



HISTORY OF ANALYTIC PHILOSOPHY



NEW ANTI-KANT

Edited by Sandra Lapointe
and Clinton Tolley



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New Anti-Kant

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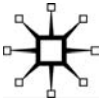
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Series Editor's Foreword

During the first half of the twentieth century, analytic philosophy gradually established itself as the dominant tradition in the English-speaking world, and over the last few decades it has taken firm root in many other parts of the world. There has been increasing debate over just what 'analytic philosophy' means, as the movement has ramified into the complex tradition that we know today, but the influence of the concerns, ideas, and methods of early analytic philosophy on contemporary thought is indisputable. All this has led to greater self-consciousness among analytic philosophers about the nature and origins of their tradition, and scholarly interest in its historical development and philosophical foundations has blossomed in recent years, with the result that history of analytic philosophy is now recognized as a major field of philosophy in its own right.

The main aim of the series in which the present book appears, the first series of its kind, is to create a venue for work on the history of analytic philosophy, consolidating the area as a major field of philosophy and promoting further research and debate. The 'history of analytic philosophy' is understood broadly, as covering the period from the last three decades of the nineteenth century to the start of the twenty-first century, beginning with the work of Frege, Russell, Moore, and Wittgenstein, who are generally regarded as its main founders, and the influences upon them, and going right up to the most recent developments. In allowing the 'history' to extend to the present, the aim is to encourage engagement with contemporary debates in philosophy, for example, in showing how the concerns of early analytic philosophy relate to current concerns. In focusing on analytic philosophy, the aim is not to exclude comparisons with other – earlier or contemporary – traditions, or consideration of figures or themes that some might regard as marginal to the analytic tradition but which also throw light on analytic philosophy. Indeed, a further aim of the series is to deepen our understanding of the broader context in which analytic philosophy developed, by looking, for example, at the roots of analytic philosophy in neo-Kantianism or British idealism, or the connections between analytic philosophy and phenomenology, or discussing the work of philosophers who were important in the development of analytic philosophy but who are now often forgotten.

One of the undoubted common sources of both the analytic and phenomenological traditions is the work of Immanuel Kant (1724–1804). His *Critique of Pure Reason*, published in 1781, is arguably the most important philosophical text of the modern period. Distinguishing between analytic and synthetic, a priori and a posteriori, necessary and contingent propositions, Kant argued that while logical truths are analytic, a priori and necessary, mathematical truths are synthetic, a priori and necessary. Frege and Russell were to criticize this view, arguing instead that arithmetical truths could be derived from pure logic – implying that they could therefore be regarded as analytic. (Frege agreed with Kant about geometrical truths; Russell thought that these, too, could be reduced to logic.) Moore was also to criticize Kant's views, especially his idealism. As the main founder of the phenomenological tradition, Edmund Husserl, too, was influenced by Kant. Like Frege and Russell, Husserl was trained as a mathematician and came to philosophy through concern with the foundations of mathematics. In his later work, like Moore, he engaged deeply with Kant's idealism.

Between Kant and the generally acknowledged founders of the analytic and phenomenological traditions, however, lived one philosopher whose work bridges the historical gap more than any other single figure – Bernard Bolzano (1781–1848). Indeed, Bolzano was born in the year that the *Critique of Pure Reason* was published and died in the year that Frege was born. Bolzano read Kant's first *Critique* when he was just 18 years old, and his work can be seen in many ways as a profound critical response to Kant's ideas. In particular, he subjected Kant's distinctions between analytic and synthetic, a priori and a posteriori, and concept and intuition, to sustained critique. Like Frege, Russell and Husserl, he also had a deep knowledge of mathematics, and it was his sense of the inadequacy of Kant's treatment of mathematics that fuelled his philosophy.

Bolzano's engagement with Kant's philosophy informs his *magnum opus*, the *Wissenschaftslehre*. But there is also a work that crystallizes his critique of Kant that has remained untranslated into English until now. This is a work written not by Bolzano himself but by František Příhonský (1788–1859), one of Bolzano's collaborator. He wrote it, however, with the support of Bolzano and it was approved shortly before Bolzano died. Its full title is '*New Anti-Kant, Or Examination of the Critique of Pure Reason According to the Concepts Laid Down in Bolzano's Theory of Science*'. It is this book that now appears for the first time in English translation in the present volume. Sandra Lapointe published a French translation of the *Neuer Anti-Kant* in 2006, and also wrote an excellent

book on Bolzano's theoretical philosophy, which appeared in this series on the history of analytic philosophy in 2011. So I am delighted that she teamed up with Clinton Tolley, a Kant scholar, to produce this long-overdue English translation.

As well as an introduction to the text written by the translators, this volume also contains four essays that help both contextualize Bolzano's contribution to philosophy and demonstrate its relevance to analytic philosophy today. Tolley examines Bolzano's critique of Kant's conception of space, focusing on the role of 'outer intuitions'. Nicholas F. Stang explores Kant's and Bolzano's views on the formality of logic, and Timothy Rosenkoetter discusses Kant's and Bolzano's moral theories. Lapointe and Chloe Armstrong locate Bolzano's logic and philosophy of mathematics in the broader tradition of work on logic in Germany in the eighteenth and nineteenth centuries.

Bolzano may not have directly influenced Frege, Russell, or Moore, but there are many points of similarity and connection in their criticisms of Kant. If anything, Bolzano's critique is far more powerful and penetrating than the criticisms made by the early analytic philosophers, and there is increasing recognition today of just how relevant Bolzano's ideas are to contemporary philosophy. In the 1930s the analytic tradition was regarded as having originated in Russell's and Moore's rebellion against British idealism. In the 1970s the analytic tradition was backdated to include Frege as one of its founders. Another forty years on there are grounds for backdating it further to include Bolzano as well. Certainly, the bigger story of analytic philosophy requires recognition of Bolzano's work, and this volume will both contribute to this bigger story and provide an essential resource in understanding Bolzano.

Michael Beaney
May 2014

Acknowledgements

Sandra Lapointe: When I translated the Anti-Kant into French almost a decade ago, my career as a ‘Bolzanistin’ was in its very first stages and I was lucky to benefit from the intellectual support of Jan Sebestik, Edgar Morscher, and Kevin Mulligan. I am still thankful today and the present book would not have come into existence without their initial encouragements. The project is also the outcome of a series of workshops on Bolzano and Kant I have organized since 2008 in various locations and in which Clinton Tolley, Nick Stang, and Timothy Rosenkoetter have continued to take part. I have gained much from interacting with them intellectually, especially with Clinton, who co-signs this translation as well as the Introduction. Katharina von Radziewsky was a godsend as an assistant, collaborator, and (at times) task-master – Clinton and I are extremely thankful to her.

Clinton Tolley: My first thanks must go to Sandra, whose tireless efforts and strength of vision kept this project from remaining a mere shadow behind far-distant clouds. And Katharina: see above! At the University of California, San Diego, David Brink, Don Rutherford, Craig Callender (Department Chairs), and Seth Lerer (Dean, Arts and Humanities) secured financial/logistical support for research and travel. For intellectual support, encouragement, collaborative research, and helpful feedback, very warm thanks to Jocelyn Benoist, Tim Jankowiak, James Messina, Waldemar Rohloff, Timothy Rosenkoetter, Jan Sebestik, Ben Sheredos, Nick Stang, and Eric Watkins. Particularly deep debts to Samantha Matherne for patient, constructive criticism and countless rich, insightful conversations. Thanks, finally, to: Jason Diller and Karolina Hübner for such friendly company around marathon translation sessions; these cafes for housing/caffeinating our work: Mulberry St., Ezra’s Pound, Gene, Subterranean, and Royal; and Erin Glass for helping me find Bolzano’s house.

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Notes on Contributors

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Contributors

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Part I

1

Introduction*

Sandra Lapointe and Clinton Tolley

§1 A unique philosophical dialogue

The present volume contains an altogether remarkable document in the history of nineteenth-century philosophy: a critical commentary on the most influential systematic work (the *Critique of Pure Reason*) of one major philosopher (Kant), written from the point of view of another major systematic philosopher (Bolzano), just decades after the former's publication. Bolzano, at the height of his powers, and with his mature philosophical views having fully taken shape with the publication of his *Theory of Science* (1837), undertakes the project of engaging, key point by key point, with Kant's masterwork. In collaboration with Bolzano, František Příhonský (who would ultimately publish the final record of this work in 1850, shortly after Bolzano's death) both compiles a comprehensive and thorough summary of the main definitions, theses, and arguments in Kant's book, and then proceeds to bring to light the most important unclarities, confusions, and fallacies that he finds each step along the way. The result, *New Anti-Kant*, is not only an extremely useful and even-handed overview of the entire first *Critique* itself – including parts often neglected by even Kant's most sympathetic readers – but also a catalogue of philosophically insightful and textually well-grounded challenges to signature Kantian doctrines. This work helps us to see anew the overarching contours of Kant's philosophy, and brings a fresh focus onto deep points of tension within Kant's system – all the while serving to introduce us, through instructive contrast, to

* Our thanks go to Edgar Morscher for his extraordinary generosity and inspiring encouragements. We owe much to his input and his detailed introduction of the new edition of the *Neuer Anti-Kant*.

the powerful alternative perspective that Bolzano develops in his own systematic philosophy.

The value of *New Anti-Kant* rests at once in the fabulous richness of Bolzano's own theories as well as in the diligence and insight with which he approached Kant's *Critique*. After a 'Preface' that motivates the need for such a project, the volume begins with an 'Introduction' that contains a concise and very helpful summary of those of Bolzano's doctrines that are most relevant to the criticisms that follow. The main part, the 'Treatise', consists of a systematic examination of the entirety of the first *Critique*. Towards the end, it also engages in an extensive discussion of the moral theory found in Kant's other writings. Throughout, the 'Treatise' alternates between extremely economical, well-articulated little digests of Kant's main doctrines following the order in which they appear in the *Critique*, followed by 'Objections' and 'Remarks' in which Příhonský deftly recapitulates the essence of Bolzano's concerns with the Kantian philosophy and thereby assesses Kant's views in light of Bolzano's own theories.

Beyond the originality and intelligence of its critical perspective, *New Anti-Kant* thus has the merit of offering an exhaustive discussion of Kant's philosophy, one which devotes its attention to both the doctrines of the *Critique* that have traditionally been of most interest to Kant scholars (e.g. the distinction between concepts and intuitions, the distinction between analytic and synthetic judgements, the possibility of synthetic *a priori* cognition, the nature of space and time, the transcendental deduction, the nature and status of transcendental Ideas, the Antinomies), as well as those doctrines that are too often overlooked (e.g. the discussions in the 'Doctrine of Method' concerning definitions or those concerning the differences between opinion, belief, and knowledge). By thoughtfully engaging with the full scope of Kant's views, *New Anti-Kant* also introduces the reader to Bolzano's own positions in both theoretical and practical philosophy, as well as his philosophy of religion. Moreover, it does so in a way that, for readers more familiar with Kant's views than with Bolzano's (which will perhaps be most of its readers), promises to make Bolzano's positions more accessible insofar as they are introduced by reference to Kant's better-known terminology.

§2 A long and vigorous intellectual engagement

Bolzano's relationship to Kant is complicated and, perhaps for this reason, it is also often misunderstood. Part of the difficulty resides in the fact that the context in which Bolzano's theories evolved – and

in particular the circumstances that ultimately contributed to isolate Austrian Academia from the rest of the German-speaking world – is little known. At the turn of the nineteenth century, Kant's theories constituted the mainstream in Germany, and a vast portion of German intellectual life was recognizably “post-Kantian”, with lively debate for and against him, in a non-trivial sense. In Austria, by contrast, the rampant antagonism to which Kant's theories became subject in the 1790s, the subsequent imposition of bans on his books and the interdiction against including the study of his work in philosophy curricula in the years that followed, all made for a considerable lack of continuity with the kind of philosophy that was being done in neighbouring countries.¹ By the time Bolzano was admitted at Charles University in the late 1790s, Austrian philosophy students were being trained according to a rigid, state-imposed programme designed predominantly to promote Leibniz–Wolffian doctrines (with the help of official textbooks!). Given such a context, the vast interest Bolzano took in Kant's philosophy early on and the diligence with which he studied his theories are thus even more remarkable. Less surprising is the fact that this should have caused Bolzano to be singled out as a trouble-maker: a few months after he took up the chair of “Science of the (Catholic) Religion” in Prague in 1805, Bolzano was accused of being a “Kantian”, a label that would cause the professional demise of a number of his colleagues under the reign of Francis II. He could remain in his position only after he had ‘officially’ vindicated himself. Even this, however, was short-lived, as in 1819 Bolzano was eventually discharged for ‘heresy’.

Judged from the perspective of Bolzano's early publications, but especially in light of his own mature positions, one could wonder whether such charges of ‘Kantianism’ made much sense. But Bolzano's intellectual engagement with Kant's thought began quite early – in fact, years earlier than his appointment in 1805 – and proved to be deeply influential on the course of Bolzano's intellectual career in ways that ran much deeper than academic politics. In his *Autobiography*, we learn that Bolzano began to study the *Critique of Pure Reason* in 1799, when he was 18 years old.² From the outset Bolzano found himself in fundamental accord with Kant over the importance of the distinction between analytic and synthetic judgements, between *a priori* and *a posteriori* judgements, as well as between intuitions and concepts, among other things (*ibid.*).

¹ For a more complete picture, see Lapointe (2011, 11–17).

² *Lebenbeschreibung des Dr Bernard Bolzanos...* BBGA 1 10, 67–68.

Yet already by the time of his seminal *Contributions to a Better-Founded Exposition of Mathematics* (1810), Bolzano had also arrived at some of the basic commitments that would motivate his later criticisms of Kant's works. Indeed, the 'Appendix' to this work directly targets 'the Kantian theory of construction of concepts through intuitions' on the grounds (among others) that Kant's underlying doctrine of pure intuition is incoherent.³ Such criticisms were expanded extensively in Bolzano's writings throughout the 1820s and 1830s, his *Science of Religion* (1834), and especially in his *Theory of Science* (1837), with Bolzano now challenging Kant not just on his philosophy of mathematics but on all fronts (logic, epistemology, metaphysics, morals, religion, etc.). All of this ultimately coalesced in Bolzano's discussions with Přihonský in the late 1830s and 1840s, which eventually were to yield *New Anti-Kant*.

§3 The text

When he died in 1848, Bolzano left behind a considerable stock of unpublished writings, the bulk of which has now appeared in the *Bernard Bolzano Gesamtausgabe* (BBGA). František Přihonský was one of three of Bolzano's former pupils and/or collaborators to which the management of the latter was assigned.⁴ His contribution to Bolzano studies, though humble, was nonetheless the most significant among them. Perhaps most notably, we owe to him the publication of the *Paradoxes of the Infinite* (1851), a work that had considerable influence on the philosophy of mathematics and the emergence of set theory at the turn of the century, and which was both republished and then translated (into English⁵ and French⁶).

We also owe to Přihonský the publication of the thoughts captured in *New Anti-Kant*. Unlike the *Paradoxes*, however, this work is not to be counted as part of the *Nachlass*. Neither is it a disciple's tribute to the memory of his teacher. Rather, *New Anti-Kant* is the outcome of a collaboration between Bolzano and Přihonský that lasted over a decade.

³ See Laz (1993, 171–182) for a French translation of this Appendix, and Rusnock (2000, 198–204) for an English one.

⁴ The other two were Josef Fesl and Robert Zimmermann. The latter would be Franz Brentano's colleague in Vienna, which may partly explain the interest almost all of Brentano's most prominent students took in Bolzano, including Twardowski, Husserl Höfler, and Meinong.

⁵ By D.A. Steele (Ithaca, Yale University Press, 1950).

⁶ By H. Sinaceur (Paris, Seuil, 1999).

The project took form in early 1837 at Bolzano's initiative. Bolzano had heard of a French literary competition inviting submissions presenting critically the Kantian philosophical system and its idealist offshoots. The two philosophers who collaborated by correspondence were unable to meet the initial deadline, and even though the competition was protracted until the following year – for lack of high-quality submissions – Bolzano resolved not to take part in it. There were apparently two reasons for this decision: on the one hand, Bolzano was not particularly interested in spending time presenting the basics of Kant's ideas – what was supposed to be the first part of the essay. On the other hand, the idea of having to publish in French or in Latin (the official languages of the competition) was likely to prevent him from reaching the German-speaking philosophical public he was targeting.⁷

The project of a criticism of Kant and the idealists was not, however, abandoned. Bolzano and Příhonský divided the labour between themselves: Příhonský, with Bolzano's help, would deal with Kant; he ultimately delivered *New Anti-Kant*. For his part, Bolzano wrote three essays on idealism that were published posthumously by Příhonský,⁸ one year after *New Anti-Kant*. Bolzano also provided constant input into Příhonský's work, even offering to draft parts of the book. The manuscript, which was finished in 1847, was granted Bolzano's approval, though Bolzano succumbed to his lifetime struggle with pulmonary illness the following year, and so was unable to see the document through to publication.

§4 The reception of *New Anti-Kant*

New Anti-Kant's literary fate turned out not to meet what had been Bolzano's hopes and expectations: to contain “as much as he could – through the diffusion of distinct notions – the terrible disaster Kant unwittingly brought about in Germany through his philosophy”.⁹ The initial distribution of the book was limited and its success dismal. Though we can find occasional references to the work – perhaps most notably by Hans Vaihinger in his *Commentary* on the first *Critique*, and by Benno Kerry, one of Brentano's students¹⁰ – few copies survived.

⁷ Morscher (2003) gives a more detailed account.

⁸ See Bolzano (1977).

⁹ In Bolzano's will. See Laz (1993, 7).

¹⁰ See Vaihinger (1922); we owe knowledge of Kerry's reference to Künne (1997, 32).

There is a record of a projected new edition of *New Anti-Kant* in the 1930s by Walter Dubislaw and Heinrich Scholz,¹¹ though this unfortunately never materialized. Rather, all the way up through the 1990s, the work continued to languish in neglect. Evidence of this is that the only monograph (Laz 1993; in French) devoted to Bolzano's criticism of Kant actually never consulted *New Anti-Kant*: the author presumed that it was lost.

In recent years, the situation has much improved. A key factor here is Edgar Morscher's excellent new critical edition (in the *Beyträge zur Bolzano-Forschung*, vol. 7), with its meticulous revision of the passages quoted from Kant's works and its detailed introduction, all of which forms a tremendous resource for continuing research. Its appearance has dovetailed with *New Anti-Kant* beginning to garner more of the attention that it deserves within the circles of Bolzano studies. Yet even if its reception-history has begun to take a happy turn for the better, the depths of this work have by no means been plumbed, with existing discussion remaining neither exhaustive nor definitive.¹² Our translation of this little book is meant to bring it even further into the spotlight. What is more, we hope to make it accessible to an even broader readership: to English-readers primarily interested in Bolzano's own thought; those who are interested in his influence on the history of analytical philosophy at large and that of phenomenology; those who are already invested in Kant's thought and his context; and those who are looking for a fresh perspective to bring to bear on Kant scholarship and the history of German thought more broadly.

This work will add a new voice to the recent growth of interest in tracing out the Kantian themes in the history of theoretical philosophy in the analytic tradition (epistemology, philosophy of logic, mathematics, and science).¹³ Readers will find in *New Anti-Kant* a repository of ideas to which they can compare their own assessment of the fate of Kant's doctrines. And though some will be struck primarily by the confirmation of their own opinions or those of other canonical figures *avant la lettre*, there are many original insights and theories contained in it that deserve to be studied on their own merit. This is perhaps unsurprising

¹¹ See Morscher (2003, 191–201).

¹² E.g. Laz (1993); Rusnock (2011, 2013). See also the recent publication of a special issue of *Grazer Philosophische Studien* devoted to Bolzano and Kant (Lapointe 2012).

¹³ See Hanna (2001). See also Rockmore (2006).

insofar as there has been increasing interest of late in the extent to which Bolzano's own views might actually be preferable to those of his more well-known successors (e.g. Frege and Tarski) on several core issues in analytic philosophy.¹⁴

New Anti-Kant also contains much that will be of interest beyond the audience of analytical philosophers. Most directly, the work has a clear significance for anyone already engaging in Kant studies, and especially for anyone interested in how Kant's programme was received by those working outside of the now-canonical German idealist tradition. There were of course other *Anti-Kants* in the years following the first *Critique* – e.g. that of Stattler (1788) and then eventually Bollinger (1882) – but nothing that can be compared to Příhonský's economical and yet systematic examination of Kant's philosophy through the lens of Bolzano's systematic philosophical programme. Indeed, what distinguishes Bolzano from almost all of Kant's other official 'commentators' is the fact that his views are rooted in a fully worked out and highly accessible theoretical alternative, and one that on both fronts had few peers for most of the century that followed.

Finally, Bolzano's unique position in the history of philosophy – engaging both Kantians and analytical philosophers – has much to teach us as we continue to rethink the relative boundaries of what count as two distinct philosophical traditions. For, as has been recently well documented, in addition to his many anticipations of analytical philosophers Bolzano also stands as one of the major influences on the founder of the phenomenological movement, Edmund Husserl.¹⁵ What is more, Bolzano actually emerges as a point of reference for several of the correspondents Husserl and Frege had in common.¹⁶ The present work can therefore provide another occasion to better understand the extent to which the sources of ostensibly diverging contemporary traditions ultimately have considerable overlap (not least by both tracing back to Kant).¹⁷

¹⁴ On the comparison with Frege, see Küne (1997); on the comparison with Tarski, see Siebel (2002) and Rusnock and Burke (2010).

¹⁵ See Husserl (1900).

¹⁶ See Küne (2009). See also Lapointe (2011, 139–157).

¹⁷ Rethinking Husserl's relation to Bolzano in light of *New Anti-Kant* represents an especially salient opportunity here for furthering of the dialogue between the history of analytic philosophy and that of phenomenology, insofar as Husserl's own thought, too, became increasingly oriented around his engagement with Kant and the neo-Kantians.

Perhaps above all, though, *New Anti-Kant* can serve as the occasion to acquaint oneself with the thoughts of a brilliant philosopher whose tragic literary fate is nothing less than a historical injustice. For, as it contains many pages of philosophically perceptive and insightful discussions of topics of both historical and contemporary significance, the work promises to connect up with any number of debates, in several different kinds of traditions, that have grown up in the aftermath of Kant's philosophical interventions. For all of these reasons, the editors of the present translation share the conviction of Dubislav, Scholz, and Morscher – and, ultimately, that of Příhonský himself: that Bolzano's criticism of Kant has much to teach, even today.

§5 Complementing the text

In addition to the text of *New Anti-Kant* itself, the present volume also includes four essays, each of which investigates a different aspect of Bolzano's critical engagement with Kant, and which thereby serves to link up the discussions in Příhonský's text with the broader context of ongoing debate over the nature and significance of Bolzano's and Kant's positions themselves. To be sure, they are by no means exhaustive treatments of their topics. Instead they are intended to help better foreground what is at stake – both philosophically, but also interpretively – on a number of the fundamental issues that bring the two authors into dialogue, in an order that follows out the structure of the work itself: the nature of our sensible representations (Tolley), the nature of logic as the science of our intellectual (conceptual) representations (Stang), the proper conception of practical reason and the fundamental moral law (Rosenkoetter), and the very nature of Bolzano's putative “anti-Kantianism” (Lapointe and Armstrong).

These treatments centre around the three ideas that immediately come to mind when one sets out to situate Bolzano among Kant's numerous early commentators and to highlight what makes his criticisms unique and his own programme distinctive: Bolzano's sustained engagement with mathematics, his conviction in the necessity of a radical logical reform, and his commitment to a utilitarian conception of morality. Much of what Kant has to say in the *Critique* bears on mathematics: the distinction between analytic and synthetic judgements, between *a priori* and *a posteriori* cognition, the fundamental difference in method between philosophy and mathematics due to the necessity of constructing mathematical concepts in pure intuition, his views on axioms and definitions, and so on. The fact that Bolzano was himself

deeply invested in mathematics and made foundational contributions to it gives him an internal perspective on all of these issues, and so one that is highly valuable, since Bolzano had the obvious advantage of having (unlike Kant) an intimate, first-hand knowledge of advanced mathematical practice. Trained in the discipline, Bolzano spent the first decades of his career working on mathematical proofs and writing on foundational questions, before turning more directly to philosophy. It is unsurprising, therefore, that Kant's remarks on the philosophical foundations of mathematics and his contrast between mathematical and philosophical method would have functioned as a continuous point of departure for Bolzano's engagement with Kant's views. Indeed, his continued interest and circumspect examination of Kant's philosophy should be understood as continuous with his seminal contribution to mathematics and its philosophy.

What do we find in this examination? What's noteworthy when comparing their views is as much their consensus as their disagreement. Take, for instance, the nature of mathematical knowledge. On the one hand, Bolzano agreed with Kant that mathematical knowledge is not analytic. On the other hand, Bolzano radically rejected Kant's appeal to pure intuition, in both arithmetic and geometry. In this respect, Bolzano's project in the philosophy of mathematics can be understood as an attempt to provide an alternative account of synthetic *a priori* cognition. In his contribution, Tolley explores what consequences the rejection of pure intuition has for Bolzano's views about space and spatial representation, and, in particular, what role (if any) such representations have to play in our intuitions of outer objects. Since Bolzano rejects the notion of pure intuition, he also rejects the idea that our most original representation of space takes the form of such a pure intuition. Bolzano's reasons for this departure also require Bolzano to give up on the idea that space can function as the general form of outer intuitions – and indeed, as Tolley argues, must give up on the idea that spatial representation can be involved in outer intuitions *at all*. Even so, Bolzano nevertheless joins Kant in accepting that space, and with it the subject-matter of geometry is an ideal (non-actual) object. What is more, Tolley shows that Bolzano also accepts that spatial representation does not represent any actual properties of outer objects, but only has a representation-internal role to play. According to Tolley, Bolzano's arguments for these conclusions draw on what Bolzano himself takes to be deeper commitments actually shared by Kant, all of which serves to point up a tension in Kant's doctrine of intuition which is still being explored today in the debates over Kant's alleged conceptualism.

Bolzano's departure from Kant on mathematical knowledge is also one firmly anchored in the conviction that the key to understanding logical and conceptual relations is the rejection of certain core features of the (Leibnizian, Kantian) conceptions of analysis and truth that were paradigmatic at the time. In their essay, Lapointe and Armstrong seek to situate Bolzano's contribution to logic and mathematics within the rationalist tradition – between Leibniz and Kant – and document the fact that in spite of his many reservations in points of detail, Bolzano's programme remains eminently post-Kantian. That a reform of logic is central to Bolzano's entire project, and by extension to his criticism of Kant's theories can hardly be overplayed. It serves as a very fruitful starting point for a reconstruction of the debate between the two philosophers. For Bolzano disagreed with Kant on a number of fundamental logical issues and endeavoured to produce new theories that would better suit the purpose of logicians. These include accounts of the nature of truth and representation, the analytic/synthetic distinction, the definition of what counts as a valid inference, and the proper definition of modal notions (necessity, possibility, and contingency).¹⁸ Yet even while Bolzano is surely critical of Kant, he also repeatedly affirms the conviction that Kant's philosophy has effectively much of interest and value to say about these topics. Indeed, contrary to a commonly held belief about Bolzano's sympathies, Lapointe and Armstrong demonstrate that Bolzano develops his own revisionary views on logic in the *Wissenschaftslehre* by engaging more directly and sustainedly with Kant and the Kantian logicians, than with Leibniz or his successors.

Despite his productive engagement with Kant and the Kantians, Bolzano did find himself in fundamental opposition to Kant on the issue of whether logic itself can undergo any further scientific progress. Bolzano not only believed (against Kant) that such progress was possible, but also viewed such progress in logic as a necessary pre-condition for lasting scientific progress in philosophy as a whole. While Kant's *Lectures on Logic* show, for instance, that Kant was interested in logic and its theory, he also famously declared that logic was "closed" and "finished" (1781, Bviii). Nonetheless, Kant devoted much effort to the philosophy of logic, especially to the proper characterization of its subject-matter. As Stang's essay documents at length, Kant's views on what is distinctive of logic, its *formality*, aroused much interest, not least in Bolzano himself. Yet though Bolzano agrees with Kant that there is a sense in which logic

¹⁸ These notions are all expediently presented by Příhonský in his remarkably concise and informative Introduction, to which we refer the reader.

is 'formal', Bolzano also is deeply critical of Kant's conception of what the relevant 'forms' are, and is critical in particular of Kant's distinction between the form and matter of a judgement. While acknowledging that Bolzano's own conception of the forms at issue in logic is a rich and fruitful one, Stang argues, first, that Kant can (and does) actually accept the core insight behind Bolzano's own characterization of logic as 'formal' (in its focus on substitutability). Stang argues, furthermore, that it nevertheless fails to capture something important that Kant was picking up on in *his* use of 'form', concerning what ultimately unifies the subject-matter of logic. Stang contends that Kant takes logic's principles to be formal in the deeper sense that their validity is grounded in the end or purpose set for our understanding as a capacity for thinking and cognizing – namely, that our understanding is oriented towards the truth.

The broader question of the ends, goals, and purposes of existence is the main focus of Rosenkoetter's essay, which explores the foundations of Bolzano's moral philosophy from the point of view put forward in Kant's *Groundwork*. As Rosenkoetter shows, Bolzano not only had a fully worked out broadly consequentialist theory of what it would mean to abide by this principle, he also uses this alternative to formulate serious and probing criticisms of Kant's deontological approach. Yet while Rosenkoetter also highlights the extent to which Bolzano's preferred version of consequentialism is one which is tailor-made, so to speak, to avoid Kant's most frequent criticisms of doctrines that ground morality in a duty to happiness alone (because Bolzano emphasizes the general – even impersonal – happiness at large, rather than any one person's individual happiness), Rosenkoetter concludes that Bolzano's position is ultimately unstable, and should either be developed in a direction towards the hybrid form of consequentialism later put forward by Moore, or should actually embrace something closer to Kant's position. Rosenkoetter's focus helps to remind us that, while Bolzano's views on logic and mathematics certainly take centre stage in many of his works, Bolzano's reasons for choosing to devote his life to teaching "Religious Sciences", as opposed to mathematics, were in great part rooted in what he took to be his moral obligation to bring about the greatest sum total of happiness in the world.

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2

Translators' Note

Editions, Pagination, Key Terms

We have based our translation on the original 1850 German edition (*Neuer Anti-Kant, oder Prüfung der Kritik der reinen Vernunft nach den in Bolzano's Wissenschaftslehre niedergelegten Begriffen*, von Dr F. Příhonský, A. Weller, Bautzen, 1850). The numbers in the margin give the pagination of the original 1850 edition. In addition to including a glossary giving our translation choices for the more central terms, we have judged it helpful in certain cases to indicate the German expression we are translating; in such cases we insert the German terms within square brackets.

Quotations, Additions, Corrections and Footnotes

New Anti-Kant contains a considerable number of quotations, many of which are inexact. In many cases, letters and even words have been omitted or changed by Příhonský, phrases have been added or are missing. These modifications are often insignificant, but they may also in certain cases betray substantial exegetical choices on Příhonský's (and/or Bolzano's) part. For this reason, we have chosen to follow Příhonský's text to the letter and we refer readers who wish to track the discrepancies to the editorial notes and insertions in Morscher's excellent new critical edition (cf. *Neuer Anti-Kant*, Beiträge zur Bolzano-Forschung vol. 9, Sankt-Augustin, Academia, 2003). Numbered footnotes are by the translators; those with asterisks are Příhonský's own.

Use and Mention

Throughout the text Příhonský deploys various devices to keep track of the distinction between use and mention – or, perhaps more broadly

put, to effect a semantic ascent, by using words (along with changing the typeface, or marking them off by a colon, etc.) to indicate either the words themselves or the representations that provide them with their sense or significance, rather than to refer to the objects usually represented by them. Yet though his use of these devices is crucial in many respects, Příhonský himself (following Bolzano) does not make use of a single, dedicated, systematic notation. Because the context makes matters clear in the vast majority of these cases, which shows that Příhonský is not confused about the distinction, we have chosen not to attempt to 'improve' on the original text in this respect, as the attentive reader will easily avoid misunderstanding.

Neuer
Anti-Kant

oder

Prüfung

der

Kritik der reinen Vernunft

nach den

in Volzano's Wissenschaftslehre niedergelegten Begriffen

von

Dr. F. Prihonsky.

Was durch Gründe der Vernunft
gewiß ist, kann eine entgegenstehende
Erfahrung nicht widerlegen.

Volzano's Wissenschaftslehre
§. 283 Nr. 5.

Baunzen, 1850.

In Commission bei A. Weller.

3

New

Anti-Kant,

Or

Examination

of the

Critique of Pure Reason

According to

the Concepts Laid Down in Bolzano's *Theory of Science*

by

František Příhonský

(Translated by Sandra Lapointe and Clinton Tolley)

*What is certain through grounds of
reason cannot be overturned by
an opposing experience.*

Bolzano's *Theory of Science*

§283, No. 5

Bautzen, 1850

In Commission with A. Weller

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Preface

Even today, an *examination of Kantian philosophy* would not prove to be a belated endeavour. *Two* different *editions* of *Kant's works** have appeared in just the past few years, which amply testifies to the interest one still takes in the philosophical research of this thinker. Kant is not a forsaken celebrity history would have now consigned to oblivion. He still counts a not insignificant number of disciples who at least in part applaud his views. His principles have provided the basis for many of the subsequent philosophical creations. Many of his most distinctive doctrines have passed into the new systems and blend into them to such an extent that it is not an error to say that they permeate contemporary philosophy. That is why *Hartenstein* can write (in the Preface to the new edition of *Kant's Works*, viii):

XIV

He (Kant) has the fortune ... that subsequent systems are for the most part developments of seeds that can be more or less determinately demonstrated in his writings, even if from time to time inner deformations are concealed by the illusion of a luxurious abundance. Fichte's idealism, which rests exclusively on the pure concept of the Ich; Schelling's older identity-theory, which makes intellectual intuition the source of knowledge; Hegel's dialectic; Herbart's monadology, which is grounded on the concept of being as absolute positing and on the recognition of the given; notwithstanding the peculiarities of each of these thinkers, all these things bring to mind thoughts, questions, and problems that Kant either first moved clearly into the light of philosophical consideration, or without allowing them the least applicability for human thinking, designated them as possible sources of cognition for other intelligences, or finally held them as essential and necessary corrections of deeply rooted errors.

XV

Marbach (in his *Lehrbuch der Geschichte der Philosophie*) even says, somewhat exaggeratedly, of our philosopher:

Throughout his long life, Kant devoted his exceptional profoundness to the progress of philosophy – it is no surprise then that we, his

* 1. *Immanuel Kant's Werke*, carefully revised complete edition in ten volumes. With a preface from *Hartenstein*, Professor of Philosophy at the University of Leipzig, 1838, Modes und Baumann.
 2. *Immanuel Kant's sämtliche Werke*, edited by Karl Rosenkranz und Friedr. Wilh. Schubert, Leipzig 1838, Leopold Voß.

posterity, could until today only further develop his view. Let only a half-century pass and then the history of philosophy be written; we will all be closely gathered around him, united under the rays of his mind whose light will make most of our singularities disappear.

Likewise, though more mildly, Dr Friedrich *Francke*, extraordinary professor of philosophy at the University of Rostock, also writes:

- XVI I freely admit that, in my opinion, it is not Schelling and Hegel or another mind of their kin but rather Kant and Fries, along with the greatest thinkers of centuries past and present, by whom all progress of healthy philosophical speculation now and henceforth must be oriented. (*Zur Theorie und Kritik der Urtheilskraft*, Leipzig 1838, preface, VI)

Prof. E.S. *Mirbt*, who is in almost every respect a Kantian, except for the few exceptions in which he opposes his master, explains in short: “The material for these (post-Kantian) systems is drawn for the most part from Kant’s works” (*Kant und seine Nachfolger*, Jena 1841, 171). The same holds for, Prof. J.E. *Erdmann*, who treats the Kantian system as “the seed in which all of the intellectual edifices of the modern period are contained *implicite*” (*Geschichte der neuern Philosophie*, Leipzig 1848, vol. 3, section 1, p. 24). Finally, the loyalty that the *Friesian* school, and at its head Dr *Apelt*, maintains to Kant’s main doctrines is well-known. – Is anything more required to justify the *investigation* that we are proposing?

- XVII If, in what follows, we not infrequently oppose the authority of a figure as well-known as the Sage of Königsberg, this may appear to some to be blameworthy audacity. This would be the case if we could not support our diverging assertions with sufficient grounds. But we believe to be in a position to do so, and indeed, to do so with the help of certain positions on which we will report later. Whether and to what extent we succeed is a question we must of course leave to the judgement of our reader. But we cannot help asking him not to conclude from the title of our small book that we intend to start up a passionate polemic. We chose the name *Anti-Kant* merely in order to designate our work concisely. The epithet *new* is meant, in part, to refer to the nature of the reasons on the basis of which we challenge Kant, and, in part, to distinguish this book from another already existing *Anti-Kant* (Stattler’s well-known work). We would therefore hope to be trusted when we declare that our efforts have no other intent than the advancement of truth. But in order to

eliminate any thought to the contrary and to convince our readers of how far we are from diminishing in the least the merits acquired by the great man as regards philosophical science, we would like to underscore here some of the most important and the most unquestionable of these merits. XVIII

It is undeniable that the period that immediately precedes the Kantian one was one of tepidity and stagnation for philosophical research, and that it was the Sage of Königsberg who once again awoke the spirit of reflection and stimulated and reanimated the enthusiasm for investigations of this kind. Indeed it would not be wrong to say that what has been accomplished in the field of philosophy since Kant's appearance has happened almost solely through his intervention and at his instigation. The new positions he presented set minds into motion in a variety of ways. Some took them up approvingly and sought to make them their own. Others who were less congenial spoke out against them. He himself, the initiator of this intellectual stimulation, never showed himself on the battleground, content with the fact that his students had taken it upon themselves to forcefully defend the contested assertions of their master. Admittedly they did not always succeed and astute opponents soon discovered points of vulnerability, which they used to attack him all the more successfully. In turn, Kant's friends saw themselves forced to fill the gaps that had become manifest, to correct the mistakes, to let go of what had become untenable, or even to attempt the construction of new systems. The fresh and rich life that necessarily awoke through all of this is certainly an uncommon merit Kant won for the philosophical sciences. Not only that: Kant also undisputedly assisted philosophy in fighting, not without success, the extreme methods of an undue dogmatism and a scepticism all too bold, methods that impair steady research and which had until then usually been observed in philosophical expositions. While attempting to investigate the grounds on which human cognition rests, he insisted that one neither decisively assert anything the truth of which one has not insured as much as possible beforehand, nor deny or doubt in a foolhardy way that for which no sufficient reason presents itself. He thus aspired to introduce the critical method in philosophy. It recommends and encourages scrutiny and modest research, and sets due limits to arrogance, whether in asserting or in doubting and denying. – But Kant gained even greater merit, not just for philosophy alone but for humanity as a whole, in virtue of the fact that he supported ethics with a purer foundation and freed it from egoistic motivations. Before him, moralists for the most part paid homage to the principle of personal happiness [*Selbstbeglückung*], a principle as false as XX

it is pernicious, which they not only sought to make valid in science but also to introduce into everyday life through popular writings. Now, it is easy to understand that men should have eagerly embraced and kept hold of a principle that so flattered their wishes, and it truly required Kant's entire, weighty authority to wrest it away from them, and to convince them of its falsehood and its deleteriousness; a task in which

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the great man fully succeeded. The previous inertness vanished gradually from the domain of morals and a praiseworthy earnestness took its place and penetrated ethics as a whole. – It is our deepest conviction that these are Kant's most praiseworthy accomplishments with respect to the philosophical science in general. But they presuppose that he has also applied himself to the particulars of it and that he, there also, accomplished great things. And indeed, how many interesting questions has Kant not aroused and attempted to answer! We bring to mind only the following as examples: Can one make synthetic judgements about supersensible objects? Are space and time actual things? Do their representations belong to the class of concepts or intuitions? What are we in a position to cognize about things? What is the nature of our knowledge? Is it merely subjective, and what limits are in general set for it? How many and which simple concepts of the understanding – categories –

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are there? And so on. – Finally, how many important and fruitful truths did our philosopher find and discover in almost all parts of philosophy, including logic! Indeed, had he accomplished nothing more than – I won't say grasping with full rigour, but rather – only pointing out and bringing to attention the distinction between *a priori* propositions and propositions of experience, between intuitions and concepts, between analytic and synthetic judgements – and this he did incontestably – this alone would secure him a name in the history of philosophy for all eternity.

As is well-known, Kant did not venture to advance a completed philosophical system of his own. Rather, he contented himself with demonstrating what was mistaken and unsatisfactory with the systems of his predecessors and offering mere indications and instructions for the construction of a new one. He did this primarily in the works which, because of the method he observes in it and which we praised above, he called *Critiques*. The most acclaimed and most complete of them is uncontroversially the *Critique of Pure Reason*. In it he laid down the most important results of his philosophical research, so that one can very well say that someone has examined Kant's philosophy if he has put under

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investigation the validity of the positions and assertions that are specifically comprised in this work alone. For this reason, we do not believe

ourselves to be committing a mistake when we confine our examination solely to this illustrious *Critique*. For what is specific to the *Critique of the Power of Judgement*, that of *Practical Reason* and other such writings of Kant's can easily be assessed by anyone who has followed our investigations with attention and to a certain extent appropriated the concepts we use here. We have borrowed them at least in part from Bolzano's remarkable writings, primarily from his *Theory of Science* or *Logic*. To be sure, more than a decade has passed since the latter work was published (Sulzbach 1837). Nonetheless, the most important concepts we find in it have received (oddly enough*) such a limited diffusion that they may still be considered as new, and that we can expect that only few of our readers will be acquainted with them.** For this reason, we have considered it necessary for a better understanding of the following discussions to start our treatise, *as a manner of introduction*, with a brief summary of some of the most exceptional of these concepts of which we mean to make the most use.

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* But not inexplicably; see Bolzano's *Wissenschaftslehre (Logik) und Religionswissenschaft in einer beurtheilenden Uebersicht*, Sulzbach, Seidel, 1841, 7ff.

** All the more that philosophers by profession seem to know nothing of the existence of a logic by Bolzano. See for instance, Dr Carl Prantl's *Bedeutung der Logik etc.*, München 1849, which omits almost none of the relevant modern writings with the exception of the most contentful and extensive one of all.

Introduction

The aspiration of any genuine philosopher must incontestably be directed at the cognition of the pure truth, and indeed, of the truth demonstrable to human beings. But here the question arises whether such truth exists and what we, in general, understand by *truth*. We thus believe that we ought to begin with an understanding concerning the meaning [*Verständigung*] of this remarkable concept.

We openly admit that we would not want to allow ourselves to lend to such an important word as that of truth a signification that would not be justified by linguistic usage. However, since the linguistic usage up till now has connected many significations with this word, it is likely to be necessary for us to determine more precisely the one according to which we intend to take this word here. It seems to us that this signification arises with sufficient distinctness from the following words:

- 2 “Among the three propositions: There were never any winged snakes on Earth; or they are now extinct, or there are still some, it is necessary that one be true or a truth.”

When one speaks in this way, it is obvious that by what is true or the truth one is thinking of a given *proposition in itself* [*Satz an sich*], whether there is anyone who holds this proposition to be true and expresses it, or indeed only represents it to himself, or whether there is no such being. We take the word truth, then, in the sense according to which there are *truths in themselves* and therefore also propositions in themselves which no one (except God) knows, and indeed which no one even represents to himself. So we say, for example: Before anyone had ever asked the question as to what is the digit of the thousandth decimal in the number π , one of the ten propositions we think when we declare that this digit must be a 0, or a 1, or a 2, and so on, is true or a truth. – Hence, if we speak of truths in this signification (Bolzano calls it the objective [signification]; see his *Theory of Science* §24), it becomes immediately clear that we presuppose that there are *truths and propositions in themselves* that must be distinguished from *thought* truths and thought propositions.¹ The latter occur in the mind of a thinking being, while the former cannot be ranked among thoughts; they are not thoughts at all – neither actual nor

¹ By adding the modifier ‘thought’ to, e.g. ‘propositions’, Bolzano has in mind something like ‘propositions [which are] thought’, etc.

possible. Thought truths and thought propositions therefore have an existence but one may not ascribe such an existence to either truths in themselves or to propositions in themselves. Truths in themselves have no *existence* (*Esse*), even less so an eternal existence. Admittedly, one says sometimes that the truths of religion and morals are eternal truths. But this is merely an improper way of speaking and has the sense that such truths consist in propositions that do not contain any temporal determinations as the condition of their validity, while the direct opposite is the case for certain other truths, for example, that a bushel of grain costs three imperial thalers, and so on. For, if the latter proposition is to be true, a temporal determination such as *at present* or something similar must be thought along with it. But for this very reason truths in themselves, propositions in themselves and thought truths, thought propositions cannot properly be considered as two *species* of truths and propositions *in general*. It would also be a mistake to declare propositions and truths in themselves to be something like propositions and truths **in abstracto**. The proposition in itself is not a judgement in abstraction from the judging being, for it would also in this case remain something actual, just as a watch in abstraction from the owner who wears it still remains something actual (something made out of metal, and so on). Propositions in themselves stand to thought propositions somewhat like flowers stand to painted flowers or paintings of flowers, only with the difference that flowers are something that exists, but propositions, as we said, are not something that exists.*

Were someone to ask what then is a proposition in itself or a truth in itself, if it is not something that *exists*, we would answer that not every something must be an existing something, just as we consider what is merely possible, to which surely there belongs no actuality (no existence), always to be a something and not to be a nothing. But, one may object further, if truths in themselves have no existence, in what sense can you still claim that *there are* truths in themselves? What is this *there are* supposed to mean, if it does not mean existence, and with what right can you still say, if truths in themselves do not exist, that some propositions *are* true – and others *are* not true? Don't you ascribe an *existence* to them, when you say the latter? – To this we reply that we are not the first to observe that the linking word (**copula**) **Est** or the **Est tertii adjecti**

* The reason why we here group propositions together with truths finds sufficient explanation in the fact that truths in our sense are propositions, i.e. true propositions, such that, whatever holds for propositions in general must hold for truths as well.

that occurs in propositions does not have the signification of proper existence at all, as the old Schoolmen made such an observation long before us. Hence, for example, in the proposition: The possible as such *is* nothing actual, the word *is* surely is not meant to indicate *existence* (Esse). If one asks: What signification does this copula actually have?, we answer: essentially no other than that of the *having* of a property. For

5 every proposition of the form: A is B has in fact no other sense than: A *has* the property B; for example, God is omnipotent, means in fact only: God has omnipotence; The possible – is – not actual, means nothing else than: The possible – has – no actuality. Now, when we claim in particular that a given proposition *x* is *true*, we say nothing other than that this proposition ascribes to the object with which it is concerned a property which belongs to the object, or which the object *has*. And when we in general say *there are truths*, this has essentially only the sense that not all propositions are false, or we advance with it the following claim: *the proposition that all propositions are false is itself false* (that is, has no truth).

We take the concept of propositions and truths in themselves to be so important that we cannot urge our readers strongly enough to make it their own and to keep hold of it. All will be lost if they cannot grant us this concept, if they keep representing truths in terms of certain *thoughts*, appearances in the mind of a thinking being, or if they claim, after we forbid this, that they do not know what they are supposed to think by truths in themselves if not the *thing itself*, about which one judges. This, it is absolutely not. We believe that we must well distinguish the thing itself and the proposition that asserts something

6 about this thing, that either ascribes or denies it a property. There is often only one single thing or object with which a proposition is concerned, but there can be innumerable propositions, true and false, about it. Likewise there are other cases in which there are infinitely many objects with which the proposition is concerned, while there is only one single proposition; for example: Every equilateral triangle is also equiangular. But why should we worry that the reader could not understand us or that he could not agree with us? For we here insist on the recognition of a concept that has been acknowledged and advanced by so many philosophers and which even comes up often enough in everyday life. What the ancient logicians called **veritatem objectivam, transcendentalem, metaphysicam** and defined as that **quæ nemine cogitante dicit, uti res est** is nothing other than the truth in itself in the signification advanced here. – And how often do we not hear in the mouth of the common man the words: if only one knew what is

true! Does one here think by what is true an assertion, a judgement or even only the mere representation of a proposition that is to be found in the consciousness of some thinking being (be it even that of God's, that of the Omniscient one, to whom all truths are certainly familiar)? Certainly not. One presupposes rather that there are truths which, with the exception of God, no one else knows. 7

One does not want to suggest, with this presupposition, that no one is in position to *cognize* what is true, i.e. some one truth or some truths. No, the common human understanding, from which stems the wish that one only knew the truth, does not stray so far as to deny all cognizing. One who would advance this proposition and would thus claim: We humans cannot cognize one truth, or: All our judgements are false, would contradict himself. But if one sees himself forced to admit that at least one truth is cognizable for him, an easy inference will lead him to conclude that he must cognize many truths, indeed infinitely many, or that *truths are in general cognizable*.

If one grants us that the concept of a truth in itself, and therefore also that of a proposition in itself, is not an empty one, and that we are in a position to cognize truths, then we claim further that every *proposition in itself*, not merely every *thought proposition*, is composed of certain parts that are not themselves propositions. For every proposition is concerned with a given object, but how could this happen if there were not, in this proposition, something that *represents* this object? Furthermore, in every proposition something is *asserted* about the object with which it is concerned, or to speak with more precision, a given property is either ascribed to or denied of this object (the latter meaning nothing else than that the lack of this property, which is likewise a kind of property, is ascribed to this object). How would this be possible if we did not find in the proposition something that represents this property? – Let us call *representations* all the parts of a proposition that are not all by themselves propositions, however else they may be constituted. Just as a thought proposition consists of *thought representations*, a proposition in itself consists of *representations in themselves*. Admittedly, by its etymology, the word *representation* evokes, not only in German but also in other languages, certain associated concepts that are irrelevant. One must abstract from these and keep only to the given definition. The case is not different with the names of other very general concepts. What awkward associated concepts do not the words: square, cube, root, power, exponent, factor and hundreds of other designations carry with themselves! But we nonetheless use them in arithmetic without any disadvantage. – 8

9 If the proposition in itself is not something that exists, then its components, the representations in themselves, should not be considered to be something that exists either. In insisting on this understanding of the concept, we are not doing anything unheard of. Other philosophers have taken, if perhaps not the word *representation*, in any case the word *concept* quite often in the signification in which we take it; for example, when they answer the question whether there are concepts that are wholly identical with one another in the negative, and indeed for the subsidiary reason that what one could see as two or more identical concepts would be nothing other than the same concept *thought* twice or many times, or designated by two or more synonymous *words*, for example, **triangulum** and **trigonum**.

One must not confuse the representation with the *object* that is represented through it. Thus the representation: Human, is in itself not in the least something actual, nor something alive endowed with a body and a soul, etc., though all of this surely holds of the object that this representation represents. Moreover, several and even infinitely many objects are often represented by one and the same representation. Thus the representation: human, has doubtless many objects; the representation: triangle doubtless infinitely many. But there are also representations that have only one single object. We call them for this reason *singular representations*; for instance, the representations: God, universe, lowest prime number, and so on.

10 As is shown by the examples of representations we've just introduced: human, God, universe, there are representations whose object is something actual (existing). But there are also representations whose object is not something existing. The *representation of a representation in itself* is of the kind of which we just demonstrated that its object is nothing that exists; so is the *representation of a proposition in itself, of a truth in itself*, among many others. – Indeed, we even dare claim that there are also representations that have no object at all, and which we call for this reason *objectless* representations. In contrast to these we call the remaining representations *objectual*. The representation: nothing, is doubtless an objectless representation; likewise, the representations: round square, 0, $\sqrt{-1}$, **log**(-1), among many others. Some have at times doubted that objectless (imaginary) representations such as $\sqrt{-1}$ deserve the name of genuine representations. Some claim that these are just words assembled like a representation. Hence one has often declared them to be **voces sine sensu**. But expressions such as 0, $\sqrt{-1}$, **log**(-1), among others, are not senseless words like: **abracadabra**, etc. We learn to connect them with certain very distinct thought

concepts and indeed connect them with concepts we also use in mathematics with the greatest success for the discovery of the most important truths. If the representation: a solid that is delimited by 20 equal surfaces, is called a genuine representation, why should the representation: a solid which is delimited by 24 equal surfaces not be a genuine representation? That no object corresponds to the latter is admittedly true. But must we, with the word *representation*, necessarily think an object that is represented? It is in any case indisputable that such representations can also occur as components in true and important propositions. Indeed, the representation in a proposition that forms the subject-representation can never be objectless (the proposition would otherwise have no object with which it is concerned; how could it be true?). Just as little can the representation that is the so-called predicate-representation (the representation of the property, which the proposition ascribes to the subject) be objectless (otherwise the proposition would not ascribe a property to its subject). Nonetheless, it is beyond doubt that imaginary representations can also occur in a proposition, namely only as components in its subject- or predicate-representation. So, for instance, the proposition: The representation of a round square is objectless or does not have an object – is certainly a truth. But in this proposition, the subject is: the representation of a round square, and the subject-representation is: the representation of this representation. The latter, however, is an objectual representation, and its object is precisely the objectless representation whose objectlessness the proposition asserts.

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Every proposition in itself is necessarily composed of certain parts (a subject-representation, a predicate-representation and a copula which is the concept of having). Likewise, not all but certainly *many representations* are *composed* of certain other parts which, following the definition we have already given, insofar as they are not themselves whole propositions, we must again call representations. So, for example, the representation of an *equilateral triangle* is certainly composed of the representations of a triangle and of certain other representations. The representation: equilateral triangle, is namely entirely the same as the one expressed by the several words: A triangle, whose sides are all equal. For this reason, we take it that, aside from the representation: triangle, this representation still contains the representations which the words: side, equal, among others, designate. Likewise, we believe that the representation *nothing* is composed of the representations: something and not, or nothing = not-something [*Nichtetwas*]. Let us therefore be allowed to call representations *complex* when they consist of several others, and

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simple, by contrast, when they do not have any further components. There must doubtlessly also be simple representations. The representation designated by the word *not*, the representation *something* should, without objection, be recognized as simple.

- 13 If a given object has the properties **a, b, c,...**, then through the connection of the representations **a, b, c,...** (and some others) we can very easily compose a representation: Something which has the properties **a, b, c,...**, which, if it does not apply exclusively to this object alone, at least comprises it with some others, and can therefore in any case be called a *representation of this object* (or, in the latter case, of this object and some others). This should not however lead us to believe that all objectual representations originate in such a way, much less that all properties of an object must be found as components in the representation of the object, even when this property relates to this object exclusively, nor that every component that is found in a complex objectual representation represents a property of that object. Neither the one nor the other. For instance, the representation: equilateral triangle, contains the concept of equilaterality, which is a property that belongs to its object, but many other properties also belong to this object despite there being no mention of them in this representation, for example, equiangularity, equality of all perpendiculars that obtain from the angles to the opposing sides, and so on. – It is even more obvious that in every complex representation there are several components which cannot be said in the least to designate properties of the objects that stand under them. So, in the representation: non-equilateral triangle, the representation of equilaterality is found
- 14 connected with the concept of negation, but equilaterality is so little an attribute of a non-equilateral triangle that it is denied of the latter in the very concept. – Finally, it is also worth noting that while there is always only a modest number of components in a complex representation, its object can have innumerable many properties. Only the following few components, for example, make up the representation of a circumference: It is namely the line, the points of which are all equidistant from two points (the poles of the circle); there is, however, an infinite multitude of properties of the circumference, such as: it is a curved line, a closed line; all its parts rest on a plane; it has a centre, has a diameter, a radius, and so on. – To specify the *components* of a complex representation is to *define* it.

Following these preliminaries, we are in a position to make our reader familiar with an objective distinction between representations in themselves that is of the greatest significance. Obviously, in relation to their

extension, the most noteworthy representations are those that have only one *single object*; in relation to their *content*, however, the most noteworthy are those that are *simple*. – Let us now take these two properties together and form the concept of a representation that unites these two properties, that is, the concept of a representation which is simple with respect to its content and which has only one object with respect to its extension. We can then consider that if there were in fact any such representations, they would be of an especially noteworthy species. If we add with Bolzano, even if this addition does not seem necessary to us, that this one object is an *actual* one, the importance of this kind of representation increases all the more. Bolzano gives them a name of their own: *intuitions*, and shows that the concept that Kant and other German philosophers of modern times have associated with this word essentially fits these representations entirely. By contrast, every representation that is neither an intuition nor contains intuitions as components he calls a *pure concept* (see *Theory of Science* §72f.). But one might doubt that there are representations such as the ones we have described as intuitions. If a representation is supposed to represent only one single object, and furthermore, one that is actual, then one might believe that it must be composed of a great number of parts, because it is only through the specification of very many of the properties that belong to this object that one can compose a representation that fits exclusively this one object and no other. How, then, could a representation that is not composed of many parts, that is absolutely simple, nonetheless represent only one single object? – This would indeed be impossible if, as one often imagines, every property of an object had to be thought together in a representation that pertains to that object uniquely. But we have already noted above that this would be mistaken. With this, of course, the existence of simple representations that also have one single object is not demonstrated. Rather, this follows from the following consideration. There are certain subjective representations that arise in us initially and immediately every time we turn our attention to the changes in our soul that are brought about by a body that presents itself to our senses, for example, a lemon. The representations that are thereby produced initially and immediately must be simple, as complex representations can only appear mediately, through the connection of several simple ones. They must furthermore have something entirely singular as an object, only the change that is happening right now in our soul. Representations that have several objects may also arise on such an occasion, such as, for example, the

17 representation of yellow-green, if we feel ourselves pressed to form the judgement: *This* (what I see presently) – is – of the colour yellow-green, etc. But it is obvious that we only arrive at the latter representation *mediately* and only through the simple representation that we designate with the little word: *This*. There are thus at least subjective representations which, regardless of their simplicity, represent nonetheless only one single object. Hence there are *subjective* or *thought intuitions* to which must naturally correspond *objective* representations of the same kind (intuitions in themselves). All representations which, in language, we indicate with the word *this* thus belong in the class of intuitions. It is true that in oral communication we typically add some other determinations, and say, for instance: This, which I presently see, smell, feel, etc. But we do this merely in order to make ourselves clearer to the hearer, and by no means because this addition seems necessary to the formation of the representation itself.

This objective distinction between representations, by means of which we can divide them into intuitions and pure concepts, also puts us in a position to form an objective distinction between propositions. Propositions that are composed solely of pure concepts are to be distinguished from those that contain one or more intuitions. We call the former *pure conceptual propositions*, while the rest may receive the name of *empirical propositions*.

18 Applied to *truths*, this distinction leads to the determination of some universally accepted concepts, namely: *possibility*, *contingency*, and *necessity*. We wish to close our introduction with the definition of these concepts. But we see ourselves forced to first present another concept of even greater importance and to decompose it into its components: that of the *deducibility* of propositions from one another.

Bolzano made a discovery whose correctness will be difficult for anyone to deny: in our representations as well as in our propositions, we sometimes take certain components to be variable and consider the behaviour that these propositions and representations observe when, in the place of those parts that are considered as variable, we put others, whatever they may be. Of a particularly great significance is the case in which the propositions **A**, **B**, **C**, **D**, ... and **M**, in which the components **i**, **j**, **v**, ... are supposed to be variable, stand in such a relation to each other that any determination of these variable parts that make the propositions **A**, **B**, **C**, ... true, also make the proposition **M** true. In this case we say namely that the proposition **M** is *deducible* from the propositions **A**, **B**, **C**, ..., that it can be *concluded* or *inferred* from them;

or we call the propositions **A**, **B**, **C**, ... the *premises*, and **M** a *conclusion* that results from them. (See the *Theory of Science* §155.) The following example will serve as an illustration. We say: If Bautzen is located to the east of Dresden, then the sun comes up earlier in Bautzen than in Dresden. This, however, has no other signification than: the two propositions stand in such a relation to one another that whenever one allows oneself to exchange the representations Bautzen and Dresden so that the first proposition stays true, the second one must also be true. One may put in the place of Bautzen and Dresden whatever other town one likes, as long as the first proposition is true; that is, as long as one puts in the place of Bautzen a town that actually lies to the east, for example, Wroclaw, Warsaw, Petersburg, etc., and in the place of Dresden: Jena, Cologne, Paris, etc., the other proposition will also be true and thereby one that is deduced from the first. The usual expression for this relation is, as the example shows: If **A**, **B**, **C**, ..., then **M**, and one has since time immemorial noted that the words: *If – then* mean just about the same as the words: *Each time* the propositions **A**, **B**, **C**... become true, the proposition **M** also becomes true. However, it was not recognized or stated with sufficient distinctness that the propositions **A**, **B**, **C**, ... and **M** could not become *sometimes true, sometimes false* if no components in them were considered variable, and that one should consider instead of these individual propositions, the entire genus of propositions that appear when one puts in the place of these variable parts others that are arbitrary.

Now at last we can define with complete distinctness what one understands when one asserts of an actual object *x* that its existence is *necessary* or when, now and then, one ascribes to mere propositions and truths the predicate of *necessity*, that is, to things that do not have existence; namely nothing other than that the proposition: the object *x* exists, is the consequence of pure conceptual truths. *Necessary* is: what follows from pure conceptual truths. *Contingent*, by contrast, is an existence which is not necessary, i.e. which is not deducible from any pure conceptual truth. *Possible*, in the strict sense, finally, is what does not contradict any pure conceptual truth, i.e. that whose non-existence is not deducible from pure conceptual truths. – We also often use the phrases: That is possible, That could be, and so on, in a much broader sense, one which is not unimportant for science, namely when we want to indicate that the existence of this thing does not, as far as we know, contradict any of the *pure conceptual truths that are known* to us, or that its non-existence is not deducible in any manner known to us from any conceptual truth known to us.

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Treatise

Kant opens his *Critique* with an *Introduction* (35–56/B1–B30)* and he begins this by claiming that there are assuredly two kinds of cognitions that need to be distinguished; namely, *pure* or *a priori* cognitions and *empirical cognitions* or *cognitions of experience*. If he otherwise wanted to provide acceptance and recognition for his further claim that philosophy is a decidedly *a priori* science, everything depended on his casting this distinction into proper light and placing it beyond doubt. For at the time, this still had not been fully comprehended with the necessary degree of distinctness and it is still not understood today, except in Germany. Philosophers in England and France still have not been able to decide in favour of such a separation of human cognitions. Indeed, 22 nowadays, even Germans seem to lose sight of this important difference from time to time and hold that philosophical cognition is at least not *purely a priori*. *Beneke* and his followers, for example, want to derive all philosophy from experience.** *Kant's* old supporter, *Fries*, who has many admirers among German scholars, claims that empirical psychology is the most profound foundation for all other philosophical sciences.*** Finally, *Schelling's* new school, led to this belief by their master's pronouncements in his famous preface to *Victor Cousin's* preface on the topic, expects from the connection of both kinds of cognition nothing 23 other than the completion of philosophy.

The importance of the distinction between *a priori* and empirical cognitions for philosophy was so evident to our philosopher that he did not hesitate to place it at the head of his *Critique*. Nonetheless, he nowhere felt inclined to provide a more exact definition of this distinction. Rather, he was content to give one or two examples and add a few phrases just as obscure as what they were supposed to define. Assuredly, we read (on 35/B2 of the same introduction) that “all our cognition begins *with* experience, but does not, however, for this reason originate

* 2nd volume of *Gesamtausgabe*, Leipzig 1838, *Modes und Baumann*, which will here be cited throughout.

** When he reviews *Hartenstein's* “*Die Probleme und Grundlehren der allgemeinen Metaphysik*” – in the *Ergänzungsblättern zur Haller allgemeinen Literatur-Zeitung vom Jahre 1837*, *Beneke* declares that the main difference between his and *Herbart's* philosophy is that *Herbart* thinks that philosophical cognitions consist of mere concepts (*a priori* concepts) while his philosophy springs from experiences of the mind (he calls these inner experiences).

*** *Handbuch der psychischen Anthropologie*, vol. 1, 2nd edition. 1837.

from experience”, for it could well be that “our faculty for cognition, merely prompted by sensible impressions, provides something of itself” to our cognition, and this is what he understands under the name of *a priori* cognition. However, soon after (36/B2), and as though this were the same thing, our author claims that *a priori* cognition is completely *independent* of the impressions of the senses, and indeed he seems to believe that he has expressed himself in the most exact manner by merely adding: “not independent of this or that experience, but *absolutely* independent of all experience”. “Those cognitions”, he continues, “are empirical which are possible only through experience”; by contrast, pure *a priori* cognitions are those “in which nothing empirical from experience is intermixed”. [B3] For example, the proposition “every change has a cause” is “an *a priori* proposition, only not pure, since every change is a concept that can only be drawn from experience.” [B3] 24

At this point everyone will immediately ask why the concept designated by the word *experience* is supposed to be already known and in no need of a definition. Would it not have been befitting, where the distinction between *a priori* and *a posteriori* cognitions is defined, to also define the concept of experience itself? According to our position, the latter concept is in fact more complex than the former, and we define what experience is only by first having defined what conceptual propositions and intuitional propositions are. – Further, how are we to understand the claim that a certain cognition begins *with* experience with respect to time, but does not originate *from* experience? Doubtless, we are to represent through this that *a priori* cognitions are not *grounded in* experience? But is this true? Is the arising of our *a priori* cognitions, for example, our mathematical cognitions (and Kant very aptly ranks them among *a priori* cognitions) not in fact – not even *partially* – grounded in experience? Surely no one would or can actually claim this: how else could it be that we come to *a priori* cognitions only after we have had experiences? Why does all *a priori* cognition commence with experience if it is not *conditioned* by it and thereby at least partially grounded in it? – Finally, how can Kant say that *a priori* cognitions are *independent* of experience, indeed of all experience, *absolutely independent*? For does he not and must he not admit that experience is necessary at least for the *instigation* of them or as the condition of their arising? – Our confusion is complete when he adds the example according to which the proposition: Every change has its cause, is supposed to be an *a priori* proposition, but not purely *a priori* because *change* is a concept that can only be drawn from experience. In our opinion, the concept of change is as pure 25

as any other. For to say that a thing *changes* is to say nothing other than that it has different conflicting properties at different times. Therefore the concept of change contains absolutely no other components other than the concepts: substance, property, time, etc. It is composed exclusively of pure concepts and does not comprise in itself a single empirical representation or intuition.

As if he had felt that his definition of the distinction between *a priori* and *a posteriori* cognitions was not sufficient enough, our philosopher begins with this subject again in section II of his Introduction, even though according to the title (it reads: “We are in possession of certain *a priori* cognitions, and even the common understanding is never without them”), he should not actually have been engaging in the determination of these concepts anymore. He says: “At issue here is a *mark* by means of which we can securely distinguish a pure cognition from an empirical one.” (36/B3) – But we think that if the distinction between *a priori* and *a posteriori* propositions had been traced down to its innermost essence, there would be no need for further marks to differentiate the two, or, rather, we could find the clearest marks among the components of which these concepts themselves are composed. Kant gives two marks by which *a priori* cognitions are meant to be distinguished from *a posteriori* cognitions, namely *necessity* and *universality*. “*Firstly*, if a proposition is thought with necessity, then it is an *a priori* judgement; *Secondly*: experience never gives its judgements true or strict universality, it only gives them assumed and comparative universality (through induction). Hence one must actually say: As far as we have perceived up to now, there is no exception to this or that rule. Thus if a judgement is thought with strict universality, in such a way that no exception at all is allowed to be possible, then it is not derived from experience, but is rather valid absolutely *a priori*.” (37/B3f.) What now shall we make of these two marks?

