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Economics of Accounting

Volume II
Performance Evaluation

Peter O. Christensen
Gerald A. Feltham

Springer

ECONOMICS OF ACCOUNTING

Volume II – Performance Evaluation

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ECONOMICS OF ACCOUNTING
Volume II – Performance Evaluation

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To
Kasper, Esben, and Anders
Tracy, Shari, and Sandra

CONTENTS

PART E
PERFORMANCE EVALUATION IN SINGLE-PERIOD/SINGLE-AGENT SETTINGS

PART F
PRIVATE AGENT INFORMATION AND RENEGOTIATION IN SINGLE-PERIOD/SINGLE-AGENT SETTINGS

PART G
CONTRACTING IN MULTI-PERIOD/SINGLE-AGENT SETTINGS

PART H
CONTRACTING WITH MULTIPLE AGENTS IN SINGLE-PERIOD SETTINGS

Foreword to Volume I

Joel S. Demski

It has long been recognized that accounting is a source of information. At the same time, accounting thought has developed with a casual if not vicarious view of this fundamental fact, simply because the economics of uncertainty was not well developed until the past four decades. Naturally, these developments in our understanding of uncertainty call for a renewed look at accounting thought, one that formally as opposed to casually carries along the information perspective.

Once this path is entered, one is struck by several facts: Information is central to functioning of organizations and markets, the use to which information is put becomes thoroughly endogenous in a well crafted economic analysis, and uncertainty and risk sharing are fundamental to our understanding of accounting issues.

This is the path offered by the remarkable Christensen and Feltham volumes. Their path takes us through equity and product markets (Volume I) and labor markets (Volume II), and offers the reader a wide-ranging, thorough view of what it means to take seriously the idea that accounting is a source of information. That said, this is not academic technology for technology's sake. Rather it cuts at the very core of the way we teach and research accounting. Once we admit to multiple sources and multiple uses of information, we are forced to test whether our understanding of accounting is affected seriously by ignoring those other sources and uses of information, both in terms of combining information from various sources for some particular use and in terms of reactive response to other sources when one, the accounting source, is altered. It is here that the importance of thinking broadly in terms of the various sources and uses comes into play, and the message is unmistakable: accounting simply cannot be understood, taught, or well researched without placing it in its natural environment of multiple users and multiple sources of information.

The challenge Peter and Jerry provide is not simply to master this material. It is to digest it and act upon it, to offer accounting thought that is matched, so to speak, to the importance of accounting institutions.

We are deeply indebted to Peter and Jerry. That debt will go unattended until we significantly broaden and deepen our collective understanding of accounting.

Preface to Volume I

In 1977, Tom Dyckman, then Director of Research for the American Accounting Association (AAA) encouraged Joel Demski and Jerry Feltham to submit a proposal for a monograph in the AAA Research Monograph series, "on the state of the art in information economics as it impacts on accounting." Joel and Jerry prepared a proposal entitled:

"Economic Returns to Accounting Information in a Multiperson Setting"

The proposal was accepted by the AAA in 1978, and Joel and Jerry worked on the monograph for the next few years, producing several of the proposed chapters. However, the task went more slowly and proved more daunting than expected. They were at separate universities and both found that, as they wrote and taught, they kept finding "holes" in the literature that they felt "needed to be filled" before completing the monograph. This, plus the rapid expansion of the field, meant they were continually chasing an elusive goal.

In the early nineties, Joel and Jerry faced up to the fact that they would never complete the monograph. However, rather than agree to total abandonment, Jerry "reserved the right" to return to the project. While, at that time, he did not expect to do so, he did have 500 pages of lecture notes that had been developed in teaching two analytical Ph.D. seminars in accounting: "Economic Analysis of Accounting Information in Markets," and "Economic Analysis of Accounting Information in Organizations."

Over the years, Jerry had received several requests for his teaching notes. These notes had the advantage of pulling together the major work in the field and of being done in one notation. However, they were very terse and mathematical, having been designed for use in class where Jerry could personally present the intuition behind the various models and their results. To produce a book based on the notes would require integration of the "words" and "graphs" used in the lectures into the notes (and there were still holes to fill).

Peter Christensen had been a student in one of Jerry's classes in 1986. In 1997, Peter asked Jerry if he was going to write a book based on his lecture notes. When Jerry stated it was too big a task to tackle alone, Peter indicated his willingness to become a coauthor. This was an important factor in Jerry's decision to return to the book, since he had worked effectively with Peter in publishing several papers over the preceding 10 years. Also of significance was our assessment that young researchers and Ph.D. students would benefit from a book that provides efficient access to the basic work in the field. The book need not try to provide all the latest results and it need not "fill the holes". The objective is to lay an integrated foundation that provides young researchers with the tools necessary to insightfully read the latest work in the field, and to develop their own theoretical analyses.

Parallel to Jerry's two Ph.D. courses, the book is divided into two volumes.

Economics of Accounting: Volume I – Information in Markets
Economics of Accounting: Volume II – Performance Evaluation

Chapter 1 gives an overview of the content of Volume I, while Chapter 16 gives an overview of the content of Volume II. Each volume is divided into several parts.

Volume I – Information in Markets
- Part A. Basic Decision-Facilitating Role of Information
- Part B. Public Information in Equity Markets
- Part C. Private Investor Information in Equity Markets
- Part D. Disclosure of Private Owner Information in Equity and Product Markets

Volume II – Performance Evaluation
- Part E. Performance Evaluation in Single-Period/Single-Agent Settings
- Part F. Disclosure of Private Management Information in Single-Period/Single-Agent Settings
- Part G. Contracting in Multi-Period/Single-Agent Settings
- Part H. Contracting with Multiple Agents

The three chapters in Part A are foundational to both volumes. However, with occasional exceptions, one can read the material in Volume II without having read Parts B, C, and D of Volume I. Jerry begins both of his Ph.D. courses by ensuring all students understand the fundamental concepts covered in Part A, since these courses are offered in alternate years and the students differ with respect to which course they take first.

Students often seem to find it easier to grasp the material in Volume II, so there is some advantage to doing it first. However, conceptually, we prefer to cover the information in markets material first, and then consider management incentives. The advantage of this sequence is that management incentive models assume the manager contracts with a principal acting on behalf of the owners. The owners are investors, and Volume I explicitly considers investor preferences with respect to the firm's operations. Furthermore, while most principal-agent models implicitly assume incentive risks are firm-specific, there are models that recognize that incentive risks are influenced by both market-wide and firm-specific factors. To fully understand the impact of the market-wide factors on management incentives, one needs to understand how the manager can personally invest in the market so as to efficiently share market-wide risks with other investors. The first volume provides the necessary background for this type of analysis.

Acknowledgments
Our greatest debt is to Joel Demski. Joel and Jerry were colleagues at Stanford from 1967 to 1971, and collaborated on some of the early information economics research in accounting. Their initial work focused on the role of accounting information in facilitating management decisions, and culminated in the book, *Cost Determination: A Conceptual Approach.* In that book they recognized that accounting had both a decision-facilitating and a decision-influencing role, but the book focused on the former. While completing that book, Joel and Jerry were exposed to work in economics which explicitly considered information asymmetries with respect to management's information and actions. They recognized that this type of economic analysis had much to contribute to our knowledge about the decision-influencing role of accounting. In 1978 they published a paper in *The Accounting Review*, "Economic Incentives in Budgetary Control Systems," which would later receive the AAA 1994 Seminal Contribution to Accounting Literature Award. One of Joel's many Ph.D. students, John Christensen, was instrumental to Peter's interest in accounting research. In recent years, Peter, as with Jerry, has had the opportunity to learn much from working with Joel on joint research.

We also want to acknowledge our debt to other coauthors who have significantly contributed to our knowledge through the joint research process. These include Joy Begley, Hans Frimor, Jack Hughes, Jim Ohlson, Jinhan Pae, Martin Wu, and Jim Xie. Their names are mentioned frequently throughout the two volumes, as we describe some of the models and results from the associated papers.

As noted above, Jerry's Ph.D. lecture notes provide the foundation for much of the material in our two volumes. Jerry acknowledges that he has learned much from preparing the notes for his students and interacting with them as they sought to learn how to apply economic analysis to accounting. The accounting Ph.D. students who have been in Jerry's classes as he developed the notes include Amin Amershi, Derek Chan, Peter Clarkson, Lucie Courteau, Hans Frimor, Pat Hughes, Jennifer Kao, Claude Laurin, Xiaohong Liu, Ella Mae Matsumura, Jinhan Pae, Suil Pae, Florin Sabac, Jane Saly, Mandira Sankar, Mike Stein, Pat Tan, Martin Wu, and Jim Xie. Some have been Jerry's research assistants, some have been his coauthors (see above), and Jerry has supervised the dissertations of many of these students. In addition to the accounting Ph.D. students, Jerry's Ph.D. seminars have been attended by graduate students in economics, finance, and management science, as well as a number of visiting scholars. All have contributed to the development of the material used in this book.

We are particularly appreciative of colleagues who have read some draft chapters and given us feedback that directly helped us to improve the book. These include Hans Frimor, Jim Ohlson, Alex Thevaranjan, and Martin Wu. Recently, Anne Adithipyangkul, Yanmin Gao, and Yinghua Li (three current Ph.D. students) have served as Jerry's research assistants and have carefully read through the recent drafts of all of the chapters. We are thankful for their diligence and enthusiasm. We are grateful to Peter's secretary, Lene Holbæk, for her substantial editorial assistance.

Jerry's research has been supported by funds from the American Accounting Association, his Arthur Andersen Professorship, and the Social Sciences and Humanities Research Council of Canada. Peter's research has been supported by funds from the Danish Association of Certified Public Accountants, and the Social Sciences Research Council of Denmark.

The writing of a book is a time consuming process. Moreover, every stage takes more time than planned. One must be optimistic to take on the challenge, and then one must constantly refocus as various self imposed deadlines are past. We are particularly thankful for the loving patience and good humor of our wives, Else and June, who had to put up with our constant compulsion to work on the book. Also, Peter has three sons at home, Kasper, Esben, and Anders. They had to share Peter's time with the book, but they also enjoyed a sabbatical year in Vancouver.

Peter O. Christensen

Gerald A. Feltham

Preface to Volume II

As we stated in the preceding "Preface to Volume I," Volume II focuses on accounting's decision-influencing role in the form of providing performance measures that are useful for incentive contracting. Part A of Volume I contains three chapters that provide foundational material on the decision-facilitating role of information: single-person decision making under uncertainty, decision-facilitating information, and risk sharing, congruent preferences, and information in partnerships. If the reader is not familiar with the basics, you are encouraged to read those three chapters before reading this second volume.

While it is helpful to have read Parts C, D. and E of Volume I before reading Volume II, it is not necessary for the vast majority of topics. The exceptions are the few sections in Volume II in which we consider either private investor information or the impact of economy-wide versus firm-specific risks, assuming only the latter are diversifiable.

Chapter 16 gives an overview of the content of Volume II, which is now divided into the following four parts.

- Part E. Performance Evaluation in Single-period/Single-agent Settings
- Part F. Private Agent Information and Renegotiation in Single-period/ Single-agent Settings
- Part G. Contracting in Multi-period/Single-agent Settings
- Part H. Contracting with Multiple Agents in Single-period Settings

Acknowledgments

This second volume is a direct outgrowth of the work Joel Demski and Jerry started in their 1978 *Accounting Review* paper, "Economic Incentives in Budgetary Control Systems." This paper later received the AAA 1994 Seminal Contribution to Accounting Literature Award. Joel is referenced many times throughout this volume because he has produced a number of significant papers dealing with agency theory. Other co-authors of papers referenced in this volume are Hans Frimor, Christian Hofmann, Florin Şabac, Martin Wu, and Jim Xie. We are also very thankful to Hans Frimor, Christian Hofmann, and Florin Şabac for their detailed comments on recent drafts of several chapters. Earlier drafts were read by Alex Thevaranjan, and Martin Wu, as well as by three Ph.D. students who are currently finishing their dissertations: Anne Adithipyangkul, Yanmin Gao, and Yinghua Li. We are thankful to all who have contributed to the two volumes, and we are grateful to Peter's secretary, Lene Holbæk, for her substantial editorial assistance.

Jerry's research on the second volume has been supported by funds from his Arthur Andersen Professorship, his Deloitte and Touche Professorship, and the Social Sciences and Humanities Research Council of Canada. Peter's research has been supported by funds from the Danish Association of Certified Public Accountants, and the Social Sciences Research Council of Denmark.

Our wives, Else and June, have endured the long, and often consuming, process as we worked to complete a second volume of over 600 pages. We again thank them for their loving care and good humor. Also, Peter has three sons, Kasper, Esben, and Anders, who have had to share Peter's time with the book.

Peter O. Christensen

Gerald A. Feltham

CHAPTER 16

INTRODUCTION TO PERFORMANCE EVALUATION

The following are excerpts from Chapter 1 of Volume I of the *Economics of Accounting*. These introductory remarks are applicable to both volumes.

> In their book on cost determination, Demski and Feltham (1977) characterize accounting as playing both *decision-facilitating* and *decision-influencing* roles within organizations. In its decision-facilitating role, accounting reports provide information that affects a decision maker's beliefs about the consequences of his actions, and accounting forecasts may be used to represent the predicted consequences. On the other hand, in its decision-influencing role, anticipated accounting reports pertaining to the consequences of a decision maker's actions may influence his action choices (particularly if his future compensation will be influenced by those reports).
>
> We adopt these two themes, but broaden the perspective to consider the impact of accounting on investors, as well as managers. We view accounting as an economic activity – it requires the expenditure of resources, and affects the well-being of those who participate in the economy. Obviously, to understand the economic impact of accounting requires economic analysis.
>
> The relevant economic analysis is often referred to as *information economics*. It is a relatively broad field that began to develop in the nineteen-fifties, with significant expansion in the nineteen-eighties. Much of information economic analysis makes no explicit reference to accounting reports. In fact, even the information economic analyses conducted by accounting researchers often do not model the specific form of an accounting report. Nonetheless, many generic results apply to accounting reports. Furthermore, the impact of accounting reports depends on the other information received by the economy's participants. Hence, it is essential that accounting researchers have a broad understanding of the impact of publicly reported information within settings in which there are multiple sources of public and private information.
>
> In our two volumes, we consider the fundamentals of a variety of economic analyses of the decision-influencing and decision-facilitating roles of information. While many of these analyses do not model the details

> of accounting reports, our choices reflect our convictions as to the analyses that are relevant for understanding the economic impact of accounting.
>
> While the two volumes contain many references to recent research, we do not seek to comprehensively cover recent research. Information economic research has grown significantly, and our focus is on fundamentals. New researchers, particularly Ph.D. students, find it difficult to find time to read the fundamental work in the field, and this makes it difficult for them to fully grasp the recent work. Our two volumes stem from two Ph.D. seminars at The University of British Columbia. The first considers economic analyses that are pertinent to the examination of the role of accounting information in capital markets. The second considers economic analyses that are pertinent to the examination of the role of accounting information in motivating managers. Hopefully, by developing an understanding of the fundamentals in these two areas, new researchers will be able to gain a broad understanding of the field, and then will be able to efficiently read and understand the recent work that is of interest to them. (Christensen and Feltham, 2003, p. 1-2)

The focus in the first volume is on the decision-facilitating role of information, with emphasis on the impact of public and private information on the equilibria and investor welfare in capital and product markets. The focus of this second volume is on the decision-influencing role of contractible information (e.g., verified, public reports) that is used to influence management and employee behavior.

> A key distinction between the analyses in the two volumes is that in (the) first volume, managers of firms are not explicitly modeled as economic agents – they do what they are told by shareholders, and do not require any incentives to do so. In the second volume, managers are economic agents with personal preferences, and the theme is the role of information for performance evaluation. (Christensen and Feltham, 2003, p. 2)

The two volumes are each divided into four parts. Part A (Chapters 2, 3, and 4) of the first volume sets the stage for both volumes. Chapter 2 reviews the basics of representing beliefs, preferences, and decisions under uncertainty. Chapter 3 reviews the basics of representing decision-facilitating information in a single decision maker context. Basic concepts of efficient risk sharing are discussed in a partnership setting in Chapter 4. If you are not familiar with the concepts discussed in Chapters 2, 3, and 4, then we recommend that you read those chapters before beginning to read this second volume.

The four parts of this second volume are as follows. Part E has five chapters (17 through 21) that discuss various aspects of the contract between a principal and a single agent in a single-period setting. The three chapters of Part F

(22 through 24) extend the single-agent/single-period model to consider post-contract/pre-decision information, pre-contract/pre-decision information, and renegotiation of the contract before it is terminated. The four chapters (25 through 28) in Part G consider several types of multi-period models, while the final two chapters (29 and 30) in Part H consider some multi-agent models.

With the exception of the basic material in Part A of Volume I, most of the content of Volume II can be read without having read Volume I. The exceptions to this occur in a few sections in which we explicitly consider market risk (i.e., economy-wide, non-diversifiable risk) in settings in which we emphasize the role of investors as owners of the firm or as sources of information that is impounded in the market price of a firm's equity.

In this introductory chapter we first provide a simple depiction of a principal-agent relationship. Then we briefly describe the content of the various chapters in each part. These descriptions also provide some perspective on why we have included the topics contained in these chapters, and how they relate to each other.

16.1 AN ILLUSTRATION OF A PRINCIPAL-AGENT RELATIONSHIP

Stimulated by a paper on sharecropping by Stiglitz (1974), Demski and Feltham (1978) introduced agency theory to accounting.[1] At a subsequent conference sponsored by the Clarkson Gordon Foundation, Atkinson and Feltham (1981) presented a non-mathematical paper that discusses the economic analysis of the role of accounting reports in evaluating and motivating worker effort. The conference was attended by both academics and professional accountants. To help the audience to understand the fundamental nature of agency theory, Feltham prepared an overhead of the cartoon described on the following page. It is designed to capture the key elements of an agency relationship.

There are three individuals in the cartoon. The man in the top hat is an *investor* who provides the initial capital, including the farm, required for production. The man in the straw hat is a *farmer* who provides the effort necessary to manage and operate the farm. The man in the "green eye shade" is an *accountant* who is hired to provide an independent report of information that is relevant to the contract between the investor and the farmer. The events depicted in the cartoon are as follows.

---

[1] Demski and Feltham received the American Accounting Association's 1994 Seminal Contribution to Accounting Literature Award for this paper.

Panel 1: The investor (a *principal*) provides capital "$" to the farmer (an *agent*), in return for a contract "C" that specifies the terms of their relationship.

Panel 2: The farmer uses the investor's capital and his own effort in production.

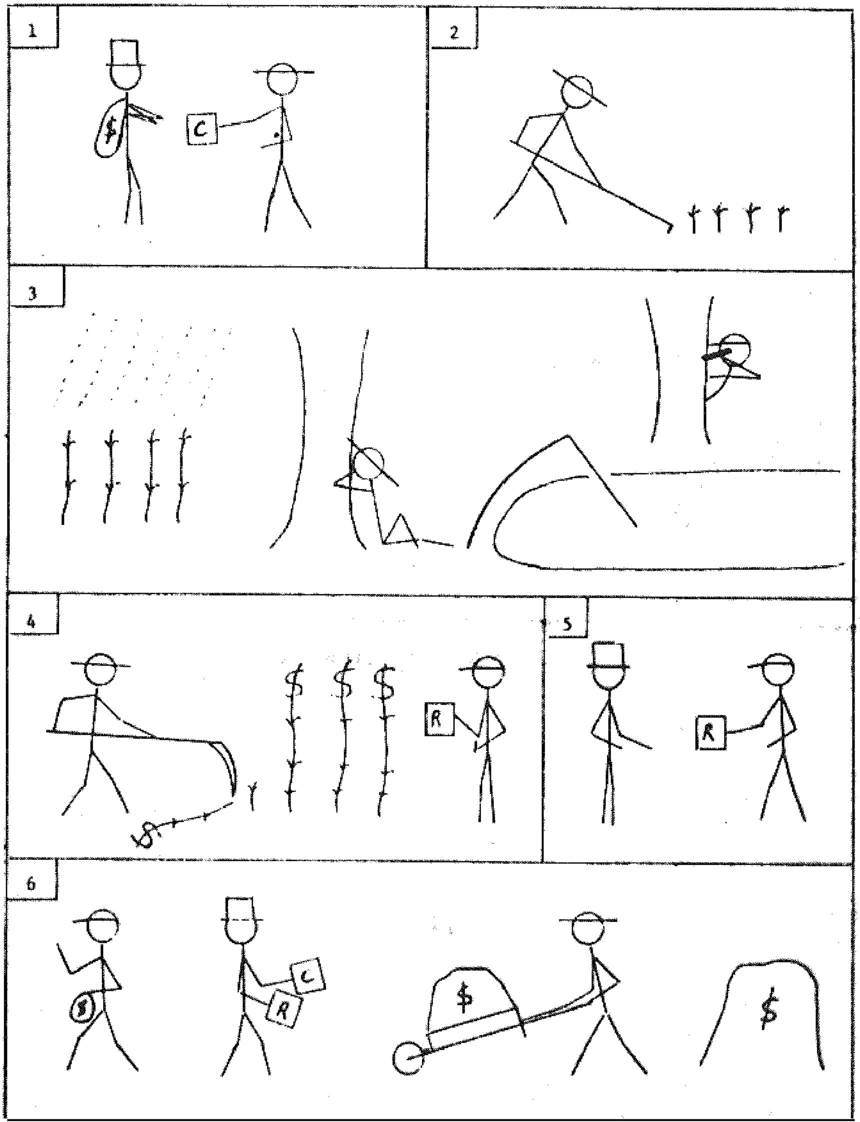

Figure 16.1: The investor, the farmer, and the accountant.

Panel 3: The outcome from the capital and effort is also influenced by random events, such as rain. The farmer is resting from his labor at the fishing pond. The accountant (a monitor) is "spying on" the farmer – is the farmer shirking or merely getting a "second wind"?

Panel 4: The farmer harvests the outcome from the capital, effort, and random events. The accountant is there to record the size of the harvest, and gives a copy of his report "R" (on the farmer's fishing and harvest, and perhaps the rain) to the farmer.

Panel 5: The accountant also gives a copy of his report to the investor.

Panel 6: It is now time to settle up. The accountant collects his fee, the investor collects his share of the harvest (the "$" in the wheelbarrow) based on the contract "C" and the auditor's report "R". The farmer retains the remainder of the harvest (the stack of "$" behind him).

The terms of the contract will depend on a variety of factors. The following questions and comments identify some of those factors.

– The contract must be acceptable to both parties. What factors affect their preferences? Both the investor and the farmer are likely to prefer more $ to less, but they may differ in their aversion with respect to variations in the $ they may receive? In addition, the investor has preferences with respect to the terminal (i.e., end-of-contract) value of his farm.

– The farmer has preferences with respect to the effort expended in operating the farm. In what tasks is this effort expended and does the mix affect the value of the harvest and the terminal value of the farm? Also, how does the mix affect the "cost" of the effort to the farmer?

– Are there other farmers the investor can hire, and are there other investors (farm owners) who would be willing to hire the farmer? What would be the terms of these alternative contracts? Who has the bargaining power with respect to any gain (surplus) from the investor contracting with the farmer instead of each contracting with the next best alternative?

– What contractible information will be available when the contract is settled? Will there be an accurate or noisy count of the harvest? What about its value (which depends on quality as well as quantity)? Will there be direct information about the farmer's level of effort in the various tasks the farmer undertakes? Will there be direct information about random uncontrollable factors that affect the harvest, such as the weather or infestations of locusts?

– Does the contractible information include the market values of the unsold harvest, the land, and the equipment as at the start and at the end of the contract?

– Will the farmer receive information that will affect his beliefs about the consequences of his effort choices? Is this information private, or does the investor receive the same information? Is it received before or after the contract is signed? If it is received after the contract is signed, is it received before or after the farmer expends his effort? If only the farmer receives this information, does he communicate it to the investor? Is he motivated to report truthfully?

– Can the investor and farmer credibly commit not to renegotiate the contract before harvesting?

– Will the investor employ the farmer for more than one period? If yes, will the initial contract be for one period or for multiple periods? What long-term commitments are enforceable? For example, can the investor and farmer preclude future revisions to the contract that are mutually acceptable at the time the initial contract is renegotiated? Can the farmer leave at the end of a period even though the contract continues beyond that date?

– Will the investor contract with other farmers, i.e., does he own other farms? Will the contract with one farmer be influenced by information about other farms and farmers, e.g., because they are affected by correlated uncontrollable events? Will the farmers coordinate their effort levels? Can the farmers collude and share their aggregate compensation differently than specified by the investor?

– What are the accountant's preferences and what form of contract does he have with the investor? Will the accountant diligently collect the desired information? Will he report it truthfully, or will he collude with the farmer? Can an independent auditor be hired to verify the accountant's report? What factors affect the diligence and truthfulness of an independent auditor?

The implications of many of the issues raised above are explored in subsequent chapters. We now briefly describe the content of those chapters.

16.2 BASIC SINGLE-PERIOD/SINGLE-AGENT SETTINGS

Part E, which consists of Chapters 17 through 21, considers a variety of issues within the context of simple settings in which a principal contracts with one agent for one period, and all reports are received by both parties at the end of the contracting period. We do not model the source of those reports, i.e., the characteristics of the information provided by an accountant's reports are exogenously specified.

16.2.1 Optimal Contracts

In much of our analysis we assume the principal owns a firm that consists of a production technology that requires input from an agent to produce an outcome that is beneficial to the principal. The principal hires the agent from a competitive labor market by offering the agent an employment contract. The agent accepts the contract if, and only if, his expected utility from this contract is at least as great as his expected utility from the next best alternative. The latter is referred to as his *reservation utility* level.

The agent's input into the firm is often referred to as *effort*. In Chapters 17, 18, and 19 we assume the set of alternative effort levels is either finite or single dimensional, and the effort alternatives are ordered such that "more" effort directly reduces the agent's expected utility (i.e., is more costly to him) and increases the firm's outcome (i.e., cash flow or terminal market value). The outcome varies with both the agent's effort level and random, uncontrollable events.

The Basic Model

In Chapter 17 we assume the outcome (e.g., the value of the realized net operating cash flow over the firm's lifetime) is contractible information (e.g., an independently verified public report of the outcome is issued at the end of the period). In Chapter 18 we relax that assumption and consider performance measures that may not include the outcome. This is the case, for example, if the outcome is not fully realized until some date subsequent to the termination of the contract, or is only reported to the principal. Accounting reports can play a particularly important role in this setting since they provide interim measures of the final outcome.

As is standard in the agency theory literature, we generally assume that only the agent knows what actions he has taken, i.e., his actions are not contractible information. Therefore, while the principal can choose the actions he would like the agent to take, the principal often cannot directly force the agent to take those actions. Instead, the principal offers the agent a contract that induces the agent to take the desired actions. A contract and the desired actions are *incentive* *compatible* if the contract is acceptable to the agent and induces him to choose the desired actions.

It is useful to view the agent as both the supplier of a factor of production, e.g., effort, and potentially a partner in the sharing of the outcome risk. Ideally, the agent would be paid the market price for his effort and the two partners would efficiently share the outcome risk. This is possible, for example, if there is a costless monitor who provides a contractible report of both the agent's actions and the outcome produced. Throughout the book we frequently determine the optimal contract assuming such a report is produced. The resulting contract and outcome are referred to as *first-best*, and this serves as a useful benchmark against which we compare *second-best* contracts. A second-best contract is the contract that maximizes the principal's expected utility given the available contractible information, the agent's preferences, and the agent's reservation utility.

First-best Contracts

Section 17.2 identifies four different conditions under which the first-best outcome can be achieved with a contractible report of the outcome, but no report of the agent's actions. To avoid achieving first-best so as to give scope for exploring the impact of alternative performance measures (which can include accounting reports), we assume there is no direct contractible report of the agent's actions. In Chapter 17 we assume the outcome is the only contractible information, the principal is the owner of the production technology, and has all the bargaining power. In Chapter 18 we introduce alternative performance measures, and also consider settings in which the agent owns the production technology and has all the bargaining power.

Second-best Contracts

In Chapter 17 we briefly consider settings in which both the principal and agent are risk averse, and explore the relationship in these settings relative to the partnership relation in Chapter 4 (Volume I) in which the agent has no direct preferences with respect to his effort. The principal's decision problem consists of choosing a compensation contract and the actions which maximize the principal's expected utility subject to three types of constraints. First, there is a *contract acceptance constraint* (which many papers call the *reservation utility constraint*). It requires the contract and desired actions to provide the agent with an expected utility that is at least as large as his expected utility from his next-best alternative employment. Second, there are one or more *incentive compatibility constraints*, which ensure that the agent's expected utility from implementing the actions desired by the principal is at least as large as the agent's expected utility from implementing any other actions. Third, there is a set of constraints that ensures that the compensation paid given each possible outcome level is at least as large as the agent's minimum compensation level.

If the set of alternative actions is finite (see Section 17.1), then the set of incentive compatibility constraints is finite. On the other hand, if the set of possible actions is an interval on the real line (see Section 17.2), then there are an infinite number of incentive compatibility constraints. To facilitate our analysis, we identify sufficient conditions for all incentive constraints to be satisfied if a single local incentive constraint is satisfied. If the set of actions is an interval on the real line, then these latter conditions permit us to use a first-order approach to characterize the agent's effort choice.

It is important to note that if the principal is risk neutral and the agent is risk averse, then, from a risk sharing perspective, it is optimal for the principal to bear all the outcome risk. However, if the agent's actions are non-contractible, the outcome is contractible, and the outcome is influenced by the agent's costly actions, then it is optimal to offer the agent a contract in which his compensation varies with the outcome if it is optimal to induce more than minimal effort. That is, in this setting, the agent bears outcome risk because the outcome is informative about the agent's actions, not because it has value to the principal. This point is highlighted by the fact that the optimal contract varies with the *likelihood ratio* associated with each outcome level. This characterization highlights the fact that it is the relative probabilities that determine the compensation level, not the relative value of the outcomes.

The Mirrlees Problem

The support of an outcome probability distribution is the set of outcome levels that has a positive probability of occurring for a given action. The first-best result can be achieved if the support of a performance measure changes with the effort level such that there is a set of performance levels that has a positive probability of occurrence if, and only if, the agent provides less than the first-best level of effort. The agent is paid the first-best fixed wage if those performance levels do not occur, and is threatened with a severe penalty if they do. This is not possible if the support is constant. However, if the support is constant, severe penalties for very low performance levels may be used to get arbitrarily close to the first-best results. If conditions are such that this occurs (see Section 17.3.3), then a second-best contract does not exist and we have what is called the *Mirrlees Problem*. Throughout the book we either assume this problem does not exist (e.g., the severity of the possible penalties is limited) or the penalty contract is not allowed (e.g., contracts must be linear).

Randomized Contracts

A randomized contract consists of a set of two contracts (one preferred by the principal and the other preferred by the agent) from which one is randomly chosen after the set is accepted by the agent. Virtually all of the literature assumes it is optimal for the principal to offer the agent a non-randomized contract. Section 17.3.4 briefly discusses the fact that there are conditions under which the principal strictly prefers to offer a randomized contract. Throughout the remainder of the book we adopt the standard approach in the literature and assume a non-randomized contract is optimal.

Agent Risk Neutrality and Limited Liability

Most agency theory models assume the agent is risk averse and, hence, must be paid a risk premium if his contract imposes incentive risk. Assuming the agent is risk neutral simplifies the analysis, but it removes the risk premium and often results in a setting in which the first-best result can be obtained by selling or leasing the firm to the agent. While the first-best contract is a useful benchmark, it is not an interesting setting in which to explore the role of accounting reports.

Section 17.4 demonstrates that implementation of the first-best result with a risk neutral agent is avoided if there is limited liability (i.e., the principal cannot receive more than the outcome), and the amount the principal receives must be a monotonic function of the firm's gross outcome. In that case, a debt contract is optimal, and the debt is risky, so that the agent does not bear all the risk and does not implement the first-best effort level.

Throughout the book, we avoid achieving first-best by assuming the agent is risk averse. Chapter 23 is an exception. In that chapter, we assume the agent is risk neutral, but the first-best result is not achieved because, prior to contracting, the agent receives private information about the random events affecting the outcome from his effort.[2]

16.2.2 *Ex Post* Reports

In Chapter 17 we assume the firm's outcome is contractible and, in much of the Chapter 17 analysis, it is the only contractible information. Chapter 18 considers multiple measures, including non-outcome measures. The analyses provide insights into how performance measure characteristics affect the principal's expected net outcome. These insights are applicable to both accounting- and non-accounting-based measures.

If the outcome from the agent's action is not contractible, then the role of performance measures depends on who "owns" (i.e., consumes) the residual net outcome and whether the "owner" is risk neutral or risk averse. The agent, for example, is deemed to "own" the outcome if he physically controls it and there is no contractible report of how much outcome he has. On the other hand, the principal "owns" the outcome if he will receive it, even if that occurs some time after the termination of the agent's contract. Sections 18.1 and 18.3 assume

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[2] Several chapters, including Chapter 17, have technical appendices. We do not, in general, mention them in this introduction.

"outcome ownership" by a principal who is, respectively, risk neutral versus risk averse. Section 18.2, on the other hand, assumes "outcome ownership" by the risk and effort averse agent.

If a risk neutral principal owns the outcome (as in Chapter 17), then his primary concern is efficiently motivating the agent's effort. An effort-informative performance measure is required, and a noise-informative report can be valuable because it reduces the incentive risk premium paid to the agent. If the principal is risk averse, then an outcome-informative report can be valuable in facilitating risk sharing.

If a risk averse agent owns the outcome, then an outcome-informative report can be valuable in facilitating risk sharing. If the primary report is influenced by the agent's action, then a moral hazard problem is induced and an action-informative report can be valuable.

Some of the analysis in Chapter 18 can be viewed as an extension of Blackwell's informativeness result for decision-facilitating information (see Chapter 3 of Volume I). In Chapter 18, our measures of informativeness are applied to *ex post* (i.e., post-decision) reports, and focus on action (incentive) and state (insurance) informativeness. A report is action (incentive) informative if it is influenced by the agent's actions, and it is state (insurance) informative if it is correlated with the uncontrollable events that influence either the outcome or action informative reports.

The likelihood measure is a useful tool in assessing the relative value of alternative reporting systems and in representing reports in settings in which the reports are used strictly to provide efficient effort incentives. If a proposed report will not change the likelihood measure obtained with the existing reports, then the proposed report has no value. On the other hand, a statistic (that provides a less detailed description of the reports) is as valuable as the detailed contents of the reports if all sets of reports that result in the same statistic have the same likelihood measure. We refer to this as a sufficient implementation statistic or a sufficient incentive statistic.

The likelihood measure is a random variable and Section 18.1.2 establishes that one reporting system is more valuable than another if the likelihood measure distribution function for the latter system dominates the former based on second-order stochastic dominance. That is, greater variability of the likelihood measure is valuable since it results in a lower risk premium for incentive risk.

Accounting reports generally report a linear aggregation of detailed information in the accounting system. Section 18.1.4 identifies settings in which a sufficient implementation statistic is a linear function of the detailed information.

A risk averse principal or a risk averse agent may be able to share risks with others by trading in the capital market. In Chapter 5 of Volume I, we consider a single-period model of efficient risk sharing in a competitive capital market. Section 18.3.1 uses results from that analysis to consider a setting in which the firm's outcome and performance measures are affected by both economy-wide and firm-specific events. The principal represents well-diversified investors who are effectively risk averse with respect to the economy-wide risks, but risk neutral with respect to the firm-specific risks. He offers the agent a contract that uses firm-specific incentive risk to induce the desired effort level, but provides the agent with his efficient level of economy-wide risk. A key point in this analysis is that economy-wide risks are efficiently shared because the agent can adjust his exposure to that risk by trading event-securities for the economy-wide events. However, the firm-specific risks are not avoidable by the agent and are imposed by the principal as a means of dealing with the moral hazard problem created by the non-contractibility of the agent's actions.

Costly Conditional Acquisition of Additional Performance Information

Management accounting often includes reports that compare actual performance to some standard, with the expectation that the system will generate additional information to explain any "significant" differences. This led several authors to develop agency theory models in which there is a primary report and a secondary report. The primary report is generated each period, but the costly secondary report is only generated if the primary report falls within some pre-specified "investigation set." Section 18.4 examines models of this type.

Likelihood measures and the shape of the agent's utility function play key roles in determining the set of performance measures that trigger investigations. For example, in one setting, the gross benefit of investigation is independent of the likelihood measure if the agent has a square-root utility function. Hence, for any given cost it is optimal to either always or never investigate. On the other hand, if the agent has an exponential or logarithmic utility function, then the gross benefit of an investigation is decreasing in the likelihood measure. Hence, for any given cost it can be optimal to investigate reports with low likelihoods, but not those with high likelihoods. The low likelihood events may be low probability events, but that is not necessarily the case.

It is important to recognize that a conditional investigation strategy is only effective if the agent acts in the belief that the investigation strategy will be implemented. Hence, the principal must be able to make a credible *ex ante* commitment to implement the proposed strategy. Otherwise, once the agent has taken his action it will not be optimal for the principal to pay the cost of the secondary report.

16.2.3 Linear Contracts

Chapter 19 considers linear contracts in settings in which a risk-neutral principal "owns" the outcome. The chapter begins by demonstrating that the optimal contract is linear if the agent has logarithmic utility for consumption and the distribution function for the performance report is from the one-parameter exponential family (e.g., a normal distribution with known variance). The second section begins with the fact that any contract based on a binary performance measure (i.e., there are two possible reports, e.g., one or zero) can be expressed as a linear function. This is then extended to a repeated agency problem with independent binary performance measures. The optimal incentive contract is a linear function of the number of "ones" that are reported if the agent's utility function is exponential with a monetary effort cost. A one-dimensional Brownian motion is a natural extension of the repeated binary model to a setting in which the agent generates a continuum of binary outcomes. This setting can be represented as a one-period model in which there is a single normally distributed performance measure (based on the aggregate outcome from the one-dimensional Brownian motion) for which the optimal contract is linear.

Extension of these results to multiple signals is problematic. For example, two binary performance measures do not, in general, yield an optimal incentive contract that is a linear function of the number of "ones" for each signal. Furthermore, two performance measures that are individually represented by a one-dimensional Brownian motion measure must be represented as a three-dimensional Brownian motion when used together. Hence, the optimal contract cannot be represented as a linear function of two normally distributed random variables.

An implication of the analyses described above is that the optimal contract is not linear except in some very limited cases. Nonetheless, many analyses in the past decade have restricted the contracts to be linear and have, therefore, identified the optimal linear contract rather than the optimal contract. Section 19.1 discusses the basics of the *LEN* model, in which *L* refers to linear contracts, *E* refers to exponential agent utility (with a monetary effort cost), and *N* refers to normally distributed performance measures. The likelihood measure is linear, but the optimal contract is concave, not linear. However, restricting the contract to be linear significantly simplifies the analysis. In particular, the agent's certainty equivalent is a linear function of the mean and variance of his compensation minus his effort cost.

Extensions of the basic *LEN* model are used extensively throughout the book. For example, Chapter 20 considers a *LEN* model with multiple tasks and multiple performance measures, Chapter 21 considers a normally distributed market price as a performance measure in a *LEN* model, Chapters 25 through 28 consider a variety of multi-period *LEN* models, and Chapter 29 considers a multi-agent *LEN* model. We caution the reader to constantly keep in mind that, in these settings, linear contracts are not optimal. Using linear approximations has a long tradition in accounting. However, we should be watchful for conditions under which it is a poor approximation or yields misleading results. For example, in Chapter 27 we introduce a *QEN* contract that uses quadratic functions to implement some useful indirect incentives that are overlooked in the standard linear contract.

16.2.4 Multiple Tasks

In Chapters 17 through 19 we assume that the agent's actions can be interpreted as the level of effort expended in a single task. That is, the actions can be ordered by the intensity of effort required, with the assumption that more effort is more costly to the agent and provides a higher outcome. In Chapter 20 we introduce multi-task models in which the agent's action is represented as a vector which describes the effort expended in each task. More effort in any given task is more costly to the agent and provides a higher outcome, but there can be many effort vectors that incur the same cost, but result in different outcome levels. Hence, in choosing a reporting system and the contract on the available contractible reports, the principal must choose both the aggregate level of induced agent effort and the allocation of that effort across tasks. Of course, while the principal is concerned with how the allocation of effort affects the outcome, the agent allocates his effort based on how that allocation affects his compensation, and that depends on how that allocation affects the performance measures used in the agent's compensation contract.

Multi-task LEN Model

Section 20.1 describes a basic multi-task model that uses the first-order approach to determine the optimal contract. The insights that can be generated by that model are limited and, hence, we base our subsequent analysis on a multi-task *LEN* model. The expected outcome is a linear function of the effort vector and the agent's cost is an additive quadratic function of the effort in each task. There are multiple normally distributed performance measures, for which the means are linear functions of the vector of effort levels. Closed form solutions are derived for the optimal incentive rates for each performance measure and for the optimal effort level in each task. The relative allocation of effort across tasks will not, in general, be the same as the first-best allocation.

If a single performance measure is used, then the relative allocation of effort depends on the relative sensitivities of that performance measure to the effort in various tasks. A performance measure is defined to be *perfectly congruent* if its relative sensitivities are the same as the relative benefits, which implies that the first-best allocation of effort is achievable with that single performance measure. However, it will not be optimal to induce the first-best levels of effort unless the performance measure contains no noise or the agent is risk neutral.

Multiple Performance Measures

In single-task models, an additional performance measure can have value if it reduces the risk premium paid to the agent to compensate him for his incentive risk. This result also applies to multi-task models, but in these models an additional performance measure can also have value if it helps overcome the incongruity of the first measure.

In single task models with unit variance in the performance measures, the relative incentive rates applied to two performance measures depend on their relative sensitivity to the agent's effort, adjusted for the correlation in the two measures. In multi-task models, adjustment must also be made for the lack of alignment between the performance and the relative benefits of the tasks, and for the lack of alignment between the two performance measures.

Insights are provided by examining several special cases. These include settings in which both measures are perfectly congruent with the outcomes, one is a sufficient statistic for the two measures, one is perfectly congruent and the other is purely insurance-informative (i.e., it has zero sensitivity to the agent's effort), the two performance measures are independent and myopic (i.e., influenced by the effort in different tasks), one measure is perfectly congruent and the other is myopic, and effort in one task has positive benefit whereas effort in the other task is merely "window dressing" (i.e., it is costly and influences the first performance measure but produces zero benefit). A common theme in the analysis of the special cases is that an additional measure can be valuable even if the first measure is perfectly congruent (due to incentive risk reduction), and if the first measure is not perfectly congruent, a second non-congruent measure can have value because a better allocation of effort can be achieved using two non-congruent measures instead of one.

Induced Moral Hazard

In most agency models there is a moral hazard problem with respect to each task since each action is assumed to be non-contractible and personally costly to the agent. However, there are many settings in which the agent takes actions that are not personally costly, e.g., the choice of investment projects funded by the principal. We illustrate how the existence of some actions that are personally costly, and some that are not, can give rise to moral hazard problems with respect to both types of actions. The latter are termed induced moral hazard problems, and they arise if both types of actions influence performance measures that are used to motivate the agent's choice of the first type of action. We provide a single performance measure example to illustrate this point, and then identify sufficient conditions for two performance measures to induce first-best investment choice while inducing the second-best effort choice. Then we explore the inducement of under- and over-investment if the conditions for inducing first-best investment are not satisfied.

A key point of this analysis is to demonstrate that performance measures are often influenced by a variety of actions. The incentives may be focused on actions that are personally costly to the agent, but care must be taken due to the inefficient spill-over effects on the choices of actions for which the agent has no direct preferences.

Incentive Implications of Non-separable Effort Costs
Section 20.3 describes some multi-task models that emphasize the incentive implications of the nature of the effort costs, rather than the performance measures. The first class of models considers settings in which the agent's actions have both personal benefits and personal costs. In this case, incentive compensation supplements the agent's personal effort incentives. The second class of models consider personal costs that are constant below some threshold level of aggregate effort. Examples are provided in which the lack of performance measures for some tasks can make it optimal to pay a fixed wage, so that the agent can be requested to undertake the threshold level of effort and then allocate it to the tasks that will yield the largest expected outcome to the principal.

Log-linear Incentive Functions
The final analysis in Chapter 20 demonstrates that the analysis of a multi-task *LEN* model can be employed in a setting in which the agent has a multiplicatively separable exponential utility function and the performance measures are log-normally distributed. The key is to restrict the compensation function to be a linear function of the log of each performance measure.

An appealing aspect of this model is that the support of a log-normal distribution is bounded below at zero, whereas normally distributed random variables can be negative.

16.2.5 Stock Prices and Accounting Numbers

Chapter 21 explores the use of the firm's end-of-contract stock price as a contractible performance measure. That price reflects the investors' end-of-contract information, which can include public contractible reports, such as published financial statements, and non-contractible information that is common knowledge to all or some investors.

Stock Price as an Aggregate Performance Measure
If all information is common knowledge, then the stock price efficiently reflects the information in those reports with respect to the firm's future cash flows. However, that does not imply that the stock price is an efficient aggregate performance measure. Section 21.2 examines the conditions under which the stock price is an efficient aggregate performance measure, i.e., the conditions under which the relative weights placed on a pair of reports in the stock price are the same as the relative weights placed on those reports in an optimal linear contract. For example, the stock price is an efficient aggregate performance measure if there is a single performance measure or if there is a single task and two performance measures whose relative correlations with the outcome equal their relative sensitivities to the agent's effort. As suggested by the second

example, there are conditions under which contracting only on the stock price is efficient, but those conditions do not generally hold.

Stock Price as a Proxy for Non-contractible Investor Information

Section 21.3 considers settings in which the stock price reflects both non-contractible investor information and public reports, such as published financial statements. Generally, both are reflected in the optimal compensation contract even though the public information is impounded in the stock price. In examining contracting in this setting it is useful to replace the price with a statistic that removes the effect of the public information and reflects only the investors' non-contractible information (plus noise). Interestingly, if the contract is written on the stock price and the accounting report, then it is quite possible the coefficient on the accounting report will be negative even though it would be positive if it was used with the statistic representing the non-contractible report.

The investors' non-contractible information may be known by all investors or only by those who pay to acquire it. In the latter case, we endogenously determine the fraction of investors who are informed using a rational expectations model similar to the model in Section 11.3 of Volume I. Endogenizing the information acquisition can have a significant effect on comparative statics. For example, an increase in the noise in the price process reduces the informativeness of the price with respect to that report if the fraction informed is exogenous. However, if the fraction informed is endogenous, then the increase in noise results in an increase in the fraction informed and no change in the informativeness of the price.

Options versus Stock Ownership in Incentive Contracts

In the standard *LEN* model, the agent's effort only affects the mean of the performance measure distribution, not the variance. The optimal contract in that setting is concave (except at the lower bound). As a result, a linear contract is a better approximation to the optimal contract than is a convex, piecewise linear contract. Hence, it is not surprising that the analysis in Section 21.4 establishes that the principal's expected net outcome is higher if he uses stock grants instead of option grants as components of the agent's incentive contract in a setting with exponential utility and normally distributed outcomes with constant variance.

To obtain insight into the role of stock options in incentive contracts, we examine the shape of the optimal contract in a setting in which the agent's effort increases the mean and the variance of the outcome. If the impact on the variance is sufficiently strong, then the optimal contract is convex in the "middle" and then concave in the two "tails." The image is such that we refer to it as a butterfly contract. Now a convex, piece-wise linear contract is a better approximation to the optimal contract than is a linear contract. Hence, an option contract may dominate a linear contract. The key here is that if the variance is increasing in effort, then more effort will result in an increase in the probabilities in both tails. Consequently, the threat of low compensation when there is a low outcome can deter the agent from working hard if he owns stock. Options prevent this deterrence.

16.3 PRIVATE AGENT INFORMATION AND RENEGOTIATION IN SINGLE-PERIOD SETTINGS

Chapters 22 through 24 consider models that serve as bridges between the single-period models of Chapters 17 through 21 and the multi-period models of Chapters 25 through 28. In multi-period models, the end-of-period information for one period is pre-decision information for subsequent periods. Furthermore, contracts may be renegotiated at the start of each period. Chapters 22 and 23 consider single-period models in which the agent receives pre-decision information, while Chapter 24 considers single-period models in which there is post-decision renegotiation of contracts.

16.3.1 Some General Comments

Revelation Principle

In all three chapters we invoke the *Revelation Principle.* This principle states that under appropriate conditions (e.g., full commitment by the principal and unrestricted agent communication), there exists an optimal contract that induces the agent to fully and truthfully report his private information. Hence, the principal can focus on contracts that induce the agent to reveal his private information before the outcome is realized. The standard mechanism for accomplishing this is for the contract offered by the principal to contain what is called a menu of contracts. In Chapter 22, the agent accepts the contract, observes his information, chooses from the menu, and then chooses his action. In Chapter 23, the agent observes his information, simultaneously accepts the contract and chooses from the menu, and takes his action. In one scenario in Chapter 24, the agent accepts the contract, randomly chooses an action, and then chooses from the menu (at the renegotiation date).

Communication of Perfect versus Imperfect Private Information

The principal is never worse off with agent communication. However, agent communication has zero value if he has perfect information about the end-of-contract performance that will be reported given each possible action. On the other hand, we provide examples in which communication of imperfect information has strictly positive value. For example, we provide a model in which the agent is paid a fixed wage if he reports bad news, whereas if he communicates good news, he is given a risky contract to induce positive effort.

16.3.2 Post-contract, Pre-decision Information

The Value of an Informed Agent
The example mentioned above illustrates that pre-decision information can be valuable because it facilitates more efficient effort choices. On the other hand, the information can have a negative effect because it facilitates shirking by the agent. The discussion in Section 22.5 illustrates that, in general, comparing the results with private pre-decision information versus no pre-decision information involves subtle trade-offs.

Delegated Information Acquisition
Instead of treating the information system as exogenous, Section 22.6 considers endogenous information acquisition by the agent. Information acquisition is personally costly, but the information is used to make an investment choice that is not personally costly. The incentives used to motivate information acquisition may create an *induced moral hazard* problem with respect to the investment choice. Subtle issues arise in setting the optimal contract when there is communication and an induced moral hazard problem. For example, it can be useful to induce the agent to choose investments that increase the informativeness of the outcome with respect to the agent's information acquisition activity.

The Optimal Timing of Reports
Section 22.7 considers two pre-decision information acquisition dates and explores the impact of the timing of when the private reports are received and when they are communicated to the principal. Under sequential communication, the agent reports his observations when they are made, whereas with simultaneous communication he reports both observations after he makes the second observation. Sequential communication is always weakly preferred to simultaneous communication, and in some cases the preference is strict.

Examples are used to illustrate a variety of effects including exogenous probabilistic verification of a report and the role of early imperfect information in predicting future perfect information.

Contracting on Market Prices and Management Disclosure
Finally, Section 22.8 examines a setting in which non-contractible investor information is reflected in a firm's market price unless the agent issues a more informative report. The Revelation Principle does not apply since the principal cannot commit the investors to ignore the agent's report. The manager can manipulate his report, but not the investors' other information. Interestingly, a model is considered in which full disclosure by the manager dominates (is

dominated by) no disclosure if the informativeness of the investor's private signal is low (high). Furthermore, an example is provided in which it is optimal for the contract to induce the manager to only partially reveal his private information, and thereby permit partial indirect contracting on the investors' private signal through the market price.

16.3.3 Pre-contract Information

Agent risk aversion plays a key role in most of the models examined in this book. However, in pre-contract information models it is common to assume that the agent is risk neutral.[3] In this setting, the principal cannot achieve first-best by selling or renting the firm to the agent since the efficient selling price varies with the agent's information. To induce the agent to accept the contract and communicate his information, the menu must be such that the agent earns information rents (i.e., his expected compensation exceeds his reservation wage) unless he has the worst possible information.

Imperfect Private Information

While communication can be valuable if the agent's private information is imperfect with respect to the outcome, this need not be the case if the agent is risk neutral. This is illustrated in Section 23.3 in a setting in which the number of possible outcomes is at least as large as the number of possible private signals, and a *spanning condition* is satisfied. An example is used to illustrate this point, and to then illustrate that spanning is not sufficient if the agent is risk averse.

Mechanism Design Problems

In the models discussed above the cost of an agent's action is common knowledge, but there is uncertainty about the outcome that will result. Private pre-contract information affects the agent's belief about the likelihood of the outcome resulting from his action choices. In *mechanism design problems* the agent chooses the outcome, but is uncertain about the cost he will incur in producing the chosen outcome. His private pre-contract information affects the agent's beliefs about the cost he will incur.

The initial section on mechanism design problems discusses model assumptions that are sufficient to yield a contract that induces an outcome function that is monotonically increasing with respect to the agent's private information.

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[3] At the time of contracting, the agent is an informed player and the principal is uninformed. We assume that the uninformed principal offers a contract, or a menu of contracts, to the informed agent. Hence, the analysis is significantly different than in signaling games (see Chapter 13 of Volume I) in which the informed agent offers a contract to the uninformed principal.

This is followed by an analysis of a setting in which there is a positive probability the agent is not informed, after which we consider a setting in which the agent endogenously decides whether to become informed prior to contracting.

Impact of a Public Report on Resource Allocation

Section 23.4.4 discusses a mechanism design model that is used to explore the impact of public and private information on investment decisions. In the basic model with no public report, the principal supplies capital to the agent in return for some contracted outcome level. The amount of capital required to produce a given outcome level is equal to the outcome times a random fraction that is revealed to the agent prior to contracting. The agent personally retains the difference between the capital supplied and the capital used. The optimal contract is characterized by a "hurdle" such that if the reported investment cost parameter is greater than the hurdle, zero capital is provided. On the other hand, if the agent reports a cost parameter below the hurdle, the capital provided equals the amount required to produce the maximum output if the cost parameter equals the hurdle.

The analysis then introduces a public report that is received prior to the agent receiving his private signal (and before he selects from the menu of contracts). The information system partitions the set of possible private signals, reducing the set of possible private signals the agent might receive. This reduces the expected information rent the principal will have pay to the agent and increases the set of signals for which the principal induces positive investment. Therefore, the public information generally has positive value to the principal, but negative value to the agent.

The latter result differs from the reporting of public information in a post-contract, pre-decision information setting. In that case the principal is often better off with the public report, but the agent is indifferent since he will reject the contract if he does not expect to receive his reservation utility.

Early versus Delayed Reporting of Private Information

Section 23.4.5 uses a mechanism design model to explore the impact of the agent's report to the principal in a setting in which the agent receives imperfect information before contract acceptance followed later by the receipt of perfect information. The analysis is similar to the analysis of the timing of reports in a post-contract, pre-decision information model in Section 22.7. The principal strictly prefers to receive an early report, but there is a loss in social welfare because the expected reduction in the agent's information rents more than offsets the principal's expected gain. We again use an example to provide insights into the factors that yield the key results.

16.3.4 Intra-period Renegotiation

In Chapters 22 and 23, the agent receives private pre-decision information. In the former, the agent receives the information after contracting and cannot quit the firm after observing his private signal. In the latter, the agent either receives the information prior to contracting or can quit after observing his signal. Hence, the differences in the two chapters illustrate the impact of differences in commitment to a contract. Chapter 24 explores the impact of other commitment limitations in single-period models.

Most agency theory models assume that the principal and the agent cannot make a mutually acceptable change in (i.e., renegotiate) the contract after it has been signed. However, it is frequently the case that the principal and agent will prefer to renegotiate the contract after the agent has taken his action if the original contract was based on the assumption of no renegotiation. Furthermore, the ability to renegotiate often makes the principal worse off, from an *ex ante* perspective, which is why it is often exogenously precluded.

Renegotiation-proof Contracts

Section 24.1 considers a standard single-period agency model, but with the added dimension of contract renegotiation after the agent has taken his action. If a risk neutral principal conjectures that a risk and effort averse agent has been induced to take some specific action, then after the action has been taken, there will be an *ex post* Pareto improvement if the principal agrees to pay the agent a fixed amount in return for absorbing all of the agent's incentive risk. Of course, if this is anticipated by the agent, he will take his least cost action, and if this is anticipated by the principal, then the initial contract will be a fixed amount that is sufficient to compensate the agent for his least cost action. Consequently, the inability to exogenously preclude renegotiation makes the principal worse off.

Section 24.1 considers a renegotiation-proof contract that contains a menu from which the agent chooses after he has taken his action. The contract is designed to induce the agent to take a randomized action strategy and the menu is designed to induce him to truthfully reveal his action choice. Hence, the contract is similar to the pre-decision contracts in Chapters 22 and 23, and is also similar to the signaling contracts considered in Chapter 13 of Volume I.

Agent-reported Outcomes

Section 24.2 extends the analysis to consider a sequence of two actions with contract renegotiation between the first action and the first outcome, which precedes the second action and second outcome. In the basic setting, the two outcomes are contractible information and the agent is induced to randomly choose his first action and then reveal his action by his choice from a menu of contracts (as in Section 24.1). The analysis is then extended to consider a setting in which the agent issues unverified reports of the period-specific outcomes, subject to the constraint that the total reported for the two periods cannot exceed the actual total (i.e., there is a limited audit). Interestingly, with agent reporting, there exists a renegotiation-proof contract that does not involve randomized first-period actions. Furthermore, the principal strictly prefers to contract on agent-reported outcomes with a limited audit instead of fully audited outcome reports.

Renegotiation Based on Non-contractible Information

Renegotiation can be beneficial if it takes place after the principal has observed the agent's action or after the principal and agent have observed an imperfect signal about the agent's action. This benefit holds even if the principal's observations are not contractible. In fact, the principal can achieve the first-best result if there is anticipated renegotiation after he makes a non-contractible observation of the agent's action. The key to this result is that the principal can offer to replace the agent's incentive contract with a fixed payment that accurately reflects the agent's information about the forthcoming compensation. Hence, in the end, the agent bears no incentive risk.

Principal Is Privately Informed

The analysis in Section 24.3 assumes both the principal and the agent make a non-contractible observation of the imperfect performance measure. In Section 24.4, only the principal makes this observation. Renegotiation is now replaced with a menu of contracts which is used to induce the principal to truthfully reveal his private information. In this setting, incentive issues are associated with both the principal and the agent, and the budget balancing constraint restricts the effectiveness of the incentives. To "break" this constraint, a risk neutral third party is introduced.

Resolving a Double Moral Hazard Problem

Chapter 24 concludes by considering a simple model in which both the risk neutral principal and the risk averse agent take personally costly non-contractible actions. There are no contractible performance measures. However, the ownership of the firm is tradeable and the principal observes the agent's action. In this setting, the principal offers the agent a contract that specifies a wage and a buyout price, with the stipulation that after the principal observes the agent's action, the principal will choose whether to retain ownership and pay the agent the wage or sell the ownership to the agent for the buyout price. Interestingly, despite the fact there are no contractible performance measures, the principal can achieve the first-best result.

16.4 MULTI-PERIOD/SINGLE-AGENT SETTINGS

We now consider models in which the agent takes a sequence of actions possibly following a sequence of periodic reports. Consumption and compensation can occur at the end of each period, but their timing can differ through borrowing and saving. Chapters 25 through 27 assume there is full commitment so that the principal and the agent can preclude contract renegotiation throughout the term of a contract, and they can preclude early termination of the contract. Chapter 28 considers settings in which there is limited commitment.

16.4.1 Full Commitment with Independent Periods

Chapter 25 examines several basic multi-period issues when there is full commitment. To simplify the analysis, we consider a sequence of periods with independent, period-specific performance measures.

Agent Preferences

Most of our analyses are based on either time-additive (*TA*) consumption preferences (i.e., the sum of a sequence of period-specific utility functions), or aggregate-consumption (*AC*) preferences (i.e., a single utility function defined over an aggregate measure of consumption). The agent's "cost" of effort is represented by either an effort-disutility (*ED*) function which is deducted from the utility for consumption or an effort-cost (*EC*) function which is deducted from the agent's consumption.

Exponential AC-EC preferences are simple to use since there is no wealth effect, and the timing of information, compensation, and consumption is irrelevant. Exponential *TA-EC* preferences also have no wealth effect, and the timing of compensation is irrelevant if there is borrowing and lending. However, the timing of information is relevant, since the agent is motivated to smooth consumption.

On the other hand, there are wealth effects with *TA-ED* and *AC-ED* preferences. For example, the cost of inducing a given level of effort increases with the wealth of the agent. As a result, the timing of reports matters.

Multi-period LEN Model

A single-period, single-task *LEN* model (i.e., linear compensation, exponential utility, and normally distributed performance measures) was introduced in Chapter 19 and extended to consider multiple tasks in Chapter 20. Multi-period *LEN* models are used extensively throughout Chapters 25 through 28.

We consider both *AC-EC* and *TA-EC* preferences. As we demonstrate, both are tractable, but the latter provides more interesting insights. For example, in our *TA-EC* models we allow the agent to have a consumption horizon beyond his employment contract – it can even be infinite to reflect the agent's bequest preferences. During the life of the agent's compensation contract, performance measure noise results in random variations in compensation, which are spread over all future periods. Interestingly, the agent's inter-temporal trades do not affect either his effort choices or the principal's optimal contract choice.

The actions induced by a linear contract are the same for both *TA-EC* and *AC-EC* preferences, and the basic form of the optimal contract is the same in both cases. The key difference is in the form of the nominal risk aversion parameter used to compute the agent's certainty equivalent.

In the *TA-EC* model the risk aversion used to calculate his certainty equivalent reflects the agent's ability to spread random variations over future periods. Hence, his effective risk aversion increases as he becomes older, if he has a finite consumption planning horizon.

T Agents versus One

Section 25.5 explores the benefits and costs to the principal of retaining the same agent for all periods. There are no wealth effects with exponential *TA-EC* or *AC-EC* preferences and, as a consequence, there is no benefit to replacing an agent. However, we demonstrate that with *TA-ED* and *AC-ED* preferences it is optimal to retain agents who earn low compensation in the first period and replace those who earn high compensation. This, of course, requires interim reporting.

16.4.2 Timing and Correlation of Reports in a Multi-period *LEN* Model

Chapter 26 extends the basic multi-period *LEN* model introduced in Section 25.4 by allowing performance measures to be stochastically and technologically interdependent. Our primary focus is on the impact of inter-period correlation of performance measure noise in a setting in which the agent has *TA-EC* preferences. However, we also consider *AC-EC* preferences and the impact of aggregation of reports.

Earlier reporting of a signal that is informative about random variations in compensation can be valuable because it facilitates more extensive smoothing of consumption. Of course, while smoothing is valuable with *TA-EC* preferences, it has no value with *AC-EC* preferences.

A performance measure is "action informative" if it is influenced by the agent's actions, "insurance informative" if the noise in the report is correlated with the noise in an action-informative report, and "purely insurance informative" if the performance measure is insurance informative but not action informative. While early reporting of any of these reports can be valuable to the agent given exogenous incentive rates, the issues are more subtle if the incentive rates are optimally chosen by the principal.

A single-action, multiple-reporting-date example with an action informative measure and a pure insurance informative measure is used to illustrate that, with optimal incentive rates, there is positive value to reporting the action informative report as soon as possible and to reporting the pure insurance measure no later than the action informative report. Notably, there is no value to reporting the pure insurance informative measure earlier than the action informative report.

The key to this last result is that in an optimal contract the principal uses the pure insurance measure to remove some of the risk associated with the action informative measure. This can be done when the latter is reported and does not involve the agent smoothing consumption.

On the other hand, it is costly to the principal to delay the report of the pure insurance informative measure beyond the report date for the action informative measure. The problem with the late report of a pure insurance informative measure is that when the action informative measure is reported, the agent cannot distinguish between its insurable and uninsurable components.

Early reporting is often achieved by reporting less precise measures. With *AC-EC* preferences, the principal prefers preciseness – timing is immaterial, and there is no demand for an imperfect interim report. However, with *TA-EC* preferences there is a tradeoff between timeliness and preciseness, and an imperfect interim report can be valuable.

Two Agents versus One

In Section 25.5 we establish that with full commitment, interim reporting, and independent performance measures, the principal is indifferent between hiring one agent for two periods or two agents each for one period. If the noise in the two performance measures are correlated, then disaggregate reporting permits the principal to use a performance measure for motivating one agent and insuring the other. Hence, two agents are preferred to one. The contracts are identical if the agents are identical and they have *AC-EC* preferences. However, that is not the case if they have *TA-EC* preferences, since the first agent is able to smooth his incentive compensation over two periods, whereas the second agent cannot. Comparisons are also made for settings in which agents differ in their productivity.

16.4.3 Full Commitment with Interdependent Periods

Chapter 27 considers settings in which there is stochastic and technological interdependence across periods. This occurs if the uncontrollable events are correlated across periods and the actions in one period affect performance measures beyond the current period. Also, we consider settings in which the performance measure in one period is informative about the marginal productivity of effort in a subsequent period.

Orthogonalized and Normalized Performance Statistics

In Chapter 27 we use orthogonalization and normalization to modify the representation of normally distributed performance measures. Orthogonalization transforms stochastically interdependent reports into stochastically independent performance statistics. The resulting statistic for each period only reveals the "new" information provided in that period. Interestingly, the orthoganalized statistics are generally technologically interdependent even if the initial representations of the reports were technologically independent.

Normalization uses the principal's conjecture with respect to the agent's actions to construct performance statistics that have zero mean if the agent's actions are equal to the principal's conjecture. That is, a normalized statistic is effectively equal to the difference between the realized value of a report and a standard or budget that is equal to its (conditional) expected value if the agent takes the conjectured action. In equilibrium, the agent's action choice equals the principal's conjecture, but in choosing his action the agent considers the possibility of deviating from the principal's conjecture.

The induced effort in any given period depends on direct and indirect incentives. The former refer to the incentive rates applied to the statistics directly affected by the action. The indirect incentives arise from the fact that the agent's action affects the reports that will be used in producing the orthogonalized statistics and in determining the posterior means used in producing the normalized statistics.

Information Contingent Actions

In the basic multi-period *LEN* model the only source of uncertainty is the additive noise in the performance measures. This additive structure plus the lack of a wealth effect (due to exponential *AC-EC* preferences), and the restriction to linear contracts, results in second-period incentive rates and, thus, second-period actions that are independent of the first-period performance reports. In section 27.3 we first use a first-order approach to obtain insight into the characteristics of an optimal contract based on the stochastically independent performance statistics (not constrained to be linear). Even though there are no wealth effects and the first-period performance report is uninformative about the second-period effort productivity, the characterization of the optimal contract shows that in contrast to the multi-period *LEN* model the second-period action varies with the first-period performance report. This is due to the fact that the variation in the second-period contract induces positive indirect first-period incentives. The characterization of the optimal first-period action shows that it is influenced by direct first-period incentives and two types of indirect incentives. The first type is referred to as an indirect "posterior mean" incentive. It is due to the impact of the first-period action on the principal's beliefs about the second-period performance measure when the two performance measures are correlated. The second type is referred to as an indirect "covariance" incentive. This incentive is due to the impact of the first-period action on the covariance between the first-period performance report and the agent's conditional expected utility of the second-period contract. The latter incentive is only present if the second-period contract varies with the first-period performance report.

Second, we consider modifications to the linear contracts that capture these key aspects of the optimal contract, yet are analytically tractable. The central element in these changes is to allow the second-period incentive rate to vary linearly with the first-period report. This causes the second-period effort cost and risk premium to vary, creating effort-cost risk and risk-premium risk. Two *quadratic* functions based on the agent's conjectured actions are introduced to insure the agent against these two risks. We refer to this as a *QEN* contract.

Varying the second-period incentives with the first-period report creates costs in the second period (i.e., the effort-cost and risk-premium risks introduced above), but those costs are offset by the benefits of the indirect first-period covariance incentives created by this variation. Interestingly, while increased positive covariance between the two performance measures has a negative effect with a *LEN* contract, it has a positive effect with a *QEN* contract.

Learning about Effort Productivity

In Section 27.4 we consider two settings in which the first-period report is informative about the output productivity of the agent's second-period effort. The first is an extension of the *LEN* model, which we call the *QEN-P* model. The preferences and performance measures are the same as in the *LEN* model, but the second-period productivity is random and correlated with the first-period report. A *QEN* contract is used. In this case there are two reasons for letting the second-period incentive rate vary with the first-period report. First, it creates indirect first-period covariance incentives of the type described above. Second, it provides more efficient direct second-period incentives, i.e., the induced second-period effort is positively correlated with its second-period output productivity.

Our second setting uses a two-period model in which the first-period action influences the information revealed by the first-period report about the second-period productivity. Optimal contracts (that are not constrained to be linear) are identified. A key feature of this example is that the optimal first-period effort reflects both its output productivity and its impact on the informativeness of the first-period report about the second-period productivity.

16.4.4 Inter-period Renegotiation

The analyses in Chapters 25, 26, and 27 assume that the principal and agent can fully commit to a long-term contract. Chapter 28 assumes there is limited commitment and, at the end of a period, the principal can make a take-it-or-leave-it offer to the agent to change the terms of the contract.

Section 28.1 identifies conditions that are sufficient for a sequence of short-term contracts to replicate the results that could be achieved by a long-term contract with full commitment. These conditions include, for example, preferences, technology, and public information (not necessarily contractible) such that, at the start of each period, the principal knows the agent's beliefs about the outcomes from his actions and his induced action choices for any possible contract. The multi-period exponential utility functions introduced in Chapter 25 play a key role in these results.

Section 28.2 examines the impact of inter-period contract renegotiation in a two-period model. The renegotiation takes place after the first-period reports have been issued and the first-period compensation has been paid. We characterize both optimal contracts and optimal linear contracts with contract renegotiation, and compare those characterizations to the full-commitment contracts examined in Chapter 27. The performance measures and payoffs are linear and normally distributed, and can be stochastically and technologically interdependent. However, the contracts are based on stochastically independent performance statistics that may be technologically interdependent (from the agent's perspective). Furthermore, the first-period performance measure may be informative about the marginal productivity of the second-period action.

A key feature of inter-period renegotiation is that at the renegotiation date the principal bases his contract offer strictly on his posterior beliefs at that date. He ignores *ex ante* considerations, which play a central role in full-commitment contracts. Only direct incentives apply to the second-period action choice in a two-period model. However, as with full-commitment contracts the agent's choice of first-period effort is influenced by direct first-period incentives and the two types of indirect incentives introduced above. In contrast to full commitment, the indirect covariance incentive only occurs with renegotiation if the first-period performance measure is correlated with the second-period performance measure and the second-period marginal productivity of effort. If the correlation between performance measures has the same sign as the correlation between the first-period performance measure and the second-period productivity, then the correlations are defined to be congruent. If they are congruent, then the payoffs from the optimal renegotiation-proof and full-commitment contracts are very similar. However, if they are incongruent, then full-commitment strongly dominates renegotiation because the former can make much more effective use of the indirect covariance incentives.

The analysis is extended to consider two variations in the basic renegotiation model. In the first variation, the agent can always choose to leave at the end of the first period. In the second variation, the principal can commit to either retain or to replace the first-period agent. Deferred compensation can be used to retain an agent for two periods in a setting in which the agent would be otherwise motivated to act strategically in the first-period and then leave. Switching costs can also serve to deter termination of the contract. If switching costs are zero, the principal will prefer to retain (terminate) the initial agent if the indirect incentives are positive (negative).

16.5 MULTIPLE AGENTS IN SINGLE-PERIOD SETTINGS

In Chapters 17 and 18 we focus on single-agent, single-task, single-period agency models. Chapter 20 introduces multiple tasks performed by a single agent in a single period. Then, Chapter 25 through 28 consider multiple tasks performed by a single agent over multiple periods, with a possible change of agent at the end of a period. In Chapters 29 and 30 we very briefly consider some key issues that arise when multiple agents perform multiple tasks within a single period. Chapter 29 considers multiple productive agents, whereas Chapter 30 considers settings in which one agent is productive, while the other is a monitor of the productive agent.

16.5.1 Multiple Productive Agents

We begin Chapter 29 by revisiting the partnership model introduced in Chapter 4 of Volume I. The original model focuses on risk sharing and assumes that either the partners' actions are contractible information or they do not incur any personal costs in taking those actions. Now we assume all partners provide effort that is personally costly and non-contractible. "Budget balancing" and "free rider" problems occur if the aggregate outcome is the only contractible information. These problems can be partially dealt with by committing to give away some of the aggregate outcome if the performance information indicates that all partners should be penalized. Introducing partner-specific performance measures is also shown to be useful, as is the addition of a general partner who does not provide effort, but provides additional risk sharing capacity and, more importantly, permits the partnership to avoid the "budget balancing" constraint with respect to the productive partners.

In Section 29.2 we move from the partnership interpretation of multiple effort averse agents with an effort neutral general partner to an agency interpretation. The general partner is now called a principal. To focus on incentive issues and simplify the risk sharing issues, we assume the principal is both risk and effort neutral, and offers incentive contracts to risk and effort averse agents who operate the principal's firm. We view the principal as a *Stackleberg leader* who specifies the payoffs for a subgame played by the agents. If the performance measures are correlated or are jointly affected by the actions of multiple agents, then the incentive compatibility constraints are potentially much more subtle than in single-agent settings. The agents choose their actions in a simultaneous play game and to be incentive compatible, their action choices must constitute a Nash equilibrium. However, there may be multiple Nash equilibria and the agents' choice may differ from the equilibrium preferred by the principal.

For example, consider a setting in which there are separate action-informative performance measures for two agents. If the performance measures are correlated, then using the measure for one agent as a standard in the contract with the other agent can reduce the incentive risk premia. Assume that the contracts are such that one agent finds it optimal to provide high effort if he believes the other agent is providing high effort, i.e., this is a Nash equilibrium. However, there may be other Nash equilibria which the agents prefer, e.g., both agents provide low effort and claim their poor outcomes are due to bad economic conditions.

One mechanism for dealing with the joint shirking problem described above is to offer one agent an optimal "single-agent" contract based on his own performance measure (so he will not benefit from joint shirking). Then his performance measure can be used as a relative performance measure in contracting with other agents.

In the basic multi-agent model (e.g., in Section 29.2), the principal contracts directly with every agent. In Section 29.3 we consider a setting in which the principal contracts with one agent (the branch manager) who in turn contracts with a second agent (the worker). In effect the principal sets the terms of the size of the pie (the compensation pool) and allows the manger to determine how the pie will be divided. This can be viewed as descriptive of either decentralized contracting or centralized contracting subject to agent renegotiation or collusion.

In the partnership setting introduced in Chapter 4, in which the partners are risk averse and effort neutral, the efficient partnership contract gives each partner a linear share of the total outcome if all partners have HARA utility functions with identical risk cautiousness. In that setting, centralized and decentralized contracting produce the same results. As established in Chapter 4, in this setting efficient contracts produce congruent preferences among the partners.

In an agency with a risk and effort neutral principal and risk and effort averse agents, the optimal centralized contract will assign all risk to the principal, except for the incentive risk that is assigned to each agent. A key feature of a decentralized contract is that the risk averse manager will choose to take on some of the worker's incentive risk and to assign some of the manager's incentive risk to the worker. In addition, while the marginal impact of the agents' actions on the principal's payoff plays a central role in the agents' incentive rates chosen by the principal, the manager will ignore the principal's payoff in his contract choice, unless the principal's payoff is also the performance measure used in the agents' contracts. Centralized and decentralized contracting produce identical results if the agents are identical and contracting is based on the principal's aggregate payoff. However, more generally, decentralized contracts involve inefficient risk sharing and inefficient allocation of effort among the agents. Nonetheless, the principal may prefer decentralized versus centralized contracting if the manager has "local" information about the worker's performance that is not available to the principal.

In Section 29.4 we consider settings in which the agents have private pre-contract information. Recall that in Chapter 23 we consider single-agent models with private pre-contract information. In this type of model, the agents can be risk neutral since information rents replace risk premia as the central focus. Some of the insights generated by the pre-contract information models are similar to insights provided by the basic principal/multi-agent models described above. However, there are differences.

The Revelation Principle applies and the agents are offered menus of contracts that induce them to truthfully report their information. The cost incurred by an agent depends on the outcome he produces and an agent-specific state (i.e., the models considered are mechanism design problems). The states are correlated so that it can be optimal to use both agents' outcomes in specifying the compensation for each agent. We consider two formulations of the principal's problem. In the first, the principal is assumed to induce each agent to report truthfully under the assumption the other agent is motivated to report truthfully. In the second, the principal is assumed to induce each agent to report truthfully even if he believes the other agent will lie (i.e., truthful reporting is a dominant strategy). With risk neutrality, it is possible to attain first-best using the two performance measures. However, that is not possible if the agents are risk averse, since the agents must bear risk, for which they are compensated.

If the states are correlated, then subgame issues arise in this setting just as they did in the basic multi-agent model. Care must be taken specifying the truth-telling constraints. It is not sufficient to require truth-telling to be an optimal response given that the other agent is telling the truth. The principal must also ensure that the agents cannot benefit by colluding in what they report. Similarly to the basic multi-agent model, one way to accomplish this is to offer one agent a contract in which truthful reporting is an undominated strategy, and then use his truthful report in contracting with the other agent.

16.5.2 A Productive Agent and a Monitor

In the final chapter of the book, we introduce a monitor (e.g., a supervisor or an auditor) as an agent who provides information that is useful in contracting with a productive agent. Our coverage is relatively brief, and is restricted to models in which the principal offers outcome- and report-contingent contracts to both the monitor and the productive agent. Hence, we do not consider settings in which the auditor's incentives stem from threats of litigation or from reputation effects, i.e., we consider internal auditors as opposed to external auditors hired on a fixed fee basis.

In the models considered in this chapter, the cost of the worker's action (e.g., the output produced) is random and the worker has private pre-contract information with respect to his cost. Hence, the models are similar to the models in the mechanism design problems considered in Section 23.4. As in Chapter 23, the privately informed worker earns information rents if he has "good news". The key difference is the introduction of an internal monitor who reports private information he obtains about the agent's information, which is used to reduce information rents (and improve production efficiency). In these settings both the worker and the monitor are induced to truthfully report their private information. As in the prior chapter, care must be taken in specifying the incentive compatibility constraints. The contracts induce each agent to report truthfully and to take the actions desired by the principal, considering both unilateral choices by each agent and coordinated actions by the two agents.

In this chapter we introduce *indirect mechanisms* for dealing with the subgame issues associated with coordinated actions by the two agents. Section 30.1 considers a basic model in which there is an informed worker and a costly monitor. The cost of the worker's action is affected by a random state variable, which he observes. The monitor can also observe the state, but only if he incurs a cost. An indirect ("whistle blowing") mechanism is introduced for inducing the worker to report truthfully and for inducing the monitor to incur the information cost and report truthfully. The monitor reports first and then the worker has three choices: accept, reject, or counter-propose (with a pre-specified "side-bet"). In equilibrium, the monitor will acquire the information and report truthfully, and then the worker will accept the contract. With risk neutrality, this mechanism can achieve first-best.

The preceding model is extended to a setting in which the monitor's information is imperfect – it partitions the worker's information. An indirect "whistle blowing" mechanism is again used, but does not achieve first-best.

Section 30.2 considers variations on a model in which the worker has perfect information about the state that influences the costs he will incur in producing a given level of output and the monitor can obtain imperfect information about the state. Both agents are risk neutral and have limited liability (i.e., there is a lower bound on the compensation they can receive). Two benchmark cases are considered, one has a perfect monitor and the other has no monitor. No information rent is paid and the output is efficient if the monitor is perfect, whereas information rent and inefficient output are used to motivate the worker if there is no monitor. Extensive analysis is then provided for a model in which the monitor's information is costless (he is employed for other purposes) but imperfect, and he will report truthfully because he has no incentive to lie. The worker is induced to produce a high output in the good state and low (possibly inefficient) output in the bad state, and the monitor is instructed to obtain and report his imperfect information if the worker produces the low output. The worker receives a base pay for the output produced and is penalized for a low output if the monitor makes a type II error (i.e., the principal incorrectly rejects the worker's claim that his low output is due to a bad state). The worker must be compensated for the expected cost of this incorrect penalty, but using a penalty based on the monitor's imperfect report allows the principal to reduce (and possibly eliminate) the information rent and increase the low output level that are used to motivate the worker's effort. Comparative statics provide insights into how the quality of the monitor's information affects the low productivity output and the information rent received by the worker if he has good news. Two measures of information quality are considered. One assumes there are no type II errors and varies the probability of type I errors (i.e., erroneously accepting a claim by the worker that his low output is due to a bad state). The other measure of quality assumes the errors are symmetric, i.e., both types of error are equally likely. Interestingly, in both settings, first-best results can be achieved with less than perfect information. Of course, the risk neutrality of the worker is crucial for this result. Also, the size of the penalty that can be imposed affects the quality of the information necessary to achieve first-best.

We define collusion to involve side-payments between agents for the purpose of inducing coordinated actions that differ from the actions that would be induced by a contract if there were no side-payments. Hence, collusion goes beyond the coordinated actions that created the subgame problems discussed in Chaper 29. Of course, as stated earlier, delegated contracting (see Section 29.3) can be viewed as equivalent to a model with collusion.

The impact of collusion between the worker and the monitor is explored in Sections 30.2.3 and 30.2.4. We refer to a monitor as collusive if there is a potential for collusion. The fact that a monitor is collusive does not mean collusion occurs. Recall that in settings where contract renegotiation is possible, it does not occur if the principal offers a renegotiation-proof contract (see Chapters 24 and 28). Similarly a collusive monitor will not engage in collusion if the principal offers a collusion-proof contract. Nonetheless, as the analysis demonstrates, collusiveness can destroy the value of the monitor, partially reduce his value, or have zero impact on his value. As we demonstrate, there are three factors that affect the loss of value due to collusiveness. The first is the set of feasible lies the monitor can tell. The second is the restrictiveness of the monitor's limited liability. The third is the probability of a type II error – is it positive or zero?

Two types of mechanisms for controlling collusion are considered: a reward option and a penalty option. In our example, if the principal ignores the possibility of collusion, then the manager will bribe the monitor not to issue a report that would result in the manager being penalized. The principal can counter this collusion by offering a reward to the monitor for issuing a negative report.

The chapter, and the book, concludes with a model in which a costly external monitor (with exogenous incentives, e.g., the threat of litigation or loss of reputation) is hired to audit the report of a costless, collusive internal monitor (whose collusiveness is costly to the principal). The external monitor is only hired with positive probability if the worker's outcome is low and the internal monitor accepts the worker's claim that his low outcome is due to a poor state. The manager and internal monitor are penalized if the external monitor reveals that the internal monitor lied.

16.6 CONCLUDING REMARKS

The reader should keep in mind that accounting reports have both decision-facilitating and decision-influencing roles. This volume focuses on their decision-influencing roles, but at times considers information that is decision-facilitating. Most of our representations of information are relatively generic and do not encompass the institutional and structural details of accounting numbers. However, our choice of topics is based on our judgment as to the fundamentals of information economic analysis that are particularly relevant for accounting researchers who are interested in management incentives.

The agency theory literature began by focusing on single-task/single-period /single-agent models. These establish the fundamentals. However, the multi-task, multi-period, and multi-agent models that have been developed more recently, provide more scope for insights into the characteristics of accounting that affect its value in influencing decisions.

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PART E

PERFORMANCE EVALUATION IN SINGLE-PERIOD/SINGLE-AGENT SETTINGS

CHAPTER 17

OPTIMAL CONTRACTS

We now introduce a model of a two person "partnership" known as the principal-agent model. It introduces incentive issues by assuming that *actions are unobservable* and the contracting parties may have *direct preferences with respect to actions*, as opposed to the standard partnership in which actions are observable and preferences are defined over monetary outcomes (see Volume I, Chapter 4). The basic principal-agent model assumes that the principal owns a production technology. In order for the technology to be productive he must hire an agent to perform a task. How the agent performs the task is unobservable to the principal, but it affects the probability distribution of the monetary outcome of the production technology. The incentive problem is caused (in part) by assuming that the agent has direct preferences with respect to what he does in the task (usually interpreted as the agent's effort), as well as his compensation (i.e., his share of the monetary outcome), while the principal is only concerned about the monetary outcome (net of the compensation paid to the agent). If the monetary outcome is the only contractible information available, then the sharing rule between the principal and the agent can only depend on the monetary outcome. Furthermore, the sharing rule based on the monetary outcome is the only mechanism available to the principal for inducing the agent to make action choices that are consistent with the principal's preferences. More generally, other performance measures may exist, and the monetary outcome may not be reported within the time frame of the contract, but we leave exploration of such settings until Chapter 18.

In this chapter we assume the principal and agent share the outcome x from the production technology operated by the agent, and cannot share the risks associated with that outcome with any other parties. The principal can represent a sole proprietor or a set of partners who own and finance the production technology, and hire the agent. Alternatively, as explored in Chapter 18, the agent can own and operate the production technology, and the principal can represent a set of investors who contract to share the agent's risk and provide investment capital. The capital market is not explicitly considered. However, the results obtained here are consistent with those obtained when the agency operates in a capital market, provided all risks are firm-specific, and therefore cannot be mitigated by appropriate investments in other firms (e.g., the market portfolio). The impact of economy-wide risk within a market setting is examined in Chapter 18.

The model examined in this chapter has an initial date at which the contract is signed and the agent exerts effort in a single task, and a terminal date at which the outcome x is realized and shared by the principal and the agent. The principal and the agent have the same information prior to signing the contract, and there is no additional information until the outcome is realized. In later chapters we extend the basic model to settings in which there are other performance measures at the contract termination date, the agent allocates effort among a number of tasks, the agent receives private information prior to taking his action and possibly prior to accepting the contract, and there is a sequence of action and consumption dates.

In this chapter we first (Section 17.1) introduce the basic principal-agent model, and provide a general discussion of the optimal contract when the agent has a finite number of alternative actions. In Section 17.2 we characterize first-best contracts, which, for example, apply if the principal can observe the agent's action. Section 17.3 explores the impact of the agent's risk and effort aversion on the characteristics of second-best contracts, which apply if the principal cannot observe the agent's action. Finally, Section 17.4 explores the characteristics of the second-best contract if the agent is risk neutral, but has limited liability constraints. Brief concluding remarks are provided in Section 17.5.

17.1 BASIC PRINCIPAL-AGENT MODEL

17.1.1 Basic Model Elements

As in the partnership model (see Volume I, Chapter 4), the outcome $x \in X \subseteq \mathbb{R}$ is determined by the action $a \in A$ (which, in this case, is an unobserved choice by the agent) and the outcome adequate events $\theta \in \Theta$. The principal and the agent have homogeneous beliefs about θ and those beliefs are denoted by a generalized probability density function $\varphi(\theta)$. However, it is useful in this analysis to suppress θ and focus on x as a random variable whose distribution depends on a. For example, if Θ is finite, then the generalized probability density function for x given a is

$$\varphi(x|a) = \sum_{\Theta(x,a)} \varphi(\theta),$$

where $$\Theta(x,a) = \{ \theta \mid \boldsymbol{x}(\theta,a) = x, \theta \in \Theta \}.$$

The principal's share of x is denoted π and the agent's share is c, so that $\pi = x - c$. We generally assume that the principal has unlimited resources so that $\Pi = \mathbb{R}$ is the set of possible values of π, but we assume (unless stated otherwise) that the agent cannot be paid less than a finite lower bound $\underline{c}$, so that $C = [\underline{c}, \infty)$ is the set of possible compensation levels for the agent.

A compensation scheme (contract) specifies the amount to be paid to the agent at the contract settlement date. To be enforceable, the payment specified by the contract must be either fixed or at most vary with the *contractible information* available at the contract settlement date. To be contractible, the information must be acceptable to the court, or whatever institution is used to enforce the contract. In the basic model, it is assumed that the outcome x is *the only contractible information*. Hence, the agent's compensation scheme in this setting is $\boldsymbol{c}: X \to C$ and the set of possible compensation functions is denoted $\boldsymbol{C}$.

The principal's preferences are assumed to be a function of only his share of x, i.e., it is represented by a utility function u^p: $\Pi \to \mathbb{R}$, where Π is the set of possible values of π. The principal has no direct preferences with respect to the agent's action a. However, the agent's preferences may depend on both his consumption c and his action a, i.e., his preferences are represented by a utility function u^a: $C \times A \to \mathbb{R}$.

The *agent's utility function* is generally assumed to be *separable*, by which we mean it can be expressed as $u^a(c,a) = u(c)k(a) - v(a)$, with $k(a) > 0$. We consider three basic forms of separability:

(a) *Additive separability:* $u^a(c,a) = u(c) - v(a)$ (i.e., $k(a) = 1$);

(b) *Multiplicative separability:* $u^a(c,a) = u(c)k(a)$ (i.e., $v(a) = 0$);

(c) *Effort neutrality:* $u^a(c,a) = u(c)$ (i.e., $k(a) = 1$ and $v(a) = 0$).

The principal's and agent's preferences with respect to consumption are assumed to be increasing and concave, i.e., $u^{p\prime} > 0$, $u^{p\prime\prime} \leq 0$, $u' > 0$ and $u'' \leq 0$. If $k(a)$ is not constant, we assume $u(c)$ is non-positive, so that increases in both $k(a)$ and $v(a)$ reduce the agent's utility, thereby representing more costly effort.

The exponential utility function with a monetary cost of effort $\kappa(a)$ is an important example of a multiplicatively separable utility function.

Lemma 17.1

If the agent has a negative exponential utility for consumption and effort imposes a personal cost $\kappa(a)$ in the form of a reduction of consumption, then the utility function is multiplicatively separable, i.e.,

$$u^a(c,a) = -\exp[-r(c - \kappa(a))] = u(c)k(a),$$

where $\quad u(c) = -\exp[-rc]$ and $k(a) = \exp[r\kappa(a)]$, and r is a parameter representing the agent's risk aversion.

17.1.2 Principal's Decision Problem

In our discussion of partnerships we provided a general characterization of Pareto efficient sharing rules. In the analysis presented here we adopt a slightly different perspective. The principal is assumed to "own" the production technology that generates x, and he hires an agent from a market for agents. To entice an agent to accept his contract, the principal must offer a contract that provides the agent with an expected utility at least as great as the agent's "reservation utility" $\bar{U}$, which is the expected utility the agent could obtain from his next best alternative.

Observe that this approach assumes that the principal has all the bargaining power. In Section 17.4 and in Chapter 18 we consider settings in which the agent owns the technology and has the bargaining power, and he contracts with the principal to share risk (and possibly obtain capital). The principal's expected utility from sharing the risk (and providing investment capital) must be at least as great as from his next best alternative. Interestingly, the basic character of the optimal contract is the same in both settings.

In specifying the principal's decision problem, we view him as selecting both the contract $\boldsymbol{c}$ that he offers to the agent, and the action a he will induce the agent to select. Of course, it is the agent who selects the action. Hence, the contract must be such that it induces the agent to accept the contract and to select the specified action a. In agency theory we typically assume the agent will select the action a specified by the principal *if, and only if,* the agent cannot increase his expected utility by doing otherwise. Hence, the principal's decision problem is

Principal's Decision Problem:

$$\underset{\boldsymbol{c}\in \boldsymbol{C}, a\in A}{\text{maximize}} \quad U^p(\boldsymbol{c},a) \equiv \int_X u^p(x - \boldsymbol{c}(x))\, d\Phi(x|a), \tag{17.1}$$

$$\text{subject to} \quad U^a(\boldsymbol{c},a) \equiv \int_X u^a(\boldsymbol{c}(x),a)\, d\Phi(x|a) \geq \bar{U},$$

$$\text{(contract acceptance)} \tag{17.2}$$

$$U^a(\boldsymbol{c},a) \geq U^a(\boldsymbol{c},\hat{a}), \quad \forall\, \hat{a} \in A, \text{ (incentive compatibility)} \tag{17.3}$$

$$\boldsymbol{c}(x) \geq \underline{c}, \quad \forall\, x \in X. \quad \text{(feasible consumption)} \tag{17.4}$$

In (17.1) the principal maximizes his expected utility of his share of the outcome $\pi(x) = x - \boldsymbol{c}(x)$ that will result from his choice of compensation scheme $\boldsymbol{c} \in \boldsymbol{C}$ and induced action $a \in A$. His choice of $\boldsymbol{c}$ and a must satisfy the constraints (17.2)-(17.4). Constraint (17.2) is often referred to as the agent's *participation* or *individual rationality constraint*, and it ensures that the agent has no incentive not to accept the contract (in which case we assume he accepts). Constraints (17.3) (one for each $\hat{a}$) are usually referred to as the agent's *incentive compatibility constraints*. They ensure that given the compensation scheme $\boldsymbol{c}$, the agent has no incentive not to take the action a specified by the principal. That is, the action specified by the principal must be at least weakly preferred by the agent over all other actions, i.e., the induced action maximizes the agent's expected utility given the accepted compensation scheme, which can be expressed equivalently as[1]

$$a \in \underset{\hat{a} \in A}{\operatorname{argmax}} \; U^a(\boldsymbol{c}, \hat{a}).$$

Finally, constraint (17.4) ensures that the agent gets his minimum wage for all outcomes.

The principal's decision problem represents a subgame perfect Nash equilibrium to the sequential game shown in Figure 17.1.[2] The game starts with the principal proposing a contract $z = (\boldsymbol{c}, a)$, which the agent then accepts or rejects. If the agent accepts the proposed contract, he then chooses his action. Constraints (17.2) and (17.3) represent the sequential equilibrium conditions stating that it is incentive compatible for the agent to accept the contract and take the action specified by the principal.

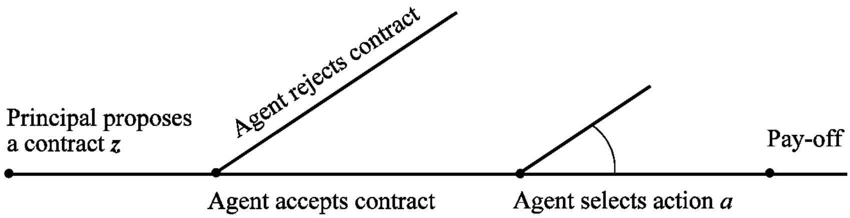

Figure 17.1: Principal's decision problem as a sequential game.

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[1] *Argmax* is the set (of actions in this case) that maximizes the following objective function. If there is a unique optimum, then the set is a singleton, but the notation allows for the possibility of multiple optima so that the set contains more than one element.

[2] The concept of sequential equilibria is discussed in Volume I, Chapter 13.

17.1.3 Optimal Contract with a Finite Action and Outcome Space

We now consider settings in which A and X are finite sets. With A finite, the incentive compatibility constraints can be written as a set of $|A| - 1$ incentive constraints,[3]

$$U^a(\boldsymbol{c},a) \geq U^a(\boldsymbol{c},\hat{a}), \quad \forall\, \hat{a} \in A \backslash \{a\}, \tag{17.3f}$$

and, similarly, with X finite, the consumption feasibility constraints are a set of $|X|$ constraints,

$$\boldsymbol{c}(x) \geq \underline{c}, \quad \forall\, x \in X. \tag{17.4f}$$

Given this formulation, the Lagrangian for the principal's decision problem is:

$$\mathcal{L} = U^p(\boldsymbol{c},a) + \lambda\,[\,U^a(\boldsymbol{c},a) - \bar{U}\,]$$

$$+ \sum_{\hat{a} \in A\backslash\{a\}} \mu(\hat{a})\,[U^a(\boldsymbol{c},a) - U^a(\boldsymbol{c},\hat{a})] + \sum_{x \in X} \xi(x)\,[\boldsymbol{c}(x) - \underline{c}], \tag{17.5}$$

where λ, $\{\mu(\hat{a})\}_{\hat{a}\in A\backslash\{a\}}$, and $\{\xi(x)\}_{x\in X}$ are the multipliers associated with the constraints (17.2), (17.3f) and (17.4f), respectively.

For a given action a to be induced, the principal's compensation scheme choice can be viewed as consisting of $|X|$ choice variables, i.e., the compensation level $\boldsymbol{c}(x)$ for each possible outcome $x \in X$. Differentiating with respect to each $\boldsymbol{c}(x)$, $x \in X$, provides the following first-order conditions characterizing the optimal compensation scheme:

$$- u^{p\prime}(x - \boldsymbol{c}(x))\,\varphi(x|a) + \lambda\, u'(\boldsymbol{c}(x))\,k(a)\,\varphi(x|a)$$

$$+ \sum_{\hat{a} \in A\backslash\{a\}} \mu(\hat{a})\,u'(\boldsymbol{c}(x))\,[\,k(a)\,\varphi(x|a) - k(\hat{a})\,\varphi(x|\hat{a})\,] + \xi(x) = 0.$$

The multiplier $\xi(x)$ is zero if $\boldsymbol{c}(x) > \underline{c}$, in which case the compensation $\boldsymbol{c}(x)$ satisfies

[3] $|A|$ is referred to as the cardinality of the set and represents the number of elements, i.e., in this case, the number of alternative actions in the set.

$$M(x, \boldsymbol{c}(x)) \equiv \frac{u^{p\prime}(x - \boldsymbol{c}(x))}{u'(\boldsymbol{c}(x))} = k(a)\left[\lambda + \sum_{\hat{a} \in A \backslash \{a\}} \mu(\hat{a})\, L(x|\hat{a}, a)\right] > 0, \quad (17.6)$$

where

$$L(x|\hat{a}, a) \equiv 1 - \frac{k(\hat{a})\,\varphi(x|\hat{a})}{k(a)\,\varphi(x|a)}.$$

The left-hand side expression, $M(x,c)$, is the ratio of the principal's and agent's marginal utilities. If the agent has no direct preference for actions, as in the partnerships examined in Volume I, Chapter 4, the principal is only concerned with efficient risk sharing, and the ratio is a constant. However, if the agent has direct preferences for his actions (and some of the incentive constraints are binding), then the right-hand side of (17.6) varies with the outcome x.

The $L(x|\,\hat{a}, a)$ function on the right-hand side reflects the relative likelihood that outcome x will occur given the "desired" action a versus the undesired action $\hat{a}$. Since probabilities must sum to one for all actions, it follows that if there is an outcome x' that is more likely with a than with $\hat{a}$, then there must be another outcome x'' for which the reverse holds. Consequently, $L(x|\,\hat{a}, a)$ is likely to be positive for some outcomes, but negative for others, and this is definitely the case if $k(a) = 1$ for all $a \in A$.

Since the principal's and agent's marginal utilities for their outcome shares are positive, the ratio $M(x,c)$ is non-negative. Hence, the left-hand side of (17.6) is always non-negative. However, the preceding comment implies that the right-hand side can be positive or negative. This creates the possibility of a corner solution. In particular, if for any $x \in X$ the multipliers λ and $\mu(\hat{a})$, $\hat{a} \in A\backslash\{a\}$ are such that

$$k(a)\left[\lambda + \sum_{\hat{a} \in A \backslash \{a\}} \mu(\hat{a}) L(x|\hat{a}, a)\right] < M(x, \underline{c}),$$

then $\xi(x) > 0$ and $\boldsymbol{c}(x) = \underline{c}$, i.e., the agent is paid his minimum compensation.

17.2 FIRST-BEST CONTRACTS

If none of the incentive constraints (17.3) are binding (so that $\mu(\hat{a}) = 0$ for all $\hat{a} \in A$), then we say the contract is *first-best*. In that setting, there is "no incentive problem" and the optimal contract achieves fully Pareto efficient action choice and risk sharing. If some of the incentive constraints (17.3) are binding, then there is a non-trivial incentive problem and we say the optimal incentive contract is *second-best* (with respect to risk sharing).

Definition *First-best Contracts*

A contract $z^* = (\boldsymbol{c}^*, a^*)$ is first-best if it maximizes (17.1) subject to (17.2) and (17.4) in the principal's decision problem.

The following proposition characterizes the efficient risk sharing given the first-best action choice.

Proposition 17.1

If the contract $z = (\boldsymbol{c}, a)$ is first-best, there exists a multiplier λ such that $\boldsymbol{c}(x)$ satisfies

$$M(x, \boldsymbol{c}(x)) = \lambda k(a), \quad \text{if } M(x, \underline{c}) \leq \lambda k(a), \text{ or}$$

$$\boldsymbol{c}(x) = \underline{c}, \quad \text{if } M(x, \underline{c}) > \lambda k(a).$$

If the *principal is risk neutral*, then $u^P(\pi) = \pi$ and $u^{P\prime}(\pi) = 1$. In that case, $M(x,c)$ is independent of x and we write it as

$$M(c) \equiv \frac{1}{u'(c)} = w'(u(c)),$$

where $w(\cdot) \equiv u^{-1}(\cdot)$ denotes the inverse of the agent's utility for consumption.[4] Hence, $w(u)$ is the cost to the principal of providing the agent a utility of u, and $M(c)$ is the principal's marginal cost of increasing the agent's utility at the compensation c.

Proposition 17.2

If the principal is risk neutral and the agent is risk averse with a separable utility function, then the first-best compensation scheme is a constant wage for all outcomes that occur with positive probability given a^*, i.e.,

$$\boldsymbol{c}^*(x) = w\left(\frac{\overline{U} + v(a^*)}{k(a^*)}\right) \equiv c^*, \quad \forall\, x \in X \text{ for which } \varphi(x|a^*) > 0.$$

Of course, if the principal is risk neutral and the agent is risk averse, efficient risk sharing calls for the principal to carry all the risk.

Grossman and Hart (GH) (1983) identify some conditions under which the first-best result is achieved.

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[4] Observe that $w(u(c)) = c$. Differentiating both sides yields $w'(u(c))\, u'(c) = 1$, which implies $w' = 1/u'$.

Proposition 17.3 (GH, Prop. 3)

Assume the agent's utility function is separable and the outcome x is contractible. The first-best result can be achieved if one of the following conditions holds.

(a) The *agent is effort neutral* and the two utility functions u^p and u belong to the HARA class with *identical risk cautiousness*.

(b) The *principal is risk neutral* and the agent is either *effort neutral* or the *first-best action is his least cost action*, i.e,

$$a^* \text{ minimizes } w((\bar{U} + v(a))/k(a)).$$

(c) The *agent is risk neutral* and has sufficient wealth.

(d) Shirking is detected with a sufficiently large positive probability, i.e., if the agent takes an action that is less costly to him than a^*, there is a sufficiently large positive probability that x will reveal that he has not taken a^*.

Given effort neutrality, result (a) follows directly from our discussion of partnerships in Volume I, Chapter 4, and is the case examined by Ross (1973). Recall that if the partners have HARA utilities with identical risk cautiousness, then the Pareto efficient sharing rules are linear and they induce identical preferences over actions. Note also that given the first-best action, first-best risk sharing can be obtained without restricting the two utility functions, i.e., the restriction to the HARA class with identical risk cautiousness is to create identical preferences over actions.

Result (b) identifies two settings in which it is optimal to pay the agent a fixed wage. The principal is risk neutral, and hence efficiently bears all risk, and paying a constant wage to the agent does not cause an incentive problem either because the agent is effort neutral or the principal fortunately desires to induce the action the agent will select if he bears no incentive risk.

Result (c) establishes that agent risk neutrality (with or without effort neutrality) is sufficient to achieve first-best as long as $x - \pi^* \geq \underline{c}$ (for all x which have a strictly positive probability of occurring given a^*), where π^* is the fixed amount paid to the risk averse principal. In this case, the firm is sold (or leased) to the agent, who in effect bears all risk. He will then make the optimal effort selection a^*. Section 17.4 examines the impact of binding limitations on the agent's ability to bear risk.

Result (d) is generally referred to as a setting in which there is "moving support," where the support for the distribution given action a is the set of outcomes that have a strictly positive probability of occurring.

Definition

$X(a) \equiv \{ x \mid \varphi(x|a) > 0, x \in X \}$ is the *support* of $\varphi(x|a)$. The support is *constant* (nonmoving) if $X(a) = X, \forall\, a \in A$.

Penalties cannot be used to achieve the first-best result if the support is constant. On the other hand, it may be possible if we have moving support, i.e., if $X(a)$ varies with a. However, to achieve first-best using the threat of penalties, we must have the following:

(*i*) $X(a) \backslash X(a^*) \neq \emptyset$ for all $a \in A^\dagger \equiv \{ a \in A \mid U^a(\boldsymbol{c}^*, a^*) < U^a(\boldsymbol{c}^*, a) \}$, i.e., there are outcomes that have a positive probability of occurring if the agent selects an action a that is less costly to him than a^*, but have zero probability if he selects a^*;

(*ii*) $u^a(c^*,a)\,[1 - \Phi(a)] + u^a(\underline{c},a)\,\Phi(a) < u^a(c^*,a^*), \quad \forall\, a \in A^\dagger,$

where
$$\Phi(a) = \sum_{x \in X(a) \backslash X(a^*)} \varphi(x|a).$$

If the above conditions hold, then the first-best result can be achieved by paying the first-best wage, $\boldsymbol{c}(x) = c^*$, for each outcome that has a positive probability with a^* (i.e., $x \in X(a^*)$) and threatening to pay the minimum wage, $\boldsymbol{c}(x) = \underline{c}$, for any outcome that has zero probability with a^* (i.e., $x \in X \backslash X(a^*)$). Observe that the payment of $\underline{c}$ is merely a threat – it will never be paid (given that the agent is induced to select a^*). Of course, this is only possible if $\Phi(a) \geq [u^a(c^*,a) - u^a(c^*,a^*)] \,/\, [u^a(c^*,a) - u^a(\underline{c},a)]$ for all $a \in A^\dagger$. This will tend to hold if either the probability of detection, $\Phi(a)$, is relatively large, or the loss in utility when shirking is detected, $u^a(c^*,a) - u^a(\underline{c},a)$, is relatively large.

17.3 RISK AND EFFORT AVERSION

We now focus on settings in which first-best contracts cannot be achieved, i.e., at least some of the incentive compatibility constraints are binding. Consequently, we assume the *agent is both risk averse and effort averse*, i.e., $u' > 0$, $u'' < 0$, and either $k' > 0$ with $u < 0$ or $v' > 0$. On the other hand, we do not view an agent's risk bearing ability as a significant part of most incentive contracts. Therefore, we generally assume that the *principal is risk neutral*, i.e., $u^p(\pi) = \pi$, so that he would bear all risk in a first-best contract. This ensures that any risk borne by the agent in a second-best contract is for incentive purposes. Incentive risk is costly to the agent, but he is compensated for that cost and, hence, it is indirectly costly to the principal.

To exclude the possibility of using moving support (in combination with sufficient penalties) to achieve the first-best result, we generally assume the support is constant across the alternative actions. Furthermore, we assume the optimal action is not the agent's least cost action.

Our maintained assumptions in this section can be summarized as follows:

(a) the principal is risk neutral;

(b) the agent is both risk and effort averse, with a separable utility function;

(c) there is constant support;

(d) the optimal action to be induced is not the agent's least cost action.

These assumptions are sufficient to ensure that the first-best result is not achievable.

17.3.1 Finite Action Space

In this section we further make the following regularity assumptions:

– $A = \{ a_1, ..., a_M \}$, with $\boldsymbol{c}^*(a_\ell) < \boldsymbol{c}^*(a_j)$ if $\ell < j$,

where $$\boldsymbol{c}^*(a) \equiv w\left(\frac{\bar{U} + v(a)}{k(a)}\right),$$

i.e., the set of actions is finite and the actions are ordered in terms of the fixed wage that would be required to compensate the agent for his effort;

– $X = \{x_1, ..., x_N\}$, with $x_h < x_i$ if $h < i$, i.e., the set of possible outcomes is finite and ordered in terms of increasing outcomes.

A Two-stage Optimization Approach

The assumption that the principal is risk neutral permits us to separate the principal's decision problem into two stages. In the *first stage*, we determine, for each action, the contract that induces the particular action at the lowest possible expected cost – the expected cost for action a is denoted $\bar{\boldsymbol{c}}^\dagger(a)$. In the *second stage* we determine the optimal action by maximizing the principal's expected profit.

The contract used to induce an action can be stated as $\mathbf{c} = \{c_1, ..., c_N\}$, where $c_i = \boldsymbol{c}(x_i)$. However, GH state the principal's decision problem for a given action in terms of $\mathbf{u} = \{u_1, ..., u_N\}$, where $u_i = u(c_i)$. The advantage of this approach is that the objective function is now convex (since $u(c)$ is concave and, hence, $w(u)$ is convex) and the constraints are linear. Hence, the Kuhn-Tucker conditions yield necessary and sufficient conditions for optimality.

The Principal's Decision Problem for Inducing Action a_j:

$$\bar{c}^\dagger(a_j) = \underset{\mathbf{u}}{\text{minimize}} \quad \sum_{i=1}^{N} w(u_i)\,\varphi(x_i|a_j), \tag{17.1f'}$$

$$\text{subject to} \quad U^a(\mathbf{u},a_j) \equiv k(a_j)\sum_{i=1}^{N} u_i\,\varphi(x_i|a_j) - v(a_j) \geq \bar{U}, \tag{17.2f'}$$

$$U^a(\mathbf{u},a_j) \geq U^a(\mathbf{u},a_\ell), \quad \forall\ \ell = 1, ..., M,\ \ell \neq j, \tag{17.3f'}$$

$$u_i \geq u(\underline{c}), \quad \forall\ i = 1, ..., N. \tag{17.4f'}$$

In general, all actions may not be implementable, i.e., there may not exist a feasible solution to the program for inducing a_j in which case we set $\bar{c}^\dagger(a_j) = \infty$. However, note that there is at least one action that can be implemented at a finite expected cost, namely the least cost action which can be implemented at its first-best cost.

Also, we cannot rule out the possibility of a corner solution in which (17.4f′) is binding for some x_i. GH avoid this by assuming that $C = (\underline{c},\infty)$ with[5]

$$\lim_{c \to \underline{c}} u(c) = -\infty.$$

Given this assumption, (17.4f′) is redundant, and we can restrict our attention to interior solutions. Hence, the Lagrangian for the above constrained minimization problem is

$$\mathcal{L} = \sum_{i=1}^{N} w(u_i)\,\varphi(x_i|a_j) - \lambda\left(k(a_j)\sum_{i=1}^{N} u_i\,\varphi(x_i|a_j) - v(a_j) - \bar{U}\right)$$

$$- \sum_{\substack{\ell=1 \\ \ell\neq j}}^{M} \mu_\ell \left(\sum_{i=1}^{N} u_i[k(a_j)\,\varphi(x_i|a_j) - k(a_\ell)\,\varphi(x_i|a_\ell)] - [v(a_j) - v(a_\ell)]\right).$$

[5] Note that for efficient risk sharing in partnerships (with no personal costs), the weaker condition $\lim_{c \to \underline{c}} u'(c) = \infty$ is sufficient to preclude corner solutions (see Volume I, Chapter 4).

Differentiating with respect to u_i provides the following characterization of an interior solution

$$w'(u_i) = k(a_j)\left[\lambda + \sum_{\substack{\ell=1 \\ \ell \neq j}}^{M} \mu_\ell L(x_i|a_\ell, a_j)\right]. \tag{17.6'}$$

Let $\mathbf{c}_j^\dagger = \{c_{1j}^\dagger, ..., c_{Nj}^\dagger\}$, where $c_{ij}^\dagger = w(u_i)$ represents the optimal second-best contract for implementing action a_j, as determined by the solution to the above problem.

The *second stage* is to identify the optimal second-best action $a^\dagger$ by comparing the cost of each possible action to the expected gross outcome it will generate, i.e.,

$$a^\dagger \in \underset{a_j \in A}{\text{argmax}}\ \mathrm{E}[x|a_j] - \bar{c}^\dagger(a_j). \tag{17.7'}$$

The optimal second-best contract for implementing $a^\dagger$ is denoted $\mathbf{c}^\dagger$.

Proposition 17.4 (GH, Prop. 1 and 2)

Given the above assumptions with either additively or multiplicatively separable agent utility, there *exists* a second-best optimal action $a^\dagger$ and compensation plan $\mathbf{c}^\dagger$, and that solution is such that the participation constraint is binding, i.e.,

$$U^a(\mathbf{c}^\dagger, a^\dagger) = \bar{U}.$$

The existence of a solution to the principal's cost minimization problem for a given action is ensured by the fact that the cost is bounded below (by the first-best cost of implementing the action), the set of constraints (17.2f′)-(17.3f′) form a closed set, and there are a finite number of alternative actions. The key to the participation constraint (17.2f′) being binding is the assumption that the agent's utility of consumption is unbounded from below. To see this, suppose, to the contrary, that there is a solution $\mathbf{u}$ to the principal's decision problem for inducing some action a_j for which the participation constraint is not binding. Since the agent's utility of consumption is unbounded, there is another contract $\mathbf{u}'$ in the *additively separable case*, defined as

$$u_i' = u_i - \varepsilon, \quad i = 1, ..., N, \quad \varepsilon > 0,$$

which satisfies the participation constraint and is less costly to the principal. The contract $\mathbf{u}'$ clearly satisfies the incentive compatibility constraint since

$$U^a(\mathbf{u}',a_j) = \sum_{i=1}^{M} u_i' \, \varphi(x_i|a_j) - v(a_j) = U^a(\mathbf{u},a_j) - \varepsilon.$$

Similarly, in the *multiplicatively separable case* a feasible less costly contract $\mathbf{u}'$ can be found as

$$u_i' = u_i\,(1 + \varepsilon), \quad i = 1, ..., N, \quad \varepsilon > 0,$$

with
$$U^a(\mathbf{u}',a_j) = k(a_j)\sum_{i=1}^{M} u_i' \, \varphi(x_i|a_j) = U^a(\mathbf{u},a_j)\,(1 + \varepsilon).$$

Hence, a contract **u** for which the participation constraint is not binding cannot be an expected cost minimizing contract for inducing a_j. However, note that if the agent's utility of consumption is bounded below, for example, by zero for the square-root utility function, the participation constraint may be a non-binding constraint. Intuitively, the reason is that inducing a given action is based on the difference in utility associated with "rewards" for good outcomes and penalties for "bad" outcomes. If the lower bound constrains the utility for "bad" outcomes, then the utility for "good" outcomes necessary to induce the desired action can result in an expected utility greater than the agent's reservation utility. On the other hand, if the optimal contract is such that the utility for "bad" outcomes strictly exceeds the lower bound, then the participation constraint will be binding, based on the same reasoning as for the case with unbounded utility of consumption.

Characteristics of Optimal Second-best Compensation Contracts

The optimal contract for inducing an action, including the second-best action, is characterized by (17.6′). The parameters λ, μ_1, ..., μ_{j-1}, μ_{j+1}, ..., μ_N are non-negative Lagrange multipliers, and $\mu_\ell > 0$ only if the agent is indifferent between a_j and a_ℓ at the optimum.

Proposition 17.5 (GH, Prop. 6)

If the second-best action $a^\dagger = a_j$, with $j > 1$, then there is at least one less costly action a_ℓ, $\ell < j$, such that $\mu_\ell > 0$ and $U^a(\mathbf{c}^\dagger,a^\dagger) = U^a(\mathbf{c}^\dagger,a_\ell)$.

The key for this result is that if all the incentive constraints for less costly actions a_ℓ, $\ell < j$, are redundant in the optimal solution, those actions can be omitted from the principal's decision problem, i.e., we can set the action space to be $A' = \{a_j, ..., a_M\}$ without changing the optimal solution. Since the second-best action a_j is now the least costly action for the agent, it is optimally implemented by a fixed wage compensation scheme. However, a fixed wage will induce a_1

(and not a_j) in the original problem and, therefore, some of the incentive constraints for the less costly actions must be binding in the optimal solution.

An important point to recognize from (17.6′) is that the amount paid to the agent for a given outcome does not depend on the value of the outcome to the principal. Instead, the amount paid depends on the inverse of the likelihood of the outcome given the action to be induced relative to other actions between which the agent is indifferent (i.e., actions for which the incentive constraints are binding). That is, while the value of the outcomes to the principal affect which action he chooses to induce, the use of the outcome to induce the desired action reflects its *information content* about the agent's unobservable action, not its value to the principal. However, while the concept of information content is similar to its use in discussing inferences about random variables, it is important to recall that the principal at the compensation date is not trying to draw inferences from the outcomes about what action the agent selected. At that date, he knows precisely (i.e., has a rational conjecture with respect to) the action the agent selected given the contract that was offered to the agent at the contracting date. Instead, the interpretation is that an optimal contract that induces a given action and shares the risk efficiently is related to the information content of the outcomes about the agent's action.

In Volume I, Chapter 2, we identified the relation between the *monotone likelihood ratio property* (MLRP) and first-order stochastic dominance for an arbitrary set of parameters $\omega \in \Omega$. We now introduce MLRP into the agency context.

Definition *Monotone Likelihood Ratio Property*

The probability distribution for x given a satisfies the monotone likelihood ratio property (MLRP) if for any pair of outcomes (x_h, x_i), $h < i$, and any pair of actions (a_ℓ, a_j), $\ell < j$, it holds that

$$\frac{\varphi(x_h|a_\ell)}{\varphi(x_h|a_j)} > \frac{\varphi(x_i|a_\ell)}{\varphi(x_i|a_j)}.$$

That is, lower outcomes are more likely with lower cost actions. Recall that MLRP implies first-order stochastic dominance (FSD), which implies $\mathrm{E}[x|a_\ell] \leq \mathrm{E}[x|a_j]$. Given our previous assumption about the ordering of the action indices, both the expected outcome $\mathrm{E}[x|a_j]$ and the first-best cost $c^*(a_j)$ are increasing in j.

MLRP implies that the likelihood ratio $\varphi(x_i|a_\ell)/\varphi(x_i|a_j)$ is decreasing in i if $\ell < j$, so that $L(x_i|a_\ell, a_j)$ is increasing in i. Hence, in implementing any action a_j it follows immediately from (17.6′) that the agent receives higher compensation for higher outcomes *if* the binding incentive constraints consist only of actions that are less costly than a_j.

Proposition 17.6

Consider the optimal second-best contract $\mathbf{c}_j^\dagger$ for implementing a_j. MLRP implies that $c_{ij}^\dagger$ is nondecreasing in x_i if $\mu_\ell = 0$ for all $\ell > j$.

Is MLRP sufficient for $\mu_\ell = 0$ for all $\ell > j$? NO! GH provide an example satisfying MLRP in which $M = 3, j = 2$ is the second-best optimum, the constraints for both $\ell = 1$ and $\ell = 3$ are binding, and the resulting compensation contract is nonmonotonic and we provide a similar example at the end of this section. This example illustrates that while MLRP implies that a higher outcome is "good news" when comparing a more costly action to a less costly action, it does not imply that the optimal second-best compensation contract pays more for a higher outcome. The reason, of course, is that a compensation scheme that always pays more for higher outcomes may induce the agent to exert more effort than the principal prefers. In so doing, the agent would "earn" an expected utility higher than $\underline{U}$ and not provide sufficient return to the principal to pay for this additional compensation.

The following condition is sufficient to ensure that the optimal second-best compensation contract never pays less for higher outcomes.

Definition *Spanning Condition*

The spanning condition (SC) is satisfied if there exists a pair of probability functions $\boldsymbol{\varphi}^L$ and $\boldsymbol{\varphi}^H$ on X such that

(a) $\varphi^L(x_i), \varphi^H(x_i) \geq 0, \quad \forall\, i = 1, ..., N;$

(b) for each $a_j \in A$ there exists a weight $\zeta(a_j) \in [0,1]$, for each action j, such that

$$\varphi(x_i|a_j) = \zeta(a_j)\varphi^H(x_i) + (1 - \zeta(a_j))\varphi^L(x_i), \quad \forall\, i = 1, ..., N;$$

(c) $\boldsymbol{\varphi}^L$ and $\boldsymbol{\varphi}^H$ satisfy MLRP (i.e., $\varphi^L(x_i)/\varphi^H(x_i)$ is nonincreasing in i).

Observe that if there are only two outcomes (i.e., $N = 2$), then spanning is always satisfied (i.e., we can let $\varphi^L(x_1) = \varphi^H(x_2) = 1$ and $\zeta(a_j) = \varphi(x_2|a_j)$). Furthermore, if $N \geq 2$ and the spanning condition is satisfied, then we can view a_j as determining the probability of obtaining one of two fixed gambles, which makes it effectively equivalent to a two-outcome setting. In fact, many results that are easy to prove in the two-outcome setting can be readily extended to the N (or even infinite) outcome settings with SC. Observe that SC implies the MLRP for the distributions induced by the alternative actions.

Proposition 17.7 (GH, Prop. 7)

If the agent is strictly risk averse, then SC implies that the second-best optimal contract is nondecreasing in i, i.e., $c_{1j}^{\dagger} \leq c_{2j}^{\dagger} \leq ... \leq c_{Nj}^{\dagger}$.

Proof: Let $k_j = k(a_j)$, $\varphi_{ij} = \varphi(x_i|a_j)$, and $\zeta_j = \zeta(a_j)$. The first-order condition (17.6′) for an expected cost minimizing compensation contract can be expressed as

$$w'(u_i) = k_j\lambda + k_j \sum_{\ell \in J} \mu_\ell - \left[\sum_{\ell \in J} \omega_\ell \frac{\varphi_{i\ell}}{\varphi_{ij}}\right]\left[\sum_{h \in J} k_h \mu_h\right],$$

where $$\omega_\ell = \frac{\mu_\ell k_\ell}{\sum_{h \in J} \mu_h k_h},$$

and J is the set of actions for which the incentive constraints are binding. The first two expressions on the right-hand side are constant, while the third varies with x_i.

SC (which includes MLRP) implies that for any set of actions $J \subset \{1,...,M\}$ and any normalized, non-negative weights ω_ℓ, $\Sigma_{\ell \in J}\, \omega_\ell = 1$,

$$\sum_{\ell \in J} \omega_\ell \frac{\varphi_{i\ell}}{\varphi_{ij}} = \frac{\bar{\zeta}\varphi_i^H + (1-\bar{\zeta})\varphi_i^L}{\zeta_j\varphi_i^H + (1-\zeta_j)\varphi_i^L}$$

is *either* nondecreasing ($\bar{\zeta} \geq \zeta_j$) *or* nonincreasing ($\bar{\zeta} \leq \zeta_j$) in x_i, where

$$\bar{\zeta} = \sum_{\ell \in J} \omega_\ell \zeta_\ell.$$

Of course, a cost minimizing compensation contract cannot be nonincreasing (unless it induces the least cost action in which case it is a fixed wage). Hence, it must be nondecreasing. **Q.E.D.**

Proposition 17.5 establishes that the incentive constraint for at least one less costly action is binding. If there are only two alternatives (i.e., $M = 2$) and a_2 is to be implemented, there is one binding incentive constraint. If there are more than two alternatives (i.e., $M > 2$) and $a_j, j \geq 2$, is implemented, then there may be multiple binding constraints. However, there are settings in which only the incentive constraint for a_{j-1} is binding. If that is the case, then MLRP implies that the optimal contract is nondecreasing in i.

Appendix 17A considers a condition known as the *concavity of distribution condition* (CDFC) which, with MLRP, is sufficient for the incentive constraint for a_{j-1} to be the only binding constraint and the compensation contract to be nondecreasing in i. Unfortunately, the examples provided in the literature of distributions that satisfy the CDFC condition seem very contrived. Based on Jewitt (1988), Appendix 17A also considers an alternative set of conditions that are satisfied by most "standard" distributions, but requires the agent's utility of compensation, $u \circ \mathbf{c}(\cdot)$, to be a concave function of x_i for the second-best compensation contract. However, in any case, those conditions are sufficient, and not necessary, conditions for a single, adjacent incentive constraint to be binding.

A Finite Action/Outcome Example

We now illustrate the above analysis using a simple numerical example with three possible outcomes and three possible actions. The three outcomes are good, moderate and bad, represented by $x_g > x_m > x_b$, and the agent's compensation for the corresponding outcomes are c_g, c_m, c_b. The three actions are high, medium, and low effort, represented by a_H, a_M, and a_L, with $v_H > v_M > v_L$ representing the corresponding disutility levels. Panel A of Table 17.1 specifies the outcome probabilities for each action. Consistent with the outcome and action labels, a_H *FS*-dominates a_M, which in turn *FS*-dominates a_L.

We assume the agent has additively separable preferences and we use the following data for our numerical example.

$$u(c) = c^{½}; \; v_H = 55, \; v_M = 40, \; v_L = 0; \; \bar{U} = 200.$$

The magnitudes of the outcomes affect which action the principal chooses to induce, but they are immaterial to the determination of the optimal incentive contract for inducing a given level of effort.

If it is optimal for the principal to induce only a low level of effort, then it is optimal to pay the agent a fixed wage of $u^{-1}(\bar{U} + v_L) = 200^2 = 40{,}000$. If he chooses to induce either high or medium effort, then the principal must impose incentive risk on the agent. The principal's problem for determining the optimal incentive contract for inducing medium effort is

$$\underset{c_b, c_m, c_g}{\text{minimize}} \;\; .20\, c_b + .60\, c_m + .20\, c_g,$$

$$\text{subject to} \;\; .20\, c_b^{½} + .60\, c_m^{½} + .20\, c_g^{½} - 40 \geq 200,$$

$$.20\, c_b^{½} + .60\, c_m^{½} + .20\, c_g^{½} - 40 \geq .54\, c_b^{½} + .40\, c_m^{½} + .06\, c_g^{½},$$

$$.20\, c_b^{½} + .60\, c_m^{½} + .20\, c_g^{½} - 40 \geq .06\, c_b^{½} + .40\, c_m^{½} + .54\, c_g^{½} - 55.$$

This problem can be readily solved using a program like "Solver" in Excel. To do so, we transform the problem by using the utility levels u_b, u_m, u_g as the decision variables so that the objective function is convex and the constraints are linear. In the constraints we also collect terms so that all decision variables are on the left-hand side and all constants are on the right.

$$\begin{aligned} \underset{u_g, u_m, u_b}{\text{minimize}} \quad & .20\, u_b^2 + .60\, u_m^2 + .20\, u_g^2, \\ \text{subject to} \quad & .20\, u_b + .60\, u_m + .20\, u_g \geq 240, \\ & -.34\, u_b + .20\, u_m + .14\, u_g \geq 40, \\ & .14\, u_b + .20\, u_m - .34\, u_g \geq -15. \end{aligned}$$

		x_b	x_m	x_g	
Panel A: Probabilities $\varphi(x_i \mid a_j)$					
	a_H	.06	.40	.54	
	a_M	.20	.60	.20	
	a_L	.54	.40	.06	
Panel B: Optimal compensation $\boldsymbol{c}(x_i)$ to induce a_j					$\bar{\boldsymbol{c}}^{\dagger}(a_j)$
	a_H	23,066	65,025	71,000	65,734
	a_M	21,805	70,225	67,492	59,850
	a_L	40,000	40,000	40,000	40,000
Panel C: Likelihood ratios $L(x_i \mid a_\ell, a_M)$			$\lambda = 480$		μ_ℓ
	a_H	.7	1/3	− 1.7	27.257
	a_L	− 1.7	1/3	.7	122.743
Panel D: Likelihood ratios $L(x_i \mid a_\ell, a_H)$			$\lambda = 510$		μ_ℓ
	a_M	− 7/3	− .5	17/27	0
	a_L	− 8	0	8/9	25.781

Table 17.1: Probabilities, optimal contracts, and likelihoods for finite action/outcome example.

The solution to this problem is presented in Panel B of Table 17.1, along with the optimal contract for inducing the high level of effort. Insight into the shape of the compensation functions for inducing a_M and a_L can be obtained by considering the likelihood ratios reported in Panels C and D in Table 17.1. The optimal contract for inducing a high level of effort is relatively straightforward. The fact the multiplier μ_M equals zero while μ_L is positive tells us that the incentive constraint for moderate effort is not binding. That is, if the incentives are sufficient to deter low effort, then they also deter moderate effort. With only the incentive constraint for a_L binding and $w'(u_i) = 2u_i = 2c_i^{½}$, we have, for example,

$$c_g = [½(\lambda + \mu_L L(x_g|a_L,a_H))]^2 = [½(510 + 8/9\times 25.781)]^2 = 71{,}000.$$

The fact that the likelihood function is increasing with x_i implies that the compensation is increasing in x_i. Again it is important to point out that, given that the principal is risk neutral, the compensation increases with x_i because large outcomes are more likely with high effort than with low effort, not because the amount available is larger.

This latter point is highlighted by the optimal compensation contract for inducing moderate effort. Observe that both incentive constraints are binding, which results in positive values for both μ_L and μ_H. The latter implies that if the principal offers a contract that focuses on inducing the agent to choose a_M instead of a_L, then the contract will induce the agent to work "too hard", i.e., to choose a_H. If the principal does not want the agent to work too hard, then he must, in a sense, penalize the agent for getting a high outcome instead of a moderate outcome. This is illustrated as follows:

$$c_g = [½(\lambda + \mu_L L(x_g|a_L,a_M) + \mu_H L(x_g|a_H,a_M))]^2$$

$$= [½(480 + 122.743\times 0.7 - 27.257\times 1.7)]^2 = 67{,}492.$$

The deviation from the base pay of $(½\times 480)^2 = 57{,}600$ reflects a bonus because this outcome is more likely with a_M than a_L less a penalty since it is less likely with a_M than with a_H (the likelihoods are $+0.7$ and -1.7, respectively).

Observe that the monotone likelihood property is satisfied by the example, but not the spanning condition. Hence, due to the lack of spanning, we can have two binding incentive constraints, and this can lead to a non-monotonic compensation for inducing a_M (which is less than maximum effort). Furthermore, even if there is a single binding incentive constraint, it need not be the adjacent constraint (as in the contract for inducing a_H). To illustrate the result with spanning, see Table 17.2 in which we have changed the probabilities for moderate effort to $\varphi(x_i|a_M) = \varphi(x_i|a_L)\times 1/6 + \varphi(x_i|a_H)\times 5/6$.

Only the likelihood ratio for the adjacent incentive constraint is reported for each induced action, since only that incentive constraint is binding. This, plus the monotone likelihood property, then implies that the compensation is monotonically increasing.

		x_b	x_m	x_g	
Panel A: Probabilities $\varphi(x_i \mid a_j)$					
	a_H	.06	.40	.54	
	a_M	.14	.40	.46	
	a_L	.54	.40	.06	
Panel B: Optimal compensation $\boldsymbol{c}(x_i)$ to induce a_j					$\bar{\boldsymbol{c}}^{\dagger}(a_j)$
	a_H	7,439	65,025	74,939	66,923
	a_M	26,678	57,600	69,344	58,673
	a_L	40,000	40,000	40,000	40,000
Panel C: Likelihood ratios $L(x_i \mid a_\ell, a_M)$			$\lambda = 480$		μ_ℓ
	a_L	$-$ 20/7	0	20/23	53.667
Panel D: Likelihood ratios $L(x_i \mid a_\ell, a_H)$			$\lambda = 510$		μ_ℓ
	a_M	$-$ 4/7	0	4/27	253.125

Table 17.2: Probabilities, optimal contracts, and likelihoods for finite action/outcome example with spanning.

17.3.2 Convex Action Space

The preceding analysis assumed that the set of actions A is finite. In this section we relax that assumption. To keep things simple, we assume that the action a is unidimensional, the agent's utility function is *additively separable*, X is finite, and the principal is risk neutral.

The key change is that the set of actions is now an interval on the real line, i.e.,

$$A = [\underline{a}, \bar{a}] \subseteq \mathbb{R},$$

and $\varphi(x_i|a)$ has constant support and is twice differentiable with respect to $a \in A$, $\forall\, x_i \in X$:

$$\varphi_a(x_i|a) \equiv \partial\varphi(x_i|a)/\partial a \quad \text{and} \quad \varphi_{aa}(x_i|a) \equiv \partial\varphi_a(x_i|a)/\partial a.$$

While multiplicatively separable functions can be readily handled, much of the initial literature focused on additively separable utility functions. Hence, we assume

$$u^a(c,a) = u(c) - v(a), \quad \text{with } u' > 0,\ u'' < 0,\ v' > 0,\ v'' \geq 0,$$

with $C = [\underline{c}, \infty)$ representing the set of feasible consumption levels.[6]

The principal's and agent's expected utilities are now differentiable with respect to a, and we introduce the following notation:

$$U_a^p(\boldsymbol{c},a) = \sum_{i=1}^{N} [x_i - c_i]\, \varphi_a(x_i|a),$$

$$U_a^a(\boldsymbol{c},a) = \sum_{i=1}^{N} u(c_i)\, \varphi_a(x_i|a) - v'(a),$$

$$U_{aa}^a(\boldsymbol{c},a) = \sum_{i=1}^{N} u(c_i)\, \varphi_{aa}(x_i|a) - v''(a).$$

Principal's Decision Problem

In the basic formulation of the principal's problem in the introduction to this chapter, incentive constraint (17.3) is stated in its generic form. Note that in this formulation (17.3) represents an infinite (and even an uncountable) number of constraints. However, if A is convex, then a *necessary condition* for inducing action a is that it be a local optimum for the agent given the contract $\boldsymbol{c}$. Given that $\varphi(x_i|a)$ is twice differentiable, this implies that to satisfy incentive constraint (17.3), $\boldsymbol{c}$ must satisfy the following two conditions:

$$U_a^a(\boldsymbol{c},a) = 0 \text{ and } U_{aa}^a(\boldsymbol{c},a) \leq 0.$$

While these two conditions are necessary, they are not sufficient to ensure that the agent will select a (since there may be other local optima). Of course, if $\boldsymbol{c}$

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[6] The measure used to represent the level of effort is inherently arbitrary. For example, we can always define it to be the level of disutility v, so that the agent's utility function is written as $u^a(c,v) = u(c) - v$ and the probability of outcome x_i is expressed as $\hat{\varphi}(x_i|v)$. The relation of this revised model to the initial representation can be characterized as follows: $\hat{\varphi}(x_i|v) = \varphi(x_i|v^{-1}(v))$ and the set of alternative "actions" is $[\underline{v}, \overline{v}]$ with $\underline{v} = v^{-1}(v(\underline{a}))$ and $\overline{v} = v^{-1}(v(\overline{a}))$. Of course, other representations are possible as well, e.g., in some settings it is useful to assume a is the probability of the good outcome in a binary outcome model or the expected outcome.

is such that $U^a_a(\boldsymbol{c},a) = 0$ and $U^a_{aa}(\boldsymbol{c},a') \leq 0., \forall a' \in A$, then the agent's decision problem is globally concave and he will implement action a.

In the analysis that follows, we adopt the approach that was common in most of the early agency theory literature. In particular, we assume that the single first-order condition for the agent's incentive constraint is a sufficient representation of the infinite number of incentive constraints (17.3), i.e.,

$$U^a_a(\boldsymbol{c},a) = 0. \qquad (17.3c)$$

In that case, the Lagrangian for the principal's decision problem can be restated as

$$\mathfrak{L} = U^p(\boldsymbol{c},a) + \lambda\,[U^a(\boldsymbol{c},a) - \bar{U}] + \mu\, U^a_a(\boldsymbol{c},a) + \sum_{i=1}^{N} \xi_i\,[c_i - \underline{c}], \qquad (17.5'')$$

and the associated first-order conditions are:

$$c_i\!: \; -\varphi(x_i|a) + \lambda\, u'(c_i)\,\varphi(x_i|a) + \mu\, u'(c_i)\,\varphi_a(x_i|a) + \xi_i = 0,$$

$$a\!: \; U^p_a(\boldsymbol{c},a) + \lambda\, U^a_a(\boldsymbol{c},a) + \mu\, U^a_{aa}(\boldsymbol{c},a) = U^p_a(\boldsymbol{c},a) + \mu\, U^a_{aa}(\boldsymbol{c},a) = 0,$$

since $$U^a_a(\boldsymbol{c},a) = 0.$$

Let the "local" likelihood ratio be defined as

$$L(x_i|a) \equiv \frac{\varphi_a(x_i|a)}{\varphi(x_i|a)}.$$

This brings us to a key expression that characterizes the optimal incentive contract. If $a \in (\underline{a},\bar{a})$ and $c_i > \underline{c}$, i.e., we have a strictly interior solution, then $\xi_i = 0$ and the above first-order conditions imply:[7, 8]

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[7] If the principal is risk averse, then $M(c)$ is replaced by $M(x,c) = u^{p\prime}(x - c)/u'(c)$. In this setting we could also consider a lower bound on $x - c$ (reflecting the principal's limited wealth or limited liability considerations).

[8] With multiplicative separability of the agent's utility function the first-order condition for the optimal incentive contract is given by

$$M(c_i) = k(a)\big[\lambda + \mu\,[k'(a)/k(a) + L(x_i|a)]\big].$$

$$M(c_i) = \lambda + \mu L(x_i|a), \tag{17.6''}$$

and
$$\mu = -\frac{U_a^p(\mathbf{c},a)}{U_{aa}^a(\mathbf{c},a)}. \tag{17.7''}$$

Note that (17.6″) is applicable to the contract used to implement any action, whereas condition (17.7″) applies only to the second-best optimal action. Observe that $M(c_i) \geq 0$ for all $c_i \geq \underline{c}$, and

$$\lim_{c_i \to \underline{c}} M(c_i) = 0, \quad \text{if, and only if,} \quad \lim_{c_i \to \underline{c}} u'(c_i) = +\infty.$$

The likelihood ratio $L(x_i|a)$ must be positive for some outcomes and negative for others (since its expected value is equal to zero). If for some x_i, $L(x_i|a)$ is sufficiently negative (and μ is positive) that

$$\lambda + \mu L(x_i|a) < M(\underline{c}),$$

then consumption constraint (17.4) is binding and $c_i = \underline{c}$.

Since the incentive constraint is an equality constraint, it is conceivable that μ could be positive, negative, or zero. Many papers assume that μ is positive, or use indirect arguments to establish that it is positive. Jewitt (1988) provides a "direct" argument for the case in which the principal is risk neutral.

Proposition 17.8 (Jewitt 1988, Lemma 1)
If the principal is risk neutral, then μ satisfying $U_a^a(\mathbf{c},a) = 0$ and (17.6″) is positive.

Proof: Solve (17.6″) for $\varphi_a(x_i|a)$,

$$\varphi_a(x_i|a) = \frac{1}{\mu}\varphi(x_i|a)\big[M(c_i) - \lambda\big]. \tag{17.8}$$

Substituting into $U_a^a(\mathbf{c},a) = 0$ gives

$$\sum_{i=1}^{N} u(c_i)\big[M(c_i) - \lambda\big]\varphi(x_i|a) = \mu v'(a). \tag{17.9}$$

Summing both sides of (17.8) across all $i = 1, ..., N$ and recognizing that $\Sigma_i\, \varphi_a(x_i|a) = 0$, establishes that

$$\mathrm{E}[M(c_i)] = \lambda.$$

Hence, (17.9) can be interpreted as stating that the covariance of $u(c_i)$ and $M(c_i)$ is equal to $\mu v'(a)$.[9] Since $M(c_i)$ and $u(c_i)$ "move" in the same direction, they have nonnegative covariance, and since $v'(a) > 0$ by assumption, it follows that $\mu \geq 0$. We can rule out $\mu = 0$, since, with a risk neutral principal, (17.6″) would imply that c_i is a constant and a constant wage cannot satisfy $U_a^a(c,a) = 0$ if $v'(a) > 0$. **Q.E.D.**

A Hurdle Model Example

We now introduce the basic agency version of what we call the "hurdle" model.[10] It is a simple model with two possible outcomes for the principal and a convex action space for the agent. This model is extended and used several times throughout the book to illustrate some of the reported results.

The agent's action is depicted as jumping over a hurdle of random height h, which is uniformly distributed over the interval [0,1]. The agent's action is $a \in [0,1]$, which represents the height he jumps and is equal to the *ex ante* probability he will clear the hurdle. If he clears the hurdle, there is a high probability (represented by $1 - \varepsilon$) he will generate a good outcome x_g. On the other hand, if he fails to clear the hurdle, there is a high probability he will generate a bad outcome $x_b < x_g$. More specifically, the outcome probabilities given the agent's action a and hurdle height h is

$$\varphi(x_g|a,h) = \begin{cases} 1 - \varepsilon & \text{if } a \geq h, \\ \varepsilon & \text{if } a < h, \end{cases} \qquad \text{where } \varepsilon \in [0,½).$$

Hence, the prior probabilities and their derivatives for the two outcomes given a are

$$\varphi(x_g|a) = a(1 - 2\varepsilon) + \varepsilon, \quad \varphi_a(x_g|a) = (1 - 2\varepsilon),$$

$$\varphi(x_b|a) = a(2\varepsilon - 1) + 1 - \varepsilon, \quad \varphi_a(x_b|a) = -(1 - 2\varepsilon),$$

and the expected outcome given a and its sensitivities to a and ε are

$$\mathrm{E}[x|a] = (x_g - x_b)\varphi(x_g|a) + x_b,$$

$$\mathrm{E}_a[x|a] = (x_g - x_b)(1 - 2\varepsilon) > 0,$$

[9] $\mathrm{Cov}[u,M] = \mathrm{E}[(u - \mathrm{E}[u])(M - \mathrm{E}[M])] = \mathrm{E}[u(M - \lambda)] - \mathrm{E}[u]\times\mathrm{E}[M - \lambda] = \mathrm{E}[u(M - \lambda)]$, since $\mathrm{E}[M - \lambda] = \mu\mathrm{E}[L] = 0$.

[10] The hurdle model was introduced in Volume I, where it was used to illustrate decision making under uncertainty (Chapter 2) and the value of decision-facilitating information (Chapter 3).

$$E_\varepsilon[x|a] = (x_g - x_b)(1 - 2a) \geq 0, \quad \text{for } a \leq ½,$$

$$E_{a\varepsilon}[x|a] = -2(x_g - x_b) < 0.$$

Observe that the expected outcome is a linear increasing function of a with a marginal productivity decreasing in ε. The expected outcome is a linear increasing function of ε with a slope decreasing in a for $a \leq ½$. With $a = ½$, the expected outcome is independent of ε.

The likelihood ratios for the two outcomes are

$$L(x_g|a) = \frac{1 - 2\varepsilon}{a(1-2\varepsilon) + \varepsilon} > 0, \quad L(x_b|a) = \frac{2\varepsilon - 1}{a(2\varepsilon - 1) + 1 - \varepsilon} < 0,$$

since $\varepsilon < ½$, and $a \leq 1$. Note that as ε increases, the likelihood ratio for the good outcome decreases, whereas the likelihood ratio for the bad outcome increases. Hence, both the marginal productivity of the agent's action and the information content of the outcomes about the agent's action decrease in ε.

In the following numerical example we assume the agent has additively separable preferences and use the following data:

$$u(c) = \ln(c); \; v(a) = a/(1 - a); \; \bar{U} = 0; \; x_g = 20; \; x_b = 10.$$

The optimal contracts are shown in Table 17.3 for varying values of ε. The optimal jump size is decreasing in ε as both the marginal productivity of the agent's action and the information content of the outcomes about the agent's action decrease in ε. Although the agent jumps lower for higher values of ε, the expected profit to the principal is higher due to the expected outcome being an increasing function of ε.

	$U^p(c,a)$	c_g	c_b	a
$\varepsilon = 0.00$	10.526	5.775	0.868	0.274
$\varepsilon = 0.05$	10.560	5.606	0.823	0.239
$\varepsilon = 0.10$	10.612	5.405	0.775	0.198
$\varepsilon = 0.15$	10.686	5.168	0.722	0.148
$\varepsilon = 0.20$	10.788	4.887	0.665	0.086
$\varepsilon = 0.25$	10.925	4.551	0.603	0.005

Table 17.3: Optimal contracts for varying values of ε.

In order to focus on the impact of the information content of the outcomes about the agent's action we consider how the expected cost minimizing contract for inducing $a = ½$ varies with ε. Note that with $a = ½$, the expected outcome is not affected by ε. In particular, the principal's expected profit is solely determined in this case by the risk premium paid to the agent for the expected cost minimizing contract that induces him to select $a = ½$.[11] The expected compensation cost can be written as the sum of the risk premium, $\tau(\varepsilon)$, and the agent's certainty equivalent, $CE(a)$, i.e.,

$$\mathrm{E}[c(x)|a] = \tau(\varepsilon) + CE(a),$$

where the certainty equivalent satisfies the agent's participation constraint,

$$u(CE(a)) - v(a) = \bar{U}.$$

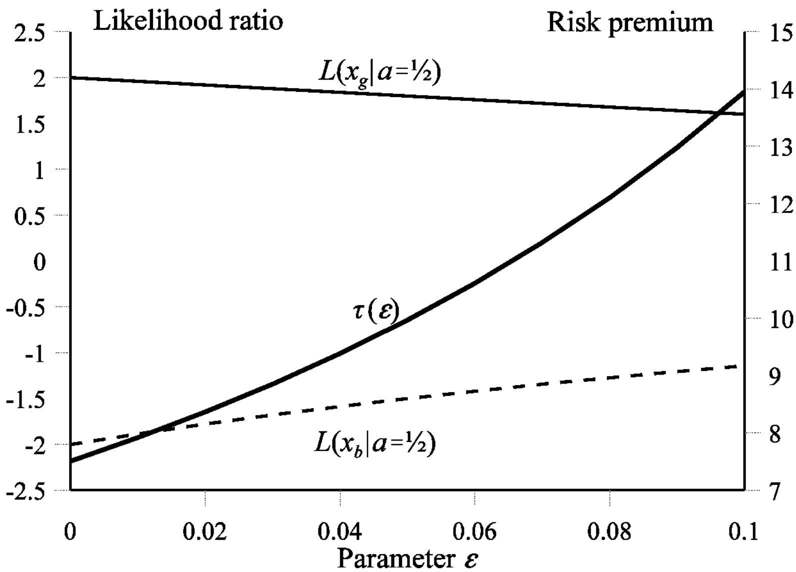

Figure 17.2: Likelihood ratios and risk premium for inducing $a = ½$ for varying values of ε.

The risk premium for inducing $a = ½$ and the likelihood ratios for the two outcomes are shown in Figure 17.2 for varying values of ε. Note that the risk

[11] Recall from Chapter 2, Volume I, that the risk premium is defined as the agent's expected compensation minus the certainty equivalent of his compensation.

premium and the variation in the likelihood ratios are inversely related, i.e., the more variation there is in the likelihood ratios, the lower is the risk premium. Of course, this is due to the fact that it is the variation in the likelihood ratios that determines the information content of the outcomes about the agent's unobserved action. We will return to this issue in Chapter 18.

Sufficient Conditions for Using a First-order Incentive Constraint

The preceding discussion assumed that imposing $U_a^a(\mathbf{c},a) = 0$ as the incentive compatibility constraint is sufficient to result in a contract that will induce the agent to select a. This is not always the case, since local incentive compatibility does not imply global incentive compatibility (see Appendix 17A). However, Jewitt (1988) identifies conditions that are sufficient for that to be the case.

Proposition 17.9 (Jewitt 1988, Theorem 1)

If the principal is risk neutral and $u^a(c,a) = u(c) - v(a)$ with $u' > 0$, $u'' < 0$, $v' > 0$, $v'' \geq 0$, then the first-order approach is valid if the following conditions (a)-(d) hold.

(a) (*i*) $G_i(a) \equiv \sum_{\iota=1}^{i} \Phi(x_\iota|a)$ is nonincreasing and convex in a for each value of $i = 1, ..., N$.

(*ii*) $\mathrm{E}[x|a]$ is nondecreasing and concave in a.

(b) $L(x_i|a)$ is nondecreasing and concave in x_i for each value of a.

(c) The function $u \circ M^{-1}(m)$ is concave.

(d) The optimal incentive contract in the first-order problem is interior, i.e., $c_i > \underline{c}$.

Proof: Let $\mathbf{c}$ solve the associated first-order problem. By Proposition 17.8, $\mu > 0$. (17.6″), conditions (b) and (d) imply that $M(c_i) = \lambda + \mu L(x_i|a)$ is nondecreasing and concave for all i.

Condition (c) implies $u(c)$ is a concave transformation of $M(c_i)$. Hence, the above implies that $u(c_i)$ is nondecreasing and concave in x_i for all i.

The final step is to prove that $U^a(\mathbf{c},a)$ is concave preserving (to ensure global concavity), and Jewitt (1988) claims that condition (a) is necessary and sufficient for

$$\Omega(a) \equiv \sum_{i=1}^{N} \omega(x_i)\, \varphi(x_i|a)$$

to be a nondecreasing concave function of action a for any nondecreasing, concave function $\omega(x_i)$, such as $u(\mathbf{c}(x_i))$. **Q.E.D.**

Condition (a) ensures that an action a second-order stochastically dominates a randomized action strategy with the same expected action. The conditions (b)-(d) ensure that the agent's utility is a concave function of x_i. Convexity of $v(a)$, then implies that the agent prefers not to randomize between actions. Hence, the incentive constraints cannot be binding for several distinctly different actions, since the agent then could select a randomized strategy over these actions and obtain the same expected utility.[12]

Jewitt demonstrates that a sufficient condition for (a) is that the production technology $x = f(a,\theta)$ is a concave function of a for each state of nature θ, which is a very natural assumption in a production context. Jewitt suggests that condition (b), i.e., $L(x_i|a)$ is nondecreasing concave in x_i for each value of a, can be interpreted as the variations in output at higher levels being relatively less useful in providing "information" on the agent's effort than they are at lower levels of output. For many "standard" distributions the likelihood ratio is a linear increasing (and thus concave) function of x_i (see below and Appendix 2B). As demonstrated in Appendix 17C, condition (c) is satisfied for all HARA utility functions with risk cautiousness less than or equal to 2 (which includes the square-root, the negative exponential, and the logarithmic utility functions). Jewitt does not include condition (d) because he does not impose a lower bound on the compensation in the statement of the principal's decision problem.

Exponential Family of Distributions

Jewitt (1988) states that any member of the *exponential family of distributions* satisfies his condition (a) (he actually uses a stronger condition) in an *appropriate parameterization*, provided the expected outcome is concave in a. In particular, any density which can be written in the form[13]

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[12] Note the similarities between these conditions and the sufficient conditions for the local incentive constraint being the only binding incentive constraint with a finite action space.

[13] These densities are an important class since they are those possessing sufficient statistics (see Appendix 18A). Appendix 2B characterizes a number of the classical members of the one-parameter exponential family. Observe that it includes distributions with X finite (binomial), X countably infinite (Poisson), and absolutely continuous distributions over $X = [0, \infty)$ (exponential and gamma) and over $(-\infty, +\infty)$ (Normal).

$$\varphi(x|a) = \theta(x)\beta(a)\exp[\alpha(a)\psi(x)], \tag{17.10}$$

with α and β nondecreasing, satisfies condition (a(i)) of Proposition 17.9.

Observe that for this class of distributions

$$L(x|a) = \alpha'(a)\psi(x) + \frac{\beta'(a)}{\beta(a)}.$$

Hence, satisfaction of condition (b) of Proposition 17.9 requires $\psi(\cdot)$ to be concave.

Corollary (Jewitt 1988, Corollary 1)

Let the outcome density satisfy (17.10) with $\psi(x)$ concave. Then conditions (a) and (b) of Proposition 17.9 are satisfied, provided only that E[$x|a$] is concave in a.

Appendix 17B provides examples that satisfy the above conditions and demonstrates that they satisfy conditions (a) and (b) of Proposition 17.9.

17.3.3 Convex Outcome Space – The Mirrlees Problem

The prior analysis has assumed that X is finite, although our discussion of the exponential family introduced distributions that were absolutely continuous on an interval in the real line. We now focus on absolutely continuous distributions and assume $X = (\underline{x},\overline{x})$, with the possibility that the lower bound can be $-\infty$ and the upper bound $+\infty$.

Much of the prior analysis, where A can be either finite or convex, can be extended to the case in which $X \subseteq \mathbb{R}$ is convex. However, Mirrlees (1975) has identified a potential problem in this case.

We know that if there is moving support, so that $X(a)\backslash X(a^*) \neq \emptyset$, and sufficiently severe penalties can be imposed, then the first-best solution can be obtained by paying a fixed wage for $x \in X(a^*)$ and threatening to impose a severe penalty if an "unacceptable" outcome occurs. The key here is that the penalties need never be imposed, provided the agent takes first-best action a^*.

To ensure that there is an "incentive problem," i.e., the first-best solution cannot be achieved, we usually assume constant support, i.e., $X(a) = X, \forall\, a \in A$. However, under some conditions there may be no solution to the second-best problem. Instead, it may be possible to get "arbitrarily close" to the first-best solution by imposing "severe penalties" on a "small" set of "bad" outcomes.

To provide insight into this issue, consider the following distributions and utility functions:

Distribution:

Exponential: $\varphi(x|a) = \frac{1}{a}\exp\left[-\frac{x}{a}\right]$, $X = [0,\infty)$,

$L(x|a) = \frac{1}{a^2}(x-a)$, $\Rightarrow\ L \in [-\frac{1}{a},\infty)$.

Normal: $\varphi(x|a) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left[-\frac{1}{2\sigma^2}(x-a)^2\right]$, $X = (-\infty,+\infty)$,

$L(x|a) = \frac{1}{\sigma^2}(x-a)$, $\Rightarrow\ L \in (-\infty,+\infty)$.

Utility Function:

Log: $u(c) = \ln(c)$, $C = (0,\infty)$,

$M(c) = c$, $\Rightarrow M \in (0,\infty)$.

Square-root: $u(c) = \sqrt{c}$, $C = [0,\infty)$,

$M(c) = 2\sqrt{c}$, $\Rightarrow M \in [0,\infty)$.

Observe that with the exponential distribution $\lambda + \mu L$ is positive for all $x \in (0,\infty)$ if, and only if, $a > \mu/\lambda$, which would result in an interior solution for $\boldsymbol{c}(x)$ for all x with either the log or the square-root utility functions. Since $u'(c) \to \infty$ as $c \to 0$, it is likely that this condition will be satisfied. It will certainly be satisfied with the log utility function since $u(c) \to -\infty$ as $c \to 0$. With the square-root utility function, we have a corner solution if $a < \mu/\lambda$, i.e.,

$$\boldsymbol{c}(x) = 0 \text{ for } x \in \left(0, \frac{a}{\mu}[\mu - \lambda a]\right).$$

On the other hand, with the normal distribution, $\lambda + \mu L$ is negative for all $x < a - \lambda\sigma^2/\mu$. This can be handled with a square-root utility function by letting let $\boldsymbol{c}(x) = 0$ for those values of x, but that is not possible with the log utility function. Hence, we have a problem with normal distributions and the log utility function, since $\mu > 0$ implies $\lambda + \mu L < 0$ for some values of x and M must be positive. In fact, a solution to the second-best problem does not exist unless we impose a positive lower bound on consumption, i.e., $C = [\underline{c},\infty)$ with $\underline{c} > 0$.

The following theorem (due to Mirrlees, 1975) characterizes the nonexistence of a second-best solution when $L \to -\infty$ as $x \to \underline{x}$ and $u(c) \to -\infty$ as $c \to \underline{c}$.

Proposition 17.10 (Mirrlees 1975, Theorem 1)

Assume MLRP holds with

(a) $\lim_{x \to \underline{x}} L(x|a) = -\infty$,

(b) $u^a(c,a) = u(c) - v(a),\ v'(a) > 0,\ v''(a) \geq 0$,

(c) $\lim_{c \to \underline{c}} u(c) = -\infty,\ \ u'(c) > 0,\ u''(c) < 0$,

(d) $u^p(x - c) = x - c$, i.e., risk neutral principal.

Under these assumptions it is possible to approximate arbitrarily closely, but not attain, the first-best optimum.

Proof: Let (c^*, a^*) denote the first-best contract, where c^* is the first-best fixed wage, and consider an $x^p > \underline{x}$ such that $\Phi_a(x^p|a) < 0$. Given x^p, consider a contract that pays a fixed penalty c^p for outcomes below x^p, and another fixed wage $\bar{c}$ for outcomes above x^p (with $\underline{c} < c^p < c^* < \bar{c}$). The two wages are such that the agent gets the same expected utility as with the first-best contract, and such that the first-best action a^* is incentive compatible. That is, c^p and $\bar{c}$ satisfy the following two conditions:

$$\text{agent expected utility:}\quad u(\bar{c}) - [u(\bar{c}) - u(c^p)]\,\Phi(x^p|a^*) = u(c^*), \tag{17.11}$$

$$\text{agent action choice:}\quad -[u(\bar{c}) - u(c^p)]\,\Phi_a(x^p|a^*) - v'(a^*) = 0, \tag{17.12}$$

i.e.,
$$\bar{c} = u^{-1}\left(u(c^*) - v'(a^*)\frac{\Phi(x^p|a^*)}{\Phi_a(x^p|a^*)}\right) > c^*,$$

and
$$c^p = u^{-1}\left(u(c^*) + v'(a^*)\frac{1 - \Phi(x^p|a^*)}{\Phi_a(x^p|a^*)}\right) < c^*,$$

which is possible since $u(c^p)$ can range between $u(\bar{c})$ and $-\infty$. Observe that (17.11) and Jensen's inequality imply

$$\bar{c} > \mathrm{E}[c|a^*] = \bar{c} - (\bar{c} - c^p)\Phi(x^p|a^*) > c^*.$$

For any large number $K > 0$, we can choose x^p so small that $L(x|a^*) < -K$, $\forall\, x < x^p$. Hence,

$$\frac{1}{K} > -\frac{\Phi(x^p|a^*)}{\Phi_a(x^p|a^*)},$$

$$\Rightarrow u^{-1}\left(u(c^*) + \frac{1}{K}v'(a^*)\right) > \bar{c},$$

$$\Rightarrow \lim_{K\to\infty} \bar{c} = c^* \text{ and } \lim_{K\to\infty} \mathrm{E}[c|a^*] = c^*.$$ **Q.E.D.**

17.3.4 Randomized Contracts

The vast majority of the principal-agent literature ignores the possibility of randomized strategies. However, there are a few papers, most notably Fellingham, Kwon, and Newmann (1984) and Arya, Young, and Fellingham (AYF) (1993), that have considered randomized contracts. In a randomized contract, the principal offers a pair of contracts with a stipulation of a randomization process that will choose between the two contracts after the agent accepts the randomized contract but before he selects his action.[14] Why might randomization be valuable to the principal? The agent's *ex ante* expected utility will equal his reservation utility. However, if the maximum expected utility the principal can achieve with non-random contracts for alternative agent reservation utility levels is convex in the region of the agent's reservation utility,[15] then it will be optimal for the principal to offer a pair of contracts such that the agent's expected utility is greater than his reservation utility if he gets the "good contract" and less than his reservation utility if he gets the "bad contract." This type of situation is depicted in Figure 17.3(a).

AYF focus on additive and multiplicatively separable utility functions in which the utility for compensation is negative exponential, i.e., we have either

(a) Additive separability: $u^a(c,a) = u(c) - v(a)$,

---

[14] This type of randomization is termed *ex-ante* randomization. Gjesdal (1982) considers *ex-post* randomization between incentive contracts after the agent has selected his action. He shows that separability of the utility function is sufficient to ensure that *ex-post* randomization is not optimal.

[15] Stated alternatively, the set of possible principal and agent utilities that can be achieved with alternative non-random contracts is not convex in the region of the agent's reservation utility level.

(b) Multiplicative separability: $u^a(c,a) = u(c)v(a)$,

with $u(c) = -\exp[-c/\rho]$, $v'(c) > 0$, and $v''(c) \geq 0$.

AYF assume that the incentive constraint is characterized by the first-order condition for the agent's choice problem. If $\boldsymbol{c}(x)$ induces the agent to select action a, then

(a) $\mathrm{E}_a[u(\boldsymbol{c}(x))|a] = v'(a)$,

(b) $\mathrm{E}_a[u(\boldsymbol{c}(x))|a]v(a) = \mathrm{E}[u(\boldsymbol{c}(x))|a]v'(a)$.

Now observe that if $\boldsymbol{c}(x)$ is increased by a fixed amount $k > 0$, then $u(\boldsymbol{c}(x) + k) = -\exp[-\boldsymbol{c}(x)/\rho]\exp[-k/\rho]$ with $\exp[-k/\rho] < 1$, from which it follows that

(a) $\exp[-k/\rho]\mathrm{E}_a[u(\boldsymbol{c}(x))|a] < v'(a)$,

(b) $\exp[-k/\rho]\mathrm{E}_a[u(\boldsymbol{c}(x))|a]v(a) = \exp[-k/\rho]\mathrm{E}[u(\boldsymbol{c}(x))|a]v'(a)$.

From (b) we observe that the compensation level has no impact on the action choice when there is multiplicative separability. This implies that the second-best action is independent of the reservation utility level and that $\mathrm{E}[x - \boldsymbol{c}(x,\bar{U})|a(\bar{U})]$ is a decreasing, concave function of the reservation utility. The latter implies that there are no gains to randomization (AYF, Prop. 2).

From (a) we observe that the compensation level affects the action choice when there is additive separability – the larger k the less the effort induced by $\boldsymbol{c}(x)$. This implies that the larger the reservation utility, the more expensive it is to induce a given effort level, and hence the less the effort that will be induced.

AYF (Prop. 1) prove that randomization is beneficial in case (a) if $\varphi_{aa}(x|a) = 0$ (e.g., if $\varphi(x|a) = a\varphi^1(x) + (1-a)\varphi^2(x)$), and

$$[v'(a)]^2 > -v''(a)[\bar{U} + v(a)].$$

The key here is that under the assumed conditions, while $\mathrm{E}[x - \boldsymbol{c}(x,\bar{U})|a(\bar{U})]$ is decreasing in the reservation utility, it is convex around the specified U (resulting in a non-convex set).

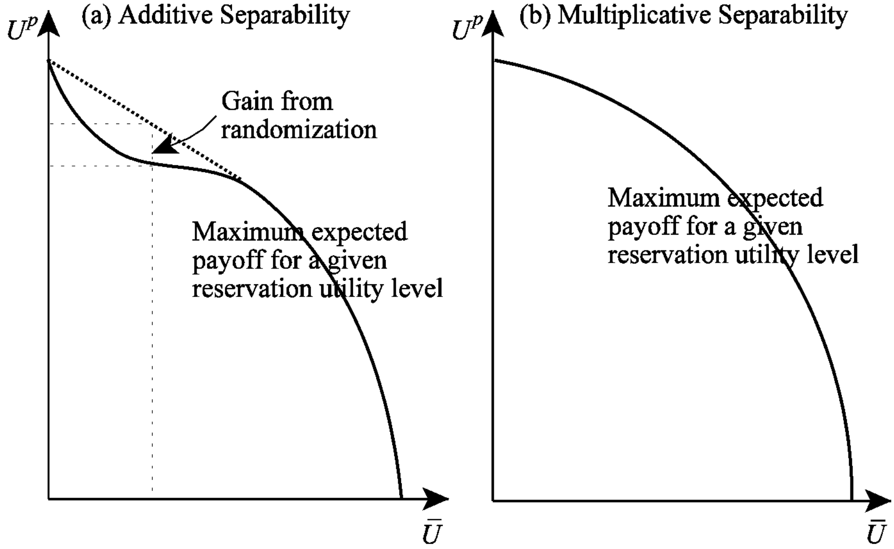

Figure 17.3: Expected utility frontiers with additive and multiplicative separable negative exponential utility.

17.4 AGENT RISK NEUTRALITY AND LIMITED LIABILITY

If the agent is risk neutral, the first-best result can be attained provided all risk can be shifted to the agent (see Proposition 17.3(c)). There are essentially two mechanisms for shifting the risk to the agent. First, the agent can "purchase" the firm, i.e., the agent makes a lump-sum payment to the principal in return for ownership of the outcome x. Of course, this can only be achieved if the agent has sufficient capital to purchase the firm. Second, the agent can "rent" the firm, i.e., the agent agrees to pay the principal a fixed amount after the outcome has been realized. This requires that the agent has other resources that he can use to make up the shortfall between a low value of x and the amount of the rent.

This result was recognized early on, so that virtually all of the initial principal-agent models assumed the agent is risk averse. This has shifted somewhat in recent times. A model is much easier to analyze if the agent is risk neutral. Hence, a researcher will typically make that assumption as long as there is something else in the model assumptions that precludes implementation of the first-best result. There are two such factors: the agent does not have sufficient resources to implement the first-best result or he has private information at the time of contracting (a setting we will consider in Chapter 23).

Innes (1990) provides an analysis in which the agent is assumed to be risk neutral and does not have sufficient resources to implement the first-best result. He makes the following assumptions:

– the agent is an entrepreneur who owns a production technology, but has no investment capital;

– implementation of the production technology requires the agent's effort a and investment of q units of capital;

– the principal (investors) will provide the amount q if offered a contract in which the expected payment equals q (given the assumption of investor risk neutrality and a zero interest rate);

– limited liability precludes contracts in which the principal makes payments to the agent at the end of the period;[16]

– $\varphi(x|a)$ satisfies MLRP, $A = [0,\bar{a}]$, and $X = [0,\infty)$;

– $u^a(c,a) = c - v(a)$, with $v'(a) > 0$, $v''(a) > 0$, i.e., the agent is risk neutral and effort averse.[17]

Unlike the previously discussed models, the agent owns the production technology and has the bargaining power. He offers a contract to the principal (investors) that provides an expected return on the capital invested that is equivalent to the return that could be obtained in the market. Let $\boldsymbol{\pi}(x)$ represent the amount paid to the principal. Hence, the agent's consumption is $\boldsymbol{c}(x) = x - \boldsymbol{\pi}(x)$. The agent's decision problem is

$$\underset{\boldsymbol{\pi},\, a \in A}{\text{maximize}} \quad \mathrm{E}[x - \boldsymbol{\pi}(x)|a] - v(a),$$

$$\text{subject to} \quad \mathrm{E}[\boldsymbol{\pi}(x)|a] \geq q,$$

$$0 \leq \boldsymbol{\pi}(x) \leq x, \quad \forall\, x \in X,$$

$$\mathrm{E}[x - \boldsymbol{\pi}(x)|a] - v(a) \geq \mathrm{E}[x - \boldsymbol{\pi}(x)|a'] - v(a'), \quad \forall\, a' \in A,$$

---

[16] Both debt and equity financing generally have limited liability in the sense that the holders of these claims cannot be required to pay for the firm's liabilities.

[17] Innes allows for a slightly more general form of utility function, $u^a(c,a) = k(a)c - v(a)$.

where the last constraint is the incentive compatibility constraint that ensures that the agent is at least weakly motivated to provide the effort a given the payoff function offered to investors.

Monotonic Contracts

Innes introduces the following *monotonicity constraint*:

$$\boldsymbol{\pi}(x+\varepsilon) \geq \boldsymbol{\pi}(x), \quad \forall\ (x,\varepsilon) \in \mathbb{R}_+^2.$$

He argues that this constraint can be viewed as the result of the principal's and agent's ability to "sabotage" non-monotonic contracts. For example, after observing a perfect signal about the firm's profits, investors may be in a position to reduce the firm's actual profits, or the agent may supplement the profits (by borrowing on a personal account).

In a *debt contract*, $\boldsymbol{\pi}(x) = \min\{x,D\}$ where D is the designated nominal amount to be paid to the principal in return for q. That is, if the outcome x is insufficient to meet the obligation to pay D, then the outcome x is paid to the principal.

Consider a monotonic contract $\boldsymbol{\pi}(x)$ that induces action a, i.e., $\mathrm{E}_a[x\,|\,a] = \mathrm{E}_a[\boldsymbol{\pi}(x)\,|\,a] + v'(a)$, and identify the debt contract that provides the principal with the same expected return, i.e.,

$$\int_0^D x\,\varphi(x|a)\,dx + D\,\Phi(D|a) = \mathrm{E}[\boldsymbol{\pi}(x)\,|\,a].$$

Innes' Lemma 1 proves that

$$\int_0^D x\,\varphi_a(x|a)\,dx + D\,\Phi_a(x|a) < \mathrm{E}_a[\boldsymbol{\pi}(x)\,|\,a],$$

which implies $a(D) > a$ (Innes' Lemma 2), i.e., the debt contract will induce a higher action than any arbitrary monotonic contract. As depicted in Figure 17.4, the key here is that moving to a debt contract reduces the amount the agent receives for low values of x and increases what he receives for high values, i.e., the agent has a call option on x with strike price D. This gives him stronger incentives to achieve the high outcomes.

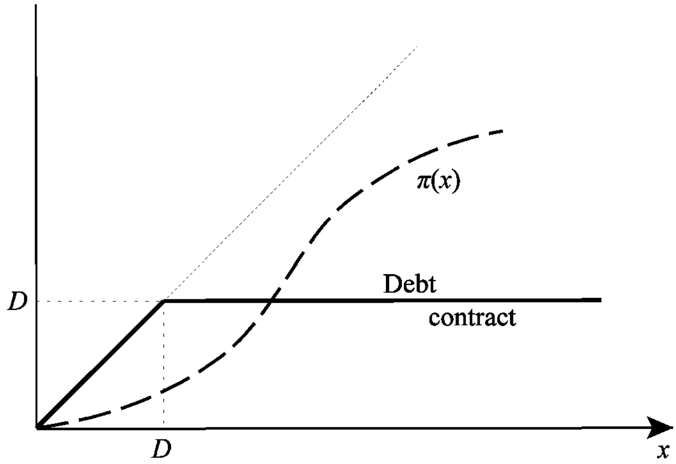

Figure 17.4: Debt contract and general monotonic contract.

Let $a(D)$ represent the action induced by debt contract D. Innes' Corollary 2 demonstrates that $a(D)$ is a continuous function and he notes that increasing the action from a to the induced action $a(D)$ will make both the principal and the agent better off. Hence, it immediately follows that the optimal monotonic contract is a debt contract.

Proposition 17.11 (Innes 1990, Prop. 1)
A solution to the agent's problem (with a monotonicity constraint) exists and has the following properties:

(a) $\pi(x) = \min\{x, D\}, \quad \forall\, x \in X,$

(b) $\mathrm{E}[\pi(x) \mid a] = q,$

(c) $a < a^* \equiv$ first-best effort choice.

That is, while a debt contract is the best monotonic contract, the optimal debt contract will induce less than the first-best level of effort.

Non-monotonic Contracts
If the monotonicity constraint is dropped, then there is a greater range of feasible contracts. Innes identifies two possibilities here. First, it may be possible to obtain the first-best result. Second, if the first-best result cannot be attained,

then it is optimal to use what Innes calls a *live-or-die contract*. In that contract, there is an outcome cut-off $x^\dagger$ such that $x < x^\dagger$ goes to the principal and $x > x^\dagger$ goes to the agent, i.e.,

$$\pi(x) = \begin{cases} x & \forall\ x \le x^\dagger, \\ 0 & \forall\ x > x^\dagger. \end{cases}$$

The argument for the optimality of the live-or-die contract is similar to the argument for a debt contract when there is a monotonicity constraint.

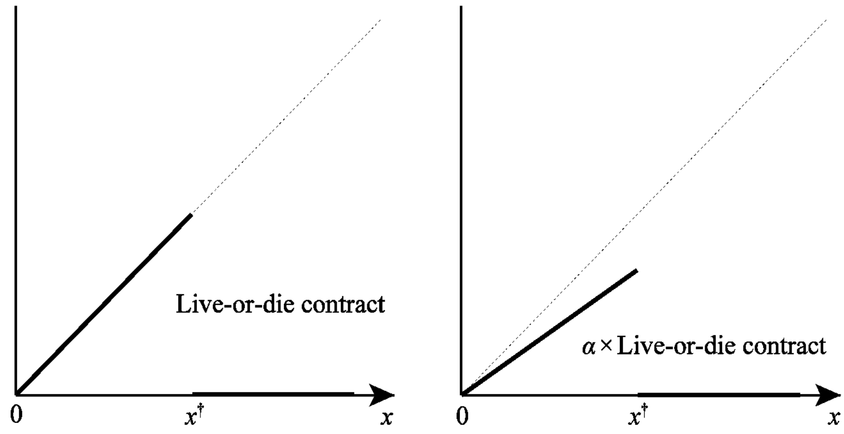

Figure 17.5: Live-or-die contracts.

A live-or-die contract is used as a starting point for assessing whether the first-best result can be achieved. Observe that the first-best action a^* is such that $E_a[x|a^*] = v'(a^*)$. The first step is to identify an outcome x^* such that

$$\int_0^{x^*} x\, d\Phi_a(x|a^*) = 0,$$

i.e., a^* maximizes the expected payment to the principal in a live-or-die contract in which x^* is the cutoff. If x^* is also such that

$$\int_0^{x^*} x\, d\Phi(x|a^*) \ge q,$$

the first-best can be achieved (see Innes' Propositions 3 and 4) with the following contract:

$$\boldsymbol{\pi}^*(x) = \alpha^* \begin{cases} x & \forall\ x \le x^*, \\ 0 & \forall\ x > x^*, \end{cases}$$

where
$$\alpha^* = q \div \left[\int_0^{x^*} x\, d\Phi(x|a^*)\, dx \right].$$

The key here is that

$$\mathrm{E}_a[x|a^*] = \mathrm{E}_a[\pi^*(x)|a^*] + v'(a^*) = v'(a^*),$$

because π^* has been constructed so that $\mathrm{E}_a[\pi^*(x)|a^*] = 0$ and $\mathrm{E}[\pi^*(x)|a^*] = q$.

Now consider the second case, which occurs if x^* is such that

$$\int_0^{x^*} x\, d\Phi(x|a^*) < q.$$

The first-best result cannot be achieved here and the optimal second-best contract is a live-or-die contract in which the induced effort is less than a^* (see Innes' Proposition 2) and the principal receives an expected return of q. The argument for the optimality of the live-or-die contract is the same as the argument for the optimality of a debt contract when there is a monotonicity constraint. The live-or-die contract provides the maximum incentive for the agent to expend effort.

17.5 CONCLUDING REMARKS

This chapter lays a foundation for the analysis in subsequent chapters. The agency model in this chapter is simple in that there is a single agent, who performs a single task for a single period, and the outcome from his effort is the only contractible information. For most of the analysis, the principal is risk neutral and the agent is risk and effort averse. The outcome from the agent's action belongs to the principal, and he must compensate the agent for his reservation wage, the cost of his effort, and a risk premium associated with the incentive risk used to motivate the desired intensity of effort.

The following can be viewed as the key results of the chapter. First, Proposition 17.1 characterizes a first-best contract and Proposition 17.3 identifies conditions under which it can be achieved. Second, Sections 17.3.1 and 17.3.2 identify several characteristics of second-best contracts, including identification of conditions under which the agent's compensation is an increasing function of our likelihood measure, which is an increasing function of the firm's outcome (see Propositions 17.6, 17.7, 17.8, and 17.9).

APPENDIX 17A: CONTRACT MONOTONICITY AND LOCAL INCENTIVE CONSTRAINTS

Several agency papers introduce a *concavity of distribution function condition* (CDFC) to facilitate the analysis by providing a set of conditions that are sufficient for the optimal incentive contract to be characterized by an increasing compensation scheme and a single binding incentive constraint which only considers the next most costly action relative to the second-best action. Unfortunately, this condition is not intuitively appealing because it is not satisfied by "standard" distributions. Based on Jewitt (1988), we provide a less restrictive condition on the distribution function which is satisfied by most standard distributions. For example, it is satisfied if the production function exhibits decreasing marginal productivity of effort for each state of nature (see Jewitt, 1988). On the other hand, since the Jewitt condition is based on second-order stochastic dominance (as opposed to first-order stochastic dominance for CDFC) additional conditions are needed on the utility function and the likelihood ratio to ensure that the agent's equilibrium utility of compensation is a concave function of the outcome.

Monotonicity and Local Incentive Constraints with a Finite Action Set

The following analysis examines a setting in which the set of actions is finite and the agent's preferences are additively separable. We let $\varphi_{ij} = \varphi(x_i|a_j)$ and $v_j = v(a_j)$ with $v_1 < v_2 < ... < v_M$, i.e., the actions are strictly increasing in their "cost" to the agent (and, hence, $c_j^* < c_\ell^*$ for $j < \ell$).

The following sufficient conditions for the optimal incentive contract to be nondecreasing are also sufficient conditions for a single binding incentive constraint which only considers the next most costly action relative to the second-best action. Therefore, we consider a relaxed version of the principal's decision problem for inducing an action a_j in which the incentive constraints (17.3f′) are substituted with a single "local" incentive constraint that only considers the next most costly action a_{j-1}, i.e., $\hat{A} = \{a_{j-1}, a_j\}$ and

$$U^a(\mathbf{u},a_j) \geq U^a(\mathbf{u},a_{j-1}). \tag{17.3f″}$$

The optimal incentive contract in the relaxed program is denoted $\hat{\mathbf{c}}_j$.

In the relaxed version of the principal's decision problem (and in any setting in which only the adjacent incentive constraint is binding), the optimal contract is characterized by the likelihood ratio $L(x_i|a_{j-1},a_j)$. The distribution is defined to satisfy the "local" MLRP, if $L(x_i|a_{j-1},a_j)$ is nondecreasing in i.

Definition *Concavity of Distribution Function Condition*

If the agent's preferences are additively separable, the *concavity of distribution function condition* (CDFC) is satisfied if for any $h, j, \ell \in \{1, ..., M\}$ there exists a $\xi \in [0,1]$ such that[18]

$$v_j = \xi v_h + (1-\xi) v_\ell \quad \Rightarrow \quad \Phi_{ij} \leq \xi \Phi_{ih} + (1-\xi)\, \Phi_{i\ell}, \quad \forall\, i = 1, ..., N,$$

where
$$\Phi_{ij} = \sum_{\imath=1}^{i} \varphi_{\imath j}\,.$$

That is, CDFC is satisfied if the "utility" cost of a_j is expressed as a weighted average of the utility cost of a_h and a_ℓ, and a_j FS-dominates a gamble between a_h and a_ℓ, with probabilities equal to the utility cost weights. Hence, if the compensation function is nondecreasing, the agent (weakly) prefers a_j to a randomized strategy over a_h and a_ℓ with an equal expected utility cost.

The following proposition identifies sufficient conditions for the second-best incentive contract for inducing a_j to be such that it is nondecreasing in the outcome, and such that only the incentive constraint for a_{j-1} is binding.

Proposition 17A.1 (GH, Prop. 8)

If the agent is strictly risk averse with additively separable preferences, then "local" MLRP and CDFC imply that the optimal second-best contract $\mathbf{c}_j^\dagger$ for inducing a_j satisfies

$$M(c_{ij}^\dagger) = \lambda + \mu\, L(x_i|a_{j-1},a_j), \quad i = 1, ..., N,$$

with λ, $\mu = \mu_{j-1} > 0$, and is nondecreasing.

Proof: The first-order condition in (a) characterizes the second-best contract for inducing a_j with $\hat{A}$ (by Proposition 17.6), and $U^a(\hat{\mathbf{c}}_j,a_{j-1}) = U^a(\hat{\mathbf{c}}_j,a_j) = \underline{U}$ by Proposition 17.4. Now show that there does not exist an lower cost action a_ℓ, $\ell < j-1$, such that $U^a(\hat{\mathbf{c}}_j,a_\ell) > U^a(\hat{\mathbf{c}}_j,a_j)$. Assume the contrary.

Let ξ be such that $v_{j-1} = \xi v_\ell + (1-\xi) v_j$. Local MLRP and CDFC imply that

[18] Note that the condition is more appropriately termed a "convexity of distribution function condition" since the distribution function at any given outcome is a convex function of the "utility" cost of effort.

$$U^a(\hat{\mathbf{c}}_j, a_{j-1}) = \sum_{i=1}^{N} u_i \varphi_{i,j-1} - v_{j-1} \geq \sum_{i=1}^{N} u_i \big(\xi \varphi_{i\ell} + (1-\xi)\varphi_{ij}\big) - v_{j-1}$$
$$= \xi\, U^a(\hat{\mathbf{c}}_j, a_\ell) + (1-\xi)\, U^a(\hat{\mathbf{c}}_j, a_j)$$
$$= \xi\, U^a(\hat{\mathbf{c}}_j, a_\ell) + (1-\xi)\, U^a(\hat{\mathbf{c}}_j, a_{j-1}),$$

which violates the assumed condition.

Now show that there does not exist a more costly action a_h, i.e., $c_h^* > c_j^*$, that would be preferred to a_j for the chosen contract, i.e., $U^a(\hat{\mathbf{c}}_j, a_h) > U^a(\hat{\mathbf{c}}_j, a_j)$. Assume the contrary.

Proposition 17.4 implies that $U^a(\hat{\mathbf{c}}_j, a_{j-1}) = U^a(\hat{\mathbf{c}}_j, a_j) = \bar{U}$. Let ξ be such that $v_j = \xi v_{j-1} + (1-\xi) v_h$. Local MLRP and CDFC imply that

$$U^a(\hat{\mathbf{c}}_j, a_j) = \sum_{i=1}^{N} u_i \varphi_{ij} - v_j \geq \sum_{i=1}^{N} u_i \big(\xi \varphi_{i,j-1} + (1-\xi)\varphi_{ih}\big) - v_j$$
$$= \xi\, U^a(\hat{\mathbf{c}}_j, a_{j-1}) + (1-\xi)\, U^a(\hat{\mathbf{c}}_j, a_h)$$
$$= \xi\, U^a(\hat{\mathbf{c}}_j, a_j) + (1-\xi)\, U^a(\hat{\mathbf{c}}_j, a_h),$$

which violates the assumed condition.

Hence, $\hat{\mathbf{c}}_j$ implements a_j not only with the action set $\hat{A}$, but also with the full action set A, and is no more costly than the optimal contract for inducing a_j with the full action set A, so $\hat{\mathbf{c}}_j = \mathbf{c}_j^\dagger$.

Monotonicity of $\mathbf{c}_j^\dagger$ follows immediately from the first-order condition in (a) and local MLRP with $\mu > 0$. **Q.E.D.**

As noted, the CDFC is not satisfied by "standard" distributions, and may therefore be difficult to justify. However, note that the proof of the proposition is based on an FSD argument. Since the agent is risk averse, it is natural to consider weaker conditions on the distribution function combined with an SSD argument. However, even though the agent is risk averse, the second-best compensation scheme may be sufficiently "convex" so that the composite function $u \circ c(x)$ may not be a concave function of x. Hence, in addition to a condition on the distribution function, we also need conditions to ensure that $u \circ c(x)$ is concave in order to use a second-order stochastic dominance argument.

Let

$$G_{ij} \equiv \sum_{\iota=1}^{i} \Phi(x_\iota | a_j), \quad i = 1, ..., N; j = 1, ..., M,$$

denote the accumulated distribution function.

Definition *Convexity of Accumulated Distribution Function Condition*
If the agent's preferences are additively separable, the *convexity of accumulated distribution function condition* (CADFC) is satisfied if for any $h, j, \ell \in \{1, ..., M\}$ there exists a $\xi \in [0,1]$ such that

$$v_j = \xi v_h + (1 - \xi) v_\ell \quad \Rightarrow \quad G_{ij} \leq \xi G_{ih} + (1 - \xi) G_{i\ell}, \quad \forall\, i = 1, ..., N.$$

That is, CADFC is satisfied if the "utility" cost of a_j is expressed as a weighted average of the utility cost of a_h and a_ℓ, and a_j SS-dominates a gamble between a_h and a_ℓ, with probabilities equal to the utility cost weights. Hence, if the agent's utility of compensation $u \circ \mathbf{c}(x)$ is nondecreasing and concave, the agent (weakly) prefers a_j to a randomized strategy over a_h and a_ℓ with equal expected utility cost.

Note that CDFC implies CADFC, whereas the converse does not necessarily hold, i.e., CADFC is a weaker condition on the distribution than CDFC.

Definition *Concavity of Utility of Compensation Condition*
If the agent's preferences are additively separable, the *concavity of utility of compensation condition* (CUCC) is satisfied if $u \circ \hat{\mathbf{c}}_j(x)$ is a concave function of x.

Note that the condition is on the optimal incentive contract in the relaxed program and, thus, satisfaction of the condition depends on endogenously determined Lagrange multipliers. However, the following lemma provides sufficient conditions for CUCC in terms of (almost) exogenous characteristics of the problem.

Lemma 17A.1
If the agent's preferences is additively separable, CUCC is satisfied if the following conditions hold:

(a) The local likelihood ratio $L(x_i | a_{j-1}, a_j)$ is increasing and concave in x_i.

(b) The function $u \circ M^{-1}(m)$ is concave.

(c) The optimal incentive contract in the relaxed program is interior, i.e., $\hat{c}_{ij} > \underline{c}$.

Proof: The optimal incentive contract in the relaxed program is characterized by

$$M(\hat{c}_{ij}) = \lambda + \mu L(x_i | a_{j-1}, a_j),$$

with $\mu > 0$ (by Proposition 17.6) whenever $\hat{c}_{ij} > \underline{c}$. The agent's utility of compensation can therefore be written as

$$u \circ \hat{\mathbf{c}}_j(x) = u \circ M^{-1}(\lambda + \mu L(x | a_{j-1}, a_j)),$$

which is concave in x by (a) and (b). (c) ensures that there is no "flat" part of the utility of compensation for low outcomes (in which case it would be impossible to satisfy CUCC). **Q.E.D.**

Condition (a) is satisfied by most standard distributions (see Appendix 2B), while condition (b) is satisfied for all HARA utility functions with risk cautiousness less than or equal to 2 (see Appendix 17C – this includes the square-root, the negative exponential, and the logarithmic utility functions). If the risk cautiousness is above 2, CUCC may still be satisfied if the local likelihood ratio is sufficiently concave and μ is sufficiently high. Whether condition (c) is satisfied depends on the optimal solution, but in most analyses one wants to make sure that it is satisfied.

Proposition 17A.2

If the agent is strictly risk averse with additively separable preferences, then "local" MLRP, CADFC, and CUCC imply that the optimal second-best contract $\mathbf{c}_j^\dagger$ for inducing a_j satisfies

$$M(c_{ij}^\dagger) = \lambda + \mu L(x_i | a_{j-1}, a_j), \quad i = 1, ..., N,$$

with λ, $\mu = \mu_{j-1} > 0$, and is nondecreasing.

Proof: The proof is the same as for Proposition 17A.1 except that the SSD argument is used instead of an FSD argument. Local MLRP, i.e., $L(x_i | a_{j-1}, a_j)$ is nondecreasing in i, and CUCC imply that u_i is nondecreasing and concave in i for $\hat{\mathbf{c}}_j$. Hence, CADFC implies for any $\ell < j < h$ and $\xi \in [0,1]$ that

$$\sum_{i=1}^{N} u_i \varphi_{ij} \geq \sum_{i=1}^{N} u_i \big(\xi \varphi_{i\ell} + (1-\xi)\varphi_{ih}\big).$$

The proof then proceeds as in the proof of Proposition 17A.1. **Q.E.D.**

Observe that in the convex action space case, Jewitt assumes $\mathrm{E}[x | a]$ is nondecreasing and concave in a (see Proposition 17.9). This along with SSD ensures that $U^a(c,a)$ is globally concave, which is sufficient for the first derivative to identify the action a that is induced. Proposition 17A.2 is based on conditions that ensure that a local optimum is a global optimum without ensuring global concavity.

Monotonicity and Local Incentive Constraints with a Convex Action Set
The following analysis now considers settings in which the set of actions is a convex interval on the real line. The conditions for contract monotonicity are basically the same as in the case with a finite action space, i.e., the local incentive constraint is a sufficient representation of the full set of incentive constraints.

Recall from Volume I, Chapter 2, that MLRP implies that $L_x(x_i|a) \equiv \partial L(x_i|a)/\partial x_i \geq 0$, with strict inequality for some x_i, and $\Phi_{ia}(a) \leq 0$ (i.e., FSD), where

$$\Phi_i(a) = \sum_{\iota=1}^{i} \varphi_\iota(a) \quad \text{and} \quad \varphi_\iota(a) = \varphi(x_\iota|a).$$

Lemma 17A.2
Assume MLRP and $\mu \geq 0$, then c_i is a nondecreasing function of x_i.

Proof: If $\mu = 0$, then we have Pareto efficiency, which implies c_i is nondecreasing in x_i. If $\mu > 0$, then $\lambda + \mu L(x_i|a)$ is increasing in x_i, which in turn implies that c_i is nondecreasing in x_i, since $M(c_i)$ can only be increased for larger x_i if c_i is increased. **Q.E.D.**

Lemma 17A.3
Assume MLRP and $a \in (\underline{a}, \bar{a})$, then $\mu \neq 0$.

Proof: Assume $\mu = 0$. This implies $x_i - c_i$ is strictly increasing in x_i. The latter implies that $U_a^p(\mathbf{c}, a) > 0$ for all $a \in A$, due to FSD implied by MLRP. However, for (17.7″) to hold with $\mu = 0$, we require $U_a^p(\mathbf{c}, a) = 0$. A contradiction. **Q.E.D.**

Definition
If the agent's preferences are additively separable with $v'(a) > 0$ and $v''(a) \geq 0$, the probability function $\varphi_i(a)$ satisfies the *concave distribution function condition* (CDFC) if

$$\Phi_{iaa}(a) \geq 0, \quad \forall\, x_i \in X,\, a \in A.$$

While MLRP requires the distribution function $\Phi_i(a)$ to decrease as a increases, CDFC requires it to decrease at a decreasing rate. The MLRP condition is satisfied by most "standard" distributions if we view disutility as an increasing function of the mean of those distributions. However, those "standard" distributions do not satisfy CDFC. Rogerson (1985) provides a "contrived" distribution that does satisfy these two conditions:

$$\Phi_i(a) = \left(\frac{x_i}{x_N}\right)^{a-\underline{a}}.$$

In using MLRP and CDFC to characterize the second-best optimal contract, Rogerson introduces three specifications of the principal's decision problem:

(*i*) *Unrelaxed:*	max (17.1)	subject to (17.2), (17.3: $a \in \text{argmax}\ U^a(\mathbf{c}, a')$), and (17.4);
(*ii*) *Relaxed:*	max (17.1)	subject to (17.2), (17.3c: $U_a^a(\mathbf{c}, a) = 0$), and (17.4);
(*iii*) *Double Relaxed:*	max (17.1)	subject to (17.2), (17.3r: $U_a^a(\mathbf{c}, a) \geq 0$), and (17.4).

Proposition 17A.3 (Rogerson 1985, Prop. 1)

If a solution to (*iii*) exists and a solution to (*i*) exists with $a^\dagger > \underline{a}$, then MLRP and CDFC imply that if $(\mathbf{c}^\dagger, a^\dagger)$ is a solution to (*iii*), then

(a) it is also a solution to (*i*), with $c_i^\dagger$ nondecreasing in x_i,

(b) if $a^\dagger < \bar{a}$, it is also a solution to (*ii*), and the principal would prefer the agent to provide more "effort," i.e., $U_a^P(\mathbf{c}^\dagger, a^\dagger) \geq 0$.

Recall that in the finite action case we used MLRP and CDFC (from GH) to establish that only one incentive constraint is binding – the constraint for the action that is the next most costly action to the agent. Thus, it is not surprising that these conditions are also sufficient to permit us to replace the set of incentive constraints in (*i*) with the "local" first-order condition in (*ii*).

The alternative set of conditions based on Jewitt (1988) is considered in the text.

APPENDIX 17B: EXAMPLES THAT SATISFY JEWITT'S CONDITIONS FOR THE SUFFICIENCY OF A FIRST-ORDER INCENTIVE CONSTRAINT

Jewitt (1988) provides the following examples which satisfy his sufficient conditions (see Proposition 17.9) for the use of a first-order incentive constraint. In the first example, the set of possible outcomes is binary, whereas in the second that set is a convex set of the real line. In both examples, effort is represented as the expected outcome from the agent's actions, i.e., $E[x|a]$. This representation is always possible, and it ensures that Jewitt's condition (a(*ii*)) is satisfied. Of course, he also requires that this definition of a results in $v(a)$ such that $v'(a) > 0$ and $v''(a) > 0$, which is a restrictive assumption.

A Binary Outcome Example

In the binary outcome example, $X = \{x_1, x_2\}$ and $a \in A = [x_1, x_2]$, with

$$\varphi(x_1|a) = \frac{x_2 - a}{x_2 - x_1} \quad \text{and} \quad \varphi(x_2|a) = 1 - \varphi(x_1|a) = \frac{a - x_1}{x_2 - x_1}.$$

This formulation can be used for any two-outcome example in which $\varphi(x_1|a)$ is a decreasing function. Interestingly, as the following demonstrates, this representation satisfies Jewitt's conditions (a(*i*)) and (b).

(a(*i*)) $G_1(a) = \Phi(x_1|a) = \varphi(x_1|a)$

$$\Downarrow$$

$$G_{1a}(a) = -\frac{1}{x_2 - x_1} < 0 \quad \Rightarrow \quad G_{1aa}(a) = 0,$$

$$G_2(a) = \Phi(x_1|a) + \Phi(x_2|a) = G_1(a) + 1,$$

which has the same properties as $G_1(a)$.

(b)

$$\varphi_a(x_1|a) = -\frac{1}{x_2 - x_1} \quad \Rightarrow \quad L(x_1|a) = -\frac{1}{x_2 - a} < 0,$$

$$\varphi_a(x_2|a) = \frac{1}{x_2 - x_1} \quad \Rightarrow \quad L(x_2|a) = \frac{1}{a - x_1} > 0.$$

Therefore, $L(x_1|a) \le 0 \le L(x_2|a)$, $\forall\, a \in A$, and the concavity condition is automatically satisfied because there are only two possible values of x_i.

In addition to satisfying Jewitt's condition (a), this example satisfies MLRP (since $L(x_1|a) < L(x_2|a)$) and CDFC (since $\Phi_{1aa}(a) = 0$).

An Exponential Distribution Example
In this example, $x \in [0,\infty)$ and $\varphi(x|a)$ has an exponential distribution (which belongs to the exponential family with $\theta(x) = 1$, $\beta(a) = \alpha(a) = 1/a$, and $\psi(x) = x$):

$$\varphi(x|a) = \frac{1}{a}\exp\left[-\frac{x}{a}\right].$$

In this case,

$$L(x|a) = \frac{1}{a^2}(x - a) \qquad L_x(x|a) = \frac{1}{a^2} > 0 \qquad L_{xx}(x|a) = 0,$$

which satisfies MLRP and Jewitt's condition (b). Furthermore,

$$\Phi(x|a) = \int_0^x \frac{1}{a}\exp\left[-\frac{\tau}{a}\right]d\tau = -\exp\left[-\frac{\tau}{a}\right]\Bigg|_0^x = 1 - \exp\left[-\frac{x}{a}\right],$$

$$\Phi_a(x|a) = -\frac{x}{a^2}\exp\left[-\frac{x}{a}\right] < 0,$$

$$\Phi_{aa}(x|a) = \frac{x}{a^4}(2a - x)\exp\left[-\frac{x}{a}\right] \begin{cases} < 0 & \text{if } x \in (2a,\infty), \\ \ge 0 & \text{if } x \in (0,2a]. \end{cases}$$

Hence, the distribution satisfies FSD, but does *not* satisfy CDFC for $x > 2a$. To test Jewitt's condition (a(i)) we compute

$$G(x|a) \equiv \int_0^x \Phi(y|a)\,dy = x + a\exp\left[-\frac{x}{a}\right] - a,$$

$$G_a(x|a) = \left(1 + \frac{x}{a}\right)\exp\left[-\frac{x}{a}\right] - 1 < 0, \qquad \forall\, x \in (0,\infty),$$

$$G_{aa}(x|a) = \frac{x^2}{a^3}\exp\left[-\frac{x}{a}\right] > 0, \quad \forall\ x \in (0,\infty).$$

Hence, $G(x|a)$ is decreasing and convex.

