Third Edition

Soils and Foundations

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To Kimmie, Jonathan, and Michele Liu
and
Linda, Susan, Scott, Sarah, and Sallie Evett

Contents

Preface

We have attempted to prepare an introductory, practical textbook for soil mechanics and foundations, which emphasizes design and practical applications that are supported by basic theory. Written in a simple and direct style that should make it very easy to read, understand, and grasp the subject matter, this book contains an abundance of both example problems in each chapter and work problems at the end of each chapter. Additionally, there are ample diagrams, charts, and illustrations throughout to help explain the subject matter better. In summary, we have tried to extract the salient and essential aspects of soils and foundations and to present these in a simple and straightforward manner.

The preceding paragraph, slightly modified, began the preface of both the first and second editions of *Soils and Foundations*, and we feel that it aptly relates to our basic philosophy in preparing this, the third edition. We have, however, updated material where applicable and added substantial amounts of new and essential material to the third edition. We believe the result is a much stronger, more comprehensive, and therefore better book.

The major feature of the third edition is the addition of substantial new material. Most significantly, we have included a totally new Chapter 5, entitled "Water in Soil." This much-needed chapter was suggested by a number of users of our first and second editions and covers flow of water in soils, capillary rise in soils, frost action in soils, and flow nets and seepage. We feel that this new chapter truly makes our book comprehensive in its coverage of soils and foundations.

We urge students using this book to review each illustration as it is cited and especially, to study each example problem very carefully. Believing that example problems are an extremely effective means of learning a subject such as soils and foundations, we have included an abundance of example problems, and we believe they will be very useful in mastering the material in the book.

We wish to express our sincere appreciation to Carlos G. Bell, formerly of The University of North Carolina at Charlotte, and to W. Kenneth Humphries, Dean of Engineering at the University of South Carolina, who read our original manuscript and offered many helpful suggestions. Also, we thank Donald Steila of the Department of Geography and Earth Science at The University of North Carolina at Charlotte, who reviewed Chapter 1. Finally, we thank those users of the previous editions of our book who communicated suggestions for improvements in the new edition.

We hope you will enjoy using the book. We would be pleased to receive your comments, suggestions, and/or criticisms.

Cheng Liu
Jack B. Evett
Charlotte, North Carolina

1

Formation of Natural Soil Deposits

1-1 INTRODUCTION

Soil is more or less taken for granted by the average person. It makes up the ground on which we live, it is for growing crops, and it makes us dirty. Beyond these observations, most people are not overly concerned with soil. There are, however, some people who *are* deeply concerned. These include certain engineers as well as geologists, contractors, hydrologists, farmers, agronomists, soil chemists, and others.

Most structures of all types rest either directly or indirectly upon soil, and proper analysis of the soil and design of the structure's foundation are necessary to ensure a safe structure free of undue settling and/or collapse. A comprehensive knowledge of the soil in a specific location is also important in many other contexts. Thus study of soils should be an important component in the education of civil engineers.

Chapter 1 relates the formation of natural soil deposits; it describes the sources of soil. Chapter 2 introduces and defines various engineering properties of soils. Subsequent chapters deal with evaluation of these properties and with essential interrelationships of soil with structures of various types.

1-2 ROCKS—THE SOURCES OF SOILS

Soil is comprised of particles, large and small, and it may be necessary to include as "soil" not only solid matter but also air and water. Normally, the particles are the result of weathering (disintegration and decomposition) of rocks and decay of vegetation. Some soil particles may, over a period of time, become consolidated under the weight of overlying material and become rock. Even cycles of rock disintegrating to form soil, soil being consolidated under great pressure and heat to form rock, rock disintegrating to form soil, and so on, have occurred repeatedly throughout geologic time. The differentiation between soil and rock is not sharp; but from an engineering perspective, if material can be removed without blasting, it is usually considered to be "soil," whereas if blasting is required, it might be regarded as "rock."

Rocks can be classified into three basic groups that reflect their origin and/or method of formation: *igneous, sedimentary,* and *metamorphic.*

Igneous Rocks

Igneous rocks form when magma (molten matter) such as that produced by erupting volcanoes cools sufficiently to solidify. Volcanic action, normally referred to as *volcanism,* can occur beneath or upon the earth's surface. Volcanoes probably produced the minority of earth's igneous rocks, however. During the earth's formative stages, its surface may well have been largely molten, thus not requiring magma to move to the surface from great depths. It is likely that great amounts of Precambrian rock formed in this fashion.

Igneous rocks can be coarse-grained or fine-grained depending on whether cooling occurred slowly or rapidly. Relatively slow cooling occurs when magma is trapped in the crust below the earth's surface (such as at the core of a mountain range), while more rapid cooling occurs if the magma reaches the surface while molten (e.g., lava flow).

Of coarse-grained igneous rocks, the most common is *granite,* a hard rock rich in quartz, widely used as a construction material and for monuments. Others are *syenites, diorites,* and *gabbros.* Most common of the fined-grained igneous rocks is *basalt,* a hard, dark-colored rock rich in ferromagnesian minerals and often used in road construction. Others are *rhyolites* and *andesites.*

Being generally hard, dense, and durable, igneous rocks often make good construction materials. Also, they typically have high bearing capacities and therefore make good foundation material.

Sedimentary Rocks

Sedimentary rocks comprise the great majority of rocks found on the earth's surface. They are formed when mineral particles, fragmented rock particles, and remains of certain organisms are transported by wind, water, and ice (with water being the predominant transporting agent) and deposited, typically in layers, to form sediments. Over a period of time as layers accumulate at a site, pressure on lower layers resulting from the weight of overlying strata hardens the deposits, forming sedimentary rocks. Additionally, deposits may be solidified and cemented by certain minerals (e.g., silica, iron oxides, calcium carbonate). Sedimentary rocks can be identified easily when their layered appearance is observable. The most common sedimentary rocks are *shale, sandstone, limestone,* and *dolomite.*

Shale, the most abundant of the sedimentary rocks, is formed by consolidation of clays or silts. Organic matter or lime may also be present. Shales have a laminated structure and often exhibit a tendency to split along laminations. They can become soft and revert to clayey or silty material if soaked in water for a period of time. Shales vary in strength from soft (may be scratched with a fingernail and easily excavated) to hard (requiring explosives to excavate). Shales are sometimes referred to as "claystone" or "siltstone," depending on whether they were formed from clays or silts, respectively.

Sandstone, consisting primarily of quartz, is formed by pressure and the cementing action of silica (SiO_2), calcite (calcium carbonate, $CaCO_3$), iron oxide, or clay. Strength and durability of sandstones vary widely depending on the kind of cementing material and degree of cementation as well as the amount of pressure involved.

Limestone is sedimentary rock comprised primarily of calcium carbonate hardened underwater by cementing action (rather than pressure); it may contain some clays or organic materials within fissures or cavities. Like shales and sandstones, the strength of limestones varies considerably from soft to hard (and therefore durable), with actual strength depending largely on the rock's texture and degree of cementation. (A porous texture means lower strength.) Limestones occasionally have thin layers of sandstone and often contain fissures, cavities, and caverns, which may be empty or partly or fully filled with clay.

Dolomites are similar in grain structure and color to limestones and are, in fact, limestones in which the calcite ($CaCO_3$) interbonded with magnesium. Hence, the principal ingredient of dolomites is calcium magnesium carbonate [$CaMg(CO_3)_2$]. Dolomites and limestones can be differentiated by placing a drop of dilute hydrochloric acid on the rock. A quick reaction forming small white bubbles is indicative of limestone; no reaction, or a very slow one, means that the rock is dolomite.

As indicated above, the degrees of strength and hardness of sedimentary rocks are variable, and engineering usage of such rocks varies accordingly. Relatively hard shale makes a good foundation material. Sandstones are generally good construction materials. Limestone and dolomite, if strong, can be both good foundation and construction materials.

Metamorphic Rocks

Metamorphic rocks are much less common at the earth's surface than sedimentary ones. They are produced when sedimentary or igneous rocks literally change their texture and structure as well as mineral and chemical composition, as a result of heat, pressure, and shear. Granite metamorphoses to *gneiss,* a coarse-grained, banded rock. *Schist,* a medium- to coarse-grained rock, results from high-grade metamorphism of both basalt and shale. Low-grade metamorphism of shale produces *slate,* a fine-textured rock that splits into sheets. Sandstone is transformed to *quartzite,* a highly weather-resistant rock; and limestone and dolomite change to *marble,* a hard rock capable of being highly polished. Gneiss, schist, and slate are *foliated* (layered); quartzite and marble are *nonfoliated.*

Metamorphic rocks can be hard and strong if unweathered. They can be good construction materials—marble is often used for buildings and monuments—but foliated metamorphic rocks often contain planes of weakness that can diminish strength. Metamorphic rocks sometimes contain weak layers between very hard ones.

1-3 ROCK WEATHERING AND SOIL FORMATION

As related in the preceding section, soil particles are the result of weathering of rocks and organic decomposition. Weathering is achieved by *mechanical* (*physical*) and *chemical* means.

Mechanical weathering disintegrates rocks into small particles by temperature changes, frost action, rainfall, running water, wind, ice, abrasion, and other physical phenomena. These cause rock disintegration by breaking, grinding, crushing, and so on. The effect of temperature change is especially important. Rocks subjected to large temperature variations expand and contract like other materials, possibly causing structural deterioration and eventual breakdown of rock material. When temperatures drop below the freezing point, water trapped in rock crevices freezes, expands, and can thereby break rock apart. Smaller particles produced by mechanical weathering maintain the same chemical composition as the original rock.

Chemical weathering causes chemical decomposition of rock, which can drastically change its physical and chemical characteristics. This type of weathering results from reactions of rock minerals with oxygen, water, acids, salts, and so on. It may include such processes as oxidation, solution (strictly speaking, "solution" is a physical process), carbonation, leaching, and hydrolysis. These cause chemical weathering actions that can (1) increase volume of material, thereby causing subsequent material breakdown, (2) dissolve parts of rock matter, yielding voids that make remaining matter more susceptible to breaking, and (3) react with the cementing material, thereby loosening particles.

The type of soil produced by rock weathering is largely dependent on rock type. Of igneous rocks, granites tend to decompose to silty sands and sandy silts with some clays. Basalts and other rocks containing ferromagnesian minerals (but little or no silica) decompose primarily to clayey soils. With regard to sedimentary rocks, decomposed shales will produce clays and silts, while sandstones again become sandy soils. Weathered limestones can produce a variety of soil types, with fine-grained ones being common. Of metamorphic rocks, gneiss and schist generally decompose to form silt-sand soils, while slate tends more to clayey soils. Weathered marble often produces fine-grained soils; quartzite decomposes to more coarse-grained soils, including both sands and gravels.

1-4 SOIL DEPOSITS

Soils produced by rock weathering can be categorized according to where they are ultimately deposited relative to the location of the parent rock. Some soils remain where they were formed simply overlying the rock from which they came. These are known as *residual soils.* Others are transported from their place of origin and deposited elsewhere. They are called *transported soils.*

Residual soils have general characteristics that depend in part on the type of rock from which they came. Particle sizes, shapes, and composition can vary widely, as do depths of residual soil deposits—all depending on amount and type of weathering. The actual depth of a residual soil deposit depends on the rate at which rock weathering has occurred at the location and the presence or absence of any erosive agents that would have carried soil away.

Transported soils are formed when rock weathers at one site and the particles are moved to another location. Some common transporting agents for particles are (1) gravity, (2) running water, (3) glaciers, and (4) wind. Transported soils can therefore be categorized with regard to these agents as *gravity deposits, alluvial deposits, glacial deposits,* and *wind deposits.*

Gravity Deposits

Gravity deposits are soil deposits transported by the effect of gravity. A common example is the landslide. Gravity deposits, which are not generally carried very far, tend to be loosely compacted and otherwise exhibit little change in the general character of soil material as a result of being transported.

Alluvial Deposits

Alluvial deposits, having been transported by moving water, are found in the vicinity of rivers. Rainwater falling on land areas runs overland, eroding and transporting soil and rock particles as it goes, and eventually enters a creek or river. Continuously moving water can carry particles and deposit them a considerable distance from their former location. All soils carried and deposited by flowing water are called *alluvial deposits.* Lack of vegetation may allow enormous amounts of erosion leading to vast alluvial deposits (e.g., the Mississippi Delta).

Rivers are capable of transporting particles of all sizes, ranging from very fine silts in suspension to, in some cases, large boulders. The greater the velocity of river flow, the larger will be the size of particles that can be carried.

Hence, a sluggish creek may carry only fine-grained sediment while a flooding river transports all particle sizes, including large rocks. The relationship between river velocity and size of particle carried also affects the manner in which particles are deposited. As river velocity decreases, relatively larger particles settle and are deposited first. If the velocity decreases further, the next-larger-size particles settle out.

Alluvial deposits are often comprised of various soil types because different types of soil tend to mix as they are carried downstream. They do, however, tend to be layered, since settling rates are proportional to particle size.

The nature of soil can be greatly influenced by past alluvial transport and deposits. For example, at a location where a river's velocity decreases, such as when the channel widens significantly or its slope decreases substantially, coarser soil particles settle, forming submerged, flat, triangular deposits, known as *alluvial fans.* When flooding rivers, which normally carry heavy sediment load, overflow their banks, the overflowing water experiences a decrease in velocity. Larger particles, such as sands and gravels, tend to settle more quickly; their deposits can form *natural levees* along river banks (see Fig. 1-1). (These natural levees may someday be washed away by a more severe flood.) Smaller particles, such as silts and clays, settle less quickly, forming *floodplain deposits* in areas beyond the levees (Fig. 1-1). (However, smaller rivers can have floodplain deposits without forming levees.)

Another type of alluvial deposit occurs when rivers meander (i.e., follow a winding and turning course). As water moves through a channel bend, velocity along the inside edge decreases while that along the outer one increases. Consequently, particle erosion may occur along the outer edge with deposition along the inner edge. This action can, over a period of time, increase the amount of bend and significantly alter the river channel and adjacent land area. Eventually, the river may cut across a large bend, as shown in Fig. 1-2, leaving the old channel bend isolated. Water remaining in the isolated bend forms an *oxbow lake,* which can eventually fill in with floodplain deposits (usually silty and organic materials). Ultimately, the entire filled-in oxbow lake may be covered by additional floodplain deposits, leaving a hidden deposit of undesirable, high-plastic and/or organic silt, silty clay, and peat.

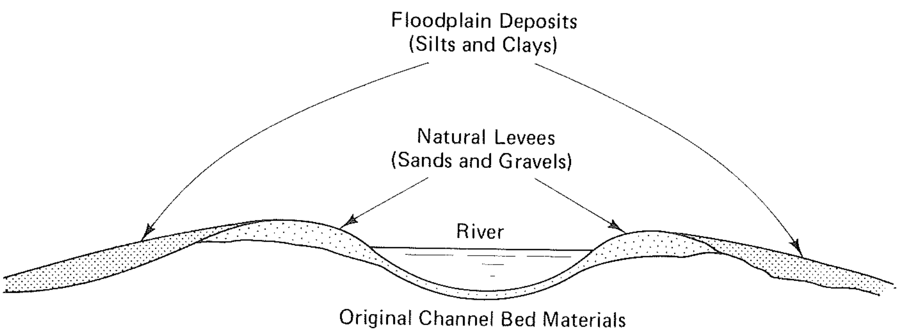

FIGURE 1-1 Natural levees and floodplain deposits.

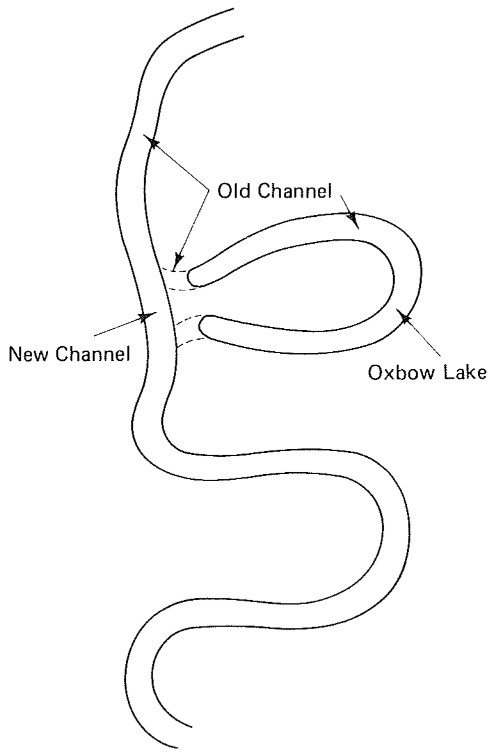

FIGURE 1-2 Oxbow lake.

Sediments deposited at the mouths of creeks and rivers flowing into lakes, bays, or seas are known as *deltas.* Those deposited in lakes and seas are called *lacustrine* and *marine deposits,* respectively. These deposits tend to be loose and compressible and may contain organic material. They are therefore generally undesirable from an engineering point of view.

Glacial Deposits

Glacial deposits result, of course, from the action of glaciers. Many years ago (over 10,000), glaciers, unimaginably enormous sheets of ice, moved southward across much of the northern United States (as well as Europe and other areas). As they progressed, virtually everything in their paths, including soils and rocks ranging in size from the finest clays to huge boulders, was picked up and transported. As they were being carried by glaciers, soils and rocks were mixed together, thrashed about, broken, crushed, and so on, by enormous internal glacial pressures. Consequently, glacial deposits can contain all types of soils.

Some soil particles were directly deposited by moving glaciers; others were taken from glaciers by water flowing from the ice to be deposited in lakes or transported in rivers flowing away from the ice; still others were deposited *en masse* when glaciers ultimately melted and disappeared. Direct glacial deposits, known as *moraines,* are heterogeneous mixtures comprised of all sizes of particles (from boulders to clay) that the ice accumulated as it traveled. *Eskers* are ridges or mounds of boulders, gravel, and sand formed when such materials flowing in streams on, within, or beneath glaciers were deposited as the stream's bedload.

The quality of soils in glacial deposits as foundations and construction materials is somewhat variable because of the different types of soils found in them. Often they make good such materials because of the intense compaction they have undergone, although ones containing mostly clays are not as strong, are often compressible, and may therefore cause problems if used for foundations or construction materials.

Wind Deposits

Wind deposits (also known as *aeolian deposits*) obviously have wind as the transporting agent. Wind is a very important agent in certain areas and has the potential to move soil particles over large distances.

Winds can move sandy soil particles by rolling them along the ground as well as sending them short distances through the air. Wind-deposited sands are known as *dunes,* and they tend to occur in sandy desert areas and along sandy beaches on the downwind side. Sands from dunes can be used for certain construction purposes.

Fine-grained soils can be airborne over long distances by winds. Silty soils are more amenable than clayey ones to wind transport, however, because the latter's bonding or cohesion reduces their wind erosion. A wind-deposited silt is known as *loess,* significant deposits of which are found in the general vicinity of the Mississippi and Missouri Rivers in the United States and in Europe and Asia (especially northern China). Loess is generally a hard and stable soil when unsaturated because of cementation from calcium carbonate and iron oxide. It tends to lose its cementation when wetted, however, and become soft and mushy. Loessial deposits typically have a yellow-brown (buff) color, low density, and relatively uniform grain size. They are generally able to stand on vertical cuts and exhibit high vertical permeability. Because of low strength when wet, special care must be taken when designing and constructing foundations over loessial deposits.

Ashes from erupting volcanoes can also produce wind deposits. Consisting of fine-sized igneous rock fragments, volcanic ash is light and porous and deposits tend to decompose quickly, often changing into plastic clays. The great Mt. St. Helens eruption produced not lava but ash.

It should be noted in concluding this section that soil deposits seldom occur in nature in neat "packages"—that is, a soil of exactly the same type at all depths throughout a construction site. An area with "original" glacial deposits may subsequently have been overlain by alluvial deposits possessing different characteristics. Even if all the soil at a given job site is of the same deposit, its properties may vary from place to place throughout the site.

For these reasons, subsurface investigation of an area is extremely important. One cannot just look at the surface and know what is beneath. Using quantitative results obtained from subsurface investigation together with qualitative knowledge of the origins of the soil(s) at the site, soils engineers can produce an adequate foundation design to ensure against failure or undue settling of a structure. (Subsurface investigation is covered in Chap. 3.)

2

Engineering Properties of Soils

2-1 SOIL TYPES

Soils may be separated into three very broad categories: *cohesionless, cohesive,* and *organic* soils. In the case of cohesionless soils, the soil particles do not tend to stick together. Cohesive soils are characterized by very small particle size where surface chemical effects predominate. The particles do tend to stick together—the result of water–particle interaction and attractive forces between particles. Cohesive soils are therefore both sticky and plastic. Organic soils are typically spongy, crumbly, and compressible. They are undesirable for use in supporting structures.

Three common types of cohesionless soil are *gravel, sand,* and *silt.* Gravel has particle sizes greater than about 2 millimeters (mm); whereas particle sizes for sand range from about 0.1 to 2 mm. Both gravel and sand may be further divided into "fine" (as fine sand) and "coarse" (as coarse sand). Gravel and sand can be classified according to particle size by sieve analysis. Silt has particle sizes that range from about 0.005 to 0.1 mm.

The common type of cohesive soil is clay, which has particle sizes less than about 0.005 mm. Clay soils cannot be separated by sieve analysis into size categories because no practical sieve can be made with openings so small; instead, particle sizes may be determined by observing settling velocities of the particles in a water mixture.

Soils can also be categorized strictly in terms of grain size. Two such categories are *coarse-grained* and *fine-grained.* Gravel and sand, with soil grains coarser than 0.075 mm or a No. 200 sieve size, are coarse-grained; silt and clay, with soil grains finer than 0.075 mm, are fine-grained.

Some more precise classifications of these soil types by particle size according to two systems—the American Association of State Highway and Transportation Officials (AASHTO) system and the Unified Soil Classification System (USCS)—are given in Table 2-1. It is clear from variations between these classifications that boundaries between soil types are more or less arbitrary.

In most applications in this book, soils are categorized as cohesionless or cohesive, with cohesionless generally implying a sandy soil and cohesive, a clayey soil. Some soils encountered in practice are mixtures of both types and therefore exhibit characteristics of both.

2-2 GRAIN-SIZE ANALYSIS AND ATTERBERG LIMITS

Never will a natural soil be encountered in which all particles are exactly the same size and shape. Both cohesionless and cohesive soils as well as mixtures of the two will always contain particles of varying sizes. Properties of a soil are greatly influenced by the sizes of its particles and distribution of grain sizes throughout the soil mass. Hence, in many engineering applications, it is not sufficient to know only that a given soil is clay, sand, rock, gravel, or silt. It is also necessary to know something about the distribution of grain sizes of the soil.

In the case of most cohesionless soils, distribution of grain size can be determined by sieve analysis. A sieve is similar to a cook's flour sifter. It is an apparatus containing a wire mesh with openings the same size and shape. When soil is passed through a sieve, soil particles smaller than the opening size of the sieve will pass through while those larger than the opening size will be retained. Certain sieve-size openings between 4.75 and 0.075 mm are designated by U.S. Standard Sieve Numbers, as given in Table 2-2. Thus grain sizes within this range can be classified according to U.S. Standard Sieve Numbers.

In practice, sieves of different opening sizes are stacked, with the largest opening size at the top and a pan at the bottom. Soil is poured in at the top, and soil particles pass downward through the sieves until they are retained on a particular sieve (see Fig. 2-1). The stack of sieves is mechanically agitated during this procedure. At the end of the procedure, the soil particles retained on each sieve can be weighed and the results presented graphically in the form of a grain-size distribution curve. This is normally a semilog plot with grain size (diameter) along the abscissa on a logarithmic scale and percentage passing that grain size along the ordinate on an arithmetic scale. Example 2-1 illustrates the analysis of the results of a sieve test, including the preparation of a grain-size distribution curve.

TABLE 2-1 Soil classification based on grain size.[1]

	Coarse-Grained			Fine-Grained	
Agency	*Gravel*	*Coarse Sand*	*Fine Sand*	*Silt*	*Clay*
AASHTO	75–2.00 (3 in.–No.10 sieves)	2.00–0.425 (No. 10–No. 40 sieves)	0.425–0.075 (No. 40–No. 200 sieves)	0.075–0.002	<0.002
USCS	Coarse: 75–19.0 (3 in.–¾ in. sieves)	4.75–2.00 (No. 4–No. 10 sieves)	0.425–0.075 (No. 40–No. 200 sieves)	Fines <0.075 (silt or clay)	
	Fine: 19.0–4.75 (¾ in.–No. 4 sieves)	Medium sand: 2.00–0.425 (No. 10–No. 40 sieves)			

[1]All grain sizes are in millimeters.

TABLE 2-2 U.S. Standard Sieve Numbers and their sieve openings.

U.S. Standard Sieve Numbers	*Sieve Opening (mm)*
4	4.75
10	2.00
20	0.850
40	0.425
60	0.250
100	0.150
200	0.075

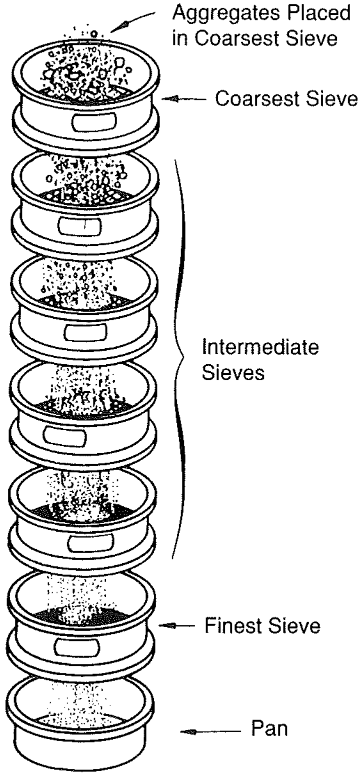

FIGURE 2-1 Sieve analysis. [1]*

EXAMPLE 2-1

Given

An air-dry soil sample weighing 2000 grams (g) is brought to the soils laboratory for mechanical grain-size analysis. The laboratory data are as follows:

* Numbers in brackets refer to the references at the end of each chapter.

U.S. Sieve Size	Size Opening (mm)	Weight Retained (g)
¾ in.	19.0	0
⅜ in.	9.50	158
No. 4	4.75	308
No. 10	2.00	608
No. 40	0.425	652
No. 100	0.150	224
No. 200	0.075	42
Pan	—	8

Required

A grain-size distribution curve for this soil sample.

Solution

To plot the gradation curve, percentage retained on each sieve, cumulative percentage retained, and percentage passing through each sieve must be calculated and the results tabulated as shown in Table 2-3.

$$\text{Total sample weight} = 2000 \text{ g}$$

1. The percentage retained on each sieve is obtained by dividing the weight retained on each sieve by the total sample weight. Thus,

$$\text{Percentage retained on } \tfrac{3}{4}\text{-in. sieve} = \frac{0 \text{ g}}{2000 \text{ g}} \times 100\% = 0\%$$

$$\text{Percentage retained on } \tfrac{3}{8}\text{-in. sieve} = \frac{158 \text{ g}}{2000 \text{ g}} \times 100\% = 7.9\%$$

$$\text{Percentage retained on No. 4 sieve} = \frac{308 \text{ g}}{2000 \text{ g}} \times 100\% = 15.4\% \qquad \text{etc.}$$

Therefore,

$$\text{Column (4)} = \frac{\text{column (3)}}{\text{total sample weight}} \times 100\%$$

2. Cumulative percentage retained on each sieve is obtained by summing percentage retained on all coarser sieves. Thus,

$$\text{Cumulative percentage retained on } \tfrac{3}{4}\text{-in. sieve} = 0\%$$

$$\text{Cumulative percentage retained on } \tfrac{3}{8}\text{-in. sieve} = 0\% + 7.9\% = 7.9\%$$

$$\text{Cumulative percentage retained on No. 4 sieve} = 7.9\% + 15.4\% = 23.3\%$$

$$\text{Cumulative percentage retained on No. 10 sieve} = 23.3\% + 30.4\% = 53.7\% \qquad \text{etc.}$$

3. Percentage passing through each sieve is obtained by subtracting from 100% the cumulative percentage retained on the sieves. Thus,

   Percentage passing through ¾-in. sieve = 100% − 0% = 100%

   Percentage passing through ⅜-in. sieve = 100% − 7.9% = 92.1%

   Percentage passing through No. 4 sieve = 100% − 23.3%
   = 76.7% etc.

   Therefore, column (6) = 100% − column (5).

4. Upon completion of these calculations, the grain-size distribution curve is obtained by plotting column (2), sieve opening (mm), versus column (6), percentage passing through, on semilog paper. Percentage passing is always plotted as the ordinate on arithmetic scale and sieve opening as the abscissa on log scale (see Fig. 2-2).

Several useful parameters can be determined from grain-size distribution curves. The diameter of soil particles at which 50% passes (i.e., 50% of the soil by weight is finer than this size) is known as the *median size* and denoted by D_{50}. The diameter at which 10% passes is called the *effective size* and denoted by D_{10}. Two "coefficients" used only in the Unified Soil Classification System (see Sec. 2-3) are the *coefficient of uniformity* (C_u) and the *coefficient of curvature* (C_c), which are defined as follows:

$$C_u = \frac{D_{60}}{D_{10}} \tag{2-1}$$

$$C_c = \frac{(D_{30})^2}{D_{60}D_{10}} \tag{2-2}$$

TABLE 2-3 Sieve analysis data.

(1) Sieve Number	(2) *Sieve Opening (mm)*	(3) *Weight Retained (g)*	(4) *Percentage Retained*	(5) *Cumulative Percentage Retained*	(6) *Percentage Passing*
¾ in.	19.0	0	0	0	100
⅜ in.	9.50	158	7.9	7.9	92.1
No. 4	4.75	308	15.4	23.3	76.7
No. 10	2.00	608	30.4	53.7	46.3
No. 40	0.425	652	32.6	86.3	13.7
No. 100	0.150	224	11.2	97.5	2.5
No. 200	0.075	42	2.1	99.6	0.4
Pan	—	8	0.4	100.0	—

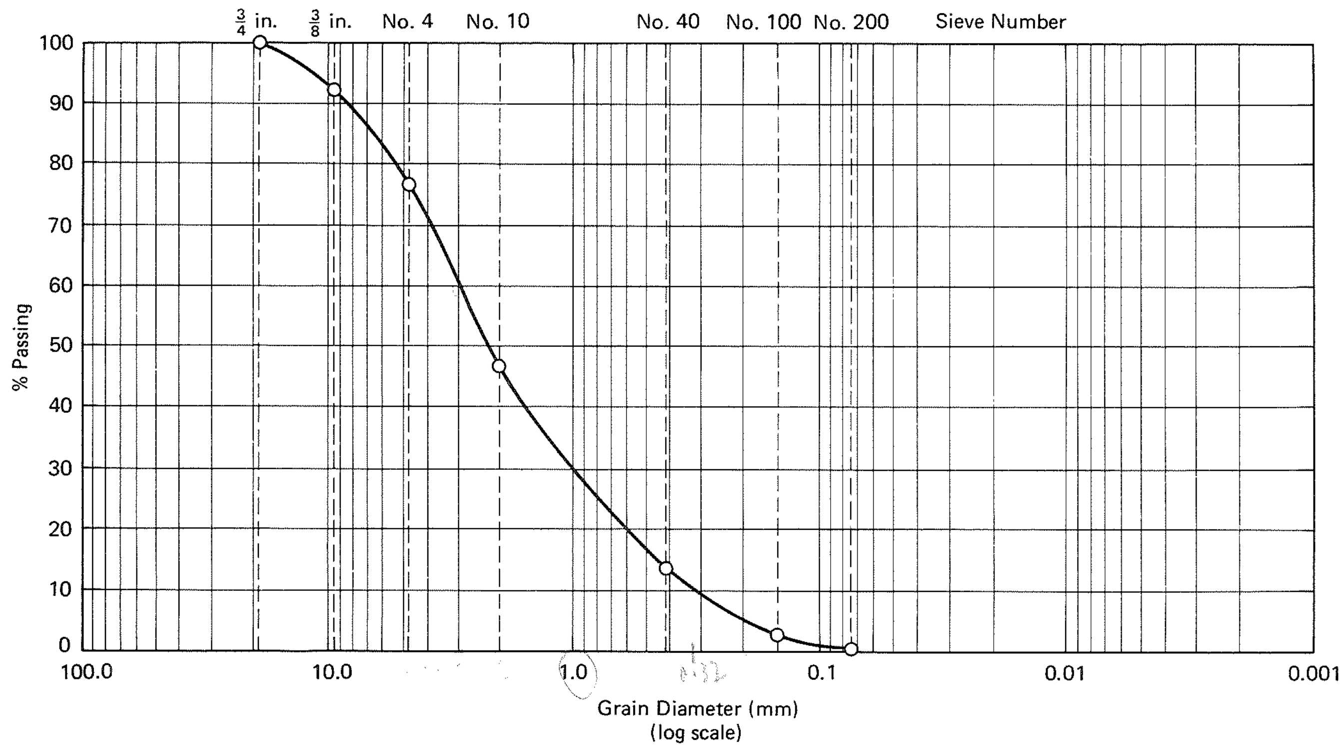

FIGURE 2-2 Grain-size distribution curve.

where D_{60} and D_{30} are the soil particle diameters corresponding to 60% and 30%, respectively, passing on the cumulative grain-size distribution curve.

Median size gives an "average" particle size for a given soil sample; other parameters offer some indication of particle size range. Effective size gives the maximum particle diameter of the smallest 10% of soil particles. It is this size to which permeability and capillarity are related. C_u and C_c have little or no meaning when more than 5% of the soil is finer than a No. 200 sieve opening (0.075 mm).

In the case of cohesive soils, distribution of grain size is not determined by sieve analysis because the particles are too small. Particle sizes may be determined by the hydrometer method, which is a process for indirectly observing the settling velocities of the particles in a soil–water mixture. Another valuable technique for analyzing cohesive soils is by use of *Atterberg limits,* which will be described in the remainder of this section.

Atterberg [2, 3] defined four states of consistency for cohesive soils. (Consistency refers to their degree of firmness.) These states are *liquid, plastic, semisolid,* and *solid* (Fig. 2-3). The dividing line between liquid and plastic states is the *liquid limit*; the dividing line between plastic and semisolid states is the *plastic limit*; and the dividing line between semisolid and solid states is the *shrinkage limit* (see Fig. 2-3). If a soil in the liquid state is gradually dried out, it will pass through the liquid limit, plastic state, plastic limit, semisolid state, and shrinkage limit, and will reach the solid state. The liquid, plastic, and shrinkage limits are quantified therefore in terms of water content. For example, the liquid limit is reported in terms of the water content at which soil changes from the liquid state to the plastic state. The difference between the liquid limit (LL) and plastic limit (PL) is the *plasticity index* (PI), that is,

$$PI = LL - PL \tag{2-3}$$

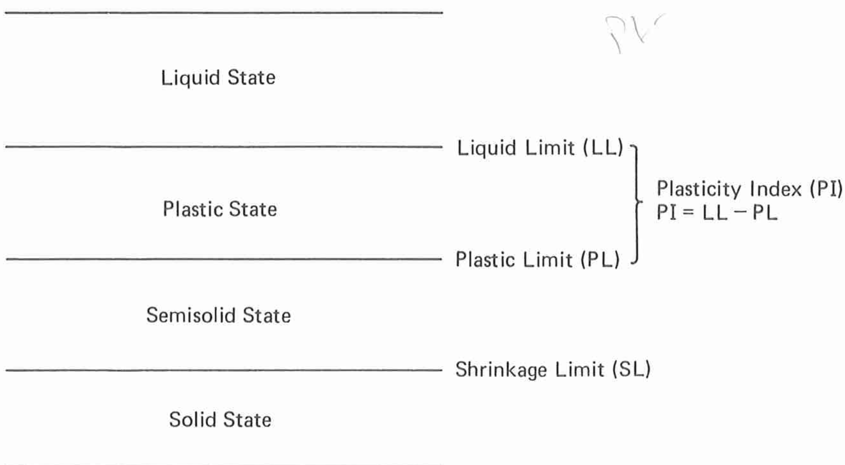

FIGURE 2-3 Atterberg limits. [3]

The liquid, plastic, and shrinkage limits and the plasticity index are useful parameters in classifying soils and in making judgments as to their applications.

Standard laboratory test procedures are available to determine Atterberg limits. Although Atterberg defined the four states of consistency for cohesive soils, his original consistency limit tests were somewhat arbitrary and did not yield entirely consistent results. Subsequently, Casagrande standardized the tests, thereby increasing reproducibility of test results.

Casagrande developed a *liquid limit device* for use in determining liquid limits. As shown in Fig. 2-4, it consists essentially of a "cup" that is raised and dropped 10 mm by a manually-rotated handle. In performing a liquid limit test, a standard groove is cut in a remolded soil sample in the cup using a standard grooving tool. The liquid limit is defined as that water content at which the standard groove will close a distance of ½ in. (12.7 mm) along the bottom of the groove at exactly 25 blows (drops) of the cup. Since it is difficult to mix the soil with the precise water content at which the groove will close ½ in. at exactly 25 blows, tests are usually run on samples with differing water contents and a straight-line plot of water content versus the logarithm of the number of blows required to close the groove ½ in. is prepared. From this plot, which is known as a *flow curve,* the particular water content corresponding to 25 blows is read and reported as the liquid limit.

The plastic limit is evaluated quantitatively in the laboratory by finding the water content at which a thread of soil begins to crumble when it is manually rolled out on a glass plate to a diameter of ⅛ in. and breaks up into segments about ⅛ to ⅜ in. (3 to 10 mm) in length. If threads can be rolled to smaller diameters, the soil is too wet (i.e., it is above the plastic limit). If

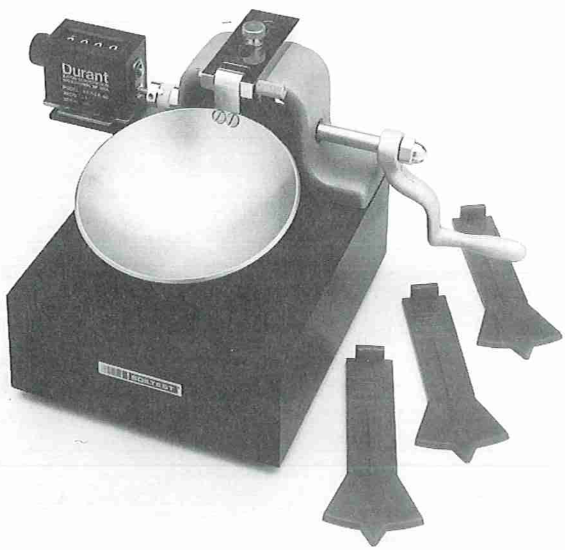

FIGURE 2-4 Liquid limit device. (Courtesy of Soiltest, Inc.)

threads crumble before reaching the ⅛-in. diameter, the soil is too dry and the plastic limit has been surpassed.

Detailed procedures for laboratory determinations of liquid and plastic limits as well as the shrinkage limit are given in *Soil Properties: Testing, Measurement, and Evaluation,* 2nd edition, by Liu and Evett [4].

2-3 SOIL CLASSIFICATION SYSTEMS

In order to be able to describe, in general, a specific soil without listing values of its many soil parameters, it would be convenient to have some kind of generalized "classification system." In practice, there have evolved a number of different classification systems, most of which were developed to meet specific needs of the particular group that developed a given system. Today, however, only two such systems—the American Association of State Highway and Transportation Officials (AASHTO) system and the Unified Soil Classification System (USCS)—are widely used in engineering practice.

The AASHTO system is widely used in highway work and is followed by nearly all state departments of highways and/or transportation in the United States. Most federal agencies (such as the U.S. Army Corps of Engineers and the U.S. Department of the Interior, Bureau of Reclamation) use the USCS; it is also utilized by many engineering consulting companies and soil-testing laboratories in the United States. Both of these classification systems are presented in detail later in this section.

Some years ago, the Federal Aviation Administration (FAA) had its own soil classification system, known appropriately as the FAA classification system, for designing airport pavements. Now, however, the FAA uses the USCS. If one needs information about the FAA classification system, it can be found in the first two editions of this book.

AASHTO Classification System [5]

In the late 1920s, the U.S. Bureau of Public Roads (currently the Federal Highway Administration) developed a soil classification system known as the BPR classification system. It was subsequently revised several times, with the 1945 revision becoming what is basically today's AASHTO system.

Required parameters for classification by the AASHTO system are grain-size analysis, liquid limit, and plasticity index. With values of these parameters known, one enters the first (left) column of Table 2-4 and determines whether or not known parameters meet the limiting values in that column. If they do, then the soil classification is that given at the top of the column (A-1-a, if known parameters meet the limiting values in the first column). If they do not, one enters the next column (to the right) and determines whether or not known parameters meet the limiting values in that column. The procedure is repeated until the *first* column is reached in which known parameters meet the limiting values in that column. The soil classification for the given soil is indicated at the top of that particular column.

Once a soil has been classified using Table 2-4, it can be further described using a *group index*. It utilizes the percent of soil passing a No. 200 sieve, the liquid limit, and the plasticity index. With known values of these parameters, the group index is computed from the equation

$$\text{Group index} = (F - 35)[0.2 + 0.005(LL - 40)] + 0.01(F - 15)(PI - 10) \qquad (2\text{-}4)$$

where F = percentage of soil passing a No. 200 sieve
LL = liquid limit
PI = plasticity index

The group index computed from Eq. (2-4) is rounded off to the nearest whole number and appended in parentheses to the group designation determined from Table 2-4. If the computed group index is either zero or negative, the number zero is used as the group index and should be appended to the group designation. If preferred, Fig. 2-5 may be used instead of Eq. (2-4) to determine the group index.

As a general rule, the value of soil as a subgrade material is in inverse ratio to its group index (i.e., the lower the index, the better the material). Table 2-5 gives some general descriptions of the various classification groups according to the AASHTO System.

EXAMPLE 2-2

Given

A sample of soil was tested in the laboratory and results of the laboratory tests were as follows:

1. Liquid limit = 42.3%.
2. Plastic limit = 15.8%.
3. The following sieve analysis data:

U.S. Sieve Size	*Percentage Passing*
No. 4	100
No. 10	93.2
No. 40	81.0
No. 200	60.2

TABLE 2-4 Classification of soils and soil-aggregate mixtures by AASHTO Classification System [5].

General Classification	Granular Materials (35% or less passing 0.075 mm)							Silt-Clay Materials (more than 35% passing 0.075 mm)			
	A-1			*A-2*							*A-7*
Group Classification	*A-1-a*	*A-1-b*	*A-3*	*A-2-4*	*A-2-5*	*A-2-6*	*A-2-7*	*A-4*	*A-5*	*A-6*	*A-7-5, A-7-6*
Sieve analysis: percent passing:											
2.00 mm (No. 10)	50 max.	—	—	—	—	—	—	—	—	—	—
0.425 mm (No. 40)	30 max.	50 max.	51 min.	—	—	—	—	—	—	—	—
0.075 mm (No. 200)	15 max.	25 max.	10 max.	35 max.	35 max.	35 max.	35 max.	36 min.	36 min.	36 min.	36 min.
Characteristics of fraction passing 0.425 mm (No. 40)											
Liquid limit	—		—	40 max.	41 min.	40 max.	41 min.	40 max.	41 min.	40 max.	41 min.
Plasticity index	6 max.		N.P.	10 max.	10 max.	11 min.	11 min.	10 max.	10 max.	11 min.	11 min.[1]
Usual types of significant constituent materials	Stone fragments, gravel, and sand		Fine sand	Silty or clayey gravel and sand				Silty soils		Clayey soils	
General ratings as subgrade	Excellent to good					Fair to poor					

[1]Plasticity index of A-7-5 subgroup is equal to or less than LL minus 30. Plasticity index of A-7-6 subgroup is greater than LL minus 30.

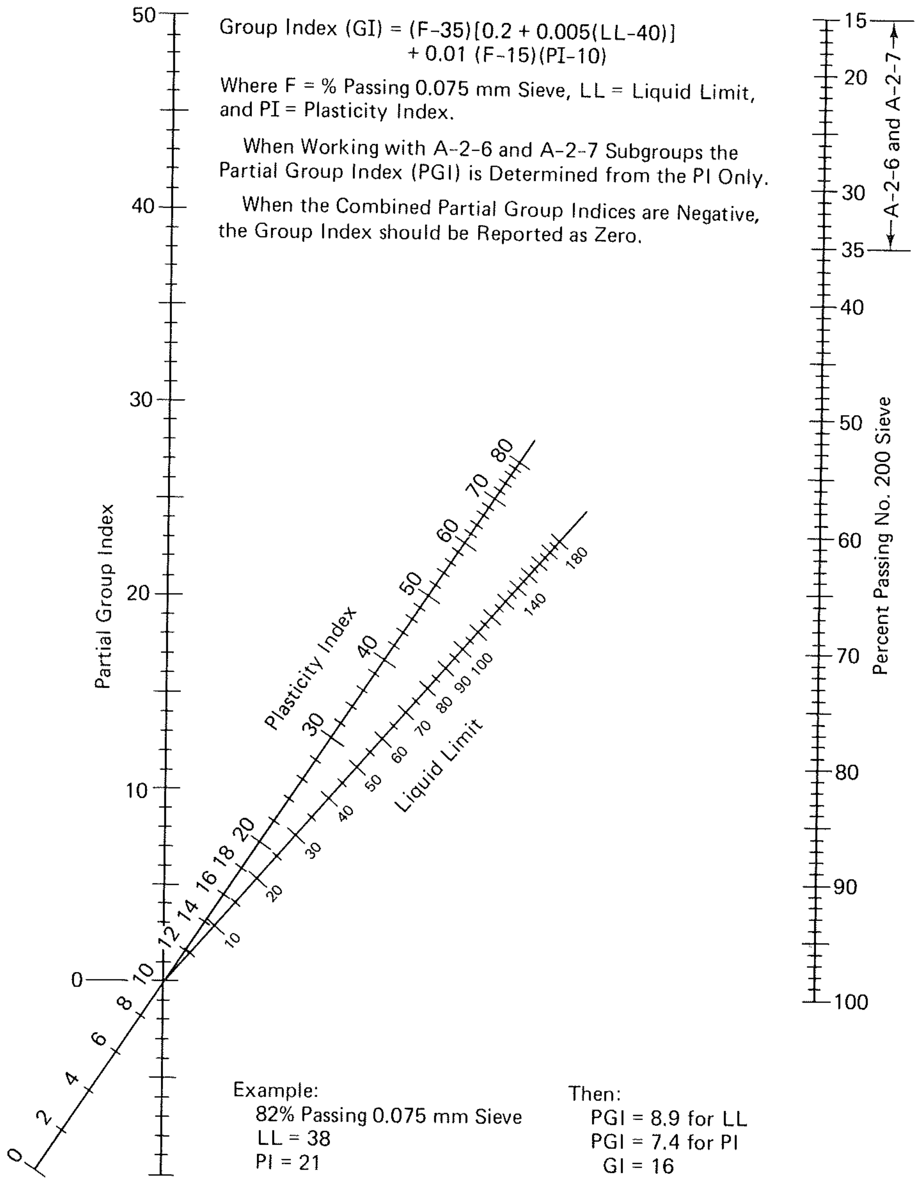

FIGURE 2-5 Group index chart. [5]

Required

Classify the soil sample by the AASHTO classification system.

Solution

By the AASHTO classification system:

$$\text{Plasticity index } (PI) = \text{liquid limit } (LL) - \text{plastic limit } (PL)$$
$$PI = 42.3\% - 15.8\% = 26.5\%$$

From Table 2-4, the sample is classified as A-7. According to the AASHTO classification system, the plasticity index of the A-7-5 subgroup is equal to or less than the liquid limit minus 30, and the plasticity index of the A-7-6 subgroup is greater than the liquid limit minus 30 (see footnote under Table 2-4).

$$LL - 30\% = 42.3\% - 30\% = 12.3\%$$
$$[PI = 26.5\%] > [LL - 30\% = 12.3\%]$$

Hence, this is A-7-6 material.

From Fig. 2-5 (group index chart), with $LL = 42.3\%$ and percentage passing No. 200 sieve = 60.2%, partial group index for $LL = 5.3$. With $PI = 26.5\%$ and percentage passing No. 200 sieve = 60.2%, partial group index for $PI = 7.5$. Hence,

$$\text{Total group index} = 5.3 + 7.5 = 12.8$$

Hence, the soil is A-7-6 (13), according to the AASHTO classification system.

Unified Soil Classification System [6, 7, 8]

The Unified Soil Classification System was originally developed by Casagrande [6] and is utilized by the Corps of Engineers. In this system, soils fall within one of three major categories: coarse-grained, fine-grained, and highly organic soils. These categories are further subdivided into 15 basic soil groups. The following group symbols are used in the Unified System:

G	Gravel
S	Sand
M	Silt
C	Clay
O	Organic
PT	Peat
W	Well graded
P	Poorly graded

Normally, two group symbols are used to classify soils. For example, SW indicates well-graded sand. Table 2-6 lists the 15 soil groups, including each one's name and symbol as well as giving specific details for classifying soils by this system.

In order to classify a given soil by the Unified System, its grain-size distribution, liquid limit, and plasticity index must first be determined. With these values known, the soil can be classified using Table 2-6 and Fig. 2-6. The Unified Soil Classification System is published as ASTM D 2487-85.

TABLE 2-5 Descriptions of AASHTO Classification Groups [5].

(1) *Granular Materials.* Containing 35% or less passing 0.075 mm (No. 200) sieve, Note 1.

(1.1) *Group A-1:* The typical material of this group is a well-graded mixture of stone fragments or gravel, coarse sand, fine sand and a nonplastic or feebly plastic soil binder. However, this group includes also stone fragments, gravel, coarse sand, volcanic cinders, etc. without soil binder.

(1.1.1) Subgroup A-1-a includes those materials consisting predominantly of stone fragments or gravel, either with or without a well-graded binder of fine material.

(1.1.2) Subgroup A-1-b includes those materials consisting predominantly of coarse sand either with or without a well-graded soil binder.

(1.2) *Group A-3:* The typical material of this group is fine beach sand or fine desert blow sand without silty or clay fines or with a very small amount of nonplastic silt. The group includes also stream-deposited mixtures of poorly-graded fine sand and limited amounts of coarse sand and gravel.

(1.3) *Group A-2:* This group includes a wide variety of "granular" materials which are border-line between the materials falling in Groups A-1 and A-3 and silt-clay materials of Groups A-4, A-5, A-6, and A-7. It includes all materials containing 35% or less passing the 0.075 mm sieve which cannot be classified as A-1 or A-3, due to fines content or plasticity or both, in excess of the limitations for those groups.

(1.3.1) Subgroups A-2-4 and A-2-5 include various granular materials containing 35% or less passing the 0.075 mm sieve and with a minus 0.425 mm (No. 40) portion having the characteristics of the A-4 and A-5 groups. These groups include such materials as gravel and coarse sand with silt contents or plasticity indexes in excess of the limitations of Group A-1, and fine sand with nonplastic silt content in excess of the limitations of Group A-3.

(1.3.2) Subgroups A-2-6 and A-2-7 include materials similar to those described under Subgroups A-2-4 and A-2-5 except that the fine portion contains plastic clay having the characteristics of the A-6 or A-7 group.

Note 1: Classification of materials in the various groups applies only to the fraction passing the 75 mm sieve. Therefore, any specification regarding the use of A-1, A-2, or A-3 materials in construction should state whether boulders (retained on 3-in. sieve) are permitted.

EXAMPLE 2-3

Given

A sample of soil was tested in the laboratory with the following results:

1. Liquid limit = 30.0%.
2. Plastic limit = 12.0%.

TABLE 2-5 *(Continued)*

(2) *Silt-Clay Materials.* Containing more than 35 percent passing the 0.075 mm sieve.

(2.1) *Group A-4:* The typical material of this group is a nonplastic or moderately plastic silty soil usually having the 75% or more passing the 0.075 mm sieve. The group includes also mixtures of fine silty soil and up to 64% of sand and gravel retained on 0.075 mm sieve.

(2.2) *Group A-5:* The typical material of this group is similar to that described under Group A-4, except that it is usually of diatomaceous or micaceous character and may be highly elastic as indicated by the high liquid limit.

(2.3) *Group A-6:* The typical material of this group is a plastic clay soil usually having 75% or more passing the 0.075 mm sieve. The group includes also mixtures of fine clayey soil and up to 64% of sand and gravel retained on the 0.075 mm sieve. Materials of this group usually have high volume change between wet and dry states.

(2.4) *Group A-7:* The typical material of this group is similar to that described under Group A-6, except that it has the high liquid limits characteristic of the A-5 group and may be elastic as well as subject to high volume change.

(2.4.1) Subgroup A-7-5 includes those materials with moderate plasticity indexes in relation to liquid limit and which may be highly elastic as well as subject to considerable volume change.

(2.4.2) Subgroup A-7-6 includes those materials with high plasticity indexes in relation to liquid limit and which are subject to extremely high volume change.

Note 2: Highly organic soils (peak or muck) may be classified as an A-8 group. Classification of these materials is based on visual inspection, and is not dependent on percentage passing the 0.075 mm (No. 200) sieve, liquid limit or plasticity index. The material is composed primarily of partially decayed organic matter, generally has a fibrous texture, dark brown or black color and odor of decay. These organic materials are unsuitable for use in embankments and subgrades. They are highly compressible and have low strength.

3. Sieve analysis data:

U.S. Sieve Size	*Percentage Passing*
⅜ in.	100
No. 4	76.5
No. 10	60.0
No. 40	39.7
No. 200	15.2

Required

Classify the soil by the Unified Soil Classification System.

TABLE 2-6 Soil classification chart by Unified Soil Classification System [8].

Criteria for Assigning Group Symbols and Group Names Using Laboratory Tests[A]				Soil Classification: *Group Symbol*	Soil Classification: *Group Name*[B]
Coarse-grained soils: More than 50% retained on No. 200 sieve	*Gravels:* more than 50% of coarse fraction retained on No. 4 sieve	*Clean gravels:* less than 5% fines[C]	$C_u \geq 4$ and $1 \leq C_c \leq 3$[E]	GW	Well-graded gravel[E]
			$C_u < 4$ and/or $1 > C_c > 3$[E]	GP	Poorly graded gravel[E]
		Gravels with fines: more than 12% fines[C]	Fines classify as ML or MH	GM	Silty gravel[F,G,H]
			Fines classify as CL or CH	GC	Clayey gravel[F,G,H]
	Sands: 50% or more of coarse fraction passes No. 4 sieve	*Clean sands:* less than 5% fines[D]	$C_u \geq 6$ and $1 \leq C_c \leq 3$[E]	SW	Well-graded sand[I]
			$C_u < 6$ and/or $1 > C_c > 3$[E]	SP	Poorly graded sand[I]
		Sands with fines: more than 12% fines[D]	Fines classify as ML or MH	SM	Silty sand[G,H,I]
			Fines classify as CL or CH	SC	Clayey sand[G,H,I]
Fine-grained soils: 50% or more passes the No. 200 sieve	*Silts and clays* Liquid limit less than 50	Inorganic	PI > 7 and plots on or above "A" line[J]	CL	Lean clay[K,L,M]
			PI < 4 or plots below "A" line[J]	ML	Silt[K,L,M]
		Organic	$\frac{\text{Liquid limit—oven dried}}{\text{Liquid limit—not dried}} < 0.75$	OL	Organic clay[K,L,M,N] / Organic silt[K,L,M,O]
	Liquid limit 50 or more	Inorganic	PI plots on or above "A" line	CH	Fat clay[K,L,M]
			PI plots below "A" line	MH	Elastic silt[K,L,M]
		Organic	$\frac{\text{Liquid limit—oven dried}}{\text{Liquid limit—not dried}} < 0.75$	OH	Organic clay[K,L,M,P] / Organic silt[K,L,M,Q]
Highly organic soils	Primarily organic matter, dark in color, and organic odor			PT	Peat

[A]Based on the material passing the 3-in. (75-mm) sieve.
[B]If field sample contained cobbles or boulders, or both, add "with cobbles or boulders, or both" to group name.
[C]Gravels with 5 to 12% fines require dual symbols:
GW-GM, well-graded gravel with silt
GW-GC, well-graded gravel with clay
GP-GM, poorly graded gravel with silt
GP-GC, poorly graded gravel with clay
[D]Sands with 5 to 12% fines require dual symbols:
SW-SM, well-graded sand with silt
SW-SC, well-graded sand with clay
SP-SM, poorly graded sand with silt
SP-SC, poorly graded sand with clay

[E]$C_u = D_{60}/D_{10}$, $C_c = \dfrac{(D_{30})^2}{D_{10} \times D_{60}}$.

[F]If soil contains ≥ 15% sand, add "with sand" to group name.
[G]If fines classify as CL-ML, use dual symbol GC-GM or SC-SM.
[H]If fines are organic, add "with organic fines" to group name.
[I]If soil contains ≥ 15% gravel, add "with gravel" to group name.

[J]If Atterberg limits plot in hatched area, soil is a CL-ML silty clay.
[K]If soil contains 15 to 29% plus No. 200, add "with sand" or "with gravel," whichever is predominant.
[L]If soil contains ≥ 30% plus No. 200, predominantly sand, add "sandy" to group name.
[M]If soil contains ≥ 30% plus No. 200, predominantly gravel, add "gravelly" to group name.
[N]PI ≥ 4 and plots on or above "A" line.
[O]PI < 4 or plots below "A" line.
[P]PI plots on or above "A" line.
[Q]PI plots below "A" line.

Solution

Since the percentage retained on the No. 200 sieve (100 − 15.2, or 84.8%) is more than 50%, go to the block labeled "coarse-grained soils" in Table 2-6. The sample consists of 100 − 15.2, or 84.8%, coarse-grain sizes, and 100 − 76.5, or 23.5%, was retained on the No. 4 sieve. Thus, the percentage of coarse fraction retained on the No. 4 sieve is (23.5/84.8) (100), or 27.7%, and the percentage of coarse fraction that passed the No. 4 sieve is 72.3%. Since 72.3% is greater than 50%, go to the block labeled "sands" in Table 2-6. The soil is evidently a sand. Since the sample contains 15.2% passing the No. 200 sieve, which is greater than 12% fines, go to the block labeled "sands with fines—more than 12% fines." Refer next to the plasticity chart (Fig. 2-6). With a liquid limit of 30.0% and plasticity index of 18.0% (recall that the plasticity index is the difference between the liquid and plastic limits, or 30.0 − 12.0), the sample is located above the A-line, and the fines are classified as CL. Return to Table 2-6, and go to the block labeled "SC." Thus, this soil is classified SC, according to the Unified Soil Classification System.

EXAMPLE 2-4

Given

A sample of soil was tested in the laboratory with the following results:

1. Liquid limit = NP (nonplastic).
2. Plastic limit = NP (nonplastic).

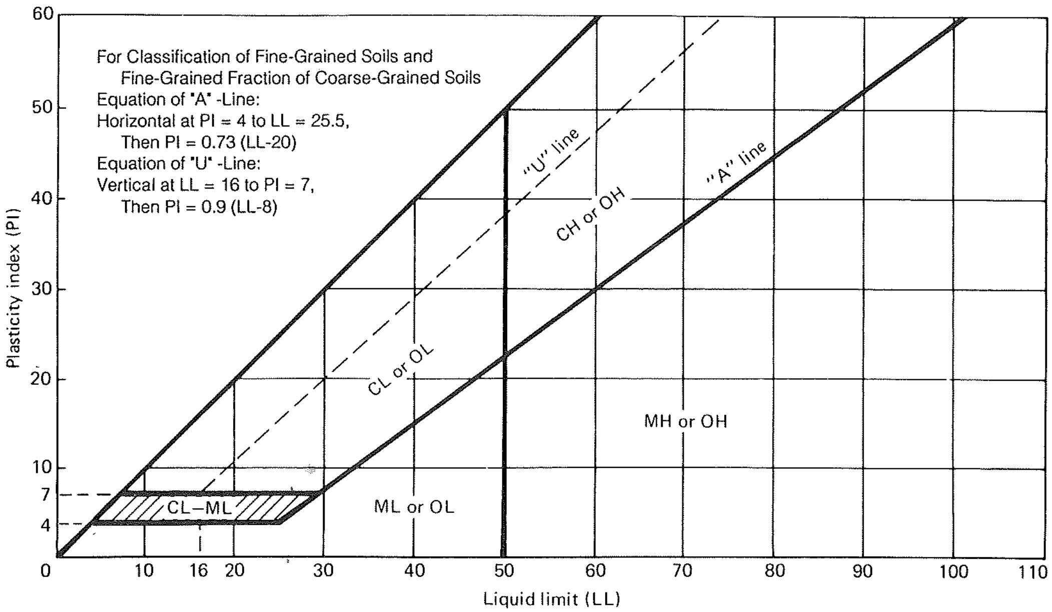

FIGURE 2-6 Plasticity chart. [8]

3. Sieve analysis data:

U.S. Sieve Size	*Percentage Passing*
1 in.	100
¾ in.	85
½ in.	70
⅜ in.	60
No. 4	48
No. 10	30
No. 40	16
No. 100	10
No. 200	2

Required

Classify the soil by the Unified Soil Classification System.

Solution

Since the percentage retained on the No. 200 sieve (100 – 2, or 98%) is more than 50%, go to the block labeled "coarse-grained soils" in Table 2-6. The sample consists of 100 – 2, or 98%, coarse-grain sizes, and 100 – 48, or 52%, was retained on the No. 4 sieve. Thus, the percentage of coarse fraction retained on the No. 4 sieve is 52/98, or 53.1%. Since 53.1% is greater than 50%, go to the block labeled "Gravels" in Table 2-6. The soil is evidently a gravel. Since the sample contains 2% passing the No. 200 sieve, which is less than 5% fines, go to the block labeled "clean gravels—less than 5% fines." The next block indicates that the coefficients of uniformity (C_u) and curvature (C_c) must be evaluated.

$$C_u = \frac{D_{60}}{D_{10}} \tag{2-1}$$

$$C_c = \frac{(D_{30})^2}{D_{60}D_{10}} \tag{2-2}$$

Values of D_{60}, D_{30}, and D_{10} are determined from the grain-size distribution curve (see Fig. 2-7) to be 9.5 mm, 2.00 mm, and 0.150 mm, respectively. Hence,

$$C_u = \frac{9.5\text{ mm}}{0.150\text{ mm}} = 63.3$$

$$C_c = \frac{(2.00\text{ mm})^2}{(9.5\text{ mm})(0.150\text{ mm})} = 2.8$$

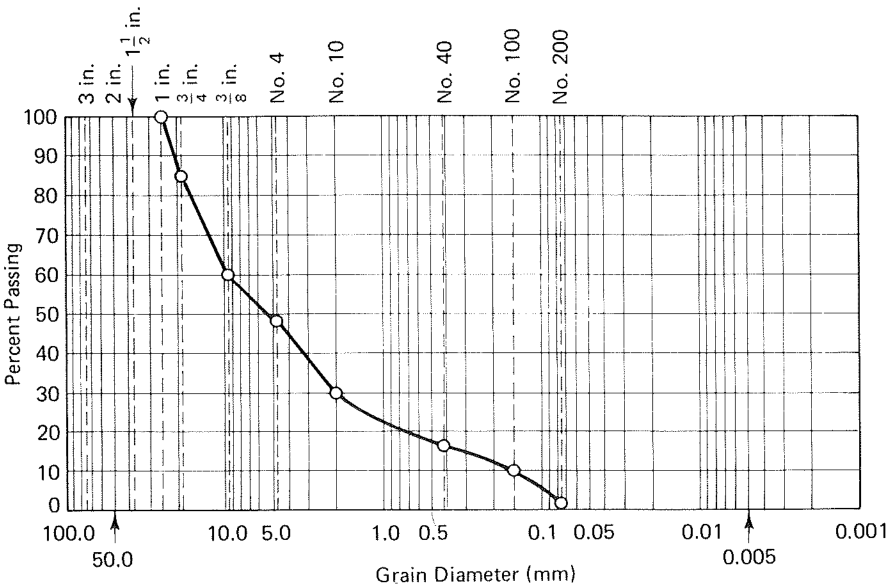

FIGURE 2-7 Grain-size distribution curve for Example 2-4.

Since C_u (63.3) is greater than 4 and C_c (2.8) is between 1 and 3, this sample meets both criteria for a well-graded gravel. Hence, from Table 2-6 the soil is classified GW (i.e., well-graded gravel), according to the Unified Soil Classification System.

EXAMPLE 2-5

Given

A sample of inorganic soil was tested in the laboratory with the following results:

1. Liquid limit = 42.3%.
2. Plastic limit = 15.8%.
3. Sieve analysis data:

U.S. Sieve Size	*Percentage Passing*
No. 4	100
No. 10	93.2
No. 40	81.0
No. 200	60.2

Required

Classify the soil sample by the Unified Soil Classification System.

Solution

Since the percentage passing the No. 200 sieve is 60.2%, which is greater than 50%, go to the lower block (labeled "fine-grained soils") in Table 2-6. The liquid limit is 42.3%, which is less than 50%, so go to the block labeled "silts and clays, liquid limit less than 50." Now, since the sample is an inorganic soil and the plasticity index is 42.3 − 15.8, or 26.5%, which is greater than 7, refer next to the plasticity chart (Fig. 2-6). With a liquid limit of 42.3% and plasticity index of 26.5%, the sample is located above the A-line. Return to Table 2-6 and go to the block labeled "CL." Thus, the soil is classified CL according to the Unified Soil Classification System.

2-4 COMPONENTS OF SOILS

Soils contain three components, which may be characterized as solid, liquid, and gas. The solid components of soils are weathered rock and (sometimes) decayed vegetation. The liquid component of soils is almost always water (often with dissolved matter), and the gas component is air. The volume of water and air combined is referred to as the *void.*

Figure 2-8 gives a block diagram showing the components of a soil. These components may be considered in terms of both their volumes and their weights/masses. In Fig. 2-8, terms V, V_a, V_w, V_s, and V_v represent total volume and volume of air, water, solid matter, and voids, respectively. Terms W, W_a, W_w, and W_s stand for total weight and weight of air, water, and solid matter, respectively. Similarly, terms M, M_a, M_w, and M_s denote total mass and mass of air, water, and solid matter, respectively. The weight and mass of air (W_a and M_a) are both virtually zero.

2-5 WEIGHT/MASS AND VOLUME RELATIONSHIPS

A number of important relationships exist among the components of soil in terms of both weight/mass and volume. These relationships define new parameters that are useful in working with soils

In terms of volume, the following new parameters are important—*void ratio, porosity,* and *degree of saturation.* Void ratio (e) is the ratio (expressed as a decimal fraction) of volume of voids to volume of solids.

$$e = \frac{V_v}{V_s} \tag{2-5}$$

Porosity (n) is the ratio (expressed as a percentage) of volume of voids to total volume.

$$n = \frac{V_v}{V} \times 100\% \tag{2-6}$$

Degree of saturation (S) is the ratio (expressed as a percentage) of volume of water to volume of voids.

$$S = \frac{V_w}{V_v} \times 100\% \tag{2-7}$$

In terms of weight/mass, the new parameters are *water content, unit weight, dry unit weight, unit mass* (or *density*), *dry unit mass* (or *dry density*), and *specific gravity of solids.* (*Note*: The terms "unit weight" and "unit mass" imply "wet unit weight" and "wet unit mass." If "dry unit weight" or "dry unit mass" is intended, the adjective "dry" is indicated explicitly.) Water content (w) is the ratio (expressed as a percentage) of weight of water to weight of solids or the ratio of mass of water to mass of solids.

$$w = \frac{W_w}{W_s} \times 100\% = \frac{M_w}{M_s} \times 100\% \tag{2-8}$$

Unit weight (γ) is total weight (weight of solid plus weight of water) divided by total volume (volume of solid plus volume of water plus volume of air).

$$\gamma = \frac{W}{V} \tag{2-9}$$

Dry unit weight (γ_d) is weight of solids divided by total volume.

$$\gamma_d = \frac{W_s}{V} \tag{2-10}$$

Unit mass (ρ) is total mass divided by total volume.

$$\rho = \frac{M}{V} \tag{2-11}$$

Dry unit mass (ρ_d) is mass of solids divided by total volume.

$$\rho_d = \frac{M_s}{V} \tag{2-12}$$

Specific gravity of solids (G_s) is the ratio of unit weight of solids (weight of solids divided by volume of solids) to unit weight of water or of unit mass of solids (mass of solids divided by volume of solids) to unit mass of water.

$$G_s = \frac{W_s/V_s}{\gamma_w} = \frac{W_s}{V_s\gamma_w} \tag{2-13}$$

$$G_s = \frac{M_s/V_s}{\rho_w} = \frac{M_s}{V_s\rho_w} \tag{2-14}$$

where γ_w and ρ_w are the unit weight and unit mass of water, respectively.

The unit weight of water varies slightly with temperature; but at normal temperatures, it has a value of around 62.4 pounds per cubic foot (lb/ft^3) or 9.81 kilonewtons per cubic meter (kN/m^3). The unit mass (density) of water is 1000 kilograms per cubic meter (kg/m^3) or 1 gram per cubic centimeter (g/cm^3). A useful conversion factor is: 1 lb/ft^3 = 0.1571 kN/m^3, or 1 kN/m^3 = 6.366 lb/ft^3.

The soils engineer must be proficient in determining these parameters based on laboratory evaluations of weight/mass and volume of the components of a soil. Use of a block diagram (as shown in Fig. 2-8) is recommended to help obtain answers more quickly and accurately. Five example problems follow.

EXAMPLE 2-6

Given

1. The weight of a chunk of moist soil sample is 45.6 lb.
2. The volume of the soil chunk measured before drying is 0.40 ft^3.
3. After being dried out in an oven, the weight of dry soil is 37.8 lb.
4. The specific gravity of solids is 2.65.

Required

1. Water content.
2. Unit weight of moist soil.
3. Void ratio.
4. Porosity.
5. Degree of saturation.

Solution

See Fig. 2-9. (Boldface data on the figure indicate given information. Other data are calculated in the solution of the problem.)

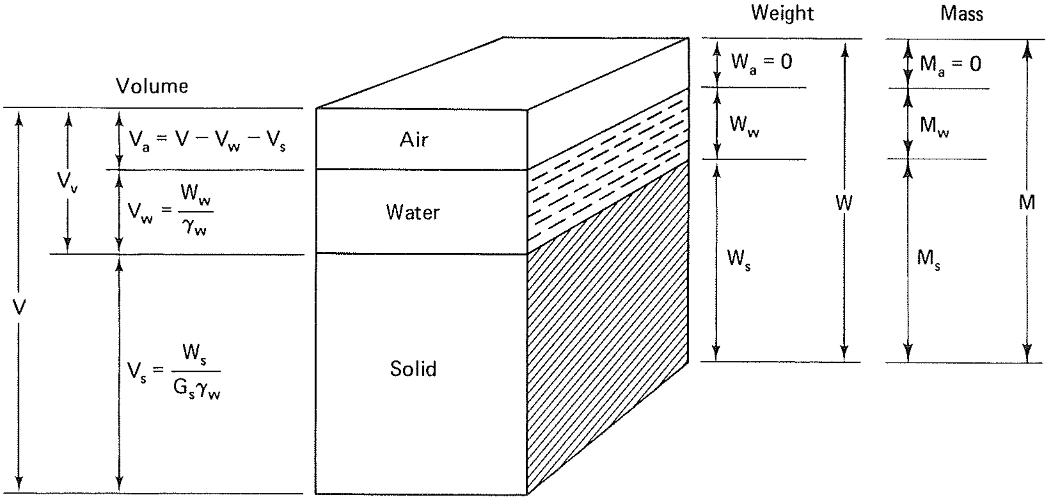

FIGURE 2-8 Block diagram showing components of soil.

1. Water content $(w) = \frac{W_w}{W_s} \times 100\% = \frac{45.6 \text{ lb} - 37.8 \text{ lb}}{37.8 \text{ lb}} \times 100\%$
$= 20.6\%$

2. Unit weight of moist soil $(\gamma) = \frac{W}{V} = \frac{45.6 \text{ lb}}{0.40 \text{ ft}^3} = 114.0 \text{ lb/ft}^3$

3. $V_w = \frac{W_w}{\gamma_w} = \frac{45.6 \text{ lb} - 37.8 \text{ lb}}{62.4 \text{ lb/ft}^3} = 0.13 \text{ ft}^3$

$V_s = \frac{W_s}{G_s\gamma_w} = \frac{37.8 \text{ lb}}{(2.65)(62.4 \text{ lb/ft}^3)} = 0.23 \text{ ft}^3$

$V_a = V - V_w - V_s = 0.40 \text{ ft}^3 - 0.13 \text{ ft}^3 - 0.23 \text{ ft}^3 = 0.04 \text{ ft}^3$

$V_v = V - V_s = 0.40 \text{ ft}^3 - 0.23 \text{ ft}^3 = 0.17 \text{ ft}^3$

or

$V_v = V_a + V_w = 0.04 \text{ ft}^3 + 0.13 \text{ ft}^3 = 0.17 \text{ ft}^3$

Void ratio $(e) = \frac{V_v}{V_s} = \frac{0.17 \text{ ft}^3}{0.23 \text{ ft}^3} = 0.74$

4. Porosity $(n) = \frac{V_v}{V} \times 100\% = \frac{0.17 \text{ ft}^3}{0.40 \text{ ft}^3} \times 100\% = 42.5\%$

5. Degree of saturation $(S) = \frac{V_w}{V_v} \times 100\% = \frac{0.13 \text{ ft}^3}{0.17 \text{ ft}^3} \times 100\% = 76.5\%$

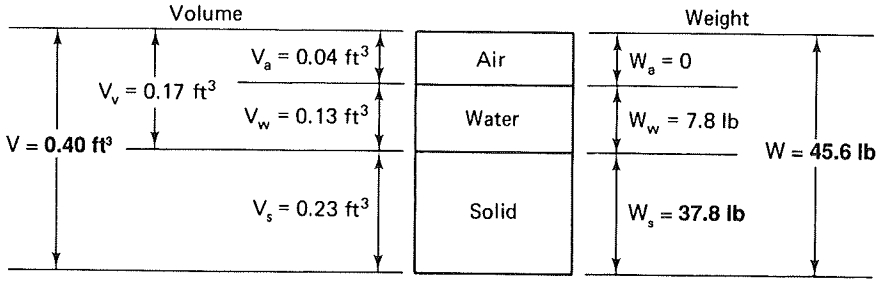

FIGURE 2-9

EXAMPLE 2-7

Given

1. The moist mass of a soil specimen is 20.7 kg.
2. The specimen's volume measured before drying is 0.011 m^3.
3. The specimen's dried mass is 16.3 kg.
4. The specific gravity of solids is 2.68.

Required

1. Void ratio.
2. Degree of saturation.
3. Wet unit mass.
4. Dry unit mass.
5. Wet unit weight.
6. Dry unit weight.

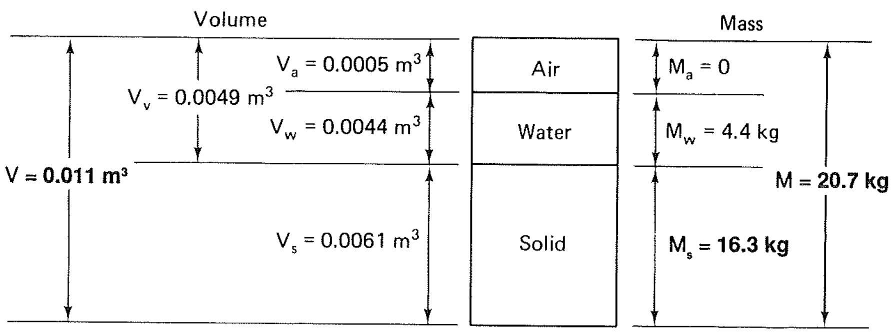

FIGURE 2-10

Solution

See Fig. 2-10.

1. $$V_s = \frac{M_s}{G_s\rho_w} = \frac{16.3 \text{ kg}}{(2.68)(1000 \text{ kg/m}^3)} = 0.0061 \text{ m}^3$$

$$V_w = \frac{M_w}{\rho_w} = \frac{20.7 \text{ kg} - 16.3 \text{ kg}}{1000 \text{ kg/m}^3} = 0.0044 \text{ m}^3$$

$$V_a = V - V_w - V_s = 0.011 \text{ m}^3 - 0.0044 \text{ m}^3 - 0.0061 \text{ m}^3 = 0.0005 \text{ m}^3$$

$$V_v = V - V_s = 0.011 \text{ m}^3 - 0.0061 \text{ m}^3 = 0.0049 \text{ m}^3$$

or

$$V_v = V_a + V_w = 0.0005 \text{ m}^3 + 0.0044 \text{ m}^3 = 0.0049 \text{ m}^3$$

$$\text{Void ratio } (e) = \frac{V_v}{V_s} = \frac{0.0049 \text{ m}^3}{0.0061 \text{ m}^3} = 0.80$$

2. $$\text{Degree of saturation } (S) = \frac{V_w}{V_v} \times 100\% = \frac{0.0044 \text{ m}^3}{0.0049 \text{ m}^3} \times 100\% = 89.8\%$$

3. $$\text{Wet unit mass } (\rho) = \frac{M}{V} = \frac{20.7 \text{ kg}}{0.011 \text{ m}^3} = 1882 \text{ kg/m}^3$$

4. $$\text{Dry unit mass } (\rho_d) = \frac{M_s}{V} = \frac{16.3 \text{ kg}}{0.011 \text{ m}^3} = 1482 \text{ kg/m}^3$$

5. $$\text{Wet unit weight } (\gamma) = \rho g = (1882 \text{ kg/m}^3)(9.81 \text{ m/s}^2) = 18{,}460 \frac{\text{kg·m}}{\text{s}^2}/\text{m}^3 = 18{,}460 \text{ N/m}^3 = 18.46 \text{ kN/m}^3$$

6. $$\text{Dry unit weight } (\gamma_d) = \rho_d g = (1482 \text{ kg/m}^3)(9.81 \text{ m/s}^2) = 14{,}540 \frac{\text{kg·m}}{\text{s}^2}/\text{m}^3 = 14{,}540 \text{ N/m}^3 = 14.54 \text{ kN/m}^3$$

EXAMPLE 2-8

Given

An undisturbed soil sample has the following data:

1. Void ratio = 0.78.
2. Water content = 12%.
3. Specific gravity of solids = 2.68.

Required

1. Wet unit weight.
2. Dry unit weight.
3. Degree of saturation.
4. Porosity.

Solution

See Fig. 2-11. Since the void ratio (e) = 0.78,

$$\frac{V_v}{V_s} = 0.78; \quad V_v = 0.78V_s \tag{A}$$

$$V_v + V_s = V = 1 \text{ m}^3 \tag{B}$$

(A volume of 1 m³ is assumed.)
Substitute Eq. (A) into Eq. (B).

$$0.78V_s + V_s = 1 \text{ m}^3$$

$$V_s = 0.56 \text{ m}^3$$

$$V_v = 1 \text{ m}^3 - 0.56 \text{ m}^3 = 0.44 \text{ m}^3$$

$$V_s = \frac{W_s}{G_s\gamma_w}; \qquad 0.56 \text{ m}^3 = \frac{W_s}{(2.68)(9.81 \text{ kN/m}^3)}$$

$$W_s = 14.72 \text{ kN}$$

From given water content, $\frac{W_w}{W_s} = 0.12$,

$$W_w = 0.12W_s = (0.12)(14.72 \text{ kN}) = 1.77 \text{ kN}$$

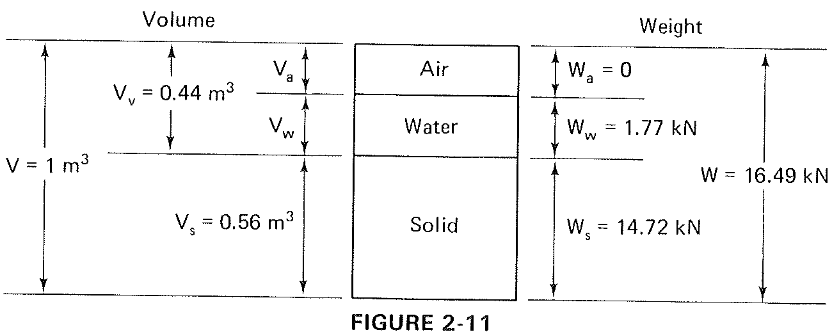

FIGURE 2-11

1. Wet unit weight $(\gamma) = \dfrac{W}{V} = \dfrac{W_w + W_s}{V} = \dfrac{1.77\text{ kN} + 14.72\text{ kN}}{1\text{ m}^3}$
$= 16.49\text{ kN/m}^3$

2. Dry unit weight $(\gamma_d) = \dfrac{W_s}{V} = \dfrac{14.72\text{ kN}}{1\text{ m}^3} = 14.72\text{ kN/m}^3$

3. $V_w = \dfrac{W_w}{\gamma_w} = \dfrac{1.77\text{ kN}}{9.81\text{ kN/m}^3} = 0.18\text{ m}^3$

   Degree of saturation $(S) = \dfrac{V_w}{V_v} \times 100\% = \dfrac{0.18\text{ m}^3}{0.44\text{ m}^3} \times 100\% = 40.9\%$

4. Porosity $(n) = \dfrac{V_v}{V} \times 100\% = \dfrac{0.44\text{ m}^3}{1\text{ m}^3} \times 100\% = 44.0\%$

EXAMPLE 2-9

Given

1. A 100% saturated soil has a wet unit weight of 120 lb/ft³.
2. The water content of this saturated soil was determined to be 36%.

Required

1. Void ratio.
2. Specific gravity of solids.

Solution

See Fig. 2-12.

$$W_w + W_s = 120\text{ lb} \qquad \text{(A)}$$

$$\frac{W_w}{W_s} = 0.36 \qquad \text{(B)}$$

From Eq. (B), $W_w = 0.36W_s$; substitute into Eq. (A).

$$0.36W_s + W_s = 120\text{ lb}$$
$$W_s = 88.2\text{ lb}$$
$$W_w = 0.36W_s = (0.36)(88.2\text{ lb}) = 31.8\text{ lb}$$

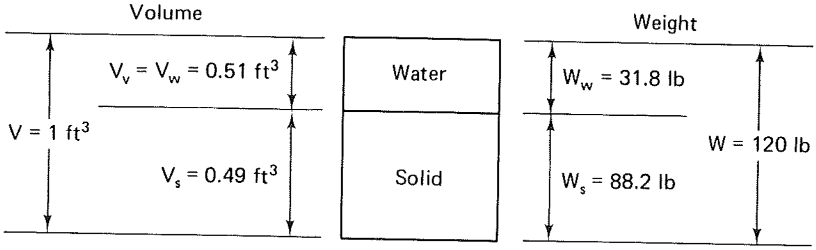

FIGURE 2-12

1. $$V_w = \frac{W_w}{\gamma_w} = \frac{31.8 \text{ lb}}{62.4 \text{ lb/ft}^3} = 0.51 \text{ ft}^3$$

$$V_s = V - V_w = 1 \text{ ft}^3 - 0.51 \text{ ft}^3 = 0.49 \text{ ft}^3$$

$$e = \frac{V_v}{V_s} = \frac{V_w}{V_s} = \frac{0.51 \text{ ft}^3}{0.49 \text{ ft}^3} = 1.04$$

Note: In this problem, because the soil is 100% saturated, $V_v = V_w$.

2. $$V_s = \frac{W_s}{G_s \gamma_w}; \qquad 0.49 \text{ ft}^3 = \frac{88.2 \text{ lb}}{(G_s)(62.4 \text{ lb/ft}^3)}$$

$$G_s = 2.88$$

EXAMPLE 2-10

Given

A soil sample has the following data:

1. Void ratio = 0.94.
2. Degree of saturation = 35%.
3. Specific gravity of solids = 2.71.

Required

1. Water content.
2. Unit weight.

Solution

See Fig. 2-13. From given void ratio, $e = \dfrac{V_v}{V_s} = 0.94$

$$V_v = 0.94 V_s \tag{A}$$

$$V_v + V_s = 1 \text{ m}^3 \tag{B}$$

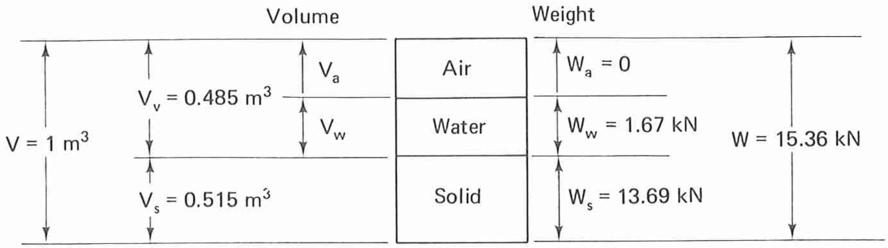

FIGURE 2-13

Substitute Eq. (A) into Eq. (B).

$$0.94V_s + V_s = 1 \text{ m}^3$$
$$V_s = 0.515 \text{ m}^3$$
$$V_v = 0.485 \text{ m}^3$$

From given degree of saturation, $S = \dfrac{V_w}{V_v} = 0.35$,

$$V_w = 0.35V_v$$
$$V_w = (0.35)(0.485 \text{ m}^3) = 0.170 \text{ m}^3$$
$$W_w = (0.170 \text{ m}^3)(9.81 \text{ kN/m}^3) = 1.67 \text{ kN}$$
$$W_s = (V_s)(G_s)(\gamma_w) = (0.515 \text{ m}^3)(2.71)(9.81 \text{ kN/m}^3) = 13.69 \text{ kN}$$

1. Water content $(w) = \dfrac{W_w}{W_s} \times 100\% = \dfrac{1.67 \text{ kN}}{13.69 \text{ kN}} \times 100\% = 12.2\%$

2. Unit weight $(\gamma) = \dfrac{W}{V} = \dfrac{W_w + W_s}{V} = \dfrac{1.67 \text{ kN} + 13.69 \text{ kN}}{1 \text{ m}^3}$

$$= 15.36 \text{ kN/m}^3$$

2-6 PERMEABILITY, CAPILLARITY, AND FROST HEAVE

As indicated in Sec. 2-4, water is a component of soil, and its presence in a given soil may range from virtually none to saturation—the latter case occurring when the soil's void space is completely filled with water. Soil properties and characteristics are influenced by changes in water content. This section introduces three phenomena that are directly related to water in soil: permeability, capillarity, and frost heave. These as well as other factors pertaining to water in soil are discussed in more detail in Chap. 5.

Permeability refers to the movement of water within soil. Actual water movement is through the voids, which might be thought of as small, interconnected, irregular conduits. Since such water movement can have profound effects on soil properties and characteristics, it is an important consideration in certain engineering applications. Construction procedures, as well as the behavior of completed structures, can be significantly influenced by water movement within soil. The type, manner, and practical effects of water movement are discussed in Chap. 5. The flow of water through soil is governed by Darcy's law, which is also covered in Chap. 5.

Capillarity refers to the rise of water (or other liquids) in a small-diameter tube inserted into the water, the rise being caused by both cohesion of the water's molecules (surface tension) and adhesion of the water to the tube's wall. The amount of rise of water in the tube above the water level surrounding the tube is inversely proportional to the tube's diameter. With soils, capillarity occurs at the water table (see Sec. 3-4) when water rises from saturated soil below into dry or partially saturated soil above the water table. The "capillary tubes" through which water rises in soils are actually the void spaces among soil particles. Since the voids interconnect in varying directions (not just vertically) and are irregular in size and shape, accurate calculation of the height of capillary rise is virtually impossible. It is known, however, that the height of capillary rise is associated with the mean diameter of a soil's voids, which is in turn related to average grain size. In general, the smaller the grain size, the smaller the void space, and consequently the greater will be the capillary rise. Thus, clayey soils, with the smallest grain size, should theoretically experience the greatest capillary rise, although the rate of rise may be very slow because of the characteristically low permeability of such soils. In fact, the largest capillary rise for any particular length of time generally occurs in soils of medium grain sizes (such as silts and very fine sands).

It is well known from physics that water expands when it is cooled and freezes. When the temperature in a soil mass drops below water's freezing point, water in the voids freezes and therefore expands, causing the soil mass to move upward. This vertical expansion of soil caused by freezing water within is known as *frost heave*. Serious damage may result from frost heave when structures such as pavements and building foundations supported by soil are lifted. Since the amount of frost heave (i.e., upward soil movement) is not necessarily uniform in a horizontal direction, cracking of pavements and building walls and/or floors may occur. When the temperature rises above the freezing point, frozen soil thaws from the top downward. Since resulting melted water near the surface cannot drain through underlying frozen soil, an increase in water content of the upper soil, a decrease in its strength, and subsequent settlement of structures occur. Clearly, alternate lifting and settling of pavements and structures as a result of frost heave are undesirable, may cause serious structural damage, and should be avoided or at least minimized.

2-7 COMPRESSIBILITY

When soil is compressed, its volume is decreased. This decrease in volume results from reduction in voids within the soil and consequently can be expressed as a reduction in void ratio (e). Soil compression, which results from loading and causes reduction in the volume of voids (or decrease in void ratio), is usually brought on by the extruding of water and/or air from the soil. If saturated soil is subjected to the weight of a building and water is subsequently squeezed out or otherwise lost, resulting soil compression can cause undue building settlement. If water is added to the soil, soil expansion may occur causing building uplift.

Compressibility is more pronounced in the case of cohesive soils, where soil moisture plays a part in particle interaction. The ultimate volume decrease may not occur until some time after loading. With cohesionless soils, compressibility is less, and ultimate volume decrease occurs at or immediately after loading.

The preceding discussion of compressibility of soil is presented here to give a brief introduction to this subject, since it is the purpose of this chapter to introduce various engineering properties of soils. A more comprehensive treatment of compressibility is given in Chap. 7.

2-8 SHEAR STRENGTH [9]*

Shear strength of soil refers to its ability to resist shear stresses. Shear stresses exist in a sloping hillside or result from filled land, weight of footings, and so on. If a given soil does not have sufficient shear strength to resist such shear stresses, failures in the forms of landslides and footing failures will occur.

Soil gains its shear strength from two sources—internal friction and cohesion. (The internal friction includes sliding and rolling friction and resistance offered by interlocking action among soil particles.) This may be exhibited in equation form by the Coulomb equation:

$$s = c + \bar{\sigma} \tan \phi \tag{2-15}$$

where s = shear strength
c = cohesion
$\bar{\sigma}$ = effective intergranular normal (perpendicular to the shear plane) pressure
ϕ = angle of internal friction
$\tan \phi$ = coefficient of friction

This equation is represented graphically by line A in Fig. 2-14.

In the case of cohesionless soil (such as sand), there is virtually no cohesion ($c = 0$) and Eq. (2-15) reverts to: $s = \bar{\sigma} \tan \phi$. This is represented graphically by line B in Fig. 2-14. With cohesive soil (such as clay), the angle of internal friction (ϕ) can be taken to be zero for many foundation design problems. If ϕ is zero, Eq. (2-15) reverts to: $s = c$. This is represented graphically by line C in Fig. 2-14.

* Wayne C. Teng, *Foundation Design*, © 1962. Reprinted by permission of Prentice-Hall, Inc. (This footnote applies to all succeeding citations to this reference in this book.)

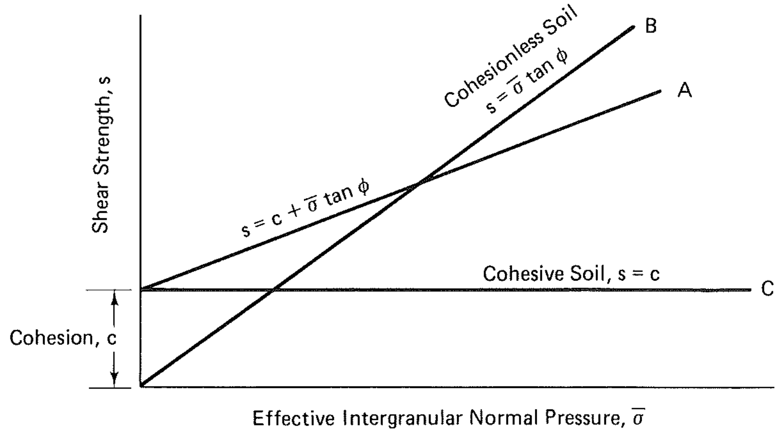

FIGURE 2-14 Shear strength diagram. [9]

The preceding discussion of shear strength of soil is presented here to give an introduction to this subject. A more comprehensive treatment of shear strength of both cohesionless and cohesive soils, including certain long-term effects on shear strength of cohesive soil, is given in Chap. 8.

The shear strength parameters, c and ϕ, in Eq. (2-15) can be determined directly or indirectly by standard field or laboratory tests (see Chap 8).

2-9 COMPACTNESS—RELATIVE DENSITY

In granular soils, compressibility and shear strength (covered in the preceding sections) are related to the compactness of the soil grains. For a soil in its densest condition, its void ratio is the lowest and it exhibits the highest shear strength and the greatest resistance to compression. Conversely, in its loosest condition, void ratio is the highest and shear strength and resistance to compression are the lowest. Soils in a natural state generally exhibit characteristics somewhere between these two extremes. *Compactness* refers to the relative condition of a given soil between these two extremes.

Compactness is expressed quantitatively by the *relative density* (D_r), which can be evaluated by the equation

$$D_r = \frac{e_{max} - e_0}{e_{max} - e_{min}} \times 100\% \tag{2-16}$$

where e_{max} = highest void ratio possible for a given soil (void ratio of the soil in its loosest condition)
e_0 = void ratio of the soil in-place
e_{min} = lowest void ratio possible for the soil (void ratio of the soil in its densest condition)

Relative density can also be evaluated in terms of maximum, minimum, and in-place dry unit weights (γ_{max}, γ_{min}, and γ, respectively) by the equation

$$D_r = \frac{\gamma_{max}(\gamma - \gamma_{min})}{\gamma(\gamma_{max} - \gamma_{min})} \times 100\% \tag{2-17}$$

This equation is generally more convenient to use because it is easier to evaluate dry unit weights than void ratios.

Values of γ_{min} or e_{max} for a given soil can be determined by performing standard laboratory tests on a quantity of the soil that has been dried, pulverized, and poured slowly from a small height through a funnel into a container. Values of γ_{max} or e_{min} can be found in the laboratory by prolonged vibration of the soil under a vertical load.

Clearly, the relative density of any soil varies between 0 and 100%. Soils having relative densities less than 15% are considered to be "very loose," while those with values between 15 and 35% are "loose." "Medium dense" soils have relative densities between 35 and 65%, while "dense" ones have values between 65 and 85%. Soils with relative densities greater than 85% are considered to be "very dense."

EXAMPLE 2-11

Given

1. A fine, dry sand with an in-place unit weight of 18.28 kN/m³.
2. The specific gravity of solids is 2.67.
3. The void ratio at its densest condition is 0.361.
4. The void ratio at its loosest condition is 0.940.

Required

Relative density of the sand.

Solution

From Eq. (2-13),

$$V_s = \frac{W_s}{G_s\gamma_w} = \frac{18.28 \text{ kN}}{(2.67)(9.81 \text{ kN/m}^3)} = 0.6979 \text{ m}^3$$

$$V_v = V - V_s = 1 \text{ m}^3 - 0.6979 \text{ m}^3 = 0.3021 \text{ m}^3$$

$$e_0 = \frac{V_v}{V_s} = \frac{0.3021 \text{ m}^3}{0.6979 \text{ m}^3} = 0.433$$

From Eq. (2-16),

$$D_r = \frac{e_{max} - e_0}{e_{max} - e_{min}} \times 100\% \qquad (2\text{-}16)$$

$$D_r = \frac{0.940 - 0.433}{0.940 - 0.361} \times 100\% = 87.6\%$$

2-10 PROBLEMS

2-1 Draw a gradation curve and find the median size, effective size, and coefficients of uniformity and of curvature for a soil sample that has the following test data for mechanical grain-size analysis:

U.S. Sieve Size	*Size Opening (mm)*	*Weight Retained (g)*
⅜ in.	9.50	0
No. 4	4.75	42
No. 10	2.00	146
No. 40	0.425	458
No. 100	0.150	218
No. 200	0.075	73
Pan	—	63

2-2 A sample of soil was tested in the laboratory and test results were listed as follows. Classify the soil by both the AASHTO and Unified soil classification systems.

1. Liquid limit = 29%.
2. Plastic limit = 19%.
3. Mechanical grain-size analysis:

U.S. Sieve Size	*Percentage Passing*
1 in.	100
¾ in.	90
⅜ in.	82
No. 4	70
No. 10	65
No. 40	54
No. 200	25

2-3 An undisturbed chunk of soil has a wet weight of 62 lb and a volume of 0.56 ft³. When dried out in an oven, the soil weighs 50 lb. If the specific gravity of solids is found to be 2.64, determine the water content, wet unit weight of soil, dry unit weight of soil, void ratio, porosity, and degree of saturation.

2-4 A 72-cm³ sample of moist soil weighs 141.5 g. When it is dried out in an oven, it weighs 122.7 g. The specific gravity of solids is found to be 2.66. Find the water content, void ratio, porosity, degree of saturation, and wet and dry unit weights.

2-5 A soil specimen has a water content of 18% and a wet unit weight of 118.5 lb/ft³. The specific gravity of solids is found to be 2.72. Find the dry unit weight, void ratio, and degree of saturation.

2-6 An undisturbed soil sample has a void ratio of 0.56, water content of 15%, and specific gravity of solids of 2.64. Find the wet and dry unit weights in lb/ft³, porosity, and degree of saturation.

2-7 A 100% saturated soil has a wet unit weight of 112.8 lb/ft³, and its water content is 42%. Find the void ratio and specific gravity of solids.

2-8 A 100% saturated soil has a void ratio of 1.33 and a water content of 48%. Find the unit weight of soil in lb/ft³ and specific gravity of solids.

2-9 The water content of a 100% saturated soil is 35% and the specific gravity of solids is 2.70. Determine the void ratio and unit weight in lb/ft³.

2-10 A soil sample has the following data:

1. Degree of saturation = 42%.
2. Void ratio = 0.85.
3. Specific gravity of solids = 2.74.

Find its water content and unit weight in lb/ft³.

2-11 A 0.082-m³ sample of soil weighs 1.445 kN. When it is dried out in an oven, it weighs 1.301 kN. The specific gravity of solids is found to be 2.65. Find the water content, void ratio, porosity, degree of saturation, and wet and dry unit weights.

2-12 The wet unit weight of a soil sample is 18.55 kN/m³. Its specific gravity of solids and water content are 2.72 and 12.3%, respectively. Find the dry unit weight, void ratio, and degree of saturation.

2-13 A fine sand has an in-place unit weight of 18.85 kN/m³ and a water content of 5.2%. The specific gravity of solids is 2.66. Void ratios at densest and loosest conditions are 0.38 and 0.92, respectively. Find the relative density.

2-14 Derive an expression for $e = f(n)$, where e is void ratio and n is porosity.

2-15 Derive an expression for $n = f(e)$, where n is porosity and e is void ratio.

2-16 A sand sample has a porosity of 38% and specific gravity of solids of 2.66. Find the void ratio and wet unit weight in lb/ft^3 if the degree of saturation is 35%.

2-17 A proposed earth dam will contain 5,000,000 m^3 of earth. Soil to be taken from a borrow pit will be compacted to a void ratio of 0.78. The void ratio of soil in the borrow pit is 1.12. Estimate the volume of soil that must be excavated from the borrow pit.

2-18 A soil sample with a water content of 14.5% and unit weight of 128.2 lb/ft^3 was dried to a unit weight of 118.8 lb/ft^3 without changing its void ratio. What is its new water content?

2-19 The unit weight, relative density, water content, and specific gravity of solids of a given sand are 17.98 kN/m^3, 62%, 7.6%, and 2.65, respectively.
(a) If the minimum void ratio for this soil is 0.35, what would be its maximum void ratio?
(b) What is its unit weight in the loosest condition?

2-20 A soil sample has a degree of saturation of 30.4% and void ratio of 0.85. How much water must be added per cubic foot of soil to increase the degree of saturation to 100%?

2-21 A soil sample has the following properties:

1. $e_{max} = 0.95$.
2. $e_{min} = 0.38$.
3. $D_r = 47\%$.
4. $G_s = 2.65$.

Find dry and saturated unit weights in both lb/ft^3 and kN/m^3.

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3

Soil Exploration

3-1 INTRODUCTION

In Chap. 2, various engineering properties of soils were presented. An evaluation of these properties is absolutely necessary in any rational design of structures resting on, in, or against soil. In order to evaluate these properties, it is imperative that soils engineers visit proposed construction sites and collect and test soil samples, in order to evaluate and record results in a useful and meaningful form.

Chapter 3 deals with evaluation of soil properties, including reconnaissance, steps of soil exploration (boring, sampling, and testing), and the record of field exploration. Although different types of soil tests are discussed in this chapter, detailed test methods are outside the scope of this book. For specific step-by-step procedures, the reader is referred to *Soil Properties: Testing, Measurement, and Evaluation,* 2nd edition, by Liu and Evett (Prentice-Hall, Inc., 1990).

3-2 RECONNAISSANCE

A reconnaissance is a preliminary examination or survey of a job site. Usually, some useful information on the area (e.g., maps or aerial photographs) will already be available, and an astute person can learn much about surface conditions and get a general idea of subsurface conditions by simply visiting the site, observing thoroughly and carefully, and properly interpreting what is seen.

A first step in the preliminary soil survey of an area should be to collect and study any pertinent information that is already available. This could include general geological and topographical information available in the form of geological and topographic maps, obtainable from federal, state, and local governmental agencies (e.g., U.S. Geological Survey, Soil Conservation Service of the U.S. Department of Agriculture, and various state geological surveys).

Aerial photographs can provide geologic information over large areas. Proper interpretation of them may reveal land patterns, sinkhole cavities, landslides, surface drainage patterns, and the like. Such information can usually be obtained on a more widespread and thorough basis by aerial photography than by visiting the project site. Specific details on this subject are, however, beyond the scope of this book. For more information, the reader is referred to the many books available on aerial photo interpretation.

After carefully collecting and studying available pertinent information, the soils engineer should then visit the site in person, observe thoroughly and carefully, and interpret what is seen. The ability to do this successfully requires considerable practice and experience; however, a few generalizations are given next.

To begin with, significant details on surface conditions and general information about subsurface conditions in an area may be obtained by observing general topographic characteristics at the proposed job site and at nearby locations where soil was cut or eroded (such as railroad and highway cuts, ditch and stream erosion, and quarries), thereby exposing subsurface soil strata.

The general topographic characteristics of an area can be of significance. Any unusual conditions (e.g., swampy areas or dump areas, such as sanitary landfill) deserve particular attention in soil exploration.

Since the presence of water is often a major consideration in working with soil and associated structures, several observations regarding water may be made during reconnaissance. Groundwater tables may be noted by observing existing wells. Historical high water marks may be recorded on buildings, trees, and so on.

Often, valuable information can be obtained by talking with local inhabitants of an area. Such information could include flooding history, erosion patterns, mud slides, soil conditions, depths of overburden, groundwater levels, and the like.

One final consideration is that the reconnoiterer should take numerous photographs of the proposed construction site, exposed subsurface strata, adjacent structures, and so on. These can be invaluable in subsequent analysis and design processes and in later comparisons of conditions before and after construction.

The authors hope the preceding discussion in this section has made the reader aware of the importance of reconnaissance with regard to soil exploration at a proposed construction site. In addition to providing important information, results of reconnaissance help determine the necessary scope of subsequent soil exploration.

At some point prior to beginning any subsurface exploration (Sec. 3-3), it is important that underground utilities (water mains, sewer lines, etc.) be located to assist in planning and carrying out subsequent subsurface exploration.

3-3 STEPS OF SOIL EXPLORATION

After obtaining all possible preliminary information as indicated in the preceding section, the next step is the actual subsurface soil exploration. It should be done by experienced personnel, using appropriate equipment. Much of soil mechanics practice can be successful only if one has long experience with which to compare each new problem.

Soil exploration may be thought of as consisting of three steps—boring, sampling, and testing. *Boring* refers to drilling or advancing a hole in the ground; *sampling* refers to removing soil from the hole; and *testing* refers to determining characteristics or properties of the soil. These three steps appear simple in concept but are quite difficult in good practice and are discussed in detail in the remainder of this section.

Boring

Some of the more common types of borings are auger borings, wash borings, test pits, and core borings.

An *auger* (Fig. 3-1) is a screwlike tool used to bore a hole. Some augers are operated by hand; others are power operated. As the hole is bored a short distance, the auger may be lifted to remove soil. Removed soil can be used for field classification and laboratory testing, but it must not be considered as an undisturbed soil sample. It is difficult to use augers in either very soft clay or coarse sand because the hole tends to refill when the auger is removed. Also, it may be difficult or impossible to use an auger below the water table because most saturated soils will not cling sufficiently to the auger for lifting. Hand augers may be used for boring to a depth of about 20 ft (6 m); power augers may be used to bore much deeper and quicker.

Wash borings (Fig. 3-2) consist of simultaneous drilling and jetting action. To begin with, a casing is usually driven into the ground. A chopping bit attached to the end of a drilling rod (or wash pipe) is driven by hammer, thereby breaking up the soil in the casing. Jetting action is accomplished by pumping water downward through the drilling bit. Water emerges at the chopping bit and further serves to break up the soil. Returning water transports soil to the ground surface, where samples can be collected for examination and classification. Such samples are, of course, disturbed samples whose water content has been increased.

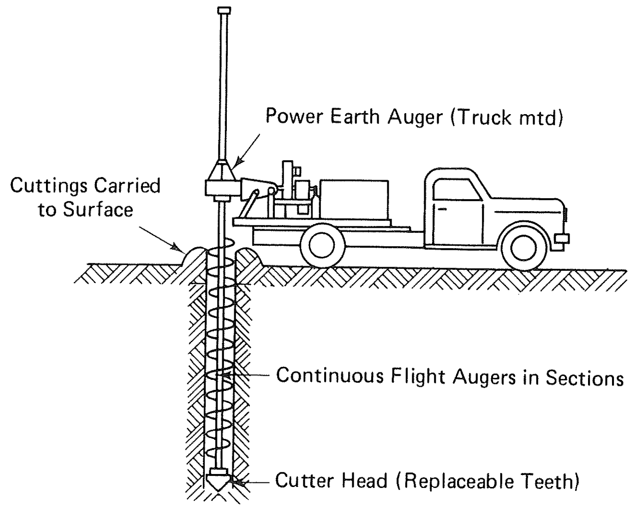


FIGURE 3-1 Auger boring. (Courtesy of Acker Drill Co.) [1]

Test pits (Fig. 3-3) are excavated either manually or by equipment (backhoe or bulldozer). They are generally bulky, expensive (when done manually), and limited to shallow exploration; nevertheless, they do have certain advantages. One is that, by observing the sides of the pit, an observer can inspect and take color photographs of the soil in its natural condition. Another advantage is that they can be used to obtain undisturbed samples. (The reader should note that even here the sample cannot be completely undisturbed.)

Core borings, used for drilling through rock, are often done using a diamond core drill in a core barrel sampler. The drill bit and core barrel sampler, attached to rods, are rotated by the drill; while water is simultaneously circulated (pumped) through the rods and barrel, emerging at the bit. The core remains in the core barrel and may be removed for examination by bringing the barrel to the surface.

Preceding paragraphs have discussed some of the more common types of borings. Once a means of boring has been decided upon, the question arises as to how many borings should be made. Obviously, the more borings made, the better the analysis of subsurface conditions should be. Borings are expensive, however, and a balance must be made between the cost of additional borings and the value of information gained from them.

As a rough guide for initial spacing of borings, the following are offered: for multistory buildings, 50 to 100 ft (15 to 30 m); for one-story buildings, earth dams, and borrow pits, 100 to 200 ft (30 to 60 m); and for highways (subgrade), 500 to 1000 ft (150 to 300 m). These spacings may be increased if soil conditions are found to be relatively uniform and must be decreased if found to be nonuniform.

Once means of boring and spacing have been decided upon, a final question arises as to how deep the borings should be. In general, borings should ex-

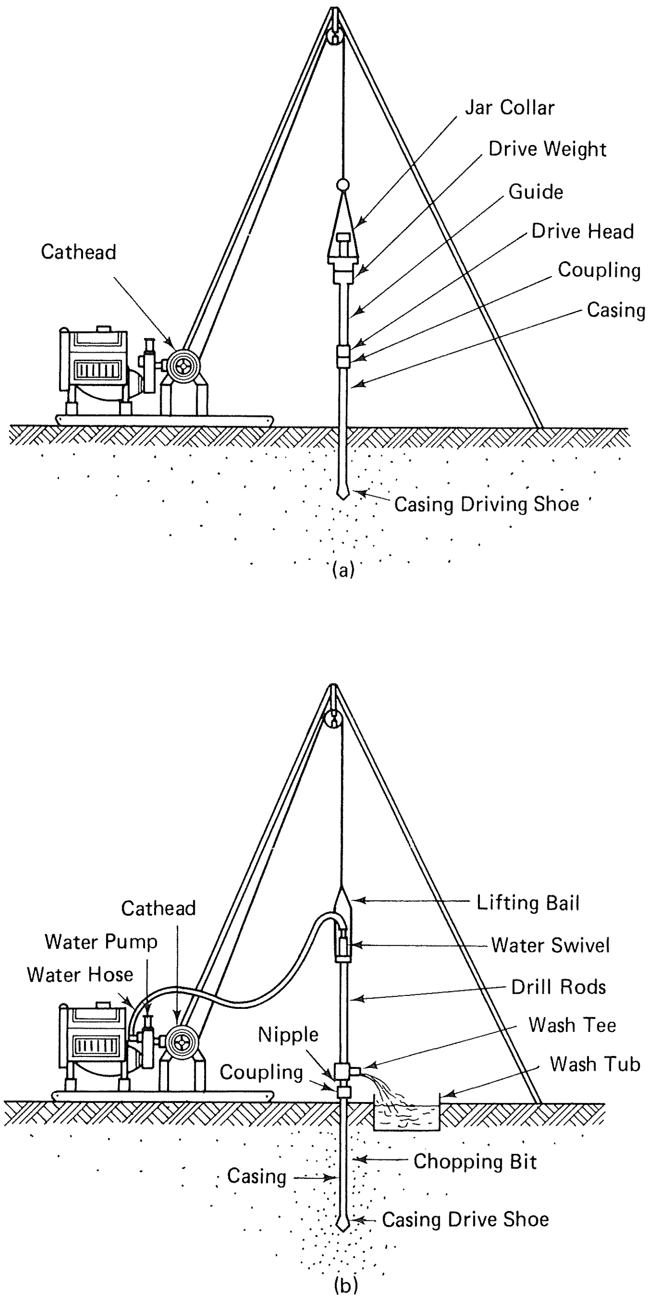


FIGURE 3-2 Typical setup for wash boring: (a) driving casing; (b) chopping and jetting. (Courtesy of Acker Drill Co.) [1]

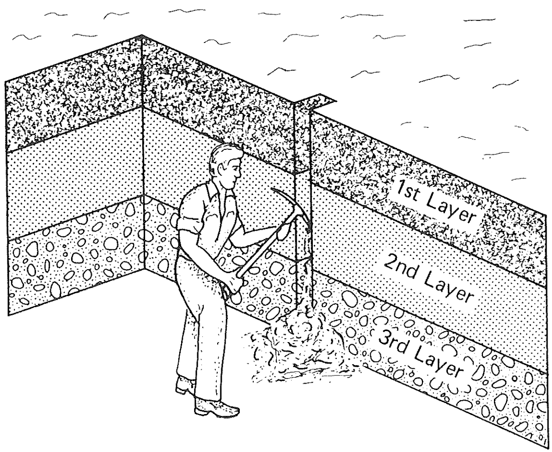


FIGURE 3-3 Test pit. [2]

tend through any unsuitable foundation strata (unconsolidated fill, organic soils, compressible layers such as soft, fine-grained soils, etc.) until soil of acceptable bearing capacity (hard or compact soil) is reached. If soil of acceptable bearing capacity is encountered at shallow depths, one or more borings should extend to sufficient depth to ensure that an underlying weaker layer, if found, will have negligible effect on surface stability and settlement. In compressible fine-grained strata, borings should extend to a depth at which stress from superimposed load is so small that surface settlement is negligible. In the case of very heavy structures, including tall buildings, borings in most cases should extend to bedrock. In all cases, it is advisable to investigate drilling at least one boring to bedrock.

The preceding discussion presented some general considerations regarding boring depths. A more definitive criterion for determining required minimum depths of test borings in cohesive soils is to carry borings to a depth where the increase in stress due to foundation loading (i.e., weight of the structure) is less than 10% of the effective overburden pressure. Figures 3-4, 3-5, and 3-6 were developed [3] to determine minimum depths of borings based on the 10% increase in stress criterion for cohesive soils. Figure 3-4 is for a continuous footing (such as a wall footing). Figure 3-5 is for a square footing with a design pressure between 1000 and 9000 lb/ft^2, and Fig. 3-6 is for a square footing with a design pressure between 100 and 1000 lb/ft^2. If the groundwater table is at the footing's base, the buoyant weight (submerged unit weight) of the soil should be used in these figures. If the groundwater table is lower than distance B below the footing (B is the footing's width), the wet unit weight should be used. For intermediate conditions, an interpolation can be made between required depths of boring for shallow and deep groundwater conditions, or the groundwater table can be conservatively assumed to be at the footing's base. It should be noted that on the left sides of Figs. 3-4 through 3-6 two scales are given, for footing width and minimum test boring depth. In each figure footing widths given on one side of the width scale correspond with boring depths given on the same side of the boring depth scale [3].

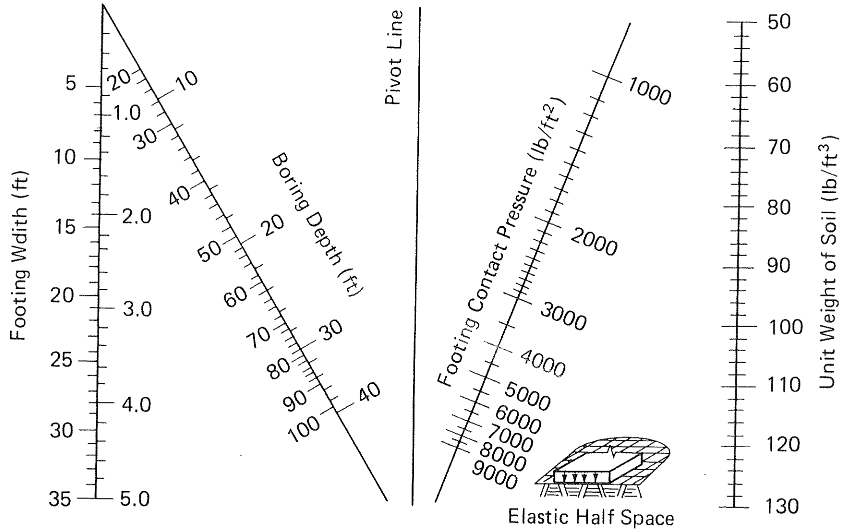


FIGURE 3-4 Infinite strip loading—Boussinesq-type solid (1 ft = 0.3048 m; 1 lb/ft^2 = 47.88 N/m^2; 1 lb/ft^3 = 0.1571 kN/m^3). [3]

EXAMPLE 3-1 [3]

Given

1. An 8-ft square footing is subjected to a contact pressure of 4000 lb/ft^2.
2. The wet unit weight of the soil supporting the footing is estimated to be 120 lb/ft^3.
3. The water table is estimated to be 30 ft beneath the footing.

Required

The minimum depth of test boring.

Solution

Since the water table is estimated to be 30 ft beneath the footing and the footing's width is 8 ft, the soil's wet unit weight should be used. From Fig. 3-5 with a wet unit weight of 120 lb/ft^3, contact pressure between footing and soil equal to 4000 lb/ft^2, and width of footing equal to 8 ft, the minimum depth of test boring is determined to be 22 ft.

Figures 3-4 through 3-6 are quite useful for estimating minimum required test boring depths in cohesive soils. In the final analysis, however, the depth of a specific boring should be determined by the soils engineer based on his or her knowledge, experience, judgment, and general knowledge of the specific area. Also, in some cases, depth (and spacing) of borings may be specified by local codes or company policy.

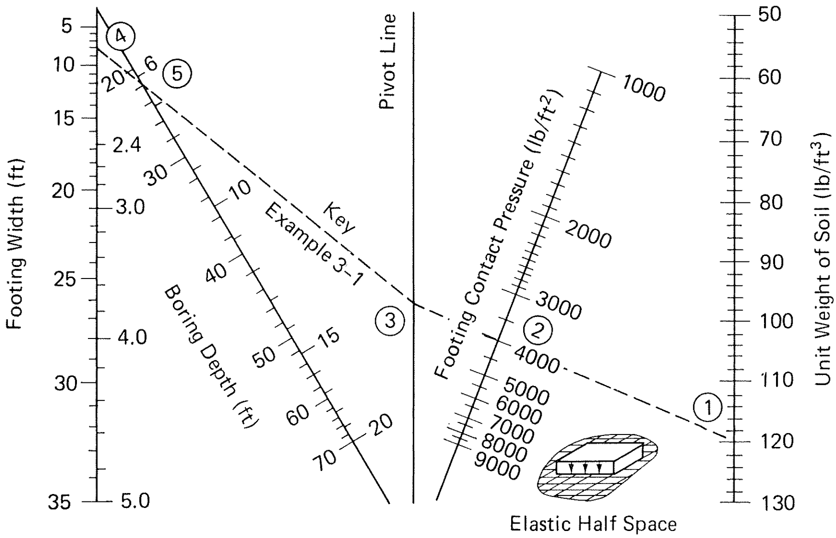


FIGURE 3-5 Square loading—Boussinesq-type solid (1 ft = 0.3048 m; 1 lb/ft^2 = 47.88 N/m^2; 1 lb/ft^3 = 0.1571 kN/m^3). [3]

Sampling

Sampling refers to the taking of soil or rock from bored holes. Samples may be classified as either disturbed or undisturbed.

As mentioned previously in this section, in both auger borings and wash borings, soil is brought to the ground surface, where samples can be collected. Such samples are obviously *disturbed samples,* and thus some of their characteristics are changed. (Split-spoon samples, described in Sec. 3-5, also provide

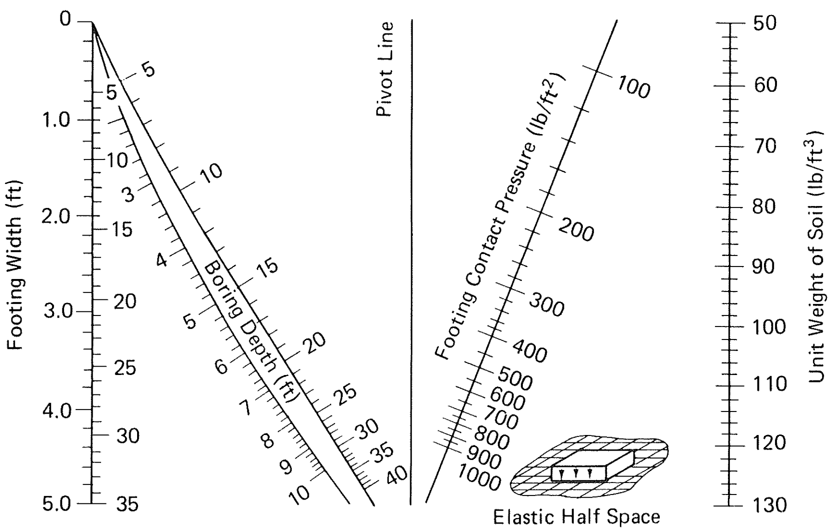


FIGURE 3-6 Square loading (low-pressure)—Boussinesq-type solid (1 ft = 0.3048 m; 1 lb/ft^2 = 47.88 N/m^2; 1 lb/ft^3 = 0.1571 kN/m^3). [3]

disturbed samples.) Disturbed samples should be placed in an airtight container (plastic bag or airtight jar, for example) and should, of course, be properly labeled as to date, location, bore-hole number, sampling depth, and so on. Disturbed samples are generally used for soil grain-size analysis, determination of liquid and plastic limits and specific gravity of soil, as well as for other tests, such as the compaction and CBR (California bearing ratio) tests.

For determination of certain other properties of soils, such as strength, compressibility, and permeability, it is necessary that the collected soil sample be exactly the same as it was when it existed in place within the ground. Such a soil sample is referred to as an *undisturbed sample.* It should be realized, however, that such a sample can never be completely undisturbed (i.e., be exactly the same as it was when it existed in place within the ground).

Undisturbed samples may be collected by several methods. If a test pit is available in clay soil, an undisturbed sample may be obtained by simply carving a sample very carefully out of the side of the test pit. Such a sample should then be coated with paraffin wax and placed in an airtight container. This method is often too tedious, time consuming, and expensive to be done on a large scale, however.

A more common method of obtaining an undisturbed sample is to push a thin tube into the soil, thereby trapping the (undisturbed) sample inside the tube and then to remove the tube and sample intact. The ends of the tube should be sealed with paraffin wax immediately after the tube containing the sample is brought to the ground surface. The sealed tube should then be sent to the soils laboratory, where subsequent tests can be made on the sample, with the assumption that such test results are indicative of the properties of the soil as it existed in place within the ground.

The thin tube mentioned above is called a *Shelby tube.* It is a 2- to 3-in. (51 to 76 mm)-diameter 16-gauge seamless steel tube, which is preferably pushed into the soil by static force rather than being driven by hammer.

Normally, samples (both disturbed and undisturbed) are collected at least every 5 ft (1.5 m) in depth of the boring hole. If, however, any change in soil characteristics is noted within 5-ft intervals, additional samples should be made when a change is noted.

The importance of properly and accurately identifying and labeling each sample cannot be overemphasized.

After a boring has been made and samples taken, an estimate of the groundwater table can be made. It is common practice to cover the hole (for example, with a small piece of plywood) for safety reasons, mark it for identification, leave it overnight, and return the next day to record the groundwater level. The hole should then be filled in to avoid subsequent injury to people or animals (see Sec. 3-4).

Testing

There exist a large number of tests that can be made to evaluate various soil properties. These include both laboratory and field tests. Some of the most common tests are listed in Table 3-1. As indicated at the beginning of

TABLE 3-1 Common types of testing [2].

Property of Soil	*Type of Test*	*ASTM Designation*	*AASHTO Designation*
	(a) *Laboratory testing of soils*		
Grain-size distribution	Mechanical analysis	D421, D422, D1140	T88
Consistency	Liquid limit (LL)	D4318	T89
	Plastic limit (PL)	D4318	T90
	Plasticity index (PI)	D4318	T90
Unit weight	Specific gravity	D854	T100
Moisture	Natural water content		
	Field moisture equivalent	D2216	T93
	Centrifuge moisture equivalent	D425	
Shear strength	Unconfined compression	D2166	T208
	Direct shear	D3080	T236
	Triaxial	D2850	T234
Volume change	Shrinkage factors	D427	T92
Compressibility	Consolidation	D2435	T216
Permeability	Permeability	D2434	T215
Compaction characteristics	Standard proctor	D698	T99
	Modified proctor	D1557	T180
California bearing ratio (CBR)		D1883	T193
	(b) *Field testing of soils*		
Compaction control	Moisture-density relations	D698	T99, T180
	In-place density	D1556	T191
		D2167	T205
Shear strength (soft clay)	Vane test	D2573	T223
Relative density (granular soil)	Penetration test	D1586	T206
Permeability	Pumping test		
Bearing capacity			
Pavements	CBR		
	Plate bearing	D1195	T221
		D1196	T222
Footings	Plate bearing	D1194	T235
Piles (vertical load)	Load test	D1143	
Batter piles	Lateral load test		

this chapter, the reader is referred to *Soil Properties: Testing, Measurement, and Evaluation,* 2nd edition, by Liu and Evett (Prentice-Hall, Inc., 1990) for specific step-by-step procedures involving these tests. Three tests—the standard penetration test, cone penetration test, and vane test—are described in some detail in Secs. 3-5 through 3-7.

3-4 GROUNDWATER TABLE

The term *groundwater table* (or just *water table*) has been mentioned several times earlier in this chapter. Section 3-4 presents more detailed information about this important phenomenon as it relates to the study of soils.

The location of the water table is a matter of importance to soils engineers, particularly when it is near the ground surface. For example, a soil's bearing capacity (see Chap. 9) can be reduced when the water table is at or near a footing. The location of the water table is not fixed at a particular site; it tends to rise and fall during periods of wet and dry weather, respectively. Fluctuations of the water table may result in reduction of foundation stability; in extreme cases structures may float out of the ground. Accordingly, foundation design and/or methods of construction may be affected by the location of the water table. Position of the water table is also very important when siting hazardous waste and sanitary landfills, to avoid contaminating groundwater.

The water table can be located by measuring down to the water level in existing wells in an area. It can also be determined from boring holes. The level to which groundwater rises in a boring hole is the groundwater elevation in that area. If adjacent soil is pervious, the water level in a boring hole will stabilize in a short period of time; if the soil is relatively impervious, it may take much longer for this to happen. General practice in soil surveying is to cover the boring hole (e.g., with a small piece of plywood) for safety reasons, leave it for at least 24 hours to allow the water level to rise in the hole and stabilize, and return the next day to locate and record the groundwater table. The hole should then be filled to avoid subsequent injury to people or animals.

3-5 STANDARD PENETRATION TEST

The Standard Penetration Test (SPT) is widely used in the United States. Relatively simple and inexpensive to perform, it is useful in determining certain properties of soils, particularly of cohesionless soils, for which undisturbed samples are not easily obtained.

The SPT utilizes a *split-spoon sampler* (Fig. 3-7). It is a 2-in. (51 mm)-O.D. 1⅜-in. (35 mm)-I.D. tube, 18 to 24 in. (457 to 610 mm) long, that is split longitudinally down the middle. The split-spoon sampler is attached to the bottom of a drilling rod and driven into the soil with a drop hammer. Specifi-

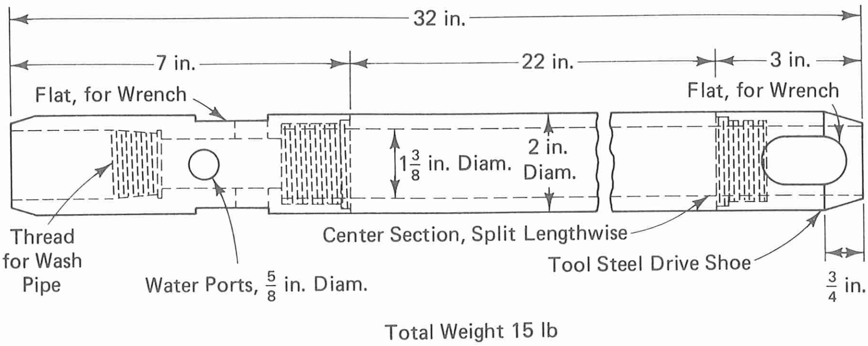


FIGURE 3-7 Split-spoon sampler for the standard penetration test. [4]

cally, a 140-lb (623 N) hammer falling 30 in. (762 mm) is used to drive the split-spoon sampler 18 in. (457 mm) into the soil. As the sampler is driven the 18 in. (457 mm) into the soil, the number of blows required to penetrate each of the three 6-in. (152 mm) increments is recorded separately. The standard penetration resistance value (or N-value) is the number of blows required to penetrate the last 12 in. (305 mm). Thus, the N-value represents the number of blows per foot (305 mm). After blow counts have been obtained, the split-spoon sampler can be removed and opened (along the longitudinal split) to obtain a disturbed sample for subsequent examination and testing [5].

SPT results (i.e., N-values) are influenced by overburden pressure (effective weight of overlying soil) at locations where blow counts are made. Several methods have been proposed to correct N-values to reflect the influence of overburden pressure. Two of these methods are presented here.

One method [6] utilizes the following equations to evaluate C_N, a correction factor to be applied to the N-value determined in the field:

$$C_N = 0.77 \log_{10} \frac{20}{\bar{p}} \qquad (\bar{p} \text{ in tons/ft}^2) \qquad (3\text{-}1)$$

$$C_N = 0.77 \log_{10} \frac{1915}{\bar{p}} \qquad (\bar{p} \text{ in kN/m}^2) \qquad (3\text{-}2)$$

where $\bar{p}$ is the effective overburden pressure at the elevation of the SPT. These equations are not valid if $\bar{p}$ is less than 0.25 ton/ft² (24 kN/m²). Figure 3-8 gives a graphical relationship, based in part on Eq. (3-1), for determining a correction factor to be applied to the N-value recorded in the field. If $\bar{p}$ is greater than or equal to 0.25 ton/ft², the correction factor may be determined using either Eq. (3-1) or Fig. 3-8. If $\bar{p}$ is less than 0.25 ton/ft², the correction factor should be taken from the figure.