As regards the expression: A judgement has necessity; this can be taken in different senses. In a certain sense, we understand the necessity of a judgement to be a relation between the latter and our power of judgement in virtue of which we feel *compelled* to make it, i.e. we try in vain to hold its opposite to be true. In this sense, many a judgement that Kant counts and must count among *a priori* cognitions – if he doesn’t want to weaken his distinction to the point that it becomes useless for its purpose – has no necessity. The judgement that the relation of the diameter to the circumference is irrational, in this understanding, would not in the least be an *a priori* judgement and made with necessity, for it is known that there were mathematicians who doubted it. Nonetheless,

Kant counts all mathematical judgements among cognitions *a priori*, and certainly very correctly so. In yet another sense, Kant himself also distinguished apodictic judgements, or judgements asserting a necessity, from others, the problematic or merely assertoric ones, in such a way that, for instance: the proposition: God *must* reward the good, is an apodictic or necessary judgement; the proposition: Every finite being can sin, is a problematic judgement; and finally the proposition: Adam actually sinned, is an assertoric judgement. But it is obvious that pure conceptual truths can belong to any of these three kinds of propositions; for example: The hypotenuse must always be longer than each of the other two sides; Its relation to these two sides can be either rational or irrational; In this triangle, the relation is actually irrational. However, there is one signification which allows us to rightfully claim that necessity is a mark, and at once a universal and exclusive mark of *a priori* truths – namely, if we take the word *necessity* in its most distinctive signification. However, this signification is of so little use for distinguishing *a priori* propositions and truths from others by means of this mark, that rather, conversely, we can only judge whether something is necessary or not if we can judge whether a proposition that lies before us is *a priori* or not. In this sense something is called *necessary* if and insofar as it is a pure conceptual truth or if it can be concluded from a pure conceptual truth or (what is really the same) if its opposite contradicts a pure conceptual truth.

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The second mark, being also ambiguous, does not fare much better. In a certain sense, universality pertains to all empirical and even to all propositions in general. By contrast, in another sense, even *a priori* truths might themselves lack universality. If by the universality of a proposition we understand merely that its predicate applies to all objects that are contained under its subject-representation, then every true proposition that is expressed correctly has universality; the property that constitutes its proper predicate-representation must always inhere in all of the objects that stand under its proper subject-representation. This is because every proposition is contained under the form: **A** – has – **b**, where **A** is the subject-representation, **b** is the predicate-representation that ascribes a certain property (or, with negative propositions, the lack of the latter) to the subject of the proposition. Thus, universality is not a peculiarity that pertains only and exclusively to *a priori* truths; for example, the empirical proposition: Niobe's children all died before their mother, has universality, so long as it is true. – But in another signification, according to which universality is ascribed only to those propositions whose subject-representation comprises *several* objects under

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itself, there are *a priori* truths that fail to have the mark of universality; for example: God is omnipotent, The highest moral law is a practical truth. And finally (in a third signification) whoever opposes universal propositions to particular propositions, like: Some people are vicious, etc., cannot claim that universality, so understood, is an attribute that pertains to all *a priori* truths. For the pure conceptual truths: Some triangles are equilateral, among others, would have to be ranked among the particular ones in this latter signification.

30 However, we have to admit that we do not accept this last distinction between propositions, according to which they are divided into universal and particular ones, if this has the sense, namely, that the predicate-representation is supposed to pertain to all objects of the subject-representation in some propositions, and only to some objects of the subject-representation in others. Instead we believe that in every true proposition the predicate-representation is rather to be related to all objects of the subject, and indeed also in the so-called particular propositions. For here the subject is not, as one generally believes, "some A". Rather, they are to be grasped under the form: **A cum B** has objectuality. For example, the proposition: Some humans are learned, certainly does not mean that the predicate of learnedness pertains to every collection of some humans (that would certainly be false); instead it means that *being human* and *being learned* form a representation that possesses objectuality.

Even though we cannot reconcile ourselves with the way in which Kant distinguishes between *a priori* and *a posteriori* cognitions, we have to acknowledge that such a distinction actually obtains, and it is Kant's merit to have at least pointed it out with greater distinctness. However, another consideration convinces us that his conception of this very important difference is inappropriate.

31 The manner in which Kant attempts to distinguish *a priori* propositions from empirical ones is essentially tied to the circumstance that there are *thought propositions* or *judgements* which this or that person makes and holds to be true, or that there are true judgements, i.e. *cognitions*. This can only be applied to cognitions, i.e. to true judgements alone; for only of these can one assert, in a certain sense, that necessity and universality pertain to them. If we consider the matter more closely, then it becomes obvious that not only true, but also false judgements, and indeed, not judgements alone, but propositions in general, whether they are taken to be true by someone or not, and whether or not they have ever been grasped in the mind of any thinking being, can be distinguished from one another in a way that

deserves attention and that can very well be the basis for a division of *a priori* and *a posteriori* propositions. Not only the true propositions: Every finite rational being is beset by vices, The sphere is the most perfect body, Every proposition must have a *copula*, and so on, but also the false propositions, like: All created substances have arisen in time, The diameter of a circle stands in a relation of 1 to 20 to its circumference, Truth consists in the agreement of the representation with its object, etc. are essentially distinguishable from the following true and false propositions: This plant belongs to the genus of lilies, Aldebaran is a double star, There are also tailless monkeys, etc. If we are to determine what the genuine distinction between these two kinds of propositions consists in, then we believe we would have to place it in the fact that the former (the *a priori* propositions) consist of *nothing but concepts*, while the latter, by contrast, also contain *intuitions*. If Kant had known this distinction, and if he had proceeded from it, then it is unlikely that he would have fallen prey to such empty and incorrect definitions and marks as we have seen him fall prey to. It almost seems as if he had wanted to make the determination of whether a cognition is *a priori* or *a posteriori* dependent upon the merely contingent circumstance of how we gain the cognition, whether in the course of experience or through mere reflection. But how unsteady is such a distinction! How uncertain when it comes to making a decision on the question of whether a cognition belongs to one or the other of the two classes! Must one not admit that we reach most, and to a certain extent, all of our cognitions, even the *a priori* cognitions, by means of experience? – How completely different do things stand according to the conception we have given of this distinction! For even though the scientific definition of an intuition and a concept is subject to some difficulties, it is unlikely that there will be anyone who is unable to distinguish an intuition from a concept and vice versa, and who therefore could not determine whether a proposition is composed solely out of concepts, or whether it contains one or more intuitions among its components. Furthermore, as we said, Kant relates everything only to mere cognitions. By contrast, our distinction has the advantage of being a thoroughly objective one, one which does not rest upon certain external circumstances but rather is grounded in an internal attribute that depends on the object itself and which already pertains to the proposition in themselves, not merely to their appearances in the mind (to judgements and cognitions). This is why we would almost wish that one would let the confusing designations: *a priori*

and *a posteriori* drop away entirely and instead of them speak only of *conceptual* propositions and *intuitional* propositions.

After having acquainted us with the essence of *a priori* cognitions, Kant advances the doctrine that we are in possession of certain *a priori* cognitions, and strengthens this by offering examples from mathematics and pure physics, with which we declare ourselves to be completely in agreement. For there really is a multitude of pure conceptual propositions, both among propositions in themselves as well as among the judgements we make, that we assert with complete confidence as true and correct and that we consider to be cognitions because all humans endowed with reason will agree to their assertion at all times. Everyone can also agree with Kant's claim that "philosophy needs a science that determines both the possibility, as well as the principles and the extension of all cognitions *a priori*" (39/B6), assuming that one has convinced oneself of the correctness of the distinction between cognitions that we

34 have given.

However, Kant gives expression to another important and so far underappreciated distinction that takes place among cognitions, or rather, truths (true propositions). (42f./B10) For he remarks: "in all judgements, there is a twofold relation between subject and predicate. Either the predicate **B** belongs to the subject **A** as something that is (covertly) contained in this concept **A**; or **B** lies entirely outside the concept **A**, though it surely stands in connection with it." (B10) He calls the former judgement an *analytic* one, the latter a *synthetic* one. He cites the proposition: All bodies are extended, as an example of an analytic judgement. For in order to connect extension with the subject: body, it is in no way necessary to go beyond the concept of the body, because in it I indeed encounter this predicate as soon as I decompose this concept. By contrast, the predicate: heavy, is something entirely different from what I think in the mere concept of a body in general.

We note that, although we hold the division of judgements into analytic and synthetic ones to be one of the most fortunate and influential discoveries that has ever been made in the domain of philosophical research, it still would seem to us that Kant did not grasp it with the necessary degree of distinctness. Even if we were prepared to overlook the fact that he ignores a well-known rule of logic and includes metaphors in his conceptual determination when he uses expressions like: "covertly" and "entirely outside the concept", we would still have to criticize his definition of analytic judgements for being too broad.

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According to the latter, propositions that no one would rank among the

analytic ones, for example: Alexander's father, king of Macedonia, was king of Macedonia; a triangle that is similar to an equilateral triangle, is itself equilateral, etc., would have to belong to the analytic ones. This mistake could be avoided by saying that analytic judgements are those in which the predicate is one of the essential components from which the concept of the subject is composed. Unfortunately, that would just mean that the definition is flawed in another way: it would be too narrow. It would assuredly fit analytic judgements of the form: "A, which is B, is B", but not at all judgements of the form: "A, which is B, is A", or similarly: "Every object is either B or not B", even though these are obviously of an analytic nature. Kant's definition of analytic judgements, and accordingly also that of synthetic judgements which he opposes to them, is not satisfactory, and this all the less that it completely neglects the genuine essence, the distinction that must have been most important to the philosopher in advancing this division, namely that the truth or falsity of certain propositions (and these are none other than the analytic ones) is no way dependent upon those individual representations of which these propositions consist. Rather, they remain true or false, no matter which changes we undertake with one or the other of their representations, just so long as the proposition itself retains its objectuality. The two propositions: "A being that is conditioned is not a God", and "A being that is conditioned is a God", are accordingly analytic propositions, the former true and the latter false. They remain true and false, no matter which change one might undertake with the representation: being, and no matter which representations one puts in its place: angel, human, animal, plant, or whichever other representation, just so long as the propositions so formed still retain their objectuality. 36

By contrast, propositions in which this is not the case, which do not consist of variable representations of this sort, like, for example, the propositions: God is omniscient, There is a highest moral law, The sun warms, and the like are called synthetic. According to this conception of analytic and synthetic propositions, their existence is beyond doubt. This is not the case with Kant's determination. For, since Kant declared that synthetic propositions are those in which the predicate is not contained in the subject, it did not suffice in order to prove that there are such propositions that he point to their existence in mathematics and pure natural sciences. Rather, he should have shown that there are properties which pertain to all of the objects that stand under a certain concept, without it being the case that the concept of these properties is contained as a component in the concept of these objects. For it is this 37

proposition that has up to now escaped many logicians and this has in return moved them to deny the existence of synthetic propositions.*

Kant however disregards all this, and instead hurries on at once to answer the question that is so important to his critique of reason: *How are synthetic judgements formed?* To this end, he distinguishes between synthetic judgements that are at the same time of an empirical nature or judgements of experience, and synthetic *a priori* judgements.

As regards *judgements of experience*, which are definitely synthetic, it seems that Kant believed that their arising can be explained without *difficulty*. For, in this kind of judgement, the predicate, even if it is not contained in the subject, can still be found in an *intuition* that is connected with the subject, and that is why we are justified in ascribing it to the subject. So, for example, the judgement: All bodies are heavy, comes about when we find the mark of heaviness, which is obviously not contained in the concept of body, in the intuition of the body and ascribe this to the latter (43/B12). – But this does not suffice to explain the arising of judgements of experience. Can one *single intuition* which we have determine us to make the judgement: All bodies are heavy? At most, I could perhaps be caused by this to utter the judgement: *This body is heavy*. But how do we get to a universal judgement by means of one single intuition? Kant does not answer this anywhere. Indeed, even the manner of arising of the singular judgement: *This body is heavy*, is not explained by the Kantian specification. For if it actually were as Kant claims, then the predicate “heavy” would be contained in the intuition which we have of a body, and therefore in that representation which forms the subject-representation of the judgement. And then the judgement would be merely analytic, which it is not supposed to be, according to Kant’s own claim (44/B12), and which, in fact, it is not. In the judgement: *This body is heavy*, the subject-representation: *this body*, does not at all contain a representation of heaviness, but rather intuitions of a different kind, such as: of colour, of smell, of shape, etc. and accordingly it is of the following form: *the object that is the cause of the intuitions of colour, smell, etc.* is. However, the judgement tells us that the same object is also “heavy” and thereby means that it *is also the cause of certain other intuitions* that I have, like,

* See Dr Fr. E. Beneke’s *System der Logik*, Berlin 1842, 1. Theil, 156f.; Dr H. Ulrici’s *Geschichte und Kritik der Principien der neueren Philosophie*, Leipzig 1845, 304; Dr Prantl’s *Bedeutung der Logik*, München 1849, 15, among others, where the division into analytic and synthetic judgements is directly called “an empty one”.

for example, the pressure that I feel, etc. It is therefore obvious that the formation of even only this merely singular judgement already presupposes many intuitions that have been repeated several times, i.e. that I must have repeatedly had the intuitions that belong to the concepts: colour, smell, etc. at the same time as the intuition that is contained under the concept of the pressure of the body or its heaviness if I am to be allowed to infer with probability that the same object that is the cause of certain earlier intuitions which I have had is also the cause of certain succeeding ones. All synthetic *a posteriori* judgements are therefore of the following form: *The same object x that is the cause of the intuitions that belong to the concept A is probably also the cause of the intuitions that are contained under the concept B.* We believe that we have hereby at once explained the genuine manner of arising of these judgements in general.

But we could perhaps expect that our philosopher was more successful with the definition of *synthetic a priori judgements*, even though he himself considers the latter to be vastly more difficult? – A large part of the *Critique* at any rate deals with this definition. Whether he succeeded is what we shall now investigate. Meanwhile, Kant just wants to introduce us to the answer to his main question about the arising of synthetic *a priori* cognitions, and he does so with the following words: “It all comes down to” (he says (45/B13)) “what the *unknown X* is, that the understanding leans on when it believes to have found a predicate that is completely different from the concept *A*, outside of this concept, which nonetheless seems to it to be connected with *A*.” But it seems to us by contrast that this is a fabricated difficulty, and that the matter is not nearly as abstruse as Kant pretends. There is no other cause why our understanding assigns a predicate *B* to a certain subject *A* that is not contained in the concept of *A*, besides that we *have* and *know* the concepts *A* and *B*. From the mere fact that we have concepts we are also in a position to make certain judgements about them. For, to say: We have the concepts *A*, *B*, *C*,... is surely to say nothing other than: We know and distinguish them. But to say: We know and distinguish them, means the same as to say: We assert something of one of them that we do not want to assert of the other, or we *judge* about them. This conclusion, because it is universally valid, will also hold if the concepts are simple, in which case our judgements will certainly be *synthetic*. By means of synthetic judgements, however, whose concepts are simple, like, for example, *A*, *B*, *C*..., we will doubtlessly be able to make various synthetic judgements about complex concepts that arise through the combination of the simple ones *A*, *B*, *C*... either with one another or with other concepts. And thereby we will also be able to

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judge synthetically about every object for which we have a concept by means of this very concept.

We have to admit that there is a world of difference between our explanation of the arising of synthetic judgements *a priori* or conceptual judgements and that of Kant's. For, as we will soon see, Kant, in his *Critique*, strives to demonstrate that even in this kind of judgement, the predicate, since it is not contained in the subject, can be found in an *intuition* that is connected to its concept. That unknown *x* that the understanding is supposed to lean on, when it forms synthetic *a priori* propositions, is accordingly an intuition.

However, if, as we have shown above, it is not possible to explain the arising of synthetic *a posteriori* judgements through the connection of the subject with an intuition, then one can make even less use of this means with respect to *a priori* judgements. For here, too, one can respond: the concept of the subject in these judgements almost always contains countless objects, and not infrequently an infinite multitude. But since the intuition that we connect with it can represent only one single object, the inference from one single of these objects to all of them is, on the one hand, surely invalid and, on the other hand, it doesn't have the degree of conviction of the impossibility of the opposite which we usually extol in a judgement of this kind. – Of course, Kant would reply: The intuitions through which synthetic *a priori* judgements are mediated are of a kind entirely of their own; they are not empirical, but *pure intuitions*, and have the effect that the judgement to which they provide a ground is not only probable but certain and even necessary. However, we have to confess that, among the totality of intuitions that we humans have (see further below), we are not at all aware of a distinction of the kind which Kant describes to us. Even if there were such a distinction, it is obvious that our philosopher has not fulfilled his duty by simply assuring us that pure intuitions provide the ground for all correct synthetic *a priori* judgements. He should have also proven how these judgements arise from those intuitions. But, as we shall soon show, to the best of our knowledge, Kant nowhere does this.

By contrast, we have to agree with him when he claims that “synthetic *a priori* judgements are contained as principles in all theoretical sciences of reason” (46/B14). However, judgements of this kind are not only to be found in mathematics, the pure natural sciences, and metaphysics, as Kant demonstrates uncontestably. They are also to be found in logic, and indeed not only among those doctrines which only belong to it according to a broader concept of the discipline, i.e. if one conceives of it, with Bolzano, as a theory of science, but even in that part of it which

is called analytic and which has been worked on since Aristotle. So, for example, the theorems that every judgement has a copula, or that there are 19 forms of syllogism, are synthetic judgements.

At the conclusion of the Introduction, Kant explains how the prior considerations led him to the idea of a distinct science that has to investigate the question “if and to what extent synthetic cognitions *a priori* are possible for us humans, and what value they have.” (49f./B25f.) He gives this science the name *Critique of Pure Reason* and divides it into a *Transcendental Doctrine of Elements* and a *Transcendental Doctrine of Method*. (52f./B29) He calls these parts of the *Critique*, like his philosophy in general, *transcendental* because they contain cognitions that relate to objects of which an experience is possible, in contrast to a *transcendent* philosophy that deals with objects which do not present themselves to us in any experience. Kant divides the *Doctrine of Elements* into a *Doctrine of the Senses* (Transcendental Aesthetic) and a *Doctrine of Thought* (Transcendental Logic); because *sensibility* and *understanding* are the two stems of human cognition. Through the former objects are *given* to us, while they are *thought* through the latter. (56/B32f.) – Here, however, we believe we must recall that we cannot let the presupposition hold that “objects are *given* to us by means of sensibility, but they are *thought* through the understanding” [B32]. One might very well say of sensibility, which we, like Kant, understand as the faculty of having intuitions, that it *gives* us objects, if one wants to interpret this as: we are allowed to infer from every (subjective) *intuition* which we have, to the presence of a certain object, which has brought about this simple representation that relates to (represents) it alone. However, it would in our opinion be most incorrect to hold that intuitions are the *only* representations that have an object, or *give* us an object, i.e. that put us in a position to judge something about it in agreement with the truth. There are innumerable many other representations which we have and of which we can assure ourselves just as completely that they have certain objects that stand under them as we can about intuitions, and indeed, these objects are sometimes of such a kind that no intuition introduces us to them or could do so, for example, the concepts: something, truth, representation, etc. However, we will return to this important point several times in the course of our investigations and we will have the opportunity to share our views about it in greater detail with the reader. – Therefore, we will proceed without delay to Kant’s transcendental *Doctrine of Elements*, namely, to the first part, the *Transcendental Aesthetic*.

Is the establishment of the *Transcendental Aesthetic* indispensable, as a particular science that has to deal with the *forms of sensibility* (with time and space), or are not rather all those doctrines which Kant wants to see assigned to this science better integrated into one that already exists? Anyhow, this is a question that we can let rest, and this all the more that its answer will emerge on its own without anything further, through the following considerations. Doubtlessly, it is more important and it better corresponds to our purpose that we highlight the *most distinguished claims* Kant advances in his *Transcendental Aesthetic*, present them point by point, and accompany them with our *objections*. Accordingly, this is what we want to do at once, and what we mean to do is to use this mode of presentation here as well as in the entire investigation in general.

First, Kant exerts himself:

1. to determine the concept of an *intuition* and the distinction between *empirical* and *pure intuitions*, as well as to make distinct what he means by the expression: *form of sensibility*. But what Kant says here sounds so peculiar to us that our reader should see it for himself: “In whatever way”, he begins his presentation, “and through whatever means a cognition may relate to objects, that through which it relates *immediately* to them, and at which all thought as a means is directed as an end, is *intuition*. This, however, takes place only insofar as an object is given to us; but this in turn, at least for us humans, is only possible through the mind’s being affected by it in a certain way. The capacity (receptivity) to receive representations through the way in which we are affected by objects is called *sensibility*. Objects are therefore given to us by means of sensibility, and it alone provides us with *intuitions*; but they are thought through the understanding, and from it originate *concepts*. But all thinking, whether directly (*directe*) or through a detour (*indirecte*), must ultimately relate by means of certain marks to intuitions, and thus, in us, to sensibility, because no object can be given to us in any other way. The effect of an object on the capacity of representation, insofar as we are affected by it, is *sensation*. That intuition which is related to the object through sensation is *empirical*. The undetermined object of an empirical intuition is *appearance*. I call that in the appearance which corresponds to sensation the *matter* of the latter, but that which makes it so that the manifold of appearance can be ordered in certain relations I call the *form* of appearance. Since that within which sensations alone can be ordered and placed in a certain form cannot itself be in turn sensation, the matter of all appearance is thus indeed given *a posteriori*, but its form has to lie ready for all of them *a priori* in

the mind, and can therefore be considered separately from all sensation. I call all representations pure (in the transcendental sense) in which nothing is to be encountered that belongs to sensation. Accordingly, the pure form of sensible intuitions in general is to be encountered in the mind *a priori*, wherein all the manifold of appearances is intuited in certain relations. This *pure form* of sensibility itself will also be *pure intuition*. So, if I separate from the representation of a body that which the understanding thinks by it, such as substance, force, divisibility, etc., as well as that which belongs to sensation, such as impenetrability, hardness, colour, etc., there remains for me from this empirical intuition still something further, namely extension and shape. These belong to pure intuition which takes place *a priori*, even without an actual object of the senses or sensation, as a mere form of sensibility in the mind." (59f./B33f.) Kant goes on to declare that he will deal with the *forms* of *sensibility* in greater detail and remarks in advance that it will be shown that "there are no more and no less than two, *time* and *space*". (59/B36)

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Objections. With the words that have just been quoted, our philosopher advances a distinction among representations to which he has paid hardly any attention so far, and by virtue of which we call them either *intuitions* or *concepts*. But even though the custom of distinguishing these two kinds of representations or denominations has become generally prevalent since Kant, at least in Germany, it still has to be doubted very much whether the concepts that one connects to these expressions have always remained the same. Indeed, it is really not such an easy matter even to determine what Kant himself thought by these expressions. For here again he failed to have a distinct definition. What seems to be the most certain thing that we can take away from the above sections and many others in which this subject-matter is discussed is that, by *intuitions*, Kant has always thought of just those representations that refer to *one single* and indeed *actual* object. For this precise reason, he would have declared a representation that has several objects, like: human, triangle, etc. to be a *common concept* of them, and not an intuition. It is not less certain that he has always thought by an intuition a representation from whose presence we are at once justified in inferring to the presence of an object which corresponds to it (which is represented through it). For it is obvious that this is just what he wants to tell us by the phrase: intuition *gives* us the object, *gives it immediately*, and indeed no object can be given to us in any other way (at least immediately). However, if we investigate the circumstances under which we are justified in inferring from the mere presence of an intuition to the presence of an object

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which corresponds to it, then it will quickly be shown that this will only be the case if this representation is an absolutely *simple* one. For if you only compose a representation from several others, doubtlessly only through the proper competence of your own mind, then you are not allowed to presuppose at once that it also has a corresponding object. Rather, such a representation can very often also be objectless, like the representations: round square, and other similar ones. It is this consideration, most of all, that determines us to agree with Bolzano's definition of the concept of an intuition, that it is a representation which, *for all its simplicity, still has only one single object*. This not only turns out to have the advantage that we can agree entirely with Kant, and indeed claim with even greater right than he does that through each intuition

50 an object is *given, immediately given*. Rather, with this we also gain the even greater advantage that we have a definition of intuitions by means of which these representations let themselves be distinguished from all others in an *objective* way, and through which it even becomes possible to ground a distinction between *a priori* and *a posteriori* propositions that is completely objective and does not bear merely on their relation to our faculty of cognition, but on their inner essence. – If Bolzano's definitions of intuition and those of the concepts that are derived from it are correct (see Introduction, 15), and if it is possible to claim that these distinctions state, if not exactly, then at least almost the same as what Kant might have thought by them, then it is indeed remarkable how extremely imperfect the definitions he advances are. He does not even explicitly mention the mark that an intuition can only have one single object, that is, that it has to be a singular representation. And there is not even the faintest trace to be found of the other mark that a pure intuition (one mixed with no concept) must be a simple representation, neither in his work nor in that of any of the later philosophers who have adopted and maintained Kant's distinction between intuitions and concepts. Instead, in what follows we find that Kant speaks very often

51 of the *manifold* which lies in intuition. Meanwhile, according to our concept, a manifold can only be found in one *singular* representation if it was first brought about through a combination of several *simple* representations, a combination that can only come about through the mediation of some representations which are not intuitions but are rather concepts. For mere intuitions like: "this red, this pleasant smell, this sweet taste, etc." do not yet form one single representation, even if we have them all at the same time. Rather, they can be united into such a representation only through a concept, such as the one designated by the particle: *and*, or any one like it. What pernicious consequences this

very faulty definition of intuition has had, how later philosophers have been misled by this flaw into introducing *a distinct intellectual faculty of intuition*, to the greatest disadvantage to science – to speak of this would lead us too far from our undertaking. Accordingly, it will suffice to highlight only those erroneous claims that Kant is guilty of making in the short section we have presented to our readers.

Here Kant claims yet again that there is no other way in which an object can be *given* to us but through intuitions. However, we have already shown above how incorrect this is, if it is supposed to have the sense that we can be assured, for a concept, that there is an object corresponding to it, only if we have some intuition that stands under this concept. Every mathematician assumes, completely rightly, of the concept of a straight line that extends to infinity, either to both or even only to one side, that it is an *objectual* concept, i.e. that there are certain objects corresponding to it. And yet, no one has or can have an intuition that stands under this concept, for certainly an infinite line can never be intuited, even in the broadest and vaguest sense one could ever give to the word *intuition*. – Kant's claim, that the *pure form of sensibility* (i.e. that which in the appearance makes it so that its manifold can be ordered in certain relations) can itself also be called *pure intuition*, shows how very unsteady the concept Kant himself connected to this word must have been. If we follow him, a *pure intuition* would be something essentially *complex*, would contain a manifold which it would combine into a certain unity. Yet at the same time it is certain, as Kant himself acknowledges elsewhere (e.g. 136/B142), that combination is a feat of the understanding, i.e. comes about only through concepts. Hence intuition would then be a complex representation that comprises concepts in itself as components. But is there really a point in taking the concept of an intuition in such a broad, vague signification? Isn't confusion in science necessarily going to originate from this? – If a *pure intuition* is really supposed to be (as one must suppose) an *intuition*, then one should in any case not have claimed straightforwardly, but first *demonstrated* that the form of sensibility is an actual intuition. This does not follow from the definition Kant himself gives, that intuitions are those representations through which an object is *given immediately*. For the representations of the form of the object (of the relations in which the manifold of the latter can be ordered) belong in no way to the representations through which it is given to us, let alone given to us *immediately*. This follows even less from the definition which, in our opinion, is the only correct one. – But perhaps the example that Kant here thought useful to introduce for the purpose of clarification will

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give us a better picture? According to the latter, the (representation of) “*extension and shape*” [B35] of a body intuited by us should be a pure intuition because this representation remains if we remove everything the *understanding* thinks by it, such as substance, force, divisibility, as well as everything that belongs to *sensation* (matter), such as impenetrability, hardness, colour, and so on. Here we must candidly admit that the first thing of which this example convinces us completely is that Kant has connected the word *intuition* to such an *indeterminate* and

54 *broad* concept that no true advantage can come from his division of representations into intuitions and concepts. Impenetrability, hardness, extension and shape are supposed to be properties or determinations of a body that are opposed to substantiality, force, divisibility, etc. in such a way that the latter can be cognized through the understanding (the faculty of concepts) and the former, by contrast, through sensibility (the faculty of intuitions). We can only admit this opposition to the extent that it is understood that we have the capacity to make the judgments: Each body consists of substances, has certain forces, is divisible, etc. without having any intuition of it; while, by contrast, the judgments that this body is impenetrable, is hard, has this specific shape, e.g. that of a sphere, can only be made if we have had certain intuitions that relate to the same object. However, we absolutely cannot admit that the representations: impenetrability, hardness, sphericity, etc. are in themselves intuitions or even just mixed representations (representations that comprise in themselves an intuition as a component), as Kant seems to believe. They are concepts, concepts no less pure than those of substantiality, force, divisibility, etc. The distinction is only: substantiality is an attribute that pertains to all bodies, while sphericity is an attribute that does not pertain to all bodies, but only to some. Whether,

55 therefore, the latter pertains to the one that lies directly in front of us must be first ascertained through its own intuitions. But there is no difference between the way this is ascertained – that is, there is no such difference between the way in which we experience whether a body that lies in front of us belongs to the *hard* ones, and the way in which we experience whether it belongs to the *round* ones – that would allow us to call the first attribute an *empirical* representation and the other one a *pure* representation (let alone intuition). If we were to present him with this objection, Kant would perhaps have responded that he did not hold the representation: *sphericity* as such, to be an intuition, but indeed the representations: *this particular spherical shape, this particular place, this particular distance*, etc. That and why we cannot admit this will soon transpire more distinctly. For if the charge of confusion can be made

towards any of Kant's doctrines, then it is towards his doctrine of *spatial* objects and of *space* and *time* in general, which forms the sole content of his Transcendental Aesthetic.

In order to bring a little light into this obscurity, we consider it necessary to tell our readers which concepts of time and space at least we hold to be the correct ones. These are, as they will already suspect, Bolzano's concepts. If they cannot agree with us, they remain at liberty to embrace either the Kantian theory or yet another one.

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Time and *space* are without doubt a pair of objects so closely related that it is no surprise that Kant presents their doctrines in unison; indeed, the connection between the two had often been recognized before him. But when it comes to ascertaining their proper essence, it seems to us more appropriate to start not with the investigation of space, but with that of time. For, since everything that is in space must for that very reason also be in time, but not the other way around (thoughts and sensations for instance are to be sure something temporal, but certainly not something that fills a space), it may be suspected at the outset that the nature of space cannot be recognized with full distinctness without first having ascertained that of time. – As is well-known, we put everything that is actual at a certain time, with the exception of the highest being, and indeed in such a way that we cannot ascribe any property to an actually existing object truly without also the addition of a certain temporal determination. This claim holds so universally that we may say even of the divine attributes that they pertain to God at a certain time, namely at all times. Therefore, every proposition whose subject-representation relates to an actual thing and that has therefore the form: "The actual A has (the property) b" has truth only insofar as one can connect a time with its subject-representation. Propositions like, for instance: I hear the clock strike two, I feel joy, The sun is shining, among others, are not perfectly true if their subject-representations fail to have the determination of time, and if they have another sense than: I, at this moment, hear the clock strike two, feel joy, The sun shines during the day, etc. From this it follows that we have to understand by the word *time* a *determination of something actual, which contains the condition under which we can ascribe certain properties to it truly*. The fact that all properties of time can be derived without great difficulty from this concept already speaks in favour of it. Let us test this with the following theorem: Several attributes that contradict one another can pertain to one and the same substance only under the condition of a different time. This follows immediately from the fact that propositions with contradictory predicate-representations can only be true if

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they have different subject-representations. Therefore, two contradictory attributes (e.g. blooming and not blooming) cannot be predicated of one and the same substance or even a collection of substances (a tree, for example) at the same time, since otherwise two propositions which, while they have the same subject-representation, have contradictory predicates would have to be true. – Regarding the concept of *space*, everybody will be sure admit that it is the *collection of all places*. But the question arises as to what one understands by places that are filled with actual things. It is undoubtedly true that everything actual has effects and insofar as it is finite both undergoes certain changes itself and brings them about in other surrounding objects. But what these changes consist in depends manifestly first only on *the forces* that the thing itself as well as its surroundings have and second on the *places* in which both are found. From this it follows that places are those *determinations of actually existing things that we have to assume in addition to their forces in order to explain all changes that they [these things] effect, one in the other*. It appears to us that this proposition expresses the correct concept of space once again because all properties of space, that are taught in geometry can be derived from it.

Whoever finds this definition of time and space to be correct will also easily agree with us when we claim that the *representations* of time and space are a pair of *pure concepts*. For the components from which we just composed these representations are without doubt all pure concepts. However, the question remains whether there are not among the *remaining* representations we call *temporal or spatial representations* some that either are intuitions or at least contain them as components? As regards those representations relating to time and space that have more than one object, for instance the representations of an instant in general, a duration in general, a point, a distance, a line, a plane and a body in general: assuredly, no one will doubt that these representations are not pure intuitions, for a pure intuition has only one single object, while these, by contrast, have infinitely many objects. Doubt could here only arise about concepts such as the following: the whole infinite time, the whole infinite space, this determinate moment (in which I presently find myself), this determinate duration (e.g. an hour), this determinate point (e.g. the centre of the Earth), this determinate distance (e.g. from the Earth to the Moon), etc. Every one of these representations has only one single object; if they were in addition simple as well then according to our own conception of the concept we would have to count them among the pure intuitions. Meanwhile, we should recall that every (subjective) intuition that arises in our soul has a determinate actually

existing object, and from this alone it is to be concluded that the aforementioned representations are not intuitions. As a matter of fact we may decide as we please as regards whether they are complex or simple. For their object *is not an existing one*. In order to find the latter evident, it will be enough that we possess these representations and understand the signification of the aforementioned terms. Whoever merely knows which concepts mathematicians connect with the words *time* and *space* must admit without reserve that only *the things* that find themselves *in time and space are actual*, but *not times and spaces themselves*. For if one wanted to declare times and spaces to be actual things, then he would have to claim that they act or bring something about. But what would the products be? – Of course, we do not seldom say that time makes so many things happen, for instance that it makes the roses bloom, or that it heals sorrows and wounds, etc. However, who fails to see that this is only an improper way of speaking used to indicate that in the course of time changes happen through causes that we come to experience sooner or later. It is therefore not time, but the forces of things that produce all effects and changes, though sure enough at certain times. The expression that a narrow space causes feelings of discomfort is to be understood in the same way. It is not the narrow space that does this, but the cramped, closed rooms, the lack of fresh air, etc. – Furthermore, if time and space were something that exists, then their existence would be either conditioned or unconditioned. Were they to be the latter, then they would be God himself (for only God is something unconditionally actual); were they to be the former, then they would be creatures and subject to change. But only things that are in time and space change, but not time and space – and whether time and space are created or not created, about this, only the Schoolmen could seriously dispute. – If time and space actually exist, then two instants or two durations, two points or two distances cannot be intrinsically alike or similar, for there are no two actual things that would be completely similar to one another. But according to the consensus of all mathematicians, it is not only two instants or two points, but rather all instants and all points that are perfectly similar to one another. – Finally, if time and space were something actual, there would have to be something actual that lacks a sufficient ground, namely the existence of a thing at this determinate time and at this determinate place. For why a thing finds itself at this time and not at another or why it should be at exactly this place and not at another, for this there would be no ground to offer whatsoever, for indeed these places and times are intrinsically completely similar to one another. – If thus the representations: this moment, this duration, this

point, this distance, etc. are not representations of an actually existing object, then they cannot be intuitions either. However, if no moment in time and no point in space is something actual: then surely neither
 62 is the collection of all instants or all points; the whole time, the whole space. Accordingly, we cannot declare either of those two representations to be intuitions, even though they have only one single object (for there is only one infinite time, only one infinite space). They therefore have to be ranked among the pure concepts. For surely no one will believe that they should contain an intuition as a component. However, if we see ourselves compelled to hold that the representations of the whole infinite time and the whole infinite space are pure concepts, then we also have to admit that the representations of an instant, a duration, a point, a distance, etc. belong in general to the class of the pure concepts. For if the representations of the whole infinite time and the whole infinite space contain nothing from intuition, then how should the representations of certain parts of the latter possibly be called intuitions? (See Bolzano, *Theory of Science* §79).

Let us now see how Kant

2. presents and proves his doctrine of *time* and *space*. What is most important concerning the latter can be found at page 63f. [B39] in the *Transcendental Aesthetic*. "Space", we read there [B39], "is *not a discursive*, or, as one says, *general concept* of relations of things in general, but
 63 rather a *pure intuition*.* For, first, one can only represent one single space to himself, and if one speaks of many spaces, then one understands by that only the parts of one and the same unique space. Similarly, these parts cannot precede the single all-encompassing space as its components (from which its composition would be possible). Rather, they can only be thought in it. It is essentially unitary; the manifold in it, and therefore also the general concept of spaces in general, rests merely on limitations. From this follows that in respect to it an *a priori* intuition (which is not properly empirical) lies at the basis of all concepts of space. Thus all geometrical propositions, e.g. that in a triangle two sides are greater than the third, are also never derived from the general concepts of line and triangle, but from intuition and indeed *a priori* with apodictic certainty. – Space is represented as an *infinite* given magnitude. Now one would have indeed to think of every concept as a representation that is

* Actually Kant here should have talked about spatial and temporal representations, and not of the objects of these representations (of time and space themselves).

contained in an infinite set of different possible representations (as their common mark), and which therefore contains these under itself. But no concept, as such, can be thought as though it contained an infinite set of representations in itself. But space is thought precisely in such a way (for all the parts of space to infinity are simultaneous). Therefore the original representation of space is an *a priori* intuition and not a concept" (64/B39) – Kant makes a similar pronouncement about time: "Time is not a *discursive, general concept* but a form of sensible intuition. Different times are only parts of one and the same time. The representation which can only be given through a single object is however an intuition. Furthermore, the proposition that different times cannot be simultaneous could not be derived from a general concept. The proposition is synthetic, and cannot arise from concepts alone. Thus, it is therefore immediately contained in the intuition and representation of time. – The *infinity* of time signifies nothing more than that every determinate magnitude of time is possible only through limitations of a unitary underlying time. The original representation *time* must therefore be given as without limitations. But if the parts themselves and every magnitude of an object can be determinately represented only through limitation, then the entire representation cannot be given through concepts (for the latter only contain part-representations). Rather an immediate intuition must lie at their basis." [B47f.]. 64

Objections. What Kant declares here is not that every representation of time and space is a *pure intuition*, but that the *whole* infinite *space* and the *whole* infinite *time* are. His main reason is: "because there is only one single space, only one single time; because if one speaks of several spaces and times, these are only the parts of one unique space, one unique time; and because a representation, which can only be given through a single object, is an intuition." [B39f, B63] – But do not also the representations: God, universe, highest moral principle, first prime number in the series of the natural numbers, and thousands of others have only one single object? But who will declare them intuitions just because of that? Yet what Kant introduces as a manner of proof for the uniqueness of space, "that namely its parts are only thought in it as limitations and can in no way precede it as components from which its composition is possible" [B39] is not well suited to convince us of his claim either. For were this to be right, we would have to represent the whole infinite space merely in order to get to the representation of even one single point. However, who would consider this necessary? Who would even be aware in the slightest that he actually encounters 65

the latter when representing a part of space? It is not even true that the representation of a line is required for the representation of a point, the representation of a plane or even of a body for the representation of a line. Assuredly one generally defines the point as the boundary of the line and the line as the boundary of the plane, etc. But are these definitions really the true definitions? If they were, then we would admittedly have to allow that we cannot represent a point without representing a line, etc. But who does not feel that this is not the case? Are there not lines (the double curved lines) for which it would be difficult to determine the plane they delimit? – One should much rather assume that the representation of a point appears as a component in the representations: line, surface, body, etc. and that the whole space is to be thought and defined as the collection of all points. By contrast we should reject as completely unfounded the claim that the concept of finite spaces (for instance: triangles in general) rests merely on limitations of the infinite space. For while there are indeed parts of a whole of such a kind that their concept contains the concept of the whole, like that of half a cubit, there are also parts of another kind for which this is not the case, like the parts of a clock, a steam-engine, etc. And the parts of space which one calls points, lines, etc. belong to this latter kind. It is thus false that the parts of space are so constituted that they comprise in themselves the whole infinite space as a component and it is not at all adequate for proving the proposition that space is an intuition. – We also deem it incorrect “that every concept can be thought as a representation that is contained in an infinite set of others as their common mark, and which thus comprise them under itself”. [B39f.] It is not the case that every concept has to have several, even infinitely many objects; and if it does not have these, how should there be several, or infinitely many representations that are contained under it? – “But that no concept can be thought as if it contained an infinite set of representations in itself” [B39f.], that, we ourselves also maintain if it means only that no concept thought by a finite understanding can be composed from an infinite multitude of components; but this holds not just of concepts but of all representations. Therefore we also do not believe that space is a representation composed of infinitely many parts. The ground “that all the parts of space to infinity are simultaneous” [B39] does not prove this. For in order to form the representation of “a whole composed of infinitely many parts” [B39] I do not need to represent these parts individually; rather I have thought such a whole as soon as I have thought the concepts that make up its concept in the appropriate composition. But as has already been proven (see *Introduction*) there is

always only a finite number of these concepts. – Kant also claims about time, as he does about space, that “every determinate magnitude of time is only possible through limitations of a unitary (infinite) underlying time.” [B47f.] This ought to have the sense that the parts of time are represented only through limitations of (the whole infinite) time, and that every representation of a determinate part of time contains the representation of the whole infinite time as a component. But we can respond here what we have already objected against the similar claim in respect to space and its parts, namely that we can assuredly represent an instant, a duration without thinking of the representation of the whole infinite time. By contrast, in order to think the latter we must already have the representation of an instant. For the infinite time is just the collection of all instants. Just as false is the premise Kant uses here in order get to the conclusion that time is an intuition: the proposition “that an object whose parts can only be represented through its limitations, can never be represented through concepts”. [B47f.] For there are enough objects whose parts can only be represented in relation to the whole (limitations of the latter in precisely the sense in which it can hold of time) but which we nonetheless still cognize through pure concepts. What the *terminus medius* is in a syllogism will only be recognized by considering the whole, but the representation of the syllogism is nonetheless a pure concept. 68

But Kant believes

3. to have found a confirmation for the correctness of his position that space is a pure intuition in the claim that “a science, geometry, exists that determines the attributes of space synthetically and yet *a priori*. For from a mere concept one cannot draw a proposition that goes beyond the concept, which however, happens in geometry.” (65/B40) One can gather how much weight our philosopher puts on his presentation and how much he trusts his proofs from the passage with which he concludes his Transcendental Aesthetic. It says: “Here we now have one of the requisite pieces for the solution of the general task of transcendental philosophy: how are *synthetic a priori propositions possible?* namely through pure *a priori* intuitions, space and time, in which we find, when we want to go beyond the concept given in a judgement *a priori*, that which is to be discovered *a priori*, not in the concept but in the intuition that corresponds to it, and which can be synthetically connected with it. But for this reason such judgements never extends beyond the objects of the senses and can hold only for objects of possible experience.” (87/B73) 69

Objections. That from mere concepts without the aid of an intuition no synthetic propositions are deducible is the *proton pseudos* of the *Critique* and nowhere in it is the latter proven. In order to support his claim, Kant

70 appeals here to the mathematical sciences, in particular to geometry for which, it seems to him, it is settled that its theorems are mediated by pure intuitions. Still, we have seen how awkward his doctrine of pure intuitions turns out to be and this very fact allows us to understand why our philosopher could never successfully establish how synthetic *a priori* judgements can in fact arise through this kind of intuition, however often he tried in the *Critique*. On page 159 [B180f.?] he exerts himself to draw the reader's attention to the (in his opinion) extremely important difference "that obtains between an intuition that corresponds to a certain concept, like that of a triangle, and that arises from an actual drawing of the object and an intuition that arises in the mind through the faculty of imagination and is *constructed*" [B741f.?] "The former", he claims, "is an empirical intuition and it has neither universality nor necessity. The latter, by contrast, is pure and has both attributes." [B746?] But why the representation of a triangle should essentially change just because we imagine it instead of drawing it on a board, that escapes us. Kant himself seems to have felt this, for he sometimes requires that this construction happens, not through the imagination, but through the *pure* imagination. Is this really more than hiding behind obscure words? – Matters do

71 not become any clearer when Kant further recalls that "the imagination in general" (he calls it productive) "produces mere *images*, by contrast, the pure imagination produces *schemata*; and a schema is supposed to be more the representation of a method or rule for providing a concept with its image than the image itself." (160) [B180f. and B160] But here it is frankly hard to comprehend how the representation of a method can be called an intuition or how this representation can be distinguished from the method for providing every concept with its image. If we are to think of something determinate by this method, then we have to understand by it the so-called *genetic definition* of a concept. But who will rank the latter among intuitions? If the schema of a circle is the representation of the method for providing this concept with an object, then it can assuredly be nothing but the way in which the circle arises, i.e. the commonly known genetic definition of a circle, or the concept of a line delineated by a point moving on a plane, such that it always maintains one and the same distance from a certain other point. What this means is that whoever cognizes the truth of a synthetic judgement by examining the schema of its subject, cognizes it merely through the examination of a concept and not through an intuition. – In several

places in the *Critique* we find the claim that we cannot think a relation within space without *constructing* it. “We cannot think” (Kant says e.g. on 143/B154) “a line without drawing it in thought, a circle without delineating it, we cannot represent the three dimensions of space at all without *placing* three lines perpendicular to each other at the same point. We cannot even represent time itself without *drawing* a straight line (which is to be the external figurative representation of time), and without attending thus merely to the action of the synthesis of the manifold through which we successively determine the inner sense, and so, without attending to the succession of this determination in inner sense.” [B154] Is this not overdone? Such constructions or images might well increase the distinctness and vividness of our representations, and they might therefore cause us to get used to them and to call them forth involuntarily. Nonetheless, neither are they necessarily connected to our concepts nor do they occur everywhere, as examples will prove. Even the novice mathematician will surely understand what we want to say when we define the dodecahedron as a body that is bound by 12 equal and similar surfaces. As soon as he hears these words, he will connect a concept with them and will represent by them something true. However, he will not at once understand how to construct the dodecahedron, he will not know of what kind of sides it is composed, and lesser still whether the sides are triangles or pentagons. Rather one is to suppose that he must have grasped the concept of a dodecahedron distinctly before he is able to construct one. We do not first gain the concept through the drawing of some spatial thing, we rather already have this concept before we can draw the object corresponding to it even in imagination. – Indeed, when mathematicians in general use certain signs and figures in the course of demonstrating their theories in arithmetic as well as geometry, they do not do this because these propositions could not be proven without this aid, but because through this they save a lot of words and these illustrations lighten their audience’s effort toward comprehension. After close consideration, everybody will have to think that Kant’s doctrine of the symbolic construction of algebraic concepts is rather forced. One will absolutely not understand what it means to say, as it is said somewhere in the *Critique*, that mathematical concepts are presented in intuition through the signs x , y , $+$, $-$, etc. if this is supposed to be something peculiar to algebra. For doesn’t the logician and even the metaphysician sometime use certain sensible signs in order to illustrate the concepts that occur in their science too? Does not the former, for instance, use circles or lines to explain the relations of subordination among concepts? – How awkward is thus the claim that arithmetic

- 74 propositions are mediated and constructed through intuitions! If one asks what kind of an intuition this might be? then we are told that (47/B15) “that we cannot think of the concept of 12 as a unification of $7 + 5$ without going beyond this concept itself and taking an intuition, for example of five fingers, as an aid”. But this is obviously not a pure intuition. And that is also why it says at other places that the pure intuition of time lies at the basis of arithmetical propositions, to the extent that when we count, we add up units bit by bit. But if this were the case, time would have to be at the basis of all our judgements. We are not in a position to grasp a series of thoughts and present it in words otherwise than bit by bit. What then would the difference between an arithmetical and any other kind of exposition be? There is really no essential difference to be found; in arithmetic just as in any other scientific presentations one has to derive the theorems from concepts. – Kant’s claim that intuitions, namely of space, lie at the basis of geometrical propositions has a somewhat greater appearance of plausibility, for here one usually deduces the conclusions from drawn figures. However, whether this is necessary remains an open question. We know of one kind of geometric presentation following which all the truths of geometry can be concluded from mere concepts without the aid of figures.
- 75 Kant may well declare that only those synthetic cognitions are correct whose concepts relate to intuitions; he may, as often as he wants, repeat that concepts without a corresponding intuitions do not yield cognitions, are empty, etc.: we have to disagree and ask the question: *what are the pure intuitions that support these and similar claims that our philosopher passes off as truths and therefore as knowledge and which are apparently not analytic but synthetic propositions?*

* * *

According to Kant, the *Transcendental Logic*, the other part of the doctrine of elements, has two divisions: an *Analytic*, which merely presents the elements of the pure cognition of the understanding and the principles without which no object can be thought; and a *Dialectic* which uncovers the false illusion of the hyperphysical use of reason. (88–98/B87–B98)

The *Transcendental Analytic* is divided into two books (99/B89f.), one of which deals with the *concepts*, the second with the *principles* of pure understanding.

At the very beginning, our philosopher reminds us (book 1, 99/B89) that “it is important for the analysis of the entirety of our *a priori*

cognition into the elements of the pure cognition of the understanding
 a) that the concepts be *pure*; b) that they be *elementary concepts*, and 76
 clearly distinguished from those which are derived or composed from
 them; finally c) that the table of them be *complete*, and that they fill
 out the entire field of pure understanding". Indeed one cannot, it says
 further, assume this completeness with reliability from the rough esti-
 mation of an aggregate put together by mere trial and error. Nonetheless,
 one is in a position to obtain a complete table of elementary concepts,
 which Kant also calls *categories*, in the following manner: "namely if
 we abstract from all content of a judgement, and pay attention only
 to the mere form of the understanding in it, we find that the function
 of thinking therein can be brought under *four* headings, each of which
 contains under itself *three* moments and both (headings and moments)
 can be represented in the following table." (103/B95)

1. *Quantity of Judgements: Universal, Particular, Singular*
2. *Quality: Affirmative, Negative, Infinite*
3. *Relation: Categorical, Hypothetical, Disjunctive*
4. *Modality: Problematic, Assertoric, Apodictic*

"The same function (activity)", we read on page 110 [B104f.], "that gives
 unity to the different representations in a judgement also gives unity to
 the mere synthesis of different representations in an intuition which is 77
 called the pure concept of understanding. Thus, the same understanding
 and namely through just the same actions through which it brings the
 logical form of a judgement into concepts by means of the analytical
 unity, also brings a transcendental content into its representations by
 means of the synthetic unity of the manifold in intuition in general.
 For this reason they are called pure concepts of the understanding that
 pertain *a priori* to objects. This can never be accomplished by general
 logic. There arise in this way exactly as many pure concepts of the
 understanding, which pertain *a priori* to objects of intuition in general,
 as there are logical functions of all possible judgements in the previous
 table. For the understanding is completely exhausted and its capacity
 entirely measured by these functions. Following Aristotle we want to
 call these concepts categories, for our aim is basically identical with his
 although very distant from it in execution."

Thus the following *table of categories* is supposed to follow from that
 of judgements:

1. *Category of Quantity: Unity, Plurality, Totality*

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2. *Category of Quality: Reality, Negation, Limitation*
 3. *Category of Relation: Inherence and Substance, Causality and Dependence* (cause and effect), *Community* (reciprocity between agent and patient)
 4. *Category of Modality: Possibility – Impossibility, Existence – Non-existence, Necessity – Contingency* [B106].

Objections. Here, Kant obviously tries to establish a completely new science which he calls *transcendental logic* and which, in its *analytical part*, is supposed to discover the pure *concepts* of the understanding (the categories) and the pure *principles* and, in its *dialectical part*, is supposed to teach to uncover the false illusions of the use of the understanding. Again, we will leave to one side whether the creation of a distinct science is necessary for such a purpose or not and we merely want to engage in a small discussion of what appears obscure and dubious to us in this analytical part, and above all in the doctrine of the *categories*.

The first thing we notice is that Kant

1. does not at the outset define the concept of a *category*. He tells us, only in what follows and as if it was an afterthought, that: “the categories are concepts of an object in general, by means of which its intuition is regarded as *determined* with regard to one of the *logical functions* of judging.” (126/B126) This definition suffers at least from the mistake that it does not give the *internal* difference between the categories and other concepts. Rather it merely notes in which relation these concepts stand to our power of thinking. – We do not deny that the categories are pure concepts because we actually consider all concepts without exception to be pure or *a priori*. However, Kant claims that the categories are “*elementary concepts*” (99/B89) “*original and primitive concepts*” (112/B108) and “*root concepts*” (*ibid.*). What he means by these names is not evident. One is almost tempted – led astray by these denominations – to believe that he considers the categories to be *simple* concepts from which all others arise through composition. And he does really talk this way about the concepts that are derived from them (*ibid.*). But from concepts, mere concepts, no new one can be built or deduced in any other way than by mere composition. Against this speaks the fact that Kant claims, on the very same page, that he knows the definitions of the individual categories, but will omit them studiously in this treatise. But definitions in the strict sense of the word can only be given for complex concepts. Accordingly Kant must hold that the categories are
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complex. And in fact they are almost all of such a nature that they can be analysed into simpler concepts and can therefore be defined. Why therefore should they be called elementary concepts, or what are we to think in general by the denomination *category* in the Kantian sense? 80

But we also cannot agree with the claim that
 2. the *table of the categories is deducible from that of judgements* and that there are as many categories as there are different forms of judgements. Even the mere manner of the deduction is something with which we can't agree. For even if we were to allow that the concept of plurality is contained in particular judgements, or at least in some of them, how would it be possible to demonstrate that the concept of totality lies in the universal judgements and the concept of unity in the singular ones? The universal judgement: All glass is breakable, says nothing else than: Glass is breakable, and the former is therefore equivalent to the latter. Where can we find the concept of totality in the latter? Just as little will one find the concept of unity in the singular judgement: Berlin is the capital of Prussia; for the two representations: the concept of *One* object and the representation of the city of Berlin are after all two very different representations. – It is even harder for us to be in a position to see how the three categories of relation are supposed to come out of the three corresponding forms of judgement? In respect to its relation, every judgement is supposed to be either categorical or hypothetical or disjunctive. The following examples are certainly categorical judgements: The point is what is simple in space, the line is bounded by two points, what is possible is not yet actual, every concept is a kind of representation. We do not comprehend how the concepts of substance and accident are contained in the subject- and predicate-representations of these propositions or indeed even how they could be subsumed under them. One is just as little in a position to deduce the categories of community or reciprocity following logical rules from the mutual restriction that, according to Kant's opinion, the spheres of subject and predicate exert on each other in a disjunctive judgement. 81

Furthermore, it is false

3. to *divide all categories* according to the *four moments* of quantity, quality, relation and modality. For the properties of every object, and therefore of the categories, can be conveniently brought under the two rubrics of *attributes* and *relations*, thus they belong either to *quality* or to *relation* and it is a mistake to divide them further into quantity and modality. For, is it not true that the magnitude of an object is one of its attributes? And does not its modality, which, according to Kant, is

the relation between the concept and our cognitive capacity, obviously belong to the moment of relation? Indeed, at least here, there would be no point in treating the two (the quantity and modality) separate from their superordinate concepts.

Finally

4. Kant indeed claimed but nowhere proved that the table of the categories is “*complete* and that it entirely fills out the entire field of pure understanding” [B89] and that therefore there *must be as many categories as forms of judgements*. For the statement that the understanding is active in the same manner whether in the formation of the categories or in the configuration the forms of judgements can certainly not take the place of a proof. The understanding certainly acts in a different way when it combines representations into concepts (namely complex ones) and when it connects representations in such a way that judgements come out of it.

* * *

After the exposition of the categories, Kant proceeds to his “*Deduction of the Pure Concepts of the Understanding*”. (118f./B116)

He understands by this *deduction* “the proof that the pure concepts of the understanding (categories) are rightfully applied by us, yet only in respect to the objects of possible experience”. ([B116 and] 124/B125) According to him, the intuitions of time and space are not supposed to need a particular deduction; “because they contain the conditions of the possibility of objects as appearances *a priori*”. [B118] This is different in the case of the categories “for even without thinking the objects they can still be given to us in intuition” [B118] – Kant believes that he can remove this difficulty by remarking that “all the manifold of sensible intuition must necessarily belong under the *original synthetic unity of apperception*, i.e. it has to be grasped under the representation: *I think* or, more distinctly, it has to be possible to accompany it with the consciousness that it is our representation. [B132] However, this can only happen through *judgements*; but the logical forms of all judgements are fixed in the categories. Thus all of the manifold in a given intuition necessarily stands under the categories. [B143]. – By contrast, without application to intuition the categories do not deliver any cognition, rather they are *empty*. Thus they have *no other use* for cognition than *only in application to objects of a possible experience*. In general there are only two ways in which a *necessary agreement of experience with the concepts of its objects* can be thought: either the experience makes these concepts possible or

these concepts make the experience possible. The first is not the case, for the categories are *a priori* concepts, hence independent of experience. Consequently, only the second remains namely that the categories contain the grounds of the possibility of all experience." [B166]

Objections. 1. That the judgement: *I think* (for: I think, is a judgement and not a mere representation) could accompany every one of our representations, may be true. However we believe that it would be more correct to say that we are in a position to form a new representation of every representation we have, and that the latter is a representation that relates only to it and is indeed even an *intuition* of it. But the claim that every one of our representations *must* be accompanied by the judgement: "I think" (this representation to myself) is false. For this happens really only whenever we raise some representation to a *clear* one or if we intuit it. Furthermore, given that the judgement: *I think* this representation, is only of one single kind, it's not clear how *all* categories, i.e. all concepts, which lie at the basis of every conceivable *form* of judging, must find an application to intuitions because of this one judgement. We are otherwise happy to admit that all the manifold of an intuition has to *stand under the categories*, if this only means that no intuition that we have had so far or that we will ever get, whatever its kind, can contradict one of our categories. Representations can never get into a contradiction with one another; only judgements (propositions). *And we will never be justified in making a judgement from certain intuitions that we have had if it contradicts a pure conceptual truth.* We always not so much intuit as believe that we intuit only what is complex; that bodies presents themselves as porous to our senses; that we continuously perceives only what is finite; and so on, etc. But all this must never mislead us into making judgements that contradict pure conceptual truths, such as: There are no simple substances, Matter does not fill space continuously, There is nothing infinite. Once the existence of a pure conceptual truth is decided, either because it is in and by itself self-evident as an axiom or because it has been deduced from other known conceptual truths through correct inference, then it cannot lose its reliability merely because the eye suggests judgements of other kinds. Rather, the impression must be corrected using the truths of reason on which we agree.*

2. We absolutely cannot accept that the categories are *empty* and *blind* without intuitions and do not have objects. Whether a representation

* *Leibnitz' Monadologie* by Dr Robert Zimmermann, Wien 1847, bei Braumüller und Seidel, 172f.

86 has objects does not depend in the least on whether we know those objects and whether we know them through intuition. Indeed we can also often know that a representation has an object, even though we have not met with it through any intuition. For instance, we know that the concept of a thousandfold sorites has objectuality, even though we do not have an intuition that stands under it. – At many places in the *Critique* (e.g. 240 [B300]) one reads the claim that without any *conditions of sensibility*, the *categories* or the pure concepts of the understanding do not represent any objects at all because the *conditions* for their *objective reality* are missing. Nothing is more incorrect than this thought. For if we omit certain *conditions* (what here can only mean *certain further determinations*) from a concept, then it becomes *broader*; and if the narrower concept to which these conditions still attach represents certain objects, then the *broader* one must do so even more.

3. That experience agrees with the concepts of its objects, and agrees with necessity, rests only on the fact that all experiences are nothing else than *conclusions* that we have deduced from pure conceptual truths by means of certain minor premises of the form: I have the intuitions x, y, z... How could such a deduction yield a conclusion that would contradict a pure conceptual truth? –

87 4. The claim that “all our categories can be validly applied only to objects of a *possible experience*” [B132] presupposes the concept of possibility; it is therefore necessary to know what to understand by objects of a possible experience before we can understand and use them [the categories]. However, Kant himself counts the concept of *possibility* to the genus of categories. Thus in order to know which objects the category or the concept of possibility ranges over, i.e. and what is possible or not, one should actually already know what is possible and therefore understand how to apply this concept absolutely (i.e. without limitation to the mere sphere of possibility). If the definition of the possible which we deem to be the only correct one, namely that: that is possible which *does not contradict any pure conceptual truth*, is true, then the appraisal of what is possible or not already presupposes knowledge, if not of all, then at least of very many pure conceptual truths, especially *synthetic* conceptual propositions. For it is this kind of truths, and in no way the analytic or even identical ones, that come here into consideration. The claim therefore that the categories are applicable only within the sphere of possible experience would actually mean: pure conceptual truths can relate only to objects such that to think of them as perceived does not contradict any pure conceptual truths. Thus in order to experience the

boundaries of the applicability of the pure conceptual truths, one must first apply the pure conceptual truths without boundaries!! –

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In the second book of his Analytic, *The Analytic of Principles*, Kant (157/B176) sets out from the claim that in all subsumptions of an object under a concept the representation of the former must be *of the same kind* as the latter, i.e. the concept must contain that which is represented in the object that is to be subsumed [B176]. “Now”, he says, “pure *categories* and empirical intuitions however are of entirely different kinds. [B176] Thus there must be a third thing, which is of the same kind as the category on the one hand and the appearance on the other. This mediating representation must be *intellectual* on the one hand and *sensible* on the other. Such is the *transcendental schema*.” (158 [B177]) “A transcendental temporal determination is, on the one hand, of the same kind as the category because it rests on a rule *a priori*, and sensible on the other. Hence an application of the category to appearances will be possible – by means of the transcendental temporal determination, which is therefore the schema of the former.” (ibid.) “This schema is not an image, rather it first makes images possible, it is merely the representation of a general procedure of the imagination for providing a concept with its image (159/B179f.). The schema for all objects in general is *time*, for all magnitudes *number*, the representation of the successive addition of one to one [B182]. The schema of actuality is existence at a determinate time. The schema of necessity is existence at all times etc.” (161f./B179f.). [B184]

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Objections. We must openly admit that we hold this doctrine of the *Schematism of the Pure Concepts of the Understanding* to be utterly incorrect and we believe to be strengthened in our judgements in part by the fact that several friends of the critical philosophy have themselves passed a similar judgement about it.

1. We know nothing whatsoever of any *sameness in kind* that would have to obtain between the representation of an object and the concept under which we want to subsume it. Kant himself answers the question as to what this sameness in kind consists of only with the words: “the concept would have to *contain* that which is represented in the object” [B175] – How is this supposed to be possible? The concept *something*, for instance, is completely simple but it nonetheless comprises all objects. But according to Kant’s claim this concept (as Lambert, too, once incorrectly thought) would have to be the *most complex* one, and, because for

instance this rose bush can also be subsumed under it, it would have to contain as its components everything that I represent, and can represent, in respect to this rose bush!!

2. Neither can we admit that every pure concept of the understanding and every intuition are a pair of things so different in kind that the latter could never stand under the former. Isn't the concept of an intuition itself a pure concept of the understanding under which every intuition necessarily stands?
- 90 3. Furthermore, how should there be a third thing which mediates this subsumption, if the category and the intuition are so different in kind that the latter does not stand under the former? We would think that if the representation x can be subsumed under the representation M , and if the latter can be subsumed under the representation A , then logic teaches that the representation x also can be immediately subsumed under representation A .
4. That the transcendental temporal determinations are both intellectual as well as sensible, the latter namely insofar as they contain the representation of time (an intuition); this seems to be a mistake. We have already demonstrated that the representation of time does not belong in any way to the genus of intuition and therefore is to be ranked solely among concepts.
5. Likewise, as we have sufficiently demonstrated, the name *schema* can surely mean nothing else than the *genetic* definition of some concept. Mediating two objects through the schema has to mean as much as to bring this about with the aid of a concept, or to use a concept for subordinating an intuition under a concept. (See above, 71)
- 91 6. Finally, we cannot allow at all that the concept of a quantum or a *magnitude* just as little as that of a *number* comprehends in itself that of time. The *Theory of Science* §87 tells us what are the true components of these two concepts: magnitude as well as number.

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Kant advances the well-known principle of *contradiction* as the *supreme principle* of all *analytic* judgements (166f./B189). However, the principle of contradiction is not and cannot be either a principle in the proper sense of the word, or a proposition from which all analytic judgements can be deduced as consequences from their grounds. Analytic judgements cannot be deduced from this principle; rather they are merely special

examples of the latter. Furthermore, Kant himself does not seem to be taking this matter too seriously. Rather, in his elucidations he merely points to the contradiction that we would commit if we wanted to deny that a predicate belongs to a certain subject when the predicate is already contained in it as a component. And against this there is indeed nothing to object. But Kant should not have been, as he actually was, of the opinion that with this he had already explained the arising of analytic judgements and demonstrated that we are justified in making them. For from the fact alone that we do not go beyond the concept of the subject-representation in the formation of its judgement it cannot be inferred that this judgement *must* necessarily be a *true* one. If the subject-representation is not objectual, then an analytic judgement does not have truth either. Circular areas are geometrical figures, is an analytical and indisputably true proposition. But if there were no circular areas, then it would not be true that they are geometrical figures either. 92

Kant expresses the *supreme principle* of all *synthetic judgements* as follows: "every object stands under the necessary conditions of the synthetic unity of the manifold of intuitions in a possible experience" (171/B197).

Synthetic *a priori* judgements are thus possible for him "if we relate to a possible cognition of experience in general the formal conditions of *a priori* intuition, the synthesis of the imagination, and its necessary unity in a transcendental apperception". (ibid.) The above expositions make clear enough what we think about Kant's claims. We may therefore only say a few short words. Kant's supreme principle of synthetic judgements says nothing else than that every object of our cognition is conditioned by an *intuition* that we connect with it, or that we only then arrive at a cognition, only then can make a true judgement, if it is possible to combine an intuition with the subject that represents the object of the judgement. However, we have already demonstrated that this assumption does not make clear *how judgements of experience* arise, let alone sufficiently defines *synthetic a priori judgements*. According to Kant, the *possibility* of the latter also rests on intuitions, but not on empirical, rather on *a priori* or *pure intuitions* and these are the representations of *time* and *space*. We have seen above how false this doctrine is and that Kant never succeeded by means of it in explaining the arising of even one *a priori* judgement. The attempt to do this by means of the so-called *schema* of the concept and the *synthesis* of both with the aid of *pure imagination* has to be considered a complete failure. Likewise talk of a *synthetic unity of apperception* through the representation: I think, has to be considered absolutely unjustified. 93

Kant wants to order the *synthetic principles* as he did earlier the categories and devotes a special section to their *Systematic Classification* (1[7]1f. [B198f.]): “The table of the categories”, he says (173/B200), “gives us the table of principles, because these are after all merely the rules of the objective use of the former. All principles of the pure understanding are, therefore: 1. *Axioms of Intuition*; 2. *Anticipations of Perception*; 3. *Analogies of Experience*; 4. *Postulates of Empirical Thinking in general*”; and they read as follows:

1. “The Principle of the *Axioms of Intuition* is: *All intuitions are extensive magnitudes.*” (174/B201)
2. “The Principle of the *Anticipations of Perception* is: In all appearances the real, which is an object of sensation, has intensive magnitude, i.e. a degree.” (178/B207)
3. “The Principle of the *Analogy of Experience* is: Experience is possible only through the representation of a necessary connection of perceptions.” (186/B219) To this belong:
 - a) “Principle of the *Persistence of Substance*: In all change of appearances substance persists, and its quantum is neither increased nor diminished in nature.” (190/B224)
 - b) “Principle of *Temporal Sequence According to the Law of Causality*: All changes happen in accordance with the law of the connection of cause and effect.” (195/B232)
 - c) “Principle of *Simultaneity, According to the Law of Interaction, or the Community*: All substances, insofar as they can be perceived in space as simultaneous, are in thoroughgoing interaction” (211/B256)
- 95 4. “The *Postulates of Empirical Thinking in General*:”
 - a) “Whatever agrees with the formal conditions of experience (in accordance with intuition and concept) is *possible*.”
 - b) “That which is connected with the material conditions of experience (of sensation) is *actual*.”
 - c) “That whose connection with the actual is determined in accordance with general conditions of experience is (exists) *necessarily*.” (217/B265f.)

We would be hard pressed to give a summary of the proofs and elucidations that Kant adds to these principles. But we believe that the following **remarks** will put the reader in a position to make a judgement about the value of the pure principles advanced by our philosopher.