APPENDIX 17C: CHARACTERISTICS OF OPTIMAL INCENTIVE CONTRACTS FOR HARA UTILITY FUNCTIONS

HARA utility functions were introduced in Volume I, Chapter 2. If the agent's utility for consumption is HARA, then

$$u(c) \sim \begin{cases} -\beta\, e^{-c/\beta} & \text{if } \alpha = 0,\ \beta > 0, \\ \ln(c+\beta) & \text{if } \alpha = 1,\ c+\beta > 0, \\ \dfrac{1}{\alpha - 1}[\alpha c + \beta]^{1-1/\alpha} & \text{if } \alpha \neq 0,1,\ \alpha c + \beta \geq 0, \end{cases}$$

where α is the agent's risk cautiousness. The analysis in this chapter establishes that, if the principal is risk neutral, optimal contracts take the general form:

$$c(m(x)) = \begin{cases} M^{-1}(m(x)) & \text{if } m(x) > M(\underline{c}), \\ \underline{c} & \text{otherwise}, \end{cases}$$

where $m(x)$ is a linear function of the likelihood ratios for x given the induced action a relative to the alternative actions for which the incentive constraints are binding (see, for example, (17.6), (17.6′), and (17.6″)).

Observe that with HARA utility functions:

$$M(c) = \frac{1}{u'(c)} = \begin{cases} e^{c/\beta} & \text{if } \alpha = 0,\ \beta > 0,\ c \geq \underline{c}, \\ c + \beta & \text{if } \alpha = 1,\ c \geq \underline{c} > -\beta, \\ [\alpha c + \beta]^{1/\alpha} & \text{if } \alpha \neq 0, 1,\ c \geq \underline{c} \geq -\beta/\alpha. \end{cases}$$

Hence, for $m = m(x) \geq M(\underline{c})$,

$$M^{-1}(m) = \begin{cases} \beta \ln m & \text{if } \alpha = 0,\ \beta > 0,\ m \geq e^{\underline{c}/\beta} > 0, \\ m - \beta & \text{if } \alpha = 1,\ m \geq \underline{c} + \beta > 0, \\ \alpha^{-1}(m^{\alpha} - \beta) & \text{if } \alpha \neq 0{,}1,\ m \geq [\alpha\underline{c} + \beta]^{1/\alpha} \geq 0. \end{cases}$$

Furthermore, the relation between the agent's utility for consumption and the likelihood measure m is

$$u(M^{-1}(m)) = \begin{cases} -\beta m^{-1} & \text{if } \alpha = 0,\ \beta > 0,\ m \geq e^{\underline{c}/\beta}, \\ \ln m & \text{if } \alpha = 1,\ m \geq \underline{c} + \beta > 0, \\ \dfrac{1}{\alpha - 1} m^{\alpha - 1} & \text{if } \alpha \neq 0{,}1,\ m \geq [\alpha\underline{c} + \beta]^{1/\alpha} \geq 0. \end{cases}$$

From the above we can readily characterize how the agent's compensation and utility vary with the likelihood measure m for $m \geq M(\underline{c})$. Of course, for $m \leq M(\underline{c})$, the compensation is equal to $\underline{c}$.

Proposition 17C.1

If the agent has separable utility with HARA utility $u(c)$ for consumption, then for $m \geq M(\underline{c})$:

(a) the agent's compensation is a strictly concave (convex) function of the likelihood measure m if the agent's risk cautiousness α is less (more) than 1, and is linear if $\alpha = 1$;

(b) the agent's utility is a strictly concave (convex) function of the likelihood measure m if the agent's risk cautiousness α is less (more) than 2, and is linear if $\alpha = 2$.

Proof: In the proof we assume that the set of possible values of m is a convex set on the real line, so that $c(m)$ and $u \circ M^{-1}(m)$ are continuously differentiable functions. The results also hold if the set of possible values of m is finite.

(a): Recall that $c(m) = M^{-1}(m)$.

$$c'(m) = \begin{cases} \beta m^{-1} & \text{if } \alpha = 0,\ \beta > 0,\ m \ge e^{\underline{c}/\beta}, \\ 1 & \text{if } \alpha = 1,\ m \ge \underline{c} + \beta > 0, \\ m^{\alpha-1} & \text{if } \alpha \ne 0,1,\ m \ge [\alpha\underline{c} + \beta]^{1/\alpha} \ge 0. \end{cases}$$

$$c''(m) = \begin{cases} -\beta m^{-2} & \text{if } \alpha = 0,\ \beta > 0,\ m \ge e^{\underline{c}/\beta}, \\ 0 & \text{if } \alpha = 1,\ m \ge \underline{c} + \beta > 0, \\ (\alpha - 1) m^{\alpha-2} & \text{if } \alpha \ne 0,1,\ m \ge [\alpha\underline{c} + \beta]^{1/\alpha} \ge 0. \end{cases}$$

(b): $$\frac{du(M^{-1}(m))}{dm} = \begin{cases} \beta m^{-2} & \text{if } \alpha = 0,\ \beta > 0,\ m \ge e^{\underline{c}/\beta}, \\ m^{-1} & \text{if } \alpha = 1,\ m \ge \underline{c} + \beta > 0, \\ m^{\alpha-2} & \text{if } \alpha \ne 0,1,\ m \ge [\alpha\underline{c} + \beta]^{1/\alpha} \ge 0. \end{cases}$$

$$\frac{d^2u(M^{-1}(m))}{dm^2} = \begin{cases} -2\beta m^{-3} & \text{if } \alpha = 0,\ \beta > 0,\ m \ge e^{\underline{c}/\beta}, \\ -m^{-2} & \text{if } \alpha = 1,\ m \ge \underline{c} + \beta > 0, \\ (\alpha - 2) m^{\alpha-3} & \text{if } \alpha \ne 0,1,\ m \ge [\alpha\underline{c} + \beta]^{1/\alpha} \ge 0. \end{cases}$$

Q.E.D.

Observe that if there exist likelihood measures $m < M(\underline{c})$, then the compensation and utility levels are flat, with $c = \underline{c}$ and $u(c) = u(\underline{c})$ for those values of m. This does not disturb the convexity of either $c(m)$ or $u \circ M^{-1}(m)$. However, the linear cases become piecewise linear, and the concave functions are not concave over the entire range.

Most analytical research is based on a general concave utility function or assumes the utility function is either exponential or square-root. The exponential utility function has $\alpha = 0$, which implies that the optimal compensation and utility functions are strictly concave functions of the likelihood measure for $m \ge M(\underline{c})$. The square-root utility function, on the other hand, has $\alpha = 2$ (since $1 - ½ = ½$), which implies the optimal compensation is a strictly convex function of the likelihood measure, while the utility function is linear (or piecewise linear if there exists $m < M(\underline{c})$).

In the first-stage of the GH approach we minimize the expected compensation cost to induce a given action. This is equivalent to minimizing the risk premium paid to the agent, since the risk premium is given by

$$\pi(\boldsymbol{c},a) = \mathrm{E}[c\,|\,a] - CE(\boldsymbol{c},a),$$

where the certainty equivalent is given by the participation constraint as the first-best cost of implementing a (provided the participation constraint is binding), i.e.,

$$CE(\boldsymbol{c},a) = w\left(\frac{\bar{U} + v(a)}{k(a)}\right),$$

where $w(\cdot) = u^{-1}(\cdot)$ denotes the inverse of the agent's utility for consumption.

In subsequent analyses, *with additive separable utility functions* of the HARA class, we use properties of the change in risk premium that occurs when the level of utility is increased by the same amount for all outcomes. That is, for a given compensation contract $\boldsymbol{c}$ that implements a we consider another compensation contract $\boldsymbol{c}^\lambda$ defined by

$$u(\boldsymbol{c}^\lambda(x)) = u(\boldsymbol{c}(x)) + \lambda, \quad \forall\, x \in X.$$

Clearly, if $\boldsymbol{c}$ implements a, $\boldsymbol{c}^\lambda$ also implements a since[19]

$$\underset{\hat{a}\in A}{\operatorname{argmax}} \int_X u(\boldsymbol{c}(x))\, d\Phi(x\,|\,a) - v(\hat{a}) = \underset{\hat{a}\in A}{\operatorname{argmax}} \int_X [u(\boldsymbol{c}(x)) + \lambda]\, d\Phi(x\,|\,a) - v(\hat{a}).$$

The risk premium paid to the agent for contract $\boldsymbol{c}^\lambda$ is given by

$$\pi(\boldsymbol{c}^\lambda,a) = \int_X w(u(\boldsymbol{c}(x)) + \lambda)\, d\Phi(x\,|\,a) - w\left(\int_X [u(\boldsymbol{c}(x)) + \lambda]\, d\Phi(x\,|\,a)\right).$$

Increasing the level of utility, increases the variance of the compensation and, therefore, one might think that the risk premium paid to the agent also increases. However, due to wealth effects on the agent's risk aversion, the relation between the utility level and the risk premium is more complicated than suggested by this intuition. The following proposition demonstrates that the risk premium increases with λ if the agent's utility is a concave function of the likelihood measure (or, equivalently, the derivative of the inverse utility function, i.e.,

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[19] In this analysis we do not consider the impact on the participation constraint. In subsequent applications we consider cases in which the level of utility is increased for outcomes that are affected by the agent's action and decreased correspondingly for outcomes that are not affected by the agent's action. The variation is such that it leaves both incentives and the agent's expected utility unchanged.

$w'(\cdot)$,[20] is convex). On the other hand, if the agent's utility is a convex function of the likelihood measure, the risk premium decreases as the utility increases.

Proposition 17C.2

If the agent has an additively separable utility function, the risk premium $\pi(c^\lambda, a)$ is increasing (decreasing) in λ if the agent's utility is a concave (convex) function of the likelihood measure m.

Proof: From the definition of the risk premium and Jensen's inequality we get that

$$\frac{\partial \pi(c^\lambda, a)}{\partial \lambda} = \int_X w'(u(c(x)) + \lambda)\, d\Phi(x|a) - w'\Big(\int_X [u(c(x)) + \lambda]\, d\Phi(x|a)\Big) >(<)\ 0,$$

if, and only if, $w'(\cdot)$ is convex (concave). Now recall that $w'(u(c(m))) = M(c(m)) = m$. Hence, $w'(\cdot)$ is the inverse function of $u \circ c(\cdot)$ so that $w'(\cdot)$ is convex (concave) if, and only if, $u \circ c(\cdot)$ is concave (convex). **Q.E.D.**

Of course, if the agent's utility for consumption is HARA we can use Proposition 17C.1 to obtain the result that the risk premium is increasing (decreasing) in λ if the risk cautiousness is less (more) than 2, and independent of λ if $\alpha = 2$.

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[20] Note that the derivative of the inverse utility function is equal to the marginal cost of providing utility to the agent.

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CHAPTER 18

EX POST PERFORMANCE MEASURES

In Chapter 17 we assume that the action a and the event θ are not observable, but there is a verified report of the final outcome x. Hence, incentive contracts can be based on the reported outcome. In this chapter we allow for the possibility that the outcome may not be contractible. If it is not, then inducing more than the agent's least cost action will require the use of incentives based on some alternative performance measure that is contractible and is influenced by the agent's action. Furthermore, it is potentially valuable to use more than one performance measure. This chapter explores the relation of the characteristics of alternative performance measures to the principal's expected payoff, and the form of the optimal incentive contract.

We continue to focus on a single-task setting, so that the key benefit from a superior set of performance measures takes the form of a reduction in the risk premium the agent must be paid for taking a given level of induced effort. Of course, a reduction in the risk premium may lead the principal to offer a contract that induces more effort.

Since the outcome x is not necessarily contractible information, it is important to designate whether the principal or the agent is the residual "owner" of that payoff. That "ownership" may derive from legal or physical considerations. For example, the principal may own the production technology and will ultimately receive the final payoff, even though that payoff may not be realized until sometime subsequent to the termination of the compensation contract with the agent. On the other hand, the agent may physically control the payoff such that he can consume the difference between the outcome received and the amount he is contracted to pay to the principal.

We first (Section 18.1) consider the setting in which a risk neutral principal "owns" the outcome. In that setting all risk is ideally borne by the principal and a performance measure is beneficial if it permits the principal to impose less risk on the agent while still inducing a given action (or permits inducement of a more preferred action). In Section 18.2, a risk averse agent "owns" the outcome. In this setting a performance measure has two potential roles: as a mechanism to facilitate the sharing of the agent's outcome risk, and as a mechanism to provide incentives for the agent's action. Section 18.3 considers the setting in which a risk averse principal "owns" the outcome. This provides results similar to those in Section 18.2. However, in that section we focus on a setting in which there are both economy-wide and firm-specific risks and the principal is a partnership of well-diversified shareholders. While well-diversified shareholders are risk neutral with respect to firm-specific risk, they are risk averse with respect to the economy-wide risk. We show how their risk preferences with respect to economy-wide risk can be represented in a simple way by using risk-adjusted probabilities for the economy-wide events, and illustrate how this translates into an optimal compensation scheme. Section 18.4 considers optimal costly acquisition of a secondary performance measure conditional on a primary performance measure. Section 18.5 concludes the chapter with some brief remarks.

18.1 RISK NEUTRAL PRINCIPAL "OWNS" THE OUTCOME

The simplest case to consider is one in which the principal is risk neutral and he ultimately receives the output x, so that there are no risk sharing concerns – only incentive issues. In Section 18.3 we consider the setting in which the principal is risk averse.

Basic Elements of the Model

The agent again chooses an action $a \in A$, which generates an outcome $x \in X$. The contractible information is denoted $y \in Y$, which is the outcome of an *information or performance measurement system* η. It can be multi-dimensional and may include x, but we allow for the possibility that x may not be part of y. We assume X and Y are finite sets to avoid potential technical problems associated with convex sets. However, given suitable regularity, the analysis can be extended to settings in which X and Y are convex sets – and much of the literature assumes that to be the case.

The joint probability function over $X \times Y$ given action a and performance measurement system η is denoted $\varphi(x,y|a,\eta)$, and the marginal probability functions are $\varphi(x|a)$ and $\varphi(y|a,\eta)$. We assume the cost of the information system is separable, so that η does not affect the gross payoff x.

The principal is assumed to be risk neutral, while the agent is risk and effort averse with an additively separable utility function:[1]

$$u^p(\pi) = \pi = x - c, \quad u^a(c,a) = u(c) - v(a), \;\; u' > 0, u'' < 0, v' > 0, v'' \geq 0.$$

Principal's Decision Problem

For the main part of the analysis we focus on the first stage of the Grossman and Hart (1983) (GH) approach in which we identify the least expected cost contract

[1] The analysis can be readily extended to consider a multiplicatively separable utility function.

for inducing an action a that is not the agent's least costly action, i.e., there is at least one other action $\underline{a} \in A$ such that $v(\underline{a}) < v(a)$.

The principal's decision problem is essentially the same as in Chapter 17, except that in this setting the agent's compensation contract is defined over the anticipated contractible information y, i.e., $\mathbf{c}$: $Y \rightarrow C \equiv [\underline{c}, \infty)$, and the probability function over the contractible signals depends on the performance measurement system that is used.

$$\bar{\mathbf{c}}^{\dagger}(a,\eta) = \underset{c(y)}{\text{minimize}} \sum_{y \in Y} c(y)\, \varphi(y|a,\eta), \tag{18.1}$$

subject to

$$U^a(\mathbf{c},a,\eta) \equiv \sum_{y \in Y} u(c(y))\, \varphi(y|a,\eta) - v(a) \geq \bar{U}, \tag{18.2}$$

$$a \in \underset{a' \in A}{\text{argmax}}\; U^a(\mathbf{c},a',\eta), \tag{18.3}$$

$$c(y) \geq \underline{c}, \quad \forall\, y \in Y. \tag{18.4}$$

We assume that A is an interval on the real line, i.e., $A = [\underline{a}, \bar{a}]$, and that (18.3) can be represented by the first-order condition for the agent's decision problem, i.e.,[2]

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[2] Jewitt (1988) identifies conditions under which the first-order incentive constraint is appropriate in a setting in which $y = (y_1, y_2)$ and $\varphi(y|a) = \varphi(y_1|a)\varphi(y_2|a)$, i.e., the two signals are independent (see his Theorems 2 and 3). Theorem 3 invokes conditions (b) and (c) from his Theorem 1 (see Proposition 17.9) and requires that $\Phi(y_i|a)$ be quasi-convex in (y_i, a), $i = 1, 2$. Quasi-convexity implies that if $\Phi(y_{i1}|a^1) \leq \Phi(y_{i2}|a^2)$, then $\Phi(\lambda y_{i1} + (1-\lambda)y_{i2}|\lambda a^1 + (1-\lambda)a^2) \leq \Phi(y_{i2}|a^2)$ for all y_{i1}, $y_{i2} \in Y_i$, $a^1, a^2 \in A$, and $\lambda \in [0,1]$.

Sinclair-Desgagne (1994) identifies conditions under which the first-order incentive constraint is appropriate in a setting in which $y = (y_1, ..., y_m)$, x is a function of y, and Y_i is finite and ordered. Sinclair-Desgagne demonstrates that the use of the first-order incentive constraint results in identification of the second-best contract and action if the following conditions on $\varphi(y|a)$ hold:

(a) MLRP: $\varphi(y|a^1)/\varphi(y|a^2)$ is nondecreasing in y whenever $a^1 > a^2$.

(b) SDC: For at least one dimension h (with $y = (y_h, y_{-h})$)

$$Q(y_h, y_{-h}|a) = \sum_{t \subset Y_h,\, t \geq y_h} \varphi(t, y_{-h}|a)$$

is nondecreasing in a, i.e., $Q_a \geq 0$ at every a, y_h, and y_{-h}, where Q is the *upper cumulative probability* of signal y_h at y_{-h}.

(continued...)

$$U_a^a(\mathbf{c},a,\eta) = 0. \tag{18.3c}$$

The first-order condition characterizing the optimal incentive contract (for $\mathbf{c}(y) > \underline{c}$) is[3]

$$M(\mathbf{c}(y)) = \lambda + \mu\, L(y|a,\eta), \tag{18.5}$$

where $\quad M(c) = \dfrac{1}{u'(c)}, \quad$ and $\quad L(y|a,\eta) = \dfrac{\varphi_a(y|a,\eta)}{\varphi(y|a,\eta)}.$

18.1.1 *A*-informativeness

Observe that in this setting the principal is not concerned about risk sharing (since he is risk neutral). His only concern is to minimize the expected cost of motivating the agent to accept the contract and select action a. Therefore, he wants to select a performance measurement system η that facilitates this objective. This means he is concerned about the relation between the agent's set of alternative actions and the set of possible performance measures.

Definition *A-informativeness*

Performance measurement system η^2 is *at least as A*-informative as performance measurement system η^1 if there exists a Markov matrix **B** (or Markov kernel) such that

$$\boldsymbol{\eta}^1 = \boldsymbol{\eta}^2\mathbf{B} \quad \left(\text{or } \varphi(y^1|a,\eta^1) = \int_{Y^2} b(y^1|y^2)\, d\Phi(y^2|a,\eta^2)\right),$$

where $\quad \boldsymbol{\eta} \equiv [\,\varphi(y|a,\eta)\,]_{|A|\times|Y|}$ and $\mathbf{B} \equiv [\,b(y^1|y^2)\,]_{|Y^2|\times|Y^1|}$.

Note that the likelihood functions used in the above definition describe the relation between the performance measures y and the action a, whereas *the relation of y to x is immaterial.* The usefulness of this definition is demon-

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2 (...continued)

(c) CDFC: For at least one dimension h, $Q(y_h,y_{-h}|a)$ is concave in a, i.e., $Q_{aa} \le 0$ at all a, y_h, and y_{-h}.

3 If A is finite, then (18.3) is expressed as $U^a(\mathbf{c},a,\eta) \ge U^a(\mathbf{c},a',\eta)$, $\forall\, a' \in A$, $a' \ne a$, and the first-order condition for $\mathbf{c}(y)$ is expressed as

$$M(\mathbf{c}(y)) = \lambda + \sum_{\substack{a' \in A \\ a' \ne a}} \mu(a)\, L(y|a',a,\eta), \quad \text{where } L(y|a',a,\eta) \equiv 1 - \frac{\varphi(y|a',\eta)}{\varphi(y|a,\eta)}.$$

strated by the following basic result from GH, Gjesdal (1982) (Gj), and Holmström (1982) (H82).[4]

Proposition 18.1 (GH, Prop. 13; Gj, Corr. 1, and H82 p. 334)
If the principal is risk neutral and η^2 is at least as A-informative as η^1, then η^2 is *at least as preferred* as η^1 for implementing any $a \in A$, i.e.,

$$\bar{\boldsymbol{c}}^{\dagger}(a,\eta^1) \geq \bar{\boldsymbol{c}}^{\dagger}(a,\eta^2), \quad \forall\, a \in A.$$

Furthermore, if $u'' < 0$ and $\mathbf{B} \gg 0$ (i.e., all elements are positive) and a is not the agent's least cost action, then η^2 is *strictly preferred* to η^1 for implementing a.

Proof: Let $\boldsymbol{c}^1$ be the cost minimizing incentive contract for implementing a with η^1. An "equivalent" contract can be constructed for η^2 if we introduce a randomized compensation plan where $\boldsymbol{c}^1(y^1)$ is paid with probability $b(y^1|y^2)$ if η^2 reports y^2. This randomized contract has the same incentive properties as $\boldsymbol{c}^1$ and has the same expected cost.

Strict preference given $\mathbf{B} \gg 0$ follows from defining a contract $\boldsymbol{c}^2$ that for each y^2 pays the certainty equivalent of the above randomized contract given y^2, i.e.,

$$c^2(y^2) = u^{-1}\left(\sum_{y^1 \in Y^1} u(\boldsymbol{c}^1(y^1))\, b(y^1|y^2) \right).$$

The contract $\boldsymbol{c}^2$ induces the same agent preferences over actions and the same expected utility to the agent. Since the induced action is not the agent's least cost action, $\boldsymbol{c}^1$ is not a constant wage. Hence, $\mathbf{B} \gg 0$ and Jensen's inequality imply that

$$\boldsymbol{c}^2(y^2) < \sum_{y^1 \in Y^1} \boldsymbol{c}^1(y^1)\, b(y^1|y^2),$$

for each y^2, so that the expected compensation cost for $\boldsymbol{c}^2$ is strictly lower than for $\boldsymbol{c}^1$. **Q.E.D.**

The proposition implies that the relation between y and x or θ is irrelevant – only the relation between the performance measure y and a matters. Further-

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[4] The *maximum value of information* in this context is $(\mathrm{E}[x|a^*] - \boldsymbol{c}^*(a)) - (\mathrm{E}[x|a^o] - \boldsymbol{c}^*(a^o))$, where a^o minimizes $\boldsymbol{c}^*(a)$ on A. That is, the best the principal can do is the first-best and he need do no worse than pay a fixed amount which induces a^o.

more, if the signals from η^1 are a "strictly garbled" version of the signals from η^2, then the latter is strictly preferred. Note that this is a stronger result than in Chapter 3, where we considered (Blackwell) Θ-informativeness with respect to *ex ante* information used in single person decision making. The key here is that, when the agent is strictly risk averse and implementation of a requires a risky incentive contract, strictly better information always permits *strict improvement* by "fine-tuning" the contract so that its "riskiness" and, thus, the risk premium, is reduced.

Now consider replacing an information system with one that reports "less" information, but with no loss in value. The following sufficient statistic result is the same as the factorization theorem in Chapter 3 (Proposition 3.2).

Lemma 18.1

Let η^ψ represent performance measurement system $\boldsymbol{\psi}$: $Y \to \Psi$. Performance measure $\boldsymbol{\psi}(y)$ is a *sufficient statistic for y with respect to a* if, and only if, there exist functions $\varphi(\psi|a,\eta^\psi)$ and $g(y)$ such that

$$\varphi(y|a,\eta) = g(y)\, \varphi(\boldsymbol{\psi}(y)|a,\eta^\psi), \quad \forall\, y \in Y, a \in A. \tag{18.6}$$

Note that if $\boldsymbol{\psi}(y)$ is a sufficient statistic for y with respect to a, then η^ψ is at least as A-informative as η with the Markov matrix defined by

$$b(y|\psi) = \begin{cases} g(y) & \text{if } \boldsymbol{\psi}(y) = \psi, \\ 0 & \text{otherwise.} \end{cases}$$

Of course, contracting on ψ cannot be more valuable than contracting on y, since the sufficient statistic is a function of y and the principal can commit to ignore the additional information in y. Hence, we get the following result.

Proposition 18.2 (H82, Theorem 5)

If $\boldsymbol{\psi}(y)$ is a sufficient statistic for y with respect to a, then

$$\bar{\boldsymbol{c}}^\dagger(a,\eta) = \bar{\boldsymbol{c}}^\dagger(a,\eta^\psi).$$

Hence, there is no loss of value of substituting performance measure y with a performance measure ψ that provides the same information about the agent's unobserved action. The additional information in y (about θ or x) compared to ψ cannot be used to reduce the cost of implementing a.

In particular, note that if $\boldsymbol{\psi}(y)$ is a sufficient statistic for a, the likelihood ratios are the same whether the performance measures ψ or y are used, i.e., from (18.6) it follows that

$$L(y|a,\eta) = \frac{g(y)\,\varphi_a(\psi(y)|a,\eta^\psi)}{g(y)\,\varphi(\psi(y)|a,\eta^\psi)} = \frac{\varphi_a(\psi(y)|a,\eta^\psi)}{\varphi(\psi(y)|a,\eta^\psi)}.$$

Therefore, given the characterization of the optimal incentive contract in (18.5), it is not surprising that performance measure ψ is as good as y.

On the other hand, if $\psi(y)$ is not a sufficient statistic for y with respect to a, does that imply that it is less costly to implement a with η than with η^ψ, i.e., $\bar{c}^\dagger(a,\eta) < \bar{c}^\dagger(a,\eta^\psi)$? Not necessarily. The problem is that (18.6) is too strong a condition in the sense that it must *hold for all a*. For example, if $\psi(y)$ equals the likelihood ratio for the action being implemented, (18.6) may not hold for all a, and yet there is no scope for a strict reduction in the cost of implementing the particular action. Therefore, we introduce a concept of a statistic sufficient for implementing a particular action a. That concept focuses directly on the likelihood ratio.[5]

Definition *Sufficient Implementation Statistic*

Performance measure $\psi(y)$ is a *sufficient implementation statistic* for action a if for all $\psi \in \Psi$ it holds that

$$L(y'|a,\eta) = L(y''|a,\eta), \quad \forall\, y', y'' \in Y(\psi) \equiv \{\, y \in Y \,|\, \psi(y) = \psi \,\}. \qquad (18.7)$$

Note that if (18.7) holds for all $a \in A$, then (18.7) implies (18.6).[6] The definition is satisfied if all y with the same statistic have the same likelihood ratio for the particular action.

Proposition 18.3 (H82, Theorem 6)

Assume ψ: $Y \to \Psi$ is *not* a sufficient implementation statistic for a, and let $\mathbf{c}_a^\psi$: $\Psi \to C$ be a nonconstant interior compensation contract that implements a. Then there exists another compensation contract $\mathbf{c}_a^y$: $Y \to C$ that strictly reduces the expected cost of implementing a.

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[5] Note that for any statistic, (18.6) can always be satisfied for a particular a. However, the likelihood ratio is the key characteristic of an optimal incentive contract. Therefore, the definition of a sufficient implementation statistic for a particular a has to focus on the likelihood ratio instead of a factorization of the probability function.

[6] To see this, observe that if ψ reveals the likelihood ratio for all a, then, since $d[\ln\varphi(\psi|a,\eta)]/da = L(\psi|a,\eta)$, it follows that

$$\varphi(\psi|a,\eta^\psi) = \exp\left[\int_{\underline{a}}^{a} L(\psi|\alpha,\eta^\psi)\, d\alpha\right].$$

Proof: In this proof we construct a variation in the contract $\boldsymbol{c}_a^\psi$ based on y such that the expected cost and the agent's incentives are unchanged, but the agent is better off due to reduced risk. Since the contract is interior, the expected cost can then be reduced by subtracting a constant "utility" amount from the agent's compensation for each signal y.

Since $\boldsymbol{\psi}$ is *not* a sufficient implementation statistic for a, there exists a statistic ψ_1 and disjoint sets of positive measure $Y_{11}, Y_{12} \subset Y_1 \equiv \{ y \mid \boldsymbol{\psi}(y) = \psi_1 \}$ such that

$$L(Y_{11}|a,\eta) \neq L(Y_{12}|a,\eta). \tag{18.8}$$

Since $\boldsymbol{c}_a^\psi$ is not constant (in which case it would implement the least cost action), there exists another statistic $\psi_2 \neq \psi_1$ such that $\boldsymbol{c}_a^\psi(\psi_1) > \boldsymbol{c}_a^\psi(\psi_2)$ and $Y_2 = \{ y \mid \boldsymbol{\psi}(y) = \psi_2 \}$ is of positive measure.

Define the following variation[7]

$$\boldsymbol{c}_a^y(y) = \boldsymbol{c}_a^\psi(\boldsymbol{\psi}(y)) + I_{11}(y)\delta_{11} + I_{12}(y)\delta_{12} + I_2(y)\delta_2,$$

where $I_{11}(y) = 1$ if $y \in Y_{11}$ and 0 otherwise, and similarly for $I_{12}(y)$ and $I_2(y)$, with δ_{11}, δ_{12}, and δ_2 representing numbers (changes in the compensation contract) that we choose. Condition (18.8) ensures that we can select $\delta_2 > 0$ and δ_{11}, δ_{12} such that the variation leaves the principal indifferent, i.e., there is no change in the expected compensation,

$$\Delta \bar{c} \equiv \delta_{11}\varphi(Y_{11}|a) + \delta_{12}\varphi(Y_{12}|a) + \delta_2\varphi(Y_2|a) = 0,$$

and induces the agent to take the same action, i.e., there is no change in the agent's marginal utility,

$$\Delta U_a^a \equiv u'(\boldsymbol{c}_a^\psi(\psi_1))\,[\delta_{11}\varphi_a(Y_{11}|a) + \delta_{12}\varphi_a(Y_{12}|a)]$$

$$+ u'(\boldsymbol{c}_a^\psi(\psi_2))\,\delta_2\,\varphi_a(Y_2|a) = 0.$$

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[7] When we construct a variation of compensation contract $\boldsymbol{c}_a^\psi(\cdot)$ using $\delta(\cdot)$, think of δ as specifying the "direction" of the change in the contract. More specifically, we can think of the new contract as

$$\boldsymbol{c}_a^\psi(\cdot) + \delta(\cdot)\,\varepsilon,$$

where $\varepsilon > 0$. The *marginal impact* of the change is then assessed by taking the derivative with respect to ε and then evaluating the derivative at $\varepsilon = 0$. Hence, we evaluate the change at $\boldsymbol{c}_a^\psi(\cdot)$.

The change in the agent's expected utility is

$$\Delta U^a \equiv u'(\mathbf{c}_a^\psi(\psi_1))\,[\delta_{11}\varphi(Y_{11}|a) + \delta_{12}\varphi(Y_{12}|a)] + u'(\mathbf{c}_a^\psi(\psi_2))\,\delta_2\,\varphi(Y_2|a).$$

The fact that $\mathbf{c}_a^\psi(\psi_1) > \mathbf{c}_a^\psi(\psi_2)$ implies $u'(\mathbf{c}_a^\psi(\psi_1)) < u'(\mathbf{c}_a^\psi(\psi_2))$ and, hence, given $\delta_2 > 0$,

$$\Delta U^a > u'(\mathbf{c}_a^\psi(\psi_1))\,[\delta_{11}\varphi(Y_{11}|a) + \delta_{12}\,\varphi(Y_{12}|a) + \delta_2\,\varphi(Y_2|a)]$$

$$= u'(\mathbf{c}_a^\psi(\psi_1))\,\Delta\bar{c} = 0.$$

Q.E.D.

Hence, if performance measure $\psi(y)$ is not a sufficient implementation statistic for a, the risk premium paid to the agent can be reduced by using performance measure y instead of $\psi(y)$. The reason, of course, is that y is more informative about the agent's action than $\psi(a)$, and that additional information is useful for reducing the risk premium paid to the agent.

The above analysis has focused on the cost minimizing contract for implementing a particular action. Of course, if performance measure $\psi(y)$ is not a sufficient implementation statistic for the optimal action, then contracting on $\psi(y)$ leads to a strictly inferior solution compared to contracting on y. Determining the latter depends on identifying the optimal action. More generally, it is obvious that if $\psi(y)$ is not a sufficient implementation statistic for any action, then it must also hold for the optimal action. Hence, we use the following definition in obtaining the subsequent result.

Definition

Performance measure ψ: $Y \rightarrow \Psi$ is *globally "incentive" sufficient* if (18.7) is true for all $a \in A$ and all $\psi \in \Psi$. On the other hand, ψ is *globally "incentive" insufficient* if for some ψ (18.7) is false for *all* $a \in A$.

Note that if $\psi(y)$ is globally "incentive" sufficient, performance measurement system η^ψ is at least as A-informative as η, but $\psi(y)$ is a function of y, so contracting on performance measure $\psi(y)$ is as good as contracting on y directly, but no better. On the other hand, if $\psi(y)$ is globally "incentive" insufficient for y, then contracting on y is strictly better than contracting on $\psi(y)$.

Proposition 18.4 (H82, Theorem 6)

Assume ψ: $Y \to \Psi$ is *globally "incentive" insufficient* for y. Let $\mathbf{c}^\psi$: $\Psi \to C$ be an optimal nonconstant compensation contract such that the agent's optimal action choice $a^\psi \in (\underline{a},\bar{a}]$ is unique. Then there exists another compensation contract $\mathbf{c}^y$: $Y \to C$ that yields a *strict Pareto improvement.*

The proof is basically the same as for Proposition 18.3. While this proposition focuses on a Pareto improvement, and the proof is constructed to make the agent better off, continuity of the utility functions and an assumption of interior optimal contracts imply that the principal can be made better off (by a small fixed "utility" reduction in the agent's compensation).

Adding a Signal

Holmström (1979) (H79) considers the special case of comparing the reporting of x to the reporting of x and an additional performance measure y. We will present the essence of his analysis, but will do so by comparing a system that reports a single signal y_1 to one that *also* reports a second signal y_2. He introduces the following definition.

Definition

Signal y_2 is defined to be *informative about a given* y_1 when there *does not exist* a function $b(y_2|y_1)$ such that, for all $a \in A$,

$$\varphi(y_1,y_2|a) = b(y_2|y_1)\varphi(y_1|a), \quad \text{for almost every } (y_1,y_2). \tag{18.9}$$

Otherwise, signal y_2 is defined to be *noninformative* given y_1.

If signal y_2 is noninformative about a given y_1, there exists a function $b(y_2|y_1)$ such that (18.9) holds for all $a \in A$, which implies that, for all $a \in A$,

$$\varphi_a(y_1,y_2|a) = b(y_2|y_1)\varphi_a(y_1|a) \text{ and } L(y_1,y_2|a) = L(y_1|a). \tag{18.10}$$

Hence, the statistic ψ: $Y_1 \times Y_2 \to \Psi$ defined by $\psi(y_1,y_2) = y_1$ is globally "incentive" sufficient for (y_1,y_2) and, therefore, the additional performance measure is not valuable. On the other hand, if signal y_2 is informative about a given y_1, the statistic $\psi(y_1,y_2) = y_1$ is globally insufficient for (y_1,y_2) and,[8] therefore, we get

[8] Footnote 21 of H79 restricts the analysis to distributions for which (18.9) is true for either *all a* <u>or</u> *no a*. This is essential for (18.10) to imply (18.9) (by integration).

from Proposition 18.4 that y_2 is a valuable performance measure in addition to y_1.[9] Hence, we get the following result.

Proposition 18.5 (H79, Prop. 3)

Let $\boldsymbol{c}(y_1)$ be an optimal compensation contract based on y_1 for which the agent's action choice is unique and interior in A. There exits a compensation contract $\boldsymbol{c}(y_1,y_2)$ which strictly Pareto dominates $\boldsymbol{c}(y_1)$ if, and only if, (18.9) is false. That is, an additional signal y_2 is valuable in addition to y_1 if, and only if, it is informative about a given y_1.

Based on Amershi and Hughes (1989) we further discuss in Appendix 18A the relation between sufficient statistics and the information used in constructing an optimal incentive contract.

18.1.2 Second-order Stochastic Dominance with Respect to the Likelihood Ratio

In this section we summarize some results from Kim and Suh (1991) and Kim (1995). They focus on comparing the distribution function for the likelihood ratio L for alternative information structures. The agent is penalized (i.e., paid low values of c) if L is small (i.e., very negative) and receives large bonuses (i.e., is paid high values of c) if L is large. The point of their analysis is that greater variability in L permits more effective use of penalties and bonuses. In particular, if the distribution function for L with η^1 second-order stochastically dominates that with η^2, then η^2 is preferred to η^1 by the principal for implementing a.

Kim (1995) provides an analysis in which he considers a set of information systems $H = \{\eta\}$ in which each system produces a signal y.[10] His setting is one in which the outcome x is "owned" by the principal, but is not contractible information. The action set A is an interval of the real line and the incentive constraint is assumed to be characterized by the first-order condition of the agent's decision problem.

The optimal contract for inducing the agent to select action a using information system η is characterized by (18.5) with explicit recognition of the fact that the optimal contract and the Lagrange multipliers depend on the information system in place, i.e., if $\boldsymbol{c}(y,\eta) > \underline{c}$,

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[9] H79 uses a different approach to prove the necessity part of the proposition. He constructs a variation that induces an increase in the agent's effort without increasing the cost.

[10] Kim (1995) allows for multi-dimensional performance measures. However, this is of no real consequence in his analysis since the likelihood ratio is always single dimensional.

$$M(\boldsymbol{c}(y,\eta)) = \lambda(a,\eta) + \mu(a,\eta)\, L(y|a,\eta), \tag{18.11}$$

where
$$L(y|a,\eta) = \frac{\varphi_a(y|a,\eta)}{\varphi(y|a,\eta)}.$$

The likelihood ratio L plays a key role in the analysis. When viewed across the different performance measures, the likelihood ratio is a random variable. Let $l \equiv L(y|a,\eta)$ denote this random variable, and let $\Phi(l|a,\eta)$ denote the probability distribution function for l, i.e.,

$$\Phi(l|a,\eta) = \Pr\{L \le l|a,\eta\} = \sum_{y \in Y(l,a,\eta)} \varphi(y|a,\eta),$$

where
$$Y(l,a,\eta) = \{\, y \mid L(y|a,\eta) \le l \,\}.$$

Proposition 18.6 (Kim 1995, Prop. 1)

Assuming the first-order approach is valid, performance measurement system η^2 is strictly preferred to performance measurement system η^1 for implementing any $a \in (\underline{a}, \bar{a}]$, if $\Phi(l|a,\eta^1)$ strictly dominates $\Phi(l|a,\eta^2)$ in the sense of second-order stochastic dominance.

Proof: Consider a setting in which the principal will use performance measurement system η^2 with probability $\alpha \in [0,1]$ and η^1 with probability $1-\alpha$ (see Section 18.4 for a similar setting in which this approach is further developed). That is, we can formulate the principal's decision problem as in the basic model except that

$$U^p(\boldsymbol{c},a|\alpha) = \alpha\, U^p(\boldsymbol{c}^2,a|\eta^2) + (1-\alpha)\, U^p(\boldsymbol{c}^1,a|\eta^1),$$

$$U^a(\boldsymbol{c},a|\alpha) = \alpha\, U^a(\boldsymbol{c}^2,a|\eta^2) + (1-\alpha)\, U^a(\boldsymbol{c}^1,a|\eta^1),$$

where $\boldsymbol{c}^1$ and $\boldsymbol{c}^2$ are the incentive contracts for η^1 and η^2, respectively. The Lagrangian in this setting is

$$\mathcal{L} = \alpha[U^p(\boldsymbol{c}^2,a|\eta^2) + \lambda U^a(\boldsymbol{c}^2,a|\eta^2) + \mu U^a_a(\boldsymbol{c}^2,a|\eta^2)]$$
$$+ (1-\alpha)[U^p(\boldsymbol{c}^1,a|\eta^1) + \lambda U^a(\boldsymbol{c}^1,a|\eta^1) + \mu U^a_a(\boldsymbol{c}^1,a|\eta^1)] - \lambda \bar{U}.$$

The first-order conditions for the optimal incentive contracts with either η^1 or η^2 are similarly characterized by (18.5):[11]

$$M(\mathbf{c}^i(y_i)) = \lambda + \mu L(y_i|a,\eta^i), \quad i = 1,2,$$

if the compensation is interior for y. Otherwise, $\mathbf{c}^i(y_i) = \underline{c}$.

Note that both the principal's and the agent's expected utilities are linear in the probability α. This implies that the optimal probability α will always be a *corner solution*, i.e., $\alpha = 0$ or $\alpha = 1$. Differentiating the Lagrangian with respect to α yields the following expected marginal benefits of increasing α:

$$\begin{aligned} B \equiv \partial\mathcal{L}/\partial\alpha &= [U^p(\mathbf{c}^2,a|\eta^2) + \lambda U^a(\mathbf{c}^2,a|\eta^2) + \mu U_a^a(\mathbf{c}^2,a|\eta^2)] \\ &\quad - [U^p(\mathbf{c}^1,a|\eta^1) + \lambda U^a(\mathbf{c}^1,a|\eta^1) + \mu U_a^a(\mathbf{c}^1,a|\eta^1)] \\ &= \sum_{y_1\in Y_1} [\, \mathbf{c}^1(y_1) - u(\mathbf{c}^1(y_1))\, M(\mathbf{c}^1(y_1))\,]\, \varphi(y_1|a,\eta^1) \\ &\quad - \sum_{y_2\in Y_2} [\, \mathbf{c}^2(y_2) - u(\mathbf{c}^2(y_2))\, M(\mathbf{c}^2(y_2))\,]\, \varphi(y_2|a,\eta^2), \end{aligned}$$

where the equality is obtained by collecting terms and substituting in the $M(\cdot)$ based on the preceding first-order condition. Hence, it is optimal to choose $\alpha = 1$ (which is what we need to show) if, and only if, $B > 0$.

Note that the incentive contracts only depend on the performance measurement system η^i and the signals y^i through the likelihood ratio, i.e., we can write $\mathbf{c}^i(m(l)) = \mathbf{c}(m(l))$ where the likelihood measure m is defined by $m(l) = \lambda + \mu l$ (and $m(l) = M(\underline{c})$ on the lower bound for c). Hence, if we define the function $f(\cdot)$ by

$$f(l) \equiv \mathbf{c}(m(l)) - u(\mathbf{c}(m(l)))m(l),$$

we can write B as follows

$$B = \sum_l f(l)\, \varphi(l|a,\eta^1) - \sum_l f(l)\, \varphi(l|a,\eta^2).$$

The function $f(\cdot)$ is a strictly concave function since

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[11] We assume that the agent knows α when he selects a, but he does not yet know η^i. Hence, there is a single incentive constraint, implying that μ is not dependent on η^i. However, η^i is contractible information, so that $\mathbf{c}$ depends on both y and η^i.

$$f'(l) = c'(m(l))\mu - [u'(c(m(l)))c'(m(l))\mu m(l) + u(c(m(l)))\mu]$$

$$= - u(c(m(l)))\mu,$$

$$f''(l) = - u'(c(m(l)))c'(m(l))\mu^2 < 0,$$

where $f' = -u\mu$ follows from the fact that $m = 1/u'$ and $f'' < 0$ follows from the fact that $u'' < 0$ implies $c'(\cdot) > 0$ (for any interior compensation). Since $f(\cdot)$ is strictly concave and $\Phi(l|a,\eta^1)$ strictly dominates $\Phi(l|a,\eta^2)$ in the sense of second-order stochastic dominance, we get that[12]

$$B = \sum_l f(l)\,\varphi(l|a,\eta^1) - \sum_l f(l)\,\varphi(l|a,\eta^2) > 0,$$

and, thus, $\alpha = 1$ is optimal. **Q.E.D.**

Note that the random variable l always has a mean of zero. Hence, $\Phi(l|a,\eta^1)$ SS-dominates $\Phi(l|a,\eta^2)$ if, and only if, the probability function $\varphi(l|a,\eta^2)$ differs from $\varphi(l|a,\eta^1)$ by adding mean-preserving spreads.[13] This illustrates the point that more variation in the likelihood ratio is desirable – it implies that there is a wider range of information upon which to efficiently place penalties and rewards (see also the hurdle model example in Chapter 17). Interestingly, the risk imposed on the agent (as indicated by the risk premium he is paid) decreases as the variation in the likelihood ratio increases.

In relating his analysis to that of GH, Gjesdal (1982) and Holmström (1979, 1982), Kim (1995) obtains the following results (see Kim, 1995, for proofs).

Proposition 18.7 (Kim 1995, Prop. 2, 4, and 5)

(a) If η^2 is more A-informative than η^1, then $\Phi(l|a,\eta^1)$ SS-dominates $\Phi(l|a,\eta^2)$. However, the converse is not necessarily true.

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[12] Strictly speaking, we here ignore the fact that the SS-dominance relation may be due to likelihood ratios where the compensation is on the boundary.

[13] Kim (1995) states his proposition in terms of mean-preserving spreads, and then interprets that to mean that $\Phi(l|a,\eta^1)$ second-order stochastically dominates $\Phi(l|a,\eta^2)$. The equivalence between the two concepts is given by the following result.

Lemma (Rothschild and Stiglitz 1970, Theorem 2)

The distribution for a random variable $\tilde{X}$ second-order stochastically dominates the distribution for another random variable $\tilde{Y}$ with the same mean, if, and only if, the probability function for $\tilde{Y}$ differs from the probability function for $\tilde{X}$ by adding mean-preserving spreads.

(b) Let $y = (y_1,y_2)$ and assume η^1 only generates y_1, while η^2 generates both y_1 and y_2. Then $\Phi(l|a,\eta^1)$ SS-dominates $\Phi(l|a,\eta^2)$ for all $a \in (\underline{a},\bar{a}]$, and $\Phi(l|a,\eta^1)$ strictly SS-dominates $\Phi(l|a,\eta^2)$ for some $a \in (\underline{a},\bar{a}]$ *if, and only if*, y_1 is not a sufficient statistic for $y = (y_1,y_2)$.

These results demonstrate that the SSD relation between the distribution functions of the likelihood ratios is a weaker condition than A-informativeness but, nevertheless, is sufficient for ranking the information systems by Proposition 18.6. In the case of the conditional value of an additional signal, the SSD relation is equivalent to the Holmström (1979) result that the additional signal is incrementally informative about a.

Kim (1995) also relates his results to the Blackwell-theorem (see Proposition 3.7) in the sense of A-informativeness. Proposition 18.1 establishes the *sufficiency* of the Blackwell relation in an agency setting. The question raised by Kim (1995) is whether it is necessary for such a Markov kernel to exist to ensure that η^2 is preferred over η^1 for all agency preferences. In addressing this issue, recall that the structure of the agency problem is restricted to be one in which the principal is risk neutral and the agent is risk averse with additively separable preferences. Hence, Kim loses some of the generality that pertains to the original Blackwell result – which considered *any* payoff function. Given that limitation in the analysis, he then claims that the "necessary part of Blackwell's theorem does not hold in the agency model" (see Kim 1995, Proposition 3). He provides an example as his proof. Essentially the point is that, by Proposition 18.7(a), the SSD relation can hold even if the Blackwell relation does not and, by Proposition 18.6, the SSD relation is sufficient for η^2 to be less costly than η^1 for all agency problems (that satisfy Kim's basic assumptions).

The above results provide a partial ordering between information systems that are distinguished by A-informativeness or mean-preserving spreads of the probability functions for the likelihood ratios. Kim and Suh (1991) seek a complete ordering in terms of a simple measure such as the variance of the likelihood ratio. They accomplish this by either restricting the agent's utility for compensation (to be a square-root function) or by restricting the underlying probability functions (to normal, log-normal, or Laplace families).

Proposition 18.8 (KS, Prop. 1)

(a) If $u(c) = 2c^{1/2}$ and the optimal incentive contracts are interior, then η^2 is more valuable than η^1 in inducing action a if, and only if, $\mathrm{Var}(l|a,\eta^1) < \mathrm{Var}(l|a,\eta^2)$.

(b) Assume that $\varphi(y|a,\eta^1)$ and $\varphi(y|a,\eta^2)$ belong to the normal, log-normal, or Laplace families. Then η^2 is more valuable than η^1 in inducing action a if, and only if, $\mathrm{Var}(l|a,\eta^1) < \mathrm{Var}(l|a,\eta^2)$.[14]

Proof:

(a): A key feature of the square-root utility function is that it results in a compensation function that is the square of a linear function of l (see Appendix 17C), i.e., $M(c) = c^{½} \Rightarrow c(l,a,\eta) = [\lambda(a,\eta) + \mu(a,\eta)l]^2$. Since l has zero mean, it follows that

$$\mathrm{E}[c|a,\eta] = [\lambda(a,\eta)]^2 + [\mu(a,\eta)]^2\mathrm{Var}(l|a,\eta).$$

Since the optimal incentive contracts are assumed to be interior, the agent's participation constraint is satisfied as an equality. Using again that l has mean zero, it follows from the participation constraint that $\lambda(a,\eta) = ½[\bar{U} + v(a)]$, and from the incentive compatibility constraint that $\mu(a,\eta)\mathrm{Var}(l|a,\eta) = ½v'(a)$ for $\eta = \eta^1$, η^2. Hence,

$$\mathrm{E}[c|a,\eta] = \frac{1}{4}\left[\bar{U}_1 + v(a)\right]^2 + \frac{1}{2}v'(a)\mu(a,\eta).$$

Finally, $\mu(a,\eta^1) > \mu(a,\eta^2) \Leftrightarrow \mathrm{Var}(l|a,\eta^1) < \mathrm{Var}(l|a,\eta^2)$.

(b): KS demonstrate that for these distributions, $\mathrm{Var}(l|a,\eta^1) < \mathrm{Var}(l|a,\eta^2)$ if, and only if, $\Phi(l|a,\eta^1)$ strictly SS-dominates $\Phi(l|a,\eta^2)$, and then Proposition 18.6 gives the result. **Q.E.D.**

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[14] The three classes of distributions have the following characteristics:

		Normal	Log-normal	Laplace
φ	~	$\exp\left[-\frac{1}{2\sigma^2}(y-\bar{y}(a))^2\right]$	$\frac{1}{y}\exp\left[-\frac{1}{2\sigma^2}(\ln y-\bar{y}(a))^2\right]$	$\exp\left[-\frac{1}{\beta}\lvert y-\bar{y}(a)\rvert\right]$
l	=	$\frac{\bar{y}'(a)}{\sigma^2}[y-\bar{y}(a)]$	$\frac{\bar{y}'(a)}{\sigma^2}[\ln y-\bar{y}(a)]$	$\pm\frac{1}{\beta}\bar{y}'(a)$
$\mathrm{Var}(l)$	=	$\left[\frac{\bar{y}'(a)}{\sigma}\right]^2$	$\left[\frac{\bar{y}'(a)}{\sigma}\right]^2$	$\left[\frac{\bar{y}'(a)}{\beta}\right]^2$
$\bar{y}(a)$	=	$\mathrm{E}[y\|\mathrm{a}]$	$\mathrm{E}[\ln y\|a]$	$\mathrm{E}[y\|a]$

18.1.3 A Hurdle Model Example

The hurdle model provides a simple setting in which we can illustrate the value of alternative performance measures. Recall (see Section 17.3.2) that the action space is continuous with $a \in [0,1]$, and there are two outcomes $x_g > x_b$. For simplicity, we assume there is zero probability of the bad outcome if the agent clears the hurdle, i.e., $\varepsilon = 0$. Note that in this case the outcome can be written as a function of the hurdle and the agent's action, i.e.,

$$\boldsymbol{x}(h,a) = \begin{cases} x_g & \text{if } a \geq h, \\ x_b & \text{if } a < h. \end{cases}$$

That is, the hurdle represents the underlying state. The hurdle is uniformly distributed over the interval $[0,1]$. Hence, the prior probability for the good outcome is simply the height of the agent's jump, i.e., $\varphi(x_g|a) = a$.

In our numerical examples we use the following data:

$$u(c) = c^{½}; \quad v(a) = a/(1-a); \quad \bar{U} = 2.$$

Only the Outcome Is Contractible Information

If the outcome is the only contractible performance measure, the agent is paid outcome-contingent wages dependent on the likelihood ratio for each outcome, i.e.,

$$L(x_g|a) = 1/a; \quad L(x_b|a) = -1/(1-a).$$

The expected cost minimizing contract for inducing $a = ½$ is shown in the first row of Table 18.1.

An Additional Contractible Performance Measure

Suppose now there is an additional ex post performance measure y that can take one of two equally likely values y_L and y_H. Performance measure y is informative about the hurdle, and the posterior density function for the hurdle is given by

$$\varphi(h|y) = \begin{cases} (1+k) - 2kh & \text{if } y = y_L, \\ (1-k) + 2kh & \text{if } y = y_H, \end{cases}$$

with $k \in [0,1]$. The signal y is uninformative about the hurdle if $k = 0$, and its information content increases with k (see Figure 18.1).[15]

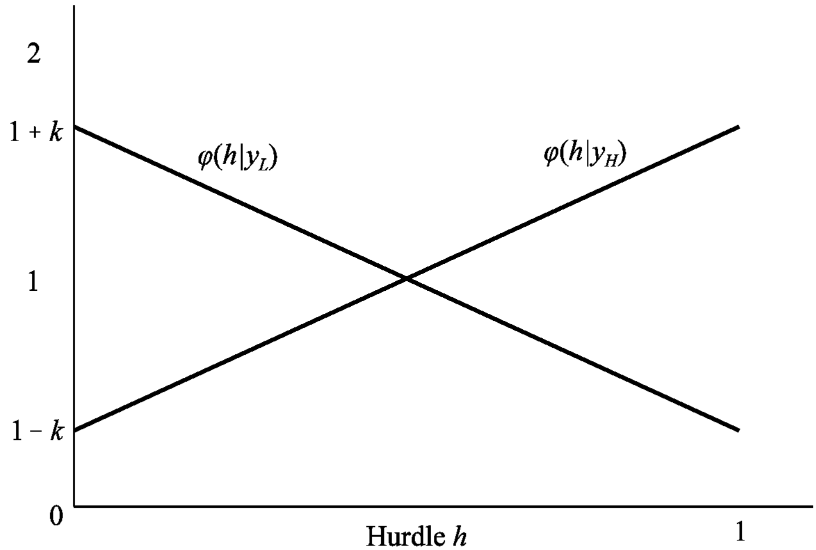

Figure 18.1: Posterior density function for hurdle given performance measure y with $k > 0$.

Note that the agent's action does not affect the likelihood of the two signals y_L and y_H (nor the informativeness of y about the hurdle). Hence, we can write the joint probability function for x and y given a as

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[15] To see this note that

$$\varphi(y|h;k) = ½\varphi(h|y;k),$$

and consider two values of k with $k' < k''$. We can then find a Markov matrix **B** with

$$\mathbf{B} = \begin{bmatrix} b(y_L|y_L) & b(y_H|y_L) \\ b(y_L|y_H) & b(y_H|y_H) \end{bmatrix} = ½ \begin{bmatrix} 1 + \dfrac{k'}{k''} & 1 - \dfrac{k'}{k''} \\ 1 - \dfrac{k'}{k''} & 1 + \dfrac{k'}{k''} \end{bmatrix},$$

such that

$$\varphi(y|h;k') = \varphi(h|y_L;k'')\, b(y|y_L) + \varphi(h|y_H;k'')\, b(y|y_H), \quad y = y_L, y_H, \quad \forall\, h \in [0,1].$$

$$\varphi(x,y|a) = \varphi(x|y,a)\varphi(y),$$

where

$$\varphi(x|y,a) = \int_0^1 \varphi(x|h,a)\varphi(h|y)\,dh = \begin{cases} \Phi(h{=}a|y) & \text{if } x = x_g, \\ 1 - \Phi(h{=}a|y) & \text{if } x = x_b. \end{cases}$$

Hence, the likelihood ratios are given by

$$L(x_g,y\,|\,a) = \frac{\varphi(h{=}a|y)}{\Phi(h{=}a|y)}, \quad L(x_b,y|a) = -\frac{\varphi(h{=}a|y)}{1 - \Phi(h{=}a|y)}.$$

Substituting in $\Phi(h{=}a|y_L) = a(\varphi(h{=}a|y_L) + ka)$ and $\Phi(h{=}a|y_H) = a(\varphi(h{=}a|y_H) - ka)$, we get

$$L(x_g,y_L|a) = \frac{1}{a}\,\frac{\varphi(h{=}a|y_L)}{\varphi(h{=}a|y_L) + ka} < \frac{1}{a},$$

$$L(x_b,y_L|a) = -\frac{\varphi(h{=}a|y_L)}{1 - a(\varphi(h{=}a|y_L) + ka)} < -\frac{1}{1-a},$$

$$L(x_g,y_H|a) = \frac{1}{a}\,\frac{\varphi(h{=}a|y_H)}{\varphi(h{=}a|y_H) - ka} > \frac{1}{a},$$

$$L(x_b,y_H|a) = -\frac{\varphi(h{=}a|y_H)}{1 - a(\varphi(h{=}a|y_H) - ka)} > -\frac{1}{1-a}.$$

Clearly, the outcome x is not a sufficient statistic for (x,y) with respect to a for $k > 0$ (since the likelihood ratios vary with y). Hence, the additional performance measure y is valuable in addition to x. This occurs even though the agent's action does not influence the characteristics of y. The key here is that y is informative about the state and, thus, the principal can use it to insure (i.e, remove) some of the incentive risk that is necessary if x is the only contractible performance measure.

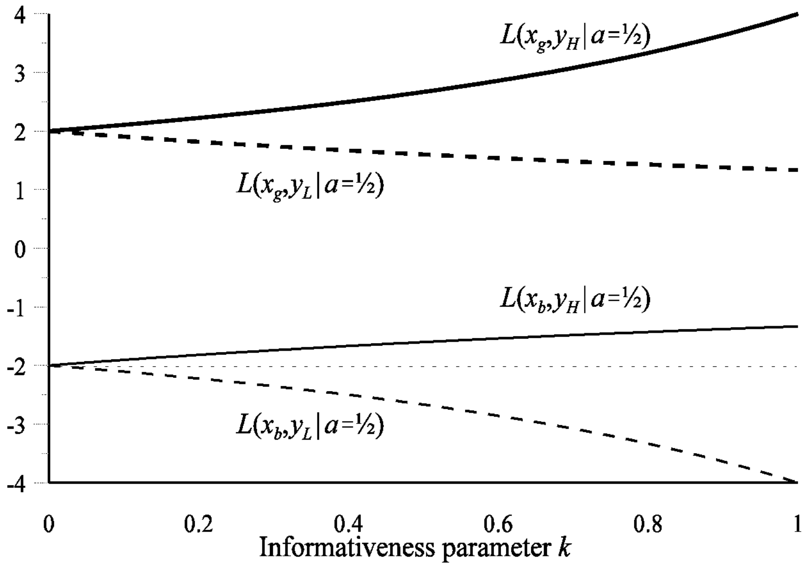

Figure 18.2: Likelihood ratios for a = ½ with varying informativeness parameter k.

Figure 18.2 shows how the likelihood ratios for inducing a = ½ vary with the informativeness of y about the state (hurdle). Note that an optimal compensation contract will reward the agent for the "high hurdle signal" y_H and punish the agent for the "low hurdle signal" y_L. Note also from Figure 18.2 that a higher k, implies that the mean-preserving spreads of the likelihood ratios also increase (since all four likelihood ratios have a probability of ¼ when $\varphi(x_g|a) = a$ = ½ and $\varphi(y)$ = ½). Hence, the variation in the likelihood ratios on which the rewards and punishments can be based increases with the informativeness of y about the state.

Table 18.1 shows the optimal contracts and Lagrange multipliers for varying values of the informativeness parameter k with $k = 0$ corresponding to the case with contracting only on the outcome. Of course, the expected compensation costs (or equivalently the risk premium) decrease with k, since the additional information in y about a increases with k. However, note that even though the variations in the likelihood ratios on which the rewards and punishments are based increase, the compensation scheme becomes less risky (since the risk premium goes down) as k increases. This is reflected by the fact that the sensitivity of the agent's compensation with respect to variations in the likelihood ratio (i.e., $\mu(k)$) goes down as k increases.[16]

k	$\bar{c}^{\dagger}(a,k)$	$c(x_g,y_L)$	$c(x_b,y_L)$	$c(x_g,y_H)$	$c(x_b,y_H)$	$\lambda(k)$	$\mu(k)$
0.00	13.000	25.000	1.000	25.000	1.000	6.0	2.000
0.25	12.937	22.562	0.562	27.562	1.563	6.0	1.969
0.50	12.750	20.250	0.250	30.250	2.250	6.0	1.875
0.75	12.437	18.062	0.063	33.063	3.062	6.0	1.719
1.00	12.000	16.000	0.000	36.002	3.999	6.0	1.500

Table 18.1: Optimal incentive contracts for inducing a = ½ for varying informativeness of y.

18.1.4 Linear Aggregation of Signals

A demand for aggregation of signals in performance evaluation may arise because reporting all basic transactions or signals about performance may be too costly and impracticable. Aggregation is particularly common in accounting information systems. Observe that in inducing a particular action a, we can always replace a multi-dimensional signal with a single-dimensional representation without losing any valuable information. This is because the likelihood ratio $L \in \mathbb{R}$, and the optimal second-best compensation contract is a function of y only through L. The issue, therefore, is how the aggregation is performed. Banker and Datar (1989) (BD) identify necessary and sufficient conditions on the joint density function of signals $y = (y_1, ..., y_n)$ under which *linear aggregation of the signals is optimal.*[17] That is, these conditions are such that there exists a *sufficient implementation statistic* that is a linear function of the signals.

BD assume that $L(y|a)$ is continuously differentiable with respect to each y_i, $i = 1, ..., n$, and that $\varphi(y|a)$ has constant support (all $a \in A$) and satisfies

[16] Note that λ is not affected by k. This is due to the fact that with square-root utility it follows from the characterization of optimal incentive contracts and the participation constraint that $\lambda = 2(\bar{U} + v(a))$. Of course, this presumes that the participation constraint is binding which in turn is assured by an optimal interior contract. Even though the optimal contract is not interior for $k = 1$, the participation constraint is binding in the example.

[17] Amershi, Banker, and Datar (1990) relate this analysis to the Amershi and Hughes (1989) analysis discussed in Appendix 18A.

MLRP with respect to each of the elements of y. The optimal compensation contract is then assumed to be characterized by[18]

$$M(\boldsymbol{c}(y)) = \lambda + \mu L(y|a) \tag{18.12}$$

subject to boundary conditions in which $\boldsymbol{c}(y) = \underline{c}$ if $\lambda + \mu L(y|a) \leq M(\underline{c})$.

Definition

The optimal compensation contract is based on *a linear aggregate of (the elements of) y* (where $y \in Y \subset \mathbb{R}^n$) if there exist weights $\delta_1, ..., \delta_n$ and a contract $\boldsymbol{c}^\psi: \mathbb{R} \to C$ such that

$$\boldsymbol{c}(y) = \boldsymbol{c}^\psi(\psi(y)) \quad \text{and} \quad \psi(y) \equiv \sum_{i=1}^{n} \delta_i y_i. \tag{18.13}$$

BD are particularly interested in settings in which the signal weights are independent of the utility function $u(c)$, although they can depend upon the action a that is to be implemented.[19]

Proposition 18.9 (BD, Prop. 1)

When the principal is risk neutral, a sufficient condition for the optimal compensation contract for inducing a to be based on a linear aggregation of the signals y, represented by

$$\psi(y,a) = \sum_{i=1}^{n} \boldsymbol{\delta}_i(a)\, y_i,$$

with $\psi(\cdot)$ independent of the agent's utility function, is that the joint density function is of the form:

$$\varphi(y|a) = \exp\left[\int_{\underline{a}}^{a} g\left(\psi(y,\alpha),\alpha\right) d\alpha + t(y)\right], \tag{18.14}$$

where $g(\cdot)$, $\boldsymbol{\delta}_1(\cdot), ..., \boldsymbol{\delta}_n(\cdot)$, and $t(\cdot)$ are arbitrary functions. Further, in this case,

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[18] For ease of notation, we suppress the dependence on the performance measurement system η, which is kept constant in this analysis.

[19] We refer to BD for proofs.

$$\frac{\partial c(y)/\partial y_i}{\partial c(y)/\partial y_j} = \frac{\delta_i(a)}{\delta_j(a)}.$$

The key characteristic of distributions satisfying (18.14) is that the likelihood ratio can be expressed as a function of a linear function of the signals, i.e.,

$$L(y|a) = g(\psi(y,a),a),$$

so that the optimal compensation contract is based on a linear aggregate of y. Note that this does not imply that the compensation contract itself is a linear function of y. A broad subclass of joint density functions satisfying (18.14) is given by:

$$\varphi(y|a) = \exp\left[\sum_{i=1}^{n} \Delta_i(a)y_i - r(a) + t(y)\right]. \tag{18.15}$$

This subclass includes, for example, a multivariate normal distribution in which a influences the means of the distributions of each variable.

Corollary

If $\varphi(y|a)$ satisfies (18.15), then the optimal compensation contract for inducing a can be written as $c^{\psi}(\psi)$, where $\psi(\cdot)$ is a linear function of y (and the action to be implemented).

Proof: (18.15) is a special case of (18.14) if $\delta_i(a) = \Delta_i'(a)$, $g(\psi,a) = \psi - r'(a)$, and $t(y)$ is the constant of integration. **Q.E.D.**

The following proposition shows that the joint density satisfying (18.14) is also a necessary condition for the optimal compensation contract to be based on a linear aggregate of y if the result must hold for all actions in A.[20]

Proposition 18.10 (BD, Prop. 2)

A *necessary* (as well as sufficient) condition for the optimal compensation contract to be written as $c^{\psi}(\psi(y,a))$, $\psi(y,a) = \delta_1(a)y_1 + ... + \delta_n(a)y_n$ for inducing all $a \in A$, is that the joint density function is of the form in (18.14).

In the above analysis the weights on the signals can depend on the action to be implemented. BD also consider the conditions under which $\delta_i(a) = \delta_i$, $\forall\, a \in A$.

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[20] BD give an example in which the optimal compensation contract for inducing the *optimal action* is based on a linear aggregate of y even though the joint density does not satisfy (18.14).

This holds if $\psi(y) = \Sigma_i\, \delta_i y_i$ is a sufficient statistic for y with respect to a, i.e., there exists a function $g(y)$ such that

$$\varphi(y|a) = g(y)\varphi(\psi(y)|a).$$

Relative Signal Weights

BD examine the relation between a signal's "precision" and "sensitivity" and the relative weight it is given in the linear aggregation of the signals.

Definition

The *precision* of signal y_i is $h_i(a) = 1/\mathrm{Var}(y_i|a)$ and its *sensitivity* is $\bar{y}_{ia}(a) = \partial \mathrm{E}[y_i|a]/\partial a$.

Proposition 18.11 (BD, Prop. 3)

If the joint density function of $y = (y_1, ..., y_n)$ is of the form

$$\varphi(y|a) = \exp\left[\sum_{i=1}^{n} \big(\Delta_i(a)\, y_i + t_i(y_i)\big) - r(a)\right], \tag{18.16}$$

then
$$\frac{\delta_i(a)}{\delta_j(a)} = \frac{h_i(a)\,\bar{y}_{ia}(a)}{h_j(a)\,\bar{y}_{ja}(a)}.$$

That is, if the y_i's are independent (which is implied by (18.16)), then the relative weights assigned to a pair of signals is equal to the relative value of the precision of the signal times its sensitivity to changes in a, where the precision and sensitivity are evaluated at the action to be implemented. The following proposition considers a case in which the signals are correlated.

Proposition 18.12 (BD, Prop. 4 and Corr. 2)

If the joint density function of $y = (y_1,y_2)$ is of the form

$$\varphi(y|a)$$

$$= \exp\big[\Delta_1(a)y_1 + \Delta_2(a)y_2 + t_1(y_1) + t_2(y_2 - \gamma y_1) - r(a)\big], \quad \gamma \neq 0, \tag{18.17}$$

then
$$\frac{\delta_1(a)}{\delta_2(a)} = \frac{h_1(a)\,[\bar{y}_{1a}(a) - \gamma_2(a)\bar{y}_{2a}(a)]}{h_2(a)\,[\bar{y}_{2a}(a) - \gamma_1(a)\bar{y}_{1a}(a)]}, \tag{18.18}$$

where
$$\gamma_i(a) = \frac{\mathrm{Cov}(y_1,y_2|a)}{\mathrm{Var}(y_i|a)} = \mathrm{Corr}(y_1,y_2|a)\,\frac{\sigma_1(a)}{\sigma_2(a)},$$

and $$\sigma_i^2(a) = \text{Var}(y_i|a),$$

which implies that $\gamma_i(a) = \gamma$.

BD refer to the expressions in the square brackets in (18.18) as the *adjusted sensitivity* of the signals. It reflects the fact that the information contained in one signal is partially reflected in the other signal if the signals are not independent.

Now consider the special case in which $\bar{y}_{1a}(a) > 0$ and $\bar{y}_{2a}(a) = 0$, i.e., the action influences the first signal but not the second.

Proposition 18.13 (BD, Prop. 5)
If the joint density function of $y = (y_1,y_2)$ belongs to the class in (18.17), and $\bar{y}_{1a}(a) > 0$, $\bar{y}_{2a}(a) = 0$, $h_1(a) > 0$, and $h_2(a) > 0$, then

$$\frac{\boldsymbol{\delta}_2(a)}{\boldsymbol{\delta}_1(a)} = -\frac{\text{Cov}(y_1,y_2|a)}{\text{Var}(y_2|a)} = -\text{Corr}(y_1,y_2|a)\,\frac{\sigma_1(a)}{\sigma_2(a)}.$$

Observe that $\boldsymbol{\delta}_2(a)$ is nonzero if y_1 and y_2 are correlated. Hence, even though y_2 reveals nothing about a directly, it is used in deriving the optimal performance measure because it is informative about the uncontrollable factors influencing y_1 (which is influenced by the action a). Further observe that if y_1 and y_2 are positively (negatively) correlated, then y_2 will be given negative (positive) weight. That is, if the two signals are positively correlated, the agent will receive higher compensation if he obtains a high value of y_1 with a low value of y_2 than with a high value of y_2. This is consistent with the concept of basing compensation on how well the agent does relative to some other "standard" or other measure that reveals whether the uncontrollable factors were favorable or unfavorable. That is, the agent receives higher compensation if he obtains a "high" outcome in "bad" times than in "good" times and, conversely, he is not penalized as severely for a "low" outcome in "bad" times as he is for a "low" outcome in "good" times.

BD make the observation that two signals, y_1 and y_2, should be aggregated into a single measure $y_1 + y_2$ if, and only if, the intensity (sensitivity times precision) of the individual components are equal.

Impact of Changes in the Scale of a Signal
Consider a pair of signals $y = (y_1,y_2)$, and assume that the second signal is replaced by a linear transformation of that signal, i.e., $y^s = (y_1,y_2^s)$ where $y_2^s = ky_2 + b$. *Observe that changing the scale of a signal does not change its informativeness.* In particular, it is relatively straightforward to prove that using y^s in-

stead of y will result in *precisely the same action choice and compensation cost* – the optimal incentive contracts will have the following relation:

$$c^s(y_1,y_2^s) = c(y_1,(y_2^s - b)/k).$$

Observe that transforming y_2 will change both the precision and the sensitivity of the second signal. In particular,

$$h_2^s(a) = h_2(a)/k^2$$

and

$$\bar{y}_{2a}^s(a) = k\,\bar{y}_{2a}(a).$$

Furthermore, the transformation will change the relative weight assigned to the two signals:

$$\frac{\delta_1^s(a)}{\delta_2^s(a)} = k\frac{\delta_1(a)}{\delta_2(a)}.$$

If $k > 1$, then the contract based on the transformed signal will place relatively more weight on the first signal – but that is merely an offsetting adjustment. The transformed second signal is more sensitive than the untransformed signal, but that is offset by the decreased precision.

18.2 RISK AVERSE AGENT "OWNS" THE OUTCOME

Now consider a setting in which the agent "receives" or "owns" outcome x. This may be a setting in which the principal owns the technology that generates x but cannot directly observe the x that is produced, so that the agent can consume any amount not paid to the principal. Alternatively, this may be a setting in which the agent owns the technology that produces x and he seeks to obtain capital from and share his risk with the risk neutral principal. In this setting we have two roles for a performance measure y: as a mechanism to facilitate the sharing of the agent's risk from x; and as a mechanism to provide incentives for the agent's action.

In this setting the contract is $\boldsymbol{\pi}$: $Y \rightarrow \mathbb{R}$, which specifies a payment π from the agent to the principal. To simplify the analysis, we assume there is no lower bound on the agent's consumption, which is $c = x - \pi(y)$.[21] Hence, the principal's decision problem is:

$$\underset{\boldsymbol{\pi}(y),\, a}{\text{maximize}} \;\; U^p(\boldsymbol{\pi},a,\eta) = \sum_{y \in Y} \boldsymbol{\pi}(y)\, \varphi(y|a,\eta), \tag{18.1$'$}$$

$$\text{subject to} \;\; U^a(\boldsymbol{\pi},a,\eta) \equiv \sum_{x \in X} \sum_{y \in Y} u(x - \boldsymbol{\pi}(y))\, \varphi(x,y|a,\eta) - v(a) \geq \bar{U}, \tag{18.2$'$}$$

$$a \in \underset{a' \in A}{\text{argmax}} \;\; U^a(\boldsymbol{\pi},a',\eta), \tag{18.3$'$}$$

We assume that A is convex and constraint (18.3$'$) can be represented by

$$U_a^a(\boldsymbol{\pi},a,\eta) = 0. \tag{18.3c$'$}$$

Forming the appropriate Lagrangian and differentiating with respect to $\boldsymbol{\pi}(y)$ provides:

$$\sum_{x \in X} u'(x - \boldsymbol{\pi}(y)) \left[\lambda \frac{\varphi(x,y|a,\eta)}{\varphi(y|a,\eta)} + \mu \frac{\varphi_a(x,y|a,\eta)}{\varphi(y|a,\eta)} \right] = 1. \tag{18.5$'$}$$

Observe that if x and y are independent, i.e., $\varphi(x,y|a,\eta) = \varphi(x|a,\eta)\varphi(y|a,\eta)$, then y reveals nothing about x and cannot be used for risk sharing. In that setting, $\boldsymbol{\pi}(y) = \pi^o$, a constant, and the induced action is

$$a^o \in \underset{a' \in A}{\text{argmax}} \;\; U^a(\pi^o,a',\eta^o) = \sum_{x \in X} u(x - \pi^o)\, \varphi(x|a') - v(a'),$$

i.e., the result is the same as if there is no contractible information.

Pure Insurance Informativeness

We first consider information that reveals nothing about the agent's action, but is informative about the uncontrollable events that influence the outcome x. We assume that events $\theta \in \Theta$ define an outcome adequate partition on the state space S, so that we can express the outcome as a function $x = \boldsymbol{x}(\theta,a)$.

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[21] If there is a lower bound $\underline{c}$ and y does not reveal x, we must either restrict $\boldsymbol{\pi}(y)$ to be such that $x - \boldsymbol{\pi}(y) \geq \underline{c}$ for all x and y such that $\varphi(x,y|a) > 0$ *or* we must introduce the possibility that the agent can declare bankruptcy if $x - \boldsymbol{\pi}(y) < \underline{c}$, possibly with a deadweight bankruptcy cost being borne by the principal.

Definition *Exclusively Θ-informative*

Performance measurement system η is *exclusively Θ-informative* if

$$\varphi(y|\theta,a) = \varphi(y|\theta), \quad \forall\, a \in A,$$

i.e., conditional on θ the action does not influence the signal y,

and $$\varphi(y|\theta) \neq \varphi(y), \quad \text{for some } (y,\theta),$$

i.e., the signal y is not independent of θ.

Recall that in Chapter 3 we introduced the concepts of an *outcome relevant partition* of the state space S (the coarsest outcome adequate partition) and the informativeness relation between two information systems. We now introduce the concepts of payoff relevance and θ informativeness for a given action.

Definition *Payoff Relevance for Action a*

$\Theta(a)$ is a payoff relevant partition of S for action a if it is the coarsest partition such that for each $\theta \in \Theta(a)$

$$\boldsymbol{x}(s^1,a) = \boldsymbol{x}(s^2,a), \quad \forall\, s^1,s^2 \in \theta.$$

Definition *At least as Θ(a)-informative*

Performance measurement system η^2 is *at least as Θ(a)-informative* as η^1 if, and only if, there exists a Markov matrix **B** such that

$$\boldsymbol{\eta}^1 = \boldsymbol{\eta}^2\mathbf{B},$$

where $\boldsymbol{\eta} \equiv [\,\varphi(y|\theta)\,]_{|\Theta(a)|\times|Y|}$ and $\mathbf{B} \equiv [\,b(y^1|y^2)\,]_{|Y^2|\times|Y^1|}$.

Proposition 18.14

If the agent "owns" x and is strictly risk averse, a system that is *exclusively Θ-informative* has positive value (relative to no information). Furthermore, if a^1 would be implemented with η^1 and system η^2 is *at least as Θ(a^1)-informative* as η^1, then η^2 is at least as preferred as η^1, with strict preference if η^1 is not at least as $\Theta(a^1)$-informative as η^2.

Proof: Set $\boldsymbol{\pi}(y^2)$ such that

$$\sum_{\theta\in\Theta(a)} u(\boldsymbol{x}(\theta,a) - \boldsymbol{\pi}(y^2))\,\varphi(\theta|y^2)$$

$$= \sum_{y^1 \in Y^1} \sum_{\theta \in \Theta(a)} u(\boldsymbol{x}(\theta,a) - \boldsymbol{\pi}^1(y^1))\, b(y^1|y^2)\varphi(\theta|y^2).$$

This new contract has the same incentive properties as $\boldsymbol{\pi}^1$ and provides $\bar{U}$ to the agent. By Jensen's inequality, it provides the principal with at least the same level of utility. Strict preference follows if $b(y^1|y^2) \in (0,1)$ for some y_1^1, y_2^1 such that $\boldsymbol{\pi}^1(y_1^1) \neq \boldsymbol{\pi}^1(y_2^1)$. Alternatively, if y^1 is a function of y^2 (i.e., η^1 is a collapsing of η^2) and y_1^2, y_2^2 are two signals such that $y^1(y_1^2) \neq y^1(y_2^2)$ and $\varphi(x|y_1^2,a) \neq \varphi(x|y_2^2,a)$ for some $x \in X$, then $\boldsymbol{\pi}^1(y^1)$ cannot satisfy the first-order conditions for both $y_1{}^2$ and y_2^2 (except in anomalous cases). **Q.E.D.**

The key here is that η provides a basis for insurance without raising any moral hazard problems (e.g., *hail insurance*).[22] There is no need here for the agent to be effort averse to obtain the above result.

Insurance/Incentive Informativeness

Now consider the case in which η is not a pure insurance reporting system (i.e., it is not exclusively Θ-informative). If x (i.e., θ) is revealed by y, then $\varphi(x|y,a) = 1$ if $x = \boldsymbol{x}(y,a)$, and the first-order condition becomes

$$M(\boldsymbol{x}(y) - \boldsymbol{\pi}(y)) = \lambda + \mu \frac{\varphi_a(y|a)}{\varphi(y|a)}.$$

Hence, if two systems both reveal x, then we can compare those systems on the basis of their relative A-informativeness, and we will get the same results as if a risk neutral principal "owns" the outcome. Therefore, we focus here on cases in which y does not fully reveal x. Of course, the system must reveal something about x, otherwise it has no value.

Definition *Insurance/Incentive Informativeness*

Performance measurement system η is *Xa-informative* (insurance/incentive informative) if $\varphi(x|y,a) \neq \varphi(x|a)$, for at least some $y \in Y$, and is *XA-informative* if it is *Xa*-informative for all $a \in A$.

η^2 is *at least as XA-informative* as η^1 if, and only if, there exists a Markov matrix **B** such that

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[22] Hail storms are a major risk for the crops on the prairies, but farmers can insure themselves against that risk by buying hail insurance. The contract is such that the farmer buys insurance for a nominal amount per acre, for example, \$1,000 per acre. In case of a hail storm, the contract is settled by paying the farmer the nominal amount per acre times the number of acres insured times the average percentage of the crop destroyed in those acres. A key feature of this contract is that the insurance payment is independent of the value of the crop and, hence, the payment is independent of the farmer's skills and effort.

$$\boldsymbol{\eta}^1 = \boldsymbol{\eta}^2\mathbf{B},$$

where $\boldsymbol{\eta} \equiv [\,\varphi(y|x,a,\eta)\,]_{(|X|\times|A|)\times|Y|}$ and $\mathbf{B} \equiv [\,b(y^1|y^2)\,]_{|Y^2|\times|Y^1|}$.

Proposition 18.15 (Gjesdal 1982, Prop. 2)
If the agent "owns" x and is strictly risk and effort averse, then η has positive value if it is XA-informative (and has zero value if it is not Xa-informative for any a). Furthermore, if η^2 is at least as XA-informative as η^1, then η^2 is at least as preferred as η^1, with strict preference if η^1 is not at least as XA-informative as η^2.

Proof: Set $\boldsymbol{\pi}(y^2)$ such that

$$\sum_{x\in X} u(x-\boldsymbol{\pi}(y^2))\,\varphi(x|y^2,a) = \sum_{y^1\in Y^1}\sum_{x\in X} u(x-\boldsymbol{\pi}^1(y^1))\,b(y^1|y^2)\,\varphi(x|y^2,a).$$

This new contract has the same incentive properties as $\boldsymbol{\pi}^1$ and provides $\bar{U}$ to the agent. By Jensen's inequality, it provides the principal with at least the same level of utility.

Strict preference follows if $b(y^1|y^2) \in (0,1)$ for some y_1^1, y_2^1 such that $\boldsymbol{\pi}^1(y_1^1) \neq \boldsymbol{\pi}^1(y_2^1)$. Alternatively, if y^1 is a function of y^2 and y_1^2, y_2^2 are two signals such that $y^1(y_1^2) \neq y^1(y_2^2)$ and $\varphi(x|y_1^2,a) \neq \varphi(x|y_2^2,a)$ for some $x \in X$, then $\boldsymbol{\pi}^1(y^1)$ cannot satisfy the first-order conditions for both y_1^2 and y_2^2 (except in anomalous cases).
Q.E.D.

Observe that informativeness about the outcome is crucial, because the primary purpose of the contract is to reduce the risk that must be borne by the agent. However, if a signal used for risk sharing is influenced by the agent's action, then a comparison of one signal to another must simultaneously include both X- and A-informativeness.

18.3 RISK AVERSE PRINCIPAL "OWNS" THE OUTCOME

If the principal is risk averse and "owns" the outcome x, but there is no report of x that can be used in contracting with the agent, then the situation is very similar to the preceding case. In particular, there is both an insurance and an incentive demand for information. However, in this case the system is valuable if it is A-informative even if it is not Θ-informative (the principal wants to motivate the agent's action choice even if he cannot share his risk with the agent). The similarity to the preceding case follows from the fact that both the insurance and incentive properties of the reports are relevant when comparing systems.[23]

Proposition 18.16

If a risk averse principal "owns" x, there is no verified report of x, and the agent is risk and effort averse, then performance measurement system η has positive value if it is either A-informative or X-informative. Furthermore, if η^2 is at least as XA-informative as η^1, then it is at least as preferred, with strict preference, if η^1 is not at least as XA-informative as η^2.

18.3.1 Economy-wide and Firm-specific Risks[24]

We now consider a setting in which the production technology is "owned" by a "principal" who is a partnership of well-diversified shareholders in an economy where there are both economy-wide risk and firm-specific risk (see Section 5.4.2). It follows from the analysis in Section 5.4.2 that the principal's (i.e., the shareholders') preferences can be represented as if he is risk neutral with respect to the diversifiable firm-specific risk, whereas he is risk averse with respect to economy-wide risk.

We assume the economy-wide event $\theta_e \in \Theta_e$ is contractible information and is not influenced by the agent's action, i.e., $\varphi(\theta_e|a) = \varphi(\theta_e)$. The outcome relevant firm-specific events are not contractible, but the contractible performance measure y is influenced by both the firm-specific and economy-wide events as well as the agent's action, as represented by the joint conditional probability function $\varphi(y,x|a,\theta_e)$. If we ignore the possibility of a lower bound on compensation, the compensation contract is a function $\boldsymbol{c}$: $Y \times \Theta_e \rightarrow \mathbb{R}$, where both Y and Θ_e are assumed to be finite sets.

The objective of the principal is to maximize the market value of the firm net of compensation to the manager (agent). If the capital market is "effectively complete" with respect to the economy-wide events, there exists a unique risk-adjusted probability function for the economy-wide events $\hat{\varphi}(\theta)$ such that the market value of the firm is given by (see Section 5.4.2):

$$U^p(\boldsymbol{c},a,\eta) = \sum_{\theta_e \in \Theta_e} \sum_{y \in Y} \sum_{x \in X} [x - \boldsymbol{c}(y,\theta_e)]\,\varphi(y,x|a,\theta_e)\hat{\varphi}(\theta_e).$$

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[23] The proof is basically the same as for Proposition 18.15.

[24] Ideally, the reader will have studied Volume I, Chapter 5 (or will have studied finance theory that deals with efficient risk sharing when there is diversifiable and non-diversifiable risk) before studying this section. That background would help you understand the assumptions made in this section. However, the material in this section can be read without that background.

That is, the market value is the risk-adjusted expected value of the conditional expected residual payoff to the shareholders given the economy-wide event. The key here is that the risk adjustment of the probability function only pertains to the economy-wide events – the firm-specific risk can be diversified and, therefore, well-diversified shareholders do not require a risk premium for taking on that type of risk. We assume the capital market is large and competitive such that the agent's action has no impact on the risk-adjusted probabilities.

In a market setting the agent may also be able to trade. We assume that he is not able to trade in claims for his own firm. This would enable the manager to (partly) undo the firm-specific risk in his compensation and, thus, be detrimental to incentives provided through his compensation. Of course, if the firm-specific events are publicly observable and the agent can trade in a complete set of firm-specific and economy-wide event claims, the first-best solution can be obtained by selling the firm to the manager and let him insure his risk through trading in the capital market. However, the capital market is typically incomplete with respect to firm-specific claims.

On the other hand, it is unreasonable to assume that the agent cannot trade in diversified portfolios. Consequently, we assume that the agent can trade in a complete set of event claims for the economy-wide events. Hence, when designing the optimal compensation contract, the principal must consider both the agent's action choices and his trading in economy-wide event claims. The payoff from the portfolio acquired by the agent is denoted $w = \boldsymbol{w}(\theta_e)$. We assume that the agent has no initial wealth so that the agent's portfolio problem given the compensation scheme $\boldsymbol{c}$ and action a can be formulated as

$$\underset{\boldsymbol{w}(\theta_e)}{\text{maximize}} \quad U^a(\boldsymbol{c}+\boldsymbol{w}, a, \eta) = \sum_{\theta_e \in \Theta_e} \sum_{y \in Y} u^a(\boldsymbol{c}(y,\theta_e)+\boldsymbol{w}(\theta_e), a)\,\varphi(y|a,\theta_e)\,\varphi(\theta_e),$$

$$\text{subject to} \quad \sum_{\theta_e \in \Theta_e} \sum_{y \in Y} [\boldsymbol{c}(y,\theta_e)+\boldsymbol{w}(\theta_e)]\,\varphi(y|a,\theta_e)\,\hat{\varphi}(\theta_e)$$

$$= \sum_{\theta_e \in \Theta} \sum_{y \in Y} \boldsymbol{c}(y,\theta_e)\,\varphi(y|a,\theta_e)\,\hat{\varphi}(\theta_e).$$

The first-order condition for the agent's position in the event claim for economy-wide event θ_e is given by

$$U_w^a(\boldsymbol{c}+\boldsymbol{w}, a|\theta_e,\eta)\,\varphi(\theta_e) - \gamma\hat{\varphi}(\theta_e) = 0, \quad \forall\theta_e,$$

where γ is the Lagrange multiplier for the agent's budget constraint and

$$U_w^a(\boldsymbol{c}+\boldsymbol{w},a|\theta_e,\eta) \equiv \sum_{y \in Y} u_w^a(\boldsymbol{c}(y,\theta_e)+\boldsymbol{w}(\theta_e),a)\,\varphi(y|a,\theta_e), \quad \forall \theta_e.$$

If there is no firm-specific risk, so that the outcome x can be written as a function of the agent's action and the economy-wide event θ_e, i.e., $x = \boldsymbol{x}(a,\theta_e)$, the first-best solution can be obtained by selling the firm to the agent.[25] In that case, the agent obtains $\boldsymbol{x}(a,\theta_e) - V^*$, where V^* is the first-best market value of the firm. It then follows from the agent's first-order condition for his portfolio choice that the agent's optimal portfolio of economy-wide event claims will be such that

$$u_w^a(\boldsymbol{x}(a,\theta_e) - V^* + \boldsymbol{w}(\theta_e),a)\,\varphi(\theta_e) = \gamma\hat{\varphi}(\theta_e), \quad \forall \theta_e.$$

Consequently, for an optimal portfolio the agent's marginal utility of consumption is proportional to the risk-adjusted probabilities for the economy-wide events. That is, the sharing of the economy-wide risk is efficient and the agent's action choice is first-best since the agent bears all the costs and benefits of his action. Therefore, *there must be firm-specific risk for an incentive problem to exist*!

Suppose $(\boldsymbol{c},\boldsymbol{w},a)$ is an optimal contract. Now consider the compensation contract $\boldsymbol{c}^\dagger$ defined by

$$\boldsymbol{c}^\dagger(y,\theta_e) = \boldsymbol{c}(y,\theta_e) + \boldsymbol{w}(\theta_e).$$

This contract gives the agent the same consumption possibilities in all contingencies (y,θ_e) and, therefore, leaves the agent's action incentives and expected utility unchanged compared to $(\boldsymbol{c},\boldsymbol{w})$. Moreover, it follows from the agent's budget constraint that $\boldsymbol{c}$ and $\boldsymbol{c}^\dagger$ are equally costly to the principal. However, since $\boldsymbol{w}$ solves the agent's portfolio problem given the compensation scheme $\boldsymbol{c}$, the agent's optimal portfolio choice with $\boldsymbol{c}^\dagger$ is not to trade in any of the event claims. Hence, *we can assume without loss of generality that the principal chooses among compensation contracts for which the agent has no incentive to trade.* Note that this does not imply that the agent's portfolio choice is a nonbinding incentive constraint.

We can now formulate the principal's decision problem for inducing a particular action a as follows, assuming the first-order conditions are sufficient conditions for the agent's incentive constraints.

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[25] Alternatively, in this case, the first-best solution might also be achievable with a penalty contract that severely punishes the agent if the outcome reveals that he has not taken the first-best action.

$$\hat{c}^\dagger(a,\eta) = \underset{c}{\text{minimize}} \sum_{\theta_e \in \Theta_e} \sum_{y \in Y} c(y,\theta_e)\varphi(y|a,\theta_e)\hat{\varphi}(\theta_e), \tag{18.1″}$$

$$\text{subject to } U^a(\boldsymbol{c},a,\eta) \geq \bar{U}, \tag{18.2″}$$

$$U^a_w(\boldsymbol{c},a,\eta|\theta_e)\varphi(\theta_e) - \gamma\hat{\varphi}(\theta_e) = 0, \ \forall\, \theta_e \in \Theta_e, \tag{18.3p″}$$

$$U^a_a(\boldsymbol{c},a,\eta) = 0. \tag{18.3c″}$$

where $\hat{c}^\dagger(a,\eta)$ is the market value of the market value minimizing compensation contract that implements a given performance measurement system η.

There are two main differences between this decision problem and those considered earlier with a risk neutral principal. Firstly, the principal and the agent use different probabilities for the economy-wide events. The principal uses the risk-adjusted probabilities for the economy-wide events reflecting the risk premiums attached to those events. The agent uses the unadjusted probabilities, since his marginal utility of consumption is affected by the firm-specific risk and is, therefore, not proportional to the risk-adjusted probabilities. Secondly, there is an additional incentive constraint for the agent's portfolio choice. This may be a binding constraint, since the agent has the possibility of mitigating the impact of the economy-wide events on his compensation through his portfolio choice of economy-wide event claims.

Assuming the agent has a separable utility function, the first-order condition for an optimal compensation contract is given by[26]

$$M(\boldsymbol{c}(y,\theta_e)) = k(a)\frac{\varphi(\theta_e)}{\hat{\varphi}(\theta_e)}\left[\lambda + \delta(\theta_e)\frac{u''(\boldsymbol{c}(y,\theta_e))}{u'(\boldsymbol{c}(y,\theta_e))} + \mu\left(\frac{k'(a)}{k(a)} + \frac{\varphi_a(y|a,\theta_e)}{\varphi(y|a,\theta_e)}\right)\right], \tag{18.19}$$

where λ, $\delta(\theta_e)$, and μ are the Lagrange multipliers for the corresponding constraints in the principal's decision problem. The impact of the agent's no-trading constraint appears as a term related to his risk aversion, whereas the impact of the differences in the beliefs for the economy-wide events enters as a simple multiple of the ratio between the agent's probability and the investors' risk-adjusted probability (i.e., the inverse of the valuation index for the econ-

[26] Note that $\varphi(\theta_e)$ and $\hat{\varphi}(\theta)$ have the same support (see Chapter 5).

omy-wide events). When the risk-adjusted probability is relatively low, i.e., aggregate consumption is relatively high, the agent receives a relatively high compensation, and vice versa. That is, the principal sets the compensation such that it is positively "correlated" with aggregate consumption. This occurs for two related reasons. Firstly, the market value of a compensation contract is lower, the more positively correlated it is with aggregate consumption, *ceteris paribus*. Secondly, since the agent can trade in economy-wide event claims and the compensation contract must be such that he has no incentive to trade, he must have a relatively low conditional expected marginal utility for the economy-wide events for which the event prices are relatively low.

In order to disentangle these two effects and to abstract from the effects of variations in the agent's risk aversion, we assume that the agent has an exponential utility function which is either additively or multiplicatively separable, i.e.,

$$u^a(c,a) = -\exp[-r(c-\kappa(a))] - v(a), \text{ so that } k(a) = \exp[r\kappa(a)]$$

with *multiplicatively separable:* $\kappa', \kappa'' > 0$ and $v(a) = 0$,

additively separable: $\kappa(a) = 0$ and $v' > 0,\ v'' > 0$.

This implies that

$$M(c) = r^{-1}\exp[rc], \quad \frac{u''(c)}{u'(c)} = -r, \quad \text{and} \quad \frac{k'(a)}{k(a)} = r\kappa'(a).$$

Hence, by taking logs of both sides of (18.19) and rearranging terms, the first-order condition becomes

$$c(y,\theta_e) = \frac{1}{r}\left[\ln\left(\frac{\varphi(\theta_e)}{\hat{\varphi}(\theta_e)}\right) + \ln\left(\bar{\lambda}(a,\theta_e) + \mu\frac{\varphi_a(y|a,\theta_e)}{\varphi(y|a,\theta_e)}\right) + \bar{\kappa}(a)\right], \quad (18.20)$$

where $\bar{\lambda}(a,\theta_e) \equiv \lambda - r\delta(\theta_e) + r\mu\kappa'(a); \quad \bar{\kappa}(a) \equiv \ln(r) + r\kappa(a).$

Proposition 18.17

Suppose the agent has either an additively or multiplicatively separable exponential utility function. If y and θ_e are independent, i.e., $\varphi(y|a,\theta_e) = \varphi(y|a)$, then

(a) the agent's no-trading constraint (18.3p″) is not binding, and

(b) the agent's compensation is additively separable in y and θ_e.

Proof: To show (a) suppose $(\mathbf{c}, a)$ is an optimal contract for the principal's decision problem in which the agent cannot trade in economy-wide event claims. The optimal compensation contract is determined by an equation similar to (18.20) except that the $\delta(\theta_e)$-term is fixed at zero so that $\underline{\lambda}(a,\theta_e)$ does not depend on θ_e. We now show that this contract leaves the agent with no incentive to trade, i.e., there exists a Lagrange-multiplier γ independent of the economy-wide event such that the agent's no-trade constraint (18.3p″) is satisfied. Inserting the structure of the optimal contract given by (18.20) using the assumption that y and θ_e are independent, we get that

$$U_w^a(\mathbf{c}, a, \eta \mid \theta_e) = \sum_{y \in Y} r \frac{\hat{\varphi}(\theta_e)}{\varphi(\theta_e)} \left(\bar{\lambda}(a) + \mu \frac{\varphi_a(y \mid a)}{\varphi(y \mid a)} \right)^{-1} \exp(-\bar{\kappa}(a) + r\kappa(a))\, \varphi(y \mid a)$$

$$= \frac{\hat{\varphi}(\theta_e)}{\varphi(\theta_e)} \sum_{y \in Y} \left(\bar{\lambda}(a) + \mu \frac{\varphi_a(y \mid a)}{\varphi(y \mid a)} \right)^{-1} \varphi(y \mid a).$$

Hence, defining γ by

$$\gamma \equiv \sum_{y \in Y} \left(\bar{\lambda}(a) + \mu \frac{\varphi_a(y \mid a)}{\varphi(y \mid a)} \right)^{-1} \varphi(y \mid a),$$

shows that the agent has no incentive to trade. Since the principal can do at least as well with the imposition of a no-trading constraint as with permitting agent trading, and $(\mathbf{c}, a)$ is feasible with agent trading, $(\mathbf{c}, a)$ is also optimal with trading permitted.

(b) follows immediately from (18.20), given independence and (a).

Q.E.D.

The proposition demonstrates that if the economy-wide event is not informative about either the agent's action or the agent's conditional marginal utility of consumption given θ_e, the variation in the agent's compensation due to the economy-wide events is solely derived from an efficient risk sharing of the economy-wide risk between the principal and the agent. That is, the sharing of the economy-wide risk and the provision of incentives through the firm-specific risk are separable. In order to minimize the market value of the compensation contract the principal chooses the compensation so that it is highly correlated with aggregate consumption. If the agent cannot trade, he requires a risk premium for taking on that type of risk. That tradeoff is precisely such that the marginal rates of substitutions for the economy-wide events are equated for the well-diversified shareholders and the agent so that the agent has no incentive to do any additional trading in economy-wide event claims.

At first glance it may seem surprising that the no-trading constraint is not binding when the agent has additively separable exponential utility since this utility function exhibits wealth effects. However, recall that the agent has only one consumption date. Hence, his trading only affects the variation in his consumption across the economy-wide events and not the level of consumption. If the agent has an initial consumption date (at the contracting date) so that he can shift the level of consumption between consumption dates, the no-trading constraint will be binding for the additively separable exponential utility function. In that case there will be a tension between the intertemporal allocation of consumption and optimal incentives (which we explore in Chapter 24). However, the no-trading constraint will still be non-binding for the multiplicatively separable exponential utility function with multiple consumption dates since the level of consumption has no impact on action choices for this utility function.

In general, we expect the performance measure y to be correlated with the economy-wide event, and to be influenced by the agent's action. Consequently, the economy-wide event is expected to be insurance informative. For example, knowledge of the economy-wide event can be helpful in making inferences about whether a good outcome is due to the agent working hard or to favorable market conditions. In such cases, there will be tension between the sharing of economy-wide risk, the agent's trading, and optimal incentives. Intuitively, if the agent can trade in economy-wide event claims, the principal cannot as efficiently allocate incentive bonuses and penalties across the economy-wide events as would be possible if the agent could not trade in these claims. When the agent can trade in these claims, he will have an incentive to "insure" (i.e., "smooth") these bonuses and penalties through his trading. We illustrate this in the following section using the hurdle model.

18.3.2 Hurdle Model with Economy-wide and Firm-specific Risks

The hurdle model provides a simple setting in which we can illustrate the impact of economy-wide risk and agent trading of economy-wide event claims. Recall from Section 17.3.2 the action space is continuous with $a \in [0,1]$ and there are two outcomes $x_g > x_b$. We now introduce two economy-wide events, $\Theta_e \in \{\theta_g, \theta_b\}$, which we refer to as the good and the bad events, respectively. The hurdle h is a firm-specific event which is independently uniformly distributed over the interval $[0,1]$. If the agent clears the hurdle, i.e., $a \geq h$, *and* the good event obtains, then the good outcome x_g occurs. Otherwise, the bad outcome x_b occurs. Hence, the probability of the good outcome given a is $\varphi(x_g|a) = a\varphi(\theta_g)$, whereas the probability of the bad outcome given a is $\varphi(x_b|a) = (1 - a)\varphi(\theta_g) + \varphi(\theta_b)$.

The good event is associated with "large" aggregate consumption compared to the bad event. Hence, the risk-adjusted probability for the good event is less than or equal to the original probability for the good event, i.e., $\hat{\varphi}(\theta_g) \le \varphi(\theta_g)$, and vice versa for the bad event.

In the following we assume that the agent has an additively separable exponential utility function and consider the optimal contract for inducing $a = ½$. We use the following data:

$$u(c) = -\exp[-c]; \quad v(a) = .1a/(1-a); \quad \bar{U} = -1;$$

$$\varphi(\theta_g) = \varphi(\theta_b) = ½.$$

Risk Neutral Shareholders and No Agent Trading

Note that the outcome is only informative about the agent's action in the good event – the bad outcome obtains with certainty in the bad event. In order to illustrate the impact of the differences in information content for the two events, we assume initially that the shareholders are risk neutral so that the risk-adjusted probabilities are equal to the original probabilities for the two economy-wide events. Furthermore, the agent is exogenously precluded from trading. The optimal contract is shown in Table 18.2 along with the agent's expected marginal utilities conditional on the economy-wide events.

$\bar{c}^\dagger(a)$	$c(x_g,\theta_g)$	$c(x_b,\theta_g)$	$c(x_b,\theta_b)$
0.155	0.542	−0.323	0.200
$U_c^a(c,a\|\theta_i)$	0.982		0.818

Table 18.2: Optimal contract for inducing $a = ½$ with risk neutral shareholders and agent exogenously precluded from trading.

We can view compensation as imposing two types of risk on the agent: outcome risk and event risk. In this example, the outcome risk only occurs if the good event occurs, and is required to induce the agent to select $a = ½$. Event risk is imposed if the agent's expected marginal utility in the good event differs from his expected marginal utility in the bad event. Since the shareholders are risk neutral, there are no risk sharing reasons for imposing event risk. However, Table 18.2 reveals that event risk is imposed. The reason for this is that with additive utility, the outcome risk premium required to induce a given action a can be reduced if the compensation in the good event is reduced (see Appendix

17C). Of course, this reduction must be offset by an increased compensation in the bad event (so the participation constraint is satisfied), which creates event risk for the agent. The greater the reduction in the good event compensation, the lower is the outcome risk premium, but the greater is the event risk premium. The contract in Table 18.2 makes an optimal tradeoff between these two types of risk premia.[27] If event claims are available, the compensation contract is such that the agent has an incentive to buy claims for the good event and sell claims for the bad event (since the conditional expected marginal utility is higher in the good event than in the bad event).[28]

Risk Averse Shareholders and No Agent Trading

Now consider the setting in which the shareholders are risk averse and, therefore, require a risk premium for taking economy-wide risk. This is depicted as the risk-adjusted probability for the good event being less than the original probability for that event. For the purpose of our numerical example we set $\hat{\varphi}(\theta_g) = .4$ $(< \varphi(\theta_g) = ½)$. Suppose again that the agent is exogenously precluded from trading. Table 18.3 shows the optimal contract along with the agent's expected marginal utilities conditional on the economy-wide events times the ratio of the original and risk-adjusted event probabilities.

$\hat{c}^\dagger(a)\,(\bar{c}^\dagger(a))$	$c(x_g,\theta_g)$	$c(x_b,\theta_g)$	$c(x_b,\theta_b)$
0.146 (0.173)	0.832	- 0.211	0.036
$U_c^a(c,a\,\vert\,\theta_i) \times \varphi(\theta_i)/\hat{\varphi}(\theta_i)$	$0.835 \times .5/.4 = 1.044$		$0.965 \times .5/.6 = 0.804$

Table 18.3: Optimal contract for inducing $a = ½$ with risk averse shareholders ($\hat{\varphi}(\theta_g) = .4$) and agent exogenously precluded from trading.

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[27] Chapter 25 uses a similar argument in an intertemporal setting where utility levels are shifted between multiple consumption dates.

[28] It can be shown that if the agent's action does not affect the probabilities for the outcome in the bad event, the following relation holds between the optimal compensations in the good and the bad event (see Christensen and Frimor, 1998),

$$M(c(x_b,\theta_b)) = \mathrm{E}[M(c)\,|\,a,\theta_g].$$

Since $M(\cdot) = 1/u'(\cdot)$ and $1/u'$ is a convex function, Jensen's inequality implies that

$$U_c^a(c,a\,|\,\theta_b) < U_c^a(c,a\,|\,\theta_g),$$

so that the agent has incentive to buy claims for the good event in return for selling claims for the bad event.

Note that the agent is paid more in the good event and less in the bad event compared to the setting in which there is no risk adjustment of the probabilities for the economy-wide events (see equations (18.19) and (18.20)). This is a consequence of the fact that paying compensation in the good event has a lower market value than paying the same amount in the bad event – the expected compensation (0.173) is higher, but the market value of the compensation contract in Table 18.3 (0.146) is lower than that of the compensation contract in Table 18.2 (0.164). However, there is a tradeoff between shifting compensation (and, thus, utilities) from the "expensive" bad event to the "less costly" good event and a higher risk premium paid in the good event to induce the agent to jump. This tradeoff is such that the agent's marginal utility conditional on the events is now lower in the good event than in the bad event. However, the agent still has an incentive to buy claims for the good event and sell claims for the bad event since the claim for the good event is relatively cheap, i.e. the conditional expected marginal utility times the ratio of the original and risk-adjusted probabilities is higher for the good than for the bad event.[29]

Risk Averse Shareholders and Agent Trading

Now consider the setting in which the shareholders are risk averse and the agent can trade in claims for the two economy-wide events. Without loss of generality the optimal contract is determined such that the agent has no incentive to trade, i.e., subject to the constraint (18.3p″). Table 18.4 shows the optimal contract along with the agent's expected marginal utilities conditional on the economy-wide events times the ratio of original and risk-adjusted event probabilities.

In order to eliminate the agent's incentive to trade, the agent's conditional expected marginal utility must be reduced for the good event and increased for the bad event compared to the contract in Table 18.4. This is achieved by paying the agent less in the bad event, and more in the good event but with a higher variability (to maintain inducement of $a = ½$). This tends to further increase the

[29] As in the risk neutral shareholder setting (see footnote 28), it can now be shown that

$$M(\mathbf{c}(x_b,\theta_b)) \times \hat{\varphi}(\theta_b)/\varphi(\theta_b) = \mathrm{E}[M(c)|a,\theta_g] \times \hat{\varphi}(\theta_g)/\varphi(\theta_g).$$

Jensen's inequality now implies that

$$U_c^a(\mathbf{c},a|\theta_b) \times \varphi(\theta_b)/\hat{\varphi}(\theta_b) < U_c^a(\mathbf{c},a|\theta_g) \times \varphi(\theta_g)/\hat{\varphi}(\theta_g),$$

so that the agent has incentive to buy (sell) claims for the good (bad) event. More generally, the optimal contract is such that the agent has incentives to buy claims for the event in which the outcome is most informative about the agent's action, and sell claims for the other event.

outcome risk premium paid in the good event to induce the agent to jump.[30] Hence, the agent's trading opportunities make it more costly to induce him to jump because of the higher risk premium he must be paid in the good event (where his utility level is relatively higher than without trading opportunities).

$\hat{c}^\dagger(a)\,(\bar{c}^\dagger(a))$	$c(x_g,\theta_g)$	$c(x_b,\theta_g)$	$c(x_b,\theta_b)$
0.159 (0.218)	1.139	-0.113	-0.077
$U_c^a(c,a\|\theta_i)\times\varphi(\theta_i)/\hat{\varphi}(\theta_i)$	$0.720\times.5/.4 = 0.900$		$1.080\times.5/.6 = 0.900$

Table 18.4: Optimal contract for inducing $a = ½$ with risk averse shareholders ($\hat{\varphi}(\theta_g) = .4$) and agent trading.

18.4 COSTLY CONDITIONAL ACQUISITION OF INFORMATION

The preceding analyses have focused on determining the value and use of alternative information with no explicit consideration of information system costs. The separate examination of the value of information has a long tradition in the information economics literature. The basic perspective is that information will not be acquired unless it is valuable, so identifying information characteristics that give it value is the first step in any analysis. Second, it is useful to assess the relative value of information systems so that we know whether one system *might* be more desirable than another from a value perspective. However, the final choice of the system must ultimately be based on a comparison of the alternative systems' relative values and costs.

In this section we explicitly consider a setting in which the acquisition of *additional information* is *conditional* on the primary signal that is reported. In this setting, the cost of acquiring the additional information plays a central role because the decision as to which primary signals induce the acquisition of additional information is based on a comparison of the acquisition cost versus the expected benefit from that acquisition for each possible primary signal. Several papers have examined this issue, e.g., Baiman and Demski (1980a,b), Lambert (1985), and Young (1986). The following analysis is based primarily on the paper by Lambert. In most papers the primary signal is the outcome x, but we adopt a more general approach and treat the former as a special case. The prin-

[30] This is reflected by the fact that the optimal Lagrange multiplier for the agent's action constraint is $\mu = 0.446$ for the contract with agent trading in Table 18.4 as opposed to $\mu = 0.298$ for the contract without agent trading in Table 18.3.

cipal is assumed to be risk neutral so the key aspect of the primary signal is its informativeness about the agent's action.

Basic Model Elements

We return to a setting in which a risk neutral principal "owns" the outcome $x \in X$, which is the result of the action $a \in A$ implemented by a risk and effort averse agent with an additively separable utility function $u^a(c,a) = u(c) - v(a)$, with $u' > 0$, $u'' < 0$, and $v' > 0$.

There are two possible signals, $y_1 \in Y_1$ and $y_2 \in Y_2$. The primary signal y_1 is always reported, whereas the secondary signal y_2 is only reported if the principal pays a cost κ. The decision to incur the cost κ may be contingent on the observed primary signal y_1.

$\varphi(y_1,y_2|a)$ is the joint probability of the two signals given action a, and $\varphi(y_i|a)$ is the marginal probability of signal y_i, $i = 1, 2$. The principal's posterior belief about y_2 given y_1 and a is given by $\varphi(y_2|a,y_1) = \varphi(y_1,y_2|a)/\varphi(y_1|a)$.[31]

The principal's investigation strategy is denoted $\boldsymbol{\alpha}$: $Y_1 \to [0,1]$, where $\boldsymbol{\alpha}(y_1)$ is the probability that the principal "investigates," i.e., he pays to have y_2 reported, given the primary signal y_1. It is important to observe that the analysis assumes that the principal *commits to a particular investigation strategy* $\boldsymbol{\alpha}$ at the time he contracts with the agent. That is, the investigation is based on a fixed investigation rule, and is not based on an *ex post* decision by the principal. Otherwise, it would be rational for the principal, *ex post*, not to investigate after the agent has taken his action (under the assumption that the investigation strategy $\boldsymbol{\alpha}$ will be implemented).

We represent the compensation contract as consisting of two components: $\boldsymbol{c}(y_1,y_2) \equiv \boldsymbol{c}(y_1) + \boldsymbol{\delta}(y_1,y_2)$, where

$\boldsymbol{c}$: $Y_1 \to [0,\infty)$ specifies the basic amount that is paid if only the primary signal is reported, i.e., there is no investigation.

$\boldsymbol{\delta}$: $Y_1 \times Y_2 \to [-\boldsymbol{c}(y_1),\infty)$ specifies the "bonus" (possibly negative) that is paid if both y_1 and y_2 are reported, i.e., if y_1 is reported and an investigation to determine y_2 is made.

Principal's Decision Problem

We again focus on the first stage of the GH approach and determine the least cost strategy and compensation contract for inducing the agent to take an arbitrary action a (which is not his least cost action). The principal's first-stage

[31] Baiman and Demski (1980a,b) assume independence, i.e., $\varphi(y_1,y_2|a) = \varphi(y_1|a)\varphi(y_2|a)$, but Lambert (1985) permits the two signals to be correlated.

decision problem is as follows, assuming that A is convex and the incentive constraint can be replaced by the agent's first-order condition:[32]

$$\bar{\boldsymbol{c}}^{\dagger}(a) = \underset{\boldsymbol{\alpha},\boldsymbol{c},\boldsymbol{\delta}}{\text{minimize}}\ \bar{\boldsymbol{c}}(a,\boldsymbol{\alpha},\boldsymbol{c},\boldsymbol{\delta})$$

$$= \sum_{y_1 \in Y_1} \Big[(1 - \boldsymbol{\alpha}(y_1))\, \boldsymbol{c}(y_1)$$

$$+ \boldsymbol{\alpha}(y_1) \sum_{y_2 \in Y_2} [\boldsymbol{c}(y_1) + \boldsymbol{\delta}(y_1,y_2) + \kappa]\, \varphi(y_2|a,y_1) \Big] \varphi(y_1|a),$$

subject to $U^a(a,\boldsymbol{\alpha},\boldsymbol{c},\boldsymbol{\delta}) \geq \bar{U},$

$$U^a_a(a,\boldsymbol{\alpha},\boldsymbol{c},\boldsymbol{\delta}) = 0,$$

$$\boldsymbol{c}(y_1) \geq 0, \quad \forall\, y_1 \in Y_1, \text{ if } \boldsymbol{\alpha}(y_1) < 1,$$

$$\boldsymbol{c}(y_1) + \boldsymbol{\delta}(y_1,y_2) \geq 0, \quad \forall\, (y_1,y_2) \in Y_1 \times Y_2, \text{ if } \boldsymbol{\alpha}(y_1) > 0,$$

$$\boldsymbol{\alpha}(y_1) \in [0,1], \quad \forall\, y_1 \in Y_1,$$

where $U^a(a,\boldsymbol{\alpha},\boldsymbol{c},\boldsymbol{\delta})$

$$= \sum_{y_1 \in Y_1} \Big[[1 - \boldsymbol{\alpha}(y_1)]\, u(\boldsymbol{c}(y_1))$$

$$+ \boldsymbol{\alpha}(y_1) \sum_{y_2 \in Y_2} u(\boldsymbol{c}(y_1) + \boldsymbol{\delta}(y_1,y_2))\, \varphi(y_2|y_1,a) \Big] \varphi(y_1|a) - v(a).$$

The Lagrangian (omitting constants and boundary conditions) for this decision problem is:

$$\mathcal{L} = \bar{\boldsymbol{c}}(a,\boldsymbol{\alpha},\boldsymbol{c},\boldsymbol{\delta}) - \lambda\, U^a(a,\boldsymbol{\alpha},\boldsymbol{c},\boldsymbol{\delta}) - \mu\, U^a_a(a,\boldsymbol{\alpha},\boldsymbol{c},\boldsymbol{\delta}).$$

The first-order conditions that characterize the two components of the optimal compensation contract (assuming an interior solution) are

$$\textit{no investigation:}\ M(\boldsymbol{c}(y_1)) = \lambda + \mu L(y_1|a),$$

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[32] See Jewitt (1988) for a discussion of conditions under which this can be done in the conditional investigation case considered by Baiman and Demski (1980).

$$\textit{investigation:} \qquad M(\boldsymbol{c}(y_1,y_2)) = \lambda + \mu L(y_1,y_2|a).$$

Observe that when there is investigation, the likelihood ratio can be written as

$$L(y_1,y_2|a) = \frac{\varphi_a(y_1,y_2|a)}{\varphi(y_1,y_2|a)} = \frac{\varphi_a(y_1|a)}{\varphi(y_1|a)} + \frac{\varphi_a(y_2|y_1,a)}{\varphi(y_2|y_1,a)} = L(y_1|a) + L(y_2|y_1,a).$$

Note also that $\mathrm{E}[L(y_2|y_1,a)|y_1] = 0$, which implies $\mathrm{E}[L(y_1,y_2|a)|y_1] = L(y_1|a)$ so that the likelihood ratio with investigation is a mean-preserving spread of the likelihood ratio without investigation. Lambert interprets this as implying that the additional incentive information provided by an investigation is not systematically favorable or unfavorable with respect to the agent's action.

Optimal Investigation Policy

In the principal's decision problem, the objective function and the participation and incentive constraints are all linear in $\boldsymbol{\alpha}(y_1)$, for each y_1. This implies that the probability of investigation will always be a *corner solution*, i.e., for each y_1 we have either $\boldsymbol{\alpha}(y_1) = 0$ or $\boldsymbol{\alpha}(y_1) = 1$.[33] Differentiating the Lagrangian for the principal's decision problem with respect to $\boldsymbol{\alpha}(y_1)$ yields:

$$-[B(y_1) - \kappa]\varphi(y_1|a),$$

where $B(y_1) = \boldsymbol{c}(y_1) - \sum_{y_2\in Y_2} \boldsymbol{c}(y_1,y_2)\varphi(y_2|y_1,a) - u(\boldsymbol{c}(y_1))\,[\lambda + \mu L(y_1|a)]$

$$+ \sum_{y_2\in Y_2} u(\boldsymbol{c}(y_1,y_2))\,[\lambda + \mu L(y_1,y_2|a)]\varphi(y_2|y_1,a).$$

The optimal investigation policy is determined by a trade-off between the cost and benefits of an investigation, so that α is either zero or one.

Proposition 18.18

The gross benefit of an investigation is positive for each y_1, i.e., $B(y_1) \geq 0$, and strictly positive if y_1 is not a sufficient statistic for (y_1,y_2) with respect to the agent's action. The optimal investigation policy is to investigate if, and only if, $B(y_1) > \kappa$.

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[33] This assumes that the optimal compensation contract is interior – otherwise, it may be optimal to use a randomized investigation strategy. In the subsequent analysis we assume the optimal compensation contract is interior.

Proof: It follows immediately from minimizing the Lagrangian that it is optimal to investigate with probability one if, and only if, $B(y_1) > \kappa$. Otherwise, no investigation is optimal.[34]

Next, show that $B(y_1) \geq 0$. Let $m(l) = \lambda + \mu l$ ($= 1/u'(c(m(l)))$) denote the likelihood measure, and let (as in the proof of Proposition 18.6) the function $f(\cdot)$ be defined by

$$f(l) = c(m(l)) - u(c(m(l)))\, m(l).$$

The gross benefits from an investigation can then be written as

$$B(y_1) = f(L(y_1|a)) - \sum_{y_2 \in Y_2} f(L(y_1,y_2|a))\, \varphi(y_2|a,y_1).$$

As is shown in the proof of Proposition 18.6, $f(\cdot)$ is a strictly concave function of l. Hence, Jensen's inequality and $\mathrm{E}[L(y_1,y_2|a)|y_1] = L(y_1|a)$ imply that $B(y_1) \geq 0$, with a strict inequality if $L(y_1,y_2|a)$ varies with y_2. **Q.E.D.**

Of course, if an investigation is costless (i.e., $\kappa = 0$), it is optimal to investigate for all signals y_1, since, at worst, the additional information in the secondary signal can be ignored. If y_1 is not a sufficient statistic for (y_1,y_2) with respect to the agent's action, there is a strict gain to an investigation. Hence, there is a non-trivial tradeoff between the gross benefits and the cost of an investigation. This tradeoff depends on the factors affecting the gross benefits and, of course, on the acquisition cost. These factors are the agent's utility function, the likelihood ratio for the primary signal, $L(y_1|a)$, and the informativeness of the secondary signal about the agent's action given y_1.

Informativeness of Secondary Signal Independent of Primary Signal

Initially, we consider the case in which the informativeness of the secondary signal about the agent's action is independent of y_1. Let $\Phi(l_2|a,y_1)$ denote the conditional distribution function for the likelihood ratio for the secondary signal, $l_2 \equiv L(y_2|a,y_1)$, given the primary signal.

Proposition 18.19

Assume the informativeness of the secondary signal about a is independent of the primary signal, i.e., $\Phi(l_2|a,y_1)$ is independent of y_1. The gross benefit of an investigation depends only on y_1 through $l_1 \equiv L(y_1|a)$, and it is de-

[34] Note, however, that the benefit function itself depends on the optimal investigation strategy through the impact of this strategy on the multipliers λ and μ. Hence, if there is some subset of primary signals for which $B(y_1) = \kappa$, the optimal investigation strategy may be a non-trivial randomized strategy with $\boldsymbol{\alpha}(y_1) \in (0, 1)$.

creasing (increasing) in l_1 if the optimal incentive contract with investigation is such that the agent's utility, $u \circ \boldsymbol{c} \circ m(\cdot)$, is a concave (convex) function of the likelihood ratio, and independent of l_1 if $u \circ \boldsymbol{c} \circ m(\cdot)$ is a linear function of the likelihood ratio.

Proof: When the conditional distribution of $l_2 \equiv L(y_2|a,y_1)$ is independent of y_1, the gross benefit of an investigation can be written as

$$B(y_1) = f(l_1) - \sum_{y_2 \in Y_2} f(l_1 + L(y_2|a,y_1))\, \varphi(y_2|a,y_1)$$

$$= f(l_1) - \sum_{l_2 \in L_2} f(l_1 + l_2)\, \varphi(l_2|a,y_1)$$

$$= f(l_1) - \sum_{l_2 \in L_2} f(l_1 + l_2)\, \varphi(l_2|a),$$

where $f(\cdot)$ is defined as in the proof of Proposition 18.18. Hence, the gross benefit of an investigation depends only on y_1 through l_1, and

$$B'(l_1) = f'(l_1) - \sum_{l_2 \in L_2} f'(l_1 + l_2)\, \varphi(l_2|a),$$

where
$$f'(l) = -\, u(\boldsymbol{c}(m(l)))\, \mu.$$

Since any μ satisfying the incentive compatibility constraint on the agent's action choice and the first-order condition for an optimal incentive contract is positive (see Proposition 17.8), the claim follows from using Jensen's inequality and the fact that $\mathrm{E}[l_1 + l_2|a] = l_1$. **Q.E.D.**

In this case the additional information provided by an investigation about the agent's action is independent of y_1. Hence, the benefit of an investigation is highest for those y_1 where the risk premium for imposing incentive risk on the agent is lowest. This risk premium depends on the utility function and the level of expected utility given y_1. If the agent's utility, $u \circ \boldsymbol{c} \circ m(\cdot)$, is a concave (convex) function of the likelihood ratio, this risk premium is increasing (decreasing) in the level of expected utility (see Proposition 17C.2).[35] Moreover, the

[35] Note that if the agent's utility is a concave function of l, an investigation is "bad news" for the agent, since the additional risk in the likelihood ratio, $L(y_2|a)$, caused by the investigation is a fair gamble. On the other hand, if the agent's utility is a convex function of l, an investigation is "good news" for the agent.

level of expected utility given y_1 is increasing in the likelihood ratio for the first signal, $L(y_1|a)$. Hence, the risk premium is lowest (highest) for small values of $L(y_1|a)$ if the agent's utility is a concave (convex) function of the likelihood ratio. However, if the agent's utility is a linear function of the likelihood ratio, the benefit of an investigation is independent of $L(y_1|a)$ (and, thus, also of y_1).

If MLRP holds for $L(y_1|a)$, then the above result can be applied directly to y_1. In this setting, we have "lower-tailed" investigation when $u \circ \mathbf{c} \circ m(\cdot)$ is concave in the likelihood ratio, and "upper-tailed" investigation when $u \circ \mathbf{c} \circ m(\cdot)$ is convex in the likelihood ratio.[36] If $u \circ \mathbf{c} \circ m(\cdot)$ is linear in the likelihood ratio, the optimal investigation policy is independent of the primary signal (i.e., only the total probability of investigation matters).[37]

Note that *the investigation region has nothing to do with whether the values of y_1 are unusual or not.* MLRP is merely a condition on the likelihood ratios. While the upper and lower tails represent unusual events for a normal distribution, one can construct distributions in which much of the mass is in one of the tails and yet the MLRP condition holds.

If the agent's utility function is a member of the HARA class, we can use Proposition 17C.1 to relate the benefits of an investigation to the agent's risk cautiousness.[38]

Corollary

If the agent's utility function for consumption is a member of the HARA class, then the gross benefit of an investigation is decreasing (increasing) in $L(y_1|a)$ if the agent's risk cautiousness is less (more) than 2, and independent of $L(y_1|a)$ if the agent's risk cautiousness is equal to 2 (i.e., square-root utility).

Informativeness of Secondary Signal Depends on Primary Signal

When the informativeness of y_2 depends on y_1, an optimal investigation is not only determined by the likelihood ratio for the primary signal as in the previous analysis, but also by how the informativeness of the investigation varies with the primary signal. Note that by Proposition 18.6, we can rank the informativeness of an investigation for different primary signals y_1 by a SSD relation between the conditional distributions for the likelihood ratio for the secondary sig-

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[36] Young (1986) considers two utility functions for which the agent's utility is a concave function of l for small l and a convex function of l for large l resulting in a "two-tailed" investigation policy.

[37] This can include null and full investigation, but also a randomized investigation strategy independent of the primary signal (see the hurdle model example below).

[38] Proposition 17C.1 is stated in terms of the likelihood measure $m(l) = \lambda + \mu\, l$ instead of directly in terms of the likelihood ratio l. However, note that $m(l)$ is linear such that $u \circ \mathbf{c} \circ m(l)$ is concave (convex) in l if, and only if, $u \circ \mathbf{c}(m)$ is concave (convex) in m.

nal given the primary signal. Of course, if the agent has square-root utility, the benefits of an investigation do not depend on the likelihood ratio $L(y_1|a)$ *per se*, but only on how the informativeness of y_2 about a given y_1 varies with y_1.[39] In general, the two effects interact and the optimal investigation policy is determined by the relative sizes of those effects. However, if the two effects go in the "same direction," lower- or upper-tailed investigation can be sustained as optimal investigation policies.

Proposition 18.20

Suppose MLRP holds for $L(y_1|a)$, and let $\Phi(l_2|a,l_1)$ denote the conditional distribution function for the likelihood ratio for the secondary signal, $l_2 = L(y_2|a,y_1)$, given the likelihood ratio for the first signal $l_1 = L(y_1|a)$.

(a) If $u \circ \boldsymbol{c} \circ m(\cdot)$ is a concave function of the likelihood ratio, and $\Phi(l_2|a,l_1'')$ SS-dominates $\Phi(l_2|a,l_1')$ for all $l_1' < l_1''$, lower-tailed investigation is optimal.

(b) If $u \circ \boldsymbol{c} \circ m(\cdot)$ is a convex function of the likelihood ratio, and $\Phi(l_2|a,l_1')$ SS-dominates $\Phi(l_2|a,l_1'')$ for all $l_1' < l_1''$, upper-tailed investigation is optimal.

(c) If $u \circ \boldsymbol{c} \circ m(\cdot)$ is a linear function of the likelihood ratio, and $\Phi(l_2|a,l_1'')$ SS-dominates $\Phi(l_2|a,l_1')$, the benefit of investigation is higher for l_1' than for l_1''.

Proof: We only show (a) since the proofs of (b) and (c) are similar. Since MLRP holds for $L(y_1|a)$ there is a one-to-one correspondence between y_1 and $l_1 = L(y_1|a)$ so we can write the benefits of an investigation as

$$B(l_1) = f(l_1) - \sum_{l_2} f(l_1 + l_2)\, \varphi(l_2|a,l_1),$$

where $f(\cdot)$ is defined as in the proof of Proposition 18.18. For $l_1' < l_1''$ we get

$$B(l_1') - B(l_1'')$$

$$= f(l_1') - f(l_1'') - \left[\sum_{l_2} f(l_1' + l_2)\, \varphi(l_2|a,l_1') - \sum_{l_2} f(l_1'' + l_2)\, \varphi(l_2|a,l_1'') \right]$$

[39] Note that in this setting, Proposition 18.8 implies that the benefits of an investigation is increasing in the conditional variance of the likelihood ratio for the second signal given y_1 (see also Lambert 1985, Prop. 3).

$$= \left[f(l_1') - \sum_{l_2} f(l_1' + l_2)\, \varphi(l_2|a,l_1') \right] - \left[f(l_1'') - \sum_{l_2} f(l_1'' + l_2)\, \varphi(l_2|a,l_1') \right]$$

$$+ \sum_{l_2} f(l_1'' + l_2)\, \varphi(l_2|a,l_1'') - \sum_{l_2} f(l_1'' + l_2)\, \varphi(l_2|a,l_1')$$

$$\geq \sum_{l_2} f(l_1'' + l_2)\, \varphi(l_2|a,l_1'') - \sum_{l_2} f(l_1'' + l_2)\, \varphi(l_2|a,l_1') \geq 0,$$

where the first inequality follows from Proposition 18.19, and the second inequality follows from $\Phi(l_2|a,l_1'')$ SS-dominating $\Phi(l_2|a,l_1')$ and the fact that $f(\cdot)$ is concave as shown in the proof of Proposition 18.6. **Q.E.D.**

A Hurdle Model Example

The hurdle model in Section 18.1.3 provides a setting in which we can illustrate the preceding results. The primary signal is the outcome $x \in \{x_g,x_b\}$ with $\varphi(x_g|a) = a$, and the secondary signal is one of two equally likely signals $y \in \{y_L,y_H\}$, with posterior distributions for the hurdle given by

$$\varphi(h|y) = \begin{cases} (1 + k) - 2kh & \text{if } y = y_L, \\ (1 - k) + 2kh & \text{if } y = y_H, \end{cases}$$

with the informativeness parameter $k \in [0,1]$. The likelihood ratio for the secondary signal given the primary signal is $L(y|x,a) = L(x,y|a) - L(x|a)$, and are shown in Figure 18.3 for $a = ½$.

Note that $L(y_H|x_g,a=½) = -L(y_L|x_b,a=½)$, $L(y_H|x_b,a=½) = -L(y_L|x_g,a=½)$, and $\varphi(y_H|x_g,a=½) = \varphi(y_L|x_b,a=½)$. Hence, the distribution of the likelihood ratio for the secondary signal given x_g, is the same as the distribution of minus the likelihood ratio for the secondary signal given x_b. This implies that the informativeness of the secondary signal is independent of the outcome x such that Proposition 18.19 applies.

In order to illustrate the results in Proposition 18.19 and its corollary we use a power utility_function, $u(c) = c^p$, with risk cautiousness equal to $1/(1-p)$. Furthermore, $\bar{U} = 2$, $v(a) = a/(1-a)$, and the informativeness parameter is $k = ½$. Table 18.5 summarizes the optimal investigation policies for varying values of the utility power p and the investigation cost κ.

Note that for $a = ½$, both outcomes are equally likely. Hence, consistent with the corollary to Proposition 18.19, the gross benefit from investigation is decreasing (increasing) in the outcome for $p = .45$ ($p = .55$), whereas it is the same for both outcomes in the square-root utility case. If the investigation cost is low ($\kappa = .15$), full investigation is optimal with $p = .45$, whereas upper-tail investigation is optimal with $p = .55$.

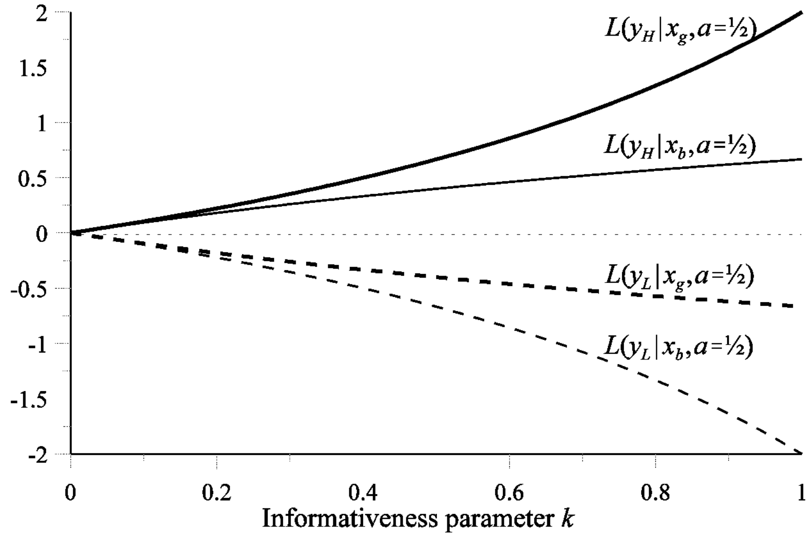

Figure 18.3: Likelihood ratios for secondary signal for a = ½ with varying informativeness parameter k.

$\bar{c}^\dagger(a,\alpha)$	$\alpha(x_g) = 0$ $\alpha(x_b) = 0$	$\alpha(x_g) = 0$ $\alpha(x_b) = 1$	$\alpha(x_g) = 1$ $\alpha(x_b) = 0$	$\alpha(x_g) = 1$ $\alpha(x_b) = 1$
p = 0.45, κ = 0.15	18.375	18.171	18.255	*18.069*
p = 0.45; κ = 0.25	18.375	*18.346*	18.430	18.419
p = 0.50; κ = 0.15	13.000	12.946	12.946	*12.900*
p = 0.50; κ = 0.25	13.000	*12.996*	*12.996*	13.000
p = 0.55; κ = 0.15	9.829	9.837	*9.815*	9.827
p = 0.55; κ = 0.25	*9.829*	9.887	9.865	9.927

Table 18.5: Minimum expected compensation cost, $\bar{c}^\dagger(a,\alpha)$, for inducing a = ½ for varying investigation policies, utility functions, and information costs.

On the other hand, if the investigation cost is high (κ = .25), lower-tail investigation is optimal with p = .45, and null investigation is optimal with p = .55. In the square-root utility case, full investigation is optimal for a low investigation cost, whereas for a high investigation cost lower- and upper-tail investigation dominates both null and full investigation. In the latter case, only the total probability of an investigation matters, i.e., $\alpha(x_g) + \alpha(x_b)$, and not how this total probability is split between the two outcomes.

In all the reported cases in Table 18.5, the optimal compensation is strictly positive for all signals and, thus, the contract is interior. Hence, the optimality of lower-tail versus upper-tail investigation depends exclusively on whether the agent's utility is a concave or convex function of the likelihood ratio for the primary signal. Table 18.6 summarizes the optimal investigation policy in a setting in which the non-negativity constraint on the agent's compensation is binding for the bad outcome (p = .45; κ = 1.25; $\bar{U}$ = 0; k = ½).

$\bar{c}^\dagger(a,\alpha)$	$\alpha(x_g)$	$\alpha(x_b)$	$c(x_g)$	$c(x_b)$	$c(x_g,y_H)$	$c(x_g,y_L)$
10.884	0.441	0.000	20.765	0.000	35.038	13.840

Table 18.6: Optimal contract with binding boundary conditions.

First, note that there is upper-tail investigation even though the risk cautiousness is less than 2. Of course, the reason is that investigation for the bad outcome cannot impose any additional penalties on the agent (but only reward the agent). Secondly, a non-trivial randomized investigation policy is used for the good outcome.

When the induced jump is a = ½, the informativeness of the secondary signal is independent of the primary signal, but that only holds for this particular induced action. Figure 18.4 shows the likelihood ratios for the secondary signal given the primary signal for a = ¼ and varying informativeness parameter k.

The distributions of the likelihood ratios for the two primary signals cannot be ranked on the basis of SSD. However, it appears that the likelihood ratio for y_H given the good outcome is significantly higher than minus the likelihood ratio for y_L given the bad outcome, so that there is more variation in the likelihood ratio for the good outcome than the bad outcome. Table 18.7 summarizes the optimal investigation policies for k = 1, κ = 0.15, and varying values of the utility power p.

Note that upper-tail investigation is optimal with square-root utility. In this setting, the cost of imposing additional risk on the agent is independent of the primary signal and, hence, the additional variation in the likelihood ratio for the good outcome compared to the bad outcome makes it optimal to investigate for the good outcome and not for the bad outcome. For a risk cautiousness moderately less than 2 (p = .45), upper-tail investigation still dominates lower-tail investigation even though the cost of imposing additional risk on the agent is higher for the good outcome than the bad outcome. If the risk cautiousness is decreased further (p = .40), the impact of the varying cost of imposing additional risk on the agent dominates the impact of the variation in the likelihood ratios such that lower-tail investigation dominates upper-tail investigation. When the risk cautiousness is higher than 2, the two effects both go in the direction of upper-tail investigation (even though no investigation is optimal when the agent has a sufficiently high risk cautiousness).

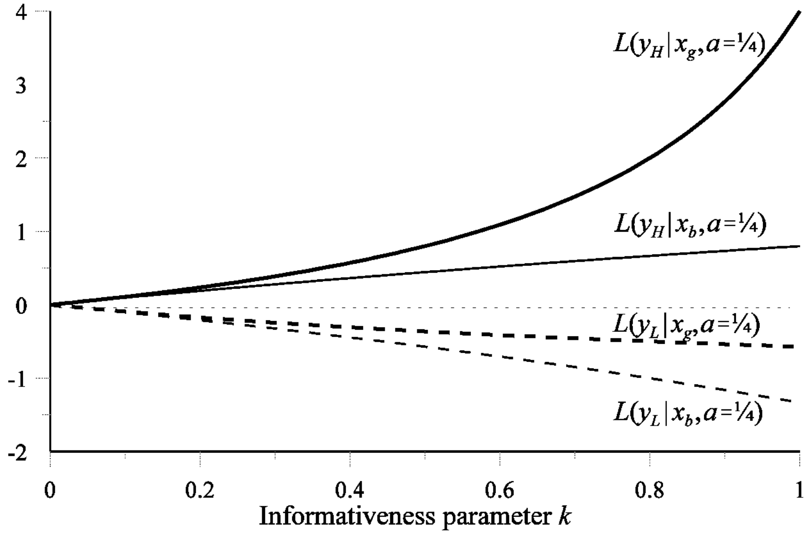

Figure 18.4: Likelihood ratios for secondary signal for $a = ¼$ with varying informativeness parameter k.

$\bar{c}^{\dagger}(a,\alpha)$	$\alpha(x_g) = 0$ $\alpha(x_b) = 0$	$\alpha(x_g) = 0$ $\alpha(x_b) = 1$	$\alpha(x_g) = 1$ $\alpha(x_b) = 0$	$\alpha(x_g) = 1$ $\alpha(x_b) = 1$
$p = 0.40$	10.114	9.947	9.999	*9.874*
$p = 0.45$	7.568	7.540	7.515	*7.510*
$p = 0.50$	6.037	6.072	*6.017*	6.066
$p = 0.55$	5.038	5.105	*5.037*	5.113
$p = 0.60$	*4.344*	4.429	4.356	4.446

Table 18.7: Minimum expected compensation cost, $\bar{c}^{\dagger}(a,\alpha)$, for inducing $a = ¼$ for varying investigation policies and utility functions.

18.5 CONCLUDING REMARKS

Chapter 17 began the analysis in this volume by examining the characteristics of optimal contracts in principal-agent settings in which there is one agent performing a single task in one period and the only contractible information is the principal's payoff from the agent's action. Chapter 18 has continued the focus on single-agent, single-task settings, but has explored the impact of alternative *ex post* performance measures. As we have seen, if the payoff is not contractible, then the characteristics that make a performance measure valuable depend upon whether the principal or the agent "owns" (consumes) the residual payoff, and whether they are risk averse or risk neutral.

Risk sharing issues arise if both the principal and agent are risk averse or if the agent "owns" the payoff and is risk averse. In those cases a performance measure can be valuable because it facilitates risk sharing, i.e., it is informative about (correlated with) the payoff. Risk sharing is of only secondary interest throughout much of this volume. Hence, to focus on incentive issues, most of the analysis in this volume assumes the principal owns the payoff and is risk neutral, while the agent is effort and risk averse. In that case, a set of performance measures is valuable only if at least one of the measures is action informative (i.e., the probability distribution function is influenced by the agent's action choice). Additional measures have value if they facilitate reducing the risk premium that is due to incentive risk. For example, a measure can be valuable even if it is not influenced by the agent's action choice, as long as it is informative about (correlated with) the noise in an action-informative performance measure. Observe that while, for investment purposes, investors value information about the noise in a firm's payoff (e.g., terminal dividend), that information has no value in contracting with the firm's manager if the payoff is not contractible (e.g., his contract will be terminated before the firm is terminated). Hence, value-relevant information is not necessarily incentive-relevant.

The preceding remarks highlight the fact that in a single-task setting, the focus is on motivating the intensity of effort with the least amount of incentive risk (in the sense of the smallest risk premium). A report of the payoff is only of interest in contracting to the extent the payoff is informative about intensity of the agent's effort. Thus, an agent's compensation will vary with the likelihood ratio for a given payoff, not its dollar value.

However, a somewhat different perspective occurs if the agent has multiple tasks. As explored in Chapter 20, when there are multiple tasks, the principal is concerned about motivating both the intensity of effort, and the direction (i.e., mix) of that effort. This raises issues of performance measure congruity in addition to performance measure noise. The congruity of a measure depends on its alignment with the payoff.

Other factors affecting the value of performance measures arise in settings in which the tasks are performed sequentially over time (see the multi-period models in Chapters 25 through 28) or by more than one agent (see the multi-agent models in Chapters 29 and 30).

APPENDIX 18A: SUFFICIENT STATISTICS VERSUS SUFFICIENT INCENTIVE STATISTICS

The following results come from Amershi and Hughes (1989) (AH) and we refer to that paper for proofs. AH focus on settings in which there is a vector of possible performance measures $y = (y_1,...,y_m)$. They examine *whether (and under what conditions) the principal is necessarily worse off if he receives less than a sufficient statistic for y.* Proposition 18.2 establishes that the principal is never worse off with a sufficient statistic than with any other representation of the information. This follows from the fact that

$$L(\psi|a,\eta^{\psi}) = \frac{\varphi_a(\psi|a,\eta^{\psi})}{\varphi(\psi|a,\eta^{\psi})} = L(y|a,\eta) \quad \text{if } \psi = \boldsymbol{\psi}(y),$$

which implies that y and ψ result in the same compensation levels (i.e., all y that result in the same statistic ψ also result in the same optimal compensation level).

Observe that the likelihood ratio $L(\cdot)$ (and the induced compensation function $\boldsymbol{c}(\cdot)$) is a statistic, and we call it a sufficient "incentive" statistic since it provides all the information necessary for specifying the optimal compensation level for a given action a. The question of whether a sufficient statistic is necessary for implementing the optimal compensation plan is equivalent to asking whether $L(\cdot|a,\eta^{\psi})$ is a minimal sufficient statistic (or whether $L(\cdot|a,\eta^{\psi})$ is invertible with respect to a minimal sufficient statistic).

Observe that the likelihood ratio $L(\cdot)$ is a one-dimensional statistic, i.e., L: $Y\times A \rightarrow \mathbb{R}$. Consequently, it can only be a sufficient statistic for a family of distributions $\{\varphi, A\}$ if the minimal sufficient statistic for that family is one-dimensional.

In examining the "necessity" of a sufficient statistic, AH pay particular attention to the exponential family of distributions (see Volume I, Chapter 2 for a characterization of the one-parameter family of exponential distributions).

Definition

A family of distributions $\{\varphi,A\}$ is a member of the *exponential family of rank* $r \geq 1$ *in* Y if:

(a) There exist real-valued statistics ψ_i: $Y \rightarrow \mathbb{R}$ and parametric functions α_i: $A \rightarrow \mathbb{R}$, $i = 1, ..., r$, such that $\varphi(y|a)$ is of the form

$$\varphi(y|a) = \theta(y)\,\beta(a)\exp\left[\sum_{i=1}^{r} \alpha_i(a)\,\psi_i(y)\right].$$

(b) The systems of functions $\{1, \psi_1(y), ..., \psi_r(y)\}$ and $\{1, \alpha_1(a), ..., \alpha_r(a)\}$ are linearly independent over Y and A, respectively, where $\beta(a)$ is a scaling function that makes $\varphi(y|a) = 1$ under integration over Y. The functions $\alpha_i(a)$ are called distribution parameters.

Observe that the rank of an exponential family is not the dimension of the signal $y = (y_1, ..., y_m)$, but rather the dimension of the minimal sufficient statistic $\boldsymbol{\psi}(y) = (\psi_1(y), ..., \psi_r(y))$. Effectively, this dimension depends on the number of "unknown parameters." For example, the normal distribution in which a influences only the mean is a member of exponential family of dimension 1, whereas the normal distribution in which a influences both the mean and the variance is a member of the exponential family of dimension 2. These characterizations hold for a signal that consists of sample size m, for any $m \geq 1$. (See Volume I, Section 3.1.4.)

Proposition 18A.1 (AH, Prop. 1)

If $\{\varphi, A\}$ belongs to the exponential family of rank one, then the principal *strictly prefers* every sufficient statistic to all nonsufficient statistics.

Proposition 18A.2 (AH, Theorem 1)

Assume $\{\varphi, A\}$ is such that the density functions $\varphi(y|a)$, $a \in A$, are continuous in y with fixed support Y. For all $a \in$ A, the likelihood ratios $L(\psi(y)|a,\eta^{\psi})$ are strictly monotone in some one-dimensional minimal sufficient statistic $\boldsymbol{\psi}(y)$ if, and only if, $\{\varphi, A\}$ belongs to the exponential family of rank one.

The monotonicity of $L(\psi(y)|a,\eta^{\psi})$ establishes its invertibility. Without monotonicity we have a setting in which two statistics ψ^1 and ψ^2 can induce the same likelihood ratio (which implies the same compensation level) and, hence, the compensation function is based on "less" than a minimal sufficient statistic. The above theorem establishes that of all the continuous distributions one can imagine, *only those in the exponential family of rank one have one-dimensional sufficient statistics that result in monotone likelihood ratios.*

Proposition 18A.3 (AH, Prop. 2)

If all actions in the set A are "relevant," then in any agency characterized by $\{\varphi, A\}$, the principal strictly prefers a sufficient statistic to all nonsufficient statistics if, and only if, $\{\varphi, A\}$ belongs to the exponential class of rank one.

Proposition 18A.4 (AH, Theorem 2)

Assume $Y = Y_1 \times ... \times Y_m$ and the signals are independent, identically distributed random variables with densities $\varphi(y_i|a)$, $i = 1, ..., m$, that are continuous in y_i with fixed support for all $a \in A$. If there exists a one-dimensional sufficient statistic, then $\{\varphi, A\}$ belongs to the exponential family of rank one.

Corollary (AH, Corollary 1)

Assume $\{\varphi, A\}$ satisfies the assumptions of Proposition 18A.2. Every non-sufficient statistic is also *globally "incentive" insufficient* (see the earlier Holmström definition) if, and only if, $\{\varphi, A\}$ belongs to the exponential family of rank one.

Corollary (AH, Prop. 3)

For every agency with statistical structure $\{\varphi, A\}$ that belongs to the exponential family of rank $r > 1$, an optimal incentive contract is always a non-sufficient statistic.

The key factor that leads to the last result is that if more than one parameter is influenced by the agent's action a, the sufficient statistic has more than one dimension. However, while the likelihood ratio is monotonic in each component of that sufficient statistic, there is more than one sufficient statistic that results in the same likelihood ratio. Hence, neither the compensation level nor the likelihood ratio that generated it is sufficient to infer even a minimal sufficient statistic. That is, in this setting, the principal *does not use all the "information" provided by a sufficient statistic in constructing an optimal compensation contract.* Of course, he can *always* use a sufficient statistic in constructing the optimal compensation contract, since he can "ignore" any information he does not require.

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CHAPTER 19

LINEAR CONTRACTS

A compensation contract is defined to be linear if there exists a constant f and another constant or vector v such that $\boldsymbol{c}(y) = f + v \cdot y$, whenever $f + v \cdot y \in C = [\underline{c}, \infty)$ and $\boldsymbol{c}(y) = \underline{c}$ otherwise, where y is perhaps a multi-dimensional performance measure and $\underline{c}$ is an exogenously imposed lower bound on consumption. The use of linear contracts in agency theory is appealing on two grounds. First, restricting our analysis to linear contracts makes some analyses more tractable and some results more intuitive. Second, many contracts observed in the "real world" appear to be linear or at least piecewise linear.

There are two basic approaches to the use of linear contracts in the agency theory literature. The first approach is to restrict the analysis to the set of linear contracts, whether the optimal contract is linear or not. In the early years of agency theory this would have been viewed as a major flaw in any research paper. However, as we have learned more about the implications of optimal contracts, the perspective has shifted such that it is now the view of many researchers (including ourselves) that a simplification to the set of linear contracts is justified, if the analysis provides insights that are believed not to be confined to settings with linear contracts. In Section 19.1 we review this approach in the simple setting in which both the performance measure and the agent's action are single-dimensional. In Chapter 20 we consider settings in which the performance measure and the agent's action are both multi-dimensional. Chapter 21 reviews models in which one of the performance measures is a market based performance measure such as the stock price. The second approach is to restrict the analysis to conditions under which the optimal contract is linear. However, as we shall see in Section 19.2, this only occurs under highly specialized conditions on preferences, beliefs, technology, and information.

Sufficient Conditions for the Optimal Contract to Be Linear

Before we proceed into the main analysis of settings with linear contracts, it is useful to review sufficient conditions for linear compensation contracts to be optimal within the basic principal-agent model. Of course, if the performance measure is binary, any compensation function can be expressed as a linear function of the performance measure. Hence, we generally assume that the performance measure has more than two possible signals. In the analysis that follows we further assume that the principal owns the outcome and is risk neutral, the agent's utility function is $u^a(c,a) = u(c)k(a) - v(a)$, with $u'(c) > 0$, $u''(c) < 0$, $k(a) > 0$, $k'(a) < 0$, $v'(a) > 0$, and either $k(a) = 1$ or $v(a) = 0$, for all a. The agent's consumption set is $C = [\underline{c}, \infty)$, the set of actions A is a convex set on the real line, i.e., $A = [\underline{a}, \bar{a}]$, and we assume the agent's incentive constraint can be represented by its first-order condition. Given these assumptions, and considering $a \in (\underline{a}, \bar{a})$, the optimal contract is characterized by (see Chapter 17)

$$M(c(y)) = \lambda k(a) + \mu[k(a)L(y|a) + k'(a)],$$

where

$$L(y|a) = \frac{\varphi_a(y|a)}{\varphi(y|a)}, \tag{19.1}$$

whenever the right-hand side is such that $M(c(y)) \geq M(\underline{c})$.

Proposition 19.1

Sufficient conditions for the compensation contract to be linear are that $u(c) = \ln(\alpha c + \beta)$, $\alpha c + \beta > 0$, $\alpha > 0$, and $L(\cdot|a)$ is a linear function of y (e.g., $\varphi(y|a)$ is from the one-parameter exponential family).

The proof is straightforward, given (19.1) and the fact that with log-utility $u'(c) = \alpha/(\alpha c + \beta)$, which implies that $M(c) = c + \beta/\alpha$. However, with log-utility we must be careful if the signal space Y is convex in which case the first-best solution might be approximated arbitrarily closely if, for example, the performance measure is normally distributed (see Proposition 17.10).

Even though optimal contracts are linear with log-utility and linear likelihood ratios, this provides no significant advantage in terms of analytical tractability since there is no simple representation of the agent's expected utility. As we will see in the following section, linear contracts combined with exponential utility, on the other hand, is the "magic" combination for providing analytical tractability.

19.1 LINEAR SIMPLIFICATIONS

In this section we consider a setting sometimes referred to as the *LEN* framework which stands for "Linear contracts", "Exponential utility", and "Normally distributed performance measure."[1] That is, compensation contracts are exogenously restricted to the class of linear contracts, the agent's preferences are represented as a multiplicatively separable exponential utility function in c and

[1] The following analysis is similar to that found in Hughes (1988).

a, and the performance measure is normally distributed. The search is for an optimal contract within the class of linear contracts. No mention is made of overall optimality at this point. An attractive feature of this approach is that it gives a very simple characterization of the "optimal" contract. To summarize, our basic assumptions in this section are:

(a) The principal must choose the compensation contract within the class of linear contracts, i.e., within the class of functions

$$\boldsymbol{LC} = \{\, \boldsymbol{c}: Y \to C \mid \exists\, v, f: \boldsymbol{c}(y) = f + vy,$$

$$\text{if } f + vy \in C \text{ and } \boldsymbol{c}(y) = \underline{c} \text{ otherwise } \}.$$

(b) The principal is risk neutral and the agent has a multiplicatively separable, negative exponential utility function with $\underline{c} = -\infty$, i.e.,

$$u^a(c,a) = -\exp[-r(c - \kappa(a))] = u(c)k(a),$$

with $u(c) = -\exp[-rc]$, $k(a) = \exp[r\kappa(a)]$, $\kappa'(a) > 0, \kappa''(a) > 0, \kappa'''(a) \geq 0$, which implies

$$M(c) = \frac{1}{r}\exp[rc].$$

(c) The contractible performance measure is normally distributed with mean a and variance σ^2 (which is in the one-parameter exponential family of distributions), i.e., $y \sim \mathrm{N}(a,\sigma^2)$, which implies that[2]

$$L(y|a) = \frac{1}{\sigma^2}(y - a).$$

In general, an optimal compensation contract is characterized by

$$M = \lambda k(a) + \mu[k(a)L + k'(a)].$$

[2] Note that if the agent's action only affects the mean of a normally distributed performance measure, we can always express the action as equal to the mean of that measure and adjust the agent's cost function accordingly. That is, there is an indeterminancy in how we express the agent's cost function and how the action affects the mean of the performance measure.

Assumptions (b) and (c) imply that[3]

$$c(y) = \kappa(a) + \frac{1}{r}\ln\left[r\left(\lambda + \mu[r\kappa'(a) + \frac{1}{\sigma^2}(y-a)]\right)\right].$$

Hence, an optimal contract with assumptions (b) and (c) is a *strictly concave function*. On the other hand, if we also impose the linear contract assumption (a), we obtain a significant simplification in the analysis.[4] The key here is that if $\mathbf{c}$ is a linear function of y and y is normally distributed, then c is normally distributed. Hence, we can then use a slightly generalized version of the relation in Proposition 2.7 to obtain

$$U^a(\mathbf{c},a) = -\exp[-rCE(v,f,a)],$$

where the agent's certainty equivalent is given by

$$CE(v,f,a) = va + f - \tfrac{1}{2}rv^2\sigma^2 - \kappa(a).$$

The sum of the first two terms in the certainty equivalent is the expected compensation, the third term is the risk premium, and the fourth term is the monetary cost of effort.

With a linear contract, the agent's incentive constraint can be expressed as the first-order condition based on the certainty equivalent $CE(\cdot)$, i.e.,

$$CE_a(v,f,a) = v - \kappa'(a) = 0 \quad \Rightarrow \quad v = \kappa'(a). \tag{19.2}$$

Assume the agent's reservation utility has a certainty equivalent of zero, i.e., $\bar{U} = -1$. Hence, given a and v, the contract acceptance constraint can be expressed as the requirement that

$$f = \kappa(a) + \tfrac{1}{2}rv^2\sigma^2 - va. \tag{19.3}$$

The principal's decision problem is

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[3] If $\underline{c} = -\infty$, the Mirrlees condition discussed in Proposition 17.10 applies even with a multiplicatively separable utility function. That is, an optimal contract does not exist and a penalty contract can be used to obtain a result that is arbitrarily close to the first-best result. We exogenously exclude this possibility here, for example by assuming that the lower bound on consumption is finite.

[4] Note that when we restrict the contracts to be linear, we cannot use penalty contracts as in the Mirrlees argument to get arbitrarily close to first-best even though $\underline{c} = -\infty$. That is, there generally exists an optimal contract within the class of linear contracts which is not first-best.

$$\underset{v,f,a}{\text{maximize}} \; U^p(\boldsymbol{c},a) = b(a) - (va + f) \quad \text{subject to (19.2) and (19.3),}$$

where $b(a)$ is the expected outcome to the principal, i.e., $b(a) = \mathrm{E}[x|a]$, and we assume $b''(a) \le 0$. Of course, if the performance measure is the outcome, then $b(a) = a$, but that does not generally have to be the case.

Incentive constraint (19.2) specifies the incentive wage parameter v required to induce a given action a, and participation constraint (19.3) specifies the fixed component f required to satisfy the agent's reservation utility.[5] Hence, we can substitute them into the objective function to simplify the principal's decision problem to an unconstrained optimization problem,

$$\underset{a}{\text{maximize}} \quad b(a) - \tfrac{1}{2} r[\kappa'(a)]^2 \sigma^2 - \kappa(a).$$

Differentiating with respect to a provides

$$b'(a) - \kappa'(a) - r\kappa'(a)\kappa''(a)\sigma^2 = 0,$$

and the optimal action satisfies

$$\kappa'(a)\left[1 + r\kappa''(a)\sigma^2\right] = b'(a). \tag{19.4}$$

A common cost function used in the literature is $\kappa(a) = \tfrac{1}{2}a^2$. In this case $\kappa'(a) = a$ and $\kappa''(a) = 1$, which implies that the optimal action (given linear contracts) is given by

$$a = v = b'(a)/(1 + r\sigma^2).$$

Proposition 19.2 (Hughes 1988, Prop. 8.1 and 8.2)
An agent who either faces more risk or is more risk averse will be induced to work less hard and will get a lower incentive wage.

Proof: Let $q = r\sigma^2$, i.e., the agent's risk times his risk aversion. Totally differentiating first-order condition (19.4) with respect to a and q yields

$$\left(\kappa''(a)[1 + q\kappa''(a)] + q\kappa'(a)\kappa'''(a) - b''(a)\right) da + \kappa'(a)\kappa''(a)\, dq = 0.$$

---

[5] Note that the assumption of a multiplicatively separable exponential utility function is of utmost importance for this simple separation of incentive and contract acceptance concerns, since it is the only concave utility function that has no wealth effects.

With the assumptions that $\kappa'(a) > 0$, $\kappa''(a) > 0$, $\kappa'''(a) \geq 0$, and $b''(a) \leq 0$, this implies that

$$\frac{da}{dq} < 0,$$

which by (19.2) implies that

$$\frac{dv}{dq} < 0.$$ **Q.E.D.**

Of course, the key here is that as $q = r\sigma^2$ increases so does the risk premium for a fixed incentive wage. Hence, the principal's trade-off between providing incentives for action choices and the risk premium paid to the agent changes such that it is optimal to lower the induced action in order to reduce the risk premium. However, note that if the risk in the performance measure, σ^2, increases with the risk aversion r fixed, the agent's compensation risk, $v^2\sigma^2$, and, thus, the risk premium may decrease or increase. This depends on the specifics of the problem.[6]

19.2 OPTIMAL LINEAR CONTRACTS

In this section we draw upon the important work of Holmström and Milgrom (1987) (HM). The setting is very similar to the one examined in the previous section, but some key differences result in a setting in which a linear contract is optimal. We simplify their setting slightly, e.g.,

- the principal is risk neutral (HM permit him to be risk averse);

- $\kappa(a)$ is independent of y (HM permit κ to be a function of both y and a).

The key features of their model are exponential utility and the agent chooses a sequence of actions with each of those actions generating independent binary signals. In Section 19.2.4 we extend the model to a setting in which the agent by his action choices continuously controls the drift of a continuous-time Brownian motion for the outcome. We start our analysis with a simple one-action/binary-signal model (in which case we know that any contract can be written as a linear function of the performance measure).

[6] With the cost function given by $\kappa(a) = ½a^2$, the agent's compensation risk increases as σ^2 increases.

19.2.1 Binary Signal Model

We consider a simplified version of HM's basic model, and adapt their propositions to this context. Suppose that

$Y = \{y_b, y_g\}$,	two contractible signals with $y_g > y_b$;
$A = [0,1]$,	the action set could be any interval or finite subset of $[0,1]$, but we will assume that a can be any amount between 0 and 1;
$\varphi_g(a) = 1 - \varphi_b(a) = a$,	the action is expressed in terms of the probability of the good signal y_g;
$u^a(c,a) = -\exp[-r(c - \kappa(a))]$,	multiplicatively separable exponential utility with $\underline{c} = -\infty$;
$\kappa(a)$,	cost of action a, with $\kappa'(a) > 0$ and $\kappa''(a) > 0$;
$\mathbf{c} = (c_b, c_g)$,	compensation contract, where c_i is the compensation paid for signal y_i, $i = b, g$.

Definition

A compensation contract $\mathbf{c}$ implements action a with certainty equivalent $CE(\mathbf{c},a) = w$ if

$$U^a(\mathbf{c},a) \equiv \sum_{i=b}^{g} u(c_i - \kappa(a))\varphi_i(a) = u(w), \text{ and } a \in \underset{a' \in A}{\operatorname{argmax}}\ U^a(\mathbf{c},a').$$

Let $\boldsymbol{C}(a,w)$ be defined as the set of contracts $\mathbf{c}$ that implement action a with certainty equivalent w, and denote the set of actions that can be implemented with certainty equivalent w

$$A^o(w) \equiv \{\, a \in A \mid \boldsymbol{C}(a,w) \neq \emptyset \,\}.$$

As a direct consequence of the multiplicatively separable exponential utility assumption we get the following result.

Proposition 19.3 (HM, Theorem 2)
For any $\mathbf{c}$, w and $a \in A^o(w)$:

(a) $\mathbf{c} \in \boldsymbol{C}(a,w)$ if, and only if, $\mathbf{c} - w\mathbf{e} \in \boldsymbol{C}(a,0)$, where $\mathbf{e} = (1,1)$;

(b) $A^o(w) = A$, i.e, the set of actions that can be implemented is independent of w.

It is useful to view the compensation contract $\mathbf{c}$ as consisting of a *variable component* $\boldsymbol{\delta}$ and a *fixed component* w, where $\mathbf{c}$ implements a with certainty equivalent w and $\delta_i \equiv c_i - w$. The key result here is that changing the "required" certainty equivalent changes the fixed component of the contract that implements a, but it does not change the variable component. This is because with the multiplicatively separable exponential utility, *wealth does not influence the agent's choice among gambles* (the variable component). Hence, the least cost contract for implementing action $a \in A$ with certainty equivalent w, $\mathbf{c}^\dagger(a,w)$, can be obtained as follows:

$$\mathbf{c}^\dagger(a,w) = \boldsymbol{\delta}^\dagger(a) + w, \;\; \text{where} \;\; \boldsymbol{\delta}^\dagger(a) \in \underset{\boldsymbol{\delta} \in \boldsymbol{C}(a,0)}{\text{argmin}} \; \delta_b\varphi_b(a) + \delta_g\varphi_g(a).$$

The key is that with exponential utility

$$\mathrm{E}[u(\delta_i + w - \kappa(a)) \,|\, a] = \exp[-rw]\, \mathrm{E}[u(\delta_i - \kappa(a)) \,|\, a].$$

In our simple binary signal setting, all actions are implementable for any w including $w = 0$. Therefore, for action a, the optimal compensation contract is given by

$$U^a(\boldsymbol{\delta}^\dagger, a) = u(\delta_b^\dagger - \kappa(a))(1-a) + u(\delta_g^\dagger - \kappa(a))a = u(0) = -1,$$

$$U_a^a(\boldsymbol{\delta}^\dagger, a) = [u(\delta_g^\dagger - \kappa(a)) - u(\delta_b^\dagger - \kappa(a))]$$

$$+ r\kappa'(a)[u(\delta_b^\dagger - \kappa(a))(1-a) + u(\delta_g^\dagger - \kappa(a))a] = 0,$$

which imply that

$$\delta_g^\dagger = \kappa(a) - \frac{1}{r}\ln\Big(1 - r(1-a)\kappa'(a)\Big), \tag{19.5a}$$

$$\delta_b^\dagger = \kappa(a) - \frac{1}{r}\ln\Big(1 + ra\kappa'(a)\Big). \tag{19.5b}$$

19.2.2 Repeated Binary Signal Model

Now consider a T-period stochastically independent repetition of the binary signal model described in the previous section.[7] Let us further assume

- y_t and a_t represent the performance measure and action at date t, respectively, and let y represent the aggregate performance measure, i.e., $y = \sum_{t=1}^{T} y_t$;

- the set of possible actions is the same at each date, and the performance measure in any period is independent of the action in any other period;

- the history of contractible performance measures and unobservable actions for periods 1, ..., t are denoted $\overleftarrow{\mathbf{y}}_t = \{y_1, ..., y_t\}$ and $\overleftarrow{\mathbf{a}}_t = \{a_1, ..., a_t\}$, respectively; the agent observes $\overleftarrow{\mathbf{y}}_t$ before choosing a_{t+1};

- the agent is paid compensation c and consumes only at date T, and that compensation is a function of the history of signals $\overleftarrow{\mathbf{y}}_T$; however, it will be useful to view the compensation contract as taking the following form:

$$c(\overleftarrow{\mathbf{y}}_T) = w + \delta_1(y_1) + \delta_2(y_2|y_1) + ... + \delta_T(y_T|\overleftarrow{\mathbf{y}}_{T-1}),$$

  i.e., the agent receives a basic wage w and then receives an increment δ_t for the performance measure in period t, given the history to that date;

- the agent's utility function is

$$u^a(c, \overleftarrow{\mathbf{a}}_T) = -\exp\left[-r\left(c - \sum_{t=1}^{T} \kappa(a_t)\right)\right]$$

$$= -\exp[-rc]\exp[r\kappa(a_1)] \cdots \exp[r\kappa(a_T)].$$

Observe that the utility function is multiplicatively separable with respect to c and the costs for each action – this is an important assumption. We assume the agent only consumes at date T, and the compensation and effort costs are stated in date T dollars. The assumption that the effort costs are the same at each date

[7] In Chapters 25 - 28 we analyze more general multi-period principal-agent relationships.

effectively implies that the timing of the costs is irrelevant. Hence, the analysis in this chapter is more appropriate for short-term horizons than for long-term horizons. Time value of money issues within multi-period principal-agent relationships is considered in Chapters 25 and 26.

Now consider a two-period model, i.e., $T = 2$, and consider the second period. Assume that the agent has "earned" compensation $\boldsymbol{c}_1(y_1) = w + \boldsymbol{\delta}_1(y_1)$ and has taken action a_1 in the first period. Further assume that the principal can select the contract for period 2, i.e., he can select the second-period action to be implemented and the final "payment" $\boldsymbol{\delta}_2(y_2|y_1)$ that will induce that action, subject to the requirement that the contract provides a certainty equivalent of *zero*.[8]

Observe that the agent's expected utility at this point is

$$U^a(\boldsymbol{c}_1(y_1), \boldsymbol{\delta}_2, a_1, a_2)$$

$$= \sum_{i=b}^{g} u\big(\boldsymbol{c}_1(y_1) + \boldsymbol{\delta}_2(y_{i2}|y_1) - \kappa(a_1) - \kappa(a_2)\big)\varphi_i(a_2)$$

$$= -\exp\big[-r\,(\boldsymbol{c}_1(y_1) - \kappa(a_1))\big] \sum_{i=b}^{g} \exp\big[-r\big(\boldsymbol{\delta}_2(y_{i2}|y_1) - \kappa(a_2)\big)\big]\,\varphi_i(a_2).$$

Hence, it is obvious from the above that y_1 and c_1 have no impact on the choice of a_2 and $\boldsymbol{\delta}_2$. The optimal choice in the final period is the same as the optimal choice in the single-period model.[9] Of course, the exponential utility with its lack of wealth effects is very crucial here.

Now consider the first period, assuming $a_2 = a^\dagger$ and $\boldsymbol{\delta}_2 = \boldsymbol{\delta}^\dagger$. The agent's expected utility is

$$U^a(w, \boldsymbol{\delta}_1, \boldsymbol{\delta}^\dagger, a_1, a^\dagger)$$

$$= \sum_{i=b}^{g} \sum_{\imath=b}^{g} u\big(w + \boldsymbol{\delta}_1(y_{i1}) + \boldsymbol{\delta}^\dagger(y_{\imath 2}) - \kappa(a_1) - \kappa(a^\dagger)\big)\varphi_i(a_1)\varphi_\imath(a^\dagger)$$

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[8] There is no loss of generality here. The principal can choose the continuation contract at the end of each period since the model assumptions are such that there are no benefits to a commitment to a "long-term" contract (see Chapter 28).

[9] Note that this implies that it is not important whether the principal can observe $\overleftarrow{\mathbf{y}}_t$ before choosing the contract for the second period.

$$= \exp[-rw]\left[\sum_{i=b}^{g} u\big(\boldsymbol{\delta}_1(y_{i1}) - \kappa(a_1)\big)\,\varphi_i(a_1)\right]$$

$$\times\left[\sum_{i=b}^{g} \exp\!\left[-r\big(\boldsymbol{\delta}^{\dagger}(y_{i2}) - \kappa(a^{\dagger})\big)\right]\varphi_i(a^{\dagger})\right]$$

$$= \exp[-rw]\sum_{i=b}^{g} u\big(\boldsymbol{\delta}_1(y_{i1}) - \kappa(a_1)\big)\,\varphi_i(a_1).$$

Again, if $\boldsymbol{\delta}_1$ is to provide a certainty equivalent of zero, the optimal contract for the first period is the same as in the one-period model (and the second period of the two-period model).

Consequently, the optimal two-period contract will induce the selection of a in both periods and the optimal contract can be characterized as

$$\boldsymbol{c}(y_1,y_2) = w + \boldsymbol{\delta}^{\dagger}(y_1) + \boldsymbol{\delta}^{\dagger}(y_2),$$

where w is set such that $u(w) = \overline{U}$ and $\boldsymbol{\delta}^{\dagger} \in \boldsymbol{C}(a^{\dagger},0)$. These arguments extend in an immediate fashion to $T > 2$ periods.

Proposition 19.4 (HM, Theorem 5)

Under the assumed conditions, if $a^{\dagger}$ is the optimal action in the single-period model, then it is the optimal action to induce in each of the T periods. And if $\boldsymbol{\delta}^{\dagger} \in \boldsymbol{C}(a^{\dagger},0)$ is the optimal variable component of the single-period contract for inducing a, the optimal multi-period contract can be characterized as

$$\boldsymbol{c}(\overleftarrow{\mathbf{y}}_T) = w + \sum_{t=1}^{T} \boldsymbol{\delta}^{\dagger}(y_t).$$

Let $T_i(\overleftarrow{\mathbf{y}}_T)$ denote the total number of periods in which signal y_i occurs in the signal history $\overleftarrow{\mathbf{y}}_T$ so that the aggregate performance measure y is

$$y = \sum_{t=1}^{T} y_t = \sum_{i=b}^{g} y_i T_i(\overleftarrow{\mathbf{y}}_T).$$

Now observe that the optimal compensation contract can be restated as

$$c(\overleftarrow{\mathbf{y}}_T) = w + \sum_{i=b}^{g} \delta_i^{\dagger} T_i(\overleftarrow{\mathbf{y}}_T).$$

Furthermore, this contract can be written as a linear contract of the aggregate performance measure y, i.e.,

$$c(y) = vy + f,$$

where $$v = \frac{\delta_g^{\dagger} - \delta_b^{\dagger}}{y_g - y_b} \quad \text{and} \quad f = w + T \times [\delta_b^{\dagger} - v y_b].$$

Hence, following HM, we have identified conditions sufficient for the optimal contract to be linear in the aggregate performance measure – a multi-stage setting in which each stage is identical and independent, and has only two possible signals.

19.2.3 Multiple Binary Signals

The binary signal case provides a nice linear result. What happens when there are multiple possible signals? This can occur because at each stage there is a single performance measure that can generate more than two signals or because there are multiple performance measures each with two possible signals. We consider the setting in which there are two performance measures y_{t1} and y_{t2} each with binary signals $y_{ti} \in \{y_{bi}, y_{gi}\}$, $i = 1, 2$. In this setting, there are effectively four possible signals at each stage: $\psi_0 = (y_{b1}, y_{b2})$, $\psi_1 = (y_{g1}, y_{b2})$, $\psi_2 = (y_{b1}, y_{g2})$, and $\psi_3 = (y_{g1}, y_{g2})$. Let $y_1 = \Sigma_t y_{t1}$ and $y_2 = \Sigma_t y_{t2}$ represent the aggregate "performance" for the two performance measures. If only one of the performance measures is used, the optimal contract could be expressed as a linear function of the aggregate for that measurement. However, if both performance measures are used, can the optimal contract be expressed as a linear function of y_1 and y_2? In general, *the answer is NO!*

We emphasize the above point because there is some confusion in the literature. To see the source of this confusion, consider the setting in which there are two binary performance measures, which are represented by four possible signals. The agent's action is again single-dimensional a, and the optimal single-period contract is $c(\psi)$, which has four different compensation levels.[10]

[10] Note that in this setting the four compensation levels *cannot* be determined by the contract acceptance constraint and the incentive constraint for the single-dimensional action alone, as is (continued...)

In a T-period context, in which the agent has exponential utility for total compensation minus the total cost of effort, and $\kappa(a_t)$ and $\varphi(\psi_t|a_t)$ are constant across periods, the optimal compensation can be expressed as

$$c(\overleftarrow{\psi}_T) = w + \sum_{i=0}^{3} \delta_i^\dagger T_i(\overleftarrow{\psi}_T),$$

where $\overleftarrow{\psi}_T$ is the signal history over the T periods and $T_i(\overleftarrow{\psi}_T)$ is the total number of periods in which signal ψ_i occurs in the signal history $\overleftarrow{\psi}_T$. That is, the optimal contract is a linear function of the number of periods in which each signal occurs. Hence, the aggregate performance for the two performance measures can be written as

$$y_1 = y_{b1}[T_0(\overleftarrow{\psi}_T) + T_2(\overleftarrow{\psi}_T)] + y_{g1}[T_1(\overleftarrow{\psi}_T) + T_3(\overleftarrow{\psi}_T)],$$

$$y_2 = y_{b2}[T_0(\overleftarrow{\psi}_T) + T_1(\overleftarrow{\psi}_T)] + y_{g2}[T_2(\overleftarrow{\psi}_T) + T_3(\overleftarrow{\psi}_T)].$$

However, the optimal contract *cannot* be expressed as a linear function of y_1 and y_2 unless there exist v_1 and v_2 such that

$$v_1 y_{b1} + v_2 y_{b2} = \delta_0^\dagger, \qquad v_1 y_{g1} + v_2 y_{b2} = \delta_1^\dagger,$$

$$v_1 y_{b1} + v_2 y_{g2} = \delta_2^\dagger, \qquad v_1 y_{g1} + v_2 y_{g2} = \delta_3^\dagger.$$

These conditions are satisfied if, and only if, $\delta_1^\dagger - \delta_0^\dagger = \delta_3^\dagger - \delta_2^\dagger$, in which case

$$v_1 = \frac{\delta_2^\dagger - \delta_0^\dagger}{y_{g1} - y_{b1}} \quad \text{and} \quad v_2 = \frac{\delta_3^\dagger - \delta_1^\dagger}{y_{g2} - y_{b2}}.$$

Those conditions are satisfied only in "knife-edge" cases! Hence, although the compensation contract can be expressed as a linear function of enumeration aggregates for each of the four signals it can, in general, *not* be written as a linear function of aggregate performance for the two performance measures.

There are two basic reasons why the latter is not possible, in general. First, the uncertainty has three dimensions (since ψ can take on four values), whereas (y_1,y_2) is only two-dimensional. Second, even if the uncertainty with respect to optimal compensation can be reduced to two dimensions, the optimal contract

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[10] (...continued)
the case in (19.5) with only one binary performance measure.

cannot be written as a linear function of aggregate performance for the two performance measures. To see this, consider the setting in which the two performance measures are *identically and independently distributed* with the agent's action representing the probability of the good signal for both measures. In that case, there is clearly no need to distinguish between the signals $\psi_1 = (y_{g1}, y_{b2})$ and $\psi_2 = (y_{b1}, y_{g2})$ – it is optimal to pay the agent the same compensation for both signals, i.e., $\delta_1^\dagger = \delta_2^\dagger \equiv \delta_{12}^\dagger$, since variations in that compensation would impose unnecessary risk on the agent. Hence, there are only three different optimal compensation levels of which one is fixed by the contract acceptance constraint, i.e., the dimension of the uncertainty in compensation is two as opposed to three. In general, in order to determine the optimal compensation it is sufficient to know the total number of good signals for the two measures, and the total number of signals where the two measures have different signals, i.e., $T_3(\overleftarrow{\psi}_T)$ and $T_1(\overleftarrow{\psi}_T) + T_2(\overleftarrow{\psi}_T)$. However, those numbers *cannot* be inferred from y_1 and y_2. On the other hand, if $\delta_{12}^\dagger - \delta_0^\dagger = \delta_3^\dagger - \delta_{12}^\dagger$ (such that there are effectively only two compensation levels that have to be determined), we only need to know the total number of good signals, $T_1(\overleftarrow{\psi}_T) + T_2(\overleftarrow{\psi}_T) + 2T_3(\overleftarrow{\psi}_T)$, to determine optimal compensation – but, unfortunately, it takes a very exceptional case to satisfy that condition.

19.2.4 Brownian Motion Model

HM examine a setting in which the agent controls the drift of a continuous-time Brownian motion, over some fixed unit time interval $[0,1]$. The significant advantage of this approach is that, under certain conditions, the optimal contract in the dynamic agency problem may be found as the optimal linear contract in the basic agency model with the agent's action representing the mean of a normally distributed performance measure. Not only does this simplify the calculation of an optimal contract, but the dynamic model avoids the Mirrlees Problem with normal distributions. Here we review their model as the limiting case of the repeated binary signal model in Sections 19.2.2 and 19.2.3 as the length of the periods goes to zero. The analysis in this section is based on Hellwig and Schmidt (2002).

One-dimensional Brownian Motion

Let the unit interval be divided into $1/\Delta$ time periods each of length Δ, and let $\tau = 0, 1, ..., 1/\Delta$ be the time index. In each period, there is either a good or a bad signal (represented by the numbers y_g^Δ and y_b^Δ, respectively) and the agent takes an action, which is represented by the probability a^Δ of obtaining the good signal. Here we appeal to Proposition 19.4, which shows that when periods are independent and the agent has negative exponential utility with no wealth effects, it is optimal to pay the agent period-by-period compensations (each depending on the outcome in that period) such that the same action is implemented in every period independent of the signal history. Let $T_g(\tau)$ denote the number of times the good signal has occurred in the first τ periods. Obviously, $T_g(\tau)$ is increasing over time. Hence, in order to get to a Brownian motion (that can both increase and decrease) we compare $T_g(\tau)$ to $\hat{T}_g(\tau) = \hat{a}\tau$, which is the expected number of good signals given some "standard" probability for the good outcome, $\hat{a}$. We assume that the bad signal is negative, i.e., $y_b^\Delta < 0$,[11] and fix the standard probability so that the expected performance is zero, i.e.,

$$\hat{a} y_g^\Delta + (1 - \hat{a}) y_b^\Delta = 0. \tag{19.6}$$

Let $\hat{y}_g^\Delta$ denote the "excess performance" from obtaining the good signal as opposed to the bad signal, i.e., $\hat{y}_g^\Delta = y_g^\Delta - y_b^\Delta$. We now define a "performance account" by

$$Z^\Delta(\tau) = \hat{y}_g^\Delta (T_g(\tau) - \hat{a}\tau), \tag{19.7}$$

which is simply the aggregate performance up to that date, i.e.,

$$\begin{aligned} Z^\Delta(\tau) &= y_g^\Delta (T_g(\tau) - \hat{a}\tau) - y_b^\Delta (T_g(\tau) - \hat{a}\tau) \\ &= y_g^\Delta T_g(\tau) + y_b^\Delta (\tau - T_g(\tau)) - (y_g^\Delta - y_b^\Delta)\hat{a}\tau - y_b^\Delta \tau \\ &= y_g^\Delta T_g(\tau) + y_b^\Delta (\tau - T_g(\tau)). \end{aligned} \tag{19.8}$$

The performance account is a candidate to be represented by a one-dimensional Brownian motion as the length of the intervals Δ goes to zero. However, before we can specify this limit, we must specify how the excess performance and the deviation of a^Δ from the standard probability $\hat{a}$ depend on the length of the period. This specification is designed so that the expected performance and effort costs over the unit interval are independent of the length of the time intervals Δ if the effort is constant. To achieve this, we assume the excess performance in each period is proportional to $\Delta^{1/2}$, i.e.,

$$\hat{y}_g^\Delta = \hat{y}_g \Delta^{1/2}.$$

[11] Note that this is a necessary condition if we want the aggregate performance over the unit interval to be normally distributed. In fact this is not just a matter of subtracting an arbitrary constant from each signal, since the aggregate performance when substituting that constant back in can only be normally distributed if the untransformed signals have some negatives.

The expected performance over the unit interval from choosing a^Δ instead of the standard probability $\hat{a}$ is defined to be

$$\mu^\Delta \equiv \frac{1}{\Delta}[a^\Delta y_g^\Delta + (1 - a^\Delta)y_b^\Delta] = \frac{1}{\Delta^{½}}(a^\Delta - \hat{a})\,\hat{y}_g. \qquad (19.9)$$

We want the total expected performance over the unit interval to be independent of the length Δ of each time interval we consider. Given the specification of μ^Δ in (19.9), this requires that the agent chooses deviations from the standard of the order of magnitude $\Delta^{½}$. Therefore, the agent's cost over an interval of length Δ of taking action a^Δ relative to taking action $\hat{a}$ is expressed as:

$$\kappa^\Delta(a^\Delta) = \Delta\,\kappa\left(\hat{a} + \frac{a^\Delta - \hat{a}}{\Delta^{½}}\right).$$

If a^Δ is taken over the entire unit interval, then the total effort cost is

$$\frac{1}{\Delta}\kappa^\Delta(a^\Delta) = \kappa\left(\hat{a} + \frac{a^\Delta - \hat{a}}{\Delta^{½}}\right).$$

Hence, if we let $\alpha \equiv [a^\Delta - \hat{a}]/\Delta^{½}$ represent the order of magnitude of the action difference, then the total effort cost over the unit interval depends on α, but is independent of Δ.

Note that $T_g(\tau)$ is generated by a binomial process with $T_g(\tau+1) - T_g(\tau) \in \{0,1\}$ and $E_\tau[T_g(\tau+1) - T_g(\tau)|a^\Delta] = a^\Delta$.[12] Hence, the expected change in aggregate performance takes the following simple form:

$$E_\tau[Z^\Delta(\tau+1) - Z^\Delta(\tau)\,|a^\Delta] = E_\tau[\hat{y}_g^\Delta(T_g(\tau+1) - T_g(\tau) - \hat{a})|a^\Delta] = \mu^\Delta\Delta,$$

and we can write the $Z^\Delta(\tau)$ process as:

$$Z^\Delta(\tau+1) - Z^\Delta(\tau) = \mu^\Delta\Delta + \hat{y}_g \times \begin{cases} +\,(1 - a^\Delta)\Delta^{½} & \text{with probability } a^\Delta, \\ -\,a^\Delta\Delta^{½} & \text{with probability } 1 - a^\Delta. \end{cases} \qquad (19.10)$$

Note that the "drift-term," i.e., the expected performance in a time interval of length Δ is of the order of magnitude Δ, while the variation around the mean is

[12] The symbol $E_\tau[\cdot]$ denotes the expected value operator conditional on the information available at date τ.

of the order of magnitude $\Delta^{1/2}$. This latter characteristic ensures that the variance, i.e.,

$$\mathrm{Var}_\tau[Z^\Delta(\tau+1) - Z^\Delta(\tau)\,|\,a^\Delta] = \hat{y}_g^2 a^\Delta(1 - a^\Delta)\Delta,$$

is also of the order of magnitude Δ. Hence, neither the drift component nor the variance component of the process for $Z^\Delta(\tau)$ dominates as Δ goes to zero. It is now relatively straightforward to show that for Δ approaching zero, the binomial process for $Z^\Delta(\tau)$ converges to a continuous-time Brownian motion $Z(t)$, $t \in [0,1]$,[13] with instantaneous drift μ and diffusion parameter σ, which we formally write as

$$dZ(t) = \mu dt + \sigma dB(t), \tag{19.11}$$

where $$\sigma = \hat{y}_g\sqrt{\hat{a}(1 - \hat{a})},$$

$B(t)$ is a standard Brownian motion with $B(0) = 0$, independent increments and $B(t') - B(t) \sim \mathrm{N}(0, t' - t)$ for $t < t'$.

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[13] The limit can be derived from the Central Limit Theorem as the increments in the performance account are identically and independently distributed given a constant action choice.

The Central Limit Theorem (Billingsley 1986, Theorem 27.1)
Suppose that $X_1, X_2, ..., X_n$ is an independent sequence of random variables having the same distribution with mean c and finite positive variance s^2. If $S_n = X_1 + X_2 + ... + X_n$, then

$$\frac{S_n - nc}{s\sqrt{n}} \rightarrow N(0,1).$$

Define $$X_\tau = Z^\Delta(\tau) - Z^\Delta(\tau-1); \quad \Rightarrow \quad c = \mu^\Delta\Delta, \quad s = \sigma\Delta^{1/2}.$$

The performance account at $t = 1$ ($\tau = 1/\Delta$) is

$$Z^\Delta(1/\Delta) = Z^\Delta(1) - Z^\Delta(0) + Z^\Delta(2) - Z^\Delta(1) + ... + Z^\Delta(1/\Delta) - Z^\Delta(1/\Delta - 1)$$

$$= X_1 + X_2 + ... + X_{1/\Delta} = S_{1/\Delta}.$$

Hence, $$\frac{Z^\Delta(1/\Delta) - 1/\Delta\,\mu^\Delta\Delta}{\sigma\Delta^{1/2}\sqrt{1/\Delta}} = \frac{Z^\Delta(1/\Delta) - \mu^\Delta}{\sigma} \rightarrow N(0,1),$$

or $$Z^\Delta(1/\Delta) \rightarrow N(\mu^\Delta, \sigma^2).$$

Hence, the account process $Z(t)$ starts at zero, has independent increments, and over any finite interval $[t,t']$ the increment in the account is normally distributed with

$$Z(t') - Z(t) \sim \mathrm{N}((t'-t)\mu, (t'-t)\sigma^2).$$

Note that the diffusion parameter σ depends neither on time t nor on the agent's choice of action – it depends only on the excess performance over the unit interval, $\hat{y}_g$, and the standard probability, $\hat{a}$, that gives a zero expected performance. This is a consequence of the fact that a^Δ converges to $\hat{a}$ as the length of each time interval Δ goes to zero (although it does so at the rate $\Delta^{½}$). The instantaneous drift μ, on the other hand, depends on the agent's action.

We now derive the compensation contract that implements a constant action. To do this it is useful to think of the agent choosing (a constant) μ^Δ in each period of length Δ, which then determines an associated action $a^\Delta(\mu^\Delta)$ by (19.9), i.e.,

$$a^\Delta(\mu^\Delta) = \hat{a} + \mu^\Delta \frac{\Delta^{½}}{\hat{y}_g}. \tag{19.12}$$

Since a^Δ is a probability, we must restrict the agent's choice of μ^Δ by

$$-\frac{\hat{y}_g \hat{a}}{\Delta^{½}} \le \mu^\Delta \le \frac{\hat{y}_g(1-\hat{a})}{\Delta^{½}}.$$

However, note that as Δ goes to zero, the bounds on μ^Δ become trivial, and μ^Δ can be chosen to be any real number by the agent. Similarly, we can express the agent's cost function in terms of μ^Δ,

$$\Delta\,\hat{\kappa}(\mu^\Delta) = \kappa^\Delta(a^\Delta(\mu^\Delta)) = \Delta\,\kappa\left(\hat{a} + \frac{a^\Delta(\mu^\Delta) - \hat{a}}{\Delta^{½}}\right).$$

Let δ_i^Δ denote the period-by-period compensation for obtaining signal $i = b,g$ that gives the agent a certainty equivalent equal to zero given μ^Δ. A zero certainty equivalent in each period and incentive compatibility of μ^Δ implies that (compare to (19.5))

$$\delta_g^\Delta = \Delta\,\hat{\kappa}(\mu^\Delta) - \frac{1}{r}\ln\left(1 - r\hat{\kappa}'(\mu^\Delta)\hat{y}_g\Delta^{½} + ra^\Delta\hat{\kappa}'(\mu^\Delta)\hat{y}_g\Delta^{½}\right),$$

$$\delta_b^\Delta = \Delta\,\hat{\kappa}(\mu^\Delta) - \frac{1}{r}\ln\left(1 + ra^\Delta\hat{\kappa}'(\mu^\Delta)\hat{y}_g\Delta^{½}\right).$$

Using a Taylor series expansion of the logarithmic term, the required compensations are given by

$$\delta_g^\Delta = \Delta\,\hat{\kappa}(\mu^\Delta) + \left(1 - a^\Delta\right)\hat{\kappa}'(\mu^\Delta)\,\hat{y}_g\Delta^{½}$$

$$+ ½r\left(1 - a^\Delta\right)^2\left(\hat{\kappa}'(\mu^\Delta)\,\hat{y}_g\right)^2\Delta + O(\Delta^{3/2}),$$

$$\delta_b^\Delta = \Delta\,\hat{\kappa}(\mu^\Delta) - a^\Delta\hat{\kappa}'(\mu^\Delta)\hat{y}_g\Delta^{½} + ½r\,{a^\Delta}^2\left(\hat{\kappa}'(\mu^\Delta)\hat{y}_g\right)^2\Delta + O(\Delta^{3/2}).$$

The accumulated compensation "earned" in the first τ periods is

$$C^\Delta(\tau) \equiv T_g(\tau)\delta_g^\Delta + (\tau - T_g(\tau))\,\delta_b^\Delta.$$

Hence, the expected incremental compensation is

$$\mathrm{E}_\tau[\,C^\Delta(\tau+1) - C^\Delta(\tau)\,|\,a^\Delta] = a^\Delta\delta_g^\Delta + (1 - a^\Delta)\delta_b^\Delta$$

$$= \Delta\,\hat{\kappa}(\mu^\Delta) + ½r\,a^\Delta(1 - a^\Delta)\left(\hat{\kappa}'(\mu^\Delta)\hat{y}_g\right)^2\Delta + O(\Delta^{3/2}),$$

and the difference between the actual incremental compensation and the expected incremental compensation is

$$C^\Delta(\tau+1) - C^\Delta(\tau) - \mathrm{E}_\tau[\,C^\Delta(\tau+1) - C^\Delta(\tau)\,|\,a^\Delta]$$

$$= \hat{\kappa}'(\mu^\Delta)\,\hat{y}_g \times \begin{cases} +\,(1 - a^\Delta)\Delta^{½} + O(\Delta) & \text{with probability } a^\Delta, \\ -\,a^\Delta\Delta^{½} + O(\Delta) & \text{with probability } 1 - a^\Delta. \end{cases}$$

The variance of the incremental compensation is

$$\mathrm{Var}_\tau[\,C^\Delta(\tau+1) - C^\Delta(\tau)\,|\,a^\Delta] = a^\Delta(1 - a^\Delta)\left(\hat{\kappa}'(\mu^\Delta)\hat{y}_g\right)^2\Delta + O(\Delta^{3/2}).$$

Since $\Delta^{3/2}$ goes faster to zero than Δ, the $O(\Delta^{3/2})$-terms can be ignored in both the expected incremental compensation and the variance of that incremental. Hence, as Δ goes to zero, the process for the accumulated compensation $C^\Delta(\tau)$ converges to a continuous-time Brownian motion $C(t)$, $t \in [0,1]$, on the form

$$dC(t) = \Big(\hat{\kappa}(\mu) + \tfrac{1}{2} r \big(\hat{\kappa}'(\mu)\hat{y}_g\big)^2 \hat{a}(1 - \hat{a})\Big) dt$$

$$+ \hat{\kappa}'(\mu)\hat{y}_g\sqrt{\hat{a}(1 - \hat{a})}\, dB(t). \tag{19.13}$$

The drift-term has two components. The first component is compensation for the incurred effort cost and the second component is a risk premium the agent is paid to compensate him for the incentive risk, i.e., the diffusion-term. The key here is that these payments are fixed such that the agent gets a certainty equivalent of zero.

The relation between the compensation and the performance measure follows from substituting the performance account process $Z(t)$ from (19.11) into (19.13):

$$dC(t) = \Big(\hat{\kappa}(\mu) + \tfrac{1}{2} r \big(\hat{\kappa}'(\mu)\hat{y}_g\big)^2 \hat{a}(1 - \hat{a}) - \hat{\kappa}'(\mu)\,\mu\Big) dt + \hat{\kappa}'(\mu)\, dZ(t).$$

Hence, the total compensation at $t = 1$ is (by "integrating both sides" and noting that $Z(0) = 0$)

$$C(1) = \hat{\kappa}(\mu) + \tfrac{1}{2} r \big(\hat{\kappa}'(\mu)\hat{y}_g\big)^2 \hat{a}(1 - \hat{a}) - \hat{\kappa}'(\mu)\,\mu + \hat{\kappa}'(\mu)\, Z(1),$$

where $Z(1)$ is normally distributed with

$$Z(1) \sim N(\mu,\sigma^2),\ \sigma = \hat{y}_g\sqrt{\hat{a}(1 - \hat{a})}.$$

Since the performance account at $t = 1$ is equal to the aggregate performance at $t = 1$, i.e., $y = Z(1)$, we may write the optimal compensation contract as a linear function of a normally distributed aggregate performance measure, i.e.,

$$c(y) = f + vy, \quad y \sim N(\mu,\sigma^2), \tag{19.14}$$

where
$$f = w + \hat{\kappa}(\mu) + \tfrac{1}{2} r \big(\hat{\kappa}'(\mu)\sigma\big)^2 - \hat{\kappa}'(\mu)\,\mu,$$

$$v = \hat{\kappa}'(\mu).$$

The fixed component of the compensation consists of the agent's reservation wage, a compensation for the incurred effort cost, a risk premium for the incentive risk, minus the expected incentive wage. The incentive wage is determined by the agent's marginal cost of providing expected aggregate performance μ.

Note that the characterization of the optimal contract in (19.14) is precisely the same as the characterization (19.3) of the optimal linear contract in Section 19.1. Hence, the optimal contract in a setting where the agent continuously controls the drift of a Brownian motion for aggregate performance may be found as the optimal linear contract in a static setting, where the agent's action is the mean of a normally distributed performance.[14]

Multi-dimensional Brownian Motion

We now turn to the multi-dimensional Brownian version of the setting with two performance measures considered in Section 19.2.3. In each period of length Δ there are two binary performance measures y_i^Δ which may take values y_{bi}^Δ and y_{gi}^Δ, $i = 1, 2$. As in Section 19.2.3, there are four possible signals: $\psi_0^\Delta = (y_{b1}^\Delta, y_{b2}^\Delta)$, $\psi_1^\Delta = (y_{b1}^\Delta, y_{g2}^\Delta)$, $\psi_2^\Delta = (y_{g1}^\Delta, y_{b2}^\Delta)$, and $\psi_3^\Delta = (y_{g1}^\Delta, y_{g2}^\Delta)$. We represent the four signals by numbers also denoted ψ_i^Δ, $i = 0, 1, 2, 3$.[15]

The agent's action is the probability of each of these four performance signals, φ_i^Δ, $i = 0, 1, 2, 3$. In the subsequent analysis we assume the agent's action space is the simplex determined by $\Sigma_i \varphi_i^\Delta = 1$, $\varphi_i^\Delta \geq 0$. Hence, the agent's action *cannot* be represented as a single-dimensional choice variable that affects the probabilities of the good signals for both performance measures. The problem with a single-dimensional choice variable is that the compensations for the four different signals cannot be determined from the incentive constraints and the contract acceptance constraint alone. However, that is possible if we represent the set of choices as a simplex, and we can perform the analysis as in the previous section except that the agent now controls a multinomial instead of a binomial process. In particular, we fix a standard probability vector $\hat{\boldsymbol{\varphi}}$ as the probabilities that give an expected performance of zero, i.e.,

$$\sum_{i=0}^{3} \hat{\varphi}_i \psi_i^\Delta = 0, \tag{19.15}$$

[14] Note that the Brownian motion model avoids the Mirrlees "problem" even though the aggregate performance measure is normally distributed. This has to do with the fact that the agent can effectively avoid the penalties by his action choices as he observes the performance measure continuously. If he only observes the normally distributed performance measures at discrete points in time, the Mirrlees argument applies no matter how small the intervals are between his observations (see Müller, 2000, for a formal development of this point). However, it is not important how often the principal observes the aggregate performance measure.