The other method [7] utilizes the equations

$$N = \frac{4N'}{1 + 2p_0} \qquad [p_0 \text{ in kips per square foot (kips/ft}^2)] \qquad (3\text{-}3)$$

$$\text{if } p_0 \leq 1.5 \text{ kips/ft}^2$$

$$N = \frac{4N'}{3.25 + 0.5p_0} \qquad (p_0 \text{ in kips/ft}^2) \text{ if } p_0 \geq 1.5 \text{ kips/ft}^2 \tag{3-4}$$

$$N = \frac{4N'}{1 + 0.0418p_0} \qquad (p_0 \text{ in kN/m}^2) \text{ if } p_0 \leq 72 \text{ kN/m}^2 \tag{3-5}$$

$$N = \frac{4N'}{3.25 + 0.0104p_0} \qquad (p_0 \text{ in kN/m}^2) \text{ if } p_0 \geq 72 \text{ kN/m}^2 \tag{3-6}$$

where N = corrected N-value
N' = N-value determined in the field
p_0 = effective overburden pressure

These two methods give comparable results. It will be noted that the first method [Eqs. (3-1) and (3-2)] results in no adjustment of N-value at a depth where the effective overburden pressure is 1 ton/ft^2 (96 kN/m^2); while the second method [Eqs. (3-3) through (3-6)] results in no adjustment at a depth where the effective overburden pressure is 0.75 ton/ft^2, or 1.5 kips/ft^2 (72 kN/m^2).

The reader is cautioned that, although the standard penetration test is widely used in the United States, results are highly variable and thus difficult to interpret. Nevertheless, it is a useful guide in foundation analysis. Much experience is necessary to properly apply the results obtained. Outside the United States, other techniques are used. For example, in Europe the Cone Penetration Test (Sec. 3-6) is often preferred.

EXAMPLE 3-2

Given

An SPT was performed at a depth of 20 ft in sand of unit weight 135 lb/ft^3. The blow count was 40.

Required

The corrected N-value by each of the methods presented above.

Solution

1. By Eq. (3-1),

$$C_N = 0.77 \log_{10} \frac{20}{\bar{p}} \tag{3-1}$$

$$\bar{p} = \frac{(20 \text{ ft})(135 \text{ lb/ft}^3)}{2000 \text{ lb/ton}} = 1.35 \text{ tons/ft}^2$$

$$C_N = 0.77 \log_{10} \frac{20}{1.35 \text{ tons/ft}^2} = 0.901$$

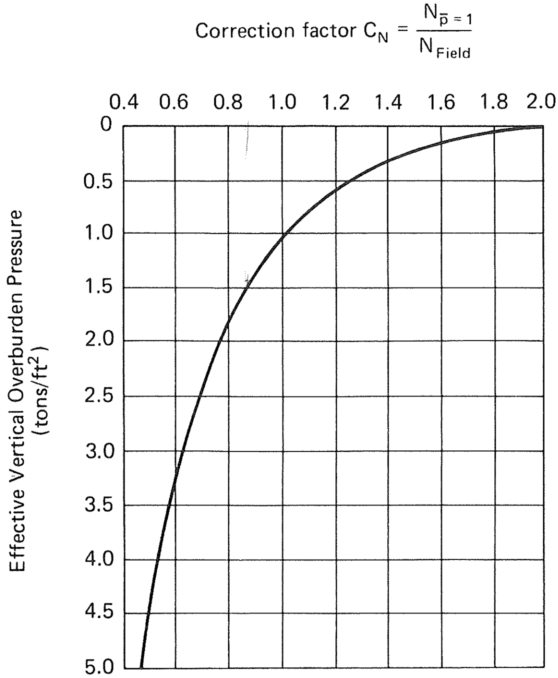


FIGURE 3-8 Chart for correction of *N*-values in sand for influence of overburden pressure (reference value of effective overburden pressure, 1 ton/ft²). [6]
Note: 1 ton/ft² = 95.76 kN/m².

(This value of 0.901 for C_N can also be obtained using Fig. 3-8 by entering 1.35 tons/ft² along the ordinate, moving horizontally to the curved line, and then moving upward to obtain the correction factor, C_N.) Therefore,

$$N_{\text{corrected}} = (40)(0.901) = 36.$$

2. By Eq. (3-3) or (3-4),

$$p_0 = \frac{(20 \text{ ft})(135 \text{ lb/ft}^3)}{1000 \text{ lb/kip}} = 2.70 \text{ kips/ft}^2$$

Since [p_0 = 2.70 kips/ft²] > 1.5 kips/ft², use Eq. (3-4).

$$N = \frac{4N'}{3.25 + 0.5p_0} \tag{3-4}$$

$$N = \frac{(4)(40)}{3.25 + (0.5)(2.70 \text{ kips/ft}^2)}$$

$$N_{\text{corrected}} = 35$$

EXAMPLE 3-3

Given

An SPT test was performed at a depth of 8.5 m in sand of unit weight 20.04 kN/m³. The blow count was 38.

Required

The corrected N-value by each of the methods presented above.

Solution

1. By Eq. (3-2),

$$C_N = 0.77 \log_{10} \frac{1915}{\bar{p}} \tag{3-2}$$

$$\bar{p} = (8.5 \text{ m})(20.04 \text{ kN/m}^3) = 170.3 \text{ kN/m}^2$$

$$C_N = 0.77 \log_{10} \frac{1915}{170.3 \text{ kN/m}^2} = 0.809$$

Therefore,

$$N_{\text{corrected}} = (38)(0.809) = 31$$

2. By Eq. (3-6) (since $p_0 > 72$ kN/m²),

$$N = \frac{4N'}{3.25 + 0.0104p_0} \tag{3-6}$$

$$N = \frac{(4)(38)}{3.25 + (0.0104)(170.3 \text{ kN/m}^2)}$$

$$N_{\text{corrected}} = 30$$

TABLE 3-2 Penetration resistance and soil properties on basis of the standard penetration test [6].

SANDS (FAIRLY RELIABLE)		CLAYS (RATHER UNRELIABLE)	
Number of Blows per Foot, N	*Relative Density*	*Number of Blows per Foot, N*	*Consistency*
		Below 2	Very soft
0–4	Very loose	2–4	Soft
4–10	Loose	4–8	Medium
10–30	Medium	8–15	Stiff
30–50	Dense	15–30	Very stiff
Over 50	Very dense	Over 30	Hard

EXAMPLE 3-4

Given

Same data as given in Example 3-2 except that the water table is located at a depth of 5 ft below the ground surface.

Required

The corrected N-value by each of the methods presented above.

Solution

1. By Eq. (3-1),

$$C_N = 0.77 \log_{10}\frac{20}{\bar{p}} \tag{3-1}$$

$$\bar{p} = \frac{(5\text{ ft})(135\text{ lb/ft}^3) + (15\text{ ft})(135\text{ lb/ft}^3 - 62.4\text{ lb/ft}^3)}{2000\text{ lb/ton}}$$

$$= 0.882\text{ ton/ft}^2$$

$$C_N = 0.77 \log_{10}\frac{20}{0.882\text{ ton/ft}^2} = 1.04$$

Therefore,

$$N_{\text{corrected}} = (40)(1.04) = 42$$

2. By Eq. (3-3) or (3-4),

$$p_0 = \frac{(5\text{ ft})(135\text{ lb/ft}^3) + (15\text{ ft})(135\text{ lb/ft}^3 - 62.4\text{ lb/ft}^3)}{1000\text{ lb/kip}} = 1.76\text{ kips/ft}^2$$

Since $[p_0 = 1.76\text{ kips/ft}^2] > 1.5\text{ kips/ft}^2$, use Eq. (3-4).

$$N = \frac{4N'}{3.25 + 0.5p_0} \tag{3-4}$$

$$N = \frac{(4)(40)}{3.25 + (0.5)(1.76\text{ kips/ft}^2)}$$

$$N_{\text{corrected}} = 39$$

Through empirical testing, correlations between (corrected) SPT N-values and several soil parameters have been established. These are particularly useful for cohesionless soils but are less reliable for cohesive soils. Table 3-2 gives correlations of relative density of sands with SPT N-value and of consistency of clays with SPT N-value. Figure 3-9 gives a graphical relationship between angle of internal friction of cohesionless soil and SPT N-value. Figure 3-9 also gives graphical relationships between certain bearing capacity factors for cohesionless soil and SPT N-value. These will be utilized in Chap. 9.

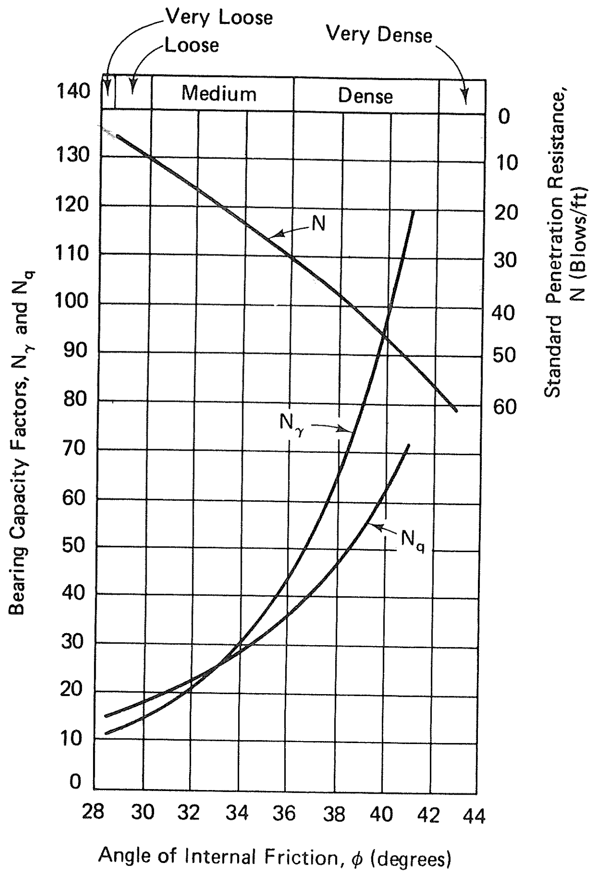


FIGURE 3-9 Curves showing the relationship between bearing capacity factors and φ, as determined by theory, and rough empirical relationship between bearing capacity factors or φ and values of standard penetration resistance *N*. [6]

3-6 CONE PENETRATION TEST

The Cone Penetration Test (CPT) has been widely used in Europe for many years but is now gaining favor in the United States. It has the advantage of accomplishing subsurface exploration rapidly without taking soil samples.

There are two types of mechanical cone penetrometers—the *mechanical cone penetrometer* (see Fig. 3-10) and the *mechanical friction-cone penetrometer* (Fig. 3-11). Both types have a conical point with a point angle of 60° and a base diameter of 35.7 mm (1.41 in.), giving a base area of 1000 mm^2 (1.55 in.^{2}). The main difference between the two is that in addition to cone resistance the friction-cone penetrometer allows for determination of side (sleeve) resistance as the penetrometer is advanced through the soil.

Penetrometers are either pushed (by a hydraulic jack, for example) or driven (such as by blows of a drop hammer) into and through soil. When pushed, the test is known as a *static cone test* (sometimes referred to as a *Dutch cone test*); when driven, it is called a *dynamic cone test*. In all cases, the penetrometer's resistance to being advanced through the soil is measured and recorded as a function of depth of soil penetrated.

The static test is sensitive to small differences in soil consistency. Since the penetrometer is pushed (rather than driven) in a static test, the procedure probably tends not to significantly alter soil structure for loose sands and sensitive clays. The dynamic test covers a wider range of soil consistencies; and since the penetrometer is driven, penetrations of gravels and soft rock are possible.

When using a mechanical cone penetrometer, the penetrometer's tip is advanced to the required depth by applying sufficient thrust on the push rods (see Fig. 3-10a). Then the tip is extended by applying sufficient thrust on the inner rods (see Fig. 3-10b). Cone resistance is obtained at some specific point during the downward movement of the inner rods relative to the stationary push rods. By repeating this two-step procedure again and again, cone resistance data are obtained as a function of depth. Each increment of depth through which the penetrometer is advanced should not normally exceed 8 in. (203 mm), and the rate of penetration should be approximately 2 to 4 ft/min (10 to 20 mm/s).

The procedure for using a mechanical friction-cone penetrometer is the same as that just described for a mechanical cone penetrometer, except that after cone resistance during the initial phase of the extension is determined, a separate resistance value of cone plus sleeve friction is also measured. Sleeve resistance is obtained by subtracting cone resistance from total resistance.

CPT data are ordinarily presented as plots of cone resistance, friction resistance, and friction ratio (ratio of friction resistance to cone resistance) versus depth (see Fig. 3-12), thereby giving a "picture" of the variation in soil

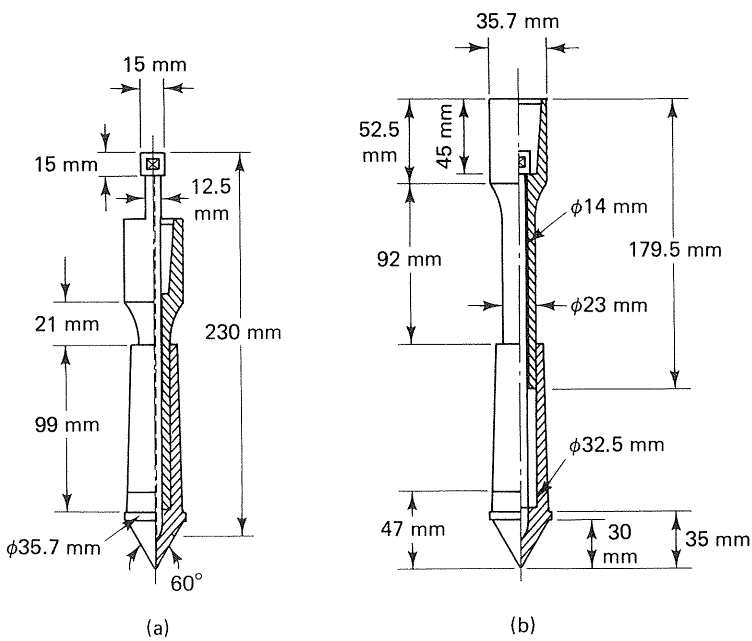


FIGURE 3-10 Mechanical cone penetrometer tip (Dutch mantle cone): (a) collapsed; (b) extended. [8]

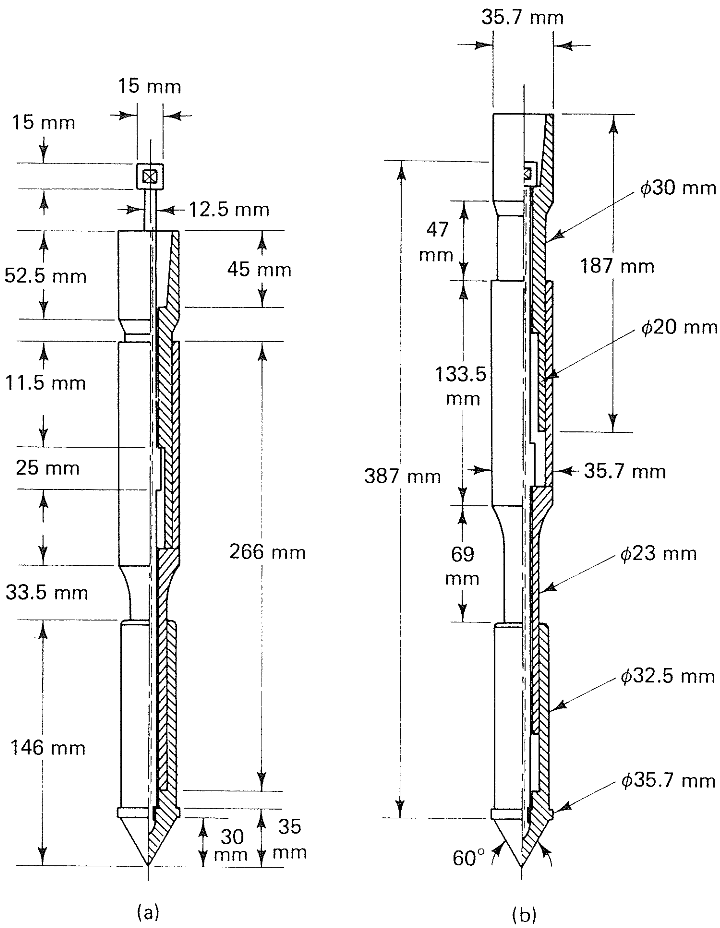


FIGURE 3-11 Mechanical friction-cone penetrometer tip (Begemann friction cone): (a) collapsed; (b) extended. [8]

types at different depths at a test site. In general, the ratio of sleeve resistance to cone resistance is higher in cohesive soils than in cohesionless soils; hence, this ratio can be used to estimate the type of soil being penetrated. However, since no soil samples are taken during a cone penetration test, supplemental boring is normally required to obtain samples in order to identify definitively the soil strata that were penetrated by the cone test.

3-7 VANE TEST

The field vane test is a fairly simple test that can be used to determine in-place shear strength for soft clay soils—particularly those clay soils that lose part of their strength when disturbed (sensitive clays)—without taking an undisturbed sample. A vane tester (Fig. 3-13) is made up of two thin metal

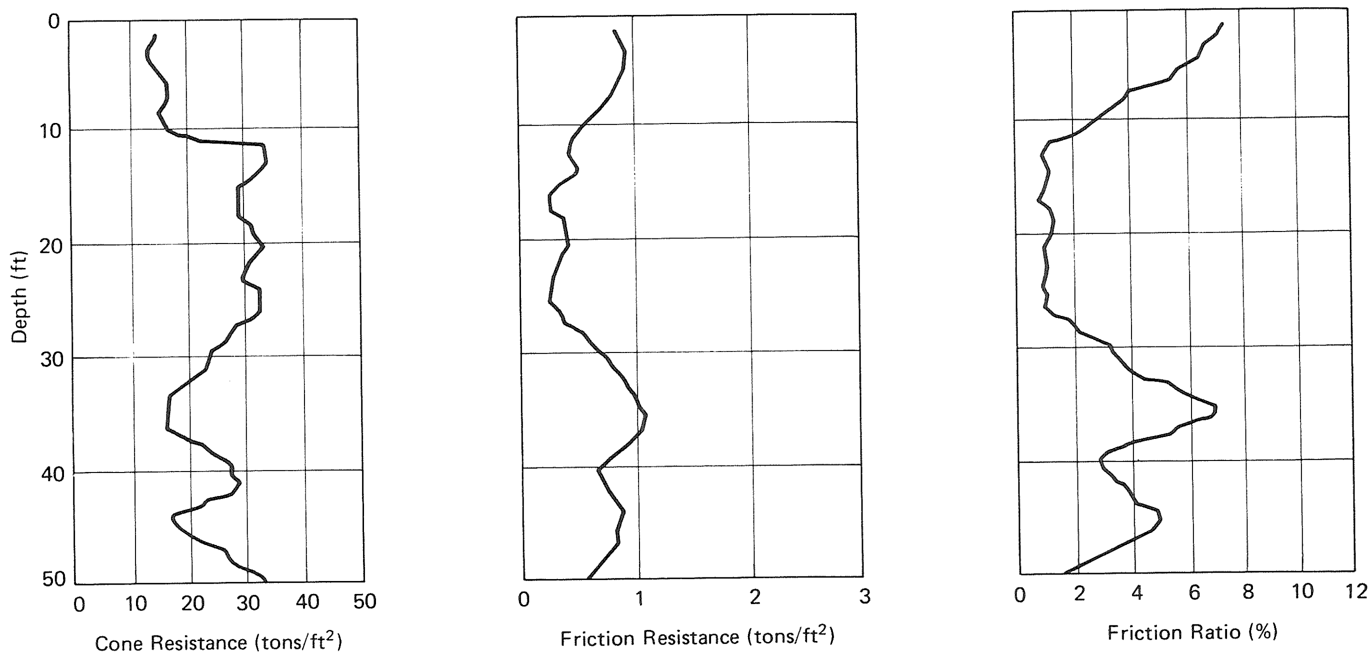


FIGURE 3-12 Cone penetration plots.

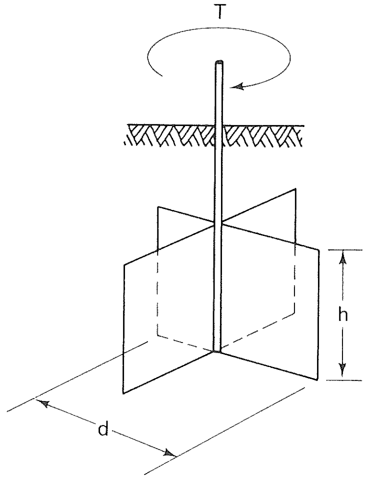


FIGURE 3-13 Vane tester. [2]

blades attached to a vertical shaft. The test is carried out by pushing the vane tester into the soil and then applying a torque to the vertical shaft. The clay's cohesion can be computed using the formula [2, 9]

$$c = \frac{T}{\pi[(d^2h/2) + (d^3/6)]} \tag{3-7}$$

where c = cohesion of the clay, lb/ft² or N/m²
T = torque required to shear the soil, ft-lb or m·N
d = diameter of vane tester, ft or m
h = height of vane tester, ft or m

Bjerrum [10] found a tendency of the vane test to overestimate cohesion in high plasticity clays and developed an empirical relationship for determining a correction factor. This relationship is shown in Fig. 3-14, where a correction factor, μ, can be determined if the clay's plasticity index is known.

It should be emphasized that the field vane test is suitable only for use in soft or sensitive clays. Also, no soil sample is obtained for subsequent examination and testing when a field vane test is performed.

EXAMPLE 3-5

Given

A vane tester with diameter and height of 3.625 in. (0.3021 ft) and 7.25 in. (0.6042 ft), respectively, requires a torque of 17.0 ft-lb to shear a clayey soil, the plasticity index of which is 48%.

Required

This soil's cohesion.

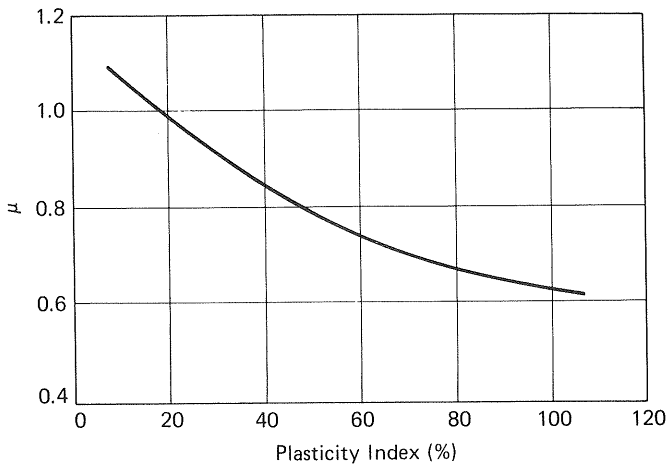


FIGURE 3-14 Correction factor for vane shear test. [10]

Solution

By Eq. (3-7),

$$c = \frac{T}{\pi[(d^2h/2) + (d^3/6)]} \tag{3-7}$$

$$c = \frac{17.0 \text{ ft-lb}}{\pi\left[\dfrac{(0.3021 \text{ ft})^2(0.6042 \text{ ft})}{2} + \dfrac{(0.3021 \text{ ft})^3}{6}\right]} = 168 \text{ lb/ft}^2$$

From Fig. 3-14, with a plasticity index of 48%, a correction factor, μ, of 0.80 is obtained. Hence,

$$c_{\text{corrected}} = (0.80)(168 \text{ lb/ft}^2) = 134 \text{ lb/ft}^2$$

3-8 GEOPHYSICAL METHODS OF SOIL EXPLORATION

Borings and test pits (Sec. 3-3) afford definitive subsurface exploration. They can, however, be both time-consuming and expensive. Additionally, they give subsurface conditions only at boring or test pit locations, leaving vast areas in between for which conditions must be interpolated, estimated, or whatever.

Geophysical methods, which are widely used in highway work and in other applications, can be done more quickly and less expensively and can cover greater areas more thoroughly. They tend, however, to yield less definitive results requiring more subjective interpretation by the user. Accordingly, a number of borings are still required to obtain soil samples from which accurate determinations of soil properties can be made in order to verify and complement results determined by geophysical methods.

Two particular geophysical methods—*seismic refraction* and *electrical resistivity*—are discussed in this section. In the former, resistance to flow of a seismic wave through soil is measured; in the latter, resistance of soil to movement of an electrical current is determined. Using values obtained therefrom, a specialist can interpret the depth to and thickness of different soil strata and estimate, with the aid of supplemental borings, some of the engineering properties of the subsurface material.

Seismic Refraction Method

The seismic refraction method is based on the fact that velocities of seismic waves traveling through soil and rock material are related to the material's density and elasticity. In general, the denser the material, the greater will be the velocity of seismic waves moving through it. In carrying out this method, seismic (sound or vibration) waves are created within the soil at a particular location. Ordinarily, these waves are produced either by exploding small charges of dynamite several feet below the ground surface or by striking a heavy hammer against a steel plate. A detector, known as a *geophone,* placed some known (or measurable) distance from the shock source, detects the presence of a wave; and a timing device measures the time required for the wave to travel from the point of impact to the point of detection.

In conducting a seismic refraction field survey, a series of geophone readings is obtained at different distances along a straight line from the point of impact. For detection points relatively close to the impact point, the first shock to reach the geophones travels from the impact point through more direct surface routes to the detection points (see Fig. 3-15).

When a harder layer, say rock, underlies the surficial soil layer, a seismic wave traveling downward from the point of impact into the rock layer is refracted to travel longitudinally through the upper part of the rock layer and eventually back to the ground surface (through the surficial layer) to be recorded by the geophones (see Fig. 3-15). Since seismic wave velocity is much greater through the rock layer than through the surficial soil, for geophones located relatively far from the impact point, the refracted wave will reach the geophone more quickly than the direct wave. The time required for the first shock to reach each geophone is plotted as a function of distance from the shock source, as in Fig. 3-16. The wave to the first few geophones closer to the shock source travels directly through the surficial layer; therefore, the slope of the time versus distance graph is inversely equivalent to velocity—that is,

$$v_1 = \frac{L_2 - L_1}{t_2 - t_1} \tag{3-8}$$

where v_1 = wave's velocity through the surficial soil layer (i.e., reciprocal of the slope of line 1 as shown in Fig. 3-16)

L_1 and L_2 = distances from shock source to geophones Nos. 1 and 2, respectively (see Fig. 3-15)

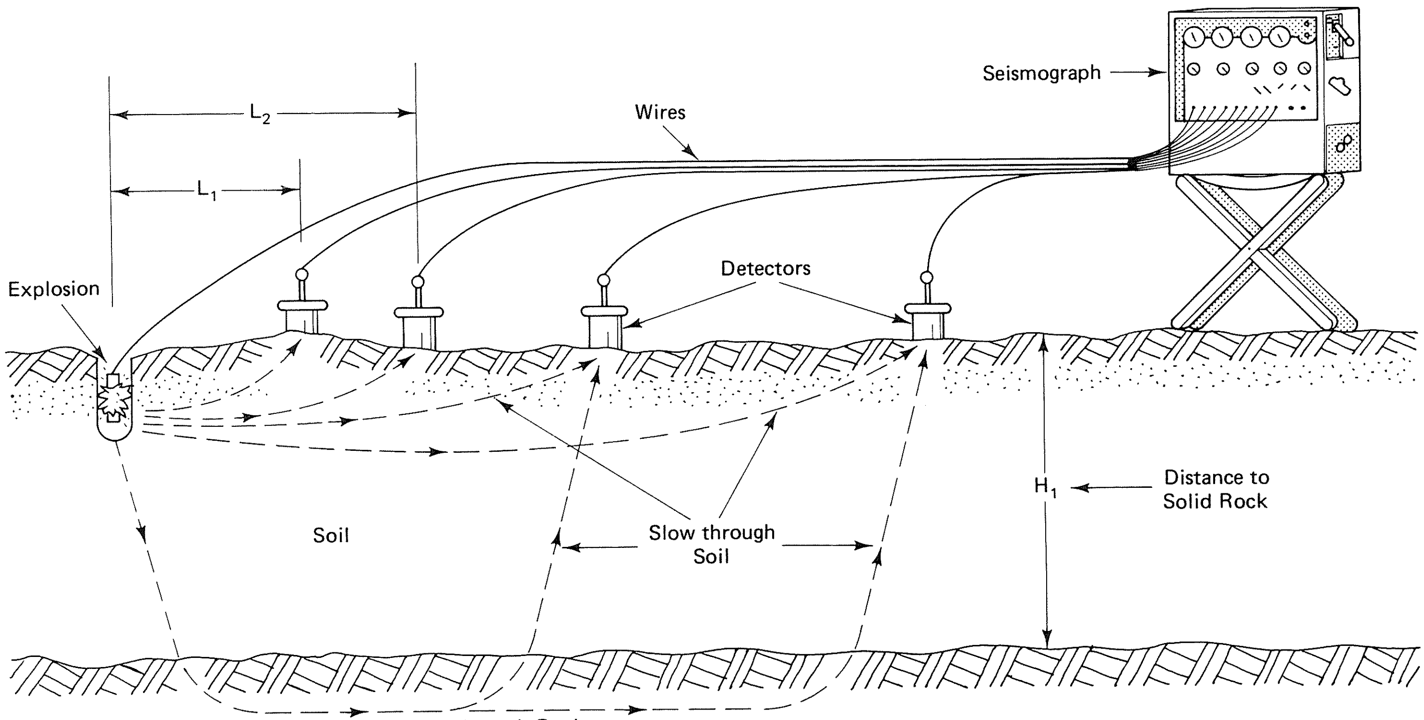
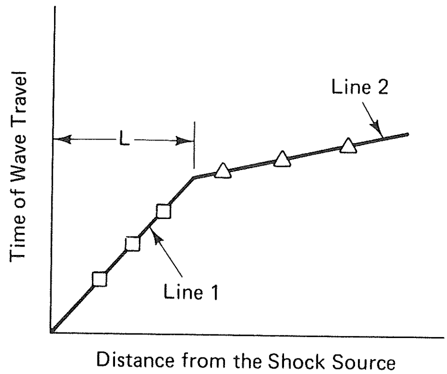


FIGURE 3-15 Seismic refraction test. [11]



Note: v_1 = Reciprocal of the Slope of Line 1
v_2 = Reciprocal of the Slope of Line 2 **FIGURE 3-16**

t_1 and t_2 = times required for the first shock wave to reach geophones Nos. 1 and 2, respectively

Similarly, v_2 is the reciprocal of the slope of line 2 as shown in Fig. 3-16. The thickness of stratum H_1 is given by

$$H_1 = \frac{L}{2}\sqrt{\frac{v_2 - v_1}{v_2 + v_1}} \tag{3-9}$$

where H_1 = depth of thickness of the upper layer
L = distance taken from the time versus distance graph where the two slopes intersect (see Fig. 3-16)

As indicated in Table 3-3, wave velocities range from about 800 ft/sec (244 m/s) in loose sand above the water table to 20,000 ft/sec (6096 m/s) in granite and unweathered gneiss. This wide range makes possible a general assessment of the characteristics of material encountered.

Seismic refraction can be used to estimate depths to successively harder strata, but it will not determine softer strata below harder strata. It can also be used to find depth to groundwater and to locate sinkholes. However, where boundaries are irregular or poorly defined, interpretation of the results of seismic refraction may be questionable.

Electrical Resistivity Method

As indicated initially in this section, resistance to movement of an electrical current through soil is determined in the electrical resistivity method. The premise for using this technique in subsurface investigations is that electrical resistance varies significantly enough among different types of soil and rock materials to allow identification of specific types if their resistivities are known.

A soil's resistivity generally varies inversely with its water content and dissolved ion concentration. Since clayey soils exhibit high dissolved ion concentrations, wet clayey soils have the lowest resistivities of all soil materials—as low as 5 ohm-ft (1.5 ohm·m). Coarse dry sand and gravel deposits and massive bedded and hard bedrocks have highest resistivities—over 8000 ohm-ft (2438 ohm·m). Table 3-4 gives resistivity correlation for various types of soil materials.

One specific procedure for conducting an electrical resistivity field survey utilizes four equally spaced electrodes. (This is known as the *Wenner method.*) The four electrodes are placed in a straight line spaced distance D

TABLE 3-3 Representative velocity values (ft/sec).[1,2]

Unconsolidated materials	
Most unconsolidated materials	Below 3000
Soil	
Normal	800–1500
Hard packed	1500–2000
Water	5000
Loose sand	
Above water table	800–2000
Below water table	1500–4000
Loose mixed sand and gravel, wet	1500–3500
Loose gravel, wet	1500–3000
Consolidated materials	
Most hard rocks	Above 8000
Coal	3000–5000
Clay	3000–6000
Shale	
Soft	4000–7000
Hard	6000–10,000
Sandstone	
Soft	5000–7000
Hard	6000–10,000
Limestone	
Weathered	As low as 4000
Hard	8000–18,000
Basalt	8000–13,000
Granite and unweathered gneiss	10,000–20,000
Compacted glacial tills, hardpan, cemented gravels	4000–7000
Frozen soil	4000–7000
Pure ice	10,000–12,000

[1]Courtesy of Soiltest, Inc.
[2]Occasional formations may yield velocities that lie outside these ranges. 1 ft/sec = 0.3048 m/s.

TABLE 3-4 Resistivity correlation.[1]

Ohm-ft	*2π ohm·cm (For Barnes Method)*	*Types of Materials*
5–10	1000–2000	Wet to moist clayey soils
10–50	3000–15,000	Wet to moist silty clay and silty soils
50–500	15,000–75,000	Moist to dry silty and sandy soils
500–1000	30,000–100,000	Well-fractured to slightly-fractured bedrock with moist soil filled cracks
1000	100,000	Sand and gravel with silt
1000–8000	100,000–300,000	Slightly fractured bedrock with dry soil filled cracks; sand and gravel with layers of silt
8000 (plus)	300,000 (plus)	Massive bedded and hard bedrock; coarse dry sand and gravel deposits

[1]Courtesy of Soiltest, Inc.

apart, as illustrated in Fig. 3-17. An electrical current is supplied (by a battery or small generator) through the outer electrodes (see Fig. 3-17); its value is measured by an ammeter. The voltage drop in the soil material within the zone created by the electrodes' electric field is measured between the two inner electrodes by a voltmeter (see Fig. 3-17). The soil material's electrical resistivity can be computed by the equation

$$\rho = 2\pi D \frac{V}{I} = 2\pi DR \tag{3-10}$$

where ρ = resistivity of the soil material, ohm-ft or ohm·m
D = electrode spacing, ft or m
V = voltage drop between the two inner electrodes, volts
I = current supplied through the outer electrodes, amperes
R = resistance, ohms

The zone created by the electrodes' electrical field extends downward to a depth approximately equal to the electrode spacing (i.e., D in Fig. 3-17). Consequently, the depth of subsurface material included in a given measurement is approximately equal to the spacing between electrodes. The resistivity determined by this method [computed by Eq. (3-10)] is actually a weighted mean value of all soil material within the zone.

A single application of the procedure just outlined would give an indication of the "average" type of subsurface material within the applicable zone.

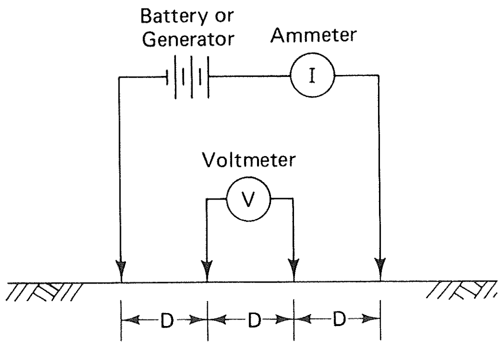


FIGURE 3-17 Electrode configuration for electrical resistivity test.

To determine depths of strata of different resistivities, the procedure is repeated for successively increasing electrode spacings (see Fig. 3-18). Since the applicable zone's depth varies directly with electrode spacing, data obtained from successively increasing electrode spacings should indicate changes in resistivity with depth, which in turn serves to locate different soil strata.

Resistivity data can be analyzed by plotting $\Sigma\rho$ (summation of soil resistivity values) versus electrode spacing (D) for increasing electrode spacings. Such a plotting is illustrated in Fig. 3-19. A straight-line plot indicates a constant soil resistivity (and therefore the same soil type) within the depth range for which the plot is straight. Furthermore, the slope of the straight line is equal to ρ_1/D, and ρ_1 gives the resistivity in the upper layer. Using this value of resistivity, one can estimate the type of soil within this layer. If a different soil type is encountered as additional tests are performed at increasing electrode spacings, a second straight-line plot should result with a slope equal to ρ_2/D, with ρ_2 giving the resistivity of the lower layer, from which, type of soil can be evaluated. Furthermore, the intersection of the two straight lines gives the approximate depth of the boundary between the two layers (see Fig. 3-19).

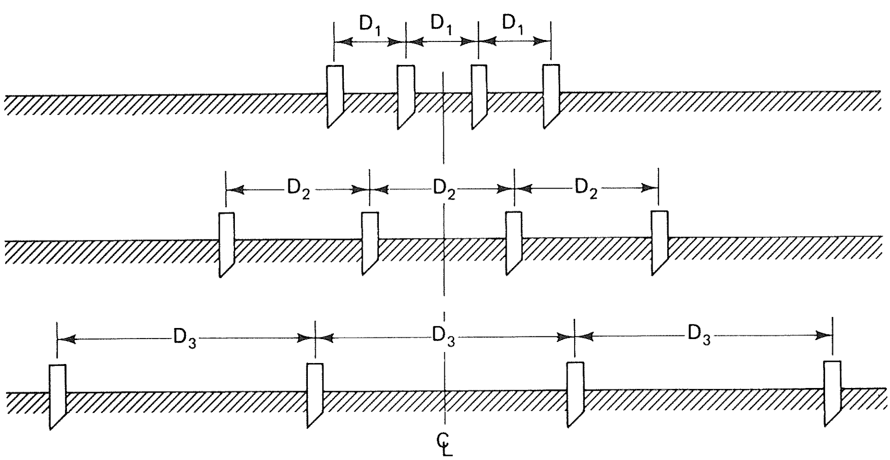


FIGURE 3-18 Representative electrode positions during a sequence of sounding measurements (the position of the center of the spread is fixed). (Courtesy of Soiltest, Inc.)

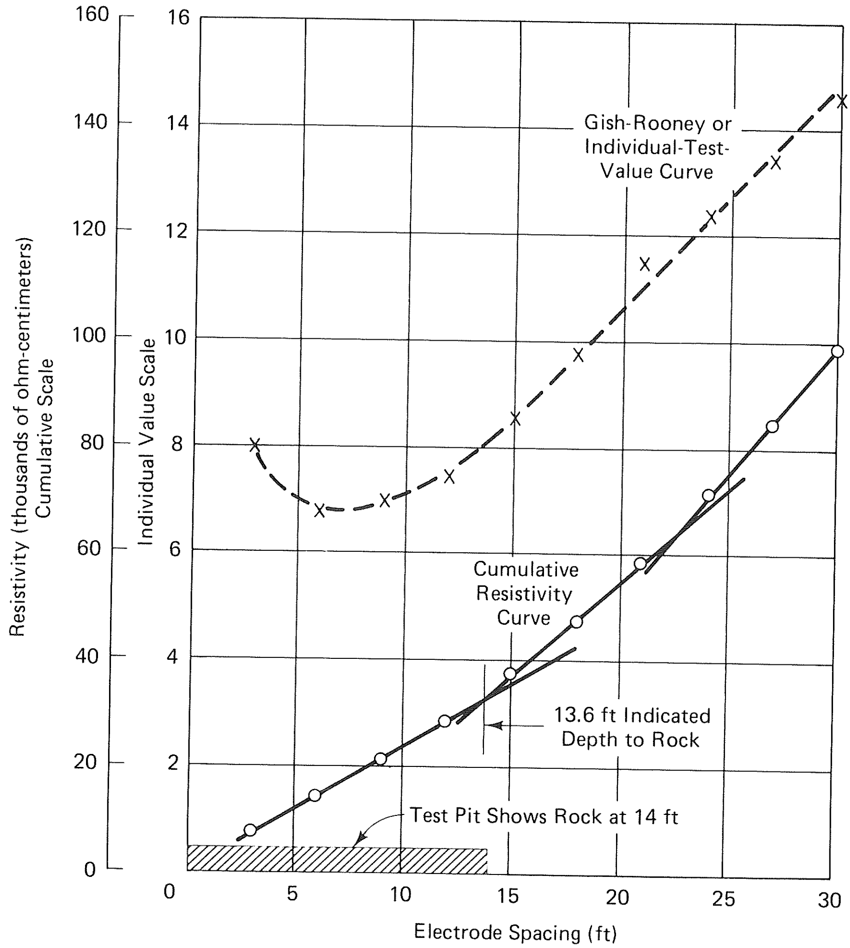


FIGURE 3-19 Typical resistivity data and method of analysis using the cumulative resistivity curve. [11]

The electrical resistivity method can be used to indicate subsurface variations where a hard layer underlies a soft layer; but unlike the seismic refraction method, it can also be used where a soft layer underlies a hard layer. The electrical resistivity method can be used not only to estimate depth to strata of different resistivities but also to find depth to groundwater and to locate masses of dry sands, gravels, and rock. It should be realized that errors in interpretation can occur because of the fact that soil resistivity varies with moisture content and identifies soil only indirectly. Hence, the electrical resistivity method should always be used with confirmatory drilling.

As related at the beginning of this section, geophysical methods afford relatively rapid and low-cost subsurface exploration as compared to test borings. However, dependable results from geophysical methods require experienced and skillful interpretation of test data. Geophysical methods have some disadvantages. The greatest is that, because of the subjectivity involved in analyzing, interpreting, and drawing conclusions from collected data, the resulting picture of the area's subsurface features may not be entirely accurate. Accordingly, geophysical methods should always be used in conjunction with test borings—either using sufficient test borings to verify results of geophysical methods or using geophysical methods to provide intermediate subsurface information between adjacent test borings.

3-9 RECORD OF SOIL EXPLORATION

It is of utmost importance that complete and accurate records be kept of all data collected. Boring, sampling, and testing are often costly undertakings, and to fail to keep good, accurate records is not only senseless, but it may also be dangerous.

To begin with, a good map giving specific locations of all borings should be available. Each boring should be identified (by number, for example), and its location documented by measurement to permanent features. Such a map is illustrated in Fig. 3-20.

For each boring, all pertinent data should be recorded in the field on a boring log sheet. Normally, these would be preprinted forms containing blanks for filling in appropriate data. An example of a boring log is given in Fig. 3-21.

Soil data obtained from a series of test borings can best be presented by preparing a geologic profile, which shows the arrangement of various layers of soil as well as the groundwater table, existing and proposed structures, and soil properties data (SPT, for example). Each bore hole is identified and indicated on the geologic profile by a vertical line. An example of a geologic profile is shown in Fig. 3-22.

A geologic profile is prepared by indicating on each bore hole on the profile (i.e., each vertical line representing a bore hole) the data obtained by boring, sampling, and testing. From these data, soil layers can be sketched in.

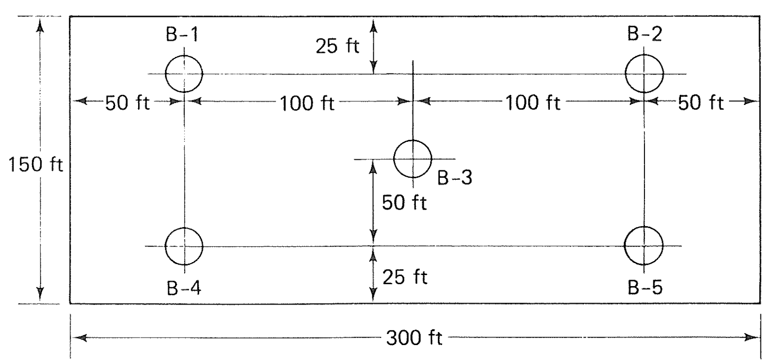


FIGURE 3-20 Example map showing boring locations on 150-ft by 300-ft construction site.

Sheet 1 of 1

ABC DRILLING COMPANY, INC.
NEWARK, NEW YORK

BORING NO. 5
ORD. ELEV. 372.4

PROJECT: Job No. 459
Name Eureka Warehouse
Address Illion, New York

CASING (Size & Type) 2 1/2" Drive Pipe
SAMPLE SPOON (Size & Type) 2" O.D.S.S.
HAMMER (Csg): Wt. 250 lbs., Drop 24 in.
(Spoon): Wt. 140 lbs., Drop 30 in.
DATE: Started 7/28/– Completed 7/29/– Driller Henry James

GROUND WATER OBSERVATIONS

Date	Time	Depth	Casing at
7/29/--	3:00 PM	18'3"	15'0"
"	4:00 PM	12'0"	10'0"
"	4:30 PM	8'0"	5'0"
7/30/--	8:30 AM	7'0"	Out

DEPTH FT.	BLOWS CSG.	BLOWS SPOON	N	Samples	
0–1	2				Black and grey moist FILL: cinders, brick and silt
1–2	16	11 / 8			
2–3	9	4	12	S #1, 1'-2'6"	3'0"
3–4	3	1 / 1			
4–5	3	2	3	S #2, 3'-4'6"	Black PEAT
5–6	3	P / 1			6'0"
6–7	5	1	2	S #3, 5'-6'6"	
7–8	6	3 / 6			
8–9	8	5	11	S #4, 7'-8'6"	Grey moist SILT with embedded fine gravel, trace of fine sand
9–10	9				
10–11	3	4 / 8			
11–12	8	6	14	S #5, 10'-11'6"	12'6"
12–13	15	15			Weathered SHALE
13–14	32	18 / 21			
14–15	78		39	S #6, 12'6"-14'	TOP OF ROCK 15'0"
15–20				Core Boring Series M– double tube core barrel, 2 in. diam. bit.	Weathered grey SHALE Run #1, 15'0" - 20'0" Recovered 30" - 50% Lost water @ 16'6" 20'0"
20–25					SHALE and SANDSTONE Run #2, 20'0" - 25'0" Recovered 56" - 93% Steady resistance while drilling 25'0"

FIGURE 3-21 Boring log. [12]

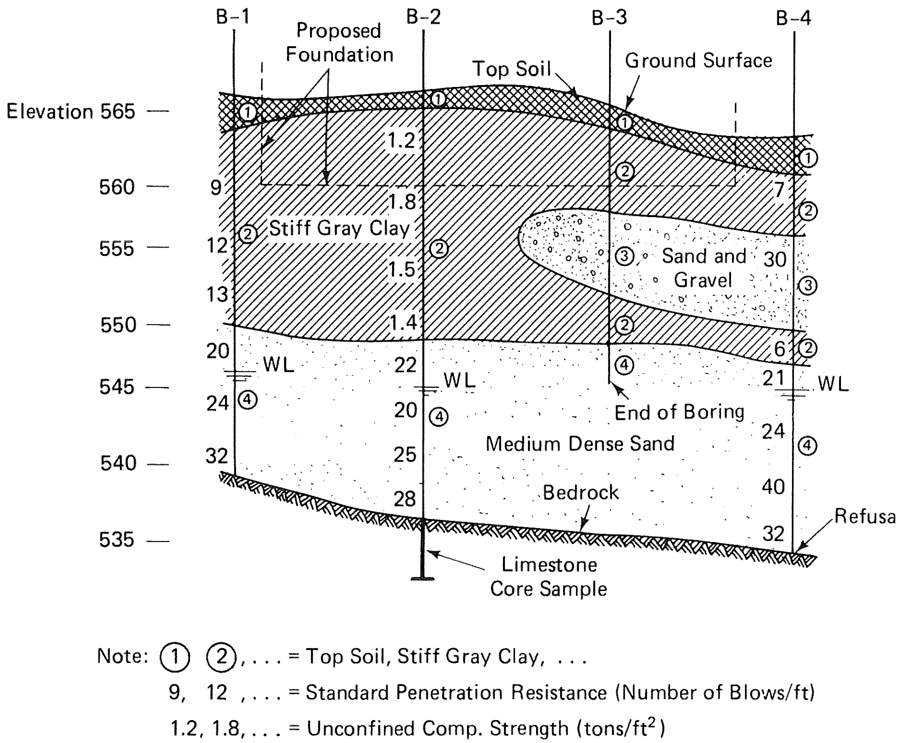


FIGURE 3-22 Example of geologic profile. [2]

Obviously, the more bore holes and the closer they are spaced, the more accurate the resulting geologic profile will be.

3-10 CONCLUSION

The subject of this chapter should be considered as one of the most important in this book. Analysis of soil and design of associated structures are of questionable value if the soil exploration data are not accurately determined and reported.

The authors hope this chapter will give the reader an effective introduction to actual soil exploration. However, learning to conduct soil exploration well requires much practice and varied experience under the guidance of experienced practitioners. Not only is it a complex science, it is a difficult art.

3-11 PROBLEMS

3-1 A 4-ft square footing is subjected to a contact pressure of 6000 lb/ft^2. The wet unit weight of the cohesive soil supporting the footing is estimated to be 118 lb/ft^3, and groundwater is known to be at great depth. Determine the minimum depth of test boring based on the criterion that test borings in cohesive soils should be carried at least to a depth where increase in stress due to the foundation loading is less than 10% of effective overburden pressure.

3-2 A Standard Penetration Test (SPT) was performed at a depth of 10 ft in sand of unit weight 120 lb/ft^3. The N-value was found to be 26. Determine the corrected N-value by both methods presented in this chapter.

3-3 Rework Problem 3-2 if groundwater is located 8 ft below the ground surface.

3-4 An SPT was performed at a depth of 7 m in sand of unit weight 20.40 kN/m^3. The N-value was found to be 22. Compute the corrected N-value by both methods presented in this chapter.

3-5 Rework Problem 3-4 if groundwater is located 2 m below the ground surface.

3-6 A field vane test was performed in a soft, sensitive clay layer. The vane tester's diameter and height are 4 in. and 8 in., respectively. The torque required to shear the clay was 61 ft-lb. Determine the clayey soil's cohesion if its plasticity index is known to be 40%.

3-7 Soil exploration was conducted at a construction site by seismic refraction, with field readings obtained as listed below:

Distance (ft)	*Time (msec)*
20	21
40	42
60	62.25
80	83
100	86.75
120	88.25
140	89.25
160	90.75
180	93

Estimate the thickness and type of material of the first soil layer and type of material in the underlying second layer.

3-8 Soil exploration was conducted at a construction site by the electrical resistivity method with field data obtained as listed below:

Electrode Spacing (ft)	*Resistance Readings (ohms)*
10	12.73
20	2.79
30	1.46
40	1.15
50	1.05
60	0.84
70	1.21
80	1.00
90	0.97
100	0.95

Estimate the thickness and type of material of the first soil layer and type of material in the underlying second layer.

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4

Soil Compaction

4-1 DEFINITION AND PURPOSE OF COMPACTION

The general meaning of the verb "compact" is to press closely together. In soil mechanics, it means to press the soil particles tightly together by expelling air from the void space. Compaction is normally produced deliberately and proceeds rapidly during construction, often by heavy compaction rollers. This is in contrast to "consolidation" (Chap. 7), which also results in a reduction of voids but which is caused by extrusion of water (rather than air) from the void space. Also, consolidation is not rapid.

Compaction of soil increases its density and produces three important effects: (1) an increase in the soil's shear strength, (2) a decrease in future settlement of the soil, and (3) a decrease in its permeability [1]. These three effects are beneficial for various types of earth construction, such as highways, airfields, and earth dams; and, as a general rule, the greater the compaction, the greater these benefits will be. Compaction is actually a rather cheap and effective way to improve the properties of a given soil.

Compaction is quantified in terms of a soil's dry unit weight, γ_d, which can be computed in terms of wet unit weight, γ, and moisture content, w (expressed as a decimal), by

$$\gamma_d = \frac{\gamma}{1 + w} \tag{4-1}$$

In most cases, dry soils can be best compacted (and thus a greater density achieved) if for each soil a certain amount of water is added to it. In effect, water acts as a lubricant and allows soil particles to be packed together better. If, however, too much water is added, a lesser density results. Thus, for a given compactive effort, there is a particular moisture content at which dry unit weight is greatest and compaction best. This moisture content is called the "optimum moisture content," and the associated dry unit weight is known as the "maximum dry unit weight."

Usual practice in a construction project is to perform laboratory compaction tests (covered in Sec. 4-2) on representative soil samples from the construction site to determine the soil's optimum moisture content and maximum dry unit weight. This maximum dry unit weight is used by the designer in specifying design shear strength, resistance to future settlement, and permeability characteristics. The soil is then compacted by field compaction methods (covered in Sec. 4-3) until the laboratory maximum dry unit weight (or an acceptable percentage of it) has been achieved. In-place soil density tests* (covered in Sec. 4-5) are used to determine if and when the laboratory maximum dry unit weight (or an acceptable percentage thereof) has been reached. Section 4-6 covers field control of compaction.

4-2 LABORATORY COMPACTION TESTS

As related in the preceding section, laboratory compaction tests are performed to determine a soil's optimum moisture content and maximum dry unit weight. Compaction test equipment, shown in Fig. 4-1, consists of a base plate, collar, and mold, in which soil is placed, and of a hammer that is raised and dropped freely onto the soil in the mold. The mold's size and the hammer's weight and drop distance are standardized, with several variations in size and weight available (see Table 4-1).

To carry out a laboratory compaction test, a soil sample from the field is allowed to dry until it becomes friable under a trowel. The soil sample may be dried in air or a drying oven. If an oven is used, its temperature should not exceed 60°C (140°F). After drying, a series of at least four specimens is prepared by adding increasing amounts of water to each sample so that the moisture contents will bracket the optimum moisture content. After a specified curing period, each prepared specimen is placed, in turn, in a compaction mold (with collar attached) and compacted in layers by dropping the hammer onto the specimen in the mold a certain distance and specified number of uniformly distributed blows per layer. This results in a specific energy exertion per unit volume of soil. Upon completion of each compaction, the attached collar is removed and the compacted soil trimmed until it is even with the top of the mold. The compacted soil specimen's wet unit weight is then determined by dividing the weight of compacted soil in the mold by the soil specimen's vol-

* Customarily, the name of this test is *in-place soil density.* Actually, it is an in-place unit weight test.

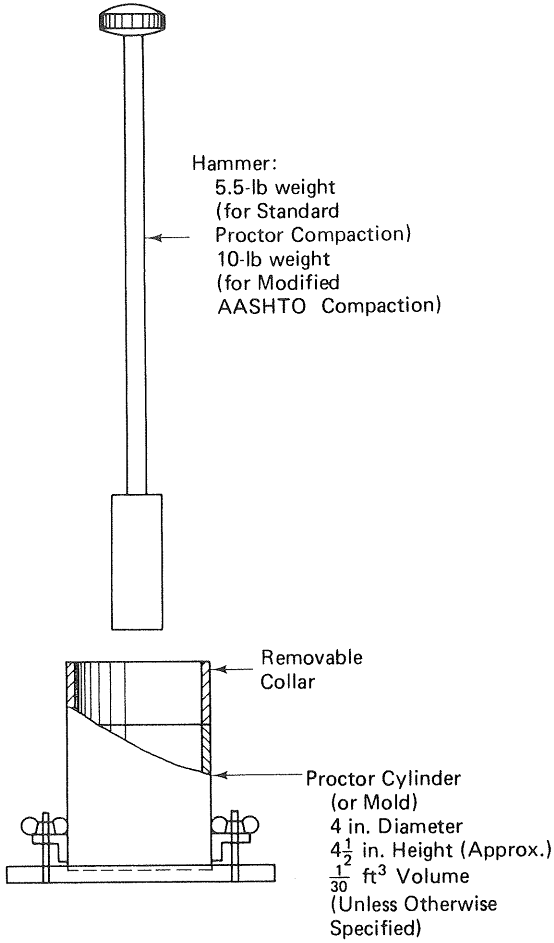

FIGURE 4-1 Compaction test equipment. [2]

ume, which is the volume of the mold. The compacted soil is subsequently removed from the mold, and its moisture content determined. With the compacted soil's wet unit weight and moisture content known, its dry unit weight is computed using Eq. (4-1).

A plot made of the soil's moisture content versus dry unit weight for the data collected as described above will be of a form similar to the curve shown in Fig. 4-2. The coordinates of the point at the curve's peak give the soil's maximum dry unit weight and optimum moisture content. Presumably, this gives maximum expected dry unit weight—the dry unit weight to be used by the designer and to be striven for in the field compaction. To achieve this maximum dry unit weight, field compaction should be done at or near the optimum moisture content.

In Fig. 4-2, the right side of the moisture content versus dry unit weight curve roughly parallels the dashed line labeled "zero air voids." This line represents the dry unit weight when saturation is 100% (i.e., the soil's entire vol-

TABLE 4-1 Summary of specifications for compaction testing equipment, compaction effort, and sample fractionation [2].[1]

	Test Designation							
	AASHTO: T 99 ASTM D 698				AASHTO: T 180 ASTM D 1557			
	Method A^2	*Method* *B*	*Method* C^3	*Method* *D*	*Method* A^4	*Method* *B*	*Method* C^3	*Method* *D*
Hammer weight (lb)	5.5	5.5	5.5	5.5	10	10	10	10
Drop (in.)	12	12	12	12	18	18	18	18
Size of mold								
Diameter (in.)	4	6	4	6	4	6	4	6
Height (in.)	4.58	4.58	4.58	4.58	4.58	4.58	4.58	4.58
Volume (ft^3)	$\frac{1}{30}$	$\frac{1}{13.33}$	$\frac{1}{30}$	$\frac{1}{13.33}$	$\frac{1}{30}$	$\frac{1}{13.33}$	$\frac{1}{30}$	$\frac{1}{13.33}$
Number of layers	3	3	3	3	5	5	5	5
Blows per layer	25	56	25	56	25	56	25	56
Fraction tested	−No. 4	−No. 4	−¾ in.	−¾ in.	−No. 4	−No. 4	−¾ in.	−¾ in.

[1] 1 lb = 4.448 N; 1 in. = 25.4 mm; 1 ft^3 = 0.02832 m^3.
[2] This is the original "Standard Proctor" test.
[3] Note 2 of this method provides for "stone substitution," a means of evaluating the effect of stone sizes up to 2-in diameter. (See complete specification.)
[4] This is the original "modified AASHTO" test.

ume is water and solids). This line actually represents, in theory, the upper limit on unit weight at any moisture content. For this reason, the zero air voids line is often included on moisture content versus dry unit weight curves. It can be determined from the equation

$$\gamma_{ZAV} = \frac{G_s\gamma_w}{1 + wG_s} \tag{4-2}$$

where γ_{ZAV} = dry unit weight at zero air voids
G_s = specific gravity of solids
γ_w = unit weight of water
w = moisture content (expressed as a decimal)

As indicated at the beginning of this section, the mold's size and the hammer's weight and drop distance are standardized for compaction tests, with several variations in size and weight available. Table 4-1 summarizes these variations. The four methods on the left side of Table 4-1 are designated AASHTO T 99 and ASTM D 698. Method A under these designations is known as the original *Standard Proctor* compaction test. The four methods on the right side of Table 4-1 are designated AASHTO T 180 and ASTM D 1557. Method A under these is known as the original *Modified AASHTO* compaction test and was developed subsequent to the Standard Proctor to obtain higher values of dry density. It was developed in response to the need for higher densities of airfield pavement subgrades, embankments, earth dams, and so on, and for compacted soil that is to support large and heavy structures.

Example 4-1 illustrates computation of unit weight of a specimen of a laboratory compacted soil. Example 4-2 illustrates determination of maximum dry unit weight and optimum moisture content, as the result of a laboratory compaction test.

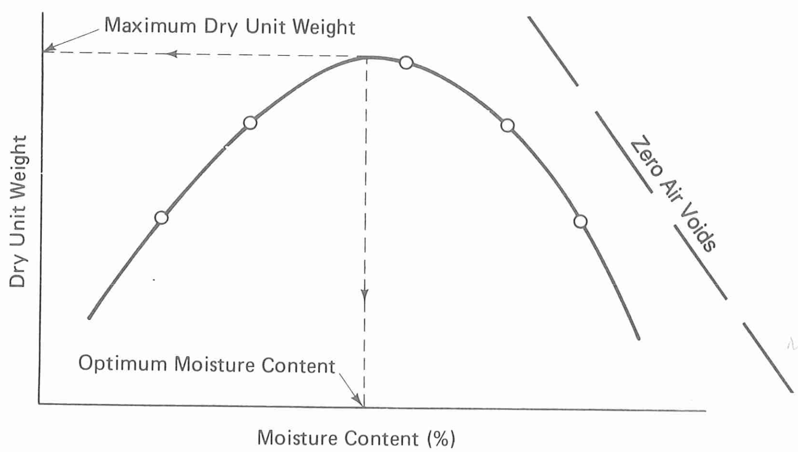

FIGURE 4-2

EXAMPLE 4-1

Given

1. The combined weight of a mold and the specimen of compacted soil it contains is 8.63 lb.
2. The mold's volume is $\frac{1}{30}$ ft^3.
3. The mold's weight is 4.35 lb.
4. The specimen's water content is 10%.

Required

1. Wet unit weight of the specimen.
2. Dry unit weight of the specimen.

Solution

1. From Eq. (2-9),

$$\gamma = \frac{W}{V} \tag{2-9}$$

$$\gamma = \frac{8.63\ \text{lb} - 4.35\ \text{lb}}{\frac{1}{30}\ \text{ft}^3} = 128.4\ \text{lb/ft}^3$$

2. From Eq. (4-1),

$$\gamma_d = \frac{\gamma}{1 + w} \tag{4-1}$$

$$\gamma_d = \frac{128.4\ \text{lb/ft}^3}{1 + 0.10} = 116.7\ \text{lb/ft}^3$$

EXAMPLE 4-2

Given

A set of laboratory compaction test data and results is tabulated below. The test was conducted in accordance with ASTM D 698 Standard Proctor test.

Determination Number	1	2	3	4	5
Dry Unit Weight (lb/ft^3)	112.2	116.7	118.3	115.2	109.0
Moisture Content (%)	7.1	10.0	13.4	16.7	20.1

Required

1. Plot a Proctor curve (i.e., dry unit weight versus moisture content).

2. Determine the soil's maximum dry unit weight and optimum moisture content.

Solution

1. See Fig. 4-3.
2. From Fig. 4-3,

$$\text{Maximum dry unit weight} = 118.5 \text{ lb/ft}^3$$

$$\text{Optimum moisture content} = 12.5\%$$

Type of soil is the primary factor affecting maximum dry unit weight and optimum moisture content for a given compactive effort and compaction method. Maximum dry unit weights may range from about 60 lb/ft³ for organic soils to about 145 lb/ft³ for well-graded granular material containing just enough fines to fill small voids. Optimum moisture contents may range from around 5% for granular material to about 35% for elastic silts and clays. Higher optimum moisture contents are generally associated with lower dry unit weights. Higher dry unit weights are associated with well-graded granular materials. Uniformly graded sand, clays of high plasticity, and organic silts and clays typically respond poorly to compaction [3].

Table 4-2 presents some general compaction characteristics of various soil types along with their values as embankment, subgrade, and base mate-

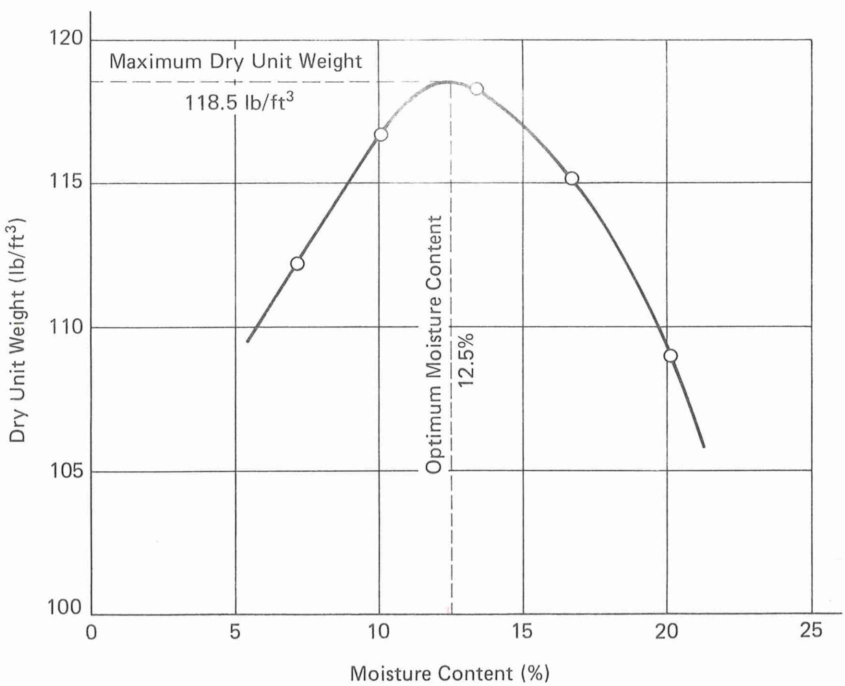

FIGURE 4-3

TABLE 4-2 Compaction characteristics and ratings of Unified soil classification classes for soil construction [3,4]

Class	*Compaction Characteristics*	*Maximum Dry Density Standard AASHTO (lb/ft^3)*[1]	*Compressibility and Expansion*	*Value as Embankment Material*	*Value as Subgrade Material*	*Value as Base Course*
GW	Good: tractor, rubber-tired, steel wheel, or vibratory roller	125–135	Almost none	Very stable	Excellent	Good
GP	Good: tractor, rubber-tired, steel wheel, or vibratory roller	115–125	Almost none	Reasonably stable	Excellent to good	Poor to fair
GM	Good: rubber-tired or light sheepsfoot roller	120–135	Slight	Reasonably stable	Excellent to good	Fair to poor
GC	Good to fair: rubber-tired or sheepsfoot roller	115–130	Slight	Reasonably stable	Good	Good to fair
SW	Good: tractor, rubber-tired or vibratory roller	110–130	Almost none	Very stable	Good	Fair to poor
SP	Good: tractor, rubber-tired or vibratory roller	100–120	Almost none	Reasonably stable when dense	Good to fair	Poor
SM	Good: rubber-tired or sheepsfoot roller	110–125	Slight	Reasonably stable when dense	Good to fair	Poor
SC	Good to fair: rubber-tired or sheepsfoot roller	105–125	Slight to medium	Reasonably stable	Good to fair	Fair to poor
ML	Good to poor: rubber-tired or sheepsfoot roller	95–120	Slight to medium	Poor stability, high density required	Fair to poor	Not suitable
CL	Good to fair: sheepsfoot or rubber-tired roller	95–120	Medium	Good stability	Fair to poor	Not suitable

[1] 1 lb/ft^3 = 0.1571 kN/m^3.

TABLE 4-2 (continued)

Class	*Compaction Characteristics*	*Maximum Dry Density Standard AASHTO (lb/ft³)*[1]	*Compressibility and Expansion*	*Value as Embankment Material*	*Value as Subgrade Material*	*Value as Base Course*
OL	Fair to poor: sheepsfoot or rubber-tired roller	80–100	Medium to high	Unstable, should not be used	Poor	Not suitable
MH	Fair to poor: sheepsfoot or rubber-tired roller	70–95	High	Poor stability, should not be used	Poor	Not suitable
CH	Fair to poor: sheepsfoot roller	80–105	Very high	Fair stability, may soften on expansion	Poor to very poor	Not suitable
OH	Fair to poor: sheepsfoot roller	65–100	High	Unstable, should not be used	Very poor	Not suitable
PT	Not suitable	—	Very high	Should not be used	Not suitable	Not suitable

TABLE 4-3 General guide to selection of soils on basis of anticipated embankment performance [3, 5].

AASHTO Classification	*Visual Description*	*Maximum Dry-Weight Range (lb/ft³)*[1]	*Optimum Moisture Range (%)*	*Anticipated Embankment Performance*
A-1-a A-1-b	Granular material	115–142	7–15	Good to excellent
A-2-4 A-2-5 A-2-6 A-2-7	Granular material with soil	110–135	9–18	Fair to excellent
A-3	Fine sand and sand	110–115	9–15	Fair to good
A-4	Sandy silts and silts	95–130	10–20	Poor to good
A-5	Elastic silts and clays	85–100	20–35	Unsatisfactory
A-6	Silt-clay	95–120	10–30	Poor to good
A-7-5	Elastic silty clay	85–100	20–35	Unsatisfactory
A-7-6	Clay	90–115	15–30	Poor to fair

[1] 1 $lb/ft^3 = 0.1571\ kN/m^3$.

rial for soils classified according to the Unified Soil Classification System. Table 4-3 gives anticipated embankment performance for soils classified according to the AASHTO system.

4-3 FIELD COMPACTION

Normally, soil is compacted in layers. An approximately 8-in. (203 mm) loose horizontal layer of soil is often spread from trucks and then compacted to a thickness of about 6 in. (152 mm). The moisture content can be increased by sprinkling water over the soil if it is too dry and thoroughly mixing the water into the uncompacted soil by disk plowing. If the soil is too wet, its moisture content can be reduced by aeration (i.e., by spreading the soil in the sun and turning it with a disk plow to provide aeration and drying.) Actual compaction is done by *tampers* and/or *rollers* and is normally accomplished with a maximum of 6 to 10 complete coverages by the compaction equipment. The surface of each compacted layer should be scarified by disk plowing or other means to provide bonding between layers. Various kinds of field compaction equipment (i.e., tampers and rollers) are discussed briefly in this section.

Tampers are devices that compact soil by delivering a succession of relatively light, vertical blows. Tampers are held in place and operated by hand. They may be powered either pneumatically or by gasoline-driven pistons. Tampers are limited in scope and compacting ability. Therefore, they are most useful in areas not readily accessible to rollers, in which case soil may be placed in loose horizontal layers not exceeding 6 in. and then compacted with tampers.

Rollers come in a variety of forms, such as the smooth wheel roller, sheepsfoot roller, pneumatic roller, and vibratory roller. Some of these are self-propelled, while others are towed by tractors. Some are more suited to certain types of soil. Rollers can easily cover large areas relatively quickly and with great compacting pressures. Brief descriptions of the four types of rollers mentioned above follow.

A *smooth wheel roller* (Fig. 4-4) employs two or three smooth metal rollers. It is useful in compacting base courses and paving mixtures and is also used to provide a smooth finished grade. Generally, smooth wheel rollers are self-propelled and equipped with a reversing gear so they can be driven back and forth without turning. A smooth wheel roller provides compactive effort primarily through its static weight.

A *sheepsfoot roller* (Fig. 4-5) consists of a drum with metal projecting "feet" attached. Since only the projecting feet contact the soil, the area of contact between roller and soil is smaller (than for a smooth wheel roller) and therefore a greater compacting pressure results (generally more than 200 $lb/in.^2$). A sheepsfoot roller provides kneading action and is effective for compacting fine-grained soils (such as clays and silts).

A *pneumatic roller* (Fig. 4-6) consists of a number of rubber tires, highly inflated. They vary from small rollers to very large and heavy ones. Most large pneumatic rollers are towed while some smaller ones are self-propelled. Some have boxes mounted above their wheels, to which sand or other material can be added for increased compacting pressure. Clayey soils and silty soils

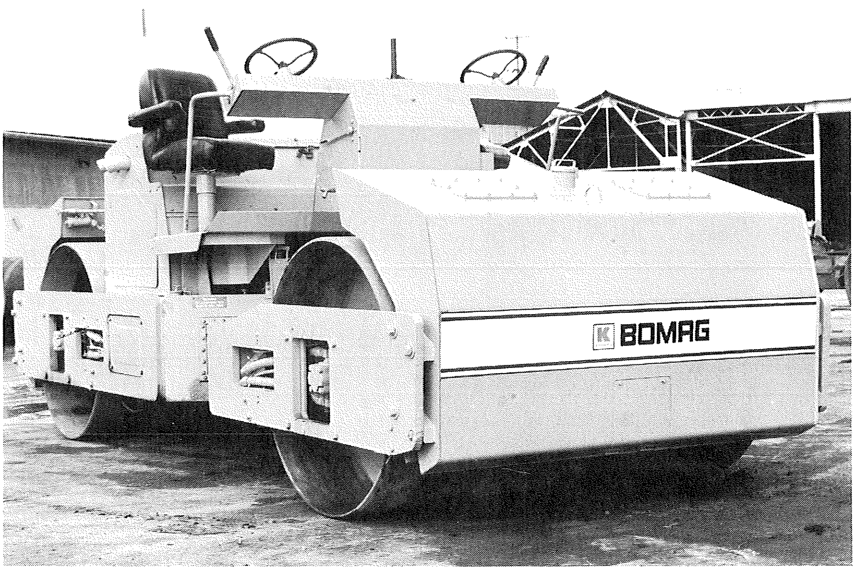

FIGURE 4-4 Smooth wheel roller. (Courtesy of BOMAG [U.S.A.], Inc.)

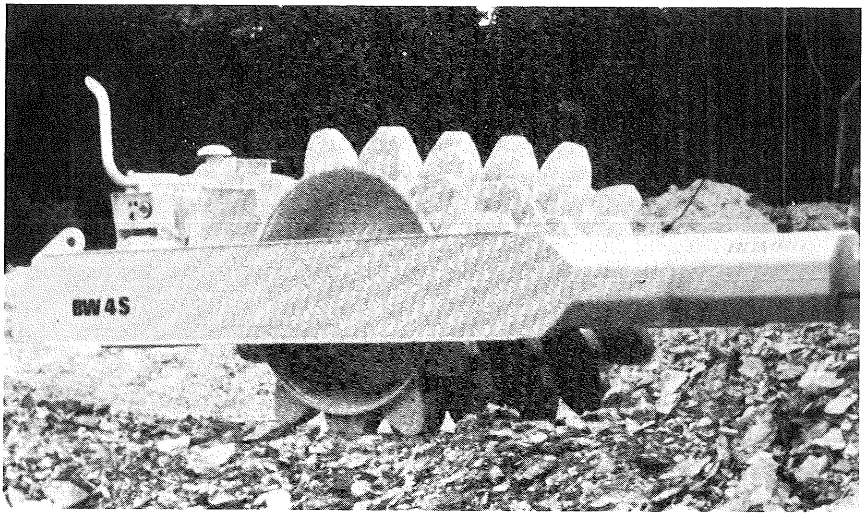

FIGURE 4-5 Sheepsfoot roller. (Courtesy of BOMAG [U.S.A.], Inc.)

may be compacted effectively by pneumatic rollers. They are also effective in compacting granular material containing some small amount of fines.

A *vibratory roller* (Fig. 4-7) contains some kind of vibrating unit that imparts an up and down vibration to the roller as it is pulled over the soil. Vibrating units can supply frequencies of vibration at 1500 to 2000 cycles per

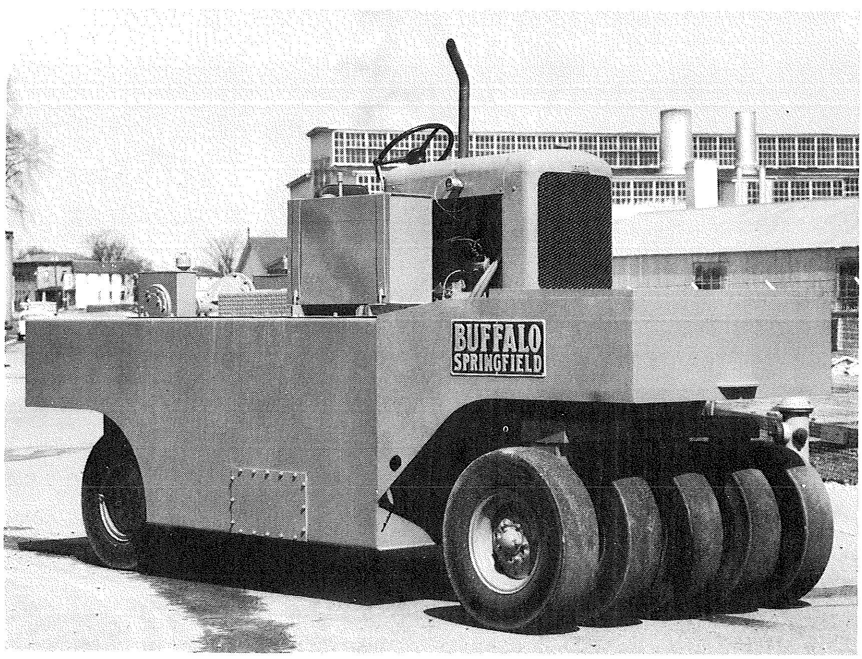

FIGURE 4-6 Pneumatic roller. (Courtesy of BOMAG [U.S.A.], Inc.)

FIGURE 4-7 Vibratory roller. (Courtesy of Hyster Company, Construction Equipment Division.)

minute, depending on compacting requirements. They are effective in compacting granular materials—particularly clean sands and gravels.

Two means (or possibly a combination of the two) may be used to specify a particular compaction requirement. One is to specify the procedure to be followed by the contractor, such as the type of compactor (i.e., roller) to be used and the number of passes to be made. The other is to simply specify the compacted soil's required final dry unit weight. The first method has the advantage that little testing is required, but it has the disadvantage that the specified procedure may not produce the required result. The second method requires much field testing, but it ensures that the required dry unit weight is achieved. In effect, the second method specifies the required final dry unit weight but leaves it up to the contractor as to how that unit weight is achieved. This (i.e., the second) method is probably more commonly employed.

4-4 DYNAMIC COMPACTION

In cases where existing surface or near-surface soil is poor with regard to foundation support, a field procedure known as *dynamic compaction* may be employed to improve the soil's properties. This method is carried out essentially by repeatedly dropping a very heavy weight onto the soil from a relatively great height. The dropped weight may be an ordinary steel wrecking ball, or it may be a mass especially designed for the dynamic compaction procedure. Typical weights range from 2 to 20 tons or higher while dropping distances range from 20 to 100 ft. Generally, the heavier the weight and the greater the dropping distance, the greater the compactive effort will be. For a given situation, however, weight and dropping distance used may depend on lifting equipment (such as a crane) available.

Dynamic compaction may be used for both cohesive and cohesionless soils. It can also be utilized to compact buried refuse fill areas. In cohesive soils, reduction of settlements due to dynamic compaction is more distinct than the increase in bearing capacity. The tamping produces a true presettlement of the soil, well beyond the settlement that would have occurred as a result of construction weight only, without any preliminary consolidation [6]. For cohesionless soils, dynamic compaction densifies loose soil.

Dynamic compaction should not be done by dropping weight randomly. Instead, a closely spaced grid pattern is selected for a given compaction site (see Fig. 4-8). Preliminary work is done to determine grid spacing and weight, height, and number of drops. Typically, five to ten drops are made on each grid point. Figure 4-9 shows a photograph of a dynamic-compaction site.

The approximate depth of influence of dynamic compaction (D) may be determined in terms of weight (W) and distance dropped (h). For cohesionless soils [7],

$$D = 0.5\sqrt{Wh} \tag{4-3}$$

For cohesive soils [6],

$$D = \sqrt{Wh} \tag{4-4}$$

These equations give the depth of zone (D) receiving improvement in meters if W is in metric tons (1000 kg) and h in meters. The extent of improvement is greatest near the surface and diminishes with depth. Improvement increases with the number of drops made up to some limit—typically from five to ten drops—beyond which additional drops afford little or no additional improvement.

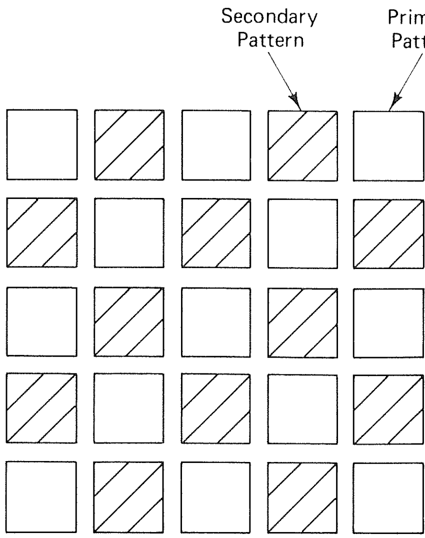

FIGURE 4-8 Drop pattern. [7]

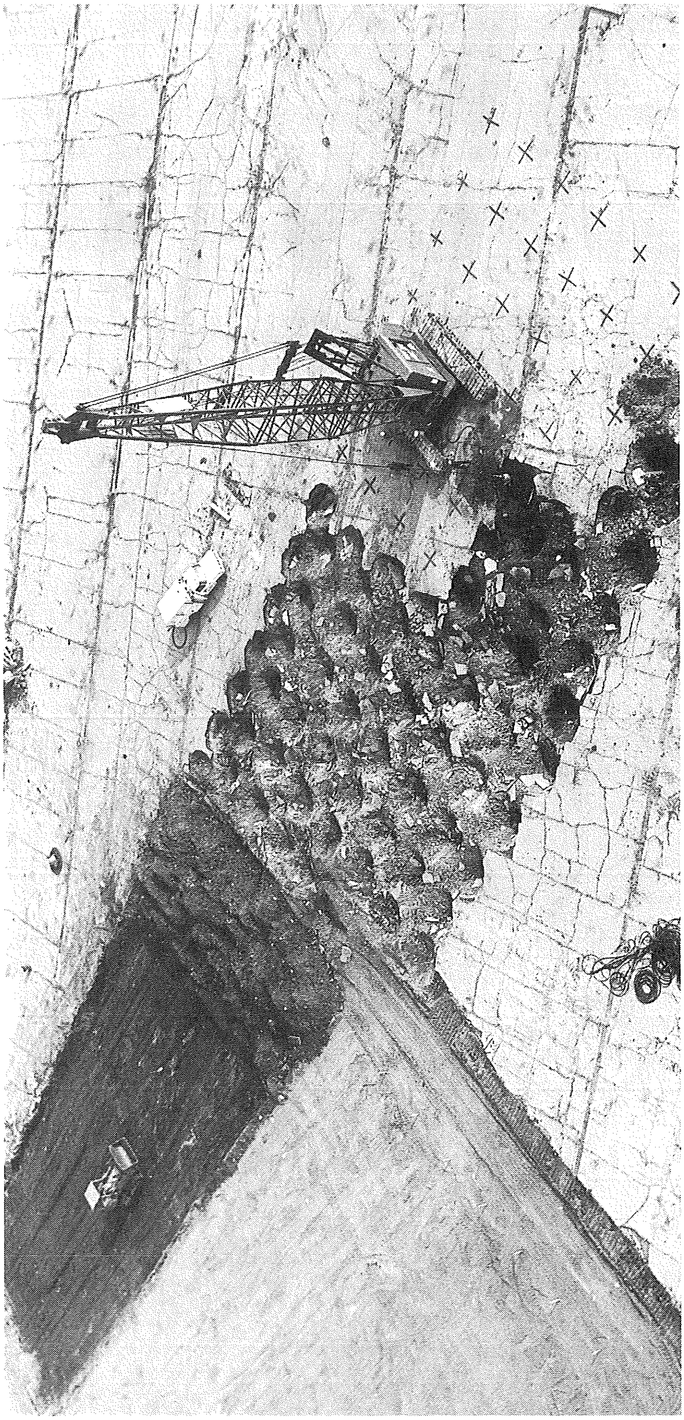

FIGURE 4-9 Dynamic-compaction site. (Courtesy of Hayward Baker Inc.)

With saturated, fine-grained soils, satisfactory results may be obtained by performing a series of droppings at intervals of one or several days, the purpose being to provide time for dissipation of pore water pressures created by the previous compaction.

It should be noted that a soil surface may become cratered as a result of dynamic compaction. This is particularly true of "loose" soils. When this happens, the craters must be backfilled and compacted by other means (such as those described in Sec. 4-3).

4-5 IN-PLACE SOIL DENSITY TEST*

As related previously, after a fill layer of soil has been compacted by the contractor, it is important that the compacted soil's in-place dry unit weight be determined in order to ascertain whether the maximum laboratory dry unit weight has been attained. If maximum dry unit weight (or an acceptable percentage thereof) has not been attained, additional compaction effort is required.

There are several methods for determining in-place unit weight. As a general rule, the weight and volume of an in-place soil sample are determined, from which unit weight can be computed. Measurement of the sample's weight is straightforward, but there are several methods for determining its volume. For cohesive soils, a thin-walled cylinder may be driven into the soil to remove a sample. The sample's volume is known from the cylinder's volume. This method is known as *density of soil in-place by the drive cylinder method* and is designated as ASTM D 2937 or AASHTO T 204. The drive cylinder method is not applicable for very hard soil that cannot be easily penetrated. Neither is it applicable for low plasticity or cohesionless soils, which are not readily retained in the cylinder.

For low plasticity or cohesionless soils, a hole can be dug and its volume determined by filling it with loose, dry sand of uniform density (such as Ottawa sand). An alternative method is to fill the hole with water (utilizing a rubber membrane, or balloon). In either case, the hole's volume, and thus the volume of soil removed, is measured by the volume of material (sand or water) added. Volume of hole determined by filling it with sand is called *density of soil in-place by the sand-cone method* and is designated as ASTM D 1556 or AASHTO T 191. Volume of hole determined by filling it with water is called *density of soil in-place by the rubber-balloon method* and is designated as ASTM D 2167 or AASHTO T 205.

In-place unit weight of soil can also be determined through the use of nuclear equipment, which utilizes radioactive materials. This method is called *density of soil and soil-aggregate in-place by nuclear methods* and is designated as ASTM D 2922 or AASHTO T 238.

* Customarily, the name of this test is *in-place soil density.* Actually, it is an in-place unit weight test.

In addition to determining in-place wet unit weight of soil, it is also necessary to determine the soil's moisture content in order to compute the compacted soil's dry unit weight. Although moisture content can be determined by oven drying, this method is often too time-consuming, since test results are commonly needed quickly. Drying of a soil sample can be accomplished by putting it in a skillet and placing the skillet over the open flame of a camp stove. The Speedy Moisture Tester (Fig. 4-10) can also be used to determine moisture content quickly with fairly good results. Because of the rather small amount of sample utilized in this test, the Speedy Moisture Tester may not be appropriate for use in coarser materials.

Step-by-step details of all the above test procedures except for the "nuclear methods" are given in *Soil Properties: Testing, Measurement, and Evaluation,* 2nd edition by Liu and Evett (Prentice-Hall, Inc., 1990).

EXAMPLE 4-3

Given

During construction of a soil embankment, a sand-cone in-place density test was performed in the field. The following data were obtained:

1. Weight of sand used to fill test hole and funnel of sand-cone device = 867 g.
2. Weight of sand to fill funnel = 319 g.
3. Unit weight of sand = 98.0 lb/ft^3.
4. Weight of wet soil from the test hole = 747 g.

FIGURE 4-10 Speedy moisture tester. (Courtesy of Soiltest, Inc.)

5. Moisture content of soil from test hole as determined by Speedy Moisture Tester = 13.7%.

Required

Dry unit weight of the compacted soil.

Solution

Weight of sand used in test hole
= weight of sand to fill test hole and funnel − weight of sand to fill funnel
= 867 g − 319 g = 548 g

$$\text{Volume of test hole} = \frac{\dfrac{548\ \text{g}}{453.6\ \text{g/lb}}}{98.0\ \text{lb/ft}^3} = 0.0123\ \text{ft}^3$$

$$\text{Wet unit weight of soil in-place} = \frac{\dfrac{747\ \text{g}}{453.6\ \text{g/lb}}}{0.0123\ \text{ft}^3} = 133.9\ \text{lb/ft}^3$$

From Eq. (4-1),

$$\gamma_d = \frac{\gamma}{1 + w} \tag{4-1}$$

$$\gamma_d = \frac{133.9\ \text{lb/ft}^3}{1 + 0.137} = 117.8\ \text{lb/ft}^3$$

4-6 FIELD CONTROL OF COMPACTION

As related previously, after a fill layer of soil has been compacted, an in-place soil density test is usually performed to determine whether the maximum laboratory dry unit weight (or an acceptable percentage thereof) has been attained. It is common to specify a required percent of compaction, which is "the required in-place dry unit weight" divided by "the maximum laboratory dry unit weight" expressed as a percentage, in a contract document. Thus, if the maximum dry unit weight obtained from ASTM or AASHTO compaction in the laboratory is 100 lb/ft^3 and required percent of compaction is 95% according to a contract, an in-place dry unit weight of 95 lb/ft^3 (or higher) would be acceptable. In theory, this is simple enough to do; but there are some practical considerations that must be taken into account. For example, the type of soil or compaction characteristics of soil taken from borrow pits may vary from one location to another. Also, the degree of compaction may not be uniform throughout.

To deal with the problem of nonuniformity of soil from borrow pits, it is necessary to conduct ASTM or AASHTO compaction tests in the laboratory to establish maximum laboratory dry unit weight along with optimum moisture content for each type of soil encountered in a project. Then, as soil is transported from the borrow pit and subsequently placed and compacted in the fill area, it is imperative that results of each in-place soil density test (i.e., in-place dry unit weight) be checked against the maximum laboratory dry unit weight of the respective type of soil.

To deal with the problem of variable degree of field compaction of a soil, it is common practice to specify a minimum number of field density tests. For example, for a dam embankment, it might be specified that one test be made for every 2400 yd^3 (loose measure) of fill placed.

To ensure that required field density is achieved by the field compaction, a specifications contract between owner and contractor is prepared. The contract will normally specify the required percent of compaction and minimum number of field density tests required. For compaction adjacent to a structure, where settlement is a serious matter, a higher percent of compaction and a higher minimum number of tests may be specified than for compaction, for example, of the foundation of a parking lot. The specifications contract may also include additional items, such as maximum thickness of loose lifts (layers) prior to compaction, methods to obtain maximum dry unit weight (e.g., ASTM D 698 or AASHTO T 99), methods to determine in-place unit weight (e.g., ASTM D 1556 or AASHTO T 191), and so on.

As the owner's representative, a soils engineer is responsible for seeing that contract provisions are carried out precisely and completely. He or she is responsible for the testing and must see that the required compacted dry unit weight is achieved. If a particular test indicates the required compacted dry unit weight has not been achieved, he or she must require additional compaction effort, possibly including an adjustment in moisture content. In addition, he or she must be knowledgeable and capable of dealing with field situations that arise which may go beyond the "textbook procedure."

EXAMPLE 4-4

Given

1. Soil from a borrow pit to be used for construction of an embankment gave the following laboratory results when subjected to ASTM D 698 Standard Proctor test (from Example 4-2).

$$\text{Maximum dry unit weight} = 118.5 \text{ lb/ft}^3$$
$$\text{Optimum moisture content} = 12.5\%$$

2. The contractor, during construction of the soil embankment, achieved the following (from Example 4-3):

$$\text{Dry unit weight reached by field compaction} = 117.8 \text{ lb/ft}^3$$
$$\text{Actual water content} = 13.7\%$$

Required

Percent of compaction achieved by the contractor.

Solution

Percent of Standard Proctor compaction achieved

$$= \frac{\text{In-place dry unit weight}}{\text{Maximum laboratory dry unit weight}} \times 100 = \frac{117.8 \text{ lb/ft}^3}{118.5 \text{ lb/ft}^3} \times 100 = 99.4\%$$

4-7 PROBLEMS

4-1 A compaction test was conducted in a soils laboratory and the Standard Proctor Compaction Procedure (ASTM D 698) was used. The weight of a compacted soil specimen plus mold was determined to be 3815 g. The volume and weight of mold were 1/30 ft^3 and 2050 g, respectively. The water content of the specimen was 9.1%. Compute both wet and dry unit weights of the compacted specimen.

4-2 A soil sample was taken from the site of a proposed borrow pit and sent to the laboratory for a Standard Proctor test (ASTM D 698). Results of the test are as follows:

Determination Number	1	2	3	4	5
Dry Density (lb/ft^3)	107.0	109.8	112.0	111.6	107.3
Moisture Content (%)	9.1	11.8	14.0	16.5	18.9

Plot a moisture content versus dry unit weight curve and determine the soil's maximum dry density and optimum moisture content.

4-3 During construction of a highway project, a soil sample was taken from compacted earth fill for a sand-cone in-place density test. The following data were obtained during the test:

1. Weight of sand used to fill test hole and funnel of sand-cone device = 845 g.
2. Weight of sand to fill funnel = 323 g.
3. Unit weight of sand = 100 lb/ft^3.
4. Weight of wet soil from test hole = 648 g.
5. Moisture content of soil from test hole = 16%.

Calculate dry unit weight of the compacted earth fill.

4-4 A soil sample was taken from a proposed cut area in a highway construction project and sent to a soils laboratory for a compaction test, using the Standard Proctor Compaction Procedure. Results of the test are as follows:

$$\text{Maximum dry unit weight} = 112.6 \text{ lb/ft}^3$$
$$\text{Optimum moisture content} = 15.5\%$$

The contractor, during construction of the soil embankment, achieved the following:

$$\text{Dry unit weight reached by field compaction} = 107.1 \text{ lb/ft}^3$$
$$\text{Actual water content} = 16.0\%$$

Determine the percent compaction achieved by the contractor.

4-5 Soil having a void ratio of 0.68 as it exists in a borrow pit is to be excavated and transported to a fill site where it will be compacted to a void ratio of 0.45. The volume of fill required is 2500 m^3. Find the volume of soil that must be excavated from the borrow pit to furnish the required volume of fill.

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5

Water in Soil

5-1 INTRODUCTION

As indicated in Chap. 2, water is a component of soil, and its presence in a given soil may range from virtually none to *saturation,* the latter case occurring when the soil's void space is completely filled with water. When the voids are only partially filled with water, a soil is said to be *partially saturated.* Any soil's characteristics and engineering behavior are greatly influenced by its water content. This is especially true for fine-grained soils. A clayey soil may be "hard as a rock" when dry but become soft and plastic when wet. In contrast, a very sandy soil, such as is found on a beach, may be relatively loose when dry but rather hard and more stable when wet. It may be somewhat ironic that one can generally walk and drive rather easily on dry clay and wet sand but more difficultly on saturated clay and very dry, loose sand.

The effects of water in soil are very important in the study of soil mechanics. Cohesive soils in particular tend to shrink when dry and swell when wet—some types of clay expanding greatly when saturated. Additionally, fine-grained soils are significantly weakened at high water contents. Such factors must be considered in most soil engineering problems and foundation design.

The effects of water movement within soil are also very important in many geotechnical engineering applications. Factors such as highway subdrainage, wells as a source of water supply, capillary and frost action, seepage flow analysis, and pumping water for underground construction all require the consideration of in-soil water movement.

5-2 FLOW OF WATER IN SOILS

As indicated above, water movement within soil is an important consideration in geotechnical engineering. The facility with which water flows through soil is an engineering property known as *permeability.* Since water movement within soil is through interconnected voids, in general, the larger a soil's void spaces, the greater will be its permeability. Conversely, the smaller the void spaces, the lesser will be its permeability. Thus, coarse-grained soils such as sand commonly exhibit high permeabilities, while fine-grained soils like clay ordinarily have lower permeabilities.

Flow of water in soil between two points occurs as a result of a pressure (or *hydraulic head*) difference between two points, with the direction of flow being from the higher to the lower pressure. Furthermore, the velocity of flow varies directly with the magnitude of the difference between hydraulic heads as well as with soil permeability.

Flow of water in soil can be analyzed quantitatively using *Darcy's law,* which was developed by Darcy in the eighteenth century based on experiments involving flow of water through sand filters. Figure 5-1 illustrates Darcy's experiment in which water moves through a soil sample contained in a cylindrical conduit. His tests indicated that flow rate through the soil in the conduit varied directly with both hydraulic head difference (h in Fig. 5-1) and cross-sectional area of the soil and inversely with the length over which the hydraulic head difference occurred (L in Fig. 5-1). Accordingly,

$$q \propto \frac{hA}{L}$$

where q = flow rate (volume per unit time)
h = hydraulic head difference (between points A and B in Fig. 5-1)
A = soil sample's cross-sectional area
L = length of soil sample (between points A and B)

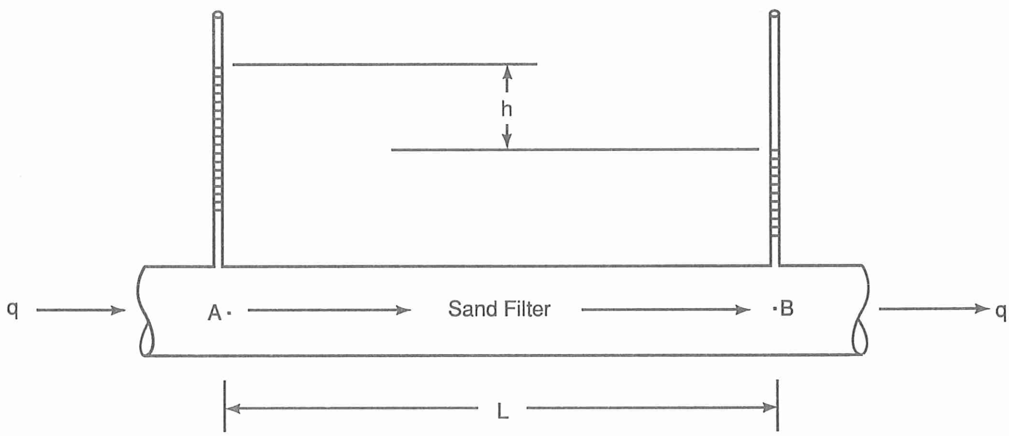

FIGURE 5-1 Illustration of Darcy's experiment.

If a constant of proportionality, k, is supplied, the proportionality above becomes

$$q = k \frac{h}{L} A \tag{5-1}$$

The constant of proportionality (k) in Eq. (5-1) is known as the coefficient of permeability and has the same units as velocity. The hydraulic head difference divided by the length of soil sample (h/L) is known as the *hydraulic gradient* and denoted by i. With this substitution, Eq. (5-1) can be rewritten as

$$q = kiA \tag{5-2}$$

If velocity of flow is desired, since $q = Av$,

$$v = ki \tag{5-3}$$

This velocity is an average velocity, since it represents flow rate divided by gross cross-sectional area of the soil. This area, however, includes both solid soil material and voids. Since water moves only through the voids, the actual (interstitial) velocity is

$$v_{\text{actual}} = \frac{v}{n} \tag{5-4}$$

Since $n = e/(1 + e)$, where e is the soil's void ratio,

$$v_{\text{actual}} = \frac{v(1 + e)}{e} \tag{5-5}$$

EXAMPLE 5-1

Given

1. Water flows through the sand filter shown in Fig. 5-1.
2. The cross-sectional area and length of the soil mass are 0.250 m^2 and 2.00 m, respectively.
3. The hydraulic head difference is 0.160 m.
4. The coefficient of permeability is 6.90×10^{-4} m/s.

Required

Flow rate of water through the soil.

Solution

From Eq. (5-2),

$$q = kiA \tag{5-2}$$

$$i = \frac{h}{L} = \frac{0.160 \text{ m}}{2.00 \text{ m}} = 0.0800$$

$$q = (6.90 \times 10^{-4} \text{ m/s})(0.0800)(0.250 \text{ m}^2) = 1.38 \times 10^{-5} \text{ m}^3\text{/s}$$

EXAMPLE 5-2

Given

In a soil test, it took 16.0 min for 1508 cm^3 of water to flow through a sand sample, the cross-sectional area of which was 50.3 cm^2. The void ratio of the soil sample was 0.68.

Required

1. Velocity of water through the soil.
2. Actual (interstitial) velocity.

Solution

1. v = volume/time/area
   $v = 1508 \text{ cm}^3/16.0 \text{ min}/50.3 \text{ cm}^2 = 1.874$ cm/min, or 0.0312 cm/s

2. $$v_{\text{actual}} = \frac{v(1 + e)}{e} \tag{5-5}$$

   $$v_{\text{actual}} = \frac{(0.0312 \text{ cm/s})(1 + 0.68)}{0.68} = 0.0771 \text{ cm/s}$$

In predicting the flow of water in soils, it becomes necessary to evaluate the coefficient of permeability for given soils. Both laboratory and field tests are available for doing this.

Laboratory Tests for Coefficient of Permeability

Laboratory tests are relatively simple and inexpensive to carry out and are ordinarily performed following either the constant-head method or the falling-head method. Brief descriptions of each of these methods follow.

The constant-head method for determining the coefficient of permeability can be used for granular soils. It utilizes a device known as a *constant-head permeameter,* as depicted in Fig. 5-2. The general test procedure is to allow water to move through the soil specimen under a stable head condition while

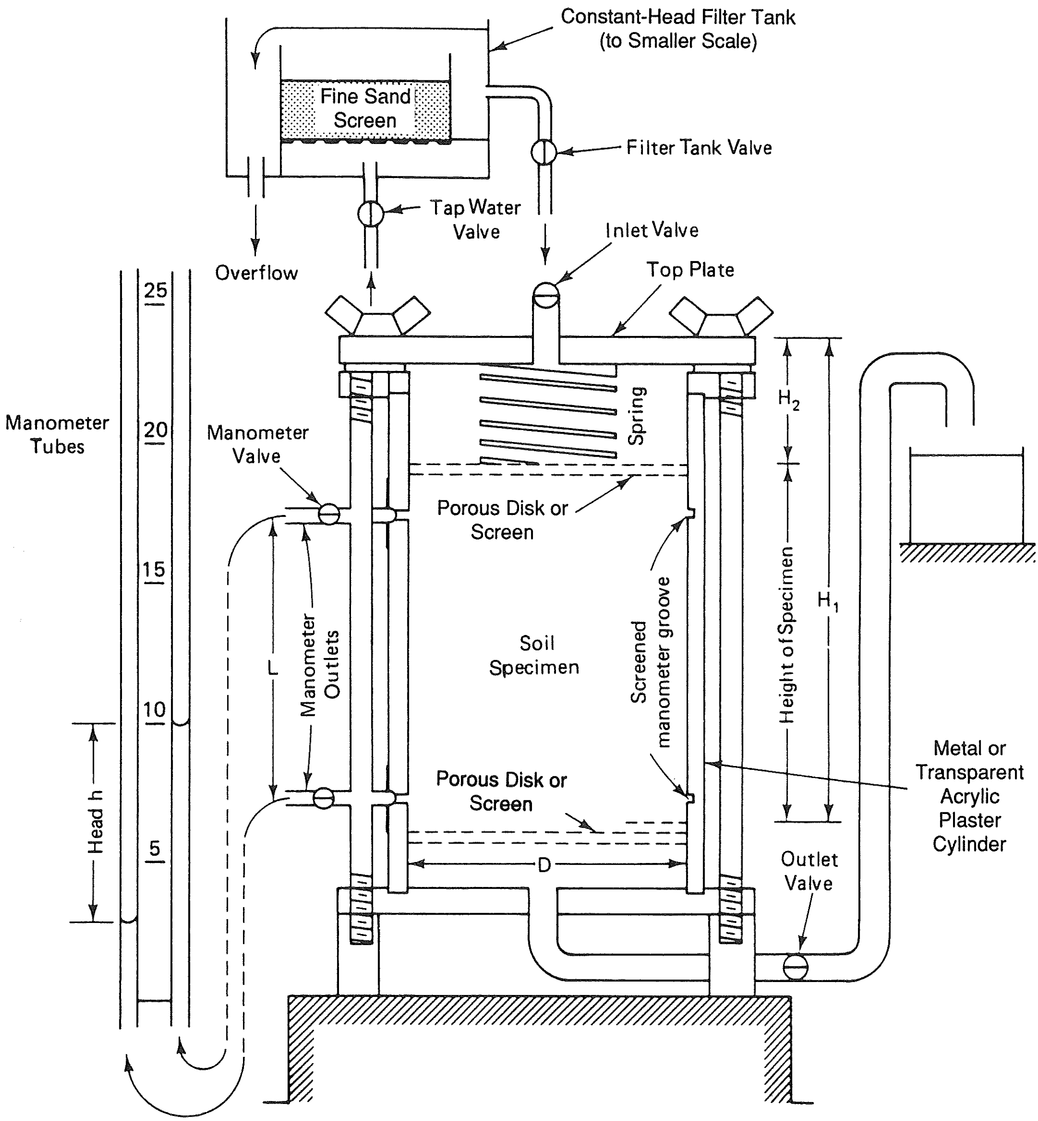

FIGURE 5-2 Constant-head permeameter. [1]

determining and recording the time required for a certain quantity of water to pass through the soil specimen. By measuring and recording the quantity (volume) of water discharged during a test (Q), length of specimen (distance between manometer outlets) (L), cross-sectional area of specimen (A), time required for quantity of water Q to be discharged (t), and head (difference in manometer levels) (h), the coefficient of permeability (k) can be derived as follows:

$$Q = Avt \tag{5-6}$$

Since $v = ki$ [from Eq. (5-3)] and $i = h/L$,

$$Q = A \frac{kh}{L} t \tag{5-7}$$

Solving for k gives

$$k = \frac{QL}{Ath} \tag{5-8}$$

The falling-head method can be used to find the coefficient of permeability for both fine-grained soils and coarse-grained, or granular, soils. It utilizes a permeameter like that depicted in Fig. 5-3. The general test procedure does not vary a great deal from the constant-head method. The specimen is first saturated with water. Water is then allowed to move through the soil specimen under a falling-head condition (rather than a stable-head condi-

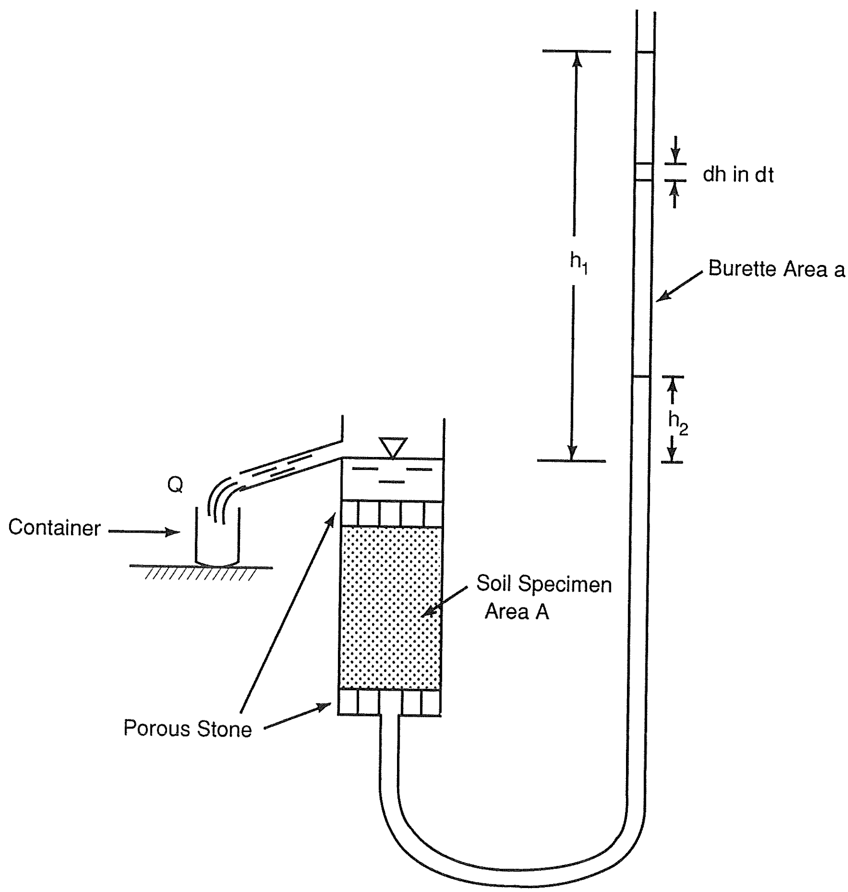

FIGURE 5-3 Schematic of the falling-head permeability setup.

tion) while the time required for a certain quantity of water to pass through the soil specimen is determined and recorded. If a is the cross-sectional area of the burette and h_1 and h_2 are the hydraulic heads at the beginning and end of the test, respectively (see Fig. 5-3), the coefficient of permeability can be derived as follows:

As shown in Fig. 5-3, the velocity of fall in the burette is given by $v = -dh/dt$, with the minus sign used to indicate a falling (and therefore decreasing) head. The flow of water into the specimen is therefore $q_{in} = -a(dh/dt)$, and the flow through and out of the specimen is, from Eq. (5-1), $q_{out} = k(h/L)A$. Equating q_{in} and q_{out} gives

$$-a \frac{dh}{dt} = k \frac{h}{L} A \tag{5-9}$$

$$-a \frac{dh}{h} = k \frac{A}{L} dt \tag{5-10}$$

$$-a \int_{h_1}^{h_2} \frac{dh}{h} = k \frac{A}{L} \int_{t_1}^{t_2} dt \tag{5-11}$$

$$-a \, [\ln h]_{h_1}^{h_2} = k \frac{A}{L} [t]_0^t \tag{5-12}$$

$$a \ln \frac{h_1}{h_2} = k \frac{A}{L} t \tag{5-13}$$

Therefore,

$$k = \frac{aL}{At} \ln \frac{h_1}{h_2} \tag{5-14}$$

or

$$k = \frac{2.3aL}{At} \log \frac{h_1}{h_2} \tag{5-15}$$

The coefficient of permeability as determined by both methods is the value for the particular water temperature at which the test was conducted. This value is ordinarily corrected to that for 20°C by multiplying the computed value by the ratio of the viscosity of water at the test temperature to the viscosity of water at 20°C.

Permeability determined in a laboratory may not be truly indicative of the *in situ* permeability. There are several reasons for this in addition to the fact that the soil in the permeameter does not exactly duplicate the structure of the soil *in situ,* particularly of nonhomogeneous soils and granular materials. For one thing, the flow of water in the permeameter is downward, whereas flow in the soil *in situ* may be more nearly horizontal or in a direction between horizontal and vertical. Indeed, the permeability of a natural soil in the horizontal direction can be considerably greater than in its vertical direction. For another thing, naturally occurring strata in the *in situ* soil will not be duplicated in the permeameter. Also, the relatively smooth walls of the permeameter afford different boundary conditions from the *in situ* soil. Finally, the hydraulic head in the permeameter may differ from the field gradient.

Another concern with the permeability test is any effect from entrapped air in the water and test specimen. To avoid this, the water to be used in the test should be de-aired by boiling distilled water and keeping it covered and nonagitated until used.

EXAMPLE 5-3

Given

A constant-head permeability test was conducted in a laboratory on a brown sand with a trace of mica. For the constant-head permeameter (see Fig. 5-2), the following data were obtained:

1. Quantity of water discharged during the test = 250 cm^3.
2. Length of specimen between manometer outlets = 11.43 cm.
3. Time required for given quantity of water to be discharged = 65.0 s.
4. Head (difference between manometer levels) = 5.5 cm.
5. Temperature of water = 20°C.
6. Diameter of specimen = 10.16 cm.

Required

Coefficient of permeability.

Solution

From Eq. (5-8),

$$k = \frac{QL}{Ath} \tag{5-8}$$

$$A = \frac{(\pi)(10.16\ \text{cm})^2}{4} = 81.07\ \text{cm}^2$$

$$k = \frac{(250\ \text{cm}^3)(11.43\ \text{cm})}{(81.07\ \text{cm}^2)(65.0\ \text{s})(5.5\ \text{cm})} = 0.0986\ \text{cm/s}$$

EXAMPLE 5-4

Given

A falling-head permeability test was conducted in a laboratory on a silty soil. For the falling-head apparatus (see Fig. 5-3), the following data were obtained:

1. Length of specimen = 15.80 cm.
2. Diameter of specimen = 10.16 cm.
3. Cross-sectional area of burette = 1.83 cm^2.
4. Hydraulic head at beginning of test (h_1) = 120.0 cm.
5. Hydraulic head at end of test (h_2) = 110.0 cm.
6. Time required for water in the burette to drop from h_1 to h_2 = 20.0 min (1200 s).
7. Temperature of water = 20°C.

Required

Coefficient of permeability.

Solution

From Eq. (5-15),

$$k = \frac{2.3aL}{At} \log \frac{h_1}{h_2} \tag{5-15}$$

$$A = \frac{(\pi)(10.16 \text{ cm})^2}{4} = 81.07 \text{ cm}^2$$

$$k = \frac{(2.3)(1.83 \text{ cm}^2)(15.80 \text{ cm})}{(81.07 \text{ cm}^2)(1200 \text{ s})} \log \frac{120.0 \text{ cm}}{110.0 \text{ cm}} = 2.58 \times 10^{-5} \text{ cm/s}$$

Field Tests for Coefficient of Permeability

As noted above, permeability determined in a laboratory may not be truly indicative of the *in situ* permeability. Thus, field tests are generally more reliable than laboratory tests for determining soil permeability, the main reason being that field tests are performed on the undisturbed soil exactly as it occurs *in situ* at the test location. Other reasons are that soil stratification, overburden stress, location of the groundwater table, and certain other factors that might influence permeability test results are virtually unchanged with field tests, which is not the case for laboratory tests.

There are several field methods for evaluating permeability, such as pumping, borehole, and tracer tests. The latter use dye, salt, or radioactive tracers to find the time it takes a given tracer to travel between two wells or borings; by finding the differential head between the two, the coefficient of permeability can be determined. The pumping method is detailed below.

Figure 5-4 illustrates a well extending downward through an impermeable layer and then a permeable layer (an aquifer) to another impermeable layer. If water is pumped from the well at a constant discharge (q), flow will enter the well only from the aquifer and the piezometric surface will be drawn down toward the well as shown in Fig. 5-4. At some time after pumping begins,

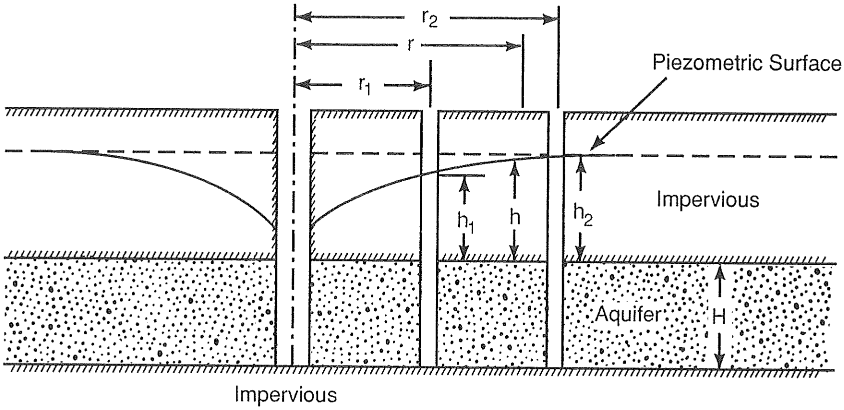

FIGURE 5-4 Flow of water toward pumping well (confined aquifer).

an equilibrium condition will be reached. The piezometric surface can be located by auxiliary observation wells located at distances r_1 and r_2 from the pumping well (see Fig. 5-4). The piezometric surface is located at distance h_1 above the top of the aquifer at point r_1 from the pumping well and at distance h_2 at point r_2. All parameters noted above and on Fig. 5-4 can be measured during a pumping test, and from these data the coefficient of permeability can be computed, as follows. It should be noted that the permeability so determined is that of the soil in the aquifer in the direction of flow (i.e., in horizontal radial directions).

Equation (5-2) can be applied to the equilibrium pumping condition in Fig. 5-4. Hydraulic gradient i in the equation is given for any point on the piezometric surface by dh/dr. The soil's cross-sectional area at any point on the piezometric surface through which water flows [A in Eq. (5-2)] is that of a cylinder with radius r and height H (see Fig. 5-4). Substituting these into Eq. (5-2) gives

$$q = kiA = k\frac{dh}{dr}2\pi rH \tag{5-16}$$

$$\int_{r_1}^{r_2} q\frac{dr}{r} = \int_{h_1}^{h_2} 2\pi kH\,dh \tag{5-17}$$

Integrating gives

$$q\,[\ln r]_{r_1}^{r_2} = 2\pi kH[h]_{h_1}^{h_2} \tag{5-18}$$

$$q\ln\frac{r_2}{r_1} = 2\pi kH(h_2 - h_1) \tag{5-19}$$

Solving for k yields

$$k = \frac{q\ln(r_2/r_1)}{2\pi H(h_2 - h_1)} \tag{5-20}$$

Figure 5-5 illustrates a pumping well located in an unconfined, homogeneous aquifer. In this case, the piezometric surface lies within the aquifer. The analysis of this type of well is the same as that for the confined aquifer (i.e., Fig. 5-4) except that the A term in Eq. (5-2) becomes $2\pi rh$. Hence,

$$q = k\frac{dh}{dr}2\pi rh \tag{5-21}$$

$$\int_{r_1}^{r_2} q\frac{dr}{r} = \int_{h_1}^{h_2} 2\pi kh\ dh \tag{5-22}$$

$$q\,[\ln r]_{r_1}^{r_2} = 2\pi k\left[\frac{h^2}{2}\right]_{h_1}^{h_2} \tag{5-23}$$

$$q\ln\frac{r_2}{r_1} = \pi k(h_2^2 - h_1^2) \tag{5-24}$$

$$k = \frac{q\ln(r_2/r_1)}{\pi(h_2^2 - h_1^2)} \tag{5-25}$$

EXAMPLE 5-5

Given

A pumping test was performed in a well penetrating a confined aquifer (Fig. 5-4) to evaluate the coefficient of permeability of the soil in the aquifer. When equilibrium flow was reached, the following data were obtained:

1. Equilibrium discharge of water from the well = 200 gal/min.
2. Water levels (h_1 and h_2) = 15 ft and 18 ft at distances from the well (r_1 and r_2) of 60 ft and 180 ft, respectively.
3. Thickness of impermeable aquifer = 20 ft.

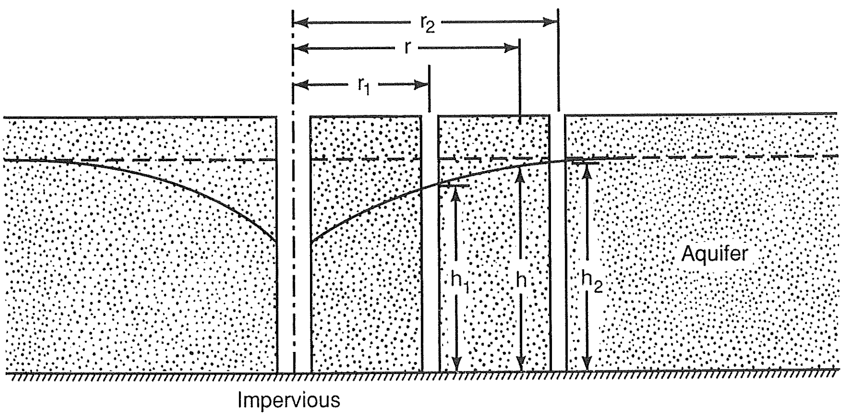

FIGURE 5-5 Flow of water toward pumping well (unconfined, homogenous aquifer).

Required

Coefficient of permeability of the soil in the aquifer.

Solution

From Eq. (5-20),

$$k = \frac{q \ln (r_2/r_1)}{2\pi H(h_2 - h_1)} \tag{5-20}$$

$$q = (200 \text{ gal/min})(1 \text{ ft}^3/7.48 \text{ gal})(1 \text{ min}/60 \text{ sec}) = 0.4456 \text{ ft}^3/\text{sec}$$

$$k = \frac{(0.4456 \text{ ft}^3/\text{sec}) \ln (180 \text{ ft}/60 \text{ ft})}{(2)(\pi)(20 \text{ ft})(18 \text{ ft} - 15 \text{ ft})} = 0.00130 \text{ ft/sec}$$

EXAMPLE 5-6

Given

Same conditions as in Example 5-5 except that the well is located in an unconfined aquifer (see Fig. 5-5).

Required

Coefficient of permeability of the soil in the aquifer.

Solution

From Eq. (5-25),

$$k = \frac{q \ln (r_2/r_1)}{\pi (h_2^2 - h_1^2)} \tag{5-25}$$

$$k = \frac{(0.4456 \text{ ft}^3/\text{sec}) \ln (180 \text{ ft}/60 \text{ ft})}{(\pi) [(18 \text{ ft})^2 - (15 \text{ ft})^2]} = 0.00157 \text{ ft/sec}$$

The coefficient of permeability for uniform sands in a loose state can be estimated by an empirical formula proposed by Hazen as follows [2]*:

$$k = C_1 D_{10}^2 \tag{5-26}$$

where k = coefficient of permeability (cm/s)
C_1 = 100 to 150 (1/cm·s)
D_{10} = effective grain size (soil particle diameter corresponding to 10 percent passing on the grain-size distribution curve; see Sec. 2-2)(cm)

Table 5-1 gives some information with regard to the range of the coefficient of permeability, drainage characteristics, and the most suitable methods for determining coefficients of permeability for various soils.

* From Karl Terzaghi and Ralph B. Peck, *Soil Mechanics in Engineering Practice*, 2nd ed., Copyright © 1967 by John Wiley & Sons, Inc., New York.

TABLE 5-1 Permeability and drainage characteristics of soils[a] [2].*

	Coefficient of Permeability k (cm/s) (log scale)											
	10^2	10^1	1.0	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}
Drainage	Good						Poor		Practically Impervious			
Soil types	Clean gravel		Clean sands, clean sand and gravel mixtures			Very fine sands, organic and inorganic silts, mixtures of sand silt and clay, glacial till, stratified clay deposits, etc.				"Impervious" soils, e.g., homogeneous clays below zone of weathering		
					"Impervious" soils modified by effects of vegetation and weathering							
Direct determination of k	Direct testing of soil in its original position—pumping tests; reliable if properly conducted; considerable experience required											
	Constant-head permeameter; little experience required											
Indirect determination of k			Falling-head permeameter; reliable; little experience required			Falling-head permeameter; unreliable; much experience required			Falling-head permeameter; fairly reliable; considerable experience necessary			
	Computation from grain-size distribution; applicable only to clean cohesionless sands and gravels									Computation based on results of consolidation tests; reliable; considerable experience required		

[a]After Casagrande and Fadum (1940).

* From Karl Terzaghi and Ralph B. Peck, *Soil Mechanics in Engineering Practice*, 2nd ed., Copyright © 1967 by John Wiley & Sons, Inc., New York.

Permeability in Stratified Soils

The preceding discussion in this section has assumed soil to be homogeneous, with the same value of permeability k throughout. In reality, natural soil deposits are often nonhomogeneous, and the value of k varies, sometimes greatly, within a given soil mass. To try to analyze permeability in a nonhomogeneous soil, a simplification can be made to consider an aquifer consisting of layers of soils with differing permeabilities. Figure 5-6 depicts such a case with layers of soils having permeabilities $k_1, k_2, k_3, \ldots, k_n$ and thicknesses $H_1, H_2, H_3, \ldots, H_n$. The general procedure is to find and use an average value of k. Since flow can occur in either the horizontal or vertical (x or y) direction, each of these cases is considered separately below. (Of course, the flow could be in some oblique direction as well, but that case is not considered here.)

Consider first the case where flow is in the y direction (Fig. 5-6). Since the water must travel successively through layers 1, 2, 3, . . ., n, the flow rate and velocity through each layer must be equal. If i denotes the overall hydraulic gradient, $i_1, i_2, i_3, \ldots, i_n$ represent gradients for each respective layer, and k_y is the average permeability of the entire stratified soil system in the y direction, application of Eq. (5-3) gives

$$v_y = k_y i = k_1 i_1 = k_2 i_2 = k_3 i_3 = \cdots = k_n i_n \tag{5-27}$$

Since total head loss is the sum of head losses in all layers,

$$iH = i_1 H_1 + i_2 H_2 + i_3 H_3 + \cdots + i_n H_n \tag{5-28}$$

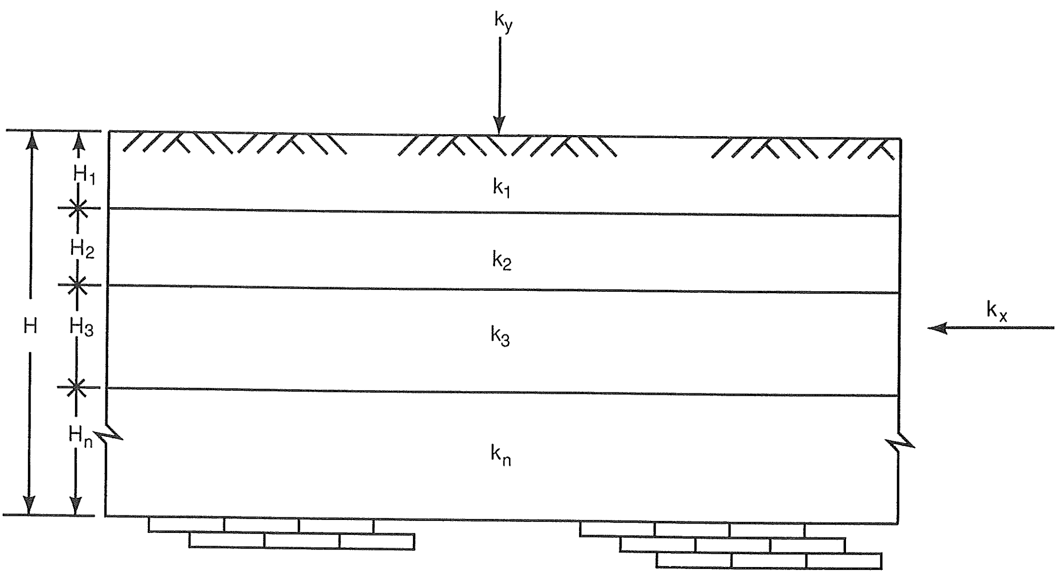

FIGURE 5-6 Stratified soil consisting of layers with various permeabilities.

or

$$i = \frac{i_1H_1 + i_2H_2 + i_3H_3 + \cdots + i_nH_n}{H} \tag{5-29}$$

From Eq. (5-27),

$$i_1 = k_yi/k_1;\ i_2 = k_yi/k_2;\ i_3 = k_yi/k_3;\ \cdots\ i_n = k_yi/k_n \tag{5-30}$$

Substitute these values of i_1, i_2, i_3, $\cdots$ i_n into Eq. (5-29).

$$i = \frac{(k_yi/k_1)\ H_1 + (k_yi/k_2)\ H_2 + (k_yi/k_3)\ H_3 + \cdots\ (k_yi/k_n)H_n}{H} \tag{5-31}$$

$$i = \frac{k_yi\ (H_1/k_1 + H_2/k_2 + H_3/k_3 + \cdots\ H_n/k_n)}{H} \tag{5-32}$$

Therefore,

$$k_y = \frac{H}{(H_1/k_1) + (H_2/k_2) + (H_3/k_3) + \cdots + (H_n/k_n)} \tag{5-33}$$

For flow in the x direction, let k_x denote the average permeability of the entire stratified soil system in that direction. In this case, total flow is the sum of the flows in all layers. Applying Eq. (5-2) and using H for the A term,

$$q = k_xiH = (k_1H_1 + k_2H_2 + k_3H_3 + \cdots + k_nH_n)i \tag{5-34}$$

or

$$k_x = \frac{k_1H_1 + k_2H_2 + k_3H_3 + \cdots + k_nH_n}{H} \tag{5-35}$$

In stratified soils, average horizontal permeability (k_x) is greater than average vertical permeability (k_y).

5-3 CAPILLARY RISE IN SOILS

As introduced in Chap. 2, capillarity refers to the rise of water (or other liquid) in a small-diameter tube inserted into the water, the rise being caused by both cohesion of the water's molecules and adhesion of the water to the tube's walls. The height of rise of water above the water level surrounding the tube is inversely proportional to the tube's diameter and can be computed using the equation

$$h = \frac{4T}{d\gamma} \tag{5-36}$$

where
h = height of rise
T = surface tension
d = tube diameter
γ = unit weight of water

Equation (5-36) is applicable only to the rise of pure water in clean glass tubes. At 20°C, the values of surface tension and unit weight of water are approximately 0.0728 N/m and 9790 N/m³, respectively. If these values are substituted into Eq. (5-36), the resulting equation is

$$h = \frac{0.030}{d} \tag{5-37}$$

where h is in meters and d in millimeters. Equation (5-37) is, of course, valid only for water at 20°C, but that is roughly room temperature and the equation gives generally adequate results for temperatures between 0 and 30°C.

