headaches for the people involved in the design and maintenance of the power transformers used to supply them.

The transformers need to meet certain needs in order to prevent high temperatures, electrical noise, transients and so forth.

To meet the temperature requirements we need to design surrounding them with a series of constraints in order to not lead them to high prices because the natural tendency is over-dimensioning.

handle harmonics is not so simple. The majority of techniques used in ordinary projects are not applicable here. For instance: if you have a problem with high temperature increasing the size of the copper, many times the temperature rises also. This is crazy, but is real. You need to know exactly to do to solve such problem.

We need to treat the problem through losses:

In windings with non-sinusoidal currents, the behavior is totally different. We can assume that the total power Pt is actually, the sum of the power of each individual harmonic. As for each frequency, there is a different resistance in the winding, to calculate, we must define the power of the harmonics as the square of the rms current of each harmonic multiplied by the winding resistance at the frequency of that harmonic.

$$P_t = I_o^2 \times R_{wo} + I_1^2 \times R_{w1} + I_2^2 \times R_{w2} + I_3^2 \times R_{w3} + I_n^2 \times R_{wn}$$

Where I_n = Current of Harmonic 'n'.

$R_{w\,n}$ = R at the frequency of that harmonic 'n'.

$$R_{wn} = Ftn \times R_{wo}$$

And

F_{tn} = Fr at the frequency of that harmonic 'n'.

Doing substitutions we make Pt:

$$P_t = R_{wo} \times \left(I_o^2 \times F_{to} + I_1^2 \times F_{t1} + I_2^2 \times F_{t2} + I_3^2 \times F_{t3} + I_4^2 \times F_{t4} + I_n^2 \times F_{tn} \right)$$

A effective resistance "R_{we}" can be defined as:

$$R_{we} = \frac{P_t}{I_{rms}^2}$$

Where Irms, is the total RMS current of the coil.

$$I_{rms}^2 = I_o^2 + I_1^2 + I_2^2 + I_3^2 + I_n^2$$

$$R_{we} = \frac{R_{wo} \cdot \left(I_o^2 \cdot F_{to} + I_1^2 \cdot F_{t1} + I_2^2 \cdot F_{t2} + I_3^2 \cdot F_{t3} + I_4^2 \cdot F_{t4} + I_n^2 \cdot F_{tn} \right)}{I_{rms}^2}$$

Handling:

$$\frac{R_{we}}{R_{wo}} = \frac{\left(I_o^2 \cdot F_{to} + I_1^2 \cdot F_{t1} + I_2^2 \cdot F_{t2} + I_3^2 \cdot F_{t3} + I_4^2 \cdot F_{t4} + I_n^2 \cdot F_{tn} \right)}{I_{rms}^2}$$

To visualize the overall effect of varying the thickness of the conductor in the winding losses, enabling simple calculation of effective resistance, we must establish normalized values and in the specific case introduce the Kr factor.

$$K_r = \frac{R_{we}}{R_{wl}} \qquad l_r = \frac{d}{d}$$

$$R_{wl} = R_{wo} \quad \text{For} \quad l_r = 1 \quad \left(\frac{d}{d} = 1 \right) \quad \text{for } n=$$

Rώ1 is the DC resistance of the winding when the conductor has a thickness of one skin depth in the fundamental frequency.

$$R_{wo} = \frac{R_{wl}}{l_r}$$

$$R_{wl} = l_r = R_{wo}$$

Thus:

$$K_r = \frac{R_{we}}{I_r \cdot R_{wo}} = \frac{P_t}{I_r \cdot R_{wo} \cdot I_{rms}^2} = \frac{R_{wo} \cdot \left(I_o^2 \cdot F_{to} + I_1^2 \cdot F_{t1} + I_2^2 \cdot F_{t2} + I_3^2 \cdot F_{t3} + I_4^2 \cdot F_{t4} + I_n^2 \cdot F_{tn}\right)}{I_r \cdot R_{wo} \cdot I_{rms}^2}$$

Finally:

$$K_r = \frac{\left(I_o^2 \cdot F_{to} + I_1^2 \cdot F_{t1} + I_2^2 \cdot F_{t2} + I_3^2 \cdot F_{t3} + I_4^2 \cdot F_{t4} + I_n^2 \cdot F_{tn}\right)}{I_r \cdot I_{rms}^2}$$

Example for rectangular Waveform:

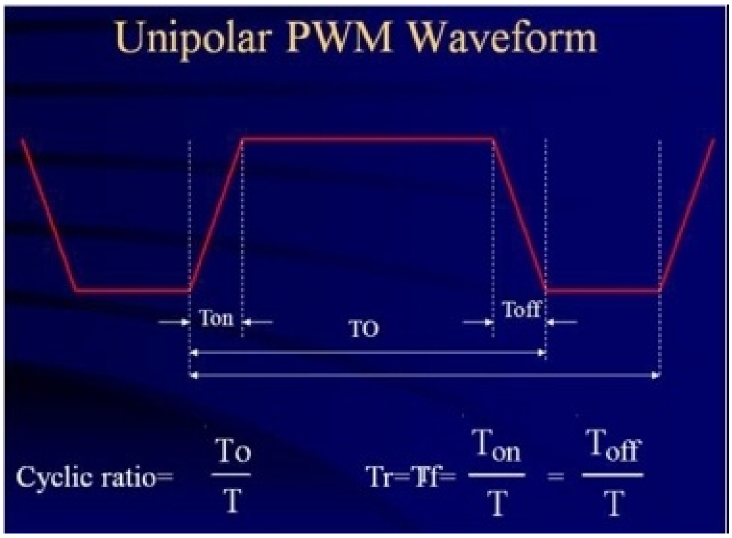


Fig. 105 Unipolar PWM waveform

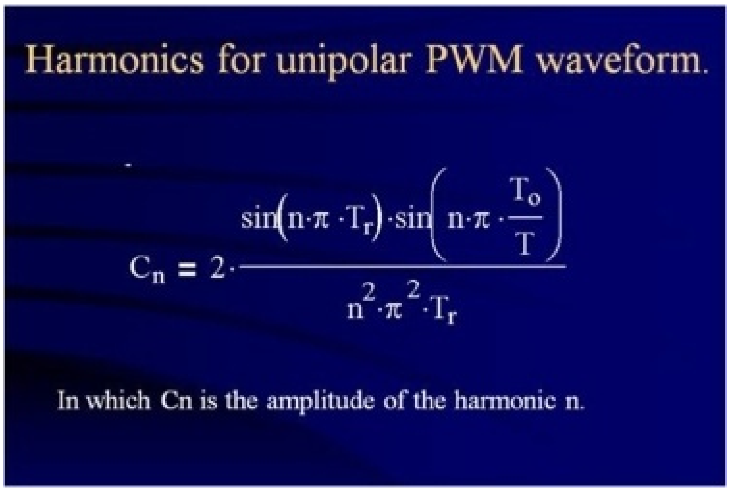


Fig. 106

Determination of the Kr Curve.

Calculating the amplitude of each harmonic, for a duty cycle pulse = 0.5.

Plotting a curve Kr as a function of normalized thickness of the conductor, we now have the figure in the following slide:

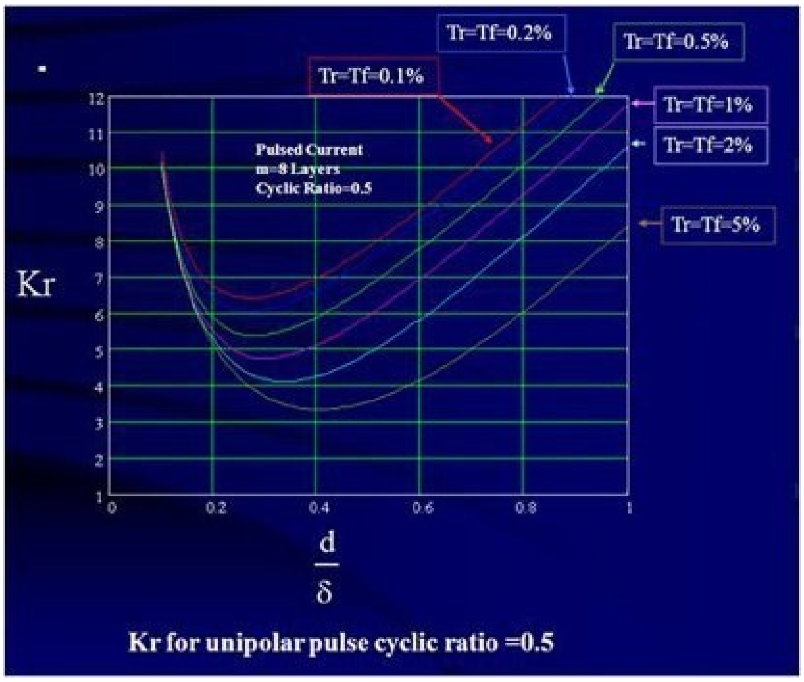


Fig. 107

Using Kr for loss calculation.

- Determine the layer effective thickness "d"
- Find the skinDepth d at the fundamental frequency.
- Calculate d/ d ;
- Calculate "R wl ";
- Find Kr for the waveform, number of layers "m" and "d / d ";
- Calculate the effective winding resistance R w e = (Kr) / (R wl);
- From "Irms", calculate the losses P=(Irms)^2*(R w e)

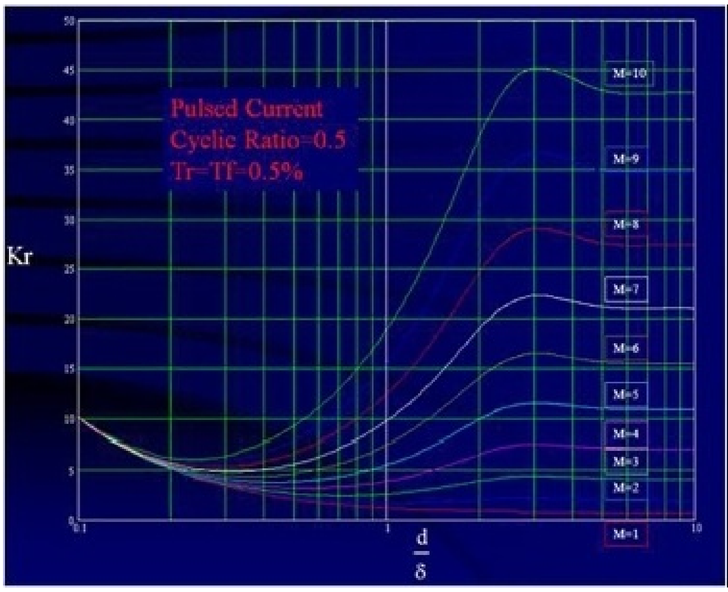


Fig. 108

Fig. 109

Further simplification with harmonics

We can further simplify these studies by applying the truncation of the results of the series of Mr and Dr functions and final function of Fr. In our previous study for the simplification of calculation of Fr, we had:

$$Fr = 1 + \Psi \cdot \frac{Ir^4}{3}$$

Where:

$$\Psi = \frac{5 \cdot \pi^2 - 1}{15}$$

We can expand the current waveform in Fourier series:

$$i(t) = I_{dc} + \sum_{n=1}^{\infty} A_n \cdot \cos(n \cdot \omega t) + \sum_{n=1}^{\infty} b_n \cdot \sin(n \cdot \omega t)$$

The Sin and Cos terms can be combined In an alternative way:

$$i(t) = Idc + \sum_{n=1}^{\infty} C_n \cdot \cos(n \cdot \omega t + \varphi_n)$$

As the losses being:

$$Ploss = Idc^2 \cdot Rdc + Rdc \cdot \sum_{n=1}^{\infty} Fr_n \cdot Irms_n^2$$

We can do:

$$Ploss = Reff \cdot Irms^2$$

$$Reff \cdot Irms^2 = Idc^2 \cdot Rdc + Rdc \cdot \sum_{n=1}^{\infty} Fr_n \cdot I_n^2$$

If:

$$\frac{R_{eff} \cdot I_{rms}^2}{R_{dc} \cdot I_{rms}^2} = \frac{I_{dc}^2 \cdot R_{dc} + R_{dc} \cdot \sum_{n=1}^{\infty} Fr_n \cdot I_n^2}{R_{rc} \cdot I_{rms}^2}$$

Then:

$$\frac{R_{eff}}{R_{dc}} = \frac{I_{dc}^2 + \sum_{n=1}^{\infty} Fr_n \cdot I_n^2}{I_{rms}^2}$$

As:

$$Fr = 1 + \frac{Y}{3} \cdot \lambda r^4$$

Making d defined for the fundamental and λr for each harmonic, we have:

$$\lambda r = \frac{d \cdot \sqrt{n}}{d}$$

Replacing:

$$Fr = 1 + \frac{Y}{3} \cdot \lambda r^4 \cdot n^2$$

$$\frac{R_{eff}}{R_{dc}} = \frac{I_{dc}^2 + \sum_{n=1}^{\infty} I_n^2 + \frac{Y}{3} \cdot \lambda^4 \cdot \sum_{n=1}^{\infty} n^2 \cdot I_n^2}{I_{rms}^2}$$

We can easily identify the first 2 terms as the RMS Current:

$$I_{rms}^2 = I_{dc}^2 + \sum_{n=1}^{\infty} I_n^2$$

Coming back to:

$$i(t) = Idc + \sum_{n=1}^{\infty} C_n \cdot \cos(n \cdot \omega \cdot t + \varphi_n)$$

Deriving we have:

$$\frac{di}{dt} = \sum_{n=1}^{\infty} -C_n \cdot \sin(n \cdot \omega \cdot t + \varphi_n) \cdot n \cdot \omega$$

The RMS value of the derivative of the current is:

$$Ider_{rms}^2 = \omega^2 \cdot \sum_{n=1}^{\infty} \frac{n^2 \cdot C_n^2}{2} = \omega^2 \cdot \sum_{n=1}^{\infty} n^2 \cdot I_n^2$$

Soon, Replacing:

$$\frac{R_{eff}}{R_{dc}} = \frac{I_{rms}^2 + \frac{\psi}{3} \cdot I \cdot r^4 \cdot \frac{1}{\omega^2} \cdot Ider_{rms}^2}{I_{rms}^2}$$

Simplifying, Finally we have:

$$\frac{R_{eff}}{R_{dc}} = 1 + \frac{\psi}{3} \cdot I \cdot r^4 \cdot \left(\frac{1}{\omega} \cdot \frac{Ider_{rms}}{I_{rms}}\right)^2$$

We see that in this case we do not need to calculate the harmonics of the waveform, just knowing the RMS value of a waveform and the RMS value of its derivative.

Final Formulas:

$$\frac{R_{eff}}{R_{dc}} = 1 + \frac{Y}{3} \cdot l_r^4 \cdot \left(\frac{1}{w} \cdot \frac{Ider_{rms}}{I_{rms}}\right)^2$$

Where:

$$Y = \frac{5 \cdot p^2 - 1}{15}$$

$$l_r = \frac{d}{d}$$

First Example: Calculation of a 4 Layers transformer with Copper sheet in the two coils.

Let’s calculate the AC resistance of a coil with 4 layers of copper sheet

the sheet of copper has thickness 0.1 mm and width 26mm

The total length is 300mm.

The frequency is 100Khz.

The waveform is monopolar with Cyclic ratio of 0.50 and Tf=Tr=0.05T

First the calculation of the RMS of the current:

To calculate the RMS value and the RMS value of the derivative of the waveform, we can make the frequency=1Hz, thus normalizing the values and keeping w=2*pi.

Then:

$I := 1$

$D := 0.5$

$T := 1$

$$Irms = \sqrt{\frac{2}{T} \cdot \int_0^{0.05} \left(\frac{I}{0.05}\right)^2 \cdot t^2\,dt + \frac{1}{T} \cdot \int_{0.05}^{(D-0.05)} I^2\,dt}$$

$$I_{rms} = .5774$$

Calculating the RMS Current of the Derivative:

$$DerIrms = \sqrt{\frac{2}{T} \cdot \int_0^{0.05} \left(\frac{1}{0.05}\right)^2 dt}$$

$$Ider_{rms} = 6.32$$

Calculation of the skin depth at 100°C:

$$\square\square \frac{758}{\sqrt{1000\square}}$$

Calculating the λr in relation to the skin depth:

$$l_r := \frac{d}{d} = \frac{0.1}{0.24}$$

$$l_r = 0.417$$

Doing:

$w := 2 \times p$

$p := 4$

$$Y := \frac{5 \times p^2 - 1}{15}$$

$$Y = 5.267$$

And finally:

$$Fr := 1 + \frac{Y}{3} \times l_r^4 \times \left(\frac{1}{w} \times \frac{I_{derms}}{I_{rms}} \right)^2$$

$Fr = 1.16$

$a_{20} := 0.00393$

$r_{20} := 1.724 \times 10^{6}$ ohms Cn

Temp=10

$$r_{temp} = r_{20} \cdot \left[1 + a_{20} \cdot (Temp - 20)\right]$$

Conductor cross sectional surface calculation:

$$A_{cu} := 0.1 \times 26 \quad mm^2$$

$$l_{cu} := 300 \quad mm$$

Resistance of the conductor in ohms:

$$R_{cnd} := \frac{\frac{l_{cu}}{10}}{\frac{A_{cu}}{100}} \cdot r_{temp}$$

$$R_{cnd} = 2.615 \cdot 10^{-3}$$

AC resistance of the conductor:

$$Rac := Fr \cdot R_{cnd}$$

$$R_{ac} = 3.036 \cdot 10^{-3}$$

Second example: Calculating ht losses:

What would be the value of the losses if the peak current were100A.

$$P_{tot} := Irms^2 \times Rac$$

$$I_{rms} := 0.577 \times 100$$

$$P_{tot} := I_{rms}^2 \times R_{ac} \qquad P_{tot} = 10.12$$

Another method for the calculation of Fr

Now we can facilitate the Fr calculation with the introduction of a new methodology. The application of this methodology is huge and can automatically calculate many parameters that were extremely difficult to be made with conventional methods.

Now the method is simple, but when added to other structures can be shown its complexity.

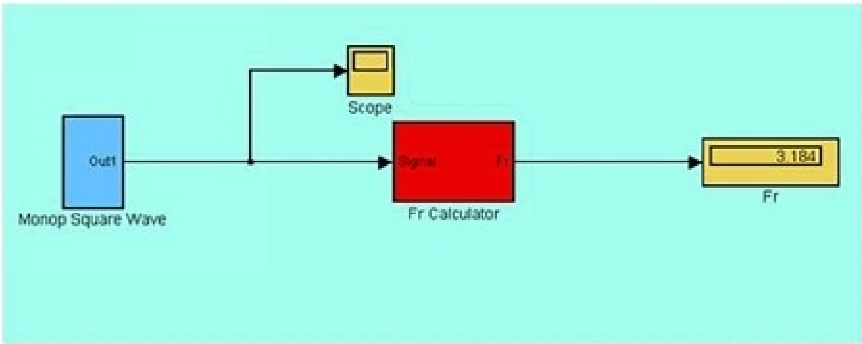


Fig. 110

Now we can see the figure above. The red block is the Fr calculation implemented through Simulink mathematical blocks. In the figure below we can see the implementation inside the block:

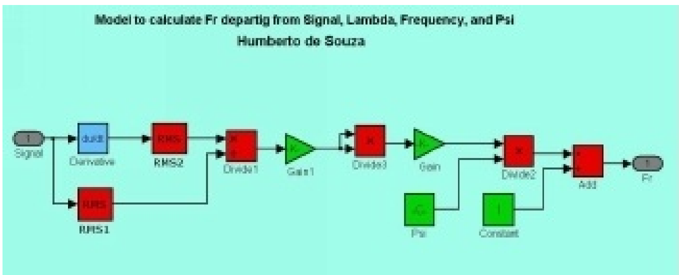


Fig. 111 The Fr calculation through Simulink.

At the left side of the figure we can see the derivative block du/dt that performs the mathematical derivative of the signal applied in its input. The signal in its output is applied to the RMS block that performs the Root Mean Square calculation of the signal. The operation of each block is demonstrated in the figure below.

The importance of this kind of system is that we can put anywhere in simulations in more complex system doing implementations including thermal models and temperature feedbacks, in order to simulate complete systems. We do not need to transport any part of the system to include another model for simulation.

We have used such models even in the project of inductors, large power transformers for UPS, railway rectifier, and many other power electronics equipments.

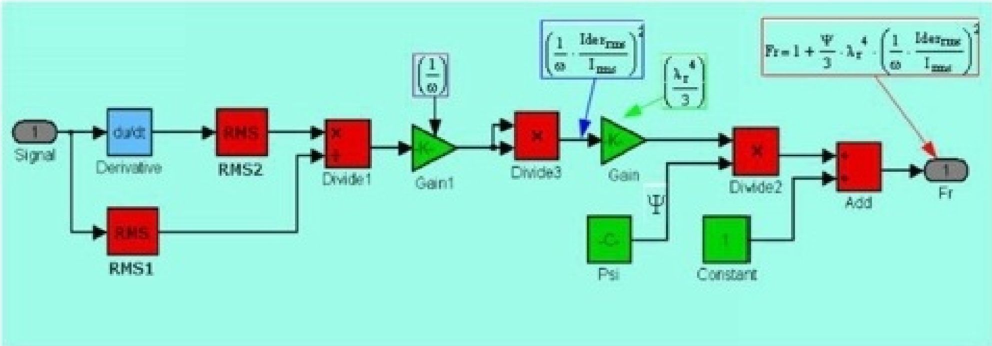


Fig. 112 In the blocks can be seen the calculation performed individually by the blocks and their results

Let us use it in the same problem we have calculated in the last example. First of all we need to make an oscillator with the same specifications of the model used in the calculations we have made.

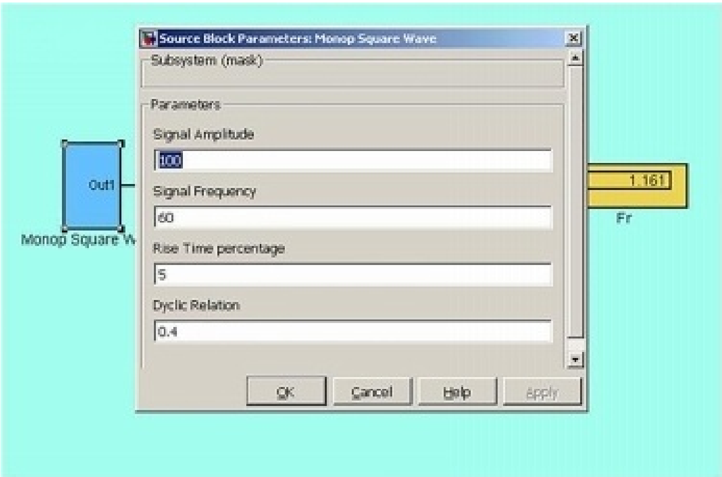


Fig. 113 Filling in the mask with the requested values

In the oscillator we are adjusting the signal amplitude, the frequency the rise and fall time as a percentage of the total period, and the cyclic ratio.

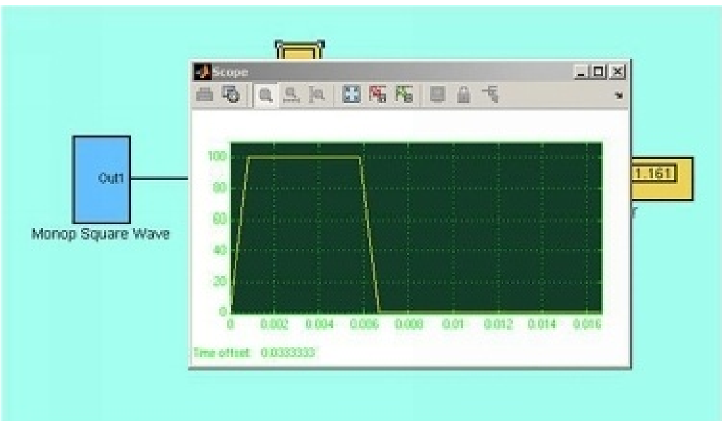


Fig. 114 The oscillator waveform

We got this:

We are setting up the values for 100A the signal amplitude, 100 Khz, rise time of 5% and cyclic ratio for 0.4.

Now we will setup our Fr calculator:

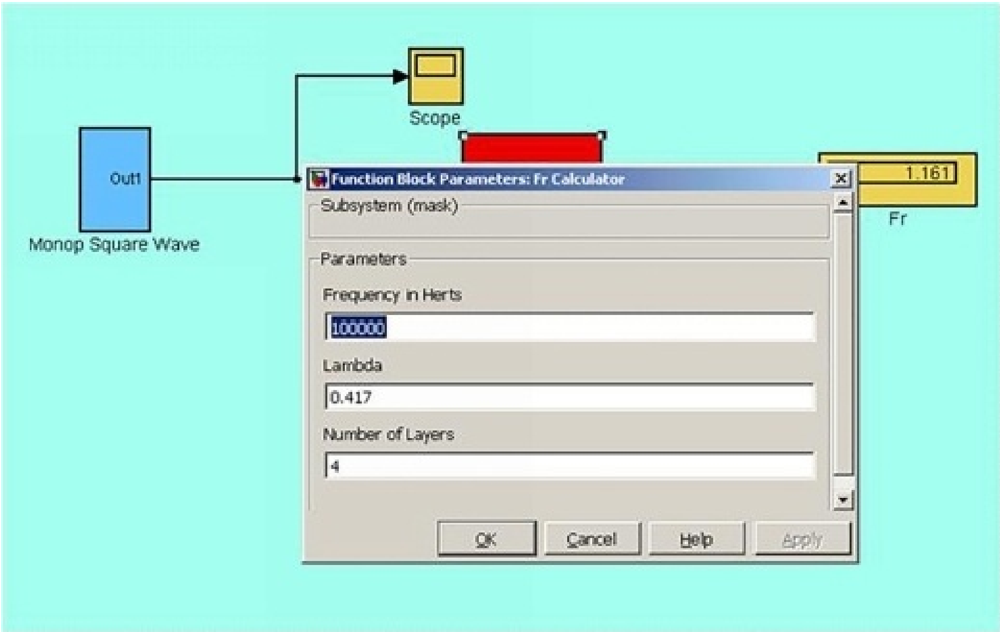


Fig. 115 Filling in the values for the calculator

Below we can see a partial view of the working of some parts:

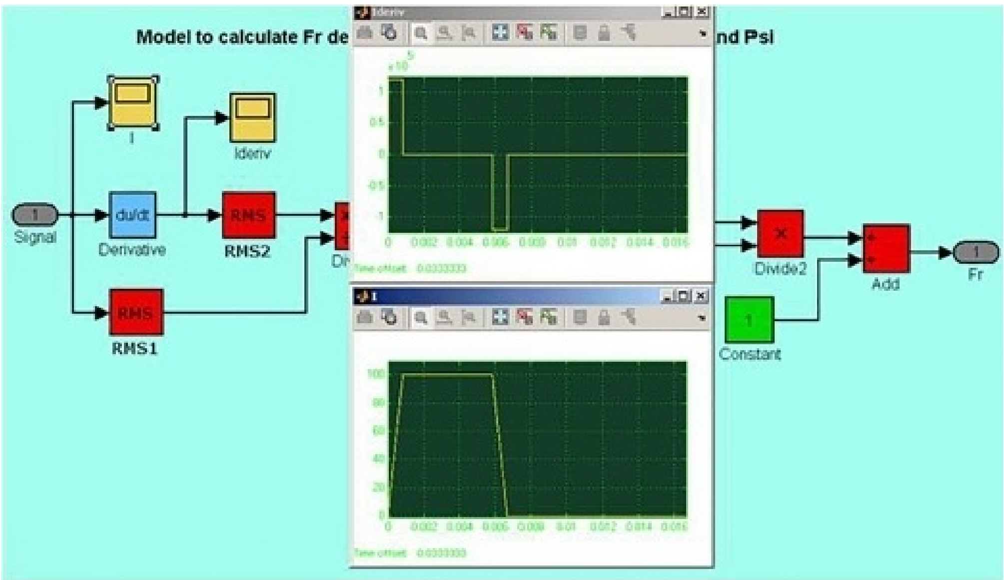


Fig. 116 The waveform of the function and the waveform of the derivative of the function

In less than 10 seconds we have our result:

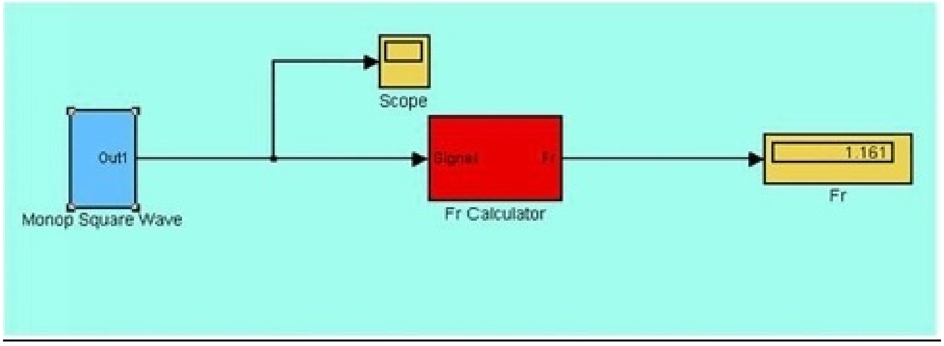


Fig. 117

In the system diagram of fig. 117, can be included other variables, constants and functions, in order to calculate more a complete task, like in fig. 118 were the system is calculating the power loss, departing from the included resistance of the sheet, and like in fig 119, departing from the geometry of the sheet and resistivity of the material.

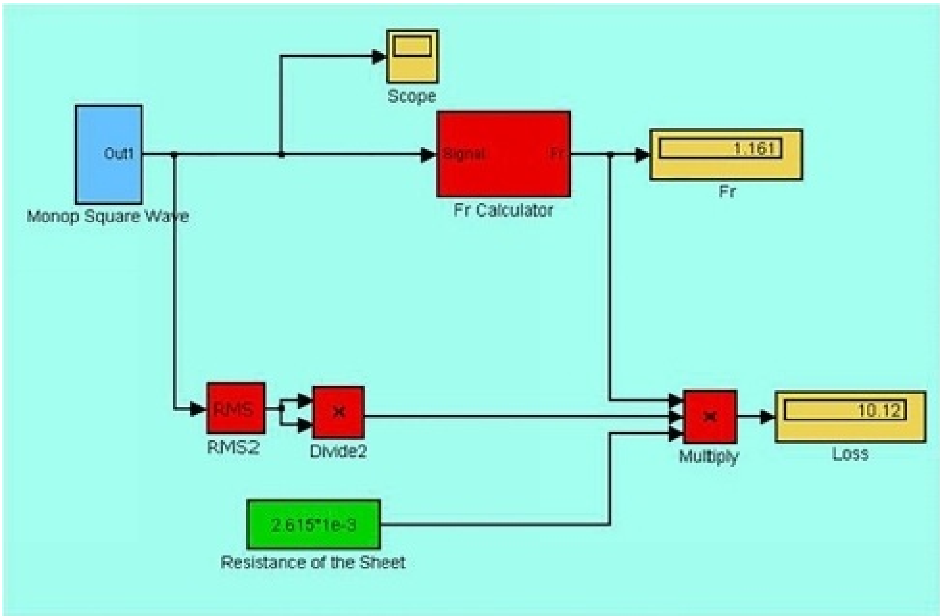


Fig. 118

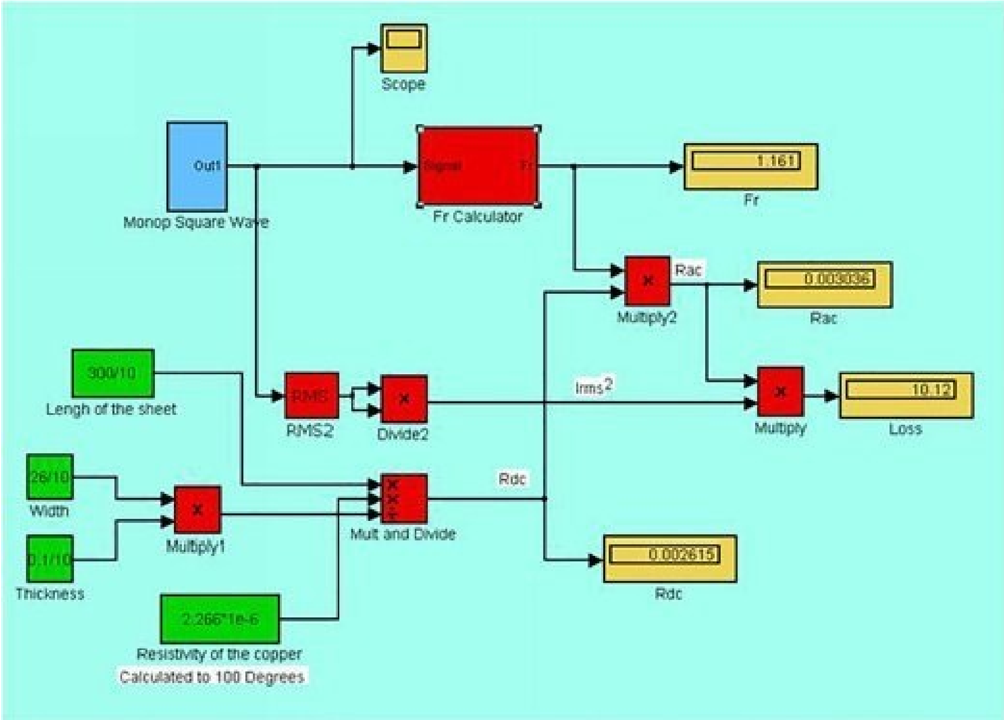


Fig. 119

The fig above can be used in calculation including the resistance of the coil. Its bullshit but it serves to demonstrate potentiality. Mainly if combining many other different aspects of circuitry.