[15] At this point, these numbers are generic representations of the information in each period. Below we consider the case in which these numbers are linear aggregates of the two performance measures.

and let $\hat{\psi}_i^\Delta$ denote the "excess performance" from obtaining signal i as opposed to the signal 0, i.e., $\hat{\psi}_i^\Delta = \psi_i^\Delta - \psi_0^\Delta$, $i = 1, 2, 3$. We can now define the "performance account" for each of the signals $i = 1, 2, 3$ by

$$Z_i^\Delta(\tau) = \hat{\psi}_i^\Delta (T_i(\tau) - \hat{\varphi}_i \tau), \quad i = 1, 2, 3, \tag{19.16}$$

where $T_i(\tau)$ is the number of times signal i has occurred in the first τ periods. Note that these accounts are *not* independent since only one account can change value in each period. If we take the sum of the performance accounts at any date τ using (19.14), we get the aggregate performance, i.e.,

$$\sum_{i=1}^{3} Z_i^\Delta(\tau) = \sum_{i=0}^{3} \psi_i^\Delta T_i(\tau). \tag{19.17}$$

Again, we let the "excess performance" in each period be proportional to $\Delta^{½}$, i.e.,

$$\hat{\psi}_i^\Delta = \hat{\psi}_i \Delta^{½}, \quad i = 1, 2, 3,$$

and define
$$\mu_i^\Delta \equiv \hat{\psi}_i \frac{\varphi_i^\Delta - \hat{\varphi}_i}{\Delta^{½}}, \quad i = 1, 2, 3. \tag{19.18}$$

Note that by using (19.15) the total expected performance over the unit interval is

$$\frac{1}{\Delta} \sum_{i=0}^{3} \varphi_i^\Delta \psi_i^\Delta = \sum_{i=1}^{3} \mu_i^\Delta,$$

such that μ_i^Δ can be interpreted as the contribution to the aggregate expected performance over the unit interval from shifting probability to signal i, $i = 1, 2, 3$, from signal zero.

Note that each $T_i(\tau)$ is generated by a binomial process with $T_i(\tau+1) - T_i(\tau) \in \{0, 1\}$ and $\mathrm{E}_\tau[T_i(\tau+1) - T_i(\tau) | \boldsymbol{\varphi}^\Delta] = \varphi_i^\Delta$. Hence,

$$\mathrm{E}_\tau[Z_i^\Delta(\tau+1) - Z_i^\Delta(\tau) | \boldsymbol{\varphi}^\Delta]$$

$$= \mathrm{E}_\tau[\hat{\psi}_i^\Delta (T_i(\tau+1) - T_i(\tau) - \hat{\varphi}_i) | \boldsymbol{\varphi}^\Delta] = \mu_i^\Delta \Delta, \quad i = 1, 2, 3,$$

so that we can write the process $Z_i^\Delta(\tau)$ as follows:

$$Z_i^{\Delta}(\tau+1) - Z_i^{\Delta}(\tau)$$

$$= \mu_i^{\Delta}\Delta + \hat{\psi}_i \times \begin{cases} +(1-\varphi_i^{\Delta})\Delta^{1/2} & \text{with prob. } \varphi_i^{\Delta}, \\ -\varphi_i^{\Delta}\Delta^{1/2} & \text{with prob. } 1-\varphi_i^{\Delta}, \end{cases} \qquad i = 1,2,3. \quad (19.19)$$

The variance of the change of account i is

$$\mathrm{Var}_{\tau}[Z_i^{\Delta}(\tau+1) - Z_i^{\Delta}(\tau) \,|\, \boldsymbol{\varphi}^{\Delta}] = \hat{\psi}_i^2 \varphi_i^{\Delta}(1-\varphi_i^{\Delta})\Delta, \quad i = 1,2,3,$$

and the covariance between any two accounts is

$$\mathrm{Cov}_{\tau}[Z_i^{\Delta}(\tau+1) - Z_i^{\Delta}(\tau),\, Z_j^{\Delta}(\tau+1) - Z_j^{\Delta}(\tau) \,|\, \boldsymbol{\varphi}^{\Delta}]$$

$$= -\hat{\psi}_i \hat{\psi}_j \varphi_i^{\Delta}\varphi_j^{\Delta}\Delta, \quad i,j = 1,2,3;\ i \neq j.$$

As in the one-dimensional case it is now relatively straightforward to show that for Δ approaching zero, the process for the three performance accounts converges to a three-dimensional continuous-time Brownian motion $\mathbf{Z}(t)$, $t \in [0,1]$, with instantaneous drift vector $\boldsymbol{\mu}$ and diffusion matrix $\boldsymbol{\Sigma}^{1/2}$ which we formally write as

$$d\mathbf{Z}(t) = \boldsymbol{\mu}dt + \boldsymbol{\Sigma}^{1/2}d\mathbf{B}(t), \tag{19.20}$$

where

$\mathbf{B}(t)$is a standard three-dimensional Brownian motion with $\mathbf{B}(0) = 0$, independent increments and $\mathbf{B}(t') - \mathbf{B}(t) \sim \mathrm{N}(0,(t'-t)\mathbf{I})$ for $t < t'$.

$$\boldsymbol{\Sigma} = \begin{Bmatrix} \hat{\psi}_1^2\hat{\varphi}_1(1-\hat{\varphi}_1) & -\hat{\psi}_1\hat{\psi}_2\hat{\varphi}_1\hat{\varphi}_2 & -\hat{\psi}_1\hat{\psi}_3\hat{\varphi}_1\hat{\varphi}_3 \\ -\hat{\psi}_2\hat{\psi}_1\hat{\varphi}_2\hat{\varphi}_1 & \hat{\psi}_2^2\hat{\varphi}_2(1-\hat{\varphi}_2) & -\hat{\psi}_2\hat{\psi}_3\hat{\varphi}_2\hat{\varphi}_3 \\ -\hat{\psi}_3\hat{\psi}_1\hat{\varphi}_3\hat{\varphi}_1 & -\hat{\psi}_3\hat{\psi}_2\hat{\varphi}_{3_2} & \hat{\psi}_3^2\hat{\varphi}_3(1-\hat{\varphi}_3) \end{Bmatrix}.$$

Hence, the account process $\mathbf{Z}(t)$ starts at zero, has independent increments, and over any finite interval $[t,t']$ the increments in the accounts are jointly normally distributed with

$$\mathbf{Z}(t') - \mathbf{Z}(t) \sim \mathrm{N}((t'-t)\boldsymbol{\mu},(t'-t)\boldsymbol{\Sigma}).$$

In order to derive the compensation contract that implements the (constant) drift vector $\boldsymbol{\mu}$ we express the agent's cost as a function of $\boldsymbol{\mu}^{\Delta}$ recognizing the relation between the drift rates and the associated action given by (19.18), i.e., as a function $\hat{\kappa}(\boldsymbol{\mu}^{\Delta})$. Following steps similar to those in the one-dimensional case, it can be shown that the process for accumulated compensation (with a zero certainty equivalent) converges to a (one-dimensional) continuous-time Brownian motion $C(t)$, $t \in [0,1]$, on the form[16]

$$dC(t) = \left(\hat{\kappa}(\boldsymbol{\mu}) + \tfrac{1}{2} r \hat{\boldsymbol{\kappa}}'(\boldsymbol{\mu})^t \boldsymbol{\Sigma} \hat{\boldsymbol{\kappa}}'(\boldsymbol{\mu}) - \hat{\boldsymbol{\kappa}}'(\boldsymbol{\mu})\,\boldsymbol{\mu}\right) dt + \hat{\boldsymbol{\kappa}}'(\boldsymbol{\mu})\, d\mathbf{Z}(t),$$

where $\hat{\boldsymbol{\kappa}}'(\boldsymbol{\mu})$ denotes the vector of the partial derivatives of the agent's cost function with respect to the drift rates, μ_i, in each account. The total compensation at $t = 1$ is

$$C(1) = \hat{\kappa}(\boldsymbol{\mu}) + \tfrac{1}{2} r \hat{\boldsymbol{\kappa}}'(\boldsymbol{\mu})^t \boldsymbol{\Sigma} \hat{\boldsymbol{\kappa}}'(\boldsymbol{\mu}) - \hat{\boldsymbol{\kappa}}'(\boldsymbol{\mu})\,\boldsymbol{\mu} + \hat{\boldsymbol{\kappa}}'(\boldsymbol{\mu})\, \mathbf{Z}(1),$$

where $\mathbf{Z}(1)$ is normally distributed with

$$\mathbf{Z}(1) \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}).$$

Hence, we can write the optimal compensation contract as a constant plus a linear function of the three jointly normally distributed performance accounts at $t = 1$, i.e.,

$$c(\mathbf{z}) = f + \mathbf{v}^t \mathbf{z}, \quad \mathbf{z} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \tag{19.21a}$$

where

$$f = w + \hat{\kappa}(\boldsymbol{\mu}) + \tfrac{1}{2} r \hat{\boldsymbol{\kappa}}'(\boldsymbol{\mu})^t \boldsymbol{\Sigma} \hat{\boldsymbol{\kappa}}'(\boldsymbol{\mu}) - \hat{\boldsymbol{\kappa}}'(\boldsymbol{\mu})\,\boldsymbol{\mu}, \tag{19.21b}$$

$$\mathbf{v} = \hat{\boldsymbol{\kappa}}'(\boldsymbol{\mu}). \tag{19.21c}$$

The fixed component has an interpretation similar to the one-dimensional case, i.e., it consists of the agent's reservation wage, compensation for the incurred effort cost, and a risk premium for the incentive risk, minus the expected incentive wage. The incentive wage is determined by the agent's vector of marginal costs of providing expected aggregate performance μ_i in each of the different performance accounts. Hence, the optimal contract in a setting in which the agent continuously controls the drift vector of a Brownian motion for aggregate performance accounts may be found as the optimal linear contract in a

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[16] Compare to HM, Theorems 6 and 7, and Hellwig and Schmidt (2002, Theorem 1).

static setting, where the agent's action is the mean vector of jointly normally distributed performance accounts.

It is useful to note that the agent's compensation is independent of the numbers assigned to the four signals. To illustrate, assume that all numbers are multiplied by a scalar λ. This will result in a mean vector $\boldsymbol{\mu}^{\lambda} = \lambda\boldsymbol{\mu}$, a covariance matrix $\boldsymbol{\Sigma}^{\lambda} = \lambda^2\boldsymbol{\Sigma}$, a vector of account totals $\mathbf{z}^{\lambda} = \lambda\mathbf{z}$, and a marginal cost vector $\boldsymbol{\kappa}^{\lambda\prime}(\boldsymbol{\mu}^{\lambda}) = \boldsymbol{\kappa}'(\boldsymbol{\mu})/\lambda$. Substituting these relations into (19.21) readily establishes that the new optimal contract is characterized by $f^{\lambda} = f$ and $\mathbf{v}^{\lambda} = \mathbf{v}/\lambda$, so that

$$c^{\lambda}(\mathbf{z}^{\lambda}) = f^{\lambda} + \mathbf{v}^{\lambda t}\mathbf{z}^{\lambda} = f + (\mathbf{v}^t/\lambda)(\lambda\mathbf{z}) = f + \mathbf{v}^t\mathbf{z} = c(\mathbf{z}).$$

Can we express the optimal compensation contract as a linear function of the aggregate performance for the two performance measures y_1 and y_2? In general, the answer is NO!

We can define the numbers that represent the four signals by a linear aggregate of the two basic performance measures, i.e.,

$$\psi^{\Delta} \equiv g_1 y_1^{\Delta} + g_2 y_2^{\Delta},$$

for some constants g_1 and g_2. For example, if y_1^{Δ} and y_2^{Δ} are revenue and cost measures, then ψ^{Δ} is a profit measure if $g_1 = 1$ and $g_2 = -1$. The aggregate of the performance accounts in (19.17) is then given by

$$\sum_{i=1}^{3} Z_i^{\Delta}(\tau) = \sum_{i=0}^{3} \psi_i^{\Delta} T_i(\tau)$$

$$= g_1\left[y_{b1}^{\Delta}\big(T_0(\tau) + T_2(\tau)\big) + y_{g1}^{\Delta}\big(T_1(\tau) + T_3(\tau)\big)\right]$$

$$+ g_2\left[y_{b2}^{\Delta}\big(T_0(\tau) + T_1(\tau)\big) + y_{g2}^{\Delta}\big(T_2(\tau) + T_3(\tau)\big)\right],$$

which is equal to a linear function of the aggregate performances for each of the two basic performance measures. Hence, in the limiting continuous-time setting the aggregate of the performance accounts is a linear function of the aggregate performances for each of the two basic performance measures, i.e,

$$\sum_{i=1}^{3} Z_i(1) = g_1 y_1 + g_2 y_2. \tag{19.22}$$

However, the optimal incentive wage is

$$\mathbf{v}^t\mathbf{z} = \sum_{i=1}^{3} \hat{\kappa}_i(\boldsymbol{\mu}) Z_i(1), \tag{19.23}$$

where $\hat{\kappa}_i(\boldsymbol{\mu})$ is the agent's marginal cost of providing expected aggregate performance μ_i on performance account i. Hence, *unless* these marginal costs are the same for all accounts, we cannot express the optimal compensation contract as a linear function of y_1 and y_2. The problem is that with two binary performance measures, the necessary Brownian motion to describe optimal compensation is not two-dimensional, it is three-dimensional. In general, with N "binary" performance measures, the Brownian motion must be of dimension $N^2 - 1$.

HM consider two special cases of the multi-dimensional Brownian motion model in which the optimal compensation is in fact a linear function of the aggregate performance for the two performance measures y_1 and y_2. The first case is a setting in which the agent's cost function depends only on the expected aggregate performance, i.e., the cost function can be written as

$$\hat{\kappa}_i(\boldsymbol{\mu}) = \hat{k}_i\left(\sum_{i=1}^{3} \mu_i\right).$$

In that setting, the agent's marginal costs are the same for all accounts and the result follows from (19.22) and (19.23). Unfortunately, as demonstrated by Hellwig (2001), this Brownian motion model *cannot* be derived as the limit of a discrete-time model since the corresponding cost function in discrete time implies that it is optimal for the principal only to induce positive probability for two "neighboring signals" in order to reduce the risk premium to the agent. Hence, the first-best solution can "almost" be implemented. This does not occur in the continuous-time Brownian motion setting, since the covariance matrix can be exogenously specified in that setting.

The second special case examined by HM is a setting in which the principal does not observe the vector of performance accounts $\mathbf{Z}(t)$ but only some linear aggregate of those accounts (see HM, Theorem 8), for example the sum of the accounts, i.e.,

$$Y(t) = \sum_{i=1}^{3} Z_i(t).$$

In that case, the optimal contract is a linear function of $Y(1)$ as in (19.21). Of course, contracting on Y is less desirable than contracting on $\mathbf{Z}$.[17] The principal loses if he throws away information. It is more costly to implement a given action vector $\boldsymbol{\mu}$, and only action vectors in which the agent has the same marginal costs for each account can be implemented. Hence, using a linear contract based on the aggregate performance Y is sub-optimal if more detailed information is available. Hellwig and Schmidt (2002) also note that this continuous-time Brownian motion setting has no immediate discrete-time analog. The key is that in a discrete-time setting, observing the process for aggregate performance $Y^{\Delta}(\tau)$ enables the principal to infer the processes of the individual accounts (by observing the size of the increments). Hellwig and Schmidt (2002) develop a discrete-time setting in which "approximately" optimal sharing rules are linear in aggregate performance. They impose two key assumptions: the principal only observes the final aggregate performance $Y^{\Delta}(1)$ and not the time path, and the agent can understate aggregate performance such that the sharing rule has to be non-decreasing.

19.3 CONCLUDING REMARKS

The analysis in Section 19.2 establishes that if the agent has exponential utility and there is a single normally distributed performance measure, then restricting the contract to be linear is not a simplification if the performance measure is interpreted to be the result of a one-dimensional Brownian motion. However, that result does not extend to settings in which there are two or more performance measures. Consequently, when we use the *LEN* model we are generally doing this to simplify the analysis, and to facilitate comparative statics with respect to concepts such as risk, sensitivity, and risk aversion. That is, the optimal linear contract is not the optimal contract when there are two or more performance measures.

The basic *LEN* model is introduced in Section 19.1. Extensions are considered in several of the subsequent chapters. For example, Section 20.2 considers multiple tasks, Sections 21.2 and 21.3 consider a firm's market price as an aggregate performance measure and as a proxy for non-contractible investor information, various sections in Chapters 25 through 27 consider multi-period *LEN* models with full commitment and various degrees of inter-dependence, Chapter 28 revisits those multi-period models but with the assumption that the principal

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[17] Of course, if the principal observes a linear aggregate of the accounts with weights $\hat{\kappa}_i(\boldsymbol{\mu})$ for the optimal action vector $\boldsymbol{\mu}$ with $\mathbf{Z}$ observable, there is no loss of efficiency. This is similar to the likelihood ratio being a sufficient statistic for optimal compensation in the multiple signals setting in Section 18.1.4.

and agent cannot preclude renegotiation of long-term contracts after a report has been released, and Section 29.3 includes a hierarchical multi-agent *LEN* model with delegated contracting.

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CHAPTER 20

MULTIPLE TASKS AND MULTIPLE PERFORMANCE MEASURES

In the preceding chapters we assumed that the set of actions A from which the agent must choose is either a finite or continuous set that is ordered such that more "effort" increases the "benefit" to the principal and increases the "cost" to the agent. We now recognize that an agent often performs multiple tasks and, therefore, his effort is more realistically described as a vector of actions $\mathbf{a}$ = $(a_1, ..., a_m)^t$ such that a_j refers to the "effort" expended in task j. In this setting, there can be two actions $\mathbf{a}'$ and $\mathbf{a}''$ that have the same "cost" to the agent but different "benefits" to the principal. This shifts the focus from being only concerned about the optimal "intensity" of effort, to being concerned about both the "intensity" and "allocation" of the effort expended by the agent. When "intensity" is the only concern, the key criterion in selecting performance measures is the extent to which they help minimize the risk imposed on the agent to induce the desired intensity. However, when "allocation" of effort is also of concern, the choice of performance measures must consider both the "allocation" induced and the risk that must be imposed to induce a particular level of "intensity" and "allocation."

A performance measure that aligns the agent's effort allocation with the benefits to the principal is referred to as being *perfectly congruent* with the principal's objectives. A single performance measure's lack of perfect congruency or inclusion of noise can result in other performance measures having value because they complement the first measure either by improving the allocation of effort or by reducing the risk imposed to induce a particular allocation.

In Section 20.1 we introduce the elements of the basic multi-task model, which are similar to the formulation by Gjesdal (1982). Section 20.2 explores multi-dimensional versions of the *LEN* model introduced in Chapter 19. This simplified model provides a number of insights into the role of multiple performance measures in motivating multi-dimensional effort and identifies the factors that affect the relative weights placed on the performance measures. Finally, in Section 20.3, we explore some simple settings introduced by Holmström and Milgrom (1991) (HM), in which the form of the agent's cost function significantly influences the nature of the optimal incentives and effort allocation.

20.1 BASIC MULTI-TASK MODEL

We assume that the principal is risk neutral and "owns" the outcome x, while a risk and effort averse agent implements an m-dimensional action $\mathbf{a} \in A \subseteq \mathbb{R}^m$. The agent's preferences are represented by a separable utility function $u^a(c,\mathbf{a}) = u(c)k(\mathbf{a}) - v(\mathbf{a})$, which is defined over the agent's compensation $c \in C$ and his action $\mathbf{a}$. As before, the agent's action $\mathbf{a}$ is assumed to be non-observable by the principal. The contractible performance measures generated by system η consist of a vector of n performance measures, denoted , $\mathbf{y} = (y_1,...,y_n)^t \in Y \subseteq \mathbb{R}^n$, which may or may not include the outcome x. The compensation contract offered to the agent by the principal is $\boldsymbol{c}$: $Y \rightarrow C$. The expected outcome (benefit) to the principal given action $\mathbf{a}$ is given by $b(\mathbf{a}) \equiv \mathrm{E}[x|\mathbf{a}]$ and the homogeneous signal beliefs are represented by the probability distribution function $\Phi(\mathbf{y}|\mathbf{a},\eta)$.

20.1.1 General Formulation of the Principal's Problem

The principal's decision problem is formulated as follows.

$$U^p(\eta) = \underset{\boldsymbol{c},\mathbf{a}}{\text{maximize}} \quad U^p(\boldsymbol{c},\mathbf{a},\eta) \equiv b(\mathbf{a}) - \int_Y \boldsymbol{c}(\mathbf{y})\, d\Phi(\mathbf{y}|\mathbf{a},\eta), \tag{20.1}$$

$$\text{subject to} \quad U^a(\boldsymbol{c},\mathbf{a},\eta) \equiv \int_Y u^a(\boldsymbol{c}(y),\mathbf{a})\, d\Phi(\mathbf{y}|\mathbf{a},\eta) \geq \bar{U}, \tag{20.2}$$

$$\mathbf{a} \in \underset{\mathbf{a}' \in A}{\text{argmax}} \; U^a(\boldsymbol{c},\mathbf{a}',\eta), \tag{20.3}$$

$$\boldsymbol{c}(\mathbf{y}) \in C, \quad \forall\, \mathbf{y} \in Y. \tag{20.4}$$

Nothing of substance has changed from the general formulation in Chapter 18, except that we have replaced a with $\mathbf{a}$ and y with $\mathbf{y}$, which merely emphasizes that they are vectors. The significance of the multi-dimensional effort becomes more obvious if we assume that the set of possible actions is a convex set of the form $A = [\underline{a}_1,\bar{a}_1] \times ... \times [\underline{a}_m,\bar{a}_m]$. In that case, if the first-order conditions for the agent's decision problem characterize his action choice, then incentive constraint (20.3) is replaced by an $m \times 1$ vector of incentive constraints of the form,

$$\nabla_{\mathbf{a}} U^a(\boldsymbol{c},\mathbf{a},\eta) = \mathbf{0}, \tag{20.3'}$$

where $\nabla_{\mathbf{a}}$ denotes the gradient with respect to $\mathbf{a}$ (i.e., the $m \times 1$ vector of derivatives with respect to the elements of $\mathbf{a}$) and $\mathbf{0}$ is an $m \times 1$ vector of zeros. With separable utility,

$$\nabla_{\mathbf{a}} U^a(c,\mathbf{a},\eta) = \int_Y [\nabla_{\mathbf{a}} k(\mathbf{a})\, \varphi(\mathbf{y}|\mathbf{a}) + k(\mathbf{a}) \nabla_{\mathbf{a}} \varphi(\mathbf{y}|\mathbf{a})]\, u(c(\mathbf{y}))\, d\mathbf{y} - \nabla_{\mathbf{a}} v(\mathbf{a}).$$

The Lagrangian in this setting is

$$\mathcal{L} = U^p(c,\mathbf{a},\eta) + \lambda[U^a(c,\mathbf{a},\eta) - \bar{U}] + \boldsymbol{\mu}^t \nabla_{\mathbf{a}} U^a(c,\mathbf{a},\eta)$$
$$+ \int_Y \xi(\mathbf{y})\, [c(\mathbf{y}) - \underline{c}]\, \varphi(\mathbf{y}|\mathbf{a})\, d\mathbf{y},$$

where $\boldsymbol{\mu}$ is an $m \times 1$ vector of Lagrange multipliers for the m incentive constraints. Differentiating $\mathcal{L}$ with respect to $c(\mathbf{y})$, and assuming $c(\mathbf{y}) > \underline{c}$, yields the following characterization of the optimal incentive contract:

$$M(c(\mathbf{y})) = k(\mathbf{a})\{\lambda + \boldsymbol{\mu}^t\, [\mathbf{K}(\mathbf{a}) + \mathbf{L}(\mathbf{y}|\mathbf{a})]\}, \tag{20.5}$$

where
$$\mathbf{K}(\mathbf{a}) = \frac{1}{k(\mathbf{a})} \nabla_{\mathbf{a}}\, k(\mathbf{a}),$$

$$\mathbf{L}(\mathbf{y}|\mathbf{a}) = \frac{1}{\varphi(\mathbf{y}|\mathbf{a})} \nabla_{\mathbf{a}} \varphi(\mathbf{y}|\mathbf{a}).$$

If the agent's utility function is additively separable, then $k(\mathbf{a}) = 1$ and $\mathbf{K}(\mathbf{a}) = \mathbf{0}$. In that case, we see that the key factors affecting the compensation $c(\mathbf{y})$ are the likelihood ratios for each task and the endogenously determined weights in $\boldsymbol{\mu}$ for each task. This structure is particularly simple if there is a separate independent performance measure for each action (i.e., $\mathbf{y}$ has m elements and $\varphi(\mathbf{y}|\mathbf{a}) = \varphi(y_1|a_1) \times \ldots \times \varphi(y_m|a_m)$).

If the agent's utility function is multiplicative negative exponential (i.e., $v(\mathbf{a}) = 0$ and $k(\mathbf{a}) = \exp[r\kappa(\mathbf{a})]$), then $\mathbf{K}(\mathbf{a}) = r\nabla_{\mathbf{a}}\kappa(\mathbf{a})$, and the optimal contract takes the following form:

$$c(\mathbf{y}) = \kappa(\mathbf{a}) + \frac{1}{r} \ln\Big(r\big(\lambda + \boldsymbol{\mu}^t[r\nabla_{\mathbf{a}}\kappa(\mathbf{a}) + \mathbf{L}(\mathbf{y}|\mathbf{a})]\big)\Big). \tag{20.6}$$

Hence, the compensation covers the agent's personal cost $\kappa(\mathbf{a})$ and provides incentives that depend on the marginal cost of the effort in each task and the likelihood ratio for each task. Observe that the incentive component of the optimal contract is *concave* with respect to the likelihood ratios, which vary with **y**.

Two types of personal costs of effort are common in the literature. One, which we refer to as the *aggregate cost model*, assumes that the cost of effort depends only on the aggregate effort, not the tasks *per se*, i.e., the cost function can be expressed as $\kappa(a^+)$, where $a^+ = a_1 + ... + a_m$, and all the elements of the vector $\nabla_{\mathbf{a}}\kappa(\mathbf{a})$ equal $\kappa'(a^+)$. This is descriptive of settings in which the cost of effort depends on total hours worked, but is independent of the specific tasks undertaken. The other type, which we refer to as the *separable cost model*, assumes the cost is a quadratic function of the form $\kappa(\mathbf{a}) = \frac{1}{2}\mathbf{a}^t\mathbf{\Gamma}\mathbf{a}$, where $\mathbf{\Gamma}$ is an $m \times m$ symmetric positive definite matrix. In this setting, $\nabla_{\mathbf{a}}\kappa(\mathbf{a}) = \mathbf{\Gamma}\mathbf{a}$, which takes a particular simple form if $\mathbf{\Gamma}$ is an identity matrix (implying that total cost is the sum of independent convex functions for each task). It is representative of settings in which, for example, the agent finds it "painful" to spend too much time on any one task.

In general, if the aggregate cost model is used, then the benefit function $b(\mathbf{a})$ is assumed to be strictly concave, so that there is an interior first-best optimum level of effort in each task. However, if the quadratic separable cost model is used, the benefit function $b(\mathbf{a})$ can be linear and still yield an interior first-best optimum level of effort in each task.

20.1.2 Exponential Utility with Normally Distributed Compensation

Participation constraint (20.2) and incentive constraint (20.3′) take particularly simple forms if the agent has a multiplicative negative exponential utility function and his compensation is normally distributed with a mean and variance that depend on his actions. To illustrate, assume $c = \mathbf{c}(\mathbf{y}) \sim \mathrm{N}(\bar{c}(\mathbf{a}), \sigma_c^2(\mathbf{a}))$. It then follows from Proposition 2.7 that

$$U^a(\mathbf{c},\mathbf{a},\eta) = -\exp[-r(\bar{c}(\mathbf{a}) - \kappa(\mathbf{a}) - \tfrac{1}{2}r\sigma_c^2(\mathbf{a}))]. \tag{20.7}$$

In that setting, if $\bar{U} = -\exp[-rc^o]$, the participation and incentive constraints can be restated as:

$$\bar{c}(\mathbf{a}) - \kappa(\mathbf{a}) - \tfrac{1}{2}r\sigma_c^2(\mathbf{a}) = c^o, \tag{20.2′}$$

$$\nabla_{\mathbf{a}}\bar{c}(\mathbf{a}) = \nabla_{\mathbf{a}}\kappa(\mathbf{a}) + \tfrac{1}{2}r\nabla_{\mathbf{a}}\sigma_c^2(\mathbf{a}). \tag{20.3″}$$

From (20.6) we observe that the compensation given the optimal contract will only be normally distributed if $\ln(\lambda + \boldsymbol{\mu}^t[r\nabla_{\mathbf{a}}\kappa(\mathbf{a}) + \mathbf{L}(\mathbf{y}|\mathbf{a})])$ is normally distributed. This condition is not satisfied by any "standard" distribution of **y**, including the normal or log-normal distributions. However, we can achieve the simplified representation in (20.7) with the normal or lognormal distributions of **y** if the contracts considered are linear functions of **y** or ln **y**, respectively. (Observe that the normal distribution with linear contracts is the *LEN* model introduced in Section 19.1.)

Lemma 20.1

The agent's compensation c is normally distributed with mean $f + \mathbf{v}\bar{\mathbf{g}}(\mathbf{a})$ and variance $\mathbf{v}^t\boldsymbol{\Sigma}(\mathbf{a})\mathbf{v}$ if $C = (-\infty, +\infty)$ and there exists a function $\mathbf{g}(\mathbf{y})$ such that

$$\mathbf{g}(\mathbf{y}) \sim \mathrm{N}(\bar{\mathbf{g}}(\mathbf{a}), \boldsymbol{\Sigma}(\mathbf{a})) \text{ and } c(\mathbf{y}) = f + \mathbf{v}^t\mathbf{g}(\mathbf{y}). \tag{20.8}$$

This condition is satisfied if:

(a) the performance measures are normally distributed and the compensation function is linear, i.e.,

$$\mathbf{g}(\mathbf{y}) = \mathbf{y} \sim \mathrm{N}(\bar{\mathbf{g}}(\mathbf{a}), \boldsymbol{\Sigma}(\mathbf{a})) \text{ and } c(\mathbf{y}) = f + \mathbf{v}^t\mathbf{y}; \text{ or}$$

(b) the performance measures are log-normally distributed and the compensation is a linear function of the logs of the performance measures, i.e.,[1]

$$\mathbf{g}(\mathbf{y}) = \begin{bmatrix} \ln y_1 \\ \vdots \\ \ln y_n \end{bmatrix} \sim \mathrm{N}(\bar{\mathbf{g}}(\mathbf{a}), \boldsymbol{\Sigma}(\mathbf{a})) \text{ and } c(\mathbf{y}) = f + v_1 \ln y_1 + \ldots + v_n \ln y_n.$$

The compensation functions in the above lemma are not optimal – they are merely tractable. This tractability has led a number of authors to use linear contracts with normally distributed performance measures, and the results from some of these papers are discussed in the next section. Hemmer (1996) considers log-normal performance measures with "log" compensation functions. His focus on log-normal measures arises from his observation that many performance measures do not take on negative values. In finance, market prices are often assumed to have a log-normal distribution, which follows from the assumption that the continuously compounded rate of return is normally distributed.

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[1] In this setting, $\bar{g}_i(\mathbf{a}) = \mathrm{E}[\ln y_i | \mathbf{a}]$ and $\sigma_{ih}(\mathbf{a}) = \mathrm{Cov}[\ln y_i, \ln y_h | \mathbf{a}]$.

20.2 ALLOCATION OF EFFORT AMONG TASKS WITH SEPARABLE EFFORT COSTS

We now examine the multi-dimensional *LEN* model introduced in Section 19.1 and expanded upon above. Much of the discussion is based on analyses found in Feltham and Xie (1994) (FX) and Feltham and Wu (2000) (FW). FX consider an arbitrary number of tasks and an arbitrary number of performance measures, and focus on the value of additional performance measures. FW, on the other hand, restrict their analysis to two tasks and two performance measures, and focus on the factors affecting the relative incentive weights applied to the two performance measures. Other papers of interest in this area include Banker and Thevaranjan (1998) and Datar, Kulp, and Lambert (2001).

20.2.1 A "Best" Linear Contract

As in the basic multi-task model, **a** is an $m \times 1$ vector of actions. However, we introduce a number of assumptions that simplify the analysis, but retain sufficient complexity to provide the basis for obtaining interesting insights into the role of multiple performance measures in a multi-task setting.

First-best Solution

The expected benefit to the principal from the agent's actions is assumed to be linear, i.e., $\mathrm{E}[x|\mathbf{a}] = \mathbf{b}^t\mathbf{a}$, while the cost of the actions to the agent is assumed to be separable and quadratic, i.e., $\kappa(\mathbf{a}) = \frac{1}{2}\mathbf{a}^t\mathbf{a}$.[2] If the agent's reservation utility is $\underline{U} = -\exp[-rc^o]$, then in the first-best setting (i.e., **a** is contractible information), the principal must pay the agent $c^o + \kappa(\mathbf{a})$ and will select **a** so as to maximize

$$U^p(\mathbf{a}) = \mathbf{b}^t\mathbf{a} - \{c^o + \tfrac{1}{2}\mathbf{a}^t\mathbf{a}\}. \tag{20.9a}$$

Differentiating (20.9a) with respect to **a** gives the first-order condition for the first-best action choice:

$$\mathbf{a}^* = \mathbf{b}, \tag{20.9b}$$

i.e., the marginal cost to the agent equals the marginal benefit to the principal for each task. Substituting (20.9b) in (20.9a) gives the principal's first-best expected utility:

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2 We could readily use a more general quadratic function $\kappa(\mathbf{a}) = \frac{1}{2}\mathbf{a}^t\boldsymbol{\Gamma}\,\mathbf{a}$, but that would merely complicate the presentation of the analysis and provide little in the way of additional insights.

$$U^{p*} = \tfrac{1}{2}\mathbf{b}^t\mathbf{b} - c^o. \tag{20.9c}$$

Observe that in the two-task setting, (20.9b) indicates that the first-best relative allocation of effort is aligned with the relative benefit to the principal, i.e.,[3]

$$\frac{a_1^*}{a_2^*} = \frac{b_1}{b_2}. \tag{20.9d}$$

Second-best Contract

If $\mathbf{a}$ is not contractible, the contract is based on performance measures generated by system η. These measures are represented by an $n\times 1$ vector $\mathbf{y}$ that is normally distributed with mean $\mathbf{M}\mathbf{a}$ and covariance matrix $\boldsymbol{\Sigma}$ (which is assumed to be independent of $\mathbf{a}$), where $\mathbf{M}$ is an $n\times m$ matrix and $\boldsymbol{\Sigma}$ is an $n\times n$ matrix of parameters.

Given linear contract $f + \mathbf{v}^t\mathbf{y}$, the agent's certainty equivalent from action $\mathbf{a}$ and system η is

$$CE(f,\mathbf{v},\mathbf{a},\eta) = f + \mathbf{v}^t\mathbf{M}\mathbf{a} - \tfrac{1}{2}\mathbf{a}^t\mathbf{a} - \tfrac{1}{2}r\mathbf{v}^t\boldsymbol{\Sigma}\mathbf{v}. \tag{20.10a}$$

The assumption that the agent's action does not influence the covariance matrix $\boldsymbol{\Sigma}$ is common in the literature since it simplifies the analysis (see, however, Section 21.4 for analyses of a setting in which the agent's action affects the variance of a normally distributed performance measure). In particular, as revealed by the following first-order condition for the agent's action choice given f and $\mathbf{v}$, the agent's action choice is independent of his risk:

$$\mathbf{a} = \mathbf{M}^t\mathbf{v}, \tag{20.10b}$$

assuming $\mathbf{M}^t\mathbf{v} \geq \mathbf{0}$.

The principal's problem for a given information structure η is

$$\underset{f,\mathbf{v},\mathbf{a}}{\text{maximize}} \quad U^p(f,\mathbf{v},\mathbf{a},\eta) = \mathbf{b}^t\mathbf{a} - (f + \mathbf{v}^t\mathbf{M}\mathbf{a}), \tag{20.11a}$$

$$\text{subject to} \quad CE(f,\mathbf{v},\mathbf{a},\eta) = c^o, \tag{20.11b}$$

first-order condition (20.10b),

[3] Note that if we use the more general cost function $\kappa(\mathbf{a}) = \tfrac{1}{2}\mathbf{a}^t\boldsymbol{\Gamma}\mathbf{a}$, the vector of first-best actions is no longer proportional to the vector of benefits $\mathbf{b}$ unless $\boldsymbol{\Gamma}$ is a diagonal matrix with equal elements in the diagonal.

where c^o is the agent's reservation wage (i.e., $\bar{U} = -\exp[-rc^o]$). Substituting (20.11b) and (20.10b) into (20.11a) restates the principal's expected utility in terms of the variable incentive rates:

$$\underset{\mathbf{v}}{\text{maximize}}\, U^p(\mathbf{v},\eta) \equiv \mathbf{b}^t[\mathbf{M}^t\mathbf{v}] - \{c^o + \tfrac{1}{2}[\mathbf{v}^t\mathbf{M}][\mathbf{M}^t\mathbf{v}] + \tfrac{1}{2}r\mathbf{v}^t\mathbf{\Sigma}\mathbf{v}\}$$

$$= \mathbf{b}^t[\mathbf{M}^t\mathbf{v}] - \{c^o + \tfrac{1}{2}\mathbf{v}^t[\mathbf{M}\mathbf{M}^t + r\mathbf{\Sigma}]\mathbf{v}\}. \quad (20.12)$$

Differentiating (20.12) with respect to **v** and solving for the second-best values of **v** and **a** (assuming an interior solution) yields:

$$\mathbf{v}^\dagger = \mathbf{Q}\mathbf{M}\mathbf{b}, \quad (20.13a)$$

$$\mathbf{a}^\dagger = \mathbf{M}^t\mathbf{v}^\dagger = \mathbf{M}^t\mathbf{Q}\mathbf{M}\mathbf{b}, \quad (20.13b)$$

where $$\mathbf{Q} = [\mathbf{M}\mathbf{M}^t + r\mathbf{\Sigma}]^{-1}.$$

Substituting (20.13a) into (20.12) yields the principal's optimal expected utility from system η is

$$U^{p\dagger}(\eta) = \tfrac{1}{2}\mathbf{b}^t\mathbf{M}^t\mathbf{Q}\mathbf{M}\mathbf{b} - c^o. \quad (20.14a)$$

Comparing the first- and second-best expected utility levels for the principal gives the *loss due to imperfect performance measures*:

$$L(\eta) = U^{p*} - U^{p\dagger}(\eta) = \tfrac{1}{2}\mathbf{b}^t[\mathbf{I} - \mathbf{M}^t\mathbf{Q}\mathbf{M}]\mathbf{b}. \quad (20.14b)$$

Observe that if the agent is risk neutral (i.e., $r = 0$), then $\mathbf{Q} = [\mathbf{M}\mathbf{M}^t]^{-1}$, which implies

$$U^{p\dagger}(\eta) = \tfrac{1}{2}\mathbf{b}^t\mathbf{M}^t[\mathbf{M}\mathbf{M}^t]^{-1}\mathbf{M}\mathbf{b},$$

$$L(\eta) = \tfrac{1}{2}\mathbf{b}^t[\mathbf{I} - \mathbf{M}^t[\mathbf{M}\mathbf{M}^t]^{-1}\mathbf{M}]\mathbf{b}.$$

Obviously, if $\mathbf{M}^t[\mathbf{M}\mathbf{M}^t]^{-1}\mathbf{M} = \mathbf{I}$, the first-best result is achieved and there is no loss in contracting on **y** instead of **a**. However, this need not be the case (for reasons that will become apparent in the next section). Hence, agent risk neutrality is not sufficient for achieving the first-best in a multi-task setting.[4] This stands in contrast to the result in Chapter 17 (Section 17.2) in which we established that agent risk neutrality is sufficient for achieving the first-best result in a single task setting in which the outcome x is contractible information.

Single Performance Measure Precision and Congruity

We now focus on a setting in which there is a single performance measure y_i, with $\mathbf{M} = \mathbf{M}_i = [M_{i1}, ..., M_{im}]$. From (20.9d) we know that the first-best relative effort between two tasks a_j and a_k is proportional to the ratio of the benefits b_j/b_k. However, from (20.13b) it follows that with a single performance measure, the relative second-best effort is

$$\frac{a_j^\dagger}{a_k^\dagger} = \frac{M_{ij}}{M_{ik}}. \tag{20.15}$$

If the outcome is contractible information (i.e., $y_i = x$ and $\mathbf{M}_i = \mathbf{b}^t$), the relative effort between two tasks will be the same in the first- and second-best cases, although the intensity of that effort will be less in the second-best case if the outcome is uncertain (i.e., $\sigma_i^2 = \sigma_x^2 > 0$) and the agent is risk averse (i.e., $r > 0$). However, if $\mathbf{M}_i$ is not proportional to $\mathbf{b}$, the relative effort in the first- and second-best cases will differ.

The above comments led FX to introduce the concept of congruity of a performance measure relative to the principal's expected benefit. In this discussion it is useful to distinguish between performance measures that are *action dependent*, i.e., $\mathbf{M}_i \neq \mathbf{0}$, and those that are *action independent,* i.e., $\mathbf{M}_i = \mathbf{0}$.[5]

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[4] Here we assume that ownership of x cannot be transferred to the agent. If x is contractible, then the first-best can be achieved if the agent is risk neutral.

[5] Datar *et al.* (2001) introduce the following aggregate measure of non-congruity in a setting with n performance measures and two tasks:

$$N_0(\mathbf{v}) = \sum_{i=1}^{n} \left[b_i - (v_1 M_{i1} + v_2 M_{i2})\right]^2.$$

They demonstrate that if the agent is risk neutral, then the optimal contract minimizes N_0. They state (p. 9) that "as in multiple regression, the weight assigned to a performance measure is not simply a function of its own 'congruence' with the outcome, but also on how it interacts with other variables in the contract."

If the agent is risk averse, the optimal contract minimizes the sum of the non-congruity and a measure of the agent's cost of risk:

$$\text{minimize } N_0(\mathbf{v}) + r\,[v_1^2\sigma_1^2 + v_2^2\sigma_2^2 + 2v_1v_2\sigma_{12}].$$

Definition *Congruity with a Single Performance Measure*

A single action-dependent performance measure y_i is *perfectly congruent* (with the principal's expected benefit) if, and only if, there exists a parameter λ such that $\mathbf{b}^t = \lambda \mathbf{M}_i$.

If there are two tasks (i.e., $m = 2$), the following is the *measure of non-congruity* of performance measure i relative to principal's expected benefit:

$$N_{0i} = b_1 M_{i2} - b_2 M_{i1},$$

where $N_{0i} > (<) 0$ implies y_i places greater (less) relative weight on a_2 versus a_1 than does the expected benefit to the principal.

Clearly, in the two-task setting, if y_i is action-dependent, $N_{0i} = 0$ implies the measure is *perfectly congruent*. The above discussion and definition lead to the following result.

Proposition 20.1 (FX, Prop. 1)

A single action-dependent performance measure y_i induces the *first-best relative effort* levels if it is perfectly congruent. Furthermore, a perfectly congruent performance measure induces the *first-best intensity of effort* if, and only if, $r\sigma_i^2 = 0$ (i.e., either the agent is risk neutral or the performance measure is noiseless).

20.2.2 The Value of Additional Performance Measures

Proposition 20.1 identifies conditions under which the first-best relative effort levels and effort intensity are achieved. Clearly, an additional performance measure cannot have value unless the first measure is either non-congruent or noisy (and the agent is risk averse). Observe that a second performance measure can have value either because it permits the principal to reduce the risk imposed on the agent (the source of value in the single-dimensional effort setting) or because it permits the principal to mitigate the non-congruity of the first performance measure.

The general expression for the value of system η^2 relative to η^1 is

$$\pi(\eta^2,\eta^1) = U^p(\eta^2) - U^p(\eta^1) = \tfrac{1}{2}\mathbf{b}^t[\mathbf{M}^{2t}\mathbf{Q}^2\mathbf{M}^2 - \mathbf{M}^{1t}\mathbf{Q}^1\mathbf{M}^1]\mathbf{b}, \quad (20.16)$$

where $\mathbf{M}^i$ and $\mathbf{Q}^i$ specify the characteristics of performance measurement system η^i, $i = 1, 2$. FX provide some general analysis based on (20.16). However, we simplify the analysis by only comparing a system (η^1) that generates a single performance measure to a system (η^2) that generates the first performance measure plus a second measure. In addition, we assume the performance measures have been scaled so that they have *unit variance*. The mean for the first system is $\mathbf{M}_1\mathbf{a}$, and the mean and covariance matrix for the second system is $\mathbf{M}\mathbf{a}$ and $\boldsymbol{\Sigma}$, where

$$\mathbf{M} = \begin{bmatrix} \boldsymbol{M}_1 \\ \boldsymbol{M}_2 \end{bmatrix} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix},$$

where ρ is the correlation between y_1 and y_2.

In Chapter 18 we identified the fact that with a risk neutral principal who owns the outcome x, adding a second performance measure has no value if y_1 is a sufficient statistic for (y_1,y_2) with respect to the agent's action. FX identify sufficient conditions for this to be the case.

Lemma 20.2 (FX, Lemma 1)

If $\mathbf{M}_1 \neq \mathbf{0}$, then performance measure y_1 is a sufficient statistic for (y_1,y_2) with respect to the set of actions $\mathbf{a} \in A$ if, and only if, $\mathbf{M}_2 = \rho\mathbf{M}_1$.

This result follows from the fact that if $\mathbf{M}_2 = \rho\mathbf{M}_1$, the performance measures can be viewed as having the following structure: $y_1 = \mathbf{M}_1\mathbf{a} + \varepsilon_1$ and $y_2 = \rho y_1 + \varepsilon$, where $\varepsilon_1 \sim \mathrm{N}(0,1)$ and $\varepsilon \sim \mathrm{N}(0,1-\rho^2)$, with $\mathrm{Cov}(\varepsilon_1,\varepsilon) = 0$. That is, y_2 is a scaled value of y_1 plus noise.

Proposition 20.2 (FX, Prop. 3)[6]

If y_1 and y_2 are noisy, then

$$\pi(\eta^2,\eta^1) = 0$$

if, and only if, $\mathbf{M}_2 = \rho\mathbf{M}_1$.

The first-best result cannot be achieved with η^1 (due to the noisiness of y_1), and this provides scope for y_2 to have value. However, the value is zero if y_1 is a sufficient statistic for (y_1,y_2).

In Section 20.2.4 we consider some special cases in which a second signal y_2 has positive value. Before considering those special cases, we follow FW and provide a characterization of the optimal relative incentive weights for two performance measures.

[6] The FX proposition is somewhat more general in that it considers the case in which y_1 can be noiseless, but N_{01} is nonzero. In this analysis we exclude noiseless performance measures.

20.2.3 Relative Incentive Weights

Banker and Datar (1989) (BD) consider the relative incentive weights for two performance measures in a setting in which the effort choice is unidimensional and the stochastic characteristics of the performance measures are such that the optimal contract is a function (possibly non-linear) of an aggregate statistic that is a linear function of the performance measures. In this chapter we restrict our analysis to contracts that are linear functions of the performance measures, and consider multi-dimensional effort.

In the analysis that follows we consider FW's analysis of a setting in which there are two actions with separable costs and two performance measures with unit variance. Focusing on the relative weights permits us to ignore components of v_1 and v_2 that are common to both.

FW introduce a measure of the nonalignment of two performance measures.

Definition *Non-alignment of Performance Measures*

The *non-alignment* of performance measure y_k relative to y_i is represented by

$$N_{ki} = M_{k1}M_{i2} - M_{k2}M_{i1},$$

where $N_{ki} > (<) \ 0$ implies that y_i places greater (less) weight on a_2 than on a_1 relative to y_k.

The expression for the second-best incentive rates, $\mathbf{v}^\dagger$, is given by (20.13). The following proposition follows directly from that expression.

Proposition 20.3 (FW, Prop. 1)

The *incentive ratio* (i.e., relative weights on the two performance measures) is:

$$IR \equiv \frac{v_1}{v_2} = \frac{\zeta_1}{\zeta_2}, \tag{20.17}$$

where $\zeta_i = r\big[\mathbf{M}_i - \rho\mathbf{M}_k\big]\mathbf{b} - N_{0k}N_{ki}, \quad i \neq k, i, k = 1, 2.$

FW refer to ζ_i as the *extended sensitivity* of performance measure y_i. In comparing this to the BD result reported in Proposition 18.12, we must keep in mind that BD focus on single-dimensional effort, whereas we consider two-dimensional effort. Furthermore, we have scaled our performance measures so that the precision of each measure (i.e., 1/variance) equals one.

In the *single-dimensional action case*, $\mathbf{M}_1 - \rho\mathbf{M}_2$ and $\mathbf{b}$ are scalars, and the incentive ratio in (20.17) is

$$IR = \frac{\mathbf{M}_1 - \rho\mathbf{M}_2}{\mathbf{M}_2 - \rho\mathbf{M}_1},$$

which is equivalent to BD's result in Proposition 18.12. However, in the two-dimensional action case, $\mathbf{M}_1 - \rho\mathbf{M}_2$ is a vector. It is converted into a scalar by multiplying by $\mathbf{b}$ – thereby weighting the sensitivity of each performance measure relative to the two dimensions of the effort by their marginal impact on the benefit to the principal.

In the two-dimensional effort case, the sensitivity of performance measure y_i must be further adjusted to take into consideration the non-alignment of y_k with y_i and the non-congruity of y_k. This adjustment is unnecessary if y_k is perfectly congruent (i.e., $N_{0k} = 0$), or the two performance measures are perfectly aligned (i.e., $N_{ki} = 0$).

As was illustrated in the single-dimensional effort case, the first parts of ζ_1 and ζ_2 pertain to the desire to minimize risk, and they disappear if the agent is risk neutral. The second parts pertain to the desire to align the agent's preferences with those of the principal, and this becomes the entire focus if the agent is risk neutral. In particular, if the agent is risk neutral, neither performance measure is perfectly congruent, and they are not perfectly aligned, then

$$\frac{v_1}{v_2} = \frac{N_{02}\,N_{21}}{N_{01}\,N_{12}},$$

which will induce the first-best level of effort. That is, the two measures can be used to "span" the space of all possible relative effort levels, including the first-best. To illustrate, consider the benefits and performance measures depicted in Figure 20.1. In setting (i), $\mathbf{b}$ lies in between $\mathbf{M}_1$ and $\mathbf{M}_2$, so that both v_1 and v_2 are positive (with $N_{01} > 0 > N_{02}$ and $N_{12} = -N_{21} > 0$). On the other hand, in setting (ii), $\mathbf{M}_1$ lies in between $\mathbf{b}$ and $\mathbf{M}_2$ such that v_1 is positive and v_2 is negative (with $N_{02} > N_{01} > 0$ and $N_{12} = -N_{21} < 0$).

20.2.4 Special Cases

The preceding sections have provided general characterizations of the optimal linear contract, the optimal induced effort, the value of an additional performance measure, and the incentive ratio for two performance measures. As in FX and FW, we now provide additional insights by considering some interesting special cases.

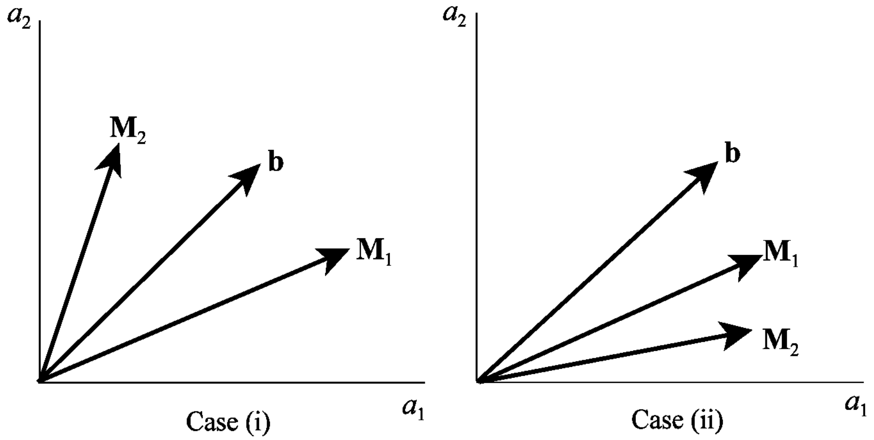


Figure 20.1: Non-congruency and non-alignment of two performance measures with two-dimensional effort.

Perfectly Congruent Performance Measures

Bushman and Indjejikian (1993) (BI) consider a model in which there are two contractible performance measures that equal the final outcome x plus noise. In this model, both performance measures are perfectly congruent (i.e., $N_{01} = N_{02} = 0$) and, hence, from (20.17) it follows that there are no alignment adjustments in the incentive ratio. This also occurs if the two performance measures are non-congruent but perfectly aligned (i.e., $N_{12} = N_{21} = 0$). However, we focus on the perfect congruity case (as depicted in Figure 20.2(i)). The basic questions are: if there are two congruent performance measures and one is more precise (i.e., more sensitive), will it be optimal to use only the more precise measure, or both; and, if both, what determines their relative use (i.e., incentive rate). The answers are: use both unless the former is a sufficient statistic; and select the relative incentive rates so as to minimize the risk premium paid to the agent.

Proposition 20.4

Assume $\mathbf{M}_1 = \alpha_1 \mathbf{b}^t$, $\mathbf{M}_2 = \alpha_2 \mathbf{b}^t$, and $r > 0$, with $\alpha_1 \geq \alpha_2 > 0$, so that y_1 is at least as sensitive as y_2.

(a) $\pi(\eta^2, \eta^1) = \frac{1}{2}(\mathbf{b}^t\mathbf{b})^2 \dfrac{r(\alpha_2 - \rho\alpha_1)^2}{(\mathbf{b}^t\mathbf{b}\alpha_1^2 + r)\,|\mathbf{Q}^{-1}|/r} > 0$ if, and only if, $\alpha_2 \neq \alpha_1\rho$,

where $|\mathbf{Q}^{-1}| = r\left(\mathbf{b}^t\mathbf{b}[\alpha_1^2 + \alpha_2^2 - 2\alpha_1\alpha_2\rho] + r(1-\rho^2)\right)$;

$$\text{(b)}\ v_1 = \frac{\mathbf{b}^t\mathbf{b}[\alpha_1 - \alpha_2\rho]}{|\mathbf{Q}^{-1}|/r};$$

$$\text{(c)}\ IR = \frac{\alpha_1 - \alpha_2\rho}{\alpha_2 - \alpha_1\rho}, \quad \text{for } \alpha_2 \neq \alpha_1\rho. \tag{20.18}$$

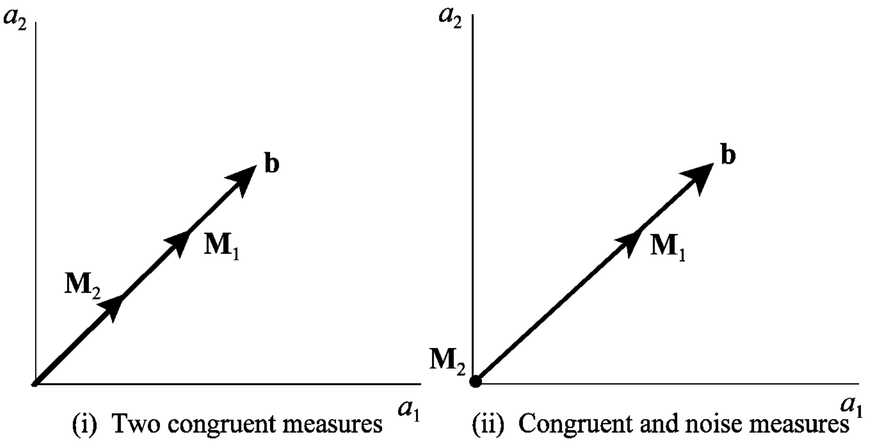


Figure 20.2: Two congruent performance measures and a congruent measure with a noise measure (informative about uncontrollable events).

If $\alpha_2 = \alpha_1\rho$, performance measure y_1 is a sufficient statistic for (y_1,y_2) with respect to the set of actions $\mathbf{a} \in A$ by Lemma 20.2 and, hence, there is no scope for y_2 to be valuable in addition to y_1. If the performance measures are uncorrelated (i.e., $\rho = 0$), the incentive ratio is equal to the relative sensitivities of the two performance measures. Stronger incentives are applied to the first measure, since it is more sensitive, i.e., $\alpha_1 > \alpha_2$ implies $v_1 > v_2$. The incentive weights for both measures are positive if $\alpha_2 > \alpha_1\rho$, but otherwise v_1 is positive and v_2 is negative. The former can be viewed as spreading the incentives over the two relatively independent performance measures so as to diversify the agent's risk. The latter occurs because, for highly correlated signals, the first can be given a high positive weight and the second a high negative weight such that the contract positively motivates congruent effort but with reduced risk (relative to just using the first measure). The negative weight on the risk in the second significantly directly offsets the positive weight on the risk in the first.

Zero correlation does not occur in the BI model since they assume the noise in the outcome is a common noise component in the two performance measures. The BI model can be viewed as taking the form:

$$x = \mathbf{b}^t\mathbf{a} + \varepsilon_x, \qquad \varepsilon_x \sim N(0,\sigma_x^2), \quad \sigma_x^2 > 0,$$

$$\hat{y}_i = \mathbf{b}^t\mathbf{a} + \varepsilon_x + \varepsilon_i, \qquad \varepsilon_i \sim N(0,\sigma_i^2), \quad \sigma_i^2 > 0, \ i = 1,2.$$

Assuming ε_x, ε_1, and ε_2 are independent, we can transform the BI model into our notation as follows:

$$\alpha_i = [\sigma_x^2 + \sigma_i^2]^{-½},$$

$$\rho = \alpha_1\alpha_2\sigma_x^2.$$

Observe that if we do not scale the performance measures to have unit variance, then the incentive weight for measure $\hat{y}_i$ is $\hat{v}_i = \alpha_i v_i$, and the incentive ratio is

$$\hat{IR} = \frac{\sigma_2^2}{\sigma_1^2},$$

i.e., the incentive ratio is determined as the relative precision of the two measures about the outcome. Of course, this is due to the fact that the non-scaled performance measures have the same sensitivities ("mean vectors").

Before leaving this special case we note that the agent's risk aversion does not affect the relative incentive weights – they depend strictly on the relative information content of the two performance measures. However, the agent's risk aversion does affect the value of the additional performance measure and the strength of the incentives. It is straightforward to demonstrate that

$$\partial\pi(\eta^2,\eta^1)/\partial r > 0, \ |\partial v_1/\partial r| < 0, \text{ and } |\partial v_2/\partial r| < 0.$$

That is, the larger the agent's risk aversion, the more valuable is the risk reduction role of the second performance measure.

Information about Uncontrollable Events

In their examination of relative incentive weights in a single-task setting, Banker and Datar (1989) (BD) consider the case in which the agent's effort influences the first performance measure but not the second (see Section 18.1.4). The noisiness of first measure forces the principal to pay the agent a risk premium if incentives are used. The second measure can have value even if it is not influenced by the agent's action provided it is informative about the uncontrollable events affecting the first measure. The key is that this informativeness permits the principal to reduce the risk premium that must be paid to induce a given level of effort. As the following analysis demonstrates, the single-task result extends to the multi-task setting.

Assume that $\mathbf{M}_2 = \mathbf{0}$ (as depicted in Figure 20.2(ii)). This implies that the direction of induced effort depends entirely on $\mathbf{M}_1$. However, the second measure has value provided $\rho \neq 0$, since it can be used to reduce the risk imposed on the agent. Hence, the value of the second measure stems entirely from risk reduction, as in the single task case. Observe that $\mathbf{M}_2 = \mathbf{0}$ implies $N_{02} = N_{12} = N_{21} = 0$, so that from (20.17) it follows that there are no alignment adjustments in computing the incentive ratio IR.

Proposition 20.5 (FX, Prop. 4, and FW, Prop. 4)

If $\mathbf{M}_1 > \mathbf{0}$, $\mathbf{M}_2 = \mathbf{0}$, and $r > 0$, then

(a) $$\pi(\eta^2, \eta^1) = \tfrac{1}{2}(\mathbf{b}^t\mathbf{b})(\mathbf{M}_1^t\mathbf{M}_1)\frac{r\rho^2}{(\mathbf{M}_1\mathbf{M}_1^t + r(1-\rho^2))(\mathbf{M}_1\mathbf{M}_1^t + r)} > 0$$

if, and only if, $\rho \neq 0$;

(b) $$v_1 = \frac{\mathbf{M}_1\mathbf{b}}{\mathbf{M}_1\mathbf{M}_1^t + r(1-\rho^2)};$$

(c) $IR = -1/\rho$.

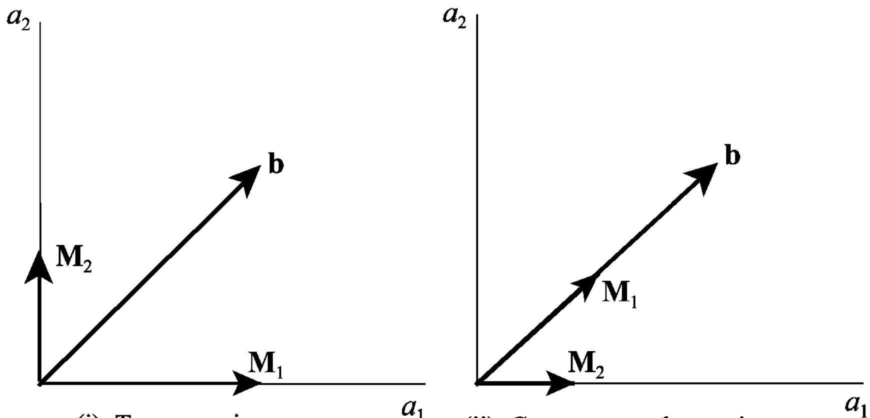
The second performance measure is useless unless it is correlated with the first, in which case the second can be used to strictly reduce the risk premium paid to the agent. If the two measures are correlated, it follows from (a) that the value is strictly increasing in ρ^2. Furthermore, the first-best result can be achieved (i.e., $L(\eta^2) = 0$) if the first performance measure is perfectly congruent with the principal's benefit ($N_{01} = 0$) and the two measures are perfectly correlated ($\rho^2 = 1$).

The incentive weight on the first performance measure, i.e., v_1, indicates the strength of the effort incentives in this setting. Result (b) establishes that the strength of those incentives decreases with r and increases with ρ^2. That is, not surprisingly, the less risk averse the agent and the more that he can be shielded from incentive risk through the second measure, the stronger are the effort incentives. Result (c) is precisely the same as in BD (see Proposition 18.13). Hence, in both the single and multi-dimensional effort cases, the relative weight placed on a second measure, which is used strictly for risk reduction, is equal to its correlation with the first measure. Interestingly, these last two results apply even if the first measure is not perfectly congruent with the marginal benefit of the agent's action to the principal.

Independent, Myopic Performance Measures

Accounting numbers are often viewed as inadequate performance measures because they report only the short-run impact of a manager's actions. Such a measure is clearly not perfectly congruent, and is described as being myopic. A second measure can be of value if it is more congruent than the myopic measure, or because it provides information about the "other" consequences of the agent's actions.[7]

To depict the latter case in stark terms, assume that each action only influences one performance measure (as depicted in Figure 20.3(i)) and those measures are uncorrelated. This effectively results in two independent decision problems. The value of having two measures instead of one is merely the expected net return to the principal from inducing effort in the second activity, since without the second measure the agent will not expend any effort in the second activity.



(i) Two myopic measures (ii) Congruent and myopic measures

Figure 20.3: Two myopic performance measures and a congruent measure with a myopic measure.

[7] The preceding comments reflect the fact that the principal and agent contract for only one period. In a multi-period contract (which we explore in Chapters 25 through 28), the accounting numbers reported in any period reflect the short-term consequences of the current actions and the long-term consequences of prior actions.

Proposition 20.6 (FX, p. 442, and FW, Prop. 4)

If b_1, $M_{11} > 0$, b_2, $M_{22} > 0$, $M_{12} = M_{21} = 0$, and $\rho = 0$,[8] then

$$\text{(a)}\ \ \pi(\eta^2;\eta^1) = \frac{1}{2}\left[1 - \frac{r}{M_{22}^2 + r}\right] b_2^2;$$

$$\text{(b)}\ \ v_i = \frac{M_{ii}\, b_i}{r + M_{ii}^2}, \quad i = 1, 2;$$

$$\text{(c)}\ \ IR = \frac{M_{11} b_1 / [r + M_{11}^2]}{M_{22} b_2 / [r + M_{22}^2]}.$$

Result (a) implies that the value of the second signal increases with the economic importance of the second task (b_2), the sensitivity of the second signal (M_{22}), and the risk tolerance of the agent ($1/r$). From (b) we observe that the weight placed on performance measure y_i increases with the economic importance of the task a_i, and decreases with the agent's risk aversion r. The impact of the sensitivity of the performance measure M_{ii} is more subtle. Increasing M_{ii} increases (decreases) v_i if $r >$ (<) M_{ii}^2. The key here is that if r is large, then increasing M_{ii} makes it optimal to use stronger incentives (and induce a larger a_i) since the risk effect has been reduced (keep in mind that the variance of y_i is equal to one). On the other hand, if r is small, the risk effect is less important and the dominant effect is that if M_{ii} is increased it takes less v_i to induce the same effort level a_i.

Combining Perfectly Congruent and Myopic Performance Measures

BI also consider a model in which the first measure is perfectly congruent, while the second is myopic (as depicted in Figure 20.3(ii)).[9] The former is representative of information obtained by investors (which is impounded in the market price) that reflects both the "short" and "long" run consequences of the agent's actions, while the latter is representative of accounting numbers (e.g., accounting earnings) which reflect only the "short" run consequences of the agent's actions. More specifically, the BI model takes the following form:

$$x = x_1 + x_2, \qquad x_j = b_j a_j + \varepsilon_{xj}, \quad \varepsilon_{xj} \sim \mathrm{N}(0, \sigma_{xj}^2),$$

---

[8] This case is considered by FX on p. 442. However, since we scale the signals so that $\sigma_1 = \sigma_2 = 1$, we cannot also have $b_1 = M_{11}$ and $b_2 = M_{22}$.

[9] Datar *et al.* (2001) also examine this setting.

$$\hat{y}_1 = x_1 + x_2 + \varepsilon_1, \qquad \varepsilon_1 \sim N(0,\sigma_1^2),$$

$$\hat{y}_2 = x_1 + \varepsilon_2, \qquad \varepsilon_2 \sim N(0,\sigma_2^2).$$

In our model this setting is represented by

$$\mathbf{M}_1 = \alpha_1 \mathbf{b}^t, \qquad \alpha_1 = [\sigma_{x1}^2 + \sigma_{x2}^2 + \sigma_1^2]^{-½},$$

$$\mathbf{M}_2 = \alpha_2[b_1, 0], \qquad \alpha_2 = [\sigma_{x1}^2 + \sigma_2^2]^{-½},$$

$$\rho = \alpha_1 \alpha_2 \sigma_{x1}^2.$$

The obvious question in this setting is whether the myopic measure has value given that the first measure is perfectly congruent. The answer is obviously no if the agent is risk neutral. However, the second measure does have value if $r > 0$, even if the two measures are uncorrelated. In particular, it will be optimal to use the second measure so as to reduce the risk imposed using the first measure alone, even though the use of the second measure will induce the agent to "mis-allocate" his effort, i.e., $a_1/a_2 \neq b_1/b_2$. The incentive ratio is

$$\frac{v_1}{v_2} = \frac{\sigma_2^2 b_2^2 - \sigma_x^2(b_1^2 + b_2^2)}{\sigma_1^2(b_1^2 + b_2^2) - \sigma_x^2 b_1^2 + \frac{1}{r} b_1^2 b_2^2}.$$

Window Dressing

Performance measures are often subject to manipulation in the sense that the agent can take actions that improve his performance measure but contribute little or nothing to the principal's gross benefit. FX refer to this as window dressing, and represent it by M_{11}, $b_1 > 0$ and $M_{12} > b_2 = 0$. A rather strange aspect of this setting is that the principal must compensate the agent for the agent's cost of undertaking the window dressing since the agent must receive his reservation wage plus effort cost plus risk premium to obtain his services. Of course, the principal would like to design a performance measure that is not subject to window dressing. Alternatively, the principal would like to have information he can use to punish the agent for any window dressing. Following FX, we consider both types of measures (which are depicted in Figure 20.4 as adding either a "carrot" or a "stick" to the primary measure).

In case (i), FX introduce a perfectly congruent second measure. While window dressing could be totally avoided by only using this measure, risk reduction makes it optimal to use both measures. In case (ii), FX introduce a second measure that provides information about the window dressing activity.

This measure only has value when used with the first, in which case it is used to deter window dressing.

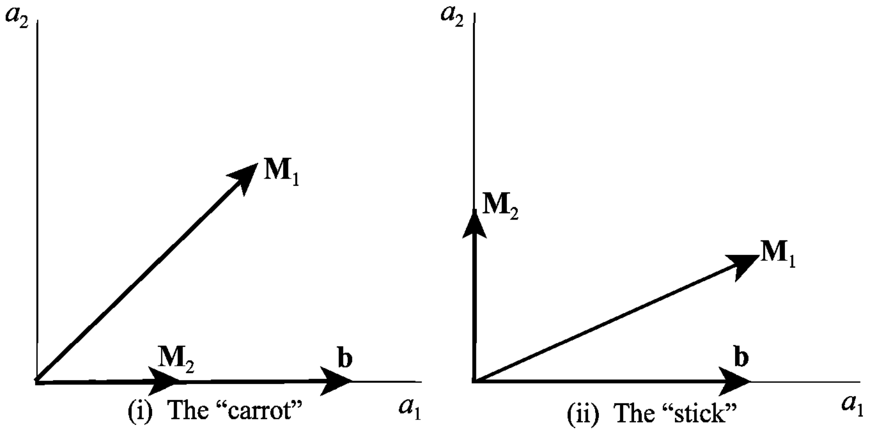


Figure 20.4: Window dressing – with a "carrot" and with a "stick."

Proposition 20.7 (FX, p. 442)

Assume M_{11}, $b_1 > 0$ and $M_{12} > b_2 = 0$, along with either (i) $M_{21} > 0$ and $M_{22} = 0$, or (ii) $M_{21} = 0$ and $M_{22} > 0$.

$$\pi(\eta^2,\eta^1) = \begin{cases} \dfrac{1}{2} b_1^2 \dfrac{[1 - M_{11}^2 Q]^2}{1 + r M_{21}^{-2} - M_{11}^2 Q}, & \text{case (i)} \\[2ex] \dfrac{1}{2} b_1^2 \dfrac{M_{11}^2 M_{12}^2 Q^2}{1 + r M_{22}^{-2} - M_{12}^2 Q}, & \text{case (ii)} \end{cases}$$

where $$Q = [b_1^2 + M_{12}^2 + r]^{-1}.$$

Observe that in both cases the value of the second performance measure is strictly positive and is increasing in the benefit of the first action (b_1) and the sensitivity of the first performance measure (M_{11}).

20.2.5 Induced Moral Hazard

The window dressing example illustrates a setting in which there are two types of tasks – both affect the primary performance measure and are costly to the agent, but only one type is beneficial to the principal. Now we consider a two-type task setting in which both types affect the primary performance measure and both are beneficial to the principal, but one is costly to the agent and the other is costly to the principal.

To simplify the analysis, we assume there are only two tasks. The first activity, represented by a_1, pertains to the effort expended by the agent, whereas the second, represented by a_2, pertains to the investment of additional capital into the project operated by the agent. The expected incremental gross benefit to the principal of operating the project is a linear function of the agent's unobserved effort and investment choices, i.e., $b(\mathbf{a}) = b_1 a_1 + b_2 a_2$, and the direct costs to the agent and principal are $\kappa^a(\mathbf{a}) = ½a_1^2$ and $\kappa^p(\mathbf{a}) = ½a_2^2$, respectively.[10]

The agent must be compensated for his direct costs. Hence, from the principal's perspective, the first-best actions maximize $b_1 a_1 - ½a_1^2 + b_2 a_2 - ½a_2^2$ and are characterized by[11]

$$a_1^* = b_1, \quad a_2^* = b_2.$$

A Single Congruent Performance Measure

We assume all performance measures are linear functions of the agent's effort, the principal's investment, and the cost of the principal's investment. That is, any measure y_i can be expressed as

$$y_i = M_{i1} a_1 + M_{i2} a_2 - M_{i3} ½a_2^2 + \varepsilon_i, \tag{20.19}$$

where $\varepsilon_i \sim N(0,1)$.

Definition

A performance measure is defined to be *congruent with respect to the investment decision* if it gives equal weight to the expected gross benefit

[10] The units used to measure the agent's actions are arbitrary. For example, the investment activity could be represented by the dollars invested, i.e., $\hat{a}_2 = ½a_2^2$. In that case, the gross benefit is a strictly concave function, $b_2\sqrt{2\hat{a}_2}$. Our approach simplifies the discussion.

One could question the assumed unobservability of the amount invested since it is provided by the principal. However, we envisage a setting in which the agent manages the principal's investment capital and $½a_2^2$ represents the amount invested in the agent's project instead of being invested in a riskless asset. The principal cannot observe the investment mix and his outcome includes the return from the investment in the riskless asset. We have deducted a constant from his outcome, so that the benefit equals the incremental benefit in excess of the amount the principal would receive if all of his capital was invested in the riskless asset.

Also note that the project risk is independent of both a_1 and a_2. This implies the project is operated even if the additional capital invested equals zero.

[11] If the second task is measured in the dollars invested by the principal (see prior footnote), the first-best action is $\hat{a}_2^* = ½b_2^2$.

from the investment and its cost. That is, if the weight on the cost is M_{i3}, then the same weight is placed on the benefit, i.e., $M_{i2} = M_{i3} b_2$.

We first consider a setting in which the only performance measure is the *ex* compensation market value of the firm plus noise. That is, the performance measure equals the principal's expected gross benefit minus his direct cost, plus noise. The performance measure is normalized so that its noise has unit variance, i.e., there exists a parameter $\xi > 0$ such that

$$y_1 = \xi[b_1 a_1 + b_2 a_2 - \tfrac{1}{2} a_2^2] + \varepsilon_1,$$

where $\varepsilon_1 \sim N(0,1)$. Hence, $M_{11} = \xi b_1$, $M_{12} = \xi b_2$, and $M_{13} = \xi$, which implies this measure is congruent with respect to the investment choice.

If the principal offers the agent a linear contract $c = f + v_1 y_1$, the agent will be motivated to choose $a_1 = \xi b_1 v_1$ and $a_2 = b_2$. That is, the agent chooses the first-best investment level for all v_1, but he only chooses the first-best level of effort if $v_1 = 1/\xi$. The latter is optimal if the agent is risk neutral, but if he is risk averse, the principal chooses v_1 so as to maximize his expected utility:

$$U^p(v_1,\eta) = \xi v_1 b_1^2 + \tfrac{1}{2} b_2^2 - \tfrac{1}{2}(\xi v_1 b_1)^2 - \tfrac{1}{2} r (v_1)^2 - c^o,$$

given the substitution of $f = c^o + \tfrac{1}{2} a_1^2 + \tfrac{1}{2} r v_1^2 - v_1 \xi[b_1 a_1 + b_2 a_2 - \tfrac{1}{2} a_2^2]$ with induced actions $a_1 = \xi b_1 v_1$ and $a_2 = b_2$. Hence, the principal's incentive rate choice and the induced actions are

$$v_1 = \frac{\xi b_1^2}{\xi^2 b_1^2 + r}, \quad a_1 = b_1 \frac{\xi^2 b_1^2}{\xi^2 b_1^2 + r}, \quad a_2 = b_2.$$

Observe that ξv_1 is strictly less than one if the agent is strictly risk averse. Therefore, the agent receives a fraction of the principal's expected gross benefit and is charged a fraction of the principal's cost. A key factor in inducing the first-best investment level is the fact that the fractions for both components are identical.

The agent incurs all of the direct cost of his effort, but receives only a fraction $\xi v_1 < 1$ of the incremental expected gross benefit. Consequently, the induced effort level is less than first-best. The first-best level could be induced by setting $v_1 = 1/\xi$, but that would result in the agent incurring more than the optimal level of risk – for which he must be compensated.

First-best Investment with Multiple Performance Measures

We now identify some conditions under which the first-best investment level is induced when there are two performance measures. We then illustrate the distortion in induced investment that can occur when these conditions are not satisfied.

Assume there are two performance measures (of the form in (20.19)) with noise terms that have unit variances and $\text{Cov}(\varepsilon_1, \varepsilon_2) = \rho$. The linear compensation contract $c(y_1,y_2) = f + v_1 y_1 + v_2 y_2$ induces the agent to choose

$$a_1 = \alpha_1(\mathbf{v}) \equiv M_{11} v_1 + M_{21} v_2, \tag{20.20a}$$

$$a_2 = \alpha_2(\mathbf{v}) \equiv \frac{M_{12} v_1 + M_{22} v_2}{M_{13} v_1 + M_{23} v_2}. \tag{20.20b}$$

Substituting

$$f = c^o + \tfrac{1}{2} a_1^2 + \tfrac{1}{2} r[v_1^2 + v_2^2 + 2\rho(1-\rho) v_1 v_2]$$
$$- v_1[M_{11} a_1 + M_{12} a_2 - \tfrac{1}{2} M_{13} a_2^2] - v_2[M_{21} a_1 + M_{22} a_2 - \tfrac{1}{2} M_{23} a_2^2]$$

and (20.20) into the principal's expected utility provides the following unconstrained decision problem for the selection of the incentive rates:

$$\underset{\mathbf{v}}{\text{maximize}} \ U^p(\mathbf{v},\eta) \equiv b_1 \alpha_1(\mathbf{v}) + b_2 \alpha_2(\mathbf{v}) - \tfrac{1}{2}\alpha_2(\mathbf{v})^2$$
$$- \{c^o + \tfrac{1}{2}\alpha_1(\mathbf{v})^2 + \tfrac{1}{2} r[v_1^2 + v_2^2 + 2\rho(1-\rho) v_1 v_2]\}.$$

The first-order conditions are

$$M_{11}[b_1 - \alpha_1(\mathbf{v})] + \alpha_{21}(\mathbf{v})[b_2 - \alpha_2(\mathbf{v})] - r[v_1 + \rho(1-\rho) v_2] = 0, \tag{20.21a}$$

$$M_{21}[b_1 - \alpha_1(\mathbf{v})] + \alpha_{22}(\mathbf{v})[b_2 - \alpha_2(\mathbf{v})] - r[v_2 + \rho(1-\rho) v_1] = 0, \tag{20.21b}$$

where $\alpha_{2j}(\mathbf{v}) \equiv \partial \alpha_2(\mathbf{v})/\partial v_j$.

If the agent is paid a fixed wage, then the agent will be willing to make the first-best investment choice, but he will not expend any effort. If the principal chooses a non-zero incentive rate for either performance measure and induces other than the first-best investment level, i.e., $a_2 \neq b_2$, then we refer to this as the result of *induced moral hazard.* Note that there is no inherent moral hazard problem with respect to the agent's choice of investment, since this action is costless to the agent. The incentive problem with respect to the investment is strictly due to the fact that the principal is offering the agent an incentive contract to induce the agent's effort, and this contract may *induce* an incentive problem with respect to the investment, i.e., the incentive constraint with respect to a_2 is a non-redundant constraint.[12] The following proposition identifies some conditions under which there is no induced moral hazard.

Proposition 20.8

The second-best contract induces the first-best investment level in the following cases:

(a) both performance measures are congruent with respect to the investment choice;

(b) one performance measure is congruent with respect to the investment choice and the other measure is independent of the investment;

(c) one performance measure is congruent with respect to the investment choice and the other measure is independent of the effort choice and uncorrelated with the other performance measure.

The three results are intuitively appealing. We earlier established that a single congruent performance measure induces the first-best investment, so it is not surprising that two congruent measures also induce the first-best investment. Mathematically, result (a) follows directly from the fact that substituting $M_{12} = b_2 M_{13}$ and $M_{22} = b_2 M_{23}$ into the right-hand-side of (20.20b) yields $a_2 = b_2$ irrespective of $\mathbf{v}$.

It is also not surprising that the first-best investment is induced if only one performance measure is influenced by the investment, and that measure is congruent. Mathematically, result (b) follows directly from the fact that if, for example, the second measure is not affected by the investment and the first is congruent, then $M_{22} = M_{23} = 0$, and $M_{12} = b_2 M_{13}$. Substituting these expressions into (20.20b) yields $a_2 = b_2$ irrespective of $\mathbf{v}$.

Result (c) is more subtle since we have one congruent measure and one non-congruent measure. If the non-congruent measure is used, then the first-best investment level will not be induced, i.e., we will have induced moral hazard with respect to investment. However, under the conditions assumed in (c), the non-congruent measure will not be used since it is not informative about the agent's effort and it cannot be used to reduce the risk incurred in using the congruent measure. Mathematically, let the first measure be congruent and the second be non-congruent, so that $M_{12} = b_2 M_{13}$ and $M_{22} \neq b_2 M_{23}$. Condition (c) then assumes that $M_{11} \geq 0$, $M_{21} = 0$, and $\rho = 0$. Substituting the above into (20.20) yields

[12] In Section 22.6 we consider induced moral hazard in a delegated private information acquisition setting – the setting in which induced moral hazard was first introduced.

$$\alpha_1(\mathbf{v}) = M_{11} v_1,$$

$$\alpha_2(\mathbf{v}) = \frac{b_2 M_{13} v_1 + M_{22} v_2}{M_{13} v_1 + M_{23} v_2}.$$

The latter implies that the first-best investment is induced if $v_2 = 0$. The key issue is whether that is optimal. Under the assumed conditions, the v_2 first-order condition (20.21b) for a given v_1 is

$$\alpha_{22}(\mathbf{v})[b_2 - \alpha_2(\mathbf{v})] - r v_2 = 0.$$

This condition is satisfied by $v_2 = 0$, since that implies $\alpha_2(\mathbf{v}) = b_2$. That is, it is optimal to use only the congruent performance measure, which will induce the first-best investment. From (20.21a) and $v_2 = 0$, we obtain

$$v_1 = \frac{b_1 M_{11}}{M_{11}^2 + r}.$$

Therefore, the incentive rate used for the congruent measure is based strictly on inducing the second-best level of effort. The investment decision is irrelevant.

Induced Under- and Over-investment

Proposition 20.8(c) imposes two conditions on the non-congruent performance measure: it is not influenced by the agent's effort and the noise in the two performance measures are uncorrelated. We first consider a setting in which the latter is violated, and then consider a setting in which neither performance measure is congruent.

Assume that the first performance measure is based on the *ex* compensation market value, with $M_{11} = \xi b_1$, $M_{12} = \xi b_2$, and $M_{13} = \xi$. The second is a noisy measure of the future benefits from the investment, with noise that is correlated with the noise in the first measure. The second measure is not influenced by the agent's effort. Hence, $M_{21} = 0$, $M_{22} > 0$, $M_{23} = 0$, and $\rho \neq 0$. This results in the following characterization of the agent's action choices given $\mathbf{v}$:

$$a_1 = \alpha_1(\mathbf{v}) = \xi b_1 v_1,$$

$$a_2 = \alpha_2(\mathbf{v}) = b_2 + \frac{M_{22} v_2}{\xi v_1}.$$

Obviously, for all $v_1 > 0$, the above implies

$$a_2\ (<, =, >)\ b_2 \text{ if } v_2\ (<, =, >)\ 0.$$

That is, the first-best investment is again induced if, and only if, $v_2 = 0$. However, the latter is not optimal if $\rho \neq 0$. To see this, consider the "v_2"-first-order condition (20.21b) for a fixed v_1:

$$-\frac{M_{22}^2}{\xi^2 v_1^2} v_2 - r[v_2 + \rho(1-\rho)v_1] = 0,$$

which implies

$$v_2 = -\frac{r\xi^2 v_1^3 \rho(1-\rho)}{M_{22}^2 + r\xi^2 v_1^2}.$$

Hence, for all $v_1 > 0$,

$$a_2\ (<, =, >)\ b_2 \text{ if } \rho\ (>, =, <)\ 0,$$

i.e., there is over- or under-investment if the two measures are negatively or positively correlated, respectively. This result occurs because the second measure is used to reduce the agent's incentive risk and this induces a non-congruity with respect to the investment decision by placing either too much or too little weight on the future benefits from the investment. The zero correlation case is equivalent to the condition in Proposition 20.8(c).

Next, we consider a setting in which the non-congruent performance measure is influenced by the agent's effort. The second performance measure is a myopic accounting number that includes a fraction of the gross benefit of the agent's effort and a fraction of the cost of the principal's investment, but does not include any of the future benefit from the investment. Hence, $M_{21} > 0$, $M_{22} = 0$, and $M_{23} > 0$.[13] We further assume the noise in the accounting number is uncorrelated with the noise in the first measure, with $M_{11} = \xi b_1$, $M_{12} = \xi b_2$, and $M_{13} = \xi$. This results in the following characterization of the agent's action choices given $\mathbf{v}$:

$$a_1 = \alpha_1(\mathbf{v}) = \xi b_1 v_1 + M_{21} v_2,$$

[13] We can interpret this setting as a setting in which the accounting is such that none of the future benefits of investments are recognized, $M_{22} = 0$, but there is a depreciation charge, $M_{23} > 0$, i.e., the non-congruity of the accounting measure is due to the revenues and the cost of the investment not being properly "matched." The stock price, on the other hand, fully recognizes both the cost and the future benefits of investments. See Dutta and Reichelstein (2003) for a related analysis.

$$a_2 = \alpha_2(\mathbf{v}) \equiv \frac{\xi b_2 v_1}{\xi v_1 + M_{23} v_2}.$$

Obviously, for all $v_1 > 0$, the latter implies a_2 (<, =, >) b_2 if v_2 (>, =, <) 0. That is, the first-best investment is induced if, and only if, $v_2 = 0$. However, the latter is not optimal. The first-order conditions (20.21) imply that

$$\xi b_1 [b_1(1 - \xi v_1) - M_{21} v_2] + \frac{\xi b_2^2 M_{23}^2 v_2^2}{(\xi v_1 + M_{23} v_2)^3} - r v_1 = 0,$$

$$M_{21} [b_1(1 - \xi v_1) - M_{21} v_2] - \frac{\xi b_2^2 M_{23}^2 v_1 v_2}{(\xi v_1 + M_{23} v_2)^3} - r v_2 = 0.$$

These conditions cannot be simultaneously satisfied by $v_2 = 0$ unless $r = 0$, in which case first-best effort and investments are induced. In fact, $v_2 > 0$, so that there is always under-investment. To see this, consider a contract in which v_2 is negative. This contract is dominated by a contract with the same incentive rate on the first measure and a zero weight on the accounting measure – more effort and first-best investments are induced at a lower risk premium (since the performance measures are uncorrelated).[14]

Of course, the key in this setting is that the accounting measure is used to more efficiently induce the agent's effort, but at same time, this induces under-investment due to the non-congruity of the accounting measure. The induced under-investment problem implies that the accounting measure is used to a lesser extent than if the investment could be chosen by the principal, i.e., the incentive ratio is higher than the ratio of effort sensitivities, $IR = v_1/v_2 > \xi b_1/M_{21}$.[15]

We conclude this section by considering a setting in which there are two non-congruent measures that could be combined to obtain a congruent measure, but it is not optimal to do so. The first measure is influenced by the gross bene-

[14] If $\xi v_1 > 1$, the increased effort may not be beneficial to the principal. However, in this case the initial contract is dominated by a contract in which $v_2 = 0$, and first-best effort is induced, i.e., $v_1 = 1/\xi$, at a lower risk premium.

[15] The first-order conditions (20.21) imply that

$$M_{21} v_1 - \xi b_1 v_2 = \frac{\xi b_2^2 M_{23}^2 v_2 \alpha_1(\mathbf{v})}{r(\xi v_1 + M_{23} v_2)^3} > 0.$$

fit from the investment and the second is influenced by the cost of that investment. To simplify the analysis we assume the agent's effort only influences the second measure and the two measures are uncorrelated. Hence, $M_{11} = 0$, $M_{21} > 0$, $M_{12} > 0$, $M_{22} = 0$, $M_{13} = 0$, $M_{23} > 0$, and $\rho = 0$, which implies that the induced actions are

$$a_1 = \alpha_1(\mathbf{v}) = M_{21} v_2,$$

$$a_2 = \alpha_2(\mathbf{v}) \equiv \frac{M_{12} v_1}{M_{23} v_2}.$$

The first-best investment can be induced by choosing the two incentive rates such that $v_1 = v_2 b_2 M_{23}/M_{12}$. However, to see that the optimal choice of v_1 is less than $v_2 b_2 M_{23}/M_{12}$, consider the v_1 first-order condition (20.21a) for a given v_2:

$$\frac{M_{12}}{M_{23} v_2}\left[b_2 - \frac{M_{12} v_1}{M_{23} v_2} \right] - r v_1 = 0,$$

which implies

$$v_1 = \frac{M_{12}}{M_{23} v_2} b_2 \left[\frac{M_{12}^2}{M_{23}^2 v_2^2} + r \right]^{-1} < v_2 b_2 \frac{M_{23}}{M_{12}}, \quad \text{if } r > 0.$$

Consequently, while first-best would be achieved if the agent is risk neutral, his risk aversion leads to less than a congruent incentive rate for the first performance measure, thereby resulting in under-investment. At the margin, the gain from reducing the risk premium paid to the agent exceeds the loss due to under-investment.

In concluding this section we point out that induced moral hazard is pervasive, but it is seldom modeled. In many models of management choice the manager's preference function is exogenously imposed instead of being endogenously derived. For example, in Chapter 14 of Volume I we examine a number of disclosure models. In those models it is common to assume that the manager seeks to maximize either the market value or intrinsic value of the firm at the disclosure choice date. The manager's action, choosing between disclosure and non-disclosure, is not directly costly to him. Therefore, a question arises as to why the owners do not pay him a fixed wage and commit him to make the disclosure choice that will maximize the *ex ante* value of the firm. A typical response is that his incentive to maximize the disclosure date value arises from an incentive contract associated with other actions he must take. That is, it is an induced moral hazard problem. However, since that problem is not modeled, one wonders whether it would take the form exogenously assumed in the analysis of disclosure choice.

20.3 ALLOCATION OF EFFORT AMONG TASKS WITH NON-SEPARABLE EFFORT COSTS

Holmström and Milgrom (1991) (HM) consider several models in which the form of the agent's cost function, as well as the available performance measures, play important roles in determining the form of the contract. As in the models discussed above, the principal is risk neutral, the agent has negative exponential utility for his consumption minus a personal cost $\kappa(\mathbf{a})$, the performance measures are normally distributed, and the analysis is restricted to linear contracts. Unlike the preceding models, HM begin their analysis with the assumption that there is a separate performance measure for each task, i.e., $\mathbf{y}$ and $\mathbf{a}$ both have dimension m, although some measures may be infinitely noisy. We limit our discussion to their basic model in which general cost functions are considered, plus two "threshold cost" models that relate most closely to the discussion in this chapter. The "threshold cost" models are such that $\kappa(\mathbf{a}) = \kappa(a^+)$, where $a^+ = a_1 + ... + a_m$ represents aggregate effort, and the cost function has the following characteristics: $\kappa'(a^+) = 0$ for $a^+ \in [0,a^o)$, and $\kappa'(a^+) > 0$, $\kappa''(a^+) \geq 0$ for $a^+ \geq a^o$.

Task Specific Performance Measures and a General Agent Cost Function

Prior to considering models with a cost threshold, HM examine a simple model in which there is a separate performance measure for each task with $\mathbf{y} \sim N(\mathbf{a},\boldsymbol{\Sigma})$, the benefit function $b(\mathbf{a})$ is concave, and the agent's cost function $\kappa(\mathbf{a})$ is strictly convex. In their analysis, they permit the cost function to be such that the agent will exert effort in some tasks even if there are no monetary incentives, i.e., $\kappa(\mathbf{a})$ is the net of the agent's personal cost minus his personal benefit from the effort expended in each task and there exists a strictly positive vector of effort levels $\mathbf{a}^o$ such that $\nabla_{\mathbf{a}}\kappa(\mathbf{a}^o) = \mathbf{0}$, where $\nabla_{\mathbf{a}}\kappa(\mathbf{a})$ is the $m \times 1$ vector of first-derivatives of the agent's cost function. If the effort to be induced is strictly positive, the incentive constraint is

$$\mathbf{v} = \nabla_{\mathbf{a}}\kappa(\mathbf{a}). \tag{20.22}$$

Hence, the principal's problem can be expressed as

$$\underset{\mathbf{a}}{\text{maximize}} \;\; b(\mathbf{a}) - [\kappa(\mathbf{a}) + \tfrac{1}{2} r \nabla_{\mathbf{a}}\kappa(\mathbf{a})^t \boldsymbol{\Sigma}\, \nabla_{\mathbf{a}}\kappa(\mathbf{a})]. \tag{20.23}$$

Differentiating (20.23) with respect to **a** yields the following first-order conditions for the principal's problem:

$$\nabla_{\mathbf{a}} b(\mathbf{a}) - \nabla_{\mathbf{a}} \kappa(\mathbf{a}) - r[\kappa_{jk}(\mathbf{a})]\,\boldsymbol{\Sigma}\,\nabla_{\mathbf{a}} \kappa(\mathbf{a}) = 0, \tag{20.24}$$

where $\nabla_{\mathbf{a}} b(\mathbf{a})$ is the $m \times 1$ vector of first-derivatives of $b(\mathbf{a})$ and $[\kappa_{jk}(\mathbf{a})]$ is the $m \times m$ matrix of second-derivatives of $\kappa(\mathbf{a})$. Solving for **v**, using (20.22) and (20.24) identifies the variable incentive rates for each task:

$$\mathbf{v} = (\mathbf{I} + r\boldsymbol{\Sigma}[\kappa_{jk}(\mathbf{a})])^{-1}\nabla_{\mathbf{a}} b(\mathbf{a}). \tag{20.25}$$

Of particular note is the fact that the cross partials of the agent's cost function $\kappa(\mathbf{a})$, but not those of the principal's expected benefit function $b(\mathbf{a})$, enter into the determination of the optimal incentives.

HM illustrate the importance of the shape of the cost function by considering a simple two-task setting in which there is a performance measure for the first task, but no performance measure for the second, i.e., $\sigma_2^2 = \infty$ and $\sigma_{12} = 0$. In this setting,

$$v_1 = [b_1(\mathbf{a}) - b_2(\mathbf{a})\kappa_{12}(\mathbf{a})\kappa_{22}(\mathbf{a})][1 + r\sigma_1^2(\kappa_{11}(\mathbf{a}) - \kappa_{12}(\mathbf{a})^2/\kappa_{22}(\mathbf{a}))]^{-1}, \tag{20.26}$$

where $b_j(\mathbf{a})$ denotes the partial derivative of $b(\mathbf{a})$ with respect to a_j. To illustrate the implications of (20.26) we assume the principal's expected benefit function is linear, i.e., $b(\mathbf{a}) = \mathbf{b}^t\mathbf{a}$, and the agent's personal cost is quadratic and his personal benefit is linear, i.e, $\kappa(\mathbf{a}) = \frac{1}{2}\mathbf{a}^t\boldsymbol{\Gamma}\mathbf{a} - \mathbf{a}^t\mathbf{g}$, where

$$\boldsymbol{\Gamma} = \begin{bmatrix} 1 & \gamma \\ \gamma & 1 \end{bmatrix},$$

$\gamma \in (-1, +1)$, and $\mathbf{g} \gg \mathbf{0}$. Observe that if there are no incentives (i.e., $v_1 = 0$), then the agent will implement

$$a_j^o = \frac{1}{1 - \gamma^2}[g_j - \gamma g_k], \quad j, k = 1, 2, j \neq k,$$

which we assume is positive. From (20.26) we obtain the following expression for the optimal incentive rate on the available performance measure,

$$v_1 = [b_1 - b_2\gamma][1 + r\sigma_1^2(1 - \gamma^2)]^{-1},$$

and the induced effort is

$$a_1^\dagger = \frac{1}{1-\gamma^2}[v_1 + g_1 - \gamma g_2],$$

$$a_2^\dagger = \frac{1}{1-\gamma^2}[g_2 - \gamma(v_1 + g_1)].$$

Observe that the incentive rate v_1 decreases with an increase in $r\sigma_1^2$, i.e., more agent risk aversion or more performance measure noise result in weaker incentives, which is the same as in the single task setting. The key difference here is that while increasing v_1 results in more induced effort in the first task, it results in less (more) induced effort in the second task if γ is positive (negative). That is, stronger incentives on the performance measure for the first task have a negative impact on the effort in the second task if effort in the two tasks are complements in the agent's cost function ($\gamma \in (0,1)$), and have a positive impact if effort in the two tasks are substitutes in the agent's cost function ($\gamma \in (-1,0)$).

An increase in the principal's benefit from the first task, b_1, has the opposite effect to an increase in $r\sigma_1^2$, whereas the impact of an increase in b_2 is more subtle. If $\gamma \in (0,1)$, then increasing the principal's benefit from the second task results in a lower incentive rate and less effort in the first task, with more effort in the second task. However, if $\gamma \in (-1,0)$, then increasing b_2 results in a higher incentive rate and more effort in both the first and second tasks. Hence, knowing whether the effort across tasks are complements or substitutes in the agent's cost function is important for understanding the impact of differences in the other model parameters.

Dominance of No Incentives over Strong Incentives in Motivating the Allocation of "Basic" Effort

The preceding model illustrates that the strength of the incentives placed on a non-congruent performance measure (e.g., one that focuses on a single task) can depend significantly on the side-effect of those incentives on the effort expended in another task for which there is no performance measure. HM starkly illustrate this in a setting in which it is *optimal to provide no incentives.*

There are three key features of the model in this setting. First, there is a "primary" task in which positive effort is critical to obtaining a positive profit, and there is a "secondary" task in which effort increases the profit if effort in the primary task is strictly positive. Second, there is a single performance measure that is influenced by effort in the secondary task, but not the primary task. Third, the cost function is represented by a "threshold cost" model, as described above. Incentive compensation can be used to motivate more effort in the secondary task, but this will motivate the agent to put all his effort into that task instead of allocating some effort to the primary task. Hence, the use of strictly positive incentives on the non-congruent performance measure is undesirable.

Proposition 20.9 (HM, Prop. 1)

Assume:

- there are two actions, $\mathbf{a} = (a_1, a_2)$;
- some effort in task 1 is necessary for a positive benefit, i.e., $b(\mathbf{a}) = 0$ if $a_1 = 0$, and $\nabla_{\mathbf{a}} b(\mathbf{a}) > \mathbf{0}$ if $a_1 > 0$;
- the agent's personal cost is a function of aggregate effort $a^+ = a_1 + a_2$, and $\kappa(a^+)$ is nonincreasing for $a^+ \in [0, a^o]$ and strictly increasing for $a^+ > a^o$;
- there exist effort levels $\mathbf{a}$ such that $a^+ = a^o$, $b(\mathbf{a}) > \kappa(a^+)$;
- there is a single performance measure, and it is independent of the effort in the first task, so that $y \sim \mathrm{N}(M(a_2), \sigma^2)$.

The efficient compensation contract pays a fixed wage and contains no incentive component.

The agent can be paid a fixed wage sufficient to satisfy the participation constraint, and asked to select $\mathbf{a}$ so as to maximize $b(\mathbf{a})$ subject to $a^+ = a^o$. If incentive compensation is paid on the basis of y, the agent will focus all his effort on a_2 and set $a_1 = 0$, resulting in zero gross benefit to the principal.

"Asset" Ownership Choice

HM also consider a setting in which the principal owns two projects that are to be operated by the agent. Let x_i and a_i represent the cash flow from project i and the effort expended by the agent in operating that project, $i = 1, 2$. The cash flow from the first project will be produced prior to the termination of the contract and is contractible – it can be shared. The cash flow from the second project will not be generated until after the termination of the contract and, hence, it is not contractible and cannot be shared. However, ownership of the second project can be transferred so that the agent, instead of the principal, receives the future cash flows. The second project will not generate any cash flow unless the agent expends positive effort on that project.

The outcomes from the two actions are risky and are represented by

$$x_i \sim \mathrm{N}(b_i(a_i), \sigma_i^2), \quad i = 1, 2,$$

with $\mathrm{Cov}(x_1, x_2) = 0$. The agent's personal cost depends on aggregate effort a^+ and that cost is nonincreasing for $a^+ \leq a^o$, but strictly increasing for $a^+ > a^o$. The only contractible information is $y = x_1$. We assume that $b_i(a_i)$, $i = 1, 2$, and $\kappa(a^+)$

are concave and convex, respectively, and are such that we can characterize optimal choices using first-order approaches.

Two types of contracts are considered.

(i) *Service contract*: Under a service contract ownership of the second project is transferred to the agent and he contracts to provide services to the principal for the first project. The principal's and agent's net consumption can be expressed as $\pi_s = (1 - v_s)x_1 + f_s$ and $c_s = v_s x_1 + x_2 - f_s - \kappa(a_1 + a_2)$, where v_s and f_s are contract parameters. The agent's risk premium is $\frac{1}{2}r[v_s^2\sigma_1^2 + \sigma_2^2]$.

(ii) *Employment contract*: Under the employment contract the principal is assigned ownership of the second project and he pays a wage (contingent on y) to the agent to operate both projects. The principal's and agent's net consumption can be expressed as $\pi_e = (1 - v_e)x_1 + x_2 - f_e$ and $c_e = v_e x_1 + f_e - \kappa(a_1 + a_2)$, where v_e and f_e are contract parameters. The agent's risk premium is $\frac{1}{2}r v_e^2\sigma_1^2$.

To determine the optimal contract, we introduce three expected net return measures.

– Maximum net return if effort is expended only on the first project:

$$\Pi^1 = \max_{a_1} \; b_1(a_1) - \kappa(a_1).$$

– Maximum expected net return if effort is expended only on the second project:

$$\Pi^2 = \max_{a_2} \; b_2(a_2) - \kappa(a_2).$$

– Maximum net return from allocating "basic" effort between the two projects:

$$\Pi^{12} = \max_{a_1 \in [0, a^o]} \; b_1(a_1) + b_2(a^o - a_1) - \kappa(a^o).$$

Proposition 20.10 (HM, Prop. 2)

Assume $\Pi^{12} \geq \Pi^1, \Pi^2$. In the optimal employment contract, the agent is paid a fixed wage ($v_e = 0$) and instructed how best to allocate his basic effort. In the optimal service contract, a "high powered incentive" is paid (i.e., $v_s >$

0). Furthermore, there exist values of r, σ_1 and σ_2 for which an employment contract is optimal and others for which a service contract is optimal.

Proof: Within an employment contract, the principal can set $v_e = 0$ and ask the agent to optimally allocate his basic effort in return for a fixed payment of $f_e = \kappa(a^o)$, which will yield an expected net return to the principal of Π^{12}, and the principal will bear all the risk. If, on the other hand, the principal sets $v_e > 0$, then the agent bears some of the risk of the first project, and will set a_1 so that $v_e = \kappa'(a_1)/b_1'(a_1)$ with $a_1 > a^o$ and $a_2 = 0$. In this case, $f_e = \kappa(a_1) + \frac{1}{2}rv_e^2\sigma_1^2 - v_e b_1(a_1)$, which yields an expected net return to the principal of $b_1(a_1) - \kappa(a_1) - \frac{1}{2}rv_e^2\sigma_1^2 < \Pi^1 \leq \Pi^{12}$. Hence, it is best to set $v_e = 0$ if an employment contract is used.

With a service contract, $v_s = 0$ induces $a_1 = 0$ and a_2 such that Π^2 is maximized, resulting in a net expected return to the principal of $\Pi^2 - \frac{1}{2}r\sigma_2^2$. Hence, this cannot be optimal. On the other hand, $v_s > 0$ imposes risk on the agent and induces him to set a_1 and a_2 so that $v_s = b_2'(a_2) = \kappa'(a_1 + a_2)$. Let $a_i^\dagger(v_s)$ represent the effort induced by v_s. In this case, $f_s = \kappa(a_1 + a_2) + \frac{1}{2}r[v_s^2\sigma_1^2 + \sigma_2^2] - v_s a_1 - b_2(a_2)$, which yields an expected net return to the principal of

$$b_1(a_1^\dagger(v_s)) + b_2(a_2^\dagger(v_s)) - \kappa(a_1^\dagger(v_s) + a_2^\dagger(v_s)) - \tfrac{1}{2}r[v_s^2\sigma_1^2 + \sigma_2^2].$$

The principal will select $v_s > 0$ to maximize his net expected return. If $\sigma_1^2 = \sigma_2^2 = 0$, the first-best is achieved by setting $v_s = 1$, and the service contract dominates the employment contract (which cannot achieve first-best). Increasing $r\sigma_2^2$ decreases the principal's expected net return without limit, so that for large values of $r\sigma_2^2$, the employment contract dominates the service contract.

Q.E.D.

The key here is that under an employment contract, the risk neutral principal bears the risk of x_2 but must avoid high powered incentives based on current cash flows in order to induce the agent to efficiently allocate his "basic" effort among the two tasks. Under the service contract, the agent receives the benefits from a_2 due to "asset" ownership and from a_1 through "high powered incentives" based on current cash flows. However, he is risk averse and cannot share the risks associated with x_2.

We do not go through the details, but merely note that HM consider two other models. In one, the agent allocates effort among tasks that are directly beneficial to the principal and tasks that are directly beneficial to the agent. It is assumed that the contract can preclude effort in one or more of the tasks directly beneficial to the agent, but cannot otherwise directly influence the level of effort in those tasks. Precluding effort in tasks beneficial to the agent reduces the marginal cost of the effort expended in the tasks beneficial to the principal. However, such restrictions increase the compensation the agent must receive from the principal (since the agent forgoes personal benefits). Hence, it can be optimal to preclude effort in some tasks that are personally beneficial to the agent, but not others.

The other model examines the optimal allocation of tasks between two identical agents. The performance measures for each task differ with respect to their noisiness. The interesting feature of the solution in this setting is that, even though the agents have equal effort costs, it is optimal to have one agent specialize in tasks that are hard to monitor (i.e., very noisy performance measures) and to have the other specialize in tasks that are easily monitored.

20.4 LOG-LINEAR INCENTIVE FUNCTIONS

In Section 20.1.2 we observed that compensation is normally distributed if the performance measures are log-normally distributed and compensation is a linear function of the log of performance measures. In this section we briefly explore a simple model in which the performance measures are log-normal. As noted by Hemmer (1996), the advantage of exploring performance measures that are log-normal is that this distribution is defined over positive values, which is representative of many performance measures, particularly non-financial measures and measures based on stock price.

The Basic Model

The agent's action again consists of two tasks, $\mathbf{a} = (a_1, a_2) \in A = [0,\infty)\times[0,\infty)$. The agent is risk and effort averse and has exponential utility with a separable, quadratic monetary cost of effort:

$$u^a(c,\mathbf{a}) = -\exp[-r(c - \kappa(\mathbf{a}))],$$

$$\kappa(\mathbf{a}) = \tfrac{1}{2}(a_1^2 + a_2^2).$$

The principal is risk neutral, and the expected benefit to the principal of the agent's effort is represented by $\mathbf{b}^t\mathbf{a}$. We assume there are separate, independent, and log-normally distributed performance measures for each task:

$$\psi_i = \bar{\psi}_i(a_i)\,\varepsilon_i, \quad i = 1, 2,$$

where
$$\bar{\psi}_i(a_i) = \exp[M_i a_i],$$

$$\ln(\varepsilon_i) \sim \mathrm{N}(0,1).$$

Now consider a contract of the form:

$$c(\boldsymbol{\psi}) = f + v_1 \ln(\psi_1) + v_2 \ln(\psi_2).$$

As noted in Lemma 20.1, the compensation is normally distributed. In fact, we can interpret the contract as a linear function of normally distributed representations of the two performance measures, with

$$y_i = \ln(\psi_i) \sim \mathrm{N}(M_i a_i, 1), \quad i = 1, 2.$$

This, of course, allows us to apply the analysis in Section 20.2. For example, from (20.13) it follows that

$$v_i^\dagger = \frac{M_i b_i}{M_i^2 + r},$$

and $$a_i^\dagger = v_i M_i.$$

Alternative Representations of the Performance Measures

Hemmer (1996) effectively begins with a performance measure that is the product of the two basic independent performance measures.[16] We represent that measure as

$$\psi_0 = \psi_1 \psi_2 = \bar{\psi}_0(\boldsymbol{a})\, \varepsilon_0,$$

where $$\bar{\psi}_0(\boldsymbol{a}) = \exp[\mathbf{Ma}],$$

$$\ln(\varepsilon_0) \sim \mathrm{N}(0, 2).$$

In this case, the transformed performance measure is

$$y_0 = \ln(\psi_0) \sim \mathrm{N}(\mathbf{Ma}, 2).$$

If y_0 is the only available performance measure, then

$$v_0^\ddagger = \frac{\mathbf{Mb}}{\mathbf{MM}^t + 2r}.$$

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[16] Hemmer assumes $x = x_1 + x_2$, where x_1 is observed and x_2 is not. Furthermore, x_1 is influenced by both a_1 and a_2, whereas x_2 is only influenced by a_2. In particular, $x_1 = \mathbf{Ma} + \ln(\varepsilon_1) + \ln(\varepsilon_2)$, so that $b_1 = M_1$ and $b_2 = M_2 + \mathrm{E}[x_2 | a_2]/a_2$.

We assume that $\mathbf{M} \neq \mathbf{b}^t$, so that y_0 is not perfectly congruent, resulting in a loss in value relative to first-best even if the agent is risk neutral.

If y_0 is used with either y_1 or y_2, the result is the same as using y_1 and y_2. For example, if y_0 and y_1 are used, the optimal contract is such that $v_0 = v_2^\dagger$, $v_1 = v_1^\dagger - v_2^\dagger$, and $\mathbf{a}^\dagger$ is induced at the same cost as using y_1 and y_2. In that case, the first-best result is achieved if $r = 0$.

20.5 CONCLUDING REMARKS

This chapter is pivotal in that it moves the reader from focusing on the induced intensity of effort and the associated risk premium, to also considering the induced allocation of effort among diverse activities undertaken by the typical agent. Incentive risk is still important, but now we also consider the congruity of the induced allocation of effort relative to the first-best allocation. While the congruity of the allocations induced by a single performance measure is important if there are strong incentives based on that measure, we observe that induced congruity can be facilitated by using a diverse set of performance measures. The key is to set the relative incentive weights after considering the expected outcome and effort cost associated with alternative induced effort allocations, in addition to considering the resulting aggregate incentive risk.

The recognition of multiple dimensions of effort permits the model to encompass some special cases that cannot be represented in models with single dimensional effort. These include:

- window dressing – an activity that is costly to the agent and improves his reported performance, but provides no benefit to the principal;
- myopia – a performance measure that provides strong incentives for some activities and weak incentives for others;
- induced moral hazard – an activity that has benefits and costs to the principal, is costless to the agent, but differentially affects the performance measures.

Issues of myopia are often associated with accounting-based performance measures, versus stock price-based performance measures which reflect investors' beliefs about both the short and long-run consequences of current actions. Chapter 21 considers multi-task, multi-performance measure models in which one measure is the stock price and another is representative of a myopic accounting measure.

In the present chapter, all activities are undertaken simultaneously or, at least, the agent does not receive any information between the time at which he chooses an "early" action and the time at which he chooses a "late" action. Multi-period models are examined in Chapters 25 through 28. They can be viewed as multi-task models in which the agent receives information between decision dates, and there can be multiple activities chosen at each decision date.

In this chapter, all activities are undertaken by a single agent. Multi-agent models are examined in Chapters 29 and 30. They can be viewed as multi-task models in which different subsets of the activities are chosen by different agents, each with their own preferences. This permits examination of the impact of performance measures and incentives on collusion and coordination.

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CHAPTER 21

STOCK PRICES AND ACCOUNTING NUMBERS AS PERFORMANCE MEASURES

CEO's often have stock price based incentives. The two primary forms of these incentives are stock ownership and stock option grants. Stock prices could serve as the only incentive information, and in some firms that is the case. However, we often see the use of both stock prices and accounting numbers. The argument in favor of stock prices instead of accounting earnings is that accounting earnings are inherently myopic – they only report the impact of the agent's actions on the short-term cash flows of the firm – whereas stock prices inherently reflect both the short and long run effects of an agent's actions. This would appear to justify using only the stock price as a performance measure. However, a careful look at this issue reveals reasons why a firm might use both accounting earnings and stock prices as performance measures. Our analysis is not an exhaustive examination of this issue. We merely discuss some insights that follow from our prior analysis.

The literature in this area has focused on the use of the end-of-period stock price as an *ex post* performance measure.[1] A key element of this analysis is that the firm's terminal value is not contractible because it is not observed by the principal until some date subsequent to the termination of the agent's contract. Hence, the stock price at the contract termination date (which we refer to as the *ex post* stock price) is based on the investors' imperfect information about the terminal value of the firm. Some of the investors' information may come from public reports, e.g., published financial statements, whereas other information may come from private information acquisition activities.

The stock price aggregates the investors' information into a single number. Under standard capital market assumptions, the stock price efficiently aggregates the investors' public information for valuation purposes. However, as discussed in Chapter 11 of Volume I, the stock price will not fully reflect the investors' private information if there is some form of noise in the price process. An issue of central concern in this chapter is whether the stock price efficiently

[1] In Chapter 22 we consider a setting in which the agent has post-contract, pre-decision private information. In that setting, the "*ex ante*" stock price may play a role as a contractible aggregate for non-contractible investor information and management disclosures of private information (see Section 22.8).

aggregates the investors' information for incentive purposes. If it does not, then agency costs may be reduced by using both the stock price and other contractible information, such as accounting earnings, as performance measures.

21.1 *EX POST* EQUILIBRIUM STOCK PRICE

In this section we consider how the *ex post* equilibrium stock price, i.e., the stock price at the end of the contracting period, is influenced by the investors' information at that date. We view the risk and the investors' information as firm-specific such that investors are effectively risk neutral with respect to the information. The analysis is similar to that in the last section of Feltham and Xie (1994) (FX). Assume that there are only two tasks and two signals. Let **a** denote the action chosen by the agent, and let $\hat{\mathbf{a}}$ represent the investors' conjecture (belief) about the agent's action (in a rational expectations equilibrium they, of course, attach probability one to the action induced by the contract in place). Assume investors receive signals $\mathbf{y} = (y_1,y_2)^t$, which are normalized so that they have unit variance. The terminal value of the firm (before deducting the agent's compensation) is denoted x. However, a key element of this analysis is that x is *not* observed until after the contract termination date. We assume $(x,\mathbf{y})$ is jointly normally distributed, i.e.,

$$\begin{bmatrix} x \\ \mathbf{y} \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} \mathbf{b}^t\mathbf{a} \\ \mathbf{M}\mathbf{a} \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & \boldsymbol{\Sigma}_{xy} \\ \boldsymbol{\Sigma}_{yx} & \boldsymbol{\Sigma} \end{bmatrix} \right),$$

where $$\mathbf{M} = \begin{bmatrix} \mathbf{M}_1 \\ \mathbf{M}_2 \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{bmatrix}, \quad \boldsymbol{\Sigma}_{yx} = \boldsymbol{\Sigma}_{xy}^t = \begin{bmatrix} \rho_{x1}\sigma_x \\ \rho_{x2}\sigma_x \end{bmatrix}.$$

The (gross) stock price at the contract termination date is equal to the investors' expectation about x conditional on $\mathbf{y}$ and $\hat{\mathbf{a}}$, i.e.,[2]

$$P_1(\mathbf{y},\hat{\mathbf{a}}) = \mathrm{E}[x|\mathbf{y},\hat{\mathbf{a}}] = \Omega(\hat{\mathbf{a}}) + \boldsymbol{\omega}^t\mathbf{y}, \tag{21.1}$$

where $$\Omega(\hat{\mathbf{a}}) = [\mathbf{b}^t - \boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}^{-1}\mathbf{M}]\hat{\mathbf{a}} \quad \text{and} \quad \boldsymbol{\omega}^t = \boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}^{-1}.$$

[2] We assume without loss of generality that there is only one share of the stock outstanding so that the stock price is equal to the market price of the firm.

Observe that $\Omega(\cdot)$ is a constant that depends on the conjectured level of effort, while $\boldsymbol{\omega}^t\mathbf{y}$ varies with $\mathbf{y}$ and is influenced by the actual level of effort. The vector $\boldsymbol{\omega}^t = (\omega_1, \omega_2)$ represents the weights assigned to the two signals by the investors in their valuation of the stock. The relative weight assigned to the two signals by investors is

$$\frac{\omega_1}{\omega_2} = \frac{\rho_{x1} - \rho_{x2}\rho_{12}}{\rho_{x2} - \rho_{x1}\rho_{12}}. \tag{21.2}$$

That is, the key factors determining the relative weight placed on the two normalized signals are the correlations ρ_{x1}, ρ_{x2}, and ρ_{12}, which are the key factors determining the information content of the signals about the firm's terminal value x.

If both signals are uncorrelated with x (i.e., $\rho_{x1} = \rho_{x2} = 0$), then neither signal will be used by the investors. The signals could be very informative about the agent's action $\mathbf{a}$ (as represented by the vector of sensitivities $\mathbf{M}$), but would be ignored by investors because they are uninformative about x.

If the two signals are uncorrelated (i.e., $\rho_{12} = 0$) but they are both correlated with x, then

$$\frac{\omega_1}{\omega_2} = \frac{\rho_{x1}}{\rho_{x2}}. \tag{21.3}$$

That is, the weights depend on their relative information content about x.

From the above it is obvious that if the two signals are uncorrelated and one is not correlated with x, then the latter receives zero weight. However, this does not occur if a signal is not correlated with x but is correlated with the other signal. That is, assume $\rho_{x1} \neq 0$, $\rho_{12} \neq 0$, and $\rho_{x2} = 0$, then

$$\frac{\omega_2}{\omega_1} = -\rho_{12}. \tag{21.4}$$

In this case, the correlation of x and y_1 is immaterial to the relative weight assigned to the two signals. The key factor is the correlation between the two signals. The adjustment is negative if ρ_{12} is positive, because y_2 is effectively used to remove some of the noise in y_1.

21.2 STOCK PRICE AS AN AGGREGATE PERFORMANCE MEASURE

The investors are not directly concerned about management incentives when they trade the firm's equity at the contract termination date. They trade the equity based on their expectations about the terminal value of the firm x and, hence, the resulting market price is not set with the objective of providing the most useful performance measure for the agent. However, the weights used by the investors will implicitly determine the weights used in the agent's incentive contract if his compensation is a function of the market price.

Our analysis is based on a model similar to the one used in Section 20.2 with the simplifying assumption of only two tasks and two signals as in the previous section. The compensation contract is restricted to be a linear function of the *ex post* stock price, i.e.,

$$c(P_1) = f + vP_1.$$

The agent has multiplicatively separable exponential utility, i.e., $u^a(c,\mathbf{a}) = -\exp[-r(c - \kappa(\mathbf{a}))]$, with a separable and quadratic cost function $\kappa(\mathbf{a}) = \frac{1}{2}\mathbf{a}^t\mathbf{a}$ and reservation utility $\underline{U} = -\exp[-rc^o]$. The agent's certainty equivalent given that the investors hold a conjecture $\hat{\mathbf{a}}$ of the agent's action is

$$CE(f,v,\hat{\mathbf{a}},\mathbf{a},\eta) = f + v[\Omega(\hat{\mathbf{a}}) + \boldsymbol{\omega}^t\mathbf{M}\mathbf{a}] - \tfrac{1}{2}\mathbf{a}^t\mathbf{a} - \tfrac{1}{2}rv^2\boldsymbol{\omega}^t\boldsymbol{\Sigma}\boldsymbol{\omega},$$

so that the agent's choice given f, v, and $\hat{\mathbf{a}}$ is

$$\mathbf{a} = v\mathbf{M}^t\boldsymbol{\omega}, \tag{21.5}$$

assuming $\mathbf{M}^t\boldsymbol{\omega} \geq \mathbf{0}$. Note that the agent's choice is independent of the investors' conjecture of his action. However, that conjecture affects his certainty equivalent through the constant $\Omega(\hat{\mathbf{a}})$. Of particular note is also the fact that only actions proportional to the vector $\mathbf{M}^t\boldsymbol{\omega}$ can be implemented with a stock based compensation scheme, i.e., the space of implementable actions is single-dimensional and exogenously given.

Given the assumption that all risk and information is firm-specific, we view the shareholders as a risk neutral principal with

$$U^p(f,v,\mathbf{a},\eta) = \mathbf{b}^t\mathbf{a} - (f + v\mathrm{E}[P_1(\mathbf{y})|\mathbf{a}]),$$

where $\mathrm{E}[x|\mathbf{a}] = \mathbf{b}^t\mathbf{a}$. Note that if $\hat{\mathbf{a}} = \mathbf{a}$, then

$$\mathrm{E}[P_1(\mathbf{y})|\mathbf{a}] = \mathrm{E}[\mathrm{E}[x|\mathbf{y},\hat{\mathbf{a}}]\,|\,\mathbf{a}] = \mathbf{b}^t\mathbf{a}.$$

We assume that investors hold rational expectations at the contract termination date such that their conjecture of the agent's action is, in fact, equal to the action induced by the contract in place (which by (21.5) is not affected by that conjecture), i.e., $\hat{\mathbf{a}} = \mathbf{a}$. Hence, we can state the shareholders' decision problem as follows:

$$\underset{f,v,\mathbf{a}}{\text{maximize}} \quad U^p(f,v,\mathbf{a},\eta) = \mathbf{b}^t\mathbf{a} - (f + v\mathrm{E}[P_1(\mathbf{y},\mathbf{a})|\mathbf{a}]), \tag{21.6a}$$

$$\text{subject to} \quad CE(f,v,\mathbf{a},\eta)$$

$$= f + v[\Omega(\mathbf{a}) + \boldsymbol{\omega}^t\mathbf{M}\mathbf{a}] - \tfrac{1}{2}\mathbf{a}^t\mathbf{a} - \tfrac{1}{2}rv^2\boldsymbol{\omega}^t\boldsymbol{\Sigma}\boldsymbol{\omega} = c^o, \tag{21.6b}$$

$$\text{first-order condition (21.5), } \mathbf{a} = v\mathbf{M}^t\boldsymbol{\omega}, \tag{21.6c}$$

$$P_1(\mathbf{y},\mathbf{a}) = \mathrm{E}[x|\mathbf{y},\mathbf{a}] = \Omega(\mathbf{a}) + \boldsymbol{\omega}^t\mathbf{y}, \quad \forall\, \mathbf{y}, \tag{21.6d}$$

where (21.6d) reflects the assumption that investors hold rational expectations at the contract termination date. Substituting (21.6b) - (21.6d) into (21.6a) gives the following unconstrained optimization problem for the shareholder's choice of incentive rate on the stock price:

$$\underset{v}{\text{maximize}} \quad U^p(v,\eta) = v\mathbf{b}^t\boldsymbol{\omega}^t\mathbf{M} - \{c^o + \tfrac{1}{2}v^2\boldsymbol{\omega}^t\mathbf{M}\mathbf{M}^t\boldsymbol{\omega} + \tfrac{1}{2}rv^2\boldsymbol{\omega}^t\boldsymbol{\Sigma}\boldsymbol{\omega}\}$$

$$= v\mathbf{b}^t\boldsymbol{\omega}^t\mathbf{M} - \{c^o + \tfrac{1}{2}v^2\boldsymbol{\omega}^t\mathbf{Q}^{-1}\boldsymbol{\omega}\}, \tag{21.7}$$

where $\mathbf{Q}^{-1} \equiv \mathbf{M}\mathbf{M}^t + r\boldsymbol{\Sigma}$. Differentiating (21.7) with respect to v and solving for the second-best values of v and $\mathbf{a}$ (assuming an interior solution) yields:

$$v^\dagger = \frac{\mathbf{b}^t\mathbf{M}^t\boldsymbol{\omega}}{\boldsymbol{\omega}^t\mathbf{Q}^{-1}\boldsymbol{\omega}}, \tag{21.8a}$$

$$\mathbf{a}^\dagger = \frac{\mathbf{b}^t\mathbf{M}^t\boldsymbol{\omega}}{\boldsymbol{\omega}^t\mathbf{Q}^{-1}\boldsymbol{\omega}}\mathbf{M}^t\boldsymbol{\omega}. \tag{21.8b}$$

To see whether (or when) the market price is an efficient aggregate performance measure, consider the setting in Section 20.2 with two directly contractible performance measures $\mathbf{y} = (y_1,y_2)$. In that setting, the optimal second-best weights and action are given by (see (20.13a) and (20.13b))

$$\mathbf{v}^\ddagger = \mathbf{Q}\mathbf{M}\mathbf{b}, \tag{21.9a}$$

$$\mathbf{a}^\ddagger = \mathbf{M}^t\mathbf{v}^\ddagger. \tag{21.9b}$$

The key difference between (21.8) and (21.9) is that in the former, the relative allocation of effort between the two tasks is constrained to be

$$\frac{a_1^\dagger}{a_2^\dagger} = \frac{M_{11}\omega_1 + M_{21}\omega_2}{M_{12}\omega_1 + M_{22}\omega_2},$$

and only the intensity of effort is endogenously determined. On the other hand, in (21.9) the relative incentive rates on the two performance measures are determined *endogenously* so as to induce the optimal intensity and allocation of effort given the two performance measures. This results in a compensation contract that is a function of $[\mathbf{QMb}]^t\mathbf{y}$, whereas with only the stock price, the relative incentive weights on the two signals are exogenously given by the vector $\boldsymbol{\omega}^t = \boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}^{-1}$ and the optimal compensation is a function of $\boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}^{-1}\mathbf{y}$. Hence, the stock price will only be an efficient aggregate performance measure if the vector of weights on the signals in the stock price, $\boldsymbol{\omega}^t = \boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}^{-1}$, is proportional to the optimally determined vector of weights with directly contractible performance measures $\mathbf{v}^\ddagger = \mathbf{QMb}$.

Proposition 21.1 (FX, Prop. 5)

The stock price is an efficient aggregate performance measure if, and only if, there exists some parameter $\xi \neq 0$ such that

$$\boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}^{-1} = \xi[\mathbf{QMb}]^t. \tag{21.10}$$

We now consider some special cases to provide further insight into whether (or when) the stock price is an efficient aggregate performance measure. We assume that the agent is strictly risk averse, i.e., $r > 0$.

Single Performance Measure

Of course, there is an obvious reason why the stock price may not be an efficient aggregate performance measure, namely that, in general, it is better to have two separate contractible performance measures instead of an aggregate of the two. Suppose there is a single performance measure with $\mathbf{M}_1 \neq \mathbf{0}$.

Corollary

The market price is an efficient performance measure if, and only if, $\rho_{x1} \neq 0$.

The key here is that the stock price P_1 is a non-trivial linear function of performance measure y_1 if, and only if, y_1 is correlated with x. Given that $\rho_{x1} \neq 0$, the stock price can be used to accomplish the same incentives as y_1 by applying an appropriate linear transformation to P_1.

Single Task

With a stock based compensation contract the set of implementable actions is single-dimensional (and proportional to $\mathbf{M}^t\boldsymbol{\omega}$) and, hence, the optimal allocation of effort (with two directly contractible performance measures) may not be implementable with the stock price. The allocation issue does not arise in a single task setting, i.e., a setting in which $\mathbf{a} = a$ is single dimensional, with

$$\mathbf{M} = (M_1, M_2)^t, \quad \mathbf{b} = b.$$

Corollary

The market price is an efficient performance measure if, and only if, (see (21.2) and the discussion following Proposition 20.3)

$$\frac{\rho_{x1} - \rho_{x2}\rho_{12}}{\rho_{x2} - \rho_{x1}\rho_{12}} = \frac{M_1 - \rho_{12}M_2}{M_2 - \rho_{12}M_1} \quad \Leftrightarrow \quad \frac{\rho_{x1}}{\rho_{x2}} = \frac{M_1}{M_2}. \tag{21.11}$$

In this setting there are no concerns regarding congruity of the performance measure. Hence, the optimal contract puts relative weights on the two performance measures to induce the optimal action at the lowest cost (of agent risk) to the shareholders, i.e., according to the relative impact, M_1/M_2, of the action on the performance measures, which have been scaled to have unit variance. This highlights the fact that the stock price is an efficient aggregate performance measure if, and only if, the relative information content of the performance measures about the terminal value of the firm is equal to their relative information content about the agent's action. Observe that the above result holds even if the performance measures are uncorrelated (i.e., $\rho_{12} = 0$).

Information about Uncontrollable Events

In Chapters 18 and 20 we established that a performance measure unaffected by the agent's action may be useful if it is informative about uncontrollable events affecting a primary performance measure. Assume there are two performance measures with $\mathbf{M}_1 \neq \mathbf{0}$, $\mathbf{M}_2 = \mathbf{0}$.

Corollary

The market price is an efficient aggregate performance measure if, and only if, (see (21.2) and Proposition 20.5)

$$\frac{\rho_{x1} - \rho_{x2}\rho_{12}}{\rho_{x2} - \rho_{x1}\rho_{12}} = -\frac{1}{\rho_{12}} \quad \Leftrightarrow \quad \rho_{x2}(1 - \rho_{12}^2) = 0.$$

Hence, the stock price is an efficient performance measure if the second performance measure is not directly informative about the investors' terminal dividend ($\rho_{x2} = 0$). In that setting, the second measure gets a weight in the stock price that reflects its correlation with the noise in the first measure.[3]

Independent, Myopic Performance Measures

Assume there is a separate, independent performance measure for each task, with $\rho_{12} = 0$, $M_{ii} > 0$, $i = 1, 2$, and $M_{12} = M_{21} = 0$.

Corollary

The market price is an efficient aggregate performance measure if, and only if, (see (21.3) and Proposition 20.6)

$$\frac{\rho_{x1}}{\rho_{x2}} = \frac{M_{11} b_1 / [r + M_{11}^2]}{M_{22} b_2 / [r + M_{22}^2]}. \tag{21.12}$$

In this setting the two signals get relative weights in the stock price according to their correlation with the terminal value of the firm, i.e., ρ_{xi}, whereas with directly contractible performance measure they get weights according to their benefits to the shareholders, b_i, adjusted for the sensitivity of the performance measure, M_{ii}, and the agent's risk aversion r. Paul (1992) explores this special case. We see here that despite being a very simple setting, there is no reason to expect condition (21.12) to be satisfied unless the two tasks are identical in every respect.

21.3 STOCK PRICE AS PROXY FOR NON-CONTRACTIBLE INVESTOR INFORMATION

In this section we consider the joint use of the stock price and a publicly reported performance measure, which we interpret to be an accounting measure, such as accounting earnings. We are particularly interested in the signs and optimal

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[3] If the two measures are perfectly (positively or negatively) correlated, the "noise" in the first measure can be eliminated through the second measure if the two measures are used in contracting. However, contracting on the price is problematic since we must specify the off-equilibrium price that occurs if the second signal is inconsistent with the first signal given the investors' conjecture with respect to the agent's actions.

relative incentive weights on the two measures. The stock price reflects both the publicly reported accounting number, y_1, and a non-contractible signal, y_2, that is received by investors after the agent's action has been taken but before the contract is terminated. The market price is contractible information and, hence, it serves as an indirect means of contracting on the non-contractible signal y_2. Of course, in using the price to make inferences about y_2, we must recognize that the price is also influenced by y_1, which is directly contractible.

In Section 21.3.1 we consider a setting similar to Section 21.1. All investors observe both the public accounting report and the non-contractible signal. They conjecture that the agent has taken action $\hat{\mathbf{a}}$, they are well diversified, and all random variations in the information and the outcome are firm-specific. Hence, the equilibrium date 1 market price is characterized by (21.1), i.e., the price equals the posterior expected terminal value of the firm. This model provides a simple illustration of the signs and relative magnitudes of the incentive weights assigned to an accounting report and the market price given that the latter is influenced by both the accounting report and non-contractible investor information.

In Section 21.3.2 we consider a *rational expectations* model similar to Feltham and Wu (FWa) (2000).[4] The firm's shares are initially held by well-diversified, long-term investors. At date 1, some of these investors exogenously sell z shares to *rational* risk-averse investors who are willing to hold an undiversified portfolio if the market price provides an appropriate risk premium. The accounting report y_1 is received by all rational investors, but only a fraction of these investors observe the non-contractible signal y_2, i.e., it is private information for some investors. The market supply of shares z is random and unobservable. Hence, the uninformed investors cannot perfectly infer the private signal y_2 from the price and the accounting report. However, they respond rationally, using the fact that the stock price provides noisy information about the informed investors' private information.

The model in Section 21.3.1 (in which all investors observe y_2) is much simpler than the rational expectations model in Section 21.3.2 (in which only a fraction of the investors obtain y_2). The two models provide similar insights if the fraction informed is exogenous. However, some comparative statics differ significantly if the fraction informed is endogenously determined. In his discussion of Bushman and Indjejikian (1993) and Kim and Suh (1993), Lambert (1993) states that there is little benefit in an agency analysis of introducing a noisy rational expectations model unless the investors' information acquisitions are endogenously determined.

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[4] Bushman and Indjejikian (1993) and Kim and Suh (1993) provide similar rational expectations models. However, they treat the investors' private information as exogenous, whereas FWa consider the endogenous acquisition of private investor information.

21.3.1 Exogenous Non-contractible Investor Information

The information is the same as in Section 21.1, and (21.1) characterizes the date 1 equilibrium price if all investors observe $\mathbf{y} = (y_1, y_2)$ and hold action conjecture $\hat{\mathbf{a}}$. If both reports are contractible, then the optimal incentive rates and actions are characterized by (21.9). We assume that the stock price is *not* an efficient aggregate performance measure (i.e., condition (21.10) is not satisfied).

Now assume that the accounting report y_1 is received by all investors and is contractible, whereas y_2 is observed by all investors but is not directly contractible. Given (21.1), it is possible to infer y_2 from the date 1 market price P_1 and the accounting report y_1, i.e.,

$$y_2 = \frac{1}{\omega_2}[P_1 - (\Omega(\hat{\mathbf{a}}) + \omega_1 y_1)]. \tag{21.13}$$

Hence, if the price is contractible information, it can be used with the accounting report to specify a linear contract that is equivalent to any linear contract based on y_1 and y_2. The optimal fixed wage $f^\ddagger$ and incentive rates $(v_1^\ddagger, v_2^\ddagger)$ can be used to specify the optimal contract based on y_1 and P_1 as follows:

$$\begin{aligned} c(y_1, P_1) &= f^\ddagger + v_1^\ddagger y_1 + v_2^\ddagger \frac{1}{\omega_2}[P_1 - (\Omega(\hat{\mathbf{a}}) + \omega_1 y_1)] \\ &= f^P + v_1^P y_1 + v_2^P P_1, \end{aligned} \tag{21.14}$$

where $f^P = f^\ddagger - v_2^\ddagger \Omega(\hat{\mathbf{a}})/\omega_2$, $v_1^P = v_1^\ddagger - v_2^\ddagger \omega_1/\omega_2$, and $v_2^P = v_2^\ddagger/\omega_2$.

Observe that the public report y_1 influences the agent's compensation in (21.14) in two ways. First, it is directly included as an argument in the agent's compensation function. Second, it enters indirectly through its impact on the price P_1. Hence, it is clear that $v_1^P \neq v_1^\ddagger$ if $\omega_1 \neq 0$.

The relative weights assigned to the accounting report and the stock price can be represented by

$$\frac{v_1^P}{v_2^P} = \frac{\omega_2 v_1^\ddagger - \omega_1 v_2^\ddagger}{v_2^\ddagger}. \tag{21.15}$$

A key point here is that the incentive rate for the accounting report (i.e., v_1^P) can be negative even though $v_1^\ddagger$ is positive. This would occur if v_2^P indirectly places too much weight on y_1 through the stock price, and v_1^P is used to reduce that weight. On the other hand, the weight on the stock price v_2^P is negative if $v_2^\ddagger$ is negative (e.g., if the non-contractible signal is informative about uncontrollable events that affect the accounting report).

21.3.2 A Fraction of Privately Informed Investors

We now consider a setting in which the non-contractible signal is only known by some, not all, investors. The model of the price process is similar to the models in Chapter 11 of Volume I. The price is imperfectly informative about the non-contractible signal (assuming it is correlated with the final outcome) and, hence, will influence the demand for the firm's shares by rational uninformed investors. Furthermore, since the price is contractible information, the price will be used in contracting with the agent (assuming the non-contractible signal is influenced by the agent's actions).

The model in this section has three types of investors. The first type consists of long-term investors who control the firm through the principal (board of directors), who contracts with the firm's manager (agent). These investors are well diversified and will not trade – the principal seeks to maximize the expected terminal value of their shares. The second type are "liquidity traders" who randomly change their holdings of the firm's shares at date 1. As in Chapter 11 of Volume I, they are introduced merely to create noise in the price process and we do not model their preferences. The number of shares traded is exogenous and is independent of price and available information. The third type are "rational" investors with negative exponential utility. Their demand for the firm's shares at date 1 depends on their risk aversion r_I, the market price P_1, and their beliefs about the terminal value x. All rational investors receive the accounting report y_1, but a rational investor only receives the non-contractible report y_2 if he pays a cost κ.[5] The fraction who choose to obtain y_2 is denoted $\lambda \in [0,1]$. This third type act "rationally" in the sense that they maximize their expected utility and, if they have not observed y_2, they form rational beliefs about this signal based on the accounting report and the market price. However, they do not trade strategically even if they are informed – they act as price-takers. That is, the informed traders do not consider how their trades affect the beliefs of the uninformed traders (see Chapter 12 of Volume I for models in which the informed traders act strategically).

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[5] The comparative statics are simplified if we assume all rational investors have the same risk aversion and the same information costs. In equilibrium, all rational investors will be indifferent between paying κ to be informed versus being uninformed. If they differed in their risk aversion the informed investors would consist of the least risk averse, and if they differed in their information costs, the informed investors would consist of the investors with the least costs.

The number of shares sold by the liquidity traders at date 1 is represented by a random variable $z \sim N(0,\sigma_z^2)$,[6] which equals the total number of shares sold divided by the number of rational investors (negative z represents shares purchased by the liquidity traders). The terminal value of the firm is expressed as $x = \mathbf{b}^t\mathbf{a} + \varepsilon_x$, with $\varepsilon_x \sim N(0,\sigma_x^2)$.[7]

The accounting report is $y_1 \sim N(\mathbf{M}_1\mathbf{a},1)$, and the non-contractible signal is $y_2 \sim N(\mathbf{M}_2\mathbf{a},1)$. That is, both may be influenced by the agent's actions and both are scaled to have unit variance. To facilitate the use of the analysis in Section 11.3, we assume the two signals are independent (i.e., $\rho_{12} = 0$)[8] and we transform the reports using the investors' conjecture $\hat{\mathbf{a}}$ with respect to the agent's action and scale factors γ_1 and γ_2, to obtain $y_a \equiv \gamma_1(y_1 - \mathbf{M}_1\hat{\mathbf{a}})$ and $y_i \equiv \gamma_2(y_2 - \mathbf{M}_2\hat{\mathbf{a}})$.[9] Let $\gamma_j \equiv \text{Cov}[x,y_j] = \rho_{xj}\sigma_x$, $j = 1,2$, which implies $\sigma_a^2 \equiv \text{Var}[y_a] = (\rho_{x1}\sigma_x)^2$ and $\sigma_i^2 \equiv \text{Var}[y_i] = (\rho_{x2}\sigma_x)^2$.

An informed investor (i.e., an investor who has observed both y_i and y_a) has the following posterior mean and variance with respect to the terminal value x:

$$\text{E}[x|y_a,y_i,\hat{\mathbf{a}}] = \mathbf{b}^t\hat{\mathbf{a}} + y_a + y_i, \tag{21.16a}$$

$$\sigma_{x|12}^2 \equiv \text{Var}[x|y_a,y_i,\hat{\mathbf{a}}] = \sigma_x^2(1 - \rho_{x1}^2 - \rho_{x2}^2) = \sigma_x^2 - \sigma_a^2 - \sigma_i^2. \tag{21.16b}$$

If all rational investors are informed (i.e., $\lambda = 1$) and have conjecture $\hat{\mathbf{a}}$ with respect to the agent's action, then the date 1 market price is

$$P_1(y_a,y_i,\lambda = 1,\hat{\mathbf{a}}) = \mathbf{b}^t\hat{\mathbf{a}} + y_a + y_i - r_I\sigma_{x|12}^2 z. \tag{21.17}$$

Observe that this characterization of the date 1 price differs from (21.1) in three respects, First, (21.17) has a risk premium adjustment, which reflects the fact that the z shares sold by the "liquidity traders" are absorbed by a finite number

[6] As in Chapter 11, the expected supply is set equal to zero merely to simplify the model. The results would not be substantially affected if we introduced a non-zero mean.

[7] More technically, we let x equal the terminal value of the shares held by the long-term investors, from which they will pay any agent compensation. The units are scaled so that the total number of shares held by the long-term investors equals one. The liquidity traders initially own zero shares. If they sell (buy) shares, they go short (long) and the rational investors go long (short).

[8] This is without loss of generality, since the non-contractible signal can always be transformed such that it is independent of the accounting report. For example, the transformed signal $y_2' \equiv \{y_2 - \rho_{12}y_1\}/\{1 - \rho_{12}^2\}^{½}$ is uncorrelated with the accounting report y_1 and has unit variance. Of course, the vector of sensitivities for y_2' and its correlation with the terminal value of the firm is also changed compared to y_2.

[9] Note that this transformation has no impact of the informativeness of the signals with respect to the agent's actions.

of risk averse rational investors.[10] There is no risk premium adjustment in (21.1) because all investors are assumed to be well diversified.

Second, $\mathbf{b}^t\hat{\mathbf{a}}$ replaces $\Omega(\hat{\mathbf{a}})$ since the expected values of y_a and y_i equal zero, whereas the expected values of y_1 and y_2 are non-zero. Third, the weights on the accounting report and the non-contractible signal both equal one in (21.17), which is due to the scale factor used in transforming y_1 and y_2 into y_a and y_i, and the assumption that $\rho_{12} = 0$.

Equilibrium Price for a Given Fraction of Informed Rational Investors

Now consider the setting in which some rational investors are uninformed (i.e., they observe the accounting report y_a, but not the non-contractible signal y_i). The fraction informed is $\lambda \in (0,1)$ and the fraction uninformed is $1 - \lambda$. The uninformed realize that the trades of the informed are affected by y_i and, hence, the market clearing price is influenced by both the unobserved signal y_i and the unobserved supply of shares z. Consequently, the equilibrium price provides the uninformed rational investors with noisy information about the private signal y_i.

As in Chapter 11 of Volume I, the equilibrium price is conjectured to be a linear function of y_a, y_i, and z:

$$\boldsymbol{P}_1(\hat{\mathbf{a}},\lambda,y_a,y_i,z) = \pi_o + \pi_a y_a + \pi_i y_i - \pi_z z. \tag{21.18}$$

Since y_a is observed by all investors, the uninformed investors can compute the following statistic from the price and the public report:

$$\psi \equiv y_i - (\pi_z/\pi_i)z = \{P_1 - [\pi_o + \pi_a y_a]\}/\pi_i.$$

The variables x, y_a, y_i, z, P_1, and ψ are jointly normally distributed. The statistic ψ provides the uninformed investors with the same information about y_i as do P_1 and y_a. Hence, the uninformed rational investors' posterior mean and variance with respect to the outcome x, given conjecture $\hat{\mathbf{a}}$, can be expressed as[11]

$$\mathrm{E}[x|y_a,\psi,\hat{\mathbf{a}}] = \mathbf{b}^t\hat{\mathbf{a}} + y_a + \psi\frac{\sigma_i^2}{\sigma_\psi^2}, \tag{21.19a}$$

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[10] Given a positive risk premium in the stock price at date 1, the well-diversified long-term investors have an incentive to trade at date 1, but we exogenously assume that they are not active traders at this date.

[11] We use the fact that $\mathrm{Cov}[x,\psi] = \mathrm{Cov}[x,y_i] = \sigma_i^2 = (\rho_{x2}\sigma_x)^2$.

$$\sigma^2_{x|1\psi} \equiv \mathrm{Var}[x|y_a,\psi,\hat{\mathbf{a}}] = \sigma_x^2 - \sigma_a^2 - \sigma_i^2\frac{\sigma_i^2}{\sigma_\psi^2} = \sigma_x^2\left[1 - \rho_{x1}^2 - \rho_{x2}^2\frac{\sigma_i^2}{\sigma_\psi^2}\right], \quad (21.19b)$$

where $\sigma_\psi^2 \equiv \mathrm{Var}[\psi] = \sigma_i^2 + (\pi_z/\pi_i)^2\sigma_z^2.$

The transformed representations of the public and private information satisfy the assumptions made in Section 11.3 of Volume I, with the prior mean of x equal to $\mathrm{E}[x|\hat{\mathbf{a}}] = \mathbf{b}^t\hat{\mathbf{a}}$. Consequently, adapting the analysis in Section 11.3 to our current setting, the equilibrium parameters of price function (21.18) are as specified in Table 21.1.

Table 21.1
Equilibrium Price Function Parameters

$$\pi_o = \mathbf{b}^t\hat{\mathbf{a}}, \qquad \pi_a = 1,$$

$$\pi_i = \frac{1}{\bar{h}}\left[\lambda\, h_{x|12} + (1-\lambda)\, h_{x|1\psi}\frac{\sigma_i^2}{\sigma_\psi^2}\right], \qquad \pi_z = \frac{r_I}{\lambda}\,\sigma_{x|12}^2\,\pi_i,$$

where $h_{x|12} \equiv \sigma_{x|12}^{-2}, \quad h_{x|1\psi} \equiv \sigma_{x|1\psi}^{-2}, \quad \bar{h} \equiv \lambda\, h_{x|12} + (1-\lambda)\, h_{x|1\psi}.$

The price used here refers only to the gross outcome x and ignores the agent's compensation, which is introduced later. It is as if the principal, acting on behalf of the long-term investors, pays the agent directly. This approach simplifies the analysis significantly, allowing us to focus on the role of price as a means of contracting indirectly on private investor information that is partially revealed through the price.

Equilibrium Fraction of Informed Rational Investors

The equilibrium fraction informed is the same as in (11.25), i.e., if $\lambda^* \in (0,1)$, then

$$\lambda^* = r_I\,\sigma_z\sigma_{x|12}\sqrt{\frac{\sigma_i^2 - K\sigma_{x|12}^2}{K\sigma_i^2}}, \quad (21.20)$$

where $K = \exp[2r_I\kappa] - 1$.

Price-informativeness

The equilibrium price (21.18) partially reveals the informed investors' signal y_i since their demand for the stock is influenced by that information. The uninformed rational investors seek to infer the signal y_i from the price (or statistic ψ) because y_i is informative about the terminal outcome they will receive from the shares they purchase. In Chapter 11 of Volume I we computed the square of the correlation between ψ and y_i as a measure of the informativeness of ψ about y_i. If λ (0, 1] is exogenous, then the price-informativeness is

$$\mathcal{I}(\lambda) \equiv \text{Corr}^2(y_i, \psi|\lambda) = \frac{\sigma_i^2}{\sigma_i^2 + ((r_I/\lambda)\,\sigma_{x|12}^2)^2\sigma_z^2}. \tag{21.21a}$$

On the other hand, if λ is endogenous, and satisfies (21.20), with $\lambda^* \in (0,1)$, then the equilibrium price-informativeness is

$$\mathcal{I}^* = \mathcal{I}(\lambda^*) = 1 - K\frac{\sigma_{x|12}^2}{\sigma_i^2} = 1 - K\left[\frac{\sigma_x^2 - \sigma_a^2}{\sigma_i^2} - 1\right]. \tag{21.21b}$$

These expressions imply that if λ is exogenous, an increase in the noise in the price, i.e., σ_z^2, decreases the informativeness of ψ with respect to y_i. However, if λ is endogenous and $\lambda^* \in (0,1)$, then an increase in the noise results in an increase in the fraction informed. The two changes are precisely offsetting, so that $\mathcal{I}^*$ is unchanged (compare to Proposition 11.7).

On the other hand, increasing the informativeness of either the accounting report or the informed investors' private signal with respect to the outcome (i.e., $\sigma_a^2 = (\rho_{x1}\sigma_x)^2$ and $\sigma_i^2 = (\rho_{x2}\sigma_x)^2$) results in an increase in the informativeness of ψ with respect to y_i. This result holds whether λ is exogenous or endogenous.

Investors are interested in y_i because it is correlated with the terminal value of the shares they purchase. Since the uninformed investors do not observe y_i, they use ψ to imperfectly infer y_i in their formation of beliefs about x. The principal, on the other hand, is interested in y_i because it is informative about the agent's actions. However, since y_i is not contractible, he uses ψ in compensating the agent because it is contractible and informative about y_i. If we hold the informativeness of y_i about $\mathbf{a}$ constant, then the greater the price-informativeness, the more informative ψ is about y_i and, hence, the more informative ψ is about the agent's actions $\mathbf{a}$.

Agent's Incentive Contract and Induced Action

The accounting report y_a and the market price P_1 are contractible information, and they can be used to infer the statistic ψ, if $\lambda^* \in (0,1]$ and $\rho_{x2} \neq 0$. Furthermore, for any linear contract based on y_a and P_1, there is as an equivalent linear contract based on y_a and ψ (and vice-versa). To simplify the analysis, we initially characterize the optimal linear contract based on y_a and ψ,[12]

$$c = f + v_a y_a + v_\psi \psi. \tag{21.22}$$

From the investors' perspective, both y_a and ψ have prior means of zero, so that their expected compensation cost equals f.

The agent, on the other hand, chooses the action and he will consider choosing $\mathbf{a} \neq \hat{\mathbf{a}}$, even though they will be equal in equilibrium. The agent's *ex ante* certainty equivalent is

$$CE_0(\mathbf{a}, \hat{\mathbf{a}}, f, \breve{\mathbf{v}}, \lambda) = \mathrm{E}[c|\mathbf{a}, \hat{\mathbf{a}}, f, \breve{\mathbf{v}}] - ½\mathbf{a}^t\mathbf{a} - ½r\mathrm{Var}[c|\lambda], \tag{21.23}$$

where $\breve{\mathbf{v}} \equiv (v_a, v_\psi)^t$. From the agent's perspective, if he chooses action $\mathbf{a}$ when he believes the investors hold conjecture $\hat{\mathbf{a}}$, the *ex ante* mean and variance of his compensation are

$$\mathrm{E}[c|\mathbf{a}, \hat{\mathbf{a}}, f, \breve{\mathbf{v}}] = f + \breve{\mathbf{v}}^t\breve{\mathbf{M}}(\mathbf{a} - \hat{\mathbf{a}}), \tag{21.24a}$$

$$\mathrm{Var}[c|\breve{\mathbf{v}}, \lambda] = \breve{\mathbf{v}}^t\breve{\boldsymbol{\Sigma}}(\lambda)\breve{\mathbf{v}}, \tag{21.24b}$$

where $$\breve{\mathbf{M}} \equiv \begin{bmatrix} \sigma_a\mathbf{M}_1 \\ \sigma_i\mathbf{M}_2 \end{bmatrix}, \quad \breve{\boldsymbol{\Sigma}}(\lambda) \equiv \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & \mathrm{Var}[\psi|\lambda] \end{bmatrix},$$

$$\mathrm{Var}[\psi|\lambda] = \sigma_i^2 + (\pi_z/\pi_i)^2\sigma_z^2 = \sigma_i^2 + ((r_I/\lambda)\sigma_{x|12}^2)^2\sigma_z^2.$$

Note that the two performance measures are independently distributed and the variances are independent of the agent's actions. However, the variance of ψ is influenced by the fraction of informed rational investors.

The agent selects $\mathbf{a}$ to maximize (21.23). The first-order condition characterizing the optimal choice is

$$\mathbf{a}^\dagger(\breve{\mathbf{v}}) = \breve{\mathbf{M}}^t\breve{\mathbf{v}}. \tag{21.25}$$

Principal's Contract Choice

In equilibrium, the investors set their conjecture with respect to the agent's action equal to (21.25). That is, the conjectured action is a function of the

[12] If $\lambda^* = 0$, then P_1 is uninformative about y_i, and the contract is only written on the accounting report.

incentive contract offered by the principal and accepted by the agent. The principal is assumed to act in the interests of the long-term investors who are risk neutral (or well-diversified) and do not trade at date 1. They will wait to receive their share of the gross outcome x minus the cost of the agent's compensation. Hence, the principal chooses the contract terms $(f, \check{\mathbf{v}})$ so as to maximize the firm's *ex ante* intrinsic value to the long-term investors:

$$U_0^p(f, \check{\mathbf{v}}, \lambda) = \mathrm{E}[x \mid \mathbf{a}^\dagger(\check{\mathbf{v}})] - \mathrm{E}[c \mid \mathbf{a}^\dagger(\check{\mathbf{v}}), f, \check{\mathbf{v}}], \tag{21.26}$$

subject to the contract acceptance constraint. If the agent's net reservation wage is zero, the contract acceptance constraint implies that the principal sets the fixed wage so that it is just sufficient to compensate the agent for the cost of his effort and his risk premium. Hence, in equilibrium,

$$f^\dagger(\check{\mathbf{v}}, \lambda) = \tfrac{1}{2}\mathbf{a}^\dagger(\check{\mathbf{v}})^t\mathbf{a}^\dagger(\check{\mathbf{v}}) + \tfrac{1}{2}r\mathrm{Var}[c \mid \check{\mathbf{v}}, \lambda].$$

The expected incentive compensation equals zero and, hence, the expected total compensation equals $f^\dagger(\check{\mathbf{v}}, \lambda)$, and the principal's objective function in selecting $\check{\mathbf{v}}$ is

$$U_0^p(f^\dagger, \check{\mathbf{v}}, \lambda) = \mathbf{b}^t\check{\mathbf{M}}^t\check{\mathbf{v}} - \{\tfrac{1}{2}\check{\mathbf{v}}^t\check{\mathbf{M}}\,\check{\mathbf{M}}^t\check{\mathbf{v}} + \tfrac{1}{2}r\,\check{\mathbf{v}}^t\check{\boldsymbol{\Sigma}}(\lambda)\check{\mathbf{v}}\}. \tag{21.27}$$

The first-order conditions obtained by differentiating (21.27) with respect to $\check{\mathbf{v}}$ yield:

$$\check{\mathbf{v}}^\dagger(\lambda) = \check{\mathbf{Q}}(\lambda)\,\check{\mathbf{M}}\mathbf{b}, \tag{21.28a}$$

where $$\check{\mathbf{Q}}(\lambda) = [\check{\mathbf{M}}\,\check{\mathbf{M}}^t + r\check{\boldsymbol{\Sigma}}(\lambda)]^{-1}.$$

Substituting (21.28a) into (21.25) and (21.27) provides the optimal actions and the principal's expected outcome in terms of the exogenous parameters and a given fraction of informed rational investors λ:

$$\mathbf{a}^\dagger(\lambda) = \check{\mathbf{M}}^t\check{\mathbf{Q}}(\lambda)\,\check{\mathbf{M}}\mathbf{b}, \tag{21.28b}$$

$$U_0^{p\dagger}(\lambda) = \tfrac{1}{2}\mathbf{b}^t\check{\mathbf{M}}^t\check{\mathbf{Q}}(\lambda)\,\check{\mathbf{M}}\mathbf{b}. \tag{21.28c}$$

These results parallel those in Section 20.2.1.

Incentive Weights for Accounting Report and Market Price

In the preceding analysis we assumed y_a and P_1 are used to infer ψ, and then the contract is written in terms of y_a and ψ. Now assume y_a and P_1 are used. Given

$v_a^\dagger$ and $v_\psi^\dagger$ from (21.28), we can derive the incentive weights for y_a and P_1 as follows:

$$v_a^p = [v_a^\dagger - v_\psi^\dagger]/\pi_i, \quad v_P^p = v_\psi^\dagger/\pi_i. \tag{21.29}$$

In the following section we consider a setting in which the accounting report is informative about actions that have short-term economic consequences while the investors can obtain non-contractible information about actions that have long-term economic consequences. In that case, $v_a^\dagger$ and $v_\psi^\dagger$ are both positive. An empirical examination of that setting using the accounting report and the market price would find significantly positive incentives with respect to the market price, but insignificant or even significantly negative incentives with respect to the accounting report (even if $v_a^\dagger > 0$).

21.3.3 Comparative Statics

This section considers changes in the informativeness of the public report and the non-contractible signal with respect to the terminal value of the equity (as represented by σ_a^2 and σ_i^2), and changes in the noise in the price (as represented by σ_z^2). We examine the impact of these changes on the induced action and the principal's expected outcome.

The prior uncertainty with respect to the terminal value of equity (i.e., σ_x^2) is held constant. Recall that $\sigma_a^2 = (\rho_{x1}\sigma_x)^2$ and $\sigma_i^2 = (\rho_{x2}\sigma_x)^2$. Hence, changes in the informativeness of the signals with respect to the terminal value of equity are changes in the square of the correlation of the two basic signals y_1 and y_2 relative to x.[13] Changes in these correlations do not affect the informativeness of the signals y_1 and y_2 with respect to the agent's action (as represented by their sensitivity to the agent's action $\mathbf{M}_1$ and $\mathbf{M}_2$, divided by σ_1 and σ_2, which both equal one).

To simplify the analysis we assume there are two tasks, i.e., $\mathbf{a} = (a_1, a_2)$, and report y_a is only influenced by the first action (i.e., $M_{11} > 0$ and $M_{12} = 0$), while signal y_i is only influenced by the second action (i.e., $M_{21} = 0$ and $M_{22} > 0$). The first task can be viewed as having short-term consequences that are measured by the accounting system, whereas the second task has long-term consequences. The investors can obtain non-contractible information about the long-term con-

[13] Recall that the informed investors' posterior uncertainty is $\sigma_{x|12}^2 = \sigma_x^2 - \sigma_a^2 - \sigma_i^2$. In the subsequent analysis we first assume that σ_a^2 and σ_i^2 can be changed separately so that increasing either one of these implies that the posterior variance decreases. This is in contrast to the analysis in Section 11.3 of Volume I in which the posterior variance is fixed so that increasing the informativeness of the accounting report reduces the informativeness of the private signal. The latter is descriptive of a setting in which there is a common noise component that can be revealed either through the accounting report or through private information acquisition.

sequences that are not reflected in the accounting report at date 1. The market price of the firm's equity reflects both sources of investor information.

Table 21.2 provides explicit expressions for the incentive rates, actions, and the principal's expected outcome. We consider the impact of changes in σ_a^2, σ_i^2, and σ_z^2, first assuming λ is exogenous, and then assuming it is endogenous (i.e., determined by (21.18)).

Observe that the principal's expected outcome from each task is equal to the induced effort times a constant, i.e., $U_0^{p\dagger}(\lambda) = ½(b_1 a_1^\dagger(\lambda) + b_2 a_2^\dagger(\lambda))$. Hence, the comparative statics focus on the agent's actions, since these also apply directly to the principal's expected outcome.

Table 21.2
Incentives, Actions, and Outcomes with Two Independent Actions

$$\check{\mathbf{v}}^\dagger(\lambda) = \left[\frac{M_{11}\, b_1}{\sigma_a \left[M_{11}^2 + r\right]}, \frac{M_{22}\, b_2}{\sigma_i \left[M_{22}^2 + r\mathbb{I}(\lambda)^{-1}\right]} \right]^t,$$

$$\mathbf{a}^\dagger(\lambda) = \left[\frac{M_{11}^2\, b_1}{M_{11}^2 + r}, \frac{M_{22}^2\, b_2}{M_{22}^2 + r\mathbb{I}(\lambda)^{-1}} \right]^t,$$

$$U_0^{p\dagger}(\lambda) = ½ \left[\frac{M_{11}^2\, b_1^2}{M_{11}^2 + r} + \frac{M_{22}^2\, b_2^2}{M_{22}^2 + r\mathbb{I}(\lambda)^{-1}} \right].$$

Exogenous Fraction Informed

The informativeness of the accounting report with respect to the terminal value of equity (i.e., $\sigma_a^2 = (\rho_{x1}\sigma_x)^2$) does not affect either the induced effort in the first task or the expected net outcome from that task – the correlation of the accounting report with the terminal value does not change the informativeness of the report with respect to the agent's effort in the first task. The key factor determining the induced effort in the first task is the sensitivity of the accounting report to the effort in the first task if the report is scaled to have unit variance, i.e., M_{11}. However, $\check{v}_a^\dagger$ is decreasing in σ_a since this incentive rate is applied to y_a, which is y_1 scaled by σ_a.

The investors' signal y_2 is not directly contractible. However, a contractible statistic ψ can be developed from the market price and the accounting report. If there is no noise in the price process, i.e., $\sigma_z^2 = 0$ (and, thus, the price-informativeness $\mathcal{I}(\lambda) = 1$), then the result for the second task parallels the first task result, i.e., increasing σ_i^2 does not affect the informativeness of y_i with respect to the agent's effort in the second task. That is, the induced effort in the second task is $M_{22}^2 b_2/(M_{22}^2 + r)$ and the expected outcome is $\frac{1}{2}M_{22}^2 b_2^2/(M_{22}^2 + r)$, i.e., neither is affected by σ_a^2 or σ_i^2.

We observe from Table 21.2 that the effort in the second task and its related expected outcome are both increasing functions of the price-informativeness measure $\mathcal{I}(\lambda)$. While the price is used to infer y_i and the price depends upon beliefs about the outcome x, the relation between y_i and x has no direct relevance in contracting with the agent. Instead, the key issue is the noise in the relation between ψ and a_2, which depends on the correlation between ψ and y_i and the noise in the relation between y_i and a_2.

If the fraction informed is exogenous with $\lambda \in (0,1)$ and the price process is noisy, i.e., $\sigma_z^2 > 0$, then it follows directly from (21.21a) that the price-informativeness $\mathcal{I}(\lambda)$ is increasing in the fraction informed λ and decreasing in both the noise σ_z^2 and the posterior variance $\sigma_{x|12}^2$. The latter affects $\mathcal{I}(\lambda)$ because the informed investors trade more aggressively on the basis of their private information if they face less posterior uncertainty.

Recall that $\sigma_{x|12}^2 = \sigma_x^2 - \sigma_a^2 - \sigma_i^2 = (1 - \rho_{x1}^2 - \rho_{x2}^2)\sigma_x^2$. Hence, increasing the informativeness (correlation squared) of the accounting report with respect to the terminal value of the firm, σ_a^2 (ρ_{x1}^2), reduces the informed rational investors' posterior variance and, thus, increases $\mathcal{I}(\lambda)$.

Increasing the informativeness of the non-contractible signal with respect to the terminal value of the firm also increases the induced effort in the second task and the associated expected outcome to the principal. The effect of increasing σ_i^2 is even stronger than increasing σ_a^2 since increasing σ_i^2 increases the price-informativeness measure in (21.21a) for two reasons. First, the posterior variance $\sigma_{x|12}^2 = \sigma_x^2 - \sigma_a^2 - \sigma_i^2$ is decreasing in σ_i^2 and, second, increasing the information content of the private signal increases the quality of the informed investors' information relative to the uninformed investors. Both lead the informed investors to trade more aggressively on their private information such that the private signal has a larger impact on the equilibrium price (relative to the noise in the price).

Finally, it is obvious that the induced effort in the second task and the associated outcome to the principal are decreasing in σ_z^2. Of course, the reason is that increasing the variance of the liquidity trades increases the noise in the equilibrium price used to infer the non-contractible signal.

Endogenous Information Acquisition

Now assume the fraction of rational investors that are informed is endogenously determined by (21.20), such that the equilibrium price-informativeness $\mathcal{I}^*$ is given by (21.21b). The most straightforward and striking aspect of (21.21b) is that the price-informativeness (and, therefore, the induced effort in the second task) is independent of the noise in the price process. As noted above the key here is that the informativeness of the price about y_i increases with λ and decreases with σ_z^2. When information acquisition is endogenous, λ^* is increasing in σ_z^2, and the rate of change is such that the informativeness of the price about y_i is constant. Hence, the comparative static for σ_z^2 is significantly affected by whether the fraction informed is exogenous or endogenous. If λ is exogenous, then both a_2 and the associated outcome decrease as the noise increases, but they remain constant if λ is endogenous.

Differentiating (21.21b) with respect to σ_a^2 establishes that the price-informativeness (and the induced effort in the second task) is again strictly increasing in the informativeness of the public report (whenever $\lambda^* > 0$):

$$\frac{d\mathcal{I}^*}{d\sigma_a^2} = \frac{K}{\sigma_i^2} > 0.$$

The same result is obtained by totally differentiating (21.21a) with respect to σ_a^2, if $\lambda = \lambda^*$, i.e.,

$$\frac{d\mathcal{I}(\lambda^*)}{d\sigma_a^2} = \frac{\partial \mathcal{I}(\lambda^*)}{\partial \sigma_a^2} + \frac{\partial \mathcal{I}(\lambda^*)}{\partial \lambda^*}\frac{d\lambda^*}{d\sigma_a^2}.$$

It is obvious from (21.21a) that $\mathcal{I}$ is increasing in both σ_a^2 and λ. And from (21.20) we find that λ^* can be increasing or decreasing in σ_a^2. If it is decreasing, then the first effect of σ_a^2 is stronger than the latter, so that the change in the induced effort in the second task, and the associated outcome, are positive.

The preceding comments are illustrated by Figure 21.1.[14] Observe that, with a high information cost, no one acquires the non-contractible signal if the public report is not very informative. However, if the public report is sufficiently informative, some rational investors become informed and trade aggressively so that the price-informativeness is high and high effort is induced in the second task. The fraction informed is not monotonic, but the induced effort is weakly monotonic.

With a low information cost, every rational investor acquires the non-contractible signal if the public report is not highly informative. If the public report is highly informative, some rational investors do not acquire the non-contractible signal. Nonetheless, the price-informativeness is high and high effort in the second task is induced.

[14] The market parameters of the examples in Figures 21.1, 21.2, and 21.3 are essentially the same as in Figure 11.3 in Volume I: σ_x^2 = 15,000, σ_z^2 = 2,500, r_I = .0001, κ_{low} = 550, κ_{high} = 3,500, σ_a^2 = 5,000 (in 21.2), σ_i^2 = 5,000 (in 21.1), and σ_{x12} = 5,000 (in 21.3). The parameters for the second action are b_2 = 2, and M_{22} = 1, and the agent's risk aversion is r = 1.

In the setting in which a small fraction exogenously acquire the non-contractible signal, increasing the informativeness of the public report results in a steady increase in the level of effort induced in the second task.

Similar results are obtained for the informativeness of the non-contractible signal. For example, the following derivative establishes that the induced effort in the second task is strictly increasing with the informativeness of the non-contractible signal (whenever $\lambda^* > 0$).

$$\frac{d\mathcal{I}^*}{d\sigma_i^2} = K(\sigma_x^2 - \sigma_a^2)/\sigma_i^4 > 0.$$

In Figure 21.2, with both low and high information costs, the fraction of informed investors is increasing and then decreasing as the informativeness of the non-contractible signal increases. However, as established by the preceding derivative, the price-informativeness and, thus, the induced effort in the second task is strictly increasing for $\lambda^* > 0$. With a small exogenous fraction of informed investors, the induced effort is strictly monotonically increasing.

Significantly different results are obtained if the posterior variance of the informed investors, $\sigma_{x|12}^2 = \sigma_x^2 - \sigma_a^2 - \sigma_i^2$, is held constant as the informativeness of the accounting report is increased (as for the comparative statics in Section 11.3). In this case, an increase in σ_a^2 reduces σ_i^2 by the same amount. The following derivative establishes that the induced effort in the second task is strictly decreasing with the informativeness of the accounting report (whenever $\lambda^* > 0$).

$$\left.\frac{d\mathcal{I}^*}{d\sigma_a^2}\right|_{\sigma_{x|12}^2 = const.} = -K\frac{\sigma_{x|12}^2}{\sigma_i^4} < 0.$$

In Figure 11.3, with both high and low information cost, the fraction of informed investors is decreasing as the informativeness of the accounting report increases (reflecting that less information can be acquired privately). Moreover, even with a fixed fraction of informed investors, the price-informativeness decreases. Hence, both effects reduce the effort induced in the second task.

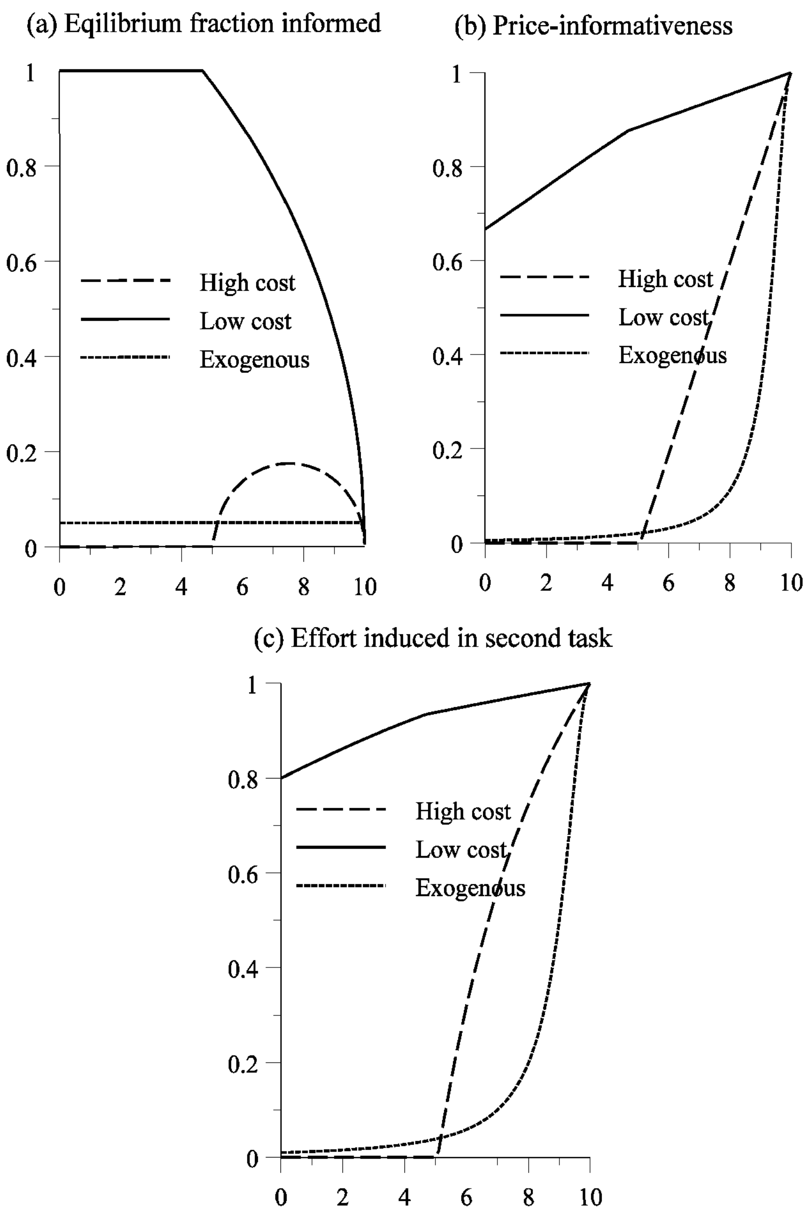

Figure 21.1: Summary statistics for varying information content of the public report. Horizontal axis = $\sigma_a^2/1{,}000$. σ_x^2 = 15,000; σ_i^2 = 5,000.

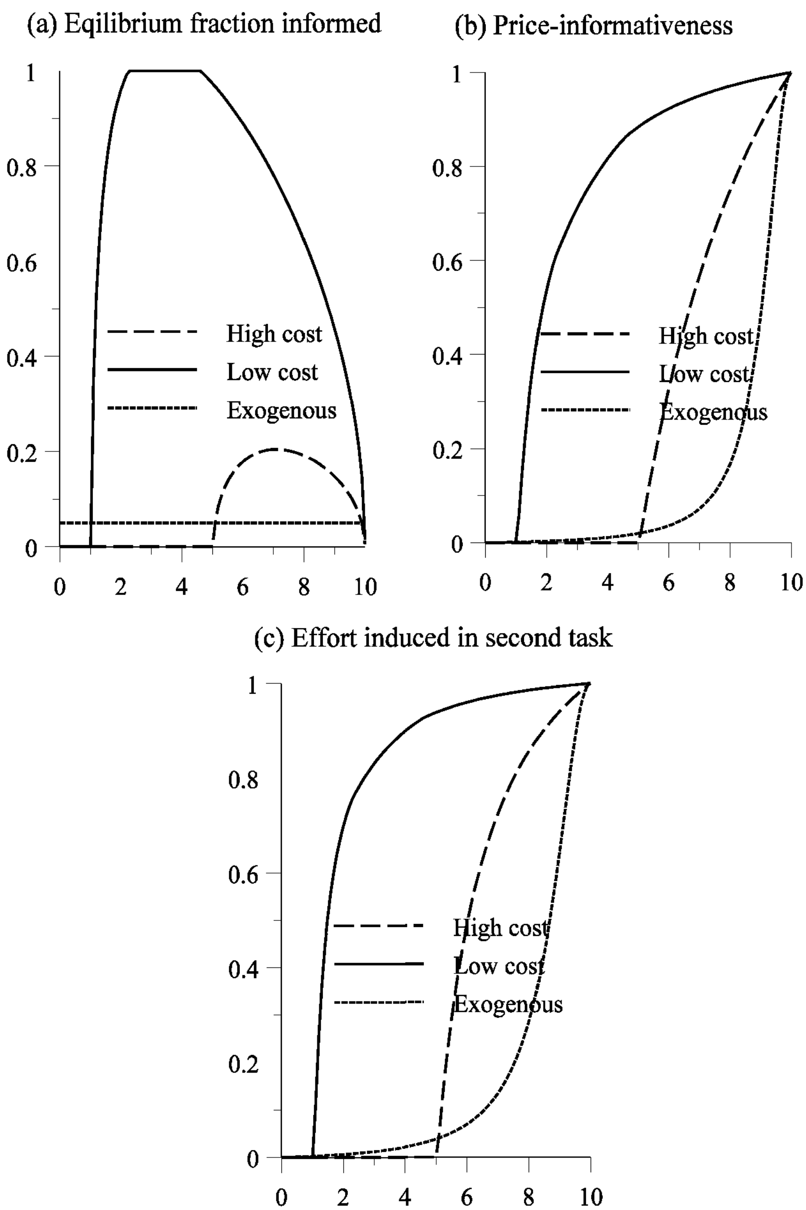

Figure 21.2: Summary statistics for varying information content of the private signal. Horizontal axis = $\sigma_i^2/1{,}000$. σ_x^2 = 15,000; σ_a^2 = 5,000.

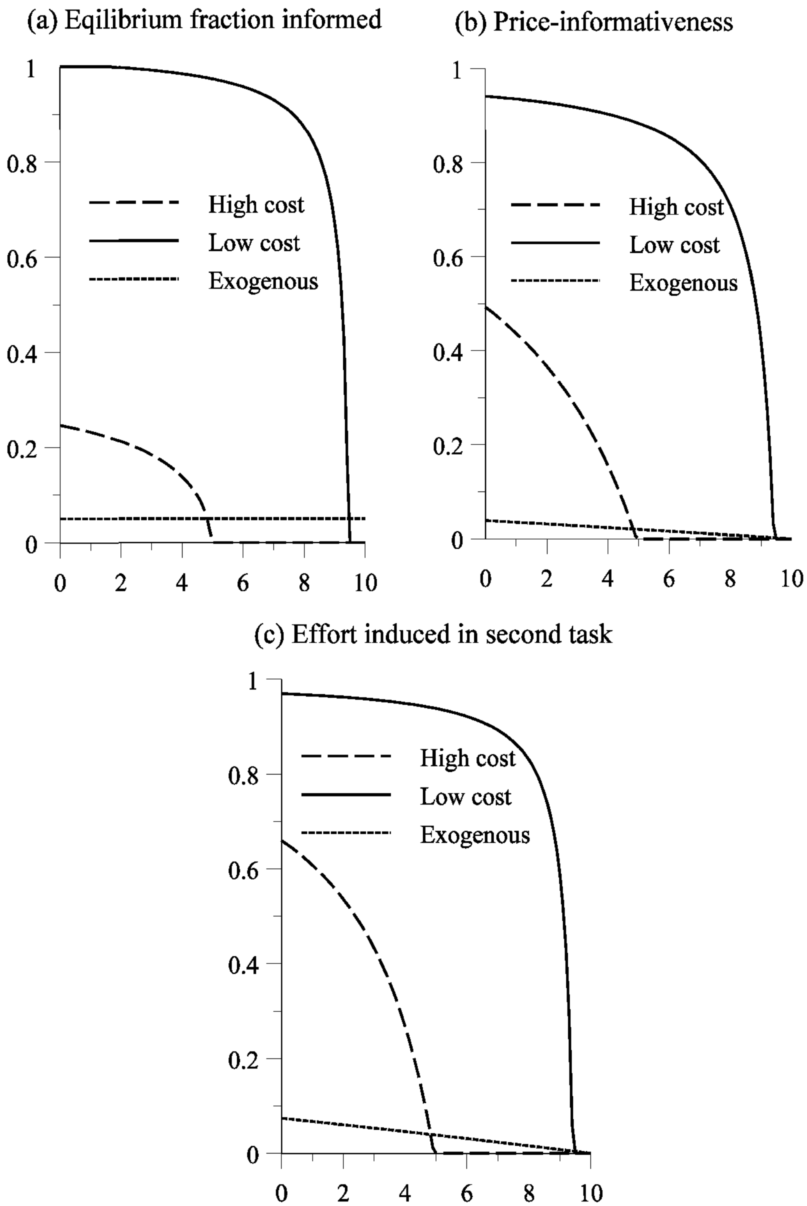

Figure 21.3: Summary statistics for varying information content of the public and private signals. Horizontal axis = $\sigma_a^2/1{,}000$. $\sigma_x^2 = 15{,}000$; $\sigma_{x|12}^2 = 5{,}000$.

21.4 OPTIONS VERSUS STOCK OWNERSHIP IN INCENTIVE CONTRACTS

In Chapter 20 and the previous sections of this chapter, much of our analysis is restricted to linear contracts, even though optimal contracts are seldom linear. Stock ownership by the agent is effectively a linear incentive contract in which the market price is the performance measure. In this section we examine whether stock options may be more efficient mechanisms for motivating agent effort. The key difference between stock and stock options is that stock options shield the agent from the down-side risk to which stock ownership exposes the agent.

We restrict our analysis to a single task setting in which the principal is risk neutral (e.g., there is no market risk and the principal represents investors who can diversify away the firm-specific risk) and the agent has negative exponential utility with a quadratic personal cost of effort, i.e., $u^a(c,a) = -\exp[-r(c - ½a^2)]$.

A key assumption in the prior analysis is that the agent's effort affects the mean but not the variance of a normally distributed performance measure. In that setting, the likelihood ratio is linear in the performance measure and combined with the negative exponential utility function this implies that the optimal incentive contract is a concave function of the performance measure (see Section 19.1).[15] Stock ownership is a linear incentive contract, whereas an incentive contract based on stock options is convex. In other words, stock ownership is "closer" to the optimal contract than stock options, and we should not expect stock options to be more efficient than stock ownership for motivating agent effort in that setting. In this section, we follow Feltham and Wu (2001) (FWb) and assume that the agent's effort not only affects the mean of a normally distributed performance measure but also its variance. If the impact on the variance is sufficiently pronounced, stock options may be more efficient than stock ownership. The key is that, in this setting, increasing effort not only increases the likelihood of good outcomes but also the likelihood of bad outcomes. Therefore, in the design of the incentive contract the principal may want to shield the agent from the down-side risk, and this is the key characteristic of stock options as opposed to stock ownership.

21.4.1 Optimal Incentive Contract

Before we introduce the specific characteristics of stock ownership and options, it is useful to derive the characteristics of the optimal incentive contract in a setting in which the agent's effort affects both the mean and the variance of a

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[15] Of course, this only holds for performance measures for which compensation is above a finite lower bound.

normally distributed performance measure. The performance measure y is normally distributed with mean $b(a)$ and variance $\sigma_y^2(a)$, which may vary with the level of effort. More specifically, as in FWb, we assume $b(a) = b + a$ and $\sigma_y^2(a) = [\sigma + \gamma a]^2$, with $\gamma \geq 0$. Of course, the variance is independent of a if $\gamma = 0$, which is the "standard" case we have considered in our prior analyses of models with exponential utility and normally distributed performance measures. If γ is strictly positive, the variance of y is strictly increasing in the level of effort a.

With negative exponential utility the optimal incentive contract for inducing a particular level of effort a has the general form (see Section 19.1),

$$c(y) = \kappa(a) + \frac{1}{r}\ln\left[r\left(\lambda + \mu\left[r\kappa'(a) + L(y|a)\right]\right)\right]. \tag{21.30}$$

Given $b(a) = b + a$ and $\sigma_y^2(a) = [\sigma + \gamma a]^2$, the likelihood ratio is a convex second-degree polynomial in the performance measure, i.e.,

$$L(y|a) = \frac{\gamma[y - b(a)]^2}{\sigma_y^3(a)} + \frac{y - b(a)}{\sigma_y^2(a)} - \frac{\gamma}{\sigma_y(a)}.$$

If $\gamma = 0$ as in our prior analysis, the likelihood ratio is a linear function of the performance measure, i.e.,

$$L(y|a) = \frac{y - b(a)}{\sigma_y^2(a)}, \quad \text{for } \gamma = 0.$$

However, if $\gamma > 0$, the likelihood ratio is a strictly convex function of the performance measure that attains a minimum value at

$$y_{min} \equiv b(a) - \frac{1}{2\gamma}\sigma_y(a) = b + a - ½(\sigma/\gamma + a).$$

The interpretation is that extreme performance measures become more likely when the agent works harder. However, since effort also has a mean effect, performance measures just below the mean are more likely when the agent shirks. The strengths of the variance and the mean effects are such that the lower is γ, the lower is the performance measure y_{min} for which the likelihood ratio attains its minimum value. Another interesting aspect of the form of the likelihood ratio is the fact that, if the likelihood ratio attains a minimum value for $\gamma > 0$, the "Mirrlees problem" no longer applies. Hence, in this setting there may exist an optimal incentive contract which is bounded away from the first-best contract even though the agent's compensation is unbounded from below and the performance measure is normally distributed.

Since optimal incentive contract (21.30) is a strictly increasing concave function of the likelihood ratio, the optimal incentive contract rewards good outcomes as well as "extremely bad" outcomes. The optimal incentive contract has the form of a "butterfly," i.e., it is symmetric around y_{min}, convex in a symmetric region around y_{min}, and concave in the tails.[16]

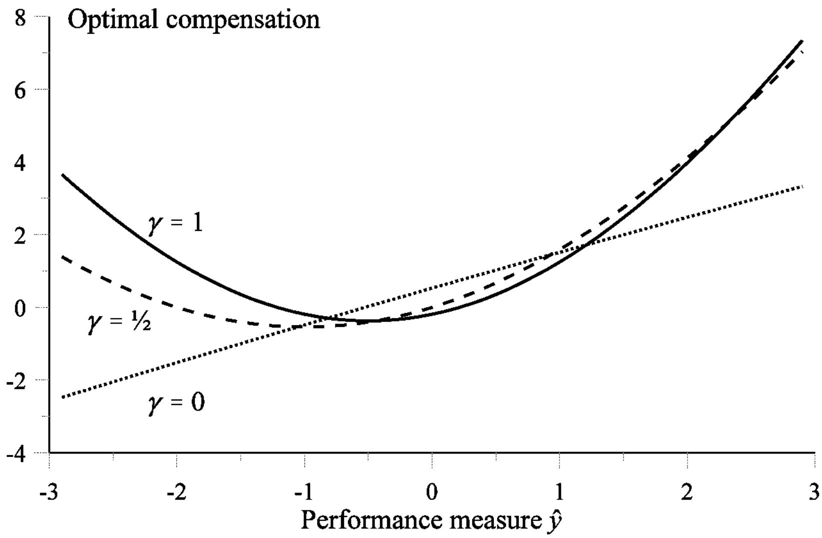

Figure 21.4: Optimal incentive contracts for inducing $a = 1$ with varying impact of effort on variance, γ. Parameters: $r = .025$ and $\sigma = 1$.

[16] Flor, Frimor, and Munk (2005) consider a similar model in which the agent must be induced to exert effort a, and to undertake capital investment q. The stock price is normally distributed and the expected stock price, $b(a,q)$, is affected by both the effort and the investment whereas the variance, $\sigma^2(q)$, is affected by the investment only. Effort is personally costly while the investment is costly to the principal but costless to the agent. As a consequence, providing incentives for effort leads to an induced moral hazard problem for the investment choice (see Section 20.2.5). Since the investment affects the variance of the stock price, the optimal contract is a "butterfly" contract. The paper provides an analysis of the agent's effort and investment choice given different contractual arrangements. For example, employing option contracts it is demonstrated that (at odds with conventional wisdom in much of the employee stock options literature) increasing the contract's sensitivity to current stock price (the "option delta") may have devastating consequences – the agent may change his effort and investment choices so that the expected future stock price decreases substantially.

Figure 21.4 depicts the optimal contracts for varying levels of γ for a given parameter set and $\kappa(a) = \frac{1}{2}a^2$. In the figure the performance measure $\hat{y}$ has been normalized to have a standard normal distribution, i.e.,

$$\hat{y} = \frac{y - b(a)}{\sigma_y(a)},$$

such that the incentive contracts are directly comparable across varying levels of γ, i.e., the compensation levels are "equally likely" given the induced effort.[17]

The optimal incentive contract is concave (but almost linear) for $\gamma = 0$ and, therefore, we should expect stock ownership to be more efficient for inducing effort than stock options in this case. However, for $\gamma = \frac{1}{2}$ and $\gamma = 1$, the optimal incentives are such that bad outcomes are "rewarded" rather than "penalized." Stock options can partly achieve this objective – at least stock options shield the agent from the down-side risk.

The optimal contracts for $\gamma = \frac{1}{2}$ and $\gamma = 1$ is such that the agent's compensation is decreasing in the performance measure for bad outcomes and, thus, the agent may have an incentive to "destroy outcome" in that region. If we view this as a realistic possibility, the compensation scheme must be restricted to be monotonically increasing in the performance measure. That is, the following constraint must be added to the incentive problem.

$$\mathbf{c}(y) \geq \mathbf{c}(y'), \quad \forall\, y' \leq y, \forall\, y \in \mathbb{R}.$$

Assigning multipliers to these constraints and deriving the first-order conditions for the incentive contract, it is relatively straightforward to show that the optimal monotonic incentive contract is such that the agent is paid a fixed wage for performance measures below some threshold above y_{min}, while he is paid according to (21.29) for performance measures above the threshold (although with different multipliers λ and μ), i.e.,

[17] The optimal incentive contract is determined by the multipliers λ and μ and the action we want to induce. The optimal incentive contract can therefore be found by (numerically) solving for a pair of multipliers such that the action we want to induce is incentive compatible for the contract determined by that pair of multipliers, and such that the agent gets his reservation utility. In the setting with multiplicatively separable exponential utility, this is a particularly simple task since only the ratio of the multipliers, μ/λ, matters for incentive compatibility. The levels of the multipliers can subsequently be found by the participation constraint.

$$c(y) = \begin{cases} \kappa(a) + \dfrac{1}{r}\ln\left[r\left(\lambda + \mu[r\kappa'(a) + L(\tilde{y}|a)]\right)\right] & \text{for } y \le \tilde{y}, \\ \\ \kappa(a) + \dfrac{1}{r}\ln\left[r\left(\lambda + \mu[r\kappa'(a) + L(y|a)]\right)\right] & \text{for } y > \tilde{y}, \end{cases}$$

where $\tilde{y} \ge y_{min}$. Figure 21.5 shows the optimal incentive contract and the optimal monotonic incentive contract for $\gamma = 1$ and the same parameters as in Figure 21.4.

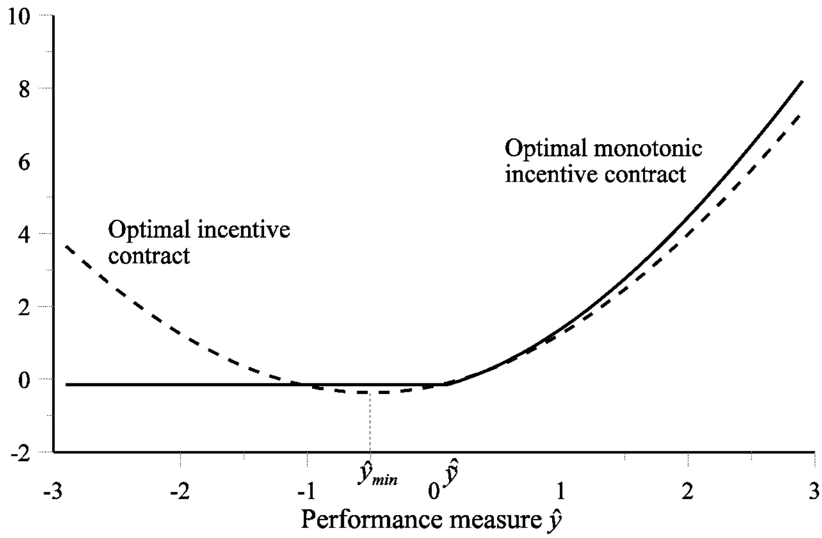

Figure 21.5: Optimal incentive contract and optimal monotonic incentive contract for inducing $a = 1$. Parameters: $r = .025$, $\sigma = 1$, and $\gamma = 1$.

Note that with the monotonic incentive contract stronger incentives have to be used in the "upper tail" since the increase in variance from increasing effort is less beneficial to the agent when he is not rewarded in the "lower tail." Another interesting aspect of the optimal monotonic incentive contract is that it does not penalize outcomes just below the mean as does the optimal incentive contract. The reason is that the monotonicity constraint implies that those penalties also have to be imposed on the "extremely" bad outcomes which are most likely when the agent has worked hard. Finally, we note the similarity between the optimal monotonic incentive contract and stock options. Hence, stock options with a strike price close to the mean of the performance measure may be a good approximation to the optimal monotonic contract in this setting, and certainly more efficient than stock ownership that does not shield the agent from the down-side risk.

21.4.2 Incentive Contracts Based on Options and Stock Ownership

We now introduce the specific characteristics of options and stock ownership as part of an incentive contract. In the prior analysis in this chapter we assumed the stock price P is normally distributed. This ignores the fact that, due to the limited liability of the owners, the price is non-negative. Ignoring this fact in our earlier analysis can be justified on the grounds that the probability of a negative value is insignificant if the expected price is three or more standard deviations from zero. However, in this section we follow Feltham and Wu (2001) (FWb) and explicitly consider the non-negativity constraint since this makes stock more comparable to options.

Representation of Stock and Options

In the analysis that follows we let x represent the underlying value of the firm (if there was no limited liability) and assume as in the previous section that it is normally distributed with a mean $b(a) = b + a$ and a variance $\sigma_x^2(a) = [\sigma + \gamma a]^2$, with $\gamma \geq 0$.

Both stock and options on the stock can be viewed as options on firm value x, so we only refer to options on x, and view stock as the special case in which the strike (exercise) price k is equal to zero. We treat the terminal value of an option as the performance measure, and view the strike price k as a parameter determining the characteristics of that performance measure, so that[18]

$$y = \max\{0, x - k\}.$$

Since x is normally distributed, it follows that y has a censored normal distribution with mean and variance

$$M(a,k) = \sigma_x(a)[n - (1 - N)\xi],$$

$$\sigma^2(a,k) = \sigma_x^2(a)[(1 - N - n^2) - \varphi(2N - 1)\xi + N(1 - N)\xi^2],$$

where $$\xi \equiv (k - b(a))/\sigma_x(a)$$

[18] We could, equivalently, view k as a parameter of the compensation function.

is the number of standard deviations between the strike price k and the mean of x,

$$n \equiv \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\xi^2\right]$$

is the standard normal density function evaluated at ξ, and

$$N \equiv \int_{-\infty}^{\xi} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\tau^2\right] d\tau$$

is the standard normal cumulative probability function at ξ (which is the probability that the option is out-of-the-money).

Management Incentives

We again restrict the incentive contract to be linear, with $c(y) = f + vy$. In this case, v is the number of options granted to the agent. The compensation is not normally distributed, but we assume that the agent's certainty equivalent can be approximated by the mean and variance of the net compensation (see Section 2.6), i.e.,

$$CE(f,v,a,k) \approx f + vM(a,k) - ½(a^2 + rv^2\sigma^2(a,k)). \qquad (21.31)$$

Hence, the incentive constraint is

$$vM_a(a,k) - a - ½rv^2\sigma_a^2(a,k) = 0, \qquad (21.32)$$

where $M_a(a,k)$ and $\sigma_a^2(a,k)$ represent the derivatives of the mean and variance of y with respect to a. It then follows that the incentive rate required to induce action a with strike price k (assuming a can be induced)[19] is

$$v(a,k) = 2a\left[M_a(a,k) + \sqrt{M_a(a,k)^2 - 2ra\sigma_a^2(a,k)}\right]^{-1}. \qquad (21.33)$$

Comparison of Stock and Options if Effort Does not Influence Risk

The strike price influences the compensation risk associated with the cost of inducing a given level effort strictly through the risk premium that must be paid

[19] FWb establish that the level of effort a that can be induced is bounded above, and the upper bound decreases as the strike price k increases. We do not discuss the details of that bound and merely restrict our analysis to an arbitrary level that can be induced.

to the agent. This risk premium depends on the compensation risk, which depends on both the number of options granted and the riskiness of each option:

$$g(a,k) \equiv v(a,k)^2 \sigma^2(a,k). \tag{21.34}$$

We first consider the simple case in which effort does not influence the riskiness of x (i.e., $\gamma = 0$) and, hence,

$$M_a(a,k) = 1 - N,$$

$$\sigma_a^2(a,k) = 2N(1 - N).$$

In this setting it is straightforward to establish that the mean and variance of a single option decreases with the strike price, i.e., $\partial M(a,k)/\partial k < 0$ and $\partial \sigma^2(a,\kappa)/\partial k < 0$. Furthermore, FWb establish (see their Lemma 2) that in this setting the number of options required to induce a given level of effort increases with the strike price, i.e., $\partial v(a,k)/\partial k > 0$. Therefore, the impact of increasing the strike price k on the risk premium $g(a,k)$ is not immediately obvious.