EXAMPLE 5-7

Given

A clean glass capillary tube with a diameter of 0.5 mm is inserted into water with a surface tension of 0.073 N/m.

Required

The height of capillary rise in the tube.

Solution

From Eq. (5-36),

$$h = \frac{4T}{d\gamma} \tag{5-36}$$

$$h = \frac{(4)(0.073\text{ N/m})}{[(0.5\text{ mm})(1\text{ m}/1000\text{ mm})](9790\text{ N/m}^3)} = 0.060\text{ m}$$

or,

$$h = \frac{0.030}{d} \tag{5-37}$$

$$h = \frac{0.030}{0.5\text{ mm}} = 0.060\text{ m}$$

With soils, capillarity occurs at the groundwater table when water rises from saturated soil below into dry or partially saturated soil above the water table. The "capillary tubes" through which water rises in soils are actually the void spaces among soil particles. Since the voids interconnect in varying directions (not just vertically) and are irregular in size and shape, accurate calculation of the height of capillary rise is virtually impossible. It is known, however, that the height of capillary rise is associated with mean diameter of a soil's voids, which is in turn related to average grain size. In general, the smaller the grain size, the smaller the void space, and consequently the greater will be the capillary rise. Thus clayey soils, with the smallest grain size, should theoretically experience the greatest capillary rise, although the rate of rise may be very, very slow because of the characteristically low permeability of such soils. In fact, the largest capillary rise for any particular length of time generally occurs in soils of medium grain sizes (such as silts and very fine sands).

A very rough approximation of the maximum height of capillary rise of water in a particular soil can be determined from the equation [3]*

$$h = \frac{C}{eD_{10}} \tag{5-38}$$

where h = maximum height of capillary rise
C = empirical coefficient
e = soil's void ratio
D_{10} = effective grain size (see Sec. 2-2)

With D_{10} expressed in centimeters and C, which depends on surface impurities and the shape of grains, ranging from 0.1 to 0.5 cm^2, the computed value of h will be in centimeters. Equation (5-38) gives the maximum height of capillary rise for smaller voids. Larger voids overlying smaller ones may interfere with the capillary process and thereby cause values of h from Eq. (5-38) to be invalid.

5-4 FROST ACTION IN SOILS

It is well known from physics that water expands when it is cooled and freezes. When the temperature in a soil mass drops below water's freezing point, water in the voids freezes and therefore expands, causing the soil mass to move upward. This vertical expansion of soil caused by freezing water within is known as *frost heave.* Serious damage may result from frost heave when structures such as pavements and building foundations supported by soil are lifted. Since the amount of frost heave (i.e., upward soil movement) is not necessarily uniform in a horizontal direction, cracking of pavements,

* From Ralph B. Peck, Walter E. Hansen, and Thomas H. Thornburn, *Foundation Engineering*, 2nd ed., Copyright © 1974 by John Wiley & Sons, Inc., New York.

building walls, and floors may occur. When the temperature rises above the freezing point, frozen soil thaws from the top downward. Since resulting melted water near the surface cannot drain through underlying frozen soil, an increase in water content of the upper soil, a decrease in its strength, and subsequent settlement of structure may occur. Clearly, such alternate lifting and settling of pavements and structures as a result of freezing and thawing of soil pore water are undesirable, may cause serious structural damage, and should be avoided or at least minimized.

The actual amount of frost heave in any particular soil is difficult to compute or even estimate or predict accurately. Although pore water freezes in soil when the temperature is low enough, the frozen water is not necessarily uniform; and ice layers, or lenses, may occur. Capillary water rising from the water table can add to an ice lens, thereby increasing its volume and causing large heaves to occur. Frost heaves of a few inches are common in the northern half of the United States and may, in extreme cases, be much greater. Figure 5-7 gives maximum depths of frost penetration for the conterminous United States.

Since frost heave is a natural phenomenon and is virtually unpreventable, the best defense against structural damage therefrom is to construct foundations deep enough to escape the effects of frost heave. A rule of thumb is to place foundations to a depth equal to or greater than the depth of frost penetration (Fig. 5-7) in a given area. In making such a judgment, one must

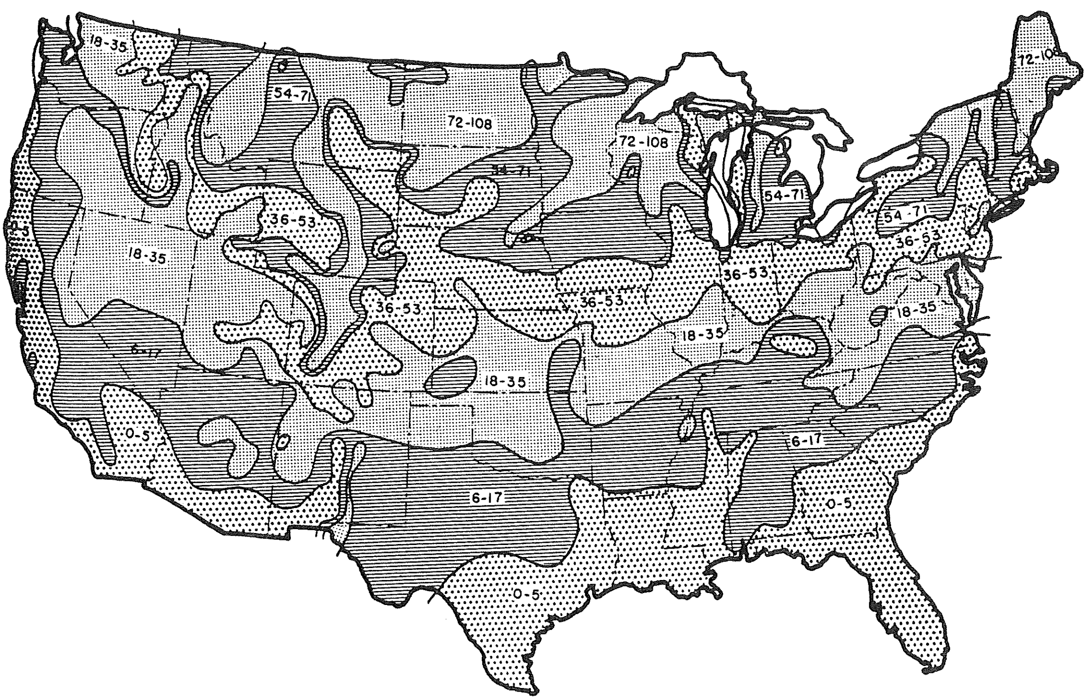

FIGURE 5-7 Maximum depth of frost penetration in the United States. [4]

remember that the location of the water table is not fixed. Of course, if a given soil is not susceptible to frost action or if no water is present (and is never expected to be present), severe frost heave problems may not occur. However, it is still good practice to construct foundations below the depth of frost penetration rather than risk structural failure resulting from possible future frost heave.

5-5 FLOW NETS AND SEEPAGE

When water flows underground through well-defined aquifers over long distances, flow rate can be computed using Darcy's law [Eq. (5-2)] if the individual terms in the equation can be evaluated. In cases where the path of flow is irregular or if the water entering and leaving the permeable soil is over a short distance, flow boundary conditions may not be so well defined and analytical solutions, such as the use of Eq. (5-2), become difficult. In such cases, flow may be evaluated using *flow nets*.

Figure 5-8 illustrates a flow net. In the figure, water seeps through the permeable stratum beneath the wall from the upstream side (left) to the downstream side (right). The solid lines below the wall are known as *flow lines*. Each flow line represents the path along which a given water particle travels in moving from the upstream side through the permeable stratum to the downstream side. The dashed lines in Fig. 5-8 represent *equipotential lines*. They connect points on different flow lines having equal total energy heads. A collection of flow lines intersecting equipotential lines, as shown in Fig. 5-8, constitutes a *flow net*, and as will be demonstrated subsequently, it is a useful tool in evaluating seepage through permeable soil.

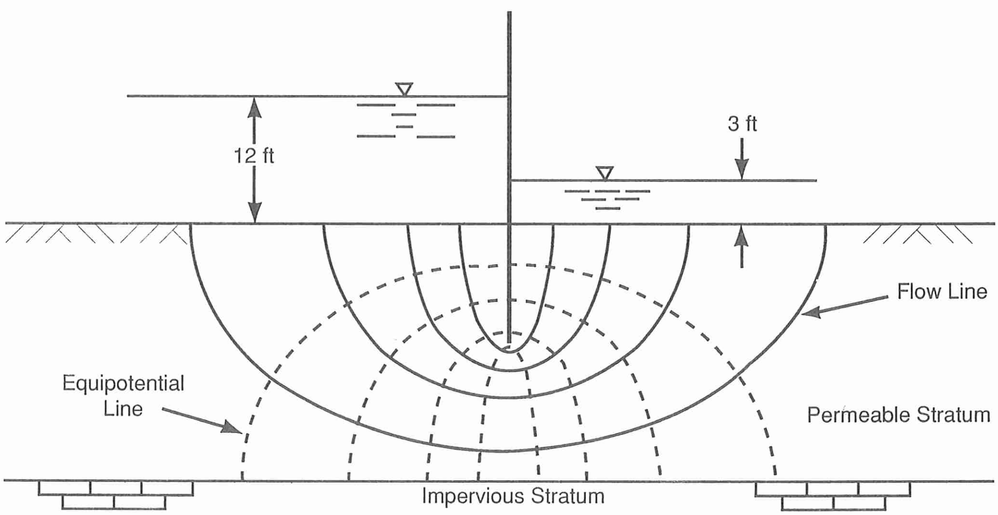

FIGURE 5-8 Flow net.

Construction of Flow Nets

Construction of a flow net requires, as a first step, a scale drawing of a cross section of the flow path, as shown in Fig. 5-9a. In addition to the pervious soil mass, the drawing shows the impervious boundaries that restrict flow and the pervious boundaries through which water enters and exits the soil.

The second step is to sketch several (generally, two to four) flow lines. As indicated above, they represent paths along which given water particles travel in moving through the permeable stratum. As shown in Fig. 5-9b, they should be drawn approximately parallel to the impervious boundaries and perpendicular to the pervious boundaries.

The next step is to sketch equipotential lines. Since they connect points on different flow lines having equal total energy heads, they should be drawn approximately perpendicular to the flow lines, as illustrated in Fig. 5-9c. Furthermore, they should be drawn to form quasi-squares where equipotential lines and flow lines intersect. In other words, intersecting equipotential lines and flow lines should form figures each of which has approximately equal lengths and widths.

Because the initial positions of the flow lines represent guesses, the first attempt at constructing a flow net will usually not be totally accurate (i.e., will not result in the necessary quasi-squares). Hence, the fourth and final step is to use the first attempted flow net as a guide to adjust the equipotential lines and the flow lines so that all figures have equal widths and lengths and all intersections are at right angles as nearly as possible. Figure 5-9d shows the final flow net achieved by adjusting the initial flow net attempt (Fig. 5-9c). It should be noted from Fig. 5-9d that the figures formed are generally not all perfect squares because their lengths and widths are not all equal and their sides are seldom straight lines and also because the lines forming them do not always intersect at precise right angles. Nevertheless, they should be drawn to approximate square figures.

Calculation of Seepage Flow

Once a suitable flow net has been prepared as described above, seepage flow can be determined by modifying Darcy's law, as follows.

$$q = kiA \tag{5-2}$$

Consider one square in a flow net—for example, the one labeled "G" in Fig. 5-10. Let $\triangle q$ and $\triangle h$ denote the flow rate and drop in head (energy), respectively, for this square. Since each square is x units wide and y units long and has unit width perpendicular to the figure, term i in Eq. (5-2) is given by $\triangle h/x$ and term A is equal to y. Hence,

$$\triangle q = k\frac{\triangle h}{x}y \tag{5-39}$$

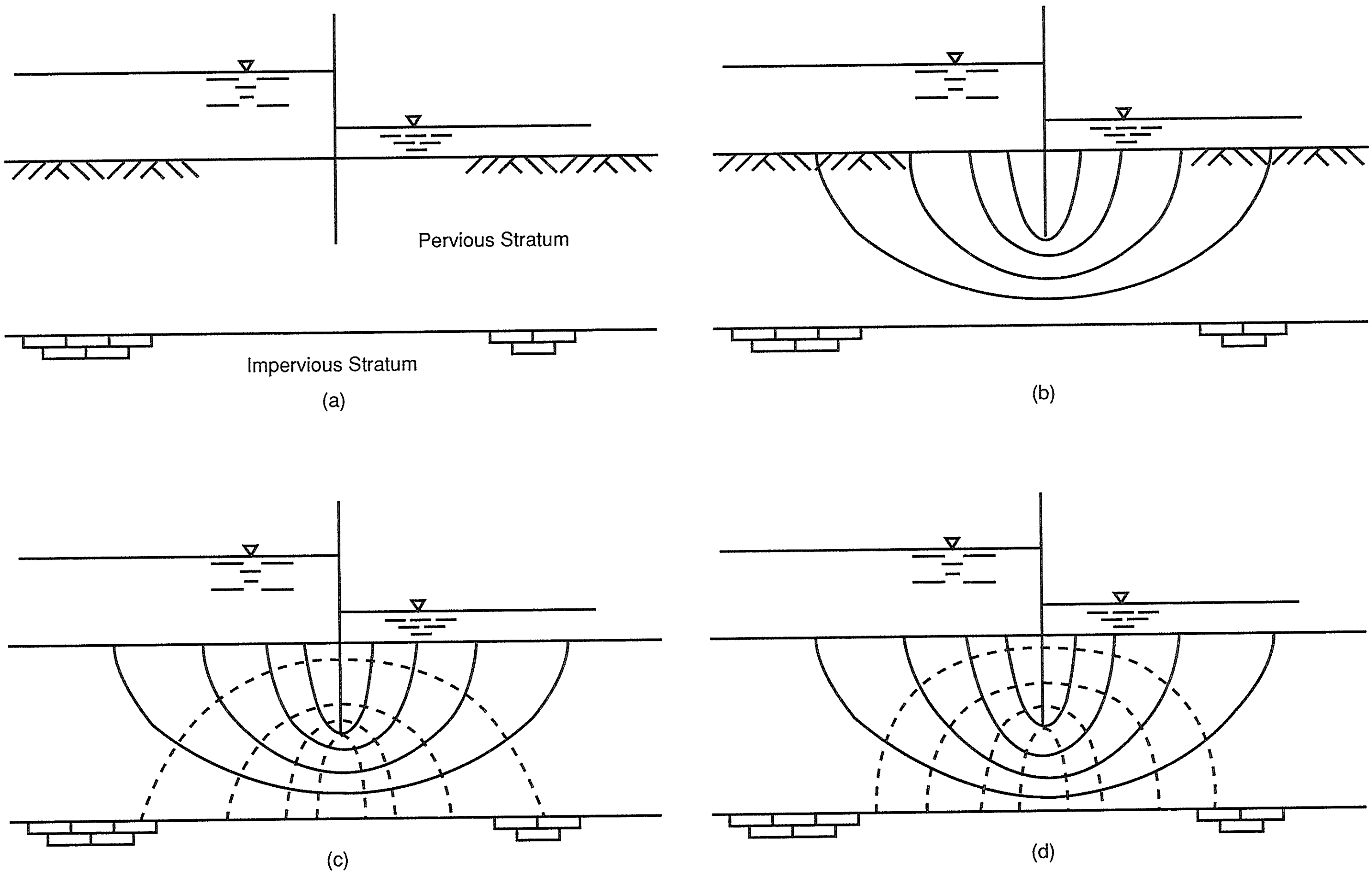

FIGURE 5-9 Construction of flow net. (a) scale drawing showing pervious and impervious boundaries; (b) flow lines; (c) equipotential lines; (d) final flow net.

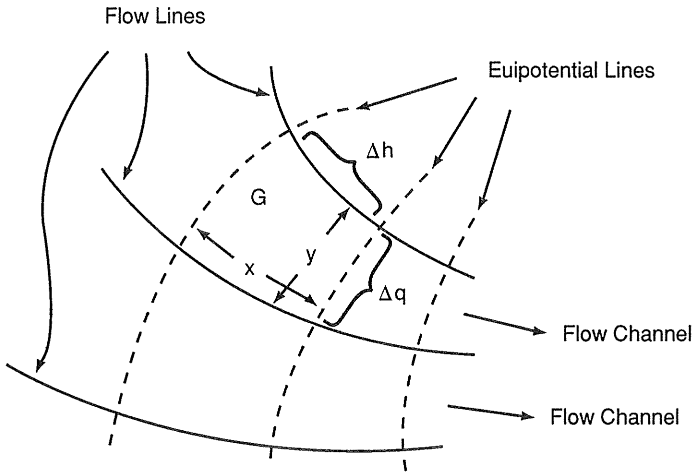

FIGURE 5-10 Flow channel and equipotential drops.

However, since the figure is square, y/x is unity and

$$\triangle q = k \triangle h \tag{5-40}$$

If N_d represents the number of equipotential increments (spaces between equipotential lines), then $\triangle h$ equals h/N_d and

$$\triangle q = \frac{kh}{N_d} \tag{5-41}$$

If N_f denotes the number of flow paths (spaces between flow lines), then $\triangle q$ equals q/N_f (where q is the total flow rate of the flow net per unit width) and

$$\frac{q}{N_f} = \frac{kh}{N_d} \tag{5-42}$$

or

$$q = \frac{khN_f}{N_d} \tag{5-43}$$

Example 5-8 illustrates the computation of seepage through a flow net using Eq. (5-43).

EXAMPLE 5-8

Given

For the flow net depicted in Fig. 5-8, the coefficient of permeability of the permeable soil stratum is 4.80×10^{-3} cm/s.

Required

The total rate of seepage per unit width of sheet pile through the permeable stratum.

Solution

From Eq. (5-43),

$$q = \frac{khN_f}{N_d} \tag{5-43}$$

$$k = (4.80 \times 10^{-3} \text{ cm/s})(1 \text{ in.}/2.54 \text{ cm})(1 \text{ ft}/12 \text{ in.})$$

$$= 1.57 \times 10^{-4} \text{ ft/sec}$$

$$h = 12 \text{ ft} - 3 \text{ ft} = 9 \text{ ft}$$

$$N_f = 5$$

$$N_d = 9$$

$$q = \frac{(1.57 \times 10^{-4} \text{ ft/sec})(9 \text{ ft})(5)}{9} = 7.85 \times 10^{-4} \text{ ft}^3\text{/sec per foot of sheetpile}$$

In the foregoing discussion of flow nets, it has been assumed that soil was isotropic—that is, equal soil permeability in all directions. In actuality, natural soils are not isotropic, but often soil permeabilities in vertical and horizontal directions are similar enough that the assumption of isotropic soil is acceptable for finding flow without appreciable error. In stratified soil deposits, however, where horizontal permeability is much greater than vertical permeability, the flow net must be modified and Eq. (5-43) altered to compute flow. For the situation where k_y and k_x (representing average vertical and horizontal coefficients of permeability, respectively) differ appreciably, the method for constructing the flow net can be modified by use of a *transformed section* to account for the different permeabilities. The modification is done when the scale drawing of the cross section of the flow path is prepared. Vertical lengths are plotted in the usual manner to fit the scale selected for the sketch, but horizontal dimensions are first altered by multiplying all horizontal lengths by the factor $\sqrt{k_y/k_x}$ and plotting the results to scale. The resulting drawing will appear somewhat distorted, with apparently shortened horizontal dimensions. The conventional flow net is then sketched on the transformed section in the manner described previously. In analyzing the resulting flow net to compute seepage flow, the k term in Eq. (5-43) must be replaced by the factor $\sqrt{k_y/k_x}$, which was used in plotting the drawing. Thus, for flow through stratified, nonisotropic soil, the seepage equation becomes

$$q = \sqrt{\frac{k_y}{k_x}}\,\frac{hN_f}{N_d} \tag{5-44}$$

5-6 PROBLEMS

5-1 Water flows through a sand filter as shown in Fig. 5-11. The soil mass's cross-sectional area and length are 400 in.^{2} and 5.0 ft, respectively. If the coefficient of permeability of the sand filter is 3.6×10^{-2} cm/s, find the flow rate of water through the soil.

5-2 A quantity of 2000 ml of water required 20 min to flow through a sand sample, the cross-sectional area of which was 60.0 cm^2. The void ratio of the sand was 0.71. Compute the velocity of water moving through the soil and the actual (interstitial) velocity.

5-3 A constant-head permeability test was conducted on a clean sand sample (see Fig. 5-2). The diameter and length of the test specimen were 10.0 cm and 12.0 cm, respectively. The head difference between manometer levels was 4.9 cm during the test, and the water temperature was 20°C. If it took 152 s for 500 ml of water to discharge, determine the soil's coefficient of permeability.

5-4 A falling-head permeability test was conducted on a silty clay sample (see Fig. 5-3). The diameter and length of the test specimen were 10.20 cm and 16.20 cm, respectively. The cross-sectional area of the standpipe was 1.95 cm^2, and the water temperature was 20°C. If it took 35 min for the water in the standpipe to drop from a height of 100.0 cm at the beginning of the test to 92.0 cm at the end, determine the soil's coefficient of permeability.

5-5 A pump test was conducted on a test well in an unconfined aquifer, with the results as shown in Fig. 5-12. If water was pumped at a steady flow of 185 gpm, determine the coefficient of permeability of the permeable soil.

5-6 A pump test was conducted on a test well drilled into a confined aquifer, with the results as shown in Fig. 5-13. If water was pumped at a steady flow of 205 gpm, determine the coefficient of permeability of the permeable soil in the aquifer.

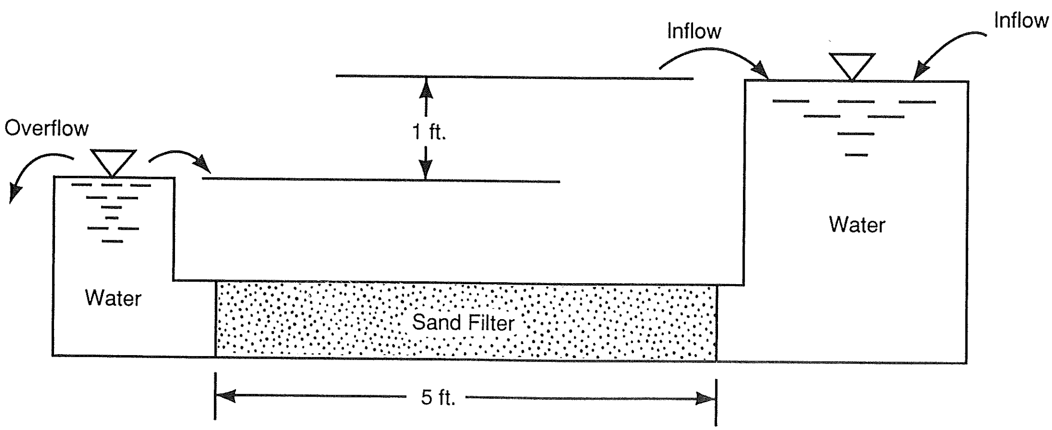

FIGURE 5-11

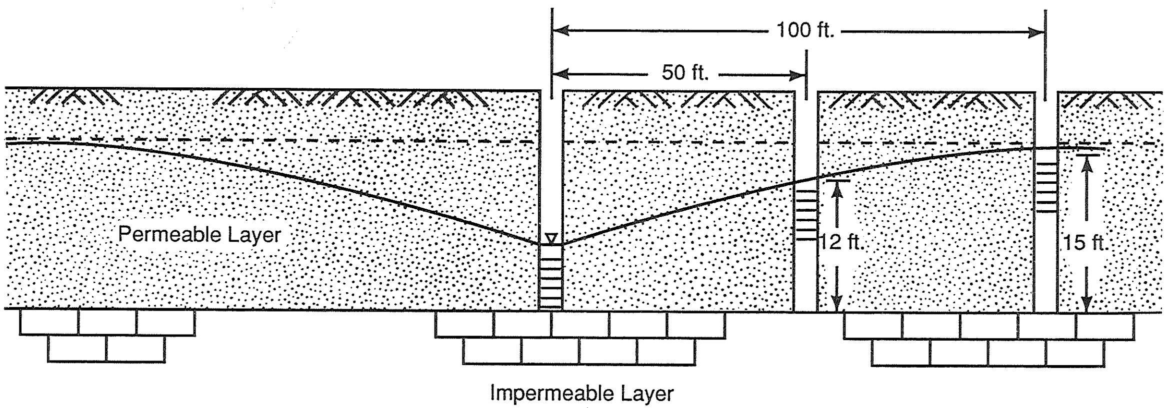

FIGURE 5-12 (not to scale)

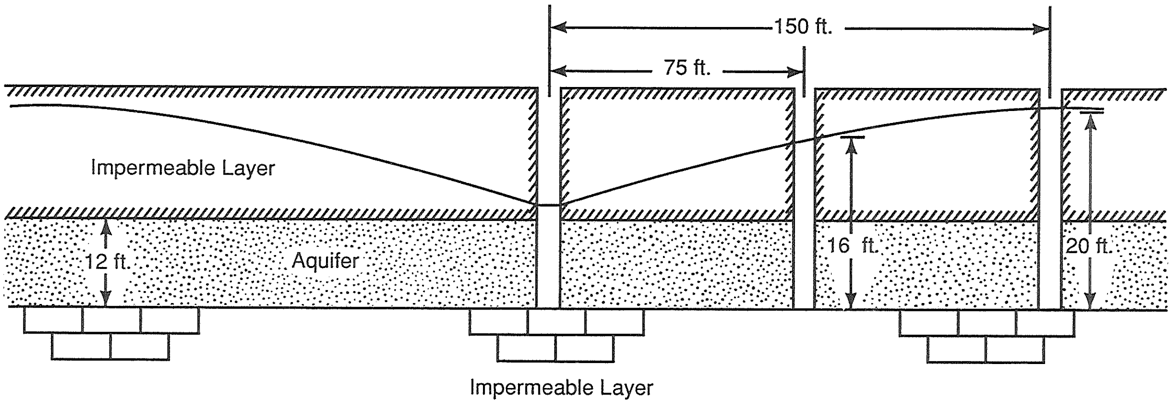

FIGURE 5-13 (not to scale)

5-7 A clean glass capillary tube having a diameter of 0.008 in. was inserted into water with a surface tension of 0.00504 lb/ft. Calculate the height of capillary rise in the tube.

5-8 A reservoir with a 35,000-ft^2 area is underlain by layers of stratified soils as depicted in Fig. 5-14. Compute the water loss from the reservoir in one year. Assume that the pore pressure at the bottom sand layer is zero.

5-9 For the reservoir described in Problem 5-8, estimate the average coefficient of permeability in the horizontal direction.

5-10 Construct a flow net for the sheetpile shown in Fig. 5-15. Estimate the seepage per foot of width of the sheetpile.

5-11 Construct a flow net for the concrete dam shown in Fig. 5-16. Estimate the seepage per foot of width of the dam.

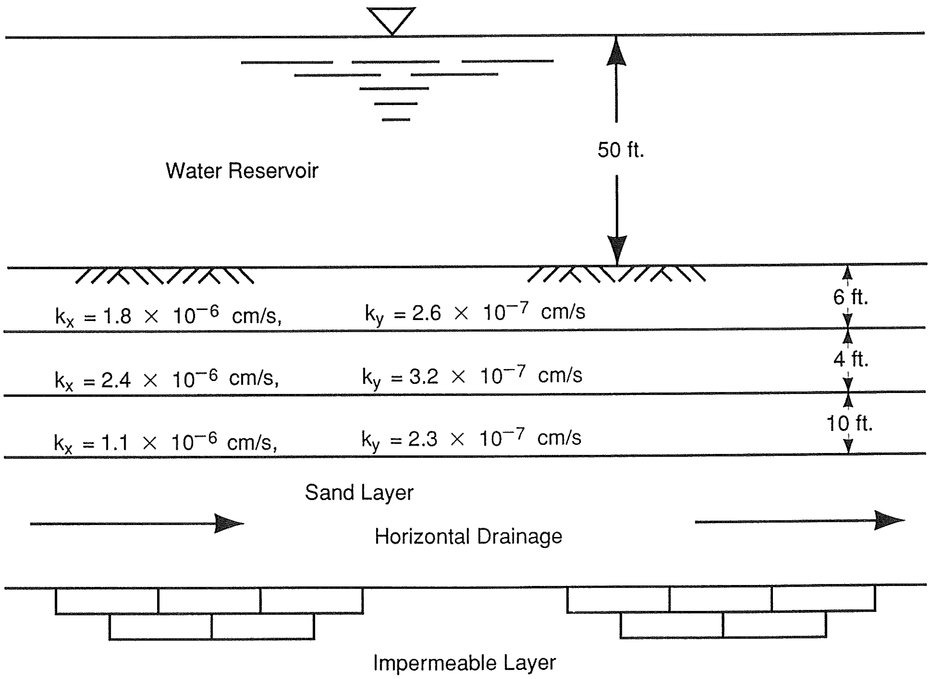

FIGURE 5-14

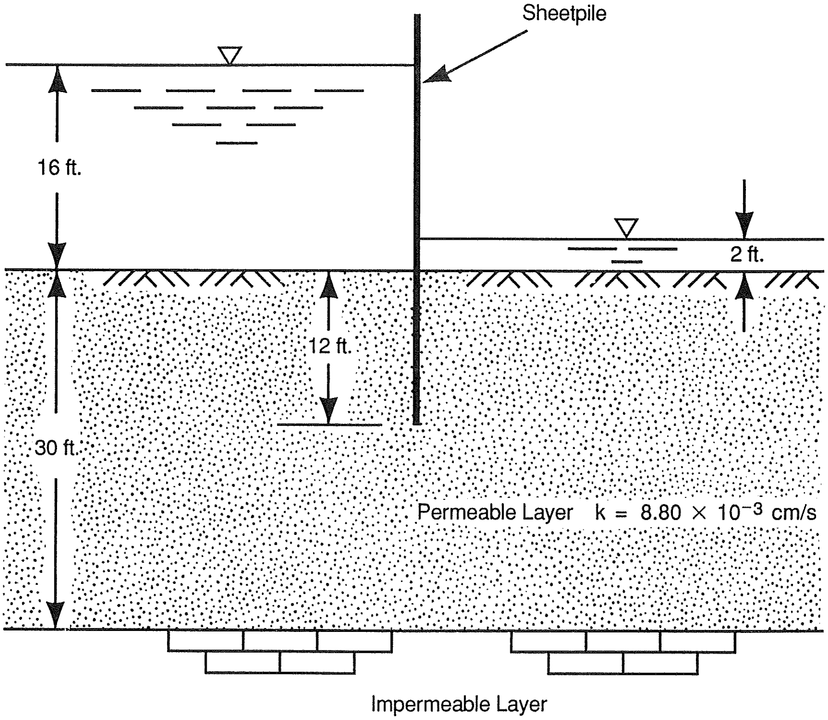
Sheetpile
16 ft.
2 ft.
12 ft.
30 ft.
Permeable Layer k = 8.80 × 10⁻³ cm/s
Impermeable Layer

FIGURE 5-15

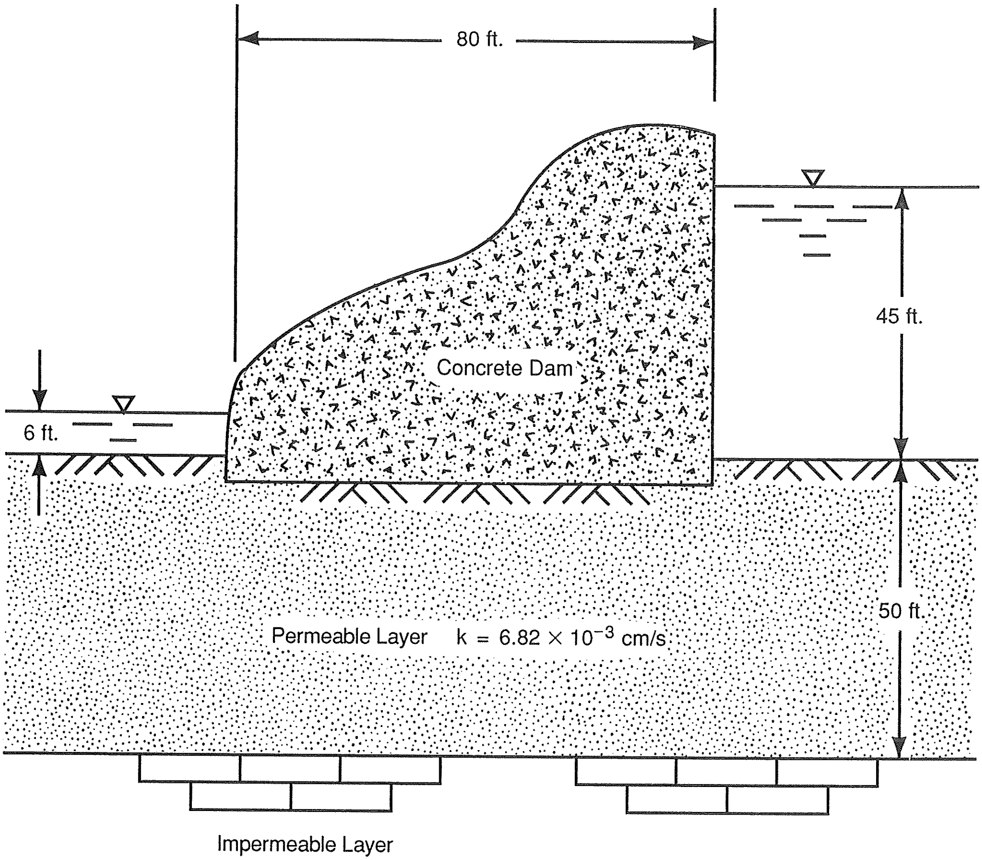

FIGURE 5-16

References

[1] *1989 Annual Book of ASTM Standards,* ASTM, Philadelphia, 1989. Copyright, American Society for Testing and Materials, 1916 Race Street, Philadelphia, PA 19103. Reprinted with permission.

[2] Karl Terzaghi and Ralph B. Peck, *Soil Mechanics in Engineering Practice,* 2nd ed., John Wiley & Sons, Inc., 1967. Copyright © 1967, John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

[3] Ralph B. Peck, Walter E. Hansen, and Thomas H. Thornburn, *Foundation Engineering,* 2nd ed., John Wiley & Sons, Inc., 1974. Copyright © 1974, John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

[4] *Frost Action in Roads and Airfields,* Highway Research Board, Special Report No. 1, Publ. 211, National Academy of Sciences—National Research Council, Washington, D.C., 1952.

6

Stress Distribution in Soil

6-1 INTRODUCTION

If a vertical load of 1 ton is applied to a column of 1 ft^2 cross-sectional area, and the column rests directly on a soil surface, the vertical pressure exerted by the column onto the soil would be, on average, 1 ton/ft^2 (neglecting the column's weight). In addition to this pressure at the area of contact between column and soil, stress influence extends both downward and outward within the soil in the general area where the load is applied. The increase in pressure in the soil at any horizontal plane below the load is greatest directly under the load and diminishes outwardly (see Fig. 6-1). The pressure's magnitude decreases with increasing depth. This is illustrated in Fig. 6-1, where pressure p_2 at depth d_2 is less than pressure p_1 at depth d_1. Figure 6-1 also illustrates the increase in the area of stress influence outward with increase in depth.

Stress distribution in soil is quite important to soils engineers—particularly with regard to stability analysis and the settlement analysis of foundations. The remainder of this chapter deals with quantitative analyses of stress distribution in soil.

6-2 VERTICAL PRESSURE BELOW A CONCENTRATED LOAD

Two methods for calculating pressure below a concentrated load are presented here—the Westergaard equation and the Boussinesq equation. Both of these result from the theory of elasticity, which assumes that stress is pro-

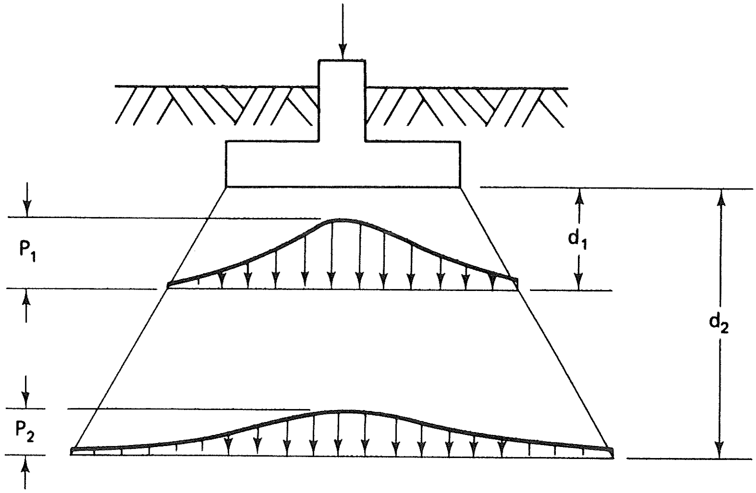

FIGURE 6-1 Distribution of pressure.

portional to strain. Implicit in this assumption is a homogeneous material, although soil is very seldom a homogeneous material. The Westergaard equation is based on alternating thin layers of an elastic material between layers of an inelastic material. The Boussinesq equation assumes a homogeneous soil throughout [1].

Westergaard Equation [1, 2]

The Westergaard equation is as follows:

$$q = \frac{Q\sqrt{(1 - 2\mu)/(2 - 2\mu)}}{2\pi z^2[(1 - 2\mu)/(2 - 2\mu) + (r/z)^2]^{3/2}} \tag{6-1}$$

where q = vertical stress at depth z
Q = concentrated load
μ = Poisson's ratio (ratio of the strain in a material in a direction normal to an applied stress to the strain parallel to the applied stress)
z = depth
r = horizontal distance from point of application of Q to point at which q is desired

Note: q, the vertical stress at depth z resulting from load Q, is sometimes referred to as the *vertical stress increment,* since it represents stress added by the load to the stress existing prior to application of the load. (The stress existing prior to application of load is the *overburden pressure.*) This equation gives stress q as a function of both the vertical distance z and horizontal distance r between the point of application of Q and the point at which q is desired (see Fig. 6-2). If Poisson's ratio is taken to be zero, Eq. (6-1) reduces to

$$q = \frac{Q}{\pi z^2[(1 + 2(r/z)^2]^{3/2}} \quad (6\text{-}2)$$

Boussinesq Equation [1]

The Boussinesq equation is as follows:

$$q = \frac{3Q}{2\pi z^2[1 + (r/z)^2]^{5/2}} \quad (6\text{-}3)$$

where the terms are the same as those in Eq. (6-1). This equation also gives stress q as a function of both the vertical distance z and horizontal distance r. For low r/z ratios, the Boussinesq equation gives higher values of q than the Westergaard equation. The Boussinesq equation is more widely used.

Examples 6-1 and 6-2 illustrate the use of the Boussinesq equation to calculate vertical stress below a concentrated load.

EXAMPLE 6-1

Given

A concentrated load of 250 tons is applied to the ground surface.

Required

The vertical stress increment due to this load at a depth of 20 ft directly below the load.

Solution

From Eq. (6-3),

$$q = \frac{3Q}{2\pi z^2[1 + (r/z)^2]^{5/2}} \quad (6\text{-}3)$$

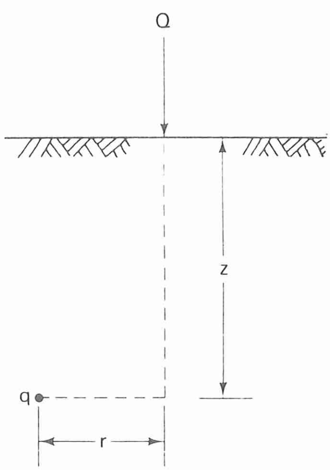

FIGURE 6-2

From given, $z = 20$ ft
$r = 0$
$Q = 250$ tons $= 500{,}000$ lb

$$q = \frac{(3)(500{,}000\ \text{lb})}{(2)(\pi)(20\ \text{ft})^2[1 + (0/20\ \text{ft})^2]^{5/2}} = 597\ \text{lb/ft}^2$$

EXAMPLE 6-2

Given

A concentrated load of 250 tons is applied to the ground surface.

Required

The vertical stress increment due to this load at a point located 20 ft below the ground surface and 16 ft from the line of the concentrated load (i.e., r = 16 ft, z = 20 ft, as illustrated in Fig. 6-3).

Solution

From Eq. (6-3),

$$q = \frac{3Q}{2\pi z^2[1 + (r/z)^2]^{5/2}} \tag{6-3}$$

From given, $z = 20$ ft
$r = 16$ ft
$Q = 250$ tons $= 500{,}000$ lb

$$q = \frac{(3)(500{,}000\ \text{lb})}{(2)(\pi)(20\ \text{ft})^2[1 + (16\ \text{ft}/20\ \text{ft})^2]^{5/2}} = 173\ \text{lb/ft}^2$$

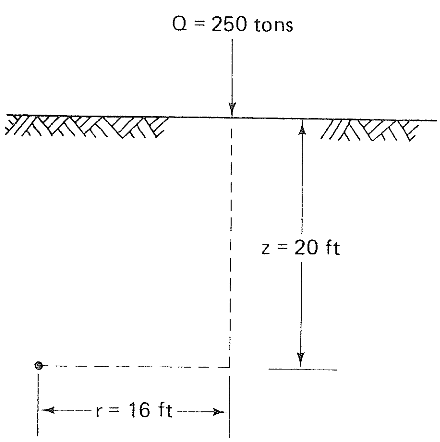

FIGURE 6-3

6-3 VERTICAL PRESSURE BELOW A LOADED SURFACE AREA (UNIFORM LOAD)

Methods presented in Sec. 6-2 deal with determination of vertical pressure below a concentrated load. Usually, however, concentrated loads are not applied directly onto soil. Instead, concentrated loads rest on footings, piers, and the like, and load is applied to soil through footings or piers in the form of a loaded surface area (uniform load). Analysis of stress distributions resulting from loaded surface areas is generally more complicated than those resulting from concentrated loads.

Two methods for computing vertical pressure below a loaded surface area will be discussed here. One of them is called the *approximate method;* the other is based on elastic theory.

Approximate Method [1]

The approximate method is based on the assumption that the area (in a horizontal plane) of stress below a concentrated load increases with depth as shown in Fig. 6-4. With the 2:1 slope shown, it is apparent that at any depth z, both L and B are increased by the amount z. Accordingly, stress at depth z is given by

$$q = \frac{Q}{(B + z)(L + z)} \tag{6-4}$$

where q = approximate vertical stress at depth z
Q = total load
B = width
L = length
z = depth

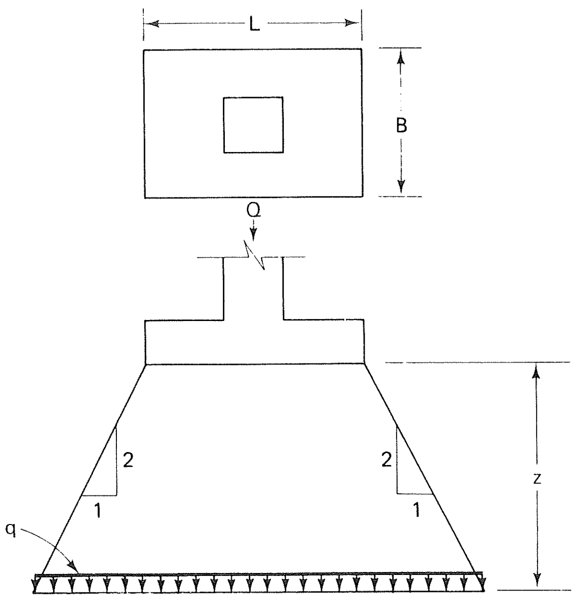

FIGURE 6-4 [1]

Since Q, L, and B are constants for a given application, it is obvious that the stress at depth z (q) decreases as depth increases. This method should be considered, at best, as being crude. It may be useful for preliminary stability analysis of footings; however, for settlement analysis the approximate method may likely not be accurate enough, and a more accurate approach based on elastic theory (discussed later in this section) may be required.

Examples 6-3 and 6-4 illustrate the use of the approximate method to calculate vertical pressure below a uniform load.

EXAMPLE 6-3

Given

A 10-ft by 15-ft rectangular area carrying a uniform load of 5000 lb/ft² is applied to the ground surface.

Required

The vertical stress increment due to this load at a depth of 20 ft below the ground surface by the approximate method.

Solution

From Eq. (6-4),

$$q = \frac{Q}{(B + z)(L + z)} \tag{6-4}$$

$$Q = (5000 \text{ lb/ft}^2)(10 \text{ ft})(15 \text{ ft}) = 750{,}000 \text{ lb}$$

$$B = 10 \text{ ft}$$

$$L = 15 \text{ ft}$$

$$z = 20 \text{ ft}$$

$$q = \frac{750{,}000 \text{ lb}}{(10 \text{ ft} + 20 \text{ ft})(15 \text{ ft} + 20 \text{ ft})} = 714 \text{ lb/ft}^2$$

EXAMPLE 6-4

Given

A 3-m by 4-m rectangular area carrying a uniform load of 200 kN/m² is applied to the ground surface.

Required

The vertical stress increment due to this load at a depth of 6 m below the ground surface by the approximate method.

Solution

From Eq. (6-4),

$$q = \frac{Q}{(B + z)(L + z)} \tag{6-4}$$

$$Q = (200\ \text{kN/m}^2)(3\ \text{m})(4\ \text{m}) = 2400\ \text{kN}$$

$$B = 3\ \text{m}$$

$$L = 4\ \text{m}$$

$$z = 6\ \text{m}$$

$$q = \frac{2400\ \text{kN}}{(3\ \text{m} + 6\ \text{m})(4\ \text{m} + 6\ \text{m})} = 26.7\ \text{kN/m}^2$$

Method Based on Elastic Theory

Uniform load on a circular area Vertical pressure below a uniform load on a circular area can be determined utilizing Table 6-1 or Fig. 6-5. Here, z and r represent, respectively, the depth and radial horizontal distance from the center of the circle to the point at which pressure is desired (these are similar to the z and r shown in Fig. 6-2); and a represents the radius of the circle on which the uniform loads acts. To calculate vertical pressure below a uniform load on a circular area, the ratios z/a and r/a are computed and an "influence coefficient" is determined from Table 6-1 or Fig. 6-5. This influence coefficient is simply multiplied by the uniform load applied to the circular area to determine the pressure at the desired point. Examples 6-5 and 6-6 illustrate this method.

TABLE 6-1 Influence coefficients for points under uniformly loaded circular area [3].

	r/a									
z/a (1)	*0* (2)	*0.25* (3)	*0.50* (4)	*1.0* (5)	*1.5* (6)	*2.0* (7)	*2.5* (8)	*3.0* (9)	*3.5* (10)	*4.0* (11)
0.25	0.986	0.983	0.964	0.460	0.015	0.002	0.000	0.000	0.000	0.000
0.50	0.911	0.895	0.840	0.418	0.060	0.010	0.003	0.000	0.000	0.000
0.75	0.784	0.762	0.691	0.374	0.105	0.025	0.010	0.002	0.000	0.000
1.00	0.646	0.625	0.560	0.335	0.125	0.043	0.016	0.007	0.003	0.000
1.25	0.524	0.508	0.455	0.295	0.135	0.057	0.023	0.010	0.005	0.001
1.50	0.424	0.413	0.374	0.256	0.137	0.064	0.029	0.013	0.007	0.002
1.75	0.346	0.336	0.309	0.223	0.135	0.071	0.037	0.018	0.009	0.004
2.00	0.284	0.277	0.258	0.194	0.127	0.073	0.041	0.022	0.012	0.006
2.5	0.200	0.196	0.186	0.150	0.109	0.073	0.044	0.028	0.017	0.011
3.0	0.146	0.143	0.137	0.117	0.091	0.066	0.045	0.031	0.022	0.015
4.0	0.087	0.086	0.083	0.076	0.061	0.052	0.041	0.031	0.024	0.018
5.0	0.057	0.057	0.056	0.052	0.045	0.039	0.033	0.027	0.022	0.018
7.0	0.030	0.030	0.029	0.028	0.026	0.024	0.021	0.019	0.016	0.015
10.00	0.015	0.015	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.011

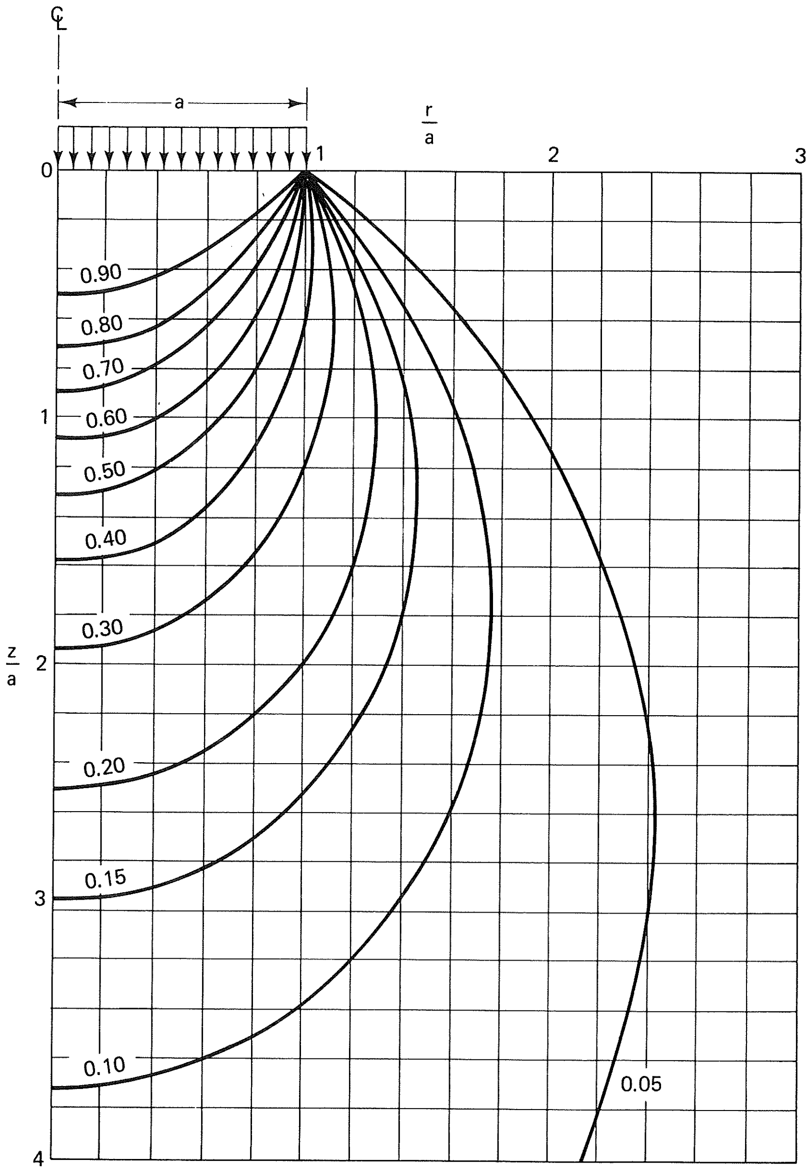

FIGURE 6-5 Influence coefficients for uniformly loaded circular area. [4]

EXAMPLE 6-5

Given

1. A circular area carrying a uniformly distributed load of 2000 lb/ft^2 is applied to the ground surface.
2. The radius of the circular area is 10 ft.

Required

The vertical stress increment due to this uniform load:

1. At a point 20 ft below the center of the circular area.
2. At a point 20 ft below the ground surface at a horizontal distance of 5 ft from the center of the circular area (i.e., $r = 5$ ft, $z = 20$ ft).
3. At a point 20 ft below the edge of the circular area.
4. At a point 20 ft below the ground surface at a horizontal distance of 18 ft from the center of the circular area (i.e., $r = 18$ ft, $z = 20$ ft).

Solution

1. q = influence coefficient multiplied by the uniform load

   With $a = 10$ ft (radius of circle)

   $r = 0$ ft

   $z = 20$ ft

$$\frac{z}{a} = \frac{20\text{ ft}}{10\text{ ft}} = 2.00$$

$$\frac{r}{a} = \frac{0\text{ ft}}{10\text{ ft}} = 0$$

   Influence coefficient from Table 6-1 or Fig. 6-5 = 0.284

$$q = (0.284)(2000\text{ lb/ft}^2) = 568\text{ lb/ft}^2$$

2. With $a = 10$ ft

   $r = 5$ ft

   $z = 20$ ft

$$\frac{z}{a} = \frac{20\text{ ft}}{10\text{ ft}} = 2.00$$

$$\frac{r}{a} = \frac{5\text{ ft}}{10\text{ ft}} = 0.5$$

   Influence coefficient from Table 6-1 or Fig. 6-5 = 0.258

$$q = (0.258)(2000\text{ lb/ft}^2) = 516\text{ lb/ft}^2$$

3. With $a = 10$ ft

   $r = 10$ ft

   $z = 20$ ft

$$\frac{z}{a} = \frac{20 \text{ ft}}{10 \text{ ft}} = 2.00$$

$$\frac{r}{a} = \frac{10 \text{ ft}}{10 \text{ ft}} = 1.00$$

Influence coefficient from Table 6-1 or Fig. 6-5 = 0.194

$$q = (0.194)(2000 \text{ lb/ft}^2) = 388 \text{ lb/ft}^2$$

4. With $a = 10$ ft

   $r = 18$ ft

   $z = 20$ ft

$$\frac{z}{a} = \frac{20 \text{ ft}}{10 \text{ ft}} = 2.00$$

$$\frac{r}{a} = \frac{18 \text{ ft}}{10 \text{ ft}} = 1.8$$

From Table 6-1:

when $\frac{z}{a} = 2.00$ and $\frac{r}{a} = 1.5$, influence coefficient = 0.127

when $\frac{z}{a} = 2.00$ and $\frac{r}{a} = 2.00$, influence coefficient = 0.073

By interpolation between 0.127 and 0.073, the desired influence coefficient for $z/a = 2.00$ and $r/a = 1.8$ is

$$0.073 + \left(\frac{0.127 - 0.073}{5}\right)(2) = 0.095$$

or

$$0.127 - \left(\frac{0.127 - 0.073}{5}\right)(3) = 0.095$$

Or, from Fig. 6-5, the influence coefficient is determined to be 0.095.

$$q = (0.095)(2000 \text{ lb/ft}^2) = 190 \text{ lb/ft}^2$$

EXAMPLE 6-6

Given

Soil with a unit weight of 16.97 kN/m^3 is loaded on the ground surface by a uniformly distributed load of 300 kN/m^2 over a circular area 4 m in diameter (see Fig. 6-6).

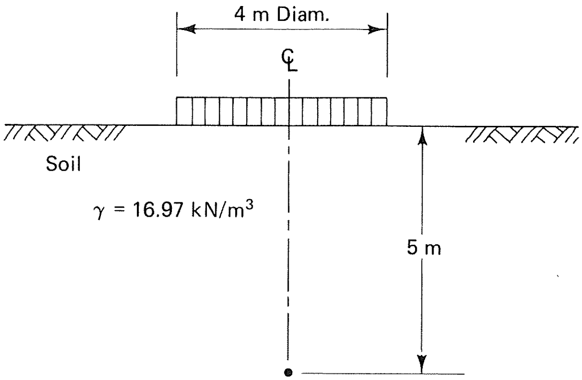

FIGURE 6-6

Required

1. The vertical stress increment due to this uniform load at a depth of 5 m below the center of the circular area.
2. The total vertical pressure at the same location.

Solution

1. With $a = 2$ m

   $r = 0$ m

   $z = 5$ m

$$\frac{r}{a} = \frac{0 \text{ m}}{2 \text{ m}} = 0$$

$$\frac{z}{a} = \frac{5 \text{ m}}{2 \text{ m}} = 2.50$$

Influence coefficient (from Table 6-1 or Fig. 6-5) = 0.200

$$q = (0.200)(300 \text{ kN/m}^2) = 60.0 \text{ kN/m}^2$$

2. Total vertical pressure = overburden pressure (p_0) + vertical stress increment (q).

   Overburden pressure (p_0) = γz = (16.97 kN/m³)(5 m) = 84.8 kN/m²

   Total vertical pressure = 84.8 kN/m² + 60.0 kN/m² = 144.8 kN/m²

Uniform load on a rectangular area Vertical pressure below a uniform load on a rectangular area can be determined utilizing Table 6-2. In the table, z, A, and B represent, respectively, depth below the loaded surface and width and length of the rectangle on which the uniform load acts. To calculate vertical pressure below a uniform load on a rectangular area, the ratios $n =$

TABLE 6-2 Influence coefficients for points under uniformly loaded rectangular areas [3, 5].

$m = A/z$ or $n = B/z$	$n = B/z$ or $m = A/z$																	
	0.1	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*1.0*	*1.2*	*1.5*	*2.0*	*2.5*	*3.0*	*5.0*	*10.0*	∞
0.1	0.005	0.009	0.013	0.017	0.020	0.022	0.024	0.026	0.027	0.028	0.029	0.030	0.031	0.031	0.032	0.032	0.032	0.032
0.2	0.009	0.018	0.026	0.033	0.039	0.043	0.047	0.050	0.053	0.055	0.057	0.059	0.061	0.062	0.062	0.062	0.062	0.062
0.3	0.013	0.026	0.037	0.047	0.056	0.063	0.069	0.073	0.077	0.079	0.083	0.086	0.089	0.090	0.090	0.090	0.090	0.090
0.4	0.017	0.033	0.047	0.060	0.071	0.080	0.087	0.093	0.098	0.101	0.106	0.110	0.113	0.115	0.115	0.115	0.115	0.115
0.5	0.020	0.039	0.056	0.071	0.084	0.095	0.103	0.110	0.116	0.120	0.126	0.131	0.135	0.137	0.137	0.137	0.137	0.137
0.6	0.022	0.043	0.063	0.080	0.095	0.107	0.117	0.125	0.131	0.136	0.143	0.149	0.153	0.155	0.156	0.156	0.156	0.156
0.7	0.024	0.047	0.069	0.087	0.103	0.117	0.128	0.137	0.144	0.149	0.157	0.164	0.169	0.170	0.171	0.172	0.172	0.172
0.8	0.026	0.050	0.073	0.093	0.110	0.125	0.137	0.146	0.154	0.160	0.168	0.176	0.181	0.183	0.184	0.185	0.185	0.185
0.9	0.027	0.053	0.077	0.098	0.116	0.131	0.144	0.154	0.162	0.168	0.178	0.186	0.192	0.194	0.195	0.196	0.196	0.196
1.0	0.028	0.055	0.079	0.101	0.120	0.136	0.149	0.160	0.168	0.175	0.185	0.193	0.200	0.202	0.203	0.204	0.205	0.205
1.2	0.029	0.057	0.083	0.106	0.126	0.143	0.157	0.168	0.178	0.185	0.196	0.205	0.212	0.215	0.216	0.217	0.218	0.218
1.5	0.030	0.059	0.086	0.110	0.131	0.149	0.164	0.176	0.186	0.193	0.205	0.215	0.223	0.226	0.228	0.229	0.230	0.230
2.0	0.031	0.061	0.089	0.113	0.135	0.153	0.169	0.181	0.192	0.200	0.212	0.223	0.232	0.236	0.238	0.239	0.240	0.240
2.5	0.031	0.062	0.090	0.115	0.137	0.155	0.170	0.183	0.194	0.202	0.215	0.226	0.236	0.240	0.242	0.244	0.244	0.244
3.0	0.032	0.062	0.090	0.115	0.137	0.156	0.171	0.184	0.195	0.203	0.216	0.228	0.238	0.242	0.244	0.246	0.247	0.247
5.0	0.032	0.062	0.090	0.115	0.137	0.156	0.172	0.185	0.196	0.204	0.217	0.229	0.239	0.244	0.246	0.249	0.249	0.249
10.0	0.032	0.062	0.090	0.115	0.137	0.156	0.172	0.185	0.196	0.205	0.218	0.230	0.240	0.244	0.247	0.249	0.250	0.250
∞	0.032	0.062	0.090	0.115	0.137	0.156	0.172	0.185	0.196	0.205	0.218	0.230	0.240	0.244	0.247	0.249	0.250	0.250

B/z and $m = A/z$ are computed and an influence coefficient is determined from Table 6-2. Either m or n can be read along the first column and the other one (n or m) is read across the top. The influence coefficient can also be determined utilizing Fig. 6-7. The influence coefficient is multiplied by the uniform load applied to the rectangular area to determine the pressure at depth z below each *corner* of the rectangle. Example 6-7 illustrates this method.

EXAMPLE 6-7

Given

A 15-ft by 20-ft rectangular foundation carrying a uniform load of 4000 lb/ft² is applied to the ground surface.

Required

The vertical stress increment due to this uniform load at a point 10 ft below the corner of the rectangular loaded area.

Solution

From Table 6-2 or Fig. 6-7, with

$$A = mz \text{ or } m = \frac{A}{z} \qquad A = 15 \text{ ft} \qquad z = 10 \text{ ft} \qquad m = \frac{15 \text{ ft}}{10 \text{ ft}} = 1.5$$

$$B = nz \text{ or } n = \frac{B}{z} \qquad B = 20 \text{ ft} \qquad z = 10 \text{ ft} \qquad n = \frac{20 \text{ ft}}{10 \text{ ft}} = 2.0$$

Influence coefficient = 0.223

$$q = (0.223)(4000 \text{ lb/ft}^2) = 892 \text{ lb/ft}^2$$

It should be emphasized that the pressure determined from the influence coefficients utilizing Table 6-2 or Fig. 6-7 (as in Example 6-7) is acting at depth z *directly below a corner of the rectangular area.* This is shown in Fig. 6-8, where such a computed stress acts at point C. It is sometimes necessary to determine the pressure below a rectangular loaded area at points other than directly below a corner of the rectangular area. For example, it may be necessary to determine the pressure at some depth directly below the center of a rectangular area, or at some point outside the downward projection of the rectangular area. This can be accomplished by dividing the area into rectangles, each of which has one corner directly above the point at which the pressure is desired at depth z. The pressure is computed for each rectangle in the usual manner and the results added or subtracted to get the total pressure. Figure 6-9 should facilitate understanding of this procedure. In each case of Fig. 6-9, the heavy dot indicates the point at which the pressure at depth z is required. Examples 6-8 through 6-11 illustrate this procedure.

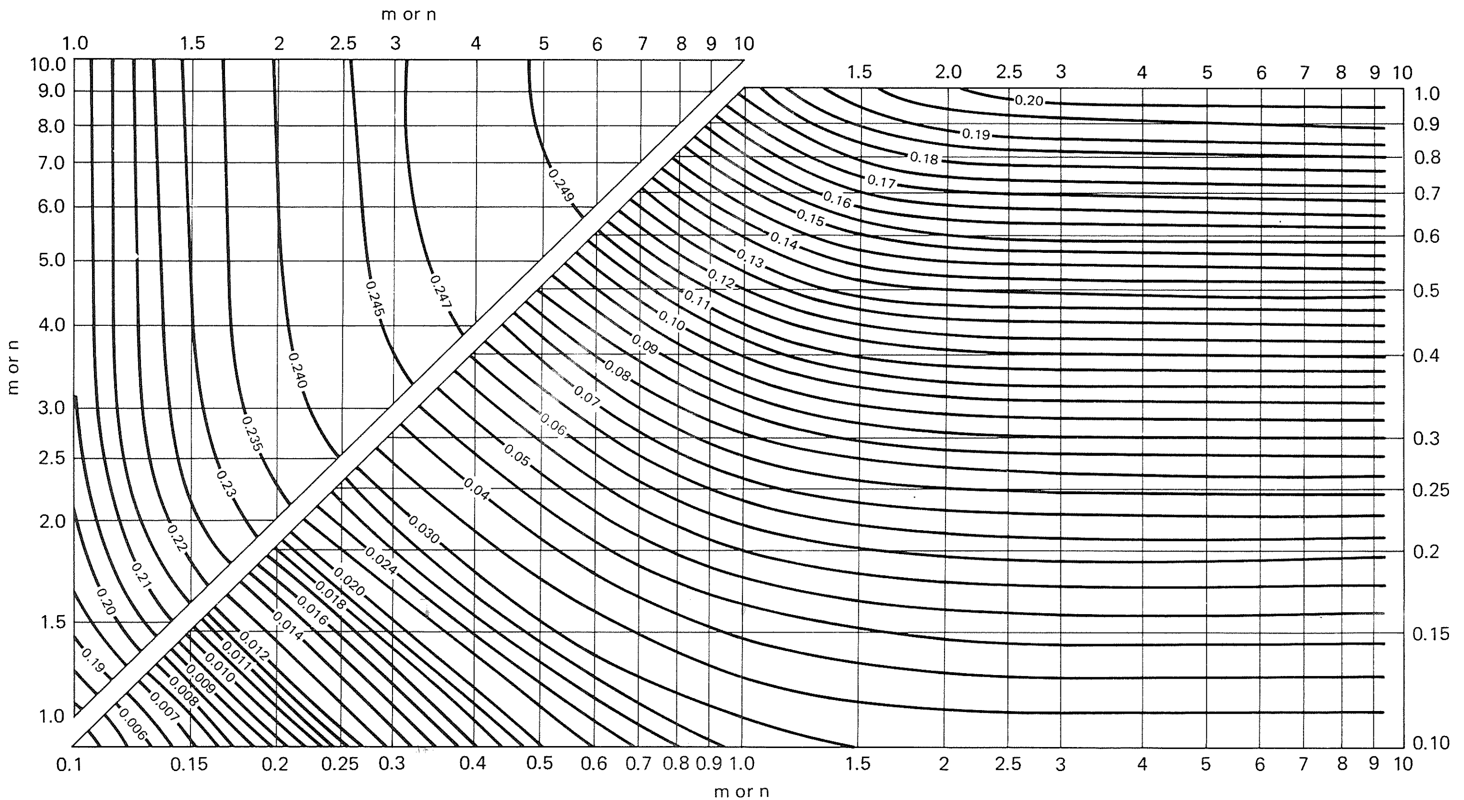

FIGURE 6-7 Chart for use in determining vertical stresses below corners of loaded rectangular surface areas on elastic, isotropic material. [4, 6]

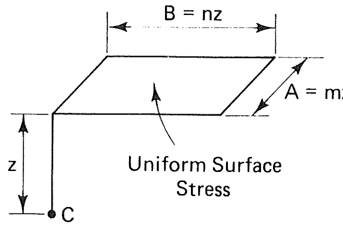

FIGURE 6-7 *(continued)*

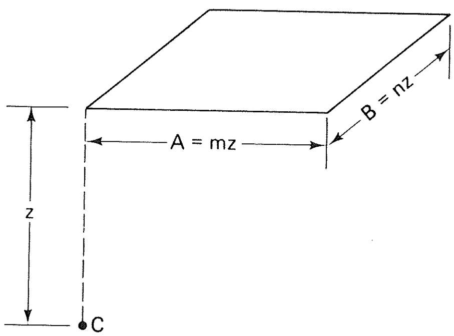

FIGURE 6-8

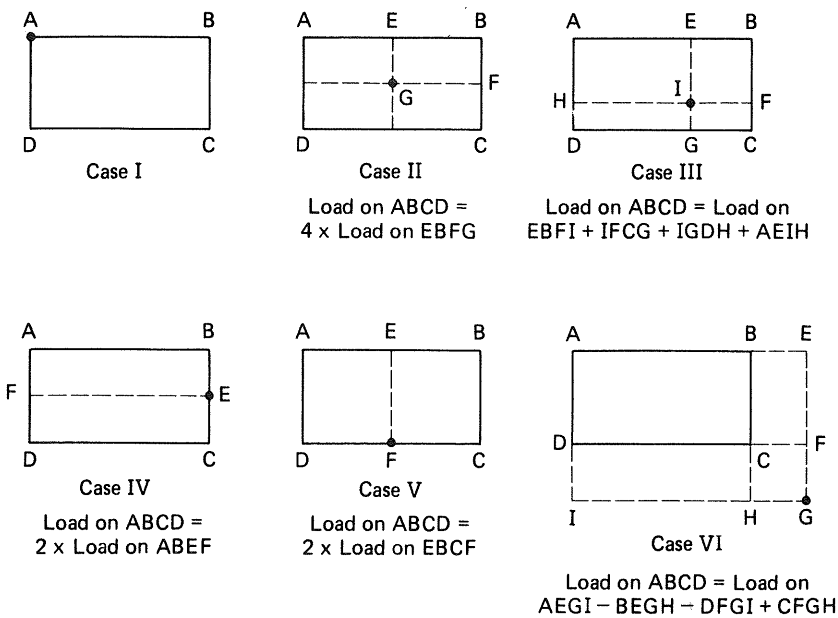

FIGURE 6-9 [3]

EXAMPLE 6-8

Given

A 20-ft by 30-ft rectangular foundation carrying a uniform load of 6000 lb/ft^2 is applied to the ground surface.

Required

The vertical stress increment due to this uniform load at a depth of 20 ft below the center of the loaded area. (See point *A* in Fig. 6-10.)

Solution

This corresponds to case II of Fig. 6-9, so the area is divided into four equal parts.

$$A = mz \text{ or } m = \frac{A}{z} \qquad A = 10 \text{ ft} \qquad z = 20 \text{ ft} \qquad m = \frac{10 \text{ ft}}{20 \text{ ft}} = 0.5$$

$$B = nz \text{ or } n = \frac{B}{z} \qquad B = 15 \text{ ft} \qquad z = 20 \text{ ft} \qquad n = \frac{15 \text{ ft}}{20 \text{ ft}} = 0.75$$

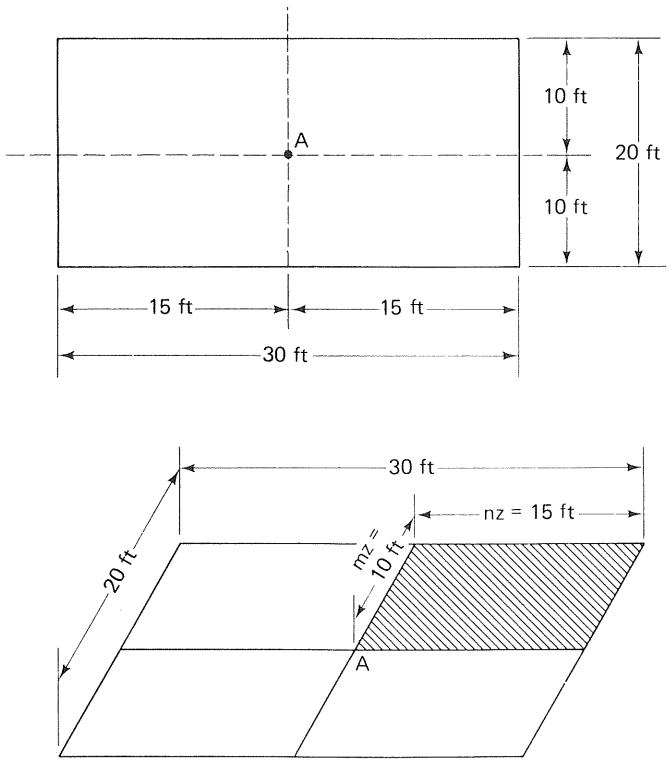

FIGURE 6-10

From Table 6-2 or Fig. 6-7, the influence coefficient = 0.107 for a 10-ft by 15-ft loaded area. Since the original area of 20 ft by 30 ft consists of four smaller equal areas of 10 ft by 15 ft and each of these four areas shares a corner at point *A*,

$$q = (4)(0.107)(6000 \text{ lb/ft}^2) = 2570 \text{ lb/ft}^2$$

EXAMPLE 6-9

Given

A 1.5-m by 1.5-m footing located 1 m below the ground surface as shown in Fig. 6-11 carries a load of 650 kN (including column load and weight of footing and soil surcharge).

Required

The net vertical stress increment due to this load at a depth of 5 m below the center of the footing (i.e., at point *A* in Fig. 6-11).

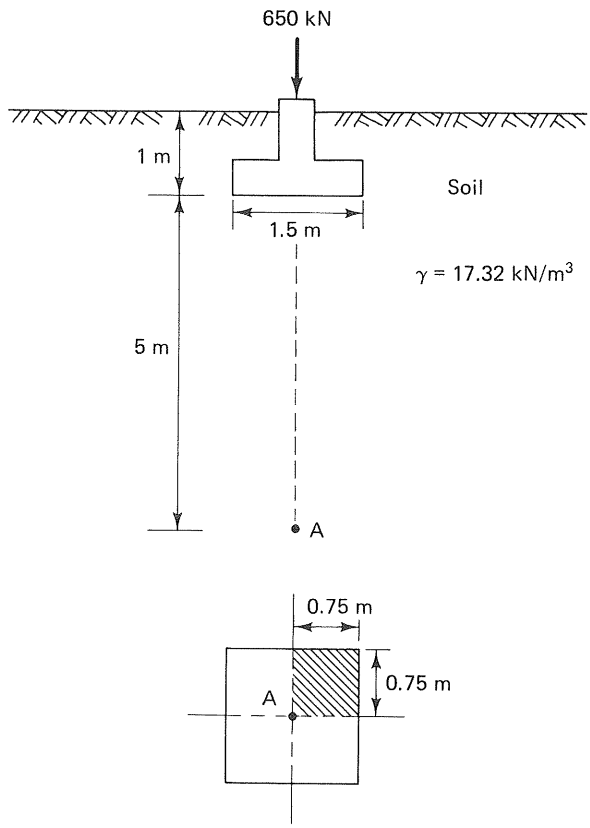

FIGURE 6-11

Solution

As in Example 6-8, the total area is divided into four equal areas, 0.75 m by 0.75 m, as shown in Fig. 6-11.

$$A = mz \text{ or } m = \frac{A}{z} \qquad A = 0.75 \text{ m} \quad z = 5 \text{ m} \qquad m = \frac{0.75 \text{ m}}{5 \text{ m}} = 0.150$$

$$B = nz \text{ or } n = \frac{B}{z} \qquad B = 0.75 \text{ m} \quad z = 5 \text{ m} \qquad n = \frac{0.75 \text{ m}}{5 \text{ m}} = 0.150$$

From Table 6-2 or Fig. 6-7, the influence coefficient = 0.0103 for a 0.75-m by 0.75-m loaded area. Since the 1.5-m by 1.5-m footing consists of four smaller equal areas of 0.75 m by 0.75 m and each of these four areas shares a corner at point *A*,

$$q = (4)(0.0103)(\text{net vertical stress increment at the footing's base})$$

Net vertical stress increment at the footing's base

$$= \frac{650 \text{ kN}}{(1.5 \text{ m})(1.5 \text{ m})} - (17.32 \text{ kN/m}^3)(1 \text{ m}) = 271.6 \text{ kN/m}^2$$

$$q = (4)(0.0103)(271.6 \text{ kN/m}^2) = 11.2 \text{ kN/m}^2$$

EXAMPLE 6-10

Given

1. An L-shaped building (in plan) shown in Fig. 6-12.
2. The load exerted by the structure is 1400 lb/ft^2.

Required

Determine the vertical stress increment due to the structure load at a depth of 15 ft below interior corner *A* of the L-shaped building. Assume that the foundation is under the entire building.

Solution

Divide the L-shaped building into three smaller areas, *ABCD, ADEF,* and *AFGH*. Note that these three areas share a common corner at point *A* (or corner *A*).

Area *ABCD*

From Table 6-2 or Fig. 6-7, with

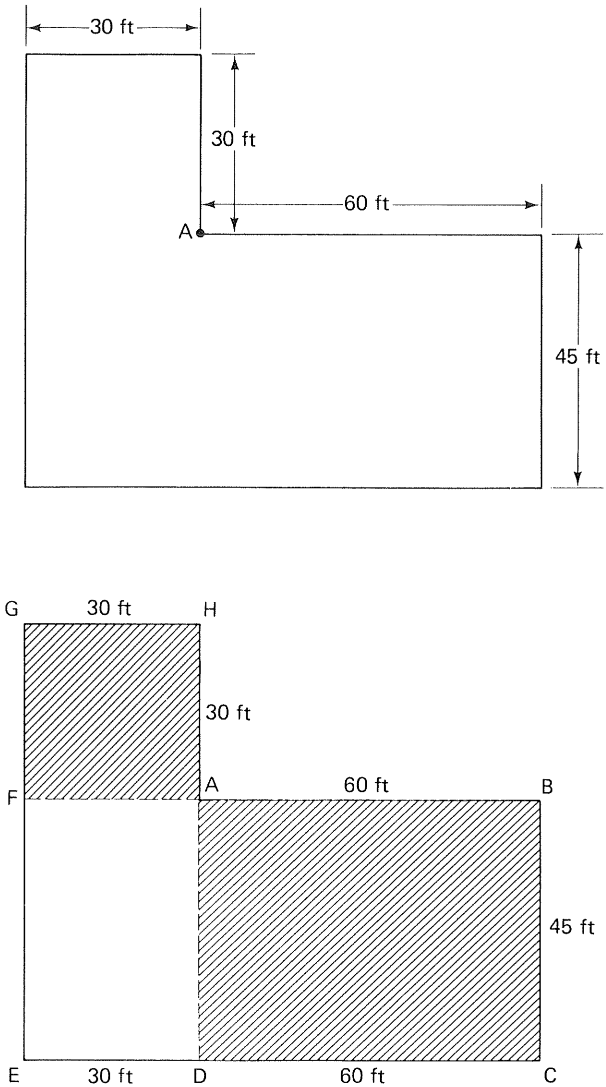

FIGURE 6-12

$$A = mz \text{ or } m = \frac{A}{z} \qquad A = 60 \text{ ft} \qquad z = 15 \text{ ft} \qquad m = \frac{60 \text{ ft}}{15 \text{ ft}} = 4$$

$$B = nz \text{ or } n = \frac{B}{z} \qquad B = 45 \text{ ft} \qquad z = 15 \text{ ft} \qquad n = \frac{45 \text{ ft}}{15 \text{ ft}} = 3$$

Influence coefficient = 0.245

Area *ADEF*

From Table 6-2 or Fig. 6-7, with

$$A = 30 \text{ ft} \qquad z = 15 \text{ ft} \qquad m = \frac{30 \text{ ft}}{15 \text{ ft}} = 2$$

$$B = 45 \text{ ft} \qquad z = 15 \text{ ft} \qquad n = \frac{45 \text{ ft}}{15 \text{ ft}} = 3$$

Influence coefficient = 0.238

Area *AFGH*

From Table 6-2 or Fig. 6-7, with

$$A = 30 \text{ ft} \qquad z = 15 \text{ ft} \qquad m = \frac{30 \text{ ft}}{15 \text{ ft}} = 2$$

$$B = 30 \text{ ft} \qquad z = 15 \text{ ft} \qquad n = \frac{30 \text{ ft}}{15 \text{ ft}} = 2$$

Influence coefficient = 0.232

$$q = [\Sigma \text{ influence coefficients}] \text{ multiplied by the uniform load}$$

$$q = (0.245 + 0.238 + 0.232)(1400 \text{ lb/ft}^2) = 1001 \text{ lb/ft}^2$$

EXAMPLE 6-11

Given

1. A rectangular loaded area *ABCD* shown in plan in Fig. 6-13.
2. The load exerted on the area is 80 kN/m^2.

Required

Vertical stress increment due to the exerted load at a depth of 3 m below point *G* (see Fig. 6-13).

Solution

This corresponds to case VI of Fig. 6-9. The influence coefficient for the vertical stress increment under point *G* due to the uniform load on area *ABCD* may be obtained from the coefficients for various rectangles as follows:

$$\text{Load on } ABCD = \text{load on } DEGI - AEGH - CFGI + BFGH$$

(*Note:* In the equation above, the last term *BFGH* is added because when *AEGH* is subtracted, area *BFGH* is included in it and when *CFGI* is subtracted, area *BFGH* is also included in it. Thus, the effect of area *BFGH* has been subtracted twice. Hence, it must be added in order that its effect be subtracted only one time.)

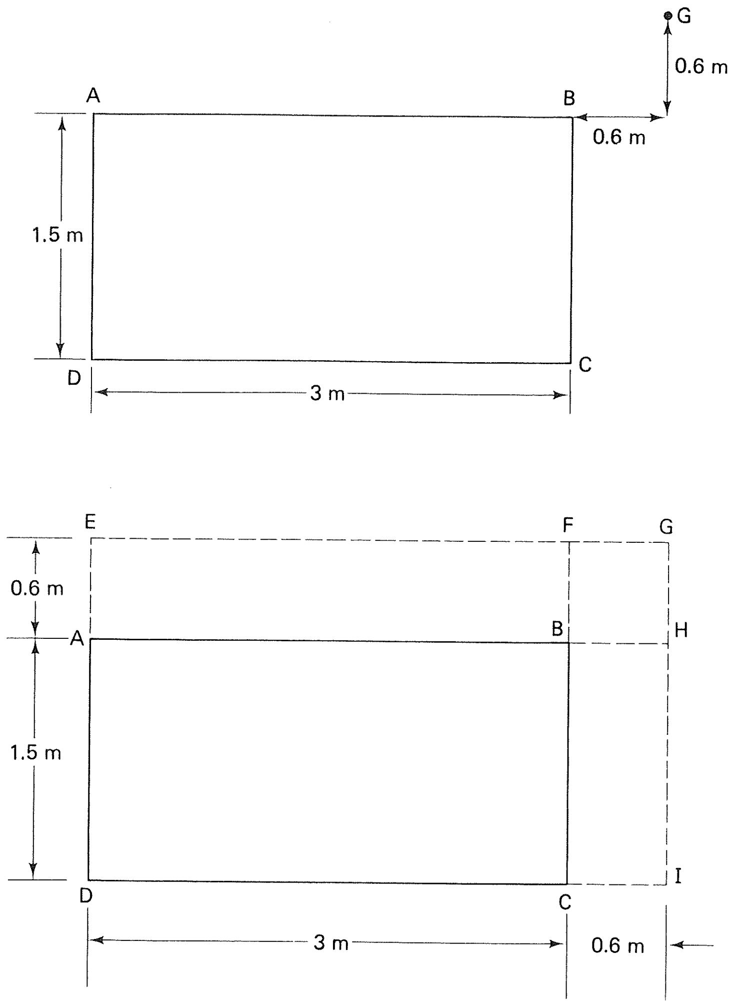

FIGURE 6-13

Area *DEGI*

From Table 6-2 or Fig. 6-7, with

$$A = mz \text{ or } m = \frac{A}{z} \qquad A = 2.1 \text{ m} \quad z = 3 \text{ m} \qquad m = \frac{2.1 \text{ m}}{3 \text{ m}} = 0.7$$

$$B = nz \text{ or } n = \frac{B}{z} \qquad B = 3.6 \text{ m} \quad z = 3 \text{ m} \qquad n = \frac{3.6 \text{ m}}{3 \text{ m}} = 1.2$$

Influence coefficient for area *DEGI* = 0.157

Area *AEGH*

From Table 6-2 or Fig. 6-7, with

$$A = 0.6 \text{ m} \qquad z = 3 \text{ m} \qquad m = \frac{0.6 \text{ m}}{3 \text{ m}} = 0.2$$

$$B = 3.6 \text{ m} \qquad z = 3 \text{ m} \qquad n = \frac{3.6 \text{ m}}{3 \text{ m}} = 1.2$$

Influence coefficient for area *AEGH* = 0.057

Area *CFGI*

From Table 6-2 or Fig. 6-7, with

$$A = 2.1 \text{ m} \qquad z = 3 \text{ m} \qquad m = \frac{2.1 \text{ m}}{3 \text{ m}} = 0.7$$

$$B = 0.6 \text{ m} \qquad z = 3 \text{ m} \qquad n = \frac{0.6 \text{ m}}{3 \text{ m}} = 0.2$$

Influence coefficient for area *CFGI* = 0.047

Area *BFGH*

From Table 6-2 or Fig. 6-7, with

$$A = 0.6 \text{ m} \qquad z = 3 \text{ m} \qquad m = \frac{0.6 \text{ m}}{3 \text{ m}} = 0.2$$

$$B = 0.6 \text{ m} \qquad z = 3 \text{ m} \qquad n = \frac{0.6 \text{ m}}{3 \text{ m}} = 0.2$$

Influence coefficient for area *BFGH* = 0.018

$$q = (0.157 - 0.057 - 0.047 + 0.018)(80 \text{ kN/m}^2) = 5.68 \text{ kN/m}^2$$

Uniform load on a strip area Vertical pressure below a uniform load on a strip area can be determined utilizing Fig. 6-14. Use of Fig. 6-14 is similar to that of Fig. 6-5 for a loaded circular area except that B and r represent strip width and radial horizontal distance from the strip footing's center line, respectively (z denotes depth in both cases).

EXAMPLE 6-12

Given

1. Soil with a unit weight of 17.92 kN/m³ is loaded on the ground surface by a wall footing 1 m wide.

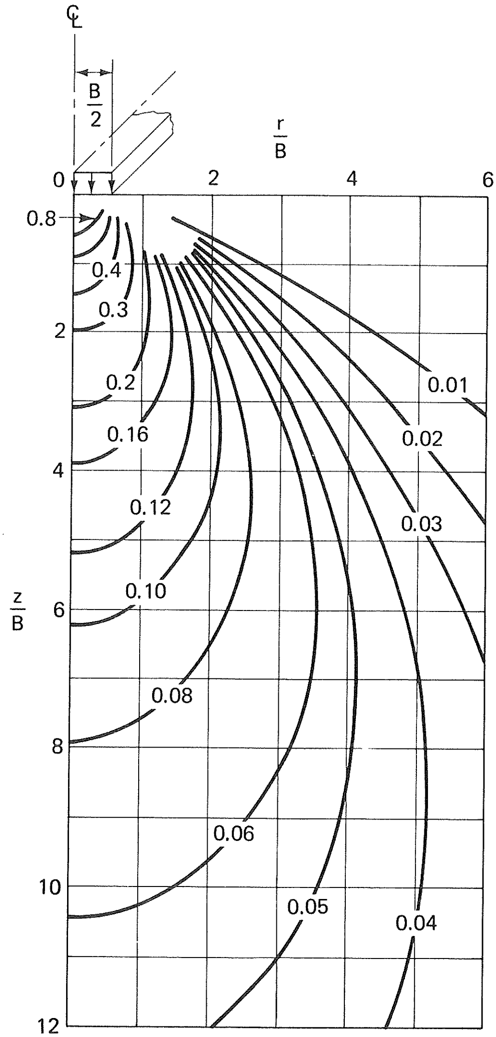

FIGURE 6-14 Influence coefficients for uniformly loaded strip area. [7]

2. The load of the wall footing is 295 kN/m of wall length.

Required

1. The vertical stress increment due to the wall footing at a point 3 m below the edge of the strip (see Fig. 6-15).
2. The total vertical load at the same location.

Solution

1. From Fig. 6-14, with

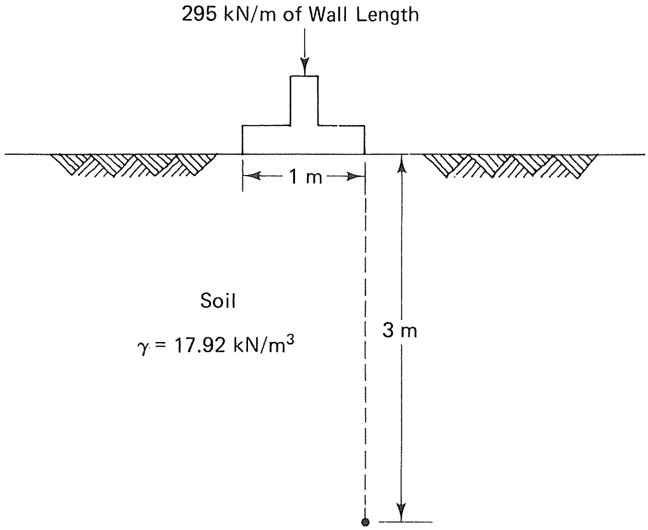

FIGURE 6-15

$$\frac{r}{B} = \frac{0.5\ \text{m}}{1\ \text{m}} = 0.5$$

$$\frac{z}{B} = \frac{3\ \text{m}}{1\ \text{m}} = 3.0$$

Influence coefficient = 0.20

$$q = (0.20)(295\ \text{kN/m}) = 59.0\ \text{kN/m of wall length}$$

2. Total vertical load = overburden pressure (p_0) + vertical stress increment (q).

Overburden pressure (p_0) = γz = (17.92 kN/m³)(3 m)
= 53.8 kN/m², or 53.8 kN/m of wall length

Total vertical load = 53.8 kN/m + 59.0 kN/m
= 112.8 kN/m of wall length

Uniform load on any area Vertical pressure below a uniform load on any area can be determined using an influence chart (Fig. 6-16) developed by Newmark [6] based on Boussinesq's equation. To utilize this method, a sketch (plan view) must be made of the loaded area on tracing paper drawn to such a scale that distance AB on Fig. 6-16 equals the depth at which pressure is desired. This sketch is placed on the chart (Fig. 6-16) so that the point below which pressure is desired coincides with the chart's center. The next step is to count the quasi-rectangles enclosed by the loaded area. The pressure at the indicated point at the desired depth is determined by multiplying the number of quasi-rectangles by the applied uniform load by 0.001. As indicated on Fig.

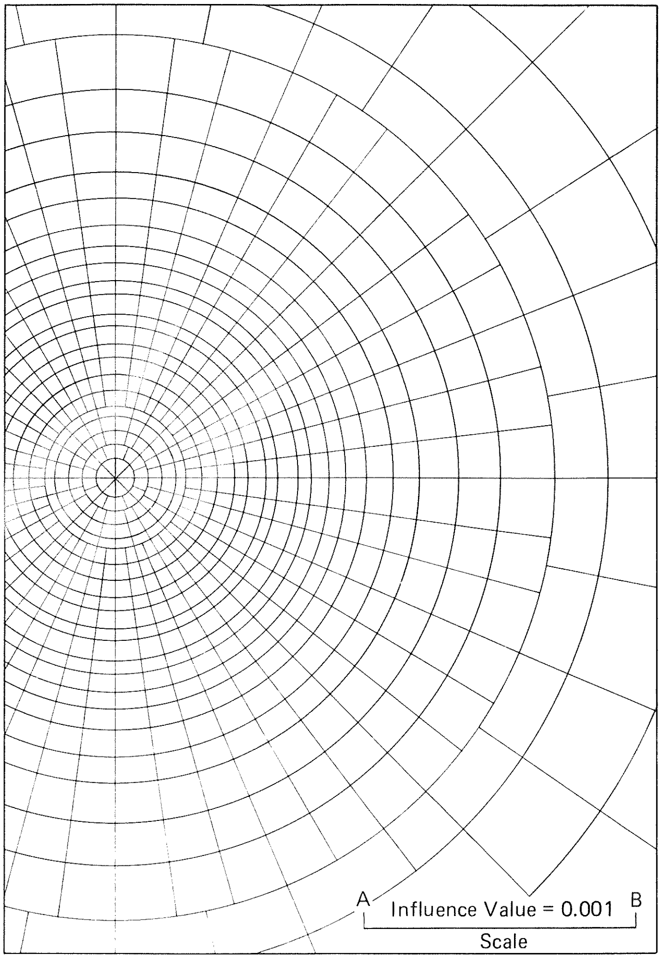

FIGURE 6-16 Newmark influence chart for computing vertical pressure. (After Corps of Engineers.) [8]

6-16, the number 0.001 is the *influence value* for this particular chart. The same sketch may be used to determine pressure at other points at the same depth by shifting the sketch until a desired point coincides with the chart's center and counting the quasi-rectangles. If, however, pressure at some other depth is required, a new sketch must be drawn to such a scale that distance *AB* on Fig. 6-16 equals the depth at which pressure is desired.

6-4 PROBLEMS

6-1 A concentrated load of 200 kips is applied to the ground surface. What is the vertical stress increment due to the load at a depth of 15 ft directly below the load?

6-2 A concentrated load of 200 kips is applied to the ground surface. What is the vertical stress increment due to the load at a point 15 ft below the ground surface at a horizontal distance of 10 ft from the line of the concentrated load?

6-3 A 10-ft by 7.5-ft rectangular area carrying a uniform load of 5000 lb/ft^2 is applied to the ground surface. Determine the vertical stress increment due to this uniform load at a depth of 12 ft below the ground surface by the approximate method (i.e., 2:1 slope method).

6-4 A rectangular area 2 m by 3 m carrying a uniform load of 195 kN/m^2 is applied to the ground surface. Determine the vertical stress increment due to the uniform load at (a) 1, (b) 3, and (c) 5 m below the area by the approximate method.

6-5 A circular area carrying a uniform load of 4500 lb/ft^2 is applied to the ground surface. The area's radius is 12 ft. What is the vertical stress increment due to this uniform load (a) at a point 18 ft below the area's center, and (b) at a point 18 ft below the ground surface at a horizontal distance of 6 ft from the area's center?

6-6 Soil with a unit weight of 16.38 kN/m^3 is loaded on the ground surface by a uniformly distributed load of 250 kN/m^2 over a circular area 3 m in diameter. Determine (a) the vertical stress increment due to the uniform load, and (b) the total vertical pressure at a depth of 3 m under the edge of the circular area.

6-7 An 8-ft by 12-ft rectangular area carrying a uniform load of 6000 lb/ft^2 is applied to the ground surface. What is the vertical stress increment due to the uniform load at a depth of 15 ft below the corner of the rectangular loaded area?

6-8 A 12-ft by 12-ft square area carrying a uniform load of 5000 lb/ft^2 is applied to the ground surface. Find the vertical stress increment due to the load at a depth of 25 ft below the center of the loaded area.

6-9 A 2-m by 2-m square footing is located 1.8 m below the ground surface and carries a load of 1000 kN. Determine the net vertical stress increment due to the uniform load at a depth of 4 m below the center of the footing (see Fig. 6-17).

6-10 The L-shaped area shown in Fig. 6-18 carries a 2000-lb/ft^2 uniform load. Find the vertical stress increment due to the structure load at a depth of 24 ft (a) below corner *A*, and (b) below corner *E*.

6-11 The square area *ABCD* shown in Fig. 6-19 carries a 2500-lb/ft^2 uniform load. Find the vertical stress increment due to the exerted load at a depth of 12 ft (a) below point *G*, and (b) below point *J*.

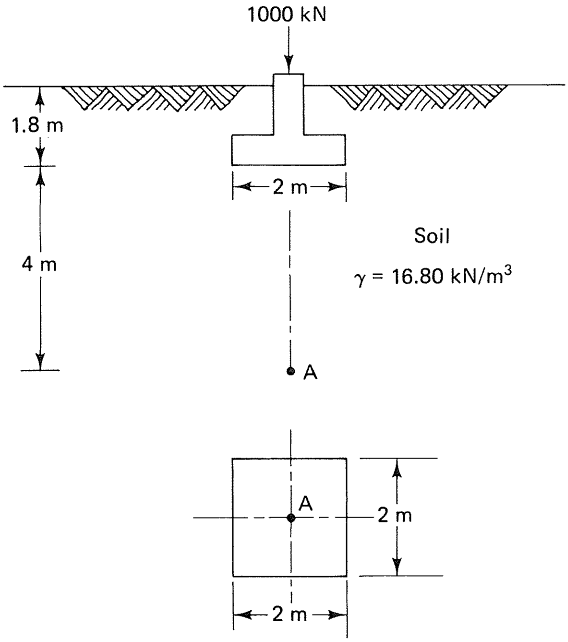

FIGURE 6-17

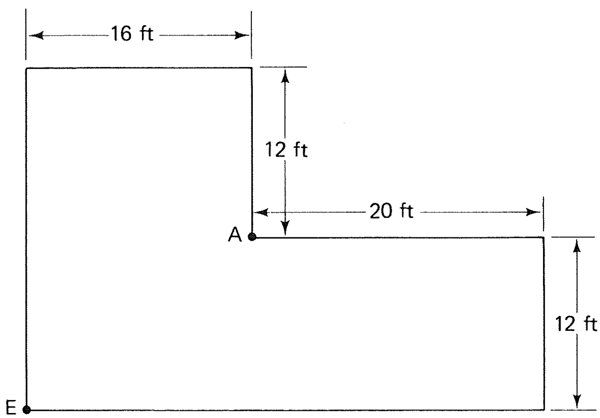

FIGURE 6-18

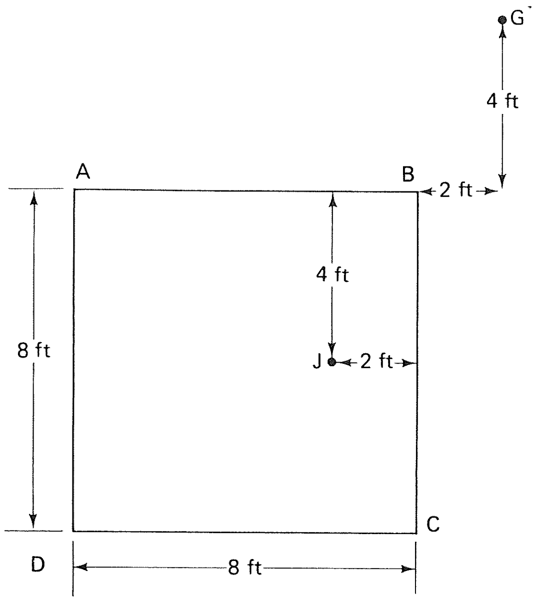

FIGURE 6-19

6-12 Soil with a unit weight of 19.65 kN/m^3 is loaded on the ground surface by a strip load 1.5 m wide. The strip load is 365 kN/m of wall length. Determine (a) the vertical stress increment due to the strip load, and (b) the total pressure—both at a point 3 m below the center of the strip load.

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[6] NATHAN M. NEWMARK, *Influence Charts for Computation of Stresses in Elastic Foundations,* Univ. Ill. Bull. 338, 1942.

[7] IRVING S. DUNN, LOREN R. ANDERSON, AND FRED W. KIEFER, *Fundamentals of Geotechnical Analysis,* John Wiley & Sons, Inc., New York, 1980. Copyright © 1980, by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

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7

Consolidation of Soil and Settlement of Structures

7-1 INTRODUCTION

Structures built on soil are subject to settlement. Some settlement is often inevitable; and, depending on circumstances, some settlement is tolerable. For example, small uniform settlement of a building throughout the floor area might be tolerable, whereas nonuniform settlement of the same building might not be. Or, settlement of a garage or warehouse building might be tolerable, whereas the same settlement (especially differential settlement) of a luxury hotel building would not be because of damage to walls, ceilings, and so on. In any event, a knowledge of the causes of settlement and a means of computing (or predicting) settlement quantitatively are important to the soils engineer.

Although there are several possible causes of settlement (e.g., dynamic forces, changes in the groundwater table, adjacent excavation, etc.), probably the major cause is compressive deformation of soil beneath a structure. Compressive deformation generally results from reduction in void volume, accompanied by rearrangement of soil grains and compression of the material in the voids. If soil is dry, its voids are filled with air; and since air is compressible, rearrangement of soil grains can occur rapidly. If soil is saturated, its voids are filled with incompressible water, which must be extruded from the soil mass before soil grains can rearrange themselves. In soils of high permeability (i.e., coarse-grained soils), this process requires a short time interval for completion, and almost all settlement occurs by the time construction is complete. However, in soils of low permeability (i.e., fine-grained soils), the process requires a long time interval for completion. The result is that the strain occurs very slowly; thus, settlement takes place slowly and continues over a long period of time. The latter case (fine-grained soil) is of more concern because of long-term uncertainty.

As indicated, the process of compression due to extrusion of water from the voids in a fine-grained soil as a result of increased loading (such as the weight of a structure above) is very slow and continues over a long period of time. This phenomenon is called *primary consolidation* [1]. Associated settlement is referred to as *primary consolidation settlement.* (These are commonly referred to simply as "consolidation" and "consolidation settlement," respectively.) After primary consolidation has ended, soil compression and additional associated settlement continue at a very slow rate, the result of plastic readjustment of soil grains due to new, changed stresses in the soil and progressive breaking of clayey particles and their interparticle bonds. This phenomenon is known as *secondary compression,* and associated settlement is called *secondary compression settlement.*

In analyzing clayey soil, it is necessary to differentiate between two types of clay—*normally consolidated* clay and *overconsolidated* clay. In the case of normally consolidated clay, the clay formation has never been subjected to any loading larger than the present effective overburden pressure. This is the case whenever the height of soil above the clay formation (and therefore the weight of the soil above, which causes the pressure) has been more or less constant through time. With overconsolidated clay, the clay formation has been subjected at some time to a loading greater than the present effective overburden pressure. This occurs whenever the present height of soil above the clay formation is less than it was at some time in the past. Such a situation could exist if significant erosion has occurred at the ground surface. (Because of the erosion, the present height of soil above the clay formation is less than it was prior to the erosion.) It might be noted that overconsolidated clay is generally less compressible. As will be related in this chapter, the analysis of clays for settlement differs somewhat depending on whether the clay is normally consolidated or overconsolidated.

This chapter deals primarily with the determination of settlement of structures. Sections 7-2 through 7-7 deal with settlement on clay. Section 7-2 covers the laboratory testing required for analyzing settlement. Section 7-3 shows how laboratory data are analyzed to determine if the clay is normally consolidated, and Sec. 7-4 shows how they are analyzed to determine if the clay is overconsolidated. Section 7-5 demonstrates the development of a "field consolidation line," which in turn is used to calculate consolidation settlement on clay (Sec. 7-6). Section 7-7 covers secondary compression and associated secondary compression settlement. Section 7-8 deals with settlement on sand.

7-2 CONSOLIDATION TEST

As a means of estimating both the amount and time of consolidation and resulting settlement, consolidation tests are run in a laboratory. For complete and detailed instructions for conducting a consolidation test, the reader is referred to *Soil Properties: Testing, Measurement, and Evaluation,* 2nd edition, by Liu and Evett (Prentice-Hall, Inc., 1990). A generalized discussion is given here.

To begin with, an undisturbed soil sample is placed in a metal ring. One porous disk is placed above the sample and another is placed beneath the sample. The purpose of the disks is to allow water to flow vertically into and out of the soil sample. This assembly is immersed in water. As a load is applied to the upper disk, the sample is compressed and deformation is measured by a dial gauge (see Fig. 7-1).

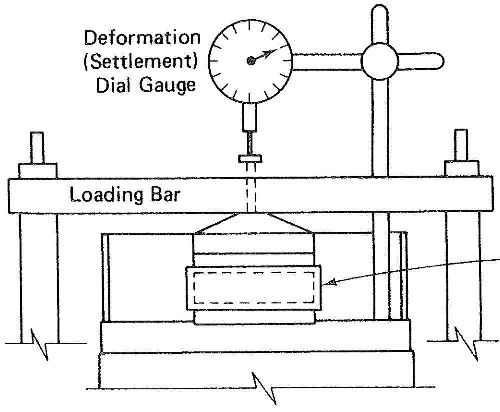
To begin a particular test, a specific pressure (e.g., ⅛ ton/ft^2) is applied to the soil sample, and dial readings (reflecting deformation) and corresponding time observations are made and recorded until deformation has nearly ceased. Normally, this is done over a 24-hour period. Then, a graph is prepared using these data with time along the abscissa on a logarithmic scale and dial readings along the ordinate on an arithmetic scale. An example of such a graph is given in Fig. 7-2.

The procedure described above is repeated after doubling the applied pressure, giving another graph of time versus dial readings corresponding to the new pressure. The procedure is repeated for additional doublings of applied pressure until the applied pressure is in excess of the total pressure to which the clay formation is expected to be subjected when the proposed structure is built. [The total pressure includes effective overburden pressure and net additional pressure (or consolidation pressure) due to the structure.]

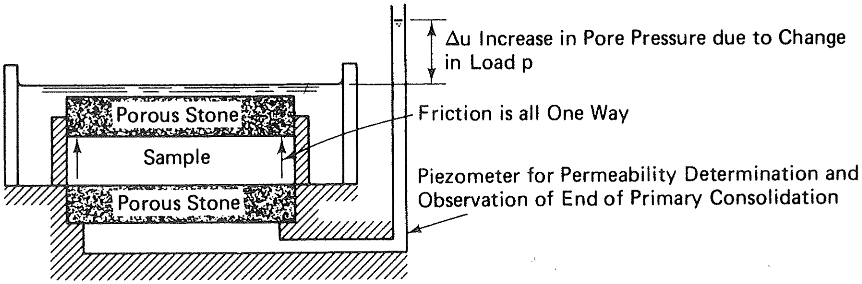
From each graph of time versus dial readings, the void ratio (e) and coefficient of consolidation (c_v) that correspond to the specific applied pressure (p) for that graph are determined using the following steps.

1. Find the deformation representing 100% consolidation for each load increment. First draw a straight line through the points representing the final readings that exhibit a straight-line trend and a flat slope. Draw a second straight line tangent to the steepest part of the deformation versus log time curve. The intersection of the two lines represents the deformation corresponding to 100% consolidation. Compression that occurs subsequent to 100% consolidation is defined as secondary compression (see Fig. 7-2).

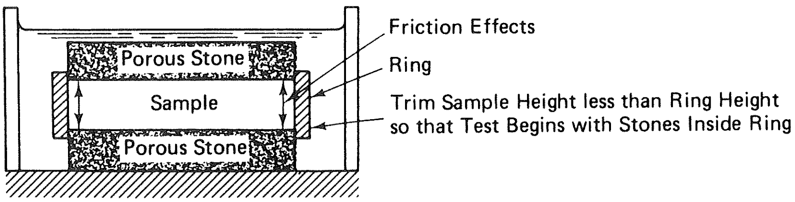
2. Find the deformation representing 0% consolidation by selecting the deformations at any two times that have a ratio of 1:4. The deformation corresponding to the larger of the two times should be greater than one-fourth but less than one-half of the total change in deformation for the load increment. The deformation corresponding to



(a) Consolidometer.



(b) Fixed-Ring Consolidometer. May be used to Obtain Permeability Information during a Consolidation Test if a Piezometer is Installed.



(c) Floating Ring Consolidometer.

FIGURE 7-1 (a) Consolidometer; (b) fixed-ring consolidometer, may be used to obtain information during a consolidation test if a piezometer is installed; (c) floating-ring consolidometer. [2]

0% consolidation is equal to the deformation corresponding to the smaller time interval less the difference in the deformations for the two selected times (see Fig. 7-2).

3. The deformation corresponding to 50% consolidation for each load increment is equal to the average of the deformations corresponding to the 0 and 100% deformations. The time required for 50% consolidation under any load increment may be found graphically from the

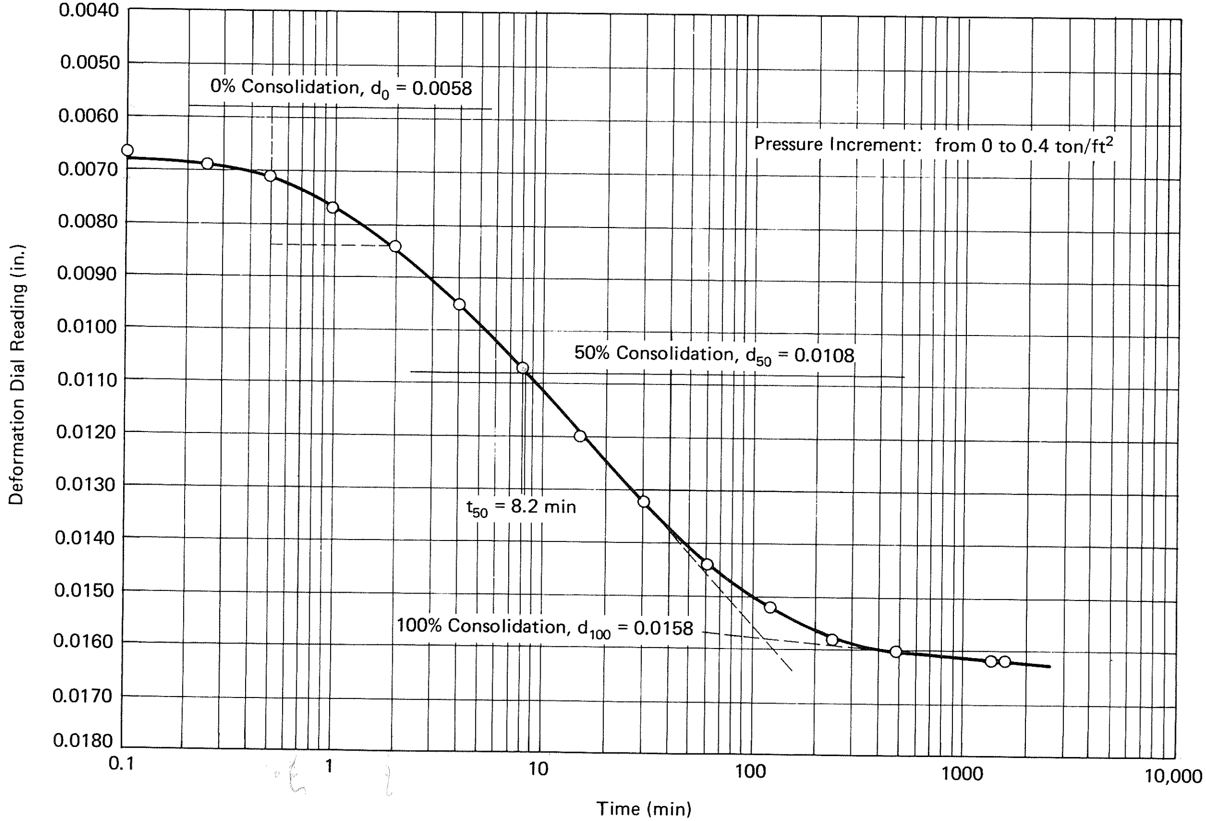


FIGURE 7-2 Dial readings versus log time curve.

deformation versus log time curve for that load increment by observing the time that corresponds to 50% consolidation [3] (see Fig. 7-2).

4. To obtain the change in thickness of the specimen, subtract the initial dial reading at the beginning of the very first loading from the dial reading corresponding to 100% consolidation for the given loading. Find the change in void ratio (Δe) for the given loading by dividing the change in thickness of the specimen by the height of solid in the specimen. Determine the void ratio (e) for this loading by subtracting the change in void ratio (Δe) from the initial void ratio (e_0).

5. Compute the coefficient of consolidation (c_v) for this loading using the equation

$$c_v = \frac{0.196H^2}{t_{50}} \tag{7-1}$$

where H = thickness of test specimen at 50% consolidation (i.e., initial height of specimen at beginning of test minus deformation dial reading at 50% consolidation). Use half the thickness if the specimen is drained on both top and bottom during the test.

t_{50} = time to 50% consolidation

By using values of e and c_v determined from the various graphs of time versus dial readings corresponding to the different test loadings, two graphs can be prepared—one of void ratio versus pressure (e–log p curve), with pressure along the abscissa on a logarithmic scale and void ratio along the ordinate on an arithmetic scale, and another of consolidation coefficient versus pressure (c_v–log p curve), with pressure along the abscissa on a logarithmic scale and coefficient of consolidation along the ordinate on an arithmetic scale. An example of an e–log p curve is given in Fig. 7-3, and an example of a c_v–log p curve is given in Fig. 7-4. As will be related subsequently, the e–log p curve is used to determine the amount of consolidation settlement, and the c_v–log p curve is used to determine the timing of the consolidation settlement.

In Fig. 7-3, the upper curve exhibits the relationship between void ratio and pressure as the pressure is increased. As will be shown in Sec. 7-5, in the case of overconsolidated clay, it is necessary to have a "rebound curve." Exhibited by the lower curve in Fig. 7-3, the rebound curve is obtained by unloading the soil sample during the consolidation test after the maximum pressure has been reached. As the sample is unloaded, the soil tends to swell, causing movement and associated dial readings to reverse direction.

The primary results of a laboratory consolidation test are (1) the e–log p curve, (2) the c_v–log p curve, and (3) the initial void ratio of the soil *in situ* (e_0).

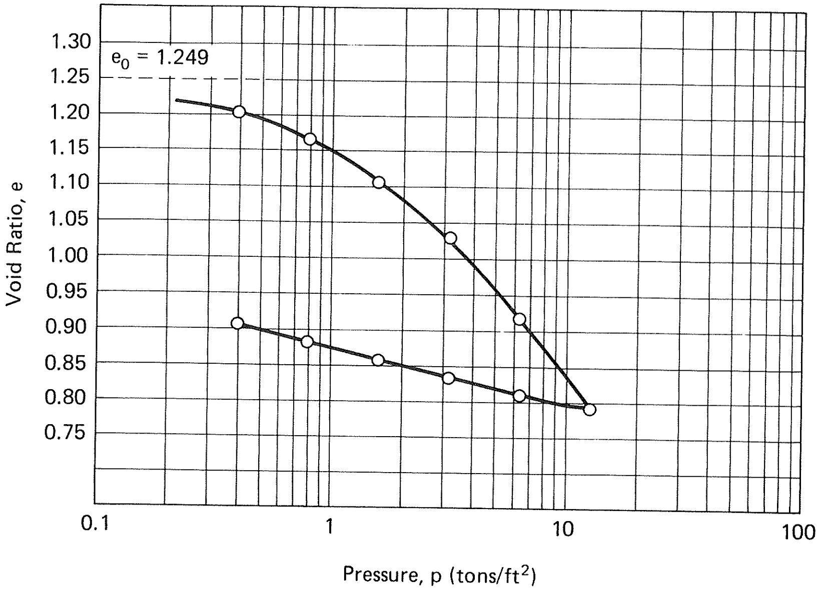


FIGURE 7-3 Void ratio versus log of pressure.

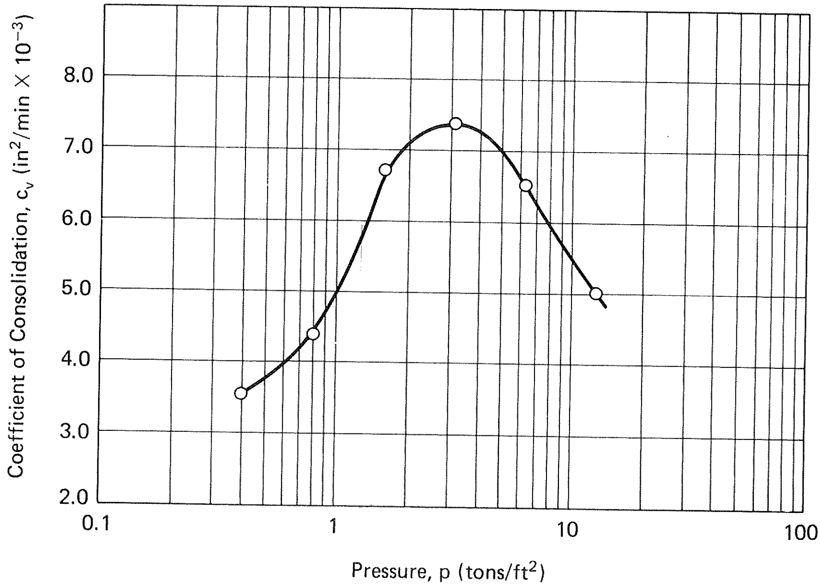


FIGURE 7-4 Coefficient of consolidation versus log of pressure.

7-3 NORMALLY CONSOLIDATED CLAY

As indicated in Sec. 7-1, in the case of a clay soil it is necessary to determine whether the clay is normally consolidated or overconsolidated. This section shows how to determine if a given clay soil is normally consolidated.

It is first necessary, however, to determine the present effective overburden pressure (p_0). This pressure is the result of the (effective) weight of soil above midheight of the consolidating clay layer. Although the reader probably knows how to calculate effective overburden pressure, the procedure is illustrated in Example 7-1.

EXAMPLE 7-1

Given

The soil profile shown in Fig. 7-5.

Required

Present effective overburden pressure (p_0) at midheight of the compressible clay layer.

Solution

$$\text{Elevation of midheight of the clay layer} = \frac{732\text{ ft} + 710\text{ ft}}{2} = 721\text{ ft}$$

$$p_0 = (132\text{ lb/ft}^3)(760\text{ ft} - 752\text{ ft}) + (132\text{ lb/ft}^3 - 62.4\text{ lb/ft}^3)(752\text{ ft} - 732\text{ ft}) + (125.4\text{ lb/ft}^3 - 62.4\text{ lb/ft}^3)(732\text{ ft} - 721\text{ ft})$$

$$p_0 = 3141\text{ lb/ft}^2 = 1.57\text{ tons/ft}^2$$

The first step in determining if a given clayey soil is normally consolidated is to locate the point designated by a pressure of p_0 (distance along the abscissa) and void ratio of e_0 (distance along the ordinate). (p_0 is the present effective overburden pressure at midheight of the compressible clay layer, and e_0 is the initial void ratio of the soil *in situ*.) This point is labeled *a* in Fig. 7-6. The next step is to project the lower right straight-line portion of the *e*–log *p* curve in a straight line upward and to the left. This is the dashed line in Fig.

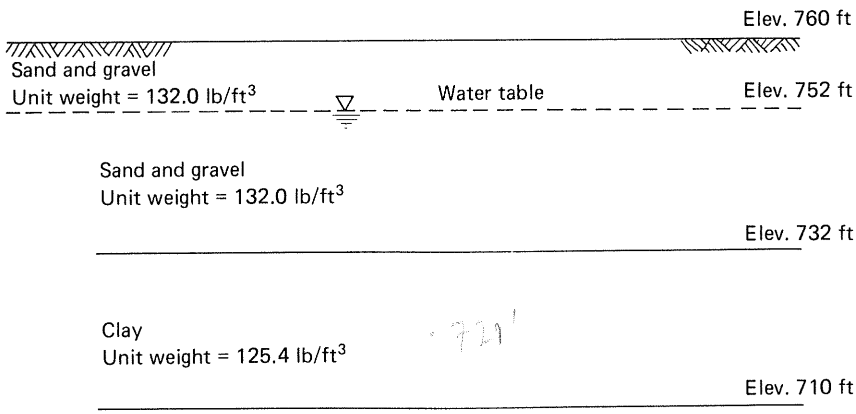


FIGURE 7-5

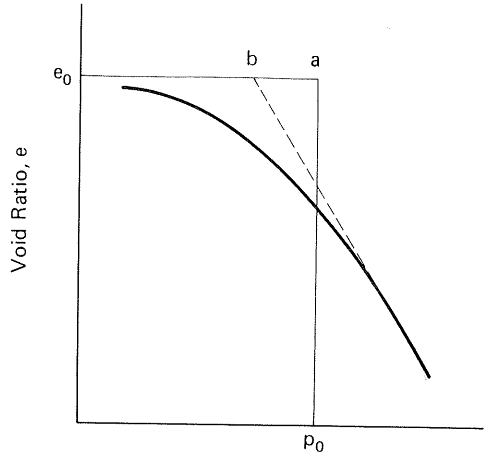


FIGURE 7-6 [4]

7-6; it will intersect a horizontal line drawn at e equal to e_0. The point of intersection of these two lines is labeled b in Fig. 7-6. If point b is to the left of point a (as in Fig. 7-6), the soil is normally consolidated clay [4].

7-4 OVERCONSOLIDATED CLAY

The procedure for determining if a given clay is overconsolidated is essentially the same as that for determining if it is normally consolidated. The point designated by a pressure of p_0 and void ratio of e_0 is located and labeled a. The lower right portion of the e–log p curve is projected in a straight line upward and to the left until it intersects a horizontal line drawn at e equal to e_0, with the point of intersection labeled b. If point b is to the right of point a (as in Fig. 7-7), the soil is overconsolidated clay [4].

If the given clay is found to be overconsolidated, it is necessary to determine (for subsequent analysis of consolidation settlement) the maximum overburden pressure at the consolidated clay layer (p_0'). The following procedure, developed by Casagrande [5], can be used to determine p_0'. The first step is to locate the point on the e–log p curve where the curvature is greatest (where the radius of curvature is smallest). This is indicated by point g in Fig. 7-8. From this point two straight lines are drawn—one horizontal line (line gh in Fig. 7-8) and one tangent to the e–log p curve (line gj in Fig. 7-8). The next step is to draw a line that bisects the angle between lines gh and gj (line gi in Fig. 7-8). The final step is to project the lower right straight-line portion of the e–log p curve in a straight line upward and to the left. This projected line will intersect line gi at a point such as k in Fig. 7-8. The value of p corresponding to point k (p coordinate of point k along the abscissa) is taken as p_0' [4].

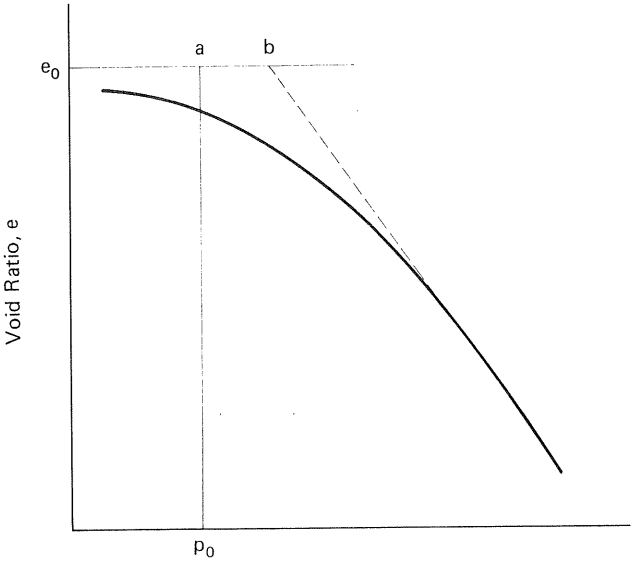


FIGURE 7-7 [4]

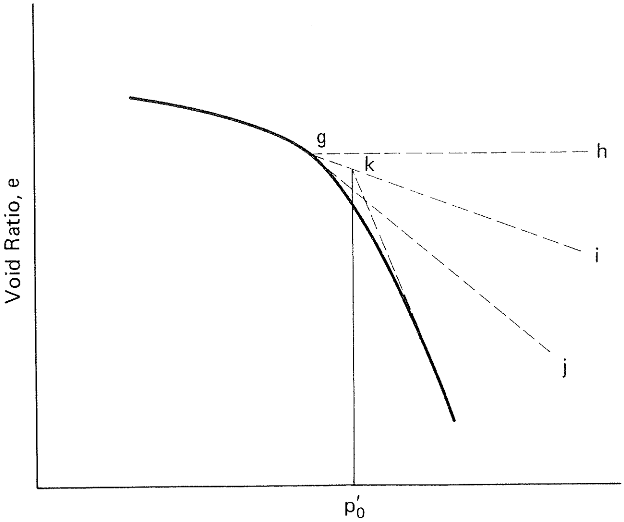


FIGURE 7-8 [4, 5]

7-5 FIELD CONSOLIDATION LINE

The e–log p curves considered in previous sections give, of course, the relationship between void ratio and pressure for a given soil. Such a relationship is used in calculating consolidation settlement. The e–log p curves of Fig. 7-3 and Figs. 7-6 through 7-8 reflect, however, the relationship between void ratio and pressure for the soil sample in the laboratory. Although an "undisturbed sample" is used in the laboratory test, it is not generally possible to duplicate soil in the laboratory exactly as it exists in the field. Thus, the e–log p curves developed from laboratory consolidation tests are modified to give an e–log p curve that is presumed to reflect actual field conditions. This modified e–log p curve is called the *field consolidation line.* Two methods for determining the field consolidation line follow—one for normally consolidated clay, the other for overconsolidated clay.

In the case of normally consolidated clay, determination of the field consolidation line is fairly simple. With the given e–log p curve developed from the laboratory test (Fig. 7-9), the point on the e–log p curve corresponding to $0.4e_0$ is determined (point f in Fig. 7-9). A straight line connecting points a and f gives the field consolidation line for the normally consolidated clay [4, 6]. (The reader will recall that, as related in Sec. 7-3 and Fig. 7-6, point a is the point designated by a pressure of p_0 and void ratio of e_0.)

For overconsolidated clay, finding the field consolidation line is somewhat more difficult. With the given e–log p curve developed from the laboratory test (Fig. 7-10), the point on the e–log p curve corresponding to $0.4e_0$ is determined (point f in Fig. 7-10). Point a (the point designated by a pressure of p_0 and void ratio of e_0) is located, and a line is drawn through point a parallel to the rebound line. This line through point a parallel to the rebound line is shown as a dashed line in Fig. 7-10; it will intersect a vertical line drawn at $p = p'_0$. (The procedure for evaluating p'_0 was given in Sec. 7-4.) This point of intersection is designated by m in Fig. 7-10. Points m and f are connected by a straight line, and points a and m are connected by a curved line that follows

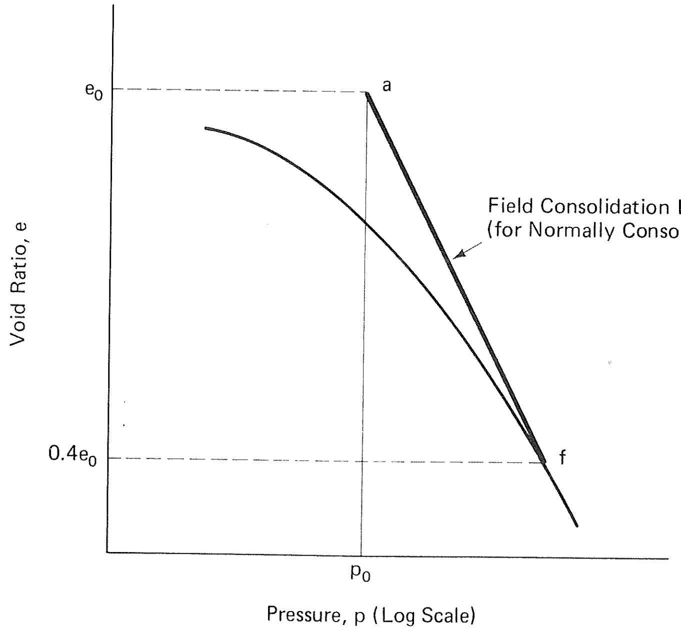


FIGURE 7-9 Field consolidation line for normally consolidated clay. [4]

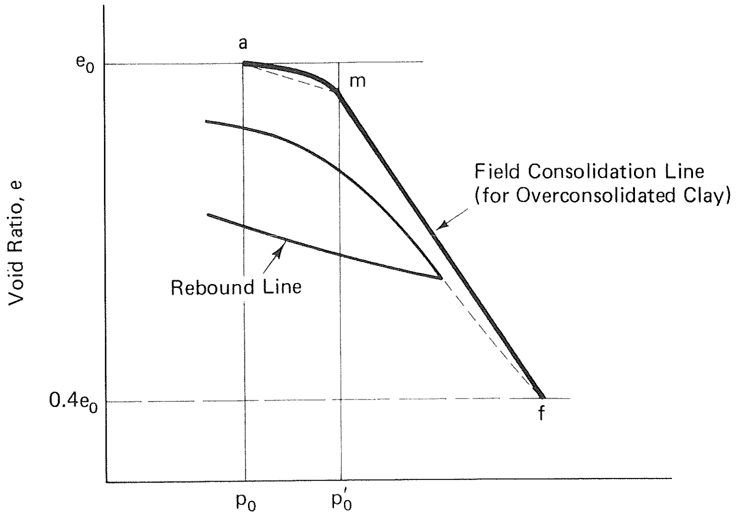


FIGURE 7-10 Field consolidation line for overconsolidated clay. [7, 8]

the same general shape of the e–log p curve. This curved line from a to m and the straight line from m to f give the field consolidation line for the overconsolidated clay [7, 8].

It is the field consolidation line—the dark line in Fig. 7-9 (normally consolidated clay) and Fig. 7-10 (overconsolidated clay)—that is used in calculating consolidation settlement. The other curves (dial readings versus time and e–log p curve) are required only as a means of determining the field consolidation line. Once the field consolidation line is established, these other curves are no longer used in determining the amount of consolidation settlement.

Of special significance is the *slope* of the field consolidation line. This slope is called the *compression index* (C_c) and may be evaluated by finding coordinates of any two points on the field consolidation line [(p_1, e_1) and (p_2, e_2)] and substituting these values into the equation [4]

$$C_c = \frac{e_1 - e_2}{\log p_2 - \log p_1} = \frac{e_1 - e_2}{\log (p_2/p_1)} \tag{7-2}$$

Skempton has shown that the compression index can be approximated in terms of the liquid limit (LL, in percent) by the equation [4, 9]

$$C_c = 0.009(LL - 10) \tag{7-3}$$

for normally consolidated clays.

It should be emphasized that the value of C_c computed from Eq. (7-2) is obtained from the field consolidation line, which is based on the results of a consolidation test, while that computed from Eq. (7-3) is based solely on liquid limit. The consolidation test is much more lengthy, difficult, and expensive to perform than the test to determine liquid limit. Also, calculation of C_c using results of a consolidation test is much more involved than calculation using liquid limit. However, calculation of C_c using liquid limit [Eq. (7-3)] is merely an approximation and should be used only when very rough values of settlement are acceptable (such as in a preliminary design).