Can be seen that the calculation are exactly the same like the values numerically calculated step by step

Let’s talk some words about optimization:

In each solution there is a layer thickness that the resistance is the minimum value. We now will concentrate in the solution to reach this mathematically.

First of all we need to normalize the equations in order to obtain values totally dimensionless to allow us perform variations to investigate the behavior of the model.

The value of d is calculated as a function of the first harmonicfrequency, temperature, resistivity, permeability and material, defining with that a reference value for the DC resistance R d as a value referenced for the first harmonic skin depth d_o.

$$\frac{R_d}{R_{dc}} = \frac{d}{d_o} = D$$

$$\frac{R_{eff}}{R_{dc}} = D \times \frac{R_{eff}}{R_d}$$

As:

$$\frac{R_{eff}}{R_{dc}} = 1 + \frac{y}{3} \times D^4 \times \left(\frac{1}{w} \times \frac{Iderms}{Irms}\right)^2$$

Doing substitutions:

$$\frac{R_{eff}}{R_d} = \frac{1}{D} + \frac{y}{3} \times D^3 \times \left(\frac{1}{w} \times \frac{Iderrms}{Irms}\right)^2$$

Let’s plot a 3D graph as a function of D and the number of layers p for a certain waveform (That one used in our last example):

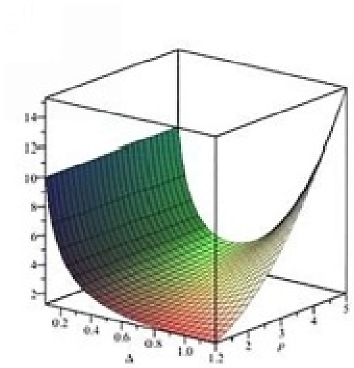


Fig. 120A surface where the minimum values (Optimal width) is the valley of the plane plotted.

As we can see above, in each solution there is a layer thickness that the resistance is the minimum value. We now will concentrate in the solution to reach this mathematically.

First of all we need to normalize the equations in order to obtain values totally dimensionless to allow us perform variations to investigate the behavior of the model.

Getting the derivative and setting it to zero:

$$0 = \frac{1}{\Delta^2} \quad \frac{\psi \Delta^2 Iderms^2}{\varpi^2 Irms^2}$$

Isolating D , we have:

$$\Delta \quad \frac{\left(\varpi^2 Irms^2 \psi^3 Iderms^2\right)^{1/4}}{\psi Iderms}$$

Simplifying,we get the value of D for the minimum value of the R_{eff} resistance:

$$\Delta opt = \frac{\sqrt{\frac{\varpi Irms}{Iderms}}}{\psi^{1/2}}$$

Combining with:

$$\frac{R_{eff}}{R_{dc}} = 1 + \frac{y}{3} \times D^4 \times \left(\frac{1}{\omega} \times \frac{I_{derms}}{I_{rms}} \right)^2$$

We get the solution, for any arbitrary waveform, but for the optimal condition:

$$\left(\frac{R_{eff}}{R_{dc}} \right)_{opt} = \frac{4}{3}$$

And finally we can get the final value for a specific D :

$$\frac{R_{eff}}{R_{dc}} = 1 + \frac{1}{3} \cdot \left(\frac{\Delta}{\Delta_{opt}} \right)^4$$

Now to normalize the entire system, we must arrange the formula to allow us see the proper format to ensure simplicity of the calculation:

$$D_{opt} = \frac{1}{Y^{\frac{1}{4}}} \times \sqrt{\frac{2 \times \pi \times f \times I_{rms}}{I_{derms}}}$$

We can see the first term from left to right as being dependent of y , and the second one dependent of the waveform and frequency. In reality this second term is dependent solely of the waveform, being the angular frequency that appears in the formula coming from the differentiation. Thus we can put the term dependent of the waveform in a table of values or even in formulas for the calculation of their values. For the sake of simplicity, if wanted, we can design a calculator of the second term in Matlab, Mathcad, Mathematica, Saber or even a program in a convenient language that could allow us calculate it easily.

Let's design one with Simulink of the Matlab. Part of it is already ready in the model we designed for the calculation of Fr. Our present design is simply a derivation of that.

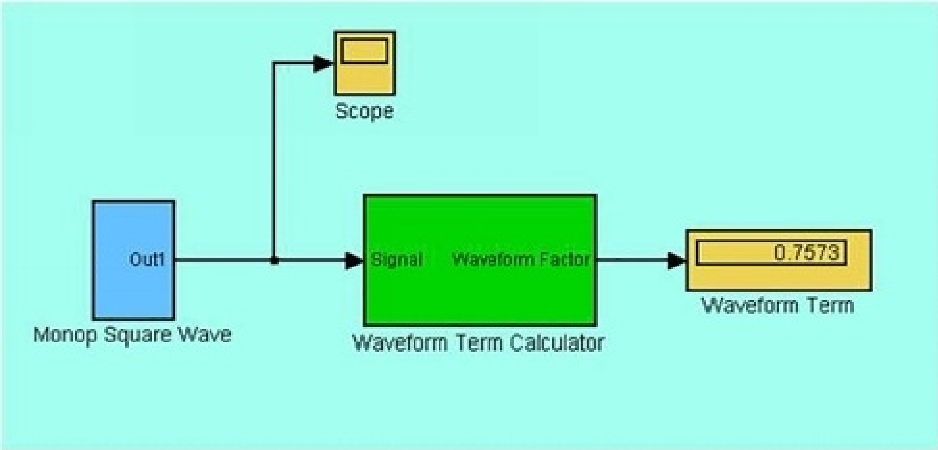


Fig. 121

Now we will calculate the optimum width of the foil from that calculation we have done above.

Firstly we can determine the values from our oscillator:

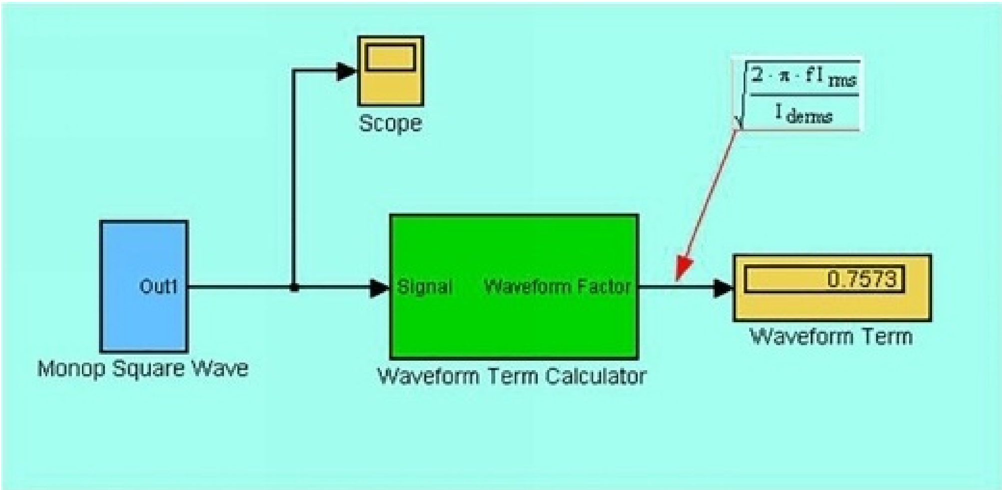


Fig. 122

$$p := 4$$

$$y := \frac{5 \cdot p^2 - 1}{15}$$

$$y = 5.267$$

$$D_{opt} := \frac{1}{y^{\frac{1}{4}}} \cdot 0.757$$

$$D_{opt} = 0.5$$

Solution other than for paper and pencil:

Today, the intense use of CAE systems do not require us spend our time simplifying formulations of mathematical calculations.

In the past due to the use of the calculation rulers and even manual calculators, the simplification of mathematical formulas, used to be very important. Today with the use of computerized tools such as CAE Systems (MathCad, Math, Maple, Matlab, etc.) or even programming in medium or high-level language such as Fortran, C, PLM, Basic, greatly facilitate our work.

Over the past 20 years, we have employed more widely the combination of symbolic and numerical methods for automatically solving complex mathematical systems.

In our work of Power electronics with the time, we developed some topological Matrix systems that we enter the incidences in hybrid matrices solving a lot of problems, but practically not involving any complex intelligence we are doing about. The complex intelligence was spent in the development of that tool developed in Maple.

Now we are presenting the simplified version of an old process for calculation of powder core like m Kool of Magnetics for our high frequency inductors.

Calculation in the design of a 20Kva inverter filter inductor

Cálculation of an Inductor for UPS Inverter of 20K

192-03-030007

$L = 50 \times 10^{-6}$

$Lfin := 51 \times 10^{-6}$

Max harmonic number:

Max:=2000 The maximum harmonic number (Up to 2000th

n:=0..Max

Thickness Interval

$X1 := 20$

This is the conductor thickness multipl

$X2 := 80$

Fundamental frequency:

$Fund := 60$

Temp

100 Degrees Centigrates.

$Constant_c := 75.764$ (At 100 Degrees)

Resistivity of copper at 100 Degrees Centigrade

$r := 2.268 \cdot 10^{-6}$

$X1 := 20$

This is the conductor thickness multipl

$X2 := 80$

Fundamental frequency:

$Fund := 60$

Temp

100 Degrees Centigrates.

$Constant_c := 75.764$ (At 100 Degrees)

Resistivity of copper at 100 Degrees Centigrade

$r := 2.268 \cdot 10^{-6}$

Height of each layer (Width of copper conductor)

$x := X1..X2$ Variation of X1, since x1 up to X2

Relação da espessura do condutor para a película de con

$$d = \frac{Constante_d}{\sqrt{f}}$$

And:

$f = n \cdot Fund$ Harmonic number times fundamental frequer

$$\frac{d}{d}_{n,x} = \frac{x \cdot \sqrt{n \cdot Fund}}{100 \cdot Constante_d}$$

$$lr_{n,x} := \frac{x \cdot \sqrt{n \cdot Fund}}{100 \cdot Constante_d}$$ At 100 Degree

$$M_{n,x} := lr_{n,x} \cdot \frac{(\sinh(2 \cdot lr_{n,x}) + \sin(2 \cdot lr_{n,x}))}{(\cosh(2 \cdot lr_{n,x}) - \cos(2 \cdot lr_{n,x}))}$$

Dowell Realat

$$D_{n,x} := 2 \cdot lr_{n,x} \cdot \frac{(\sinh(lr_{n,x}) - \sin(lr_{n,x}))}{(\cosh(lr_{n,x}) + \cos(lr_{n,x}))}$$

$m := 10$ Number of layers

$$Fr_{n,x} := M_{n,x} + \frac{m^2 - 1}{3} \cdot D_{n,x}$$ Calculation of F.r

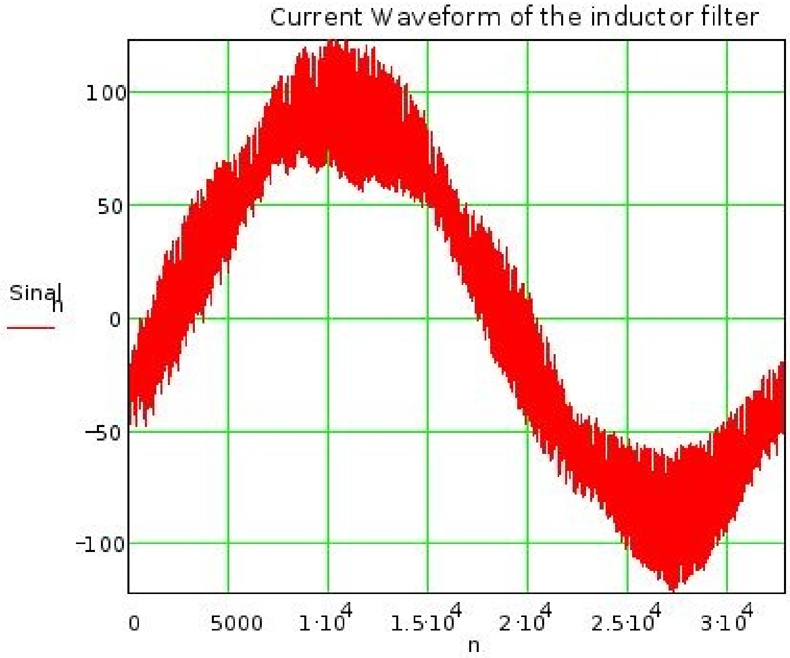
Calculation of each Harmonic::

Sinal = READPRN("Arquivo1.d)

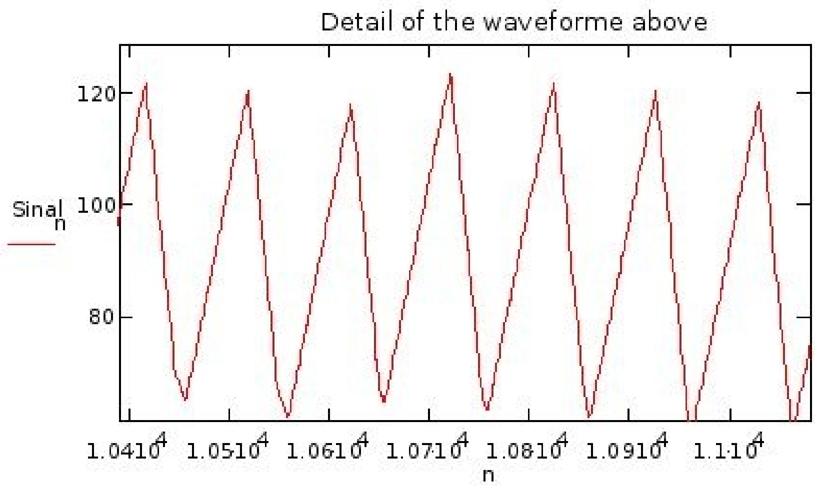
$n := 0..last(Sinal)$

$Const = last(Sinal)$

The waveform plotted matemátically from a mathematical model of an thre
20Kva. This is the current in the filter inductor:



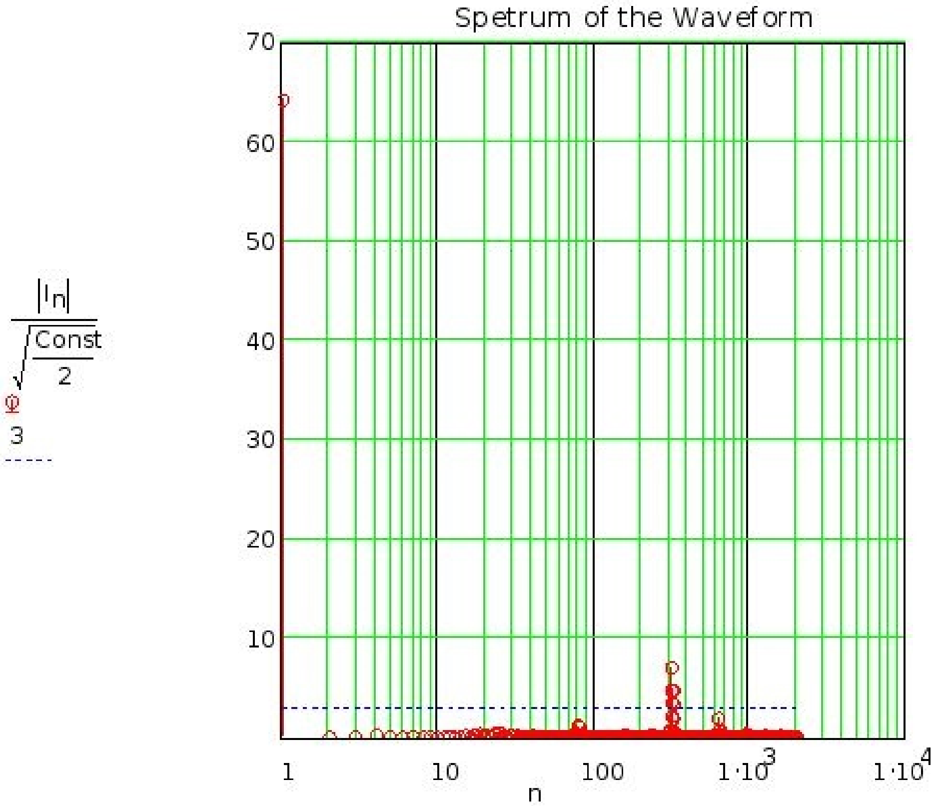
An excerpt from the upper waveform



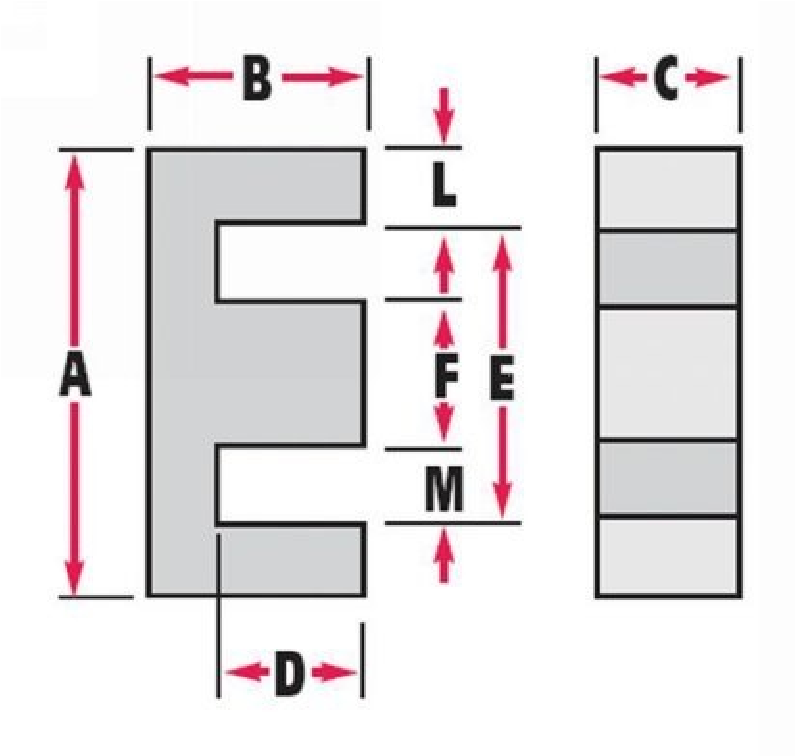
$I_{max} := max(Sinal)$

$I_{max} = 123.5$

$I := fft(Sinal)$



PART NO.		A	B	C	D (min.)	E (min.)	F	L (nom.)	M (min.)
00K1207E	in	.500±.010	.252±.004	.140±.006	0.178	0.35	.140±.005	0.07	0.104
(EF 12.6)	(mm)	(12.70)	(6.40)	(3.56)	(4.42)	(8.89)	(3.56)	(1.78)	(2.64)
00K1808E	in	.760±.012	.319±.007	.188±.006	0.218	0.548	.188±.005	0.094	0.183
(EI-187)	(mm)	(19.30)	(8.10)	(4.78)	(5.54)	(13.90)	(4.78)	(2.39)	(4.65)
00K2510E	in	1.000±.015	.375±.007	.250±.004	0.245	0.74	.250±.005	0.125	0.246
(E-2425)	(mm)	(25.40)	(9.53)	(6.53)	(6.22)	(18.80)	(6.22)	(3.17)	(6.25)
00K3007E	in	1.185±.018	.591±.009	.278±.006	0.376	0.768	.274±.008	0.201	0.254
(DIN 30/7)	(mm)	(30.10)	(15.01)	(7.06)	(9.70)	(19.50)	(6.96)	(5.11)	(6.46)
00K3515E	in	1.380±.020	.557±.009	.368±.007	0.378	0.995	.367±.008	0.175	0.31
(EI-375)	(mm)	(34.54)	(14.10)	(9.35)	(9.65)	(25.30)	(9.32)	(4.45)	(7.87)
00K4017E	in	1.687±.025	.830±.013	.424±.010	0.587	1.195	.468±.010	0.234	0.365
(EE 42/11)	(mm)	(42.80)	(21.10)	(10.80)	(15.00)	(30.40)	(11.90)	(5.95)	(9.27)
00K4020E	in	1.687±.025	.830±.013	.608±.010	0.587	1.195	.468±.010	0.234	0.365
(DIN 42/15)	(mm)	(42.80)	(21.10)	(15.40)	(15.00)	(30.40)	(11.90)	(5.95)	(9.27)
00K4022E	in	1.687±.025	.830±.013	.788±.010	0.587	1.195	.468±.010	0.234	0.365
(DIN 42/20)	(mm)	(42.80)	(21.10)	(20.00)	(15.00)	(30.40)	(11.90)	(5.95)	(9.27)
00K4317E	in	1.609±.024	.650±.011	.493±.007	0.409	1.115	.493±.008	0.238	0.31
(EI-21)	(mm)	(40.90)	(16.50)	(12.50)	(10.40)	(28.30)	(12.50)	(6.00)	(7.90)
00K5528E	in	2.16±.032	1.085±.016	.812±.015	0.729	1.476	.660±.015	0.33	0.405
(DIN 55/21)	(mm)	(54.90)	(27.60)	(20.60)	(18.50)	(37.50)	(16.80)	(8.38)	(10.30)
00K5530E	in	2.16±.032	1.085±.016	.969±.015	0.729	1.476	.660±.015	0.33	0.405
(DIN 55/25)	(mm)	(54.90)	(27.60)	(24.61)	(18.50)	(37.50)	(16.80)	(8.38)	(10.30)
00K6527E	in	2.563±.050	1.279±.150	1.063±.016	.874	1.740	.775±.012	.394	.476
(Metric E65)	(mm)	(65.10)	(32.50)	(27.00)	(22.20)	(44.20)	(19.70)	(10.00)	(12.10)
00K7228E	in	2.850±.043	1.100±.020	.750±.015	0.699	2.072	.750±.015	0.375	0.665
(F11)	(mm)	(72.39)	(27.94)	(19.05)	(17.78)	(52.63)	(19.05)	(9.52)	(16.89)
00K8020E	in	3.150±.047	1.500±.025	.780±.015	1.103	2.334	.780±.015	0.39	0.78
(Metric E80)	(mm)	(80.01)	(38.10)	(19.81)	(28.14)	(59.28)	(19.81)	(9.91)	(19.81)



Dados do Núcleo: K5530-E060

$A := 54.9$ $B := 27.60$

$C := 24.61$ $F := 16.8$ $M := 10.30$ $l_{fe} := 12.3$ $m_r := 6($

$N_{esp} := 10$

$Num := 2$

$A_{ef} := 4.17 \cdot Num$

$$l_{cu} := \frac{2 \cdot (Num \cdot C + 2 \cdot M + F)}{10}$$

$l_{cu} = 17.324$

$$Leght_{Window} := 18.5 \cdot 2 - 4$$

$$Leght_{Window} = 33$$

$$Acum_x := \frac{\frac{x}{100} \cdot Leght_{Window}}{100}$$

$$Acum_x := \frac{1}{10000} \cdot x \cdot Leght_{Window}$$

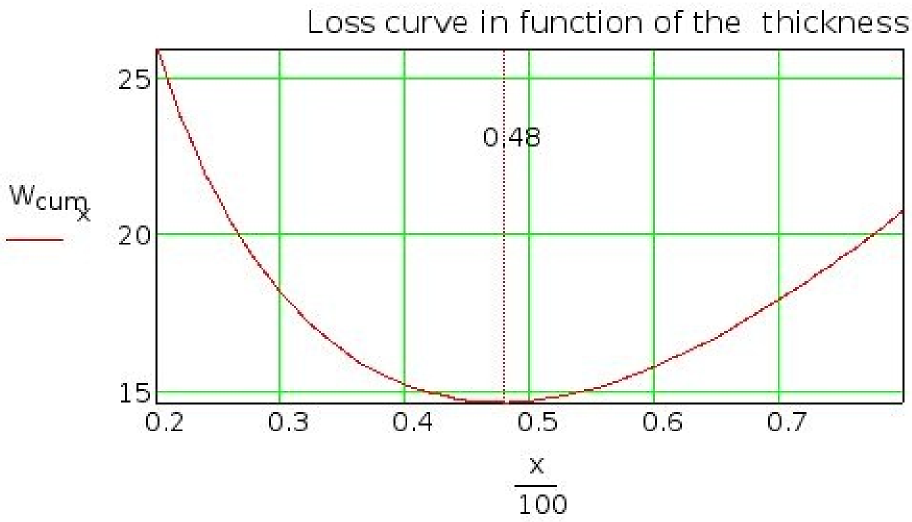
Conductor DC resistance:

$$Rdcm_x := \frac{l_{cu} \cdot Nesp_x}{Acum_x} \cdot \rho$$

Conductor resistance for each frequency:

$$Rm_{n,x} := Rdcm_x \cdot Fr_{n,x}$$

$$Wcum_x := \left[\left(\frac{|I_0|}{\sqrt{\frac{Const}{2}}} \right)^2 \cdot Rdcm_x + \sum_{n=1}^{2000} Rm_{n,x} \cdot \left[\left(\frac{|I_n|}{\sqrt{\frac{Const}{2}}} \right)^2 \right] \right]$$



$$W_{cum_x} := \left[\left(\frac{|I_0|}{\sqrt{\frac{Const}{2}}}\right)^2 \cdot Rdc_{m_x} + \sum_{n=1}^{2000} Rm_{n,x} \cdot \left[\left(\frac{|I_n|}{\sqrt{\frac{Const}{2}}}\right)^2\right]\right]$$

$W_{cum_x} = 14.653$ Copper losses

$$B_{max} := \frac{0.4\pi \cdot N_{esp} \cdot I_{max} \cdot \mu_r}{l_{fe}} \qquad B_{max} = 7.57 \times 10^3$$

$$L := \frac{0.4\pi \cdot N_{esp}^2 \cdot \mu_r \cdot A_{ef}}{l_{fe} \cdot 10^8} \qquad L = 5.112 \times 10^{-5}$$

AHeight of the coil:

$$Henr := \frac{N_{esp} \cdot \left(\frac{x}{100} + 0.05\right) + 2}{0.85}$$

$Henr = 8.588$ Less than $M = 10.3$

We can calculate the RMS of derivative of the current waveform used in the example above and calculate the optimum exactly like the example of chapter xx in the calculation of the K factor transformer.

Design of a ferrite 3Kva 200Khz power transformer for a ZVS-QRC telecommunication SMPS.

One another example is the project of a power transformer for a ZVS Quasi Resonant Converter – 52.8 Vdc/50A 200Khz SMPS:

This project was done in 1993 for a modular telecommunications power supply. in two versions: one module of 50A/52.8Vdc and another of 100A/52.8Vdc. The photo of the equipment can be seen in the end of this design procedure.