1. We admit that every *subjective intuition*, indeed every representation in general, has a magnitude in respect to its *duration* as well as its *liveliness*. But that is not what Kant claims in his Axioms of Intuition. All he actually wants to say here is that every duration in time and also every distance in space possesses a *magnitude*, an *extensive magnitude*, and that is incontrovertible. However, we cannot admit that these magnitudes are *intuitions*. For the *duration* of a second or the *distance* between the tips of my two fingers is and can be an object of my intuition just as little as the duration of a millennium or the distance between this fixed star and the disc of the moon that floats above it. What we intuit are merely certain colours, sounds and tones, etc. and we *conclude* from their presence, often with many steps in between, that a certain amount of time has passed and that certain objects are present that bring about these representations in us, and which stand in these or those spatial relations to each other.* 96

2. The principle that every *sensation* has a certain *magnitude* (namely an intensive one) is perfectly correct; for we certainly have to think of the powers of beings, and therefore also of their powers of sensation and their effects, the sensations, as being intensive magnitudes.

3. Nor is there anything to remark against the three anticipations of *perceptions*. Only, it seems to us that Kant did not recognize distinctly enough how all causes and effects are always simultaneous, because he claims (for example 206/B284) only of the *greater part* of the effective causes that they are simultaneous. But in fact things are as follows. Every cause that brings about a finite effect consists of a *change* and can therefore only come into being within a certain time. *As long as it lasts* (as long as something changes), *the cause*, namely the complete cause, must be present and *effective*. However, in the common use of language, we very often call something a *cause* which is only a *partial cause*, i.e. which is merely one of the objects that can bring about the effect, and only in combination with certain other ones. Of such partial causes it certainly 97

* We cannot help but to think with approval of the presentation that Dr Theodor Waitz gives of this topic in his *Grundlegung zur Psychologie* (Hamburg und Gotha 1846, Perthes, 87ff.): "It has to be denied that the soul arrives at the representation of a *surface originally* with and at the same time through visual impressions" (87); "The representations of a surface in general can be given *immediately* through vision just as little as this is possible with the representation of *continuity*" (89); "*Location* and *magnitude* of the objects are not immediately seen, rather *judged* and *estimated*" (90); and so on.

holds that they often are present a very long time before the actual effect comes into being and also sometimes outlast them. An example of this is stone and steel. They exist before the spark does, of which one commonly says that it is brought about by them, is their effect; therefore those two are its cause. Ultimately, stone and steel are merely partial causes of the spark. Because in order to produce the latter a force is also required that hits them against each other; air is also required and, in short, the whole complete cause which only first comes into existence at the same time as the effect. The effect outlasts the cause just as little. Rather, it ceases as soon as the cause stops being effective. There is no sign of a spark when stone and steel are not hit against each other. It is futile to object here that the building long outlasts its builder or that all bodies, once they have been set into motion by some force, continue to move in the same direction without stopping and at the speed that they acquire in the last instant in which the force has its effect on them and if they do not encounter a hindrance on their way, etc. For here, in these examples, something is taken to be the effect of a cause which really is not its effect but rather the product of many other factors. For instance, that the building lasts is in no way the effect of the builder, but rather the effect of the forces of the material the building consists of. For a body to change its place or move, speed is necessary, which lies in the body as the proper, immediate cause of the movement and a force that produces a change in the body's speed by affecting it. As long as this force takes effect, the speed persists, i.e. it grows. When the force stops being active, the effect, i.e. the change or the growth of the speed, stops too. Thus the speed that the body received at the last moment remains unchanged, and because the speed is the proper cause of the change of place, it also persists and the body moves without stopping. – These examples do not at all refute our above claim that cause and effect are simultaneous, rather they confirm it. Incidentally, this theorem is not without significance for science; for instance when it comes to deciding the metaphysical question: whether the world in its *eternal* existence can be taken to be an effect of God?

4. Finally we cannot avoid declaring the *three* so-called *Postulates of Empirical Thinking* completely incorrect. It is our conviction that all those and only those things which do not contradict a pure *conceptual truth* are *possible*. We claim that a plane circumscribed by two straight lines (*bilinium*) is not possible, but rather impossible because such a plane contradicts a pure conceptual truth; not, as Kant thinks, an intuition

(namely the pure intuition of space). As long as we proceed with the appropriate caution when forming our judgements, there is absolutely no reason to worry that this assumption should ensnare us in figments of the mind. – That no contradictory marks like **b** and **non b** are found in the composition of a concept is indeed not enough for presupposing the *objectuality* of that concept. But that we can only be assured of its objectuality if *experience* introduces us to an object that stands under this concept, that is also an exaggeration. This is the consequence of the false claim, which we have already refuted, that we are always only justified in forming a synthetic judgement if we have connected its subject-representation to a pure or empirical intuition and with it the predicate which we want to ascribe to the subject.

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In the same book of the *Analytic* we also find a section with the heading: *On the Ground of the Distinction of all Objects in General into phenomena and noumena* (236ff./B294ff.). Here it is repeatedly assured that “we cannot cognize any other objects than those that can be sensibly intuited and are called *phenomena*. The concept of a thing that would not be the object of a sensible intuition, *noumenon*, is a merely *problematic* one, of which we cannot know whether an object corresponds to it. For the object cannot be given otherwise than through intuitions, and we do not have intuitions other than sensible ones, say intellectual ones. We do not even know whether they exist.” (247ff./B310ff.) [B305–B331] Nonetheless, Kant grounds many an important doctrine on this problematic distinction between things, for instance that of human freedom, as we will see later!

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This chapter is accompanied by an “Appendix: On the Amphiboly of the Concepts of Reflection Through Confusion of the Empirical Use of the Understanding with the Transcendental” (254ff./B316ff.). – All judgements, it says here, first require a preliminary reflection, i.e. a comparison of the relevant concepts. It is either merely logical, if it regards strictly the relation of the concepts, or transcendental, if it regards the kind of cognition to which the concepts belong. The concepts which are compared can stand in the following relations to each other; identity and difference, agreement and opposition, the inner and the outer, the determinable (matter) and the determination (form) [B317]. If the concepts are compared merely logically, then one runs the risk of establishing a relation of this kind between them that has no validity from

the transcendental perspective. Kant calls such a violation an amphiboly. He advances the following rules:

- 102 1. *Identity and difference.* An object with the same internal determinations is always the same object if it is an object of pure understanding (*noumenon*). However, if it is a thing of appearance (*phenomenon*) then a mere difference in places at the same time can be reason enough to assume *several* of them. Leibniz's *Principium identitatis indiscernibilium* (that no two things can be completely identical with one another) therefore holds for *things of the understanding* but not for *things of appearance*, even though he wanted it to apply to them as well.
2. *Agreement and opposition.* Realities of the pure understanding as mere logical relations can never conflict with each other. However, as appearances, they can. – Again, Leibniz's claim that evil is a mere limitation does not hold of appearances.
3. The *inner* and the *outer*. An object of pure understanding must have internal determinations that do not have any relation to something that is different from it. By contrast, the internal determinations of a thing of appearance are nothing but relations. This caused Leibniz to assume *monads* as substances that *differ internally* through their peculiar representations. However their existence cannot be proven because they are not supposed to be appearances.
- 103 4. *Matter and form.* Matter precedes form in the concept of the pure understanding. That is why Leibniz first assumed things with powers of representation (monads) and first based their outer relations on this. However, if we determine all objects as appearances only in sensible intuitions, then the form of intuition, space and time, precedes all matter (the sensations).

Objections. 1. We cannot admit that every judgement is *preceded* by a *reflection* which compares its representations. For such a comparison, a cognition of identity or difference etc., is itself already a judgement and therefore would again presuppose another comparison or a judgement and so forth *ad infinitum*.

2. In general, we do not think that the four pairs of concepts Kant has collected under the name of concepts of reflection genuinely form a whole of the kind he boasts. Kant namely derives these concepts from the familiar four moments under which he places all forms of judgements. However, because this division of judgements seems incorrect to us, we cannot approve it for the concepts of reflection either. They are

meant to represent the mere relations in which *concepts* or *representations* in general stand to each other. By contrast, it is certain that when we claim that two representations are generated by one and the same actual object as their cause, i.e. when we claim that that which we are presently perceiving is the same as that which we already perceived at another time, we do not assert the relation of two representations. Rather, we intend to represent a relation in which two representations stand to an actual object when we apply the two concepts of *being identical* and *being different* in the way mentioned. The concepts of *agreement* and *opposition*, however, do not initially and immediately relate to representations, but to propositions. Only of the latter and not the former can one say that they agree or conflict with each other. We can say of certain representations **A** and **B** that they conflict only insofar as the two propositions: **x is A** and **x is B** conflict or cannot be true simultaneously. The concepts of the *outer* and the *inner* express two kinds of properties that are not in themselves representations but can merely be the objects of representations. Finally, *form* and *matter* are concepts that are indeed also applicable to representations, but to a hundred other objects as well. For, do we not say that gold, silver, marble, wood, etc. are the matter from which a sculpture is made? By contrast, we usually call the manner in which those materials are shaped into a whole the form of the sculpture. – “But even”, someone could say on Kant’s behalf: “yes even if these concepts were to relate to many different objects, it is still only when they are applied to representations and judgements, only then, that they form a whole that belongs to logic. The understanding produces a universal or particular judgement insofar as the relation in which the objects of the subject- and predicate-representations stand to each other appears to it as a relation of identity or difference. The understanding affirms or denies to the extent that it realizes that the subject and the predicate agree with each other or that they conflict. The understanding judges categorically or hypothetically or disjunctively to the extent that the relation between subject and predicate is an internal or an external one. Finally, the understanding asserts a problematic, assertoric, or apodictic judgement depending on whether it believes that the ground of the combination of the two representations mentioned must be found in the form or in the matter.” – Against this, we have to object that even if we were to accept the way in which the origin of the universal and particular, the affirmative and the negative judgement is being explained here (even though it sounds a bit strange to say that the understanding produces a universal or particular judgement by discerning that the subject-representation stands to the predicate-representation in a relation of either identity or difference), still,

we cannot allow that the difference between categorical, hypothetical and disjunctive judgements lies in the fact that in the former an internal relation between subject and predicate occurs, while in the two latter, it is an external one. For wouldn't one have to use the concepts of the inner and the outer in a truly arbitrary manner if one claimed that in the proposition: The planet Mercury is closest to the sun, an internal relation is expressed, while by contrast in the proposition: If Mercury is closest to the sun, then the intensity of light it receives is greater than that which the Earth receives; or: Mercury receives either a stronger or a weaker light than the Earth, an external relation is expressed? We can only agree even less with the claim that we make a problematic judgement if the ground of the combination between subject and predicate is given merely in the form, while an assertoric or apodictic one is made if the ground is given in the matter. The ground why I am allowed to make the judgement: The moon has no rivers, even problematically is not found through the consideration of the form alone, rather, it lies in the matter.

3. While Leibniz's principle of the *identity of indiscernibles* is not strictly proven by the mere fact that two substances, in order to be completely identical with one another, would have to experience the very same fate for all eternity in addition to having entirely the same original conditions, the latter still gives it an extremely high, indeed an infinitely high degree of probability.
4. That logical affirmations can never conflict with each other is an error that comes from the idea that there are no truths other than analytical ones, or that the proposition **A** has **b** can be true only if the representation **b** lays in the representation **A** as a component. If the opposite occurs, then the propositions: Every **A** has **b**, and: No **C** has **b**, can both be true even if the representations: **A** and **C** do not contain any negation; for then the representation of a thing that would be **A** and **C** at the same time is a contradictory one even though it is composed merely of affirmations. For this reason we do not at all, as indeed others have done before, define the most perfect being as a being that unites every conceivable power in itself, rather only as a being that unites all those powers that are possible alongside one another, and that it unites them to the highest possible degree in which they can subsist alongside one another.
5. That all determinations of the things of appearance consist only in relations, this we already deny for the reason that we humans must indeed be counted among the things of appearance as well, and the

property of thinking which each one of us perceives within oneself is admittedly not a mere relation but an inner determination.

6. The question whether matter precedes form or form matter surely cannot be understood in the proper meaning of a precedence in time, and therefore has probably only the sense of the question: which of the two grounds or conditions the other? – If, therefore, we understand by matter, as is necessary to understand here, the substances, and by form their relations in time and space, then we can agree neither with the opinion that Kant ascribes to Leibniz, namely, that the temporal and spatial relations in which the substances are found are grounded in the inner determinations of the latter, nor also with the Kantian verdict. For, from the internal properties of a substance follows neither at which time it has these properties, nor in what place it is to be found. Rather according to our opinion the places of things are those relations that we have to think of in addition to their forces, in order to be able to explain why they exert this and that influence on each other at this and that time.

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The Transcendental *Dialectic* forms the second part of the Transcendental Logic. It is prefaced by the doctrine of *transcendental illusion* (276ff./B349ff.), then by the doctrine of *pure reason* as the seat of transcendental illusion (280ff./B355ff.). For Kant's dialectic is nothing other than "the logic of illusion" [B349]. Accordingly, he begins by posing the question: from whence do illusion and error in general arise?, and answers: not through the senses; for the senses do not judge at all. Not through the understanding alone, because as long as the understanding proceeds merely according to its own laws, the judgements it makes necessarily must be in accordance with these laws, and then they will not be erroneous but correct. *It follows that error must be brought about only through the unnoticed influence of sensibility upon the understanding*; through this it happens namely that "*the subjective grounds of the judgement flow together with the objective ones and make the latter deviate from their destination.*" (277/B350f.) Kant further remarks that it will not be *empirical* but *transcendental* illusion which comes into investigation here. The first arises merely due to a failure of attention to the rules of logic and disappears as soon as one pays attention; the second, by contrast, will not cease even if one has already detected it and has already had distinct insight into its nullity through the transcendental critique. The cause of this is that in our reason there lies fundamental rules and maxims for its

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use, which look entirely like *objective principles*, and on account of this it comes to pass that the *subjective* necessity of a certain connection of our concepts for the benefit of the understanding is taken for an *objective* necessity of the determination of things in themselves. (278f./B353)

We remark:

1. We cannot approve of the manner in which Kant tries to explain the origin of error. For the question of *how* sensibility exercises its *influence* on the understanding and brings about the *flowing together* of the subjective grounds with the objective ones, is here left unanswered. Indeed, 110 it is not even clear to us how we should actually represent this flowing together.
2. If, in order to explain error, one presupposes that there are certain *maxims* of the use of reason which have the (false) *look* of objective principles, then the presence of a certain error would indeed already lie at the basis of this presupposition itself, namely the error in virtue of which we hold the merely subjective necessity of a connection of concepts for an objective necessity.
3. Bolzano approaches the subject in a completely different and, as we deem it, more correct manner. He starts by bringing to attention that, just as there is a relation of deducibility, there is also a relation of *mere probability* between propositions in themselves or propositions in the objective signification, i.e. entirely irrespective of our power of judgement. For instance, the proposition that Cajus has drawn a black ball from this urn, stands in a relation of probability to the proposition: The urn contains 90 black and 10 white balls, such that the degree of this probability is = 9/10, and it does so completely irrespective of whether or not there is someone who holds the one or the other propositions to be true or not (*Theory of Science* §309). Furthermore, Bolzano claims that it is a peculiarity of our power of judgement that, if a proposition **M** is recognized by us to stand in a relation of uppermost probability to certain other propositions **A, B, C, D...** which we hold to be true (i.e. 111 if we regard the probability to be greater than $\frac{1}{2}$), then *one will not only make the judgement that M is probable, but also the judgement M itself*, and this with a degree of *confidence* that is in accord with the perceived degree of probability. Nobody can deny that this actually is the case; nor that we do not necessarily err by *judging* this way (by expecting that which has a higher degree of probability). But how the *possibility of erring* comes about through this peculiarity of our mind is thus certainly comprehensible. For the *probable*, no matter how high its degree of

probability, does not for this reason have to be the *true*. Thus by seeing ourselves compelled to *expect* that which has a high degree of probability, and therefore to hold it to be true, we sometimes hold something to be true which is in fact not true, i.e. *we err*.

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Of *pure reason* (the seat of transcendental illusion) Kant gives us the definition that it is the faculty of *principles* (synthetic cognition from pure concepts) (280/B355f.). The aspiration of reason, especially when drawing inferences, is to bring the great manifold of cognition under the smallest number of principles and thereby effect the highest unity (284/B362f.). The highest principle of pure reason is the proposition: "If the conditioned is given, then the whole series of conditions, or the unconditioned, is also given." (286/B364) The question, however, is whether this principle has *objective* validity or whether it is a merely logical prescription to bring the highest possible unity in our cognitions. Kant examines this in the two books of his Transcendental Dialectic, the first of which deals with the *concepts* of reason, the second with its *inferences*. 112

The *concepts of pure reason* that go beyond the possibility of experience are what Kant wants to call *Ideas*, and he claims that Plato, too, used the word in this sense (289/B370), something which we do not want to dispute even though we believe that there is a more appropriate signification for this word. Kant says that just as the forms of judgement deliver the categories, so too can we expect that the forms of the *inferences of reason* will offer us the transcendental Ideas (295/B378f.). The series of *prosyllogisms* or the ascending series of conditions, whether it is bounded or unbounded, must namely always be presupposed by reason as *complete, unconditioned, or absolutely true*. (301f./B387f.) – Kant derives the transcendental Ideas themselves from the following consideration. (302ff./B390f.) Every relation of a representation is twofold, either to its subject or to its object, the latter of which can either be mere appearances or objects of thinking. Therefore, *every relation of representations* is threefold: (1) to the subject, (2) to the manifold of the object in appearance, and (3) to all things in general. Consequently, all transcendental Ideas can be brought under three classes: (1) the unconditioned unity of the thinking subject (Idea of the *soul*); (2) the absolute unity of the series of conditions of appearance (Idea of the *world*); (3) the absolute unity of the condition of all objects of thinking in general (Idea of *God*). 113

Remarks. Could there be anything more forced than this deduction of the *three Ideas: Soul, World, God*? And this liberty in the relations or

relationships among representations? Is it not forced and incorrect! – A relation to the thinking *subject* will obtain only in representations in the subjective signification of the word, i.e. in the appearances in the mind of a thinking being. By contrast, to an *object*, *representations in themselves* are also related. Furthermore, it should not be forgotten that not every representation (whether we understand the word in its subjective or its objective signification) represents an object, e.g. the representation of nothing, $\sqrt{-1}$, etc. Finally we should remember that the division of objects into *appearances* and *objects of thinking* is in general extremely illogical, because the first member is wholly lodged
 114 in the second. – However, even more peculiar is how one is supposed to derive the two Ideas: World and God from the latter two relations. Is the World really the collection of *only* the things of appearance? Not of *all finite* substances? It is just as incorrect to define God as a thing (being) that contains the highest condition of the possibility of all that can be thought. For *first*, not all objects of thinking are something that *exists*, like for example truths in themselves. However, it is false to make the possibility of truths in themselves, above all that of the necessary *a priori* truths dependent on God's thinking and cognizing. God cognizes conceptual truths, pure conceptual truths, the law of gravity, the highest moral principle, etc. because they are true. But they are not truths because God cognizes them. Rather, they are completely independent of the will of God, the Creator. *What is more*, it is not true that everything that exists has a ground for its existence and its possibility that lies in God Himself. For at least God Himself has to be excluded from this.

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As regards the dialectical *Inferences of Pure Reason*, there are supposed to be *three classes* of the latter (307/B397), just as there are three Ideas which result from their conclusions. In the first class, from the concept of a subject that contains no manifold, the absolute unity of this subject
 115 is inferred. Kant calls this inference the *Paralogism* of pure reason. The second class concerns itself with the absolute totality of the series of conditions for a given unity. Kant calls these the *Antinomies*. In the third class, one infers a Being of all beings. He calls this inference of reason the *Ideal* of pure reason. Incidentally, it was already stated on pages 283 [B360f.], and even repeated on page 296 [B379] that these three kinds of dialectical inference correspond to the three kinds of syllogisms: the *categorical*, *hypothetical*, and *disjunctive*.

Remarks. Were we to want to provide here a complete refutation of Kant's doctrine of the dialectical inferences, proving how lacking Kant's doctrine of inferences in general is would take us too far. For this reason, the following alone must suffice. – The very division of the inferences into *immediate* and *mediate* ones, which we find in Kant's *Logic* and which is maintained by his school, does not seem appropriate to us insofar as inferences with one premise are counted among the *former* and inferences with several among the *latter*. For if the classification of inferences is grounded on the number of their premises, one can't see why one should assign a distinct class to inferences with one premise, but bring inferences with two, three etc. premises all into one single class? One could at best do this if one could find no remarkable, simple and peculiar inferences among the inferences with more than two premises. But that is not the case. And with what right does one call some inferences immediate and others mediate? Does not every conclusion result, to the contrary, immediately from its premises? – Further, we hold the classification of judgments according to the four familiar moments of quantity, quality etc. to be incorrect. Thus we also cannot admit its use for the classification of inferences. How extremely arbitrary and in many ways forced does its application to the classification of immediate inferences appear to be! Here, in immediate inferences, each moment is supposed to have its own mode of inference, whereas with mediate inferences, by contrast, it is a different story, since the moment of relation alone forms the ground for the classification! Even if we granted that the inferences according to the moment of quantity do not yield an appropriate classification, it would still not be clear why the quality of the propositions in an inference could not ground a useful classification. The various *modi* of syllogism which the logicians introduce in fact align themselves to the quality of the premises. And what are these *modi* but members of a classification? But as regards modality, after all, the same differences that apply to inferences must apply to propositions in general insofar as the modality of a proposition is namely understood to be the higher or lower degree of certainty with which one holds it to be true. – Why the syllogism is the only simple (mediate) kind of inference and why there cannot be more of the latter is just as unclear. Undisputable examples teach us that among them are several kinds of inferences with two premises that do not belong to syllogisms. How can we, for instance, subordinate the following argument to the class of syllogistic inferences? From the following two premises:

“What has **a** also has **b**, and
What has **b** also has **a**”

one can deduce the conclusion: "Every object of one of the representations A and B" (A and B designate the concreta of the abstract representations **a** and **b**) "is an object of both" or "A and B are reciprocal representations"* – how could this be classified as a syllogistic mode of inference? Shouldn't one much rather admit that this is a wholly simple kind of inference which is fully different from syllogism? – The same also holds for the inference:

"What has **a** has **b**,
 What doesn't have **a** has **b**,
 Therefore everything has **b**."

118 and for many other inferences, that not only rest on two premises, but also of those that rest on three and more. – Finally, we consider the division of syllogisms into categorical, hypothetical and disjunctive to be incorrect. The so-called hypothetical syllogism is merely one species of categorical syllogism, namely the *modus Barbara* if one infers *modo ponente*, and *modus Camestres* if one infers *modo tollente*. It should therefore not be coordinated with the categorical form, but rather subordinated to it. Why the kind of inference called the disjunctive syllogism can be counted as a syllogism at all is completely incomprehensible. The *medius terminus*, which is essential to all syllogistic inferences, is here missing completely, and there is also something incorrect with the presence of the two *termini extremi* that are supposed to be united in the conclusion. For if one infers *modo ponente*, then from the two premises, "Among the propositions A, B, C, ... only one is true", and "A is true", we obviously derive the conclusion: "Therefore B, C, ... are false"; and *modo tollente*, from the premises: "Among the propositions A, B, C, ... only one is true." and "A is false" the conclusion results: "Therefore there is also only one true proposition among the propositions B, C..." Yet where is the *terminus medius* and where are the two *extremi* that would be united in the conclusion? Indeed, the representation A occurs in both premises, but not as the subject- or predicate-representation in the main premise and therefore not as the *terminus medius* (see Bolzano's *Theory of Science* §223ff.).

119 One who ponders on all this and acquires from it the conviction that the common doctrine of inference is flawed and incorrect, namely their

* The following can serve as an example: What has equilaterality also has equiangularity; what has equiangularity also has equilaterality; thus *what is equiangular* and *what is equilateral* are reciprocal representations.

classification into categorical, hypothetical and disjunctive ones, will not be especially inclined to see in the circumstance that each of “the three dialectical inferences” should “*correspond* to the three inferences of reason” [B396f.] a special guarantee for their deeper grounding in the nature of the human capacity of cognition.

* * *

Let us now examine what Kant teaches about the three classes of dialectical inferences, taken one by one, and indeed starting with what he claims about the *Paralogisms of Pure Reason* (308ff./B399ff.).

The (transcendental) concept: *I* (we read on 309/B399) is missing from the table of the transcendental concepts, *because* it is the vehicle of all concepts in general; the *I think* is the sole text from which rational psychology is to develop its entire wisdom (310/B499f.). Nonetheless, Kant repeatedly declares that this: *I think*, is itself already an *empirical* proposition (323, footnote/B422, footnote). – When the table of categories is applied to the *I* (= soul), one obtains the following four propositions: (1) The soul is *substance*; (2) In its quality, *simple*; (3) In the different times at which it exists, numerically identical, i.e. *unity*; (4) in relation to *possible* objects in space. These are the four *paralogisms* of the transcendental doctrine of the soul (311/402f.). For

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1. that the *I* always has to be considered as the subject in thinking is an *identical* proposition, but that this *I* is a *substance* is not and demands altogether different data. (314/B407)
2. that the *I* is a *singular* thing is likewise a merely *analytic* proposition, but that it is a *simple substance* is a *synthetic* proposition, one we are not justified to establish through mere sensible intuitions. (ibid./B407f.)
3. likewise, the identity of the subject of which *I* am conscious in all my representations does not concern the intuition through which it is given as an object, and thus cannot prove the identity of the substance either; for the latter various synthetic propositions would be necessary. (315/B408f.)
4. that *I* distinguish myself from certain things outside of me is likewise an analytic proposition; but *I* do not thereby know whether *I* could exist without these things outside of me. (ibid./B409).

After this, Kant tries to prove that there are fallacious inferences in the common proofs of the substantiality of the soul and, likewise, in Mendelssohn’s proof of the persistence of the soul (316ff./B413ff.).

121 **Remarks.** Our readers already know that Kant's claim that we are only justified in making analytic judgements about supersensible objects but in no way in making synthetic ones, and therefore that we will get caught up in empty paralogisms if we attempt to judge in this way, is one that we hold to be itself a paralogism, and so we can therefore conveniently restrict ourselves to the following remarks.

1. In the Introduction to this part of the *Critique*, it is proposed that what these paralogisms of rational psychology have in common is that they infer the absolute *unity* of the subject from the transcendental concept of the subject which contains no manifold. Yet, by contrast, in the discussion it turns out that *only a single one* of these (four) paralogisms concludes to the *unity of the soul*, and this from a completely different ground than the one just indicated, namely from the identity of consciousness. The other three paralogisms have completely different conclusions.

2. That there is a *substance* in which thinking takes place can probably be claimed with the same certainty with which we infer to the presence of any substances whatsoever, as long as we leave it undecided whether it is a simple one or a collection of several.

3. However, that our soul is a *simple* substance, this is definitely not taught to us by our *intuitions* immediately, just as little as they teach us immediately that the things surrounding us are *complex*.

122 4. We also admit that certain synthetic judgements are needed for demonstrating the *identity* of the soul. But we have shown that Kant failed to demonstrate the impossibility of such synthetic cognitions.

5. It is not a matter of knowing whether I could exist without things outside of me, rather only of knowing whether my soul is a substance that is *numerically* distinct from the substances of these things.

6. Mistakes can certainly be found in the common proofs, specifically also in Mendelssohn's proof, even if they do not contain the ones of which Kant accuses them. However, one would like to see demonstrated which fallacies have been committed in the proofs for the substantiality, simplicity, identity, and imperishability of the soul that Bolzano conducted in *Athanasia* (Sulzbach 1838, Seidel, 2nd edition).

* * *

The *Critique* pronounces itself about the *Antinomies* of pure reason in the following manner (330f./B432f.). If reason is applied to the *objective*

synthesis of appearances, then something special happens, namely that it soon sees itself entangled in contradictions and an *antithetic* of pure reason ensues. In turn reason in its progress necessarily encounters propositions that carry with them not a merely artificial, but a natural and unavoidable illusion. – If we now examine the table of the categories and highlight those that carry with them a series in the synthesis of the manifold, then it turns out that there are no more than the following *four cosmological Ideas*, namely: The absolute completeness (1) of the *composition* of the given whole of all appearances, (2) of the *division* of a given whole in the appearance, (3) of the *arising* of an appearance in general, (4) of the *dependence of the existence* of what can change in appearance. (336/B443) These Ideas lead to the following antinomies (conflicting propositions).

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First Antinomy. Thesis: The world has a beginning in time, and it is enclosed in boundaries with respect to space. *Antithesis:* The world has no beginning and no bounds in space, but rather it is infinite with regard to both time and space. (344f./B454f.)

Second Antinomy. Thesis: Every composite substance in the world consists of simple parts, and nothing exists anywhere except the simple or what is composed of it. *Antithesis:* No composite thing in the world consists of simple parts, and nowhere in it does there exist anything simple. (350f./B462f.)

Third Antinomy. Thesis: Causality in accordance with laws of nature is not the only one from which all the appearances of the world can be derived. It is also necessary to assume a causality through freedom in order to explain them. *Antithesis:* There is no freedom, but everything in the world happens solely in accordance with laws of nature. (358f./B472f.)

Fourth Antinomy. Thesis: To the world belongs something that, either as a part of it or as its cause, is an absolutely necessary being. *Antithesis:* There is no absolutely necessary being existing anywhere, either in the world or outside the world as its cause. (364f./B480f.)

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Kant provides proofs for each of these *eight* propositions in which *no* other *fallacy* is supposed to be demonstrable if it is not that one presumes the right to judge synthetically about supersensible objects.

In merely extract form, these proofs read as follows:

1. For the *first thesis*. The world necessarily has a beginning in time; for if it had no beginning, then an eternity would have elapsed up to every given point in time, and hence an infinite series of states following one another would have passed by in the world. However, an infinite

series of states of things following one another is not possible because the infinity of a series consists precisely in the fact that it can *never be completed through a successive synthesis*. – However neither is the world infinite in respect to space. Rather it is enclosed in boundaries. For the infinite aggregate of actual things cannot be viewed as a given whole, and therefore it cannot be viewed as given *simultaneously*. For in order to think as a whole the world that fills all space, the successive synthesis of the parts of an infinite world would have to be viewed as completed, i.e. *an infinite time would have to be viewed as elapsed in the enumeration of all co-existing things*, which is impossible.

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For its *antithesis*. The world does not have a beginning. For suppose it has a beginning, since the beginning is an existence before which a time has passed in which the thing is not yet, there must be a preceding time in which the world was not, i.e. an *empty time*. But now in an empty time, no arising of any thing whatsoever is possible. Because no part of such a time has in itself prior to any other part any distinguishing condition of its existence rather than of its non-existence. Thus many series of things can indeed begin in the world, but the world itself cannot have any beginning, and so in respect to time is infinite. – However, the world cannot have boundaries in space either. For let's assume that the world is finite and bounded with respect to space, then it follows that it finds itself in an *empty space* which is not bounded. There would thus be not only a relation between things in space, but also a relation of things to space. Such a relation, however, and hence also the boundedness of the world by an empty space, is a non-entity, therefore the world is not bounded with respect to space. [B454ff.]

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2. For the *second thesis*. Everything composite consists of simple parts. For assuming that the composite does not consist of simple parts, then, if one were to think away all compositeness, there would be no composite part left over, and since there are also no simple parts, also no simple part, and thus nothing at all would be left over.

For its *antithesis*. No composite thing consists of simple parts. For every part of something composite has to take up a space. *Space*, however, does *not* consist of *simple parts*, but again only of spaces. Thus the simple, if there were such a thing, would take up some space. But then it could no longer be simple, rather it would have to be composite. [B462ff.]

3. For the *third thesis*. If there is a causality according to mere laws of nature or of necessity, or if everything happens according to the laws of nature alone, then at every time there is only a subordinate but never a first beginning and thus no completeness of the series on the side of the

causes descending one from another at all, because every current state of things can only be an effect of some preceding state. Thus the claim that there is only a causality in accordance with laws of nature contradicts itself, and there must therefore be still another kind of causality, namely a ground of causes which has *absolute spontaneity* and begins from itself the series of the appearances that proceed according to laws of nature, and this is a *transcendental freedom*.

For its *antithesis*. There is no freedom, but everything happens in accordance with mere laws of nature. For suppose there was a freedom 127
in the transcendental sense, as a special kind of causality, in accordance with which the occurrences of the world could follow, namely a capacity of absolutely beginning a state, and hence also a series of its consequences, then not only would *one series* begin absolutely through this spontaneity, but the determination of the spontaneity itself to produce the series, i.e. the *causality, would begin absolutely, so that nothing precedes it* through which this occurring action is determined in accordance with constant laws. This, however, is contrary to the law of the connection of causes. [B472ff.]

4. For the *fourth thesis*. The world of sense is a series of changes, i.e. of conditioned things. Now everything that is conditioned presupposes, with respect to its existence, a complete series of conditions up to the absolutely unconditioned, which alone is absolutely necessary. Thus there exists necessarily something absolutely necessary. This necessary being however belongs itself to the world of sense. For supposing it is outside of it, the series of changes in the world would then derive its beginning from a necessary cause which does not belong to the world of sense, and that is impossible. *For since the beginning of a time-series can be determined only through what precedes it in time, the supreme condition of the beginning of a series of changes must exist in the world when the series was not yet.* Hence the *necessary cause* of the changes *itself* belongs 128
to time, i.e. to *appearance*, and cannot be thought as separate from the world of sense.

For its *antithesis*. There is no absolutely necessary being existing anywhere, neither in the world nor outside the world. *Neither in the world.* For suppose the world itself were a necessary being or there were such a being in it, then in the series of its changes there either would be a beginning that would be unconditionally necessary and hence without a cause, which is contradictory; or else the series itself would be without any beginning, and, although contingent and conditioned in all its parts, it would nevertheless be absolutely necessary and unconditioned

as a whole, which contradicts itself. *Nor outside the world.* For suppose there were an absolutely necessary cause of the world outside the world, then this cause, as the supreme member in the *series of causes* of changes in the world, would first begin the existence of these changes and their series. But then it would have to begin to act, and its causality would belong in time and for this very reason in the collection of appearances, i.e. in the world. Consequently it itself, the cause, would not be outside the world, which contradicts what was presupposed. [B480f.]

Kant explains this very striking appearance of the *antinomies* in the following way (394/B525).

129 If the conditioned is given, then a regress of the conditions up to the unconditioned is not also always *thereby given* but only *given as a problem*. The unconditioned is given together with the conditioned only if both are *things in themselves*. If, however, we are speaking of *appearances*, then it is not necessary for the complete series of conditions to also be given together with the conditioned; for this could only be first given as a sensible thing if the regress to the unconditioned had actually been completed, which is impossible if the series is infinite. – The transcendental inferences of reason take in their *major premise* the conditioned in its transcendental signification as a thing in itself and in this case the unconditioned would in fact have to be given. However, in the *minor premise* we are speaking only of mere appearances. Therefore, the error that is made here is really a so-called *sophisma figurae dictionis*. Thus the contradiction that obtains in the antinomies can be resolved if we note that here the idea of absolute totality is applied to appearances which exist only in representation, even though it holds only as a condition of things in themselves. – After this, Kant presents us with the following *resolutions of the cosmological antinomies* (407ff./B545ff.).

130 The first two, he says, have the peculiarity that the two propositions that conflict with one another are false to a certain extent; while the last two, by contrast, have the distinctive feature that the two propositions that conflict with one another are *true* to a certain extent, namely:

1. *The idea of the totality of the composition of appearances of a world-whole* (407/B545). The world has *no beginning* in time and *no boundaries* in space but nonetheless, it is *not infinite*. The first; because the perception of such a *beginning* and such a *boundary* is something impossible, for empty time and empty space cannot be perceived. The second; because an infinite magnitude is not the object of a possible perception either.

2. *The idea of the totality of the division of a given whole in intuition* (411/B551). When I divide a whole that is given in intuition, the possibility

of division has to go on to infinity; but I am nonetheless not allowed to say that the whole consists of *infinitely many parts*.

3. *The idea of the totality of the derivation of occurrences in the world from their causes* (416/B560). One and the same effect can be viewed with respect to its intelligible (i.e. not its appearing) cause as free, and yet at the same time with respect to appearances as a succession from them according to the necessity of nature. "These appearances, because they in themselves are not things, must be grounded in a transcendental object which determines them as mere representations. Therefore nothing hinders us from ascribing to this, apart from the property through which it appears, also a *causality* that is not *appearance* even though its effect is encountered in appearance [B566f.]. Now this acting subject would not stand under temporal determinations. Hence it would not be subject to the law of all temporal determinations that everything that happens must find its cause in the appearances [ibid.]. It could from itself begin its effects in the sensible world, and as appearances these would nonetheless still have their empirical conditions in the preceding time." (42[2]f./B566f.) – Such a case is to be found in us *humans*. Our reason possesses a peculiar causality that springs from its *imperatives*. The *ought* expresses a possible action whose ground does not lie in the things of experience. Thus our actions follow *necessarily* from empirical causes, on the one hand, and, on the other hand, from *freedom*. ([4]26f./B575f.)

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4. *The idea of the totality of dependence of appearances according to their existence in general* (434/B587). All things of the sensible world are contingent. However, the series as a whole has a non-empirical condition. There exists therefore an unconditionally necessary being, but not as a member of the world of sense, but as an intelligible being. (435/B588)

Objections.

1. It is certainly strange that our human capacity for cognition should be constituted in such a way that when we engage in a reflection that we do not pursue arbitrarily but rather treat as a necessary and inescapable task in the development of our powers of mind we should *encounter four propositions that conflict with one another*.

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2. We probably do not need to point out how forced is the deduction of the cosmological Ideas from the table of the categories. Who would have guessed that the *fourth* one arises from the concept of *contingency* and not from that of *dependence* or *causality*? Who would have suspected that the *second* one belongs to the categories of *quality* (reality, negation, limitation)?

3. Those who do not want to deprive our reason of the right to an assessment of supersensible things will be aware of the fact that the proofs that Kant has thought up for his antinomies contain mistakes of a wholly different sort.

a) The proof of the *first thesis*

133 α) implicitly assumes that there cannot be a *multitude* present without someone present who has *counted through* it, or at least *could count through it* (cf. 346, note/B456). If this were the case, then of course it would already be decided that no *infinite* multitude could be present. For an infinite multitude is one that is greater than any number, and therefore could not be enumerated. However, why should it belong to the possibility of a certain multitude that someone be there who counts through it and who represents individually all the units of which it is composed? Doesn't one declare with this claim also that all irrational relations are impossible? For instance, could there be even *one* square, given that every square displays the magnitude $\sqrt{2}$, i.e. an infinite multitude of fractions, in its diagonal? One sees that Kant's claim brings to an end the whole of mathematics.

β) It is just as incorrect to presuppose that there can only be *series in time* and that the *infinity* of a series consists in the fact that it *can never be completed through the successive synthesis of its members*. However, the concept of a series does not comprise that of time at all and it is possible that there are countlessly many finite as well as infinite series that have nothing to do with time. Of this kind is the series which expands infinitely in both directions

...., -4, -3, -2, -1, 0, +1, +2, +3, +4, ...

(see *The Theory of Science* §85 and §87 footnote)

134 γ) Finally, the claim that *an infinite time cannot have elapsed* is so entirely false that we should instead claim that an infinite time has elapsed prior to every instant. For as every mathematician knows, time as a whole can be divided into two parts at every instant, one of which is past, the other future, and both are completely the same in respect to all their internal properties. Therefore, if one wanted to claim that an infinite time has not elapsed, one would also have to claim that there is no infinite future time.

δ) The thesis as a whole is as false as the proof Kant provides for it. By contrast, it seems to us that the *antithesis*, according to which the world has no beginning in time and no boundaries in space, is not only completely correct but also demonstrable, if only through grounds other than the ones Kant introduces and which rest upon the false presupposition of an empty time and an empty space.

b) The *thesis* of the *second antinomy*: “Every composite substance consists of simple parts, and nothing exists anywhere except what is simple and what is composed of simples.” [B462] – seems to us to be true; the *antithesis*, by contrast, seems to us to be false: “No composite consists of simple parts.” [B463] Kant derives the latter from the claim that space does not consist of simple parts but only of spaces, i.e. of manifolds that are external to each other. Now this is false if it is taken in the sense in which it must here be taken. For there are two different kinds of parts of composite things, namely *homogeneous* ones, i.e. ones that belong to the concept to which the whole is related, and *heterogeneous* ones. For instance, if we grind a piece of saltpetre, then we get homogeneous parts; however, if we produce hydrochloric acid and potash, out of whose combination saltpetre is composed, from the saltpetre by precipitation, then we get heterogeneous components. Hence when we are talking about parts which are homogeneous with the whole, then there is no doubt that all space (everything which is extended) consists of mere spaces (of extended things), a line of lines, a plane of planes, etc. If however we are speaking about parts in general and indeed in such a way that we understand by this heterogeneous ones as well, then we have to admit that every space, whether it is a line or a plane or a body, also consists of simple parts, namely of points. For space is actually nothing other than a collection of points. To make this even more evident, we note that all mathematicians teach that there are points in every line, plane, and body, which means no more and no less than that points make up their parts. Indeed, someone might object that these are not components (*partes integrantes*) because it is impossible to produce a line, plane, or body from points, no matter how many one might use. Certainly, we reply, if there is only a finite multitude of points; and we will not deny that not every infinite multitude of points thereby forms an extended thing, but rather an extended thing is formed only through that collection of points which is so constituted that, for every point in it and every distance, even for the smallest one, there is one or more points that in fact have this distance. Only such a system of points, whether it is a line, a plane, or a body, is a true continuum.*

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* The concept of continuity is therefore the concept of that attribute of a spatial thing according to which every one of its points has a neighbouring one for every sufficiently small distance or, as one can also put it, it has so many neighbouring ones that none of them is *closest* to it. See Robert Zimmermann's *Leibniz's Monadologie*, 167ff., for a adept proof of the impossibility of closest spatial points and thereby for the continuity of space.

Thus, whoever speaks about parts in general without understanding by this a specific kind of part, e.g. the homogeneous ones alone, makes an error when he claims that space does not consist of simple parts, namely infinitely many of them.

- c) The theses of the *third antinomy* are extremely obscure even in the way they are expressed. In the proof of the antithesis it is said that “by the term freedom is here understood a capacity for *absolutely beginning* a state and with it a series of effects, such that nothing precedes it through which this occurring action would be determined in accordance with constant laws”. [B474] Here we can only wonder why such a capacity should be called causality, given that this word designates only a causal nexus, which does not occur in this case. In addition it is presupposed that every cause precedes its effect, and we have already shown this to be false. Even more important however is the fact that this definition defines something altogether different from what one usually understands by the word *freedom* in the common use of language. For, instead, we consistently understand by it a certain property of the capacity for willing. If we now take the concept in the sense that comes closest to this definition (i.e. in the sense of indeterminism), then freedom is the possibility of a decision of will without a determining ground, indeed such that the same decision under the same circumstances could have failed to be made. In this sense it can be said that every free decision of the will begins a series of effects, but it can never be claimed that freedom can be found wherever a series of effects begins absolutely. A series of effects begins absolutely when a cause is active that lacks any further cause, which is the highest being or God and to Whom one can certainly not ascribe freedom in the sense of indeterminism. – In the proof of the thesis it is claimed that “there can be no completeness of the series on the side of causes descending one from another if everything happens according to the laws of nature” [B472], i.e. if everything that happens has a cause that is itself something that happened. Yet from this hypothesis it follows merely that the series of events must be infinite, not that it must be incomplete and cannot exist in actuality. For as we have noted above, infinite series can exist and have gone by. – Furthermore, if an infinite series consists of causes in the strict sense, then it is simultaneous with its effects. If the members are conditions rather than causes, then every preceding member must be a condition of the one that follows after it, but no infinite time is necessary for them to elapse. Every movement through which a body covers even the smallest distance gives us an example of a series consisting of infinitely many members, a series within which the preceding one is the condition of

the one that follows and has to precede it in time, even though the whole span of time during which the members continue to move can be very short.

d) Finally, the proof of the *fourth antinomy* presupposes wrongly that the series of events in the world has had a beginning in time if it is grounded in something else. But it should rather have been presupposing that *the substances of the world exist for all time*, but only as the effects of that substance which is unconditionally actual. – Also premature is the conclusion that a substance that is effective in time belongs to appearances and therefore to the sensible world. No simple substance can be counted among sensible things, all the less the substance responsible for Creation. Nonetheless, even though they are supersensible, substances are effective in time, as for instance our soul is. God, too, can be effective in time even though he is not an object of the senses.

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4. No matter how many essential mistakes these proofs may contain, they are free of the only one Kant wants to admit they have, namely that of a *fallacia figurae dictionis* that consists of speaking of things sometimes as mere *appearances* and sometimes as *things in themselves*. For things as mere appearances are never mentioned at all in any of these antinomies, either in the propositions themselves or in their proofs. Only such propositions that assert relations between things and our sensory apparatus, for instance: Sugar is sweet, and so on, speak of things as *appearances*. Where do we find propositions of this kind in the proofs?

5. This is also why we cannot be satisfied with the *resolutions* Kant gives for this fourfold conflict.

a) As we have already said, we can only admit the proposition that the world has no beginning in time and no boundaries in space. But we have to reject the other one: that it is *also not infinite*, together with the ground given for it, that an *infinite magnitude cannot be the object of any possible perception*. For it is *false* that something cannot exist if it cannot be perceived. Furthermore, Kant runs into contradiction with himself with the regulative principle of reason on which he depends here, “that in the empirical regress *no experience of an absolute boundary* can be encountered”. (407/B545) Wherefrom will he prove this impossibility if not from *pure conceptual truths* which can be applied more generally than to experience, can be applied *unconditionally*?

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b) As regards to the second antinomy we would like to claim exactly the opposite, namely that an infinite division of every whole given in intuition does not have to be possible, at least not if we are speaking of

a possibility in all respects (namely, also with reference to our limited powers and instruments). However, we can indeed convince ourselves through *a priori* grounds, that *all bodies large enough to bring about a sensible intuition in us are composed from an infinite multitude of simple parts*. To say against this, as Kant does here, that the possibility of division goes to infinity but that nonetheless the whole does not consist of infinitely many parts, is and remains a striking contradiction. For if
 141 the multitude of parts is only finite, then the business of dividing must reach an end after a certain number of repetitions.

c) Nothing can be more unfortunate than Kant's attempted unification of *freedom* with *natural necessity* through the distinction between the human being as *an appearance* and as an *intelligible being*. That which is in itself, i.e. in fact and in truth without a cause can appear only *in a mistaken manner* as brought about through a cause.* One would believe that whoever has no preconceptions would soon sense how unsatisfactory this theory is. The fact that there were nonetheless a not insignificant number of German intellectuals who adhered to it proves only that we humans will often get used to a highly incorrect explanations and will even end up persuading ourselves that things behave this way. – Furthermore, the presupposition made here of the existence of *intelligible things* is, according to the principles of critical philosophy, an entirely unjustified presupposition. For from what else do we infer
 142 to the existence of *things in themselves* grounding certain appearances, if not from an inference from an effect to its cause? And yet it is nevertheless stated that this inference is to be admissible only within the world of appearances.

d) The same inference, unjustified according to the principles of the *Critique*, is also made in the assumption of the existence of God as an intelligible being that is the absolute condition of the existence of the entire sensible world. What's more, we are supposed to think of God not only as the cause of the sensible world but also that of intelligible beings, things in themselves.

* * *

* Indeed, Kant claims that it is *possible* "that one and the same appearance is the effect of an intelligible cause, without its connection to the natural causes being interrupted in the least". (420/B572) However, on which *intuition* does this obviously *synthetic* judgement rest? And how are we justified in applying the category of *possibility* here to something that certainly is not an object of a *possible* experience?

By an *Ideal* Kant understands (as he puts it) “an Idea not just *in concreto* but *in individuo*, i.e. an individual thing which is determinable, or even determined, through an Idea alone” (440/B596). He is probably trying to say: the representation of an individual that can be determined through *mere concepts a priori* according to a rule. But as it turns out unclarity is one of Kant’s very unfortunate flaws.

Our philosopher claims further that the *only genuine Ideal* human reason is capable of is “the concept of a thing that unites all realities, *omnitudo realitatis*, God in the transcendental sense.” (445f./B603)

We very much doubt that it is possible to define God as a being which unites *all realities*, especially if one (as Kant seems to do) understands by realities all such properties which can be represented by a purely positive concept (one that does not comprise any negation). At the very least it is certain that God has many an attribute which can be represented by a negative concept since the attribute of God on the basis of which all the others can most easily be cognized, namely the attribute that he has no ground for his existence, is obviously negative. We feel that Bolzano proceeds more correctly and more carefully in his *Science of Religion* (vol. 1, §74), by substituting the concept of the *most perfect* Being for that of the most real Being, and by conceiving of the former (as has already been noted) so that it is understood to be the attribute on account of which God possesses all those powers that can exist alongside of one another, and possesses them to the highest degree in which they can exist alongside one another.

* * *

After the concept of God, Kant discusses “*the grounds of proof of speculative reason for inferring the existence of a highest Being*” (451/B612) and submits them to his criticism. Without demonstrating it, he claims that there are *only three* proofs for God’s existence (455/B618), the *ontological* proof, which infers the existence of a highest cause from mere concepts, the *cosmological* proof, which puts only some existing thing as its basis, and the *physico-theological* proof which proceeds from the specific constitution of our sensible world. He wants to prove that *all three* of them are indefensible.

The impossibility of an ontological proof of God’s existence (456/B620) supposedly results from the following manner of inference:

Something is necessary if its existence follows from mere concepts, or if its non-existence is impossible, i.e. contradicts itself. For instance,

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145 it is necessary that a triangle has three angles. However, these and other examples are only examples of necessary *judgements*, not of *things*. The unconditioned necessity of a judgement, however, is only a conditioned necessity of the predicate. This means that, if there is a triangle, then it is also necessary that it has three angles [B622]. But if we cancel the subject, no contradiction arises. Things are similar in respect to the concept of an *absolutely necessary being*. It is surely a contradiction to say: The absolutely necessary being is not all-powerful. But it is no contradiction to say: The absolutely necessary being is not." [B625] – If someone was to say that he thinks of God as the most real being and therefore already includes actuality in the concept and so could not negate it without contradiction, then Kant would answer, "It is already a contradiction to bring the existence of a thing into its concept. Every existential proposition is synthetic. [B625] Being is not a real predicate of a thing, but it is merely its positing through which no new property is added to the thing. Through the proposition: *God* is, I add no new predicate to the concept of God. A hundred actual thalers do not contain the least bit more than a hundred possible thalers, otherwise they would not be thalers. [B627] Our concept of an object may contain whatever it might, we still have to *go beyond* it to grant it existence. Finally, from this it follows only that the realities (positings) do not contain a contradiction, not even the *possibility* of God. For there is still the question of whether the *connection* (synthesis) of these realities has possibility, and this cannot be judged in respect to an object which lies outside of all experience. (456–464/B620ff.)

146 **Remarks.** 1. When Kant defined *absolute necessity as an existence from mere concepts*, he was in fact close to elevating himself to a distinct concept of necessity. Had he only brought to mind that nothing follows from *concepts in themselves*, but only from *conceptual truths*! Likewise, when he defines that which is necessary as that whose non-existence is self-contradictory, he should have recalled that a *contradiction* can only genuinely arise between *statements*, i.e. *propositions*, and therefore that one should actually say that an object is *necessary* if the proposition: This object *is not*, conflicts with some pure conceptual truth.

2. A *judgement* or a proposition is called *necessary* only if it itself is a mere conceptual truth; like the judgement that a triangle has three angles.

3. However, it is false that the unconditioned necessity of the judgement implies only a conditioned necessity of the predicate, namely only

under the condition that the object is actually there. If the subject-representation in a proposition has no object, then the whole proposition is false. If there are no triangles, then the proposition that all triangles have three angles is not true.

4. Further, it is false that no contradiction ever arises if the subject of a judgement is cancelled, i.e. if the proposition: This subject is not present, is established. For instance, it is in fact a contradiction to claim: There are no triangles, or There is no truth, etc. Kant says: "where then is the contradiction supposed to come from? *Outside* it there is nothing that would contradict it, for the thing is not supposed to be externally necessary; and nothing *internally* either, for by cancelling the thing itself, you have thereby at the same time cancelled everything internal" (458/B623). – Here it can be seen clearly that Kant has not made for himself a correct concept of what actually is involved in a contradiction. The contradiction caused by the *cancellation* of an object A that truly exists, i.e. through the proposition: A does not exist, or The representation: A has no objectuality, does not spring from something outer or something inner. Rather the contradiction arises solely because there is a truth of the form: A exists, or *The representation: A has objectuality*.

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5. Accordingly, the proposition: There is no God, is in fact contradictory. It contradicts the truth that a God exists.

6. Whoever would draw the inference that God necessarily has actuality because he had already put actuality into the concept of God would of course be committing a fallacy. For the fact that a certain property **a** is thought in the subject-concept **M** = Something that has the properties **a**, **b**, **c**... does not immediately allow the inference that the proposition: **M** has **a**, is true. For this inference also involves the premise: *The representation of something that has the properties a, b, c... is an objectual representation*. Thus Kant was completely right when he claimed that the actuality of God cannot be derived from the fact that one already has the concept of actuality in the concept of God.

7. However, he went too far when he claimed that we already commit a contradiction whenever we even only incorporate existence in the concept of a thing. On the contrary, there are countless genuine concepts that contain the concept of actuality, even the concept of a substance, i.e. of something actual that is not a property; like the concepts: human, animal, plant, body, etc.

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8. Therefore it is also false that every existential proposition must be synthetic, if one understands by this a proposition of the form: A *has*

actuality or *A is*. But indeed, usually, when we assert the being of an object and expressly state the proposition: there is an A, we really have in mind the following proposition: *The representation A has objectuality*; and that proposition is indeed synthetic.

9. Furthermore it is false that being or actuality is not a predicate of a thing but merely its *positing*. Actuality is just as much a property of certain things, for instance of humans, animals, etc. as *non-actuality* is an property of certain other objects, e.g. of truths in themselves. We're just as correct when we say: Cajus has actuality, as when we say: A proposition in itself – has – no actuality.

10. It is true that this actuality is not always a new property that is not already thought in the subject-representation, as in the proposition: A human is something actual; because actuality already lies in the concept of human. However, it does not follow from this that such propositions
149 are always true and even less that those propositions always have to be false. Whether they are depends on whether the subject-representation is objectual or not.

11. However, in what sense is it supposed to be true that a hundred actual thalers do not contain anything more than a hundred possible thalers? A hundred actual thalers are something completely different than a hundred merely possible thalers. For the former are something existing, corporeal, etc. But the latter are certainly not! But perhaps Kant just wanted to say that the *representation* of a hundred actual thalers does not contain anything more, anything else besides the representation of a hundred possible thalers? However, this is not true either. For precisely because the objects of these two representations are so different, because a hundred actual thalers are certainly not the same as a hundred merely possible thalers, the representations have to be different as well. For the same representations also have the same objects.

12. Just as strange is the expression that we have to go *beyond* the concept of an object in order to provide it with existence, or even (what here without doubt is meant) to make sure that it has existence. This is true only if it is supposed to say as much as that one cannot gather from the mere form of a concept (Nr. 6) whether an object corresponds to it or not.

13. However, we do indeed admit that from the mere fact that every reality does not in itself contain a negation, it cannot yet be concluded
150 that the concept of a Being which unites all realities in itself does not contain a contradiction. However we do believe that we can judge,

partially from experiences, and partially from mere concepts whether certain realities like, for instance, understanding, will and power are compatible with each other or not. And if the concept of the complete perfection of God is conceived of in the way we indicated above, then it should be possible to demonstrate with sufficient rigour that there is also an object corresponding to this concept, i.e. a completely perfect Being. And yet, from the mere fact that such a Being cannot be the object of an experience, it certainly does not follow that we are not able to assure ourselves of its existence. One sees that Kant's whole line of reasoning turns again on the false proposition *that we cannot judge synthetically about supersensible things*.

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The cosmological proof of the existence of God is supposedly impossible as well and it reads: "If something exists, then something absolutely necessary must also exist. Now I myself, at least, exist. Therefore, something absolutely necessary exists. The necessary Being can be determined only in one single way, so therefore it must be thoroughly determined through its concept. Now there is only one single concept of a thing that thoroughly determines the thing *a priori*, that of the most real Being. Therefore, this is the necessary thing. Thus a most real Being exists" (465/B632f.).

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The first thing that is said of this proof is "that it is dependent on the ontological proof, because it presupposes that the absolutely necessary Being is the most real Being, which would imply conversely that the most real Being is also absolutely necessary, as is claimed in the ontological proof. Therefore the appeal to experience, with which the cosmological proof begins, is superfluous and mistaken" (466f./B634f.).

In addition the proof supposedly contains several further mistakes: (a) "applying to a supersensible object the transcendental principle of inferring from something contingent to a cause"; (b) "the inference from the impossibility of an infinite series of causes to a first cause"; (c) "one takes the fact that one cannot comprehend anything further to be the completion of the concept"; (d) "because one finds no logical contradiction in denying all realities, one infers the transcendental possibility" (468f./B637f.).

Remarks. 1. Firstly, we do not comprehend what right Kant has to claim that the appeal to experience in the cosmological proof is superfluous because it borrows something from the ontological proof in its process. When examining the ontological proof, Kant claimed that its weakness

152 consists in that it is not possible to demonstrate that an actual object corresponds to the concepts: *an absolutely necessary Being* and *a most real Being*. However, it is precisely this deficit that is remedied by the appeal to experience in the cosmological proof. – In the examination of the ontological proof, the inference from absolute necessity to complete perfection and vice versa is not reproached with even one word. Only here, when examining the cosmological proof, do we find out that this inference is false and that it constitutes the entire essence of the ontological proof.

2. We cannot admit (for reasons already known) that the inference from the contingent to a cause cannot be applied to supersensible objects. And how, if not by this inference, does Kant himself suppose that there are certain *things in themselves grounding* the sensible appearances?

3. We are so far from claiming that an infinite series of causes is impossible that we admit without hesitation the opposite, namely the existence of countless such series. For every present state of a finite thing is at least partially grounded in a preceding one, and so forth without end. However, this does not in any way contradict the assumption that all these infinite series are themselves grounded in a Being that itself has no further ground for its existence, in God.

153 4. We do not want to argue about the fallacy Kant criticizes above (point c) because we do not know definitively whether we understand it correctly. And in fact not much hangs on whether this or that previously conducted proofs for the existence of God is completely without mistakes or not, as long as it is not demonstrated that it is impossible to find a better one. But that at least Kant has not demonstrated.

5. We ourselves believe that one cannot immediately infer the possibility of a thing from the fact that no formal contradiction (**a** and **non a**) appears in the composition of its concept. That is why we consider it necessary to grasp the concept of the absolute perfection of God in a way different from the usual one.

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Under the heading: “*Discovery and Explanation of the Dialectical Illusion in all transcendental proofs of the existence of a necessary Being*” (471/B642), Kant claims that this illusion arises because we take a merely *regulative principle* of reason for a *constitutive* one because of “an unavoidable subreption” [B649].

Indeed, it is easy to understand the distinction between a regulative principle, for instance the proposition which comes into question here: Search for the cause of any change, and a constitutive principle, like the proposition: Everything contingent (changing) is grounded in a necessary (unchanging) Being. It seems to us that there is all the less danger of confusing these two propositions when we conduct the proof for the existence of God, and of using the merely regulative as the constitutive one. Indeed, even though Kant claims that this mistake is unavoidable, this will in fact happen to almost no one. Rather it is probably the case that whoever uses the proposition: Everything contingent is grounded in something necessary, in a proof for the existence of God, does this because he is convinced of its truth on the grounds of reason. 154

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We can admit for the most part what Kant remarks against the *physico-theological proof*, which infers the existence of an infinitely wise and powerful Author from the purposiveness of the arrangement of the world, namely that only the existence of a *very* wise and powerful Being can be inferred from it and that – in order to conclude to the existence of an infinite Being – certain premises borrowed from the ontological and cosmological proofs are needed. – However, it in no way follows from this that the physico-theological inferences are utterly useless for conducting a proof of the existence of God. We should definitely not expect of them what their nature prevents them from doing. Their true relation to the ontological argument consists not, as is commonly believed, in providing a preparation for the latter, but on the contrary, in *confirming* the result of the ontological proof, as it were, from experience. 155

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The *Critique of all Theology from Speculative Principles of Reason* (483/B659) does not contain anything new, but rather only the repetition of the unproven principle that we cannot judge about God either affirmatively or negatively.

In the *Appendix to the Transcendental Dialectic* (490/B670) it is explained that there are three *regulative principles* (maxims) of reason: (1) the *Principle of Sameness of Kind* (by which reason requires that we unite every given manifold under a higher concept of genus); (2) the *Principle of Variety* (by which we are always supposed to seek a further distinction between given things); (3) the *Principle of Affinity* (by which we are supposed to seek a continuous transition (an intermediate species) from

any species to any other); and that one has often taken these principles [*Principien*] falsely for *constitutive* principles [B685f.].

In this we can only agree with our philosopher. But we do not believe, as he does, that in these principles a *specific arrangement of nature* is presupposed.

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156 An excerpt from Kant's doctrine of the *Final Aim of the Natural Dialectic of Human Reason* (508/B697):

The ideas of pure reason have a very good vocation, and only through misuse do they produce deceptive illusion. [B697] These ideas are mere *schemata*, for which no object is *admitted directly, not even hypothetically*, but they serve only to represent other objects by means of the relation to them in accordance with their systematic unity, that is, *indirectly* [B698]. They have no corresponding objects themselves, but that does not preclude us from being able to use them as *regulative principles* for bringing unity to our cognition [B698] by deriving as it were the object of experience *from the imagined object of the idea as its ground* [B699]. We should consider them merely as *schemata*, as *analogues* of actual things, and we should consistently remain conscious of the fact that through them we think of *an indeterminate something* about which we have no concept at all of what it itself is in itself. [These are] *merely ideal beings, beings in Idea* [B702] which we do not assume *absolutely*, but only relative to the sensible world, which we assume only for making the greatest possible *empirical use* of our reason without nonetheless expanding it beyond the limits of experience. Therefore we assign only those *attributes* to the *indeterminate something* which we think in these ideas that are analogous to the concepts of the understanding in empirical use. [B703] Even the concepts of *reality, substance, causality, necessity*, which we ascribe to these beings lose all *signification* and are *empty titles for concepts without any content* when with them we dare venture outside the field of the sensible world [B705f.]. These beings of reason are thus mere *Ideas* and are laid down not as something *actual* but rather only as something *problematic* [B709].

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Specifically, if we assume

1. a *soul* or a *simple self-sufficient intelligence* in the first Idea, then we should do this merely for considering all representations as in one

single subject, for being able to derive all our powers from one unique fundamental power, and so on. For all this can *at best, indeed only arise* through such a schema as if it were an actual being. However, we shall not claim that such a simple substance is *actually* the ground of all our representations, and so on. Lesser still may we allow hypotheses about the arising, and so on, of our soul. [B711]

2. Similarly, we shall use the second Idea of a *world-whole* only in order to be able to approach the explanation of the given appearances as if the series was infinite in itself. But if reason itself is considered as a determining cause (in the case of freedom), i.e. in respect to practical principles, then we have to proceed as if what we had before us was not a sensible object but an object of pure understanding, in which case the series of states can be considered as having begun absolutely (through an intelligible cause). All this proves that the cosmological Ideas are merely regulative principles. [B712f.]

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3. Likewise, in respect to the third Idea (of God), we do not have the least reason to suppose such a Being absolutely, but only in order to consider all connections of the world according to principles of a systematic unity." [B713f.]

However, if we use these ideas not merely regulatively but constitutively, several errors will arise, namely:

a) "The *lazy reason*, if we regard our investigation into nature as already completed instead of seeking the cause in the laws of the mechanism of matter". [B718]

b) "The *perverted reason*, if an end of God is determined anthropomorphically and imposed on nature, instead of searching for it". [B720]

Finally, Kant asks *a number of questions*:

1. "Is there anything different from the world which contains the ground of the world order? *Answer: Without a doubt*; for there must in any case be some transcendental ground." [B724]

2. "Is this being a substance, of complete perfection, necessary, etc.? *Answer: This question has no significance at all*. For all the categories are only of empirical use." [B724]

3. "May we at least think this Being according to an *analogy* with the things of appearance? *Answer: by all means*, but only as an object in idea." [B724]

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4. "Can we then assume a wise and all-powerful Author? *Answer: Without any doubt*, we must presuppose Him." [B725]

5. "But do we not thereby extend our cognition beyond the field of experience? *Answer: By no means*, for we have only presupposed a

something, of which we have no concept at all of what it is in itself.” [B725f.]

6. “Can we still make use of this presupposition? *Answer*: Yes, that is the very reason why we laid down this Idea. [B726] You are even allowed to ascribe liking, disliking and desire to this Being.” [B726ff.]

Objections. 1. No matter how many words Kant devotes to explaining how he wants us to use the Ideas of pure reason, we still don’t know wholly definitely and distinctly what he wants. He speaks of a (regulative) use of these Ideas for the mere purpose of bringing our knowledge of nature into a systematic unity. Obviously, this is possible only through inferences. However, inferences cannot be drawn from mere concepts, but only from *propositions*. Thus Kant must actually have thought of his Ideas as certain propositions, presumably the propositions: There is a soul, a world-whole, a God. Because he also said in addition that we should not ascribe reality to these Ideas, he seems to have wanted that we view these propositions indeed not as demonstrated *truths*, but that we nonetheless use them as premises for the deduction of certain conclusions about the systematic unity. – If that was really all he meant, how much more briefly and clearly could he not have expressed himself!

2. Presupposing that this was really Kant’s opinion, we further remark that it is indeed true that it can sometimes be safe to use false or at least *unproven propositions* as premises in inferences of a certain kind. However, we would not want to believe that this is *the best, or even the only way* to arrive at these conclusions, as Kant here claims. In fact, why should we not try and be able to succeed at deriving all the powers we can find in our I from one unique fundamental power, even without presupposing the substantial simplicity of this I? Why should we not seek and find a preceding state for every state in nature as its condition, indefinitely without presupposing that the series of these conditions is in fact infinite? Etc.

3. Furthermore, what does it mean that the concepts: reality, substance, causality, necessity, etc. lose all signification and become empty titles for concepts without any content when we dare venture outside the field of the sensible world with them? – Does a concept change even in the slightest just because we apply it as predicate to some arbitrary object, even an inappropriate one? If we assert of a circle that it is square, does the concept of being square lose its signification, or doesn’t rather the falsity of the proposition rest on the very fact that it retains it?

4. We do not at all see how the Idea of God must yield the mistakes Kant is concerned with if it is used not merely regulatively but constitutively,

i.e. if we, say, consider the proposition: There is a God, as true and proven and with that assume uncritically *all the consequences* that result from it. Which minor premise together with the proposition: There is a God, yields a conclusion that demands us to regard our investigation into nature as already completed instead of searching ever further in the mechanism of matter, if we otherwise can? However, considering the search for natural purposes, it is definitely true that the presupposition of the existence of God provides us with the certainty that *every arrangement* and every event in the universe which is not absolutely necessary, i.e. a mere consequence from purely theoretical conceptual truths such as mathematical ones, etc., has as an end the greatest possible happiness of living creatures. However, when we engage in a teleological investigation, we are not content with this merely general assurance. Rather here we want to see that a *determinate, manifestly beneficent effect* it brings about *is demonstrated in experience* for any given arrangement of nature or any occurrence that does not seem absolutely necessary to us. We explain the obliquity of the ecliptic teleologically, if we demonstrate first that at least among all the pure conceptual truths (the laws of mechanics) we know so far, there is not a single one from which this obliquity follows with absolute necessity, and then go on to demonstrate how this arrangement yields the beneficent effect that a much bigger part of the surface of the earth is hospitable to humans and animals than it would have been the case if the axis of the world had been straight on the ecliptic. Everyone should see that such demonstrations always have to be made in the same way, whether we assume the existence of God as certain, problematic or not at all.