EXAMPLE 7-2

Given

When the total pressure acting at midheight of a consolidating clay layer is 200 kN/m², the corresponding void ratio of the clay is 0.98. When the total pressure acting at the same location is 500 kN/m², the corresponding void ratio decreases to 0.81.

Required

The void ratio of the clay if the total pressure acting at midheight of the consolidating clay layer is 1000 kN/m².

Solution

From Eq. (7-2),

$$C_c = \frac{e_1 - e_2}{\log (p_2/p_1)} \tag{7-2}$$

$$e_1 = 0.98$$

$$e_2 = 0.81$$

$$p_2 = 500 \text{ kN/m}^2$$

$$p_1 = 200 \text{ kN/m}^2$$

$$C_c = \frac{0.98 - 0.81}{\log \left(\dfrac{500 \text{ kN/m}^2}{200 \text{ kN/m}^2}\right)} = 0.427$$

Substituting the computed valued of C_c and the same values of e_1 and p_1 into Eq. (7-2) gives

$$0.427 = \frac{0.98 - e_2}{\log \left(\dfrac{1000 \text{ kN/m}^2}{200 \text{ kN/m}^2}\right)}$$

$$e_2 = 0.68$$

EXAMPLE 7-3

Given

A normally consolidated clay has a liquid limit of 51.2%.

Required

Estimate the compression index (C_c).

Solution

From Eq. (7-3),

$$C_c = 0.009(LL - 10) \tag{7-3}$$

$$C_c = 0.009(51.2 - 10) = 0.371$$

7-6 SETTLEMENT OF LOADS ON CLAY DUE TO PRIMARY CONSOLIDATION

Once the field consolidation line has been defined for a given clayey soil, total expected primary consolidation settlement of load on the clay can be determined. Consider Fig. 7-11, where a mass of soil is depicted before (Fig. 7-11a) and after (Fig. 7-11b) consolidation settlement has occurred in a clay layer of initial thickness H. From Fig. 7-11,

$$\frac{\Delta H}{H} = \frac{\Delta H}{H_s + (H_v)_0} \tag{7-4}$$

where ΔH represents the amount of clay settlement and H_s and $(H_v)_0$ denote the height of solids and initial height of voids, respectively. By definition, the initial void ratio, e_0 in Fig. 7-11b, is given by

$$e_0 = \frac{(V_v)_0}{V_s} \tag{7-5}$$

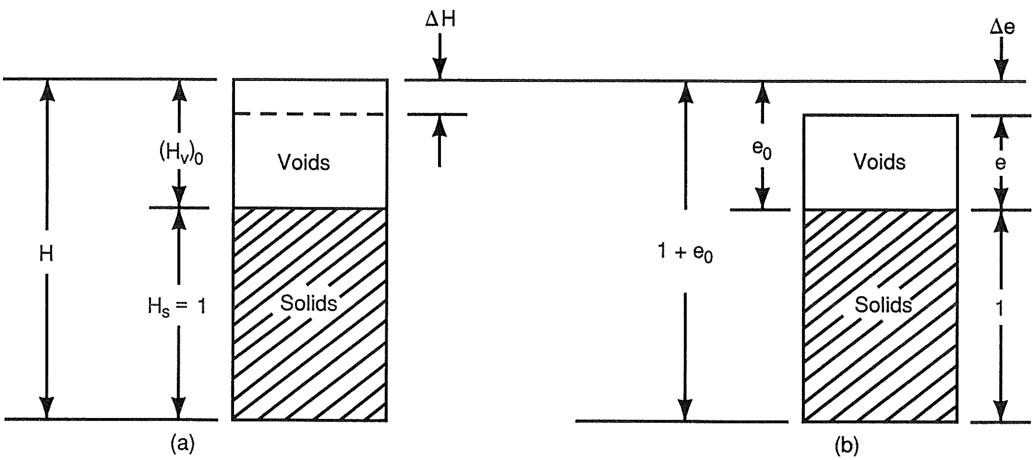


FIGURE 7-11 Settlement of a mass of soil.

where $(V_v)_0$ and V_s represent the original volume of voids and the volume of solids, respectively. Since each volume can be replaced by the soil's cross-sectional area (A) times height of soil, this equation can be modified as follows:

$$e_0 = \frac{(A)(H_v)_0}{(A)(H_s)} = \frac{(H_v)_0}{H_s} \tag{7-6}$$

Also

$$\Delta e = \frac{\Delta H}{H_s} \tag{7-7}$$

where Δe represents the change in void ratio as a result of consolidation settlement. If we let the height of solids (H_s) equal unity, then Eqs. (7-4), (7-6), and (7-7) become

$$\frac{\Delta H}{H} = \frac{\Delta H}{1 + (H_v)_0} \tag{7-8}$$

$$e_0 = (H_v)_0 \tag{7-9}$$

$$\Delta e = \Delta H \tag{7-10}$$

Substituting Eqs. (7-9) and (7-10) into Eq. (7-8) yields

$$\frac{\Delta H}{H} = \frac{\Delta e}{1 + e_0} \tag{7-11}$$

or

$$\Delta H = \frac{\Delta e}{1 + e_0}[H] \tag{7-12}$$

Since $\Delta e = e_0 - e$ and $\Delta H =$ settlement S,

$$S = \frac{e_0 - e}{1 + e_0}[H] \tag{7-13}$$

where S = total settlement due to primary consolidation
e_0 = initial void ratio of the soil *in situ*
e = void ratio of the soil corresponding to the total pressure (p) acting at midheight of the consolidating clay layer
H = thickness of the consolidating clay layer

In practice, the value of e_0 is obtained from the laboratory consolidation test, and the value of e is obtained from the field consolidation line based on total pressure (i.e., effective overburden pressure plus net additional pressure due to structure—both at midheight of the consolidating clay layer). The value of H is obtained from soil exploration (Chap. 3).

An alternative equation for computing total expected consolidation settlement using the compression index (i.e., slope of the field consolidation line) can be derived by recalling Eq. (7-2):

$$C_c = \frac{e_1 - e_2}{\log (p_2/p_1)} \tag{7-2}$$

Since (p_1, e_1) and (p_2, e_2) can be the coordinates of any two points on the field consolidation line, let

p_1 = present effective overburden pressure at midheight of the consolidating clay layer (i.e., p_0)

e_1 = initial void ratio of the soil *in situ* [i.e., e_0 in Eq. (7-13)]

p_2 = total pressure acting at midheight of the consolidating clay layer [$p_0 + \Delta p$ (i.e., p)]

e_2 = void ratio of the soil corresponding to the total pressure (p) acting at midheight of the consolidating clay layer [i.e., e in Eq. (7-13)]

Making these substitutions in Eq. (7-2) gives

$$C_c = \frac{e_0 - e}{\log (p/p_0)} \tag{7-14}$$

Rearranging this equation,

$$e_0 - e = C_c[\log (p/p_0)] \tag{7-15}$$

Substituting Eq. (7-15) into Eq. (7-13),

$$S = \frac{C_c[\log (p/p_0)]}{1 + e_0}[H] \tag{7-16}$$

or

$$S = C_c\left(\frac{H}{1 + e_0}\right) \log \frac{p}{p_0} \tag{7-17}$$

where C_c = slope of the field consolidation line (compression index)

p = total pressure acting at midheight of the consolidating clay layer ($= p_0 + \Delta p$)

p_0 = present effective overburden pressure at midheight of the consolidating clay layer

Δp = net additional pressure at midheight of the consolidating clay layer due to structure

The value of C_c can be determined by evaluating the slope of the field consolidation line [Eq. (7-2)] or approximated based on liquid limit [Eq. (7-3)]. If the latter method is used, computed settlement should be considered as a rough approximation.

The time rate of settlement due to consolidation can be computed from the equation [1, 7]

$$t = \frac{T_v}{c_v} H^2 \tag{7-18}$$

where t = time to reach a particular percent of consolidation; percent of consolidation is defined as the ratio of the amount of settlement at a certain time during the process of consolidation to the total settlement due to consolidation

T_v = time factor, a coefficient depending on the particular percent of consolidation

c_v = coefficient of consolidation corresponding to the total pressure ($p = p_0 + \Delta p$) acting at midheight of the clay layer

H = thickness of the consolidating clay layer [however, if the clay layer *in situ* is drained on both top and bottom, half the thickness of the layer should be substituted for H in Eq. (7-18)]

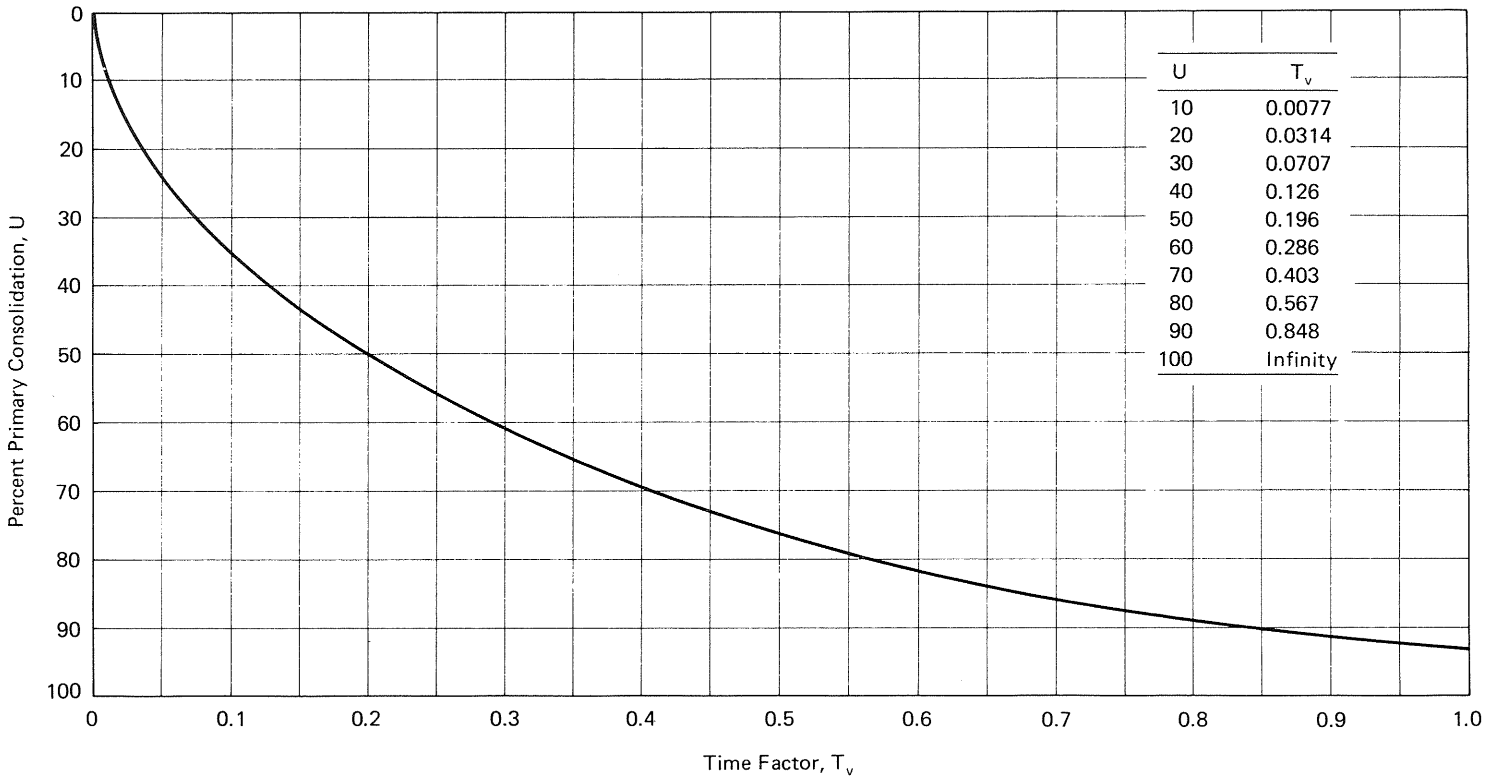
In practice, the value of T_v is determined from Fig. 7-12, based on the desired percent of consolidation (U), and the value of c_v is determined from the c_v–log p curve (e.g., Fig. 7-4) based on the total pressure acting at midheight of the clay layer. It will be recalled that the c_v–log p curve is a product of the laboratory consolidation test.

In summary, either Eq. (7-13) or Eq. (7-17) can be used to compute total settlement due to consolidation. Then Eq. (7-18) can be used to find the time required to reach a particular percentage of that consolidation. For example, if total settlement due to consolidation is computed to be 3.0 in., the time required for the structure to settle 1.5 in. could be determined from Eq. (7-18) by substituting a value of T_v of 0.196 (along with applicable values of c_v and H). The value of 0.196 is obtained from Fig. 7-12 for a value of U of 50%. U is 50% because the particular settlement being considered (1.5 in.) is 50% of total settlement (3.0 in.).

EXAMPLE 7-4

Given

A compressible normally consolidated clay layer is 7.40 m thick and has an initial void ratio *in situ* of 0.988. Consolidation tests and subsequent compu-



U	T_v
10	0.0077
20	0.0314
30	0.0707
40	0.126
50	0.196
60	0.286
70	0.403
80	0.567
90	0.848
100	Infinity

FIGURE 7-12 Time factor as a function of percentage of consolidation. [1]

tations indicate that the void ratio of the clay layer corresponding to the total pressure acting at midheight of the consolidating clay layer after construction of a building is 0.942.

Required

Expected consolidation settlement.

Solution

From Eq. (7-13)

$$S = \frac{e_0 - e}{1 + e_0}[H] \tag{7-13}$$

$$e_0 = 0.988$$

$$e = 0.942$$

$$H = 7.40 \text{ m}$$

$$S = \frac{0.988 - 0.942}{1 + 0.988}[7.40 \text{ m}] = 0.171 \text{ m}$$

EXAMPLE 7-5

Given

1. A sample of normally consolidated clay was obtained by a Shelby tube sampler from the midheight of a compressible clay layer (see Fig. 7-13).
2. A consolidation test was conducted on a portion of this sample. Results of the consolidation test are as follows:
   (a) Natural (or initial) void ratio of the clay existing in the field (e_0) is 1.65.
   (b) Pressure–void ratio relations are as follows:

p (*tons/ft*²)	*e*
0.8	1.50
1.6	1.42
3.2	1.30
6.4	1.12
12.8	0.94

3. A footing is to be located 5 ft below ground level, as shown in Fig. 7-13. The base of the square footing is 10 ft by 10 ft and it exerts a total load of 250 tons, which includes column load, weight of footing, and weight of soil surcharge on the footing.

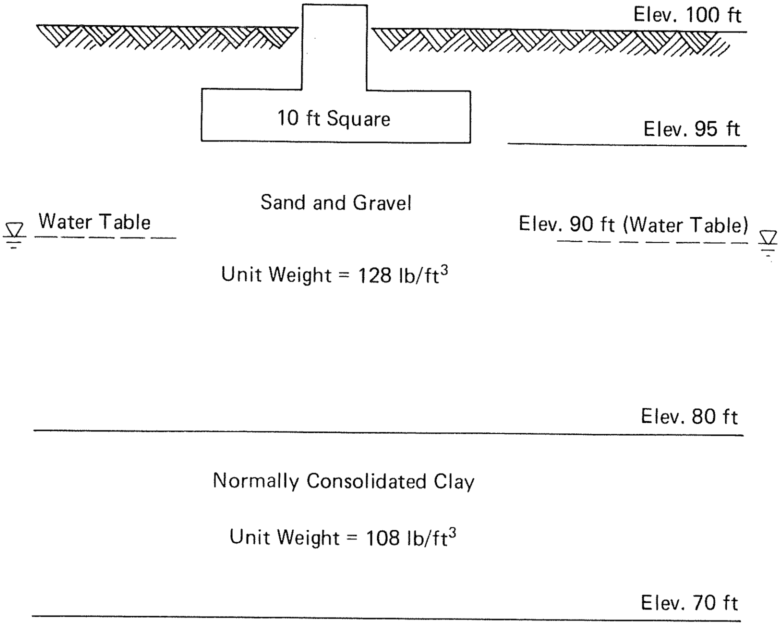


FIGURE 7-13

Required

1. From results of the consolidation test given above, prepare an e–log p curve and construct a field consolidation line, assuming that point f is located at $0.4e_0$.
2. Compute total expected consolidation settlement for the clay layer.

Solution

1. Present effective overburden pressure (p_0) at midheight of clay layer

$$= (128 \text{ lb/ft}^3)(100 \text{ ft} - 90 \text{ ft}) + (128 \text{ lb/ft}^3 - 62.4 \text{ lb/ft}^3) \times (90 \text{ ft} - 80 \text{ ft}) + (108 \text{ lb/ft}^3 - 62.4 \text{ lb/ft}^3)\left(\frac{80 \text{ ft} - 70 \text{ ft}}{2}\right)$$

$$p_0 = 2164 \text{ lb/ft}^2, \text{ or } 1.08 \text{ tons/ft}^2$$

$$e_0 = 1.65 \text{ (given)}$$

$$0.4e_0 = (0.4)(1.65) = 0.66$$

The e–log p curve is shown in Fig. 7-14 together with the field consolidation line.

2. Effective weight of excavation = (128 lb/ft³)(5 ft) = 640 lb/ft², or 0.32 ton/ft². Net consolidation pressure at the base of footing

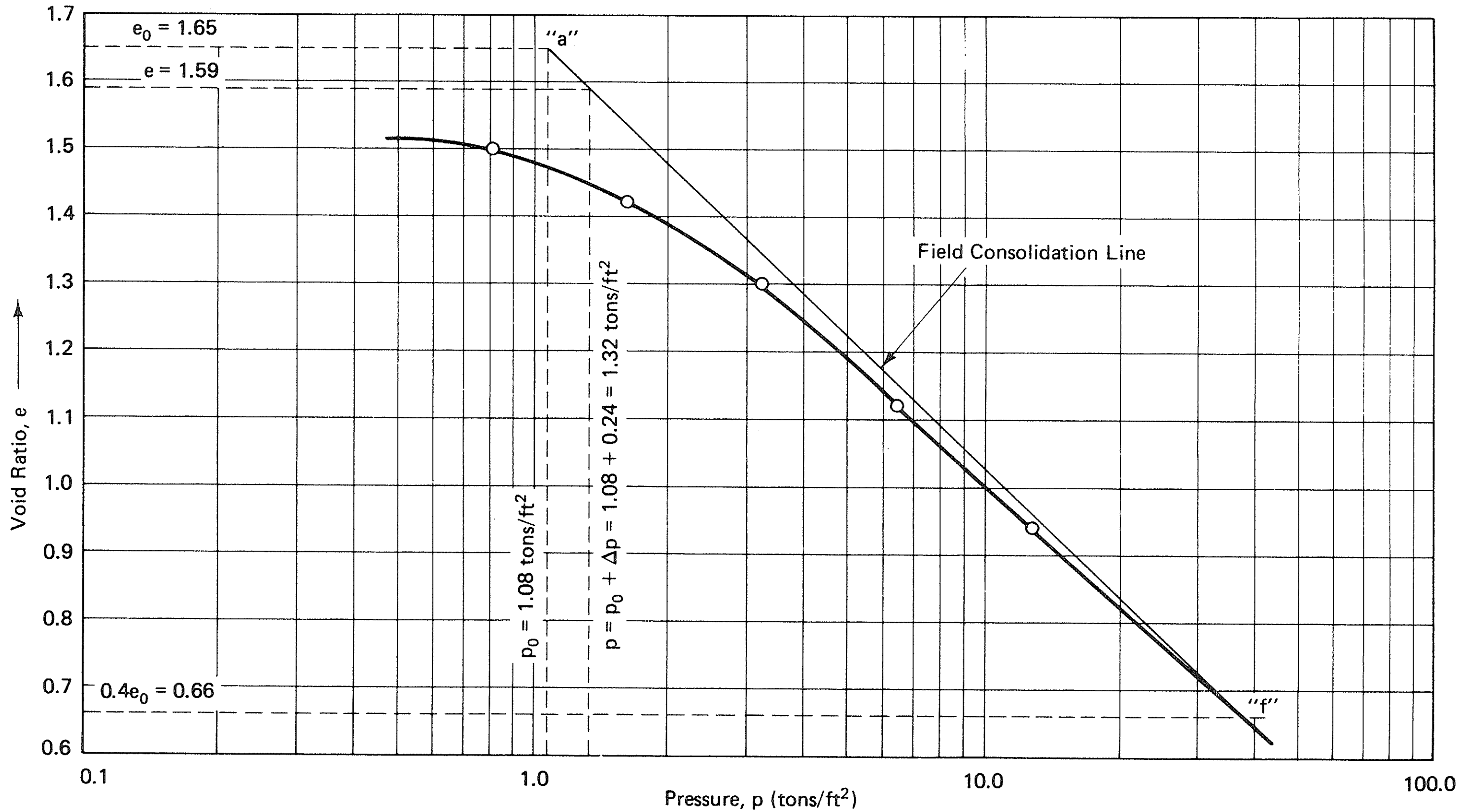


FIGURE 7-14 *e*–log *p* curve for Example 7-5.

$$= \frac{250 \text{ tons}}{(10 \text{ ft})(10 \text{ ft})} - 0.32 \text{ ton/ft}^2 = 2.18 \text{ tons/ft}^2$$

To determine net consolidation pressure at midheight of the clay layer under the center of the footing, it is necessary to divide the base of the footing into four equal 5-ft by 5-ft square areas. Since each of these square areas has a common corner at the footing's center, desired net consolidation pressure at midheight of the clay layer can be calculated upon determining an influence coefficient using either Table 6-2 or Fig. 6-7. Referring to Fig. 6-7, we see that

$$mz = 5 \text{ ft} \qquad z = 95 \text{ ft} - \frac{80 \text{ ft} + 70 \text{ ft}}{2} = 20 \text{ ft}$$

$$m = \frac{5 \text{ ft}}{20 \text{ ft}} = 0.25$$

$$nz = 5 \text{ ft} \qquad z = 20 \text{ ft} \qquad n = \frac{5 \text{ ft}}{20 \text{ ft}} = 0.25$$

From Fig. 6-7, the influence coefficient = 0.027. Net consolidation pressure at midheight of the clay layer under the center of the footing

$$\Delta p = (4)(0.027)(2.18 \text{ tons/ft}^2) = 0.24 \text{ ton/ft}^2$$

Final pressure at midheight of clay layer

$$p = p_0 + \Delta p = 1.08 \text{ tons/ft}^2 + 0.24 \text{ ton/ft}^2 = 1.32 \text{ tons/ft}^2$$

Enter $p = 1.32$ tons/ft² along the abscissa of the e–log p curve (Fig. 7-14) and move upward vertically until the "field consolidation line" is intersected. Then turn left and move horizontally to read a void ratio e of 1.59 on the ordinate of the e–log p curve. With

$$e_0 = 1.65$$
$$e = 1.59$$
$$H = 10 \text{ ft} = 120 \text{ in.}$$

substitute into Eq. (7-13):

$$S = \frac{e_0 - e}{1 + e_0}[H] \tag{7-13}$$

$$S = \frac{1.65 - 1.59}{1 + 1.65}[120 \text{ in.}] = 2.72 \text{ in.}$$

Total expected consolidation settlement is 2.72 in.

EXAMPLE 7-6

Given

1. Same as Example 7-5; total consolidation settlement = 2.72 in.
2. Results of the laboratory consolidation test also indicated that the coefficient of consolidation (c_v) for the clay sample is 3.28×10^{-3} in.^{2}/min for the pressure increment from 0.8 to 1.6 tons/ft^2.

Required

Time of consolidation settlement if the clay layer is underlain by:

1. Permeable sand and gravel (double drainage).
2. Impermeable bedrock (single drainage).

Take U at 10% increments and plot these values on a settlement–log time curve.

Solution

1. *Clay layer is underlain by permeable sand and gravel (double drainage).* Use Eq. (7-18):

$$t = \frac{T_v}{c_v} H^2 \tag{7-18}$$

where $c_v = 3.28 \times 10^{-3}$ in.^{2}/min

$$H = \frac{10}{2}\text{ ft} = 5\text{ ft} = 60\text{ in. (double drainage)}$$

(a) When $U = 10\%$ (i.e., 10% of total settlement, $S_{10} = 2.72\text{ in.} \times 0.10 = 0.27\text{ in.}$),

$$T_v = 0.0077 \text{ (from Fig. 7-12)}$$

$$t_{10} = \frac{(0.0077)(60\text{ in.})^2}{3.28 \times 10^{-3}\text{ in.}^2/\text{min}} = 8451\text{ min} = 0.016\text{ yr}$$

This indicates the footing will settle approximately 0.27 in. in 0.016 yr.

(b) When $U = 20\%$ (i.e., 20% of total settlement, $S_{20} = 0.54$ in.),

$$T_v = 0.0314 \text{ (from Fig. 7-12)}$$

$$t_{20} = \frac{(0.0314)(60\text{ in.})^2}{3.28 \times 10^{-3}\text{ in.}^2/\text{min}} = 34{,}463\text{ min} = 0.066\text{ yr}$$

This indicates the footing will settle approximately 0.54 in. in 0.066 yr.

(c) When $U = 30\%$ (i.e., 30% of total settlement),

$$T_v = 0.0707 \text{ (from Fig. 7-12)}$$

$$t_{30} = \frac{(0.0707)(60 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 77{,}598 \text{ min} = 0.15 \text{ yr}$$

(d) When $U = 40\%$ (i.e., 40% of total settlement),

$$T_v = 0.126 \text{ (from Fig. 7-12)}$$

$$t_{40} = \frac{(0.126)(60 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 1.383 \times 10^5 \text{ min} = 0.26 \text{ yr}$$

(e) When $U = 50\%$ (i.e., 50% of total settlement),

$$T_v = 0.196 \text{ (from Fig. 7-12)}$$

$$t_{50} = \frac{(0.196)(60 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 2.151 \times 10^5 \text{ min} = 0.41 \text{ yr}$$

(f) When $U = 60\%$ (i.e., 60% of total settlement),

$$T_v = 0.286 \text{ (from Fig. 7-12)}$$

$$t_{60} = \frac{(0.286)(60 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 3.139 \times 10^5 \text{ min} = 0.60 \text{ yr}$$

(g) When $U = 70\%$ (i.e., 70% of total settlement),

$$T_v = 0.403 \text{ (from Fig. 7-12)}$$

$$t_{70} = \frac{(0.403)(60 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 4.423 \times 10^5 \text{ min} = 0.84 \text{ yr}$$

(h) When $U = 80\%$ (i.e., 80% of total settlement),

$$T_v = 0.567 \text{ (from Fig. 7-12)}$$

$$t_{80} = \frac{(0.567)(60 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 6.223 \times 10^5 \text{ min} = 1.18 \text{ yr}$$

(i) When $U = 90\%$ (i.e., 90% of total settlement),

$$T_v = 0.848 \text{ (from Fig. 7-12)}$$

$$t_{90} = \frac{(0.848)(60 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 9.307 \times 10^5 \text{ min} = 1.77 \text{ yr}$$

2. *Clay layer is underlain by impermeable bedrock (single drainage).* Eq. (7-18) is still applicable.

$$t = \frac{T_v}{c_v} H^2 \tag{7-18}$$

where $c_v = 3.28 \times 10^{-3}$ in.^{2}/min
H = 10 ft = 120 in. (single drainage)

(a) When $U = 10\%$,

$$T_v = 0.0077$$

$$t_{10} = \frac{(0.0077)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 33{,}805 \text{ min} = 0.064 \text{ yr}$$

(b) When $U = 20\%$,

$$T_v = 0.0314$$

$$t_{20} = \frac{(0.0314)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 1.379 \times 10^5 \text{ min} = 0.26 \text{ yr}$$

(c) When $U = 30\%$,

$$T_v = 0.0707$$

$$t_{30} = \frac{(0.0707)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 3.104 \times 10^5 \text{ min} = 0.59 \text{ yr}$$

(d) When $U = 40\%$,

$$T_v = 0.126$$

$$t_{40} = \frac{(0.126)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 5.532 \times 10^5 \text{ min} = 1.05 \text{ yr}$$

(e) When $U = 50\%$,

$$T_v = 0.196$$

$$t_{50} = \frac{(0.196)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 8.605 \times 10^5 \text{ min} = 1.64 \text{ yr}$$

(f) When $U = 60\%$,

$$T_v = 0.286$$

$$t_{60} = \frac{(0.286)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 1.256 \times 10^6 \text{ min} = 2.39 \text{ yr}$$

TABLE 7-1 Computed time–settlement relation

Fraction of Total Consolidation Settlement, U (%)	*Consolidation Settlement (in.)*	*TIME (YR)*	
		Double Drainage	*Single Drainage*
10	0.27	0.016	0.064
20	0.54	0.066	0.26
30	0.82	0.15	0.59
40	1.09	0.26	1.05
50	1.36	0.41	1.64
60	1.63	0.60	2.39
70	1.90	0.84	3.37
80	2.18	1.18	4.74
90	2.45	1.77	7.08
100	2.72	∞	∞

(g) When $U = 70\%$,

$$T_v = 0.403$$

$$t_{70} = \frac{(0.403)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 1.769 \times 10^6 \text{ min} = 3.37 \text{ yr}$$

(h) When $U = 80\%$,

$$T_v = 0.567$$

$$t_{80} = \frac{(0.567)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 2.489 \times 10^6 \text{ min} = 4.74 \text{ yr}$$

(i) When $U = 90\%$,

$$T_v = 0.848$$

$$t_{90} = \frac{(0.848)(120 \text{ in.})^2}{3.28 \times 10^{-3} \text{ in.}^2/\text{min}} = 3.723 \times 10^6 \text{ min} = 7.08 \text{ yr}$$

The results of these computations are tabulated in Table 7-1 and are shown graphically by a settlement–log time curve in Fig. 7-15.

EXAMPLE 7-7

Given

1. An 8-ft clay layer beneath a building is overlain by a stratum of permeable sand and gravel and is underlain by impermeable bedrock.

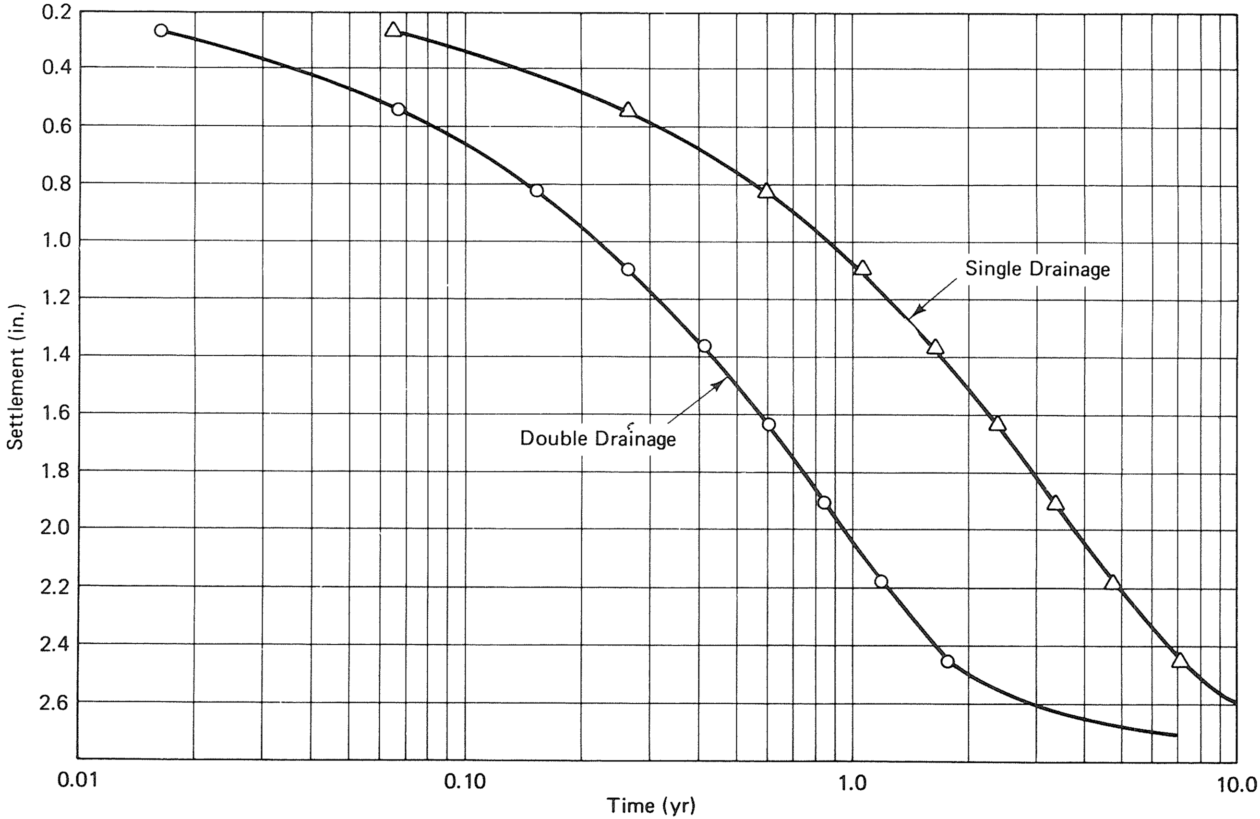


FIGURE 7-15 Settlement–log time curves for Examples 7-5 and 7-6.

2. The total expected consolidation settlement for the clay layer due to the footing load is 2.50 in.
3. The coefficient of consolidation (c_v) is 2.68×10^{-3} in.^{2}/min.

Required

1. How many years will it take for 90% of the total expected consolidation settlement to take place?
2. Compute the amount of consolidation settlement that will occur in 1 yr.
3. How many years will it take for consolidation settlement of 1 in. to take place?

Solution

1. From Eq. (7-18),

$$t = \frac{T_v}{c_v} H^2 \qquad (7\text{-}18)$$

$$T_v = 0.848 \text{ (for } U = 90\%\text{; see Fig. 7-12)}$$

$$c_v = 2.68 \times 10^{-3} \text{ in.}^2\text{/min (given)}$$

$$H = 8 \text{ ft} = 96 \text{ in. (single drainage)}$$

$$t_{90} = \frac{(0.848)(96 \text{ in.})^2}{2.68 \times 10^{-3} \text{ in.}^2\text{/min}} = 2.916 \times 10^6 \text{ min} = 5.55 \text{ yr}$$

2. From Eq. (7-18),

$$t = \frac{T_v}{c_v} H^2 \qquad (7\text{-}18)$$

$$t = 1 \text{ yr}$$

$$c_v = 2.68 \times 10^{-3} \text{ in.}^2\text{/min}$$

$$H = 8 \text{ ft} = 96 \text{ in.}$$

$$1 \text{ yr} = \frac{T_v}{2.68 \times 10^{-3} \text{ in.}^2\text{/min}} (96 \text{ in.})^2 \times \frac{1}{(60 \text{ min/hr})(24 \text{ hr/day})(365 \text{ days/yr})}$$

$$T_v = 0.15$$

From Fig. 7-12, with $T_v = 0.15$, $U = 43\%$.

Amount of consolidation settlement that will occur in 1 yr

= total consolidation settlement multiplied by $U\%$

= (2.50 in.)(0.43) = 1.08 in.

3. $U\%$ = fraction of total settlement

$$U = \frac{1 \text{ in.}}{2.50 \text{ in.}} \times 100 = 40\%$$

From Fig. 7-12, with $U = 40\%$, $T_v = 0.126$.

From Eq. (7-18),

$$t = \frac{T_v}{c_v} H^2 \tag{7-18}$$

$$t = \frac{(0.126)(96 \text{ in.})^2}{2.68 \times 10^{-3} \text{ in.}^2/\text{min}} = 4.333 \times 10^5 \text{ min} = 0.82 \text{ yr}$$

EXAMPLE 7-8

Given

1. A foundation is to be constructed at a site where the soil profile is as shown in Fig. 7-16.
2. The base of the foundation is 3 m by 6 m, and it exerts a total load of 5400 kN, which includes the weight of the structure, foundation, and soil surcharge on the foundation.
3. The initial void ratio *in situ* (e_0) of the compressible clay layer is 1.38.
4. The compression index (C_c) of the clay layer is 0.68.

Required

Expected consolidation settlement of the clay layer.

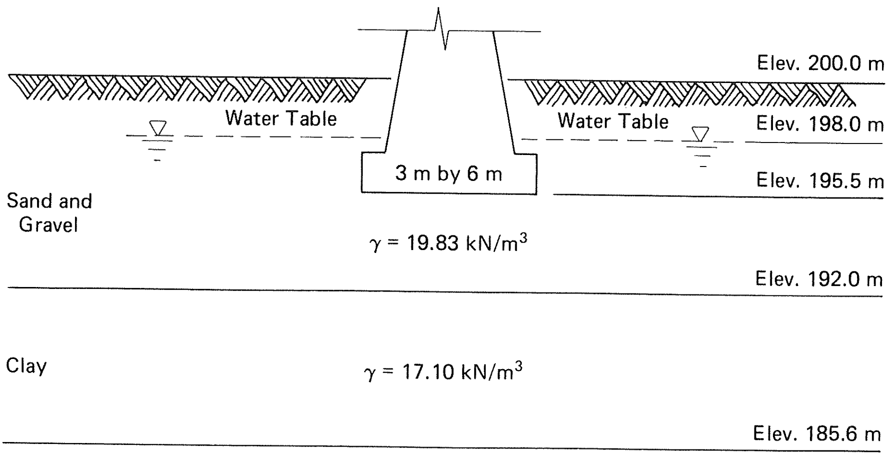


FIGURE 7-16

Solution

Present effective overburden pressure (p_0) at midheight of the clay layer

$$= (19.83\ \text{kN/m}^3)(200.0\ \text{m} - 198.0\ \text{m}) + (19.83\ \text{kN/m}^3 - 9.81\ \text{kN/m}^3)$$

$$\times (198.0\ \text{m} - 192.0\ \text{m}) + (17.10\ \text{kN/m}^3 - 9.81\ \text{kN/m}^3)$$

$$\times \left(\frac{192.0\ \text{m} - 185.6\ \text{m}}{2}\right) = 123.1\ \text{kN/m}^2$$

Effective weight of excavation $= (19.83\ \text{kN/m}^3)(200.0\ \text{m} - 198.0\ \text{m})$
$+ (19.83\ \text{kN/m}^3 - 9.81\ \text{kN/m}^3)(198.0\ \text{m} - 195.5\ \text{m}) = 64.7\ \text{kN/m}^2$

Net consolidation pressure at the foundation's base =

$$\frac{5400\ \text{kN}}{(3\ \text{m})(6\ \text{m})} - 64.7\ \text{kN/m}^2 = 235.3\ \text{kN/m}^2$$

To determine net consolidation pressure at midheight of the clay layer under the center of the foundation, it is necessary to divide the foundation's base into four equal 1.5 m by 3.0 m rectangular areas. Since each of these areas has a common corner at the foundation's center, desired net consolidation pressure at midheight of the clay layer can be calculated by determining an influence coefficient using either Table 6-2 or Fig. 6-7.

$$mz = 1.5\ \text{m} \qquad nz = 3.0\ \text{m}$$

$$z = 195.5\ \text{m} - \frac{192.0\ \text{m} + 185.6\ \text{m}}{2} = 6.7\ \text{m}$$

$$m = \frac{1.5\ \text{m}}{6.7\ \text{m}} = 0.224 \qquad n = \frac{3.0\ \text{m}}{6.7\ \text{m}} = 0.448$$

From Fig. 6-7, the influence coefficient is 0.04. Therefore,

Net consolidation pressure at midheight of the clay layer under the center of the foundation (Δp) $= (4)(0.04)(235.3\ \text{kN/m}^2) = 37.6\ \text{kN/m}^2$

Final pressure at midheight of the clay layer (p) $= p_0 + \Delta p$

$$= 123.1\ \text{kN/m}^2 + 37.6\ \text{kN/m}^2 = 160.7\ \text{kN/m}^2$$

From Eq. (7-17)

$$S = C_c\left(\frac{H}{1 + e_0}\right)\log\frac{p}{p_0} \qquad (7\text{-}17)$$

$C_c = 0.68$ (given)

$H = 192.0\ \text{m} - 185.6\ \text{m} = 6.4\ \text{m}$

$$e_0 = 1.38 \text{ (given)}$$
$$p = 160.7 \text{ kN/m}^2$$
$$p_0 = 123.1 \text{ kN/m}^2$$
$$S = (0.68)\left(\frac{6.4 \text{ m}}{1 + 1.38}\right) \log\left(\frac{160.7 \text{ kN/m}^2}{123.1 \text{ kN/m}^2}\right) = 0.212 \text{ m}$$

EXAMPLE 7-9

Given

1. Same data as for Example 7-8, including the computed consolidation settlement of 0.212 m.
2. Coefficient of consolidation (c_v) is 4.96×10^{-6} m²/min.

Required

How long will it take for half the expected consolidation settlement to take place if the clay layer is underlain by:

1. Permeable sand and gravel?
2. Impermeable bedrock?

Solution

1. *Clay layer underlain by permeable sand and gravel:*
   From Eq. (7-18),

$$t = \frac{T_v}{c_v} H^2 \tag{7-18}$$

From Fig. 7-12, for $U = 50\%$, $T_v = 0.196$.

$$H = \frac{192.0 \text{ m} - 185.6 \text{ m}}{2} = 3.2 \text{ m}$$

$$t_{50} = \left(\frac{0.196}{4.96 \times 10^{-6} \text{ m}^2/\text{min}}\right)(3.2 \text{ m})^2 = 404{,}645 \text{ min, or } 0.77 \text{ yr}$$

2. *Clay layer underlain by impermeable bedrock:*
   Equation (7-18) is still applicable with $T_v = 0.196$ and $c_v = 4.96 \times 10^{-6}$ m²/min, but with $H = 192.0 \text{ m} - 185.6 \text{ m} = 6.4 \text{ m}$.

$$t_{50} = \left(\frac{0.196}{4.96 \times 10^{-6} \text{ m}^2/\text{min}}\right)(6.4 \text{ m})^2 = 1{,}618{,}581 \text{ min, or } 3.08 \text{ yr}$$

7-7 SETTLEMENT OF LOADS ON CLAY DUE TO SECONDARY COMPRESSION

After primary consolidation has ended (i.e., all water has been extruded from the voids in a fine-grained soil) and all primary consolidation settlement has occurred, soil compression (and additional associated settlement) continues very slowly at a decreasing rate. This phenomenon is known as *secondary compression* and perhaps results from plastic readjustment of soil grains due to new stresses in the soil and progressive breaking of clayey particles and their interparticle bonds.

Figure 7-17 gives a plot of void ratio as a function of the logarithm of time. Clearly, as void ratio decreases, settlement increases. Secondary compression begins immediately after primary consolidation ends; it appears in Fig. 7-17 as a straight line with a relatively flat slope. The void ratio corresponding to the end of primary consolidation (or the beginning of secondary compression) can be determined graphically as the point of intersection of the secondary compression line extended backward and a line tangent to the primary consolidation curve (i.e., point A in Fig. 7-17).

Secondary compression settlement can be computed from the equation [10]

$$S_s = C_\alpha H \log \frac{t_s}{t_p} \tag{7-19}$$

where
- S_s = secondary compression settlement
- C_α = coefficient of secondary compression
- H = (initial) thickness of the clay layer
- t_s = life of the structure (or time for which settlement is required)
- t_p = time to completion of primary consolidation

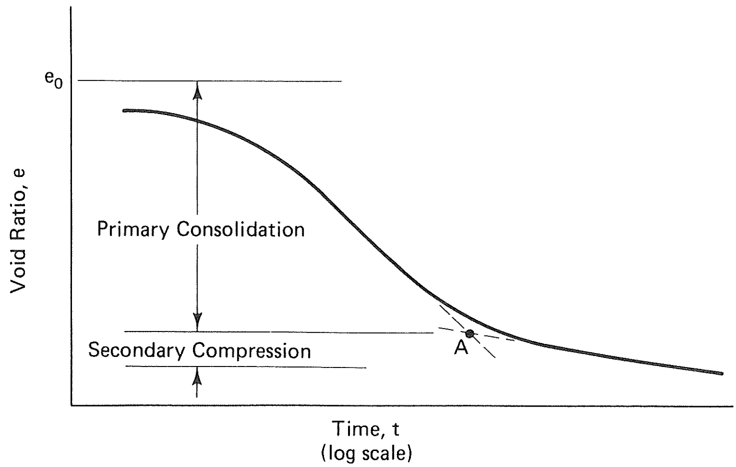


FIGURE 7-17

The coefficient of secondary compression (C_α) varies with the clay layer's natural water content and can be determined from Fig. 7-18.

The amount of secondary compression settlement may be quite significant for highly compressible clays, highly micaceous soils, and organic materials. On the other hand, it is largely insignificant for inorganic clay with moderate compressibility.

EXAMPLE 7-10

Given

1. A foundation is to be built on a sand deposit underlain by a highly compressible clay layer 5.0 m thick.
2. The clay layer's natural water content is 80%.
3. Primary consolidation is estimated to be complete in 10 yr.

Required

Secondary compression settlement expected to occur from 10 yr to 50 yr after construction of the foundation.

Solution

From Eq. (7-19),

$$S_s = C_\alpha H \log \frac{t_s}{t_p} \tag{7-19}$$

$$C_\alpha = 0.015$$

(from Fig. 7-18 with a natural water content of 80%)

$$H = 5.0 \text{ m}$$
$$t_s = 50 \text{ yr}$$
$$t_p = 10 \text{ yr}$$
$$S_s = (0.015)(5.0 \text{ m}) \log \left(\frac{50 \text{ yr}}{10 \text{ yr}}\right) = 0.052 \text{ m}$$

EXAMPLE 7-11

Given

1. Same data as for Example 7-8.
2. Assume that primary consolidation will be complete in 15 yr.
3. Natural water content of the clay layer is 50%.

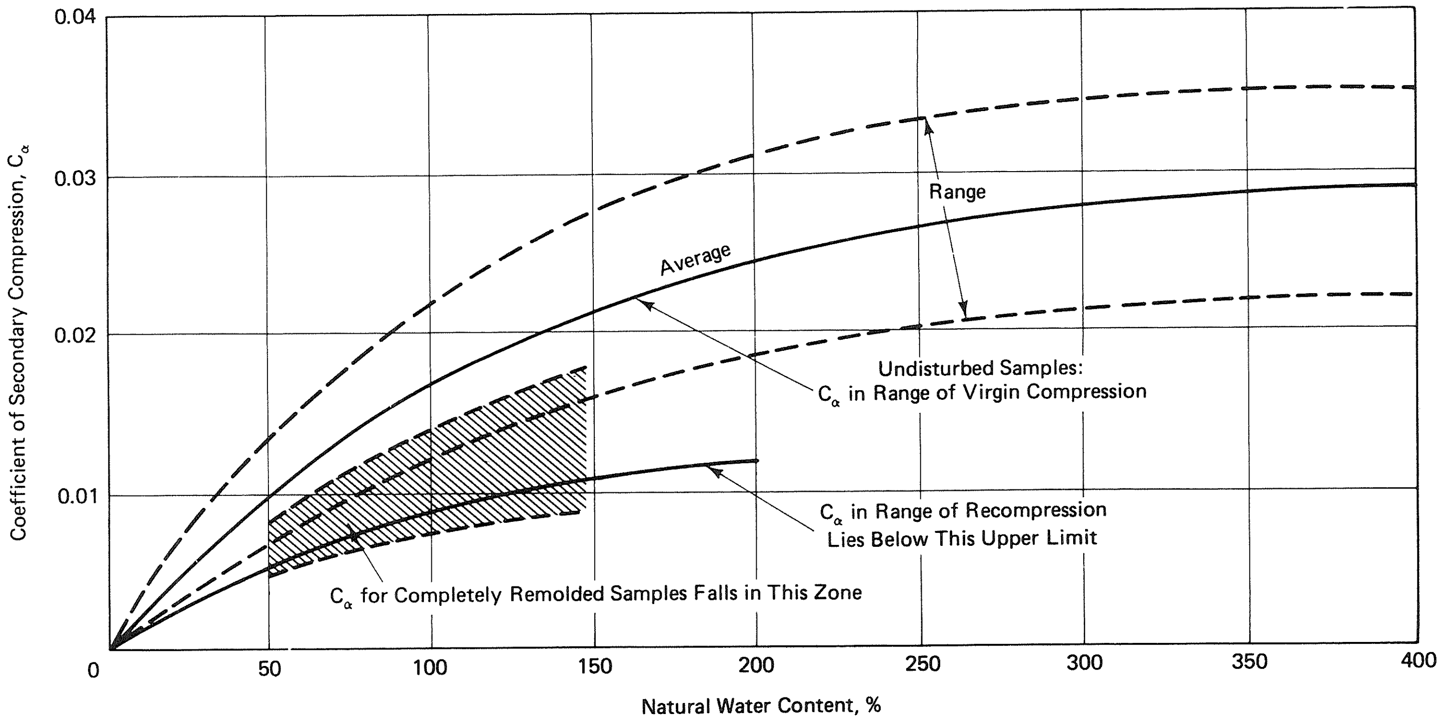


FIGURE 7-18 Coefficient of secondary compression; C_α = ratio of decrease in sample height to initial sample height for one cycle of time on log scale following completion of primary consolidation. [10]

Required

Estimated total settlement 50 yr after construction.

Solution

From Eq. (7-19),

$$S_s = C_\alpha H \log \frac{t_s}{t_p} \tag{7-19}$$

From Fig. 7-18, with the natural water content of the clay layer of 50%,

$$\begin{aligned}
C_\alpha &= 0.010 \\
H &= 192.0\text{ m} - 185.6\text{ m} = 6.4\text{ m} \\
t_s &= 50\text{ yr} \\
t_p &= 15\text{ yr} \\
S_s &= (0.010)(6.4\text{ m}) \log \left(\frac{50\text{ yr}}{15\text{ yr}}\right) = 0.033\text{ m}
\end{aligned}$$

Estimated total settlement is the sum of primary consolidation settlement (determined in Example 7-8) and secondary compression settlement (S_s). Hence,

$$\text{Estimated total settlement} = 0.212\text{ m} + 0.033\text{ m} = 0.245\text{ m}$$

7-8 SETTLEMENT OF LOADS ON SAND

Most of the settlement of loads on sand has occurred by the time construction is complete. Thus, time rate of settlement is not a factor as it is with clay. Settlement criteria rather than ultimate bearing capacity (see Chap. 9) commonly govern allowable bearing capacity for footings on sand; furthermore, settlement on sand is not amenable to solution based on laboratory consolidation tests. Instead, settlement on sand is generally calculated by empirical means.

One empirical method is based on the standard penetration test (SPT), which was discussed in Sec. 3-5. To determine settlement on sand, SPT determinations are made at various depths at the test site, normally at depth intervals of 2½ ft, (0.76 m), beginning at a depth corresponding to the proposed footing's base. The SPT N-values must be corrected for overburden pressure (see Chap. 3). The next step is to compute the average corrected N-value for each boring for the sand between the footing's base and a depth B below the base, where B is the footing's width. The lowest of the average corrected N-values for all borings at the site is noted and designated N_{lowest}. Maximum settlement can then be computed from the equation [11]

$$s_{max} = \frac{2q}{N_{lowest}} \left[\frac{2B}{1 + B} \right]^2 \quad (7\text{-}20)$$

where s_{max} = maximum settlement on dry sand, inches
q = applied pressure, tons/ft²
B = width of footing, ft

Equation (7-20) is applicable to settlement on dry sand. If the groundwater table is located at a depth below the base of the footing less than half the footing's width, the settlement computed from Eq. (7-20) should be corrected by multiplying it by x_B, where [11]

$$x_B = \frac{p_d}{p_w} \quad (7\text{-}21)$$

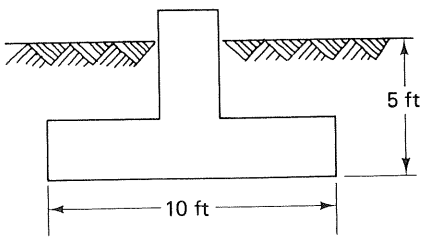
where p_d = effective overburden pressure at depth $B/2$ below the footing's base, assuming the groundwater table is not present
p_w = effective overburden pressure at the same depth with the groundwater table present

Examples 7-12 through 7-15 demonstrate the calculation of settlement on sand.

EXAMPLE 7-12

Given

1. A 10-ft by 10-f .ooting carrying a total load of 280 tons is to be constructed on sand as shown in Fig. 7-19.
2. Standard penetration tests were conducted on the site. Test results were corrected for overburden pressure (see Chap. 3), and the corrected N-values are listed below.



Medium to Coarse Sand

Unit Weight = 124 lb/ft³

No Groundwater Was Encountered

FIGURE 7-19

Depth (ft)	*Corrected N-Values*
5.0	31
7.5	36
10.0	30
12.5	28
15.0	35
17.5	33
20.0	31

Required

Maximum expected settlement of this footing.

Solution

Average corrected *N*-values

The average corrected N-value is determined for each boring for the soil located between the level of the footing's base and a depth B below this level, where B is the footing's width. In this example, appropriate depths for calculating average corrected N-values are 5 to 15 ft. The average corrected N-value is a cumulative average down to the depth indicated.

For a depth of 5 ft,

$$\text{Average corrected } N\text{-value} = 31$$

For a depth of 7.5 ft,

$$\text{Average corrected } N\text{-value} = \frac{31 + 36}{2} = 33$$

For a depth of 10.0 ft,

$$\text{Average corrected } N\text{-value} = \frac{31 + 36 + 30}{3} = 32$$

For a depth of 12.5 ft,

$$\text{Average corrected } N\text{-value} = \frac{31 + 36 + 30 + 28}{4} = 31$$

For a depth of 15.0 ft,

$$\text{Average corrected } N\text{-value} = \frac{31 + 36 + 30 + 28 + 35}{5} = 32$$

Lowest average corrected N-value for design

Subsurface soil conditions generally vary somewhat at most construction sites. The N-value selected for design is usually the lowest average corrected N-value, which in this example is 31 (at depth 12.5 ft).
From Eq. (7-20),

$$s_{max} = \frac{2q}{N_{lowest}} \left[\frac{2B}{1+B}\right]^2 \tag{7-20}$$

$$q = \frac{280 \text{ tons}}{(10 \text{ ft})(10 \text{ ft})} = 2.8 \text{ tons/ft}^2$$

$$N_{lowest} = 31$$

$$B = 10 \text{ ft}$$

$$s_{max} = \frac{(2)(2.8 \text{ tons/ft}^2)}{31} \left[\frac{2 \times 10 \text{ ft}}{1 + 10 \text{ ft}}\right]^2 = 0.60 \text{ in. on dry sand}$$

EXAMPLE 7-13

Given

Same conditions as in Example 7-12 except that the groundwater table is located 7 ft below ground level (see Fig. 7-20).

Required

Maximum expected settlement of the footing.

Solution

From Example 7-12,

$$s_{max} = 0.60 \text{ in. on dry sand}$$

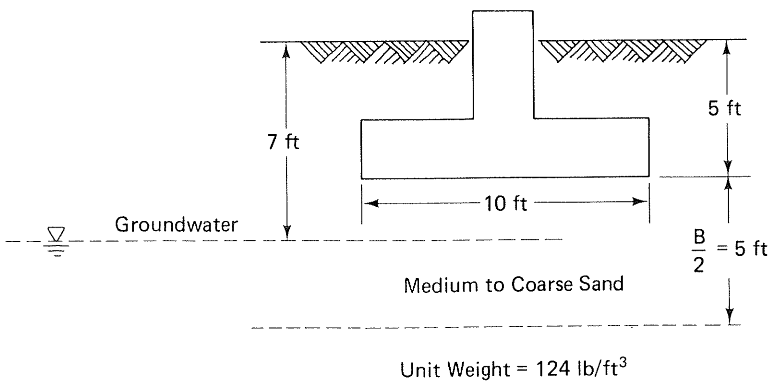


FIGURE 7-20

From Eq. (7-21),

$$x_B = \frac{p_d}{p_w} \tag{7-21}$$

$$p_d = (124 \text{ lb/ft}^3)\left(5 \text{ ft} + \frac{10 \text{ ft}}{2}\right) = 1240 \text{ lb/ft}^2$$

$$p_w = (124 \text{ lb/ft}^3)(7 \text{ ft}) + (124 \text{ lb/ft}^3 - 62.4 \text{ lb/ft}^3)\left(5 \text{ ft} + \frac{10 \text{ ft}}{2} - 7 \text{ ft}\right)$$
$$= 1053 \text{ lb/ft}^2$$

$$x_B = \frac{1240 \text{ lb/ft}^2}{1053 \text{ lb/ft}^2} = 1.178$$

$$s_{max} = (0.60 \text{ in.})(1.178) = 0.71 \text{ in. on wet sand}$$

EXAMPLE 7-14

Given

1. A square footing 8 ft by 8 ft located 5 ft below ground level is to be constructed on sand.
2. Standard penetration tests were conducted on the site. Test results were corrected for overburden pressures, and the lowest average corrected N-value was determined to be 41.
3. Groundwater was not encountered.

Required

Allowable soil pressure for a maximum settlement of 1 in.

Solution

From Eq. (7-20),

$$s_{max} = \frac{2q}{N_{lowest}}\left[\frac{2B}{1+B}\right]^2 \tag{7-20}$$

$$s_{max} = 1 \text{ in.}$$
$$N_{lowest} = 41$$
$$B = 8 \text{ ft}$$

$$1 \text{ in.} = \frac{2q}{41}\left[\frac{2 \times 8 \text{ ft}}{1 + 8 \text{ ft}}\right]^2$$

$$q = 6.49 \text{ tons/ft}^2$$

EXAMPLE 7-15

Given

Same conditions as in Example 7-14 except that the groundwater table is located 6 ft below ground level and the sand's unit weight is 128 lb/ft³ (see Fig. 7-21).

Required

Allowable soil pressure for a maximum settlement of 1 in.

Solution

From Eq. (7-21),

$$x_B = \frac{p_d}{p_w} \tag{7-21}$$

$$p_d = (128\ \text{lb/ft}^3)\left(5\ \text{ft} + \frac{8\ \text{ft}}{2}\right) = 1152\ \text{lb/ft}^2$$

$$p_w = (128\ \text{lb/ft}^3)(6\ \text{ft}) + (128\ \text{lb/ft}^3 - 62.4\ \text{lb/ft}^3)(3\ \text{ft}) = 964.8\ \text{lb/ft}^2$$

$$x_B = \frac{1152\ \text{lb/ft}^2}{964.8\ \text{lb/ft}^2} = 1.194$$

From Example 7-14, allowable soil pressure (q) is 6.49 tons/ft² for a settlement of 1 in. when no groundwater is encountered. When the groundwater table is at a depth below the base of the footing less than $B/2$, s_{max} computed from Eq. (7-20) should be multiplied by x_B. Therefore, in this example an allowable soil pressure of 6.49 tons/ft² will produce a settlement of 1.194 × 1 in., or 1.194 in. Since settlement varies directly with bearing pressure,

$$\frac{6.49\ \text{tons/ft}^2}{1.194\ \text{in.}} = \frac{\text{allowable soil pressure for a settlement of 1 in.}}{1\ \text{in.}}$$

Allowable soil pressure for a settlement of 1 in. = 5.44 tons/ft².

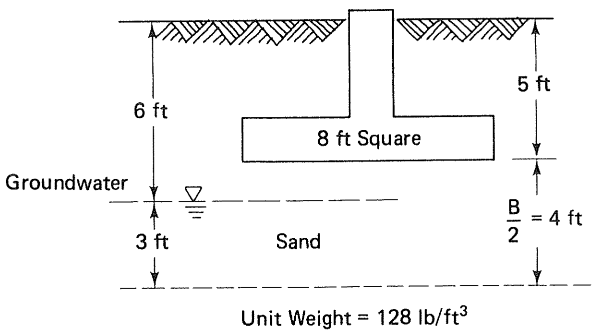


FIGURE 7-21

7-9 PROBLEMS

7-1 Determine the present effective overburden pressure at midheight of the compressible clay layer in the soil profile shown in Fig. 7-22.

7-2 When the total pressure acting at midheight of a compressible clay layer is 100 kN/m², the corresponding void ratio is 1.09. When the total pressure increases to 400 kN/m², the corresponding void ratio decreases to 0.89. What would be the void ratio for a total pressure of 800 kN/m²?

7-3 A compressible clay layer 10.0 m thick has an initial void ratio *in situ* of 1.026. Tests and computations show that the final void ratio of the clay layer after construction of a structure is 0.978. Determine the estimated primary consolidation settlement of the structure.

7-4 A foundation is to be constructed at a site where the soil profile is as shown in Fig. 7-23. The base of the foundation, which is 2 m square, exerts a total load (weight of structure, foundation, plus soil surcharge on the foundation) of 1000 kN. Initial void ratio *in situ* of the compressible clay layer is 1.058, and its compression index is 0.60. Find the estimated primary consolidation settlement for the clay layer.

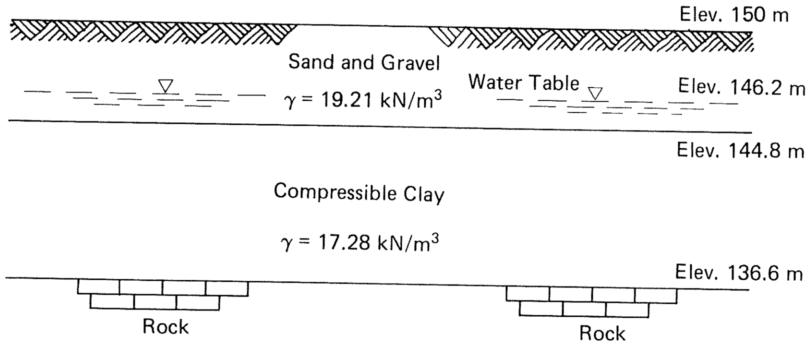


FIGURE 7-22

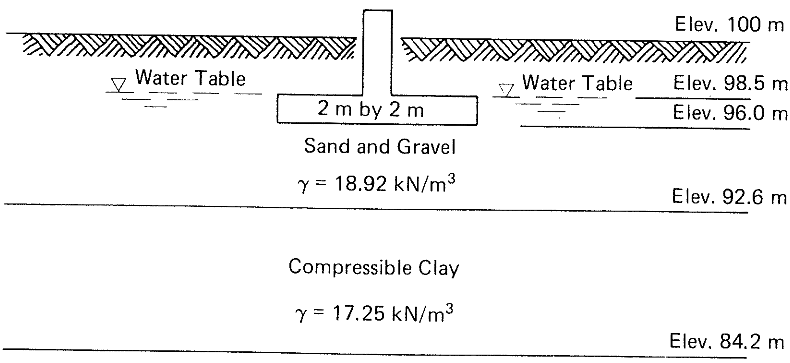


FIGURE 7-23

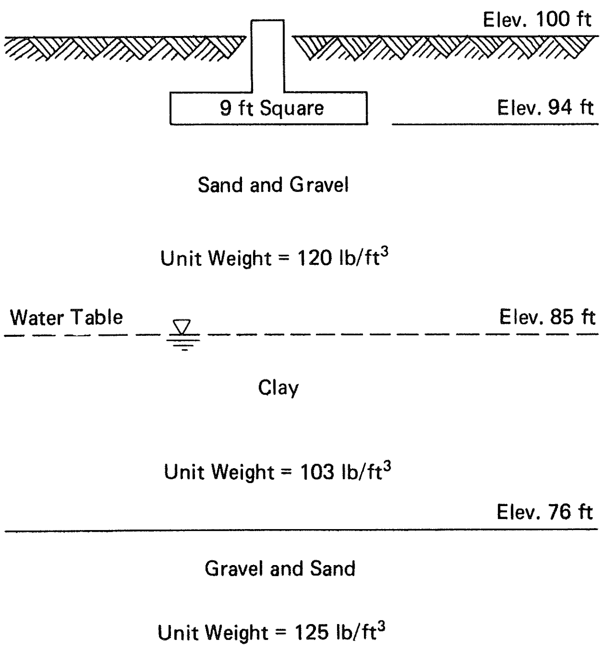


FIGURE 7-24

7-5 Continuing Problem 7-4, tests and computations indicate that the coefficient of consolidation is 6.98×10^{-6} m²/min. Compute the time required for 90% of the expected primary consolidation settlement to take place if the clay layer is underlain by (a) permeable sand and gravel, and (b) impermeable bedrock.

7-6 A sample of normally consolidated clay was obtained by a Shelby tube sampler from the midheight of a compressible clay layer (see Fig. 7-24). A consolidation test was conducted on a portion of this sample, results of which are given below:

1. Natural (initial) void ratio of the clay existing in the field (e_0) = 1.80.
2. Pressure–void ratio relationships are as follows:

p (*tons/ft*²)	*e*
0.250	1.72
0.500	1.70
1.00	1.64
2.00	1.51
4.00	1.34
8.00	1.15
16.00	0.95

A footing is to be constructed 6 ft below ground surface, as shown in Fig. 7-24. The base of the footing is 9 ft by 9 ft, and it carries a total load of 200 tons, which includes the column load, weight of footing, and weight of soil surcharge on the footing.

(a) From consolidation test results, prepare an e–log p curve and construct a field consolidation line assuming point f is located at $0.4e_0$.
(b) Compute the total expected primary consolidation settlement of the compressible clay layer.

7-7 Continuing Problem 7-6, test results also indicated that the coefficient of consolidation (c_v) of the clay is 2.18×10^{-3} in.^{2}/min for the pressure increment from 1 to 2 tons/ft^2. Compute the time of primary consolidation settlement. Take U at 10% increments and plot these values on a settlement–log time curve.

7-8 A compressible 12-ft clay layer beneath a building is overlain by a stratum of sand and gravel and underlain by impermeable bedrock. The total expected primary consolidation settlement of the compressible clay layer due to building load is 4.60 in. The coefficient of consolidation (c_v) is 9.04×10^{-4} in.^{2}/min.

(a) How long will it take for 90% of the expected total primary consolidation settlement to take place?
(b) Compute the amount of primary consolidation settlement that will occur in 1 yr.
(c) How long will it take for primary consolidation settlement of 1 in. to take place?

7-9 Continuing Problem 7-4, assume 100% primary consolidation will be complete in 14 yr. If the clay layer's natural water content is 35%, compute the estimated secondary compression settlement that would occur from 14 to 40 yr after construction.

7-10 A 9-ft by 9-ft square footing to carry a total load of 300 tons is to be installed 6 ft below ground surface on a sand stratum. Standard penetration tests were conducted on the site. Test results were corrected for overburden pressures, and the corrected N-values are listed below:

Depth (ft)	*Corrected N-Values*
2.5	25
5.0	28
7.5	27
10.0	30
12.5	28
15.0	23
17.5	24
20.0	28

No groundwater was encountered during subsurface exploration. Estimate the maximum expected settlement of the footing.

7-11 Assume the same conditions as in Problem 7-10 except that the groundwater table is located 8 ft below ground level and the sand's unit weight is 130 lb/ft^3. Estimate the maximum expected settlement of the footing.

7-12 A square footing 6 ft by 6 ft is to be installed 6 ft below ground level on a sand stratum. Standard penetration tests were conducted on the construction site. Test results were corrected for overburden pressures, and the lowest average corrected N-value was determined to be 18. Assuming that groundwater was not encountered, determine allowable soil pressure for a maximum settlement of 1 in.

7-13 Assume the same conditions as in Problem 7-12 except that the groundwater table is located 8 ft below ground level and the sand's unit weight is 118 lb/ft^3. Determine allowable soil pressure for a maximum settlement of 1 in.

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8

Shear Strength of Soil

8-1 INTRODUCTION

As a structural member, a piece of steel is capable of resisting compression, tension, and shear. Soil, however, like concrete and rock, is not capable of resisting high tension stresses (nor is it required to do so). It is capable of resisting compression to some extent; but in the case of excessive (failure producing) compression, failure usually occurs in the form of shearing along some internal surface within the soil. Thus, structural strength of soil is primarily a function of its shear strength, where shear strength refers to the soil's ability to resist sliding along internal surfaces within a mass of the soil.

Since the ability of soil to support an imposed load is determined by its shear strength, the shear strength of soil is of great importance in foundation design (Chap. 9), lateral earth pressure calculations (Chap. 12), slope stability analysis (Chap. 14), and many other considerations. As a matter of fact, shear strength of soil is of such great importance that it is a factor in most soil problems. Determination of shear strength is one of the most frequent, important problems in soil mechanics.

As explained in Sec. 2-8, soil gains its shear strength from two sources—internal friction and cohesion—as indicated by the Coulomb equation, Eq. (2-15), which is repeated here [1].

$$s = c + \overline{\sigma} \tan \phi \tag{2-15}$$

where s = shear strength
c = cohesion
$\overline{\sigma}$ = effective intergranular normal (perpendicular to the shear plane) pressure
ϕ = angle of internal friction
$\tan \phi$ = coefficient of friction

Cohesion (c) refers to strength gained from ionic bond between grain particles and is predominant in clayey (cohesive) soils. Angle of internal friction (ϕ) refers to strength gained from internal frictional resistance (including sliding and rolling friction and the resistance offered by interlocking action among soil particles) and is predominant in granular (cohesionless) soils. Cohesion (c) and angle of internal friction (ϕ) might be referred to as the *shear strength parameters.* They can be evaluated for a given soil by standard laboratory and/or field tests (Sec. 8-2), thereby defining the relationship for shear strength (s) as a function of effective intergranular normal pressure ($\overline{\sigma}$). The latter term ($\overline{\sigma}$) is not a soil parameter; it refers instead to the magnitude of applied load.

As indicated above, the same two parameters affect shear strength of both cohesive and cohesionless soils. However, the predominant parameter differs depending on whether a cohesive soil or a cohesionless soil is being considered. Accordingly, study and analysis of shear strength of soil are normally done separately for cohesive and cohesionless soils.

Field and laboratory methods for determining shear strength parameters, from which shear strength can be evaluated, are presented in Sec. 8-2. Study and analysis of shear strength of cohesionless soils are presented in Sec. 8-4, and those of cohesive soils in Sec. 8-5.

8-2 METHODS OF INVESTIGATING SHEAR STRENGTH

There are several methods of investigating shear strength of soil. Discussed here are the (1) unconfined compression test, (2) vane test, (3) direct shear test, and (4) triaxial compression test. The unconfined compression test can be used only to investigate cohesive soils, and the vane test can be used to investigate soft clays—particularly sensitive clays. The direct shear test and the triaxial test can be used to investigate both cohesive and cohesionless soils. As done previously in this book, only generalized discussions of the various test procedures are presented here.

Unconfined Compression Test

The unconfined compression test is perhaps the simplest, easiest, and least expensive test for investigating shear strength. It is quite similar to the usual determination of compressive strength of concrete, where crushing a concrete cylinder is carried out solely by measured increases in end loading. A cylindrical cohesive soil specimen is cut to a length of between 2 and 2½ times its diameter. It is then placed in a compression testing machine (Fig. 8-1) and subjected to an axial load. The axial load is applied to produce axial strain at a rate of ½ to 2% per min, and resulting stress and strain are measured.

As load is applied to the specimen, its cross-sectional area will increase a small amount. For any applied load, the cross-sectional area, A, can be computed by the equation

$$A = \frac{A_0}{1 - \varepsilon} \tag{8-1}$$

where A_0 is the specimen's initial area. The load itself, P, can be determined by multiplying the proving ring dial reading by the proving ring calibration factor, and the load per unit area can be found by dividing the load by the corresponding cross-sectional area. The axial unit strain, ε, can be computed by dividing the change in length of the specimen, ΔL, by its initial length, L_0. In equation form,

$$\varepsilon = \frac{\Delta L}{L_0} \tag{8-2}$$

The value of ΔL is given by the deformation reading, provided the deflection dial is set to zero initially.

The largest value of the load per unit area or the load per unit area at 15% strain, whichever occurs first, is known as the *unconfined compressive strength*, q_u, and cohesion [c in Eq. (2-15)] is taken as one-half the unconfined compressive strength (i.e., $q_u/2$).

In the unconfined compression test, since there is no lateral support, the soil specimen must be able to stand alone in the shape of a cylinder. A cohesionless soil (such as sand) cannot generally stand alone in this manner without lateral support; hence, this test procedure is usually limited to cohesive soils.

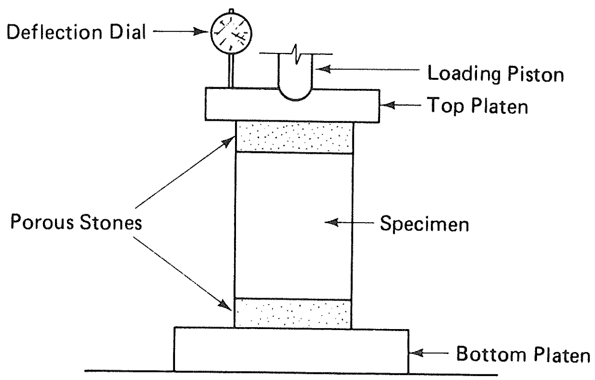


FIGURE 8-1 Unconfined compression test apparatus. [2]

EXAMPLE 8-1

Given

A clayey soil subjected to an unconfined compression test fails at a pressure of 2540 lb/ft^2 (i.e., q_u = 2540 lb/ft^2).

Required

Cohesion of this clayey soil.

Solution

$$\text{Cohesion} = \frac{\text{unconfined compressive strength}}{2}$$

or

$$c = \frac{q_u}{2}$$

$$c = \frac{2540 \text{ lb/ft}^2}{2} = 1270 \text{ lb/ft}^2$$

Vane Test

The vane test, which was discussed in Sec. 3-7, can also be used to determine shear strength of cohesive soils. This test can be used in the field to determine *in situ* shear strength for soft clay soil—particularly for sensitive clays (those that lost part of their strength when disturbed). The test can also be carried out in the laboratory on a cohesive soil sample.

Direct Shear Test

To carry out a direct shear test, a soil specimen is placed in a relatively flat box, which may be round or square (Fig. 8-2). A normal load of specific (and constant) magnitude is applied. The box is "split" into two parts hori-

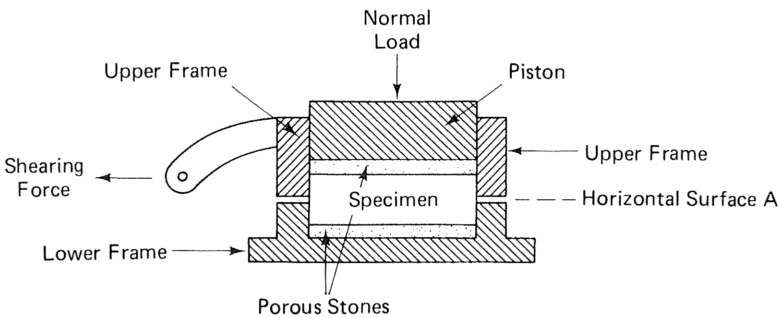


FIGURE 8-2 Typical direct shear box for single shear. [3]

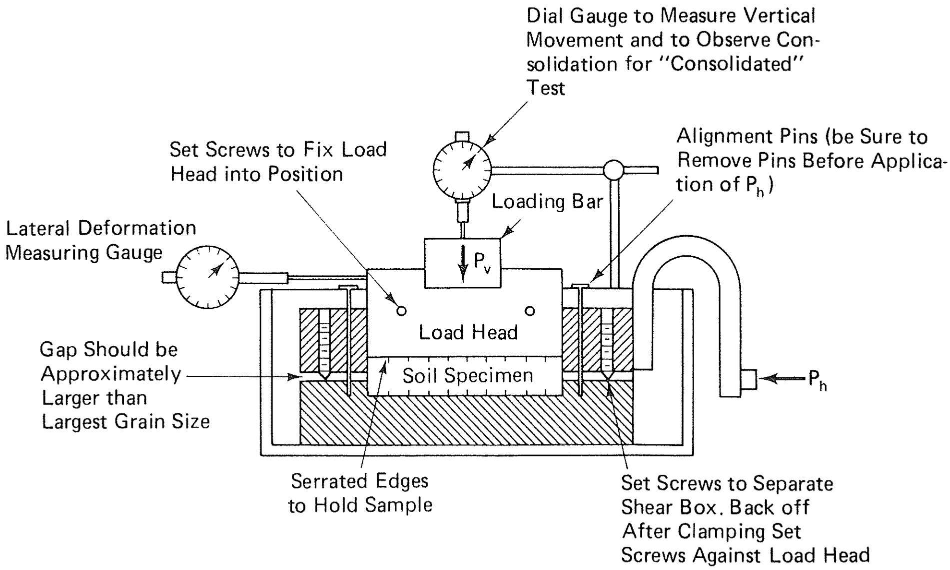


FIGURE 8-3 Direct shear apparatus. [2]

zontally (see Fig. 8-2); and if half the box is held while the other half is pushed with sufficient force, the soil specimen will experience shear failure along horizontal surface *A*. This procedure is carried out in a direct shear apparatus (Fig. 8-3) and the particular normal load and shear stress that produced shear failure are recorded. The soil specimen is then removed from the shear box and discarded, and another specimen of the same soil sample is placed in the shear box. A normal load differing from (either higher or lower) the one used in the first test is applied to the second specimen, and a shearing force is again applied with sufficient magnitude to cause shear failure. The normal load and shear stress that produced shear failure are recorded for the second test.

Results of these two tests are plotted with normal stress (which is total normal load divided by the specimen's cross-sectional area) along the abscissa and the shear stress that produced failure of the specimen along the ordinate (see Fig. 8-4). (The same scale must be used along both the abscissa and ordinate.) A straight line drawn connecting these two plotted points is extended to intersect the ordinate. The angle between this straight line and a horizontal line (ϕ in Fig. 8-4) is the angle of internal friction [ϕ in Eq. (2-15)], and the shear stress where the straight line intersects the ordinate (c in Fig. 8-4) is the cohesion [c in Eq. (2-15)]. These values of ϕ and c can be used in Eq. (2-15) to determine the given soil's shear strength for any load (i.e., for any effective intergranular normal pressure, $\overline{\sigma}$).

In theory, it is adequate to have only two points to define the straight-line relationship of Fig. 8-4. In practice, however, it is better to have three (or more) such points through which the best fitting straight line can be drawn. This means, of course, that three (or more) separate tests must be made on three (or more) specimens from the same soil sample.

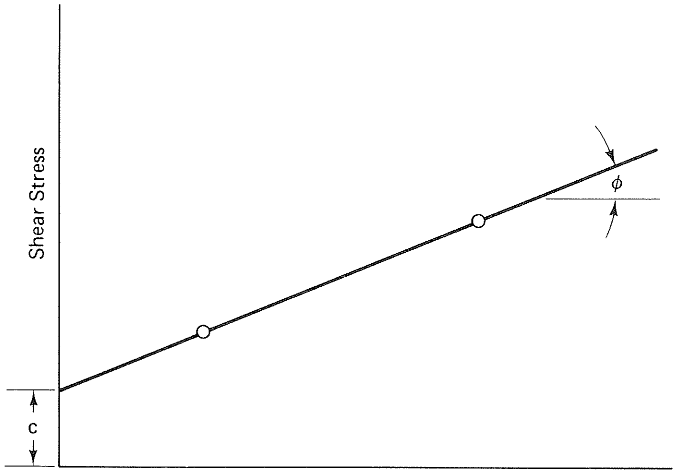


FIGURE 8-4 Shear diagram for direct shear test.

The direct shear test is a relatively simple means of determining shear strength parameters of soils. However, in this test shear failure is forced to occur along or across a predetermined plane (surface *A* in Fig. 8-2), which is not necessarily the weakest plane of the soil specimen tested. Since development of the much better triaxial test (discussed next), use of the direct shear test has decreased.

EXAMPLE 8-2

Given

A series of direct shear tests was performed on a soil sample with each test carried until the soil specimen experienced shear failure. Test data are listed below.

Specimen Number	*Normal Stress (lb/ft²)*	*Shearing Stress (lb/ft²)*
1	604	1522
2	926	1605
3	1248	1720

Required

The soil's cohesion and angle of internal friction.

Solution

Given data are plotted on a shear diagram (see Fig. 8-5). (Note that both ordinate and abscissa scales are the same.) Connect the plotted points by the best-fitting straight line and note that it makes an angle of 17° with the horizontal and intersects the ordinate at 1340 lb/ft². Therefore, cohesion (c) = 1340 lb/ft² and angle of internal friction (ϕ) = 17°.

EXAMPLE 8-3

Given

A specimen of dry sand subjected to a direct shear test was carried until the specimen sheared. A normal stress of 96.0 kN/m² was imposed for the test, and shear stress at failure was 65.0 kN/m².

Required

This sand's angle of internal friction.

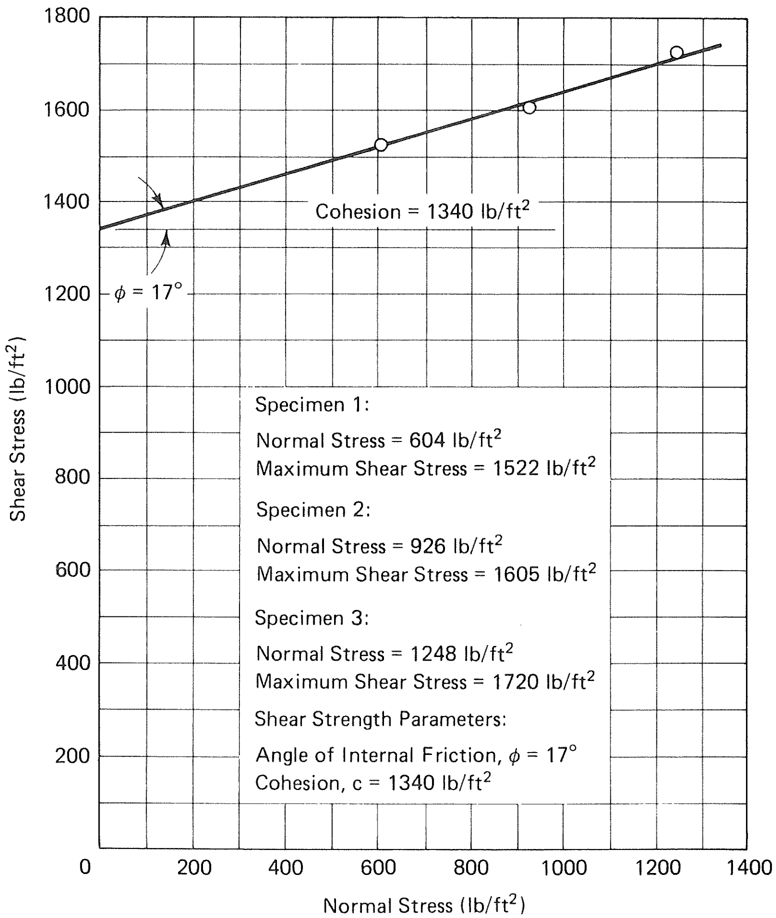


FIGURE 8-5 Maximum shear stress versus normal stress curve for Example 8-2.

Solution

Given data are plotted on a shear diagram (Fig. 8-6). (Note that both ordinate and abscissa scales are the same.) Since cohesion is virtually zero for dry sand, the shear plot passes through the origin. Hence, draw a line through the plotted point and the origin. The angle between this line and the horizontal is measured to be 34°. Therefore, the sand's angle of internal friction (ϕ) is 34°. This value can also be determined by direct computation.

$$\tan \phi = \frac{65.0 \text{ kN/m}^2}{96.0 \text{ kN/m}^2} = 0.6771$$
$$\phi = 34^\circ$$

Triaxial Compression Test

The triaxial compression test is carried out in a manner somewhat similar to the unconfined compression test in that a cylindrical soil specimen is subjected to a vertical (axial) load. The major difference is that, unlike the unconfined compression test where there is no confining (lateral) pressure, the triaxial test is carried out with confining (lateral) pressure present. Lateral pressure is made possible by enclosing the specimen in a chamber (see Fig. 8-7) and introducing water or compressed air into the chamber to surround the soil specimen.

To carry out a test, a cylindrical soil specimen having a length between 2 and 2½ times its diameter is wrapped in a rubber membrane and placed in the triaxial chamber, and a specific (and constant) lateral pressure is applied by

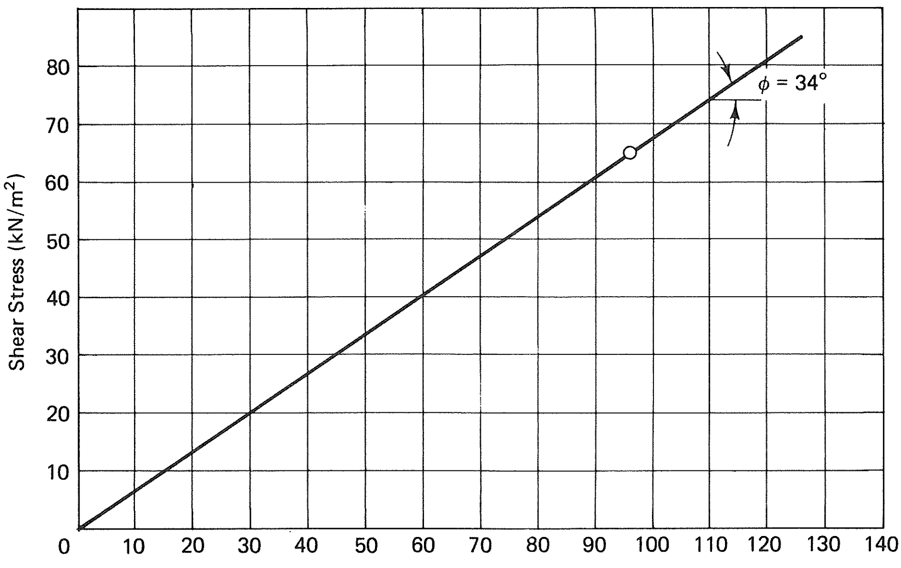


FIGURE 8-6 Shear diagram for Example 8-3.

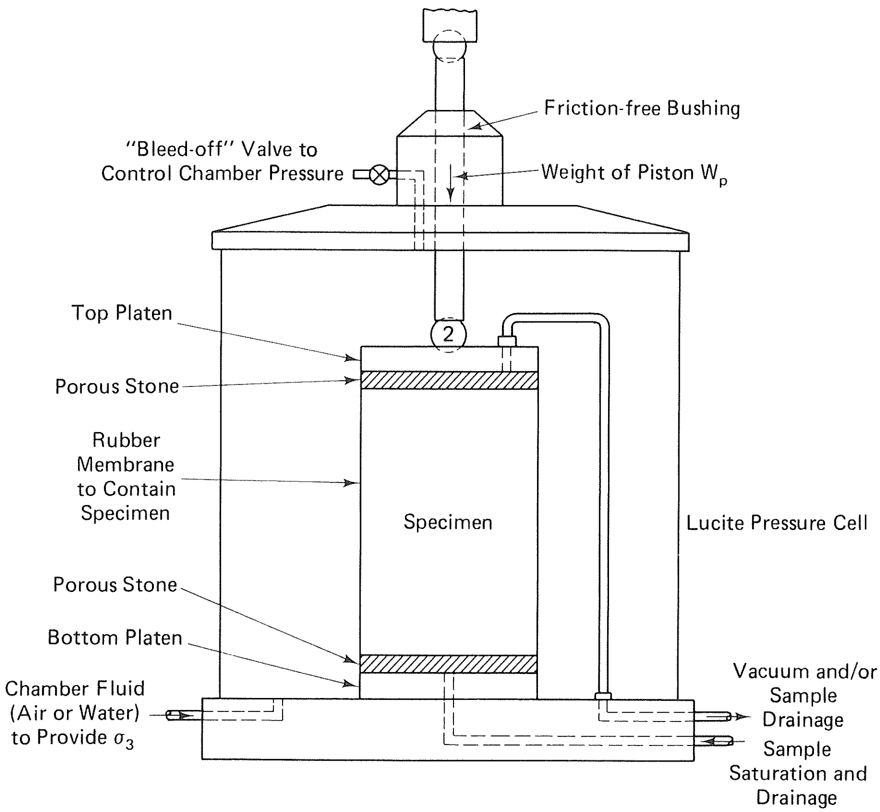


FIGURE 8-7 Schematic diagram of triaxial chamber. [2]

means of water or compressed air within the chamber. A vertical (axial) load is then applied externally and steadily increased until the specimen fails. The externally applied axial load that caused the specimen to fail and the lateral pressure are recorded. As in the direct shear test, it is necessary to remove the soil specimen and discard it, and then to place another specimen of the same soil sample in the triaxial chamber. The procedure described above is repeated for the new specimen for a different (either higher or lower) lateral pressure. The axial load at failure and the lateral pressure are recorded for the second test.

Lateral pressure is designated as σ_3. However, it is applied not only to the specimen's sides but also to its ends. It is therefore called the *minor principal stress.* Externally applied axial load at failure divided by cross-sectional area of the test specimen is designated as Δp, and it is called the *deviator stress at failure.* Total vertical (axial) pressure causing failure is the sum of the minor principal stress (σ_3) and the deviator stress (Δp) at failure. This total vertical (axial) pressure at failure is designated as σ_1, and it is called the *major principal stress.* In equation form,

$$\sigma_1 = \sigma_3 + \Delta p \tag{8-3}$$

Results of triaxial compression tests can be plotted in the following manner. Using results of one of the triaxial tests, locate a point along the abscissa at distance σ_3 from the origin. This point is denoted by *A* in Fig. 8-8, and it is indicated as being located along the abscissa at distance $(\sigma_3)_1$ from the origin. It is also necessary to locate another point along the abscissa at distance σ_1 from the origin. This point can be located by measuring either distance σ_1 from the origin or Δp from point *A* (the point located at distance σ_3 from the origin). This point is denoted by *B* in Fig. 8-8 and indicated as being located along the abscissa at distance $(\Delta p)_1$ from point *A*. Using *AB* as a diameter, construct a semicircle as shown in Fig. 8-8. (This is known as *Mohr's circle.*) The entire procedure is repeated using the data obtained from the triaxial test on the other specimen of the same soil sample. Thus point *C* is located along the abscissa at distance $(\sigma_3)_2$ from the origin, and point *D* along the abscissa at distance $(\Delta p)_2$ from point *C*. Using *CD* as a diameter, construct another semicircle. The final step is to draw a straight line tangent to the semicircles, as shown in Fig. 8-8. This straight line is called the *strength envelope, failure envelope,* or *Mohr's envelope.* As in the direct shear test (Fig. 8-4), the angle between this straight line (the strength envelope) and a horizontal

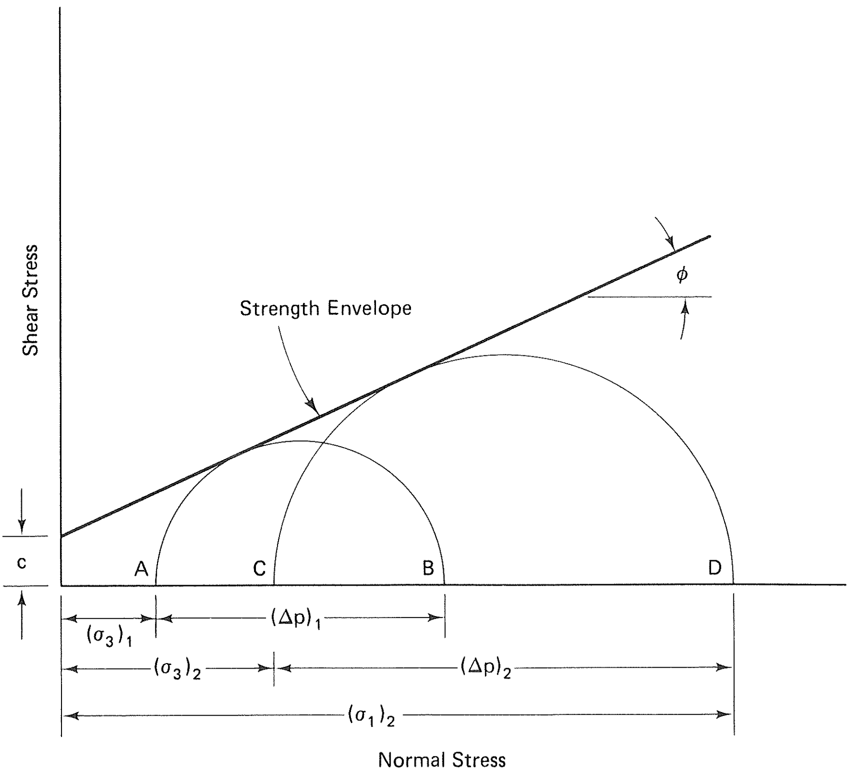


FIGURE 8-8 Shear diagram for triaxial compression test.

line (ϕ in Fig. 8-8) is the angle of internal friction [ϕ in Eq. (2-15)], and the shear stress where the straight line intersects the ordinate (c in Fig. 8-8) is the cohesion [c in Eq. (2-15)]. The same scale must be used along both the abscissa and ordinate.

As in the direct shear test, it is adequate, in theory, to have only two Mohr's circles to define the straight-line relationship of Fig. 8-8. In practice, however, it is better to have three (or more) Mohr's circles that can be used to draw the best strength envelope. This means, of course, that three (or more) separate tests must be performed on three (or more) specimens from the soil sample. In actuality, the strength envelope for both sand and clay will seldom be perfectly straight except perhaps at low lateral pressures; therefore, it requires some interpretation to draw a best-fitting strength envelope of Mohr's circles.

EXAMPLE 8-4

Given

Triaxial compression tests on three specimens of a soil sample were performed with each test carried until the specimen experienced shear failure. Test data are tabulated below.

Specimen Number	*Minor Principal Stress, σ_3 (Confining Pressure) (kips/ft^2)*	*Deviator Stress at Failure, Δp (kips/ft^2)*
1	1.44	5.76
2	2.88	6.85
3	4.32	7.50

Required

The soil's cohesion and angle of internal friction.

Solution

As shown in Fig. 8-9, draw three Mohr's circles. Each one starts at a minor principal stress (σ_3) and has a diameter equal to the deviator stress at failure (Δp). Then draw the strength envelope tangent as nearly as possible to all three circles. The soil's cohesion is indicated by the intersection of the strength envelope and the ordinate where a value of 1.8 kips/ft^2 is read. The soil's angle of internal friction, which is the angle between the strength envelope and the horizontal, is 17°.

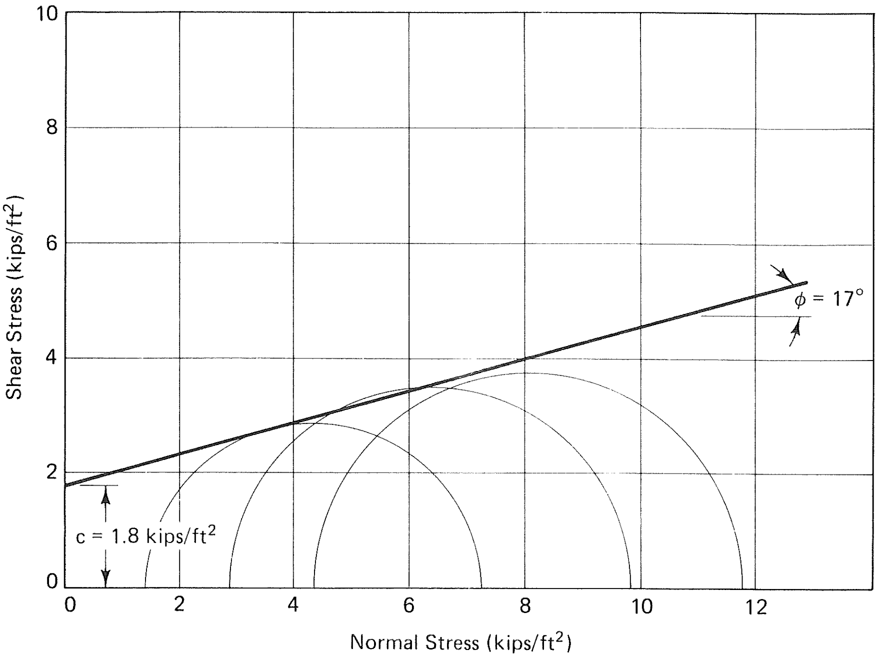


FIGURE 8-9 Mohr's circles for Example 8-4.

EXAMPLE 8-5

Given

A sample of dry, cohesionless soil subjected to a triaxial compression test was carried until the specimen failed at a deviator stress of 105.4 kN/m². A confining pressure of 48.0 kN/m² was used for the test.

Required

This soil's angle of internal friction.

Solution

Given data are plotted on a shear diagram (Fig. 8-10). (Note that both ordinate and abscissa scales are the same.) Point *A* is located along the abscissa at 48.0 kN/m² (the confining pressure—σ_3) and point *B* at 48.0 kN/m² + 105.4 kN/m², or 153.4 kN/m² (confining pressure plus deviator stress at failure—$\sigma_3 + \Delta p$). The Mohr's circle is drawn with a center along the abscissa at 100.7 kN/m² $\left(\text{i.e., } 48.0\ \text{kN/m}^2 + \dfrac{105.4\ \text{kN/m}^2}{2}\right)$ and a radius of 52.7 kN/m². Since cohesion is virtually zero for dry, cohesionless soil, a line is drawn through the origin and tangent to the Mohr's circle. The angle between this line and the horizontal is measured to be 32°. Therefore, the soil's angle of internal friction (ϕ) is 32°.

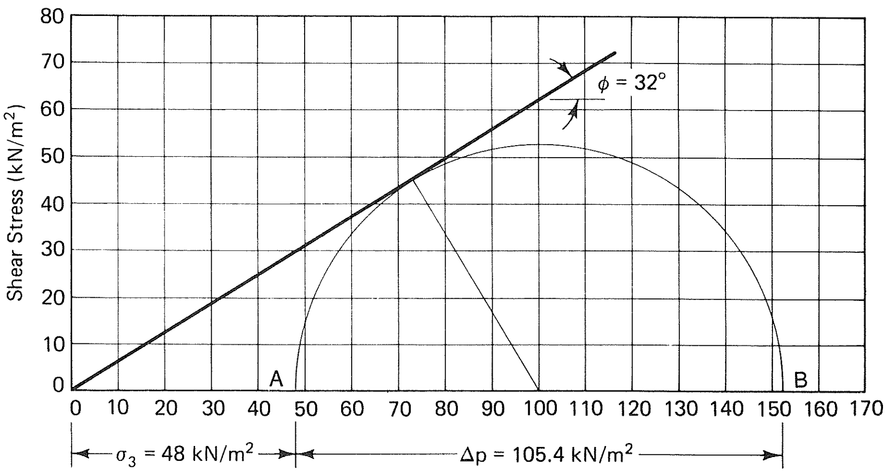


FIGURE 8-10 Mohr's circle for Example 8-5.

EXAMPLE 8-6

Given

A sample of dry cohesionless soil whose angle of internal friction is 37° is subjected to a triaxial test.

Required

If the minor principal stress (σ_3) is 14 lb/in.^{2}, at what values of deviator stress (Δp) and major principal stress (σ_1) will the test specimen fail?

Solution

All samples of dry cohesionless soils have cohesions of zero. Therefore, the Mohr's envelope must go through the origin. Draw a strength envelope starting at the origin for $\phi = 37°$. Then draw the Mohr's circle, starting at a minor principal stress (σ_3) of 14 lb/in.^{2} and tangent to the strength envelope (see Fig. 8-11). It can now be determined that the deviator stress at failure (Δp) is 42.3 lb/in.^{2} (deviator stress at failure equals the diameter of the Mohr's circle), and the major principal stress at failure ($\sigma_1 = \Delta p + \sigma_3$) is 42.3 lb/in.^{2} + 14 lb/in.^{2} or 56.3 lb/in.^{2}

This problem can also be solved analytically, using a sketch (i.e., drawing not made to scale). From Fig. 8-11 with given values of ϕ of 37° and σ_3 of 14 lb/in.^{2},

$$\sin 37° = \frac{R}{14 + R}, \text{ or } (14 + R) \sin 37° = R$$

$$8.4254 + 0.6018R = R$$

$$R = 21.16$$

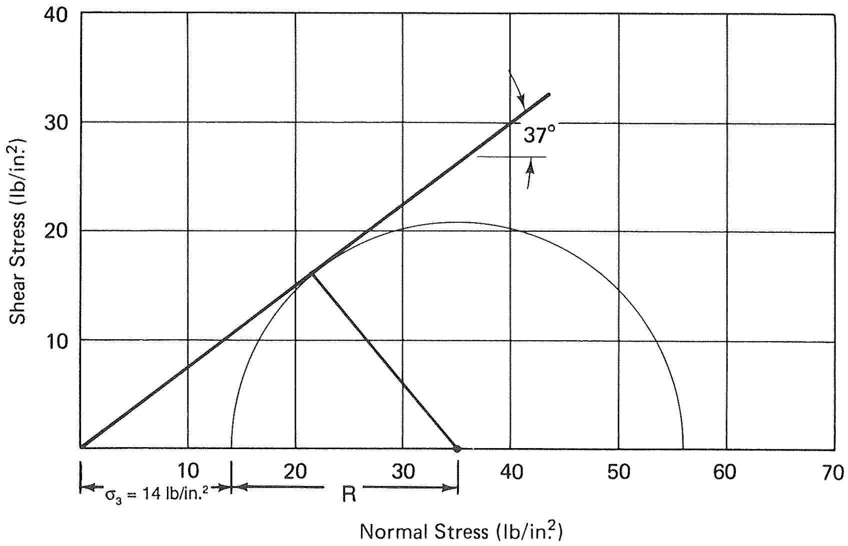


FIGURE 8-11 Mohr's circle for Example 8-6.

$$\Delta p = 2R = (2)(21.16) = 42.3 \text{ lb/in.}^2$$

$$\sigma_1 = \Delta p + \sigma_3 = 42.3 \text{ lb/in.}^2 + 14 \text{ lb/in.}^2 = 56.3 \text{ lb/in.}^2$$

As will be shown in Chap. 9, the angle of internal friction (ϕ) can be approximated for cohesionless soils, based on results of a standard penetration test (SPT).

Variations in Shear Test Procedures

There are three basic types of shear test procedures as determined by the sample drainage condition: unconsolidated undrained (UU), consolidated undrained (CU), and consolidated drained (CD). These can be defined as follows. Although these three types apply to both direct shear and triaxial compression tests, they are explained below for the triaxial test only.

The unconsolidated undrained (UU) test is carried out by placing the specimen in the chamber and introducing lateral (confining) pressure without allowing the specimen to consolidate (drain) under the lateral pressure. Axial load is then applied without allowing drainage of the sample. The UU test can be run rather quickly because the specimen is not required to consolidate under the lateral pressure or drain during application of axial load. Because of the short time required to run this test, it is often referred to as the quick, or Q, test.

The consolidated undrained (CU) test is performed by placing the specimen in the chamber and introducing lateral pressure. The sample is then allowed to consolidate under the lateral pressure by leaving the drain lines open (see Fig. 8-7). The drain lines are then closed and axial stress increased without allowing further drainage.

The consolidated drained (CD) test is similar to the CU test except that the specimen is allowed to drain as axial load is applied so that high excess pore pressures do not develop. Since the permeability of clayey soils is low, axial load must be added very slowly during CD tests so that excess pore pressure can be dissipated. CD tests may take considerable time to run because of time required for both consolidation under the lateral pressure and drainage during application of axial load. Inasmuch as the time requirement is long for low-permeability soils, it is often referred to as the slow, or S, test.

The specific type of test (UU, CU, or CD) to be used in any given case depends largely on field conditions to be simulated. For example, if field loading on a particular soil during construction of, say, an earth dam is expected to be slow so that excess pore water will have drained by the end of construction, the slow (CD) test might be most appropriate. On the other hand, the quick (UU) test might be called for if loading during construction is to be very rapid. The CU test might be considered in practice as a compromise between the slow and quick tests.

In the final analysis, the type of test to be used may be based on the engineer's judgment of the problem at hand, the type of soil involved, and so on.

8-3 CHARACTERISTICS OF THE FAILURE PLANE

Whenever homogeneous soils are stressed to failure in unconfined and triaxial compression tests, failure tends to occur along a distinct plane, as shown in Fig. 8-12a. The precise position of the failure plane is located at angle θ with the horizontal, which, as will be shown, is a function of the soil's angle of internal friction (ϕ). Figure 8-12b gives schematically the stresses acting on the failure plane, and Fig. 8-12c shows the Mohr's circle and strength envelope for the given soil. Since the sum of the interior angles of a triangle is 180°,

$$(180^\circ - 2\theta) + 90^\circ + \phi = 180^\circ \tag{8-4}$$

Therefore,

$$\theta = 45^\circ + \frac{\phi}{2} \tag{8-5}$$

The normal stress and shear stress on the failure plane (see Fig. 8-13) can be calculated using the following equations, which result from the principles of solid mechanics.

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta \tag{8-6}$$

$$s = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta \tag{8-7}$$

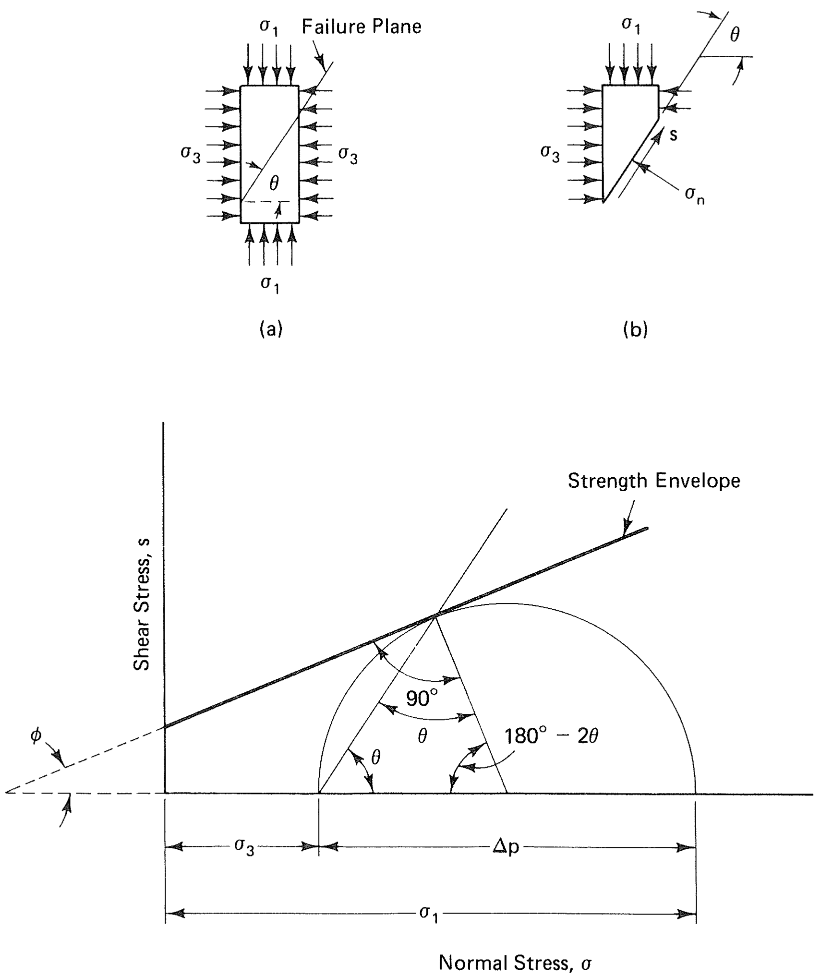


FIGURE 8-12 Relationship between angle of internal friction (ϕ) and orientation of failure plane (θ): (a) failure plane; (b) stresses acting on the failure plane; (c) Mohr's circle.

where s = shear stress on the failure plane
σ_n = normal stress on the failure plane
σ_1 = major principal stress
σ_3 = minor principal stress
θ = angle between failure plane and horizontal plane (see Fig. 8-13)

Normal stress and shear stress can also be determined graphically. In Fig. 8-13, points of tangency (e.g., point D in Fig. 8-13) represent stress conditions on the failure plane in the test specimen. From the point where the strength envelope is tangent to Mohr's circle (point D), a line drawn vertically

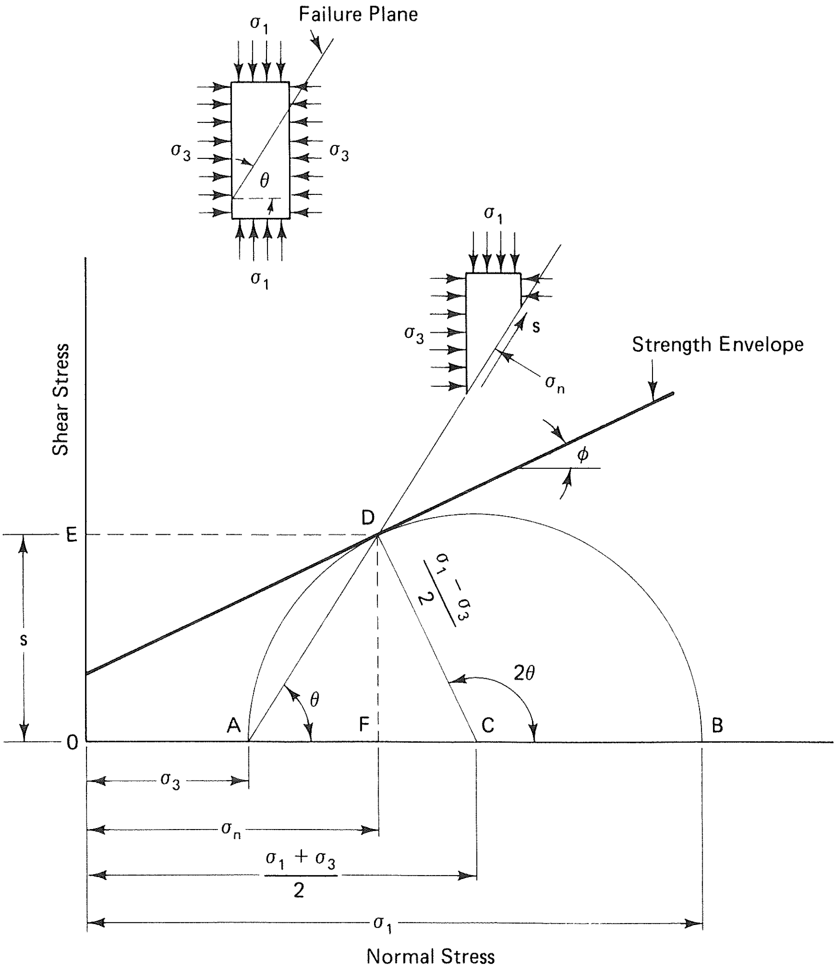


FIGURE 8-13 Normal stress and shear stress on failure plane.

downward intersects the abscissa at point F and one drawn horizontally leftward intersects the ordinate at point E. With the coordinate system's origin denoted by O in Fig. 8-13, OF is the normal stress (σ_n) on the failure plane and OE is the shear stress (s). Furthermore, the angle between the abscissa and a line drawn from point A (the point located at distance σ_3 from the origin, see Fig. 8-13) through the point of tangency (point D)—that is, angle DAB or θ in Fig. 8-13—gives the orientation of the failure plane (i.e., angle θ in Fig. 8-12a).

EXAMPLE 8-7

Given

The same conditions as given for Example 8-4.

Required

Angle of the failure plane and shear stress and normal stress on the failure plane for test specimen No. 1.

Solution

From Example 8-4, the following data are known:

$$\left.\begin{aligned} c &= 1.8 \text{ kips/ft}^2 \\ \phi &= 17^\circ \end{aligned}\right.$$

$$\left.\begin{aligned} \sigma_3 &= 1.44 \text{ kips/ft}^2 \\ \Delta p &= 5.76 \text{ kips/ft}^2 \\ \sigma_1 &= 7.20 \text{ kips/ft}^2 \end{aligned}\right\} \text{test specimen No. 1}$$

These data are plotted, as shown in Fig. 8-14, according to procedures described previously. Equation (8-5) may be used to find the angle of the failure plane (θ).

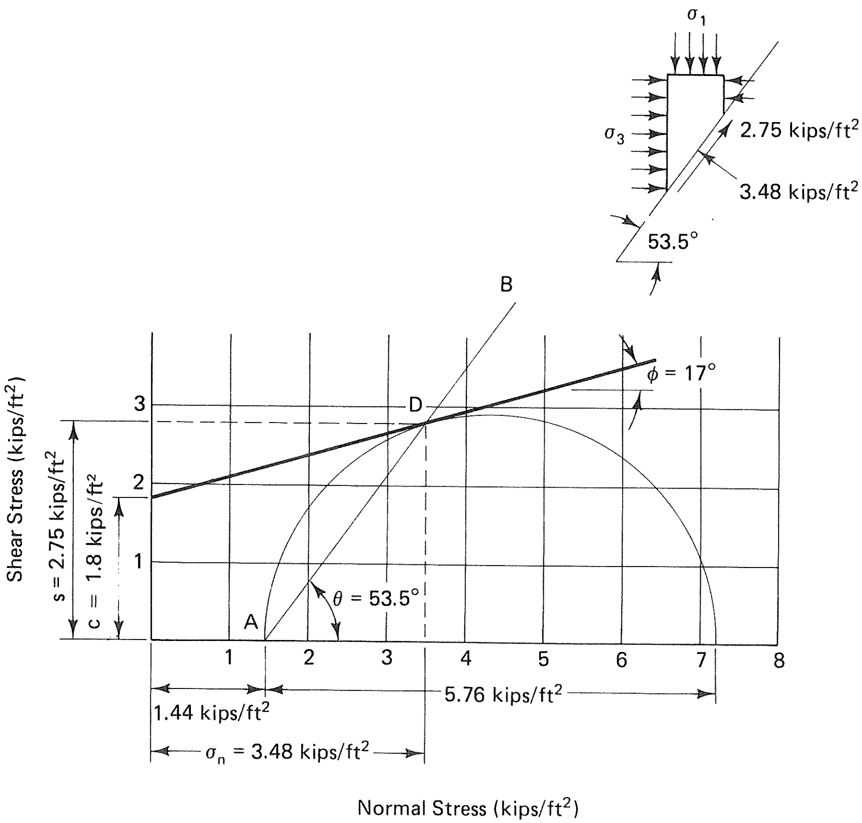


FIGURE 8-14 Mohr's circle for Example 8-7.

$$\theta = 45° + \frac{\phi}{2} \quad (8\text{-}5)$$

$$\theta = 45° + \frac{17°}{2} = 53.5°$$

From point A, line AB is drawn at an angle of 53.5° (see Fig. 8-14), intersecting Mohr's circle and the strength envelope at point D. The horizontal and vertical distances from the origin to point D are determined to be 3.48 kips/ft^2 and 2.75 kips/ft^2, respectively. Hence, the specimen's normal stress on the failure plane is 3.48 kips/ft^2 and its shear stress is 2.75 kips/ft^2.

These stresses can also be determined by computation. From Eq. (8-6),

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta \quad (8\text{-}6)$$

$$\sigma_n = \frac{7.20 \text{ kips/ft}^2 + 1.44 \text{ kips/ft}^2}{2} + \frac{7.20 \text{ kips/ft}^2 - 1.44 \text{ kips/ft}^2}{2}\cos[(2)(53.5°)] = 3.48 \text{ kips/ft}^2$$

From Eq. (8-7),

$$s = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta \quad (8\text{-}7)$$

$$s = \frac{7.20 \text{ kips/ft}^2 - 1.44 \text{ kips/ft}^2}{2}\sin[(2)(53.5°)] = 2.75 \text{ kips/ft}^2$$

8-4 SHEAR STRENGTH OF COHESIONLESS SOILS

Because of relatively large particle size, all mixtures of pure silt, sand, and gravel possess virtually no cohesion. This is because large particles have no tendency to stick together. Large particles do, however, develop significant frictional resistance, including sliding and rolling friction as well as interlocking of the grains. This gives significant values of the angle of internal friction (ϕ); and with no cohesion ($c = 0$), Eq. (2-15) reverts to

$$s = \overline{\sigma} \tan \phi \quad (8\text{-}8)$$

Since most of a cohesionless soil's shear strength results from interlocking of grains, values of ϕ differ little whether the soil is wet or dry. Extrusion of water from void space is an extremely slow process for cohesive soils. Accordingly, the most critical condition with regard to shear strength usually occurs at construction time or upon application of load. With cohesionless soils, any water contained in void space at construction time or upon application of load will be driven out almost immediately, because of the high permeability of cohesionless soils. Thus, shear strength of cohesionless soils remains more or less constant throughout a structure's life.

The angle of internal friction (ϕ) of cohesionless soils can be obtained from laboratory or field tests (Sec. 8-2). However, ϕ can also be estimated based on the correlation between corrected SPT (standard penetration test) N-values and ϕ given by Peck et al [4]. This correlation is shown in Fig. 3-9 (page 64). To use this graph, one enters at the upper right with the corrected N-value, moves horizontally to the curve marked N, then vertically downward to the abscissa where the value of ϕ is read.

8-5 SHEAR STRENGTH OF COHESIVE SOILS

As related at the beginning of this chapter, cohesive (clay or clayey) soils gain shear strength from ionic bond between grain particles. Actual shear strength of a given clay deposit is related to its water content and type of clay mineral as well as the consolidation pressure experienced by the soil in the past (i.e., whether it is normally consolidated or overconsolidated clay). Shear strengths of clays may also differ enormously depending on whether a sample is undisturbed or remolded (as in fill).

Possible variation in a clay's shear strength is affected not only by the aforementioned factors but also by pore water drainage that can occur during shearing deformation. Most clays in their natural state are at or near saturation; their relatively low permeabilities tend to inhibit pore water drainage that tries to occur during shearing. Thus drainage considerations are important in evaluation of shear strength of cohesive soils.

Normally Consolidated Clay

Strength in drained shear If a saturated clay specimen is allowed to consolidate in a triaxial chamber under a lateral (confining) pressure equal to or greater than the maximum *in situ* pressure experienced by the clay, and if axial load is slowly applied and increased and drainage allowed at both ends of the sample, then a shear diagram similar to that shown in Fig. 8-15 will be obtained. In the diagram, Mohr's circles are plotted for stress conditions at failure for three different lateral pressures and the strength envelope drawn tangent to the Mohr's circles.

The strength envelope shown in Fig. 8-15 is sometimes referred to as the *effective stress strength envelope,* because it is based on effective stresses at failure. Since points of tangency represent stress conditions on the failure plane in each sample, results of consolidated drained (CD) triaxial tests on normally consolidated clays can be expressed by Coulomb's equation [Eq. (2-15)] with $c = 0$. Thus,

$$s = \bar{\sigma} \tan \phi \tag{8-8}$$

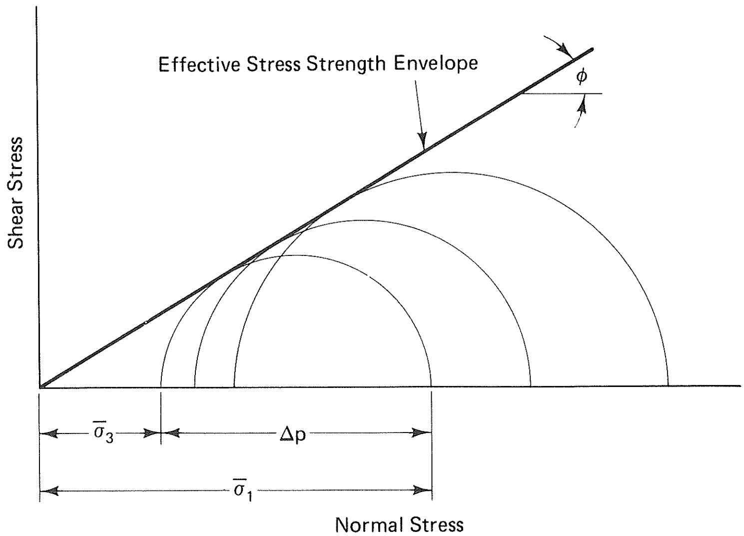


FIGURE 8-15 Results of consolidated-drained triaxial tests on normally consolidated clay.

Consolidated undrained shear The consolidated undrained (CU) test is performed by placing a saturated clay specimen in the chamber, introducing lateral (confining) pressure, and allowing the specimen to consolidate under the lateral pressure by leaving the drain lines open. Drain lines are then closed and axial load applied at a fairly rapid rate without allowing further drainage. With no drainage during axial load application, a buildup of excess pore pressure will result. [Initial excess pore pressure (μ_i) equals applied lateral pressure (σ_3) minus the pressure to which the sample had been consolidated ($\bar{\sigma}_c$)—that is, $\mu_i = \sigma_3 - \bar{\sigma}_c$. Hence, if σ_3 equals $\bar{\sigma}_c$, initial excess pore pressure in the specimen will be zero. If σ_3 is greater than $\bar{\sigma}_c$, initial pore pressure will be positive; if it is less than $\bar{\sigma}_c$, initial pore pressure will be negative.] The pore pressure (μ) during the test must be measured to obtain the effective stress needed to plot Mohr's circle [effective stress ($\bar{\sigma}$) equals total pressure (σ) minus pore pressure (μ)—that is, $\bar{\sigma} = \sigma - \mu$] (see Fig. 8-16). Pore pressure measurement can be accomplished by a pressure-measuring device connected to the drain lines at each end of the specimen.

Results of a CU test are also commonly presented with Mohr's circles plotted in terms of total stress (σ). The strength envelope in this case is referred to as the *total stress strength envelope.* Both the effective stress strength envelope and total stress strength envelope obtained from a CU test are shown in Fig. 8-16. It will be noted that Mohr's circle has equal diameters for total stresses and effective stresses but the Mohr's circle for effective stresses is displaced leftward by an amount equal to the pore pressure at failure (μ_f) (see Fig. 8-16).

If several CU tests are performed on the same clay initially consolidated under different lateral pressures (σ_3), the total stress strength envelope is ap-

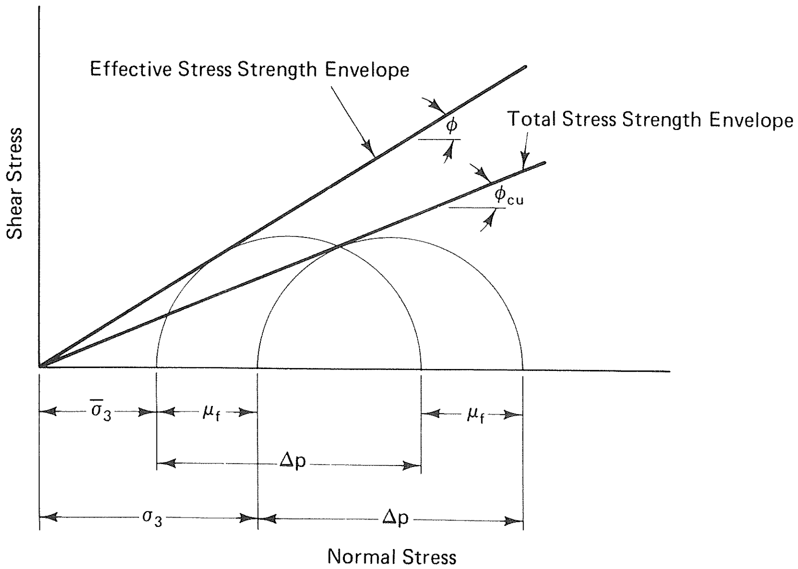


FIGURE 8-16 Results of consolidated-undrained triaxial tests on normally consolidated clay.

proximately a straight line passing through the origin (see Fig. 8-16). Hence, results of CU triaxial tests on normally consolidated clays can be expressed by Coulomb's equation [Eq. (2-15)] as

$$s = \sigma \tan \phi_{CU} \tag{8-9}$$

where ϕ_{CU} is known as the consolidated undrained angle of internal friction.

Undrained shear The unconsolidated undrained (UU) shear test is performed by placing a specimen in the chamber and introducing lateral (confining) pressure without allowing the specimen to consolidate (drain) under the lateral pressure. Axial load is then applied without allowing drainage of the specimen.

Three Mohr's circles resulting from three UU tests run under different lateral pressures on an identical normally consolidated, saturated clay are plotted in Fig. 8-17 and labeled *A, B,* and *C*. It will be noted that all of the circles have equal diameters; hence the strength envelope is a horizontal line, which represents the undrained shear strength. Roughly the same effective stress at failure would result (see Mohr's circle *E* in Fig. 8-17) for all three tests if pore pressures were measured and subtracted from total pressures (see Fig. 8-17). Hence, in terms of effective stresses, all undrained tests are represented by Mohr's circle *E* in Fig. 8-17. When total stresses are plotted, the undrained test yields a series of Mohr's circles all having the same diameter, and the strength envelope for these forms a horizontal line (see Mohr's circles *A, B,* and *C* in Fig. 8-17).

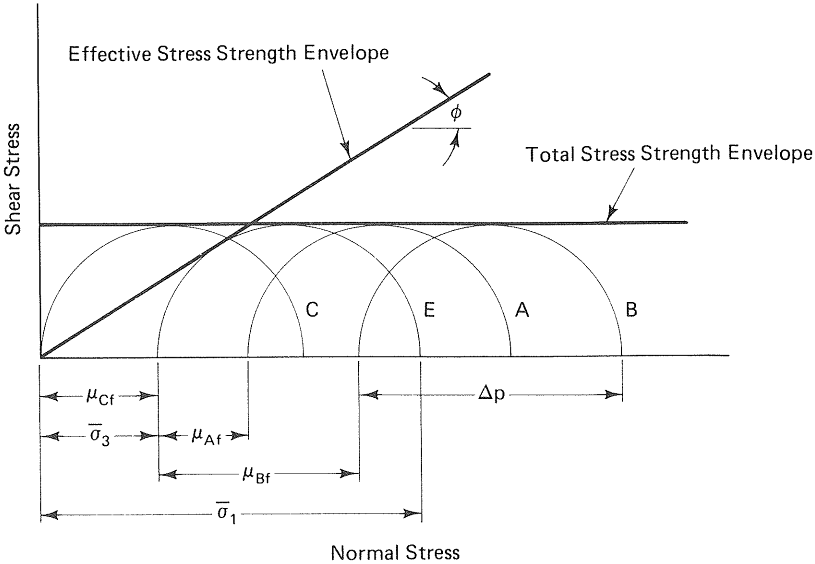


FIGURE 8-17 Results of unconsolidated-undrained triaxial tests on normally consolidated clay.

Mohr's circle C in Fig. 8-17 is a special case of the UU test where the total minor stress (σ_3) is zero. In other words, this test was performed without any lateral pressure; hence, this special case of the UU test is the "unconfined compression test" that was discussed in Sec. 8-2. The diameter of Mohr's circle C is equal to the applied axial vertical stress at failure; it is referred to as the *unconfined compressive strength* (q_u). Since the Mohr's circle is tangent to a horizontal strength envelope, the undrained shear strength under $\phi = 0$ conditions may be evaluated on the basis of unconfined compression tests as

$$s = c = \frac{q_u}{2} \tag{8-10}$$

[This phenomenon was stated previously (in Sec. 8-2) without detailed explanation at that point.]

When load is applied to a saturated, or nearly saturated, normally consolidated cohesive soil (most clays in their natural condition are close to full saturation), water in the soil's voids carries the load first and consequently prevents the relatively small soil particles from coming into contact to develop frictional resistance. At that time, the soil's shear strength consists only of cohesion (i.e., $s = c$). As time goes on, water in the voids of cohesive soils is slowly expelled, and soil particles come together and offer frictional resistance. This increases the shear strength from $s = c$ to $s = c + \bar{\sigma} \tan \phi$ [see Eq. (2-15)]. Because permeability of cohesive soil is very low, the process of water being expelled or extruded from the voids is very slow, perhaps occurring over a period of years (i.e., the water content of clay does not change significantly for an appreciable time after application of a stress). What all of this means is that, immediately after a structure is built (i.e., immediately upon load application), the shear strength of a saturated normally consolidated cohesive soil consists only of cohesion. Therefore, in foundation design problems, the bearing capacity of normally consolidated cohesive soil should be estimated based on the assumption that soil behaves as if the angle of internal friction (ϕ) is equal to zero, and shear strength is equal to cohesion (the $\phi = 0$ concept). Such a design practice should be adequate at construction time, and subsequent increase in shear strength should give an added factor of safety to the foundation [1].