SMPS 48V 50A - ZVS - QRC 1993 MOSFET APT5020BNF

Project main specifications:

$Eo_{max} := 57.6$ $rel_{trasnf} := \frac{1}{5}$

Nominal input $Vin_{nom} := 220$

efficiency $h := 0.92$

$Io_{max} := 50$ $D_d := 0.10$ Apparent duty cycle reduction due to res inductance

$f := 200000$ $a_{max} := 0.98$ Max duty Cycle

Input power facto

$Pf := 0.99$

$Ein_{min} := 400$ $a_{tot} := a_{max} - D_d$

$a_{tot} = 0.88$

TRANSFORMER FOR SMS 50A/52.8 Vdc PROJECT-CORE TDK PC40

Max temperature Var | Max Amb Temperature | Form Factor of Voltage

$D_{tmax} := 60$ $T_a := 40$ $FF := 1$

Secondary Voltage : Secondary Current: Maximum current den

$Es_{rms} := 70.682$ $Is_{rms} := \frac{50}{\sqrt{2}}$ $J_a := 6.91$

Primary Voltage: Primary current:

$Ep_{rms} := 400$ $Ip_{rms} := 50 \cdot rel_{trasnf}$

Transformation relation: $Ip_{rms} = 10$

$Rel := rel_{trasnf}$

Core selected: E65BAR39

Core external dissipation surface:

$Se_{fe} := 145.9$

$Se_{fe} = 145.9$

Maximum dissipation per unit area:

$$Diss := \frac{20.2}{4} \cdot 10^{-6} \cdot (T_a + 100) \cdot D_{tmax}^{\left(\frac{6}{5}\right)}$$

$Diss = 0.096$

Maximum power that we can have within the core. This is the dissi
comes from the temperature and is multiplied by the total dissipati

$W_{femaxima} := Se_{fe} \cdot Diss$ $W_{femaxima} = 14.036$

Core Volume:

$Vol_{efinal} = 117.3$ This comes from the geometry

$Vol_{efinal} = 117.3$

Maximum core loss density in watts/cm^3

$$J_{vemax} = \frac{W_{femaxima}}{Vol_{efinal}}$$ $J_{vemax} = 0.12$

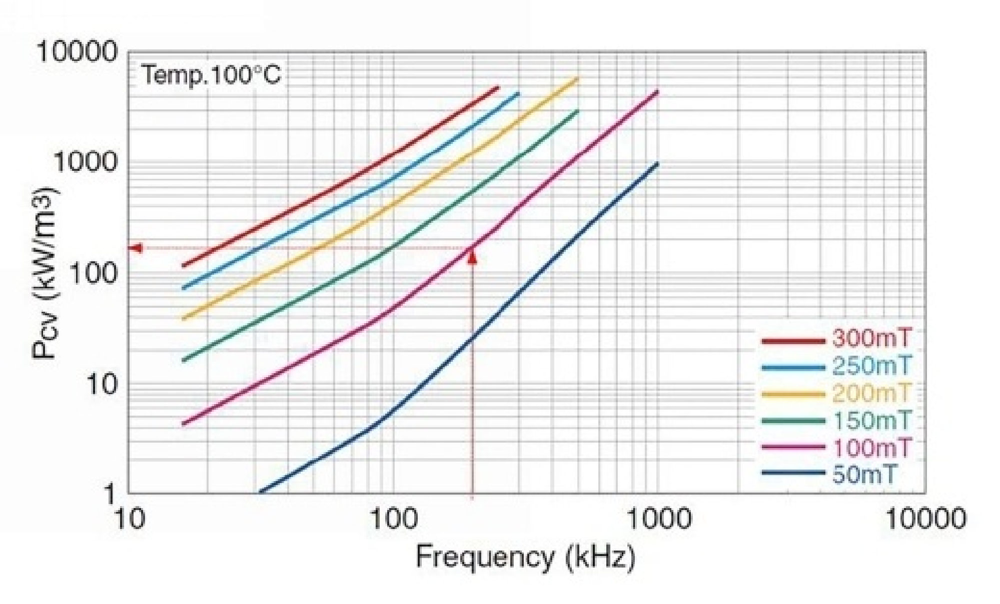


Fig. 123

Actual ferrite loss density:

$J_{vfe} := 0.15$

Core weight in grams::

$P_{fe} := 614$

$P_{fe} = 614$

Magnetic dimensional features:

Magnetic cross sectional area:

$A_{ef} := 7.98$

$A_{ef} = 7.98$

Magnetic mean path lenght:

$L_{fe} := 14.7$

$L_{fe} = 14.7$

Bobbin Dimensional features:

Useful length of layer-Bed:

$L_{cam} := 4$

$L_{cam} = 4$

Maximum useful height

$H_{bob} := 1.085$

$H_{bob} = 1.085$

Selection and configuration of conductors

Skin depth at maximun temperature:

$$d_o := \frac{75.8}{\sqrt{f}}$$

$d_o = 0.169$ in milimeters

For Secondary:

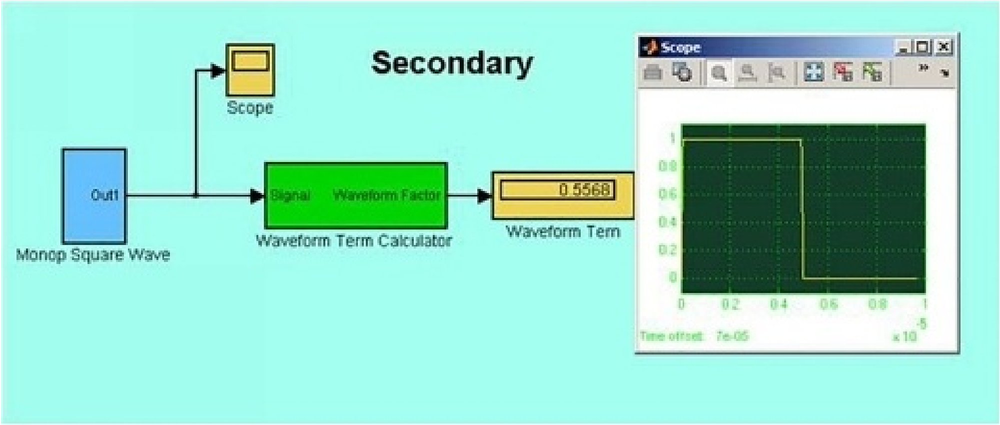


Fig. 124

$$ST_s := 0.5567$$

Number of layers:

$$p_s := 1$$

$$y_s := \frac{5 \cdot p_s^2 - 1}{15}$$

$$y_s = 0.267$$

So, the optimal D should be:

$$Ds_{opt} := \frac{1}{y_s^{\frac{1}{4}}} \cdot ST_s \qquad Ds_{opt} = 0.775$$

$$ds_{opt} := Ds_{opt} \cdot d_o$$

$$ds_{opt} = 0.131$$

SECONDARY

With copper sheet

Calculating the secondary cross sectional area in fuction of curr

$$A_{cu1sec} := \frac{Is_{rms}}{J_a}$$

$$A_{cu1sec} = 5.117$$

$n := 1..10$

$$D_{sec} := 0.12$$

$$ds_{opt} = 0.131$$

The thickness D_{sec} must be close to ds_{opt} If it is impossible, other me
must be taken

Conductor cross sectional area:

$$A_{cusec} := D_{sec} \cdot (10 \cdot k_{cam} - 2)$$

$$A_{cusec} = 4.56$$

$$Ds_{fin} := \frac{D_{sec}}{d_o}$$

$$Ds_{fin} = 0.708$$

Calculation of The relation between Reff and Rdc:

$$\frac{R_{eff}}{R_{dc}} = 1 + \frac{y_s}{3} \cdot Ds_{fin}^{4} \cdot \left(\frac{1}{ST_s}\right)^4$$

$$Rs_{elres} := 1 + \frac{y_s}{3} \cdot Ds_{fin}^{4} \cdot \left(\frac{1}{ST_s}\right)^4$$

$$Rs_{elres} = 1.233$$

$Rs_{elres} = 1.233$

Secondary number of turns:

$$N_{espsec} := \frac{Es_{rms} \times 10^{8}}{4 \times KF \times f \times B_{max} \times A_{ef}}$$

$N_{espsec} = 1.007$

$N_{espsec} := 1$

Number of turns per layer:

$Esp_{porcamsec} := 1$

$N_{camsec} := N_{espsec}$

Mean turn lenght:

$LCU_{med} := 17.58$ (Catalog Data)

$LCU_{med} = 17.58$

$Lcu_{totsec} := N_{espsec} \times LCU_{med}$ Total conductor lenght centimeters

$Lcu_{totsec} = 17.58$

Weight in grams and Conductor losses:

Copper volume calculation:

$$Vol_{cusec} := Lcu_{totsec} \times \frac{A_{cusec}}{100}$$

Total weight in grams:

$P_{cusec} := 8.96 \times Vol_{cusec}$

$P_{cusec} = 7.183$

$$Pcu_{totalsec} := \frac{P_{cusec}}{1000}$$ Becomes Kg

Secondary total losses: $Pcu_{totalsec} = 7.183 \times 10^{-3}$

Copper resistivity:

$T := 100$

$r := 1.724 \times 10^{-6} \cdot [1 + 0.00393 \cdot (T - 20)]$

$r = 2.266 \times 10^{-6}$

Copper density:

$d_{cu} := 8.96$ g/cm^3

Real current density in amperes per centmeter square:

$$J_{realsec} := \frac{Is_{rms}}{A_{cusec}} \cdot 100$$

$J_{realsec} = 775.336$ In amps per cm^2

Conductor losses considering proxinity and skin effects:

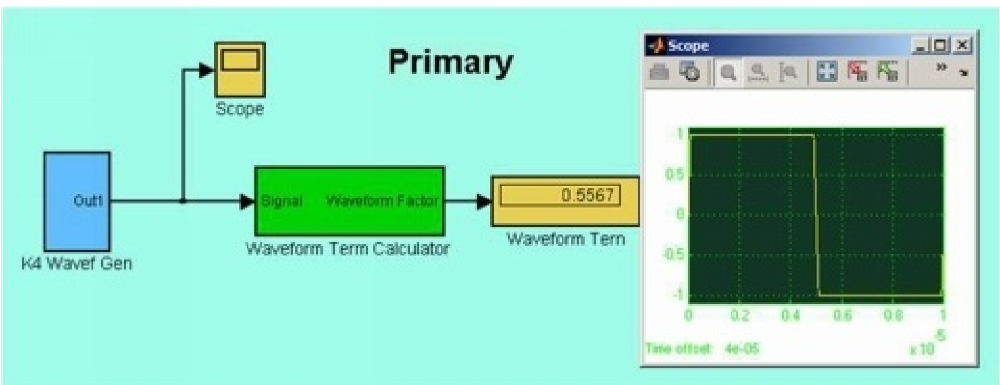
$$W_{cusec} := \frac{Lcu_{totsec}}{(L_{cam} \cdot 2) \cdot \frac{D_{sec}}{10}} \cdot r \cdot Is_{rms}^2 \cdot Rs_{elres}$$

$W_{cusec} = 2.557$

The secondary is divided in two windings of 2 turns each and curled tc
insulated bv an insulator of 0.1 mm.

Primary

For the Primary:



$$ST_p := 0.5567$$

Number of layers:

$$p_p := 5$$

$$y_p := \frac{5 \cdot p_p^2 - 1}{15}$$

$$y_p = 8.267$$

So, the optimal would be:

$$Dp_{opt} := \frac{1}{y_p^{\frac{1}{4}}} \cdot ST_p$$

$$Dp_{opt} = 0.328$$

$$dp_{opt} := Dp_{opt} \cdot d_o$$

$$dp_{opt} = 0.056$$

$dp_{opt} = 0.056$

Calculating the secondary cross sectional area in fuction of current de

$$A_{cu1prim} := \frac{Ip_{rms}}{J_a}$$

$n := 1..10$

$A_{cu1prim} = 1.447$

Copper sheet chosen => $D_{prim} := 0.05$

The thickness D_{prim} must be close to dp_{opt}. If it is impossible, other measures must be taken

Conductor Cross sectional area:

$$A_{cuprim} := D_{prim} \cdot (10 \cdot k_{cam} - 2)$$

$A_{cuprim} = 1.9$

$$Dp_{fin} := \frac{D_{prim}}{d_o}$$

$Dp_{fin} = 0.295$

Calculation of The relation between Reff and Rdc:

$$\frac{R_{eff}}{R_{dc}} = 1 + \frac{y_p}{3} \cdot Dp_{fin}^4 \cdot \left(\frac{1}{ST_p}\right)^4$$

$$Rp_{elres} := 1 + \frac{y_p}{3} \cdot Dp_{fin}^4 \cdot \left(\frac{1}{ST_p}\right)^4$$

FF = Form Factor. For sinusoidal wave is 1.11

$Rp_{elres} = 1.217$

Primary number of turns:

$$N_{esprim} := \frac{Ep_{rms} \cdot 10^8}{4 \cdot FF \cdot f \cdot B_{max} \cdot A_{ef}}$$

$N_{esprim} = 5.696$

Number of turns per layer:

$Esp_{porcamprim} = 1$

$N_{camprim} = N_{esprim}$

Mean turn lenght and conductor (Copper Foil) total lenght:

$LCU_{med} = 17.58$

$LCU_{med} = 17.58$

$Lcu_{totprim} = N_{esprim} \cdot LCU_{med}$ em centímetros

Weight in Grams and Conductor Loss :

Copper volume calculation:

$$Vol_{cuprim} = Lcu_{totprim} \cdot \frac{A_{cuprim}}{100}$$

Total weight in Grams:

$P_{cuprim} = 8.96 \cdot Vol_{cuprim}$

$P_{cuprim} = 14.964$

$$Pcu_{totalprim} = \frac{P_{cuprim}}{1000}$$ Becomes Kg

$Pcu_{totalprim} = 0.015$

Primary copper loss:

Real current density in amps/Cm^2::

$$J_{realprim} = \frac{Ip_{rms}}{A_{cuprim}} \cdot 100$$

$J_{realprim} = 526.316$

Perdas no condutor em watts considerando o efeito pelicular é de::

$$W_{cuprim} = \frac{Lcu_{totprim}}{\frac{A_{cuprim}}{100}} \cdot Ip_{rms}^{2} \cdot Rp_{elres}$$

$W_{cuprim} = 1.276$

Height of coil:

Height of coil:

In the transformer there are two Dielectric shields for semiconductors p
EMI/EMC purposes.

$eisol1 = 0.05$ Insulating
$eisol2 = 0.1$ Insulating
$Diel_{pb} := 1.0$ Insulating
$Diel_{bb} := 1.2$ Insulating
$Diel_{bs} := 0.2$ Insulating
$Sh1_{electro} = 0.05$ Copper sheet
$Sh22_{electro} = 0.05$ Copper sheet

$$Shield_{electro} = Diel_{pb} + Sh1_{electro} + Diel_{bb} + Sh22_{electro} + Diel_{bs}$$

$$Shield_{electro} = 2.5$$

In the present case we have 2 windings for the secondary, That are cone
tap conection.

$$H_{forma} := \frac{N_{camsec} \cdot (D_{sec} + eisol2) \cdot 2 + Shield_{electro} \cdot 2 + N_{camprim} \cdot (D_{prim} + eisol1) + 0.1}{0.86}$$

H shape has to be less than H coil:

$H_{forma} = 7.023$ $\quad$ $10 \times H_{bob} = 10.85$

Capacitance between windings and shields::

$A := \frac{L_{cam}}{100} \times \frac{LCU_{med}}{100} \times m^2$ $\quad$ $d := \frac{Diel_{bb}}{1000} \times m$

$e_0 := 8.854187817 \cdot 10^{-12} \times \frac{farad}{m}$ Permissivity of vácum

$e := 2.9$ Permissivity relactive to mylar

$$C := \frac{e \times e_0 \times A}{d}$$

$C = 1.505 \cdot 10^{-10} farad$

$Wcu_{total} := W_{cuprim} + W_{cusec} \cdot 2$

$Pcu_{total} := Pcu_{totalprim} + Pcu_{totalsec} \cdot 2$

$$Vq_{rms} := \frac{Wcu_{total}}{Is_{rms} \cdot Es_{rms}} \qquad Vq_{rms} = 2.557 \cdot 10^{-3}$$

Area externa da bobina:

$Sex_{cu} := 51.73$

$Sex_{cu} = 51.73$

Weight in grams and loss of the core :

$Vol_{fefinal} = 117.3$

$J_{vfe} = 0.15$

$$Wfe_{tot} := \frac{Vol_{fefinal} \cdot J_{vfe}}{1000} \qquad Wfe_{tot} = 0.018$$

Peso do Núcleo em Kg:

$$Pfe_{total} := \frac{P_{fe}}{1000} \qquad Pfe_{total} = 0.614$$

Temperature rise over the ambie

$$\frac{Wcu_{total} + Wfe_{tot}}{0.2 \cdot \left(Pcu_{total} + Pfe_{total}\right)^{\frac{2}{3}}} = 42.998 \qquad \text{Ok}$$

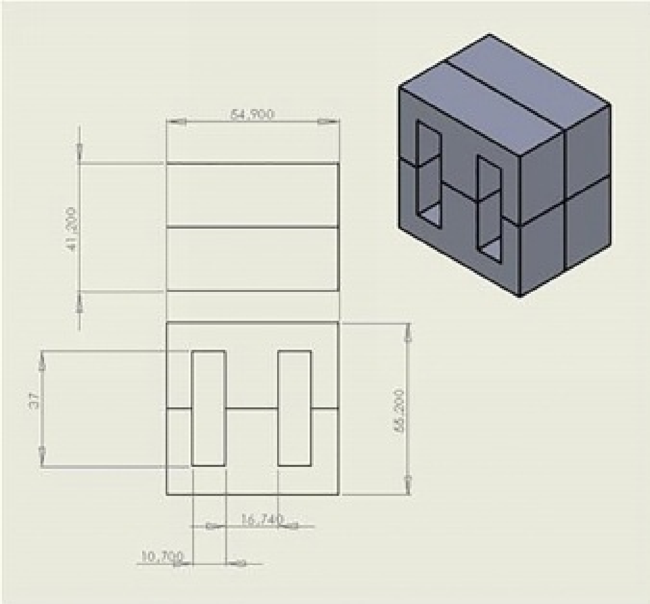


Fig. 125

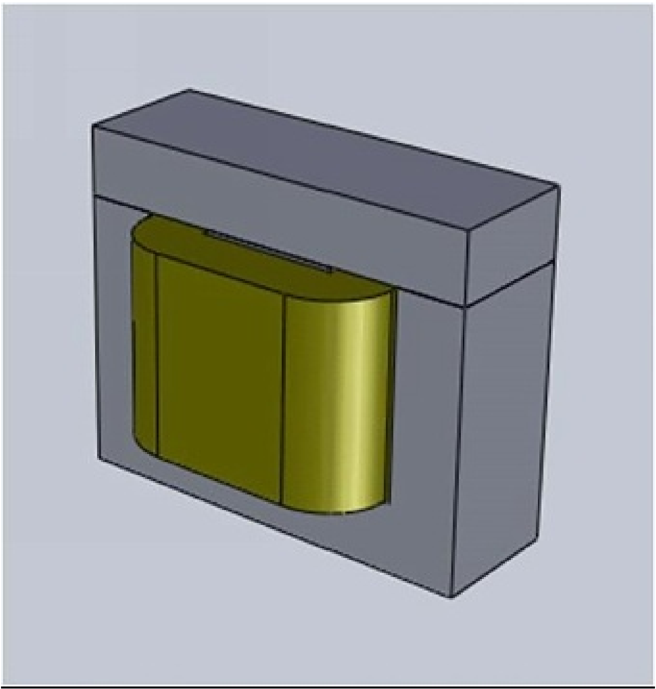


Fig. 126

Fig. 127

Telecommunications power supply of 4000A/52.8 Vdc composed of 40 SMPS of 100/52.8 V the internal link for current sharing and information exchange is done through fiber optics token ring network. It was microcontroller controlled and the concept for reliability was the need of “three simultaneous events” to fail. The power transformer used is that one we have calculated. Project 1995.

The Gap Losses - The approach

The visual effect of the gaps

Let´s consider a core with three gaps with a certain frequency:

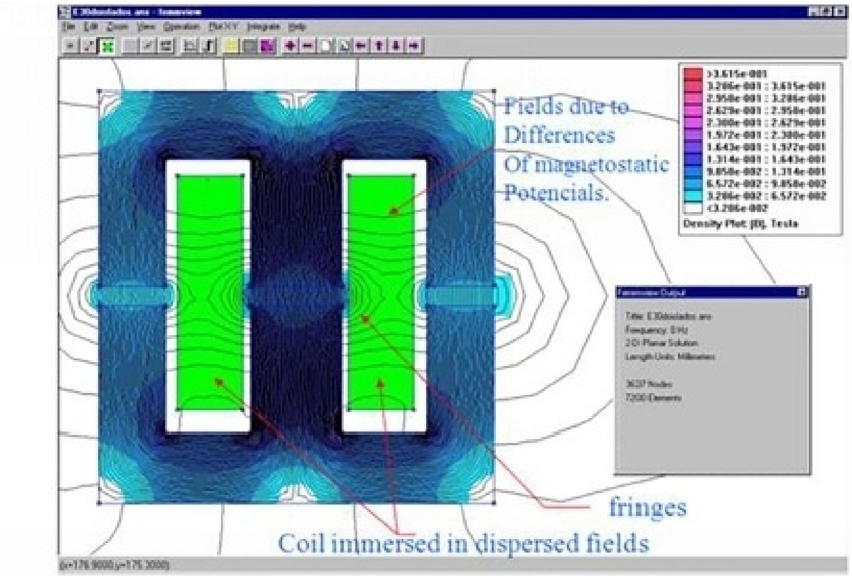


Fig. 128

Vortex of induced currents in a cylindrical conductor due to dispersed fields.

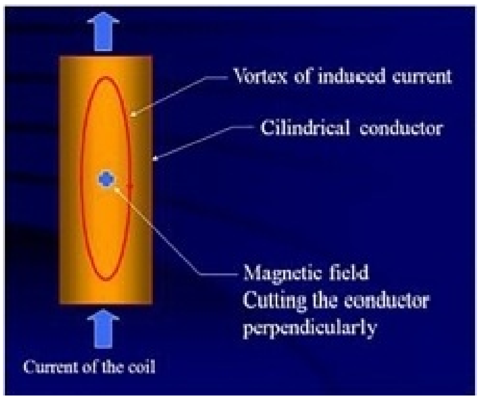


Fig. 129

Loss on a cylindrical conductor subjected to a field perpendicular to its axis

For cylindrical conductors of small diameter d compared with d, the loss by eddy current (proximity effect) due to an ac field amplitude B, perpendicular to the axis of conductor and w frequency is:

Where d is the diameter l is the conductor length and r_c is the resistivity

For the complete winding the total loss is:

$$P_{pc} = \frac{\pi \cdot \omega^2 \cdot F_p}{128 \cdot \rho_c} \cdot l \cdot d^2 \cdot \iint \left|B^2\right| dA = \frac{\pi \cdot \omega^2 \cdot F_p}{128 \cdot \rho_c} \cdot l_t \cdot d^2 \cdot A_u \cdot \overline{\left(|B|\right)^2}$$

Where l_t is the length of one turn, Au is the portion of the window area that is really being used. $\overline{\left(|B|\right)^2}$ is the spatial mean square of the magnetic field in that region and Fp is the packing factor $F_p = \frac{n \cdot N \cdot \pi \cdot d^2}{4 \cdot A_u}$ with N being the number of turns and n the number of conductors per turn. From the equation we can conclude that the loss by proximity effect is proportional to area times the square of the field in that area.

Determination of losses by finite elements:

When we make the magnetic measurement in 2D window using FEMM, we are getting the energy in Joules / m.

We know that the energy stored in the magnetic field is:

$$W = \frac{B \cdot H}{2} \cdot V$$

$$Joules = \frac{B \cdot H}{2} \cdot V = \frac{B \cdot H}{2} \cdot A_c \cdot Compri$$

$$\frac{Joules}{M} = \frac{B \cdot H}{2} \cdot A_c$$

At present, we need the B square average over the area:

$$\frac{Joules \cdot \mu_0}{M} = \frac{B \cdot H \cdot \mu_0}{2} \cdot A_c = \frac{B^2}{2} \cdot Ac$$

How the energy is proportional to the sum of B^2:

$$A_u \cdot \overrightarrow{(|B|)}^2 = \frac{B^2}{2} \cdot Ac \cdot 2 = \frac{B \cdot H \cdot \mu_0}{2} \cdot A_c \cdot 2 = \frac{Joules \cdot \mu_0}{M} \cdot 2$$

We can simply multiply the energy by 2 and mo and we have the product, and in this way, replace it in the equation of loss of the conductor.

$$\frac{\pi \cdot \omega^2 \cdot F_p}{128 \cdot \rho_c} \cdot l_t \cdot d^2 \cdot A_u \cdot \overrightarrow{(|B|)}^2 = \frac{\pi \cdot \omega^2 \cdot F_p}{128 \cdot \rho_c} \cdot l_t \cdot d^2 \cdot \left(\frac{Joules \cdot \mu_0}{M} \cdot 2 \right)$$

Making $\frac{Joules}{M} = Ene$, We have:

$$P_c = \frac{\pi \cdot w^2 \cdot F_p}{128 \cdot r_c} \cdot l_t \cdot d^2 \cdot (Ener \cdot m_0 \cdot 2)$$

Although the differential equations that describe the phenomena are relatively compact,

It is very difficult to get closed solutions, except for extremely simple geometries. Here is where the idea of the finite element, which breaks the complex geometry in a large number of simple design elements.

The advantage of breaking the complex geometry in a large number of simple elements, is that we turn a small problem but difficult to solve in a big problem, but of relatively simple solution. Specifically, triangulation serves this purpose, as it reduces the case to a simple solution of linear algebra, with perhaps tens of thousands of unknowns.

The FEMM uses triangular elements. About each element, the solution is approximated by linear interpolation of the values in the three vertices of the triangle. The linear algebra problem is formed by the choice of solution based on minimizing the total energy.

The calculation of losses in Litz Wires

We designed two cores DIN E30 of a 48V/10A power supply resonant inductor. We designed them in Solidworks, in two versions, one with a central gap, and another with gap on all legs and through DXF we transferred it for FEMM, setting the ferrite core with parameters (magnetic curve, resistivity, etc.) and created a copper block with a current density to establish a similar field H created by the actual reel. The test is magnetostatic 2D (two-dimensional) and the triangulation was made in order to provide us with a precision compatible with our current and simple goals to avoid lengthy calculations.

The energy was measured in FEMM in one of the copper blocks (in green in the figure above) and we 1.0642 * 10-2 Joules per meter, then:

$$\frac{Energia}{m} = 1.0642 \cdot 10^{-2}$$

$$\mu_0 := 0.4 \cdot \pi \cdot 10^{-6}$$

Number of conductors in the Litz Wire:

$n := 13$

Number of turns:

$N := 4$

Area Utilized:

$$A_u := 9 \times 10^{-5}$$

Mean turn:

$$l_t := \frac{7}{100}$$

Angular frequency:

$$\omega := 2 \cdot \pi \cdot 20000$$

Packing Factor:

$$F_p := \frac{n \cdot N \cdot \pi \cdot d^2}{4 \cdot A_u}$$

$$F_p = 0.191$$

Copper resistivity:

$$\rho_c := 1.673 \times 10^{-8}$$

$$P_c := \frac{\pi \cdot \omega^2 \cdot F_p}{128 \cdot \rho_c} \cdot l_t \cdot d^2 \cdot (\mathrm{Ener} \cdot m_0 \cdot 2)$$

The conclusion is that the loss in copper for the 130 conductors #42 litz wire is:

$$P_c = 3.327$$

We will vary the wire gauge keeping the total area of the conductor. All the anterior values will be valid for the present calculation:

$$\acute{A}rea_{tot} := \left(\frac{d}{2}\right)^2 \cdot \pi \cdot n$$

$$\acute{A}rea_{tot} = 4.096 \times 10^{-7}$$

As:

$$\acute{A}rea_{tot} = \left(\frac{d}{2}\right)^2 \cdot \pi \cdot n$$

Arranging:

$$n = 4 \cdot \frac{\acute{A}rea_{tot}}{\left(d^2 \cdot \pi\right)}$$

Ranging the gauge (AWG) from #30 Up to #50:

$$AWG := 30, 31 .. 50$$

$$d_{AWG} := \frac{\frac{25.4}{200}\cdot 92^{\left(\frac{36-AWG}{39}\right)}}{1000}$$

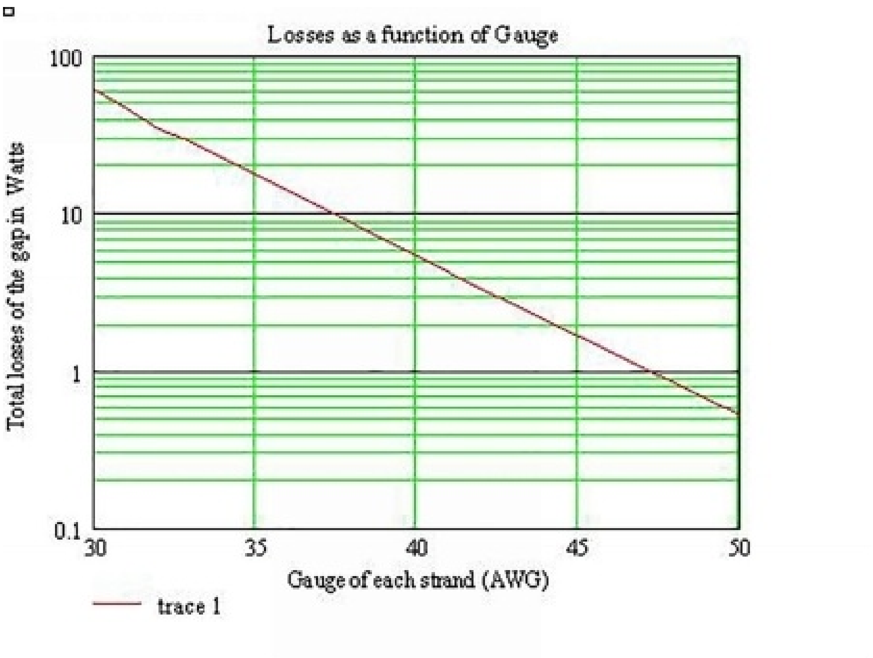
$$n_{AWG} := \mathrm{ceil}\left[4\cdot\frac{Area_{tot}}{\left(d_{AWG}\right)^2\cdot\pi}\right]$$

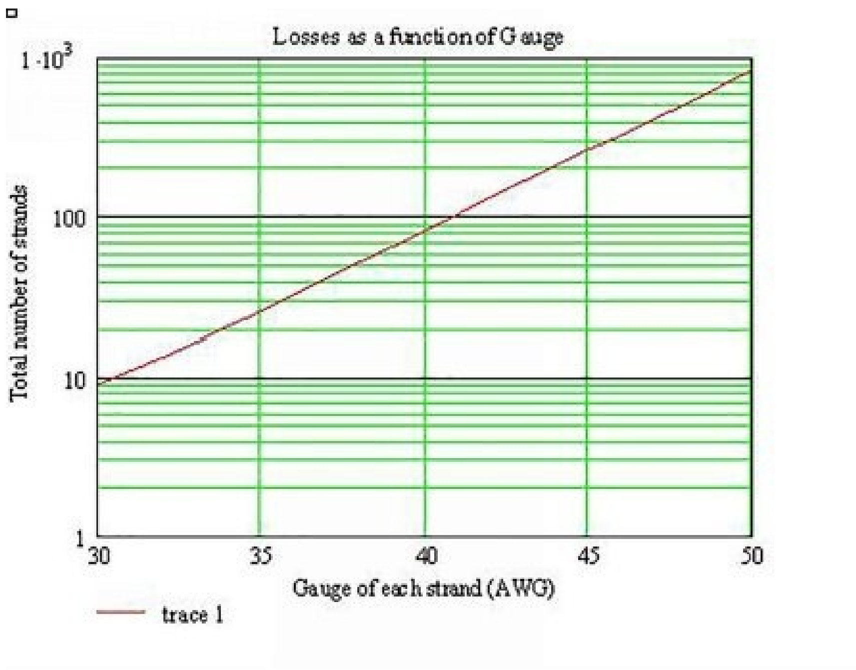
Packing Factor:

$$Fp_{AWG} := \frac{n_{AWG}\cdot N\cdot\pi\cdot\left(d_{AWG}\right)^2}{4\cdot A_u}$$

$$Pc_{AWG} := \frac{\pi\cdot w^2\cdot Fp_{AWG}}{128\cdot\rho_c}\cdot l_t\cdot\left(d_{AWG}\right)^2\cdot\left(Ener\cdot\mu_0\cdot 2\right)$$

Plotting:





Conductor Diameter:

Like defined in the American Wire and Gauge the diameter of the wire in accordance with the AWG number in inches is:

$$d = \frac{1}{200} \cdot 92^{\left(\frac{36 - AWG}{39}\right)}$$

To get in millimeters multiply by 25.4:

$$d = \frac{25.4}{200} \cdot 92^{\left(\frac{36 - AWG}{39}\right)}$$

To get in meters, like our present need:

$$d := \frac{\frac{25.4}{200} \cdot 92^{\left(\frac{36 - AWG}{39}\right)}}{1000}$$

Some other considerations were made, but the most interesting is to prevent the occupation of area where the borders (most intense fields) are more frequent. With this objective in mind we will have a staggered winding wire of litz wire conductors 130 # 42 more or less as follows:

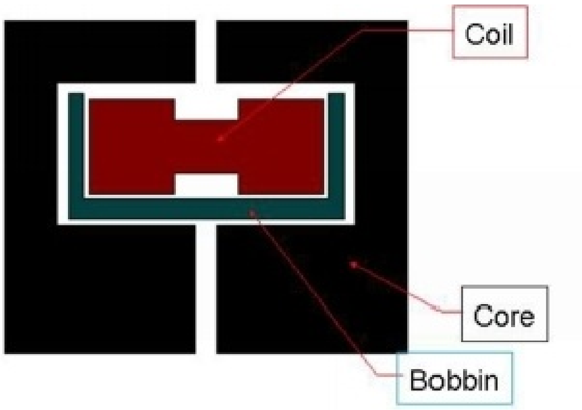


Fig. 132 -

Another approach of inductors is the confrontation of the high reluctance regions nearby the gaps. This effect combined with the skin effect produces a result where the winding configuration with respect to the gap position is very important. In the following figures we can observe the high current regions with curled plates.

The following sequence of figures will give us an idea of this effect.

