629 LECTURE NOTES IN ECONOMICS AND MATHEMATICAL SYSTEMS

Michaela Isabel Höhn

Relational Supply Contracts

Optimal Concessions in Return Policies for Continuous Quality Improvements

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Foreword

This book considers a supply chain consisting of one buyer and one supplier. The parties interact over a long time horizon. While the horizon may not be infinite, the parties do not know exactly how long their relationship will last. In each period, the buyer faces stochastic demand and must procure materials from the supplier before demand is realized. Inventory does not carry over from one period to another. The returns from this business depend on the quality of the product, which evolves randomly. However, the parties have some influence on how the quality changes by exerting effort. All benefits from higher quality accrue to the downstream buyer. One issue to consider then is how to induce the supplier to exert effort to improve quality when she does not reap a direct benefit.

Transactions between the parties are governed by an explicit contract that calls for the buyer to pay a predetermined amount per unit and for the supplier to compensate the buyer for unsold units – i.e., returns are allowed. Two contract forms are considered, which differ in how returns are managed. The first is a quantity flexibility contract which calls for the supplier to refund the purchase price on returned units fully, but limits what fraction of the order can be returned. The second is a buy-back contract under which any quantity can be returned but the buyer will only receive a partial refund of the purchase price. Thus, they are both partial returns policies: Quantity flexibility contracts allow returns of part of the order for the full price, while buy backs allow return of the entire order for part of the purchase price.

It is assumed that regardless of the contract form, these explicit contracts are court-enforceable. Effectively, this means that a third party can verify the amount purchased by the buyer and the amount sold to end customers. The third party could consequently determine the financial transfers required by the explicit contract and further verify whether the appropriate transfers have been made.

These assumptions on contract enforceability are fairly reasonable in reality and commonly made in the literature. A partial returns contract, however, does nothing to induce quality-related effort by the supplier. Furthermore, it may be hard for a third party to evaluate whether the buyer or supplier exerted effort in a given period or whether the failure to improve effort was due to chance, despite both players exerting heroic amounts of effort. An outsider may also be unable to properly measure the supply chain's gains from improved effort. Note that these may all be obvious to the buyer and supplier. The important factor is that they cannot be easily measured by a disinterested outsider. It is these concerns that keep the parties from being able to sign an explicit formal contract on exerting effort in sharing the resulting gains.

This leads to the introduction of relational contracts. These are informal agreements that potentially call for transfers or other considerations between the parties that are not enforceable by court. Payments, however, will get made because the parties find it in their interests to do so. Stated another way, a relational contract creates a set of deviations from the formal contract that the parties are willing to carry out because they prefer continuing the informal arrangement to keeping to the formal agreement or abandoning the relationship altogether.

We see three real strengths in this book. The first is the analysis of relational contracts. This is an understudied topic in the supply chain literature. This is not the first research in this area. As mentioned in the book, Plambeck and Taylor were the real pioneers in this area. That said, there has been surprisingly little follow work. Not every interaction within a supply chain is specified by a formal contract vetted by lawyers. Many things are agreed to on an informal basis and it is worth exploring how these informal side agreements relate to formal contracts.

A second feature of this work is the attempt to place its findings in the context of industry practice. To some extent, this is related to the proceeding observation that real supply chains rely on trust and partnerships. The industry studies discussing how firms interact are valuable and do a very solid job motivating the following model.

Finally, we appreciate the comparison of quantity flexibility and buy-back contracts. Again, this is something the literature has been lacking. Too much research has focused on one contract form over another and not carefully considered whether their intricacies matter. It is easy to show that either contract works well in a one period newsvendor setting. That rough equivalence may not carry over to more complex settings but little is known about how the setting impacts the relative performance of the contracts.

Overall, the thesis presents a significant contribution to the Operations Management and Supply Chain Management literature as well as industry practice as outlined in the case study chapter.