For most normally consolidated cohesive soils, shear strength is estimated from results of unconfined compression tests. Only for large projects and research work are the other types of shear tests generally justified. However, for soft and/or sensitive clays, shear strength is commonly obtained from results of field or laboratory vane tests (Sec. 8-2).

Overconsolidated Clay

As mentioned previously, overconsolidated clay has been subjected at some time in the past to pressure greater than that presently existing. If identical specimens of overconsolidated clay are sheared in a triaxial test under drained conditions, the resulting plots of data are as shown in Fig. 8-18. The intersection of the first strength envelope with the ordinate is the cohesion, or the cohesive shear strength. The greater the overconsolidated pressure, the higher will be both the line labeled ϕ_1 and the cohesion.

The slope of this line (ϕ_1) represents the degree of relaxation of shear strength after removal of the overconsolidated pressure. No strength is retained in sands, so ϕ_1 is steep and equal to ϕ, and c (cohesion) is zero. Considerable strength may be retained in clays, however. Therefore, ϕ_1 is flatter and c may be quite high. For stress combinations up to the overconsolidated pres-

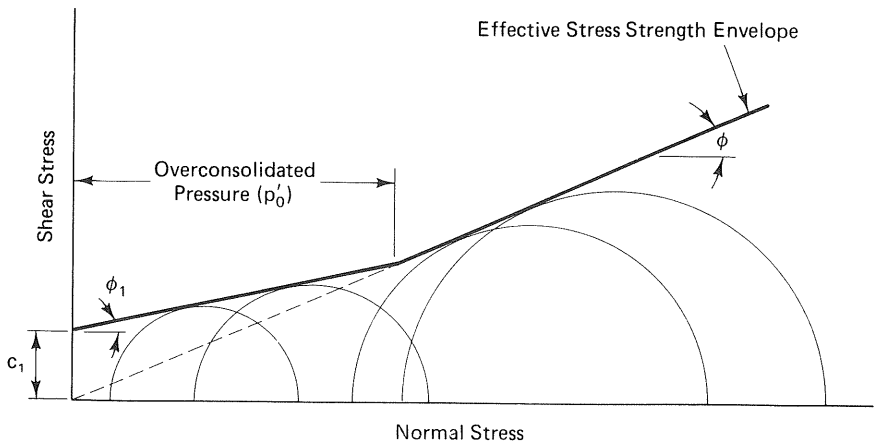


FIGURE 8-18 Results of consolidated-drained triaxial tests on overconsolidated clay.

sure (p_0'), a cohesion parameter (c_1) and reduced ϕ_1 are produced (see Fig. 8-18). Beyond p_0', the soil behaves as a normally consolidated clay.

Shear strength characteristics for an overconsolidated clay under drained conditions can be expressed by the following equations:

1. For effective normal pressure less than overconsolidated pressure [i.e., $\bar{\sigma} < p_0'$]

$$s = c_1 + \bar{\sigma} \tan \phi_1 \tag{8-11}$$

2. For effective normal pressure greater than overconsolidated pressure [i.e., $\bar{\sigma} > p_0'$]

$$s = \bar{\sigma} \tan \phi \tag{8-12}$$

The relative amount of overconsolidation is usually expressed as the *overconsolidation ratio* (OCR). It is defined as the ratio of overconsolidation pressure (p_0') to present overburden pressure (p_0). Hence,

$$\text{OCR} = \frac{p_0'}{p_0} \tag{8-13}$$

Under undrained conditions, the strength of an overconsolidated clay may be either smaller or larger than that under drained conditions, depending on the value of the OCR. If the OCR is in the range between 1 and about 4 to 8, the clay's volume tends to decrease during shear and, like that of normally consolidated clay, the undrained strength is less than the drained strength. If the OCR is greater than about 4 to 8, however, the clay's volume tends to increase during shear, pore water pressure decreases, and the undrained strength is greater than the drained strength.

For high OCRs, the undrained strength may be very high. However, clays with high OCRs exhibit strong negative pore pressures, which tend to draw water into the soil causing it to swell and lose strength. Accordingly, undrained shear strength cannot be depended upon. Furthermore, in most practical problems, to apply the $\phi = 0$ concept for an overconsolidated clay would lead to results on the unsafe side, whereas for a normally consolidated clay the $\phi = 0$ and $c > 0$ concept would lead to errors in the conservative direction. Hence, except for OCRs as low as possibly 2 to 4, the $\phi = 0$ concept should not be used for overconsolidated clays [5].

Sensitivity

Cohesive soils often lose some of their shear strength if disturbed. A parameter known as *sensitivity* indicates the amount of strength lost by soil as a result of thorough disturbance. To determine a soil's sensitivity, unconfined compression tests are performed on an undisturbed soil sample and on a remolded specimen of the same soil. Sensitivity (S_t) is the ratio of the unconfined compressive strength (q_u) of the undisturbed clay to that of the remolded clay. Hence,

$$S_t = \frac{(q_u)_{\text{undisturbed clay}}}{(q_u)_{\text{remolded clay}}} \tag{8-14}$$

Values of S_t for most clays range between 2 and about 4. For sensitive clays, they range from 4 to 8, and extrasensitive clays are encountered with values of S_t between 8 and 16. Clays with sensitivities greater than 16 are known as *quick clays* [5].

8-6 PROBLEMS

8-1 A specimen of dry sand was subjected to a direct shear test. A normal stress of 120.0 kN/m² was imposed on the specimen. The test was carried out until the specimen sheared, with a shear stress at failure of 75.0 kN/m². Determine the sand's angle of internal friction.

8-2 A series of direct shear tests was performed on a soil sample. Each test was carried until the specimen sheared (failed). Laboratory data for the tests are tabulated below. Determine the soil's cohesion and angle of internal friction.

Specimen Number	*Normal Stress (lb/ft²)*	*Shearing Stress (lb/ft²)*
1	200	450
2	400	520
3	600	590
4	1000	740

8-3 The data shown below were obtained in triaxial compression tests of three identical soil specimens. Find the soil's cohesion and angle of internal friction.

Specimen Number	*Minor Principal Stress,* σ_3 *(lb/in.²)*	*Major Principal Stress,* σ_1 *(lb/in.²)*
1	5	23.0
2	10	38.5
3	15	53.6

8-4 A cohesionless soil sample was subjected to a triaxial test. The sample failed when the minor principal stress (confining pressure) was 1200 lb/ft² and the deviator stress was 3000 lb/ft^2. Find the angle of internal friction for this soil.

8-5 A triaxial test was performed on a dry cohesionless soil under a confining pressure of 144.0 kN/m^2. If the sample failed when the deviator stress reached 395.8 kN/m^2, determine the soil's angle of internal friction.

8-6 A sample of dry cohesionless soil has an angle of internal friction of 35°. If the minor principal stress is 15 psi, at what values of deviator stress and major principal stress is the sample likely to fail?

8-7 A cohesive soil sample is subjected to an unconfined compression test. The sample fails at a pressure of 3850 lb/ft^2 [i.e., unconfined compressive strength (q_u) = 3850 lb/ft^2]. Determine the soil's cohesion.

8-8 Determine the orientation (angle θ) of the failure plane and the shear stress and normal stress on the failure plane of specimen No. 2 of Problem 8-3.

References

[1] Wayne C. Teng, *Foundation Design,* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1962.

[2] Joseph E. Bowles, *Engineering Properties of Soils and Their Measurement,* 2nd ed., McGraw-Hill Book Company, New York, 1978.

[3] *Standard Specifications for Transportation Materials and Methods of Sampling and Testing,* Part I, *Specifications,* 12th ed., AASHTO, 1978.

[4] Ralph B. Peck, Walter E. Hansen, and Thomas H. Thornburn, *Foundation Engineering,* 2nd ed., John Wiley & Sons, Inc., New York, 1974. Copyright © 1974, by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

[5] Karl Terzaghi and Ralph B. Peck, *Soil Mechanics in Engineering Practice,* John Wiley & Sons, Inc., New York, 1967. Copyright © 1967, by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

9

Shallow Foundations

9-1 INTRODUCTION

The word *foundation* might be defined in general as "that which supports something." Many universities, for example, have an "athletic foundation," which supports in part the school's sports program. In the context of this book, foundation normally refers to something that supports a structure, such as a column or wall, along with the loads carried by the structure.

Foundations may be characterized as shallow or deep. *Shallow foundations* are located just below the lowest part of the superstructures they support; *deep foundations* extend considerably down into the earth. In the case of shallow foundations, the means of support is usually either a *footing,* which is often simply an enlargement of the base of the column or wall it supports, or a *mat* or *raft foundation,* in which a number of columns are supported by a single slab. This chapter deals with shallow foundations—primarily footings. For deep foundations, the means of support is usually either a pier, caisson, or group of piles. These will be covered in Chaps. 10 and 11.

An individual footing is shown in Fig. 9-1a. For purposes of analysis, a footing such as this may be thought of as a simple flat plate or slab, usually square in plan, acted on by a concentrated load (the column) and a distributed load (soil pressure) (see Fig. 9-1b.). The enlarged size of footing (compared to the column it supports) gives an increased contact area between footing and soil; the increased area serves to reduce pressure on the soil to an allowable amount, thereby preventing excessive settlement or bearing failure of the foundation.

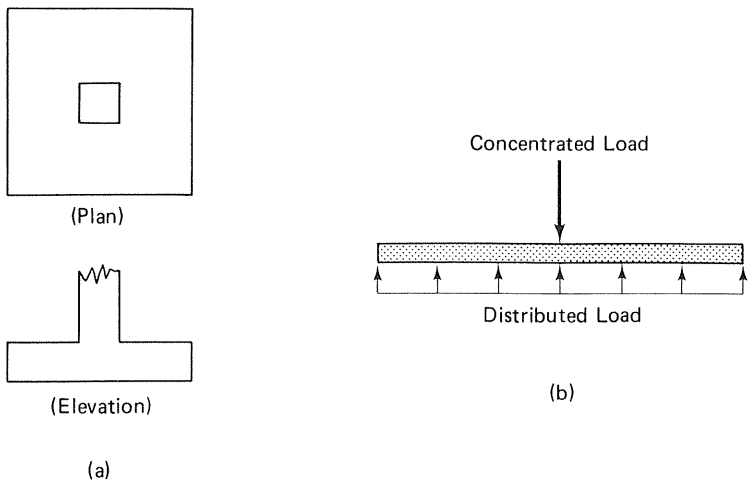


FIGURE 9-1 Individual footing.

Footings may be classified in several ways. For example, the footing depicted in Fig. 9-1a is an *individual footing.* Sometimes one large footing may support two or more columns, as shown in Fig. 9-2a. This is known as a *combined footing.* A footing extended in one direction to support a long structure such as a wall is called a *continuous footing*, or *wall footing* (Fig. 9-2b). Two or more footings joined by a beam (called a *strap*) are called a *strap footing* (Fig. 9-2c). A large slab supporting a number of columns not all of which are in a straight line is called a *mat* or *raft foundation* (Fig. 9-2d).

Foundations must be designed to satisfy three general criteria:

1. They must be located properly (both vertical and horizontal orientation) so as not to be adversely affected by outside influences.
2. They must be safe from bearing capacity failure (collapse).
3. They must be safe from excessive settlement.

Specific procedures for designing footings are given in the remainder of this chapter. For initial orientation and future quick reference, the following steps are offered at this point:

1. Calculate loads acting on the footing—Sec. 9-2.
2. Obtain soil profiles along with pertinent field and laboratory measurements and testing results—Chap. 3.
3. Determine depth and location of the footing—Sec. 9-3.
4. Evaluate bearing capacity of the supporting soil—Sec. 9-4.
5. Determine the size of footing—Sec. 9-5.

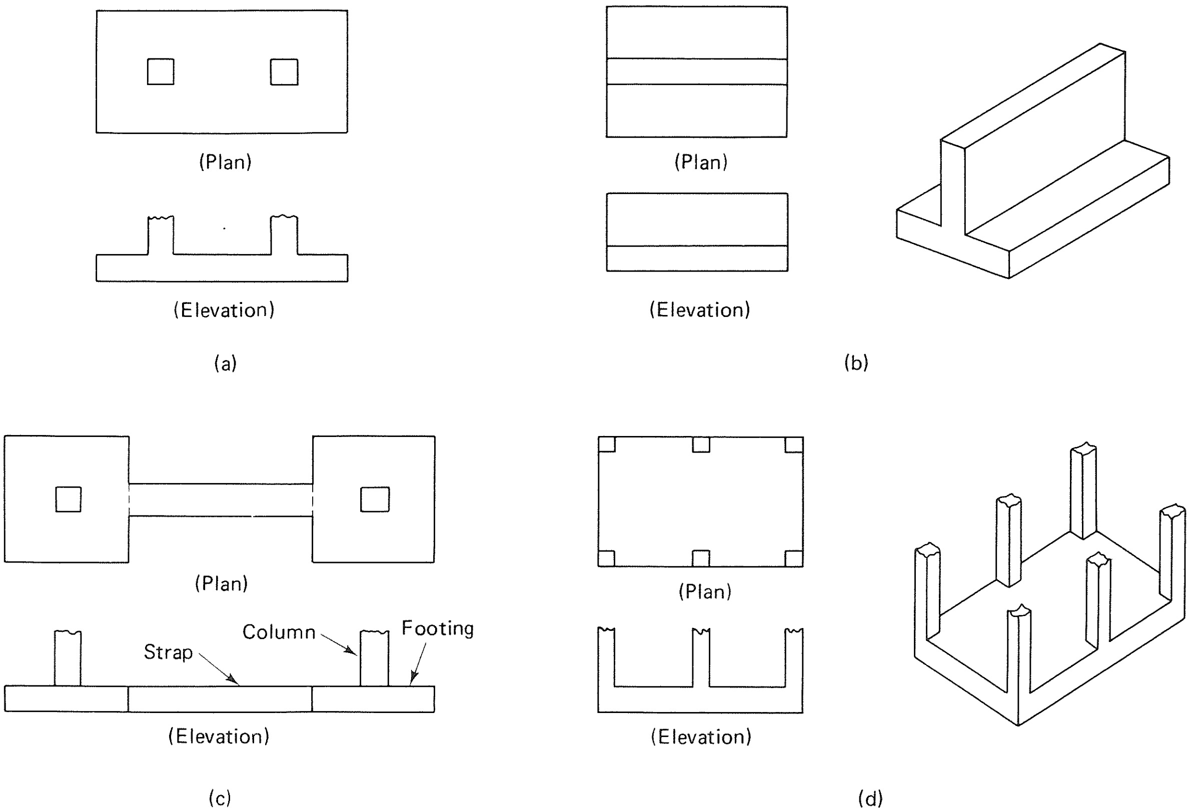


FIGURE 9-2 Classification of footings: (a) combined footing; (b) wall footing; (c) strap footing; (d) mat or raft foundation.

6. Compute footing contact pressure and check stability against sliding and overturning—Sec. 9-6.
7. Estimate total and differential settlements—Chap. 7 and Sec. 9-7.
8. Design the footing structure—Sec. 9-8.

9-2 LOADS ON FOUNDATIONS [1]

When designing any structure, whether it is a steel beam or column, a floor slab, a foundation, or whatever, it is of basic and utmost importance that an accurate estimation (computation) of all loads acting on the structure be made. In general, a structure may be subjected upon construction or sometime in the future to some or all of the following loads, forces, and pressures: (1) dead load, (2) live load, (3) wind load, (4) snow load, (5) earth pressure, (6) water pressure, and (7) earthquake forces. These are discussed in this section.

Dead Load

Dead load refers to the overall weight of a structure itself. It includes weight of materials permanently attached to the structure (such as flooring) and fixed service equipment (such as air-conditioning equipment). Dead load can be calculated if sizes and types of structural material are known. This presents a problem, however, because a structure's weight is not known until its size is known, and its size cannot be known until it has been designed based (in part) on its weight. Normal procedure is to estimate dead load initially, use the estimated dead load (along with live load, wind load, etc.) to size the structure, and then compare the sized structure's weight with the estimated dead load. If the sized structure's weight differs appreciably from the estimated dead load, the design procedure should be repeated, using a revised estimated weight.

Live Load

Live load refers to weights of applied bodies that are not permanent parts of a structure. These may be applied to the structure during part of its useful life (such as people, warehouse goods) or during its entire useful life (e.g., furniture). Because of the nature of live load, it is virtually impossible in most cases to calculate live load directly. Instead, live loads to be used in structural design are usually specified by local building codes. For example, a state building code might specify a minimum live loading of 100 lb/ft^2 for restaurants and 80 lb/ft^2 for office buildings.

Wind Load

Wind load, which is not considered as live load, may act on all exposed surfaces of structures. Additionally, overhanging parts of buildings may be subject to uplift pressure as a result of wind. Like design live loads, design wind loads are usually calculated based on building codes. For example, a building code might specify a design wind loading for a particular locality of 15 lb/ft^2 for buildings less than 30 ft tall and 40 lb/ft^2 for buildings taller than 1200 ft, with a sliding scale in between.

Snow Load

Snow load results from accumulation of snow on roofs and exterior flat surfaces. The unit weight of snow varies, but it averages about 6 lb/ft^3. Thus, an accumulation of several feet of snow over a large roof area results in a very heavy load. (Two feet of snow over a 50-ft by 50-ft roof would be about 15 tons.) Design snow loads are also usually based on building codes. A building code might specify a minimum snow loading of 30 lb/ft^2 for a specific locality.

Earth Pressure

Earth pressure produces a lateral force that acts against the portion of substructure lying below ground or fill level (see Fig. 9-3a). It is normally treated as dead load.

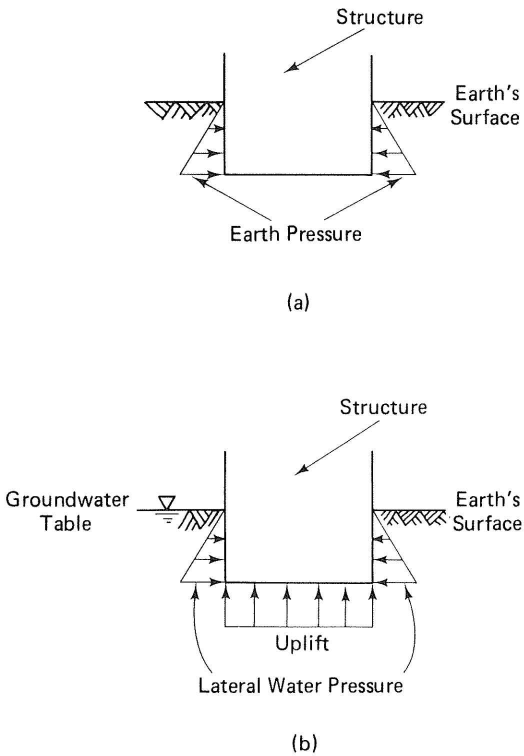


FIGURE 9-3 (a) Earth pressure; (b) water pressure.

Water Pressure

Water pressure may produce a lateral force similar in nature to that produced by earth pressure. Water pressure may also produce a force that acts upward (hydrostatic uplift) on the bottom of a structure. These forces are illustrated in Fig. 9-3b. Lateral water pressure is generally balanced, but hydrostatic uplift is not. It must be counteracted by the structure's dead load or else some provision must be made to anchor the structure.

Earthquake Forces

Earthquake forces may act laterally, vertically, or torsionally on a structure in any direction. A building code should be consulted for the specification of earthquake forces to be used in design.

9-3 DEPTH AND LOCATION OF FOUNDATIONS [2]

As related previously (Sec. 9-1), foundations must be located properly (both vertical and horizontal orientation) so as not to be adversely affected by outside influences. Outside influences include adjacent structures; water, including frost and groundwater; significant soil volume change; and underground defects (caves, for example). Thus, depth and location of foundations are dependent on the following factors:

1. Frost action.
2. Significant soil volume change.
3. Adjacent structures and property lines.
4. Groundwater.
5. Underground defects.

These factors are discussed in this section.

Frost Action

In areas where air temperature falls below the freezing point, moisture in the soil near the ground surface will freeze. When the temperature subsequently rises above the freezing point, any frozen moisture will melt. As soil moisture freezes and melts, it alternately expands and contracts. Repeated expansion and contraction of soil moisture beneath a footing may cause it to be lifted during cold weather and dropped during warmer weather. Such a sequence generally cannot be tolerated by the structure.

Frost action on footings is prevented by placing the foundation below the depth of soil that is ever expected to be penetrated by frost. Depth of frost penetration varies from 4 ft (1.2 m) or more in some northern states (Maine, Minnesota) to zero in parts of some southern states (Florida, Texas). Since frost penetration varies with location, local building codes often dictate minimum depths of footings.

Significant Soil Volume Change

Some soils, particularly certain clays having high plasticity, shrink and swell significantly upon drying and wetting, respectively. This volume change is greatest near the ground surface and decreases with increasing depth. The specific depth and volume change relationship for a particular soil is dependent on type of soil and level of groundwater. Volume change is usually insignificant below a depth of from 5 to 10 ft (1.5 to 3.0 m) and does not occur below the groundwater table. Generally speaking, soil beneath the center of a structure is more protected from sun and precipitation; therefore, moisture change and resulting soil movement are smallest there. On the other hand, soil beneath the edges of a structure is less protected, and moisture change and consequent soil movement are greatest there.

As in the case of frost action, significant soil volume change beneath a footing may cause alternate lifting and dropping of the footing. Possible means of avoidance include placing the footing below all strata that are subject to significant volume changes (those soils with plasticity indices over 30%), placing it below the zone of volume change, and placing it below any objects that could affect moisture content unduly (such as roots, steam lines, etc.).

Adjacent Structures and Property Lines

Adjacent structures and property lines often affect the horizontal location of a footing. Existing structures may be damaged by construction of new foundations nearby as a result of vibration, shock resulting from blasting, undermining by excavation, or lowering of the water table. After new foundations have been constructed, the (new) load they place on the soil may cause settlement of previously existing structures as a result of new stress patterns in the surrounding soil.

Since damage to existing structures by new construction may result in liability problems, new structures should be located and designed very carefully. In general, the deeper the new foundation and the closer to the old structure, the greater will be the potential for damage to the old structure. Accordingly, old and new foundations should be separated as much as is practical. This is particularly true if the new foundation will be lower than the old one. A general rule is that a straight line drawn downward and outward at a 45° angle from the end of the bottom of any new (or existing) higher footing should not intersect any existing (or new) lower footing (see Fig. 9-4).

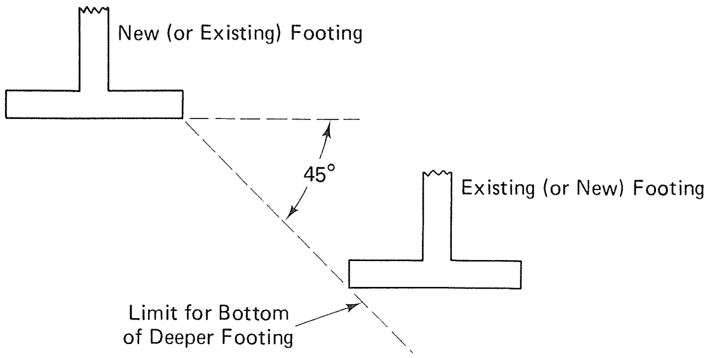


FIGURE 9-4 [2]

Special care must be exercised in placing a footing at or near a property line. One reason is that, since a footing is wider than the structure it supports, it is possible for part of the footing to extend across a property line and encroach on adjacent land, although the structure supported by the footing does not do so (see Fig. 9-5). Also, excavation for a footing at or near a property line may have a harmful effect (cave-in, for example) on adjacent land. Either of these cases could result in liability problems; hence, much care should be exercised when footings are required near property lines.

Groundwater

The presence of groundwater within soil immediately around a footing is undesirable for several reasons. First, footing construction below groundwater level is difficult and expensive. Generally, the area must be drained prior to construction. Second, groundwater around a footing can reduce the strength of soils by reducing their ability to carry foundation pressures. Third, groundwater around a footing may cause hydrostatic uplift problems; fourth, frost action may increase; and fifth, if groundwater reaches a structure's lowest floor, waterproofing problems are encountered. For these reasons, footings should be placed above the groundwater level whenever practical to do so.

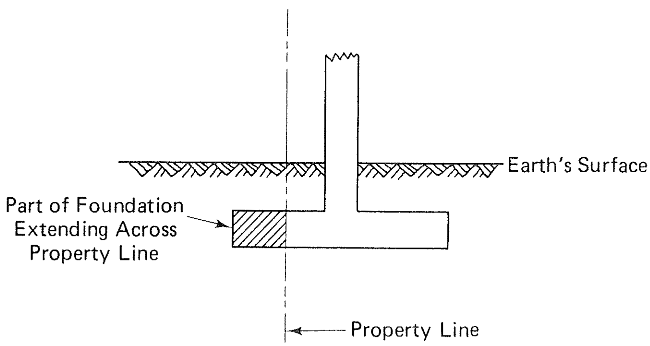


FIGURE 9-5

Underground Defects

Footing location is also affected by the presence of underground defects, including faults, caves, and mines. Additionally, man-made discontinuities such as sewer lines and underground cables and utilities must be considered when locating footings. Minor breaks in bedrock seldom are a problem unless they are active. Structures should never be built on or near tectonic faults that may slip. Certainly, foundations placed directly above a cave or mine should be avoided if at all possible. The man-made discontinuities cited above are often encountered, and generally foundations should not be placed above them. When they are encountered where a footing is desired, either they or the footing should be relocated. As a matter of fact, a survey of underground utility lines should be made prior to excavation for a foundation in order to avoid damage (or even an explosion) to the utility lines during excavation.

9-4 BEARING CAPACITY ANALYSIS

The conventional method of designing foundations is based on the concept of bearing capacity. One meaning of the verb "to bear" is to support or hold up. Generally, therefore, bearing capacity refers to the ability of a soil to support or hold up a foundation and structure. The *ultimate bearing capacity* of a soil refers to the loading per unit area that will just cause shear failure in the soil. It is given the symbol q_{ult}. The *allowable bearing capacity* (symbol q_a) refers to the loading per unit area that the soil is able to support without unsafe movement. It is the "design" bearing capacity. Allowable load is equal to allowable bearing capacity multiplied by area of contact between foundation and soil. Allowable bearing capacity is equal to ultimate bearing capacity divided by the factor of safety. A factor of safety of from 2.5 to 3 is commonly applied to the value of q_{ult}. Care must be taken to ensure that a footing design is safe with regard to (1) foundation failure (collapse) and (2) excessive settlement.

The basic principles governing bearing capacity theory as developed by Terzaghi [3] can be better followed by referring to Fig. 9-6. As load (Q) is ap-

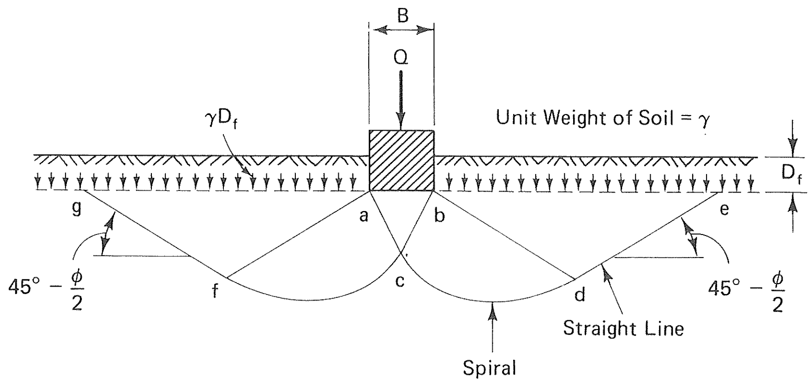


FIGURE 9-6

plied, the footing undergoes a certain amount of settlement as it is pushed downward; and a wedge of soil directly below the footing's base moves downward with the footing. The soil's downward movement is resisted by shear resistance of the foundation soil along slip surfaces *cde* and *cfg* and by the weight of the soil in sliding wedges *acfg* and *bcde*. For each set of assumed slip surfaces, the corresponding load Q that would cause failure can be determined. The set of slip surfaces giving the least applied load Q (that would cause failure) is the most critical; hence, the soil's ultimate bearing capacity (q_{ult}) is equal to the least load divided by the footing's area.

The following equations for calculating ultimate bearing capacity were developed by Terzaghi [3]:

Continuous footings (width B):

$$q_{\text{ult}} = cN_c + \gamma D_f N_q + 0.5\gamma B N_\gamma \tag{9-1}$$

Circular footings (radius R):

$$q_{\text{ult}} = 1.2cN_c + \gamma D_f N_q + 0.6\gamma R N_\gamma \tag{9-2}$$

Square footings (width B):

$$q_{\text{ult}} = 1.2cN_c + \gamma D_f N_q + 0.4\gamma B N_\gamma \tag{9-3}$$

The terms in these equations are

q_{ult} = ultimate bearing capacity

c = cohesion of soil

N_c, N_q, N_γ = Terzaghi's bearing capacity factors

γ = effective unit weight of soil

D_f = depth of footing, or distance from ground surface to base of footing

B = width of continuous or square footing

R = radius of a circular footing

The Terzaghi bearing capacity factors (N_c, N_q, N_γ) are functions of the soil's angle of internal friction (ϕ). The term in each equation containing N_c cites the influence of the soil's cohesion on its bearing capacity; the term containing N_q reflects the influence of surcharge; and that containing N_γ shows the influence of soil weight and foundation width or radius. Values of these factors for different values of ϕ are obtainable from Fig. 9-7.

Equations (9-1) through (9-3) are applicable for both cohesive and cohesionless soils. Dense sand and stiff clay produce what is called *general*

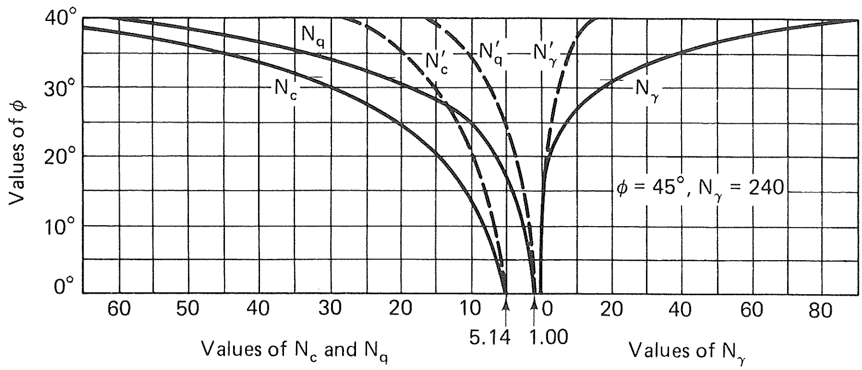


FIGURE 9-7 Chart showing relationship between ϕ and bearing capacity factors (values of N_γ after Meyerhof 1955). [3, 4]

shear (see Fig. 9-8), and the solid lines of Fig. 9-7 are used along with the angle of internal friction (ϕ) to determine values of N_c, N_q, and N_γ. On the other hand, loose sand and soft clay produce what is called *local shear* (see Fig. 9-8), and the dashed lines of Fig. 9-7 are used to determine values of N'_c, N'_q, and N'_γ. In the latter case (loose sand and soft clay), the term c (cohesion) in Eqs. (9-1) through (9-3) is replaced by c', which is equal to $\frac{2}{3}c$. Thus for loose sand and soft clay, terms c', N'_c, N'_q, and N'_γ are used in Eqs. (9-1) through (9-3) in place of the respective unprimed terms.

With cohesive soils, shear strength is most critical just after construction or as load is first applied, at which time shear strength is assumed to consist only of cohesion. In this case, ϕ (angle of internal friction) is taken to be zero [1]. There are several means of evaluating cohesion [c terms in Eqs. (9-1) through (9-3)]. One is to use the unconfined compression test for ordinary sensitive or insensitive normally consolidated clay. In this test, c is equal to half the unconfined compressive strength (i.e., $\frac{1}{2}q_u$) (see Chap. 8). For sensitive clay, a field vane test may be used to evaluate cohesion (see Chap. 3).

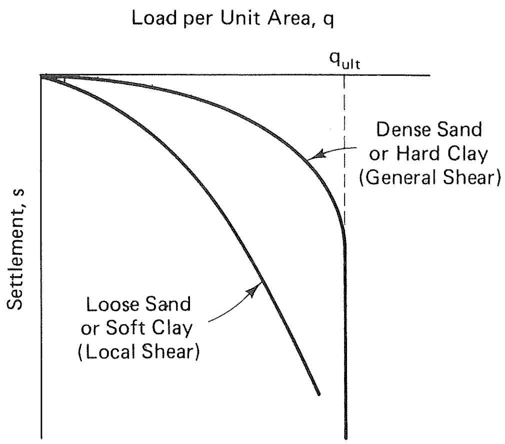


FIGURE 9-8

In the case of cohesionless soils, the c term in Eqs. (9-1) through (9-3) is zero. The value of ϕ may be determined by several methods. One is to use corrected standard penetration test values (see Chap. 3) and the curves shown in Fig. 9-9. One enters the graph at the upper right with a corrected SPT N-value, moves horizontally to the curve marked N, then vertically downward to the abscissa to read the value of ϕ. This value of ϕ can be used with the curves in Fig. 9-7 to determine values of N_q and N_γ. Or, values of N_q and N_γ may be determined using Fig. 9-9 by projecting vertically downward from the curve marked N to the curves marked N_q and N_γ, then projecting horizontally over to the ordinate to read values of N_q and N_γ, respectively. It is not necessary to determine a value of N_c, since c is zero for cohesionless soils, and thus the cN_c terms of Eqs. (9-1) through (9-3) are zero.

The four example problems that follow demonstrate the application of the Terzaghi bearing capacity formulas [that is, Eqs. (9-1) through (9-3)]. Example 9-1 deals with a wall footing in stiff clay. Example 9-2 has a square footing in a stiff cohesive soil. A circular footing on a mixed soil is covered in Example 9-3, and a square footing in a dense cohesionless soil is considered in Example 9-4.

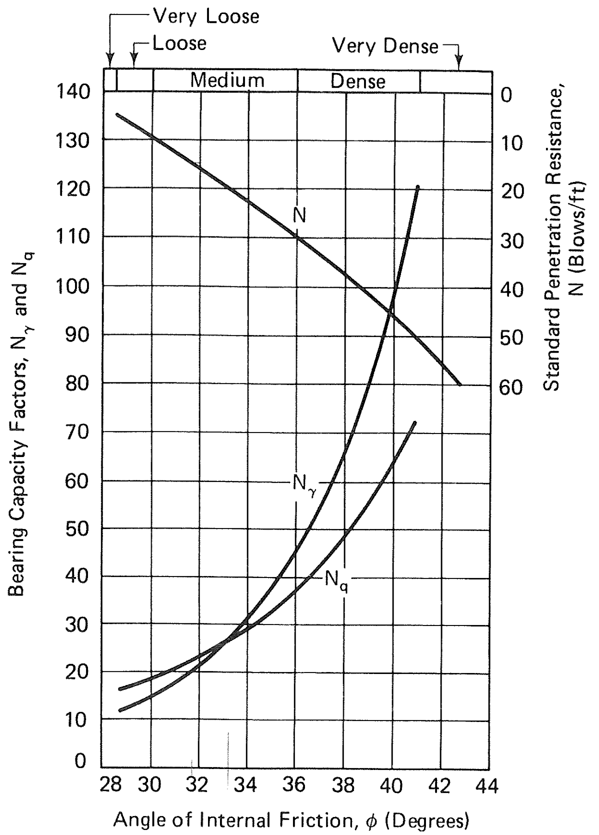


FIGURE 9-9 Curves showing the relationship between bearing capacity factors and ϕ, as determined by theory, and rough empirical relationship between bearing capacity factors or ϕ and values of standard penetration resistance N. [5]

EXAMPLE 9-1

Given

1. A strip of wall footing 3.5 ft wide is supported in a uniform deposit of stiff clay (see Fig. 9-10).
2. Unconfined compressive strength of this soil (q_u) = 2.8 kips/ft².
3. Unit weight of the soil (γ) = 130 lb/ft³.
4. Groundwater was not encountered during subsurface soil exploration.
5. Depth of wall footing (D_f) = 2 ft.

Required

1. Ultimate bearing capacity of this footing.
2. Allowable wall load, using a factor of safety of 3.

Solution

Since the supporting stratum is stiff clay, general shear condition is evident in this case.

1. For a continuous wall footing,

$$q_{\text{ult}} = cN_c + \gamma D_f N_q + 0.5\gamma BN_\gamma \tag{9-1}$$

$$c = \frac{q_u}{2} = \frac{2.8 \text{ kips/ft}^2}{2} = 1.4 \text{ kips/ft}^2$$

$$\gamma = 0.130 \text{ kip/ft}^3$$

$$D_f = 2 \text{ ft}$$

$$B = 3.5 \text{ ft}$$

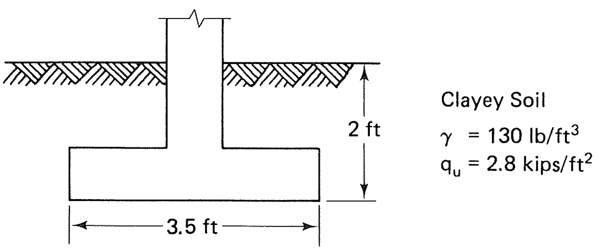


FIGURE 9-10

Using $c > 0$, $\phi = 0$ analysis for cohesive soil, when $\phi = 0$, Fig. 9-7 gives

$$N_c = 5.14$$
$$N_q = 1.0$$
$$N_\gamma = 0$$
$$q_{\text{ult}} = (1.4\ \text{kips/ft}^2)(5.14) + (0.130\ \text{kip/ft}^3)(2\ \text{ft})(1.0) + (0.5)(0.130\ \text{kip/ft}^3)(3.5\ \text{ft})(0) = 7.46\ \text{kips/ft}^2$$

2. $q_a = \dfrac{7.46\ \text{kips/ft}^2}{3} = 2.49\ \text{kips/ft}^2$

Allowable wall loading $= q_a \times B$

$= (2.49\ \text{kips/ft}^2)(3.5\ \text{ft}) = 8.72$ kips/ft of wall length

EXAMPLE 9-2

Given

1. A square footing with 5-ft sides is located 4 ft below the ground surface (see Fig. 9-11).
2. The groundwater table is at great depth and its effect may be ignored.
3. Subsoil consists of a thick deposit of stiff cohesive soil, with unconfined compressive strength (q_u) equal to 3000 lb/ft².
4. Unit weight (γ) of soil is 120 lb/ft³.

Required

Allowable bearing capacity using a factor of safety of 3.0.

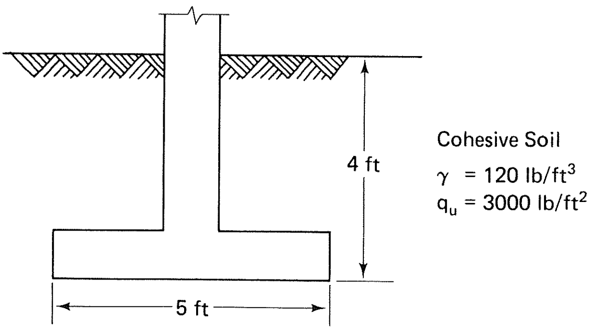


FIGURE 9-11

Solution

Since the supporting stratum is stiff clay, general shear condition is evident in this case. For a square footing,

$$q_{ult} = 1.2cN_c + \gamma D_f N_q + 0.4\gamma B N_\gamma \tag{9-3}$$

$$c = \frac{q_u}{2} = \frac{3000\ \text{lb/ft}^2}{2} = 1500\ \text{lb/ft}^2$$

$$\gamma = 120\ \text{lb/ft}^3$$

$$D_f = 4\ \text{ft}$$

$$B = 5\ \text{ft}$$

Using $c > 0$, $\phi = 0$ analysis for cohesive soil, when $\phi = 0$, Fig. 9-7 gives

$$N_c = 5.14$$

$$N_q = 1.0$$

$$N_\gamma = 0$$

$$q_{\text{ult}} = (1.2)(1500\ \text{lb/ft}^2)(5.14) + (120\ \text{lb/ft}^3)(4\ \text{ft})(1.0) + (0.4)(120\ \text{lb/ft}^3)(5\text{ft})(0) = 9730\ \text{lb/ft}^2$$

$$q_a = \frac{9730\ \text{lb/ft}^2}{3} = 3240\ \text{lb/ft}^2$$

EXAMPLE 9-3

Given

1. A circular footing with a 1.52-m diameter is to be constructed 1.22 m below the ground surface (see Fig. 9-12).
2. Subsoil consists of a uniform deposit of dense soil having the following strength parameters:

   Angle of internal friction = 25°

   Cohesion = 48.0 kN/m²

3. The groundwater table is at great depth, and its effect can be ignored.

Required

The total allowable load (including column load and weight of footing and soil surcharge) the footing can carry, using a factor of safety of 3.

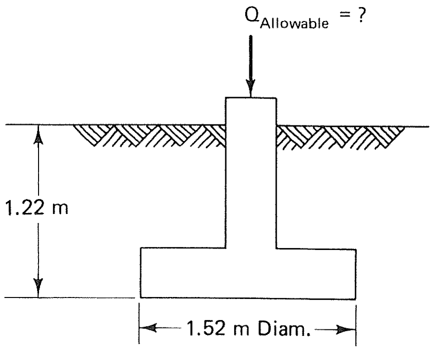


FIGURE 9-12

Solution

Since the soil supporting the footing is dense soil, general shear condition is evident. For a circular footing,

$$q_{\text{ult}} = 1.2cN_c + \gamma D_f N_q + 0.6\gamma R N_\gamma \tag{9-2}$$

$$c = 48.0 \text{ kN/m}^2$$

$$\gamma = 20.12 \text{ kN/m}^3$$

$$D_f = 1.22 \text{ m}$$

$$R = \frac{1.52 \text{ m}}{2} = 0.76 \text{ m}$$

From Fig. 9-7, with $\phi = 25°$,

$$N_c = 21$$

$$N_q = 10$$

$$N_\gamma = 6$$

Therefore,

$$q_{\text{ult}} = (1.2)(48.0 \text{ kN/m}^2)(21) + (20.12 \text{ kN/m}^3)(1.22 \text{ m})(10) + (0.6)(20.12 \text{ kN/m}^3)(0.76 \text{ m})(6) = 1510 \text{ kN/m}^2$$

$$q_a = \frac{1510 \text{ kN/m}^2}{3} = 503 \text{ kN/m}^2$$

Therefore,

$$Q_{\text{allowable}} = A \times q_a = \frac{(\pi)(1.52 \text{ m})^2}{4}(503 \text{ kN/m}^2) = 913 \text{ kN}$$

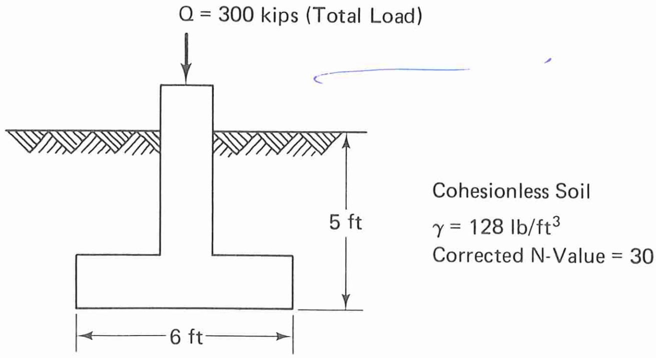


FIGURE 9-13

EXAMPLE 9-4

Given

1. A column footing 6 ft by 6 ft is buried 5 ft below the ground surface in a dense cohesionless soil (see Fig. 9-13).
2. Results of laboratory and field tests on the soil are as follows:
   (a) Unit weight of soil (γ) = 128 lb/ft^3.
   (b) Average corrected SPT N-value beneath the footing = 30.
   (c) Groundwater was not encountered during subsurface soil exploration.
3. The footing is to carry a total load of 300 kips, including column load, weight of footing, and weight of soil surcharge.

Required

The factor of safety against bearing capacity failure.

Solution

Since the supporting stratum is dense cohesionless soil, general shear condition is evident. Hence, the Terzaghi bearing capacity formula for a square footing is used with $c = 0$, $\phi > 0$.
For a square footing,

$$q_{ult} = 1.2cN_c + \gamma D_f N_q + 0.4\gamma B N_\gamma \qquad (9\text{-}3)$$

$$c = 0 \text{ (cohesionless soil)}$$

$$\gamma = 128 \text{ lb/ft}^3$$

$$D_f = 5 \text{ ft}$$

$$B = 6 \text{ ft}$$

From Fig. 9-9, with corrected N-value = 30, $\phi = 36°$. Then from Fig. 9-7 with $\phi = 36°$ the following bearing capacity factors are obtained:

$$N_q = 37$$

$$N_\gamma = 42$$

$$q_{\text{ult}} = (1.2)(0)(N_c) + (128\ \text{lb/ft}^3)(5\ \text{ft})(37) + (0.4)(128\ \text{lb/ft}^3)(6\ \text{ft})(42)$$

$$= 36{,}600\ \text{lb/ft}^2 \text{ or } 36.6\ \text{kips/ft}^2$$

$$q_{\text{actual}} = \frac{Q}{A} = \frac{300\ \text{kips}}{6\ \text{ft} \times 6\ \text{ft}} = 8.33\ \text{kips/ft}^2$$

$$\text{Factor of safety against bearing capacity failure} = \frac{q_{\text{ult}}}{q_{\text{actual}}} = \frac{36.6\ \text{kips/ft}^2}{8.33\ \text{kips/ft}^2}$$

$$= 4.4 > 3.0 \text{ O.K.}$$

Effect of Water Table on Bearing Capacity [6]

Heretofore in this discussion of bearing capacity, it has been assumed that the water table was well below the footings and thus did not affect the soil's bearing capacity. This is not always the case, however. Depending on where the water table is located, two terms in Eqs. (9-1) through (9-3)—the γBN_γ (or γRN_γ) term and the $\gamma D_f N_q$ term—may require modification.

If the water table is at or above the footing's base, the soil's submerged unit weight (unit weight of soil minus unit weight of water) should be used in the γBN_γ (or γRN_γ) terms of Eqs. (9-1) through (9-3). If the water table is at distance B (note that B is the footing's width) or more below the footing's base (see Fig. 9-14), the water table is assumed to have no effect, and the soil's full unit weight should be used. If the water table is below the base of the footing but less than distance B below the base, a linearly interpolated value of effective unit weight should be used in the γBN_γ (or γRN_γ) terms. (That is, the soil's effective unit weight is considered to vary linearly from submerged unit

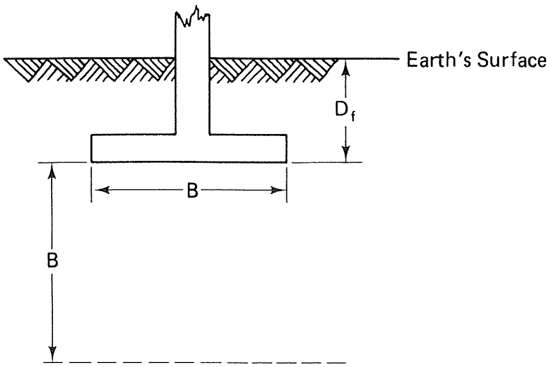


FIGURE 9-14

weight at the footing's base to full unit weight at distance B below the footing's base.)

If the water table is at the ground surface, the soil's submerged unit weight should be used in the $\gamma D_f N_q$ terms of Eqs. (9-1) through (9-3). If the water table is at or below the footing's base, the soil's full unit weight should be used in these terms. If the water table is between the footing's base and the ground surface, a linearly interpolated value of effective unit weight should be used in the $\gamma D_f N_q$ terms. (That is, the soil's effective unit weight is considered to vary linearly from submerged unit weight at the ground surface to full unit weight at the footing's base.)

Example 9-5 deals with a square footing in soft, loose soil with the groundwater table located at the ground surface.

EXAMPLE 9-5

Given

1. A 7-ft by 7-ft square footing is located 6 ft below the ground surface (see Fig. 9-15).
2. The groundwater table is located at the ground surface.
3. Subsoil consists of a uniform deposit of soft, loose soil. Laboratory test results are as follows:

$$\text{Angle of internal friction} = 20^\circ$$
$$\text{Cohesion} = 300\ \text{lb/ft}^2$$
$$\text{Unit weight of soil} = 105\ \text{lb/ft}^3$$

Required

Allowable (design) load that can be imposed on this square footing, using a factor of safety of 3.

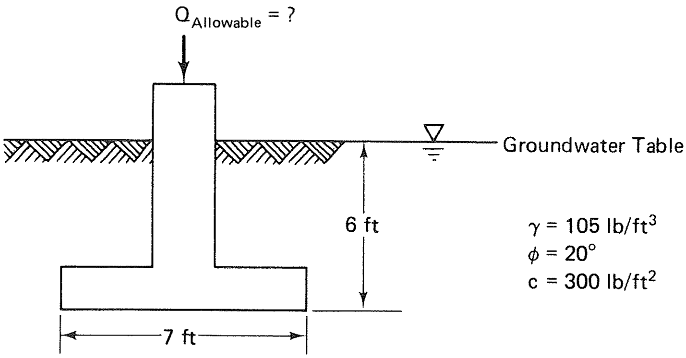


FIGURE 9-15

Solution

Since the footing is resting on soft, loose soil, Eq. (9-3) must be modified to reflect a local shear condition.

$$q_{\text{ult}} = 1.2c'N'_c + \gamma D_f N'_q + 0.4\gamma B N'_\gamma$$

$$c' = \tfrac{2}{3}c = \tfrac{2}{3} \times 300 \text{ lb/ft}^2 = 200 \text{ lb/ft}^2$$

With $\phi = 20°$, Fig. 9-7 gives

$$N'_c = 10$$

$$N'_q = 3$$

$$N'_\gamma = 2$$

$$B = 7 \text{ ft}$$

$$D_f = 6 \text{ ft}$$

$\gamma = 105 \text{ lb/ft}^3 - 62.4 \text{ lb/ft}^3 = 42.6 \text{ lb/ft}^3$ (with the water table at the ground surface, the soil's submerged unit weight must be used)

$$q_{\text{ult}} = (1.2)(200 \text{ lb/ft}^2)(10) + (42.6 \text{ lb/ft}^3)(6 \text{ ft})(3) + (0.4)(42.6 \text{ lb/ft}^3)(7 \text{ ft})(2) = 3410 \text{ lb/ft}^2$$

$$q_a = \frac{3410 \text{ lb/ft}^2}{3} = 1140 \text{ lb/ft}^2$$

$$Q_{allowable} = q_a \times \text{area of footing} = (1140 \text{ lb/ft}^2)(7 \text{ ft})(7 \text{ ft}) = 55{,}900 \text{ lb} = 55.9 \text{ kips}$$

Inclined Load

If a footing is subjected to an inclined load (Fig. 9-16), the inclined load can be resolved into vertical and horizontal components. The vertical component can then be used for bearing capacity analysis in the same manner as described previously. After the bearing capacity has been computed by the normal procedure, it must be corrected by an R_i factor, which can be obtained from Fig. 9-17. The footing's stability with regard to the inclined load's horizontal component must be checked by calculating the factor of safety against sliding (see Sec. 9-6).

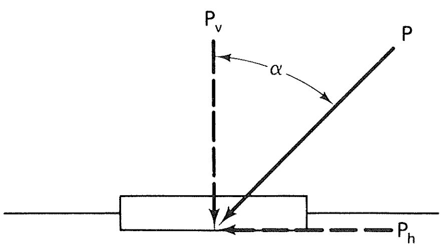


FIGURE 9-16 Footing subjected to an inclined load.

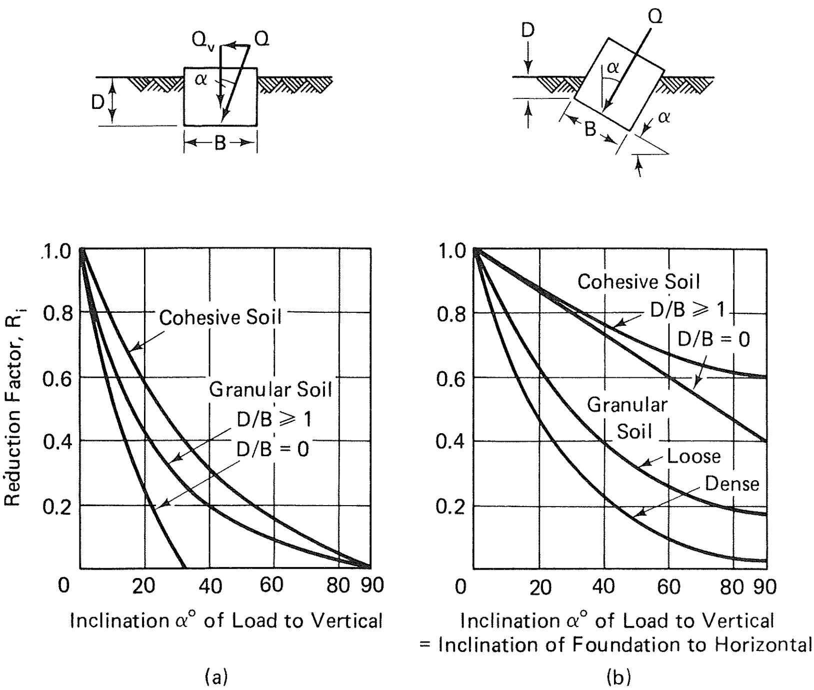


FIGURE 9-17 Inclined load reduction factors: (a) horizontal foundation [7]; (b) inclined foundation [8]. [1]

EXAMPLE 9-6

Given

A square footing (5 ft by 5 ft) is subjected to an inclined load as shown in Fig. 9-18.

Required

The factor of safety against bearing capacity failure.

Solution

For a square footing,

$$q_{\text{ult}} = 1.2cN_c + \gamma D_f N_q + 0.4\gamma B N_\gamma \qquad (9\text{-}3)$$

$$c = \frac{q_u}{2} = \frac{3600 \text{ lb/ft}^2}{2} = 1800 \text{ lb/ft}^2$$

$$\gamma = 130 \text{ lb/ft}^3$$

$$D_f = 5 \text{ ft}$$

$$B = 5 \text{ ft}$$

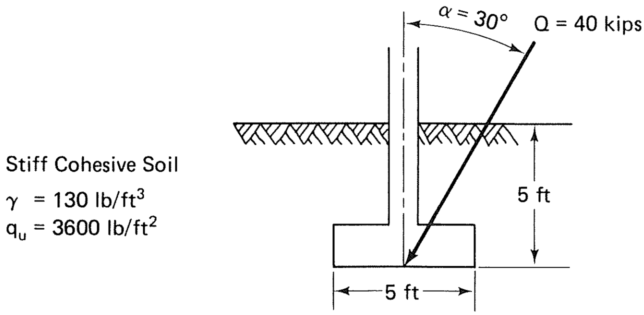


FIGURE 9-18

Use $c > 0$, $\phi = 0$ analysis for cohesive soil. Fig. 9-7 gives

$$N_c = 5.14$$
$$N_q = 1.0$$
$$N_\gamma = 0$$
$$q_{\text{ult}} = (1.2)(1800\ \text{lb/ft}^2)(5.14) + (130\ \text{lb/ft}^3)(5\ \text{ft})(1.0)$$
$$+ (0.4)(130\ \text{lb/ft}^3)(5\ \text{ft})(0) = 11{,}800\ \text{lb/ft}^2 = 11.8\ \text{kips/ft}^2$$

From Fig. 9-17, with $\alpha = 30°$ and cohesive soil, the reduction factor for inclined load = 0.42.

$$\text{Corrected } q_{\text{ult}} \text{ for inclined load} = (0.42)(11.8\ \text{kips/ft}^2) = 4.96\ \text{kips/ft}^2$$
$$Q_v = Q \cos 30° = (40\ \text{kips})(0.866) = 34.6\ \text{kips}$$

$$\text{Factor of safety} = \frac{Q_{\text{ult}}}{Q_v} = \frac{(4.96\ \text{kips/ft}^2)(5\ \text{ft} \times 5\ \text{ft})}{34.6\ \text{kips}} = 3.6$$

Eccentric Load [1]

Design of a footing is somewhat more complicated if it must support an eccentric load. Eccentric loads result from loads applied somewhere other than the footing's centroid or from applied moments, such as those resulting at the base of a tall column from wind loads on the structure. Footings with eccentric loads may be analyzed for bearing capacity by two methods: (1) the concept of "useful width" and (2) application of "reduction factors."

In the useful width method, only that part of the footing that is symmetrical with regard to the load is used to determine bearing capacity by the usual method, with the remainder of the footing being ignored. Thus, in Fig. 9-19, with (eccentric) load applied at the point indicated, the shaded area is symmetrical with regard to the load, and it is used to determine bearing capacity. That area is equal to $L \times (B - 2e_b)$ in this example.

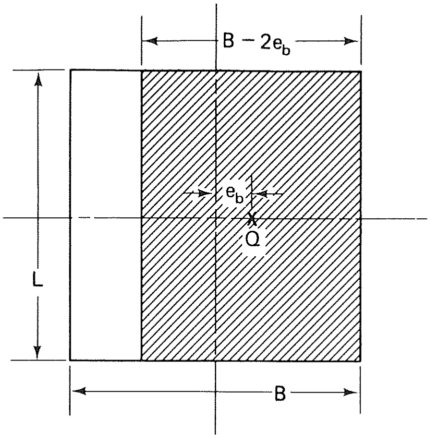


FIGURE 9-19 [1]

Upon reflection, it will be observed that this method means mathematically that the bearing capacity decreases linearly as eccentricity (distance e_b in Fig. 9-19) increases. This linear relationship has been confirmed in the case of cohesive soils. With cohesionless soils, however, a more nearly parabolic bearing capacity reduction has been determined [8]. The linear relationship for cohesive soils and the parabolic relationship for cohesionless soils are illustrated in Fig. 9-20. Since the useful width method is based on a linear bearing capacity reduction, it is recommended that this method be used only with cohesive soils.

To use the reduction factors method, bearing capacity is computed by the normal procedure on the assumption that the load is applied at the centroid of the footing. The computed value of bearing capacity is then corrected for eccentricity by multiplying by a reduction factor (R_e) obtained from Fig. 9-21.

Example 9-7 shows how bearing capacity can be calculated for an eccentric load in a cohesive soil by each of the methods described above.

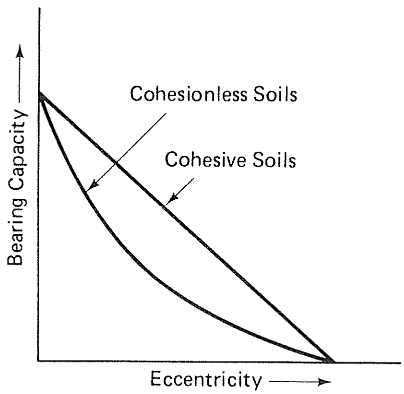


FIGURE 9-20

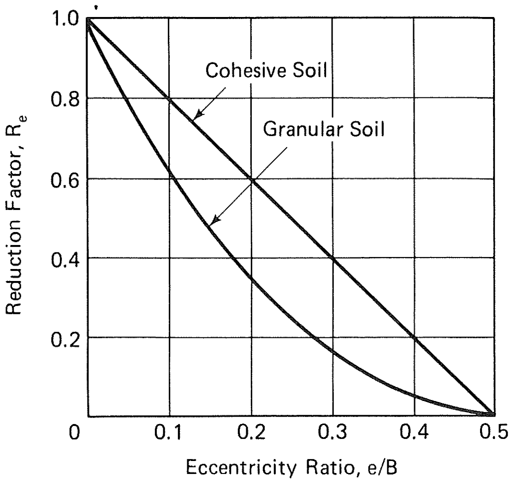


FIGURE 9-21 Eccentric load reduction factors. [1, 7]

EXAMPLE 9-7

Given

1. A 5-ft by 5-ft square footing is located 4 ft below the ground surface.
2. The footing is subjected to an eccentric load of 75 kips (see Fig. 9-22).
3. Subsoil consists of a thick deposit of cohesive soil with q_u = 4.0 kips/ft^2 and γ = 130 lb/ft^3.
4. The water table is at great depth and its effect on bearing capacity may be ignored.

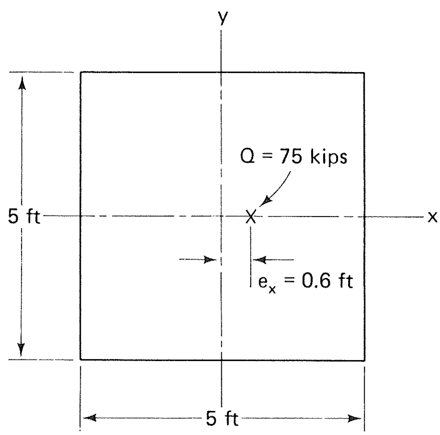


FIGURE 9-22

Required

The factor of safety against bearing capacity failure:

1. By the concept of useful width.
2. Using a reduction factor from Fig. 9-21.

Solution

1. *The concept of useful width:*
   From Fig. 9-23, the useful width is 3.8 ft.

$$q_{ult} = 1.2cN_c + \gamma D_f N_q + 0.4\gamma B N_\gamma \tag{9-3}$$

$$c = \frac{q_u}{2} = \frac{4.0 \text{ kips/ft}^2}{2} = 2.0 \text{ kips/ft}^2$$

Use $c > 0$, $\phi = 0$ analysis for cohesive soil. From Fig. 9-7,

$$\begin{aligned}
N_c &= 5.14\\
N_q &= 1.0\\
N_\gamma &= 0\\
\gamma &= 0.130 \text{ kip/ft}^3\\
B &= \text{useful width} = 3.8 \text{ ft}\\
q_{ult} &= (1.2)(2.0 \text{ kips/ft}^2)(5.14) + (0.130 \text{ kip/ft}^3)(4 \text{ ft})(1.0)\\
&\quad + (0.4)(0.130 \text{ kip/ft}^3)(3.8 \text{ ft})(0) = 12.9 \text{ kips/ft}^2
\end{aligned}$$

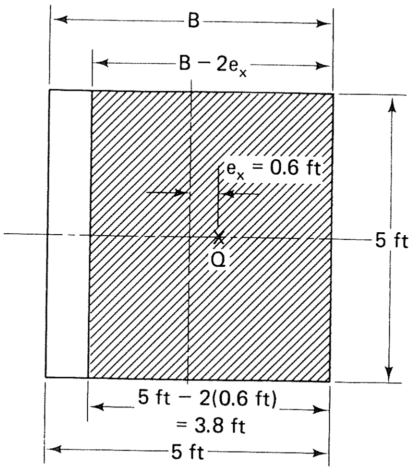


FIGURE 9-23

$$\text{Factor of safety} = \frac{12.9 \text{ kips/ft}^2}{\left(\dfrac{75 \text{ kips}}{3.8 \text{ ft} \times 5 \text{ ft}}\right)} = 3.27$$

2. *Using a reduction factor from Fig. 9-21:*

$$\text{Eccentricity ratio} = \frac{e_x}{B} = \frac{0.6 \text{ ft}}{5 \text{ ft}} = 0.12$$

For cohesive soil, Fig. 9-21 gives $R_e = 0.76$. Here q_{ult} is computed based on actual $B = 5$ ft.

$$q_{\text{ult}} = 1.2cN_c + \gamma D_f N_q + 0.4\gamma BN_\gamma \tag{9-3}$$

$$q_{\text{ult}} = (1.2)(2.0 \text{ kips/ft}^2)(5.14) + (0.130 \text{ kip/ft}^3)(4 \text{ ft})(1.0) + (0.4)(0.130 \text{ kip/ft}^3)(5 \text{ ft})(0) = 12.9 \text{ kips/ft}^2$$

$$q_{\text{ult}} \text{ corrected for eccentricity} = q_{\text{ult}} \times R_e = (12.9 \text{ kips/ft}^2)(0.76) = 9.80 \text{ kips/ft}^2$$

$$\text{Factor of safety} = \frac{9.80 \text{ kips/ft}^2}{\left(\dfrac{75 \text{ kips}}{5 \text{ ft} \times 5 \text{ ft}}\right)} = 3.27$$

Footings on Slopes

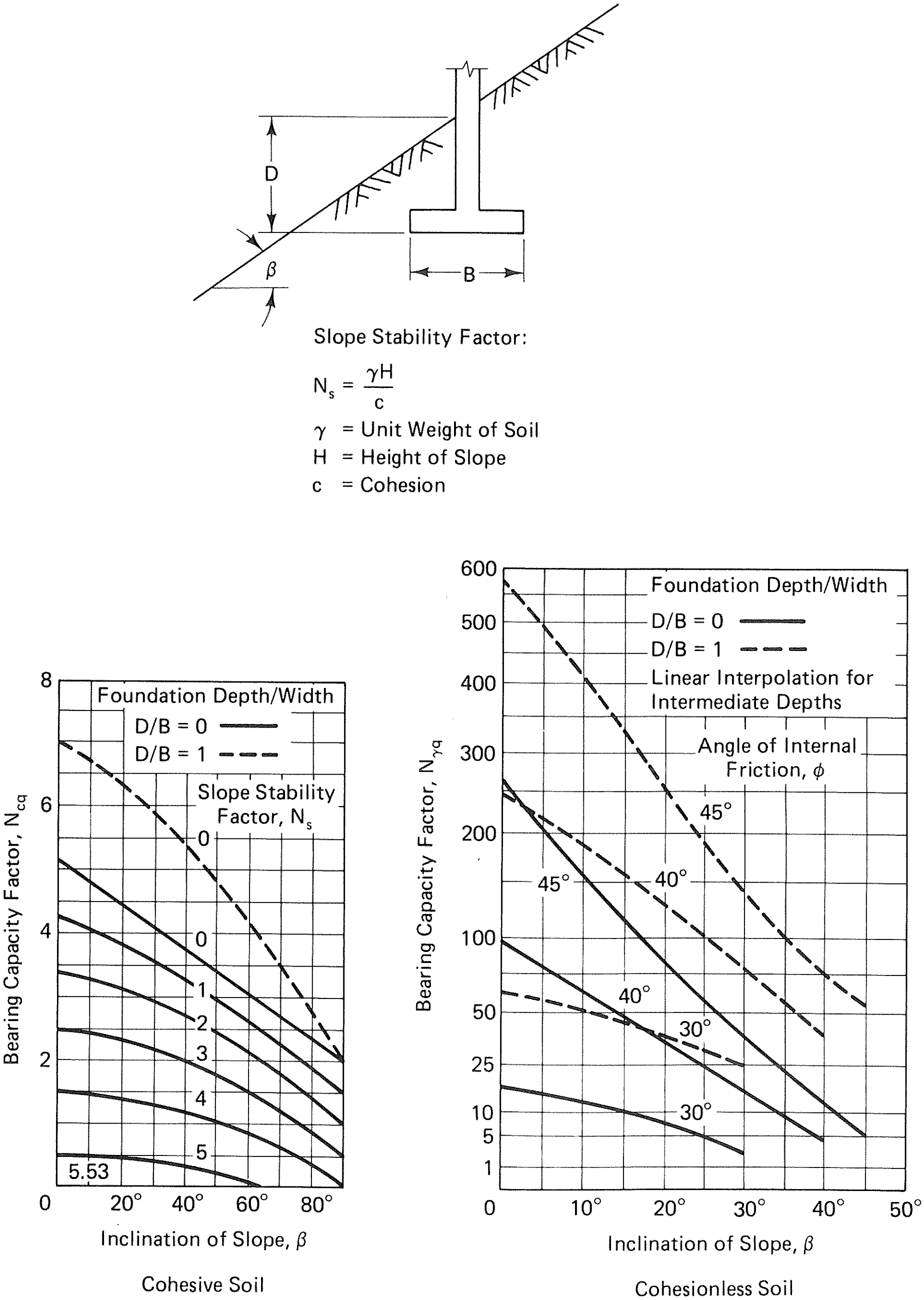
If footings are on slopes, their bearing capacities are less than if the footings were on level ground. In fact, bearing capacity of a footing is inversely proportional to ground slope.

Ultimate bearing capacity for continuous footings on slopes can be determined from the equation [9]

$$q_{\text{ult}} = cN_{cq} + \tfrac{1}{2}\gamma BN_{\gamma q} \tag{9-4}$$

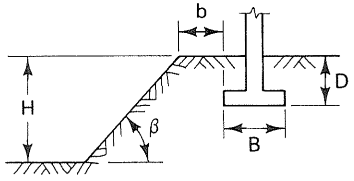
where N_{cq} and $N_{\gamma q}$ are the bearing capacity factors for footings on slopes and other terms are as defined previously for Eqs. (9-1) through (9-3). Bearing capacity factors for use in Eq. (9-4) can be determined from Fig. 9-24.

For circular or square footings on slopes, it is assumed that the ratios of their bearing capacities on the slope to their bearing capacities on level ground are in the same proportions as the ratio of bearing capacities of continuous footings on slopes to the bearing capacities of the continuous footings on level ground. Hence, their ultimate bearing capacities can be evaluated by first computing q_{ult} by Eq. (9-4) (i.e., as if the given footing on a slope were a continuous footing) and then multiplying that value by the ratio of q_{ult} computed from Eq. (9-2) or (9-3) (as if the given circular or square footing were on level ground) to q_{ult} determined from Eq. (9-1) (continuous footing on level ground). This may be expressed in equation form as [10]



(a)

FIGURE 9-24 Bearing capacity factors for continuous footing on (a) face of slope, and (b) top of slope. [9]



Note:

If B ⩽ H:

(1) Obtain N_{cq} from Diagram Using the Curves for $N_s = 0$.

(2) Interpolate for Values of $0 < D/B < 1$.

If B > H:

(1) Obtain N_{cq} from Diagram Using the Curves for the Calculated Slope Stability Factor N_s.

(2) Interpolate for Values of $0 < D/B < 1$.

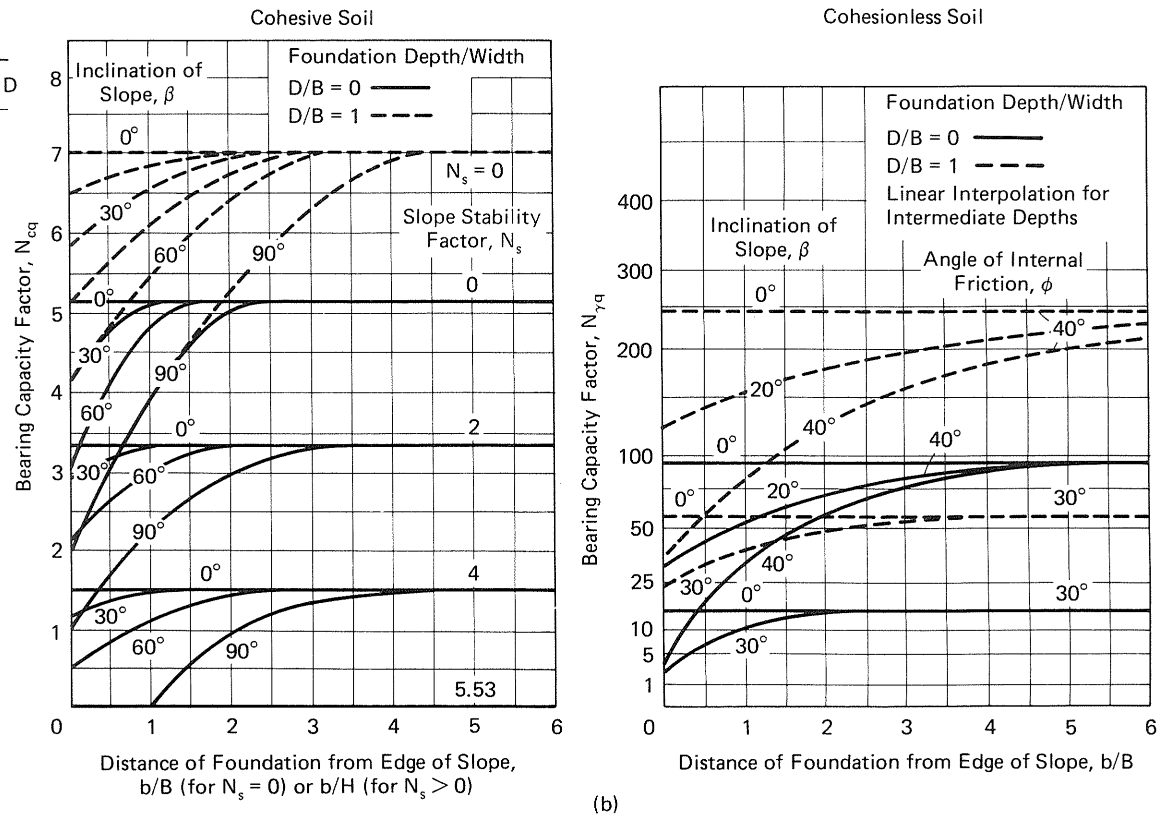


FIGURE 9-24 *(continued)*

$(q_{\text{ult}})_{c \text{ or } s \text{ footing on slope}}$

$$= (q_{\text{ult}})_{\text{continuous footing on slope}} \left[\frac{(q_{\text{ult}})_{c \text{ or } s \text{ footing on level ground}}}{(q_{\text{ult}})_{\text{continuous footing on level ground}}} \right] \quad (9\text{-}5)$$

Note: "*c or s*" footing denotes either circular or square footing.

Examples 9-8 and 9-9 consider footings on slopes.

EXAMPLE 9-8

Given

A bearing wall for a building is to be located close to a slope as shown in Fig. 9-25. The groundwater table is located at great depth.

Required

Allowable bearing capacity, using a factor of safety of 3.

Solution

From Eq. (9-4),

$$q_{\text{ult}} = cN_{cq} + \tfrac{1}{2}\,\gamma B N_{\gamma q} \quad (9\text{-}4)$$

$$c = 0$$

$$\gamma = 19.50 \text{ kN/m}^3$$

$$B = 1.0 \text{ m}$$

From Fig. 9-24b, with $\phi = 30°$,

$$\beta = 30°$$

$$\frac{b}{B} = \frac{1.5 \text{ m}}{1.0 \text{ m}} = 1.5$$

$$\frac{D_f}{B} = \frac{1.0 \text{ m}}{1.0 \text{ m}} = 1.0 \text{ (use the dashed line)}$$

$$N_{\gamma q} = 40$$

Therefore,

$$q_{\text{ult}} = (0)(N_{cq}) + (\tfrac{1}{2})(19.50 \text{ kN/m}^3)(1.0 \text{ m})(40) = 390 \text{ kN/m}^2$$

$$q_a = \frac{390 \text{ kN/m}^2}{3} = 130 \text{ kN/m}^2$$

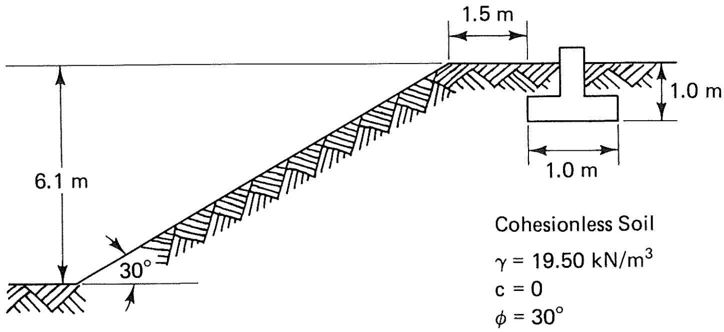


FIGURE 9-25

EXAMPLE 9-9

Given

Same conditions as Example 9-8, except that a 1.0-m by 1.0-m square footing is to be constructed on the slope.

Required

Allowable bearing capacity, using a factor of safety of 3.

Solution

From Eq. (9-5),

$$(q_{\text{ult}})_{\text{square footing on slope}} = (q_{\text{ult}})_{\text{continuous footing on slope}} \left[\frac{(q_{\text{ult}})_{\text{square footing on level ground}}}{(q_{\text{ult}})_{\text{continuous footing on level ground}}} \right] \qquad (9\text{-}5)$$

From Example 9-8,

$$(q_{\text{ult}})_{\text{continuous footing on slope}} = 390 \text{ kN/m}^2$$

From Eq. (9-3),

$$(q_{\text{ult}})_{\text{square footing on level ground}} = 1.2cN_c + \gamma D_f N_q + 0.4\,\gamma B N_\gamma \qquad (9\text{-}3)$$

From Fig. 9-7, with $\phi = 30°$,

$$N_c = 30$$
$$N_q = 18$$
$$N_\gamma = 16$$

$$(q_{\text{ult}})_{\text{square footing on level ground}} = (1.2)(0)(30) + (19.50 \text{ kN/m}^3)(1.0 \text{ m})(18) + (0.4)(19.50 \text{ kN/m}^3)(1.0 \text{ m})(16) = 475.8 \text{ kN/m}^2$$

From Eq. (9-1),

$$(q_{\text{ult}})_{\text{continuous footing on level ground}} = cN_c + \gamma D_f N_q + 0.5\gamma B N_\gamma \qquad (9\text{-}1)$$

$$(q_{\text{ult}})_{\text{continuous footing on level ground}} = (0)(30) + (19.50\ \text{kN/m}^3)(1.0\ \text{m})(18) + (0.5)(19.50\ \text{kN/m}^3)(1.0\ \text{m})(16) = 507.0\ \text{kN/m}^2$$

Therefore, substituting into Eq. (9-5),

$$(q_{\text{ult}})_{\text{square footing on slope}} = (390\ \text{kN/m}^2)\left[\frac{475.8\ \text{kN/m}^2}{507.0\ \text{kN/m}^2}\right] = 366\ \text{kN/m}^2$$

$$(q_a)_{\text{square footing on slope}} = \frac{366\ \text{kN/m}^2}{3} = 122\ \text{kN/m}^2$$

9-5 SIZE OF FOOTINGS

After the soil's allowable bearing capacity has been determined, the footing's required area can be determined by dividing footing load by allowable bearing capacity.

The following three example problems illustrate the sizing of footings based on allowable bearing capacity.

EXAMPLE 9-10

Given

The footing shown in Fig. 9-26 is to be constructed in a uniform deposit of stiff clay and must support a wall that imposes a loading of 152 kN/m of wall length.

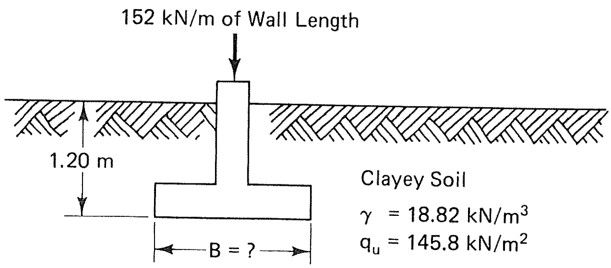


FIGURE 9-26

Required

The width of footing, using a factor of safety of 3.

Solution

From Eq. (9-1),

$$q_{\text{ult}} = c\text{N}_c + \gamma D_f N_q + 0.5\gamma B N_\gamma \tag{9-1}$$

$$c = \frac{q_u}{2} = \frac{145.8 \text{ kN/m}^2}{2} = 72.9 \text{ kN/m}^2$$

Using $c > 0$, $\phi = 0$ analysis for cohesive soil, when $\phi = 0$, Fig. 9-7 gives

$$N_c = 5.14$$
$$N_q = 1.0$$
$$N_\gamma = 0$$

$$q_{\text{ult}} = (72.9 \text{ kN/m}^2)(5.14) + (18.82 \text{ kN/m}^3)(1.20 \text{ m})(1.0) + (0.5)(18.82 \text{ kN/m}^3)(\text{B})(0) = 397.3 \text{ kN/m}^2$$

$$q_a = \frac{397.3 \text{ kN/m}^2}{3} = 132.4 \text{ kN/m}^2$$

$$\text{Required width of wall} = \frac{152.0 \text{ kN/m}}{132.4 \text{ kN/m}^2} = 1.15 \text{ m}$$

EXAMPLE 9-11

Given

1. A square footing rests on a uniform thick deposit of stiff clay with unconfined compressive strength (q_u) of 2.4 kips/ft².
2. The footing is located 4 ft below the ground surface and is to carry a total load of 250 kips (see Fig. 9-27).
3. The clay's unit weight is 125 lb/ft³.
4. Groundwater is at great depth.

Required

The necessary square footing dimension, using a factor of safety of 3. Also, find the necessary diameter of a circular footing, using a factor of safety of 3, if the footing is located 5 ft below the ground surface and is to carry a total load of 300 kips, and if q_u = 2.6 kips/ft².

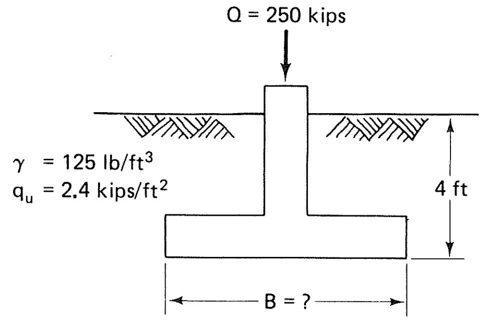


FIGURE 9-27

Solution

Since the supporting stratum is stiff clay, a condition of general shear governs this case.

$$q_{\text{ult}} = 1.2cN_c + \gamma D_f N_q + 0.4\gamma B N_\gamma \tag{9-3}$$

$$c = \frac{q_u}{2} = \frac{2.4 \text{ kips/ft}^2}{2} = 1.2 \text{ kips/ft}^2$$

Assuming $\phi = 0$, from Fig. 9-7,

$$N_c = 5.14$$
$$N_q = 1.0$$
$$N_\gamma = 0$$
$$\gamma = 0.125 \text{ kip/ft}^3$$
$$D_f = 4 \text{ ft}$$

$$q_{\text{ult}} = (1.2)(1.2 \text{ kips/ft}^2)(5.14) + (0.125 \text{ kip/ft}^3)(4 \text{ ft})(1.0) + (0.4)(0.125 \text{ kip/ft}^3)(B)(0) = 7.90 \text{ kips/ft}^2$$

$$q_a = \frac{7.90 \text{ kips/ft}^2}{3} = 2.63 \text{ kips/ft}^2$$

$$\text{Required footing area} = \frac{250 \text{ kips}}{2.63 \text{ kips/ft}^2} = 95.1 \text{ ft}^2$$

Therefore,

$$B^2 = 95.1 \text{ ft}^2$$
$$B = 9.75 \text{ ft}$$

A 10-ft by 10-ft square footing would probably be specified.

For a circular footing,

$$q_{\text{ult}} = 1.2cN_c + \gamma D_f N_q + 0.6\gamma R N_\gamma \tag{9-2}$$

$$c = \frac{q_u}{2} = \frac{2.6\ \text{kips/ft}^2}{2} = 1.3\ \text{kips/ft}^2$$

Assuming $\phi = 0$, from Fig. 9-7,

$$N_c = 5.14$$
$$N_q = 1.0$$
$$N_\gamma = 0$$
$$\gamma = 0.125\ \text{kip/ft}^3$$
$$D_f = 5\ \text{ft}$$

$$q_{\text{ult}} = (1.2)(1.3\ \text{kips/ft}^2)(5.14) + (0.125\ \text{kip/ft}^3)(5\ \text{ft})(1.0) + (0.6)(0.125\ \text{kip/ft}^3)(R)(0) = 8.64\ \text{kips/ft}^2$$

$$q_a = \frac{8.64\ \text{kips/ft}^2}{3} = 2.88\ \text{kips/ft}^2$$

$$\text{Required footing area} = \frac{300\ \text{kips}}{2.88\ \text{kips/ft}^2} = 104.2\ \text{ft}^2$$

Therefore,

$$\pi D^2/4 = 104.2\ \text{ft}^2$$
$$D = 11.5\ \text{ft}$$

EXAMPLE 9-12

Given

1. A uniform soil deposit has the following properties:

$$\gamma = 130\ \text{lb/ft}^3$$
$$\phi = 30^\circ$$
$$c = 800\ \text{lb/ft}^2$$

2. A proposed footing to be located 5 ft below the ground surface must carry a total load of 600 kips (see Fig. 9-28).
3. The groundwater table is at great depth and its effect may be ignored.

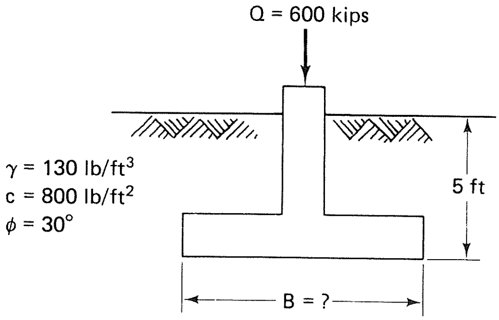


FIGURE 9-28

Required

Determine the required dimension of a square footing to carry the proposed total load of 600 kips using general shear condition and a factor of safety of 3.

Solution

$$q_{\text{ult}} = 1.2cN_c + \gamma D_f N_q + 0.4\gamma B N_\gamma \qquad (9\text{-}3)$$

$$c = 800 \text{ lb/ft}^2$$

$$\gamma = 130 \text{ lb/ft}^3$$

$$D_f = 5 \text{ ft}$$

$$\phi = 30°$$

From Fig. 9-7,

$$N_c = 30$$

$$N_q = 18$$

$$N_\gamma = 17$$

First trial

Assume that $B = 10$ ft.

$$q_{\text{ult}} = (1.2)(800 \text{ lb/ft}^2)(30) + (130 \text{ lb/ft}^3)(5 \text{ ft})(18) + (0.4)(130 \text{ lb/ft}^3)(10 \text{ ft})(17) = 49{,}300 \text{ lb/ft}^2$$

$$q_a = \frac{49{,}300 \text{ lb/ft}^2}{3} = 16{,}400 \text{ lb/ft}^2$$

$$\text{Required footing area} = \frac{600{,}000 \text{ lb}}{16{,}400 \text{ lb/ft}^2} = 36.6 \text{ ft}^2$$

$$B^2 = 36.6 \text{ ft}^2$$

$$B = 6.05 \text{ ft}$$

Second trial

Assume that $B = 6$ ft.

$$q_{ult} = (1.2)(800\ \text{lb/ft}^2)(30) + (130\ \text{lb/ft}^3)(5\ \text{ft})(18) + (0.4)(130\ \text{lb/ft}^3)(6\ \text{ft})(17) = 45{,}800\ \text{lb/ft}^2$$

$$q_a = \frac{45{,}800\ \text{lb/ft}^2}{3} = 15{,}300\ \text{lb/ft}^2$$

$$\text{Required footing area} = \frac{600{,}000\ \text{lb}}{15{,}300\ \text{lb/ft}^2} = 39.2\ \text{ft}^2$$

$$B^2 = 39.2\ \text{ft}^2$$

$$B = 6.26\ \text{ft}$$

A 6.5-ft by 6.5-ft square footing would probably be specified.

A footing sized in the manner just described and illustrated should be checked for settlement (see Chap. 7). If settlement is excessive (see Sec. 9-7), the size of footing should be revised.

9-6 CONTACT PRESSURE

The pressure acting between a footing's base and the soil below is referred to as *contact pressure.* A knowledge of contact pressure and associated shear and moment distribution is important in footing design.

Contact pressure can be computed using the flexural formula:

$$q = \frac{Q}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y} \tag{9-6}$$

where q = contact pressure
Q = total axial vertical load
A = area of footing
M_x, M_y = total moment about respective x and y axes
I_x, I_y = moment of inertia about respective x and y axes
x, y = distance from centroid to the point at which the contact pressure is computed along respective x and y axes

In the special case where moments about both x and y axes are zero, contact pressure is simply equal to total vertical load divided by the footing's area. In theory, contact pressure in this special case is uniform; but in practice, it tends to vary somewhat because of distortion settlement. It is generally assumed to be uniform, however, for design purposes.

Use of the flexural formula to determine contact pressure is illustrated by the following example problems. Example 9-13 illustrates computation of contact pressure when no moment is applied to either the x or y axis. Examples 9-14 and 9-15 illustrate the computation when moment is applied to one axis.

EXAMPLE 9-13

Given

1. A 5-ft by 5-ft square footing as shown in Fig. 9-29.
2. Centric column load on the footing = 50 kips.
3. Unit weight of soil = 120 lb/ft³.
4. Unit weight of concrete = 150 lb/ft³.
5. Cohesive soil with unconfined compressive strength = 3000 lb/ft².

Required

1. Soil contact pressure.
2. Factor of safety against bearing capacity failure.

Solution

1. *Soil contact pressure:*

$$q = \frac{Q}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y} \tag{9-6}$$

Since the column load is imposed on the centroid of the footing, $M_x = 0$ and $M_y = 0$.

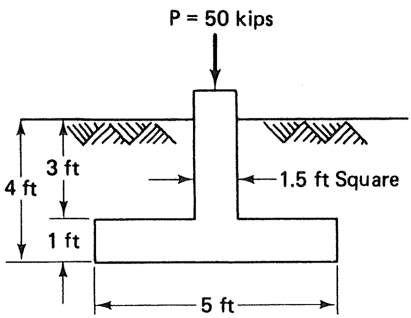


FIGURE 9-29

Q = total axial vertical load on the footing's base

Q = column load + weight of footing's base pad + weight of footing's pedestal + weight of backfill soil

$$\text{Column load} = 50 \text{ kips (given)}$$

$$\text{Weight of footing's base} = (5 \text{ ft})(5 \text{ ft})(1 \text{ ft})(0.150 \text{ kip/ft}^3) = 3.75 \text{ kips}$$

$$\text{Weight of footing's pedestal} = (1.5 \text{ ft})(1.5 \text{ ft})(3 \text{ ft})(0.150 \text{ kip/ft}^3) = 1.01 \text{ kips}$$

$$\text{Weight of backfill soil} = [(5 \text{ ft})(5 \text{ ft}) - (1.5 \text{ ft})(1.5 \text{ ft})](3 \text{ ft}) \times (0.120 \text{ kip/ft}^3) = 8.19 \text{ kips}$$

$$Q = 50 \text{ kips} + 3.75 \text{ kips} + 1.01 \text{ kips} + 8.19 \text{ kips} = 62.95 \text{ kips}$$

$$A = (5 \text{ ft})(5 \text{ ft}) = 25 \text{ ft}^2$$

$$q = \frac{62.95 \text{ kips}}{25 \text{ ft}^2} = 2.52 \text{ kips/ft}^2$$

Thus, soil contact pressure = 2.52 kips/ft² (see Fig. 9-30).

2. *Factor of safety against bearing capacity failure:*

From Eq. (9-3),

$$q_{ult} = 1.2\, cN_c + \gamma D_f\, N_q + 0.4\gamma BN_\gamma \tag{9-3}$$

$$c = \frac{q_u}{2} = \frac{3000 \text{ lb/ft}^2}{2} = 1500 \text{ lb/ft}^2 = 1.50 \text{ kips/ft}^2$$

From Fig. 9-7 using $c > 0$, $\phi = 0$ analysis,

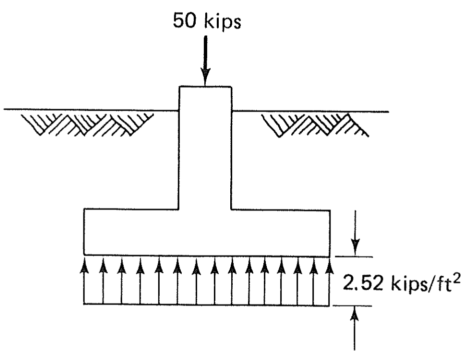


FIGURE 9-30

$N_c = 5.14$

$N_q = 1.0$

$N_\gamma = 0$

$D_f = 4 \text{ ft}$

$q_{ult} = (1.2)\ (1.50 \text{ kips/ft}^2)\ (5.14) + (0.120 \text{ kip/ft}^3)\ (4 \text{ ft})\ (1.0) + 0$

$= 9.73 \text{ kips/ft}^2$

$$\text{F.S.} = \frac{9.73 \text{ kips/ft}^2}{2.52 \text{ kips/ft}^2} = 3.86$$

EXAMPLE 9-14

Given

1. A 6-ft by 6-ft square column footing as shown in Fig. 9-31.
2. The column's base is hinged.
3. Load on the footing from the column (P) = 60 kips.

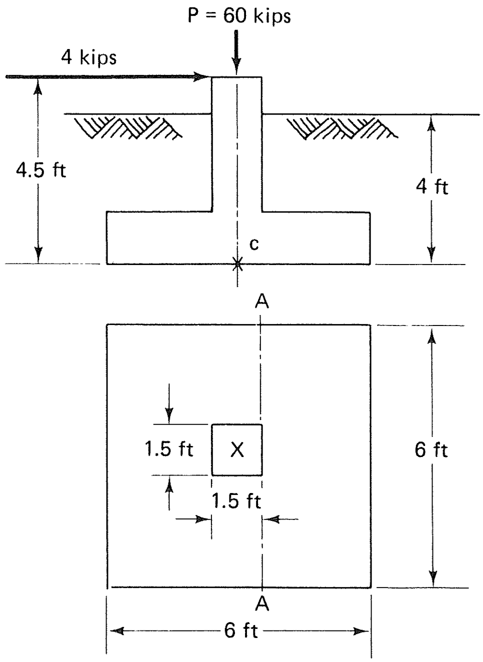


FIGURE 9-31

Weight of concrete footing including pedestal and base pad (W_1) = 9.3 kips.

Weight of backfill soil (W_2) = 11.2 kips.

4. Horizontal load acting on the base of the column = 4 kips.
5. Allowable bearing capacity of the supporting soil = 3.0 kips/ft².

Required

1. Contact pressure and soil pressure diagram.
2. Shear and moment at section A–A (see Fig. 9-31).
3. Factor of safety against sliding if the coefficient of friction between footing base and supporting soil is 0.40.
4. Factor of safety against overturning.

Solution

1. *Contact pressure and soil pressure diagram:*

$$q = \frac{Q}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y} \tag{9-6}$$

$$Q = P + W_1 + W_2 = 60 \text{ kips} + 9.3 \text{ kips} + 11.2 \text{ kips} = 80.5 \text{ kips}$$

$$A = 6 \text{ ft} \times 6 \text{ ft} = 36 \text{ ft}^2$$

$$M_y = 4 \text{ kips} \times 4.5 \text{ ft} = 18 \text{ ft-kips (take moment at point } C\text{; see Fig. 9-31)}$$

$$x = \frac{6 \text{ ft}}{2} = 3 \text{ ft}$$

$$I_y = \frac{(6 \text{ ft})(6 \text{ ft})^3}{12} = 108 \text{ ft}^4$$

$$M_x = 0$$

$$\frac{M_x y}{I_x} = 0$$

$$q = \frac{80.5 \text{ kips}}{36 \text{ ft}^2} \pm \frac{(18 \text{ ft-kips})(3 \text{ ft})}{108 \text{ ft}^4} = 2.24 \text{ kips/ft}^2 \pm 0.50 \text{ kip/ft}^2$$

$$q_{\text{right}} = 2.24 \text{ kips/ft}^2 + 0.50 \text{ kip/ft}^2 = 2.74 \text{ kips/ft}^2 < 3.0 \text{ kips/ft}^2 \quad \therefore \text{ O.K.}$$

$$q_{\text{left}} = 2.24 \text{ kips/ft}^2 - 0.50 \text{ kip/ft}^2 = 1.74 \text{ kips/ft}^2 < 3.0 \text{ kips/ft}^2 \quad \therefore \text{ O.K.}$$

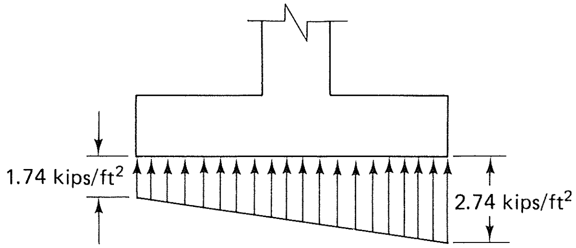


FIGURE 9-32

The pressure diagram is shown in Fig. 9-32.

2. *Shear and moment at section A–A:*

From Fig. 9-33, $\triangle FDG$ and $\triangle EDH$ are similar triangles. Therefore,

$$\frac{DE}{DF} = \frac{EH}{FG}$$

$$DF = 2.74 \text{ kips/ft}^2 - 1.74 \text{ kips/ft}^2 = 1.0 \text{ kip/ft}^2$$

$$EH = \frac{6 \text{ ft}}{2} - \frac{1.5 \text{ ft}}{2} = 2.25 \text{ ft (see Figs. 9-31 and 9-33)}$$

$$\text{FG} = 6 \text{ ft}$$

$$\frac{DE}{1.0 \text{ kip/ft}^2} = \frac{2.25 \text{ ft}}{6 \text{ ft}}$$

$$DE = 0.375 \text{ kip/ft}^2$$

$$\text{Shear at A–A} = (2.25 \text{ ft})(2.365 \text{ kips/ft}^2)(6 \text{ ft}) + (1/2)(2.25 \text{ ft})(0.375 \text{ kip/ft}^2)(6 \text{ ft})$$

$$= 31.93 \text{ kips} + 2.53 \text{ kips} = 34.46 \text{ kips}$$

$$\text{Moment at A–A} = (31.93 \text{ kips})\left(\frac{2.25 \text{ ft}}{2}\right) + (2.53 \text{ kips})(2/3 \times 2.25 \text{ ft})$$

$$= 39.7 \text{ ft-kips}$$

3. *Factor of safety against sliding:*

Factor of safety against sliding

$$= \frac{\text{total vertical load times coefficient of friction between base and soil}}{\sum \text{horizontal forces}}$$

$$= \frac{(60 \text{ kips} + 9.3 \text{ kips} + 11.2 \text{ kips})(0.40)}{4 \text{ kips}} = 8.05$$

4. *Factor of safety against overturning:*

See Fig. 9-34. By taking moments at point K, the factor of safety against overturning can be computed as follows:

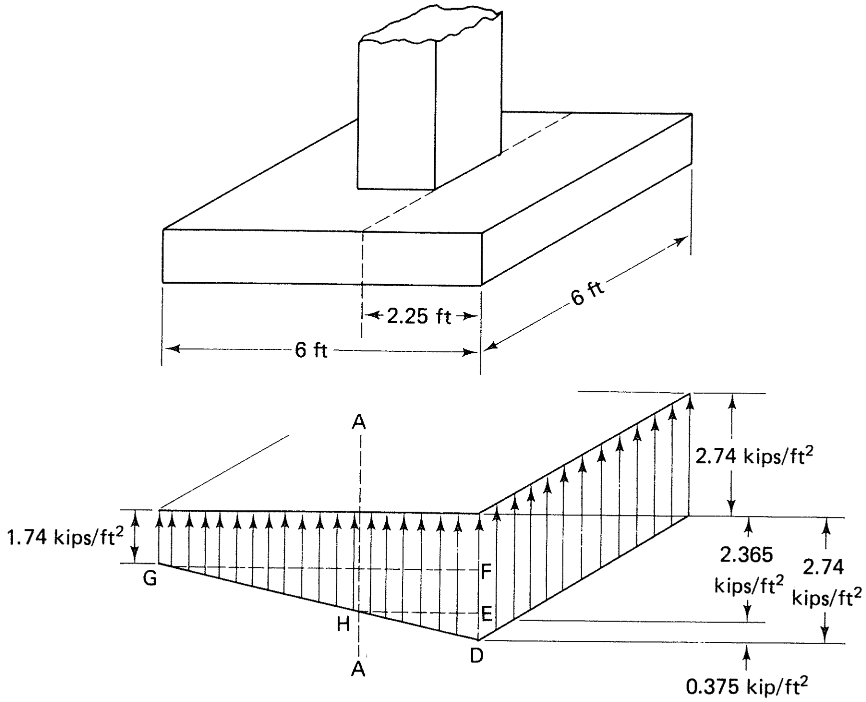


FIGURE 9-33

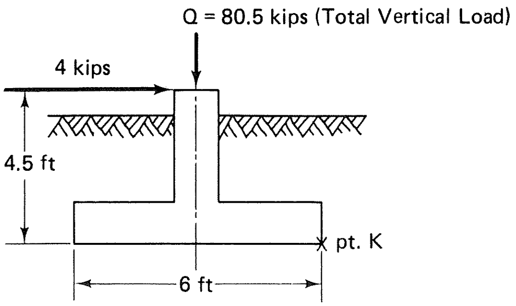


FIGURE 9-34

$$\text{F.S.} = \frac{\text{moment to resist turning}}{\text{turning moment}} = \frac{(80.5 \text{ kips})(6 \text{ ft}/2)}{(4 \text{ kips})(4.5 \text{ ft})} = 13.4$$

EXAMPLE 9-15

Given

1. A 7.5-ft by 10-ft rectangular column footing as shown in Fig. 9-35.
2. The column's base is fixed into the foundation.
3. Load on the footing from the column (P) = 50 kips.

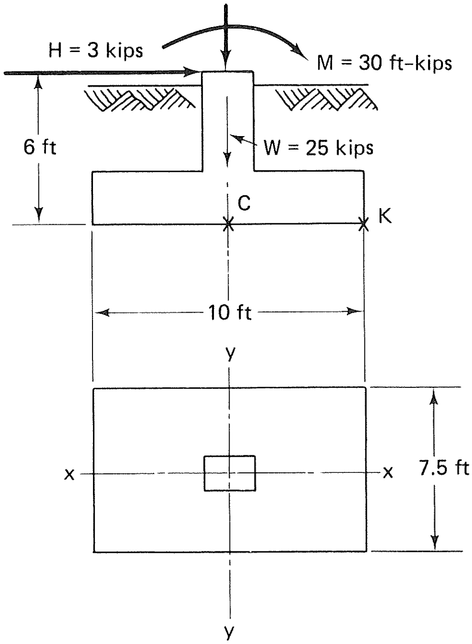


FIGURE 9-35

Weight of the concrete footing and weight of the backfill soil (W) = 25 kips.

Horizontal load acting on the column's base (H) = 3 kips.

Moment acting on the foundation (M) = 30 ft-kips.

4. Allowable bearing capacity of the soil = 2 kips/ft².

Required

1. Contact pressure and soil pressure diagram.
2. Factor of safety against overturning.

Solution

1. *Contact pressure and soil pressure diagram:*

$$q = \frac{Q}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y} \tag{9-6}$$

$$Q = 50 \text{ kips} + 25 \text{ kips} = 75 \text{ kips}$$

$$A = 7.5 \text{ ft} \times 10 \text{ ft} = 75 \text{ ft}^2$$

$$M_y = (3 \text{ kips})(6 \text{ ft}) + 30 \text{ ft-kips} = 48 \text{ ft-kips}$$

(take moments at point C; see Fig. 9-35)

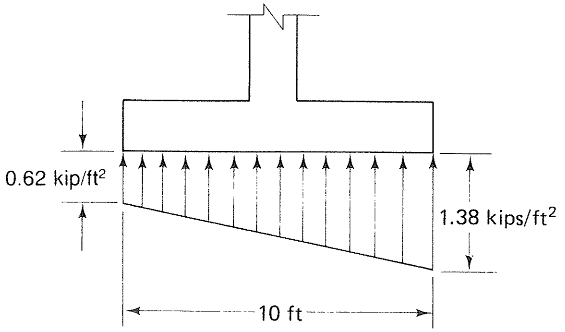


FIGURE 9-36

$$x = \frac{10 \text{ ft}}{2} = 5 \text{ ft}$$

$$I_y = \frac{(7.5 \text{ ft})(10 \text{ ft})^3}{12} = 625 \text{ ft}^4$$

$$M_x = 0$$

$$q = \frac{75 \text{ kips}}{75 \text{ ft}^2} \pm \frac{(48 \text{ ft-kips})(5 \text{ ft})}{625 \text{ ft}^4} = 1.00 \text{ kip/ft}^2 \pm 0.38 \text{ kip/ft}^2$$

$$q_{\text{right}} = 1.38 \text{ kips/ft}^2 < 2 \text{ kips/ft}^2 \quad \therefore \text{O.K.}$$

$$q_{\text{left}} = 0.62 \text{ kip/ft}^2 < 2 \text{ kips/ft}^2 \quad \therefore \text{O.K.}$$

The pressure diagram is shown in Fig. 9-36.

2. *Factor of safety against overturning:*

By taking moments at point K (Fig. 9-35),

$$\text{F.S.} = \frac{\text{moment to resist turning}}{\text{turning moment}} = \frac{(50 \text{ kips} + 25 \text{ kips})(10 \text{ ft}/2)}{(3 \text{ kips})(6 \text{ ft}) + (30 \text{ ft-kips})} = 7.8$$

Under certain conditions, such as very large applied moments, Eq. (9-6) may give a negative value for the contact pressure. This implies tension between footing and soil. Soil cannot furnish any tensile resistance; hence, the flexural formula is not applicable in this situation. Instead, contact pressure may be calculated according to the basic equations of statics in the following manner.

Referring to Fig. 9-37, by summing all forces in the vertical direction and all moments about point C and setting both sums equal to zero, the following two equations are obtained.

$$\sum V = 0 \uparrow_+$$

$$\left(\frac{q}{2}\right)(d)(L) - P - W = 0 \qquad (9\text{-}7)$$

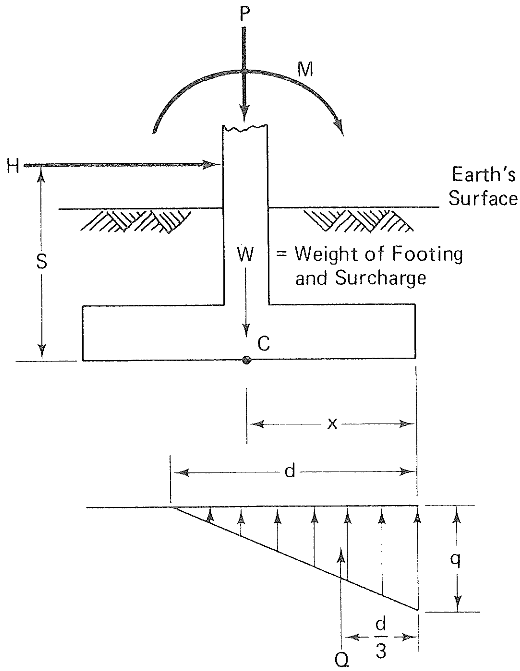


FIGURE 9-37

$$\sum M_c = 0 \; + \curvearrowright$$

$$M + (H)(S) - \left(\frac{q}{2}\right)(d)(L)\left(x - \frac{d}{3}\right) = 0 \qquad (9\text{-}8)$$

Since all terms in Eqs. (9-7) and (9-8) are known except q and d, the two equations may be solved simultaneously to determine q and d. With q and d both known, the soil pressure diagram may be drawn. This technique is illustrated by Example 9-16.

EXAMPLE 9-16

Given

A rectangular footing 5 ft by 7.5 ft loaded as shown in Fig. 9-38.

Required

Compute contact pressure and draw the soil pressure diagram.

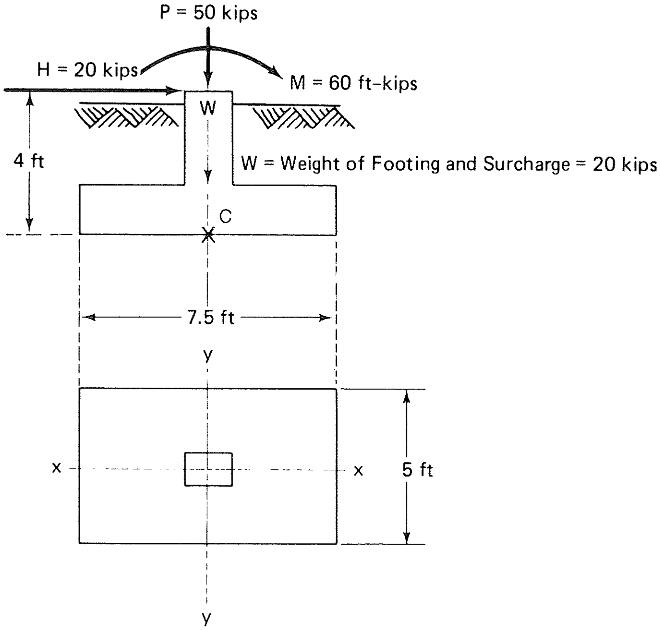


FIGURE 9-38

Solution

By the flexural formula,

$$q = \frac{Q}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y} \tag{9-6}$$

$$Q = 50 \text{ kips} + 20 \text{ kips} = 70 \text{ kips}$$

$$A = 5 \text{ ft} \times 7.5 \text{ ft} = 37.5 \text{ ft}^2$$

$$M_x = 0$$

$$M_y = (4 \text{ ft})(20 \text{ kips}) + 60 \text{ ft-kips} = 140 \text{ ft-kips}$$

(take moments at point *C*; see Fig. 9-38)

$$x = \frac{7.5 \text{ ft}}{2} = 3.75 \text{ ft}$$

$$I_y = \frac{(5 \text{ ft})(7.5 \text{ ft})^3}{12} = 176 \text{ ft}^4$$

$$q = \frac{70 \text{ kips}}{37.5 \text{ ft}^2} \pm \frac{(140 \text{ ft-kips})(3.75 \text{ ft})}{176 \text{ ft}^4} = 1.87 \text{ kips/ft}^2 \pm 2.98 \text{ kips/ft}^2$$

$$q_{\text{right}} = +4.85 \text{ kips/ft}^2$$

$$q_{\text{left}} = -1.11 \text{ kips/ft}^2$$

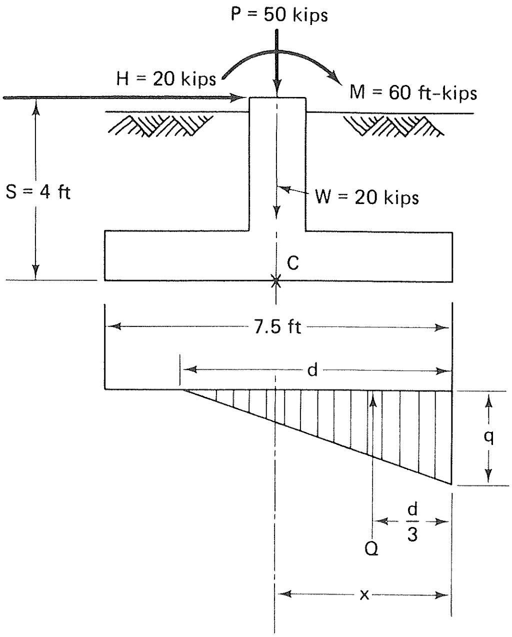


FIGURE 9-39

Since q_{left} has a negative value, the flexural formula is not applicable in this case. Solve this problem by $\Sigma V = 0$ and $\Sigma M_c = 0$ [i.e., Eqs. (9-7) and (9-8)]. Referring to Figs. 9-38 and 9-39,

$$\left(\frac{q}{2}\right)(d)(L) - P - W = 0 \tag{9-7}$$

$$\left(\frac{qd}{2}\right)(5\text{ ft}) = 70\text{ kips} \tag{A}$$

$$M + (H)(S) - \left(\frac{q}{2}\right)(d)(L)\left(x - \frac{d}{3}\right) = 0 \tag{9-8}$$

$$60\text{ ft-kips} + (20\text{ kips})(4\text{ ft}) - (70\text{ kips})\left(\frac{7.5\text{ ft}}{2} - \frac{d}{3}\right) = 0 \tag{B}$$

[Note that $(qd/2)(L)$ equals 70 kips, from Eq. (A).]
From Eq. (B),

$$60\text{ ft-kips} + 80\text{ ft-kips} - 262.5\text{ ft-kips} + \frac{70\text{ kips}}{3}d = 0$$

$$d = 5.25\text{ ft}$$

Substitute $d = 5.25$ ft into Eq. (A):

$$\left(\frac{q}{2}\right)(5.25\text{ ft})(5\text{ ft}) = 70\text{ kips}$$

$$q = 5.33\text{ kips/ft}^2$$

The pressure diagram is shown in Fig. 9-40.

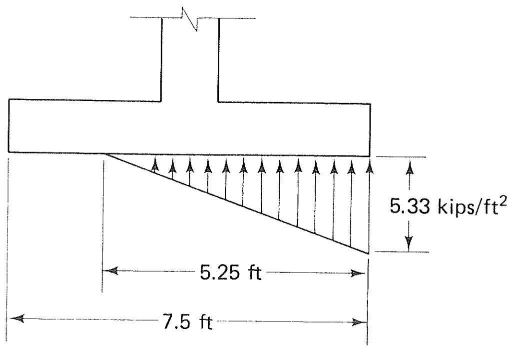


FIGURE 9-40

9-7 TOTAL AND DIFFERENTIAL SETTLEMENT

Previous material in this chapter has dealt primarily with bearing capacity analysis and prevention of bearing capacity failure of footings. Footings may also fail as a result of excessive settlement; thus, after the size of footing has been determined by bearing capacity analysis, footing settlement should be calculated and the design revised if the calculated settlement is considered to be excessive.

Calculation of settlement has already been covered (Chap. 7). Maximum permissible settlement depends primarily on the nature of the superstructure. Some suggested maximum permissible settlement values are given in Table 9-1.

TABLE 9-1 Maximum permissible settlement [11]

Limiting Factor or Type of Structure	Maximum permissible settlement	
	Differential[1]	*Total (in.)*
Drainage of floors	0.01–0.02L	6–12
Stacking, warehouse lift trucks	0.01L	6
Tilting of smokestacks, silos	0.004B	3–12
Framed structure, simple	0.005L	2–4
Framed structure, continuous	0.002L	1–2
Framed structure with diagonals	0.0015L	1–2
Reinforced concrete structure	0.002–0.004L	1–3
Brick walls, one-story	0.001–0.002L	1–2
Brick walls, high	0.0005–0.001L	1
Cracking of panel walls	0.003L	1–2
Cracking of plaster	0.001L	1
Machine operation, noncritical	0.003L	1–2
Crane rails	0.003L	
Machines, critical	0.0002L	

[1] *L* is the distance between adjacent columns; *B* is the width of base.

9-8 STRUCTURAL DESIGN OF FOOTINGS

As has been noted in Sec. 9-5, the required base area of a footing may be determined by dividing column load by allowable bearing capacity. Determining the thickness and shape of the footing and amount and location of reinforcing steel and performing other details of the actual structural design of footings are, however, ultimately the responsibility of a structural engineer.

In general, a soils engineer furnishes the contact pressure diagram and the shear and moment at a section (in the footing) at the face of the column, pedestal, or wall. This was demonstrated in Example 9-14 when the contact pressure diagram and the shear and moment at section A–A were determined. From this information, the structural engineer can do the actual structural design of the footing.

9-9 PROBLEMS

9-1 A strip of wall footing 3 ft wide is located 3.5 ft below the ground surface. Supporting soil has a unit weight of 125 lb/ft^3. Results of laboratory tests on the soil samples indicate that the supporting soil's cohesion and angle of internal friction are 1200 lb/ft^2 and 25°, respectively. Groundwater was not encountered during subsurface soil exploration. Determine allowable bearing capacity, using a factor of safety of 3.

9-2 A square footing with a size of 10 ft by 10 ft is located 8 ft below the ground surface. Subsoil consists of a thick deposit of stiff cohesive soil with unconfined compressive strength equal to 3600 lb/ft^2. The soil's unit weight is 128 lb/ft^3. Compute the ultimate bearing capacity.

9-3 A circular footing with a 1.22-m diameter is to be constructed 1.07 m below the ground surface. Subsoil consists of a uniform deposit of dense soil having a unit weight of 21.33 kN/m^3, an angle of internal friction of 20°, and a cohesion of 57.6 kN/m^2. The groundwater table is at great depth, and its effect can be ignored. Determine the safe total load (including column load and weight of footing and soil surcharge), using a factor of safety of 3.

9-4 A footing 8 ft by 8 ft is buried 6 ft below the ground surface in a dense, cohesionless soil. Results of laboratory and field tests on the supporting soil indicate that the soil's unit weight is 130 lb/ft^3 and the average corrected SPT N-value beneath the footing is 37. Compute the allowable (design) load that can be imposed onto this footing, using a factor of safety of 3.

9-5 A square footing with a size of 8 ft by 8 ft is to carry a total load of 40 kips. The depth of footing is 5 ft below the ground surface and groundwater is located at the ground surface. The subsoil consists of a uniform deposit of soft clay, the cohesion of which is 500 lb/ft^2. The soil's unit weight is 110 lb/ft^3. Compute the factor of safety against bearing capacity failure.

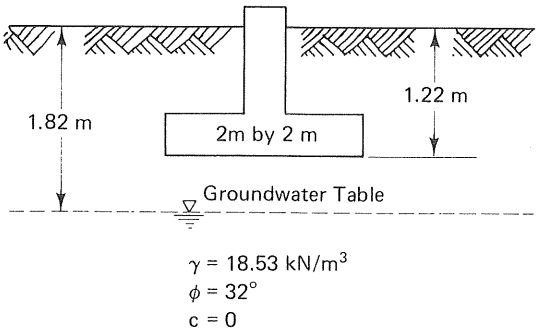


FIGURE 9-41

9-6 A square footing 2 m by 2 m is to be constructed 1.22 m below the ground surface, as shown in Fig. 9-41. The groundwater table is located 1.82 m below the ground surface. Subsoil consists of a uniform, medium dense, cohesionless soil with the following properties:

Unit weight of soil = 18.53 kN/m^3

Angle of internal friction = 32°

Cohesion = 0

Determine the foundation soil's allowable bearing capacity if a factor of safety of 3 is used.

9-7 A square footing is to be constructed on a uniform thick deposit of clay with unconfined compressive strength of 3 kips/ft^2. The footing will be located 5 ft below the ground surface and is designed to carry a total load of 300 kips. Unit weight of the supporting soil is 128 lb/ft^3. No groundwater was encountered during soil exploration. Considering general shear, determine the square footing dimension, using a factor of safety of 3.

9-8 A proposed square footing carrying a total load of 500 kips is to be constructed on a uniform thick deposit of dense cohesionless soil. The soil's unit weight is 135 lb/ft^3 and its angle of internal friction is 38°. The depth of footing is to be 5 ft. Determine the dimension of this proposed footing, using a factor of safety of 3.

9-9 A bearing wall for a building is to be located close to a slope, as shown in Fig. 9-42. The groundwater table is at great depth. Determine the foundation soil's allowable bearing capacity for the wall if a factor of safety of 3 is used.

9-10 Solve Problem 9-9 if the proposed footing is to be a 1.22-m by 1.22-m square footing (instead of a wall).

9-11 A wall footing is to be constructed on a uniform deposit of stiff clay, as shown in Fig. 9-43. The footing is to support a wall that imposes 130 kN/m of wall length. Determine the required width of footing if a factor of safety of 3 is used.

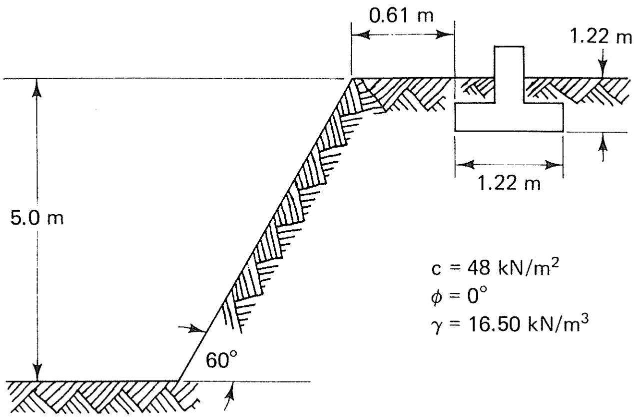


FIGURE 9-42

9-12 Compute and draw soil pressure diagrams for the footing shown in Fig. 9-44 for the following loads:

(a) $P = 70$ kips and $H = 20$ kips
(b) $P = 70$ kips and $H = 10$ kips

9-13 Considering general shear, compute the safety factor against a bearing capacity failure for each of the two loadings in Problem 9-12 if the bearing soil is:

(a) Cohesionless
$\phi = 30°$
$\gamma = 110$ lb/ft^3
$c = 0$

(b) Cohesive
$\phi = 0°$
$\gamma = 110$ lb/ft^3
$c = 3000$ lb/ft^2

In each case, groundwater is 10 ft below the base of the footing.

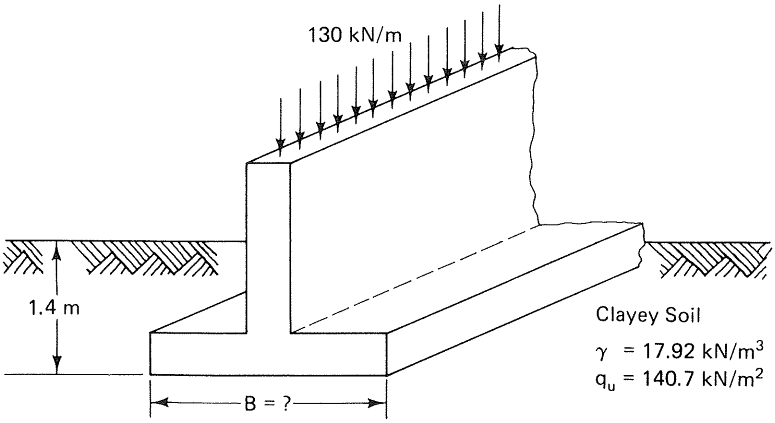


FIGURE 9-43

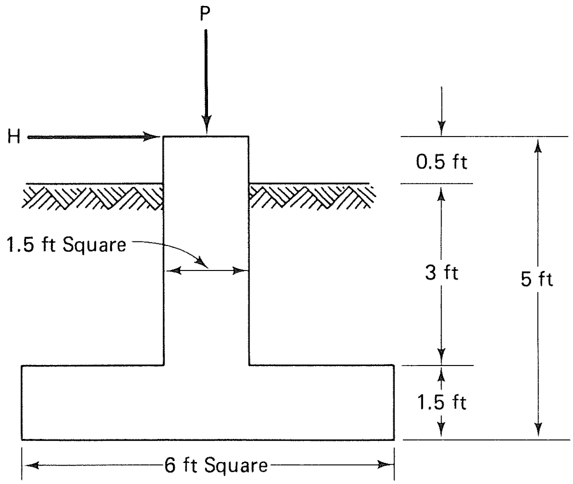


FIGURE 9-44

9-14 Same as Problem 9-13 except that groundwater is located at the ground surface.

9-15 For the footing shown in Fig. 9-45, vertical load, including column load, surcharge weight, and weight of footing, is 120 kips. Horizontal load is 10 kips and a moment of 50 ft-kips (clockwise) is also imposed on the foundation.

(a) Compute soil contact pressure and draw the soil contact pressure diagram.
(b) Compute the shear on section a–a (Fig. 9-45).
(c) Compute the moment on section a–a (Fig. 9-45).
(d) Compute the factor of safety against overturning.
(e) Compute the factor of safety against sliding, if the coefficient of friction between soil and base of footing is 0.60.
(f) Compute the factor of safety against bearing capacity failure if the ultimate bearing capacity of the soil supporting the footing is 5.4 tons/ft^2.

9-16 A 6-ft by 6-ft square footing is buried 5 ft below the ground surface. The footing is subjected to an eccentric load of 200 kips. The eccentricity of the 200-kip load (e_x) is 0.8 ft. Supporting soil has values of $\phi = 38°$, $c = 0$, and $\gamma = 135$ lb/ft^3. Calculate the factor of safety against bearing capacity failure by a reduction factor from Fig. 9-21.

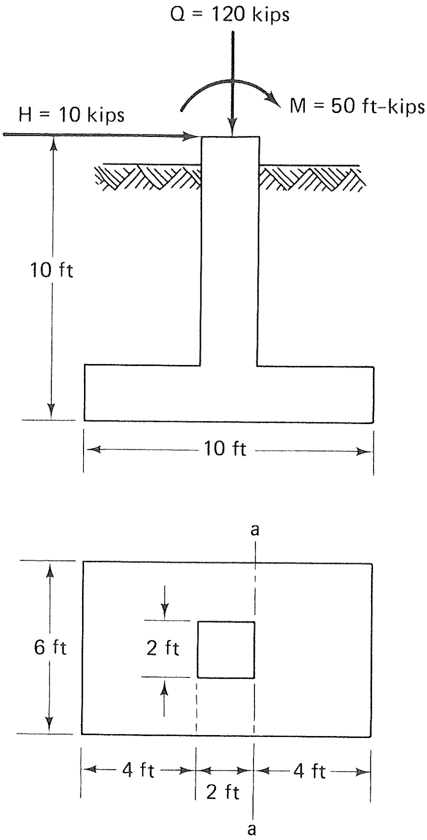


FIGURE 9-45

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10

Pile Foundations

10-1 INTRODUCTION

Chapter 9 covered shallow foundations. Sometimes, however, the soil upon which a structure is to be built is of such poor quality that a shallow foundation would be subject to bearing capacity failure and/or excessive settlement. In such cases, *pile foundations* may be used to support the structure (i.e., to transmit the load of the structure to firmer soil, or rock, at greater depth below the structure).

A pile foundation is a relatively long and slender member that is forced or driven into the soil or it may be poured in place. If a pile is driven until it rests on a hard, impenetrable layer of soil or rock, the load of the structure is transmitted primarily axially through the pile to the impenetrable layer. This type of pile is called an *end-bearing* pile. With end-bearing piles, care must be exercised to ensure that the hard, impenetrable layer is adequate to support the load. If a pile cannot be driven to a hard stratum of soil or rock (e.g., if such a stratum is located too far below the ground surface), the load of the structure must be borne primarily by skin friction or adhesion between the surface of the pile and adjacent soil. Such a pile is known as a *friction* pile.

In addition to simply supporting the load of a structure, piles may perform other functions, such as densifying loose cohesionless soils, resisting horizontal loads, anchoring structures subject to uplift, and so on. The emphasis in this book, however, is on piles that support the load of a structure.

10-2 TYPES OF PILES

Piles may be classified according to the types of materials of which they are made. Virtually all piles are made of timber, concrete, or steel (or a combination of these). Each of these is discussed in general terms in this section.

Timber piles have been used for centuries and are still widely used. They are made relatively easily by delimbing tall, straight tree trunks. They generally make economical pile foundations. Timber piles have certain disadvantages, however. They have less capacity to carry load than do concrete or steel piles. Also, the length of a timber pile is limited by the height of tree available. Timber pile length is generally limited to around 60 ft (18 m), although longer ones are available in some locales. Timber piles may be damaged in the pile-driving process. In addition, they are subject to decay and attack by insects. This generally is not a problem if the pile is both in soil and always below the water table; if above the water table, timber piles can be treated chemically to increase their life.

Concrete piles can be either *precast* or *cast-in-place.* Precast concrete piles may be manufactured with circular, square, octagonal, or other cross-sectional shapes. They can be made of uniform cross section (with a pointed tip), or they may be tapered. Precast piles can be made of prestressed concrete. The main disadvantages of precast concrete piles have to do with problems of manufacturing and handling of the piles (space needed, time required for curing, heavy equipment necessary for handling and transporting, etc.).

Cast-in-place concrete piles may be *cased* or *uncased.* The cased type can be made by driving a shell containing a core into the soil, removing the core, and filling the shell with concrete. The uncased type can be made in a similar manner, except that the shell is withdrawn as concrete is poured. Cast-in-place concrete piles have several advantages over precast ones. One is that, since the concrete is poured in place, damage due to pile driving is eliminated. Also, the length of pile is known at the time concrete is poured. (With a precast pile, the exact length of pile to be cast must be known initially. If a given pile turns out to be too long or too short, extra cost is involved in cutting off the extra length of pile or adding to it.)

Concrete piles generally have a somewhat larger capacity to carry load than do timber piles. They are usually not very susceptible to deterioration, except possibly by seawater and strong chemicals.

Steel piles are commonly either pipe-shaped or H-sections. Pipe-shaped steel piles may be filled with concrete after being driven. H-shaped steel piles are strong and capable of being driven to great depths through stiff layers. Steel piles are subject to damage by corrosion. They generally have a somewhat larger capacity to carry load than do timber piles or concrete piles.

Table 10-1 gives some customary design loads for different types of piles.

TABLE 10-1 Customary design loads for piles [1].

Type of Pile	*Allowable Load (tons)*[1]
Wood	15–30
Composite	20–30
Cast-in-place concrete	30–50
Precast reinforced concrete	30–50
Steel pipe, concrete filled	40–60
Steel H-section	30–60

[1] 1 ton = 8.896 kN.

10-3 LENGTH OF PILES

In the case of end-bearing piles, required pile length can be found fairly accurately, as it is the distance from the structure being supported by the pile to the hard, impenetrable layer of soil or rock on which the pile rests. This distance is established from soil boring tests.

With friction piles, required pile length is determined indirectly. Friction

TABLE 10-2 Available lengths of various pile types [2].