If b/σ is sufficiently large, then $N \approx 0$, $M_a(a,0) \approx 1$, and $\sigma_a^2(a,0) = 0$, which implies that the number of units of stock required to induce effort a is

$$v(a,0) \approx a. \tag{21.35}$$

Options are often issued with a strike price that is at-the-money, which implies $k \approx b + \hat{a}$, where $\hat{a}$ is the effort level conjectured by the market when it sets the initial market price (and assuming the interest rate is zero). In this case, $\xi \approx 0$, $n \approx 1/\sqrt{2\pi}$, $N \approx ½$,

$$\sigma^2(a,k) \approx \sigma^2[1 - N - n^2] = \sigma^2 \frac{1}{2}\frac{\pi - 1}{\pi} > \frac{1}{4}\sigma^2, \tag{21.36}$$

and the number of options required to induce a is

$$v(a,k) = 2a\left[\frac{1}{2} + \sqrt{\frac{1}{4} - 2ar\sigma\frac{1}{\sqrt{2\pi}}}\right]^{-1}. \tag{21.37}$$

Observe that if the agent is risk neutral, i.e., $r \to 0$, then

$$v(a,k) = 2a. \tag{21.38}$$

That is, it takes twice as many units of at-the-money options as units of stock to induce a given level of effort a if the agent is risk neutral. From (21.37) it is obvious that $\partial v(a,k)/\partial r > 0$, so that with a risk averse agent it takes more than twice as many options as stock. Furthermore, with $k = b + a$, the agent's compensation risk is

$$g(a,k) = v(a,k)^2\sigma^2(a,k) > 4a^2\sigma^2(a,k) > a^2\sigma^2 \approx g(a,0).$$

Hence, the number of at-the-money options required to induce a given level of effort is sufficiently larger than the number of units of stock so that the compensation risk is larger with the options (even though the risk associated with one option is much smaller than for one unit of stock).

FWb establish that the above relation holds for all options with a strike price greater than zero. That is, consistent with the characteristics of the optimal incentive contract derived in the previous section, stock is more efficient than options for inducing implementable effort a if effort does not affect the riskiness of the outcome.

Proposition 21.2 (FWb, Prop. 1)

If the agent is strictly risk averse and $x \sim \mathrm{N}(b + a, \sigma^2)$, then the compensation risk for inducing implementable action $a > 0$, $g(a,k)$, is strictly increasing in the strike price k. Hence, the optimal strike price is zero.

Comparison of Stock and Options if Risk Increases with Effort

Options are often proposed as incentive mechanisms in settings in which the agent's risk aversion induces him to under-invest in risky projects. Hence, we now consider the simple setting examined by FWb in which $\sigma_x^2(a) = [\sigma + \gamma a]^2$ and, hence, with $\gamma > 0$ the agent's action choice influences risk. If γ is large, so that effort has a significant influence on risk, options with a positive strike price are more efficient than stock in inducing a given level of effort.

To illustrate this effect, first observe that in our simple setting,

$$M_a(a,k) = (1 - N) + \gamma n.$$

Hence, if the agent is risk neutral ($r = 0$), the number of options required to induce the agent to implement a (see (21.32)) is

$$v(a,k) = a[(1 - N) + \gamma n]^{-1}, \tag{21.39a}$$

$$v(a,0) \approx a, \tag{21.39b}$$

$$v(a,k = b + a) \approx a\left[\frac{1}{2} + \frac{\gamma}{\sqrt{2\pi}}\right]. \tag{21.39c}$$

In this setting, the difference in compensation risk with strike price $k = 0$ versus $k = b + a$ is

$$g(a,0) - g(a,k = b + a) = a^2[\sigma + \gamma a^2]^2\left(1 - \frac{\pi - 1}{\pi}\left[\frac{1}{2} + \frac{\gamma}{\sqrt{2\pi}}\right]^{-2}\right),$$

which is positive (negative) if $\gamma > (<) \sqrt{\pi - 1} - \sqrt{\pi/2}$. That is, the compensation risk is larger with stock than options if γ is sufficiently large.

To simplify the preceding analysis we considered the number of units of stock and at-the-money options required to induce a given level of effort a if the agent is risk neutral. Of course, in that case the compensation risk is immaterial, since the agent does not have to be paid a risk premium. FWb also analyze settings in which r is strictly positive. Numerical analysis indicates that for small γ stock dominates all options, but if the effect of effort on risk is sufficiently large, the optimal strike price is strictly positive and increasing in γ. They are unable to prove this result in general, but they do prove the following result for at-the-money options.

Proposition 21.3 (FWb, Prop. 2)
If the agent is risk averse and $x \sim N(b + a, [\sigma + \gamma a]^2)$, then there exists a cutoff $\gamma^\dagger$ such that for $\gamma > (<) \gamma^\dagger$, $g(a,0) > (<) g(a,k = b + a)$.

FWb also consider the impact of γ on the optimal level of induced effort. We will not go into the details, but merely report that their analysis indicates that for small γ the optimal effort with stock is greater than for at-the-money options, while the converse holds for large γ.

In the preceding analysis we interpret x as the value of the firm and y as the value of an option on firm value. This model could also be used to represent incentive contracts in which bonuses are based on accounting numbers. In that case, x would be some measure of net income while y is the difference between net income and the bonus threshold k, assuming that incentive compensation is proportional to that difference. Many bonus schemes have a maximum bonus, which might be effective if the upper component of the optimal "butterfly" contract is sufficiently concave. We leave exploration of such contracts to the reader.

21.5 CONCLUDING REMARKS

Capital market efficiency implies that the market price of publicly traded stock efficiently reflects all information about future dividends that is common knowledge. However, the stock price is unlikely to be an efficient aggregate performance measure. That is, the relative weights used in pricing are very likely to differ from the relative weights that would be used in an optimal incentive contract. Nonetheless, a stock grant may be a useful incentive device, particularly if the stock price reflects decision-influencing information that is not directly contractible. The key is to also use other contractible reports, even though information in those reports is impounded in the market price. Furthermore, if the agent's action influences both the mean and variance of the outcome, then option grants may be more efficient than stock grants.

In Chapters 25 through 28 we consider multi-period contracts. Those chapters essentially ignore market prices. Given that market prices reflect investors' current information about future dividends, it would be potentially beneficial to extend the single-period model in this chapter to a multi-period model, with multiple tasks in each period.

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PART F

PRIVATE AGENT INFORMATION AND RENEGOTIATION IN SINGLE-PERIOD/ SINGLE-AGENT SETTINGS

CHAPTER 22

POST-CONTRACT, PRE-DECISION INFORMATION

In this chapter we review a variation of the principal-agent model in which the agent (as opposed to the principal) receives private information *after a binding contract has been signed* but before he takes his action. In the following chapter, we consider the cases where the agent either is endowed with private information before signing the contract or can leave the firm after observing his private information.

Throughout this chapter we assume that the agent's utility is defined with respect to his terminal compensation. Chapters 25 and 26 considers settings in which the agent's utility is defined over consumption at more than one date and communication can affect both the amount and timing of the agent's compensation. Timing is not an issue in this chapter.

Private agent information is found in many business contexts. The information may, for example, pertain to cost of production, productivity of capital, and market conditions for the firm's products. It can be claimed that comparative advantage in information acquisition is one of the prime skills of successful managers. As we saw in Chapters 3, 4, and 8 additional pre-decision information may improve economic welfare through changes in production choices. The same phenomenon occurs in a principal-agent context. However, the agent's information about his performance measure also improves and, thus, the incentive problem may be more severe due to private information. The questions are whether the agent should be motivated to acquire private information, and whether economic welfare can be improved by letting the agent report his private information to the principal conditioning the agent's compensation on that report. In that case, the agent's information acquisition, actions and reports must be motivated by the principal through the compensation scheme.

22.1 THE BASIC MODEL AND THE REVELATION PRINCIPLE

In this analysis, we generally assume that the outcome $x \in X$ is contractible information and is the only *ex post* performance measure. The signal $y \in Y$ from information system η is privately observed by the agent *after* he has signed the contract with the principal, but before he takes his action. The contract, once accepted, is assumed to bind the agent to the firm so that he cannot leave after he has observed y. Since the agent observes y before he selects $a \in A$, his action choice may well depend on what he observes, i.e., the agent's action strategy is a function $\boldsymbol{a}$: $Y \rightarrow A$. This is one of the key differences between this setting and those considered previously. A key issue is whether it is optimal to allow the agent to influence the contract based on his unverified report of the signal he has observed. That report, if made, is termed his message and is represented by $m \in M$, i.e., the message strategy is a function $\boldsymbol{m}$: $Y \rightarrow M$.[1] We assume, unless otherwise specified, that the message space M is the same as the signal space Y if there is agent communication. If there is no agent communication, then M = $\emptyset$. The basic notation is the same as in previous chapters.

$\boldsymbol{c}$: $X \times M \rightarrow C$ is the compensation contract expressed as a function of the outcome $x \in X$ and possibly of the agent's message $m \in M$,

$u^p(x - c) = x - c$ is the utility of a risk neutral principal,

$u^a(c,a) = u(c) - v(a)$ is the additively separable utility of the agent with $u(\cdot)$ strictly increasing and strictly concave and $v(\cdot)$ strictly increasing and convex (agent is risk and effort averse).

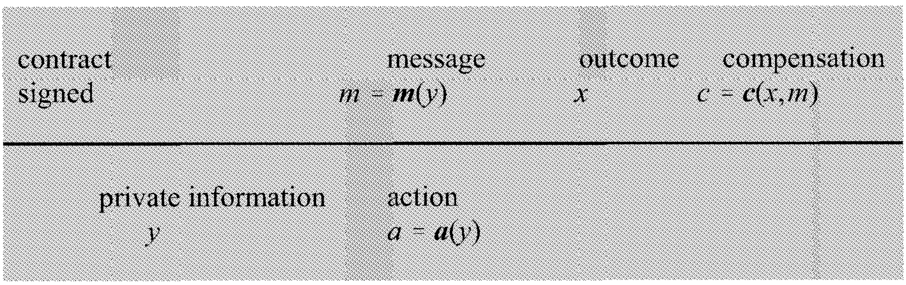

Figure 22.1: Timeline for incentive problem with post-contract, pre-decision information.

[1] Note that we can assume w.l.o.g. that the agent does not randomize over actions and messages since he (as the last mover in the sequential game) will only randomize over choices with equal conditional expected utilities. That is, action and message strategies can be represented as *functions* from the signal space to A and M, respectively. Of course, this does not rule out the possibility that it is optimal for the principal to induce randomization in the contract by including a contractible randomization variable in the compensation scheme. However, in formulating the incentive problems we generally assume that this is not optimal.

The timeline for the incentive problem with post-contract, pre-decision information is shown in Figure 22.1. With agent communication $m \in Y$, and with no agent communication $m \in \emptyset$. The programs defining Pareto optimal contracts with and without agent communication are as follows.

Principal's Decision Problem without Agent Communication:[2]

$$\underset{\boldsymbol{c},\boldsymbol{a}}{\text{maximize}} \quad U^p(\boldsymbol{c},\boldsymbol{a},\eta) \equiv \int_Y \int_X [x - \boldsymbol{c}(x)]\, d\Phi(x|\boldsymbol{a}(y),y)\, d\Phi(y), \tag{22.1}$$

$$\text{subject to} \quad U^a(\boldsymbol{c},\boldsymbol{a},\eta) = \int_Y U^a(\boldsymbol{c},\boldsymbol{a}(y)|y,\eta)\, d\Phi(y) \geq \bar{U}, \tag{22.2}$$

$$U^a(\boldsymbol{c},\boldsymbol{a}(y)\,|y,\eta) \geq U^a(\boldsymbol{c},a|y,\eta), \quad \forall\ a \in A, y \in Y, \tag{22.3}$$

$$\text{where} \quad U^a(\boldsymbol{c},a\,|\,y,\eta) \equiv \int_X u(\boldsymbol{c}(x))\, d\Phi(x|a,y) - v(a).$$

The key feature of this decision problem, compared to a problem in which the agent does not have private information, is that the incentive constraints (22.3) must be specified for each of the agent's possible signals, reflecting the fact that the principal is inducing an *action strategy* $\boldsymbol{a}(y)$ instead of a single action a.

Principal's Decision Problem with Agent Communication:

$$\underset{\boldsymbol{c},\boldsymbol{a},\boldsymbol{m}}{\text{maximize}} \quad U^p(\boldsymbol{c},\boldsymbol{a},\boldsymbol{m},\eta) = \int_Y \int_X [x - \boldsymbol{c}(x,\boldsymbol{m}(y))]\, d\Phi(x|\boldsymbol{a}(y),y)\, d\Phi(y), \tag{22.1$'$}$$

$$\text{subject to} \quad U^a(\boldsymbol{c},\boldsymbol{a},\boldsymbol{m},\eta) = \int_Y U^a(\boldsymbol{c}(\boldsymbol{m}(y)),\boldsymbol{a}(y)|y,\eta)\, d\Phi(y) \geq \bar{U}, \tag{22.2$'$}$$

$$U^a(\boldsymbol{c}(\boldsymbol{m}(y)),\boldsymbol{a}(y)\,|y,\eta) \geq U^a(\boldsymbol{c}(m),a\,|y,\eta), \forall\ a \in A,\ m,y \in Y, \tag{22.3$'$}$$

$$\text{where} \quad U^a(\boldsymbol{c}(m),a\,|\,y,\eta) \equiv \int_X u(\boldsymbol{c}(x,m))\, d\Phi(x|a,y) - v(a).$$

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[2] The expected utilities depend on the information structure η through its impact on the probability functions involving y. While η is explicitly recognized in U^p and U^a, we simplify the notation by leaving η as implicit in probability functions involving y, unless its explicit recognition is important.

In this problem, $\boldsymbol{c}$ is a function of the outcome x and the agent's message m, and $\boldsymbol{c}(m)$ refers to the compensation contract over the outcome x for a given agent message m. The agent is free to choose any message m he wants when he observes y. However, in (22.3′) both the action strategy $\boldsymbol{a}$ and the message strategy $\boldsymbol{m}$ are required to be incentive compatible. The principal optimizes not only over compensation schemes and action strategies but also over message strategies. However, there is an indeterminancy in the choice of message strategy since many message strategies are informationally equivalent, i.e., induce the same partitions on the set of signals. For example, if $Y = \{good, bad\}$, then saying *good* when *bad* is observed and saying *bad* when *good* is observed is informationally equivalent to telling the truth (assuming the principal knows whether the agent is motivated to lie or tell the truth). The *Revelation Principle* eliminates this indeterminancy.

Proposition 22.1 *The Revelation Principle*

For any optimal contract $z = (\boldsymbol{c}, \boldsymbol{a}, \boldsymbol{m})$ based on communication by the agent, there is an equivalent contract z' that (weakly) induces full and truthful disclosure of the agent's private information, i.e., $\boldsymbol{m}'(y) = y$ for all $y \in Y$.

Proof: Define the contract $z' = (\boldsymbol{c}', \boldsymbol{a}', \boldsymbol{m}')$ as $\boldsymbol{c}'(y) = \boldsymbol{c}(\boldsymbol{m}(y))$, $\boldsymbol{a}'(y) = \boldsymbol{a}(y)$, and $\boldsymbol{m}'(y) = y$ for all $y \in Y$. Clearly, z' gives both the principal and the agent the same expected utilities as z. Incentive compatibility of z' follows from

$$\begin{aligned} U^a(\boldsymbol{c}'(m), a \,|\, y, \eta) &= U^a(\boldsymbol{c}(\boldsymbol{m}(m)), a \,|\, y, \eta) && \forall\ a \in A,\ m, y \in Y \\ &\le U^a(\boldsymbol{c}(\boldsymbol{m}(y)), \boldsymbol{a}(y) \,|\, y, \eta) && \forall\ a \in A,\ m, y \in Y \\ &= U^a(\boldsymbol{c}'(y), \boldsymbol{a}'(y) \,|\, y, \eta) && \forall\ a \in A,\ m, y \in Y, \end{aligned}$$

where the equalities follow from the definition of z' and the inequality follows from the incentive compatibility of z. **Q.E.D.**

The Revelation Principle implies that we can restrict our analysis to finding the best contract that induces full and truthful disclosure, i.e., $\boldsymbol{m}(y) = y$ for all $y \in Y$. We do not need to consider contracts that induce the agent to either withhold information or tell lies. The trick used in the Revelation Principle is to construct a new compensation scheme that pays the agent the same amount for telling the truth as he is paid for telling an optimal lie with the original contract.

Consider the following example. Assume that the agent has three possible actions, $\{a_1, a_2, a_3\}$, and four possible signals, $\{y_1, y_2, y_3, y_4\}$. Let $\boldsymbol{c}$ denote an optimal compensation scheme that induces the following action and message strategies.

Signal observed	Message reported	Action taken
y_1	$m = y_1$	a_1
y_2	$m = y_1$	a_2
y_3	$m = y_4$	a_2
y_4	$m = y_3$	a_3

The compensation scheme $\boldsymbol{c}$ induces lying and withholding of information. Now consider an alternative contract $\boldsymbol{c}'$ which is defined as follows:

$$\boldsymbol{c}'(x,y_1) = \boldsymbol{c}(x,y_1),$$

$$\boldsymbol{c}'(x,y_2) = \boldsymbol{c}(x,y_1),$$

$$\boldsymbol{c}'(x,y_3) = \boldsymbol{c}(x,y_4),$$

$$\boldsymbol{c}'(x,y_4) = \boldsymbol{c}(x,y_3).$$

Observe that under the new contract the agent will have no incentive to lie (based on what he would choose under the old contract) and the new contract will provide both the principal and the agent with precisely the same payoffs as under the old contract. Note also that inducing the agent to reveal y_2 under $\boldsymbol{c}'$ is obtained by paying the agent the same whether he reports y_2 or y_1. That is, the principal is not using the additional information reported under $\boldsymbol{c}'$ against the agent. This illustrates the fact that although in the search of an optimal contract we can restrict the analysis to truth-inducing contracts, the Revelation Principle by no means implies that it is costless to induce truthful reports of the agent's private information. Motivating truthtelling may involve using the reports to a lesser extent than they would have been used if there was a verifiable report of that information or it may involve simply ignoring the agent's report in his compensation scheme.

Invoking the Revelation Principle, the program defining Pareto optimal contracts with truthful agent communication is as follows.

Principal's Decision Problem with Truthful Agent Communication:

$$\underset{\boldsymbol{c},\boldsymbol{a}}{\text{maximize}} \quad U^p(\boldsymbol{c},\boldsymbol{a},\eta) = \int_Y \int_X [x - \boldsymbol{c}(x,y)]\, d\Phi(x|\boldsymbol{a}(y),y)\, d\Phi(y), \qquad (22.1'')$$

subject to $$U^a(\boldsymbol{c},\boldsymbol{a},\eta) \;=\; \int\limits_Y U^a(\boldsymbol{c}(y),\boldsymbol{a}(y)\,|\,y,\eta)\; d\Phi(y) \;\geq\; \bar{U}, \qquad (22.2'')$$

$$U^a(\boldsymbol{c}(y),\boldsymbol{a}(y)\,|\,y,\eta) \;\geq\; U^a(\boldsymbol{c}(m),a\,|\,y,\eta), \quad \forall\, a \in A;\, m,y \in Y, \quad (22.3'')$$

where $$U^a(\boldsymbol{c}(m),a\,|\,y,\eta) \;\equiv\; \int\limits_X u(\boldsymbol{c}(x,m))\; d\Phi(x\,|\,a,y) \;-\; v(a).$$

Observe that the Revelation Principle permits elimination of reference to the message strategy since $\boldsymbol{m}(y) = y$, and (22.3″) ensures that $\boldsymbol{c}(y)$ induces the agent to tell the truth, i.e., to report $m = y$ if he observes y, as well as to induce the chosen action strategy $\boldsymbol{a}(y)$. We assume that if the agent has no incentive to lie, then he will tell the truth.

It may seem as if the Revelation Principle is a general result that will always hold. However, we have made a number of implicit assumptions that are crucial for the Revelation Principle to apply. The Revelation Principle applies only if the principal can *commit* to how the agent's message will affect his compensation. Given the ability to commit, the principle can be seen to hold by recognizing that any contract that induces either the withholding of information or lying can be restated so that it induces full and truthful reporting. In this setting, that commitment takes the form of the contract $\boldsymbol{c}(x,m)$ – it specifies how the agent's compensation will be influenced by the outcome and by what he says. If such a commitment is not possible, then the Revelation Principle may not hold. In section 22.8, we consider a variation of this problem in which the agent is compensated on the basis of the firm's market price. Since competitive investors are unable to commit to ignoring the agent's report, the Revelation Principle may not apply. Other settings in which the Revelation Principle does not apply are those in which there is contract renegotiation or a limited message space. If there is contract renegotiation (see Chapter 24), the principal cannot commit to ignoring the information he receives prior to the renegotiation stage (even though he would like to be able to do so). A limited message space exists if the cardinality of M is less than the cardinality of Y. For example, the agent cannot be induced to fully reveal which of three or more signals he has observed if he only has a binary message space (e.g., he can only report either *good* or *bad*).

Unless explicitly stated otherwise, we assume that the Revelation Principle applies. The following basic result is a straightforward extension of Proposition 22.1.

Proposition 22.2

The principal is never worse off with agent communication (i.e., $M = Y$) than with no communication (i.e., $M = \emptyset$).

Proof: Observe that if $z^o = (c^o, a^o)$ is an optimal solution to the principal's problem with no communication, then it is a feasible solution to his problem with communication. In particular, if we set $c(x,m) = c^o(x)$, $\forall\, m \in Y$, then the agent will have no incentive to lie and will be motivated to implement a^o. **Q.E.D.**

The key feature in this proposition is that the principal can always (weakly) motivate the agent to truthfully report his private information if the report is ignored in compensating the manager. However, note that the principal's ability to commit to ignoring the agent's report is crucial for this result.

Christensen (1981, 1982) was the first to explore communication of private pre-decision information in a principal-agent model. He assumed the action and signal spaces are convex and replaced incentive compatibility constraint (22.3″) with its corresponding first-order conditions, i.e.,

$$\int_X u(\mathbf{c}(x,y))\, d\Phi_a(x|\mathbf{a}(y),y) \;-\; v'(\mathbf{a}(y)) \;=\; 0, \quad \forall\, y \in Y,$$

$$\int_X u'(\mathbf{c}(x,y))\, \mathbf{c}_y(x,y)\, d\Phi(x|\mathbf{a}(y),y) \;=\; 0, \qquad \forall\, y \in Y.$$

Assuming that the first-order approach is applicable, the first-order condition characterizing the optimal compensation scheme is

$$M(\mathbf{c}(x,y)) \;=\; \lambda \;-\; \boldsymbol{\delta}'(y) \;+\; [\boldsymbol{\mu}(y) - \boldsymbol{\delta}(y)\,\mathbf{a}'(y)]\, L_a(x|\mathbf{a}(y),y)$$
$$-\; \boldsymbol{\delta}(y)\, L_y(x|\mathbf{a}(y),y),$$

where λ is the multiplier for (22.2″), $\boldsymbol{\mu}$ and $\boldsymbol{\delta}$ are the multipliers for the incentive compatibility constraints, L_a and L_y are the likelihood ratios for $\mathbf{a}(y)$ and y, respectively, and $M(\cdot)$ is the marginal cost to the principal of increasing the agent's utility, i.e., $M(\mathbf{c}(x,y)) = 1/u'(\mathbf{c}(x,y))$. Although the characterization has a structure similar to the compensation scheme characterization in the basic principal-agent model, it is, in general, not useful for identifying the conditions for which communication is strictly valuable.[3] However, Christensen (1981) provides an interesting example with $u(c,a) = 2c^{½} - a^2$, and an exponential joint distribution of (x,y), i.e.,

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[3] Analysis of this problem is difficult, in part because the first-order characterization includes the derivative of a multiplier, $\boldsymbol{\delta}(y)$, which is an endogenous function of y. Hence, it yields few general results.

$$\varphi(x,y|a) = \frac{1}{a+y}\frac{1}{\hat{y}}\exp\{-(x/a+y)\}\exp\{-y/\hat{y}\},$$

where $\hat{y}$ is the prior mean of y. The posterior mean of x is $a + y$. The interesting aspect of this example is that the optimal contract is such that the agent's compensation is a function of the difference between the final outcome and its posterior mean, i.e.,

$$\sqrt{c(x,y)} = \lambda - \boldsymbol{\delta}'(y) + \boldsymbol{\delta}(y)/\hat{y}^2 + \boldsymbol{a}(y)\{x - (\boldsymbol{a}(y) + y)\}.$$

Hence, the agent is compensated on the basis of his report and the deviation of the final outcome from the expected or budgeted outcome, i.e., the "budget variance" as it appears in standard accounting textbooks. Here the budget, $\boldsymbol{a}(y) + y$, is not exogenously specified but is calculated on the basis of the agent's report, such that the agent is evaluated with the self-reported budget as the base. This has a natural "budget participation" interpretation.

22.2 THE HURDLE MODEL

In this section we reintroduce what we have called the "hurdle model." This model has a structure that permits us to understand the role of private post-contract, pre-decision information and, in particular, the role of communication of that information. The model has been used by Christensen and Feltham (CF) in a number of papers on communication in agencies.

There are two possible outcomes, $X = \{x_g, x_b\}$ with $x_g > x_b$. The agent's set of alternative actions is convex with $A = [0,1]$. There is a hurdle $h \in H = [0,1]$. If the agent selects $a \geq h$, i.e., "clears the hurdle," then he has a high probability of obtaining the good outcome, but if he selects $a < h$, i.e., he "does not clear the hurdle," then there is a high probability of obtaining the bad outcome, i.e.,

$$\varphi(x_g|a,h) = \begin{cases} 1-\varepsilon & \text{if } a \geq h, \\ \varepsilon & \text{if } a < h, \end{cases} \qquad \text{where } \varepsilon \in [0,½).$$

That is, h is the minimum effort level required to clear the hurdle, and clearing the hurdle results in a high probability of obtaining the good outcome. The agent's and principal's prior beliefs are that the hurdle h is uniformly distributed on $H = [0,1]$.

The agent has an additively separable utility function $u^a(c,a) = u(c) - v(a)$ with $v(0) = 0$, $v(1) = \infty$, $v'(a) > 0$, $v'(1) = \infty$, and $v''(a) > 0$. The principal is risk neutral. In numerical examples we use the following data:

$$u(c) = \sqrt{c},\ c \geq 0;\ v(a) = a/(1-a);\ \bar{U} = 2;\ x_g = 20;\ x_b = 10;\ \varepsilon = 0.05.$$

If the agent has no pre-decision information denoted η^o, i.e., he does not observe "how high he must jump before he jumps," then he must choose a fixed action a^o. In that case, he is paid outcome-contingent wages $\boldsymbol{c}(x_g) = c_g^o > \boldsymbol{c}(x_b) = c_b^o$, and the probability of obtaining the high wage is

$$\varphi(x_g|a) = a - 2a\varepsilon + \varepsilon.$$

Given our numerical data, the optimal contract and the principal's expected payoff are shown in Table 22.1.

$U^p(\boldsymbol{c},a,\eta^o)$	c_g^o	c_b^o	a^o
6.756	11.74	3.583	0.149

Table 22.1: Optimal contract for no information, η^o.

In the following sections we use this model to illustrate the economic insights of the general analysis of pre-decision, post-contract information in various settings. In those settings the agent observes how high he must jump before he jumps, but after signing the contract, i.e., $y = h$.

22.3 PERFECT PRIVATE INFORMATION

In models with imperfect information it is useful to work with conditional outcome probabilities $\varphi(x|a,y)$. Many papers have considered the special case where the agent gets perfect information before choosing his action, i.e., the agent's signal y reveals the state so that the outcome from each action is known to the agent with certainty given y. Then $\varphi(x|a,y) = 0$ or 1. In this case, it is useful to view the outcome as a function of a and y, i.e., $x = \boldsymbol{x}(a,y)$. The two decision problems can be restated as follows.

Principal's Decision Problem without Agent Communication of Perfect Information:

$$\underset{\boldsymbol{c},\boldsymbol{a}}{\text{maximize}} \quad U^p(\boldsymbol{c},\boldsymbol{a},\eta) \equiv \int_Y [\boldsymbol{x}(\boldsymbol{a}(y),y) - \boldsymbol{c}(\boldsymbol{x}(\boldsymbol{a}(y),y))]\, d\Phi(y), \tag{22.1P}$$

$$\text{subject to} \quad U^a(\boldsymbol{c},\boldsymbol{a},\eta) = \int_Y U^a(\boldsymbol{c},\boldsymbol{a}(y)\,|\,y,\eta)\, d\Phi(y) \geq \bar{U}, \tag{22.2P}$$

$$U^a(\mathbf{c},\mathbf{a}(y)\,|\,y,\eta) \;\geq\; U^a(\mathbf{c},a\,|\,y,\eta), \quad \forall\; a \in A, y \in Y, \tag{22.3P}$$

where $\quad U^a(\mathbf{c},a\,|\,y,\eta) \;\equiv\; u(\mathbf{c}(\mathbf{x}(a,y))) \;-\; v(a).$

Principal's Decision Problem with Truthful Agent Communication of Perfect Information:

$$\underset{\mathbf{c},\mathbf{a}}{\text{maximize}} \quad U^p(\mathbf{c},\mathbf{a},\eta) \;\equiv\; \int_Y [\mathbf{x}(\mathbf{a}(y),y) \;-\; \mathbf{c}(\mathbf{x}(\mathbf{a}(y),y),y)]\, d\Phi(y), \tag{22.1P$''$}$$

$$\text{subject to} \quad U^a(\mathbf{c},\mathbf{a},\eta) \;=\; \int_Y U^a(\mathbf{c}(y),\mathbf{a}(y)\,|\,y,\eta)\, d\Phi(y) \;\geq\; \bar{U}, \tag{22.2P$''$}$$

$$U^a(\mathbf{c}(y),\mathbf{a}(y)\,|\,y,\eta) \geq U^a(\mathbf{c}(m),a\,|\,y,\eta),\; \forall\; a \in A;\; m,y \in Y, \tag{22.3P$''$}$$

where $\quad U^a(\mathbf{c}(m),a\,|\,y,\eta) \;\equiv\; u(\mathbf{c}(\mathbf{x}(a,y),m)) \;-\; v(a).$

Proposition 22.3

If the agent receives perfect information, there is no value to communication.

Proof: Let $\mathbf{z} = (\mathbf{c},\mathbf{a})$ denote the solution to the principal's problem with communication. Observe that to be incentive compatible, this contract must be such that $\mathbf{c}(x,y') = \mathbf{c}(x,y'')$ if $x = \mathbf{x}(\mathbf{a}(y'),y') = \mathbf{x}(\mathbf{a}(y''),y'')$. That is, any two signals that induce the same outcome must pay the same compensation for that outcome. Otherwise if, for example, $\mathbf{c}(x,y') > \mathbf{c}(x,y'')$, then the agent will be better off if he reports $m = y'$ when he has observed y''.

Given the above characteristics of $\mathbf{z}$, we can construct the following contract based on no communication: $\mathbf{c}^o(x) = \mathbf{c}(x,y)$ for any y such that $\mathbf{x}(\mathbf{a}(y),y) = x$. Given that $\mathbf{c}$ induced the implementation of $\mathbf{a}$ (as well as truthful reporting), it follows that $\mathbf{c}^o$ will induce the implementation of $\mathbf{a}$ without any communication.

Q.E.D.

When the agent gets perfect information about the relation between his action and the final outcome, he also gets perfect information about the compensation he is going to get whether there is communication or not. Hence, if there is communication, he can simultaneously choose his action and report so as to maximize his compensation. Truthful reporting then implies that there can be no latitude for the principal to vary the agent's compensation based on the report in addition to the outcome.

Note that even though there is no value to communication, the principal might still find it valuable to have the agent receive perfect private information prior to taking his action due to the decision-facilitating role of that information. However, cases also exist in which this information reduces the principal's expected utility due to the greater severity of the incentive problem (see Section 22.5).

In the hurdle model the agent's information is perfect if he observes $y = h$ before taking his action and if $\varepsilon = 0$. That is, if the agent clears the hurdle, $a \geq h$, then the good outcome occurs with certainty, whereas the bad outcome is obtained if $a < h$. Since communication is not useful when the agent has perfect information, the optimal compensation scheme is similar to the scheme without any information, i.e., the agent is paid an outcome-contingent wage $\boldsymbol{c}(x_g) = c_g^P > \boldsymbol{c}(x_b) = c_b^P$. However, the action strategy is quite different. Since the agent observes the hurdle before taking his action, he can adjust his action according to his private information. The optimal action strategy is characterized by a cut-off $\hat{h}^P$ such that

$$\boldsymbol{a}(h) = \begin{cases} h & \text{if } h \leq \hat{h}^P, \\ 0 & \text{if } h > \hat{h}^P. \end{cases}$$

That is, if the hurdle is sufficiently low, the agent jumps exactly high enough to clear the hurdle, whereas if the hurdle is above the cut-off, he does not jump at all. The optimal contract is shown in Table 22.2, and the corresponding optimal contract with no information and $\varepsilon = 0$ is shown in Table 22.3.

$U^P(\boldsymbol{c},\boldsymbol{a},\eta^P)$	c_g^P	c_b^P	$\hat{h}^P$
10.103	8.266	2.179	0.583

Table 22.2: Optimal contract for perfect information, η^P.

$U^P(\boldsymbol{c},a,\eta^o)$	c_g^o	c_b^o	a^o
6.549	12.026	3.784	0.19

Table 22.3: Optimal contract for no information, η^o and $\varepsilon = 0$.

Comparing the two contracts demonstrates that in this model the principal is better off if the agent receives perfect hurdle information before choosing his action versus not receiving that information. The value of the hurdle information arises primarily because it permits implementation of more efficient action choices, i.e., in the producing region $[0, \hat{h}^P]$ the agent provides just enough effort to clear the hurdle and get the good outcome and, in the non-producing region $(\hat{h}^P, 1]$, he provides no effort. In the no information case, he provides the same effort for all hurdles. This makes him clear higher hurdles with perfect information than with no information, i.e., $\hat{h}^P > a^o$.

22.4 IMPERFECT PRIVATE INFORMATION

In this section we provide insights into the potential roles of imperfect post-contract, pre-decision information and the communication of such information. In section 22.4.1 we consider some benchmarks, and in section 22.4.2 we consider the hurdle model with imperfect private information ($\varepsilon > 0$) to illustrate the potential role of communication.

22.4.1 Some Benchmarks

We consider two benchmark settings: the first-best case, in which y and a are contractible information; and a second-best case in which a is not observable by the principal but y is contractible information (i.e., the principal receives a verified report of y).

The First-best Contract

If x, y, and a are all contractible information, then the principal's decision problem is

$$\underset{\mathbf{c},\mathbf{a}}{\text{maximize}} \quad U^p(\mathbf{c},\mathbf{a},\eta) = \int_Y \int_X [x - \mathbf{c}(x,y)]\, d\Phi(x|\mathbf{a}(y),y)\, d\Phi(y),$$

$$\text{subject to} \quad U^a(\mathbf{c},\mathbf{a},\eta) = \int_Y U^a(\mathbf{c}(y),\mathbf{a}(y)|y,\eta)\, d\Phi(y) \ \geq\ \bar{U}.$$

Proposition 22.4

With additive separability of the utility function, i.e., $u^a(c,a) = u(c) - v(a)$, the first-best compensation scheme is a fixed wage independent of y and x, i.e., $\mathbf{c}(x,y) = c$ for all $x \in X$, $y \in Y$, whereas the first-best action strategy, in general, will vary with y. If the first-best action strategy varies non-trivially with y, then post-contract, pre-decision information is strictly valuable compared to η^o.

Proof: The proof follows immediately from the first-order conditions characterizing the optimal contract and Blackwell's Theorem. **Q.E.D.**

Note that with additive separable utility, the first-best contract only insures the agent perfectly against compensation risk and not against disutility risk.[4] In the hurdle model, the optimal contract is characterized by a fixed wage $\boldsymbol{c}(x,y) = c^*$ and a cut-off $\hat{h}^*$ such that

$$\boldsymbol{a}^*(h) = \begin{cases} h & \text{if } h \leq \hat{h}^*, \\ 0 & \text{if } h > \hat{h}^*. \end{cases}$$

Given the data for our numerical example (with $\varepsilon = 0.05$) the optimal solution to the principal's decision problem is shown in Table 22.4.

$U^p(\boldsymbol{c},\boldsymbol{a},\eta^*)$	c^*	$\hat{h}^*$
10.591	5.776	0.652

Table 22.4: Optimal first-best contract, η^*.

Verified Report of Imperfect Private Information

Now consider a setting in which the agent's action is not observable, but y is contractible information (e.g., there is independent verification of the truthfulness of the agent's report). Verification allows us to relax constraint (22.3″) in the principal's decision problem with communication by eliminating its truth-telling component. That is, this constraint becomes

$$U^a(\boldsymbol{c}(y),\boldsymbol{a}(y)\,|\,y,\eta) \geq U^a(\boldsymbol{c}(y),a\,|\,y,\eta), \quad \forall\ a \in A,\ y \in Y. \qquad (22.3\text{V}'')$$

In this setting, a verified report of y can be useful because it is informative about the uncontrollable events influencing x given a and, hence, can be used to reduce the risk imposed on the agent. This is the role y would play if it was observed after the agent implemented his action (see Chapter 18). Moreover, if y is informative about the productivity of the agent's action, then y may be valuable because it permits the agent to make a better production decision (i.e., a better choice of a).

To illustrate these two roles of a verified report of y we consider two cases of the hurdle model, (a) the agent observes the hurdle *after* choosing his action,

[4] If the agent's disutility for actions has the form of a monetary cost, i.e., $u^a(c,a) = u(c - \kappa(a))$, then it will be optimal to set $\boldsymbol{c}(x,y) = c + \kappa(\boldsymbol{a}(y))$. That is, the agent's compensation will vary with y so as to provide the agent with a constant level of *net* consumption c, thereby insuring the agent against variations in his cost of effort.

and (b) the agent observes the hurdle *before* choosing his action. Note that it is of no consequence when the principal observes the report.

(a) Post-decision verified report: In this case the agent must choose a fixed action a^a independent of h, as in the no information case. However, the post-decision verified report of h can be used to eliminate the risk imposed on the agent for signals where the agent does not clear the hurdle given a^a (i.e., in the cases where the outcome does not provide any information about the agent's action given h and a^a). Furthermore, optimal risk sharing implies that the agent is paid outcome-contingent wages for signals where the agent clears the hurdle given a^a. That is, the optimal compensation scheme has the form

$$c(x,h) = \begin{cases} c_g^a & \text{if } h \le a^a \text{ and } x = x_g, \\ c_b^a & \text{if } h \le a^a \text{ and } x = x_b, \\ c_o^a & \text{if } h > a^a. \end{cases}$$

The optimal contract is shown in Table 22.5. Compared to the no information case in Table 22.1, the gain from eliminating the risk imposed on the agent for high hurdles makes it optimal to induce the agent to take a higher action, i.e., $a^a > a^o$. Hence, the value of a post-decision verified report of the size of the hurdle is not only due to improved risk sharing. The improved risk sharing also leads to a more efficient action choice.

$U^p(c,a,\eta_{ver}^a)$	c_g^a	c_b^a	c_o^a	a^a
7.26	5.998	0.185	5.514	0.258

Table 22.5: Optimal contract for post-decision verified report η_{ver}^a.

(b) Pre-decision verified report: If the agent observes the hurdle before choosing his action, the optimal action strategy is (as for the first-best contract) characterized by a cut-off $\hat{h}^b$ such that

$$a^b(h) = \begin{cases} h & \text{if } h \le \hat{h}^b, \\ 0 & \text{if } h > \hat{h}^b. \end{cases}$$

However, unlike the first-best case, the incentive constraint (22.3V″) implies that risk must be imposed on the agent for signals for which the principal wants to induce the agent to clear the hurdle. The optimal compensation scheme is such that

$$c(x,h) = \begin{cases} c_g^b(h) & \text{if } h \le \hat{h}^b \text{ and } x = x_g, \\ c_b^b(h) & \text{if } h \le \hat{h}^b \text{ and } x = x_b, \\ c_o^b & \text{if } h > \hat{h}^b, \end{cases}$$

where $c_g^b(h) > c_b^b(h)$ are such that the agent is indifferent between clearing the hurdle and not jumping at all, i.e.,

$$(1-\varepsilon)u(c_g^b(h)) + \varepsilon u(c_b^b) - v(h) = \varepsilon u(c_g^b(h)) + (1-\varepsilon)u(c_b^b), \quad \forall\ h \le \hat{h}^b.$$

As in the post-decision case, the agent is paid a fixed compensation when the hurdle is above the cut-off. Moreover, a cost minimizing allocation of utility levels over different hurdles implies that

$$M(c_o^b) = (1-\varepsilon)\, M(c_g^b(h)) + \varepsilon M(c_b^b(h)), \quad \forall\ h \le \hat{h}^b.$$

That is, the expected marginal cost to the principal of increasing the agent's utility level must be the same for all hurdles that are to be cleared. In our numerical example, with a square-root utility function, the condition implies the agent's conditional expected utility of compensation is independent of h. The optimal contract is shown in Table 22.6 and graphically in Figure 22.2. Note that in order to induce $a = h$ in the production-region, the risk imposed on the agent increases as the hurdle increases.[5]

$U^P(c,a,\eta_{ver}^b)$	$c_g^b(h)$	$c_b^b(h)$	c_o^b	$\hat{h}^b$
10.473	$\left\{\sqrt{c_o^b} + v(h)\frac{\varepsilon}{1-2\varepsilon}\right\}^2$	$\left\{\sqrt{c_o^b} - v(h)\frac{1-\varepsilon}{1-2\varepsilon}\right\}^2$	5.568	0.627

Table 22.6: Optimal contract for pre-decision verified report η_{ver}^b.

[5] In Section 22.4.2 we derive the agent's reporting strategies if this contract is offered to the agent in a setting with unverified reporting.

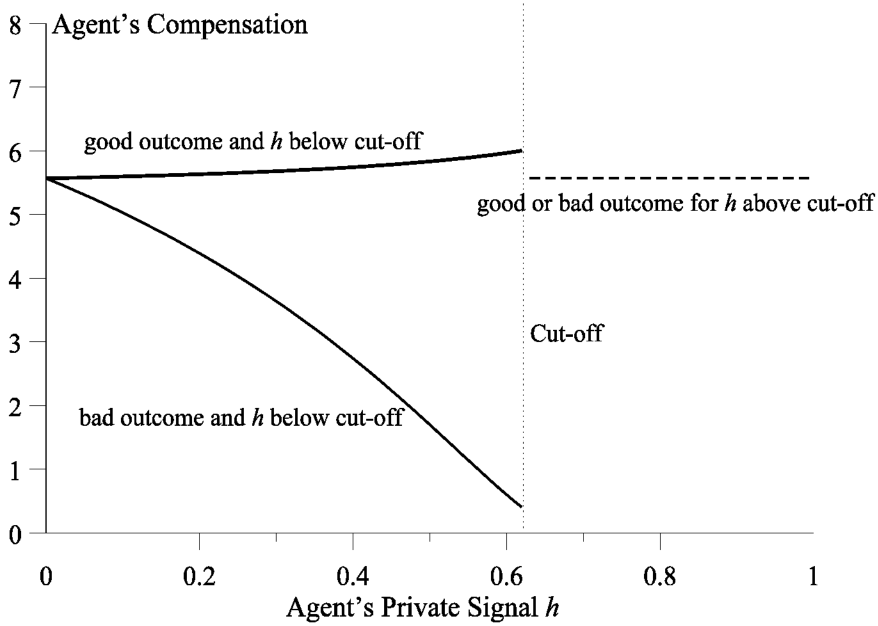

Figure 22.2: Optimal contract with verified report.

22.4.2 Examples of Private Imperfect Information with and without Communication

When there is no verified report of the agent's private information, the agent's compensation can only depend on that information if he is motivated to report his information. Hence, without communication the compensation scheme only depends on the outcome x. However, the agent's action strategy can depend on his signal whether there is communication or not. In the hurdle model, the action strategies with and without communication are characterized by cut-offs such that the agent clears the hurdle if he observes a hurdle below the cut-off and provides no effort above the cut-off, i.e., the action strategies are similar to those in the first-best contract and the contract with pre-decision verified reports.

Without communication the outcome-contingent wages, $c(x_g) = c_g^n > c(x_b) = c_b^n$, and the cut-off, $\hat{h}^n$, are determined such that the agent is indifferent between clearing the hurdle and providing no effort if he observes a hurdle equal to the cut-off, i.e.,

$$(1-\varepsilon)u(c_g^n) + \varepsilon u(c_b^n) - v(\hat{h}^n) = \varepsilon u(c_g^n) + (1-\varepsilon)u(c_b^n). \qquad (22.4)$$

Since $v(h)$ is increasing in h, the agent has strict incentives to clear the hurdle if the hurdle is below the cut-off, and strict incentives to provide no effort if the hurdle is above the cut-off. The optimal no communication contract is shown in Table 22.7.

$U^p(\boldsymbol{c},\boldsymbol{a},\eta^n)$	c_g^n	c_b^n	$\hat{h}^n$
9.937	8.204	2.283	0.549

Table 22.7: Optimal contract for no communication, η^n.

With communication the compensation scheme may (as in the case with pre-decision verified reports) depend on the agent's signal through his (truthful) report of that information. However, truthful reporting as well as the action strategy must be motivated simultaneously.

If the agent observes a hurdle in the production region, $h \in [0,\hat{h}^c]$, and makes an effort sufficient to clear the hurdle, $a = h$, then truthtelling implies that the agent's conditional expected utility of compensation must be independent of his report in that region, i.e.,

$$(1-\varepsilon)u(\boldsymbol{c}(x_g,h)) + \varepsilon u(\boldsymbol{c}(x_b,h)) - v(h)$$
$$= (1-\varepsilon)u(\boldsymbol{c}(x_g,m)) + \varepsilon u(\boldsymbol{c}(x_b,m)) - v(h), \quad \forall\ h,m \in [0,\hat{h}^c].$$

Moreover, the simultaneous choice of truthful reporting and clearing the hurdle in the producing region also implies that

$$(1-\varepsilon)u(\boldsymbol{c}(x_g,h)) + \varepsilon u(\boldsymbol{c}(x_b,h)) - v(h)$$
$$\geq \varepsilon u(\boldsymbol{c}(x_g,m)) + (1-\varepsilon)u(\boldsymbol{c}(x_b,m)), \quad \forall\ h,m \in [0,\hat{h}^c].$$

Since $v(h)$ is increasing in h, combining the two constraints we get that

$$u(\boldsymbol{c}(x_g,m)) - u(\boldsymbol{c}(x_b,m)) \geq \frac{v(\hat{h}^c)}{1-2\varepsilon}, \quad \forall\ m \in [0,\hat{h}^c].$$

Therefore, efficient risk sharing implies that the agent is paid output-contingent wages independent of the agent's report in the production region, i.e., $\boldsymbol{c}(x_g,h) = c_g^c > \boldsymbol{c}(x_b,h) = c_b^c$ for all $h \in [0,\hat{h}^c]$, where the spread in compensations are just sufficient to induce the agent to clear the hurdle at the cut-off, i.e.,

$$(1-\varepsilon)u(c_g^c) + \varepsilon u(c_b^c) - v(\hat{h}^c) = \varepsilon u(c_g^c) + (1-\varepsilon)u(c_b^c). \qquad (22.5)$$

Similarly, in the non-producing region, $h \in (\hat{h}^c, 1]$, the agent's conditional expected utility must be independent of his report in that region, i.e.,

$$\varepsilon u(\mathbf{c}(x_g,m)) + (1-\varepsilon)u(\mathbf{c}(x_b,m))$$
$$= \varepsilon u(\mathbf{c}(x_g,h)) + (1-\varepsilon)u(\mathbf{c}(x_b,h)), \quad \forall\ m,h \in (\hat{h}^c, 1].$$

Therefore, efficient risk sharing implies that the agent is paid a fixed wage in the non-producing region, i.e., $\mathbf{c}(x_g,h) = \mathbf{c}(x_b,h) = c_o^c$ for all $h \in (\hat{h}^c, 1]$.

Since $v(h)$ is increasing in h, the simultaneous choice of truthful reporting and action choices across the producing and non-producing regions now implies that

$$(1-\varepsilon)u(c_g^c) + \varepsilon u(c_b^c) - v(\hat{h}^c) = u(c_o^c). \tag{22.6}$$

Comparing the structure of the optimal contracts with unverified reports to those with verified reports yields the following main differences. Firstly, the unverified reports cannot be used to reduce the risk imposed on the agent to the level that precisely induces him to clear the hurdle for each signal in the producing region, i.e., the truthtelling constraints imply less efficient risk sharing. Secondly, with unverified reports, the truthtelling constraints across the production and non-production regions imply that the agent must be paid a lower fixed wage in the non-producing region than he is with verified reports. That is, the risk sharing across the production and the non-production regions is less efficient.

Table 22.8 shows the optimal communication contract for our numerical example.

$U^p(\mathbf{c},\mathbf{a},\eta^c)$	c_g^c	c_b^c	c_o^c	$\hat{h}^c$
9.977	8.27	2.241	2.452	0.554

Table 22.8: Optimal contract with communication, η^c.

The main difference between contracts with and without communication is that communication facilitates the elimination of risk when the agent makes no effort.[6, 7] The improved risk sharing also leads to more efficient action choices,

[6] Penno (1984) considers a model in which the gain from communication also is due to risk elimination in a region for the private signal where the agent makes no effort. In his model the private signal and the agent's effort are perfect substitutes in terms of the impact on the dis-

(continued...)

i.e., $\hat{h}^c > \hat{h}^n$. However, due to the truthtelling constraints, the risk sharing is less efficient than with pre-decision verified reports. In the hurdle model, pre-decision information permits the agent to make improved action choices, and that information is valuable to the principal whether or not there is communication. Of course, due to the improved risk sharing, the information is more valuable if there is communication.

22.5 IS AN INFORMED AGENT VALUABLE TO THE PRINCIPAL?

The hurdle model illustrates a setting in which the principal is better off if the agent has pre-decision information, whether that information is verified or not. The key to the value of the pre-decision information is that it permits the agent to more efficiently select his action (i.e, it reveals the minimum effort required to make the good outcome highly likely). Baiman and Sivaramakrishnan (1991) provide another setting in which increasing the informativeness of the agent's signal about a productivity parameter is strictly valuable.

In Chapter 18 it is demonstrated that verified post-decision information is valuable if the signals are informative about the agent's action (given the reported outcome). However, the impact of pre-decision information is more subtle. It can directly impact the agent's action choices, and that impact can be positive (as in the hurdle model) or negative. The negative result occurs if the pre-decision information permits the agent to more easily "shirk" because it is informative about the resulting performance measures.[8] This type of setting is illustrated in the following proposition.

Proposition 22.5

Let $z^o = (c^o, a^o)$ be an optimal contract based on no information, η^o, and let η_{ver} be a verified information system in which the agent's action has no impact on the likelihood of the signals, i.e., $d\Phi(y|a) = d\Phi(y)$, $\forall\, y, a$. Further-

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[6] (...continued)
tribution for the outcome, i.e., $d\Phi(x\,|\,a,y) = d\Phi(x\,|\,ay)$. Hence, the productivity of effort is low when y is low. Therefore, a communication contract can be constructed that pays a fixed wage if the agent reports "low," whereas it pays the optimal no-communication compensation if he reports "high" such that it strictly dominates the no-communication contract. However, it is not clear that the optimal communication contract has a fixed wage component.

[7] Dye (1983) provides sufficient conditions for strictly valuable communication of post-decision private information.

[8] Christensen (1981) shows, by use of an example, that the principal may be worse off if the agent privately observes a pre-decision signal about the state.

more, suppose the action strategy $\boldsymbol{a}(y) = a^o, \forall\, y$, can be implemented if y is pre-decision information.[9] Then,

(a) the minimal cost of implementing a^o with y is no greater if y is reported after a is selected than if it is reported prior to selecting a;

(b) if the signals y are uninformative about the agent's action a, i.e., $d\Phi(x,y|a) = g(x,y)k(x,a)$, and z^o is not incentive compatible with pre-decision information, then the minimal cost of implementing a^o is strictly smaller with no information than with pre-decision information.

Proof: (a): Let $\boldsymbol{c}'$ be a cost-minimizing pre-decision information compensation scheme that implements a^o, so that

$$U^a(\boldsymbol{c}'(y),a^o|\,y,\eta) \ \geq\ U^a(\boldsymbol{c}'(y),a'|\,y,\eta), \quad \forall\ a' \in A,\ y \in Y.$$

Taking the expectation with respect to y on both sides establishes that

$$U^a(\boldsymbol{c}',a^o,\eta) \ \geq\ U^a(\boldsymbol{c}',a',\eta), \quad \forall\ a' \in A.$$

Hence, $\boldsymbol{c}'$ also implements a^o if y is post-decision information.

(b): Since y is uninformative about a, no information and post-decision information are equivalent. Furthermore, the uninformativeness of y about a implies that $d\Phi(y|x,a) = d\Phi(y|x)$. Let $\boldsymbol{c}'$ be the cost-minimizing pre-decision information compensation scheme that implements a^o. If $\boldsymbol{c}'$ is independent of y, then $\boldsymbol{c}'$ also implements a^o without y, i.e., $(\boldsymbol{c}',a^o)$ is feasible with no information. Since an optimal no information contract z^o is not incentive compatible with pre-decision information, $\boldsymbol{c}'$ is strictly more costly than $\boldsymbol{c}^o$. On the other hand, if $\boldsymbol{c}'$ depends non-trivially on y, we define a compensation scheme $\boldsymbol{c}''$ as

$$u(\boldsymbol{c}''(x)) \ =\ \int_Y u(\boldsymbol{c}'(x,y))\ d\Phi(y|x), \quad \forall\, x \in X.$$

Given a^o, this contract gives the agent the same expected utility as $\boldsymbol{c}'$ but is strictly less costly to the principal (due to Jensen's inequality and the agent's risk aversion). Furthermore, it implements a^o, since

[9] This condition is not satisfied by the hurdle model since the agent will never choose $a > h$ if he knows h, whereas a^o is fixed in the no information setting.

$$\int_X u(\boldsymbol{c}''(x))\, d\Phi(x|a^o) - v(a^o)$$

$$= \int_X \int_Y u(\boldsymbol{c}'(x,y))\, d\Phi(y|x)\, d\Phi(x|a^o) - v(a^o)$$

$$= \int_Y \int_X u(\boldsymbol{c}'(x,y))\, d\Phi(x|y,a^o)\, d\Phi(y) - v(a^o)$$

$$\geq \int_Y \int_X u(\boldsymbol{c}'(x,y))\, d\Phi(x|y,a')\, d\Phi(y) - v(a') \qquad \forall\ a' \in A$$

$$= \int_X u(\boldsymbol{c}''(x))\, d\Phi(x|a') - v(a') \qquad \forall\ a' \in A\,.$$

where the inequality follows from the incentive compatibility of a^o for $\boldsymbol{c}'$. Hence, $(\boldsymbol{c}'',a^o)$ is a feasible no information contract which is strictly less costly than $(\boldsymbol{c}',a^o)$. **Q.E.D.**

If the information system is informative about the agent's action, then post-decision information can be used to strictly decrease the cost to the principal of implementing the optimal no information action, a^o. However, if the optimal no information action has to be implemented after the agent has observed the information, then the cost to the principal is higher than for post-decision information. The reason is that the agent knows more about the performance measure when choosing his action, and $\boldsymbol{c}^o$ induces him to select an action other than a^o for some y even though the principal would prefer the agent to select a^o irrespective of y. In that setting, a pre-decision contract $\boldsymbol{c}'$ that induces the selection of a^o imposes strictly more ex ante risk on the agent and, hence, is more costly to the principal than is $\boldsymbol{c}^o$.

In general, comparing pre-decision information with no information involves subtle trade-offs. There is no loss, and there may be a gain, if the information is verified and reported after the agent selects his action – the gain will occur if the information permits implementation of a less risky contract to induce a given action. That gain may also be available if the information is verified and reported prior to the agent's action choice, and there may also be a gain due to improved action choice (as illustrated in the hurdle model). However, these gains may be offset by the "loss" that occurs when the agent has improved information about the performance measures (e.g., outcome) that will result from his actions. Of course, if the pre-decision information is privately observed by the agent, then it is, in general, even more costly for the principal to induce the optimal no information action (if it is implementable) whether there is communication or not.

In Proposition 22.5 the comparison is for a fixed action choice. The comparison between no information and pre-decision information must also take into account that it may be optimal to vary the agent's action with the signals. Clearly, this increases the value of pre-decision information relative to no information.

The hurdle model can be used to illustrate the potential negative value of pre-decision information relative to no information. Instead of viewing the parameter ε as a fixed constant, let it be a random variable with two outcomes ε_H and ε_L of equal probability, such that the expected value is $E[\varepsilon] = 0.05$. It is clear from the analysis in Section 22.4 that it is valuable that the agent observes the hurdle before he jumps. Therefore, we consider two *verified* information systems termed "hurdle information" and "hurdle + ε information". With hurdle information the agent observes only the hurdle before choosing his action, i.e., $y = h$. With hurdle + ε information the agent observes both the hurdle and the parameter ε prior to taking his action, i.e., $y = (h, \varepsilon)$. Hence, the optimal contract with hurdle information is as reported in Table 22.6. With hurdle + ε information, the optimal contracts have a similar structure contingent on the observation of ε. However, efficient risk sharing implies that the fixed wages in the non-producing regions must be the same for both values of ε, i.e., $\mathbf{c}(x_g, h, \varepsilon_i) = \mathbf{c}(x_b, h, \varepsilon_i) = c_o^b$ for $h > \hat{h}_i^b$, $i = H, L$. Incentive compatibility of the action choices in the producing regions implies that the spread in utilities is increasing in ε, i.e.,

$$u(\mathbf{c}(x_g, h, \varepsilon_i)) - u(\mathbf{c}(x_b, h, \varepsilon_i)) = \frac{v(h)}{1 - 2\varepsilon_i}, \quad \forall\, h \in [0, \hat{h}_i^b], \quad i = H, L.$$

Table 22.9 reports the cost minimizing contract that implements the optimal hurdle information action strategy with hurdle + ε information,

$$\mathbf{a}^b(h) = \begin{cases} h & \text{if } h \le \hat{h}^b, \\ 0 & \text{if } h > \hat{h}^b, \end{cases}$$

i.e., the induced production cut-offs are the same for both values of ε. It appears from Table 22.9 that the cost of implementing the optimal hurdle information action strategy increases as the spread between the two values of ε increases. The increased cost is due to the additional risk that has to be imposed on the agent for $\varepsilon = \varepsilon_H$ in order to motivate $a = h$ for h below the cut-off.

$(\varepsilon_H, \varepsilon_L)$	$U^P(\boldsymbol{c}, \boldsymbol{a}^b, \eta^b_{ver})$	c^b_o	$\hat{h}^b$
(0.05,0.05)	10.473	5.568	0.627
(0.06,0.04)	10.473	5.556	0.627
(0.07,0.03)	10.470	5.556	0.627
(0.08,0.02)	10.466	5.556	0.627
(0.09,0.01)	10.460	5.556	0.627

Table 22.9: Cost-minimizing contract for pre-decision verified report of ε with $\hat{h}^b_H = \hat{h}^b_L = \hat{h}^b$.

Table 22.10 reports the optimal contracts with verified reports of h and ε. Allowing the action strategy to depend on the agent's information now increases the value of the principal's decision problem as the spread between the two values of ε increases. The higher risk imposed for $\varepsilon = \varepsilon_H$ implies that the cut-off decreases, whereas the cut-off increases for $\varepsilon = \varepsilon_L$ where the risk imposed is lower.

$(\varepsilon_H, \varepsilon_L)$	$U^P(\boldsymbol{c}, \boldsymbol{a}, \eta^b_{ver})$	c^b_o	$\hat{h}^b_H$	$\hat{h}^b_L$
(0.05,0.05)	10.473	5.568	0.627	0.627
(0.06,0.04)	10.475	5.570	0.617	0.637
(0.07,0.03)	10.479	5.575	0.608	0.647
(0.08,0.02)	10.486	5.585	0.598	0.657
(0.09,0.01)	10.496	5.599	0.588	0.668

Table 22.10: Optimal contracts for pre-decision verified report of ε.

In this example, the hurdle + ε information makes it more expensive to motivate the agent's actions, but this cost is outweighed by more efficient action choices. However, other examples could easily be constructed in which more efficient action choices do not offset the increased cost of motivating those actions; for example, in a model with binary actions where the principal always wants to induce the "high" action, there is no gain from improved actions. Of course, the potential negative value of pre-decision information is more pronounced when it is unverifiable.

22.6 DELEGATED INFORMATION ACQUISITION

In the preceding analysis the agent's private information is exogenously determined. However, information acquisition is an important management activity, and one can argue that it is a manager's ability to acquire and process information efficiently that makes him an effective manager, i.e., allows him to be better at selecting actions. Our discussion of this topic is based on Demski and Sappington (DS) (1987). Lambert (1986) is another interesting, and frequently referenced, paper in this area. The setting considered by DS is a setting in which an "expert" is hired who is uniquely qualified to acquire information and subsequently use this information in the choice of a productive act. That is, they consider a multi-task setting, one of which is information acquisition, and analyze the interaction between the two tasks.

The basic elements of the DS model are as follows. The agent's action has two dimensions, $a = (q,\eta) \in A = Q \times H$, where $\eta \in H$ is his pre-decision information system choice (planning) and $q \in Q$ is a productive act (implementation) based on the privately acquired information $y \in Y$ from η. The outcome is assumed to be a function of the state $\theta \in \Theta$ and productive act $q \in Q$, i.e., the outcome function is $\boldsymbol{x}$: $\Theta \times Q \rightarrow \mathbb{R}$.

The choice of η does not affect the outcome x, but is *personally costly* to the agent, whereas q influences x but is *not personally costly* to the agent. Hence, we let $u^a(c,\eta) = u(c) - v(\eta)$ represent the agent's utility function for compensation c and information structure η where $v(\eta) > v(\eta^o) = 0$ for $\eta \neq \eta^o$.

The prior beliefs with respect to event θ are denoted $d\Phi(\theta)$, and the likelihood of signal y given event θ and information system η is $d\Phi(y|\theta,\eta)$. The marginal distribution for signal y given information structure η is

$$d\Phi(y|\eta) = \int_{\Theta} d\Phi(y|\theta,\eta)\, d\Phi(\theta),$$

and the induced probability of outcome x given productive act q and signal y from system η is $d\Phi(x|q,y,\eta)$. The agent's production strategy, i.e., his productive act for each signal y, is $\boldsymbol{q}$: $Y \rightarrow Q$ and his action strategy is $\boldsymbol{a} = (\boldsymbol{q},\eta) \in \boldsymbol{A}$, where $\boldsymbol{A}$ is the set of possible production strategies and information structures. The compensation plan, if there is no communication and x is the only contractible information, is $\boldsymbol{c}$: $X \rightarrow C$.

Principal's Decision Problem without Agent Communication:

$$\underset{\boldsymbol{c},\boldsymbol{a} = (\boldsymbol{q},\eta)}{\text{maximize}} \quad U^p(\boldsymbol{c},\boldsymbol{a}) = \int_Y \int_X [x - \boldsymbol{c}(x)]\, d\Phi(x|\boldsymbol{q}(y),y,\eta)\, d\Phi(y|\eta),$$

$$\text{subject to} \quad U^a(\boldsymbol{c},\boldsymbol{a}) = \int_Y U^a(\boldsymbol{c},\eta|y,\boldsymbol{q}(y))\, d\Phi(y|\eta) \ \geq \ \bar{U},$$

$$\boldsymbol{a} \in \underset{\hat{\boldsymbol{a}} \in A}{\text{argmax}} \ U^a(\boldsymbol{c},\hat{\boldsymbol{a}}),$$

$$\text{where} \quad U^a(\boldsymbol{c},\eta\,|\,y,q) \equiv \int_X u(\boldsymbol{c}(x))\, d\Phi(x|q,y,\eta) \ - \ v(\eta).$$

Observe that the incentive constraint ensures that the agent implements the "suggested" production strategy $\boldsymbol{q}$ for the "suggested" information system η and will not select a different information system with some other production strategy. Hence, the agent has the appropriate incentives both at the planning stage (when he selects his information system η) and at the implementation stage (when he selects his productive act q given a particular signal).

Furthermore, observe that there is *no moral hazard problem if η is publicly reported*, even if q is not observable. This follows from the fact that the agent has no direct preferences with respect to q and, hence, will pick the optimal q for each y if he is paid a fixed wage (and threatened with penalties if the reported η is inconsistent with the contract).

The key feature analyzed in DS is the interaction between the agent's two choices η and q. There is no inherent moral hazard problem associated with the agent's choice of productive act, but the moral hazard problem associated with the choice of the information system may distort the production choices in order to affect the information in x about the agent's choice of η. DS divide the incentive constraints into two sets:

$$(q) \quad \boldsymbol{q}(y) \in \underset{q \in Q}{\text{argmax}} \ U^a(\boldsymbol{c},\eta|y,q), \quad \forall\, y \in Y,$$

$$(\eta) \quad U^a(\boldsymbol{c},\boldsymbol{a}) \geq U^a(\boldsymbol{c},\hat{\boldsymbol{a}}), \qquad \forall\ \hat{\boldsymbol{a}} = (\hat{\boldsymbol{q}},\hat{\eta}) \in \boldsymbol{A},\ \hat{\eta} \neq \eta.$$

Definition[10]

Induced moral hazard is present if the set of constraints (q) is *not* redundant, i.e., the solution to the principal's decision problem has binding q-constraints for the optimal information system.

If there is no induced moral hazard, then the agent has the same incentives as the principal when he selects his productive act (given a particular signal y). In

[10] Compare to the analysis in Section 20.2.5.

this case, planning (information acquisition η) and implementation (choice of productive act q) do not interact, and the implementation problems can be virtually ignored in the contract design. However, if induced moral hazard is present, the planning and implementation segments of the control problem interact. This occurs because the outcome of the implementation phase provides a signal about the agent's preceding planning activities. The agent may wish to influence the signal about his information acquisition activities.

DS provide two settings in which there are no induced moral hazard. They assume in their analysis that the outcome, signal and action spaces are discrete.

Proposition 22.6 (DS, Prop. 2)

Suppose the outcome set is binary, $X = \{x_1, x_2\}$, with $x_1 < x_2$. Then the optimal contract is such that if $\eta \neq \eta^o$, then $\mathbf{c}(x_1) < \mathbf{c}(x_2)$, and there is *no induced moral hazard.*

Proof: If $\eta = \eta^o$ (the null information system), then the irrelevance of the q-constraints is trivial to establish (the agent gets a fixed wage). If $\eta \neq \eta^o$, then it is obvious that $\mathbf{c}(x_1) \neq \mathbf{c}(x_2)$. If $\mathbf{c}(x_1) < \mathbf{c}(x_2)$, then the agent will choose q to maximize $\varphi(x_2|q,y,\eta)$ for each y, which is the first-best optimal choice given y and η. (Alternatively, if $\mathbf{c}(x_1) > \mathbf{c}(x_2)$, the agent will choose q to minimize $\varphi(x_2|q,y,\eta)$ for each y, and that cannot be an optimal strategy since the principal would be better off paying a fixed wage for the agent to select η^o.) **Q.E.D.**

The same congruence of productive act preferences will occur if $\mathbf{c}$ is increasing in x and given any signal y from any system η, the set of productive acts Q can be ordered by first-order stochastic dominance.

Proposition 22.7 (DS, Prop. 3)

Let $\varphi^*(x|\eta)$ represent the probability of outcome x given information system η and the associated first-best production strategy. Suppose the following conditions hold.

(a) The productive acts $q \in Q$ can be ranked by first-order stochastic dominance given any signal $y \in Y$ from any system η.

(b) $\varphi^*(x|\eta)$ satisfies:

(i) MLRP: $v(\eta^1) \leq v(\eta^2)$ implies that $\varphi^*(x|\eta^1)/\varphi^*(x|\eta^2)$ is non-increasing in x;

(ii) CDFC: $v(\eta) = \delta v(\eta^1) + (1-\delta)v(\eta^2)$ for some $\delta \in [0,1]$ implies that $\Phi^*(x|\eta) \leq \delta\Phi^*(x|\eta^1) + (1-\delta)\Phi^*(x|\eta^2)$.

Then induced moral hazard is absent if the first-best production strategy is induced in the solution to the principal's decision problem.

The key to these two results is that, if it is optimal for the principal to induce the first-best implementation strategy for the optimal information system and if $\varphi^*(x|\eta)$ satisfies MLRP and CDFC, then the optimal compensation scheme that motivates the agent to choose η is increasing in x. On the other hand, when the compensation scheme is increasing in x and the productive acts can be ordered according to first-order stochastic dominance for each signal, then it is optimal for the agent to choose the first-best production strategies, since only the monetary returns matter to the agent. Hence, the agent's and the principal's preferences over productive acts coincide at the implementation stage.

However, it may not be optimal for the principal to induce the first-best production strategy. DS provide a numerical example (DS, Example 3.3), in which they illustrate the inducement of a second-best production strategy. The idea is that the production strategy affects the probability distribution over outcomes $d\Phi(x|\mathbf{q},\eta)$ and, thereby, the informativeness of the outcome x about the agent's choice of η. Hence, in addition to the monetary returns the production strategy is also chosen so as to provide information that is useful in motivating the agent's choice of η. This creates induced moral hazard and illustrates how a moral hazard problem in one area of agent choice can create a moral hazard problem in another area of choice.

The analysis above assumes that the outcome x is the only contractible information. DS also consider cases in which the agent communicates the signals y and cases in which the productive act is directly contractible. That analysis is focused on a "binary environment:"

$X = \{x_1, x_2\}, \quad x_1 < x_2,$

$Q = [\underline{q}, \overline{q}],$

$H = \{\eta^o, \eta\}$, with $Y = \emptyset$ for η^o and $Y = \{y_1, y_2\}$ for η,

$\varphi(x_2|q, y_j)$:

- strictly concave in q with an interior maximum at q_j^*, i.e., q_j^* is the first-best productive act given signal y_j,
- greater for y_2 than for y_1 (y_2 is "good news") except $\varphi(x_2|\underline{q}, y_j) = 0, j = 1, 2,$
- $\varphi_q(x_2|q, y_2) > \varphi_q(x_2|q, y_1),$

$c_{ij} = \mathbf{c}(x_i, m_j), \quad$ for $x_i \in X$ and $m_j \in Y$.

Furthermore, the optimal solution to the principal's problem is assumed to select η over η^o in both the first- and second-best solutions, with and without communication.

Proposition 22.8 (DS, Prop. 5 - 8)
In the binary environment the following relations hold.

(a) If q is observable and m is communicated, then $c_{21} < c_{11}$, $c_{12} < c_{22}$ and $q_j > q_j^*, j = 1, 2$.

(b) If q is not observable and m is communicated, then $c_{11} < c_{21}$, $c_{12} < c_{22}$ and $q_j = q_j^*, j = 1, 2$.

(c) If q is not observable and m is not communicated, then $c_1 < c_2$ and $q_j = q_j^*, j = 1, 2$.

The principal's preferences for the settings (a), (b), and (c) are such that (a) $\succ$ (b) $\succ$ (c).

If q is observable (setting (a)), the optimal compensation plan rewards the agent for his "prediction" of x by paying the largest compensation for the outcomes that are the most "consistent" with the agent's message ("prediction"). That is not optimal if q is not observable, since in that setting (with or without communication) the compensation for x_2 is greater than for x_1 for both messages.

An interesting aspect of the solution in setting (a) is that it is optimal to induce the agent to select q other than that which maximizes the probability of x_2. This arises because inducing a sub-optimal productive act results in a relationship between x and y that is more conducive for efficiently motivating the selection of η over η^o.[11]

22.7 SEQUENTIAL PRIVATE INFORMATION AND THE OPTIMAL TIMING OF REPORTING

In this section we extend the basic model to acknowledge that the agent may acquire information at a sequence of dates prior to taking his action, and that it may be beneficial to the firm's owners to induce him to report his information when he receives it rather than waiting until he is about to take his action. That is, we consider the value of *sequential communication* (reporting information

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[11] Note that in setting (a), *communication is redundant*. The same contract can be written as a function of the outcome x and the productive act q.

as it is received) versus *simultaneous communication* (reporting all information when the manager takes his action). Our discussion of this topic is based on Christensen and Feltham (CF) (1997). Sequential reporting of private information occurs, for example, when managers make earnings forecasts or in participative budgeting.

In this setting $y = (y_1,y_2) \in Y = Y_1 \times Y_2$ and y_1 is observed before y_2, and y_2 is observed when the agent selects his action $a \in A$. The representation of no communication and simultaneous communication of y at the time the action is selected is the same as in the basic model (which did not specify the dimensionality of y). The key issue addressed is whether there is any benefit from having the agent communicate y_1 before he observes y_2. Let $m = (m_1,m_2) \in M = Y_1 \times Y_2$ represent the two sets of possible messages.

Principal's Decision Problem with Sequential Communication:

$$\underset{\boldsymbol{c},\boldsymbol{a}}{\text{maximize}} \quad U^p(\boldsymbol{c},\boldsymbol{a},\eta) = \int_Y \int_X [x - \boldsymbol{c}(x,y)]\, d\Phi(x|\boldsymbol{a}(y),y)\, d\Phi(y), \tag{22.1^S}$$

$$\text{subject to} \quad U^a(\boldsymbol{c},\boldsymbol{a},\eta) = \int_Y U^a(\boldsymbol{c}(y),\boldsymbol{a}(y)|y,\eta)\, d\Phi(y) \geq \bar{U}, \tag{22.2^S}$$

$$y_1 \in \underset{m_1 \in Y_1}{\text{argmax}} \int_{Y_2} U^a(\boldsymbol{c}(m_1,\boldsymbol{m}_2(y,m_1)),\boldsymbol{a}(y,m_1)\,|\,y,\eta)\, d\Phi(y_2|y_1), \quad \forall\, y_1 \in Y_1, \tag{22.3^{S1}}$$

$$(\boldsymbol{a}(y,m_1),\boldsymbol{m}_2(y,m_1)) \in \underset{a \in A,\, m_2 \in Y_2}{\text{argmax}} U^a(\boldsymbol{c}(m_1,m_2),a|y,\eta), \quad \forall\, y \in Y,\, m_1 \in Y_1, \tag{22.3^{S2}}$$

$$\boldsymbol{m}_2(y,y_1) = y_2, \quad \forall\, y \in Y, \tag{22.3^{S3}}$$

where $$U^a(\boldsymbol{c}(m),a\,|\,y,\eta) \equiv \int_X u(\boldsymbol{c}(x,m),a)\, d\Phi(x|a,y).$$

Incentive constraints (22.3^{S1}) and (22.3^{S2}) ensure that the agent tells the truth about y_1 by ensuring that the truth is optimal given any subsequent rational choice of a and m_2. In specifying those constraints we recognize that the agent might choose to lie about y_1 and then follow up with a lie about y_2, or the selection of some action other than $\boldsymbol{a}(y)$, i.e., what can be descriptively referred to as "double" or even "triple" shirking. Incentive constraint (22.3^{S3}) ensures that the agent truthfully reports y_2 if he has truthfully reported y_1. The constraints do not require $\boldsymbol{m}_2(y,m_1) = y_2$ and $\boldsymbol{a}(y,m_1) = \boldsymbol{a}(y)$ if $m_1 \neq y_1$ – that is, there is no requirement that the agent tells the truth or selects the optimal action "off the equilibrium path." Some authors have failed to recognize the possibility of "double" shirking and have implicitly (and incorrectly) assumed that imposing second stage communication or action incentives on the equilibrium path is sufficient to ensure that the contract will induce these same choices even if the agent lies in the first stage. Such errors can result in solutions that overstate the value of the program.

If truthtelling is incentive compatible for a simultaneous communication contract, then the same compensation scheme induces truthtelling in a sequential communication contract. Moreover, it induces the same action strategy. The key is that in a simultaneous communication contract, it must be incentive compatible to report the first signal, y_1 (as well as the second signal y_2) truthfully *for each* of the second set of signals, whereas in a sequential communication contract it is only required that y_1 be truthfully reported given the agent's conditional expected utility with respect to both the second signal and the final outcome.[12] These arguments lead to the following result.

Proposition 22.9 (CF, Prop. 1)

Sequential communication is weakly preferred to simultaneous communication.

Of course, a truth-inducing contract for sequential communication may not induce truthful reporting with simultaneous communication. Hence, sequential communication may be strictly preferred to simultaneous communication.

In general, it is more expensive to motivate truthtelling of y_1 where the agent has also observed y_2, than where the agent does not know y_2. CF consider two settings to demonstrate the potential benefits of the agent sequentially reporting his private information. In the first setting, y_1 is a sufficient statistic for y with respect to x for any choice of effort a, and in the second, y_2 is a sufficient statistic for y with respect to x for any choice of effort a.

First Signal y_1 Is a Sufficient Statistic for y

If y_1 is a sufficient statistic for y with respect to x and messages are unverified, then sequential and simultaneous communication are equivalent. That is, early communication has no benefit if nothing new about x is learned later. The key point is that since y_2 provides no new information its only possible role is to reduce the cost of inducing truthful reporting of y_1. That is not possible if y_2 is privately observed by the agent. An unverified report of y_2 cannot be used to

[12] This is basically the same argument as used in the proof of Proposition 22.5(a).

reduce the cost of inducing truthful reporting of y_1, i.e., we cannot expect the agent to turn himself in.

The result changes, however, if m_1 is unverified but m_2 is verified (so that $m_2 = y_2$ is exogenously imposed). In this setting the incentive constraints in the principal's two problems are:

Simultaneous: $$(\boldsymbol{a}(y), y_1) \in \underset{a \in A,\, m_1 \in Y_1}{\operatorname{argmax}} \; U^a(\boldsymbol{c}(m_1, y_2), a \,|\, y, \eta), \quad \forall\, y \in Y, y_1 \in Y_1,$$

Sequential: $$y_1 \in \underset{m_1 \in Y_1}{\operatorname{argmax}} \int_{Y_2} U^a(\boldsymbol{c}(m_1, y_2), \boldsymbol{a}(y, m_1) \,|\, y, \eta)\, d\Phi(y_2 | y_1), \quad \forall y_1 \in Y_1,$$

$$\boldsymbol{a}(y, m_1) \in \underset{a \in A}{\operatorname{argmax}} \; U^a(\boldsymbol{c}(m_1, y_2), a \,|\, y, \eta), \quad \forall\, y \in Y, m_1 \in Y_1.$$

Sequential reporting can be valuable because now the uncertainty about the forthcoming verified report of y_2 can be used to discipline the early reporting of the more informative unverified signal y_1.

For example, if the information system is such that the support for the verified signal depends on y_1, then a truthful report of y_1 may be induced by imposing a threat of penalties if y_2 reveals that the agent lied about y_1. To explore this more formally, let $Y_2(y_1)$ denote the support of y_2 given y_1, i.e., $Y_2(y_1) = \{\, y_2 \in Y_2 \mid \varphi(y_2 | y_1) > 0 \,\}$. Furthermore, let $Y_2(y_1, m_1)$ represent the set of verified signals that reveals that m_1 is a lie if the agent's true signal is y_1, i.e.,

$$Y_2(y_1, m_1) = \{\, y_2 \in Y_2 \mid y_2 \in Y_2(y_1) \text{ and } y_2 \notin Y_2(m_1) \,\}.$$

If the set $Y_2(y_1, m_1)$ has positive measure for all $m_1 \neq y_1$ and all y_1, then there is a positive probability that any lie will be detected and one can achieve, with sufficiently large penalties, agent reporting, and the verified report of y_2, precisely the same result as in the case in which y_1 is also verified.

Of course, one need not have a positive probability of detecting all lies to achieve the above result. In particular, the verified report need only have a positive probability of detecting lies the agent would choose if there was zero probability of detection. Let $\boldsymbol{z}^v = (\boldsymbol{c}^v, \boldsymbol{a}^v)$ represent the optimal contract for the setting in which both y_1 and y_2 are publicly reported (i.e., the full verification setting) and define for that contract the set of messages (lies) the agent would strictly prefer to telling the truth if he has observed y_1:

$$M_1(y_1) \equiv \left\{ m_1 \in Y_1 \,\middle|\, U^a(\boldsymbol{c}^v(y_1), \boldsymbol{a}^v(y_1)) < \max_a U^a(\boldsymbol{c}^v(m_1), a) \right\}.$$

Proposition 22.10 (CF, Prop. 3)

Suppose y_1 is a sufficient statistic for y with respect to the outcome x for any choice of effort a, and that y_2 is publicly reported.

(a) Sequential communication is equivalent to full verification of y if, and only if, for every $y_1 \in Y_1$, $Y_2(y_1,m_1)$ has positive measure for every $m_1 \in M_1(y_1)$, and the principal can impose a sufficiently large penalty if the public report y_2 reveals that the agent has lied.

(b) Simultaneous communication is equivalent to full verification if, and only if, for every $y_1 \in Y_1$, $Y_2(y_1) \cap Y_2(m_1)$ has zero measure for all $m_1 \in M_1(y_1)$, and the principal can impose sufficiently large penalties if the public report y_2 reveals that the agent has lied.

In order to achieve the full verification solution, there need only be a positive probability that any preferred lie will be detected with sequential communication, whereas for simultaneous communication any preferred lie must be detected with certainty.

Of course, sequential communication can be valuable even if it is not equivalent to full verification, and this is illustrated by CF using the hurdle model. Assume that the agent first observes the hurdle $y_1 = h$ and subsequently there is a public report y_2 which will probabilistically reveal over-reporting of the effort required to be productive but will not reveal under-reporting, i.e., $Y_2(h,m_1)$ has positive measure for $m_1 > h$ and zero measure otherwise.

In our numerical example with a square-root utility function, the optimal contract with pre-decision verification of h is such that above the cut-off $\hat{h}^b$ no risk is imposed and below the cutoff the risk imposed is increasing in h and the conditional expected utility of compensation is independent of h (cf. Section 22.4.1). Using this contract when h is unverified would induce the sets of preferred lies:

– $M(h) = \emptyset$ for $h \in 0 \cup (\hat{h}^b, 1]$, i.e., if the hurdle is zero or above the non-productive cut-off;

– $M(h) = [0, h] \cup (\hat{h}^b, 1]$ for $h \in (0, \hat{h}^b]$, i.e., for a positive hurdle in the production region, the agent prefers to either claim his hurdle is lower or is in the non-productive region, and in either case would take zero effort.

The agent's incentive to understate the hurdle follows from the fact that, for $m < h < \hat{h}^b$,

$$\begin{aligned}(1-\varepsilon)u(c_g^b(h)) + \varepsilon u(c_b^b(h)) - v(h) &= (1-\varepsilon)u(c_g^b(m)) + \varepsilon u(c_b^b(m)) - v(h)\\ &< (1-\varepsilon)u(c_g^b(m)) + \varepsilon u(c_b^b(m)) - v(m)\\ &= \varepsilon u(c_g^b(m)) + (1-\varepsilon)u(c_b^b(m)).\end{aligned}$$

With partial verification, a public report y_2 which probabilistically reveals over-reporting can be used to prevent that the agent states that the hurdle is above the cut-off, but not that he understates the hurdle. Hence, the optimal contract with partial verification must satisfy a similar incentive constraint as (22.5) for the unverified case, i.e., the agent is paid outcome-contingent wages in the producing region, (c_g^p, c_b^p), such that

$$(1-\varepsilon)u(c_g^p) + \varepsilon u(c_b^p) - v(\hat{h}^p) = \varepsilon u(c_g^p) + (1-\varepsilon)u(c_b^p),$$

where $\hat{h}^p$ is the production cut-off. Hence, partial verification does not prevent this inefficiency of imposing excess incentives on the agent for $h \in (0, \hat{h}^p)$. However, partial verification eliminates the incentive constraints across the production and non-production regions, i.e., constraint (22.6), such that the agent's consumption can be allocated efficiently over the two regions, i.e.,

$$(1-\varepsilon)u(c_g^p) + \varepsilon u(c_b^p) = u(c_o^p).$$

where c_o^p is the fixed wage in the non-production region. That is, the fixed wage in the non-production region is increased compared to the unverified case.

Second Signal y_2 Is a Sufficient Statistic for y

Now we consider a setting in which the agent's information increases in informativeness about x given a as he moves from his prior beliefs to receiving y_1 and then to receiving y_2. The key, in this case, is that the only role of reporting y_1 early is to provide information about the beliefs the agent will hold after receiving y_2. If y_1 is uninformative about those beliefs, then y_1 is pure noise and cannot be used to reduce the cost of motivating a truthful report of y_2. However, if y_1 is indeed informative about those beliefs, then making an early although imprecise report of those beliefs when observing y_1 may be valuable.

CF provide two hurdle model examples in which sequential communication dominates simultaneous communication. In the first example, $\varepsilon = 0$, and $y_2 = h$ reveals the hurdle, i.e., the agent has perfect information when selecting his action. Hence, simultaneous communication has no value over no communication. The first signal y_1 is imperfectly informative about the hurdle $y_2 = h$. In particular, $Y_1 = \{y_1^1, y_1^2\}$ where y_1^1 reveals that the hurdle is in a middle interval, i.e., $y_1^1 = [h_L, h_H]$, whereas y_1^2 reveals that the hurdle is in the "tails", i.e., $y_1^2 =$

$[0,h_L) \cup (h_H,1]$. Sequential communication permits the use of a compensation scheme $\{c(x_j,y_1^i)\}_{ij}$ where reporting $m_1 \in \{y_1^1,y_1^2\}$ before the agent observes the hurdle effectively selects a pair of outcome-contingent compensation levels $\{c(x_j,m_1)\}_j$. A numerical example is provided where this is strictly valuable. The key is that the compensation scheme for the "tail" report $m_1 = y_1^2$ is less risky than for the "middle" report $m_1 = y_1^1$. The outcome-contingent compensation scheme for the tail report only need provide sufficient incentives to induce effort for $h \le h_L < \hat{h}^o$, where $\hat{h}^o$ is the cut-off for the simultaneous (or equivalently the no) communication contract.

In the second example, $\varepsilon > 0$, $y_1 \in [0,1]$ reveals the hurdle rate, and $y_2 \in \{y_2^1,y_2^2\}$ affects the probability of the good outcome given the hurdle is cleared (i.e., $\varphi(x_g|a,y_1,y_2^i) = 1 - \varepsilon^i$ if $a \ge y_1$, $i = L,H$). In the numerical example, the solutions to both the simultaneous and sequential communication programs yield menus of four distinct outcome-contingent compensation contracts, which vary with whether y_1 is less than or greater than a cutoff $\hat{h}^i$ and the report of y_2^i. However, the menus differ, and the sequential communication program yields a higher expected payoff to the principal. Forcing the agent to choose between $m_1 \in [0,\hat{h}^i]$ or $m_1 \in (\hat{h}^i,1]$ before observing y_2^i is more valuable than permitting him to make the selection of m_1 after observing y_2^i. Again, this value is due to a more efficient tailoring of the risk imposed on the agent conditional on his report taking into account the difference in sufficient incentives for the two values of ε.

22.8 IMPACT OF DISCLOSURE ON THE INFORMATION REVEALED BY THE MARKET PRICE

Most top management compensation schemes contain market based performance measures such as equity claims and stock options. At the same time managers make disclosures of unverifiable private management information, for example through their choice of accruals in published financial reports, management earnings forecasts, reports to analysts, or through their choice of financial policies. Clearly, if the agent discloses his private information and investors believe that he is reporting truthfully, then the market price of the firm at the disclosure date will be influenced by that disclosure. Moreover, market prices may also depend on *non-contractible investor information* acquired from other sources such as through personal information acquisition. Market prices, however, are contractible and, therefore, market prices may be a useful device for contracting indirectly on otherwise non-contractible investor information.

Dye (1985) and Christensen and Feltham (CF) (2000) demonstrate that it may not be optimal to motivate the manager to fully and truthfully reveal his information in such settings, i.e., the Revelation Principle may not apply. The reason is that market prices are set in a competitive market and there is no mechanism for investors to agree to set a price that only reflects their investor information (i.e., the principal has limited ability to commit to the way disclosed information is used in the agent's performance measure). Hence, if the manager discloses his information, he may affect the informational characteristics of the contractible information. Dye (1985) considers verified disclosure of a post-decision signal in a setting in which the contractible information with disclosure is less informative about the agent's action than it is without disclosure, i.e., disclosure has negative value compared to no disclosure. CF consider unverified disclosure of pre-decision information. In that setting the choice between a commitment to no or full disclosure is a choice between the use of a "hard" performance measure only influenced by investor information versus a "soft" performance measure which is influenced by the manager's unverifiable disclosure but which is more informative. Thus, the contracting friction is the incentive constraints on the manager's disclosure. An interesting aspect of their analysis is that partial disclosure is optimal.

Suppose the investors receive a signal $y_I \in Y_I$ at the same time as the manager observes (and possibly discloses) a signal $y_m \in Y_m$. The manager is assumed to have at least as much information as the investors. That is, y_m is a sufficient statistic for $y = (y_m, y_I)$ with respect to x for all a, i.e., $\Phi(x|y,a) = \Phi(x|y_m,a)$ for all a. Neither signal is contractible. Furthermore, assume that the market price at the disclosure date P_1, the final outcome x as well as the agent's report of y_m (if there is disclosure) are all contractible information. If well diversified investors believe that the agent is induced to choose an action strategy $\boldsymbol{a}$ and that y_m is truthfully disclosed, then the rational expectations equilibrium market price of the firm (gross of the agent's compensation) is[13]

$$P_1(y_m) = \int_X x \, d\Phi(x|y_m, \boldsymbol{a}).$$

That is, the market price will be unaffected by the investors' direct information. Consequently, there is no mechanism by which y_I can be used in specifying the agent's compensation scheme.

On the other hand, if the agent does not disclose his information, then the investors will use their direct information in trading. If they believe that the agent will implement an action strategy $\boldsymbol{a}$, then the market price is

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[13] Note that the rational expectations equilibrium market value of the firm at the contracting date is the expected gross market price at the disclosure date minus the expected compensation to the agent taken the equilibrium contract as given. That is, the decision problem is equivalent to maximizing the market value of the firm at the contracting date. CF consider an alternative formulation in which it is assumed that the agent is endowed with the production technology and that he sells the firm at the contracting date to well diversified investors.

$$P_1(y_I) = \int_{Y_m} \int_X x \, d\Phi(x|y_m,\mathbf{a}) \, d\Phi(y_m|y_I).$$

Hence, if the agent does not disclose his information, then y_I may be at least partially revealed by the market price $P_1(y_I)$, and the price can be used in the compensation contract to contract on y_I indirectly.

The choice between no disclosure and full disclosure of y_m is, therefore, a choice between compensation schemes based on $\mathbf{q}^o = (P_1(y_I),x)$ and $\mathbf{q}^d = (y_m,x)$, respectively. Note that the market price is a redundant contracting variable when the report of y_m is itself contractible.[14] Even though y_I is less informative about x than y_m, contracts based on $\mathbf{q}^o$ may be preferred to contracts based on $\mathbf{q}^d$. We explore how this can occur in two distinct settings. In the first, y_I and y_m are observed after the agent has taken his action (i.e., post-decision information) and before the outcome x is reported – if the agent discloses y_m, that disclosure is verified. In the second, y_I and y_m are observed before the agent has taken his action (i.e., pre-decision information) – if the agent discloses y_m, that disclosure is not verified.

Verified Disclosure of Post-decision Information

If the investors and the agent observe their signals after the agent has selected his action, the agent's action may affect the probability distributions of the signals as well as the final outcome, i.e., $\Phi(y,x|a)$. Suppose the investors' signal is *exclusively* informative about the agent's action, i.e., $\Phi(x|y_I,a) = \Phi(x|a)$. That is, for example, y_I is an observation of the agent's action with noise independent of the events influencing the outcome.[15]

We first consider a setting in which there is no communication. Hence, the agent's information is irrelevant.

Proposition 22.11

Let $z = (\mathbf{c},a)$ be an optimal contract with no agent disclosure where the agent's compensation scheme may depend on the contractible information $\mathbf{q}^o = (P_1,x)$. Suppose the support of the probability distribution for the investors' signal is independent of the agent's action and that the investors' signal is exclusively informative about the agent's action. Then an equivalent no disclosure contract exists in which the agent's compensation only depends on the final outcome.

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[14] CF also consider the case where the agent's report is not directly contractible. In that case, the reports may be indirectly contractible through the market price.

[15] Baiman and Verrecchia (1996) consider the case where $y_I = a$.

Proof: If the support of $\varphi(y_I|a)$ is independent of the agent's action and the investors hold rational expectations about the agent's equilibrium action, then investors do not revise their beliefs about the agent's action based on y_I. Since y_I is exclusively informative about the agent's action, investors do not revise their beliefs about x either. Hence, the market price is independent of y_I, and the result follows. **Q.E.D.**

The key aspect in this proposition is that in a rational expectations equilibrium, the investors will only revise their beliefs about the agent's action if they observe a signal which is inconsistent with the conjectured equilibrium action. On the other hand, if such signals are possible, contracting on the market price may be useful to control for off-equilibrium actions. For example, if $y_I = a$ and the market price $P_1(a)$ is an invertible function of a, the first-best solution can be obtained if sufficient penalties can be imposed on the agent.

Proposition 22.11 establishes that the market price cannot have value in contracting unless either the support of y_I depends on a or y_I is influenced by events correlated with the events affecting the outcome x. The following proposition considers the second case and identifies sufficient conditions for the market price to be strictly valuable in contracting.

Proposition 22.12

Let $z = (\mathbf{c},a)$ be an optimal contract with no agent disclosure where the agent's compensation scheme may depend on the contractible information $\mathbf{q}^o = (P_1,x)$. Suppose the probability distributions of x given the investors' signal and a, i.e., $\Phi(x|y_I,a)$, can be ordered by first-order stochastic dominance, and that x is *not* a sufficient statistic for (y_I,x) with respect to the agent's action. Then z is strictly more valuable than any contract that only depends on x.

Proof: Since $\Phi(x|y_I,a)$ can be ordered by first-order stochastic dominance, the market price is an invertible function of y_I. Hence, contracting on P_1 is equivalent to contracting directly on y_I. The result then follows from y_I being informative about the agent's action given x. **Q.E.D.**

If the investor signal has a unique impact on the market price such that $P_1(y_I)$ and y_I are informationally equivalent, then a market based compensation scheme is useful when the investors' signal provides information about the agent's action in addition to the other contractible information x.

Proposition 22.13

Let $z = (\mathbf{c},a)$ be an optimal contract with no agent disclosure where the agent's compensation scheme may depend on the contractible information $\mathbf{q}^o = (P_1,x)$. Suppose the probability distributions of x given the investors' signal and a, i.e., $\Phi(x|y_I,a)$, can be ordered by first-order stochastic dominance, and that x is *not* a sufficient statistic for (y_I,x) with respect to the agent's action.

(a) If x is a sufficient statistic for (y_m,x) with respect to the agent's action, then no disclosure is strictly valuable compared to disclosure of the agent's superior information y_m.

(b) If (y_m,x) is a sufficient statistic for (y,x) with respect to the agent's action, then disclosure of the agent's superior information y_m is weakly more valuable than no disclosure.

Proof: (a): Since $\Phi(x|y,a) = \Phi(x|y_m,a)$ for all a, the market price depends only on y_m. Furthermore, since x is a sufficient statistic for (y_m,x) with respect to the agent's action, x is also a sufficient statistic for $(P_1(y_m),y_m,x)$ with respect to the agent's action. Therefore, it follows from Proposition 18.5, that any optimal compensation scheme with disclosure of y_m only depends on x. Proposition 22.12 then gives the result.

(b): If (y_m,x) is a sufficient statistic for (y,x) with respect to the agent's action, then a contract based on (y_m,x) dominates weakly any contract based on (y,x), i.e., a contract in which both y_m and y_I are directly contractible, which in turn weakly dominates a contract based on $(P_1(y_I),y_m)$. **Q.E.D.**

Condition (a) demonstrates the potential negative value of disclosure compared to no disclosure. If y_m provides no information about the agent's action in addition to x, then y_m is not useful in contracting with the agent in addition to x. Hence, if y_I is useful in contracting in addition to x, either because the support of y_I depends on the agent's action or the events affecting y_I are correlated with the events affecting x, then disclosing y_m destroys otherwise useful contractible information. The condition of y_m is satisfied if, for example, y_m is a function of x and events independent of events affecting x. Dye (1985) considers the special case where $y_m = x$. On the other hand, disclosure is valuable if (y_m,x) is more informative about the agent's action than (y_I,x).

Unverified Disclosure of Pre-decision Information

If the investors and the agents observe their signals before the agent has selected his action, the action strategy may, in general, be a function of both y_I and y_m, i.e., $\mathbf{a}$: $Y_I \times Y_m \to A$. However, with full disclosure of y_m the action strategy is only a function of y_m, since y_m is more informative about the events influencing x than is y_I. Furthermore, note that when both signals are observed before the action is taken, the action has no impact on the probability distributions of the signals and, therefore, directly contracting on (y_m,x) is always at least weakly preferred to directly contracting on (y_I,x). That is, verified disclosure of y_m is weakly preferred to no (or partial) disclosure of y_m.

The potential negative value of disclosure in the setting with unverified disclosures is readily shown in the case where the agent gets perfect information. If there is disclosure, then the market price is a function of y_m only.[16] Hence, the contractible information is $\boldsymbol{q}^d = (y_m,x)$. However, it follows from Proposition 22.3 that there is an equivalent contract where the compensation scheme only depends on the outcome x. That is, the choice between no disclosure and full disclosure is a choice between contracts with contractible information $\boldsymbol{q}^o = (P_1(y_I),x)$ and $\boldsymbol{q}^d = x$, respectively. Therefore, if the market price is performance relevant, i.e.,[17]

$$\exists\, x,y_I,y_I': \boldsymbol{c}(x,P_1(y_I)) \neq \boldsymbol{c}(x,P_1(y_I')),$$

then no disclosure is strictly preferred to full disclosure.

Proposition 22.14 (CF, Prop. 1)

Suppose y_m provides the agent with perfect information about the relation between his action choice and the outcome. Then no disclosure is weakly preferred to full disclosure. If the market price for the optimal no disclosure contract is performance relevant, then no disclosure is strictly preferred to full disclosure.

If the market price is a valuable contracting variable with no disclosure, then y_m is also a valuable contracting variable if it were directly contractible since it is more informative than the market price. However, with unverified disclosures the incentive constraints on the agent's disclosure of y make it a redundant contracting variable (in addition to x) and, moreover, truthful disclosure of y_m makes the valuable investor information y_I unaccessible for contracting indirectly through the market price.

If the agent's information is imperfect, then there is a non-trivial trade-off between no and full disclosure. The informativeness of the investors' signal and the incentive problems involved in motivating full disclosure play key roles in that trade-off. To see this, consider varying the informativeness of the investors' signal (about the agent's private information). When the informativeness is low (or uninformative), then the advantage of indirectly using y_I in the no dis-

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[16] We assume throughout this section that the support of the conditional distribution of y is independent of y_I. Otherwise, we would have to specify off-equilibrium beliefs for the investors if they observe a report inconsistent with their signal y_I.

[17] This means first of all that y_I must be valuable in contracting if it were directly contractible. Otherwise, the case would be moot. Secondly, the market price must at least partially reveal some of that information.

closure contract is also low (or zero). Hence, if the gain from an unverified report is sufficiently positive, recognizing the incentive problems involved in motivating full disclosure, then full disclosure dominates no disclosure. As the informativeness of y_I increases, the advantage of indirectly using y_I in the no disclosure contract also increases. However, the value of the full disclosure contract is independent of the informativeness of y_I, since y_I cannot be used when there is full disclosure. In the extreme, where y_I is perfectly informative about the agent's private information, no disclosure clearly dominates full disclosure (provided that y_I is revealed through the market price). Hence, there is a threshold for the informativeness of y_I for which no disclosure dominates full disclosure above the threshold, whereas the opposite relation holds below the threshold.

CF consider a variation of the hurdle model to examine this trade-off more closely. The agent observes the hurdle before choosing his action, i.e., $y_m = h$. If the agent fully and truthfully discloses his private information, then the optimal contracts are as derived in Section 22.4.2. The investors observe an imperfect signal about the hurdle with a probability distribution $\Phi(y_I)$ and conditional probability distribution $\Phi(y_m|y_I)$. There are two investor signals y_I^H and y_I^L with probabilities $\varphi(y_I^H) = \varphi(y_I^L) = ½$. The conditional densities for the hurdle are

$$\varphi(y_m|y_I) = \begin{cases} (1 + k) - 2ky_m & \text{if } y_I = y_I^L, \\ (1 - k) + 2ky_m & \text{if } y_I = y_I^H, \end{cases}$$

with $k \in [0,1]$. The signal y_I is uninformative about the hurdle if $k = 0$, and its information content increases with k.

Suppose the investors' signal is revealed by the market price with no disclosure. Then the optimal compensation scheme with no disclosure consists of a pair of outcome-contingent payments $c_{gi}^o > c_{bi}^o$, $i = H, L$, for each investor signal y_I and the optimal production cut-offs are determined such that

$$(1 - \varepsilon)u(c_{gi}^o) + \varepsilon u(c_{bi}^o) - v(\hat{h}_i^o) = \varepsilon u(c_{gi}^o) + (1 - \varepsilon)u(c_{bi}^o), \quad i = H, L.$$

The market price is

$$P_1(y_I^i) = \Phi(\hat{h}_i^o|y_I^i)\,[(1 - \varepsilon)x_g + \varepsilon x_b]$$

$$+ [1 - \Phi(\hat{h}_i^o|y_I^i)]\,[\varepsilon x_g + (1 - \varepsilon)x_b], \quad i = H, L.$$

Since the cut-off is always higher for the "low hurdles signal" y_I^L than for the "high hurdles signal" y_I^H (except in the uninformative case, $k = 0$), and $\Phi(y_m|y_I^H)$ first-order stochastically dominates $\Phi(y_m|y_I^L)$, the market price is also higher for y_I^L than for y_I^H. Hence, the investors' signal is revealed by the market price and contracting on that market price is equivalent to contracting on the investors' signal directly.

For our numerical example introduced in Section 22.2, the optimal cut-offs and market prices for the investors' two possible signals are shown in Figure 22.3 as functions of the informativeness parameter k. Observe that $\hat{h}_L^o$ is increasing in k, while $\hat{h}_H^o$ is decreasing for low and moderate informativeness of y_I and slightly increasing for high informativeness, indicating the significant use of the investors' signal in contracting with the agent if that signal is informative. The fact that $P_1(y_I^L)$ and $P_1(y_I^H)$ diverge for $k > 0$ demonstrates that the market price can be used to contract on y indirectly.

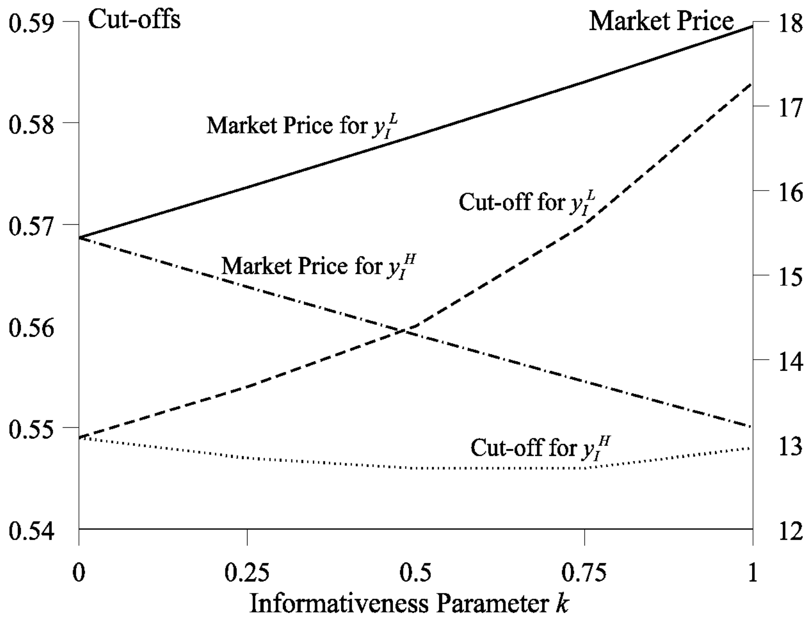

Figure 22.3: Characteristics of no disclosure contract with varying k.

Figure 22.4 depicts the optimal value of the principal's (i.e., the investors') decision problem for the verified, full and no disclosure contracts for varying informativeness parameter k.

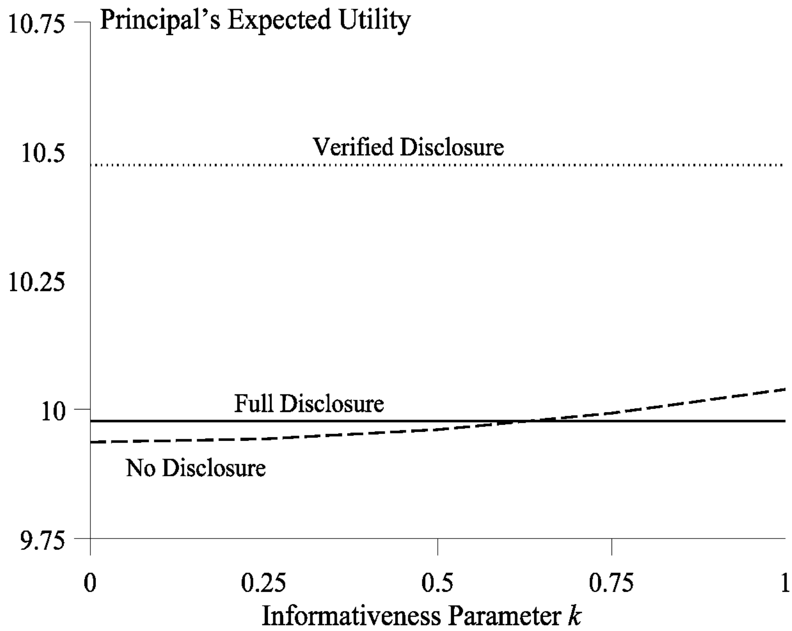

Figure 22.4: Comparison of verified, full, and no disclosure with varying k.

The values for verified and full disclosure are independent of k, and verified disclosure clearly dominates both full and no disclosure. The value for no disclosure is increasing in k, with full disclosure dominating no disclosure for $k \leq \frac{1}{2}$, and the latter dominating if $k \geq \frac{3}{4}$. Hence, this numerical example demonstrates the trade-off between the value of being able to implicitly contract on the investors' signal (through $P_1(y_I)$) versus the value of full disclosure, which permits payment of a fixed wage for non-productive effort. The less informative the investors' signal, the more likely it is that full disclosure dominates, whereas no disclosure dominates if the investors' signal is highly informative.

The preceding discussion focuses on a comparison of no versus full disclosure of the agent's private information. However, the optimal disclosure policy may involve partial disclosure (if the agent does not receive perfect information). Under full disclosure $M = Y_m$ and $\boldsymbol{m}(y) = y_m$, while under partial disclosure $\boldsymbol{m}$ defines a non-trivial y_I-contingent partition of Y_m for which $Y_m(y_I,m) = \{ y_m \in Y_m \mid \boldsymbol{m}(y_I,y) = m \}$. Observe that if $Y_m(y_I,m)$ contains more than one signal y_m or varies with y_I, then the investors' information can influence the investors' beliefs about y_m (since $\Phi(y_m|m,y_I) = \Phi(y_m|y_I)/\Phi(Y_m(y_I,m)|y_I)$) for $y \in Y_m(y_I,m)$ and thereby may influence the market price. Hence, with partial disclosure, there can be at least partial indirect contracting on the investors' signal through the market price. Of particular notice is the fact that while the market price is only used to induce the agent's action choice under no disclosure, the market price is used to induce the agent's disclosure as well as his action with partial disclosure.

CF illustrate the value of partial disclosure compared to no and full disclosure using the hurdle model. The action choices are characterized by investor-signal-contingent cut-offs, $\hat{h}_L^{pd}$ and $\hat{h}_H^{pd}$. The message space is binary, $M = \{m_\ell, m_h\}$, and the disclosure policy is characterized by investor-signal-contingent cut-offs, $\tilde{h}_L^{pd}$ and $\tilde{h}_H^{pd}$, such that, given y_I^i, the agent reports m_ℓ if $h \in [0, \tilde{h}_i^{pd}]$ and m_h otherwise. We assume $\tilde{h}_H^{pd} = \hat{h}_H^{pd}$, so that for y_I^H the agent's message m reports whether he will provide productive effort $a = h$ or zero effort. We further assume $\tilde{h}_L^{pd} \in (\hat{h}_L^{pd}, 1]$, and consider two types of contracts.[18]

Type (a): In this contract, $\tilde{h}_L^{pd} = 1$, so that m is uninformative about h or the agent's effort if the investors' signal is y_I^L. Observe that this contract imposes no risk on the agent if y_I^H is observed and zero effort is provided, i.e., $h \in (\hat{h}_H^{pd}, 1]$, but if y_I^L is observed, then the contract imposes risk on the agent whether he provides effort or not, i.e., for all $h \in [0,1]$. In effect, a type (a) contract can be viewed as a randomization between a no disclosure contract (if y_I^L is observed) and a full disclosure contract (if y_I^H is observed).

Type (b): In this contract, $\tilde{h}_L^{pd} = \hat{h}_L^{pd} + \delta$, for $\delta > 0$ arbitrarily small. In this contract, m_h again reveals that the agent will not provide productive effort and, hence, no risk is imposed on the agent. If y_I^H is observed then m_ℓ reveals that h is such that productive effort $a = h$ is provided. However, if y_I^L is observed, then m_ℓ is reported primarily if h is such that productive effort $a = h$ is provided, but it is also reported for some h for which productive effort is not provided, i.e., for $h \in (\hat{h}_L^{pd}, \tilde{h}_L^{pd})$. The contract imposes risk on the agent even though he provides zero effort for these latter agent signals, but this serves to induce $P_1(y_I^L, m_\ell) < P_1(y_I^H, m_\ell)$ so that the market price reveals the investors' signal if m_ℓ is reported. Hence, the risky incentives used to induce productive effort can vary with the investors' signal.

The interesting aspect of these two partial disclosure contracts is that (a) dominates no disclosure and (b) dominates full disclosure, so that some form of partial disclosure is optimal.

Proposition 22.15 (CF, Prop. 2)

The following two results hold in the hurdle model:

(a) The type (a) partial disclosure contract strictly dominates no disclosure for all $\varepsilon \in (0, ½)$ and $k \in [0,1]$, whereas it is equivalent to no disclosure for $\varepsilon = 0$ or ½.

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[18] CF demonstrate that if $\tilde{h}_L^{pd} \in (\hat{h}_L^{pd}, 1]$ the contract can be contingent on y_I^i through v_1 if the agent reports m_ℓ but not if he reports m_h.

(b) The type (b) partial disclosure contract strictly dominates full disclosure for all $\varepsilon \in [0, \frac{1}{2})$ and $k \in (0, 1]$, whereas it is equivalent to full disclosure if $\varepsilon = \frac{1}{2}$ or $k = 0$.

Hence, a partial disclosure contract is always optimal.

The dominance of the type (a) partial disclosure contract over no disclosure follows from the fact that the outcome-contingent wages for the optimal no disclosure contract satisfy the incentive constraints with partial disclosure, but the partial disclosure contract eliminates outcome-contingent risk for y_I^H and m_h. The dominance of the type (b) partial disclosure contract over full disclosure follows from the fact that full disclosure precludes contracting on the investors' signal in both the producing and non-producing regions, whereas the investors' signal is useful in the producing region with partial disclosure.

Figure 22.5 depicts the impact of investor-signal informativeness (k) on the optimal value of the principal's decision problem for no, full, and the two types of partial disclosure contracts (CF also consider the impact of varying ε). Observe that type (b) partial disclosure is optimal for low investor-signal informativeness ($k \le \frac{1}{4}$), whereas the type (a) partial disclosure is optimal for high investor-signal informativeness ($k \ge \frac{1}{2}$).

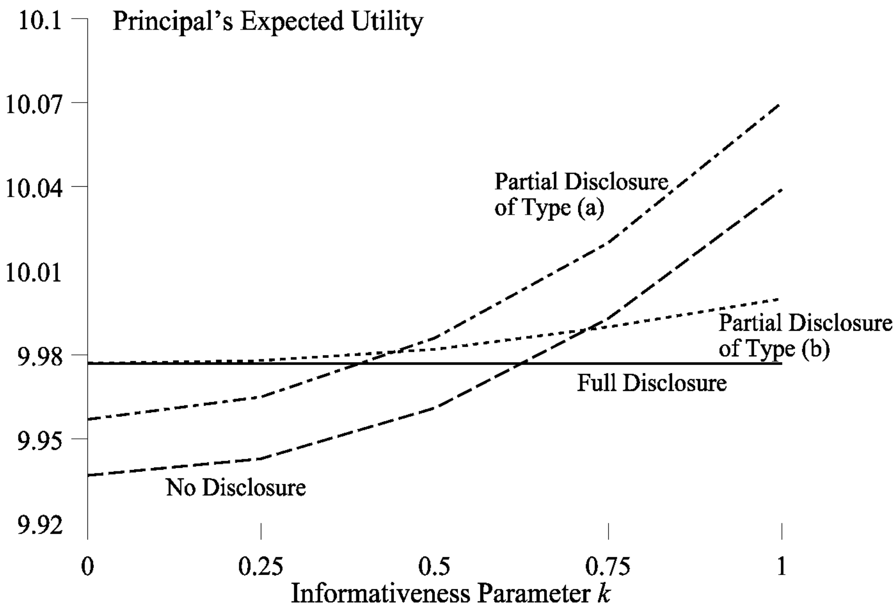

Figure 22.5: Comparison of full, no, and partial disclosure with varying k.

The trade-offs involved can be summarized as follows. No disclosure allows implicit contracting on the investors' signal for all hurdles. So does partial disclosure of type (a), but it also facilitates risk reduction in the non-producing region for investor-signal y_I^H (where the conditional probability for high hurdles is high). Partial disclosure of type (b) only allows implicit contracting on the investors' signal in the producing regions, but it facilitates risk reduction for the agent in the "non-producing region" for both investor-signals. Full disclosure also facilitates risk reduction in the non-producing regions, but does not allow implicit contracting on the investors' signal for any hurdles.

22.9 CONCLUDING REMARKS

The models in this chapter assume that the principal and agent sign a contract before the agent receives private information. The terms of the contract are conditional on what the agent reveals about his information after he receives it. We assume throughout the analysis that the principal wishes to continue employing the agent no matter what he observes and the agent is committed to stay no matter what he observes. The latter is a non-trivial assumption since it will be the case that the agent's expected utility conditional on the signal received will be less than his reservation utility for some signals and greater for others, so that the agent's *ex ante* expected utility equals his reservation utility. The models in this chapter could be easily modified to consider settings in which it is optimal for the initial contract to specify termination of employment given the report of some "bad" signals by the agent. On the other hand, if the agent cannot commit to stay, then the analysis should be based on the pre-contract, pre-decision models considered in Chapter 23.

The analysis in this chapter is based on single-period models. Chapters 25 through 28 consider multi-period models. The information reported at the end of a period naturally becomes pre-decision information with respect to the next period, particularly if the periods are stochastically interdependent. However, our analysis of multi-period agency relations is largely restricted to public information. There is definitely scope for more analysis of multi-period models with private agent information. See the analysis in Chapter 28 for a discussion of some of the problems that occur in analyzing these types of models.

Chapters 13, 14, and 15 consider disclosure of management information in settings in which the manager is either an entrepreneur seeking to sell shares in his firm or a manager with exogenously specified preferences. The models in those earlier chapters generally assume that truthful reporting is exogenously induced, and that the agent often withholds information. In fact, Chapter 14 specifically explores the impact of various incentives on the existence of and nature of partial disclosure. The contracts are not endogenous and the Revelation Principle is not applied. Section 22.8 considers the impact of management disclosure on market prices in a setting in which management disclosure incentives are endogenously determined and the market price is used in incentive contracting. There is scope for more research that considers management disclosure in capital and product markets with endogenously determined management disclosure incentives. The introduction of frictions that preclude application of the Revelation Principle is likely to be of particular interest.

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CHAPTER 23

PRE-CONTRACT INFORMATION – UNINFORMED PRINCIPAL MOVES FIRST

The analysis in the preceding chapters has already demonstrated that differences in the timing and contractibility of information can have a significant impact on the optimal contract between a principal and his agent. The preceding chapter assumed that the agent obtained private information after he had signed a contract with the principal and the agent could not break that contract after he observed his private signal. We now consider the case in which the principal contracts with an agent who has already received private information. The principal is fully aware (i.e., it is *common knowledge*) that the agent has private pre-contract information, but the principal does not know which signal the agent has received.

In this setting we assume the principal offers a contract (or a menu of contracts) to the agent. That is, this is a game in which the uninformed player moves first and *commits* to a contract. This permits us to invoke the Revelation Principle, as we did in the previous chapter (which considered post-contract/pre-decision information). In some settings, such as an initial public offering (IPO), the informed player moves first, i.e., the agent offers a contract to the principal. This is a radically different game – the Revelation Principle does not (necessarily) apply here and it is frequently referred to as a signaling game. We examined signaling games in Chapter 13.

23.1 BASIC MODEL

Two basic models are considered. In the first, a single contract is offered, i.e., there is no communication of the agent's private information. In the second, a menu of contracts is offered and the agent's choice from that menu reveals his private information.

The notation is the same as in the setting with post-contract/pre-decision information considered in Chapter 22. However, the timeline is different, with the key difference being that the agent observes his private information before accepting a contract offered by the principal. We depict contract acceptance and communication of message m as two distinct steps in the process so that it encompasses both no communication ($M = \emptyset$) and communication in the form of selecting one of the elements from the menu of contracts.

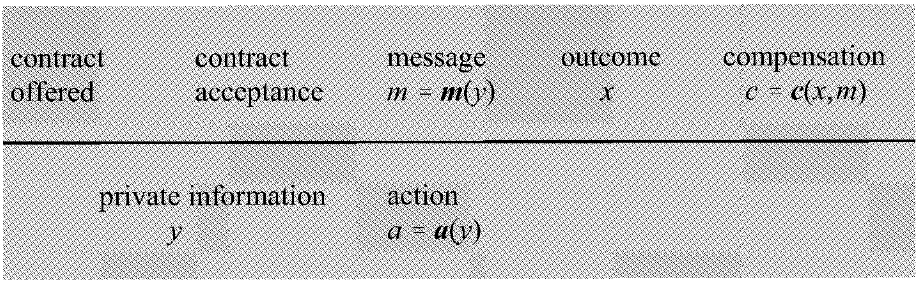

Figure 23.1: Timeline for incentive problem with pre-contract information.

In the formulation of the communication program we directly appeal to the Revelation Principle, which has the same formulation and proof as in Chapter 22.

Proposition 23.1 *The Revelation Principle*

For any optimal contract $\boldsymbol{z} = (\boldsymbol{c}, \boldsymbol{a}, \boldsymbol{m})$ based on communication by the agent, there is an equivalent contract $\boldsymbol{z}'$ that (weakly) induces full and truthful disclosure of the agent's private information, i.e., $\boldsymbol{m}'(y) = y$ for all $y \in Y$.

The programs defining the Pareto optimal contracts with and without agent communication can be formulated as follows.

Principal's Decision Problem without Agent Communication:

$$\underset{\boldsymbol{c},\boldsymbol{a}}{\text{maximize}} \quad U^p(\boldsymbol{c},\boldsymbol{a},\eta) = \int_Y \int_X [x - \boldsymbol{c}(x)]\, d\Phi(x|y,\boldsymbol{a}(y))\, d\Phi(y), \tag{23.1}$$

$$\text{subject to} \quad U^a(\boldsymbol{c},\boldsymbol{a}(y)|y,\eta) = \int_X u(\boldsymbol{c}(x))\, d\Phi(x|y,\boldsymbol{a}(y)) - v(\boldsymbol{a}(y)) \geq \bar{U}, \quad \forall\, y \in Y, \tag{23.2}$$

$$U^a(\boldsymbol{c},\boldsymbol{a}(y)|y,\eta) \geq U^a(\boldsymbol{c},a|y,\eta), \quad \forall\, a \in A,\ y \in Y. \tag{23.3}$$

Principal's Decision Problem with Truthful Agent Communication:

$$\underset{c,a}{\text{maximize}} \quad U^p(\boldsymbol{c},\boldsymbol{a},\eta) = \int_Y \int_X [x - \boldsymbol{c}(x,y)]\, d\Phi(x|y,\boldsymbol{a}(y))\, d\Phi(y), \tag{23.1'}$$

$$\text{subject to} \quad U^a(\boldsymbol{c}(y),\boldsymbol{a}(y)|y,\eta) = \int_X u(\boldsymbol{c}(x,y))\, d\Phi(x|y,\boldsymbol{a}(y)) - v(\boldsymbol{a}(y)) \geq \bar{U},$$

$$\forall\ y \in Y, \tag{23.2'}$$

$$U^a(\boldsymbol{c}(y),\boldsymbol{a}(y)|y,\eta) \geq U^a(\boldsymbol{c}(m),a|y,\eta), \quad \forall\ a \in A,\ m,y \in Y. \tag{23.3'}$$

The key difference between these programs and the corresponding programs with post-contract information in Chapter 22 is that now there is a *participation constraint for each private signal y* since the agent has the option to reject the contract (or menu of contracts) after observing y. In some settings, the principal may prefer to have the agent reject the menu of contracts for some signals $y \in Y$, e.g., the news is sufficiently bad that the benefit to the principal of hiring the agent is less than the cost. While we could generalize the model to permit that possibility (with or without communication), we adopt the simplifying assumption that, in the settings considered, the principal prefers to offer a menu of contracts such that, for each signal y, at least one of the offered contracts is acceptable to the agent. We also assume here that the agent's reservation utility is independent of his private information. While this may not be particularly realistic, it is a common assumption in the literature since it is difficult to specify a set of signal-contingent reservation utility levels that would be consistent with some more general equilibrium model that considered the demand and supply of managers with private information.

In principal-agent models with no pre-contract information, the agent accepts a contract if it provides him with an *ex ante* expected utility equal to or greater than his reservation utility $\bar{U}$, and equality will hold if the principal has all the *bargaining power*.[1] However, the situation changes dramatically when the agent has pre-contract information. As we demonstrate, private pre-contract information enables the agent to collect "*information rents*." In particular, while an agent with "bad news" may receive only his reservation utility, an agent with "good news" will typically receive more than his reservation utility.

[1] Characterization of the resulting contracts is relatively insensitive to who has the bargaining power if there is no pre-contract information. However, the analysis changes considerably if there is pre-contract information. In this chapter we consider settings in which the principal has the bargaining power and offers take-it-or-leave-it contracts to the agent. In Chapter 13 we consider signaling games in which the informed agent moves first and offers a contract (which varies with his information) to competing principals (i.e., investors).

Another interesting feature of this setting is that *incentive problems do not disappear if the agent is risk neutral*. With no private pre-contract information and agent risk neutrality (and no limited liability problems), the principal can sell (rent) the firm to the agent for a price equal to the first-best expected return to the principal. Now, however, the "first-best" price will vary with the agent's private information and the principal does not know that information. Consequently, many papers in the "communication" literature consider pre-contract information and assume the agent is risk neutral. They focus on the "information rents" rather than on the risk premiums that must be paid to risk averse agents who accept risky incentive contracts.

Assuming the agent is risk neutral often facilitates closed form solutions of the principal's decision problem. However, communication of pre-contract information may also be useful for reducing the risk imposed on risk averse agents and for improving action choices, as is the case with post-contract information. This latter point will be illustrated using the hurdle model in a pre-contract information setting.

The focus of Sections 23.2 and 23.3 is on the value of communicating pre-contract information. As in the post-contract information setting, the following result is straightforward when the principal can commit to how he will use the agent's message to determine the agent's compensation. The key is that the principal always has the option to commit to ignore the message.

Proposition 23.2

The principal is never worse off with agent communication than with no communication.

In Section 23.4 we review models in which the agent's private information pertains to the personal cost of providing a given outcome (mechanism design).

23.2 PERFECT PRIVATE INFORMATION

Communication involves selecting from among a menu of contracts *before* observing the contractible performance measures influenced by his action choice. If the agent's private pre-contract information is imperfect about variations in the performance measures that affect his compensation, then he is uncertain about the compensation that will result from his action and menu choices. However, if he has perfect information about the performance measures that will result from his action choice, then he knows the compensation that will result from his action when he selects from the menu of contracts. That makes *ex ante* selection from the menu unnecessary. Hence, as in the post-contract information setting, there is no value to communication when the agent gets perfect information about the contractible performance measures that will result from his action choice.

Proposition 23.3

If the agent receives perfect information about the contractible performance measures that will result from his action choice, then there is no value to communication.

Proof: Let $z = (\boldsymbol{c}, \boldsymbol{a})$ denote the solution to the principal's problem with communication and assume, for simplicity, that the outcome x is the only contractible performance measure. Observe that to be incentive compatible, this contract must be such that $\boldsymbol{c}(x,y') = \boldsymbol{c}(x,y'')$ if $x = \boldsymbol{x}(\boldsymbol{a}(y'),y') = \boldsymbol{x}(\boldsymbol{a}(y''),y'')$. That is, any two signals that induce the same outcome must pay the same compensation for that outcome. Otherwise, if, for example, $\boldsymbol{c}(x,y') > \boldsymbol{c}(x,y'')$, then the agent will be better off if he reports $m = y'$ when he has observed y''.

Given the above characteristics of z, we can construct the following contract based on no communication: $\boldsymbol{c}^o(x) = \boldsymbol{c}(x,y)$ for any y such that $\boldsymbol{x}(\boldsymbol{a}(y),y) = x$. Given that $\boldsymbol{c}$ induced the implementation of $\boldsymbol{a}$ (as well as truthful reporting), it follows that $\boldsymbol{c}^o$ will induce the implementation of $\boldsymbol{a}$ without any communication. Moreover, since $\boldsymbol{c}(y)$ and $\boldsymbol{a}(y)$ satisfy the participation constraint for each y, so will $\boldsymbol{c}^o$ and $\boldsymbol{a}$. **Q.E.D.**

Contractible Agent Productivity

The key aspect of Proposition 23.3 is that the agent has perfect information about the relation between his action/message choices and the compensation he gets, and not that he has perfect information about the outcome (although the latter implies the former). Melumad and Reichelstein (MR) (1989) consider a setting (with a risk neutral agent) in which the agent receives perfect information about his compensation although he has only imperfect information about the outcome. MR introduce a "productivity" measure, which we denote by γ, which is a function of the agent's action a and his private information y. More specifically, they assume

$$d\Phi(x|y,a) = d\Phi(x|\gamma(y,a)), \quad \forall\, x \in X,$$

i.e., the productivity measure γ is a sufficient statistic for (x,γ) with respect to (y,a). In other words, given γ, the conditional distribution for x does not depend on either y or a. If the productivity measure is *directly contractible*, the compensation contract is a function of γ instead of x (whether there is communication or not), and there is no value to communication. In developing this point more formally, let $\Gamma(\boldsymbol{a}) = \{\, \gamma \mid \gamma = \gamma(y,\boldsymbol{a}(y))$ for some $y \in Y \,\}$, i.e., the set of productivity measures that might result from action plan $\boldsymbol{a}$.

Proposition 23.4

Suppose γ is directly contractible, and let $\mathbf{z} = (\mathbf{c},\mathbf{a})$ be an optimal contract in which the compensation may depend on x (in addition to γ and y if there is communication).

(a) There is an equivalent optimal contract $\mathbf{z}' = (\mathbf{c}',\mathbf{a}')$ with $\mathbf{a}' = \mathbf{a}$ and $\mathbf{c}'$ defined as the agent's certainty equivalent of $\mathbf{c}$ given γ and y, i.e.,

$$u(\mathbf{c}'(\gamma,y)) = \int_X u(\mathbf{c}(x,\gamma,y))\, d\Phi(x|\gamma), \quad \forall\, y \in \mathrm{Y}, \gamma \in \Gamma(\mathbf{a}).$$

(b) There is no value to communication.

Proof: (a): We only present the proof for an optimal communication contract, since the proof for an optimal no communication contract can be performed as a special case. Clearly, $\mathbf{z}'$ gives the agent the same expected utility as does $\mathbf{z}$ given the same action choices and truthful reporting of y. If the agent is strictly risk averse and $\mathbf{c}$ were to depend non-trivially on x, then the expected compensation costs to the principal are lower for $\mathbf{z}'$ than $\mathbf{z}$ due to Jensen's inequality. Incentive compatibility of $\mathbf{z}'$ can be seen as follows,

$$U^a(\mathbf{c}'(y), \mathbf{a}(y)\,|\,y,\eta)$$

$$= u(\mathbf{c}'(\gamma(y,\mathbf{a}(y)),y)) - v(\mathbf{a}(y))$$

$$= \int_X u(\mathbf{c}(x,\gamma(y,\mathbf{a}(y)),y))\, d\Phi(x\,|\,\gamma(y,\mathbf{a}(y))) - v(\mathbf{a}(y))$$

$$\geq \int_X u(\mathbf{c}(x,\gamma(y,a),m))\, d\Phi(x\,|\,\gamma(y,a)) - v(a) \quad \forall\, a \in A,\ m \in Y$$

$$= u(\mathbf{c}'(\gamma(y,a),m)) - v(a) \quad \forall\, a \in A,\ m \in Y$$

$$= U^a(\mathbf{c}'(m), a\,|\,y,\eta) \quad \forall\, a \in A,\ m \in Y,$$

where the equalities come from the definition of the contract and the inequality comes from incentive compatibility of $\mathbf{z}$. The contract $\mathbf{z}$ might be such that large penalties are used to preclude some messages given γ. In that case, the choice of (a,m) must be consistent with γ given y, i.e., actions and reports that avoid the penalty are such that $\gamma(y,a) = \gamma(m,\mathbf{a}(m))$.

(b): Let z' be given as in (a). Observe that to be incentive compatible, this contract must be such that $\mathbf{c}'(\gamma,y') = \mathbf{c}'(\gamma,y'')$ if $\gamma = \boldsymbol{\gamma}(y',\mathbf{a}(y')) = \boldsymbol{\gamma}(y'',\mathbf{a}(y''))$. That is, any two signals that induce the same productivity measure must pay the same compensation for that productivity. Otherwise if, for example, $\mathbf{c}'(\gamma,y') > \mathbf{c}'(\gamma,y'')$, then the agent will be better off if he reports $m = y'$ when he has observed y''.

Given the above characteristics of z', we can construct the following contract z^o based on no communication: $\mathbf{c}^o(\gamma) = \mathbf{c}'(\gamma,y)$ for any y such that $\boldsymbol{\gamma}(y,\mathbf{a}(y)) = x$, and $\mathbf{a}^o = \mathbf{a}'$. Feasibility of z^o follows easily. **Q.E.D.**

Given a report of the agent's productivity, the outcome x provides no additional information about either the agent's action or his private information. Hence, if the productivity γ is contractible information, the outcome x is useless contractible information. In fact, contracting on x would only impose additional risk on the agent (which is harmful when he is risk averse). When the compensation does not depend on x, communication is not useful because the agent knows with certainty what his performance measure will be when he selects his action. Hence, when the agent's productivity is directly contractible, an optimal compensation contract can be written as $\mathbf{c}(\gamma)$ whether there is communication or not.

To illustrate the above, we return to the hurdle model (see Section 22.2), but now assume the agent privately observes the hurdle, $y = h$, before he contracts with the principal.[2] The productivity parameter γ represents whether the agent clears the hurdle or not, i.e., $\gamma \in \{0,1\}$, where $\gamma = \boldsymbol{\gamma}(h,a) = 1$ if $a \geq h$, and 0 otherwise. If the agent clears the hurdle, i.e., $\gamma = 1$, then the high probability of the good outcome is obtained. If γ is contractible information, then the optimal compensation contract is "jump"-contingent, with $\mathbf{c}(\gamma = 1) > \mathbf{c}(\gamma = 0)$, and the agent will choose to "jump" if $h \in [0, \hat{h}^\gamma]$, where the cut-off $\hat{h}^\gamma$ is such that

$$v(\hat{h}^\gamma) = u(\mathbf{c}(\gamma = 1)) - u(\mathbf{c}(\gamma = 0)).$$

Observe that the contract cannot be improved by having the agent communicate his private information. To see this, assume to the contrary that there exists an optimal contract $\mathbf{c}(\gamma,m)$ that varies with m and induces $m = y$. If the agent

[2] For another example see MR and Kirby, Reichelstein, Sen, and Paik (KRSP) (1991). They analyze a setting with risk neutral agents and $\boldsymbol{\gamma}(y,a) = y + a$. If there is communication, the agent is penalized (i.e., receives the minimum possible compensation) if $\gamma = y + a < m + \mathbf{a}(m)$. Hence, for any given y, the agent effectively chooses the realized value of γ that will maximize $\mathbf{c}(\gamma) - v(\gamma - y)$, where $a = \gamma - y$ is the action level required to attain γ given y.

KRSP focus on the minimization of production costs (instead of maximizing the output) and interpret the action as the amount of budgetary "slack," and consider a personal benefit (instead of a personal cost) to the agent of increasing the slack (instead of the effort).

selects a given y, then this will result in $\gamma = \gamma(y,a)$ and the agent will choose the message m that maximizes $c(\gamma,m)$, irrespective of the y he observed. Hence, we have a contradiction to the assumed truthtelling unless $c(\gamma,m)$ is independent of m.

The direct contractibility of whether the agent clears the hurdle eliminates the risk imposed on the agent when he jumps (as well as when he does not jump). If it is profitable for the principal to hire the agent whether he jumps or not, the compensation for not jumping must be such that every non-jumping agent (i.e., every $h \in (\hat{h}^\gamma, 1])$ gets his reservation utility, i.e.,

$$u(c(\gamma = 0)) = \bar{U}.$$

The agent with a hurdle at the cut-off also just gets his reservation utility for jumping the hurdle, i.e.,

$$u(c(\gamma = 1)) - v(\hat{h}^\gamma) = \bar{U},$$

while all agents with hurdles strictly below the cut-off collect "information rents", i.e.,

$$u(c(\gamma = 1)) - v(h) - \bar{U} = v(\hat{h}^\gamma) - v(h) \geq 0, \quad \forall\, h \in [0, \hat{h}^\gamma].$$

That is, the agents with the lowest hurdles collect the highest information rents, and the magnitude of the rents depends on the highest hurdle the principal would want an agent to jump.[3]

When the agent is risk averse, an optimal contract is independent of the uncertain outcome x given the contractible productivity parameter γ. However, independence of x is unnecessary if the agent is risk neutral. In fact, in risk neutral agent settings there is a multiplicity of optimal compensation schemes that depend on γ and x. The key requirement is that if $c(\gamma)$ is an optimal contract, then $c'(x,\gamma)$ is also an optimal contract if

$$(1 - \varepsilon)\, c'(x_g, \gamma = 1) + \varepsilon\, c'(x_b, \gamma = 1) = c(\gamma = 1),$$

$$\varepsilon\, c'(x_g, \gamma = 0) + (1 - \varepsilon)\, c'(x_b, \gamma = 0) = c(\gamma = 0).$$

[3] The structure of the optimal contract is similar if the principal prefers not to hire the agent if he does not clear the hurdle. In that case, $c(\gamma = 0) = 0$ and $c(\gamma = 1)$ is the same as above. This induces the agent to reject the contract if $h \in (\hat{h}^\gamma, 1]$, but accept it if $h \in [0, \hat{h}^\gamma]$.

It then follows directly that instead of contracting on γ, the same result can be achieved by contracting on the agent's report of h and the outcome x. This is achieved with any contract $\boldsymbol{c}''(x,h)$ such that [4]

$$(1-\varepsilon)\,\boldsymbol{c}''(x_g,h) + \varepsilon\,\boldsymbol{c}''(x_b,h) = \boldsymbol{c}(\gamma{=}1), \quad \forall\, h \le \hat{h}^{\gamma},$$

$$\varepsilon\,\boldsymbol{c}''(x_g,h) + (1-\varepsilon)\,\boldsymbol{c}''(x_b,h) = \boldsymbol{c}(\gamma{=}0), \quad \forall\, h > \hat{h}^{\gamma}.$$

In the next section we return to this setting and demonstrate that communication is unnecessary and contracting on x yields the same results as contracting on γ if the agent is risk neutral.

23.3 IMPERFECT PRIVATE INFORMATION

The preceding section has established that there is no value to communication if the agent has perfect information about his compensation when he selects his action and message. Two examples were provided: one in which the agent has perfect information about the outcome and one in which his productivity is directly contractible. When the agent has only imperfect information about his compensation, there is more scope for communication to be valuable as in the post-contract information setting in Chapter 22. However, if agents are risk neutral, contracting on the outcome might achieve the same solution as if the agent's productivity is directly contractible.

Proposition 23.5 (MR, Prop. 1)

Let $\boldsymbol{z} = (\boldsymbol{c},\boldsymbol{a})$ denote the optimal contract given that γ is directly contractible, i.e., $c = \boldsymbol{c}(\gamma)$. If both the *principal and the agent are risk neutral*, and there exists a compensation contract only dependent on the outcome, i.e., $c = \boldsymbol{c}'(x)$, that satisfies the *spanning condition*,

$$\boldsymbol{c}(\gamma) = \int_X \boldsymbol{c}'(x)\, d\Phi(x|\gamma), \quad \forall \gamma \in \Gamma(\boldsymbol{a}),$$

then

(a) the contract $\boldsymbol{z}' = (\boldsymbol{c}',\boldsymbol{a}')$ with $\boldsymbol{a}' = \boldsymbol{a}$ is an equivalent contract to $\boldsymbol{z}$, and

(b) communication has no value.

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[4] KRSP use this type of multiplicity of contracts with risk neutral agents to show that in their model there exists a menu of linear contracts that implements an optimal non-linear compensation contract only dependent on γ.

Proof: (a): Clearly, z' gives both the principal and the agent the same expected utilities (i.e., expected payments) as does z. Incentive compatibility of z' can be seen as follows:

$$\begin{aligned}
U^a(\mathbf{c}', \mathbf{a}(y)|y,\eta) &= \int_X \mathbf{c}'(x)\, d\Phi(x|\gamma(y,\mathbf{a}(y))) - v(\mathbf{a}(y)) \\
&= \mathbf{c}(\gamma(y,\mathbf{a}(y))) - v(\mathbf{a}(y)) \\
&\geq \mathbf{c}(\gamma(y,a)) - v(a) \quad \forall\, a \in A \\
&= \int_X \mathbf{c}'(x)\, d\Phi(x|\gamma(y,a)) - v(a) \quad \forall\, a \in A \\
&= U^a(\mathbf{c}', a|y,\eta) \quad \forall\, a \in A,
\end{aligned}$$

where the equalities come from the definition of z' and the inequality comes from incentive compatibility of z.

(b): Clearly, a communication contract in which y is directly contractible is weakly preferred to a communication contract in which only the outcome (and the message) are contractible. Since communication has no value when y is directly contractible and a no communication contract based on x implements it, communication has no value when only the outcome is contractible.

Q.E.D.

The spanning condition can be satisfied if the family of distributions $\{d\Phi(x|\gamma), \gamma \in \Gamma(\mathbf{a})\}$ is sufficiently rich. If X and $\Gamma(\mathbf{a})$ are finite sets, with $|X| = N$ and $|\Gamma(\mathbf{a})| = M$, the following result is immediate.

Lemma

If X and $\Gamma(\mathbf{a})$ are finite, then the probability function $\varphi(x_i|\gamma_j)$ admits *spanning* if the matrix

$$\boldsymbol{\varphi} = [\ \varphi(x_i|\gamma_j)\ ]_{N \times M}$$

has rank M.

MR point out that spanning is a more complicated condition when X and Γ are continuous sets. They introduce the concepts of *approximate spanning* and *communication has no distinct value*.[5]

The key aspect of the spanning condition is that it facilitates an outcome-contingent contract that implements the optimal solution with a contractible productivity measure. The outcome-contingent contract generally imposes more risk on the agent, but since the agent is risk neutral this is not costly to the principal (as it would be if the agent were risk averse).

A Hurdle Model Example with Spanning and a Risk Neutral Agent

The hurdle model with $y = h$ provides an example of an agency problem in which there is spanning. In this case $X = \{x_g, x_b\}$, $\Gamma(\mathbf{a}) = \{0, 1\}$, and $\boldsymbol{\varphi}$ is given by

$$\boldsymbol{\varphi} = \begin{Bmatrix} 1-\varepsilon & \varepsilon \\ \varepsilon & 1-\varepsilon \end{Bmatrix}.$$

With a risk neutral agent, the outcome-contingent compensation $c_g > c_b$ satisfying the equations,

$$\begin{Bmatrix} 1-\varepsilon & \varepsilon \\ \varepsilon & 1-\varepsilon \end{Bmatrix} \begin{Bmatrix} c_g \\ c_b \end{Bmatrix} = \begin{Bmatrix} \mathbf{c}(\gamma = 1) \\ \mathbf{c}(\gamma = 0) \end{Bmatrix},$$

implements the solution of the optimal contract with a contractible productivity measure. Moreover, there is no value to communication. Note that the outcome-contingent contract imposes risk on the agent both when he jumps and when he does not jump.

A Hurdle Model Example with Spanning and a Risk Averse Agent

If the agent is risk averse, there is scope for communication to be valuable. The probability function $\varphi(x|\gamma)$ still admits *spanning* in terms of certainty equivalents, i.e., there exist outcome-contingent compensations $c_g > c_b$ such that

$$\begin{Bmatrix} 1-\varepsilon & \varepsilon \\ \varepsilon & 1-\varepsilon \end{Bmatrix} \begin{Bmatrix} u(c_g) \\ u(c_b) \end{Bmatrix} = \begin{Bmatrix} u(\mathbf{c}(\gamma = 1)) \\ u(\mathbf{c}(\gamma = 0)) \end{Bmatrix}.$$

This outcome-contingent compensation scheme implements the optimal action strategy for the contract with γ observable and satisfies the participation constraints. However, by Jensen's inequality, it does so at a higher expected com-

[5] See also Amershi, Datar, and Hughes (1989) for an extension of this analysis.

pensation cost to the principal since there is outcome risk for both jumping and non-jumping agents. With communication, the risk for non-jumping agents can be eliminated. That is, the optimal communication contract is similar to the optimal communication contract in the post-contract information setting, i.e., the compensation is outcome-contingent, with $c_g^c > c_b^c$, if the agent reports a hurdle below a cut-off $\hat{h}^c$ and is a fixed wage c_o^c if he reports a hurdle above the cut-off. As in the post-contract information setting the risk imposed on jumping agents is just sufficient to make the agent with $h = \hat{h}^c$ jump, i.e.,

$$(1 - \varepsilon)u(c_g^c) + \varepsilon u(c_b^c) - v(\hat{h}^c) = \varepsilon u(c_g^c) + (1 - \varepsilon)u(c_b^c).$$

The fixed wage for a non-jumping agent is such that he gets his reservation utility, i.e., $u(c_o^c) = \bar{U}$, and the level of the outcome-contingent compensation is such that an agent with $h = \hat{h}^c$ is indifferent between reporting his hurdle truthfully and reporting a hurdle above the cut-off, i.e.,

$$(1 - \varepsilon)u(c_g^c) + \varepsilon u(c_b^c) - v(\hat{h}^c) = u(c_o^c).$$

Of course, as in the case with a contractible productivity measure, agents with hurdles below the cut-off obtain information rents, which are determined by the highest hurdle the principal would want an agent to jump. However, in this case the information rents in terms of expected compensation cost to the principal are higher, *ceteris paribus*, due to the risk imposed on jumping agents.

To illustrate the above, we examine an example using the following data:

$$u(c) = c^{1/2}; \; v(a) = a/(1 - a); \; \bar{U} = 2; \; x_g = 20, x_b = 10; \; \varepsilon = 0.15.$$

The optimal contracts with and without communication as well as the optimal contract with a contractible productivity measure are shown in Table 23.1. Note that the highest hurdle the principal would want an agent to jump increases as we go from no communication to communication and from communication to a contractible γ.

	$U^p(c,a,\eta)$				$\hat{h}$
η^n	9.020	$c_g^n = 6.938$		$c_b^n = 3.565$	0.343
η^c	9.068	$c_g^c = 7.058$	$c_b^c = 3.550$	$c_o^c = 4.000$	0.351
η^γ	9.096	$c(\gamma = 1) = 6.546$		$c(\gamma = 0) = 4.000$	0.358

Table 23.1: Optimal contracts with and without communication, and contractible γ for a risk averse agent.

A Hurdle Model Example with a Risk Neutral Agent, but No Spanning
The hurdle model can also be used to illustrate the potential value of communication with risk neutral agents when the spanning condition is not satisfied. Suppose the agent not only knows the hurdle before contract acceptance but also has private information about the distribution of outcomes given that he jumps, i.e., $y = (h, \varepsilon)$ where $\varepsilon \in \{\varepsilon_H, \varepsilon_L\}$ with $\varepsilon_H > \varepsilon_L$ and prior distribution $\varphi(\varepsilon_H) = \varphi(\varepsilon_L)$. In this case the productivity measure can take four distinct values, i.e., $\Gamma(\boldsymbol{a}) = \{\gamma_{1H}, \gamma_{1L}, \gamma_{0H}, \gamma_{0L}\}$. The spanning condition is clearly not satisfied, since there are only two outcomes and, thus, the optimal solution with contractible γ cannot be implemented by an outcome-contingent contract.

If γ is not contractible, the *optimal no communication contract* with outcome-contingent compensation $c_g^n > c_b^n$ induces cut-offs $\hat{h}_H^n$ and $\hat{h}_L^n$ for ε_H and ε_L, respectively. If the principal wants to contract with all types of agents, then both types of non-jumping agents must obtain their reservation value, i.e.,

$$\varepsilon_i c_g^n + (1 - \varepsilon_i) c_b^n \geq \bar{U}, \qquad i = H, L.$$

Since the expected utility of a non-jumping agent (i.e., the left-hand side) is increasing in ε_i, a non-jumping agent with $\varepsilon = \varepsilon_H$ gets strictly positive information rents, whereas a non-jumping agent with $\varepsilon = \varepsilon_L$ gets no information rents.[6] The cut-off $\hat{h}_i^n$, $i = H, L$, is characterized by

$$v(\hat{h}_i^n) = (1 - 2\varepsilon_i)(c_g^n - c_b^n).$$

Since $v(\cdot)$ is an increasing function and the right-hand side is decreasing in ε_i, the cut-off is higher for ε_L than for ε_H. That is, the more productive the agent's effort is (i.e., the lower ε), the higher the hurdles that are cleared.

Using the following data

$$v(a) = a/(1 - a); \;\; \bar{U} = 2; x_g = 20, x_b = 10; \varepsilon_H = 0.30, \varepsilon_L = 0.05,$$

the optimal contract is as shown in Table 23.2 and the agent's information rents are shown in Figure 23.2.

---

[6] If the principal does not want to contract with an agent who will not clear the hurdle, then

$$\varepsilon_i c_g^n + (1 - \varepsilon_i) c_b^n \leq \bar{U}, \quad i = L, H,$$

and the cut-offs are characterized by

$$v(\hat{h}_i^n) = (1 - \varepsilon_i) c_g^n + \varepsilon_i c_b^n - \bar{U}, \quad i = L, H.$$

$U^P(\mathbf{c},\mathbf{a},\eta^n)$	c_g^n	c_b^n	$\hat{h}_H^n$	$\hat{h}_L^n$
12.538	4.123	1.888	0.472	0.668

Table 23.2: Optimal contract for no communication and ε-information, η^n.

Note that the information rents in this case are not only determined by the maximum hurdle the principal would want an ε-type agent to jump, but also by the information rents to a non-jumping agent of that ε-type.

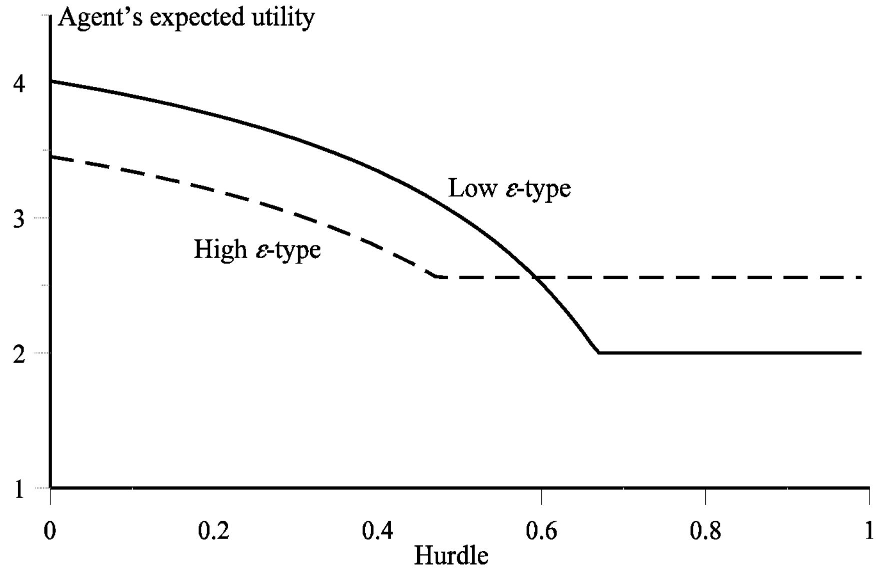

Figure 23.2: Information rents with no communication and ε-information.

The *optimal communication contract* takes a very simple form in which the agent effectively reports only whether he is going to jump or not – other details of his report of $y = (h, \varepsilon)$ are irrelevant. The key in determining this contract is to recognize that paying an agent for reporting that he is not going to jump a fixed wage c_o^c equal to his reservation utility, i.e., $c_o^c = \underline{U}$, eliminates the information rents for both ε-types of non-jumping agents. The action strategy is, as in the no communication contract, characterized by separate cut-offs for the two ε signals, $\hat{h}_i^c$, $i = H, L$. Agents reporting that they are going to jump, i.e., $m = (h, \varepsilon_i)$ with $h \leq \hat{h}_i^c$, are paid outcome-contingent compensation $c_g^c > c_b^c$. The truthtelling constraints in the jumping regions imply that this outcome-contingent compensation cannot depend on the reported ε. Hence, the cut-offs are determined by[7]

$$v(\hat{h}_i^c) = (1 - \varepsilon_i)c_g^c + \varepsilon_i c_b^c - c_o^c, \quad i = L, H,$$

and the compensation satisfies

$$\varepsilon_i c_g^c + (1 - \varepsilon_i)c_b^c \leq c_o^c, \quad i = L, H.$$

Again, it is readily verified that the cut-off is higher for the ε_L-type than for the ε_H-type agent.

Using the same data as for the no communication contract, the optimal communication contract is shown in Table 23.3, and the information rents for the two ε signals are shown in Figure 23.3.

$U^P(\mathbf{c},\mathbf{a},\eta^c)$	c_g^c	c_b^c	c_o^c	$\hat{h}_H^c$	$\hat{h}_L^c$
12.852	4.348	0.643	2.000	0.553	0.684

Table 23.3: Optimal contract for communication and ε-information, η^c.

Note that both types of non-jumping agents get zero information rents, and the cut-offs for both types of agents are higher with communication than without communication. The jumping ε_L-types of agents get higher information rents with communication than without communication due to the fact that the maximum hurdle the principal wants ε_L-types to jump has increased and the non-jumping agents of this type get no information rents both with and without communication.

Proposition 23.6

Consider a hurdle model setting in which a risk neutral agent observes $y = (h, \varepsilon)$ prior to contract acceptance, with $\varepsilon \in \{\varepsilon_H, \varepsilon_L\}$. In this setting, communication is strictly valuable.

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[7] These equations come from the truthtelling constraints for reporting hurdles above and below the cut-off, while the inequalities ensure that the agent prefers to announce he is not going to jump if he is not going to do so.

Observe that while communication has value given that the principal wishes to contract with the agent even if he is not going to clear the hurdle, this does not hold if the principal prefers not to contract with a non-jumping agent. In this latter case, the acceptance (rejection) of the offered contract is equivalent to the agent announcing that he is (not) going to jump.

The proof is reported in Appendix 23A. It demonstrates that there exists a communication contract which induces the same action choices as the optimal no communication contract but at a lower expected compensation cost to the principal, i.e., communication reduces the information rents (for the ε_H-type). However, as the numerical example preceding the proof demonstrates, communication can also be used to improve action choices.

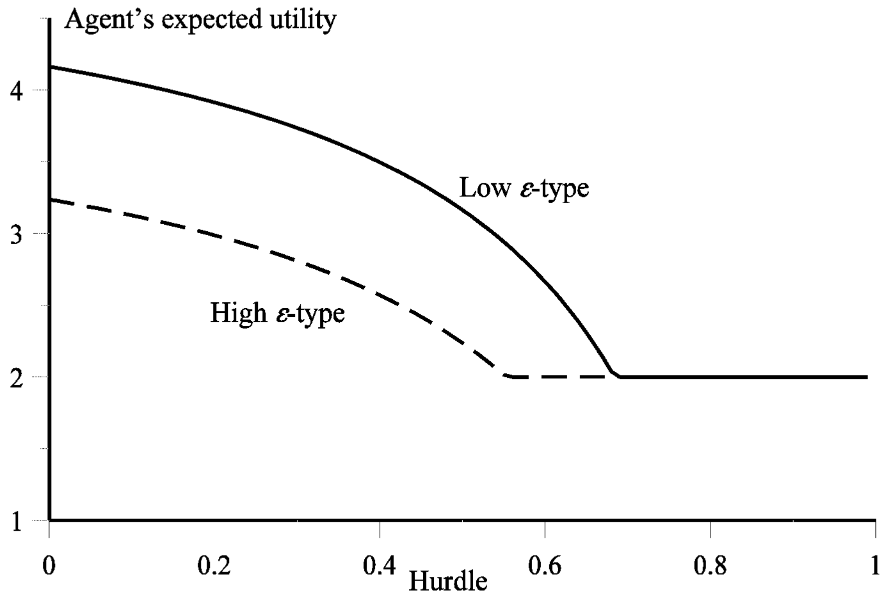

Figure 23.3: Information rents with communication and ε-information.

Examples with a Continuum of Productivity Measures

MR provide another setting in which communication is strictly valuable. In this setting, the productivity measure γ takes on a continuum of values, while there are only two outcomes. As in the hurdle model, the productivity measure shifts the probability of the good outcome x_g, and we let $\varphi(x_g|\gamma) = p(\gamma)$. The productivity measure is assumed to have the simple linear form $\gamma(y,a) = y + a$, so that effort and the private signal are perfect substitutes in terms of their impact on the probability of the good outcome.

Proposition 23.7 (MR, Prop. 4)

Let $X = \{x_g,x_b\}$, $A = [\underline{a},\overline{a}]$, and $Y = [\underline{y},\overline{y}]$. *Communication is valuable* if

(a) $\gamma(y,a) = y + a$;

(b) $p(y)$ is increasing and strictly concave in y;

(c) $v(a)$ is increasing and strictly convex in a; and

(d) $\Phi(y)$ has full support Y.

Given principal and agent risk neutrality, the first-best effort level for each y provides the maximum expected surplus to be shared by the principal and the agent. It is characterized by marginal costs equal marginal benefits, i.e., $v'(\boldsymbol{a}^*(y)) = (x_g - x_b)p'(y + \boldsymbol{a}^*(y))$, if $\boldsymbol{a}^*(y) < \bar{a}$, and $\boldsymbol{a}^*(y)$ is weakly decreasing with y. If y is contractible, the first-best effort level can be induced with a contract of the form $x - k$. Furthermore, the principal can retain all the surplus by setting $k = x_b + (x_g - x_b)p(y + \boldsymbol{a}^*(y)) - v(\boldsymbol{a}^*(y)) - \bar{U}$, which is increasing in y. However, since neither y nor a are contractible, the principal can contract only on x or, if there is communication, on x and $m \in Y$. If he uses $\boldsymbol{c}(x) = x - k$, then action choices will be first-best but satisfying the participation constraint for all $y \in Y$ requires $k = x_b + (x_g - x_b)p(\underline{y} + \boldsymbol{a}^*(\underline{y})) - v(\boldsymbol{a}^*(\underline{y})) - \bar{U}$, and provides the agent with information rents of

$$(x_g - x_b)[p(y + \boldsymbol{a}^*(y)) - p(\underline{y} + \boldsymbol{a}^*(\underline{y}))] + [v(\boldsymbol{a}^*(\underline{y})) - v(\boldsymbol{a}^*(y))].$$

MR consider two cases. In their first case, the optimal no-communication contract $\boldsymbol{c}^n(x)$ induces an action $\boldsymbol{a}^n(y)$ that is less than $\bar{a}$ for at least some y. MR show that this action is also less than the first-best $\boldsymbol{a}^*(y)$, which reduces the total surplus to be shared (relative to using $x - k$), but reduces the information rents sufficiently so as to increase the principal's net surplus. With communication, MR consider offering a menu with both $\boldsymbol{c}^n(x)$ and $\boldsymbol{c}(x) = x - k$. For some k, the agent will select $\boldsymbol{c}^n(x)$ for low values of y and $\boldsymbol{c}(x) = x - k$ for high values of y. The agent's net payoff will be higher for the latter, but the increase in expected compensation is less than the expected gain to the principal from the improved action choices, thus making the menu advantageous to the principal.

In MR's second case, $\boldsymbol{a}^n(y) = \boldsymbol{a}^*(y) = \bar{a}$ for all y. There cannot be an improvement in the induced action choice (or total surplus) in this setting, but MR prove that there exists a communication contract $\boldsymbol{c}^c(x,m)$ that reduces the agent's information rents, thereby increasing the principal's net surplus (see MR).

An upper bound on the value of communication is provided by the difference between the principal's expected payoff when y is publicly reported and the principal's expected payoff when there is no communication. If this is zero, then communication has no value. If this is positive, then communication may be at least as valuable as having y publicly reported, but it cannot be more valuable. MR identify conditions under which this upper bound can be achieved. In the following, let $\boldsymbol{c}^*(y)$ denote the optimal compensation contract for the setting in which y is publicly reported.

Proposition 23.8 (MR, Theorem 2 and Corollary)
Assume

(a) $\Gamma = [\underline{\gamma}, \overline{\gamma}] \subset \mathbb{R}$;

(b) $\boldsymbol{c}^*(\gamma)$ is increasing and convex; and

(c) there exists a bounded and measurable function $\boldsymbol{w}(x)$ such that $\bar{\boldsymbol{w}}(\gamma) \equiv \mathrm{E}[w(x)|\gamma]$ is monotone and concave.

Then the *upper bound on the value of communication can be attained.* Furthermore, it can be attained with a *menu of linear compensation contracts* if $w = x$ satisfies condition (c).

The theorem applies to settings in which the impact of y and a on x can be represented by a *one-dimensional* statistic γ. Condition (c) is directly satisfied with $w = x$ if $\mathrm{E}[x|\gamma] = \gamma$ – KRSP consider such a setting. In this setting, a menu of linear compensation contracts in which the agent reports $m \in \Gamma$,

$$\boldsymbol{c}(x,m) = \boldsymbol{c}^*(m) + \boldsymbol{c}^{*\prime}(m)[x - m],$$

will attain the upper bound on the value of communication. This can be seen as follows. Given γ, the agent's expected compensation when he reports m is

$$\int_X \boldsymbol{c}(x,m)\, d\Phi(x|\gamma) = \boldsymbol{c}^*(m) + \boldsymbol{c}^{*\prime}(m)[\gamma - m].$$

If the agent reports truthfully, i.e., $m = \boldsymbol{\gamma}(y,\boldsymbol{a}(y))$, then his expected compensation given y and $\boldsymbol{a}(y)$ is equal to his optimal compensation with γ directly observable, i.e.,

$$\int_X \boldsymbol{c}(x,\boldsymbol{\gamma}(y,\boldsymbol{a}(y)))\, d\Phi(x|\boldsymbol{\gamma}(y,\boldsymbol{a}(y))) = \boldsymbol{c}^*(\boldsymbol{\gamma}(y,\boldsymbol{a}(y))).$$

Moreover, since $\boldsymbol{c}^*$ is convex, truthtelling is incentive compatible with the menu of contracts $\boldsymbol{c}$ (see Figure 23.4), i.e.,

$$\boldsymbol{c}^*(\boldsymbol{\gamma}(y,\boldsymbol{a}(y))) \geq \boldsymbol{c}^*(m) + \boldsymbol{c}^{*\prime}(m)[\boldsymbol{\gamma}(y,\boldsymbol{a}(y)) - m].$$

Hence, the menu of linear contracts implements the optimal contract with γ directly observable at the same cost to the principal.

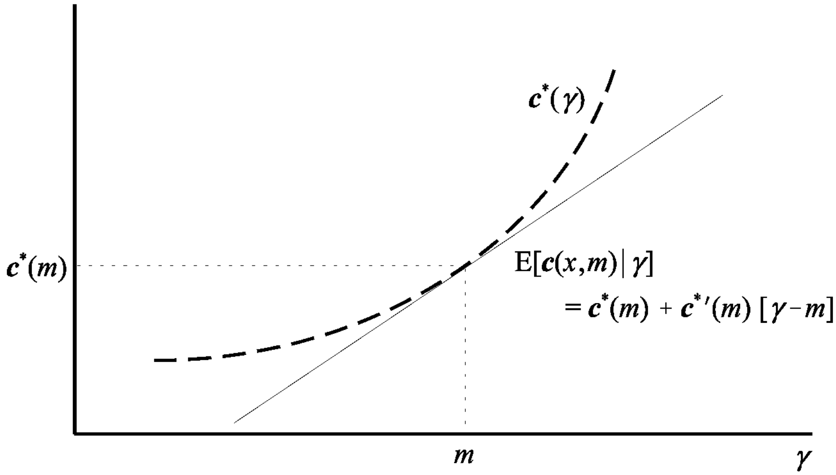

Figure 23.4: Incentive compatibility of menu of linear compensation contracts.

The general case considered in the proposition can be given a similar interpretation by making a transformation of the performance measure x and the productivity measure y. Observe that if there exists a function $\mathbf{w}(x)$ such that the performance measure x can be restated as $w = \mathbf{w}(x)$, and the expected performance measure given y, $\bar{\mathbf{w}}(y) \equiv E[w(x)|y]$, is monotone and concave, then $\bar{\mathbf{w}}(y)$ is effectively a restatement of the productivity measure y. In terms of the transformed productivity measure the optimal contract with y directly observable can be restated as $\bar{c}(\bar{w}) \equiv c^*(\bar{\mathbf{w}}^{-1}(\bar{w}))$ and is convex in $\bar{w}$ since c^* is convex and $\bar{\mathbf{w}}$ is concave. Hence, if the agent reports $m \in \bar{W} = \{\bar{w} \mid \bar{w} = \bar{\mathbf{w}}(y), y \in \Gamma\}$, the menu of linear contracts (in $w(x)$)

$$c(w(x), m) = \bar{c}(m) + \bar{c}'(m)[w(x) - m]$$

implements the optimal contract with y directly observable at the same cost to the principal (for the same reasons as for the special case considered above).

23.4 MECHANISM DESIGN

In this section we examine a class of problems often referred to in the economics literature as mechanism design problems. Our discussion of the basic mechanism design problem is based on Guesnerie and Laffont (1984) and Fudenberg and Tirole (1992, Chapter 7).

23.4.1 Basic Mechanism Design Problem

The structure of a mechanism design problem is slightly different from the basic model considered in the previous sections. The private information $y \in Y$ is represented as pertaining to the agent's personal cost $\kappa(y,x)$ of providing alternative outcome levels $x \in X$.

The agent's cost for a given outcome x is assumed to increase with y and the cost is an increasing convex function of x given y, i.e.,

$$\frac{\partial \kappa}{\partial y} > 0, \qquad \text{and} \qquad \frac{\partial \kappa}{\partial x} > 0, \; \frac{\partial^2 \kappa}{\partial x^2} > 0.$$

Note that, similar to the hurdle model, a high y is bad news.

The outcome x is contractible and obtained with certainty by the agent, so there is no value to communication. However, we can appeal to the Revelation Principle and assume that at the time the contract is signed, the agent commits to an outcome schedule $\boldsymbol{x}(m)$ contingent on his subsequent message m. Both the principal and the agent are risk neutral. The principal's utility from outcome x is an increasing concave function $V(x)$.[8]

The Principal's Mechanism Design Problem:

$$\underset{\boldsymbol{c},\boldsymbol{x}}{\text{maximize}} \quad U^p(\boldsymbol{c},\boldsymbol{x},\eta) = \int_Y \big[V(\boldsymbol{x}(y)) - \boldsymbol{c}(y) \big] d\Phi(y), \tag{23.1$''$}$$

$$\text{subject to} \quad U^a(\boldsymbol{c}(y),\boldsymbol{x}(y)\,|\,y,\eta) = \boldsymbol{c}(y) - \kappa(y,\boldsymbol{x}(y)) \geq \bar{U} = 0, \quad \forall\; y \in Y, \tag{23.2$''$}$$

$$U^a(\boldsymbol{c}(y),\boldsymbol{x}(y)\,|\,y,\eta) \geq U^a(\boldsymbol{c}(m),\boldsymbol{x}(m)\,|\,y,\eta), \quad \forall\; y,m \in Y. \tag{23.3$''$}$$

where we have normalized the agent's personal cost so that his reservation utility is zero (w.l.o.g.). We assume in the general analysis that $Y = [\underline{y},\bar{y}]$ and $x \in [0,\bar{x}]$, and that suitable differentiability conditions are satisfied.

Before we characterize an optimal solution to this program, we provide a necessary condition for the outcome schedule to be implementable, i.e., there exists some compensation scheme $\boldsymbol{c}(y)$ such that $\boldsymbol{x}(y)$ satisfies the truthtelling constraints.

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[8] For simplicity, the principal's utility from x depends only on x and not on y. However, in Section 23.4.5 we consider a setting in which the principal's utility depends on the private signal as well, i.e., the utility is expressed as $V(x,y)$.

Proposition 23.9 (Guesnerie and Laffont 1984, Theorem 1)
An outcome schedule $\boldsymbol{x}(y)$ is implementable only if

$$\frac{\partial^2 \kappa}{\partial y\, \partial x}(y, \boldsymbol{x}(y))\, \boldsymbol{x}'(y) \leq 0, \quad \forall\, y \in Y.$$

Proof: The first- and second-order conditions for the truthtelling constraint are

$$\boldsymbol{c}'(y) - \frac{\partial \kappa}{\partial x}\boldsymbol{x}'(y) = 0, \quad \forall\, y \in Y,$$

$$\boldsymbol{c}''(y) - \frac{\partial^2 \kappa}{\partial x^2}(\boldsymbol{x}'(y))^2 - \frac{\partial \kappa}{\partial x}\boldsymbol{x}''(y) \leq 0, \quad \forall\, y \in Y.$$

Differentiating the first-order condition yields that

$$\boldsymbol{c}''(y) = \frac{\partial^2 \kappa}{\partial y\, \partial x}\boldsymbol{x}'(y) + \frac{\partial^2 \kappa}{\partial x^2}(\boldsymbol{x}'(y))^2 + \frac{\partial \kappa}{\partial x}\boldsymbol{x}''(y), \quad \forall\, y \in Y.$$

Substituting this expression into the second-order condition gives the result.
Q.E.D.

The proposition implies that if, for example, the agent's marginal cost of producing x increases with y (which we assume), then the induced outcome schedule $\boldsymbol{x}(y)$ must be non-increasing in order to satisfy the truthtelling constraints. Conversely, if we want to examine settings in which the principal induces "good" types to produce more than "bad" types in equilibrium, the proposition shows that it is sufficient to assume that the marginal cost of x increases with y.

The proposition is illustrated in Figure 23.5. The agent's indifference curve in (x,c)-space, for a given y, reflects his trade-off between c and $\kappa(y,x)$. It is increasing and convex functions with a slope equal to the marginal cost of producing x, i.e., $\partial\kappa/\partial x$. If this marginal cost is increasing in y, i.e., $\partial^2\kappa/\partial y \partial x > 0$, the indifference curve for y'' is steeper than the indifference curve for y' when $y'' > y'$. That is, the indifference curves cross only once and, hence, the condition $\partial^2\kappa/\partial y \partial x > 0$ is commonly referred to as the *single-crossing condition.* Let $z(y') = (\boldsymbol{x}(y'), \boldsymbol{c}(y'))$ denote the allocation for y', and let k' and k'' equal the expected utility levels generated by that contract given y' and y'', respectively. Allocations for y'' must be below the indifference curve for y', i.e., allocations in regions B and C are excluded, since otherwise an agent with signal y' would claim that he has the higher costs y''. Similarly, the allocations must be above the indifference curve for y'' passing through $z(y')$, i.e., regions C and D are excluded, since otherwise an agent with y'' would claim that he has the lower costs

y'. Hence, only allocations for y'' in the shaded region A are consistent with truthtelling for both types of agents. This implies that an incentive compatible outcome schedule must be decreasing in y, i.e., $x(y'') < x(y')$.

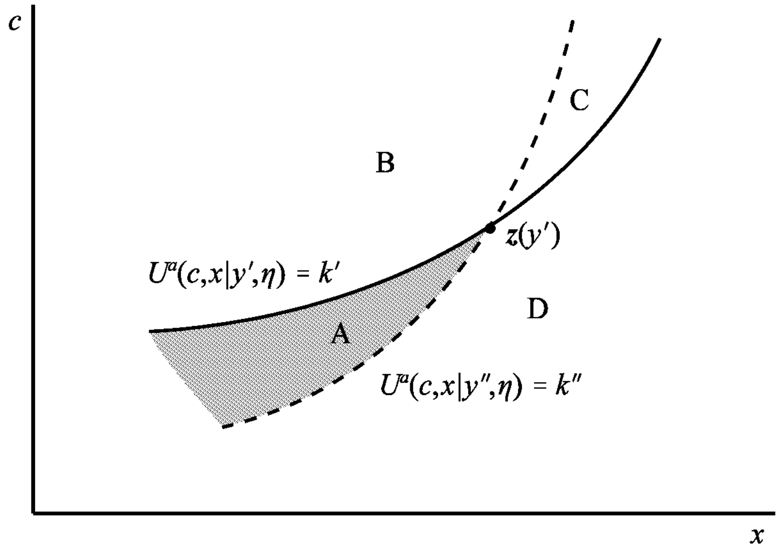

Figure 23.5: Monotonicity of outcome schedule, $y'' > y'$.

Moreover, if there are only two types of agents y' and y'' (or no types between y' and y''), then, from the principal's perspective, the optimal allocation for y'', $z(y'')$ must be on the upper boundary of the shaded region. That is, the binding incentive constraint is such that the low cost agent does not overstate his costs. This is illustrated in Figure 23.6. Suppose, to the contrary, that the optimal allocation to type y'' is strictly below the upper boundary, e.g., at $z'(y'')$. Then there is another allocation $z'(y')$ for type y' along the indifference curve for type y' that passes through $z'(y'')$, which has the same outcome $x(y')$ as $z(y')$ but has a lower compensation to the agent, i.e., $c'(y') < c(y')$. The allocations $z'(y'')$ and $z'(y')$ are incentive compatible, and $z'(y')$ satisfies the contract acceptance constraint for type y' since $z'(y'')$ satisfies this constraint for type y'' and type y' has a lower cost than type y''. This contradicts the assumption that $z(y')$ and $z'(y'')$ are optimal allocations for y' and y''.

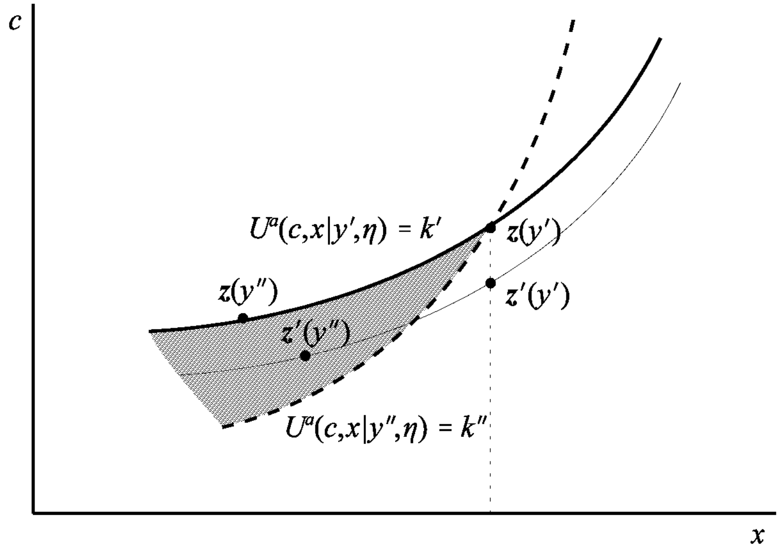

Figure 23.6: Optimal allocations for $y'' > y'$.

Now we return to the characterization of the optimal solution to the principal's program. The first-order condition for the truthtelling constraints implies that

$$\mathbf{c}'(y) = \frac{\partial \kappa}{\partial x}\mathbf{x}'(y), \quad \forall\, y \in Y. \tag{23.4}$$

Suppose initially that (23.4) is also a sufficient condition for the truthtelling constraints. (23.4) implies

$$\frac{dU^a}{dy} = \mathbf{c}'(y) - \left[\frac{\partial \kappa}{\partial y} + \frac{\partial \kappa}{\partial x}\mathbf{x}'(y)\right] = -\frac{\partial \kappa}{\partial y}. \tag{23.5}$$

It then follows, since the agent's cost of providing x increases with y, that his utility is decreasing in y and this, in turn, implies that the participation constraint need only be satisfied for the worst possible type, $\bar{y}$. We can obtain U^a by integrating (23.5) from y to $\bar{y}$ and setting the constant of integration equal to $\underline{U}$. Hence, truthtelling implies that $\mathbf{c}(y)$ must be such that the agent's utility given y equals his reservation utility plus the integral of the marginal costs incurred by the "worse types" from which he must be separated:

$$U^a(\boldsymbol{c}(y),\boldsymbol{x}(y)\,|\,y,\eta) \;=\; \bar{U} \;+\; \int_{y}^{\bar{y}} \frac{\partial\kappa}{\partial\tilde{y}}(\tilde{y},\boldsymbol{x}(\tilde{y}))\,d\tilde{y}.$$

Using the definition of U^a and substituting for $\boldsymbol{c}(y)$, we can formulate the principal's mechanism design problem as the following unconstrained optimization problem:

$$\underset{\boldsymbol{x}}{\text{maximize}}\;\; U^p(\boldsymbol{c},\boldsymbol{x},\eta) = \int_{\underline{y}}^{\bar{y}} \Big[\, V(\boldsymbol{x}(y)) - \kappa(y,\boldsymbol{x}(y)) - \int_{y}^{\bar{y}} \frac{\partial\kappa}{\partial\tilde{y}}(\tilde{y},\boldsymbol{x}(\tilde{y}))\,d\tilde{y}\,\Big]\, d\Phi(y),$$

which after integration by parts is equivalent to

$$\underset{\boldsymbol{x}}{\text{maximize}}\;\; U^p(\boldsymbol{x},\eta) = \int_{\underline{y}}^{\bar{y}} \Big[\, V(\boldsymbol{x}(y)) - \kappa(y,\boldsymbol{x}(y)) - \frac{\Phi(y)}{\varphi(y)}\frac{\partial\kappa}{\partial y}(y,\boldsymbol{x}(y))\Big]\, d\Phi(y). \quad (23.6)$$

Note that the two first terms, i.e., $V(x(y)) - \kappa(y,\boldsymbol{x}(y))$, can be interpreted as the agency's total expected surplus, whereas the last term is the expected information rent paid to the agent.[9] The optimal outcome for each y is obtained by differentiating (23.6) with respect to x for each y:

$$V'(\boldsymbol{x}(y)) \;=\; \frac{\partial\kappa}{\partial x}(y,\boldsymbol{x}(y)) \;+\; \frac{\Phi(y)}{\varphi(y)}\frac{\partial^2\kappa}{\partial x\,\partial y}(y,\boldsymbol{x}(y)), \quad \forall\, y\in Y. \qquad (23.7)$$

That is, the optimal outcome schedule is such that the marginal benefit to the principal is equal to the agent's marginal cost plus the marginal information rents (to an agent of type y and all lower types). The optimal compensation scheme that implements $\boldsymbol{x}(y)$ can now be found by computing the sum of the agent's reservation utility plus his personal cost and information rent:

$$\boldsymbol{c}(y) \;=\; \bar{U} \;+\; \kappa(y,\boldsymbol{x}(y)) \;+\! \int_{y}^{\bar{y}} \frac{\partial\kappa}{\partial\tilde{y}}(\tilde{y},\boldsymbol{x}(\tilde{y}))\,d\tilde{y}, \quad \forall\, y\in Y. \qquad (23.8)$$

[9] The last two terms are sometimes referred to as the virtual costs, i.e., the agent's true costs plus information rents.

We have assumed that the first order conditions for the truthtelling constraints are also sufficient conditions for those constraints. The following proposition provides conditions under which this is the case.

Proposition 23.10 (Fudenberg and Tirole 1992, Theorem 7.4)
Let $\boldsymbol{x}(y)$ be an optimal solution to the principal's unconstrained program (23.6). If

(a) $\dfrac{\partial^2 \kappa}{\partial y\, \partial x}(y,\boldsymbol{x}(y)) \ \geq\ 0$ (*single-crossing*),

(b) $\dfrac{\partial}{\partial y}\left(\dfrac{\Phi(y)}{\varphi(y)}\right) \ \geq\ 0$ (*monotone inverse hazard rate*),

(c) $\dfrac{\partial^3 \kappa}{\partial x^2\, \partial y}(y,\boldsymbol{x}(y)) \ \geq\ 0$ and $\dfrac{\partial^3 \kappa}{\partial x\, \partial y^2}(y,\boldsymbol{x}(y)) \ \geq\ 0,$

then $\boldsymbol{x}(y)$ is non-increasing, and the first-order condition for the truthtelling constraint is both necessary and sufficient.

Proof: Firstly, totally differentiating (23.7) with respect to x and y, and re-arranging terms yields

$$\frac{dx}{dy}\left[\frac{d^2V}{dx^2} - \frac{\partial^2\kappa}{\partial x^2} - \frac{\partial^3\kappa}{\partial x^2\,\partial y}\frac{\Phi(y)}{\varphi(y)}\right] = \frac{\partial^2\kappa}{\partial y\,\partial x}\left[1 + \frac{d}{dy}\frac{\Phi(y)}{\varphi(y)}\right] + \frac{\partial^3\kappa}{\partial x\,\partial y^2}\frac{\Phi(y)}{\varphi(y)}.$$

Hence, (a) - (c) (in addition to $V(\cdot)$ concave and $\kappa(\cdot)$ convex) are sufficient conditions for $\boldsymbol{x}(y)$ to be non-increasing. Secondly, we show that the single-crossing condition and $\boldsymbol{x}(y)$ non-increasing are sufficient for the local truthtelling constraint (23.4) to imply global truthtelling. Suppose, to the contrary, that for some signal y it is optimal for the agent to report $y' > y$, i.e.,

$$\boldsymbol{c}(y') - \kappa(y,\boldsymbol{x}(y')) > \boldsymbol{c}(y) - \kappa(y,\boldsymbol{x}(y)).$$

Hence,

$$\int_y^{y'} \frac{\partial}{\partial \tilde{y}}\Big(\boldsymbol{c}(\tilde{y}) - \kappa(y,\boldsymbol{x}(\tilde{y}))\Big)d\tilde{y} = \int_y^{y'}\Big(\boldsymbol{c}'(\tilde{y}) - \frac{\partial\kappa}{\partial x}(y,\boldsymbol{x}(\tilde{y}))\,\boldsymbol{x}'(\tilde{y})\Big)d\tilde{y} > 0.$$

However, the single-crossing condition implies

$$\frac{\partial \kappa}{\partial x}(y, \boldsymbol{x}(\tilde{y})) < \frac{\partial \kappa}{\partial x}(\tilde{y}, \boldsymbol{x}(\tilde{y})) \qquad \text{for } \tilde{y} > y,$$

such that $\boldsymbol{x}(y)$ non-increasing implies

$$\int_{y}^{y'} \left(c'(\tilde{y}) - \frac{\partial \kappa}{\partial x}(\tilde{y}, \boldsymbol{x}(\tilde{y}))\, \boldsymbol{x}'(\tilde{y}) \right) d\tilde{y} > 0.$$

The integrand is equal to zero for all signals by (23.4) – so a contradiction is obtained. A similar argument shows that the agent will not understate his signal.

Q.E.D.

As Proposition 23.9 demonstrates, the single-crossing condition (a) implies that only non-increasing outcome schedules can be implemented, i.e., "good" types produce more than "bad" types (which seems to be a natural characteristic). Therefore, the first-order conditions (23.4) are only sufficient conditions for the truthtelling constraints (23.3″) if the optimal solution to the principal's unconstrained program (23.6) is such that the outcome schedule is non-increasing. Otherwise, (23.6) has to be solved subject to the constraint that the outcome schedule is non-increasing. Conditions (b) and (c) ensure that this constraint is satisfied by the optimal solution to the unconstrained program (given the single-crossing condition). The monotone inverse hazard rate condition is satisfied by a wide range of standard probability distributions such as the uniform, normal, and exponential distributions. Condition (c) is difficult to justify, in general, since it involves third order derivatives of the cost function. However, it is satisfied by many simple functions, such as $\kappa(y,x) = yx$, which is used in several papers discussed below.[10]

The single-crossing condition is a standard assumption in the signaling literature that dates back to Mirrlees (1971) and Spence (1974). Given that the optimal solution to the principal's unconstrained program (23.6) is such that the outcome schedule is non-increasing, the single-crossing condition (a) implies that the local truthtelling constraints (23.4) are sufficient to imply global truthtelling. This is illustrated in Figure 23.7 for a case with three types of agents $y''' > y'' > y'$. As illustrated in Figure 23.6, optimal allocations are such that the allocation for a given type is at the upper boundary of the incentive compatibility region for that type and the type just below it (as reflected by the locations of $z(y')$, $z(y'')$, and $z(y''')$). In particular, note that type y' is indifferent between reporting y' and y'', and type y'' is indifferent between reporting y'' and

[10] Guesnerie and Laffont (1984) provide a general analysis for the case in which the optimal solution to the principal's unconstrained program is not monotonic.

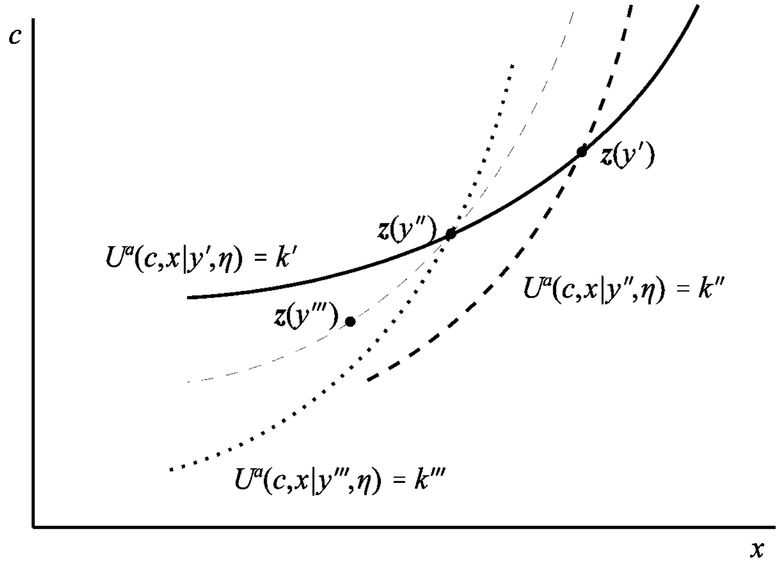

Figure 23.7: Sufficiency of local incentive constraints, $y''' > y'' > y'$.

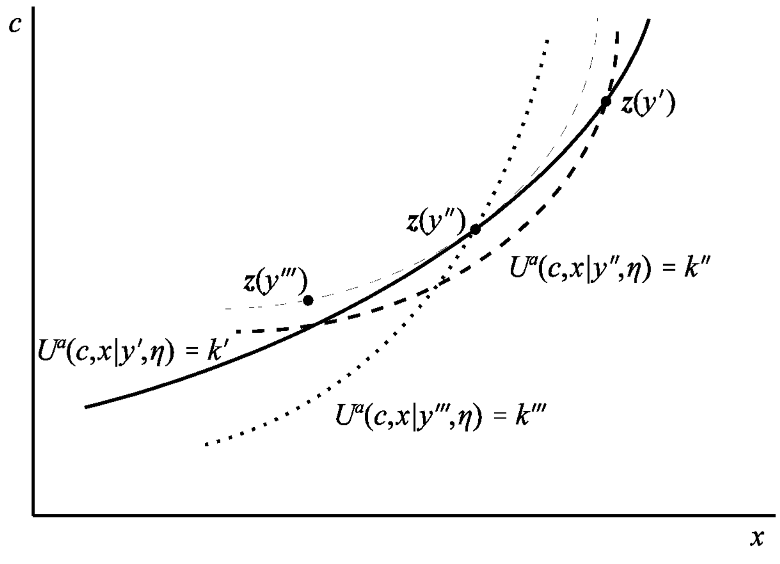

Figure 23.8: Violation of global incentive constraints without the single crossing condition, $y''' > y'' > y'$.

y''', whereas type y' would be strictly worse off by reporting y'''. Hence, for an agent of a given type the incentive constraint for reporting a type just above it is binding, whereas the incentive constraints for reporting types further above are not binding.

Figure 23.8 illustrates a similar setting in which the single-crossing condition is not satisfied. Although all the adjacent ("local") incentive constraints are satisfied by the allocations $z(y')$, $z(y'')$, and $z(y''')$, the low cost type y' has an incentive to report the highest cost y''' violating the "global" incentive constraints.

23.4.2 A Possibility of No Private Information

Lewis and Sappington (LS) (1993) extend the basic mechanism design problem to consider a setting in which there is a positive probability p that the agent has received no private information about his cost of providing a given outcome x, versus being perfectly informed with probability $1 - p$.[11] In particular, LS assume the agent observes $y \in Y = y^o \cup [\underline{y}, \bar{y}]$, where the probability that $y = y^o$ is p and the probability that $y \in [\underline{y}, \bar{y}]$ is $(1 - p)$. Conditional on the fact that the agent is informed, the probability of observing $y \in [\underline{y}, \bar{y}]$ is characterized by a density function $\varphi(y)$ defined on that set. This probability function is assumed to satisfy the inverse hazard rate condition (b) in Proposition 23.10:

$$\frac{d}{dy}\left\{\frac{\Phi(y)}{\varphi(y)}\right\} \geq 0, \quad \forall\, y \in [\underline{y}, \bar{y}].$$

The agent and principal are both risk neutral and the agent's cost function is $\kappa(y,x) = yx$, which satisfies conditions (a) and (c) in Proposition 23.10. Observe that, given the cost function, $y \in [\underline{y}, \bar{y}]$ represents perfect information about the cost of producing x and, given that y^o is to represent no information, it follows that

$$y^o = \int_{\underline{y}}^{\bar{y}} y \, d\Phi(y),$$

[11] In general, a model that permits information to be imperfect (as in the basic models earlier in the chapter) can readily include the possibility of no information by representing this as the receipt of an uninformative signal y^o for which the posterior belief is the same as the prior belief. However, this is a "different kind" of signal and its possibility generally affects the structure of the optimal contract.

so that $E[\kappa(y,x)|y^o] = y^o x$. If $p = 0$, then Proposition 23.10 implies that the optimal outcome schedule denoted $\boldsymbol{x}^0(y)$ is non-increasing and given by (23.7).

Proposition 23.11 (LS, Lemma 1)
If $p = 0$, and the principal wants to contract with all agents, then the solution to the principal's problem is

(a) $\boldsymbol{x}^0(y) > 0$ for all $y \in Y$;

(b) $\boldsymbol{x}^0(y)$ is non-increasing and satisfies $V'(x) = A(y) \equiv y + \Phi(y)/\varphi(y)$ for all $y \in Y$.

Property (a) implies that the firm always operates. Property (b) states that the induced output equates the marginal benefit of output x with the adjusted marginal cost of production, i.e., including the marginal information rents. Observe that there is "no pooling" and that $\boldsymbol{x}^0(y)$ is a continuous function.

If $p \in (0,1)$, there is a possibility that the agent is uninformed (LS refer to this as being *ignorant*), and there are four fundamental changes in the optimal menu of contracts:

(i) pooling arises (i.e., the agent accepts the same contract for a subset of signals $y \in Y$);

(ii) the induced outcome schedule is discontinuous;

(iii) severe outcome distortions are induced over a range of high values of y;

(iv) shutdown ($\boldsymbol{x}(y) = 0$) may occur for a range of high values of y.

Observe that the efficient (i.e., first-best) production schedule $\boldsymbol{x}^*(y)$ satisfies $V'(x) = y$. This provides the maximum surplus for each y.

Proposition 23.12 (LS, Prop. 1 – see Fig. 23.9)
If $p \in (0,1)$, then there exists $y_p \in (\underline{y},y^o)$ such that:

(a) $\boldsymbol{x}(y) = \boldsymbol{x}^0(y)$, $\quad \forall\, y \in [\underline{y},y_p]$,

(b) $\boldsymbol{x}(y) = \boldsymbol{x}(y^o) = \boldsymbol{x}^0(y_p) \in (\boldsymbol{x}^0(y^o), \boldsymbol{x}^*(y^o))$, $\quad \forall\, y \in [y_p,y^o]$,

(c) $\boldsymbol{x}(y) = \boldsymbol{x}^p(y) < \boldsymbol{x}^0(y)$, $\quad \forall\, y \in (y^o, \min\{y^+,\bar{y}\}]$,

(d) $\boldsymbol{x}(y) = 0$, $\quad \forall\, y \in [\min\{y^+,\bar{y}\},\bar{y}]$, where $x^p(y)$ satisfies $V'(x) = A(y) + I(y)$,

$I(y) = p/[(1 - p)\varphi(y)]$,

and y^+ is defined by $V(x^p(y^+)) = [A(y^+) + I(y^+)]x^p(y^+)$.

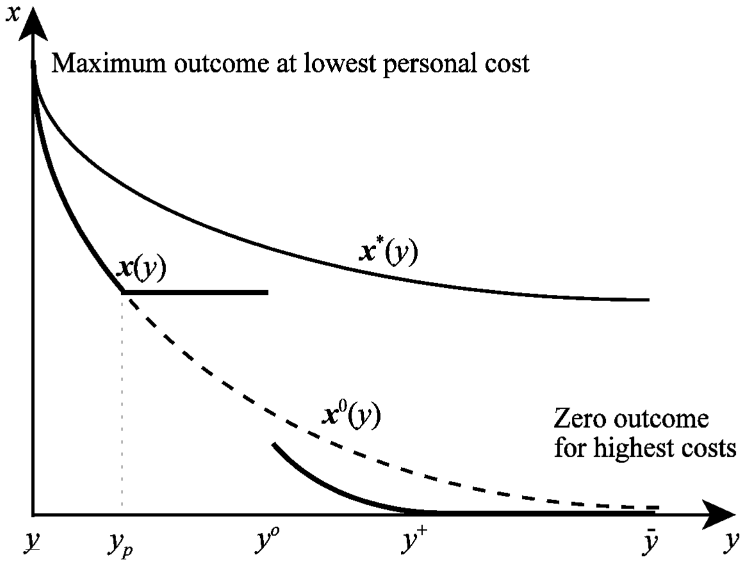

Figure 23.9: Optimal outcome schedules (LS, Figure 1).

A truth-inducing contract must motivate the agent not to exaggerate his expected marginal cost. Implementation of the first-best output schedule $x^*(y)$ would provide the largest possible surplus, but would require paying the agent significant information rents. The principal can reduce those rents by inducing outcomes below the efficient level for $y > \underline{y}$. The distortions are particularly severe for $y > y^o$, due to the probability mass at the latter point. For $y > y^o$, the induced outcome level equates the marginal benefit of the outcome to the sum of the adjusted marginal cost $A(y)$ and the cost $I(y)$ of being "ignorant" (i.e., uninformed). Observe that $I(y)$ increases with p – thus the more pronounced the mass point at y^o, the more severe the distortions induced for $y > y^o$ in order to limit the uninformed agent's incentive to exaggerate his expected costs.

Proposition 23.13 (LS, Prop. 2)[12]
If $p \in (0,1)$, then

(a) $d\boldsymbol{x}(y^o)/dp > 0$; (b) $dy_p/dp < 0$;

(c) $d\boldsymbol{x}(y)/dp < 0, \quad \forall\, y \in (y^o, y^+)$; (d) $dy^+/dp \le 0$.

23.4.3 To Be or Not to Be Informed Prior to Contracting

Crémer and Khalil (CK) (1992) extend the basic mechanism design problem to consider a setting in which the agent chooses whether to become informed prior to contract acceptance. The principal moves first and offers a menu of contracts. The agent will observe y before he selects from the menu, but he has a choice as to whether he immediately accepts/rejects the menu or pays a cost K to observe y before making his accept/reject decision. That is, in both cases the agent has pre-decision information, but he has a choice between whether it is post-contract or pre-contract information, where the latter is more costly.

CK assume there are n possible signals, denoted by index $I = \{1,...,n\}$. The probability of signal i is φ_i and the agent's personal cost given the i^{th} signal and outcome x is the same as in LS, i.e., $\kappa_i(x) = y_i x$, with $0 < y_1 < ... < y_n$. Let x_i and c_i represent the output and compensation paid if $m = i$. Invoking the Revelation Principle, incentive compatibility requires the menu of contracts to be such that

$$c_i - y_i x_i \ge c_m - y_i x_m, \quad \forall\, i,m \in I.$$

If the agent does not expend K to acquire y before contract acceptance (denoted as choosing η^o), then he will accept the contract if

$$U^a(\boldsymbol{c},\boldsymbol{x}|\eta^o) = \sum_{i=1}^{n} \varphi_i\,[c_i - y_i x_i] \ \ge\ \bar{U} = 0.$$

If the agent expends K to acquire y before contract acceptance (denoted as choosing η^1), then he will accept the contract given signal i if

$$c_i - y_i x_i \ge 0.$$

If $m > i$, then incentive compatibility and $y_m > y_i$ imply

$$c_i - y_i x_i \ge c_m - y_i x_m > c_m - y_m x_m,$$

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[12] See LS for the proof.

i.e., the agent earns a greater net return if he has lower costs. CK allow for the possibility that the agent may choose to reject the menu given some y_i and, hence, there is a "cut-off" signal $\imath \in I$ such that

$$c_i - y_i x_i \geq (<) \; 0 \text{ if } i \leq (>) \; \imath,$$

i.e., the agent will accept the contract if, and only if, $i \leq \imath$.

The agent's *ex ante* expected net return from acquiring the signal early is

$$U^a(\mathbf{c},\mathbf{x}|\eta^1) = \sum_{i=1}^{\imath} \varphi_i \, [c_i - y_i x_i] \; - \; K.$$

The principal's net return from the agent's two choices (assuming $U^a(\mathbf{c},\mathbf{x}|\eta^o) \geq 0$) is

$$U^p(\mathbf{c},\mathbf{x}|\eta^o) = \sum_{i=1}^{n} \varphi_i \, [V(x_i) - c_i],$$

$$U^p(\mathbf{c},\mathbf{x}|\eta^1) = \sum_{i=1}^{\imath} \varphi_i \, [V(x_i) - c_i].$$

Proposition 23.14 (CK, Lemma 1)

For every menu of contracts that induces the agent to choose η^1, there exists a menu that induces the agent to choose η^o and makes the principal better off.

The key to this result is that for any menu $\mathbf{z}^1 = \{c_i, x_i\}_{i-1,...,n}$ that induces the agent to choose η^1, the menu $\mathbf{z}'$ in which the principal commits to terminate production whenever the agent reports a signal $i > \imath$, i.e.,

$$c_i' = c_i, \text{ and } x_i' = x_i, \;\; i \leq \imath,$$

$$c_i' = 0, \text{ and } x_i' = 0, \;\; i > \imath,$$

induces the agent to choose η^o (as well as truthtelling). The menu $\mathbf{z}'$ makes the agent strictly better off since he is not spending K, and the principal is equally well off. The agent's saved costs can then be transferred to the principal without affecting incentives by choosing a contract $\mathbf{z}''$ with $c_i'' = c_i' - K$ for all signals i, since with η^o the agent's contract acceptance constraint has only to be satisfied as an expectation over signals.