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5. We think that it is inconsistent (as we already pointed out earlier) that Kant answers the first of the questions above positively, and indeed because “there must be some transcendental ground”. [B724] Why should the concept of *ground* be the only one that can reach beyond the domain of the *world of experience* and tell us of the existence of a supersensible cause of this world. However little we determine this “*something* different from the world which contains its ground” [B724f.] we still apply some of our concepts to it, e.g. that of a *something* in general, that of a *something different* from the world, finally that of a *ground* of the world. The question becomes why are we allowed to apply these concepts transcendently but not others? And how can Kant say that we have no *concept of how this being is in itself*, given that we call it a being different from the world which grounds it? Is this not already a determination of it? – By contrast, we believe that Kant in fact allows too

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much when he wants to give us permission to ascribe *desires and dislikes* to this Being.

* * *

Kant claims that the *Transcendental Doctrine of Method* (535f./B733f.) is “the determination of the formal conditions of a complete system of pure reason” [B735f.], and he divides this second part of his *Critique* into four main sections: a *discipline*, a *canon*, an *architectonic*, and a *history* of pure reason, without stating in more detail why these subdivisions are

164 appropriate here.

The *discipline* of pure reason is according to page 538 [B739] its own negative legislation, built from the nature of reason and the objects of its pure uses in the system of caution and self-examination. However, these prescriptions are not supposed to relate to the content (about which enough has earlier been said), but rather only to the *method*. – The fortunate progress the *mathematical* sciences have made could awaken the hope that one would achieve the same apodictic certainty in philosophy if one just imitated the *mathematical method* very closely. However, Kant assures us that this hope is in vain. Philosophy is a cognition of reason from *mere concepts*. Mathematics, by contrast, is a cognition from the construction of concepts through pure intuitions. Mathematics can adhere to this method of construction because its objects (space, time, and magnitudes in general) allow it, or rather (Kant believes that) has these objects because they are the only ones to which its method of construction through pure intuitions can be applied. However, this method can in no way be applied to the objects with which philosophy has to occupy itself because there are no pure intuitions for them (539f./B743f.). – The thoroughness of mathematics rests essentially on

165 *definitions, axioms, and demonstration*; none of these three items can be applied in philosophy (548/B754f.).

1. According to Kant (549/B755), *to define* means to display the exhaustive concept of a thing within its boundaries *originally* (i.e. in such a way that this display does not itself need a proof). It is impossible to define firstly (a) an *empirical* concept, because with the latter one can never be sure whether the displayed marks are enough to differentiate it; as well as (b) an *a priori* concept, e.g. substance, cause, right, fairness, etc., because we can never be wholly sure that we have brought all the marks which we might only represent obscurely in the concept to distinct consciousness. Therefore, only *concepts thought arbitrarily* remain for which we certainly must know what we want to think by them. However, we

can still not know *a priori* whether an object corresponds to such an arbitrarily composed concept if the synthesis which is contained in the concept cannot be constructed *a priori*. This is possible only in mathematics. Therefore it is the only one with actual definitions. By contrast, in philosophy, one should never want (a) to imitate mathematics in its procedure of putting definitions first. Rather we should more appropriately put them at the end, for that we have formulated them correctly can only come out reliably at the end of our treatise. (b) We should never forget that philosophical definitions can never achieve the unmistakability of mathematical ones. 166

2. *Axioms* (552/B760) are synthetic *a priori* principles, insofar as they are immediately certain. However, no concept can be synthetically (i.e. if it is not already hidden in it as its component), yet *immediately* combined with another. Rather a third *mediating cognition* is always necessary for this. In philosophy, therefore, as a cognition of reason through mere concepts, we will not encounter a principle that deserves the name of an *axiom*. Mathematics, by contrast, is capable of delivering axioms, because by means of the construction of concepts in the intuition of the object it can connect the predicates of the latter *a priori* and immediately. Thus we should never advance axioms in philosophy, rather we have to justify every principle first through a thorough deduction.

3. *Demonstrations* (553/B762) are proofs that do not only show that something is, but also that it *cannot be* different than it is (apodictic), if in addition to that they also have intuitiveness (evidence). Proofs from mere concepts can never achieve intuitive certainty. Thus philosophy can never deliver demonstrations but has to leave them to mathematics, which constructs its concepts in intuition.

Remarks. 1. In our opinion, the damage our philosopher has wrought and still continues to wreak in the domain of philosophy through the claims of which he has made himself guilty in this section is incalculably great. For even those who did not take up anything else he said still enthusiastically adopted and maintained the claim that *in philosophical investigations, nothing can be exactly defined, nothing can be rigorously demonstrated*. And from then on it became common custom – (Kant himself certainly cannot have wanted this to happen) – in German philosophy, the one science which requires the most careful determination of its concepts and the most circumspect proofs, with a constant consideration of all the false grounds that stand against it; and from then on, it became common to mock these procedures as pedantry, to burden 167

the reader with the task of guessing from the mere context of discourse and without having been provided with any proper agreement on their meaning, which concepts one connects with one's expressions, which reasons have prompted one to make one's claims, and which reasons one could still advance for them. Indeed, things have come so far that in a certain school one claims that the concepts themselves would have to change during the philosophical exposition and that one proposition would have to sublate another because none could be completely true all by itself. This fashion or rather this deplorable style deserves the most emphatic admonitions, and we cannot find enough words for cautioning against it.

- 168 2. As regards the distinction Kant assumes between *philosophy* and *mathematics* when he declares the first to be a cognition from *pure concepts* and the latter a cognition through the *construction of concepts* in pure intuition: we must characterize it as utterly false and misleading. It is indeed true, and we have already noted this at other places, that the geometer often resorts in his proofs to the aid of what one could call mere *visual inspection*. But in our opinion it is absolutely not necessary that he does this. Rather it is possible to deduce all truths of geometry from the correct definition of space, without ever allowing even once a conclusion that has no other ground for justification in favour of it besides what visual inspection teaches. The well-known analytical geometry provides us with examples of this procedure that show at least how many geometrical truths which visual inspection teaches can also be brought out without any appeal to it, through mere inferences. In his mathematical treatises which he wrote and partly published more than 30 years ago, Bolzano's goal was to prepare the way for such a method and bring it to the attention of the learned world. But his written
- 169 *Nachlass** still contains a lot more, indeed, perhaps even the basis for the completed whole of such a new presentation of mathematics and only awaits the skilful hand that will be in a position to order and unify what it contains into a system.
3. The concepts Kant on this occasion offers of the essence of *definitions* and other logical terms are very flawed. He obviously confuses the mere *agreement on the meaning of a word*, i.e. what is involved when we

* Bolzano died on 18 December 1848, after successfully finishing one of his most astute treatises "about the paradoxes in mathematics and philosophy" shortly before he died. It will be published soon.

connect a concept with a certain word, let it arise in the reader's soul and make him realize that this is the one we want to designate by the word, and the *definition of a concept*, i.e. the specification that this concept is simple or complex, and, in the latter case, from what components and in what manner it is composed. The former is a necessary task in respect to every word that could be even slightly confusing for the reader. It is a very wrong belief that the philosopher does not need to produce such agreements on the meaning of terms; he is the one for whom this becomes necessary most often. It is also false that he does not have the means to do it and that he is permitted to burden his reader with having to guess laboriously from the context of discourse, and if luck permits it, which concept he has connected to which word. Further, it is false that there is no other means to agree on the meaning of a term than the definition of the concept; Bolzano's *Theory of Science* §668, describes a whole number of ways in which agreement on the meaning of a term can be achieved. Kant says that a definition is an *exhaustive* concept and, after that, that definitions often have the flaw of neglecting certain marks (components) which are only obscurely thought in the concept. But this is obviously true only of definitions; however, in this case he should not have demanded that a definition should not require a proof. For, in fact, the definition of a given concept, i.e. the claim that a certain concept that is already known is simple or that it arises from the connection of these and these components almost always requires a separate proof. And not only philosophy, but also mathematics possess a great amount of such given concepts about which, to this day, mathematicians do not know how to define, like sameness, similarity, opposite, magnitude, number, sum, product, difference, quotient, power, point, distance, direction, angle, extension, line, plane, body, etc. It is also false that the definition of these concepts does not belong to mathematics, or that it cannot contribute to its perfection. For instance, just to hint at one single example, the correct definition of the concept of similarity provides the easiest proof for three very important theorems which, with the concepts we have to date, could rarely be proven even under the input of several hundred inferences: that similar lines, planes and bodies behave to similar lines, planes and bodies that have been derived from them in a similar manner (for instance the surface areas of two triangles like the square of the two corresponding sides, etc.). Incidentally, Kant's claim that not one concept can reliably be defined because it is always possible that we neglect a mark obscurely thought in it is probably an exaggeration. Shouldn't the existence of such a component become apparent in the end if we inspect the concept often and

thoroughly and make use of it? For instance, can we not be sure enough that the concept: *something* is simple? – But suppose we could in fact never be sure that the concepts to which we connect with the words (which Kant offers as examples) *substance*, *cause*, etc. are really exactly those we posit in a certain definition: it is enough to convince ourselves that the combination of representations we suggested is a concept worth
 172 positing. Then we could rightfully demand that a word (for example one from common language with a similar signification) be assigned to this concept as its sign. For instance, suppose someone were to want to connect the word *substance* to the auxiliary representation of something *stable* which *carries* the properties assigned to the substance: we could rightfully demand that this auxiliary representation be removed or at least detached from the concept we teach is connected to this word, because it does not belong to the subject matter. That *empirical* concepts cannot be defined at all, and this because one thinks these, the other those marks as belonging to objects like water, gold, and because it is impossible to be sure that these marks are enough for distinguishing them: this is a claim we have to reject. For if it is true that different people think of different marks with the same words, like *water*, *gold*, etc., then they really connect different *concepts* to these words; and it is very important and not at all inappropriate to get rid of the differences between the concepts, or at least, to inform the reader about the concepts we ourselves connect to these words. However, it is a self-contradictory claim that the marks contained in a concept are not *enough* for distinguishing the object; for if we can find the marks we posit in our concept,
 173 e.g. gold, in platinum too, then platinum, and not gold, is the object of our concept. A new mistake transpires when Kant continues to say that such words are only *denominations* and not *concepts of the object*. Can we *designate* a thing without first having a concept or at least a *representation* of it? – Our readers already know that we have to object to Kant's claim that it is impossible to know that an arbitrarily constructed concept is objectual unless one has a corresponding intuition. In mathematics, too, it is not *intuitions* that assure us of the reality of the concepts: infinitely big, infinite line, chiliagon, $\sqrt{2}$, and hundreds other similar ones. As regards the *place* where definitions have to be advanced, the same rule holds for philosophy as for mathematics. In both sciences, *agreement on the meaning of the word* is necessary before the word is used; in both sciences, *definitions* of a concept can be sometimes neglected, sometimes added later; however, neglecting them is obviously worse in philosophy than in mathematics. Experience also refutes the claim that the mathematician never errs in his definitions, even in those that are

174 arbitrary. For how often have mathematicians found that the concepts they advanced turned out to be useless, or even contradictory and objectless? An example is the concept of the angle of contact.

4. If it were true that no concept can be connected immediately with another (that is not contained in it as a component), i.e. that no synthetic judgement can be immediately appreciated: then the cognition of synthetic judgements would be impossible altogether, namely because every one of them would require another one for its generation and so forth to infinity. Bolzano teaches much more appropriately (*Theory of Science* §300), that there are and have to be many immediate judgements, empirical as well as pure conceptual judgements. Those judgements, by contrast, which Kant thinks are not mediated through any other judgement but only through a mere *intuition* are certainly all *mediated*, even though it might be true that we are seldom or never fully aware of the premises we derive them from. For we would like to know how a judgement of the form: **A is B**, if **A** and **B** are pure concepts, should be mediated by a certain intuition, i.e. by a pure simple representation that has only one object, and how we are to understand the nature of the relationship between the representation **x** and the representations **A** and **B**. We are told that the representation **x** can mediate the judgement: **A is B**, if we are aware that the representation **x** is subsumed under **A**, on the one hand, and if we find that **B** is one of its marks, on the other hand. However, granted that this is sufficient, aren't the perceptions: **x is A**, and: **x is B** actual judgements? Can one thus say that the judgement: **A is B** is mediated by other judgements, given that its mediation proceeds in this way? Furthermore, we have already said that we cannot accept that these two judgements should be sufficient for the derivation of the universal: All **A** are **B**.

5. We also concede that philosophical proofs seldom or never have as a high degree of reliability and vividness as many mathematical proofs; however we believe that this is merely a quantitative difference, and that there are certain philosophical theorems which rather than being inferior to mathematical theorems in this respect, even exceed them. For has not many a *logical* theorem, for instance, in the doctrine of inference (though it cannot be demonstrated from intuitions), been proven more perspicuously than many mathematical truths, for instance that of the irrationality of the relation between the diameter and the circumference?

Since the *Critique of Pure Reason* teaches us that we do not know anything in its pure and speculative use, the question becomes whether we are at least permitted to form *hypotheses*? (577/B797) And the *Critique* answers: “By no means, if they are meant to be propositions which we are permitted *to assume with a certain degree of probability*; for such a hypothesis requires at least that its *possibility* has been proven and that it is sufficient for explaining the given.” [B797ff.] However:

1. first, Kant continues, we cannot claim the possibility of any object which is supposed to have a new property that is not given empirically, e.g. an understanding which intuitively without sensibility, or a duration which is not in time, or our soul as a simple substance, etc. One who uses a mere Idea of reason to explain a thing of nature tries to explain that which he does not sufficiently understand through something he does not understand at all. The wildest hypothesis, as long as it is physical, is more tolerable than the hyperphysical one, e.g. the appeal to a divine Author. (578f./B780f.)

177 2. A hypothesis that is inadequate and in need of new auxiliary hypotheses arouses the suspicion of being mere invention. This is the case in respect to the hypothesis of the limitlessly perfect Cause, which requires still further hypotheses for the explanation of the many evils and imperfections of the world. This is the case for the hypothesis of the simple self-sufficiency of our soul, if we have to explain the waxing and waning of our powers etc. (580/B802). Furthermore, attempting to make the actuality of the transcendental Ideas merely *probable* is an absurd resolution; it would be as though one thought to prove a proposition of geometry as merely probable. Pure reason can cognize everything only *a priori* and necessarily, or not at all (581/B803). Although in merely speculative questions of pure reason no hypotheses are allowed to ground propositions, they are nevertheless admissible to defend them, i.e. in the *polemical use*, when the *Ideas that have been assumed in some respect other than the speculative one* are being attacked (ibid./B804). If, for instance, the objection is made that the waxing and waning of the mental powers seems to refute the immaterial nature of the soul: then you can advance the hypothesis that the body is nothing but the fundamental appearance to which the entire faculty of sensibility and therewith all thinking are related as its condition (583/B806). Against the objection taken from the contingency of conception you can advance the hypothesis that all life is really only intelligible, not subject to temporal changes at all; that our
178 earthly life is nothing but a mere appearance (sensible representation) of the purely spiritual life (584/B808). However, we shall never adopt

these hypotheses as our genuine opinions, rather we have to abandon them as soon as we are finished with the dogmatic self-conceit of our opponent (585/B809).

Remarks. 1. It is true that in order for a hypothesis to be advanced it is always required that the object assumed in it has at least *problematic possibility*, i.e. that its assumption does not conflict with any of the truths *known* to us. However, it is a very incorrect claim that this assumption has to be recognized as *possible in all regards* (*absolutely possible*), i.e. that we first have to assure ourselves that there exists no truth at all, not even only one *pure conceptual truth* that conflicts with our assumption before we may dare advance it with a degree of *probability* which is sometimes higher, sometimes lower depending on the nature of the circumstances. For instance, I can assume that the hundredth decimal place in the number π is 9 with a high degree of probability because I find it stated in the works of the mathematicians that deal with the topic, though I do not in fact know if I hereby assume something absolutely impossible. Given that Kant himself admits, indeed explicitly and repeatedly claims at this very place, that the assumption of the transcendental Idea does not stand in contradiction with even one single truth known to us (or that the impossibility of God, the soul, etc. cannot be demonstrated), the first of the reasons he advances for why the existence of God or of a simple substantial soul cannot even be assumed as a hypothesis ceases to apply entirely.

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2. The concept of a duration which is not in time is self-contradictory because we understand a duration as nothing else than an existence that holds for a certain stretch of time. Such a duration would indeed be a reprehensible hypothesis.

3. That everytime we use a transcendental Idea (e.g. God or the soul) to explain an appearance in nature we explain something that we do not understand sufficiently through something else that we don't understand *at all*, that is an accusation which we cannot concede to our philosopher (and our reader already knows why).

4. However, there is something true in the claim that the wildest hypothesis for the explanation of an appearance in nature, as long as it is physical, is more tolerable than the appeal to God. An appeal to God, which consists in trying to establish the proximate cause of a certain appearance in the immediate influence of God: that is always definitely objectionable. And any hypothesis, however improbable, that places this proximate cause in certain natural forces is assuredly more admissible. For

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since we do not know *all* natural forces, we are never permitted to claim of an appearance, even if it cannot be explained by any forces known to us, that it is produced by an immediate effect of God. The definitions of revelation and wonder as *immediate* lessons and effects of God, such as those theologians usually advance, must be rejected. However, it must be noted that we can assume without deciding anything in the slightest about the physical cause that brought about an event, that it has an *effect* which is *intended* by God, e.g. that there is a *lesson* to be imparted to us through it. For more on this, see the *Theory of Science* (§379 note) and *Science of Religion* (vol. 1, §174 and elsewhere).

5. That every hypothesis which is not sufficient to fully explain an appearance and which rather requires certain auxiliary hypotheses (further assumptions) becomes so suspicious that we have in turn to abandon it, i.e. that every admissible explanation has to consist of only one single simple assumption: that is a demand whose exaggeration Kant probably felt himself, as the expressions he uses here betray. A light suddenly fills my chamber and vanishes just as fast. Immediately after I perceive a loud bang that dissipates slowly. Is it not a very probable hypothesis that it was thunder? Then, only moments later, I see a neighbouring tower in bright flames. Shouldn't I be allowed to add the new auxiliary hypothesis that lightning struck the tower to my former hypothesis? Another second and the whole tower explodes with a horrible bang right before my eyes. Can I not now dare add the third hypothesis that this tower which is now in the grips of the fire was used to store powder? –

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6. Furthermore, it is not at all true that the assumption of an absolutely perfect Being and of a simple substantiality of the soul ensnares us in difficulties which can only be removed by seeking aid in new hypotheses. We can sufficiently explain the many evils we encounter in the world and the phenomenon of the waxing and waning of the powers of our soul without any hypotheses we would not be justified, indeed compelled, to make by other circumstances, for instance, that every created being has and can only have a limited amount of powers, that they gradually become more perfect, that an interaction obtains between them, etc. No one will demand that we here discuss this until it is distinct, but they will be content if we instead refer to Bolzano's *Science of Religion* (vol. 1, 286) and *Athanasia* (Second edition, 84ff.).

182 For now, we will assess whether the hypotheses Kant finally allows for merely *polemical* (apologetic) ends grant us more satisfaction than what is said in the books indicated above.

7. That it is an absurd enterprise to give a transcendental Idea or a pure conceptual proposition in general *mere probability* because reason must be in a position to decide about propositions of this kind either with necessity or not at all, that is not at all convincing. Doesn't Kant himself after all claim that we can neither affirm nor deny the existence of God, etc. for *a priori* reasons. Then this is precisely a case where inferences about probabilities are at their right place. And doesn't he emphatically say (529/B727): *you have as much confirmation* for the rectitude of your Idea (of a highest Intelligence) as you encounter purposiveness in the world. And (477/B651f.) that the proof from the purposiveness of the world always deserves to be named with respect and elevates the belief in a highest Author to *irresistible conviction*. And (618/B857) that the question (of the existence of God) is merely a *task of speculation* for someone who recognizes no moral law, and that it may not be supported by grounds such that the most *obstinate scepticism* would have to surrender, but that it is still supported by *strong grounds from analogy*. A confirmation admitting of degrees is obviously nothing other than an *inference of probability*. Thus the Sage of Königsberg explicitly accepts here what he calls absurd elsewhere (581/B803). It is not even absurd to sometimes also allow proofs of mere probability in geometry and the other mathematical sciences. Who will not immediately think of the beautiful theorems of the doctrine of prime numbers that *Fermat* discovered by mere trial and error and made probable through mere examples? The truth is rather that in this science one should not be content with mere proofs of probability. Rather one must search for other kinds of proofs because we can assume in advance that a diligent search will ultimately lead us to a decisive proof for or against the truth of the proposition. However, as has already been said, in respect to the doctrine of God, the complete opposite is supposed to be already certain. Why then, we ask again, should we spurn grounds of mere probability if they are available?

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Kant teaches the following about the nature of the proofs of pure reason (586ff./B810ff.):

1. The first rule is to attempt no transcendental proof without first having considered whence one can derive the principles (premises) for these proofs, and with what right one can expect to succeed with these inferences (588/B814). But if such putative proofs are already present, then oppose the *non liquet* of your mature power of judgement against

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their deceptive conviction, and even if you cannot yet penetrate their deception (589/B815), you still do not need to concern yourself with the development and refutation of every groundless illusion. But you can reject all at once the entire heap of these tricks of the inexhaustible dialectic in the court of a critical reason that demands laws (ibid./B815).

2. Only a single proof can be found for each transcendental proposition. For every such proposition proceeds solely from one concept, and posits the synthetic condition of the possibility of the object according to this concept. The ground of the proof can therefore only be unique, *since* outside this concept there is nothing further by means of which the object could be determined. If I am to prove, not from concepts but rather from the intuition which corresponds to a concept, then this intuition offers me a manifold of material for synthetic propositions that I can connect in more than one way, thus allowing me to reach the same proposition by different paths (589/B815f.).

185 3. The transcendental proofs of pure reason must never be *apagogic* but always *ostensive* because only these combine the conviction of the truth simultaneously with the insight into its sources. Proofs of the apagogic kind can be allowed only in those sciences where it is impossible to substitute that which is *subjective* in our representations for that which is *objective*, namely the cognition of what is in the object. Where the latter dominates, it must often transpire that the opposite of a certain proposition either simply contradicts the subjective conditions of thought, but not the object, or else that both propositions contradict each other only under a subjective condition (that is falsely taken to be objective) and that in fact both of them can be false.

Remarks. 1. It is very good advice and every scholar should take to heart that, before we engage in the proof or even more generally the examination of a proposition, we should first investigate whether it not possible to see in advance that the knowledge necessary for this examination is at our disposal. However, it is not always possible to see this in advance. And therefore it is certainly asking too much, to ask that we not begin the examination of the proposition until we can expect that we will finish it successfully. However, one must completely reject the advice to oppose every proof of a transcendental proposition with an educated: *non liquet* before it has even been heard, and to reject these inferences
186 as a heap without even considering them. Kant himself (566/B781) says: “When I hear that an uncommon mind has demonstrated away the

freedom of the human will, the hope of a future life, and the existence of God, I am eager to read the book. For I expect that his talent will advance my insights." – Should this not hold in the opposite case?

2. We do not want to claim that transcendental propositions can always only be proven in a single way if what is supposed to be understood by proof is a mere certification, not a grounding [*Begründung*]. The reason from which Kant infers this does not, in our view, prove anything. It is our conviction that all proofs, even mathematical proofs, can be conducted from pure concepts. And even those proofs that relate to *visual inspection* arise in a completely different way than Kant presents it. Further, if every proposition allows for only one single proof from pure concepts, why then does Kant himself distinguish two proofs, namely *apagogic* and *ostensive* ones, and criticizes the first only for failing to provide insight into the *sources* of truth, but not for failing to give certainty?

3. If by *apagogic* proofs one is supposed to understand those that derive the truth of a proposition from the demonstration of the absurdity to which its contrary opposite leads (and what Kant says on 591 [B818f.] forces us to conclude that Kant does not connect any other concept with this denomination), then we claim (supported by what Bolzano has demonstrated in the *Theory of Science* §530) that a very small change in all apagogic proofs can easily avoid them and turn them into direct ones. However, we very much doubt that it is possible to boast of every *direct*, i.e. non-apagogic proof that it provides insight into the sources of truth (this must mean, insight into their Why?). The proof of the first proposition of Euclid's *Elements* (the possibility of an equilateral triangle) is a direct proof. But who would claim that this proof gives us insight as to *why* an equilateral triangle has possibility, i.e. why for every given two points **a**, **b** there is one, indeed, infinitely many, third point **c** of such a constitution that the distances **ac**, **bc** are the same as the distance **ab**? Who would fancy that the ground for the presence of such a third point lies in the circumstance that two circles around **a** and **b** lying on the same plane and with the radius **ab** intersect? Who cannot see that, quite to the contrary, the latter is the consequence of the former? –

* * *

"I understand by a Canon of Pure Reason", Kant explains (595/B824), "the collection of the *a priori* principles of the correct use of certain cognitive faculties in general. Now according to the proofs that have previously been given, all synthetic cognition of pure reason in its speculative

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use is entirely impossible. There is thus no canon for its speculative use [B824]. Consequently, if there is to be any correct use of pure reason and any canon of it, this will concern only the practical use of reason, which we will now investigate" [B825].

We **remark**: Everyone should have noticed that, after having *unconditionally* denied our reason the ability to have synthetic cognition of supersensible objects almost everywhere, and even after speaking of it in the most laborious manner, and after having given reasons which, if they proved anything at all, would demonstrate the impossibility of such a cognition absolutely, Kant nonetheless adds the qualifying phrase: "in its *speculative use*" [B824], here and also earlier on in the section highlighted. The fact that this expression, itself obscure, has never been defined in detail makes this even stranger. Properly speaking, we should understand the *speculative use* of pure reason as its application for the discovery of *speculative* truths and the *practical use* as its application for the discovery of *practical* truths. We shall now see whether the following confirms or refutes this conjecture.

* * *

"In the end", Kant says, (in the first section of the *Ultimate End of the Pure Use of Reason*) "the final aim of the speculation of reason in its transcendental use amounts to a concern with three objects: the freedom of the will, the immortality of the soul, and the existence of God" (596/B826). Thereafter he attempts to show that the merely *speculative* interest of reason in regard to all three of these questions is very small, since the discoveries that might be made about this could not at all be used in the investigation of nature. Therefore, their importance can really only pertain to the *practical*. However, the practical is everything that is possible through *freedom*. And *freedom* in the *practical sense* is the faculty for being determined through motives of reason and independently of sensible impulses. This practical freedom is proven through *experience*, because we very often overcome sensual desires and, for instance, decide to do something which only promises to be useful as a distant prospect. However, the question regarding *transcendental* freedom (i.e. the question whether reason can begin a series of appearances without any determining causes in the sensible world) is not of any practical interest. Therefore, only the two questions remain with regard to which a canon must be possible: Is there a God? and is there a future life?

Remark. What Kant understands by the *practical use* of pure reason is here still left wholly undetermined. By contrast, we do want to concede

to him that the decision of the three questions about freedom, immortality, and God are not of speculative interest, at least insofar as they do not have any influence on our *investigation into nature*. Furthermore, we also admit all of the following: that the question regarding the presence of a freedom in the sense in which Kant defines his *transcendental* freedom does not have any practical importance; that everything depends on the presence of what he calls *practical* freedom and that this is sufficiently demonstrated through experience itself; that therefore only two objects actually remain about which one can dispute: God and immortality, and which are at once of the greatest practical importance.

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The second section: *On the Ideal of the Highest Good as a Ground for the Determination of the Ultimate End of Pure Reason*, says: All speculative and practical interest of reason is united in the three questions: (1) What can I know? (2) What should I do? (3) What may I hope? The first is merely speculative, the second merely practical, and the third is at once practical and theoretical (601f./B832f.). – There is (as everyone’s consciousness testifies) a moral law which commands not merely hypothetically but unconditionally that something ought to happen. This moral law demands that we act in such a way as to be worthy of happiness (603/B836f.). – Now the question arises: If I act so as to be worthy of happiness, may I hope thereby to partake in it? I (Kant) say that just as the moral principles are necessary in accordance with reason in its practical use, it is equally necessary to assume in accordance with reason in its theoretical use that everyone has to hope for happiness to the degree he has made himself worthy of it [B836f.]. However, this can only be hoped for if a highest reason which commands according to moral laws, i.e. an Ideal of the highest good, is at once laid down as the cause of nature and if a future world is presupposed for us. For we do not find such a connection between virtue and happiness in our present life. Thus God and a future life are two presuppositions that are not to be separated from the obligation that pure reason imposes on us (604f./B839). From this (Kant holds) result the attributes of God, e.g. omnipotence, so that all of nature and its relation to morality in the world are subordinated to Him (608/B842), and so on. Thus pure reason has, though only in its practical use, the merit of connecting with our highest interests a cognition that mere speculation can only think up but never make obtain, and through this make it indeed not into a demonstrated dogma, but

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still an absolutely necessary presupposition for its most essential ends (610/B846).

Remarks. 1. We certainly now see that by the cognitions which pure reason gains through its practical use, Kant does not at all understand certain practical propositions, as we suspected. Rather he understands by them the same *theoretical* propositions about God and the soul about which he had claimed that *they* themselves are inaccessible to it in its *speculative use*. A cognition, he says here, is gained through the *practical use of reason* if it is in itself a theoretical truth, yet has been derived from a practical premise. And he calls it inaccessible through *purely speculative use* if there is no way of deriving it from purely theoretical premises. The question here is first and foremost: How can such a claim be compatible with the ground for the impossibility of synthetic cognition about supersensible objects that was introduced earlier? This impossibility
193 rests merely on the circumstance that we do not have intuitions of such objects. Does the addition of a practical premise eliminate this ground? Certainly not. But how could it be then that through this aid a cognition would nevertheless be possible?

2. It is true that the moral commands are not hypothetical if that means that they are not bound to the condition "*insofar as we please*". Nonetheless, there is another condition under which they stand, namely: we are only required *to want* and *to try what we hold to be possible and beneficial for the common good*. Thus the command depends on the presupposition of the problematic *possibility* of an action and on our discernment of the expected *success*. This is a claim (especially the last part) which Kant would definitely never concede. However, we believe that mere common sense (to which, after all, Kant himself has appealed on account of the existence of a moral law) already fully decides in our favour. For when it determines the good nature or wickedness of an action to be performed, it usually weighs the advantages and disadvantages that can reasonably be expected from it, i.e. its influence on the *welfare* of the living. One sees here at once that common sense
194 acknowledges the *command* to promote the *common good* or *common happiness* as the foundation and condition of all moral conduct or as the *highest moral command*. Now, Kant declares himself against all *principles of happiness* in the *Critique of Practical Reason*. However, his grounds actually only concern the *principle of personal happiness*, and this is indeed a principle as false as it is objectionable. But Kant has nothing valid to bring against the principle of general happiness. For he claims that it is not appropriate as a law of practical volition because it rests on prescriptions derived

from mere *data of experience* and is therefore *unstable* and dependent on the diverse concepts of happiness which everyone forms for oneself. Therefore, these prescriptions cannot hold for *all times*, for *everyone*, nor *universally*, as is demanded of the moral laws (*Critique of practical Reason*, [Hartenstein] vol. 4, 138 [part 1, book 1, chapter 1, §8]). – But we reply: Indeed, it is true and not to be denied that in order to distinguish whether some conduct is in fact compatible with general happiness or not, several insights are presupposed which rest for the most part on experience, i.e. that one or more premises of experience are therefore always needed in order to derive a moral prescription from the law of general happiness or common good. However, this circumstance does not make the principle incorrect or useless, insofar as we do not find experience in general and its judgements to be objectionable. But when it comes to what our duty is in each single case, hardly anything can be determined through Kant's own principle otherwise than by drawing from equally many experiences! For in order to satisfy its demand, i.e. in order to always choose the action which *can be established as a universal law*, is it not necessary to know to which action pertains such a property? This cannot be determined by the mere consideration of a certain action, at least not always. There are few actions that obligate everyone at once and under all circumstances (it is, then, that they are commanded by the highest moral law). Rather an action is always only a duty for all those who find themselves in a similar situation. Therefore we have to know the situation, the particular circumstances in which it turns out that everyone who finds himself in this situation has to follow the same law. But how else than through experience could we ascertain this? We will soon see that Kant's attempt to decide this matter *a priori*, from a contradiction into which we fall whenever we proceed against a command of duty is highly unfortunate. That the moral commands become unstable if we assume the principle of the common good, because different people have different representations of happiness and its objects, is an over-hasty conclusion. The deficient concepts of what makes us truly happy have to be rectified, and people generally ever only err in this respect if they are blinded by passion. – The claim that the commands of duty are such that each necessarily holds at all times and for everyone, is also false. On the contrary, there are only *a few* moral laws which are binding universally and without exception.

Our readers might be interested in examining whether and how the principle of the *common good*, which common sense acknowledges with so much certainty through an intimate feeling of truth, can be derived from distinct premises? – Bolzano attempted such a derivation in his

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Science of Religion (1st part, §87) and we believe it was not unhappy. He starts from two indisputably correct concepts: that the ground of all moral obligation has to be a conduct that falls to us *unconditionally*, without exceptions, and that it must also be *possible*. Thus if we examine in turn all the different manners of acting (of finite beings), then among them we will surely find the one that is generally, i.e. for us humans, possible and has unconditional, exceptionless validity. The latter attribute will serve us as the mark by which we can easily

197 recognize the highest moral law. All conceivable manners of acting possible to us range over mere changes we bring about sometimes in *inanimate*, sometimes in *animate* beings, and in the case of the latter, they can only consist in effects on their *faculty of cognition*, or *desire*, or *sensation*. There are of course multiple commands as regards our effects on the inanimate world. But everyone sees that these are only conditional. For whether we ought to cause or to refrain from causing certain changes in the inanimate world, this must only ever happen on account of the influence these changes have on the animate beings. Therefore which conduct is the ultimate ground of all moral duty and which is commanded by the highest moral law is to be found only amongst the effects on the animate beings. But there is not even one that is unconditionally commanded to us, not even as regards effects on the faculty of cognition. For that we strive to promote the recognition of truth and steer clear of error (which seems to be closest to an unconditional rule) has a ground, and this comes to light when we consider that we are instructed to avoid spreading the less useful truths, but not the more useful ones. Spreading the truth is thus required on account of the benefits it can cause. There is certainly only one wholly exceptionless duty as regards the effects on a being's faculty of desire:

198 act in such a way that everyone would want the morally good and not the morally bad. But this rule cannot be viewed as the highest moral law because it is merely an identical proposition. For what's morally good and bad is just that which we ought to do or ought not to do. Furthermore, we would have to presuppose a further proposition in order to be able to derive any duty from the latter, namely, the determination that this or that is morally good or bad. Therefore, only one single kind of effect remains, the one that relates to a being's faculty of sensation. Here and nowhere else, therefore, is there an unconditional ought. Now we can act on a being's faculty of sensation only in two ways: we can excite in them pleasant sensations or unpleasant ones. No one will say that to excite unpleasant sensations is an unconditional command. Thus it must be *to excite pleasant sensations*. – Admittedly, we

will here immediately encounter the objection that even this rule has its exceptions, because there are certain pleasant sensations that we deny ourselves and others, and certain unpleasant ones that we must induce. However, a little reflection will at once illuminate the fact that this exception – as sure as there is an ought – can only be apparent. For duty forbids a pleasant sensation only when it cannot be imparted without causing even greater unpleasant sensations to oneself or others. And it commands to cause an unpleasant one only if it is the means for prohibiting an even greater displeasure. Thus even in these cases of apparent exceptions we still ought to act only in such a way as to bring about the *greatest possible sum of pleasant sensations (of pleasure) or of happiness*. Now we must still ask whether the highest moral principle demands that we all merely engage in promoting the increase of our own happiness or of the happiness of *all* (as far as this is possible). If the former were the case, there would be no difference between duty and preference, and the difference between virtue and vice could at most consist in the more or less prudent choice of a means to one and the same end. Thus the highest moral law certainly demands that we promote not merely our own, but rather the happiness of all, as far as we can. We could therefore call it the principle of *the promotion of the common good*. Confusing the latter with the principle of *eudemonism* or *of personal happiness*, by which one understands the error we just refuted that one has a duty to care exclusively for one's own happiness, is most unfair. – But it is wholly correct that one acts completely in accordance with the principle of the common good that, whenever several courses of action are possible, one always chooses to act in such a way that the sum of pleasure to be brought about, no matter which being experiences it, is the *greatest*. Understandably, what matters here is not the kind, but rather the degree of pleasure or happiness that is supposed to be produced, and this is the reason why this law can also rightfully be called the law of the *greatest pleasure*, the *principle of the maximum*.* – Other formulas of the highest moral law are sometimes merely *identical* propositions, like: Do what is good; Follow reason; Live according to nature (the Stoics); Strive for perfection (Plato, Leibniz); Act in such a way that you can reasonably want the maxim of your will to become a universal law (Kant), etc. – Sometimes they do not specify the proper (objective) *ground* of our obligations, like: Follow God's will; Emulate God, etc.

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* See G. Th. Fechner's: *Ueber das höchste Gut*, Leipzig 1846.

We now want to devote closer attention to the Kantian principle. It reads: Act only according to that maxim whereby you can will that it should become a universal law (*Grundlegung zur Metaphysik der Sitten*, [Hartenstein] vol. 4, 43, und *Kritik der praktischen Vernunft*, vol. 4, 130 [part 1, vol. 1, §7]). We claimed that it is identical. For the words: universal law mean the same as: That which *everyone ought* to do. Therefore this principle must be graspable through the following expression: That which you ought to do should be of such a nature that everyone ought to do it. However, from such a tautological proposition not one single fruitful truth, let alone all practical truths or duties, can be derived. It is therefore not possible that it be the highest moral principle. Furthermore, it is true only if it is stated with a certain qualification and indeed in the following manner: That, which you ought to do in a certain situation and under certain circumstances ought to be of such a nature that everyone who finds himself in the same situation and under the same circumstances ought to do it. But expressed in this way, it presupposes the practical truth: In this or that situation, you ought to do this and that, and therefore even when modified this way it cannot be recognized as the highest law. – Kant nonetheless believed that all duties result from this identical principle, and he tried to show this with a number of examples (*Grundlegung zur Metaphysik der Sitten*, vol. 4, 44 f., und *Kritik der praktischen Vernunft*, vol. 4, 126). It should suffice that we examine the derivation that we find in the *Critique of Practical Reason*, where he attempts to demonstrate the duty not to keep borrowed goods, from the ground that the opposed maxim would have to contradict itself if it were advanced as a universal law because no one would then ever lend anything to anyone. If this were correct, then the proposition that would state the permission to keep borrowed goods would certainly be false and its direct opposite would be true. However, there exists not one single proposition of duty that is of such a nature that its contradictory opposite would have to be called an actually self-contradictory proposition. The contradiction Kant has in mind is merely apparent. His claim that there would be no borrowed goods if it wasn't a duty to give them back is not even true. For as long as it is not a duty to keep them, many a person could hope to retrieve what they had lent to others. Further, assume there were in fact no more borrowed goods. The rule: you shall not keep borrowed goods, would still contain no inner contradiction. For it merely asserts: We shall not keep a good that has been lent to us. However, it does not command not to keep a good that has been lent to us if it has not been lent to us. It would only come into contradiction with itself in

the latter case. In addition, this rule would make it possible to demonstrate the falsest propositions as duties. For instance, the proposition: You shall, as far as you can, destroy everything around you. For no matter how much effort people would devote to proceed according to this rule, the world would nonetheless persist for a long time. Suppose, however, that there would finally be nothing left to destroy. Then this proposition would still not contain an inner contradiction because it merely demands that we destroy for as long as there is something to destroy. But enough of this.

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3. In the same section (*On the Ideal of the Highest Good*), Kant declares as necessary, *that it may be hoped that everyone receive happiness to a degree commensurate to his virtue*, without further explaining *why* this is necessary. All he says is (606/B840): “It is *necessary* that the entire course of our life be subordinated to moral maxims. But it is at the same time *impossible* for this to happen if reason does not connect with the moral law an *efficient cause* which determines for conduct an outcome that precisely corresponds to it” [B840f.]. Otherwise, he continues (607/B841) “the magnificent ideas of morality” would be “object of approbation and admiration but not *incentives for resolve and realization.*” – From this we should conclude that Kant held it to be actually *impossible* to satisfy the demands of the moral law if one cannot hope to be rewarded for it; something he denies in several other writings, and something that is also in itself incorrect. – Elsewhere, he therefore derives an essentially different conclusion (e.g. in the *Critique of Practical Reason*, vol. 4, 257) which reads as follows: “It is a fact that cannot be denied, because we are conscious of them in our innermost, that there are certain *duties*, or that something *ought* to happen through us. But what we *ought* to do must be possible; or, we also have to believe with all our conviction that it is possible that what practical reason demands should happen. But now practical reason demands the *realization of the highest Good*. Thus the realization of the highest Good has to be possible for us. But it would not be possible if our *will* was not *free*, our *soul* not *immortal*, and if there was no most *powerful, wise, and holy Being* as the ultimate ground of the world and its arrangement. Thus it is certain that our will must be free, that the soul must be immortal and that there must be a most powerful, wise and holy God.” Kant designates this kind of derivation with the name of a *demand* or a *postulate of practical reason*.

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Even if we abstract from the fact that Kant here commits a contradiction against his own theory – because he suddenly concedes to reason

the right to make correct synthetic judgements – even if we abstract from that: is this *kind of inference* otherwise faultless? We believe that it is not and that it deserves the reproach that it presupposes what is to be proven and moves in a circle. For if, with Kant, we understand the highest good to be virtue and happiness commensurate to the degree of virtue achieved (*Critique of Practical Reason*, 245), then it is obvious that we can only be permitted to say that reason demands the highest good, that it should be brought about, *if we have demonstrated that this highest good is something possible*. For we have no obligation to what is impossible. However, if one tries to demonstrate the possibility of the highest good, i.e. to demonstrate that there is a happiness corresponding to every degree of virtue in the world, then it will soon become obvious that this proof can be conducted in no other way than if we presuppose the existence of God, immortality and freedom (see Bolzano's *Science of Religion*, vol. 1, §167).

4. Kant boasts that the *moral theology* he has outlined here (608/B842) has the *peculiar advantage* over the speculative one that it inexorably leads to the concept of a *unique, most perfect, and primordial rational Being* and, for instance, shows that the uniqueness, omnipotence, omniscience, omnipresence, and eternity of God result from his theory. And he finishes (611/B847) with the remark, “that certain actions are not considered obligatory because they are God's commands, but rather conversely we can hold them to be divine commands because we are obligated by them.” – While we agree wholeheartedly with the last claim, we cannot but very much doubt that a distinct concept and a convincing proof of God's perfections can be deduced from those practical postulates without seeking the aid of several synthetic propositions that concern a supersensible object. For instance, what is the essence of omnipotence? Certainly that God can do everything that is not impossible in itself? But how are we to judge that something is impossible in itself without knowing certain synthetic truths that do not merely concern objects of experience?

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Excerpt from the 3rd Section. On Having an Opinion, Knowing, and Believing (611/B848).

Having an opinion is (according to Kant) taking something to be true, whose grounds we ourselves recognize as insufficient not only *objectively* (i.e. in relation to everyone) but also *subjectively* (i.e. for ourselves). *Believing* is taking something to be true, whose grounds

we recognize as sufficient subjectively but not objectively. *Knowing*, finally, is taking something to be true, whose grounds are sufficient both subjectively and objectively. – In respect to the transcendental use of reason, having an opinion is not enough and knowing is too much. Therefore we cannot judge at all in the merely speculative regard, but we can very well *believe* in respect to the practical. For if, namely, some end that we have prescribed to ourselves is supposed to be fulfilled, if therefore we are supposed to *act*, then it is necessary to consider certain means as appropriate for attaining it. This holding to be true is only subjectively but not also objectively necessary, if we do not ourselves know of any other means. It will therefore be a belief that is merely contingent, even in our own view. Kant calls such a contingent belief, one that lies at the basis of the use of the means to certain actions, a *pragmatic* belief. – However, even if we are unable to undertake anything at all in relation to an object, and therefore if the holding to be true is merely *theoretical*, if there are such means, we can still grasp in thought what would constitute an attempt to conclude the matter. Thus there is in mere theoretical judgements an *analogue* to practical judgements, and we can call this holding to be true a *doctrinal* belief. “And the doctrine of God and immortality belong to such a doctrinal belief. [B854f.] For presupposing a wise Author of the world is the condition for an aim which is, to be sure, contingent but yet not insignificant, namely that of having a guide for the investigation of nature. The outcome of my experiments also often confirms the usefulness of this presupposition, namely that I would say too little if I called my holding to be true merely *having an opinion*. Rather even in this theoretical relations it can be said that *I firmly believe in a God*. As regards God’s wisdom, the magnificent equipment of human nature and the shortness of life which is so ill suited to it, one finds likewise just as sufficient a ground for the *doctrinal belief in a future life*” (615f./B854f.) – However, there is something wavering in itself about this doctrinal belief. And the case of the *moral belief* is entirely different. For there it is absolutely *necessary* that something *must* happen, namely that I follow the moral law in all points. And according to all my insights only one single condition is possible under which this end can be achieved, namely, that there be a God and a future world. I also know for sure that no one else knows of any other condition. Thus I will inexorably believe in the existence of God and an eternal life. Therefore no one can say *he knows* that there is a God, not even that it is *morally certain* that there is a God, but surely *that he is morally certain that there is a God*. (617/B856f.)

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Remarks. 1. The definitions of the terms: *having an opinion*, *believing*, and *knowing* that Kant assumes here correspond neither to the common use of language nor to the ends of science. Something someone gets from immediate perception, e.g. a pain he just now feels, he cognizes from a ground that is completely sufficient for him, but that he cannot communicate to others and which is therefore not objective. Thus we would have to say, wholly against the common use of language, that he merely *believes* to feel pain. How much more correct are the definitions given in the *Theory of Science* (§321)! –

209 2. It is a new error that belief should generally only take place in regard to *practical* matters (i.e. where we are supposed to act). And because Kant could not deny that a belief can take hold also of doctrines that do not have any influence on actions, e.g. regarding the question whether other planets are inhabited, he tried to explain this with the assumption that, in these cases, one at least *imagines* that one grounds actions in this holding to be true, for instance, that we could accept a bet based on the latter. Who does not see how forced this explanation is! –

3. But the worst is that what our philosopher is so determined to presuppose with his so-called *pragmatic belief* that he invents at least an analogue for it in the *doctrinal* belief: that is something that in all clarity never occurs. Our intention to attain a certain end nowhere provides, nor can it provide, a *ground* for the belief that this or that means is the most suitable or the only one suitable for this end. Like every other holding to be true that is not an immediate judgement, belief only originates from grounds that consist in other judgements, and the mere *will* has only a mediate influence upon it, insofar as it determines us to direct our attention to these grounds. But not only can our wish to attain a certain end never immediately produce the belief that a certain means to that end is the most suitable one, this wish cannot, at least if we are rational, corrupt with our judgement about the suitability of this means. Rather, the more fervently someone wishes to attain a certain end, the more fervently will he also wish to become aware of the means that are the correct ones and guard himself against obstinately favouring the one over the other. The example which Kant himself introduces refutes him. “The doctor”, it is said on page 614 [B852], “must do something for a patient who is in danger, but he does not recognize the illness. He observes the appearances and judges, because he does not know any better, that it is consumption. His belief is, even in *his own judgement*, merely contingent, and another could perhaps do better.” – This is correct, the doctor should not persuade himself that he has a case of consumption in front of him,

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if he is still uncertain that it is such a case. Rather, he must remain aware of the fact that another, perhaps even more probable, evil is to be found. We see from this example that although we sometimes express ourselves in this manner, it is unlikely “that we have to make a decision in order to be able to act”. Rather, in our behaviour we not seldomly encounter dispositions for cases which, even if we don’t consider them to be probable, we do not consider to be completely impossible either.

4. If things were, in fact, as Kant claims they are, such that the belief in God and immortality is, for us, necessary for virtue (but we hold that this twofold belief in virtue is merely *beneficial*), then from this nothing else would follow but that it is our duty to direct all of our mind’s attention to the grounds that speak in favour of God and immortality in order to strengthen our belief as much as possible. But we have to doubt very much that through this effort a belief as strong and steadfast as Kant describes the *moral* one would come about if it is to be otherwise true that theoretical and speculative reason does not offer us decisive grounds for these two truths. For indeed, mere will does not increase the weight of these grounds: or then it would have to happen through a kind of self-deception. 211

5. It would definitely be otherwise if “the *necessity*” to obey the moral law that Kant mentions here were to occur in the strictest sense. For what is necessary in the strictest sense of the word also has actuality. We would thus all in fact obey the moral law, and because this (as Kant says) is only possible if we believe in God and immortality, this belief too would indeed have to be (without us being able to explain the way in which it came about) found in fact in us all.

6. Finally, the distinction between the two propositions: *It is morally certain that there is a God*, and: *I am morally certain that there is a God*, seems to us to be an empty subtlety. That of which I am completely certain, I consider for this very reason to be something which is also certain in itself. But if the latter expression is merely supposed to mean something whose certainty can be universally *communicated* (namely: certain for all humans), then we admit indeed that there are certainties that cannot be communicated. But the doctrines that are discussed here do not appear to us to belong to these kinds of truths. For even if we suppose that the certainty of God’s existence and of our immortality arises only in the way that Kant teaches here, namely from our *conscience*; doesn’t Kant also say with absolute right that this conscience is something *all* humans share? Why then should we not be able to expect everyone to 212

have the same certainty we have of those two truths, in the very same way in which we expect from everyone that they recognize as true the same geometrical theorems we do, because we can presuppose the same representation of space in them too?

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- 213 The *Architectonic* of pure reason is (according to 619/B860) the art of systems or of the sciences. A system, however, is the unity of the manifold cognitions under an Idea of the form of a whole, a concept of reason, insofar as the extension of the manifold as well as the place of the parts among themselves is determined *a priori* through it. – Here, Kant merely wanted to draft the architectonic of *pure reason*, i.e. of the *a priori* cognitions. All cognition of reason is either cognition *from concepts* or from *the construction* of concepts. The first is called *philosophical*, the second *mathematical*. The system of all philosophical cognition is *philosophy* according to the *scholastic concept*. According to the *cosmopolitan concept* (i.e. the one that concerns that which is of interest to everyone) one can define philosophy as the science of the relation of all cognition to the *essential* ends of human reason. Now the highest, essential end is none other than the whole vocation of the human being and the philosophy that concerns the latter is called *morals*. Philosophy (Kant says further [B868]) has two objects: Nature and freedom. The philosophy of nature concerns everything that *is*, that of freedom or morals concerns that which *ought to be*. All philosophy is either cognition from pure concepts, *pure philosophy*, or cognition of reason from empirical principles, *empirical philosophy*. The philosophy of pure reason is either *propedeutic* (preliminary exercise), which investigates the capacity of reason in regard to all pure cognitions *a priori*, and is called *critique*; or the system of pure reason, the whole philosophical cognition from pure reason and systematic connection, and is called *metaphysics* [B868f.].
- 214 It is divided into that of the *speculative* and the *practical* use of pure reason, and therefore is either *metaphysics of nature*, i.e. metaphysics in the *narrower sense*; or *metaphysics of morals* [B869], i.e. *pure morality*. *Metaphysics in the narrower sense* consists of *transcendental philosophy*, which considers only understanding and reason itself in a system of all concepts and principles that relates to *objects in general* without assuming any given objects (*ontology*); and of the *physiology* of pure reason which considers the collection of the given objects, although only rationally. The use of reason in this rational consideration of nature however is either *immanent*, i.e. it pertains to nature as far as its cognition can be

applied in experience; or *transcendent*, i.e. it pertains to the connection of the objects of experience which supersedes all experience. This *transcendental physiology* has either an *internal* or an *external* connection to its object; the former is *transcendental cognition of the world*, the latter *transcendental cognition of God*. The *immanent* physiology considers the concept of all sensible objects, but only according to the conditions *a priori* under which they can be given to us at all. It has two objects: (1) those of the *external* senses, the *corporal* nature, (2) the object of the *inner* sense, the *soul*. Thence the two sciences: *rational physics* and *psychology*. Accordingly, the whole system of metaphysics consists of four main parts: (1) *ontology*, (2) *rational physiology*, (3) *rational cosmology*, (4) *rational theology*, while the rational physiology still divides into the rational physics and rational psychology [B874ff.]. 215

Remarks. 1. That a system or a science is a collection of cognitions, which is determined by an Idea in respect to its content as well as its form, is indeed true. But here the nature of this Idea should also be depicted in more detail. For not every collection of cognitions ordered according to any arbitrary Idea constitutes a science. Doesn't Kant himself close the Architectonic with the following very beautiful and memorable words: "That it (metaphysics) as mere speculation better serves to prevent errors than to broaden cognitions does not diminish its value but rather gives it a dignity and eminence, through the office of a censor, which secures the general order and harmony, indeed the well-being of the *overall scientific enterprise* and prevents its courageous and fruitful efforts from *straying from the main end, general happiness*". [B879] Here it is thus clearly admitted that the *main end* of all sciences has to be general happiness. Should not a closer determination of the Idea according to which a whole of cognitions has to be formed in order to have a rightful claim to the name of a science result from this? Compare what Bolzano argued in the *Theory of Science* (§395). 216

2. As our readers already know, we do not concede that *mathematics* and *philosophy* are divided in the way Kant indicates. Nonetheless, we too separate the two sciences in a way which, it appears to us, is completely in accordance with the use hitherto, by defining mathematics as a *doctrine of magnitude*. And we count not only the *doctrine of time and space*, but also *statics, mechanics, hydraulics, and optics* and other similar sciences among the *mathematical* ones only because in all these sciences there is more or less to calculate (i.e. determinations of magnitudes).

3. Kant's claims that only mathematics can be taught, never philosophy and that as regards the latter one has to be content with teaching

217 “*philosophizing*” (623/B866) is probably somewhat exaggerated, as is likewise the claim that the use of reason in mathematics, because it happens in pure intuition, excludes all *illusion and error* (ibid./[B865]). Mathematicians have never, thank God, committed the vanity of declaring themselves infallible. And if they had, one could soon refute them through the history of their science. However, given the more than babylonian confusion that reigns in the domain of philosophy to this day, that no one can praise the system that appears to be the right one to him with confidence, that therefore everyone has to be content with saying he teaches, if not philosophy, at least philosophizing; that would be spoken from our soul. Nonetheless, we would not dare claim that the situation is also bound to stay this way for all future times. For even if it is hard, we still do not consider it impossible for people to eventually reach consensus on the doctrines of philosophy (in particular, the practical ones) as they reach consensus now on the doctrines of mathematics. But one will not at all comprehend how Kant could have written the claim that one cannot achieve in the *mathematical* doctrines, as is the case in the *philosophical* ones, merely *historical* knowledge (with no insight into the grounds). For he himself certainly possessed merely historical knowledge of a great part of the mathematical doctrines, e.g. of astronomy.

4. It is also very strange, that no explicit mention of *logic* is made in this enumeration of the philosophical sciences, notwithstanding the fact that we see from page 624 [B867] that Kant ranked this science among the philosophical ones.

218 5. It seems a bit forced that *rational theology* should be presented here as a branch of the rational *examination of nature* (as an examination of given objects), namely as that branch of the examination of nature that examines the connection of the whole of nature with a Being *above* nature. How can one declare the examination of a Being that stands above nature to be a branch of the examination of nature?

6. The objection Kant made to himself: “How can I expect an *a priori* cognition, and therefore metaphysics, of objects insofar as they are given to our senses, thus given *a posteriori*?”, he answered in the following manner: “We take from experience nothing more than what is necessary to give ourselves an *object*, partly of the outer, and partly of the inner sense (matter and thinking beings). But we abstain entirely from all empirical principles that would add any sort of experience beyond these concepts in order to judge, on that basis, something about these objects” [B875]. We would rather say that the concepts: *matter* and *mind*

even though they are like all our representations prompted in us by experience, are still in no way empirical but *pure*, i.e. concepts such that they do not contain one single intuition as a component. This will be self-evident to anyone who knows our definition of the concept of an *intuition*, namely that it is this kind of simple representation that has only one single object. For which simple representation that has only one single object should possibly lie in the representations *matter* and *mind*? That Kant overlooked this and, in addition to these two, took quite a number of other pure concepts for empirical ones, e.g. the concept of *change*, similarly the concepts of *pleasure* and *pain*, etc. comes from the fact that the concept he connected with the word *intuition* failed to have the mark of *simplicity*.

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Kant closes his *Critique* with a short "*History of Pure Reason*". (639ff./B880ff.) Since he himself says that this title is only in his book in order to mark a place that needs to be filled in the future, we too certainly have no cause to linger on the little he presents here in the course of these four pages. He is satisfied with bringing the most notable changes that metaphysics has experienced under three viewpoints.

1. *As regards the object* of all our cognitions of reason, it says, some were merely *sensual* others merely *intellectual philosophers*. The former claimed that actuality and truth are only in the sensible *objects*, the latter only in the *intelligible* objects which we cognize through a peculiar kind of intuition, not accompanied by any sense (intellectual?).
2. *As regards the Origin of the Pure Cognitions of Reason*, the *empiricists* claimed that they are derived merely from experience, the *noologists* that they have their source in reason.
3. *As regards the Method*, the *naturalists* advanced the principle that more can be done in regard to the most sublime questions of metaphysics without science through common reason, than through speculation. Those observant of the scientific method however proceeded in part *dogmatically*, in part *sceptically*. "*The critical path*" (Kant closes his *Critique* with these words) "*alone is still open*. If the reader had the kindness and patience of travelling in my company, he can now judge whether or not he pleases to contribute his own in order to turn this narrow path into a highroad, in order to attain that which many centuries could not accomplish before the end of the present one, namely to bring human reason to full satisfaction

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in that which has always, though until now in vain, engaged its curiosity." [B883]

Remarks. 1. If *intellectual intuition* is supposed to be understood as nothing else but that we generally have the capacity to form a concept even of objects that we do not perceive through any sense, and to make many a correct synthetic judgement about them, then we ourselves would not
221 want to deny that there is such intellectual intuition. No one, however, should allow themselves to present such judgements as *immediate* cognitions, if they have not first demonstrated that they are judgements that have not been deduced in any way.

2. If the empiricists had examined the nature of our experiential cognitions more thoroughly, then it would have soon become clear to them that most judgements which they consider to be immediate experiences or perceptions are deduced judgements. And if they had given closer consideration to the way in which this deducing happens, then they would have found that to the latter belong premises which are themselves not perceptual judgements, but rather pure conceptual propositions.

3. We agree with the opinion that mere common sense does not give incorrect answers to those questions of theoretical and practical philosophy which are by far the most important. And we have the conviction that at least statements about which all humans on earth think the same way, even though they are of such a content that they do not flatter the sensibility of humans, but rather disrupt it, have the highest degree of reliability. But it follows in no way that one does not have to engage in the investigation of the premises on which this kind of judgements are based and how they are to be demonstrated in systematic connection.
222 Such a scientific effort will always be of great use.

4. However, regarding the *critical* method: it has to this day not generated the success which its author did not consider impossible, namely "to bring human reason to full satisfaction in that which has always, though until now in vain, engaged its curiosity". [B883] As regards the prohibition which Kant issues to reason: that it not venture into the domain of the supersensible, it must we believe be considered as a completely failed attempt toward the end he indicates. Nonetheless, we are not loath to still repeat even at the end of our investigations what we have asserted right at the beginning in the most determinate manner. Far from denying the merits Kant has gained for himself in philosophy, we rather consider that there is not one single philosophical discipline

which he has not enriched with new and important doctrines through his insightful remarks. His critical works, especially the *Critique of Pure Reason*, which rightfully became famous, remain, in spite of the manifold confusions to which its writer fell prey, forever a rich repository for anyone interested in philosophical knowledge.

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Appendix

We believe that we do not do a disagreeable favour to our readers by summarizing the most exceptional doctrines of the *Critique of Pure Reason*, which are scattered over these pages following the order of Kant's extensive work, in a simple overview and accompany this with a conclusion.

The main tendency of the *Critique* undisputedly goes towards pointing out determinate *boundaries* for our *cognizing* and limiting it to mere objects of experience, which it believes to be able to accomplish in the following ways. First it examines the question: *under which conditions our cognitions, i.e. our judgements have validity*, or, as Kant puts it, the question *regarding the arising of our judgements* in general. On this occasion it makes the certainly noteworthy discovery that our cognition does not consist only of *analytic* judgements but also of *synthetic* ones, and that the latter are of just as great a significance for science since they alone serve its true enrichment and expansion. Accordingly, one understands why the *Critique* is almost solely concerned with explaining the origin of synthetic cognition. At the same time, the important distinction that obtains among the synthetic judgements and which has to come into consideration here does not escape Kant's astuteness: the distinction that grounds the division of judgements into *a priori* and *a posteriori*, that is, into *conceptual* and *experiential* judgements. If we favour the latter denomination, we do so because it not only marks the nature of these two kinds of judgements more precisely, but also because it additionally serves to eliminate certain misleading auxiliary representations which are excited by the expressions *a priori* and *a posteriori* and through which the opinion will easily arise that there are judgements which are present in the soul before any and every experience and indeed even perception.

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As regards *synthetic experiential judgements**, their validity, our philosopher believes, *obviously* depends on an *intuition*. Here a predicate is

* In the strict sense all experiential judgements are synthetic.

ascribed to a subject because he who judges encounters it in an intuition which is related to the subject. In contrast, we believe to have demonstrated that things are somewhat different from what Kant pretends even as regards the arising of those judgements which seem to be the ones most likely to allow for such a definition. However Kant was so convinced of the correctness of his definition that he strove to extend it to all *synthetic* judgements, even to the *a priori* ones and advanced the following as their highest principle: Every object of our cognition is conditioned by an intuition which we connect to it. He tried in every possible way to eliminate the difficulties he encountered in the application of this principle. The judgements of pure mathematics already caused him no slight trouble. Their validity, this he had to admit to himself, is clear to everyone. But still no one, when he makes experiential judgements, can appeal to intuitions of the sort to which Kant refers. Accordingly, had he not found an escape route by assuming certain *pure intuitions*, Kant would have had to give up on his theory of intuition as soon as he considered mathematical judgements. However, this is a most indefensible route. For even though the usual manner of exposition in geometry seemed to justify his opinion to a certain extent, it still would not fit arithmetical propositions in any way. However, Kant had grown completely accustomed to the thought that all synthetic judgements rest upon an intuition. And therefore he pursued it with all perseverance. And even though his indeed very flawed proofs in fact related only to experiential judgements and among the *a priori* ones, merely brushed against mathematical ones, he still could not be deterred by this in the least, and he concluded, just as if he had demonstrated the matter universally: "*All synthetic judgements, including all a priori ones, must if they are to be correct be grounded upon intuitions, and when this is not the case, if intuitions are not present, as in judgements whose subject represents a supersensible object, there, our judgements are absolutely unwarranted and no one is justified in making them. Thus, the objects most important to the human being: God, freedom and immortality, lie completely outside the circle of his cognition, and if he dares to transgress this circle and to pronounce himself synthetically about them, then he will soon entrap himself in paralogisms and gets into antinomies or propositions that can be proven as readily as their direct opposite. Human beings necessarily fall prey to such a state of confusion of the mind in the constitutive use of their theoretical reason, and only through practical reason can he still be rescued, which secures the belief in God, freedom, and immortality through postulates!*"

These claims have – who would have believed it? – been repeated by so many scholars, and Kant has been praised as the philosopher who first succeeded in *presenting the boundaries of our cognition with determination*. He himself often times (for instance, 518/B708, 572/B760, among others) speaks of this as the goal of his efforts. – However, who would ever be able to measure the limits of the mind? Who could determine: to here and no further must humans advance? That is why Hegel and Schelling opposed this, but nowhere distinctly developed the reasons for their opposition. Others, among them even Herbart and Beneke, stuck to this Kantian doctrine precisely because of this, though with certain modifications. One very praiseworthy thinker of most recent times, Dr Theodor Waitz, presents the subject in a slightly different manner and wants to see our philosopher credited with the immortal merit “of having, for the first time, captured the thought, that the insight into the *process of the formation* of our concepts is the only means and at the same time the only possible *guarantee* for the validity and objectivity of cognition itself”.* Now, it would still be possible to argue as to whether this thought, as Waitz supposes, is indeed peculiar to Kant and “is at the basis of his enterprise to criticise reason”. But in any case it contains a mistake. For even though cognition depends on concepts which we have of the objects, what Waitz concludes from it in no way follows, namely that the insight into the process of the formation of our concepts is the only possible *guarantee for the validity of our cognition*. For the insight gained here into the process of formation of our concepts is certainly no guarantee for the validity of our cognition, i.e. the truth of our judgements, since we can *never* reach certainty in this way. For every *process of formation* can only be presented, and recognized as presented correctly, through a long chain of judgements which need a *new guarantee*. However, even the mere cognition of a truth, i.e. any judgement that we make about any object of our consideration, is often already guarantee enough for its validity. I hold this or that to be true, e.g. that I just now feel a pain or hear a sound, without demanding a proof, a guarantee that I am not erring. This is the case with all immediate judgements. Of course, things are definitely different in respect to the mediated (experiential as well as conceptual) judgements which are based on shorter or longer chains of inferences. In order to be convinced of the correctness of these inferences, I do indeed have several means at my disposal, the most important of which Bolzano cites in his *Theory of Science* §300ff. The Kantian

* See his *Grundlegung der Psychologie*, p. 114f.

229 “analysis of the faculty of cognition”, however it be conducted, cannot effect this in the least. Sensibility, understanding, reason, imagination and all the many powers of which the *Critique* speaks, however they may be called, can in our opinion conveniently be left aside. To speak of powers and faculties where there should only be talk of *representations*, *propositions*, or *judgements* themselves causes only misunderstandings. The question which is being investigated by Kant: Are our judgements in accordance to truth? All or some? Are those whose object is a supersensible one, true and demonstrably true as well? etc. – all these questions need not consider these powers for their answer. Rather, the examination of these powers is only an *allotriion*, which is very tempting insofar as one knows nothing else of these powers but that they are the *causes* which bring about those *effects*. Nonetheless by hypostatizing them, by arbitrarily separating them, contrasting them, etc. one concludes something more from these names than one could have concluded without them. The path therefore which the *Critique* pursues for deciding the question about the validity of our cognition is not at all the right one. And, in order to gain clarity about this, a completely different one has to be pursued. The faculty of cognition of human beings or of thinking beings in general must not be investigated and analysed first. Rather it is necessary to look into the nature of the truths in themselves or, to speak more generally, of the propositions in themselves, their components and kinds, their connection amongst themselves and similar relations of them. Only after this has happened can one usefully deal with cognizing and the conditions of cognizing. It must not be forgotten that the object of cognition is at first only truths and not actually existing objects. The latter are cognizable to us only mediated by means of the truths we cognize. For whether an actual object belongs to one of our representations or not, and how this object is otherwise constituted, that we can only learn from certain truths which assert something about it, e.g. when inferring the existence of sensible objects from perceptual judgements, etc. But the truths which are about sensible things are not the only ones that we are able to cognize. By means of the concepts which we attain little by little, an unsurveyable realm of truth opens up for our cognition, into which we constantly penetrate further, but which we will never be able to exhaust. – The cognition of objects (more correctly: the cognizability of truths) always caused philosophers great difficulty. The more recent ones often declare this business “to be a darkness that lies over the deepest secret of philosophy, the riddle of the connection between the ideal and the real world or the world as it is in itself independent of all cognition and how it mirrors itself in the brain

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as the organ of cognition". Now, in order to somewhat disperse this darkness and, as someone once said, go beyond every dualism (even the Kantian one between the thing in itself and the appearance) one resorted to the idea of the *identity* of the subject–object. But neither the *Doctrine of Identity*, nor the soon following *Dialectical Method*, nor certain other attempts brought the desired light. – Was it indeed true that darkness is, as it were, essential here and that one is bound to fail to escape it? In no way, we reply. Rather you overcome it without difficulty, insofar as you only embrace the assumption that has repeatedly been recommended in the course of these pages: "There are infinitely many *truths in themselves*, we are in a position to *cognize* a significant multitude of them and to divide and order the cognized ones into their own sciences." A further discussion of this subject matter would lead us too far from our goal. Accordingly, we refer to that which we have already said about it in the Introduction, and to Bolzano's *Theory of Science* §19ff., and close our treatise with some words regarding Kant's distinction between the *things in themselves* and their *appearances*. 231

A distinct definition of what the *thing in itself* is cannot be found in the *Critique*. Rather it assumes such things in contrast to the appearances in order to support certain doctrines, in just the way in which it invented the pure intuitions of time and space in order to justify the doctrine of the necessity of intuition for the grounding of synthetic conceptual truths. It seems not to have sensed the contradiction in which it gets with itself through the assumption of an object that is supersensible and without further ado it advances the claim *that we do not cognize the things in themselves but only their appearances*. In what sense is this supposed to be true? We can concede this proposition only if we are permitted to interpret it to mean that we do not cognize the actual things *immediately* but *infer* their actuality and other attributes only from certain representations (appearances in the mind). But Kant hardly connected this meaning to his statement. For indeed he straightforwardly denies us every cognition, and thus also every mediated cognition of the things in themselves. Indeed, in the first edition of the *Critique* we read that "the appearances are nothing but *representations*" (see vol. 2 of the *Gesamtausgabe*, 246/A250). However, if these words are not supposed to have a sceptical but a truly critical meaning, then Kant, when he uttered them, had to suppose that certain actual objects outside of us correspond to these appearances or representations in us (at least to many of them) and that these actual objects, because after all they must not be things in themselves, are *properties* or attributes of these things. For outside of this there is nothing truly actual. Accordingly, Kant would have conceded 232 233

to us the cognition of the properties or attributes of things by means of representations. However, how he then can deny us the cognition of the things in themselves is not comprehensible. Things in general cannot at all be cognized other than through the cognition of attributes (thus also things in themselves, if indeed, they are things), neither by us humans nor by any other cognizing being. In the subsequent editions, Kant uses the word “appearance” in a completely different sense and declares it to be equivalent to the words “objects of possible experience” (see vol. 2 of the *Gesamtausgabe*, 234/B298), “empirical objects” (239/B299), “beings of the senses” (24[5]/B307). According to this, the proposition: We do not cognize things in themselves, but only their appearances, would only be a different expression for the generally known, unsavoury result of the *Critique*, “that we are in a position to judge only about sensible objects correctly”, a result which, if it were true, would at once put an end to all metaphysics. But we believe that we have sufficiently demonstrated the falsity of the Kantian principles, from which he concluded this claim.