Pile Type	*Comment, Available Maximum Length*[1]
Timber	Depends on wood (tree) type. Lengths in the 50- to 60-ft range are usually available in most areas; lengths to about 75 ft are available but in limited quantity; lengths up to the 100-ft range are possible but very limited.
Steel H and pipe	Unlimited length; "short" sections are driven and additional sections are field-welded to obtain a desired total length.
Steel shell, cast-in-place	Typically to between 100 and 125 ft, depending on shell type and manufacturer-contractor.
Precast concrete	Solid, small cross-section piles usually extend into the 50- to 60-ft length, depending on cross-sectional shape, dimensions, and manufacturer. Large-diameter cylinder piles can extend to about 200 ft long.
Drilled shaft, cast-in-place concrete	Usually in the 50- to 75-ft range, depending on contractor equipment.
Bulb-type, cast-in-place concrete	Up to about 100 ft.
Composite	Related to available lengths of material in the different sections. If steel and thin-shell cast-in-place concrete are used, the length can be unlimited; if timber and thin-shell cast-in-place concrete are used, lengths can be on the order of 150 ft.

[1] 1 ft = 0.3048 m.

piles must be driven to such a depth that adequate lateral surface area of the pile is in contact with soil in order that sufficient skin friction or adhesion can be developed.

Table 10-2 gives available lengths of various types of piles.

10-4 PILE CAPACITY

The capacity of a single pile may be evaluated by the structural strength of the pile and by the supporting strength of the soil.

Pile Capacity as Evaluated by the Structural Strength of the Pile

Obviously, a pile must be strong enough structurally to "carry" the load imposed upon it. A pile's structural strength depends on its size and shape as well as the type of material of which it is made.

Allowable structural strengths of different types of pile are specified by a number of building codes. Table 10-3 shows allowable stress in various types of pile according to one code.

Pile Capacity as Evaluated by the Supporting Strength of the Soil

In addition to the strength of the pile itself, pile capacity is limited by the soil's supporting strength. As mentioned previously, load carried by a pile is ultimately borne by either or both of two ways. Load is transmitted to soil surrounding the pile by friction or adhesion between the soil and the sides of the pile, and/or load is transmitted directly to the soil just below the pile's tip. This can be expressed in equation form as

$$Q_{\text{ultimate}} = Q_{\text{friction}} + Q_{\text{tip}} \tag{10-1}$$

where Q_{ultimate} = ultimate (at failure) bearing capacity of a single pile
Q_{friction} = bearing capacity furnished by friction or adhesion between the soil and the sides of the pile
Q_{tip} = bearing capacity furnished by the soil just below the pile's tip

The term Q_{friction} in Eq. (10-1) can be evaluated by multiplying the unit skin friction or adhesion between the soil and the sides of the pile (f) by the pile's surface (skin) area (A_{surface}). The term Q_{tip} can be evaluated by multiplying the ultimate bearing capacity of the soil at the tip of the pile (q) by the area of the tip (A_{tip}). Hence, Eq. (10-1) can be expressed as

$$Q_{\text{ultimate}} = f \cdot A_{\text{surface}} + q \cdot A_{\text{tip}} \tag{10-2}$$

TABLE 10-3 Allowable stress in piles [3].

(a) *Timber piles*

Species	*Compression Parallel to Grain (psi)*	*Bending (psi)*	*Shear Horizontal (psi)*	*Compression Perpendicular to Grain (psi)*	*Modulus of Elasticity (psi)*
Douglas fir (all varieties)	1150	2300	110	225	1,500,000
Southern yellow pine (market weighted averages)	1150	2300	110	225	1,400,000
Southern red oak	950	2400	110	325	1,100,000

(b) *Steel piles*

The design load shall not cause a stress in the steel greater than 12,600 psi and a stress in any concrete used to fill piles, driven either open or closed end, greater than 25% of its ultimate 28-day compressive strength.

(c) *Concrete piles*

Cast-in-place piles: The stress in concrete shall not exceed 25% of the ultimate 28-day strength of the concrete.

Prestressed concrete piles: The maximum allowable compressible stress in precast piles due to externally applied load shall not exceed

$$f_c = 0.33f'_c - 0.27f_{pe}$$

where f'_c is the 28-day compressive strength of concrete and f_{pe} is the effective prestress stress on the gross section.

TABLE 10-4 Coefficient of friction between sand and pile materials [2].

Material	*Tan δ*
Concrete	0.45
Wood	0.4
Steel (smooth)	0.2
Steel (rough, rusted)	0.4
Steel (corrugated)	Use tan ϕ of sand

In the case of end-bearing piles, the term Q_{tip} of Eq. (10-1) or $q \cdot A_{tip}$ of Eq. (10-2) will be predominant; whereas with friction piles, the term $Q_{friction}$ of Eq. (10-1) or $f \cdot A_{surface}$ of Eq. (10-2) will be predominant.

Equations (10-1) and (10-2) are generalized and therefore applicable for all soils. The manner in which some of the terms of Eq. (10-2) are evaluated differs, however, depending on whether the pile is driven in sand or clay. It is convenient, therefore, to consider separately piles driven in sand and those driven in clay.

Piles driven in sand In the case of piles driven in sand, skin friction between the soil and the sides of the pile [$f \cdot A_{surface}$ in Eq. (10-2)] can be evaluated by multiplying the coefficient of friction between sand and pile surface (tan δ) by the total horizontal soil pressure acting on the pile. The coefficient of friction between sand and pile surface can be obtained from Table 10-4. The total horizontal soil pressure acting on the pile is a function of effective vertical (overburden) pressure of soil adjacent to the pile. Soil pressure normally increases as depth increases. In the special case of piles driven in sand, however, it has been determined that the effective vertical (overburden) pressure of soil adjacent to a pile does not increase without limit as depth increases. Instead, effective vertical pressure increases as depth increases until a certain depth of penetration is reached. Below this depth, which is called the critical depth and denoted D_c, effective vertical pressure remains more or less constant. The critical depth is dependent on the field condition of the sand and the pile's size. Tests indicate that critical depth ranges from about 10 pile diameters for loose sand to about 20 pile diameters for dense compact sand [2]. Thus, effective vertical pressure of soil adjacent to a pile varies with depth as illustrated in Fig. 10-1.

The term $f \cdot A_{surface}$ of Eq. (10-2) can now be determined for a pile by multiplying the pile's circumference by the area under the p_v versus depth curve (Fig. 10-1) by the coefficient of lateral earth pressure (K) by the coefficient of friction between sand and pile surface (tan δ). The coefficient of lateral earth pressure is assumed to vary between 0.60 and 1.25, with lower values used for silty sands and higher values for other deposits [7].

The bearing capacity at the pile tip [q in Eq. (10-2)] can be calculated using bearing capacity equations for cohesionless soil, which were developed by Terzaghi and Peck [1].

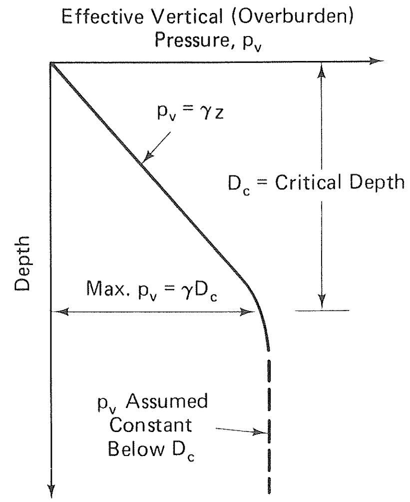


FIGURE 10-1 [2, 4, 5, 6]

$$q_{tip} = \gamma D_f N_q + 0.6\gamma R N_\gamma \quad \text{(for circular piles)} \tag{10-3}$$

$$q_{tip} = \gamma D_f N_q + 0.4\gamma B N_\gamma \quad \text{(for square piles)} \tag{10-4}$$

where q_{tip} = bearing capacity at pile tip
γ = unit weight of soil
D_f = embedded length of pile
N_γ, N_q = bearing capacity factors (see Fig. 9-7)
R = radius of pile tip (for circular piles)
B = width of pile tip (for square piles)

It will be noted that these equations have the same general form as the bearing capacity equations given in Chap. 9 for shallow foundations. However, as indicated previously, the magnitude of effective vertical (overburden) pressure of soil adjacent to a pile is more or less constant below the critical depth. Thus, for design purposes, the term $\gamma D_f N_q$ in Eqs. (10-3) and (10-4) should be replaced by the term $p_v N_q$, where p_v is the effective vertical pressure adjacent to the pile at the pile tip (see Fig. 10-1) [2].

In most cases, driven piles are relatively small in cross section, and therefore the terms in Eqs. (10-3) and (10-4) involving R and B are small compared to the other term in the equations. Thus, for many cases, Eqs. (10-3) and (10-4) may be approximated as

$$q_{tip} = p_v N_q \tag{10-5}$$

and the term $q \cdot A_{tip}$ of Eq. (10-2) can be evaluated by multiplying the value of q_{tip} from Eq. (10-5) by the area of the pile tip.

The value of N_q is related to the angle of internal friction (ϕ) of the sand, and it should, of course, be based on the value of the angle of internal friction of the sand located in the general vicinity of where the pile tip will ultimately rest. The angle of internal friction of the sand at this location can be determined by laboratory tests on a sample taken from the specified location or by correlation with penetration resistance tests in a boring hole (i.e., corrected SPT N-value) (see Figs. 3-9 and 9-9).

To summarize the method described in this section for computing pile capacity for piles driven in sand, Eq. (10-2) is used with the term $f \cdot A_{\text{surface}}$ evaluated by multiplying the pile's circumference by the area under the p_v versus depth curve (Fig. 10-1) by the coefficient of lateral earth pressure (K) by the coefficient of friction between sand and pile surface ($\tan \delta$) and the term $q \cdot A_{\text{tip}}$ evaluated by multiplying the value of q_{tip} obtained from Eq. (10-5) by the area of the pile tip. Pile capacity thus determined represents the ultimate load that can be applied to the pile. In practice, it is common to apply a factor of safety of 2 to determine the (downward) design load for the pile [2].

Examples 10-1 and 10-2 illustrate the procedure for calculating pile capacity for piles driven in sand.

EXAMPLE 10-1

Given

1. A concrete pile is to be driven into a medium dense to dense sand.
2. The pile's diameter is 12 in. and its embedded length is 25 ft.
3. Soil conditions are shown in Fig. 10-2.
4. No groundwater was encountered, and the groundwater table is not expected to rise during the life of the structure.

Required

The pile's axial capacity if the coefficient of lateral earth pressure (K) is assumed to be 0.95 and the factor of safety is 2.

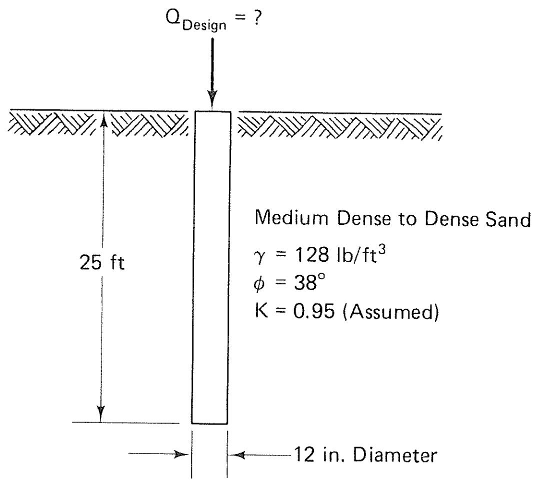


FIGURE 10-2

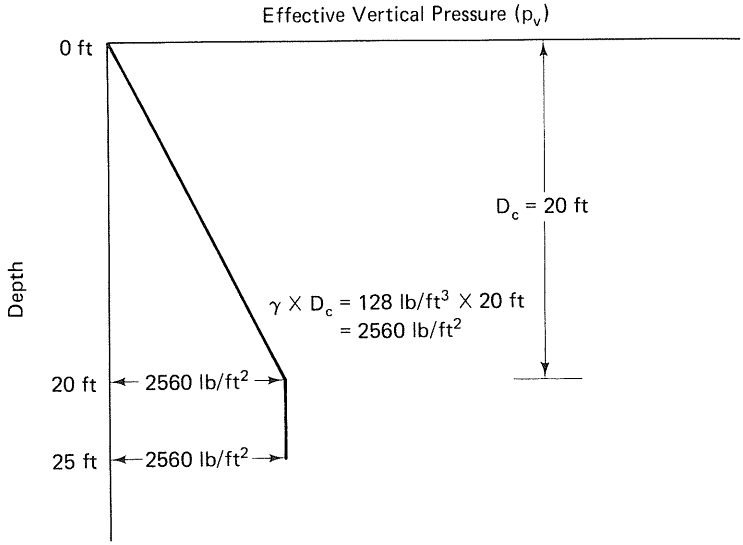


FIGURE 10-3

Solution

For dense sand,

$$D_c = 20 \text{ times the pile's diameter} = 20 \times 1 \text{ ft} = 20 \text{ ft} \quad \text{(see Fig. 10-3)}$$

From Eq. (10-2),

$$Q_{\text{ultimate}} = f \cdot A_{\text{surface}} + q \cdot A_{\text{tip}} \tag{10-2}$$

$$f \cdot A_{\text{surface}} = (\text{circumference of pile})(\text{area of } p_v \text{ diagram})(K)(\tan \delta)$$

$$\text{Circumference of pile} = \pi d = (\pi)(1 \text{ ft}) = 3.14 \text{ ft}$$

$$\text{Area of } p_v \text{ diagram} = (1/2)(2560 \text{ lb/ft}^2)(20 \text{ ft}) + (2560 \text{ lb/ft}^2)(25 \text{ ft} - 20 \text{ ft})$$

$$= 38{,}400 \text{ lb/ft}$$

$$K = 0.95 \qquad \text{(given)}$$

$$\tan \delta = 0.45 \qquad \text{(see Table 10-4 for concrete pile)}$$

$$f \cdot A_{\text{surface}} = (3.14 \text{ ft})(38{,}400 \text{ lb/ft})(0.95)(0.45)$$

$$= 51{,}500 \text{ lb} = 51.5 \text{ kips}$$

From Eq. (10-5),

$$q_{\text{tip}} = p_v N_q \tag{10-5}$$

$$p_v = 2560 \text{ lb/ft}^2 \qquad \text{(see Fig. 10-3)}$$

$$N_q = 50 \qquad (\text{from Fig. 9-7 for } \phi = 38°)$$

$$q_{tip} = (2560 \text{ lb/ft}^2)(50) = 128{,}000 \text{ lb/ft}^2$$

$$A_{tip} = \frac{\pi d^2}{4} = \left(\frac{\pi}{4}\right)(1 \text{ ft})^2 = 0.785 \text{ ft}^2$$

$$q \cdot A_{tip} = (128{,}000 \text{ lb/ft}^2)(0.785 \text{ ft}^2) = 100{,}500 \text{ lb} = 100.5 \text{ kips}$$

$$Q_{ultimate} = 51.5 \text{ kips} + 100.5 \text{ kips} = 152.0 \text{ kips}$$

$$Q_{design} = \frac{Q_{ultimate}}{\text{F.S.}} = \frac{152.0 \text{ kips}}{2} = 76.0 \text{ kips}$$

EXAMPLE 10-2

Given

The same conditions as in Example 10-1, except that groundwater is located 10 ft below the ground surface (see Fig. 10-4).

Required

The pile's axial capacity if K is 0.95 and a factor of safety of 2 is used.

Solution

$$D_c = 20 \times 1 \text{ ft} = 20 \text{ ft} \qquad \text{(see Fig. 10-5)}$$

$$f \cdot A_{surface} = (\text{circumference of pile})(\text{area of } p_v \text{ diagram})(K)(\tan \delta)$$

$$\text{Circumference of pile} = \pi d = (\pi)(1 \text{ ft}) = 3.14 \text{ ft}$$

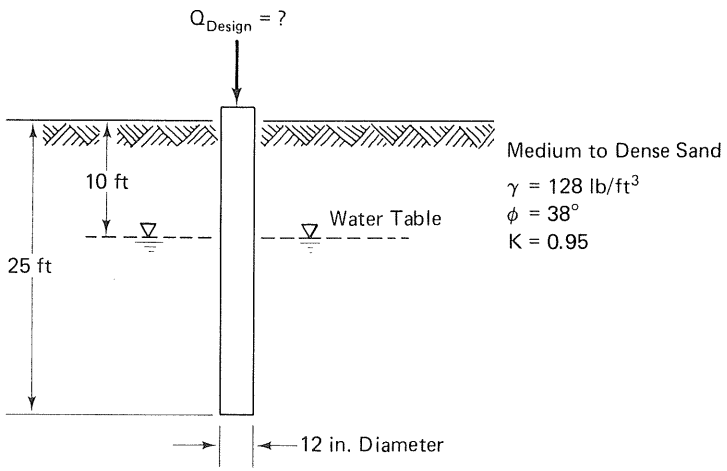


FIGURE 10-4

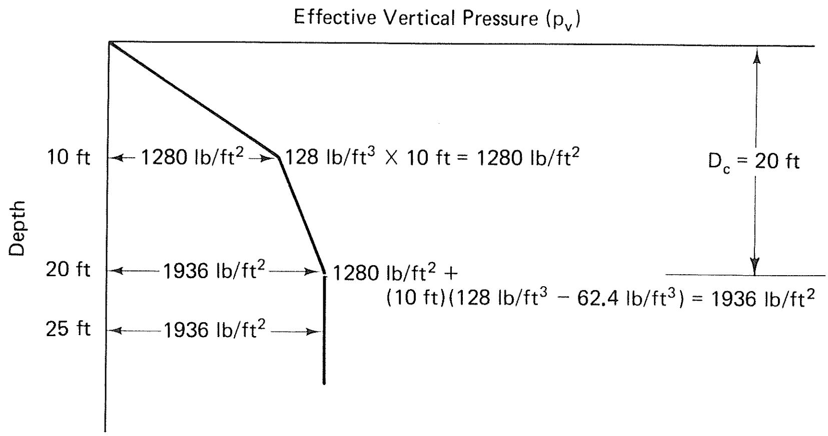


FIGURE 10-5

$$\text{Area of } p_v \text{ diagram} = (\tfrac{1}{2})(1280\ \text{lb/ft}^2)(10\ \text{ft}) + (\tfrac{1}{2})(1280\ \text{lb/ft}^2 + 1936\ \text{lb/ft}^2)(10\ \text{ft}) + (1936\ \text{lb/ft}^2)(5\ \text{ft})$$

$$= 32{,}200\ \text{lb/ft}$$

$$K = 0.95$$

$$\tan\delta = 0.45$$

$$f \cdot A_{\text{surface}} = (3.14\ \text{ft})(32{,}200\ \text{lb/ft})(0.95)(0.45) = 43{,}200\ \text{lb} = 43.2\ \text{kips}$$

$$q_{\text{tip}} = p_v N_q \qquad (10\text{-}5)$$

$$N_q = 50$$

$$q_{\text{tip}} = (1936\ \text{lb/ft}^2)(50) = 96{,}800\ \text{lb/ft}^2$$

$$A_{\text{tip}} = 0.785\ \text{ft}^2$$

$$q \cdot A_{\text{tip}} = (96{,}800\ \text{lb/ft}^2)(0.785\ \text{ft}^2) = 76{,}000\ \text{lb} = 76.0\ \text{kips}$$

$$Q_{\text{ultimate}} = 43.2\ \text{kips} + 76.0\ \text{kips} = 119.2\ \text{kips}$$

$$Q_{\text{design}} = \frac{119.2\ \text{kips}}{2} = 59.6\ \text{kips}$$

Piles driven in clay For piles driven in clay, unit adhesion between the soil and the sides of the pile [f in Eq. (10-2)] can be evaluated by multiplying the cohesion of the clay (c) by the adhesion factor (α). The adhesion factor can be determined using Fig. 10-6. The term $f \cdot A_{\text{surface}}$ of Eq. (10-2) can thus be evaluated by multiplying the (undisturbed) cohesion of the clay (c) by the adhesion factor (α) by the surface (skin) area of the pile (A_{surface}).

With soft clays, there is a tendency for the clay to contact the pile, in which case adhesion is assumed equal to cohesion (meaning $\alpha = 1.0$). In the case of stiff clays, pile driving disturbs surrounding soil and may cause a small open space to develop between clay and pile. Thus adhesion is smaller than cohesion (meaning $\alpha < 1.0$).

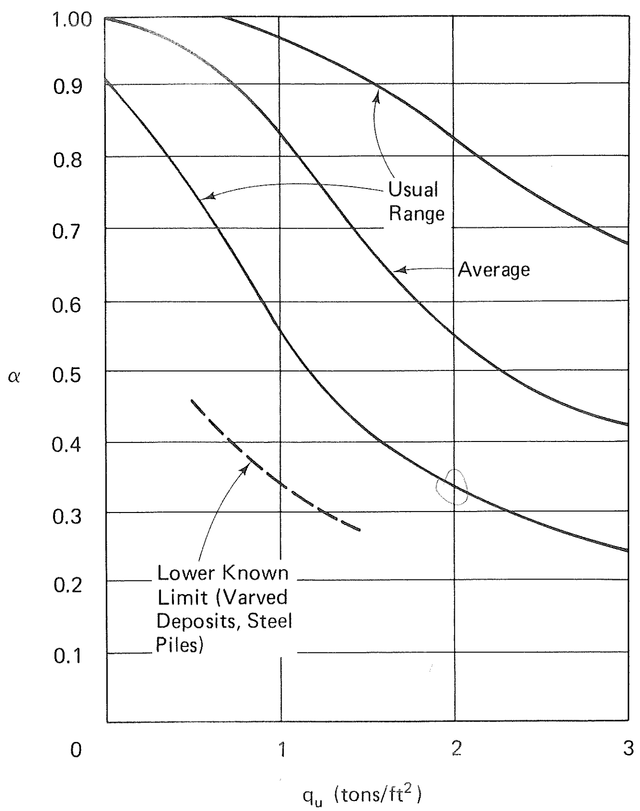


FIGURE 10-6 Relationship between adhesion factor α and unconfined compressive strength q_u (1 ton/ft^2 = 95.76 kN/m^2). [8]

Bearing capacity [q in Eq. (10-2)] at the pile tip can be calculated using [2]

$$q_{\text{tip}} = cN_c \tag{10-6}$$

where q_{tip} = bearing capacity at pile tip
c = cohesion of the clay located in the general vicinity of where the pile tip will ultimately rest
N_c = bearing capacity factor and has a value of about 9 [2]

Thus, the term $q \cdot A_{\text{tip}}$ of Eq. (10-2) can be evaluated by multiplying the value of q_{tip} from Eq. (10-6) by the area of the pile tip.

To summarize the method described in this section for computing pile capacity for piles driven in clay, Eq. (10-2) is used with the term $f \cdot A_{\text{surface}}$ evaluated by multiplying the cohesion of the clay (c) by the adhesion factor (α) by the surface (skin) area of the pile and the term $q \cdot A_{\text{tip}}$ evaluated by multiplying the value of q_{tip} obtained from Eq. (10-6) by the area of the pile tip. Pile capacity thus determined represents the ultimate load that can be applied to the pile. In practice, it is common to apply a factor of safety of 2 to determine the (downward) design load for a pile [2].

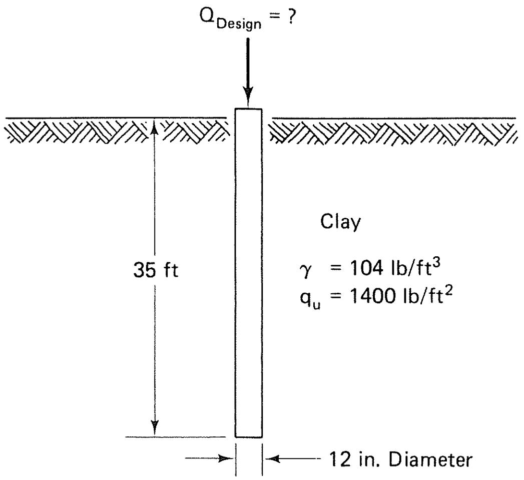


FIGURE 10-7

Examples 10-3 through 10-5 illustrate the procedure for calculating pile capacity for piles driven in clay.

EXAMPLE 10-3

Given

1. A 12-in.-diameter concrete pile is driven at a site as shown in Fig. 10-7.
2. The embedded length of pile is 35 ft.

Required

Design capacity of the pile, using a factor of safety of 2.

Solution

From Eq. (10-2),

$$Q_{\text{ultimate}} = f \cdot A_{\text{surface}} + q \cdot A_{\text{tip}} \tag{10-2}$$

$$f = \text{adhesion} = \alpha c$$

$$q_u = 1400 \text{ lb/ft}^2 = 0.7 \text{ ton/ft}^2$$

$$\alpha = 0.9 \qquad \text{(see Fig. 10-6 with } q_u = 0.7 \text{ ton/ft}^2\text{)}$$

$$c = \frac{q_u}{2} = \frac{1400 \text{ lb/ft}^2}{2} = 700 \text{ lb/ft}^2$$

$$f = (0.9)(700 \text{ lb/ft}^2) = 630 \text{ lb/ft}^2$$

$$A_{\text{surface}} = (\pi d)(L) = (\pi)(1 \text{ ft})(35 \text{ ft}) = 110 \text{ ft}^2$$

$$q_{\text{tip}} = cN_c \tag{10-6}$$

$$q_{\text{tip}} = (700 \text{ lb/ft}^2)(9) = 6300 \text{ lb/ft}^2$$

$$A_{\text{tip}} = \frac{\pi d^2}{4} = \frac{\pi}{4}(1\text{ ft})^2 = 0.785\text{ ft}^2$$

$$Q_{\text{ultimate}} = (630\text{ lb/ft}^2)(110\text{ ft}^2) + (6300\text{ lb/ft}^2)(0.785\text{ ft}^2) = 74{,}200\text{ lb}$$

$$= 74.2\text{ kips}$$

$$Q_{\text{design}} = \frac{74.2\text{ kips}}{2} = 37.1\text{ kips}$$

EXAMPLE 10-4

Given

A 12-in.-diameter concrete pile is driven at a site as shown in Fig. 10-8.

Required

Design capacity of the pile, using a factor of safety of 2.

Solution

From Eq. (10-1),

$$Q_{\text{ultimate}} = Q_{\text{friction}} + Q_{\text{tip}} \tag{10-1}$$

$$Q_{\text{friction}} = f \cdot A_{\text{surface}} = f_1 \cdot A_{\text{surface}1} + f_2 \cdot A_{\text{surface}2}$$

From Fig. 10-6 with $q_{u1} = 1400\text{ lb/ft}^2 = 0.7\text{ ton/ft}^2$, $\alpha_1 = 0.9$.

$$c_1 = \frac{q_{u1}}{2} = \frac{1400\text{ lb/ft}^2}{2} = 700\text{ lb/ft}^2$$

$$f_1 = c_1\alpha_1 = (700\text{ lb/ft}^2)(0.9) = 630\text{ lb/ft}^2$$

$$A_{\text{surface}1} = (\pi d)(L_1) = (\pi)(1\text{ ft})(20\text{ ft}) = 62.8\text{ ft}^2$$

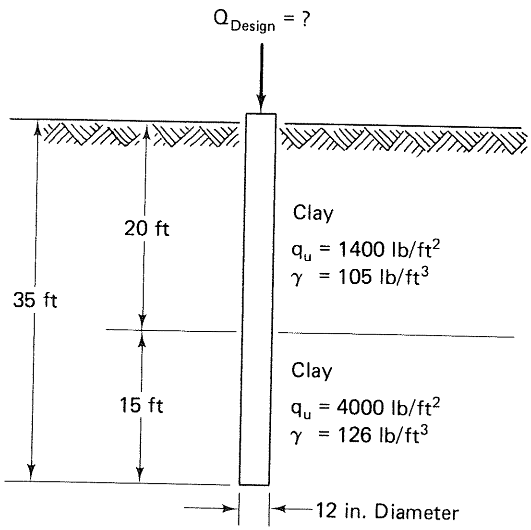


FIGURE 10-8

From Fig. 10-6 with q_{u2} = 4000 lb/ft² = 2.0 tons/ft², α_2 = 0.56.

$$c_2 = \frac{q_{u2}}{2} = \frac{4000 \text{ lb/ft}^2}{2} = 2000 \text{ lb/ft}^2$$

$$f_2 = c_2\alpha_2 = (2000 \text{ lb/ft}^2)(0.56) = 1120 \text{ lb/ft}^2$$

$$A_{\text{surface2}} = (\pi d)(L_2) = (\pi)(1 \text{ ft})(15 \text{ ft}) = 47.1 \text{ ft}^2$$

$$Q_{\text{friction}} = (630 \text{ lb/ft}^2)(62.8 \text{ ft}^2) + (1120 \text{ lb/ft}^2)(47.1 \text{ ft}^2) = 92{,}300 \text{ lb}$$

$$= 92.3 \text{ kips}$$

$$q_{\text{tip}} = cN_c \qquad (10\text{-}6)$$

$$q_{\text{tip}} = (2000 \text{ lb/ft}^2)(9) = 18{,}000 \text{ lb/ft}^2$$

$$A_{\text{tip}} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (1 \text{ ft})^2 = 0.785 \text{ ft}^2$$

$$Q_{\text{tip}} = (18{,}000 \text{ lb/ft}^2)(0.785 \text{ ft}^2) = 14{,}100 \text{ lb} = 14.1 \text{ kips}$$

$$Q_{\text{ultimate}} = 92.3 \text{ kips} + 14.1 \text{ kips} = 106.4 \text{ kips}$$

$$Q_{\text{design}} = \frac{106.4 \text{ kips}}{2} = 53.2 \text{ kips}$$

EXAMPLE 10-5

Given

1. A 0.36-m square prestressed concrete pile is to be driven in a clayey soil (see Fig. 10-9).

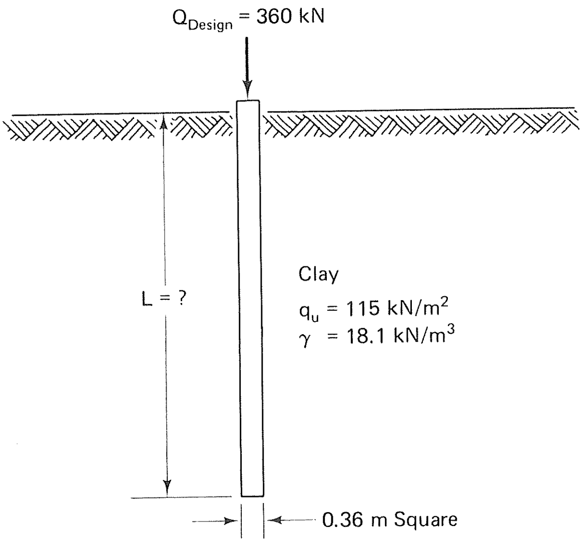


FIGURE 10-9

2. Design capacity of the pile is 360 kN.

Required

The necessary length of pile if the factor of safety is 2.

Solution

$$Q_{\text{design}} = 360 \text{ kN}$$
$$Q_{\text{ultimate}} = \text{F.S.} \times Q_{\text{design}} = (2)(360 \text{ kN}) = 720 \text{ kN}$$
$$c = \frac{115 \text{ kN/m}^2}{2} = 57.5 \text{ kN/m}^2$$
$$q_{\text{tip}} = cN_c = (57.5 \text{ kN/m}^2)(9) = 518 \text{ kN/m}^2$$
$$Q_{\text{tip}} = (518 \text{ kN/m}^2)(0.36 \text{ m})(0.36 \text{ m}) = 67.1 \text{ kN}$$

From Eq. (10-1),

$$Q_{\text{ultimate}} = Q_{\text{friction}} + Q_{\text{tip}} \tag{10-1}$$
$$Q_{\text{friction}} = Q_{\text{ultimate}} - Q_{\text{tip}}$$
$$Q_{\text{friction}} = 720 \text{ kN} - 67.1 \text{ kN} = 652.9 \text{ kN}$$
$$Q_{\text{friction}} = f \cdot A_{\text{surface}} = \alpha c A_{\text{surface}}$$

From Fig. 10-6, with $q_u = 115 \text{ kN/m}^2$,

$$\alpha = 0.76$$
$$652.9 \text{ kN} = (0.76)(57.5 \text{ kN/m}^2)(4 \times 0.36 \text{ m})(L)$$
$$L = 10.4 \text{ m}$$

The required length of the 0.36-m square pile is 10.4 m.

Soft clays adjacent to piles may lose a large portion of their strength as a result of being disturbed by pile driving. Propitiously, the disturbed clay gains strength rapidly after driving stops. The original clay's full strength is usually regained within a month or so after pile driving has terminated. Ordinarily, this is not a problem, since piles are not usually loaded immediately after driving, and thus the clay has time to regain its original strength prior to being loaded. In cases where piles are to be loaded immediately after driving, the effect of decreased strength must be taken into account by performing laboratory tests to determine the extent of strength reduction and rate of strength recovery [9].

Slender piles driven in soft clay have a tendency to buckle when loaded. Ultimate load for buckling of slender steel piles in soft clay can be estimated by the equation [10]

$$Q_{ult} = \lambda\sqrt{cEI} \tag{10-7}$$

where Q_{ult} = ultimate bearing capacity of a single slender pile for buckling in soft clay
λ = 8 for very soft clay; 10 for soft clay
c = cohesion of the soil
E = modulus of elasticity of the steel
I = moment of inertia of the cross section of the pile

Heavy steel, timber, and concrete piles do not tend to buckle if embedded in the soil for their entire lengths.

10-5 PILE-DRIVING FORMULAS

In theory, it seems possible to calculate pile capacity based on the amount of energy delivered to a pile by the hammer and resulting penetration of the pile. Intuitively, the greater the resistance required to drive a pile, the greater will be the capacity of the pile to carry load. Hence, many attempts have been made to develop *pile-driving formulas* by equating energy delivered by the hammer to work done by the pile as it penetrates a certain distance against a certain resistance, with allowance made for energy losses.

Generally, no pile-driving formula has been developed that gives accurate results for pile capacity. Soil resistance does not remain constant during and after the pile-driving operation. In addition, pile-driving formulas give varying results. While pile-driving formulas are not generally used to determine pile capacity, they may be used to determine when to stop driving a pile so its bearing capacity will be the same as that of a test pile or of other piles driven in the same subsoil. To accomplish this, piles should be driven until the number of blows required to drive the last inch is the same as that of the test piles that furnished the information for evaluating the design load. However, piles driven in soft silt or clay should all be driven to the same depth, rather than driven a certain number of blows [1]. Penetration resistance can also be used to prevent pile damage due to overdriving.

One simple and widely used pile-driving formula is known as the *Engineering-News formula.* It is given by [11]

$$Q_a = \frac{2W_rH}{S + C} \tag{10-8}$$

where Q_a = allowable pile capacity, lb
W_r = weight of ram, lb
H = height of fall of ram, ft
S = amount of pile penetration per blow, in./blow
C = 1.0 for drop hammer
C = 0.1 for steam hammer

For use with SI units, Eq. (10-8) may be expressed as

$$Q_a = \frac{1000 W_r H}{6(S + C)} \tag{10-9}$$

with Q_a computed in kN if W_r is in kN, H in m, S in mm/blow, and $C = 25$ for drop hammers and 2.5 for steam hammers. The Engineering-News formula has a built-in factor of safety of 6. Tests have shown that this formula is not really reliable for computing pile loads, and it should be avoided except as a rough guide [2].

EXAMPLE 10-6

Given

The design capacity of a 0.3-m-diameter concrete pile is 160 kN. The pile is driven by a drop hammer with a manufacturer's hammer energy rating of 40 kN·m.

Required

Average penetration of the pile from the last few driving blows.

Solution

From Eq. (10-9),

$$Q_a = \frac{1000 W_r H}{6(S + C)} \tag{10-9}$$

$$Q_a = 160 \text{ kN}$$

$$W_r H = 40 \text{ kN·m}$$

$$C = 25 \text{ (for a drop hammer)}$$

Therefore,

$$160 \text{ kN} = \frac{(1000)(40 \text{ kN·m})}{(6)\ (S + 25)}$$

$$S = 17 \text{ mm/blow}$$

Another pile-driving formula is known as the *Danish formula.* It is given by [2]

$$Q_{\text{ultimate}} = \frac{e_h(E_h)}{S + \frac{1}{2}S_0} \tag{10-10}$$

where
Q_{ultimate} = ultimate capacity of the pile
e_h = efficiency of pile hammer (see Table 10-5)
E_h = manufacturer's hammer energy rating (see Table 10-6)
S = average penetration of the pile from the last few driving blows
S_0 = elastic compression of the pile
$S_0 = \left[\frac{2e_h E_h L}{AE}\right]^{1/2}$
L = length of pile
A = cross-sectional area of pile
E = modulus of elasticity of pile material

Statistical studies indicate that a factor of safety of 3 should be used with the Danish formula.

Example 10-7 demonstrates how the Danish formula can be used as a field control during pile driving to indicate when desired pile capacity has been obtained.

EXAMPLE 10-7

Given

1. The design capacity of a 12-in. steel pipe pile is 100 kips.
2. The pile's modulus of elasticity is 29,000 kips/in.^{2}
3. The pile's length is 40 ft.
4. The pile's cross-sectional area is 16 in.^{2}
5. The hammer is a Vulcan 140 C with a weight of pile hammer ram of 14,000 lb and manufacturer's hammer energy rating of 36,000 ft-lb.
6. Hammer efficiency is assumed to be 0.80.

TABLE 10-5 Pile hammer efficiency [7].

Type of Hammer	*Efficiency, e_h*
Drop hammer	0.75–1.00
Single-acting hammer	0.75–0.85
Double-acting hammer	0.85
Diesel hammer	0.85–1.00

TABLE 10-6 Properties of selected impact pile hammers [8].[1]

Rated Energy (ft-lb)	*Make*	*Model*	*Type*[2]	*Blows per Minute*[3]	*Stroke at Rated Energy (in.)*	*Weight Striking Parts (lb)*
7,260	Vulcan	2	S	70	29	3,000
8,750	MKT[4]	9B3	DB	145	17	1,600
13,100	MKT	10B3	DB	105	19	3,000
15,000	Vulcan	1	S	60	36	5,000
15,100	Vulcan	50C	DF	120	$15\frac{1}{2}$	5,000
16,000	MKT	DE-20	DE	48	96	2,000
18,200	Link-Belt	440	DE	86–90	$36\frac{7}{8}$	4,000
19,150	MKT	11B3	DB	95	19	5,000
19,500	Raymond	65C	DF	100–110	16	6,500
19,500	Vulcan	06	S	60	36	6,500
22,400	MKT	DE-30	DE	48	96	2,800
22,500	Delmag	D-12	DE	42–60		2,750
24,375	Vulcan	0	S	50	39	7,500
24,400	Kobe	K13	DE	45–60	102	2,870
24,450	Vulcan	80C	DF	111	16	8,000
26,000	Vulcan	08	S	50	39	8,000
26,300	Link-Belt	520	DE	80–84	$43\frac{1}{6}$	5,070
32,000	MKT	DE-40	DE	48	96	4,000
32,500	MKT	S10	S	55	39	10,000
32,500	Vulcan	010	S	50	39	10,000
32,500	Raymond	00	S	50	39	10,000
36,000	Vulcan	140C	DF	103	$15\frac{1}{2}$	14,000
39,700	Delmag	D-22	DE	42–60		4,850
40,600	Raymond	000	S	50	39	12,500
41,300	Kobe	K-22	DE	45–60	102	4,850
42,000	Vulcan	014	S	60	36	14,000
48,750	Vulcan	016	S	60	36	16,250

[1] 1 ft-lb = 1.356 N·m; 1 in. = 25.4 mm; 1 lb = 4.448 N.
[2] S, single-acting steam; DB, double-acting steam; DF, differential-acting steam; DE, diesel.
[3] After development of significant driving resistance.
[4] For many years known as McKiernan–Terry.

Required

1. What should be the average penetration of the pile from the last few driving blows?
2. How many blows/ft for the last foot of penetration are required for the design capacity, using the Danish formula?

Solution

1. From Eq. (10-10),

$$Q_{\text{ultimate}} = \frac{e_h(E_h)}{S + \tfrac{1}{2}S_0} \tag{10-10}$$

$$S + \tfrac{1}{2}S_0 = \frac{e_h(E_h)}{Q_{\text{ultimate}}}$$

$$S = \frac{e_h(E_h)}{Q_{\text{ultimate}}} - \tfrac{1}{2}S_0$$

$$Q_{\text{design}} = \frac{Q_{\text{ultimate}}}{\text{F.S.}} = \frac{Q_{\text{ultimate}}}{3}$$

$$Q_{\text{ultimate}} = 3 \times Q_{\text{design}} = 3 \times 100 \text{ kips} = 300 \text{ kips}$$

$$S_0 = \left[\frac{2e_hE_hL}{AE}\right]^{1/2}$$

$$e_h = 0.80$$

$$E_h = 36{,}000 \text{ ft-lb} = 36 \text{ ft-kips}$$

$$L = 40 \text{ ft}$$

$$A = 16 \text{ in.}^2$$

$$E = 29{,}000 \text{ kips/in.}^2$$

$$S_0 = \left[\frac{(2)(0.80)(36 \text{ ft-kips})(40 \text{ ft})}{(16 \text{ in.}^2)(29{,}000 \text{ kips/in.}^2)}\right]^{1/2} = 0.070 \text{ ft} = 0.84 \text{ in.}$$

$$S = \frac{(0.80)(36 \text{ ft-kips})(12 \text{ in./ft})}{300 \text{ kips}} - (\tfrac{1}{2})(0.84 \text{ in.}) = 0.73 \text{ in./blow}$$

2. Number of blows required for the last foot of penetration

$$= \frac{12 \text{ in./ft}}{0.73 \text{ in./blow}} = 16 \text{ blows/ft}$$

10-6 PILE LOAD TESTS

Load tests are performed on-site on test piles to determine or verify the design capacity of piles. Normally, piles are designed initially by analytical or other methods, based on estimated loads and soil characteristics. Pile load tests are performed on test piles during the design stage to check the design capacity. Should load test results indicate possible bearing failure or excessive settlement, the pile design should be revised accordingly. Also, data collected from pile load tests help develop criteria for the foundation installation.

To carry out pile load tests, the first step is to drive test piles. They should be driven at a location where soil conditions are known (such as near a bore hole) and where soil conditions are relatively poor. Both test piles and method of driving them should be exactly the same as will be used in the construction project. A penetration record should be kept as each test pile is driven.

The next step is to load the test piles. For reasons explained previously in this chapter, test piles in clays should not be loaded until some time (at least several weeks) has passed after the piles were driven. Test piles in sands, however, may be loaded several days after they were driven. Test piles may be loaded by adding dead weight or by hydraulically jacking (against a fixed platform, for example). (Figure 10-10 illustrates schematically how test piles can be loaded by these methods.) Total load on test piles should be 200% of proposed design load. Load should be applied to the pile in increments of 25% of total test load. For specific details regarding loading, the reader is referred to the ASTM Book of Standards. In any event, a record of load and corresponding settlement must be kept as each test pile is loaded and unloaded.

The next step is to plot a load versus settlement graph, as shown in Fig. 10-11. From this graph, the relationship between load and net settlement can be obtained. Ordinates along the loading curve of Fig. 10-11 give gross settlement. Subtracting the final settlement upon unloading (point A in Fig. 10-11) from ordinates along the unloading curve gives the rebound. Net settlement can then be determined by subtracting rebound from corresponding gross settlement.

Allowable pile load is generally determined based on criteria specified by applicable building codes. There are many building codes and therefore many criteria for determining allowable pile loads based on pile tests. It is, of course, the responsibility of the soils engineer to follow criteria specified by the applicable building code. Examples 10-8 and 10-9, in addition to illustrating determination of allowable pile load, give two possible building code criteria for determining pile capacity by the pile load test.

EXAMPLE 10-8

Given

1. A 12-in.-diameter pipe pile with a length of 50 ft was subjected to a pile load test.
2. Test results were plotted and the load-settlement curve is shown on page 312 in Fig. 10-12.
3. The local building code states that allowable pile load is taken as one-half of that load which produces a net settlement of not more than 0.01 in./ton, but in no case more than 0.75 in.

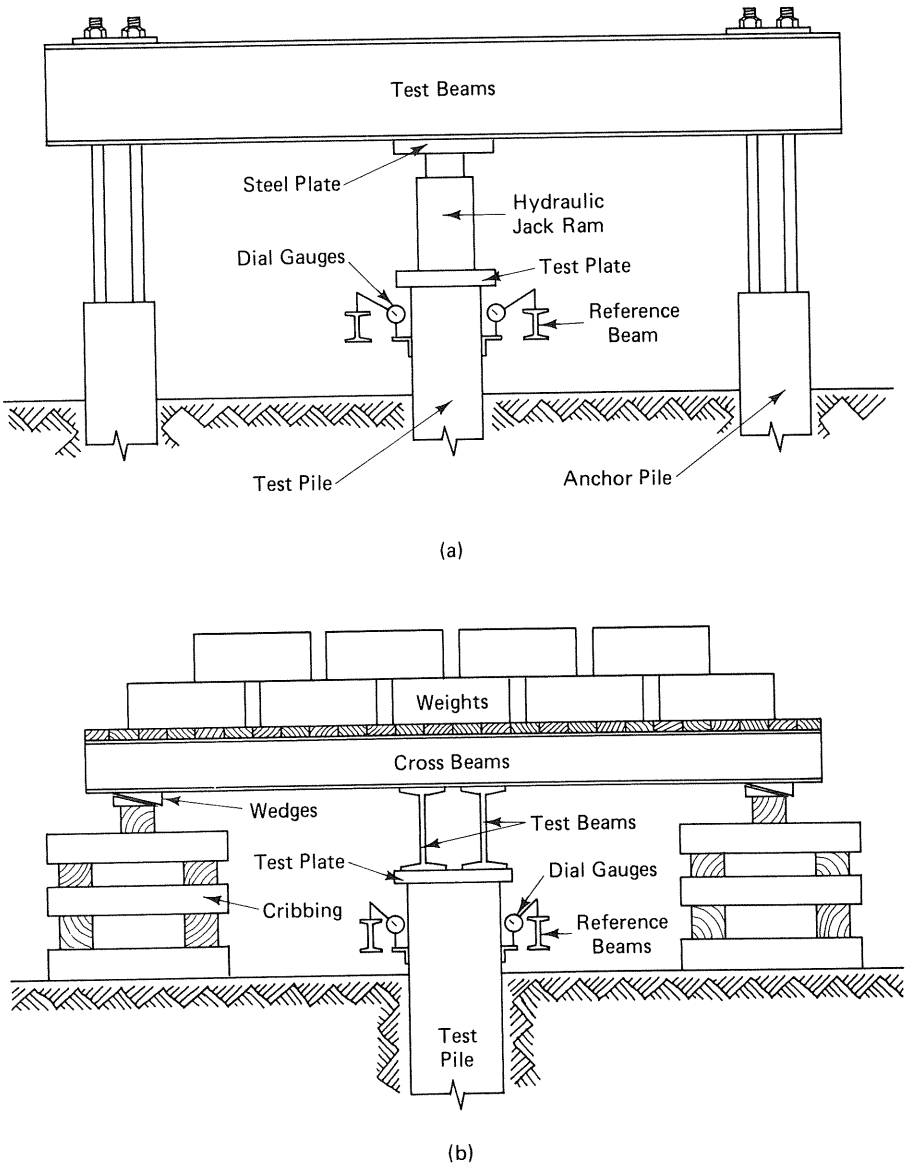


FIGURE 10-10 Schematic setup for test pile loading: (a) using hydraulic jack acting against anchored reaction frame; (b) using weighted platform. [12]

Required

Allowable pile load.

Solution

Net settlement = gross settlement − rebound

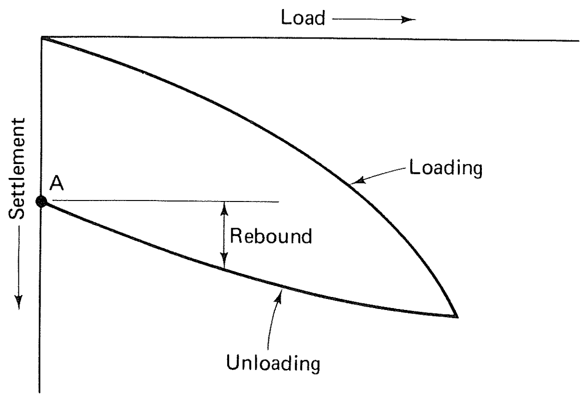


FIGURE 10-11 Typical load versus settlement graph.

Test Load (kips)	Test Load (tons)	Gross Settlement (in.)	Rebound (in.)	Net Settlement (in.)	Building Code Maximum Allowable Settlement (in.)
100	50	0.20	2.39 − 2.20 = 0.19	0.20 − 0.19 = 0.01	< 0.5
200	100	0.45	2.54 − 2.20 = 0.34	0.45 − 0.34 = 0.11	< ~~1.0~~ (use 0.75)
300	150	0.76	2.64 − 2.20 = 0.44	0.76 − 0.44 = 0.32	< ~~1.5~~ (use 0.75)
400	200	1.25	2.73 − 2.20 = 0.53	1.25 − 0.53 = 0.72	< ~~2.0~~ (use 0.75)
500	250	2.80	2.80 − 2.20 = 0.60	2.80 − 0.60 = 2.20	> ~~2.5~~ (use 0.75)

Since a test load of 200 tons produces a net settlement of 0.72 in. and maximum allowable settlement is 0.75 in.,

$$\text{Allowable pile load} = \frac{200 \text{ tons}}{2} = 100 \text{ tons}$$

EXAMPLE 10-9

Given

The same conditions as in Example 10-8 except that another local building code is to be applied as follows: "The allowable pile load shall be not more than one-half of that test load which produces a net settlement per ton of test load of not more than 0.01 in. but in no case more than one-half inch."

Required

Allowable pile load.

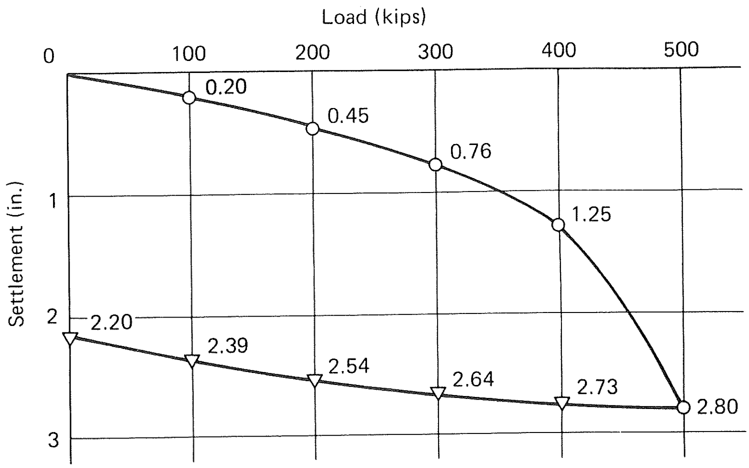


FIGURE 10-12

Solution

From Example 10-8:

Test Load (tons)	*Net Settlement (in.)*	*Building Code Maximum Allowable Settlement (in.)*
50	0.01	< 0.5
100	0.11	< ~~1.0~~ (use 0.5)
150	0.32	< ~~1.5~~ (use 0.5)
200	0.72	> ~~2.0~~ (use 0.5)
250	2.20	> ~~2.5~~ (use 0.5)

Since a test load of 150 tons produces a net settlement of 0.32 in. and maximum allowable settlement is 0.5 in.,

$$\text{Allowable pile load} = \frac{150 \text{ tons}}{2} = 75 \text{ tons}$$

Some building codes use a “breaking in the curve” or the point defined by tangents drawn on either side of a break of a load-settlement graph. One building code [3] states that

> the design load on piles may be determined by the designer based on an analysis of the results of pile load tests performed in accordance with ASTM D-1143. The allowable pile load shall be determined by the application of a safety factor of 2 to the ultimate pile capacity as determined by the intersection of the initial and final tangents to a curve fitted to the plotted results of the pile load test. The fitted curve shall not extend to any point at which the pile continued to move under the applied load. . . .

EXAMPLE 10-10

Given

Results of a pile load test are as follows:

Load (kN)	*Settlement (mm)*
250	2.7
500	5.8
750	9.3
1000	12.5
1250	16.2
1500	20.0
1750	44.0
2000	80.0

Required

Assuming the building code given just prior to this example is applicable, find the allowable load on the pile.

Solution

Load test data are shown plotted in Fig. 10-13. Initial and final tangents to the plotted curve intersect at a load of 1600 kN. Hence, according to the code, allowable load on the pile is 1600 kN/2, or 800 kN.

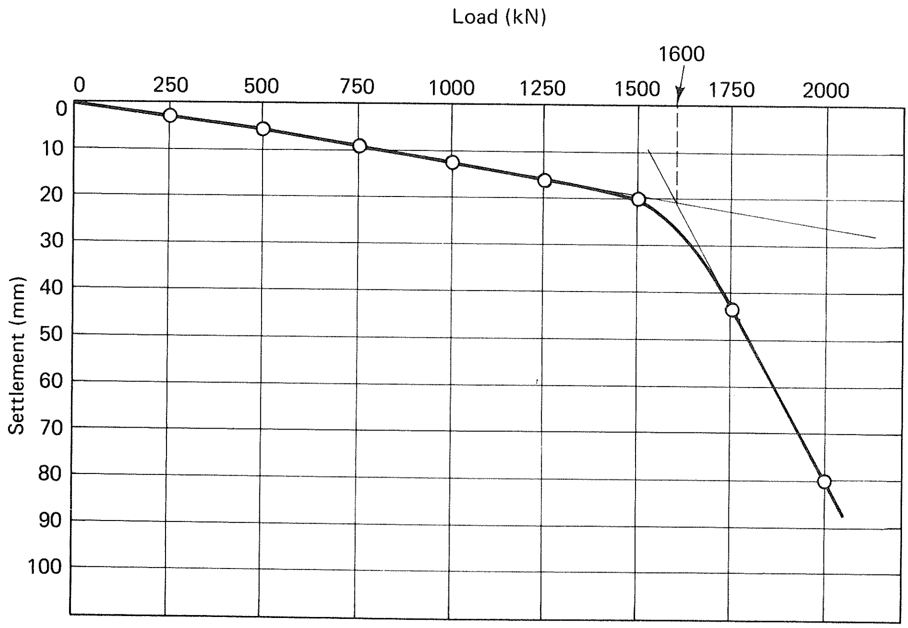


FIGURE 10-13 Plot of load test data for Example 10-10.

10-7 NEGATIVE SKIN FRICTION (DOWNDRAG)

As related throughout this chapter, piles depend, in part at least, on skin friction for support. Under certain conditions, however, skin friction may develop that causes downdrag on a pile rather than support. Skin friction that causes downdrag is known as *negative skin friction.*

Negative skin friction may occur if soil adjacent to a pile settles more than the pile itself. This is most likely to happen when a pile is driven through compressible soil, such as soft to medium clay or soft silt. Subsequent consolidation of the soil (caused by newly placed fill, for example) can cause negative skin friction as soil adjacent to the pile moves downward while the pile, restrained at the tip, remains fixed. A similar phenomenon may occur as a result of lowering the water table at the site.

Negative skin friction is, of course, detrimental with regard to a pile's ability to carry load. Hence, if conditions at a particular site suggest that negative skin friction may occur, its magnitude should be determined and subtracted from the pile's load-carrying ability.

10-8 PILE GROUPS AND SPACING OF PILES

Heretofore in this chapter, discussion has pertained to a single pile. In reality, however, piles are almost always arranged in groups of three or more. Furthermore, the group of piles is commonly tied together by a pile cap, which is attached to the head of individual piles and causes the several piles to act together as a pile foundation. Figure 10-14 illustrates some typical pile grouping patterns.

If two piles are driven close together, soil stresses caused by the piles tend to overlap; and bearing capacity of the pile group consisting of two piles is less than the sum of individual capacities. If the two piles are moved farther apart, so that individual stresses do not overlap, bearing capacity of the pile group is not reduced significantly from the sum of individual capacities. Thus, it would appear that piles should be spaced relatively far apart. This consideration is offset, however, by the unduly large pile caps that would be required for the wider spacing.

Minimum allowable pile spacing is often specified by applicable building codes. For example, a building code may state that "the minimum center-to-center spacing of piles not driven to rock shall be not less than twice the average diameter of a round pile, nor less than 1.75 times the diagonal dimension of a rectangular or rolled structural steel pile, nor less than 2 ft 6 in. (0.76 m). For piles driven to rock, the minimum center-to-center spacing of piles shall be not less than twice the average diameter of a round pile, nor less than 1.75 times the diagonal dimension of a rectangular or rolled structural steel pile, nor less than 2 ft 0 in. (0.61 m)" [3].

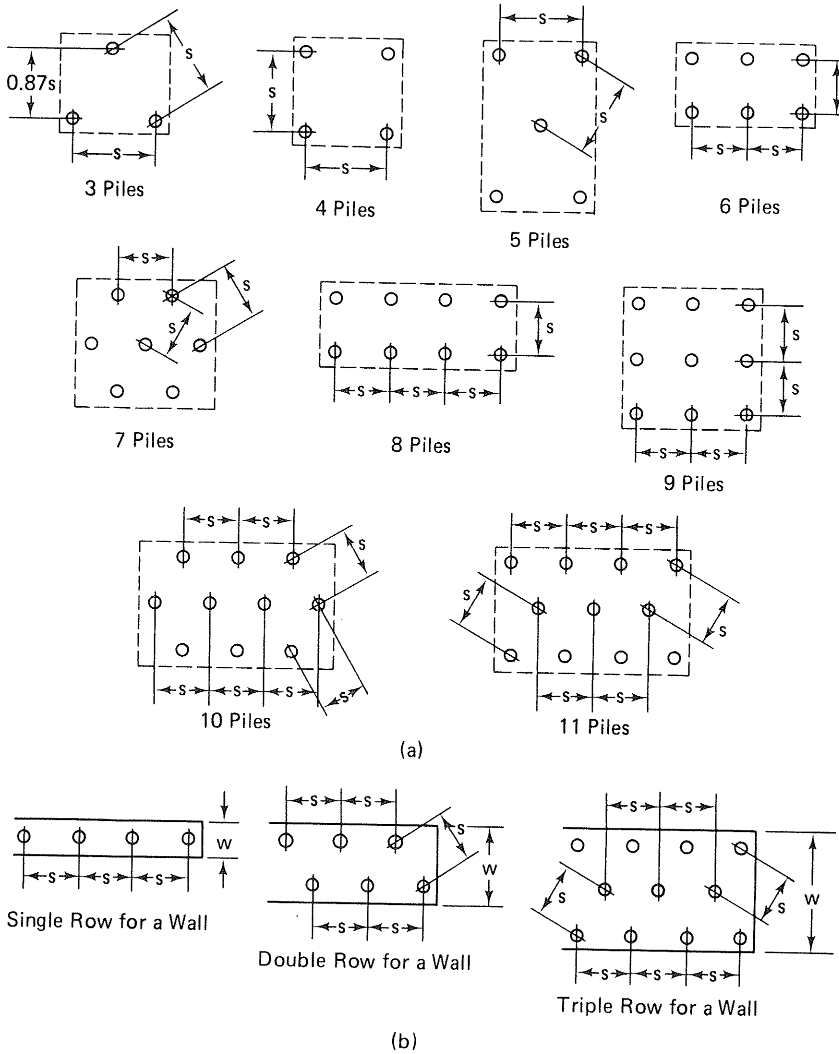


FIGURE 10-14 Typical pile group patterns for (a) single footings, and (b) foundation walls. [7]

10-9 EFFICIENCY OF PILE GROUPS

As related in the last section, the capacity of a pile group may be less than the sum of the individual capacities of the piles making up the group. Inasmuch as it would be convenient to estimate the capacity of a group of piles based on the capacity of a single pile, attempts have been made to determine the efficiency of pile groups. (Efficiency of a pile group is the capacity of a pile group divided by the sum of the individual capacities of the piles making up the group.)

In the case where a pile group is comprised of end-bearing piles resting on bedrock (or on a layer of dense sand and gravel overlying bedrock), an efficiency of 1.0 may be assumed [13]. (In other words, the group of n piles will carry n times the capacity of a single pile.) An efficiency of 1.0 is also often assumed by designers for friction piles driven in cohesionless soil. For a pile group comprised of friction piles driven in cohesive soil, an efficiency of less than 1.0 is to be expected because stresses from individual piles build up and reduce the capacity of the pile group.

One equation that has been used to compute pile group efficiency is known as the *Converse–Labarre equation* [13]:

$$E_g = 1 - \theta \frac{(n-1)m + (m-1)n}{90mn} \tag{10-11}$$

where E_g = pile group efficiency
θ = arctan d/s, deg
n = number of piles in a row
m = number of rows of piles
d = diameter of piles
s = spacing of piles, center to center, in same units as pile diameter

Example 10-11 illustrates the application of the Converse–Labarre equation.

EXAMPLE 10-11

Given

1. A pile group consists of 12 friction piles in cohesive soil (see Fig. 10-15).
2. Each pile's diameter is 12 in. and center-to-center spacing is 3 ft.
3. By means of a load test, the ultimate load of a single pile was found to be 100 kips.

Required

Design capacity of the pile group, using the Converse–Labarre equation.

Solution

$$E_g = 1 - \theta \frac{(n-1)m + (m-1)n}{90mn} \tag{10-11}$$

$$\theta = \arctan \frac{d}{s} = \arctan \frac{1}{3} = 18.4^\circ$$

$$E_g = 1 - (18.4)\frac{(4-1)(3) + (3-1)(4)}{(90)(3)(4)} = 0.710$$

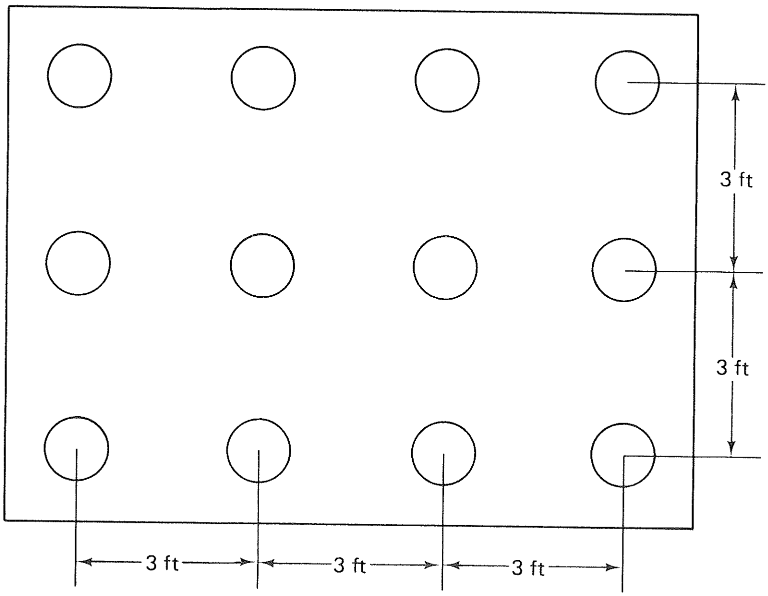


FIGURE 10-15

$$\text{Allowable bearing capacity of a single pile} = \frac{100 \text{ kips}}{2} = 50 \text{ kips}$$

$$\text{Design capacity of the pile group} = (0.710)(12)(50 \text{ kips}) = 426 \text{ kips}$$

For friction piles driven in cohesive soil, Coyle and Sulaiman suggested that pile group efficiency may be assumed to vary linearly from a value of 0.7 at a pile spacing of 3 times the pile diameter to a value of 1.0 at a pile spacing of 8 times the pile diameter [14, 15]. For pile spacings less than 3 times the pile diameter, group capacity may be considered as block capacity and total capacity can be estimated by treating the group as a pier and applying the equation [1, 14]

$$Q_g = 2D(W + L)f + 1.3 \times c \times N_c \times W \times L \qquad (10\text{-}12)$$

where Q_g = ultimate bearing capacity of pile group
D = depth of pile group
W = width of pile group
L = length of pile group
f = unit adhesion developed between cohesive soil and pile surface (equal to αc)
α = ratio of adhesion to cohesion (see Fig. 10-6)
c = cohesion
N_c = bearing capacity factor for a shallow rectangular footing (see Fig. 9-7)

A pile group can be considered safe against block failure if total design load (i.e., "safe design load" per pile multiplied by the number of piles) does not exceed $Q_g/3$. If the total design load exceeds $Q_g/3$, the foundation design must be revised.

Figure 10-16 gives a summary of criteria for pile group capacity.

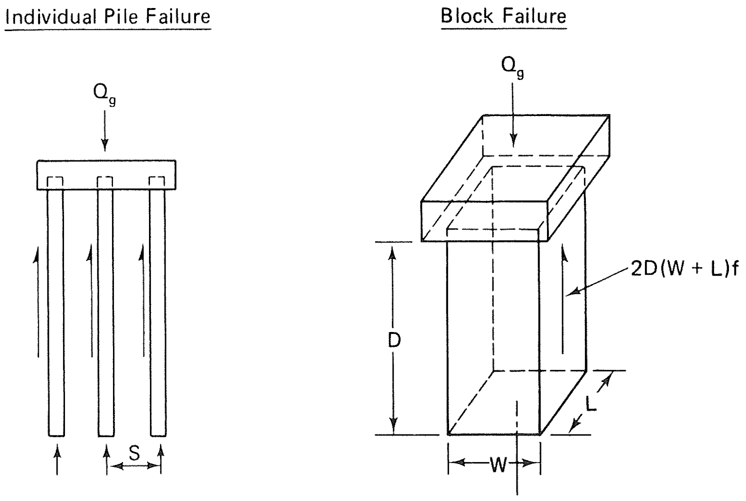
EXAMPLE 10-12

Given

1. A pile group consists of four friction piles in cohesive soil (see Fig. 10-17).
2. Each pile's diameter is 12 in. and center-to-center spacing is 2.5 ft.

Required

1. Block capacity of the pile group. Use a factor of safety of 3.
2. Allowable group capacity based on individual pile failure. Use a factor of safety of 2, along with the Converse–Labarre equation for pile group efficiency.



In Cohesionless Soils

$Q_g = n \times Q_u$

In Cohesive Soils

For S ⩾ 3 Diameters,
$Q_g = E_g \times n \times Q_u$
E_g Varies Linearly from 0.7 at S = 3 Diameters to 1.0 at S ⩾ 8 Diameters

In Cohesive Soils

For S < 3.0 Diameters,
$Q_g = 2D(W + L)f + 1.3 \times c \times N_c \times W \times L$

FIGURE 10-16 Summary of criteria for pile group capacity. [14, 15]

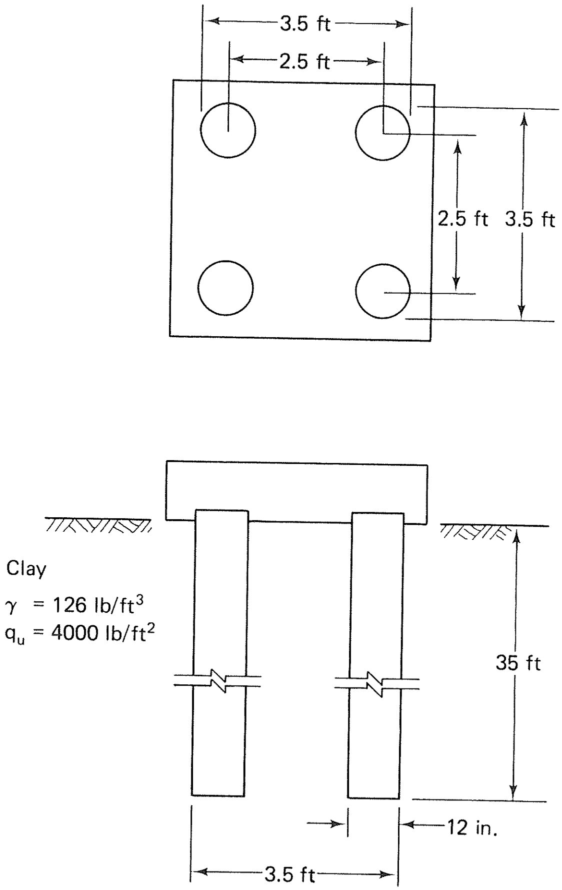


FIGURE 10-17

3. Design capacity of the pile group.

Solution

1. *Block capacity:*
   Since center-to-center spacing of the piles is 2.5 ft, which is less than 3 ft (i.e., 3 diameters), according to criteria suggested by Coyle and Sulaiman [15], block capacity of the pile group can be estimated by Eq. (10-12).

$$Q_g = 2D(W + L)f + 1.3 \times c \times N_c \times W \times L \tag{10-12}$$

$$D = 35 \text{ ft}$$

$$W = 2.5 \text{ ft} + 0.5 \text{ ft} + 0.5 \text{ ft} = 3.5 \text{ ft}$$

$$L = 2.5 \text{ ft} + 0.5 \text{ ft} + 0.5 \text{ ft} = 3.5 \text{ ft}$$

$$f = \alpha c$$

$$q_u = 4000 \text{ lb/ft}^2 = 2.0 \text{ tons/ft}^2$$

$$c = \frac{4000 \text{ lb/ft}^2}{2} = 2000 \text{ lb/ft}^2 = 2 \text{ kips/ft}^2$$

From Fig. 10-6 with $q_u = 2.0$ tons/ft^2,

$$\alpha = 0.56$$
$$f = (0.56)(2000 \text{ lb/ft}^2) = 1120 \text{ lb/ft}^2 = 1.12 \text{ kips/ft}^2$$
$$N_c = 5.14 \qquad \text{(from Fig. 9-7 with } \phi = 0° \text{ for clay)}$$
$$Q_g = (2)(35 \text{ ft})(3.5 \text{ ft} + 3.5 \text{ ft})(1.12 \text{ kips/ft}^2)$$
$$+ (1.3)(2 \text{ kips/ft}^2)(5.14)(3.5 \text{ ft})(3.5 \text{ ft}) = 713 \text{ kips}$$
$$\text{Allowable block capacity} = \frac{713 \text{ kips}}{3} = 238 \text{ kips}$$

2. *Group capacity based on individual pile:*

$$Q_{\text{ultimate}} = Q_{\text{friction}} + Q_{\text{tip}} \tag{10-1}$$
$$Q_{\text{friction}} = f \cdot A_{\text{surface}}$$
$$f = 1.12 \text{ kips/ft}^2 \text{ [from (1) above]}$$
$$A_{\text{surface}} = (\pi d)(L) = (\pi)(1 \text{ ft})(35 \text{ ft}) = 110.0 \text{ ft}^2$$
$$Q_{\text{friction}} = (1.12 \text{ kips/ft}^2)(110.0 \text{ ft}^2) = 123 \text{ kips}$$
$$Q_{\text{tip}} = cN_cA_{\text{tip}} = (2 \text{ kips/ft}^2)\ (9)\left(\frac{\pi}{4}\right)(1 \text{ ft})^2 = 14 \text{ kips}$$
$$Q_{\text{ultimate}} = 123 \text{ kips} + 14 \text{ kips} = 137 \text{ kips}$$
$$Q_a = \frac{137 \text{ kips}}{2} = 68.5 \text{ kips (allowable load for an individual pile)}$$
$$E_g = 1 - \theta\frac{(n-1)m + (m-1)n}{90mn} \tag{10-11}$$
$$\theta = \arctan\frac{d}{s} = \arctan\frac{1}{2.5} = 21.8°$$
$$n = 2$$
$$m = 2$$
$$E_g = 1 - (21.8)\,\frac{(2-1)(2) + (2-1)(2)}{(90)(2)(2)} = 0.758$$

$$\text{Allowable } Q = (68.5 \text{ kips})(4)(0.758) = 208 \text{ kips}$$
(allowable load for pile group)

3. *Design capacity of the pile group:*
This is the smaller group capacity of (1) and (2), which is 208 kips.

10-10 DISTRIBUTION OF LOADS IN PILE GROUPS

The load on any particular pile within a pile group may be computed using the elastic equation [9]:

$$Q_m = \frac{Q}{n} \pm \frac{M_y x}{\sum(x^2)} \pm \frac{M_x y}{\sum(y^2)} \tag{10-13}$$

where Q_m = axial load on any pile m
Q = total vertical load acting at the centroid of the pile group
n = number of piles
M_x, M_y = moment with respect to x and y axes, respectively
x, y = distance from pile to y and x axes, respectively

(Both x and y axes pass through the centroid of the pile group and are perpendicular to each other.) It should be noted that shears and bending moments can be determined for any section of pile cap by using elastic and static equations.

EXAMPLE 10-13

Given

1. A pile group consists of 9 piles as shown in Fig. 10-18.
2. A column load of 450 kips acts vertically on point A.

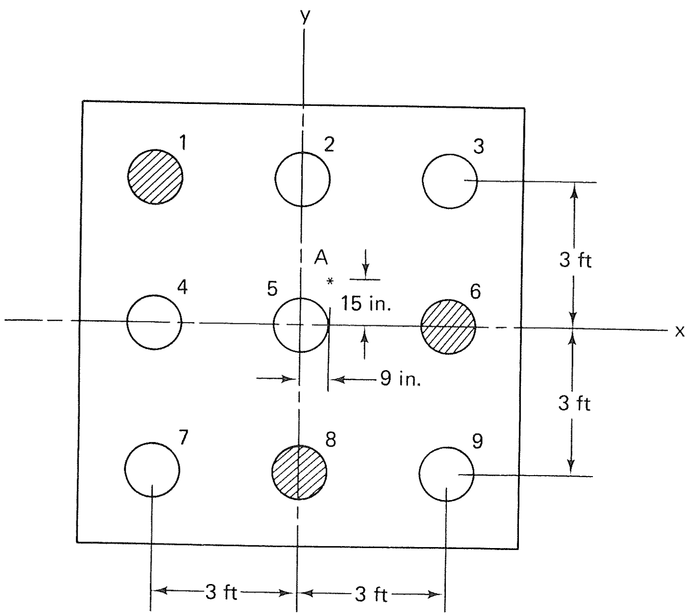


FIGURE 10-18

Required

Load on piles 1, 6, and 8.

Solution

From Eq. (10-13)

$$Q_m = \frac{Q}{n} \pm \frac{M_y x}{\sum(x^2)} \pm \frac{M_x y}{\sum(y^2)} \tag{10-13}$$

$$Q = 450 \text{ kips}$$

$$n = 9$$

$$\sum(x^2) = (6)(3 \text{ ft})^2 = 54 \text{ ft}^2$$

$$\sum(y^2) = (6)(3 \text{ ft})^2 = 54 \text{ ft}^2$$

$$M_x = (450 \text{ kips})\left(\frac{15 \text{ in.}}{12 \text{ in./ft}}\right) = 562.5 \text{ kip-ft}$$

$$M_y = (450 \text{ kips})\left(\frac{9 \text{ in.}}{12 \text{ in./ft}}\right) = 337.5 \text{ kip-ft}$$

Load on pile no. 1

$$Q_1 = \frac{450 \text{ kips}}{9} + \frac{(337.5 \text{ kip-ft})(-3 \text{ ft})}{54 \text{ ft}^2} + \frac{(562.5 \text{ kip-ft})(+3 \text{ ft})}{54 \text{ ft}^2} = 62.5 \text{ kips}$$

Load on pile no. 6

$$Q_6 = \frac{450 \text{ kips}}{9} + \frac{(337.5 \text{ kip-ft})(+3 \text{ ft})}{54 \text{ ft}^2} + \frac{(562.5 \text{ kip-ft})(0)}{54 \text{ ft}^2} = 68.8 \text{ kips}$$

Load on pile no. 8

$$Q_8 = \frac{450 \text{ kips}}{9} + \frac{(337.5 \text{ kip-ft})(0)}{54 \text{ ft}^2} + \frac{(562.5 \text{ kip-ft})(-3 \text{ ft})}{54 \text{ ft}^2} = 18.8 \text{ kips}$$

EXAMPLE 10-14

Given

1. Figure 10-19 shows a pile foundation consisting of five piles.
2. The pile foundation is subjected to a 200-kip vertical load and a moment with respect to the y axis of 140 kip-ft (see Fig. 10-19).

Required

Shear and bending moment on section a–a due to the pile reacting under the pile cap.

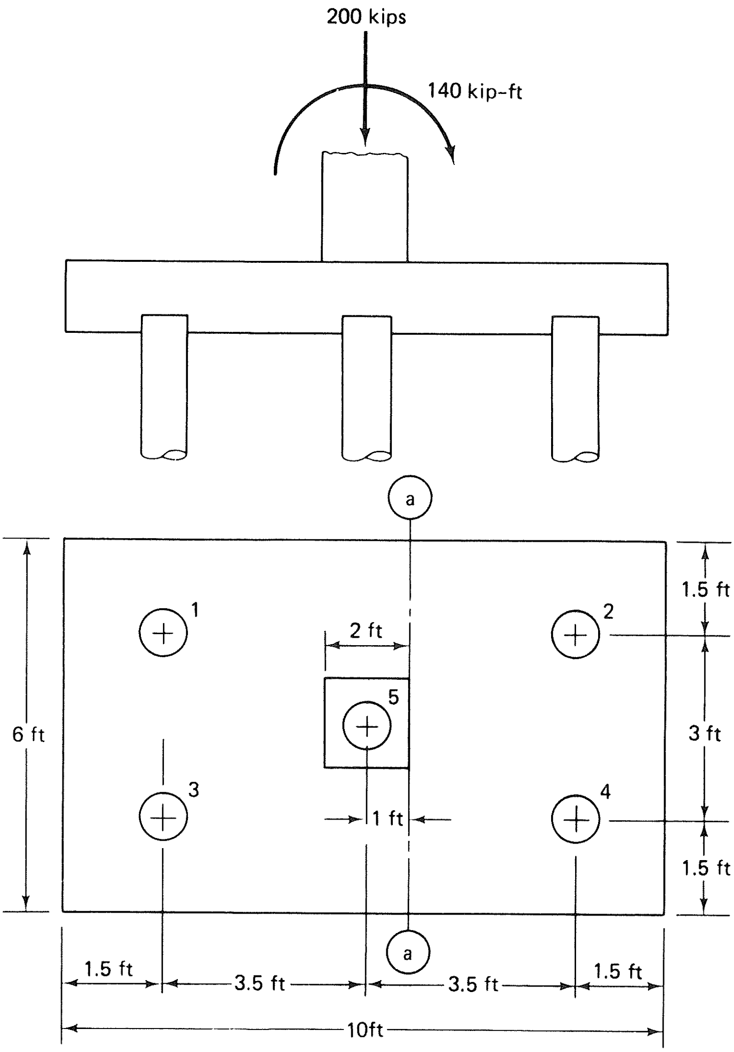


FIGURE 10-19

Solution

From Eq. (10-13),

$$Q_m = \frac{Q}{n} \pm \frac{M_y x}{\sum(x^2)} \pm \frac{M_x y}{\sum(y^2)} \tag{10-13}$$

$$Q = 200 \text{ kips}$$

$$n = 5$$

$$M_y = 140 \text{ kip-ft}$$

$$M_x = 0$$

$$\sum(x^2) = (4)(3.5 \text{ ft})^2 = 49 \text{ ft}^2$$

$$Q_2 = Q_4 = \frac{200 \text{ kips}}{5} + \frac{(140 \text{ kip-ft})(3.5 \text{ ft})}{49 \text{ ft}^2} + \frac{(0)y}{\sum(y^2)} = 50 \text{ kips}$$

$$\text{Shear at section a–a} = (50 \text{ kips})(2) = 100 \text{ kips}$$

$$\text{Moment at section a–a} = (2)(50 \text{ kips})(3.5 \text{ ft} - 1 \text{ ft}) = 250 \text{ kip-ft}$$

10-11 SETTLEMENT OF PILE FOUNDATIONS

Like shallow foundations, pile foundations must be analyzed to predict their settlement to ensure that it is tolerable. Unfortunately, universally accepted methods for predicting pile settlements are not available today. The following give some possible methods for predicting pile settlement for end-bearing piles on bedrock, piles in sand, and piles in clay.

Settlement of End-Bearing Piles on Bedrock

A well-designed and constructed pile foundation on hard bedrock generally will not experience an objectionable amount of settlement. The amount of settlement of pile foundations on soft bedrock is very difficult to predict accurately and can only be estimated by judging from the characteristics of rock core samples. Local experience, if available, should be employed as guidance [9].

Settlement of Piles in Sand

Settlement of a pile group in sand may be estimated by using the empirical relationship of Fig. 10-20. This relationship gives the ratio of settlement of

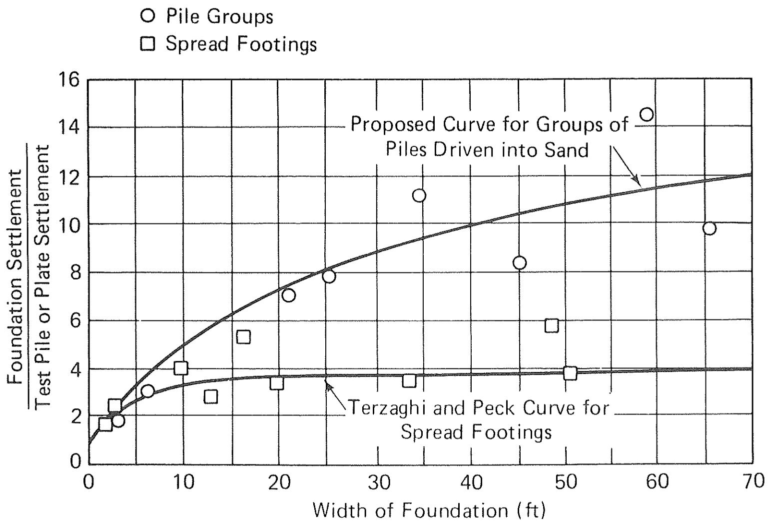


FIGURE 10-20 Settlement ratio versus width of foundations on sand (1 ft = 0.3048 m). [14, 16]

pile group to settlement of single test pile as a function of width of the pile group.

Settlement of a pile group is substantially larger than that of a single test pile, which can readily be determined by a pile load test. For example, according to Fig. 10-20 settlement of a pile group with a 15-ft width would be about 6.4 times that of a single test pile.

Settlement of Piles in Clay

Prediction of pile settlements in deep clay requires first an estimate of load distribution in the soil, followed by settlement calculation in accordance with consolidation theory. One method of estimating load distribution is to assume the load is applied to an equivalent flexible mat (i.e., an imaginary mat) at some selected level and then to compute the distribution of load from that imaginary mat. For friction piles in deep clay, the equivalent (imaginary) mat may be assumed at a plane located at two-thirds the pile depth [1, 14] (see Fig. 10-21a). Consolidation of soil below that plane is then computed as if the piles are no longer present. If piles pass through a layer of very soft clay to a firm bearing in a layer of stiff clay, an equivalent mat may be placed at the level of the pile tips, assuming eventual concentration of load at that level (see Fig. 10-21b).

Settlement analysis is then performed, based on consolidation test results, to predict the expected, approximate settlement that would occur for an ordinary (unpiled) foundation if the foundation were a mat of the same depth and dimensions at the same plane. In such cases the method of settlement analysis of pile-supported foundations is the same as that used for shallow foundations. From Chap. 7, based on consolidation test results, the amount of settlement due to consolidation can be calculated for a layer of compressible soil by the equation [1, 9]

$$S = \frac{e_0 - e}{1 + e_0}[H] \qquad (7\text{-}13)$$

or

$$S = C_c\left(\frac{H}{1 + e_0}\right)\log\frac{p_0 + \Delta p}{p_0} \qquad (7\text{-}17)$$

where S = consolidation settlement
e_0 = initial void ratio (void ratio *in situ*)
e = final void ratio
H = thickness of layer of compressible soil
C_c = compression index (slope of field e–log p curve)
p_0 = effective overburden pressure (effective weight of soil above midheight of the consolidating layer)
Δp = consolidation pressure (net additional pressure)

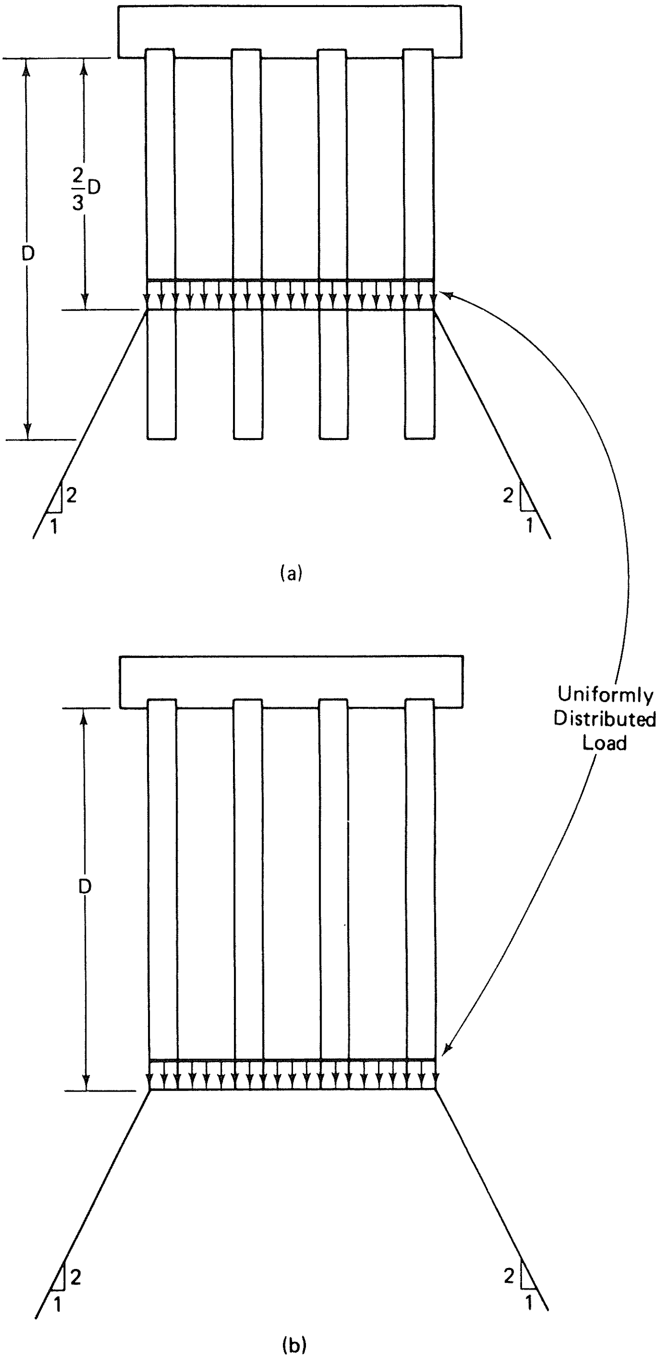


FIGURE 10-21 Friction piles: (a) in deep clay; (b) through soft clay into stiff clay. [14]

Example 10-15 illustrates computation of approximate total settlement of a pile foundation in deep clay.

EXAMPLE 10-15

Given

1. A group of friction piles in deep clay is shown in Fig. 10-22.
2. Total load on the piles reduced by the weight of soil displaced by the foundation is 300 kips.

Required

Approximate total settlement of the pile foundation.

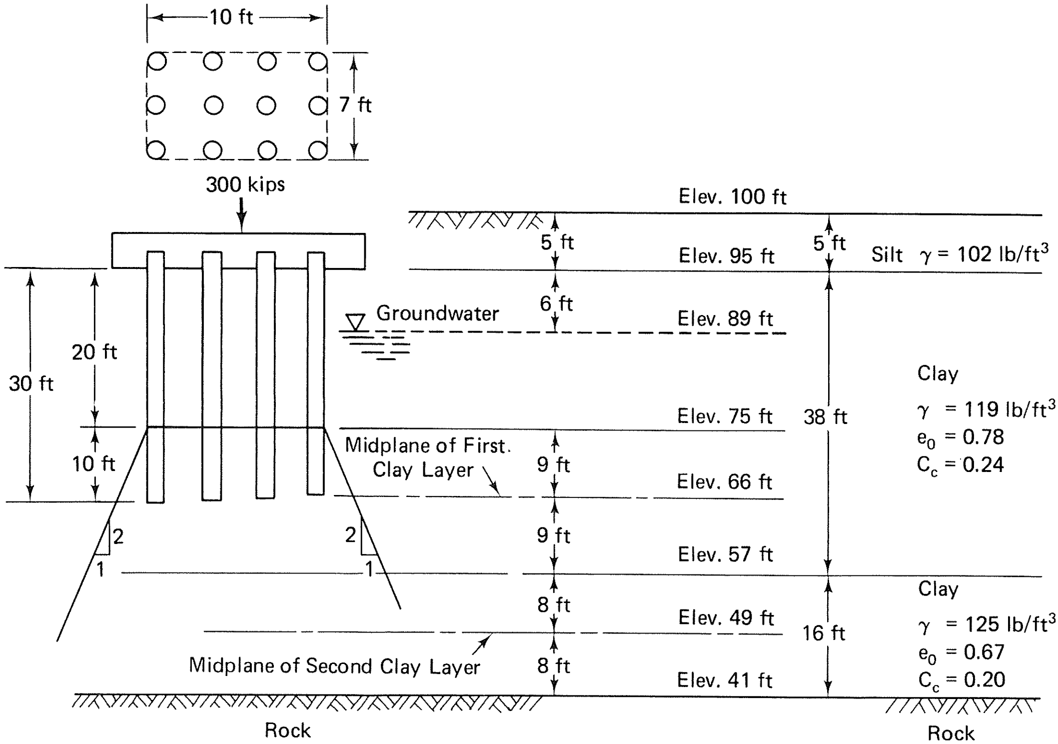


FIGURE 10-22

Solution

Computation of effective overburden pressures (p_0)

$$p_0 \text{ at elev. 66 ft} = (100\text{ ft} - 95\text{ ft})(102\text{ lb/ft}^3) + (95\text{ ft} - 89\text{ ft})(119\text{ lb/ft}^3) + (89\text{ ft} - 66\text{ ft})(119\text{ lb/ft}^3 - 62.4\text{ lb/ft}^3)$$
$$= 2526\text{ lb/ft}^2\text{, or } 2.53\text{ kips/ft}^2$$

$$p_0 \text{ at elev. 49 ft} = (100\text{ ft} - 95\text{ ft})(102\text{ lb/ft}^3) + (95\text{ ft} - 89\text{ ft})(119\text{ lb/ft}^3) + (89\text{ ft} - 57\text{ ft})(119\text{ lb/ft}^3 - 62.4\text{ lb/ft}^3) + (57\text{ ft} - 49\text{ ft})(125\text{ lb/ft}^3 - 62.4\text{ lb/ft}^3)$$
$$= 3536\text{ lb/ft}^2\text{, or } 3.54\text{ kips/ft}^2$$

Computation of Δp

$$\text{Area at elev. 66 ft} = [10\text{ ft} + (2)(75\text{ ft} - 66\text{ ft})(\tfrac{1}{2})] \cdot [7\text{ ft} + (2)(75\text{ ft} - 66\text{ ft})(\tfrac{1}{2})] = 304\text{ ft}^2$$

$$\Delta p \text{ at elev. 66 ft} = \frac{300\text{ kips}}{304\text{ ft}^2} = 0.99\text{ kip/ft}^2$$

$$\text{Area at elev. 49 ft} = [10\text{ ft} + (2)(75\text{ ft} - 49\text{ ft})(\tfrac{1}{2})] \cdot [7\text{ ft} + (2)(75\text{ ft} - 49\text{ ft})(\tfrac{1}{2})] = 1188\text{ ft}^2$$

$$\Delta p \text{ at elev. 49 ft} = \frac{300\text{ kips}}{1188\text{ ft}^2} = 0.25\text{ kip/ft}^2$$

Settlement computations

From Eq. (7-17),

$$S = C_c \frac{H}{1 + e_0} \log \frac{p_0 + \Delta p}{p_0} \tag{7-17}$$

Elev. 75 to 57 ft:

$$S = (0.24)\left(\frac{18\text{ ft}}{1 + 0.78}\right) \log \frac{2.53\text{ kips/ft}^2 + 0.99\text{ kip/ft}^2}{2.53\text{ kips/ft}^2} = 0.35\text{ ft}$$

Elev. 57 to 41 ft:

$$S = (0.20)\left(\frac{16\text{ ft}}{1 + 0.67}\right) \log \frac{3.54\text{ kips/ft}^2 + 0.25\text{ kip/ft}^2}{3.54\text{ kips/ft}^2} = 0.06\text{ ft}$$

Approximate total settlement = 0.35 ft + 0.06 ft = 0.41 ft = 4.9 in.

10-12 CONSTRUCTION OF PILE FOUNDATIONS

Construction of pile foundations consists of installing the piles (see Fig. 10-23) (usually by driving) and constructing pile caps. Pile caps are often made of concrete, and their construction is usually a relatively simple structural problem.

With regard to pile installation, most piles are driven by a device called a *pile hammer.* Simply speaking, a pile hammer is a weight that is alternately raised and dropped onto the top of a pile to drive the pile into the soil. Hammer weights vary considerably. As a general rule, a hammer's weight should be at least half the weight of the pile being driven, and the driving energy should be at least 1 ft-lb for each pound of pile weight [9]. The hammer itself is contained within a larger device, with the hammer operated between a pair of parallel steel members known as *leads.*

Several types of pile hammers are available. *Drop hammers* consist of a heavy ram that is raised by a cable and hoisting drum and dropped onto the pile. For *single-acting hammers,* the ram is raised by steam or compressed air and dropped onto the pile. With *double-acting hammers,* the ram is both raised and accelerated downward by steam or air. *Differential-acting hammers* are similar to double-acting hammers. *Diesel hammers* use gasoline for fuel,

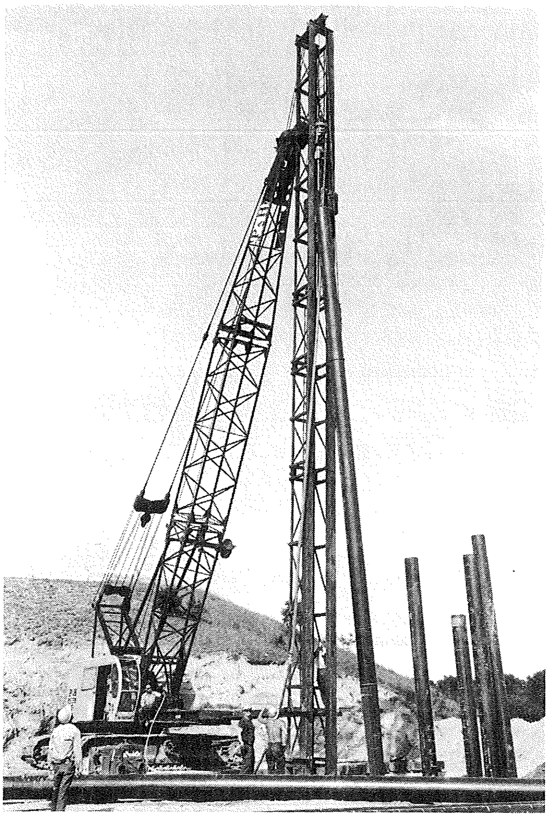


FIGURE 10-23 Pile installation. (Courtesy of Associated Pile & Fitting Corporation of New Jersey.)

which causes an explosion that advances the pile and lifts the ram. The total driving energy delivered to the pile includes both the impact of the ram and the energy delivered by explosion. Table 10-6 (in Sec. 10-5) gives more specific information on various pile hammers.

Selection of a pile hammer for a specific job depends on a number of factors. Table 10-7 gives data for selection of pile hammers for various conditions.

Repeated striking of a pile by a pile hammer's heavy ram can damage the pile. A wood pile's fibers at its head (top) may be crushed by the ram (an action known as *brooming*), causing the pile to split near its top end. Brooming and splitting can be minimized by putting a heavy steel ring over the pile's head while it is being driven into the soil. Any damaged part of the pile must be cut off and removed prior to loading the pile. (Hence, a somewhat longer wood pile than is ultimately needed should be used at the beginning to allow for length of pile that must be cut off.) Precast concrete piles may be protected by placing a metal cap over the pile's head with laminated layers of wood beneath the cap (i.e., between the cap and the pile's head) and a block of hardwood above the cap—all of this to help protect the pile as it is being driven by cushioning the ram's blow.

The other end of a pile—the tip—also needs protection—particularly if the pile is being driven through very hard soil or boulders. Such protection is provided by *driving points* (sometimes referred to as *driving shoes*). Figure 10-24 illustrates some commercially available driving points (or shoes) for

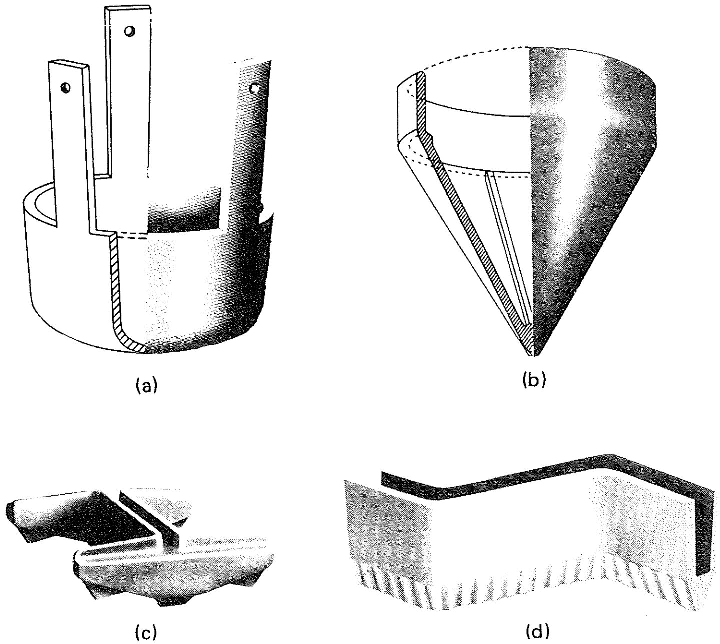


FIGURE 10-24 Driving points (or shoes): (a) timber pile shoes; (b) pipe pile point; (c) H-pile point; (d) sheet pile protector. (Courtesy of Associated Pile & Fitting Corporation of New Jersey.)

TABLE 10-7 Data for selection of pile hammers for driving concrete, timber, and steel sheetpiling under average and heavy driving conditions [9].[1,2]

Length of Pile (ft)	Depth of Penetration (%)	Sheet Pile[3]			Timber Pile		Concrete Pile	
		Light	Medium (ft-lb per blow)	Heavy	Light (ft-lb per blow)	Heavy	Light (ft-lb per blow)	Heavy
		Driving through earth, sand, loose gravel—normal frictional resistance						
25	50	1000–1800	1000–1800	1800–2500	3600–4200	3600–7250	7250–8750	8750–15,000
	100	1000–3600	1800–3600	1800–3600	3600–7250	3600–8750	7250–8750	13,000–15,000
50	50	1800–3600	1800–3600	3600–4200	3600–8750	7250–8750	8750–15,000	13,000–25,000
	100	3600–4200	3600–4200	3600–7500	7250–8750	7250–15,000	13,000–15,000	15,000–25,000
75	50		3600–7500	3600–8750		13,000–15,000		19,000–36,000
	100			3600–8750		15,000–19,000		19,000–36,000
		Driving through stiff clay, compacted gravel—very resistant						
25	50	1800–2500	1800–2500	1800–4200	7250–8750	7250–8750	7250–8750	8750–15,000
	100	1800–3600	1800–3600	1800–4200	7250–8750	7250–8750	7250–15,000	13,000–15,000
50	50	1800–4200	3600–4200	3600–8750	7250–15,000	7250–15,000	13,000–15,000	13,000–25,000
	100		3600–8750	3600–13,000		13,000–15,000		19,000–36,000
75	50		3600–8750	3600–13,000		13,000–15,000		19,000–36,000
	100			7500–19,000		15,000–25,000		19,000–36,000
Weight (per lin. ft)		20 lb	30 lb	40 lb	30 lb	60 lb	150 lb	400 lb
Pile size (approx.)		15 in.	15 in.	15 in.	13 in. diam	18 in. diam	12 in.^{2}	20 in.^{2}

[1]Tennessee Valley Authority.
[2]1 ft-lb = 1.356 N·m; 1 in. = 25.4 mm; 1 lb = 4.448 N.
[3]Energy required in driving single-sheet pile. Double these when driving two piles at a time.

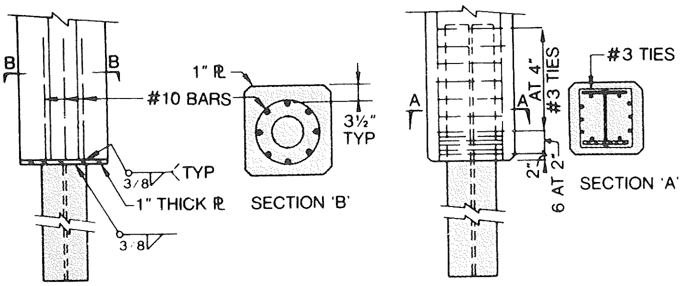
various types of piles. If hard driving is anticipated for precast concrete piles, driving points (or shoes) may be cast at the tips of the piles (see Fig. 10-25).

10-13 PROBLEMS

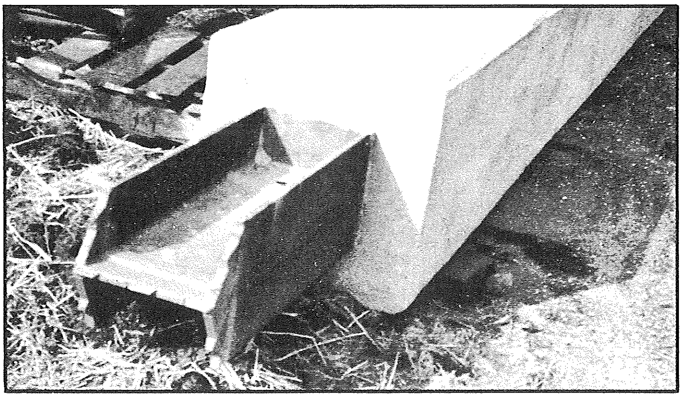
10-1 A 12-in. square concrete pile is driven into a loose sand to a depth of 30 ft. Soil conditions are shown in Fig. 10-26. Find the pile's axial capacity if K is assumed to be 0.7 and the factor of safety is 2.

10-2 Rework Problem 10-1 assuming the groundwater table is located 5 ft below the ground surface.

10-3 A 0.5-m-diameter steel pile is driven into dense sand. The pile is driven with the tip closed by a flat plate. The closed end, steel pipe pile is



The Prestressed Concrete Institute Standard for 10 to 36 in. piles has these details. HARD-BITE™ or Pluyn Points will protect the vulnerable corners of the H and assure penetration into dense and boulder-filled soils. The H and points prevent damage to the tip of the precast concrete pile.



H extends 4 ft into concrete; plate in web of H adds to bond strength. PILETIPS July - August 1983.

FIGURE 10-25 Driving points (or shoes) cast at the tips of the piles. (Courtesy of Associated Pile & Fitting Corporation of New Jersey.)

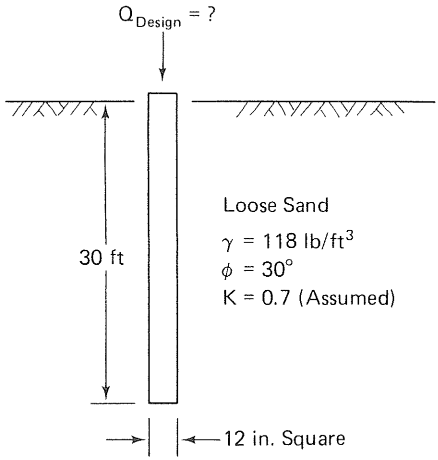


FIGURE 10-26

filled with concrete after driving. The embedded length of pile is 20 m. Soil conditions are as shown in Fig. 10-27. Determine the design capacity of the pile, using a factor of safety of 2.

10-4 A 14-in. square concrete pile is driven at a site as shown in Fig. 10-28. The embedded length of pile is 40 ft. Determine the pile's design capacity, using a factor of safety of 2.

10-5 A 12-in.-diameter concrete pile is driven at a site as shown in Fig. 10-29. What is the pile's design capacity if the factor of safety is 2?

10-6 A 0.5-m-diameter steel pile is driven into a varved clay deposit. The pile is driven with the tip closed by a flat plate. The closed-end, steel pipe pile is filled with concrete after driving. The embedded length of pile is 15 m. The clay deposit has a unit weight of 17.92 kN/m^3 and an unconfined compressive strength of 120 kN/m^2. Determine the design capacity of the pile, using a factor of safety of 2.

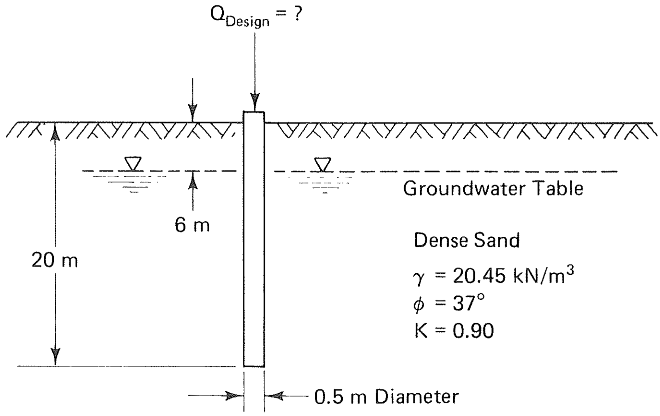


FIGURE 10-27

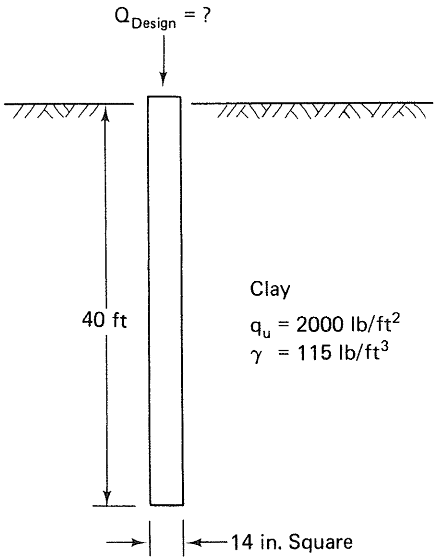


FIGURE 10-28

10-7 Rework Problem 10-6 if the embedded length of pile is 20 m and the clay deposit's unit weight and unconfined compressive strength are 17.29 kN/m³ and 96 kN/m², respectively.

10-8 A 12-in.-diameter concrete pile is to be driven into a clay soil as shown in Fig. 10-30. The pile's design capacity is 30 tons. Determine the pile's required length if the factor of safety is 2.

10-9 Design capacity of a steel pile is 250 kN. The pile is driven by a steam hammer with a manufacturer's hammer energy rating of 36 kN·m. Deter-

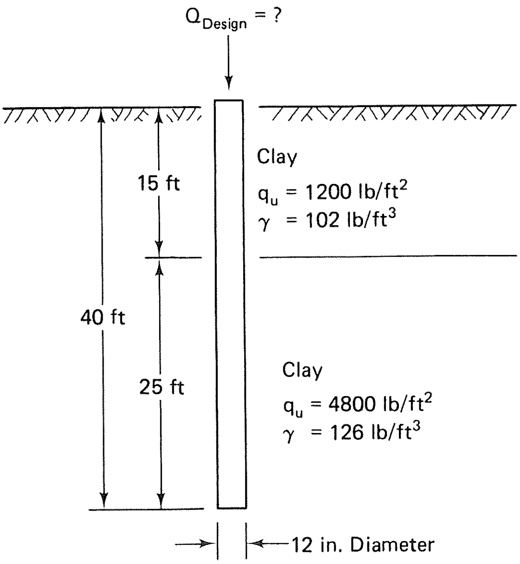


FIGURE 10-29

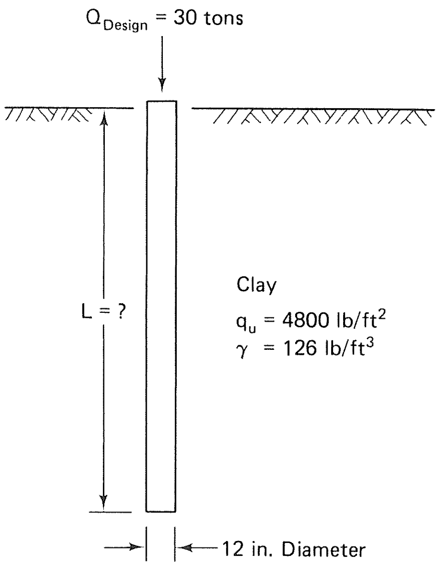


FIGURE 10-30

mine the average penetration of the pile from the last few driving blows. Use the Engineering-News formula.

10-10 A steel pipe pile is to be driven to an allowable load (design load) of 35 tons capacity by an MKT-11B3 double-acting steam hammer. The steel pipe has a net cross-sectional area of 17.12 in.^{2} and a length of 45 ft. The Danish pile-driving formula is to be used to control field installation of the piles. How many blows per foot are required for the last foot of penetration?

10-11 Rework Problem 10-10 using the Engineering-News formula.

10-12 A pile load test produced the settlement and rebound curves given in Fig. 10-31. The pile has a 12-in. diameter and is 25 ft long. Determine the allowable load for this pile using a local building code which states that:

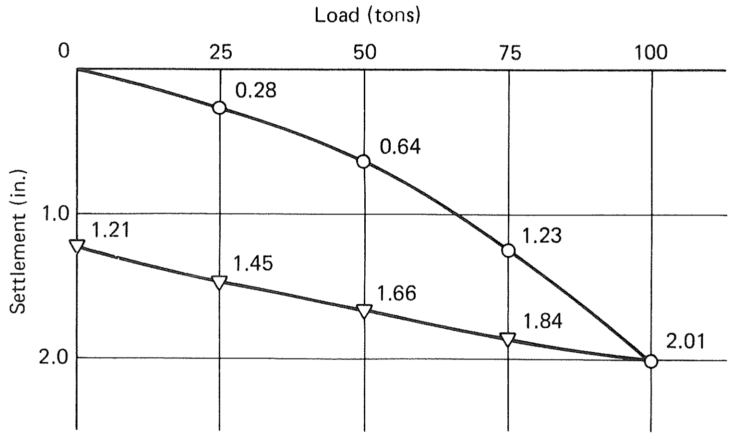


FIGURE 10-31

"The allowable load shall not be more than one-half of that test load which produces a net settlement per ton of test load of not more than 0.01 in. but in no case more than 0.75 in."

10-13 Rework Problem 10-12, except that the local building code is changed to read as follows: "The allowable pile load is taken as one-half of that load which produces a net settlement of not more than 0.01 in./ton of test load, but in no case more than 0.5 in."

10-14 A pile group consists of nine friction piles in clay soil (see Fig. 10-32). The diameter of each pile is 16 in. and their embedded length is 30 ft each. Center-to-center pile spacing is 4 ft. Soil conditions are shown in Fig. 10-32. Find the pile group's design capacity if the factor of safety is 2. Use the Converse–Labarre equation.

10-15 A concrete pile with a diameter of 0.3 m and length of 20 m was subjected to a pile load test, with the following results:

Load (kN)	*Settlement (mm)*
250	5.0
500	9.1
750	12.6
1000	16.2
1250	20.0
1500	32.0
1750	48.0
2000	67.1

Determine the allowable load for this pile using the building code cited on page 312.

10-16 A nine-pile group consists of 12-in. diameter friction concrete piles 30 ft long. The piles are driven into clay, the unconfined compressive strength of which is 6000 lb/ft^2 and the unit weight of which is 125 lb/ft^3. Pile spacing is 2½ diameters. Find: (a) block capacity of the pile group using a factor of safety of 3, (b) allowable group capacity based on individual pile failure using a factor of safety of 2 along with the Converse–Labarre equation for pile group efficiency, and (c) design capacity of the pile group.

10-17 A pile group consists of 12 piles as shown in Fig. 10-33. A vertical load of 480 kips acts vertically on point A. Determine the load on piles 2, 4, 7, and 9.

10-18 A pile group consists of four friction piles in cohesive soil. Each pile's diameter is 0.4 m, and center-to-center spacing is 1.5 m. The ultimate capacity of each pile is 453 kN. Estimate the design capacity of the pile group, using a factor of safety of 2 and the criteria suggested by Coyle and Sulaiman (Fig. 10-16).

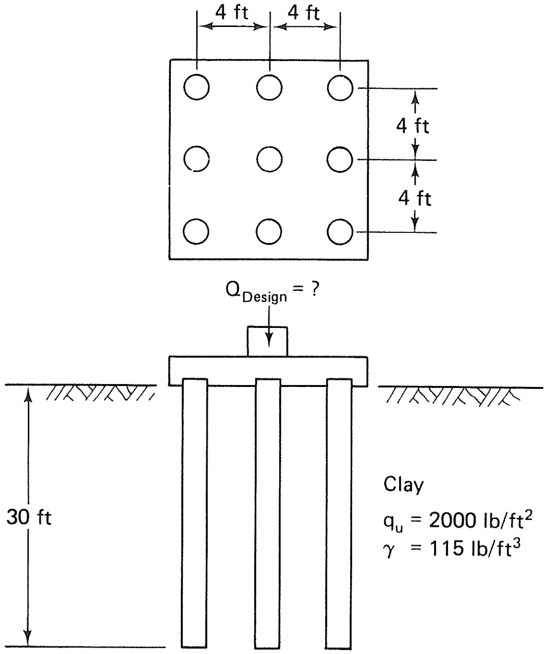


FIGURE 10-32

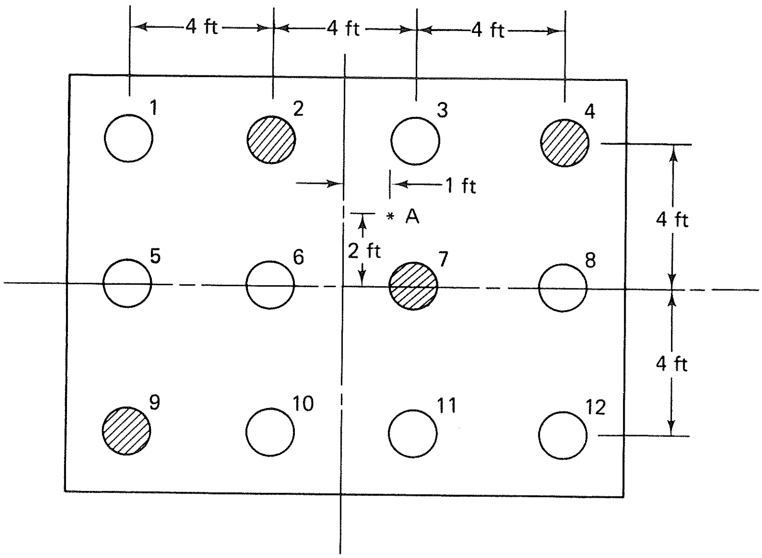
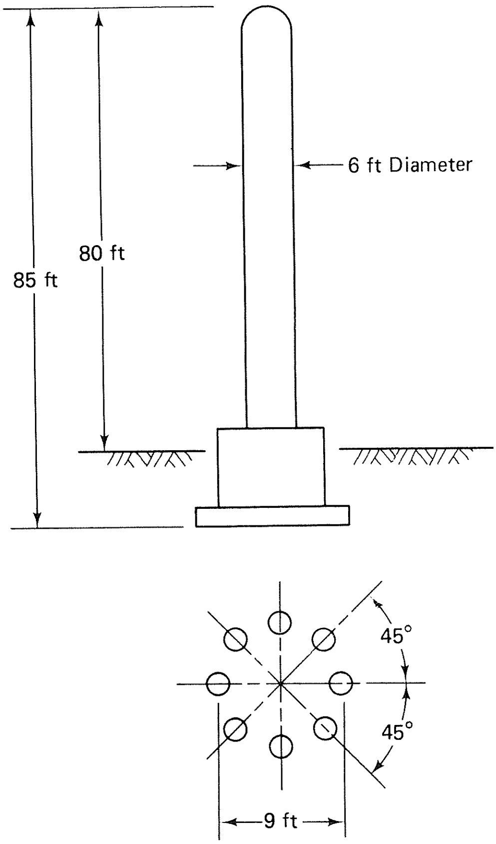


FIGURE 10-33

10-19 A pile group consists of nine friction piles in cohesive soil. Each pile's diameter is 0.3 m, and center-to-center spacing is 1.2 m. The ultimate capacity of each pile is 300 kN. Estimate the design capacity of the pile group, using a factor of safety of 2 and the criteria suggested by Coyle and Sulaiman (Fig. 10-16).



8 Piles Equally Spaced on a Circle of 9-ft Diameter **FIGURE 10-34**

10-20 A tower shown in Fig. 10-34 is subjected to a wind pressure of 25 lb/ft^2 on its projected area. The tower and foundation weigh 320 kips. Determine maximum and minimum pile reactions for the layout shown.

10-21 A group of friction piles in deep clay is shown in Fig. 10-35. Total load on the piles reduced by the weight of soil displaced by the foundation is 400 kips. Find the expected total settlement of the pile foundation.

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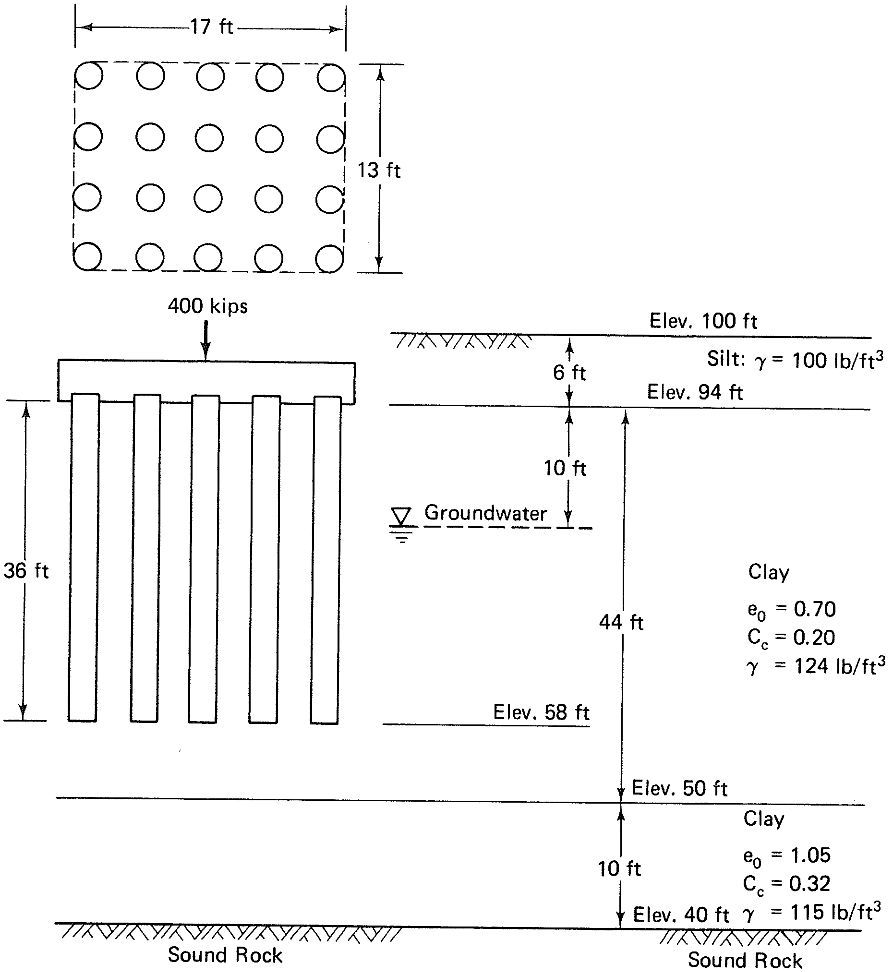

FIGURE 10-35

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11

Drilled Caissons

11-1 INTRODUCTION

A drilled caisson is a type of deep foundation that is constructed in place by drilling a hole into the soil, often to bedrock or a hard stratum, and subsequently placing concrete in the hole. The concrete may or may not contain reinforcing steel. Some drilled caissons have straight sides throughout (straight-shaft caissons); others are constructed with enlarged bases (belled caissons) (see Fig. 11-1). The enlarged base area results in a decreased contact pressure (soil pressure) at the caisson's base.

The purpose of a drilled caisson is to transmit a structural load to the caisson's base, which may be bedrock or other hard stratum. In essence, a drilled caisson is primarily a compression member with an axial load applied at its top, a reaction at its bottom, and lateral support along its sides.

Drilled caissons are constructed by using auger drill equipment to form the hole in the soil. Soil is removed from the hole during drilling in contrast to the driven pile, which only compresses soil aside. Thus, such problems as shifting and lifting of driven piles do not occur with drilled caissons. In some cases, such as in dry, strong cohesive soil, the hole may be drilled dry and without any side support. In this case, concrete placed in the hole makes direct contact with the soil forming the sides of the hole. If cohesionless soil and/or groundwater is encountered, a bentonite slurry may be introduced during drilling to prevent the soil from caving in. (Protective casing may also be used to prevent cave in.) In this case concrete is placed from the bottom up so as to displace the slurry. If a casing is used, it is slowly removed as concrete

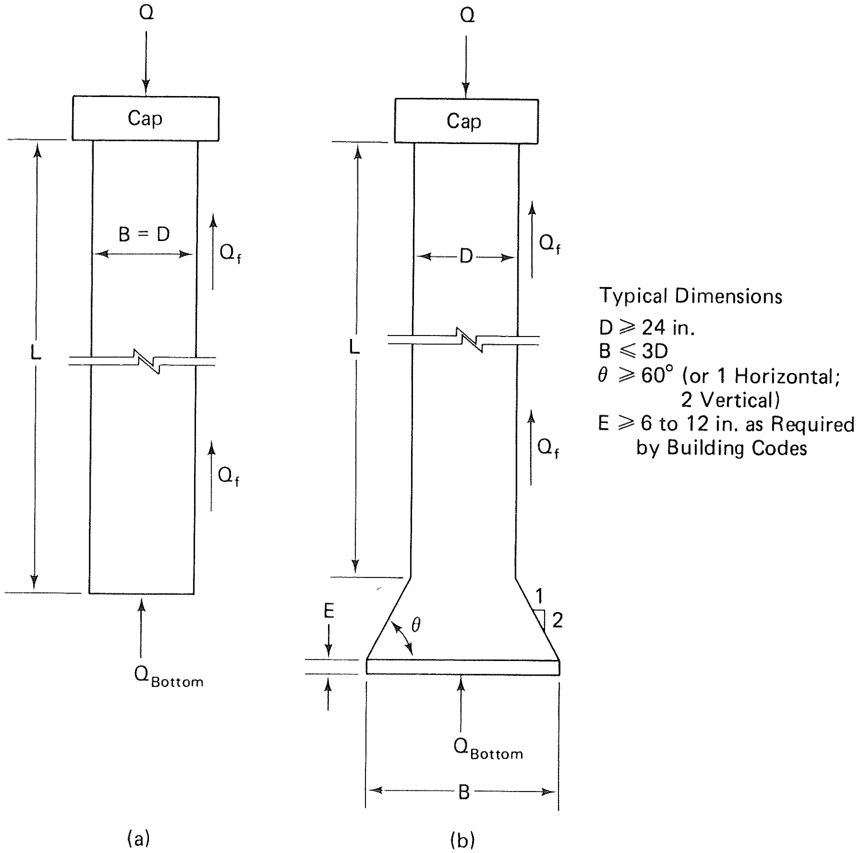

FIGURE 11-1 Caissons: (a) straight-shaft; (b) belled.

is placed, and the operator makes sure that soil does not fall into the excavated hole and mix with the concrete.

Drilled caissons are a popular type of deep foundation for several reasons. Drilling equipment is relatively light and easy to use compared to piledriving equipment. Drilling equipment is much quieter than pile drivers and does not cause massive ground vibrations that can adversely affect adjacent piles. Finally, drilled holes afford better (visual) inspection of the subsoil encountered.

11-2 BEARING CAPACITY OF DRILLED CAISSONS

As with a pile, a caisson gets its supporting power from two sources—skin friction and bearing capacity at the caisson's base. Thus, at failure the load on a drilled caisson may be expressed (as for a pile) by Eq. (10-1), which is reproduced here.

$$Q_{\text{ultimate}} = Q_{\text{friction}} + Q_{\text{tip}} \qquad (10\text{-}1)$$

Since caissons are not driven (as are piles), they do not make tight contact with surrounding soil (as do piles). Consequently, the supporting power for caissons provided by skin friction is relatively small.

To evaluate bearing capacity, it is helpful to consider separately drilled caissons in cohesive soils, in sands, and on bedrock.

Drilled Caissons in Cohesive Soils

Analysis of drilled caissons in cohesive soils is similar to that of piles in that the caisson's total bearing capacity results from resistance provided by its end bearing and skin friction, in accordance with Eq. (10-1). The term Q_{tip} of Eq. (10-1) can be evaluated by multiplying the cohesion (c) of the soil at the caisson's bottom by the bearing capacity factor (N_c) and this by the area of the caisson's bottom. The term $Q_{friction}$ of Eq. (10-1) can be evaluated by multiplying unit adhesion or skin friction developed between shaft surface and soil (f) by the shaft's surface area (A_{shaft}) (obtained by multiplying the circumference of the caisson shaft by the depth of caisson from ground surface to the top of the bell). Making these substitutions in Eq. (10-1) gives [1]

$$Q_{total} = cN_c A_{bottom} + fA_{shaft} \tag{11-1}$$

Thus far in this discussion, the analysis has been approximately the same as that for piles driven in clay. There is a significant difference between the two, however, and that is in determining the bearing capacity factor (N_c) and the unit adhesion or skin friction (f) of Eq. (11-1). For drilled caissons, the value of the bearing capacity factor (N_c) can be obtained from Table 11-1, and the value of unit adhesion or skin friction (f) can be obtained from Table 11-2. Adhesion or skin friction that develops along the shaft is related to the clay's cohesion and the manner in which the caisson is drilled.

TABLE 11-1 Bearing capacity factors for drilled caissons [2].

Ratio of Depth of Caisson to Diameter of Caisson Bottom	N_c
0	6.2
0.5	7.1
1.0	7.7
1.5	8.1
2.0	8.4
2.5	8.6
3.0	8.8
4.0 and over	9.0

TABLE 11-2 Adhesion or skin friction values for drilled caisson foundations in clay [1].

Foundation Type and Drilling Method Utilized	*Adhesion or Skin Friction, f**	*Upper Limit on f Value (kips/ft²)†*
Straight shaft, excavation drilled dry	$0.5c$	1.8
Straight shaft, drilled with slurry	$0.3c$	0.8
Belled, drilled dry	$0.3c$	0.8
Belled, drilled with slurry	$0.15c$	0.5

**c* is soil cohesion determined from triaxial testing, not *in situ* vane shear tests.
†1 kip/ft^2 = 47.88 kN/m^2.

A factor of safety of 3 is recommended for Q_{bottom} [i.e., the term cN_cA_{bottom} in Eq. (11-1)] [1]. Thus,

$$Q_{\text{allowable}} = \tfrac{1}{3}cN_cA_{\text{bottom}} + fA_{\text{shaft}} \tag{11-2}$$

EXAMPLE 11-1

Given

1. A 3-ft-diameter plain concrete drilled caisson is constructed in clay.
2. Soil conditions and a sketch of the caisson are shown in Fig. 11-2.
3. The excavation is drilled dry.

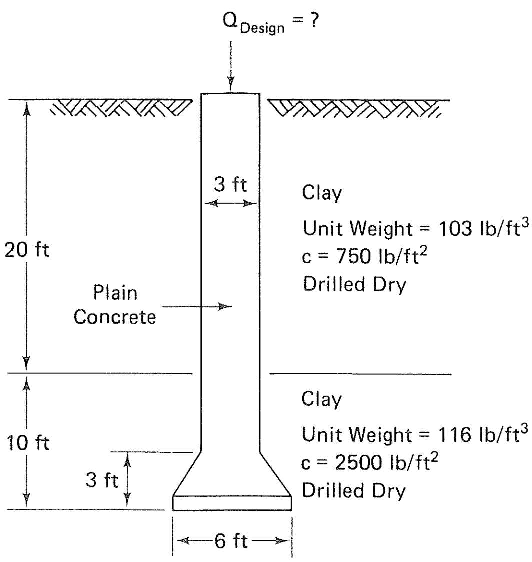

FIGURE 11-2

4. A local building code states: "The shafts of caissons shall be designed as concrete columns with continuous lateral support. The unit compressive stress in the concrete shall not exceed 33% of its ultimate 28-day compressive strength nor 1200 psi. No steel reinforcement is required in concrete-filled, drilled piers or caissons unless required by the load imposed thereon."
5. The caisson will be made of concrete with f_c' of 4000 psi.