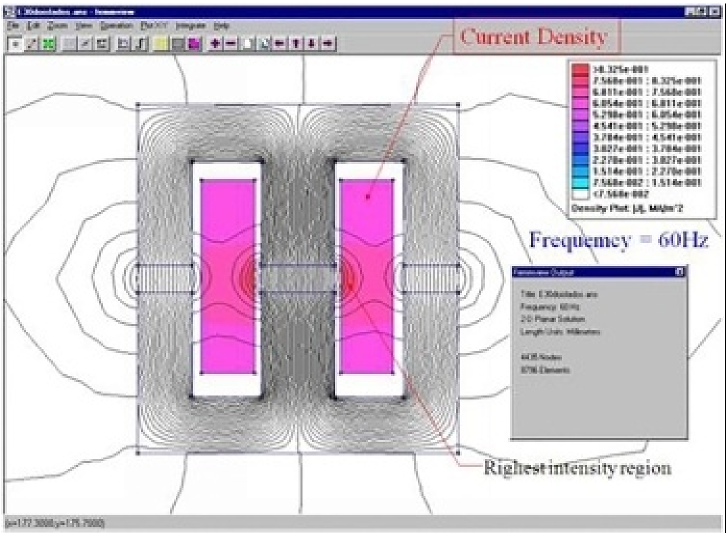


Fig. 133 DEisperse field at the frequency of 60Hz

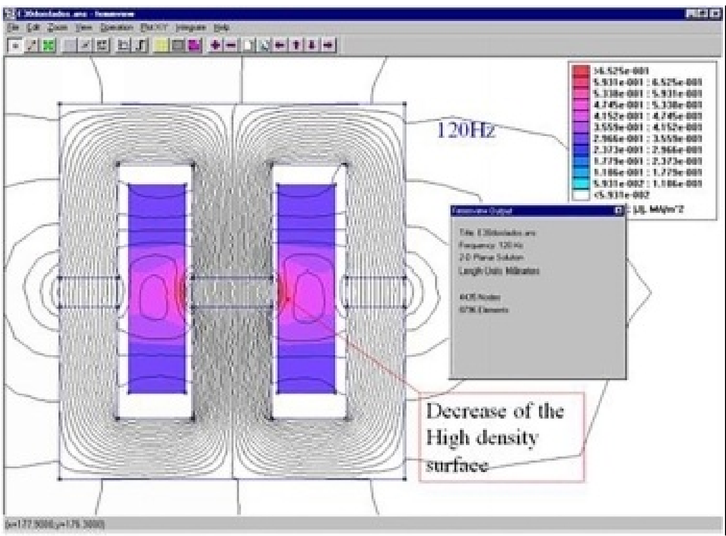


Fig. 134 Frequeny at 120Hz

Note that with increasing frequency, there was a much more apparent decrease in high current density regions. Getting completely clear the effect of high reluctance of the gap in combination with the skin effect decreases the high density region.

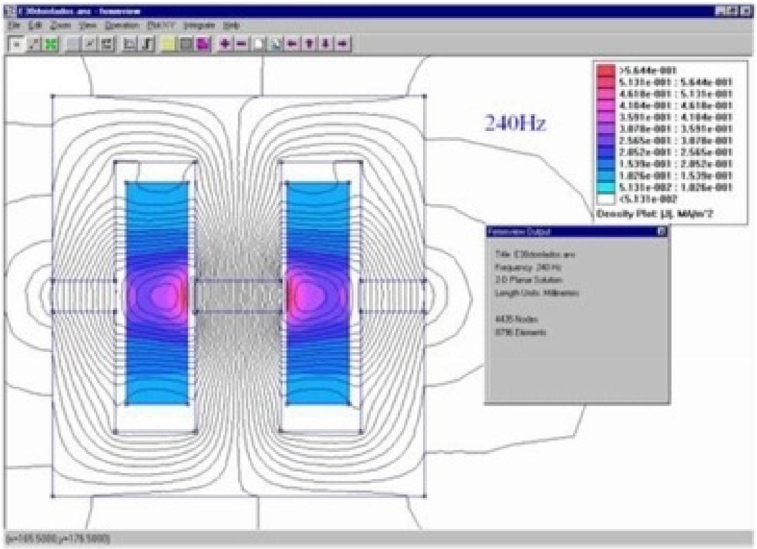


Fig. 135 Frequency at 240Hz

Now For 1000 Hz:

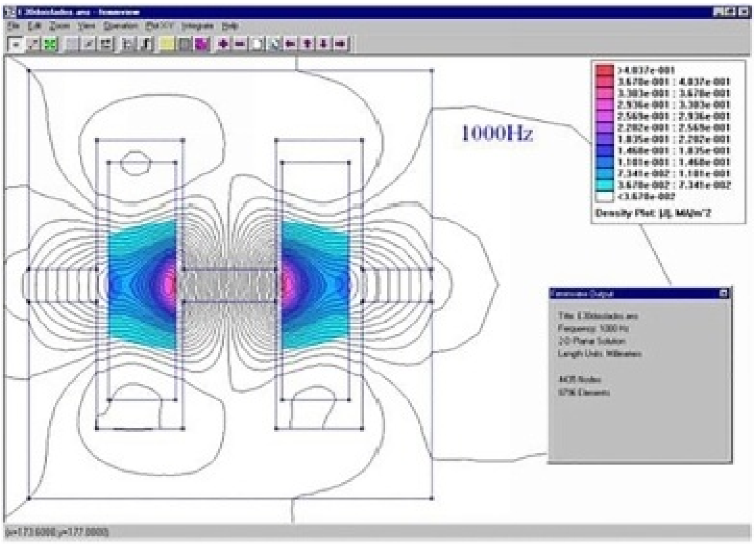


Fig. 136 Frequency at 1000Hz

Effect for 100Khz:

Note in the figure the thin area around the high reluctance of the gap region

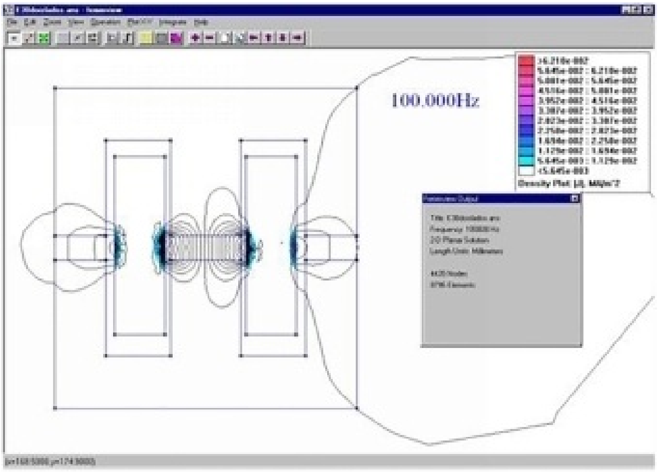


Fig. 137 Frequency at 100.000 Hz

Changing the position of the winding:

Returning to 10,000Hz and shifting the position of the winding, we can see the highest density film forming toward high reluctance region. Likewise, it can be noted the effect of a spreading reluctance. This is due to the reluctance behavior, spreading the extent to which departs. Behaving in different ways exactly like a light source

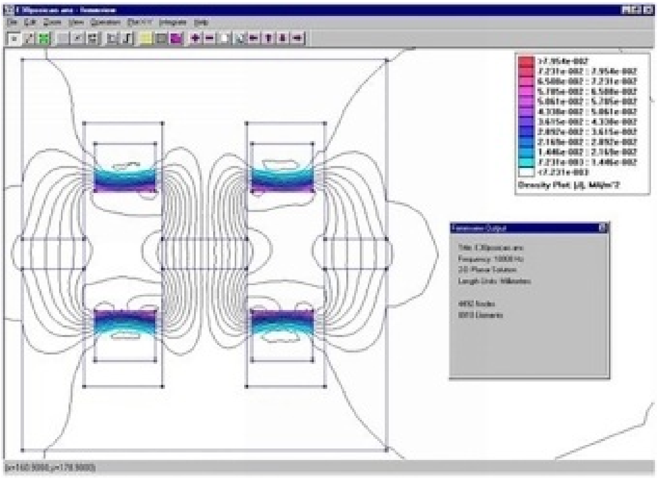


Fig. 138

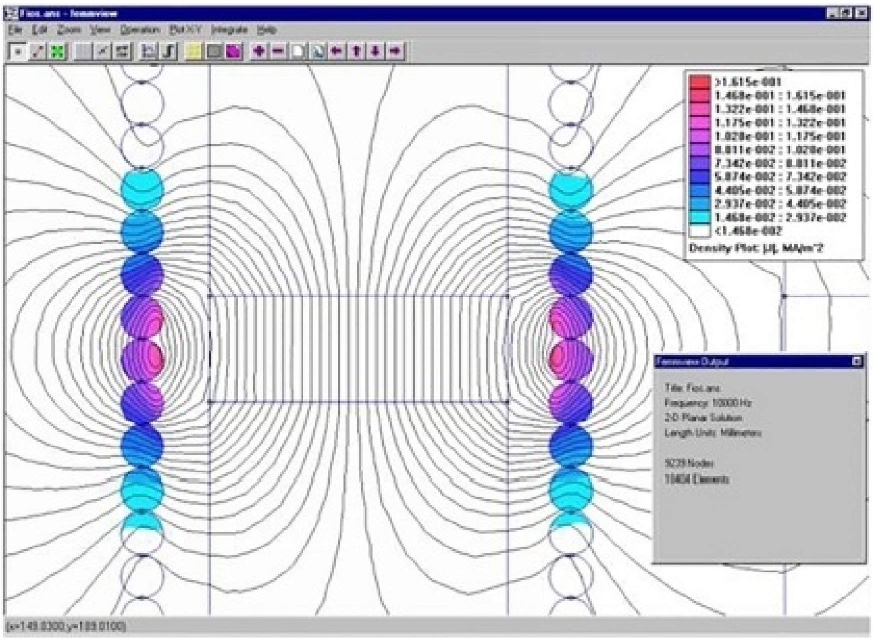


Fig. 139

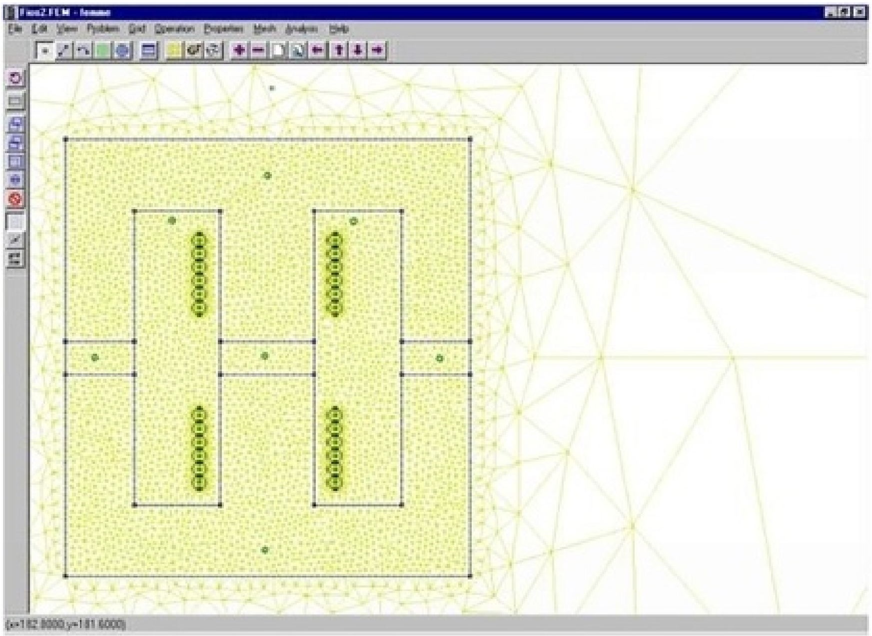


Fig. 140

Now at 1000 Hz:

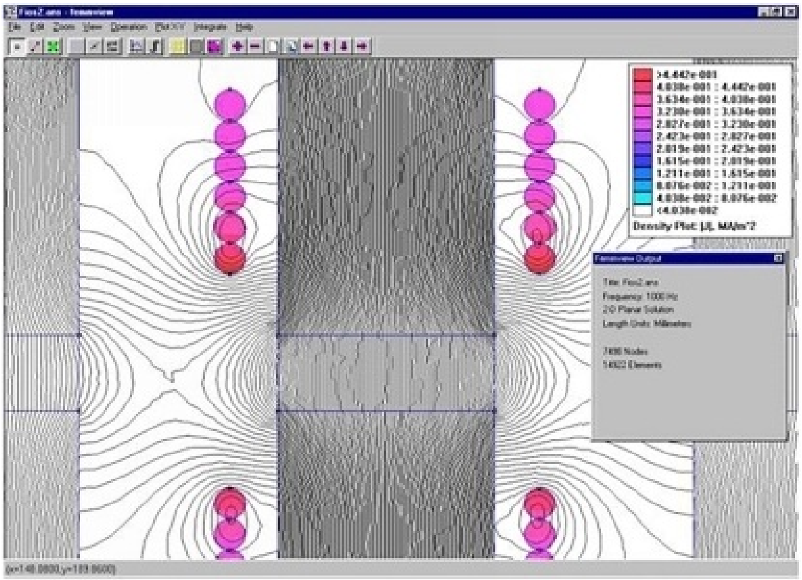


Fig. 141

At 10000 Hz.

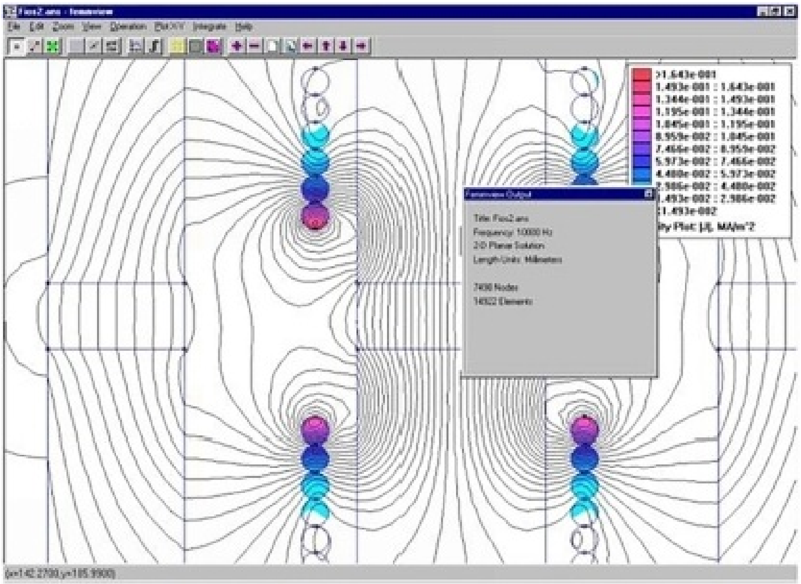


Fig. 142

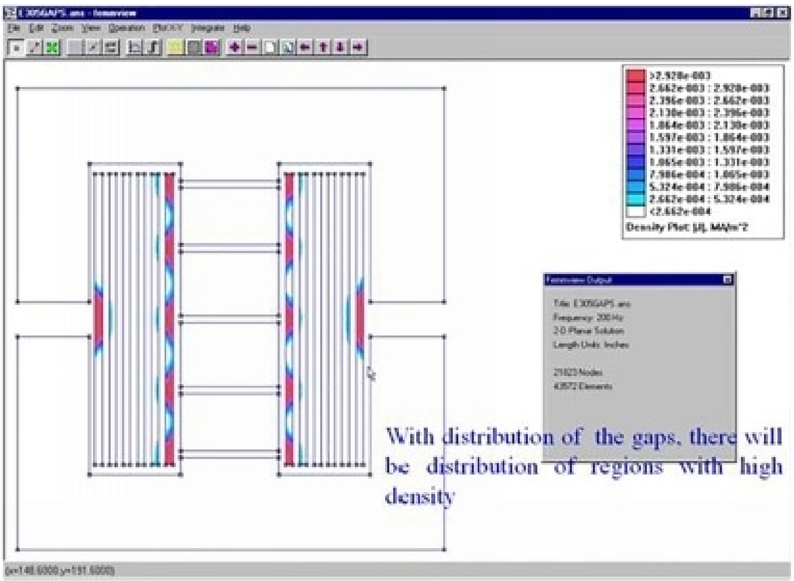


Fig. 143

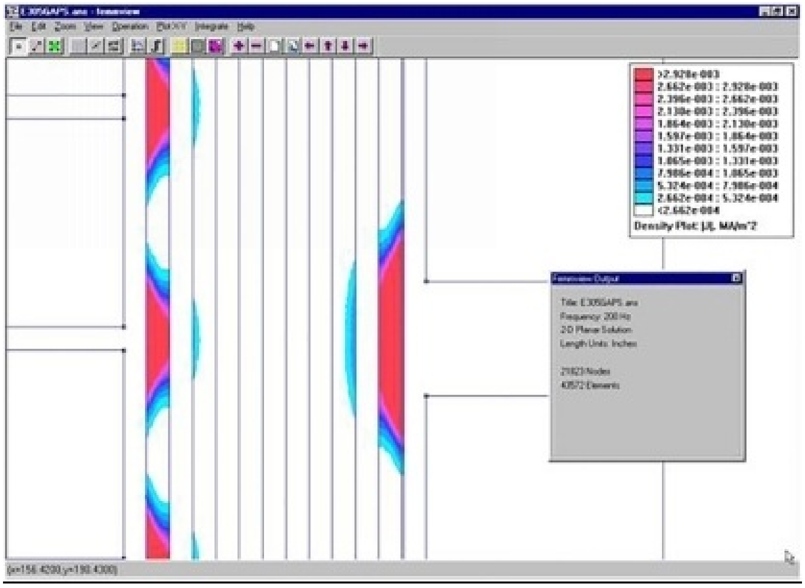


Fig. 144

With the move away of the conductor, we can see the blurring of the high current density region. This makes more evident the divergence of the effects of high reluctance region, behaving as a light source.

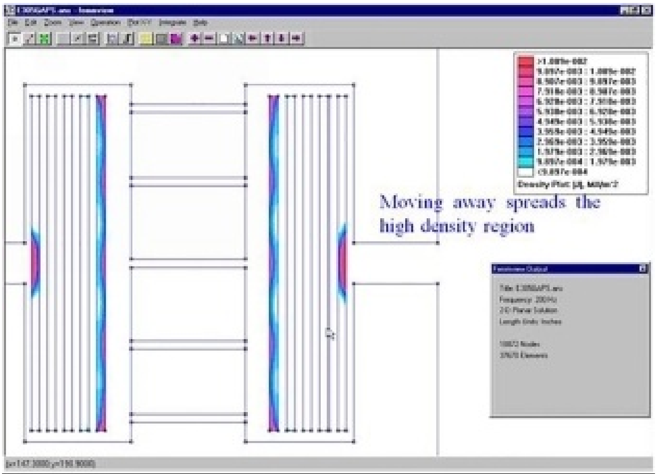


Fig. 145

We can see in the figure, that if we move away the conductor to an appropriate distance, we have the high current density region distributed along the width of the bar, using it in a far more interesting way. This is due to what is called the blurring of reluctance. It is about a gap reluctance of a certain myopia effect.

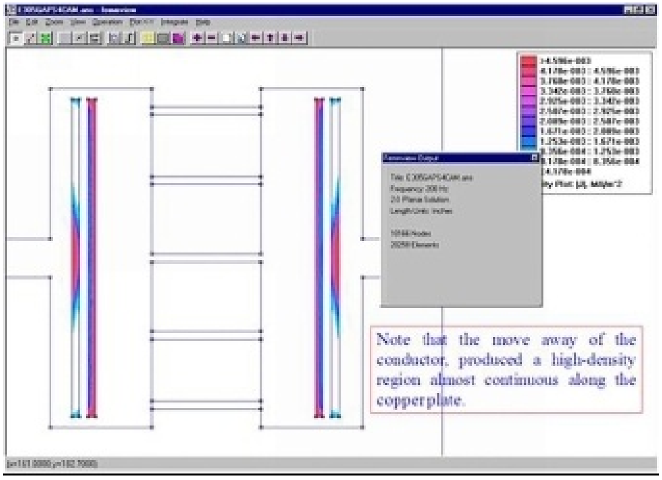


Fig. 146

Results of the observation:

We had the opportunity to observe in the previous sequence that the winding position on the high reluctance regions is very important for the current distribution along the conductor.

Skin effect, the thickness of the gap and the gap clearance distances between the conductor and the core length should be considered,

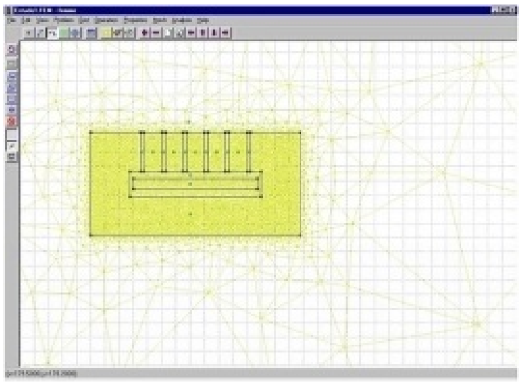


Fig. 147

Using FEA we can adjust the distances trying to get a good diffusion of current in the copper sheet.

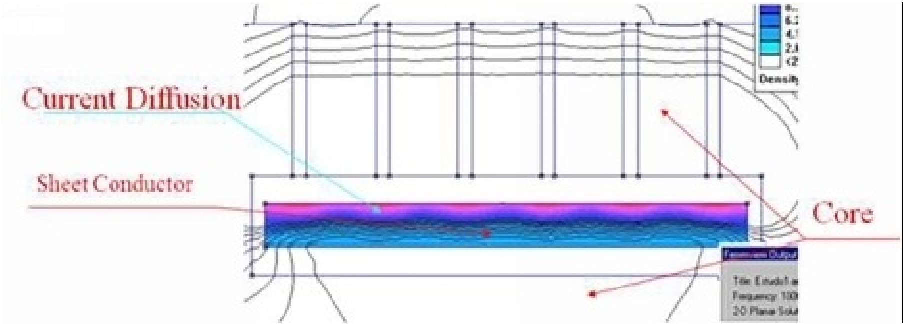


Fig. 148

This model has been created in FEMM, departing from the normalization of dimensions as a function of the skin depth of the frequency under test.

Sullivan and Jinkun [10] investigated this phenomenon and through simulation plus curve fitting, they determined a relationship that with a good precision for practical purposes, describes its behavior.

They used five basic dimensions as can be seen from the figure.

It can be seen that with the movement away, there was a greater distribution of the surface, thereby increasing the higher density regions reducing the overall resistance of the winding.

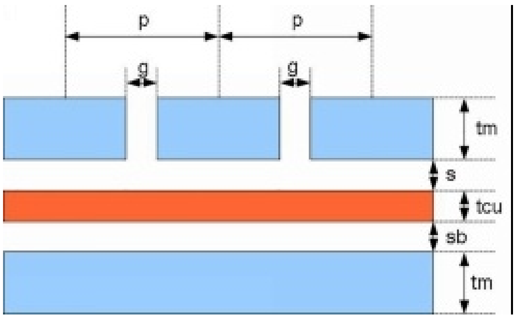


Fig. 149

The framework was reduced through a simplification of the structure of the problem, doing a infinite number of gaps distributed in an infinite length sheet. Thus the problem can be analyzed in one of those gaps, considering the boundaries of the core to be infinite. With those assumptions the number of variables was reduced for 3, The gap pitch p, the gap length g, and the spacing between the sheet and the gap. The thickness of the core are not important, so as the distance between the copper sheet and the botton part of the cores due obviously to the fact that being the reluctance of the core low, the diffusion of the current in this region of the copper sheet tends to be zero. Unless there is a possible DC component.

The parameters that were changed during the simulation process were p, g and s because the width of the copper sheet was maintained constant in two skin depth due to the fact that thicker conductor could only contribute for the DC component.

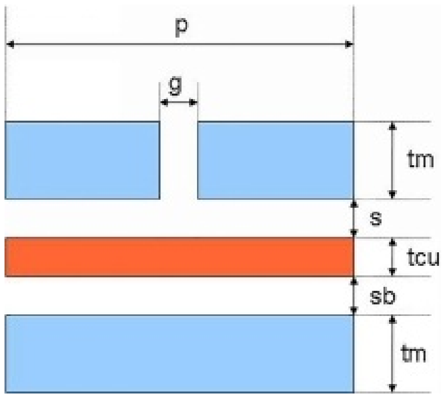


Fig. 150

The g parameter was maintained constant also to keep the simulation easier, but also because if we change p we are automatically changing g.

The simulations results were plotted in curves giving resistance factors Fr, for various values of p and s in a 3 D surface like shown in the figure xx.

$$F_r = \frac{R_{ac}}{R_{dc}}$$

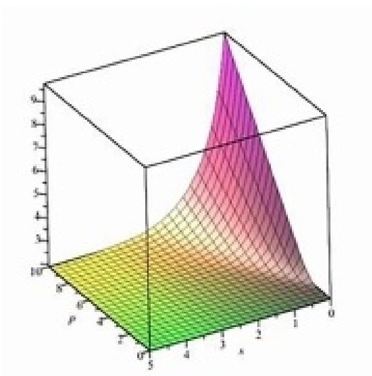


Fig. 151

The curves plotted by finite elements simulations, were used to curve fitting and the result was an analytical approximation formula that facilitates calculation due to the without need to use tables or everything else to design quasi distributed inductors.

$$Fr(s,p) := \frac{k}{\left(b^{n}+p^{n}\right)^{\frac{1}{n}}} + k \cdot p + 1.9;$$

Where:

$$n := 5.4;$$

$$k := \frac{0.95}{0.95 - 1.4 \cdot s}$$

$$b := 3.3 \cdot s + 2.14;$$

Like informed by Sullivan and Jinkun in [10] the expression approximation and the simulation presented an error less than 4.5% and an absolute error in Fr less than 0.08. And that the expression is maintained accurate even with large values of s and p.

For large s, k and b become:

$$K := \frac{0.68}{s}$$

$$b := 3.33 \cdot s;$$

And the expression is rewritten to:

$$Fr := \frac{0.68}{\left(3.33 + \left(\frac{p}{s}\right)^{n}\right)^{\frac{1}{n}}} + 0.68 \cdot \left(\frac{p}{s}\right) + 1.9;$$

If the ratio of $\frac{p}{s}$ is much greater than 3.33, the equation will enter a roughly linear region and it can be simplified as:

$$Fr = \frac{0.68 \cdot p}{s} - 0.36$$

By the other side, if the ratio of $\frac{p}{s}$ is much smaller than 3.33 the equation is enter in a constant region with Fr=1.9.

We can get good results and get Fr near that one obtained in distributed inductors (Fr = 2.5) if we maintain the following conditions:

$$\frac{P}{s} < 4$$

$$P < 2.5 \times d$$

If the gap length is high, it can also reduce the ac resistance. But it needs to be kept substantially low when compared to the gap pitch.

Like initially presented to us, this problem is first of all a geometric puzzle and the guidelines were presented. The solution for the rest of the problem is like the others.

The End Effect

Another kind of scattered field inside window is the “End Effect” that is the responsible for the heat of the ends of coil in transformers. This will be studied in volume 2.

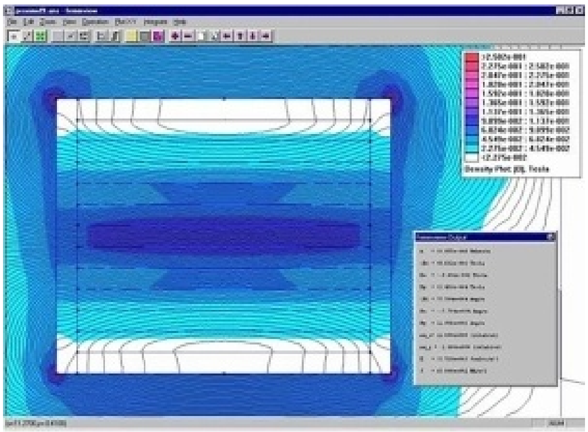


Fig. 152 There are perpendicular H lines in the coil

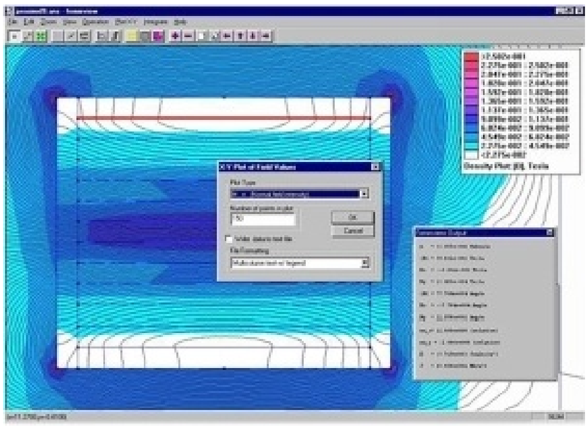


Fig. 153 The red line at the top is draw for the calculatio of the normal elements.

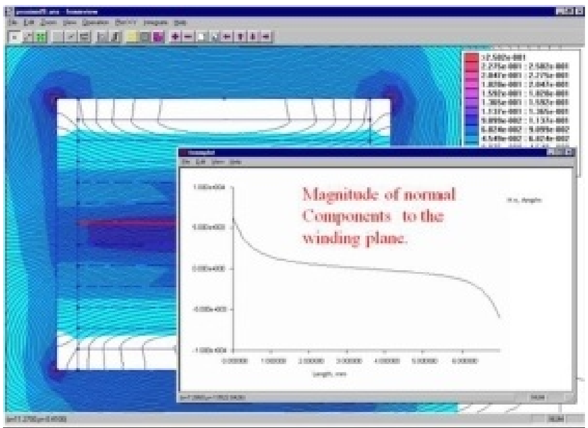


Fig. 154 The values of normal elements

The problem of the size

The size of a transformer is constrained by many factors: Temperature, power loss, cost, size, Induction, voltage regulation, Impedance, Inductance, harmonics and others.

The determination of the dimensions of the transformer is a quite complex task, because there are infinite possibilities of dimensions and also infinite relations between them.

The definition of the equation that determines the flux density B, departing from frequency, voltage and magnetic cross sectional area of the core:

First of all, we need to determine the relation between voltage and flux density because this is the dominant factor of a transformer:

In Section I we have defined the relation between a constant voltage and the evolution of the flux density for a determined number of turns, cross sectional area, voltage and time. The relation is shown in the block diagram of simulink below:

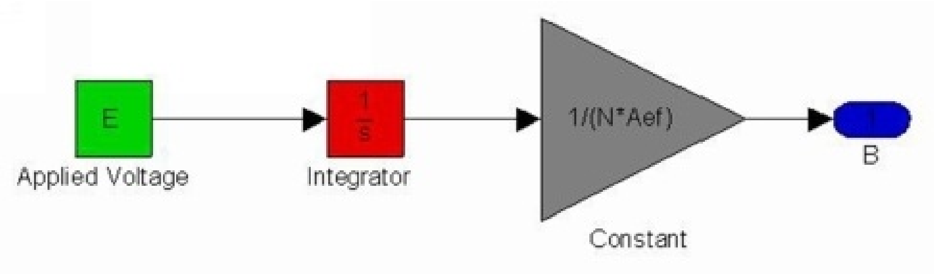


Fig. 155

Let us consider now for the sinusoidal case. We have the model for Constant voltage applied at the input, and we are going to employ it in the same dynamic system at the present case. We need change the excitation only:

$$E \times \cos(w \times t)$$

$$E \times \frac{s}{\left(s^2 + w^2\right)} \times \frac{1}{s} \times Const$$

$$E \times \frac{Const}{w} \times \sin(w \times t)$$

$$B_{max} \quad \text{for} \quad \sin(w \times t) = 1$$

$$w = 2 \times p \times f$$

$$e_{rms} \times \sqrt{2} \times \frac{1}{N \times A_{ef} \times f \times 2 \times p}$$

Making:

$$\frac{2 \times p}{\sqrt{2}} = 4.443$$

$$B_{max} = \frac{e_{rms}}{4.44 \times N \times A_{ef} \times f}$$

With this formula above we can calculate the number of turns of the transformer:

Aef - cross sectional area of the core:

Bmax – Amplitude of the flux density.

F – Frequency of operation.

erms – RMS voltage of the coil or applied to it..

4.44 - constant derived from 4*ff where ff (form factor) means RMS/Avg of one determined waveform. For instance sinusoidal ff=1.11. Then 4*1.11. For square wave ff=1 then 4*1=4; For other waveform we need to determine ff and multiply by 4.