Glossary

German-English

- Ableitbarkeit*: deducibility
Ableitung: deduction
ableiten: deduce
Anschauung: intuition
Anschauungssatz: intuitional proposition
Art: species
Augenschein: visual inspection
aussagen: assert
Bedeutung: signification
Begriff: concept
Begriffsfassung: conception
Begriffssatz: conceptual proposition
Begriffswahrheit: conceptual truth
Begründung: grounding
Beschaffenheit: property
Bestandteil: component
bestimmen: determine
Bestimmung: determination
Beweis: proof

beweisen: prove
bezeichnen: designate
Bezeichnung: designation
bilden: form
Dasein: existence
Deduction: deduction
definieren: define
Deutlichkeit: distinctness
Eigenschaft: attribute
Einfluß: influence
einschliessen: comprise
Einteilung: division, classification
einwirken: affect, act upon
Einzelvorstellung: singular representation
enthalten: contain
enthalten unter: contained under
entstehen: arise
entspringen: originate
Erfahrungsseelenlehre: empirical psychology
Erfahrungssatz: proposition of experience
Erfahrungsurteil: judgement of experience, experiential judgement
Erfahrungserkenntnisse: cognitions of experience
erkennen: cognize
Erkenntnis: cognition
erklären: define
Erklärung: definition
Etwas: something
Existenz: existence
folgern: to conclude
Fürwahrhalten: holding to be true
Gedanke: thought
gedachter Satz: thought proposition
gegenständlich: objectual
gegenstandlos: objectless
Gemüthe: mind
Gewissmachung: certification
Glaube: belief
gleichbedeutend: synonymous
Grundsatz: principle
Inhalt: content
kennen: know

Kenntnis: knowledge
Kraft: force
Mangel: lack
Menge: multitude
Merkmal: mark
mittelbar: mediate
nachweisen: demonstrate
Nebenbegriff: associated concept
Nichts: nothing
die Sache selbst: the thing itself
Satz: proposition
Sein: existence, being
Selbstbeglückung: personal happiness
Sinn: sense
Teilvorstellung: part-representation
Übereinstimmung: agreement
Umfang: extension
umfassen: comprise
unmittelbar: immediate
Veränderung: change
verbinden: combine, connect
Verbindung: combination, connection
verknüpfen: connect
Verknüpfung: connection
vermitteln: mediate
Verständigung: agreement on the meaning of a term
Vorstellung: representation
Wahrheitsgefühl: feeling for truth
Wahrnehmungsurteil: perceptual judgement
wechseln: exchange
Wechselwirkung: interaction
wirken: act, have effects
wirklich: actual
Wirklichkeit: actuality
wissen: know
Wissen: knowledge
Wunsch: preference
Zeitbestimmung: temporal determination
zergliedern: decompose

English-German

act: *wirken* (see also: have effects)
 act upon: *einwirken* (see also: affect)
 actual: *wirklich*
 actuality: *Wirklichkeit*
 affect: *einwirken* (see also: act upon)
 agreement: *Übereinstimmung*
 agreement on the meaning of a term: *Verständigung*
 arise: *entstehen*
 arising: *Entstehen*
 assert: *aussagen*
 associated concept: *Nebenbegriff*
 attribute: *Eigenschaft*
 being: *Sein* (see also: existence)
 belief: *Glaube*
 certification: *Gewissmachung*
 change: *Veränderung*
 classification: *Einteilung* (see also: division)
 cognition: *Erkenntnis*
 cognitions of experience: *Erfahrungserkenntnisse*
 cognize: *erkennen*
 combination: *Verbindung* (see also: connection)
 combine: *verbinden* (see also: connect)
 component: *Bestandteil*
 comprise: *einschliessen, umfassen*
 concept: *Begriff*
 conception: *Begriffsfassung*
 conceptual proposition: *Begriffssatz*
 conceptual truth: *Begriffswahrheit*
 conclude: *folgern*
 connect: *verknüpfen, verbinden* (see also: combine)
 connection: *Verknüpfung, Verbindung* (see also: combination)
 contain: *enthalten*
 contained under: *enthalten unter*
 content: *Inhalt*
 decompose: *zergliedern*
 deducibility: *Ableitbarkeit*
 deduction: *Ableitung, Deduction*
 deduce: *ableiten*
 define: *definieren, erklären*

definition: *Definition, Erklärung*
demonstrate: *nachweisen*
designate: *bezeichnen*
designation: *Bezeichnung*
determination: *Bestimmung*
determine: *bestimmen*
distinct: *deutlich*
distinctness: *Deutlichkeit*
division: *Einteilung* (see also: classification)
have effects: *wirken* (see also: act)
empirical psychology: *Erfahrungseelenlehre*
exchange: *wechseln*
existence: *Sein, Dasein, Existenz*
experiential judgement: *Erfahrungsurteil* (see also: judgement of experience)
extension: *Umfang*
feeling for truth: *Wahrheitsgefühl*
force: *Kraft*
form: *bilden*
grounding: *Begründung*
holding to be true: *Fürwahrhalten*
immediate: *unmittelbar*
influence: *Einfluß*
interaction: *Wechselwirkung*
intuition: *Anschauung*
intuitional proposition: *Anschauungssatz*
judgement of experience: *Erfahrungsurteil* (see also: experiential judgement)
know: *wissen, kennen*
knowledge: *Wissen, Kenntnis*
lack: *Mangel*
multitude: *Menge*
mark: *Merkmal*
mediate: *mittelbar*
mind: *Gemüthe*
nothing: *Nichts*
objectless: *gegenstandlos*
objectual: *gegenständlich*
origin: *Ursprung*
originate: *entspringen*
part-representation: *Teilvorstellung*

perceptual judgement: *Wahrnehmungsurteil*
personal happiness: *Selbstbeglückung*
preference: *Wunsch*
principle: *Grundsatz*
proof: *Beweis*
prove: *beweisen*
property: *Beschaffenheit*
proposition: *Satz*
proposition in itself: *Satz an sich*
proposition of experience: *Erfahrungssatz*
representation: *Vorstellung*
representation in itself: *Vorstellungen an sich*
sense: *Sinn*
signification: *Bedeutung*
singular representation: *Einzelvorstellung*
something: *Etwas*
species: *Art*
synonymous: *gleichbedeutend*
temporal determination: *Zeitbestimmung*
the thing itself: *die Sache selbst*
thought: *Gedanke*
thought proposition: *gedachter Satz*
unitary: *einig*
visual inspection: *Augenschein*

Part II

4

Bolzano and Kant on Space and Outer Intuition*

Clinton Tolley

§1 Introduction: rethinking the relation between space and outer intuitions

In his *Critique of Pure Reason*, Kant famously argues for what he calls the ‘transcendental ideality’ of space. A key step in Kant’s argument is his attempted proof in the Transcendental Aesthetic that our most ‘original’ representation of space must be an *intuition* rather than a concept, and moreover, must be one that is *pure*, insofar as it must be in the mind *a priori*, prior to all actual ‘empirical’ (sensation-involving) intuitions of external objects, what Kant calls ‘outer intuitions’. Kant thinks this intuition of space must be present (or ‘occur’) in the mind *a priori* since spatial representation is universally and necessarily involved in all of our outer intuitions. Kant then goes on to argue (briefly in the first *Critique* but then at length in the *Prolegomena*) that accepting his account of the pure intuition of space is also necessary in order to make sense of how it is possible that we could come to have the *a priori* cognition of space in pure geometry that Kant, along with most of his contemporaries, assumes that we possess.

Though a handful of Kant’s most influential successors in the philosophy of mathematics have accepted the broad outlines of these claims about the role of pure intuition in geometry,¹ many of Kant’s readers – even many of his most sympathetic ones – have been sharply critical of

* I would like to thank Samantha Matherne and Sandra Lapointe for their very helpful comments on an earlier draft of this essay.

¹ Perhaps two of the most well-known are Frege and the early Carnap; for Frege, see his 1924/1925 ‘Erkenntnisquellen der Mathematik und der mathematischen Naturwissenschaften’; for Carnap, see his 1922 *Der Raum*.

this component of Kant's doctrine of space and spatial representation. Especially after Dedekind and Hilbert, it became common, even among self-styled neo-Kantians, to reject the idea that any appeal to intuition is necessary in order to account for the knowledge of space provided in pure geometry.²

As has now been increasingly appreciated, one of Kant's earliest critics on this point was Bernard Bolzano.³ Challenges to Kant's account of geometry appear already in some of Bolzano's earliest publications (cf. Bolzano 1810), and are developed more sustainedly in his later discussions of Kant in the 1837 *Wissenschaftslehre* ('WL') and those recorded by Přihonský in the 1850 *New Anti-Kant* ('NAK'). Bolzano argues, against Kant, that it is possible to define the representation of space through mere concepts alone, without this definition including any representations whatsoever drawn from intuition (cf. WL §79.6, I.366; §79 Anm, I.369–370; NAK 74). In this respect, Bolzano thereby puts forward a form of geometrical 'logicism' *avant la lettre*.⁴ In fact, Bolzano's criticisms go considerably further, insofar as he argues that the very idea of a pure intuition is essentially incoherent (as we will see below, cf. §§4–5).

Yet while existing treatments of Bolzano's criticism of Kant on space have focused primarily on Bolzano's contrasting account of knowledge in geometry and mathematics more broadly, much less attention has been paid to the consequences that Bolzano's rejection of pure intuition has for Bolzano's own account of our intuitions of external objects – representations that Bolzano himself also calls 'outer intuitions'.⁵ This will be my focus in what follows.

What will emerge is that the position Bolzano is led to on the nature and structure of outer intuitions is considerably different from Kant's, from the ground up, as it were. Bolzano's rejection of a pure intuition of space turns out to be intimately connected with his denial that outer intuitions contain *any spatial representation whatsoever*. This is because Bolzano rejects the idea that the content of our outer intuitions has *any*

² For the rejection of pure intuition in geometry by the neo-Kantians, cf. Friedman 2000, 28, and Coffa 1991, 57f.

³ See Coffa 1991; Laz 1993; Rusnock 2000, 45–50 and 131–140; and Sebestik 2003.

⁴ Cf. Coffa 1991, 27f.; Sebestik 2003, 54f.; cf. Palagyi 1902, iii.

⁵ An early start on this topic can be found in Palagyi 1902, chapter VI (esp. §18). Some more recent helpful treatments of related topics can be found in George 2003 and Rosenkoetter 2012. For a discussion of Bolzano's rejection of Kant's doctrine of the pure intuition of time that is in key ways complementary to what follows, see George 1987.

universal or necessary 'form' whatsoever. A fortiori, Bolzano also rejects the idea that such a form is provided by a representation of space. Rather, on Bolzano's account, the content of each outer intuition is *essentially simple*, and so does not contain anything 'manifold' in itself (such as the manifold Kant thought was provided by sensation) that would need to be unified by such a form – and so they do not contain anything that would do such unifying either (such as the representation of space itself).

Bolzano will thus be seen to depart from Kant at a quite fundamental level concerning the nature of our sensible representations of external objects. As we will also see, however, Bolzano takes the grounds for his departure to lie in commitments that, at least as he reads him, *Kant himself* explicitly affirms. Especially important here, for Bolzano, are Kant's remarks that link representational unity to intellectual acts of synthesis and combination.

What is more, though one might suspect that Bolzano's rejection of pure intuition would be part and parcel of a rejection of idealism about space as well – given the role that the doctrine of the pure intuition of space plays in Kant's own argument for the ideality of space – Bolzano actually agrees with Kant (and Leibniz before him) that space itself is not an 'actual [*wirklich*]' object in its own right, and also agrees – more surprisingly – that spatial representation has 'ideal' contents, in something close to Kant's sense of the term. Or so I will argue.

In several respects, then, Bolzano's alternative account of outer intuitions can be seen to take shape as a kind of internal challenge to Kant's account.⁶ In effect, Bolzano's alternative itself provides us with a competitor form of idealism developed from Kantian commitments.

My discussion will proceed as follows. I will begin in §2 by presenting the basics of Kant's account of space, spatial representation, and outer intuition, as it is developed in the *Transcendental Aesthetic*. In §3 I will then turn to Bolzano's account of intuition in general and outer intuition in particular, noting the extent to which he means for it to accord with Kant's own officially stated position on intuitions. In §4 I shift the focus to Bolzano's main departures from Kant on outer intuitions, departures made on the grounds that Kant's talk of intuitions containing a 'manifold' entails that synthetic intellectual activity (and

⁶ In its focus on Kant's remarks on synthesis especially, Bolzano's criticisms of Kant's doctrine of intuition can be seen to directly anticipate points made by various 'conceptualist' revisions to Kant's views on intuitions, of both the neo-Kantian variety as well as contemporary philosophers inspired by Kant (cf. Tolley 2013).

hence, concepts) are involved in the constitution of intuitions – though at the same time I also show how Bolzano takes these to be grounds that Kant himself actually should accept. In §5 I then show how the foregoing parallels and divergences on outer intuition furnish Bolzano with the basic material for his criticisms of Kant’s account of pure intuition in particular, highlighting how Bolzano’s criticism here again actually draws upon an important shared commitment – this time concerning the ontological ideality of space. Perhaps more controversially, I also argue that Bolzano ultimately agrees with Kant on the more straightforwardly transcendental idealist thesis that the representation of space represents something which is broadly representation-dependent, even if it is not intuition-dependent. In the concluding section (§6), I will take up the question of whether a defender of Kant’s account might have any grounds for resisting Bolzano’s criticism of Kant on the nature of outer intuitions and the pure intuition of space, both in light of claims Kant makes elsewhere which Bolzano doesn’t consider, as well as in light of reflection on the psychology and phenomenology of such intuitions.

§2 Kant’s account of space and outer intuition in the Transcendental Aesthetic

Let us begin by laying out Kant’s doctrine of space and outer intuition as it is found in the Transcendental Aesthetic. This section contains one of the most well-known and controversial conclusions Kant thinks he has established in the first *Critique* – namely, that space (the object) is something that ‘exists’ *only* ‘in the representation of it’ (A375n). Kant thinks he has demonstrated here that space ‘exists’ only as a ‘form’ of the contents of our sensible representations of objects which are ‘outside of’ or ‘external to [*ausser*]’ us (B42–43), rather than existing as something ‘actual [*wirklich*]’ in its own right, or existing as a determination of the way things are ‘in themselves’, independently of our sensibly representing them via intuitions (B37). As Kant ultimately puts this point, space is something that is ‘transcendentally *ideal*’ (B44). In support of this conclusion, Kant first sets out to demonstrate that the most fundamental, ‘originary [*ursprüngliche*]’ representation that we have of space is an kind of ‘*intuition*’ itself, rather than a concept (B39), and an intuition, moreover, that is ‘in’ us *a priori*, and is therefore ‘pure’ (B38–39). I will start with this preliminary argument.

§2.1 The originary representation of space

Kant begins his argument for the ideality of space from the ‘exposition’, or ‘distinct representation’, of what ‘belongs to’ our ordinary concept

of space (B38). At the outset, Kant assumes that we understand space to be something that is related in some way to our sensible representations of objects which are ‘outside us’, objects which we represent by means of our ‘outer sense’ (B37). More specifically, we represent ‘all’ objects of outer sense ‘as *in* space’ (B37; my italics). Furthermore, Kant takes us to understand space as that in which the shape and magnitude of external objects, and their relations to one another (e.g. distance), are ‘determined’, or at least ‘determinable’ (B37). Finally, Kant takes space to be something that we don’t intuit ‘in’ us, in the sense that when we do represent our own mind and its states in intuition, these are not represented as ‘in’ space but as only in time (B37).

As a key step in his argument for the ideality of space so understood, Kant sets out to establish, first, that the ‘original [*ursprüngliche*]’ representation that we possess of space must be ‘in’ the mind *a priori*, prior to all actual ‘sensation [*Empfindung*]’, ‘intuition [*Anschauung*]’, and ‘experience [*Erfahrung*]’ of external objects, and so cannot be an ‘empirical’ representation, or one drawn from these experiences (B38–39). This representation must be present in the mind prior to all actual outer intuition because it contains the universal and necessary ‘form’ of the contents of all such intuitions, and so is what makes such intuitions possible in the first place (more on this in a moment). In Kant’s words, the representation of space must be the ‘ground’ of these outer intuitions and their contents (B38). And since experience arises out of the synthesis of intuitions via concepts in judgment (cf. *Prolegomena* §20, 4: 300f.), the representation of space must therefore lie at the ground of outer experiences as well.

Kant then sets out to establish, second, that this original representation of space must nevertheless also be a special kind of ‘intuition [*Anschauung*]’ itself, rather than a general, common, or discursive concept (B39–40). Kant’s arguments here depend on the consideration of the special nature of this universal and necessary form of the contents of outer intuition – most importantly, that this content represents an object that is ‘essentially unitary [*einig*]’ (B39), even though this content in some sense also ‘contains within itself’ an infinity of further representations (B40).

The former point about essential unitariness leads Kant to insist that all the further representations we form of space (e.g. of parts of space, spaces, points, shapes, distances) arise due to acts of abstraction from a more original representation which first gives this single object itself immediately as the essentially unitary whole that it is. This abstraction takes the form of an intellectual delimitation of what are essentially non-independent parts of space, by ‘thinking’ these parts ‘into’ the ‘single

all-encompassing space' (B39). Since Kant takes intuition to be both the only 'representation that can be given prior to all thinking' (B132), and also to be the only representation that can 'give' its object 'immediately' (B33), Kant concludes that this more original representation of space must itself be a kind of intuition.

Concerning the latter point about infinity: Kant takes it for granted that no concept could contain 'in itself' (in its content) the possibility for an infinity of additional representations (B40). Yet the original representation of space can and does serve as the ground for an infinity of further representations – namely, the representations of all of the different kinds of parts (delimitations, shapes) of space and relations in space (B40). Kant sees this point about content as confirming his previous conclusion that the original representation cannot be a concept, but must be an intuition.

Combining these two theses (apriority and intuitionality), Kant then concludes that the original representation of space must be what he has earlier called a *pure intuition*, one which would 'occur [*stattfindet*] *a priori*', and so be non-empirical, and which would give its individual object (space) 'immediately' (cf. B33 and B41), all at once, as the essentially unitary object that it is (B34–35).

§2.2 Space as the form of outer appearances

Nevertheless, when viewed in relation to the 'outer' intuitions of external objects that it makes possible, Kant holds that this original representation of space must ultimately contain only the 'form' of the content of these outer intuitions, without containing any of the 'matter' eventually provided through sensation (B34). This points up the fact that, for Kant, what is 'contained in' an outer intuition is a composite of form and matter. The whole content is what Kant calls an 'appearance [*Erscheinung*]', which serves as the immediate 'object' of an outer intuition (B34). Space is what provides the 'form of all appearances of outer sense' (B42). The 'matter' is provided by the contents of sensation, e.g. colour, impenetrability, hardness (B34–5). This is then what fills in space itself, a matter which is then 'ordered' according to spatial relations (of 'extension [*Ausdehnung*]', 'figure [*Gestalt*]', etc.) (B35). And since Kant takes the matter from sensation that is 'contained' in every intuition to be 'something manifold [*ein Mannigfaltiges*]', the order that space provides to this matter is what brings a kind of 'unity [*Einheit*]' to the content of outer intuitions (A99).

Finally, it is this whole outer appearance – the unity which arises from sensory contents being ordered in a certain spatial configuration – which itself represents the further thing which is responsible for

bringing about the outer intuition in the first place. This further thing is what Kant at times describes as ‘something = x ’ (cf. A104, A250), since we don’t have any insight into what it is like, except through its effects on our sensibility – namely, the sensory contents which get ordered in a spatial form.

While this is how things stand with ‘empirical’ outer intuitions (ones which involve sensation), all that a *pure* outer intuition would contain, by contrast, is ‘that within which’ whatever ‘matter’ sensations will deliver ‘can be ordered in certain relations’ – i.e. the mere form of outer appearances (B34). Indeed, it is by containing *only* this form that such an intuition can occur *a priori*, ‘without any actual [*wirkliche*] object of the senses’ being yet encountered through it affecting our sensibility (B35). This is possible because Kant thinks that, in order to do such ordering of sensory material, the ‘form’ itself is something that ‘must lie ready for [the matter] in the mind *a priori*’ (B34).

§2.3 The ideality of space itself

With all of this in place,⁷ Kant then draws his famous conclusions concerning the ontological standing of space itself. Kant assumes that

⁷ Because of my focus here (cf. §1), I have skipped over the part of the Transcendental Aesthetic where Kant then turns briefly to the relation between the foregoing and our knowledge in ‘pure’ (*a priori*) geometry. There Kant points out that the syntheticity and the apodicticity of such knowledge provides separate confirmation of the correctness of his claim that the original representation of space needs to be a pure intuition. Concerning apodicticity, Kant argues that it is only by accepting that we have a pure (*a priori*) representation of space that we can account for how we can know *a priori* that geometrical propositions will be universally and necessarily true of everything we encounter in space (B41). Yet since in geometrical reasoning, we use construction (drawing) to come to know that certain predicate-concepts are truly related to some subject-concepts, despite the fact that the contents of these predicate-concepts ‘go beyond’ what is contained in the subject-concepts (which makes the relevant judgments synthetic), Kant thinks that this pure representation cannot itself be merely another concept (B40–41).

The argument from geometry for the necessity of a pure intuition of space, as well as for the ideality of space, receives much fuller development in Kant’s *Prolegomena*, where Kant uses the ‘analytic’ method to demonstrate the truth of transcendental idealism, which proceeds from the accepted fact of *a priori* cognition in pure mathematics, to the ideality of space as a condition for the possibility of such cognition. In the first *Critique*, by contrast, Kant proceeds according to the ‘synthetic’ method, which moves from accepted facts about the elements of all our cognitions, to an inventory of what *a priori* cognition is possible given these elements. (Kant discusses the difference between these two methods at the outset of the *Prolegomena*, cf. 4: 263–264.)

'neither absolute nor relative determinations [of things] can be intuited prior to the existence of the things to which they pertain, thus cannot be intuited *a priori*' (B42). And yet he also takes himself to have just shown that space itself *can and must* be intuited *a priori*. Kant therefore concludes:

Space represents no property at all of any things in themselves nor any relation of them to each other, i.e., no determination of them that attaches to objects themselves and that would remain even if one were to abstract from all subjective conditions of intuition. (B42)

Instead of being something 'actual', with a representation-independent existence 'in itself', space exists only 'in' our intuitions and in our experiences, and is therefore dependent for its existence on the 'subjective constitution' of our senses being the way it is:

We therefore assert...[space's] transcendental ideality, i.e., that it is nothing as soon as we leave aside the condition of the possibility of all experience, and take it as something that grounds the things in themselves. (B44)

[I]f we remove our own subject or even only the subjective constitution of the senses in general, then all constitution, all relations of objects in space and time, indeed space and time themselves would disappear, and as appearances they cannot exist in themselves, but only in us. (B49)

Note, finally, that Kant concludes as well that all relations of objects in space are also 'ideal'. This entails that outer appearances as a whole are themselves ideal as well, insofar as Kant believes that 'nothing is given to us through outer sense except mere representations of relation' (B67).

§3 Kantian themes in Bolzano's account of outer intuitions

When we now turn to Bolzano's criticisms of Kant's account of space and spatial representation, we should first take care to determine how the terminology lines up between the two authors, especially concerning the term 'intuition' and 'outer' intuition in particular. On the one hand, Bolzano makes clear that he means to be taking over some of the key elements of Kant's analysis of 'intuition [*Anschauung*]'. Most importantly,

Bolzano accepts Kant's claims that outer intuitions are representations that 'give' their object *immediately*, involve *sensation*, and are essentially *singular*, in that they necessarily represent only one individual object. On the other hand, as we will see in the next section (cf. §4), Bolzano thinks that these two facts about intuitions entail that intuitions bear two further marks, at least the second of which Bolzano recognizes that Kant does *not* seem to accept: first, the object of an intuition must be something '*actual* [*wirklich*]'; second, intuitions must have a content that is *simple*.

§3.1 The immediacy of intuition

Bolzano says he means to be using 'intuition' in a way that picks up on Kant's use (cf. NAK 15).⁸ The first respect in which he means to be following Kant is that he, too, accepts that intuitions are 'immediate'. As Bolzano sees it, for a representation to be 'immediate' in the sense in which Kant has in mind – for it to 'give' its object to the mind (as Kant puts it; B33) – the occurrence of this sort of representation must entail the existence of an actual object to which it is related: '[Kant's] expression, intuition gives the object, seems to have the sense that we are justified in concluding, from the possession of an intuition, that there must exist an object which brought it about' (WL §77.2, I.346; cf. NAK 44). Understood this way, Bolzano thinks 'there seems to be something very true in this': 'We can indeed infer from the possession of a subjective intuition to the existence of an object corresponding to it, which brought it about through its influence upon our faculty of representation' (WL §77.2, I.347).

Though Bolzano accepts that the 'arising' of 'all other representations which appear in our consciousness' must also have 'an appropriate cause', Bolzano thinks that the case of an intuition is special: 'The difference is that from the presence of [an intuition], we can infer to a cause which is itself the one and the same actual [*wirkliche*] object *which we are representing*' (WL §77.2, I.347; my italics). Thus Bolzano identifies the object represented by an intuition with the cause of the intuition.

What is more, in order to preserve what is special about intuitions, Bolzano thinks we must restrict the application of the label 'intuition'

⁸ This is so, even though Bolzano is much more explicit (and persistent) about the distinction between the act of intuiting and the content thereby intuited. Bolzano calls the former 'subjective intuitions', and calls (something closer to) what Kant means by the latter an intuition 'in itself [*an sich*]' (cf. WL §76.2, I.342).

only to those representations which do, in fact, come about due to the influence of an *actual* object which the representation thereby represents:

It does seem quite correct to me...to say that an intuition (that is, a subjective one) always concerns an actual [*wirkliches*], and indeed, if you will, a present [*gegenwärtiges*] (that is, acting on us at the time) individual thing [*Einzelding*]...and that the content of the intuition is not applied to anything other than this thing. (WL §77.8, I.352; my italics)

This feature is present in Bolzano's primary example of a representation that is 'immediate' in this sense of 'giving' an actual object – namely, the representation which occurs in our soul when we direct our attention to a 'change' that is also in our soul:

As soon as we direct the attention of our mind upon the change that is brought about in our soul by some external body that is brought before our senses, e.g., a rose, the next [*nächste*] and immediate [*unmittelbare*] effect of this attending is that a representation of this change arises in us. (WL §72, I.326; cf. NAK 16)

A few sections later, Bolzano makes explicit that he takes the inclusion of a relation to an actual object to be a feature not just of these examples but to characterize *all* human intuitions:

The intuitions that I gave in §72 as examples were all constituted in such a way that the objects which corresponded to them all belonged in the realm of actuality [*Wirklichkeit*], since these were throughout certain changes occurring in our soul. Now, I am of the opinion that this holds of all intuitions, at least those of which we humans are capable, i.e., I believe that the object of any humanly attainable (subjective) intuition must be an actual [*wirkliches*] thing. (WL §74.1, I.331)

One might wonder whether all of the foregoing entails that Bolzano thinks we can only have 'inner' intuitions ('inner' representations of changes among the representations 'in' our soul). There is, in fact, something to the thought that, for Bolzano, the immediate objects of all intuitions are 'inner'. Bolzano states explicitly that intuitions are distinguished from all other representations due to the fact that 'they appear

as the next and immediate *effect* of certain changes just having occurred *in us* (my italics), changes in us ‘which are therefore the object represented through them [*der durch sie vorgestellte Gegenstand*]’ (WL §286.1, III.84–85). And since we have seen that intuitions are special in that they always represent their causes, the represented objects of every (human) intuition must therefore be things ‘in the soul’.

§3.2 The involvement of sensation

Even so, Bolzano means to follow Kant in accepting that there is reason to distinguish certain intuitions as especially ‘inner’ and others as ‘outer’. For Bolzano, the difference between inner and outer intuitions rests upon a difference in the kind of ‘change in us’ which is ‘immediately’ represented – namely, whether this change is *itself* already a representation or whether it is not:

We can, however, distinguish two kinds of intuition, depending on whether or not this change itself is already a representation (or even a judgment). Intuitions which have as their object another representation equally present in our mind may be called inner [*innere*] intuitions. Those, by contrast, which concern a change which is not itself a representation, may be called outer [*äußere*]. (WL §286.1, III.85)

Now, we might wonder what Bolzano is thinking of by referring to a change which is ‘in our soul’ and yet which is not itself a representation, but which nevertheless is to function as the object of the representation that he calls an ‘outer’ intuition (since, as we have just seen, *all* intuitions have ‘changes in us’ as their objects). A promising proposal here has been made by Rolf George, who has argued that Bolzano is thinking of mere ‘sensations [*Empfindungen*]’ as the objects of outer intuitions, as opposed to other ‘mental appearances [*Erscheinungen*]’ which are properly called ‘representations’ (cf. George 2003, 21f.). Now, like Kant, Bolzano’s examples of outer intuitions do, in fact, involve representations of sensory qualities, e.g. of red (cf. WL §286.1, III.85). What is more, Bolzano claims that it is outer intuitions that we ‘subsume’ under ‘the common representations of colors, sounds, odors, etc.’, which he identifies as the ‘common sensible [*sinnliche*] representations’ (WL §286.8, III.88). All of this suggests that sensations are what is represented by outer intuition.

Are sensations, however, not themselves a form of representation? We saw above that Kant takes them to provide contents which factor into

outer appearances and so serve to represent external objects.⁹ In *WL* §143, however, when Bolzano is classifying ‘what occurs in our own inner sphere [was in *unserem eigenen Innern vorgehet*]’, he does in fact distinguish sensation from both representations and judgments, with sensation being said only to ‘accompany’ many of our representations (II.67). Elsewhere when Bolzano again explicitly distinguishes the sensations we have from both the representations that we have of them and the judgments we make about sensations, he also argues that we cannot make a judgment about a sensation until we have formed a representation of them:

We can only make judgments about sensations if we first represent them. The sensation itself is one thing, the representation of it is quite another, and the judgment about it (e.g., that it is pleasant or unpleasant, is constituted thus and so, etc.) is still something further. (*WL* §35.8, I.163)

This might be thought to give some evidence for the related, though somewhat indirect argument made by George. George points out that, given Bolzano’s explanation of the concept of a representation, Bolzano in principle restricts the term ‘representation’ to that which can function as a component in a proposition (content of a judgment) but which is not itself a proposition (cf. *WL* §52, I.228; *NAK* 8). Bolzano’s claim that we need to first form representations of sensations in order to make judgments about them might be taken to suggest that it is only representations of sensations, rather than the sensations themselves, that can function as components in the contents of judgments. But, then, if sensations themselves cannot become components of propositions (as contents of judgments), they would thereby be ruled out from being representations.¹⁰

⁹ Kant seems to explicitly classify sensations as a kind of representation on the so-called ‘Stufenleiter’ (cf. B376–377). What is more, Kant also at times explicitly describes sensations as having an ‘objective’ representational function (as an ‘objective representation of the senses’; cf. *Critique of the Power of Judgment* §3). Both of these speak against George’s claim (in George 1981) that sensations for Kant do not represent anything.

¹⁰ This is so, even if Bolzano of course allows that judgments (propositions) can be *about* sensations (cf. again *WL* §35.8, and *WL* §143.3, II.69). What functions, for example, as the predicate-representation in a proposition of the form ‘A has the sensation D’ just needs to be understood as a representation *of* the sensation D, not the sensation D itself. In *WL* §42, Bolzano identifies these components of judgments with intuitions of objects that ‘obtain outside of representations’ (I.181).

Such an argument is not conclusive, however, because here Bolzano is talking explicitly only about the possibility of judgments *about* sensations, not the possibility of judgments about other things *through* representational contents which might include sensations. For this same point holds for intuitions and concepts as well: we cannot make a judgment *about* an intuition or a concept until we first form a representation *of* the intuition or concept. Nevertheless, while Bolzano does regularly talk about intuitions and concepts both functioning as components in the contents of judgments, he nowhere (to my knowledge) talks about sensations functioning as possible contents (rather than objects) of judgments. For this reason, George's suggestion seems to be on the right track.

Yet if sensations are something 'inner', and if outer intuitions are immediate representations of sensations, and thereby have something inner as their immediate object, why should these representations of sensations still be called 'outer' intuitions?¹¹ Bolzano here points us to what is involved in the causal origin, or the bringing about, of such intuitions. Every intuition of a sensation is, Bolzano claims, 'a representation that has its existence immediately due to a change in our soul *produced by the action [Einwirkung] of an external [äußeren] object*' (WL §288.1, III.104; my italics). These representations are called 'outer', therefore, because the nature of their 'arising' is such that it 'leads [*leitet*] us immediately to the presupposition [*Voraussetzung*] of an external object that, through its action [*Einwirkung*], must have brought about this change in our soul' (WL §286.1, III.85).

Matters are complicated further, however, by the fact that Bolzano takes the *full* cause of the change in outer intuitions to also include other changes '*in me*', rather than solely being ascribed to the action upon me of an external object:

For each change which precedes my intuitions which are related to external objects, I must presuppose as cause a change either in me, or in some external objects, indeed either in their inner properties or in their spatial relations to me, or (what is more correct) in *all of*

¹¹ To be fair, Bolzano very occasionally seems to suggest that the object of an outer intuition is actually a change that is 'external' to us – in the following passage, for example: '[E]very subjective intuition has its own object, namely, the change to be found *outside of* or in us which is the immediate cause of its arising' (WL §75.1, I.334; my italics). On balance, however, it would seem that the account given above is his considered view.

these things at once, only in varying degrees. Because my intuitions, when they *are related to* [*sich beziehen auf*] an external object, *represent* [*vorstellen*] changes which an external object has brought about in me through its action [*Einwirkung*], their nature [*Beschaffenheit*] is determined by my nature and the nature of the external object and through the spatial relation between us. (WL §303.21, III.151; my italics)

As is evident from this passage, then, Bolzano therefore actually means to distinguish (a) the total cause of the outer intuition, which involves not just the external object but factors ‘in me’, (b) the effects of this cause, i.e. the sensations which are ‘in me’, and which are actually the immediate objects that the intuition ‘represents [*vorstellt*]’, and (c) the external object to which the intuition ultimately ‘is related [*sich bezieht*]’. What the outer intuition *immediately* ‘represents’ are the sensations as changes ‘in me’ that are brought about by the action of an external object upon me – representing them in a way that is determined jointly by my own nature, the nature of the affecting object, and the spatial relations between us. What the outer intuition is *ultimately* ‘related to’, however, is the external affecting object alone, and is thereby taken to be an intuition ‘of’ that object.

This ‘relating’ to the external object, then, is something we associate with the intuition only secondarily, as a ‘presupposition’ to which we are ‘led’ by the presence of the intuition itself (cf. WL §286.1, III.85, quoted above). Indeed, later in the same passage Bolzano describes the secondary act of ‘relating’ an outer intuition to an external object as one that occurs through my ‘inferring [*schliessen*]’ (WL §303.21, III.151). For Bolzano, then, sensations are not directly or autonomously representational – at least not in the same way that other representations (intuitions, concepts) are – since our minds must judge or infer them to ‘be related to’ a further object, rather than simply being conscious of this further object (as it appears) through a grasping of the sensation itself. Hence, though Bolzano agrees with Kant on the involvement of sensations in outer intuition, sensation plays a quite different role in Bolzano’s account.

§3.3 The singularity of intuition

Finally, Bolzano also takes himself to agree with Kant on the essential *singularity* of intuitions (cf. NAK 50). Though it is not stated explicitly in the discussion of space in the Transcendental Aesthetic, Kant later (and elsewhere) makes clear that he takes one of the marks of an intuition is

that it is a 'singular [*einzelne*]' representation, that it is a representation of an individual object (cf. B376–377, *Jäsche Logic* §1, 9: 91). One might also think that this is at least implicit in his argument that only an intuition could relate us to an 'essentially unitary' and hence individual object (one with no 'real' parts), in order for us then to be able to 'think' parts 'into' the object (cf. §2 above).

For his part, Bolzano arrives at the singularity of intuitions by asking us to reflect further on what occurs in our soul when we direct our attention to some change in us which is occurring due to the influence of an external object – e.g. to what occurs when our attention is directed to the change that takes place in our soul (i.e. the sensation) when a rose is brought before our senses (cf. *WL* §72, I.326). As we saw above, Bolzano thinks that the representation of the sensation which thereby arises in us is 'immediate' because it comes about due to an *actual* immediate effect 'in us' (namely, the sensation as change in our soul), an actual effect which it also thereby represents. In other words, we can always infer from the presence of the representation of the change in our soul to the existence of its object (the sensation itself). Yet Bolzano also takes it to be evident that the representation which arises as the 'immediate effect' of our 'attending' to a sensation is one which can have only one *single* object:

Now, this representation is an objectual [*gegenständliche*] one; its object is, namely, the change that has just occurred in our soul, and nothing else; therefore, a singular [*einzelner*] object. Thus, we can say that this representation is a singular representation [*Einzelvorstellung*]. (*WL* §72, I.326; cf. *NAK* 16)

As Bolzano also puts the point, the 'content' of such representations 'is related to *nothing* distinct from this thing [*ihr Inhalt sich auf nichts von diesem Dinge Verschiedenes beziehe*]' (*WL* §77.8, I.352; my italics).

What Bolzano has in mind would seem to be the following: if we are able to limit our attention solely to the 'next and immediate' change in our soul (rather than some indeterminate collection (series) of changes), then we can be sure that our attention (intuition) has one and only one object – namely, just that one change in our soul and nothing else. It is *this* representation-relation (intuition-to-sensation) that Bolzano thinks must be singular – even if the content of an outer intuition is *also* 'related to' something distinct from this sensation as a change 'in us', when it is related in judgment or inference to the single external object which is (partly) responsible for determining the nature of the intuition by

(partly) bringing about the relevant change. Still, it is only because the initial relation is indexed, as it were, to the particular change brought about – more specifically, to one particular sensation – that Bolzano takes the singularity of each outer intuition to be secured.¹²

§4 Bolzano's 'Kantian' departure from Kant on the simplicity of intuitions

On the points covered so far – with the notable exception of the precise role of sensations – Bolzano's account of outer intuitions is broadly in line with what Kant himself held about such intuitions. As noted above, however, Bolzano takes his account to involve two further commitments that he recognizes might not square directly with the letter of Kant's positions. First, whereas Kant seemed to hold that the 'content [*Inhalt*]' of an outer intuition involves both a matter, consisting in a (*complex*) manifold of sensation, and a form, consisting in space, Bolzano holds that the content of an outer intuition must be something entirely 'simple [*einfach*]'. Second, whereas Kant's account of the pure intuition of space forces him to accept the possibility of an outer intuition of an *ideal*, non-actual object, Bolzano insists, by contrast, that the object of every outer intuition must be something '*actual* [*wirklich*]'.

Yet if Bolzano recognizes that he is here departing from Kant's official position, what is striking is that in both cases Bolzano takes his departure from Kant to be justifiable by reference to commitments that Bolzano thinks *Kant himself* actually held. In this way, these departures from Kant can be viewed as criticisms from 'within' a broadly Kantian framework.

§4.1 The simplicity of the content of intuitions

By the 'content [*Inhalt*]' of a representation, Bolzano means 'the *sum* of the components of which this representation consists', which Bolzano contrasts both with the *object* that the (whole) representation itself represents, but also with 'the *way* in which these parts are connected with one another' (*WL* §56, I.244), which he later associates with the '*form*' of the representation (*WL* §81.1, I.389).

¹² Bolzano's argument for the singularity of inner intuitions follows in a parallel fashion, since (subjective) representations themselves are also among the changes in our soul (along with sensations), and so can be attended to as 'next and immediate' (cf. *WL* §75, I.334–335).

As Bolzano sees it, given the nature of the arising of intuitions and their immediate objects, Bolzano thinks that we must conclude that the content of every intuition must be ‘*simple [einfach]*’ (WL §72, I.327), in the sense of having *no* parts. Bolzano’s argument for this claim runs as follows:

[I]f they were composed of parts, they would not be the next and immediate effect that arises from the observation of the change just having occurred in our soul; rather, the singular representations which would form the parts of any such complex representation would have been produced earlier and more immediately. (WL §72, I.327; cf. NAK 16)

Bolzano is here clearly assuming that no bringing-together or composition of representations into a complex one can occur *as immediately as* the representation produced by the observation or attention to a just-occurring change in our soul. Why can’t anything complex be the immediate effect of our attention or observation? Bolzano takes all such complexity to be the result of a further compositional or synthetic ‘activity’ of our soul, something over and above the mere directing of our attention toward something. Bolzano makes this further premise explicit later on in the *Wissenschaftslehre*, in a critical discussion of the alleged possibility – one with clear Kantian echoes – that outer intuitions are ‘infinitely complex’ representations:

Should one not already find something impossible in itself in such a representation composed out of infinitely many parts, it can in no way be assumed that such representations could occur merely due to the immediate action [*Einwirkung*] of an external object on our soul. Rather, it seems indisputable to me that any such composite representation requires a special activity on the side of our soul to bring it about out of the simple representations which are called its parts. Even supposing that the representations: ‘red, pleasantly fragrant, prickly’, etc., arise immediately through the action of an external object upon me, the representation which is supposed to be composed of all of them (the representation, perhaps, of an object which has in itself the collection of all of these properties) would not yet be present, but would rather require a special activity of my soul for its arising. (WL §287.3, III.94–95)

What is more, Bolzano thinks that, to actually form the representation: an object (being) which has in itself this collection of properties, this

'special activity' would need to avail itself of certain special connective representations which he also thinks cannot be viewed as 'immediately produced' by the object: '[Such composition] indeed would also require certain representations which most certainly were not immediately produced by that object, such as the concept of a being [*Wesen*], that of having [*Haben*] certain properties, of a collection, etc.' (*WL* §287.3, III.95; cf. *NAK* 51). In fact, Bolzano thinks that the representations – red, pleasantly fragrant, etc. – involved in such a complex representation are actually *general concepts* as well, for they are functioning as predicates under which we 'subsume' the immediate simple representation (*ibid.*).

For these reasons, Bolzano insists that the more accurate expression for the content of intuitions would be simply the demonstrative 'this', as comes out in his discussion in *WL* §42 of judgments which 'contain intuitions of certain objects that obtain outside of [our] representation'. Bolzano takes the canonical expression for this type of judgment to be: 'this (what I see here now) is red', such that 'the subject-representation of the proposition is a simple intuition (this) and the predicate-representation is a concept (red)', with the parenthetical expression 'what I see here now' therefore not being essential to the expression of the content of the subject-representation but 'redundant' (*WL* §42, I.181; cf. *NAK* 16–17). Later Bolzano writes similarly that when 'complete judgments are made...about the change itself that has just occurred in us', these can be expressed, for example, as: 'this (what I right now see) is the sensation or representation red; this (what I now smell) is a pleasant fragrance; this (what I just feel upon touching a thorn with the tips of my fingers) is a painful sensation, etc.' – with the word 'this' in each case serving to 'designate [*bezeichnen*]' a representation which is *simple* as to its content, one which relates immediately and directly to some presently noticed sensation as a 'next and immediate' change in us (*WL* §72, I.326).¹³

¹³ Cf. *WL* §73.3, I.331 and §278, III.22. For more discussion, see George 2003 and Rosenkoetter 2012: §6.

To head off a possible misunderstanding, let me here note that Bolzano accepts that the ultimate *object* (a sensation) that an outer intuition represents, in virtue of having the content that it does, does not have to be simple – though it must be an individual: '[L]et us first note that the expression *individuum* may be misunderstood, since if no further explanation is forthcoming it can be interpreted as if the object of an intuition had to be simple, which is by no means the case' (*WL* §77.1, I.344).

§4.2 The Kantian motives behind Bolzano's simplicity condition

Now, Bolzano recognizes that, with the condition of simplicity of content, his account of intuition is departing not just from Kant but from many of his predecessors and contemporaries, however close it otherwise might be:

By the word *intuition* almost all modern logicians have in mind a representation that has only a *single* and *actual* [*wirklich*] object, and depart from me therefore only in that they ...do not require as firmly as I do that a [mere] intuition must be a thoroughly *simple* representation. (WL §76.2, I.342; my italics)

With respect to Kant in particular, however, Bolzano thinks that both of the two key points that he marshals in favour of this departure consist in aligning himself *with what should have been Kant's own considered position*. For Bolzano thinks he has fairly straightforward evidence from the *Critique* itself that Kant, too, holds, first, that all synthesis (combination) requires a special act of the mind (an act of understanding) over and above any act of mere sensibility, and holds, second, that all synthesis of the manifold given in intuition requires the use of concepts, and so cannot consist in mere intuitions.¹⁴

Concerning the former point, Kant claims explicitly that the synthesis (combination) of the manifold in intuition is something done by our understanding, not our sensibility:

all combination, whether we are conscious of it or not, whether it is a combination of the manifold of intuition or of several concepts, and in the first case either of sensible or non-sensible intuition, is an *action of the understanding*, which we would designate with the general title *synthesis* ... (B130; my italics)

Concerning the latter point, Kant again claims quite explicitly that the means by which our understanding achieves the synthesis of the manifold in intuition is not some further intuition but is a 'function' which

¹⁴ In this respect Bolzano's criticism of Kant prefigures both neo-Kantian intellectualist revisions to Kant's doctrine of intuition (cf. Friedman 2000, 31f. and 89f.), as well as more recent 'conceptualist' interpretations of Kant's views on intuitions (cf. Tolley 2013).

he associates with a 'pure' concept or category: 'The same function that gives unity to the different representations in a judgment also gives unity to the mere synthesis of different representations in an intuition, which, expressed generally, is called the pure *concept* of understanding' (B104–105; my italics).¹⁵ Finally, Kant even seems to explicitly and directly reject the idea that such combination can arise from intuition itself, or from what is 'given' in them directly from objects:

[T]he combination (*conjunctio*) of a manifold in general can never come to us through the senses, and therefore cannot already be contained in the pure form of sensible intuition...Among all representations combination is the only one that is not given through objects... (B129–130)

These passages (and others) suggest to Bolzano that Kant, too, should agree that anything which involves combination or synthesis through concepts cannot be the *most immediate* representation of what is present in our mind due to the affection of our sensibility by an external object. For in order for such combination to be possible, Kant too seems to hold that we must first have simple representations of the elements in this manifold itself, with these representations being there prior to any combinatory or synthetic act by the mind.

Despite Bolzano's sense of accord with Kant on the premises for his conclusion that intuitions per se (in Bolzano's words: 'pure' (i.e. mere) intuitions) must have simple contents, Bolzano acknowledges that neither Kant himself nor his followers anywhere explicitly accept this conclusion:

[In Kant's writings] there is not even the faintest trace to be found of the other mark that a pure intuition (one mixed with no concept) must be a simple representation, neither in his work nor in that of any of the later philosophers who have adopted from Kant, and maintained, the distinction between intuitions and concepts. (NAK 50; cf. NAK 219)

¹⁵ Already in his initial discussion of synthesis, Kant appears to allude to the necessary involvement of 'concepts [*Begriffe*]': 'By synthesis in the most general sense, however, I understand the action [*Handlung*] of putting different representations together with each other and *comprehending* [*begreifen*] their manifoldness [*Mannigfaltigkeit*] in one cognition' (B103).

What is worse, as we have already anticipated, Bolzano thinks that Kant's official position is actually incompatible with this conclusion. This is due to the fact that, as we saw in §2, 'Kant speaks very often of the manifold which lies in intuition' (NAK 50). Yet based on the passages here just cited, Bolzano thinks Kant himself actually accepts that

a manifold can be found in a singular representation *only if* it was first brought about through a combination of several simple representations, a combination that can *only* come about through the mediation of some representations which are not intuitions but are rather concepts. (NAK 50–51; my italics)

From this, Bolzano concludes that Kant's account of intuition as immediate and yet a unity of a manifold is not fully consistent.

§4.3 From simplicity to the rejection of form

Now, as was also seen above, what Kant officially takes to 'unify' the manifold of an outer intuition is not an act of the understanding or any concepts, but is *space itself*. This is because space itself is to function as the 'form' of the content of an outer intuition (a form of our sensibility), as something that 'orders' the 'matter' provided by sensation into a single whole, according to how such matter fills out the places or locations in space. What is more, as we also saw above, Kant argues that we must possess *a priori* a 'pure' intuition of this order-giving form itself, since it must lie ready 'in the mind' prior to the reception of any matter from sensation.

As we might now suspect, Bolzano takes his argument for the simplicity of the content of intuitions in general, along with Kant's own claims about combination, to also demonstrate the untenability of these aspects of Kant's analysis of pure intuitions in particular.

[A] pure intuition would be accordingly something essentially complex, would contain a manifold which it would combine into a certain unity. Yet at the same time, it is certain, and Kant himself acknowledges elsewhere..., that combination is a feat [*Werk*] of the understanding, i.e. comes about only through concepts. In this way, intuition would then be a complex representation that comprises concepts in itself as components. (NAK 52)

Having shown, then, that Kant himself should conclude that whatever is responsible for such a unity is something that necessarily involves

concepts (e.g. the concept <and>), Bolzano then challenges Kant to explain why this representation should nevertheless be called an *intuition*: ‘But is there really a point in taking the concept of an intuition in such a broad, vague signification?’ (NAK 52–53).

§4.4 Rethinking outer intuitions

This departure has the following important consequences for how Bolzano himself views the nature of our outer intuitions. Since the content of every intuition is simple, Bolzano himself must conclude that what an intuition contains cannot have *any form* whatsoever, in the sense of something which would unify its parts (*matter*). Intuitions contain no parts, and hence contain nothing at all to unify. But then, *a fortiori*, the contents of outer intuitions cannot all share *space* as their *universal and necessary form*. Conversely, if the content of intuitions is in fact simple, then they cannot ‘contain’ a manifold of sensations as the *matter* (component parts) unified by space as a form. Indeed, as we saw above (cf. §3.2), on Bolzano’s view, a single sensation corresponds to each outer intuition as its immediate *object* rather than as a component of its content, as Kant would have it. The content itself is, again, the wholly ‘simple’ representational correlate of the demonstrative ‘this’.

In fact, Bolzano thinks that Kant’s willingness to apply the term ‘intuition’ to the representation which would perform such a unifying function just shows that Kant does not have a clear conception of what an intuition is in the first place:

Kant’s claim, that the pure form of sensibility (i.e., that in the appearance which makes it so that its manifold can be ordered in certain relations) can also be called pure intuition, shows especially how vacillating the concept which Kant himself connected to this word must have been. (NAK 52)

Since no intuition can contain anything that unifies *at all*, there can be no intuition which has a mere ‘form’ of unity as its content. But then there can be no pure intuition of space in particular as an intuition of the form that unifies each outer appearance.

The absence of form in each outer intuition forces Bolzano to conclude that no outer intuition contains *any* spatial representation or spatially extended content (such as distance) whatsoever:

[T]he distance between the tips of my two fingers is and can be an object of my intuition just as little as... the distance between this fixed

star and the disc of the moon that floats above it. What we intuit are merely certain colors, sounds and tones etc. and we conclude from their presence, often with many steps in between... that certain objects are present that bring about these representations in us, and which stand in these or those spatial relations to each other. (NAK 95–96)

Here even more explicitly than before, we see Bolzano insisting that the spatial relation to an external object comes into our representations *only* with an act of *inference* (a ‘conclusion’), rather than being already present in the immediate sensible representation itself – i.e. in the intuition.

§5 Bolzano against Kant on intuitions of ‘ideal’ (non-actual) objects

While this disagreement over the simplicity of intuitions surely provides *one* key platform for Bolzano’s criticism of Kant’s doctrine of the pure intuition of space, Bolzano takes himself to have a *second* platform from which to criticize Kant’s doctrine. What is more, just as in the previous case, Bolzano takes this second platform to be grounded in commitments that Kant himself ultimately shares. In fact, as we will see in this section, Bolzano actually thinks that he can marshal an argument against Kant’s doctrine of the pure intuition of space *on the basis of their shared commitment to the ideality of space itself*. For, as Bolzano sees it, Kant must not have thought through what his commitment to the ‘immediacy’ of intuitions actually requires, and concludes that Kant must have lost sight of this commitment when he tries to introduce the possibility of an intuition of *non-actual*, ‘ideal’ object, like space.

§5.1 The actuality of the object of every intuition

Given the conditions on being an intuition that we have set out above (that the representation give its object immediately, involve sensation, and have a simple content), Bolzano concludes that all human intuitions must have ‘*actual things*’ as their objects:

It seems to me that this already follows from the mere concept of an intuition alone, as a simple singular representation. For if a representation is to represent merely one object, despite all of its simplicity, then it must have something so peculiar (something exclusively relating to only this object) that its arising in our mind can hardly be explained in any other way than through the assumption that it

is related to this object as an effect is to its cause. From this it follows at once that this object must be something actual, since as a cause it should show itself to be efficacious [*wirksam*]. (WL §74, I.331)

And as we have also already anticipated, here again Bolzano appears to think that Kant implicitly accepts this condition, something Bolzano thinks is indicated by Kant's aforementioned talk of intuitions 'giving' their objects 'immediately':

[B]y intuitions, Kant has always thought of just those representations that refer to one single and indeed actual object. ... [H]e has always thought by an intuition a representation from whose presence we are at once justified in inferring the presence of an object which corresponds to it (which is represented through it). For it is obvious that this is just what he wants to tell us by the phrase: intuition gives us the object, gives it immediately, and indeed no object can be given to us in any other way (at least immediately). (NAK 48–49)

Bolzano is confused, then, as to why Kant would want to call a representation of a *non-actual* or *ideal* object like space an intuition, since by Kant's own lights – and, as we will see now, by Bolzano's, too – space is not an actual object that can ever actually be present to our senses.

§5.2 The non-actuality (ideality) of space

Bolzano gives several arguments for why space is not something 'actual' [*wirklich*]. One key argument concludes that space (and time as well) is not something actual from the fact that space itself is not 'active' in the sense that it brings about no effects:

I ask anyone who knows what mathematicians understand by the words time and space, whether he must not concede that only the objects that are found in time and space are something actual, but not the times and spaces themselves. And if someone wanted to define time and space as something actual, then he would have to claim that they also have certain effects [*wirken*]. And what could these be? (WL §79.2, I.362)

In addition to this argument from the non-efficaciousness of space, Bolzano gives several further arguments for the ideality of space, many of which echo those Leibniz gives in his correspondence with Clarke.

One argues in the form of a dilemma about whether space is changeable or not:

If time and space were something actual, then their actuality would have to be one of the two, either unconditioned or conditioned [*unbedingt oder bedingt*]. In the first case they would be God, in the second, they would be created things that are subject to change. Now, nobody can really say either that space and time are God himself or that they are subject to change, since only the things that are in time and space change but not time and space themselves. (WL §79.2, I.363)

Another argues for space's ideality from the non-equality of every actual thing:

If time and space were something actual, then no two moments or durations, or two points or distances could be exactly equal [*gleich*] to one another, since among actual things there are not two that equal one another exactly. But this is quite contrary to the concepts that mathematicians have about these objects. (WL §79.2, I.363)

A third argues for space's ideality from the equality of all of its parts in conjunction with the principle of sufficient reason:

[I]f two moments or two points are exactly equal (as has been asserted by all mathematicians for eternity), and if time and space were something actual, then the existence of a thing at this determinate time and this determinate place would have to be something actual that has no ground. For there would be entirely no ground for why this thing should be in just this particular state at this particular time and place and no other, not just none for us humans to give, but there would be none available in itself, since these places and times are completely equal internally [*innerlich völlig gleich*]. (WL §79.2, I.363)

And Bolzano accepts, as does Kant, that, ontologically speaking, what is true of the parts of space is true of space as a whole:

If no individual moment and so also no individual point is, for itself, something actual, then neither can we hold the collection of all moments, i.e., the whole infinite time, nor the collection of all points, i.e., the whole infinite space, to be something actual. (WL §79.3, I.364)

§5.3 The impossibility of intuiting space

Yet while Bolzano therefore shares Kant's views that space as a whole is not an 'actual [*wirklich*]' object, Bolzano takes this to entail that there can be *no intuition* of space, whether of its parts (places, distances) or as a whole. As we saw above, Bolzano holds that *all* intuitions must have something actual as their object – whether a sensation or another representation or judgment:

Recall that every (subjective) intuition that appears in our mind must have an existing object. This gives us a means for proving that the aforementioned representations [of space and time] are not intuitions, and indeed without having to decide anything about whether they are complex or simple, merely on the ground that the objects that are represented through them since the objects they represent are not at all something actual (something existing). (*WL* §79.2, I.362)

Note that this argument is supposed to proceed independently of whether the representation of space is something *simple* or complex.

Bolzano also takes his conclusion to be independent of the fact – which he too acknowledges, again, in agreement with Kant – that space itself, as a whole, is an individual object, and so its representation will be *singular*. For even though space is an individual, it is still not actual:

If no individual moment and so also no individual point is, for itself, something actual, then neither can we hold the collection of all moments, i.e., the whole infinite time, nor the collection of all points, i.e., the whole infinite space, to be something actual. And thus also neither of these two representations [of time and space] can be called intuitions, even though both of them have only one single object (since there is only one infinite time and only one absolutely infinite space). (*WL* §79.3, I.364)¹⁶

Since they are not intuitions, Bolzano concludes that these representations must be *concepts* (cf. *NAK* 58f.).¹⁷ What is more, Bolzano thinks

¹⁶ See *NAK* 59f. for a review of all of these arguments for the ideality of space.

¹⁷ Bolzano adds the following further argument for the representation of space as a whole being a concept rather than an intuition, which picks up on the Leibnizian points made above: '[J]ust like *Leibniz*, I find a proof that space is not actual, and that its representation is therefore not an intuition, in the fact that we cannot determine a single point in space through mere concepts, since they are all completely equal to each other' (*WL* §79.6 Anm, I.375).

that this is a conclusion Kant himself can and should accept. To be sure, Bolzano recognizes that at times Kant says things that suggest he thinks that only intuitions can be ‘singular’ representations, in the sense of representing individual objects (cf. B376–377). Bolzano also thinks, however, that this cannot be Kant’s considered view, because Kant accepts (cf. B596, B603) that our representation of God is both a representation of an individual and (obviously) not an intuition (cf. NAK 65).

§5.4 Spatial representations as conceptual ‘determinations’ of actual objects

Because it represents something non-actual, Bolzano concludes that the basic representation of space must be a concept, rather than an intuition. Even so, Bolzano still retains a key part of the spirit of Kant’s account of the nature of space itself, insofar as he accepts that space is ideal rather than actual. This raises a question about whether Bolzano is also an idealist about space in the further sense that he thinks that space has no existence (or rather, subsistence) independently of the representations of it.

As I will now show, Bolzano does in fact seem to accept that the distinctive role spatial representations play within our representations of external objects is one that contrasts in a very important way with the role played by other conceptual representations, and that it contrasts in a way that does seem to share a core affinity with Kant’s thesis of the dependence of space upon spatial representation.

Bolzano takes the class of conceptual representations of objects to be one which includes both representations of actual ‘*properties* [*Beschaffenheiten*]’ of objects, but also other representations that function to direct our representations to objects but do not themselves represent properties of these objects. To count as a representation of an actual property of an object, Bolzano holds that the conceptual representation must be able to occur as the *predicate*-representation in a proposition:

The representation that appears in the place of b [in: A has b] (the predicate-representation) must, if the proposition is to be true, always be a genuine [*echte*] representation of a property; and conversely, every genuine representation of a property must be able to function as the predicate-representation in a true proposition. (WL §80.2, I.380)

Other conceptual representations, however, help to ‘determine [*bestimmen*]’ which object the proposition is about (i.e. the object of

the subject-representation) in a fundamentally different way – namely, by qualifying only the *subject*-representation in ways that do not ultimately track any real or actual properties of the object itself, and, in fact, by being representations that can never function as predicate-representations:

[T]here are representations that serve for the determination [Bestimmung] of an object without being properties of it. These representations have the peculiarity that they can never occur in the place of the predicate-representation (b) but only as parts of the subject-representation (A) itself. (WL §80.2, I.380–381)¹⁸

For our purposes, what is relevant about this distinction is that, immediately after introducing it, Bolzano goes on to state explicitly that the representations of space (and time) belong to the class of mere determinations *rather than* representations of properties:

Of this sort [i.e., mere determinations] are especially the spatial and temporal determinations of existing things. For the time in which an actual thing is to be found, during which a certain property can with truth be attributed to it, *is not a property of this thing*, and, for this reason, the representation of this time does not occur in the predicate-, but in the subject-representation of the proposition. The same thing holds also of the determinations of the places of things. (WL §80.2, I.381; my italics)

When viewed in light of Bolzano's commitment to the ideality of space (and time), it becomes clear why Bolzano wants to distinguish what function spatial representations have in a proposition about actual things from the function of other conceptual representations. Since

¹⁸ Bolzano actually uses the term 'determination' to pick out the 'broader concept' which includes both the representation of a property of the object and 'mere' determinations: though all representations of properties also determine the objects of the subject-proposition (when the proposition is true), not all determinations of objects are representations of properties of the object (WL §80.2, I.380); some are mere determinations. For some scepticism about whether Bolzano is consistent throughout all of his works in his usage of 'determination' – see Schnieder 2009, 58f.; cf. Morscher 1973, 73f. I am assuming here only that he is consistent across these few sections of the WL (§§79–80), and that the same usage is in play in the NAK. For scepticism about whether the resulting view of temporal representations is consistent, see George 1987, 454f.

these representations do not represent anything actual, they cannot represent actual properties of the object in question.

Nevertheless, Bolzano does accept that using spatial (and temporal) determinations in the subject-representation is a necessary condition for counting as true the 'ascription' of certain predicates to the relevant actual object – i.e. a condition for asserting, thinking truly about actual objects. Here is Bolzano making this point about temporal determinations:

[W]e place everything that is actual, perhaps with the exception of the single being of divinity, in a certain time; and *if we want to ascribe [beilegen] a property with truth of something actual, then we must always add a certain time at which this property is supposed to pertain to it.* (WL §79.5, I.364–365; my italics)

If we examine the matter more closely, it becomes apparent, as I believe, that by the word *time* we think nothing other than that particular determination [*Bestimmung*] of something actual which is the condition which must take place *so that we can ascribe a certain property in truth.* (WL §79.5, I.365; my italics; cf. NAK 57)

And here is Bolzano making the same point concerning space:

As concerns the concept of space, it will be admitted first of all that by space in general we represent nothing other than the collection of all possible locations, and so the only question is what we think of as the locations of things.... [L]ocations of (actual) things are those determinations [*Bestimmungen*] of these things that we must think in addition to their forces in order to comprehend [*begreifen*] the changes that they bring about in one another. (WL §79.6, I.365–366; cf. NAK 58)

What we find, then, is Bolzano ascribing a hybrid nature to spatial representation, one that brings them much closer to the role they have in Kant's system. Despite not being sensible representations (intuitions), Bolzano's spatial representations are nevertheless like Kant's in that they serve only to help pick out the subject of judgments without actually representing features that that object has 'in itself'. In this sense, for Bolzano as well, the representations of space and spatial relations have only an 'internal' role to play, laterally, within the context of other representations, rather than serving to represent anything in the actual

world as it is in itself. In this further sense, too, Bolzano would seem therefore to be an idealist about spatial representation.¹⁹

§6 Conclusion: exploring replies on Kant's behalf

We have thus seen, first, that Bolzano means for his doctrine of *outer intuition* to agree with much of Kant's official published account of intuitions. Like Kant, Bolzano, too, takes intuitions to be singular representations that give their objects immediately, and are related to external objects that we represent through spatial representations. We have seen, second, that Bolzano joins Kant in taking *spatial representation* itself not to track any actual properties in the outer objects themselves.

Against Kant, however, Bolzano thinks all spatial representation is *conceptual*, because all intuitions have *actual* objects, and space is not something actual. Furthermore, Bolzano holds, against Kant, that all outer intuitions are *non-spatial*, since they have entirely *simple* contents, because they immediately represent inner sensations as objects, rather than being representations that 'contain' an infinitely divisible 'manifold' within themselves. All of these differences lead directly to Bolzano's rejection of Kant's account of the nature and structure of outer intuition, as well as the grounds for Kant's postulation of an allegedly pure (*a priori*) intuition of space itself, since Bolzano takes himself to have shown that the representation of space is actually *not* a condition for, or a constituent in, outer intuition (cf. §2.1).

Nevertheless, we also saw that these departures from Kant's views are departures that Bolzano takes to be required by commitments that he and Kant both share. What we should turn to now, in conclusion, is what Kant might say in response to such challenges, especially Bolzano's claim that they arise from within the Kantian framework itself.

§6.1 Kant on unity without synthesis

A key step in Bolzano's argument against intuitions containing complexity was his thesis that no representation which has a content that is composite could be an immediate representation, because its arising would depend on a further intellectual act of mind which would synthesize or combine the relevant manifold of parts into a whole. What is more, Bolzano takes Kant to likewise hold that all combination

¹⁹ The possibility of this deeper parallel with Kant is touched upon by Palagyi 1902, 110f.; cf. as well Benoist 2003, 147–148.

or synthesis is the result of a further act of our understanding, and so is not present in what is given. And as we saw above, Bolzano is able to point to several texts which push in this direction.

What is not captured in Bolzano's reporting of Kant's views, however, is the fact that Kant appears to posit a *separate* kind of unity or belonging-together that he explicitly distinguishes from the kind of combination or synthesis that he assigns to the activity of understanding. This is the unity that accrues to the sensible manifold prior to any synthetic or combinatory activity of the understanding, a unity that consists in what Kant calls the '*synopsis of the manifold a priori through sense*' (B127; my italics). Though this synopsis accords a unity to intuition that 'corresponds to' the one which results from synthesis (combination) by the understanding (A97), it is present *prior to* such activity, as a kind of primitive and 'absolute unity' that every intuition has in the 'moment' it arises (A99). It is a primitive kind of seeing altogether (*syn-opsis*), all at once, though not in the way that the mind 'runs through and takes together' the various parts of this manifold and so becomes conscious of them as different (A99).²⁰

If this is right, then Kant would simply seem to reject Bolzano's claim that every representation that contains parts (is composite) requires a separate act of understanding which unifies the relevant manifold of parts by means of concepts. With the synopsis of the sensible manifold, we have a mental content which contains parts (a manifold) and yet that arises immediately, in each moment, without any act of combination of synthesis by our understanding.