Required

Design capacity of the drilled caisson.

Solution

Supporting strength of soil

From Eq. (11-2),

$$Q_{\text{allowable}} = \tfrac{1}{3}cN_cA_{\text{bottom}} + fA_{\text{shaft}} \qquad (11\text{-}2)$$

$$c = 2500\ \text{lb/ft}^2 = 2.5\ \text{kips/ft}^2$$

$$\frac{\text{Depth of caisson}}{\text{Diameter of caisson bottom}} = \frac{30\ \text{ft}}{6\ \text{ft}} = 5$$

From Table 11-1, $N_c = 9.0$.

$$A_{\text{bottom}} = \frac{\pi(6\ \text{ft})^2}{4} = 28.27\ \text{ft}^2$$

$$\tfrac{1}{3}cN_cA_{\text{bottom}} = \frac{(2.5\ \text{kips/ft}^2)(9.0)(28.27\ \text{ft}^2)}{3} = 212\ \text{kips}$$

From Table 11-2, with belled caisson and drilled dry, adhesion or skin friction (f) = 0.3c but not more than 0.8 kip/ft².

$$f_1 = (0.3)(750\ \text{lb/ft}^2) = 225\ \text{lb/ft}^2 = 0.225\ \text{kip/ft}^2 < 0.8\ \text{kip/ft}^2$$

$$f_2 = (0.3)(2500\ \text{lb/ft}^2) = 750\ \text{lb/ft}^2 = 0.750\ \text{kip/ft}^2 < 0.8\ \text{kip/ft}^2\ \text{O.K.}$$

A_{shaft} = circumference of caisson shaft multiplied by effective length of shaft in developing skin friction

$$A_{\text{shaft}_1} = (\pi)(3\ \text{ft})(20\ \text{ft}) = 188.5\ \text{ft}^2$$

$$A_{\text{shaft}_2} = (\pi)(3\ \text{ft})(10\ \text{ft} - 3\ \text{ft}) = 66.0\ \text{ft}^2$$

$$fA_{\text{shaft}} = (0.225\ \text{kip/ft}^2)(188.5\ \text{ft}^2) + (0.750\ \text{kip/ft}^2)(66.0\ \text{ft}^2) = 92\ \text{kips}$$

$$Q_{\text{allowable}} = 212\ \text{kips} + 92\ \text{kips} = 304\ \text{kips}$$

Supporting strength of concrete shaft

According to the local building code given,

$$(4000\ \text{lb/in.}^2)(0.33) = 1320\ \text{lb/in.}^2 > 1200\ \text{lb/in.}^2$$

Therefore, use $q_a = 1200\ \text{lb/in.}^2$

$$Q_{\text{allowable}} = 1200\ \text{lb/in.}^2 \times \frac{\pi(36\ \text{in.})^2}{4} = 1{,}221{,}000\ \text{lb} = 1221\ \text{kips}$$

Design capacity of the caisson is the smaller of the allowable capacities which is 304 kips.

EXAMPLE 11-2

Given

The excavation for a caisson to carry a total load of 1200 kN is to be drilled with slurry in the soil profile shown in Fig. 11-3. The caisson will be made of concrete with f'_c of 27,500 kN/m². It is specified that the unit compressive stress of the caisson shaft shall not exceed 33% of its ultimate 28-day compressive strength nor 8274 kN/m².

Required

The diameter of the plain concrete straight-shaft caisson.

Solution

Diameter of caisson as determined by the soil's supporting strength

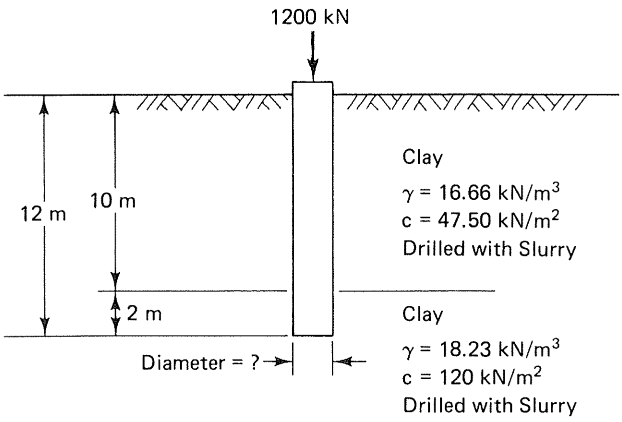

FIGURE 11-3

From Eq. (11-2),

$$Q_{\text{allowable}} = \tfrac{1}{3}cN_cA_{\text{bottom}} + fA_{\text{shaft}} \qquad (11\text{-}2)$$

$$c = 120\ \text{kN/m}^2$$

Assume ratio of depth of caisson to diameter of caisson bottom > 4. From Table 11-1, $N_c = 9.0$.

$$A_{\text{bottom}} = \frac{\pi d^2}{4}$$

$$\tfrac{1}{3}cN_cA_{\text{bottom}} = \tfrac{1}{3}(120\ \text{kN/m}^2)(9.0)\left(\frac{\pi d^2}{4}\right) = 282.7d^2\ \text{kN/m}^2$$

From Table 11-2, for a straight shaft caisson, drilled with slurry, adhesion or skin friction (f) is $0.3c$ but not more than (0.8)(47.88 kN/m²), or 38.3 kN/m².

$$f_1 = (0.3)(47.5\ \text{kN/m}^2) = 14.2\ \text{kN/m}^2 < 38.3\ \text{kN/m}^2 \qquad \text{O.K.}$$

$$f_2 = (0.3)(120\ \text{kN/m}^2) = 36.0\ \text{kN/m}^2 < 38.3\ \text{kN/m}^2 \qquad \text{O.K.}$$

$$(A_{\text{shaft}})_1 = (\pi d)(10\ \text{m}) = 31.42d\ \text{m}$$

$$(A_{\text{shaft}})_2 = (\pi d)(2\ \text{m}) = 6.283d\ \text{m}$$

$$fA_{\text{shaft}} = (14.2\ \text{kN/m}^2)(31.42d\ \text{m}) + (36.0\ \text{kN/m}^2)(6.283d\ \text{m})$$

$$= 672.4d\ \text{kN/m}$$

Since $Q_{\text{allowable}} = 1200$ kN, substituting into Eq. (11-2) gives

$$1200\ \text{kN} = 282.7d^2\ \text{kN/m}^2 + 672.4d\ \text{kN/m}$$

Solving this quadratic equation gives

$$d = 1.19\ \text{m}$$

Diameter of caisson as determined by the concrete shaft's strength

$$(0.33)(27{,}500\ \text{kN/m}^2) = 9075\ \text{kN/m}^2 > 8274\ \text{kN/m}^2$$

Therefore, use $q_a = 8274$ kN/m².

$$Q_{\text{allowable}} = Aq_a$$

$$1200\ \text{kN} = \left(\frac{\pi d^2}{4}\right)(8274\ \text{kN/m}^2)$$

$$d = 0.430\ \text{m}$$

The required caisson diameter is the larger of the values determined above, which is 1.19 m. (Since the ratio of depth of caisson to diameter of bottom is greater than 4, the use of $N_c = 9.0$ is valid.)

Drilled Caissons in Sands

The analysis of drilled caissons in sands is also similar to that of piles, in accordance with Eq. (10-1). The term Q_{tip} of Eq. (10-1) can be evaluated by multiplying effective vertical pressure (p_v) considering the limits imposed by the concept of critical depth by the bearing capacity factor (N_q) and this by the area of the bottom of the caisson (A_{bottom}). The term Q_{friction} of Eq. (10-1) can be evaluated by multiplying the coefficient of lateral earth pressure of the soil at rest (K_0) by effective vertical pressure (p_v) by the coefficient of friction between sand and concrete ($\tan \delta$) by the skin area of the caisson shaft (A_{shaft}). Making these substitutions in Eq. (10-1) gives [1]

$$Q_{\text{ultimate}} = p_v N_q A_{\text{bottom}} + (K_0 p_v \tan \delta) A_{\text{shaft}} \qquad (11\text{-}3)$$

The value of the coefficient of lateral earth pressure at rest (K_0) ranges from about 0.4 for dense sand to 0.5 for loose sand [3]. The value of the coefficient of friction between sand and concrete ($\tan \delta$) can be taken to be the value of the coefficient of friction among sand particles ($\tan \phi$) if the excavation has been drilled dry. If the excavation is drilled using a slurry, some reduction should be applied to the value of $\tan \phi$ used [1].

When a drilled caisson in sand is designed by the procedure given above, a factor of safety of from 2 to 3 is recommended.

Example 11-3 illustrates the computation of allowable bearing capacity for a drilled caisson in sand.

EXAMPLE 11-3

Given

1. A 3-ft-diameter straight-shaft caisson is constructed in sand.
2. Soil conditions and a sketch of the caisson are shown in Fig. 11-4.
3. The excavation is drilled dry.

Required

Allowable bearing capacity of the caisson as determined by the soil's supporting strength.

Solution

D_c = critical depth = 10 times the caisson's diameter (for loose sand)

$D_c = 10 \times 3 \text{ ft} = 30 \text{ ft}$ (see Fig. 11-5)

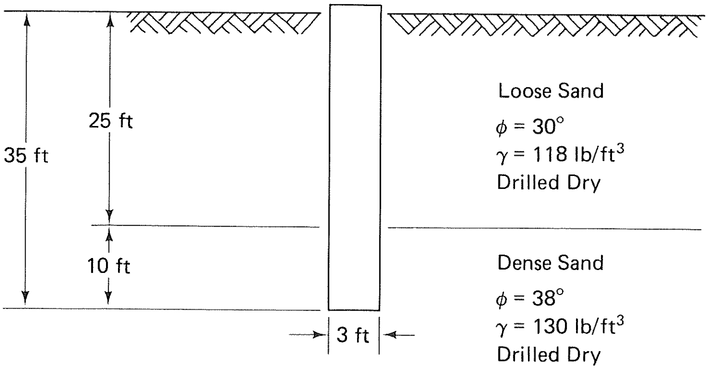

FIGURE 11-4

From Eq. (11-3),

$$Q_{\text{ultimate}} = p_v N_q A_{\text{bottom}} + (K_0 p_v \tan\delta) A_{\text{shaft}} \tag{11-3}$$

$$p_v = 3600 \text{ lb/ft}^2 \qquad \text{(see Fig. 11-5)}$$

$$N_q = 50 \qquad \text{(from Fig. 9-7 for } \phi = 38°)$$

$$A_{\text{bottom}} = \frac{\pi}{4}(3 \text{ ft})^2 = 7.07 \text{ ft}^2$$

$$Q_{\text{bottom}} = (3600 \text{ lb/ft}^2)(50)(7.07 \text{ ft}^2) = 1{,}273{,}000 \text{ lb} = 1273 \text{ kips}$$

$$Q_{\text{friction}} = (K_0)(\text{area of } p_v \text{ diagram})(\text{circumference of caisson shaft})(\tan\delta)$$

$$K_0 = 0.5 \text{ (for loose sand)}$$

$$K_0 = 0.4 \text{ (for dense sand)}$$

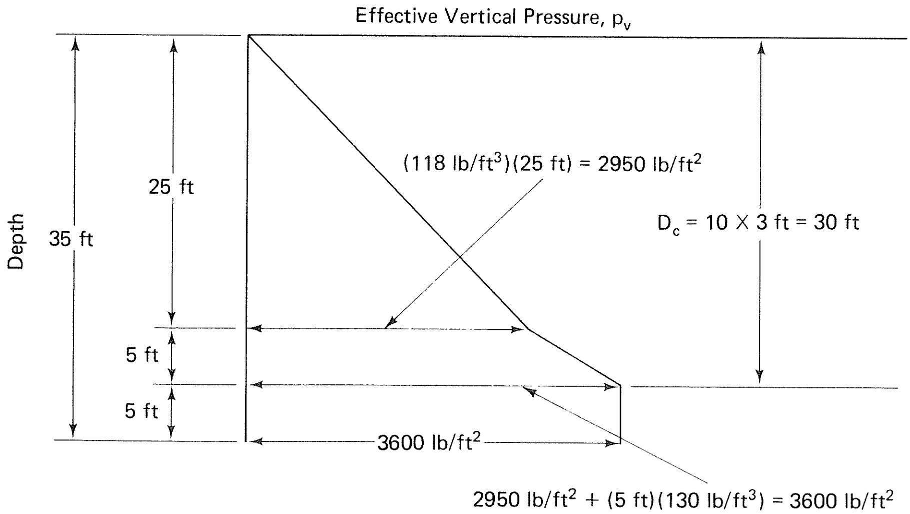

FIGURE 11-5

$$\tan \delta = \tan 30^\circ \text{ (for upper layer of loose sand)}$$

$$\tan \delta = \tan 38^\circ \text{ (for lower layer of dense sand)}$$

$$Q_{\text{friction}} = (0.5)(\tfrac{1}{2} \times 2950 \text{ lb/ft}^2 \times 25 \text{ ft})(\pi \times 3 \text{ ft})(\tan 30^\circ)$$
$$+ (0.4)[(2950 \text{ lb/ft}^2)(5 \text{ ft}) + (\tfrac{1}{2})(5 \text{ ft})(3600 \text{ lb/ft}^2 - 2950 \text{ lb/ft}^2)] \times (\pi \times 3 \text{ ft})(\tan 38^\circ)$$
$$+ (0.4)(5 \text{ ft} \times 3600 \text{ lb/ft}^2)(\pi \times 3 \text{ ft})(\tan 38^\circ)$$
$$= 202{,}000 \text{ lb} = 202 \text{ kips}$$

$$Q_{\text{ultimate}} = 1273 \text{ kips} + 202 \text{ kips} = 1475 \text{ kips}$$

$$Q_{\text{allowable}} = \frac{1475 \text{ kips}}{3} = 492 \text{ kips}$$

Drilled Caissons on Bedrock

Clear-cut procedures do not exist for determining design capacity for drilled caissons on bedrock. The designer usually goes by the applicable local building code, which is often based on past experience in the area. Such codes may give criteria with regard to the concrete's structural strength and/or the bedrock's supporting strength. Unconfined compression tests may be performed on rock samples. Using a factor of safety of 5 to 8, allowable bearing pressure can then be evaluated [2]. However, if allowable bearing capacity of rock specified by a local building code is less than the allowable bearing pressure evaluated by unconfined compression tests, the allowable bearing capacity specified by the building code should be used.

Example 11-4 illustrates computation of design capacity for a drilled caisson on bedrock. This example gives a sample of a possible local building code's specification regarding drilled caissons on rock.

EXAMPLE 11-4

Given

1. A 3-ft-diameter plain concrete drilled caisson is constructed on massive limestone, the unconfined compressive strength of which is found to be 2500 lb/in.^{2}
2. Soil conditions and a sketch of the caisson are shown in Fig. 11-6.
3. The excavation is drilled dry.
4. A local building code states:

   (a) "The shafts of caissons shall be designed as concrete columns with continuous lateral support. The unit compressive stress in the con-

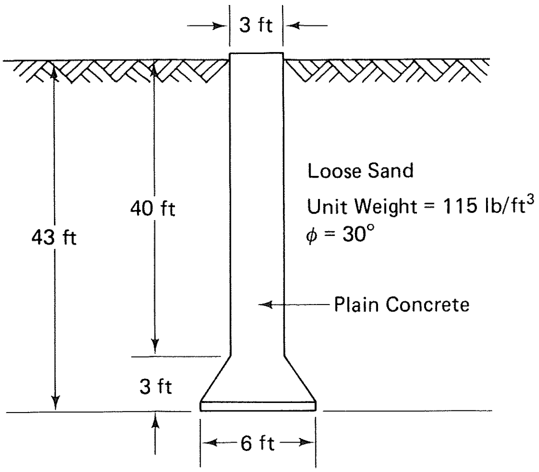

FIGURE 11-6

crete shall not exceed 33% of its ultimate 28-day compressive strength nor 1200 psi. No steel reinforcement is required in concrete filled, drilled piers or caissons unless required by the load imposed thereon."

(b) "Allowable bearing capacity of rock shall not exceed the following:

Massive igneous or metamorphic rock	100 tons/ft^2
Massive sedimentary rock	20 tons/ft^2"

5. The caisson will be made of concrete with f'_c of 4000 psi.

Required

Design capacity of the caisson (neglect skin friction of the caisson shaft).

Solution

Allowable bearing capacity of the caisson as determined by the structural strength of the concrete

$$(4000 \text{ lb/in.}^2)(0.33) = 1320 \text{ lb/in.}^2 > 1200 \text{ lb/in.}^2$$

Therefore, use $q_a = 1200$ lb/in.^{2}

$$Q_{\text{allowable}} = \frac{(\pi)(3 \text{ ft})^2}{4}(1200 \text{ lb/in.}^2)(144 \text{ in.}^2/\text{ft}^2)$$
$$= 1{,}221{,}000 \text{ lb} = 611 \text{ tons}$$

Allowable bearing capacity of the caisson as determined by the supporting strength of the rock

$$q_a = \left(\frac{2500 \text{ lb/in.}^2}{8}\right)\left(\frac{144 \text{ in.}^2/\text{ft}^2}{2000 \text{ lb/ton}}\right) = 22.5 \text{ tons/ft}^2$$

(This uses a factor of safety of 8 for unconfined compressive strength of rock.) However, the local building code specifies q_a shall not exceed 20 tons/ft². Therefore, q_a of 20 tons/ft² should be used.

$$Q_{\text{allowable}} = \frac{\pi}{4}(6 \text{ ft})^2(20 \text{ tons/ft}^2) = 565 \text{ tons}$$

The design capacity of the caisson is the smaller of the allowable capacities, which is 565 tons.

11-3 SETTLEMENT OF DRILLED CAISSONS

Settlement of drilled caissons in clay depends largely on the load history of the clay. This is similar to settlement of footings. Since drilled caissons are uneconomical in normally consolidated clays and settlement thereon is excessive, drilled caissons should be used only in overconsolidated clays. Long-term settlement analysis in clay soils can be performed using consolidation theory and assuming the drilled caisson's bottom to be a footing [3].

Settlement of drilled caissons in sand "at any depth is likely to be about one-half the settlement of an equally loaded footing covering the same area on sand of the same characteristics" [3]. Generally, such settlement will not be detrimental, since the caisson will normally be found on dense sand and settlement will be small. Settlement in sand can be computed using procedures given in Chap. 7 for footings on sand. It should be kept in mind, however, that settlement of the caisson should be about one-half the settlement computed for the equivalent footing.

Settlement of drilled caissons on bedrock should be very small if the rock is dry. However, water may be found at the bottom of some caissons, and it can cause some settlement—sometimes large if soft rocks disintegrate upon soaking. It is, therefore, desirable that the water be pumped out and the caisson thoroughly cleaned during the last stage of drilling [2].

11-4 CONSTRUCTION AND INSPECTION OF DRILLED CAISSONS

Construction of drilled caissons consists for the most part of excavation of soil and placement of concrete (perhaps with reinforcing steel). As related in

Sec. 11-1, drilled caissons generally are excavated using an auger drill or other type of drilling equipment. An auger is a screwlike device that is attached to a shaft and rotated under power. The rotating action digs into the soil and raises it to the surface. If a caisson is to have a bell at the bottom, the bell is made using a reamer.

While excavation is being done, soil is exposed in the walls. Soil at the caisson's bottom and exposed in the walls should be examined (and records kept) whenever possible to check the adequacy of the supporting soil at the caisson's bottom and to determine the depth to, and thickness of, different soil strata. Sometimes a person may be able to descend in the shaft for inspection.

After excavation of the soil, the concrete must be of acceptable quality and properly placed. It is preferable that concrete not strike the sides of the hole as it is being poured. A casing, if used, is generally removed as the concrete is poured. Normally, only the concrete in the upper part of the shaft is vibrated. It is always best to pour the concrete in the dry; but if water is present, the concrete can be placed under water. Installation of the reinforcing steel (if specified) should be carefully checked prior to placing the concrete.

One final aspect of the overall construction process is inspection. A drilled caisson should be inspected for accuracy of the caisson's alignment and dimensions, for bearing capacity of the soil at the caisson's bottom, for proper placement of reinforcing steel and concrete, and so on. Normally, the owner's representative should be present during construction to ensure that the construction of the caisson is done properly and according to specifications.

11-5 PROBLEMS

11-1 A plain concrete drilled caisson is to be constructed in clay soils. The caisson shaft's diameter is 4 ft, and the belled bottom is 8 ft in diameter. The drilled caisson is extended to a total depth of 36 ft. Soil conditions are illustrated in Fig. 11-7. Compute the caisson's allowable bearing capacity if (a) the excavation is drilled dry, and (b) the foundation is to be drilled with bentonite slurry, and a factor of safety of 3 is to be used. (*Note:* The maximum allowable compressive stress of plain concrete is assumed to be 1200 lb/in.^{2})

11-2 The excavation for a caisson to carry a total load of 1500 kN is to be drilled dry in the soil profile shown in Fig. 11-8. The caisson will be made of concrete with f'_c of 27,500 kN/m^2. It is specified that the unit compressive stress of the caisson shaft shall not exceed 33% of its ultimate 28-day compressive strength nor 8274 kN/m^2. Find the required diameter of the plain concrete straight-shaft caisson.

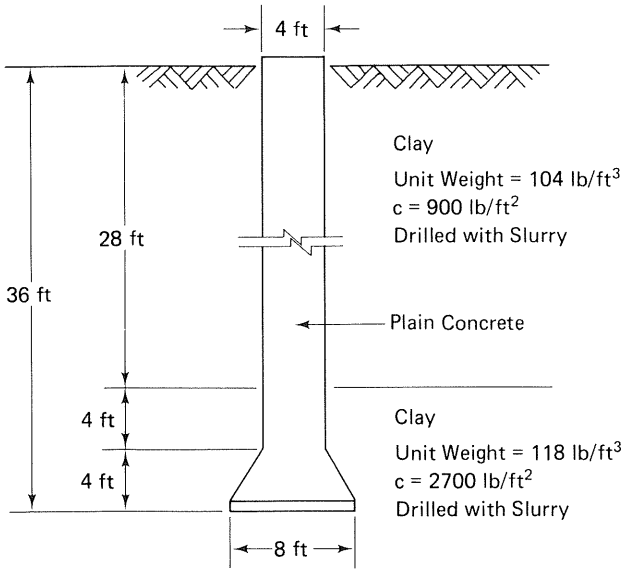

FIGURE 11-7

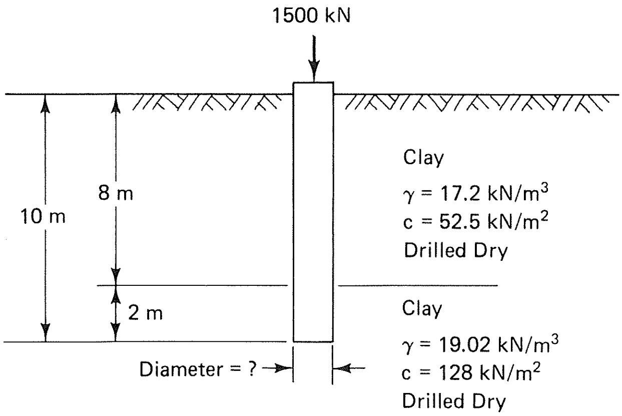

FIGURE 11-8

11-3 A drilled caisson 4 ft in diameter and supported by a bell end is to be constructed of plain concrete in sand. Soil conditions and a sketch of the caisson are shown in Fig. 11-9. Compute the caisson's design capacity if the excavation is slurry drilled and the factor of safety is 3. Assume the coefficient of friction between sand and concrete is tan $\frac{2}{3}\phi$ for this bentonite-slurry-drilled caisson. The maximum allowable compressive stress of plain concrete is assumed to be 1200 lb/in.^{2}

11-4 A straight drilled caisson, 4 ft in diameter and made of reinforced concrete, rests on a horizontal bedded granite (massive igneous rock). The unconfined compressive strength of the intact granite sample is 20,000 lb/in.^{2} Determine the safe design load on the caisson if skin friction of the caisson shaft is neglected (see Fig. 11-10). A local building code states:

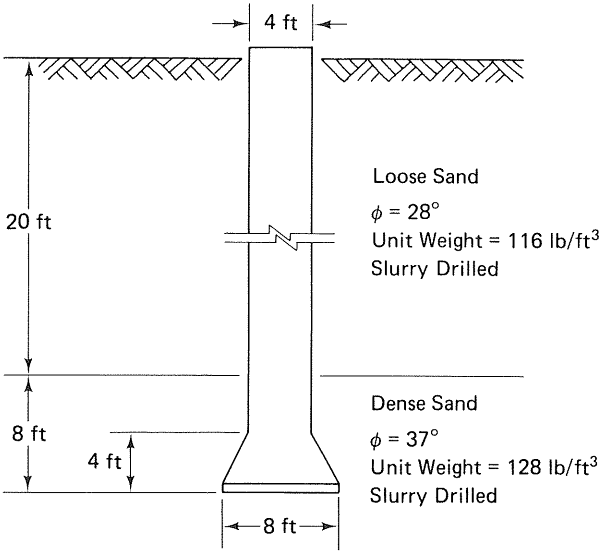

FIGURE 11-9

1. "The unit allowable compressive stress shall not exceed 1200 lb/in.^{2} for plain concrete."
2. "The allowable bearing value of the rock shall not exceed the following:

Massive igneous or metamorphic rock	100 tons/ft^2
Massive sedimentary rock	20 tons/ft^2"

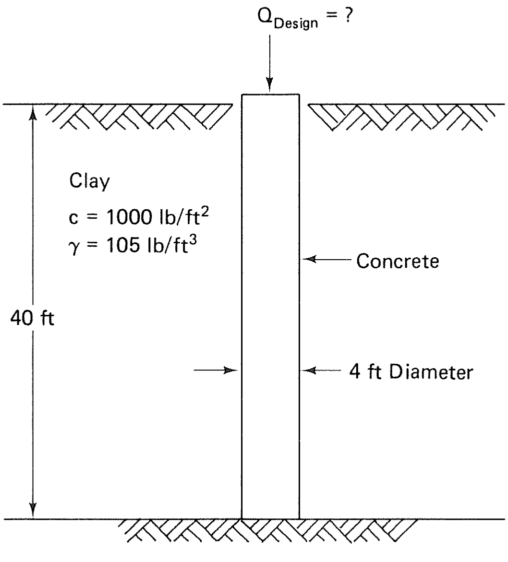

FIGURE 11-10

References

[1] David F. McCarthy, *Essentials of Soil Mechanics and Foundations,* Reston Publishing Company, Inc., Reston, Va., 1977.

[2] Wayne C. Teng, *Foundation Design,* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1962.

[3] Karl Terzaghi and Ralph B. Peck, *Soil Mechanics in Engineering Practice,* John Wiley & Sons, Inc., New York, 1967. Copyright © 1967, by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.

12

Lateral Earth Pressure

12-1 INTRODUCTION

The word "lateral" means "to the side" or "sideways." Thus, lateral earth pressure means pressure to the side, or sideways pressure. Analysis and determination of lateral earth pressure are necessary to design retaining walls and other earth retaining structures, such as bulkheads, abutments, and the like. Obviously, the magnitude and location of lateral earth pressure must be known in order to design a retaining wall or other retaining structure that can withstand applied pressure with an adequate safety margin. Almost always engineers calculate earth pressures and forces on a unit (1-ft or 1-m) section of the retaining wall.

There are three categories of earth pressure—*earth pressure at rest, active earth pressure,* and *passive earth pressure.* Earth pressure at rest refers to lateral pressure caused by earth that is prevented from lateral movement by an unyielding wall. In actuality, however, some retaining wall movement often occurs, resulting in either active or passive earth pressure as explained below.

If a wall moves away from soil, as sketched in Fig. 12-1, the earth surface will tend to be lowered, and lateral pressure on the wall will be decreased. If the wall moves far enough away, shear failure of the soil will occur, and a sliding soil wedge will tend to move forward and downward. The earth pressure exerted on the wall at this state of failure is known as active earth pressure (P_a), and it is at minimum value.

If, on the other hand, a wall moves toward soil, as shown in Fig. 12-2, the earth surface will tend to be raised, and lateral pressure on the wall will be in-

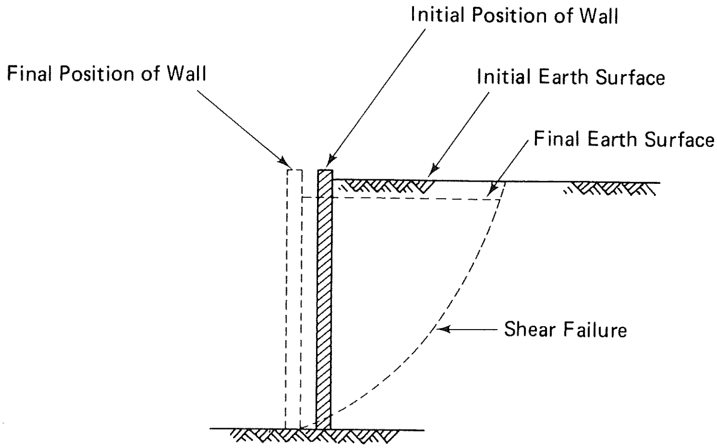

FIGURE 12-1 Active earth pressure. (For illustrative purposes, assume that the wall yields by moving outward away from the soil with its surface remaining vertical.)

creased. If the wall moves far enough toward the soil, shear failure of the soil will occur, and a sliding soil wedge will tend to move backward and upward. The earth pressure exerted on the wall at this state of failure is known as passive earth pressure (P_p), and it is at maximum value.

Section 12-2 discusses earth pressure at rest, while Secs. 12-3 and 12-4 cover determination of active and passive earth pressures according to Rankine and Coulomb theory, respectively. Effects of surcharge load on active thrust is discussed in Sec. 12-5. Culmann's graphical solution for finding active earth pressure is presented in Sec. 12-6. Lateral earth pressure on braced sheeting is considered in Sec. 12-8.

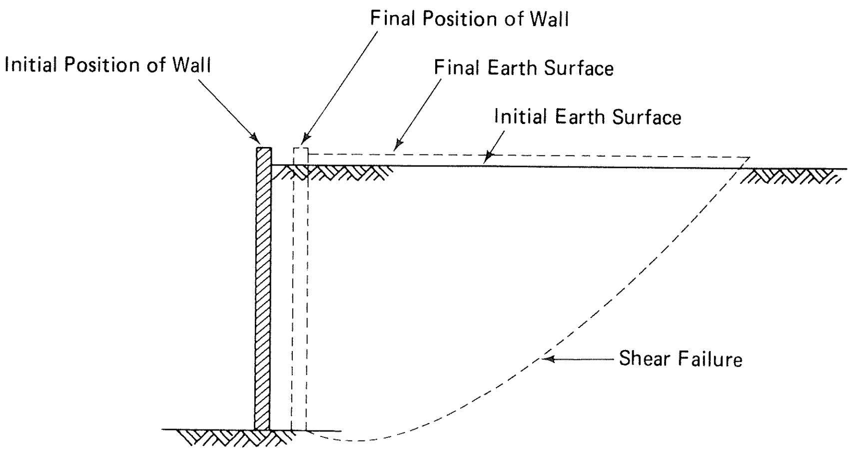

FIGURE 12-2 Passive earth pressure. (For illustrative purposes, assume that the wall moves backward toward the soil with its surface remaining vertical.)

12-2 EARTH PRESSURE AT REST

As noted in Sec. 12-1, earth pressure at rest refers to lateral pressure caused by earth that is prevented from lateral movement by an unyielding wall. Such a condition can occur, for example, when earth rests against the outer sides of a building's basement walls. With virtually no wall movement, soil in contact with the wall does not undergo lateral strain and does not therefore develop its full shearing resistance. In this case, the magnitude of earth pressure on the wall (i.e., the earth pressure at rest) falls somewhere between the active and passive pressures.

To analyze earth pressure at rest, consider the stress conditions on an element of soil at depth z (see Fig. 12-3). Although the element can deform vertically when loaded, it cannot deform laterally, since the element is confined by the same soil under the identical loading condition. This configuration is equivalent to soil resting against a smooth, immovable wall (see Fig. 12-4), and the soil is in a state of elastic equilibrium. In this case, pressure at the base of the wall and the resultant force per unit length of wall can be determined for dry soil by

$$p_0 = K_0 \gamma H \tag{12-1}$$

$$P_0 = \tfrac{1}{2} K_0 \gamma H^2 \tag{12-2}$$

where p_0 = pressure at base of the wall
P_0 = resultant force per unit length of wall for earth pressure at rest
K_0 = coefficient of earth pressure at rest (defined below)
γ = unit weight of the soil
H = height of the wall

For the zero lateral strain condition, lateral and vertical stresses (p_h and p_v in Fig. 12-3) are related by Poisson's ratio, μ, as follows:

$$\frac{p_h}{p_v} = \frac{\mu}{1 - \mu} \tag{12-3}$$

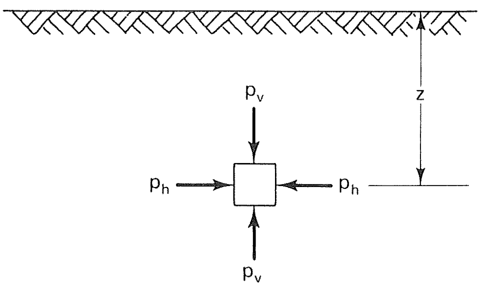

FIGURE 12-3 Subsurface stresses in a soil mass at depth *z*.

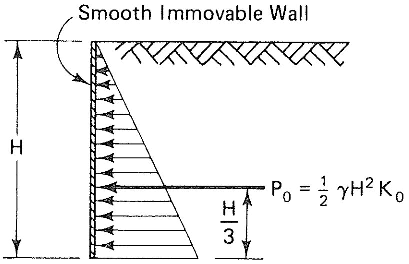

FIGURE 12-4 Earth pressure at rest for dry soil.

The ratio of p_h to p_v in a soil mass is known as the *coefficient of earth pressure at rest* and is denoted by K_0. Hence,

$$K_0 = \frac{p_h}{p_v} \tag{12-4}$$

K_0 has been observed by experiments to be dependent on a soil's angle of internal friction (ϕ) and plasticity index as well as its stress history.

As stated in Chap. 11, for granular soils the coefficient of lateral earth pressure at rest ranges from about 0.4 for dense sand to 0.5 for loose sand. K_0 can also be determined for sands by the following empirical relationship [1]:

$$K_0 = 1 - \sin\phi \tag{12-5}$$

For normally consolidated clays, the following empirical equation can be used to estimate K_0 [2]:

$$K_0 = 0.19 + 0.233 \log (PI) \tag{12-6}$$

where *PI* is the soil's plasticity index. For overconsolidated clays, values of K_0 tend to be larger than those of normally consolidated clays. Figure 12-5 gives an empirical relationship for determining K_0 as a function of the overconsolidation ratio (OCR) (see Sec. 8-5).

When some or all of the wall in question is below the groundwater table, hydrostatic pressure acting against the submerged section of wall must be added to the effective lateral soil pressure. From Fig. 12-6, it can be observed that the lateral earth pressure at rest at the water table (p_1) is given by

$$p_1 = K_0\gamma z_1 \tag{12-7}$$

while that at the base of the wall (p_2) is

$$p_2 = K_0\gamma z_1 + K_0\gamma_{sub}z_2 + \gamma_w z_2 \tag{12-8}$$

(γ, γ_{sub}, and γ_w represent the unit weight of soil, submerged unit weight of soil, and unit weight of water, respectively.) The resultant force per unit length of

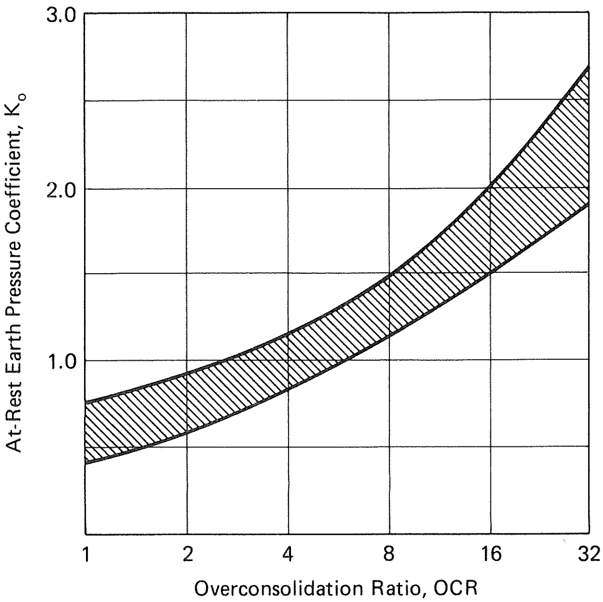

FIGURE 12-5 Relationship between K_0 and OCR. [3]

wall (P_0) can be determined by finding the area under the lateral earth pressure diagram.

$$P_0 = \frac{p_1 z_1}{2} + \frac{p_1 + p_2}{2}(z_2) \tag{12-9}$$

EXAMPLE 12-1

Given

A smooth, unyielding wall retains a dense, cohesionless soil with no lateral movement of soil (i.e., "at rest condition" is assumed), as shown in Fig. 12-7.

Required

1. Diagram of lateral earth pressure against the wall.
2. Total lateral force acting on the wall.

Solution

From Eq. (12-5),

$$K_0 = 1 - \sin \phi \tag{12-5}$$

$$K_0 = 1 - \sin 37° = 0.398$$

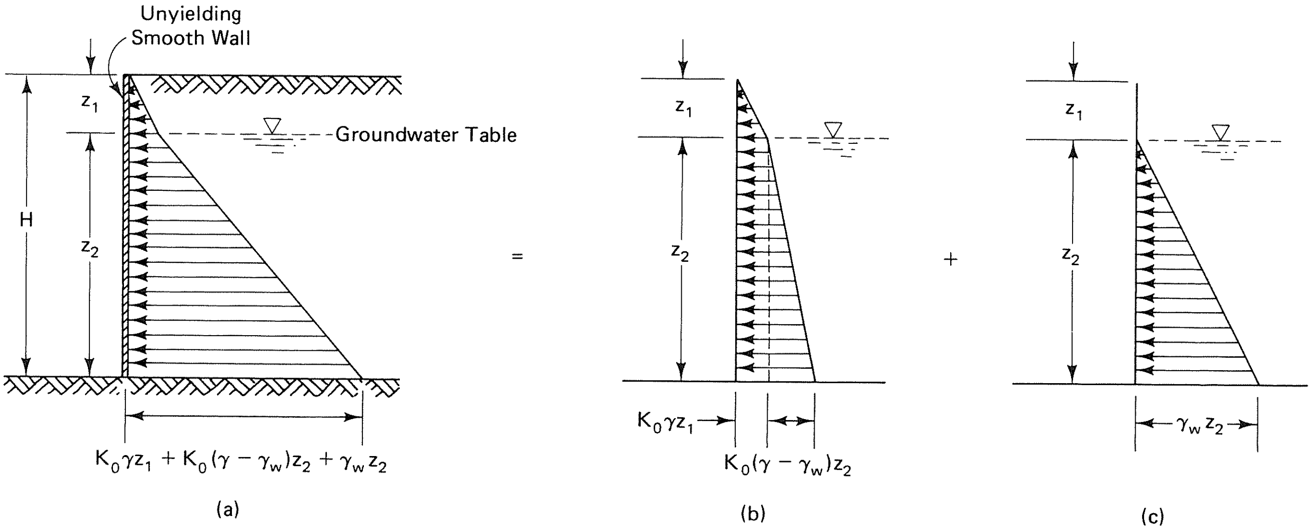

FIGURE 12-6 Lateral pressure acting against submerged wall; (a) unyielding smooth wall with groundwater table present at depth z_1 below ground surface; (b) effective lateral soil pressure; (c) lateral water pressure.

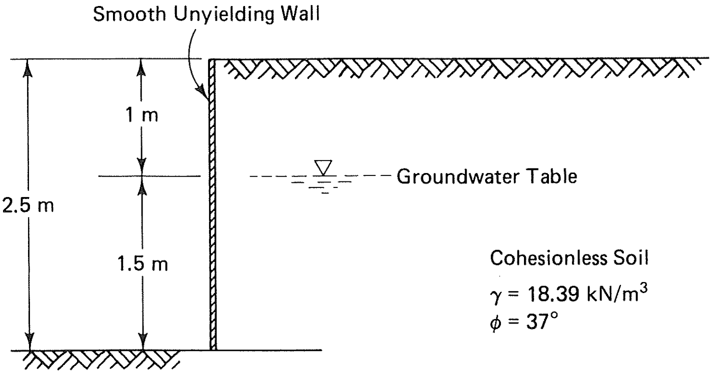

FIGURE 12-7

1. *Pressure at 1 m depth (at the water table):*
   From Eq. (12-7),

$$p_1 = K_0 \gamma z_1 \tag{12-7}$$

$$p_1 = (0.398)(18.39\ \text{kN/m}^3)(1.00\ \text{m}) = 7.32\ \text{kN/m}^2, \text{ or } 7.32\ \text{kPa}$$

   Pressure at 2.5 m depth (at the wall base):
   From Eq. (12-8),

$$p_2 = K_0\gamma z_1 + K_0\gamma_{sub} z_2 + \gamma_w z_2 \tag{12-8}$$

$$\begin{aligned} p_2 &= (0.398)(18.39\ \text{kN/m}^3)(1.00\ \text{m}) \\ &\quad + (0.398)(18.39\ \text{kN/m}^3 - 9.81\ \text{kN/m}^3)(1.5\ \text{m}) \\ &\quad + (9.81\ \text{kN/m}^3)(1.5\ \text{m}) \\ &= 27.16\ \text{kN/m}^2, \text{ or } 27.16\ \text{kPa} \end{aligned}$$

   The required diagram of lateral earth pressure against the wall is shown in Fig. 12-8.

2. From Eq. (12-9),

$$P_0 = \frac{p_1 z_1}{2} + \frac{p_1 + p_2}{2}(z_2) \tag{12-9}$$

$$P_0 = \frac{(7.32\ \text{kN/m}^2)(1.00\ \text{m})}{2} + \frac{7.32\ \text{kN/m}^2 + 27.16\ \text{kN/m}^2}{2}(1.5\ \text{m})$$

$$= 29.52\ \text{kN/m of wall}$$

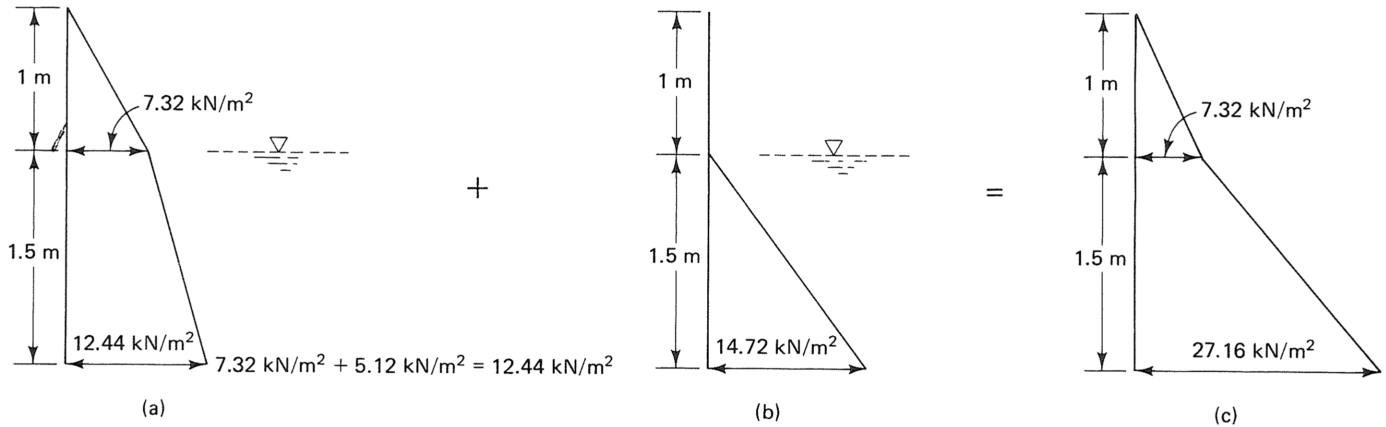

FIGURE 12-8 (a) Effective lateral soil pressure; (b) lateral water pressure; (c) total lateral pressure.

12-3 RANKINE EARTH PRESSURES

The Rankine theory for determining lateral earth pressures is based on several assumptions. The primary one is that there is no adhesion or friction between wall and soil (i.e., the wall is smooth). In addition, lateral pressures computed from Rankine theory are limited to vertical walls. Failure is assumed to occur in the form of a sliding wedge along an assumed failure plane defined as a function of the soil's angle of internal friction (ϕ), as shown in Fig. 12-9. Lateral earth pressure varies linearly with depth (see Fig. 12-10) and resultant pressures are assumed to act at a distance up from the base of the wall equal to one-third the vertical distance from the heel at the wall's base to the surface of the backfill (see Fig. 12-10). The direction of resultants is parallel to the backfill surface.

The primary assumption stated above (i.e., the wall is smooth) is not valid. Nevertheless, equations derived based on this assumption are widely used for computing lateral earth pressures; and, propitiously, results obtained using these equations may not differ appreciably from results based on more accurate and sophisticated analyses. In fact, results based on Rankine theory generally give slightly larger values, causing a slightly larger wall to be designed, thus giving a small additional safety factor.

The equations for computing lateral earth pressure* based on Rankine theory are as follows [4]:

$$P_a = \tfrac{1}{2}\gamma H^2 K_a \tag{12-10}$$

where

$$K_a = \cos\beta \, \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \tag{12-11}$$

$$P_p = \tfrac{1}{2}\gamma H^2 K_p \tag{12-12}$$

where

$$K_p = \cos\beta \, \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}} \tag{12-13}$$

where P_a = active earth pressure
γ = unit weight of the backfill soil
H = height of the wall (see Fig. 12-10)
K_a = coefficient of active earth pressure
β = angle between backfill surface line and a horizontal line (see Fig. 12-10)

* P_a and P_p are actually forces per unit length of wall; however, they are commonly referred to as the lateral earth pressure.

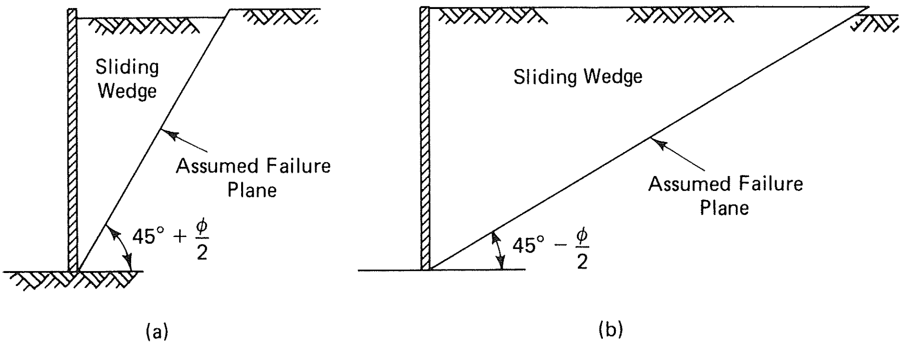

FIGURE 12-9 Assumed failure plane for Rankine theory: (a) Rankine active state; (b) Rankine passive state.

ϕ = angle of internal friction of the backfill soil
P_p = passive earth pressure
K_p = coefficient of passive earth pressure

If the backfill surface is level, angle β is zero, and Eqs. (12-11) and (12-13) revert to

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} \tag{12-14}$$

$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi} \tag{12-15}$$

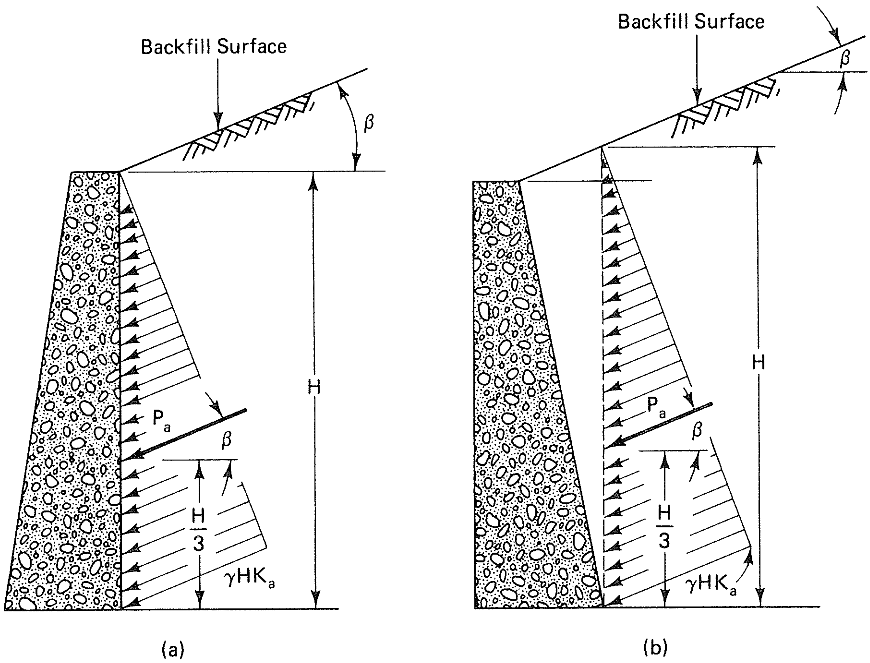

FIGURE 12-10 Lateral earth pressure for Rankine theory: (a) back side vertical, (b) back side inclined.

Example 12-2 illustrates the computation of lateral earth pressure for a level backfill surface, and Example 12-3 illustrates the computation for a sloping backfill surface. Example 12-4 gives a technique for computing lateral earth pressure based on Rankine theory for a retaining wall with a back side that is not vertical.

EXAMPLE 12-2

Given

The retaining wall shown in Fig. 12-11.

Required

Total active earth pressure per foot of wall and its point of application.

Solution

From Eqs. (12-10) and (12-14) (for level backfill),

$$P_a = \tfrac{1}{2}\gamma H^2 K_a \qquad (12\text{-}10)$$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} \qquad (12\text{-}14)$$

$$K_a = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = 0.333$$

$$P_a = (\tfrac{1}{2})(110\ \text{lb/ft}^3)(30\ \text{ft})^2(0.333) = 16{,}500\ \text{lb/ft}$$

Point of application of the total earth pressure $(\bar{y}) = H/3 = 30\ \text{ft}/3 = 10\ \text{ft}$ from the base of the wall.

EXAMPLE 12-3

Given

The retaining wall shown in Fig. 12-12.

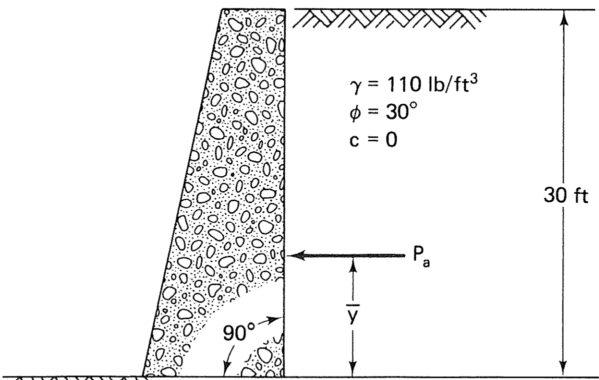

FIGURE 12-11

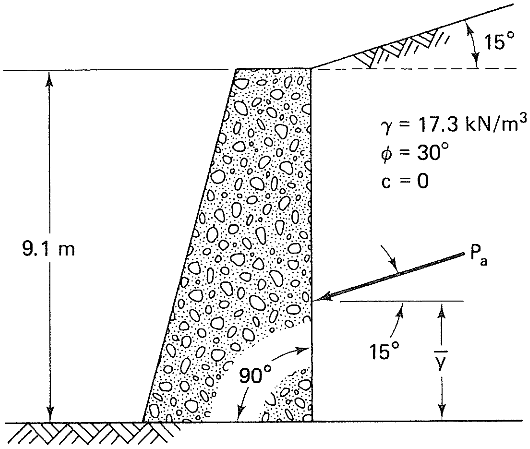

FIGURE 12-12

Required

Total active earth pressure per foot of wall and its point of application.

Solution

From Eqs. (12-10) and (12-11),

$$P_a = \tfrac{1}{2}\gamma H^2 K_a \tag{12-10}$$

$$K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \tag{12-11}$$

$$K_a = (\cos 15°) \frac{\cos 15° - \sqrt{\cos^2 15° - \cos^2 30°}}{\cos 15° + \sqrt{\cos^2 15° - \cos^2 30°}} = 0.373$$

$$P_a = (1/2)(17.3\ \text{kN/m}^3)(9.1\ \text{m})^2(0.373) = 267\ \text{kN/m}$$

$\bar{y} = H/3 = 9.1\ \text{m}/3 = 3.03\ \text{m}$ from the base of the wall (see Fig. 12-12).

EXAMPLE 12-4

Given

The retaining wall shown in Fig. 12-13.

Required

Total active earth pressure per foot of wall.

Solution

As shown on Fig. 12-13,

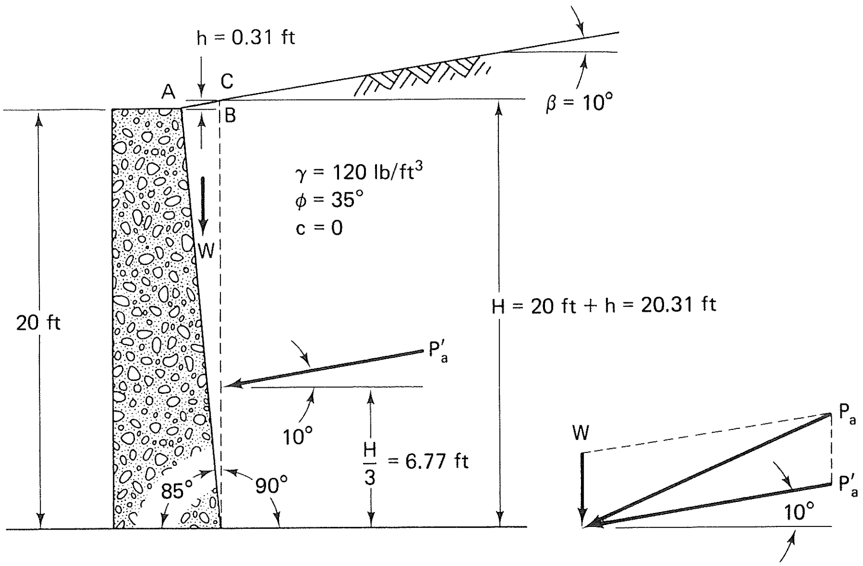

FIGURE 12-13

$$\tan 5^\circ = \frac{AB}{20\text{ ft}}$$

$$AB = (20\text{ ft})(\tan 5^\circ) = 1.75\text{ ft}$$

Also,

$$\tan 10^\circ = \frac{BC}{AB} = \frac{h}{1.75\text{ ft}}$$

$$h = (1.75\text{ ft})(\tan 10^\circ) = 0.31\text{ ft}$$

From Eqs. (12-10) and (12-11),

$$P'_a = \tfrac{1}{2}\gamma H^2 K_a \qquad (12\text{-}10)$$

$$K_a = \cos\beta\,\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \qquad (12\text{-}11)$$

$$\gamma = 120\text{ lb/ft}^3$$

$$H = 20.31\text{ ft}$$

$$\beta = 10^\circ$$

$$\phi = 35^\circ$$

$$K_a = (\cos 10^\circ)\,\frac{\cos 10^\circ - \sqrt{\cos^2 10^\circ - \cos^2 35^\circ}}{\cos 10^\circ + \sqrt{\cos^2 10^\circ - \cos^2 35^\circ}} = 0.282$$

$$P'_a = (1/2)(120\text{ lb/ft}^3)(20.31\text{ ft})^2(0.282) = 6979\text{ lb/ft}$$

$$W = (½)(\gamma)(AB)(H)$$

$$W = (½)(120\ \text{lb/ft}^3)(1.75\ \text{ft})(20.31\ \text{ft}) = 2133\ \text{lb/ft}$$

$$P_h = P'_a \cos\beta = (6979\ \text{lb/ft}) \cos 10° = 6873\ \text{lb/ft}$$

$$P_v = P'_a \sin\beta = (6979\ \text{lb/ft}) \sin 10° = 1212\ \text{lb/ft}$$

$$\sum V = W + P_v = 2133\ \text{lb/ft} + 1212\ \text{lb/ft} = 3345\ \text{lb/ft}$$

$$\sum H = P_h = 6873\ \text{lb/ft}$$

$$\text{Total active earth pressure } (P_a) = \sqrt{(\Sigma V)^2 + (\Sigma H)^2}$$

$$= \sqrt{(3345\ \text{lb/ft})^2 + (6873\ \text{lb/ft})^2}$$

$$= 7640\ \text{lb/ft}$$

Equations (12-10) through (12-15) are applicable for cohesionless soils. The generalized lateral earth pressure distribution for soils that have both cohesion and friction is, based on Rankine theory, as shown in Fig. 12-14. Figure 12-14a gives the pressure distribution for active pressure, and Fig. 12-14b gives that for passive pressure. It will be noted that active pressure acts only over the lower part of the wall (see Fig. 12-14a). The pressure distribution for a particular case can be ascertained by substituting appropriate parameters into the equations indicated on Fig. 12-14. Example 12-5 illustrates this method.

EXAMPLE 12-5

Given

The retaining wall shown in Fig. 12-15.

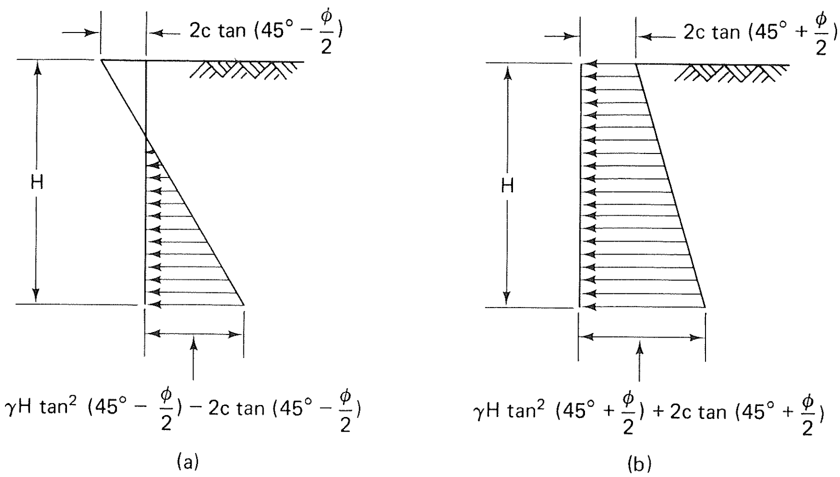

FIGURE 12-14 (a) Active earth pressure; (b) passive earth pressure. [5]

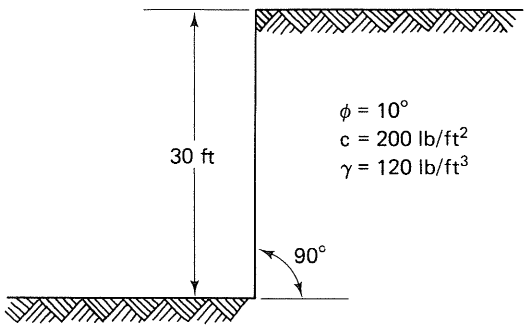

FIGURE 12-15

Required

The active earth pressure diagram.

Solution

From Fig. 12-14a,

$$2c \tan\left(45° - \frac{\phi}{2}\right) = (2)(200\ \text{lb/ft}^2)\tan\left(45° - \frac{10°}{2}\right) = 336\ \text{lb/ft}^2$$

$$\gamma H \tan^2\left(45° - \frac{\phi}{2}\right) - 2c\tan\left(45° - \frac{\phi}{2}\right) = (120\ \text{lb/ft}^3)(30\ \text{ft})\tan^2\left(45° - \frac{10°}{2}\right)$$

$$- (2)(200\ \text{lb/ft}^2)\tan\left(45° - \frac{10°}{2}\right) = 2200\ \text{lb/ft}^2$$

Thus, the pressure distribution is as shown in Fig. 12-16. By direct proportion of two triangles,

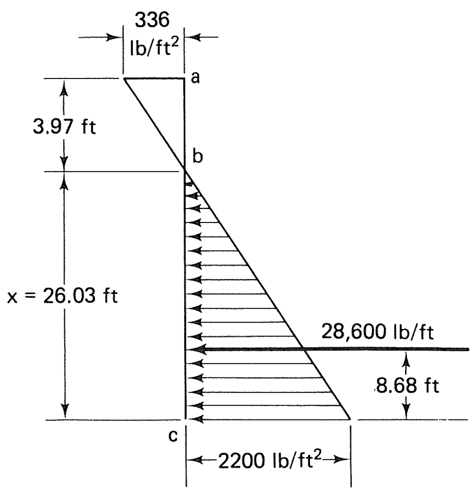

FIGURE 12-16

$$\frac{336\ \text{lb/ft}^2}{2200\ \text{lb/ft}^2} = \frac{30\ \text{ft} - x}{x}$$

$$x = 26.03\ \text{ft}$$

$$\text{Resultant} = (\tfrac{1}{2})(2200\ \text{lb/ft}^2)(26.03\ \text{ft}) = 28{,}600\ \text{lb/ft}$$

$$\bar{y} = x/3 = 26.03\ \text{ft}/3 = 8.68\ \text{ft above the base of the wall.}$$

12-4 COULOMB EARTH PRESSURES

The Coulomb theory for determining lateral earth pressure, developed nearly a century before the Rankine theory, assumes that failure occurs in the form of a wedge and friction occurs between wall and soil. The sides of the wedge are the earth side of the retaining wall and a failure plane that passes through the heel of the wall (see Fig. 12-17). Resultant active earth pressure acts on the wall at a point where a line through the wedge's center of gravity and parallel to the failure plane intersects the wall (see Fig. 12-18). The resultant's direction at the wall is along a line that makes an angle δ with a line normal to the back side of the wall, where δ is the angle of wall friction (see Fig. 12-19).

Equations for computing lateral earth pressure based on Coulomb theory are as follows [4]:

$$P_a = \tfrac{1}{2}\gamma H^2 K_a \tag{12-10}$$

where

$$K_a = \frac{\sin^2(\alpha + \phi)}{\sin^2\alpha \sin(\alpha - \delta)\left[1 + \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi - \beta)}{\sin(\alpha - \delta)\sin(\alpha + \beta)}}\right]^2} \tag{12-16}$$

$$P_p = \tfrac{1}{2}\gamma H^2 K_p \tag{12-12}$$

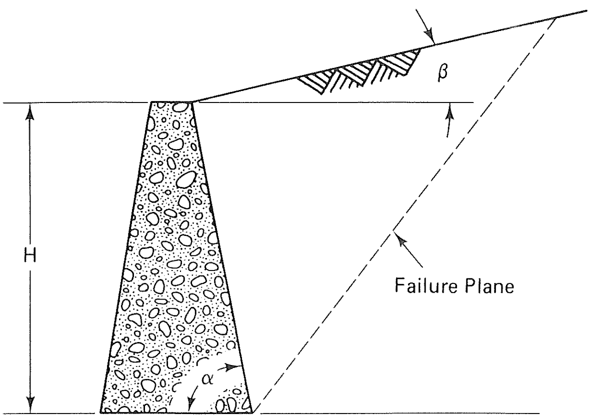

FIGURE 12-17

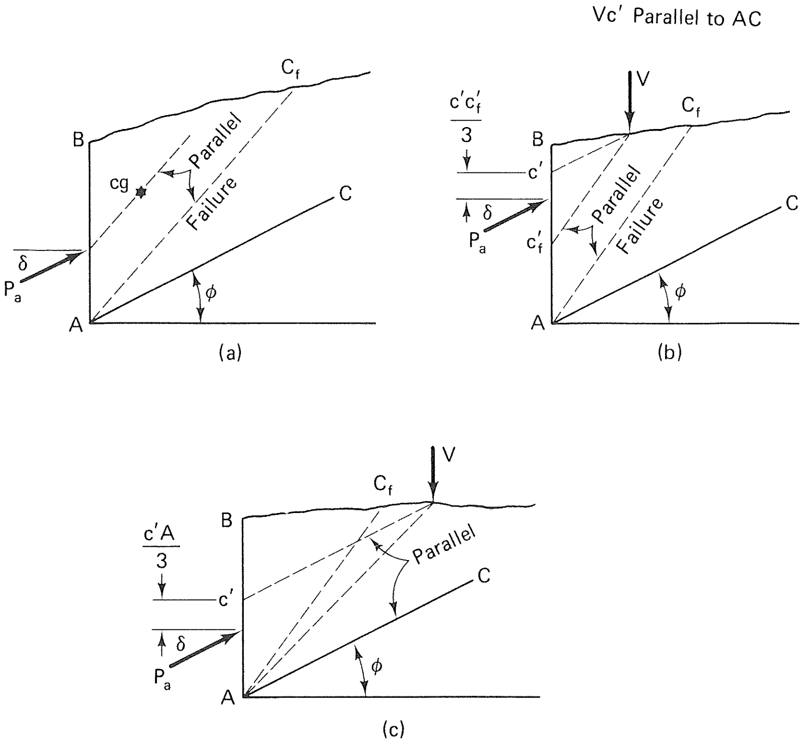

FIGURE 12-18 Procedures for location of point of application of P_a: (a) irregular backfill; (b) concentrated or line load inside failure zone; (c) concentrated or line load outside failure zone (but inside zone *ABC*). [4]

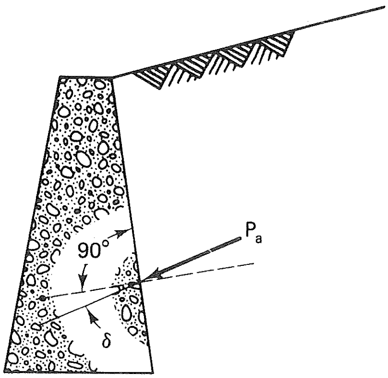

FIGURE 12-19

where

$$K_p = \frac{\sin^2(\alpha - \phi)}{\sin^2\alpha \sin(\alpha + \delta)\left[1 - \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi + \beta)}{\sin(\alpha + \delta)\sin(\alpha + \beta)}}\right]^2} \qquad (12\text{-}17)$$

where P_a = active earth pressure
γ = unit weight of the backfill soil
H = height of the wall (see Fig. 12-17)
K_a = coefficient of active earth pressure
α = angle between back side of wall and a horizontal line (see Fig. 12-17)
ϕ = angle of internal friction of the backfill soil
δ = angle of wall friction
β = angle between backfill surface lines and a horizontal line (see Fig. 12-17)
P_p = passive earth pressure
K_p = coefficient of passive earth pressure

In the case of a smooth, vertical wall with level backfill, δ and β are each zero and α is 90°; and if these values are substituted into Eqs. (12-16) and (12-17), the equations revert to Eqs. (12-14) and (12-15), respectively. The latter two equations are the Rankine equations for the conditions stated (i.e., smooth, vertical wall with level backfill).

Table 12-1 gives some typical values of angles of internal friction, angles of wall friction, and unit weights of common types of backfill soil.

Examples 12-6 and 12-7 illustrate the computation of lateral earth pressure based on Coulomb theory.

EXAMPLE 12-6

Given

Same conditions as in Example 12-2, except that the angle of wall friction between backfill and wall (δ) is 25° (see Fig. 12-20).

Required

Total active earth pressure per foot of wall by Coulomb theory.

Solution

From Eqs. (12-10) and (12-16),

$$P_a = \tfrac{1}{2}\gamma H^2 K_a \qquad (12\text{-}10)$$

TABLE 12-1 Friction angles and unit weights for backfill soil [6].

Number	Description of Soil	Angle of Internal Friction, ϕ		Angle of Wall Friction, δ		Unit Weight, γ (lb/ft^3)	
		Dry	*Moist*	*Dry*	*Moist*	*Dry*	*Moist*
1	Coarse to medium sand, trace fine gravel	36°00′	27°30′	27°30′	26°10′	—	91
2	Coarse to fine sand, trace + silt (7.5%)	37°40′	27°50′	32°10′	26°20′	101	95
3	Coarse to fine sand, trace + (7.5%) fine gravel	38°40′	30°00′	27°10′	26°20′	106	94
4	Coarse to fine sand	36°30′	30°00′	28°50′	27°10′	95	80
5	Medium to fine sand, some silt (29%), trace fine gravel	35°10′	29°10′	25°10′	21°30′	99	82
6	Fine sand, trace silt	37°50′	29°20′	29°40′	26°20′	94	82
7	Fine sand, some silt	35°00′	30°20′	28°00′	28°00′	103	96
8	Coarse silt, fine sand (45%)	34°50′	26°10′	27°50′	25°40′	94	80
9	Silt, some coarse to fine sand, trace + clay (7%)	—	31°20′	—	28°50′	—	75

$$K_a = \frac{\sin^2(\alpha + \phi)}{\sin^2\alpha \sin(\alpha - \delta)\left[1 + \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi - \beta)}{\sin(\alpha - \delta)\sin(\alpha + \beta)}}\right]^2} \tag{12-16}$$

$\gamma = 110$ lb/ft^3

$H = 30$ ft

$\alpha = 90°$

$\phi = 30°$

$\delta = 25°$

$\beta = 0°$ (level backfill)

$$K_a = \frac{\sin^2(90° + 30°)}{\sin^2(90°)\sin(90° - 25°)\left[1 + \sqrt{\dfrac{\sin(30° + 25°)\sin(30° - 0°)}{\sin(90° - 25°)\sin(90° + 0°)}}\right]^2}$$

$K_a = 0.296$

$P_a = (½)(110 \text{ lb/ft}^3)(30 \text{ ft})^2(0.296) = 14{,}700$ lb/ft

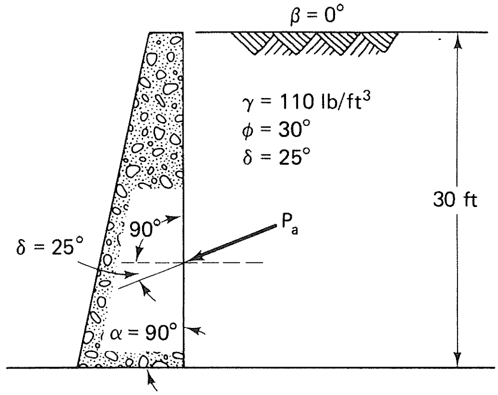

FIGURE 12-20

EXAMPLE 12-7

Given

Same conditions as Example 12-4 except that the angle of wall friction between backfill and wall (δ) is 20° (see Fig. 12-21).

Required

Total active earth pressure per foot of wall by Coulomb theory.

Solution

From Eqs. (12-10) and (12-16),

$$P_a = \tfrac{1}{2}\gamma H^2 K_a \qquad (12\text{-}10)$$

$$K_a = \frac{\sin^2(\alpha + \phi)}{\sin^2\alpha \sin(\alpha - \delta)\left[1 + \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi - \beta)}{\sin(\alpha - \delta)\sin(\alpha + \beta)}}\right]^2} \qquad (12\text{-}16)$$

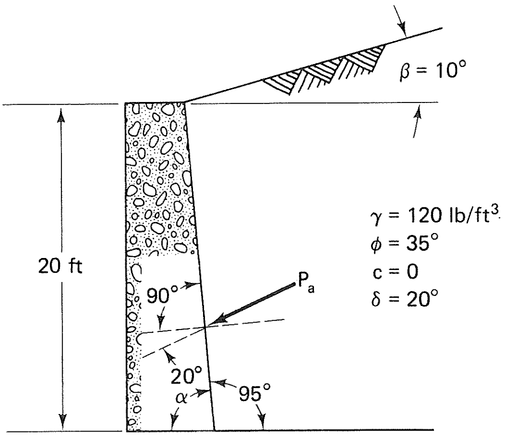

FIGURE 12-21

$$\gamma = 120 \text{ lb/ft}^3$$
$$H = 20 \text{ ft}$$
$$\alpha = 180° - 95° = 85°$$
$$\phi = 35°$$
$$\delta = 20°$$
$$\beta = 10°$$

$$K_a = \frac{\sin^2(85° + 35°)}{\sin^2(85°)\sin(85° - 20°)\left[1 + \sqrt{\dfrac{\sin(35° + 20°)\sin(35° - 10°)}{\sin(85° - 20°)\sin(85° + 10°)}}\right]^2}$$

$$K_a = 0.318$$
$$P_a = (\tfrac{1}{2})(120 \text{ lb/ft}^3)(20 \text{ ft})^2(0.318) = 7630 \text{ lb/ft}$$

12-5 EFFECTS OF SURCHARGE LOAD UPON ACTIVE THRUST

Sometimes backfill resting against a retaining wall is subjected to a surcharge. A surcharge, which is simply a uniform load and/or concentrated load imposed on the soil, adds to the lateral earth pressure exerted against the retaining wall by the backfill. This added pressure must, of course, be considered when designing the retaining wall.

Additional pressure exerted against a retaining wall as a result of a surcharge in the form of a uniform load can be computed from the following equation (see Fig. 12-22) [7]:*

$$P' = qHK_a \tag{12-18}$$

where P' = additional active earth pressure as a result of uniform load surcharge
q = uniform load (surcharge) on backfill
H = height of wall
K_a = coefficient of active earth pressure [determined from Eq. (12-14)]

Example 12-8, which follows, illustrates the computation of pressure due to a surcharge in the form of a uniform load. Example 12-11 in Sec. 12-6 illustrates the treatment (graphical solution) of a surcharge in the form of a concentrated load.

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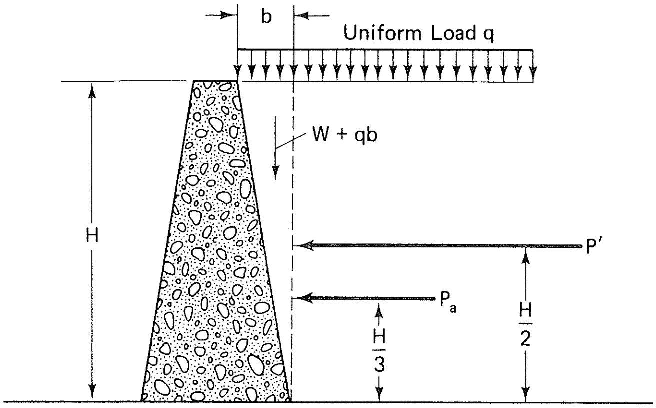

FIGURE 12-22 [7]*

EXAMPLE 12-8

Given

1. A smooth vertical wall is 20 ft high and retains a cohesionless soil with $\gamma = 120$ lb/ft^3 and $\phi = 28°$.
2. The top of the soil is horizontal and level with the top of the wall.
3. The soil surface carries a uniformly distributed load of 1000 lb/ft^2 (see Fig. 12-23).

Required

1. Total active earth pressure on the wall per linear foot of wall.
2. Point of action of the total active earth pressure by Rankine theory.

Solution

From Eqs. (12-10) and (12-14) (for level backfill),

$$P_a = \tfrac{1}{2}\gamma H^2 K_a \tag{12-10}$$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} \tag{12-14}$$

$$K_a = \frac{1 - \sin 28°}{1 + \sin 28°} = 0.361$$

$$P_a = (\tfrac{1}{2})(120 \text{ lb/ft}^3)(20 \text{ ft})^2(0.361) = 8660 \text{ lb/ft}$$

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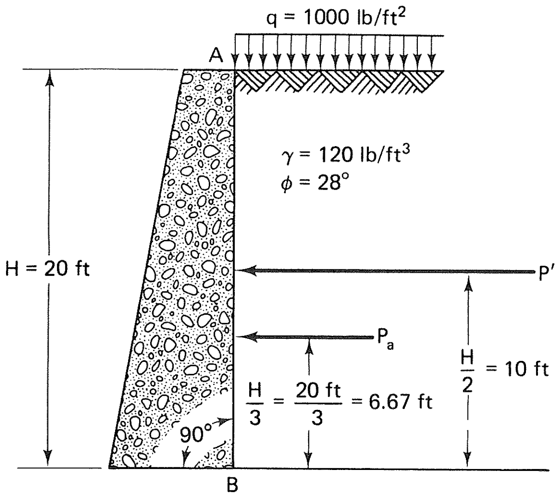

FIGURE 12-23

Point of action for $P_a = H/3 = 20\text{ ft}/3 = 6.67$ ft from the base of the wall. From Eq. (12-18),

$$P' = qHK_a \tag{12-18}$$

$$P' = (1000\text{ lb/ft}^2)(20\text{ ft})(0.361) = 7220\text{ lb/ft}$$

Point of action for $P' = H/2 = 20\text{ ft}/2 = 10$ ft from the base of the wall.

1. Total active earth pressure $= P_a + P' = 8660\text{ lb/ft} + 7220\text{ lb/ft}$ $= 15{,}880\text{ lb/ft}$.
2. Let the point of application of the total active earth pressure be h ft above the base of the wall. h is obtained by taking moments of forces (i.e., P_a and P') at the base of the wall.

$$(15{,}880\text{ lb/ft})(h) = (8660\text{ lb/ft})(6.67\text{ ft}) + (7220\text{ lb/ft})(10\text{ ft})$$

$$h = 8.18\text{ ft}$$

Hence, total active earth pressure acts at 8.18 ft above the base of the wall.

12-6 CULMANN'S GRAPHICAL SOLUTION

Several graphical methods to determine earth pressures are available, one of which is Culmann's graphical solution. The steps in carrying out a Culmann's graphical solution for active earth pressure (P_a) may be summarized as follows [4]:

1. Draw the retaining wall, backfill, and so on, to a convenient scale (see Fig. 12-24).

2. From point A (the base of the wall), lay off a line at angle ϕ (angle of internal friction) with a horizontal line. This is line AC in Fig. 12-24.

3. From point A, lay off a line at an angle θ with line AC (from step 2). Angle θ is equal to α (the angle between the back side of the wall and a horizontal line, as indicated in Fig. 12-24) minus δ (angle of wall friction). This line is AD in Fig. 12-24.

4. Draw some possible failure wedges, such as ABC_1, ABC_2, ABC_3, and so on.

5. Compute weights of the wedges (W_1, W_2, W_3, etc.).

6. Using a convenient weight scale along line AC, lay off the respective weights of the wedges, locating points w_1, w_2, w_3, and so on.

7. Through each point, w_1, w_2, w_3, and so on, draw a line parallel to line AD, intersecting the corresponding line AC_1, AC_2, AC_3, respectively.

8. Draw a smooth curve (*Culmann's line*) through the points of intersection determined in step 7 (i.e., the point of intersection of the line through point w_1 parallel to line AD and of line AC_1, the point of intersection of the line through point w_2 parallel to line AD and of line AC_2, etc.)

9. Draw a line that is both tangent to the Culmann line and parallel to line AC.

10. Draw a line through the tangent point (determined in step 9) that is parallel to line AD and intersects line AC. The length of this line applied to the weight scale gives the value of P_a (see Fig. 12-24). A line from point A through the tangent point defines the failure plane.

As discussed in Sec. 12-4, the point of application of P_a can be found by drawing a line through the center of gravity of the failure wedge and parallel to the failure plane until it intersects the wall (see Fig. 12-18). The direction of P_a is along a line that makes an angle δ (the angle of wall friction) with a line normal to the back side of the wall (see Fig. 12-19).

Examples 12-9 through 12-11 illustrate the application of Culmann's graphical solution.

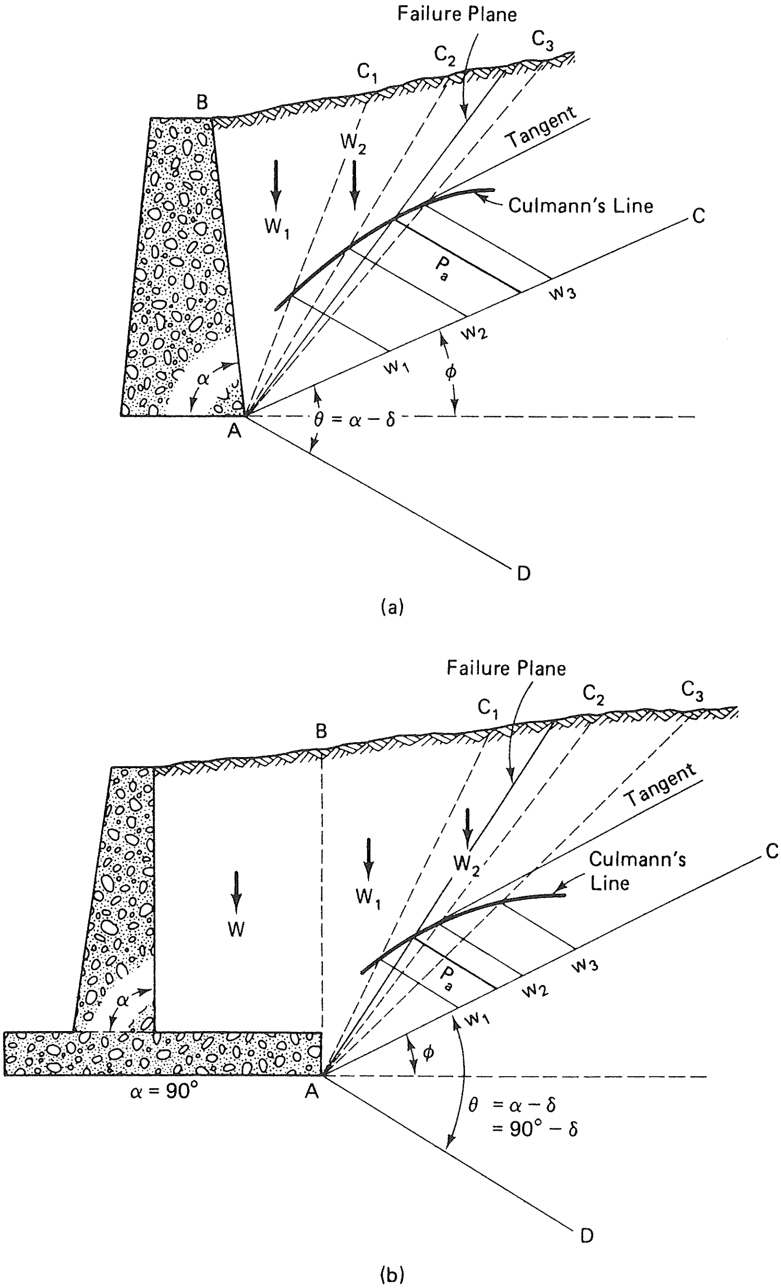

FIGURE 12-24 (a) Gravity wall, (b) Cantilever wall. [4]

EXAMPLE 12-9

Given

The same conditions as in Example 12-7 (see Fig. 12-25).

Required

Total active earth pressure per foot of wall by Culmann's graphical solution.

Solution

By following the steps outlined previously for Culmann's graphical solution, the sketch of Fig. 12-26 is prepared. The weights of the wedges (step 5) are computed as follows:

$$W_1 = (1/2)(120\text{ lb/ft}^3)(4.7\text{ ft})(21.0\text{ ft}) = 5920\text{ lb/ft}$$

$$W_2 = (1/2)(120\text{ lb/ft}^3)(4.4\text{ ft})(22.2\text{ ft}) = 5860\text{ lb/ft}$$

$$W_3 = (1/2)(120\text{ lb/ft}^3)(5.0\text{ ft})(27.2\text{ ft}) = 8160\text{ lb/ft}$$

$$W_4 = (1/2)(120\text{ lb/ft}^3)(3.5\text{ ft})(31.4\text{ ft}) = 6590\text{ lb/ft}$$

From Fig. 12-26, the value of P_a is determined to be 7600 lb/ft.

EXAMPLE 12-10

Given

Same conditions as in Example 12-8 (see Fig. 12-27).

Required

Total active earth pressure per foot of wall by Culmann's graphical solution.

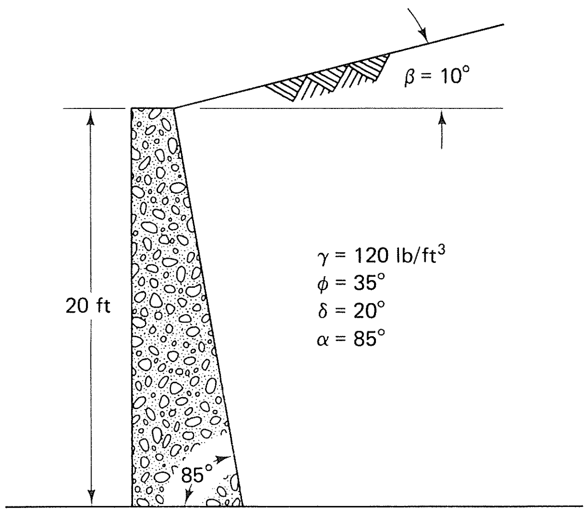

FIGURE 12-25

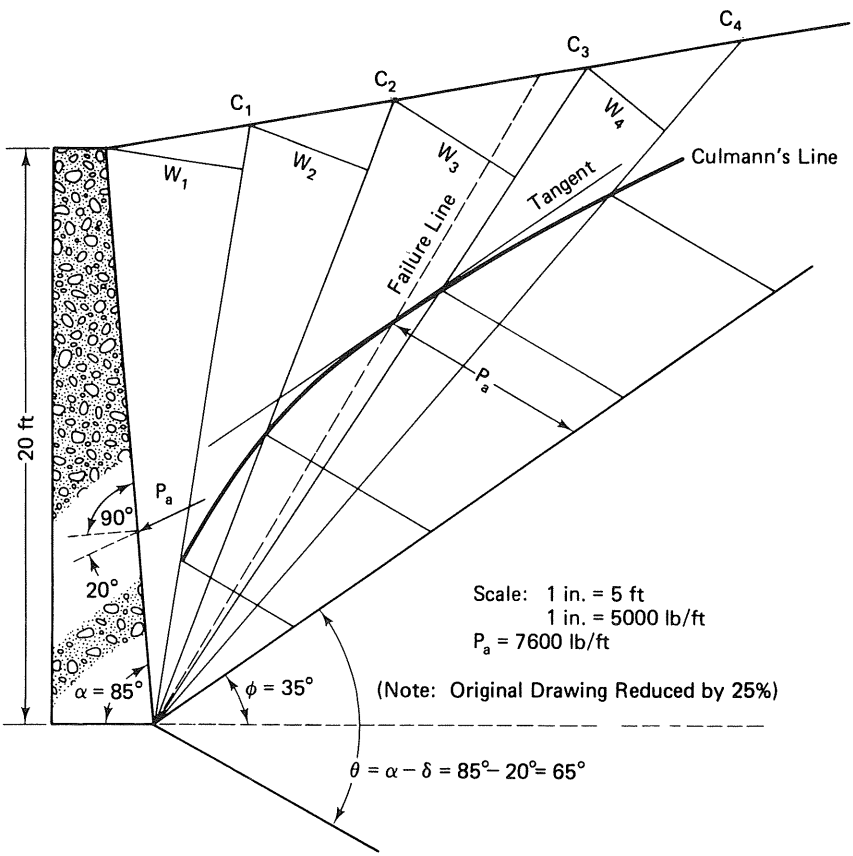

FIGURE 12-26 Culmann's solution for Example 12-9.

Solution

The effect of the surcharge uniform load of 1000 lb/ft² (such as highway loading) is taken into account by superposing an equivalent depth of fill $h = q/\gamma = (1000 \text{ lb/ft}^2)/(120 \text{ lb/ft}^3) = 8.33$ ft on each trial wedge. Then, the Culmann's graphical solution is carried out by following the steps outlined previously and preparing the sketch of Fig. 12-28. Weights of the wedges (step 5) are computed as follows:

$$W_1 = (\tfrac{1}{2})(120 \text{ lb/ft}^3)(5.0 \text{ ft})(20.0 \text{ ft}) + (120 \text{ lb/ft}^3)(8.33 \text{ ft})(5 \text{ ft}) = 11{,}000 \text{ lb/ft}$$

$$W_2 = (\tfrac{1}{2})(120 \text{ lb/ft}^3)(4.5 \text{ ft})(22.4 \text{ ft}) + (120 \text{ lb/ft}^3)(8.33 \text{ ft})(5 \text{ ft}) = 11{,}050 \text{ lb/ft}$$

$$W_3 = (\tfrac{1}{2})(120 \text{ lb/ft}^3)(4.0 \text{ ft})(25.0 \text{ ft}) + (120 \text{ lb/ft}^3)(8.33 \text{ ft})(5 \text{ ft}) = 11{,}000 \text{ lb/ft}$$

$$W_4 = (\tfrac{1}{2})(120 \text{ lb/ft}^3)(3.5 \text{ ft})(28.3 \text{ ft}) + (120 \text{ lb/ft}^3)(8.33 \text{ ft})(5 \text{ ft}) = 10{,}940 \text{ lb/ft}$$

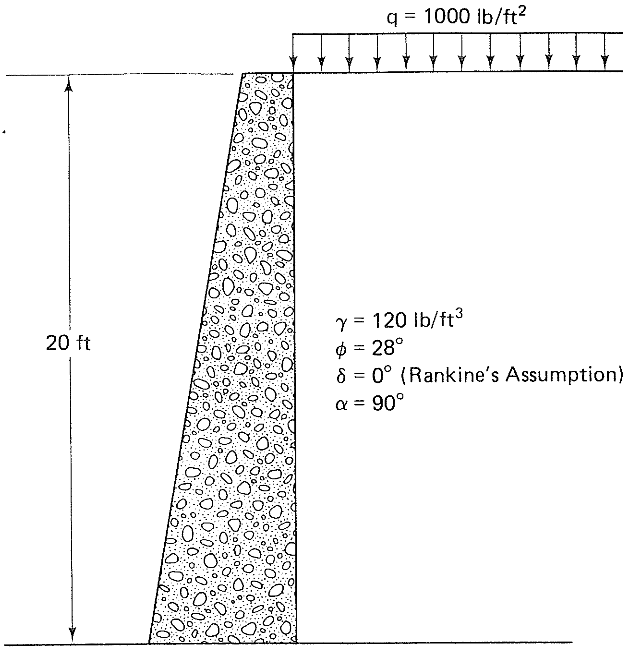

FIGURE 12-27

From Fig. 12-28, the value of P_a is determined to be 15,800 lb/ft. As computed in Example 12-8, P_a acts 8.18 ft from the base of the wall (see Fig. 12-28).

EXAMPLE 12-11

Given

The retaining wall shown in Fig. 12-29.

Required

Total active earth pressure, P_a, by Culmann's graphical solution.

Solution

By following the steps outlined previously for Culmann's graphical solution, the sketch of Fig. 12-30 is prepared. Weights of the wedges (step 5) are computed as follows:

$$W_1 = (1/2)(0.12\ \text{kip/ft}^3)(5.2\ \text{ft})(25.5\ \text{ft}) = 7.96\ \text{kips/ft}$$

$$W_2 = (1/2)(0.12\ \text{kip/ft}^3)(2.9\ \text{ft})(26.4\ \text{ft}) = 4.59\ \text{kips/ft}$$

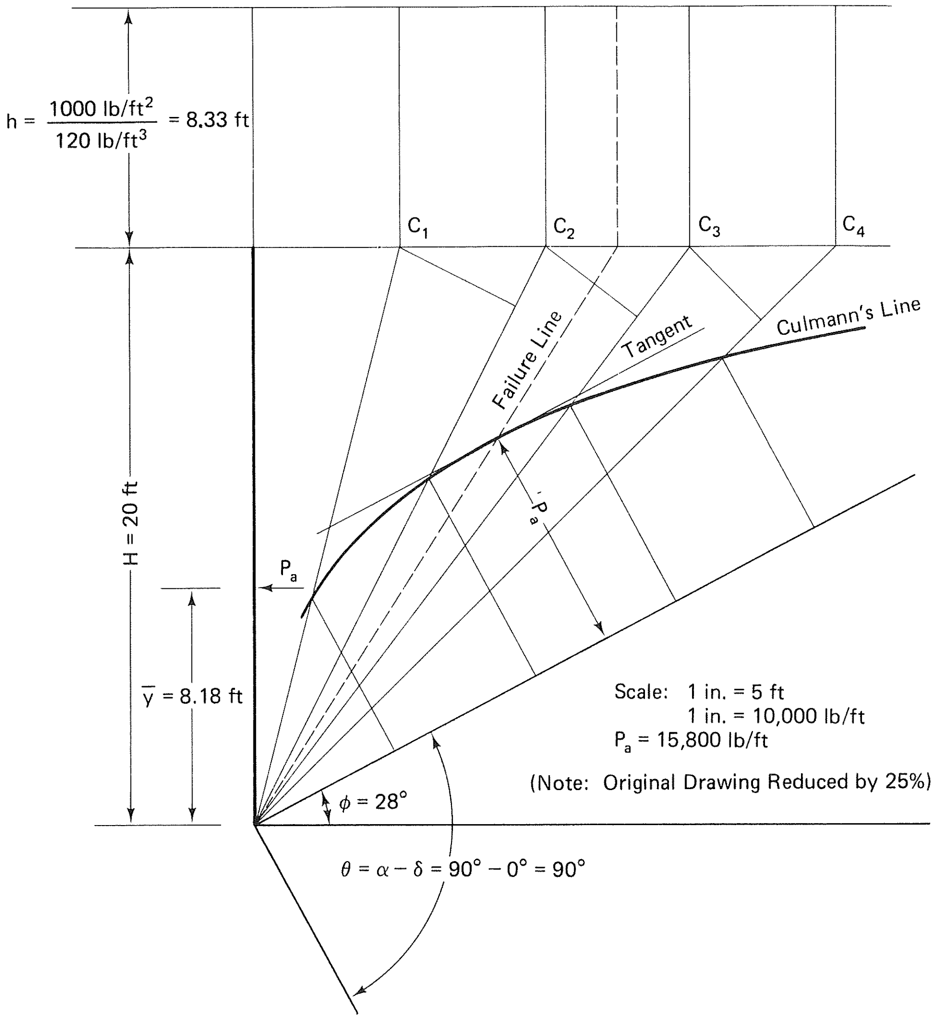

FIGURE 12-28 Culmann's solution for Example 12-10.

$$W_3 = (\tfrac{1}{2})(0.12\ \text{kip/ft}^3)(2.7\ \text{ft})(27.6\ \text{ft}) = 4.47\ \text{kips/ft}$$

$$W_{3c} = 8\ \text{kips (concentrated load)}$$

$$W_4 = (\tfrac{1}{2})(0.12\ \text{kip/ft}^3)(2.6\ \text{ft})(29.0\ \text{ft}) = 4.52\ \text{kips/ft}$$

$$W_5 = (\tfrac{1}{2})(0.12\ \text{kip/ft}^3)(3.0\ \text{ft})(31.0\ \text{ft}) = 5.58\ \text{kips/ft}$$

From Fig. 12-30, the value of P_a is determined to be 17.0 kips/ft.

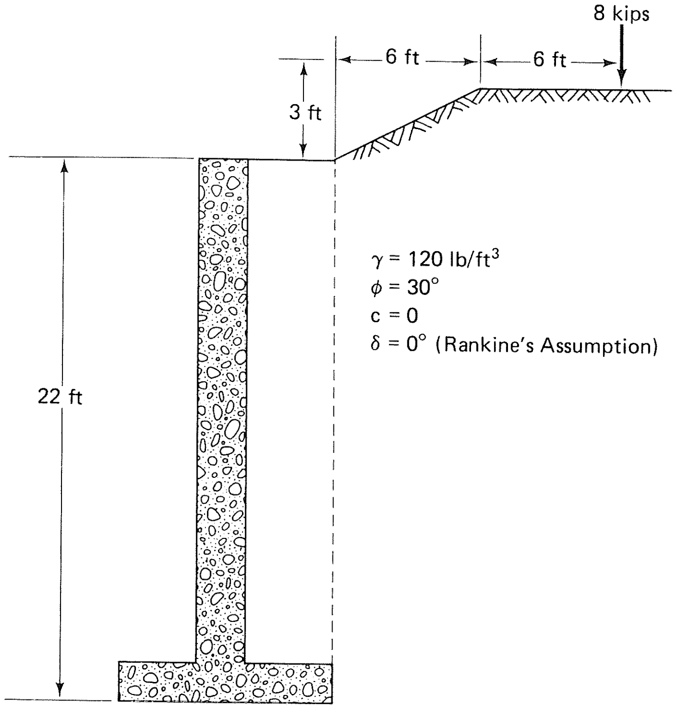

FIGURE 12-29

12-7 DESIGN CONSIDERATIONS FOR RETAINING WALLS

In designing retaining walls, the first step is to determine the magnitude and location of the active earth pressures that will be acting on the wall. These determinations can be made by utilizing any of the methods presented previously in this chapter. Active earth pressure is used to design free-standing retaining walls.

The next step is to assume a retaining wall size. Normally, the required wall height will be known; thus a wall thickness and base width must be estimated. The assumed wall is then checked for three conditions. First, the wall must be safe against sliding horizontally. Second, the wall must be safe against overturning. Third, the wall must not introduce a contact pressure on the foundation soil beneath the wall's base that exceeds the allowable bearing pressure of the foundation soil. If any of these conditions is not safe, the assumed wall size must be modified, and conditions checked again. If (when) the three conditions are met, the assumed size is used for design. If, however, the three conditions are met with plenty to spare, the size might be reduced somewhat, and conditions checked again. Obviously, this is more or less a trial-and-error procedure.

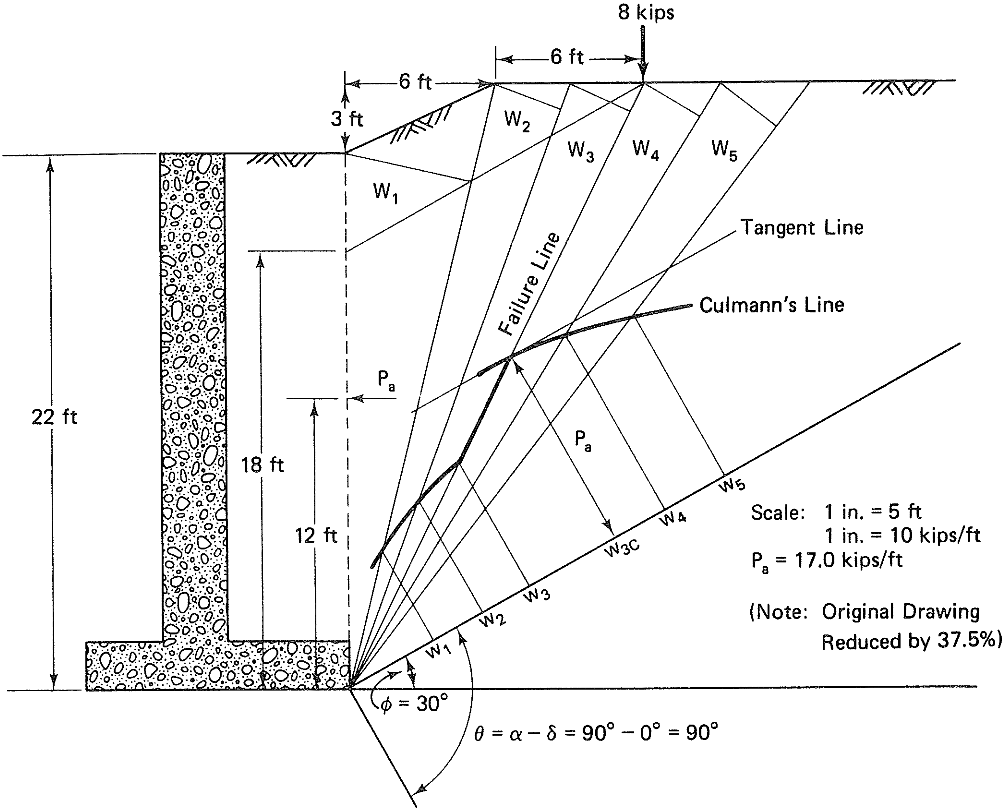

FIGURE 12-30 Culmann's solution for Example 12-11.

The preceding gives a brief preview of design considerations for retaining walls. This topic will be addressed in greater detail in Chap. 13.

12-8 LATERAL EARTH PRESSURE ON BRACED SHEETINGS

Sometimes earth cuts are retained by braced sheetings rather than the rigid walls considered heretofore in this chapter. Commonly made of wood or steel, sheetings are normally driven vertically and often used to retain earth temporarily during a construction project. A sketch of braced sheetings used to retain earth is shown in Fig. 12-31. A horizontal brace providing lateral support to resist earth pressure behind the sheeting is known as a *strut.* A continuous horizontal (longitudinal) member extending along a sheeting's face to provide intermediate sheeting support between strut locations is called a *wale.* Examples of struts and wales are shown in Fig. 12-31.

Lateral earth pressure on braced sheetings cannot ordinarily be analyzed by the Rankine, Coulomb, or other theories that are used to analyze pressures on rigid retaining walls. Those theories are based on the condition

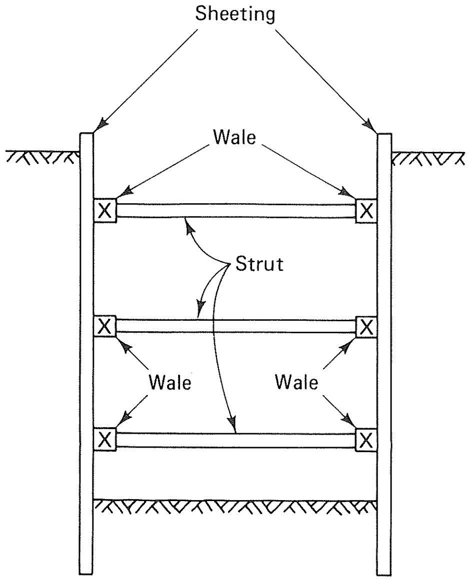

FIGURE 12-31 Braced sheetings.

that the (rigid) wall yields laterally, either by sliding sideways or rotating about the bottom of the wall, so that the soil's full shearing resistance can be developed. Braced sheetings are much more flexible; hence, they do not yield in the same manner as rigid walls, thereby giving different shearing patterns.

Braced sheeting design can be done using empirical diagrams of lateral pressure against braced sheetings. Figure 12-32 gives such diagrams for braced sheetings in sand, soft to medium clays, and stiff-fissured clays. Struts may be designed by assuming vertical members are hinged at each strut level except those at the top and bottom (see Fig. 12-33).

EXAMPLE 12-12

Given

1. A braced sheet pile for an open cut in soft to medium clay is illustrated in Fig. 12-34.
2. Struts are spaced longitudinally at 4.0 m center to center.
3. The sheet piles are pinned or hinged at strut levels *B* and *C*.

Required

1. Lateral earth pressure diagram for the braced sheet pile system.
2. Loads on struts *A, B, C,* and *D*.

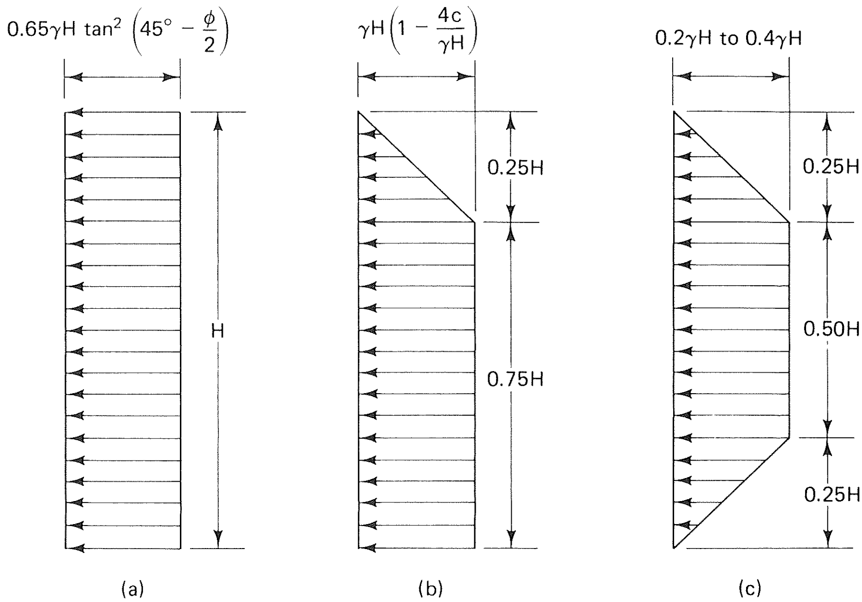

FIGURE 12-32 Diagrams of lateral pressure against braced sheetings: (a) sand; (b) soft to medium clay; (c) stiff-fissured clay. [8]

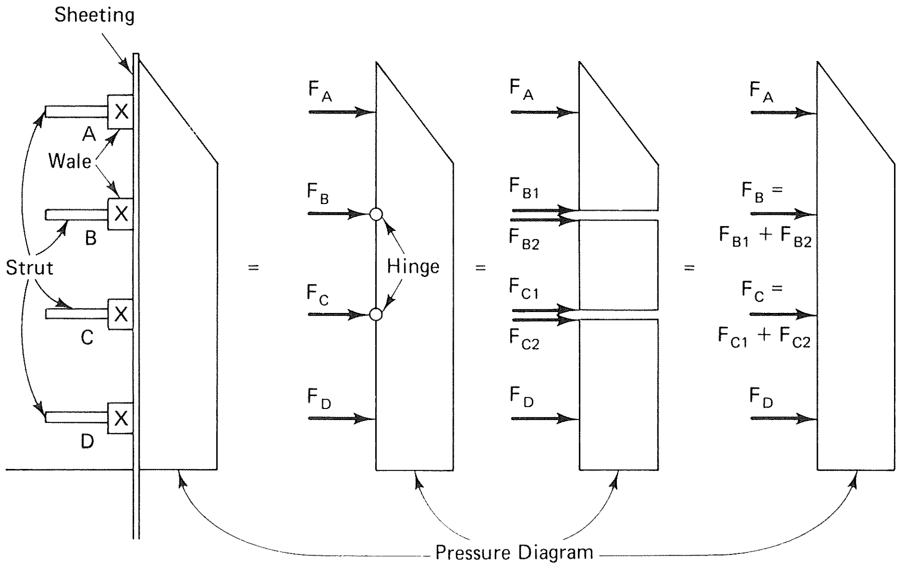

FIGURE 12-33 Forces on struts in braced sheeting.

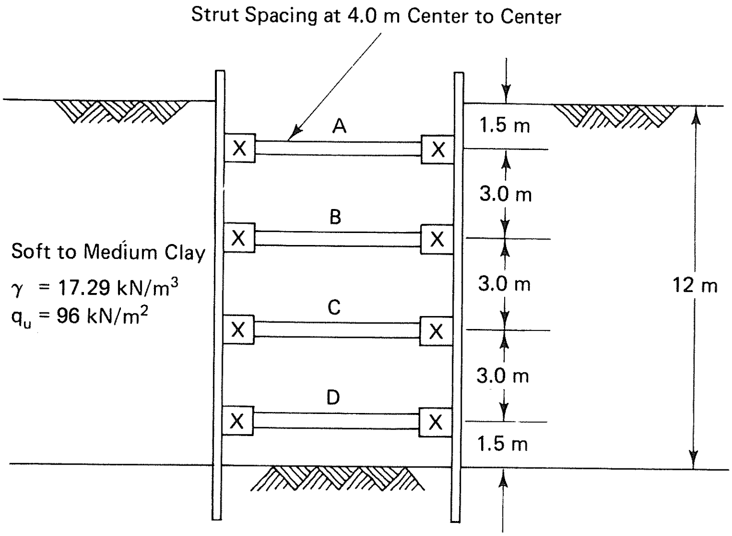

FIGURE 12-34

Solution

1. $p = \gamma H \left[1 - \frac{4c}{\gamma H}\right]$ (from Fig. 12-32)

$$c = \frac{96\ \text{kN/m}^2}{2} = 48\ \text{kN/m}^2$$

$$p = (17.29\ \text{kN/m}^3)(12\ \text{m})\left[1 - \frac{(4)(48\ \text{kN/m}^2)}{(17.29\ \text{kN/m}^3)(12\ \text{m})}\right] = 15.48\ \text{kN/m}^2$$

The lateral earth pressure diagram for the braced sheet pile system is, therefore, as shown in Fig. 12-35.

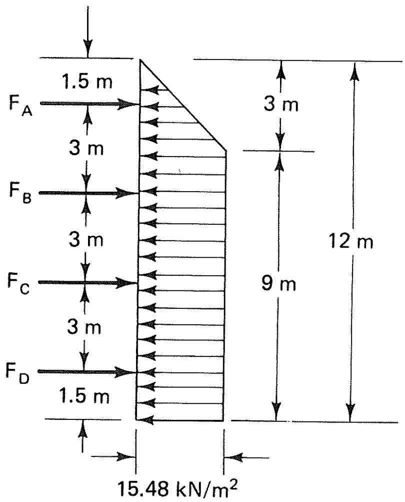

FIGURE 12-35

2. In the free body diagram of Fig. 12-36a,

$$\Sigma M_B = 0$$

$$(½)(15.48 \text{ kN/m}^2)(3.0 \text{ m})(4.0 \text{ m}) \left(1.5 \text{ m} + \frac{3.0 \text{ m}}{3}\right)$$

$$+ (1.5 \text{ m})(15.48 \text{ kN/m}^2)(4.0 \text{ m}) \left(\frac{1.5 \text{ m}}{2}\right) - (F_A)(3.0 \text{ m}) = 0$$

$$F_A = 100.6 \text{ kN}$$

$$\Sigma H = 0$$

$$F_{B1} = (½)(1.5 \text{ m} + 4.5 \text{ m})(15.48 \text{ kN/m}^2)(4.0 \text{ m}) - 100.6 \text{ kN}$$

$$= 85.2 \text{ kN}$$

In the free body diagram of Fig. 12-36b,

$$F_{B2} = F_{C1} = (½)(3.0 \text{ m})(15.48 \text{ kN/m}^2)(4.0 \text{ m}) = 92.9 \text{ kN}$$

In the free body diagram of Fig. 12-36c,

$$\Sigma M_c = 0$$

$$(F_D)(3.0 \text{ m}) - (4.5 \text{ m})(15.48 \text{ kN/m}^2)(4.0 \text{ m}) \left(\frac{4.5 \text{ m}}{2}\right) = 0$$

$$F_D = 209.0 \text{ kN}$$

$$\Sigma H = 0$$

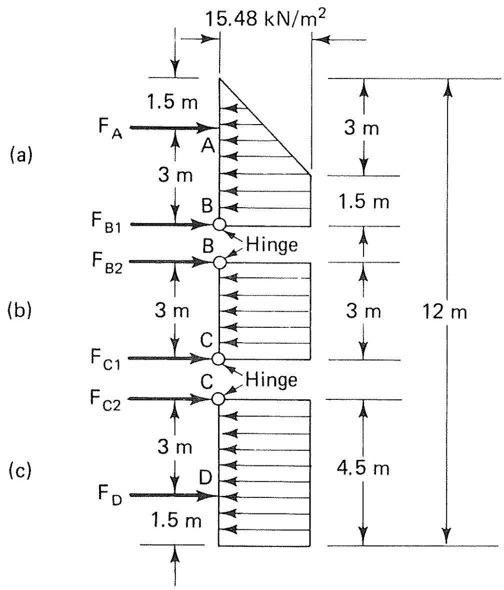

FIGURE 12-36

$$F_{C2} + F_D - (4.5 \text{ m})(15.48 \text{ kN/m}^2)(4.0 \text{ m}) = 0$$

$$F_{C2} = (4.5 \text{ m})(15.48 \text{ kN/m}^2)(4.0 \text{ m}) - 209.0 \text{ kN} = 69.6 \text{ kN}$$

Therefore,

$$F_A = 100.6 \text{ kN}$$
$$F_B = 85.2 \text{ kN} + 92.9 \text{ kN} = 178.1 \text{ kN}$$
$$F_C = 92.9 \text{ kN} + 69.6 \text{ kN} = 162.5 \text{ kN}$$
$$F_D = 209.0 \text{ kN}$$

12-9 PROBLEMS

12-1 A smooth, unyielding wall retains a loose sand (see Fig. 12-37). Assume that no lateral movement occurs in the soil mass and the at rest condition prevails. Draw the diagram of earth pressure against the wall and find the total lateral force acting on the wall if the groundwater table is located 2 m below the ground surface, as shown in Fig. 12-37.

12-2 A vertical retaining wall 25 ft high supports a deposit of sand having a level backfill. Soil properties are as follows:

$$\gamma = 120 \text{ lb/ft}^3$$
$$\phi = 35°$$
$$c = 0$$

Calculate the total active earth pressure per foot of wall and its point of application by Rankine theory.

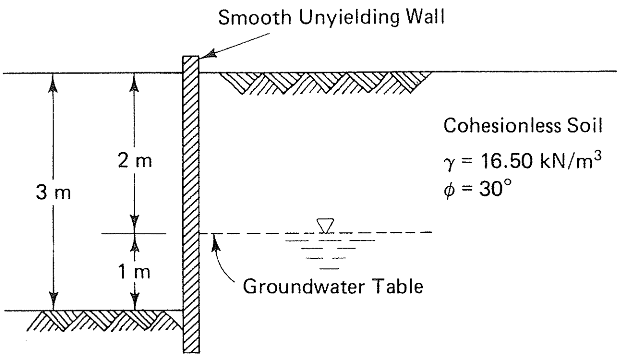

FIGURE 12-37

12-3 A vertical retaining wall 7.62 m high supports a deposit of sand with a sloping backfill. The angle of sloping backfill is 10°. Soil properties are as follows:

$$\gamma = 18.85 \text{ kN/m}^3$$

$$\phi = 35°$$

$$c = 0$$

Calculate the total active earth pressure per meter of wall and its point of application by Rankine theory.

12-4 What is the total active earth pressure per foot of wall for the wall shown in Fig. 12-38, using Rankine theory?

12-5 A vertical wall 25 ft high supports a level backfill of clayey sand. The samples of the backfill soil were tested and the following properties were determined: $\phi = 20°$, $c = 250$ lb/ft², and $\gamma = 125$ lb/ft³. Draw the active earth pressure diagram using Rankine theory.

12-6 What is the total active earth pressure per foot of wall for the retaining wall in Problem 12-2, with angle of wall friction between backfill and wall of 20°, using Coulomb theory?

12-7 What is the total active earth pressure per foot of wall for the retaining wall in Problem 12-4, with an angle of wall friction between backfill and wall of 25°, using Coulomb theory?

12-8 A smooth vertical wall is 25 ft high and retains a cohesionless soil with $\gamma = 115$ lb/ft³ and $\phi = 30°$. The top of the soil is level with the top of the wall and the soil surface carries a uniformly distributed load of 500 lb/ft².

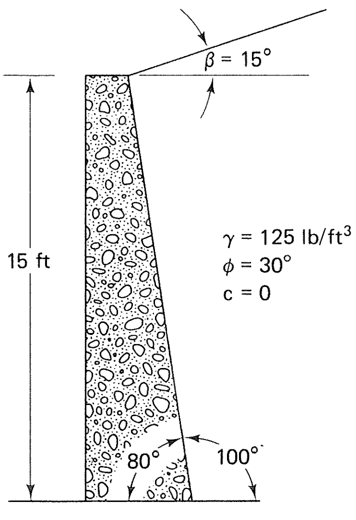

FIGURE 12-38

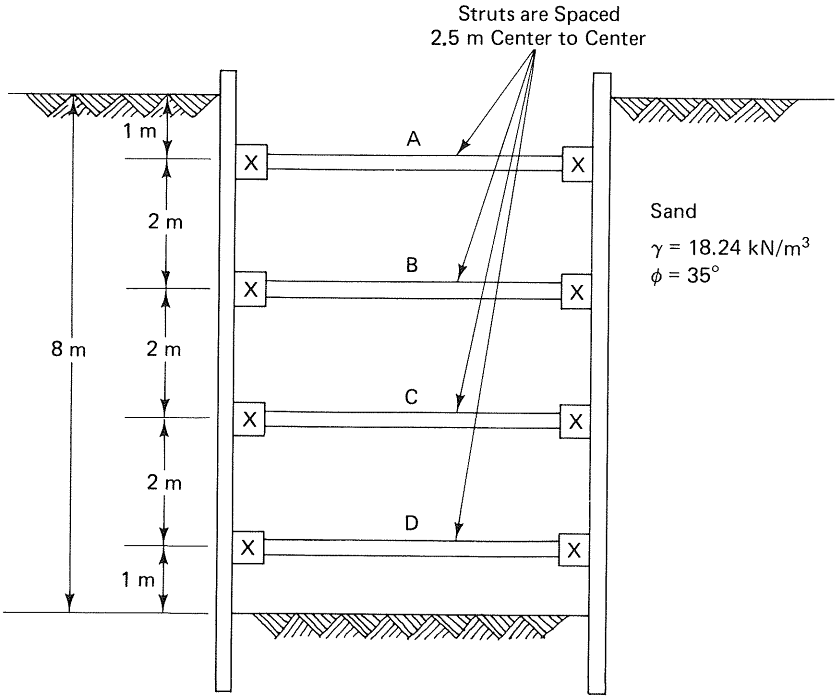

FIGURE 12-39

Calculate the total active earth pressure on the wall per linear foot of wall, and determine its point of application by Rankine theory.

12-9 Solve Problem 12-7 by Culmann's graphical solution.

12-10 Solve Problem 12-8 by Culmann's graphical solution.

12-11 A braced sheet pile to be used in an open cut in sand is shown in Fig. 12-39. Assume the sheet piles are hinged at strut levels *B* and *C*. Struts are spaced longitudinally at 2.5 m center-to-center spacing. Draw the lateral earth pressure diagram for the braced sheet pile system and compute the loads on struts *A*, *B*, *C*, and *D*.

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13

Retaining Structures

13-1 INTRODUCTION

Retaining structures are built for the purpose of retaining, or holding back, a soil mass (or other material). Probably a majority of retaining structures are concrete walls, which are covered in Secs. 13-2 through 13-6. A relatively new type of retaining structure known as "Reinforced Earth" is presented in Sec. 13-7. Slurry trench walls, specially constructed concrete walls built entirely below ground level, are described in Sec. 13-8. Anchored bulkheads, covered in Sec. 13-9, are useful when certain waterfront retaining structures are needed.

13-2 RETAINING WALLS

A simple retaining wall is illustrated in Fig. 13-1. This type of wall depends on its weight to achieve stability; hence, it is called a *gravity wall.* In the case of taller walls, large lateral pressure tends to overturn the wall, and for economic reasons *cantilever walls* may be more desirable. As illustrated in Fig. 13-2, a cantilever wall has part of its base extending underneath the backfill, and (as will be shown subsequently) the weight of the soil above this part of the base helps prevent overturning.

Gravity walls are often built of plain concrete and are bulky. Concrete cantilever walls are generally more slender and must be adequately reinforced

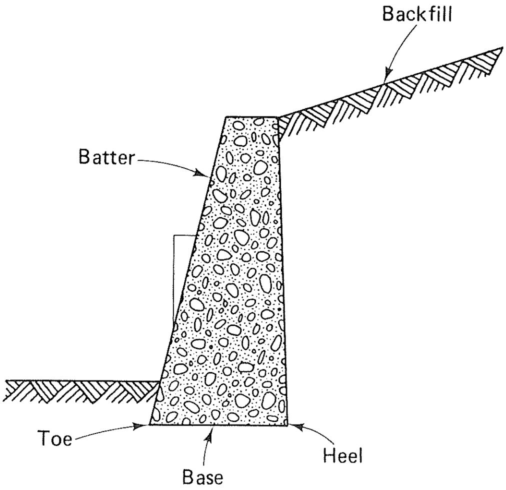


FIGURE 13-1 Gravity wall.

with steel. Although there are other types of retaining walls, these two are most common.

While retaining walls may give the appearance of being unyielding, some wall movement is to be expected. In order that walls may undergo some forward yielding without appearing to tip over, they are often built with an inward slope on the outer face of the wall, as shown in Figs. 13-1 and 13-2. This inward slope is called *batter.*

Material placed behind a retaining wall is commonly referred to as *backfill.* It is highly desirable that backfill be a select, free-draining, granular material, such as clean sand, gravel, or broken stones. If necessary, appropriate material should be hauled in from an area outside the construction site.

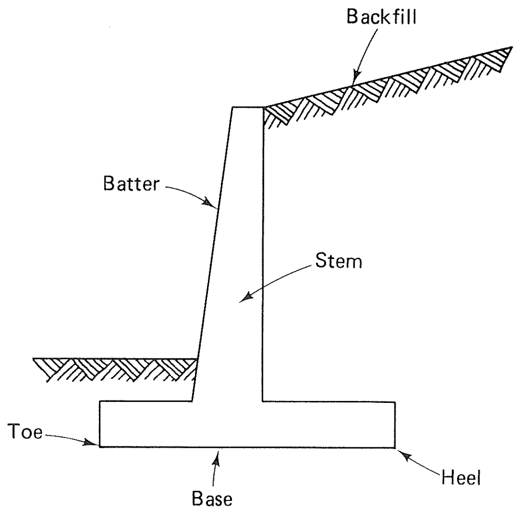


FIGURE 13-2 Cantilever wall.

Clayey soils make extremely objectionable backfill material because of excessive lateral pressure they create. The designer of a retaining wall should either (1) write specifications for the backfill and base the design of the wall on the specified backfill, or (2) be given information on the material to be used as backfill and base the design of the wall on the indicated backfill. If it is possible that the water table will rise in the backfill, special designing, construction, and monitoring must go into effect.

In Chap. 12, several methods were presented for analyzing both magnitude and location of the lateral earth pressure acting on retaining walls. For economic reasons, retaining walls are commonly designed for active earth pressure, developed by a free-draining, granular backfill acting on the wall. As related in Sec. 12-7, a retaining wall must (1) be able to resist sliding along the base, (2) be able to resist overturning, and (3) not introduce a contact pressure on the foundation soil beneath the wall's base that exceeds the allowable bearing pressure of the foundation soil. (Walls must also meet structural requirements, such as shear and bending moment; however, such considerations are not covered in this book.) Chapter 13 deals in more detail with retaining wall design.

13-3 EARTH PRESSURE COMPUTATION

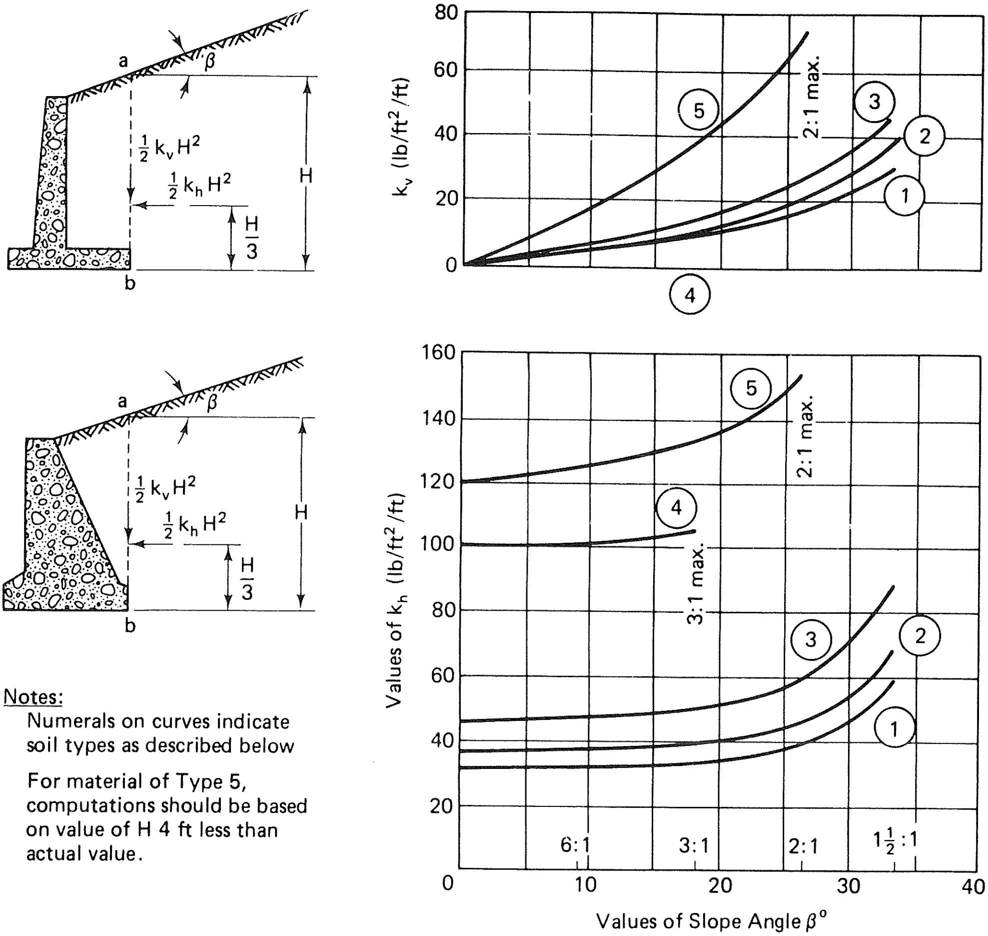
To design a retaining wall, it is, of course, necessary to determine the earth pressure acting on the wall. Analytical determinations of earth pressures—including Rankine earth pressure, Coulomb earth pressure, and Culmann's graphical solution—were covered in detail in Chap. 12. Retaining wall design is normally based on active earth pressure.

In practice, earth pressures for walls less than 20 ft (6 m) high are often obtained from graphs or tables. Almost all such graphs and tables are developed from Rankine theory. One graphical relationship is given in Fig. 13-3. Use of this approach to obtain earth pressure should be self-explanatory.

As will be noted by both the analytical methods of Chap. 12 and the graphical method of Fig. 13-3, the magnitude of earth pressure on a retaining wall depends in part upon the type of soil backfill.

13-4 STABILITY ANALYSIS

Common procedure in retaining wall design is to assume a trial wall shape and size and then to check the trial wall for stability. If it does not prove to be stable by conventional standards, the wall's shape and/or size must be revised and the new wall checked for stability. This procedure is repeated until a satisfactory wall is found.



Notes:
Numerals on curves indicate soil types as described below

For material of Type 5, computations should be based on value of H 4 ft less than actual value.