There are many possibilities of design, leading to different results of costs, size and performance

The control of the several factors that governs the transformers is of fundamental importance in the design of such apparatus.

First off all we need to have guidelines concerning certain parameters that mainly define the overall specifications of a transformer. Among many, the main constraints are: Power, Temperature, Flux density, Magnetic cross sectional Area, Copper window area, frequency and current density in copper.

There are also many different technologies and applications: High, medium and low power transformers, High frequency, High frequency medium power, planar high frequency, fly back, forward half pulse and further huge universe of possibilities, being each one a particular scenario of vast possibilities.

Even in power transformers we have many possibilities in the design. Composite laminates, cruciform cores, number of steps, 90 degree cutting, 45 degree cutting and so forth,.

We will begin with the magnetic cross sectional area Aef. We have the working frequency and the max flux density. Thus considering that our transformer have one turn, there will be a voltage in it. If besides we imagine that our one turn winding is a block of copper with same area of the entire coil of the primary (Or secondary). You must hypothetically consider the individual cross sectional area of the copper multiplied by the number of turns. This performs the total useful copper area.

Considering the figure nest page, there we have a transformer window full of windings of copper wire.

The space (Area) occupied by the copper is naturally less than the total window. Thus we have a occupation factor Kjan that is called "Space Factor" of the copper in the window.

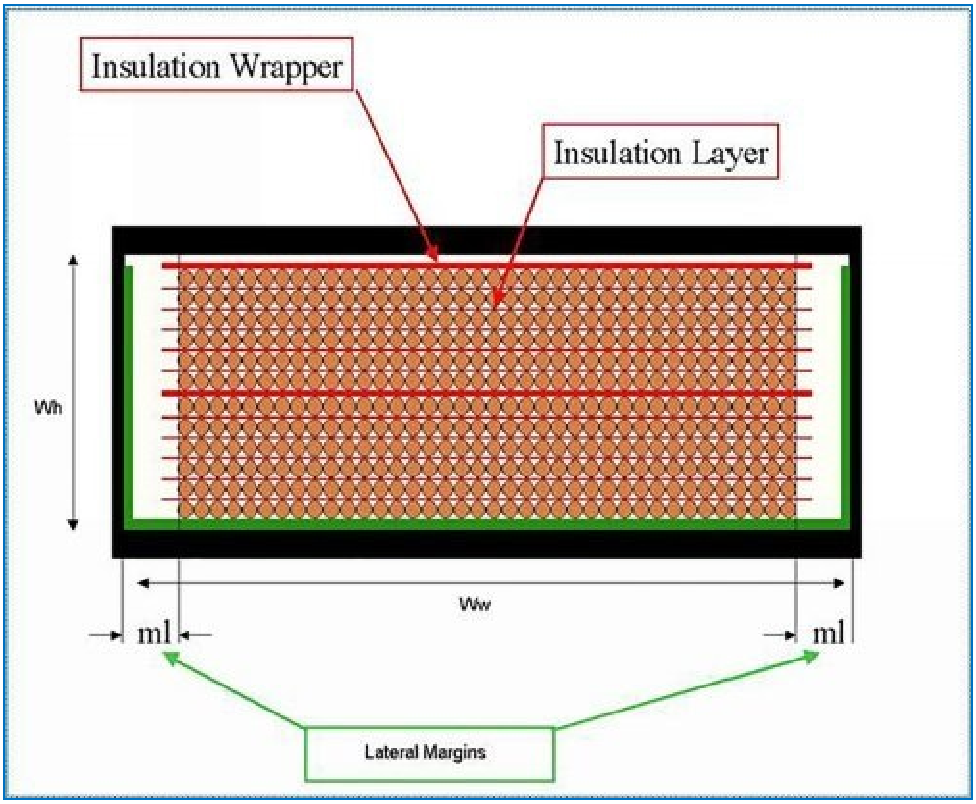


Fig. 156

First of all we will determine the formulation about the geometry of an EI transformer then we will be calculating the temperature of a transformer in order at the end determine the size and the main parameters.

Using our study, let us calculate a input transformer for a UPS of 80Kva.

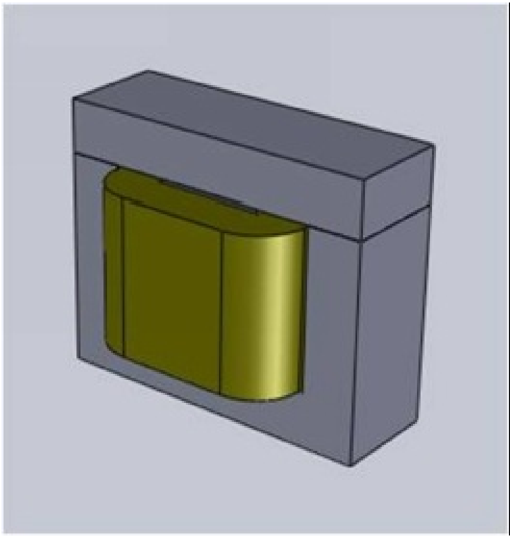
We need to determine the lowest cost and the optimum relations for a and b, thus determining the best conditions for a three phase transformer using right angle composite lamination.

For the EI transformer we must first to consider the most common normalized relation of EI lamination. There are other values that can be analyzed for each particular case.

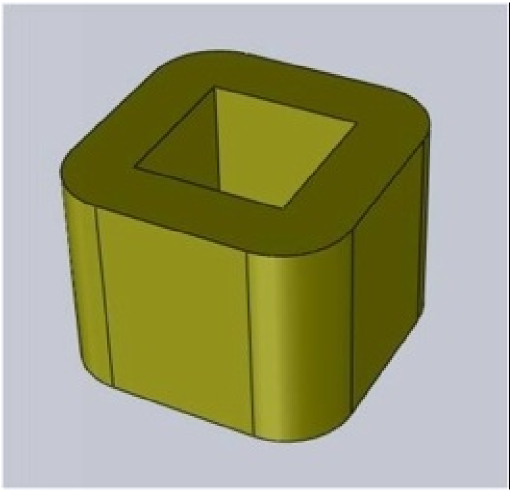
In the formulation we are treating them in normalized form giving space to introduce the factors for other standards.

EI Core transformer:

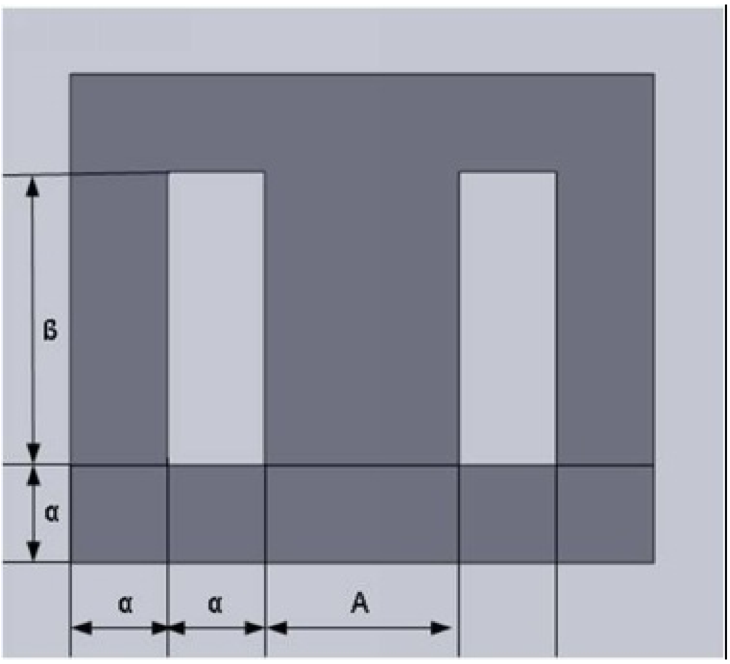
Calculating over a single Phase EI lamination transformer



The Coil



Defining the core normalized dimensions



Length of the window:

$$lw := \beta \cdot A$$

Height of window Area:

$$Hw := \alpha \cdot A$$

Core Window Area:

$$Aw := lw \cdot Hw$$

Core Width:

$$lcore := A \cdot (1 + 4 \cdot \alpha)$$

Core Height:

$$Hcore := A \cdot (\beta + 2 \cdot \alpha)$$

Lamination Surface:

$$Slam := lcore \cdot Hcore - 2 \cdot Aw$$

$$A^2\,(\beta + 2\,\alpha)\,(1 + 4\,\alpha) - 2\,\beta\,A^2\,\alpha$$

Stack:

$$Stack := 0.95 \cdot \xi \cdot A$$

Volume of the core

$$Volfe := Slam \cdot Stack$$

$$0.95 \left(A^2 (\beta + 2\alpha)(1 + 4\alpha) - 2\beta A^2 \alpha\right) \xi A$$

Iron core density:

Dfe:=7.65

Core weight in Kilograms:

$$Pfe := \frac{(Dfe \cdot Volfe)}{1000}$$

$$0.0009500000000\, Dfe \left(A^2 (\beta + 2\alpha)(1 + 4\alpha) - 2\beta A^2 \alpha\right) \xi A$$

Magnetic Cross Section:

$$Aef := 0.95\, \xi \cdot A^2$$

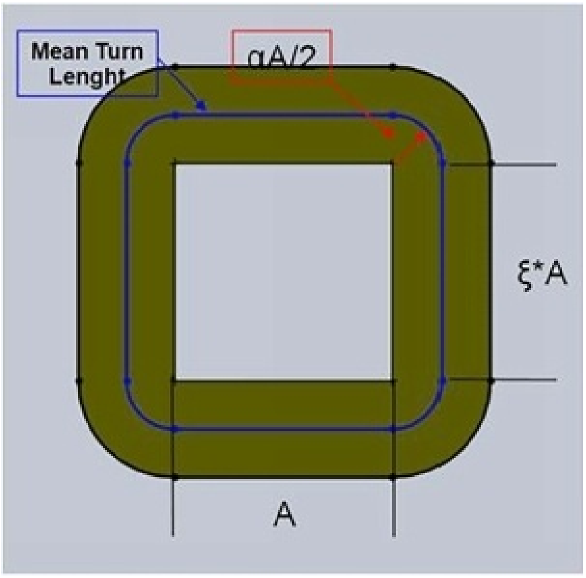


Fig. 157

Mean turn length:

$$lcuAvg := 2 \cdot A \cdot \left(1 + \xi + \frac{(\pi \cdot \alpha)}{2}\right)$$

Volume of Copper (Primary and secondary together):

$$Volcu := Aw \cdot Kjan \cdot lcuAvg$$

Density of Copper (Reference Value)

$$DensRef := 8.96$$

Weight of copper:

$$Pcu := \frac{(Denscu \cdot Volcu)}{1000}$$

$$0.005540000000\, \beta\, A^3\, \alpha\, Kjan \left(1 + \xi + \frac{1}{2} \pi\, \alpha\right)$$

Current Density:

We are defining Di as current density in (Amper’s)/Cm^2:

The current of coil is:

$$Ienr := \frac{Aw \cdot Kjan \cdot Di}{2}$$

As we have a winding of one turn, the voltage in the winding is:

$$Erms := 4.44 \cdot F \cdot Bm \cdot Aef \cdot 10^{-8}$$

$$4.218000000\,10^{-8}\,F\,Bm\,\xi\,A^2$$

We can determine the total power of this transformer as:

$$Pot.tot := Ienr \cdot Erms$$

$$2.109000000\,10^{-8}\,\beta\,A^4\,\alpha\,Kjan\,Di\,F\,Bm\,\xi$$

As an example, we can calculate the cross sectional effective area, departing from the power of the transformer, defining the lamination to be used:

Solving for $A^2$and making x =1, we get a so used calculation of the effective area departing from the power of the transformer:

$$Aeff := \mathrm{sqrt}\left(\frac{Po \cdot 0.95^2}{2.109000000\,10^{-8} \cdot \beta \cdot \alpha\,Kjan\,Di\,F\,Bm}\right);$$

Considering that for general EI proportions we have:

$$Aef := 6541.620043\sqrt{\frac{Po}{\beta\,\alpha\,Kjan\,Di\,F\,Bm}}$$

Only in another form:

$$Aef := 6541.620043\sqrt{Po}\,\sqrt{\frac{1}{\beta\,\alpha\,Kjan}}\,\sqrt{\frac{1}{Di\,F\,Bm}}$$

Making:

$\alpha := 0.5;$

$\beta := 1.5;$

$Kjan := 0.421;$

$Bm := 10000;$

$Di := 200;$

$F := 60;$

$$Aef := 1.062730176 \times \sqrt{F}$$

$$Aef := 1.063 \times \sqrt{F}$$

Now we can calculate the resistance of the coil:

$$Rcoil := \frac{(\rho \cdot lcuAvg)}{Aw \cdot Kjan}$$

The total copper losses can be calculated easily:

$$Wcu := (2 \cdot Ienr)^2 \cdot Rcoil;$$

The Leakage Inductance:

For our present interest the leakage inductance is for the number of turns=1, then:

$$Lleak := \frac{0.4 \cdot \pi \cdot 10^{-8} \cdot \pi \cdot dm}{lw} \cdot \left(\frac{0.75 \cdot \alpha \cdot A}{3} + 0.016 \cdot 0.75 \cdot \alpha \cdot A \right)$$

$\mu o := 0.4 \cdot \pi \cdot 10^{-8};$

$dm := \frac{lcuAvg}{\pi};$

By normalized parameter, we find:

$$Lleak := \frac{2.096000000\,10^{-9}\,\pi\,A\left(1+\xi+\frac{1}{2}\pi\,\alpha\right)\alpha}{\beta}$$

The reactive power in the leakage inductance in Vars is:

$$Var := (2\cdot Ienr)^2\cdot 2\cdot\pi\cdot F\cdot Lleak$$

Iron Losses:

The iron losses, depends on the flux density and frequency of use:

For a given condition we get the information from the curves of the magnetic core, the loss is in watts per kilogram:

DenspPferro (Loss density of the iron core).

Now we can get the losses in the iron:

$$Wfe := Pfe\,DensPferro$$

The temperature calculation formula is derived from a general study for temperature discussed in “Loss and weight approach” in “The problem of Temperature” in the Second Volume of this book.

The average temperature is calculated from:

$$\Delta t \quad \frac{(Wfe + Wcu)}{0.162\cdot(Pfe + Pcu)^{\frac{2}{3}}}$$

We can also determine the temperature from calculating the specific dissipation in watts/cm^2 in the area in blue, shown in the figure below.

The graph of next page can be used to determine the final temperature.

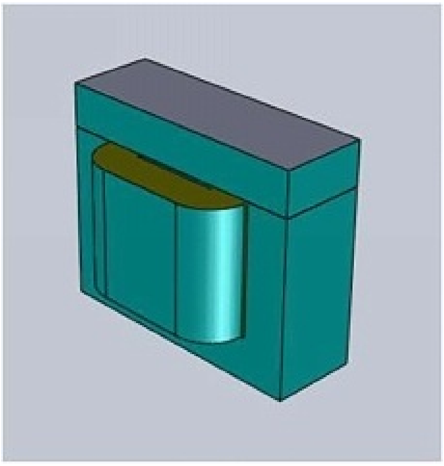
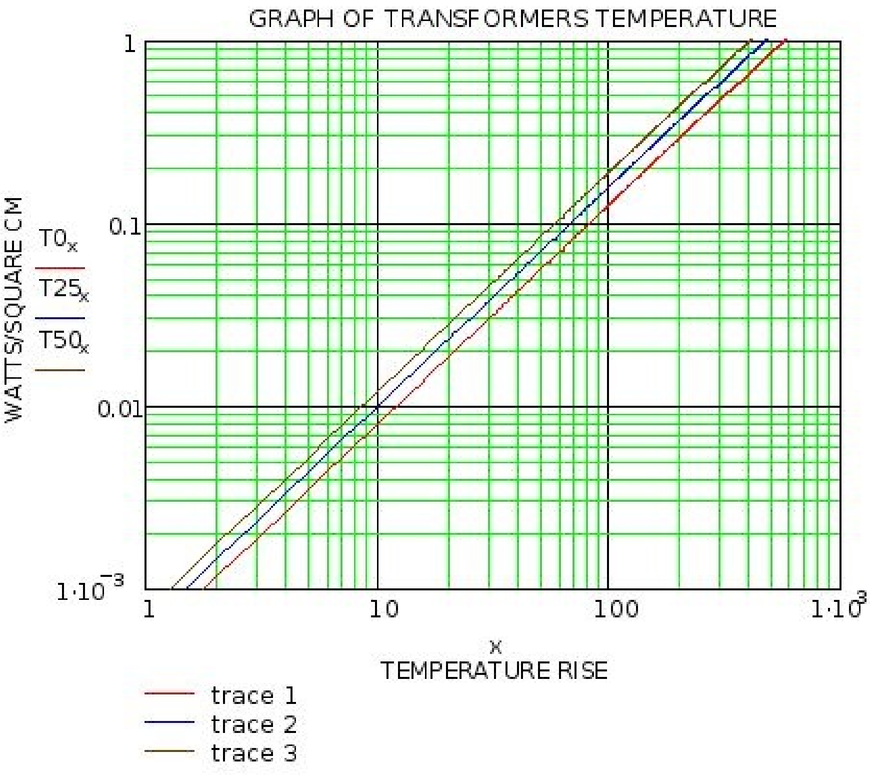


Fig. 158 The Dissipation areas of the transformer

They are the vertical areas used for heat dissipation.

Normally the areas that take part of the dissipation process are the vertical ones that work in the air convection, which is the major part of the cooling process.



T0,T25 and T50 are graphs for ambient temperatures of 0, 25 and 50 ° C respectively.

Making:

$\alpha := 0.5$

$\beta := 1.5$

$\xi := 1$

$Kjan := 0.53$

$Dfe := 7.65$

Deriving from t we can calculate de current density required for that temperature:

$$isolate\left(\Delta t = \frac{Wfe + Wcu}{0.162\,(Pfe \quad Pcu)^{\frac{2}{3}}}, Di^2 \right);$$

$$Di^2 = \frac{1}{A^3\,\rho\,(2 - 0.2500000000\,\pi)} \Big(1.583531275 \big(0.1620000000\,\Delta t\,(0.04360500000\,A^3 - 0.005658210000\,A^3\,(2 + 0.2500000000\,\pi)\big)^{2/3} - 0.04360500000\,A^3\,DensPferro\Big)\Big)$$

At last, we define the current density:

$$Density := 0.001224744871\,\sqrt{-\frac{-0.9343701700\,\Delta t\,(A^3)^{2/3} + 1.652663334\,A^3\,DensPferro}{A^3\,\rho}}$$

Now we will plot two graphs of Di as a function of the central lag width. One for A Class (105 ° C) and the second one for B Class (130 ° C).

Curve for A class (105 ° C) transformers:

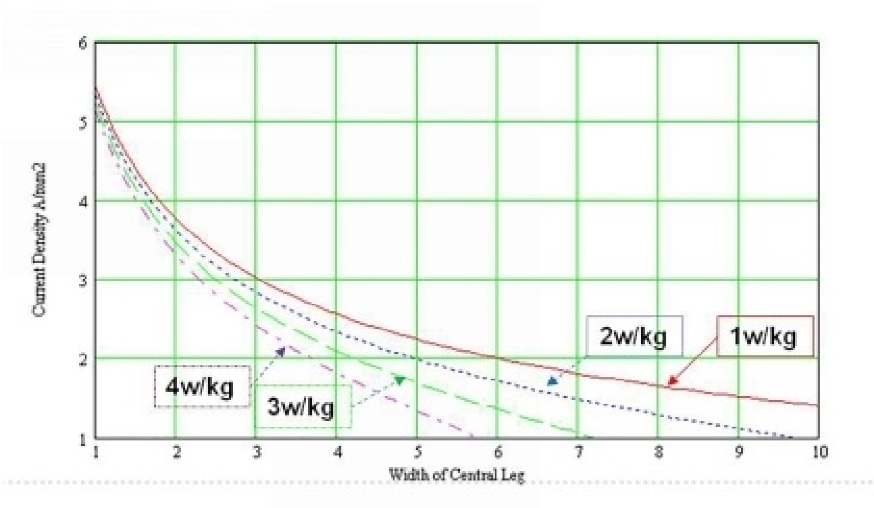


Fig. 159Current density calculation from central leg Class A (105 ° C)

The parameters of the curves are for 1Watt per Kilogram, 2Watts per Kilogram, 3Watts per Kilogram and 4 Watts per Kilogram . In the abscissa we have the central leg of the “E” lamination and in the ordinate the current density.

Curve for B class (130 ° C) Transformers:

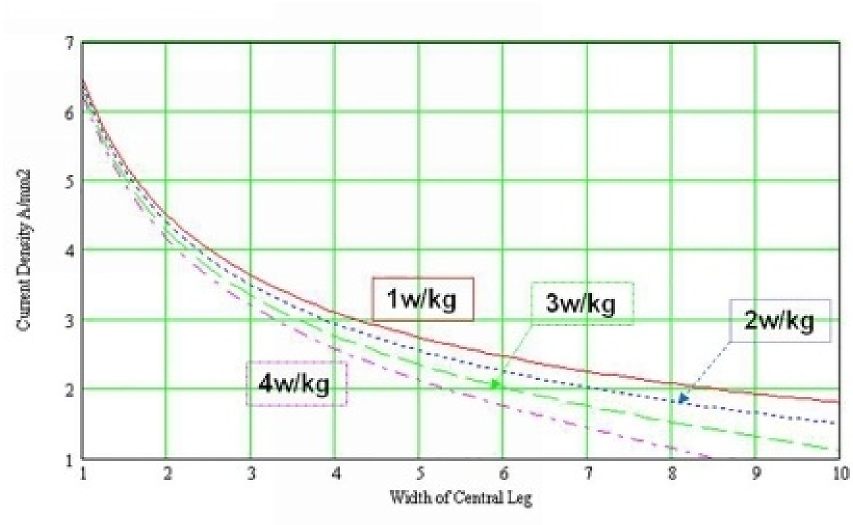


Fig. 160Current density calculation from central leg Class B (130 ° C)

Now we need to calculate the percentage of copper losses:

$$Perc\_of\_Culoss := \left| 100 \cdot \frac{Wcu}{Pottot} \right|;$$

$$2.641439700\ 10^{10} \left| \frac{Di\ \rho}{A\ F\ Bm} \right|$$

The percentage of reactive leakage power can be determined:

$$Perc\_reac\_pwer := \frac{100 \cdot Var}{Pottot};$$

$$\frac{1.501689213\ 10^{-0}\ Var}{A^4\ Di\ F\ Bm}$$

Voltage regulation of the transformer:

$$Voltage\_Regulation := 100 \cdot \frac{\mathrm{sqrt}\left(Var^2 - Wva^2\right)}{Pottot};$$

$$Voltage\_Regulation := \frac{4.748758251\ 10^{5} \sqrt{A^{6}\, Di^{4} \left(1.466721088\ 10^{-8} A^{4} F^{2} + 3.094006913\ 10^{9} \rho^{2}\right)}}{A^{4}\, Di\, F\, Bm}$$

Design of a sample E I transformer:

Values for calculation:

Voltage of secondary:

Vsec=100V

Voltage of Primary:

Vprim=100V

Output current (Current of secondary):

Isec=5.90A

Flux density:

Bm=10000 Gausses.

Relations of lamination:

A =0.5

B =1.5

Kjan=0.421

x =1

$$P := \frac{E_{sec} \times I_{sec}}{0.95}$$

$P = 621.05$

$$A_{ef} := 6541.62004 \times \sqrt{P} \times \sqrt{\frac{1}{b \times a \times K_{jan}}} \times \sqrt{\frac{1}{D_i \times F \times B_m}}$$

$A_{ef} := 25$

We can select A=5:

For 10 Kilogausses the loss density for the silicon iron we are using is:

1 Watt/Kg

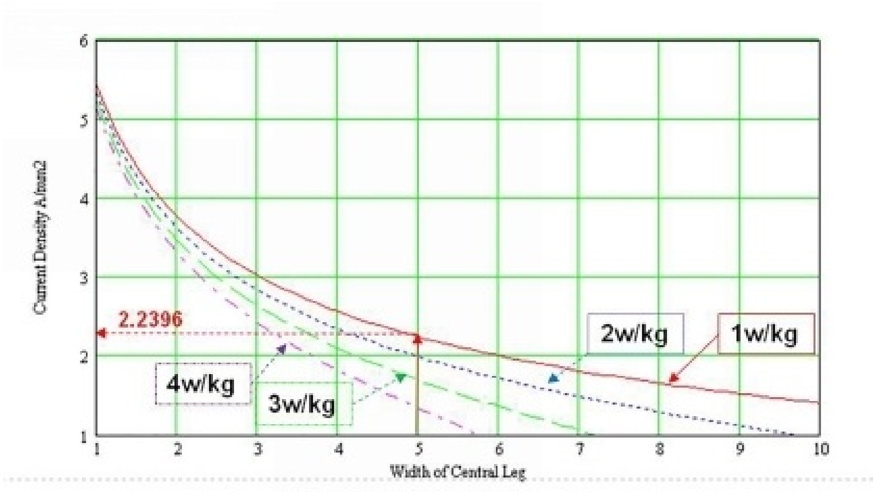


Fig. 161Current density calculation from central leg Class A (105 ° C)

Using the graph, as shown, we get:

$Di=2.24A/mm^2$

By the current density, we can get the cross sectional area of the conductor:

$$S_{wire} := \frac{I_{sec}}{\left(\frac{D_i}{100}\right)}$$

$S_{wire}=2.634$

By the AWG table we get: Scu=2.634 $Diam_{cu}$=1.88 (With insulating Varnish).

We are now calculating the voltage regulation due to leakage inductance and resistance of primary and secondary coils:

The approximate value is:

$$Por_{reg} := 4.7487582 \cdot 10^{5} \cdot \frac{\sqrt{A^{6} \cdot D_i^{4} \cdot \left(1.4667210 \cdot 10^{-8} \cdot A^{4} \cdot F^{2} + 3.0940069 \cdot 10^{9} \cdot r^{2}\right)}}{\left(A^{4} \cdot D_i \cdot F \cdot B_m\right)}$$

$Por_{reg}=7.878$

Calculation of the number of turns:

E=100V:

$$N := \frac{E \times 10^{8}}{4.44 \times F \times B_m \times A_{ef}}$$

$N = 150.15$

$N := 150$

Accommodating the wire in the core

All in millimeters

Length of the window:

$l_w := b \times A \times 10$

$ml := 1$ Lateral margi

$$Turns_{perlayer} := \frac{l_W - 2 \times ml}{Diam_{cu}} \times 0.85$$

$Turns_{perlayer} = 33.005$

$Turns_{perlayer} := 33$

$$Nof_{layers} := \frac{N}{Turns_{perlayer}}$$

$Nof_{layers} = 4.545$

$Nof_{layers} := 5$

$$\frac{1 + Nof_{layers} \times (Diam_{cu} + 0.1) + 0.3 + Nof_{layers} \times (Diam_{cu} + 0.1) + 0.2}{0.86} = 24.767$$

Now we are using centimeters:

$$l_{cuavg} := 2 \times A \times \left(1 + x + \frac{p + a}{2}\right)$$

Doing:

$x := 1$

$$l_{cuavg} := 2 \times A \times \left(1 + 1 + \frac{p + a}{2}\right)$$

For A class (105 °C)

$$r := 2.3 \times 10^{-6}$$

$$R_{tot} := \frac{l_{cuavg} \times N}{\frac{S_{wire}}{100}} \times r$$

$$R_{tot} = 0.5$$

$$W_{cu} := R_{tot} \times I_{sec}^{2} \times 2$$

$$W_{cu} = 34.842$$

$$P_{cu} := \frac{l_{cuavg} \times N \times \frac{S_{wire}}{100} \times 8.99 \times 2}{1000}$$

$$P_{cu} = 2.714$$

$$P_{fe} := \frac{\left(2.5 \times A \times 3 \times A - 1.5 \times A^{2}\right) \times x \times A \times 7.65}{1000}$$

$$P_{fe} = 5.737$$

$$W_{fe} := 1 \times P_{fe}$$

$$l_{W} := b \times A$$

$$l_{W} = 7.5$$

$$dm := \frac{l_{cuav}}{p}$$

$$dm = 12.16$$

$$L_{leak} := 0.4 \times p \times 10^{-8} \times p \times \frac{dm}{l_{W}} \times N^{2} \times \left(\frac{25}{10 \times 3} + \frac{1}{10}\right)$$

$$Var := L_{leak} \times 2 \times p \times F \times 2 \times \left(I_{sec}\right)^{2}$$

$$Var = 35.28$$

The voltage regulation calculated from the transformer is:

$$Volt\_Reg = 100 \times \frac{\sqrt{Var^2 + W_{cu}^2}}{625}$$

$$Volt\_Reg = 7.934$$

Such value is very close to the previously calculated

Temperature rise:

$$D_t := \frac{W_{cu} + W_{fe}}{0.1622 \left(P_{cu} + P_{fe}\right)^{\frac{2}{3}}}$$

$$D_t = 60.371$$

We must do something, because the temperature variation is beyond the permitted for Class A transformers:

We can increase the stacking factor E for 1.12, reducing the number of turns and consequently reducing the temperature rise that becomes:

$$D_t := \frac{W_{cu} + W_{fe}}{0.1622 \left(P_{cu} + P_{fe}\right)^{\frac{2}{3}}} \qquad D_t = 55.398$$

Determination of the better cost for a three phase transformer

Fig. 162

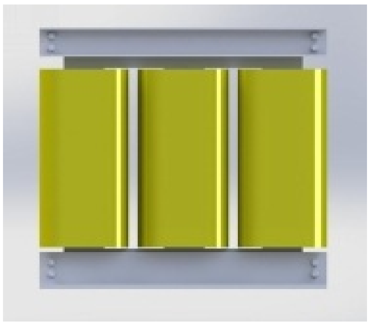


Fig. 163

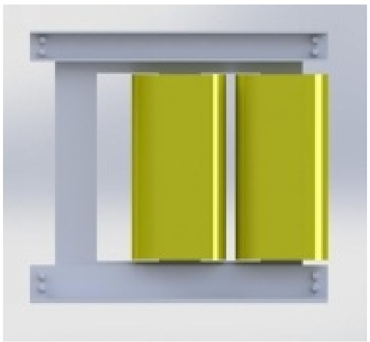


Fig. 164

Without any previous definition of size and proportions of a three phase transformer, let´s introduce our study to define the overall dimensions departing only from normalized free relations

Using our study, let us calculate a input transformer for a UPS of 80Kva.

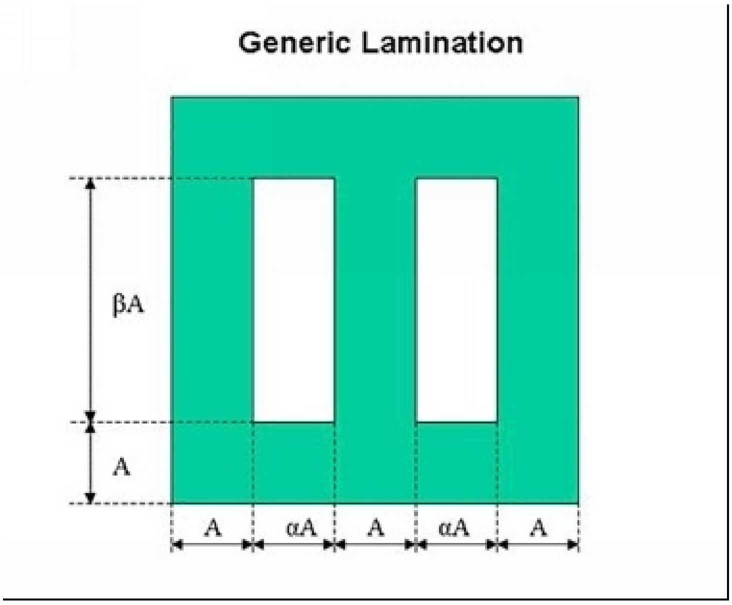


Fig. 165

We need to determine the lowest cost and the optimum relations for a and b , thus determining the best conditions for a three phase transformer using right angle composite lamination.