§6.2 Kant on phenomenal presence without actual objects

It is even more clear that Kant cannot accept Bolzano's condition that all intuitions must have *actual* things as their objects. Indeed, this condition is so straightforwardly opposed to the possibility of the intuition of ideal things, it is no wonder that Bolzano cannot find a place where Kant states it explicitly. In fact, Bolzano himself (and Příhonský too) cites a place where Kant explicitly states the opposite view – namely, in Kant's initial exposition of the notion of a pure intuition in the Aesthetic itself: '[P]ure intuition ... occurs *a priori*, even without an actual [*wirklich*] object of the senses or sensation' (B35; my italics). What is especially bizarre is that, just one page after quoting the above passage from B35 (NAK 47), we are told that 'by *intuitions*, Kant has always

²⁰ For more discussion, see Tolley 2013; cf. Allison 2004, 14–15.

thought of just those representations that refer to *one single* and indeed *actual object*' (NAK 48).

Nor is this the only place where Kant makes such a claim. In fact, in *Prolegomena* §8, *Kant himself* takes up this worry about the possibility of an intuition without an actual object as a possible objection:

How is it possible to intuit something *a priori*? An intuition is a representation of the sort which would depend immediately on the presence [*Gegenwart*] of an object. It therefore seems impossible originally to intuit *a priori*, since then the intuition would have to occur without an object being present, either previously or now, to which it could refer, and so it could not be an intuition. (4: 281–282)

To this Kant gives the same reply in the next section (*Prolegomena* §9) that he gives in the Aesthetic:

There is therefore only one way possible for my intuition to precede the actuality [*Wirklichkeit*] of the object and occur as an *a priori* cognition, namely if it contains nothing else except the form of sensibility, which in me as subject precedes all actual [*wirkliche*] impressions through which I am affected by objects. (4: 282)

§6.3 Kant on the phenomenology of our sensible representations of external objects

If this serves as a textual defence for Kant to be able to escape charges of incoherence, what should we say about the competing positions themselves? Which of the two accounts fits better with the phenomenology of our sensible representations of external objects? On Kant's account, we are immediately and sensibly aware of contents that have spatial dimensionality (extension) and are composed of a manifold of sensory qualities like colour, texture, and so on. These are the appearances of objects external to us. To this extent, the content of an outer intuition (an outer appearance) is, for Kant, something like a (partial) image of the 'something = x' which has brought about its intuiting.²¹

On Bolzano's account, by contrast, we are immediately and sensibly aware only of single sensations of colour, etc., through simple and

²¹ Cf. George 2003, 26. We have to qualify the sense in which an appearance is an image, for Kant, due to Kant's own usage of the term 'image [*Bild*]' to pick out something formed out of appearances by the 'imagination [*Einbildungskraft*]'; cf. A120.

direct contents. They are not themselves image-like depictions, because strictly speaking the contents of outer intuitions have no form whatsoever, but are absolutely simple. Any consciousness of a composite of these contents, in an ostensibly spatial array, owes its spatial aspects not to something sensible but to concepts, with the resulting spatial array itself also not being something sensible in the strict sense but a unity of contents woven together conceptually or intellectually.

This also helps point up the difference in Bolzano's position on the representation-internal role that spatial representations are to play in our representations of outer objects. This is not a role that is played *within* the direct immediate sensible representations that are associated with external objects – i.e. *within* outer intuitions. For the representations in question are subject-concepts, and moreover seem to occur only within propositions, as contents of judgments (inferences).

What, then, is the truth about our most immediate sensible representations? Studies of so-called 'early vision', for example, suggest that it at least takes the form of a two-dimensional extended array, bracketing whether or not (as Berkeley had suggested many centuries prior) the representation of depth only comes at a later stage and incorporates conceptual representations, judgment, inference, and so on.²² This might be taken as a first bit of partial evidence that our most primitive external sensible awareness is spatial.

There is a further question, however, of whether Bolzano's position is itself internally coherent. For even Bolzano's own account of the genesis of our concepts of space would seem to rely on *some* acceptance of the idea that intuitions have a common form in which certain material can vary according to different magnitudes. The variable magnitude that is most important for Bolzano pertains to the quality of the intuition, i.e. its 'liveliness': 'We admit that every *subjective intuition*, indeed every representation in general, has a magnitude in respect to its *duration* as well as its *liveliness*' (NAK 95). It is precisely from noticing the variations in this quality that Bolzano thinks we first form the concepts of something being present, absent, closer, and farther from our sense-organs, and so is that upon which he bases his own genetic account of the formation of our basic concepts of spatial representation (cf. *WL* §303.21, III.151f.). But for this to be true, Bolzano would seem to have

²² See Grush 2007. See George 1987, 464f. for worries about Bolzano's parallel account of the formation of the representations of time by way of inference or judgment rather than by way of a primitive aesthetic awareness, as Kant suggests.

to posit something as a background against which such variation can be noticed – i.e. something quite close to a common form in which such variation in intensity occurs.

Whether there is any ultimate or inescapable inconsistency here would require further research into Bolzano's genetic account. And even if Bolzano's account were shown to be inconsistent, this would not by itself be a vindication of Kant's claim that there is a primitive, *sensible*, 'given' space of appearances. Even less would it speak for the plausibility of the remainder of Kant's views on space, especially those concerning our ability to immediately, sensibly, and yet 'purely' represent space *per se*, *a priori*, or those concerning the necessary role that this intuition is supposed to have to play within geometry.

Still, seeing what sort of picture emerges of ordinary outer intuitions, once pure intuition is rejected, might be enough to give one pause and explore once again a more Kantian alternative.

* * *

In the foregoing I have aimed to bring into focus several dimensions of Bolzano's views on outer intuition that have not yet received sufficient attention, with the goal of coming to a better understanding of his own accounts of both spatial and sensible representations. What is more, the path we have followed gives us a new angle into the debate about the nature and origin of spatial representation, one that proceeds largely independently of Bolzano's views on the nature of geometrical knowledge – and, for that matter, independently of those of Kant's as well. It also allows us to see the extent to which the commitment to idealism about space (something Bolzano and Kant share) can swing free not just from one's commitments about geometrical knowledge, but also from one's views on the nature of outer intuition as well.

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5

Kant, Bolzano, and the Formality of Logic

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§1 Introduction

In §12 of his 1837 *magnum opus*, the *Wissenschaftslehre*, Bolzano remarks that “In the new logic textbooks one reads almost constantly that ‘in logic one must consider not the material of thought but the mere form of thought, for which reason logic deserves the title of a purely formal science’” (WL §12, 46).¹ The sentence Bolzano quotes is his own summary of others’ philosophical views; he goes on to cite Jakob, Hoffbauer, Metz, and Krug as examples of thinkers who held that logic abstracts from the matter of thought and considers only its form. Although Bolzano does not mention Kant by name here, Kant does of course hold that “pure general logic”, what Bolzano would consider logic in the traditional sense (the theory of propositions, representations, inferences, etc.), is formal. As Kant remarks in the Introduction to the 2nd edition of *Kritik der reinen Vernunft*, (pure general) logic is “justified in abstracting – is indeed obliged to abstract – from all objects of cognition and all of their differences; and in logic, therefore, the understanding has to do with nothing further than itself and its own form” (*KrV*, Bix).²

In recent work, both John MacFarlane and Sandra Lapointe have argued that this ‘formality thesis’ is original to Kant; according to them, no one in the pre-Kantian, Leibnizian logical tradition held that logic

¹ References to the *Wissenschaftslehre* (WL) are to Bolzano (1837); it is cited by section number and page.

² Cf. A55/B79, A56/B80, A70/B95, A131/B170. See the end of the essay for a list of abbreviations of Kant’s work, and editions and translations used. While I have mainly used the Cambridge translations, I have occasionally modified them in small ways.

is about the form of thinking.³ As MacFarlane points out, the claim that logic is formal is now so widespread that it is often simply asserted without argument. So in criticizing the formality thesis in these post-Kantian figures (whom Lapointe aptly dubs ‘Kantian logicians’) Bolzano is really targeting one of Kant’s most influential ideas in the philosophy of logic.

Bolzano raises two distinct objections to the formality thesis. First, he objects that the formality thesis, as formulated at least by Kant and the Kantian logicians, assumes that logic is about thoughts [*Gedanken*],⁴ which Bolzano takes to be, roughly, mental states with propositional content. Logic is not about thoughts so conceived, Bolzano argues throughout the *WL*, or anything else psychological, but about ‘propositions in themselves’ [*Sätze an sich*], abstract bearers of truth-values that stand in entailment relations, and ‘representations in themselves’ [*Vorstellungen an sich*], the mind-independent concepts that constitute propositions ‘*an sich*’. But this objection arises from Bolzano’s very general opposition to psychologism; it is not specifically a problem for the formality thesis. For the defender of the formality thesis could, in principle, claim that logic concerns propositions ‘*an sich*’ in respect of their form.

I want to focus, instead, on Bolzano’s second objection, which is specifically about the notion of ‘form’ in the formality thesis. Bolzano begins by pointing out that “this explanation [that logic is about the form of thought] appears to be, in part, insufficiently clear and, in part insufficiently determinate” and proceeds to explain one clear sense in which logic might be said to concern the *forms* of propositions, which Bolzano finds relatively unproblematic. Logic concerns not individual propositions, except perhaps as examples, but whole sets of propositions that share structural (formal) features. For instance, in logic we regard <All men are mortal> and <All dogs are mammals> as having the same form because they are instances of the same scheme: <All As are B>.⁵ Two propositions *p* and *q* are said to have the same form if by consistent substitution of terms in *p* we can obtain *q*. This, Bolzano remarks, is a

³ MacFarlane (2002) and Lapointe (2012).

⁴ “All of the scholars who have made the claims quoted above tacitly assume that the totality of things that constitute the subject-matter of logic fall under the concept of *thought*, i.e. that all of them, whatever else they might be, must be *thoughts*” (*WL* §12, 47).

⁵ Though Bolzano would not take this to be their form, for, on his view, the fundamental form of the proposition is <A has b>.

perfectly coherent notion of form, and in this sense logic can correctly be said to concern the *forms* of propositions.

If ‘formality’ in the formality thesis means something other than this, Bolzano claims, then Kant and the Kantian logicians have failed to explain what it is, to clearly demarcate the forms of propositions from their ‘matter’, or to explain why logic should be restricted to the formal study of propositions, in this other, specifically Kantian sense of ‘form’.⁶

The idea that logic is formal has proved very durable, and, furthermore, Bolzano’s ‘substitutional’ conception of the formality of logic has entered the mainstream of philosophical thought. Alfred Tarski’s definition of the formality of logic in “The Concept of Logical Consequence” (1936) in particular is similar to Bolzano’s, as Lapointe (2011) points out.⁷ This raises an important question: was Bolzano right about Kant’s version of the formality thesis? Is there more to Kant’s formality thesis than what Bolzano’s substitutional conception of formality allows, and, if so, can this additional Kantian conception of formality be made clear (contra Bolzano) and defensible?

In this essay I am going to argue that the answer to both questions is ‘yes’. There is more to the formality of logic in Kant’s sense than Bolzano’s ‘substitutional’ understanding of formality, and this Kantian conception of formality can be rendered reasonably clear and defensible. In Section §2 I explain Bolzano’s substitutional understanding of the formality of logic. In Section §3 I argue that Kant does think that logic is formal in Bolzano’s substitutional sense. However, in Section §4 I argue that, on Kant’s view, the fact that logic is substitution-formal is a *consequence* of the fact that logic is formal in Kant’s sense of ‘formal’; it is not what the formality of logic, for Kant, consists in. The formality of logic for Kant is closely tied up with the matter/form distinction and I distinguish several different classes of logical items to which Kant applies that distinction: concepts, judgments, and inferences [*Schlüsse*] are all said to have a matter and a form. I argue that close attention to Kant’s texts shows that the formality of logic ultimately consists in the fact that logic concerns the *form* of the capacity for thought itself, not merely the *form* of its products (concepts, judgments, inferences). The form of the capacity explains the form of its products. That logic is about the

⁶ Bolzano makes a similar objection to Kant’s theory of analyticity in terms of conceptual ‘containment’ – that it is a metaphor that Kant has not cashed out in literal terms. See *WL* §148, 87.

⁷ However, others have argued that Tarski’s conception is quite different than Bolzano’s. See Siebel (1996) and (2002); and Rusnock and Burke (2010).

form of thought explains why logic is only concerned with the forms of propositions, hence, why logic is substitution-formal in Bolzano's sense. In Section §5 I explore what Kant could mean by talking of the 'form' of a faculty (a capacity) and I propose that the form of a faculty is what Aristotle would call the 'formal cause' of that faculty: it is that which makes the faculty the faculty that it is. In claiming that logic concerns the form of understanding, Kant is claiming that logic discovers laws that apply to the faculty of understanding *in virtue of being the faculty that it is*. I then give a Kantian characterization of what it is to be the capacity for thought, and why this entails that certain laws (the laws of logic) apply to the activity of that capacity. I conclude by arguing that this explains why the products of this capacity (in particular, inferences) have the features that Bolzano identifies as their formality: validity under substitution.

§2 The substitutional account of formality: Bolzano

In *WL* §12 Bolzano explains the sense in which logic can correctly be said to be formal, which I have called the 'substitutional' conception of the formality of logic:

For these reasons logic (at least in its doctrines – it can be otherwise in the examples) never considers a fully determinate proposition [*Satz*], i.e. one in which the subject, predicate, and copula are fully specified, but, rather, a whole class [*Gattung*] of propositions, i.e. all propositions collectively, which, if some of their components are completely fixed, the remainder can be read in this or that way. [...] If one calls such classes of propositions general *forms* of propositions [...] then one can say that logic concerns only the forms of propositions, never individual propositions. If one calls that which is left indeterminate in such a class of propositions, such as **A** and **B** in the previous example, the *matter* of these propositions [...] then one can also say that logic concerns only the form, not the matter, of propositions. (*WL* §12, 48)

Although he does not directly state the definition of the form of a proposition, it is not difficult to reconstruct such a definition that would be acceptable on Bolzano's lights. I will proceed in two steps: first I will define what it is for two propositions to be formally equivalent, then I will define what the form of a proposition is. At each stage I follow Bolzano's characteristic procedure of defining the notion (being formally equivalent, having this or that logical form) relative to a designated set of terms (i, j, k...) and then defining the absolute notion.

Propositions, for Bolzano, are structured complexes of representations. To define formal equivalence of propositions we need, first, to define what is a ‘substitution’. Where p is a proposition and i, j, k, \dots is an ordered sequence of representations in p , a substitution for i, j, k, \dots in p is a function f from that sequence to a sequence, of the same length, of representations of the same syntactic type, i^*, j^*, k^*, \dots . The substitution of f in p refers to the proposition that is obtained from p by replacing i with i^* , j with j^* , k with k^* , etc. Two propositions p and q are said to be formally equivalent with respect to representations i, j, k, \dots where i, j, k, \dots are representations in p , if and only if there is a substitution f that, when applied to p , produces q . For instance, the two propositions

- (1) <Men have ears>
- (2) <Cows have ears>

are formally identical with respect to <men> because (2) can be obtained from (1) by substituting <cows> for <men>. Proposition (1) is formally equivalent to

- (3) <Dogs have fleas>

with respect to <men> and <ears>, because (3) can be obtained from (1) by substituting concepts for concepts: <dogs> for <men> and <fleas> for <ears>. We can represent all three propositions (1)–(3) as substitution instances of the abstract schema:

- (4) <A have b>

We will say that two propositions p and q are absolutely formally equivalent in Bolzano’s sense if there is a substitution f of all of the non-logical terms in p that, when applied to p , produces q .⁸ Propositions (1)–(3) are absolutely formally equivalent.

⁸ As Bolzano himself points out later in the *WL*, this definition relies on being able to distinguish the logical terms from the non-logical terms. We need to restrict the range of terms that can be substituted to ‘non-logical’ or ‘non-formal’ features, or else we will obtain the absurd result: <P & Q> has the same form as <P → Q> because we can obtain the latter from the former by substituting → for &. The plausibility of this definition of logical form depends upon drawing a principled distinction between the ‘formal’ (or logical) terms in propositions and their ‘material’ (or non-logical) terms, a problem philosophers of logic have discussed down to the present day. Cf. Woods (forthcoming) for further discussion.

Having defined a notion of formal equivalence, we can now define the logical form of a proposition as the class of all propositions with which it is formally equivalent. The form of a proposition p with respect to its terms i, j, k, \dots is the class of all propositions that are formally equivalent to p with respect to i, j, k, \dots . We can adopt the following notational convention: the form of a proposition p with respect to i, j, k, \dots will be indicated by replacing i, j, k, \dots with variables. Any substitution instance of that scheme will be a proposition that is formally equivalent to p with respect to those terms. For instance, the form of (1) with respect to $\langle \text{men} \rangle$ can be represented as

(5) $\langle A \text{ have ears} \rangle$

Similarly, we can define the absolute logical form of a proposition p as the class of all propositions that are absolutely formally equivalent to p .⁹

When Bolzano claims in §12 of the *WL* that logic is formal he means that logic concerns itself with forms of propositions in this precise sense. Logic does not study individual propositions, but whole classes of propositions that share a logical form. Logic studies classes of propositions that share a logical form because propositions have many of their most important logical properties in virtue of their logical form. Some caution is required here. It would be misleading to say that, on Bolzano's view, *all* of logic is formal in this substitutional sense. Bolzano uses 'logic' in both a wide and a narrow sense. In the wide sense, 'logic' refers to the philosophical discipline of '*Wissenschaftslehre*' itself (*WL* §6, 22), the general theory of any scientific knowledge whatsoever. For instance, logic-as-*Wissenschaftslehre* includes in its field of study the distinction between *a priori* and *a posteriori* knowledge, and how the former is possible. Since the property of being *a priori* knowable is clearly not

⁹ Trivially, absolute formal equivalence is reflexive (since every proposition is equivalent to itself under the trivial substitution that maps every representation to itself). Likewise, if there is a substitution f that takes p to q , then there is a substitution that takes q to p , so absolute formal equivalence is symmetric. Finally, if there is a substitution that takes p to q , and one that takes q to r , there is a substitution that takes p to r , so absolute formal equivalence is transitive. Consequently, the absolute logical form of a proposition is its equivalence class under the relation of absolute formal equivalence. It follows that two propositions p and q are absolutely formally equivalent if and only if they have the same absolute logical form. The logical form of propositions (1)–(3) can be represented as (4).

shared by propositions in virtue of their common logical form, Bolzano cannot mean that logic in the broader sense is formal. However, Bolzano also has a narrower conception of logic on which logic is the science of the relations between propositions, representations, inferences, etc., the domain of logic as traditionally conceived. On this narrower conception, ‘logic’ is roughly equivalent to what Kant would call ‘pure general logic’, although of course they have quite different conceptions of what that logic is and how it should be done. Henceforth, when I talk about ‘logic’ in Bolzano I mean logic in the narrow sense.

We can now formulate the Bolzano substitutional formality thesis (SFT) more precisely:

- (SFT) Property F is a proper object of study for logic if and only if propositions have property F in virtue of their logical form.

One corollary of this claim is the following:

- (C) Property F is a proper object of study for logic if and only if, for any propositions p and q that share the same form (they are formally equivalent to one another), p has property F if and only if q has property F.

I will illustrate this point in the case of Bolzano’s theory of logical ‘*Ableitbarkeit*’ because this corresponds most closely to what has come to be seen as a (if not the) central concept in logic, logical consequence, and will facilitate the comparison with Kant’s theory of logical inference.¹⁰

Since Bolzano’s own definition of logical *Ableitbarkeit* is formulated in his own, not always very natural, technical notation, I will quote, with my own slight modifications, Sandra Lapointe’s gloss on this definition in *Bolzano’s Theoretical Philosophy* (2011):

Let $P = \{p_1, p_2, p_3, \dots\}$ and $Q = \{q_1, q_2, q_3, \dots\}$ be sets of propositions. The Q propositions are logically *ableitbar* from the P propositions if and only if:

- (i) There is a substitution for the non-logical terms in P and Q that makes all of the propositions in P and all the propositions in Q true (P and Q are said to be ‘compatible’).

¹⁰ This discussion of *Ableitbarkeit* is heavily indebted to the discussion in Lapointe (2011), Chapter 6.

- (ii) For every substitution of the non-logical terms in P and Q , if all of the propositions in P are true under this substitution, then all of the propositions in Q are true under this substitution. (Lapointe 2011, 73–74; WL §155)¹¹

Since I will only be discussing Bolzano's notion of 'logical *Ableitbarkeit*' (rather than the more general notion), I will drop the modifier and refer to this as *Ableitbarkeit simpliciter*. Let P and Q be as in the definition and let $P^* = \{p^*_1, p^*_2, \dots\}$ be a set of propositions, where each p_n is absolutely formally equivalent to p^*_n . A short argument proves that Q is also logically *ableitbar* from P^* . By clause (i) of the definition of logical *Ableitbarkeit*, there is at least one way of substituting for the non-logical terminology in the P propositions and the Q propositions that make all the propositions in both sets come out true. But for every substitution of the non-logical vocabulary in p_n there is a corresponding substitution of the non-logical vocabulary in p^*_n such that these substitutions will produce the same proposition; they will not in general be the same *substitution*, for p_n and p^*_n will not have the same non-logical vocabulary. So there is at least one substitution of the non-logical terminology in the P^* propositions and the Q propositions that make all the propositions in both sets come out true. In Bolzano's terminology, these two sets of propositions are compatible. This satisfies clause (i) of the definition of the logical *Ableitbarkeit* of Q from P^* . Next, we will prove that clause (ii) obtains as well (for P^* and Q). Each p_n is absolutely formally equivalent to the corresponding p^*_n . By definition, this means that for every substitution of the non-logical terms in p^*_n , there is a corresponding substitution of non-logical terms in p_n such that these two substitutions will produce the same proposition. So for any substitution of non-logical terms that makes all of P^* true, there is a corresponding substitution of non-logical terms that makes all of P true. By assumption, for any substitution of non-logical terms in P and Q , if all of the P propositions are made true, all of the Q propositions are made true. So, for any substitution of the non-logical vocabulary in P^* and Q , if all of the P^* propositions are true, all of the Q propositions are true. So the set of propositions Q is *ableitbar* from the set of propositions P^* . Similar reasoning will show that if P is *ableitbar* from Q then P^* is *ableitbar* from

¹¹ I have modified Lapointe's presentation slightly; rather than first define *Ableitbarkeit* relative to a set of terms, then define the absolute notion of *Ableitbarkeit*, I have defined the absolute notion of *Ableitbarkeit* from the beginning.

Q as well. In other words, relations of *Ableitbarkeit* are shared by sets of propositions that have the same logical form. Or, to put it another way, relations of *Ableitbarkeit* are formal features of sets of propositions. Given the substitution-formality thesis (SFT) from above, this explains why *Ableitbarkeit* is a proper object of study for logic.

§3 The substitutional account of formality: Kant

In this section I will argue that, on this point, Kant agrees with Bolzano: Kantian pure general logic is formal in Bolzano's substitutional sense. Kant accepts the SFT from above. This is not the only coherent sense in which logic can be said to be formal, according to Kant, but my argument for that must wait until the next section. Since I have focused on the formality of *Ableitbarkeit* in Bolzano's logic, I will focus on the closest corresponding notion in Kant's logic, his theory of logical inferences [*Schlüsse*]. First, though, we must explore Kant's conception of the logical form of the judgments that figure in such inferences.

Kantian judgments are unified wholes composed of concepts. The structure or unifying principle of a judgment is called its logical form. The table of logical functions in judging characterizes the logical forms of judgments according to four 'moments': quantity, quality, relation, and modality. Under each 'moment' of the table of logical functions there are three types of judgment:

	<u>1. Quantity</u>	
	Universal	
	Particular	
	Singular	
<u>2. Quality</u>		<u>3. Relation</u>
Affirmative		Categorical
Negative		Hypothetical
Infinite		Disjunctive
	<u>4. Modality</u>	
	Problematic	
	Assertoric	
	Apodictic ¹²	

Every judgment has a quantity, a quality, a relation, and a modality. This means that every judgment is either universal, particular, or

¹² *KrV* A70/B95.

singular in quantity; affirmative, negative, or infinite in quality, etc. For reasons I do not have the space to explore here, Kant claims that within pure general logic we cannot distinguish between the third judgmental form and the first two in the moments of quantity and quality.¹³ This means that, for the purposes of pure general logic, the Kantian logic with which we are concerned here, we can ignore singular and infinite judgments. Likewise, the modality of a judgment, according to Kant, does not concern the form of the judgment itself, but the relation of that judgment to the mind;¹⁴ consequently, the modality of judgments does not affect their role in syllogistic inference, and I will forgo further discussion of it here.

The logical form of a judgment is uniquely characterized by its logical quality, quantity, relation, and modality. Two judgments have the same form if and only if, through consistent substitution of concepts, one can be transformed into the other. For instance, consider the two judgments:

- (1) <Some Greeks are Athenian>
- (2) <Some dogs are Alsations>

Through substituting <dogs> for <Greeks> and <Alsation> for <Athenian>, (1) can be transformed into (2); (1) and (2) have the same logical form: they are particular (in quantity), affirmative (in quality) categorical (in relation) judgments (ignoring for the moment their modality). If we refer to the concepts of which these judgments as composed as their matter, then we can say: judgments with the same form can differ in matter, or, equivalently, a judgment is a complex whole of some matter structured according to one of the possible logical forms.

Kant claims that in the moments of quantity and quality the third type of judgment cannot be distinguished from the first type by pure general logic (for Bolzano, 'logic' in the narrower sense), but only by 'transcendental logic', the philosophical discipline that "concerns the origin of our cognitions of objects insofar as that [cognition] cannot be ascribed to the objects" (A55–56/B80). In this essay, I am concerned exclusively with Kant's pure general logic, so I will ignore the positive claim that transcendental logic can distinguish the third logical

¹³ *KrV* A71-72/B96-97. See Stang (2012) for further discussion of why infinite judgment cannot be distinguished within pure general logic.

¹⁴ *KrV* A74/B99-100.

function of quantity and of quality and the difficulties surrounding the notion of ‘transcendental logic’¹⁵ but I will briefly comment on what it means that, within pure general logic, we cannot distinguish universal judgments from singular (quantity), or affirmative judgments from infinite (quality). Very roughly, Kant’s idea is that *with respect to inferential relations* singular judgments behave just like universal judgments (the moment of quantity) and infinite judgments behave just like affirmative judgments (the moment of quality). So from within pure general logic, the science tasked with studying the inferential relations among judgments, we cannot distinguish these two kinds of judgments. I will illustrate the point in the case of affirmative judgments and infinite judgments;¹⁶ the case of singular and universal judgments brings in additional complications.¹⁷

Consider two kinds of judgments that can be distinguished within general logic, affirmative and negative judgments, for example.

- (1) Socrates is Athenian. [Affirmative]
- (2) Socrates is not Athenian. [Negative]

In Kant’s logic, these judgments have, respectively, the logical forms

- (1*) A is B
- (2*) $\sim(A \text{ is } B)$

The negation in (2*) takes wide scope (over the whole judgment) because it applies to the copula;¹⁸ (2*) says that the predicative relationship asserted in (1*) fails to obtain. However, we might also negate, not the copula as in (1) itself, but the predicate, as in:

- (3) Socrates is non-Athenian [Infinite]

whose corresponding logical form is (3*):

- (3*) A is ($\sim B$)

Judgment (3) asserts that Socrates falls within the infinite sphere of everything outside the sphere of the concept <Athenian>. That pure

¹⁵ See, however, Stang (2012) for an explanation of why infinite judgments are distinct from affirmative judgments in transcendental logic.

¹⁶ The main discussion of this in the *KrV* is A72/B97.

¹⁷ See A71/B96.

¹⁸ Cf. Ak. 24:758.

general logic cannot distinguish (3*) means that, from the point of view of pure general logic, (3) is an affirmative judgment; it affirmatively predicates the concept <non-Athenian> of Socrates. Consequently, it has the inferential role, with respect to other judgments containing the concepts <Socrates> and <non-Athenian> that any affirmative judgment does. For instance, the following is a valid instance of the schema for categorical syllogisms:

Socrates is non-Athenian (3) [X is A]
 All non-Athenians are warlike. [All A are B]
 ∴ Socrates is warlike. [X is B]

However, (3) has a different inferential role from (1), because (1) and (3) are affirmative judgments with distinct and *unrelated* (from the point of view of general logic) predicates. Nor is the inferential role of (3) the same as the inferential role of (2), for (at least within pure general logic), substituting (2) for (3) in the syllogism above renders it invalid. The difference between (1) and (3) concerns not the form of the judgment *itself* but the form of the predicate, <~A> as opposed to <A>. Pure general logic cannot account for the difference between these predicates; to do so, we need to bring in considerations of how our *a priori* concepts obtain content (relation to an object), and this is a subject for transcendental logic.¹⁹

Kant’s theory of inference, following the Aristotelian tradition, is a theory of syllogistic inference. A syllogism contains two premises (a major and a minor) and a conclusion. He divides syllogisms into three kinds, based on the logical form of their major premise: categorical, hypothetical, and disjunctive. I will begin with the best developed part of Kant’s theory of inference, the theory of categorical syllogisms, which is encoded in this table:²⁰

MP	PM	MP	PM
SM	SM	MS	MS
SP	SP	SP	SP

¹⁹ This is Kant’s ‘official’ view about infinite judgments in pure general logic. However, there are reasons to worry that he is not completely consistent on this point. See below for more.

²⁰ *JL* §68, Ak. 9:125.

'S' stands for the subject-concept of the conclusion, 'P' for its predicate, and 'M' for its middle term. Each column represents a categorical syllogistic 'figure', that is, a kind of a logically valid categorical syllogism. In each figure, the top line represents the major premise, the middle the minor premise, and the line on the bottom, below the black line, represents the conclusion. In each judgment, the left-hand variable indicates the subject-concept, and the right-hand variable indicates the predicate concept. For instance, the third syllogistic figure, the third column from the left, indicates that syllogisms of this figure have the following form:

	<i>Subject</i>	<i>Predicate</i>
<i>Major premise:</i>	M	P
<i>Minor premise:</i>	M	S
<i>Conclusion:</i>	S	P

The logical quantity and quality of the premises and conclusions are not specified, though, because each syllogistic figure includes under it several sub-cases that fully specify the logical forms of its constituent judgments.

Kant analyses the validity of syllogisms of the other three figures by showing how they can be reduced to syllogisms of the first-figure by certain conversion rules. First, I will show that the validity of a categorical syllogism of the first-figure is substitution-formal in the sense defined earlier. Then I will show that the conversion rules that transform syllogisms of the other three figures are themselves substitution-formal. I will then argue that the validity of *all* categorical syllogisms is substitution-formal. In fact, it is a trivial (though laborious) exercise to enumerate every possible example of a valid categorical syllogism and manually check that they are all substitution-formal.

A syllogism is of the first-figure just in case (i) the concepts in its major, minor and conclusion can be obtained by substitution from the scheme in the above table, (ii) the major premise is universal (either negative or affirmative), and (ii) the minor premise is affirmative (either universal or particular). This gives us four possible sub-cases of the first-figure:

	<u>I.1</u>	<u>I.2</u>
<i>Major:</i>	All M are P	All M are P
<i>Minor:</i>	All S are M	Some S are M
<i>Conclusion:</i>	All S are P	Some S are P

	<u>I.3 (invalid)</u>	<u>I.4 (invalid)</u>
Major:	Not all M are P	Not all M are P
Minor:	All S are M	Some S are M
Conclusion:	Not all S are P	Not all S are P

Since I.3 and I.4 are clearly invalid, there are only two valid syllogistic forms of the first-figure: I.1 and I.2.

When we get to the other figures, things get slightly more complicated. First, we have to introduce a set of conversion rules, rules that allow us to convert premises in syllogisms into forms that allow the application of a syllogistic form of the first-figure. These conversion rules are:

1. No X is Y \leftrightarrow No Y is X
2. Some X is Y \leftrightarrow Some Y is X
3. All X are Y \leftrightarrow Some X are Y (\leftrightarrow Some Y are X)²¹

Kant describes rule (3) as the rule that universal affirmative judgments can be converted (predicate and subject switched) *per accidens*, which means that they can be converted *only* in one direction, to the corresponding logically weaker particular affirmative judgment. Rule (3) can be thought of as the logical product of the traditional Aristotelian rule of sub-alternation²² and rule (2), which converts particular affirmative judgments. The rule of sub-alternation has the effect of ruling out vacuous universal affirmative judgments. If the concept <A> has no objects in its extension, then no true affirmative judgments can be made with it as the subject-concept.

A syllogism is valid if and only if it can be transformed, via the conversion rules, into a categorical syllogism of one of the two kinds listed above (I.1 & I.2). Since the two kinds of valid first-figure categorical syllogisms are substitution-formal (I take it that this is obvious by inspection), and the conversion rules are substitution-formal (ditto), it follows that whether a syllogism is valid is substitution-formal. Although, I take it, the premises of this argument are obvious by inspection, it may be less obvious that the conclusion follows. Let S and S* be two syllogisms which are formally equivalent in the following sense: by consistent substitution of concepts, the premises and conclusion of S* can be transformed into those of S. Note, though, that logical forms of judgments

²¹ *JL* §53, Ak. 9:119.

²² Kant discusses sub-alternation at *JL* §46, Ak. 9:116.

are not concepts; otherwise, we could get the absurd result that <All A are B> has the same logical form as <Some A are B>. Assume that S is a valid syllogism. Then S is either already a syllogism of the first-figure, or it can be transformed into one by a series of application of conversion rules. If S is a first-figure syllogism, then so is S*, so S* is valid. If there is a series of applications of conversion rules that transforms S into a valid first-figure syllogism, then, the same series of applications of conversion rules will transform S* into a valid first-figure syllogism. So S* is a valid syllogism. I conclude that the property of being a valid categorical syllogism is substitution-formal.

It is even clearer that the property of being a valid hypothetical or disjunctive syllogism is substitution-formal, for Kant's theory of those syllogistic forms is simpler than in the case of the categorical. All hypothetical syllogisms come in one of two forms:

	<u>H.1</u>	<u>H.2</u>
<i>Major</i>	If p then q	If p then q
<i>Minor</i>	p	$\sim q$
<i>Conclusion</i>	q	$\sim p$

where p and q are any judgment whatsoever and ' $\sim p$ ' denotes the negative judgment in which the *copula* in p (not its predicate, as in infinite judgments – see above) is negated. Disjunctive syllogisms have one of two forms:²³

	<u>D.1</u>	<u>D.2</u>
<i>Major</i>	A is $(B_1 \vee B_2 \vee \dots \vee B_n)$	A is $(B_1 \vee B_2 \vee \dots \vee B_n)$
<i>Minor</i>	A is B_j	$\sim(A \text{ is } (B_1 \vee B_2 \vee \dots \vee B_{j-1} \vee B_{j+1} \vee \dots \vee B_n))$
<i>Conclusion</i>	$\sim(A \text{ is } (B_1 \vee B_2 \vee \dots \vee B_{j-1} \vee B_{j+1} \vee \dots \vee B_n))$	A is B_j

As is clear from these syllogistic forms, Kant thinks of disjunctive judgments in terms of what we would now call the 'exclusive or'. Since disjunctive judgments can be either universal or particular in quantity, we should distinguish two different forms of the major premise, and thus multiple different kinds of disjunctive syllogism. However, for the sake of simplicity, I will just assume that the major premise is universal

²³ *JL* §77.

and the minor is particular, and the conclusion retains the quantity of the major premise:

	<u>D.1U</u>	<u>D.2U</u>
<i>Major</i>	All As are $(B_1 \vee B_2 \vee \dots \vee B_n)$	All As are $(B_1 \vee B_2 \vee \dots \vee B_n)$
<i>Minor</i>	Some A is B_j	$\sim(\text{Some A is } (B_1 \vee B_2 \vee \dots \vee B_{j-1} \vee B_{j+1} \vee \dots \vee B_n))$
<i>Conclusion</i>	$\sim(\text{All A is } (B_1 \vee B_2 \vee \dots \vee B_{j-1} \vee B_{j+1} \vee \dots \vee B_n))$	All As are is B_j ²⁴

Once again, by inspection, it is clear that the property of being a valid hypothetical syllogism and the property of being a valid disjunctive syllogism are substitution-formal.

We could conclude, on Kant's behalf, that the property of being a valid syllogism is substitution-formal because being a valid syllogism means being either a valid categorical, hypothetical, or disjunctive syllogism. However, this would be too quick for two reasons. Although Kant does not make this explicit, he should also count as mixed inferences of reason those syllogisms in which one premise immediately entails a premise that, together with the other premise, entails the conclusion by one of the syllogistic figures. For instance, consider this syllogism:

All M are P
 If (some M are P) then (some R are Q)
 \therefore Some R are Q

Intuitively, this is a valid syllogism because the first premise immediately entails (according to rule 3 above) that some M are P, which, together with the second, constitutes a valid hypothetical syllogism. However, the original syllogism is not a valid syllogism of any of the officially recognized forms. So I propose the following expansion of our notion of a valid syllogism:

A pair of premises and a conclusion (P_1, P_2, C) are a valid syllogism if and only if $P_1, P_2,$ and C instantiate one of the valid syllogistic forms, or if P_1 or P_2 immediately entail one or more judgments that, together

²⁴ This is how I interpret Kant's remark that "we infer either (1.) from the truth of one member of the disjunction to the falsehood of the others, or (2.) from the falsehood of all members but one to the truth of this one" (*JL* §77, Ak. 9:130).

with C , instantiate such a form, or if P_1 , P_2 and C^* instantiate such a figure for some judgment C^* , where C^* immediately entails C .

But even taking this amendment on board, Kant has only given an account of entailment that holds between either a single premise and single conclusion (the immediate conversion rules from earlier, or “immediate inferences of reason”), or between a pair of premises and a single conclusion (syllogisms, “mediate inferences of reason”). For Kant’s logic to be adequate, it would need to account for general entailment relations among arbitrary (perhaps arbitrarily large finite) sets of premises and conclusions. Here is a sketch of how that might go:

Defn. The deductive closure of $P_1 \dots P_n$ is defined iteratively in stages:

- (i) At stage 0 we add $P_1 \dots P_n$.
- (ii) At stage $n + 1$ we add all immediate consequences of every judgment added at stage n or earlier, and all judgments C such that judgments A and B (where A and B are added in stage n or earlier) and (A, B, C) instantiate a valid syllogistic form.

Defn. $P_1 \dots P_n$ entail $C_1 \dots C_n$ if and only if the deductive closure of P_1 through P_n includes each of $C_1 \dots C_n$.

Therefore, despite the serious defects in Kant’s logic, his account of entailment is formal in precisely Bolzano’s sense: it is substitutional. Or, to put it as Bolzano does in §12 of *WL*, the part of pure general logic that studies inferences of reason does not study individual ordered triples of judgments (propositions), but sets of such ordered triples: sets of ordered triples of judgments that share a logical form, meaning they can be obtained through consistent substitution of concepts from some abstract schema. This does not show that *all* of Kantian pure general logic is substitutional; it shows, however, that a significant portion of that logic, the theory of inference, is substitutional, and this is the part of Kant’s logic that most closely corresponds to Bolzano’s theory of *Ableitbarkeit*, which, in the previous section, we saw to be substitution-formal as well.

§4 Form of the capacity versus form of the product of the capacity

Now I am going to argue that, while Kant does think that logic, paradigmatically the part of pure general logic that studies inference, is substitution-formal, there is a more basic sense in which logic is formal, and that more basic sense explains why logic is substitution-formal.

Understanding the precise meaning of Kant's formality thesis can seem a daunting task, given the sheer variety of ways in which Kant characterizes the formality of logic: logic is said to concern the form of thought, the form of reason, the form of the understanding, the formal laws of the understanding, as well as the more familiar claims that it concerns the form of concepts, the form of judgments, or the form of inferences. The different forms that are relevant to the formality of logic can be roughly separated into two classes: forms of faculties (e.g. the form of understanding) and forms of products or activations of those faculties (e.g. the form of concepts, judgments, and inferences).

As numerous scholars have pointed out, Kant applies the matter/form distinction across a wide swath of his philosophy; concepts, experience, the will, etc. are all said to have a form and a matter.²⁵ In some contexts, by distinguishing between the form and the matter of some item, Kant is claiming that the item in question (e.g. a judgment) is a complex structured whole and distinguishing within that whole between its parts (its matter) and the structure that obtains among those parts (its form), in virtue of which those parts compose that very item. To apply the matter/form distinction, in these paradigm cases, requires distinguishing a class of items called *matter* from a class of structural relations called *forms* such that, when instances of the appropriate kind of matter are structured by an appropriate kind of form, a complex entity of the relevant kind exists. For instance, to talk of the matter/form distinction with respect to judgments requires distinguishing *concepts*, the parts out of which judgments are made, from *logical forms*, the structures that when applied to these concepts produce complex wholes, *judgments*. A consequence of the matter/form distinction, when applied in paradigm cases like these, is that the very same matter could be structured by a different form, producing a different complex whole. For instance, the concepts <Greek> and <Athenian> could be structured in the judgment <All Athenians are Greek> or in the distinct judgment <Some Greeks are Athenian>. It is also a consequence of the matter/form distinction, when applied in paradigm cases like this, that the same form can produce distinct complex wholes by structuring different matter. For instance, the logical form of universal affirmative judgment structures the distinct judgments <All Athenians are Greek> and <All squares are equiangular>, because these judgments are made of different concepts (their matter is different). In these paradigm cases of the matter/form distinction, the

²⁵ My thinking about the matter/form distinction in Kant is indebted to an unpublished paper by Matt Boyle, "Kant's Hylomorphism".

distinction is ultimately between the parts (matter) of some complex whole, and the structure (form) of that complex whole. I will call these cases the ‘standard hylomorphic cases’, because the distinction between matter (*hyle*) and form (*morphe*) can be applied here in a reasonably straightforward way. I take it that Kant’s application of the matter/form distinction to judgments and inferences are standard hylomorphic cases. I have already explained this in the case of judgments. The distinction between the matter and form of syllogisms follows directly from the theory of syllogistic inference in the previous section: the form of the syllogism is the syllogistic figure, while the matter is the particular judgments (which themselves have a further form).²⁶ Kant also applies the matter/form distinction to concepts, but I will not explore that here, because it involves the complex doctrine that the form of concepts is “universality”.²⁷

One thing that leaps out about Kant’s ubiquitous use of the matter/form distinction is that some of the things to which he applies the distinction are not standard hylomorphic cases. Some of things that are said to have a form and a matter are not very naturally thought of as complex structured wholes with parts. Very roughly, we can divide the cases into two kinds (with citations to relevant texts in parentheses):

<u>Standard hylomorphic cases</u>	<u>Non-standard hylomorphic cases</u>
Judgments (Ak. 9:101)	Understanding (Bx)
Inferences (Ak. 9:121)	Reason (Bx)
Intuition (A23/B37)	Sensibility (A20/B34)
Maxims (Ak. 5:28)	Will (Ak. 4:436)
Concepts (?) (Ak. 9:91)	Thought (A59/B84)

From this table we can see that the items that are not naturally thought of as complex structured wholes (the non-standard hylomorphic cases) are faculties or capacities, while the items that are standard hylomorphic cases are products of such faculties. For instance, a judgment, according to Kant, is an activation of the faculty of the understanding. A judgment

²⁶ In *JL* §59 Kant claims that the form of a syllogism is the conclusion “insofar as it contains the *consequentia*” (Ak. 9:121). I take this to mean: the conclusion qua consequence of those premises. But if the conclusion is a valid consequence then it is a consequence in virtue of it and the two premises instantiating a syllogistic figure. So the point in the main text remains: the form is the syllogistic figure.

²⁷ *JL*, Ak. 9: 91.

is a complex structured whole, but it is very unnatural to think of the understanding *itself* as having parts and a structure. Concepts are an intermediate case because, while they are products of a capacity, they are not *standard* hylomorphic cases since, while they might be thought to consist in parts (marks) organized according to a form, Kant refers to the form of concepts, not as the logically complex manner in which they are arranged, but as their *universality*; whatever the ‘universality’ of concepts is, I take it, it is not a matter of their parts being related in some way, for even simple concepts are general and thus possess the form of universality.

Applying the hylomorphic analysis to faculties immediately leads to difficulties. What are the parts of the understanding? Are they other faculties? If so, why is understanding one of the basic faculties (rather than the faculties out of which it is composed)? If the parts of the understanding are not faculties, what could they be? I am going to take this as sufficient reason to at least *entertain the hypothesis* that when Kant talks about the *form* of the understanding (or of any other faculty) he does not mean the structure that obtains among the parts of the understanding (or of that other faculty). What, then, does he mean?

Before we address that question, though, we must first note some variation in Kant’s claims about which faculty’s *form* logic is about; although he typically describes logic as concerning the form of *thinking* (e.g. Bxxiii, A21/B36, A55/B79, etc.), he sometimes describes it as concerned with the form of *understanding* (e.g. Bix, A59/B84), sometimes with the form of reason (e.g. Bxi), and sometimes with the form of understanding *and* reason (e.g. A53/B77, A796/B824). One thing to note is that *thinking* is not a faculty in its own right, according to Kant. Kant’s lists of faculties vary somewhat from text to text, but in the first *Kritik* he seems to admit only four basic faculties: sensibility, understanding, (theoretical) reason, and the faculty of desire, practical reason (which, only briefly discussed in the first *Kritik*,²⁸ forms the central topic of the second). In the third *Kritik* he supplements this with a fifth faculty: the power of (reflecting) judgment [*Urtheilskraft*].²⁹

²⁸ The main discussion of practical reason in *KrV* is in the Canon der reinen Vernunft, A797/B825–A820/B848.

²⁹ In the A Deduction Kant appears to claim that imagination is one of “three original sources which contain the conditions of the possibility of experience and cannot themselves be derived from any other faculty of the mind” (A94; cf. A115). However, already in the A Deduction (A119) and even more so in the B Deduction (B153), the synthesizing role of the imagination seems to be attributed to the understanding itself.

Thinking is not a separate faculty; it is a very general description that applies to the activities of both the understanding and reason because they are discursive, or conceptual.³⁰ Thinking is discursive cognition in general, a point Kant makes in passing: “since merely formal logic, so conceived, abstracts from all content of cognition (whether it be pure or empirical), and concerns itself with the form of thinking (of discursive cognition) in general” (A131/B170).³¹ That understanding and reason are discursive/conceptual (and hence instances of thinking) means that activations of these faculties constitutively involve concepts: concepts themselves (both empirical and *a priori*), judgments (in the case of the understanding) and inferences (in the case of reason). In claiming that logic concerns the form of *thinking* Kant is claiming that it concerns the form of discursive representation in general, of which understanding and reason are instances. In order to mark this distinction between thinking and the particular discursive faculties (understanding, reason) I will refer to thinking as a ‘capacity’: we possess the capacity to think in virtue of possessing the more determinate faculties of understanding and reason.

To return to our earlier question, one possibility would be to interpret Kant’s talk of the form of a faculty as shorthand for talk of the form of the products of its activity (which *are* standard hylomorphic cases). So when Kant claims that logic concerns the form of thinking, what he means is that logic deals with concepts, judgments, and inferences (the products of the discursive faculties, understanding and reason) solely in virtue of their forms, that is, solely with features of, and relations among, these items that they possess in virtue of having the structure they do, and which they would share with any item that shared such a structure. On this reading, the formality of logic, for Kant, is very close to the formality of logic for Bolzano: logic does not concern itself with individual judgments or arguments, but with whole classes of

³⁰ I take it that the ‘power of judgment’ [*Urtheilskraft*] is also conceptual (though not wholly conceptual — see Ak. 5:214–217), in the minimal sense that its activities (reflecting judgments) are expressed with concepts, and hence is a faculty for thought normatively subject to logical laws (e.g. a logically contradictory aesthetic reflecting judgment is impossible). However, since by 1787 Kant had not even formulated the project of a *Kritik* of the faculty of reflecting judgment, and whether logic applies to reflecting judgment is not thematized in the *Kritik der Urtheilskraft*, this would be hard to substantiate from the text of either the first or the third *Kritik*.

³¹ See also B93 and A230/B283.

structurally (formally) indistinguishable judgments or arguments. There are passages that suggest this reading, for instance:

General logic abstracts, as we have seen, from all content of cognition, i.e. from any relation of it to the object, and considers only the logical form in the relation of cognitions to one another, i.e. the form of thinking in general. (*KrV*, A55/B79)

One reading of this passage is that Kant is claiming that logic (a) abstracts from, or ignores, the matter (constituent concepts) of judgments, what their objects are, whether they even have objects, etc.; (b) considers only the relations that judgments have to one another when this content is abstracted from; and (c) he identifies the formality of logic with (b). This suggests that by 'formal' here Kant merely means that logic is concerned with relations (e.g. inferential relations) that hold between judgments in virtue of their logical form ("the logical form in the relation of cognitions to one another"). This 'reductive' account of the form of thinking (i.e. reducing it to the form of its product) might be further encouraged by Kant's claim that "we can trace all acts of the understanding back to judgments, so that the understanding in general can be represented as a faculty [*Vermögen*] for judging" (A69/B94). Kant seems to be claiming that we can understand what the understanding is by understanding what it does: judge. Consequently, it might be argued, talk of the form of the understanding is shorthand for talk of the form of its most basic activity, judgment. Likewise, on this interpretation, talk of the form of thought is shorthand for talk of the form of its most basic activities/products: concepts and judgments (understanding) and inferences (reason).

I think we should be sceptical of this 'reductive' reading of Kant's talk of the 'form of thinking' because it leaves unexplained why Kant does not *only* talk about logic as being about forms of judgments, inferences, etc. (or about judgments, inferences, etc. in virtue of their form) but consistently and repeatedly claims that logic is about the form of thinking itself. If Kant only meant to convey that logic is about the forms of the products of this capacity why does he consistently and repeatedly claim that logic concerns the form of the capacity itself? In general, Kant does not restrict the hylomorphic analysis to the products of faculties, but consistently and repeatedly speaks of the forms of various faculties (see table above); if he meant the latter only to be a shorthand for the former, this is at least a very confusing way for him to express his point. This is a *prima facie* reason to look for another explanation of Kant's talk of the form of a faculty.

In several places in his lectures on logic Kant employs a more general notion of form:

In every cognition we must distinguish matter, i.e. the object, from form, i.e. the manner in which we cognize the object. (Ak. 24:510)

In logic one abstracts from objects and regards only the form of the understanding, i.e. *modus cognoscendi*. (Ak. 24:616)

The second quote is from the *Busolt Logik*; a few pages later Kant distinguishes between matter and form and adds this parenthetical qualification: “form (*modus cognoscendi*)” (Ak. 24:621). I call this a more ‘general’ notion of form because it encompasses the more strictly hylomorphic notion of ‘form’: the *modus cognoscendi* of, e.g., a judgment is its logical form (in the hylomorphic sense) because the manner in which judgments cognize objects (form as *modus cognoscendi*) is by combining concepts according to that (hylomorphic) form. So in these texts at least Kant identifies form *in general* with manner (or mode) of cognition. This is not, I think, a deviant or minor usage on Kant’s part. For even in the *Jäsche Logik* Kant refers to the *form* of concepts as universality (Ak. 9:91), and to an improvement in the distinctness of a cognition as a change in its logical *form* (Ak. 9:64).³² As I mentioned earlier, the *form* of concepts (universality) cannot naturally be thought of in terms of the hylomorphic notion of form: universality is not a structural relation among the parts of a concept (its marks) because those are concepts and thus possess universality in their own right. Likewise, Kant describes the distinctness of a cognition, the degree to which its parts are consciously apprehended by the subject, as a difference in the form of the cognition. This cannot be understood on the hylomorphic analysis of form because *by definition* the distinct and the indistinct cognition have the same parts organized in the same manner; the difference does not consist in the parts, or their manner of arrangement, but in the subject’s ability to consciously apprehend and differentiate them.

If we take this on board as Kant’s most general notion of form then we can say: logic concerns the *form* of thinking because it concerns the manner of cognition involved in thought. This can sound extremely vague, since thinking can be done in many ways (many ‘modes’): judging, inferring, etc. The *modus cognoscendi* of thinking is most naturally understood as the mode of cognition common to thought as such, or, what all

³² Cf. Ak. 24:418, 512, 538, and 28:229.

thinking has in common in virtue of being thinking. If it referred merely to the accidental mode of our thinking (how we happen to think) then the study of the form (*modus cognoscendi*) of thought would be what Kant calls “applied logic” (a kind of empirical psychology), rather than “pure general logic”, the science of the absolutely necessary rules of thinking as such. So logic concerns the form of thought in this sense: it concerns the mode of cognition all thinking has in virtue of being thinking. This is exactly what Kant says about logic elsewhere:

[pure general logic] contains the absolutely necessary rules of thinking, without which no use of the understanding takes place. (*KrV*, A52/B76)

All rules according to which the understanding operates are either *necessary* or *contingent*. The former are those without which no use of the understanding would be possible at all, the latter those without which a certain determinate use of the understanding would not occur. (*JL*, Ak. 9:12)

So the claim that logic concerns the form of thinking means that logic concerns the manner of cognition of thinking as such, which means it concerns the rules which thinking necessarily obeys, the rules without which thinking is not thinking. This, I take it, is what Kant means when he describes the laws of logic as the “essential” laws of the understanding (Ak. 16:44, 24:526), or as the understanding’s “own laws” (24:824). Logic does not concern the *modus cognoscendi* thinking merely happens to have; it characterizes the laws that characterize the manner of cognition thinking has in virtue of being the capacity (of discursive representation) that it is. Since understanding and reason are essentially discursive capacities, logic also studies laws that describe the manner in which these faculties cognize their objects in virtue of being the faculties they are.

Earlier, we saw that Kant claims that logical laws are necessary for thought. In the *Jäsche Logik* he makes clear that this necessity is normative not descriptive: “in logic, the question is not about *contingent* but about *necessary* rules; not how we do think, but how we ought to think” (*JL*, 9:14).³³ It is not that we necessarily *do* obey logical laws, but that we ought to. So if the form of thinking is the *modus cognoscendi* of thinking

³³ Cf. Pölitz *Logik* 24:503; Dohma-Wundlacken *Logik* 24:693; Wiener *Logik* 24:792; Refl. 1627.

itself, and this consists in laws that are normative for thought itself, then one thing the formality of logic might mean is: logic studies the normative laws that apply to thinking in virtue of being the capacity that it is. This, I take it, is what Kant means by claiming that “logic is to teach us the correct use of the understanding, i.e. that in which it agrees with itself” (*JL* 9: 14), a claim echoed in his reference in the *Kritik* to “general logical rules for the agreement of cognition with itself” (B116). Logic brings the understanding (or more generally, thinking) into agreement with itself because it teaches the normative laws that apply to that capacity in virtue of being the capacity that it is.

Since ‘law’ carries the normative connotation of guiding one’s action (or, in this case, one’s thought) when I talk about ‘laws’ I mean the normativity of logic; e.g. the logical *law* of non-contradiction is that one ought not to *judge* that $\langle A \text{ is } \sim A \rangle$ is true. When I talk about logical ‘rules’ I will mean the descriptive logical claims (e.g. that no judgment of the form $\langle A \text{ is } \sim A \rangle$ is true) that are transformed into normative claims in logical laws. Kant himself draws a similar distinction between laws and rules:³⁴ rules become laws when they are represented as necessary. Although Kant has descriptive natural laws in mind, we could adapt this to the logical context: logical *laws* are logical rules thought as normatively necessary.

I have outlined an alternative reading of one thing that Kant means by claiming that logic studies the form of thinking: it studies the *modus cognoscendi* of thinking as such and thus discovers normative laws that apply to thinking in virtue of being thinking. I have also discussed the ‘reductive’ interpretation of Kant’s talk of the form of thought, according to which Kant’s claim that logic studies the form of thinking means that logic studies the products of the capacity for thought – concepts, judgments, inferences – in virtue of their form. It is a short step to understand the formality of logic’s study of these products in terms of substitution-formality: for instance, logic only studies properties of inferences that are preserved under substitution of non-logical concepts. This reductive reading would make Kant’s claim that logic is formal very close to Bolzano’s claim that logic is substitutional. In the remainder of this section, I will explain why I think the reductive reading is untenable.

³⁴ See A126, Ak. 5:184, and Refl. 5414. Cf. Kant’s claim in the *Grundlegung* that “only law brings with it the concept of an *unconditional* and objective and hence universally valid *necessity*” (Ak. 4:416). Kant has practical laws in mind here, but I think the same could be said of logical laws.

Consider the following passage, one of the longest sustained discussions of the formality of logic in the *Jäsche Logik*:

If we now put aside all cognition that we have to borrow from *objects* and merely reflect on the use of the understanding as such, we discover those of its rules that are necessary without qualification, for every purpose and without regard to any particular objects of thought, because without them we would not think at all. Thus we can have insight into these rules *a priori*, i.e., *independent of all experience*, because they contain merely the conditions for the use of the understanding in general, *without distinction among its objects*, be that use *pure* or *empirical*. And from this follows at the same time that the universal and necessary rules of thinking concern merely its form and never its matter. Therefore the science that contains these necessary and universal rules of thinking in general is merely a science of the form of our cognition through the understanding, or of thought. (*JL*, 9:12)³⁵

Kant begins by claiming that if we ignore the particular concepts involved in judgments we can discover the rules that necessarily apply to any thinking, no matter about what object or with what kinds of concepts. These rules are normative rules, logical laws. But Kant goes on to claim that we can have *a priori* insight into these normative rules. ‘Insight’ is a technical term for Kant. It means knowledge from an explanatory ground; insight requires not merely knowledge that something is the case, but knowledge *why* it is the case.³⁶ So Kant is claiming that because the laws of logic are normative for thought as such we can have *a priori* insight into them, i.e. we can know *why* those normative rules of thought apply to all thinking as such.

He then concludes from this that these rules are *formal*. Now the reductive reading faces a dilemma: what does Kant mean when he claims that “the universal and necessary rules for thinking concern merely its form”? If he means that logic is substitution-formal, or that it concerns only properties of judgments, inferences, etc. that are preserved under substitution of non-logical concepts (matter), then in this passage Kant is

³⁵ For a very different interpretation of this passage, see MacFarlane (2002), 46.

³⁶ See *Jäsche Logik* (Ak. 9:65), *Pölitiz Logik* (Ak. 24:539), *Dohna-Wundlacken Logik* (Ak. 24:730), Refl. 1866 (Ak. 16:141), Refl. 1955 (Ak. 16:169) and Refl. 2394 (Ak. 16:342), *Kritik der praktischen Vernunft* (Ak. 5:27, 46, 47), and *Kritik der Urtheilskraft* (Ak. 5:83).

inferring the formality of logic (in this sense) from the *normativity* (necessity) of logic for all thinking as such. But we have already seen that Kant sometimes uses ‘form’ in a very general sense to refer to the *modus cognoscendi* of some faculty, so this claim (the normativity of logical laws for thought as such) is a kind of formality claim (for Kant): logic studies the *modus cognoscendi* of thinking as such and thus its essential normative laws. On this reading, Kant is inferring substitution-formality from its normative-formality. On the other hand, *if* the reductive reader of Kant interprets ‘formal’ in the second half of this passage as a restatement of logic’s normativity for thinking as such (normative-formality) then they have to admit that by the formality of logic Kant sometimes means what I take it to mean: it studies normative laws essential to thinking as such. I read the passage in the first way: Kant begins by asserting that logic studies the normative laws that apply to thinking as such (form in the sense of *modus cognoscendi*) and concludes from this that it studies the products of the capacity for thinking by abstracting from their differences in concepts and objects. Substitution-formality follows from normative-formality.

This passage from the *Pölitiz Logik* poses a similar interpretive dilemma:

The rules without which one would not think at all are necessary. The contingent rules that depend upon a particular object of explanation, are as numerous as the objects. Because the understanding, which acts according to rules, is present in every thinking there must be rules are common to all thinking, regardless of the object, and which lie a the basis of every use of the understanding, without which it would not be possible; and these are necessary. They therefore contain the form of thinking. (Ak. 24:502)

Kant here infers from the fact that logic is normative (necessary) for all thinking, regardless of its object, that logic is formal. Does formal mean substitution-formal here? If so, Kant is inferring substitution-formality from normative-formality, which is precisely my thesis. If not, then Kant is not drawing a conclusion from the normative-formality in the final line but restating that thesis in new terms: it is formal (in the normative sense). This, again, confirms my claim that the formality of logic, in its most fundamental sense, means that logic concerns the form of thinking, that is, the laws that apply to thinking in virtue of being the capacity that it is, its “essential laws”.

Readers who favour the reductive interpretation might want to object at this point: granted that Kant sometimes infers the (substitution)

formality of logic from its normativity for thought as such, why should we think that the latter (normativity) claim is ever what he means by claiming that logic studies the form of thinking? Two points in reply. First, as we have seen, Kant repeatedly refers to the forms of faculties, but this notion of form is not well captured in hylomorphic terms (because faculties, standardly conceived, do not have parts) and he does have a more general notion of form: *modus cognoscendi*. Thus, my reading gives us an explanation of why Kant talks of logic as studying the form of a faculty not just the forms of its products. Second, Kant sometimes talks about the formality of logic in a way that naturally invites my reading, but which is at best awkwardly interpreted on reductive lights: “logic is thus a self-cognition of the understanding and of reason, not in regards to their object, however, but merely in regard to form” (*JL*, Ak. 9:14). By now, it should be clear how I interpret this claim; logic is genuine self-knowledge on the part of the understanding because logical knowledge is knowledge of what norms the activity of the understanding is bound by in virtue of being the faculty it is (a discursive faculty). The reductive reader is forced to read it as follows: logic is self-knowledge of the understanding and of reason in that it is knowledge of normative laws that apply to the products of these faculties in virtue of the forms of those products. My objection is simply: this does not look like self-knowledge on the part of understanding or of reason, but of knowledge of their products. This is an instance of a wider problem with the reductive reading: where Kant talks about (the form of) a faculty, the reductive reading has to interpret him as meaning the (form of) the products of that faculty.

The connection to Aristotle, from whom the matter/form distinction ultimately derives, can help us understand why Kant might have talked about the ‘form’ of thinking in this way. While Aristotle introduces matter and form in the *Physics* in such a way that the notion of form (*eidos*) seems closely tied to the shape (*morphe*) or structure of some complex whole,³⁷ in the *Metaphysics* it is a wider notion. In its most general use, the form of a thing is its essence, the answer to the question, *what is it?*³⁸ Consequently, things without matter can nonetheless have *forms* (immaterial substances). In other words, the form of a thing makes it the thing it is. So, going back to Kant, if logic concerns the

³⁷ *Physics* 190b15, 191a10, and 193a30. Cf. *On generation and corruption* 335b6 where he seems to use *morphe* and *eidos* interchangeably.

³⁸ *Metaphysics* 1032b1–2, 1035b33–1036a2.

form of thinking, then logic concerns *what thinking is*, or, to put the point more straightforwardly, whatever rules logic uncovers are rules that are grounded in *what it is to think*. In other words, the rules of logic apply to thinking in virtue of *being the capacity it is*. They are grounded in the essence (form) of this capacity.³⁹ To bring this back to the point I put aside above, when Kant claims that logic “teaches us the correct use of the understanding, i.e., that in which it agrees with itself” I take this to mean: the rules of logic describe how the understanding operates when it is operating correctly as the faculty that it is. This also explains how we can have *a priori* insight into the rules of logic. *A priori* insight into the rules of logic is not merely knowledge that they are valid, but knowledge of *why*. We can know *why* the rules of logic are valid by understanding what capacity the capacity for thinking is and why, when it operates correctly as the capacity that it is, it obeys these rules.

The conclusion of this line of thought is that to understand what the formality of logic means for Kant, we must understand (1) what the capacity for thought is, and (2) what, if any, normative laws apply to it in virtue of what it is. If those normative laws are the laws of logic, then we will have made significant progress in understanding what it could mean that logic concerns the form of thinking.

§5 Logic and the unity of apperception

Kant’s claims that logic studies the form of thought, the form of the understanding, and the form of reason means: each of these is a faculty of discursive representation and logical norms apply to discursive representation in virtue of its form (in virtue of being the faculty it is). Why does it lie in the nature of discursive faculties that their activities are normatively subject to the laws of logic? We might expect Kant’s answer to have this structure: an explanation of why logical norms apply to discursive representation as such, which would entail that logical norms

³⁹ I am not claiming that ‘form’ generally means essence, for Kant; as we have seen, a cognition changes its *form* as it becomes more distinctly understood by a subject, but it makes no sense to say its essence has changed. The most general notion of form in logical contexts is that of *modus cognoscendi*; since thinking is a capacity, a capacity to cognize (represent) in a certain way, its *modus cognoscendi* characterizes what it is to *be* that capacity, its essence. This is not true in general of things that had a *modus cognoscendi* (e.g. a concept had by a particular subject, which can have varying degrees of distinctness).

apply to the activity of the understanding. However, this is not what Kant does. He explains why logical norms apply to the activity of the understanding. He leaves it up to us to generalize from this to an explanation of why logical norms apply to activities of discursive representation in general. So I will first give a reconstruction of Kant's explanation of why logical norms apply to the activity of the understanding (in virtue of what that faculty is), and then I will explain how this generalizes to discursive representation in general (thinking).

Some readers might be surprised at my claim that Kant "explains why logical norms apply to the activity of the understanding". Where, they will ask, does Kant do anything of the sort? At three places in the Transcendental Deduction (two of which occur in footnotes!) he claims that the unity of apperception explains the possibility of logic. In each case, Kant's remarks are brief and cryptic, so it is up to us to reconstruct how they might constitute an explanation:

[...] the logical form of all cognition necessarily rests on the relationship to apperception as a faculty. (*KrV*, A117n)

And thus the synthetic unity of apperception is the highest point to which one must attach all use of the understanding, even the whole of logic and, after it, transcendental philosophy; indeed this faculty is the understanding itself. (*KrV*, B133–134n)

The category presupposes combination. We must therefore seek this unity... someplace higher, namely in that which itself contains the ground of the unity of different concepts in judgments, and, with it, the possibility of the understanding, even in its logical use. (*KrV*, B131; underlining in all three quotes by NS)

I have argued that logic, for Kant, is a normative science. In these passages, Kant claims that the unity of apperception explains how this normative science is possible. I take this to mean two things: (i) the unity of apperception explains how it is possible to cognize logical laws; and (ii) the unity of apperception explains how it is possible that (normative) logical laws apply to us. If (ii) were false, the unity of apperception would not explain the possibility of logic: it would fail to explain why the claims of logic (claims that certain norms apply to our thinking) are true. If (i) were false, logical norms might apply to us, but we could not cognize this (nor could we cognize why they apply). I take this to entail that logic, the normative science, would not be possible. However, I will focus on (ii); the Kantian account of how logical cognition (cognition

of logical laws as normatively binding on our thought) is possible lies outside the scope of this essay.

Kant does not tell us explicitly how the unity of apperception does (i) or (ii). He leaves it up to us to reconstruct his reasoning. We need to explain why logical norms apply to us, e.g. why I stand under the norm that if I judge that p , that $p \rightarrow q$, and that $\sim q$ then I ought to revise or reject at least one of my judgments. But now ask yourself, why am I bound by that norm? Why am I normatively required not to have inconsistent judgments? Consider Kant's remark at B64 that:

For although a cognition may be in complete accord with logical form, i.e., not contradict itself, yet it can still always contradict the object. The merely logical criterion of truth, namely the agreement of a cognition with the general and formal laws of understanding and reason, is therefore certainly the *conditio sine qua non*, thus the negative condition of all truth; further, however, logic cannot go, and logic cannot discover by any touchstone the error that concerns not form but content. (*KrV*, A59/B84)

If Kant is claiming that logic concerns the negative principles about truth (i.e. no thought that violates logical laws can be true), this does not get us any closer to understanding why logic is normative for us. If logic describes a set of principles that my thoughts cannot violate and still be true, we do not get an explanation of why logic is normative for us *unless* we assume that truth (or at least non-falsehood) is the end of our thinking. This opens the space for a reconstruction of Kant's reasoning that the unity of apperception explains the possibility of logic, where logic studies the norms that apply to the understanding in virtue of the faculty the understanding is (logical laws). The explanation is: the faculty of understanding, because of the faculty it is, aims at non-falsehood. To put it another way, among the ends of the faculty of the understanding, one of them is non-falsehood. Logic describes the negative requirements that must not be violated if our thinking is going to satisfy one of its constitutive ends, non-falsehood.