CK provide additional analysis of the two signal (n = 2) case. In this analysis, CK establish that if K is sufficiently small, then increasing the cost K *increases the principal's expected return.* The key here is that increasing K reduces the agent's incentive to acquire the signal early, and thereby reduces his information rents.

CK also consider a setting in which the principal offers the menu of contracts to $q > 1$ agents and then randomly chooses an agent from the set of agents who accept the contract. CK's Theorem 4 establishes that for K small enough, the principal's expected payoff increases as the number of agents q increases. The key here is that increasing the number of agents is similar to increasing the cost K because increasing the number of agents decreases the probability that any one agent will "recover" his investment, since he does not know for certain that he will be chosen by the principal if he accepts the contract (and other agents also accept the contract).

23.4.4 Impact of a Public Report on Resource Allocation

Antle and Eppen (1985) and Antle and Fellingham (1995) study resource allocation (e.g., financing capital investments) in a setting similar to the basic mechanism design problem. In their model, $x \in [0, \bar{x}]$ is the outcome from an investment which requires capital financing of yx, where y is privately observed by the agent before he chooses from the menu of contracts offered by the principal. In this setting c is interpreted as the capital financing provided by the principal to the agent, so that the net return to the principal is $x - c$. The agent receives the capital c, but need only invest yx to produce x, leaving him with a surplus of $c - yx$, which he can consume. The agent has no personal capital, so that the financing restriction requires $c - yx \geq 0$ (which effectively plays the same role as the agent's reservation utility in the models discussed above). Observe that if the principal could contract on y, then it would be efficient to produce the maximum outcome $\bar{x}$ for all $y \in [0,1)$. In the following analysis we assume $Y = [0,1]$.

The basic resource allocation model is identical to the basic mechanism design model with $\kappa(y,x) = yx$ and $V(x) = x$. This structure is such that the objective function in the principal's unconstrained optimization problem (23.6) is a linear function of $\boldsymbol{x}(y)$, i.e.,

$$\underset{x}{\text{maximize}} \quad U^p(\boldsymbol{x},\eta) = \int_0^1 \left[1 - y - \frac{\Phi(y)}{\varphi(y)} \right] \boldsymbol{x}(y)\, d\Phi(y).$$

If the monotone inverse hazard rate condition is satisfied, the optimal outcome schedule is characterized by a "hurdle" strategy where[13]

$$\boldsymbol{x}(y) = \begin{cases} \bar{x} & \text{if } y \le \hat{y}, \\ 0 & \text{if } y > \hat{y}, \end{cases}$$

and the hurdle $\hat{y}$ is given by the highest signal y for which the true marginal cost is less than the adjusted marginal outcome to the principal, i.e.,

$$y \le 1 - \frac{\Phi(y)}{\varphi(y)}.$$

In particular, if there is an interior solution, then

$$\hat{y} = 1 - \frac{\Phi(\hat{y})}{\varphi(\hat{y})}. \tag{23.9}$$

Note that the hurdle $\hat{y}$ is independent of $\bar{x}$. The capital financing given to the agent is specified by (23.8), and in this setting takes the simple form

$$\boldsymbol{c}(y) = \begin{cases} \hat{y}\bar{x} & \text{if } y \le \hat{y}, \\ 0 & \text{if } y > \hat{y}. \end{cases}$$

Hence, if the agent's signal is above the hurdle $\hat{y}$, he does not invest and receives no information rents. However, if his signal is below the hurdle, he produces the maximum outcome and receives capital financing equal to the amount necessary to produce the maximum output $\bar{x}$ given $y = \hat{y}$. The key here is that to induce $\bar{x}$ and truthtelling for all $y \le \hat{y}$, the agent must receive capital financing independent of the y he reports. Therefore, the agent consumes slack (i.e., receives information rent) equal to $[\hat{y} - y]\bar{x}$ if his signal is strictly below the hurdle. In selecting the cut-off $\hat{y}$ to be induced, the principal must tradeoff obtaining the maximum outcome for more costly signals and paying larger information rents to induce the agent to invest.

Antle and Fellingham (AF) (1995) examine the impact of a public (contractible) signal y^c that is reported prior to the agent observing his private signal and contract acceptance. Clearly, if y^c provides perfect information about the agent's private signal, then the principal obtains the first-best investments and the agent gets no information rents. However, an interesting aspect of this

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[13] It is readily verified that the two other conditions for the validity of the first-order approach in Proposition 23.10 are satisfied.

analysis is that, even though the public signal reduces the agent's informational advantage, AF identify public signals that result in both the principal and the agent being better off *ex-ante*.

In particular, AF consider a setting in which y^c induces a partition on the set of private signals. Assume that $X = [0,1]$ and y is uniformly distributed on $Y = [0,1]$. Without the public signal, the optimal hurdle is calculated from (23.9) to be $\hat{y} = ½$. With a public signal that partitions Y, the optimal outcome schedule conditional on the public signal is also characterized by a "hurdle" strategy using the conditional distribution for y given y^c, i.e., $\Phi(y|y^c)$.

We first consider the *equal partition case* in which Y is partitioned by the public report into n equal sets of the form

$$\{ [0,h), [h,2h), ..., [(n-1)h,1] \} \quad \text{where } h = 1/n; \; n = 1, 2, 4, 8, ...$$

Let the i^{th} public report be represented by $y_i^c \equiv [(i-1)h, ih)$, $i = 1, 2, ..., n$, for a given number of partition elements n. Then the corresponding optimal hurdles are given by (23.9) using the conditional probabilities $\Phi(y|y^c)$:

$$\hat{y}_i = \max\left\{(i-1)h, \min\left\{½ + \frac{(i-1)h}{2}, ih\right\}\right\}, \quad i = 1,...,n.$$

Solving the above establishes that there is "full" investment for all elements of the partition except the last, where there is only investment for the lower half of the signals, i.e.,

$$\hat{y}_i = ih, \quad i = 1, 2, ..., n-1,$$

$$\hat{y}_n = 1 - \frac{1}{2n}.$$

Observe that the agent's information rent for a given signal y is determined by the *maximum signal in the relevant partition*, and not the maximum signal for which there is investment. Hence, the rent for signal $y \in [0,½]$ is less than or equal to the rent with no public report, but is positive (instead of zero) for $y \in (½, 1 - 1/(2n))$ and continues to be zero for $y \in [1 - 1/(2n), 1]$. Of course, the rent for y $(½, 1 - 1/(2n))$ occurs because the principal uses the reduction in information rents to induce positive instead of zero investment in this range. The resulting expected utility levels for the principal and the agent are as follows.

Proposition 23.15 (AF, Prop. 1)

Consider the AF setting in which a public report partitions Y into n equal-sized elements.

(a) The principal's expected profit is given by

$$U^p(\boldsymbol{x}|n) = \frac{1}{2} - \frac{2n-1}{4n^2},$$

which is increasing in n, with $U^p(\boldsymbol{x}|n) \rightarrow ½$ as $n \rightarrow \infty$.

(b) The agent's expected information rent is given by

$$U^a(\boldsymbol{x}|n) = \frac{4n-3}{8n^2} \rightarrow 0 \quad \text{for } n \rightarrow \infty,$$

which achieves a maximum of $U^a(\boldsymbol{x}|n) = 5/32$ at $n = 2$ and is decreasing in n for $n \geq 2$ with $U^a(\boldsymbol{x}|n) \rightarrow 0$ as $n \rightarrow \infty$.

The proposition demonstrates that the agent is generally hurt by additional public information since the principal uses that information to reduce his expected information rent. Therefore, generally, there is conflict of interest between the agent and the principal concerning the fineness of the partition. However, the agent's expected information rent is maximized at $n = 2$. To see why, observe that for all $y \in [0,½]$, the investment hurdle is ½ for both $n = 1$ and $n = 2$, resulting in the same investment decision, principal profit, and agent information rent. However, for $y \in (½,1]$, the hurdle is ½ for $n = 1$ and ¾ for $n = 2$, resulting in more investment, more principal profit, and more agent information rent if $y \in (½,¾)$ and $n = 2$.

Now consider the agent's partition preferences assuming the partition elements need not be of equal size. The above discussion indicates that there is no benefit to the agent of further partitioning $[0,½]$. However, it would be useful to further partition $(½,1]$. Partitioning this in half would not change the results for $y \in (½,¾)$, but would increase the investment, principal profit, and agent information rent for $y \in (¾, ⅞)$. Extending this argument leads to the following result.

Proposition 23.16 (AF, Prop. 2)

The partition that maximizes the agent's expected information rent is given by

$$\left\{\left[0,\frac{1}{2}\right),\left[\frac{1}{2},\frac{3}{4}\right),\left[\frac{3}{4},\frac{7}{8}\right),\dots\right\}$$

If n is the number of elements in the partition, then

$$\lim_{n\to\infty} U^p(\boldsymbol{x}|n) = \frac{1}{3},$$

$$\lim_{n\to\infty} U^a(\boldsymbol{x}|n) = \frac{1}{6}.$$

Hence, the total expected "surplus" approaches ½, which is equal to the total expected surplus when the public signal y^c gives perfect information about the private signal y.

The key aspect of this proposition is that the agent has incentives to provide some information to the principal, in order to make it optimal for the principal to induce investment above $y = ½$, which is the maximum y for which there is investment with no information. The optimal partition is such that the agent keeps the information rent for the low signals at the same time as he gets information rent for the high signals. In the limit, there is investment for all signals such that the investment strategy is the same as with perfect public information so that the total expected surplus attains its maximum value of ½.

Of course, the principal prefers a public information system that perfectly reveals the agent's signal. Assuming that he is restricted to a partition with n elements, the following proposition provides the optimal n-element partition for the principal.

Proposition 23.17 (AF, Prop. 3)

The partition *with n* elements that maximizes the principal's expected profit is

$$\left\{\left[0,\frac{1}{n+1}\right),\left[\frac{1}{n+1},\frac{2}{n+1}\right),\left[\frac{2}{n+1},\frac{3}{n+1}\right),\ldots,\left[\frac{n-1}{n+1},\frac{1}{n+1}\right)\right\},$$

and

$$U^p(\boldsymbol{x}|n) = \frac{n}{2(n+1)},$$

$$U^a(\boldsymbol{x}|n) = \frac{n}{2(n+1)^2}.$$

Letting $n = 3$ illustrates the conflict of interest over information systems. The agent's optimal 3-element partition is

$$\{\,[0,\tfrac{1}{2}),\,[\tfrac{1}{2},\tfrac{3}{4}),\,[\tfrac{3}{4},1]\,\},$$

whereas the principal's optimal 3-element partition is

$$\{ [0,\tfrac{1}{4}), [\tfrac{1}{4},\tfrac{1}{2}), [\tfrac{1}{2},1] \}.$$

In order to maintain his information rents for low signals and increase them for some higher signals, the agent has incentives to make the lower partition large and introduce finer partitions of the higher signals. Conversely, the principal prefers to have more accurate information for the low cost signals in order to reduce the information rents paid to the agent (and have no investments for more high cost signals).

23.4.5 Early Reporting

Farlee (1998) examines a setting in which the agent gets an imperfect signal about a cost parameter before contract acceptance. After contract acceptance he gets perfect information about the cost parameter prior to choosing his action. Farlee (1998) examines two types of reporting, (1) *early reporting* in which the agent communicates his imperfect signal prior to contract acceptance (by choosing from a menu of contracts offered by the principal), and (2) *delayed reporting* in which the agent communicates his private information after contract acceptance and getting perfect information about the cost parameter. Clearly, early reporting is weakly preferred by the principal to delayed reporting for basically the same reason as sequential reporting is weakly preferred to simultaneous reporting with post-contract information (see Section 22.7). That is, an optimal delayed communication contract is also a feasible early reporting contract, whereas the converse is not necessarily true. Farlee (1998) demonstrates that cases exist in which early reporting is strictly preferred by the principal to delayed communication. Moreover, an interesting aspect of his analysis is that even though the principal prefers early to delayed communication, the total expected surplus may be higher with delayed communication, i.e., the principal's benefit from early communication is lower than the agent's loss of information rents.

We illustrate these results using the hurdle model. The agent's pre-contract information is an imperfect signal y_1 about the hurdle. He can obtain one of two signals y_1^L and y_1^H with probabilities $\varphi(y_1^L) = \varphi(y_1^H) = ½$. The conditional densities for the hurdle are

$$\varphi(h|y_1) = \begin{cases} (1+k) - 2kh & \text{if } y_1 = y_1^L, \\ (1-k) + 2kh & \text{if } y_1 = y_1^H, \end{cases} \qquad (23.10)$$

with $k \in [0, 1]$.[14] The signal y_1 is uninformative about the hurdle if $k = 0$ (corresponding to no pre-contract information), and its information content increases with k. The agent observes the hurdle $y_2 = h$ after contract acceptance but prior to determining how high he wants to jump. We assume the agent commits to the contract once accepted, i.e., he cannot quit after observing the hurdle.

The principal's decision problem can be formulated and solved by extending the analysis used for the basic model in Section 23.1, but we illustrate how it can be formulated and solved within the framework of the mechanism design model. There is no value to communication in the basic mechanism design model since the agent has perfect information about the outcome prior to contract acceptance. However, in the current setting, the pre-contract information is imperfect and communication may be valuable.

We assume that both the principal and the agent are risk neutral. Moreover, the spanning condition is satisfied at the second reporting date. Hence, there is no value to communication at that date, and we may solve the problem as if the productivity measure (i.e., whether the agent jumps, $\gamma = 1$, or not, $\gamma = 0$) is observable (see Section 23.3). A cut-off $\hat{h}$ is induced by jump-contingent compensation $\boldsymbol{c}(\gamma = 1, \hat{h})$ and $\boldsymbol{c}(\gamma = 0, \hat{h})$ if, and only if, the following incentive compatibility constraint is satisfied, independent of y_1:

$$v(\hat{h}) = \boldsymbol{c}(\gamma = 1, \hat{h}) - \boldsymbol{c}(\gamma = 0, \hat{h}). \tag{23.11}$$

The principal's value function given y_1 and induced cut-off $\hat{h}$ is

$$\begin{aligned} V(y_1, \hat{h}) &= \Phi(\hat{h}\,|y_1)\,\mathrm{E}[x|\gamma = 1] + (1 - \Phi(\hat{h}\,|y_1))\,\mathrm{E}[x|\gamma = 0] \\ &= \Phi(\hat{h}\,|y_1)\,(1 - 2\varepsilon)(x_g - x_b) + \varepsilon(x_g - x_b) + x_b, \end{aligned}$$

and, similarly, the agent's cost function is[15]

$$\kappa(y_1, \hat{h}) = \int_0^{\hat{h}} v(h)\, d\Phi(h|y_1).$$

The agent's expected compensation given y_1 and induced cut-off $\hat{h}$ is

$$\bar{c}(y_1, \hat{h}) = \Phi(\hat{h}|y_1)\boldsymbol{c}(\gamma = 1, \hat{h}) + (1 - \Phi(\hat{h}|y_1))\boldsymbol{c}(\gamma = 0, \hat{h}).$$

---

[14] This information structure is the same as used in Section 22.8.

[15] It is readily verified that $\kappa(y_1, \hat{h})$ is increasing and convex in $\hat{h}$ for both private signals.

Substituting in incentive compatibility constraint (23.11) yields

$$\bar{c}(y_1,\hat{h}) = c(y=1,\hat{h}) - v(\hat{h})(1 - \Phi(\hat{h}|y_1)). \qquad (23.12)$$

With *early reporting*, the principal's mechanism design problem can be viewed as choosing an y_1-contingent cut-off schedule $\{\hat{h}(y_1^L), \hat{h}(y_1^H)\}$ from which the agent chooses (by announcing his type). The truthtelling constraints are

$$\bar{c}(y_1,\hat{h}(y_1)) - \kappa(y_1,\hat{h}(y_1)) \geq \bar{c}(y_1,\hat{h}(m)) - \kappa(y_1,\hat{h}(m)), \quad \forall\, y_1, m \in \{y_1^L, y_1^H\}.$$

Using that only the truthtelling constraint for the low cost signal y_1^L is binding, and that a high cost agent gets no information rent, the information rent for the low cost signal can be calculated as follows:

$$\begin{aligned} IR(y_1^L,\hat{h}(y_1^H)) &\equiv \bar{c}(y_1^L,\hat{h}(y_1^L)) - \kappa(y_1^L,\hat{h}(y_1^L)) - \bar{U} \\ &= \bar{c}(y_1^L,\hat{h}(y_1^H)) - \kappa(y_1^L,\hat{h}(y_1^H)) \\ &\qquad - \left[\bar{c}(y_1^H,\hat{h}(y_1^H)) - \kappa(y_1^H,\hat{h}(y_1^H))\right] \\ &= v(\hat{h}(y_1^H))\left[\Phi(\hat{h}(y_1^H)|y_1^L) - \Phi(\hat{h}(y_1^H)|y_1^H)\right] \\ &\qquad - \int_0^{\hat{h}(y_1^H)} v(h)\left[\varphi(h|y_1^L) - \varphi(h|y_1^H)\right]dh. \end{aligned}$$

Hence, the principal's unconstrained mechanism design problem, i.e., maximizing the total expected surplus minus the expected information rent to the low cost agent, can be formulated as follows:

$$\begin{aligned} \underset{\hat{h}(y_1^L),\hat{h}(y_1^H)}{\text{maximize}} \; & \sum_{y_1}\left[V(y_1,\hat{h}(y_1)) - \kappa(y_1,\hat{h}(y_1))\right]\varphi(y_1) \\ & - IR(y_1^L,\hat{h}(y_1^H))\,\varphi(y_1^L). \qquad (23.13) \end{aligned}$$

The first-order conditions determine the optimal y_1-contingent cut-off schedule:

$$(1 - 2\varepsilon)(x_g - x_b) = v(\hat{h}(y_1^L)),$$

$$(1 - 2\varepsilon)(x_g - x_b) = v(\hat{h}(y_1^H)) + v'(\hat{h}(y_1^H))\frac{\Phi(\hat{h}(y_1^H)|y_1^L) - \Phi(\hat{h}(y_1^H)|y_1^H)}{\varphi(\hat{h}(y_1^H)|y_1^H)}.$$

Note that the optimal cut-off for the low cost signal y_1^L is independent of the informativeness of the pre-contract signal, and is equal to the optimal cut-off for y_1 publicly observable (i.e., no distortions at the bottom). The cut-off for the high cost signal is lower due to its impact on the information rents to the low cost agent. Using the conditional densities in (23.10) and $v(h) = h/(1 - h)$, the optimal cut-off for the high cost signal is given by

$$(1 - 2\varepsilon)(x_g - x_b) = v(\hat{h}(y_1^H))\left[1 + \frac{2k}{1 - k + 2k\hat{h}(y_1^H)}\right]. \tag{23.14}$$

Note that the optimal cut-off for the low cost signal decreases as the pre-contract signal gets more informative, i.e., for higher values of k.

The expected compensations for the two signals are given by

$$\bar{c}(y_1^L, \hat{h}(y_1^L)) = \bar{U} + \kappa(y_1^L, \hat{h}(y_1^L)) + IR(y_1^L, \hat{h}(y_1^H)),$$

$$\bar{c}(y_1^H, \hat{h}(y_1^H)) = \bar{U} + \kappa(y_1^H, \hat{h}(y_1^H)),$$

and the jump-contingent compensations are then determined by (23.11) and (23.12).

The optimal solution to the *delayed reporting program* can be found by solving (23.13) subject to the constraint that the cut-offs must be the same for both private signals, i.e., $\hat{h}(y_1^L) = \hat{h}(y_1^H) = \hat{h}^d$. The first-order condition for this program yields that the common optimal cut-off is determined by

$$\begin{aligned}(1 - 2\varepsilon)(x_g - x_b) &= v(\hat{h}^d) + \tfrac{1}{2}v'(\hat{h}^d)\left[\Phi(\hat{h}^d|y_1^L) - \Phi(\hat{h}^d|y_1^H)\right] \\ &= v(\hat{h}^d)(1 + k).\end{aligned} \tag{23.15}$$

Since $k \le 1$ and $\hat{h}(y_1^H) < 1$, it follows that the optimal cut-off with delayed communication is between the optimal cut-offs with early communication. Given the optimal hurdle, the constraints which determine the optimal jump-contingent compensation levels are the same as those used to determined the optimal early communication contract.

The optimal contracts for early and delayed communication programs are illustrated in Table 23.4 using a numerical example based on the following numerical data:

$$v(a) = a/(1-a);\ \bar{U} = 0;\ x_g = 20, x_b = 10;\ \varepsilon = 0.15;\ k = 1.$$

	$U^p(\mathbf{c},\mathbf{a},\eta)$	$c(\gamma=1,y_1^L)$	$c(\gamma=0,y_1^L)$	$\hat{h}(y_1^L)$	$c(\gamma=1,y_1^H)$	$c(\gamma=0,y_1^H)$	$\hat{h}(y_1^H)$
Early	15.54	2.148	-4.852	0.88	2.023	-0.977	0.75
Delayed	15.492	2.23	-1.27	0.78	2.23	-1.27	0.778

Table 23.4: Optimal contracts for early and delayed communication with $k = 1$.

Note that the higher cut-off for the low cost message with early communication is obtained by subsequently severely punishing the agent if he announces that his hurdle is above the cut-off. It is incentive compatible for the agent to truthfully report that he has obtained the low cost signal since the probability that the hurdle is above the cut-off is low (and there is a premium for reporting the low cost signal if he subsequently reports that the hurdle is below the cut-off).

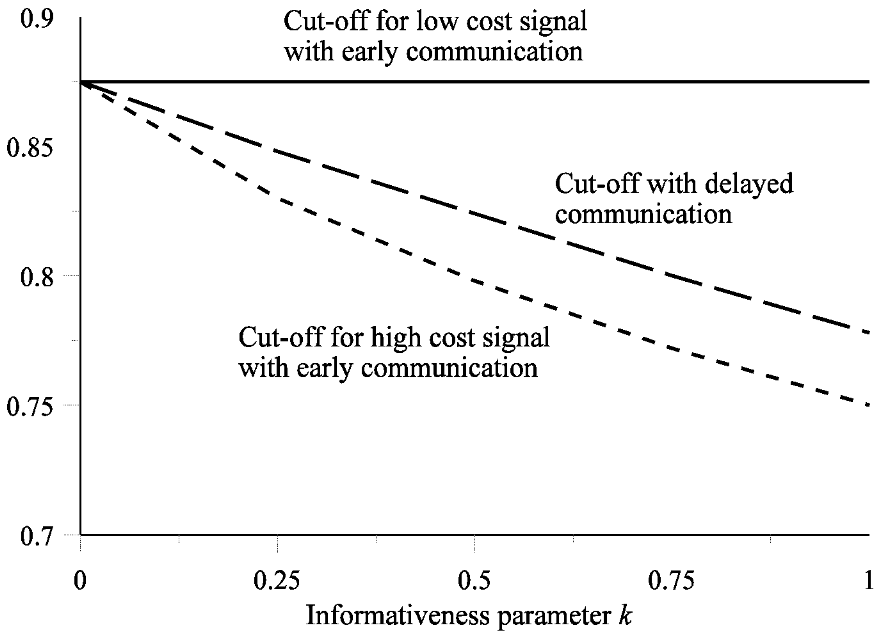

Figure 23.10: Optimal cut-offs with early and delayed communication for varying informativeness of the pre-contract signal.

Figure 23.10 shows the optimal cut-offs with early and delayed communication with varying informativeness of the pre-contract signal. With early communication, the optimal cut-off for the low cost signal is unaffected by the informativeness of the pre-contract signal, whereas the optimal cut-off for the high cost signal decreases as the informativeness of the pre-contract signal increases. This latter effect is caused by the fact that the reduction in the low cost type's information rent resulting from a reduction in the high cost signal cut-off increases as the informativeness of the pre-contract signal increases (see (23.14)). With delayed communication, the optimal cut-off also decreases with the informativeness of the pre-contract signal for basically the same reason (see (23.15)).

Figure 23.11 shows the optimal expected profits to the principal as well as the total expected surplus with early and delayed communication with varying informativeness of the pre-contract signal.

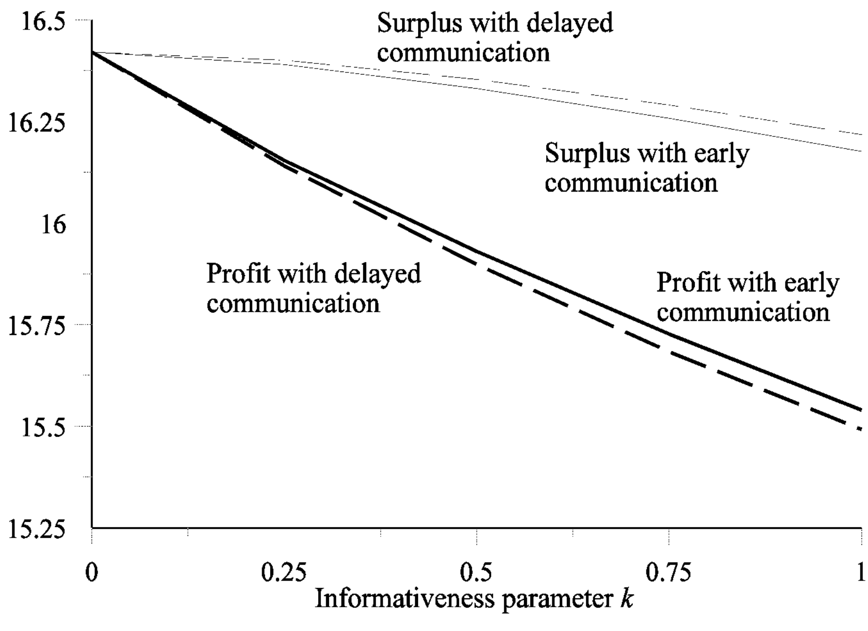

Figure 23.11: Principal's expected profit and total expected surplus with early and delayed communication for varying informativeness of the pre-contract signal.

Given that the agent is risk neutral, the principal can achieve his first-best maximum surplus if the agent has perfect post-contract hurdle information and no pre-contract information. However, if the agent also has pre-contract information, the principal must share his surplus with the agent by giving him information rent if he has the low-cost pre-contract signal. Hence, pre-contract agent information affects the division of the "pie" between the principal and the agent. Since the principal is only concerned about his share, he optimally reduces the total "pie" by inducing an inefficient cut-off so as to reduce the information rent paid to the low-cost agent. The same inefficient cut-off applies to both the low- and high-cost agents if there is delayed communication. However, with early communication the cut-off for the low-cost agent is efficient and the cut-off for the high-cost agent is even more inefficient than in the delayed communication case. This use of message contingent cut-offs reduces both the total pie and the low-cost agent's share (i.e., information rent) – the net effect is an increase in the principal's expected net payoff. The prior probability that the agent jumps is almost the same with early and delayed communication.[16] Hence, the main effect on the size of the "pie" is that the variations in the optimal cut-offs with early communication and the convexity of the agent's cost function makes the expected effort costs higher than with delayed communication.

23.5 CONCLUDING REMARKS

Chapters 22 and 23 both consider information that is received by the agent prior to the agent taking an action in a single-period model. The key difference between the two chapters is that in Chapter 22 the agent must commit to the terms of the contract before he receives his private information, whereas in Chapter 23 the agent can choose to reject the contract after he has observed his private information. The Revelation Principle applies to both settings, so that there always exists an optimal contract which induces the agent to truthfully and fully report his private information to the principal. Nonetheless, in both settings, there is zero value to inducing the agent to report his private information if that information is perfect, i.e., the agent knows the outcome (or other performance measure) that will result from each of his possible action choices.

The agents in Chapter 22 are risk averse, whereas they are often risk neutral in Chapter 23. In Chapter 22, risk neutrality (with unlimited liability) results in a first-best solution that is obtained by selling or renting the firm to the agent. On the other hand, in Chapter 23, the first-best selling price depends on the agent's information which is not known by the principal. As a result, first-best is not achievable and the principal must pay the agent information rents unless

[16] We do not report the prior probability that the agent jumps with early communication, i.e.,

$$\varphi(y_1^L)\,\Phi(\hat{h}(y_1^L)|y_1^L) + \varphi(y_1^H)\,\Phi(\hat{h}(y_1^H)|y_1^H),$$

since it is graphically indistinguishable from the optimal cut-offs with delayed communication (which is the prior probability that the agent jumps with delayed communication).

he has the worst possible signal. Assuming the agent is risk neutral is appealing in Chapter 23 because it makes the analysis more tractable and it does not significantly affect the key results.

The pre-contract information models are particularly relevant to settings in which the agent is an entrepreneur who is taking his firm public, or a setting in which the agent can leave the firm at any point in time. However, the pre-decision, post-contract model is more appropriate for settings in which the manager is committed to staying with the firm. Of course, the latter setting is more appropriately examined in a multi-period model. (See Chapters 25 through 28).

APPENDIX 23A: PROOF OF PROPOSITION 23.6

Let $\mathbf{z}^n = (\mathbf{c}^n, \mathbf{a}^n)$ be an optimal no communication contract as characterized in the text, i.e., outcome-contingent compensation $c_g^n > c_b^n$ and cut-offs $\hat{h}_i^n$, $i = L, H$, such that

$$(1 - \varepsilon_H)c_g^n + \varepsilon_H c_b^n - v(\hat{h}_H^n) = \varepsilon_H c_g^n + (1 - \varepsilon_H)c_b^n > \bar{U},$$

$$(1 - \varepsilon_L)c_g^n + \varepsilon_L c_b^n - v(\hat{h}_L^n) = \varepsilon_L c_g^n + (1 - \varepsilon_L)c_b^n = \bar{U}.$$

The proof demonstrates that there exists a communication contract of the form characterized in the text that implements $\mathbf{a}^n$ (i.e., the same cut-offs) at a lower cost to the principal. The compensation scheme with communication, $\mathbf{c}^c$, is as follows. The compensation is $c_o^c = \underline{U}$ for both ε-types when the agent reports above the respective cut-offs. The outcome-contingent compensations for reporting below the cut-offs are

$$c_g^c = c_g^n + \frac{\varepsilon_L}{1 - \varepsilon_L}\gamma,$$

$$c_b^c = c_b^n - \gamma,$$

where $\gamma > 0$ is determined such that

$$(1 - \varepsilon_H)c_g^c + \varepsilon_H c_b^c - v(\hat{h}_H^n)$$

$$= (1 - \varepsilon_H)c_g^n + \varepsilon_H c_b^n + \frac{\varepsilon_L - \varepsilon_H}{1 - \varepsilon_L}\gamma - v(\hat{h}_H^n) = c_o^c.$$

That is, the ε_L-type gets the same expected compensation as with $\boldsymbol{c}^n$ when he jumps, and the ε_H-type is indifferent between reporting above and below the cut-off if his hurdle is equal to the cut-off $\hat{h}_H^n$ (and strictly prefers to report below the cut-off if his hurdle is strictly below the cut-off).

In order to show that $\boldsymbol{a}^n$ and truthtelling is incentive compatible with $\boldsymbol{c}^c$ it only remains to be shown that both types of agents jump, i.e., $a = h$, when they report below their cut-offs. Consider first an agent who has observed ε_L and $h \le \hat{h}_L^n$:

$$\begin{aligned}
\varepsilon_L c_g^c + (1-\varepsilon_L)c_b^c &= \varepsilon_L c_g^n + (1-\varepsilon_L)c_b^n + \frac{\varepsilon_L^2 - (1-\varepsilon_L)^2}{1-\varepsilon_L}\gamma \\
&< \varepsilon_L c_g^n + (1-\varepsilon_L)c_b^n \\
&= (1-\varepsilon_L)c_g^n + \varepsilon_L c_b^n - v(\hat{h}_L^n) \\
&= (1-\varepsilon_L)c_g^c + \varepsilon_L c_b^c - v(\hat{h}_L^n),
\end{aligned}$$

where the inequality follows from $\varepsilon_L < ½$ and $\gamma > 0$, and the equalities follow from the definition of $\boldsymbol{c}^c$ and $\hat{h}_L^n$. Note that $\boldsymbol{c}^c$ creates slack in the incentive constraint for $a = h$. Similarly, consider an agent who has observed ε_H and $h \le \hat{h}_H^n$:

$$\begin{aligned}
\varepsilon_H c_g^c + (1-\varepsilon_H)c_b^c &= \varepsilon_H c_g^n + (1-\varepsilon_H)c_b^n + \frac{\varepsilon_L - (1-\varepsilon_H)}{1-\varepsilon_L}\gamma \\
&< \varepsilon_H c_g^n + (1-\varepsilon_H)c_b^n + \frac{\varepsilon_L - \varepsilon_H}{1-\varepsilon_L}\gamma \\
&= (1-\varepsilon_H)c_g^n + \varepsilon_H c_b^n - v(\hat{h}_H^n) + \frac{\varepsilon_L - \varepsilon_H}{1-\varepsilon_L}\gamma \\
&= (1-\varepsilon_H)c_g^c + \varepsilon_H c_b^c - v(\hat{h}_H^n),
\end{aligned}$$

where the inequality follows from $\varepsilon_H < ½$ and $\gamma > 0$, and the equalities again follow from the definition of $\boldsymbol{c}^c$ and $\hat{h}_H^n$. As for ε_L, $\boldsymbol{c}^c$ creates slack in the incentive constraint for $a = h$. Hence, an agent observing a hurdle below the cut-off prefers to jump the hurdle when he has reported a hurdle below the cut-off. Moreover, the definition of $\boldsymbol{c}^c$ implies that the truthtelling constraints are satisfied as well.

The contract $\boldsymbol{c}^c$ is such that both types of non-jumping agents get their reservation wages, whereas jumping agents get information rents if their hurdle is strictly below their cut-offs. Hence, the contract $(\boldsymbol{c}^c, \boldsymbol{a}^n)$ is a feasible communication contract. However, it is less costly to the principal than $\boldsymbol{z}^n$ since (i) the expected compensation to agents observing ε_L remains unchanged, (ii) non-jumping agents who have observed ε_H obtain their reservation wage for $\boldsymbol{c}^c$ as opposed to strictly positive information rents with $\boldsymbol{z}^n$, and (iii) a jumping agent who has observed ε_H gets lower expected compensation:

$$(1 - \varepsilon_H)c_g^c + \varepsilon_H c_b^c = (1 - \varepsilon_H)c_g^n + \varepsilon_H c_b^n + \frac{\varepsilon_L - \varepsilon_H}{1 - \varepsilon_L}\gamma$$

$$< (1 - \varepsilon_H)c_g^n + \varepsilon_H c_b^n,$$

since $\varepsilon_L < \varepsilon_H$. **Q.E.D.**

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CHAPTER 24

INTRA-PERIOD CONTRACT RENEGOTIATION

The basic principal-agent model assumes that the two parties establish a contract at the start of the period and there can be no changes to the contract subsequent to that date. *The two parties make a binding commitment that cannot be broken even if both parties would prefer to change the terms of the contract at some subsequent date prior to the "end of the period."* Is this assumption plausible and, in particular, is it enforceable? That is, would the courts prohibit the change in a contract if both parties agreed to that change?

The Incentive to Renegotiate

Why would a principal and an agent want to renegotiate a contract? If the initial contract is optimal, does that not mean that any change in the contract that would make one party better off would make the other worse off?

The answer depends on the timing of the potential renegotiation. The contract is optimal *ex ante*. Therefore, no Pareto-improvement is possible prior to changes in the information available to the two parties. However, once their information changes, it may be possible to make an *ex post* "improvement" in the contract.

To illustrate, consider the simple one-period principal-agent model in which the two parties have agreed to an efficient compensation contract $\mathbf{c}^I: X \rightarrow C$, where a verified report of outcome x will be generated at the end of the period. Now consider a date between when the agent implemented his action a and when the two parties receive information about the outcome x.

initial contract $\mathbf{c}^I$ is set	agent selects action a	renegotiated contract $\mathbf{c}^R$ is set	x is reported and contract $\mathbf{c}^R$ is settled

At the renegotiation date, the agent's belief about x is $\varphi(x|\hat{a})$ where $\hat{a}$ is the action he *has selected*. The principal's belief about x, on the other hand, is

$$\varphi(x|\Psi) = \int_A \varphi(x|a)\, d\Psi(a),$$

where $\Psi(a)$ is the principal's *belief about the action that was selected by the agent.*

If the principal believes with certainty that the agent took $\hat{a}$, then the two parties have homogeneous beliefs. With homogeneous beliefs and no further actions to be taken, the two parties face a classic risk sharing problem. The principal will receive $x - \mathbf{c}^I(x)$ and the agent will receive $\mathbf{c}^I(x)$ – both random amounts. Efficient risk sharing implies that they agree to a renegotiated contract $\mathbf{c}^R$ that satisfies the following conditions (where $u^p(x-c)$ and $u^a(c,a) = u(c) - v(a)$ are the principal's and agent's utility functions, respectively):

efficiency:

$$\frac{u^{p\prime}(x - \mathbf{c}^R(x))}{u'(\mathbf{c}^R(x))} = \lambda, \quad \forall\, x \in X,$$

principal's acceptance:

$$\int_X u^p(x - \mathbf{c}^R(x))\, d\Phi(x|\hat{a}) \;\geq\; \int_X u^p(x - \mathbf{c}^I(x))\, d\Phi(x|\hat{a}),$$

agent's acceptance:

$$\int_X u(\mathbf{c}^R(x))\, d\Phi(x|\hat{a}) \;\geq\; \int_X u(\mathbf{c}^I(x))\, d\Phi(x|\hat{a}).$$

The first condition indicates that the contract will be renegotiated, unless no incentive constraints were binding at the time the initial contract was established.

For example, if the principal is risk neutral, the two parties will agree to a contract in which all risk is shifted to the principal, i.e., $\mathbf{c}^R(x) = w$, where w is a fixed amount satisfying the second two inequalities. There will almost certainly be a range of w values that satisfy the above conditions. The amount selected will depend on the *relative bargaining power* of the two parties at the time of the renegotiation.

Observe that if the principal believes that the agent did not anticipate any renegotiation when he selected his action, the principal will hold belief $\varphi(x|\hat{a})$ where $\hat{a}$ is the action induced by the initial contract. However, if the agent anticipates the renegotiation, he will select action a^o, where a^o minimizes $v(a)$. Then, if the principal "knows" that the agent has selected a^o, there will be homogeneous beliefs $\varphi(x|a^o)$. Furthermore, if the principal "knows" that the agent *will anticipate* renegotiation when he selects his action, the principal can do no better than offer the agent an initial contract $\mathbf{c}^1$ that pays a fixed wage that is sufficient to compensate the agent for his minimal effort a^o. *Hence, the agent's anticipation of renegotiation makes it impossible to induce any effort above a^o and, thus, makes the principal worse off.*

A number of papers consider mechanisms that reduce the loss caused by the inability of the contracting parties to preclude contract renegotiation. In these papers the principal's knowledge (or lack of knowledge) about the history of the game at the renegotiation stage plays a crucial role. In Section 24.1 we consider a setting in which the agent randomizes across actions so that the principal at the renegotiation stage has imperfect information about the agent's action. Hence, the principal does not know the certainty equivalent of the agent's compensation and therefore he cannot offer the agent perfect insurance. In Section 24.2 we consider a two-period model in which there is renegotiation before the first-period outcome is observed. If outcomes are directly contractible, the only contracts that induce more than the lowest possible action are randomized contracts as in Section 24.1. However, if outcomes are self-reported by the agent, contracts exist that induce pure action strategies. In this case, perfect insurance is eliminated due to the fact that there has to be a premium for reporting good outcomes in order to induce truthful reporting and subsequent actions. Cases exist in which the principal prefers a setting with self-reported outcomes over the setting with directly contractible outcomes. The general lesson seems to be that the less the principal knows (or the less confidence he has) at the renegotiation stage, the better, i.e., there is an advantage to not knowing!

In Section 24.3 we consider a model with a different perspective in which renegotiation may be beneficial to the agency. In that model renegotiation facilitates contracting on unverifiable and, therefore, not directly contractible information observable to both the principal and the agent. In Sections 24.4 and 24.5 we consider models in which the principal observes (private) unverifiable information about the agent's action.

24.1 RENEGOTIATION-PROOF CONTRACTS

Fudenberg and Tirole (FT) (1990) examine the problem in which only the least costly action can be implemented as a pure strategy when there is renegotiation after the agent has taken his action. They propose an equilibrium in which the agent plays a *mixed strategy* when he selects his action. Hence, while the agent knows which action he has selected at the renegotiation date, the principal holds beliefs determined by the equilibrium mixed strategy $\psi(a)$. To induce the playing of such a mixed strategy, the principal offers an initial contract $\mathbf{c}$ that contains a *menu of contracts* of the form $\mathbf{c}(x,m)$, where $m \in A$ is an *unverified message* about the agent's action that the agent issues at the "contract selection" or "renegotiation date." Observe that if the principal holds beliefs based on $\psi(a)$, we can interpret the renegotiation stage as a contracting setting in which the principal faces pre-contract agent information (see Chapter 23). The menu of contracts provides a means by which the principal induces the agent to truthfully report his action. In general, the agent will receive a low fixed wage if he announces the selection of a^o and will receive riskier contracts with higher expected payoffs for actions that require more effort.

The *principal's ex ante decision problem* can be characterized as one in which he offers a *renegotiation-proof* menu of contracts (let $\boldsymbol{c}(m)$ denote the contract $\boldsymbol{c}(\cdot,m)$ associated with message m):

$$\underset{\Psi,\, \boldsymbol{c}}{\text{maximize}} \quad \int_A U^p(\boldsymbol{c}(a),a)\, d\Psi(a),$$

$$\text{subject to} \quad \int_A U^a(\boldsymbol{c}(a),a)\, d\Psi(a) \;\geq\; \bar{U},$$

$$U^a(\boldsymbol{c}(a),a) \;\geq\; U^a(\boldsymbol{c}(m),a'), \quad \forall\; a,m,a' \in A \text{ and } \psi(a) > 0,$$

$$\boldsymbol{c} \in \underset{\boldsymbol{c}^R}{\text{argmax}} \quad \int_A U^P(\boldsymbol{c}^R(a),a)\, d\Psi(a)$$

subject to

$$U^a(\boldsymbol{c}^R(a),a) \;\geq\; U^a(\boldsymbol{c}(a),a), \quad \forall\; a \in A \text{ and } \psi(a) > 0,$$

$$U^a(\boldsymbol{c}^R(a),a) \;\geq\; U^a(\boldsymbol{c}^R(m),a), \quad \forall\; a,m \in A.$$

The first constraint is the standard contract (menu) acceptance constraint. The second set of constraints ensures that the agent is indifferent between all actions that have a positive probability of occurrence with mixed strategy $\psi(a)$, and that these actions and truth-telling (through the menu choice) are preferred to any other action and/or lying. The third constraint ensures the contract is renegotiation-proof. In the renegotiation-proof contract the mixed strategy $\psi(a)$ is taken as given and is used to compute the principal's expected return from a menu of contracts that must satisfy two types of constraints. The first set of renegotiation-proof constraints ensures that the agent weakly prefers the proposed menu to the existing menu for each action-contingent contract – since the agent knows which action he has taken at the time of renegotiation. The second set of renegotiation-proof constraints ensures that the proposed menu would induce the agent to truthfully reveal the action he has taken.

In formulating the principal's decision problem we have assumed that the initial contract must be renegotiation-proof. FT show that there is no loss of generality in restricting the analysis to such contracts.

Proposition 24.1 (FT, Prop. 2.1) *Renegotiation-proof Contracts*

If there is a Nash equilibrium with mixed strategy $\psi(a)$ over effort levels, the initial contract is $\boldsymbol{c}^I$ and the final contract is $\boldsymbol{c}^R$, then there is an equilibrium with the same distribution over efforts where $\boldsymbol{c}^R$ is the initial as well as the final contract.

Of course, if $\boldsymbol{c}^R$ is offered as the initial contract, $\boldsymbol{c}^R$ is renegotiation-proof given $\psi(a)$. Since the agent's utility only depends on the final contract, $\psi(a)$ is also incentive compatible for the agent when $\boldsymbol{c}^R$ is offered as the initial contract.

Note that the renegotiation stage can be viewed as a setting in a "screening" game in which an uninformed insurer (the principal) offers a menu of insurance contracts to an informed insuree (the agent).

The optimal renegotiation-proof contracts are most easily illustrated in a setting with a risk neutral principal, two actions, $A = \{a_L, a_H\}$, and two outcomes, $X = \{x_g, x_b\}$, where the probability of the good outcome is higher for the high type (action) than the low type (action), i.e., $\varphi(x_g|a_H) > \varphi(x_g|a_L)$, and the least cost effort is a_L. To illustrate the characteristics of the optimal renegotiation-proof contract consider Figure 24.1, where $\boldsymbol{c}_L^I$ and $\boldsymbol{c}_H^I$ are initial L-type and H-type contracts, respectively, that are incentive compatible but not optimal. The indifference curves denoted $U^a(\boldsymbol{c},a_i) = U^a(\boldsymbol{c}_L^I,a_i)$ represent the outcome-contingent compensation that is equivalent to $\boldsymbol{c}_L^I$ given the agent has taken action a_i, $i = L, H$, and $U^a(\boldsymbol{c},a_H) = U^a(\boldsymbol{c}_L^I,a_L)$ is the H-type's indifference curve such that the agent is indifferent between choosing a_L and $\boldsymbol{c}_L^I$ versus a_H and $\boldsymbol{c}$.

The initial L-type contract is below the no-risk line, i.e., does not provide full insurance. The shaded region is the set of *ex post* truth-inducing contracts that can be offered to the H-type given $\boldsymbol{c}_L^I$, i.e., they would not be preferred by the L-type agent, but would be preferred by the H-type. The initial H-type contract $\boldsymbol{c}_H^I$ is in that set, as well as being on the indifference denoted $U^a(\boldsymbol{c},a_H) = U^a(\boldsymbol{c}_L^I,a_L)$. Hence the initial contracts are incentive compatible both with respect to the *ex ante* randomization between a_L and a_H and the *ex post* reporting of his type. However, the initial contract is not renegotiation-proof. To see this, consider $\boldsymbol{c}_L^R$, which is a no-risk contract on the L-type's indifference curve. It is less costly to the principal (by Jensen's inequality) and is acceptable to the L-type. Furthermore, the H-type strictly prefers $\boldsymbol{c}_H^I$ to $\boldsymbol{c}_L^R$, so that truth-telling is maintained. Hence, it is clear that the optimal contract will impose no risk on the agent if he selects a_L. The initial H-type contract is also below the no-risk line, and while some risk is required for incentive purposes, $\boldsymbol{c}_H^I$ imposes too much risk. The revised contract $\boldsymbol{c}_H^R$ is the minimum risk (i.e., least costly contract) that maintains truth-telling and is acceptable to the H-type with $\boldsymbol{c}_H^I$ – it is at the intersection of the indifference curves for $U^a(\boldsymbol{c}_L^I, a_L)$ and $U^a(\boldsymbol{c}_H^I, a_H)$. That is, there is sufficient risk to ensure that the L-type will not prefer to choose $\boldsymbol{c}_H^R$.

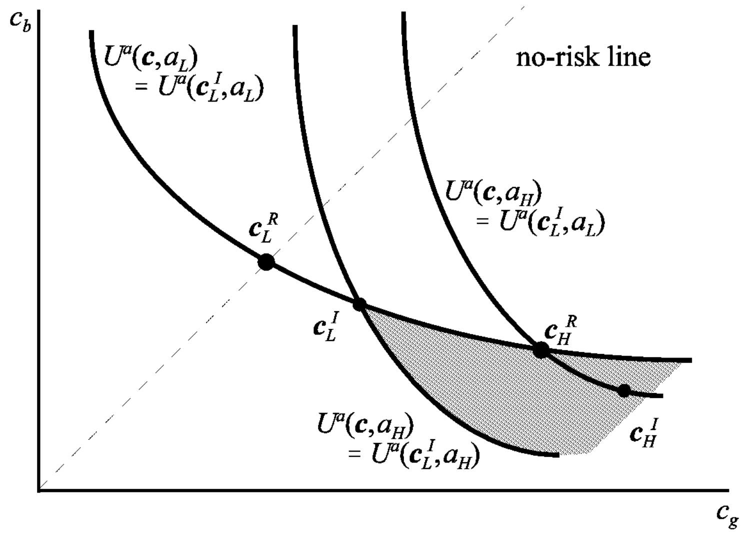

Figure 24.1: Renegotiated contracts.

Observe that while $\boldsymbol{c}_L^R$ and $\boldsymbol{c}_H^R$ induce *ex post* truthtelling, they are not incentive compatible *ex ante*. In particular, the structure is such that $U^a(\boldsymbol{c}_L^R, a_L) < U^a(\boldsymbol{c}_H^R, a_H)$, i.e., the agent will strictly prefer to choose a_H knowing that renegotiation will lead to a better result than choosing a_L. Proposition 24.2 summarizes the preceding arguments, and Figure 24.2 depicts a renegotiation-proof contract.

Proposition 24.2 (FT, Lemma 2.1)

With two actions and two outcomes, if the contract $\boldsymbol{c}$ is renegotiation-proof and consistent with distribution $\psi(a_H) \in (0,1)$, then

(a) $\boldsymbol{c}(x_g, a_L) = \boldsymbol{c}(x_b, a_L) \equiv c_L$, and $\boldsymbol{c}(x_g, a_H) > \boldsymbol{c}(x_b, a_H)$,

(b) $U^a(\boldsymbol{c}(a_L), a_L) = U^a(\boldsymbol{c}(a_H), a_L)$,

(c) $U^a(\boldsymbol{c}(a_L), a_L) = U^a(\boldsymbol{c}(a_H), a_H)$.

Of course, if $\psi(a_H) = 0$, the renegotiation-proof contract is a constant wage, and the agent always chooses the least cost effort. The conditions (a) - (c) do not impose any conditions on the distribution $\psi(a_H)$ for a renegotiation-proof contract, i.e., they are merely necessary conditions.

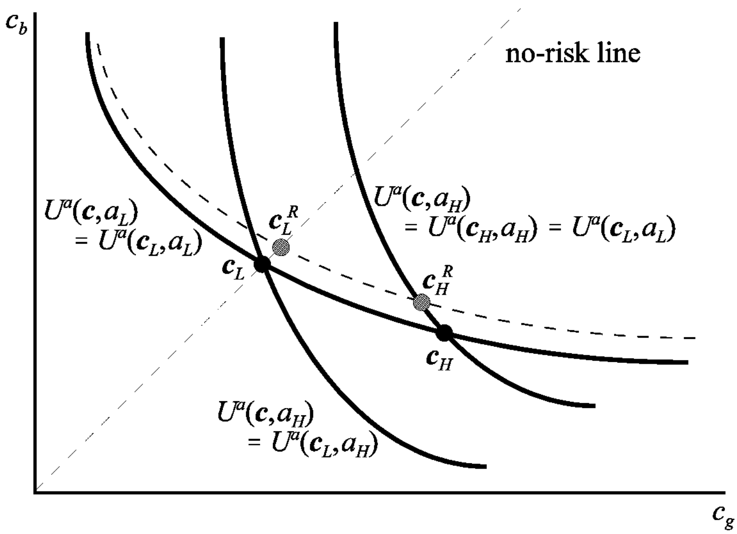

Figure 24.2: Renegotiation-proof contracts.

Of course, the principal would like the probability of the high action, $\psi(a_H)$, to be as high as possible. The key restriction on this probability comes from the principal's incentive to reduce the risk imposed on the high type at the renegotiation stage. If the risk is reduced for the high type from $\boldsymbol{c}_H$ to $\boldsymbol{c}_H^R$ (such that it is acceptable to the high type), the fixed wage for the low type must be increased from $\boldsymbol{c}_L$ to $\boldsymbol{c}_L^R$ in order for the low type not to select the high-type contract. Reducing the risk for the high type reduces the principal's expected compensation cost (by Jensen's inequality), whereas increasing the fixed wage for the low type increases the expected compensation cost. If $\boldsymbol{c}$ is renegotiation-proof for distribution $\psi(a_H)$, the total expected compensation cost must be at least as high for $\boldsymbol{c}^R$ as for $\boldsymbol{c}$, and this will be the case if $\psi(a_H)$ is not too high. Clearly, $\psi(a_H)$ must be strictly less than one since, otherwise, the principal would offer the high type full insurance.

Let a marginal change in the contract be parameterized by a marginal change, δ, in the fixed wage for the low type. The expected compensation cost for the proposed renegotiated contract is

$$(1 - \psi_H)[c_L + \delta]$$
$$+ \psi_H[\varphi(x_g|a_H)\{\boldsymbol{c}(x_g,a_H) + \delta_g(\delta)\} + \varphi(x_b|a_H)\{\boldsymbol{c}(x_b,a_H) + \delta_b(\delta)\}],$$

where the variations in the outcome-contingent wages for the high type are determined by the truth-telling constraint for the low type and the indifference constraint for the high type, respectively, i.e.,

$$u(c_L + \delta) = \varphi(x_g|a_L)u(\boldsymbol{c}(x_g,a_H) + \delta_g(\delta)) + \varphi(x_b|a_L)u(\boldsymbol{c}(x_b,a_H) + \delta_b(\delta)),$$

$$\varphi(x_g|a_H)u(\boldsymbol{c}(x_g,a_H) + \delta_g(\delta)) + \varphi(x_b|a_H)u(\boldsymbol{c}(x_b,a_H) + \delta_b(\delta))$$
$$= \varphi(x_g|a_H)u(\boldsymbol{c}(x_g,a_H)) + \varphi(x_b|a_H)u(\boldsymbol{c}(x_b,a_H)).$$

The first-order condition determining the maximum probability for the high type ψ_H^* is

$$(1 - \psi_H^*) + \psi_H^*\{\varphi(x_g|a_H)\delta_g'(0) + \varphi(x_b|a_H)\delta_b'(0)\} = 0$$

or, equivalently,

$$\frac{\psi_H^*}{1 - \psi_H^*} = - \frac{1}{\varphi(x_g|a_H)\delta_g'(0) + \varphi(x_b|a_H)\delta_b'(0)}, \tag{24.1}$$

where the marginal variations in the outcome-contingent wages for the high type are determined by

$$u'(c_L) = \varphi(x_g|a_L)u'(\boldsymbol{c}(x_g,a_H))\,\delta_g'(0) + \varphi(x_b|a_L)u'(\boldsymbol{c}(x_b,a_H))\,\delta_b'(0),$$

$$\varphi(x_g|a_H)\,u'(\boldsymbol{c}(x_g,a_H))\,\delta_g'(0) + \varphi(x_b|a_H)u'(\boldsymbol{c}(x_b,a_H)\,\delta_b'(0) = 0.$$

Proposition 24.3 (FT, Lemma 2.2)

With two actions and two outcomes, the contract $\boldsymbol{c}$ is renegotiation-proof and consistent with distribution $\psi(a_H) \in (0,1)$ if, and only if, the conditions (a) - (c) in Proposition 24.2 hold, and $\psi(a_H) \le \psi_H^*$.

FT also consider the case with a continuum of actions $a \in A = [\underline{a},\bar{a}]$ (and a continuum of outcomes). They show that a renegotiation-proof contract is characterized by a mixed strategy $\psi(a)$ over actions with support $[\underline{a},\tilde{a}]$. The mixed strategy has no mass points except possibly at the lowest possible action $\underline{a}$ (and no "gaps"),[1] and the upper bound on the support of $\psi(a)$ is strictly greater than the second-best effort level, $a^\dagger$, in an equivalent problem with no renegotia-

[1] FT restrict their analysis to additively separable preferences, i.e., $u^a(c,a) = u(c) - v(a)$. There is a mass point at $\underline{a}$ if, and only if, $v'(\underline{a}) > 0$.

tion, i.e., $\tilde{a} > a^{\dagger}$. In this case, the *ex ante* incentive compatibility constraints completely determine the form of an incentive compatible contract. The main force of the no-renegotiation constraint is that it restricts the admissible set of mixed strategies $\psi(a)$ (see their Proposition 5.1).

A key characteristic of the menu of contracts with a continuum of actions is that the contracts get "riskier" for higher levels of actions. To illustrate this, consider a *LEN* model with a single task and a single performance measure, i.e.,

$$y = a + \varepsilon, \quad \varepsilon \sim \mathrm{N}(0,\sigma^2), \text{ and}$$

$$u^a(c,a) = -\exp[-r(c - ½a^2)].$$

For a given mixed strategy $\psi(a)$ with support $\tilde{A} = [0,\tilde{a}]$, we assume that the principal is restricted to offering menus of linear contracts, i.e.,[2]

$$\mathbf{c}(y,m) = f(m) + v(m)y, \quad \text{for all } m \in \tilde{A}.$$

If the agent has taken action a and reports action m, the agent's certainty equivalent is

$$CE(m,a) = f(m) + v(m)a - ½rv(m)^2\sigma^2 - ½a^2, \quad \text{for all } a,m \in \tilde{A}.$$

Hence, the first-order condition for truthful reporting is

$$f'(a) + v'(a)a - rv'(a)v(a)\sigma^2 = 0, \quad \text{for all } a \in \tilde{A}. \tag{24.2}$$

Secondly, *ex ante* incentive compatibility of the mixed strategy $\psi(a)$ implies that the agent's certainty equivalent, given that he subsequently reports truthfully, must be the same for all $a \in \tilde{A}$. Since the are no wealth effects with the multiplicatively separable exponential utility, this common certainty equivalent is equal to the agent's reservation wage c^o, i.e.,

$$CE(a,a) = f(a) + v(a)a - ½rv(a)^2\sigma^2 - ½a^2 = c^o, \quad \text{for all } a \in \tilde{A}. \tag{24.3}$$

This implies, that

$$f'(a) + v'(a)a + v(a) - rv'(a)v(a)\sigma^2 - a = 0, \quad \text{for all } a \in \tilde{A}. \tag{24.4}$$

Substituting (24.2) into (24.4) yields

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[2] We assume like FT that there are sufficient penalties available to ensure the agent is not reporting $a \notin \tilde{A}$.

$$\tilde{v}(a) = a, \quad \text{for all } a \in \tilde{A}, \tag{24.5}$$

and substituting (24.5) into (24.3) yields

$$\tilde{f}(a) = ½[r\sigma^2 - 1]a^2 + c^o, \quad \text{for all } a \in \tilde{A}. \tag{24.6}$$

We assume that $r\sigma^2 < 1$.[3] Hence, the *ex ante* incentive compatibility constraints and zero rents with exponential utility completely determines the menu of contracts by (24.4) and (24.5) for a given support of the mixed strategy. The incentive rate is equal to the reported action and, hence, higher actions are associated with greater incentive risk. The fixed wage, on the other hand, decreases with the reported action both to induce truthful reporting and to ensure indifference between actions.

Note that there has been no mention of the no-renegotiation constraint so far, and that the menu of contracts does not depend on the mixed strategy $\psi(a)$. It is the no-renegotiation constraint which determines the admissible mixed strategies. That is, the menu of contracts given by (24.5) and (24.6) is renegotiation-proof for mixed strategy $\psi(a)$, if it minimizes the principal's expected compensation cost at the renegotiation stage subject to the *ex post* individual rationality and truth-telling constraints, i.e.,

$$\{\tilde{v}(a),\tilde{f}(a)\} \in \underset{\{f^r(a),v^r(a)\}}{\text{argmin}} \int_0^{\tilde{a}} \{f^r(a) + v^r(a)a - ½rv^r(a)^2\sigma^2\}\,\psi(a)\,da,$$

subject to

$$f^r(a) + v^r(a)a - ½rv^r(a)^2\sigma^2 \geq c^o + ½a^2, \quad \forall\, a \in [0,\tilde{a}],$$

$$f^{r\prime}(a) + v^{r\prime}(a)a - rv^{r\prime}(a)v^r(a)\sigma^2 = 0, \quad \forall\, a \in [0,\tilde{a}].$$

Of course, this problem characterizes a set of mixed strategies consistent with the no-renegotiation constraint. Hence, the principal must choose an optimal mixed strategy within this set in order to determine an optimal mixed strategy. However, we have not investigated the form the optimal mixed strategy will take.

FT provide extensive analysis of similar settings with optimal contracts, but we will not go any further into those results. While this paper is very interesting from a technical perspective, the contracting arrangements are not broadly

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[3] This ensures that a marginal increase in the incentive rate is beneficial to the agent *ceteris paribus*, i.e., the impact on the expected compensation is higher than on the risk premium.

representative of what we observe in the "real world." There are situations in which managers appear to be offered a menu of contracts from which they can choose at some subsequent date, but randomization across actions seems to be unappealing as a description. In the following section based on Christensen, Demski, and Frimor (CDF) (2002) we consider a mechanism which (partly) avoids randomization.

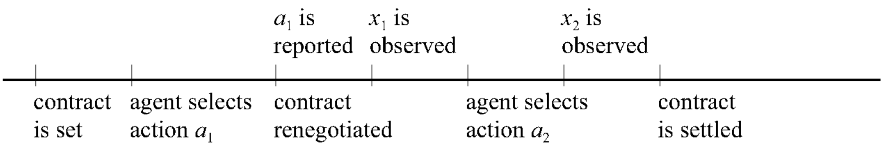
24.2 AGENT-REPORTED OUTCOMES

Consider a repeated binary moral hazard problem with two independent periods similar to the model in Section 19.2.2. As in that section, we abstract from intertemporal consumption smoothing concerns and wealth effects on action choices by assuming the agent has a domain additive exponential utility function:

$$u^a(c_1, c_2, a_1, a_2) = -\exp\left[-r(c_1 + c_2 - \kappa(a_1) - \kappa(a_2))\right].$$

In order to simplify the analysis assume that in each period there are only two possible outcomes $x_g > x_b$ and a continuum of actions, $a_t \in A = [\underline{a}, \overline{a}]$. For each period, the probability function $\varphi(x_g|a_t)$ is increasing and concave, while the cost function $\kappa(a_t)$ is increasing and convex in a_t.

Suppose there is renegotiation after the agent has selected a_1 but before x_1 is observed by either of the two parties, and assume, for simplicity, that there is no subsequent renegotiation.[4] Initially assume that both x_1 and x_2 are directly contractible through perfectly audited reports of outcomes, i.e.,



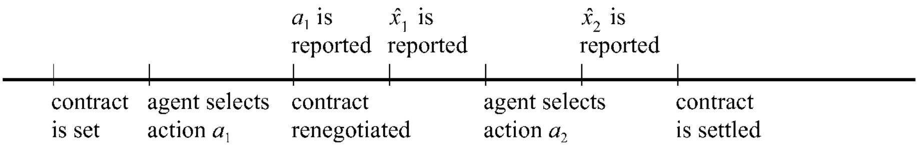
Since there is only one contract renegotiation, and there are no intertemporal dependencies, a renegotiation-proof contract must be of the form

$$c_1 + c_2 = \mathbf{c}_1(x_1, m) + \mathbf{c}_2^*(x_2), \tag{24.7}$$

[4] In a more general model with more than two outcomes later renegotiations may in the case with agent-reported outcomes lead to a breakdown of the Revelation Principle when the agent is reporting the first-period outcome (see Demski and Frimor, 1999).

where $\boldsymbol{c}_2^*(\cdot)$ is an optimal contract for the one-period agency problem for the second period. If an action strategy other than always choosing the lowest first-period action is to be induced, $\boldsymbol{c}_1(.,m)$ is a menu of contracts from which the agent chooses by truthfully reporting his first-period action selected according to a mixed strategy $\psi(a)$. FT show for the case with a continuum of actions that the mixed strategy $\psi(a)$ has no mass points except possibly at the lowest possible action $\underline{a}$. Hence, no pure strategy can be implemented (except the least costly action).

Suppose now that the two outcomes are not directly contractible but the agent personally reports $\hat{x}_1$ and $\hat{x}_2$, respectively, i.e.,



We assume the reporting technology is such that the agent cannot overstate the aggregate outcome at any given date, i.e., $\hat{x}_1 \le x_1$ and $\hat{x}_1 + \hat{x}_2 \le x_1 + x_2$. This specification presumes that there is an (imperfect) auditing technology that prevents the agent from overstating results but it allows for understatements. Thus, this technology implies that the only possible lie at $t = 1$ is for the agent to report $\hat{x}_1 = x_b$ when he as observed x_g, thereby assuring the agent that he can report $\hat{x}_2 = x_g$ even if x_b occurs in the second period.

Since there is no renegotiation after x_1 has been observed by the agent, the Revelation Principle applies. Hence, it can be assumed without loss of generality that the contract is not only renegotiation-proof but also induces the agent to truthfully report a good first-period outcome when it is observed, instead of claiming the outcome is bad and shirking in the second period.[5] We assume that it is optimal to induce $\bar{a}$ in the second period, so that the truth-telling constraint becomes

$$\forall\, a_1 \in [\underline{a},\bar{a}]:\ \varphi(x_g|\bar{a})u^a(c_{1g}(a_1) + c_{2g}, a_1, \bar{a})$$

$$+\ \varphi(x_b|\bar{a})\ u^a(c_{1g}(a_1) + c_{2b},\ a_1, \bar{a}) \ge\ u^a(c_{1b}(a_1) + c_{2g}, a_1, \underline{a}).$$

Equivalently, using the particular form of the agent's utility function, we obtain

$$\forall\, a_1 \in [\underline{a},\bar{a}]:\ c_{1g}(a_1) - c_{1b}(a_1) \ge c_{2g} - \kappa(\underline{a}) - CE(\boldsymbol{c}_2,\bar{a}), \qquad (24.8)$$

[5] Observe that there is no truthtelling constraint for the bad outcome since the auditing technology precludes the agent from reporting a good outcome when it is bad.

where

$$CE(\boldsymbol{c}_2,\bar{a}) = -\frac{1}{r}\ln\Big[\exp[-rc_{2g}]\varphi(x_g|\bar{a}) + \exp[-rc_{2b}]\varphi(x_b|\bar{a})\Big] - \kappa(\bar{a}).$$

That is, the premium for truth-telling in the first period must be at least as high as the gain in certainty equivalents that can be obtained from reporting the bad outcome and obtaining the good second-period compensation with certainty for the least costly effort. Hence, the truth-telling constraint for the good first-period outcome combined with the second-period incentive problem places a bound on how much insurance the principal can offer the agent in the renegotiation stage. This helps the principal to better commit himself in the renegotiation stage.

Proposition 24.4

Let $z = (\boldsymbol{c},\psi(\boldsymbol{a}))$ be a renegotiation-proof and truth-inducing contract with self-reported outcomes and a non-randomized second-period action strategy $\boldsymbol{a}_2 = \bar{a}$.

(a) The compensation scheme can be written as

$$\boldsymbol{c}(x_1,x_2,a_1) = \boldsymbol{c}_1(x_1,a_1) + \boldsymbol{c}_2(x_2).$$

(b) If $\kappa'(\underline{a}) = 0$, there exists a renegotiation-proof and truth-inducing contract $\hat{z} = (\hat{\boldsymbol{c}}, \hat{\boldsymbol{a}})$ with

(i) $\hat{\boldsymbol{c}}(.,a_1)$ independent of a_1, $\hat{c}_{1g} - \hat{c}_{1b} = \boldsymbol{c}_{2g} - \kappa(\underline{a}) - CE(\boldsymbol{c}_2,\bar{a})$, and a non-randomized first-period action choice $\hat{a}_1 > \underline{a}$, such that

$$\hat{a}_1 = \underset{a\in[\underline{a},\bar{a}]}{\text{argmax}} -\Big[\varphi(x_g|a)\exp\{-r(c_{1g}-\kappa(a))\}$$
$$+ \varphi(x_b|a)\exp\{-r(c_{1b}-\kappa(a))\}\Big].$$

(ii) $\hat{\boldsymbol{c}}_2(x_2) = \boldsymbol{c}_2(x_2)$, and $\hat{\boldsymbol{a}}_2 = \bar{a}$.

The key characteristic of this result is that the premium necessary to induce the agent to truthfully report the good first-period outcome precludes the principal from offering full insurance at the renegotiation stage and, thus, a non-trivial pure first-period action strategy can be sustained as part of a negotiation-proof contract. Note that the *optimal* renegotiation-proof and truth-inducing contract may not have pure first-period action strategies. However, first-period actions

$a_1 \in [\underline{a}, \hat{a}_1]$ are not in the support of an optimal randomized first-period action strategy, $\psi(a_1)$.

Another interesting aspect of this analysis is that the setting with agent-reported outcomes is strictly preferred to the setting with directly contractible outcomes, i.e., imperfect auditing of outcomes is strictly preferred to perfect auditing of outcomes.

Proposition 24.5 (CDF, Prop. 4)

Let $z = (\mathbf{c}, \psi(\mathbf{a}))$ be a renegotiation-proof contract with directly contractible outcomes and a non-randomized second-period action strategy $\mathbf{a}_2 = \bar{a}$. If $\kappa'(\underline{a}) = 0$, then there exists a renegotiation-proof and truth-inducing contract with agent-reported outcomes, $\hat{z} = (\hat{\mathbf{c}}, \hat{\psi}(\mathbf{a}))$, that strictly dominates z. In that contract the randomized strategy over first-period actions is given by

$$\hat{\Psi}(a_1) = \begin{cases} 0 & a_1 < \hat{a}_1, \\ \Psi(a_1) & a_1 \geq \hat{a}_1, \end{cases}$$

where $\hat{a}_1$ is determined as the first-period action induced by the minimal premium that induces truthful reporting of the good first-period outcome.

24.3 RENEGOTIATION BASED ON NON-CONTRACTIBLE INFORMATION

In the early principal-agent models it was typically assumed that contracts could be contingent on an event (information) *if, and only if,* that event is observable by both parties. Later a distinction was made between observability and verifiability. Since contract enforcement generally presumes the existence of an enforcement mechanism, such as the courts or "head office," observability by the two parties has been considered necessary but not sufficient for use in contracts. Verifiability is generally presumed, where verifiability refers to the ability to "convince" the enforcement mechanism (e.g., the courts) that an event has taken place. We refer to this as *contractible* information.

When we have used information in contracts, whether it has been the outcome x or some other signal y, we have implicitly assumed that a contractible report of x and/or y will be produced prior to settling up the contract. In this section we consider the potential role of non-contractible information that is common knowledge in principal-agent contracting.[6]

[6] See also the impact of non-contractible investor information in Section 22.8.

24.3.1 Renegotiation after Unverified Observation of the Agent's Action

An interesting result is obtained if the principal observes the agent's action prior to renegotiating their contract. In this setting, the initial contract cannot be contingent on the observed action (since it is not contractible), but *renegotiation permits achievement of the first-best solution.* The expected utilities of the two parties depend on who has the *bargaining power at the time the initial contract is set*, but the initial contract used to achieve the first-best depends on who will have the *bargaining power at the time of renegotiation.*

Hermalin and Katz (HK) (1991) provide the following result for a basic principal-agent model with the principal observing the agent's action prior to renegotiation. It applies to the setting in which the principal has all the bargaining power at the time of renegotiation.

Proposition 24.6 (HK, Prop. 1 and Corollary)
When the agent's action is observable but non-contractible and the principal makes a take-it-or-leave-it offer in renegotiation, then any implementable action is implementable at the *first-best cost.* Furthermore, if there is no moving support and the agent is strictly risk averse, then the principal is strictly better off implementing an action with renegotiation (except for the least-cost action).

Proof: The following focuses on the implementation of the first-best action a^*. If $\mathbf{c}^I$ is the initial contract and the agent takes action a, then the principal will offer the agent a renegotiated contract in which the agent's compensation is a constant, $c^R(a)$, equal to the certainty equivalent of $\mathbf{c}^I$ given a , i.e.,

$$u(\mathbf{c}^R(a)) = \int_X u(\mathbf{c}^I(x))\, d\Phi(x|a).$$

This is less costly to the principal (by Jensen's inequality). The key now is to offer an initial contract that satisfies the following conditions:

$$\int_X u(\mathbf{c}^I(x))\, d\Phi(x|a^*) - v(a^*) = \bar{U},$$

$$a^* \in \underset{a \in A}{\operatorname{argmax}} \int_X u(\mathbf{c}^I(x))\, d\Phi(x|a) - v(a).$$

If a solution exists, then the first-best is achieved. **Q.E.D.**

It is not essential that the principal has the bargaining power at the renegotiation stage (see HK, Propositions 3 and 4). For example, if the agent has all the bargaining power at the time of renegotiation, then (given initial contract $\mathbf{c}^I$ and observed action a) he will offer the principal a contract $\mathbf{c}^R(a)$ that is a constant such that

$$\mathbf{c}^R(a) = \int_X \mathbf{c}^I(x)\, d\Phi(x|a).$$

To achieve the first-best result in this setting (assuming the principal has the bargaining power at the initial contract stage), the principal offers the agent an initial contract that satisfies

$$u\left(\int_X \mathbf{c}^I(x)\, d\Phi(x|a^*)\right) - v(a^*) = \bar{U},$$

$$a^* \in \underset{a \in A}{\text{argmax}}\; u\left(\int_X \mathbf{c}^I(x)\, d\Phi(x|a)\right) - v(a).$$

Observe that in the first setting (in which the principal has the renegotiation bargaining power), the agent achieves his reservation utility whether there is renegotiation or not. On the other hand, if the agent has all the renegotiation bargaining power, the initial contract is such that the agent only achieves his reservation utility level if renegotiation takes place. The second setting can be implemented by having the principal "sell" the firm to the agent in the initial contract ($\mathbf{c}(x) = x - \pi^*$) and then having the agent sell the firm back to the principal at the renegotiation stage – the seller sets a take-it-or-leave-it price at each stage.

24.3.2 Renegotiation after Observing a Non-contractible, Imperfect Signal about the Agent's Action

HK extend their analysis to a setting in which the principal and agent jointly observe a non-contractible signal $y \in Y$ prior to renegotiation (and the realization of outcome x). Let $\varphi(x,y|a)$ denote the joint probability with respect to the outcome x and the signal y given action a. The outcome x is contractible information, whereas y is not. For purposes of our analysis assume there are n possible outcomes x_i and m possible signals y_k.

Sufficient Statistic Condition

An interesting special case occurs when y is a *sufficient statistic* for (y,a) with respect to x, i.e., for any pair of actions $a^1, a^2 \in A$,

$$\varphi(x|a^1,y) = \varphi(x|a^2,y), \quad \forall\, x \in X, y \in Y.$$

Observe that any risk faced by the agent at the time of renegotiation can be shifted to the principal (to the benefit of the principal if he has the bargaining power).

If y was a contractible report, the sufficient statistic condition would imply that the optimal incentive contract is a function of y only. Contracting on y instead of x would be a strict improvement if $\varphi(x|a,y) \in (0,1)$ for some x and y. HK address whether a non-contractible report of y *with* renegotiation is sufficient to achieve the result that would be obtained by contracting on a contractible report of y.

Proposition 24.7 (HK, Prop. 6)

Assume the sufficient statistic condition is satisfied and the principal makes a take-it-or-leave-it offer in renegotiation. Let $\boldsymbol{\varphi} = [\,\varphi(x_i|a,y_k)\,]_{m\times n}$, which is independent of a. Let $\boldsymbol{c}^Y\!: Y \to C$ be the optimal incentive contract if y was contractible information. If $\boldsymbol{\varphi}$ has rank m, then there exists an incentive contract $\boldsymbol{c}^X\!: X \to C$ (i.e., it is contingent solely on x) that induces the same action at the same expected cost as would $\boldsymbol{c}^Y$ (with and without renegotiation).

Proof: If $\boldsymbol{\varphi}$ has full rank, then there exists a contract $\boldsymbol{c}^X$ such that

$$\sum_{i=1}^{n} u(\boldsymbol{c}^X(x_i))\,\varphi(x_i|a,y_k) = u(\boldsymbol{c}^Y(y_k)), \quad \forall\, k = 1,\dots,m.$$

Clearly, $\boldsymbol{c}^X$ and $\boldsymbol{c}^Y$ induce the same action. Furthermore, because of the renegotiation, the principal expects to pay

$$u^{-1}\left(\sum_{i=1}^{n} u(\boldsymbol{c}^X(x_i))\,\varphi(x_i|a,y_k)\right)$$

given y_k, which is what the principal would pay if $\boldsymbol{c}^Y$ were the contract.

Q.E.D.

The key aspect of this result is that the renegotiation permits the principal to shield the agent from the additional risk in the outcome x compared to y that is not informative about the agent's action.[7] Observe that this result cannot be obtained unless $n \geq m$, i.e., there must be at least as many outcomes as signals.

Non-sufficient Statistic Condition

If y is not a sufficient statistic for (y,a) with respect to x, then an optimal contract with y contractible would depend on both y and x. Nonetheless, there might be gains to renegotiation (due to risk reduction at the renegotiation stage) even though a contract replicating a contract with y contractible can generally not be obtained. Moreover, without the sufficient statistic condition, the set of pure action strategies that can be implemented with renegotiation might be reduced compared to the implementable strategies without renegotiation. To see this, suppose $\hat{a}$ is chosen with certainty in equilibrium and $\varphi(y_k|\hat{a}) > 0$, $\forall\, k$. Then at the renegotiation stage the principal would offer the agent a compensation scheme with expected cost given y_k

$$u^{-1}\left(\sum_{i=1}^{n} u(\mathbf{c}^X(x_i))\, \varphi(x_i|\hat{a},y_k)\right).$$

The agent's incentive compatibility constraint (assuming that he only considers deviations to pure action strategies) is

$$\forall\, a \in A: \sum_{k=1}^{m}\sum_{i=1}^{n} u(\mathbf{c}^X(x_i))\, \varphi(x_i|\hat{a},y_k)\, \varphi(y_k|\hat{a}) - v(\hat{a})$$

$$\geq \sum_{k=1}^{m}\sum_{i=1}^{n} u(\mathbf{c}^X(x_i))\, \varphi(x_i|\hat{a},y_k)\, \varphi(y_k|a) - v(a).$$

It is clear that if there exists some action $\bar{a}$ with the same distribution over signals y and a lower disutility than $\hat{a}$, i.e.,

$$\varphi(y_k|\bar{a}) = \varphi(y_k|\hat{a}) \quad \text{for all } k \quad \text{and} \quad v(\bar{a}) < v(\hat{a}),$$

then $\hat{a}$ cannot be implemented. This result readily generalizes to the case in which the agent may deviate to a randomized strategy over actions.

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[7] The implementation of a pure action strategy is assured by all performance relevant information being revealed before renegotiation takes place. Observe that the renegotiated contract offers the agent full insurance conditional on y. The principal will not offer the agent full insurance because y is performance relevant information as well as information about the final outcome.

Proposition 24.8 (HK, Prop. 8)

Suppose $\varphi(y_k|\hat{a}) > 0$, $\forall\, k$, and that the principal makes a take-it-or-leave-it offer in renegotiation. Action $\hat{a}$ is implementable under renegotiation only if there is no (randomized) action strategy $\psi(a)$ that induces the same density over signals y as $\hat{a}$ and which costs less, in terms of expected disutility, than $\hat{a}$.

Note that the proposition provides a necessary condition for the implementation of the pure strategy $\hat{a}$, and not a sufficient condition.

The FT analysis can be viewed as a special case in which the y-signals are pure noise, i.e., y only informs the principal that an action has been taken. In this case, any action strategy induces the same density over signals y and, therefore only the least costly action can be implemented as a pure strategy. This suggests that in less extreme cases an optimal contract with renegotiation based on non-contractible signal y may also involve randomized action strategies.

24.3.3 Information about Outcome before Renegotiation

HK also provide some analysis of the case in which both parties observe two signals before the contract is renegotiated: a perfect signal y^a about the agent's action, and a signal y^l about the final outcome (leakage). Clearly, if y^l is pure noise, the first best solution can be implemented. At the other extreme where $y^l = x$, there is no basis for renegotiation, and the optimal contract is the same as the optimal contract based on x alone without renegotiation. The reason, of course, is that perfect revelation of x prior to renegotiation eliminates beneficial risk sharing facilitated by renegotiation based on the observation of the agent's action.

HK provide more general sufficient conditions (than pure noise) for the implementation of first-best. Frimor (1995) provides an extensive analysis of the intermediate case in which y^a and y^l are imperfect signals about the agent's action and the final outcome, respectively, and in which optimal randomized action strategies are determined. Without going into details, the general picture seems to be that renegotiation is most beneficial when y^a carries much information about the agent's action and the leakage of the final outcome is minimal.

24.4 PRIVATE PRINCIPAL INFORMATION ABOUT THE AGENT'S PERFORMANCE

We now consider the paper by Demski and Sappington (DS93) (1993). They examine a setting in which the *principal obtains non-contractible performance information before the final outcome is realized.* Renegotiation is not explicitly considered. Instead, the principal offers the agent a contract that consists of a menu from which the *principal* will choose after he obtains his *non-contractible information* about the agent's performance.

DS93 interpret their model as one in which the principal (buyer) buys an input (good or service) from an agent (outside supplier). The supplier's effort is unobservable and affects the quality of the input, which in turn affects the ultimate outcome from its use by the buyer (which is contractible information). The buyer receives private information about the quality of the input. There is no third party to verify the information about the input's quality.

Basic DS93 Model

DS93 restrict their analysis to a setting in which there are two possible outcomes ($X = \{x_b, x_g\}$), two possible actions ($A = \{a_H, a_L\}$), and two possible unverified performance signals ($Y = \{y_1, y_2\}$). Let the contract be expressed as $\boldsymbol{c}(x_i, m_\ell)$, where $m_\ell \in Y$ is the principal's "message" regarding his unverified performance signal. The prior beliefs about x and y are represented by $\varphi(x_i, y_j | a)$. Let

$$\bar{\boldsymbol{c}}(y_k, a_j) \equiv \sum_{i=b}^{g} \boldsymbol{c}(x_i, y_k)\, \varphi(x_i | y_k, a_j).$$

DS93 make the following basic assumptions (DS93 introduce a number of assumptions, but they often examine special cases in which some inequalities do not hold or are weak instead of strict):

(A1) high effort a_H is to be motivated;

(A2) the high outcome is more likely with a_H, i.e.,

$$\sum_{k=1}^{2} \varphi(x_g, y_k | a_H) > \sum_{k=1}^{2} \varphi(x_g, y_k | a_L);$$

(A3) there is no moving support, i.e., $\varphi(x_i, y_k | a_H) > 0$, $\forall\, i = b, g$ and $k = 1, 2$.

The following two assumptions pertain to the likelihood ratio,

$$L_{ik} \equiv \frac{\varphi(x_i, y_k | a_L)}{\varphi(x_i, y_k | a_H)}.$$

(M-y) signal-contingent monotone likelihood ratio property (i.e., the low outcome is more likely with low effort): $L_{bk} \geq L_{gk}$, $k = 1, 2$;

(M-x) outcome-contingent monotone likelihood ratio property (i.e., low signal is more likely with low effort): $L_{i1} \geq L_{i2}$, $i = g, b$.

The preference assumptions are:

Risk neutral principal:

$$u^p(x - c) = x - c,$$

Risk and effort averse agent:

$$u^a(c,a) = u(c) - v(a), \;\; u'(c) > 0, u''(c) < 0, v(a_L) < v(a_H).$$

Principal's Mechanism Design Problem to Implement a_H with an Unverified Signal (US):

$$\bar{c}^{US} \equiv \underset{\mathbf{c}}{\text{minimize}} \;\; \sum_{k=1}^{2} \bar{\mathbf{c}}(y_k, a_H)\, \varphi(y_k | a_H),$$

subject to

$$U^a(\mathbf{c}, a_H) \equiv \sum_{i=b}^{g} \sum_{k=1}^{2} u(\mathbf{c}(x_i, y_k))\, \varphi(x_i, y_k | a_H) - v(a_H) \geq \bar{U},$$

$$U^a(\mathbf{c}, a_H) \geq U^a(\mathbf{c}, a_L),$$

$$\bar{\mathbf{c}}(y_k, a_H) \leq \sum_{i=b}^{g} \mathbf{c}(x_i, m_\ell)\, \varphi(x_i | y_k, a_H), \quad \forall\; k, \ell = 1, 2.$$

The first constraint ensures the agent's acceptance of the contract (given that he believes the principal will tell the truth), the second constraint ensures that the agent will choose a_H, and the third set of constraints ensures that the principal tells the truth (given that he believes the agent has taken action a_H).

Benchmark Solutions

(i) *No Moral Hazard (FB):*

Agent is paid the first-best fixed wage:

$$\bar{c}^{FB} = u^{-1}\left(v(a_H) + \bar{U}\right).$$

(ii) *Principal Receives no Signal (NS):*

$$\boldsymbol{c}^{NS}(x_b) < \boldsymbol{c}^{NS}(x_g),$$

$$\bar{c}^{NS} = \sum_{i=b}^{g} \boldsymbol{c}^{NS}(x_i)\, \varphi(x_i | a_H).$$

(iii) *Principal Receives a Contractible Signal that Is A-informative given the Outcome (VS):*

$$\boldsymbol{c}^{VS}(x_b,y_1) < \boldsymbol{c}^{VS}(x_b,y_2),\ \boldsymbol{c}^{VS}(x_g,y_1) < \boldsymbol{c}^{VS}(x_g,y_2),$$

$$\bar{c}^{VS} = \sum_{i=b}^{g} \sum_{k=1}^{2} \boldsymbol{c}^{VS}(x_i,y_k)\, \varphi(x_i,y_k | a_H).$$

The ranking of the benchmark cases is as follows (given the assumptions stated above):

$$\bar{c}^{FB} < \bar{c}^{VS} < \bar{c}^{NS}.$$

The first inequality is strict because the agent is strictly risk averse, $v(a_L) < v(a_H)$, and (A3) ensures that penalties cannot be used to enforce the first-best action with a fixed wage. The second inequality is strict because M-x implies that the principal's information is strictly informative given x_i, $i = b, g$.

Perfect Monitoring

We first relax assumption (A3) and assume that y_k reveals a_j. This is essentially the case considered by HK although there is no explicit renegotiation in DS93.

Proposition 24.9 (DS93, Prop. 1)

Assume (A1) and (A2), as well as $\varphi(x_i,y_1 | a_H) = \varphi(x_i,y_2 | a_L) = 0$. Then

$$\bar{c}^{US} = \bar{c}^{FB}$$

Proof: Let $\boldsymbol{c}(x_b,m_2) = \boldsymbol{c}(x_g,m_2) = \bar{c}^{FB}$, and let $\boldsymbol{c}(x_b,m_1) = \boldsymbol{c}^{NS}(x_b) + \delta$ and $\boldsymbol{c}(x_g,m_1) = \boldsymbol{c}^{NS}(x_g) - \delta$, where $\delta > 0$ is set so that

$$\bar{c}^{FB} = \boldsymbol{c}(x_b,m_1)\, \varphi(x_b,y_2 | a_H) + \boldsymbol{c}(x_g,m_1)\, \varphi(x_g,y_2 | a_H).$$

By construction, the principal is indifferent between reporting m_1 or m_2 if he observes y_2 (i.e., the agent has taken a_H). However, he strictly prefers to report

m_1 if he observes y_1 (i.e., the agent has taken a_L), which will give the agent less than $\underline{U}$. **Q.E.D.**

Imperfect Monitoring

Proposition 24.10 (DS93, Prop. 2 & 3)
Given (A1), (A2), (A3) and (M-x):

$$\bar{c}^{FB} < \bar{c}^{VS} < \bar{c}^{US} \leq \bar{c}^{NS}.$$

The basic assumptions ensure that there is no moving support that can be exploited to achieve the first-best solution. Furthermore, under these conditions the contract for verified signals will not induce the principal to tell the truth (and, hence, at least one of the truth-telling constraints is binding, resulting in a more costly solution).

Now consider the impact of introducing a risk neutral third party who will permit the sum of the amounts received by the principal and the agent to differ from the outcome.[8]

Proposition 24.11 (DS93, Fn 10)
If the principal can contract with a risk neutral third party, then there exists a contract for the unverified signal case such that

$$\bar{c}^{US} = \bar{c}^{VS}.$$

The latter holds because the principal can be paid $x - \bar{c}^{VS}$ while the agent receives the same compensation as with the verified report. This gives the third party an expected net return of zero if the principal reports truthfully, and he will do so because his contract gives him a fixed return that is independent of what he reports (assuming that he does not collude with the agent against the third party).

Now return to our basic setting with unverified signals.

Proposition 24.12 (DS93, Prop. 5)
Given (A1), (A2), (M-x) and strict (M-y), and

$$\frac{\varphi(x_b, y_1 | a_H)}{\varphi(x_g, y_1 | a_H)} > \frac{\varphi(x_b, y_2 | a_H)}{\varphi(x_g, y_2 | a_H)},$$

[8] See Chapter 29 for a discussion of using a third party to "break" what Holmström (1982) calls the "budget constraint problem."

we obtain

$$c(x_b,y_1) \le c(x_b,y_2) < c(x_g,y_2) \le c(x_g,y_1).$$

(M-x) and (M-y) imply that both x_g and y_2 are interpreted as "good news." With verified signals we obtain

$$c^{VS}(x_b,y_1) < c^{VS}(x_b,y_2), \;\; c^{VS}(x_g,y_1) < c^{VS}(x_g,y_2).$$

However, without verification, this contract would induce the principal to report $m = y_1$ even if he observed y_2. Truth-telling requires that either $c(x_b,y_1) > c(x_b,y_2)$ or $c(x_g,y_1) > c(x_g,y_2)$. The assumed conditions in Proposition 24.12 are sufficient to make the latter optimal. The condition in Proposition 24.12 establishes that x_b is more likely to result if y_1 has been observed than if y_2 has been observed, i.e., x and y are positively correlated given a_H. However, to motivate the principal to be truthful, he is rewarded (and the agent is unavoidably punished) when x_g occurs with y_2 instead of y_1.

DS93 demonstrate that the optimal contracts can be such that the principal's private information is ignored (i.e., $c(x_i,m_\ell)$ is independent of m_ℓ). They state:

> *"Intuitively, there are two interacting control problems. Careful management of the buyer's control problem may help alleviate the supplier's problem, but often at a cost. If the cost is prohibitive, it will be optimal not to use the buyer's quality assessment. Both parties know the buyer will receive private information, and both agree in advance to ignore it."* (p. 10)

24.5 RESOLVING DOUBLE MORAL HAZARD WITH A BUYOUT AGREEMENT

Demski and Sappington (DS91) (1991) provide an interesting analysis of a simple setting in which both the principal and agent provide productive effort, the principal observes the agent's action (but it is not contractible information), and the final outcome is only observable by the "final" owner of the firm.

Let a_P and a_A represent the actions taken by the principal and the agent, respectively. These actions are expressed in terms of the personal monetary cost incurred by each individual. The terminal value of the firm is denoted x, and $\varphi(x\,|\,a_P,a_A)$ represents the probability density over the terminal value given the two actions (with $\Phi_i(x\,|\,a_P,a_A) = \partial\Phi/\partial a_i \le 0$, $i = P, A$, which implies that a first-order stochastic dominant distribution is provided by more effort). The agent takes his action first, and that action is observable by the principal before he takes his own action.

The gross return to the principal is denoted π and the gross return to the agent is denoted c. The principal is risk neutral with respect to his net return, $u^p(\pi,a_P) = \pi - a_P$, and the agent is strictly risk averse with respect to his net return, $u^a(c,a_A) = u(c - a_A)$, where $u' > 0$ and $u'' < 0$.

The principal is the initial owner of the firm and is assumed to have all the bargaining power. There is *no contractible information* except for the ownership of the firm. Let $\delta \in \{P, A\}$ denote whether the principal ($\delta = P$) or the agent ($\delta = A$) is the final owner of the firm. The principal can offer the agent an enforceable agreement in which the principal will have the right to choose between the following two options after he has taken his action:

retention option: $\delta = P$, $\pi = x - w$, and $c = w$,

buyout option: $\delta = A$, $\pi = b$, and $c = x - b$,

where w is a wage paid to the agent and b is a buyout price paid by the agent.

It may seem "unfair" that the principal sets w and b and then gets to choose which option is implemented. However, keep in mind that the agent has the right to reject the contract. He must expect to receive his reservation utility $\bar{U}$, otherwise, he will go elsewhere.

First-best Result

As a benchmark case, assume that the agent's action is contractible information. Since the agent is risk averse, we obtain the standard result that the agent is paid a fixed wage w^* such that

$$u(w^* - a_A^*) = \bar{U},$$

where a_A^* is the agent's first-best action. Observe that for any a_A, the first-best compensation is

$$w^*(a_A) = u^{-1}(\bar{U}) + a_A.$$

Hence, if a_A is observable, we formulate the principal's decision problem as

$$\underset{a_P,\, a_A}{\text{maximize}} \quad \mathrm{E}[x|a_P,a_A] - [u^{-1}(\bar{U}) + a_A] - a_P.$$

The optimal actions in this setting are characterized by the following first-order conditions:

$$a_P\text{: } \mathrm{E}_P[x|a_P^*,a_A^*] = 1,$$

$$a_A\text{: } \mathrm{E}_A[x|a_P^*,a_A^*] = 1,$$

where subscript i indicates that $\mathrm{E}[x|\cdot]$ is differentiated with respect to a_i, $i = P, A$.

Optimal Buyout Contract
The principal can do no better than the first-best result. Interestingly, in this simple setting with virtually no contractible information (other than firm ownership), the principal can achieve the first-best result.

Proposition 24.13 (DS91, Prop. 1)
The first-best result can be achieved as equilibrium behavior (a_P^*,a_A^*) using the first best wage w^* and the following buyout price:

$$b^* = \mathrm{E}[x|a_P^*,a_P^*] - a_P^* - w^*.$$

Proof: If the principal observes $a_A < a_A^*$ it will be optimal for him to select $a_P = 0$ and require the agent to buy him out at b^*. This will give the principal the same expected return as the first-best solution and will leave the agent with a utility less than his reservation utility. Consequently, it will not be optimal for the agent to select a_A less than a_A^*.

If the agent selects $a_A{}^*$, then the principal will be indifferent between retaining ownership and selecting $a_P = a_P^*$ versus selecting $a_P = 0$ and requiring the agent to buy him out. Since the latter option will leave the agent with an expected utility less than his reservation utility and the principal does not have a strong incentive to take that option, we assume the principal retains his ownership (and thereby bears all the risk).

If the agent selects $a_A > a_A^*$, then the principal will retain firm ownership (and select the optimal action given a_A) and pay the agent w^*. This will leave the agent with an expected utility less than his reservation utility. Consequently, it is not optimal for the agent to select a_A greater than a_A^*. **Q.E.D.**

24.6 CONCLUDING REMARKS

From an *ex ante* perspective, the inability to preclude renegotiation of a contract after an agent has taken an unobserved action is costly to a principal who is uninformed at the time of renegotiation. On the other hand, the ability to renegotiate after the principal has received information can be valuable even if the information received is non-contractible. In fact, if he observes the agent's action prior to renegotiation, then the first-best result can be achieved.

Since most single-period agency models assume the agent's action is unobservable and the only information received by the principal is the end-of-period contractible performance measures, these models assume (either explicitly or implicitly) that the principal can commit not to renegotiate prior to the end of the period. The mechanism for making that commitment is generally unstated – it is exogenous. The key issue is the agent's belief when he takes his action – does he believe the principal can refrain from renegotiating before the contractible performance measures are reported. In a multi-period setting in which the principal makes a series of one-period contracts, refusing to renegotiate can be an equilibrium strategy if the agent believes the principle is one of two types. The first type is our standard principal, who will renegotiate in a single-period setting, whereas the second type will always refuse to renegotiate, even if his expected payoff would be increased by renegotiating. In this setting, the first type will refuse to renegotiate so as to induce the agent to assign a higher probability to the possibility he is a second-type agent.

Chapters 25 through 28 examine incentive contracting in multi-period settings. We do not formulate the "reputation game" model described above. Instead, we exogenously assume there is no intra-period renegotiation. Furthermore, in Chapters 25 through 27 we assume there is full commitment such that there is no renegotiation at any point prior to the termination of the contract. On the other hand, Chapter 28 consders inter-period renegotiation, i.e., renegotiation can take place at the end of a period, after contractible performance measures have been issued. Full commitment is still preferred, but one must always be alert to the possibility that full commitment may not be possible.

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PART G

CONTRACTING IN MULTI-PERIOD/ SINGLE-AGENT SETTINGS

CHAPTER 25

MULTI-PERIOD CONTRACTS WITH FULL COMMITMENT AND INDEPENDENT PERIODS

In this and the following three chapters, we examine principal-agent models in which the principal owns a technology that is operated by an agent for two or more periods. The key feature of a multi-period model is that the agent takes a sequence of actions and his information may change from period to period. Furthermore, there may be consumption and/or compensation at the end of each period, and their timing may be significant.

Obviously, introducing multiple periods raises several new issues. For example, how are the agent's preferences affected by the timing of information and consumption? Can the timing of consumption differ from the timing of the compensation (i.e., does the agent have access to personal borrowing and saving)? Can a different agent be hired at the start of each period? If an agent is hired for more than one period, can the principal and agent commit to a contract for all periods? If full commitment is not feasible, what commitments (if any) are feasible, e.g., is it possible to preclude renegotiation between when an action is taken and a report is released (to avoid the Fudenberg and Tirole problem discussed in Chapter 24)? To what extent are the production and reporting systems characterized by technological or stochastic independence?

In Chapters 25, 26, and 27 we assume full commitment is feasible. That is, the principal and the agent are precluded from reneging on or renegotiating the contract. Chapter 28 then considers limited commitment, e.g., the contract cannot preclude a mutually agreeable change in the contract at the end of each period, or subsequent to release of a report, but renegotiation is precluded at other dates.

After specifying a basic model in Section 25.1, the current chapter explores full commitment contracting in settings in which the production and reporting systems are technologically and stochastically independent. This permits uncluttered exploration of the basic implications of differences in the form of the agent's preferences for time-dependent consumption, and differences in the agent's access to personal borrowing and saving. For most of our analysis we exogenously assume the principal contracts with the same agent for each period, but in Section 25.5 we discuss the desirability of hiring a new agent at the start of each period.

The multi-period model can be viewed as a variation of the multi-task models considered in Chapter 20. A key difference is that the agent takes actions sequentially, and his information may change over time. Furthermore, in the basic multi-task model there is a single consumption date. We consider sequential choice models with both single and multiple consumption dates.

In Sections 25.2 and 25.3 we consider complete, full-commitment contracts in settings in which the reporting system produces period-specific reports that are technologically and stochastically independent. The initial model (Section 25.2) assumes the agent's preferences are defined in terms of aggregate consumption and the sequence of actions. Interestingly, a sequence of identical periods generally does not result in a sequence of identical contracts. The exception is the type of setting considered in Section 19.2.2, i.e., a single consumption date model with multiplicatively separable exponential utility.

We consider time-additive preferences in Section 25.3. A key result is that even though past performance levels are uninformative about future performance, every Pareto optimal contract is such that the agent's current consumption depends on both the current and past performance. Furthermore, under those conditions, current actions may also depend on past performance.

With time-additive preferences, the terms of the contract depend on whether the agent has access to personal banking, i.e., can he borrow and save? The principal is often better off (and the agent is no worse off) if the principal can preclude the agent from borrowing and saving. If such restrictions are not feasible, then we solve for the optimal contract for which the agent will have no incentive to borrow or save. With banking, the risk averse agent smooths his consumption across periods (or the principal smooths his compensation).

Some aspects of the analysis and results in Sections 25.2 and 25.3 are further illustrated in Section 25.4, in the context of a multi-period version of the *LEN* model (linear contracts, exponential utility, and normally distributed performance measures). This analysis is further extended in Chapter 26 for settings in which there are both stochastic and technological interdependence. The differences between time-additive and aggregate consumption (i.e., single consumption date) utility functions are particularly emphasized. In the time-additive case, most of our analysis assumes the agent and market have the same rate of time-preference, but in this section we characterize the optimal consumption choice in settings in which those rates may differ. Identical rates result in flat consumption smoothing. Differences in rates result in planned "smooth" growth or decay in consumption.

In the first four sections of this chapter we assume that at date $t = 0$ the principal contracts with a single agent who will operate his firm for T periods. In Section 25.5 we examine the desirability of contracting with a new agent at the start of each period. The results are significantly affected by whether the agent's direct preferences for his actions are represented as "monetary" costs within an exponential utility for net consumption, or those preferences are represented as "disutility" for effort. Given our independence assumptions, in the "monetary" cost model there is no wealth effect and the principal is indifferent between retaining the first agent or replacing him. On the other hand, in the "disutility" model there is a wealth effect – it is less expensive to motivate a poor agent than a rich agent. As a result, the principal prefers to replace the first agent if, and only if, he obtains a bad outcome such that he has less wealth than the alternative agent.

25.1 BASIC MODEL

Actions, Performance Measures, Compensation, and Consumption

The general setup of the model is as follows. The principal-agent relationship extends over T periods. At the beginning of each period t (i.e., at date t - 1), the agent takes a vector of unobservable actions $\mathbf{a}_t \in \mathbf{A}_t$, and at the end of period t a set J_t of contractible reports is released. The set of all reports is $J = J_1 \cup ... \cup J_T$, the release date of report j is t_j, and the content of the j^{th} report is $y_j \in Y_j$. The content of all reports released at date t is represented by the vector $\mathbf{y}_t = (y_j)_{j \in J_t} \in \mathbf{Y}_t = \times_{j \in J_t} Y_j$.

At the end of period t, compensation s_t is paid to the agent based on contractible information available at that date. If the agent has access to riskless banking, he can save (lend) $\ell_t \in \mathbb{R}$ at date t (with $\ell_t < 0$ representing borrowing). The one-period riskless interest rate is constant and denoted $\imath$, the one-period return is $R = 1 + \imath$, and the one-period discount rate is $\beta = R^{-1}$.[1] Hence, the agent's consumption at date t is $c_t = s_t + R\ell_{t-1} - \ell_t$. If the agent has no access to banking, his consumption is equal to his compensation, i.e., $c_t = s_t$ and $\ell_t = 0$.

We use the following notation to denote the history of, for example, action choices:[2]

$$\overleftarrow{\mathbf{a}}_t = (a_1, ..., a_t), \quad t = 1, ..., T,$$

with similar notation for other variables.

At date t the contractible information consists of the history of public reports, $\overleftarrow{\mathbf{y}}_t$, so that the compensation function for date t =1, ..., T, is expressed as

$$s_t \colon \overleftarrow{\mathbf{Y}}_t \rightarrow C.$$

[1] In Section 25.4 we consider the case in which interest rates are time-dependent, i.e., there is a "non-flat" term structure of interest rates.

[2] If the agent retires before date T, then A_t is null during his post-retirement period. Furthermore, we can introduce a consumption planning horizon $\bar{T}$ (referred to as *T-cup*) that extends beyond T, which is the last date at which the agent receives compensation from the principal.

We also assume that the report, action, and saving histories are the only information available to the agent when he selects his action and his savings level. Hence, these choices and the resulting consumption level can be expressed as functions of those histories:

$$\boldsymbol{a}_{t+1}\colon \overleftarrow{\mathbf{Y}}_t \times \overleftarrow{\mathbf{A}}_t \times \overleftarrow{\mathbf{L}}_t \to A_{t+1},$$

$$\boldsymbol{\ell}_t\colon \overleftarrow{\mathbf{Y}}_t \times \overleftarrow{\mathbf{A}}_t \times \overleftarrow{\mathbf{L}}_{t-1} \to L,$$

$$\boldsymbol{c}_t\colon \overleftarrow{\mathbf{Y}}_t \times \overleftarrow{\mathbf{A}}_t \times \overleftarrow{\mathbf{L}}_{t-1} \to C.$$

In this chapter, we assume the principal and the agent commit to a long-term contract at date 0 with no possibility of reneging or renegotiation (see Chapters 24 and 28 for analyses with renegotiation). Hence, the timeline can be depicted as in Figure 25.1.

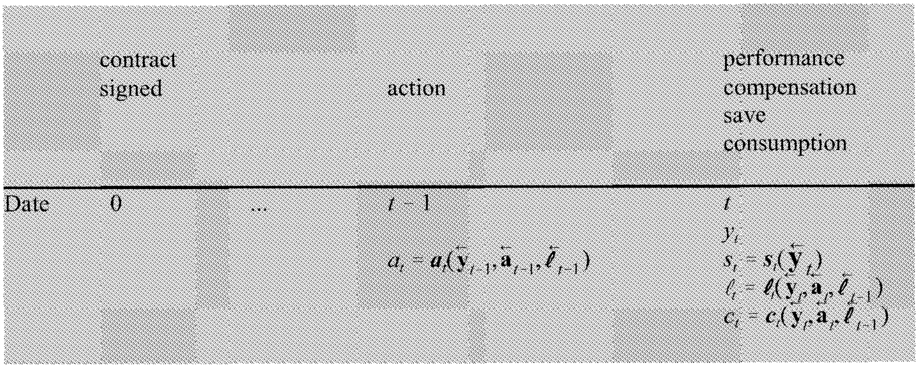

Figure 25.1: Timeline for multi-period incentive problem.

Technological and Stochastic Independence

In this chapter we focus on settings in which the sequences of outcomes and performance measures are independent across periods. More specifically, the outcomes are assumed to be *technologically independent* and the performance reports are assumed to be both *technologically and stochastically independent.*

Definition *Technological and Stochastic Independence*

The outcomes are *technologically independent* if the expected outcome attributable to period t depends only on the actions taken in period t, i.e., there exist period-specific functions $\bar{x}_t(a_t)$, $t = 1, ..., T$, (measured in date t dollars) such that the expected aggregate outcome (measured in date T dollars) can be expressed as

$$\mathrm{E}[x|\overleftarrow{\mathbf{a}}_T] = \sum_{t=1}^{T} R^{T-t}\,\overline{x}_t(a_t).$$

The performance reports are *technologically and stochastically independent* if there exist period-specific reports such that the probability distribution for the reports associated with the actions in period t are independent of the actions in other periods and are independently distributed across periods, i.e.,

$$\varphi(\overleftarrow{\mathbf{y}}_T|\overleftarrow{\mathbf{a}}_T) = \varphi(y_1|a_1)\,\varphi(y_2|a_2)\,...\,\varphi(y_T|a_T).$$

The outcomes can be correlated, since that correlation is immaterial given that the principal is risk neutral and the outcomes are not contractible.

These independence assumptions significantly simplify the analysis since they imply that past performance and actions do not affect the beliefs about the relation between future actions, performance, and payoffs. On the other hand, past performance may affect beliefs about the agent's future compensation. Consequently, the agent's action and savings choices at date t can be expressed as functions of the performance history alone, i.e.,

$$\boldsymbol{a}_{t+1}: \overleftarrow{\mathbf{Y}}_t \to A_{t+1},$$

$$\boldsymbol{\ell}_t: \overleftarrow{\mathbf{Y}}_t \to L,$$

$$\boldsymbol{c}_t: \overleftarrow{\mathbf{Y}}_t \to C.$$

Agent's Preferences

The specifications of the agent's preferences have profound effects in a multi-period context. We consider four types of preference assumptions based on two representations of preferences for period-specific consumption combined with two representations of the agent's direct preference with respect to the amount of effort he expends.

With respect to period-specific consumption $\overleftarrow{\mathbf{c}}_T$, the agent's preferences differ as to whether they are represented by a concave function of aggregate consumption, or by the sum of concave functions with respect to period-specific consumption.

Aggregate-consumption (AC) preferences:

$$u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T) = u^a\big(w, \overleftarrow{\mathbf{a}}_T\big),$$

where
$$w = \sum_{t=1}^{T} \Xi_t c_t$$

is a measure of aggregate consumption based on time-preference index Ξ_t, t = 1, ..., T. If the agent can borrow and save at the riskless return R per period, then it is essential that the agent's time-preference index have the same rate of change, i.e., $\Xi_{t-1} = R\Xi_t$, for all t. Otherwise, the agent's consumption choice will be a corner solution or unbounded (e.g., if the consumption set is unbounded, then the agent will go infinitely long in consumption in one period and infinitely short in another). In these models, c_t can be interpreted as the consumption at date t measured in nominal date t dollars, whereas $\Xi_t c_t$ converts this amount into common (valuation-date) dollars. The aggregate consumption measure w can then be interpreted as the amount of wealth expended on consumption measured in valuation-date dollars. For example, if $\Xi_t = \beta^t$, then w is the classical NPV of aggregate consumption measured in date 0 dollars, whereas with $\Xi_t = R^{T-t}$, w is measured in terminal date T dollars (in both cases, $\Xi_{t-1} = R\Xi_t$).

In several papers that use this utility function, interest rates are assumed to be zero and $w = c_1 + ... + c_T$, i.e., total consumption (which equals total compensation). These models can also be interpreted as measuring consumption, compensation, and effort costs in some common (valuation-date) dollars. That is, time-preference indices have been applied and the interest rate is implicit rather than explicit.

The utility function $u^a(w, \overleftarrow{\mathbf{a}}_T)$ is assumed to be strictly increasing and concave in w, which makes it strictly increasing and concave in each element of $\overleftarrow{\mathbf{c}}_T$. Furthermore, we assume the utility function is strictly decreasing and concave in $\overleftarrow{\mathbf{a}}_T$. Hence, the agent is both risk and effort averse.

If the interest rate is non-zero, then w depends on the valuation date. However, the difference is not substantive. The valuation date merely affects the scale of the wealth expenditure measure, and the parameters of the utility function can always be modified to adjust for a change in the valuation date. For example, assume the utility for consumption is expressed as $u(\overleftarrow{\mathbf{c}}_T) = -\exp[-r_0 w_0]$, where w_0 is measured in date 0 dollars and r_0 is the risk aversion measure given that valuation date. This utility function can be equivalently expressed, for example, in terms of the terminal value of consumption, i.e., $-\exp[-r_T w_T]$, provided the risk aversion parameter is set at $r_T = \beta^T r_0$. Not surprisingly, the risk aversion measure decreases if consumption is measured in later dollars.

As we shall see, with *AC* preferences, the timing of consumption is immaterial (as long as it is interest-rate adjusted), so that consumption can always be set equal to the compensation in each period. Of particular note is the fact that, with *AC* preferences, there is no incentive to smooth consumption. This is not the case, however, with time-additive preferences.

Time-additive (TA) preferences:

$$u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T) = \sum_{t=1}^{T} u_t^a(c_t, a_t).$$

The period-specific utility functions are each strictly increasing and concave in c_t and strictly decreasing and concave in a_t, i.e., the agent is again both risk and effort averse. They can vary across periods, and those differences will reflect the agent's personal time preferences. With strictly concave, period-specific utility functions it is not necessary to assume the agent's time-preference is the same as the financial market's – the non-linearity of the utility function will ensure the solution is bounded.

As in the single-period models, we consider two types of separability with respect to the preferences for consumption and actions (see Section 17.1.1).

Additively separable (an "effort disutility", *ED*, model)*:*

$$AC: \quad u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T) = u^a(w, \overleftarrow{\mathbf{a}}_T) = u(w) - v(\overleftarrow{\mathbf{a}}_T), \tag{25.1a}$$

$$TA: \quad u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T) = \sum_{t=1}^{T} [u_t(c_t) - v_t(a_t)]. \tag{25.1b}$$

Multiplicatively separable exponential (an "effort cost", *EC*, model)*:*

$$AC: \quad u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T) = u^a(w, \overleftarrow{\mathbf{a}}_T) = -\exp[-r(w - k(\overleftarrow{\mathbf{a}}_T))], \tag{25.2a}$$

$$TA: \quad u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T) = -\sum_{t=1}^{T} \Xi_t \exp[-r_t(c_t - \kappa_t(a_t))], \tag{25.2b}$$

where r_t is the risk aversion for consumption (or value of consumption) at date t, and $\kappa_t(a_t)$ is the agent's direct cost of action a_t expressed in date t dollars. Under *AC*, the valuation date used in measuring w must also be used for r and k, e.g.,

$$k(\overleftarrow{\mathbf{a}}_T) = \sum_{t=1}^{T} \Xi_t \kappa_t(a_t)$$

is the aggregate cost of the agent's actions over the T periods expressed in valuation-date dollars.

Post-contract Consumption

In the preceding discussion, date T is interpreted as the termination date with respect to the agent's actions, his performance reports, his compensation, and his consumption. It is possible to separate these dates. For example, we could let T^a represent the agent's retirement date (after which he takes no actions), let T represent the contract termination date (after which he receives no compensation), and let $\breve{T}$ represent the final consumption date (after which the agent – and his heirs – cease to consume). That is, $A_t = \emptyset$ for $t > T^a$, $s_t = 0, \forall\, t > T$, and $c_t = 0, \forall\, t > \breve{T}$.

Principal's Preferences

The principal takes no actions and his consumption is equal to the gross outcome from the agent's actions, minus the compensation paid to the agent. If the outcome, or any components of the outcome are contractible information, then they are included in the J reports. As in the single-period agency models, many of the early multi-period models assume there is an outcome each period, and it is publicly reported (and contractible). More recent models have, as we do, assumed that the outcomes are not necessarily contractible.

Let $x_t \in X_t$ represent the outcome "attributable" to (but not necessarily reported at) date t and measured in date t dollars, and let x represent an aggregate valuation-date measure of those outcomes, i.e.,

$$x = \sum_{t=1}^{T} \Xi_t x_t,$$

where Ξ_t is the principal's date t valuation index. The principal is assumed to be risk neutral with time-additive preferences that can be represented by

$$u^p(x,w) = x - w,$$

where
$$w = \sum_{t=1}^{T} \Xi_t s_t.$$

As with agent AC preferences, if the principal has unrestricted access to riskless borrowing and saving with one-period return R, then his time-preference indices must satisfy $\Xi_{t-1} = R\Xi_t$. The market riskless returns for the principal and agent are assumed to be identical.[3] They could use different valuation dates, but that

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[3] While equal interest rates are not entirely realistic, they keep the analysis simple and tractable. If one is examining a setting in which a difference in interest rates is an important issue, then the model would have to be expanded to introduce factors that preclude unbounded solutions, e.g.,

(continued...)

difference would not be substantive. Hence, if the agent has *AC* preferences, we use the same valuation date (i.e., time-preference index) for the principal and the agent.

With *AC* preferences, both the principal and the agent are indifferent with respect to the timing of the agent's compensation provided it is interest-rate adjusted, e.g., they are indifferent between compensation of one dollar at date t versus $R^{\tau-t}$ dollars at date τ. Hence, for any contract there is an equivalent contract (in terms of principal and agent preferences and agent incentives) in which all compensation is paid at date T. Therefore, models with *AC* preferences can be characterized as if they are single consumption date models with a sequence of actions.

With *TA* preferences, the timing of the compensation is important if the agent cannot borrow or save. However, if he can borrow and save, then the timing of the compensation is again irrelevant.

25.2 AGGREGATE CONSUMPTION PREFERENCES

In this section we assume the agent has *AC* preferences and examine how contracts and actions are affected by the form of effort preferences, and by the timing of reports. For purposes of this discussion we assume $\mathbf{a}_t$ and $\mathbf{y}_t$ are both single dimensional, MLRP holds, and that the first-order approach can be used to specify the incentive constraint for each period.

25.2.1 An "Effort Cost" Model with Exponential Utility

In the following analysis the agent's utility for consumption and effort is represented by $u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T) = -\exp[-r(w - k)]$, where r, w, and k are all measured with respect to the same date. Furthermore, consistent with our other independence assumptions, we assume the effort cost for period t depends only on a_t. We allow for the possibility of a positive interest rate, but ι could equal zero.

As stated earlier, the valuation date for r, w, and k is arbitrary. However, it is important that they are consistent and have the same valuation date throughout the analysis. The agent's utility function takes the following form:

$$u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T) = -\exp\left[-r\sum_{t=1}^{T} \Xi_t(c_t - \kappa_t(a_t))\right]$$

[3] (...continued)
the risk of personal bankruptcy.

$$= -\prod_{t=1}^{T} \exp[-r\,\Xi_t(c_t - \kappa_t(a_t))]. \tag{25.3}$$

This setting is very similar to the setting considered in Chapter 19 except that a time-preference index is introduced and that the agency problem may not be identical across periods. The index is immaterial provided $\Xi_{t-1} = R\Xi_t$. However, note that the risk aversion parameter r pertains to measures of consumption in valuation-date dollars, whereas the appropriate risk aversion parameter for measures of consumption in nominal date t dollars is $\hat{r}_t^{AC} \equiv r\Xi_t$, which we refer to as the *nominal risk aversion.*

Assume that the agent has a reservation utility of $\bar{U}$ for the T periods, and let c^o represent the aggregate reservation certainty equivalent measured in valuation-date dollars. Let $s_1^o, ..., s_T^o$ denote an arbitrary set of period-specific reservation compensation levels such that

$$\bar{U} = -\exp\Big[-r\sum_{t=1}^{T} \Xi_t s_t^o\Big].$$

Then there exists an *AC-EC* (aggregate consumption-effort cost) contract that takes the form $\mathbf{s}(\overleftarrow{\mathbf{y}}_t) = \mathbf{s}_1(y_1) + ... + \mathbf{s}_T(y_T)$, where $\mathbf{s}_t(y_t)$ solves the following single-period incentive problem.

$$\underset{\mathbf{s}_t, a_t}{\text{maximize}} \;\; \Xi_t\Big[\mathrm{E}[x_t|a_t] - \int_{Y_t} \mathbf{s}_t(y_t)\, d\Phi_t(y_t|a_t)\Big], \tag{25.4}$$

$$\text{subject to} \quad -\int_{Y_t} \exp[-r\Xi_t(\mathbf{s}_t(y_t) - \kappa_t(a_t))]\, d\Phi_t(y_t|a_t) = -\exp[-r\Xi_t s_t^o],$$

$$-\int_{Y_t} \exp[-r\Xi_t(\mathbf{s}_t(y_t) - \kappa_t(a_t))] \times \big(r\Xi_t\kappa_t'(a_t)\, d\Phi_t(y_t|a_t) + d\Phi_{ta_t}(y_t|a_t)\big) = 0.$$

The first-order conditions for problem (25.4) imply

$$\mathbf{s}_t(y_t) = s_t^o + \kappa_t(a_t) + \frac{1}{r\Xi_t}\ln\left(r\Xi_t\left[\lambda_t + \mu_t\left(r\Xi_t\kappa_t'(a_t) + \frac{d\Phi_{ta_t}(y_t|a_t)}{d\Phi_t(y_t|a_t)}\right)\right]\right). \tag{25.5}$$

That is, the contract is represented as paying, in each period, a fixed component $s_t^o + \kappa_t(a_t)$ that is sufficient to obtain contract acceptance and compensate the agent for his effort, plus a variable component that induces the optimal action and provides a risk premium due to the risk imposed to motivate the implementation of a_t. Observe that, in this setting, the long-term contract is equivalent to a series of one-period contracts that are sufficient to obtain the agent's participation and induce the optimal effort for each period. The ability to borrow and save is not an issue here.

The sequence of the one-period contracts is such that the variable component in each period is determined by the agent's *nominal risk aversion* $\hat{r}_t^{AC} = r\Xi_t$ as opposed to just r. If c_t, s_t, and κ_t are measured in valuation-date dollars instead of nominal dollars, then the explicit interest rate is set equal to zero, the time-preference index Ξ_t is set equal to one, and the agent's risk aversion is constant across periods. On the other hand, if c_t, s_t, and κ_t are measured in nominal dollars and the explicit interest rate is strictly positive, then $\Xi_t / \Xi_{t-1} = \beta < 1$ and the agent's nominal risk aversion decreases as he approaches the terminal consumption date. This is not an attractive feature of this type of utility function.

On the other hand, if the time frame is sufficiently short that a zero interest rate is a reasonable assumption, then the optimal contract is characterized by

$$s_t(y_t) = s_t^o + \kappa_t(a_t) + \frac{1}{r}\ln\left(r\left[\lambda_t + \mu_t\left(r\kappa_t'(a_t) + \frac{d\Phi_{ta_t}(y_t|a_t)}{d\Phi_t(y_t|a_t)}\right)\right]\right). \quad (25.6)$$

In that setting, as in Chapter 19, if the probability distributions, expected payoffs, and effort costs are independent and identical for each period, then the optimal compensation function is identical for each period.

In the above analysis we implicitly assume that the agent's action choice at date t is independent of his information and actions from prior periods. The proof that this holds is similar to the proof in Chapter 19 and is not developed here.[4] The key to obtaining this result is the combination of technological and stochastic independence, and *AC-EC* preferences with exponential utility (which eliminates wealth effects).

Since current actions are independent of prior information, and the timing of the compensation is incidental, it follows that the results are the same whether $\mathbf{y}_t$ is reported at date t or T.

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[4] See the proof for the repeated binary signal model in Section 19.2.2. Also see Holmström and Milgrom (1987).

25.2.2 An "Effort Disutility" Model

In this section we continue to assume independence and AC preferences, but now the agent's direct preferences with respect to his effort are expressed as additive disutility functions. To facilitate comparison with the preceding analysis, we again assume an exponential utility for aggregate consumption, and we set aside the issue of the risk aversion changing with time by assuming a zero interest rate. The key difference is that the disutility for effort is assumed to be additive instead of multiplicative. That is, $u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T) = -\exp[-rw] - v$, where $v = v_1(a_1) + ... + v_T(a_T)$, with $v_t(a_t) = \exp[r\kappa_t(a_t)]$.

Unlike an AC-EC model, the timing of the reports matters in an ED model. We first consider the case in which the performance measures are not reported until date T ("terminal" reporting). Then we consider the case in which reports are released at the end of each period ("interim" reporting).

Terminal Reporting

We initially consider the case in which all the performance reports are issued at the contract termination date T, i.e., after all actions have been selected. Action choices must, therefore, be chosen without knowledge of prior performance. Hence, the analysis is basically the same as a simultaneous multi-task model (see Chapter 20).

We again simplify the discussion by assuming the action and report in each period are both single-dimensional, and the first-order approach can be used to characterize the incentive constraints for each period's actions. Under these conditions, the risk neutral principal's problem, given reporting system η, is as follows.

Principal's Decision Problem:

$$\underset{s, \overleftarrow{\mathbf{a}}_T}{\text{maximize}} \quad U^p(s, \overleftarrow{\mathbf{a}}_T, \eta) \equiv \mathrm{E}[x|\overleftarrow{\mathbf{a}}_T] - \int_{\overleftarrow{\mathbf{Y}}_T} s(\overleftarrow{\mathbf{y}}_T)\, d\Phi(\overleftarrow{\mathbf{y}}_T|\overleftarrow{\mathbf{a}}_T), \tag{25.7}$$

$$\text{subject to} \quad U^a(s, \overleftarrow{\mathbf{a}}_T, \eta) = \int_{\overleftarrow{\mathbf{Y}}_T} [u(s(\overleftarrow{\mathbf{y}}_T)) - v(\overleftarrow{\mathbf{a}}_T)]\, d\Phi(\overleftarrow{\mathbf{y}}_T|\overleftarrow{\mathbf{a}}_T) \geq \overline{U},$$

$$\frac{\partial U^a(s, \overleftarrow{\mathbf{a}}_T, \eta)}{\partial a_t} = 0, \quad t = 1, 2, ..., T.$$

The optimal compensation contract for (25.7) is characterized by

$$M(s(\overleftarrow{\mathbf{y}}_T)) = \lambda + \sum_{t=1}^{T} \mu_t L(y_t|a_t), \tag{25.8}$$

where $M(s(\overleftarrow{\mathbf{y}}_T)) = \dfrac{1}{u'(s(\overleftarrow{\mathbf{y}}_T))}$ and $L(y_t|a_t) = \dfrac{d\Phi_{ta_t}(y_t|a_t)}{d\Phi_t(y_t|a_t)}$.

Observe that if the agent has exponential utility, i.e., $u(c) = -\exp[-rc]$, then $M(s(\overleftarrow{\mathbf{y}}_T)) = 1/r \exp[rs(\overleftarrow{\mathbf{y}}_T)]$ and

$$s(\overleftarrow{\mathbf{y}}_T) = \frac{1}{r}\ln\left[r\left(\lambda + \sum_{t=1}^{T} \mu_t \frac{d\Phi_{ta_t}(y_t|a_t)}{d\Phi_t(y_t|a_t)}\right)\right]. \tag{25.9}$$

Interestingly, even though (25.5) and (25.9) are both based on exponential utility functions for aggregate consumption, the optimal contract $s(\overleftarrow{\mathbf{y}}_T)$ is additively separable in (25.5), but not in (25.9). This is due to the fact that in an *ED* model, there is a wealth effect between the utility for consumption and the disutility for effort (even though there is exponential utility for consumption). On the other hand, if the utility for consumption is a log function, i.e., $u^a(\overleftarrow{\mathbf{c}}_T,\overleftarrow{\mathbf{a}}_T) = \ln(w + b) - v$, then $M(s(\overleftarrow{\mathbf{y}}_T)) = s(\overleftarrow{\mathbf{y}}_T) + b$ and

$$s(\overleftarrow{\mathbf{y}}_T) = \lambda - b + \sum_{t=1}^{T} \mu_t \frac{d\Phi_{ta_t}(y_t|a_t)}{d\Phi_t(y_t|a_t)}, \tag{25.10}$$

which is additively separable.

If the probability functions, expected payoffs, and effort costs are identical for each period, then the optimal compensation function can be expressed as the sum of T identical compensation functions in (25.5) and (25.6), but not in (25.9). However, the identical periods case for (25.9) is interesting because it results in a symmetric compensation function given that no report is released until date T.

Proposition 25.1

If, for problem (25.7), the outcome functions, effort cost functions, and probability functions are identical across periods, and the set of possible reports $Y_t = \{y_1,...,y_m\}$ is finite, then $s(\overleftarrow{\mathbf{y}}_T)$ can be expressed as $s(\boldsymbol{\psi})$, where $\boldsymbol{\psi} = \psi(\overleftarrow{\mathbf{y}}_T) = (\psi_1,...,\psi_m)$ and ψ_j is the number of times y_t equals y_j in $\overleftarrow{\mathbf{y}}_T$.

The proof follows directly from (25.9), and the key implication of this result is that the sequence of reports is immaterial.

Interim Reporting

Now assume the reporting system generates reports each period. Hence, action choices can be based on prior performance, i.e., $a_t = \boldsymbol{a}_t(\overleftarrow{\mathbf{y}}_{t-1})$. In this discussion, $\overleftarrow{\mathbf{a}}_T = (\boldsymbol{a}_1, ..., \boldsymbol{a}_T)$ where $\boldsymbol{a}_t$ is a function specifying the agent's choice given each possible report history $\overleftarrow{\mathbf{y}}_{t-1}$.

Principal's Decision Problem:

$$\underset{\boldsymbol{s},\overleftarrow{\mathbf{a}}_T}{\text{maximize}} \quad U^p(\boldsymbol{s},\overleftarrow{\mathbf{a}}_T,\eta) \equiv \mathrm{E}[x|\overleftarrow{\mathbf{a}}_T] - \int_{\overleftarrow{\mathbf{Y}}_T} \boldsymbol{s}(\overleftarrow{\mathbf{y}}_T)\, d\Phi(\overleftarrow{\mathbf{y}}_T|\overleftarrow{\mathbf{a}}_T),$$

$$\text{subject to} \quad U^a(\boldsymbol{s},\overleftarrow{\mathbf{a}}_T, \eta) = \int_{\overleftarrow{\mathbf{Y}}_T} [u(\boldsymbol{s}(\overleftarrow{\mathbf{y}}_T)) - v(\overleftarrow{\mathbf{a}}_T)]\, d\Phi(\overleftarrow{\mathbf{y}}_T|\overleftarrow{\mathbf{a}}_T) \geq \overline{U},$$

$$\frac{\partial U^a(\boldsymbol{s},\overleftarrow{\mathbf{a}}_T|\overleftarrow{\mathbf{y}}_{t-1})}{\partial a_t} = 0, \quad \forall\, \overleftarrow{\mathbf{y}}_{t-1} \in \overleftarrow{\mathbf{Y}}_{t-1},\ t = 1, 2, ..., T,$$

$$\text{where} \quad U^a(\boldsymbol{s},\overleftarrow{\mathbf{a}}_T|\overleftarrow{\mathbf{y}}_{t-1}) = \int_{\overleftarrow{\mathbf{Y}}_T} [u(\boldsymbol{s}(\overleftarrow{\mathbf{y}}_T)) - v(\overleftarrow{\mathbf{a}}_T)]\, d\Phi(\overleftarrow{\mathbf{a}}_T|\overleftarrow{\mathbf{a}}_T,\overleftarrow{\mathbf{y}}_{t-1}).$$

Note that even though the reports are independent, knowing $\overleftarrow{\mathbf{y}}_{t-1}$ affects the agent's beliefs about his future compensation if compensation varies with past performance. If that occurs, then the performance history may affect the agent's current action choice. The optimal compensation contract is characterized by

$$M(\boldsymbol{s}(\overleftarrow{\mathbf{y}}_T)) = \lambda + \sum_{t=1}^{T} \mu_t(\overleftarrow{\mathbf{y}}_{t-1})\, L(y_t|\boldsymbol{a}_t(\overleftarrow{\mathbf{y}}_{t-1})).$$

Of course, in the first-best case where $\overleftarrow{\mathbf{y}}_T$ and $\overleftarrow{\mathbf{a}}_T$ are both contractible and the agent is paid a fixed wage, the optimal action strategy is such that $\boldsymbol{a}_t$ is independent of the reported history $\overleftarrow{\mathbf{y}}_{t-1}$.

Matsumura (1988) considers the special case in which the principal's outcome is the reported performance measure. The following are some key points obtained in her analysis, but with y_t representing the contractible information reported at the end of period t.

Proposition 25.2 (Matsumura 1988, Prop. 7.1)
In the first-best case with a risk neutral principal and independent performance measures, the optimal action strategy is such that $\boldsymbol{a}_t$ is independent of the performance history $\overleftarrow{\mathbf{y}}_{t-1}$.

There are three basic reasons why it might be optimal to vary $\boldsymbol{a}_t$ with $\overleftarrow{\mathbf{y}}_{t-1}$.

(i) *Information effect:* If $\varphi(x_t|\overleftarrow{\mathbf{a}}_t, \overleftarrow{\mathbf{y}}_{t-1})$ is dependent on prior performance or actions, then that can influence the optimal choice of a_t.

(ii) *Principal's wealth effect:* If the principal is risk averse, his marginal utility for wealth is decreasing in wealth. If y_t is positively correlated with x_t, then high values of $\overleftarrow{\mathbf{y}}_{t-1}$ imply high values for $x_1 + ... + x_{t-1}$. This implies that the incremental utility for x_t is lower and, hence, it is not "worth" as much effort.

(iii) *Agent's wealth effect:* If the agent is risk averse with additively separable utility, his marginal utility for compensation is decreasing in total compensation. If $s(\overleftarrow{\mathbf{y}}_T)$ is increasing in $\overleftarrow{\mathbf{y}}_t$, then the incremental cost of inducing more effort in period t is increasing in $\overleftarrow{\mathbf{y}}_{t-1}$.

The independence assumption made in this section eliminates the first effect, while the risk neutral principal assumption eliminates the second. The preceding proposition establishes that contractible effort (which permits achievement of first-best) is sufficient to eliminate the third effect. The following proposition establishes that if we only have the first two conditions and not the third (i.e., effort is not contractible), then past performance affects the optimally induced effort levels.

Proposition 25.3 (Matsumura 1988, Prop. 7.2)
If the principal is risk neutral, the agent has *AC-ED* preferences, and the performance measures are independent across time, then in the second-best case ($\overleftarrow{\mathbf{x}}_T$ is contractible, but $\overleftarrow{\mathbf{a}}_T$ is not) with observability of $\overleftarrow{\mathbf{x}}_{t-1}$ by the agent prior to choosing a_t:

(a) $\mu_t(\overleftarrow{\mathbf{x}}_{t-1}) > 0$ and $s(\overleftarrow{\mathbf{x}}_T)$ is increasing in x_t, $t = 1, ..., T$;

(b) $U^a(\boldsymbol{s}, \overleftarrow{\mathbf{a}}_T|\overleftarrow{\mathbf{x}}_{t-1})$ is increasing in $\overleftarrow{\mathbf{x}}_{t-1}$; and

(c) $\boldsymbol{a}_t(\overleftarrow{\mathbf{x}}_{t-1})$ is decreasing in $\overleftarrow{\mathbf{x}}_{t-1}$ given suitable regularity on $v(\overleftarrow{\mathbf{a}}_T)$.

While there is no wealth effect with respect to the principal (due to risk neutrality), there is a wealth effect with respect to the agent. He receives higher compensation if the higher prior outcomes are observed. This increases the cost to the principal of inducing more subsequent effort. Observe that this wealth effect occurs even if $u_t(c_t)$ is negative exponential – it is strictly due to the fact that a given level of incremental compensation provides less marginal utility when the agent has more wealth (and, due to additive separability, i.e., the agent's marginal disutility of effort is unaffected by his wealth).[5]

Result (c) is illustrated by the multi-period hurdle model introduced in Section 25.5 (see Appendix 25A for analytical and numerical details). Particularly note the principal's problem formulated in Table 25A.1(b) and the numerical example in Table 25A.3(b). The optimal contract induces the agent to provide more effort in the second period if he obtains a bad versus a good outcome in first period. The key to this result is the fact the agent will receive less compensation if his first-period outcome is bad versus good.

25.3 TIME-ADDITIVE PREFERENCES

In this section we assume the agent has a *TA-ED* utility function, i.e., it is additively separable across time and with respect to consumption and effort. Compared to the previous sections, the timing of the agent's consumption now becomes a key aspect of the analysis. Initially, we assume the principal directly controls the agent's consumption because the agent does not have access to personal banking – he cannot even save from one period to the next. This is what we call the "isolated agency/perishable goods" assumption. Despite its lack of realism, a number of the classical papers on multi-period agencies have made this assumption. We view it as a useful benchmark relative to the subsequent analysis in which the agent has access to borrowing and saving.

The following analysis is based on papers by Lambert (1983), Rogerson (1985), and Christensen and Frimor (1998).[6] They assumed that the outcome is contractible and is the sole performance measure. We consider a generic performance measure, allowing for the possibility that the outcome may not be contractible.

25.3.1 No Agent Banking

As in the preceding section, the compensation scheme and action strategy are $\overleftarrow{\mathbf{s}}_T = (\boldsymbol{s}_1, ..., \boldsymbol{s}_T)$, where $\boldsymbol{s}_t$: $\overleftarrow{\mathbf{Y}}_t \rightarrow C$, and $\overleftarrow{\mathbf{a}}_T = (\boldsymbol{a}_1, ..., \boldsymbol{a}_T)$, where $\boldsymbol{a}_t$: $\overleftarrow{\mathbf{Y}}_{t-1} \rightarrow A_t$, respect-

[5] See Figure 25.2 later in the chapter for an illustration of the impact of agent wealth on the compensation contract and the resulting expected compensation costs.

[6] See also Fellingham, Newman, and Suh (1985), and Chiappori, Macho, Rey, and Salanié (1994) for general analyses of commitment, memory, and banking in multi-period agencies.

ively. That is, the compensation paid at date t may depend on the entire performance history to that date, $\overleftarrow{\mathbf{y}}_t$, whereas the action taken at the start of period t depends on the performance history $\overleftarrow{\mathbf{y}}_{t-1}$. At any given date $t-1$, the action strategy can be expressed as consisting of the strategy implemented in the past, $\overleftarrow{\mathbf{a}}_{t-1}$, and that which will be implemented in the future, denoted $\vec{\mathbf{a}}_t = (\boldsymbol{a}_t, ..., \boldsymbol{a}_T)$. Similarly, we let $\vec{\mathbf{s}}_t = (\boldsymbol{s}_t, ..., \boldsymbol{s}_T)$ represent the future compensation scheme. We assume throughout that both the compensation scheme and the action strategy are interior. The *ex ante* probability distribution over the set of possible performance reports, given that action strategy $\overleftarrow{\mathbf{a}}_T$ will be implemented, is denoted $\Phi(\overleftarrow{\mathbf{y}}_T \,|\, \overleftarrow{\mathbf{a}}_T)$.

The agent cannot borrow or save across consumption dates. That is, his consumption at date t is assumed to be equal to his compensation at date t, i.e., $c_t = s_t$. The principal may or may not be able to borrow and save. However, if he can borrow and save, his time-preferences must be equal to the market return R (i.e., $\Xi_{t-1} = R\Xi_t$) to avoid unbounded solutions to his decision problem. Therefore, we can in both cases ignore his borrowing and saving decisions and represent his preferences as maximizing the expected net value of his firm measured at the valuation date implicit in the principal's time-preference index Ξ_t.

Principal's Decision Problem:

$$\underset{\overleftarrow{\mathbf{s}}_T,\, \overleftarrow{\mathbf{a}}_T}{\text{maximize}} \quad U^p(\overleftarrow{\mathbf{s}}_T, \overleftarrow{\mathbf{a}}_T, \eta) \equiv \mathrm{E}[x \,|\, \overleftarrow{\mathbf{a}}_T] - \int_{\overleftarrow{\mathbf{Y}}_T} \sum_{t=1}^{T} \Xi_t\, s_t(\overleftarrow{\mathbf{y}}_t)\, d\Phi(\overleftarrow{\mathbf{y}}_T | \overleftarrow{\mathbf{a}}_T),$$

$$\text{subject to} \quad U^a(\overleftarrow{\mathbf{s}}_T, \overleftarrow{\mathbf{a}}_T, \eta) = \int_{\overleftarrow{\mathbf{Y}}_T} \sum_{t=1}^{T} u_t^a(s_t(\overleftarrow{\mathbf{y}}_t), \boldsymbol{a}_t(\overleftarrow{\mathbf{y}}_{t-1}))\, d\Phi(\overleftarrow{\mathbf{y}}_T | \overleftarrow{\mathbf{a}}_T) \geq \bar{U},$$

$$U_{t-1}^a(\vec{\mathbf{s}}_t, \vec{\mathbf{a}}_t \,|\, \overleftarrow{\mathbf{y}}_{t-1}) \geq U_{t-1}^a(\vec{\mathbf{s}}_t, \hat{\vec{\mathbf{a}}}_t \,|\, \overleftarrow{\mathbf{y}}_{t-1}), \quad \forall\, \hat{\vec{\mathbf{a}}}_t, \overleftarrow{\mathbf{y}}_{t-1},$$

where[7]

$$U_{t-1}^a(\vec{\mathbf{s}}_t, \vec{\mathbf{a}}_t \,|\, \overleftarrow{\mathbf{y}}_{t-1}) = \int_{\overleftarrow{\mathbf{Y}}_T} \sum_{\tau=t}^{T} [u_\tau^a(s_\tau(\overleftarrow{\mathbf{y}}_\tau), \boldsymbol{a}_\tau(\overleftarrow{\mathbf{y}}_{\tau-1}))]\, d\Phi(\overleftarrow{\mathbf{y}}_T | \vec{\mathbf{a}}_t, \overleftarrow{\mathbf{y}}_{t-1}).$$

Note that the independence between future performance and past actions implies that the agent's incentives for action choices only depend on the performance

[7] Observe that $U_{t-1}^a(\vec{\mathbf{s}}_t, \vec{\mathbf{a}}_t \,|\, \overleftarrow{\mathbf{y}}_{t-1})$ is the agent's expected future utility at date $t-1$ given future contract $\vec{\mathbf{s}}_t$, future action strategy $\vec{\mathbf{a}}_t$, and performance history $\overleftarrow{\mathbf{y}}_{t-1}$.

history and not on the action history.[8] This condition also implies that the incentive constraint need only consider "local" changes in actions, i.e.,

$$U^a_{t-1}(\vec{\mathbf{s}}_t, \vec{\mathbf{a}}_t | \overleftarrow{\mathbf{y}}_{t-1}) \geq U^a_{t-1}(\vec{\mathbf{s}}_t, (\vec{\mathbf{a}}_{t+1}, \hat{a}_t) | \overleftarrow{\mathbf{y}}_{t-1}), \quad \forall\ \hat{a}_t \in A_t,\ \overleftarrow{\mathbf{y}}_{t-1}.$$

Additive separability of preferences across time implies that in the *first-best case*, compensation as well as action choices are independent of the performance history.[9] This is not the case in the *second-best solution.*

Proposition 25.4 (Lambert 1983, Prop. 1-3)

Assume the principal is risk neutral, the agent has *TA-ED* preferences, MLRP holds for each period, and the first-order approach to incentives is valid. The optimal second-best compensation plan satisfies the following first-order conditions:

$$M_t(\boldsymbol{s}_t(\overleftarrow{\mathbf{y}}_t)) = \lambda + \sum_{\tau=1}^{t} \mu_\tau(\overleftarrow{\mathbf{y}}_{\tau-1}) L(y_\tau | \boldsymbol{a}_\tau(\overleftarrow{\mathbf{y}}_{\tau-1})),$$

where $$M_t(\boldsymbol{s}_t(\overleftarrow{\mathbf{y}}_t)) \equiv \frac{\Xi_t}{u_t'(\boldsymbol{s}_t(\overleftarrow{\mathbf{y}}_t))},$$

with $$\mu_\tau(\overleftarrow{\mathbf{y}}_{\tau-1}) > 0, \quad \forall\ \overleftarrow{\mathbf{y}}_{\tau-1},\ \tau = 1, ..., T.$$

For $T = 2$ (the case Lambert (1983) analyzes), we have

$$M_1(\boldsymbol{s}_1(y_1)) = \lambda + \mu_1 L(y_1 | a_1),$$

and $$M_2(\boldsymbol{s}_2(y_1, y_2)) = \lambda + \mu_1 L(y_1 | a_1) + \mu_2(y_1) L(y_2 | \boldsymbol{a}_2(y_1)).$$

Observe that with MLRP, s_1 is increasing in y_1 and s_2 is increasing in y_2. The fact that $\mu_1 > 0$ implies that s_2 depends on the first-period performance y_1, i.e.,

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[8] This would still hold if we were to allow for stochastic dependence across periods of the form $\varphi(y_t | \overleftarrow{\mathbf{a}}_t, \overleftarrow{\mathbf{y}}_{t-1}) = \varphi(y_t | a_t, \overleftarrow{\mathbf{y}}_{t-1})$. The key assumption is that y_t is independent of past actions $\overleftarrow{\mathbf{a}}_{t-1}$ conditional on $\overleftarrow{\mathbf{y}}_{t-1}$. If that is not the case, we would have to account for "double shirking" in the incentive constraints (see Section 27.1).

[9] The fixed wages and the action choices may, however, depend on time due to differences in time preferences for the agent and the principal as well as differences in production technology across periods.

optimal contracts have *memory*. Furthermore, it is easy to show that the agent's conditional expected utility, i.e., $U^a(\overleftarrow{\mathbf{s}}_2, \overleftarrow{\mathbf{a}}_2, a_2(y_1)|y_1)$, is an increasing function of y_1. That is, the second-period incentives are used to reinforce the first-period incentives. Hence, even though periods are independent and preferences are time-additive, the intertemporal allocation of compensation is used to mitigate the trade-off between incentives and risk sharing both within periods and across periods.

The following proposition is noted in Lambert (1983) and follows directly from the characterization of the first-order conditions in the above proposition, but is proved by Rogerson (1985) without relying on the applicability of the first-order approach – he only assumes that the compensation scheme is interior.

Proposition 25.5 (Rogerson 1985, Prop. 1 and 2)
Assume the principal is risk neutral and the agent has *TA-ED* preferences. For any optimal contract with an interior compensation scheme, the following relations hold between compensations at different dates.

(a) The unconditional expected marginal cost of agent utility is the same for all periods, i.e.,[10]

$$\int_{\overleftarrow{\mathbf{Y}}_t} M_t(s_t(\overleftarrow{\mathbf{y}}_t))\, d\Phi(\overleftarrow{\mathbf{y}}_t|\overleftarrow{\mathbf{a}}_t) = \lambda, \quad \forall\, t = 1, ..., T. \tag{25.11}$$

(b) For any date t performance history, $\overleftarrow{\mathbf{y}}_t$, the marginal cost of agent utility at date t is equal to the expected marginal cost of agent utility in all future periods conditional on the performance history $\overleftarrow{\mathbf{y}}_t$, i.e.,

$$M_t(s_t(\overleftarrow{\mathbf{y}}_t)) = \int_{\overleftarrow{\mathbf{Y}}_\tau} M_\tau(s_\tau(\overleftarrow{\mathbf{y}}_\tau))\, d\Phi(\overleftarrow{\mathbf{y}}_\tau|\overleftarrow{\mathbf{a}}_\tau, \overleftarrow{\mathbf{y}}_t), \quad \forall\, \tau = t+1, ..., T. \tag{25.12}$$

(c) If $s_t(\overleftarrow{\mathbf{y}}_t)$ depends on y_s for $s \le t$, then $s_\tau(\overleftarrow{\mathbf{y}}_\tau)$ with $\tau > t$ also depends on y_s.

The compensation scheme is constructed so that, at any date t, the *expected marginal cost of agent utility* is the same across all future periods, both for the unconditional expectation and the expectation conditional on $\overleftarrow{\mathbf{y}}_t$. That is, the principal smooths the cost of agent utility across periods. This suggests that the optimal long-term contract *smooths the agent's compensation over time*. To

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[10] Recall that $M_t(s_t) \equiv \Xi_t/u_t'(s_t)$, so that $M_t(s_t) = \Xi_t\, du_t^{-1}(v)/dv$ for $v = u_t(s_t)$ is the marginal cost of agent utility for the compensation s_t.

minimize the cost of the incentive risk imposed on the agent, the principal spreads the incentive risk based on the earlier period performances over as many future periods as possible, i.e., optimal contracts have *memory*. For example, suppose $T = 2$ and that there is no second-period incentive problem (i.e., $\mu_2(y_1) = 0$). In that case, the second-period compensation will depend on the first-period performance (and not on the performance in the second period) in such a way that the agent's marginal utility of consumption is the same in both periods. That is, the agent obtains perfect consumption smoothing over the two periods. The basic reason here is that the agent's compensation must vary with y_1 in order to motivate his choice of a_1. The risk premium required to compensate for the incentive risk based on y_1 can be reduced if the agent "shares his risk with himself" across periods.[11] However, as we shall see in Section 25.3.2, the agent does not obtain perfect consumption smoothing if there are non-trivial incentive problems in all periods. The reason is that the principal can reduce the cost of agent utility by shifting *utility levels* across periods without affecting incentives, whereas shifting *compensation levels* across periods affects incentives (see Proposition 25.9 and Christensen and Frimor (1998) for further details of the trade-offs involved in the optimal intertemporal allocation of compensation).

The following proposition relates the expected compensation across periods. It follows almost immediately from Proposition 25.5 and Jensen's inequality.

Proposition 25.6 (Rogerson 1985, Prop. 3)

Suppose the agent has *TA-ED* preferences, and his utility for date t consumption can be represented by $u_t(c_t) = \Xi_t u(c_t)$ for some function $u(\cdot)$, with $\Xi_{t-1} = R\Xi_t$.[12] Further suppose that $M_t(\cdot)$ is convex (concave). For any optimal contract with an interior compensation scheme, the following relations hold between compensation levels at different dates.

(a) The unconditional expected compensation is decreasing (increasing) through time, i.e.,

$$\int_{\overleftarrow{\mathbf{Y}}_t} s_t(\overleftarrow{\mathbf{y}}_t)\, d\Phi(\overleftarrow{\mathbf{y}}_t|\overleftarrow{\mathbf{a}}_t) \;\geq\; (\leq) \int_{\overleftarrow{\mathbf{Y}}_\tau} s_\tau(\overleftarrow{\mathbf{y}}_\tau)\, d\Phi(\overleftarrow{\mathbf{y}}_\tau|\overleftarrow{\mathbf{a}}_\tau), \quad \forall\, \tau = t+1, ..., T.$$

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[11] Of course, if there is a second-period incentive problem and it was possible to make the first-period compensation a function of both y_1 and y_2, then it would be optimal to do so.

[12] The assumption that the time-preference index satisfies $\Xi_{t-1}/\Xi_t = R$ is critical (to avoid unbounded or corner solutions) for a risk-neutral principal and an agent with *AC* preferences, but it is not crucial for an agent with *TA* preferences (see Section 25.4).

(b) For any performance history at date t, $\overleftarrow{\mathbf{y}}_t$, the compensation at date t is higher (lower) than the expected compensation in all future periods conditional on the performance history $\overleftarrow{\mathbf{y}}_t$, i.e.,[13]

$$s_t(\overleftarrow{\mathbf{y}}_t) \geq (\leq) \int_{\overleftarrow{\mathbf{Y}}_\tau} s_\tau(\overleftarrow{\mathbf{y}}_\tau)\, d\Phi(\overleftarrow{\mathbf{y}}_\tau | \overleftarrow{\mathbf{a}}_\tau, \overleftarrow{\mathbf{y}}_t), \quad \forall \tau = t+1, ..., T.$$

If $M_t(\cdot)$ is linear, i.e., $u_t(c_t) = \Xi_t \ln(c_t + b)$, the weak inequalities hold as equalities.

In the HARA class of utility functions (see Appendix 17C), $M(\cdot)$ is convex with the exponential utility function, and with $u(c) = [\alpha c + b]^{1-1/\alpha}/(\alpha - 1)$ for $\alpha < 1$, implying that the expected compensation is decreasing over time. On the other hand, $M(\cdot)$ is concave for $\alpha > 1$, implying that the expected compensation is increasing over time. Keep in mind that if the likelihood ratio is linear, then $M(\cdot)$ convex (concave) implies that $\boldsymbol{c}(\cdot)$ is concave (convex).

25.3.2 Agent Access to Personal Banking

If the agent has access to the financial markets, his consumption need not equal his compensation in each period, i.e., his consumption at date t is $c_t = s_t + R\ell_{t-1} - \ell_t$ where ℓ_{t-1} is his savings (or end of period wealth) at date $t - 1$ and R is the riskless return. Observe that this implies that the value of the agent's source of consumption is equal to the net value of his consumption plus terminal wealth, i.e.,

$$\sum_{t=1}^{T} \Xi_t s_t = \sum_{t=1}^{T} \Xi_t c_t + \Xi_T \ell_T,$$

where ℓ_T is the agent's terminal wealth measured in date T dollars. In this analysis we assume that the agent has *no desire to hold terminal wealth, and that he must repay any outstanding debt at date T.* Hence, $c_T = s_T + R\ell_{T-1}$ and $\ell_T = 0$. We also assume that the principal and agent can commit to a long-term contract and that the agent has *TA-ED* preferences.

In the basic Rogerson model, the principal can borrow and save at a rate R. That is, the principal and agent can borrow or save at the *same rate*. The principal is risk neutral and he evaluates the agency in terms of its *net value* at the valuation date implicit in his time-preference index Ξ_t.

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[13] The weak inequality holds as a strict inequality if $\Phi(\overleftarrow{\mathbf{y}}_\tau | \overleftarrow{\mathbf{a}}_\tau, \overleftarrow{\mathbf{y}}_t)$ is non-degenerate.

The introduction of borrowing and saving opportunities for the agent introduces an indeterminancy in the principal's decision problem in that there are an indefinite number of compensation schemes that solve his problem. Let $w(\overleftarrow{\mathbf{y}}_T)$ denote the net value of the agent's compensation for the performance history $\overleftarrow{\mathbf{y}}_T$, i.e.,

$$w(\overleftarrow{\mathbf{y}}_T) = \sum_{t=1}^{T} \Xi_t s_t(\overleftarrow{\mathbf{y}}_t).$$

Any borrowing and saving strategy has zero net value. Hence, any two compensation schemes that have the same net values for any performance history $\overleftarrow{\mathbf{y}}_T$ can be used to implement the same consumption strategies.

Proposition 25.7

Let $(\overleftarrow{\mathbf{s}}_T, \overleftarrow{\ell}_T)$ represent an optimal compensation scheme and an optimal saving strategy with associated consumption strategy $\overleftarrow{\mathbf{c}}_T$. Then any other compensation scheme $\overleftarrow{\hat{\mathbf{s}}}_T$ that has the same net value for any performance history $\overleftarrow{\mathbf{y}}_T$, is also optimal. In particular, a compensation scheme equal to $\overleftarrow{\mathbf{c}}_T$ is optimal.

Proof: Given compensation scheme $\overleftarrow{\mathbf{s}}_T$, let $w_t(\overleftarrow{\mathbf{y}}_T)$ denote the net value of the compensation received subsequent to date t for the realized performance history $\overleftarrow{\mathbf{y}}_T$, i.e.,

$$w_t(\overleftarrow{\mathbf{y}}_T) = \sum_{\tau=t+1}^{T} \Xi_\tau s_\tau(\overleftarrow{\mathbf{y}}_\tau),$$

and, similarly, let $\hat{w}(\overleftarrow{\mathbf{y}}_T)$ represent the net value for compensation scheme $\overleftarrow{\hat{\mathbf{s}}}_T$. Since $w(\overleftarrow{\mathbf{y}}_T) = \hat{w}(\overleftarrow{\mathbf{y}}_T)$ for any $\overleftarrow{\mathbf{y}}_T$ and

$$w(\overleftarrow{\mathbf{y}}_T) = \sum_{\tau=1}^{t} \Xi_\tau s_\tau(\overleftarrow{\mathbf{y}}_\tau) + w_t(\overleftarrow{\mathbf{y}}_T),$$

it follows that $w_t(\overleftarrow{\mathbf{y}}_T) - \hat{w}_t(\overleftarrow{\mathbf{y}}_T)$ is independent of the performance history from date $t+1$ until date T. Hence, using backward induction starting from $T-1$, it is straightforward to show that there exists a savings strategy $\overleftarrow{\hat{\ell}}_T$ that implements $\overleftarrow{\mathbf{c}}_T$ with the compensation scheme $\overleftarrow{\hat{\mathbf{s}}}_T$. Since $(\overleftarrow{\mathbf{s}}_T, \overleftarrow{\ell}_T)$ and $(\overleftarrow{\hat{\mathbf{s}}}_T, \overleftarrow{\hat{\ell}}_T)$ have the same consumption strategies, the optimal action strategy for $\overleftarrow{\mathbf{s}}_T$ remains incentive compatible for $\overleftarrow{\hat{\mathbf{s}}}_T$. Hence, since the two compensation schemes have the same expected net value to the principal, $\overleftarrow{\hat{\mathbf{s}}}_T$ is also an optimal compensation scheme.

Finally, since the value of the agent's compensation is equal to the net value of his consumption, $\overleftarrow{\mathbf{c}}_T$ is also an optimal compensation scheme. **Q.E.D.**

To eliminate the indeterminancy in the principal's decision problem, we can, without loss of generality, assume the compensation contract is such that the agent has *no incentive to borrow or save*. This is very different from exogenously precluding the agent from borrowing and saving, since the no borrowing and saving constraint on the compensation scheme is generally binding. A necessary and sufficient condition that ensures the compensation scheme is such that the agent has no incentive to borrow or save is that for any date t his marginal utility of current consumption is equal to his conditional expected marginal utility of consumption for the following date, i.e.,[14]

$$u_t'(s_t(\overleftarrow{\mathbf{y}}_t)) = \int_{\overleftarrow{\mathbf{Y}}_{t+1}} R u_{t+1}'(s_{t+1}(\overleftarrow{\mathbf{y}}_{t+1}))\, d\Phi(\overleftarrow{\mathbf{y}}_{t+1}|\overleftarrow{\mathbf{a}}_{t+1}, \overleftarrow{\mathbf{y}}_t). \tag{25.13}$$

That is, the agent gets *perfect consumption smoothing* when he can borrow and save. Note that this also implies that the compensation scheme has memory.

Comparing (25.12) to (25.13) and using Jensen's inequality, Rogerson (1985) demonstrates that for any optimal contract with the agent exogenously precluded from borrowing and saving, the agent would never want to borrow from one period to the next, but would strictly prefer to save if there is a non-trivial incentive problem in the following period.

Proposition 25.8 (Rogerson 1985, Prop. 4)

Assume the agent has *TA-ED* preferences. Given the optimal contract with the agent exogenously precluded from borrowing and saving,

(a) the agent would *not prefer to borrow* from one period to the next, even if it were possible;

(b) if the conditional probability distribution for the agent's compensation in the following period is non-degenerate, then the agent would strictly *prefer to save* from the current period to the next.

The proposition demonstrates that unless the incentive problem in the following period is trivial, saving constraint (25.13) is binding. However, the solution is unchanged if it is replaced with the following weaker constraint, which ensures that the agent has no incentive to save:

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[14] Of course, this also implies that the agent's marginal utility of current consumption is equal to his conditional expected marginal utility of consumption at all future dates.

$$u_t'(s_t(\overleftarrow{\mathbf{y}}_t)) \geq \int_{\overleftarrow{\mathbf{Y}}_{t+1}} R u_{t+1}'(s_{t+1}(\overleftarrow{\mathbf{y}}_{t+1}))\, d\Phi(\overleftarrow{\mathbf{y}}_{t+1} | \overleftarrow{\mathbf{a}}_{t+1}, \overleftarrow{\mathbf{y}}_t). \qquad (25.13')$$

The fact that this constraint would be binding implies that the principal would strictly prefer to be able to exogenously preclude the agent from saving (thereby removing binding constraint (25.13′) from his problem). It may seem reasonable to assume that the principal can restrict the agent's borrowing possibilities, but it is not appealing to assume he can restrict the agent's saving opportunities (unless the agent is at a remote site with perishable outcomes). Hence, multi-period agency models with multiple consumption dates and time-additive preferences must allow the agent to save in order to get results that do not rely on restrictions on the agent's saving. Of course, as noted earlier, restriction of borrowing and saving is not an issue if a model assumes the agent has *AC* preferences (see Sections 25.2.1 and 25.2.2).

To understand why the optimal contract without agent access to banking creates an incentive to save, assume the contrary. That is, assume the optimal contract is such that (25.13) holds even if it is not imposed. Christensen and Frimor (1998) demonstrate that such a contract cannot be optimal since there would exist a variation in the compensation scheme (equivalent to forcing the agent to borrow on personal account) that either facilitates improved risk sharing with no change in incentives or improves the second-period action choice with no change in the cost of risk.

Proposition 25.9 (Christensen and Frimor 1998, Lemma 6)

Assume the agent has *TA-ED* preferences. Let z be an interior optimal contract without personal access to banking. If the conditional probability distribution for $s_{t+1}(\overleftarrow{\mathbf{y}}_{t+1})$ given $\overleftarrow{\mathbf{y}}_t$ is non-degenerate, $s_{t+1}(\overleftarrow{\mathbf{y}}_{t+1})$ is increasing in y_{t+1}, and MLRP holds,[15] then there exists a variation in the compensation scheme of the form, for $\delta > 0$,

$$s_t^b(\overleftarrow{\mathbf{y}}_t) = s_t(\overleftarrow{\mathbf{y}}_t) + \delta,$$

$$s_{t+1}^b(\overleftarrow{\mathbf{y}}_{t+1}) = s_{t+1}(\overleftarrow{\mathbf{y}}_{t+1}) - R\delta, \quad \forall y_{t+1} \in Y_{t+1},$$

that either makes the incentive compatibility constraint non-binding (for discrete action spaces) or increases a_{t+1} (for convex action spaces).

[15] It is sufficient that $s_{t+1}(\overleftarrow{\mathbf{y}}_{t+1})$ is increasing in the likelihood ratio between the induced action, a_{t+1}, and lower cost actions.

Reducing the compensation at date $t+1$ increases the agent's date $t+1$ marginal utilities and, thus, makes it less costly to induce incentives. Hence, an optimal contract must leave the agent with an incentive to save.[16]

With agent access to banking, the principal's opportunities to allocate compensation across periods so that it equals the agent's consumption in each period is limited by the fact that the agent can (partly) undo that allocation by unobservable borrowing and saving on a personal account and, thereby, change the incentives for action choices (as demonstrated in Proposition 25.9). Hence, providing the agent with access to banking affects not only the optimal allocation of compensation across periods but also the incentives for action choices.

The multi-period hurdle model introduced in Section 25.5 and Appendix 25A illustrates the impact of the agent's access to banking. Particularly note the model formulated in Table 25A.2(b) and the numerical example in Table 25A.4(b). The principal's expected payoff is distinctly lower if the agent has access to banking, and there is a distinct difference in the induced actions. The model is a two-period model in which the agent can save (or borrow) from the first to the second period. Access to banking reduces the induced second-period actions, while the induced first-period action increases, reflecting the fact that access to banking increases the cost to the principal of inducing second-period actions.

25.4 MULTI-PERIOD *LEN* MODEL

In this section we introduce a multi-period version of the *LEN* model (see Chapter 20). That is, the contracts are restricted to be linear functions of the performance measures, the agent's utility function is exponential with either *AC-EC* or *TA-EC* preferences. The monetary costs of effort are strictly convex, and the performance measures are linear functions of the agent's effort, with normally distributed noise. The key benefit of these assumptions is that they result in relatively simple mean-variance representations of the agent's certainty equivalent. These representations yield closed form expressions for the optimal contract and actions, which in turn facilitate comparative statics.

To simplify the discussion, we assume that the consumption planning horizon $\breve{T}$ equals the contract termination date T, unless explicitly assumed otherwise.

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[16] The variation in Proposition 25.9 does not affect the expected compensation cost for fixed action choices. Furthermore, if the agent has no incentive to save or borrow, a marginal variation does not change the agent's conditional expected utility either.

25.4.1 The Agent's Preferences and Compensation

We consider two types of exponential utility functions introduced in (25.2). Both are effort cost (*EC*) models, but one is time-additive (*TA*) with respect to period-by-period net consumption, whereas the other is expressed in terms of aggregate net consumption (*AC*). More specifically, the two *EC* models are:[17]

$$TA: \quad u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T) = -\sum_{t=0}^{T} \Xi_t^a \exp\big[-r(c_t - \kappa_t(a_t))\big],$$

$$AC: \quad u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T) = -\exp\Big[-r\sum_{t=0}^{T} \Xi_t(c_t - \kappa_t(a_t))\Big],$$

where Ξ_t^a and Ξ_t are time-preference indices. In this analysis we allow for three generalizations compared to the prior analysis. First, there is consumption at the contracting date $t = 0$ (but there is no effort cost at that date, i.e., $\kappa_0 = 0$).[18] Second, the rate of change in the time-preference indices may be time-dependent, i.e., $\beta_t^a \equiv \Xi_{t+1}^a/\Xi_t^a$ and $\beta_t \equiv \Xi_{t+1}/\Xi_t$ may be time-dependent. Hence, the term structure of interest rates may be non-flat, but we continue to assume deterministic interest rates. Third, we allow for the fact that the agent's time-preference index may differ from the market's with *TA* preferences, i.e., $\beta_t^a \neq \beta_t$ for at least some dates t. If the agent's time-preference is the same as the market's, i.e., $\beta_t^a = \beta_t$ for all dates t, "flat" consumption smoothing is obtained. In fact, if compensation is deterministic, then the agent's consumption will be absolutely flat even if the compensation varies (deterministically) from period to period. Smoothing does not occur with *AC* preferences, but assuming $\beta_t^a = \beta_t$ is essential, since otherwise the solution to the agent's consumption decision problem is unbounded, i.e., he will choose to go infinitely long in consumption in one period and infinitely short in another.

Consumption, compensation, and effort costs are measured in the nominal dollars of the period in which they occur. Under *AC*, multiplying the nominal dollars at date t by Ξ_t can be interpreted as converting them to a common mea-

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[17] Accounting papers that have used *AC-EC* preferences in multi-period *LEN* models include Indjejikian and Nanda (1999), Christensen, Feltham, and Şabac (2003, 2005), and Christensen, Feltham, Hofmann, and Şabac (2004). Multi-period *LEN* models with *TA-EC* preferences are used in Dutta and Reichelstein (1999, 2003) as well as in Christensen, Feltham, Hofmann, and Şabac (2004). Preliminary analysis of a *LEN* model with *TA-EC* preferences with agent-specific time preferences was provided to us by Christian Hofmann.

[18] We could have introduced consumption at $t = 0$ in the earlier models without changing any of the substantive results. In the current analysis, having consumption at the contracting date simplifies the representation of the agent's expected utility.

sure that can be added. For example, with a flat term structure of interest rates, $\Xi_t = \beta^t$ converts nominal date t dollars into date 0 dollars, whereas $\Xi_t = R^{T-t}$ converts date t dollars into date T dollars (in both cases, $\Xi_{t+1}/\Xi_t = \beta$). Models that assume *AC* with a zero interest rate can be interpreted as having made this conversion rather than using nominal dollars.

Access to borrowing and saving is not an issue with *AC* preferences, but it is with *TA*. Hence, while we assume that the agent can borrow and save, its role is only significant with *TA*. Our main focus in this section is the nature of the consumption smoothing with *TA*, and how non-zero interest rates affect the agent's risk premium for the two types of preferences.

As in Chapter 20, we consider a setting with multiple tasks and multiple performance measures each period. The agent's action in period $t = 1, ..., T$ is an $m_t \times 1$ vector $\mathbf{a}_t \in \mathbb{R}^{m_t}$, and at date $t = 1, ..., T$, a vector of n_t (contractible) performance measures, represented by $\mathbf{y}_t$, is publicly reported. We assume in this chapter that periods are technologically and stochastically independent so that the report at date t is only influenced by the action taken in period t, and an $n_t \times 1$ random noise vector $\boldsymbol{\varepsilon}_t$ which is independent of the noise vector in the other periods, i.e., $\text{Cov}(\boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_\tau) = \mathbf{0}$ for $t \neq \tau$. The noise vector is normally distributed with a zero mean vector and a $n_t \times n_t$ covariance matrix $\boldsymbol{\Sigma}_t$ which is independent of the agent's actions. Furthermore, as in Chapter 20 we assume that the performance measures are scaled such that each performance measure has a unit variance. The mean vector of $\mathbf{y}_t$ is $\mathbf{M}_t\mathbf{a}_t$, where $\mathbf{M}_t$ is an $n_t \times m_t$ matrix of performance measure sensitivities such that we can write $\mathbf{y}_t$ as

$$\mathbf{y}_t = \mathbf{M}_t\mathbf{a}_t + \boldsymbol{\varepsilon}_t, \quad \text{for all } t = 1, 2, ..., T.$$

As in the standard single-period *LEN* model, we restrict the compensation paid at each date t to be a linear function of the performance measures reported up to date t, i.e.,

$$s_t(\overleftarrow{\mathbf{y}}_t) = f_t + \sum_{\tau=1}^{t} \mathbf{v}_{t\tau}^{\text{t}} \mathbf{y}_\tau,$$

where $\mathbf{v}_{t\tau}$ is the $n_\tau \times 1$ vector of incentive rates in date t compensation for the performance measures reported at date $\tau \leq t$, with $\mathbf{v}_{tt}$ denoted $\mathbf{v}_t$. We assume the principal and the agent can borrow and save at the market rates of interest which are reflected in the valuation index Ξ_t. Hence, since the value of the agent's compensation is a linear function of his period-by-period compensation, i.e.,

$$w(\overleftarrow{\mathbf{y}}_T) = \sum_{t=0}^{T} \Xi_t s_t(\overleftarrow{\mathbf{y}}_t),$$

that value is also a linear function of the performance measures reported up to date T and the timing of compensation is flexible.

Proposition 25.7 establishes that the optimal contract is not unique, unless we restrict the timing. That is, the principal's and agent's preferences are not affected by whether a given compensation s_t is paid at date t or $(\Xi_t/\Xi_\tau)s_t$ is paid at date τ. To obtain uniqueness, we assume, without loss of generality, that at each date $t \geq 1$ the agent is paid a variable wage $\mathbf{v}_t^t\mathbf{y}_t$ depending on the performance measures reported at that date only. A fixed wage f is paid at the contract date, $t = 0$. Hence,

$$s_0 = f, \quad s_t(\overleftarrow{\mathbf{y}}_t) = s_t(\mathbf{y}_t) = \mathbf{v}_t^t\mathbf{y}_t, \quad \text{for all } t = 1, ..., T.$$

25.4.2 The Agent's Choices

With AC preferences, the agent is clearly indifferent with respect to how his consumption is inter-temporally allocated, as long as it has the same net value. This is not the case with time-additive preferences! In this subsection, we first derive the agent's optimal consumption plan for exogenous actions and incentive rates, anticipating that future actions and incentive rates are not dependent on the reported performance measures.

The Agent's Consumption Plan with Time-additive Preferences

To understand the implications of agent borrowing and saving, and differences between the rates of change in the market's and the agent's time-preference indices, $\beta_t = \Xi_{t+1}/\Xi_t$ and $\beta_t^a = \Xi_{t+1}^a/\Xi_t^a$, it is useful to consider the agent's optimal consumption plan if his only source of funds is an initial bank balance B_0 (or riskless investments with an NPV of B_0). In this analysis, $B_t = R_{t-1}(B_{t-1} - c_{t-1})$ is the pre-consumption bank balance at date t, with $R_{t-1} = \beta_{t-1}^{-1}$, and $\beta_{\tau t} \equiv \Xi_\tau/\Xi_t$ is the price of a zero coupon bond at date t that pays one dollar at date τ.[19] Of course, if the term structure of interest rates is flat, then $\beta_{\tau t} = \beta^{\tau - t}$. Furthermore,

$$A_t \equiv \left[1 + \sum_{\tau=t+1}^{T} \beta_{\tau t}\right]^{-1}, \quad t = 0, 1, ..., T-1, \tag{25.14}$$

is an annuity factor that specifies the amount per period that can be paid from date t through date T by investing one dollar in the market at date t.

With no uncertainty, the agent's consumption plan at date $t = 0,...,T-1$ can be expressed as $\vec{\mathbf{c}}_t = (c_t, ..., c_T)$, and his decision problem can be expressed as

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[19] See Section 6.1.3 in Volume I for further analyses of zero-coupon prices.

$$V_t^{TA}(B_t) \equiv \max_{\vec{\mathbf{c}}_t} -\Xi_t^a \sum_{\tau=t}^{T} \beta_{\tau t}^a \exp[-r\, c_\tau] \qquad (25.15a)$$

subject to budget constraint

$$c_t + \beta_{t+1,t}\, c_{t+1} + ... + \beta_{Tt}\, c_T \le B_t, \qquad (25.15b)$$

where $\beta_{\tau t}^a \equiv \Xi_\tau^a / \Xi_t^a$, and $V_t^{TA}(B_t)$ is the agent's *maximum* remaining utility given the current bank balance B_t (which we refer to as *the agent's value function* at date t). The optimal consumption decision and the value function are summarized in the following proposition, and the proof is provided in Appendix 25B.

Proposition 25.10

Given an initial bank balance of B_0 and no other source of funds, the agent's optimal consumption choice and valuation function for date $t = 0, 1, ..., T$ are

$$c_t^{TA} = A_0 B_0 + \Omega_{t0} \qquad (25.16a)$$

$$= A_t(B_t + \omega_t), \qquad (25.16b)$$

$$V_t^{TA}(B_t) = -\Xi_t^a A_t^{-1} \exp[-r A_t(B_t + \omega_t)], \qquad (25.16c)$$

where $B_t = R_{t-1} B_{t-1} - c_{t-1}$, $t = 1, ..., T$,

$$\Omega_{\tau t} \equiv \frac{1}{r}\{\ln[\beta_{\tau t}^a / \beta_{\tau t}] - A_t \sum_{i=t+1}^{T} \beta_{it} \ln[\beta_{it}^a / \beta_{it}]\},$$

and

$$\omega_t \equiv -\frac{1}{r} \sum_{\tau=t+1}^{T} \beta_{\tau t} \ln[\beta_{\tau t}^a / \beta_{\tau t}].$$

If $\beta_t^a = \beta_t$ for all t, then

$$c_t = A_0 B_0 \qquad (25.17a)$$

$$= A_t B_t, \qquad (25.17b)$$

$$V_t^{TA}(B_t) = -\Xi_t^a A_t^{-1} \exp[-r A_t B_t]. \qquad (25.17c)$$

The following aspects of the optimal consumption plan are noteworthy. First, the bank balance is used to buy an annuity and this is the only component of consumption if the agent and the market have the same relative time preferences. Interestingly, the agent's consumption is constant in this latter case even if the interest rates deterministically vary across periods. The key is that with constant consumption, the marginal utility for consumption in each period is the same except for the differences in the time-preference index Ξ_t^a, and the relative difference across periods corresponds to the relative marginal cost of borrowing or saving in order to shift consumption from one period to another.

Second, if the agent's relative time preference differs from the market's, then the agent will vary his consumption over time to take advantage of the time-preference differences. The expression $\Omega_{\tau t}$ represents the net effect of the inter-temporal trades. The ratio $\beta_{\tau t}^a/\beta_{\tau t}$ represents the agent's relative preference for shifting consumption from date t to date τ and $\ln(\beta_{\tau t}^a/\beta_{\tau t})/r$, the first term of $\Omega_{\tau t}$, is the amount of that shift. The change in date τ consumption is positive (negative) if the ratio is greater (less) than one. The second term of $\Omega_{\tau t}$ reflects the NPV at date t of the increases in future consumption (due to the first terms) multiplied by the annuity factor A_t. If that NPV is positive (negative) then the agent reduces or increase his annuity to finance his inter-temporal trades.

Third, (25.16) provides two different representations of the agent's consumption choice. The first, (25.16a), takes advantage of the fact that the consumption plan is deterministic. The basic annuity is generated by the initial bank balance, and Q_{t0} is the net effect of the gross increase in date t consumption due to inter-temporal trades minus the change in the annuity used to finance all such trades. The second representation, (25.16b), uses the annuity that can be acquired with the bank balance at date t and then adjusts for the change required by shifts in consumption from date t into future periods. As demonstrated below, this approach can be used when stochastic events cause changes in the bank balance and the date t value of other sources of funds for consumption.

Fourth, the magnitude of the agent's net inter-temporal trading is independent of his bank balance and the consumption of his bank balance is independent of his time-preference – it depends on the market's time preference as reflected in the annuity factor. That is, there is a *separation* between the consumption generated by the agent's bank balance (or any other source of funds) and the consumption generated by the inter-temporal trading due to differences between his time preference and that of the market. As we demonstrate below, this separation implies that the agent's action choices and the principal's contract choice are not affected by the agent's time preference, even though those preferences affect his consumption choices.

The agent's consumption planning horizon $\check{T}$ can extend beyond the agent's contract or even his expected life, reflecting his preference to leave an endowment to his heirs. This allows for the possibility that the agent's consumption planning horizon $\check{T}$ is infinite, resulting in an annuity factor that does not vary with time if there is a flat term structure of interest rates (i.e., $\beta_t = (1+\imath)^{-1}$ for all t):[20]

$$A_t = A^* \equiv \lim_{\check{T}\to\infty}\left[\sum_{\tau=t}^{\check{T}} \beta^{\tau-t}\right]^{-1} = 1-\beta = \imath\beta, \quad \text{for all } t.$$

We now extend the above result to settings in which the agent receives risky compensation and incurs effort costs. The agent's certainty equivalent plays a major role in the analysis. It consists of his current bank balance plus terms reflecting the agent's future compensation, future effort costs, and future risk premia. The future effort costs and risk premia are known with certainty, but future expectations with respect to subsequent compensation will vary with the information received. Consequently, the certainty equivalent is a random variable, and the consumption annuity that is implemented each period varies with the information received.

The agent's consumption decision problem is solved as a dynamic programming problem starting at date T, and solving it recursively backwards to the initial consumption date 0. The value function at date t for the dynamic programming problem is $V_t^{TA}(CE_t^{TA})$ with the subscript t denoting all the information available to the agent before he chooses his date t consumption, and CE_t^{TA} denoting the agent's *post-compensation, pre-consumption* certainty equivalent at date t measured in date t dollars (as specified below).[21] The value function represents the *maximum* conditional expected utility the agent can obtain until the horizon T by choosing an optimal consumption plan for dates t until T, i.e.,

$$V_t^{TA}(CE_t^{TA}) \equiv \max_{\vec{\mathbf{c}}_t} -\mathrm{E}_t\Big[\sum_{\tau=t}^{T} \Xi_\tau^a \exp\big[-r(c_\tau - \kappa_\tau(a_\tau))\big]\Big],$$

subject to the agent's budget constraints, where the subscript on the expectation denotes that it is calculated conditional on all information available to the agent at date t. In particular, $U^a(\overleftarrow{\mathbf{s}}_T, \overleftarrow{\mathbf{a}}_T, \eta) = V_0^{TA}(CE_0^{TA})$ represents the agent's optimal expected utility at the contracting date 0 given the contract offered by the principal and the anticipated actions.

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[20] As is always the case with infinite horizon problems, we must ensure that the appropriate transversality conditions are satisfied.

[21] With technological and stochastic independence, a sufficient statistic for the information with respect to future actions is the agent's current bank balance (in addition to the anticipated non-stochastic incentive rates $\vec{\mathbf{v}}_{t+1}$). However, we use the more general notation, since the procedure described here also applies to the more general settings with technological and stochastic dependence in Chapter 26 as well as with renegotiation in Chapter 28.

The agent's *pre-consumption* bank balance at date t after receiving his date t compensation and paying his period t personal costs is

$$B_t \equiv R_{t-1}(B_{t-1} - nc_{t-1}) + s_t - \kappa_t = nc_t + \ell_t, \quad \text{for } t = 0, 1, 2, ..., T, \tag{25.18}$$

where $nc_t \equiv c_t - \kappa_t$ represents the agent's *net consumption* at date t (with $B_{-1} = nc_{-1} = 0$). The agent's bank balance is the amount available to the agent at date t for current net consumption or for saving for future consumption. Current net consumption can be greater than the current bank balance, with a negative balance representing borrowing against future compensation. Hence, the agent's only "effective" budget constraint is $nc_T \leq B_T$.

At the terminal date T, the agent's certainty equivalent is his remaining bank balance, i.e., $CE_T^{TA} = B_T$, and the agent's (or his heirs') optimal net consumption choice is $nc_T^{TA} = B_T$. Hence, $V_T^{TA}(CE_T^{TA}) = -\Xi_T^a \exp(-rCE_T^{TA})$. At all preceding dates $t = 0, ..., T-1$, the agent's consumption decision problem after $\mathbf{y}_t$ has been reported is

$$V_t^{TA}(CE_t^{TA}) = \max_{nc_t} \{ -\Xi_t^a \exp(-r\, nc_t) + \mathrm{E}_t[\, V_{t+1}^{TA}(CE_{t+1}^{TA})] \}. \tag{25.19}$$

This representation of the agent's consumption decision problem is known as the *Bellman equation*. The optimal net consumption plan is solved inductively by $nc_t^{TA} = \boldsymbol{nc}_t^{TA}(CE_t^{TA})$, where $\boldsymbol{nc}_t^{TA}(CE_t^{TA})$ denotes the solution to the Bellman equation.

The following proposition characterizes the agent's net consumption choice, as well as specifying the terms that make up his certainty equivalent. Let $W_t \equiv \Sigma_{\tau=t}^{T} \beta_{t\tau} s_\tau = s_t + \beta_t W_{t+1}$ and $K_t \equiv \Sigma_{\tau=t}^{T} \beta_{t\tau}\kappa_\tau = \kappa_t + \beta_t K_{t+1}$ represent the NPV, at date t, of current and future compensation and effort costs, respectively.

Proposition 25.11[22]

Assume *TA-EC* preferences. Given the incentive contract offered by the principal and the agent's anticipated sequence of actions, the agent's optimal net consumption plan for $t = 0, 1, ..., T$, and his *pre-consumption* certainty equivalent, nominal wealth risk premium, and expected utility for $t = 0, 1, ..., T$, are

$$nc_t^{TA} = \Lambda_t(CE_t^{TA} + \omega_t), \tag{25.20a}$$

[22] This proposition (and its proof) holds for the more general *LEN* model with both technological and stochastic dependence across periods that we consider in Chapter 26, as well as for the *LEN* model with renegotiation in Chapter 28.

$$CE_t^{TA} = B_t + \beta_t \mathrm{E}_t[W_{t+1} - K_{t+1}] - RP_t^{TA}, \tag{25.20b}$$

$$RP_t^{TA} = \tfrac{1}{2} \sum_{\tau=t+1}^{T} \beta_{\tau t} r A_\tau \mathrm{Var}_{\tau-1}[\mathrm{E}_\tau[W_\tau]], \tag{25.20c}$$

$$V_t^{TA}(CE_t^{TA}) = -\,\Xi_t^a A_t^{-1} \exp[-rA_t(CE_t^{TA} + \omega_t)]. \tag{25.20d}$$

The proof is provided in Appendix 25B. Expression (25.20a) is the same as (25.16b) in Proposition 25.10, except that the funds available for consumption are represented by the agent's certainty equivalent, not just his bank balance. The motivation for the form of (25.20a) is the same as for (25.16b) – see the discussion of Proposition 25.10. Furthermore, if $\beta_\tau^a = \beta_\tau \ \forall\ \tau \geq t$, then $\omega_t = 0$, and (25.20a) simplifies to

$$nc_t^{TA} = A_t\, CE_t^{TA}, \tag{25.20a'}$$

which corresponds to (25.17b). Observe that it is net consumption nc_t that is proportional to the certainty equivalent, not gross consumption c_t.

The certainty equivalent is specified in (25.20b), and includes W_t, K_t, and RP_t^i, which are described above, in addition to B_t. The NPV of the agent's personal costs, K_t, is not a random variable, whereas W_t is random, due to incentive compensation based on noisy performance measures. In particular, given the technological and stochastic independence assumptions,

$$\begin{aligned} \mathrm{Var}_t[\mathrm{E}_{t+1}[W_{t+1}]] &= \mathrm{Var}_t[\mathrm{E}_{t+1}[s_{t+1} + \beta_{t+1} W_{t+2}]] \\ &= \mathrm{Var}_t[s_{t+1}] = \mathbf{v}_{t+1}^{\mathrm{t}} \mathbf{\Sigma}_{t+1} \mathbf{v}_{t+1}, \end{aligned} \tag{25.21a}$$

which implies that the agent's nominal wealth risk premium is

$$RP_t^{TA} = \tfrac{1}{2} \sum_{\tau=t+1}^{T} \beta_{\tau t} r A_\tau \mathbf{v}_\tau^{\mathrm{t}} \mathbf{\Sigma}_\tau \mathbf{v}_\tau. \tag{25.21b}$$

A key feature of the *TA* result is that current net consumption equals the amount that would be paid by two annuities. One varies deterministically over time reflecting inter-temporal trades with the market. The other varies stochastically over time, reflecting the risk-adjusted value to the agent of the future random stream of compensation less the NPV of the future deterministic stream of effort costs. Since the certainty equivalent at date t depends on the compensation received at date t, the annuity will change randomly from period to period as uncertainty about current compensation is resolved. Expressions (25.20c) and (25.20d) can be interpreted as either applying the risk aversion parameter r to a measure of random nominal date t consumption, or as applying a *nominal wealth risk aversion* parameter, $\hat{r}_t^{TA} \equiv rA_t$, to a measure of nominal date t "wealth" (as measured by the agent's certainty equivalent).

As in Proposition 25.10, a striking feature of these results is that the agent's risk premium and his nominal wealth risk aversion *do not* depend on his personal time-preference index. That index affects his deterministic personal intertemporal trading in the capital market,[23] but it does not affect how he smooths his random compensation over his consumption horizon. Note that his nominal wealth risk aversion, $\hat{r}_t^{TA}$, increases over time (unless he has an infinite consumption horizon) reflecting the fact that there are fewer periods over which he can smooth compensation risk. Irrespective of the shape of the term structure of interest rates, there is "flat" smoothing of his compensation and effort costs. Increasing market interest rates also increases his nominal wealth risk aversion, reflecting the fact that it becomes more costly to borrow against future compensation.

The Agent's Consumption Plan with Aggregate Consumption Preferences

As noted earlier, with AC-preferences, the agent and the market must have the same time-preference index, Ξ_t. It converts nominal date t dollars into some common (valuation date) dollars. The risk aversion parameter r is measured relative to the common dollars, so that changing the valuation date would require a change in the risk aversion measure.

We let $V_t^{AC}(CE_t^{AC})$ represent the agent's maximum conditional expected utility at date t for consumption at t, $t+1$, ..., T, given the information available to the agent before he chooses his date t consumption, i.e.,

$$V_t^{AC}(CE_t^{AC}) = \max_{\vec{\mathbf{c}}_t} - \mathrm{E}_t\Big[\exp\Big[-r\sum_{\tau=t}^{T} \Xi_\tau(c_\tau - \kappa_\tau(a_\tau))\Big]\Big],$$

subject to the agent's budget constraints. In this case the Bellman equation takes the form

$$V_t^{AC}(CE_t^{AC}) = \max_{nc_t} \{\exp[-r\Xi_t nc_t]\, \mathrm{E}_t[V_{t+1}^{AC}(CE_{t+1}^{AC})]\}.$$

[23] Note that this "side-trading" not only affects the agent's net consumption but also the agent's certainty equivalent through the bank balance. However, the agent's certainty equivalent at the contracting date is independent of the agent's personal time-preference, since $B_0 = 0$.

The conjecture for the value function is again of the same form as the period-specific utility function, i.e.,

$$V_t^{AC}(CE_t^{AC}) = -g_t \exp[-rh_t \Xi_t CE_t^{AC}], \quad t = 0, 1, ..., T,$$

for time-dependent constants g_t and h_t with $g_T = h_T = 1$, and the conjecture for the certainty equivalent is chosen consistently (given normally distributed future certainty equivalents),

$$CE_t^{AC} = nc_t + \beta_t \{ \mathrm{E}_t[CE_{t+1}^{AC}] - \tfrac{1}{2} r h_{t+1} \Xi_{t+1} \mathrm{Var}_t[CE_{t+1}^{AC}] \}, \quad t = 0, 1, ..., T-1.$$

If we choose $g_t = h_t = 1$ for all dates t, the Bellman equation is satisfied for any net consumption choice, since

$$\begin{aligned} V_t^{AC}(CE_t^{AC}) &= -\exp[-r\Xi_t CE_t^{AC}] \\ &= -\exp[-r\Xi_t(nc_t + \beta_t \{ \mathrm{E}_t[CE_{t+1}^{AC}] - \tfrac{1}{2} r h_{t+1} \Xi_{t+1} \mathrm{Var}_t[CE_{t+1}^{AC}] \})] \\ &= \exp[-r\Xi_t nc_t] \, \mathrm{E}_t[V_{t+1}^{AC}(CE_{t+1}^{AC})]. \end{aligned}$$

Hence, the optimal net consumption plan is indeterminate, and we may, without loss of generality, assume that net consumption at date t equals the agent's compensation less the effort costs for period t.

The following proposition parallels Proposition 25.11 with the key difference that the risk aversion parameter with respect to nominal date t wealth now is equal to $\hat{r}_t^{AC} \equiv r\Xi_t$ as opposed to $\hat{r}_t^{TA} \equiv rA_t$ with *TA*-preferences.

Proposition 25.12

Assume *AC-EC* preferences. Given the incentive contract offered by the principal and the agent's anticipated sequence of actions, the agent's optimal net consumption plan for $t = 0, 1, ..., T$, and his *pre-consumption* certainty equivalent, nominal wealth risk premium, and expected utility for $t = 0, 1, ..., T$, are:[24]

$$nc_t^{AC} = s_t - \kappa_t, \tag{25.22a}$$

$$CE_t^{AC} = B_t + \beta_t \mathrm{E}_t[W_{t+1} - K_{t+1}] - RP_t^{AC}, \tag{25.22b}$$

[24] If nc_t, CE_t^{AC}, and RP_t^{AC} are measured in common dollars, while B_t, s_t, and κ_t are measured in nominal dollars, then (25.22) becomes: (a) $nc_t^{AC} = \Xi_t(s_t - \kappa_t)$; (b) $CE_t^{AC} = \Xi_t B_t + \mathrm{E}_t[\Xi_{t+1}(W_{t+1} - K_{t+1})] - RP_t^{AC}$; (c) $RP_t^{AC} = \tfrac{1}{2} r \mathrm{Var}_t[\Xi_{t+1}\mathrm{E}_{t+1}[W_{t+1}]] + RP_{t+1}^{AC}$; and (d) $V_t^{AC}(CE_t^{AC}) = -\exp[-rCE_t^{AC}]$.

$$RP_t^{AC} = \tfrac{1}{2} \sum_{\tau=t+1}^{T} \beta_{\tau t} r \Xi_\tau \mathrm{Var}_{\tau-1}[\mathrm{E}_\tau[W_\tau]], \tag{25.22c}$$

$$V_t^{AC}(CE_t^{AC}) = -\exp[-r\Xi_t CE_t^{AC}]. \tag{25.22d}$$

With AC-preferences, the agent can simply consume his current compensation without regard for his current certainty equivalent. Expressions (25.22c) and (25.22d) can be interpreted as either applying the risk aversion r to a measure of wealth expressed in common valuation-date dollars, or applying the *nominal wealth risk aversion*, $\hat{r}_t^{AC} \equiv r\Xi_t$, to a measure of wealth expressed in nominal date t dollars.

Observe that the basic risk aversion parameter r is assumed to be constant across time, but the nominal wealth risk aversion, $\hat{r}_t^{AC}$, is decreasing over time, whereas the nominal wealth risk aversion for TA-preferences, $\hat{r}_t^{TA}$, is increasing over time if the time-horizon T is finite, but it is constant if $T \to \infty$. The former occurs because the time-preference index used to restate nominal dollars in common valuation-date dollars reflects the time-value of money and, hence, decreases over time. However, note that the nominal risk premia (and the certainty equivalents) in the TA and AC cases only differ due to the nominal wealth risk aversion parameters being different in the two cases, i.e.,

$$RP_t^i = \tfrac{1}{2} \sum_{\tau=t+1}^{T} \beta_{\tau t} \hat{r}_\tau^i \mathrm{Var}_{\tau-1}[\mathrm{E}_\tau[W_\tau]], \quad i = TA, AC. \tag{25.23}$$

The Agent's Action Choices

We now consider the action choices at the start of each period for an exogenous contract. The fact that the noise vectors are normally distributed and additive implies that the agent's actions do not influence his risk premium under either TA or AC. Hence, the agent chooses $\mathbf{a}_{t+1}$ (at date t) so as to maximize

$$\mathrm{E}_t[W_{t+1}] - K_{t+1} = \mathrm{E}_t[s_{t+1} + \beta_{t+1} W_{t+2}] - [\kappa_{t+1} + \beta_{t+1} K_{t+2}].$$

Given technological and stochastic independence across periods, and separable effort costs $\kappa_{t+1}(\mathbf{a}_{t+1}) = \tfrac{1}{2}\mathbf{a}_{t+1}^{\mathrm{t}}\mathbf{a}_{t+1}$, the first-order condition with respect to $\mathbf{a}_{t+1}$ is

$$\mathbf{a}_{t+1} = \mathbf{M}_{t+1}^{\mathrm{t}} \mathbf{v}_{t+1}, \quad \text{for all } t = 0, ..., T-1. \tag{25.24}$$

Of course, since only the risk premia differ for TA and AC, the induced actions are the same.

25.4.3 The Principal's Contract Choice

The principal's expected gross payoff from actions taken in period t is represented by $\mathbf{b}_t^{\rm t}\mathbf{a}_t$, for $t = 1, 2, ..., T$. This payoff is expressed in date t dollars irrespective of when the outcome is realized. That is, the timing of the payoff is immaterial (other than making the adjustment for the time-value of money) given that we assume that it is not contractible. Of course, if the payoff or any part of the payoff is contractible, then it is also included among the performance measures, with explicit recognition of the timing of the reports.

The principal is assumed to be risk neutral with time-preference index $\Xi_t = \beta_{t0}$, i.e., the zero-coupon prices at date $t = 0$. The net present value at date $t = 0$ of the principal's expected future net payoffs is

$$U^p(\overleftarrow{\mathbf{s}}_T, \overleftarrow{\mathbf{a}}_T, \eta) = \pi_0 - \mathrm{E}_0[W_0], \quad \text{where } \pi_0 \equiv \sum_{t=1}^{T} \beta_{t0}\mathbf{b}_t^{\rm t}\mathbf{a}_t.$$

Since the fixed wage f paid at date 0 does not affect the agent's decisions and only serves to directly increase his certainty equivalent, the principal chooses f to be just sufficient to induce the agent to accept the contract that is offered. The agent's reservation wage does not have any substantive effect on the analysis, so we let it be equal to zero.[25] Hence, in selecting the contract offered at $t = 0$, the principal seeks to maximize

$$U^p(\overleftarrow{\mathbf{s}}_T, \overleftarrow{\mathbf{a}}_T, \eta) = \pi_0 - \{K_0 + RP_0^i\}, \quad i = TA, AC,$$

i.e., the NPV at date $t = 0$ of his expected gross payoffs minus the sum of the NPVs of the agent's effort costs and risk premia.

In a first-best setting, the agent is paid a fixed wage, so that there is no risk premium. Hence, in this setting we have

$$U^p(\overleftarrow{\mathbf{s}}_T, \overleftarrow{\mathbf{a}}_T, \eta) = \pi_0 - K_0 = \sum_{t=1}^{T} \beta_{t0}[\mathbf{b}_t^{\rm t}\mathbf{a}_t - \tfrac{1}{2}\mathbf{a}_t\mathbf{a}_t],$$

and the first-best actions are characterized by

$$\mathbf{a}_t^* = \mathbf{b}_t,$$

[25] Note that even though the agent's time-preference index may differ from the market's with TA-preferences, the agent's certainty equivalent at the contracting date is independent of the agent's personal time-preferences. Hence, the agent's reservation wages s_t^o do not affect the agent's "side-trading" in the market.

which is equivalent to the single-period *LEN* model in Chapter 20 (see 20.9b). However, in the second-best setting, we must recognize the risk premium. Substituting the risk premium for $t = 0$ from (25.23) and action choices (25.24) into the principal's objective function, we get the following unconstrained decision problem expressed in terms of the incentive rates:

$$U^p(\overleftarrow{\mathbf{s}}_T, \overleftarrow{\mathbf{a}}_T, \eta) = \pi_0 - \{K_0 + RP_0^i\}$$

$$= \sum_{t=1}^{T} \beta_{t0}\big[\mathbf{b}_t^{\mathrm{t}}\mathbf{M}_t^{\mathrm{t}}\mathbf{v}_t - \{\tfrac{1}{2}[\mathbf{v}_t^{\mathrm{t}}\mathbf{M}_t][\mathbf{M}_t^{\mathrm{t}}\mathbf{v}_t] + \tfrac{1}{2}\hat{r}_t^i\,\mathbf{v}_t^{\mathrm{t}}\boldsymbol{\Sigma}_t\mathbf{v}_t\}\big],$$

$$= \sum_{t=1}^{T} \beta_{t0}\big[\mathbf{b}_t^{\mathrm{t}}\mathbf{M}_t^{\mathrm{t}}\mathbf{v}_t - \tfrac{1}{2}\mathbf{v}_t^{\mathrm{t}}\mathbf{Q}_t^{i\ 1}\mathbf{v}_t\big], \quad i = A, M,$$

where $\mathbf{Q}_t^i \equiv [\mathbf{M}_t\mathbf{M}_t^{\mathrm{t}} + \hat{r}_t^i\boldsymbol{\Sigma}_t]^{-1}$. Hence, the second-best incentive rates and actions are

$$\mathbf{v}_t^{i\dagger} = \mathbf{Q}_t^i\mathbf{M}_t\mathbf{b}_t, \quad i = TA, AC, \tag{25.25a}$$

$$\mathbf{a}_t^{i\dagger} = \mathbf{M}_t^{\mathrm{t}}\mathbf{Q}_t^i\mathbf{M}_t\mathbf{b}_t, \quad i = TA, AC, \tag{25.25b}$$

which are equivalent to the single-period *LEN* model result in Chapter 20 (see 20.13).