General Considerations:

We are using the Mathematical CAE Maple in our study. We do not present the internal mathematical formulation because due to the fact that is a natural consequence of the initial assignments: The equations are quite elementary and do not need further comments. If we follow step by step the evolution of the equations, the creation of the variables in the formation of the general equations, You can determine all the components of the study.

Length of core copper window:

$$lw := \beta \cdot A$$

Height of core copper window:

$$Hw := \alpha \cdot A$$

Area of the copper Window:

$$\mathrm{Aw} := \mathrm{lw} \cdot \mathrm{Hw};$$

Width of the core:

$$Lcore := 3 \cdot A + 2 \cdot \alpha \cdot A$$

Height of the core:

$$Hcore := 2 \cdot A + \beta \cdot A$$

Surface of the lamination:

$$Slam := Lcore \cdot Hcore - 2 \cdot Aw$$

Stacking:

$$Emp := 0.95 \cdot \xi \cdot A$$

Volume of the core:

$$Volfe := Slam \cdot Emp$$

Core Density:

Dfe:=7.65

Weight of the core in Kilograms:

$$Pfe := \frac{(Dfe \cdot Volfe)}{1000}$$

Magnetic cross Sectional Area:

$$Aef := 0.95 \xi \cdot A^2$$

Mean Turn:

$$lcuAvg := 2 \cdot A \cdot \left(1 + \xi + \frac{(\pi \cdot \alpha)}{2} \right)$$

Volume of the copper coil (Primary and secondary):

$$Volcu := 3 \cdot Aw \cdot Kjan \cdot lcuAvg$$

Weight of the copper:

$$Pcu := \frac{(Denscu \cdot Volcu)}{1000}$$

Considerations about power, cores and losses:

D_i is the current density in ampere/square centimeter:

The current of the hypothetical unitary turn is:

$$Ienr := \frac{Aw \cdot Kjan \cdot Di}{2}$$

Due to the fact we have only one turn in the winding

$$Erms := 4.44 \cdot F \cdot Bm \cdot Aef \cdot 10^{-8}$$

We can define the total power of the transformer as:

$$Pot.tot := 3 \cdot Ienr \cdot Erms$$

As an example we can determine the magnetic section from the full power of the transformer:

$$Pottot - 3 \cdot 4.218000000\,10^{-8}\,\beta\,A^4\,\alpha\,Kjan\,Di\,F\,Bm\,\xi$$

From here we can get the sqrt of P to calculate the Aef, so used in calculation of transformers:

Now we can calculate the resistance of the winding:

$$Renr := \frac{(\rho \cdot IcuAvg)}{Aw \cdot Kjan}$$

We calculate the copper loss:

$$Wcu := 3 \cdot 2 \cdot Ienr^2 \cdot Renr$$

The iron losses depend on the flux density and frequency of use

For a given condition in watts per kilogram:

DenspPferro=Watts/Kilogram

The total iron loss is:

$$Wfe := Pfe\,DensPferro$$

Preliminary considerations about cost:

Cost density of Iron:

Denscustoferro:

Cost of the core:

$$CustoNucleo := DensCustoFerro \cdot Pfe$$

Cost density of copper:

DensCustoCobre

Cost of the coil:

$$CustoEnrolamento := DensCustoCobre \cdot Pcu$$

Total cost of the transformer:

$$PreçoTotal := CustoNucleo + CustoEnrolamento$$

Leakage Inductance:

We are considering that there is one turn for primary and secondary:

$$\mu o := 0.4 \cdot \pi \cdot 10^{-8};$$

$$4.000000000\,10^{-9}\,\pi$$

$$dm := \frac{lcuAvg}{\pi};$$

$$\frac{2A\left(1 + \xi - \frac{1}{2}\pi\alpha\right)}{\pi}$$

$$Lleak := \frac{0.4 \cdot \pi \cdot 10^{-8} \cdot \pi \cdot dm}{hw} \cdot \left(\frac{0.375 \cdot \alpha \cdot A}{3} + 0.016 \cdot 0.75 \cdot \alpha \cdot A\right);$$

$$\frac{1.096000000\,10^{-9}\,\pi A\left(1 - \xi + \frac{1}{2}\pi\alpha\right)\alpha}{\beta}$$

$$Lleak := \text{simplify}(\mathbf{(3.10)});$$

$$\frac{1.721592774\,10^{-9}\,A\,(2.\,\xi - 3.141592654\,\alpha + 2.)\,\alpha}{\beta}$$

Reactive power in the leakage inductance:

$$Var := 3 \cdot (2 \cdot Ienr)^2 \cdot 2 \cdot \pi \cdot F \cdot Lleak;$$

The average temperature of a transformer:

$$\Delta t = \frac{\left(\frac{2}{3} \cdot (Wfe + Wcu)\right)}{0.162 \cdot \left(\frac{2}{3} \cdot \left(Pfe + \frac{Pcu \cdot DensRef}{Denscu}\right)\right)^{\frac{2}{3}}}$$

$$\Delta i = \left(4.115226338 \left(0.0009500000000\, Dfe \left(\left(A\beta + 2A\right)\left(2A\alpha + 3A\right) - 2\beta A^2 \alpha\right) \xi A\, DensPferro - 6\beta A^3 \alpha\, Kjan\, Di^2 \rho \left(1 + \xi + \frac{1}{2}\pi\alpha\right)\right)\right) \Big/ \left(0.0006333333333\, Dfe \left(\left(A\beta + 2A\right)\left(2A\alpha - 3A\right) - 2\beta A^2 \alpha\right) \xi A - 0.03584000000\, \beta A^3 \alpha\, Kjan \left(1 + \xi + \frac{1}{2}\pi\alpha\right)\right)^{2/3} \tag{5.1}$$

Isolating Di^2:

$isolate((5.1), Di^2)$

$$Di^2 = \frac{1}{\beta A^3 \alpha\, Kjan\, \rho \left(1 + \xi + \frac{1}{2}\pi\alpha\right)} \left(0.1666666667 \left(0.2430000000\, \Delta i \left(0.0006333333333\, Dfe \left(\left(A\beta + 2A\right)\left(2A\alpha - 3A\right) - 2\beta A^2 \alpha\right) \xi A - 0.03584000000\, \beta A^3 \alpha\, Kjan \left(1 + \xi + \frac{1}{2}\pi\alpha\right)\right)^{2/3} - 0.0009500000000\, Dfe \left(\left(A\beta - 2A\right)\left(2A\alpha - 3A\right) - 2\beta A^2 \alpha\right) \xi A\, DensPferro\right)\right)$$

Calculating the core cost:

$$CoreCost := CoreCostDensity \cdot Pfe;$$

Calculating the coil cost:

$$CoilCost := CopperCostDensity \cdot Pcu$$

Total cost:

$$TotalCost := CoreCost + CoilCost$$

$$Powersquared := subs(Di^2), Pottot^2)$$

$$UnitCostSquared := \frac{(TotalCost^2)}{Powersquared}$$

Setting the parameters:

$\rho := 2.82 \cdot 10^{-6} \cdot (1 + 0.00393 \cdot (180 - 20))$

Space factor of the window:

Kjan:=0.23

Density of the iron core:

$Dfe := 7.65$

Iron core density:

$DensPferro := 1.15$

$At := 100$

Ambient Temperature:

$Ta := 45$

Stacking multiplier:

$\xi := 1$

Core cost density:

$CoreCostDensity := 9$

Operation frequency:

$F := 60$

Maximum Flux density:

$Bm := 15000$

Copper cost density:

$CopperCostDensity := 16$

Surface of cost of the solution under consideration:

A=10Cm;

$plot3d\left(\sqrt{UnitCostSquared}, \alpha = 0.2..2, \beta = 2..7\right)$

We did not give any prior relationship that the transformer must assume. The unique predetermined relationship is between the stack and the width of the blade leg that is square.

This is also the main reason for our search for optimal values of the normalized relations of the core.

In later studies we will give complete freedom to the program find the best solution for the core that will also be the optimal value.

The figure below gives a surface where the minimum point is the best cost of the transformer.

Now we will search for the minimum value of the cost by the Maple command NLPsolve.

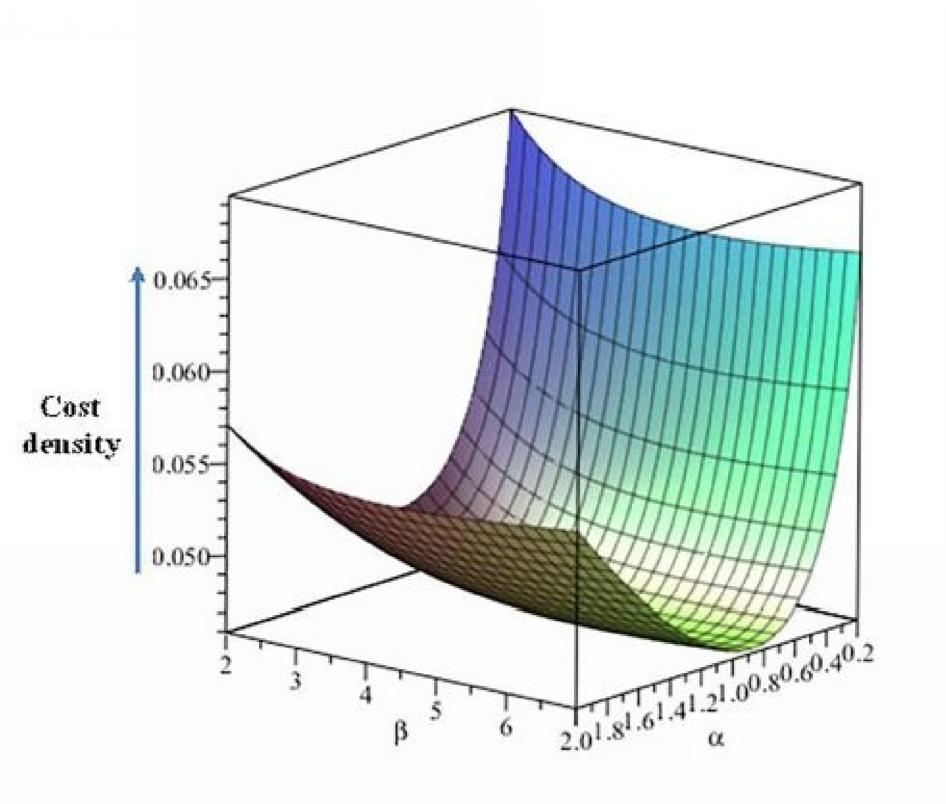


Fig. 166

Minimum cost density and Optimum Values of alpha and beta:

$>NLPSolve(p, \alpha \quad 0.2..1.5, \beta \quad 1..6)$

$[0.0459350890232343018, [\alpha \quad 0.882846008856371, \beta \quad 5.29443096085636]]$

Minimum Cost Density=0.04593508……

$$[\alpha = 0.880559950805819, \beta = 5.30383528288125]$$

Evidently this depends on unity prices in each particular country, but gives an idea of what can be done to minimize costs by improving the relationship of measurements of the iron core.

Voltage regulation due to leakage inductances and coil resistances:

$$VoltageREg := \frac{100 \cdot \mathrm{sqrt}\left(Var^2 - Wcu^2\right)}{Pottot}$$

$$1.580527896\,10^{9} \left(1.066997404\,10^{-16} \beta^2 A^{10} \alpha^6 Kjan^4 Di^4 \pi^2 F^2 (2.\,\xi - 3.141592654\,\alpha - 2.)^2 - 36\,\beta^2 A^6 \alpha^2 Kjan^2 Di^4 \rho^2 \left(1 + \xi + \frac{1}{2}\pi\alpha\right)^2\right)^{1/2}$$

$\xi := 1;$

$$1$$

$Kjan := 0.23;$

$$0.23$$

$\alpha := 0.8806;$

$$0.8806$$

$\beta := 5.3038;$

$$5.3038$$

$Di := 107.1;$

$$107.1$$

$F := 60;$

$$60$$

$Bm := 15000;$

$$15000$$

$>\rho := 2.82\,10^{-6}(1 + 0.00393\,(180 - 20));$

$$0.000004593216000$$

The voltage regulation of this transformer, considering the parameters found is:

$$VolReg := \frac{1}{A^4\,Di\,F\,Bm}\left(1.471613452\,10^{9}\sqrt{1.792181496\,10^{-16} A^{10} Di^4 \pi^2 F^2 - 11.52592676\,A^6 Di^4 \rho^2 (2 + 0.4402800000\,\pi)^2}\right);$$

$$0.001655226634 \sqrt{6.935904905\,10^5\,\pi^2 \quad 1.021097962\,10^5\,(2 \quad 0.4402800000\,\pi)^2} \quad \textbf{(5.17)}$$

$evalf((\textbf{5.17}));$

$$4.685842020$$

The resistive voltage drop:

$$ResDrop := \frac{100 \cdot Wcu}{Pottot};$$

$$1.057842876 - 0.2328841093\,\pi \quad \textbf{(5.19)}$$

$evalf((\textbf{5.19}));$

$$1.789469883$$

The inductive voltage drop as a percentage of the output voltage:

- $InductiveDrop := \frac{100 \cdot Var}{Pottot};$

$$1.378658020\,\pi \quad \textbf{(5.21)}$$

$evalf((\textbf{5.21}));$

$$4.331181908$$

We have made an introduction about optimization of the normalized relations of the iron cores. Evidently this is apartial solution, because to get the entire figure we must to parameterize the A and x , searching the minimum price. In reality there are two far values for minimum prices but this is the question for the third volume of our book where we will discuss about general conditions for optimizations.

The optimization of the core corresponds for each power. Depending structure of the transformer, for example: In one project of an UPS of 500Kva the transformer uses duct for refrigeration. This is another kind of study.

Bellow we can see the coil simplified drawing of this transformer. In the drawing, at the left side up, can be seen the refrigeration ducts. This is a special case, because this transformer needed to handle harmonics and many other factor must be accounted. We are considering this in the third volume:

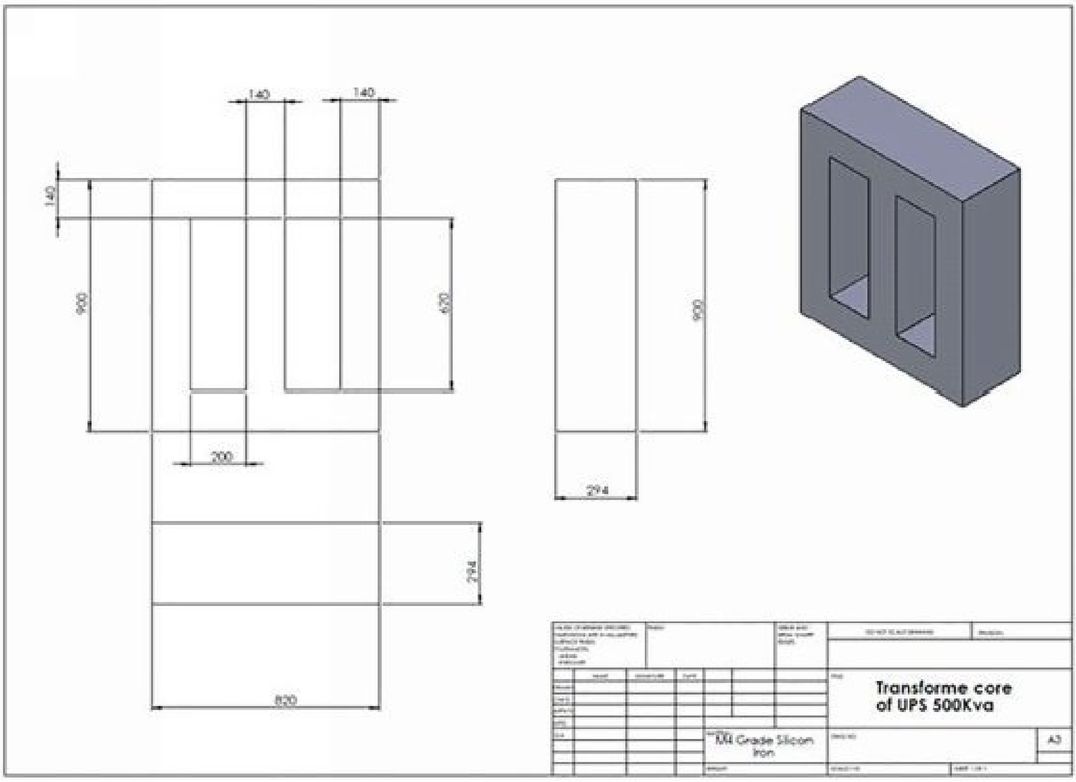


Fig. 167

For cruciform cores the solution is the same, however we must reference to the internal dimensions of the core to get Alpha and Beta. The computation of the average turn length, must be calculated as a function of the radius of the leg.

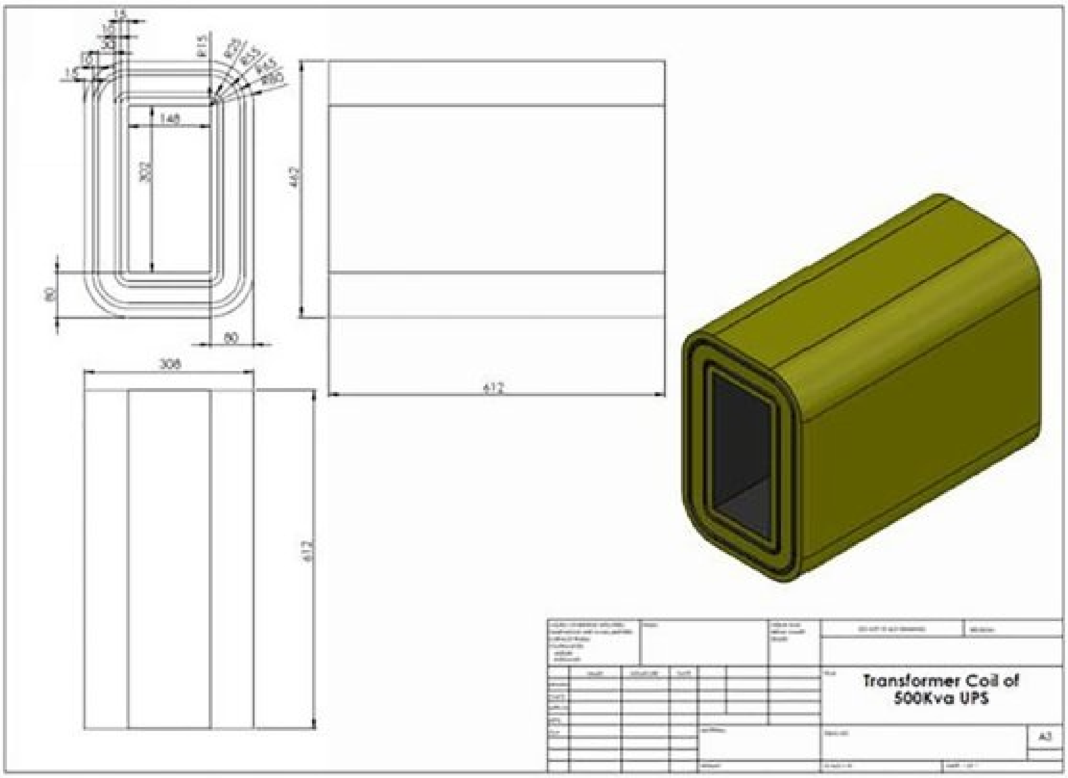


Fig. 168

K Factor and K Factor transformers

The problem of the loads and the current spectrum

Today in the industry in modern data centers, telecommunication sites and even office buildings, there are a wide use of information technology equipments and even power control equipments that require large power. Unfortunately, most of them have non-linear power supplies that inject harmonics in the power grid.

The great inconvenience of the harmonic injection is that leads not only to low power factors, but to the creation of additional losses which can often bring the electrical system to fail.

The measure of the harmonic content is the most important value in dimensioning of the energy system. There are many solutions to mitigate the harmonics in power, but their majority is expensive and in many cases impossible of being used mainly by space consumption, cost and other additional factors.

The transformers are the most sensible apparatus of the system, because the content of harmonics injected in the line, directly leads to very high dissipation.

Its detrimental effects increase with the square of the harmonic number, as the losses involving skin depth like proximity and strange effects, and other sensitive to the square of the product of the intensity of each, with their harmonic number. They create large power losses that can lead them to a fire.

Underwriters Laboratories (UL) recognized the potential safety hazardous associated with the using standard transformers with nonlinear loads.

They developed a rating system to indicate the capability of a transformer to handle a specified harmonic content. The rating described in UL 1561 is known as K factor transformers. UL derived K factor transformers from the information contained in ANSI/IEEE STD C57.110 “IEEE recommended practice for establishing transformers capability when supplying non sinusoidal load currents”

K factor transformers are designed to reduce the heating effects of the harmonic currents developed by the non linear loads.

The K Factor definition:

The K factor as defined by IEEE 110-1992 is:

$$K = \frac{\sum_{n=1}^{N} (I_n \times n)^2}{\sum_{n=1}^{N} (I_n)^2}$$

The k factor applies exclusively to transformers it is a weighted of average of harmonic current magnitudes. Increased higher order harmonics will greatly increase K factor. This represents the heating effects of harmonic induced eddy currents in winding conductors.

Different topologies of non linear loads and the K Factor:

Let us consider different topologies of non linear loads thus establishing different levels of K factors.

A simplified but full precision, Matlab-Simulink K Factor Calculator.

In order to facilitate full comprehension of the matter, we will establish as a measurement device implemented in Simulink.

As the need of get the information of harmonic content of a certain waveform, we are using the block "Fourier" of the Simulink library. The decomposition is given in the output as vector of the required list of harmonics.

The output is led to two places: One is the 2 input multiplier, multiplying by itself (squaring) each harmonic. The other is led to the multiplier, which multiply it with the number of each harmonic. At the end of this phase, the value is squared.

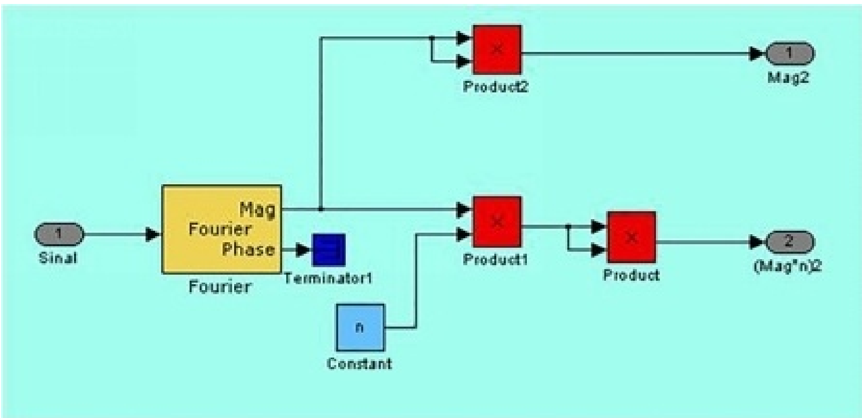


Fig. 169

The hart of the system is the standard Simulink Fourier solver block (In yelow) at left side. It can be seen internally in the figure below.

The block can solve in blocks treating the signal like a vector where each element is a harmonic.

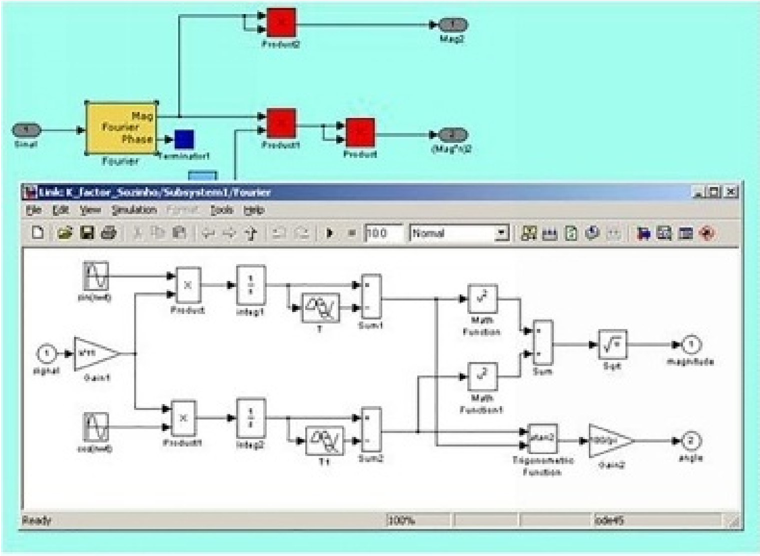


Fig. 170

To work we need to set the fundamental frequency and the harmonic number or a vector of harmonic numbers:

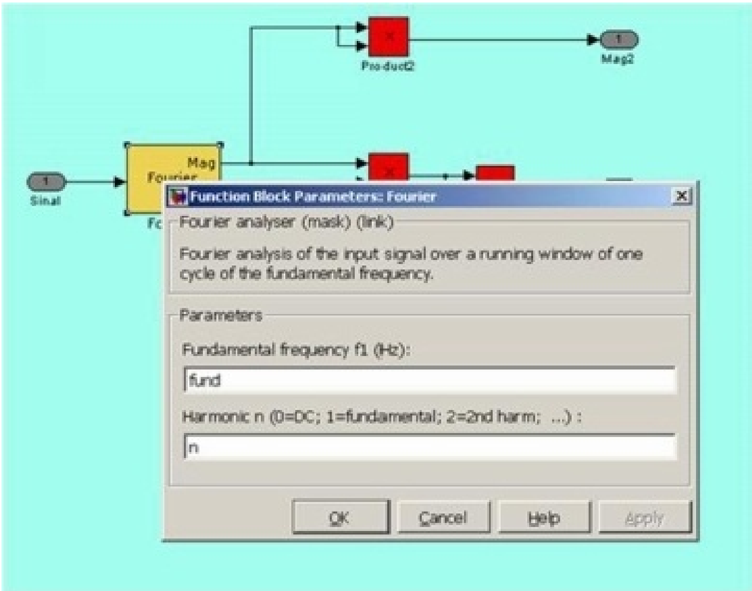


Fig. 171

We are beginning our description of the K factor calculator using resources of images. We think as spoken in the Chinese proverb: “An image worth more than a thousand of words”.

Below we can see the blocks showing the result of each output.

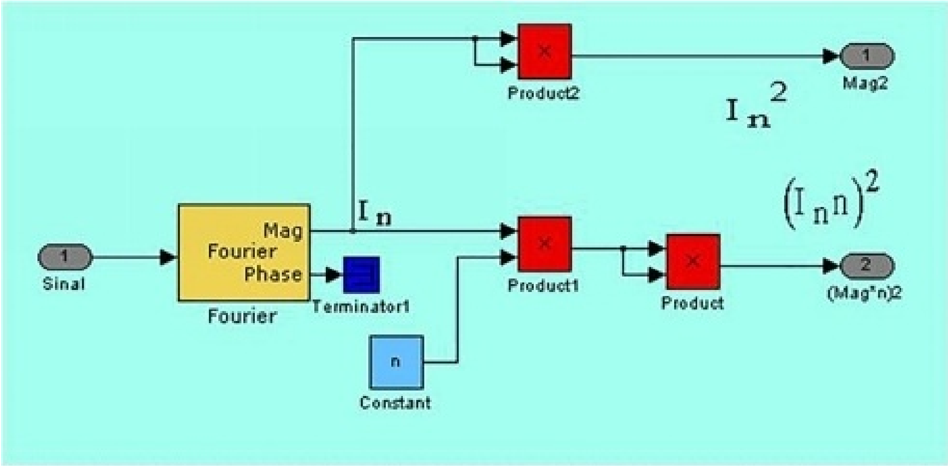


Fig. 172

We transformed the block obove in a subsystem as shown below. The mask of the subsystem includes the variables needed by the fourier block.

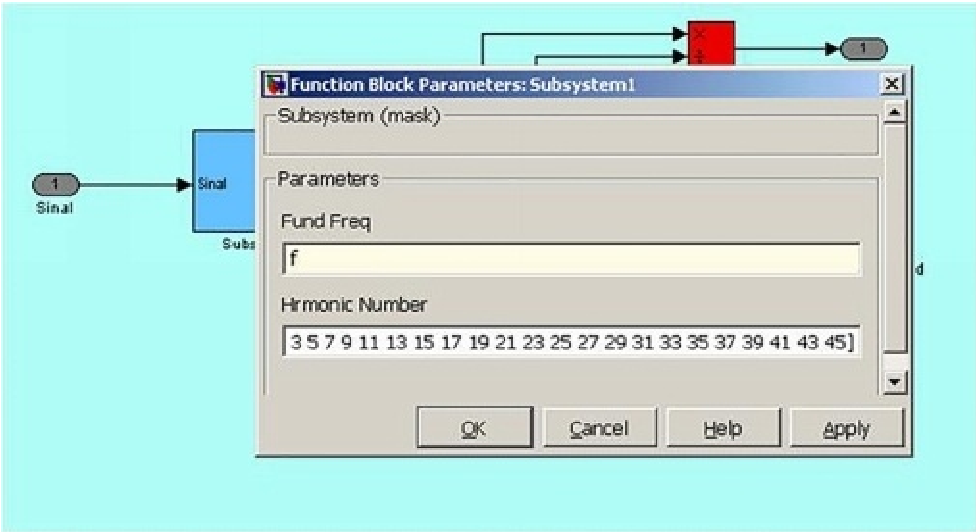


Fig. 173 The model needs to define de vector with the harmonic numbers that the module has to calculate.

As can be seen, the harmonic number is supplied as a vector with the harmonic numbers required from 1 (Hidden) up to 45nth.

Other operations are done and finally we can get the K factor:

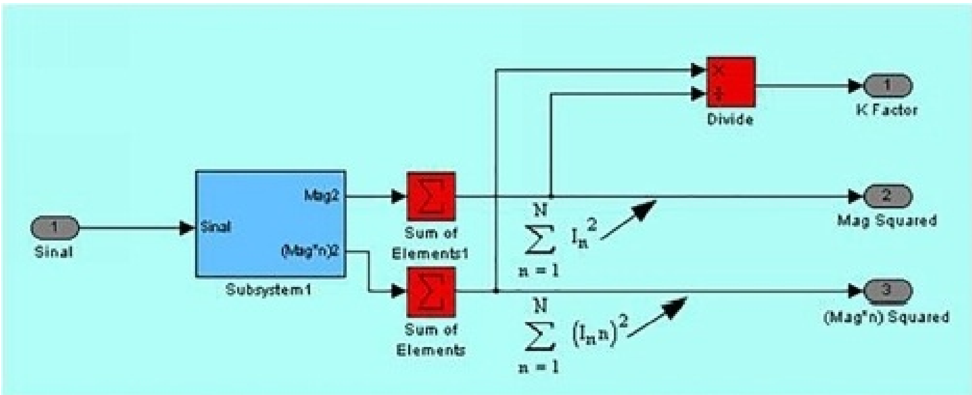


Fig. 174 In this figure is shown the values calculated for each output

We completed our instrument, implementing a Wattmeter, a VA meter a Power factor meter and a displacement factor meter. All will be used for analysis of different kinds of loads.