Types of Backfill for Retaining Walls

1. Coarse-grained soil without admixture of fine soil particles, very free-draining (clean sand, gravel or broken stone)
2. Coarse-grained soil of low permeability, owing to admixture of particles of silt size
3. Fine silty sand; granular materials with conspicuous clay content; or residual soil with stones
4. Soft or very soft clay; organic silt; or soft silty clay
5. Medium or stiff clay that may be placed in such a way that a negligible amount of water will enter the spaces between the chunks during floods or heavy rains

FIGURE 13-3 Earth pressure charts for retaining walls less than 20 ft (6.1 m) high (1 lb/ft²/ft = 0.1571 kN/m²/m). [1, 2]

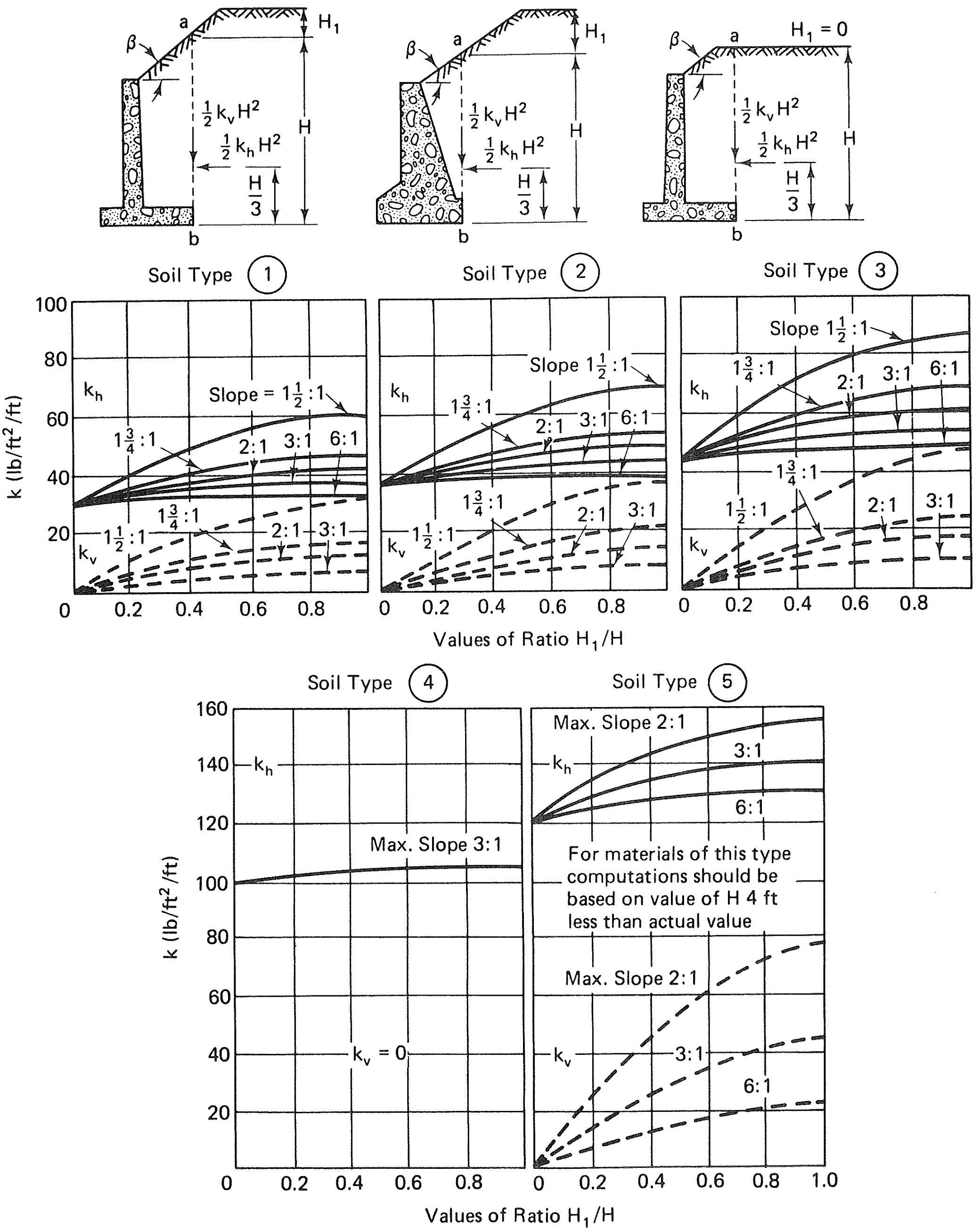


FIGURE 13-3 *(continued)*

If a wall is *stable,* it means, of course, that the wall does not move. Essentially, there are three means by which a retaining wall can move—horizontally (by sliding), vertically (by excessive settlement and/or bearing capacity failure of the foundation soil), and by rotation (by overturning). Standard procedure is to check for stability with respect to each of the three means of movement to ensure that an adequate factor of safety is present in each case. Checks for sliding and overturning hark back to the basic laws of statics. Checks for settlement and bearing capacity of foundation soil are done by settlement and bearing capacity analyses, which were presented in Chaps. 7 and 9, respectively.

The factor of safety against sliding is found by dividing sliding resistance force by sliding force. The sliding resistance force is the product of total downward force on the base of the wall and coefficient of friction (μ) between the base of the retaining wall and the underlying soil. The sliding force is typically the horizontal component of lateral earth pressure exerted against the wall by backfill material.

If an adequate factor of safety against sliding is not obtained with an ordinary flat-bottomed wall, some additional sliding resistance may be achieved by constructing a "key" into the wall's base. As shown in Fig. 13-4, soil in front of the key's vertical face provides additional resistance to sliding in the form of passive resistance (i.e., zone *BC* of the earth pressure diagram). Of course, soil in front of the wall and its base furnishes some passive resistance (zone *AB* of the earth pressure diagram of Fig. 13-4); however, since this soil may be

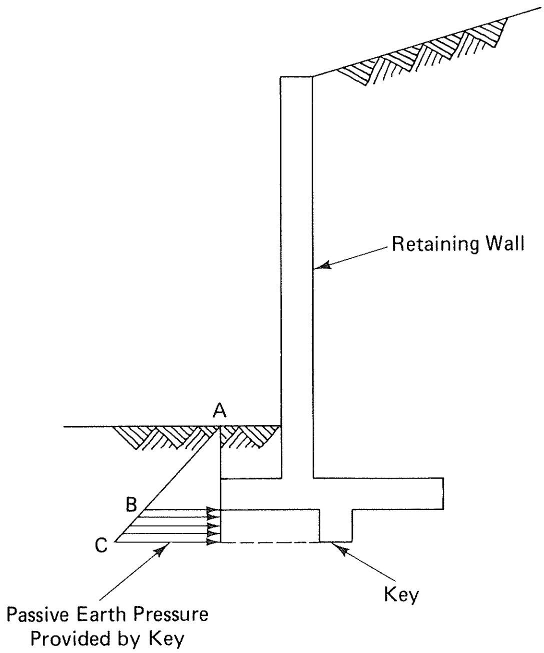


FIGURE 13-4

subsequently removed by erosion, this passive resistance is often ignored in retaining wall design. Keys are most effective in hard soil or rock.

The factor of safety against overturning is determined by dividing total righting moment by total overturning moment. Since overturning tends to occur about the front base of a wall (at the toe), righting moments and overturning moments are computed about the wall's toe.

The factor of safety against bearing capacity failure is determined by dividing ultimate bearing capacity by actual maximum contact (base) pressure. Contact pressure is computed by the methods presented in Chap. 9.

To summarize the three factors of safety with regard to stability analysis,

$$(\text{F.S.})_{\text{sliding}} = \frac{\text{sliding resistance force}}{\text{sliding force}} \tag{13-1}$$

$$(\text{F.S.})_{\text{overturning}} = \frac{\text{total righting moment about toe}}{\text{total overturning moment about toe}} \tag{13-2}$$

$$(\text{F.S.})_{\text{bearing capacity failure}} = \frac{\text{soil's ultimate bearing capacity}}{\text{actual maximum contact (base) pressure}} \tag{13-3}$$

Some common minimum factors of safety for sufficient stability are as follows:

$(\text{F.S.})_{\text{sliding}}$ = 1.5 (if the passive earth pressure of the soil at the toe in front of the wall is neglected) [3]*

= 2.0 (if the passive earth pressure of the soil at the toe in front of the wall is included) [3]*

$(\text{F.S.})_{\text{overturning}}$ = 1.5 (granular backfill soil)

= 2.0 (cohesive backfill soil) [1]

$(\text{F.S.})_{\text{bearing capacity failure}}$ = 3.0

The two example problems that follow illustrate the investigation of stability analysis for retaining walls. Example 13-1 refers to a gravity wall, and Example 13-2 to a cantilever wall.

EXAMPLE 13-1

Given

1. The retaining wall shown in Fig. 13-5 is to be constructed of concrete having a unit weight of 150 lb/ft³.

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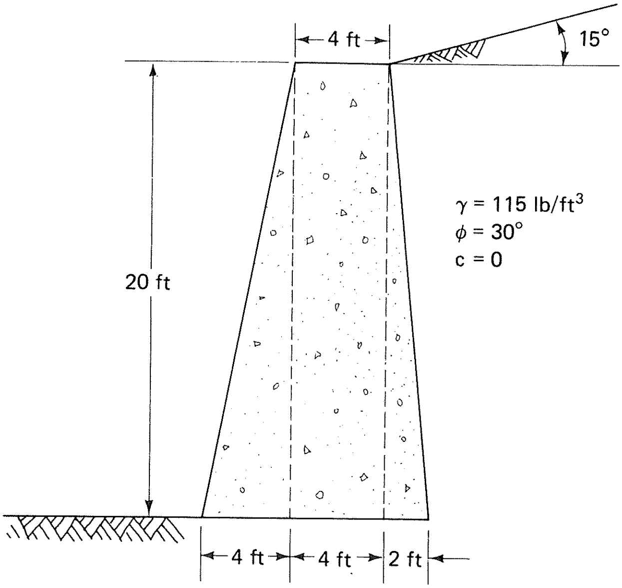


FIGURE 13-5

2. The retaining wall is to support a deposit of granular soil that has the following properties:

$$\gamma = 115 \text{ lb/ft}^3$$
$$\phi = 30^\circ$$
$$c = 0$$

3. The coefficient of base friction is 0.55.

4. The foundation soil's ultimate bearing capacity is 6.5 tons/ft^2.

Required

Check the stability of the proposed retaining wall; that is, check the factor of safety against:

1. Sliding.

2. Overturning.

3. Bearing capacity failure.

Solution

Calculation of active earth pressure on the back of the wall by Rankine theory

From Eqs. (12-10) and (12-11),

$$P_a = \frac{1}{2}\gamma H^2 \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

Referring to Fig. 13-6,

$$H = \overline{BC} = 20 \text{ ft} + (2 \text{ ft})(\tan 15^\circ) = 20.54 \text{ ft}$$

$$P_a = (\tfrac{1}{2})(0.115 \text{ kip/ft}^3)(20.54 \text{ ft})^2(\cos 15^\circ)\frac{\cos 15^\circ - \sqrt{\cos^2 15^\circ - \cos^2 30^\circ}}{\cos 15^\circ + \sqrt{\cos^2 15^\circ - \cos^2 30^\circ}}$$

$$P_a = 9.05 \text{ kips/ft}$$

P_a acts parallel to the surface of the backfill, and therefore

Horizontal component $(P_h) = P_a \cos 15^\circ = (9.05 \text{ kips/ft}) \cos 15^\circ$
$= 8.74 \text{ kips/ft}$

Vertical component $(P_v) = P_a \sin 15^\circ = (9.05 \text{ kips/ft}) \sin 15^\circ = 2.34 \text{ kips/ft}$

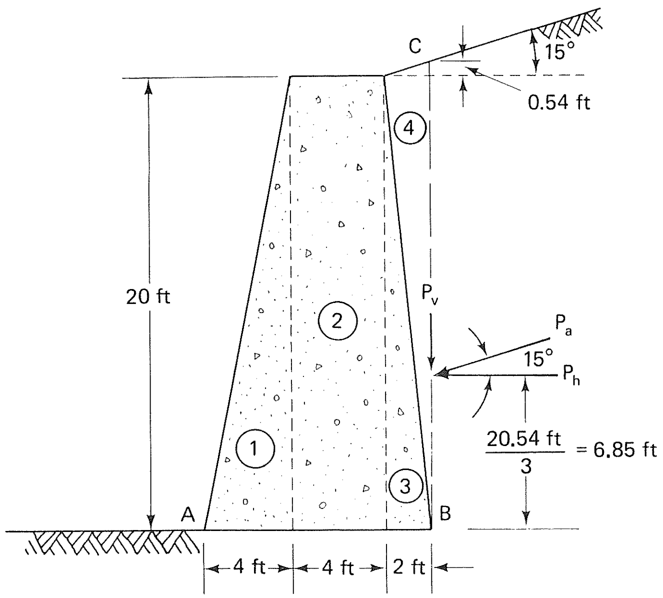


FIGURE 13-6

Calculation of righting moment (see Fig. 13-6)

Component	Weight of Component (kips/ft)		Moment Arm from A (ft)	Righting Moment about A (ft-kips/ft)
1	(0.15)(½)(4)(20) =	6.00	(⅔)(4) = 8/3	16.0
2	(0.15)(4)(20) =	12.00	4 + 4/2 = 6	72.0
3	(0.15)(½)(2)(20) =	3.00	4 + 4 + (⅓)(2) = 26/3	26.0
4	(0.115)(½)(20.54)(2) =	2.36	4 + 4 + (⅔)(2) = 28/3	22.0
P_v		2.34	4 + 4 + 2 = 10	23.4
	$\Sigma V =$	25.70	$\Sigma M_r =$	159.4

Calculation of overturning moment

Overturning moment (M_0) = (8.74 kips/ft)(6.85 ft) = 59.9 ft-kips/ft

1. From Eq. (13-1),

$$(\text{F.S.})_{\text{sliding}} = \frac{\text{sliding resistance force}}{\text{sliding force}} \tag{13-1}$$

$$(\text{F.S.})_{\text{sliding}} = \frac{(\mu)(\Sigma V)}{P_h} = \frac{(0.55)(25.70 \text{ kips/ft})}{8.74 \text{ kips/ft}}$$

$$= 1.62 > 1.5 \qquad \text{O.K.}$$

2. From Eq. (13-2),

$$(\text{F.S.})_{\text{overturning}} = \frac{\text{total righting moment about toe}}{\text{total overturning moment about toe}} \tag{13-2}$$

$$(\text{F.S.})_{\text{overturning}} = \frac{\Sigma M_r}{\Sigma M_0} = \frac{159.4 \text{ ft-kips/ft}}{59.9 \text{ ft-kips/ft}}$$

$$= 2.66 > 1.5 \text{ (for granular backfill)} \qquad \text{O.K.}$$

Base pressure calculations

Location of resultant R (= ΣV) if R acts at $\bar{x}$ from the toe (point A)

$$\bar{x} = \frac{\Sigma M_A}{\Sigma V} = \frac{\Sigma M_r - \Sigma M_0}{\Sigma V} = \frac{159.4 \text{ ft-kips/ft} - 59.9 \text{ ft-kips/ft}}{25.70 \text{ kips/ft}} = 3.87 \text{ ft}$$

$$e = \frac{4 \text{ ft} + 4 \text{ ft} + 2 \text{ ft}}{2} - 3.87 \text{ ft} = 1.13 \text{ ft} < \frac{L}{6} \text{ (i.e., } 10/6 \text{ ft, or } 1.67 \text{ ft)} \qquad \text{O.K.}$$

(i.e., R acts within the middle third of the base)

Using the flexural formula, from Eq. (9-6) (see Chap. 9),

$$q = \frac{Q}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y} \tag{9-6}$$

Here,

$$Q = \text{resultant } (R) = \Sigma V = 25.70 \text{ kips}$$
$$A = (1 \text{ ft})(10 \text{ ft}) = 10 \text{ ft}^2$$
$$M_x = 0 \text{ (one-way bending)}$$
$$M_y = Q \times e = (25.70 \text{ kips})(1.13 \text{ ft}) = 29.04 \text{ ft-kips}$$
$$x = \frac{10 \text{ ft}}{2} = 5 \text{ ft}$$
$$I_y = \frac{bh^3}{12} = \frac{(1 \text{ ft})(10 \text{ ft})^3}{12} = 83.33 \text{ ft}^4$$
$$q = \frac{25.70 \text{ kips}}{10 \text{ ft}^2} \pm \frac{(29.04 \text{ ft-kips})(5 \text{ ft})}{83.33 \text{ ft}^4}$$
$$q_L = 2.57 \text{ kips/ft}^2 + 1.74 \text{ kips/ft}^2 = 4.31 \text{ kips/ft}^2 = 2.16 \text{ tons/ft}^2$$
$$q_R = 2.57 \text{ kips/ft}^2 - 1.74 \text{ kips/ft}^2 = 0.83 \text{ kip/ft}^2 = 0.42 \text{ ton/ft}^2$$

The pressure distribution is shown in Fig. 13-7.

3. From Eq. (13-3),

$$(\text{F.S.})_{\text{bearing capacity failure}}$$
$$= \frac{\text{soil's ultimate bearing capacity}}{\text{actual maximum contact (base) pressure}} \tag{13-3}$$

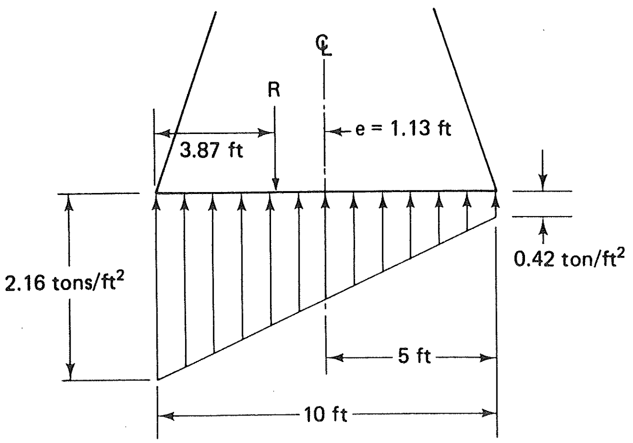


FIGURE 13-7

$$(\text{F.S.})_{\text{bearing capacity failure}} = \frac{6.5 \text{ tons/ft}^2}{2.16 \text{ tons/ft}^2} = 3.01 > 3 \qquad \text{O.K.}$$

EXAMPLE 13-2

Given

1. The retaining wall shown in Fig. 13-8.
2. Backfill material is Type 1 soil (see Fig. 13-3).
3. The unit weight and ϕ angle of the backfill material are 120 lb/ft³ and 37°, respectively.
4. The coefficient of base friction is 0.45.
5. Allowable soil pressure is 3 kips/ft².
6. The unit weight of the concrete is 150 lb/ft³.

Required

1. The factor of safety against sliding. Analyze both without and with passive earth pressure at the toe.
2. The factor of safety against overturning.
3. The safety against failure of the foundation soil.

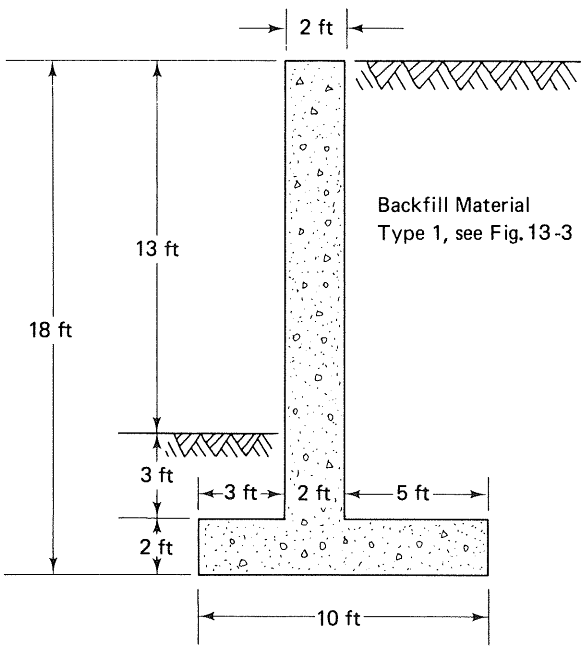


FIGURE 13-8

Solution

Calculation of the active earth pressure by Fig. 13-3

From Fig. 13-3,

$$P_h = \tfrac{1}{2}k_h H^2 \text{ with } \beta = 0° \text{ and Type 1 backfill material, } k_h = 30 \text{ lb/ft}^2\text{/ft}$$

$$P_h = (\tfrac{1}{2})(30 \text{ lb/ft}^2\text{/ft})(18 \text{ ft})^2 = 4860 \text{ lb/ft} = 4.86 \text{ kips/ft}$$

$$P_v = \tfrac{1}{2}k_v H^2 \text{ with } \beta = 0°,\ k_v = 0$$

$$P_v = 0$$

Calculation of righting moment (see Fig. 13-9)

Component	*Weight of Component (kips/ft)*		*Moment Arm from Toe (ft)*	*Righting Moment about Toe (ft-kips/ft)*
1	(0.15)(2)(13 + 3)	= 4.8	3 + 2/2 = 4.0	19.2
2	(0.15)(2)(10)	= 3.0	10/2 = 5.0	15.0
3	(0.12)(5)(13 + 3)	= 9.6	3 + 2 + 5/2 = 7.5	72.0
4	(0.12)(3)(3)	= 1.1	3/2 = 1.5	1.6
		ΣV = 18.5		ΣM_r = 107.8

Calculation of overturning moment

Overturning moment (M_0) = (4.86 kips/ft)(6 ft) = 29.16 ft-kips/ft

1. *Factor of safety against sliding:*

   (a) *Without passive earth pressure analysis (neglect passive earth pressure at the toe):*
   From Eq. (13-1),

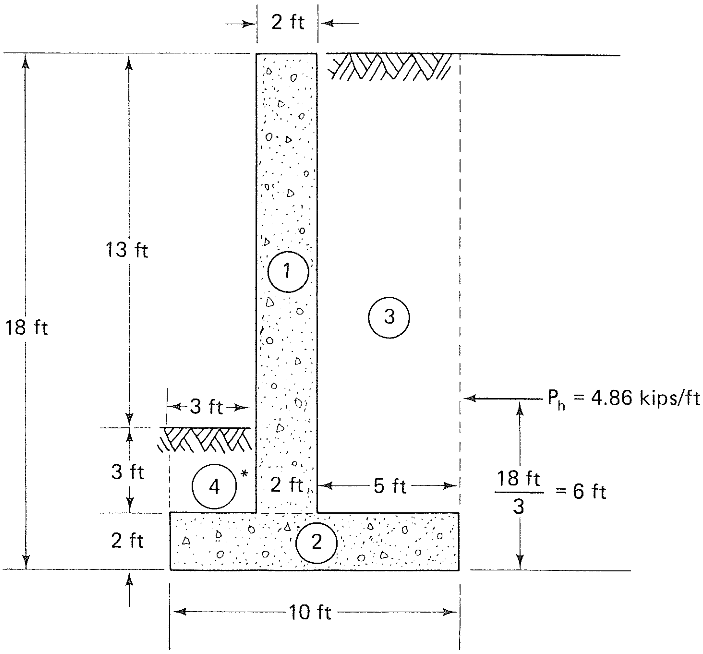
$$(\text{F.S.})_{\text{sliding}} = \frac{\text{sliding resistance force}}{\text{sliding force}} \tag{13-1}$$

$$(\text{F.S.})_{\text{sliding}} = \frac{(\mu)(\Sigma V)}{P_h} = \frac{(0.45)(18.5 \text{ kips/ft})}{4.86 \text{ kips/ft}} = 1.71 > 1.5 \qquad \text{O.K.}$$

   (b) *With passive earth pressure at the toe:*

   Sliding resistance = passive earth pressure at toe
   + friction available along the base

   According to Rankine theory for level backfill, from Eqs. (12-12) and (12-15),



* Assume the soil above the toe is also Type I backfill material.

FIGURE 13-9

$$P_p = \tfrac{1}{2}\gamma H^2 \frac{1 + \sin\phi}{1 - \sin\phi}$$

$$P_p = (\tfrac{1}{2})(0.12 \text{ kip/ft}^3)(5 \text{ ft})^2 \left(\frac{1 + \sin 37^\circ}{1 - \sin 37^\circ}\right) = 6.03 \text{ kips/ft}$$

$$(\text{F.S.})_{\text{sliding}} = \frac{(\mu)(\Sigma V) + P_p}{P_h}$$

$$(\text{F.S.})_{\text{sliding}} = \frac{(0.45)(18.5 \text{ kips/ft}) + 6.03 \text{ kips/ft}}{4.86 \text{ kips/ft}} = 2.95 > 2.0 \qquad \text{O.K.}$$

2. *Factor of safety against overturning:*
From Eq. (13-2),

$$(\text{F.S.})_{\text{overturning}} = \frac{\text{total righting moment}}{\text{total overturning moment}} \tag{13-2}$$

$$(\text{F.S.})_{\text{overturning}} = \frac{\Sigma M_r}{\Sigma M_0} = \frac{107.8 \text{ ft-kips/ft}}{29.16 \text{ ft-kips/ft}} = 3.70 > 1.5 \qquad \text{O.K.}$$

Base pressure calculations

Location of resultant R (= ΣV) if R acts at $\bar{x}$ from the toe

$$\bar{x} = \frac{\Sigma M_{\text{toe}}}{\Sigma V} = \frac{\Sigma M_r - \Sigma M_0}{\Sigma V} = \frac{107.8 \text{ ft-kips/ft} - 29.16 \text{ ft-kips/ft}}{18.5 \text{ kips/ft}} = 4.25 \text{ ft}$$

$$e = \frac{10 \text{ ft}}{2} - 4.25 \text{ ft} = 0.75 \text{ ft} < \frac{L}{6} \text{ (i.e., } {}^{10}\!/_{6} \text{ ft, or } 1.67 \text{ ft)} \qquad \text{O.K.}$$

Using the flexural formula, from Eq. (9-6),

$$q = \frac{Q}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y} \tag{9-6}$$

Here

$$Q = \text{resultant } (R) = \Sigma V = 18.5 \text{ kips}$$

$$A = (1 \text{ ft})(10 \text{ ft}) = 10 \text{ ft}^2$$

$$M_x = 0$$

$$M_y = Q \times e = (18.5 \text{ kips})(0.75 \text{ ft}) = 13.9 \text{ ft-kips}$$

$$x = \frac{10 \text{ ft}}{2} = 5 \text{ ft}$$

$$I_y = \frac{bh^3}{12} = \frac{(1 \text{ ft})(10 \text{ ft})^3}{12} = 83.33 \text{ ft}^4$$

$$q = \frac{18.5 \text{ kips}}{10 \text{ ft}^2} \pm \frac{(13.9 \text{ ft-kips})(5 \text{ ft})}{83.33 \text{ ft}^4}$$

$$q_L = 1.85 \text{ kips/ft}^2 + 0.83 \text{ kip/ft}^2 = 2.68 \text{ kips/ft}^2$$

$$q_R = 1.85 \text{ kips/ft}^2 - 0.83 \text{ kip/ft}^2 = 1.02 \text{ kips/ft}^2$$

3. Since q_L is 2.68 kips/ft², which is less than the allowable soil pressure of 3.0 kips/ft² (given), the wall is safe against failure of the foundation soil.

13-5 BACKFILL DRAINAGE

If water is allowed to permeate the soil behind a retaining wall, large additional pressure will be applied to the wall. Unless the wall is designed to withstand this large additional pressure (not the usual practice), it is imperative that steps be taken to prevent water that infiltrates the backfill soil from accumulating behind the wall.

One method of preventing water from accumulating behind a wall is to provide an effective means of draining away any water that enters the backfill soil. To accomplish this, it is highly desirable to use as backfill material a highly pervious soil such as sand, gravel, or crushed stone. To remove water

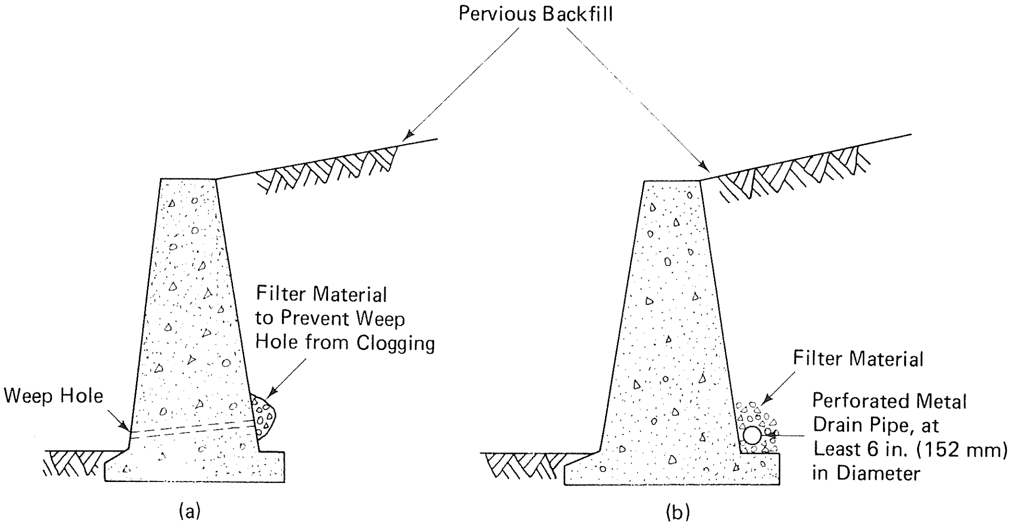


FIGURE 13-10 (a) Weep holes, (b) perforated drain pipe. [1]

from behind the wall, 4- to 6-in. (102 to 152 mm) weep holes, which are pipes extending through the wall (see Fig. 13-10a), may be placed every 5 to 10 ft (1.5 to 3 m) along the wall. A perforated drain pipe placed longitudinally along the back of the wall (Fig. 13-10b) may also be used to remove water from behind the wall. In this case, the pipe is surrounded by filter material and water drains through the filter material into the pipe and then through the pipe to one end of the wall. In both cases (weep holes and drain pipes) a filter material must be placed adjacent to the pipe to prevent clogging, and the pipes must be kept clear of debris.

If a less pervious material (silt, granular soil containing clay, etc.) has to be used as backfill because a free-draining, granular material is too expensive in the locality, it is highly desirable to place a wedge of pervious material adjacent to the wall, as shown in Fig. 13-11. If this is not possible, a "drainage blanket" of pervious material may be placed as shown in Fig. 13-12.

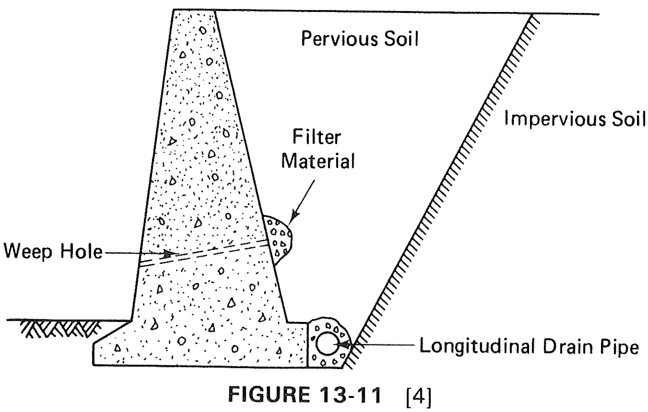


FIGURE 13-11 [4]

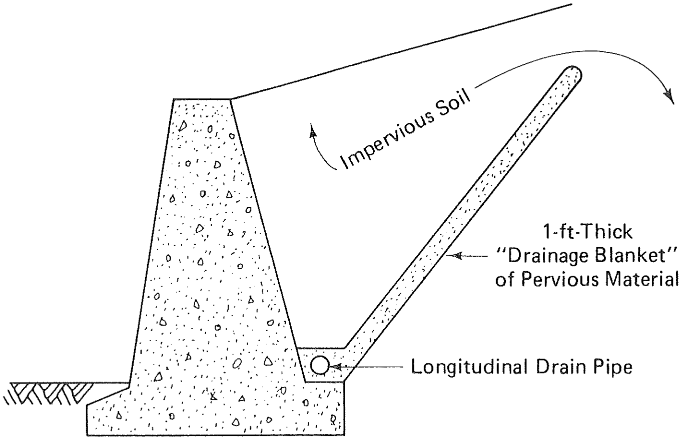


FIGURE 13-12 [5]*

A highly impervious soil (clay) is very undesirable as backfill material because, in addition to excessive lateral earth pressure it creates, it also is difficult to drain and may be subject to frost action. Also, clays are subject to swelling and shrinking. If clayey soil must be used as backfill material, it is advisable to place a wedge of pervious material adjacent to the wall between the wall and clay backfill (as shown in Fig. 13-11) [6].

13-6 SETTLEMENT AND TILTING

A certain amount of settlement by retaining walls is to be expected, just as by any other structures resting on footings or piles. In the case of retaining walls on granular soils, most of the expected settlement will have occurred by the time construction of the wall and placement of backfill have been completed. With retaining walls on cohesive soils, for which consolidation theory is applicable, settlement will occur slowly and for some period of time after construction has been completed.

The amount of settlement for retaining walls resting on spread footings can be determined using the principles of settlement analysis for footings (see Chap. 7). For walls resting on piles, the amount of settlement can be estimated using the principles of settlement analysis for pile foundations (see Chap. 10). To keep settlement relatively uniform, the resultant force must be kept near the middle of the base.

If the soil upon which a retaining wall rests is not uniform in bearing capacity along the length of the wall, differential settlement may occur along the wall, which could cause the wall to crack vertically. If soil of poor bearing capacity occurs only for a very short distance, differential settlement may not

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be a problem, as the wall tends to bridge across poor material. If, however, poor bearing capacity of the soil exists for a considerable distance along the length of the wall, differential settlement will likely happen unless the designer takes this into account and implements remedies to correct the situation. Possible remedies include improving the soil (e.g., by replacement, compaction, or stabilization of the soil) and changing the footing's width. If computed settlement is excessive, pile foundations may be used [7].

In addition to settlement, a retaining wall is also subject to tilting caused by eccentric pressure on the base of the wall. Tilting can be reduced by keeping the resultant force near the middle of the base. In many cases, walls tilt forward because the resultant force intersects the base at a point between the center and toe.

It is difficult to determine the amount of tilting to be expected, and rough estimates must suffice. If stability requirements are met in accordance with established design procedures (see Sec. 13-4), the amount of tilting may be expected to be in the order of magnitude one-tenth of 1% of the height of wall or less. However, if the subsoil consists of a compressive layer, this amount may be exceeded [1].

13-7 REINFORCED EARTH* WALLS

One alternative to the conventional retaining wall for holding soil embankments is known as a *Reinforced Earth wall,* a patented method first developed in France by Vidal. It is particularly useful where high or otherwise difficult to construct retaining structures are needed.

A typical section of a Reinforced Earth structure is shown in Figure 13-13. The wall here consists actually of precast concrete facings resting on

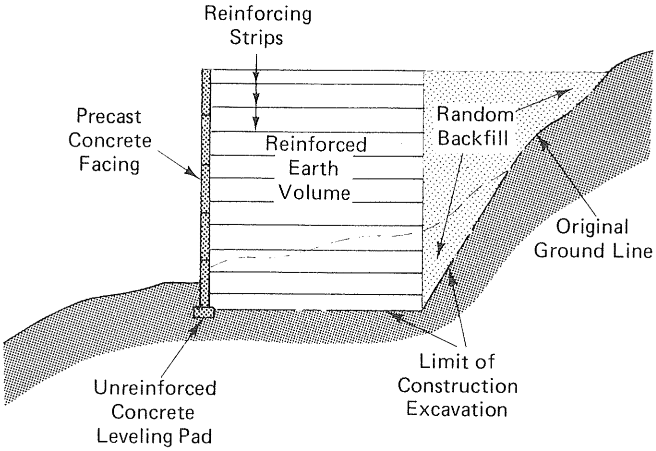

FIGURE 13-13 Typical section of a Reinforced Earth retaining wall. (Courtesy of The Reinforced Earth Company.)

* Reinforced Earth is a registered trademark of The Reinforced Earth Company, Arlington, Va.

an unreinforced concrete leveling pad. However, the "wall" does not have to be particularly strong and can therefore be constructed of thinner materials. It is sometimes referred to as the "skin." The Reinforced Earth volume is cohesionless soil, spread and compacted in layers. Reinforcing strips, commonly made of ribbed, galvanized steel, are placed atop each layer and bolted to the skin material (i.e., wall element).

Figure 13-14 gives another sketch of a Reinforced Earth wall, which depicts length, width, thickness, and horizontal and vertical spacings (L, w, t, s, and h, respectively) of the reinforcing strips. Design considerations require that (1) the skin (wall element) resist soil pressure from adjacent soil layers, (2) strip length be long enough to support the skin, and (3) the strip be strong enough to resist its internal tension. Typical strip spacings are about 1 ft (0.3 m) vertically and 2 ft (0.6 m) horizontally.

To evaluate criteria for Reinforced Earth design, consider a strip at depth z below the wall top. Here, the force against the area of wall that must be supported by a strip can be determined by the equation [8]

$$T = \gamma z K_a s h \tag{13-4}$$

where T = tensile force per strip
γ = unit weight of backfill soil

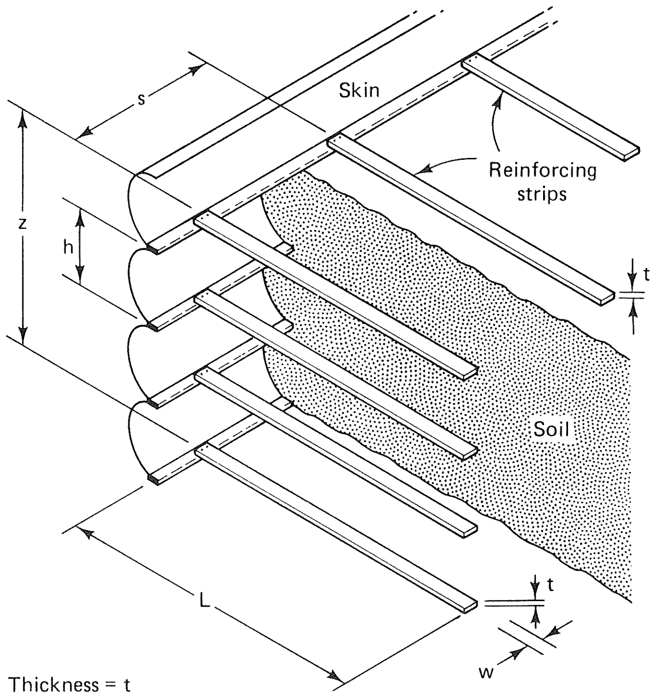

FIGURE 13-14 Component parts and key dimensions of a Reinforced Earth wall. [8]

z = depth from wall top to strip
K_a = coefficient of active earth pressure (Rankine)
s = horizontal spacing between strips
h = vertical spacing between strips

The frictional resistance of a strip at depth z (F) developed between the strip's top and bottom faces and the backfill soil is [8]

$$F = (\gamma z \tan \delta)(2Lw) \tag{13-5}$$

where δ is the angle of friction between the strip's surfaces and the backfill soil and other terms are as previously defined. Tan δ can be taken as $\tan(\phi/2)$, where ϕ is the soil's angle of internal friction.

The required minimum length of strip (L_{min}) can be evaluated by equating T [Eq. (13-4)] and F [Eq. (13-5)] and including an appropriate factor of safety against pullout (F.S., normally 1.5 to 2.0). Hence,

$$(\text{F.S.})(\gamma z K_a sh) = (\gamma z \tan \delta)(2Lw) \tag{13-6}$$

Solving for L_{min} (i.e., L),

$$L_{min} = \frac{(\text{F.S.})(K_a sh)}{2w \tan \delta} \tag{13-7}$$

L_{min} is measured beyond the zone of Rankine failure, as shown in Fig. 13-15a. That is,

$$L_{total} = L_{Rankine} + L_{min} \tag{13-8}$$

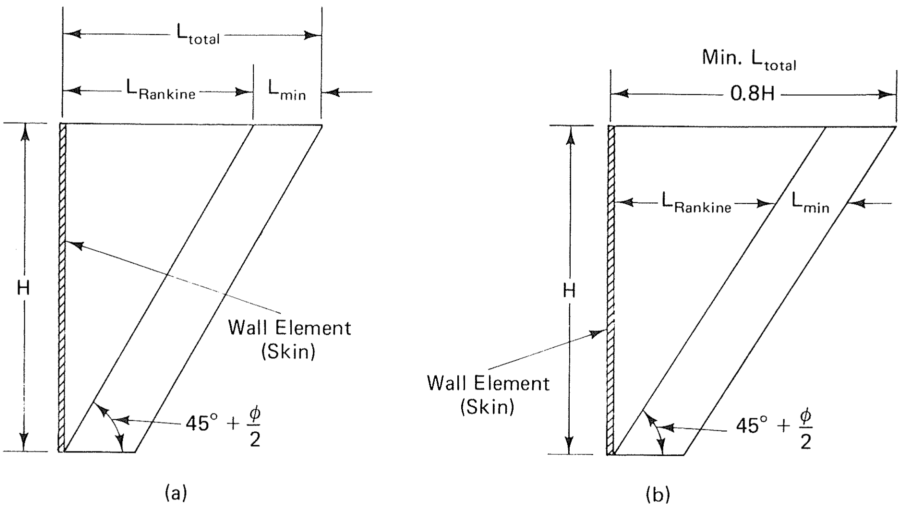

FIGURE 13-15 Minimum length of strip.

where

$$L_{\text{Rankine}} = H \tan\left(45° - \frac{\phi}{2}\right) \tag{13-9}$$

For overall stability, a minimum length (L_{total}) of 80% of wall height, H, is suggested (see Fig. 13-15b). That is,

$$(L_{\text{total}})_{\text{minimum}} = 0.80H \tag{13-10}$$

Strip thickness can be determined from the basic stress equation

$$t = \frac{T}{wf_s} \tag{13-11}$$

where t = strip thickness
T = tensile force (per strip) [from Eq. (13-4)]
w = strip width
f_s = allowable stress for strip material

It may be noted from Eq. (13-4) that the tensile force per strip is greatest at the bottom of the wall (i.e., where z is greatest). At lesser values of z, the tensile force is less, but the friction on each strip is also reduced. Accordingly, total strip area should be constant at all depths to provide the same resistance to strip pullout. Also, from Eq. (13-7) it is clear that the minimum length of strip is independent of depth. For these reasons as well as simplicity in construction, usually the same size, length (see Fig. 13-13), and spacing of strips are used throughout a Reinforced Earth structure.

It should be emphasized that Reinforced Earth structures must use cohesionless soils as backfill material because of their needed high friction.

EXAMPLE 13-3

Given

A 6-m-high Reinforced Earth wall is to be constructed with level backfill (see Fig. 13-14) and will have no surcharge on the backfill. A granular soil with a unit weight of 17.12 kN/m^3 and angle of internal friction of 34° will be used for backfill material. The steel strips' width, vertical spacing, horizontal spacing, and allowable stress are 75 mm, 0.3 m, 1.0 m, and 138,000 kN/m^2, respectively. The factor of safety against pullout is to be 1.5.

Required

1. Total length of strip required.
2. Thickness of strip required.

Solution

1. From Eq. (13-7),

$$L_{\text{min}} = \frac{(\text{F.S.})(K_a s h)}{2w \tan \delta} \tag{13-7}$$

$$\text{F.S.} = 1.5 \qquad \text{(given)}$$

From Eq. (12-14),

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} \tag{12-14}$$

$$K_a = \frac{1 - \sin 34°}{1 + \sin 34°} = 0.283$$

$$s = 1.0 \text{ m} \qquad \text{(given)}$$

$$h = 0.3 \text{ m} \qquad \text{(given)}$$

$$w = 75 \text{ mm} = 0.075 \text{ m} \qquad \text{(given)}$$

$$\delta = \frac{\phi}{2} = \frac{34°}{2} = 17°$$

Therefore, substituting into Eq. (13-7),

$$L_{\text{min}} = \frac{(1.5)(0.283)(1.0 \text{ m})(0.3 \text{ m})}{(2)(0.075 \text{ m})(\tan 17°)} = 2.78 \text{ m}$$

From Eq. (13-9),

$$L_{\text{Rankine}} = H \tan\left(45° - \frac{\phi}{2}\right) \tag{13-9}$$

$$L_{\text{Rankine}} = (6.0 \text{ m}) \tan\left(45° - \frac{34°}{2}\right) = 3.19 \text{ m}$$

From Eq. (13-8),

$$L_{\text{total}} = L_{\text{Rankine}} + L_{\text{min}} \tag{13-8}$$

$$L_{\text{total}} = 3.19 \text{ m} + 2.78 \text{ m} = 5.97 \text{ m}$$

From Eq. (13-10),

$$(L_{\text{total}})_{\text{minimum}} = 0.80H \tag{13-10}$$

$$(L_{\text{total}})_{\text{minimum}} = (0.80)(6.0 \text{ m}) = 4.80 \text{ m}$$

Therefore, use a total length of strip of 5.97 m. (In practice, one would probably specify 6 m.)

2. From Eq. (13-11),

$$t = \frac{T}{wf_s} \tag{13-11}$$

From Eq. (13-4),

$$T = \gamma z K_a s h \tag{13-4}$$

$$\gamma = 17.12 \text{ kN/m}^3 \qquad \text{(given)}$$

$$z = 6.0 \text{ m} \qquad \text{(given)}$$

$$T = (17.12 \text{ kN/m}^3)(6.0 \text{ m})(0.283)(1.0 \text{ m})(0.3 \text{ m})$$

$$= 8.72 \text{ kN}$$

$$f_s = 138{,}000 \text{ kN/m}^2 \qquad \text{(given)}$$

Therefore, substituting into Eq. (13-11),

$$t = \frac{8.72 \text{ kN}}{(0.075 \text{ m})(138{,}000 \text{ kN/m}^2)} = 0.00084 \text{ m, or } 0.84 \text{ mm}$$

13-8 SLURRY TRENCH WALLS

The *slurry trench method* of constructing a wall to retain earth is applicable for retaining walls built entirely below ground level. The procedure includes excavating a trench the width of the wall while simultaneously filling the excavation with a viscous bentonite slurry, which exerts a lateral pressure and thereby helps stabilize the excavated wall. When excavation is complete, reinforcing steel is placed in the bentonite slurry-filled trench; and, with the trench's soil walls acting as forms, concrete is poured from the bottom up. The concrete is delivered by a tremie (a long canvas tube) that is slowly raised as the excavated trench is filled with the concrete. Being displaced by the poured concrete, the lighter bentonite slurry rises to the top where it is removed and may be saved for later use. Adjacent wall sections may be keyed together by a steel beam that becomes a part of the wall.

After the concrete has hardened, soil is removed from one side to expose the face of the wall and allow installation of a tieback system (see Fig. 13-16). Depending on the height of wall, two or more levels of tiebacks may be used.

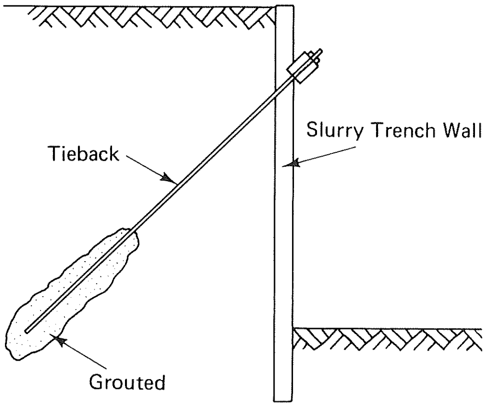

FIGURE 13-16 Tieback for slurry trench wall.

13-9 ANCHORED BULKHEADS

Anchored bulkheads are often used for various waterfront structures (e.g., wharfs and waterfront retaining walls). As illustrated in Fig. 13-17, they consist of interlocking sheet pilings driven into the soil and anchored by steel tie-rods or cables attached near the pilings' tops, extended through the ground, and anchored securely somewhere away from the sheet pile wall in firm soil. The distant anchors may be "deadmen" or braced piles.

In general, bulkheads can be constructed by one of two methods. Either the bulkhead can be built (driven) in open water and fill placed behind it, or the bulkhead can be constructed in the natural ground and earth removed from its face. The former type is known as a *fill bulkhead;* the latter, a *dredged bulkhead.*

As related in Fig. 13-18, there are several possible anchoring systems available. Concrete and sheet pile deadmen (Fig. 13-18a and 13-18b, respec-

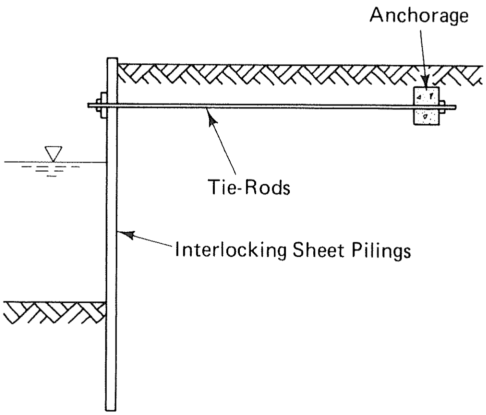

FIGURE 13-17 Anchored bulkhead.

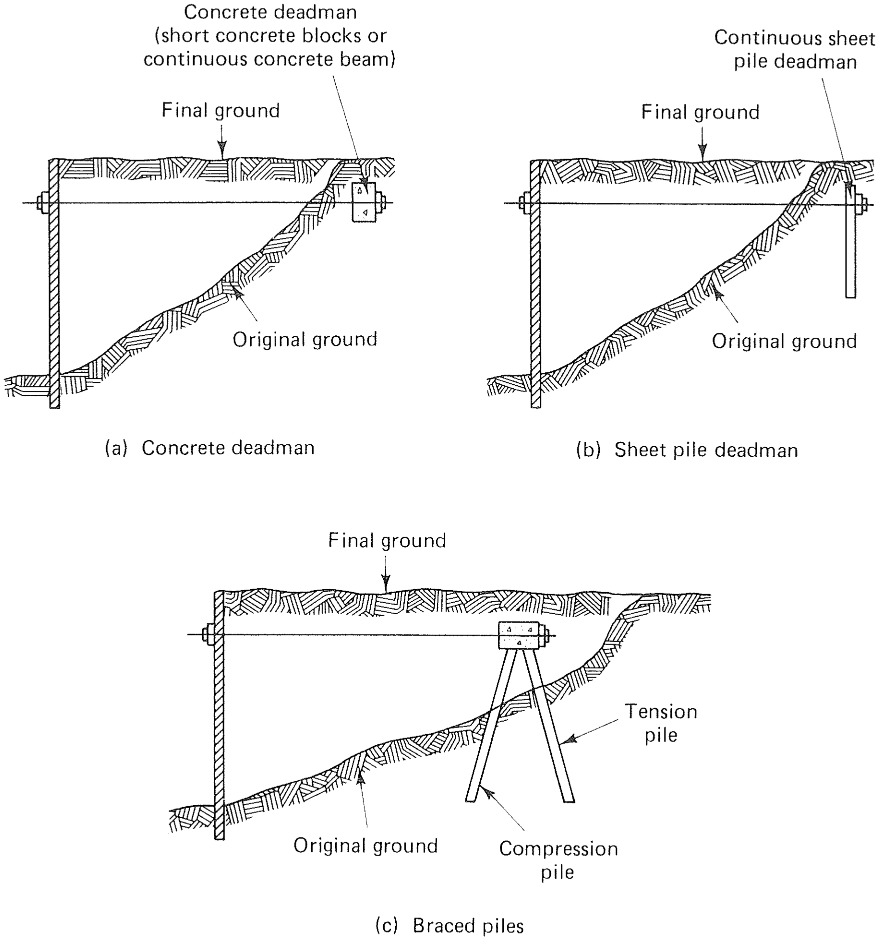

FIGURE 13-18 Alternative anchoring systems for anchored bulkheads: (a) concrete deadman; (b) sheet pile deadman; (c) braced piles.

tively) are usable in strong soils. They are, however, rather bulky and therefore require sufficient space. Where upper soils are weak or space is severely limited, braced piles (Fig. 13-18c) may be used as anchors.

The fill behind a bulkhead applies lateral pressure to it and tends to push it forward. It is restrained at its top by the anchors and at its bottom by the soil in front of the bulkhead. It is also restrained by the standing water, but this tends to be offset by groundwater behind the bulkhead. Anchored bulkheads are usually subjected to fluctuating water levels; hence, the engineer must base the design on "worst-case" conditions. (For example, tidal fluctuations produce high pressures behind the wall compared to those in front when the tide is out.)

Figure 13-19 shows the various forces acting on a typical anchored bulkhead. Actuating forces result from active pressure of the soil backfill and from water behind the wall. Resisting forces are comprised of tension in the cable

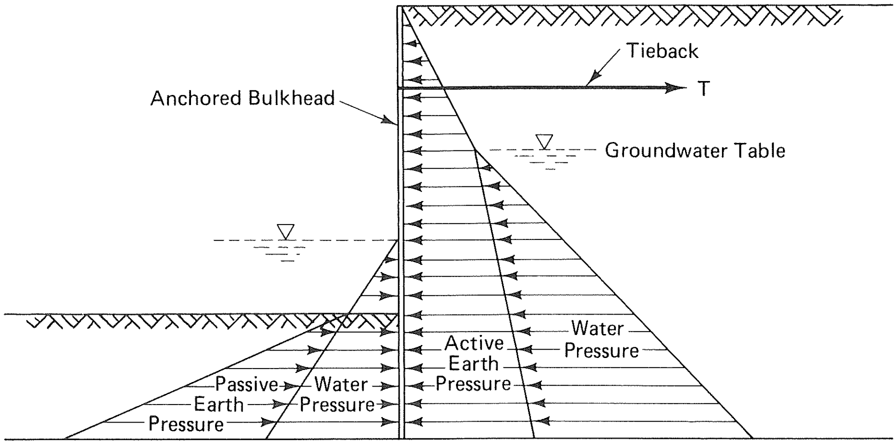

FIGURE 13-19 Forces acting on anchored bulkheads.

or tie-rod to the anchor, water pressure on the wall's front side, and passive resistance pressure of the soil within the penetrating depth of the sheet piling. The ratio of total resisting force to total actuating force gives the bulkhead's factor of safety. The factor of safety can be increased by driving the sheet piling deeper into the soil.

EXAMPLE 13-4

Given

An anchored bulkhead is to be constructed as shown in Fig. 13-20, using a factor of safety of 1.5.

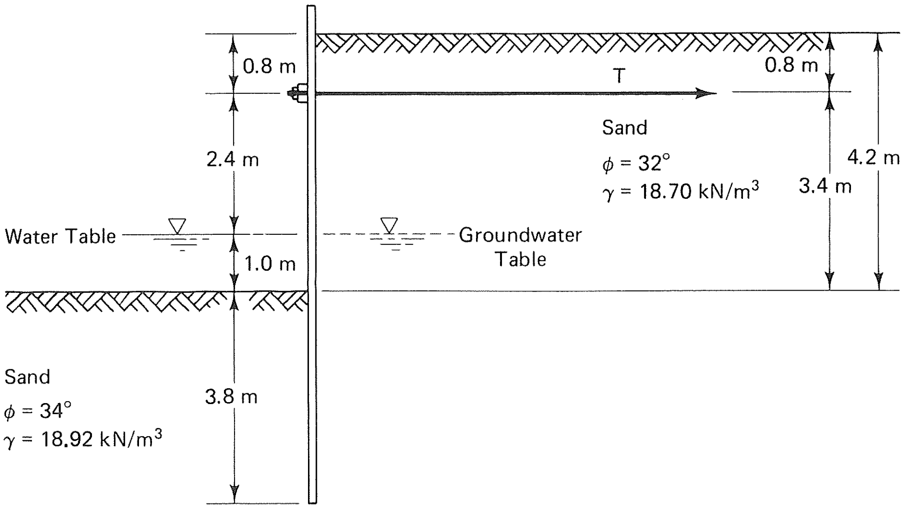

FIGURE 13-20

Required

Analyze the bulkhead system and determine the tension in the anchor rod (tie-back).

Solution

From Eqs. (12-10) and (12-12),

$$P_a = \tfrac{1}{2}\gamma H^2 K_a \tag{12-10}$$

$$P_p = \tfrac{1}{2}\gamma H^2 K_p \tag{12-12}$$

From Eq. (12-14),

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} \tag{12-14}$$

$$K_a = \frac{1 - \sin 32^\circ}{1 + \sin 32^\circ} = 0.307$$

From Eq. (12-15),

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi} \tag{12-15}$$

$$K_p = \frac{1 + \sin 34^\circ}{1 - \sin 34^\circ} = 3.54$$

Referring to Fig. 13-21, applicable pressure and moments may be determined as follows:

Component	*Active Earth or Water Pressure (kN/m)*		*Moment about Tie Point (kN·m/m)*	
1	$(\tfrac{1}{2})(18.7)(3.2)^2(0.307)$	= 29.39	$(29.39)[(3.2)(\tfrac{2}{3}) - 0.8]$ =	39.2
2	$(3.2)(18.7)(4.8)(0.307)$	= 88.18	$(88.18)[(4.8)(\tfrac{1}{2}) + 2.4]$ =	423.3
3	$(\tfrac{1}{2})(18.7 - 9.81)(4.8)^2(0.307)$	= 31.44	$(31.44)[(4.8)(\tfrac{2}{3}) + 2.4]$ =	176.1
4	$(\tfrac{1}{2})(9.81)(4.8)^2$	= 113.01	$(113.01)[(4.8)(\tfrac{2}{3}) + 2.4]$ =	632.9
		262.02		1271.5

To simplify the analysis that follows, let

R_{wp} = resistance from water pressure (left of sheet piling)(i.e., P_5)

$(P_6)_{min}$ = minimum required passive earth pressure in component 6

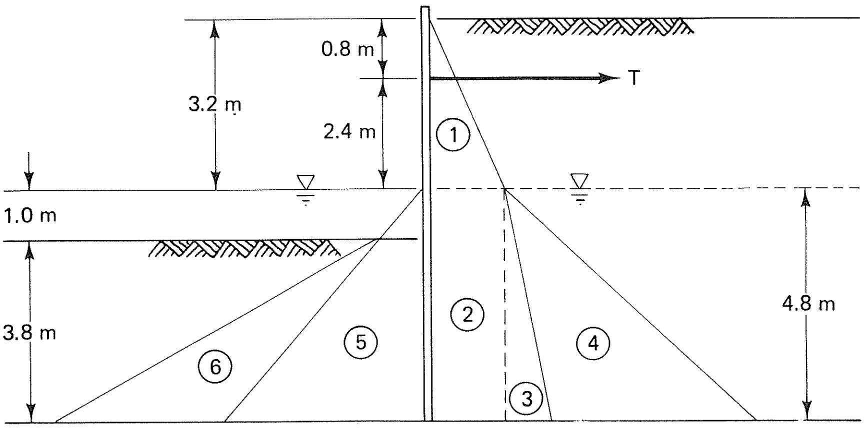

FIGURE 13-21

d_{P6} = distance from tie point to P_6

d_{Rwp} = distance from tie point to R_{wp} (i.e., d_{P5})

$(P_6)_{mm}$ = maximum mobilizable P_6

$$\sum \text{Moments}_{\text{tie point}} = 0 \; (+ \curvearrowleft)$$

$$[(P_6)_{\min}][d_{P6}] + [R_{wp}][d_{Rwp}] - \text{sum of moments of } P_1, P_2, P_3, \text{ and } P_4 = 0 \quad \text{(A)}$$

$$d_{P6} = (3.8 \text{ m})(\tfrac{2}{3}) + 1.0 \text{ m} + 2.4 \text{ m} = 5.933 \text{ m}$$

$$R_{wp} = \text{force of component 5} = \frac{\tfrac{1}{2}\gamma H^2}{\text{F.S.}}$$

$$R_{wp} = \frac{(\tfrac{1}{2})(9.81 \text{ kN/m}^3)(4.8 \text{ m})^2}{1.5} = 75.34 \text{ kN/m}$$

$$d_{Rwp} = (4.8 \text{ m})(\tfrac{2}{3}) + 2.4 \text{ m} = 5.600 \text{ m}$$

Sum of moments of P_1, P_2, P_3, and P_4 = 1271.5 kN·m/m (from tabulation above). Substituting into Eq. (A) gives

$$[(P_6)_{\min}][5.933 \text{ m}] + [75.34 \text{ kN/m}][5.600 \text{ m}] - 1271.5 \text{ kN·m/m} = 0$$

$$(P_6)_{\min} = 143.20 \text{ kN/m}$$

$$(P_6)_{mm} = \frac{\tfrac{1}{2}(\gamma_{soil} - \gamma_{\text{water}})h^2 K_p}{\text{F.S.}}$$

$$(P_6)_{mm} = \frac{(\tfrac{1}{2})(18.92 \text{ kN/m}^3 - 9.81 \text{ kN/m}^3)(3.8 \text{ m})^2(3.54)}{1.5} = 155.2 \text{ kN/m}$$

Since $[(P_6)_{mm} = 155.2 \text{ kN/m}] > [(P_6)_{\min} = 143.20 \text{ kN/m}]$, the design is O.K., as the sheet pile penetration depth is adequate to develop sufficient passive resistance.

To determine the tension (T) in the anchor rod,

$$\sum \text{Forces}_{\text{horizontal}} = 0$$

$$T + R_{wp} + (P_6)_{\min} - (P_1 + P_2 + P_3 + P_4) = 0$$

$$P_1 + P_2 + P_3 + P_4 = 262.02 \text{ kN/m} \qquad \text{(from tabulation above)}$$

$$T + 75.34 \text{ kN/m} + 143.20 \text{ kN/m} - 262.02 \text{ kN/m} = 0$$

$$T = 43.48 \text{ kN/m}$$

EXAMPLE 13-5

Given

A continuous deadman is to be designed and installed near the ground surface, as shown in Fig. 13-22. Anchor rod (tie-back) tension is to be 75 kN/m.

Required

Design the deadman, using a factor of safety against anchor resistance failure of 1.5.

Solution

$$\text{capacity of deadman} = \text{tie-back tension} = \frac{\frac{1}{2}\gamma H^2(K_p - K_a)}{\text{F.S.}}$$

From Eq. (12-14),

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} \qquad (12\text{-}14)$$

$$K_a = \frac{1 - \sin 32^\circ}{1 + \sin 32^\circ} = 0.307$$

From Eq. (12-15),

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi} \qquad (12\text{-}15)$$

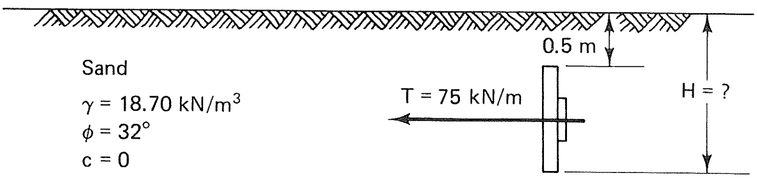

FIGURE 13-22

$$K_p = \frac{1 + \sin 32°}{1 - \sin 32°} = 3.255$$

$$75 \text{ kN/m} = \frac{(½)(18.70 \text{ kN/m}^3)(H)^2(3.255 - 0.307)}{1.5}$$

$$H = 2.02 \text{ m}$$

The deadman should be placed with its bottom 2.02 m below the ground surface.

13-10 PROBLEMS

13-1 A proposed L-shaped reinforced concrete retaining wall is shown in Fig. 13-23. The backfill material will be Type 2 soil (Fig. 13-3) and its unit weight is 125 lb/ft^3. The coefficient of base friction is estimated to be 0.48, and allowable soil pressure for the foundation soil is 4 kips/ft^2. Determine the (a) factor of safety against overturning, (b) factor of safety against sliding, and (c) safety against failure of the foundation.

13-2 Investigate the stability against overturning, sliding resistance (consider passive earth pressure at the toe), and foundation soil pressure of the retaining wall shown in Fig. 13-24. The retaining wall is to support a deposit of granular soil, which has a unit weight of 17.30 kN/m^3 and an angle of internal friction of 32°. The coefficient of base friction is 0.50. Allowable soil

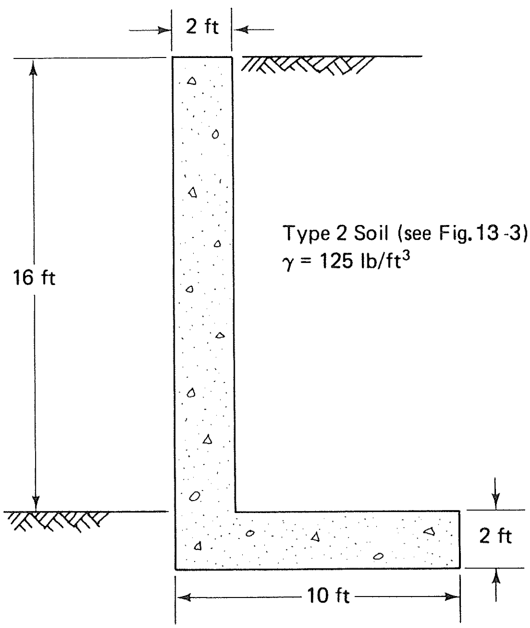

FIGURE 13-23

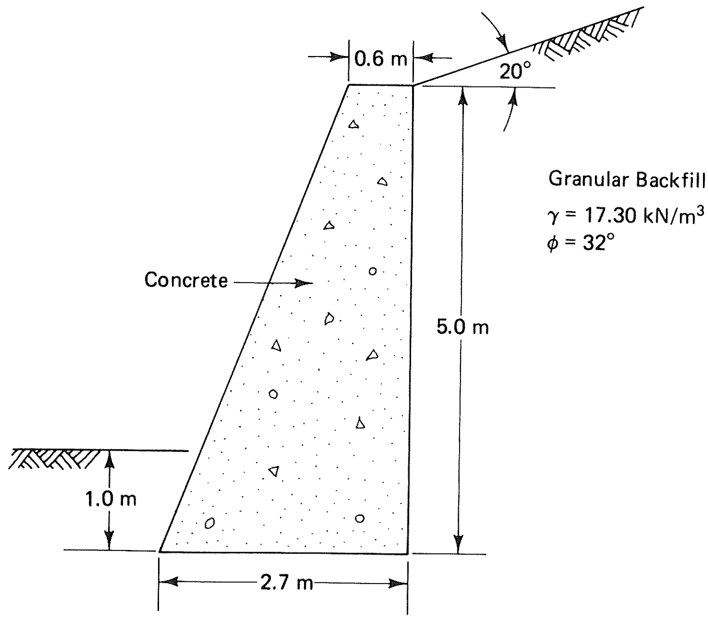

FIGURE 13-24

pressure for the foundation soil is 144 kN/m². Use Rankine theory to calculate both active and passive earth pressures.

13-3 For the retaining wall shown in Fig. 13-25, compute the factors of safety against overturning and sliding (analyze the latter both without and with passive earth pressure at the toe). Also determine the soil pressure at the base of the wall. Use the Rankine equation to compute passive earth pressure.

13-4 An 8-m-high Reinforced Earth wall is to be built with level backfill and without surcharge on the backfill. Sand with an angle of internal friction of 36° and unit weight of 17.0 kN/m³ will be used as backfill material. The steel strips are 90 mm wide and 0.762 mm thick and have an allowable stress of 138,000 kN/m². For vertical spacing of 0.4 m, determine required total length and horizontal spacing of the steel strips.

13-5 Analyze the anchored bulkhead system shown in Fig. 13-26 using a factor of safety of 1.5. Determine the anchor rod tension per unit length of sheet piling.

13-6 A continuous deadman is to be designed and constructed near ground surface, as shown in Fig. 13-27. Anchor rod tension is to be 79 kN/m. Using a factor of safety of 1.5, determine how far the bottom of the deadman should be placed below the ground surface.

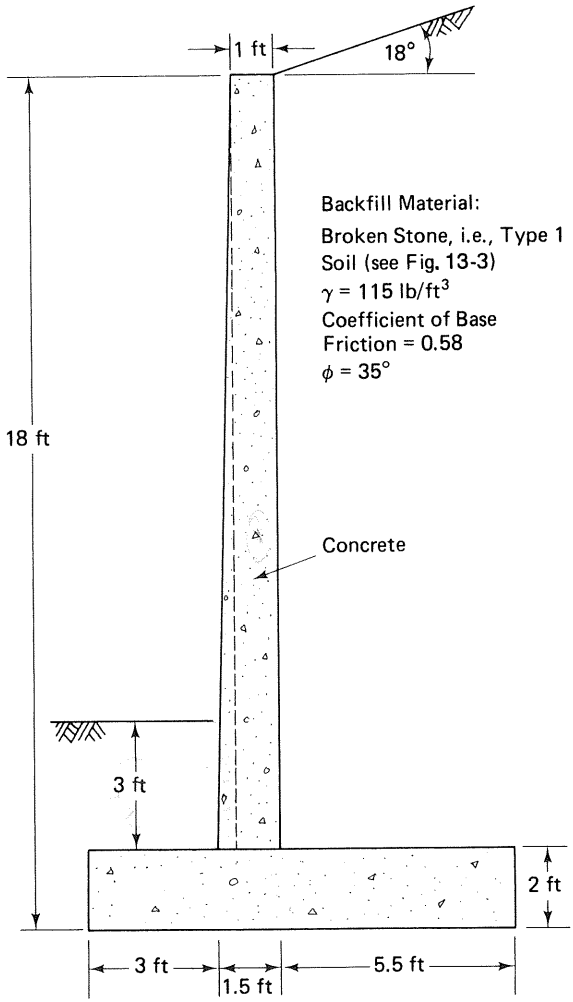

FIGURE 13-25

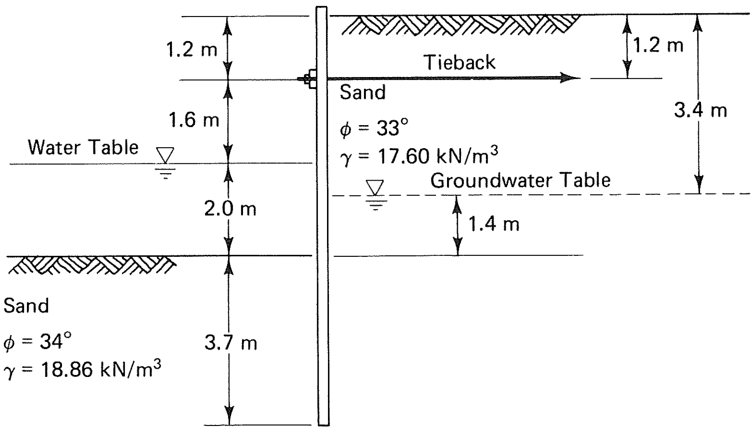

FIGURE 13-26

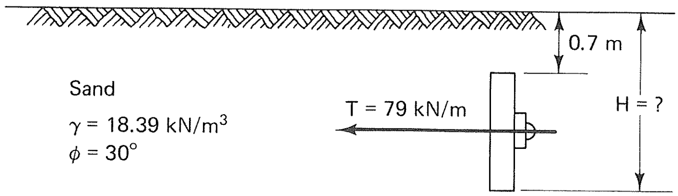

FIGURE 13-27

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14

Stability Analysis of Slopes

14-1 INTRODUCTION

Whenever a mass of soil has an inclined surface, the potential always exists for part of the soil mass to slide from a higher location to a lower one. Sliding will occur if shear stresses developed in the soil exceed the corresponding shear strength of the soil. This phenomenon is of importance in the case of highway cuts and fills, embankments, earth dams, and so on.

The principle stated above—that sliding will occur if shear stresses developed in the soil exceed the corresponding shear strength the soil possesses—is simple in theory; but certain practical considerations make precise stability analyses of slopes difficult in practice. In the first place, sliding may occur along any of a number of possible surfaces. In the second place, a given soil's shear strength generally varies throughout time, as soil moisture and other factors change. Obviously, stability analysis should be based on the smallest shear strength a soil will ever have in the future. This is difficult, if not impossible, to ascertain. It is, therefore, normal in practice to use appropriate safety factors when making slope stability analyses.

There are several techniques available for stability analysis. Section 14-2 covers the analysis of a soil mass resting on an inclined layer of impermeable soil. Section 14-3 discusses slopes in homogeneous cohesionless soils. Section 14-4 gives two methods of analyzing stability for homogeneous soils that have cohesion. The first is known as the *Culmann method.* It is only applicable to vertical, or nearly vertical, slopes. The second might be called the *stability number method.* Section 14-5 presents the *method of slices.*

14-2 ANALYSIS OF MASS RESTING ON INCLINED LAYER OF IMPERMEABLE SOIL

One situation for which slope stability analysis is fairly simple is that of a soil mass resting on an inclined layer of impermeable soil (see Fig. 14-1). There exists a tendency for the upper mass to slide downward along its plane of contact with the lower layer of impermeable soil.

The force tending to cause sliding is the component of the upper mass's weight along the plane of contact. By referring to Fig. 14-2 and considering a unit width of slope (i.e., perpendicular to wedge abc), the upper mass's weight (W) (i.e., weight of wedge abc) can be computed by

$$W = \frac{Lh\gamma}{2} \tag{14-1}$$

where γ is the unit weight of the upper mass. Hence, the force tending to cause sliding (F_s) is given by

$$F_s = W \sin \alpha \tag{14-2}$$

Forces that resist sliding result from cohesion and friction. In quantitative terms, the cohesion (i.e., adhesion) component is the product of the soil's cohesion (c) times the length of the plane of contact (L in Fig. 14-2). The friction component is obtained by multiplying the coefficient of friction between the two strata ($\tan \phi$) by the component of the upper mass's weight that is perpendicular to the plane of contact ($W \cos \alpha$). Hence, the resistance (to sliding) force, R_s, is given by

$$R_s = cL + W \cos \alpha \tan \phi \tag{14-3}$$

where ϕ is the angle of friction between the upper mass and the lower layer of impermeable soil.

The factor of safety against sliding is determined by dividing the resist-

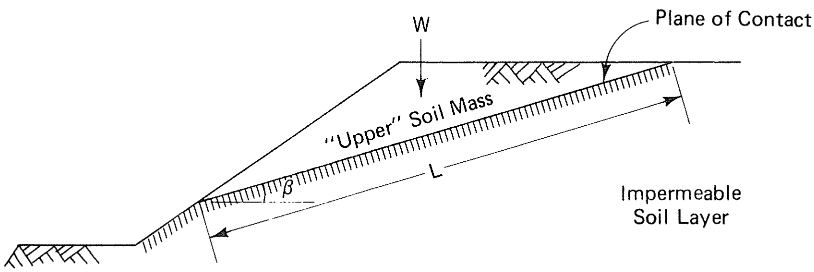

FIGURE 14-1

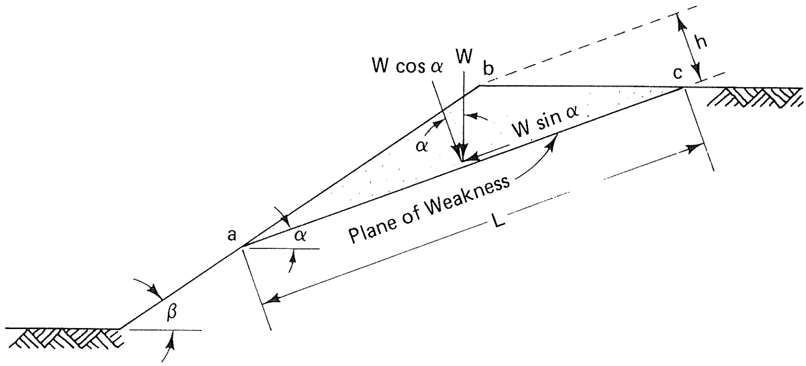

FIGURE 14-2

ance (to sliding) force, R_s [Eq. (14-3)], by the sliding force, F_s [Eq. (14-2)]. Hence,

$$\text{F.S.} = \frac{cL + W\cos\alpha\tan\phi}{W\sin\alpha} \tag{14-4}$$

Figure 14-3 gives formulation required to evaluate L and h, which are needed in applying Eqs. (14-1) and (14-4). Table 14-1 gives the significance of factors of safety against sliding for design. Example 14-1 illustrates the computation of the factor of safety for stability analysis of a soil mass resting on an inclined layer of impermeable soil.

EXAMPLE 14-1

Given

1. Figure 14-4 shows a 15-ft cut through two soil strata. The lower is a highly impermeable cohesive soil.

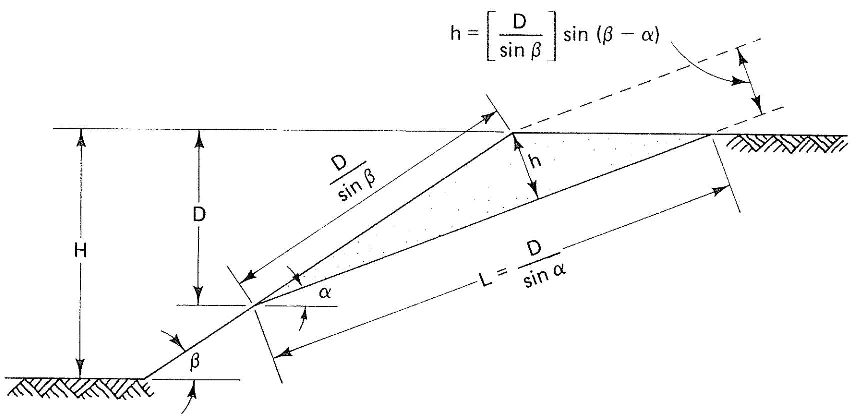

FIGURE 14-3

TABLE 14-1 Significance of factors of safety for design [1].[1]

Safety Factor	*Significance*
Less than 1.0	Unsafe
1.0–1.2	Questionable safety
1.3–1.4	Satisfactory for cuts, fills; questionable for dams
1.5–1.75	Safe for dams

[1]Reprinted with permission of Macmillan Publishing Co., Inc., from *Introductory Soil Mechanics and Foundations*, 4th Edition, by George F. Sowers. Copyright © 1979, Macmillan Publishing Co., Inc.

2. Shearing strength data between the two strata are as follows:

$$\text{Cohesion} = 150 \text{ lb/ft}^2$$

$$\text{Angle of friction} = 25°$$

$$\text{Unit weight of the upper layer} = 105 \text{ lb/ft}^3$$

3. Neglect effects of soil water between the two strata.

Required

Factor of safety against sliding.

Solution

From Fig. 14-3,

$$L = \frac{D}{\sin \alpha}$$

$$L = \frac{10 \text{ ft}}{\sin 30°} = 20.0 \text{ ft}$$

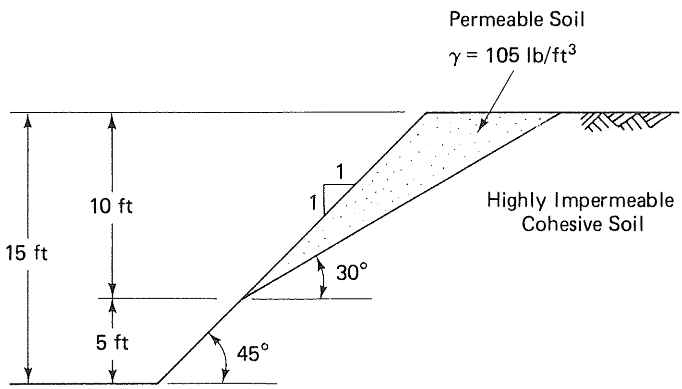

FIGURE 14-4

Again, from Fig. 14-3,

$$h = \frac{D}{\sin \beta} \sin (\beta - \alpha)$$

$$h = \frac{10 \text{ ft}}{\sin 45^\circ} \sin (45^\circ - 30^\circ) = 3.66 \text{ ft}$$

From Eq. (14-1),

$$W = \frac{Lh\gamma}{2} \tag{14-1}$$

$$W = \frac{(20.0 \text{ ft})(3.66 \text{ ft})(105 \text{ lb/ft}^3)}{2} = 3843 \text{ lb/ft}$$

From Eq. (14-4),

$$\text{F.S.} = \frac{cL + W \cos \alpha \tan \phi}{W \sin \alpha} \tag{14-4}$$

$$\text{F.S.} = \frac{(150 \text{ lb/ft}^2)(20.0 \text{ ft}) + (3843 \text{ lb/ft})(\cos 30^\circ)(\tan 25^\circ)}{(3843 \text{ lb/ft})(\sin 30^\circ)}$$

$$= 2.37 > 1.5 \qquad \text{O.K.}$$

14-3 SLOPES IN HOMOGENEOUS COHESIONLESS SOILS ($c = 0, \phi > 0$)

When the slope angle (β) of a sand slope exceeds the sand's angle of internal friction (ϕ), the sand slope tends to fail by sliding in a downhill direction parallel to the slope. This phenomenon can be inferred by visualizing individual grains of sand being blocks resting on an inclined plane at the slope angle. If the slope angle is increased, the sand grains will begin to slide down the slope when the slope angle exceeds the sand's ϕ angle. Accordingly, the greatest slope for a free-standing cohesionless soil is at an angle approximately equal to the soil's ϕ angle.

The slope angle at which a loose sand fails may be estimated by its *angle of repose,* the angle formed (with the horizontal) by sand as it forms a pile below a funnel through which it passes. A sand's angle of repose is roughly equal to its angle of internal friction in a loose condition, and sand at or near ground surface is ordinarily in a loose condition and therefore near its maximum value of ϕ.

The factor of safety for slopes in homogeneous cohesionless soils is given by

$$\text{F.S.} = \frac{\tan \phi}{\tan \beta} \tag{14-5}$$

Clearly, when slope angle β equals angle of internal friction ϕ, the factor of safety is 1. For slopes with β less than ϕ, the factor of safety is greater than 1.

14-4 SLOPES IN HOMOGENEOUS SOILS POSSESSING COHESION ($c > 0$, $\phi = 0$ and $c > 0$, $\phi > 0$)

Two methods are presented in this section for analyzing slope stability in homogeneous soils possessing cohesion. One is known as the *Culmann method;* the other might be called the *stability number method.*

Culmann Method

In the Culmann method, the assumption is made that failure (sliding) will occur along a plane that passes through the toe of the fill [2]. Such a plane is indicated in Fig. 14-5. As with the analysis of a mass resting on an inclined layer of impermeable soil (Sec. 14-2), the force tending to cause sliding is given by

$$F_s = W \sin \alpha \tag{14-2}$$

Also similarly, resistance to sliding results from cohesion and friction and is given by

$$R_s = c_d L + W \cos \alpha \tan \phi_d \tag{14-3}$$

where c_d is the developed cohesion (c/F.S.$_c$), $\tan \phi_d$ is the developed coefficient of friction ($\tan \phi$/F.S.$_\phi$), and the other terms are as defined in Fig. 14-5. (F.S.$_c$

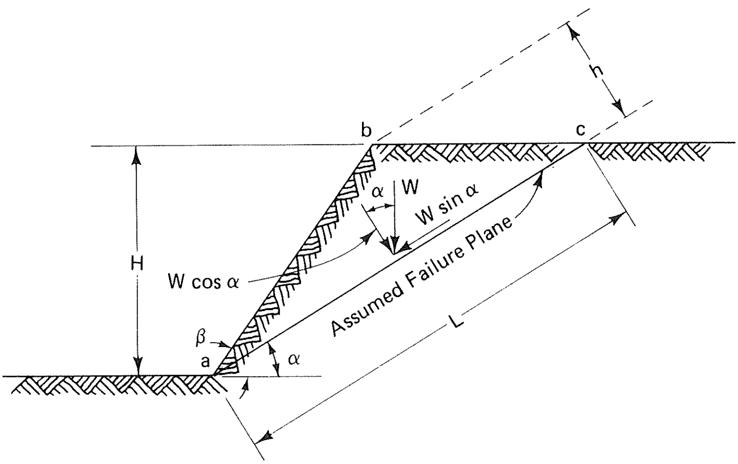

FIGURE 14-5

and $F.S._\phi$ denote factors of safety for cohesion and angle of internal friction, respectively.) As in Sec. 14-2, the weight of soil in the upper triangle *abc* (W) can be computed by

$$W = \frac{Lh\gamma}{2} \tag{14-1}$$

but h, the height of the triangle, can be evaluated by

$$h = \left[\frac{H}{\sin\beta}\right]\sin(\beta - \alpha) \tag{14-6}$$

Substituting Eq. (14-6) into Eq. (14-1) gives

$$W = \left(\frac{1}{2}\right) L \left[\frac{H}{\sin\beta}\right]\sin(\beta - \alpha)(\gamma) \tag{14-7}$$

Equating Eqs. (14-2) and (14-3) and substituting W from Eq. (14-7) into the new equation and then solving for c_d gives

$$c_d = \left[\frac{\gamma H}{2\sin\beta}\right]\left[\frac{\sin(\beta - \alpha)\sin(\alpha - \phi_d)}{\cos\phi_d}\right] \tag{14-8}$$

The critical angle for α (i.e., α_c) can be determined by equating the first derivative of c_d with respect to α to zero [i.e., $d(c_d)/d(\alpha) = 0$] and solving for α. The result of this operation is

$$\alpha_c = \frac{\beta + \phi_d}{2} \tag{14-9}$$

Substituting α_c from Eq. (14-9) into Eq. (14-8) for α gives

$$c_d = \frac{\gamma H[1 - \cos(\beta - \phi_d)]}{4\sin\beta\cos\phi_d} \tag{14-10}$$

Solving for H gives [3]

$$H = \frac{4c_d\sin\beta\cos\phi_d}{\gamma[1 - \cos(\beta - \phi_d)]} \tag{14-11}$$

where H = safe depth of cut
c_d = developed cohesion
β = angle from horizontal to cut surface (see Fig. 14-5)
ϕ_d = developed angle of internal friction of the soil
γ = unit weight of the soil

In using Eq. (14-11) to compute safe depth of cut, developed cohesion (c_d) and developed angle of internal friction (ϕ_d) are determined by dividing cohesion and tangent of the angle of internal friction by respective safety factors.

The Culmann method gives reasonably accurate results if the slope is vertical, or nearly vertical (i.e., angle β equal to, or nearly equal to, 90°) [2]. Examples 14-2 and 14-3 illustrate the Culmann method.

EXAMPLE 14-2

Given

1. A vertical cut is to be made through a soil mass.
2. The soil to be cut has the following properties:

$$\text{Unit weight } (\gamma) = 105 \text{ lb/ft}^3$$

$$\text{Cohesion } (c) = 500 \text{ lb/ft}^2$$

$$\text{Angle of internal friction } (\phi) = 21^\circ$$

Required

Safe depth of cut in this soil by the Culmann method, using a factor of safety of 2.

Solution

From Eq. (14-11),

$$H = \frac{4c_d \sin\beta \cos\phi_d}{\gamma[1 - \cos(\beta - \phi_d)]} \tag{14-11}$$

Here,

$$c_d = \frac{c}{\text{F.S.}_c} = \frac{500 \text{ lb/ft}^2}{2} = 250 \text{ lb/ft}^2$$

(F.S._c is the factor of safety with respect to cohesion)

$$\tan\phi_d = \frac{\tan\phi}{\text{F.S.}_\phi} = \frac{\tan 21^\circ}{2} = 0.192$$

(F.S._ϕ is the factor of safety with respect to $\tan\phi$)

$$\phi_d = \arctan 0.192 = 10.87^\circ$$

$$\beta = 90^\circ \qquad \text{(vertical cut)}$$

$$H = \frac{(4)(250 \text{ lb/ft}^2)\sin 90^\circ \cos 10.87^\circ}{(105 \text{ lb/ft}^3)[1 - \cos(90^\circ - 10.87^\circ)]} = 11.5 \text{ ft}$$

EXAMPLE 14-3

Given

A 1.8-m-deep vertical-wall trench is to be dug in soil without shoring. The soil's unit weight, angle of internal friction, and cohesion are 19.0 kN/m^3, 28°, and 20.2 kN/m^2, respectively.

Required

Factor of safety of this trench using Culmann's method.

Solution

From Eq. (14-11),

$$H = \frac{4c_d \sin \beta \cos \phi_d}{\gamma[1 - \cos(\beta - \phi_d)]} \tag{14-11}$$

Try $\mathbf{F.S._\phi = 1.0}$

$$\tan \phi_d = \frac{\tan \phi}{F.S._\phi} = \frac{\tan 28°}{1.0} = \tan 28°$$

Therefore, $\phi_d = 28°$

$$\beta = 90° \qquad \text{(for a vertical wall)}$$

Substituting into Eq. (14-11),

$$1.8 \text{ m} = \frac{(4)(c_d) \sin 90° \cos 28°}{(19.0 \text{ kN/m}^3)[1 - \cos(90° - 28°)]}$$

$$c_d = 5.14 \text{ kN/m}^2$$

$$F.S._c = \frac{c}{c_d} = \frac{20.2 \text{ kN/m}^2}{5.14 \text{ kN/m}^2} = 3.93$$

Since $[F.S._c = 3.93] \neq [F.S._\phi = 1.0]$, another trial factor of safety must be attempted.

Try $\mathbf{F.S._\phi = 2.0}$

$$\tan \phi_d = \frac{\tan \phi}{F.S._\phi} = \frac{\tan 28°}{2.0} = 0.2659$$

Therefore, $\phi_d = 14.89°$

$$1.8 \text{ m} = \frac{(4)(c_d) \sin 90° \cos 14.89°}{(19.0 \text{ kN/m}^3)[1 - \cos(90° - 14.89°)]}$$

$$c_d = 6.57 \text{ kN/m}^2$$

$$\text{F.S.}_c = \frac{c}{c_d} = \frac{20.2 \text{ kN/m}^2}{6.57 \text{ kN/m}^2} = 3.07$$

Since $[\text{F.S.}_c = 3.07] \neq [\text{F.S.}_\phi = 2.0]$, another trial factor of safety must be attempted.

Try F.S.$_\phi$ = 3.0

$$\tan \phi_d = \frac{\tan \phi}{\text{F.S.}_\phi} = \frac{\tan 28°}{3.0} = 0.1772$$

Therefore, $\phi_d = 10.05°$

$$1.8 \text{ m} = \frac{(4)(c_d) \sin 90° \cos 10.05°}{(19.0 \text{ kN/m}^3)[1 - \cos (90° - 10.05°)]}$$

$$c_d = 7.17 \text{ kN/m}^2$$

$$\text{F.S.}_c = \frac{c}{c_d} = \frac{20.2 \text{ kN/m}^2}{7.17 \text{ kN/m}^2} = 2.82$$

Since $[\text{F.S.}_c = 2.82] \neq [\text{F.S.}_\phi = 3.0]$, the correct factor of safety has not yet been found. Rather than continue this trial-and-error procedure, the values of F.S.$_c$ and F.S.$_\phi$ are plotted in Fig. 14-6, from which the applicable factor of safety of about 2.84 can be read.

Stability Number Method

The stability number method is also based on the premise that resistance of a soil mass to sliding results from cohesion and internal friction of the soil along the failure surface. Unlike the Culmann method, the failure surface is assumed to be a circular arc (see Fig. 14-7). A parameter called the "stability number" was introduced, which groups factors affecting stability of soil slopes. The stability number (N_s) is defined as [4]

$$N_s = \frac{\gamma H}{c} \tag{14-12}$$

where γ = unit weight of soil
H = height of cut (see Fig. 14-7)
c = cohesion of soil

For the embankment illustrated in Fig. 14-7, three types of failure surface are possible. These are shown in Fig. 14-8. For the toe circle (Fig. 14-8a), the failure surface passes through the toe. In the case of the slope circle (Fig. 14-8b), the failure surface intersects the slope above the toe. For the midpoint

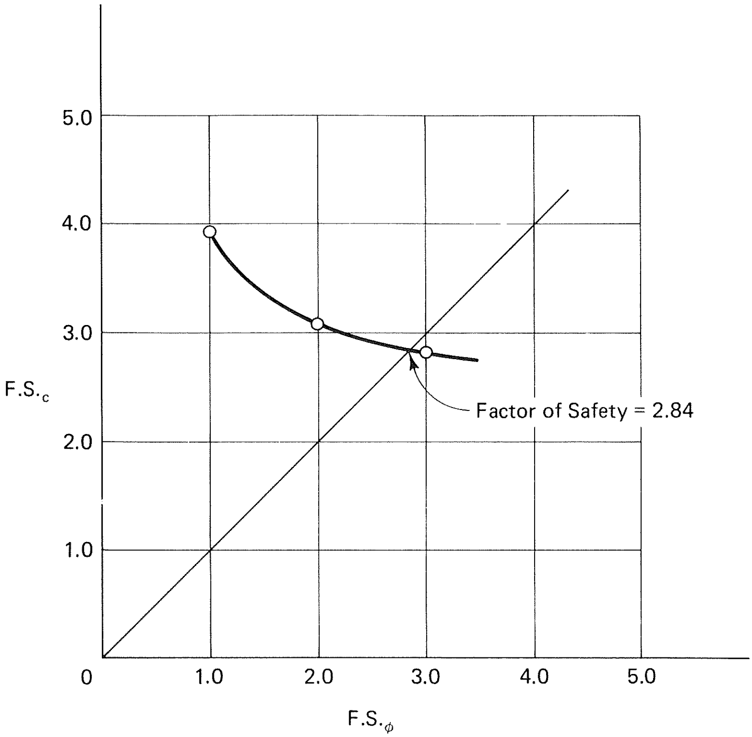

FIGURE 14-6

circle (Fig. 14-8c), the center of the failure surface is on a vertical line passing through the midpoint of the slope.

Both the type of failure surface and the stability number can be determined for a specific case based on given values of ϕ (angle of internal friction) and β (slope angle, Fig. 14-7). If the value of ϕ is zero, or nearly zero, Fig. 14-9 may be used to determine both the type of failure surface and the stability number. One enters along the abscissa at the value of β and moves upward to the line that indicates the appropriate value of n_d. (n_d is a depth factor related

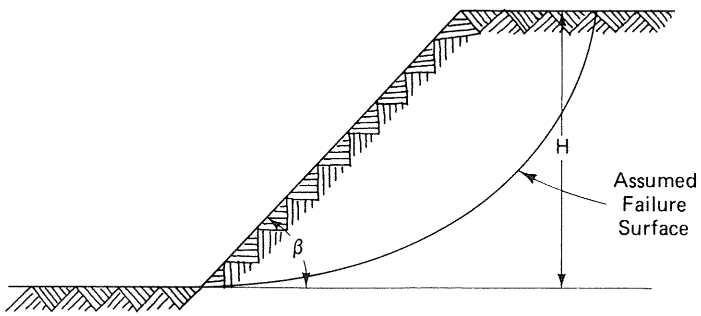

FIGURE 14-7

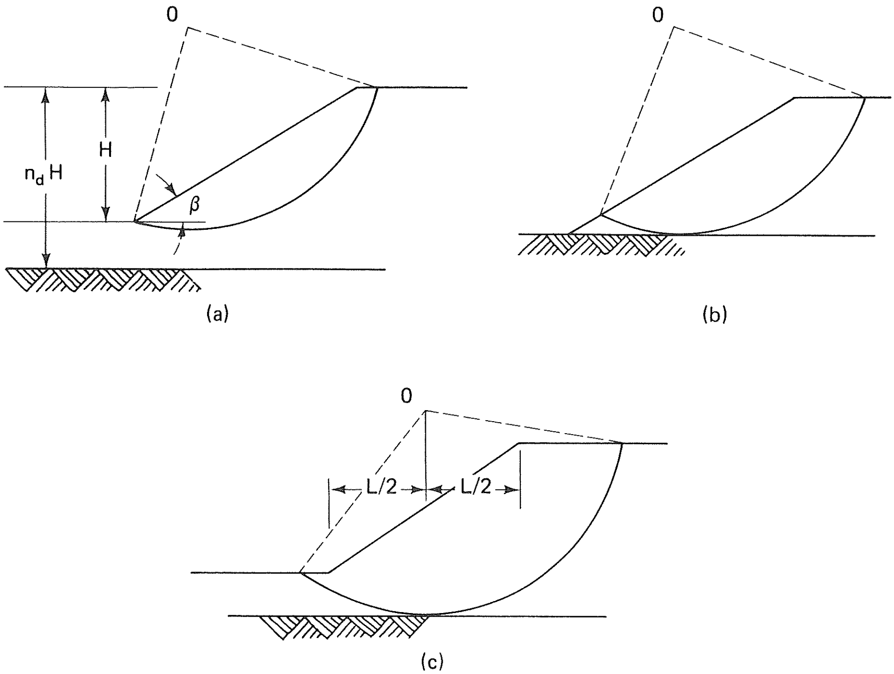

FIGURE 14-8 Types of failure surface: (a) toe circle; (b) slope circle; (c) midpoint circle. [4]*

to the distance to the underlying layer of stiff material or bedrock and is determined from the relationship indicated in Fig. 14-8a). The type of line for n_d indicates the type of failure surface, and the value of stability number is determined by moving leftward and reading from the ordinate. Observation of Fig. 14-9 indicates that if β is greater than 53°, the failure surface is always a toe circle; and if n_d is greater than 4, the failure surface is always a midpoint circle [4].

If the value of ϕ is greater than 3°, the failure surface is always a toe circle [4]. Figure 14-10 may be used to determine the stability number for different values of ϕ [5]. One enters along the abscissa at the value of β, moves upward to the line that indicates the ϕ angle, and then leftward to the ordinate where the stability number is read.

The factor of safety for highly cohesive soils (that have $\phi = 0$) can be obtained from Fig. 14-9. This is illustrated in Example 14-5. For soils possessing cohesion and having $\phi > 0$, the procedure is more complicated. One procedure is to estimate $F.S._\phi$ and determine $\phi_{required}$. Using this value and slope angle β, the stability number can be found from Fig. 14-10. With this stability number, $c_{required}$ can be computed from Eq. (14-12). $F.S._c$ is the quotient of c_{given} divided by $c_{required}$. If $F.S._\phi$ equals $F.S._c$, the overall factor of safety is equal to $F.S._\phi$ (or $F.S._c$). If $F.S._\phi$ and $F.S._c$ are not equal, additional values of $F.S._\phi$ can be

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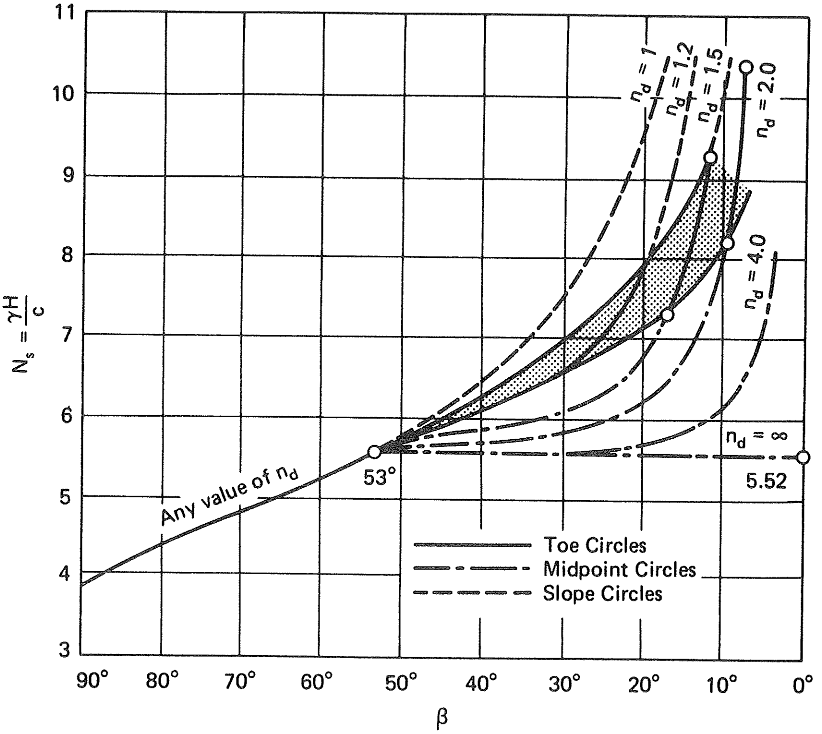

FIGURE 14-9 Stability numbers and types of slope failures for $\phi = 0$. [5, 6]

estimated and the above procedure repeated to determine corresponding values of F.S.$_c$ until the factor of safety is found where F.S.$_\phi$ equals F.S.$_c$. If the right factor of safety has not been found after several such trials, it may be expedient to plot corresponding values of F.S.$_\phi$ and F.S.$_c$ on a graph, from which the overall factor of safety (i.e., were F.S.$_\phi$ equals F.S.$_c$) can be read. This procedure is illustrated in Example 14-4.

EXAMPLE 14-4

Given

The slope and data shown in Fig. 14-11.

Required

The factor of safety against failure by the stability number method.

Solution

Since the given angle of internal friction (ϕ) of 10° is greater than 3°, the failure surface will be a toe circle.

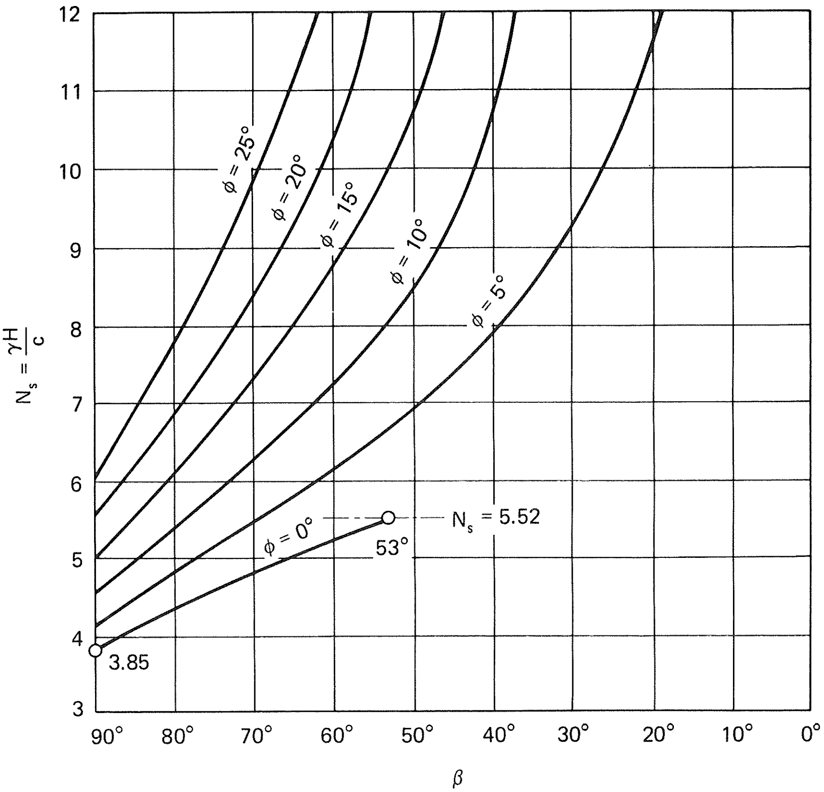

FIGURE 14-10 Stability numbers for soils having cohesion and friction. [5, 6]

Try F.S.$_\phi$ = 1

$$\tan \phi_{\text{required}} = \frac{\tan \phi_{\text{given}}}{\text{F.S.}_\phi} = \frac{\tan 10^\circ}{1}$$

$$\phi_{\text{required}} = 10^\circ$$

With $\phi_{\text{required}} = 10^\circ$ and $\beta = 45^\circ$, from Fig. 14-10,

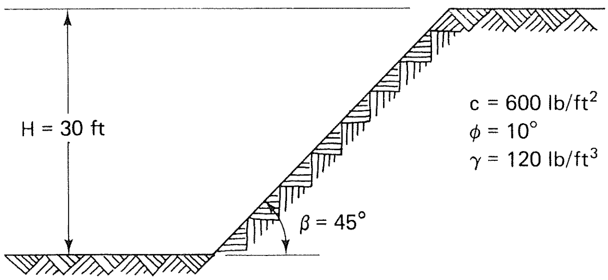

FIGURE 14-11

$$N_s = 9.2$$

$$N_s = \frac{\gamma H}{c} \tag{14-12}$$

$$\gamma = 120 \text{ lb/ft}^3$$

$$H = 30 \text{ ft}$$

$$9.2 = \frac{(120 \text{ lb/ft}^3)(30 \text{ ft})}{c_{\text{required}}}$$

$$c_{\text{required}} = 391 \text{ lb/ft}^2$$

$$\text{F.S.}_c = \frac{c_{\text{given}}}{c_{\text{required}}} = \frac{600 \text{ lb/ft}^2}{391 \text{ lb/ft}^2} = 1.53$$

Since F.S._ϕ and F.S._c are not the same value, another value of F.S._ϕ will be tried.

Try $\text{F.S.}_\phi = 1.2$

$$\tan \phi_{\text{required}} = \frac{\tan \phi_{\text{given}}}{\text{F.S.}_\phi} = \frac{\tan 10^\circ}{1.2} = 0.147$$

$$\phi_{\text{required}} = 8.36^\circ$$

With $\phi_{\text{required}} = 8.36^\circ$ and $\beta = 45^\circ$, from Fig. 14-10,

$$N_s = 8.6$$

$$c_{\text{required}} = \frac{(120 \text{ lb/ft}^3)(30 \text{ ft})}{8.6} = 419 \text{ lb/ft}^2$$

$$\text{F.S.}_c = \frac{c_{\text{given}}}{c_{\text{required}}} = \frac{600 \text{ lb/ft}^2}{419 \text{ lb/ft}^2} = 1.43$$

Again, F.S._ϕ and F.S._c are not the same value; hence, another value of F.S._ϕ will be tried.

Try $\text{F.S.}_\phi = 1.5$

$$\tan \phi_{\text{required}} = \frac{\tan \phi_{\text{given}}}{\text{F.S.}_\phi} = \frac{\tan 10^\circ}{1.5} = 0.118$$

$$\phi_{\text{required}} = 6.73^\circ$$

With $\phi_{\text{required}} = 6.73^\circ$ and $\beta = 45^\circ$, from Fig. 14-10,

$$N_s = 7.9$$

$$c_{\text{required}} = \frac{(120 \text{ lb/ft}^3)(30 \text{ ft})}{7.9} = 456 \text{ lb/ft}^2$$

$$\text{F.S.}_c = \frac{c_{\text{given}}}{c_{\text{required}}} = \frac{600 \text{ lb/ft}^2}{456 \text{ lb/ft}^2} = 1.32$$

Again, F.S.$_\phi$ and F.S.$_c$ are not the same value. Rather than continue a trial-and-error solution, plot the values computed. From Fig. 14-12, the factor of safety of the slope against failure is observed to be 1.36.

EXAMPLE 14-5

Given

1. A cut 25 ft deep is to be made in a stratum of highly cohesive soil (see Fig. 14-13).
2. The slope angle β is 30°.
3. Soil exploration indicated that bedrock is located 40 ft below the original ground surface.
4. The soil has a unit weight of 120 lb/ft^3, and its cohesion and angle of internal friction are 650 lb/ft^2 and 0°, respectively.

Required

The factor of safety against slope failure.

Solution

From Fig. 14-8a,

$$n_d H = 40 \text{ ft}$$
$$H = 25 \text{ ft}$$
$$n_d = \frac{40 \text{ ft}}{25 \text{ ft}} = 1.60$$

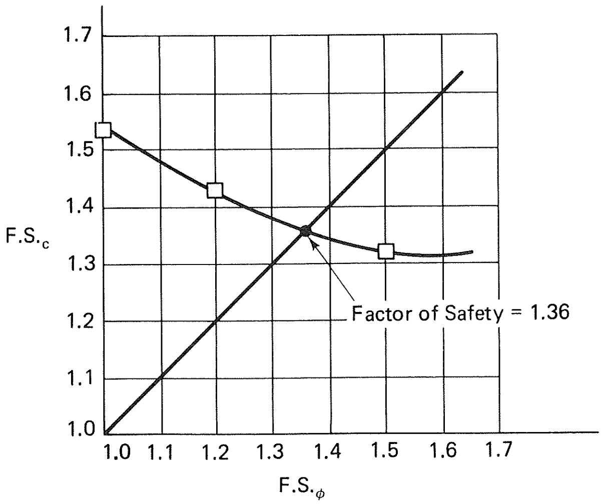

FIGURE 14-12

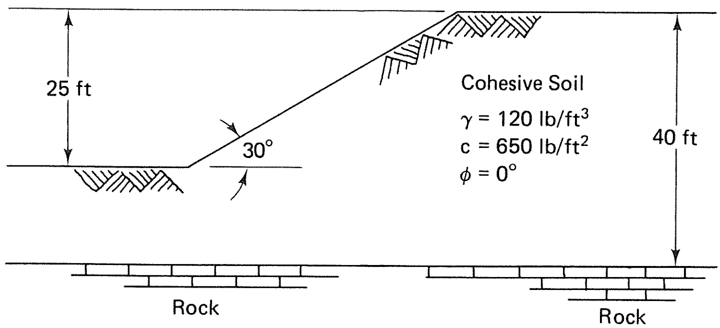

FIGURE 14-13

With $\beta = 30°$ and $n_d = 1.60$, from Fig. 14-9,

$$N_s = 6.0$$

$$N_s = \frac{\gamma H}{c_{\text{required}}} \tag{14-12}$$

$$\gamma = 120 \text{ lb/ft}^3$$

$$H = 25 \text{ ft}$$

$$6.0 = \frac{(120 \text{ lb/ft}^3)(25 \text{ ft})}{c_{\text{required}}}$$

$$c_{\text{required}} = 500 \text{ lb/ft}^2$$

$$\text{F.S.} = \frac{c_{\text{given}}}{c_{\text{required}}} = \frac{650 \text{ lb/ft}^2}{500 \text{ lb/ft}^2} = 1.30$$

EXAMPLE 14-6

Given

1. A cut 30 ft deep is to be made in a deposit of highly cohesive soil that is 60 ft thick and underlain by rock (see Fig. 14-14).
2. The properties of the soil to be cut are as follows:

$$c = 750 \text{ lb/ft}^2$$

$$\phi = 0°$$

$$\gamma = 120 \text{ lb/ft}^3$$

3. The factor of safety against slope failure must be 1.25.

Required

Estimate the slope angle (β) at which the cut should be made.

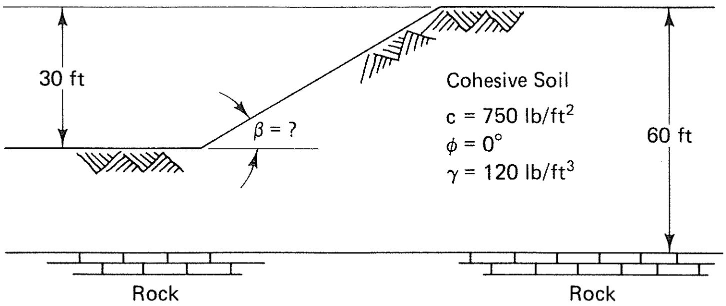

FIGURE 14-14

Solution

From Fig. 14-8a,

$$n_d H = 60 \text{ ft}$$

$$H = 30 \text{ ft}$$

$$n_d = \frac{60 \text{ ft}}{30 \text{ ft}} = 2.0$$

From Eq. (14-12),

$$N_s = \frac{\gamma H}{c_{\text{required}}} \tag{14-12}$$

$$\gamma = 120 \text{ lb/ft}^3$$

$$H = 30 \text{ ft}$$

$$c_{\text{required}} = \frac{c_{\text{given}}}{\text{F.S.}} = \frac{750 \text{ lb/ft}^2}{1.25} = 600 \text{ lb/ft}^2$$

$$N_s = \frac{(120 \text{ lb/ft}^3)(30 \text{ ft})}{600 \text{ lb/ft}^2} = 6.0$$

From Fig. 14-9, with $N_s = 6.0$ and $n_d = 2.0$,

$$\beta = 23°$$

EXAMPLE 14-7

Given

A cut 10 m deep is to be made in soil that has the following properties:

$$\gamma = 17.66 \text{ kN/m}^3$$

$$c = 19.2 \text{ kN/m}^2$$

$$\phi = 16°$$

Required

Using a factor of safety of 1.25, estimate the slope angle at which cut should be made.

Solution

$$c_d = \frac{c}{\text{F.S.}_c} = \frac{19.2 \text{ kN/m}^2}{1.25} = 15.36 \text{ kN/m}^2$$

From Eq. (14-12),

$$N_s = \frac{\gamma H}{c_d} \tag{14-12}$$

$$N_s = \frac{(17.66 \text{ kN/m}^3)(10 \text{ m})}{15.36 \text{ kN/m}^2} = 11.5$$

$$\tan \phi_d = \frac{\tan \phi}{\text{F.S.}_\phi} = \frac{\tan 16°}{1.25} = 0.2294$$

$$\phi_d = 12.9°$$

From Fig. 14-10, with $\phi_d = 12.9°$ and $N_s = 11.5$,

$$\beta = 44°$$

14-5 METHOD OF SLICES

In Sec. 14-4, the assumption was made in the Culmann method that failure (sliding) would occur along a plane that passes through the toe of the slope. It is probably more likely, and observations suggest, that failure will occur along a curved surface (rather than a plane) within the soil. Like the stability number method, the method of slices, which was developed in the 1920s by Swedish engineers, performs slope stability analysis assuming failure occurs along a curved surface.

The first step in applying the method of slices is to draw to scale a cross section of the slope such as that shown in Fig. 14-15. A trial curved surface along which sliding is assumed to take place is then drawn. This trial surface is normally approximately circular. Soil contained between the trial surface and the slope is then divided into a number of vertical slices of equal width. The weight of soil within each slice is calculated by multiplying the slice's volume by the soil's unit weight. (This problem is, of course, three-dimensional; however, by assuming a unit thickness throughout the computations, the problem can be treated as two dimensional.)

Figure 14-16 shows a sketch of a single slice. The weight of soil within the slice is a vertically downward force (W in Fig. 14-16). This force can be re-

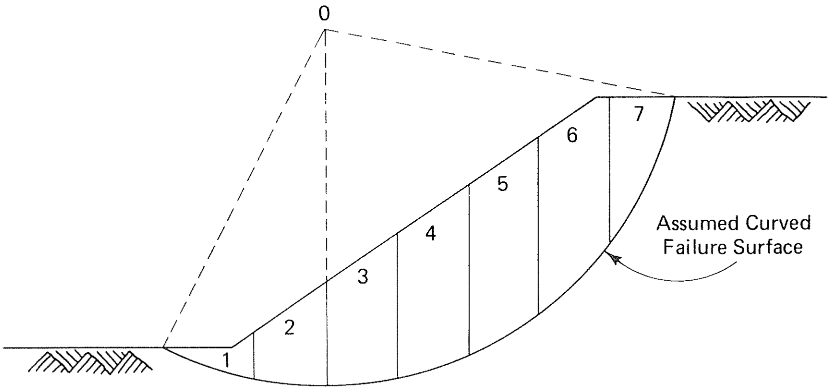

FIGURE 14-15

solved into two components—one normal to the base of the slice (W_n) and one parallel to the base of the slice (W_p). It is the parallel component that tends to cause sliding. Resistance to sliding is afforded by the soil's cohesion and internal friction. The cohesion force is equal to the product of the soil's cohesion times the length of the slice's curved base. The friction force is equal to the component of W normal to the base (W_n) multiplied by the friction coefficient ($\tan \phi$, where ϕ is the angle of internal friction).

Since W_p, the component tending to cause sliding of the slice, is equal to W multiplied by $\sin \alpha$ (see Fig. 14-16), the *total* force tending to cause sliding of the entire soil mass is the summation of products of weight of each slice times respective value of $\sin \alpha$, or $\Sigma\, W \sin \alpha$. Since W_n is equal to W multiplied by $\cos \alpha$, the *total* friction force resisting sliding of the entire soil mass is the summation of products of weight of each slice times respective value of $\cos \alpha$ times $\tan \phi$, or $\Sigma\, W \cos \alpha \tan \phi$. The *total* cohesion force resisting sliding of the entire soil mass can be computed simply by multiplying the soil's cohesion by

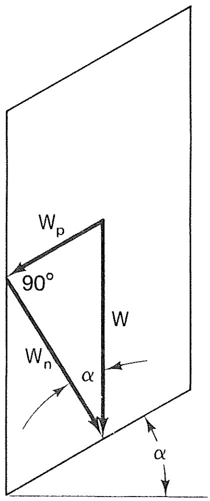

FIGURE 14-16

the (total) length of the trial curved surface, or cL. Based on the foregoing, the factor of safety can be computed by the equation

$$\text{F.S.} = \frac{cL + \sum W \cos \alpha \tan \phi}{\sum W \sin \alpha} \tag{14-13}$$

(As will be related in Example 14-8, the term $W \sin \alpha$ may be negative in certain situations.)

The method related above gives the factor of safety for the specific assumed failure surface. It is quite possible that the circular surface selected may not be the weakest, or the one along which sliding would occur. It is essential, therefore, that several different circular surfaces be analyzed until the designer is satisfied that the worst condition has been considered.

EXAMPLE 14-8

Given

1. The stability of a slope is to be analyzed by the method of slices.
2. On a particular trial curved surface through the soil mass (see Fig. 14-17), the shearing component (i.e., sliding force) and the normal component (i.e., normal to the base of each slice) of each slice's weight are as tabulated below.

Slice Number	*Shearing Component ($W \sin \alpha$) (lb/ft)*	*Normal Component ($W \cos \alpha$) (lb/ft)*
1	−63[1]	358
2	−51[1]	1450
3	86	2460
4	722	3060
5	1470	3300
6	1880	3130
7	2200	2270
8	950	91

[1]Since the trial surface curves upward near its lower end, the shearing components of the weights of slices 1 and 2 will act in a direction opposite to those along the remainder of the trial curve, resulting in a negative sign.

3. The length of the trial curved surface is 36 ft.
4. The ϕ angle of the soil is 5° and the cohesion (c) is 400 lb/ft².

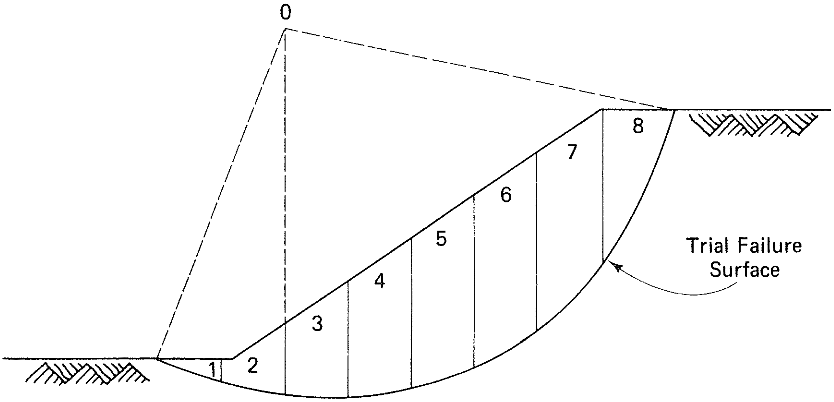

FIGURE 14-17

Required

The factor of safety of the slope along this particular trial surface.

Solution

From Eq. (14-13),

$$\text{F.S.} = \frac{cL + \sum W \cos \alpha \tan \phi}{\sum W \sin \alpha} \tag{14-13}$$

$$c = 400 \text{ lb/ft}^2$$

$$L = 36 \text{ ft}$$

$$\begin{aligned}\Sigma W \cos \alpha &= 358 \text{ lb/ft} + 1450 \text{ lb/ft} + 2460 \text{ lb/ft} + 3060 \text{ lb/ft} \\ &\quad + 3300 \text{ lb/ft} + 3130 \text{ lb/ft} + 2270 \text{ lb/ft} + 91 \text{ lb/ft} \\ &= 16{,}119 \text{ lb/ft}\end{aligned}$$

$$\phi = 5°$$

$$\begin{aligned}\Sigma W \sin \alpha &= -63 \text{ lb/ft} - 51 \text{ lb/ft} + 86 \text{ lb/ft} + 722 \text{ lb/ft} \\ &\quad + 1470 \text{ lb/ft} + 1880 \text{ lb/ft} + 2200 \text{ lb/ft} + 950 \text{ lb/ft} \\ &= 7194 \text{ lb/ft}\end{aligned}$$

$$\text{F.S.} = \frac{(400 \text{ lb/ft}^2)(36 \text{ ft}) + (16{,}119 \text{ lb/ft}) \tan 5°}{7194 \text{ lb/ft}} = 2.20$$

It should be emphasized that the computed factor of safety of 2.20 is for the given trial surface, which is not necessarily the weakest surface.

14-6 PROBLEMS

14-1 Figure 14-18 shows a 20-ft cut through two soil strata. The lower is a highly impermeable cohesive clay. Shear strength data between the two strata are as follows:

$$c = 220 \text{ lb/ft}^2$$

$$\phi = 12°$$

The unit weight of the upper layer is 110 lb/ft³. Determine if a slide is likely by computing the factor of safety against sliding. Neglect the effects of soil water.

14-2 A vertical cut is to be made in a deposit of homogeneous soil. The soil mass to be cut has the following properties: The soil's unit weight is 120 lb/ft³, cohesion is 350 lb/ft², and angle of internal friction is 10°. It has been specified that the factor of safety against sliding must be 1.50. Using Culmann's method, determine the safe depth of cut.

14-3 A 1.5-m-deep vertical-wall trench is to be cut in a soil whose unit weight, angle of internal friction, and cohesion are 17.36 kN/m³, 25°, and 20.6 kN/m², respectively. Determine the factor of safety of this trench by the Culmann method.

14-4 Determine the factor of safety against slope failure by means of the stability number method for the slope shown in Fig. 14-19.

14-5 A cut 20 ft deep is to be made in a stratum of highly cohesive soil that is 80 ft thick and underlain by bedrock. The slope of the cut is 2: 1 (i.e., 2 horizontal to 1 vertical). The clay's unit weight is 110 lb/ft³, and its c and ϕ values are 500 lb/ft² and 0°, respectively. Determine the factor of safety against slope failure.

14-6 A cut 25 ft deep is to be made in a deposit of cohesive soil with c = 700 lb/ft², ϕ = 0°, and γ = 115 lb/ft³. The soil is 30 ft thick and underlain by

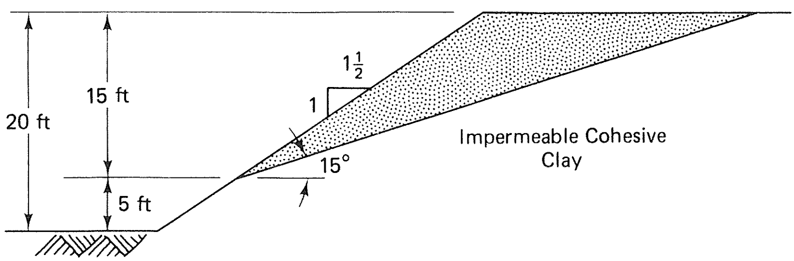

FIGURE 14-18

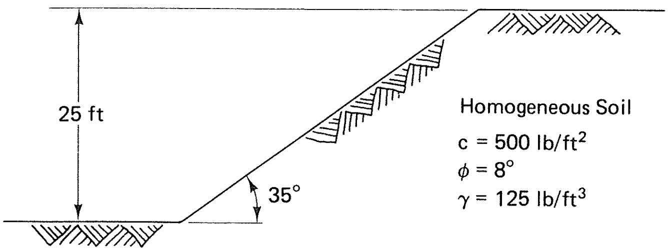

FIGURE 14-19

rock. The factor of safety of the slope against failure must be 1.50. At what slope angle should the cut be made?

14-7 A slope 8 m high is to be made in a soil whose unit weight, angle of internal friction, and cohesion are 16.7 kN/m^3, 10°, and 17.0 kN/m^2, respectively. Using an overall factor of safety of 1.25, estimate the slope angle that should be used.

14-8 The stability of a slope is to be analyzed by the method of slices. On a particular trial curved surface through the soil mass, the shearing and normal components of each slice's weight are as tabulated below. The length of the trial curved surface is 40 ft. The cohesion c and ϕ angle of the soil are 225 lb/ft^2 and 15°, respectively. Determine the factor of safety along this trial surface.

Slice Number	*Shearing Component (W sin α) (lb/ft)*	*Normal Component (W cos α) (lb/ft)*
1	−38	306
2	−74	1410
3	124	2380
4	429	3050
5	934	3480
6	1570	3540
7	2000	3210
8	2040	2190
9	766	600

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[6] D. W. TAYLOR, "Stability of Earth Slopes," *J. Boston Soc. Civil Eng.*, **24** (1937).

Answers to Selected Problems

CHAPTER 2

2-2. A-2-4 (0)
SC

2-4. $w = 15.3\%$
$e = 0.56$
$n = 36.0\%$
$S = 72.6\%$
$\gamma = 122.6$ lb/ft^3
$\gamma_d = 106.3$ lb/ft^3

2-6. $\gamma = 121.2$ lb/ft^3
$\gamma_d = 105.4$ lb/ft^3
$n = 36.0\%$
$S = 70.3\%$

2-8. $\gamma = 109.8$ lb/ft^3
$G_s = 2.77$

2-10. $w = 13.04\%$
$\gamma = 104.4$ lb/ft^3

2-12. $\gamma_d = 16.52$ kN/m^3
$e = 0.62$
$S = 54.3\%$

2-16. $e = 0.61$
$\gamma = 111.2$ lb/ft^3

2-18. 6.1%

2-20. 19.9 lb

CHAPTER 3

3-2. 30, 31

3-4. 19, 19

3-6. 382 lb/ft^2

3-8. 63 ft

CHAPTER 4

4-1. $\gamma = 116.7$ lb/ft^3
$\gamma_d = 107.0$ lb/ft^3

4-3. $\gamma = 124.2$ lb/ft^3
$\gamma_d = 107.1$ lb/ft^3

4-5. 2897 m^3

CHAPTER 5

5-1. 6.56×10^{-4} ft^3/sec

5-3. 0.103 cm/s

5-5. 0.0342 cm/s

5-7. 0.485 ft or 5.8 in.

5-9. 1.57×10^{-6} cm/s

CHAPTER 6

6-2. 169 lb/ft^2

6-4. **(a)** $q_{(1\ m)} = 97.50$ kN/m^2
(b) $q_{(3\ m)} = 39.00$ kN/m^2
(c) $q_{(5\ m)} = 20.89$ kN/m^2

6-6. **(a)** 48.50 kN/m²
(b) 97.64 kN/m²

6-8. 520 lb/ft²

6-10. **(a)** 628 lb/ft²
(b) 350 lb/ft²

6-12. **(a)** 109.5 kN/m of wall
(b) 197.9 kN/m of wall

CHAPTER 7

7-2. 0.79

7-4. 0.062 m

7-6. **(b)** 3.66 in.

7-8. **(a)** 37 yr, **(b)** 0.69 in.,
(c) 1.75 yr

7-10. 0.89 in.

7-12. 3.06 tons/ft²

CHAPTER 8

8-2. c = 380 lb/ft²
ϕ = 19.5°

8-4. 34°

8-5. 35.4°

8-7. 1925 lb/ft²

CHAPTER 9

9-2. 12,100 lb/ft²

9-4. 695 tons

9-6. 228 kN/m²

9-8. 5.5 ft by 5.5 ft

9-10. 91 kN/m²

9-13. [A] **(a)** 2.47, **(b)** 3.36
[B] **(a)** 3.58, **(b)** 4.87

9-15. **(a)** q_R = 3.5 kip/ft²
q_L = 0.5 kip/ft²
(b) 69.6 kips
(c) 148.8 ft-kips
(d) 4.0
(e) 7.2
(f) 3.1

CHAPTER 10

10-2. 17.4 kips

10-4. 83.6 kips

10-6. 265 kN

10-8. 28 ft

10-10. 20 blows/ft

10-12. 37.5 tons

10-14. 382 kips

10-16. **(a)** 557 kips, **(b)** 444 kips,
(c) 444 kips

10-18. 675 kN

10-20. Q_{max} = 70 kips
Q_{min} = 10 kips

CHAPTER 11

11-2. 1.08 m

11-4. 1086 tons

CHAPTER 12

12-1. 39.58 kN/m of wall

12-4. 6900 lb/ft of wall

12-6. 9190 lb/ft of wall

12-8. 16,150 lb/ft of wall acting at 9.41 ft above the base

12-10. 16,100 lb/ft of wall

CHAPTER 13

13-1. **(a)** 3.31
(b) 1.96
(c) q_L = 4.66 kips/ft², q_R = 0.10 kip/ft² (Note: $q_L > q_a$ of 4 kips/ft²)

13-4. $L_{total} = 6.40$ m
$s = 0.69$ m

13-6. 2.20 m

CHAPTER 14

14-2. 8.7 ft

14-4. 1.38

14-6. 38°

14-8. 1.86

APPENDIX:

Conversion Factors

1 in. = 25.40 mm	*Length*	1 mm = 0.03937 in.
1 in. = 2.540 cm		1 cm = 0.3937 in.
1 ft = 0.3048 m		1 m = 3.281 ft
1 mile = 1.609 km		1 km = 0.6214 mile
1 in.^{2} = 645.2 mm^2	*Area*	1 mm^2 = 0.001550 in.^{2}
1 in.^{2} = 6.452 cm^2		1 cm^2 = 0.1550 in.^{2}
1 ft^2 = 0.09290 m^2		1 m^2 = 10.76 ft^2
1 in.^{3} = 16,390 mm^3	*Volume*	1 mm^3 = 0.00006102 in.^{3}
1 in.^{3} = 16.39 cm^3		1 cm^3 = 0.06102 in.^{3}
1 ft^3 = 0.02832 m^3		1 m^3 = 35.31 ft^3
1 lb = 0.4535 kg	*Force*	1 kg = 2.205 lb
1 lb = 4.448 N		1 N = 0.2248 lb
1 kip = 4.448 kN		1 kN = 0.2248 kip
1 ton = 8.896 kN		1 kN = 0.1124 ton
1 lb/in.^{2} = 0.07029 kg/cm^2	*Pressure or stress*	1 kg/cm^2 = 14.23 lb/in.^{2}
1 lb/in.^{2} = 6.894 kN/m^2		1 kN/m^2 = 0.1450 lb/in.^{2}
1 lb/ft^2 = 0.04788 kN/m^2		1 kN/m^2 = 20.89 lb/ft^2
1 kip/ft^2 = 47.88 kN/m^2		1 kN/m^2 = 0.02089 kip/ft^2
1 ton/ft^2 = 95.76 kN/m^2		1 kN/m^2 = 0.01044 ton/ft^2
1 lb/ft^3 = 0.1571 kN/m^3	*Unit weight* 62.4 lb/ft^3 = 9.81 kN/m^3	1 kN/m^3 = 6.366 lb/ft^3

Note: kg is used as a unit of force in the metric system.

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