June 2009 *Arnd Huchzermeier and Martin Lariviere*

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Symbols

a_{bt}	Buyer's effort for quality in period t
a_{st}	Supplier's effort for quality in period t
b	Buy-back price
c	Production cost
c_b	Buyer's cost from effort in period t
c_s	Supplier's cost from effort in period t
D_t	Demand realization in period t
δ	Discount factor
Δ	Quantity flexibility
e_{bt}	Buyer's decision to execute the informal transaction at the end of period t
e_{st}	Supplier's decision to execute the informal transaction at the end of period t
F_{xz}	Continuous distribution function of L_t with support $[\underline{l}_{xz}, \overline{l}_{xz}]$ where $\underline{l}_{xz} < \overline{l}_{xz}$
g	Goodwill cost
G	Continuous distribution function of D_t
γ_{bt}	Buyer's outside option in period t
γ_{st}	Supplier's outside option in period t
H^t	Public history of the game at the beginning of period t
L_t	Buyer's cost benefit from quality transition in period t
$L(x, z)$	Expected value of L_t given $(X_t, X_{t+1}) = (x, z)$
p_t	Supplementary transfer payment in period t
$P_{xz}(a_s, a_b)$	Probability for a transition from state $X_t = x$ to state $X_{t+1} = z$, given efforts a_s and a_b
Q	Production quantity
r	Retail price
s	Salvage value
t	Period
τ_{bt}	Buyer's decision to transact in period t
τ_{st}	Supplier's decision to transact in period t
w_Δ	Wholesale price of the quantity flexibility contract
w_b	Wholesale price of the buy-back contract

$\mathcal{X}$	Finite, discrete state space reflecting quality
X_t	Initial quality level in period t
ξ_t	Degree of reduction of exploitable quantity flexibility in period t
ζ_t	Degree of buy-back price reduction in period t

Chapter 1
Introduction

In buyer-supplier relationships, firms often expect a level of performance and adaptability that goes well beyond contractual requirements. In fact, supply relations are often governed by so-called relational contracts. These are informal agreements sustained by the value of future cooperation. Although relational contracts persist in practice, research on these types of contracts is only emerging in Operations and Supply Chain Management. This treatise studies a two-firm supply chain, where repeated transactions via well-established supply chain contracts and continued quality-improvement efforts are governed by a relational contract. We are able to characterize an optimal relational contract, i.e., to develop policies for supplier and buyer that structure investments in quality and flexibility in a way that no other self-enforcing contract generates higher expected joint surplus. For this purpose, we study an infinite horizon dynamic game with Markovian dynamics modelling the stochastic influence of the firms' actions on quality. We examine both quantity-based (quantity flexibility contracts) and price-based returns (buy backs) mechanisms. Hence, a second goal is to compare the performance of different returns mechanisms in the context of relational contracting.

1.1 Motivation and Objectives

Repeated supply chain interactions are often governed by so-called relational contracts. These are informal agreements sustained by the value of future cooperation. Or, like Baker et al. (2002) put it, relational contracts describe "informal agreements and unwritten codes of conduct that powerfully affect the behavior of individuals". For example, firms undertaking a long-term business relationship often expect a level of performance and flexibility that goes well beyond contractual requirements. A formal contract may provide a reasonable starting point here. But the key to success will be the ability of the supply chain partners to adapt to specific circumstances without incessantly insisting on contractual agreements. In the same vein, the literature on vertical supply contracting suggests that adaptability is a key feature of successful long-term relationships (Williamson 1985).

Researchers in Supply Chain Management (SCM) have proposed a number of contracts that coordinate the supplier selling to a newsvendor in a single period

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model (see Cachon 2003 and Chen 2003 for surveys on supply chain coordination with contracts). However, long-term buyer-supplier relationships, which offer many opportunities for informal adaptations, have been neglected for a long time. Only recently, researchers have given increased attention to studying relational contracts in supply chain settings, taking into account that real-world supply chain contracting often takes place over long periods of time involving both formal and informal aspects (Debo and Sun 2004; Ren et al. 2006; Tunca and Zenios 2006; Plambeck and Taylor 2006; and Taylor and Plambeck 2007a,b).

The treatise at hand deals with formal and relational aspects of procurement. The goal is to view supply chain contracts in the context of a relational contract environment, where both parties of a two-firm supply chain repeatedly invest in quality-improvement efforts.[1] In particular, the focus lies on quantity flexibility (QF) contracts in the spirit of Tsay (1999) and Tsay and Lovejoy (1999a) and buy-back contracts (Pasternack 1985), and the question to which extent the inclusion of relational aspects alters contract design. QF and buy-back contracts are designed to optimize total supply chain profit in a one-period setting by granting to the buyer a certain right to return unsold units to the supplier. In our model, repeated interactions and joint responsibility for quality improvement introduce dynamics into the contracting model, which affect the parameters of the supply chain contract employed. That is, in each period the firms transact, the buyer may informally offer the supplier an adaptation of the supply chain contract to incentivize quality-improvement efforts. This is in sharp contrast to the one-period setting, where such informal promises could never be sustained or where, technically speaking, the only enforceable contracts are court-enforceable ones (Bolton and Dewatripont 2005). Since QF and buy-back contracts differ in the returns mechanism - quantity-based in the case of the quantity flexibility contract and price-based in the case of the buy back - a second research question is to understand how an initial contract choice affects adaptability and maintenance of the supply relationship in the future.