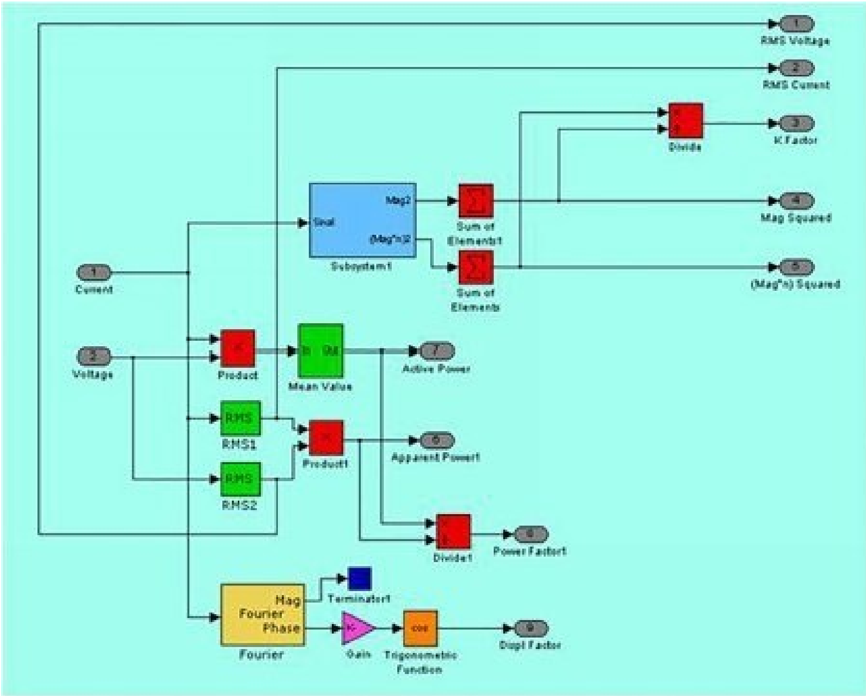


Fig. 175

We must work first with our first load. They are present in each personal computer in our houses, offices and many more other places.

There are variations of them with and input power factor converter, but they are only few among many others.

Its use is very harmful for the power mains, mainly for transformers. Fortunately the majority of the loads of a building are of almost linear nature.

But the problem can be serious when there are many distribution transformers segregated from the rest of the mains, feeding only loads of information equipment.

The IEC 62040 standard for UPS systems and the non linear input interface power supply for personal computers.

This power supply works with a single phase rectifier with the values defined by IEC62040 for each power, and a switching mode power supply bringing down the voltages to the levels and powers required by the information equipment.

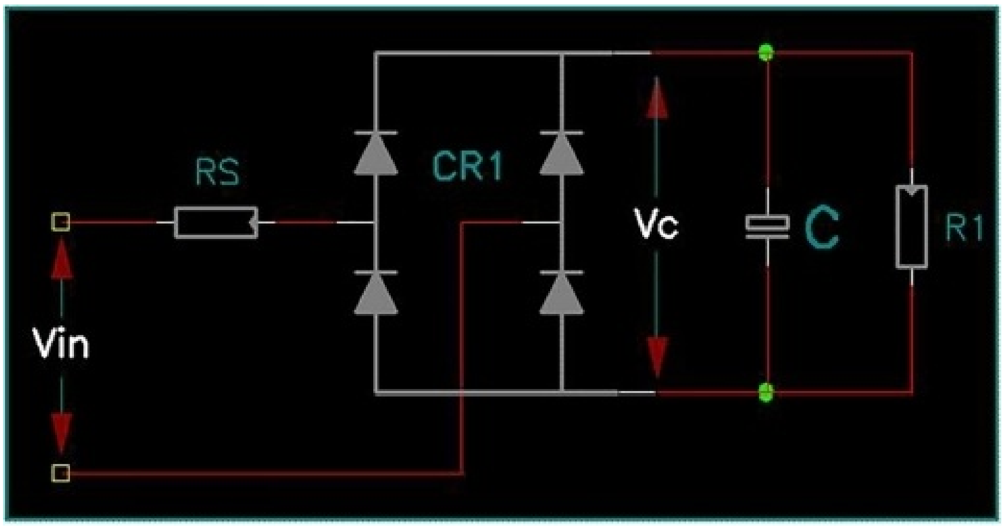


Fig. 176

We first are using a model of approximately 12Kva (To simulate a combination of many power supplies in parallel, because our main purpose is to define the K factor of them.

Values defined in the standard

$$V_c = \sqrt{2} \times (0.92 \times 0.96 \times 0.875) \times V_c = 1.22 \times V_{in} \qquad R_s := \frac{0.04 \times V_{in}^2}{P_{ap}}$$

$$T = 0.15 \qquad R \times C = T \qquad R_1 = \frac{V_c^2}{0.66 \times P_{ap}}$$

$$V_c := 1.22 \times V_{in} \qquad C = \frac{0.15}{R_1}$$

Let’s make the measurement getting this information, and as a second step we can make an arrangement of a three phase configuration to measures neutral current.

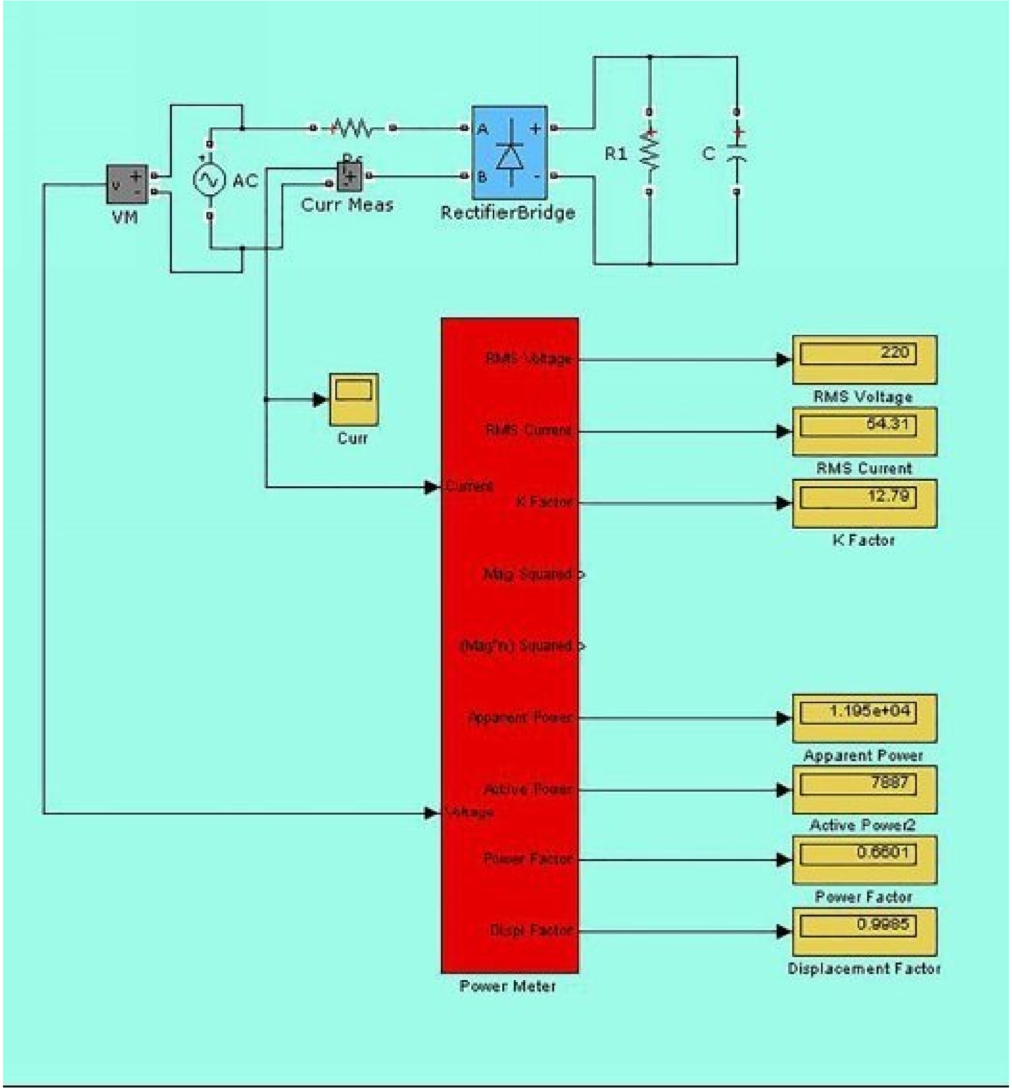


Fig. 177 System in SimPower and Simulink arranged for testing and get our values

Results of the simulation:

Input Voltage: 220 Vac

Rms Current: 54.31 A

K Factor: 12.79

The values encountered are:

Real Power: 7887 Watts

Apparent Power: 11950VA

Power Factor: 0.6601

Displacement Factor: 0.9985

The effect on the neutral current

Now we can see this same power supply distributed in three phase configuration to see the neutral current:

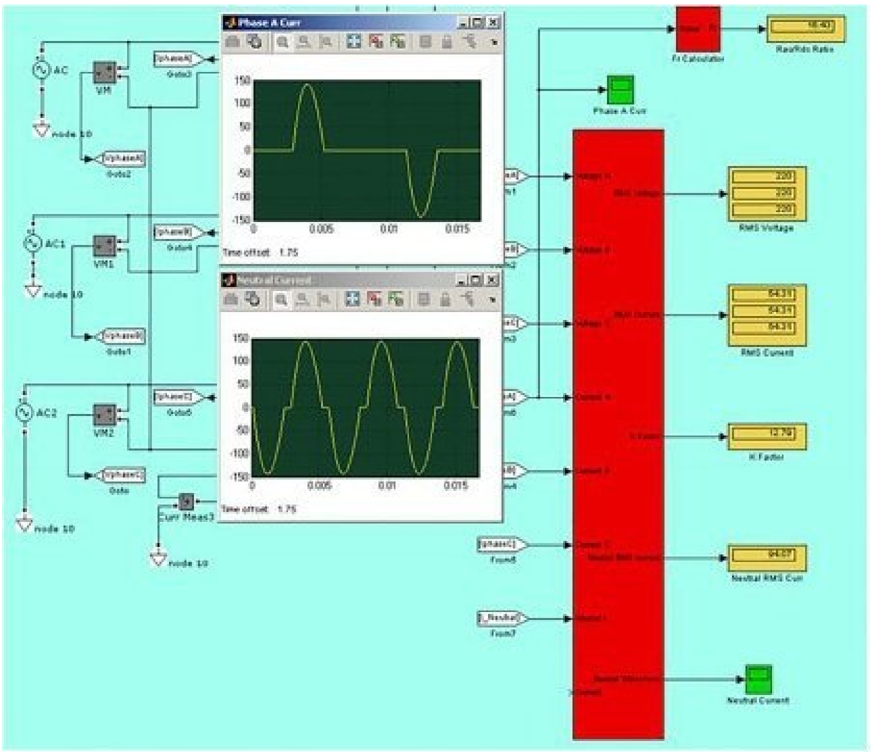


Fig. 178 The Phase and neutral current waveforms

The harmful effect of the neutral current: 94.07 A RMS of third Harmonic.

Let's consider a transformer of 100 Kva with 2.5% of total impedance supplying a system with 3 power supplies same as those presented in the anterior example. The effect over the K factor will be shown.

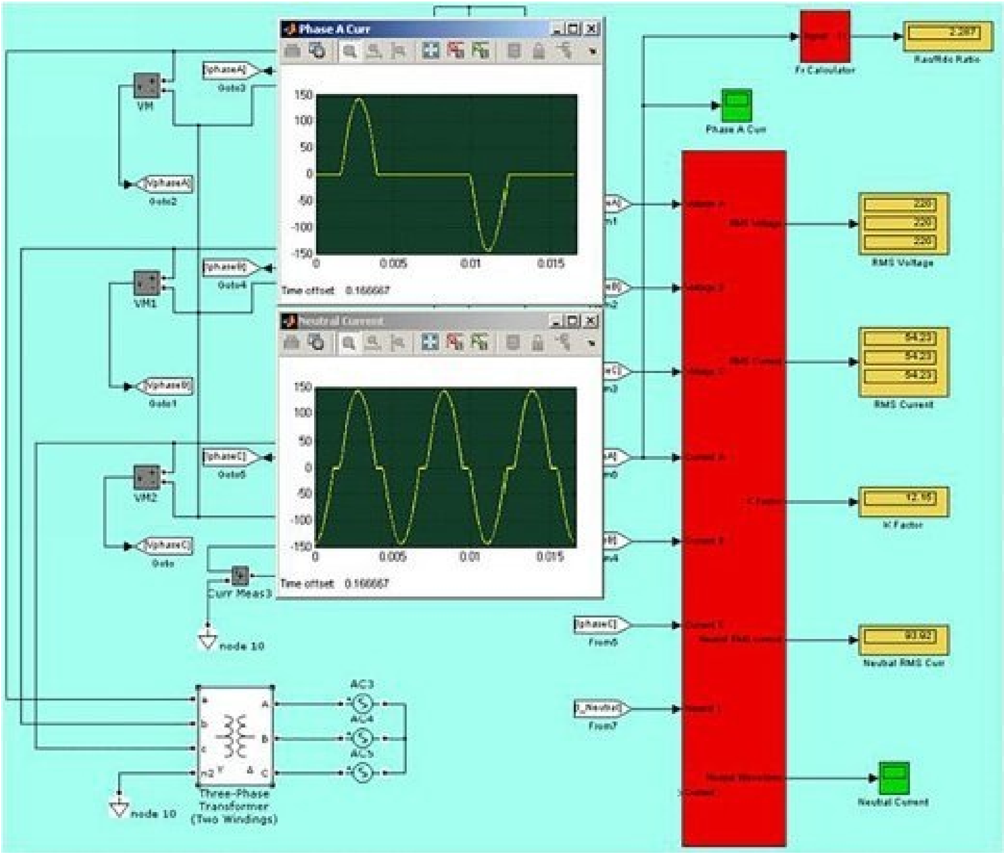


Fig. 179

Now we can see that with the effect of the transformer, even with 100 Kva and 2.5% of series impedance with mostly inductive effect, the K factor decreased for 12.15. Very close to the value considered in the major part of specifications for this kind of apparatus. Evidently in the physical installations like sites of data centers where there are many transformers, circuit breakers cables and so forth, the impedance will be significantly higher than we have arbitrated. A thing became very clear: the neutral cables in this kind of installation must be double of the size of the phase cables.

Before entering the K factor transformers project we need to consider other power apparatus used, because we need to analyze them carefully.

In most installations on sites involving power sources, there are many UPS systems that often do not consider the harmful effects that their rectifiers may pose to the commercial power grid.

I have seen many UPS rectifier of very large manufacturers, including equipment that features high reliability, but the rectifiers of its UPS are made without any knowledge of polyphase rectification. They seem to be designed by beginners, without any knowledge of power electronics. They present very poor input power factors and harmonics spectra very harmful for the commercial line.

At the moment we are entering only to talk of a few topologies. Our present concern is to obtain K factor values for our projects. But in a special section we will cover transformers for high power rectifiers; there we will talk about such subjects. In those studies we must define the current waveforms of each winding and their mobile magnetic portions that mean, descend deeper into these matters.

The portion mentioned above is that one part of the H profile that lies between zero and the maximum value of H, as defined in page 73. Following the behavior of the transformer the H profile can move depending the combination of the contribution of the field of different coils. With such movement the other parameters of the transformer can change according with the change of the portion. An method of “averaging” can be done that can affect the calculation of the transformer.

By now let´s study the K factor for other topologies, at the moment a three phase 6 and 12 pulses controlled rectifiers will be used.

First of all let’s consider the fig 180 where we have a 6 pulse non controlled rectifier with a current source simulating an infinite inductance as a load.

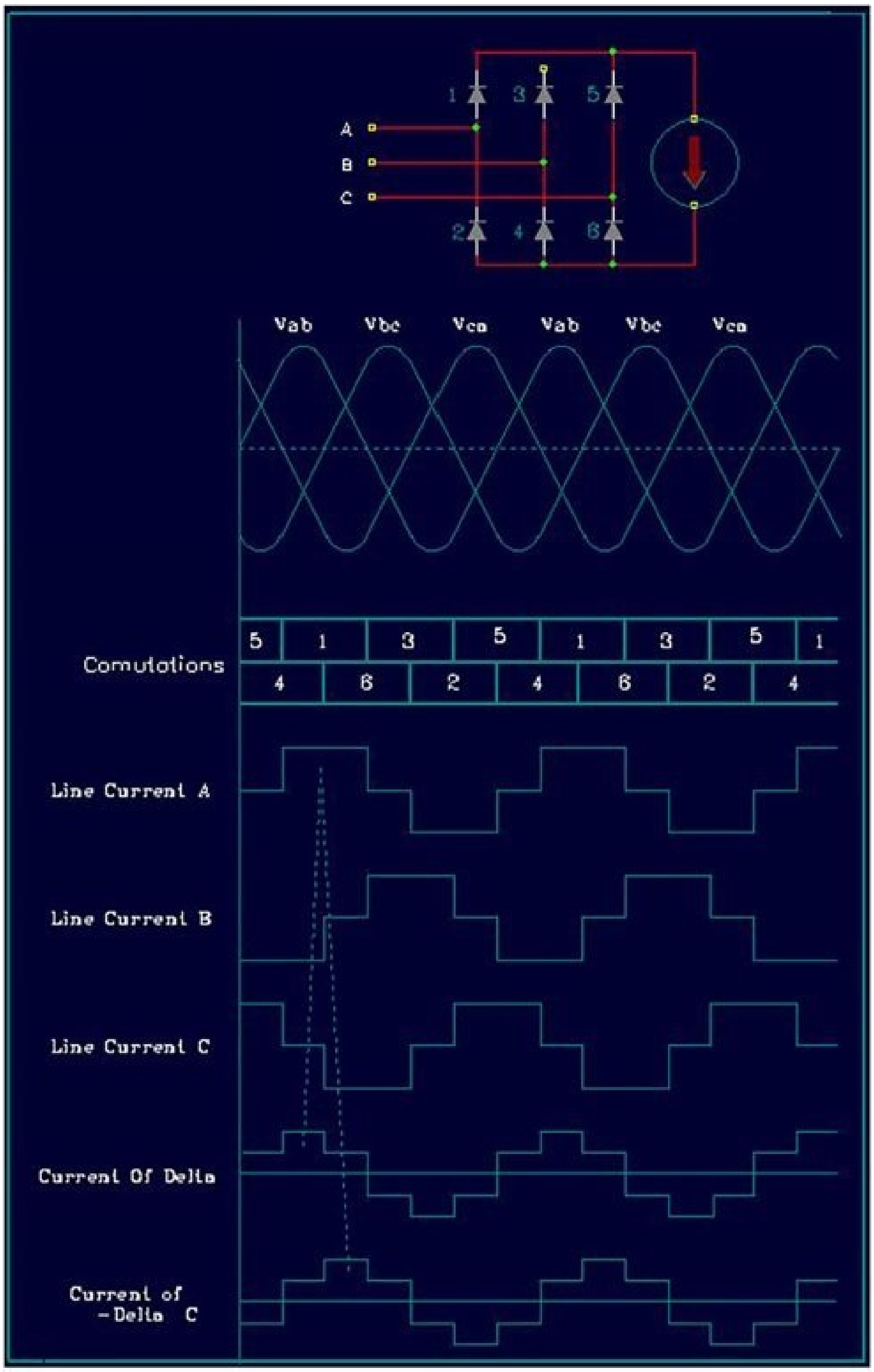


Fig. 180

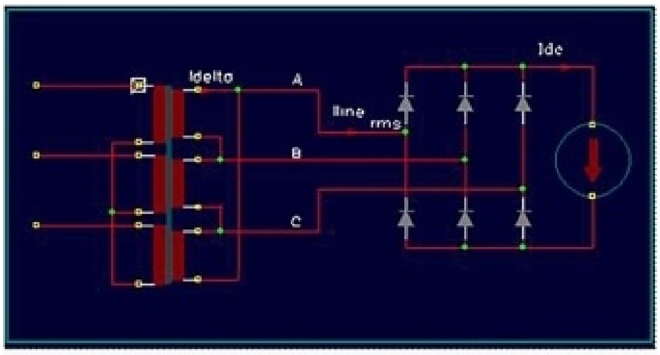


Fig. 181

The line current is:

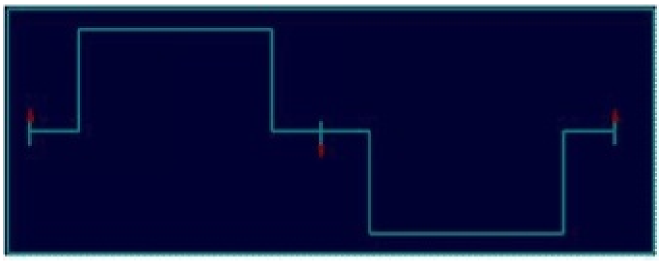


Fig. 182

The input current (Combination of the line currents by transformer windings:

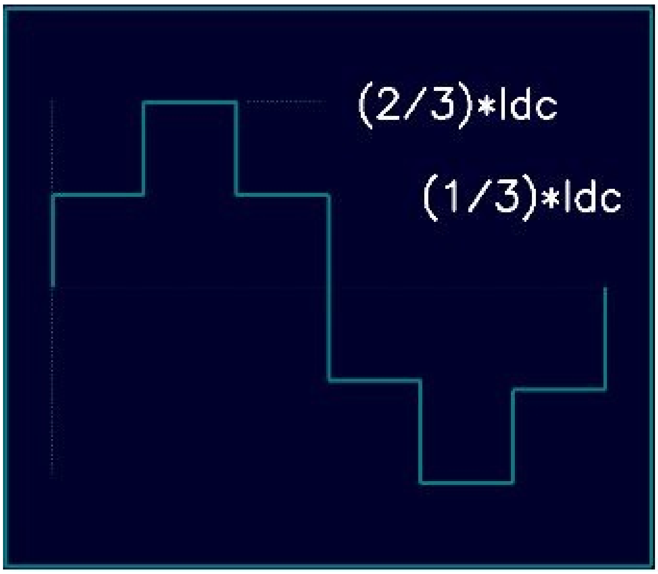


Fig. 183

Calculating Harmonics:

$$C_n = \frac{1}{\pi}\left(\int_{-\frac{5\pi}{6}}^{-\frac{\pi}{6}} -I_p \cdot \sin(n \cdot wt)\,dwt\right) + \frac{1}{\pi}\left(\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} I_p \cdot \sin(n \cdot wt)\,dwt\right)$$

$$C_n = \frac{2}{\pi} \cdot \left(\frac{\cos\left(\frac{5}{6} \cdot \pi \cdot n\right)}{n} \cdot I_p + \frac{\cos\left(\frac{1}{6} \cdot \pi \cdot n\right)}{n} \cdot I_p\right)$$

Doing:

$Max := 1001$

$n := 1, 3..Max$

$I_p := 1$

Plotting:

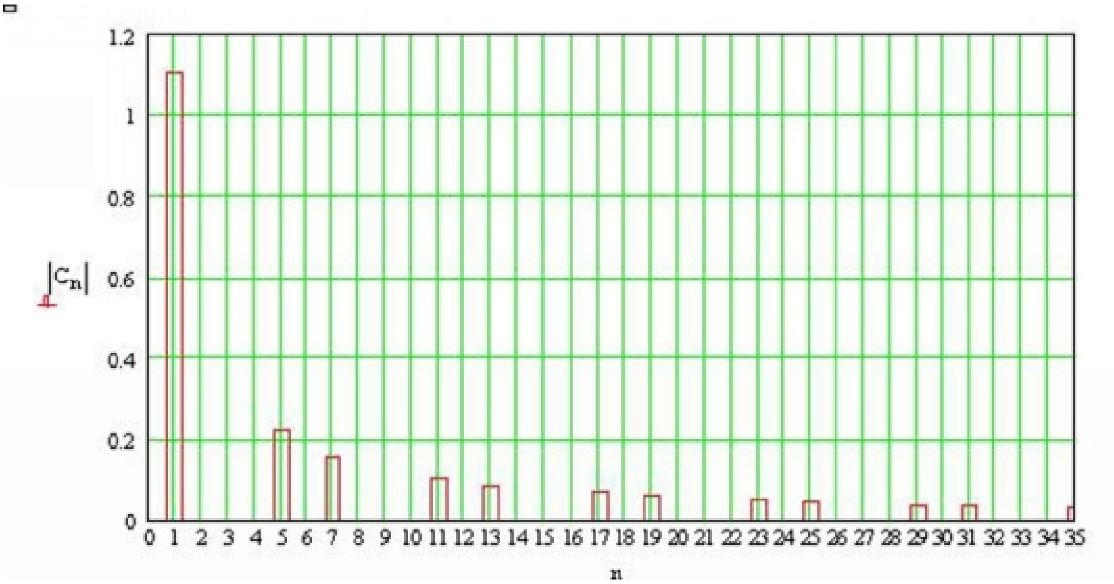


Fig. 184

Calculating K Factor:

$$\frac{\sum_{n=1}^{Max} (c_n \cdot n)^2}{\sum_{n=1}^{Max} (c_n)^2} = 304.664 \quad \text{Wrong}$$

In this case, such equation do not converges to a value, being totally dependent to the number of harmonics. This is due to the sharpness of the square wave that presents low decay of harmonics added to the product of the amplitude of the harmonic by its number. Such product tends to be constant.

We can observe that in the first graph the decay of the harmonic is dependent of 1/n. It's clear that if we multiply each one per n, the result is maintained constant up to infinity.

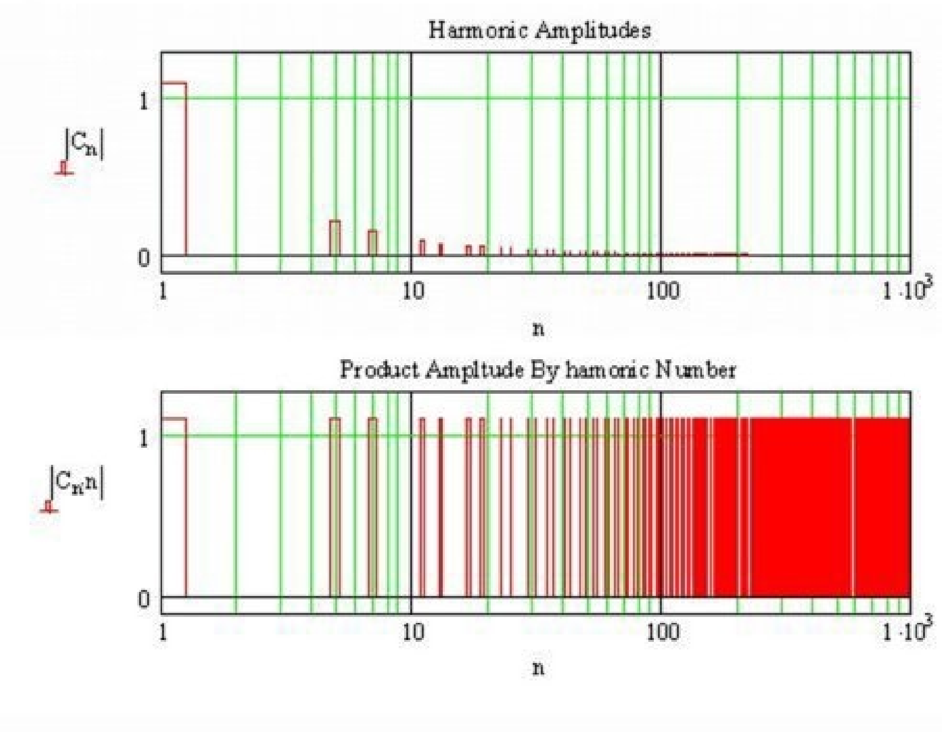


Fig. 185

We will be using this series for the study of high power rectifiers where do not exists this kind of product, due to the fact that the application do not exist need for the calculation of harmonic product by its harmonic number. In the real rectifier there is the commutation angle created by the impedance of the transformer and power line that forces the decay of harmonics, making possible the convergence of the harmonic series: We will be seeing this in the high power rectifiers.

By now we will be using simulation in Mathlab that is straight forward and sufficient precise because they are numerical solutions of the formulas involved. Even in power systems, because the solutions involve state equations and are very close to the mathematical solutions. In the past we have used state equations conjugated with Petri Nets

We are performing the simulation simultaneously in three rectifiers. One is a 12 pulses rectifier and the other two, 6 pulses rectifiers. The difference between the two is the arrangement of the secondary's where one is delta and the other is star. This promotes different current waveforms as can be observed in the two oscillograms at below right of the figure. Despite the different waveforms is maintained the equality of spectrum, and therefore is maintained same K factor.

We have used same unit impedances in the simulation of the rectifiers and such impedances are those ones employed in the characteristics rectifiers used for those purposes.

In the figure 186 we have the two rectifiers and the resultant waveforms. The oscilloscopes in the right side of the figures show the waveforms of the input currents.

Classification of K factor according to loads and lines.

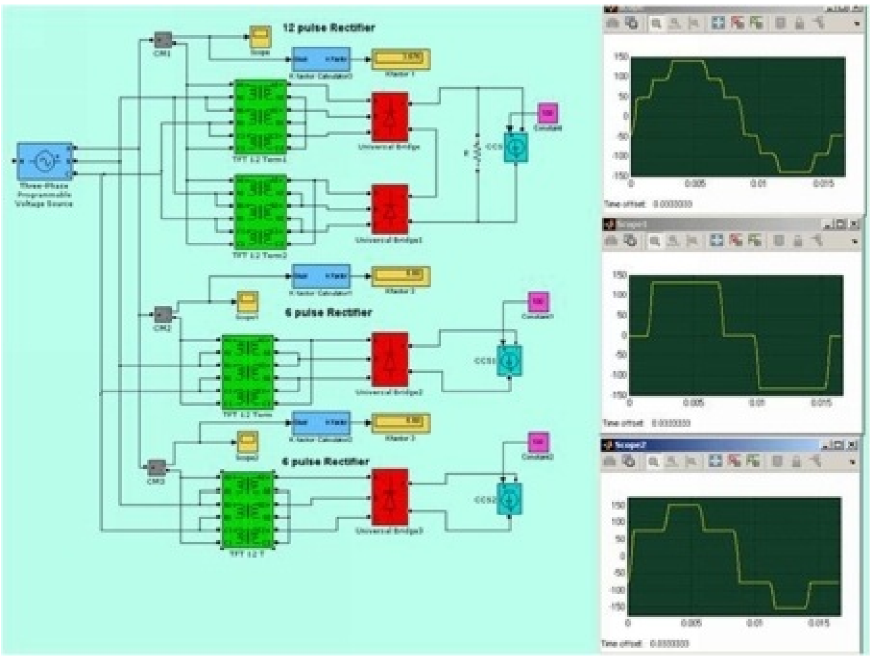


Fig. 186

The values we got for the three solutions are:

12 pulses rectifiers: K factor of 3.676 ~ 3.7

2 rectifiers of 6 pulses: K Factor 6.68 ~6.7

The four solutions match the table of specifications for the key factor observed in such applications:

Load	K factor
Incandescent lighting Electric Resistance Heating Tap Changing regulation transformers Control transformers Standard distribution transformers	K1
Electric Discharge Lighting Electric Discharge Lighting (HID) UPS system 12 Pulse Welders Induction heating equipment	K4
High power Telecomunication Equipment - Old Versions with thyristors UPS 6 pulse rectifier Multiwire standard receptacle for different areas	K13
Main frame computer loads Motor Drives Multiwire industrial receptacles	K20

Fig. 187

Transformers normally come in standard values like K4, K9, K13, K20, K30, K40 and K50.

The first problem is the definition of the spectral figure coming from a defined K factor number.

We defined a universal oscillator that can be settled for each K factor used in calculations of transformers. In reality there isn't a fixed determination of a particular "Waveform" and the own definition of the K factor do not guarantee any spectrum in particular, do not existing also any limit for the number of the harmonics. Evidently the main interest of the standard is to guarantee the limitation of the eddy currents generated inside the electrical conductors by proximity and skin depth effects.