Identical Periods

Several papers that examine dynamic *LEN* models focus on settings in which the periods are assumed to be identical (i.e., $\mathbf{b}_t = \mathbf{b}$, $\mathbf{M}_t = \mathbf{M}$, and $\boldsymbol{\Sigma}_t = \boldsymbol{\Sigma}$). Many of these papers also assume the interest rate equals zero and the agent has *AC* preferences, so that the agent's and principal's time-preference index Ξ_t equals 1 and the agent's nominal wealth risk aversion $\hat{r}_t^{AC}$ equals r for all t. In that case, if there is technological and stochastic independence, then Q_t^i is constant across periods, resulting in constant incentives and actions. However, these papers typically assume a lack of stochastic independence and are interested in how correlated performance measures affect the sequence of actions. We consider these types of settings in Chapters 26, 27, and 28.

Observe that the zero interest rate assumption implies that all amounts are measured in common valuation-date dollars. They are not identical when measured in nominal dollars. Now assume, to the contrary, that the periods are identical when measured in nominal dollars, and interest rates are positive. As noted earlier, positive interest rates imply that Ξ_t and $\hat{r}_t^{AC}$ are decreasing over time, while A_t and $\hat{r}_t^{TA}$ are increasing (if T is finite) or constant (if $\check{T}$ is infinite).

Hence, if there is a single performance measure and action in each period, then $\mathbf{Q}_t^{AC}$, a_t^{AC}, and v_t^{AC} increase over time, while $\mathbf{Q}_t^{TA}$, a_t^{TA}, and v_t^{TA} decrease (or are constant) over time if T is finite (or infinite). Of course, the reason for the differences is that greater risk aversion makes it more costly to use strong incentives, and that, in turn, makes it optimal to use weaker incentives which induce less effort.

25.5 *T* AGENTS VERSUS ONE

In the preceding analysis we have exogenously assumed that the principal hires a single agent to operate his production system for T periods. We now compare those results with the results from hiring a new agent each period. Obviously, if there are significant "change-over" costs (e.g., training costs), it will be beneficial to hire a single agent for T periods. In the following analysis, we assume the change-over costs are zero, and continue to assume technological and stochastic independence. All agents have the same utility functions and the same market opportunity in each period, represented by a net reservation wage of s_t^o (wage minus effort costs). They also have the opportunity to borrow and save. Consequently, we assume, without loss of generality, that all of the compensation for the agent in period t is paid at date t based on report $\mathbf{y}_t$, and is represented by $s_t(\mathbf{y}_t)$.

The form of the agents' utility functions affects whether there is a benefit to changing agents at the end of each period. In particular, a key issue is whether there are wealth effects. There are no wealth effects if the agent has either *AC-EC* or *TA-EC* preferences with exponential utility functions. However, there are wealth effects if the agent has either *AC-ED* or *TA-ED* preferences (see (25.1)), even if the utility for consumption is exponential.

25.5.1 Exponential *EC* Utility Functions

Section 25.2.1 considers *AC* preferences represented by an exponential utility function defined over aggregate consumption minus personal effort costs measured in common valuation-date dollars (see (25.3)). The optimal contract for a single agent is characterized (see (25.5)) as a sequence of compensation contracts in which s_t is a function of y_t, independent of all other reports and compensation levels.[26] Hence, it immediately follows that the results for the principal would be the same whether one agent or T agents are hired. This point

[26] This point is also illustrated by the analysis in Section 19.2.2, which considers a sequence of problems in which the outcome for each problem is binary.

is reinforced by our analysis of the multi-period *LEN* model with *AC-EC* preferences (Section 25.4).

The issue is more subtle if the agent has *TA-EC* preferences, with technological and stochastic independence. The lack of a wealth effect in the second period action choice implies that the optimal incremental compensation associated with y_2 in inducing a_2 will be the same whether it is a new or an old agent making the action choice. The key issue is whether the incremental first-period contract is the same whether the first-period agent will be retained or released at the end of the first period. Since the agent can borrow or save, we can view the first-period contract as making payments at both date 1 and date 2 based on y_1. These amounts will not affect the agent's second-period action choice if the second-period compensation is an additively separable function of y_1 and y_2.

We do not formally consider optimal contracts with *TA* preferences. However, there is no wealth effect, and the following proposition establishes that hiring one agent for two periods is equivalent to changing agents at the end of the first period. This assumes, of course, that the effort costs are additively separable across periods.

Proposition 25.13[27]

If the reporting system is technologically and stochastically independent, and the agents are identical, with either *AC-EC* or *TA-EC* preferences represented by exponential utility functions, then the principal is indifferent between hiring a single agent for two periods or hiring a new agent at the start of each period.

25.5.2 *ED* Utility Functions

In this section, we consider *AC-ED* and *TA-ED* (see Sections 25.2.2 and 25.3.2). The ordering of the principal's expected payoffs is very similar for *AC* and *TA* preferences, given the form of separability between the agents' utility for consumption and effort. However, the *ED* models produce very different results than the *EC* models. In the analysis that follows we use a two-period hurdle model to illustrate the benefit of interim versus terminal reporting, the costs of

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[27] We do not present a proof here, because the result follows from the analysis in Section 28.1 based on Fudenberg, Holmström, and Milgrom (1990). In that analysis we consider long-term contracts versus a sequence of short-term contracts. Fudenberg *et al.* (1990, Theorem 5) show that if there is equal access to borrowing and saving, the agent has exponential *TA-EC* preferences, and the technology is history-independent (which includes the independent periods case), then there is an optimal long-term contract in which the compensation function is a sequence of compensation contracts in which s_t is a function of y_t, independent of all other reports and compensation levels (see Proposition 28A.1). Of course, with *TA* preferences equal access to borrowing and saving is a crucial assumption.

not being able to exogenously preclude borrowing and saving by the agents, the benefit of employing two workers instead of one, and the benefit of retaining a worker if he obtains a bad first-period outcome and replacing him if he obtains a good first-period outcome.

To understand some of the results reported below it is important to realize that agent wealth has a significant effect on the cost of providing incentives in *ED* models. This holds even if the utility for consumption is negative exponential, and it stands in contrast to *EC* exponential utility functions (as in the *LEN* model) for which there is no wealth effect. We comment further on the wealth effect after introducing the hurdle model.

The Basic Elements of the Two-period Hurdle Model

In each of the two identical periods, $t = 1, 2$, there is a binary outcome $x_t \in X_t = \{x_g, x_b\}$, a hurdle $h_t \in [0,1]$, and an action $a_t \in A_t = [0,1]$, with $x_t = x_g$ if, and only if, $a_t \geq h_t$. The prior distribution for both hurdles is uniform, and they are independently distributed, so that $\varphi(x_g|a_t) = a_t$ and $\varphi(x_1,x_2|a_1,a_2) = \varphi(x_1|a_1)\times\varphi(x_2|a_2)$. The reservation wage for both agents in each period is s^o, and the interest rate is zero.

The outcomes are publicly reported and contractible. We consider both terminal and interim reporting systems. The terminal reporting system only issues reports at $t = 2$, whereas the interim reporting system reports the outcome at the end of each period, i.e., $y_t = x_t$, $t = 1, 2$.

Let $i = 1, 2$, denote the agent. Agent $i = 1$ is either hired for the first period or both periods, whereas agent $i = 2$ is either hired for the second period or not at all. With *AC* preferences, we let c_i represent agent i's total consumption for the two periods, whereas with *TA* preferences, $c_i = (c_{i1}, c_{i2})$, where c_{it} is agent i's consumption in period t. The agent's actions are $\mathbf{a}_i = (a_{i1}, a_{i2})$, where a_{it} is agent i's effort in period t (which is zero if he is not hired in that period).

Agent i's utility function takes either of the two following forms:

$$AC\text{-}ED\text{: } u_i(c_i, \mathbf{a}_i) = \ln(c_i) - a_{i1}/(1 - a_{i1}) - a_{i2}/(1 - a_{i2}),$$

$$TA\text{-}ED\text{: } u_i(c_i, \mathbf{a}_i) = \ln(c_{i1}) + \ln(c_{i2}) - a_{i1}/(1 - a_{i1}) - a_{i2}/(1 - a_{i2}).$$

The compensation paid to the agent by the principal is denoted s, with appropriate subscripts to indicate, where necessary, the agent, the period, and the reports. If an agent works for the principal in a given period, then the agent's consumption equals his compensation (assuming he is not motivated to borrow or save). The principal may pay the agent in a period in which he does not work, but in that case the agent's consumption equals his compensation plus his reservation wage of s^o.

To understand the previously mentioned wealth effect in *ED* models, consider a one-period hurdle model in which the agent's utility for compensation and effort is $u(s,a) = \ln(s+b) - a/(1-a)$, where b is the agent's initial wealth. As in the basic model, a is the probability that outcome x_g occurs instead of x_b. Assume the agent's reservation wage is s^o, so that the agent's reservation utility is $\ln(s^o + b)$. The outcome is contractible and the principal chooses the actions to be induced and the compensation s_g and s_b to be paid if x_g or x_b occur, respectively. These choices must be such that the agent will accept the contract and select the desired action, i.e., $a\ln(s_g+b) + (1-a)\ln(s_b+b) - a/(1-a) \geq \ln(s^o+b)$ and $\ln(s_g+b) - \ln(s_b+b) = 1/(1-a)^2$. Figure 25.2 provides a numerical example which varies the agent's wealth while holding the induced action a constant at .4 and assuming the reservation wage is $s^o = .5$. Observe that as the agent's wealth increases the spread between s_g and s_b increases, with a significant increase in the former and a slight decrease in the latter.[28] Consequently, there is a significant increase in the expected compensation cost, $as_g + (1-a)s_b$ (due to the increased risk). This illustrates the fact that with *ED* models, *it is more costly to motivate a wealthy agent than a poor agent.*

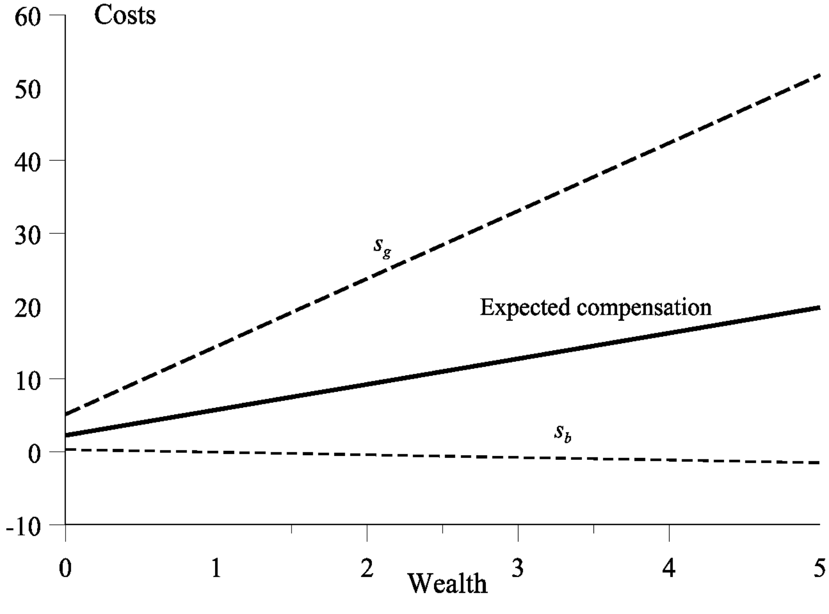

Figure 25.2: Impact of wealth on compensation cost, ($a = .4$ and $s^o = .5$).

[28] The marginal utility $1/(s+b)$ is much more sensitive to changes in s for small b than for large b.

Tables 25A.1 and 25A.2 in Appendix 25A provide formulations of the principal's problem for a series of settings that vary with respect to the form of the agent's preferences, the timing of the reports, agent access to borrowing and saving, and the number of agents hired. Solutions to the numerical examples are summarized in Tables 25A.3 and 25A.4.

Comparing cases (a) and (b) reveals that with a single agent, the principal receives a higher expected payoff with interim reporting than with terminal reporting (21.761 vs. 21.526 with *AC*, and 21.718 vs. 20.710 with *TA*). With terminal reporting, the *AC* model is effectively the same as a simultaneous choice multi-task model (see Chapter 20). Since the periods are identical, the induced actions are the same and the compensation is symmetric, i.e., $a_1 = a_2$ and $s_{gb} = s_{bg}$. This also occurs with *TA* preferences. With interim reporting, the second-period action varies with the reported result for the first-period even though the outcomes for the two periods are independently distributed. The *AC* case illustrates some of the analysis in Section 25.2.2. In particular, consistent with Proposition 25.3(c), under interim reporting, the induced second-period effort is greater if the first-period outcome is bad instead of good, i.e., a_{2b} = .2901 > a_{2g} = .0066. This is due to the wealth effect discussed above – a good report in the first period increases the agent's perceived wealth, whereas a bad report decreases it. Hence, stronger, more costly, incentives are required in the second period if the first-period outcome is good instead of bad. The tailoring of second-period incentives to the agent's interim wealth information is beneficial to the principal, and results in more induced first-period effort (e.g., a_1 equals .2290 under terminal reporting, and .2666 under interim reporting).

The above phenomena also occurs with *TA* preferences, independent of whether the agent can borrow or save. In both Tables 25A.4(a) and (b), the induced effort in both periods and the principal's expected payoff are all greater if the agent *cannot* borrow or save. That is, the principal benefits from having greater control over the agent.

Comparing cases (a) and (c) reveals that using two agents dominates using a single agent when there is terminal reporting. Under *AC* the two agents are offered the same contracts, and produce the same results. Under *TA*, the two agents receive the same contracts in substance, but the contracts differ in form since one receives s^o from another source in the first period, while the other agent receives it in the second period. The benefit of replacing the first agent in the second period again derives from the wealth effect discussed above. At the end of the first-period the first agent's expected compensation from the first-period contract under *AC* is $.2737 \times 5.275 + .7263 \times .368 = 1.711$, which is distinctly greater than the .500 the second agent has received from an external source. Since the first agent has more expected wealth it is more expensive to hire him for the second period than it is to hire the second agent. A similar result occurs under *TA*. We assume the principal can pay compensation over two periods even though the agent only provides effort in one period. Hence, the first agent's expected wealth from the first-period contract is 1.063 + (5.580 – 1.0)×.2807 + (.808 – 1.000)×.7193 = 2.487, which is distinctly greater than the wage of 1.000 the second agent received from an external source.

The wealth effect is revealed even more starkly when we compare case (d) to either (b) or (c). In (d) and (b) there is interim reporting, so that at the end of the first period the agent and principal know whether the agent will be compensated for a good or a bad first-period outcome. Based on the contract in (c), the first agent's wealth given a good outcome is 1.063 + (5.580 – 1.000) = 5.643 and given a bad outcome it is 1.063 + (.808 – 1.000) = .871. The former is greater than the second agent's 1.000, while the latter is less. Hence, it is cheaper to hire the second agent in the second period given a good first-period outcome, but it is cheaper to rehire the first agent if he is poor due to a bad first-period outcome. Hence, contingent replacement (with an expected payoff of 22.350) dominates both unconditional retention (21.718 in case (b)) or unconditional replacement (2.192 in case (c)).

25.6 CONCLUDING REMARKS

Performance measures, such as accounting earnings, are often correlated across periods and are often influenced by at least some of the actions taken in prior periods, as well as the current period. However, in this initial chapter on multi-period incentives we have assumed stochastic and technological independence. This has allowed us to focus on the characterization of the optimal contracts and the agent's consumption and action choices in a basic model with period-specific, independent performance measures and four different types of agent preferences (*TA-EC*, *AC-EC*, *TA-ED*, and *AC-ED*).

The analysis based on time-additive (*TA*) preferences has highlighted the importance of recognizing that agents can typically borrow and save, so that the timing of their consumption can differ from the timing of their compensation. Therefore, it is not necessary to have smooth compensation in order to have smooth consumption.

The analysis based on effort disutility (*ED*) preferences has highlighted that there can be wealth effects such that the principal prefers to hire poor agents or retain agents who had poor performance reports (due to random factors beyond their control).

We briefly considered the timing of the performance reports and established that early reporting is preferred if the agent has *TA* preferences, whereas timing is irrelevant if the agent has *AC-EC* preferences. This issue is explored in more depth in the next chapter.

Throughout the chapter we assumed full commitment. The agent could not leave at the end of the first period and the principal could not fire him. Furthermore, they cannot renegotiate their contract at the end of the first period even though they might find it beneficial to do so at that time. We defer the analysis of the impact of contract renegotiation until Chapter 28.

APPENDIX 25A: Two-period Hurdle Model Examples

Table 25A.1
Two-period Hurdle Model with *AC* Agent Preferences

(a) *Principal's Single-agent, Terminal Reporting Problem:*

$$U^{T1} = \underset{\substack{a\in[0,1] \\ s_{gg}, s_{gb}, s_{bb}}}{\text{maximize}} \; a_1[(2x_g - s_{gg})a_2 + (x_g + x_b - s_{gb})(1-a_2)] + (1-a_1)[(x_b + x_g - s_{bg})a_2 + (2x_b - s_{bb})(1-a_2)],$$

subject to

$$a_1[\ln(s_{gg})a_2 + \ln(s_{gb})(1-a_2)] + (1-a_1)[\ln(s_{bg})a_2 + \ln(s_{bb})(1-a_2)] - a_1/(1-a_1) - a_2/(1-a_2) \geq \ln(2s^o),$$

$$\ln(s_{gg})a_2 + \ln(s_{gb})(1-a_2) - \ln(s_{bg})a_2 - \ln(s_{bb})(1-a_2) = 1/(1-a_1)^2,$$

$$\ln(s_{gg})a_1 - \ln(s_{gb})a_1 + \ln(s_{bg})(1-a_1) - \ln(s_{bb})(1-a_1) = 1/(1-a_2)^2.$$

(b) *Principal's Single-agent, Interim Reporting Problem:*

$$U^{I1} = \underset{\substack{a_1, a_{2g}, a_{2b}\in[0,1] \\ s_{gg}, s_{gb}, s_{bg}, s_{bb}}}{\text{maximize}} \; a_1[(2x_g - s_{gg})a_{2g} + (x_g + x_b - s_{gb})(1-a_{2g})] + (1-a_1)[(x_b + x_g - s_{bg})a_{2b} + (2x_b - s_{bb})(1-a_{2b})],$$

subject to

$$a_1[\ln(s_{gg})a_{2g} + \ln(s_{gb})(1-a_{2g}) - a_{2g}/(1-a_{2g})] + (1-a_1)[\ln(s_{bg})a_{2b} + \ln(s_{bb})(1-a_{2b}) - a_{2b}/(1-a_{2b})] - a_1/(1-a_1) \geq \ln(2s^o),$$

$$\ln(s_{gg})a_{2g} + \ln(s_{gb})(1-a_{2g}) - \ln(s_{bg})a_{2b} - \ln(s_{bb})(1-a_{2b}) - a_{2g}/(1-a_{2g}) + a_{2b}/(1-a_{2b}) = 1/(1-a_1)^2,$$

$$\ln(s_{gg}) - \ln(s_{gb}) = 1/(1-a_{2g})^2, \qquad \ln(s_{bg}) - \ln(s_{bb}) = 1/(1-a_{2b})^2.$$

(c) ***Principal's Two-agent, Terminal Reporting Problem:***
(identical one-period problems are solved for each agent)

$$U^{T2} = 2 \times \underset{a \in [0,1],\, s_g,\, s_b}{\text{maximize}} \; (x_g - s_g)a + (x_b - s_b)(1 - a),$$

subject to

$$\ln(s_g + s^o)a + \ln(s_b + s^o)(1 - a) - a/(1 - a) \geq \ln(2s^o),$$

$$\ln(s_g + s^o) - \ln(s_b + s^o) = 1/(1 - a)^2.$$

(d) ***Principal's Conditional Employment, Interim Reporting Problem:***

$$U^{I2} = \underset{\substack{a_1, a_{2b} \in [0,1] \\ s_g,\, s_{bg},\, s_{bb}}}{\text{maximize}} \; a_1[(x_g - s_g) + \tfrac{1}{2}U_2^{T2}] + (1 - a_1)[(x_b + x_g - s_{bg})a_{2b} + (2x_b - s_{bb})(1 - a_{2b})],$$

subject to

$$a_1 \ln(s_g + s^o) + (1 - a_1)[\ln(s_{bg})a_{2b} + \ln(s_{bb})(1 - a_{2b})] - a_1/(1 - a_1) - (1 - a_1)a_{2b}/(1 - a_{2b}) \geq \ln(2s^o),$$

$$\ln(s_g + s^o) - \ln(s_{bg})a_{2b} - \ln(s_{bb})(1 - a_{2b}) + a_{2b}/(1 - a_{2b}) = 1/(1 - a_1)^2,$$

$$\ln(s_{bg}) - \ln(s_{bb}) = 1/(1 - a_{2b})^2.$$

Table 25A.2
Two-period Hurdle Model with *TA* Agent Preferences
(* indicates constraints that apply if the agent can borrow and save)

(a) ***Principal's Single-agent, Terminal Reporting Problem:***

$$U^{T1} = \underset{\substack{a_1, a_2 \in [0,1] \\ s_1, s_{gg}, s_{gb}, s_{bg}, s_{bb}}}{\text{maximize}} \; a_1[(x_g - s_1) + (x_g - s_{gg})a_2 + (x_b - s_{gb})(1 - a_2)] + (1 - a_1)[(x_b - s_1) + (x_g - s_{bg})a_2 + (x_b - s_{bb})(1 - a_2)]$$

subject to

$$\ln(s_1) + a_1[\ln(s_{gg})a_2 + \ln(s_{gb})(1-a_2)] + (1-a_1)[\ln(s_{bg})a_2$$

$$+ \ln(s_{bb})(1-a_2)] - a_1/(1-a_1) - a_2/(1-a_2) \geq 2\ln(s^o),$$

$$\ln(s_{gg})a_2 + \ln(s_{gb})(1-a_2) - \ln(s_{bg})a_2 - \ln(s_{bb})(1-a_2) = 1/(1-a_1)^2,$$

$$a_1[\ln(s_{gg}) - \ln(s_{gb})] + (1-a_1)[\ln(s_{bg}) - \ln(s_{bb})] = 1/(1-a_2)^2,$$

$$* \quad 1/s_1 - a_1[a_2/s_{gg} + (1-a_2)/s_{gb}] - (1-a_1)[a_2/s_{bg} + (1-a_2)/s_{bb}] = 0.$$

(b) ***Principal's Single-agent, Interim Reporting Problem:***

$$U^{I1} = \underset{\substack{a_1, a_{2g}, a_{2b} \in [0,1] \\ s_{1g}, s_{1b} \\ s_{gg}, s_{gb}, s_{bg}, s_{bb}}}{\text{maximize}} \; a_1[(x_g - s_{1g}) + (x_g - s_{gg})a_{2g} + (x_b - s_{gb})(1-a_{2g})]$$

$$+ (1-a_1)[(x_b - s_{1b}) + (x_g - s_{bg})a_{2b} + (x_b - s_{bb})(1-a_{2b})],$$

subject to

$$a_1[\ln(s_{1g}) + \ln(s_{gg})a_{2g} + \ln(s_{gb})(1-a_{2g}) - a_{2g}/(1-a_{2g})]$$

$$+ (1-a_1)[\ln(s_{1b}) + \ln(s_{bg})a_{2b} + \ln(s_{bb})(1-a_{2b}) - a_{2b}/(1-a_{2b})]$$

$$- a_1/(1-a_1) \geq 2\ln(s^o),$$

$$[\ln(s_{1g}) + \ln(s_{gg})a_{2g} + \ln(s_{gb})(1-a_{2g}) - a_{2g}/(1-a_{2g})]$$

$$- [\ln(s_{1b}) + \ln(s_{bg})a_{2b} + \ln(s_{bb})(1-a_{2b}) - a_{2b}/(1-a_{2b})] = 1/(1-a_1)^2,$$

$$\ln(s_{gg}) - \ln(s_{gb}) = 1/(1-a_{2g})^2,$$

$$\ln(s_{bg}) - \ln(s_{bb}) = 1/(1-a_{2b})^2,$$

$$* \quad 1/s_{1g} - a_{2g}/s_{gg} - (1-a_{2g})/s_{gb} = 0,$$

$$* \quad 1/s_{1b} - a_{2b}/s_{bg} - (1-a_{2b})/s_{bb} = 0.$$

(c) ***Principal's Two-agent, Terminal Reporting Problem:***

$$U_1^{T2} = \underset{a_1 \in [0,1], s_1, s_g, s_b}{\text{maximize}} \; - s_1 + (x_g - s_g)a_1 + (x_b - s_b)(1-a_1),$$

subject to

$$\ln(s_1) + \ln(s_g + s^o)a_1 + \ln(s_b + s^o)(1 - a_1) - a_1/(1 - a_1) \geq 2\ln(s^o),$$

$$\ln(s_g + s^o) - \ln(s_b + s^o) = 1/(1 - a_1)^2,$$

$$* \quad 1/s_1 - a_1 /(s_g + s^o) - (1 - a_1)/(s_b + s^o) = 0.$$

$$U_2^{T2} = \underset{a_2 \in [0,1], s_1, s_g, s_b}{\text{maximize}} \; - s_1 + (x_g - s_g)a_2 + (x_b - s_b)(1 - a_2),$$

subject to

$$\ln(s_1 + s^o) + \ln(s_g)a_2 + \ln(s_b)(1 - a_2) - a_2/(1 - a_2) \geq 2\ln(s^o),$$

$$\ln(s_g) - \ln(s_b) = 1/(1 - a_2)^2,$$

$$* \quad 1/(s_1 + s^o) - a_2/s_g - (1 - a_2)/s_b = 0.$$

(d) ***Principal's Conditional Employment, Interim Reporting Problem:***

$$U^{I2} = \underset{\substack{a_1, a_{2b} \in [0,1] \\ s_g, s_{bg}, s_{bb}}}{\text{maximize}} \; a_1[(x_g - s_{1g}) + (U_2^{T2} - s_{2g})] + (1 - a_1)[(x_b - s_{1b}) + (x_g - s_{bg})a_{2b} + (x_b - s_{bb})(1 - a_{2b})],$$

subject to

$$a_1[\ln(s_{1g}) + \ln(s_{2g} + s^o)] + (1 - a_1)[\ln(s_{1b}) + \ln(s_{bg})a_{2b} + \ln(s_{bb})(1 - a_{2b}) - a_{2b}/(1 - a_{2b})] - a_1/(1 - a_1) \geq 2\ln(s^o),$$

$$[\ln(s_{1g}) + \ln(s_{2g} + s^o)] - [\ln(s_{1b}) + \ln(s_{bg})a_{2b} + \ln(s_{bb})(1 - a_{2b}) - a_{2b}/(1 - a_{2b})] = 1/(1 - a_1)^2,$$

$$\ln(s_{bg}) - \ln(s_{bb}) = 1/(1 - a_{2b})^2,$$

$$* \quad 1/s_{1g} - 1/(s_{2g} + s^o) = 0,$$

$$* \quad 1/s_{1b} - [a_{2b}/s_{bg} + (1 - a_{2b})/s_{bb}] = 0.$$

Table 25A.3
Two-period *AC* Hurdle Model Examples

Model Parameters: $x_g = 20, x_b = 10, s^o = .5.$

(a) *Single Agent with Terminal Reporting:* $U^{T1} = 21.526$

$a_1 = a_2 = .2290$ $s_{gg} = 10.713$
$s_{gb} = s_{bg} = 5.747$
$s_{bb} = .780$

(b) *Single Agent with Interim Reporting:* $U^{I1} = 21.761$

$a_1 = .2666$ $s_{gg} = 15.490$
$a_{2g} = .0066$ $s_{gb} = 5.623$
$a_{2b} = .2901$ $s_{bg} = 5.394$
$s_{bb} = .742$

(c) *Two Agents with Terminal Reporting:* $U^{T2} = 2 \times 11.026 = 22.052$

$a_1 = a_2 = .2737$ $s_{tg} + s^o = 5.275 + .500 = 5.775$
$s_{tb} + s^o = 0.368 + .500 = .868$

(d) *Contingent Replacement with Interim Reporting:* $U^{I2} = 22.185$

$a_1 = .2875$ $s_g + s^o = 5.592 + 0.500 = 6.092$
$a_{2b} = .2937$ $s_{bg} = 5.306$
$s_{bb} = .715$

Table 25A.4
Two-period *TA* Hurdle Model Examples
(the numbers in brackets assume the agent cannot borrow or save)

Model Parameters: $x_g = 20, x_b = 10, s^o = 1.$

(a) *Single Agent with Terminal Reporting:* $U^{T1} = 20.710$ (21.519)

$a_1 = .2374$ (.3203) $s_1 = 1.100(2.443)$
$a_2 = .2274$ (.3203) $s_{gg} = 11.002(7.184)$
$s_{gb} = 5.271(3.698)$
$s_{bg} = 5.271(3.698)$
$s_{bb} = .670(\ .209)$

(b) ***Single Agent with Interim Reporting:*** U^{I1} = 21.718 (21.947)

a_1 = .3801	(.3785)	s_{1g} = 3.159	(3.543)
a_{2g} = .1238	(.1404)	s_{1b} = .879	(1.598)
a_{2b} = .3020	(.3699)	s_{gg} = 10.574	(9.774)
		s_{gb} = 2.874	(2.526)
		s_{bg} = 5.044	(3.798)
		s_{bb} = .648	(.306)

(c) ***Two Agents with Terminal Reporting:*** U_t^{T2} = 10.596 (10.881)
U^{T2} = 21.192 (21.762)

a_1 = .2807	(.3405)	s_1 = 1.063	(1.762)
		$s_{2g} + s^o$ = 5.580	(4.334)
		$s_{2b} + s^o$ = .808	(0.435)
a_2 = .2807	(.3405)	$s_1 + s^o$ = 1.063	(1.762)
		s_{2g} = 5.580	(4.334)
		s_{2b} = .808	(.435)

(d) ***Contingent Replacement with Interim Reporting:*** U^{I2} = 22.350 (22.565)

a_1 = .4154	(.4161)	s_{1g} = 3.356	(3.362)
a_{2b} = .3090	(.3804)	s_{1b} = .823	(1.543)
		$s_{2g} + s^o$ = 3.356	(3.362)
		s_{bg} = 4.874	(3.620)
		s_{bb} = .600	(.268)

APPENDIX 25B: Proofs

Lemma 25.B1 $\omega_t - R_{t-1}\omega_{t-1} = \frac{1}{r}A_t^{-1}\ln[\beta_{t-1}^{a}/\beta_{t-1}]$.

Proof:

$$\omega_t - R_{t-1}\omega_{t-1} = -\frac{1}{r}\sum_{\tau=t+1}^{T}\{\beta_{\tau t}\ln[\beta_{\tau t}^{a}/\beta_{\tau t}] - R_{t-1}\beta_{\tau t-1}\ln[\beta_{\tau t-1}^{a}/\beta_{\tau t-1}]\}$$

$$+ \frac{1}{r}R_{t-1}\beta_{t-1}\ln[\beta_{t-1}^{a}/\beta_{t-1}]$$

$$= -\frac{1}{r}\sum_{\tau=t+1}^{T}\beta_{\tau t}\{\ln[(\beta_{\tau t}^{a}/\beta_{\tau t})/(\beta_{\tau t-1}^{a}/\beta_{\tau t-1})]\} + \frac{1}{r}\beta_t\ln[\beta_{t-1}^{a}/\beta_{t-1}]$$

$$= \frac{1}{r}\left[\sum_{\tau=t}^{T} \beta_{\tau t}\right] \ln[\beta_{t-1}^{a}/\beta_{t-1}] = \frac{1}{r} A_t^{-1} \ln[\beta_{t-1}^{a}/\beta_{t-1}]. \qquad \textbf{Q.E.D.}$$

Proof of Proposition 25.10

The first-order condition for c_τ in the agent's problem (25.15) is

$$\Xi_t^a r \beta_{\tau t}^a \exp[-rc_\tau] = \lambda \beta_{\tau t}.$$

which implies
$$c_\tau = \frac{1}{r}\{\ln[\beta_{\tau t}^a/\beta_{\tau t}] - \ln[\lambda/(r\Xi_t^a)]\}, \tag{25.B1}$$

where λ is the multiplier for the budget constraint. Substitute (25.B1) for all τ into (25.15b):

$$\sum_{\tau=t}^{T} \beta_{\tau t} \frac{1}{r}\{\ln[\beta_{\tau t}^a/\beta_{\tau t}] - \ln[\lambda/(r\Xi_t^a)]\} = B_t,$$

and solve for $-\ln[\lambda/(r\Xi_t^a)]$,

$$-\ln[\lambda/(r\Xi_t^a)] = A_t\{rB_t - \sum_{\tau=t}^{T} \beta_{\tau t} \ln[\beta_{\tau t}^a/\beta_{\tau t}]\}. \tag{25.B2}$$

Substituting (25.B2) into (25.B1) for $t = 0$ yields (25.16a). Similarly, solving for an arbitrary date t and setting $\tau = t$ yields (25.16b). If $\beta_\tau^a = \beta_\tau\ \forall\ \tau \geq t$, then $\ln[\beta_{\tau t}^a/\beta_{\tau t}] = 0,\ \forall\ \tau \geq t$, and we obtain (25.17a) and (25.17b).

Substituting the agent's optimal consumption choices into (25.15a) yields the agent's value function. Note that the agent's "asset" balance at date τ can be written as

$$B_\tau + \omega_\tau = R_{\tau-1}(1 - A_{\tau-1})(B_{\tau-1} + \omega_{\tau-1}) + \omega_\tau - R_{\tau-1}\omega_{\tau-1}$$

$$= R_{\tau-1}(1 - A_{\tau-1})(B_{\tau-1} + \omega_{\tau-1}) + \frac{1}{r} A_\tau^{-1} \ln[\beta_{\tau-1}^a/\beta_{\tau-1}],$$

where the second equality follows from Lemma 25.B1. Since $A_\tau R_{\tau-1}(A_{\tau-1}^{-1} - 1)A_{\tau-1} = A_{\tau-1}$, the preceding implies, for $\tau > t$,

$$A_\tau(B_\tau + \omega_\tau) = A_{\tau-1}(B_{\tau-1} + \omega_{\tau-1}) + \frac{1}{r} \ln[\beta_{\tau-1}^a/\beta_{\tau-1}]$$

$$= A_t(B_t + \omega_t) + \frac{1}{r} \ln[\beta_{\tau t}^a/\beta_{\tau t}].$$

Substituting this into the value function yields

$$V_t^{TA}(B_t) = -\Xi_t^a \sum_{\tau=t}^{T} \beta_{\tau t}^a \exp[-rA_\tau(B_\tau + \omega_\tau)]$$

$$= -\Xi_t^a \Big[\sum_{\tau=t}^{T} \beta_{\tau t}\Big] \exp[-rA_t(B_t + \omega_t)]$$

$$= -\Xi_t^a A_t^{-1} \exp[-rA_t(B_t + \omega_t)],$$

which is (25.16c). **Q.E.D.**

Proof of Proposition 25.11

To solve the dynamic programming problem (25.19) we conjecture a specific form of the value function and the certainty equivalent, and then verify that the conjecture is a solution to the problem. The conjectured form of the value function is:

$$V_t^{TA}(CE_t^{TA}) = -g_t \Xi_t^a \exp[-rh_t(CE_t^{TA} + q_t)], \quad t = 0, ..., T, \quad (25.B3)$$

with time-dependent constants g_t, h_t, and q_t with $g_T = h_T = 1$ and $q_T = 0$.

The certainty equivalent at date $t+1$ is conjectured to be a linear function of $\mathbf{y}_t$ (which is normally distributed) and the risk aversion parameter for the value function $V_{t+1}^{TA}(\cdot)$ is rh_{t+1}, so that the conjectured date t certainty equivalent is

$$CE_t^{TA} = nc_t + \beta_t\{\mathrm{E}_t[CE_{t+1}^{TA}] - \tfrac{1}{2} rh_{t+1}\mathrm{Var}_t[CE_{t+1}^{TA}]\}, \quad t = 0, ..., T-1, \quad (25.B4)$$

with $CE_T^{TA} = nc_T = B_T$. Note that the discounting on the right-hand side occurs to ensure that everything is measured in nominal date t dollars.

Using (25.B3) and (25.B4), we can write the Bellman equation (25.19) as

$$V_t^{TA}(CE_t^{TA}) = \max_{nc_t} \{-\Xi_t^a \exp(-r\,nc_t)$$

$$- g_{t+1}\Xi_{t+1}^a \exp[-rh_{t+1}(R_t(CE_t^{TA} - nc_t) + q_{t+1})]\}, \quad (25.B5)$$

and the first-order condition for the optimal consumption choice is

$$nc_t^{TA} = \mathbf{nc}_t^{TA}(CE_t^{TA}) = \frac{R_t h_{t+1}}{1 + R_t h_{t+1}} CE_t^{TA} - \frac{\ln(\beta_t^a/\beta_t) + \ln(g_{t+1}h_{t+1})}{r(1 + R_t h_{t+1})}. \quad (25.B6)$$

Observe that (25.B3) implies that h_t and h_{t+1} must satisfy

$$h_t = \frac{R_t h_{t+1}}{1 + R_t h_{t+1}}$$

to be consistent with our conjecture. As in Proposition 25.10 this is satisfied by $h_t = \Lambda_t$:

$$\frac{R_t h_{t+1}}{1 + R_t h_{t+1}} = \frac{1}{\beta_t \Lambda_{t+1}^{-1} + 1} = \Lambda_t.$$

We now conjecture that the two other coefficients are also the same as in Proposition 25.10 and prove that they satisfy the Bellman equation.

Substitute $nc_t = h_t(CE_t^{TA} + q_t)$, $g_t = \Lambda_t^{-1}$, $h_t = \Lambda_t$, and $q_t = \omega_t$ into the right-hand side of (25.B5) to obtain

$$\begin{aligned}
&- \Xi_t^a \exp[- r\Lambda_t(CE_t^{TC} + \omega_t)] \\
&\qquad - \Lambda_{t+1}^{-1} \Xi_{t+1}^a \exp[-r\Lambda_{t+1}(R_t(CE_t^{TA} - \Lambda_t(CE_t^{TA} + \omega_t)) + \omega_{t+1})] \\
&= - \Xi_t^a \exp[- r\Lambda_t(CE_t^{TC} + \omega_t)] \\
&\qquad - \Lambda_{t+1}^{-1} \Xi_{t+1}^a \exp[-r\Lambda_t(CE_t^{TA} + \omega_t)] \exp[r\Lambda_{t+1}(R_T\omega_t - \omega_{t+1})] \\
&= - (\Xi_t^a + \Lambda_{t+1}^{-1} \Xi_{t+1}^a \beta_t^a/\beta_t) \exp[- r\Lambda_t(CE_t^{TA} + \omega_t)] \\
&= - \Xi_t^a \Lambda_t^{-1} \exp[- r\Lambda_t(CE_t^{TC} + \omega_t)].
\end{aligned}$$

The first equality rearranges terms and uses the fact that $\Lambda_{t+1}R_t(1 - \Lambda_t) = \Lambda_t$. Then the second equality uses Lemma 25.A1, and the third equality uses $\Xi_t^a = \Xi_{t+1}^a \beta_t^a$ and $1 + \Lambda_{t+1}^{-1}/\beta_t = \Lambda_t^{-1}$. The result satisfies the conjectured form of the left-hand side of the Bellman equation.

Finally, we derive the agent's certainty equivalent. It is given inductively by (25.20b) with initial condition $CE_T^{TA} = B_T$. It includes the NPV at date t of current and future compensation and effort costs, represented by $W_t \equiv \Sigma_{\tau=t}^T \beta_{\tau t} s_\tau = s_t + \beta_t W_{t+1}$ and $K_t \equiv \Sigma_{\tau=t}^T \beta_{\tau t} \kappa_\tau = \kappa_t + \beta_t K_{t+1}$, respectively. In addition, the certainty equivalent includes the *nominal weath risk premium* represented by RP_t^{TA}, and specified in (25.20c)

The specification of the certainty equivalent in (25.20b) follows by induction using (25.B4) and (25.20b). That is, assume it holds for date $t+1$ and then show it holds for date t, i.e.,

$$CE_t^{TA} = nc_t + \beta_t\{E_t[B_{t+1} + \beta_{t+1}E_{t+1}[W_{t+2} - K_{t+2}] - RP_{t+1}^{TA}]$$

$$- \tfrac{1}{2}rA_{t+1}\text{Var}_t[B_{t+1} + \beta_{t+1}E_{t+1}[W_{t+2} - K_{t+2}] - RP_{t+1}^{TA}]\}$$

$$= B_t + \beta_t\{E_t[s_{t+1} - \kappa_{t+1} + \beta_{t+1}[W_{t+2} - K_{t+2}] - RP_{t+1}^{TA}]$$

$$- \tfrac{1}{2}rA_{t+1}\text{Var}_t[s_{t+1} + \beta_{t+1}E_{t+1}[W_{t+2}]]\}$$

$$= B_t + \beta_t\{E_t[W_{t+1} - K_{t+1}] - [\tfrac{1}{2}rA_{t+1}\text{Var}_t[E_{t+1}[W_{t+1}]] + RP_{t+1}^{TA}]\},$$

where the second equality follows from the law of iterated expectations, i.e., $E_t[E_{t+1}[\cdot]] = E_t[\cdot]$, and the assumption that future effort costs are non-stochastic.

Q.E.D

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CHAPTER 26

TIMING AND CORRELATION OF REPORTS IN A MULTI-PERIOD *LEN* MODEL

This is the second of four chapters that examine multi-period principal-agent models. As in Chapter 25, we assume the principal and the agent can commit to a long-term contract without subsequent renegotiation. The key innovation in this chapter is that we relax the Chapter 25 assumptions that the performance reports are stochastically and technologically independent.

The impact of correlated noise is examined in depth using a multi-period *LEN* model.[1] This model is a relatively straightforward extension to the multi-period *LEN* model introduced in Section 25.4. We establish that the timing of performance measure reports is irrelevant if the agent has exponential *AC-EC* (aggregate-consumption/effort cost) preferences, but early reporting can have strictly positive value to the principal if the agent has exponential *TA-EC* (time-additive/effort cost) preferences. The key, of course, is whether early reporting permits the agent to more fully smooth his consumption. Interestingly, the results differ for action-informative reports (those influenced by the agent's actions) versus reports that are "purely insurance" informative (i.e., they are not influenced by the agent's productive acts but are correlated with the noise in action-informative reports). Early reporting of the former is generally valuable to the principal, whereas it is not valuable to report the latter before the insured action-informative report is issued. The analysis also considers how the inter-period correlation of the reports affects the principal's expected net payoff.

Section 26.2 explores the impact of report characteristics in a two-period setting, other than timing, on the principal's expected utility and his preference for two versus a single agent. These characteristics include the level of correlation between reports, the sensitivity of the reports, and the aggregation of reports.

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[1] Much of the analysis in this chapter is based on Christensen, Feltham, Hofmann, and Şabac (2004) (CFHS).

26.1 IMPACT OF CORRELATED REPORTS IN A MULTI-PERIOD *LEN* MODEL

In Section 25.4 we considered a *LEN* model in which there are T technologically and stochastically independent periods. We now extend that model by allowing the noise in one report to be correlated with the noise in other reports, so the stochastic independence assumption no longer holds.

The analysis in Chapter 20 considers the impact of correlation in a single-period *LEN* model, and many of the results in that chapter can be extended to the multi-period model considered here. We leave that to the reader and focus on the implications of correlation among signals released at different dates. Inter-period correlation implies that the agent's uncertainty about the noise in future reports is reduced as correlated reports are issued. There are two key issues to be examined. First, if we hold the correlations fixed, how does the timing of the reports affect the agent's consumption and action choices, the contract offered by the principal, and the principal's expected utility? Second, if we hold the timing of reports fixed, what impact does the level of correlation have?

26.1.1 Impact of Report Timing on the Agent's Utility with Exogenous Incentive Rates

While our analysis in this section emphasizes the relaxation of the stochastic independence assumption, we also relax the technological independence assumption. In particular, the general form of the j^{th} performance measure is

$$y_j = \sum_{\tau=1}^{\bar{t}_j} \mathbf{M}_{j\tau}\mathbf{a}_\tau + \varepsilon_j,$$

where $\mathbf{a}_\tau$ is the $m_\tau \times 1$ vector of actions taken at the start of period τ, $\mathbf{M}_{j\tau}$ is the $1\times m_t$ matrix of sensitivities for the j^{th} performance measure with respect to the actions $\mathbf{a}_\tau$ in period τ, $\bar{t}_j$ is the date of the latest action that impacts y_j, and $\varepsilon_j \sim$ N(0, 1) is the noise in the j^{th} performance measure. The reports issued at date t are represented by the vector $\mathbf{y}_t$, which consists of all y_j such that $j \in J_t$. Similarly, the reports issued up through date t are represented by $\overleftarrow{\mathbf{y}}_t$, which includes all y_j such that $j \in \overleftarrow{J}_t \equiv J_1 \cup ... \cup J_t$.[2] Conversely, the reports issued subsequent to date t are represented by $\vec{\mathbf{y}}_{t+1}$, which includes all y_j such that $j \in \vec{J}_{t+1} \equiv J_{t+1} \cup ... \cup J_T$.

[2] We assume date 1 is the earliest report date. CFHS consider both pre- and post-contract reports at date 0, but for simplicity we exclude these types of reports from the current analysis.

Proposition 25.7 demonstrates that if the agent has access to personal banking, then the timing of the payment of compensation is not important as long as the net present value for any complete performance history is unchanged.[3] For example, paying s_t at date t or paying s_t/β_{Tt} at date T does not affect the agent's consumption and action choices nor his expected utility. Of course, the compensation at date t must be measurable with respect to the information available at that date. Since our focus in this section is on the impact of report timing, we assume without loss generality that the fixed wage is paid at date t = 0, and the incentive wages are all paid at the consumption horizon, i.e.,

$$s_0 = f; \quad s_t = 0, \quad \text{for all } t = 1, ..., T-1; \quad s_T(\overleftarrow{\mathbf{y}}_T) = \sum_{j \in J} v_j y_j.$$

This allows us to change the report date of a given report without affecting the timing of compensation. Note that, since there are no intermediate payments, the NPV of the agent's remaining compensation as of date t is given by

$$W_t = \beta_{\tau t} W_\tau, \quad \text{for any } \tau > t, \tag{26.1}$$

with $W_T = s_T(\overleftarrow{\mathbf{y}}_T)$.

The Agent's Choices

As noted in Section 25.4, the characterization of the agent's consumption choices, certainty equivalents, and expected utility provided by Propositions 25.11 and 25.12 also apply to the settings considered here. However, the characterization of the agent's action choices and the principal's contract choices (see (25.24) and (25.25)) are only applicable to settings in which there is technological and stochastic independence. Nonetheless, the pre-consumption certainty equivalent used to characterize the agent's action choices can be readily extended to the current setting.

Section 25.4 considered time-varying interest rates and differences between the agent's and market's time-preference index. That analysis demonstrated that the consumption choice issues raised by these factors have little impact on the agent's action choice and the principal's contract choice. Hence, since action choices and contract choices are of central focus in this chapter, we simplify the analysis by assuming a flat term structure of interest rates that also characterizes the agent's time preference, i.e., $\beta_t^a = \beta_t = \beta$ for all t and $\beta_{\tau t} = \beta^{\tau - t}$.

The agent selects his actions at each date so as to maximize his certainty equivalent given his information at that date. His pre-consumption certainty

[3] Proposition 25.7 is stated for time-additive preferences but, obviously, it also holds for aggregate consumption preferences.

equivalent has four components. Three are the same for both AC and TA preferences: the current bank balance (B_t), the expected value of the NPV of future compensation, $\mathrm{E}_t[W_{t+1}]$, and the NPV of future effort costs, K_{t+1}.[4] The fourth is the NPV of the risk premia associated with future compensation risk, RP_t^{AC} or RP_t^{TA}. The risk premium differs between the two types of preferences because of the difference in the relation between consumption and compensation in the two settings. More specifically, the agent's certainty equivalent at dates $t = 0, 1, ..., T-1$, for preferences $i = AC, TA$, are (see (25.20) and (25.22)):

$$CE_t^i = B_t + \beta\{\mathrm{E}_t[W_{t+1}] - K_{t+1}\} - RP_t^i, \tag{26.2}$$

and his compensation risk premium is (see (25.23)):

$$RP_t^i = \tfrac{1}{2}\sum_{\tau=t+1}^{T} \beta^{\tau-t}\, \hat{r}_\tau^{\,i}\, \mathrm{Var}_{\tau-1}[\mathrm{E}_\tau[W_\tau]], \tag{26.3}$$

with $\hat{r}_\tau^{AC} = r\Xi_\tau$ and $\hat{r}_\tau^{TA} = rA_\tau$ representing the nominal wealth risk aversions under the two types of preferences. Recall from Section 25.4 that Ξ_τ is the agent's time-preference index under AC preferences, and in this chapter we assume it has a ratio $\Xi_{\tau+1}/\Xi_\tau$ equal to the market discount rate β. On the other hand, with TA preferences we use the annuity factor $A_\tau = [1 + \beta + ... + \beta^{T-\tau}]^{-1}$.

The bank balance and effort cost components are precisely the same as in Section 25.4. However, the other two components of the agent's certainty equivalent are more complex. The expected NPV of future compensation can be expressed as

$$\mathrm{E}_t[W_{t+1}] = \beta^{T-t-1}\sum_{\tau=1}^{T}\sum_{j\in J_\tau} v_j\left(\sum_{h=1}^{\tau} \mathbf{M}_{jh}\mathbf{a}_h + \mathrm{E}_t[\varepsilon_j]\right). \tag{26.4}$$

Calculating the expectation of W_{t+1} is straightforward in Chapter 25 because technological and stochastic independence imply that the incentive wage attributable to the reports at each date τ depends only on $\mathbf{a}_\tau$ and ε_j, $j \in J_\tau$, and $\mathrm{E}_t[\varepsilon_j] = 0$ for all $j \in J_\tau$, $\tau = t+1,...,T$.

Now consider the agent's action choices at the start of period $t+1$. Again we have a situation in which the agent's actions do not affect the compensation attributable to the noise terms. Hence, they do not impact the compensation risk premium under either TA or AC, nor do they impact the conditional expectation of the future noise terms. Furthermore, we assume the current action does not

[4] It is assumed that future effort costs are non-stochastic. This is justified in the LEN model we consider here, but it may not be justified if we allow non-linear contracts (see Chapter 27).

affect the cost of future actions, and the cost for period $t+1$ is $\boldsymbol{\kappa}_{t+1}(\mathbf{a}_{t+1}) = \frac{1}{2}\mathbf{a}_{t+1}^{t}\mathbf{a}_{t+1}$. Hence, given the incentive rates $\mathbf{v}_{t+1}, ..., \mathbf{v}_T$, the agent chooses $\mathbf{a}_{t+1}$ to maximize $E_t[W_{t+1}] - \boldsymbol{\kappa}_{t+1}(\mathbf{a}_{t+1})$. The first-order condition is

$$\mathbf{a}_{t+1} = \beta^{T-t-1} \sum_{\tau=t+1}^{T} \sum_{j \in J_\tau} v_j \mathbf{M}_{jt+1}^{t}. \tag{26.5}$$

Obviously, (25.24) is a special case in which technological independence implies $\mathbf{M}_{jt+1} = \mathbf{0}$ for all $j \in J_\tau$, $\tau > t+1$.[5] Without that independence, the agent's action choice at any given date is influenced by the incentive rates for all future reports affected by the current action. The discounting reflects that incentive wages are paid at date T, whereas the effort costs are paid at date $t+1$.

Report Timing

Consider a change in the timing of report y_j holding the incentive rate for that report constant. In particular, assume that $t_j > \underline{t}_j$, so that it is technically feasible to issue the report one date earlier, i.e., at $t_j - 1$. In that case j shifts from being a member of the set J_{t_j} to being a member of set J_{t_j-1}. Note from (26.4) and (26.5) that the timing of report y_j does not affect the expected NPV of future compensation nor the action choice.

Now consider the impact of a change in the timing of report y_j on the agent's consumption choice and compensation risk premium. Recall that W_{t+1} is the NPV at date $t+1$ of the compensation that will be paid at date T. Those payments will depend on the reports issued at each date. Hence, since the reports are affected by random noise, W_{t+1} is a random variable. The following lemma establishes that the conditional variance of W_{t+1} given the information at date t is not affected by the timing of the reports subsequent to date t.

Lemma 26.1

For any given date $t = 0, ..., T-1$,

$$\mathrm{Var}_t[W_{t+1}] = \beta^{2(T-t-1)}\,\mathrm{Var}_t[W_T] = \beta^{2(T-t-1)} \sum_{\tau=t+1}^{T} \mathrm{Var}_{\tau-1}[E_\tau[W_T]], \tag{26.6}$$

and $\mathrm{Var}_t[W_{t+1}]$ is not changed if y_j is reported at date $\tau > t$, instead of date $t_j > t$.[6]

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[5] The difference in the discounting reflects that in this chapter all incentive wages are paid at T.

[6] The first equality follows directly from (26.1), and the second equality follows from the fact that we can write W_T as

(continued...)

The lemma demonstrates that the conditional compensation variance at any given date t can be written as a sum of variances for the subsequent dates each measuring the amount of uncertainty resolved at that date. Timing affects when uncertainty is resolved, but not the total. This directly implies that the timing of a report is irrelevant if the agent has AC preferences. Note from (26.1), (26.3) and (26.6) that the agent's risk premium with AC preferences can be written as

$$RP_t^{AC} = \tfrac{1}{2} \sum_{\tau=t+1}^{T} \beta^{\tau-t} r \Xi_\tau \beta^{2(T-\tau)} \mathrm{Var}_{\tau-1}[\mathrm{E}_\tau[W_T]]$$

$$= \tfrac{1}{2} r \beta^{T-t} \Xi_T \sum_{\tau=t+1}^{T} \mathrm{Var}_{\tau-1}[\mathrm{E}_\tau[W_T]]$$

$$= \tfrac{1}{2} \beta^{T-t} \hat{r}_T^{AC} \mathrm{Var}_t[W_T].$$

Hence, the agent's compensation risk premium at date t is the discounted nominal wealth risk premium for the conditional compensation variance given the information at date t and, therefore, is not affected by the timing of the reports subsequent to date t. Of course, this is due to the fact that the timing of consumption has no impact on the agent's expected utility in this case – we may assume without loss of generality that the agent only consumes at date T.

On the other hand, consumption smoothing occurs under TA preferences, and earlier reporting may facilitate more smoothing. Therefore, it is not surprising that the following proposition establishes that, for any exogenous set of performance measures, incentive rates, and induced actions, issuing a report earlier will not reduce, and may increase the agent's certainty equivalent under TA. The increase occurs if earlier reporting permits the agent to reduce his compensation risk premium by smoothing random compensation over more periods.

We state the following proposition in terms of issuing a report one period earlier. This can be applied iteratively to consider any arbitrary reporting date that is feasible.

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[6] (...continued)

$$W_T = \sum_{\tau=t+1}^{T} \{\mathrm{E}_\tau[W_T] - \mathrm{E}_{\tau-1}[W_T]\} + \mathrm{E}_t[W_T],$$

and the fact that $M_\tau \equiv \mathrm{E}_\tau[W_T]$ is a martingale with independent increments, i.e.,

$$\mathrm{Var}_t[\mathrm{E}_\tau[W_T] - \mathrm{E}_{\tau-1}[W_T]] = \mathrm{Var}_{\tau-1}[\mathrm{E}_\tau[W_T] - \mathrm{E}_{\tau-1}[W_T]] = \mathrm{Var}_{\tau-1}[\mathrm{E}_\tau[W_T]].$$

Proposition 26.1

Consider a reporting structure η that generates a set of reports $J = J_0 \cup J_1 \cup \ldots \cup J_T$, where the set J_τ is issued at date τ, with exogenous incentive rates $\overleftarrow{\mathbf{v}}_T = (v_j)_{j\in J}$ and induced actions $\overleftarrow{\mathbf{a}}_T = (\mathbf{a}_1, \ldots, \mathbf{a}_T)$. Let η^e represent an alternative "early" reporting system in which report $h \in J$ is issued at date $t_e = t_h - 1 \geq t_h$ instead of t_h. Given the exogenous incentive rates, the change in the agent's *ex ante* certainty equivalent is

$$\Delta CE_0^{AC}(\eta^e,\eta) = 0,$$

$$\begin{aligned}\Delta CE_0^{TA}(\eta^e,\eta) &= RP_0^{TA}(\eta) - RP_0^{TA}(\eta^e) \\ &= \tfrac{1}{2} r\beta^T \beta^{T-t_h} A_{t_h} A_{t_e} \operatorname{Var}_{t_e}[\mathrm{E}_{t_e}^e[W_T]] \geq 0,\end{aligned}$$

where $\mathrm{E}_{t_e}^e$ is the expectation at t_e given early reporting of y_h.

Three of the four components of the agent's *ex ante* certainty equivalent (B_0, $\mathrm{E}_0[W_0]$, and K_0), are unaffected by early reporting. The only component that may change is his *ex ante* compensation risk premium. Lemma 26.1 implies directly that RP_0^{AC} is also unchanged. However, RP_0^{TA} is strictly reduced with early reporting if, and only if, $\operatorname{Var}_{t_e}[\mathrm{E}_{t_e}^e[W_{t_e}]] > 0$. Note that

$$RP_0^{TA} = \tfrac{1}{2}\sum_{t=1}^{T} \beta^t \hat{r}_t^{TA} \operatorname{Var}_{t-1}[\mathrm{E}_t[W_t]] = \tfrac{1}{2}\beta^T r \sum_{t=1}^{T} \beta^{T-t} A_t \operatorname{Var}_{t-1}[\mathrm{E}_t[W_T]].$$

That is, the *ex ante* risk premium is a constant times a weighted sum of the compensation uncertainty resolved at each date with weights $A_t \equiv \beta^{T-t} A_t$. Lemma 26.1 establishes that an equally weighted sum is independent of timing, but since the weights A_t are increasing over time, early resolution of compensation uncertainty is valuable with *TA* preferences, i.e., reduces the compensation risk premium.[7] That is, early reporting of y_h has positive value if it provides new information about future compensation that is not provided by $\overleftarrow{\mathbf{y}}_{t_e}$.

26.1.2 Impact of Report Timing on the Principal's Optimal Expected Net Payoff

Now consider the optimal incentive rates. The principal selects the contract and induced actions that maximize his expected utility subject to the requirement

[7] For example, the additional compensation uncertainty resolved at t_e with η^e (relative to η) is $\operatorname{Var}_{t_e}[\mathrm{E}_{t_e}^e[W_T]] = \operatorname{Var}_{t_e}[\mathrm{E}_{t_h}[W_T]] - \operatorname{Var}_{t_e}^e[\mathrm{E}_{t_h}^e[W_T]]$, and $A_{t_h} - A_{t_e} = \beta^{T-t_h} A_{t_h} A_{t_e}$.

that the contract is acceptable to the agent and the induced actions are incentive compatible (i.e., satisfy (26.5)). The contract is represented by $\overleftarrow{\mathbf{s}}_T = (f, \overleftarrow{\mathbf{v}}_T)$ which consists of the initial fixed wage f and incentive rates $\overleftarrow{\mathbf{v}}_T = (v_j)_{j\in J}$ that are applied to the set of reports J in the periods they are issued. By setting f so that the first constraint is an equality, the principal's problem can be expressed solely as a function of the incentive rates and induced actions:

$$U^p(\overleftarrow{\mathbf{v}}_T,\eta) = \pi_0 - \{K_0 + RP_0^i\}, \quad i = TA, AC, \tag{26.7}$$

where
$$\pi_0 \equiv \sum_{t=1}^{T} \beta^t\, \mathbf{b}_t^{\mathrm{t}}\mathbf{a}_t,$$

$$K_0 = \sum_{t=1}^{T} \beta^t\, \tfrac{1}{2}\mathbf{a}_t^{\mathrm{t}}\mathbf{a}_t,$$

$$RP_0^i = \tfrac{1}{2}\beta^T \sum_{t=1}^{T} \beta^{T-t}\, \hat{r}_t^i\, \mathrm{Var}_{t-1}[\mathrm{E}_t[W_T]],$$

and $\mathbf{b}_t$ is a vector of payoffs to the principal per unit of effort in each action at date t. First-order condition (26.5) can then be used to express the principal's problem strictly in terms of the incentive rates $\overleftarrow{\mathbf{v}}_T$.

The first-order condition for incentive rate v_j, given $i = AC, TA$, is

$$\sum_{t=1}^{t_j} \beta^t[\mathbf{b}_t^{\mathrm{t}} - \mathbf{a}_t^{\mathrm{t}}]\nabla_{v_j}\mathbf{a}_t = \tfrac{1}{2}\beta^T \sum_{t=1}^{T} \beta^{T-t}\, \hat{r}_t^i\, \partial\mathrm{Var}_{t-1}[\mathrm{E}_t[W_T]]/\partial v_j. \tag{26.8}$$

Using (26.5), the left-hand side of (26.8) is

$$\sum_{t=1}^{t_j} \beta^t[\mathbf{b}_t^{\mathrm{t}} - \mathbf{a}_t^{\mathrm{t}}]\nabla_{v_j}\mathbf{a}_t$$

$$= \sum_{t=1}^{t_j} \beta^t[\mathbf{b}_t^{\mathrm{t}} - \beta^{T-t}\sum_{\tau=t}^{T}\sum_{j'\in J_\tau} v_{j'}\mathbf{M}_{j'\tau}^{\mathrm{t}}]\beta^{T-t}\mathbf{M}_{jt}^{\mathrm{t}}, \tag{26.9}$$

which implies that the impact of increasing v_j on the principal's gross payoff is independent of the other incentive rates. On the other hand, the impact of increasing v_j on the agent's effort cost and the risk premium on the right-hand side are potentially affected by the incentive rates for both prior and subsequent reports. The precise form depends on the structure of the sensitivities and the underlying correlations. Examples are provided below.

Action and Insurance Informativeness

In this chapter, all actions are costly to the agent and all influence at least one performance measure. However, an action may not be beneficial to the principal. We use the following definitions in referring to actions and performance measures (similar terms are used in Chapter 20).

Definition

An action $a_{t\ell}$ (the ℓ^{th} element of $\mathbf{a}_t$) is *productive* if $b_{t\ell} > 0$ and is *window dressing* if $b_{t\ell} = 0$.

A report y_j is *action informative* with respect to $a_{t\ell}$ if $M_{jt\ell} \neq 0$, and it is not action informative if $\mathbf{M}_{jt} = \mathbf{0}$ for all t. The report is *insurance informative* if $\text{Cov}[\varepsilon_j, \varepsilon_{j'}] \neq 0$ for some report $y_{j'}$ that is informative with respect to some productive action, and it is not insurance informative if $\text{Cov}[\varepsilon_j, \varepsilon_{j'}] = 0$ for all action informative reports $y_{j'}$. Finally, a report is *purely insurance informative* if it is not action informative, but it is insurance informative with respect to some action informative report.

Recall that t_j represents the date report y_j is issued. If report y_j is action informative, then we let $\underline{t}_j$ represent the latest period in which an action influencing that report is taken. Report date t_j cannot precede $\underline{t}_j$. If report y_j is purely insurance informative, then $\underline{t}_j = 0$, and there is no restriction on the timing of the report.

26.1.3 A Single Action with Multiple Consumption Dates

We now introduce a setting in which there is a single productive action a, an action informative report y_a, and a purely insurance informative report y_i. We initially focus on the impact of the timing of the reports. Then we consider changes in the level of correlation and the impact of window dressing.

The Basic Model

The productive action a is assumed to be taken at the start of the first period. Hence, date 1 is the earliest date at which $y_a = M_a a + \varepsilon_a$ can be reported. The purely insurance informative report is $y_i = \varepsilon_i$. The noise in the two reports have unit variance and correlation ρ. Let v_a and v_i represent the incentive rates for the two reports expressed in date $T = 3$ dollars. Hence, the *ex ante* variance of the NPV of the agent's compensation is

$$\text{Var}_0[W_0] = \text{Var}_0[\beta^3 W_3] = \text{Var}_0[\beta^3(v_a y_a + v_i y_i)] = \beta^6[v_a^2 + 2\rho v_a v_i + v_i^2].$$

An information system that reports y_a at date t_a and y_i at date t_i is represented by $\eta^{t_a t_i}$, e.g., η^{21} reports y_a at date 2 and y_i at date 1.

In the analysis we assume the agent's time-preference index is $\Xi_t = \beta^t$. The timing is irrelevant, if the agent has *AC* preferences. In that case, with exogenous incentive rates, the agent's risk premium is

$$RP_0^{AC}(\eta|v_a,v_i) = \tfrac{1}{2}r\beta^6\text{Var}_0[W_3] = \tfrac{1}{2}r\beta^6[v_a^2 + 2\rho v_a v_i + v_i^2]$$

for all reporting systems.

Impact of Report Timing with TA Preferences and Exogenous Incentives

Report timing has an effect if the agent has *TA* preferences. In that case, with exogenous incentive rates, the agent's risk premium is

$$RP_0^{TA}(\eta) = \tfrac{1}{2}r\beta^3\{A_1\text{Var}_0^\eta[\text{E}_1^\eta[W_3]] + A_2\text{Var}_1^\eta[\text{E}_2^\eta[W_3]] + A_3\text{Var}_2^\eta[W_3]\},$$

where the conditional variances obviously depend on the timing, and the weights, $A_t = \beta_t A_t$, applied to these variances are such that $A_t < A_{t+1}$ and $A_3 = 1$. Table 26.1 (panel A) summarizes $RP_0^{TA}(\eta)$ for the feasible timing of reports from date 1 to date 3.

To illustrate these calculations, consider systems η^{22} and η^{21}. With system η^{22} both reports are reported at $t = 2$, i.e., all uncertainty about the agent's final compensation $W_3 = v_a y_a + v_i y_i$ is resolved at $t = 2$. Hence, $\text{Var}_0^{22}[\text{E}_1^{22}[W_3]]$ and $\text{Var}_2^{22}[W_3]$ are both equal to zero (where the superscripts represent the information system). On the other hand, $\text{E}_2^{22}[W_3] = W_3$ and given that no information is reported until $t = 2$, the conditional variance at $t = 1$ is equal to the prior variance of W_3, i.e., $\text{Var}_1^{22}[\text{E}_2^{22}[W_3]] = v_a^2 + 2\rho v_a v_i + v_i^2$. The multiple, $\tfrac{1}{2}r\beta^3 A_2$, applied to this variance reflects the agent's risk aversion, that the agent's incentive wages are paid at $t = 3$, and that the agent smooths his consumption as an annuity from $t = 2$ and onwards. Note that, for all systems in which both reports are reported simultaneously the agent's *ex ante* risk premium has the same structure – the only difference is the applied weights $A_t = \beta^{T-t}A_t$ reflecting the number of remaining periods over which the agent can smooth his compensation risk. Of course, since both β^{T-t} and A_t are increasing with the reporting date t, the agent's *ex ante* risk premium is lower for earlier reporting dates.

With system η^{21} the action informative report y_a is reported at $t = 2$, whereas the insurance informative report is reported one period earlier at $t = 1$. In this case, uncertainty can be resolved at both $t = 1$ and $t = 2$, whereas no uncertainty is resolved at the final date $t = 3$, i.e., $\text{Var}_2^{21}[W_3] = 0$. The uncertainty resolved at $t = 1$ is determined by the uncertainty in the conditional expectation at $t = 1$, $\text{E}_1^{21}[W_3]$, as viewed from $t = 0$. Using the rules for conditional expectations of normally distributed variables, we get that

$$E_1^{21}[W_3] = v_a M_a + v_i y_i + \rho v_a y_i = v_a M_a + (v_i + \rho v_a) y_i,$$

and, hence, $\text{Var}_0^{21}[E_1^{21}[W_3]] = (v_i + \rho v_a)^2$. Similarly, since the posterior variance of y_a given y_i is equal to $(1 - \rho^2)$, the remaining uncertainty resolved at $t = 2$ is $\text{Var}_1^{21}[E_2^{21}[W_3]] = v_a^2(1 - \rho^2)$. The agent's *ex ante* risk premium is obtained by a weighted sum of these variances with weights reflecting the number of remaining periods over which the agent can smooth the compensation risk resolved at each date.

TABLE 26.1
Risk Premia for Single Action, Multiple Reporting Date Example, with Time-additive Preferences

η	***Panel A*** v_a and v_i are exogenous	***Panel B*** v_i is optimal given v_a
η^{33}:	$\frac{1}{2} r\beta^3 \{v_a^2 + 2\rho v_a v_i + v_i^2\}$	$\frac{1}{2} r\beta^3 \{v_a^2(1-\rho^2)\}$
η^{32}:	$\frac{1}{2} r\beta^3 \{A_2(v_i + \rho v_a)^2 + v_a^2(1-\rho^2)\}$	$\frac{1}{2} r\beta^3 \{v_a^2(1-\rho^2)\}$
η^{31}:	$\frac{1}{2} r\beta^3 \{A_1(v_i + \rho v_a)^2 + v_a^2(1-\rho^2)\}$	$\frac{1}{2} r\beta^3 \{v_a^2(1-\rho^2)\}$
η^{23}:	$\frac{1}{2} r\beta^3 \{A_2(v_a + \rho v_i)^2 + v_i^2(1-\rho^2)\}$	$\frac{1}{2} r\beta^3 \{A_2[A_2\rho^2 + (1-\rho^2)]^{-1} v_a^2\}$
η^{22}:	$\frac{1}{2} r\beta^3 \{A_2(v_a^2 + 2\rho v_a v_i + v_i^2)\}$	$\frac{1}{2} r\beta^3 \{A_2 v_a^2(1-\rho^2)\}$
η^{21}:	$\frac{1}{2} r\beta^3 \{A_1(v_i + \rho v_a)^2 + A_2 v_a^2(1-\rho^2)\}$	$\frac{1}{2} r\beta^3 \{A_2 v_a^2(1-\rho^2)\}$
η^{13}:	$\frac{1}{2} r\beta^3 \{A_1(v_a + \rho v_i)^2 + v_i^2(1-\rho^2)\}$	$\frac{1}{2} r\beta^3 \{A_1[A_1\rho^2 + (1-\rho^2)]^{-1} v_a^2\}$
η^{12}:	$\frac{1}{2} r\beta^3 \{A_1(v_a + \rho v_i)^2 + A_2 v_i^2(1-\rho^2)\}$	$\frac{1}{2} r\beta^3 \{A_1[(A_1/A_2)\rho^2 + (1-\rho^2)]^{-1} v_a^2\}$
η^{11}:	$\frac{1}{2} r\beta^3 \{A_1(v_a^2 + 2\rho v_a v_i + v_i^2)\}$	$\frac{1}{2} r\beta^3 \{A_1 v_a^2(1-\rho^2)\}$

The optimal insurance ratio is $v_i/v_a = -\rho$ for all systems with $t_i \leq t_a$, whereas with $t_i > t_a$ it is $v_i/v_a = -\rho A_{t_a}[A_{t_a}\rho + A_{t_i}(1-\rho^2)]^{-1}$.

Impact of Report Timing with TA Preferences and Optimal Insurance Rates

Consider system η^{21}. If the incentive rates are such that $v_i = -\rho v_a$, then the insurance informative report y_i reported at $t = 1$ does not resolve any uncertainty about the agent's incentive wages, i.e., $\text{Var}_0^{21}[E_1^{21}[W_3]] = 0$. This is due to the fact that in this case $W_3 = v_a(y_a - \rho y_i)$, and the fact that y_i and $y_a - \rho y_i$ are independent, i.e, $\text{Cov}_0(y_i, W_3) = 0$. Hence, the agent's *ex ante* risk premium is the same whether y_i is reported at $t = 1$ or $t = 2$. In fact, in this case the contract is as if it is written strictly in terms of the second of two stochastically independent sufficient performance statistics (see Section 27.2.1), i.e., $\chi_1 \equiv y_i$ and $\chi_2 \equiv y_a - \rho y_i$. Similarly, if the incentive rates are such that $v_a = -\rho v_i$, there is again no difference in the agent's *ex ante* risk premium whether y_a is reported before or at the same date as y_i. In this case, the two stochastically independent sufficient performance statistics are $\chi_1 \equiv y_a$ and $\chi_2 \equiv y_i - \rho y_a$. The key difference between the two settings is that in the former the first statistic is neither insurance informative about the second statistic nor action informative, whereas both statistics are action informative in the latter. This implies that it is optimal to set $v_i = -\rho v_a$ in the former setting (since $\chi_1 \equiv y_i$ is pure noise), whereas $v_a = -\rho v_i$ will not be optimal in the latter (since it is optimal to use non-zero incentive rates on both action informative statistics).

Panel B of Table 26.1 summarizes the compensation risk premium for each of the reporting alternatives when the insurance rate v_i is chosen optimally given an exogenous incentive rate v_a. The results are striking. If the pure insurance information y_i is reported *no later than* the action informative report, then $v_i = -\rho v_a$ and the timing of the insurance report is irrelevant. On the other hand, the compensation risk premium is greater if the action informative report is delayed, i.e., it is reported at date 2 or 3 instead of date 1. Furthermore, the compensation risk premium is greater the further the insurance informative report is delayed beyond the action informative report date (e.g., $RP_0^{TA}(\eta^{13}) > RP_0^{TA}(\eta^{12}) > RP_0^{TA}(\eta^{11})$).

Of course, the key to these results is that although y_i is not informative about the agent's action, it is informative about the noise in the action informative report. Hence, y_i is strictly used to remove noise in the action informative report, and the uninsurable noise is $\varepsilon_a - \rho\,\varepsilon_i$ (compare to Proposition 20.5 for the comparable result in a single-period setting). The insurance informative report is not informative about the uninsurable noise and, hence, it is not valuable to have that information early. However, if y_i is reported after y_a, then the agent cannot distinguish between the insurable and uninsurable components of y_a when it is reported. In that case, the agent's consumption choice based on y_a is affected by insurable noise and, therefore, his consumption smoothing is less efficient – the delay of insurance informative information is costly.

Comparative Statics

We now hold the reporting system constant (using η^{11}) and consider the impact of the level of correlation. We use the fact that it is optimal to set $v_i = -\rho v_a$, and from (26.5) the induced action is

$$a = \beta^2 v_a M_a, \tag{26.10}$$

where the discount factor reflects the fact that the effort cost is in date 1 dollars and the incentive rate is in date 3 dollars. The principal's objective is to select the incentive rate v_a so as to maximize

$$\begin{aligned} U^P(v_a,\eta^{11}) &= \beta[ba - \tfrac{1}{2}a^2] - \tfrac{1}{2}r\beta^3 A_1 v_a^2(1-\rho^2) \\ &= \beta[b\beta^2 v_a M_a - \tfrac{1}{2}(\beta^2 v_a M_a)^2] - \tfrac{1}{2}r\beta^5 A_1 v_a^2(1-\rho^2). \qquad (26.11) \end{aligned}$$

The first-order condition yields

$$v_a = \frac{bM_a}{\beta^2[M_a^2 + rA_1(1-\rho^2)]}, \qquad (26.12)$$

and substituting (26.12) into (26.11) provides

$$U^P(v_a,\eta^{11}) = \beta\frac{1}{2}\,\frac{b^2 M_a^2}{M_a^2 + rA_1(1-\rho^2)}. \qquad (26.13)$$

Note that the expected net payoff is smaller for $\eta^{2\tau}$ and $\eta^{3\tau}$, for all τ.

It follows immediately from (26.13) that the principal's optimal expected net payoff is increasing in his payoff b per unit of agent effort, the sensitivity M_a of the performance measure per unit of agent effort, and the square of the correlation ρ^2 between the action informative and insurance informative report. None of these comparative statics are surprising. The value of the insurance report is zero if it is uncorrelated with the action informative report, and its usefulness in "removing" incentive risk is the same for positive and negative correlation – there is merely a difference in the sign of the insurance rate.

Timeliness versus Precision

There is often a trade-off between obtaining an earlier report and the preciseness of that report. In our model, in which reports have unit variance, the preciseness of an action informative report is represented by its sensitivity to the agent's action. To illustrate this trade-off, we consider a single-action setting in which the principal chooses between systems η^1 and η^2, which generate action informative reports $y_1 = M_1 a + \varepsilon_1$ and $y_2 = M_2 a + \varepsilon_2$, at dates t_1 and $t_2 > t_1$, respectively. The systems have the same cost, but $M_1 < M_2$, i.e., the first is less precise than the second.

Let v_1 and v_2 represent the incentive rates for the two systems, measured in date $T = t_2$ dollars. Hence, the *ex ante* risk premium for system $\eta^j, j = 1, 2,$ and preferences $i = AC, TA$, are $RP_0^i(\eta^j) = \tfrac{1}{2}\beta^{2T-\tau}\hat{r}_\tau^{\,i} v_j^2$, where $\tau = t_j$ is the report date.

The principal's gross payoff per unit of effort and the agent's effort cost are b and $\frac{1}{2}a^2$, measured in date 1 dollars. Consequently, given v_j, the induced action with system η^j is $a^j = \beta^{T-1}M_j v_j$. The optimal incentive rate and optimal net payoff to the principal, for system $\eta^j, j = 1, 2$, and agent preferences $i = TA, AC$, are

$$v_j^i = bM_j[\beta^T(RM_j^2 + R^\tau \hat{r}_\tau^{\,i})]^{-1},$$

$$U^{pi}(\eta^\tau) = \tfrac{1}{2}(bM_j)^2[RM_j^2 + R^\tau \hat{r}_\tau^{\,i}]^{-1}.$$

Given our earlier results, it is not surprising that, given $M_1 < M_2$,

$$U^{pAC}(\eta^2) = \frac{1}{2}\frac{b^2M_2^2}{RM_2^2 + r} > U^{pAC}(\eta^1) = \frac{1}{2}\frac{b^2M_1^2}{RM_1^2 + r}.$$

That is, with *AC* preferences, report timing is immaterial and, hence, the principal strictly prefers the later system if it generates a more precise (i.e., sensitive) report.

On the other hand, with *TA* preferences, *ceteris paribus*, earlier action informative reports are preferred to later reports to facilitate consumption smoothing, but more precise reports are preferred to less precise reports because of the reduced risk premium. More specifically,

$$U^{pTA}(\eta^2) - U^{pTA}(\eta^1) = \frac{1}{2}\frac{b^2M_2^2}{RM_2^2 + rR^{t_2}A_{t_2}} - \frac{1}{2}\frac{b^2M_1^2}{RM_1^2 + rR^{t_1}A_{t_1}} > 0,$$

if, and only if, $$\frac{M_2^2}{M_1^2} > R^{t_2 - t_1}\frac{A_{t_2}}{A_{t_1}}.$$

The preceding analysis compares an early, less precise report to a later, more precise report. A related question is whether there is value to having both reports. In particular, is it valuable to issue a preliminary report even though it contains measurement errors or estimates that create noise in the first report which will be corrected in the second? We do not formally analyze this setting, but CFHS establish that it will be valuable with both *AC* and *TA* agent preferences to have both reports if y_2 is not a sufficient statistic for (y_1,y_2) with respect to a (i.e., $M_1 \neq \rho M_2$). However, if $M_1 = \rho M_2$, then the first report has no incremental value if the agent has *AC* preferences, but has positive incremental value if he has *TA* preferences. Again, the key to these results is that the early report facilitates additional consumption smoothing if the agent has *TA* preferences, but there is no value to consumption smoothing if he has *AC* preferences.

26.1.4 Multiple Actions and Consumption Dates

We now examine some settings with multiple actions, multiple consumption dates, and multiple performance reports. The reports have correlated noise, so that all reports potentially play an insurance role with respect to the noise in the other reports. We assume that a single agent is hired for two periods and provides productive effort, a_1 and a_2, in periods 1 and 2, respectively. The expected gross payoffs to the principal from the agent's actions are $b_1 a_1$ and $b_2 a_2$, measured in date 1 and date 2 dollars, respectively.

There are two action informative reports, $y_1 = M_{11} a_1 + \varepsilon_1$ and $y_2 = M_{22} a_2 + \varepsilon_2$, where the noise terms ε_1 and ε_2 have zero means and unit variances with $\text{Cov}[\varepsilon_1, \varepsilon_2] = \rho$. We consider three reporting systems.

Interim reporting (η^{12}): y_1 and y_2 are issued at dates 1 and 2, respectively.

Disaggregate terminal reporting (η^{22}): both y_1 and y_2 are issued at date 2.

Aggregate terminal reporting (η^{2}): aggregate report $y = y_1 + y_2$ is issued at date 2.

Our analysis of terminal reporting can be viewed as representative of settings in which the agent takes a sequence of actions between reports. Accounting reports for a month or a year often provide only summary data when issued, although a monthly report could contain daily or weekly details.

Table 26.2 summarizes the agent's *ex ante* certainty equivalent and risk premium, given preferences $i = TA, AC$, his induced actions a_1 and a_2 given the incentive rates v_1 and v_2, the principal's choice of incentive rates, and his optimal expected net payoff for each of the three reporting systems. Note that the incentive compensation, $v_1 y_1$ and $v_2 y_2$, is expressed in date 2 dollars for all cases. Hence, if the first report is issued at date 1 and the incentive compensation is paid at that time, then the incentive compensation in date 1 dollars is $\beta v_1 y_1$. On the other hand, since the timing of the actions is held constant, we assume that the expected gross payoffs, $b_1 a_1$ and $b_2 a_2$, as well as the agent's corresponding personal costs, $\frac{1}{2} a_1^2$ and $\frac{1}{2} a_2^2$, are expressed in date 1 and date 2 dollars.

The agent's certainty equivalent reflects his expected compensation, his cost of effort, and his risk premium, which is influenced by the correlation between the two components of the agent's incentive compensation. The risk premium is not influenced by the action choices, so that the agent's choice of a_1 and a_2 depends only on v_1 and v_2, respectively. There is a time-value adjustment since the effort cost and resulting expected compensation are measured in different dollars. Let the weighted average of the agent's date-specific risk aversion parameters be defined as $\bar{r}^i \equiv \beta\hat{r}_1^i\rho^2 + \hat{r}_2^i(1-\rho^2)$, for i = *AC*, *TA*.

Recall that with *AC* preferences $\hat{r}_1^{AC} = r\beta$ and $\hat{r}_2^{AC} = r\beta^2$. Hence, $\beta\hat{r}_1^{AC} = \hat{r}_2^{AC} = \bar{r}^{AC}$ are equal, which implies that the actions, incentive rates, and the principal's expected net payoff are *precisely the same* for interim (η^{12}) and disaggregate terminal reporting (η^{22}). This, of course, is merely another illustration of the fact that the timing of reports is irrelevant if the agent has *AC* preferences. The forms of the various elements are almost identical for η^{22} and η^2, but the latter problem is constrained to apply the same incentive rate to the two performance reports. Hence, the latter cannot be greater than, and may be strictly less than, the former.

While report timing does not matter with *AC* preferences, it does matter with *TA* preferences. In that case,

$$\hat{r}_1^{TA} = r[1+\beta]^{-1} < \bar{r}^{TA} < \hat{r}_2^{TA} = r.$$

Hence, interim reporting strictly dominates disaggregate terminal reporting if there is non-zero correlation. Of course, for the reasons discussed above, disaggregate reporting dominates aggregate reporting.

TABLE 26.2
Multiple Actions and Consumption Dates

Agent's *Ex Ante* Certainty Equivalent and Action Choices:

$$CE_0^i(\eta) = \beta[\beta v_1 M_{11} a_1 - \tfrac{1}{2}a_1^2] + \beta^2[v_2 M_{22} a_2 - \tfrac{1}{2}a_2^2] - RP_0^i(\eta),$$

$$a_1 = \beta v_1 M_{11}, \quad a_2 = v_2 M_{22}.$$

Principal's Optimal Expected Net Payoff:

$$U^{pi}(\eta) = \beta[b_1 a_1 - \tfrac{1}{2}a_1^2] + \beta^2[b_2 a_2 - \tfrac{1}{2}a_2^2] - RP_0^i(\eta)$$

$$= \tfrac{1}{2}[\beta v_1(\eta) b_1 M_{11} + \beta^2 v_2(\eta) b_2 M_{22}].$$

Risk Premia and Optimal Incentive Choice:

$$\eta^{12}: \quad RP_0^i(\eta^{12}) = \tfrac{1}{2}[\beta^2\hat{r}_1^i\beta(v_1 + \rho v_2)^2 + \beta^2\hat{r}_2^i v_2^2(1-\rho^2)],$$

$$v_1^i(\eta^{12}) = \left\{ \frac{Rb_1 M_{11}}{(M_{11}^2 + \hat{r}_1^i)} - \frac{\rho\, b_2 M_{22} \hat{r}_1^i}{(M_{11}^2 + \hat{r}_1^i)(M_{22}^2 + \bar{r}^i)} \right\} \left[1 - \frac{\beta(\rho\, \hat{r}_1^i)^2}{(M_{11}^2 + \hat{r}_1^i)(M_{22}^2 + \bar{r}^i)} \right]^{-1},$$

$$v_2^i(\eta^{12}) = \left\{ \frac{b_2 M_{22}}{M_{22}^2 + \bar{r}^i} - \frac{\rho\, b_1 M_{11} \hat{r}_1^i}{(M_{11}^2 + \hat{r}_1^i)(M_{22}^2 + \bar{r}^i)} \right\} \left[1 - \frac{\beta(\rho\, \hat{r}_1^i)^2}{(M_{11}^2 + \hat{r}_1^i)(M_{22}^2 + \bar{r}^i)} \right]^{-1}.$$

$$\eta^{22}: \quad RP_0^i(\eta^{22}) = \tfrac{1}{2}\hat{r}_2^i \beta^2 [v_1 + 2\rho v_1 v_2 + v_2^2],$$

$$v_1^i(\eta^{22}) = \left\{ \frac{b_1 M_{11}}{(\beta M_{11}^2 + \hat{r}_2^i)} - \frac{\rho\, b_2 M_{22} \hat{r}_2^i}{(\beta M_{11}^2 + \hat{r}_2^i)(M_{22}^2 + \hat{r}_2^i)} \right\} \left[1 - \frac{(\rho \beta\, \hat{r}_1^i)^2}{(\beta M_{11}^2 + \hat{r}_2^i)(M_{22}^2 + \hat{r}_2^i)} \right]^{-1},$$

$$v_2^i(\eta^{22}) = \left\{ \frac{b_2 M_{22}}{M_{22}^2 + \hat{r}_2^i} - \frac{\rho\, b_1 M_{11} \hat{r}_2^i}{(\beta M_{11}^2 + \hat{r}_2^i)(M_{22}^2 + \hat{r}_2^i)} \right\} \left[1 - \frac{(\rho \beta\, \hat{r}_1^i)^2}{(\beta M_{11}^2 + \hat{r}_2^i)(M_{22}^2 + \hat{r}_2^i)} \right]^{-1}.$$

$$\eta^2: \quad RP_0^i(\eta^2) = \tfrac{1}{2}\beta^2 \hat{r}_2^i v^2 2(1+\rho), \qquad v^i(\eta^2) = \frac{b_1 M_{11} + b_2 M_{22}}{\beta M_{11}^2 + M_{22}^2 + \hat{r}_2^i 2(1+\rho)}.$$

The Impact of Correlation

Figure 26.1 illustrates the impact of the correlation ρ on the principal's expected net payoff for the three reporting systems, given that the agent has *TA* preferences. In each example, $M_{11} = M_{22} = M = 1$ and $r = 1$. The key differences are with respect to the diversity of the principal's gross payoffs. In graph (a) the payoffs are identical, with $b_1 = b_2 = b = 10$, and all amounts are measured in the same dollars, so that the interest rate is zero (i.e., $R = \beta = 1$). We refer to this as the *identical periods case*.[8] At the other extreme is graph (c) in which only the first action is productive, with $b_1 = 20$ and $b_2 = 0$. We assume the interest rate is positive, with $R = 1.10$, but the key characteristic is that the second action influences one of the performance measures but is not productive.[9] As in Feltham and Xie (1994), and Chapter 20, we refer to this as the *window dressing* case. Graph (b) is an *intermediate case* in which $b_1 = 15$ and $b_2 = 5$, with $R = 1.10$.

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[8] The periods are *nominally identical* if $b_1 = b_2 = b$ and $R > 1$.

[9] We assume that the manager is hired for two periods even though his actions in one of the periods have no incremental impact on the principal's payoff.

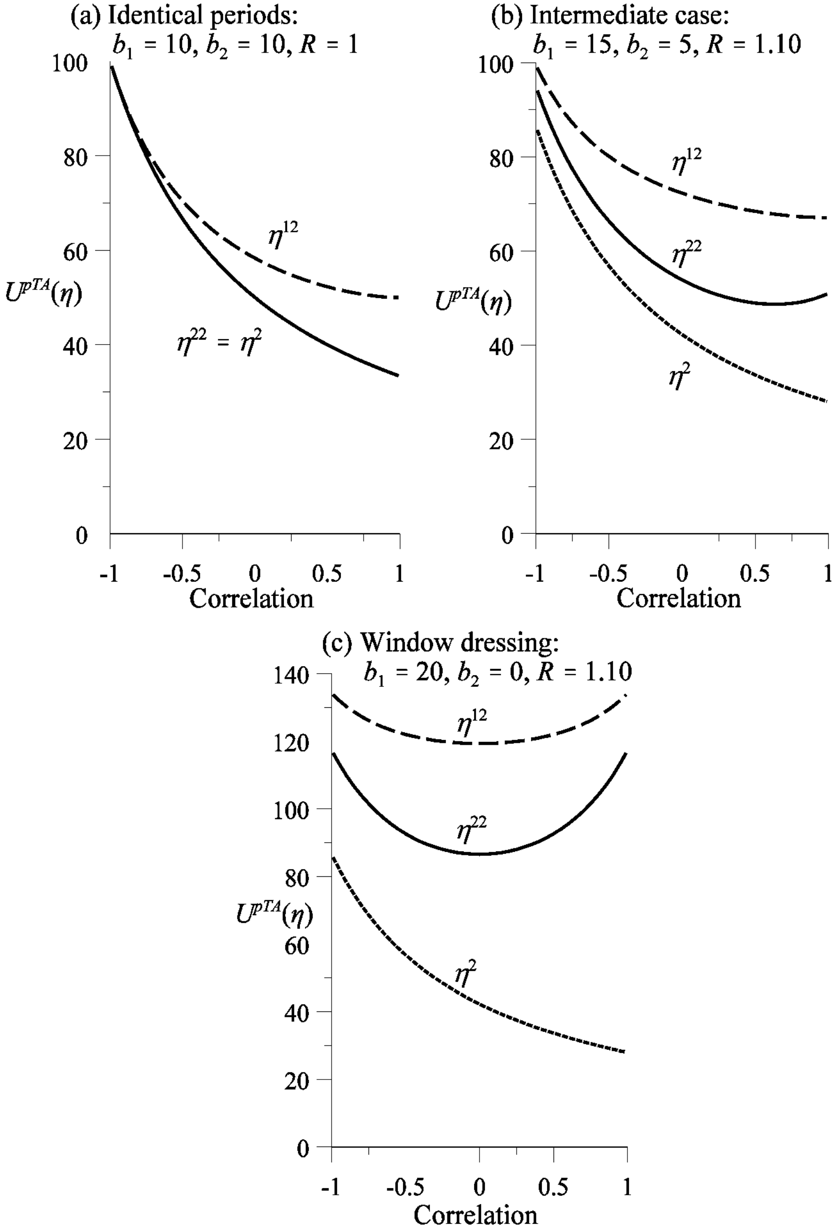

Figure 26.1: Impact of performance measure correlation with interim (η^{12}), disaggregate (η^{22}), and aggregate (η^{2}) reporting.

The following observations are noteworthy. First, interim reporting dominates both disaggregate and aggregate terminal reporting in all three graphs. This reflects the fact that, with *TA* preferences, interim reporting facilitates greater consumption smoothing.

Second, disaggregate terminal reporting dominates aggregate terminal reporting in graphs (b) and (c), but not in (a). Aggregate reporting constrains the incentive rates for the two reports to be equal, whereas disaggregate reporting does not. The equality constraint is not binding in the identical periods case, so that aggregate reporting achieves the same payoff as disaggregate reporting. However, in graphs (b) and (c) the equality constraint is binding, i.e., it is optimal to set different incentive rates for the two reports. In effect, the single aggregate performance measure is *congruent* with the principal's preferences in (a), but not in (b) and (c).

Third, the payoff from the aggregate reporting system is monotonically decreasing in ρ in all three graphs. However, while this also applies to the other two systems in graph (a) and for interim reporting in graph (b), it does not apply to disaggregate reporting in either graphs (b) or (c), or to interim reporting in graph (c). With aggregate reporting the single report is used exclusively for providing effort incentives – there is no insurance. Furthermore, the variance of the NPV of compensation is increasing in the correlation. This implies that the agent's risk premium, which must be paid by the principal, is increasing in ρ. However, when there are two separate reports, y_1 can be used to provide incentives for the agent's choice of action a_1 and insurance for the incentive risk associated with y_2. Conversely, y_2 can be used to provide incentives for the agent's choice of action a_2 and insurance for the incentive risk associated with y_1. The insurance roles are enhanced by the informativeness of one report with respect to the other, as represented by ρ^2. In graph (c), the second action is not productive, so that y_1 only plays an incentive role, while y_2 only plays an insurance role. Hence, the payoffs for interim and disaggregate reporting are "U" shaped in that graph.

In graph (b), the second action provides a positive, but relatively small, benefit to the principal. Hence, while y_2 has an incentive role, it is small relative to its insurance role. These two roles are complementary for negative values of ρ, resulting in a payoff that is decreasing in ρ for $\rho < 0$. On the other hand, the two roles are conflicting for positive values of ρ. For disaggregate terminal reporting in graph (b) it is optimal to set $v_2 < 0$ for $\rho > .65$, to obtain the insurance benefit (at the expense of forgoing the incentive benefit). Hence, the payoff is increasing in ρ for $\rho \in (.65,1)$.

26.2 TWO AGENTS VERSUS ONE

In Section 25.5 we considered whether the principal would prefer to hire one or two agents. Proposition 25.13 establishes that, with independence, the principal is indifferent between hiring one or two agents if the agents have *AC-EC* or *TA-EC* preferences. Hence, in our two-period *LEN* model, with either *AC* or *TA* preferences, the principal is indifferent between hiring one or two agents if ρ = 0. We now consider the optimal contract and the principal's optimal net payoff if the principal contracts with two agents using the interim reporting system η^{12} (from the preceding section) in settings in which $\rho \neq 0$. Then we compare those results to the optimal single-agent contract discussed above.[10]

Contracting with Two Agents Using Interim Reporting

Recall that the interim reporting system η^{12} issues y_1 at date 1 and y_2 at date 2. Given full commitment, we assume that the principal can contract with both agents at date 0. The first agent is hired to work for the principal in the first period and then leave, while the second agent is hired to work in the second period and can work elsewhere in the first period. The compensation for both agents can depend on y_1 and y_2, and it can be paid by the principal at dates 0, 1, or 2 (of course, any report contingent payment cannot be made until after the report is issued).

Both reports have an incentive role and an insurance role. If two agents are hired, then these roles can be separated. That is, y_1 can be used to motivate the first agent's choice of a_1 and to provide insurance for the second agent's incentive risk, while y_2 can be used to motivate the second agent's action choice and to provide insurance for the first agent's incentive risk.

More specifically, in the two-agent case, the ℓ^{th} agent, $\ell = 1, 2$, takes action a_ℓ in period ℓ and receives a fixed wage $f_{\ell 0}$ at $t = 0$ and incentive compensation $s_\ell = v_{\ell 1} y_1 + v_{\ell 2} y_2$ at $t = 2$. Table 26.3 presents the agents' certainty equivalents, their action choices, the principal's optimal incentive contracts for the two agents, and the principal's expected net payoff.

If the agents have *AC* preferences, then $\beta \hat{r}_1^{AC} = \hat{r}_2^{AC} = \bar{r}^{AC} = \beta^2 r$. Consequently, the relative insurance rates are $v_{12}/v_{11} = v_{21}/v_{22} = -\rho$. That is, they equal the negative of the correlation, which is the standard single-period result for a setting in which one of the performance measures is purely insurance informative. Furthermore, as we have seen in our prior analysis, the timing of reports is irrelevant with *AC* preferences.

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[10] We focus on interim reporting since that system provides agent-specific reports if two agents are hired.

TABLE 26.3
Two Actions and Two Agents

***Ex Ante* Certainty Equivalents and Risk Premia:**

$$\ell = 1: \quad CE^i_{10}(\eta) = \beta^2 v_{11} M_{11} a_1 + \beta^2 v_{12} M_{22}\hat{a}_2 - \beta\tfrac{1}{2}a_1^2 - RP^i_{10}(\eta),$$

$$\ell = 2: \quad CE^i_{20}(\eta) = \beta^2 v_{21} M_{11}\hat{a}_1 + \beta^2 v_{22} M_{22} a_2 - \beta^2\tfrac{1}{2}a_2^2 - RP^i_{20}(\eta),$$

$$\text{where} \quad RP^i_{\ell 0}(\eta^{12}) = \tfrac{1}{2}[\beta^3 \hat{r}^i_1 (v_{\ell 1} + \rho v_{\ell 2})^2 + \beta^2 \hat{r}^i_2 v^2_{\ell 2}(1-\rho^2)].$$

Action Choices: $a_1 = \beta v_{11} M_{11}, \quad a_2 = v_{22} M_{22}.$

Principal's Optimal Expected Net Payoff:

$$U^{2p}(\eta) = \beta[b_1 a_1 - \tfrac{1}{2}a_1^2] + \beta^2[b_2 a_2 - \tfrac{1}{2}a_2^2] - RP^i_{10}(\eta) - RP^i_{20}(\eta).$$

$$U^{2p}(\eta^{12}) = \tfrac{1}{2}[\beta v_{11}(\eta_{12}) b_1 m_{11} + \beta^2 v_{22}(\eta_{12}) b_2 m_{22}].$$

Optimal Incentive and Insurance Choices:

$$v_{11}(\eta^{12}) = \frac{1}{D_1} b_1 M_{11}, \qquad v_{12}(\eta^{12})/v_{11}(\eta^{12}) = -\rho\beta\hat{r}^i_1/\bar{r}^i,$$

$$D_1 \equiv \beta M^2_{11} + \hat{r}^i_2(1-\rho^2)\beta\hat{r}^i_1/\bar{r}^i,$$

$$v_{22}(\eta^{12}) = \frac{1}{D_2} b_2 M_{22}, \qquad v_{21}(\eta^{12})/v_{22}(\eta^{12}) = -\rho,$$

$$D_2 \equiv M^2_{22} + \hat{r}^i_2(1-\rho^2).$$

If the agents have TA preferences, then $\beta\hat{r}_1^{TA} \le \bar{r}^{TA} \le \hat{r}_2^{TA}$, with strict inequalities if $\rho \neq 0$ or $\rho \neq \pm 1$, respectively. Consequently, if $\rho \neq 0$, the absolute value of the first agent's insurance ratio $|v_{12}/v_{22}|$ is less than $|-\rho|$, whereas the second agent's insurance ratio v_{21}/v_{22} equals $-\rho$. Hence, the two contracts are not identical even if the payoffs, effort costs, and performance measure sensitivities are identical. The key difference is that the first agent's insurance informative report is issued after his action informative report has been issued, whereas the reverse applies to the second agent. Hence, the first agent receives his action informative report at date 1 and can therefore smooth his incentive compensation over two dates (although he cannot distinguish between the insurable and uninsurable components of y_1 at date 1). The second agent's incentive risk consists only of the uninsurable component of his action informative report, i.e., $y_2 - \rho y_1$, and y_1 is not informative about this risk. Hence, the second agent cannot smooth incentive risk over the two dates.[11]

One or Two Agents?

Figure 26.2 extends the example in Figure 26.1 to demonstrate how the correlation ρ affects the principal's expected net payoff from hiring two agents instead of one, when there is interim reporting and the agents have *TA* preferences.[12] As in Figure 26.1, $M_{11} = M_{22} = M = 1$ and $r = 1$, and the three graphs differ in their diversity of payoffs to the principal: (a) $b_1 = b_2 = b = 10$ and $R = 1$; (b) $b_1 = 15$, $b_2 = 5$, and $R = 1.10$; and (c) $b_1 = 20$, $b_2 = 0$, and $R = 1.10$.

As it appears from Figure 26.1, the principal's optimal expected net payoff from the single-agent contract is monotonically decreasing with the correlation ρ. This also applies to the two-agent contract for negative values of ρ, but changes radically for positive values of ρ. In fact, in the two-agent case, we see that the principal's optimal expected net payoff is "U" shaped and can be described as increasing with ρ^2, which is a measure of the informativeness of one report with respect to the other. This latter result follows from the fact that, with two agents, if the correlation is positive, v_{12} and v_{21} can be given negative values so as to provide insurance for the incentive risk created by positive incentive values for v_{11} and v_{22}. Hence, it is not surprising that contracting with two agents dominates contracting with a single agent if the correlation is positive, and that the benefit from doing so increases as ρ gets closer to one.

Proposition 25.13 demonstrates for optimal contracts that the principal is indifferent between contracting with one or two agents if the periods are independent and the agents have either *AC-EC* or *TA-EC* exponential preferences. This result also applies to our *LEN* model. To see this, observe that if $\rho = 0$, then we obtain from Tables 26.2 and 26.3

$$v_1(\eta^{12}) = v_{11}(\eta^{12}) = \frac{R\, b_1 M_{11}}{M_{11}^2 + r_1^i}, \qquad v_{12}(\eta^{12}) = 0,$$

$$v_2(\eta^{12}) = v_{22}(\eta^{12}) = \frac{b_2 M_{22}}{M_{22}^2 + r_2^i}, \qquad v_{21}(\eta^{12}) = 0,$$

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[11] Compare to the results in Table 26.1.

[12] The graphs for *AC* preferences are very similar.

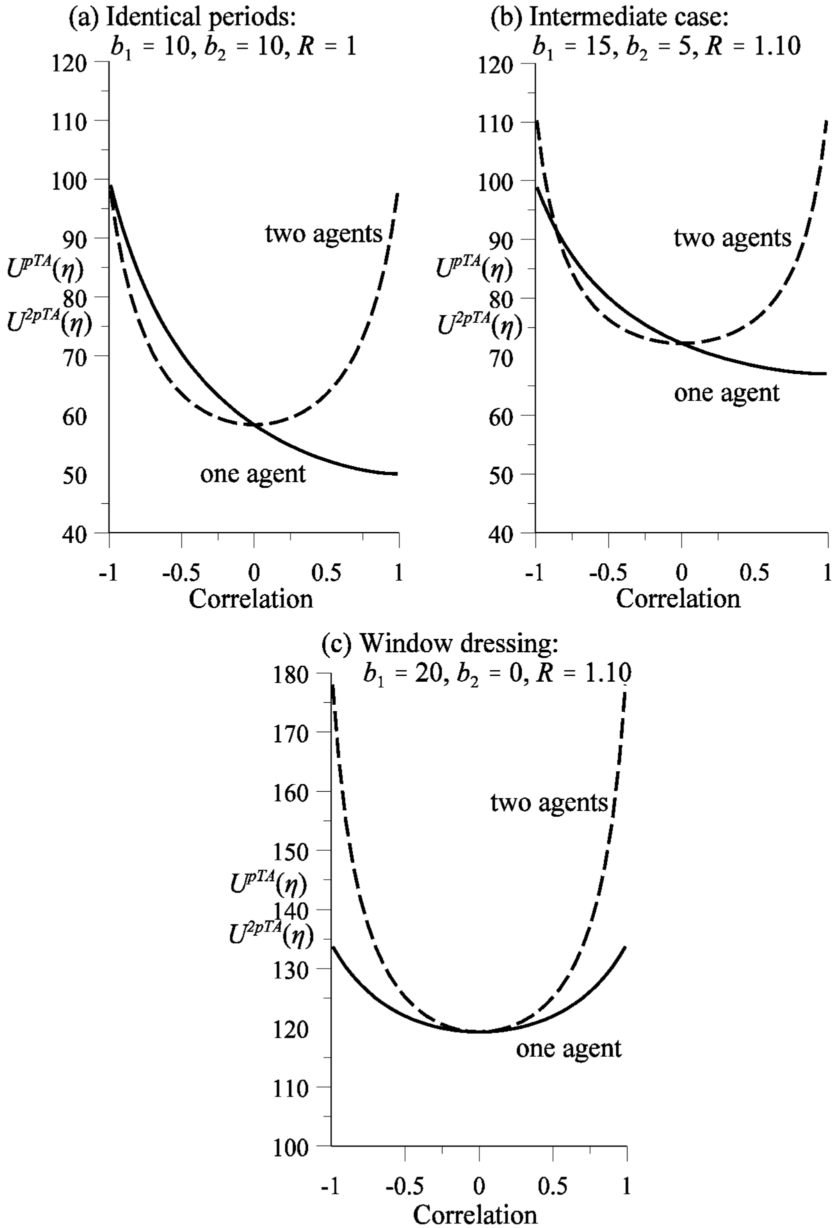

Figure 26.2: Two agents versus a single agent with interim reporting (η^{12}).

and $$U^p(\eta^{12}) = U^{2p}(\eta^{12}) = ½\left[\beta\frac{R\,b_1^2 M_{11}^2}{M_{11}^2 + r_1^i} + \beta^2\frac{b_2^2 M_{22}^2}{M_{22}^2 + r_2^i}\right].$$

Hence, $U^p(\eta^{12})$ and $U^{2p}(\eta^{12})$ either intersect or are tangent at $\rho = 0$. As illustrated by graph (c), tangency occurs in the window dressing case. The following summarizes the results illustrated by the graphs in Figure 26.2.[13]

Proposition 26.2

With interim reporting, there exists a cutoff $\hat{\rho}$ such that the principal's optimal expected net payoff $U^p(\eta^{12})$, from hiring one agent for both periods, is strictly greater than the optimal expected net payoff $U^{2p}(\eta^{12})$, from hiring two agents (one for each period) if, and only if, $\rho \in (\hat{\rho},0)$, where

(a) $\hat{\rho} = -1$ in the identical periods case,[14]

(b) $\hat{\rho} \in (-1,0)$ in the intermediate case,

(c) $\hat{\rho} = 0$ in the window dressing case.

When the correlation is negative and contracting with one agent is strictly preferred to contracting with two agents, the benefit derives from the fact that the insurance for the risk associated with one performance measure can be treated as a "free" by-product of the effort incentives associated with the other performance measure. With two agents, each performance measure must be used twice, and, if there is negative correlation, the two-agent contract only dominates the single-agent contract if the correlation is very negative and both agents are positively, but differentially productive.

In case (c), it is optimal to hire two agents but provide no incentives for the second agent. This avoids the cost of window dressing while still using y_2 to insure the first agent against his first-period incentive risk.

In case (b), the benefits of hiring a single agent when there is negative correlation are similar to case (a). However, since the periods are not identical, there is a set of very negative values of ρ for which it is optimal to hire two agents so that the second agent (who is the least productive agent) can be given a small incentive commensurate with his productivity (i.e., by setting v_{22} small)

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[13] Although Figure 26.2 is based on exponential *TA-EC*, the same results holds for exponential *AC-EC* preferences.

[14] Christensen, Feltham, and Şabac (2003) examine this case and demonstrate that it results in a preference for two agents if the reports are positively correlated and a preference for one agent if the reports are negatively correlated.

and yet the first agent is appropriately insured (by setting v_{12} large if ρ is very negative).

A Caveat

The preceding analysis assumes that only linear contracts are feasible, and shows how contracting with two agents may facilitate separation between the incentive and insurance roles of correlated reports. In Chapter 27 we consider single-agent contracts in which the second-period incentive rate is a linear function of the first-period performance report. This leads to random variations in the agent's certainty equivalent at the end of the first period, and provides indirect first-period effort incentives due to the correlated reports. With this type of contract, positive correlation strengthens the indirect incentives and, of course, these indirect first-period incentives are only obtained, if a single agent is hired for both years. In Chapter 28, we consider inter-period renegotiation of contracts. In that setting, we show that it may be impossible to sustain a second-period incentive rate that depends on the first-period report. Hence, in that setting, the choice of one versus two agents is similar to the analysis above (see Section 28.4).

26.3 CONCLUDING REMARKS

Action-informative performance measures can only be reported after the action has been taken. Incentives based on those measures derive from the fact that the agent anticipates the impact of his actions on the future performance measures and the resulting compensation. From an action incentive perspective, the timing of the report is immaterial, i.e., delays in reporting do not affect the action incentives. However, in a multi-period setting, the agent chooses the timing of his consumption as well as his action choices, and the timing of the performance reports will affect the extent to which he can smooth consumption. This is not relevant if the agent has *AC-EC* preferences, but delays in reporting can be costly if the agent has *TA-EC* preferences.

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CHAPTER 27

FULL COMMITMENT CONTRACTS WITH INTERDEPENDENT PERIODS

This is the third of four chapters that examine multi-period principal-agent models. As in Chapters 25 and 26, we assume the principal and the agent can commit to a long-term contract without subsequent renegotiation. In this chapter, as in Chapter 26, we relax the Chapter 25 assumptions that the performance reports are stochastically and technologically independent. The key innovations pertain to the exploration of the impact of transforming performance measures to achieve stochastic independence, characterization of optimal non-linear contracts, creation of indirect covariance incentives by allowing the second-period incentive rates to vary with the first-period performance reports, the use of effort cost risk insurance and risk-premium risk insurance, and the consideration of productivity information.

We begin in Section 27.1 by examining some basic issues in sequential choice. To explore these issues, in Section 27.1.1 we formulate a two-period model that is a special case of the basic model introduced in Section 25.1. This model is less general than the basic model, but it is sufficiently general to encompass both stochastic and technological interdependence. A key point in this section is that one must be careful in specifying the incentive compatibility constraints when the agent makes sequential choices. Of particular concern is the potential for "double shirking," which refers to the agent's strategy in the second period if he deviates from the planned action in the first period. The deviation takes him "off the equilibrium path," and, to be a sequential equilibrium, the incentive constraints must be such that they reflect his rational response if he finds himself on that path.

Section 27.1.2 briefly describes three special cases in which there is stochastic interdependence, so that the first-period reports are informative about both the first-period action and about future random events. The three types of random events are: additive noise, payoff productivity, and performance productivity.

Chapter 26 examines the correlated additive noise case within a *LEN* model. Section 27.2 introduces transformations of the normally distributed performance measures such that the revised representations continue to be normally distributed, but are stochastically independent. The revised measures are referred to as *stochastically independent sufficient performance statistics*. In Section 27.2.1, the transformation merely orthogonalizes the noise terms, whereas in

Section 27.2.2 the transformation normalizes the statistics so that they have zero means. While creating stochastically independent statistics can simplify the analysis, the transformation generally creates technological interdependence. As illustrated using a simple two-period *LEN* model in Section 27.2.1, orthogonalizing two technologically independent, stochastically correlated measures produces two stochastically independent but technologically interdependent performance statistics. If the linear contract is expressed in terms of the original measures, then the induced first-period action depends entirely on the first-period incentive rate. However, with the statistics, the induced first-period action depends directly on the first-period incentive rate and indirectly on the second-period incentive rate.

Section 27.2.1 examines two examples. The first is an auto-regressive process that is technologically and stochastically interdependent. It is noteworthy that, in this case, orthogonalization provides statistics that are both stochastically and technologically independent. The second example is a stock price process, for which the orthogonalized statistics are excess returns. These returns are stochastically independent, but they are not likely to be technologically independent.

Orthogonalized statistics work well in the *LEN* model in which the actions do not vary with the information received. However, if the actions vary with the information received, it is useful to normalize as well as to orthogonalize the performance measures. The normalization process described in Section 27.2.2 requires the use of the principal's conjectures with respect to the agent's actions, including the principal's conjecture with respect to how the agent's actions will vary with the information received, given the contract between the principal and the agent.

In the *LEN* model, the optimally induced actions are independent of prior information – they are constants. This is, in part, a result of the fact the *LEN* contract is constrained to be linear. Section 27.3 considers a model in which the preferences and performance measures are the same as in the *LEN* model, but the contract need not be linear. Section 27.3.1 explores the nature of the optimal contract (when the form of the contract is not constrained). Key features of the optimal contract include second-period incentives that vary with the first-period performance report, effort-cost risk insurance, and an additional indirect first-period covariance incentive not present in the *LEN* model.

The characterization of the optimal contract is complex, and does not lend itself to comparative statics. Section 27.3.2 considers a more tractable contract that permits inducement of actions that vary with the information received. The linearity constraint of the *LEN* contract is relaxed by allowing the second-period incentive rate to be a linear function of the first-period performance statistic. In addition, the second-period "fixed" wage can vary with the first-period performance statistic so as to compensate the agent for his second-period effort cost and risk premium, conditional on the first-period report. This approach provides effort-cost risk insurance and risk-premium risk insurance. This is called a *QEN* contract. Varying the second-period incentive rate with the first-period report affects the first-period effort choice through an indirect covariance incentive. Interestingly, contrary to the *LEN* contract, positive correlation between periods is desirable with a *QEN* contract because of the indirect first-period covariance incentives it provides.

Section 27.4 considers two settings in which the first-period report is informative about the second-period productivity (i.e., the rate of output per unit of effort in the second period). Section 27.4.1 again uses the *QEN* contract in a setting with *LEN* model preferences and performance statistics (which are orthogonalized and normalized). In this case, varying the second-period incentive rate with the first-period report again affects the first-period effort choice through an indirect covariance incentive, but also allows the principal to directly affect the second-period effort choice, so that it is more efficient. However, these effects do not always go in the same direction.

Section 27.4.2 analyzes a two-period hurdle model in which the first-period outcome is contractible and informative about the hurdle in the second period. A key feature of this model is that the first-period action affects the informativeness of the first-period outcome about the second-period hurdle. Hence, the optimal first-period action is chosen both for its direct effect on the first-period outcome and for the informativeness about the second-period hurdle.

27.1 BASIC ISSUES IN SEQUENTIAL CHOICE

In Chapter 25 we formulate a multi-period incentive model, and then simplify the analysis by assuming technological and stochastic independence of both the performance reports and the principal's gross payoffs. In this section we examine some implications of technological and stochastic interdependence. For simplicity, much of our analysis is done within the context of a two-period setting.

27.1.1 A Two-period Model with Interdependent Periods

We assume a risk neutral principal hires a risk and effort averse agent to take actions $\overleftarrow{\mathbf{a}}_2 \equiv (\mathbf{a}_1, \mathbf{a}_2)$ in periods one and two. There is no private information (except that the principal does not observe the agent's actions), and the public (and contractible) reports issued at date $t = 1, 2$, are denoted $\mathbf{y}_t \in \mathbf{Y}_t$. The public reports may include the principal's gross payoffs, but not necessarily.[1]

The principal is assumed to maximize the NPV of the net cash flows from his firm's operations, where the NPV is computed using the market discount factor per period of β. The gross payoff (e.g., cash from operations prior to deducting the agent's compensation) is represented by x, and is measured in date $t = 2$ dollars. The agent's gross compensation is paid at $t = 2$, and is denoted $s(\overleftarrow{\mathbf{y}}_2)$, where $\overleftarrow{\mathbf{y}}_2 \equiv (\mathbf{y}_1, \mathbf{y}_2)$ is the contractible information available at date $t = 2$.

At date 0, the agent accepts or rejects the compensation contract offered by the principal. If he accepts, then the agent chooses his first-period action. At date $t = 1$ the first-period performance report is issued, and the agent chooses his date $t = 1$ consumption c_1 (measured in date $t = 1$ dollars), followed by selection of his second-period action. The agent's date $t = 2$ consumption c_2 equals his compensation minus the debt plus interest used to finance his date $t = 1$ consumption. That is, $c_2 = s - Rc_1$, where $R = \beta^{-1}$.

The agent's utility for his consumption and actions is represented by $u^a(\overleftarrow{\mathbf{c}}_2, \overleftarrow{\mathbf{a}}_2)$, where $\overleftarrow{\mathbf{c}}_2 = (c_1, c_2)$. He is unconcerned about the principal's gross payoff unless it is part of his performance report and it influences his compensation. The agent chooses his actions sequentially (with no prior commitment). Hence, for a given incentive contract we solve for his induced consumption and actions by starting with his second-period action choice given the compensation contract accepted by the agent, his first-period action, the date $t = 1$ report, and the agent's date $t = 1$ consumption choice.

The first-period report can be influenced by the agent's first-period action. Hence, we represent his prior first-period report beliefs by the distribution function $\Phi(\mathbf{y}_1 | \mathbf{a}_1)$. At the start of the second period, the agent knows his first-period action and the date $t = 1$ report. Hence, his posterior belief with respect to the second-period report that will result from his second-period action can depend on $\mathbf{y}_1$ and $\mathbf{a}_1$, as well as $\mathbf{a}_2$. We represent that belief by the conditional distribution function $\Phi(\mathbf{y}_2 | \mathbf{y}_1, \mathbf{a}_1, \mathbf{a}_2)$.

The Agent's Induced Consumption and Action Choices

At date $t = 1$, the agent chooses his first-period consumption and his second-period action given his first-period action and the date $t = 1$ report. These choices are represented by $\boldsymbol{c}_1(\mathbf{y}_1, \boldsymbol{a}_1)$ and $\boldsymbol{a}_2(\mathbf{y}_1, \mathbf{a}_1)$, which satisfy

$$(\boldsymbol{c}_1(\mathbf{y}_1, \mathbf{a}_1), \boldsymbol{a}_2(\mathbf{y}_1, \mathbf{a}_1)) \in \underset{c_1, \mathbf{a}_2}{\operatorname{argmax}}\, U_1^a(\boldsymbol{s}, c_1, \mathbf{a}_2 | \mathbf{y}_1, \mathbf{a}_1), \quad \forall\, \mathbf{y}_1 \in \mathbf{Y}_1,\ \mathbf{a}_1 \in \mathbf{A}_1, \quad (27.1)$$

[1] The contract is signed at date 0. We could readily extend the model to include a post-contract report at date 0 (before the first action is taken), but for simplicity we exclude this type of report from the current analysis. See Christensen *et al.* (2004) for analysis of settings with both pre- and post-contract reports at date 0.

where $U_1^a(\boldsymbol{s}, c_1, \mathbf{a}_2 | \mathbf{y}_1, \mathbf{a}_1) \equiv \int_{\mathbf{Y}_2} u^a(c_1, \boldsymbol{s}(\mathbf{y}_1, \mathbf{y}_2) - Rc_1, \mathbf{a}_1, \mathbf{a}_2)\, d\Phi(\mathbf{y}_2 | \mathbf{y}_1, \mathbf{a}_1, \mathbf{a}_2)$.

When the agent selects his first-period action, he anticipates that his first-period consumption and second-period action will be consistent with (27.1). That is,

$$\mathbf{a}_1 \in \underset{\hat{\mathbf{a}}_1}{\text{argmax}}\ U_0^a(\boldsymbol{s}, \hat{\mathbf{a}}_1), \tag{27.2}$$

where $\quad U_0^a(\boldsymbol{s}, \mathbf{a}_1) \equiv \int_{\mathbf{Y}_1} U_1^a(\boldsymbol{s}, \boldsymbol{c}_1(\mathbf{y}_1, \mathbf{a}_1), \boldsymbol{a}_2(\mathbf{y}_1, \mathbf{a}_1) | \mathbf{y}_1, \mathbf{a}_1)\, d\Phi(\mathbf{y}_1 | \mathbf{a}_1)$.

We specify the principal's problem shortly. The actions and consumption induced by the optimal contract can be represented by $\mathbf{a}_1^\dagger$ and $(\boldsymbol{c}_1^\dagger(\mathbf{y}_1), \boldsymbol{a}_2^\dagger(\mathbf{y}_1)) \equiv (\boldsymbol{c}_1(\mathbf{y}_1, \mathbf{a}_1^\dagger),\ \boldsymbol{a}_2(\mathbf{y}_1, \mathbf{a}_1^\dagger))$ – this is the *equilibrium consumption/action path.* In specifying the incentive constraints in the principal's problem we must be careful to recognize that, if the agent deviates from $\mathbf{a}_1^\dagger$ in the first period by selecting $\mathbf{a}_1$, he will take his best response $(\boldsymbol{c}_1(\mathbf{y}_1, \mathbf{a}_1), \boldsymbol{a}_2(\mathbf{y}_2, \mathbf{a}_1))$ in the second. If $(\boldsymbol{c}_1(\mathbf{y}_1, \mathbf{a}_1), \boldsymbol{a}_2(\mathbf{y}_2, \mathbf{a}_1)) \neq (\boldsymbol{c}_1^\dagger(\mathbf{y}_1), \boldsymbol{a}_2^\dagger(\mathbf{y}_1))$, then we refer to this as *double shirking*.

Let $(\mathbf{a}_1^\dagger, \boldsymbol{a}_2^\dagger, \boldsymbol{c}_1^\dagger, \boldsymbol{c}_2^\dagger)$ represent the agent strategy the principal would like to induce. To ensure that the contract $\boldsymbol{s}$ induces this strategy, the incentive constraints should take the following form:

$$(\boldsymbol{c}_1^\dagger(\mathbf{y}_1),\ \boldsymbol{a}_2^\dagger(\mathbf{y}_1)) \in \underset{c_1, \mathbf{a}_2}{\text{argmax}}\, U_1^a(\boldsymbol{s}, c_1, \mathbf{a}_2 | \mathbf{y}_1, \mathbf{a}_1^\dagger) \quad \forall\, \mathbf{y}_1 \in \mathbf{Y}_1, \tag{27.1$'$}$$

$$\mathbf{a}_1^\dagger \in \underset{\mathbf{a}_1}{\text{argmax}}\, U_0^a(\boldsymbol{s}, \mathbf{a}_1). \tag{27.2$'$}$$

The key point is that (27.2′) considers the agent's optimal date $t = 1$ response to any choice of $\mathbf{a}_1 \neq \mathbf{a}_1^\dagger$.

If double shirking is not an issue,[2] then the initial incentive constraint can be expressed as

$$\mathbf{a}_1^\dagger \in \underset{\mathbf{a}_1}{\text{argmax}} \int_{\mathbf{Y}_1} U_1^a(\boldsymbol{s}, \boldsymbol{c}_1^\dagger(\mathbf{y}_1), \boldsymbol{a}_2^\dagger(\mathbf{y}_1) | \mathbf{y}_1, \mathbf{a}_1)\, d\Phi(\mathbf{y}_1 | \mathbf{a}_1). \tag{27.2$''$}$$

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[2] For example, if the first-order conditions for the agent's incentive compatibility constraints are sufficient to represent the incentive compatibility constraints, then it is straightforward to show that double shirking is not a problem.

The Principal's Problem

The agent is not directly concerned with the principal's gross payoff, but that payoff will affect the principal's contract choice. We assume full commitment on the part of the principal, so he chooses the contract s at $t = 0$, and makes no further choices. Of course, in choosing s, the principal is implicitly choosing the agent's induced strategy. Therefore, we depict the principal as choosing $(\mathbf{s}, \mathbf{a}_1, \boldsymbol{a}_2, \boldsymbol{c}_1)$, and express his decision problem as

$$U^p(\mathbf{s}^\dagger) = \underset{s,\, \mathbf{a}_1, \mathbf{a}_2, c_1}{\text{maximize}} \int_{\mathbf{Y}_1} \int_{\mathbf{Y}_2 \times X} [x - s(\mathbf{y}_1, \mathbf{y}_2)]\, d\Phi(\mathbf{y}_2, x | \mathbf{y}_1, \mathbf{a}_1, \boldsymbol{a}_2(\mathbf{y}_1))\, d\Phi(\mathbf{y}_1 | \mathbf{a}_1),$$

$$\text{subject to} \quad U_0^a(s, \mathbf{a}_1) \geq \bar{U}^a, \text{(27.1), and (27.2)},$$

where $\bar{U}^a$ is the agent's two-period reservation utility.

27.1.2 Stochastic Interdependence

The following discussion briefly describes some stochastic interdependencies, without characterizing the optimal contracts and agent strategies. Characterizations for some of the examples are explored in more detail later in the chapter.

Learning about Noise

The simplest interdependency to consider is one in which the gross payoff and the reports are jointly normally distributed as follows:

$$x = \mathbf{b}_1^t \mathbf{a}_1 + \mathbf{b}_2^t \mathbf{a}_2 + \varepsilon_x, \tag{27.3a}$$

$$\mathbf{y}_t = \mathbf{M}_t \mathbf{a}_t + \boldsymbol{\varepsilon}_t, \quad t = 1, 2, \tag{27.3b}$$

where the noise terms for the performance measures, $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$, have zero means, unit variances, and are correlated. The outcome noise, ε_x, may have a nonzero but action-independent mean, and an arbitrary variance/correlation structure.

This type of structure is assumed in *LEN* models, which we explore in Sections 25.4 and 26.1. The fact that the principal is risk neutral and the agent's compensation is not affected by the payoff x (unless it is included in the set of performance measures that are used), implies that the payoff noise ε_x is immaterial. In Section 25.4 we assumed $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ are uncorrelated (i.e., there is stochastic independence), so that $\boldsymbol{\varepsilon}_1 = \mathbf{y}_1 - \mathbf{M}_1 \mathbf{a}_1$ is uninformative about $\boldsymbol{\varepsilon}_2$. In Section 27.2, on the other hand, we assume $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ are correlated. As a result, $\boldsymbol{\varepsilon}_1 = \mathbf{y}_1 - \mathbf{M}_1 \mathbf{a}_1$ is informative about $\boldsymbol{\varepsilon}_2$, e.g., if the correlation is positive then the posterior mean of $\boldsymbol{\varepsilon}_2$ increases (decreases) if $\mathbf{y}_1 > (<)\ \mathbf{M}_1 \mathbf{a}_1$, and the posterior variance of $\boldsymbol{\varepsilon}_2$ decreases by an amount that is independent of the realized value of $\boldsymbol{\varepsilon}_1$.

In the *LEN* model, in which s is a linear function of $\mathbf{y}_1$ and $\mathbf{y}_2$ (i.e., $s = f + \mathbf{v}_1^t\mathbf{y}_1 + \mathbf{v}_2^t\mathbf{y}_2$), and the preferences are such that there is no wealth effect, the induced second-period actions are independent of the date $t = 1$ report given the incentive contract. Furthermore, the optimal second-period incentive rate is independent of $\mathbf{y}_1$. However, the principal will choose different first- and second-period incentive rates if there is a change in the correlation between $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$. With exponential utility, this change is driven entirely by the fact that the correlation affects the agent's incentive risk premium, for which he must be compensated. If a change in the correlation reduces (increases) the risk premium, then (loosely speaking) the principal will choose stronger (weaker) incentive rates.

Learning about Payoff Productivity

In the preceding analysis, we refer to ε_x, $\boldsymbol{\varepsilon}_1$, and $\boldsymbol{\varepsilon}_2$ as noise, since they are additive and do not affect the marginal impact of the agent's actions on either the principal's expected payoff or the agent's expected performance. The situation changes, for example, if

$$x = \boldsymbol{\theta}_1^t\mathbf{a}_1 + \boldsymbol{\theta}_2^t\mathbf{a}_2, \tag{27.4a}$$

$$\mathbf{y}_t = \mathbf{M}_t\mathbf{a}_t + \boldsymbol{\varepsilon}_t, \quad t = 1, 2, \tag{27.4b}$$

where $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are random productivity parameters with means $\mathbf{b}_1$ and $\mathbf{b}_2$.

Feltham, Indjejikian, and Nanda (2005) use a structure similar to (27.4).[3] In this setting, $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ are again additive noise, so that the issues that arise in the *LEN* model examined in Section 27.2 also arise here. The key difference in this model is that we have replaced ε_x with two vectors $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$, which are multiplied by the agent's actions at dates $t = 1$ and 2. This multiplicative form implies that the marginal effects of the agent's actions are now random. That randomness is not of direct importance to the principal given that he is risk neutral. However, it is important if $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\theta}_2$ are correlated, since the expected marginal impact of the second-period action on the principal's payoff varies with the date $t = 1$ report. For example, if they are positively correlated, then

[3] They consider two types of information about payoff productivity. One is similar to the model described here. In this model, they refer to $\mathbf{y}_1$ as a "dual purpose" measure because it is informative about both x and $\mathbf{y}_2$. In their other model, they assume $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\theta}_2$ are uncorrelated, but there is another first-period report that is informative about x, but not about $\mathbf{y}_2$. They demonstrate that, with renegotiation, there exist conditions under which the dual purpose report dominates two special purpose reports. The key to those results is the existence of indirect first-period incentives, which are more powerful if the dual purpose measures are used.

the principal will want to induce increased (decreased) second-period effort if $\mathbf{y}_1 >$ (<) $\mathbf{M}_1\mathbf{a}_1$. That is, if linear contracts are used, then $\mathbf{v}_2$ will be an increasing function of $\mathbf{y}_1$, whereas $\mathbf{v}_2$ is constant in the additive noise case.

Learning about Performance Productivity

While the principal is concerned with the impact of the agent's actions on the principal's payoff, the agent will be concerned about the impact of his actions on his own performance. The first-period performance report is informative about the expected marginal second-period performance if

$$\mathbf{y}_t = \boldsymbol{\Theta}_t\mathbf{a}_t, \quad t = 1, 2, \tag{27.5}$$

for which $\boldsymbol{\Theta}_1$ and $\boldsymbol{\Theta}_2$ are correlated random matrices with means $\mathbf{M}_1$ and $\mathbf{M}_2$.

While this model might yield interesting results, it is less tractable than the *LEN* model considered in Section 27.2 or even the payoff productivity model described above. The key aspect of the model is that if the random performance productivity is positively correlated, then a given incentive rate will induce more (less) second-period effort if $\mathbf{y}_1 >$ (<) $\mathbf{M}_1\mathbf{a}_1$. This may make it optimal to reduce (increase) $\mathbf{v}_2$ to partially offset these effects. However, since we are not aware of any analyses of this type of setting, the above comments must be viewed as speculative.

27.2 STOCHASTICALLY INDEPENDENT SUFFICIENT PERFORMANCE STATISTICS

In the "learning about noise" example discussed above (see (27.1.2)), the reports are stochastically interdependent, but technologically independent. As noted, the induced actions in the *LEN* model are independent of prior reports. However, with optimal (non-linear) contracts, the second-period incentives are likely to depend on prior performance so as to efficiently induce the first-period action. We demonstrate that, in a two-period model, benefits from varying the second-period incentives with first-period performance can be achieved within a modified *LEN* model. While the resulting compensation is not normally distributed from a date 0 perspective, tractability can be achieved by transforming the performance measures.

In the "learning about productivity" example, the first-period performance measure is informative about second-period productivity. This creates another reason for varying the second-period incentives with first-period performance.

In this section, we demonstrate that, if the noise terms are jointly normally distributed, then the reports can always be transformed so that the revised reports (statistics) provide the same information, are stochastically independent, and are jointly normally distributed (in equilibrium). We refer to these representations as *stochastically independent sufficient performance statistics*. First, we merely orthogonalize the performance measures so as to obtain stochastically independent statistics. These work well in a *LEN* model in which the optimal actions are independent of the prior information. Then we both orthogonalize and normalize the performance measures so as to obtain stochastically independent statistics with zero means. These are useful in contracts that induce the agent to choose actions that vary with the information received.

27.2.1 Orthogonalization: Achieving Stochastic Independence

Consider a sequence of reports $\mathbf{y}_1, ..., \mathbf{y}_T$ and actions $\mathbf{a}_1, ..., \mathbf{a}_T$ for which

$$\mathbf{y}_t = \sum_{\tau=1}^{t} \mathbf{M}_{t\tau} \mathbf{a}_\tau + \boldsymbol{\varepsilon}_t, \tag{27.6}$$

and the noise terms $\boldsymbol{\varepsilon}_1, ..., \boldsymbol{\varepsilon}_T$ are jointly normally distributed with zero means and unit variances. The noise terms may be correlated, but they can be "orthogonalized."

The *new information* provided by $\boldsymbol{\varepsilon}_t$ at date t given the noise history $\overleftarrow{\boldsymbol{\varepsilon}}_{t-1} \equiv (\boldsymbol{\varepsilon}_1, ..., \boldsymbol{\varepsilon}_{t-1})$ can be represented as:

$$\boldsymbol{\delta}_1 = \boldsymbol{\varepsilon}_1, \quad \boldsymbol{\delta}_t \equiv \boldsymbol{\varepsilon}_t - \mathrm{E}[\boldsymbol{\varepsilon}_t \mid \overleftarrow{\boldsymbol{\varepsilon}}_{t-1}], \quad \text{for all } t = 2, ..., T. \tag{27.7a}$$

That is, the new information at date t is the difference between $\boldsymbol{\varepsilon}_t$ and its orthogonal projection on the linear subspace spanned by the prior noise terms. Hence, the orthogonalized noise terms $\boldsymbol{\delta}_1, ..., \boldsymbol{\delta}_T$ are uncorrelated such that their posterior variances equal their prior variances. Furthermore, since the noise terms $\boldsymbol{\varepsilon}_1, ..., \boldsymbol{\varepsilon}_T$ are simultaneously normally distributed, we can apply the results for normal distributions in Volume I, Section 3.1.3, to calculate the posterior mean of $\boldsymbol{\varepsilon}_t$ given the noise history as a linear function of $\overleftarrow{\boldsymbol{\varepsilon}}_{t-1}$, i.e.,[4]

$$\mathrm{E}[\boldsymbol{\varepsilon}_t \mid \overleftarrow{\boldsymbol{\varepsilon}}_{t-1}] = \mathbf{E}_{t-1}^{t} \overleftarrow{\boldsymbol{\varepsilon}}_{t-1}, \quad \text{for all } t = 2, ..., T. \tag{27.7b}$$

where $\mathbf{E}_{t-1}^{t}$ is a matrix representing the covariance between $\boldsymbol{\varepsilon}_t$ and $\overleftarrow{\boldsymbol{\varepsilon}}_{t-1}$ multiplied by the inverse covariance matrix for $\overleftarrow{\boldsymbol{\varepsilon}}_{t-1}$. Hence, the orthogonalized noise terms are simultaneously and independently normally distributed with prior (and posterior) mean zero.

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[4] Note that there is no intercept term since the noise terms have zero prior means.

We can apply the same linear transformation to the reports and construct *stochastically independent sufficient performance statistics*, i.e.,

$$\chi_1 \equiv \mathbf{y}_1, \quad \chi_t \equiv \mathbf{y}_t - \mathbf{E}_{t-1}^{\,t}\overleftarrow{\mathbf{y}}_{t-1}, \qquad \text{for all } t = 2, ..., T. \tag{27.8a}$$

If the induced actions do not depend on prior reports, i.e., they are deterministic, then these statistics are also simultaneously and independently normally distributed. Furthermore, the transformation is *invertible*, such that the statistics (χ_1, ..., χ_t) are equivalent to the reports ($\mathbf{y}_1$, ..., $\mathbf{y}_t$) and, therefore, also a sufficient statistic (see Volume I, Section 3.1.4).

Note that the statistics can be written as

$$\chi_t = \sum_{\tau=1}^{t} \mathbf{M}_{t\tau}\mathbf{a}_\tau - \mathbf{E}_{t-1}^{\,t}\mathrm{E}[\overleftarrow{\mathbf{y}}_{t-1} \,|\, \overleftarrow{\mathbf{a}}_{t-1}] + \boldsymbol{\delta}_t. \tag{27.8b}$$

Hence, the noise terms for the statistics are the orthogonalized noise terms, and their prior variances are equal to their posterior variances which in turn are equal to the posterior variances of noise terms $\boldsymbol{\varepsilon}_t$, i.e., the following equalities hold:

$$\begin{aligned} \mathrm{Var}[\chi_t] &= \mathrm{Var}[\boldsymbol{\delta}_t] = \mathrm{Var}[\boldsymbol{\delta}_t \,|\, \overleftarrow{\boldsymbol{\delta}}_{t-1}] \\ &= \mathrm{Var}\big[\boldsymbol{\varepsilon}_t - \mathrm{E}[\boldsymbol{\varepsilon}_t \,|\, \overleftarrow{\boldsymbol{\varepsilon}}_{t-1}]\big] = \mathrm{Var}[\boldsymbol{\varepsilon}_t \,|\, \overleftarrow{\boldsymbol{\varepsilon}}_{t-1}]. \end{aligned} \tag{27.9}$$

It also follows from (27.8) that, even if action $\mathbf{a}_\tau$, $\tau < t$, does not affect the report $\mathbf{y}_t$, it will affect the statistic χ_t if it affects prior reports which are correlated with $\mathbf{y}_t$. Hence, if contracts are written in terms of the statistics, there will be *indirect incentives* for prior actions, if they affect reports that are correlated with later reports. We illustrate these issues in the following two subsections.

A Simple Correlated Noise Example

To illustrate the use of the independent sufficient performance statistics we return to the simple two-period "learning about noise" example in the preceding section. Recall that the performance measures are technologically independent and stochastically interdependent, with

$$\mathbf{y}_t = \mathbf{M}_t\mathbf{a}_t + \boldsymbol{\varepsilon}_t, \qquad t = 1, 2.$$

If $\boldsymbol{\Sigma}_{t\tau} \equiv \mathrm{Cov}(\boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_\tau)$, $t, \tau = 1, 2$, then the linear transformation is given by $\mathbf{E}_{2-1}^{\,2} = \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}$ and, hence, the statistics are

$$\chi_1 = \mathbf{M}_1\mathbf{a}_1 + \boldsymbol{\delta}_1; \quad \chi_2 = \mathbf{M}_2\mathbf{a}_2 - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\mathbf{M}_1\mathbf{a}_1 + \boldsymbol{\delta}_2,$$

where

$$\boldsymbol{\delta}_1 \sim \mathrm{N}(\mathbf{0},\boldsymbol{\Sigma}_{11}), \quad \boldsymbol{\delta}_2 \sim \mathrm{N}(\mathbf{0},\boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}), \quad \mathrm{Cov}(\boldsymbol{\delta}_t,\boldsymbol{\delta}_\tau) = \mathbf{0},\ t \neq \tau = 1,2.$$

From the above structure we see that in the "learning about noise" example, reports that are stochastically interdependent and technologically independent are replaced with statistics that are stochastically independent but technologically dependent.

To illustrate these differences, consider the *LEN* model with exponential *AC-EC* preferences and assume, for simplicity, that there is only a single task and a single performance measure in each period, and that there is no discounting. If the reports have unit variances, then $\mathbf{E}_{2\text{-}1}^{\,2} = \rho$, where ρ is the correlation between ε_1 and ε_2, and the variance of χ_2 is $1 - \rho^2$. The linear contract can be written in terms of the reports y_1 and y_2, or in terms of the statistics χ_1 and χ_2, i.e.,

$$s_y(y_1,y_2) = f + v_1 y_1 + v_2 y_2, \quad \text{or} \quad s_\chi(\chi_1,\chi_2) = f_\chi + \upsilon_1\chi_1 + \upsilon_2\chi_2.$$

The two approaches give the principal the same optimal expected utility (since one is an invertible transformation of the other). Nonetheless, the form of the agent's certainty equivalent at date 1 given y_1 versus χ_1, and given the first- and second-period actions a_1 and a_2, differs as follows:[5]

$$CE_1^y(y_1,a_1,a_2) = v_2[M_2 a_2 + \rho(y_1 - M_1 a_1)] - \kappa_2(a_2) - \tfrac{1}{2} r v_2^2(1-\rho^2),$$

$$CE_1^\chi(\chi_1,a_1,a_2) = \upsilon_2[M_2 a_2 - \rho M_1 a_1] - \kappa_2(a_2) - \tfrac{1}{2} r \upsilon_2^2(1-\rho^2).$$

The first-order conditions for the second-period actions are

$$\kappa_2'(a_2^y) = v_2 M_2 \quad \text{and} \quad \kappa_2'(a_2^\chi) = \upsilon_2 M_2,$$

respectively, which yields the same action choices if, and only if, $v_2 = \upsilon_2$.

On the other hand, the two types of contracts differ in their representation of both the agent's *ex ante* risk premium and his first-period action choice. The *ex ante* risk premium differs because the two incentive wages are correlated

[5] Although the timing of payments is immaterial in this setting, we implicitly assume in the calculation of the date 1 certainty equivalents that the fixed wage and the first-period incentive wage and effort cost have been paid.

with $\mathbf{s}_y$, whereas they are uncorrelated with $\mathbf{s}_\chi$. For given incentive rates, the two *ex ante* risk premia are

$$RP_0^y = \tfrac{1}{2}r[v_1^2 + v_2^2 + 2\rho v_1 v_2]; \quad RP_0^\chi = \tfrac{1}{2}r[\upsilon_1^2 + \upsilon_2^2(1-\rho^2)].$$

Clearly, if the correlation is positive, the risk premium is higher with $\mathbf{s}_y$ than with $\mathbf{s}_\chi$ if the same incentive rates are applied in both types of contracts, i.e., $v_t = \upsilon_t$, $t = 1,2$. However, while identical incentive rates will induce the same second-period effort, the induced first-period effort will be lower with $\mathbf{s}_\chi$ than with $\mathbf{s}_y$, i.e., the first-period incentive constraints differ

$$CE_0^y(a_1) = f + v_1 M_1 a_1 + v_2 M_2 a_2^\dagger - \kappa_1(a_1) - \kappa_2(a_2^\dagger) - RP_0^y,$$

$$CE_0^\chi(a_1) = f_\chi + \upsilon_1 M_1 a_1 + \upsilon_2[M_2 a_2^\dagger - \rho M_1 a_1] - \kappa_1(a_1) - \kappa_2(a_2^\dagger) - RP_0^\chi,$$

and the induced first-period actions are

$$\kappa_1'(a_1^y) = v_1 M_1 \quad \text{and} \quad \kappa_1'(a_1^\chi) = [\upsilon_1 - \rho\upsilon_2]M_1.$$

With the contract $\mathbf{s}_y$, the induced effort levels are determined by the sensitivities and the incentive rates for the reports they affect directly, i.e., $a_t^y = v_t M_t$. With the contract $\mathbf{s}_\chi$, there is a direct incentive for the first-period effort, $\upsilon_1 M_1$, as well as an indirect "posterior mean" incentive, $-\rho\upsilon_2 M_1$, due to the fact that the first-period effort also affects the mean of the second-period statistic. This orthogonalization incentive is negative for positive correlation and, thus, less first-period effort is induced if the same incentive rates are applied in the two types of contracts.

If the two types of contracts induce the same actions, the relation between the incentive rates must be

$$v_1 = \upsilon_1 - \rho\upsilon_2; \quad v_2 = \upsilon_2.$$

Substituting these incentive rates into the *ex ante* risk premium for $\mathbf{s}_y$, we get

$$RP_0^y = \tfrac{1}{2}r[(\upsilon_1 - \rho\upsilon_2)^2 + \upsilon_2^2 + 2\rho(\upsilon_1 - \rho\upsilon_2)\upsilon_2] = \tfrac{1}{2}r[\upsilon_1^2 + \upsilon_2^2(1-\rho^2)] = RP_0^\chi.$$

Hence, inducing the same actions is equally costly to the principal (in terms of the risk premium he must pay to the agent) and, therefore, the principal is indifferent between which type of contract is used. The key difference is that the impact of inter-period correlation of the incentive wages with $\mathbf{s}_y$ is substituted with indirect incentives with $\mathbf{s}_\chi$ – a difference in form but not in content!

Technological Interdependence

In Section 27.1.2, the basic models have stochastic interdependence and technological independence, e.g., $\mathbf{y}_2$ is only influenced by $\mathbf{a}_2$, and not by $\mathbf{a}_1$. The transformation introduced above results in stochastic independence and technological interdependence. We now consider an example in which the reports are stochastically and technologically interdependent, and a transformation yields statistics that are both stochastically and technologically independent.

An Auto-regressive Process

We define the performance measures as following an auto-regressive process if there exist exogenous vectors of weights $\boldsymbol{\lambda}_{t\tau}$ for all t = 1, ..., T and τ = 1,..., $t-1$, such that

$$\mathbf{y}_t = \mathbf{M}_{tt}\mathbf{a}_t + \sum_{\tau=1}^{t-1} \boldsymbol{\lambda}_{t\tau}^{\mathrm{t}} \mathbf{y}_\tau + \varsigma_t, \tag{27.10}$$

where the noise terms ς_1, ..., ς_T are independently, normally distributed with zero means. Obviously, the following transformation yields fully independent performance measures, and the analyses discussed in Chapter 25 can be applied:

$$\chi_t = \mathbf{y}_t - \sum_{\tau=1}^{t-1} \boldsymbol{\lambda}_{t\tau}^{\mathrm{t}} \mathbf{y}_\tau = \mathbf{M}_{tt}\mathbf{a}_t + \varsigma_t.$$

Observe that a random walk is a special case of (27.10) in which $\boldsymbol{\lambda}_{t\tau}$ is an identity matrix if $\tau = t-1$, and zero otherwise. In that case, the changes in the performance measures, i.e., $\chi_t = \mathbf{y}_t - \mathbf{y}_{t-1}$, is a stochastically independent sufficient statistic, if $\varsigma_1, ..., \varsigma_T$ are independently distributed.

A Price Process

If a firm's shares are publicly traded, then stock or option grants are frequently used as incentive devices for the CEO and others in top management. In this case, the principal is effectively using the change in the stock price or the return on the stock as the performance measure, and the price change is a proxy for the information received by investors during the period.

To illustrate this type of performance measure, assume the prices at dates t = 0, 1, and 2, represented by P_0, P_1, and P_2, equal the NPV of the expected gross payoff x (with discount factor β). The initial price P_0 is based on the principal's (investors') prior beliefs at date 0. In this case $\mathbf{y}_1$ and $\mathbf{y}_2$ represent non-contractible reports received by investors at dates 1 and 2. Hence, the prices are

$$P_0 = \beta^2\, \mathrm{E}[x \mid \hat{\mathbf{a}}_1, \hat{\boldsymbol{a}}_2],$$

$$P_1(\mathbf{y}_1) = \beta\, \mathrm{E}[x|\mathbf{y}_1, \hat{\mathbf{a}}_1, \hat{\boldsymbol{a}}_2],$$

$$P_2(\mathbf{y}_1,\mathbf{y}_2) = \mathrm{E}[x|\mathbf{y}_1,\mathbf{y}_2, \hat{\mathbf{a}}_1, \hat{\boldsymbol{a}}_2(\mathbf{y}_1)],$$

where $\hat{\mathbf{a}}_1$ and $\hat{\boldsymbol{a}}_2$ are the principal's conjectures with respect to the agent's first-period action and the agent's second-period action strategy (assuming the agent observes $\mathbf{y}_1$ before he selects $\mathbf{a}_2$).

The changes in prices,

$$P_1(\mathbf{y}_1) - P_0 = \iota P_0 + [P_1(\mathbf{y}_1) - R\,P_0],$$

$$P_2(\mathbf{y}_1,\mathbf{y}_2) - P_1(\mathbf{y}_1) = \iota P_1(\mathbf{y}_1) + [P_2(\mathbf{y}_1,\mathbf{y}_2) - R\,P_1(\mathbf{y}_1)],$$

reflect the "normal returns" (where $\iota = R - 1$) plus "excess returns" due to the incremental information (good or bad) received during the period. Note that the changes in prices are positively correlated (for a deterministic "normal return") due to the "normal return" on the beginning-of-period stock price. However, by construction, the "excess returns" in the two periods,

$$\chi_1 = P_1(\mathbf{y}_1) - R\,P_0,$$

$$\chi_2 = P_2(\mathbf{y}_1,\mathbf{y}_2) - R\,P_1(\mathbf{y}_1),$$

will be uncorrelated. It is unlikely that the price change (or the "excess return") will be a sufficient performance statistic for the non-contractible information received by investors. As in Chapter 21, this is because the incremental information is aggregated based on what it reveals about x, not what it reveals about the agent's actions.

Furthermore, while "excess returns" are stochastically independent, they need not be technologically independent. Of course, they will be technologically independent if the underlying incremental information is technologically independent. However, that seems unlikely since the actions taken in any given period typically have a mix of short- and long-run effects.

27.2.2 Normalization: Obtaining Zero-mean Statistics

Consider again the sequence of reports $\mathbf{y}_1, ..., \mathbf{y}_T$ and actions $\mathbf{a}_1, ..., \mathbf{a}_T$ in (27.6) for which the noise terms $\boldsymbol{\varepsilon}_1, ..., \boldsymbol{\varepsilon}_T$ are jointly normally distributed with zero means and unit variances. Assume now that the agent's induced actions may depend on prior reports, i.e., the actions are random from an *ex ante* perspective. We can again construct statistics as in (27.8). These statistics will be uncorrelated, since they are based on orthogonal projections, but they may not be normally distributed due to random means and, therefore, not stochastically independent.[6] However, we can "mean-adjust" the statistics based on the principal's *conjecture of the agent's actions* such that, in equilibrium, the normalized statistics are independently and jointly normally distributed with zero means.

The agent knows the actions he has chosen, but the principal must base his beliefs on his conjectures with respect to the past actions. Let $\hat{\mathbf{a}}_t = \hat{\boldsymbol{a}}_t(\overleftarrow{\mathbf{y}}_{t-1})$ represent the conjectured choice for period t given the reports received. In equilibrium, the conjectures equal the agent's choices, but contracts must be based on the principal's conjectures.

The mean-adjusted statistics are constructed (using the same linear transformation) as follows:

$$\boldsymbol{\psi}_1 \equiv \boldsymbol{\chi}_1 - \mathbf{M}_{11}\hat{\mathbf{a}}_1 = \mathbf{M}_{11}[\mathbf{a}_1 - \hat{\mathbf{a}}_1] + \boldsymbol{\delta}_1, \tag{27.11a}$$

$$\boldsymbol{\psi}_t \equiv \boldsymbol{\chi}_t - \Big[\sum_{\tau=1}^{t} \mathbf{M}_{t\tau}\hat{\mathbf{a}}_\tau - \mathbf{E}_{t-1}^{t}\mathrm{E}[\overleftarrow{\mathbf{y}}_{t-1} \mid \overleftarrow{\hat{\mathbf{a}}}_{t-1}]\Big]$$

$$= \sum_{\tau=1}^{t} \mathbf{M}_{t\tau}[\mathbf{a}_\tau - \hat{\mathbf{a}}_\tau] - \mathbf{E}_{t-1}^{t}\big[\mathrm{E}[\overleftarrow{\mathbf{y}}_{t-1} \mid \overleftarrow{\mathbf{a}}_{t-1}] - \mathrm{E}[\overleftarrow{\mathbf{y}}_{t-1} \mid \overleftarrow{\hat{\mathbf{a}}}_{t-1}]\big] + \boldsymbol{\delta}_t. \tag{27.11b}$$

In equilibrium, the conjectured actions are equal to the agent's actual actions, and the conjectured noise history is equal to the actual noise history. Hence, from *the perspective of the principal* the statistics,

$$\boldsymbol{\psi}_t = \boldsymbol{\delta}_t \equiv \boldsymbol{\varepsilon}_t - \mathrm{E}[\boldsymbol{\varepsilon}_t \mid \overleftarrow{\boldsymbol{\varepsilon}}_{t-1}],$$

are jointly and independently normally distributed with zero means, and

$$\mathrm{Var}[\boldsymbol{\psi}_t] = \mathrm{Var}[\boldsymbol{\delta}_t] = \mathrm{Var}[\boldsymbol{\varepsilon}_t \mid \overleftarrow{\boldsymbol{\varepsilon}}_{t-1}].$$

However, at date $t - 1$ the agent may consider choosing $\mathbf{a}_t \neq \hat{\mathbf{a}}_t$. Moreover, he knows his past actions and, therefore, also the noise history,

$$\boldsymbol{\varepsilon}_\tau = \mathbf{y}_\tau - \sum_{h=1}^{\tau} \mathbf{M}_{\tau h}\mathbf{a}_h, \quad \tau = 1, \ldots, t-1.$$

[6] Normality is only obtained if the induced actions are linear functions of the prior reports. This will be the case in the *QEN* model considered below, but not for optimal contracts.

Hence, given his information at date $t - 1$, the statistic reported at date t is normally distributed with mean

$$\sum_{\tau=1}^{t} \mathbf{M}_{t\tau}[\mathbf{a}_\tau - \hat{\mathbf{a}}_\tau] - \mathbf{E}_{t-1}^{t\prime}[\mathrm{E}[\overleftarrow{\mathbf{y}}_{t-1} \,|\, \overleftarrow{\mathbf{a}}_{t-1}] - \mathrm{E}[\overleftarrow{\mathbf{y}}_{t-1} \,|\, \overleftarrow{\hat{\mathbf{a}}}_{t-1}]]$$

and variance $\mathrm{Var}[\boldsymbol{\varepsilon}_t \,|\, \overleftarrow{\boldsymbol{\varepsilon}}_{t-1}]$. Note that the principal and the agent have the same posterior variance while their posterior means may differ due to differences in conjectured and actual actions. Of course, the latter affects the agent's incentive constraints, while the principal's expected utility is calculated on the presumption that the agent in fact chooses the conjectured actions.

From the perspective of the principal, the prior as well as the posterior distributions of the statistics are jointly normally distributed with mean zero, and the conditional distributions of one-period-ahead statistics are also normally distributed (even though conjectured and actual actions depend on the report history). Thus, we can use the results in Volume I, Section 3.1.3, to calculate the posterior mean and variance of the statistics given the report and conjectured action histories. The key point for our purposes is the fact that $\boldsymbol{\psi}_t$ (like $\boldsymbol{\chi}_t$) is an *invertible* linear transformation of the initial performance measure.

A Simple Correlated Noise Example

To illustrate the use of normalized stochastically independent sufficient performance statistics consider the simple two-period "learning about noise" example in the preceding section, i.e.,

$$\mathbf{y}_t = \mathbf{M}_t\mathbf{a}_t + \boldsymbol{\varepsilon}_t, \quad t = 1, 2,$$

with $\boldsymbol{\Sigma}_{t\tau} \equiv \mathrm{Cov}(\boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_\tau)$, $t, \tau = 1, 2$. Using (27.8) and (27.11), the normalized statistics are

$$\boldsymbol{\psi}_1 = \mathbf{M}_1(\mathbf{a}_1 - \hat{\mathbf{a}}_1) + \boldsymbol{\delta}_1,$$

$$\boldsymbol{\psi}_2 = \mathbf{M}_2(\mathbf{a}_2 - \hat{\mathbf{a}}_2) - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\mathbf{M}_1(\mathbf{a}_1 - \hat{\mathbf{a}}_1) + \boldsymbol{\delta}_2,$$

$$\boldsymbol{\delta}_1 \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma}_{11}), \quad \boldsymbol{\delta}_2 \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}), \quad \mathrm{Cov}(\boldsymbol{\delta}_t, \boldsymbol{\delta}_\tau) = \mathbf{0},\ t \neq \tau = 1, 2.$$

Note that from the *perspective of the principal*, i.e., with $\mathbf{a}_t = \hat{\mathbf{a}}_t$, the statistics are both stochastically and technologically independent with zero mean, but *from the perspective of the agent* they are technologically interdependent (since his first-period action $\mathbf{a}_1$ influences both $\boldsymbol{\psi}_1$ and $\boldsymbol{\psi}_2$). Note also that the reports and the statistics are equivalent since the statistics are invertible linear transformations of the reports (given the conjectured actions), i.e.,

$$\mathbf{y}_1 = \boldsymbol{\psi}_1 + \mathbf{M}_1 \hat{\mathbf{a}}_1,$$

$$\mathbf{y}_2 = \boldsymbol{\psi}_2 + \boldsymbol{\Sigma}_{21}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\psi}_1 + \mathbf{M}_2 \hat{\mathbf{a}}_2.$$

In analyses in which actions may depend on the report histories, it can be useful to mean-adjust the statistics by the impact of the conjectured actions. In equilibrium (i.e., from the principal's perspective) this adjustment preserves the joint normality of the statistics since the statistics merely equal the normally distributed noise terms. Of course, while, in equilibrium, the agent's action choices equal the principal's conjectures, the agent considers other actions when making his choices.

In the *LEN* model, the incentive rates with orthogonalized and normalized statistics are the same as when orthogonalized statistics are used. Hence, their relation to the incentive rates based on the performance measures is again $v_1 = \upsilon_1 - \rho\upsilon_2$; $v_2 = \upsilon_2$. Of course, with the performance measures or the orthogonalized statistics the fixed payment must include $-\{v_1 \mathrm{E}[y_1] + v_2 \mathrm{E}[y_2]\}$ plus compensation for the agent's cost of effort and his risk premium. The first term is not required with a normalized statistic since it has mean zero.

27.3 INFORMATION CONTINGENT ACTIONS

A noteworthy feature of the standard *LEN* model is that the induced actions are independent of the information received. Various aspects of the model's assumptions contribute to that fact. For example, *AC-EC* preferences ensure that there are no wealth effects. The normally distributed performance measures with fixed coefficients and additive noise ensure that the information does not affect the agent's beliefs about the marginal impact of his actions on his performance. Also, the information does not affect the principal's beliefs about the marginal impact of the agent's actions on the principal's payoffs. Constraining the agent's compensation contract to be a linear function (with non-random coefficients) of the performance statistics also implies that the marginal impact of the agent's action on his certainty equivalent is independent of prior information. In particular, linear contracts rule out the use of contracts in which the coefficients vary with the information reported.

In this section, we retain the payoff function, performance measure, and preference assumptions of the *LEN* model, but we relax the linearity constraint on the compensation contract. In Section 27.3.1 we explore the first-order characterization of an optimal contract. The contract characterization is complex, but it does reveal that it is optimal to vary the second-period incentives with the first-period report in order to induce an additional indirect first-period covariance incentive not present in the *LEN* model. The induced information contingent second-period action creates effort-cost risk on the part of the agent, but that risk is perfectly insured by the risk neutral principal. This leads us to introduce what we call a "*QEN* contract." It uses an incentive contract similar to the *LEN* contract, but allows the second-period incentive rate to vary linearly with the first-period report and it allows the second-period "fixed" compensation to equal the second-period effort cost and the second-period risk premium conditional on the first-period report. The *QEN* contract is shown to strictly dominate the *LEN* contract and, more importantly, to significantly affect the comparative statics with respect to the inter-period correlation of the performance measures.

27.3.1 Optimal Contracts

As stated in the preceding introduction, the underlying structure of the model considered in this and the following section is essentially the same as in the *LEN* model. In particular, we assume, in both sections, that the agent has exponential *AC-EC* preferences, there is a single task and a single performance measure in each period with unit variance, and there is no discounting.

Performance Statistics and Likelihood Ratios

Given the agent's action choices (a_1, a_2) and the principal's conjecture $(\hat{a}_1, \hat{a}_2)$, the performance statistics are

$$\psi_1 = M_1(a_1 - \hat{a}_1) + \delta_1 \tag{27.12a}$$

and

$$\psi_2 = M_2(a_2 - \hat{a}_2) - \rho M_1(a_1 - \hat{a}_1) + \delta_2, \tag{27.12b}$$

where $\delta_1 = \varepsilon_1$, $\delta_2 = \varepsilon_2 - \rho\,\varepsilon_1$.

Let the two prior distributions given the agent's actual and conjectured actions be represented by $\Phi(\psi_1 | a_1, \hat{a}_1)$ and $\Phi(\psi_2 | a_1, \hat{a}_1, a_2, \hat{a}_2)$, and the latter is also the posterior distribution. Increasing a_1 increases the mean of ψ_1, but decreases the mean of ψ_2 if $\rho > 0$. Let $\boldsymbol{s}(\psi_1, \psi_2)$ represent the compensation contract and let $\boldsymbol{a}_2(\psi_1, a_1)$ represent the agent's second-period action strategy.

In this section we use a first-order approach to characterize the optimal contract. As noted later, if we assume the first-order approach is applicable, then there is no double-shirking, so a_2 can be written as $\boldsymbol{a}_2(\psi_1)$. In equilibrium, $a_t = \hat{a}_t$ for $t = 1, 2$, and ψ_1 and ψ_2 both have zero means independently of the equilibrium action choices. Let $\Phi^\dagger(\psi_1) = \mathrm{N}(0,1)$ and $\Phi^\dagger(\psi_2) = \mathrm{N}(0, 1 - \rho^2)$ denote the equilibrium distributions. Hence, in equilibrium, the likelihood ratios are:

$$L_{a_1}(\psi_1) \equiv \frac{\Phi_{a_1}(\psi_1 | \hat{a}_1)}{\Phi^\dagger(\psi_1)} = M_1 \psi_1, \tag{27.13a}$$

$$L_{a_1}(\psi_2) \equiv \frac{\Phi_{a_1}(\psi_2 | \hat{a}_1, \hat{a}_2)}{\Phi^\dagger(\psi_2)} = -\frac{\rho M_1 \psi_2}{1-\rho^2}, \tag{27.13b}$$

$$L_{a_2}(\psi_2) \equiv \frac{\Phi_{a_2}(\psi_2 | \hat{a}_1, \hat{a}_2)}{\Phi^\dagger(\psi_2)} = \frac{M_2 \psi_2}{1-\rho^2}. \tag{27.13c}$$

In (27.13), we express, for example, $\Phi_{a_1}(\psi_2 | a_1, \hat{a}_1, a_2, \hat{a}_2)$ as $\Phi_{a_1}(\psi_2 | \hat{a}_1, \hat{a}_2)$ when $a_1 = \hat{a}_1$ and $a_2 = \hat{a}_2$. Similar notation is used elsewhere in this section.

Optimal Contract

The principal's decision problem is to select the agent's compensation contract $\mathbf{s}^\dagger(\psi_1,\psi_2)$ and induced effort $(a_1^\dagger, \mathbf{a}_2^\dagger)$ that maximize the principal's expected net payoff, subject to providing the agent with his reservation utility $\overline{U}^a$ and incentive compatibility for the actions to be induced.

The principal's equilibrium expected net payoff is

$$U^p(\mathbf{s}^\dagger, a_1^\dagger, \mathbf{a}_2^\dagger) = \int_{\Psi_1}\int_{\Psi_2} \left[b_1 a_1^\dagger + b_2 \mathbf{a}_2^\dagger(\psi_1) - \mathbf{s}^\dagger(\psi_1, \psi_2) \right] \times d\Phi^\dagger(\psi_2)\, d\Phi^\dagger(\psi_1).$$

The agent's expected utility at dates 0 and 1, given the compensation function $\mathbf{s}^\dagger$, the principal's equilibrium conjectures $(a_1^\dagger, \mathbf{a}_2^\dagger)$, and the agent's action choices $(a_1, \mathbf{a}_2)$, are

$$U_0^a(\mathbf{s}^\dagger, a_1, a_1^\dagger, \mathbf{a}_2, \mathbf{a}_2^\dagger) = \int_{\Psi_1} U_1^a(\mathbf{s}^\dagger, a_1, a_1^\dagger, a_2(\psi_1), \mathbf{a}_2^\dagger(\psi_1) \,|\, \psi_1) \times d\Phi(\psi_1 | a_1, a_1^\dagger),$$

$$U_1^a(\mathbf{s}^\dagger, a_1, a_1^\dagger, a_2, \mathbf{a}_2^\dagger(\psi_1) \,|\, \psi_1) = -\int_{\Psi_2} \exp\left[-r\{ \mathbf{s}^\dagger(\psi_1, \psi_2) - \kappa_1(a_1) - \kappa_2(a_2) \} \right] \times d\Phi(\psi_2 | a_1, a_1^\dagger, a_2, \mathbf{a}_2^\dagger(\psi_1)).$$

The agent's equilibrium participation constraint is

$$U_0^a(\mathbf{s}^\dagger, a_1^\dagger, \mathbf{a}_2^\dagger) \geq \overline{U}^a, \tag{27.14a}$$

Under the assumption that the incentive constraints can be represented by the agent's first-order conditions, those constraints are (given that the principal seeks to induce $a_1^\dagger$ and $\boldsymbol{a}_2^\dagger$):[7, 8]

$$\partial U_1^a(\boldsymbol{s}^\dagger, a_1^\dagger, \boldsymbol{a}_2^\dagger(\psi_1) \mid \psi_1)/\partial a_2 = -\int_{\Psi_2} \exp[-r\{\boldsymbol{s}^\dagger(\psi_1,\psi_2) - \kappa_1(a_1^\dagger) - \kappa_2(\boldsymbol{a}_2^\dagger(\psi_1))\}]$$

$$\times \Big[r\kappa_2'(\boldsymbol{a}_2^\dagger(\psi_1)) + L_{a_2}(\psi_2) \Big] d\Phi^\dagger(\psi_2) = 0, \quad \forall\, \psi_1, \qquad (27.14b)$$

$$\partial U_0^a(\boldsymbol{s}^\dagger, a_1^\dagger, \boldsymbol{a}_2^\dagger)/\partial a_1 = -\int_{\Psi_1}\int_{\Psi_2} \exp[-r\{\boldsymbol{s}^\dagger(\psi_1,\psi_2) - \kappa_1(a_1^\dagger) - \kappa_2(\boldsymbol{a}_2^\dagger(\psi_1))\}]$$

$$\times \Big[r\kappa_1'(a_1^\dagger) + L_{a_1}(\psi_1) + L_{a_1}(\psi_2) \Big] d\Phi^\dagger(\psi_2)\, d\Phi^\dagger(\psi_1) = 0. \quad (27.14c)$$

The Lagrangian for the principal's contract choice problem is

$$\mathfrak{L} = U^p(\boldsymbol{s}^\dagger, a_1^\dagger, \boldsymbol{a}_2^\dagger) - \lambda[U_0^a(\boldsymbol{s}^\dagger, a_1^\dagger, \boldsymbol{a}_2^\dagger) - \bar{U}^a]$$

$$- \int_{\Psi_1} \mu_2(\psi_1)\, \partial U_1^a(\boldsymbol{s}^\dagger, a_1^\dagger, \boldsymbol{a}_2^\dagger(\psi_1) \mid \psi_1)/\partial a_2\, d\Phi^\dagger(\psi_1) - \mu_1\, \partial U_0^a(\boldsymbol{s}^\dagger, a_1^\dagger, \boldsymbol{a}_2^\dagger)/\partial a_1,$$

where λ, $\mu_2(\psi_1)$, and μ_1 are multipliers. Differentiating $\mathfrak{L}$ with respect to $\boldsymbol{s}$, for each (ψ_1,ψ_2) for which the contract pays more than a lower bound on compensation, results in the following characterization of the optimal contract:[9]

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[7] The likelihood ratio in the first-order condition for a_2 occurs because the distribution function $\Phi(\psi_2|a_1,a_1^\dagger,a_2,\boldsymbol{a}_2^\dagger(\psi_1))$ has mean $M_2\{(a_2 - \boldsymbol{a}_2^\dagger(\psi_1)) - \rho(a_1 - a_1^\dagger)\}$ with variance $1-\rho^2$, which implies

$$d\Phi_{a_2}(\psi_2|a_1,a_1^\dagger,a_2,\boldsymbol{a}_2^\dagger(\psi_1)) = d\Phi(\psi_2|a_1,a_1^\dagger,a_2,\boldsymbol{a}_2^\dagger(\psi_1))$$

$$\times [\psi_2 - M_2\{(a_2 - \boldsymbol{a}_2^\dagger(\psi_1)) - \rho(a_1 - a_1^\dagger)\}]\, M_2/(1-\rho^2)$$

$$= d\Phi^\dagger(\psi_2)\, L_{a_2}(\psi_2),$$

in equilibrium. A similar approach is used in the first-order condition for a_1.

[8] Observe that dividing through by $\exp[r\kappa_1(a_1^\dagger)]$ removes $a_1^\dagger$ from the first-order condition. Hence, the optimal choice of a_2 is independent of a_1, implying that double-shirking is not an issue here.

[9] We assume there is a lower bound on the agent's compensation so as to avoid the Mirrlees problem (see Section 17.3.3). However, for simplicity, we do not explicitly introduce the lower bound into the notation.

$$\boldsymbol{s}^\dagger(\psi_1,\psi_2) = \{\kappa_1(a_1^\dagger) + \kappa_2(\boldsymbol{a}_2^\dagger(\psi_1))\} + \frac{1}{r}\ln[\boldsymbol{G}(\psi_1,\psi_2)], \tag{27.15a}$$

where $\boldsymbol{G}(\psi_1,\psi_2) = r\{\lambda + \mu_1 g_1(a_1^\dagger,\psi_1) + \mu_1 L_{a_1}(\psi_2)$

$$+ \mu_2(\psi_1) g_2(\boldsymbol{a}_2^\dagger(\psi_1),\psi_2)\}, \tag{27.15b}$$

$$g_t(a_t,\psi_t) = r\kappa_t{}'(a_t) + L_{a_t}(\psi_t), \quad t = 1,2.$$

Effort-cost Risk Insurance

A noteworthy aspect of the optimal contract is that the agent is compensated for his conjectured effort cost, contingent on the first-period report. If $\boldsymbol{a}_2$ varies with ψ_1, then that creates what we call *effort-cost risk*. In equilibrium, contract (27.15) provides the agent with insurance against that risk, so that it is borne by the risk neutral principal and not the risk averse agent.

Given this insurance, the agent's equilibrium realized utility for (ψ_1,ψ_2) can be represented by

$$u^\dagger(\psi_1,\psi_2) \equiv -\exp[-r\{\boldsymbol{s}^\dagger(\psi_1,\psi_2) - \kappa_1(a_1^\dagger) - \kappa_2(\boldsymbol{a}_2^\dagger(\psi_1))\}]$$

$$= -\boldsymbol{G}(\psi_1,\psi_2)^{-1}.$$

Independent Periods

From (27.13b) it follows that $L_{a_1}(\psi_2) = 0$ if the two reports y_1 and y_2 are uncorrelated, i.e., $\rho = 0$. We know from Section 25.2.1 that, with exponential *AC-EC* preferences, $\rho = 0$ implies that the optimal contract can be written as a sum of two single-period contracts, and the second-period action is independent of the first-period report.

Proposition 27.1

Assume the agent has exponential *AC-EC* preferences and the contracts are written on the stochastically independent sufficient statistics in (27.12). If $\rho = 0$, then $\boldsymbol{a}_2(\psi_1)$ is independent of ψ_1 and there exist multipliers $\bar{\lambda}_t$ and $\bar{\mu}_t$ for $t = 1,2$, such that

$$\boldsymbol{s}(\psi_1,\psi_2) = \boldsymbol{s}_1(\psi_1) + \boldsymbol{s}_2(\psi_2), \tag{27.16a}$$

where $\boldsymbol{s}_t(\psi_t) = \kappa_t(a_t^\dagger) + \frac{1}{r}\ln[r\{\bar{\lambda}_t + \bar{\mu}_t g_t(a_t^\dagger,\psi_t)\}], \quad t = 1,2, \qquad (27.16b)$

and $a_t^\dagger$ is the agent's equilibrium action choice.

To reconcile (27.16) with (27.15) we note that $\boldsymbol{G}(\psi_1,\psi_2)$ can always be expressed as the product of $\boldsymbol{G}_1(\psi_1)$ and $\boldsymbol{G}_2(\psi_1,\psi_2)$, where

$$\boldsymbol{G}_1(\psi_1) \equiv r\{\bar{\lambda}_1 + \bar{\mu}_1 g_1(a_1^\dagger,\psi_1)\}, \tag{27.17a}$$

$$\boldsymbol{G}_2(\psi_1,\psi_2) \equiv \boldsymbol{G}(\psi_1,\psi_2)/\boldsymbol{G}_1(\psi_1)$$

$$= r\{\bar{\lambda}_2 \boldsymbol{G}_1(\psi_1) + \mu_1 L_{a_1}(\psi_2) + \mu_2(\psi_1) g_2(\boldsymbol{a}_2^\dagger(\psi_1),\psi_2)\}/\boldsymbol{G}_1(\psi_1), \tag{27.17b}$$

for any arbitrary $\bar{\lambda}_2 > 0$, $\bar{\lambda}_1 = \lambda/[r\bar{\lambda}_2]$, $\bar{\mu}_1 = \mu_1/[r\bar{\lambda}_2]$.

If $\rho = 0$, then $L_{a_1}(\psi_2) = 0$, $\boldsymbol{a}_2^\dagger(\psi_1) = a_2^\dagger$, and $\mu_2(\psi_1) = \bar{\mu}_2 G_1(\psi_1)$, resulting in[10]

$$\boldsymbol{G}_2(\psi_1,\psi_2) \equiv r\{\bar{\lambda}_2 + \bar{\mu}_2 g_2(a_2^\dagger,\psi_2)\},$$

which is independent of ψ_1.[11]

The compensation function is increasing and concave in ψ_t, and the agent's action choice for period $t = 1, 2$ can be characterized by[12]

$$\kappa_t'(a_t^\dagger) = -\frac{\mathrm{Cov}(u_t^\dagger,\psi_t)}{r\mathrm{E}[u_t^\dagger]} M_t, \quad t = 1,2, \tag{27.18}$$

where
$$u_t^\dagger(\psi_t) = -[r\{\bar{\lambda}_t + \bar{\mu}_t g_t(a_t^\dagger,\psi_t)\}]^{-1}.$$

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[10] To prove that $\boldsymbol{a}_2^\dagger(\psi_1) = a_2^\dagger$, and $\mu_2(\psi_1) = \bar{\mu}_2 G_1(\psi_1)$, one can conjecture that the optimal compensation function is additively separable in ψ_1 and ψ_2, and then verify that there is a contract of this type which satisfies the first-order conditions for the principal's decision problem. Clearly, the point-wise first-order conditions for the compensation function are satisfied if $\mu_2(\psi_1) = \bar{\mu}_2 G_1(\psi_1)$, and additive separability of the compensation function with exponential *AC-EC* preferences implies that the agent's second-period effort choice is independent of ψ_1. Verification of the first-order conditions for the principal's choice of induced actions is also straightforward.

[11] It may seem strange that the second-period incentive constraint multiplier $\mu_2(\psi_1)$ varies with ψ_1 in the independent periods case. This occurs because the first-period compensation affects the scale of the incentive constraint. Hence, the variation in $\mu_2(\psi_1)$ merely represents a scale adjustment in the multipliers.

[12] Substituting the optimal contract into first-order condition (27.14b) and taking advantage of the independence allows us to restate (27.14b) as

$$-\mathrm{E}[u_1 u_2[r\kappa_2' + M_2\psi_2]] = -\mathrm{E}[u_1]\{\mathrm{E}[u_2]r\kappa_2' + \mathrm{E}[u_2\psi_2]\}$$

$$= -\mathrm{E}[u_1]\{\mathrm{E}[u_2]r\kappa_2' + \mathrm{Cov}(u_2,\psi_2)\} = 0,$$

where the covariance term reflects the fact that the statistic has been normalized so that $\mathrm{E}[\psi_2] = 0$. Dividing by $-\mathrm{E}[u_1]$ and $-r\mathrm{E}[u_2]$, and adding κ_2' to both sides yields (27.18) for $t = 2$.

The compensation contract $\boldsymbol{s}_t^\dagger$ has two components. The first $(\kappa_t(a_t^\dagger))$ compensates the agent for the cost of his conjectured effort, while the second $(\ln[r\{\bar{\lambda}_t + \bar{\mu}_t g_t(a_t^\dagger,\psi_t)\}]/r)$ provides effort incentives. The likelihood ratio $L_{a_t^\dagger}(\psi_t)$ is an increasing linear function of ψ_t, which implies that $\ln[r\{\bar{\lambda}_t + \bar{\mu}_t g_t(a_t^\dagger,\psi_t)\}]$ is an increasing concave function of ψ_t. Since increasing the agent's effort increases the mean of ψ_t and does not affect the variance, it follows that increasing a_t increases the agent's utility from his compensation, and that is traded off against the increased cost of effort. A linear compensation function with incentive rate

$$v_t = -\operatorname{Cov}(u_t^\dagger,\psi_t)/\{r\mathrm{E}[u_t^\dagger]\}$$

would induce the same action, but would impose more incentive risk.

Stochastic Interdependence

If $\rho \neq 0$, $\boldsymbol{s}(\psi_1,\psi_2)$, $\boldsymbol{G}(\psi_1,\psi_2)$, and $u^\dagger(\psi_1,\psi_2) \equiv -\boldsymbol{G}(\psi_1,\psi_2)^{-1}$ are strictly non-separable functions and $\boldsymbol{a}_2(\psi_1)$ varies with ψ_1 (except possibly in knife-edge cases), even though the two statistics are independently distributed. The optimal contract is characterized by (27.15a). As noted earlier, this contract compensates the agent for the conjectured cost of his effort in each period, conditional on the information at the date the action is chosen. Hence, the agent is insured by the principal against "effort-cost risk."

To provide insight into the factors influencing the agent's equilibrium effort choice, note that the agent's equilibrium realized utility can be decomposed as

$$u^\dagger(\psi_1,\psi_2) = -\,u_1^\dagger(\psi_1)\,u_2^\dagger(\psi_1,\psi_2), \tag{27.19}$$

where

$$u_1^\dagger(\psi_1) \equiv \int_{\Psi_2} u^\dagger(\psi_1,\psi_2)\,d\Phi^\dagger(\psi_2),$$

$$u_2^\dagger(\psi_1,\psi_2) \equiv -\frac{u^\dagger(\psi_1,\psi_2)}{u_1^\dagger(\psi_1)}.$$

Assume without loss of generality that the agent's reservation utility is equal to minus one, i.e., $\bar{U}^a = -1$, corresponding to a reservation wage equal to zero. We can then interpret the function $u_t^\dagger(\cdot)$ as a measure of the agent's period t equilibrium realized utility, $t = 1, 2$, since

$$\int_{\Psi_2} u_2^\dagger(\psi_1,\psi_2)\,d\Phi^\dagger(\psi_2) = -1, \quad \forall\,\psi_1, \tag{27.20a}$$

and

$$\int_{\Psi_1} u_1^\dagger(\psi_1)\,d\Phi^\dagger(\psi_1) = -1. \tag{27.20b}$$

In other words, the decomposition is as if the agent has zero reservation wage in each period, but note that the realized utility in the second period, $u_2^\dagger$, may depend on both ψ_2 and ψ_1. Substituting the agent's equilibrium utility for the optimal contract in (27.15) into the first-order condition (27.14b) for the second-period action, and rearranging terms so that κ_2' is on the left-hand side yields:

$$\kappa_2'(\boldsymbol{a}_2^\dagger(\psi_1)) = \frac{M_2}{r(1-\rho^2)}\mathrm{Cov}(u_2^\dagger,\psi_2\,|\,\psi_1). \tag{27.21a}$$

The expression for induced second-period action in (27.21a) has the same structure as in the independence case (see (27.18)). The induced second-period action is determined by its direct impact on the second-period performance statistic as reflected in the likelihood ratio $L_{a_2}(\psi_2) = M_2\psi_2/(1-\rho^2)$, and the likelihood ratio's covariance with the agent's second-period utility, $u_2^\dagger$. The key difference is that the agent's second-period utility in the interdependence case depends on the reported first-period statistic and, thus, the induced action will vary with the first-period report. Note that the agent's first-period utility, $u_1^\dagger$, has no impact on the agent's second-period effort choice. Hence, a linear compensation function for the second period with incentive rate

$$\upsilon_2(\psi_1) = \frac{\mathrm{Cov}(u_2^\dagger,\psi_2\,|\,\psi_1)}{r(1-\rho^2)}$$

would induce the same second-period effort choices contingent on the first-period report. This type of variation in second-period effort choices does not occur in the *LEN* model. However, in the next section we introduce an extension to the *LEN* contract, called the *QEN* contract, in which we allow the second-period incentive rate to depend linearly on the first-period report.

Using (27.20a) we can restate the first-order condition (27.14c) for the first-period action as

$$\int_{\Psi_1} u_1^\dagger(\psi_1)\big[r\kappa_1'(a_1^\dagger) + L_{a_1}(\psi_1) + q_2^\dagger(\psi_1)\big]\,d\Phi^\dagger(\psi_1) = 0,$$

where
$$q_2^\dagger(\psi_1) \equiv \mathrm{Cov}(u_2^\dagger,\psi_2\,|\,\psi_1)\,\frac{\rho M_1}{1-\rho^2}.$$

Using (27.20b), and rearranging terms yields:

$$\kappa_1'(a_1^\dagger) = \frac{1}{r}\Big[M_1\,\mathrm{Cov}(u_1^\dagger,\psi_1) - \mathrm{E}[q_2^\dagger] + \mathrm{Cov}(u_1^\dagger,q_2^\dagger)\Big]. \tag{27.21b}$$

The induced first-period action is influenced by three types of incentives. First, it is influenced by its direct impact on the first-period performance statistic, M_1, and the covariance between the first-period performance statistic and his first-period utility (as determined by the optimal compensation contract and the conjectured actions). Note that, if the two periods are stochastically independent, i.e., $\rho = 0$, then $q_2^\dagger = 0$, and, thus, the first-period action is only influenced by the direct first-period incentive.

However, if $\rho \neq 0$, the induced first-period action is influenced by two additional indirect incentives as reflected by the last two terms in (27.21b). Both are due to the impact of the first-period action on the second-period performance statistic as reflected in the likelihood ratio $L_{a_1}(\psi_2) = -\rho M_1 \psi_2/(1-\rho^2)$. In order to better understand how this likelihood ratio affects the agent's first-period effort choice, suppose $\mathrm{Cov}(u_2^\dagger, \psi_2 \mid \psi_1)$ is a constant independent of ψ_1. This would be the case, if the set of feasible compensation functions were restricted to being the set of additively separable functions of ψ_1 and ψ_2. In this setting the induced second-period action would be independent of ψ_1 (see (27.23a)), and $q_2^\dagger(\psi_1)$ would be a constant independent of ψ_1. Consequently, the last term in (27.23b) would equal zero, and (27.23b) would simplify to

$$\kappa_1'(a_1^\dagger) = \frac{1}{r}\left[M_1 \,\mathrm{Cov}(u_1^\dagger, \psi_1) - \mathrm{Cov}(u_2^\dagger, \psi_2)\,\frac{\rho M_1}{1-\rho^2} \right]. \qquad (27.21\mathrm{b}')$$

Therefore, in this setting, the induced first-period action would be determined by the covariances between the likelihood ratios $L_{a_1}(\psi_1)$ and $L_{a_1}(\psi_2)$ and the first- and second-period utilities, respectively. Since $\mathrm{Cov}(u_2^\dagger, \psi_2) > 0$,[13] the orthogonalization incentive reflected by the last term is negative (positive) if $\rho > 0$ ($\rho < 0$). A linear compensation function with constant incentive rates,

$$v_1 = \frac{\mathrm{Cov}(u_1^\dagger, \psi_1)}{r}, \qquad v_2 = \frac{\mathrm{Cov}(u_2^\dagger, \psi_2)}{r(1-\rho^2)},$$

would induce the same first- and second-period effort choices (compare to Section 27.2.1):

$$\kappa_1'(a_1^\dagger) = (v_1 - \rho v_2) M_1, \qquad \kappa_2'(a_2^\dagger) = v_2 M_2.$$

In summary, if the compensation function is restricted to be additively separable with respect to the two performance statistics, then the direct incentive and the first indirect *posterior mean* incentive (reflected by the middle term in (27.23b))

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[13] This follows from (27.23a) using that $\boldsymbol{a}_2^\dagger(\psi_1) = a_2^\dagger > 0$, and the fact that $\kappa_2' > 0$.

are similar to those in a *LEN* model that is based on the performance statistics. However, the optimal compensation function characterized in (27.15) is not additively separable in the two performance statistics, i.e., the agent's second-period utility varies non-trivially with the first-period statistic ψ_1 and, thus, there is a second indirect first-period incentive reflected in (27.21b) by the third term, $\text{Cov}(u_1^\dagger, q_2^\dagger)$. This term is non-zero only if $\text{Cov}(u_2^\dagger, \psi_2 \mid \psi_1)$ varies with ψ_1 and, thus, only if the second-period action varies with ψ_1. If the variation in $u_2^\dagger$ with respect to ψ_1 is chosen such that $\text{Cov}(u_1^\dagger, q_2^\dagger) > 0$, then the principal obtains an additional *positive* indirect first-period *covariance incentive*. This additional indirect covariance incentive is not present in the *LEN* model (even if it is formulated in terms of the performance statistics).

This raises the question: why is it optimal for the principal to write a compensation contract that is not additively separable in the two stochastically independent performance statistics? Is it because it is optimal to induce variation in the agent's second-period effort choice, or is it because it creates indirect first-period effort incentives? Although it is hard to tell from the characterization of the optimal contract, the former does not seem to be substantive – the exponential utility function is characterized by no wealth effects, variation in second-period actions increases the agent's expected second-period effort costs (since $-\kappa(a_2)$ is convex) for which he must be compensated, and the expected marginal gross payoff to the principal from the second-period effort is independent of the first-period report. Hence, the likely explanation is that it provides an indirect covariance incentive for the first-period effort choice. That then raises a question as to whether this is a first- or a second-order effect, merely reflecting a "fine-tuning" of the contract. Unfortunately, the answers to these questions are not immediately provided by the above characterizations of the optimal contract.

The next section considers what we call a *QEN* contract. It is designed to mimic some of the key characteristics of the optimal contract discussed above and, therefore, it more closely approximates an optimal contract than does a *LEN* contract. This is accomplished by using a constant first-period incentive rate v_1, a second-period incentive rate $v_2(\psi_1)$ that varies linearly with ψ_1, compensation for the conjectured cost of the agent's second-period effort that is contingent on ψ_1, plus compensation for the agent's second-period risk premium that is also contingent on ψ_1. The model is sufficiently tractable to facilitate comparative statics that explicitly quantify the impact of the indirect covariance incentive for the first-period effort choice.

27.3.2 A *QEN* Contract of Indirect Covariance Incentives

In the standard *LEN* model, the contract is constrained to be a linear function of the performance measures. Hence, the second-period incentive rates and induced actions are independent of the specific information reported at the end of the first period. However, the preceding analysis establishes that in a setting with the *LEN* model preferences and performance measures, the optimal contract is not linear and induces second-period actions that vary with the first-period performance report. Unfortunately, the characterization of the optimal contract is complex.

In this section we again consider a setting with *LEN* model preferences and performance measures. To facilitate our analysis, the form of the contract is again constrained, but in this section the linear contract is extended to permit the second-period incentive rate to vary linearly with the first-period report.[14] This leads to random variations in the second-period effort cost and the second-period risk premium. Furthermore, as in the optimal contract, the contract in this section includes "effort-cost risk insurance," and we also explicitly include "risk-premium risk insurance." Since the insurance payments are quadratic functions of the first-period report, we refer to this as a *QEN* contract.

The Preferences and Performance Measures

We assume that a single agent with exponential *AC-EC* preferences, and a reservation wage of zero, is hired at date zero to take actions $a_1, a_2 \in \mathbb{R}$ in periods 1 and 2 at a personal cost of $½(a_1^2 + a_2^2)$ expressed in date 2 dollars. The contractible information consists of two performance reports,

$$y_t = M_t a_t + \varepsilon_t, \quad t = 1, 2,$$

where $\varepsilon_t \sim \text{N}(0,1)$ and $\text{Cov}(\varepsilon_1, \varepsilon_2) = \rho$. The reports are issued at dates 1 and 2, respectively, and are represented in the compensation contract by the stochastically independent sufficient statistics given in (27.12):

$$\psi_1 = M_1(a_1 - \hat{a}_1) + \delta_1 \tag{27.22a}$$

and

$$\psi_2 = M_2(a_2 - \hat{\boldsymbol{a}}_2(\psi_1)) - \rho M_1(a_1 - \hat{a}_1) + \delta_2, \tag{27.22b}$$

where $\delta_1 = \varepsilon_1$, $\delta_2 = \varepsilon_2 - \rho\varepsilon_1$, and $\hat{a}_1$ and $\hat{\boldsymbol{a}}_2(\psi_1)$ are the principal's conjectures with respect to the agent's actions. Observe that while the second-period action is independent of the first-period report (or statistic) in the *LEN* model, the *QEN* model allows the principal to induce second-period actions that vary with the information reported at the end of the first-period. In equilibrium, $a_1 = \hat{a}_1$ and $\boldsymbol{a}_2(\psi_1) = \hat{\boldsymbol{a}}_2(\psi_1)$, so that from the perspective of the principal, the performance

[14] The introduction of the variation of the second-period incentive rate with the first-period performance measure was initially motivated by the introduction of productivity information in Feltham, Indjejikian, and Nanda (2005). We explore the impact of productivity information in Section 27.4. In this section we establish that the variation is valuable even if there is no productivity information.

statistics are independently and jointly normally distributed with zero means, i.e.,

$$\psi_1 = \varepsilon_1 \sim \text{N}(0,1),\ \psi_2 = \varepsilon_2 - \rho\,\varepsilon_1 \sim \text{N}(0,1-\rho^2), \text{ and } \text{Cov}(\psi_1,\psi_2) = 0. \quad (27.23)$$

The principal is risk neutral, and we simplify the analysis by assuming zero interest rates (or, equivalently, all cash flows are expressed in date 2 dollars).

The QEN Contract

The *QEN* contract is constrained to take the following form:

$$s(\psi_1,\psi_2) = f_1 + v_1\psi_1 + f_2(\psi_1) + v_2(\psi_1)\psi_2. \quad (27.24a)$$

The *LEN* contract is a special case in which $f_2(\psi_1)$ and $v_2(\psi_1)$ are constrained to be independent of ψ_1. In a *QEN* contract we allow v_2 and f_2 to vary with the first-period performance report. However, the variation in the second-period incentive rate is constrained to take the following linear form:

$$v_2(\psi_1) = \bar{v}_2 + \gamma\psi_1. \quad (27.24b)$$

For expositional reasons, we divide the "fixed" payment into two components. The first component, denoted f_1, is independent of ψ_1 and compensates the agent for the principal's conjecture with respect to the agent's first-period effort and his first-period risk premium, i.e.,

$$f_1 = \tfrac{1}{2}\hat{a}_1^2 + \tfrac{1}{2}rv_1^2. \quad (27.24c)$$

The second component, denoted $f_2(\psi_1)$, compensates the agent for the principal's conjecture with respect to the agent's second-period effort costs and his second-period risk premium, conditional on the first-period report, i.e.,

$$f_2(\psi_1) = \tfrac{1}{2}\hat{\boldsymbol{a}}_2(\psi_1)^2 + \tfrac{1}{2}rv_2(\psi_1)^2(1-\rho^2). \quad (27.24d)$$

Insurance

If the second-period incentive rate varies with ψ_1, i.e., $\gamma \neq 0$, then the second-period effort-cost and the second-period risk premium will vary with ψ_1. The principal could make a fixed payment to compensate the agent for these costs, but that would require paying the expected effort cost and the expected risk premium, plus a premium to compensate the agent for bearing the effort-cost risk and his risk-premium risk. Imposing these risks on the risk averse agent serves no useful purpose. It is more efficient if they are borne by the risk neutral principal. Hence, the *QEN* contract provides insurance, in the form of $f_2(\psi_1)$, so that the expected compensation paid by the principal equals the agent's expected effort cost and expected risk premium. This is consistent with the optimal contract, which clearly has effort-cost risk insurance.

The Agent's Certainty Equivalent and Action Choices

Given the agent's first-period action choice a_1 and the first-period statistic ψ_1, the agent's date 1 certainty equivalent with respect to the second-period compensation (based on contract (27.24) and second-period action choice a_2) is[15]

$$CE_1(\psi_1,a_1,a_2,\hat{a}_1,\hat{\boldsymbol{a}}_2(\psi_1)) = (\bar{v}_2 + \gamma\psi_1)[M_2(a_2 - \hat{\boldsymbol{a}}_2(\psi_1)) - \rho M_1(a_1 - \hat{a}_1)]$$
$$- \tfrac{1}{2}[a_2^2 - \hat{\boldsymbol{a}}_2(\psi_1)^2]. \quad (27.25)$$

The second-period compensation paid to the agent depends on the principal's conjectures with respect to the agent's actions. The agent can choose whatever action he prefers. Hence, his expected net incentive compensation reflects the potential difference between the agent's choice of a_1 and a_2 compared to the principal's conjectured values. Observe that, in equilibrium, the induced effort equals the conjectured effort, so that the effort costs minus the effort-cost insurance equals zero. Since the second-period risk premium is not influenced by the action choice, the risk premium minus the risk premium insurance does not appear – the difference is zero.

Differentiating (27.25) with respect to a_2, provides the following characterization of the agent's second-period effort choice:

$$\boldsymbol{a}_2(\psi_1) = (\bar{v}_2 + \gamma\psi_1)M_2. \quad (27.26)$$

Recognizing that, in equilibrium, $\hat{\boldsymbol{a}}_2(\psi_1) = \boldsymbol{a}_2(\psi_1)$, yields the agent's optimal certainty equivalent at date 1, with respect to the second-period compensation:

$$CE_1^\dagger(\psi_1,a_1,\hat{a}_1) = -(\bar{v}_2 + \gamma\psi_1)\rho M_1[a_1 - \hat{a}_1]. \quad (27.27)$$

From (27.27) we compute the agent's *ex ante* certainty equivalent with respect to the first-period compensation and effort cost plus the second-period certainty equivalent (27.27):[16, 17]

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[15] Given ψ_1, $\boldsymbol{s}(\psi_1,\psi_2)$ is a linear function of ψ_2, which is normally distributed. Hence, with exponential *AC-EC* preferences, the certainty equivalent takes the standard mean-variance structure.

[16] Expression (27.27) is a linear function of ψ_1 so that the certainty equivalent again takes the standard mean-variance form.

[17] Note that there is no double shirking issue in this setting, since the agent's second-period action choice in (27.26) does not depend on his first-period action (given the first-period statistic).

$$CE_0(a_1,\hat{a}_1) = v_1 M_1[a_1 - \hat{a}_1] - (\bar{v}_2 + \gamma M_1[a_1 - \hat{a}_1])\rho M_1[a_1 - \hat{a}_1]$$
$$- \tfrac{1}{2}[a_1^2 - \hat{a}_1^2] + \tfrac{1}{2} r v_1^2 - \tfrac{1}{2} r\{v_1 - \gamma\rho M_1[a_1 - \hat{a}_1]\}^2. \quad (27.28)$$

Differentiating with respect to a_1, and then setting $\hat{a}_1 = a_1$, yields

$$a_1 = [v_1 - \bar{v}_2\rho + r v_1 \gamma\rho] M_1. \quad (27.29)$$

Impact of γ on the Action Induced by a QEN Contract

Observe that, in (27.29), a_1 is the result of three sources of incentives. The first ($v_1 M_1$) is the direct incentive resulting from the application of the first-period incentive rate to the first-period performance statistic, which has a mean that is increasing in a_1. The second ($-\bar{v}_2 \rho M_1$) is an indirect posterior mean incentive resulting from the impact of a_1 on y_1 which in turn affects the calculation of ψ_2 through an increase in the posterior mean of y_2 (for positive correlation). These two effects also occur in the *LEN* model if it is written in terms of the performance statistics (see Section 27.2.2).

The third component ($r v_1 \gamma\rho M_1$) is an indirect incentive that arises from the covariance between $v_1\psi_1$ and the agent's date 1 certainty equivalent if the agent takes a first-period action other than the action conjectured by the principal (see (27.27)). The date 0 variance using that certainty equivalent is

$$\text{Var}[v_1\psi_1 + CE_1^\dagger(\psi_1,a_1) \,|\, a_1] = v_1^2 - 2v_1\gamma\rho M_1[a_1 - \hat{a}_1] + \{\gamma\rho M_1[a_1 - \hat{a}_1]\}^2.$$

Hence, $CE_0(a_1)$ takes the form specified in (27.28). The derivative of the variance with respect to a_1 is $-2v_1\gamma\rho M_1 + 2\{\gamma\rho M_1[a_1 - \hat{a}_1]\}$, which, in equilibrium, equals $-2v_1\gamma\rho M_1$. Hence, both the covariance and variance terms are affected by the agent's action choice, but only the marginal impact of the first-period action on the covariance is non-zero in equilibrium (i.e., from the agent's action choice perspective).