Before continuing, I want to briefly explain what I mean by claiming that thought as such aims at non-falsehood rather than truth. In the *Kritik* Kant defines truth as "the agreement of cognition with its object" (A58/B82).⁴⁰ The definite description "its object" presupposes that the

⁴⁰ This definition of truth is echoed throughout the logic lectures: Ak. 24:391, 525, 718, and 823, as well as Refl. 2162 and 2177.

cognition has an object. But not all thoughts are cognitions; some thoughts do not have objects. These thoughts are not *false*. They do not fail to agree with their object; they simply lack objects and are thus not even *apt* to be true or false (unless an object is provided for them).⁴¹ This is confirmed by Kant's corresponding account of falsity in the *Philippi Logik*:

Falsity is either: *materialis* or *formalis*. Formal falsity is when a cognition contradicts itself, or does not agree with itself [*sich nicht paßt*]. (Ak. 24:391)

'Material' falsity I take to refer to a cognition that does not agree with its object. But then what are we to make of a consistent (not formally false) thought that does not even have an object? It is not materially false, because it does not even have an object. Nor is it true, for the reasons given above.

The conditions under which an object can be given for a thought are specified in transcendental logic;⁴² in particular, the thought must involve concepts of objects in space and time, because these are the forms in which objects are given to us.⁴³ Consequently, if thought as such aimed at *truth* then the logic that studies the form of thought (and thus uncovers why logical norms apply to thought) would have to be transcendental logic.⁴⁴ But it is not; it is pure general logic. Logic is concerned with the laws that no thought can violate if it is going to be non-false. Obeying the laws of logic is obviously not sufficient to make a thought *true* (for it might still lack an object),⁴⁵ but it isn't

⁴¹ By denying the "truth-aptness" of objectless thoughts I am not talking about the contemporary notion of "truth-aptness" (that is involved in, for instance, debates about the truth-aptness of moral judgments). As I am using the term, it merely refers to the fact that such thoughts fail to satisfy the presupposition of Kant's definition of truth (agreement with their object).

⁴² A55/B80.

⁴³ In my reading, giving an object for thought (e.g. giving an object for a concept) is making that object available for thought in a way that (among other things) makes it possible to cognize that there is such an object. So it is possible that there *are* objects of some of our concepts but which cannot be *given* to us (we cannot intuit them), so we do not cognize these objects. For instance, there is a God (as Kant argues in the second *Kritik*), so the concept <God> has an object, although that object cannot be given to us (intuited). I do not have the space here to defend this interpretation.

⁴⁴ A62/B87.

⁴⁵ Kant makes this point at A59/B84 and again and again in the logic lectures.

even sufficient to make a thought truth-apt; a thought might obey all of the logical norms and still be objectless. For instance, the principle of contradiction (<No A is \sim A>) is the highest principle of logic, but not every substitution instance of it is *true*; if A is an objectless concept, <No A is \sim A> is neither true nor false. But no substitution instance of this principle is *false*.⁴⁶

Nor is it the case that all discursive faculties constitutively aim at truth (where truth is understood as the agreement of a cognition with its object). The faculty of reason, as we have seen, is a discursive faculty, but reason's constitutive end is not *truth* but to find a condition for any given conditioned object and, in pursuit of this end, the faculty of reason forms the concept of an *unconditioned* condition.⁴⁷ Since there are a variety of different relations of condition to conditioned (substance–accident, cause–effect, and totality–limitation), this generates several different representations of unconditioned objects: the unconditioned thinking substance (the object of rational psychology), the unconditioned cause of effects in space and time (the object of rational cosmology), and the unconditioned ground of all possibility (the object of rational theology).⁴⁸ But none of these representations can be given an object in intuition, the only way objects can be given to us. Consequently, these concepts cannot be involved in cognition. The constitutive activity of reason – the activity in which reason manifests itself as the capacity it is – is not about any object that can be given to us. So not only does reason *not* constitutively aim at truth, it constitutively aims at representations which are not even truth-apt, because they violate the conditions under which any representation can be a cognition (a representation to which an object is given) and thus be either a true or false representation of its object.⁴⁹

It is relatively clear that Kant thinks that (theoretical) reason has a constitutive end: to find a condition for any given conditioned object. It is more controversial to claim that understanding (*Verstand*) has a constitutive end, but this is one of the central claims of the third *Kritik*. In both the published and the unpublished ('first') Introductions to that work, he argues that the 'formal purposiveness' of nature is an *a priori* principle of reflecting judgment. He defines formal purposiveness as follows: "the correspondence of a thing with that constitution of things

⁴⁶ Cf. A258/B314.

⁴⁷ A307/B364.

⁴⁸ A327/B384.

⁴⁹ A329/B385.

that is possible only in accordance with ends is called the **purposiveness** of its form" (Ak. 5:180). To say that X is purposive for Y is to say that X is only possible if X has an end, and that end furthers the end of Y. A thing is represented as formally purposive if its form (in this context, structure) is represented as being only possible if it has some end. That reflecting judgment represents nature as formally purposive means that reflecting judgment represents nature's systematic form (e.g. its division into species and genus) as possible "only in accordance with ends", that is to say, possible only under the assumption that this system of nature has some end. For what (or whose) end does reflecting judgment represent the systematic form of nature as purposive? Our capacity for cognition (*Erkenntnisvermögen*), in other words, the understanding.⁵⁰ To represent nature's systematic form as purposive for our *Erkenntnisvermögen* means representing that systematic form as possible "only in accordance with ends", namely ends that further the ends of our cognitive capacity. So to represent nature's systematic form as purposive for our cognitive capacity presupposes that this cognitive capacity has *ends* and that the systematic form of nature furthers those ends. This is what Kant means when he writes:

since the lawful unity in a combination that we cognize as in accordance with a necessary aim (a need) [*Absicht (einem Bedürfnis)*] of the understanding, but yet at the same time as contingent in itself, is represented as a purposiveness of the objects (in this case, of nature), thus the power of judgment [...] must think of nature with regard to the latter in accordance with a **principle of purposiveness** for our faculty of cognition. (Ak. 5: 184)

The systematic form of nature is contingent for our understanding because the understanding leaves open the possibility that the multiplicity of species and genres in nature might be so great that we could never discover them (or empirical laws governing them). This systematic unity is not provided by the understanding, but it furthers a necessary aim, an end, of the understanding: to cognize empirical objects under laws (which requires species and genus concepts). This systematic form, therefore, is represented as purposive for the understanding: it serves the ends (*Zwecke*) of the understanding. That the systematic form of nature is represented by reflecting judgment as *purposive* for our understanding is

⁵⁰ Ak. 5:174.

repeated throughout the third *Kritik*.⁵¹ Something can be represented as purposive for any cognitive faculty only if that cognitive faculty has an end, so our understanding has an end: to cognize empirically given objects under empirically given concepts/laws. Clearly, understanding cannot achieve that end unless it forms logically consistent concepts and judgments. So part of understanding's constitutive aim is: non-falsehood.⁵²

When I say that 'thought as such aims at non-falsehood' I mean that non-falsehood is a constitutive end of the capacity for thought (discursive representation in general). I have argued that (theoretical) reason and understanding have non-falsehood among their constitutive ends (more precisely, that non-falsehood is a component of their constitutive ends). Since understanding and reason are the two discursive faculties, I take this to be sufficient evidence that non-falsehood is among the ends of discursive thought as such. It does not *entail* this conclusion; it is compatible, strictly speaking, with these facts that, although understanding and reason do aim at non-falsehood, this is not part of what makes them discursive faculties. I will ignore that possibility in what follows.

I have begun reconstructing Kant's explanation of how the unity of apperception explains the bindingness of logical laws upon discursive representation by arguing that discursive representation (thought) as such has a certain end – non-falsehood – and that logical laws relate to the fulfilment of this end. But in order to understand how some normative laws might apply to a certain representational activity with a given end (or 'aim') we must understand the relation between those normative laws and that end. This is perhaps the place where Kant leaves the most to his readers' reconstruction. I am going to assume what I take to be the following minimal conditions: the negative rules our thought must satisfy to achieve one of its constitutive ends (non-falsehood), described at A59/B84, become normatively binding on us (they become logical *laws*) only if (i) we are capable of following or not following them (ought implies can),⁵³ and (ii) capable of representing ourselves as

⁵¹ Cf. Kant's repeated references to the *a priori* principle of the "purposiveness of nature for our cognitive faculty (*Erkenntnisvermögen*)" (e.g. Ak. 5:182, 184, 185, 186).

⁵² It is not merely that non-falsehood is a means to the end of the understanding; the understanding aims to cognize empirical objects, that is, to cognize them *accurately* and logical consistency (non-falsehood) is a constituent of that end (see above), not merely a means towards it.

⁵³ A principle to which Kant appears to be committed at A548/B576.

either following or not following them, and (iii) allowing the capacity mentioned in (ii) to guide the activity of the capacity mentioned in (i). For a normative law to apply to the activity of some capacity, it not only has to be something we both can satisfy and can fail to satisfy, but we must be able to represent ourselves as failing to do so, and our representation of the law has to be part of the explanation of why we do or do not fail to satisfy the law. In other words, our representation of our activity as satisfying or failing to satisfy the law cannot simply be *passive*; we must be able to actively bring our activity into agreement with the law *because* of our representation of the law. If we are merely capable of *observing* whether our activity agrees with the putative law, but not of modifying that activity in light of its agreement or non-agreement with the putative law, then that law lacks normative force for our activity (it is not a 'law' in the sense defined above). Kant, as I have interpreted him, claims that the unity of apperception explains how logical laws can be normatively binding on us. Although he does not spell it out for us, it is not difficult to find a plausible story in the Transcendental Deduction about how the unity of apperception does this.

Kant states at the beginning of §16 of the B Deduction that "the *I think* must be able to accompany all of my representations".⁵⁴ To 'accompany' a representation with the 'I think' is to explicitly attend to that representation as *my* representation. This means that my representations have a certain kind of unity, albeit in a very minimal sense of 'unity' (although the unity required for self-consciousness will be strengthened significantly over the course of the Deduction): I can explicitly attend to them as *my* representations, while I cannot do so to your representations (although you can). Kant calls this 'unity of apperception'. Clearly, unity of apperception involves unity of consciousness. I can become explicitly conscious of any of my representations (accompany them with the 'I think'); I can direct my awareness at any of my representations. However, unity of apperception means more than that. For if that were all there is to the unity of apperception, I might have unity of apperception even though I am in general totally unaware of my representations but could direct my attention to them at will. Unity of apperception requires that, even when I am not consciously attending to my representations, I am still implicitly aware of them. In other words, it requires

⁵⁴ There is an extensive literature on the B Deduction, the notion of apperception, and, indeed, this very sentence. I do not have space in this essay to engage with it. However, I have learned much from Longuenesse (1998) and the discussion of apperception in Van Cleve (1999).

that were I to explicitly attend to one of them, that explicit attention would be an activation of some awareness of that representation that was already in place.

In §17 Kant extends this analysis by claiming that the representational manifold in an intuition (an immediate singular representation of an object) must have the unity of apperception; so far, this is a direct consequence of the claims of §16. He goes further, though, in claiming that if I am conscious of a unity of representations constituting an intuition of an object – as I must be if that manifold stands under the unity of apperception – then I must ‘unite’ those representations in a concept of an object. In other words, if I am consciously intuiting an object, and that intuition is composed of a manifold of representations, then I must be conscious of those representations (though perhaps not explicitly) and ‘unite’ them (though perhaps not explicitly) under a concept by thinking of them as a manifold of representations that compose an intuition of an object of a particular kind. For instance, if I consciously visually perceive a dog running across my yard, then I am conscious of that perception as composed of overlapping perceptions of the dog at different temporal intervals, and I am conscious of those overlapping perceptions as united under the concept ‘dog’. This conceptual unity among a manifold of representations is something my mind does (though I may not be explicitly attending to it). I do not passively receive this information from the senses; the fact that my total perceptual experience includes as part of its content that this manifold of representations is a manifold of representations of a *dog* is due to my mental *activity*.⁵⁵ Two points are crucial here. First, because I unify this manifold of representations under a *concept* (a general representation) I can do so incorrectly. Second, because this unity is effected by an act of my mind, I can be subject to normative criticism if I make an error. If I did not unify this manifold through a general representation (a concept) but only by some essentially indexical or demonstrative representation (e.g. *that-there*) then I could not in principle go wrong: for whatever I am intuiting is *that-there*. By unifying the manifold through a general representation, though, I open my act to the possibility of *error* because I represent the object of the intuition as having the general features constitutive of the concept; if the object turns out not to have one of those features, I have incorrectly unified the manifold. Similarly, if my senses (without any input from the understanding or apperception) represented the object

⁵⁵ B129–130.

of the intuition as a dog, then if the object turned out not to be a dog, I could be said to have misperceived, but normative criticism would be out of place. It would be wrong to say I should *not* have seen it as a dog if my senses simply deceived me. As it is, I might represent a manifold of representations of an object under the concept 'dog' but then upon receiving more information see that it is not a dog ('dog' was the wrong concept under which to unify those representations), for the object is actually a wolf. The objective unity of apperception, the unity a manifold of conscious representations is to have if it is going to be about an intuited object, brings in the possibility of normative evaluation of our following of conceptual rules.⁵⁶

For our purposes, the key text here is §19 of the B Deduction, "The logical function of all judgments consists in the objective unity of the apperception of the concepts contained therein." After an initial paragraph in which he rejects the "explanation that the logicians give of a judgment in general" Kant explains his own view in the second paragraph, beginning with the crucial claim that "a judgment is nothing other than the way to bring given cognition to the **objective** unity of apperception" (B141f). The objective unity of apperception was originally introduced by Kant as a unity among manifold representations in an intuition, but here he extends it to a manifold of concepts in a judgment. This means that Kant's claim that "a judgment is nothing other than the way to bring given cognitions to the **objective** unity of

⁵⁶ Contrary to appearances, this is not only compatible with Kant's repeated claim that the senses do not *judge*, but in fact expresses the very same point (from a different direction). One finds repeated throughout the lectures on logic a similar story about the possibility of error: the senses by themselves do not err (because they do not judge, a point made in the *Kritik* at A293/B350) and the understanding by itself would not err (e.g. Ak. 24:526–527, 720, 824). The origin of error is in the interaction of sense and understanding. But if we take seriously the claim of the Transcendental Deduction that "all manifold, insofar as it is given in one empirical intuition, is determined in regard to one of the logical functions of judgment" (B143) then some function of judging is active in any perceptual consciousness of an object that can belong to the unity of apperception, i.e. be a representation this "for me" (B132). I take this to mean that conscious perceptual awareness of objects in our environment is the joint-product of passive reception of sensory materials (sensibility, which does not judge, and thus does not err) and the synthetic activity of the understanding. If this is correct, then given that Kant locates the possibility of error in the interaction of sensibility and understanding, it follows that consciousness perceptual awareness of objects in our environment can *err* (unlike mere passive receipt of sensory matter).

apperception" means that a judgment is nothing other than the way to unite concepts together so that they possess the objective unity of apperception. Just as the manifold of an intuition is consciously united in a concept of the intuited object, Kant claims, concepts are consciously united in a judgment that asserts a relation between their domains. This is what distinguishes a judgment from mere 'association': when I judge $\langle \text{All } S \text{ are } P \rangle$ I am not merely having an associative episode involving the concepts $\langle S \rangle$ and $\langle P \rangle$. I am making a claim about objects.

The natural question is, what in the case of judgment plays the unifying role played by the concept in the case of intuitions? Kant's answer is "that is the aim of the copula *is* in [judgments]: to distinguish the objective unity of given representations from the subjective" (B141–142). Kant's reference to 'is', however, is disappointing, for we will look in vain among the table of logical functions of judgment for 'is'. I think Kant's point, though, is that all judgments *as such* are unified by the copula, and the different logical functions of judgment are different ways of determining the copula.⁵⁷ But just as in the case of the synthesis of a manifold of representations in an intuition, two points are crucial: in judging, our minds are active, not passive; and the 'copula' by which we unify concepts into a judgment introduces the possibility of *error*. Combining those two points, the fact that judgments are acts of the mind and that we can incorrectly unify concepts into judgments opens the possibility of normative criticism of our judgmental acts. That judgment is an *act* means merely that our judgmental representations (our representations whose content is judgmental) are not passively received by the senses: if the impact of the world on my sensory organs by itself made it the case that I believe that there is a dog in front of me, it would be wrong to normatively criticize me for this judgment. I might be wrong that there is a dog in front of me, but it would not be the case that I *ought not* to have judged that.

The question then is, how can the copula be wrong? In other words, how can I be incorrect in synthesizing two concepts $\langle A \rangle$ and $\langle B \rangle$ into the judgment $\langle A \text{ is } B \rangle$. Well, there are lots of ways I can be wrong! But in this essay we are interested in a very specific way I can be wrong: a way I can be wrong that does not depend upon which particular concepts $\langle A \rangle$ and $\langle B \rangle$ are. The principle of contradiction is one such rule. If $\langle B \rangle = \langle \sim A \rangle$, then $\langle A \text{ is } B \rangle$ is a false judgment, regardless of what concept

⁵⁷ Cf. B332/A266, Ak. 9:104, Ak. 17:344. See however *JL* §25 where Kant claims that in hypothetical judgments "*Consequenz*" takes the place of the copula.

<A> is. So, it might be thought, that my judgment that <A is \sim A> can now be seen to be subjective to normative criticism: the judgment is an *act* of mine (I synthesized it, rather than passively receiving it from the senses), and the judgment is *false*.

But this *only* issues in an explanation of why I am subject to normative criticism in judging <A is \sim A> if we assume that: (i) <A is \sim A> is false, regardless of what concept A is, and (ii) a judgment is subject to normative criticism if it is false. In other words, we need to assume (i) that the principle of contradiction is *true* regardless of the concepts involved, that (ii) that thought as such aims at *non-falsehood* and that thoughts that are false are subject to normative criticism in virtue of failing to be non-false. The philosophical lessons of this are that we can derive from the unity of apperception an explanation of why logical norms apply to all thought as such, but only if we assume the truth of the principle of contradiction. This means that the unity of apperception does not explain *why* the principle of contradiction is true; at most, it explains why the principle of contradiction normatively constrains all thought as such. But if we take seriously Kant's claim that the "entirety of logic" is attached to the unity of apperception and explained by it, this entails that (what Kant calls logic) does not explain why the principle of contradiction is *true*. Logic does not explain why there are no true contradictions, but why *we* are normatively bound not to have any internally contradictory thoughts. The unity of apperception explains the normativity, not the truth, of logic.

Now we can give a unified account of how the unity of apperception explains why logical norms apply to all thought as such. Thought constitutively aims at non-falsehood. Logical laws describe the negative conditions on non-falsehood; no thought can violate these principles and still be non-false. Because judgments and the concepts that compose them stand under the unity of apperception, these judgments are the product of synthesis by the judging subject (they are spontaneously generated, *not* passively received). The synthesizing of concepts into judgments involves uniting them under what Kant calls the 'copula is' in §19 of the B Deduction, which, I argued, is the genus of which different logical functions are the species (the different logical functions are determinate forms of the general copula). Consequently, judgments can be incorrect or correct in the following minimal sense: they can obey or not obey the logical principles (principles of non-falsehood) specified in terms of the various logical functions. But all of this stands under the unity of apperception; so the judging subject is aware of the judgments he or she is making, and can be aware of their agreement or non-agreement with the

logical principles. Furthermore, since these judgments themselves are spontaneous (not passive), the judging subject's consciousness (apperception) of these judgments can be efficacious in modifying or rejecting these judgmental acts. In other words, the subject can modify his or her spontaneous judgmental acts in light of her apperceptive awareness of their agreement or disagreement with logical principles. These logical principles articulate negative conditions on thought's satisfaction of its constitutive end: non-falsehood. So, I conclude, these logical rules obtain the status of norms for all thought as such. The unity of apperception explains why logical principles are normative for thought, why they are *laws*.

To bring this discussion full circle, this also explains why logical norms are formal in Bolzano's sense, namely, why they are substitution-formal. It is because the rules of logic are substitution-formal: they can be specified as abstract schemata from which determinate rules for particular concepts and judgments can be obtained through substitution of determinate concepts for concept-variables. I argued in Section §2 that the most worked-out part of Kant's logic – the theory of inference – is substitution-formal. These logical principles become norms for thought because they specify negative principles that any thought must satisfy in order to be non-false, and non-falsehood is the constitutive aim of thought as such. The unity of apperception then explains how these principles obtain the status of norms for the discursive representational acts of judging subjects, and since the principles are substitution-formal, the resulting norms are substitution-formal as well. Consequently, we can agree with Bolzano that the Kantian thesis of the formality of logic "*scheint noch nicht deutlich zu seyn*", for Kant has left to us much of the work of rendering it clear, but there is at least room to disagree whether it is "*nicht richtig genug entschieden*".⁵⁸

Abbreviations for works of Kant

A/B *Kritik der reinen Vernunft*, Ak. 3 & 4. Cited by page number in 1st edition of 1781 (A) and 2nd edition of 1787 (B). Translations generally follow Kant (1997), with occasional alterations.

⁵⁸ I'd like to thank Sandra Lapointe and Clinton Tolley for reading, and commenting on, an earlier draft of this essay. I'd also like to thank Jack Woods, Catharine Diehl, Reed Winegar, James Kreines, Franz Knappik, Tobias Rosefeldt, Peter Yong, Bianca Ancillotti, and the other members of the Colloquium for Classical German Philosophy at Humboldt-Universität zu Berlin for a very helpful discussion.

- Ak. Kant (1905). Cited by volume and page number (e.g. Ak. 29: 1034).
- KrV *Kritik der reinen Vernunft*, Ak. 3 & 4. Cited by page number in 1st edition of 1781 (A) and 2nd edition of 1787 (B). Translations generally follow Kant (1997), with occasional alterations.
- JL *Jäsche Logik*, Ak. 9. Translations generally follow Kant (1992), with occasional alterations.
- Refl. *Kants handschriftlicher Nachlass* ('Reflexionen') in Ak. 14–18. Cited by four digit number.

Where no translation is listed, translations are my own.

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6

Kant, Bolzano, and Moore on the Value of Good Willing

Timothy Rosenkoetter

§1 Overview

Bolzano's many achievements in mathematics, logic, metaphysics, and epistemology, as well as the monumental scope and ambition of his *Theory of Science* and *Theory of Magnitudes*, make it easy to overlook his relatively brief treatments of morality. Yet Bolzano took moral theory quite seriously.¹ This carries over to Bolzano's engagement with Kant. Though only a small fraction of *The New Anti-Kant* (192–202) is concerned with moral philosophy, Bolzano thinks that the correction he is able to provide there is every bit as important as his critical engagement with the central topics of the first *Critique*. Thus, one purpose of this chapter is to place that brief comparison with Kant's moral theory within the context of Bolzano's most thorough defence of utilitarianism, which is found in lectures that his students published as *Treatise on the Science of Religion*.²

An all too familiar problem when comparing deontologists and consequentialists is that the great distance between their positions virtually eliminates any real engagement. At first blush Kant and Bolzano appear to be just such a pair of intractable enemies, so that comparing them

¹ For historical background on how Bolzano endeavoured to guide his life as a philosopher and public intellectual by utilitarian principles, see Rusnock's and George's introduction to Bolzano (2007).

² This present essay is based on four sources: *The New Anti-Kant* ("NAK"), *Theory of Science*, *Science of Religion* (Bolzano 1994/1995, hereafter "R"), and "On the Right of the Clergy". The Rusnock/George translation of relevant sections of the last two are available in Bolzano (2007). I frequently depart from their translation of *Science of Religion*, using Bolzano (1994/1995) as my basis. See Künne (1996, 325ff.) for a list of inaccuracies in the latter edition.

will be profitable at most because Bolzano has some insightful criticisms of Kant. However, we will see that the almost unremittingly critical nature of Bolzano's discussion conveys a false impression. In certain respects Bolzano is still very much working within the ambit of Kant's influence.

While a number of Bolzano's criticisms are based simply on misunderstandings of Kant's sometimes cryptic pronouncements,³ his criticism of Kant for ignoring *impartial* consequentialism should worry any defender of Kant. Kant, namely, shows every sign of thinking that he can disprove consequentialism as a whole by arguing against just its egoistic variety.⁴ It is evident that Bolzano finds this irritating, and not only because he considers impartial consequentialism to be correct. As a philosopher who has a healthy respect for the epistemic role of our intuitions⁵ Bolzano finds it philosophically irresponsible that Kant repeatedly argues against a tremendously implausible position – implausible because Bolzano regards morality as self-evidently impartial⁶ – while apparently remaining silent about the position that any neutral observer should count at least among the serious contenders. The milieu in which Kant formed his views might go some way towards explaining his silence on impartial consequentialism. Still, one can see why Bolzano suspects that Kant ignores utilitarianism because he simply “has nothing valid to say against the principle of universal happiness” (NAK 194).

³ Many of these misunderstandings are understandable, given the difficulty of Kant's presentation. Here are some criticisms which I won't cover below, since responses can be found in the literature: (1) that the Formula of Universal Law (FUL) is tautological, NAK 200; (2) that FUL is circular, *R I*: 253; NAK 201; and (3) that Kant believes that an impermissible act is contradictory *in itself*, *R I*: 254; NAK 202.

⁴ The most important text in this regard is the *Critique of Practical Reason* (*KprV*) 5: 35–37. A word on terminology is in order. Though I will speak simply of “consequentialism”, this chapter has only act-consequentialism in view, viz., the position that the moral properties of acts are determined by the consequences of those acts, including the trivial consequence of that act itself occurring. Impartial consequentialism counts all bearers of goodness equally. The only form of partial consequentialism that will be mentioned is act-egoism, the position that the morality of an act by agent *S* is determined by how its consequences affect the good of *S*.

⁵ Cf. NAK 196; 221; *R I*: 244.

⁶ One hint of this comes in the Foreword's striking assertion that Kant's greatest service to “humanity as a whole” is that he “supported the doctrine of morals upon purer foundations and freed it from self-serving motives” (NAK: xx). In Bolzano's estimation Kant was completely successful in separating genuine morality from its distortion at the hands of those who would ground it in prudence.

Kant's obvious objection to consequentialism is that it rejects what are now known as "deontological constraints", i.e. act-types that are prohibited (or required) regardless of their effects. This is important, but I will concentrate here on an objection that is arguably more fundamental to Kant's theory.⁷ In order to access that objection the next section distinguishes three ways that intentions or maxims (or *willings*, in Bolzano's terminology that will be used below)⁸ might be considered relevant to moral theory. Eventually this will enable us to see that Kant has a case that impartial and egoistic consequentialism suffer from the same flaw. Both make the mistake of treating our willings first and foremost as causal interventions in the world, and in doing so consequentialism misses the true moral significance of willing. What makes Kant's case more relevant in the context of the present volume is that Bolzano actually agrees with elements of it. My goal in this essay is to uncover those points of agreement in the course of reconstructing the foundations of Bolzano's moral theory. We will see that the basic structure of Bolzano's theory is at times obscured precisely because he is approaching his defence of utilitarianism under the lingering influence of Kant.

§2 Three conceptions of willing

A. Pure Causal View. Kant and Bolzano take it as obvious that willings are distinguished from other mental states by the fact that in normal (or at least successful) cases they cause the state of affairs that they represent to become actual. Accordingly, one way for moral theory to conceive of willings is as ways of causing changes in the world. If a moral theorist views willings *exclusively* as 'causal levers',⁹ then the natural position to take is that they possess, at most, instrumental value. In particular, willings have instrumental value when they bring about intrinsic goods such as happiness, which themselves have no necessary connection to

⁷ One reason that constraints would not be especially fruitful as a guiding theme when comparing our philosophers is that Bolzano is unrealistically optimistic about his ability to use empirical truths concerning precedent effects and special pleading to mimic *some* deontological constraints (R §89.2, I: 238f.).

⁸ Bolzano's conception of willing (R §15.11, I: 44f.) appears to be directly modelled on Kantian *Willkür* (*Metaphysics of Morals* [MS] 6: 213). Kant's *Wille* is not directly relevant to this chapter, and it should not be assumed that "willings" are actualizations of *Wille*.

⁹ To use Bernard Williams' classic metaphor (1973, 115).

that willing.¹⁰ The Pure Causal View limits the moral relevance of willings to their causal role in bringing about other goods. It falls at one end of a spectrum of positions. Nobody disputes that some willings have, perhaps among other kinds of value, instrumental value. The Pure Causal View distinguishes itself by saying that willings have *only* instrumental value.

B. Fitting Attitude View. At the opposite end of this spectrum is the view that willings are properly understood, at least for purposes of moral theory, primarily as attitudes, where attitudes are not properly evaluated by what they cause, if anything, but instead by whether they fit their intentional objects. Instead of evaluating a willing according to its causal relations, this view makes intentional relations the key to the moral relevance of our willings. It asks, “Is this willing appropriate to the nature of its object?”¹¹ Since willings generally have effects, this *Fitting Attitude View* does not deny that some willings have instrumental value.¹² What distinguishes it is instead its claim that moral theory properly considers willings *in the first place* as attitudes. There are various ways to spell out the italicized phrase. However, I will limit Fitting Attitude Views to those holding that the deontic status of a willing can be determined by

¹⁰ This essay will treat “value” and “goodness” as precise synonyms. Many of its claims regarding value are naturally supplemented by analogous claims regarding disvalue. So as not to overburden the text with cases that are easily supplied by the reader, I generally dispense with separate mention of disvalue.

¹¹ It is important to distinguish this view from “fitting attitude” accounts of value, which belong to meta-axiology. Those accounts hold that what it is for x to be valuable is, roughly, for x to be the fitting object of a pro-attitude. The (A) Pure Causal View and (B) Fitting Attitude View are instead positions on which aspects of willing are relevant to normative theory. Neither is a position in meta-axiology or metaethics – at least in the standard, rather narrow sense of those terms in which deciding between various versions of realism and irrealism is their central task. Schapiro (2001) uses positions apparently similar to (A) and (B) in order to motivate a third option, which she identifies with Kant. The surface similarity is misleading, however, since Schapiro takes metaethical issues into account in carving up the logical space. The position similar to (B) is tied to non-reductive normative realism, and Schapiro thinks that a third option, unrelated to my (C), must be found for Kant because he rejects such realism (2001, 99).

¹² One might at first assume that the instrumental value of willings can be ruled out if the Fitting Attitude View is defined so that there are no goods other than fitting attitudes. That is to be avoided in the first place because it would artificially narrow the scope for Fitting Attitude Views. But it also wouldn’t work, since my adopting an attitude is instrumentally valuable (beyond any intrinsic value it might have) if it leads someone else or myself at a later time to adopt a fitting, and thus intrinsically valuable, attitude.

considering the willing solely as an intentionally directed attitude. This, I suggest, is what we find in Kant's famous claim that whether an action is obligatory, merely permissible, or prohibited does not depend even in part on its success or failure in realizing its end, but only on its "principle of willing" or maxim (*G* 4: 400). The question of what a willing 'says' is, in other words, lexically prior¹³ to the question of what it causes, even if those effects are themselves intrinsically valuable fitting attitudes.

On the reading I will be using here, Kant holds that the deontic status of each and every willing can be judged according to whether it fits, or is appropriate to, its intentional object. One barrier to seeing this is that Kant in effect claims, perhaps implausibly, that though individual willings might have various intentional objects by virtue of their disparate ends, there is one intentional object that all willings have in common, and we cannot avoid relating to it. Whatever particular effect agents might be aiming to bring about, Kant treats their maxims as standing in an intentional relation to *persons*.¹⁴ Though this proposal imports some foreign terminology, it can be recognized in Kant's treatment of the Formula of Humanity. An immoral willing treats persons – not in a causal sense of "treats", but in terms of what it judges about them¹⁵ – as if they were things to be used for the satisfaction of this or that desire, while they are really independent or self-standing.

Though they are both extreme positions, the Causal and Fitting Attitude Views have the virtue of being internally consistent and stable. I will contend that Bolzano does not manage to find a similarly stable position. He has on the one hand a deep attachment to the Pure Causal View. The fundamental question that Bolzano poses in his search for a

¹³ I.e. "an order which requires us to satisfy the first principle in the ordering before we can move on to the second, [etc.] ... A principle does not come into play until those previous to it are either fully met or do not apply" (Rawls 1971, 43).

¹⁴ Or perhaps to some of their properties. In any case, I take it that an agent's relation to *the moral law* (for instance, when acting out of respect for the law, cf. *G* 4: 400) is one and the same intentional relation, simply under a different name. A full defence of this point would require that we explain why a subject who has one of the universalization formulas as an intentional object is thereby also representing persons as "ends in themselves".

¹⁵ *G* 4: 429. Of course, the immoral willing might *also* affect another person causally, but that is neither necessary nor part of Kant's account of what is wrong with immoral maxims. Potentially confusing is also the fact that in Kant's view having a fitting attitude towards persons requires that we intend to promote certain ends (e.g. their happiness), and we cannot do so without trying to manipulate the causal levers that the world presents us. However, on the current reading these duties are explained by our more fundamental duty to have a fitting attitude.

supreme principle of morality is, “How ought the world to be changed?” If our willings were unable to make any causal difference, there would be no subject matter for moral theory. Yet on the other hand Bolzano seems to think that our willings have more than merely instrumental value, and below we will see him take positions that make willings morally significant by virtue of what they mean. In sum, I will suggest that Bolzano didn’t manage to find a stable position, but instead bounces back and forth between the two poles of the Causal and Fitting Attitude Views. The fact that these are the two extremes to which he is attracted is made more interesting if, as I claim, Kant in fact exemplifies one of them in undiluted form.

Before we begin looking at how Kant and Bolzano develop these views, it makes sense to consider a stable position that falls between the two. Neither philosopher has it in view as an option. This is significant for Bolzano because it might actually satisfy all of his desiderata. It is relevant to Kant because his case that impartial and egoistic consequentialism both treat our willings primarily as causal interventions in the world is threatened by it.

C. Moorean Compromise. One way to craft an alternative to the Fitting Attitude View is, namely, to adopt its insight that willings are at least sometimes intentionally directed attitudes whose intrinsic value varies with fit, while denying that it makes sense for moral theory to view them primarily as attitudes. Each willing is potentially an attitude *and* a cause, and both factors must be weighed as *pro tanto* reasons in determining all-things-considered duty, with neither enjoying lexical priority. We find an influential version of this approach in G.E. Moore’s claim that the two greatest goods are the attitudes of enjoyment of the beautiful and love of persons,¹⁶ which he combines with the insistence that no answer to the question of what ought to be done in particular circumstances (i.e. all-things-considered duty) will be justified unless it takes into account both the intrinsic value of possible attitudes and how each option open to us would change the world.¹⁷ Moore has additional

¹⁶ There are actually four intuitively plausible types of attitude: (1) a pro-attitude towards something valuable is itself valuable; (2) a con-attitude towards something disvaluable is itself valuable; (3) a pro-attitude towards something disvaluable is itself disvaluable; and (4) a con-attitude towards something valuable is itself disvaluable. For convenience the text will for the most part just mention (1); however, it is important to remember that some attitudes instantiate the anti-fit of (3) or (4).

¹⁷ On the former, see Moore 1903, §113f., §122; on the latter, see esp. §17.

relevance in the current context because of far-ranging similarities between his theory and Bolzano's, starting with the fact that Moore develops his insights within a maximizing consequentialist framework. Moore's view is that one ought to undertake the act with the best consequences, one of which is always the trivial consequence of one's willing that act, which can itself be an attitude of positive, negative, or neutral value. There are two aspects of this *Moorean Compromise* that we do well to highlight for present purposes.

First, in Moore's own work we find a formal difference between aesthetic appreciation and the love of persons that will prove instructive as we consider Bolzano's theory of value. According to Moore, when one has a positive attitude towards a person, this intentional object is by itself a great positive value, and it would still possess this value even if nobody ever took up an attitude towards it. This claim regarding second-order value seems to many to make sense. The attitude gets the value of its object right, and it is plausible that such attitudes have their own value in virtue of that fit. Moore argues that admiring contemplation of the beautiful differs in that beautiful things by themselves have no value.¹⁸ It follows that whatever such an attitude fits, it cannot be the intentional object's *value*. It is important to be clear that there is nothing incoherent in this position. Nonetheless, many people find it more difficult to accept, and some find it downright paradoxical. Whether or not one finds it plausible that such attitudes are intrinsically valuable, all should agree that a theory that recognizes them introduces a third fundamental category of valuable entity. First, there are *basic goods*, whose value need not depend on their being intentionally directed. These plausibly include at least some pleasures, as well as states of affairs that are realized in things which have no capacity for intentionality (e.g. a wilderness minus its living inhabitants). Second, there are what I will term *standard attitudes*, whose value consists in their fitting *the value* of an intentional object, whether that object is a basic good or itself an intrinsically valuable attitude. Third, there are *generative attitudes*, whose value consists in their fitting some property of an intentional object, though by hypothesis it cannot be that object's value. There are no relations of implication constraining which combinations of these three are posited by a theory.

¹⁸ Moore often hedges by allowing that beautiful things have "negligible" intrinsic value (1903, §113, cf. §50). In other contexts he implies that they have no intrinsic value (§121, at page 202).

Second, Moore has a more restrictive conception of intrinsically valuable attitudes than was presented in (B)'s introduction of the notion above:

1. Moore requires that there be an emotional component in any intrinsically valuable attitude.
2. Moore does not focus on cases in which a subject is not consciously aware of which object is the intentional object of her attitude.
3. As a consequence, Moore shows no signs of thinking that *willings* generally (much less always) qualify as attitudes of the relevant kind. Moore is concerned with what we can call *pure attitudes*, which will be my term for attitudes that are not partially constituted by their bearers taking them to be causally efficacious.

Though at first glance these differences appear to be major, we can ignore them in this essay. The main reason is that many versions of the Fitting Attitude View – including, I would argue, Kant's – agree in principle with Moore on (1) and (2), while simply opting for less demanding interpretations of what these requirements amount to. This is the situation with (2), since Kant claims that we all have an indistinct, non-technical grasp of the content of the moral law. Moreover, he claims that in our daily lives we are often obscurely conscious of it.¹⁹ Regarding (1) Kant thinks that a robust principle of action (which includes a specification of act-type, situation, and end) does not actually qualify as a maxim unless the agent is to some extent motivated by it. This is already a baseline emotional directedness.²⁰ We find the same basic idea applied more narrowly just to the ends of action in the work of the neo-Moorean, Thomas Hurka. Since the active pursuit of end E arises from a desire for E, willing the realization of E should count as one of the primary forms of having a positive attitude towards E.²¹ This idea, whether in its Kantian or neo-Moorean guise, shows us why treating willings as attitudes needn't be a category mistake. Moreover, acts that are not merely

¹⁹ G 4: 402; 4:424; *KprV* 5: 8; 5: 32. It could be argued from the other direction that Moore virtually ignores cases in which subjects are not themselves *fully* clear about the description under which something is the intentional object of their attitudes, and that such cases are the most common type for beings like us.

²⁰ Cf. G 4: 413; 4: 45934; and *KprV* 5: 90.

²¹ Hurka's brief case is worth quoting in full: "To 'love' *x* is to be positively oriented towards *x* in one's desires, actions, or feelings, or, more generally, in one's attitudes. This positive orientation has three main forms. One can love *x*

“legal” but are willed out of respect for the moral law rise above this and are such that Moore himself would recognize them as exemplary attitudes.²² We will see below (§7) that such attitudes play an important role in both Kant’s and Bolzano’s projects.

Now that we have surveyed three ways that moral theory can treat willings as significant, the next four sections (§§3–6) will present the core tenets of Bolzano’s position. I begin with a look at Bolzano’s basic toolkit for theorizing about the practical (§3), followed by a reconstruction of his theory of value (§4). This will put us in position to evaluate Bolzano’s argument for utilitarianism (§§5–6). These sections paint the portrait of a thinker who hews closely to the Pure Causal View. The final section (§7) focuses on Bolzano’s position on moral worth, which has all of the hallmarks of an appeal to the intrinsic value of a standard attitude. I will argue that Bolzano is conflicted as to what is really valuable. Is it intrinsically valuable willings *qua* attitudes that matter? Or is it happiness? One eminently reasonable reaction is “both”. If one further allows that it makes sense to weigh these goods on a single scale and to trade them off against one another, then one has taken the Moorean Compromise. Bolzano doesn’t take it. At this point Kant will briefly re-enter the conversation in a more prominent way. I suggest that we think of Kant as posing this question: With respect to *which* intentional objects is it plausible that a willing *qua* attitude possesses intrinsic value? We have seen Hurka propose that the ends to be effected in routine productive action fit that bill. Kant’s blanket denial of this is bold, if also somewhat worrying. He claims that no mere state of affairs (not even the greatest happiness for the greatest number) makes an attitude *intrinsically* valuable by virtue of serving as its intentional object. On this point at least Bolzano may agree with Kant.

by desiring or wishing for it when it does not obtain, by actively pursuing it to make it obtain, or by taking pleasure in it when it does obtain... Regardless of whether this action [in the second case] succeeds in its aim, its origin in a desire for x makes it a form of loving x ” (2001, 13f.). The notion of desire invoked here is thinner than the emotional component in the attitudes that Moore highlights (e.g. 1903, §114). Moore’s and Hurka’s insights can be married by conceding that all willing qualifies as an attitude in a minimal sense, while allowing that a more intense focus on x than is required for intentional action, as well as more subtle shades of emotional engagement than simply the desire requisite for any motivation, can alter the attitude’s intrinsic value. So while working to create a beautiful thing might have positive intrinsic value m , doing so while also reacting to its beauty in an emotionally attentive manner might have value $m + n$.

²² Cf. *KpV* 5: 71ff.

§3 Ought

Bolzano follows Kant in holding that practical truths are distinguished by the fact that they contain the concept OUGHT. All other truths are theoretical.²³ Of course, *any* concept (including, e.g. RUTEBEGA) could be used to partition the set of all truths, but this particular division is of special importance to our philosophers because they believe that only propositions containing OUGHT are immediately and rationally action-guiding.²⁴ Bolzano might have this concept's key role in agency in mind when he declares OUGHT to be "one of the most noteworthy in the whole sphere of human knowledge". However, this extraordinary role cannot be clarified through definition, as "it appears to be a completely *simple concept*, which cannot be further explained or analyzed".²⁵

Quite possibly under the influence of Kant, and of pivotal importance for his resulting position, Bolzano takes ought to imply can. Unlike Kant, however, Bolzano does not use the principle in order to establish the possibility of anything.²⁶ Bolzano instead uses it to winnow out what might otherwise be considered candidates for the supreme moral law, the true OUGHT-proposition which in combination with theoretical truths explains the truth of all other practical propositions. It cannot, for example, be our duty to create substances, since that is impossible for finite beings. Bolzano's most significant application of this strategy is to be found in his arguments for:

²³ For Kant's claim, see *Critique of Pure Reason (KrV)* A633; Bolzano, *R* §15.10, I: 44; cf. *Science of Logic (WL)* §144, II: 71, regarding "Kant's service". I will follow Kant and Bolzano in using "imperative" incorrectly to refer to indicative propositions that predicate OUGHT of something.

²⁴ In particular, they believe that practical reason directs our actions through practical syllogisms containing OUGHT; cf. *KprV* 5: 90; and *R* §15.10, I: 44 for Bolzano's definition of practical reason as our capacity for cognizing propositions containing OUGHT.

²⁵ *R* §15.10, I: 43; cf. *WL* §144, II: 69. Bolzano holds that "S ought to ϕ " is a confused expression of [S has an ought of ϕ -ing], so that many colloquial uses of "sollen" and "ought" express a complex concept that includes both HAS and OUGHT (II: 70f.). In declaring OUGHT itself to be simple, Bolzano anticipates by six decades a famous Moorean tenet (1903, §13f.). Contrary to how Moore often presents his claim, it is not GOOD but OUGHT that is simple, since he analyses the former as OUGHT TO EXIST (1903, §13, at page 17; cf. Zimmerman 2001, 80). I will suggest in §4 that Bolzano agrees.

²⁶ Cp. *KprV* 5: 29f.

Restriction Thesis: it is only bare willings, and not actions, that can have oughtness truly predicated of them.²⁷

Though Bolzano is confident that ought implies can, he remains undecided between very different versions of this modal condition: “What someone ought to *will* must also be *possible* for him, or he must at least regard it as possible” (R §88.1, I: 230). In either case Bolzano is decisive in cautioning that the willing of an action must not be confused with any physical process, much less with its successful completion. I can will to set a piece of paper ablaze but then suddenly find that my hand is paralysed; and the same is true, at least in principle, of my willing *not* to burn the paper. Bolzano continues:

we can never say with complete certainty that one of these two actions will be possible for us; however we can say that it will be possible for us *to will either one*. For this reason, when we set out to express practical propositions accurately, the *ought* governs not the *action itself*, the production of an outer result, but instead only the *willing* of this result; that is, we do not say “You ought to do this or that” but rather “You ought to *will* this or that.” (R §15.11, I: 44f.)

Here Bolzano has in effect rejected (C₁), a condition on imperatives which would allow that there can be a true imperative commanding subject S to perform the action itself (as opposed to the mere willing of it) so long as S happens to *believe*, however irrationally, that she can burn the paper. In its place Bolzano seems to have adopted as his working criterion something like: (C₂) what an *epistemically rational* agent can be *certain* of. Bolzano assumes that epistemically rational agents can be completely certain, at least with respect to some actions, that they can *will* them, though they cannot be certain of anything more.²⁸

²⁷ According to WL §144 (II: 70), oughts are in the final analysis always adherences of agents, though Bolzano allows propositions that assert “the presence [*Vorhandensein*] of an ought..., without specifying the being” which possesses it. The true form of such propositions is [[An ought of B] is objectual], where B represents an action or, more precisely, a willing of an action.

²⁸ Why only *some* actions? Bolzano would appear to follow Kant (MS 6: 213) in holding that if an agent does not believe that she can bring a particular end about, she can only *wish* for it, not *will* an action with that end. But if it is impossible for an agent to will a particular action, then it is also impossible for her to exercise her freedom to will or not to will that action (R §15.10f., I: 44f.). So the fact that OUGHT governs only willings does not have as its consequence that it

Thus, nothing more than willings can be obligatory. This is Bolzano's core case for the Restriction Thesis, which has as a consequence that the supreme moral law can only concern what ought to be willed, not which actions or other states of affairs ought to exist. In coming sections I will argue, contrary to the consensus in existing secondary literature²⁹, that this obscures Bolzano's true case for his core commitment, his utilitarianism.

Bolzano's C_2 , since it depends on facts about what all subjects are able to know under somewhat idealized conditions, is 'less subjective' than C_1 . However, all of the modal conditions that are suggested by Bolzano's brief statements in some way make what agents are capable of – whether it be their actual physical powers or their beliefs and knowledge regarding those powers – a limit on what can possess oughtness. One worry is that there are a variety of modest departures from C_1 and C_2 , each of which might instead be thought to restrict the range of true predications of OUGHT, and it is unclear precisely which one is correct. For instance, a somewhat 'more subjective' condition than C_2 gains support from the sense many people have that there is something unfair and pointless in requiring an action (or even the corresponding willing) of an agent if she never would have *thought* to perform it except out of 'dumb luck'. For a case of this sort consider a rescue pilot who, with time running out, is patrolling a vast sea in search of the survivors of a shipwreck. Clearly it would be best for the pilot to focus all her efforts on the small portion of the sea – quadrant D17, let us say – that unbeknownst to her as a matter of fact contains the survivors. Moreover, she *can* be certain of her ability to will "search quadrant D17 assiduously before looking elsewhere", should she think to will it. Hence, Bolzano's own C_2 does not disallow a true imperative with that content. Yet Bolzano might still be reluctant to affirm that this is what the pilot *ought* to do, and this might lead him to formulate a new condition, C_3 , designed to capture all of C_2 while eliminating problem cases of this sort.

A second worry can be introduced by considering a scenario in which the pilot, despite the fact that concentrating on D17 is not even close to the optimal search strategy, measured by what she is justified in

could be true that I ought to jump over Chicago. Unless I am delusional, not only can't I complete that act, I can't will it. I am unsure of whether Bolzano is always clear about this implication of his view.

²⁹ Cp. Morscher (2013, §5) and, much more briefly, Rusnock/George (2007: 33).

believing, does it anyway. (Perhaps she does this out of a superstitious belief in the significance of her birthday, which falls on 17 December.) The pilot in fact rescues the survivors, who go on to lead long and happy lives. A proponent of C_3 is committed to denying that the pilot ought to focus on D17, and thus is committed to holding that the pilot did not do what she ought to have done. This is a highly problematic position to take for anyone, like Bolzano, who thinks that morality is about making the world a better place. Whatever one might conclude about the agent and her liability to blame, how could performing the optimal action ever be *wrong*? Nonetheless, *expected consequences consequentialism* (ECC) says that we ought always to perform the action that is expected (or rationally expected given available information, etc.) to maximally promote the good (in this case, happiness). As we will see below, Bolzano's argument for utilitarianism instead supports only *actual consequences consequentialism* (ACC), the position that we ought always to perform the action that is in fact optimal.³⁰ ACC itself says nothing about how agents can most effectively discover what they ought to do. Nor does it specify the conditions under which an act has moral value, including when it is appropriate to praise or blame an agent, including oneself. ACC is solely a criterion of correct action. My hypothesis is that Bolzano adopts the Restriction Thesis and whichever specific modal condition he takes to support it (C_2 , C_3 , etc.) as the result of a confusion. He has intuitions about when it is proper to praise and blame agents, which he misinterprets as a conceptual truth connecting C_2 or some similar principle to OUGHT. By disentangling the former from the latter, we can make Bolzano's position on OUGHT consistent with his arguments for ACC.

A different way to formulate this hypothesis is to suppose that Bolzano is confusing two concepts. There *is* a concept OUGHT of which it is true that it governs only adherences of subjects that meet a particular subject-involving modal condition. However, we can also form thoughts using another concept, OUGHT*, which is, so far as we can tell, the same as OUGHT, but for the fact that *true predications of it are not limited by any facts about what subjects can accomplish* (or can be certain they can accomplish, etc.). So there is no reason to doubt that I ought* to burn the paper, in addition to willing to burn it. Interestingly, so far as we can tell it is OUGHT* that is the simple concept, since it would appear that we can

³⁰ For a subtle discussion of issues related to ACC and ECC, see Driver (2012, chapters 5–6). In order to make the distinction clearer, I have formulated ACC and ECC in their maximizing versions.

derive OUGHT from it simply by *adding* one or more concepts.³¹ Finally, if we are looking for a partition among concepts that isn't merely arbitrary, one that might even cut the realm of concepts in themselves at one of its joints (as RUTEBEGA surely does not), then we do better to use OUGHT* than OUGHT. For this reason it further seems that Bolzano should choose OUGHT* rather than OUGHT as the concept determining the partition between the theoretical and the practical.³²

§4 Bolzano's theory of the good

As it is usually understood, consequentialism is a position in deontic theory, which studies what ought to be, including what ought to be done (or willed) by agents. As it is usually formulated, maximizing consequentialism says that agents ought to do whatever maximizes the good. It thus depends on results from axiology, the theory of the good. At first glance Bolzano might seem to follow this familiar pattern. Looking more closely, however, we find that his arguments appear to be carefully formulated so that he never has to tell us anything about the goodness of happiness. Instead, he limits himself to the deontic claim that we ought to will to bring about happiness. The same pattern holds with respect to all of the other candidate goods that Bolzano considers. He never directly addresses the value or disvalue of knowledge, for instance, instead limiting his attention to practical propositions commanding that one bring about knowledge. Bolzano's sheer consistency in avoiding axiological claims – I'll call it the *idiosyncrasy* – should make us wary of

³¹ I will formulate many points below using the concept OUGHT*. However, nothing substantive depends on choosing this way of stating my hypothesis about Bolzano's mistake, as opposed to the version that questions his belief that some version of "ought implies can" is a conceptual truth involving OUGHT. A disadvantage of conducting our discussion in the latter way would be that if there are indeed two concepts, OUGHT and OUGHT*, parties disagreeing about whether "ought implies can" might simply be talking past each other.

³² This raises an interesting issue in the comparison of Kant and Bolzano, which I can do no more than flag here. The role of the subject and its capacities in Kant's methodology would seem to allow him to appeal to the fact that *we unavoidably use* OUGHT, rather than OUGHT*, as justification for the former's pivotal role in his practical project. The reason that there is not a separate branch of philosophy built around RUTEBEGA is that its use is not bound up with our capacity for agency. In contrast, Bolzano's commitment to semantic objectivism would seem to push him in the direction of recognizing that certain concepts are by their own natures "joint-carving", while others are not (cf. Sider 2011).

dismissing its significance with a wave of the hand.³³ But does the idiosyncrasy tell us anything important about Bolzano's position?

As we will see in §7, there is one exception to Bolzano's quietism about value. He does explicitly allow that *willings* sometimes have value – in cases, namely, in which they have *moral* value. This might lead us to wonder whether the idiosyncrasy reflects the fact that Bolzano is treating willings that produce happiness as generative attitudes. On this proposal it is not that we ought to will to cause happiness because the latter is good. Instead, we ought to actualize willings that are valuable by virtue of their relation to happiness, though happiness is not itself valuable.³⁴ The problem with this suggestion is that Bolzano's arguments for a utilitarian supreme moral law treat the relation in which willings stand to happiness as causal, not intentional. What we ought to do is to will such that happiness is produced. For purposes of determining our duty there is no consideration of how, or under what descriptions, we might be intentionally directed to our ends. So it is just not plausible that Bolzano is treating them as intentionally directed attitudes for purposes of that argument. Below we will see that subsequent to concluding that the supreme law is utilitarian, Bolzano treats willings that possess moral worth as intentionally directed attitudes. Yet it is crucial to be clear that Bolzano's theory of moral worth plays no role in his argument for the utilitarian supreme practical principle.

A more promising alternative for making sense of Bolzano's axiological quietism is to read him as holding that claims about value are unnecessary in moral theory.³⁵ True, this would be a bold step, given that it seems both evident and morally relevant that some actions and states of affairs are *better* than others. Yet Bolzano might very well at least be able to craft a consistent position by adopting eliminativism with respect to moral worth. If ascribing moral worth to S is just an unclear way of judging that others *ought* to will to reward S,³⁶ then Bolzano can maintain that there

³³ To take just one example, NAK 197–198 includes numerous uses of deontic terms, and the text is explicit that the few uses of what at first appear to be axiological or evaluative terms (e.g. “the morally good”, NAK 198) are just a shorthand for what ought/ought not to be done. So far as I can tell, previous commentators have not noticed the idiosyncrasy.

³⁴ This would not automatically commit one to holding that one ought to actualize these attitudes *because* they are valuable. However natural, that would be a further claim.

³⁵ For two examples, cf. Scanlon (1998, chapter 2) and Velleman (2008, 270).

³⁶ For brief comments supporting such a reduction, cf. R §89.4, I: 240.

are no axiological truths at the foundational level, only deontic truths about first-order promotion of happiness and second-order rewarding of that promotion. This fits much of the textual evidence pretty well. However, in order to assess readings that see Bolzano as some sort of sceptic about value (or at least about its role in moral theory), we need to gain some sense of the alternatives. This, in turn, should lead us to ask how Bolzano might reconstruct the meaning of our ordinary, common-sense judgments about value.

One possibility is that neither GOOD nor OUGHT contains the other (in which case it follows from Bolzano's partition that axiological propositions are theoretical). Of course, Bolzano could simply hazard this as a guess, but a solid reason for maintaining it would be if he had an independent reason to think that GOOD is a simple concept. Yet GOOD is not among the concepts that Bolzano tells us he suspects of being simple (HAS, SOMETHING, NON, ACTUALITY, and OUGHT).³⁷ So perhaps the two concepts are partially coincident. But then since OUGHT is simple, this can only be the case if GOOD contains OUGHT. In my view this is indeed the interpretation that makes the best sense of Bolzano's entire position, though the full case for it will not emerge for a few sections. I propose, more specifically, that in practice Bolzano recognizes a composite concept, GOOD, which is formed from the simple concepts OUGHT and ACTUALITY. The basic idea is just that to attribute value or goodness to something is to hold that it ought to be actual. For this to work the ought in question needs to be a *pro tanto* ought (*ought_p*), i.e. an ought that applies as far as it goes, though it can be outweighed:³⁸

4. GOOD = _{Def.} OUGHT_p TO BE ACTUAL

This will allow us to explain what it is for x to be better than y : the *pro tanto* ought governing x outweighs the *pro tanto* ought governing y . Now, developing this proposal fully would involve making decisions on several technical issues that it is implausible Bolzano considered, given how little he says about these issues.³⁹ Yet my claim is not at all that

³⁷ For the citations of texts in which Bolzano makes these simplicity claims, see Morscher (2013, §3.5).

³⁸ Such oughts, in contrast to *prima facie* oughts (which will play no role in this chapter), do not cease to have genuine force when they are outweighed (cf. e.g. Reisner (2013)). Though Bolzano typically has all-things-considered oughts in mind, we will see in §5 that he does make implicit use of *pro tanto* oughts in his argument for utilitarianism.

³⁹ See Zimmerman (2001, 81f.) for a sketch of some of the problems with (4).

Bolzano had a fully worked out development of (4)'s basic idea up his sleeve. My claim is instead that we can make the best overall sense of Bolzano's argument for utilitarianism, characterized as it is by a studied avoidance of axiological claims, if we take him to be depending implicitly on the thought that some things are indeed good, and that their goodness consists in their possessing an ought of actuality. We will soon see that (4) can help us understand how it can make sense to Bolzano to conduct his inquiry into utilitarianism wholly in terms of OUGHT-claims. The answer, in brief, is that the axiology that he is depending on is, at the most fundamental level, likewise composed of OUGHT-claims. It can also help us to understand how Bolzano can be a utilitarian and yet not suspect that practical truths are ultimately explained by theoretical truths, as they would be if GOOD did not contain OUGHT.

Since willings are among the many sorts of things that can be actual, this proposal allows us to attribute to Bolzano the common-sense claim that if an agent ought to ϕ , then that agent's ϕ -ing is valuable or good. The immediate problem with using (4) as the basis for Bolzano's missing axiology is that on his official view OUGHT – and thus, one assumes, its *pro tanto* version as well – cannot be properly predicated of states of affairs that are not actions.⁴⁰ It might be that my burning of the paper ought to exist, but it cannot be true, strictly speaking, that the state of affairs of its burning ought to be actual. I will take this up at the start of the next section, where we will see that Bolzano's practice in his argument for utilitarianism seems to indicate otherwise.

Assuming for now that states of affairs do not pose a problem, the remainder of this section will return to Bolzano's belief that OUGHT can only be truly predicated of actions which meet a subject-involving modal condition. The problem is that our common-sense intuitions about goodness do not make any distinction between actions and states of affairs that are within, as opposed to beyond, the capabilities of currently actual agents to cause (to pick one of various slightly different versions). So, for the proposal to be fully fruitful we must insist, as I did in §3, that Bolzano is wrong about that condition. Put differently, we must insist that (4) be replaced with:

5. GOOD = _{Def.} OUGHT*_p TO BE ACTUAL

⁴⁰ According to *WL* §144 (II: 70), the form of ought-propositions is [A has an ought of B], where [B] denotes an action. The action-requirement is not specifically discussed, much less defended, but there is no doubt that this is Bolzano's assumption.

The worry about this is that it is revisionist, as it makes use of a concept that Bolzano would seem not to recognize. However, we are now ready to examine Bolzano's analysis of the concept *RIGHTFUL*, which gives us reason to reconsider this.

Bolzano's understanding of the relation of *RIGHTFUL* to moral duties is problematic, but for current purposes it will suffice to note that someone can have a right to do something even though it is not morally correct.⁴¹ For instance, a person can have a right to use her property in a manner that does not maximize happiness. Bolzano's initial analysis is that S has a right to ϕ iff all others ought not to prevent S from ϕ -ing. Correlatively, it is unrightful for S to ϕ iff some others ought to prevent S from ϕ -ing.⁴² Bolzano immediately recognizes that on this analysis whether others can have a duty to prevent S's ϕ -ing will depend "on the accidental circumstance whether the people in the area have enough power successfully to oppose it." In order to show that this is a defect in the analysis, Bolzano appeals to a case in which S's action strikes us as clearly unrightful, yet S is sufficiently powerful and isolated that nobody can prevent S's ϕ -ing (or nobody can rationally believe that she can prevent it, etc.). Bolzano corrects this defect by making the criterion counterfactual: "we say that an action is *unrightful* if its inner character is such that there would be a duty forcibly to oppose it if a sufficiently large number of people were present..." Once one has allowed this change, it's difficult to see why the relevant counterfactual scenarios shouldn't include an omnipotent agent. This should in turn make clear that though Bolzano chose to use a counterfactual, there would have been no important difference if he had simply used *OUGHT** instead.

Bolzano finds it obvious "that by the *rightfulness* or *unrightfulness* of our actions we think of certain properties that belong to their *inner*

⁴¹ One of Bolzano's ambitions is to counter those who claim that rights and ethics can conflict, but certain claims that he makes are inconsistent with the pair of (i) agent-neutrality and (ii) *maximizing* consequentialism, both of which Bolzano embraces in *Theory of Religion*. I will suggest in §6 that Bolzano is actually wavering in his commitment to (ii). One factor that complicates the interpretative task is that Bolzano's analysis of rights is contained in a different work, "On the Right of the Clergy to obtain their Livelihood from Persons not of their Faith" (1838). All subsequent quotations in this section are taken from §1 (2007, 145f.).

⁴² Bolzano thereby brilliantly anticipates one element of Wesley Hohfeld's influential (and now standard) analysis of rights (Hohfeld 1919), namely the element that is often disambiguated as a "claim-right" against all other agents (Wenar 2011, §2.1.2).

essence and are entirely detached from the circumstance whether at this time there is more or less power of resistance". I've proposed that Bolzano is most reasonably read as presupposing a formally identical account of goodness. A state of affairs or action x is good if its "inner character" or "essence" is such that it ought to be actual, apart from whether circumstances external to that state of affairs make it possible for agents to make it actual. One way to make this vivid would be to ask what an omnipotent agent, given the choice whether to create x or to allow it never to exist, ought_p to do.⁴³

It is worth asking why we find hints that Bolzano would allow this abandonment of the subject-involving modal constraint in his treatment of rights, while similar hints are absent from his comments on oughtness itself. The answer, I speculate, is that when Bolzano is thinking about rights his attention is diverted from the issues of praise, blame and moral worth. In order to capture his thought that S is acting unrightfully most economically, he needs the thought that others *ought** to intercede, and it simply seems to him beside the point whether they are in the right location, understand the situation correctly, would have thought to do so, and the like – and, thus, whether they *ought* to intercede. This provides some evidence for the hypothesis that ended the previous section: though often our thoughts about obligation, broadly construed, include assumptions about agents' abilities, their subjective states, and related issues that we regard as relevant to praise or blame, the *simpler* thought is that the bystanders *ought** to prevent S 's outrageous behaviour. And this apparently simpler concept is one, as we will see in the next section, that Bolzano himself uses.⁴⁴

This section began by noting Bolzano's studied avoidance of axiological claims in his core treatments of the moral law. I conclude that we

⁴³ A test similar to this is among the several analyses of goodness that Moore proposes (1942, 600; cf. Zimmerman 2001, 79ff.). With more space I would argue that Bolzano, if he is to be consistent in following a methodology that does not privilege a Kantian analysis of our capacities, should join Moore in holding that the best possible world might be "composed of qualities which we cannot even imagine" (1903, §111.1). Agents in our epistemic position cannot make these qualities actual, except perhaps by accident, because they would not even *think* to do so.

⁴⁴ Our interest in whether Bolzano would recognize OUGHT* makes it worth noting how Bolzano handles an analogous worry regarding RIGHT. In response to those who would quibble with his final analysis, Bolzano argues that even if it does not capture our *word* "right", "some different name would have to be found" for the concept that he has defined, since it is "highly important" (2007, 146).

should not interpret this idiosyncrasy as the symptom of a monumental commitment, such as Bolzano's having forsworn all axiology. However, this does not mean that the idiosyncrasy is merely accidental or wholly unimportant. It should instead be seen as one element in Bolzano's Kantian tendency to place agents at the centre of morality, at least rhetorically, even when the underlying theory itself does not warrant this central placement. We will see more symptoms of this tendency in the next section.

§5 Bolzano's argument for utilitarianism

Both philosophers take moral laws to be practical truths, but Bolzano, unlike Kant, has an extensive theory of grounding that he uses to explain how the *supreme* moral law differs from other moral truths.⁴⁵ It is, namely, "a practical truth from which every other practical truth (thus, also every particular human duty) can be *objectively* derived, i.e. as a *consequence* is from its *ground*" (R §87.1, I: 228). A grounding relation (*Abfolge*) obtains between a ground and a consequence, both of which are true propositions or sets of true propositions. This makes this relation different from causality, which holds between actual things. It also differs from derivability (*Ableitbarkeit*) by virtue of being irreflexive, intransitive, asymmetric, as well as by holding only between *true* propositions. Nor is grounding an epistemic relation. A truth about the state of a thermometer is, let us suppose, a completely reliable indicator that this room is below 8° C, yet the former is not the ground of the latter, but rather the reverse. Bolzano argues that "One ought to follow God's will" is likewise fully reliable regarding particular human duties, yet it does not ground them: "For it is not because God wills (i.e., commands) this or that that it ought to be willed by us but rather the reverse. Because it *ought* to be willed by us God wills or commands it for us."⁴⁶

⁴⁵ Kant clearly attaches importance to the distinction between the "grounding law [*Grundgesetz*] of pure practical reason" (e.g. *KprV* 5: 30) and other moral laws, but he tells us much less than Bolzano does about how to understand that relation (cf. *KrV* A737). One benefit of reading Kant's and Bolzano's moral theories alongside one another is that it raises this neglected question. Unfortunately, I won't be able to pursue it here.

⁴⁶ R §90, I: 247; cf. I: 243f. for two further cases. Notice that the previous section's axiological criterion involving an omnipotent agent is parallel to this extensionally adequate, but not grounding, moral truth.

Bolzano argues that the supreme moral law is:

6. One ought to will the act which, among all possible acts, maximally promotes the happiness of all when all consequences are taken into account.⁴⁷

This practical truth is not supposed to be the *complete* ground of all other practical truths. Bolzano's idea is instead that the supreme moral law and a single, well-chosen "theoretical" instrumental bridging truth of the form [Whoever wills A, must also will B] can be used to derive any and all particular, immediately action-guiding practical truths.⁴⁸ As the partial ground of all other practical truths, the supreme moral law is at once the starting point for all scientific proofs⁴⁹ that ϕ -ing ought to be willed, as well as that which provides the partial explanation (and the full *practical* explanation) for *why* ϕ -ing ought to be willed.

Bolzano's argument falls into two parts. He first argues that there is a single, supreme moral law. Then he argues that nothing other than (6) could be that law. Bolzano makes quick work of the *first part* (as I will refer to it below). It is a deliverance of common-sense that there are at least some practical truths.⁵⁰ Assuming that there are two or more practical truths standing in a ground-consequence relation, one of them must lack any "further condition", since there cannot be an infinite

⁴⁷ R §88.5, I: 236; cf. NAK 199. Bolzano actually says "virtue and happiness", but then he clarifies that this does not alter in the least the extension of the set of obligatory willings, and that virtue is added only for the subjective purpose of reminding us that one way to bring about happiness is by influencing the choices of other agents (cf. R §88.5, I: 236; WL §447, IV: 119). However, in some passages, including the first of these (R §88.5), Bolzano does not in fact abide by this stricture. So far as I am aware, commentators have given Bolzano a pass on his attempt to have his cake and eat it too (e.g. Künne 1996, 313f.). This is another example of Bolzano encouraging us to view agents and their intentions as the very centre of his project, even when his arguments do not support this Kantian flavour.

⁴⁸ Cf. R §87.2, I: 228 for Bolzano's treatment of the bridging truth, which we will return to below. Cf. R §90.5, I: 244 for evidence that a *single* such truth is sufficient in each case. This shouldn't, of course, be confused with the suggestion that this is the customary manner in which an agent comes to believe practical truths that are sufficiently specific to be *directly* followed, as "cause as much happiness as possible" never is.