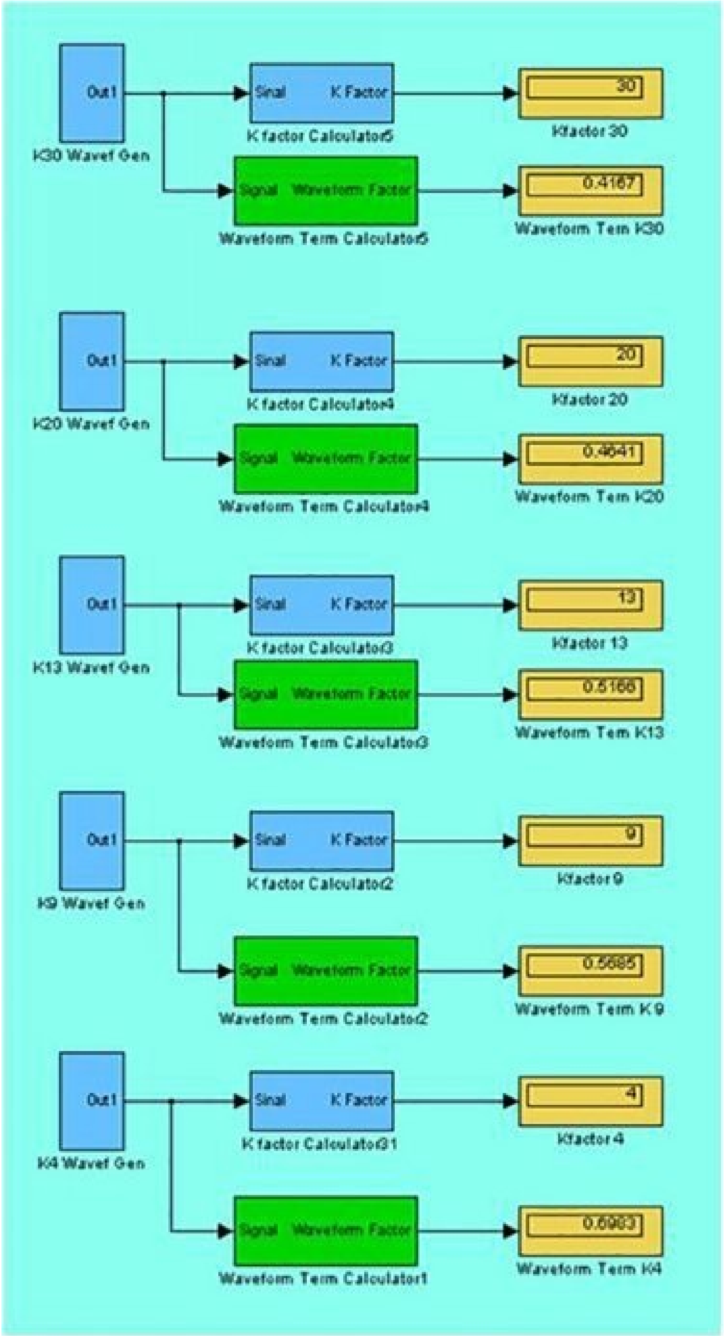
In the present case, we can define a table of certain values of Kfactor and our "Second term", that is calculated by the "Waveform Term Calculator as stated in the chapter xx.

Firstly let's define a structure for the determination of our values: In this system there are 5 waveform generators with K4,K9,K13,K20 and K30 that are the most used standard for K Factor transformers. For each one there is a K factor calculator and a waveform term calculator (Second term calculator).

With this system we are making a survey of the second term for each k factor.

K Factor	ST
30	0.4167
20	0.4641
13	0.5166
9	0.5685
4	0.6983
1	1

Fig. 188 Kfactor and Second Term Table



Out1
K30 Wavef Gen
Sinal K Factor
K factor Calculator5
30
Kfactor 30
Signal Waveform Factor
Waveform Term Calculator5
0.4167
Waveform Term K30
Out1
K20 Wavef Gen
Sinal K Factor
K factor Calculator4
20
Kfactor 20
Signal Waveform Factor
Waveform Term Calculator4
0.4641
Waveform Term K20
Out1
K13 Wavef Gen
Sinal K Factor
K factor Calculator3
13
Kfactor 13
Signal Waveform Factor
Waveform Term Calculator3
0.5166
Waveform Term K13
Out1
K9 Wavef Gen
Sinal K Factor
K factor Calculator2
9
Kfactor 9
Signal Waveform Factor
Waveform Term Calculator2
0.5685
Waveform Term K9
Out1
K4 Wavef Gen
Sinal K Factor
K factor Calculator31
4
Kfactor 4
Signal Waveform Factor
Waveform Term Calculator1
0.6983
Waveform Term K4

Fig. 189

Formulas group:

$$ST := \sqrt{\frac{\omega\, Irms}{Iderms}}$$

$$\frac{R.eff}{Rdc} := 1 + \frac{\psi}{3}\cdot\Delta^4\cdot\left(\frac{Iderms}{\varpi\cdot Irms}\right)^2;$$

Or:

$$\frac{R.eff}{Rdc} := 1 + \frac{\psi}{3}\cdot\Delta^4\cdot\left(\frac{1}{st}\right)^4;$$

$$\Delta opt = \frac{\sqrt{\frac{\varpi\, Irms}{Iderms}}}{\psi^{1/4}} = \frac{1}{\psi^{1/4}}\cdot st$$

Where:

$$\psi = \frac{(5\cdot p^2 - 1)}{15}$$

$$\frac{Reff}{Rdc} = 1 + \frac{1}{3}\cdot\left(\frac{\Delta}{\Delta opt}\right)^4$$

$$d_{opt} = D_{opt} \times d_o$$

Sample project of a 50Kva three Phase distribution transformer for

Kfactor = 13

Let's make an example of a transformer distribution of 50 Kva of Kfactor=13.

The input is a three phase line of 380 Vca delta connection.

The output voltage is a three phase 127/220 Vca star connection.

The frequency is 60Hz.

The coil of the transformer is of aluminum.

The flux density is of 15000 Gausses.

First of all we see a problem at first glance. What is the kfactor of the current in the delta primary of this distribution transformer?

Exactly the same, because the transformation is done, involving only the secondary for the primary in one leg. This, do not interfere in the spectral composition of the waveform.

$B_m := 15000 \quad a := 1.026 \quad b := 4.232 \quad x := 1.30$

With all those, let's calculate our transformer:

$$Stack = A \times x$$

We have to apply the stacking factor in order to get the net stack for the magnetic calculation.

$$Netstack = 0.95 \times Stac$$

Magnetic Cross sectional area:

$$Netstack = 0.95 \times Stac$$

$$A_{ef} := A \times Netstac$$

$$A_{ef} = 100.03$$

Secondary:

$Vsec_{rms} := 127$

$$N_S := \frac{Vsec_{rms} \times 10^8}{4.44 \times f \times B_m \times A_{ef}}$$

$N_S = 31.771$

Rounding $N_S := 32$

With k factor of 20 through the table xx of page xx, we have ST=0.46

$$ST := 0.4641$$

If we are planing to use aluminum bar, then:

Number of layers:

$$p := 4$$

$$y := \frac{5 \cdot p^2 - 1}{15}$$

$$y = 5.267$$

So, the optimal Δ would be:

$$D_{opt} := \frac{1}{y^{\frac{1}{4}}} \cdot ST \qquad D_{opt} = 0.306$$

Now let's calculate the d_0 for the first harmonic. Then for aluminum at 180 degrees::

$$d_0 := \frac{104}{\sqrt{60}}$$

$$d_0 = 13.426$$

$$d_{opt} := D_{opt} \cdot d_0$$

$$d_{opt} = 4.113$$

$$l_w := A \cdot 10 \cdot b$$

$$l_w = 380.88$$

The Bar chosen is:

$W_{bar} := 12.7 \qquad H_{bar} := 3.969$

Turns per layer:

The Side Margin is:

$$m_l := 8$$

$$D_{fin} := \frac{H_{bar}}{d_0} \qquad D_{fin} = 0.296$$

Calculation of The relation between Reff and Rdc:

$$\frac{R_{eff}}{R_{dc}} = 1 + \frac{y}{3} \cdot D_{fin}^4 \cdot \left(\frac{1}{ST}\right)^4$$

$$R_{elres} := 1 + \frac{y}{3} \cdot D_{fin}^4 \cdot \left(\frac{1}{ST}\right)^4$$

$$R_{elres} = 1.289$$

Calculation of the number of turns per layer:

Calculation of the number of turns per layer:

$$\frac{l_W - 2 \cdot m}{(W_{bar} + 0.1) \cdot 3} \cdot 0.85 = 8.077$$ 8 turns per lay

$$\frac{N_S}{8} = 4$$ 4 Layers

Calculation of the secondary current:

$$I_{sec} := \frac{P_{out}}{3 \cdot Vsec_{rms}}$$

$$I_{sec} = 131.234$$

Current density:

$$D_i := 0.893$$

$$S_{cu} := \frac{I_{sec}}{D_i}$$

$$S_{cu} = 146.958$$

Conductor cross sectional area:

$$Acus := \frac{W_{bar} \cdot H_{bar} \cdot 3}{100}$$

Mean turn leght:

$$Acus = 1.512$$

$$lcus := 64$$

Calculation of the secondary resistance, loss and weight:

$$r := 3.908 \cdot 10^{-6}$$

$$Dens_{alu} := 2.77$$

$$R_{sec} := \frac{lcus \cdot N_S \cdot Relres}{Acus} \cdot r$$

$$R_{sec} = 6.822$$

$$Wcu_S := I_{sec}^2 \cdot R_{sec}$$

$$Wcu_S = 117.4$$

$$Vol_{cus} := \frac{W_{bar} \cdot H_{bar}}{100} \cdot 3 \cdot lcus \cdot N_S$$

$$Vol_{cus} = 3.097$$

$$P_{alus} := \frac{Dens_{alu} \times Vol_{cus}}{1000}$$

$P_{alus} = 8.579$

Primary:

$Vprim_{rms} := 380$

$$N_p := \frac{Vprim_{rms} \times 10^8}{4.44 \times f \times B_m \times A_{ef}}$$

$N_p = 95.062$

Rounding $N_p := 95$

The Bar chosen is:r

$W_{bar} := 12.7$ $H_{bar} := 3.969$

Turns per layer:

Side Margin:

$m_l := 8$

$$\frac{l_w - 2 \times m_l}{(W_{bar} + 0.1)} \times 0.85 = 24.23$$ 24 turns per layer

$$\frac{N_p}{24} = 3.958$$ (4Layers)

Height of the coil:

$$\frac{4 \times (H_{bar} + 0.2) + 0.3 + 4 \times (H_{bar} + 0.2) + 0.3}{0.82} + 4 = 45.405$$

$$I_{prim} := \frac{P_{out}}{3 \times Vprim_{rms}}$$

$I_{prim} = 43.86$

$$S_{prim} := \frac{I_{prim}}{D_i}$$

$S_{prim} = 49.115$

$$Acup := \frac{W_{bar} \cdot H_{bar}}{100} \qquad \text{Ok!}$$

Calculation of the primary resistance, loss and weight::

$$lcup := 51$$

$$R_{prim} := \frac{lcup \cdot N_p \cdot R_{elres}}{Acup} \cdot r$$

$$R_{prim} = 0.048$$

$$Wcu_p := I_{prim}^2 \cdot R_{prim}$$

$$Wcu_p = 93.141$$

$$Vol_{cup} := \frac{W_{bar} \cdot H_{bar}}{100} \cdot lcup \cdot N_p$$

$$Vol_{cup} = 2.442 \cdot 10^3$$

$$P_{alup} := \frac{Dens_{alu} \cdot Vol_{cup}}{1000}$$

$$P_{alup} = 6.765$$

$$Ptot_{al} := 3 \cdot P_{alup} + 3 \cdot P_{alus}$$

There are 3 coils in a three P
transformer

$$Ptot_{al} = 46.03$$

$$W_{altot} := 3 \cdot Wcu_p + 3 \cdot Wcu_s$$

$$W_{altot} = 631.902$$

Iron Core:

$$S_{lam} := 45.5 \cdot 56 - (38 \cdot 9.25 \cdot 2)$$

$$Vol_{fe} := S_{lam} \cdot A \cdot x \cdot 0.95$$

$$P_{fe} := \frac{7.65 \cdot Vol_{fe}}{1000}$$

$$DensPferro = 1.15 \ \frac{Watts}{Kg}$$

$$P_{fe} = 156.88$$

$$W_{fe} := 1.15 \cdot P_{fe}$$

$Ptot_{al} = 46.03$

$W_{altot} := 3 \times Wcu_p + 3 \times Wcu_s$

$W_{altot} = 631.902$

Iron Core:

$S_{lam} := 45.5 \times 56 - (38 \times 9.25 \times 2)$

$Vol_{fe} := S_{lam} \times A \times x \times 0.95$

$P_{fe} := \frac{7.65 \times Vol_{fe}}{1000}$

$DensPferro = 1.15 \ \frac{Watts}{Kg}$

$P_{fe} = 156.88$

$W_{fe} := 1.15 \times P_{fe}$

$W_{fe} = 180.412$

Temperature calculation:

$$D_t := \frac{(W_{fe} + W_{altot}) \times \frac{2}{3}}{\left[0.162 \times \left(\frac{2}{3}\right) \times \left(P_{fe} + Ptot_{al} \times \frac{8.96}{2.7}\right)\right]^{\frac{2}{3}}}$$

$D_t = 96.407$

$D_{hotspot} = 30$ Temperature difference between average and the hottest spot:

$T_{fin} := D_t + 45 + D_{hotspot}$

$T_{fin} = 171.407$ Ok For H class (18

Below in the next page, can be seen the 3 figures of the transformer coil. There can be found the values of the average magnetic path of the secondary and primary. Additionally can be seen in the third drawing the side margin ML in the two sides of the coil.

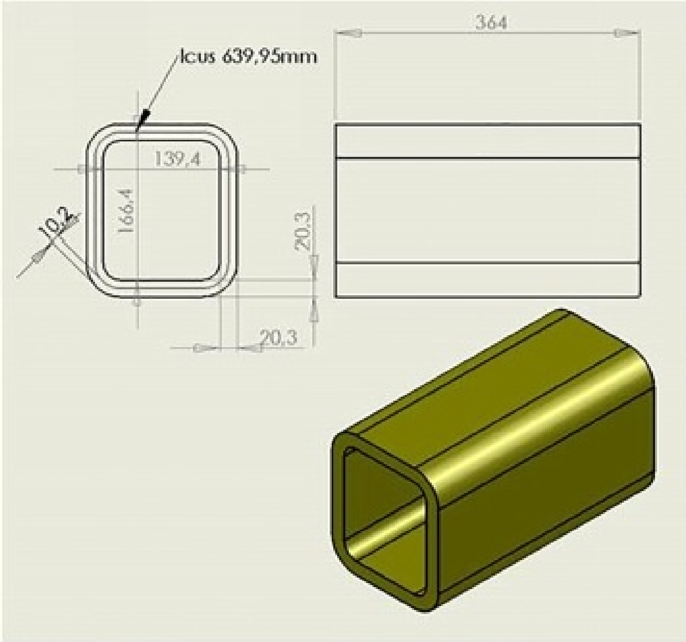


Fig. 190

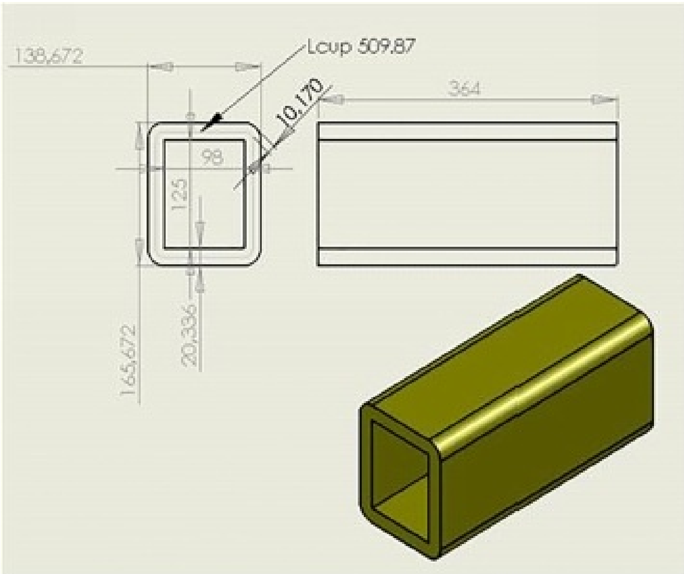


Fig. 191

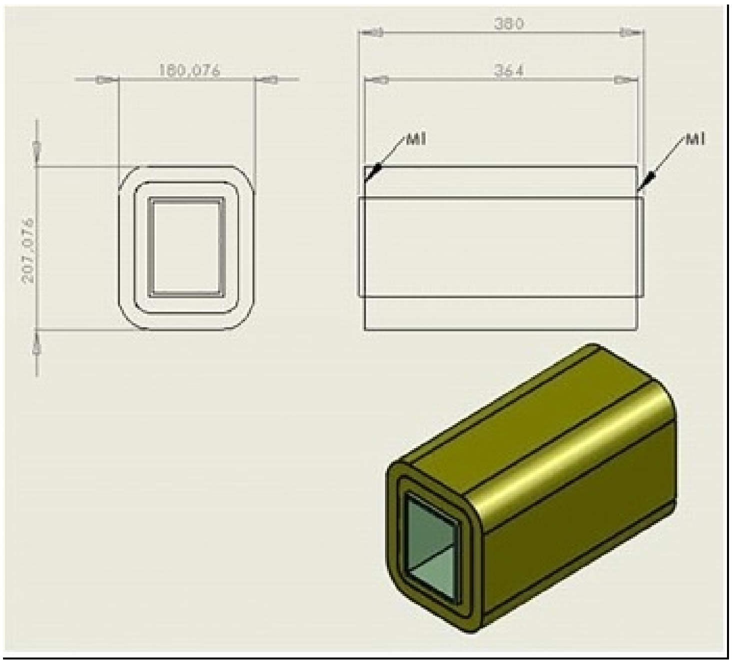


Fig. 192

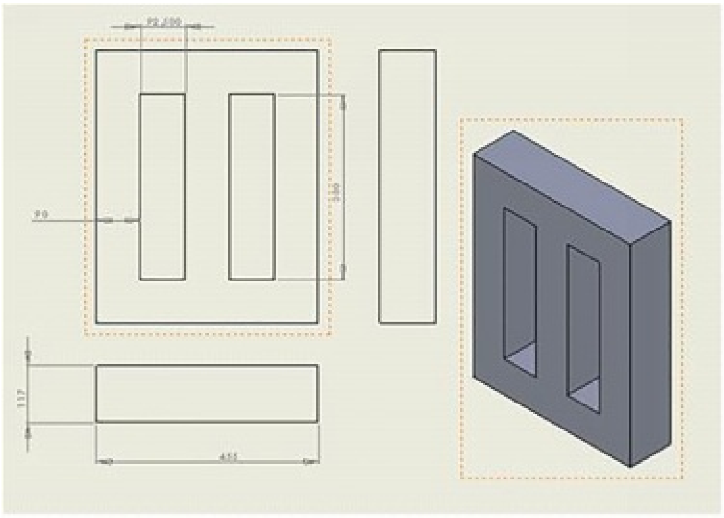


Fig. 193

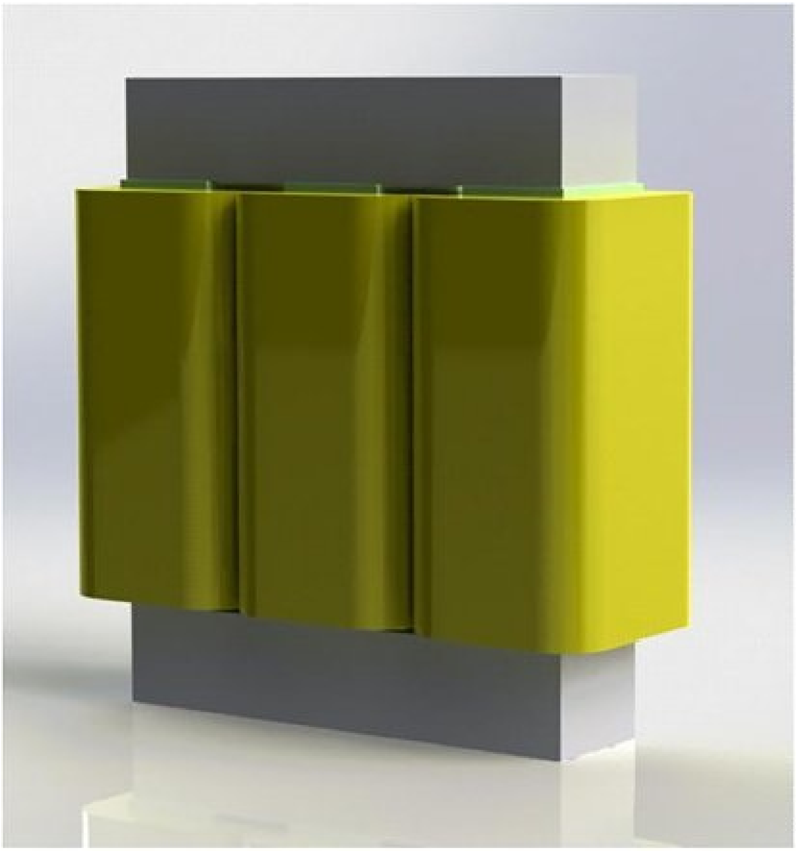


Fig. 194

Due to the impossibility of choosing an exactly thickness for a defined optimal “d” calculated, certain considerations must be done in order we get more advantageous results.

Let’s see with more accuracy our definition of the bar used in our project:

We have seen that the Dopt was 4.13 but by virtue of practicity, we must use standard bars, we have chosen a 3.969 height bar. What this means? We could have selected another one with bigger dimension. Why we have chosen this one?

To see why, we need plot the graph to see the consequences of the variation of the relation and see why we decided so.

Making:

$$\Lambda := \frac{d}{\delta o}$$

$$RelTrec := \frac{1}{\Delta} + \frac{\psi}{3} \cdot \Lambda^{4} \cdot \left(\frac{1}{ST}\right)^{-1}$$

We can plot it:

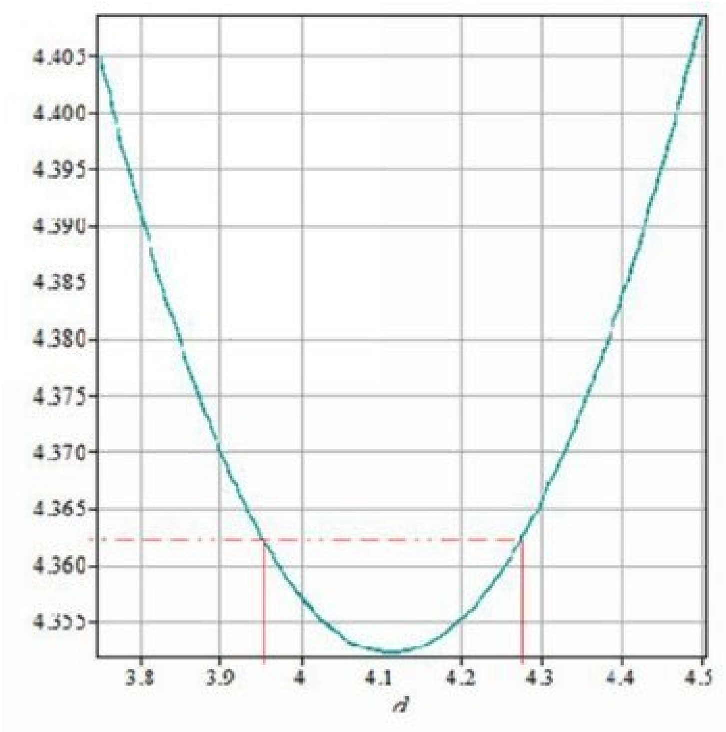


Fig. 195

Can be seen in fig 195 that we can choose in the two sides around the optimal point and the better are the left side ones, because we can use less conductor weight and consequently cheaper and also occupying less space.

Do not worry about the dimension of the vertical axis because it's only a relative measure:

Solutions for skin depth

The obvious reason about the replacement of the solid conductor by a Litz Wire:

The use of this conductor is due to the fact that of an improvement in the use of multiple areas of the conductors by individually possess better ratio d/ d . There is also the fact that multiple conductors, by twisting, to submerge several times in different field gradients H, creating eddy currents in lower locations, thereby reducing losses. The angle of twist and the settings are very important. There is Litz wire "True" and "False". The true litz wire, are twisted of twisted of twisted yarn twist … in various hierarchies, in a perfectly defined prescription, causing all conductors to pass through all positions, ensuring in addition to other effects, current equalization.

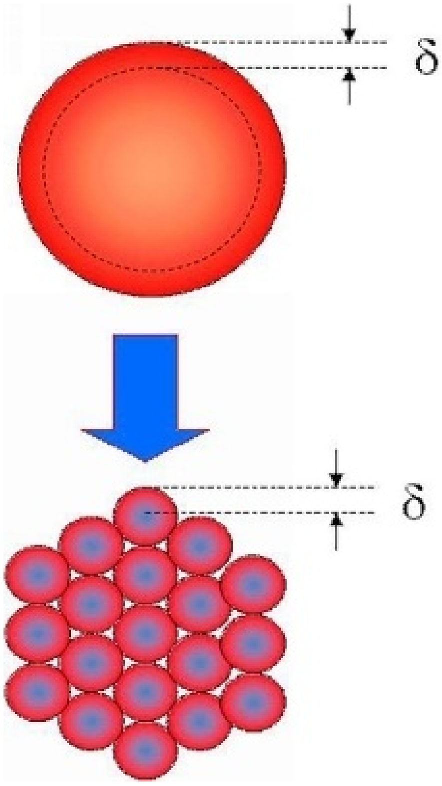


Fig. 196

False, they are only twisted and do not guarantee that all wires pass through everywhere. The simple twisted wire or "False Litz wire" is known in the English language as "bunched WIRE". In this paper these two forms will be indistinctly called "Litz wire".

Perhaps the most obvious way to reduce skin effect losses are by using the classic "Litz wire" (TWISTED WIRE).

If a circular conductor is replaced by a Litz wire twisted-n, it can be assumed that the wire was replaced by a square conductor divided by a number of layers equal to the square root of n.

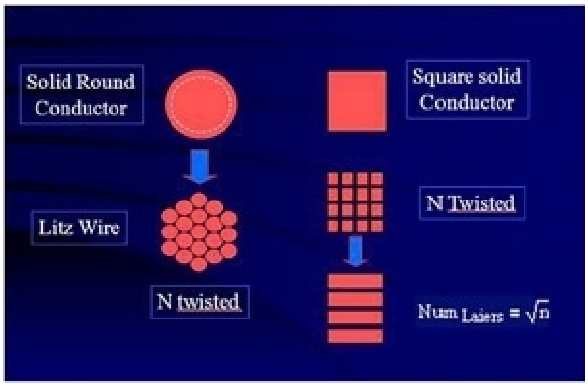


Fig. 196

Skin effect at strand level:

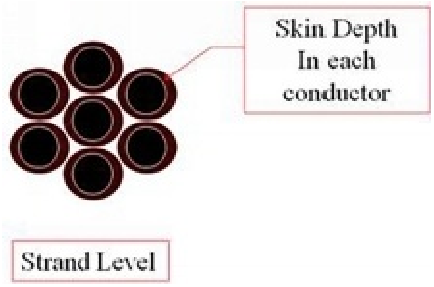


Fig. 197

We note that the effects produced in one conductor are duplicated in groups of conductors.

The skin depth effect observed in a conductor, can likewise be seen in groups of conductors, the obstruction to penetration of the magnetic field works like they were a solid conductor.

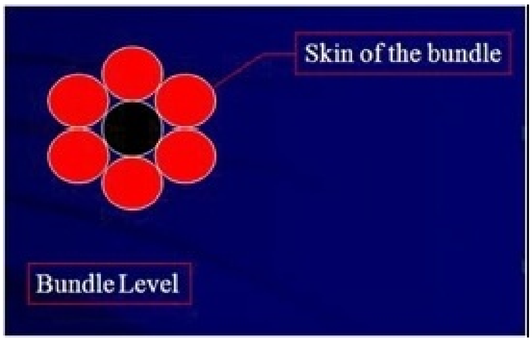


Fig. 198 – Proximity effect at bundle level

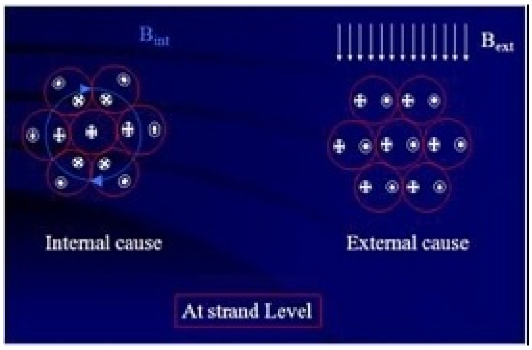


Fig. 199 – Proximity effect at strand level

Proximity effect at bundle level:

There are circulating currents (Eddy currents) in conductors produced by external f

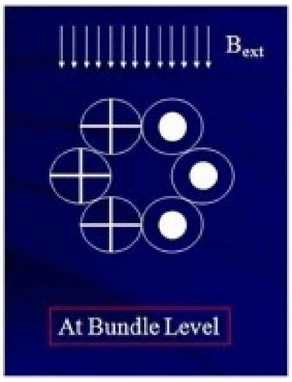


Fig. 200

The Litz wire is by far the most effective way to reduce high frequency losses for waveform close to sinusoidal. 120 twisted conductors, can reduce the losses of 42% can be obtained by solid wire for triangular waveforms to 70% square wave and pulsed current to 85%.

If the conductors of the twisted litz wire are too thick, the losses can be much larger than the solid conductors (Due to proximity effects to multiple layers)

Conductor plate or foil

The use of the copper plate is intended to reduce the thickness of the conductor, thereby maintaining its area. This decrease in thickness of the conductor plate is possible by having the width equal to the width of the window.

Using plate or foil decreases the thickness of the conductor, improving the ratio d / δ.

Winding plate or foil results in the greatest possible number of layers, for a given number of turns (assuming a single layer solid conductor), which only minimize losses if the thickness of the conductor is optimum.

The use o aluminum plate or foil is a good idea in the replacement of copper due to the fact that the increase of the skin depth produced by this metal pays off partially the effect of the resistivity but using a metal less heavy and cheaper.

Interleaving winding

Interleaving windings increases the number of “Portions” (This can create problems with available window size). The average loss (Net) can be reduced by this practice due to reduced amplitudes in the profile of FMM that reduces the field to which the portions are submitted and consequently reducing the circulating currents. This is without a doubt one of the most effective skin effect and eddy current loss reduction means at the highest levels of power.

Winding Construction for Minimum Losses:

Avoid big conductors.

Insulate attaching screws that passes through high field, including those ones that pass inside the core.

Avoid using metal parts inside the windings that do not work as coils. They produce strange effect.

Make a current confrontation where possible.

Use conductor sheet, plate or equivalent.

Use Litz wire.

Planar Windings

The planar winding is without a doubt the most suitable for very high frequency operations (In the range of megahertz) as the windings may be constructed from printed circuit plates where the copper layers are extremely thin (17 microns, 35 microns, 70 microns, or other), the windings can be intercalated with great ease, and consequently to use the conduction on both sides possibly no reverse currents (Optimum confrontation).

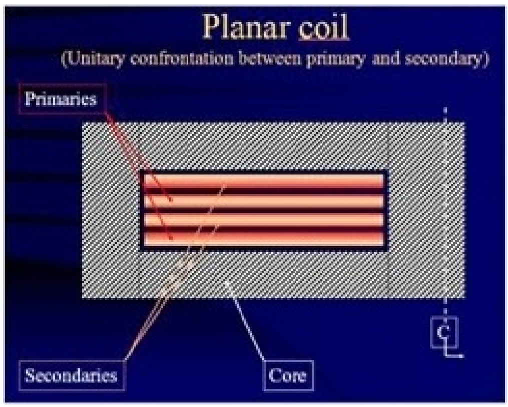


Fig. 202

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