Assuming that $v_1 > 0$, observe that the sign of the indirect covariance incentive is the same as the sign of $\gamma\rho$. Therefore, it equals zero if either γ or ρ equal zero. Furthermore, the *QEN* model induces the same actions as the *LEN* model if $\gamma = 0$.

In the standard *LEN* model (based directly on the reports y_1 and y_2), the incentive compensation in the first and second periods are correlated if $\rho \neq 0$. However, the incentive compensations (excluding the second-period "fixed" wage) are also correlated in the *QEN* model if $\gamma \neq 0$, even if $\rho = 0$. Nonetheless, there is no indirect incentive effect on the first-period action choice in that case. To see this, observe that (27.27) implies that, in equilibrium (i.e., $a_2 = \hat{\boldsymbol{a}}_2(\psi_1)$), the agent's date 1 certainty equivalent is equal to zero if $\rho = 0$. Hence, while the incentive compensation in the two periods are correlated, the correlation between the first-period compensation and $CE_1^\dagger$ equals zero. The key to this result is the principal's use of risk-premium risk insurance in the agent's second-period compensation. In particular, while the second-period incentive compensation has a risk premium that varies with ψ_1, the risk-premium risk insurance in $f_2(\psi_1)$, as its name implies, precisely offsets the variation in the second-period risk premium. It is also important here that ψ_1 only impacts the second-period compensation through $v_2(\psi_1)$, whereas ψ_1 also affects beliefs about ψ_2 if $\rho \neq 0$. *Hence, the indirect covariance incentive in (27.29) only occurs if both sources of covariance exist.*

The *QEN* contract is equivalent to the *LEN* contract if $\gamma = 0$. We now establish that, if the principal is constrained to offer a *QEN* contract, then it is optimal for him to choose $\gamma \neq 0$ if $\rho \neq 0$.

Principal's Contract Choice

The principal's *QEN* contract choice parameters are v_1, $\bar{v}_2$, and γ. He chooses those parameters so as to maximize the following *ex ante* expected net payoff:

$$U^p(v_1,\bar{v}_2,\gamma) = b_1 a_1 + b_2 \mathrm{E}[\boldsymbol{a}_2(\psi_1) \mid a_1] - \{½ a_1^2 + ½\mathrm{E}[\boldsymbol{a}_2(\psi_1)^2 \mid a_1]\}$$

$$- ½ r\{v_1^2 + \mathrm{E}[(\bar{v}_2 + \gamma\psi_1)^2(1-\rho^2) \mid a_1]\}. \quad (27.30)$$

Substituting the equilibrium induced actions from (27.26) and (27.29), and the equilibrium distributions (27.23) into (27.30), and then differentiating with respect to v_1, $\bar{v}_2$, and γ, yields the following characterization of the optimal *QEN* contract.

Proposition 27.2

In the *QEN* model described above, the optimal incentive rate parameters are characterized by the following first-order conditions:[18]

$$v_1^\dagger = \frac{1}{D}\{(b_1 M_1)[M_2^2 + r(1-\rho^2)] + (b_2 M_2)\rho M_1^2\}\{1 + r\gamma^\dagger\rho\}, \quad (27.31a)$$

$$\bar{v}_2^\dagger = \frac{1}{D}\{(b_2 M_2)[M_1^2(1 + r\gamma^\dagger\rho)^2 + r] - (b_1 M_1) r\rho\}, \quad (27.31b)$$

$$\gamma^\dagger = \frac{[r v_1^\dagger \rho M_1][b_1 - a_1^\dagger]}{M_2^2 + r(1-\rho^2)}, \quad (27.31c)$$

[18] The first-order condition for γ is necessary but not sufficient in this setting. The principal's expected utility for optimal incentive rates v_1 and $\bar{v}_2$ for a fixed γ is not a concave function of γ. Hence, the optimal contract can only be found numerically.

where $D \equiv [M_1^2(1 + r\gamma^\dagger\rho)^2 + r][M_2^2 + r(1 - \rho^2)] + r\rho^2 M_1^2.$

The expression for $\gamma^\dagger$ in (27.31c) can be viewed as determining the level of γ at which the marginal cost of increasing γ equals its marginal benefit. The numerator equals the marginal first-period net benefit of an increase in γ, whereas γ times the denominator equals the marginal second-period cost of an increase in γ. More specifically, the first term in the numerator is the marginal impact of γ on $a_1^\dagger$ (see (27.29)), while the second term is the principal's marginal gross payoff minus the agent's marginal cost of an increase in a_1. The latter is positive if less than first-best effort is induced. We assume rM_1 is positive, and v_1 is positive unless the first-period performance measure is primarily used for insurance purposes.[19] Therefore, the sign of the numerator and, hence, the sign of $\gamma^\dagger$, is the same as the sign of ρ, so that their product is positive.

The denominator reflects the fact that increasing γ increases the variability of the second-period effort and payoff. The payoff is a linear function of ψ_1, so there is no change in the expected second-period gross payoff. However, the second-period effort cost and risk premium are affected because they are quadratic functions of ψ_1. They are the two components of the denominator.

The expressions in (27.31a&b) are complex, so that it is difficult to develop insights from comparative statics. Consequently, we use numerical examples.

Figure 27.1 illustrates the impact of the performance measure correlation on the principal's expected net payoff for a setting in which the periods are identical, with $b_1 = b_2 = b = 10$, $M_1 = M_2 = M = 1$, and $r = 1$. We compare the results that occur with a *LEN* contract (in which γ is exogenously constrained to equal zero) to a *QEN* contract (in which γ is chosen so as to maximize the principal's expected payoff). Figure 27.1 also plots the optimal choice of γ in the *QEN* contract.

With the *LEN* contract, increasing the correlation results in a reduction in the principal's expected payoff. The key here is that the riskiness of the contract (and, hence, the risk premium paid to the agent) increases as ρ increases.[20]

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[19] Interestingly, if the contract is written in terms of the performance measures y_t, then in a window-dressing case with $b_1 = 0$, the first-period performance measure is used strictly for insuring the agent's second-period incentive risk, and v_1 is negative if $\rho > 0$ (see, for example, Christensen *et al.*, 2004). However, if the contract is written in terms of the performance statistics ψ_t, then the insurance role is handled by the orthogonalization used in computing ψ_2. This creates positive indirect first-period incentives if $\rho < 0$, and, hence, $v_1^\dagger$ is negative in order to offset the indirect first-period incentive (since a_1 is costly and provides no benefit). This is reflected in the term $-\rho\,\overline{v}_2^\dagger M_1 > 0$ in (27.29). In the following discussion, we assume that $v_1^\dagger[b_1 - a_1^\dagger] > 0$.

[20] This is most easily seen for the equivalent contract expressed in terms of the performance measures (see Section 26.1.4).

This is consistent with the results for the *LEN* model in the preceding chapter (see Figure 26.1).[21]

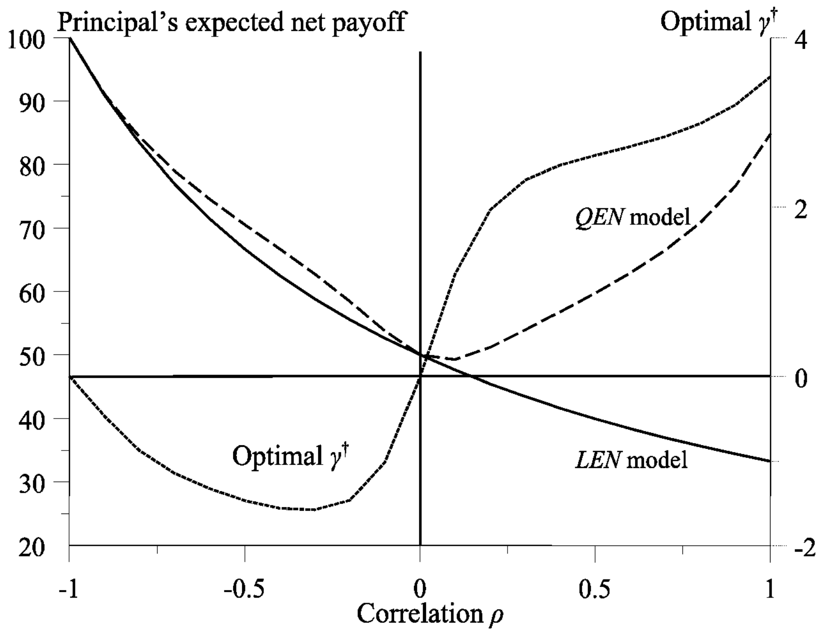

Figure 27.1: Impact of performance measure correlation in *LEN* and *QEN* models for identical periods case.

The difference between the results for the *QEN* contract and the *LEN* contract is striking! The payoffs are identical if the performance measures are uncorrelated ($\rho = 0$), since that results in the principal choosing $\gamma^\dagger = 0$ in the *QEN* contract. However, if the two performance measures are correlated (positively or negatively) it is optimal in the *QEN* contract for the principal to choose $\gamma^\dagger \neq 0$, and this yields distinctly higher expected net payoffs than the *LEN* contract (especially for $\rho > 0$). Furthermore, while the principal's payoff is again decreasing in ρ if it is negative, his payoff is increasing in ρ if it is positive (except for very low values of ρ). This may seem counterintuitive and warrants further exploration.

As discussed above, setting $\gamma \neq 0$ creates an indirect first-period incentive effect if $\rho \neq 0$ (see (27.29)). Since that is the only role of γ in the basic *QEN*

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[21] Note that we use exponential *AC-EC* preferences in Figure 27.1, whereas exponential *TA-EC* preferences are used in Figure 26.1.

model, it is always possible to set γ so that the indirect incentive effect $r\upsilon_1^\dagger\gamma\rho M_1$ is positive (i.e., $\gamma^\dagger$ and ρ have the same sign as shown in Figure 27.1). Increased first-period effort results in an increase in the expected first-period payoff and the first-period effort cost. Since the induced effort is less than first-best, the marginal net benefit from increased effort (i.e., $b_1 - a_1^\dagger$) is strictly positive. The marginal net benefit times the marginal effect of γ on $a_1^\dagger$ is $r\upsilon_1^\dagger\rho M_1[b_1 - a_1^\dagger]$. Due to the interaction of the two sources of covariance discussed above, the marginal net benefit from increasing γ is increasing in ρ if ρ is positive. On the other hand, if ρ is negative, then $\gamma^\dagger$ is negative (so that $\rho\gamma^\dagger > 0$) and increasing ρ decreases $r\upsilon_1^\dagger\gamma^\dagger\rho M_1$.[22] Consequently, the principal's expected payoff with the optimal *QEN* contract is "U"-shaped, as depicted in Figure 27.1. Of course, since first-best is obtained in the *LEN* model for the identical periods case if $\rho = -1$, first-best is also obtained in the *QEN* model with $\gamma^\dagger = 0$.

27.4 LEARNING ABOUT EFFORT PRODUCTIVITY

In the preceding section, the correlation between payoffs in the two periods is attributable to the correlation of random factors that create additive noise in the performance measure. Those random factors do not influence the impact of effort on either the principal's payoff or the measure of the agent's performance. In this section we consider two settings in which the rate of second-period productivity is random and the first-period report is informative about that rate. The first productivity model, which we call the *QEN-P* model has *LEN* model preferences and performance measures, a *QEN* contract (see Section 27.3.2), and correlation between the first-period performance measure and the second-period productivity. It is based on one of the models in Feltham, Indjejikian, and Nanda (2005). The second productivity model is a multi-period extension of the hurdle model that has been used throughout this volume. It has elements that are similar to Hirao (1993). Both are two-period models.

As in the *LEN* model, the noise in the performance reports is additive and correlated. However, unlike the *LEN* model, the noise in the first-period performance measure is correlated with the productivity of effort in the second period. This creates a direct demand to vary the agent's second-period action with the first-period performance. Hence, we use the *QEN* contract introduced in Section 27.3.2 to implement that variation. Furthermore, to simplify the analysis, we assume the *QEN-P* model performance reports do not include the principal's payoff.

In the multi-period hurdle model we assume the principal's payoff in each period is contractible and these payoffs are the only performance measures. As

[22] Note from (27.31) that $\upsilon_1^\dagger > 0$ in the identical periods case.

in previous hurdle models, we do not constrain the form of the contracts and, therefore, consider optimal contracts.

27.4.1 A *QEN-P* Model

The *QEN-P* model is the same as the basic *QEN* model in Section 27.3.2 except that the noise in the first-period performance measure is correlated with the productivity of effort in the second period. The principal's payoff and the performance measures are as described in the "learning about payoff productivity" example in Section 27.1.2, i.e.,

$$x = \theta_1 a_1 + \theta_2 a_2, \tag{27.32a}$$

$$y_t = M_t a_t + \varepsilon_t, \quad t = 1,2, \tag{27.32b}$$

where θ_1 and θ_2 are normally distributed productivity parameters with means b_1 and b_2, i.e., $\theta_t \sim N(b_t,\sigma^2)$, and $\varepsilon_t \sim N(0,1)$ and $Cov(\varepsilon_1,\varepsilon_2) = \rho_y$. We assume the first-period report is informative about the second-period productivity as reflected by $Cov(\varepsilon_1,\theta_2) = \rho_\theta\sigma$. Note that risk neutrality of the principal implies that the other covariances are irrelevant, and that the basic *QEN* model is a special case of the *QEN-P* model in which $\rho_\theta = 0$. As in the basic *QEN* model, the compensation contract is based on the statistics given in (27.12):

$$\psi_1 = M_1(a_1 - \hat{a}_1) + \delta_1 \tag{27.33a}$$

and

$$\psi_2 = M_2(a_2 - \hat{\boldsymbol{a}}_2(\psi_1)) - \rho_y M_1(a_1 - \hat{a}_1) + \delta_2, \tag{27.33b}$$

where $\delta_1 = \varepsilon_1$, $\delta_2 = \varepsilon_2 - \rho_y\varepsilon_1$, and, thus, $Cov(\delta_1,\theta_2) = \rho_\theta\sigma$.

The agent's first- and second-period action choices take the same form as in (27.29) and (27.26), but with ρ_y replacing ρ. Observe that, since the principal's payoff is not contractible, ρ_θ has no direct effect on the agent's decision problem. As the following analysis demonstrates, the only impact of ρ_θ on the agent is through the principal's choice of γ. In section 27.3.2 we established that the *QEN* contract with $\gamma \neq 0$ dominates the *LEN* contract because of the indirect first-period covariance incentives it provides. Now, with $\rho_\theta \neq 0$, we have another important reason for setting $\gamma \neq 0$. For example, if $\rho_\theta > 0$, then for the purposes of improving the second-period expected payoff, the principal will want to set $\gamma > 0$, so that the induced second-period action is an increasing function of $E[\theta_2 \mid \psi_1]$. This may be consistent with or contrary to the use of $\gamma \neq 0$ so as to provide indirect first-period covariance incentives for inducing a more efficient first-period effort.

First-best Contract

Before deriving the optimal second-best contract it is useful to consider the optimal first-best contract. In the first-best setting, the agent's marginal costs are equal to a_1 and a_2 and the principal's marginal benefits (given the information at the time the actions are chosen) are $E[\theta_1] = b_1$ and $E[\theta_2 \mid \psi_1] = b_2 + \rho_\theta \sigma \psi_1$. Hence, the first-best action choices are

$$a_1^* = b_1 \quad \text{and} \quad \mathbf{a}_2^*(\psi_1) = b_2 + \rho_\theta \sigma \psi_1.$$

Obviously, the first-best second-period action varies with ψ_1, which implies that the second-period effort cost, $½\mathbf{a}_2^*(\psi_1)^2 = ½(b_2 + \rho_\theta \sigma \psi_1)^2$, is a random variable from the perspective of date 0. The agent could be paid a fixed wage that is specified at date 0 and compensates him for his anticipated second-period effort cost, but that would require paying him for both the expected effort cost and a premium to compensate him for his effort-cost risk. That risk serves no useful purpose and is insurable by the risk neutral principal.[23] Hence, the first-best compensation contract pays the agent

$$\mathbf{s}^*(\psi_1) = ½[b_1^2 + (b_2 + \rho_\theta \sigma \psi_1)^2]$$

if he takes the first-best actions. As a result, the agent's realized net consumption is constant at zero.

The principal's first-best expected net payoff is

$$U^{p*} = ½\left[b_1^2 + b_2^2 + (\rho_\theta \sigma)^2\right].$$

Note that the principal's expected net payoff is increasing in the quality of the productivity information as measured by $(\rho_\theta \sigma)^2$. The quality of the productivity information is increasing in the prior variability in the second-period productivity of effort (σ^2) and the "preciseness" (ρ_θ^2) of the first-period information about the second-period productivity of effort.

Principal's Contract Choice

Now consider the principal's choice of the second-best incentive contract parameters v_1, $\bar{v}_2$, and γ in the *QEN-P* model. He chooses those parameters so as to maximize the following *ex ante* expected net payoff:

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[23] Note that while this is optimal for *EC*-preferences, the first-best compensation is a constant with *ED*-preferences even though the agent's second-period effort varies with the first-period statistic. The reason is that with *ED*-preferences cash compensation cannot be used to insure the effort disutility risk.

$$U^P(v_1,\bar{v}_2,\gamma) = b_1 a_1 + \mathrm{E}[\theta_2 \boldsymbol{a}_2(\psi_1)\,|\,a_1] - \{½a_1^2 + ½\mathrm{E}[\boldsymbol{a}_2(\psi_1)^2\,|\,a_1]\}$$

$$- ½r\{v_1^2 + \mathrm{E}[(\bar{v}_2 + \gamma\psi_1)^2(1-\rho_y^2)\,|\,a_1]\}. \quad (27.34)$$

Substituting the equilibrium induced actions from (27.26) and (27.29), and the equilibrium distributions (27.23) into (27.34), and then differentiating with respect to v_1, $\bar{v}_2$, and γ, yields the following characterization of the optimal incentive contract.

Proposition 27.3

In the *QEN-P* model described above, the optimal incentive rate parameters are characterized by the following first-order conditions:[24]

$$v_1^\dagger = \frac{1}{D}\{(b_1 M_1)[M_2^2 + r(1-\rho_y^2)] + (b_2 M_2)\rho_y M_1^2\}\{1 + r\gamma^\dagger\rho_y\}, \quad (27.35a)$$

$$\bar{v}_2^\dagger = \frac{1}{D}\{(b_2 M_2)[M_1^2(1 + r\gamma^\dagger\rho_y)^2 + r] - (b_1 M_1) r\rho_y\}, \quad (27.35b)$$

$$\gamma^\dagger = \frac{[r v_1^\dagger \rho_y M_1][b_1 - a_1^\dagger]}{M_2^2 + r(1-\rho_y^2)} + \frac{\rho_\theta \sigma M_2}{M_2^2 + r(1-\rho_y^2)}, \quad (27.35c)$$

where $\quad D \equiv [M_1^2(1 + r\gamma^\dagger\rho_y)^2 + r][M_2^2 + r(1-\rho_y^2)] + r\rho_y^2 M_1^2.$

Observe that the first two first-order conditions are precisely the same as in the basic *QEN* model (see 27.31a&b). The key difference occurs in (27.35c), which has a second term which is non-zero if $\rho_\theta \neq 0$. In both components of $\gamma^\dagger$, the denominator times $\gamma^\dagger$ is equal to the marginal expected second-period effort cost and risk premium resulting from an increase in γ. As discussed in Section 27.3.2, the numerator in the first component of (27.35c) is the marginal impact of γ on a_1 times the marginal impact of a_1 on the difference between the principal's first-period payoff minus the agent's first-period effort cost. It is equal to zero if the two performance measures are uncorrelated.

The numerator of the second component of $\gamma^\dagger$ equals the marginal impact of γ on the expected second-period payoff to the principal. This is increasing in the covariance between the first-period report and the marginal payoff productivity of second-period effort. If both ρ_y and ρ_θ equal zero, then $\gamma^\dagger$ equals zero and the optimal incentive rates are characterized by (27.35a) and (27.35b)

[24] As in the basic *QEN* model the first-order condition for γ is necessary but not sufficient in this setting.

and they are the same as in the *LEN* model with independent periods, i.e., $v_1^\dagger = b_1 M_1/(M_1^2 + r)$ and $\bar{v}_2^\dagger = b_2 M_2/(M_2^2 + r)$.

Interestingly, if $\rho_y = 0$ and $\rho_\theta \neq 0$, then $\gamma^\dagger = \rho_\theta \sigma M_2/(M_2^2 + r)$, but this does not impact $v_1^\dagger$ or $\bar{v}_2^\dagger$ since γ is always multiplied by ρ_y in (27.35a&b). Hence, if the performance measures are uncorrelated, then the expected second-period effort is the same as in the independent periods case, but the induced second-period effort varies around that mean so as to match it with the productivity information provided by the first-period performance measure. Furthermore, in this setting, the principal's expected net payoff is a linear increasing function of the quality of the productivity information, $(\rho_\theta \sigma)^2$, and, thus, independent of the sign of the correlation between the first-period report and the second-period productivity (as in the first-best setting). The sign of this correlation only affects the sign of $\gamma^\dagger$, but not its absolute value.

The results are subtle when both ρ_y and ρ_θ are non-zero. In this setting, the optimal γ is determined both by the productivity information and the impact of γ on the covariance incentive effect on the first-period action. In (27.35c), the first expression reflects the first-period indirect covariance incentive effect and its sign is the same as the sign of ρ_y if $v_1(b_1 - a_1) > 0$, whereas the second expression reflects the productivity effect and its sign is the same as the sign of ρ_θ. If ρ_θ and ρ_y are both positive (or both negative), then both effects call for a positive (negative) slope γ in the second-period incentive rate. On the other hand, if ρ_y is positive and ρ_θ is negative, the productivity information calls for a negative slope, whereas the indirect first-period covariance incentive calls for a positive slope (such that $v_1 \gamma \rho_y M_1$ is positive). In this case, the desired direct second-period incentive and the indirect first-period incentive work in opposite directions. We use the following language to distinguish between these cases.

Definition

The information system provides *congruent correlations* if ρ_θ and ρ_y are both positive or both negative. Otherwise, the information system is said to provide incongruent correlations.

Figures 27.2(a) and (b) depict the principal's expected net payoff and his optimal choice of γ, respectively, as functions of ρ_y for five values of ρ_θ: 0, ±½, and ±1. The $\rho_\theta = 0$ case is the same as in Figure 27.1.

If the information system provides congruent correlations, the productivity information and the first-period indirect covariance incentive work in the same direction in the determination of the optimal slope $\gamma^\dagger$. Hence, the principal's expected payoff is substantially greater using the *QEN* contract compared to the *LEN* contract. Note also that these gains are increasing in the quality of the productivity information, $(\rho_\theta \sigma)^2$.

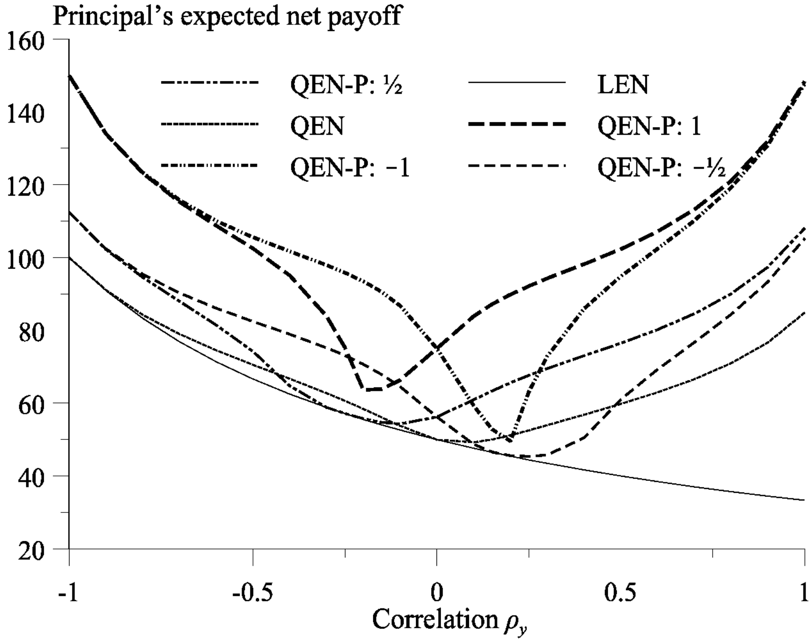

Figure 27.2(a): Impact of performance measure correlation in *LEN*, *QEN*, and *QEN-P* models with varying productivity information (ρ_θ) for identical periods case.

The situation changes when the correlations are incongruent. Consider Figure 27.2(b), which depicts the impact on γ of varying ρ_y from -1 to $+1$ for fixed ρ_θ. For $\rho_\theta = -½$ and values of ρ_y between -1 and 0, the correlations are congruent, and the desired incentives are implemented with $\gamma^\dagger < 0$. Incongruence occurs for ρ_y between 0 and $+1$. To understand what is happening in this region it is useful to recall that the action choices are characterized by[25]

$$a_1 = [v_1 - \bar{v}_2\rho_y + rv_1\gamma\rho_y]M_1, \tag{27.36a}$$

$$\boldsymbol{a}_2(\psi_1) = (\bar{v}_2 + \gamma\psi_1)M_2. \tag{27.36b}$$

We focus on two choice variables in these expressions, v_1 and γ. For a_1, v_1 is the direct incentive, while $rv_1\gamma\rho_y M_1$ is the indirect incentive of interest. For a_2, $\gamma\psi_1 M_1$ is the direct incentive of interest. When the correlations are incongruent the principal has three basic choices with respect to these two parameters.

[25] These equations are derived in (27.26) and (27.29).

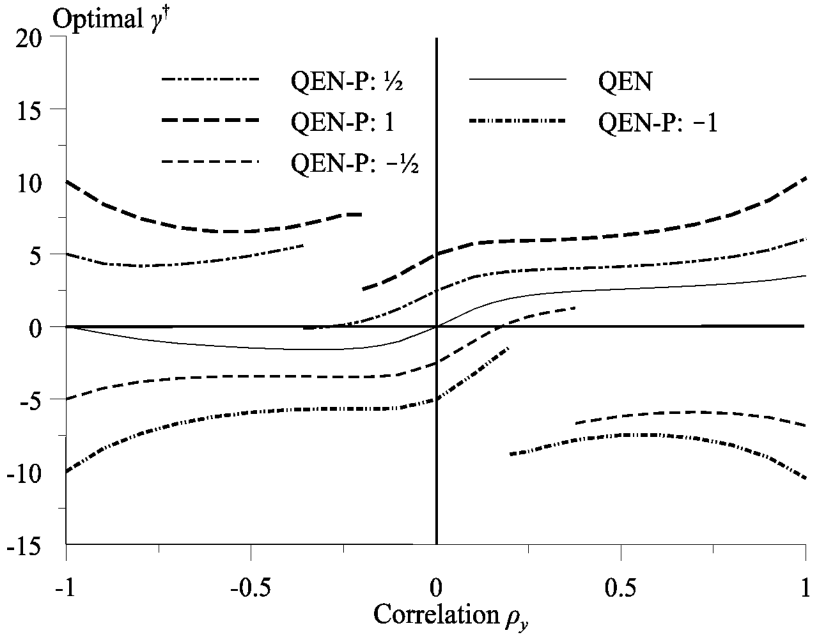

Figure 27.2(b): Impact of performance measure correlation in *LEN*, *QEN*, and *QEN-P* models with varying productivity information (ρ_θ) for identical periods case.

In the first option, the principal chooses γ so that it has the same sign as ρ_y. This results in positive direct and indirect first-period incentives with $v_1 > 0$, but induces the "incorrect" use of ψ_1 in setting the second-period incentive rate. That is, it induces high (low) second-period production when productivity is low (high).

In the second and third options, the principal chooses γ so that it has the same sign as ρ_θ. This "correctly" induces high (low) second-period production when productivity is high (low). The second and third options differ with respect to the two types of first-period incentives. Under the second option, v_1 is positive, which provides positive direct incentives, but negative indirect incentives. Under the third option, v_1 is negative, which provides negative direct incentives, but positive indirect incentives.

Now return to our example with $\rho_\theta = -½$ and ρ_y increasing from 0 to +1. For $\rho_y \in (0,.2)$, the indirect first-period incentives are relatively insignificant, so that γ is negative and v_1 is positive. That is, the direct first- and second-period incentives are "correct," while the indirect first-period incentive is "incorrect." At $\rho_y \approx .2$, γ equals zero, which is the point at which the *LEN* and *QEN* contracts are identical – the expected payoffs in Figure 27.2(a) are tangent at this point.

For $\rho_y \in (.2,.3)$ it is optimal to set $\gamma > 0$ and $v_1 > 0$, so that the direct and indirect first-period incentives are "correct," at the expense of "incorrect" second-period incentives. That "incorrectness" is relatively small since γ is close to zero. However, it becomes more significant as ρ_y increases. Consequently, at $\rho_y \approx .3$ it becomes optimal to make a significant shift in approach. In particular, for $\rho_y \in (.3, 1]$ it is optimal to set $\gamma < 0$ and $v_1 < 0$. This provides "correct" indirect first- and direct second-period incentives, at the expense of "incorrect" direct first-period incentives. The expected payoff reaches its minimum at the point of discontinuity in $\gamma^\dagger$, i.e., at $\rho_y \approx .3$.[26]

Similar patterns are observed for the other values of ρ_θ in Figure 27.2. However, note that, if the quality of the of the productivity information, $(\rho_\theta\sigma)^2$, is sufficiently high (as illustrated by $\rho_\theta = \pm 1$), then the first option with "incorrect" direct second-period incentives is never used (i.e., $\gamma^\dagger$ always has the same sign as ρ_θ). The discontinuities in $\gamma^\dagger$ reflect the shifts between the second and the third options.

In the *QEN-P* model, the *QEN* contract uniformly dominates the *LEN* contract for all values of ρ_θ and ρ_y. However, while having $\rho_\theta \neq 0$ implies that ψ_1 provides productivity information, such informativeness is not necessarily valuable. As illustrated in Figure 27.2(a), zero productivity information (i.e., $\rho_\theta = 0$) may or may not be preferred to having $\rho_\theta \neq 0$. The cases, in which productivity information destroys value, are characterized by incongruent correlations, and significant conflicting objectives in the determination of $\gamma^\dagger$.

Note that for all values of ρ_θ, first-best is attained for $\rho_y = -1$. In this case there is no second-period incentive risk. Therefore, the first-best second-period actions $\boldsymbol{a}_2^*(\psi_1)$ can be induced at the first-best expected cost using

$$\bar{v}_2 = b_2 / M_2, \quad \gamma = \rho_\theta \sigma / M_2,$$

(see (27.36b)). Moreover, if the first-period incentive rate v_1 is equal to zero, then there is no first-period risk premium, and the induced first-period action is (see (27.36a) with $\rho_y = -1$)

$$a_1 = \bar{v}_2 M_1 = b_2 M_1 / M_2.$$

Hence, the first-best first-period action $a_1^* = b_1$ is induced at first-best cost in the identical periods case, i.e., $b_1 = b_2$ and $M_1 = M_2$, irrespectively of the quality of the productivity information, $(\rho_\theta\sigma)^2$.

[26] In a region around $\rho_y = .3$ the principal's expected net payoff has two local optima as a function of γ. The discontinuity point for $\gamma^\dagger$ corresponds to the level of ρ_y where the global optimum changes from one to the other local optimum. Note that even though there is a discontinuity in $\gamma^\dagger$, the principal's optimal expected net payoff is continuous (but may not be differentiable).

It is also striking that close to first-best is obtained for ρ_y close to $+1$, irrespectively of whether the correlations are congruent or not. As with $\rho_y = -1$, there is no second-period risk premium. The first-period action is primarily induced through the first-period indirect covariance incentive using a high slope γ and a low direct first-period incentive. Actually, for $\rho_y = +1$, $\gamma^\dagger$ is numerically higher than the value that implements first-best second-period actions. This is optimal in order to induce the first-period action with a numerically lower direct first-period incentive and, thus, a lower first-period risk premium.

27.4.2 A Hurdle Model of Productivity Information

In this section we consider a two-period hurdle model in a setting similar to Hirao (1993). We assume the principal's gross payoffs, denoted b_1x_1 and b_2x_2, are contractible and period specific. These are the only performance measures, and they are influenced by a common random *productivity factor* θ. Hence, x_1 provides pre-decision productivity information with respect to the second-period action. A key characteristic of the model in this section is that the first-period action affects the information about θ. Hence, in choosing the level of first-period effort to be induced, the principal considers both the value and the information content of the first-period payoff. In the following analysis we consider a simple model in which learning about the productivity of effort helps make better decisions without increasing the costs of inducing actions.

The Basic Elements of the Two-period Hurdle Model

In each of the two periods, $t = 1, 2$, there is a binary outcome $x_t \in X_t = \{x_g, x_b\}$ with payoff b_tx_t to the principal, a hurdle $h_t \in [0,1]$, and an action $a_t \in A_t = [0,1]$, with $x_t = x_g$ if, and only if, $a_t \geq h_t$. The prior distribution for both hurdles is uniform, but they may be correlated. We consider the extreme setting in which the hurdle is the same in both periods, i.e., $h_1 = h_2$, and compare it to a benchmark setting in which the two hurdles are independent.

The principal is risk neutral, the agent has exponential *AC-EC* preferences, and the interest rate is equal to zero. In numerical examples we use the following data:

$$\kappa_t(a_t) = a_t/(1 - a_t);\;\; r = ½;\;\; c^o = 0;\;\; x_g = 20, x_b = 10;\;\; b_1 = b_2 = 1.$$

Independent Hurdles

We first consider the benchmark setting in which the hurdles are independent. Given our specification of the agent's utility function, there exists an optimal contract on the form $\mathbf{s}(x_1,x_2) = \mathbf{s}_1(x_1) + \mathbf{s}_2(x_2)$, where $\mathbf{s}_t(x_t)$ can be found by solving a single-period problem (see Section 25.2.1), i.e.,

$$s_t(x_t) = \kappa_t(a_t) + \frac{1}{r}\ln\left[r\left(\lambda_t + \mu_t[r\kappa_t'(a_t) + L(x_t|a_t)]\right)\right].$$

The optimal single-period contract for the numerical example is shown in Table 27.1, where $s_{ti}^{\dagger}$ denotes the optimal compensation for payoff i in period t, $i = g$, b and $t = 1, 2$.

$U_t^P(\mathbf{s}_t^{\dagger}, a_t^{\dagger})$	$s_{tg}^{\dagger}$	$s_{tb}^{\dagger}$	$a_t^{\dagger}$
12.439	4.009	- 0.199	0.387

Table 27.1: Optimal single-period contract with independent hurdles.

Same Hurdle in both Periods

Now consider the setting in which the hurdle is the same in both periods, i.e., $h_1 = h_2 = h$. In this setting, the second-period beliefs about the hurdle are

$$\varphi(h|x_{1g}, a_1) = \begin{cases} \dfrac{1}{a_1} & \text{for } h \in [0, a_1], \\ 0 & \text{for } h \in [a_1, 1], \end{cases}$$

$$\varphi(h|x_{1b}, a_1) = \begin{cases} 0 & \text{for } h \in [0, a_1], \\ \dfrac{1}{1 - a_1} & \text{for } h \in [a_1, 1]. \end{cases} \tag{27.37}$$

That is, if the good payoff is observed in the first period, the hurdle is less than the first-period action, whereas if the bad payoff is observed, the hurdle is above the first-period action. This information is useful for the choice of the second-period action. Note that the information about the hurdle depends on both the first-period payoff and the first-period action and, thus, the optimal choice of the first-period action may be affected by its role of providing useful pre-decision productivity information for the second-period action. For example, the first-period payoff provides no information about the hurdle if the first-period action is either $a_1 = 0$ or $a_1 = 1$, but it does provide information about the hurdle if $a_1 \in (0, 1)$.

No Direct First-period Incentive Problem

To provide a simple illustration of this point consider a setting in which the first-period payoff has no direct value to the principal, i.e., $b_1 = 0$, and the agent has no disutility for first-period effort, i.e., $\kappa_1(a_1) = 0$ for all a_1.[27] Hence, there is no direct first-period incentive problem, and the only role for the first-period action is to generate useful information about the hurdle before the second-period action is taken.

In this setting, the first-best contract is such that the first-period action is set equal to the first-best hurdle for the second period, and the agent is paid to jump that hurdle in the second period if he is successful in the first. That is, $a_2^* = a_1^*$ if $x_1 = x_g$, and $a_2^* = 0$ if $x_1 = x_b$, where

$$a_1^* \in \underset{a}{\operatorname{argmax}}\ [b_2 x_g - \kappa_2(a)]a + b_2 x_b(1 - a) - c^o.$$

Furthermore, $s^*(x_g) = c^o + \kappa_2(a_2^*)$ and $s^*(x_b) = c^o$. Interestingly, the first-best result can be obtained if a_1 is contractible, even if a_2 is not. This is accomplished by paying the first-best wage if the second-period payoff is consistent with the first (and $a_1 = a_1^*$):

$$\mathbf{s}^*(x_{1g},x_{2g}) = s_{gg}^* = c^o + \kappa_2(a_1^*), \quad \mathbf{s}^*(x_{1g},x_{2b}) = P,$$

$$\mathbf{s}^*(x_{1b},x_{2g}) = \mathbf{s}^*(x_{1b},x_{2b}) = s_{bb}^* = c^o,$$

where P is sufficiently negative. Hence, the difference in compensation for good and bad payoffs is $s_{gg}^* - s_{bb}^* = \kappa_2(a_1^*)$.

Even if neither action is contractible, the optimal contract is still such that the first-period action is equal to the hurdle the principal wants the agent to clear in the second period given a good first-period payoff. Furthermore, the form of the optimal contract is similar to the optimal contract when a_1 is contractible. In particular,

$$\mathbf{s}^\dagger(x_{1g},x_{2g}) = s_{gg}; \quad \mathbf{s}^\dagger(x_{1g},x_{2b}) = P; \quad \mathbf{s}^\dagger(x_{1b},x_{2g}) = \mathbf{s}^\dagger(x_{1b},x_{2b}) = s_{bb}; \tag{27.38a}$$

$$\mathbf{a}_2^\dagger(x_{1g}) = a_1^\dagger \quad \text{and} \quad \mathbf{a}_2^\dagger(x_{1b}) = 0. \tag{27.38b}$$

Of course, due to the moral hazard problem, the compensation and induced actions differ from those in the preceding case.

Even though there is no direct incentive problem in the first period in this setting, there may be an induced moral hazard problem for the first-period action, since the optimal second-period action may depend on both x_1 and a_1 and the agent takes that into account when selecting his first-period action. The

[27] Note that the model in this setting is similar to the delegated information acquisition model in Section 22.6.

general form of the incentive constraint for the first-period action is given in (27.2) and for the second-period action in (27.1). The key in formulating these incentive constraints is that the first-period action affects the information that can be inferred from the first-period payoff and, thus, "shirking" in the first period may affect the optimal action strategy in the second.

Given the payments in the second-best contract and the form of the posterior beliefs in (27.37), it is particularly simple to characterize the agent's optimal second-period response function in this example, i.e.,

$$\boldsymbol{a}_2(x_{1g},\hat{a}_1) = \hat{a}_1, \quad \boldsymbol{a}_2(x_{1b},\hat{a}_1) = 0, \quad \forall\, \hat{a}_1 \in A_1. \tag{27.39}$$

That is, if the good first-period payoff is obtained, the hurdle is below $\hat{a}_1$, and to avoid the risk of a penalty for a bad payoff following a good payoff, the agent has to jump at least as high in the second period as in the first period, and there is no reason to jump any higher. If the bad first-period payoff is obtained, there is no premium to the agent for clearing the hurdle, so he does not jump in the second period.[28]

Given the agent's optimal second-period response function, the first-period incentive constraint can be formulated as

$$\begin{aligned} a_1 &\in \underset{\hat{a}_1}{\operatorname{argmax}}\ \hat{a}_1 u^a(s_{gg},\hat{a}_1,\hat{a}_1) + (1-\hat{a}_1)u^a(s_{bb},\hat{a}_1,0) \\ &= \underset{\hat{a}_1}{\operatorname{argmax}}\ -\hat{a}_1 \exp[-r(s_{gg}-\kappa_2(\hat{a}_1))] - (1-\hat{a}_1)\exp[-rs_{bb}]. \end{aligned} \tag{27.40}$$

Note that this incentive constraint is similar to the incentive constraint for a single-period problem with no information about the hurdle except that there is no effort cost for the "bad" payoff. The first-order condition for this incentive constraint is

$$s_{gg} - s_{bb} = \kappa_2(a_1) + \frac{1}{r}\ln(1 + ra_1\kappa_2'(a_1)).$$

This implies that the difference between the compensation levels is larger than in the first-best setting. Of course, this is due to the induced moral hazard problem for a_1 caused by the agent's optimal second-period response to (x_1, a_1).

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[28] Note that the agent's conditional expected utility is not differentiable at $\boldsymbol{a}_2(x_{1g},\hat{a}_1) = \hat{a}_1$. Hence, the first-order approach is not applicable and, therefore, the double shirking problem must be explicitly recognized. In fact, in this model it is the double shirking problem that creates the induced moral hazard problem in the first period.

Table 27.2 shows the optimal contract for the numerical example with no direct first-period incentive problem, i.e., $b_1 = 0$, $\kappa_1(a_1) = 0$.[29] Since the only role of the first-period action in this example is to generate information about the hurdle before the second-period action is taken, this contract is directly comparable to the optimal single-period contract with independent hurdles shown in Table 27.1. Note that the information generated by the first-period action is valuable to the principal since his expected utility goes up from 12.439 to 14.887, and that the agent jumps substantially higher in the second period given a good first-period payoff than he does when he must jump for both first-period payoffs with no information about the hurdle in the second period.

$U^p(\mathbf{s}^\dagger,\mathbf{a}^\dagger)$	$s_{gg}^\dagger$	$s_{gb}^\dagger$	$s_{bg}^\dagger$	$s_{bb}^\dagger$	$a_1^\dagger$	$\mathbf{a}_2^\dagger(x_{1g},a_1^\dagger)$	$\mathbf{a}_2^\dagger(x_{1b},a_1^\dagger)$
14.887	3.116	P	-1.25	-1.25	0.65	0.646	0

Table 27.2: Optimal contract with same hurdle in both periods and $b_1 = 0$, $b_2 = 1$, and $\kappa_1(a_1) = 0$.

Incentive Problems in both Periods

We now return to the setting in which there are incentive problems in both periods. Consider a contract of the type that is optimal when there is no direct incentive problem in the first period. That is, the agent is induced to select a first-period action equal to the hurdle induced in the second period. This is accomplished by paying s_{gg} if good payoffs occur in the both periods, s_{bb} if a bad payoff occurs in the first period (independent of the second period result), and $s_{gb} = P$ if a good payoff in the first period is followed by a bad payoff in the second period. Given this contract form, the second-period response function is again given by (27.39), but the incentive constraint on the first-period action (27.40) is now replaced with

$$a_1 \in \underset{\hat{a}_1}{\text{argmax}}\ \hat{a}_1 u^a(s_{gg},\hat{a}_1,\hat{a}_1) + (1-\hat{a}_1)u^a(s_{bb},\hat{a}_1,0)$$

$$= \underset{\hat{a}_1}{\text{argmax}}\ -\hat{a}_1 \exp[-r(s_{gg} - \kappa_1(\hat{a}_1) - \kappa_2(\hat{a}_1))]$$

$$- (1-\hat{a}_1)\exp[-r(s_{bb} - \kappa_1(\hat{a}_1))]. \qquad (27.41)$$

The first-order condition is

[29] The first-best choice of a_1 is $a_1^* = .698$ with an expected utility to the principal of 15.367 and $s_{gg}^* = 2.317$, $s_{bb}^* = 0$.

$$s_{gg} - s_{bb} = \kappa_2(a_1) + \frac{1}{r}\ln\big(1 + ra_1[\kappa_1'(a_1) + \kappa_2'(a_1)]\big) - \frac{1}{r}\ln(1 - r(1 - a_1)\kappa_1'(a_1)).$$

In this case there is a direct as well as an induced moral hazard problem for the first-period action and, therefore, there is a larger spread between the two compensation levels than when there is no direct first-period incentive problem.

The key source of the value of learning in this model is that the agent need not incur effort in the second period if he has a bad payoff in the first period, implying the hurdle is higher than his first-period effort. Of course, the benefit goes to the principal who can reduce his compensation cost. To illustrate this benefit consider the setting with independent hurdles. Table 27.1 reports that, in the independent payoff setting, the optimal effort in each period is .387 and the total payoff for two periods is $2 \times 12.439 = 24.878$. Now assume that the hurdles are the same in each period and the agent is offered the cost-minimizing contract for inducing $a_1 = a_2(x_{1g}) = .387$. As reported in Table 27.3, learning permits the principal to increase his expected two-period payoff to 25.875, i.e., the value of learning is 0.997 if the principal merely induces the same actions.

$U^P(\boldsymbol{s},\boldsymbol{a})$	s_{gg}	s_{gb}	s_{bg}	s_{bb}	a_1	$\boldsymbol{a}_2(x_{1g},a_1)$	$\boldsymbol{a}_2(x_{1b},a_1)$
25.875	5.193	P	– .24	– .24	0.39	0.387	0

Table 27.3: Cost-minimizing contract to induce $a_1 = a_2(x_{1g}) = .387$ when the hurdle is the same in each period, $b_1 = b_2 = 1$, and $\kappa_1(a_1) = \kappa_2(a_1)$.

Of course, given the cost reduction, it is now optimal for the principal to induce more effort. Taking into account that the first-period action affects the information in the first-period payoff about the hurdle, it is optimal to increase the first-period action induced as shown by the contract in Table 27.4.[30] Increasing the first-period action increases the value of learning by .164, so that the total value of learning is 1.161.

$U^P(\boldsymbol{s}^\dagger,\boldsymbol{a}^\dagger)$	$s_{gg}^\dagger$	$s_{gb}^\dagger$	$s_{bg}^\dagger$	$s_{bb}^\dagger$	$a_1^\dagger$	$\boldsymbol{a}_2^\dagger(x_{1g},a_1^\dagger)$	$\boldsymbol{a}_2^\dagger(x_{1b},a_1^\dagger)$
27.039	6.309	P	– .29	– .29	0.429	0.429	0

Table 27.4: Optimal contract with the same hurdle in each period, $b_1 = b_2 = 1$, and $\kappa_1(a_1) = \kappa_2(a_1)$.

[30] We use the term "optimal" somewhat loosely here. Table 27.7 reports the optimal contract of the form considered in this analysis, i.e., $s_{gg} > s_{bg} = s_{bb} > s_{gb} = P$, so that $a_2^\dagger(x_{1g},a_1^\dagger) = a_1^\dagger$ and $a_2^\dagger(x_{1b},a_1^\dagger) = 0$. While this is the optimal contract in the numerical example considered, we have not provided a general proof that the optimal contract is of this form (although this seems likely to be the case).

In this particular example, the learning effect increases the optimal first-period action. However, it is easy to construct examples in which the learning effect would have the opposite impact on the first-period action. This would occur, for example, in a setting in which the effort costs in the second period are sufficiently higher than in the first period. Hence, the key insight from this example is that learning affects the optimal first-period action choice to provide "optimal" pre-decision information for the second-period action.

27.5 CONCLUDING REMARKS

The last three chapters have examined a variety of multi-period models in which we have assumed that the principal and the agent are able to make binding commitments not to deviate from the terms of the initial contract. In the next chapter we examine the impact of renegotiation on several of the models previously considered under the assumption of full commitment. The discussion is an extension of the Chapter 24 discussion of renegotiation in a single-period model. However, we avoid many of the issues raised in that earlier chapter by assuming that renegotiation can only take place after a report date, and before the forthcoming action is taken.

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CHAPTER 28

INTER-PERIOD CONTRACT RENEGOTIATION

Chapter 24 considers a single-period setting in which the initial contract is renegotiated after the agent has taken his action, but before the outcome has been reported. From an *ex ante* perspective (i.e., at the time of the initial contract), permitting renegotiation can be beneficial if it takes place after the principal receives non-contractible information about the agent's action.[1] However, in a setting in which all signals are directly contractible, the principal is generally better off *ex ante* if he can exclude the possibility of future renegotiation (even though both parties may prefer to renegotiate *ex post*). On the other hand, while renegotiation is generally *ex ante* inefficient, it may be difficult (or impossible) for the principal and the agent to commit themselves not to engage in *ex post* mutually beneficial renegotiation.

Chapters 25, 26, and 27 consider multi-period models in which it is assumed that full commitment is possible, i.e., renegotiation can be precluded. This chapter also considers multi-period models, but assumes full commitment is not feasible. More specifically, we assume that the principal and agent cannot preclude inter-period renegotiation of a long-term contract (i.e., at the end of a period). However, we do assume they can preclude intra-period renegotiation (i.e., prior to the end of a period). We also exogenously exclude contracts in which the agent randomizes over actions.

Section 28.1 considers a set of sufficient conditions under which a sequence of short-term contracts can provide the same expected utilities to the principal and the agent as can an efficient long-term contract with no renegotiation. One of the key conditions is that at the beginning of each period the future technological opportunities are common knowledge, i.e., conditional on public information the agent's past unobservable actions do not have any impact on the distribution of future outcomes and performance measures.

Section 28.2 examines the impact of inter-period renegotiation in a two-period model with exponential *AC-EC* preferences, normally distributed performance measures, and payoff functions similar to the *LEN* and *QEN-P* models in Chapter 27. The performance measures may be correlated across periods, and actions may have long-term effects on outcomes as well as on performance

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[1] In that setting, renegotiation facilitates implicit contracting on the otherwise non-contractible information.

measures, i.e., future technological opportunities are *not* common knowledge. The performance measures are transformed into stochastically independent performance statistics, which may be technologically interdependent (from the agent's perspective). Furthermore, the first measure (or statistic) may be informative about the productivity of the second-period action.

Initially, a first-order approach is used to characterize the optimal renegotiation-proof contract. In renegotiating the second-period contract, the principal solves a one-period problem using the information available at the start of the second period. In choosing the first-period contract the principal takes the solution to the second-period problem as given, rather than determining the two contracts simultaneously (as in the full-commitment setting). Nonetheless, as with full commitment, the induced first-period action with renegotiation is shown to be the result of up to three types of incentives. First, there is a direct incentive that applies in all settings. Second, there is an indirect "posterior-mean" incentive which applies if the performance measures are correlated, and is due to the impact of the first-period action on the second-period statistic. Third, there is an indirect "covariance" incentive that applies if the second-period contract varies with the first-period performance measure. Contrary to the full-commitment setting, the latter incentive only applies if the first-period performance measure is informative about the productivity of the second-period action.

As in Chapter 27, after characterizing the optimal contracts, we characterize the optimal linear contracts.[2] In this setting the contract for period t is restricted to being a linear function of the performance measure for period t. However, due to renegotiation, the second-period "fixed wage" and incentive rate vary with the first-period performance measure if it is informative about the productivity of the second-period action and the performance measures are correlated. This approach implicitly produces contracts similar to the *QEN-P* contracts in Chapter 27.

The correlation between the two performance measures plays a central role in determining the difference in payoffs and first-period actions given renegotiation versus full commitment. To explore these differences, we provide comparative statics for a setting in which the two periods are identical. If the first-period performance measure is uninformative about the productivity of the second-period action, the contract with renegotiation will be a renegotiation-proof *LEN* contract, i.e., the indirect first-period covariance incentive in the full-commitment setting cannot be sustained with renegotiation. In the setting with productivity information, the correlations between the two performance measures and between the first-period performance measure and the second-period productiv-

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[2] Our model is similar to the dual purpose model in Feltham *et al.* (2005). They also consider a setting in which information about the marginal productivity of second-period effort is provided by a separate report. The dual purpose report can be preferred to the special purpose report if there is renegotiation. However, the latter clearly dominates if there is full commitment.

ity are defined to be congruent (incongruent) if they have the same signs (different signs). If they are congruent, then the payoff is very similar with full commitment and renegotiation, but if they are incongruent, then full commitment clearly dominates renegotiation. In this latter case, full commitment allows the principal to make much more effective use of the indirect incentives.

In Section 28.2 we assume the principal and the agent can commit not to "break" the employment relation, even though they cannot commit not to renegotiate the terms of the contract. In Section 28.3 we continue to assume that the principal is committed to the employment relation, but the agent can always leave after the compensation at the end of the first period has been settled. We demonstrate that in this latter setting, the principal can use deferred compensation to obtain the same result as if the agent could commit not to leave.

In Section 28.4 we introduce the possibility of replacing the initial agent at the end of the first period. Key issues in this setting include whether either the agent or principal incur "switching costs" if the agent chooses to leave or is replaced. The sign and magnitude of the indirect incentives are also important. Switching costs can provide incentives for the principal and agent not to break a commitment to continue the employment relation. The principal can also use deferred compensation to induce the agent to continue. If the indirect incentives are positive (sufficiently negative), then the principal will prefer that the first agent stays (leaves) at the end of the first period.

28.1 REPLICATING A LONG-TERM CONTRACT BY A SEQUENCE OF SHORT-TERM CONTRACTS

Based on Fudenberg, Holmström and Milgrom (1990) (FHM) this section identifies a set of sufficient conditions under which a series of short-term contracts can replicate an efficient long-term contract.

An obvious advantage of a long-term contract is that it may expand the agent's ability to smooth consumption over time if he has no access to banking. FHM do not view consumption smoothing as a major reason for long-term contracts and, therefore, they assume that the agent can borrow and save on the same terms as the principal. In that setting, if at all dates of potential renegotiation the principal and the agent share the same beliefs about future outcomes and performance measures, there are no gains to long-term contracts. That is, long-term contracts only serve to prevent renegotiation under asymmetric information. In particular, the following four conditions are sufficient for a series of short-term contracts to emulate an efficient long-term contract: at the start of each period,

(i) the preferences of the principal and the agent over future action and compensation plans are common knowledge,

(ii) future technological opportunities are common knowledge,

(iii) the compensation in the period can be made contingent on all information shared by the principal and the agent, and

(iv) the efficient utility frontier given any history is downward sloping.

The precise meanings of these conditions are developed more fully in the subsequent discussion. Conditions (i) and (ii) are information conditions. They rule out any form of adverse selection at the dates at which contracts are renegotiated. Adverse selection may not only be due to exogenous private signals received by the agent, but also due to unobserved actions taken by the agent such as effort choices and personal borrowing and saving. If the agent's borrowing and saving and, thus, his wealth, are unobservable to the principal, condition (i) rules out preferences where the agent's wealth affects his preferences, i.e., effectively, the agent's preferences must be negative exponential with a monetary cost of effort. Otherwise, the agent's wealth process must be known to the principal. Condition (ii) rules out cases in which the agent's prior actions affect the distribution of future outcomes (given the information shared by the principal and the agent). In those cases, a long-term contract is valuable because it awaits the arrival of additional performance information, while a renegotiated contract would not include that information – past actions are already taken at the date of renegotiation and, therefore, there is no value to including such information in the renegotiated contract.[3]

Condition (iii) requires all joint information to be contractible at the date of occurrence (i.e., without delay) so as to avoid the loss of incentive risk reduction

[3] The example in Section 27.2.1 in which the performance measures are given by an auto-regressive process illustrates a setting where condition (ii) is met even though periods are not independent. The key in that example is that the performance measures $\mathbf{y}_t$ can be transformed into equivalent performance statistics which are both stochastically and technologically independent. Stated differently, given $\mathbf{y}_t$ the agent's prior actions do not affect the distribution of future performance measures. On the other hand, if the normalized performance statistics are technologically interdependent (from the agent's perspective) condition (ii) is not met. In that setting, the long-term contract awaits the arrival of additional information in the future performance measures about prior actions. In the two-period model of Section 27.3.1, this is reflected by the likelihood ratio $L_{a_1}(\psi_2 | \cdot)$. A renegotiated contract for the second period will not include this likelihood ratio (see Section 28.2.2).

opportunities under short-term contracting.[4] Condition (iv) ensures that given any history, the agent can be offered the same level of future expected utility using an efficient long-term contract as using any feasible, incentive compatible long-term contract. This condition is met if the agent's preferences are additively or multiplicatively separable over time.

28.1.1 Basic Elements of the FHM Model

We use the same basic notation as in Chapters 25-27. In their analysis, FHM assume that the principal and agent both have unrestricted access to riskless borrowing and saving at discount rate β. The principal is risk neutral and evaluates an outcome/compensation stream in terms of its net present value. The agent's utility is a function of his consumption and actions over the T periods, plus possibly his utility of terminal wealth, $u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T, w_T)$. Observe that his terminal wealth w_T can be computed from initial wealth, w_o, and the consumption and compensation plans $\overleftarrow{\mathbf{c}}_T$ and $\overleftarrow{\mathbf{s}}_T$:

$$w_T = R^T w_o + \sum_{t=1}^{T} R^{(T-t)} [s_t - c_t],$$

where $R \equiv \beta^{-1}$. Observe that $u^a(\cdot)$ need not be time-additive – FHM treat such functions as special cases.

We simplify the FHM model slightly by assuming that the agent receives *no private information* although his consumption (and personal borrowing and saving) as well as his actions may be private information. Public *and contractible* information, y_t, is reported at the end of period t, and it includes the outcome x_t. At the end of period t, the principal only knows the history of publicly reported information and compensation, $\omega_t^p \equiv (\overleftarrow{\mathbf{y}}_t, \overleftarrow{\mathbf{s}}_t)$, whereas the agent knows his past actions, consumption, compensation, and the publicly observed information, $\omega_t^a \equiv (\overleftarrow{\mathbf{a}}_t, \overleftarrow{\mathbf{c}}_t, \overleftarrow{\mathbf{s}}_t, \overleftarrow{\mathbf{y}}_t)$.

Efficient Strategies

The agent's action and consumption strategies are denoted $\boldsymbol{a} = \{\, \boldsymbol{a}_t(\omega_{t-1}^a) \,\}$ and $\boldsymbol{c} = \{\boldsymbol{c}_t(\omega_{t-1}^a, a_t, y_t, s_t)\}$, and the compensation plan is $\boldsymbol{s} = \{\, \boldsymbol{s}_t(\overleftarrow{\mathbf{y}}_t) \,\}$.

Definition

(a) A *long-term contract* consists of the triple $z = (\boldsymbol{s}, \boldsymbol{a}, \boldsymbol{c})$, and it is *incentive compatible* if z induces the agent to implement $(\boldsymbol{a}, \boldsymbol{c})$. The agent's and

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[4] In Section 24.3 we considered a setting in which joint information is not directly contractible. In that setting renegotiation may dominate a commitment to a long-term contract, since renegotiation may facilitate implicit contracting on otherwise non-contractible joint information.

principal's expected utilities from a long-term contract are denoted $U^p(z)$ and $U^a(z)$, respectively.

(b) A long-term contract is *efficient* if there is no other incentive compatible long-term contract that both parties weakly prefer and at least one strictly prefers.

(c) An efficient long-term contract that guarantees the principal zero expected utility (net present value) is called *optimal.*

The FHM setting can be viewed as one in which the agent chooses the contract and the principal merely acts as a competitive risk-sharer, i.e., he will accept any contract that has a non-negative net present value.[5]

Note that with equal access to borrowing and saving the agent and principal are indifferent between two compensation plans $\mathbf{s}$ and $\mathbf{s}'$ that differ in the amounts paid at various contingencies/dates but have the *same net present values along every complete path* $\overleftarrow{\mathbf{y}}_t$ (see Proposition 25.7). Consequently, the timing of the compensation does not matter if it is adjusted to provide the same net present value.

Common Knowledge Assumptions

Throughout their analysis, FHM use the assumption that future technological opportunities are common knowledge.

Common Knowledge of Technology Assumption: The history of public information is sufficient to determine how period t's actions will affect future outcomes and public reports, i.e.,

$$\varphi(y_t | \overleftarrow{\mathbf{a}}_{t-1}, a_t, \overleftarrow{\mathbf{y}}_{t-1}) = \varphi(y_t | a_t, \overleftarrow{\mathbf{y}}_{t-1}).$$

This assumption is, for example, satisfied, if periods are independent, i.e., the public information in period t only depends on the action taken in that period,

$$\varphi(y_t | \overleftarrow{\mathbf{a}}_{t-1}, a_t, \overleftarrow{\mathbf{y}}_{t-1}) = \varphi(y_t | a_t).$$

More generally, it is satisfied if the public information includes all the relevant information about the inter-period dependencies. Appendix 28A briefly comments on FHM's common knowledge of technology assumption.

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[5] FHM state that *"Our focus on optimal contracts is motivated by the idea that competition in the market for agents will force the principal to offer the agent the best zero profit contract."* This assumes, of course, that the agent is a monopolist with respect to his skills, rather than the principal being a monopolist with respect to his production technology.

Let $\vec{\mathbf{a}}_{t+1}$, $\vec{\mathbf{s}}_{t+1}$, and $\vec{\mathbf{c}}_{t+1}$ represent action, compensation, and consumption plans that will be implemented *subsequent* to period t and let $\Omega_t^a(\omega_t^p)$ be the set of agent histories ω_t^a that are *consistent with the public history* ω_t^p. The following assumption implies that the action and consumption history, $\overleftarrow{\mathbf{a}}_t$ and $\overleftarrow{\mathbf{c}}_t$, does not influence the agent's preference with respect to his future action strategies and the contract specifying future compensation. Hence, *the principal knows the agent's preferences with respect to future contracts* based on the public history ω_t^p.

Common Knowledge of Preferences Assumption: For all t and any two future action/compensation plans $(\vec{\mathbf{a}}_{t+1}^1, \vec{\mathbf{s}}_{t+1}^1)$ and $(\vec{\mathbf{a}}_{t+1}^2, \vec{\mathbf{s}}_{t+1}^2)$,

$$\{\, \omega_t^a \in \Omega_t^a(\omega_t^p) \,|\, V_{t+1}(\omega_t^a, \vec{\mathbf{a}}_{t+1}^1, \vec{\mathbf{s}}_{t+1}^1) > V_{t+1}(\omega_t^a, \vec{\mathbf{a}}_{t+1}^2, \vec{\mathbf{s}}_{t+1}^2) \,\}$$

$$\in \{\, \emptyset, \Omega_t^a(\omega_t^p) \,\}, \quad \forall\, \omega_t^p,$$

where $V_{t+1}(\omega_t^a, \vec{\mathbf{a}}_{t+1}, \vec{\mathbf{s}}_{t+1})$

$$\equiv \underset{\vec{\mathbf{a}}_{t+1}}{\text{maximize}} \quad E\big[u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T, w_{T+1}(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{s}}_T)) \mid \omega_t^a, \vec{\mathbf{a}}_{t+1}, \vec{\mathbf{s}}_{t+1}, \vec{\mathbf{c}}_{t+1}\big]$$

represents the maximal expected utility that the agent can obtain by choosing an optimal future consumption plan $\vec{\mathbf{c}}_{t+1}$, given history ω_t^a and future action/compensation plans $(\vec{\mathbf{a}}_{t+1}, \vec{\mathbf{s}}_{t+1})$.

Given the common knowledge of technology assumption, the common knowledge of preferences assumption is satisfied if *current wealth is common knowledge*[6] and the agent's utility function in the time dimension is either:

(i) additively separable,

$$u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T, w_T) = \sum_{t=1}^{T} \beta^t u_t(c_t, a_t) + u_{T+1}(w_T), \text{ or}$$

(ii) multiplicatively separable,

$$u^a(\overleftarrow{\mathbf{c}}_T, \overleftarrow{\mathbf{a}}_T, w_T) = \left[\prod_{t=1}^{T} u_t(c_t, a_t)\right] u_{T+1}(w_T).$$

[6] Common knowledge of wealth or consumption does not imply that it is contractible information – it merely implies that the principal can base his renegotiation on this information.

Even if wealth is not common knowledge, the common knowledge of preferences assumption is met if there are no wealth effects on preferences, i.e., (ii) combined with exponential *AC-EC* preferences (see Section 25.1),

$$u_t(c_t,a_t) = -\exp\{-r\Xi_t(c_t - a_t)\} \quad u_{T+1}(w_T) = -\exp\{-r\Xi_T w_T\},$$

where Ξ_t is a time-preference index with $\Xi_t/\Xi_{t-1} = \beta$, $t = 2, ..., T$. Below we comment on a setting with time-additive exponential utility.

Renegotiation[7]

FHM assume that the principal and agent *cannot commit* to a long-term contract. Instead, a long-term contract serves as a point from which the parties can mutually agree to a new contract that will apply to the remaining periods (subject to future renegotiation). Let z^I represent the initial contract and let $\vec{\mathbf{z}}_{t+1}^R$ represent a renegotiated contract at the end of period t.[8] The renegotiated contract cannot change the past, but it can change the future.

Definition

For a given history ω_t^a, a renegotiated contract $\vec{\mathbf{z}}_{t+1}^R$ is *incentive compatible* if, given the compensation plan $\vec{\mathbf{s}}_{t+1}^R$, the agent prefers the action/consumption plan $(\vec{\mathbf{a}}_{t+1}^R, \vec{\mathbf{c}}_{t+1}^R)$ to any other action/consumption plan. A long-term contract z^I is *sequentially incentive compatible* if for every t and every history ω_t^a, $\vec{\mathbf{z}}_{t+1}^I$ is incentive compatible. The set of sequentially incentive compatible long-term contracts is denoted *SIC*.

Sequential incentive compatibility means that the agent is willing, at each date t, to follow the instructions in z^I no matter what history has occurred up to that date.

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[7] In their Section 3, FHM provide two examples that illustrate how "adverse selection" prevents short-term contracts from emulating optimal long-term contracts.

Example 1: The opportunity to renegotiate the contract after the agent has taken his action and before all uncertainty has been resolved can destroy the incentive effects of the optimal "long-term" contract. (Also see Chapter 24.)

Example 2: If there is consumption both before the action is taken and subsequent to the realization of the outcome, then the opportunity to renegotiate the contract before taking the action, but after the initial consumption has occurred, can have a negative effect on the agent's incentives.

[8] Note that renegotiation can only occur at the end of periods after all uncertainty about the consequences of the period's action has been resolved. This is a critical assumption (see Chapter 24).

Since the principal cannot observe ω_t^a, he generally cannot tell how the agent values the contract z^l, nor how he himself would value that contract if he had the agent's information. Therefore, renegotiation will typically take place under asymmetric information about the value of alternative options. However, when technology and preferences are common knowledge, there is no essential information asymmetry in the bargaining process. For every history ω_t^a and every pair of sequentially incentive compatible contracts $\vec{\mathbf{z}}_{t+1}^1$ and $\vec{\mathbf{z}}_{t+1}^2$ we have

$$\{\ \omega_t^a \in \Omega_t^a(\omega_t^p) \mid U^a(\vec{\mathbf{z}}_{t+1}^1 \,|\, \omega_t^a) > U^a(\vec{\mathbf{z}}_{t+1}^2 \,|\, \omega_t^a)\ \} \in \{\ \emptyset, \Omega_t^a(\omega_t^p)\ \},$$

$$\{\ \omega_t^a \in \Omega_t^a(\omega_t^p) \mid U^p(\vec{\mathbf{z}}_{t+1}^1 \,|\, \omega_t^a) > U^p(\vec{\mathbf{z}}_{t+1}^2 \,|\, \omega_t^a)\ \} \in \{\ \emptyset, \Omega_t^a(\omega_t^p)\ \}.$$

The first implies that ω_t^p is sufficient for the principal to infer how the agent *ranks* incentive compatible renegotiated contracts for any history ω_t^a. By contrast, the second is equivalent to assuming that ω_t^p is sufficient for the principal to know the expected net present value to him of each incentive compatible renegotiated contract since contracts that give all future profits to the agent in exchange for a fixed rental fee will provide the requisite calibration.

We now characterize the principal and agent utility levels that can be achieved given a particular history at date t. The *utility possibility set* conditional on history ω_t^a is the set of feasible payoff pairs to the principal and agent:

$$UPS(\omega_t^a) = \{\ (U^p, U^a) \mid \exists\ \vec{\mathbf{z}}_{t+1} \in SIC,$$

$$\text{such that } U^p = U^p(\omega_t^a, \vec{\mathbf{z}}_{t+1}) \text{ and } U^a = U^a(\omega_t^a, \vec{\mathbf{z}}_{t+1})\ \}.$$

The *principal's utility frontier* conditional on ω_t^a is characterized by the function

$$UPF(U^a \,|\, \omega_t^a) = \text{maximize } U^p, \text{ subject to } (U^p, U^a) \in UPS(\omega_t^a).$$

The *efficient utility frontier* conditional on ω_t^a is the set of undominated feasible payoff pairs:

$$EF(\omega_t^a) \equiv \{\ (U^p, U^a) \in UPS(\omega_t^a) \mid \nexists\ (U^{p\prime}, U^{a\prime}) \in UPS(\omega_t^a),$$

$$\text{such that } (U^{p\prime}, U^{a\prime}) \geq (U^p, U^a) \text{ and } (U^{p\prime}, U^{a\prime}) \neq (U^p, U^a)\ \}.$$

Definition

Given ω_t^a, a renegotiated contract $\vec{\mathbf{z}}_{t+1}$ is *efficient* if $\vec{\mathbf{z}}_{t+1} \in SIC$ and if the payoffs from $\vec{\mathbf{z}}_{t+1}$ are on the efficient frontier $EF(\omega_t^a)$.

Decreasing Utility Frontier Assumption: For every history ω_t^a, the function $UPF(U^a\,|\,\omega_t^a)$ is *strictly decreasing* in U^a.

This condition effectively states that the efficient frontier coincides with the principal's utility frontier. This implies that one can replace any incentive compatible contract with an efficient contract without altering the agent's payoff (see Figure 28.1). Thus, the full range of agent incentives can be provided within the set of efficient contracts.

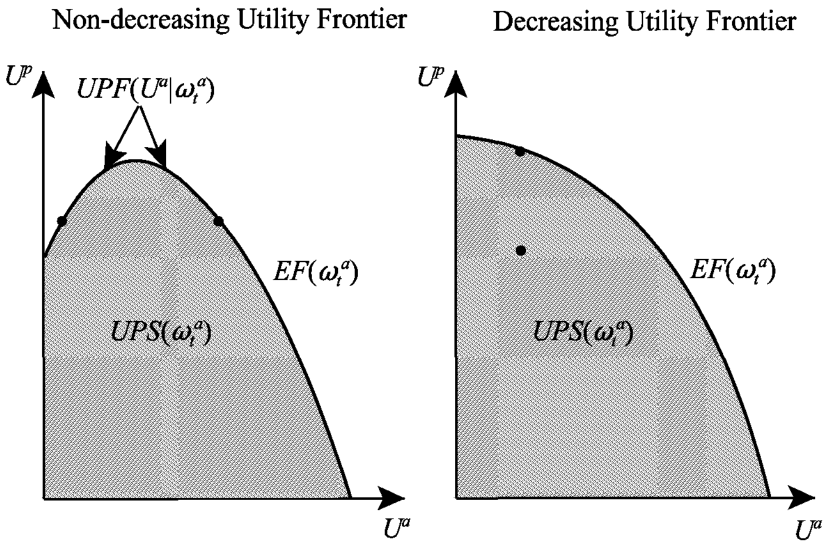

Figure 28.1: Non-decreasing and decreasing utility frontiers.

Proposition 28.1 (FHM, Theorem 1)

If agent consumption (or wealth) is observable, then the decreasing utility frontier condition is satisfied when either of the following two conditions holds:

(a) Preferences are additively separable over time and $u_{T+1}(w_T)$ is increasing, continuous and unbounded below.

(b) Preferences are multiplicatively separable over time, each $u_t(c_t, a_t)$ is positive, the function $u_{T+1}(w_T)$ is increasing and continuous, and either $u_{T+1}(w_T)$ is negative and unbounded below (e.g., negative exponential) or it is positive and has a greatest lower bound of zero (e.g., square root).[9]

The proof is essentially the same as the proof that the participation constraint is binding in single-period models with additive or multiplicative separability between consumption and effort. For example, consider case (a). Take any sequentially incentive compatible long-term contract $\vec{\mathbf{z}}_{t+1}$. Construct a new contract that subtracts k units of utility from the expected utility of $\vec{\mathbf{z}}_{t+1}$ along every complete history. The new contract preserves incentives, but it decreases the agent's expected utility, whereas the principal's expected utility increases proving that the principal's utility frontier is decreasing.

28.1.2 Main Results

A long-term contract can only be emulated by a sequence of short-term contracts if it is immune to renegotiation. In this section we review FHM's sufficient conditions for a long-term contract to be renegotiation-proof. However, before doing so we introduce FHM's concept of sequential efficiency.

Definition

A long-term contract z is *sequentially efficient* if it is efficient for every history ω_t^a.

Sequential efficiency is a strong requirement, and $\overleftarrow{\mathbf{y}}_t$ does not generally provide sufficient contractible information to maintain the payoffs on the efficient frontier in all contingencies ω_t^a. However, the following proposition identifies a set of conditions for which this is assured if the agent does not have access to financial markets.

Proposition 28.2 (FHM, Theorem 2)

Assume contractibility of $\overleftarrow{\mathbf{y}}_t$, a finite contracting horizon, *no access to financial markets*, common knowledge of technology and preferences, and a decreasing utility frontier. Then for any efficient long-term contract, there is a corresponding sequentially efficient long-term contract providing the same initial expected utility levels.

The proof is by construction.[10] Let z be a long-term efficient contract. If it is not sequentially efficient, then for some history ω_t^a, there exists a renegotiated

[9] This precludes u_{T-1} from being a log utility function.

[10] The logic is the same as the one used to argue that *ex ante* optimality implies *ex post* optimality in complete markets.

contract $\vec{\mathbf{z}}_{t+1}$ that is strictly Pareto preferred and does not change the prior incentives. Furthermore, given the decreasing utility frontier, this contract can be modified so that it has the same incentives and makes the agent indifferent between the revised contract and z. Hence, we can construct a sequentially efficient contract that is Pareto preferred to z by making these substitutions for each history ω_t^a for which a Pareto preferred contract exists.

Now we consider settings in which the agent does have access to financial markets. We also go beyond efficiency and focus on efficient contracts that yield a zero expected return to the principal.

Definition

A sequentially efficient long-term contract which gives the principal zero expected net present value ($U^p = 0$) conditional on any history ω_t^a is called *sequentially optimal.*

A sequentially optimal long-term contract has the feature that if the agent and the principal were to terminate their contract at any time and start negotiating for a new long-term contract immediately afterwards, the old contract would be accepted anew. Working backwards from date T, it is then clear that a sequentially optimal long-term contract signed at date 0 can be decomposed into a sequence of short-term contracts negotiated at the beginning of each period and only specifying payments and plans for that period. By adding an access to financial markets assumption to Proposition 28.2, sequential optimality follows from sequential efficiency by a simple rearrangement of payments.

Proposition 28.3 (FHM, Theorem 3)

Assume contractibility of $\overleftarrow{\mathbf{y}}_t$, a finite contracting period, *equal access to financial markets*, common knowledge of technology and preferences, and decreasing utility frontier. If there is an optimal long-term contract, then there is a sequentially optimal contract, which can be implemented via a sequence of short-term contracts.

Proof: Let z be an optimal, sequentially efficient long-term contract, which implies $U^p(z) = 0$. Now modify the timing of payments to make expected profits zero from each node (history) ω_t^a onwards (the finiteness of the contract implies no payments are made subsequent to date T):

$$\hat{s}_t(\overleftarrow{\mathbf{y}}_t) = s_t(\overleftarrow{\mathbf{y}}_t) + \mathrm{E}\left[\sum_{\tau=t}^{T} \beta^{\tau-t}[x_\tau - s_\tau] \,\middle|\, \psi_{t-1}^a, \vec{\mathbf{a}}_t, \vec{\mathbf{s}}_t\right]$$

$$- \mathrm{E}\left[\sum_{\tau=t+1}^{T} \beta^{\tau-t}[x_\tau - s_\tau] \,\middle|\, \psi_t^a, \vec{\mathbf{a}}_{t+1}, \vec{\mathbf{s}}_{t+1}\right].$$

By common knowledge, the right-hand side varies with $\overleftarrow{\mathbf{y}}_t$ only, so this construction is possible. For every complete history ω_t^a, the present value of the agent's compensation is the same under $\hat{s}$ as under s, so the contract $z = (a,c,\hat{s})$ is incentive compatible. By construction, the principal's expected profits are zero from each "node" onwards, so the contract is sequentially optimal. The change effectively pays the entire incremental return to the agent due to outcome y_t given $\overleftarrow{\mathbf{y}}_t$ at the end of period t. **Q.E.D.**

Time-additive Exponential Utility

In general, if the principal cannot observe the agent's wealth (or consumption), then the agent's preferences will not be common knowledge and, therefore, a commitment to a long-term contract will be of value. However, FHM suggest that unobserved wealth is not an empirically significant reason for long-term contracts. To demonstrate this, they consider exponential *TA-EC* preferences (see Section 25.1), which neutralizes wealth effects and, thereby, suggests that for other utility functions, wealth is likely to have only a secondary effect.[11]

Time-additive Exponential Utility (*TA-EC*) Assumption: The agent's utility function is

$$u^a(\overleftarrow{\mathbf{c}}_T,\overleftarrow{\mathbf{a}}_T,w_T) = -\sum_{t=1}^{T} \beta^t \exp\left[-r\,(c_t - \kappa(a_t))\right],$$
$$- \frac{\beta^{T+1}}{1-\beta} \exp\left[-r\,(1-\beta)\,w_T\right],$$

where the last term reflects an assumption that after retirement at date T the agent (or his beneficiaries) lives an infinite life consuming the interest from his wealth at retirement.

Proposition 28.4 (FHM, Theorem 4)

Assume $\overleftarrow{\mathbf{y}}_t$ is verifiable, a finite contracting horizon, equal access to financial markets, common knowledge of technology, and exponential *TA-EC* preferences. Then the agent's preferences will be common knowledge, and the utility frontier will be decreasing.

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[11] The utility for wealth at $t+1$ in the FHM model is equivalent to extending the consumption horizon to infinity in the exponential *TA-EC* preferences introduced in Section 25.1. Furthermore, the exponential *AC-EC* preferences introduced in Section 25.1 yield effectively the same results.

28.2 INTERDEPENDENT PERIODS WITH JOINT COMMITMENT TO EMPLOYMENT

The previous section identifies conditions under which there is no loss from inter-period renegotiation. For example, there is no loss if the agent has *exponential TA-EC* preferences with access to financial markets or he has exponential *AC-EC* preferences (see Chapter 25), and there is both technological and stochastic independence across periods. The following sections explore the impact of technological and stochastic dependence across periods.

We assume that, at the start of each period, it is feasible for a principal to hire either the same agent or a different agent, and it is feasible for the agent to accept employment from either the same principal or a different principal. If the principal prefers to hire the same agent for all periods, then the contract(s) between the principal and the agent will be affected by both their preferences and the commitments they can make. Commitment limitations can take a variety of forms. For example, the principal may not be able to commit to rehiring the agent at the start of each period, and the agent may not be able to commit to staying with the firm in future periods. Furthermore, even if both can commit to a long-term employment relation, they may not be able to preclude renegotiating the terms of the initial contract at some future date.

We initially assume that, *ex ante*, the principal prefers a long-term employment relation with one agent, and he can either commit to that relation or his *ex post* preferences are such that he will not change agents. The latter can occur, for example, if the principal would incur significant switching costs if he hired a different agent. We further assume the agent can commit to not leave, or he has significant switching costs that would deter him from leaving. In this section, we assume that once an initial contract is signed, both parties are committed to the employment relation for the full duration of the contract, but the principal can change the terms of the contract if the agent agrees.

Throughout the analysis in the remainder of this chapter we assume the agent has exponential *AC-EC* preferences (with a zero riskless interest rate). This prevents wealth effects and consumption smoothing concerns.[12]

28.2.1 Performance Measure and Payoff Characteristics

Our analysis is based on a two-period setting with normally distributed performance measures and payoff functions similar to the *LEN* and *QEN-P* models in Chapter 27. More specifically, we assume the performance measures and the principal's payoffs can be represented by

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[12] Christensen, Feltham, Hofmann, and Şabac (2004) examine the time-additive case in a *LEN* setting with renegotiation.

$$y_1 = M_{11}a_1 + \varepsilon_1, \quad y_2 = M_{21}a_1 + M_{22}a_2 + \varepsilon_2,$$

$$x_t = \theta_1 a_1 + \theta_2 a_2 + \varepsilon_{xt}, \quad t = 1,2.$$

The performance measures are scaled to have unit variances and correlation ρ_y, the productivity parameters, θ_t, are normally distributed with mean $\mathrm{E}[\theta_t] = b_t$, $t = 1,2$, and the covariance between the first-period report y_1 and the productivity of second-period effort is $\mathrm{Cov}[y_1,\theta_2] = \rho_\theta \sigma$. This setting is identical to the *QEN-P* model in Section 27.4.1 except that we allow the first-period action to have an impact on the second-period report. We assume the principal's payoffs are not observable until after the termination of the contract.

As in Chapter 27 we use stochastically independent sufficient performance statistics to characterize the optimal contracts. Given the agent's action choices $\mathbf{a} = (a_1,a_2)$ and the principal's conjecture $\hat{\mathbf{a}} = (\hat{a}_1,\hat{a}_2)$, the performance statistics are

$$\psi_1 = M_{11}(a_1 - \hat{a}_1) + \delta_1 \tag{28.1a}$$

and

$$\psi_2 = [M_{21} - \rho_y M_{11}](a_1 - \hat{a}_1) + M_{22}(a_2 - \hat{a}_2) + \delta_2, \tag{28.1b}$$

where $\delta_1 = \varepsilon_1$, and $\delta_2 = \varepsilon_2 - \rho_y \varepsilon_1$.

Let $\mathbf{s}$: $\Psi_1 \times \Psi_2 \to \mathbb{R}$ represent the agent's aggregate compensation function and let $\mathbf{a}_2$: $\Psi_1 \times A_1 \to A_2$ represent the agent's second-period action strategy, where Ψ_t is the set of possible date t performance statistics and A_t is the set of possible period t actions. The prior distributions for the performance statistics ψ_1 and ψ_2 given the agent's actual and conjectured actions are represented by $\Phi(\psi_1|a_1,\hat{a}_1)$ and $\Phi(\psi_2|\mathbf{a},\hat{\mathbf{a}})$, and the latter is also the posterior distribution. In equilibrium, $a_t = \hat{a}_t$ for $t = 1,2$, and, thus, ψ_1 and ψ_2 both have zero means independently of the equilibrium action choices. Let $\Phi^\dagger(\psi_1) = \mathrm{N}(0,1)$ and $\Phi^\dagger(\psi_2) = \mathrm{N}(0,1-\rho_y^2)$ denote the equilibrium distributions. Hence, the equilibrium likelihood ratios are:

$$L_{a_1}(\psi_1) \equiv \frac{\Phi_{a_1}(\psi_1|\hat{a}_1)}{\Phi^\dagger(\psi_1)} = M_{11}\psi_1, \tag{28.2a}$$

$$L_{a_1}(\psi_2) \equiv \frac{\Phi_{a_1}(\psi_2|\hat{\mathbf{a}})}{\Phi^\dagger(\psi_2)} = \frac{[M_{21} - \rho_y M_{11}]\psi_2}{1-\rho_y^2}, \tag{28.2b}$$

$$L_{a_2}(\psi_2) \equiv \frac{\Phi_{a_2}(\psi_2|\hat{\mathbf{a}})}{\Phi^\dagger(\psi_2)} = \frac{M_{22}\psi_2}{1-\rho_y^2}. \tag{28.2c}$$

In (28.2), we express, for example, $\Phi_{a_1}(\psi_2|\mathbf{a},\hat{\mathbf{a}})$ as $\Phi_{a_1}(\psi_2|\hat{\mathbf{a}})$ when $a_1 = \hat{a}_1$ and $a_2 = \hat{a}_2$.

28.2.2 Optimal Renegotiation-proof Contracts

While the performance measures and agent preferences are assumed to have characteristics similar to those in the *LEN* and *QEN-P* models in Chapter 27, the contracts considered in this chapter differ due to renegotiation at $t = 1$. In addition, we initially characterize the optimal contract (without restricting its form) using a "first-order approach" similar to the full-commitment analysis in Section 27.3.1. Later we examine the impact of renegotiation in settings in which contracts are constrained to have the same structure as the *LEN* or *QEN-P* contracts considered in Section 27.3.2.

At date 0, the principal and agent sign an initial two-period compensation contract $\mathbf{s}^I$. The agent then takes his first action a_1. At $t = 1$, the first report y_1 is issued, after which the principal can offer the agent a revised contract $\mathbf{s}^R$. If the initial contract is such that the principal does not prefer to revise it at date 1, then the initial contract is defined to be renegotiation-proof.

Inter-period Renegotiation

Let $z^I = \{\mathbf{s}^I, a_1^I, \mathbf{a}_2^I\}$ represent the initial contract signed at $t = 0$, plus the conjectured actions a_1^I and $\mathbf{a}_2^I$: $\Psi_1 \to A_2$ used in computing the performance statistics ψ_1 and ψ_2.[13]

We assume this contract is incentive compatible, so that the conjectured actions are implemented if the agent believes that $\mathbf{s}^I$ will be implemented. Consequently, at $t = 1$, after the first-period effort cost has been incurred and the first-period report has been issued, the agent's expected utility from continuing with the initial contract is

$$U_1^a(z^I,\psi_1) = -\int_{\Psi_2} \exp[-r\{\mathbf{s}^I(\psi_1,\psi_2) - \kappa_2(\mathbf{a}_2^I(\psi_1))\}]\, d\Phi(\psi_2|a_1^I,\mathbf{a}_2^I(\psi_1)).$$

The corresponding certainty equivalent is

$$CE_1(z^I,\psi_1) = -\frac{1}{r}\ln[-U_1^a(z^I,\psi_1)].$$

At $t = 1$, after the first-period report ψ_1 has been issued, the principal can propose replacing z^I with $\mathbf{z}^R(\psi_1) = \{\mathbf{s}^R(\psi_1), a_1^I, \mathbf{a}_2^R(\psi_1)\}$, where $\mathbf{s}^R(\psi_1)$: $\Psi_2 \to \mathbb{R}$ re-

[13] Assuming that the incentive compatibility constraints can be represented by their first-order conditions implies that there is no "double shirking," so that an incentive compatible second-period action can be expressed as a function of ψ_1 alone.

presents the revised compensation contract offered by the principal after ψ_1 is issued at $t = 1$, and $\boldsymbol{a}_2^R(\psi_1) \in A_2$ is the revised conjectured second-period action. If the agent rejects the principal's revision, he takes second-period action $\boldsymbol{a}_2^I(\psi_1)$ and is paid $\boldsymbol{s}^I(\psi, \psi_2)$ at $t = 2$, whereas he takes action $\boldsymbol{a}_2^R(\psi_1)$ and is paid $\boldsymbol{s}^R(\psi_1,\psi_2)$ if he accepts the principal's proposed revision.

At $t = 1$, the principal chooses the revised contract $\boldsymbol{z}^R(\psi_1)$ that maximizes his expected payoff given the report y_1 (or ψ_1) and conjectured first-period action $a_1{}^I$, subject to the revision being acceptable to the agent, and inducing the agent to implement action $\boldsymbol{a}_2^R(\psi_1)$.

The principal's decision problem at the renegotiation stage given an initial contract $\boldsymbol{s}^I$, conjectured action a_1^I, and the issuance of first-period statistic ψ_1 is stated as follows:[14]

$$\underset{\boldsymbol{s}^R, a_2}{\text{maximize}}\; U_1^P(\boldsymbol{s}^R, a_1^I, a_2, \psi_1)$$

$$= (b_2 + \rho_\theta \sigma \psi_1) a_2 - \int_{\Psi_2} \boldsymbol{s}^R(\psi_1,\psi_2)\, d\Phi^\dagger(\psi_2), \qquad (28.3a)$$

$$\text{subject to} \quad -\int_{\Psi_2} \exp[-r\{\boldsymbol{s}^R(\psi_1,\psi_2) - \kappa_2(a_2)\}]\, d\Phi^\dagger(\psi_2) \geq \bar{U}_1^I(\psi_1), \quad (28.3b)$$

$$-\int_{\Psi_2} \exp[-r\{\boldsymbol{s}^R(\psi_1,\psi_2) - \kappa_2(a_2)\}] \times [r\kappa_2'(a_2) + L_{a_2}(\psi_2)]\, d\Phi^\dagger(\psi_2) = 0, \qquad (28.3c)$$

$$\boldsymbol{s}^R(\psi_1,\psi_2) \geq CE_1(z^I, \psi_1) + \underline{s}, \quad \forall \psi_2, \qquad (28.3d)$$

$$\text{where} \quad \bar{U}_1^I(\psi_1) \equiv U_1^a(z^I, \psi_1).$$

This decision problem is a "standard single-period problem" in which the agent's reservation certainty equivalent is $CE_1(z^I, \psi_1)$, and the lower bound on the agent's compensation is $CE_1(z^I, \psi_1) + \underline{s}$.[15] Using a "first-order" approach

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[14] In formulating the principal's renegotiation problem we use that $a_1 = \hat{a}_1$ such that the distribution of ψ_2 is independent of the agent's actual or conjectured first-period action. We also renormalize the second-period performance statistic using the second-period action to be induced such that $\psi_2 = \delta_2$. In addition, the payoff and cost of the first-period action are omitted – at $t = 1$, they can no longer be affected by the principal or the agent.

[15] The lower bound is introduced to avoid the Mirrlees problem (see Section 17.3.3). Stating this bound relative to the agent's certainty equivalent is not standard, but it significantly simplifies our subsequent analysis by ensuring that the principal's decision problem at the renegotiation (continued...)

yields the following characterization of the optimal renegotiated second-period contract and induced action.

Proposition 28.5[16]

Given initial contract z^I and performance statistic ψ_1, the optimal renegotiated second-period contract and effort choice are characterized by

$$\mathbf{s}^R(\psi_1,\psi_2) = CE_1(\mathbf{s}^I,\psi_1) + \mathbf{s}_2^o(\psi_1,\psi_2), \quad \mathbf{a}_2^R(\psi_1) = \mathbf{a}_2^o(\psi_1), \qquad (28.4a)$$

$$\kappa_2'(\mathbf{a}_2^o(\psi_1)) = \frac{M_{22}}{r(1-\rho_y^2)}\,\mathrm{Cov}(u_2^o,\psi_2|\psi_1), \qquad (28.4b)$$

where $$u_2^o(\psi_1,\psi_2) \equiv -\exp[-r\{\mathbf{s}_2^o(\psi_1,\psi_2) - \kappa_2(\mathbf{a}_2^o(\psi_1))\}], \qquad (28.5a)$$

$$\mathbf{s}_2^o(\psi_1,\psi_2) = \kappa_2(\mathbf{a}_2^o(\psi_1)) + \frac{1}{r}\ln[\mathbf{G}_2(\psi_1,\psi_2)], \qquad (28.5b)$$

$$\mathbf{G}_2(\psi_1,\psi_2) \equiv r\{\lambda_2^o(\psi_1) + \mu_2^o(\psi_1)g_2(\mathbf{a}_2^o(\psi_1),\psi_2)\}, \qquad (28.5c)$$

$$g_2(\mathbf{a}_2^o(\psi_1),\psi_2) = r\kappa_2'(\mathbf{a}_2^o(\psi_1)) + L_{a_2}(\psi_2). \qquad (28.5d)$$

Since there are no wealth effects with exponential *AC-EC* preferences, the size of the agent's reservation certainty equivalent has no impact on either the variable component of an optimal renegotiated contract or the induced second-period action – it only affects the agent's "fixed wage." Hence, in characterizing the optimal renegotiated second-period contract it is useful to focus on the optimal contract and action when the second-period certainty equivalent is zero, which are denoted $\mathbf{s}_2^o$ and $\mathbf{a}_2^o$. We refer to this as problem $P_2^o(\psi_1)$.[17]

To see how renegotiation affects the induced second-period action, compare (28.4b) to (27.21a). Given the agent's equilibrium second-period utility $u_2^o(\cdot)$, the characterization of the agent's induced second-period action is the same as with full commitment. The key difference is that with renegotiation, the agent's equilibrium second-period utility, $u_2^o(\psi_1,\psi_2)$, is determined as the solution to the principal's *ex post* single-period problem $P_2^o(\psi_1)$, whereas with full commitment it is determined as part of an *ex ante* two-period problem. In particular, note that the likelihood ratio $L_{a_1}(\psi_2)$ affects the equilibrium second-period utility with full commitment, but not with renegotiation (compare (27.17) to (28.5)). At the

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[15] (...continued)
stage is homogeneous of degree one in the agent's reservation certainty equivalent.

[16] The proof is in Appendix 28B.

[17] See Appendix 28B for the formulation and the solution to this problem.

renegotiation stage, the first-period action has been taken and, hence, is not controllable when determining the renegotiated contract.

Optimal Renegotiation-proof Contracts
The agent rationally anticipates the principal's proposed contract revision at $t = 1$ when he decides whether to accept the initial contract and when he chooses his first-period action. Of course, there is no revision at $t = 1$ if the initial contract is consistent with (28.4). In that case, the initial contract is defined to be *renegotiation-proof.* Moreover, for any initial contract that results in a revision at $t = 1$, there is an alternative initial contract that yields the same results but does not require any revision. Hence, as in Chapter 24, we can, without loss of generality, restrict our attention to *renegotiation-proof contracts*.

Proposition 28.6 *Renegotiation-proof Contracts*

If there is an optimal initial contract z^I which results in revised contract z^R, then there is an optimal solution to the principal's *ex ante* decision problem with the same actions and payoffs in which z^R is the initial as well as the final contract. Moreover, an initial contract z^I is *renegotiation-proof* if, and only if,

$$\mathbf{s}^I(\psi_1,\psi_2) = CE_1(z_1^I,\psi_1) + \mathbf{s}_2^o(\psi_1,\psi_2), \qquad \mathbf{a}_2^I(\psi_1) = \mathbf{a}_2^o(\psi_1),$$

where $\mathbf{s}_2^o(\psi_1)$: $\Psi_2 \to \mathbb{R}$ and $\mathbf{a}_2^o(\psi_1) \in A_2$ are as specified in (28.5).

Given the restriction to renegotiation-proof contracts, it is useful to divide the compensation into two components of the form $\mathbf{s}(\psi_1,\psi_2) = \mathbf{s}_1(\psi_1) + \mathbf{s}_2^o(\psi_1,\psi_2)$, where s_1 is paid at $t = 1$ (prior to renegotiation) and s_2^o is paid at $t = 2$. The principal's decision problem is solved by backward induction. The first step is to solve $P_2^o(\psi_1)$ to determine $\mathbf{s}_2^o(\psi_1,\psi_2)$ and $\mathbf{a}_2^o(\psi_1)$. The principal then solves the *ex ante* decision problem in Table 28.1[18] to obtain the optimal first-period compensation $\mathbf{s}_1(\psi_1)$ and action a_1, which are characterized in the following proposition.

Proposition 28.7

For an optimal renegotiation-proof contract, the optimal first-period compensation contract and induced action given the anticipated renegotiation at $t = 1$ are characterized by

$$\mathbf{s}_1^r(\psi_1) = \kappa_1(a_1^r) + \frac{1}{r}\ln[\mathbf{G}_1(\psi_1)], \tag{28.6a}$$

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[18] We assume without loss of generality that the agent's *ex ante* reservation certainty equivalent is equal to zero.

$$\kappa_1'(a_1^r) = \frac{1}{r}\left[M_{11}\mathrm{Cov}(u_1^r,\psi_1) - \mathrm{E}[q_2^o] + \mathrm{Cov}(u_1^r,q_2^o)\right], \qquad (28.6b)$$

where
$$\boldsymbol{G}_1(\psi_1) \equiv r\{\lambda_1 + \mu_1[g_1(a_1^r,\psi_1) + q_2^o(\psi_1)]\}, \qquad (28.7a)$$

$$g_1(a_1^r,\psi_1) \equiv r\kappa_1'(a_1^r) + L_{a_1}(\psi_1), \qquad (28.7b)$$

$$q_2^o(\psi_1) \equiv \mathrm{Cov}(u_2^o,\psi_2|\psi_1)\frac{\rho_y M_{11} - M_{21}}{1-\rho_y^2}, \qquad (28.7c)$$

$$u_1^r(\psi_1) \equiv -\exp[-r\{\boldsymbol{s}_1^r(\psi_1) - \kappa_1(a_1^r)\}]. \qquad (28.7d)$$

The principal's *ex ante* decision problem in Table 28.1 is equivalent to a standard single-period problem except for the term $q_2^o(\psi_1)$ defined in (28.7). That term is equal to zero, if $M_{21} = \rho_y M_{11}$, which is satisfied if the reports y_1 and y_2 are both technologically and stochastically independent (i.e., $M_{21} = 0$ and $\rho_y = 0$), or they follow a first-order auto-regressive process with weight ρ_y (which yields independent periods in terms of the statistics, see Section 27.2.1). Hence, in these settings, the principal's *ex ante* decision problem separates into two independent single-period problems. Of course, this merely reconfirms the analysis in Section 28.1 showing that a long-term contract can be replicated by a sequence of short-term contracts in settings in which periods are independent and the agent has exponential utility with effort costs.

Observe that the characterizations of the agent's induced first-period action with full commitment, (see (27.21b)) versus renegotiation (see (28.6b)) are the same except that $q_2^\dagger$ is used in the former and q_2^o in the latter. Again, the key difference is that with renegotiation, q_2^o is determined in the principal's *ex post* single-period problem $P_2^o(\psi_1)$, whereas $q_2^\dagger$ is determined as part of an *ex ante* two-period problem with full commitment. As with full commitment the induced first-period action with renegotiation is the result of three types of incentives: a direct first-period incentive, an indirect "posterior-mean" incentive due to the impact of a_1 on the second-period statistic, and an indirect "covariance" incentive if q_2^o varies with the first-period report.

Since ψ_1 and ψ_2 are independent, expression (28.7c) implies that q_2^o only varies with ψ_1 if u_2^o varies with ψ_1, which also implies that $\boldsymbol{a}_2^o(\psi_1)$ varies with ψ_1 (see (28.4b)). Under full commitment, it is optimal to obtain the indirect covariance incentive by inducing the second-period effort to vary with ψ_1, independent of whether ψ_1 is informative about second-period productivity θ_2, or not. This changes when we consider the renegotiation setting.

Table 28.1
Principal's *Ex Ante* Decision Problem

(a) $\underset{s_1, a_1}{\text{maximize}}\ U^p(\mathbf{s}_1, a_1 | \mathbf{s}_2^o, \mathbf{a}_2^o) = b_1 a_1 - \int_{\Psi_1} \mathbf{s}_1(\psi_1)\, d\Phi^\dagger(\psi_1),$

subject to

(b) $\int_{\Psi_1} \int_{\Psi_2} u_1(\psi_1, a_1)[-u_2^o(\psi_1, \psi_2)]\, d\Phi^\dagger(\psi_2)\, d\Phi(\psi_1 | a_1)$

$$= \int_{\Psi_1} u_1(\psi_1, a_1)\, d\Phi(\psi_1 | a_1) \geq -1,$$

(c) $\int_{\Psi_1} \int_{\Psi_2} u_1(\psi_1, a_1)[-u_2^o(\psi_1, \psi_2)]$

$$\times [r\kappa_1'(a_1) + L_{a_1}(\psi_1) + L_{a_1}(\psi_2)]\, d\Phi^\dagger(\psi_2)\, d\Phi(\psi_1 | a_1)$$

$$= \int_{\Psi_1} u_1(\psi_1, a_1)[g_1(a_1, \psi_1) + q_2^o(\psi_1)]\, d\Phi(\psi_1 | a_1) = 0,$$

(d) $\mathbf{s}_1(\psi_1) \geq \underline{s}, \quad \forall \psi_1,$

where $u_1(\psi_1, a_1) \equiv -\exp[-r\{\mathbf{s}_1(\psi_1) - \kappa_1(a_1)\}].$

The participation constraint (28.3b) for the decision problem $P_2^o(\psi_1)$ implies (assuming it is binding for each ψ_1):

(e) $$\int_{\Psi_2} u_2^o(\psi_1, \psi_2)\, d\Phi^\dagger(\psi_2) = -1, \quad \forall \psi_1,$$

from which the equality in (b) follows. The incentive compatibility constraint (28.3c) for the decision problem $P_2^o(\psi_1)$ and (e) imply that

(f) $$\int_{\Psi_2} u_2^o(\psi_1, \psi_2)\, L_{a_2}(\psi_2)\, d\Phi^\dagger(\psi_2) = r\kappa_2'(\mathbf{a}_2^o(\psi_1)), \quad \forall \psi_1.$$

From (28.2) we obtain $L_{a_1}(\psi_2) = L_{a_2}(\psi_2)\,[M_{21} - \rho_y M_{11}]/M_{22}$, which, together with (e) and (f), provide the first equality in (c).

No Productivity Information

We first assume the first-period report is not informative about the productivity of the second-period action, i.e., $\rho_\theta = 0$. This results in the following corollary.

Corollary

If $\rho_\theta = 0$, then the optimal renegotiation-proof contract and induced actions are characterized as follows.

$$\boldsymbol{s}(\psi_1,\psi_2) = \boldsymbol{s}_1^r(\psi_1) + \boldsymbol{s}_2^o(\psi_2), \quad \boldsymbol{a}_2^o(\psi_1) = a_2^o, \tag{28.8a}$$

$$\kappa_1'(a_1^r) = \frac{1}{r}\left[M_{11}\mathrm{Cov}\left(u_1^r,\psi_1\right) - q_2^o \right]. \tag{28.8b}$$

Since ψ_1 is independent of both ψ_2 and θ_2, it is pure noise in the second-period problem $P_2^o(\psi_1)$ and does not affect the optimal renegotiated second-period contract $z_2^o(\psi_1)$. In other words, renegotiation-proof contracts are characterized by a compensation function that is an additively separable function of ψ_1 and ψ_2, and a second-period action choice that is independent of the first-period report.

In the full-commitment setting, the optimal contract is not additively separable and the first-period action is characterized by (27.21b), which includes the indirect covariance incentive associated with $\mathrm{Cov}(u_1^\dagger,q_2^\dagger)$. However, with renegotiation, there is no covariance incentive since $q_2^o = r\kappa_2'(a_2^o)[\rho_y M_{11} - M_{21}]/M_{22}$ is a constant and, thus, $\mathrm{Cov}(u_1^r,q_2^o) = 0$. This is the principal's optimal choice *ex post*, even though it is not optimal *ex ante*, and with renegotiation the principal cannot commit to make the optimal *ex ante* choice.

Productivity Information

Now we assume the first-period report is informative about the second-period effort productivity, i.e., $\rho_\theta \neq 0$. In this setting, the optimal second-period contract $z_2^o(\psi_1)$ varies with the first-period report in order to take advantage of the productivity information. Clearly, the contract $z_2^o(\psi_1)$ is such that $\boldsymbol{a}_2^o(\psi_1)$ is an increasing (decreasing) function of ψ_1, if $\rho_\theta > 0$ ($\rho_\theta < 0$). This creates a covariance between u_1^r and q_2^o which affects the induced first-period effort (see (28.7b)). With renegotiation, the latter effect is ignored by the principal when selecting the second-period contract, whereas it is taken into consideration when choosing the optimal full-commitment contract (e.g., see the *QEN-P* model in Section 27.4.1). That is, with renegotiation and productivity information, there is a non-trivial covariance incentive in (28.7b), i.e., $\mathrm{Cov}(u_1^r,q_2^o) \neq 0$, but it is a by-product and not part of the second-period contract choice. On the other hand, the variation in q_2^o with respect to ψ_1 affects the principal's *ex ante* choice of the first-period contract. This raises a question as to whether the indirect first-period covariance incentive is positive or negative.

It follows from (28.6b) that the covariance incentive has a positive impact on the induced first-period action if, and only if, $\text{Cov}(u_1^r, q_2^o) > 0$ (since $\kappa_1(\cdot)$ is convex). Using (28.7c) and (28.4b), we obtain

$$q_2^o(\psi_1) = \frac{r(\rho_y M_{11} - M_{21})}{M_{22}} \kappa_2'(\mathbf{a}_2^o(\psi_1)). \tag{28.9}$$

Since $\kappa_2(\cdot)$ is convex, it follows that $q_2^o(\psi_1)$ is an increasing function of the first-period report if, and only if,

$$\frac{\partial \mathbf{a}_2^o(\psi_1)}{\partial \psi_1} [\, \rho_y M_{11} - M_{21}] > 0. \tag{28.10}$$

In order to examine this relation, we generalize the definition of congruent correlations from Section 27.4.1 in order to recognize the potential impact of the first-period action on the second-period performance measure.

Definition

The information system provides congruent correlations if ρ_θ and $\rho_y M_{11} - M_{21}$ are both positive or both negative. Otherwise, the information system is said to provide incongruent correlations.

Since $\mathbf{a}_2^o(\psi_1)$ is an increasing function of ψ_1 if, and only if, $\rho_\theta > 0$, it follows that $q_2^o(\psi_1)$ is an increasing function of ψ_1 if, and only if, the information system provides congruent correlations. Note also from (28.7a) that congruent correlations imply that $u_1^r(\psi_1)$ is an increasing function of ψ_1, since both $L_{a_1}(\psi_1)$ and $q_2^o(\psi_1)$ are increasing functions of ψ_1. Hence, the covariance incentive has a positive impact on the first-period action, if the information system provides congruent correlations. However, if the correlations are incongruent, the situation is more subtle. In this case $q_2^o(\psi_1)$ is a decreasing function of ψ_1, but $u_1^r(\psi_1)$ may be an increasing, decreasing, or non-monotonic function of ψ_1 (since $L_{a_1}(\psi_1)$ is a linear increasing function of ψ_1, but $q_2^o(\psi_1)$ is decreasing). Therefore, the covariance incentive may have an ambiguous impact on the first-period action, depending on, for example, the quality of the productivity information, $(\rho_\theta \sigma)^2$, and the correlation of the performance measures, ρ_y. Interestingly, it may be optimal for the principal to choose a decreasing first-period compensation function merely to induce a significant positive covariance incentive (see the *QEN-P* model in Section 27.4.1 for further discussion).

28.2.3 Endogenous *QEN-P* Models of Inter-period Renegotiation

In this section we assume the principal is restricted to offering a linear contract at $t = 0$, but can renegotiate a revised linear contract at $t = 1$. From an *ex ante* perspective, this process results endogenously in a contract that is equivalent to a renegotiation-proof *QEN-P* contract if there is productivity information, i.e., $\rho_\theta \neq 0$.

The performance measures are represented by the performance statistics in (28.1). The initial contract has the form

$$s^I(\psi_1,\psi_2) = f^I + v_1^I\psi_1 + v_2^I\psi_2, \qquad (28.11a)$$

where f^I is the fixed wage, and v_1^I and v_2^I are the initial incentive rates. The renegotiation offer at $t = 1$ has the form

$$s^R(\psi_1,\psi_2) = f^R(\psi_1) + v_2^R(\psi_1)\psi_2, \qquad (28.11b)$$

where the "second-period fixed wage," $f^R(\psi_1)$, and the second-period incentive rate, $v_2^R(\psi_1)$, both may depend on the first-period report.

The outcome of the renegotiation encounter can be formulated as in the preceding section. That is, the optimal solution to the principal's renegotiation problem can be written as (in 28.4a):

$$s^R(\psi_1,\psi_2) = CE_1(z^I,\psi_1) + s_2^o(\psi_1,\psi_2), \quad a_2^R(\psi_1) = a_2^o(\psi_1), \qquad (28.12)$$

where $CE_1(z^I,\psi_1)$ is the agent's certainty equivalent for the contract in place, and $z_2^o(\psi_1) = \{s_2^o(\psi_1), a_2^o(\psi_1)\}$ is the optimal solution to a pre-contract information single-period problem in which the compensation contract is restricted to being a linear function of ψ_2 and the agent has a reservation wage equal to zero. Since the conditional expected second-period effort productivity equals $\mathrm{E}[\theta_2|\psi_1] = b_2 + \rho_\theta\sigma\psi_1$, the contract $z_2^o(\psi_1)$ is characterized by

$$s_2^o(\psi_1,\psi_2) = f_2^o(\psi_1) + v_2^o(\psi_1)\psi_2, \quad a_2^o(\psi_1) = M_{22}\, v_2^o(\psi_1), \qquad (28.13a)$$

where

$$v_2^o(\psi_1) = \bar{v}_2^o + \gamma^o\psi_1, \qquad (28.13b)$$

$$\bar{v}_2^o = \frac{b_2 M_{22}}{M_{22}^2 + r(1-\rho_y^2)}, \quad \gamma^o = \frac{\rho_\theta\sigma M_{22}}{M_{22}^2 + r(1-\rho_y^2)}, \qquad (28.13c)$$

$$f_2^o(\psi_1) = ½[a_2^o(\psi_1)]^2 + ½r(1-\rho_y^2)[v_2^o(\psi_1)]^2. \qquad (28.13d)$$

We have not restricted the form of the functions $f_2^o(\psi_1)$ and $v_2^o(\psi_1)$ as we did in the *QEN-P* model with full commitment. With renegotiation, the linear second-period incentive rate and the quadratic fixed wage covering effort-cost and risk premium risk insurance arise endogenously!

Note also that, if $\rho_\theta = 0$, the contract $z_2^o(\psi_1)$ is independent of the first-period report, i.e., the second-period fixed wage and incentive rate are constants, $f_2^o(\psi_1) = f_2^o$ and $v_2^o(\psi_1) = v_2^o$. Furthermore, the agent's date $t = 1$ certainty equivalent, $CE_1(z_1^I, \psi_1)$ is a linear function of ψ_1. Hence, the optimal compensation contract for the principal's renegotiation problem in (28.12) is an *ex ante* linear function of ψ_1 and ψ_2 and, thus, it can be offered as the initial contract. In this case, we can, without loss of generality, restrict the analysis to *linear renegotiation-proof contracts*.

On the other hand, if $\rho_\theta \neq 0$, there is *no* linear renegotiation-proof contract. In this case, the slope on the second-period incentive rate is non-zero ($\gamma^o \neq 0$) and, thus, both the second-period incentive rate and the second-period action depend on the first-period report. Hence, any initial linear contract (as in (28.11a)) will be renegotiated! The agent's date $t = 1$ certainty equivalent, $CE_1(z_1^I, \psi_1)$ is a linear function of ψ_1 (since the initial contract is linear). Hence, we can, without loss of generality, restrict the analysis to contracts of the form

$$s(\psi_1, \psi_2) = f_1 + v_1\psi_1 + f_2^o(\psi_1) + v_2^o(\psi_1)\psi_2, \tag{28.14}$$

where $f_2^o(\psi_1)$ and $v_2^o(\psi_1)$ are determined in (28.13). Note that this is the same as the *QEN-P* contract in which the second-period fixed wage provides effort-cost and risk premium risk insurance and is a quadratic function of the first-period report. Furthermore, note that even though a *QEN-P* contract cannot be offered as the initial contract (given the linear contract restriction), we can perform the analysis as if it could be offered as the initial contract, and it is renegotiation-proof if, and only if, it has the form as in (28.14).

Proposition 28.8 *Renegotiation-proof LEN and QEN-P Contracts*
Assume the information is given by the performance statistics in (28.1), and the principal is restricted to offering linear contracts given the information at each date.

(a) If $\rho_\theta = 0$, then we can restrict the analysis to renegotiation-proof *LEN* contracts.

(b) If $\rho_\theta \neq 0$, then we can restrict the analysis to renegotiation-proof *QEN-P* contracts.

Renegotiation-proof Contracts

Renegotiation fixes the second-period contract to be as characterized by (28.13). Hence, we begin by characterizing the agent's first-period action choice given an arbitrary first-period incentive rate v_1 and second-period contract parameters $\bar{v}_2^o$ and γ^o. While the parameter values with renegotiation versus full-commitment setting differ, the form of the agent's first-period decision problem is the same. Hence, the induced first-period action is[19]

$$a_1 = v_1 M_{11} - [\bar{v}_2^o - r v_1 \gamma^o][\rho_y M_{11} - M_{21}]. \tag{28.15}$$

As with full commitment (see 27.29), the induced first-period action is the result of three types of incentives: a direct first-period incentive ($v_1 M_{11}$); a posterior-mean incentive ($-\bar{v}_2^o[\rho_y M_{11} - M_{21}]$) resulting from the impact of a_1 on the posterior mean of ψ_2; and an indirect covariance incentive ($r v_1 \gamma^o [\rho_y M_{11} - M_{21}]$). Observe that in Chapter 27 we assume that $M_{21} = 0$, so that the posterior-mean incentive only has $\rho_y M_{11}$, which is an indirect incentive reflecting the fact that ψ_2 is orthogonalized and, hence, a_1 affects y_1 at the rate M_{11} and y_1 affects the posterior mean of ψ_2 at the rate $-\rho_y$. In this chapter we also have a direct second-period mean effect because a_1 affects both y_2 and ψ_2 at the rate M_{21}.

The key difference between the full commitment and renegotiation settings is that in the former the principal simultaneously selects all three contract parameters v_1, $\bar{v}_2$, and γ from an *ex ante* perspective, whereas in the latter, $\bar{v}_2^o$ and γ^o are the solution to his *ex post* single-period problem as given in (28.14). Consequently, with renegotiation only the first-period incentive rate v_1 is determined *ex ante*, but, of course, it is chosen to take into consideration the interaction with the second-period contract.

Given contract parameters v_1, $\bar{v}_2^o$, and γ^o, the principal's expected net pay-off can be expressed as[20]

$$\begin{aligned} U^p(v_1, \bar{v}_2^o, \gamma^o) &= b_1 a_1 + b_2 \mathrm{E}[\boldsymbol{a}_2^o(\psi_1) | a_1] - \{ \tfrac{1}{2} a_1^2 + \tfrac{1}{2} \mathrm{E}[\boldsymbol{a}_2^o(\psi_1)^2 | a_1] \} \\ &\qquad - \tfrac{1}{2} r \{ v_1^2 + \mathrm{E}[(\bar{v}_2^o + \gamma^o \psi_1)^2 (1 - \rho_y^2) | a_1] \} \\ &= b_1 [v_1 \tilde{M}_1^o - \bar{v}_2^o(\rho_y M_{11} - M_{21})] - \tfrac{1}{2}[v_1 \tilde{M}_1^o - \bar{v}_2^o(\rho_y M_{11} - M_{21})]^2 - \tfrac{1}{2} r v_1^2 \\ &\qquad + b_2 \mathrm{E}[\boldsymbol{a}_2^o(\psi_1)] - \tfrac{1}{2}\mathrm{E}[\boldsymbol{a}_2^o(\psi_1)^2] - \tfrac{1}{2} r \mathrm{E}[(\bar{v}_2^o + \gamma^o \psi_1)^2 (1 - \rho_y^2)], \end{aligned} \tag{28.16}$$

[19] See (27.27) - (27.29) and recognize the impact of a_1 on y_2, i.e., M_{21}.

[20] This is similar to (27.30) in the full-commitment setting.

where $\tilde{M}_1^o \equiv M_{11} + r\gamma^o[\rho_y M_{11} - M_{21}]$ is an "adjusted first-period sensitivity" reflecting the direct first-period incentive as well as the indirect covariance incentive. Since the second-period action is independent of v_1, the first-order condition for (28.16) with respect to the first-period incentive rate provides the following result.

Proposition 28.9

In the *QEN-P* model with renegotiation and the joint commitment to employment described above, the optimal incentive rate parameters are given by

$$v_1^r = \frac{b_1 \tilde{M}_1^o + \bar{v}_2^o \tilde{M}_1^o (\rho_y M_{11} - M_{21})}{(\tilde{M}_1^o)^2 + r}, \tag{28.17a}$$

$$\bar{v}_2^o = \frac{b_2 M_{22}}{M_{22}^2 + r(1 - \rho_y^2)}, \tag{28.17b}$$

$$\gamma^o = \frac{\rho_\theta \sigma M_{22}}{M_{22}^2 + r(1 - \rho_y^2)}. \tag{28.17c}$$

Comparison between Renegotiation and Full Commitment

Under full commitment the three parameters are determined simultaneously. Hence, the first-order conditions that characterize the optimal full-commitment *QEN-P* model parameters (see, for example, (27.35)) are more complex than the characterization of the sequentially determined, optimal renegotiation-proof parameters in (28.17).

First consider γ, which is the slope of v_2 with respect to ψ_1. With renegotiation this parameter is chosen, as reflected in (28.17c), solely to correlate the induced second-period effort with its productivity. This same term appears in the specification of the full-commitment parameter, but there is also another term in that case (see, for example, (27.35c)). The other term reflects the fact that γ affects the first-period indirect covariance incentive and this can be taken into consideration if there is full commitment. In fact, as reflected in (27.29) and (27.31c), this role exists even if there is no productivity information, i.e., $\rho_\theta = 0$. Furthermore, as illustrated in Figure 27.1, the indirect covariance incentives can provide significant benefits to the principal (if $\rho_y > 0$). However, they *cannot* be sustained with renegotiation.

Next consider $\bar{v}_2$, which is the expected second-period incentive rate. With renegotiation (see 28.17b), this solely reflects the expected second-period productivity b_2. However, as discussed in Section 27.4.1 and illustrated in (27.29), this parameter also has an indirect effect on the induced first-period action if the two performance measures are correlated (i.e., $\rho_y \neq 0$). Hence, with full-commitment, $\bar{v}_2$ is modified to take this indirect first-period incentive into consideration. In addition, it is modified to take into consideration the direct impact of a_1 on ψ_2 as reflected by M_{21}. To illustrate this, differentiate the principal's expected payoff (which has the same form as (28.16)) with respect to $\bar{v}_2$ and solve for $\bar{v}_2$ given the full-commitment values of v_1 and γ:[21]

$$\bar{v}_2^\dagger = \frac{b_2 M_{22} - (b_1 - v_1^\dagger \tilde{M}_1^\dagger)(\rho_y M_{11} - M_{21})}{M_{22}^2 + r(1 - \rho_y^2) + (\rho_y M_{11} - M_{21})^2}, \tag{28.18}$$

where $$\tilde{M}_1^\dagger \equiv M_{11} + r\gamma^\dagger[\rho_y M_{11} - M_{21}].$$

Observe that the first term in the numerator and the first two terms in the denominator of (28.18) are the same as in (28.17b). They are equivalent to the terms in the optimal incentive rate for a single-period problem based on the posterior variance for the second-period performance measure. The additional terms in the numerator and denominator in (28.18) reflect the fact that if $\rho_y M_{11} - M_{21} > 0$, then increasing $\bar{v}_2$ reduces the induced first-period effort which reduces the first-period payoff minus the first-period effort cost (but does not affect the first-period risk premium). In particular, from (28.15) we obtain

$$b_1 \frac{\partial a_1}{\partial \bar{v}_2} - ½ \frac{\partial a_1^2}{\partial \bar{v}_2} = -(b_1 - v_1 \tilde{M}_1)(\rho_y M_{11} - M_{21}) - \bar{v}_2 (\rho_y M_{11} - M_{21})^2.$$

The first two terms appear in the numerator of (28.18), while the last term appears in the denominator, since we are solving for $\bar{v}_2$.

The impact of $\bar{v}_2$ on the first-period effort stems from two factors. First, the performance statistics are orthogonalized, so the posterior mean of ψ_2 depends on y_1, which in turn is influenced by a_1; the net rate of impact is $-\rho_y M_{11}$. Second, a_1 has a direct impact on the mean of ψ_2 at the rate M_{21}. The sum of these two effects can be positive or negative and, thus, the renegotiated rate $\bar{v}_2^o$ can be less than or greater than the full-commitment rate $\bar{v}_2^\dagger$.[22] In any case, $\bar{v}_2^o$ is not as efficient as $\bar{v}_2^\dagger$, if $\rho_y M_{11} - M_{21} \neq 0$.

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[21] This characterization is equivalent to (27.35b) except for the impact of a_1 on the mean of the second-period report, M_{21}.

[22] This relation depends also on the slope of the second-period incentive rate and the first-period incentive rate.

Renegotiation-proof *QEN-P* contracts achieve the same optimal solution as full-commitment *QEN-P* contracts if $\rho_y M_{11} - M_{21} = 0$ (since $\bar{v}_2$ has no impact on the first-period action), and the indirect covariance incentive is also equal to zero irrespectively of the slope γ. Note that even if the performance reports are uncorrelated, i.e., $\rho_y = 0$, there is a loss to renegotiation if the first-period action has a long-term impact, i.e., $M_{21} \neq 0$. Hence, no loss due to renegotiation requires either both stochastic and technological independence in the performance measures, or a first-order auto-regressive process for the performance measures with weight ρ_y.

28.2.4 Comparative Statics Given Identical Periods

In order to provide some basic comparative statics, we focus on the identical periods case (i.e., $b_1 = b_2 = b$, $M_{11} = M_{22} = M$, and $M_{21} = 0$).[23] As noted above, the loss due to renegotiation derives from the fact that the second-period incentive rate parameters γ^o and $\bar{v}_2^o$ may differ from their full-commitment values, $\gamma^\dagger$ and $\bar{v}_2^\dagger$, respectively. Hence, we begin by exploring the difference between $\bar{v}_2^o$ versus $\bar{v}_2^\dagger$, followed by γ^o versus $\gamma^\dagger$. Of course, these differences affect the principal's *ex ante* choice of the first-period incentive rate.

Expected Second-period Incentive Rate

Consider first the expected second-period incentive rate. For the identical periods case, its full-commitment and renegotiated values are (see (27.35b) and (28.17b))

$$\bar{v}_2^\dagger = \frac{bM[(\tilde{M}^\dagger)^2 + r(1-\rho_y)]}{[(\tilde{M}^\dagger)^2 + r][M^2 + r(1-\rho_y^2)] + r\rho_y^2 M^2}, \tag{28.19a}$$

$$\bar{v}_2^o = \frac{bM}{M^2 + r(1-\rho_y^2)}, \tag{28.19b}$$

where $\tilde{M}^\dagger \equiv M(1 + r\gamma^\dagger\rho_y)$. Hence, the ratio between the two can be expressed as

[23] In numerical examples, we assume that $b = 10$, $M = 1$, and $r = 1$ (which is consistent with the numerical examples in Sections 27.3.2 and 27.4.1).

$$\frac{\bar{v}_2^\dagger}{\bar{v}_2^o} = \frac{(\tilde{M}^\dagger)^2 + r - r\rho_y}{(\tilde{M}^\dagger)^2 + r + r\dfrac{\rho_y^2 M^2}{M^2 + r(1-\rho_y^2)}},$$

and, thus, $\bar{v}_2^o > \bar{v}_2^\dagger$, if, and only if,

$$-\rho_y < \frac{\rho_y^2 M^2}{M^2 + r(1-\rho_y^2)}. \tag{28.20}$$

Note that this condition is independent of both the informativeness of y_1 about θ_2 and the variability of the second-period incentive rate, i.e., ρ_θ and γ. Of course, since $\bar{v}_2^\dagger$ (but not $\bar{v}_2^o$) depends on $\gamma^\dagger$, the difference between the two depends on $\gamma^\dagger$ (which depends on the productivity information ρ_θ). This leads to the following proposition.

Proposition 28.10

Assume identical periods, i.e., $b_1 = b_2 = b$, $M_{11} = M_{22} = M$, and $M_{21} = 0$. In the *QEN-P* model, the following relations hold between the expected second-period incentive rate with renegotiation and with full commitment:

(a) $\bar{v}_2^o > \bar{v}_2^\dagger$ if $\rho_y > 0$;

(b) $\bar{v}_2^o < \bar{v}_2^\dagger$ if $-1 < \rho_y < 0$;

(c) $\bar{v}_2^o = \bar{v}_2^\dagger$ if $\rho_y = 0$ or -1.

In case (c) there is no difference between renegotiation and full commitment. The performance measures are independent if $\rho_y = 0$, and there are no indirect first-period incentives (see (28.15)). First-best is obtained if $\rho_y = -1$. In cases (a) and (b), the principal's choice of the expected second-period incentive rate with renegotiation fails to reflect the indirect first-period incentive it creates, i.e., the second term in (28.15), $-\bar{v}_2^o \rho_y M$. If $\rho_y > 0$, a negative indirect first-period incentive is induced, but with renegotiation $\bar{v}_2^o$ is not modified to reflect this indirect first-period incentive and, therefore, the second-period incentive rate is "too high" with renegotiation. On the other hand, if $\rho_y < 0$, a positive indirect first-period incentive is induced, but is not reflected in $\bar{v}_2^o$ and, therefore, the second-period incentive rate is "too low" with renegotiation.

Variability of the Second-period Incentive Rate

Now consider the slope γ of the second-period incentive rate. Its full-commitment and renegotiated values are (see (27.35c) and (28.17c))

$$\gamma^\dagger = \frac{[rv_1^\dagger \rho_y M][b - a_1^\dagger]}{M^2 + r(1 - \rho_y^2)} + \frac{\rho_\theta \sigma M}{M^2 + r(1 - \rho_y^2)}, \tag{28.21a}$$

$$\gamma^o = \frac{\rho_\theta \sigma M}{M^2 + r(1 - \rho_y^2)}. \tag{28.21b}$$

To simplify the discussion of the differences between $\gamma^\dagger$ and γ^o, we first consider the case with no productivity information, and subsequently consider the case with productivity information.

Full-commitment versus Renegotiation-proof QEN Contracts with No Productivity Information

If there is no productivity information ($\rho_\theta = 0$), then, with renegotiation, $\gamma^o = 0$ (which implies the renegotiation-proof *QEN* contract reduces to a renegotiation-proof *LEN* contract).[24] However, with full commitment, $\gamma^\dagger$ is non-zero (if ρ_y is not equal to zero or minus one) and has the same sign as ρ_y in order to provide positive indirect first-period covariance incentives (see also Figure 27.1).[25] The correlation between the two performance measures, ρ_y, has a significant effect on the principal's expected payoff, the first-period incentive rate, and the induced first-period action. Figures 28.2(a) and (b) illustrate these effects for the optimal full-commitment and the renegotiation-proof *QEN* contracts. Figure 28.2(a) also includes the payoff for the full-commitment *LEN* contract (to help explain the difference in payoff between the two other contracts).

Figure 28.2(a) has two key features. First, the three types of contracts produce very similar payoffs if the performance measures are negatively correlated. Second, the payoffs are dramatically different if the performance measures are positively correlated. We focus on the latter.

Indirect covariance incentives are a major source of the difference in payoffs between the full-commitment and the renegotiation-proof *QEN* contracts. Full commitment results in $\gamma^\dagger \neq 0$ and renegotiation results in $\gamma^o = 0$, which

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[24] If the principal offered a *QEN* contract at the initial stage, the resulting contract will be a *LEN* contract. The optimal solution to the principal's *ex post* second-period problem has a constant second-period incentive rate that is independent of the first-period performance, and the agent's $t = 1$ certainty equivalent for the initial contract is a linear function of the first-period performance (see (27.27)).

[25] This follows from the fact that $v_1^\dagger > 0$, and $a_1^\dagger$ is less than the first-best effort b.

implies that there are indirect covariance incentives with full commitment but not with renegotiation. The payoff from a full-commitment *QEN* contract increases significantly as the correlation ρ_y becomes more positive since that yields stronger indirect covariance incentives. The fact that $\gamma^\dagger \neq 0$ is a major source of the difference is illustrated in Figure 28.2(a) by the fact that the full-commitment payoff is not increasing with more positive correlation if $\gamma^\dagger$ is constrained to equal zero (i.e., if it is a full-commitment *LEN* contract).

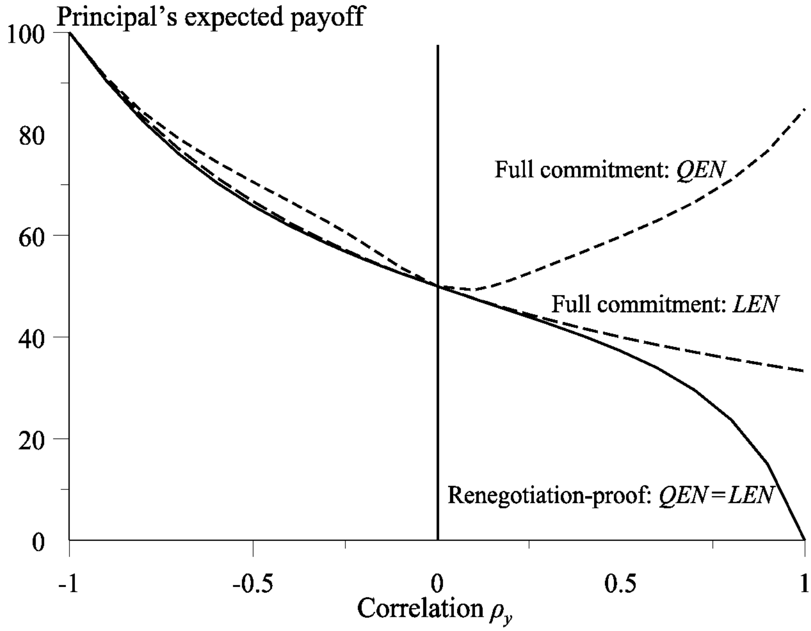

Figure 28.2(a): Impact of performance measure correlation in renegotiation-proof and full-commitment *QEN* and *LEN* contracts for identical periods case with no productivity information.

However, note that the indirect covariance incentive is not the only source of the difference in payoffs between the full-commitment and the renegotiation-proof *QEN* contracts. This is illustrated in Figure 28.2(a) by the fact that the renegotiation-proof payoff decreases more than the full-commitment *LEN* payoff as the positive correlation increases. In order to understand the difference in payoff between a full-commitment and a renegotiation-proof *LEN* contract, consider the optimal first-period incentive rate given the expected second-period incentive rate (with γ set equal to zero – see (28.17a))

$$v_1(\bar{v}_2) = \frac{M[b + \bar{v}_2 \rho_y M]}{M^2 + r}. \tag{28.22}$$

In a full-commitment *LEN* contract (see (27.31b)),

$$v_2^{LEN} = \frac{bM[M^2 + r(1 - \rho_y)]}{[M^2 + r][M^2 + r(1 - \rho_y^2)] + r\rho_y^2 M^2}, \tag{28.23}$$

whereas in a renegotiation-proof contract (see (28.17b))

$$\bar{v}_2^o = \frac{bM}{M^2 + r(1 - \rho_y^2)}.$$

Note that the relation between v_2^{LEN} and $\bar{v}_2^o$ is the same as between $v_2^\dagger$ and $\bar{v}_2^o$ given in Proposition 28.10. Substituting these relations into (28.22) yields the following results.

Proposition 28.11

Assume identical periods, i.e., $b_1 = b_2 = b$, $M_{11} = M_{22} = M$, and $M_{21} = 0$. In the *LEN* model, the following relations hold between the second- and first-period incentive rates with renegotiation versus full commitment:

(a) $\bar{v}_2^o > v_2^{LEN}$, and $v_1^r > v_1^{LEN}$ if $\rho_y > 0$;

(b) $\bar{v}_2^o < v_2^{LEN}$, and $v_1^r > v_1^{LEN}$ if $-1 < \rho_y < 0$;

(c) $\bar{v}_2^o = v_2^{LEN}$, and $v_1^r = v_1^{LEN}$ if $\rho_y = 0$ or -1.

The relations between the second-period incentive rates are (as in Proposition 28.10) due to the fact that the principal's choice of the second-period incentive rate with renegotiation does not reflect the indirect posterior-mean incentive it creates, i.e., renegotiation leads to less efficient indirect first-period incentives whether the correlation is positive or negative. This is recognized by the principal at the initial contracting stage and, therefore, it is optimal for him to increase the direct first-period incentives by choosing $v_1^r > v_1^{LEN}$ whenever ρ_y is not equal to zero or minus one.

However, note that while $\bar{v}_2^o$ > (<) v_2^{LEN} implies that the induced second-period action with renegotiation is higher (lower) than the induced second-period action with full commitment, $v_1^r > v_1^{LEN}$ does not imply that the induced first-period action is higher with renegotiation than with full commitment. This is due to the fact that the induced first-period action depends upon both direct and indirect incentives, and both types of incentives are affected by renegotiation. It follows from (28.15) and (28.22) that the induced first-period action is

$$a_1 = v_1(\bar{v}_2)M - \bar{v}_2\rho_y M = \frac{M[Mb - r\bar{v}_2\rho_y]}{M^2 + r}.$$

Furthermore, it follows from Proposition 28.11 that $v_2^{LEN}\rho_y \le \bar{v}_2^{\,o}\rho_y$ for all ρ_y (with a strict inequality for ρ_y different from zero and minus one). This provides the following result.[26]

Corollary

Consider the setting in Proposition 28.11. The following relations hold in the *LEN* model between the induced first-period action with renegotiation versus full commitment:

(a) $a_1^r < a_1^{LEN}$ if $\rho_y \neq 0$ and $\rho_y \neq -1$;

(b) $a_1^r = a_1^{LEN}$ if $\rho_y = 0$ or -1.

Hence, even though the direct incentive is higher with renegotiation than with full commitment, i.e., $v_1^r > v_1^{LEN}$, the increased direct incentive is not sufficient to fully offset the less efficient indirect posterior-mean incentive.[27] The negative effect of renegotiation in the *LEN* model with $\rho_y \neq 0$ (or -1) is discussed extensively in the accounting and economics literature.[28]

Now consider the comparison between the optimal first-period incentive rates with renegotiation and in the full commitment *QEN* contract. Again, the principal recognizes that with renegotiation, the second-period incentive rate parameters will lead to "less efficient" indirect first-period incentives whether the correlation is positive or negative. *Ceteris paribus*, this will lead the principal to increase the first-period incentive rate (in order to increase the direct first-period incentives). However, note also that the indirect first-period covariance incentive in the full commitment *QEN* contract, i.e., $r\gamma^\dagger\rho_y M v_1$, is increasing in the first-period incentive rate (since $\gamma^\dagger$ has the same sign as $\rho_y \ge 0$). That

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[26] Here we use the fact that $a_1^{LEN} > 0$ (since $a_1^{LEN} = a_2^{LEN}$ in the identical periods setting).

[27] See also Indjejikian and Nanda (1999) and Christensen, Feltham, and Şabac (2003, 2005). The latter papers express the compensation contract in terms of the correlated performance measures y_1 and y_2 instead of the stochastically independent performance statistics ψ_1 and ψ_2. In this (equivalent) formulation there is no indirect posterior-mean incentive and renegotiation results in a reduction of the first-period incentive rate.

[28] For example, in the accounting literature, see Indjejikian and Nanda (1999), Christensen *et al.* (2004, 2005)

covariance incentive is not present with renegotiation and, thus, *ceteris paribus* this effect decreases the first-period incentive rate with renegotiation relative to the full-commitment *QEN* contract. In our numerical example, the former of these two opposite effects dominates as illustrated in Figure 28.2(b). Note that even though $v_1^r \geq v_1^\dagger$ for all performance measure correlations, the induced first-period action in the full-commitment *QEN* contract is higher than the induced first-period action in the renegotiation-proof contract. Of particular note is the fact that as a positive performance measure correlation increases, the induced first-period action also increases in the full-commitment *QEN* contract, whereas the induced first-period action decreases in the renegotiation-proof contract for all performance measure correlations.

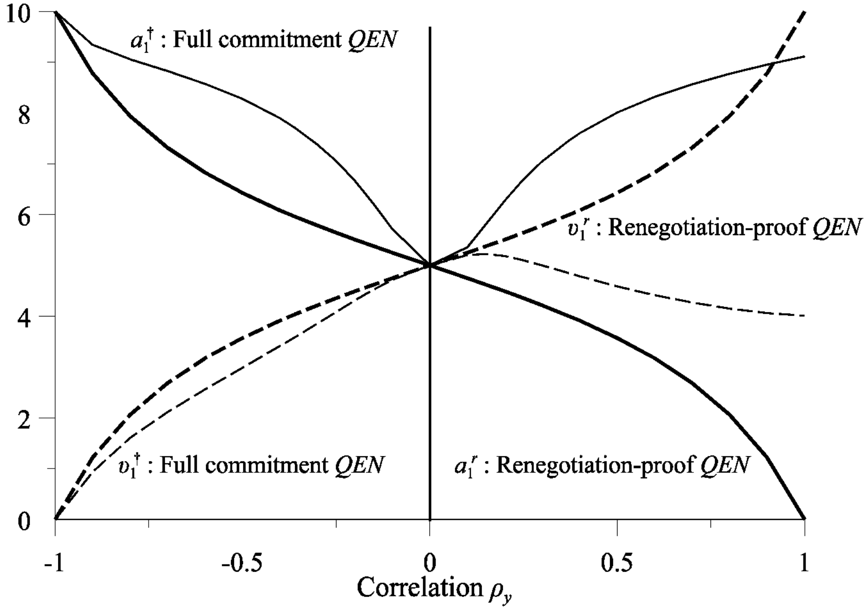

Figure 28.2(b): Impact on first-period incentive rates and induced actions of performance measure correlation in full-commitment and renegotiation-proof *QEN* contracts for identical periods case with no productivity information.

Full-commitment versus Renegotiation-proof QEN-P Contracts with Productivity Information

As discussed in Section 27.3.1, if there is productivity information (i.e., $\rho_\theta \neq 0$) and full commitment, the choice of the slope γ reflects the fact that it affects both the indirect first-period covariance incentive and the correlation between the induced second-period effort and its productivity. With renegotiation, on the other hand, the optimal renegotiation-proof level of γ only reflects the latter relation. Hence, the full-commitment contract dominates the renegotiation-proof contract.

As illustrated in Figure 28.3(a), the size of the difference in the principal's expected payoff is affected by both ρ_y and ρ_θ. In particular, the difference in the principal's expected payoff from a full-commitment versus a renegotiation-proof contract is small if the correlations are congruent, but can be large if they are incongruent. For example, the expected payoffs are almost identical if ρ_θ = ρ_y = −½ or +½, but they differ significantly if ρ_θ = $-\rho_y$ = −½ or +½. In the latter cases, the expected payoffs with renegotiation are even substantially lower than if there is no productivity information (ρ_θ = 0), i.e., with a renegotiation-proof *LEN* contract.

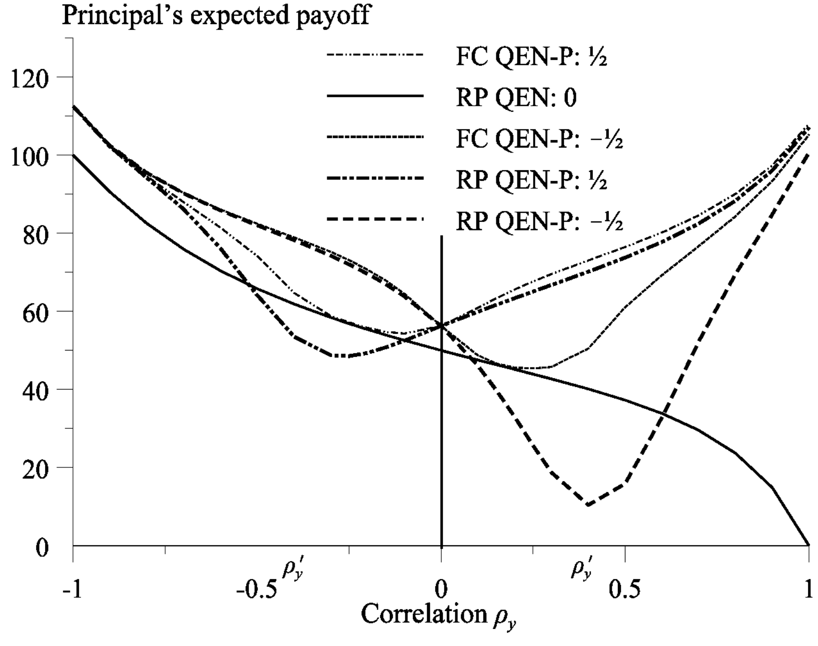

Figure 28.3(a): Impact of performance measure correlation in full-commitment and renegotiation-proof *QEN-P* contracts with varying productivity information (ρ_θ) for identical periods case.

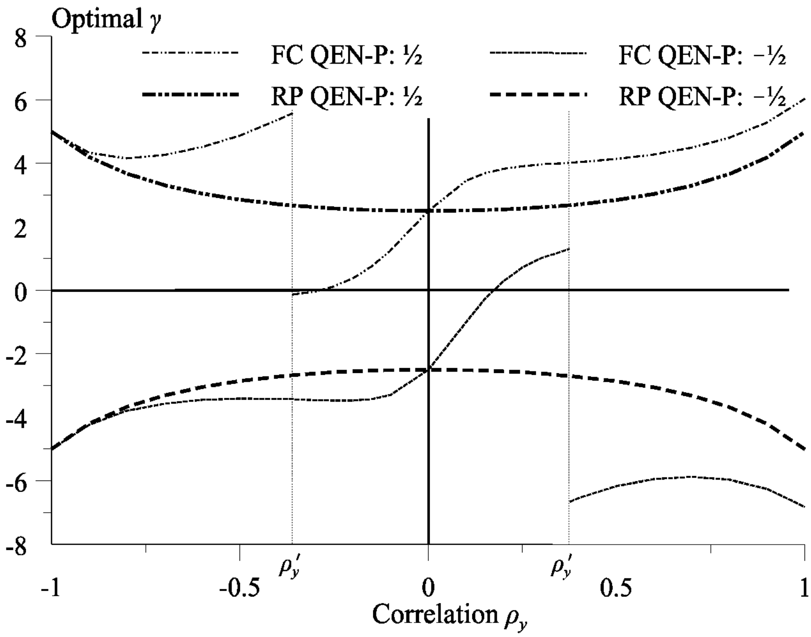

Figure 28.3(b): Impact on optimal slope of the second-period incentive rate of performance measure correlation in full-commitment and renegotiation-proof *QEN-P* contracts with varying productivity information (ρ_θ) for identical periods case.

To understand these relations see Figures 28.3(b) and (c). Figure 28.3(b) depicts the optimal slope γ with renegotiation and full commitment. In a renegotiation-proof contract, γ^o has the same sign as ρ_θ so that the second-period incentive rate and the resulting induced second-period effort are high if the second-period productivity θ_2 is high. Note from (28.21b) that γ^o only depends on ρ_θ and ρ_y^2 and, thus, is independent of the sign of ρ_y and the first-period incentive rate. This, however, ignores the indirect first-period covariance incentive created by $\gamma \neq 0$, i.e., $rM[v_1]\,[\gamma\rho_y]$. The key elements of the indirect covariance incentive are the first-period incentive rate, and the slope times the performance measure correlation, i.e., v_1 and $\gamma\rho_y$, respectively. With full commitment these elements can be chosen simultaneously but with renegotiation, the slope γ^o is independent of the sign of ρ_y as well as of the first-period incentive rate. From (28.21) it follows that

$$\gamma^\dagger\rho_y - \gamma^o\rho_y = \frac{rv_1^\dagger\rho_y^2 M[b - a_1^\dagger]}{M^2 + r(1-\rho_y^2)}. \tag{28.24}$$

The induced first-period action with full commitment is less than the first-best first-period action, i.e., $a_1^\dagger < b$. Hence, the relation between this element of the indirect covariance incentive with full commitment versus renegotiation is determined by the sign of $v_1^\dagger$. This provides the following result.

Proposition 28.12

Assume identical periods, i.e., $b_1 = b_2 = b$, $M_{11} = M_{22} = M$, and $M_{21} = 0$. In the *QEN-P* model, the following relations hold between the slope times the performance measure correlation with renegotiation versus full commitment (for $\rho_y \neq 0, -1$):

(a) With renegotiation, the slope has the same sign as ρ_θ, i.e., $\gamma^o\rho_\theta > 0$.

(b) If the correlations are congruent ($\rho_y\rho_\theta > 0$), then:

$$v_1^\dagger,\ v_1^r,\ \gamma^\dagger\rho_y,\ \gamma^o\rho_y > 0, \text{ and } \gamma^o\rho_y < \gamma^\dagger\rho_y.$$

(c) If the correlations are incongruent ($\rho_y\rho_\theta < 0$), then:

$$0 > \rho_y\gamma^o, \text{ and } \rho_y\gamma^o > \rho_y\gamma^\dagger \text{ if, and only if, } v_1^\dagger < 0.$$

If the correlations are congruent, the "correct" second-period action variability, i.e., $\gamma\rho_\theta > 0$, induces a positive indirect covariance incentive with a positive first-period incentive rate, i.e., $rM[v_1][\gamma\rho_y] > 0$. Hence, the first-period incentive rates and the indirect covariance incentives are all positive with both full commitment and renegotiation. However, the determination of the optimal slope with renegotiation fails to reflect the positive indirect covariance incentive and, thus, the covariance incentive is less efficient with renegotiation, i.e., $0 < \gamma^o\rho_y < \gamma^\dagger\rho_y$. As illustrated in Figure 28.3(c) for $\rho_\theta = ½$,[29] the less efficient indirect covariance incentive with renegotiation (for $\rho_y > 0$) is partly mitigated by increasing the first-period incentive rate, i.e., $v_1^r > v_1^\dagger$ but $a_1^r < a_1^\dagger$.

The results are more subtle if the correlations are incongruent. In this case, the "correct" second-period action variability implies that $\gamma\rho_y$ is negative. If ρ_y is large (i.e., close to one or minus one), a positive first-period incentive rate would imply a large and negative indirect covariance incentive. Hence, as discussed for the full commitment setting in Section 27.4.1, it may be optimal to use a negative direct first-period incentive, i.e., $v_1^\dagger < 0$, in order to maintain the "correct" second-period action variability and provide large and positive indirect covariance incentives. It then follows from (28.24) and (a) that $0 > \rho_y\gamma^o > \rho_y\gamma^\dagger$ (see Figure 28.3(b)). On the other hand, as the performance measure corre-

[29] The graph for $\rho_\theta = -½$ is virtually a mirror image of Figure 28.3(c).

lation gets closer to zero, the impact of $\gamma\rho_y$ on the indirect covariance incentive gets smaller, *ceteris paribus*. Hence, with full commitment there is a point ρ_y' at which the first-period incentive rate makes a discrete jump to being positive. At this point, it is optimal either to switch to using an "incorrect" second-period action variability ($\gamma\rho_\theta < 0$) to maintain a positive indirect covariance incentives, or to using a smaller "correct" second-period action variability although this yields a negative indirect covariance incentive (depending on the parameter values).[30] However, with renegotiation the principal cannot commit to using an "incorrect" or low second-period action variability and, therefore, close to point ρ_y' the first-period incentive rate increases continuously and becomes positive as ρ_y approaches zero. The principal's lack of ability to simultaneously control the sign and magnitudes of the first-period incentive rate and the slope can be very costly as illustrated in Figure 28.3(a) for performance measure correlations close to the discontinuity points ρ_y'. As illustrated in Figure 28.3(c) a significant part of that loss is due to a lower induced first-period effort.

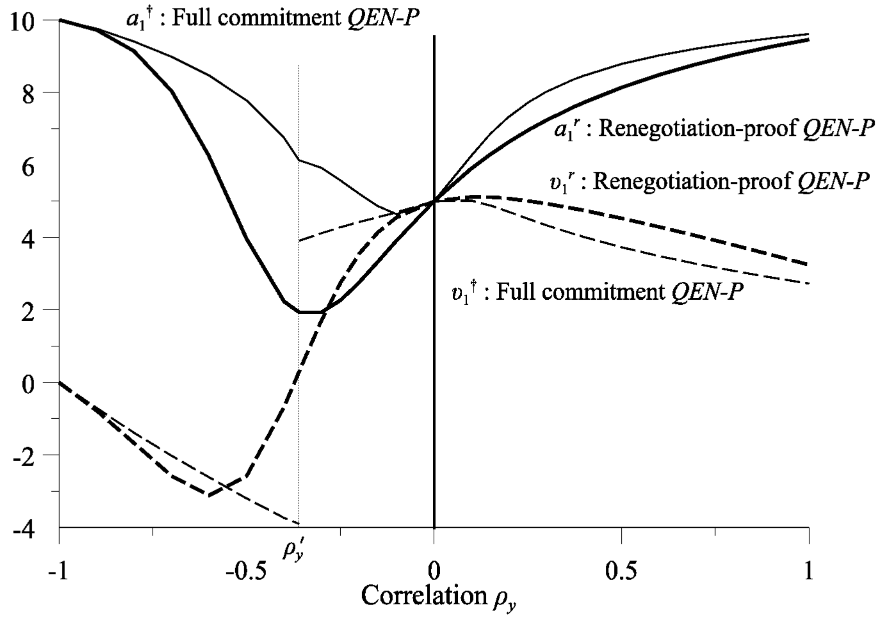

Figure 28.3(c): Impact on first-period incentive rates and induced actions of performance measure correlation in full-commitment and renegotiation-proof *QEN-P* contracts for identical periods case with productivity information: ρ_θ = ½.

[30] See the discussion of Figure 27.2(b) for further discussion of these discontinuities.

28.3 INTERDEPENDENT PERIODS WITH NO AGENT COMMITMENT TO STAY

In the previous section we assumed that once the initial contract is signed, both the agent and the principal are committed to the employment relation for the full duration of the contract (although the principal can change the terms of the contract if the agent agrees). In this section, we consider a setting in which there are no switching costs for the agent and he can accept employment from a different principal at the end of the first period. However, we assume the principal wants to induce the initial agent to stay for both periods, for example, due to high switching costs for the principal.

The Incentive for the Agent to Act Strategically and then Leave

Without loss of generality, the agent's reservation wage is assumed to be zero in each period. Of course, the agent cannot leave before settling the contract for the first period. Therefore, the initial compensation contract is divided into two period-specific components, i.e.,

$$\mathbf{s}(\psi_1, \psi_2) = \mathbf{s}_1(\psi_1) + \mathbf{s}_2(\psi_1, \psi_2),$$

where s_t is paid to the agent before he can leave at date t. Note that, if the initial contract is renegotiation-proof, and the second-period contract is the solution to the principal's basic second-period problem $P_2^o(\psi_1)$, i.e., $\mathbf{s}_2(\psi_1, \psi_2) = \mathbf{s}_2^o(\psi_1, \psi_2)$, then it may appear at first glance that the agent has no *ex post* incentive to leave at the end of the first period. The second-period contract $z_2^o(\psi_1) = \{\mathbf{s}_2^o(\psi_1), \mathbf{a}_2^o(\psi_1)\}$ gives the agent a certainty equivalent equal to zero, which is what he can get from alternative employment. However, this assertion presumes that the agent will not act strategically when he selects his first-period action. The key question is whether he can be better off by choosing a first-period action different than the principal's conjecture and then leave at the end of the first-period – a so-called "take-the-money-and-run" strategy (see Baron and Besanko, 1987, and Christensen, Feltham, and Şabac, 2003).

The benefit from acting strategically is due to the indirect first-period incentives created by the second-period contract $z_2^o(\psi_1)$. These incentives occur because the agent's first-period action affects the posterior mean of the second-period statistic, and it affects the covariance between the agent's first-period utility and his second-period certainty equivalent. However, they only influence the agent's first-period action choice if he plans to stay – they are irrelevant if he plans to leave.

More specifically, if the agent plans to stay for the second period for all ψ_1, then the first-order condition for his first-period action choice is given by (see (28.6b)):

$$\kappa_1'(a_1^r) = \frac{1}{r}\Big[\; M_{11}\text{Cov}(u_1^r,\psi_1) - \text{E}[q_2^o] + \text{Cov}(u_1^r,q_2^o)\Big]. \qquad (28.25)$$

Suppose the principal offers the agent the optimal contract z^r based on the problem in Table 28.1 – which assumes the agent will not leave. Now consider whether the agent will benefit from acting strategically and leaving for some first-period reports Ψ_1^l. Let $U_0(a_1,z^r,\Psi_1^l)$ represent the agent's *ex ante* expected utility if he accepts contract z^r, takes action a_1, and leaves if $\psi_1 \in \Psi_1^l$. If the agent leaves (i.e., $\psi_1 \in \Psi_1^l$), then he receives a second-period wage of $c_2^o = 0$, whereas he receives $s_2^o(\psi_1,\psi_2)$ if he stays.

If the agent takes the conjectured action a_1^r, then he receives his reservation utility of -1 whether he stays or leaves, since staying means the solution to the problem in Table 28.1 is implemented and leaving results in him receiving the reservation wage $c_1^o = 0$. To provide insight into the benefits of acting strategically, we consider the case in which the agent chooses the first-period action a_1^l that is optimal if he plans to leave for all first-period reports, i.e., $\Psi_1^l = \Psi_1$ and

$$a_1^l \in \underset{a_1}{\text{argmax}}\; U_0(a_1,z^r,\Psi_1) = -\int_{\psi_1}\exp[-r\{s_1^r(\psi_1) - \kappa(a_1)\}]\, d\Phi(\psi_1|a_1,a_1^r).$$

To determine whether a_1^l differs from a_1^r, we take the derivative of the agent's *ex ante* expected utility with respect to a_1 evaluated at $a_1 = a_1^r$:[31]

$$\frac{\partial U(a_1^r,z^r,\Psi_1)}{\partial a_1} = -r\kappa_1'(a_1^r) + M_{11}\text{Cov}(u_1^r,\psi_1).$$

Hence, it follows from (28.25) that

$$\frac{\partial U(a_1^r,z^r,\Psi_1)}{\partial a_1} \neq 0 \;\Leftrightarrow\; \text{E}[q_2^o] - \text{Cov}(u_1^r,q_2^o) \neq 0.$$

[31] See the proof of Proposition 28.7 in Appendix 28B for the derivation of the first-order condition (28.6). The derivative of the agent's *ex ante* expected utility is exactly the same as in that proof except that the term $q_2^o(\psi_1)$ (which reflects the indirect first-period incentives in Table 28.1(b)) is not present if the agent plans to leave.

That is, the agent has an *ex ante* incentive to deviate from a_1^r except in knife-edge cases in which $\mathrm{E}[q_2^o] = \mathrm{Cov}(u_1^r, q_2^o)$.

For example, in the no productivity information case, $q_2^o = r\kappa_2'(a_2^o)[\rho_y M_{11} - M_{21}]/M_{22}$, which is a constant. Hence, $\mathrm{Cov}(u_1^r, q_2^o) = 0$ and $\mathrm{E}[q_2^o] = \mathrm{Cov}(u_1^r, q_2^o)$ if, and only if, $M_{21} = \rho_y M_{11}$. If $\rho_y M_{11} > (<) M_{21}$, then the conjectured action a_1^r is not incentive compatible and the agent has an incentive to increase (decrease) his first-period action relative to that conjecture and then leave at the end of the first period![32]

Given that z^r induces the agent to act strategically and leave, the key question is whether the agent's total compensation can be reallocated over the two periods, such that he has no incentive to act strategically and leave. Observe that, loosely speaking, the agent is motivated to leave (stay) if, after the first-period report is issued, he anticipates low (high) second-period compensation. Hence, while deferred compensation can be effective, it must be carefully designed both to induce the conjectured first-period action, and to induce the agent to stay for all first-period reports.

Given exponential *AC-EC* preferences, deferring a fixed wage to the second period, i.e.,

$$s_1(\psi_1) = s_1^r(\psi_1) - \delta, \quad s_2(\psi_1, \psi_2) = s_2^o(\psi_1, \psi_2) + \delta,$$

has no impact on the agent's second-period action choice and the agent's *ex ante* expected utility if he stays for both periods. Moreover, if the agent takes the conjectured first-period action, the conditional certainty equivalent of the second-period contract is independent of the first-period report. However, deferring the payment of δ to the end of the second period reduces the certainty equivalent of the first-period contract and, thus, deviating from the conjectured first-period action and leaving for some first-period reports becomes less attractive to the agent, *ex ante*. In fact, if the deferred compensation is sufficiently high, the agent's *ex ante* expected utility is higher if he takes the conjectured first-period action than if he deviates and follows an optimal "leave strategy" depending on the first-period report. Determining the minimum deferred compensation which makes the conjectured first-period action incentive compatible even if the agent can leave after the first period is complex for optimal contracts. Therefore, we limit our analysis to how it can be determined for renegotiation-proof *QEN-P* contracts.

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[32] This argument presumes that the agent leaves after the first period no matter what the first-period report is going to be. However, the agent may not want to leave *ex post* for all first-period reports, if he takes a first-period action different from the conjectured action. On the other hand, allowing for this possibility makes a deviation from the conjectured first-period action even more attractive to the agent.

Renegotiation-proof Contracts with No Productivity Information and Deferred Compensation

First consider the setting in which there is renegotiation with no productivity information. In this case, the second-period contract is independent of the first-period report. Hence, the agent will either stay for all ψ_1 or leave for all ψ_1.

Let z^r represent the principal's optimal renegotiation-proof contract if the agent can commit not to leave, and let δ represent the compensation deferred from the first to the second period. Given that contract and deferral, the agent's maximum total certainty equivalent if he plans to leave, i.e., $\Psi_1^l = \Psi_1$, is[33]

$$CE_0^*(z^r, \Psi_1, \delta) = \underset{a_1}{\text{maximize}}\ f_1^r + v_1^r M_{11}[a_1 - a_1^r] - \tfrac{1}{2} a_1^2 - \tfrac{1}{2} r (v_1^r)^2 - \delta$$

$$= v_1^r M_{11}[a_1^l - a_1^r] - \tfrac{1}{2}[(a_1^l)^2 - (a_1^r)^2] - \delta, \tag{28.26a}$$

where
$$a_1^l = v_1^r M_{11} \tag{28.26b}$$

and
$$a_1^r = v_1^r M_{11} - \bar{v}_2^o [\rho_y M_{11} - M_{21}]. \tag{28.26c}$$

That is, the optimal "leave action" is based on the first-period direct incentives, whereas the optimal "stay action" is based on both the first-period direct and indirect incentives. Of course, the principal has set the direct incentives v_1^r under the assumption the agent will stay. The deferred compensation required to induce the agent to stay, i.e., to ensure that $CE_0^*(z^r, \Psi_1, \delta) \leq CE_0^*(z^r, \varnothing, \delta) = 0$, is characterized in the following proposition (see also Christensen, Feltham, and Şabac, 2003, Prop. 3).

Proposition 28.13

Given that there is no productivity information, the optimal renegotiation-proof contract z^r will be implemented if, and only if, there is deferred compensation with[34]

$$\delta \geq \delta^r \equiv v_1^r M_{11}[a_1^l - a_1^r] - \tfrac{1}{2}[(a_1^l)^2 - (a_1^r)^2]$$

$$= \tfrac{1}{2}(\bar{v}_2^o [\rho_y M_{11} - M_{21}])^2 \geq 0. \tag{28.27}$$

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[33] a_1^r is characterized by (28.15) if we let $\gamma^o = 0$, and $f_1^r = \tfrac{1}{2}(a_1^r)^2 + \tfrac{1}{2} r (v_1^r)^2$.

[34] Inserting (28.26b) and (28.26c) into the first expression yields

$$\delta = (v_1^r M_{11})\, \bar{v}_2^o [\rho_y M_{11} - M_{21}] - \tfrac{1}{2}[(v_1^r M_{11})^2 - (v_1^r M_{11} - \bar{v}_2^o [\rho_y M_{11} - M_{21}])^2].$$

Expanding and collecting terms yields (28.27).

That is, the agent has no incentive to act strategically in the first period (with the intention of leaving after the first period) if the deferred compensation is greater than or equal to δ^r, which is the difference in the expected incentive compensation minus the difference in effort costs. This amount is equal to one half the square of the indirect first-period incentive.

If the deferred compensation is less than δ^r, then $CE_0^*(z^r, \Psi_1, \delta) > 0$. Moreover, the agent's *ex post* certainty equivalent of the second-period contract given the optimal first-period deviation a_1^l and the first-period report ψ_1 is negative (see, for example, (27.27) with $\gamma = 0$):

$$CE_1(\psi_1, a_1^l, z^r, \delta) = \delta - \bar{v}_2^o\,[\rho_y M_{11} - M_{21}][a_1^l - a_1^r]$$

$$< \delta^r - \bar{v}_2^o\,[\rho_y M_{11} - M_{21}][a_1^l - a_1^r]$$

$$= -\tfrac{1}{2}\delta^r < 0.$$

Hence, for $\delta < \delta^r$ the agent has both an *ex ante* incentive to deviate from the conjectured first-period action and an *ex post* incentive to leave given the optimal first-period deviation, i.e., δ^r is the *minimum* deferred compensation which makes the renegotiation-proof contract z^r sequentially incentive compatible even if the agent can leave after the first period.

Renegotiation-proof Contracts with Productivity Information and Deferred Compensation

In this section we specify renegotiation-proof period-specific compensation contracts which induce the same actions as the renegotiation-proof *QEN-P* contract in Proposition 28.9, pays the agent the same total compensation, and induces the agent to stay for all first-period reports. Let the two-period contract, represented by (z^r, δ), be of the form

$$s_1(\psi_1) = f_1^r + v_1^r \psi_1 - \delta, \quad s_2(\psi_1, \psi_2) = \delta + f_2^o(\psi_1) + v_2^o(\psi_1)\psi_2. \tag{28.28}$$

Clearly, the contract is renegotiation-proof, and the agent's total certainty equivalent is equal to his reservation wage of zero, independent of δ, if he takes the conjectured first-period action a_1^r and stays for both periods.

Now assume the agent acts strategically and takes action $a_1 \neq a_1^r$ with the intent of possibly leaving at the end of the first period, and assume $[\rho_y M_{11} - M_{21}] \neq 0$. If report ψ_1 is received and the agent leaves, then his second-period certainty equivalent equals zero. On the other hand, if he stays, it is (see, for example, (27.27))

$$CE_1(\psi_1, a_1, z^r, \delta) = \delta - (\bar{v}_2^o + \gamma^o \psi_1)[\rho_y M_{11} - M_{21}][a_1 - a_1^r]. \tag{28.29a}$$

Therefore, for any first-period action $a_1 \neq a_1^r$ and deferral δ, there exists a performance statistic for which the agent is indifferent between leaving and staying, i.e.,

$$\psi_1^r(a_1,\delta) = \delta\{\gamma^o[\rho_y M_{11} - M_{21}][a_1 - a_1^r]\}^{-1} - \bar{\upsilon}_2^o/\gamma^o. \qquad (28.29b)$$

Consequently, the set of first-period statistics for which it is *ex post* optimal for the agent to leave, given a_1, δ, and z^r, is

$$\Psi_1^l(a_1,z^r,\delta) \equiv \begin{cases} (-\infty, \psi_1^r(a_1,\delta)) & \text{if } \gamma^o[\rho_y M_{11} - M_{21}][a_1 - a_1^r] < 0, \\ (\psi_1^r(a_1,\delta), \infty) & \text{if } \gamma^o[\rho_y M_{11} - M_{21}][a_1 - a_1^r] > 0. \end{cases} \qquad (28.30)$$

Let $\Psi_1^s(a_1,z^r,\delta)$ denote its complement. The agent's *ex ante* expected utility, given a_1,δ, and z^r, with an optimal *ex post* "leave strategy," is

$$U_0^a(a_1,z^r,\delta) \qquad (28.31)$$

$$= -\Bigg(\int_{\Psi_1^l(a_1,z^r,\delta)} \exp[-r(f_1^r + \upsilon_1^r\psi_1 - \tfrac{1}{2}a_1^2 - \delta + c_2^o]\, d\Phi(\psi_1|a_1,a_1^r)$$

$$+ \int_{\Psi_1^s(a_1,z^r,\delta)} \exp[-r(f_1^r + \upsilon_1^r\psi_1 - \tfrac{1}{2}a_1^2 - \delta$$

$$+ \delta - (\bar{\upsilon}_2^o + \gamma^o\psi_1)[\rho_y M_{11} - M_{21}][a_1 - a_1^r])]\, d\Phi(\psi_1|a_1,a_1^r)\Bigg).$$

The agent's optimal first-period action given contract (z^r,δ) is

$$a_1^l(\delta) \in \underset{a_1}{\operatorname{argmax}}\, U_0^a(a_1,z^r,\delta). \qquad (28.32)$$

Using our basic identical-periods example with $\rho_y = .25$, Figures 28.4(a) and (b) show for $\rho_\theta = \tfrac{1}{2}$ and $\rho_\theta = -\tfrac{1}{2}$, respectively, the agent's optimal first-period action (on the secondary axis), his certainty equivalent, and the probability that he leaves after the first period.[35]

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[35] Appendix 28B gives details of how to calculate the agent's *ex ante* expected utility in (28.31). The maximization problem in (28.32) must be solved by a numerical method, since the agent's *ex ante* utility is not necessarily a concave function of a_1 when δ is small.

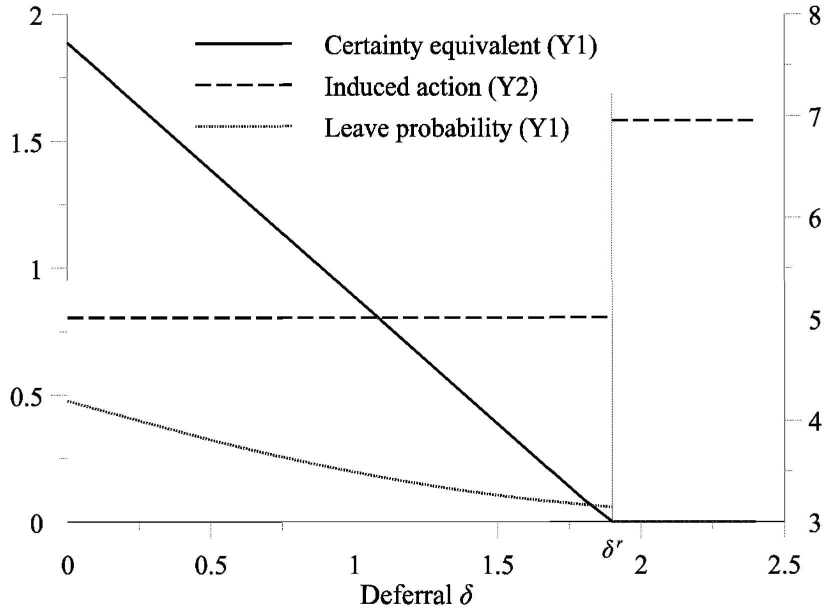

Figure 28.4(a): Impact of deferral on induced first-period action, agent's certainty equivalent, and leave probability for basic identical-periods *QEN-P* contract with $\rho_y = .25$, and $\rho_\theta = ½$.

In each case there exists a finite deferral δ^r such that for all deferrals $\delta \geq \delta^r$, the contract (z^r,δ) in (28.28) induces the agent to take $a_1^l(\delta) = a_1^r$, and stay for all ψ_1, where a_1^r is the agent's optimal action given z^r and a binding commitment to stay for the second period. Moreover, there is a discrete jump in induced first-period action at δ^r. In Figure 28.4(a), the induced first-period action with $\delta < \delta^r$ is equal to the optimal first-period action given that the agent leaves for all ψ_1, i.e., $a_1^l(\delta) = v_1^r M_{11}$ (= 5.01), and the leave probability is significant. On the other hand, in Figure 28.4(b), the induced first-period action with $\delta < \delta^r$ is greater than $v_1^r M_{11}$ (= 3.56), but the leave probability is very small.[36]

[36] All examples we have done show that there is a finite deferral δ^r such that for all $\delta \geq \delta^r$, the contract (z^r,δ) in (28.28) induces $a_1^l(\delta) = a_1^r$. However, we have not been able to explicitly characterize this level of deferral.

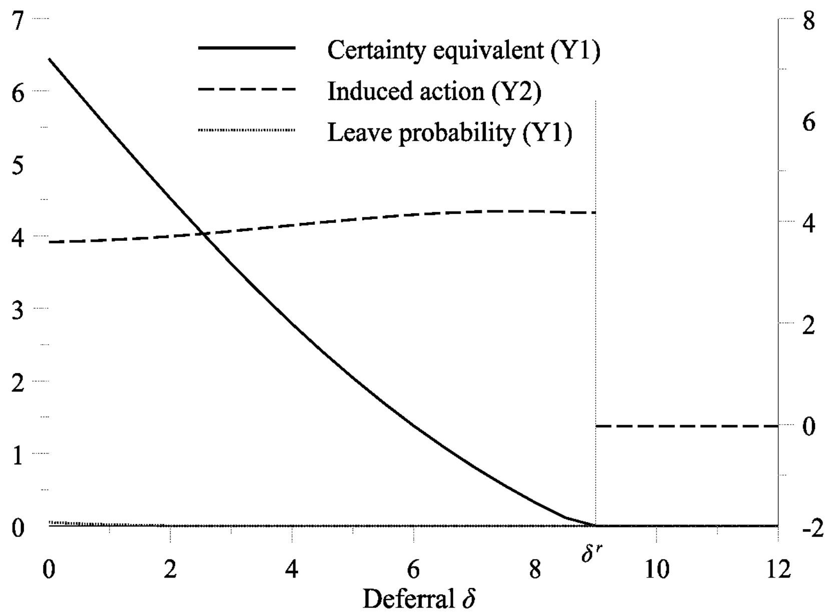

Figure 28.4(b): Impact of deferral on induced first-period action, agent's certainty equivalent, and leave probability for basic identical-periods *QEN-P* contract with $\rho_y = .25$, and $\rho_\theta = -½$.

28.4 ONE VERSUS TWO AGENTS WITH INTERDEPENDENT PERIODS

In the previous section we assumed that once the initial contract is signed, the principal is committed to the employment relation for the full duration of the contract (although he can change the terms of the contract if the agent agrees). In this section we examine the principal's preferences for one versus two agents. and examine his ability to commit to either retaining or replacing the first agent. The latter depends crucially on whether it is costly to switch agents. These switching costs could be job search costs incurred by the first agent for which he must be compensated if his employment is terminated.[37] In addition, it can include training costs incurred by the principal when he hires a new agent.

[37] Inclusion of compensation for these costs is part of the initial contract, given the requirement that it induces the agent to accept the contract.

Note that the second-period contract that the principal would offer to a new agent at the beginning of the second period is exactly the same as the second-period contract the principal would offer to the first agent at the renegotiation stage, i.e., the optimal solution to the principal's basic second-period problem, $z_2^o(\psi_1)$. Hence, the principal's basic preferences for retaining or replacing the first agent depend on whether the indirect first-period incentives created by the second-period contract (if the first agent is retained) are beneficial to the principal or not, i.e., increase or decrease the induced first-period action. The principal's *ex post* incentive to retain or replace the first agent depends, of course, on the principal's switching costs. We first examine the case with zero switching costs, and then we examine the case with strictly positive switching costs. Both cases are examined within the *QEN-P* model.

Zero Switching Costs

Note from (28.15) that for a given direct first-period incentive rate v_1, the total incremental effect of the two indirect first-period incentives is

$$ia_1(v_1) \equiv -\bar{v}_2^o\,[\rho_y M_{11} - M_{21}] + r v_1 \gamma^o [\rho_y M_{11} - M_{21}]. \qquad (28.33)$$

Clearly, there are no indirect incentives if $\rho_y M_{11} = M_{21}$ (i.e., the independent periods case) and the principal is indifferent between retaining and replacing the first agent. If $\rho_y M_{11} \neq M_{21}$, then there is an indirect posterior-mean incentive, which is independent of v_1 and if $\gamma^o \neq 0$, then there is also an indirect covariance incentive, which does depend on v_1. If there is no productivity information, then $\gamma^o = 0$ and the principal prefers to retain the first agent if, and only if, $\rho_y M_{11} < M_{21}$. In that case, the first agent will provide more first-period effort at the same risk premium if he expects to stay for both periods than if he expects to be replaced after the first period.[38] On the other hand, if there is productivity information, then $\gamma^o \neq 0$ and both types of indirect first-period incentives exist. In that case, the principal's preference for retaining or replacing the first agent is less obvious. However, if $\rho_y M_{11} < M_{21}$ and $\rho_\theta < 0$ (implying that $\gamma^o < 0$), both types of indirect first-period incentives are positive for all positive first-period incentive rates, i.e., the principal prefers to retain the first agent.

The optimal first-period incentive rate is v_1^r if the first agent is retained for both periods, whereas the optimal first-period incentive rate is

[38] We assume that $\bar{v}_2^o > 0$. If $\bar{v}_2^o = 0$ (as in a window dressing case with $b_2 = 0$), the principal is indifferent between retaining and replacing the agent. Compare to the analysis of full-commitment *LEN* contracts in Section 26.2.

$$v_1^o = \frac{b_1 M_{11}}{M_{11}^2 + r}$$

if the first agent is to be replaced after the first period. Clearly, if the total of the indirect first-period incentive is positive given v_1^o, i.e., $ia_1(v_1^o) > 0$, then the principal prefers to retain the first agent (because offering one agent the two-agent contract will induce more first-period effort at the same risk premium). On the other hand, if $ia_1(v_1^r)$ is negative, the principal prefers to switch agents. However, $ia_1(v_1^o)$ may be negative while $ia_1(v_1^r)$ is positive and, in that case, the tradeoff is more complicated. The key in this case is that even though $ia_1(v_1^o)$ is negative, it may be possible to adjust the first-period incentive rate and, thus, change the indirect first-period covariance incentive, such that the principal gets a higher net-payoff from retaining the first agent as opposed to replacing him. Of course, this can only occur if $ia_1(v_1^r)$ is positive.

We illustrate these results in Figure 28.5 for the identical periods case with $b_1 = b_2 = b = 10$, $M_{11} = M_{22} = M = 1$, $M_{21} = 0$, $r = 1$, and $\rho_\theta = -½$. With $\rho_y < 0$, the correlations are congruent and both types of indirect first-period incentives are positive for all positive first-period incentive rates. Thus, in this region the principal prefers to pay the first agent a positive direct first-period incentive and retain him for both periods. Note that because the indirect first-period incentive is positive, it is optimal for the principal to use a lower first-period direct incentive than would be the case if the first agent is expected to be replaced at the end of the first period.

In this example (with $M_{21} = 0$ and $\rho_\theta = -½$), the correlations are incongruent if $\rho_y > 0$ and both types of indirect first-period incentives are negative for all positive first-period incentive rates. On the other hand, if the first-period incentive rate is negative, then the indirect covariance incentive is positive, while the indirect posterior-mean incentive continues to be negative. If the correlation is high, i.e., $\rho_y > \rho_y''$, then it is optimal to use a negative first-period incentive and retain the first agent to obtain a large positive indirect covariance incentive. On the other hand, for moderate levels of positive correlation, i.e., $\rho_y \in (0,\rho_y'')$ it is optimal to use a positive first-period direct incentive and replace the first agent at the end of the first-period, thereby avoiding the negative indirect first-period incentive.

Figure 28.5 includes a plot of the optimal indirect incentive for both the one-agent and two-agent contracts. For $\rho_y \in (0,\rho_y')$ the indirect incentive for the optimal one-agent contract $ia_1(v_1^r)$ is negative and, thus, it is clearly optimal to replace the first agent. On the other hand, for $\rho_y \in (\rho_y', 1]$, $ia_1(v_1^r)$ is positive while $ia_1(v_1^o)$ is negative. Two agents are preferred to one agent for $\rho_y \in (\rho_y',\rho_y'')$, while it is optimal to retain the first agent for $\rho_y > \rho_y''$ – even though $ia_1(v_1^o)$ is negative, it is only optimal to adjust the first-period incentive rate sufficiently to make one agent preferred to two agents for $\rho_y > \rho_y''$.

Proposition 28.14

Assume renegotiation-proof *QEN-P* contracts with no switching costs for the principal. The following relations hold for the principal's preference for retaining or replacing the first agent with an identical agent after the first period:

(a) If there is no productivity information, then one agent is preferred to two agents if, and only if, $\rho_y M_{11} < M_{21}$.

(b) If there is productivity information, then one agent is preferred to two agents if $ia_1(v_1^o) > 0$ (which, for example, is the case with $\rho_y M_{11} < M_{21}$ and $\rho_\theta < 0$).

(c) If there is productivity information, then two agents are preferred to one agent if $ia_1(v_1^\dagger) < 0$.

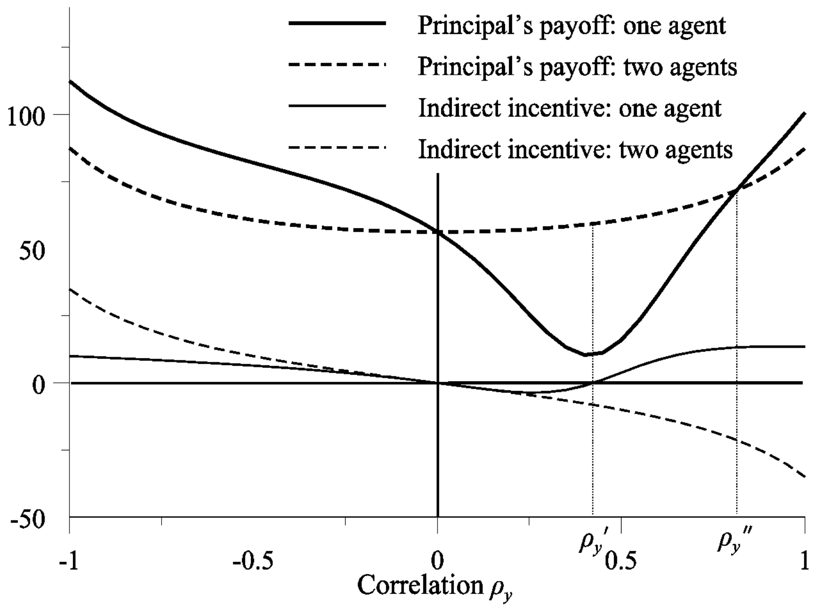

Figure 28.5: One versus two agents in renegotiation-proof *QEN-P* contracts for identical periods case with $\rho_\theta = -½$.

Strictly Positive Switching Costs

The preceding analysis assumes that the principal can costlessly replace the first agent after the first period with an identical agent. This implies that *ex post* the principal is indifferent between retaining or replacing the first agent – one agent is as good as any other agent. However, that is not the case if the principal incurs a switching cost if he replaces the first agent: *ex post* the principal has an incentive to retain the first agent even though his *ex ante* preferences are to replace the first agent (due to negative indirect first-period incentives).

If the principal can make an *ex ante* commitment to replace the first agent after the first period, strictly positive switching costs pose no problems. In this case, the gain from avoiding negative indirect first-period incentives if the first agent is retained must be compared to the cost of switching agents after the first period. However, if the principal cannot commit to switching agents, the first agent will anticipate the principal's *ex post* incentive to retain him for the second period and, therefore, his first-period action will recognize the indirect first-period incentives created by the second-period contract.[39]

Proposition 28.15

Assume renegotiation-proof *QEN-P* contracts with strictly positive switching costs for the principal. If the principal cannot commit to switching agents after the first period, then the first agent is retained for both periods (by paying sufficient deferred compensation).

28.5 CONCLUDING REMARKS

Perhaps the most noteable aspect of the analysis in this and the previous chapter is the fact that if incentives are based on stochastically independent performance statistics, then there are potentially three types of incentives for the agent's first-period action choice. First, there is a direct first-period incentive if the first-period action influences the first-period performance measure. Second, there is an indirect posterior-mean incentive if the first-period action influences the first-period performance measure which is correlated with the second-period performance measure, or if the first-period action directly influences the second-period performance measure. Third, there is an indirect covariance incentive if

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[39] If the agent does not incur switching costs and there is no deferred compensation, then the agent might take a first-period action, a_1^l, with the intention of leaving after the first period. However, these incentives are recognized by the principal and, therefore, the second-period contract offered by the principal at $t = 1$ will be acceptable to the first agent given the conjectured first-period action a_1^l. Hence, if there are strictly positive switching costs, the principal can retain the first agent, but this is inconsistent with the agent's *ex ante* intention to leave after the first period. That is, if there are strictly positive switching costs for the principal, there is no equilibrium in which the principal does not pay deferred compensation and replaces the first agent after the first period (see also Christensen, Feltham, and Şabac, 2003, Prop. 1).

the first-period performance measure is correlated with the marginal productivity of the second-period effort.

A key difference between full commitment (Chapter 27) and renegotiation (this chapter) is that all three types of incentives are present in the former whether there is productivity information or not, while the indirect covariance incentive can only be sustained in the latter if there is productivity information.

The indirect incentives create incentives for the agent to act strategically when he takes his first-period action anticipating that he may wish to leave after the first period. However, deferred compensation can be used to eliminate the incentives to act strategically.

The indirect incentives can be positive or negative. If they are positive with a two-agent contract, the principal will prefer to hire the same agent for both periods. If they are negative (with a two-agent contract), the principal may prefer to terminate the first agent at the end of the first period unless there are sufficiently large switching costs.

Our analysis and results may depend significantly on the fact we assume the performance measures are normally distributed and the agent has exponential *AC-EC* preferences (which prevents wealth effects and removes incentives for consumption smoothing). Normal distributions and the lack of wealth effects are the key assumption underlying our result that the renegotiation-proof second-period contract is independent of the first-period performance (except for possible productivity information). This result would not hold even with exponential *AC-ED* preferences (see, for example, the analysis of one versus two agents in Chapter 25 with *AC-ED* preferences). The wealth effects would also be avoided if the agent has exponential *TA-EC* preferences, but would create a demand for consumption smoothing. Of course, if we allow the agent to borrow and save, the consumption smoothing issue will likely have only a limited effect (see, for example, the analysis in Chapters 25 and 26). Interestingly, the timing of reports is irrelevant if there is full commitment and the agent has exponential *AC-EC* preferences, but the timing is not irrelevant when there is renegotiation. For example, in the model considered in this chapter, the first-period performance measure would become significantly less useful if it was not issued until the end of the second period and renegotiation continued to occur at the end of the first period. This latter result occurs because of what is often called the Fudenberg and Tirole (1990) problem, which we discussed in Section 24.1. Observe that the timing issues that arise with renegotiation also apply if there are exponential *TA-EC* preferences and differ significantly from the timing issues that arise with *TA-EC* preferences with full commitment.

With full commitment more informative performance measures are generally preferred. There is a growing literature showing that this may not be the case when there is renegotiation.[40] The key in these cases is that less informative first-period performance measures may result in the principal offering a second-period that is more closely aligned with an optimal second-period contract from an *ex ante* perspective. That is, if the principal chooses a more informative information system, and if he cannot commit not to renegotiate, then he shoots himself in the foot!

APPENDIX 28A: FHM PRODUCTION TECHNOLOGY ASSUMPTIONS

FHM use the following *common knowledge of technology assumption* throughout their analysis:

$$\varphi(y_t | \overleftarrow{\mathbf{a}}_{t-1}, a_t, \overleftarrow{\mathbf{y}}_{t-1}) = \varphi(y_t | a_t, \overleftarrow{\mathbf{y}}_{t-1}),$$

i.e., the history of public information is sufficient to determine how period t's actions will affect future outcomes and public reports. In the latter part of their paper they introduce the following more restrictive assumptions.

History-independent Technology:

$$\varphi_t(y_t | \overleftarrow{\mathbf{a}}_{t-1}, \overleftarrow{\mathbf{y}}_{t-1}, a_t) = \varphi_t(y_t | a_t), \quad t = 1, ..., T.$$

Stationary Technology:

$$\varphi_t(y_t | \overleftarrow{\mathbf{a}}_{t-1}, \overleftarrow{\mathbf{y}}_{t-1}, a_t) = \varphi(y_t | a_t), \quad t = 1, ..., T.$$

Proposition 28A.1 (FHM, Theorem 5)

Assume $\overleftarrow{\mathbf{y}}_t$ is contractible, a finite contracting horizon, equal access to financial markets, a history-independent technology, exponential *TA-EC* preferences, and existence of an optimal long-term contract. Then there is an optimal contract for which

(a) current actions and compensation do not depend on past performance:

$$\boldsymbol{a}_t(\overleftarrow{\mathbf{y}}_{t-1}) = a_t, \text{ and } \boldsymbol{s}_t(\overleftarrow{\mathbf{y}}_t) = \boldsymbol{s}_t(y_t);$$

(b) the principal's expected net profit in every period is zero; and

[40] See, for example, Indjejikian and Nanda (1999), Christensen, Feltham, and Şabac (2005), and Feltham, Indjejikian and Nanda (2005).

(c) action and compensation plans are identical to those in the optimal contract that would be offered in the "one-period problem" in which the agent retires at the end of the period and the available technology is that of period t.

Corollary

Given the same assumptions as in Proposition 28A.1, but with a stationary technology, there is an optimal contract such that $\boldsymbol{a}_t(\overleftarrow{\mathbf{y}}_{t-1}) = a_1$ and $\boldsymbol{s}_t(y_t) = \boldsymbol{s}_1(y_t)$. Thus, the net present value of the agent's total compensation when he retires with history ω_t^a is

$$\sum_{t=1}^{T} \beta^t \boldsymbol{s}_1(y_t).$$

The optimal contract requires no "memory," and the ability to provide optimal incentives in this model is not enhanced by having the agent write a long-term contract (or have a long-term relationship) with the principal. FHM emphasize that these "one-period contracts" are not the same as those which would be optimal if the agent *lived* for only one period – even when the agent works for only one period, he lives (and consumes) for an infinite number of periods. Conditions (a) and (b) of Proposition 28A.1 hold even if the agent has a finite life, but condition (c) does not hold because the agent's preferences over contracts depend on the length of his remaining life.

Small Discount Rates

If β is close to one, then the agent can spread variations in compensation in one period over many periods, so that his consumption becomes almost constant. Hence, he becomes almost risk neutral and can achieve a result close to first-best. FHM provide a result for the case in which $T \to \infty$ and they never allow the agent to borrow (to avoid the possibility of infinite negative debt), i.e., $w_t \geq 0$.

Proposition 28A.2 (FHM, Theorem 6)

Assume $\overleftarrow{\mathbf{y}}_t = \overleftarrow{\mathbf{x}}_t$ is contractible, the contracting horizon is infinite, the agent can save but not borrow (and can consume the minimum possible level of x), a stationary technology, and exponential *TA-EC* preferences. Let the principal pay the agent $\boldsymbol{s}_t(x_t) = x_t$ in every period. Then, for every $\varepsilon > 0$, there exists a discount rate $\beta(\varepsilon) < 1$, such that the agent can ensure himself a utility level $u(c^*, a^*) - \varepsilon$ for all $\beta > \beta(\varepsilon)$, where a^* is the first-best action and $c^* = \mathrm{E}[x|a^*]$.

The proof constructs a strategy which guarantees, with high probability, that the agent is able to consume approximately the mean output in every period after a finite number of periods. The strategy specifies that the agent chooses an efficient effort level and consumes close to the expected output unless his wealth falls below a critical level in which case he consumes the minimum output. This result is related to the literature on "folk theorems" in infinitely repeated principal-agent models, see Dutta and Radner (1994) for a review.

APPENDIX 28B: PROOFS OF PROPOSITIONS

Proof of Proposition 28.5:

Let $P_2^o(\psi_1)$ represent the principal's problem (28.3) for the special case in which $CE_1(z_1^I,\psi_1) = 0$ and, hence, the reservation utility at $t = 1$ is $\underline{U}_1^I(\psi_1) = -1$. The solution to that problem is represented by the contract $\boldsymbol{z}_2^o(\psi_1) = \{\boldsymbol{s}_2^o(\psi_1), \boldsymbol{a}_2^o(\psi_1)\}$.

The Lagrangian for $P_2^o(\psi_1)$ (in which we do not explicitly introduce the lower bound in the notation) is

$$\mathcal{L} = (b_2 + \rho_\theta \sigma \psi_1) a_2 - \int_{\Psi_2} \boldsymbol{s}^o(\psi_1,\psi_2)\, d\Phi^\dagger(\psi_2)$$

$$- \mu^o(\psi_1) \left\{ \int_{\Psi_2} \exp[-r\{\boldsymbol{s}^o(\psi_1,\psi_2) - \kappa_2(a_2)\}]\, d\Phi^\dagger(\psi_2) + 1 \right\}$$

$$- \lambda^o(\psi_1) \int_{\Psi_2} \exp[-r\{\boldsymbol{s}^o(\psi_1,\psi_2) - \kappa_2(a_2)\}][r\kappa_2'(a_2) + L_{a_2}(\psi_2)]\, d\Phi^\dagger(\psi_2).$$

Differentiating the Lagrangian for the basic second-period problem $P_2^o(\psi_1)$ with respect to the agent's compensation yields

$$- d\Phi^\dagger(\psi_2) + r\lambda_2^o(\psi_1) \exp[-r\{\boldsymbol{s}_2^o(\psi_1,\psi_2) - \kappa_2(\boldsymbol{a}_2^o(\psi_1))\}]\, d\Phi^\dagger(\psi_2) +$$

$$r\mu_2^o(\psi_1) \exp[-r\{\boldsymbol{s}_2^o(\psi_1,\psi_2) - \kappa_2(\boldsymbol{a}_2^o(\psi_1))\}][r\kappa_2'(\boldsymbol{a}_2^o(\psi_1)) + L_{a_2}(\psi_2)]\, d\Phi^\dagger(\psi_2) = 0.$$

Hence,

$$\boldsymbol{s}_2^o(\psi_1,\psi_2) = \kappa_2(\boldsymbol{a}_2^o(\psi_1)) + \frac{1}{r} \ln[r\{\lambda_2^o(\psi_1) + \mu_2^o(\psi_1)[r\kappa_2'(\boldsymbol{a}_2^o(\psi_1)) + L_{a_2}(\psi_2)]\}],$$

which is restated in (28.5b) using g_2 and $\boldsymbol{G}_2$ (as defined in (28.5d) and (28.5c)). For the reasons discussed in the text, the optimal contract with a reservation certainty equivalent $CE_1(z_1^I,\psi_1) \neq 0$ is equal to (28.4a).

Substituting (28.5a) and (28.4a) into (28.3c) provides

$$\mathrm{E}[u_2^o|\psi_1]r\kappa_2' + \mathrm{E}[u_2^o\psi_2|\psi_1]M_{22}/(1-\rho_y^2) = 0.$$

Then, since $\mathrm{E}[u_2^o|\psi_1] = -1$, $\mathrm{E}[\psi_2|\psi_1] = 0$, and $\mathrm{E}[u_2^o\psi_2|\psi_1] = \mathrm{Cov}(u_2^o,\psi_2|\psi_1)$ and solving for κ_2' yields (28.4b). **Q.E.D.**

Proof of Proposition 28.7:
We represent the participation and incentive compatibility constraints (b) and (c) in Table 28.1 in their simplified versions (by using the fact that $z_2^o(\psi_1)$ is a solution to $P_2^o(\psi_1)$ for each ψ_1). Then, forming the Lagrangian for the problem in Table 28.1, using multipliers λ_1 and μ_1 for (b) and (c), respectively, and differentiating with respect to the first-period compensation for each ψ_1 yields first-order condition (28.6a) for the optimal first-period compensation function.

Define (28.7d) to be the agent's first-period equilibrium utility. Substituting (28.6a) back into (b) yields (28.6b) as the first-order condition for the agent's first-period action (using that $\mathrm{E}[\psi_1] = 0$ and that the participation constraint is binding such that $\mathrm{E}[u_1^r] = -1$).

Characterization of (28.31):
Note that the agent's *ex ante* expected utility in (28.31) is determined as the sum of "truncated" expected utilities of two linear contracts. Using the same technique as in Appendix 3A of Volume I it is straightforward to prove the following result, which can be used to calculate this type of expected utilities.

Lemma 28B.1
Assume $x \sim \mathrm{N}(\mu,\sigma^2)$, and let $B \subseteq \mathbb{R}$. Then

$$-\int_B \exp[-r(\alpha+\beta x)]d\mathrm{N}(\mu,\sigma^2)$$
$$= -\exp[-r(\alpha+\beta\mu - \tfrac{1}{2}r\beta^2\sigma^2)]\ \mathrm{Prob}\,(y \in B;\beta),$$

where $y \sim \mathrm{N}(\mu - r\beta\sigma^2,\sigma^2)$.

Hence, the truncated expected utility can be calculated as the corresponding "standard" expected utility of a linear contract times a mean-adjusted probability. Note that the mean-adjustment only depends on the slope of the linear contract – the higher the slope, the lower the adjusted mean.

Recall that $U_0^a(a_1,z^r,\Psi_1)$ and $U_0^a(a_1,z^r,\varnothing)$ denote the agent's *ex ante* expected utility, given contract z^r, if he takes action a_1, and leaves or stays for all ψ_1, respectively. That is,

$$U_0^a(a_1,z^r,\Psi_1) \equiv -\int_{\mathbb{R}}\exp\big[-r(f_1^r + v_1^r\psi_1 - \tfrac{1}{2}a_1^2 - \delta\big]\,d\Phi(\psi_1|a_1,a_1^r),$$

$$U_0^a(a_1,z^r,\emptyset) \equiv -\int_{\mathbb{R}} \exp\big[-r(f_1^r + v_1^r\psi_1 - \tfrac{1}{2}a_1^2$$

$$- (\bar{v}_2^o + \gamma^o\psi_1)[\rho_y M_{11} - M_{21}][a_1 - a_1^r])\big]\, d\Phi(\psi_1|a_1,a_1^r),$$

which, of course, can also be written in terms of certainty equivalents. Using Lemma 28B.1 we can write the agent's *ex ante* expected utility in (28.31) as

$$U_0^a(a_1,z^r,\delta) = U_0^a(a_1,z^r,\Psi_1)\,\mathrm{Prob}(y_l \in \Psi_1^l(a_1,z^r,\delta);\, a_1, leave)$$

$$+ U_0^a(a_1,z^r,\emptyset)\,\mathrm{Prob}(y_s \in \Psi_1^s(a_1,z^r,\delta);\, a_1, stay), \qquad (28B.1)$$

where the distributions for the normally distributed random variables y_l and y_s depend on whether we are using the "leave-contract" or the "stay-contract."

Let $K(a_1) \equiv [\rho_y M_{11} - M_{21}][a_1 - a_1^r]$. Note that the slope of the "leave-contract" is $\beta_l \equiv v_1^r$ while the slope of the "stay-contract" is $\beta_s(a_1) \equiv v_1^r - \gamma^o K(a_1)$. Hence,

$$\mathrm{Prob}(y_l \in \Psi_1^l(a_1,z^r,\delta);\, a_1, leave) = \int_{\Psi_1^r(a_1,z^r,\delta)} d\mathrm{N}(M_{11}[a_1 - a_1^r] - r v_1^r, 1),$$

$$\mathrm{Prob}(y_s \in \Psi_1^s(a_1,z^r,\delta);\, a_1, stay)$$

$$= \int_{\Psi_1^s(a_1,z^r,\delta)} d\mathrm{N}(M_{11}[a_1 - a_1^r] - r[v_1^r - \gamma^o K(a_1)], 1).$$

Note from (28.39) that the sign of $\gamma^o K(a_1)$ determines the integration bounds. Moreover, if $\gamma^o K(a_1) > (<)\ 0$, then y_l has a lower (higher) mean than y_s. This implies that

$$\mathrm{Prob}(y_l \in \Psi_1^l(a_1,z^r,\delta);\, a_1, leave) + \mathrm{Prob}(y_s \in \Psi_1^s(a_1,z^r,\delta);\, a_1, stay) < 1.$$

Hence, the agent's *ex ante* expected utility in (28B.1) is not a "simple" weighted average of the "leave" and "stay" expected utilities.

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PART H

CONTRACTING WITH MULTIPLE AGENTS IN SINGLE-PERIOD SETTINGS

CHAPTER 29

CONTRACTING WITH MULTIPLE PRODUCTIVE AGENTS

Up to this point, our discussion of contracting has focused on settings in which there is one principal (possibly acting on behalf of investors) and one agent. In this and the following chapter, we move to settings in which there are multiple agents. The models in this area are diverse, but we limit our discussion to a few basic issues. In this chapter, we consider settings in which there are multiple productive agents, while Chapter 30 considers settings in which there is a single productive agent, and a non-productive agent who is hired by the principal merely to monitor the productive agent.