⁴⁹ On the relation of science to grounding, cf. WL §221, III: 388; Sebestik (2012, §7); and Lapointe (2011, 91ff.).

⁵⁰ Bolzano further suggests that the simplicity of OUGHT makes it especially safe to trust common-sense in this case. His idea, versions of which can be found among the classic British empiricists, is that empty subjective representations are

regress of grounds. If there turn out to be two or more non-coincident ground-consequence chains, then the supreme moral law will simply take the form of a conjunction, with one conjunct corresponding to the unconditioned practical truth at the head of each chain: [A and B (and...) ought to be willed] (*R* §87.2, I: 229f.). This argument suffers from a non-obvious flaw that is best introduced after further preparation. In contrast, Bolzano's silent assumption that no set of theoretical truths can be the complete ground of a practical truth is already sufficiently clear. Bolzano joins a long tradition, including Kant, in assuming that no *is* can fully explain an *ought*.⁵¹ A second point to notice is that this argument does not actually suffice to show that there is a single practical truth that grounds *every* other practical truth. Because grounding is intransitive, [A and B ought to be willed] might be the unconditioned first link in a chain of (partial) grounds and consequences that stretches all the way to quite specific, action-guiding practical truths such as [Jones-at- t_1 ought to will to help her neighbour shovel the driveway], without the former *grounding* the latter. Bolzano's confidence that a single practical truth grounds all other practical truths apparently depends on his assumption that (6), once combined with a single well-chosen instrumental premise, suffices to explain any practical truth, no matter how specific.

The starting point for the deceptively difficult second part of the argument is Bolzano's by now familiar belief that only that which agents can actually bring about need to be considered in determining what ought to be done. Now, in one sense the bare minimum that an agent can realize are pure attitudes such as enjoyment of the beautiful, i.e. attitudes that are in relevant respects causally inert. Yet, crucially, Bolzano does not consider the possibility that we ought to adopt pure attitudes such as the love of a person. Instead, he limits his attention to willings that are intended to be causally efficacious. So far the only problem is that Bolzano fails to consider one candidate act-type (pure attitudes). However, his next step is to sort willings into act-types according to their *actual* effects. For example, acts causing happiness are distinguished from acts causing knowledge. This amounts to an unacknowledged abandonment of the Restriction Thesis (§3). Subject to certain

always the result of infelicitous combinations of simple subjective representations (cf. *NAK* 49). Though one might of course predicate a simple subjective representation such as OUGHT of the wrong thing, there must be something that has oughtness, since otherwise we would not possess the subjective representation.

⁵¹ "...it is most reprehensible to derive the laws concerning what I *ought to do* from what *is done*, or to want to limit the former by the latter" (*KrV* A319).

conditions, agents *can* control whether they will with the intention of causing some bit of happiness to exist. Yet whether this happiness actually results is a different matter, as Bolzano's own example of willing to burn paper is designed to show. What this shows is that Bolzano is in effect making use of OUGHT*, rather than OUGHT, when setting up the second part of his argument.

Before we get into the details of that argument, let's consider the conception of action that underlies it. What we find, I submit, is that throughout this argument Bolzano is treating willing, and action more generally, as *transparent* with respect to its effects. He "looks completely *through* agency" to the states of affairs that it brings about.⁵² This is initially somewhat surprising in a thinker who is as concerned as is Bolzano to emphasize that oughts do not govern physical processes. Yet Bolzano's commitment to transparency is evident, first, in the way that he sorts acts into different types. Rather than distinguishing acts according to agents' intentions or expectations (neither of which is even mentioned), Bolzano classifies acts by what they actually cause. For a stark alternative we can look to Kant, who assumes that each action has a maxim which makes it the act that it is. For purposes of moral theory (in contrast to history and various other pursuits), an act with maxim M would not change types even if it were embedded in a different world and had different effects. In contrast, the very notion of an act-type remains strictly speaking undefined within Bolzano's argument, since he leaves it indeterminate how far from the act (e.g. how far into the future) consideration properly extends. The same act might be both a net cause of happiness and a net cause of unhappiness, depending on whether our view is ten minutes or ten years. One sensible fix would be to mandate that the proper viewpoint takes all effects whatsoever into account, thereby eliminating the need to defend an apparently arbitrary cut-off. The important point, though, is that Bolzano himself feels no pressure to make such a move. Neither does Bolzano take the opposite tack of separating what belongs to the act itself from its external effects. What matters morally – at least for purposes of discovering the supreme moral law – are just changes to the external world, beginning with changes in the agent's body. So there is no need to carve out a space for something that is distinctively act.⁵³

⁵² This is Darwall's evocative description of Cumberland's position (1995, 97, also quoted at Schapiro 2001, 94).

⁵³ Another relevant fact about acts, as Bolzano conceives of them, is that it does not matter who 'does' them. This commitment to agent-neutrality emerges

Now it should be clearer why allowing OUGHT* to be predicated not just of actions but also of states of affairs, as must done if the account of goodness contained in (4)–(5) is to be plausible, fits Bolzano's underlying practice, even though it conflicts with his stated views about "ought". My suggestion, in brief, is that often when Bolzano writes about "actions" he really just means "cause of a change in the world". The contexts in which these terms are *not* interchangeable are, as we will see in §7, precisely the contexts in which he is considering whether a willing, quite apart from whether it is morally correct, possesses moral worth. Why, then, does Bolzano insist that oughts govern willings, and willings alone? What is special about willings, which propels them to their central nominal role within Bolzano's argument, is just that they are subject to our *free* control, so that it is appropriate to hold us responsible when those causal levers are not engaged. Nonetheless, while Bolzano is inquiring into the content of the moral law he is still always asking which causal levers ought* to be pulled.⁵⁴

The transparency of willing leads to the following important correction to Bolzano's position. What ought* to be actual by virtue of its intrinsic properties are states of happiness, not willings of a certain type. Any willing that ought* to exist has this property *because* states of happiness ought* to exist. But then no proposition that concerns willings alone, such as (6), can qualify as the *supreme* practical proposition. This is a straightforward consequence of the following relation: the truth that happiness ought* to be actual *explains* the truth that willings that cause happiness ought* to be actual. That relation is not merely inferential or evidential. Rather, the truth about states of happiness is the ground of the truth about willings. Thus, already from the set-up of the argument it would appear that Bolzano in fact treats:

most clearly when Bolzano argues that a duty to affect how people will cannot be any part of the supreme moral law. He reasons that [one ought to endeavour to bring it about that others will what ought to be willed] is simply equivalent to [It ought to be willed that what ought to be willed, be willed], and is thus an identical proposition (NAK 197f.; R §88.3, I: 233f.). By itself this is not enough to demonstrate that Bolzano treats action as transparent. However, agent-neutrality follows trivially from transparency, so it is reasonable to take agent-neutrality as further evidence of Bolzano's commitment to transparency.

⁵⁴ Morscher misses this, which leads him to interpret Bolzano as making the deontic status of a willing dependent at least on the agent's intention, and perhaps also on whether it is done because it is moral (2013, §5.3). Cf. Rusnock/Rose (2007, 33) for a similar mistake.

7. [Happiness ought*_p to be actual]

as the supreme practical truth. In order to see how strange it would be for Bolzano not to be making unacknowledged use of the goodness of happiness, consider generative attitudes. Some find it too strange to suppose that an attitude can be intrinsically valuable, though its intentional object is neither valuable nor disvaluable. (This is just what a generative attitude is.) But it is just plain bizarre to suppose that something that is *understood simply as the cause of x* itself ought* to be, though *x* is neither valuable nor ought* to be. Rather than attribute this bizarre commitment to Bolzano, it is more reasonable to suppose that he found it difficult to *formulate* the axiological thought reconstructed as (7), though he made use of it nonetheless. One explanation is that Bolzano's acceptance of the Restriction Thesis (along with the associated modal condition) blinded him to the possibility of reconstructing goodness as, at base, a deontic property. Were GOOD instead a concept that did not contain any ought-concepts, the transparency of willing would tend to indicate that (6)'s complete ground is a *theoretical* truth, i.e. that an *is* can fully explain an *ought*. Bolzano would not be alone among philosophers in finding this to be patently false. Now we are ready to examine the second part of Bolzano's argument, in part to see whether it provides any further evidence for the hypothesis that Bolzano treats (7) as the supreme practical truth.

Bolzano begins the second part of his argument by proposing an exhaustive division of what are treated as act-types. Some actions create substances; others have effects on the condition of lifeless beings, or on the condition of living beings. The last category is then divided into acts that have effects on a living being's will, capacity for knowledge, capacity for desire, or capacity for sensation. Within the final category Bolzano then effects a division into unpleasant and pleasant sensations (or "happiness"),⁵⁵ this time without any appeal to the differences between capacities. The ensuing argument takes the form of an argument by elimination. In a series of negative arguments all of the candidates aside from happiness are considered and rejected. That is, there is no "original" duty to change the state of lifeless beings, nor an

⁵⁵ Bolzano rejects speciesism, telling us that he includes "the happiness of every being that is capable of sensation," including "merely animal beings" (*R* §88.3, I: 234; cf. also *NAK* 198f.). That said, §88.4 (I: 235) leaves open the possibility that the pleasures of less developed animals might count for less than those of humans.

“unconditioned” duty to promote knowledge, etc. Insofar as any of us ever ought to cause a change in a lifeless thing or spread some knowledge, these practical truths are explained by a more original duty, a fact that can be seen from the exceptions to which these derivative duties are subject. With these negative arguments (to which we will return shortly) complete, and having already argued that there is a supreme moral law, Bolzano is then ready to state:

Intermediate Conclusion: “The promotion of happiness is thus the only genuine original command of reason...”. (*R* §88.4, I: 235; cf. *NAK* 198)⁵⁶

Though the texts tend to obscure this fact, the Intermediate Conclusion marks a turning point in Bolzano’s argument. It is a result concerning non-derivative *pro tanto* duties. (Though investigation has turned up only one such duty, it could in principle have identified two or more, each of which would apply as far as it goes.) Only afterwards does Bolzano shift his focus to all-things-considered duty. One indication of this is that only after the Intermediate Conclusion is it made clear that in an exclusive choice between two acts, both of which cause happiness, one ought (i.e. all-things-considered ought) to will the act that causes the *most* happiness. It is likewise only at this point that Bolzano asserts that morality demands maximization.

⁵⁶ Bolzano presents his case for utilitarianism quite pointedly as an argument by elimination (cf. *NAK* 198; *R* §88.3, I: 234). This is a problem, for while an argument from elimination works just fine when only extensional considerations are relevant, it is not a reliable means of identifying a *ground*. This can be made clear if we consider a formally identical argument purporting to show that the promotion of rubeegas is “the only genuine original command of reason”. We would begin by making the extensional observation that all possible actions can be divided into those with effects on rubeegas, eggplants, leeks, and other types of beings. Then, after eliminating the candidate duties to affect eggplants, leeks, and all other types of beings, we would be ready to conclude that there must be a duty to affect rubeegas, since otherwise there would be no original duty, contrary to the first part’s conclusion. What this shows is that an argument by elimination will not accomplish Bolzano’s task unless we can be sure that the space of options has been divided up so that at least one of them contains an original truth. Yet if we knew *that* we would have no need for the argument by elimination in the first place. Getting clear about this can help us avoid the illusion, encouraged by this argument form, that there is a reliable method for isolating the precise factor(s) that ground all practical truths. As noted above (fn. 43), we may even lack epistemic access to the truly original factors.

§6 Assessing the argument

We can begin our assessment of the argument by noting an ambiguity that pervades the series of negative arguments. In brief, Bolzano fails to distinguish between instrumental and intrinsic value. As a result, he does not distinguish between two types of exceptions, one of which is probative and the other of which is irrelevant to his inquiry. If Bolzano were to find even a single instance of happiness that does not possess *intrinsic* value, then he could conclude, given the assumed transparency of agency, that there is no *pro tanto* duty to produce happiness. Accordingly, what he needs to argue is that nothing other than happiness possesses intrinsic value. Yet all of the evidence he adduces concerns our all-things-considered duties, and those truths also take account of the instrumental value or disvalue of particular changes to the world. Even on Bolzano's theory I very often ought* (all things considered) to cause some pain, since its downstream effects outweigh the more immediate bad of that pain. Yet then the fact that there are exceptions to a putative (all things considered) duty to bring about knowledge does not suffice to distinguish it from happiness.

Just as important as noting this error is that we ask why Bolzano falls prey to such a straightforward mistake. I suggest that the culprit is his decision to frame his discussion exclusively as an inquiry into (transparent) *willings*. This leaves him with no way to isolate the crucial question, Are the *states of affairs* that they cause intrinsically valuable? This in turn helps us to see that the truth that Bolzano needs to state, and whose exceptionless nature would actually distinguish it from *all* other axiological truths, is (7), rather than any truth about willings *per se*.⁵⁷ By way of comparison, had Bolzano held with Kant that some willings are intrinsically valuable (and that this is relevant to inquiry into the content of the moral law), there would have been some exceptionless truths concerning the intrinsic value of willings to be found. As it is, Bolzano is looking for exceptionless truths in a place that his model guarantees he will not find them.⁵⁸

⁵⁷ For a clear-eyed discussion of these points, see Moore (1903, §§15–17). Ultimately, Bolzano agrees with Moore that judgments of intrinsic value, “if true at all, ... are all of them *universally* true” (§17, italics added). This is a crucial insight for axiology, and it ends up playing a central role in Moore's project. It is only the willing-centred framework of Bolzano's discussion that interferes with his clear expression of this principle.

⁵⁸ Why, then, does Bolzano conclude that there are exceptionless truths involving happiness? Doesn't my reading predict that Bolzano would conclude

I will limit my case for Bolzano's dependence on (7) to one further consideration, which can be introduced by noting a subtle flaw in the first part of Bolzano's overall argument, i.e. the part showing that this supreme practical proposition has the form:

8. [A (and...) ought to be willed],

where [A] and any conjuncts are supposed to represent act-types. Consider that for all Bolzano has demonstrated in the first part of his argument, W.D. Ross might be correct that moral inquiry shows there to be an original *pro tanto* duty of fidelity to others, as well as an original *pro tanto* duty of non-maleficence. The problem is that these two duties might stand in various relations to one another *without either one grounding the other*. For instance, the duty of non-maleficence might enjoy lexical priority with respect to the duty to fidelity, even though the former does not *explain* the latter, even partially.⁵⁹ Yet in this case [fidelity to others and non-maleficence ought to be willed] is not by itself the *supreme* practical proposition.⁶⁰ So Bolzano has not shown that the supreme moral law has the form of (8).

Why doesn't Bolzano see this? The answer, I think, is that already in the first part of his argument, prior to canvassing the different possible ways we can will (i.e. one associated with knowledge, another with happiness, etc.), Bolzano has in effect limited the scope of alternatives to just a single act-type: the causing of effects. With this assumption in

that there are no exceptionless OUGHT-truths? The answer is that sometimes Bolzano, inconsistently and without announcing it, in effect switches over to asking about states of affairs (e.g. NAK 198).

⁵⁹ Comparison of the second part of Bolzano's argument with Ross is likewise instructive. For instance, Ross is also committed to there being *no* possible case in which maleficence fails to count against an action (1930, chapter 2). Famously, though, Ross does not think that lexical priority is a correct account of how these various duties are related.

⁶⁰ The supreme proposition would need to include a further clause such as [non-maleficence ought to be willed instead of fidelity whenever the two conflict]. In order to avoid technicalities, I have set this up as a "flaw" in Bolzano's first part, but he might be able to escape it by reinterpreting the two duties as a *single* duty with respect to a disjunctive act-type: we ought to will acts of fidelity-*or*-non-maleficence-except-when-they-conflict-in-which-case-only-non-maleficence. However, this fix creates problems in the second part, as Bolzano's method of noting exceptions would naturally begin by noting the two simpler duties and the conditioning relation between them, instead of beginning with that complex, 'single' duty. See fn. 56.

place it makes sense to Bolzano that the only remaining question is a technical question of efficacy, How to bring the goods that are identified into existence? The only remaining question is a causal question, which is a merely theoretical issue.

The key to answering that question, on Bolzano's model, is the instrumental principle that he glosses as "whoever wills E, must also will M". Though Bolzano provides precious few details, I take it to have the form [If E ought*_p to be actual, and that E is actual entails that M is actual, then M ought*_p to be actual]. This raises a number of interesting issues, which I lack the space to treat. The most important point for our purposes is that Bolzano takes the instrumental principle to do no more than transfer a *pro tanto* ought from an end to something else that is its causal presupposition – or perhaps to anything that at least promotes it causally.⁶¹ Now, in the case at hand it is "happiness" that ought* to be actual. Yet that is potentially misleading. There is not one thing, happiness, but countless possible instances of happiness, each of which ought_p to be actual. For some of these instances there are corresponding willings, which would promote them causally. Each of these willings ought, so far as it goes, to exist. Moreover, in cases in which an agent cannot will two acts, each of which ought*_p to exist because it is an instrument for producing happiness, the agent ought* to will the act whose effect is the greater good. Each of the acts ought* to be done, so far as it goes, but one *pro tanto* ought* outweighs the other.

Thus we can see why (7) underwrites an all-things-considered duty to cause as much happiness to exist as we can. At any given moment I have the most reason to will what leads to the best consequences. That said, the theory that has now been reconstructed does not support a maximizing moral duty in a familiar, more stringent sense. On a natural reading of (6), namely, if an agent wills an act whose actual consequences have 90% of the value of those that would result from the optimal willing,

⁶¹ Bolzano's entire discussion of the instrumental principle occurs in a few lines at R §87, I: 229. If Bolzano is following Kant, then he holds that the instrumental principle justifies only this: taking means without which the end *cannot* be actualized (cf. G 4: 417). Kant might be drawn to this formulation because he can make the case that this principle is then analytic. There is no reason to expect that Bolzano will feel himself to be under analogous strictures, so he may envision a conceptual truth that transfers a *pro tanto* ought to *any* means that would further the realization of the end. Bolzano does seem to follow Kant in holding that substitution instances of his instrumental principle are trivial corollaries of natural science (*Critique of the Power of Judgment* [KU] 5: 172f.). That is, the fact that they contain "ought" does not make them genuinely practical.

she is *just as morally incorrect* as if she wills an act whose consequences have 10% of the optimal value. For in both cases the agent has failed to perform the act which maximally promotes happiness.⁶² It might at first be assumed that the failure of the present reading to explain Bolzano's right to a strict conception of duty counts against it as an interpretation. Looking closer, however, there is a strong case that this reading provides a better explanation of Bolzano's actual practice. For instance, *Theory of Science* provides a conceptual analysis of supererogatory willings, without Bolzano giving any hint that he regards this concept as objectless, as it must be for any maximizing consequentialist of the stringent mold.⁶³ Second, Bolzano just does not seem especially concerned that we will be doing wrong if we fail to identify and will the one optimal set of acts. Thus, I conclude that recognizing (7) as the supreme practical truth, in addition to helping us make sense of various other obscure features of Bolzano's argument, can explain why Bolzano does not attach a significance to the difference between the best and the slightly sub-optimal which is out of line with the underlying axiological facts.

§7 Bolzano and Kant on good willing

The previous section suggested that Bolzano's arguments for utilitarianism make more sense if we take him to be treating the truth about what agents ought to will as grounded in an axiological truth – albeit an axiological truth that, as such, is also a practical truth. The reason why we ought to will according to the utilitarian standard is that willings are a means for producing something else, happiness, which ought to exist by virtue of its intrinsic properties. Thus, if we restrict our view to this portion of Bolzano's theory, willings are only instrumentally valuable. Yet Bolzano provides us with more than just an argument for a standard of morally correct willing. He also commits himself to a theory of moral worth, which might attribute *intrinsic* value to some willings.

Bolzano's most important treatment of this topic comes in response to the imagined objection that his candidate for the moral law would render the moral worth of our actions dependent on accidents that finite agents are unable to anticipate. Bolzano could have defused the objection by clarifying that the supreme law requires only that we perform

⁶² See Norcross (2006) for a case that consequentialists should abandon a maximizing duty in this stringent sense.

⁶³ *WL* §144, II: 69.

the action with the best *expected* consequences, had he been inclined towards ECC.⁶⁴ Instead, Bolzano parries the objection by introducing a manner of evaluating willings that is distinct from their moral correctness: “the *moral goodness* of an action (i.e., its claim to be worthy of praise) always depends only on [i] whether the action was undertaken in the *opinion* that it was in accord with this law, and was [ii] undertaken *for this reason alone*, as well as on [iii] the greater or smaller advantages that the agent sacrificed for the sake of his duty” (R §89.4, I: 240). Here Bolzano identifies two factors that must be met for a willing to have moral value at all, as well as a third factor that can, presumably, increase its moral value above some minimum threshold. Let’s call a willing that satisfies the first two factors a *morally directed willing*.

The first question we face is whether directed willings have intrinsic value. There is *prima facie* reason to doubt this, as Bolzano soon asserts that all or part of the above-quoted claim “follows quite naturally” from the supreme moral law. The brief comments that follow offer a sketch of a standard utilitarian rationale for the practice of praising those who act morally, as well as for praising them more when the moral act was difficult for the agent. If Bolzano intends this to be a *full* account of what it is to attribute moral worth, then his view should be that morally directed willings have no intrinsic value. Yet this utilitarian foundation looks to be little more than part of an attempt to justify the posit of moral worth with multiple, redundant arguments. First, it is highly doubtful that all aspects of Bolzano’s doctrine concerning moral worth could be justified by its long-term contribution to happiness. Why, for instance, should a willing need to be undertaken *only* for the sake of its compliance with the law in order to be rewarded? Instead of attributing to Bolzano a sloppy and unconvincing utilitarian foundation for singling out certain willings for reward, I suggest that Bolzano regards it as a conceptual truth. Elsewhere Bolzano uses the fact that “morally good actions...deserve a reward” as an example of a relation that cannot change and is infallibly evident to common reason (R §15.14, I: 48), neither of which would be the case if the reason to reward agents were based on our contingent psychological tendency to act better in the future when we’ve been praised. I conclude that Bolzano is most reasonably interpreted as

⁶⁴ The one passage that is most naturally read as an expression of ECC is NAK 193. That said, a close reading of all of Bolzano’s formulations makes it seem most likely that Bolzano did not have a firm grasp of the difference between ECC and ACC. Part of what I tried to show in §§5–6 is that Bolzano’s *arguments* support ACC.

holding that morally directed willings have intrinsic value. Moreover, Bolzano shows signs of understanding morally directed willings as attitudes whose intrinsic value consists in their fitting their intentional object(s). One question that this raises is how Bolzano's account of these intentional object(s) compares to Kant's account, as summarized in §2. However, before we close by touching on that question, it makes sense to compare the overall structure of Kant's and Bolzano's theories, now in light of what we have learned about Bolzano's two conceptions (instrumental and intrinsic) of the moral significance of willing.

If morally directed willings have intrinsic value, then one element of Bolzano's theory cannot be explained by the goodness of happiness (i.e. by (7)), after all. The fact that happiness ought* to be actual by virtue of its intrinsic nature cannot explain the fact that willings-that-are-willed-in-the-belief-that-they-are-moral-and-only-because-they-are-moral ought* to be actual by virtue of their intrinsic natures. As we observed in §2, though standard attitudes make sense to many people (especially when compared to generative attitudes), the value of a standard attitude's *object* does not actually explain or ground the intrinsic value of the attitude itself. A theory that recognizes the value of standard attitudes includes an additional posit that does not simply follow from the value of the basic good that serves as the attitude's intentional object. So if Bolzano attributes intrinsic value to morally directed willing as well, it would seem that he in effect recognizes the following conjunctive practical truth:

9. [Happiness and morally directed willing ought*_p to be actual].

Now, there is nothing wrong with holding that there are two distinct types of value. However, if this is Bolzano's *de facto* position, then he needs to take a stand on the question of what ought* to be actual when either a particular state of happiness, H_1 , or a particular directed willing, W_1 , but not both, can be made actual. One answer is that one or the other type has lexical priority, in which case (9) by itself cannot be the supreme practical truth. It would at least need to be supplemented with this priority relation. Another approach is to hold that trade-offs between the two types make sense, so that the answer to the question will depend on the relative values of H_1 and W_1 . This is the Moorean Compromise. True, Moore disagrees on some of the details. (For one, he takes the unusual position that happiness is neither intrinsically good, nor intrinsically bad; only pain is intrinsically bad.) However, the second approach is in formal respects precisely Moore's position. For instance,

Moore believes that when we can avoid causing pain only by forgoing an intrinsically valuable attitude, then our all-things-considered duty depends on the particular quantities of disvalue and value involved.

Each of these two paths would be a way to make Bolzano's *entire* moral theory – the utilitarian standard of correct willing together with his doctrine of moral worth – into a unified, consistent whole. However, I see no signs that Bolzano takes either of these paths. It seems to me that Bolzano instead maintains two different models for thinking about the moral significance of willing, without managing to incorporate them into a single science. Bolzano's theory of grounding helps us to state this failure precisely. Namely, Bolzano does not accept (9), if we take a proposition of that form, as Bolzano himself does, to ground truths about whether to forsake H_1 in order to realize W_1 , and the like. Bolzano is clearly in favour of there being as much happiness as possible; and he has the same attitude toward moral worth. Yet these commitments are not integrated. Their precise relation to one another remains undefined.

We saw in §2 that Kant solves a similar problem by insisting on the lexical priority of good willing (i.e. fitting attitudes) over any other intrinsically valuable factors. Of course, there are a few important differences. For Kant the second intrinsically valuable factor is *deserved* happiness, rather than all happiness; and Kant's criterion for morally correct willing is completely different from Bolzano's utilitarian standard. However, in formal respects our philosophers face the same question. Kant's choice for lexical priority, while extreme, yields a single, unified moral theory. One way that Kant justifies this choice is reminiscent of the schizophrenic treatment of willings that we find in Bolzano – and it is just as unsatisfying. Namely, Kant sometimes simply asserts, without further argument, that in some contexts it is appropriate to consider the will as a cause, while in other contexts the will needs to be considered in itself.⁶⁵ What is different and enables Kant to avoid introducing this rift *into his moral theory* is his claim that morality properly considers the will only as it is in itself. This is a move that we may sensibly question,

⁶⁵ One clear and discrete instance of Kant making this assumption can be found in §1 of the second *Critique*: “[imperatives] either determine the conditions of the causality of a rational being as an efficient cause, merely with respect to the effect and its adequacy to it, or they determine only the will, whether or not it is sufficient for the effect” (5: 20). Two things should strike us as questionable about this way of distinguishing between hypothetical and categorical imperatives. First, if there are genuine imperatives commanding us to take some means to a presupposed end, it is unclear why the criterion for whether I have followed

and I have suggested that we look to the Moorean Compromise as an available alternative.

Now that we have sketched the structural differences between these two paths, we can state succinctly the argument against utilitarianism, which Bolzano and Přihonský accuse Kant of lacking (NAK 194): whether in its egoistic or impartial form, *consequentialism wrongly treats the will as a cause*. Interestingly, one of Kant's main points when he is advancing this argument is that it takes no special technical expertise or knowledge of the world in order to do precisely what morality requires: "what is to be done in accordance with the principle of the autonomy of will [*Willkür*] is seen quite easily and without hesitation by the most common understanding." In contrast, what is to be done when one is promoting happiness "is difficult to see and requires knowledge of the world".⁶⁶ Clearly, Kant does not see any space for ECC, whether in its egoistic or impartial form. Kant thinks of *any* consequentialist position as treating the will simply as a cause, but then what should matter is solely what the will *actually* causes.

Turning now to look at willings as attitudes, we can observe first that the two philosophers largely agree when it comes to the 'structure' of moral worth. At least in the first place, both attribute moral worth to willings, rather than to states of admiration or other pure attitudes (i.e. attitudes that are not partially constituted by the fact that their bearer takes them to be causally efficacious). Moreover, Kant might well have been the model for the two conditions that Bolzano places on directed willings. The agent of a Kantian good willing [i] believes that her maxim is morally correct, and she has [ii] adopted it solely because of its moral correctness. However, if this Kantian aspect of Bolzano's conception of willing is to be developed and made determinate, one thing we need to ask is what range of attitudes in a willing is consistent with it still qualifying as *fitting*. In closing I will raise just one set of such questions.

Kant does not think that there can be intrinsic value in an attitude whose intentional object is simply a state of affairs. We see this reflected

them should lie exclusively in whether I actually succeed in bringing about the effect. Why isn't it also (or instead) relevant whether I act with a *rationaly justifiable* intention, by taking what I believe to be adequate means? Second, it is unclear why morality properly treats the will as if it were causally inert. After all, the mere fact of morality's taking the effects of a willing into account would not preclude it from also recognizing the moral significance of willings as attitudes, as the examples of Moore and Hurka amply demonstrate.

⁶⁶ *KprV* 5: 36.

in his familiar insistence that actions be judged morally by their maxims, which are principles that refer to more than simply a state of affairs to be produced.⁶⁷ This separates him from someone who holds that if an agent is aiming at a state of affairs that morality tells us it would be correct to aim at, though the agent does not aim at it *under a description involving morality*, the attitude still has intrinsic value.⁶⁸ Bolzano agrees with Kant here. Both think that in some sense a willing must have the moral law as its intentional object in order to have the intrinsic value of moral worth. But it is important to ask what would suffice to qualify as such. Kant believes that all persons in fact grasp the propositional content of the grounding law of morality, though they do so obscurely and indistinctly, and as a result sometimes misapply it.⁶⁹ This content includes the equal status of all persons.

Because morality for Bolzano is about making the world a happier place, and because in his view that is a property with which we are all familiar, it is interesting to ask what, above and beyond the mere causing of happiness for happiness's sake, is required in order for a willing to have moral worth. More specifically, would it be enough for an agent's willing to have moral worth if she (i) believes that she is causing some happiness to be actual; and (ii) is willing the action exclusively because she expects it to cause some happiness? I think it is reasonably clear that Bolzano would answer this question negatively. For all that we have been told, the agent in question is willing instances of *her own* happiness. This is of course, morally speaking, a beneficial thing to be doing, since any instance of happiness counts the same as any other. However, Bolzano's brief comments on moral worth have a sufficiently Kantian flavour that it seems likely he would object to what could be a merely accidental extensional overlap with what morality requires. What would be required in order to 'convert' this scenario into one in which the agent's willing does in fact possess moral worth? My general suggestion would be that the agent needs to be causing happiness in a way that is governed (perhaps counterfactually) by an intention to cause it impartially. The agent needs to recognize and be motivated by the thought that happiness is equally valuable no matter which sentient being possesses it. It is

⁶⁷ "An action from duty has its moral worth *not in the purpose* that is to be attained by it, but in the maxim according to which it is resolved upon, and thus it does not depend on the actuality of the object of the action..." (G 4: 399f.).

⁶⁸ Hurka is an example of such a theorist (see §2, esp. fn. 21). Note that one could adopt this position without being committed to the Moorean Compromise.

⁶⁹ Cf. *KpV* 5: 8.

appropriate to close on this observation, since impartiality is likewise at the centre of Kant's account of what must be built into the content that makes any of our willings have moral worth.

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7

Bolzano, Kant, and Leibniz*

Sandra Lapointe and Chloe Armstrong

§1

The few decades that extend from the publication of the *Critique of Pure Reason* (1781) to what some consider the ‘high water mark’ of German Idealism, Hegel’s elaboration of his philosophical system in the *Encyclopedia of all philosophical sciences* (1817), make for a fascinating period in intellectual history. While it has been extensively studied by scholars of post-Kantian Idealism, this scholarly literature has been, in important respects, one-sided. In contrast to what is often assumed, the history of the influence of Kant’s critical philosophy in the German-speaking world is not merely the story of the development of idealism in the works of Fichte, Schelling, and Hegel.¹ It is also the story of the development of logic at the turn of the nineteenth century. While Kant himself is not usually considered to have made a substantial contribution to logic itself, his work was seminal and his influence – both positive and negative – on the logic of his time considerable. There were, on the one hand, those who attempted to devise logics based on the *Critique*. There was, on the other hand, Bernard Bolzano.² Bolzano engaged with

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¹ In *The Fate of Reason, German Philosophy from Kant to Fichte*, Frederick Beiser tells a compelling story that has become authoritative and a mandatory read for anyone interested in the history of German philosophy. Nonetheless, the scope of Beiser’s narrative, while it is quite broad, leaves out important aspects of the early Kant-reception. In particular, it leaves out an account of the way in which Kant’s philosophy informed the development of logic at the turn of the nineteenth century.

² One could remark here that Fichte’s *Wissenschaftslehre* and Hegel’s *Wissenschaft der Logik* also engage with Kant’s views on logic. I leave them aside here

Kant and the Kantian logicians, sought to determine what is distinctive in their approach and their overall conception of the role of logic in epistemology at large. He also subjected their views to a withering criticism.

The idea that the *Critique of Pure Reason* has a place in the history of logic, and indeed had a significant impact, is bound to be controversial. As we understand the term today, Kant was not a logician. Despite the fact that Kant did not entirely avoid logical issues and even innovated to a certain extent – one might consider the criticism of the traditional theory of syllogism in *Von der falschen Spitzfindigkeit der vier syllogistischen Figuren* (1762), or the classification of judgements he put forward in the section of the *Critique of Pure Reason* entitled “Of the Logical Function of the Understanding in Judgements” – his treatment of logical questions per se has often been deemed comparatively inconsequential (see for instance Kneale and Kneale 1962, 354ff.). However, Kant’s dealings with questions concerning the subject matter of logic, its scope and its place within a theory of rationality should in any case be seen as pioneering. It is true that in the *Critique* Kant discussed logic directly mainly to distinguish it from ‘transcendental’ logic (in the Introduction to the eponymous section), that is, what Kant conceived as the investigation of the conditions of possibility of our knowledge of objects. But this discussion – a discussion that is further developed in the first few pages of the Introduction to the *Logic* (1800) – turns out to be substantial. What’s more, it was considerably influential. In the years that followed the publication of the (second edition of the) *Critique* (1787), many authors sought to work Kant’s ideas into their presentations of logic. The result is an interesting collection of logical *Systeme*, *Anfangsgründe*, *Grundrisse*, etc. whose authors all have in common a more or less explicit commitment to Kantian doctrines. They include Ludwig Heinrich von Jakob (1759–1827), Wilhelm Traugott Krug (1770–1842), Johann Christoff Hoffbauer (1766–1827), Johann Gottfried Karl Christian Kiesewetter (1766–1819), and Gottlob Wilhelm Gerlach (1786–1864).³

Both historically and philosophically Bolzano’s contribution to philosophy is to be understood within the context of the reception of Kant’s critical philosophy, or so we will argue. This claim is also

since their contribution cannot be properly seen as a contribution to logic understood traditionally.

³ See Lapointe (2012).

likely to be controversial. Bolzano's contribution to philosophy, and in particular his contribution to the epistemology of logic and mathematics, is more often than not positioned in stark opposition to Kant's, in the intellectual lineage of Leibniz. What we are proposing is deliberately meant to upset this picture. Bolzano's relationship to critical philosophy is far more complex than what is generally assumed. For one thing, Bolzano's relationship to Kantian philosophy is not exhausted by his relationship to Kant. Bolzano paid close attention to the logical theories of those who followed in Kant's stride, the "new logicians" (Bolzano's term), and he discussed their views in at least as much depth as he did Kant's. What's more, Bolzano sought to determine what is distinctive of the "new logic" and thus offered a philosophical reflexion that is still, even today, enlightening when it comes to understanding this aspect of the reception of Kant's first *Critique*. Bolzano thought, for instance, that one thing that discriminates the Kantian school of logic is the idea that what is distinctive of logic is its formality. He discussed the thesis in detail, ultimately rejecting the theoretical assumptions on which the Kantian notion of formality rests and proposing his own alternative theory of logical form.⁴ Furthermore, while Bolzano disagreed with Kant on a range of crucial issues, his own theories are more often than not determined by problems that have their sources in critical philosophy. For instance, Bolzano considered the problem of synthetic *a priori* knowledge to be of fundamental importance for epistemology – despite scrupulously rejecting Kant's own solution (the doctrine of pure intuition). But the notion of synthetic *a priori* knowledge was a thoroughly Kantian innovation that presupposed a series of distinctions (between analytic and synthetic propositions, between intuitions and concepts, between *a priori* and *a posteriori* cognitions) Bolzano somehow had to work into his system. Consequently, Bolzano's relationship to Kantian philosophy, in contrast to what is ordinarily assumed, was not purely antagonistic. It is more apt to think of Bolzano's theories as distinctively 'post-Kantian', and not merely, as the title of Příkladný's (1850) book somewhat misleadingly suggests, as anti-Kantian.⁵ Bolzano preserved some of Kant's most fundamental distinctions and this ought to be kept in mind when examining Bolzano's own views.

⁴ On this topic, see Lapointe (2012). See also Tolley (2012) and Rusnock (2011).

⁵ See Příkladný (1850, XVII) for an apologetic rationale for the title.

§2

On the standard understanding of Bolzano's place within the history of German philosophy, what is distinctive of Bolzano, other than his assumed distance from Kant, is his putative proximity to Leibniz.⁶ If anything, brandishing the idea that Bolzano's theories are indebted to Leibniz's is an effective rhetorical strategy: the association emphasizes Bolzano's rationalist leanings as well as his predilection for logic thus stressing the connection with post-Fregean theories. For this reason, it is likely to be attractive to analytical philosophers. The problem with this idea is not that it is fundamentally wrong: Bolzano's approach to philosophical questions is similar to Leibniz's (and to any theory on which logic should play a fundamental part in epistemology). The problem with this picture is that it suggests a historical dependence that cannot in fact be documented. In particular, it presupposes that Bolzano had some sustained interest in Leibniz's philosophical programme, a presumption that is hard to reconcile with historical evidence.

Of course, there are connections between Bolzano and Leibniz. But Bolzano discusses Leibniz's work in fact comparatively rarely. We find over the some 2400 pages of the *Theory of Science* (1837) a mere 30 references to Leibniz, mostly to the *Nouveaux essais* (1704) – compare this with the some 150 references to Kant, and some 200 to Kiesewetter. It is not only that the number of references is small, but also that many references are in footnotes, even in the sections entirely devoted to discussion of the views of other philosophers. Furthermore, many of the references to the *Nouveaux essais* focus on Leibniz's polemic against Lockean doctrines but fall short of explicitly endorsing Leibniz's views.⁷ Bolzano often concludes that Leibniz's own views are ultimately either inadequate or incomplete as they stand.⁸ Bolzano shares with Leibniz a dissatisfaction with certain empiricist doctrines, but this in itself is insufficient to support the idea that Bolzano sought to pursue an even

⁶ See, for instance, Morscher (2003, xxi). One may also consider J. Danek's (1969, 1975) suggestion that Bolzano's logic is a mere development of Leibniz and Mugnai (1992).

⁷ For example, Bolzano endorses Leibniz's criticisms of Locke's treatment of knowledge (*Theory of Science* vol. 1 (78 and 251)), truth bearers (vol. 1 (97)), the nature of innate ideas (vol. 3 (119)), and the relation between temporality and objects in time (vol. 3 (121)). This last reference is a good example of a place where Bolzano endorses Leibniz's critical views of Locke while also criticizing Leibniz's positive views for being incomplete.

⁸ See our discussion of propositions below.

broadly Leibnizian programme. Bolzano often draws upon Leibniz's work to help clear the way for his own views but the idea that these views were themselves 'Leibnizian' is misleading.

One might want to argue that, while its scope was limited, the significance of Leibniz's influence on Bolzano's theory was nonetheless decisive. One could contend, as Church (1956, 1) does, that Bolzano's notion of a "proposition in itself" (*Satz an sich*) – the basis for Bolzano's logical realism and the fundamental feature of his logic – is indebted to Leibniz. According to Bolzano, the primary bearers of truth and falsity are structured, mind- and language-independent entities to the characterization of which he devotes the first two volumes of the *Theory of Science*. Bolzanian propositions are *mind- and language-independent* to the extent that they are non-actual (*nicht wirklich*), that is, to the extent that they are neither in space nor in time and indeed do not stand in causal relations of any kinds. Bolzanian propositions are furthermore *structured* to the extent that they are composed of parts – which Bolzano calls 'representations in themselves' (*Vorstellungen an sich*) – that correspond to terms of a definite syntactic type in the sentences that express them. As Bolzano sees it, a representation is any part of a proposition that is not itself a proposition (cf. Bolzano 1837, §52, 228)⁹ and every proposition is composed of three representations that can themselves be complex and indeed structured syntactically (cf. Bolzano 1837, §127, 9ff.). Every proposition in Bolzano's account is composed of (i) an "object-representation" corresponding to the subject term and which Bolzano symbolizes by means of upper-case letters, e.g. 'A'; (ii) a "property-representation" that corresponds to the predicate term and which Bolzano symbolizes by means of lower-case letters, e.g. 'b'; and (iii) a (simple) representation Bolzano designates by the sentence-forming operator 'has' and whose role is to connect the ideas that correspond to 'A' and 'b' to form propositions of the form 'A has b' (cf. Bolzano 1837, §128, 18).

With respect to the non-actuality of propositions in themselves, the connection to Leibniz is explicit. Bolzano (1837, §21, 85) gives credit to Leibniz for having anticipated the notion of a *Satz an sich* in *Dialogus de connexione inter verba et res* (1677).¹⁰ However, there are at least two reasons to be wary of the claim that Leibniz effectively influenced Bolzano on this point. First, Leibniz is not the only philosopher whom Bolzano credits with having anticipated the concept of a *Satz an sich*.

⁹ In the same section, Bolzano criticizes what he takes to be Leibniz's attempt at a definition of the notion of representation (cf. Bolzano 1837, §52, 229).

¹⁰ Henceforth *Dialogus*.

The list of those who, according to Bolzano, had a clear conception of the notion includes, besides Leibniz, a number of philosophers, some belonging to the Kantian School: Mehmel and Metz who both prefer the term ‘Satz’ to ‘Urtheil’, Herbart who argues in favour of a distinction between the logical and the psychological and Gerlach who draws a distinction between a subjective and objective understanding of judgements (cf. Bolzano 1837, §21, 85). Indeed, when the notion of proposition is introduced at other places (see for instance Přihonský 1850, 6) no mention is made of Leibniz; rather the Stoics are evoked.

Second, as he makes clear in a footnote to section §27 of the *Theory of Science*, Bolzano did not uncritically take over the Leibnizian notion of a *propositio*. As Bolzano understands it, while he admits that Leibniz must have had the same kind of entity in mind, he also objects to what he takes to be Leibniz’s definition of ‘proposition’. According to Bolzano, Leibniz characterizes the proposition as “something that can be thought, i.e. that can constitute the content of a thought” (cf. Bolzano 1837, §21, 84f.). To be fair, Leibniz himself does not offer a systematic treatment of the nature of propositions in his work. Most of Leibniz’s discussion of propositions is devoted to explaining what propositions are not. In the passages Bolzano knew – in the *Nouveaux essais* (1704)¹¹ – Leibniz examined competing candidates to the role of primary truth-bearers. In *Dialogus* Leibniz claims that neither mere thoughts, nor objects, nor indeed sentences can be the primary truth-bearers. Because what is true remains true even when we are no longer consciously considering it, Leibniz argues, truths are not mere transitory psychological episodes. Moreover, on Leibniz’s account, what bears the predicate ‘is true’ cannot be states of affairs or things since this supposition doesn’t account for falsehoods.¹² Finally, since the same truth can be expressed in German, Greek, and Latin, Leibniz further claims that the primary truth-bearers are not mere *sentences*.¹³

¹¹ Hereafter *NE*.

¹² Leibniz’s reasoning here is not forthcoming. “But can we ever call any *thing* false? No, I suppose only thoughts or statements about a thing are called false” (*Dialogus*, 279 in Leibniz 1956) It’s not clear whether Leibniz is appealing to linguistic intuitions about the inappropriateness of describing things as true or false, or whether he is gesturing to a deeper problem. He could, for instance, be thinking about whether false propositions can be composed of real things (can the false proposition “The table is red” just be the blue table?).

¹³ “How can anyone be so irrational as to hold truth to be arbitrary and make it depend on names, when surely the same geometrical truth is expressible in

While what Leibniz says ultimately is not sufficient to be qualified as a definition, much less a theory, in *Dialogus* Leibniz does claim that “truth really belongs to the class of thoughts which are possible” (1956, 279, 282). Whether Leibniz thought of ‘thinkability’ as a defining, or merely as a characteristic, feature of propositions is not clear. So Leibniz might very well have been sympathetic to Bolzano’s criticism, namely that while the possibility of being thought is a property – and even an essential property – of propositions, the concept of a proposition does not include this property.¹⁴

Bolzano had misgivings about what he took the Leibnizian definition of propositions to be. Nonetheless, Bolzano assumed that he and Leibniz were committed to the same kind of objective, abstract entities, and consequently to the same kind of ontology of logic. While they both subscribe to a form of logical realism, determining the extent of the similarity requires further assessment.¹⁵ The fact that Bolzano and Leibniz make similar ontological commitments teaches us rather little about the details of their respective theories and in fact conceals great theoretical disparity. Whatever the degree of influence of Leibniz on Bolzano, the basic principles of Bolzano’s logic are fundamentally opposed to those of Leibniz’s logic. Assuredly, Bolzano may have shared some of Leibniz’s theoretical goals and starting points – for instance the demonstration that mathematical knowledge is purely conceptual and the idea that concepts are not images. Because of this, it may be tempting to think of Bolzano as pursuing a “Leibnizian programme” in logic. However, one should resist drawing this conclusion. By his own standard, Bolzano’s success required him to reject some of the key features of Leibniz’s logical theories. When it comes to such issues as

Greek, Latin, and German?” (*Dialogus*, in Leibniz 1956, 280–281). Also see: “But what is least to my liking in your definition of truth is that it looks for truth among words, so that if the same sense is expressed in Latin, German, English, and French it will not be the same truth; and we shall have to say with Mr Hobbes that truth depends upon the good pleasure of men...” (*NE* IV.v.396)

¹⁴ As Bolzano sees it, the concept of being “thinkable” is not necessary for our understanding the concept of a proposition, Bolzano thought, and hence does not belong to its definition. On this point, and though Bolzano is not categorical, the position he is most likely to have held is that the notion of proposition cannot be defined explicitly because it is primitive (cf. Bolzano 1837, §128, 18).

¹⁵ Lapointe (forthcoming), for instance, develops a framework in which different types of logical realism can be compared and argues that Bolzano was first to adopt the form of “semantic descriptivism” that is characteristic of his approach.

truth, meaning, and reference, Bolzano's views differ most radically from those of Leibniz. For one thing, Leibniz's conceptual containment theory of truth is a distinctive feature of his treatment of logical issues, and it is one that Bolzano adamantly rejects.

Truth, for Leibniz, is a matter of the relations between the objects of representations. These relations among objects coincide with containment relations among our concepts:

It would be better to assign truth to the relationships amongst the objects of the representations, by virtue of which one representation is or is not included within another. That does not depend on languages, and is something we have in common with God and the angels.... So it is to these relationships that truth should be assigned... (*NE IV.v.397*)

This putative coincidence between objects and representations explains that we can establish the truth of a proposition on the basis of the containment relations that effectively hold between the concepts that compose it:

in any true proposition the predicate or consequent is always in the subject or antecedent. It is just this – as Aristotle observes – that constitutes the nature of truth in general, or the true-making connection between the terms of a statement. In identities the connection of the predicate with the subject (its inclusion in the subject) is explicit; in all other true propositions it is implicit, and has to be shown through the analysis of notions... (“First Truths”, Leibniz, 1956, 412)

If we follow Leibniz, whether or not a proposition is true depends on whether or not the predicate-concept is contained in the subject-concept. When we decompose the subject-concept in a true proposition, we find therein the concept expressed by the predicate. One proves the truth of a proposition or judgement by making explicit the containment relation between the subject and the predicate, that is, by “analysing” the subject; in this sense, true judgements are “analytic”¹⁶ – some of them “infinitely” so. In Leibniz's theory, analysis is the process by which we in principle can render the containment of the predicate in the subject explicit:

For to demonstrate is merely, by an analysis of the terms of a proposition and the substitution of the definition or a part of it, for the

¹⁶ Although Leibniz himself does not employ the term “analytic”.

thing defined, to show a kind of equation or coincidence of predicate and subject in a reciprocal proposition, or, in other cases, at least the inclusion of one in the other, so that what was concealed in the proposition or was contained in it only potentially, is rendered evident or explicit by the demonstration. For example, if we understand by a ternary, a senary, and a duodenary, numbers divisible by 3, 6, and 12, respectively, we can demonstrate this proposition: Every duodenary is a senary. For every duodenary is a binary-binary-ternary, since this reduction of a duodenary into its prime factors ($12 = 2 \times 2 \times 3$), or the definition of a duodenary. But every binary-binary-ternary is a binary-ternary (this is an identical proposition), and every binary-ternary is a senary (by the definition of a senary, since $6 = 2 \times 3$). Therefore every duodenary is a senary (12 is the same as $2 \times 2 \times 3$, and $2 \times 2 \times 3$ is divisible by 2×3 , and 2×3 is the same as 6; therefore 12 is divisible by 6). (“On Freedom”, Leibniz, 1956, 407)

Bolzano was thoroughly critical of the containment view of concepts and the theory of truth it underlies. As he sees it, the idea that in some propositions the concept of the predicate is “part of” or “contained in” or “repeated in” or “identical with” the concept of the subject are mere metaphors (*bildliche Redensarten*) that fail to define the notion at hand and allow for too broad an interpretation (1837, §148, 87). Naturally, if Bolzano is right the implications for a decompositional account of truth of the type Leibniz (and others) advocate are problematic. As Bolzano points out, in the absence of a clear definition or at least some further specification of what containment is, the idea that a proposition is true if the predicate-concept is contained in the subject-concept remains underdetermined and thus inadequate.

Bolzano’s main target when he discusses decompositional conceptions of analysis is the assumption that *the content of a concept coincides in some substantial way with features of the object it “represents”*. Let us call this broad assumption (R). (R) played an important role in the semantics of the time. As we’ve seen from the quotations above, (R) is indeed something Leibniz presupposed.¹⁷ Without (R) Leibniz would have been incapable of explaining how we can, on the basis of conceptual analysis, acquire knowledge of objects. That we can follows, however, from (R), i.e. the assumption that what is to be found in a concept are representations that correspond to features of the object in question.

¹⁷ Cf. *NE* IV.v.397.

Bolzano's characterization of (R) is general enough to pick out a range of positions, Leibniz's being only one of them. For instance, Bolzano was particularly dissatisfied with the idea found in a number of his predecessors (cf. Bolzano 1837, §53, 230) that representations are "pictures" of the objects they represent and he put some effort toward discarding the view.¹⁸ Interestingly, Bolzano's criticisms of the picture-theory of representations are consistent with some of Leibniz's remarks. Like Bolzano, Leibniz denied in certain places that concepts are images. As we have seen above, Leibniz's exemplary cases of conceptual analysis involve mathematical concepts, such as 'multiple of 12' or 'multiple of 6'.¹⁹ According to Leibniz, while these concepts can be expressed using symbols – and while we can indeed have mental pictures of these symbols – the concept is not the same as an image of the symbol.²⁰ On Leibniz's view, an analysis of 'multiple of 12' does not proceed by analysing the structure of an image.

Nonetheless, Leibniz did subscribe to a more general version of (R). In his view, representation allows us to infer from the relations between concepts to the relations between objects, which presupposes precisely the kind of correspondence (R) defines:

there are various kinds of expression; for example, the model of a machine expresses the machine itself, the projective delineation on a plane expresses a solid, speech expresses thoughts and truths, characters express numbers, and an algebraic equation expresses a circle or some other figure. *What is common to all these expressions is*

¹⁸ Bolzano argues that the picture-theory of representations rests on a misunderstanding concerning the nature of pictures as well as the nature of representations. In Bolzano's account pictures are (i) objects that subsist independently of the thing of which they are images both with respect to their origin and to their persistence, (ii) objects that may "stand for" the objects of which they are pictures by virtue of their resemblance with them and (iii) that may also be used for given cognitive purposes. A picture of my son, for instance, may be used to remind me of him while I'm travelling. But concepts or more generally representations can fulfil none of conditions (i)–(iii). Bolzano concludes: "[...] it results that one should not call the representation of an object in the stronger sense an image of the latter; for it is indeed no object that we can consider instead of it, but it is that which arises in our minds, when we consider this object itself" (cf. 1837, §52, 230).

¹⁹ See Leibniz "On Freedom" (1679, 407) in Leibniz (1956).

²⁰ Further see *Dialogus* 280–283 in Leibniz (1956).

that we can pass from a consideration of the relations in the expression to a knowledge of the corresponding properties of the thing expressed. ("What is an idea?" (1678) in Leibniz 1956, 318–319; emphasis added)

Leibniz supposes that containment relations among our representations tell us something about the relationships that hold amongst their objects.²¹ This presumed coincidence is what explains, on Leibniz's account, that we can gain knowledge of truth by merely conceptual means: by looking at conceptual containment relations, we can discover which states of affairs obtain or, *via negativa*, which do not.

Bolzano's rejection of (R) and the decompositional conception of analysis it supports is rooted in a theoretical alternative whose import is still vastly underestimated. For Bolzano, what allows us to establish the truth of a proposition are not relations between concepts themselves but relations between their "extensions" (*Umfänge*), a notion Bolzano attempted to define systematically. As Bolzano sees it, a representation's having an extension consists in its having determinate object(s), i.e. in it being the case that at least one object "falls under" it. In this sense, the extension of a concept B is the *collection* of objects that have the corresponding property b.²² Bolzanian extensions are not sets in the contemporary sense: sets can be empty, but Bolzanian collections cannot. If a concept does not have an object, Bolzano also considers that it does not have an extension. (1837, §66, 298) As Bolzano conceives of it, concepts either are objectual (*gegenständlic*), in which case they have an extension (*Umfang*), or they are objectless (*gegenstandlos*), in which case they do not. Bolzano thought of this aspect of his theory as particularly crucial and devoted a substantial part of the first volume of the *Theory of Science* to defining the notions of objectuality and its derivatives (Bolzano 1837, §§66–68) as well as the various types of relations that hold among extensions: subordination, equivalence, etc. (Bolzano 1837, §§91–108). In Bolzano's account, whether a proposition is true depends on two factors. On the one hand, it depends on whether the concepts it involves are objectual. On the other hand, it depends on whether the subject-concept is subordinated to the predicate-concept – which as he understands

²¹ Also see NE IV.v.397.

²² Uses of upper-case and lower-case imply syntactic differences that correspond to ontological determinations. Bolzano uses the upper-case to pick out objects such as horses and triangles. He uses the lower-case to refer to the corresponding properties: equinity, triangularity.

it implies that the object(s) denoted by the subject have the property described by the predicate (cf. Bolzano 1837, §196, 329ff.).

Bolzano's alternative to Leibniz's theory of analysis and truth is also rooted in a novel way to think of logical syntax. In addition to having an extension, representations also have a content (*Inhalt*) on Bolzano's account. This content can be simple. Such is the case of the (singular) representations corresponding to demonstratives such as 'this' and which Bolzano calls "intuitions", and of the (general) concept that is expressed by the word "something" (*Etwas*). But representations can also be complex. When Bolzano speaks of what it means for a concept to be complex what he has in mind are not (unstructured) collections of representations of properties of objects but types of sequences of representations that have a syntactic structure. Consequently, what it is for a concept to be included or contained in other concepts for Bolzano is not a mereological matter but a syntactic one. In his account, the representation $[\beta_1]$ is contained in the representation $[\alpha]$ if and only if $[\alpha]$ is a construct that results from the connection of $[\beta_1]$ with other representations $[\beta_2], \dots, [\beta_n]$ according to determinate rules Bolzano lays out throughout the first two volumes of the *Theory of Science*.²³ Bolzano claims, for instance, that the representation designated by the word 'Earthling' is complex. We know that it is complex, Bolzano explains, because we think by 'Earthling' precisely what we think by 'A creature, that lives on Earth' – all of these components and in this order – that is, what corresponds to a name that is formed by the apposition of an attributive clause (cf. Bolzano 1837 §56, 243). What this means, for one thing, is that a representation, if it is not simple, is syntactically structured. This implies that propositions involve a great deal of syntactic complexity beyond the predicative structure they all share.

Bolzano's syntactic resources are vastly richer than what is often assumed. This is an important point to keep in mind. The ultimate problem for Bolzano was to explain how the kind of knowledge we acquire in arithmetic and geometry, for instance, can both extend our knowledge and remain purely conceptual at once. The main obstacle to Bolzano's programme was the limitation of the theory of logic that was available at the time. In order to provide an account of deductive knowledge that would not resort to extra-conceptual resources, Bolzano had to show that logic could effectively reflect the structure of deductive knowledge. The

²³ See Lapointe (2011, 29ff.).

first step was to do away with the decompositional conception of analysis and truth that provided the paradigm explanation of conceptual knowledge at the time and which, Bolzano argued, cannot.²⁴ While Bolzano's theories might, in fact, serve some of Leibniz's programmatic aims, the proximity with Leibniz does not appear to be an intentional, or at least an important philosophical motivation for Bolzano.

§3

The thesis that Bolzano's place within the history of philosophy is determined by his "proximity" to Leibniz, despite some similarities in their theories, ought to be rejected. Likewise, the idea that Bolzano's position is to be determined by his "distance" from Kant is misleading. Historically, Bolzano stands in a close relationship to Kant. Bolzano's work evolved in important ways as a response to the *Critique of Pure Reason* and the theories of those who followed in Kant's stride. Rather than its "distance" from Kant, what characterizes Bolzano's philosophy is the place it occupies within the reception of Kant's critical system. This place is distinctive in many respects. For one thing, Bolzano's discussion of Kantian theories is systematic in the following sense: it is rooted in a fully worked out alternative philosophical system whose sophistication and significance had no match at the time. Besides, Bolzano's main theoretical focus – the epistemology of logic and mathematics – was virtually unique among Kant's commentators, at least until the twentieth century.²⁵ Both factors concur to deliver what counts as the

²⁴ One could summarize Bolzano's views by saying that the main problem with the decompositional theory of analysis is that it tries to safeguard a logical syntax based on the idea that all propositions have a subject–predicate structure, and that no such logical syntax can do the work when it comes to accounting for deductive reasoning as it is found in sciences such as arithmetic and geometry. But if that is the case Bolzano would seem to face the same predicament. We believe that this appearance is misleading. See Lapointe (2011, 2014).

²⁵ Kant's views on mathematics – in particular the idea that mathematical knowledge requires the support of putatively pure intuitions – played an important role in subsequent discussion on the foundations of arithmetic and geometry. As Coffa puts it, some of the major developments of semantics were motivated by a shared rejection of Kant's epistemology of *a priori* knowledge – the "common enemy" – and it makes sense to say that in this sense, what has come to be known as "analytical philosophy" is distinctively post-Kantian. Bolzano is among the few logicians and mathematicians of the turn of the nineteenth century to have engaged with Kantian theories and to have asked, as a logician and mathematician, whether critical philosophy is apt to provide a foundation to scientific knowledge.

single most original and extensive discussion of critical philosophy at the turn of the nineteenth century. In this respect, it is fair to say that the prolonged neglect of Bolzano's contribution to Kant scholarship has been detrimental to our understanding of the Kant-reception and that this is a situation that needs to be redressed.

Bolzano consistently devoted entire passages and sections of his published philosophical work to his discussion of Kant. Detailed analyses are to be found, in particular, in the "Appendix" to the *Contributions to a Better Founded Exposition of Mathematics* (1810), the *Science of Religion* (1834), and the *Theory of Science* (1837). The latter are summarized in *New Anti-Kant* (Příhonský 1850) in a remarkably accurate manner. Bolzano's discussion ranges over dozens of issues that are painstakingly inventoried over some ten pages in the table of content of the latter. They include: the distinction between *a priori* and *a posteriori* cognitions, the distinction between analytic and synthetic propositions, Kant's demonstration of the possibility of synthetic *a priori* cognition, the distinction between concepts and intuitions, the doctrine of space and time, the theory of pure intuition, the theory of the categories, the origin of error, the theory of definition, the supreme moral principle and the distinction between philosophy and mathematics, to name only a few.

Assuredly, some of these issues stand out as particularly central and enlightening when it comes to understanding the gist of Bolzano's criticism of Kant's theories. The question of the possibility of synthetic *a priori* knowledge is one of them. Bolzano accepted the Kantian view that what characterizes mathematical truths is the fact that, while they are not analytic (they are synthetic), they are nonetheless *a priori*. This implies that he agreed with Kant that there is a distinction to be made between analytic and synthetic judgements, that some synthetic judgements are *a priori*, and that truths of the latter sort are constitutive of disciplines such as arithmetic and geometry. In this he distinguishes himself from other contemporary critics of Kant such as Eberhard and Maaß who rejected the notion of the synthetic *a priori*. Yet, Bolzano's criticism of Kant's views on synthetic *a priori* knowledge is radical. Part of the disagreement comes from Bolzano's conviction that Kant failed to provide a correct analysis of the notions of analyticity, syntheticity, apriority, aposteriority, intuition and concept, among others. But the core of the dispute rests on Bolzano's belief that Kant failed to recognize that what makes a discipline *a priori* is the fact that it proceeds by means of pure concepts, on the basis of deductions in which no intuitions intrude, as is the case according to Bolzano, in geometry (cf. Příhonský 1850, 74). Indeed, in Bolzano's account, the notion of a priority is, if not

defined by, then at least equivalent to the notion of purely conceptual cognition.²⁶

Kant, of course, had reasons to believe that mathematics is not purely conceptual. In Kant's account, analytic cognition is purely conceptual since analyticity is the property of judgements whose truth can be discovered by purely conceptual means. Kant conceived of analyticity as the property of judgements whose truth can be established on the basis of the putative containment relations that hold among concepts. In Kant's view, since the truths of mathematics are not such that the concept of the predicate is included in the concept of the subject, they are not analytic. Kant drew two related conclusions from this. On the one hand, he concluded that the truths of mathematics could not be purely conceptual. On the other hand, he concluded that what makes a judgement synthetic is precisely the fact that it depends upon an intuition (in which the object of cognition is given). Kant's theory culminates in the argument that what makes the judgements of mathematics synthetic, yet *a priori* is the fact that they depend on a putatively "pure intuition". Bolzano could never dismiss this idea vehemently enough.

As Bolzano sees it, one can provide an account of mathematical knowledge only to the extent that one has an account of the structure of conceptual knowledge in general. What's distinctive of disciplines such as arithmetic and geometry is also common to all other *a priori* disciplines, on Bolzano's account, and it is the fact that the (purely conceptual) propositions they contain stand as "grounds" to their "consequences": what characterizes conceptual truths of a certain kind, say the truths of geometry, is not simply the fact that they do not contain "intuitions" but the fact that they stand in determinate relations within a deductive structure. The structure in question is defined on the basis of a consequence-like relation Bolzano calls "grounding" (*Abfolge*).²⁷ Their being embedded in a grounding structure, explains both why conceptual truths are necessary and why we can know *a priori* that they are. Conceptual truths derive their necessity from the grounding relations in which they stand. We can know conceptual truths *a priori*, on Bolzano's account, because we can know *a priori* that they stand in the grounding relations in which they stand. In the case of theorems – and granting that grounding preserves truth as a matter of necessity, as Bolzano himself assumes – there is little that is mysterious about this. In the case

²⁶ See Textor (2013).

²⁷ On *Abfolge*, see Tatzel (2002).

of axioms, what's involved on Bolzano's view – a metaduction of their axiomatic status and a metainduction of their truth²⁸ – is somewhat more intricate and perhaps also more perplexing, but it is of a piece with the view that key to an account of *a priori* knowledge is an adequate modelling of logical relations.

§4

In this respect – but this is not the only one – there is a great deal of continuity between Bolzano's epistemological programme and that of Frege in the *Foundations of Arithmetic* (1884). Both philosophers sought to find ways to extend what counts as conceptual knowledge to arithmetic – and, in Bolzano's case, to geometry as well – and they did so by improving our methods of logical deduction. Bolzano's and Frege's respective criticisms of Kant are rooted in the same complaint and consists in saying that arithmetical knowledge does not require extra-conceptual reasoning. In this sense, Bolzano's endeavour – the same can be said about Frege's – is a distinctively “post-Kantian” affair.

Bolzano and Frege would have agreed that arithmetical concepts can be defined in terms of purely logical concepts and that the truths of arithmetic follow from these definitions by means of logic alone.²⁹ Hence the fact that Bolzano conceives of deductive knowledge under the heading of “synthetic *a priori*” could appear to be a mere terminological oddity. One could conclude that his views in fact align with those of Frege. But there are reasons to resist this conclusion. For one thing, Bolzano did maintain an (all be it) very narrow conception of what he took analyticity to be and this raises a number of questions, including the following: if the notion of synthetic *a priori* in Bolzano is meant to play the role played by the notion of analyticity in Frege, what purpose is the Bolzanian notion of analyticity supposed to serve?³⁰ At first glance, the fact that Bolzano maintains a narrow conception of analyticity would seem to put him in the Kantian tradition:

In my opinion not even one principle in logic, or in any other science, should be a merely analytic truth. For I look upon merely analytic

²⁸ See Lapointe (2011, 111ff.).

²⁹ See, for instance, Simons (1999) for a reconstruction of Bolzano's definition of numbers on the basis of what Bolzano would argue are purely logical resources. Whether this means that Bolzano was engaged in a “logician” project that can be assimilated to Frege's and his successors' is a question we will leave open.

³⁰ See Lapointe 2011, 43–72 for a suggestion.

propositions as much too unimportant to be laid down in any science as proper theorems of it. Who would want to fill up geometry, for example, with propositions like: an equilateral triangle is a triangle, or is an equilateral figure, etc.? (1837, §12, 51–52)

Bolzano agreed with Kant that analytic cognition, “logically analytic” cognition in particular, seems comparatively trifling and unimportant. Ironically, this itself is the result of Bolzano having thoroughly misunderstood the intuition that underlies Kant’s own theory of analyticity. Bolzano thought that what is important about analytic propositions is the fact that “their truth does not depend on the individual representations of which they are composed” and “remains the same no matter how we vary certain of their parts” (1837, §148...). Bolzano indeed completely overlooked the fact that analytic truths in Kant’s sense are also always knowable by virtue of knowing the meaning of the terms they involve – or, as Kant himself would put it, by virtue of conceptual analysis (as he understands it) – and therefore knowable *a priori*.³¹ In order to cash out the idea that arithmetical truths are *a priori*, Bolzano consequently subscribed to the Kantian view according to which “in all the theoretical sciences of reason synthetic *a priori* judgements are involved as axioms [*Grundsätze*]” (Příhonský 1850, 42). This includes logic:

we find judgments of this sort not only in mathematics, in the pure natural sciences and in metaphysics, as Kant proves it incontestably, but they are also to be found in logic, namely not merely among the theorems that belong to this discipline if we understand it, with Bolzano, according to a wider concept, but in the very part of it which one calls analytic and which has been worked on since Aristotle. (Příhonský 1850, 42, 43)

What effectively distinguishes Frege and Bolzano is the programme they chose to pursue. Frege understood (correctly) the intuition that underlies the traditional (Leibnizian, Kantian) theory of analyticity – the idea that some truths are *a priori* knowledge in virtue of knowledge of the relations among concepts – and accordingly sought to enrich the notion using new, more fruitful logical resources. Bolzano, following Kant, sidelined the notion of analyticity and set out to develop a new theory of

³¹ See Lapointe (2014) for a discussion of Bolzano’s views on analyticity.

synthetic *a priori* knowledge that would deliver an acceptable alternative to Kant's views on the epistemic status of, among others, arithmetical and geometrical claims. Because Bolzano's views on synthetic *a priori* knowledge and deductive reasoning are substantial, his rejection of the idea that logic is analytic (in his, Leibniz's or Kant's sense) was enormously fruitful. As we've seen above, it coincides with a rejection of the (Leibnizian) decompositional conception of analysis, i.e. with the paradigmatic conception of conceptual knowledge at the time and with the elaboration of a new logic that anticipated some of the most important developments of the twentieth century. Kant's theories were instrumental in Bolzano reaching the conclusions he reached. They opened the path for a criticism of logic at large, and one in which Bolzano was among the first to engage.

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