In Section 29.1 we consider a partnership setting in which the contracting parties are both agents and principals. This is an extension of our Chapter 4 discussion of risk sharing in partnerships. In that earlier discussion we assumed that the partners had no direct preferences with respect to their action choices – their preferences in that setting depend only on their share of the firm's outcome. Furthermore, if all partners had HARA utility functions with identical risk cautiousness and homogeneous beliefs, then the efficient risk sharing contract gives each partner a linear share of the firm's aggregate outcome and they have the same preferences over action choices. The form of the efficient contract changes significantly if the partners have direct preferences with respect to their actions, e.g., disutility for effort. We briefly explore the form of the efficient contract in a setting in which each partner is risk and effort averse. A key issue in this setting is whether the partnership contract is based solely on the firm's aggregate outcome, or whether there are partner-specific performance measures, such as a partner-specific component of the firm's outcome. As pointed out by Holmström (1982), a key issue in the first setting is what has been termed the "budget balancing" constraint, i.e., any reduction in one partner's share necessarily results in an increase in some other partner's share. This constraint is much less significant if there are partner-specific performance measures.

Most of the analysis in this chapter considers settings in which a principal (who does not take a costly action) contracts with multiple risk and effort averse agents. Obviously, if the principal is risk neutral and agents generate independent outcomes and performance measures, then the principal can separately solve the incentive contracting problem for each agent. This changes somewhat if the principal is risk averse, since then it is optimal for the principal to share risks with the agents, as well as impose risk on them for incentive purposes. We briefly characterize the optimal contract with a risk averse principal and independent agents, and then assume in the remainder of the chapter that the principal is risk neutral.

Multi-agent issues arise even if the principal is risk neutral, provided the agents' performance measures are not independent. A key feature of these settings is that an agent's compensation is a function of performance measures that are influenced by both agents' actions. This raises some interesting incentive compatibility issues, which are explored in Section 29.2. The principal's problem can be described as selecting an optimal strategy in a game in which he moves first (specifying the terms of the agents' compensation contact) and the agents then play either a simultaneous or sequential move game among themselves. In anticipating the outcome of the game, the principal must consider how his choices will affect the choices made by the agents. Those choices are assumed to be a Nash equilibrium in the second-stage game, and if there are multiple Nash equilibria in that game, then the principal must predict which equilibrium the agents will choose.

In Section 29.3 we shift from assuming contracting is centralized to also considering decentralized contracting. Under centralized contracting, the principal contracts directly with both agents. On the other hand, with decentralized contracting, the organization is hierarchical. The principal acts on behalf of the owners, the first agent is a branch manager, and the second manager is a branch worker. Only the latter two take productive actions. The principal offers the branch manager a contract that specifies how the branch compensation pool will be determined. The branch manager then offers the branch worker a contract that species the worker's share of the branch compensation pool. To avoid the subgame issues explored in Section 29.2, we assume in Section 29.3 that the outcomes from the effort of the two agents are stochastically independent. Section 29.3.1 establishes that decentralized contracting provides the same result as centralized contracting if the agents do not have direct preferences with respect to their actions. On the other hand, Section 29.3.2 demonstrates that decentralized contracting is less efficient than centralized contracting if the agents have direct effort preferences. These incentives create incentive risk. The loss of efficiency occurs because the manager allocates the compensation risk between the two agents, thereby reducing each agent's effort incentives.

Initially, we assume disaggregate performance measures are available for both contracts. In Section 29.3.3 we assume only an aggregate performance measure is available. Then, in Section 29.3.4, we assume the branch compensation pool must be based on an aggregate performance measure, whereas disaggregate information is available for contracting at the branch level. We establish that centralized contracting is strictly preferred if it is based on disaggregate performance measures. However, decentralized contracting is preferred to centralized contracting if the former permits the use of disaggregate information in the worker's contract, whereas the latter limits it to aggregate information.

An alternative interpretation of the centralized versus decentralized contracting analysis is to view it as a comparison of full- versus limited-commitment contracting.[1] Under full-commitment contracting the principal fully specifies the contract with each agent and the agents make binding commitments not to collude, i.e., not to make mutually agreeable changes in how their aggregate compensation is allocated between them. Under limited commitment, the "no reallocation" commitment is not enforceable. The changes they will make in that setting will yield the same results as decentralized contracting.

In Section 29.4 we explore a multi-agent model similar to the model in Demski and Sappington (1984). In this model, the agents have correlated pre-contract information and separate contractible outcomes. Section 29.4.1 provides first- and second-best benchmark contracts in which agent i's contract depends only on his outcome. Section 29.4.2 then explores the benefits and problems associated with contracts that use messages and outcomes from both agents in contracting with each agent. Two versions of the principal's problem are considered. The first identifies the optimal contract given truthtelling constraints in which one agent is induced to tell the truth if he believes the other agent is truthful. The second finds the optimal contract given truthtelling constraints in which telling the truth is a dominant strategy (i.e., preferred no matter what the other agent says.

An important part of the analysis in Section 29.4.2 is the demonstration that in settings with correlated information, the agents may be able to engage in coordinated strategies that are beneficial to them, but detrimental to the principal. As demonstrated near the end of the section, the principal can use asymmetric contracts to mitigate the negative effects of coordinated agent strategies.

29.1 PARTNERSHIPS AMONG AGENTS

In this section we consider a partnership consisting of two agents.

29.1.1 Basic Partnership Model

The firm's aggregate gross outcome is represented by $x \in X \subseteq \mathbb{R}$. It is assumed to be contractible, and any additional contractible performance measures are represented by $y \in Y$. The vector of actions taken by the agents is represented by $\mathbf{a} = (a_1, a_2)$, where $a_i \in A_i$ is agent i's action, $i = 1, 2$, and the probability

[1] This approach is discussed by Feltham and Hofmann (2005a).

distribution function for the outcome and performance measures is $\Phi(x,y|\mathbf{a})$. Agent i's compensation function is $\boldsymbol{c}_i(x,y)$, and economic feasibility requires that $\boldsymbol{c}_1(x,y) + \boldsymbol{c}_2(x,y) \leq x$ for all y. Observe that the latter allows the partners to throw some of the outcome away (e.g., give it to charity).

The agents are risk and effort averse, and agent i's utility function is $u_i^a(c_i,a_i)$, $i = 1, 2$. In characterizing the efficient partnership contracts, we maximize a weighted sum of the agents' expected utility levels subject to two sets of constraints. First, the agents' actions are not contractible and, hence, the choice of actions and compensation for both agents are subject to incentive constraints. Second, the compensation payments must be economically feasible. Hence, the partnership problem takes the following basic form.

Partnership Decision Problem:

$$\underset{c,\,a}{\text{maximize}} \quad \lambda_1 U_1^a(\boldsymbol{c}_1,\mathbf{a}) + \lambda_2 U_2^a(\boldsymbol{c}_2,\mathbf{a}), \tag{29.1}$$

$$\text{subject to} \quad U_1^a(\boldsymbol{c}_1,\mathbf{a}) \geq U_1^a(\boldsymbol{c}_1,(\hat{a}_1,a_2)), \quad \forall\, \hat{a}_1 \in A_1,$$

$$U_2^a(\boldsymbol{c}_2,\mathbf{a}) \geq U_2^a(\boldsymbol{c}_2,(a_1,\hat{a}_2)), \quad \forall\, \hat{a}_2 \in A_2, \tag{29.2}$$

$$\boldsymbol{c}_1(x,y) + \boldsymbol{c}_2(x,y) \leq x, \qquad \forall\, x \in X, y \in Y, \tag{29.3}$$

$$\text{where} \quad U_i^a(\boldsymbol{c}_i,\mathbf{a}) \equiv \int_X \int_Y u_i^a(\boldsymbol{c}_i(x,y),a_i)\, d\Phi(x,y|\mathbf{a}).$$

First-best Contract

Assume that the agents' utility functions are additively separable, i.e., $u_i^a(c_i,a_i) = u_i(c_i) - v_i(a_i)$, and consider the setting in which y reveals $\mathbf{a}$. In that case, $\boldsymbol{c}_i(x,y)$ can be separated into a first-best risk sharing contract based on x, and a penalty contract based on y. (Whether agent i pays his penalty to a third party – e.g., charity – or the other agent is immaterial here, since the penalty is merely a threat that never has to be imposed.) Let $\boldsymbol{c}_1(x)$ and $\boldsymbol{c}_2(x) = x - \boldsymbol{c}_1(x)$ represent the risk-sharing components of the contracts. Differentiating (29.1) with respect to $\boldsymbol{c}_1(x)$ yields the following characterization of the efficient risk sharing contract:

$$\frac{u_1'(\boldsymbol{c}_1(x))}{u_2'(x - \boldsymbol{c}_1(x))} = \frac{\lambda_1^{-1}}{\lambda_2^{-1}}. \tag{29.4}$$

This is, of course, equivalent to efficient risk-sharing condition (4.9), and the other efficient risk-sharing results in Chapter 4 also apply here.

For example, if both agents have logarithmic utility (with no fixed components), then (see Table 4.1)

$$c_i^*(x) = \frac{\lambda_i}{\lambda_1 + \lambda_2} x. \tag{29.5}$$

As in (4.18), we can express agent i's utility as a function of x and a_i:

$$w_i(x,a_i) = \ln\left[\frac{\lambda_i}{\lambda_1 + \lambda_2} x\right] - v_i(a_i) = \ln(x) + \ln\left[\frac{\lambda_i}{\lambda_1 + \lambda_2}\right] - v_i(a_i)$$

$$\sim \ln(x) - v_i(a_i). \tag{29.6}$$

From (29.6) we derive a partnership utility function

$$w_o(x,\mathbf{a}) = \lambda_1 w_1(x,a_1) + \lambda_2 w_2(x,a_2)$$

$$\sim (\lambda_1 + \lambda_2)\ln(x) - \lambda_1 v_1(a_1) - \lambda_2 v_2(a_2). \tag{29.7}$$

Hence, the first-best actions can be characterized as

$$\mathbf{a}^* \in \underset{\mathbf{a}}{\text{argmax}} \ (\lambda_1 + \lambda_2)\int_X \ln(x)\, d\Phi(x|\mathbf{a}) - \lambda_1 v_1(a_1) - \lambda_2 v_2(a_2). \tag{29.8}$$

In subsequent discussions we assume the action space is convex, with $A_i \in [0,1]$, and for most of the analysis we further assume the optimal action choices are characterized by the first-order conditions. Consequently, $\mathbf{a}^*$ is characterized by

$$\lambda_1 \partial U_1^a(\mathbf{c}_1^*,\mathbf{a}^*)/\partial a_i + \lambda_2 \partial U_2^a(\mathbf{c}_2^*,\mathbf{a}^*)/\partial a_i = 0, \quad i = 1,2. \tag{29.9}$$

With additively separable logarithmic utility this becomes

$$(\lambda_1 + \lambda_2)\int_X \ln(x)\, d\Phi_{ai}(x|\mathbf{a}^*) = \lambda_i v_i'(a_i^*), \quad i = 1,2. \tag{29.10}$$

29.1.2 Second-best Contract Based on Aggregate Outcome

We now consider the setting in which x is the only contractible information, i.e., a is not contractible and $Y = \emptyset$. Agent i's personal action choice is assumed to be characterized by the following first-order condition:

$$\partial U_i^a(\mathbf{c}_i, \mathbf{a})/\partial a_i = 0, \quad i = 1,2. \tag{29.11}$$

Hence, with additively separable logarithmic utility functions, the first-best sharing rule will induce partner *i* to select action $a_i^\dagger$ such that

$$\int_X \ln(x)\, d\Phi_{ai}(x|\mathbf{a}^\dagger) = v_i'(a_i^\dagger). \tag{29.11$'$}$$

Compare the first-best and personal action choices implied by (29.10) and (29.11′). In the first-best case, it is recognized that while an action is personally costly to one partner, the resulting outcome has value to both partners. However, in the personal-choice case, a partner chooses his action based only on the value of the outcome to him, and his personal action cost. Hence, the effort levels in the personal-choice case will be distinctly lower than in the first-best case. We refer to this as the"free-rider" problem – each agent chooses to provide less effort than would be beneficial to the partnership, even though they would prefer to agree to have each provide more effort.

Independent Two-agent Hurdle Model

To illustrate the preceding comments we consider a two-agent hurdle model in which each agent has two possible observable outcome levels, represented by $X_i = \{x_{ig}, x_{ib}\}$, $x_{ig} > x_{ib}$. More specifically, the agents face two independent uncertain uniformly distributed hurdles $h_i \in [0,1]$, $\varphi(h_i) = 1$, for $i = 1,2$, which, if cleared, yield the good outcomes. Hence, the outcome probability for agent *i* given a_i and h_i is

$$\varphi(x_{ig}|a_i,h_i) = \begin{cases} 1 & \text{if } a_i \geq h_i, \\ 0 & \text{if } a_i < h_i. \end{cases}$$

The agents do not directly observe their hurdles h_i either before or after the fact, and the prior outcome probabilities are $\varphi(x_{ig}|\mathbf{a}) = a_i$, and $\varphi(x_{ib}|\mathbf{a}) = 1 - a_i$. We assume that the possible outcomes are the same for the two agents, i.e., $x_{1g} = x_{2g}$ and $x_{1b} = x_{2b}$, so that there are three possible aggregate outcome levels, i.e., $X = \{x_L, x_M, x_H\}$, where $x_L = 2x_{ib}$, $x_M = x_{ib} + x_{ig}$, and $x_H = 2x_{ig}$. The individual outcome probabilities are assumed to be independent so that $\varphi(x_L|\mathbf{a}) = (1 - a_1)(1 - a_2)$, $\varphi(x_M|\mathbf{a}) = a_1(1 - a_2) + (1 - a_1)a_2$, and $\varphi(x_H|\mathbf{a}) = a_1 a_2$. The agents are risk and effort averse with additive preferences and compensation utility $u_i(c_i) = \ln(c_i)$, for $c_i > 0$, and effort disutility $v_i(a_i) = \gamma a_i/(1 - a_i)$, where $\gamma > 0$ is a scaling parameter.

The agents (who are identical) are given equal weight, e.g., $\lambda_1 = \lambda_2 = 1$. Hence, the first-best sharing rule is $\mathbf{c}_i^*(x) = \frac{1}{2}x$, and the first-best effort $\mathbf{a}^*$ is such that $a_1^* = a_2^*$, with a_i^* characterized by

$$2\{(1 - a_i^*)[\ln(x_M) - \ln(x_L)] + a_i^*[\ln(x_H) - \ln(x_M)]\} = v_i'(a_i^*). \qquad (29.12)$$

On the other hand, the actions $\mathbf{a}^\dagger$ in the personal-choice setting are a Nash equilibrium in which $a_1^\dagger = a_2^\dagger$, with $a_i^\dagger$ characterized by

$$(1 - a_i^\dagger)[\ln(x_M) - \ln(x_L)] + a_i^\dagger[\ln(x_H) - \ln(x_M)] = v_i'(a_i^\dagger). \qquad (29.13)$$

We illustrate these results with an example in which $\gamma = .2$ and the possible outcomes are $x_{ig} = 40$, and $x_{ib} = 20$. In this case the first-best action is .466 while the personal action choice is only .269. Of course, the reason is that with personal choice, each partner only gets half the outcome while incurring the full cost of their personal effort. As a consequence, while an agent's first-best expected utility is 3.174 it drops to 3.132 if each agent implements his personal action choice. (These amounts are summarized later in Table 29.1.)

Second-best Contract

A question arises as to whether the first-best risk sharing contract and the induced personal choice actions $\mathbf{a}^\dagger$ constitute a second-best solution. In particular, are the partners better off if they agree to "give away" some of their "low" outcomes (making them even lower) if the probability of these outcomes would be significantly increased by exerting "low" effort?

Holmström (1982) explores this issue first in a setting in which there is outcome certainty and then in a setting with uncertainty in which the "Mirrlees condition" holds. Holmström demonstrates that the first-best result can be achieved in the certainty setting, provided it is possible to commit to a penalty on all agents (i.e., give some of the aggregate outcome away) if the outcome is less than first-best.[2] The key here is that individual agents are deterred from shirking by ensuring that the decrease in their utility due to the reduced outcome is greater than the reduction in their disutility for effort. That is, the penalty threat removes the "free-rider" problem when there is individual choice.

A similar result can be achieved in the uncertainty setting if there is moving support such that less than first-best effort has a positive probability of yielding an aggregate outcome that has zero probability of occurring with first-best effort

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[2] Holmström (1982) initially assumes strict "budget balancing" (i.e., the agents' total compensation must equal the aggregate outcome), and demonstrates that, even under certainty, the first-best cannot be achieved if this condition is imposed.

(assuming the outcome can be reduced such that an agent's utility is very low).[3] The Mirrlees Problem, which is discussed in Section 17.3.3, applies if there is constant support. Holmström demonstrates that, given this condition, one can obtain a result arbitrarily close to the first-best result by committing to a severe penalty for outcomes that have a low probability of occurrence if every agent exerts first-best effort, but for which the probability increases significantly if *any* agent exerts less effort.

We assume that the conditions sufficient for implementing the first-best result do not hold, and that is the case in our hurdle model. If a penalty δ is imposed on x_L in the basic hurdle model (with effort cost function $v_i(a_i) = \gamma a_i/(1 - a_i)$), then the induced effort will increase for both agents. This has the positive effect of increasing the probability of x_H, but has the negative effect of reducing the agents' compensation if x_L occurs (since they will now share $x_L - \delta$) and they will incur a higher disutility for effort. In our numerical examples, the negative effects outweigh the positive effects, implying that it is not beneficial to introduce a penalty in this setting.

The situation potentially changes when we introduce a "setup cost" $\kappa > 0$ into the agent's disutility for effort, i.e., $v_i(a_i) = \kappa + \gamma a_i/(1 - a_i)$ if $a_i \in (0,1]$ and 0 if $a_i = 0$. The setup cost can be such that in the individual choice setting, with efficient risk sharing, each agent chooses zero effort, even though they would choose positive effort in the first-best setting. Introducing a penalty can then induce individual choice of positive effort for which the benefits exceed the costs. The second-best penalty $\delta^\dagger > 0$, and second-best action $\mathbf{a}^\dagger > \mathbf{0}$ constitute a Nash equilibrium if the following two conditions hold:

$$[1 - a_i^\dagger][\ln(x_M) - \ln(x_L - \delta^\dagger)] + a_i^\dagger[\ln(x_H) - \ln(x_M)] = v_i'(a_i^\dagger), \quad (29.14a)$$

$$[a_i^\dagger]^2 \ln(x_H) + 2a_i^\dagger[1 - a_i^\dagger]\ln(x_M) + [1 - a_i^\dagger]^2 \ln(x_L - \delta^\dagger)$$
$$- \kappa - \gamma a_i^\dagger/(1 - a_i^\dagger) \geq a_i^\dagger \ln(x_M) + [1 - a_i^\dagger]\ln(x_L - \delta^\dagger). \quad (29.14b)$$

Condition (29.14a) is the "local" incentive compatibility constraint corresponding to (29.11), while (29.14b) is the "global" incentive compatibility constraint stating that any individual partner does not prefer deviating to providing zero effort (and, thus, save the setup cost as well as the variable effort cost).

To illustrate this setting again let $\gamma = .2$, $x_{ig} = 40$, and $x_{ib} = 20$, and assume the setup cost is $\kappa = .05$. The first-best action is the same as with no setup cost, i.e., $a_i^* = .466$, but each partner's first-best expected utility is reduced by the setup cost, i.e., $U_i(\mathbf{a}^*) = 3.124$ (which is higher than the expected utility with

[3] See Section 18.2 for a formal discussion of the role of moving support in permitting the implementation of first-best in a single agent problem. The approach is similar here.

zero effort for both partners, $U_i(\mathbf{0}) = 2.996$). If there is no penalty, i.e., $\delta = 0$, then there is no action $\mathbf{a} > \mathbf{0}$ that satisfies both (29.14a) and (29.14b), implying that $\mathbf{a} = \mathbf{0}$ is the only Nash equilibrium in the individual-choice setting with $\delta = 0$. However, allowing δ to be positive, the second-best solution satisfying (29.14) is $\delta^\dagger = 4.725$, and $a_i^\dagger = .333$ with second-best expected utility $U_i(\mathbf{a}^\dagger,\delta^\dagger) = 3.048$. Of course, the expected utility is lower than first-best, but still higher than with zero effort.

The so-called "budget-breaking mechanism" assumes that the two partners commit to give away some of the low outcome and, *ex ante*, this is an efficient thing to do in the individual choice setting to ensure that each partner provides positive effort. However, both partners have an incentive to renege on that commitment *ex post* and, thus, there is a question whether the commitment can be enforced. If the partners anticipate that the commitment cannot be imposed, then we are back in the inefficient solution in which no partner provides any effort. In Section 29.1.4 we introduce a third party, called a general partner or principal, who effectively works as a "budget-breaking mechanism" in the sense that the agents' aggregate compensation does not have to be equal to or less than the aggregate outcome.

29.1.3 Second-best Contract Based on Disaggregate, Independent Outcomes

An obvious way to reduce the incentive problems that occur with second-best contracts based on the aggregate outcome is to obtain additional performance measures. Partner-specific measures, such as partner-specific outcomes are likely to be particularly useful. If these measures are used, then the sharing rule serves to provide incentives for influencing the partners' individual action choices, as well as to share risk.

We assume the two agents generate independent outcomes, $x_i \in X_i$ and the outcome sets are finite. Hence, $\varphi(x_i|a_i)$ is the probability of outcome x_i, and $\varphi(x_1,x_2|\mathbf{a}) = \varphi(x_1|a_1)\varphi(x_2|a_2)$. The performance measure consists of the two outcomes, i.e., $y = (x_1,x_2)$. Since $x = x_1 + x_2$, the aggregate outcome is now redundant, and we can express the sharing rules solely in terms of y, $\boldsymbol{c}_i(y)$. The optimal sharing rules and actions are solutions to the partnership decision problem in (29.1) - (29.3). We assume the incentive compatibility constraints (29.2) can be characterized by the following first-order conditions:

$$\partial U_i^a(\boldsymbol{c}_i,\mathbf{a})/\partial a_i = 0, \quad i = 1,2. \tag{29.15}$$

Under these conditions, the optimal sharing rules for implementing a given action pair $\mathbf{a}$ can be determined by differentiating the Lagrangian by $\boldsymbol{c}_i(y)$, which yields

$$\frac{\xi(y)}{u_i'(\boldsymbol{c}_i(y))} = \lambda_i + \mu_i L_i(x_i|a_i), \quad i = 1,2, \forall\, y \in Y, \tag{29.16}$$

where μ_i is the incentive constraint multiplier for agent i, $\xi(y)$ is the economic feasibility multiplier for each pair of outcomes y, and the likelihood ratio is

$$L_i(x_i|a_i) \equiv \frac{\varphi_{ia_i}(x_i|a_i)}{\varphi_i(x_i|a_i)}.$$

Assuming $\xi(y) > 0$ for all y (i.e., the partners do not give any outcome away), it follows from (29.16) that

$$\frac{u_2'(\boldsymbol{c}_2(y))}{u_1'(\boldsymbol{c}_1(y))} = \frac{\lambda_1 + \mu_1 L_1(x_1|a_1)}{\lambda_2 + \mu_2 L_2(x_2|a_2)}, \quad \forall\, y \in Y. \tag{29.17}$$

Observe that (29.17) and $\boldsymbol{c}_1(y) + \boldsymbol{c}_2(y) = x$ imply that if the partners are identical (i.e., same preferences, same beliefs, and same induced actions, with $\lambda_1 = \lambda_2$ and $\mu_1 = \mu_2$), then the two partners each receive half of the aggregate outcome if they have the same outcomes, i.e., $\boldsymbol{c}_1(x_1,x_2) = \boldsymbol{c}_2(x_1,x_2) = \frac{1}{2}x$ if $x_1 = x_2$.[4] Furthermore, (29.17) implies that the partner with the outcome with the highest likelihood ratio receives the highest share of the aggregate outcome, i.e., $\boldsymbol{c}_1(x_1,x_2) > \boldsymbol{c}_2(x_1,x_2)$ if $L_1(x_1|a_1) > L_2(x_2|a_2)$.

To illustrate this result, we again consider our basic hurdle model in which $x_{ig} = 40$, $x_{ib} = 20$, $\gamma = .20$, and $\kappa = 0$. The likelihood ratios are

$$L_i(x_{ig}|a_i) = \frac{1}{a_i}, \quad L_i(x_{ib}|a_i) = -\frac{1}{1 - a_i}. \tag{29.18}$$

Table 29.1 summarizes the contracts and results for the first-best case, the second-best case with contracting on aggregate outcome, and the second-best case with contracting on the disaggregate outcomes. This table also includes contracts which include a general partner, who is introduced in Section 29.1.4. The relevant rows in this section are those in which the general partner is designated as "none".

Not surprisingly, the disaggregate outcome contract is preferred to the second-best aggregate outcome contract, but is not as desirable as first-best.

[4] Note that this may not hold for $\lambda_1 \neq \lambda_2$. In that setting, the share of the aggregate outcome may depend on x, i.e., the efficient linear risk sharing contract (with HARA-utility) may not apply even for identical outcomes for the two partners.

The key to the first relation is that instead of merely receiving $x_M = ½(40 + 20) = 30$ if there is a good and a bad outcome, greater effort can be induced by paying a partner more than 30 if he has a good outcome when the other partner has a bad outcome, but paying him less than 30 if the outcomes are reversed. That is, in order to mitigate the free-rider problem, the two agents effectively make a "bet" in which agent 2 makes a "side-payment" of 5.507 to agent 1 if good/bad occurs, with the reverse if bad/good occurs.

Panel A: First-best contracts						
general partner	c_{1H}	c_{1M}		c_{1L}	a_1^*	U_1^*
none	40	30		20	0.466	3.174
risk neutral	29.194	29.194		29.194	0.46	3.204
risk averse	39.475	29.931		20.386	0.466	3.175
Panel B: Second-best contracts with aggregate outcome						
general partner	c_{1H}	c_{1M}		c_{1L}	$a_1^\dagger$	$U_1^\dagger$
none	40	30		20	0.269	3.132
risk neutral	42.653	30.684		18.716	0.326	3.138
risk averse	40.38	30.223		19.712	0.283	3.133
Panel C: Second-best contracts with disaggregate outcomes						
general partner	c_{1gg}	c_{1gb}	c_{1bg}	c_{1bb}	$a_1^\ddagger$	$U_1^\ddagger$
none	40	35.507	24.493	20	0.392	3.158
risk neutral	36.563	36.563	22.033	22.033	0.372	3.162
risk averse	39.759	35.57	24.285	20.17	0.392	3.159

Table 29.1: Optimal partnership contracts in the basic hurdle model with independent outcomes.

29.1.4 Second-best Contract with a General Partner

The partners face two issues in their partnership: the sharing of risk and the inducement of effort. Due to the economic feasibility requirements, they cannot pay each other a bonus for two good outcomes. The only option they have for increasing individual incentives is to commit to penalize each other if they both receive bad outcomes. However, as discussed above, such penalties may not be efficient.

Of course, if they can contract with a general partner (who does not contribute to the firm's outcome), then they can mitigate both the risk sharing and incentive issues. The former is mitigated merely by introducing a third party with whom to share risks. This effect is seen most clearly if we compare the first-best result with and without a risk-neutral general partner. Without the general partner, the risk-averse agents must share the outcome risk, but they will avoid all of that risk if they contract with a risk-neutral general partner (and they can credibly commit to a specific action).

Interestingly, in the second-best setting, introducing a general partner can result in either a decrease or an increase in induced effort. The risk borne by the original partners, whom we now refer to as agents, will induce their second-best level of effort. If the risk-sharing benefit dominates, then introducing a general partner will reduce the agents' risk and reduce their induced effort. On the other hand, there is a "free-rider problem" in that when an agent chooses his action, he only considers the expected utility of his share of the outcome relative to his disutility for effort. This can be mitigated by imposing *more incentive risk* through giving the agents more than their outcome for good outcomes and less than their outcome for bad outcomes.

We provide a general formulation of the partnership problem and then illustrate the mitigation of the free-rider problem in the hurdle model. Introducing the general partner into partnership problem (29.1) - (29.3) is relatively straightforward (if the subgame issues discussed in the next section do not arise). The general partner is effectively a principal, so we represent his preferences with a utility function $u^p(\pi)$, where $\pi = x - c_1 - c_2$ is the general partner's return from the firm. Consequently, his expected utility, given return function $\boldsymbol{\pi}(x,y) = x - [\boldsymbol{c}_1(x,y) + \boldsymbol{c}_2(x,y)]$, is

$$U^p(\boldsymbol{c},\mathbf{a}) \equiv \int_X \int_Y u^p(x - [\boldsymbol{c}_1(x,y) + \boldsymbol{c}_2(x,y)])\, d\Phi(x,y|\mathbf{a}).$$

Let $\bar{U}^p$ represent the general partner's expected utility if he does not join the agents' partnership. We then modify the partnership problem by replacing the economic feasibility requirement (29.3) with a general partner participation constraint,

$$U^p(\boldsymbol{c},\mathbf{a}) \geq \bar{U}^p. \tag{29.3$'$}$$

Contracting on Aggregate Outcome

First consider the setting in which only the aggregate outcome x is contractible, and X is a finite set for which $\varphi(x|\mathbf{a})$ is the probability of outcome x given actions $\mathbf{a}$. Assume that agent i's utility function is additively separable and incentive constraint (29.2) can be represented by the first-order condition (29.11). It then follows that differentiating the Lagrangian by $\boldsymbol{c}_i(x)$ yields

$$\frac{u^{p\prime}(\boldsymbol{\pi}(x))}{u_i'(\mathbf{c}_i(x))} = \frac{\lambda_i + \mu_i L_i(x|\mathbf{a})}{\lambda^p}, \quad i = 1,2, \tag{29.19}$$

where
$$L_i(x|\mathbf{a}) \equiv \frac{\varphi_{a_i}(x|\mathbf{a})}{\varphi(x|\mathbf{a})}, \tag{29.20}$$

and λ^p is the multiplier on the principal's participation constraint (29.3′). Condition (29.19) then implies

$$\frac{u_2'(\mathbf{c}_2(x))}{u_1'(\mathbf{c}_1(x))} = \frac{\lambda_1 + \mu_1 L_1(x|\mathbf{a})}{\lambda_2 + \mu_2 L_2(x|\mathbf{a})}, \quad \forall y \in Y. \tag{29.21}$$

Not surprisingly, (29.21) implies that if the agents are identical, then for every aggregate outcome level x, they receive equal compensation. There is no basis for distinguishing one agent from the other, so they share the incentive risk equally. However, unlike the setting in which the agents are the only partners, identical agents do not receive ½x. For example, if the agents are identical, with $\lambda_1 = \lambda_2$, $a_1 = a_2$, and $u_i(c_i) = \ln(c_i)$, and the general partner is risk neutral with $\bar{U}^p = 0$, then (29.19) implies that an agent's compensation depends on the likelihood of x and not its dollar amount, i.e.,

$$\mathbf{c}_i(x) = \frac{\lambda_i + \mu_i L_i(x|\mathbf{a})}{\lambda^p}. \tag{29.22}$$

That is, the agent bears only incentive risk.

Of course, if the principal is risk averse, then the agent continues to share in the outcome risk as well as bearing incentive risk. For example, assume the principal has initial wealth w, with $u^p(\pi) = \ln(w + \pi)$ and $\bar{U}^p = \ln(w)$, then

$$\mathbf{c}_1(x) = \frac{\lambda_1 + \mu_1 L_1(x|\mathbf{a})}{\lambda^p + \lambda_1 + \mu_1 L_1(x|\mathbf{a})}(w + x - \mathbf{c}_2(x)), \tag{29.23}$$

with a similar expression for $\mathbf{c}_2(x)$. Effectively, the general partner adds his wealth to the outcome to be shared and then each agent receives a share that reflects his multiplier λ_i (as in Table 4.1) plus an adjustment for the likelihood of the outcome. Expression (29.23) depicts the first agent and the general partner as sharing the outcome risk minus the second agent's compensation.

Now return to our hurdle model. The likelihood ratios for the three aggregate outcomes are

$$L_i(x_H|\mathbf{a}) = \frac{1}{a_i}, \quad L_i(x_M|\mathbf{a}) = \frac{1}{a_1(1-a_2) + (1-a_1)a_2}, \quad L_i(x_L|\mathbf{a}) = -\frac{1}{1-a_i}.$$

Table 29.1 reports the results for the basic hurdle model with no general partner, a risk-neutral general partner, and a risk-averse general partner (with $w = e$, so that $\ln(w) = 1$).

The first-best effort and utility increase with the addition of a general partner, reflecting a pure risk-sharing benefit, which is greatest if the general partner is risk neutral. In the second-best setting with contracting based on the aggregate outcome, we observe that introducing a risk-neutral general partner results in more risk being imposed on the agents (compensation for two good outcomes increases from 40.000 to 42.653, while the compensation for two bad outcomes decreases from 20.000 to 18.716), thereby inducing increased effort, which is beneficial in this setting. As illustrated in Table 29.1, the changes are qualitatively similar, but less dramatic if the general partner is risk averse.

Contracting on Disaggregate Outcome

We consider agent contracting on the disaggregate outcomes in Section 29.1.3. Adding a general partner to that setting has an interesting effect. First observe that (29.16) is replaced with an expression similar to (29.19):

$$\frac{u^{p\prime}(\boldsymbol{\pi}(y))}{u_i'(\boldsymbol{c}_i(y))} = \frac{\lambda_i + \mu_i L_i(x_i|a_i)}{\lambda^p}, \quad i = 1,2, \tag{29.24}$$

which implies that (29.17) holds. The latter implies that identical agents receive identical compensation if their outcomes have the same likelihood ratio.

Furthermore, if the general partner is risk neutral, then $u^{p\prime}(\pi) = 1$ and (29.24) implies that $\boldsymbol{c}_i(y)$ varies only with x_i, i.e., it is independent of the other agent's outcome, so that it can be expressed as $\boldsymbol{c}_i(x_i)$. One can view this setting as equivalent to one in which the agents sign separate partnership contracts with the risk-neutral general partner. The first agent's contract is illustrated in Table 29.1. Interestingly, the introduction of a risk neutral partner reduces the induced effort from 0.392 to 0.372, but increases the agent's expected utility from 3.158 to 3.162. That is, without the general partner, the outcome risk induces more effort than if the outcome risk can be shared.

As illustrated in Table 29.1, the effect on the agents' actions and their expected utility is similar, but less pronounced, if the general partner is risk averse. More notable is the fact that with a risk-averse general partner, the compensation of the first agent now varies with the outcome of the second agent. This is implied by (29.24). To see this, assume $u^p(\pi) = \ln(w + \pi)$ and $u_i(c_i,a_i) = \ln(c_i) - v_i(a_i)$, $i = 1,2$. In this setting, (29.24) is

$$\frac{c_i(y)}{x - c_1(y) - c_2(y)} = \frac{\lambda_i + \mu_i L_i(x_i | a_i)}{\lambda^p}, \quad i = 1,2, \tag{29.25}$$

which implies

$$c_1(y) = \frac{\lambda_1 + \mu_1 L_1(x_1 | a_1)}{\lambda^p + \lambda_1 + \mu_1 L_1(x_1 | a_1)} (w + x_1 + x_2 - c_2(x)), \tag{29.26}$$

which is very similar to (29.23). Here we can interpret the first agent's compensation as a share of the general partner's wealth plus the agent's gross outcome and the net outcome from the second agent. The agent's share varies only with the likelihood of his outcome.

29.2 BASIC PRINCIPAL/MULTI-AGENT MODEL

The preceding analysis began with a partnership between two agents and then considered the value of adding a general partner who could absorb some of the outcome risk. The general partner is effectively a principal, and we now assume a risk-neutral principal offers the contracts to the agents. As discussed in Chapter 18, varying who offers the contract does not change the fundamental characteristics of the optimal contract, but assuming it is a risk-neutral principal who offers has three advantages. First, this assumption is common in the principal-agent literature. Second, this perspective facilitates consideration of a model in which the principal receives the outcome $x \in X$ which may not be contractible. More specifically, the contractible performance measures are represented by $y \in Y$, and y may not include x. Third, risk-sharing issues are set aside and the principal/multi-agent perspective facilitates discussion of incentive issues, including issues that arise in the agents' post-contract acceptance subgame. We ignored the subgame issues in the preceding section, but they can arise there as well.

29.2.1 The Principal's Problem

The vector of actions taken by the agents is again represented by $\mathbf{a} = (a_1, a_2)$, where $a_i \in A_i$ is agent i's action, $i = 1,2$, and the probability distribution function for the outcome and performance measures is $\Phi(x,y | \mathbf{a})$. Agent i's compensation function is $\mathbf{c}_i(y)$ and the risk-neutral principal's net payoff is $\pi = x - c_1 - c_2$. The agents are strictly risk and effort averse, with preferences represented by $u_i^a(c_i, a_i)$, which we assume to be additively separable, i.e., $u_i^a(c_i, a_i) = u_i(c_i) - v_i(a_i)$, $i = 1,2$. Agent i's reservation utility is $\underline{U}_i$, $i = 1,2$.

We can interpret the principal and agents as playing a sequential game in which the principal acts as a *Stackleberg leader* by offering contracts to the two agents. The agents then decide whether to participate. If they accept the contracts, they participate in a simultaneous play subgame in which they choose and implement their actions. Finally, the principal receives the firm's outcome and pays the agents in accordance with the accepted contracts, based on the reported performance measures. In formulating the principal's problem it is straightforward to specify the objective function and agents' participation constraints. However, the incentive compatibility constraints in the agents' subgame are more subtle. The following provides a succinct statement of the problem.

Principal's Decision Problem:

$$\underset{c,\,\mathbf{a}}{\text{maximize}} \quad U^p(c,\mathbf{a}) \equiv \int_Y \int_X (x - c_1(y) - c_2(y))\, d\Phi(x,y|\mathbf{a}), \tag{29.27}$$

$$\text{subject to} \quad U_i^a(c_i,\mathbf{a}) \equiv \int_Y u_i^a(c_i(y),a_i)\, d\Phi(y|\mathbf{a}) \geq \bar{U}_i, \quad i = 1,2, \tag{29.28}$$

Incentive compatibility constraints: (29.29)

$$U_1^a(c_1,\mathbf{a}) \geq U_1^a(c_1,(\hat{a}_1,a_2)), \quad \forall\, \hat{a}_1 \in A_1,$$

$$U_2^a(c_2,\mathbf{a}) \geq U_2^a(c_2,(a_1,\hat{a}_2)), \quad \forall\, \hat{a}_2 \in A_2,$$

There does not exist any other action pair $\bar{\mathbf{a}}$ such that:

$$U_1^a(c_1,\bar{\mathbf{a}}) > U_1^a(c_1,\mathbf{a}),$$

$$U_2^a(c_2,\bar{\mathbf{a}}) > U_2^a(c_2,\mathbf{a}),$$

$$U_1^a(c_1,\bar{\mathbf{a}}) \geq U_1^a(c_1,(\hat{a}_1,\bar{a}_2)), \quad \forall\, \hat{a}_1 \in A_1,$$

$$U_2^a(c_2,\bar{\mathbf{a}}) \geq U_2^a(c_2,(\bar{a}_1,\hat{a}_2)), \quad \forall\, \hat{a}_2 \in A_2.$$

The first two incentive compatibility constraints ensure that neither agent will prefer to select a different action if they think the other agent will select the action specified by the principal, i.e., the specified actions constitute a Nash equilibrium in the second-stage subgame played by the two agents. In some settings, that may be all that is required. However, the principal must also be certain that there does not exist another Nash equilibrium in the agents' subgame which both agents prefer. Observe that this specification presumes that an agent will not "defect" from an equilibrium if he is no better off, i.e., he puts the principal's welfare ahead of the welfare of the other agent. Unfortunately, the complexity of the incentive compatibility constraints often makes it difficult to characterize the optimal contract.

The above formulation assumes that the agents can make a binding commitment to the principal that they will not make a mutually agreeable side-contract to reallocate their aggregate compensation, i.e., they cannot collude with respect to how they will share their aggregate compensation.[5] Furthermore, the above formulation assumes the principal cannot introduce messages, which effectively expands the agents' action choices and the information upon which the contract can be based. We later briefly explore this variation in the model.

29.2.2 Independent Performance Measures

The simplest multi-agent setting is one in which the performance measures are independently distributed, i.e., $\Phi(y|\mathbf{a}) = \Phi(y_1|a_1)\Phi(y_2|a_2)$, where y_i is the performance measure reported for agent i, $i = 1, 2$. The set of possible reports Y_i is assumed to be finite, so that $\varphi(y_i|a_i)$ is the probability of report y_i. Furthermore, we again assume the action space is convex (with $A_i = [0,1]$), and the basic incentive compatibility constraints can be characterized by the following first-order condition

$$\partial U_i^a(\boldsymbol{c}_i, \mathbf{a})/\partial a_i = 0, \quad i = 1, 2, \tag{29.30}$$

which is essentially the same as (29.15).

Under these conditions, the optimal compensation contract for implementing a given action pair **a** can be characterized by differentiating the Lagrangian by $\boldsymbol{c}_i(y)$, which yields

$$\frac{1}{u_i'(\boldsymbol{c}_i(y))} = \lambda_i + \mu_i L_i(y_i|a_i), \quad i = 1, 2, \tag{29.31}$$

where λ_i and μ_i are the participation and incentive constraint multipliers for agent i,[6] and

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[5] Section 29.3 considers decentralized contracting. It is essentially equivalent to centralized contracting with collusion by the agents. Also see Chapter 30 for a discussion of collusion between a productive agent and a monitor.

[6] Observe that (29.31) is very similar to (29.19), but with $u^{p\prime} = 1$ and λ^p is exogenously set equal to one. The former follows from the assumption of principal risk neutrality, and the latter occurs (continued...)

$$L_i(y_i|a_i) \equiv \frac{\varphi_{ia_i}(y_i|a_i)}{\varphi_i(y_i|a_i)}.$$

Condition (29.31) implies that, as in the partnership setting, the optimal contract for agent *i* depends only on the report for that agent – it is independent of the report for the other agent. That is, the optimal contract is the same as would be offered to a single agent – there are no multi-agent effects. Hence, all the single-agent results derived in earlier chapters apply here.

29.2.3 Contracting with Agents with Correlated Outcomes

In Chapter 18 we examined the use of multiple performance measures, including measures that are not influenced by the agent's actions, such as the performance of other firms in the same industry. The latter type of performance measure has no value if it is the only performance measure. However, such a performance measure can be valuable if it supplements a performance measure (such as the outcome) that is influenced by the agent's action and the noise in the two measures are correlated. For example, using the performance of other firms in the industry as a relative performance measure is valuable if some of the uncertain factors affecting the principal's and competitors' outcomes are the same.

Implicitly, our analysis in Chapter 18 assumes that the characteristics of the relative performance measure are not influenced by the agent's contract and actions. That may be a reasonable assumption if the relative performance measure is based on reports from other firms. However, the assumption may not hold if the principal contracts with two agents and uses both reports in compensating each agent. An obvious problem arises if the two agents explicitly or implicitly collude. To understand the general nature of this threat, consider a two-agent setting in which the performance measures for the two agents are positively correlated (they are influenced by similar uncontrollable events). Further assume that if the principal believes there will be no collusion, then (see Proposition 18.13) this results in a compensation contract in which one agent's compensation increases with his reported performance, but decreases with the other agent's report. That is, an agent is paid higher compensation for good reports in "bad times" than in "good times," where the report of the other agent is evidence with respect to whether it is "bad" or "good" times. The key issue is that such a contract may create incentives for the agents to collude and put in minimal effort – they will both have bad reports but will not be penalized

[6] (...continued)
since $U^P(\boldsymbol{\pi}, \mathbf{a})$ is now in the objective function. Conversely, agent *i*'s multiplier λ_i has changed from being exogenously specified in the objective function to being endogenously associated with agent *i*'s participation constraint.

because it will appear as if the two bad reports are merely due to "bad times." Hence, the identification of an optimal contract is a complex issue when there is potential collusion.

Demski and Sappington (1984) (DS84) provide an insightful analysis of a setting of this latter type. Rather than examining a "standard" two-agent problem, they consider a setting in which each agent has pre-contract information, and their pre-contract information is correlated. We illustrate most of the key insights from their analysis using our hurdle model, with no pre-contract information.

29.2.4 Two-agent Model with Perfectly Correlated Hurdles

We again consider a two-agent hurdle model and assume the performance measures consist of the disaggregate output measures, i.e., $y_i = x_i \in X_i = \{x_{ig}, x_{ib}\}$ and $x = x_1 + x_2$. However, unlike the model in Section 29.1.4, we now assume the heights of the two hurdles are perfectly correlated. More specifically, the agents face a common, uncertain uniformly distributed hurdle $h \in [0,1]$, $\varphi(h) = 1$, which, if cleared, yields the good outcome. Hence, the outcome probability for agent i given a_i and h is

$$\varphi(x_{ig}|a_i,h) = \begin{cases} 1 & \text{if } a_i \geq h, \\ 0 & \text{if } a_i < h. \end{cases}$$

The principal and agents do not directly observe h, either before or after the fact, and the prior outcome probability is $\varphi(x_{ig}|a_i) = a_i$. Hence, the likelihood ratios for agent i's two possible outcomes are the same as in (29.18), and the optimal contract based solely on the agent's own outcomes is characterized in (29.22) and illustrated by the risk-neutral general partner case in Panel C of Table 29.1. This is the second-best result given independent contracting. However, with correlated hurdles the principal may be better off if he offers contracts in which the compensation of one or both of the agents is a function of both agents' outcomes.

A First-best Relative Performance Contract

To explore this issue, assume that the principal and agent 1 conjecture that agent 2 will implement action $\hat{a}_2$. This implies that the joint distribution for the two agents' outcomes given action a_i is

$$\varphi(x_{1g},x_{2g}|a_1,\hat{a}_2) = \min\{a_1,\hat{a}_2\}, \quad \varphi(x_{1b},x_{2b}|a_1,\hat{a}_2) = 1 - \max\{a_1,\hat{a}_2\},$$

$$\varphi(x_{1b},x_{2g}|a_1,\hat{a}_2) = \max\{0,\hat{a}_2 - a_1\}, \quad \varphi(x_{1g},x_{2b}|a_1,\hat{a}_2) = \max\{0,a_1 - \hat{a}_2\}.$$

Observe that if both agents are induced to take the same actions, then the probabilities in the second row are both zero. This fact raises a question as to whether the principal can achieve the first-best result by paying the first-best wage (for the first-best action a_i^*) if the agents have identical outcomes and severely penalizing an agent if he obtains a bad outcome when the other agent obtains a good outcome, i.e.,

$$c_{igg} = c_{ibb} = c_i^* \equiv \exp[\bar{U}_i + \gamma a_i^*/(1 - a_i^*)], \quad i = 1, 2,$$
$$c_{1bg} = c_{2gb} = \underline{c}^* > 0, \tag{29.32}$$

with some appropriate specification of c_{1gb} and c_{2bg}.

Observe that, with identical agents, if both select the first-best effort level, then they both receive the first-best wage with certainty. An agent will be punished (with a sufficiently small compensation $\underline{c}^* > 0$) if he selects a lower effort level and obtains a bad outcome when the other agent receives a good outcome. This threat is sufficient to deter either agent from unilaterally selecting less than the first-best effort level if the difference in expected utility, i.e.,

$$\{[1 - a_i^* + a_i]\ln(c_i^*) + [a_i^* - a_i]\ln(\underline{c}^*) - \gamma a_i/(1 - a_i)\} - \bar{U}_i$$
$$= \gamma a_i^*/(1 - a_i^*) - \gamma a_i/(1 - a_i) - [a_i^* - a_i]\{\ln(c_i^*) - \ln(\underline{c}^*)\}, \tag{29.33}$$

is negative for all $a_i \in [0, a_i^*]$. This condition is clearly satisfied if $\underline{c}^*$ is chosen sufficiently close to zero (and, therefore, $\ln(\underline{c}^*)$ is sufficiently negative). Of course, to ensure that the first two incentive constraints in (29.29) are satisfied, we must also ensure that agent i will not unilaterally put in more effort than a_i^*. This is ensured for the first agent if

$$(a_1 - a_1^*)\{\ln(c_{1gb}) - \ln(c_1^*)\} - \gamma a_i/(1 - a_i) + \gamma a_i^*/(1 - a_i^*) \le 0,$$
$$\forall\, a_i \in [a_i^*, 1]. \tag{29.34}$$

Since the left-hand side of (29.34) is a concave function of a_i, differentiating with respect to a_i and evaluating it at $a_i = a_i^*$ establishes that (29.34) holds if

$$\ln(c_{1gb}) - \ln(c_1^*) \le v_1'(a_1^*) = \gamma(1 - a_1^*)^{-2}, \tag{29.35}$$

i.e., the maximum marginal increase in expected utility from putting in slightly more effort is equal to the marginal disutility of effort, $v_1'(a_1^*)$. The "off-equilibrium" compensation c_{1gb} is not particularly significant here, but it is significant when we consider subgame issues.

Examining the Subgame

The preceding analysis establishes that the agents' first-best actions constitute a Nash equilibrium in the agents' subgame. Now the question is whether there is another Nash equilibrium that would be preferred by the two agents. The obvious alternative to consider is an agreement between the two agents to both provide zero effort. With perfect correlation, the outcomes will be either both good or both bad. Hence, while the probability of two bad outcomes is now increased to one and the probability of two good outcomes is reduced to zero, the agents receive, nevertheless, the first-best wage with certainty. Since zero effort is much less costly than first-best effort, it is obvious that both agents will be strictly better off if they both provide zero effort instead of first-best effort.

The question remaining is whether zero effort by both agents is a Nash equilibrium, i.e., will either agent defect from their agreement to provide zero effort? Consider agent 1. He will defect from the zero effort agreement with the second agent if there exists an effort level $a_1 \in (0,1]$ such that

$$a_1\{\ln(c_{1gb}) - \ln(c_1^*)\} - \gamma a_1/(1 - a_1) > 0. \qquad (29.36)$$

Since the left-hand side of (29.36) is a concave function of a_i, differentiating with respect to a_i and evaluating at $a_i = 0$ establishes that (29.36) holds if

$$\ln(c_{1gb}) - \ln(c_1^*) > v_1'(0) = \gamma, \qquad (29.37)$$

i.e., the marginal increase in expected utility is higher than the marginal disutility of effort at zero effort. Obviously, there exists a compensation level c_{1gb} such that both (29.34) and (29.36) are satisfied. This implies that there exists a first-best contract, with off-equilibrium compensation c_{ijk}, for $j \neq k$, for which the first-best effort levels are a Nash equilibrium in the agents' subgame, while the zero effort levels are not.

Applying the same argument to other pairs of equal effort levels between zero and a_i^* establishes that if c_{1gb} is chosen such that (29.34) holds as an equality, then there is no other Nash equilibrium in the agents' subgame that is preferred by both agents. Consequently, there is no "subgame problem" for the principal if he carefully chooses the off-equilibrium compensation.

Again consider the basic hurdle model in which $x_{ig} = 40$, $x_{ib} = 20$, and $v_i(a_i) = \gamma a_i/(1 - a_i)$ with $\gamma = .2$. Assume agent i's reservation utility is $\underline{U}_i = 0$. In this case, the first-best effort and wage are $a_i^* = .8345$ and $c_i^* = 2.7405$. The first-best result is a Nash equilibrium if $c_{igg} = c_{ibb} = c_i^* = 2.7405$ and $c_{1bg} = c_{2gb} = .00001$, and the subgame problem is avoided by letting

$$c_{1gb} = c_{2bg} = \exp[\ln(c_1^*) + \gamma(1 - a_1^*)^{-2}] = 4{,}048.5.$$

Interestingly, to avoid inducing the agents to collusively select less than the first-best effort, they must be offered a very large bonus for obtaining a good outcome when the other agent obtains a bad outcome.

Subgame Problem Induced by a Setup Cost

The lack of a subgame problem in the basic hurdle model (with perfectly correlated hurdles) is attributable to the fact that the marginal disutility of effort is increasing in the effort level. Hence, setting c_{1gb} at the maximum amount, for which the first-best action is a Nash equilibrium, is sufficient to induce defection from any Nash equilibrium consisting of a pair of smaller effort levels. However, introducing a non-convexity into the disutility function can result in a subgame problem.

For example, assume agent i's disutility function has a setup cost of κ, and assume the first-best effort a_i^* is strictly positive, so that the first-best wage is

$$c_i^* \equiv \exp[\,\overline{U}_i + \kappa + \gamma a_i^*/(1 - a_i^*)], \qquad i = 1, 2.$$

As in the basic model, the first-best effort levels constitute a Nash equilibrium in the agents' subgame if the contract is such that

$$\begin{aligned} c_{igg} = c_{ibb} = c_i^*, \qquad i = 1, 2, \\ c_{1bg} = c_{2gb} = \underline{c}^*, \quad c_{1gb} = c_{2bg} = \exp[\ln(c_i^*) + v_i'(a_i^*)], \end{aligned} \tag{29.38}$$

with $\underline{c}^*$ sufficiently close to zero.

In the subgame created by the preceding contract, the agents again prefer to jointly choose zero effort instead of first-best effort. However, with a sufficiently large setup cost, the zero effort levels now constitute a Nash equilibrium. More specifically, the following condition is sufficient for the first agent not to defect from the zero effort agreement with the second agent:

$$\kappa > a_1\{\ln(c_{1gb}) - \ln(c_1^*)\} - v_1(a_1), \qquad \forall\, a_1 \in (0,1]. \tag{29.39}$$

Substituting for c_{1gb} from (29.38) establishes that (29.39) holds if

$$\kappa > a_1 v_1'(a_1^*) - v_1(a_1), \qquad \forall\, a_1 \in (0,1]. \tag{29.40}$$

Observe that, given the convexity of $v_1(a_1)$, the right-hand side of (29.40) is maximized at $a_1 = a_1^*$. Hence, the zero effort agreement is a Nash equilibrium (and the first-best result cannot be implemented using relative performance measures) if

$$\kappa > a_1^* v_1'(a_1^*) - v_1(a_1^*). \tag{29.41}$$

For example, if $\kappa = 2$, the first-best action and compensation are $a_1^* = .6677$ and $c_1^* = 11.0434$. Consequently,

$$a_1^* v_1'(a_1^*) - v_1(a_1^*) = .8073 < \kappa = 2,$$

which implies the principal has a subgame problem, since the zero effort agreement between the two agents is a Nash equilibrium.

Differential Contracts

What can the principal do in this setting? The simplest approach is to use individual contracts in which agent i's contract depends only on x_i. However, it is possible to achieve a better result by using contracts that make partial use of the relative performance information and avoid the subgame problem. For example, one approach, similar to the one suggested by DS84, is to use an individual contract with one agent and a relative performance contract with the other.

The following provides a formulation of the two-agent hurdle model problem in which an individual contract $\boldsymbol{c}_2 = (c_{2g}, c_{2b})$ is used for the second agent and a relative performance contract $\boldsymbol{c}_1 = (c_{1gg}, c_{1gb}, c_{1bg}, c_{1bb})$ is used for the first agent. We assume both agents are induced to provide the same level of effort, denoted a. The first agent can be induced to provide a at the "first-best" wage for that action if his outcome matches the second agent's action, and $\underline{c}^*$ if it is less. More specifically, for a given action $a \in (0,1]$, the first agent's contract is $c_{1gg} = c_{1bb} = \boldsymbol{c}_1^*(a) = \exp[\bar{U}_1 + \kappa + \gamma a/(1-a)]$, $c_{1bg} = \underline{c}^*$, and c_{1gb} arbitrarily set equal to c_{1gg}.

Given this structure, the principal's decision problem is (assuming there is an interior solution for which $a \in (0,1)$)

$$\underset{c_{2g},\, c_{2b},\, a}{\text{maximize}} \quad [x_{1g} + x_{2g} - c_{2g}]a + [x_{1b} + x_{2b} - c_{2b}](1-a) - \exp[\bar{U}_1 + \kappa + \gamma a/(1-a)],$$

$$\text{subject to} \quad \ln(c_{2g})a + \ln(c_{2b})(1-a) - \kappa - \gamma a/(1-a) \geq \bar{U}_2,$$

$$\ln(c_{2g}) - \ln(c_{2b}) = \gamma/(1-a)^2,$$

$$\ln(c_{2b}) \leq \bar{U}_2,$$

where the second constraint is the "local" incentive compatibility constraint, and the third constraint ensures that the second agent does not provide zero effort (and, thus, does not incur the setup cost κ).

The optimal interior contract based on our basic example with a setup cost of κ = 2 is shown in Table 29.2. The principal's expected payoff decreased to 40.5741 from the first-best payoff of 44.6205, even though the induced effort has increased to .7597 as opposed to the first-best effort level of .6677.

More effort is induced with the differential contract due to the setup cost of effort. The second incentive constraint (the third constraint in the principal's problem) ensures that the second agent will not choose zero effort and thereby avoid the setup cost. This constraint is binding, implying that if only local shirking was of concern, then c_{2b} would be set less than $\underline{U}_2$ and less effort would be induced. However, the marginal cost of inducing more effort is reduced given that c_{2b} is restricted to be equal to $\underline{U}_2$.

	(x_{1g},x_{2g})	(x_{1b},x_{2g})	(x_{1g},x_{2b})	(x_{1b},x_{2b})
c_{1jk}	13.9079	$\underline{c}^*$	13.9079	13.9079
c_{2jk}	31.9736	31.9736	1	1

Table 29.2: Optimal interior two-agent contract with individual contracting for agent 2 (a = .7597 and U^P = 40.5741).

Inducing more than First-best Effort

The preceding analysis demonstrates that it is possible for the principal to deal with the "subgame problem" by using limited relative performance evaluation (as suggested by the DS84 analysis). On the other hand, it is so costly to the principal since he must compensate the second agent for his incentive risk.

The proposed use of a differential contract stemmed from the fact that while contracts could be offered for which the first-best actions and results are a Nash equilibrium in the agents' subgame, the setup cost κ resulted in zero effort being a preferred Nash equilibrium in their subgame. An alternative approach is to use the first type of contract, but to motivate more than the first-best level of effort. If that effort level is sufficiently high, zero effort will not be a Nash equilibrium in the agents' subgame.

Recall that zero effort is a preferred Nash equilibrium in the agent's subgame for the "first-best" contract if condition (29.41) holds. The right-hand side of (29.41) is increasing in a_1^*. Hence, there exists an action $a^\ddagger > a^*$ such that

$$\kappa = a_1^\ddagger v_1'(a_1^\ddagger) - v_1(a_1^\ddagger).$$

Now consider the following contract (in which $a^\dagger$ is slightly greater than $a^\ddagger$)[7]

$$\begin{aligned} c_{igg} &= c_{ibb} = c_i^*(a^\dagger), \qquad i = 1,2, \\ c_{1bg} = c_{2gb} = \underline{c}^*, \quad c_{1gb} &= c_{2bg} = \exp[\ln(c_i^*(a^\dagger)) + v_i'(a^\dagger)]. \end{aligned} \tag{29.42}$$

Using the same arguments as in the first-best case, it follows that both agents providing $a^\dagger$ is a Nash equilibrium in the agent's subgame. Furthermore, both agents providing zero effort is not a Nash equilibrium in the agent's subgame.

Clearly, even though the agents are paid the first-best wage for the induced action, i.e., $\mathbf{c}^*(a^\dagger)$, the net payoff to the principal is less than in the first-best case because "too much" costly effort is provided.

In our example, with $\kappa = 2$, $a^\ddagger = .7597$, which significantly exceeds the first-best effort level $a^* = .6677$ and is approximately equal to the optimal effort using the differentiated contract. Obviously, the payment of $\mathbf{c}^*(a^\dagger)$ to implement $a^\dagger$ (close to $a^\ddagger$) is more beneficial to the principal than the differentiated contracts, since the second agent must be paid a risk premium in the differentiated contract case. The principal's payoff from the first-best contract is 44.6205, from implementing $a^\dagger$ paying $\mathbf{c}^*(a^\dagger)$ it is 42.5741, and the optimal differentiated contract yields 40.5741.

The preceding analysis demonstrates that in contracting with multiple agents, "subgame problems" may or may not exist and, if they do exist, then there are several ways for the principal to adjust the contracts such that there is no "subgame problem." Independent contracting with both agents and independent contracting with one agent and a relative performance evaluation contract with the other agent always does the job, but as illustrated by our hurdle model example there may be cheaper ways of dealing with a subgame problem.

29.3 HIERARCHICAL AGENCIES WITH DECENTRALIZED CONTRACTING

The preceding analysis and much of the remaining analysis in Chapters 29 and 30 assume the principal (representing the owners of the firm) directly specifies the contracts offered to all agents. We now briefly consider settings in which there is a principal and two levels of agents. The principal contracts directly with the first agent, who subsequently contracts with the second agent. This is representative of a decentralized firm in which the principal is the CEO (acting

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[7] The induced level of effort must be strictly higher than $a_i^\ddagger$, such that the maximum "utility bonus" gives the agents strict incentives to unilaterally deviate from a collusion with zero effort for both agents.

on behalf of the firm's owners), the first agent is a branch manager, and the second agent is a worker at the branch. Alternatively, our model can be viewed as representative of a decentralized partnership in which the principal is the senior head office partner (representing all other partners), the first agent is the managing partner of a local office, and the second agent is a junior partner in the local office. We assume the CEO or senior partner have no direct contact with the worker or junior partner and have no direct control or knowledge of the worker's or junior partner's contract that is implemented.[8]

The gross payoff to the principal from the branch's operations is represented by x. The principal does not take any actions that directly affect the branch's payoff. However, the actions of the branch manager and his worker, denoted a_1 and a_2, respectively, with $\mathbf{a} = (a_1, a_2)$, do influence the branch's gross payoff, as represented by the distribution function $\Phi(x|\mathbf{a})$. The contractible information available to the principal and the branch manager is $\mathbf{y}_1$, and total branch compensation pool is $s(\mathbf{y}_1)$. The information available for contracting between the branch manager and his worker is $\mathbf{y}_2$ and the compensation paid to the worker is $c_2(\mathbf{y}_2)$. Hence, the principal's net share of the branch's gross payoff is $c_0(x, \mathbf{y}_1) = x - s(\mathbf{y}_1)$ and the branch manager's net consumption is $c_1(\mathbf{y}_1, \mathbf{y}_2) = s(\mathbf{y}_1) - c_2(\mathbf{y}_2)$.

Note that, given the branch compensation pool function $s(\cdot)$, decentralized contracting between the branch manager and his worker is similar to the partnership settings examined in Sections 29.1.2 and 29.1.3 without a general partner. The branch manager offers his worker a contract that both shares the risk in the branch compensation pool and provides the manager and worker with effort incentives. Hence, decentralized contracting induces a free-rider problem due to the budget-balancing constraint $c_1(\mathbf{y}_1, \mathbf{y}_2) + c_2(\mathbf{y}_2) = s(\mathbf{y}_1)$. Interestingly, we can view decentralized contracting as the "flip-side" of introducing a general partner. The key difference is that in the hierarchy, the principal can endogenously choose the total branch compensation pool to mitigate the free-rider problem (instead of being limited to sharing the gross payoff x).

29.3.1 Efficient Contract Delegation

The simplest case to consider is a geographically disbursed partnership similar to that examined in Chapter 4 and Section 29.1. In this setting, the principal and the two local partners are all risk averse, and they have no direct preferences with respect to their actions.

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[8] The results for decentralized contracting are the same as those obtained in a limited-commitment setting in which the agents can collude to make side-payments contingent on their performance reports. These side-contracts effectively reallocate the agents' aggregate consumption. See, for example, Feltham and Hofmann (2005a) for this approach to the analysis.

We assume the utility functions for the three partners depend only on their consumption and are represented by $u_i(c_i) = -\exp[-r_i c_i]$, $i = 0,1,2$, which belongs to the HARA class with identical risk cautiousness (equal to zero). Agent i has reservation utility $\underline{U}_i = -\exp[-r_i c_i^o]$, $i = 1,2$.

Assume $\mathbf{y}_1 = \mathbf{y}_2 = x$, i.e., the outcome is the only contractible information and it is reported to all partners at date 1. At date 0, the principal contracts with the local manager, who in turn contracts with the worker. The two agents then take their actions so as to maximize their expected utilities. At date 1, the outcome x is realized, the principal pays $\mathbf{s}(x)$ to the first agent, who in turn pays $\mathbf{c}_2(x)$ to the second agent.

From Chapter 4, we know that the first-best partnership contract is linear, i.e., $\mathbf{c}_i^*(x) = f_i^* + v_j^* x$, $i = 0,1,2$, with $v_i^* = r_o/r_i$, $i = 0,1,2$, $f_0^* = -(f_1^* + f_2^*)$, and

$$f_i^* = -\frac{1}{r_i}\ln(\mathrm{E}[\exp[-r_i v_i^* x]|\mathbf{a}]) + c_i^o, \quad i = 1,2,$$

where $r_o = [r_0^{-1} + r_1^{-1} + r_2^{-1}]^{-1}$. That is, the partners share the output risk in proportion to their risk tolerances r_i^{-1}.

The following describes a sequential contracting process that results in the first-best partnership contract. In the first stage, the principal offers the manager a contract that leaves the principal with his efficient share of the output, i.e.,

$$\mathbf{s}(x) = g + \upsilon x,$$

where $g = -f_0^*$ and $\upsilon = 1 - v_0^*$.

In the second stage, the manager offers the worker the optimal contract for sharing $\mathbf{s}(x)$. From the analysis in Chapter 4, it follows directly that this contract will be a linear function of $\mathbf{s}(x)$, and will take the following form:

$$\mathbf{c}_2(x) = f_2 + v_2 \mathbf{s}(x),$$

with $v_2 = r_{12}/r_2$ and

$$f_2 = -\frac{1}{r_2}\ln(\mathrm{E}[\exp[-r_2 v_2 \mathbf{s}(x)]]) + c_2^o,$$

where $r_{12} = [r_1^{-1} + r_2^{-1}]^{-1}$.

The net result of these two contracts is

$$\mathbf{c}_2(x) = f_2 + v_2\{-f_0^* + (1 - v_0^*)x\} = f_2^* + v_2^* x,$$

and
$$\mathbf{c}_1(x) = \mathbf{s}(x) - \mathbf{c}_2(x) = f_1^* + v_1^* x.$$

That is, the principal can achieve first-best risk sharing with the two agents if he retains his first-best share and contracts with the manager to allocate the remainder between himself and the worker. Of course, this assumes that the risk aversion and reservation utility of the second agent are common knowledge to the principal and first agent.

Recall, from Chapter 4, that the three partners have congruent, and efficient, preferences among the alternative actions. Key factors in this result are that the agent's have no direct preferences with respect to their actions, and their utility functions for consumption belong to the HARA class with identical risk cautiousness.

29.3.2 Inefficient Contract Delegation

Now assume that the actions are directly costly to the agents and are non-contractible. This implies that the optimal partnership contract does not merely share the outcome risk, but must also impose incentive risk. To focus on the distortions created by incentive risk in decentralized contracts, we assume the principal is risk neutral. Hence, he would bear all the risk if the agents had no direct preferences with respect to their actions. Furthermore, to simplify the analysis we adopt a *LEN* model approach (which was introduced in Chapter 19 and used in several subsequent chapters). That is, the contracts are restricted to be linear, the agents have exponential utilities with effort costs, and the performance measures are normally distributed.[9]

The principal is risk neutral, so that his utility function is $u_0(c_0) = c_0$. Agent i's utility function is $u_i(c_i,a_i) = -\exp[-r_i(c_i - \tfrac{1}{2}a_i^2)]$, $i = 1, 2$.

Independent Agents – Optimal Centralized Linear Contracts

The simplest case to consider is one in which the payoffs from the two agents are contractible and independently distributed. That is, $\mathbf{x} = (x_1,x_2)$ is contractible, $x_i = b_i a_i + \varepsilon_i$, $\varepsilon_i \sim \mathrm{N}(0,\sigma_i^2)$, and $\mathrm{Cov}(\varepsilon_1,\varepsilon_2) = 0$.

If the principal can contract directly with each agent on the basis of x_1 and x_2, then the optimal second-best linear contract and action for agent i is[10]

$$c_i^\dagger(\mathbf{x}) = f_i^\dagger + v_i^\dagger x_i,$$

where
$$v_i^\dagger = b_i^2[b_i^2 + r_i\sigma_i^2]^{-1}, \tag{29.43}$$

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[9] Note that the linear contract restriction makes it impossible to use large penalties for low outcomes in order to avoid the free-rider problem as discussed in Section 29.1.2.

[10] This solution is adapted from (20.13).

and the fixed cost is such that the agent's expected utility is equal to his reservation utility.[11] The induced effort is $a_i^\dagger = b_i v_i^\dagger$.

A key feature of the optimal centralized contract is that, due to the independence assumption, each agent's compensation is independent of the other agent's payoff.[12] That is, there is no risk sharing between the agents. The risk neutral principal bears all the risk except for the incentive risk imposed on the agents.[13]

The principal's expected net payoff with disaggregate reporting (represented by η^x) and centralized contracting is

$$U^{p\dagger}(\eta^x) = b_1 a_1^\dagger + b_2 a_2^\dagger - \{\tfrac{1}{2}(a_1^\dagger)^2 + \tfrac{1}{2}(a_2^\dagger)^2$$

$$+ \tfrac{1}{2} r_1 (v_1^\dagger)^2 \sigma_1^2 + r_2 (v_2^\dagger)^2 \sigma_2^2\}, \qquad (29.44)$$

where the incentive rates are as characterized in (29.43), and $a_i^\dagger = b_i v_i^\dagger$, $i = 1,2$.

Independent Agents – Decentralized Contracts

Now consider the same setting but assume the principal offers the following compensation pool contract to the manager:

$$\mathbf{s}(\mathbf{x}) = g + \upsilon_1 x_1 + \upsilon_2 x_2,$$

where υ_1 and υ_2 are the variable compensation pool rates. The manager then offers the following contract to the worker:

$$\mathbf{c}_2(\mathbf{x}) = f_2 + v_{21} x_1 + v_{22} x_2,$$

where v_{22} is the worker's incentive rate and v_{21} is his risk-sharing rate with respect to the compensation pool risk associated with the manager's output.

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[11] In this setting, $f_i^\dagger = \tfrac{1}{2}(a_i^\dagger)^2 + \tfrac{1}{2} r_i (v_i^\dagger)^2 \sigma_i^2 - v_i^\dagger b_i^2 a_i^\dagger + c_i^o$. In the subsequent discussion we omit the details regarding the fixed costs. In general, the fixed wage compensates the agent for his effort cost, his risk premium due to incentive risk, and his reservation wage, and it is decreased by the agent's expected incentive wage.

[12] The subgame issues discussed in Section 29.2.3 and illustrated in Section 29.2.4 would occur in the current setting if the payoffs (which are used as performance measures) were correlated. We avoid those issues by assuming the payoffs are stochastically independent. This simplifies the analysis, but without removing the key risk sharing issues that are the focus of the analysis in this section.

[13] Note that the centralized setting considered here is equivalent to the setting with a general risk neutral partner examined in Section 29.1.4 (except we now imposed the *LEN* conditions).

The manager's and worker's certainty equivalents, given their action choice and their conjecture with respect to the other agent's action (denoted by a "hat") are:

$$CE_1(a_1,\hat{a}_2) = g - f_2 + (\upsilon_1 - v_{21})b_1a_1 + (\upsilon_2 - v_{22})b_2\hat{a}_2 - ½a_1^2$$
$$- ½r_1\{(\upsilon_1 - v_{21})^2\sigma_1^2 + (\upsilon_2 - v_{22})^2\sigma_2^2\}, \qquad (29.45a)$$

$$CE_2(\hat{a}_1,a_2) = f_2 + v_{21}b_1\hat{a}_1 + v_{22}b_2a_2 - ½a_2^2$$
$$- ½r_2\{v_{21}^2\sigma_1^2 + v_{22}^2\sigma_2^2\}. \qquad (29.45b)$$

Differentiating with respect to the actions yields the following characterizations of the agents' action choices:

$$a_1^‡ = b_1(\upsilon_1 - v_{21}), \qquad (29.46a)$$

$$a_2^‡ = b_2v_{22}. \qquad (29.46b)$$

Given rational expectations, $\hat{a}_1 = a_1^‡$ and $\hat{a}_2 = a_2^‡$. Substituting (29.46) into the certainty equivalent (29.45b) and setting f_2 such that $CE_2 = c_2^o$ provides f_2. Substituting (29.46) and f_2 into (29.45a) provides an unconstrained manager decision problem in terms of his choice of contract parameters v_{21} and v_{22}, given υ_1 and υ_2, and g:

$$\underset{v_{21},\, v_{22}}{\text{maximize}} \quad CE_1 = g + \upsilon_1b_1^2(\upsilon_1 - v_{21}) + \upsilon_2b_2^2v_{22}$$
$$- ½b_1^2(\upsilon_1 - v_{21})^2 - ½r_1\{(\upsilon_1 - v_{21})^2\sigma_1^2 + (\upsilon_2 - v_{22})^2\sigma_2^2\}$$
$$- ½b_2^2v_{22}^2 - ½r_2\{v_{21}^2\sigma_1^2 + v_{22}^2\sigma_2^2\} - c_2^o. \qquad (29.47)$$

Differentiating (29.47) provides first-order conditions that characterize the manager's optimal decentralized contract choice, from which we derive the decentralized incentive rates for the two agents:

$$v_{11}^‡ = \upsilon_1H_{11}, \quad H_{11} = [b_1^2 + r_2\sigma_1^2]Q_1, \quad Q_1 \equiv [b_1^2 + (r_1 + r_2)\sigma_1^2]^{-1}, \qquad (29.48a)$$

$$v_{21}^‡ = \upsilon_1H_{21}, \quad H_{21} = r_1\sigma_1^2Q_1, \qquad (29.48b)$$

$$v_{12}^‡ = \upsilon_2H_{12}, \quad H_{12} = r_2\sigma_2^2Q_2, \qquad Q_2 \equiv [b_2^2 + (r_1 + r_2)\sigma_2^2]^{-1}, \qquad (29.48c)$$

$$v_{22}^‡ = \upsilon_2H_{22}, \quad H_{22} = [b_2^2 + r_1\sigma_2^2]Q_2. \qquad (29.48d)$$

Observe that if the principal offers the manager a contract in which $g = f_1^\dagger + f_2^\dagger$, $v_1 = v_1^\dagger$, and $v_2 = v_2^\dagger$, then the optimal centralized contracts could be implemented by the manager. However, (29.47) implies that the manager will not choose to do so. In particular, observe that $v_{11}^\ddagger < v_1$, $v_{12}^\ddagger > 0$, $v_{21}^\ddagger > 0$ and $v_{22}^\ddagger < v_2$. These results imply that, in the decentralized contract, the manager and worker share each other's risks, whereas that does not occur in the optimal centralized contract. Furthermore, since some of each agent's risk is shifted to the other agent, this reduces the action incentives for each agent. Hence, in this setting, decentralized contracting leads to a loss of efficiency relative to centralized contracting. The fundamental cause is that when allowed to contract among themselves, the agents will share the incentive risk which the risk neutral principal would prefer to impose on them.[14]

In the preceding discussion we have interpreted the setting as one in which contracting with the worker is delegated to the manager because the principal does not have direct contact with the worker. Interestingly, similar results occur if the principal directly contracts with both agents, but he cannot preclude them from renegotiating these contracts before they take their actions. In this case, if $s(\mathbf{y}_1)$ is the sum of the two individual centralized contracts, then renegotiation yields the same result as decentralized contracting if the initial contract provides the worker with his reservation utility and the manager has the bargaining power in the renegotiation.

In Chapter 30 we consider collusion between a worker and a monitor. Collusion involves side-payments between the players, and we can interpret renegotiation as a form of collusion. Hence, decentralized contracting, centralized contracting with renegotiation between agents, and centralized contracting with collusion by the agents are effectively the same.[15]

The Principal's Contract Choice

The preceding analysis has treated $s(\mathbf{x})$ as exogenous. To demonstrate the loss of efficiency due to delegation we only needed to consider the manager's contract choice if $s(\mathbf{x})$ equals the sum of the agent's optimal centralized contracts. However, that will not be the principal's optimal choice given the anticipated response of the manager.

The principal's contract choice problem is

$$U^{p\ddagger}(\eta^x) = \underset{v_1,\, v_2}{\text{maximize}}\ b_1 a_1^\ddagger + b_2 a_2^\ddagger - \{ \tfrac{1}{2}(a_1^\ddagger)^2 + \tfrac{1}{2}(a_2^\ddagger)^2$$

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[14] Of course, this phenomenon is exactly the same as identified in the partnership setting without a general partner in section 29.1.3.

[15] Feltham and Hofmann (2005a) consider contract characteristics in a similar model and point out the equivalence of decentralized contracting and centralized contracting with collusion.

$$+ \tfrac{1}{2} r_1 [(v_{11}^{\ddagger})^2 \sigma_1^2 + (v_{12}^{\ddagger})^2 \sigma_2^2] + \tfrac{1}{2} r_2 [(v_{21}^{\ddagger})^2 \sigma_1^2 + (v_{22}^{\ddagger})^2 \sigma_2^2]\}, \qquad (29.49)$$

where the incentive rates are functions of υ_1 and υ_2 as specified by (29.48), and $a_i^{\ddagger} = v_{ii}^{\ddagger} b_i$. Differentiating with respect to υ_1 and υ_2 yields the first-order conditions characterizing the optimal choice of these two contract parameters:

$$\upsilon_1^{\ddagger} = b_1^2 H_{11} [H_{11}^2 (b_1^2 + r_1 \sigma_1^2) + r_2 \sigma_1^2 H_{21}^2]^{-1}, \qquad (29.50a)$$

$$\upsilon_2^{\ddagger} = b_2^2 H_{22} [H_{22}^2 (b_2^2 + r_2 \sigma_2^2) + r_1 \sigma_2^2 H_{12}^2]^{-1}. \qquad (29.50b)$$

Under centralized contracting, each agent receives all the variable compensation associated with his output (represented by $v_i^{\dagger} x_i$). Under decentralized contracting, $\upsilon_i x_i$ represents the total variable compensation associated with agent i's output. He only receives a fraction $H_{ii} \in (0,1)$, so there is a free-rider problem (the other agent receives a fraction $1 - H_{ii}$). In particular, agent i's incentive rate is $v_{ii}^{\ddagger} = H_{ii} \upsilon_i^{\ddagger}$, which is strictly less than the variable compensation pool rate $\upsilon_i^{\ddagger}$. The following proposition summarizes the fact that it is optimal for the principal to partially, but not fully, mitigate the loss of production due to the free-rider problem, by setting $\upsilon_i^{\ddagger} > v_i^{\dagger}$, but such that $v_{ii}^{\ddagger} < v_i^{\dagger}$. These results follow directly from comparing (29.43) to (29.50). For example,[16]

$$v_i^{\dagger} = b_i^2 [b_i^2 + r_i \sigma_i^2]^{-1}$$

$$> v_{ii}^{\ddagger} = \upsilon_i^{\ddagger} H_{ii} = b_i^2 [b_i^2 + r_i \sigma_i^2 + r_j \sigma_i^2 H_{ji}^2 / H_{ii}^2]^{-1}, \quad i = 1,2, j \neq i.$$

Proposition 29.1

The decentralized *LEN* hierarchy with independent agents and disaggregate information has the following properties relative to a centralized *LEN* hierarchy.

(a) Decentralization is costly to the principal.

(b) Decentralization reduces effort incentives: $a_i^{\ddagger} = b_i v_{ii}^{\ddagger} < a_i^{\dagger} = b_i v_i^{\dagger}$.

(c) Decentralization increases branch compensation pool risk: $\upsilon_i^{\ddagger} > v_i^{\dagger}$.

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[16] The proof that $\upsilon_i^{\ddagger} > v_i^{\dagger}$ is somewhat more tedious.

29.3.3 Centralized versus Decentralized Contracting with an Aggregate Performance Measure

Now consider the same setting as above, but assume that $\mathbf{y}_1 = \mathbf{y}_2 = x = x_1 + x_2$ is the only performance measure (represented by η^x), and let $\sigma^2 \equiv \sigma_1^2 + \sigma_2^2$. The optimal second-best incentive rate for agent i with centralized contracting is

$$v_i^\dagger = b_i^2[b_i^2 + r_i\sigma^2]^{-1}. \tag{29.51}$$

The induced effort is $a_i^\dagger = b_i v_i^\dagger$, and the principal's expected net payoff is

$$U^{p\dagger}(\eta^x) = b_1 a_1^\dagger + b_2 a_2^\dagger - \{½(a_1^\dagger)^2 + ½(a_2^\dagger)^2$$

$$+ ½r_1(v_1^\dagger)^2\sigma^2 + r_2(v_2^\dagger)^2\sigma^2\}, \tag{29.52}$$

where the incentive rates are as characterized in (29.51).

The key difference between this contract and the contract based on disaggregate reporting is the noise in the performance measure. The aggregate noise exceeds the noise of the agent-specific payoff, so that the incentive rate for each agent is lower under aggregate reporting.

Now consider decentralized contracting and assume the manager is offered a compensation pool $s(x) = g + \upsilon x$. The manager offers the worker the linear contract $c_2 = f_2 + v_2 x$. This induces $a_1 = b_1(\upsilon - v_2)$ and $a_2 = b_2 v_2$. Substituting these expressions into the worker's certainty equivalent to obtain f_2 and substituting the results into the manager's certainty equivalent provides the manager's unconstrained decision problem in terms of his choice of contract parameters v_2, given υ:

$$\underset{v_2}{\text{maximize}} \quad CE_1 = g + \upsilon[b_1^2(\upsilon - v_2) + b_2^2 v_2] - ½b_1^2(\upsilon - v_2)^2$$

$$- ½b_2^2 v_2^2 - ½r_1(\upsilon - v_2)^2\sigma^2 - ½r_2 v_2^2\sigma^2.$$

Taking the derivative with respect to v_2 yields the first-order conditions

$$v_1^\ddagger = \upsilon - v_2^\ddagger = \upsilon[b_2^2 + r_2\sigma^2][b_1^2 + b_2^2 + (r_1 + r_2)\sigma^2]^{-1}, \tag{29.53a}$$

$$v_2^\ddagger = \upsilon[b_1^2 + r_1\sigma^2][b_1^2 + b_2^2 + (r_1 + r_2)\sigma^2]^{-1}. \tag{29.53b}$$

The principal's optimal net payoff in this setting is

$$U^{p\ddagger}(\eta^x) = \underset{\upsilon}{\text{maximize}} \; [b_1 a_1^\ddagger + b_2 a_2^\ddagger] - \{½(a_1^\ddagger)^2 + ½(a_2^\ddagger)^2$$

$$+ \tfrac{1}{2}r_1(v_1^\ddagger)^2\sigma^2 + r_2(v_2^\ddagger)^2\sigma_2^2\}, \tag{29.54}$$

where $a_i^\ddagger = b_i v_i^\ddagger$ and $v_i^\ddagger$ is a function of υ as characterized in (29.53).

Observe that if the manager and worker have identical risk aversion (i.e., $r_1 = r_2 = r$) and identical productivity (i.e., $b_1 = b_2 = b$), then $v_1^\ddagger = v_2^\ddagger = \tfrac{1}{2}\upsilon$. Hence, if the two agents are identical, then decentralized contracting obtains the same result as centralized contracting if $\upsilon = v_1^\dagger + v_2^\dagger$.[17] However, decentralized contracting is inefficient (except in knife-edge cases) if the manager and worker differ in their risk aversion or productivity.

29.3.4 Disaggregate Local Information

In our preceding analysis we assume the contractible information is the same for centralized and decentralized contracting. As a result, centralized contracting weakly dominates decentralized contracting and, if there is incentive risk, the dominance is strict except in some special cases. However, a common argument in favor of delegation is that lower level managers have local information not available to senior management. To illustrate the benefits of local information, we consider an example comparing centralized contracting based on an aggregate report $x = x_1 + x_2$ versus decentralized contracting in a setting in which $\mathbf{y}_1 = x$ and $\mathbf{y}_2 = \mathbf{x}$.[18] That is, the principal only receives a report of the branch's aggregate output, but the two agents can contract on a disaggregate report $\mathbf{x} = (x_1,x_2)$.

In the latter case, the principal offers the manager $\boldsymbol{s}(x) = g + \upsilon x$, and the manager, in turn, offers the worker $\boldsymbol{c}_2(\mathbf{x}) = f_2 + v_{21}x_1 + v_{22}x_2$. Hence, the manager receives $\boldsymbol{c}_1(\mathbf{x}) = f_1 + v_{11}x_1 + v_{12}x_2$, where $f_1 = g - f_2$, $v_{11} = \upsilon - v_{21}$, and $v_{12} = \upsilon - v_{22}$. The manager's contract choice is very similar to his choice with disaggregate reporting and decentralized contracting (see (29.48)), but with $\upsilon_1 = \upsilon_2 = \upsilon$, i.e.,

$$v_{11}^\ddagger = \upsilon H_{11},\; v_{12}^\ddagger = \upsilon H_{12}, \quad v_{22}^\ddagger = \upsilon H_{22},\; v_{21}^\ddagger = \upsilon H_{21}. \tag{29.55}$$

[17] Note that, if the two agents are identical, the optimal compensation to each of the agents with decentralization will be one half of the branch compensation pool even with optimal contracts, i.e., $\boldsymbol{c}_i^\dagger(x) = \tfrac{1}{2}\boldsymbol{s}(x)$. Hence, if the branch compensation pool is chosen as the sum of the optimal centralized compensation to each of the agents (which would also be identical), i.e., $\boldsymbol{s}(x) = \boldsymbol{c}_1^\dagger(x) + \boldsymbol{c}_2^\dagger(x)$, then the centralized solution will be implemented. That is, there is no loss to decentralization even with optimal contracts if the agents are identical.

[18] See Feltham and Hofmann (2005b) for analysis of the impact of alternative reporting systems in limited commitment, multi-agent settings. They consider how the distribution of information affects the effectiveness of centralized versus decentralized contracting, and full- versus limited-commitment contracting.

The induced actions are $a_1^\ddagger = b_1 v_{11}^\ddagger$ and $a_2^\ddagger = b_2 v_{22}^\ddagger$, and the principal's optimal expected net payoff is

$$U^{p\ddagger}(\eta^{x,x}) = \underset{v}{\text{maximize}}\ b_1 a_1^\ddagger + b_2 a_2^\ddagger - \{½(a_1^\ddagger)^2 + ½(a_2^\ddagger)^2 + ½r_1[(v_{11}^\ddagger)^2\sigma_1^2 + (v_{12}^\ddagger)^2\sigma_2^2] + ½r_2[(v_{21}^\ddagger)^2\sigma_1^2 + (v_{22}^\ddagger)^2\sigma_2^2]\}. \quad (29.56)$$

Figure 29.1 plots the principal's optimal payoffs with centralized contracting based on both disaggregate and aggregate reporting, as well as decentralized contracting based on aggregate reporting to the principal, but disaggregate reporting to the agents. We assume that, for exogenous reasons, the agents cannot communicate their disaggregate information to the principal. In this example, each agent takes a single action, they have identical risk aversion ($r_1 = r_2 = 1$), and they have identical payoff noise ($\sigma_1^2 = \sigma_2^2 = 50$). However, the productivity of the worker (b_2) is varied from 10 to 20, while holding the total productivity constant at 20 (i.e., $b_1 = 20 - b_2$).

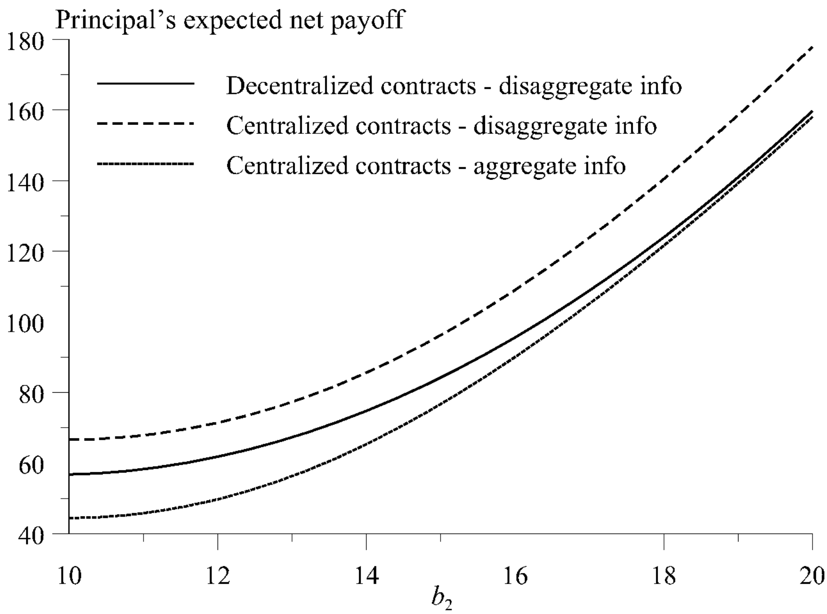

Figure 29.1: Centralized versus decentralized contracting with a more informative branch manager.

The principal's expected net payoff is uniformly larger with centralized contracting if it is based on disaggregate information. However, decentralized contracting with disaggregate agent information dominates centralized contracting if it is based solely on aggregate information. Observe that the latter difference decreases as b_1 and b_2 diverge even though both payoffs are increasing. That is, using the better informed local manager to contract with the worker is strictly valuable, and that value is largest when the two agents are identical. Table 29.3 depicts the contract differences for the settings in which (a) $b_1 = b_2 = 10$ and (b) $b_1 = 0$ and $b_2 = 20$.

			Centralized contracts with aggregate (disaggregate) reports			Decentralized contracts with a more informed manager				
	b_1	b_2	v_1	v_2	U^p	v_{11}	v_{12}	v_{21}	v_{22}	U^p
(a)	10	10	.667 (.667)	.667 (.667)	66.67 (44.44)	0.77	0.255	0.255	0.766	56.79
(b)	0	20	0 (0)	.889 (.889)	177.78 (158.02)	0.48	0.1	0.477	0.858	159.74

Table 29.3: Value of delegating to a more informed branch manager.

In case (a), the agents are identical and their contracts are symmetric. With decentralized contracting using disaggregate information, the manager is able to use stronger effort incentives for both himself and the worker, and at the same time reduce the incentive risk since, for example v_{11} is applied to x_1, whereas v_1 is applied to x. The incentive rates in the centralized contracts are essentially the same with disaggregate and aggregate information, but the risk premium is much less in the former since v_1 and v_2 are applied to x_1 and x_2, respectively, instead of to x.

In case (b), the centralized contract is effective because the principal can limit the imposition of incentive risk to the productive agent and impose zero risk on the non-productive agent. Nonetheless, decentralization using disaggregate information is slightly more effective. In this case, x_1 is pure noise and the manager is non-productive. The manager and worker equally share the risk associated with the pure noise, and the manager bears a small a fraction of the risk associated with the worker's output x_2.

29.4 MULTIPLE AGENTS WITH PRE-CONTRACT INFORMATION

As noted earlier, our analysis of the basic principal-agent model is similar to the Demski and Sappington (1984) (DS84) analysis of a model in which there is a risk neutral principal and two weakly risk averse, strictly effort averse agents, each with private pre-contract information. Their analysis can be interpreted as an extension of the single-agent, pre-contract information model in which the uniformed principal moves first (see Chapter 23). We briefly describe the DS84 model and explore some of the analysis, particularly focusing on insights that are not provided by the discussion of the basic principal-agent model in Section 29.2.

Each agent operates his own technology, and the contractible outcome from one agent's technology is independent of the action taken by the other, i.e., there are no interactive (synergistic) effects. Hence, the principal always has the option of contracting with each agent on the basis of his own outcome, in which case the form of the optimal contract is the same as in a single-agent setting. However, the setting is such that the uncontrollable events influencing the agents' outcomes are correlated. This implies that the outcome of one agent is informative about the uncontrollable events influencing the outcome of the other agent. Hence, it may be optimal to write contracts in which the compensation of one agent depends on the outcomes for *both* agents.

The outcome for agent i is denoted $x_i \in X_i = [0,\infty)$, $i = 1, 2$, and it is represented as a function of his action, $a_i \in A_i \in [0,\infty)$, and an agent-specific uncertain state of nature, $\theta_i \in \Theta_i = \{\theta_{i\ell}, \theta_{ih}\}$, i.e., $x_i = \boldsymbol{x}_i(a_i, \theta_i)$. We assume $\boldsymbol{x}_i(\cdot,\cdot)$ is increasing and concave in a_i and the outcome is zero if effort is zero, independent of the state, i.e., $\boldsymbol{x}_i(0,\theta_i) = 0$, $\forall\ \theta_i \in \Theta_i$. For positive effort, the outcome is greater in the high state than in the low state, i.e., $\boldsymbol{x}_i(a_i,\theta_{i\ell}) < \boldsymbol{x}_i(a_i,\theta_{ih})$, $\forall\ a_i > 0$.

The joint probability for the two states is represented by $\varphi(\theta)$, where $\theta = (\theta_1,\theta_2)$, and the marginal probability of state θ_{ij}, $i = 1, 2, j = \ell, h$, is $p_{ij} = \varphi(\theta_i = \theta_{ij})$. The two states are assumed to be positively correlated so that the conditional probabilities are such that $\varphi(\theta_{2h}|\theta_{1h}) > \varphi(\theta_{2h}|\theta_{1\ell})$. In Section 29.2 we focused on settings in which outcomes are perfectly correlated, but in this section we allow the correlations to be imperfect.

There is a single consumption date and agent i's compensation function is c_i: $X_1 \times X_2 \rightarrow C_i$, which recognizes that both outcomes can be used in contracting with a given agent. The principal is assumed to be risk neutral, while each agent has an additively separable utility function $u_i^a(c_i, a_i) = u_i(c_i) - v_i(a_i)$ such that an agent is weakly risk averse, i.e., $u_i' > 0$ and $u_i'' \leq 0$, and strictly effort averse, i.e., $v_i' > 0$ and $v_i'' > 0$. Agent i's reservation utility is $\underline{U}_i$.

Agent i is assumed to observe θ_i prior to contracting and selecting his action. Hence, this is a setting in which the agents have perfect pre-contract information with respect to their own outcome (but not the state or outcome for the other agent). The agents know the amount of disutility they must incur to provide a given outcome, and we represent that disutility by

$$\kappa_i(x_i, \theta_i) \equiv v_i(a_i = \boldsymbol{x}_i^{-1}(x_i, \theta_i)).$$

The previous assumptions imply that $\kappa_i(\cdot)$ is increasing and strictly convex in x_i, with $\kappa_i(x_i, \theta_{i\ell}) > \kappa_i(x_i, \theta_{ih})$.

We know from our analysis in Chapter 23 that there is no value to communication when the agent has perfect information about his contractible outcome.[19] However, it is useful to use truth-telling to refer to the selection of the outcome that the principal chooses to induce for the state observed by the agent. The principal is assumed to have all the bargaining power. Hence, he offers a menu of contracts to each agent, and we can invoke the Revelation Principle.

29.4.1 Independent Contracts

First-best Independent Contracts

As a benchmark, consider the setting in which θ_i is contractible information (and, hence, the first-best result can be achieved in each state). The first-best contract has the following characteristics for each agent $i = 1, 2$, where c_{ij}^* and x_{ij}^* represent agent i's compensation and outcome in state θ_{ij}, $j = \ell, h$. It is straightforward to obtain the following characterization of the first-best contract.

Proposition 29.2

In the setting described above, contractible information about the state θ permits achievement of the following first-best results.

(a) *No rents:* Agent i receives a payment that depends on his state (because his effort is state dependent) that is just sufficient to cover his reservation utility plus his disutility for effort:

$$u_i(c_{ij}^*) = \overline{U}_i + \kappa_i(x_{ij}^*, \theta_{ij}), \quad \text{for } j = \ell, h.$$

(b) *Efficient production:* Agent i's marginal utility for compensation equals his marginal disutility for increasing the outcome he produces:

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[19] An agent does not have completely perfect information in that he does not know the other agent's state or outcome. However, the perfect information implication of no value of communication applies since his outcome is directly controlled by the agent given his information. The other agent's outcome is independent of his action.

$$u_i'(c_{ij}^*) = \partial\kappa_i(x_{ij}^*,\theta_{ij})/\partial x_i, \quad \text{for } j = \ell, h.$$

(c) The outcome is higher in the high productivity state:

$$x_{i\ell}^* < x_{ih}^*.$$

Second-best Independent Contracts
As another benchmark DS84 consider the optimal independent contracts, i.e., agent i's compensation depends only on x_i, in a setting in which θ_i is not contractible information. The agent will produce a specific outcome for each state. The principal must choose the state-contingent outcome plan he wants to induce using an outcome-contingent compensation. Let $\mathbf{x}_i = (x_{i\ell}, x_{ih})$ represent the state-contingent production plan for agent i, and let $\mathbf{c}_i = (c_{i\ell}, c_{ih})$ represent the outcome-contingent compensation, where c_{ij} is the amount paid to agent i if x_{ij} is produced, $j = \ell, h$. (A severe penalty is imposed if the agent produces an outcome other than $x_{i\ell}$ or x_{ih}.)

The principal's decision problem for agent i is as follows.

Principal's Independent Contracting Problem P1:

$$\underset{\mathbf{x}_i,\, \mathbf{c}_i}{\text{maximize}} \ [x_{i\ell} - c_{i\ell}]p_{i\ell} + [x_{ih} - c_{ih}]p_{ih},$$

$$\text{subject to} \ \ u_i(c_{ij}) - \kappa_i(x_{ij},\theta_{ij}) \geq \bar{U}_i, \qquad j = \ell, h,$$

$$u_i(c_{ij}) - \kappa_i(x_{ij},\theta_{ij}) \geq u_i(c_{ik}) - \kappa_i(x_{ik},\theta_{ij}), \qquad j,k = \ell, h.$$

The first constraint ensures that the agent will accept the state-contingent contract for both states (i.e., there are two participation constraints), whereas the second constraint ensures that the agent will "truthfully report" his state. The characteristics of the solution to this problem are essentially the same as those in Chapter 23.

Proposition 29.3 (DS84, Finding 1)
The optimal independent contract has the following properties.

(a) If the agent is in the low productivity state, he receives no rent,

$$u_i(c_{i\ell}) - \kappa_i(x_{i\ell},\theta_{i\ell}) = \bar{U}_i,$$

and his production is not Pareto efficient, i.e.,

$$u_i'(c_{i\ell}) > \partial\kappa_i(x_{i\ell},\theta_{i\ell})/\partial x_i.$$

(b) If the agent is in the high productivity state, he receives a positive rent and is indifferent between providing the low or high productivity outcome,

$$u_i(c_{ih}) - \kappa_i(x_{ih},\theta_{ih}) = u_i(c_{i\ell}) - \kappa_i(x_{i\ell},\theta_{ih}) > \bar{U}_i,$$

and his production is Pareto efficient, i.e.,

$$u_i'(c_{ih}) = \partial\kappa_i(x_{ih},\theta_{ih})/\partial x_i.$$

(c) The output is again higher in the high productivity state, i.e.,

$$x_{i\ell} < x_{ih}.$$

As noted in Chapter 23, the inducement of less than Pareto efficient production in the low state is used to reduce the rent paid in the high state. The payment of information rent to the agent with high productivity makes the asymmetric information setting undesirable from the principal's perspective.

29.4.2 Contracting on the other Agent's Outcome

Now consider the setting in which the contract for agent 1 can be based on both x_1 and x_2 under the assumption that x_2 reveals θ_2. We consider two formulations of the principal's decision problem, and in each case focus on the problem for agent 1 – the problem for agent 2 is the same. In the first, the principal is assumed to induce agent 1 to tell the truth *under the assumption* that the agent believes that x_2 truthfully reveals θ_2. Let x_{1j} represent the outcome chosen by agent 1 if he observes θ_{1j} and let $c_{1j\tau}$ represent agent 1's compensation if he reports θ_{1j} and x_2 reveals $\theta_{2\tau}$.

Principal's Relative Performance Measure Problem P2a:

$$\underset{\mathbf{x}_i,\,\mathbf{c}_i}{\text{maximize}} \quad \sum_{j\in\{\ell,h\}} \sum_{\tau\in\{\ell,h\}} [x_{1j} - c_{1j\tau}]\,\varphi(\theta_{1j},\theta_{2\tau}),$$

$$\text{subject to} \quad \sum_{\tau\in\{\ell,h\}} u_1(c_{1j\tau})\,\varphi(\theta_{2\tau}|\theta_{1j}) - \kappa_1(x_{1j},\theta_{1j}) \geq \bar{U}_i, \qquad \text{for } j = \ell, h,$$

$$\sum_{\tau\in\{\ell,h\}} u_1(c_{1j\tau})\,\varphi(\theta_{2\tau}|\theta_{1j}) - \kappa_1(x_{1j},\theta_{1j}) \geq$$

$$\sum_{\tau \in \{\ell,h\}} u_1(c_{1k\tau})\, \varphi(\theta_{2\tau}|\theta_{1j}) - \kappa_1(x_{1k},\theta_{1j}), \text{ for } j,k = \ell, h.$$

The optimal solution for this problem is similar to the solution for P1. The key difference is that $u_1(c_{1j})$ is replaced with $u_1(c_{1j\ell})\,\varphi(\theta_{2\ell}|\theta_{1j}) + u_1(c_{1jh})\,\varphi(\theta_{2h}|\theta_{1j})$, and we obtain $c_{1\ell\ell} > c_{1\ell h}$ and $c_{1h\ell} > c_{1hh}$.

Problem P2a presumes that agent 1 believes that agent 2 will truthfully reveal his type. Given that belief, agent 1 is induced to truthfully reveal his type. Now we consider the problem in which agent 1 is induced to tell the truth even if agent 2 lies. This is referred to as inducing agent 1 to *truthfully reveal his information as a dominant strategy.*

Principal's Dominant Truthful Reporting Problem P2b:

$$\underset{\mathbf{x}_i,\, \mathbf{c}_i}{\text{maximize}} \quad \sum_{j \in \{\ell,h\}} \sum_{\tau \in \{\ell,h\}} [x_{1j} - c_{1j\tau}]\, \varphi(\theta_{1j},\theta_{2\tau}),$$

$$\text{subject to} \quad \sum_{\tau \in \{\ell,h\}} u_1(c_{1j\tau})\, \varphi(\theta_{2\tau}|\theta_{1j}) - \kappa_1(x_{1j},\theta_{1j}) \geq \bar{U}_i, \quad \text{for } j = \ell, h,$$

$$u_1(c_{1j\tau}) - \kappa_1(x_{1j},\theta_{1j}) \geq u_1(c_{1k\tau}) - \kappa_1(x_{1k},\theta_{1j}), \quad \text{for } j,k,\tau = \ell, h.$$

Observe that Problems P2a and P2b differ only in their incentive compatibility constraints. In P2a we use the expectation for $\theta_{2\tau}$ given θ_{1j}, whereas in P2b we consider $\theta_{2\ell}$ and θ_{2h} separately. Consequently, the solution to Problem P2b is a feasible contract for Problem P2a (but the reverse does not necessarily hold).

The relative performance contract is risky to the agent – he does not know which state the other agent has observed. If the states are uncorrelated, it is obvious that there is no benefit from including agent 2's state in determining agent 1's compensation. However, if the states are correlated, we know that (x_1,θ_2) is more A-informative than x_1 and the principal can benefit from including θ_2 in agent 1's contract.

If the *agent is risk neutral*, then θ_2 can be used to achieve the first-best result.

Proposition 29.4 (DS84, Prop. 1)

Assume that θ_1 and θ_2 are imperfectly correlated, agent 1 observes θ_1, but not θ_2, before contracting, and *agent 1 is risk neutral.* Then the principal can achieve the full information efficient solution for agent 1.

Proof: The result for agent 1 is achieved if we set $c_{1j\tau}$ such that it solves P2b for x_{1j}^*, i.e.,

$$c_{1j\ell}\varphi(\theta_{2\ell}|\theta_{1j}) + c_{1jh}\varphi(\theta_{2h}|\theta_{1j}) = c_{1j}^*, \quad j = \ell, h, \tag{29.57}$$

$$c_{1j\tau} - \kappa_{1jj}^* \geq c_{1k\tau} - \kappa_{1kj}^*, \quad j,k,\tau = \ell, h, \tag{29.58}$$

where
$$\kappa_{1\tau j}^* = \kappa_1(x_{1\tau}^*, \theta_{1j}).$$

The inequalities in (29.58) can be satisfied by selecting two numbers $\delta_h < \delta_\ell$ such that

$$\kappa_{1hh}^* - \kappa_{1\ell h}^* \leq \delta_h \leq \delta_\ell \leq \kappa_{1h\ell}^* - \kappa_{1\ell\ell}^*,$$

and then setting the compensation such that $c_{1hj} = c_{1\ell j} + \delta_j, j = \ell, h$. Substituting for c_{1hh} and $c_{1h\ell}$ into the equalities in (29.57) provides a system of two equations in two unknowns which is readily solved if θ_1 and θ_2 are positively correlated.

Q.E.D.

Corollary 1

Under the conditions assumed in Proposition 29.4, the agents prefer to have uncorrelated states (so that they can obtain positive rents).

Corollary 2

Under the conditions assumed in Proposition 29.4 and positively correlated states, agent 1 is no better off than if he is uninformed (i.e., zero rents).

If the *agents are risk averse*, the principal must compensate them for any risk due to varying the contract with the other agent's state. DS84 focus on incentive schemes in which *truthtelling is a dominant strategy*, i.e., Problem P2b, and obtain the following result for agent 1 (as the representative agent).

Proposition 29.5 (DS84, Prop. 2)

Assume θ_1 and θ_2 are imperfectly and positively correlated, agent 1 observes θ_1, but not θ_2, before contracting, and agent 1 is strictly risk averse. Then, among all incentive schemes in which truth-telling is a dominant strategy, the contract most preferred by the principal has the following properties:

(a) If agent 1 observes $\theta_{1\ell}$, he will receive no rents and his marginal rate of substitution between compensation c and output x is strictly less than unity.

(b) If agent 1 observes θ_{1h}, he may receive rents and his marginal rate of substitution between compensation c and output x is exactly unity.

(c) Agent 1 will face a lottery regardless of his private information.

(d) Agent 1 will be induced to produce more if he observes θ_{1h} than if he observes $\theta_{1\ell}$.

(e) Agent 1 will receive greater compensation if agent 2's state is $\theta_{2\ell}$ than if it is θ_{2h}.

DS84 prove the above characterization of the optimal contract by applying the usual Kuhn-Tucker analysis to Problem P2b. They note that, as in the independent contract, the binding constraints are the contract acceptance constraint for $\theta_{1\ell}$ and the incentive constraints to truthfully report θ_{1h}.

Observe the strong similarities between the result in Proposition 29.5, which uses θ_2 in motivating agent 1, and Proposition 29.2, which does not use θ_2. The principal gains from using θ_2 in contracting with agent 1. These gains stem from using compensation lotteries based on θ_2 that permit the principal to *reduce the agent's information rents* when he observes θ_{1h}.

Corollary 3

The principal strictly prefers a contract based on (x_1, θ_2) to one based only on x_1.

This corollary follows from condition (c) in Proposition 29.5, which establishes that the optimal contract (in which truth-telling is a dominant strategy) involves using lotteries based on θ_2.

Subgame Undominated Equilibria

We have interpreted the preceding analysis in terms of contracting on (x_1, θ_2) under the assumption that θ_2 is contractible information. However, DS84 interpret the analysis in terms of contracting on (x_1, x_2). These are equivalent if both agents select output levels that reveal their state.

Requiring truth-telling to be a dominant strategy is not restrictive if the agents are risk neutral – the first-best is achieved. However, requiring truth-telling to be a dominant strategy in Proposition 29.5 is restrictive. The optimal solution to Problem P2 would not generally satisfy this condition if θ_2 is contractible information. However, requiring truth-telling to be a dominant strategy when θ_2 is not contractible (and is replaced by x_2) has the advantage of ensuring that the agents will "tell the truth" when they play their "subgame."

To understand this point, observe that the multi-agent model can be viewed as a two-stage game. In the first stage, the principal is a *Stackleberg leader* who sets the terms of the second-stage game by offering a menu of contracts to each agent, where the menu for agent i is $\{z_{i\ell} = (x_{i\ell}, c_{i\ell\ell}, c_{i\ell h}),\ z_{ih} = (x_{ih}, c_{ih\ell}, c_{ihh})\}$ and, for example, $c_{1j\tau} = \mathbf{c}_1(x_{1j}, x_{2\tau})$. In the second-stage game, the agents make *simul-*

taneous moves in which each agent selects from the menu offered to him and then takes an action (that provides the outcome specified by his menu choice). Since compensation now depends on $x = (x_1, x_2)$, the outcome to agent 1 depends on both his own choice and the choice of agent 2. The principal must be careful in specifying the agents' simultaneous play game. There may be multiple equilibria in that game, and the equilibrium the principal would prefer may not be the equilibrium the agents will choose.

Each contract is essentially determined by the output that is produced, so we describe each player's strategy in terms of the outcome he chooses given the state he has observed. Each agent has four possible strategies, $\boldsymbol{\alpha}_i: \{\theta_{i\ell}, \theta_{ih}\} \to \{x_{i\ell}, x_{ih}\}$. Let $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2)$ denote the pair of strategies for the two agents. Agent 1's expected utility given strategy pair $\boldsymbol{\alpha}$ is

$$U_1^a(\boldsymbol{\alpha}, \theta_{1j}) = u_1(\boldsymbol{c}_1(\boldsymbol{\alpha}_1(\theta_{1j}), \boldsymbol{\alpha}_2(\theta_{2\ell})))\, \varphi(\theta_{2\ell}|\theta_{1j})$$

$$+ \; u_1(\boldsymbol{c}_1(\boldsymbol{\alpha}_1(\theta_{1j}), \boldsymbol{\alpha}_2(\theta_{2h})))\, \varphi(\theta_{2h}|\theta_{1j}).$$

Definition

(a) A pair of strategies $\boldsymbol{\alpha}$ are a *(Nash) equilibrium* if for agent 1 (and similarly for agent 2):

$$\boldsymbol{\alpha}_1(\theta_{1j}) \in \underset{x_1 \in \{x_{1\ell}, x_{1h}\}}{\text{argmax}} \; U_1^a(x_1, \boldsymbol{\alpha}_2, \theta_{1j}), \quad j = \ell, h.$$

(b) Equilibrium $\boldsymbol{\alpha}$ in the agents' subgame is *subgame undominated* if there does not exist another equilibrium $\hat{\boldsymbol{\alpha}}$ such that both agents weakly prefer their expected utilities for $\hat{\boldsymbol{\alpha}}$ than for $\boldsymbol{\alpha}$, given every θ_1 and θ_2, and there is at least one strict inequality.

(c) Equilibrium $\boldsymbol{\alpha}$ in the agents' subgame is *subgame dominated* if it is not subgame undominated.

A key result is that it is not sufficient to merely require truth-telling to be an optimal response given that the other agent is telling the truth.

Proposition 29.6 (DS84, Prop. 3)

Suppose the conditions in Proposition 29.5 hold, and consider the optimal incentive scheme in which truth-telling for each agent is constrained (only) to be an equilibrium response to truth-telling by the other agent (i.e., the solution to P2a). The resulting truth-telling equilibrium is *subgame dominated.* In particular, both agents prefer the equilibrium in which they *claim to have always observed* $\theta_{1\ell}$ and $\theta_{2\ell}$.

The proof is constructed by first applying the Kuhn-Tucker conditions to Problem P2a to characterize its optimal solution. It is then demonstrated that given this contract, it is optimal for agent 1 to always (i.e., for both $\theta_{1\ell}$ and θ_{1h}) choose $x_{1\ell}$ if he believes that agent 2 always chooses $x_{2\ell}$. The converse also applies, so that both are better off if they always produce the outcome associated with low productivity. Hence, a myopic focus on truth-telling will not suffice when one recognizes that the agents will rationally play their subgame, taking the actions which they prefer rather than those that the principal prefers.

DS84 provide insight into the optimal menu of truth-inducing contracts that avoid the subgame domination problem. The key here is to offer one agent a menu for which truth-telling is a dominant strategy, and then to offer the other agent a menu that is optimal given that the first agent's state is contractible information.

Proposition 29.7 (DS84, Prop. 4)

Among all incentive schemes which guarantee that the equilibrium in which both agents tell the truth is subgame undominated, the one preferred by the principal is the one in which one agent is induced to report truthfully as a dominant strategy (with the scheme described in Proposition 29.5) and the other agent is induced to report truthfully as an equilibrium response to truth-telling by the first agent (with the scheme described in the proof of Proposition 29.6).

An interesting aspect of this solution to the principal's problem is that even if two agents face identical problems, it is not optimal to offer them the same contract. One is given stronger incentives (more rents) to tell the truth so the information from his actions can be reliably used in contracting with the other agent (who will receive less rents). Of course, this is essentially the same result we obtained with the basic principal-agent model with multiple agents (see Section 29.2.4.

29.5 CONCLUDING REMARKS

An unstated assumption in the last result is that we (and DS84) have only considered what is called *direct mechanisms*. In particular, we have only considered mechanisms in which the message space is restricted to be the set of possible types. In single-agent settings in which the Revelation Principle applies, there always exists an optimal solution which induces truth-telling using a direct mechanism. However, in multi-agent settings there can be gains from expanding the message space, i.e., specifying a contract that depends on specified possible statements beyond what the agents have observed. These mechanisms are generally complex, sometimes involving infinite message spaces. Hence, we do not explore indirect mechanisms in detail, and merely refer the reader to papers that discuss these mechanisms, such as Ma (1988), Ma, Moore, and Turnbull (1988), Demski, Sappington, and Spiller (1988), and Glover (1994). Examples of indirect mechanisms are provided in the following chapter.

Limited-commitment contracting due to either decentralization or collusion in multi-agent settings is similar to inter-period renegotiation in a multi-period setting (see Chapter 28). The latter involves end-of-period contract renegotiation between the principal and the agent, whereas the former involves "second-stage" negotiation between the two agents. Exploration of the differences and similarities is potentially interesting. For example, in Chapter 28 we derived many of the results in terms of orthogonalized and normalized performance statistics which result in one direct and two types of indirect incentives. The use of a similar approach in the multi-agent setting could be interesting.

At the end of Chapter 28 we compared the results from hiring two agents (one for each period) versus one agent for both periods. In the two agent analysis we ignored the possibility of agent collusion. Is such collusion possible? If so, how does the timing sequence affect the results?

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CHAPTER 30

CONTRACTING WITH A PRODUCTIVE AGENT AND A MONITOR

The preceding chapter focuses on settings in which all agents are productive. In this chapter we consider some settings in which there is an agent who is not directly productive, but is hired by the principal to monitor a productive agent. The monitor can represent a supervisor, an internal auditor, or an external auditor. A supervisor and an internal auditor are employees of the principal's firm and, hence, their compensation can vary with performance measures in much the same way as the compensation of a productive agent. Institutional restrictions typically preclude paying external auditors performance contingent compensation. Instead, their incentives come from the threat of litigation and the resulting penalties, or from reputation effects.

In general, monitoring may pertain to verifying the content of reports issued by a privately informed productive agent (which is the classical role of an auditor) or to observing the activities and consequences of productive agents (which is the classical role of a supervisor). The simple models we consider can be given either interpretation. We refer to the productive agent as the "worker" and the non-productive agent as the "monitor."

In each model considered in this chapter, we assume the worker has pre-contract private information (as in Chapter 23 and Section 29.4). Consequently, he has the potential to earn "information rents" that are costly to the principal and result in the principal inducing less than efficient (i.e., first-best) worker effort. In Section 29.3, those rents and inefficiency are reduced by using relative performance measures for two productive agents. In this chapter, the worker's rents and inefficiency are reduced by using information provided by the monitor.

In Section 30.1, the model is similar to Demski and Sappington (1989) (DS89). In this model, the worker knows his "state", which affects both the cost of his effort and the probability of the outcome from his effort. The monitor expends costly effort to acquire information about what the worker knows. The principal offers the worker and monitor contracts that motivate them both to work and to induce the monitor to report truthfully. The subgame issues that arose in Chapter 29 with two productive agents also arise here and indirect mechanisms are again used to deal with those subgame issues. Of course, these mechanisms differ somewhat because they focus on inducing truthful reporting by the monitor.

In Section 30.2, the model is similar to Kofman and Lawarree (1993) (KL). In this model, the worker's information is perfect and the monitor's imperfect information is costless. The principal offers contracts to induce worker effort and to induce truthful reporting by the monitor. The monitor does not expend costly effort, so the subgame issues that arise in Section 30.1 do not occur here. However, in this model, we assume the worker and monitor can collude. In particular, the worker can bribe the monitor to lie and issue reports that avoid the imposition of penalties on the worker. We identify conditions under which the ability to collude (a) destroys the value of a collusive monitor (relative to an exogenously truthful monitor), (b) partially reduces that value, and (c) has no impact on the monitor's value.

Finally, in Section 30.4.2 we extend the prior analysis by considering the use of a costly, truthful external monitor to partially counter the negative effects of collusion between the worker and a costless internal monitor.

30.1 CONTRACTING WITH AN INFORMED WORKER AND A COSTLY MONITOR

As in DS89, the model in this section focuses on the subgame issues that arise in a setting in which a productive worker expends effort to increase the principal's payoff and the monitor expends effort to obtain information about the worker's pre-contract information.

30.1.1 The Basic Worker Model

A risk neutral principal owns a technology that will produce one of two possible outcomes, $x_g > x_b$, at date 1.[1] The probability of generating the good outcome is an increasing function of the worker's action $a_w \in A_w = [0,1]$.

The Worker

The worker is risk neutral with respect to the compensation c_w he receives from the principal, minus a cost κ_w that he incurs in providing action a_w, i.e.,

$$u_w(c_w,\kappa_w) = c_w - \kappa_w.$$

[1] DS89 develop the outcome in their model in terms of a direct cost incurred by the principal. To maintain coherence with the analyses in Section 29.3, we represent their model using outcome x.

The worker's effort cost, represented by $\kappa_w(a_w,\theta)$, varies with both his effort level a_w and his "state" $\theta \in \Theta$. For each state, the worker's cost is increasing and convex with respect to his effort, i.e., $\kappa_{wa}(a_w,\theta) \equiv \partial\kappa_w(a_w,\theta)/\partial a_w > 0$ and $\kappa_{waa}(a_w,\theta) > 0$.

At date 0 (the contract date) the state is known to the worker, but not to the principal. More specifically, at date 0, the principal believes the worker's state can be one of N possible values, i.e., $\Theta = \{\theta_1, ..., \theta_N\}$, and assigns probability p_j to state θ_j.

In addition to influencing the worker's effort cost, his state influences his belief with respect to the likelihood of generating the good outcome given each effort level. The conditional probability that the good outcome x_g will occur given the worker's action and state $\theta \in \Theta$ is denoted $\varphi(a_w,\theta)$. DS89 assume that $\varphi(a_w,\theta)$ and $\varphi_a(a_w,\theta) \equiv \partial\varphi(a_w,\theta)/\partial a_w$ are both positive and increasing in θ, and $\varphi_{aa}(a_w,\theta) < 0$. That is, more effort increases the probability of the good outcome, but at a decreasing rate.

Furthermore, both the first and second derivatives of the agent's cost function with respect to his action are smaller for higher numbered states. Hence, the form of φ and κ_w are such that higher θ connotes higher productivity, in the sense that the outcome lottery is more favorable and the worker's direct cost is lower. The worker's reservation utility, denoted $\bar{U}_w$, is assumed to be independent of the state (and, thus, we may assume it is equal to zero without loss of generality).

The First-best Worker Contract

The principal is assumed to be risk neutral with respect to his net payoff, which equals his gross payoff minus the compensation he pays to his agents. In our basic model, the worker is the only agent, so that the principal's net payoff is

$$\pi = x - c_w.$$

In the first-best setting, the principal can contract on the agent's action and the state that is known to the agent when he takes that action. There is no need to have the compensation vary with the outcome. Hence, the first-best action and the first-best compensation can be represented as functions of the state.

Let a_j^* and c_{wj}^* represent the optimal action and compensation given that θ_j is observed, and let

$$\pi_j^* \equiv x_g\varphi(a_j^*,\theta_j) + x_b[1 - \varphi(a_j^*,\theta_j)] - \kappa_w(a_j^*,\theta_j) - \bar{U}_w, \quad j = 1, ..., N,$$

i.e., the first-best expected net payoff to the principal for θ_j (given that the agent is paid for the cost of his effort and his reservation wage).

Proposition 30.1

Under the assumed conditions, the first-best contract is characterized as follows (for $j = 1, ..., N$).

(a) Efficient production: $[x_g - x_b]\varphi_a(a_j^*, \theta_j) = \kappa_{wa}(a_j^*, \theta_j)$.

(b) No rents:[2] $\mathrm{E}[c_w^* | a_j^*, \theta_j] = \bar{U}_w + \kappa_w(a_j^*, \theta_j)$.

(c) Increasing effort: $a_1^* < a_2^* < ... < a_N^*$.

(d) Increasing payoffs: $\pi_1^* < \pi_2^* < ... < \pi_N^*$.

In this setting the agent does not earn any information rents and the optimal action is *efficient*, i.e., it maximizes the principal's net payoff.

At the time of contracting (date 0), the principal's expected net payoff is

$$\Pi^* = \sum_{j=1}^{N} \pi_i^* p_j.$$

Second-best Worker Contract

Now assume a_w and θ are not contractible, but the worker's compensation can be contingent on the gross outcome x (which is contractible). Initially, we ignore the possibility of communication by the worker, and let c_{wg} and c_{wb} represent the worker's compensation for the good and bad outcomes, respectively, and let a_j represent the induced action if the agent has observed θ_j. To choose the optimal compensation contract the principal solves the following "second-best worker" problem.

Principal's SBW Problem:

$$\Pi^\dagger = \underset{\{a_w, c_w\}}{\text{maximize}} \quad \sum_{j=1}^{N} p_j \{ [x_g - c_{wjg}]\varphi(a_j, \theta_j) + [x_b - c_{wjb}][1 - \varphi(a_j, \theta_j)] \},$$

$$\text{subject to} \quad \mathrm{E}[c_w | a_j, \theta_j] - \kappa_w(a_j, \theta_j) \geq \bar{U}_w, \qquad \text{all } j,$$

[2] This condition is expressed in terms of the expected compensation:

$$\mathrm{E}[c_w^* | a_j^*, \theta_j] = c_{wjg}^* \varphi(a_j^*, \theta_j) + c_{wjb}^* [1 - \varphi(a_j^*, \theta_j)].$$

Due to the worker's risk neutrality, there are an infinite number of pairs (c_{wjg}^*, c_{wjb}^*) that will satisfy this condition.

$$a_j \in \underset{a_w \in A}{\text{argmax}}\ \mathrm{E}[c_w|a_w,\theta_j] - \kappa_w(a_w,\theta_j), \quad \text{all } j,$$

where $$\mathrm{E}[c_w|a_j,\theta_j] \equiv c_{wjg}\,\varphi(a_j,\theta_j) + c_{wjb}[1 - \varphi(a_j,\theta_j)].$$

Three important characteristics of the solution to the above problem are provided below.

Proposition 30.2

Under the assumed conditions, the second-best contract is characterized as follows (for $j = 1, ..., N$).

(a) Inefficiency: $a_j^\dagger \leq a_j^*$, with strict inequality for $j < N$.

(b) Rents: $\mathrm{E}[c^\dagger|a_j^\dagger,\theta_j] \geq \bar{U}_w + \kappa_w(a_j^\dagger,\theta_j)$, with strict inequality for $j > 1$.

(c) Reduced expected payoff: $\Pi^\dagger < \Pi^*$.

These are standard results in agency models with pre-contract information (e.g., see Chapter 23).

Worker Reports the State

The worker knows the state θ at the time of contracting. He could issue a message m_w at the time of contracting and the principal could offer the worker a compensation contract in which the payoff contingent compensation varies with the worker's message. If the message is unverified, then Proposition 23.5 implies that the worker's message will have no value,[3] i.e., it does not affect his compensation and the principal's expected payoff equals $\Pi^\dagger$.

On the other hand, if the principal can costlessly verify the worker's message, then the first-best result can be obtained, i.e., the principal's expected payoff equals Π^*. This is achieved by setting the worker's compensation for the good and bad outcomes, given θ_j, equal to $c_{wgj}^* = x_g - \pi_j^* + \bar{U}_w$ and $c_{wbj}^* = x_b - \pi_j^* + \bar{U}_w$ (i.e., essentially sell the firm to the worker at the price π_j^*).

30.1.2 Contracting with a Worker and a Fully Informable Monitor

We now consider the possibility that a second employee, called the monitor, observes the state and makes a report to the principal regarding his observation.

[3] With only two outcomes, and the assumptions made with regard to φ, we have a setting in which the spanning condition in Chapter 23 is satisfied (see Section 23.3).

Initially, we assume the monitor does not incur any cost in observing the state, and then we assume he only observes the state if he incurs a monitoring cost. The monitor is risk neutral and there is a lower bound $\underline{c}_m$ on the compensation he can receive.

Costless Information Acquisition

To provide insight into the fundamental role of a monitor, we consider a setting in which the monitor is hired for other purposes (so that his reservation utility $\overline{U}_m$ equals zero) and observes θ as a costless by-product of his other work. The principal can offer a contract to the monitor that does not vary with the message he sends, but does require him to report what he observes. The monitor has no incentive to lie. If, as is common in the agency literature, we assume that the monitor acts in the best interests of the principal unless he has personal incentives to do otherwise, then the monitor will report truthfully. The principal can then offer a contract to the worker in which the worker's compensation varies with the outcome and the state reported by the monitor. The result is equivalent to having a verified report of the state and, hence, the first-best result can be achieved.

Costly Information Acquisition

We now consider a model in which the monitor can observe the state, but only if he incurs a personal cost κ_m. In this setting, DS89 refer to the monitor (called the "boss") and the worker as being "informationally balanced."

Assume that the worker and monitor simultaneously issue messages to the principal giving the state they claim to have observed. Their compensation contracts can be represented by c_{wijk} and c_{mijk} if the payoff is x_i, $i = g, b$, the worker reports $m_w = \theta_j$, and the monitor reports $m_m = \theta_k$.

Observe that the worker's first-best action and truthful reporting by both the worker and the monitor are a Nash equilibrium in the worker and monitor subgame if they receive the following compensation (again assuming $\overline{U}_m = 0$):

$$c_{wijk} = \begin{cases} x_i - \pi_j^* + \overline{U}_w, & \text{if } j = k, \\ \underline{c}_w, & \text{if } j \neq k. \end{cases}$$

$$c_{mijk} = \begin{cases} \kappa_m, & \text{if } j = k, \\ \underline{c}_m, & \text{if } j \neq k. \end{cases}$$

In these contracts, the worker's and monitor's minimum compensation levels are less than their reservation utility, i.e., $\underline{c}_w < \overline{U}_w$ and $\underline{c}_m < 0$. Payment of these minimum levels penalizes the two agents if their reports do not match. The penalties are never imposed, and the result is the same as for costless verification of the worker's report, except that a cost κ_m is incurred.

Unfortunately, the above contract induces many subgame Nash equilibria for which the worker and monitor are better off than in the equilibrium that yields the principal's first-best result (minus κ_m). The choice most beneficial to the worker is for both the worker and the monitor to report that the state is θ_1, for the worker to take the first-best action a_j^* if he has observed θ_j, and for the monitor not to work. The worker benefits from only "paying" the principal $\pi_1^* - \underline{U}_w$ for "ownership" of the payoff, instead of $\pi_j^* - \underline{U}_w$. The monitor, on the other hand, benefits from being paid $\kappa_m + \underline{U}_m$ even though he has not incurred the cost κ_m.

An Indirect Mechanism

DS89 provide what is often called an "indirect mechanism" for dealing with the subgame problem described above. The key here is to deter the monitor from shirking (i.e., not incurring the cost κ_m) by inducing the informed worker to "blow the whistle" if he knows the monitor is lying. DS89 accomplish this by having the principal offer a contract which specifies the following sequence of events and payoffs.

1. The monitor announces his observation of θ – let θ_k be the state he reports.

2. The principal gives the worker (who has observed θ_j) the option of:

   (a) accept – the monitor receives $c_m^* = \kappa_m$ and the worker receives $c_{wik}^* = x_i - \pi_k^* + \underline{U}_w$;

   (b) reject – the monitor receives $\underline{c}_m$ and the worker receives $\overline{U}_w$ from working elsewhere;

   (c) counterpropose – the monitor receives $\underline{c}_m$ and the worker receives $c_{wik} = x_i - \delta_k + \varepsilon_{ik} - \pi_k^* + \underline{U}_w$, if payoff x_i occurs.

The worker will accept if $j = k$. The counterproposal involves a "side-bet" between the worker and the principal with respect to the outcome given the monitor's reported state. In this side-bet, the worker makes an up front payment δ_k and receives ε_{ik} if outcome x_i occurs. The parameters δ_k and ε_{ik} are set such that the counterproposal is strictly preferred if, and only if, the worker knows $\theta_j > \theta_k$, and quits if $\theta_j < \theta_k$.

The preceding mechanism permits implementation of the first-best solution (at a cost κ_m) as a unique equilibrium in the worker-monitor subgame. Rejection and counterproposals are never observed in this game. They are off-equilibrium strategies. The monitor is motivated to acquire information and report truthfully because he faces the threat that the worker will "blow the whistle" on him.

If the monitor is not hired, then the principal's expected payoff is $\Pi^\dagger$, whereas it increases to Π^* if the monitor is hired. Of course, it will not be optimal to hire the monitor unless

$$\Pi^* - \Pi^\dagger > \kappa_m.$$

That is, the benefit must exceed the cost.

30.1.3 Contracting with a Worker and a Partially Informable Monitor

The preceding analysis assumes the monitor can observe the state. Now consider a setting in which the monitor can only be partially informed about the state.

Costless Partial Information Acquisition

To understand the potential role of a partially informed monitor we consider a setting in which the monitor observes a signal $y_h \in Y$, where Y partitions Θ such that if $\theta_j \in y_h$ and $\theta_k \in y_\ell$, then $j > k$ if $h > \ell$. That is, the monitor's signals have the same ordering as the states but he has less detailed information than the worker. For example, DS89 consider an example in which $N = 4$ and the set of possible signals the monitor can acquire is $Y = \{y_L = \{\theta_1, \theta_2\}, y_H = \{\theta_3, \theta_4\}\}$. That is, the monitor observes whether the worker has observed one of the two "low" states or one of the two "high" states.

In the following discussion it is useful to let $J = \{1, ..., N\}$ represent the set of possible states and to let $J_h = \{j_h^-, j_h^- + 1, ..., j_h^+\}$, where $\theta_{j_h^-}$ and $\theta_{j_h^-}$ are the worst and best possible states in y_h.

If the monitor's information is costless, then he has no incentive not to acquire the information and no incentive to lie about what he observed. In that case, if there are M possible signals the monitor may receive, then the principal can be viewed as solving M separate SBW problems in which J_h is the set of possible states for each problem $h \in \{1, ..., M\}$. In problem SBW_h, the prior probability of state j is replaced by the monitor's posterior probability

$$p_j(y_h) = \begin{cases} p_j / P_h, & \text{if } j \in J_h, \\ 0, & \text{if } j \notin J_h, \end{cases}$$

and

$$P_h = \sum_{j \in J_h} p_j.$$

The solutions to the principal's *MSBW* problems (which are identified with the superscript ‡) are characterized by the following proposition.

Proposition 30.3

Under the assumed conditions, the second-best contracts, given the monitor's signals are truthfully reported, have the following characteristics (for $j = 1, ..., N$ and $h = 1, ..., M$) relative to the *SBW* problem with no monitor and to first-best.

(a) Less inefficiency: $a_j^{\dagger} \leq a_j^{\ddagger} \leq a_j^{*}$,

with equality for $j = N$ and $a_j^{\ddagger} = a_j^{*}$ for $j = j_h^{+}$, $h = 1, ..., M$.

(b) Lower rents: $\mathrm{E}[c_w^{\dagger}|a_j^{\dagger},\theta_j] \geq \mathrm{E}[c_w^{\ddagger}|a_j^{\ddagger},\theta_j] \geq \bar{U}_w + \kappa_w(a_j^{\ddagger},\theta_j)$,

with equality for $j = 1$ and $\mathrm{E}[c_w^{\ddagger}|a_j^{\ddagger},\theta_j] = \bar{U}_w + \kappa_w(a_j^{\ddagger},\theta_j)$, for $j = 1, j_2^{-}, ..., j_M^{-}$.

(c) Increased expected payoff: $\Pi^{\dagger} < \Pi^{\ddagger} < \Pi^{*}$.

The key here is that the principal's *SBW* problem is now divided into *M* smaller *SBW* problems. The induced production is efficient for the best state in each of the smaller problems, and no rents are paid for the worst state in each of the smaller problems. In addition, the inefficiencies and rents are reduced for each intermediate state within a smaller problem.

Costly Partial Information Acquisition

The setting in Section 30.1.2, in which the monitor is fully informable, can be viewed as a special case of the partially informable monitor. The key difference is that if the monitor is not fully informable, then the first-best result cannot be achieved even though the fully informed worker is optimally induced to truthfully report the state he has observed. The best that can be achieved with the partially informed monitor is the optimal result that is obtained with exogenously supplied partial information (i.e., the results characterized in Proposition 30.3) minus the monitor's information cost κ_m.

Truthful reporting by the worker and monitor, with implementation of $a_j^{\ddagger}$ if the worker has observed θ_j, is a Nash equilibrium in the worker-monitor subgame if they report simultaneously and are penalized if their reports are inconsistent. That is, the worker and monitor receive $\underline{c}_w$ and $\underline{c}_m$ if $m_w = \theta_j \notin m_m = y_h$. On the other hand, if they are consistent, the monitor is compensated for his costs (i.e., $c_m = \kappa_m$) and the agent receives payoff contingent compensation $c_{wij}^{\ddagger}$.

However, we again have a setting in which there are multiple Nash equilibria in the worker-monitor subgame. The monitor will prefer to not acquire any information (thus avoiding the cost κ_m) and report y_1, while the worker reports $\theta_1 \in y_1$. That is, both the worker and the monitor will claim they have received the worse possible news.

An Indirect Mechanism

In an approach similar to the approach in the fully informable monitor case, truthful reporting by the worker and the monitor can be induced by using a "whistle blowing" mechanism such as the following.

1. The monitor announces his observation of y_h.

2. The principal gives the worker (who has observed θ_j) the option of:

   (a) accept – the monitor receives $c_m^\ddagger = \kappa_m$ and the worker receives $c_{wih}^\ddagger$ if payoff x_i is realized;

   (b) reject – the monitor receives $\underline{c}_m$ and the worker receives $\bar{U}_w$ (from working elsewhere);

   (c) counterpropose – the monitor receives $\underline{c}_m$ and the worker receives $c_{wih} = c_{wih}^\ddagger - \delta_h + \varepsilon_{ijh}$ if payoff x_i occurs.

The worker accepts if $\theta_j \in y_h$, quits if $\theta_j < \theta_{j_h}$, and counterposes if $\theta_j > \theta_{j_h}$.

30.2 CONTRACTING WITH A PRODUCTIVE AGENT AND A COLLUSIVE MONITOR

Coordination and collusion by agents are always potential problems in multi-agent settings. In the models examined in the preceding section we considered the use of indirect ("whistle blowing") mechanisms to avoid coordinated actions that would implement Nash equilibria in the agents' subgame that differ from the Nash equilibrium preferred by the principal. In that section, as in Chapter 29, we implicitly assume that the agents cannot collude. For example, the worker cannot bribe the monitor to lie. In this section we refer to the monitor as collusive if collusion between the worker and the monitor is possible. Collusion does not take place in these settings since we assume the principal offers a collusion-proof contract. However, collusiveness is costly since the principal's expected payoff from a collusion-proof contract is less than for a contract with an exogenously truthful monitor.

We consider two types of collusion-proof contracts. The first involves rewards for "whistle blowing" and the second involves the use of penalties based on information provided by a costly, truthful external monitor. However, the reward mechanism in this section differs from the indirect mechanism considered in Section 30.1.

Interestingly, there are conditions under which the reward mechanism is ineffective, partially effective, and fully effective. If it is ineffective and an external monitor is too costly to use, then the collusive monitor has no value, i.e., the worker's contract is the same as his contract with no monitor. If the reward mechanism is fully effective, the worker's contract is the same as his contract when the costless monitor is exogenously truthful and the monitor's contract has a net expected cost of zero. Key factors affecting the effectiveness of the reward mechanism are the set of feasible lies the monitor can tell, the existence of "type II errors" in the monitor's information system, and the restrictiveness of the monitor's limited liability.

Section 30.2.1 describes the basic model and examines the no monitor and perfect monitor benchmark settings. This is followed in Section 30.2.2 with the benchmark case in which the costless monitor is exogenously motivated not to collude and, therefore, to always report truthfully. Collusion is introduced in Section 30.2.3 and the reward mechanism is used to provide a collusion-proof contract. Finally, the use of a costly, truthful external monitor is examined in Section 30.2.4.

30.2.1 The Basic Model

As in Section 30.1, the principal contracts with a worker and a monitor.[4] They are all risk neutral, the worker has *pre-contract information*, and the contracts with the worker and monitor are constrained by *limited liability* (i.e., lower bounds of $\underline{c}_w$ and $\underline{c}_m$ on their compensation). Unlike the model in Section 30.1, the worker has perfect information about the output from his productive action and the monitor's information is costless.

The Worker

The worker can produce any outcome $x \geq 0$ at a personal cost $\kappa_w(x,\theta_j) = \frac{1}{2}(x - \theta_j)^2$ if $x \geq \theta_j$ and zero otherwise, where $\theta_j \in \{\theta_1, \theta_2\}$ is a binary state known to the worker prior to contracting. That is, the worker chooses the output to produce and incurs a personal cost if $x \geq \theta_j$, but can costlessly dispose of excess output if $\theta_j > x$.

The principal's prior probability that the agent has observed state θ_1 is represented by p. This is the low productivity state, i.e., $\theta_1 < \theta_2$, and the differ-

[4] Our model has a number of components that are similar to components of the model examined by Kofman and Lawarree (1993) (KL).

ence in productivity in the two states is represented by $\Delta = \theta_2 - \theta_1$. We assume $\Delta < 1$.

The worker's compensation and personal cost are represented by c_w and κ_w, respectively. He is risk neutral, so that his utility is $u_w(c_w,\kappa_w) = c_w - \kappa_w$. The worker's compensation is scaled so that his reservation utility is $\underline{U}_w = 0$ and $\underline{c}_w < 0$ is the minimum he can be paid.[5]

If a monitor is not hired, then the principal's net payoff is $\pi = x - c_w$.

A Perfect Monitor

We now consider two benchmark cases, which we refer to as the perfect and no monitor cases. In the perfect monitor case, the information system, denoted η^*, costlessly reveals the state and the monitor truthfully reports this information to the principal.

The monitor's report of θ_j combined with the outcome x (which is assumed to be contractible) permits implementation of the first-best contract. The contract consists of outcome/state contingent payments $\mathbf{c}_w^* = (c_{w1}^*, c_{w2}^*)$ and outcomes $\mathbf{x}^* = (x_1^*, x_2^*)$, where x_j^* is the output to be produced if the worker's productivity parameter is θ_j. The worker receives $\underline{c}_w$ if his output is not consistent with the state.

With a perfect monitor, the effort levels are *efficient*, and the worker receives *no information rent*, i.e.,

$$x_j^* = 1 + \theta_j, \text{ and } c_{wj}^* = \kappa_w(x_j^*,\theta_j) = ½, \quad j = 1,2.$$

Hence, the principal's first-best expected payoff is

$$\Pi(\eta^*) = p[x_1^* - c_1^*] + (1-p)[x_2^* - c_2^*] = ½ + \theta_2 - p\Delta.$$

No Monitor

In the no monitor case, the information system, denoted η^o, is uninfomative about the state, so that the monitor has nothing to report. In this setting, the principal contracts with the worker strictly on the basis of the outcome. Since the worker has perfect information about the outcome that will be generated by his action, the optimal contract specifies an outcome for each state and the compensation that will be paid for each outcome. Let x_j denote the outcome to be produced if the worker has observed θ_j and let c_{wj} denote the compensation paid to the worker if he produces x_j. He is paid $\underline{c}_w$ if he produces any other output. The principal's expected payoff is

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[5] The minimum compensation is measured relative to the reservation wage, which is normalized to zero (e.g., a constant has been deducted). Hence, $\underline{c}_w < 0$ does not mean the minimum wage is literally negative. Instead, it measures the minimum wage relative to the worker's reservation wage.

$$\Pi(\eta^o) = p[x_1^o - c_{w1}^o] + (1-p)[x_2^o - c_{w2}^o].$$

To be acceptable to the worker in both states the contract must be such that

$$c_{wj} \geq \kappa_w(x_j,\theta_j) = \begin{cases} \tfrac{1}{2}(x_j - \theta_j)^2, \text{ if } x_j \geq \theta_j, \\ 0, \qquad\qquad \text{otherwise,} \end{cases} \qquad j = 1,2.$$

And to induce the desired output in each state (which is equivalent to truthful reporting of the state by the worker), the contract must be such that

$$c_{w1} - \kappa_w(x_1,\theta_1) \geq c_{w2} - \kappa_w(x_2,\theta_1),$$

and

$$c_{w2} - \kappa_w(x_2,\theta_2) \geq c_{w1} - \kappa_w(x_1,\theta_2).$$

As we have seen in other models with pre-contract information (see Chapter 23), the optimal contract $(\mathbf{c}^o,\mathbf{x}^o,\mathbf{a}^o)$ has the following characteristics.

(a) In the low productivity state, the outcome is less than efficient and the worker is compensated for his effort:

$$x_1^o < x_1^* = 1 + \theta_1, \text{ and } c_{w1}^o = \kappa_w(x_1^o,\theta_1) < \tfrac{1}{2}.$$

(b) In the high productivity state, the outcome is efficient and the worker receives positive information rent:

$$x_2^o = x_2^* = 1 + \theta_2, \quad c_{w2}^o = \tfrac{1}{2} + [\kappa_w(x_1^o,\theta_1) - \kappa_w(x_1^o,\theta_2)].$$

The worker's net payoff (i.e., information rent) if he observes θ_2 and produces x_2^o is $c_{w2}^o - \kappa_w(x_2^o,\theta_2) = [\kappa_w(x_1^o,\theta_1) - \kappa_w(x_1^o,\theta_2)]$, which is the difference in the cost of providing x_1^o if the agent has observed θ_2 instead of θ_1. The principal's expected payoff is

$$\Pi(\eta^o) = p\{x_1^o - \kappa_w(x_1^o,\theta_1)\} + (1-p)\{\tfrac{1}{2} + \theta_2 - [\kappa_w(x_1^o,\theta_1) - \kappa_w(x_1^o,\theta_2)]\}. \quad (30.1)$$

Differentiating with respect to x_1 provides the first-order condition

$$x_1^o = \begin{cases} \theta_1 + p + (1-p)(1 - \Delta/p), & \text{if } p > \Delta, \\ \theta_1 + p, & \text{if } p < \Delta. \end{cases} \quad (30.2)$$

Hence, x_1^o is an increasing, continuous function of p, with $\partial x_1/\partial p = \Delta/p^2$ if $p > \Delta$, and $\partial x_1/\partial p = 1$ if $p < \Delta$.

30.2.2 A Costless, Truthful, Imperfect Monitor

In this section we consider a setting in which the monitor's information system η is costless and imperfectly informative about the state. The system generates one of two signals, y_ℓ or y_h, and is characterized by $\eta = (q_{1\ell}, q_{1h}, q_{2\ell}, q_{2h})$, where q_{jk} is the conditional probability that the monitor observes signal y_k given that the worker has observed θ_j, for $j = 1, 2$ and $k = \ell, h$. We assume the signals are labeled such that $q_{1\ell} \geq q_{1h} = 1 - q_{1\ell}$, $q_{2h} \geq q_{2\ell} = 1 - q_{2h}$, and $q_{2h} > q_{1h}$.

The worker's contract specifies two possible output levels, $\mathbf{x} = (x_1, x_2)$, and induces him to produce x_j if he has observed θ_j, $j = 1, 2$. As in Kofman and Lawarree (1993) (KL), the principal only asks the monitor to obtain and report his information if the worker chooses the low-productivity output x_1.[6] Hence, the compensation contract can be represented by $\mathbf{c} = (c_{w1}, c_{w2}, \zeta_w)$, where c_{wj} is the base pay given production of output x_j, $j = 1, 2$, and ζ_w is a penalty that is imposed on the worker if he produces x_1 and the monitor reports y_h. In this latter case, the worker's net compensation is $c_{w1} - \zeta_w$, and we assume that this amount cannot be less than $\underline{c}_w$.[7]

Characterizing the Optimal Contract

In this setting the monitor is already employed by the firm, he incurs no cost in obtaining his information, and he requires no motivation to report his information truthfully. The principal must choose the output the worker is to produce given the state observed, and then he must choose a compensation contract that induces the worker to implement the desired production plan. The compensation paid to the worker will consist of his base pay given the output produced plus the expected penalty, which will only occur if the monitor's information system makes a type II error (i.e., the principal incorrectly rejects the worker's claim that his output is low because he has a bad state).

Let $\bar{c}_{1j}$ denote the worker's expected compensation if his productivity is θ_j and he produces output x_1, so that

$$\bar{c}_{11} = c_{w1} - q_{1h}\zeta_w, \text{ and } \bar{c}_{12} = c_{w1} - q_{2h}\zeta_w.$$

The probability of the high report y_h is more likely with the high state θ_2, i.e., $q_{2h} > q_{1h}$. Therefore, $\bar{c}_{11} > \bar{c}_{12}$.

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[6] We could have the monitor always report his information, but in this model there is no benefit to issuing the report if the worker produces x_2.

[7] Our formulation differs slightly from the KL model. They assume there is a maximum penalty ζ_w^{max} that can be imposed, whereas we assume that there is a minimum level of compensation $\underline{c}_w$ that can be paid.

Principal's Problem with a Costless, Truthful, Imperfect Monitor:

$$\underset{x_1, x_2, c_{w1}, c_{w2}, \zeta_w}{\text{maximize}} \quad p[x_1 - \bar{c}_{11}] + (1 - p)[x_2 - c_{w2}], \tag{30.3}$$

subject to (a) $\bar{c}_{11} - \kappa_w(x_1, \theta_1) \geq 0,$

(b) $c_{w2} - \kappa_w(x_2, \theta_2) \geq 0,$

(c) $\bar{c}_{11} - \kappa_w(x_1, \theta_1) \geq c_{w2} - \kappa_w(x_2, \theta_1),$

(d) $c_{w2} - \kappa_w(x_2, \theta_2) \geq \bar{c}_{12} - \kappa_w(x_1, \theta_2),$

$c_{w1} - \zeta_w \geq \underline{c}_w,\ c_{w2} \geq \underline{c}_w,\ \zeta_w \geq 0.$

Constraints (a) and (b) ensure that the worker will accept the contract, whether he has observed θ_1 or θ_2. Constraints (c) and (d) ensure that the worker will produce x_j, if he has observed θ_j, $j = 1, 2$. The final constraints recognize the worker's limited liability.

The optimal production plan again involves producing the first-best output for the high productivity state, and setting the worker's compensation in the low productivity state equal to the cost of the effort expended (plus the expected cost of the type II errors), i.e., $x_2^\dagger = x_2^*$ and $c_{w1}^\dagger = \kappa_w(x_1^\dagger, \theta_1) + q_{1h}\zeta_w$. On the other hand, the output in the low productivity state may be different from the first-best output and the information rent can be equal to or greater than zero.

The Use of Penalties to Reduce Information Rents

The principal wants to induce the first-best output $x_2^* = \theta_2 + 1$ in the high productivity state, for which the agent's cost is $\kappa_w(x_2^*, \theta_2) = ½$. If he seeks to induce output x_1 in the low productivity state, then the principal must compensate the worker for his expected cost $\kappa_w(x_1, \theta_1) + q_{1h}\zeta_w$, and he must ensure that the worker will not under-produce in the high productivity state (i.e., produce x_1 after observing θ_2). If there is no monitor, then the latter is accomplished by paying information rent, i.e., $c_{w2} \geq \kappa_w(x_2^*, \theta_2) + rent$, where

$$rent = c_{w1} - \kappa_w(x_1, \theta_2) = \kappa_w(x_1, \theta_1) - \kappa_w(x_1, \theta_2).$$

With the costless, truthful monitor, the information rent for θ_2 is reduced by the agent's expected penalty, i.e., $q_{2h}\zeta_w$, increased by the expected cost of the type II errors when he has observed θ_1, i.e., $q_{1h}\zeta_w$, but recall that $q_{2h} > q_{1h}$. Hence, the information rent to ensure that the worker will not under-produce in the high productivity state is

$$rent = c_{w1} - \kappa_w(x_1,\theta_2) = \kappa_w(x_1,\theta_1) + q_{1h}\zeta_w - q_{2h}\zeta_w - \kappa_w(x_1,\theta_2).$$

In fact, if the rent is non-positive, i.e.,

$$q_{2h}\zeta_w \geq \kappa_w(x_1,\theta_1) + q_{1h}\zeta_w - \kappa_w(x_1,\theta_2), \tag{30.4}$$

then (x_1,x_2^*) can be induced without paying information rent, i.e., $c_{w2} = \kappa_w(x_2^*,\theta_2)$. Clearly, there is no reason to impose larger penalties than necessary. Hence, setting (30.4) to be an equality and solving for ζ_w yields

$$\zeta_w = [\kappa_w(x_1,\theta_1) - \kappa_w(x_1,\theta_2)]/[q_{2h} - q_{1h}]. \tag{30.5}$$

Substituting (30.5) into (30.3) implies

$$\begin{aligned} c_{w1} &= \kappa_w(x_1,\theta_1) + q_{1h}[\kappa_w(x_1,\theta_1) - \kappa_w(x_1,\theta_2)]/[q_{2h} - q_{1h}] \\ &= [q_{2h}\kappa_w(x_1,\theta_1) - q_{1h}\kappa_w(x_1,\theta_2)]/[q_{2h} - q_{1h}], \end{aligned} \tag{30.6}$$

and

$$c_{w1} - \zeta_w = -[q_{2\ell}\kappa_w(x_1,\theta_1) - q_{1\ell}\kappa_w(x_1,\theta_2)]/[q_{2h} - q_{1h}]. \tag{30.7}$$

The latter is restricted to be greater than or equal to $\underline{c}_w$. Hence, x_1 and x_2^* can be induced with zero information rent if, and only if,

$$-[q_{2\ell}\kappa_w(x_1,\theta_1) - q_{1\ell}\kappa_w(x_1,\theta_2)]/[q_{2h} - q_{1h}] \geq \underline{c}_w. \tag{30.8}$$

Proposition 30.4

Assume the contract is based on the outcome and a truthful *ex post* report from system η. Output (x_1,x_2^*) can be induced with zero information rent if, and only if, (30.8) is satisfied. Furthermore, there exists a threshold minimum compensation level

$$\underline{c}_w' = -\tfrac{1}{2}[q_{2\ell} - q_{1\ell}(1-\Delta)^2]/[q_{2h} - q_{1h}], \tag{30.9}$$

such that the first-best result can be obtained if, and only if, $\underline{c}_w \leq \underline{c}_w'$.

If η is such that (x_1^*,x_2^*) cannot be induced without paying information rent, then there exist threshold output levels $x_1'(\eta) \in (\theta_1,\theta_2)$ and $x_1''(\eta) > \theta_2$ such that (x_1,x_2^*) can be induced without paying information rent if, and only if, $x_1 \leq x_1'(\eta)$ or $x_1 \geq x_1''(\eta)$. These thresholds reflect two options that could potentially be used to deter the high productivity worker from choosing the low output without paying information rent. First, if the low output $x_1 < \theta_2$, then $\kappa_w(x_1,\theta_2) = \tfrac{1}{2}[\max\{0,x_1 - \theta_2\}]^2 = 0$ and, hence, the deterrence is based solely on the expected penalty $q_{2h}\zeta_w$. In that case, the threshold $x_1'(\eta)$ is determined by (see (30.8)):

$$q_{2\ell}\kappa_w(x_1',\theta_1) = -\underline{c}_w[q_{2h} - q_{1h}]. \tag{30.10a}$$

Secondly, if the low output $x_1 > \theta_2$, then $\kappa_w(x_1,\theta_2) > 0$, and the deterrence is based both on the expected penalty $q_{2h}\zeta_w$ and a positive effort cost of producing the low output, i.e., the threshold $x_1''(\eta)$ is determined implicitly by the following condition:[8]

$$q_{2\ell}\kappa_w(x_1'',\theta_1) - q_{1\ell}\kappa_w(x_1'',\theta_2) = -\underline{c}_w[q_{2h} - q_{1h}]. \tag{30.10b}$$

It may not be optimal for the principal to choose the low output such that the information rent is zero – information rents for the high productivity worker must be compared to output inefficiencies for the low productivity worker. Of course, if $x_1^* \le x_1'(\eta)$ or $x_1^* \ge x_1''$, then first-best is obtained. However, if $x_1^* \in (x_1'(\eta),x_1''(\eta))$, it may be optimal to induce $x_1 \in (x_1'(\eta),x_1''(\eta))$, and in this case there are both information rents and output inefficiencies.

The Impact of Information Quality on the Principal's Expected Payoff

To explore the impact of information quality, we consider two special cases in which the information quality is represented by a single parameter q.

(a) *Asymmetric system (no type II errors)*: The first system (η^a) is characterized by $q_{1\ell} = 1$, $q_{1h} = 0$, $q_{2\ell} = 1 - q$, and $q_{2h} = q$, with $q \in (0,1]$. It is representative of a performance report in which the monitor may make a type I error and erroneously accept a claim by the worker that his low output is due to poor uncontrollable events (the low state) rather than low effort, i.e., $q_{2\ell} = 1 - q \ge 0$. On the other hand, the monitor does not make type II errors, i.e., he will not report that the worker has a good state if it is poor. The type II errors may be avoided because, if the monitor's initial evidence indicates that the state is good when it is in fact poor, the worker will provide the monitor with additional evidence so as to avoid being incorrectly penalized.

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[8] Given the form of the cost functions, the left-hand side of (30.8) is a convex quadratic function and (30.10b) has at most two roots larger than θ_2, $x_{1a}'' < x_{1b}''$. Hence, the information rent is zero, if $x_1 \ge x_{1b}''$ or $x_1 \in [\theta_2,x_{1a}'']$. In the discussion above we focus on the bigger root x_{1b}''. For some information systems, there is no solution to (30.10b), implying that the information rent is zero for all $x_1 \ge \theta_2$, such that first-best is obtained (since $x_1^* = 1 + \theta_1 > \theta_2$).

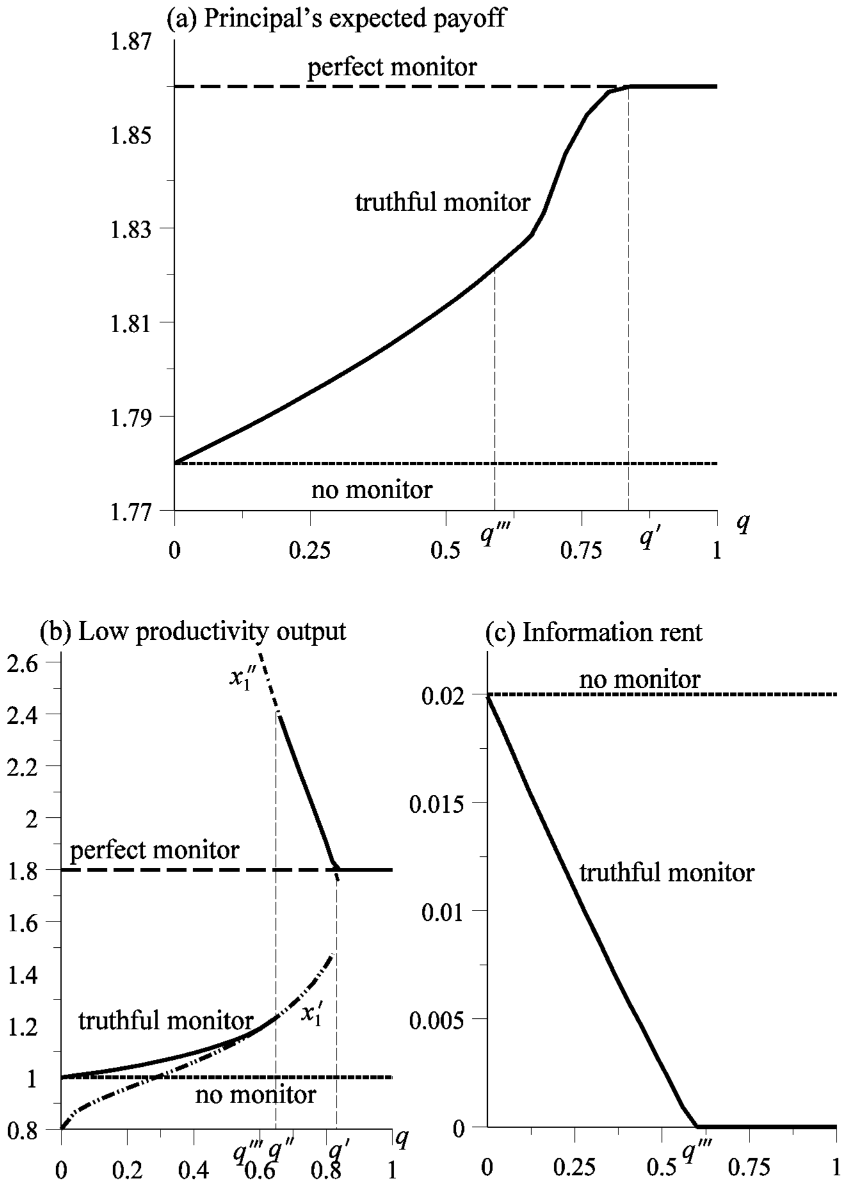

Figure 30.1: Asymmetric information system.

Figure 30.1 illustrates,[9] for system η^a, the impact of $q \in [0, 1]$ on (a) the principal's payoff, (b) the worker's output x_1 with low productivity, and (c) the worker's information rent. In each case the perfect and no monitor cases are provided as benchmarks. Increasing q represents a reduction in the probability of a type I error. Observe that both graphs (a) and (b) depict the fact that the perfect monitor result can be obtained with values of q strictly less than one. Substituting the η^a probabilities into (30.9) yields

$$\underline{c}_w^{a\prime} = -\tfrac{1}{2}[1 - q - (1-\Delta)^2]/q. \tag{30.11}$$

Observe that $\underline{c}_w^{a\prime}$ is an increasing function of q. Hence, the higher the information quality, the less severe is the penalty necessary to implement first-best. Inverting (30.11) yields the following result.

Proposition 30.5

Assume the contract is based on the outcome and a truthful *ex post* report from system η^a. With $\underline{c}_w < 0$, there exists a threshold information quality level

$$q^{a\prime} = [1 - (1-\Delta)^2]/[1 - 2\underline{c}_w], \tag{30.12}$$

such that the first-best is achieved if, and only if, $q \geq q^{a\prime}$.

System η^a is characterized by q and Figure 30.1 depicts three thresholds, q''', q'' and q'. The threshold outcomes x_1' and x_1'' defined in (30.10) are depicted in (b). For all $q \in (0, q''')$, the optimal low productivity outcome $x_1^\dagger \in (x_1', x_1'')$ and, hence, there are both output inefficiency and information rent. Increasing the information quality, increases the principal's expected payoff by reducing both output inefficiency and information rent. For $q \geq q'''$, there is no information rent, and increasing information quality increases the principal's expected payoff strictly by reducing output inefficiency. As noted above there are two options for deterring the high productivity worker from choosing the low output without paying information rent. One option is to set $x_1 < \theta_2$ and depend solely on the threat of a penalty to deter the high productivity worker from producing x_1. The second option is to set $x_1 > \theta_2$ and to use the effort cost plus the threat of a penalty to deter the high productivity worker from producing x_1.

In the region $q \in [q''', q'')$, the first option is adopted, i.e., the optimal low productivity outcome $x_1^\dagger$ is equal to the lower threshold outcome x_1'. At $q = q''$, it is optimal to switch to the other option with $x_1^\dagger = x_1''$. At q'', the threshold x_1'' is strictly greater than x_1'. Hence, the optimal low productivity output $x_1^\dagger$ is a

[9] The parameter values for these figures are $p = .2$, $\theta_1 = .8$, $\theta_2 = 1.5$, and $\underline{c}_w = -.05$.

discontinuous function of q at $q = q''$. Nonetheless, the principal's payoff is a continuous function of q (although it may not be differentiable at $q = q''$).

As the information quality increases, the threshold x_1'' decreases (since the expected penalties become more effective). The first-best is obtained at the information quality q for which $x_1'' = x_1^*$, i.e., at $q = q' = q^{a\prime}$. Increasing the information quality q beyond this point has no impact on the principal's expected payoff, since, for $q \in (q', 1]$, $x_1^\dagger(\eta) = x_1^* > x_1''$.[10]

(b) *Symmetric system (both type I and type II errors)*: The second system, which is examined by KL, is denoted η^s and is characterized by $q_{1\ell} = q_{2h} = q$ and $q_{1h} = q_{2\ell} = 1 - q$, with $q \in [½, 1]$. It can be viewed as representative of imperfectly correlated relative performance information. The worker's state tends to be high (low) when the states for other workers are high (low). This system results in both type I and type II errors.

Substituting the probabilities for η^s into (30.9) yields

$$\underline{c}_w^{s\prime} = -½[1 - q - q(1 - \Delta)^2]/[2q - 1]. \tag{30.13}$$

Inverting (30.13) again provides a threshold value for q.

Proposition 30.6

Assume the contract is based on the outcome and a truthful *ex post* report from system η^s. With $\underline{c}_w < 0$, there exists a threshold information quality level

$$q^{s\prime} = [1 - 2\underline{c}_w]/[1 + (1 - \Delta)^2 - 4\underline{c}_w], \tag{30.14}$$

such that the first-best is achieved if, and only if, $q \geq q^{s\prime}$.

Figure 30.2 depicts (a) the principal's expected payoff, (b) the worker's output x_1 with low productivity, and (c) the worker's information rent with information system η^s. The graphs for the perfect and no monitor cases are the same as in Figure 30.1, and the graphs for the truthful monitor using η^s are similar to the η^a graphs in Figure 30.1. The nature of the thresholds q', q'', and q''' are the same as in Figure 30.1.

However, in Figure 30.2 we also depict the results for the case where the monitor is collusive, i.e., he is willing to lie if the worker makes it in his interest to do so. We discuss the impact of collusion in the next section. Interestingly, while collusion affects the results for η^s, it does not affect the results for η^a.

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[10] For q slightly above q', there is no solution to (30.10b), such that any $x_1 > \theta_2$ can be induced without information rent.

Hence, there is no distinction between a truthful versus a collusive monitor in Figure 30.1.

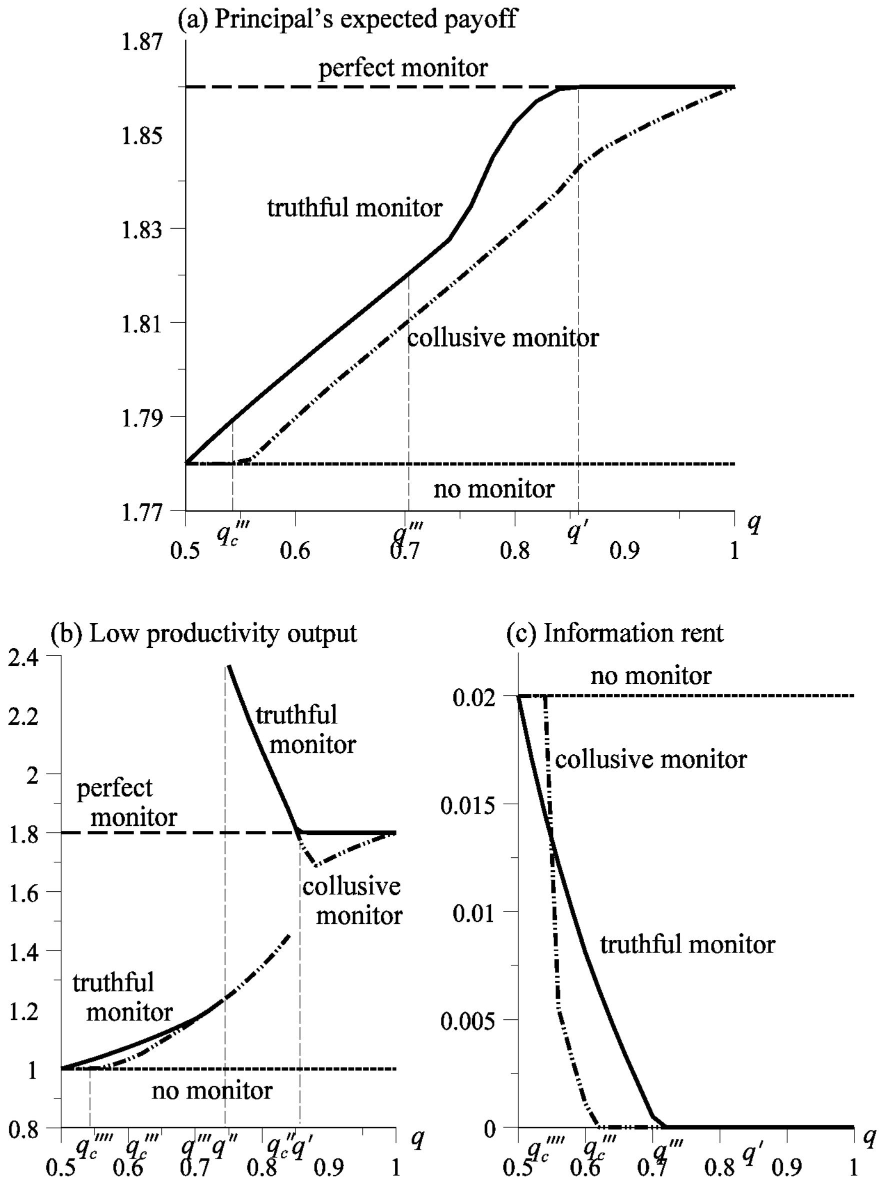

Figure 30.2: Symmetric information system.

30.2.3 Collusion and the Reward Mechanism

The analysis in the preceding section can be interpreted as considering settings in which a monitor is paid a fixed wage and costlessly acquires the information which he reports truthfully. Now we assume the worker and monitor can collude. In particular, if the worker produces x_1 and the monitor observes y_h, the worker can bribe the monitor to report y_ℓ so as to avoid the penalty ζ_w.

Collusion differs from the subgame issues addressed in Section 29.2. In particular, in this model, collusion is possible if the worker can reliably commit to make a *side-payment* to the monitor to lie about what he has observed, and the monitor cannot reliably commit to the principal not to accept the side-payment. We refer to a monitor as *collusive* if there is a *potential* for collusion. This potential may not result in side-payments because the principal offers collusion-proof contracts that eliminate the incentives for collusion. Nonetheless, the potential to collude may be costly to the principal because the collusion-proof contracts differ from those that would be offered if side-payments between the worker and monitor were exogenously precluded.

The maximum value of a monitor is the difference between the principal's first-best expected payoff and his expected payoff with no monitor (see the discussion of the perfect and no monitor cases in Section 30.2.1). The maximum value may not be achieved either because the monitor's information system is imperfect or because he is collusive. We view collusiveness as costly if the worker's and monitor's ability to collude reduces the principal's expected payoff relative to his optimal expected payoff if the monitor is exogenously truthful.

There are three possibilities. First, the potential to collude may totally destroy a monitor's value, i.e., the principal's expected payoff is the same as with no monitor. Second, the potential to collude may partially reduce the monitor's value, i.e., the principal's expected payoff is greater than with no monitor but less than with an exogenously truthful monitor. Third, the potential to collude may have zero impact on the monitor's value, i.e., the principal's expected payoff is the same as with an exogenously truthful monitor.

As we demonstrate in the following analysis, there are three factors that significantly affect the loss of value due to monitor collusiveness. First, are the monitor's possible lies restricted or unrestricted? Second, is the limited liability of the monitor restrictive or non-restrictive? Third, is the monitor's information system such that the probability of a type II error is zero or positive?

Settings in which Collusion Reduces Monitor Value

Lying by the monitor is unrestricted if he can report either y_ℓ or y_h irrespective of what he has observed. In contrast, lying is restricted if, for example, the monitor can report either y_ℓ or y_h if he has observed y_h, but he can only report y_ℓ if he has observed y_ℓ. KL assume the monitor's lying is restricted in this manner. They justify this restriction by assuming that the monitor must provide evidence with his report. A low report can be issued when he has observed y_h merely by withholding evidence, but he cannot provide supporting evidence for a high report if he has observed y_ℓ.

Potential collusion with unrestricted lying totally destroys the value of a monitor. On the other hand, as KL demonstrate, potential collusion with restricted lying can result in a partial loss of value. To demonstrate these two results, we make the following assumptions, which are similar to explicit or implicit assumptions in KL.

Let c_{m1k} represent the incremental compensation paid by the principal to the monitor if the worker produces x_1, and the monitor reports y_k, $k = \ell, h$. The payment to the monitor is constrained to be greater than or equal to $\underline{c}_m$. KL implicitly assume $\underline{c}_m = 0$, and we make that assumption in this section.

We assume the monitor's information system η is such that $q_{jk} > 0$, for $j = 1, 2$, and $k = \ell, h$. Hence, it is subject to both type I and type II errors. The symmetric system η^s considered by KL has this property if $q \in (\frac{1}{2}, 1)$.

With truthful reporting (i.e., collusion is not possible), the optimal contract (see Section 30.2.3) offered to the worker consists of two output levels (x_1, x_2), two corresponding basic compensation levels (c_{w1}, c_{w2}), and a penalty ζ_w that is imposed if the worker produces x_1 and the monitor reports y_h. The monitor's compensation is fixed in that setting.

Now we introduce potential collusion. If the principal ignores the potential collusion and offers the truthful reporting contract, then the worker can produce x_1 irrespective of the state and avoid the penalty ζ_w by bribing the monitor to report y_ℓ even if he has observed y_h. The maximum bribe the worker would be willing to pay is obviously ζ_w. As KL point out, the principal can counter the bribe by paying the monitor a reward $c_{m1h} \geq \zeta_w$ if the monitor reports y_h and the worker has produced x_1. In this setting, the monitor receives no other incremental compensation, but for purposes of subsequent analysis we recognize that the monitor's compensation for a low report, $c_{m1\ell}$, could be non-zero.

Observe that the assumed restrictions on lying are important here. The monitor can only report y_h if he has observed y_h, even though he can report y_ℓ independent of what he has observed.

The following formulates the principal's decision problem given the worker's and monitor's potential for collusion, restricted lying by the monitor, non-negative incremental payments to the monitor, and monitor information that is subject to both type I and type II errors.

Principal's Problem with Collusion, Restricted Lying, Non-negative Monitor Compensation, and Type II Errors:

$$\underset{x_1,x_2,c_{w1},c_{w2},\zeta_w,\zeta_{m1h}}{\text{maximize}} \quad p[x_1 - c_{w1} + q_{1h}(\zeta_w - c_{m1h}) - q_{1\ell}c_{m1\ell}] + (1-p)[x_2 - c_{w2}], \quad (30.15)$$

subject to

(a) $c_{w1} - q_{1h}\zeta_w - \kappa_w(x_1,\theta_1) \geq 0,$

(b) $c_{w2} - \kappa_w(x_2,\theta_2) \geq 0,$

(c) $q_{1h}c_{m1h} + q_{1\ell}c_{m1\ell} > 0,$

(d) $c_{w1} - q_{1h}\zeta_w - \kappa_w(x_1,\theta_1) \geq c_{w2} - \kappa_w(x_2,\theta_1),$

(e) $c_{w2} - \kappa_w(x_2,\theta_2) \geq c_{w1} - q_{2h}\zeta_w - \kappa_w(x_1,\theta_2),$

(f) $c_{m1h} \geq \zeta_w,$

$c_{w1} - \zeta_w \geq \underline{c}_w,\ c_{w2} \geq \underline{c}_w,\ \zeta_w \geq 0,\ c_{m1\ell} \geq \underline{c}_m,\ c_{m1h} \geq \underline{c}_m.$

Constraints (a) and (b) ensure that the worker will accept the contract, whether he has observed θ_1 or θ_2. Constraint (c) ensures that the monitor will accept the principal's proposed change in his contract. Constraints (d) and (e) ensure that the worker will produce x_j, if he has observed $\theta_j, j = 1, 2$. KL refer to constraint (f) as the *Coalition Incentive Compatibility* (*CIC*) constraint – it ensures that the worker is not able to avoid the penalty by bribing the monitor to lie. The final constraints recognize the worker's and monitor's limited liability.

If the monitor is exogenously truthful, then collusion is not a threat. In that case, the principal can set c_{m1h} equal to zero and drop the third and fourth constraints. This implies that the principal receives and retains the penalty imposed on the worker. However, with the threat of collusion and $\underline{c}_m = 0$, the optimal contract has $c_{m1h} = \zeta_w$ and $c_{m1\ell} = 0$. That is, the penalty imposed on the worker is received by the principal, but is then transferred to the monitor. Hence, if there is no other change in the contract, the worker's production choices and compensation are unchanged and the threat of collusion reduces the principal's expected payoff by $pq_{1h}\zeta_w$. Of course, the principal may be able to reduce the cost of collusion by reducing the induced output for low productivity and thereby reduce the penalty that is imposed on the worker.

Figure 30.2(a) depicts the principal's loss of payoff due to collusion with symmetric system η^s and $\underline{c}_m = 0$. The basic reason for the difference between the payoffs with a truthful versus a collusive monitor is the expected payment made by the principal to the monitor to deter his acceptance of a bribe from the worker. As depicted in Figures 30.2(b) the low productivity output differs for a wide range of values of q, while there is zero information rent in both cases with high values of q.

For values of $q \in (½, q_c'''')$, the report of the collusive monitor is ignored, so that the output and the information rent are equal to the no monitor levels. On the other hand, for values of $q \in [q_c''', 1)$, the information rent is equal to zero, and the induced low productivity output is equal to one of the output thresholds x_1' and x_1'' determined in (30.10).[11] However, note that while the first-best low productivity output can be induced with zero information rent for $q \in (q', 1)$, both with a truthful and a collusive monitor, this is not optimal for the principal if the monitor is collusive. Of course, the reason is that the principal has to transfer the penalties for type II errors to the monitor to avoid collusion, and that these penalties are increasing in the induced output (see (30.5)).

Observe that in Figures 30.1(a) and 30.2(a) the principal's payoff with a truthful monitor is strictly greater than with no monitor for all values of q. On the other hand, with a collusive monitor and symmetric information there exists a threshold q_c'''' such that the monitor has zero value if q is less than q_c'''', and positive value otherwise. This reflects the fact that the expected payment to the collusive monitor is equal to

$$p q_{1h} \zeta_w = p(1 - q)[\kappa_w(x_1, \theta_1) - \kappa_w(x_1, \theta_2)]/[2q - 1].$$

If we hold the induced output x_1 constant, then this cost is infinite for $q \to ½$ and decreases to zero as $q \to 1$. Hence, there is a threshold $q_c'''' \in (½, 1)$ at which the cost of collusion decreases to the value of a truthful monitor. KL obtain the following results.

Proposition 30.7 (KL, Prop. 1)

Assume the monitor is costless and collusive, his information system is symmetric, his lies are restricted, and $\underline{c}_m = 0$.

(a) The monitor has positive value if, and only if, $q > q_c'''' = 1/(2 - p)$.

(b) The first-best is not achieved unless the monitor is perfectly informed (i.e., $q = 1$), no matter how low $\underline{c}_w$ is.

Result (a) formalizes our prior discussion. In Figure 30.2(a), $q_c'''' = 1/(2-p) = .5556$. Result (b) states that the first-best cannot be achieved if $q \in (½, 1)$. This follows from the fact that to avoid collusion, the principal transfers any penalty imposed on the worker to the monitor. The expected cost of that transfer is

[11] For $q \geq .87$, there is no solution to (30.10b) implying that any $x_1 \geq \theta_2 = 1.5$ can be induced with zero information rent.

positive if there is a positive probability of a type II error, i.e., the worker is penalized for producing x_1 if the monitor reports y_h even though the worker had observed θ_1. As noted above, the expected cost of that transfer also leads the principal to induce less than first-best output for q close to one.

Total Destruction of Monitor Value

Observe that if there is no restriction on lying, the contract discussed above will induce the monitor to report y_h whenever the worker produces x_1. Hence, to avoid receiving the minimum wage $\underline{c}_w < 0$, the worker will reject the contract if he observes θ_1. This is clearly not optimal, and the principal will be better off if he offers the worker the optimal no monitor contract.

Settings in which Collusion Does not Reduce Monitor Value

We assume the monitor's lying is restricted and consider two special cases in which collusiveness does not reduce the value of the monitor. In the first case, the probability of a type II error is equal to zero, and in the second case the monitor's minimum incremental compensation is significantly negative.

There is no type II error if $q_{1h} = 0$, which is the case in the asymmetric system η^a introduced above. Observe that in the principal's problem (30.15), setting $q_{1h} = 0$, implies that the expected payment to the monitor is zero, and the worker's incentive and contract acceptance constraints are the same as in the truthful monitor setting. We continue to have the monitor's contract acceptance constraint and the *CIC* constraint, since they continue to ensure that the worker cannot bribe the monitor to lie. The key here is that while the threat of a penalty (which would be transferred to the monitor) motivates the worker, the fact that there is no type II error implies that the worker is never penalized and the monitor is never rewarded. Hence, the results with a collusive monitor are the same as with an exogenously truthful monitor. That implies that the truthful monitor results plotted in Figure 30.1 are also the collusive monitor results.

Now assume that there is a strictly positive probability of a type II error (as in the symmetric system), but the monitor's minimum compensation $\underline{c}_m$ is less than $-q_{1h}\zeta_w^\dagger$, where $\zeta_w^\dagger$ is the optimal penalty with a truthful monitor. It is optimal to offer the monitor a contract which rewards him for reporting y_h if x_1 is reported but to also deduct a constant so that the monitor's expected incremental compensation is zero. That is, in this setting, the monitor's optimal compensation is

$$c_{m1\ell} = -q_{1h}\zeta_w^\dagger, \quad c_{m1h} = \zeta_w^\dagger - q_{1h}\zeta_w^\dagger = q_{1\ell}\zeta_w^\dagger,$$

and the worker's output and compensation are the same as in the truthful monitor setting.

Proposition 30.8

Assume the monitor is costless and collusive, and his lies are restricted. Then there is no loss of monitor value (relative to an exogenously truthful monitor) if either (a) $q_{1h} = 0$ or (b) $\underline{c}_m \le -q_{1h}\zeta_w^\dagger$.

For values of $\underline{c}_m$ less than zero and greater than $-q_{1h}\zeta_w$, collusiveness will cause a loss in monitor value on the order of $p[q_{1h}\zeta_w + \underline{c}_m]$ (assuming no changes in output). Table 30.1 illustrates the cost of having $\underline{c}_m$ less than zero, but greater than $-q_{1h}\zeta_w^\dagger$. The information system is symmetric and the basic parameter values are again $\theta_1 = .8$, $\theta_2 = 1.5$, $p = .2$, and $\underline{c}_w = -.05$. We consider two levels of information quality: $q = .6$ and $q = .9$. The last row is the same as the output, penalty, and payoffs with a costless, truthful monitor since $-(1-q)\zeta_w^\dagger > \underline{c}_m$, i.e., the monitor's limited liability constraint is not binding with the optimal truthful monitor penalty. The first-best (perfect monitor) result is obtained with $q = .9$ and $\underline{c}_m = -.06$, i.e., there is no cost of collusion and no cost of imperfect information. On the other hand, while there is no cost of collusion with $q = .6$ and $\underline{c}_m = -.06$, there is a cost of imperfect information – due to inefficient production. The first row is the KL model and represents the maximum cost of collusion.

$q = .60$				$q = .90$			
$\underline{c}_m$	x_1	ζ_w	payoff	$\underline{c}_m$	x_1	ζ_w	payoff
.00	1.031	0.13	1.790	.00	1.712	0.49	1.849
-.01	1.031	0.13	1.792	-.01	1.712	0.49	1.851
-.02	1.031	0.13	1.794	-.02	1.712	0.49	1.853
-.03	1.031	0.13	1.796	-.03	1.712	0.49	1.855
-.04	1.031	0.13	1.798	-.04	1.712	0.49	1.857
-.05	1.031	0.13	1.800	-.05	1.721	0.50	1.859
-.06	1.073	0.15	1.801	-.06	1.800	0.57	1.860

Table 30.1: Impact of monitor limited liability on the cost of collusion.

Note that for both values of q, the output and the penalty are unchanged for a range of values of $\underline{c}_m$ below zero. This reflects the fact that if there is slack in the monitor's acceptance constraint, a marginal reduction in $\underline{c}_m$ will not change the induced output or penalty, but only the monitor's base wage (which is equal to $\underline{c}_m$). With $q = .9$, and $\underline{c}_m = -.05$, the monitor's acceptance constraint and his limited liability constraint are both binding, and a marginal reduction in $\underline{c}_m$ will increase the principal's payoff as well as the production and the penalty.

30.2.4 The Use of an External Monitor to Deter Lying by an Internal Monitor

Monitors can be either employees of the firm, or independent contractors. The former include "bosses" and "internal auditors," whereas the latter include external members of the board of directors and external auditors. The initial agency models that considered the monitoring role of auditors assumed the principal can write contingent contracts with the auditor.[12] However, while this is an appropriate approach if the auditor is an employee of the firm, i.e., an internal auditor, it is less appropriate for examining the role of external auditors. They typically are paid a fixed fee, perhaps contingent on the work done, but not directly contingent on the firm's outcome or other performance measures.

Consequently, more recent agency models of the role and incentives for external auditors have assumed that their incentives stem from exogenous sources, such as the threat of litigation and reputation effects.[13] Due to limited time and space, we do not explore the role of litigation and reputation in motivating external auditors. Instead, we explore the role of an exogenously motivated, costly monitor in a setting in which the principal employs an internal monitor whose collusiveness is costly to the principal.

The Basic Model

Many of the elements of the model in this section are the same as in Section 30.2.3. To provide a setting in which an internal monitor's collusiveness is costly, we adopt an approach similar to KL and assume the internal monitor has an information system that is subject to both type I and type II errors, and his lies are restricted. Furthermore, the internal monitor's minimum incremental compensation $\underline{c}_m$ is equal to zero. However, if the principal has evidence of fraud, he can take the monitor (and the worker) to court, resulting in an aggregate penalty of ζ_f. We treat the size of that penalty as exogenously set by the court. Also, we do not specify how the penalty would be distributed. The threat of the penalty is important because of its deterrence of fraud, but, in equilibrium, no fraud is committed so that no penalty is imposed.

Since the internal monitor is costless, we assume he always issues a report if the worker produces x_1. (There is no value to reporting if the worker has produced x_2.) The cost of hiring the external monitor is κ_e and he is hired with

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[12] See, for example, Antle (1982), Baiman *et al.* (1987), and Baiman *et al.* (1991).

[13] See, for example, Chan and Pae (1998), Pae and Yoo (2001), and Narayanan (1994).

probability $\gamma \in [0,1]$ if the worker produces x_1 and the internal monitor reports y_ℓ. The probability γ is chosen by the principal, and he never hires the external monitor if the internal monitor reports y_h.

If the external monitor is hired, he issues a statement that either accepts or rejects the y_ℓ report issued by the internal monitor. Let $\delta \in \{0,1\}$ represent the external monitor's statement, where $\delta = 0$ is reject and $\delta = 1$ is accept. The external monitor never rejects a report of y_ℓ if the internal monitor has observed y_ℓ, but if he has observed y_h, the rejection probability is $q_e \in (0,1]$. This structure implies that the external monitor does not provide any additional information about the state θ_j. He only checks the truthfulness of the internal monitor's report relative to the information available to him. KL effectively assume that $q_e = 1$, i.e., the external monitor observes the information received by the internal monitor.

For a given information system η, we assume the principal can choose between two options in contracting with the worker and a collusive internal monitor. Under the *monitor-reward* option (see Section 30.2.3), the internal monitor is paid ζ_w if he reports y_h when the worker produces x_1. On the other hand, under the *monitor-penalty* option, the principal commits to hire the external monitor with probability $\gamma > 0$ if the worker produces x_1 and the internal monitor reports y_ℓ.

As in the monitor-reward option, the worker's base pay is c_{wj} if he produces x_j, $j = 1, 2$, and a penalty ζ_w is deducted from his base pay if he produces x_1 and the internal monitor reports y_h. The penalty is also imposed if the internal monitor reports y_ℓ and the external monitor reveals that this is a lie.

The external monitor is always truthful and receives a fixed fee $c_e = \kappa_e$ if he is hired. The internal monitor receives no incremental compensation, but he and the worker are penalized ζ_f by the courts if the external monitor reveals that the internal monitor has lied (i.e., colluded with the worker to commit fraud).

The principal must choose between the use of a reward or a penalty. His optimal payoff from the reward option is provided by the solution to (30.15). The solution to the following problem provides the principal's optimal payoff from the penalty option.

Principal's Problem with Internal and External Monitors:

$$\underset{x_1, x_2, \gamma, \zeta_w, c_{w1}, c_{w2}}{\text{maximize}} \quad p[x_1 - c_{w1} + q_{1h}\zeta_w - q_{1\ell}\gamma\kappa_e] + (1-p)[x_2 - c_{w2}], \tag{30.16}$$

subject to (a) $c_{w1} - \kappa_w(x_1,\theta_1) - q_{1h}\zeta_w \geq 0,$

(b) $c_{w2} - \kappa_w(x_2,\theta_2) \geq 0,$

(c) $c_{w1} - \kappa_w(x_1,\theta_1) - q_{1h}\zeta_w \geq c_{w2} - \kappa_w(x_2,\theta_1),$

(d) $c_{w2} - \kappa_w(x_2,\theta_2) \geq c_{w1} - q_{2h}\zeta_w - \kappa_w(x_1,\theta_2)$,

(e) $\gamma q_e(\zeta_w + \zeta_f) \geq \zeta_w$,

(f) $c_{w1} - \zeta_w \geq \underline{c}_w$, $\zeta_w \geq 0$, $c_{w2} \geq \underline{c}_w$.

The first two constraints (a) and (b) ensure that the worker will accept the contract after observing either of the two states. There is no contract acceptance constraint for the internal monitor since there is no change in his compensation given that he is induced to report truthfully. The third and fourth constraints (c) and (d) ensure that the worker will produce x_j if he has observed θ_j, $j = 1, 2$ (and he expects the internal monitor to report truthfully). The fifth constraint (e) is a *CIC* constraint. It ensures that, if the worker produces x_1 and the monitor observes y_h, the worker and the monitor are jointly better off if the monitor truthfully reports y_h instead of lying and reporting y_ℓ. The final constraints (f) ensure that the worker's limited liability restrictions are satisfied.

The constraints are similar to those in the principal's problem with only the internal monitor. The key difference is that the *CIC* constraint now reflects the fact that truthfully reporting y_h after producing x_1 results in the worker being penalized ζ_w but avoids the possibility the coalition will be penalized ζ_f and ζ_w. The probability the latter will occur is the probability the external monitor will be hired times the probability he will uncover the lie.

The fraud penalty ζ_f and the detection probability q_e are exogenous, whereas the output penalty ζ_w and the probability of hiring the external monitor γ are endogenous. Obviously, for any penalty ζ_w, the probability of hiring the external monitor will be set so that the *CIC* constraint holds as an equality, i.e.,

$$\gamma = \zeta_w/[q_e(\zeta_f + \zeta_w)]. \tag{30.17}$$

This minimizes the expected cost of the external monitor while inducing truthful reporting. Consequently, the *CIC* constraint can be dropped by substituting (30.17) into the objective function. Now the constraints are the same as in the truthful reporting case, but the objective function has an additional cost term:

$$\Pi = p\big[x_1 - c_{w1} + q_{1h}\zeta_w - q_{1\ell}\kappa_e\zeta_w/[q_e(\zeta_f + \zeta_w)]\big] + (1-p)\big[x_2 - c_{w2}\big]. \tag{30.18}$$

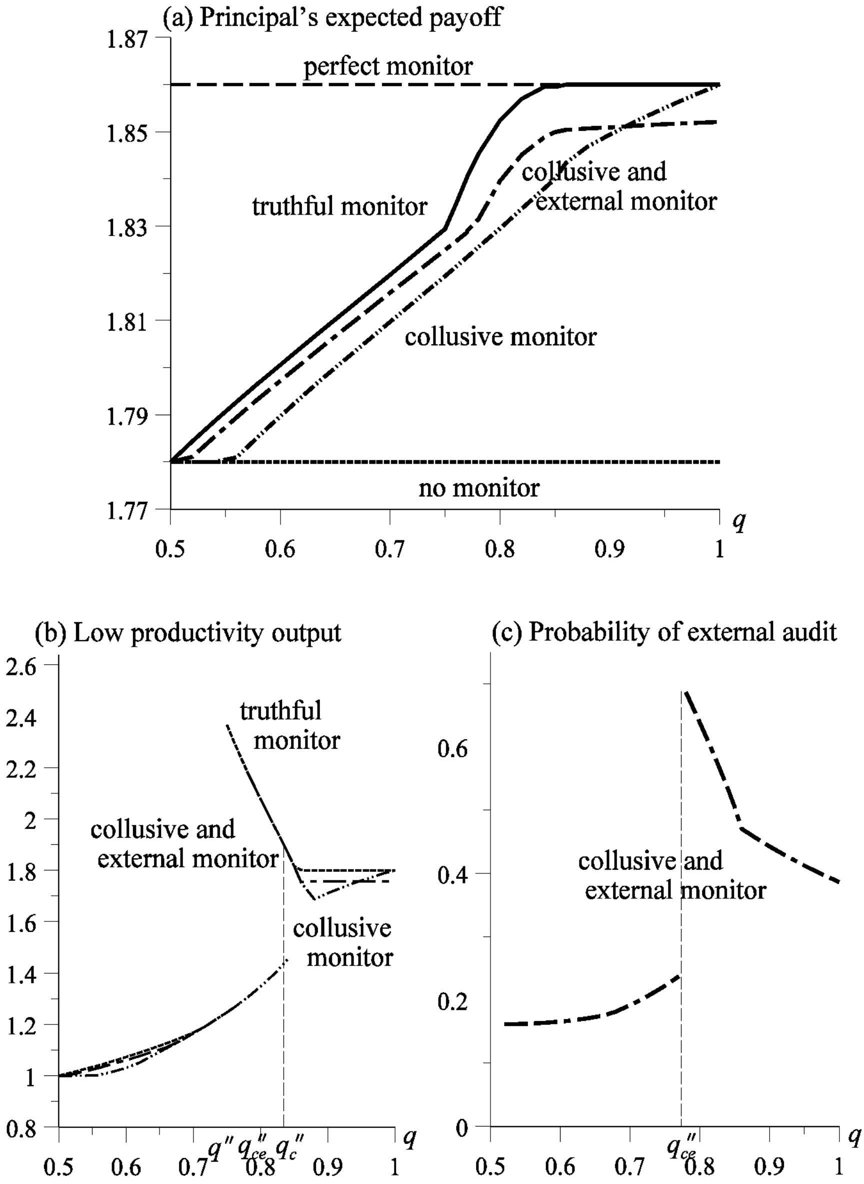

Figure 30.3: Impact of external monitor with symmetric information system.

Figure 30.3 depicts (a) the principal's expected payoff, (b) the low productivity output level for the monitor reward and penalty options as well as for the truthful monitor, and (c) the probability for an external audit.[14] In this example, the penalty option dominates the reward option except for values of q close to one. In that region, the optimal reward option payoff approaches first-best because the probability of a type II error shrinks to zero as q goes to one. That occurs in the penalty option as well, but in that setting, the expected cost of the external monitor and the *CIC* constraint are independent of q except for its second-order effect on the worker's penalty ζ_w. Hence, as depicted in (b), the low productivity output is almost constant with the penalty option (but below first-best) for q close to one.

The probability of hiring the external monitor is depicted in (c). Observe that, in this example, it is strictly positive and strictly less than one. This probability is increasing in q up to the point q''_{ce} at which the low productivity output jumps from the lower outcome threshold x_1' to the upper outcome threshold x_1''. For $q > q''_{ce}$ the external audit probability decreases. As noted above, the external audit probability γ is determined by the *CIC* constraint as reflected in (30.15). Hence, the external audit probability is increasing (decreasing) in q if, and only if, the optimal worker penalty ζ_w is increasing (decreasing) in q. Note that the principal's expected payoff is almost constant for q close to one even though the expected external audit cost conditional on a low internal report, $\gamma\kappa_e$, is decreasing in q for $q > q''_{ce}$. However, as q increases, the probability of a low internal report also increases.

While our model differs slightly from the KL model, the following results from their Proposition 2 also apply here.

(a) There is a set of parameter values for which it is optimal not to use any monitor.

(b) There is a set of parameter values for which it optimal to use only the internal monitor.

(c) If used at all, the external monitor is used with a probability strictly less than one.

(d) If the external monitor is used, the internal monitor is threatened with a penalty if he lies and this is sufficient to deter that lie, which implies he does not receive a bonus for truthfully reporting y_h.

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[14] The example uses the symmetric information system η^s with the same parameters as in Figure 30.2. In addition, $\kappa_e = .10$, $q_e = .90$, and $\zeta_f = .80$.

If monitor information is of poor quality, it is optimal to not use either monitor, whereas if the monitor information is of high quality, it is optimal to only use the internal monitor. For middle ranges of monitor information quality, both monitors are used – in this setting it is less expensive to use the threat of the external monitor instead of rewards to deter collusion between the worker and the internal monitor.

30.3 CONCLUDING REMARKS

Our discussion of models involving monitors has been limited. The models in this area (including the ones we have discussed) are rather idiosyncratic in character. Hence, it is difficult to identify general results.

It is obviously important to distinguish between models of internal versus external monitors. The contract between the principal and an internal monitor is similar to the contract between the principal and a "worker" in that these contracts are the primary determinants of these agents' incentives. This can also apply to some external monitors, such as a private security agency. However, it does not apply to external auditors, who are required to be hired on a fixed fee basis. The first type of external monitor is hired strictly for the benefit of the principal. The second type, on the other hand, has a responsibility to third parties, as well as to the principal. As a result, an external auditor's incentives are attributable to external forces, such as threats of litigation and loss of reputation (which leads to loss of clients).

In our analysis, we focused on internal monitors, and only introduced an external monitor in the final model. In that model the external monitor's incentives are exogenous. As mentioned in the chapter, there are a few papers that explore how the threat of litigation provides monitor incentives, but we have not included those models in this book.

Models that consider the threat of litigation and loss of reputation are clearly relevant when considering external auditors. In addition, these threats may be relevant when examining the incentives of managers, particularly senior managers. That is, while incentive compensation may be a major determinant of manager incentives, those incentives will also be influenced by the manager's personal threat of litigation and the impact of the market for managers. Agency theory has largely ignored litigation against managers and given only limited attention to the market for managers. On the other hand, the recent interest in "corporate governance" has led or will lead to consideration of these issues in research on management incentives. Ideally, this research will recognize that a principal optimally considers the existing external (exogenous) incentives when endogenously determining the internal monitoring of a manager and his incentive compensation.

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Author Index

Subject Index