According to these research questions, the contributions of our research head for two directions. A first goal is to provide a theoretical framework to study the interplay of formal and informal aspects of a supply relationship. This involves describing optimal action strategies and the impact on the economic outcome for each party involved. Second, the developed framework should inform optimal contract choice.

- **Contribution 1** *Provide a framework for optimal design of QF and buy-back contracts in a relational contract setting.*
- **Contribution 2** *Compare the performance of QF and buy-back contracts in the context of relational contracting.*

The results are established in a general and broadly applicable setting. Dynamic game theory helps enlighten the effect of repeated interaction on contract design,

[1] Zhu et al. (2007) highlight that the buyer cannot cede the responsibility of quality improvement to the supplier in many cases.

building on work by Levin (2003) and Plambeck and Taylor (2006). In a first step, recent progress in the fields of supply chain coordination and relational contracting are reviewed in detail. In a second step, a new perspective on relational contracts is developed. The structure chosen to achieve the aforementioned objectives is detailed in the following section.

1.2 Structure

The treatise is organized as follows. The remainder of Chap. 1 introduces the reader to the concept of relational procurement and aims at further motivating our approach by taking up recent developments in the consumer goods and automotive industry. Chapter 2 reviews literature on modelling supply chain contracts in a one-period setting. Special attention is given to the supply chain contracts of interest, the QF and the buy-back contract. Chapter 3 is devoted to relational contracts. The first part of the chapter shows how economists have tried to integrate the concept of relational contracting in their models. The second part demonstrates how relational contracts have found their way into Operations and Supply Chain Management in the last years by reviewing recent research papers in the field. The chapter closes with a discussion of game-theoretic as well as dynamic programming concepts relevant for the analysis of relational contracts. Chapter 4 presents the actual model and the model analysis. Here, two firms interact repeatedly on the basis of a QF contract, making informal adaptations to the contract with the objective to incentivize quality-improvement efforts and to maintain the supply relationship. An optimal relational contract is determined. Throughout the chapter, interpretations of the employed equations and proofs of the main results are provided. In Chap. 5, the developed model is adapted to integrate buy-backs instead of QF contracts. Again, an optimal relational contract can be characterized. In Chap. 6, differences between QF and buy-back contracts when applying them in long-term buyer-supplier relationships are analyzed. Numerical examples highlight the implications contract choice may have on the leeway for contract adaptation, additional transfer payments needed to sustain the relationship, and therewith the continuance of the supply relationship. Chapter 7 presents a case study on Volkswagen Group, founder of the often-cited marketplace VWGroupSupply.com, showing how Volkswagen Group shapes the interface to its suppliers. The conclusion summarizes managerial insights and shows directions of future research to better understand and optimize the design of supply relationships.

1.3 Transactional vs. Relational Procurement

As an introduction to buyer-supplier relationships, which are in the focus of our analysis, we first discuss differences between transactional and relational procurement. We then proceed to provide a concise overview of the characteristics

Table 1.1 Different terms for the continuum of governance modes (source: Schramm-Klein and Morschett 2006)

Author(s)	Governance modes
MacNeil (1978)	Discrete exchange – relational exchange
Håkansson (1982)	Exchange episodes – relationships
Williamson (1985)	Markets – relational contracting
Shapiro (1985)	Traditional adversarial approach – new adversarial approach – buyer-supplier partnership – "conduit for innovation"
Heide (1994)	Market governance – non-market governance (unilateral/hierarchical vs. bilateral)
McIvor et al. (1998)	Adversarial – collaborative
Spekman et al. (1998)	Open market negotiations – cooperation – coordination – collaboration

of buyer-supplier relationships. Prerequisites, goals, and obstacles connected with successful buyer-supplier relationships are given. Finally, we show how the presented characteristics of buyer-supplier relationships are translated into our model.

One observes that the terms buyer-supplier relationship, buyer-supplier partnership, bilateral relationship, relational contract, or collaboration are often employed interchangeably. In fact, researchers studying the management of business relations have developed several frameworks to classify buyer-supplier relationships and to separate those from other governance modes. Table 1.1 gives an overview of terms used to differentiate between transactional and relational business interactions. Note that these terms generally represent a continuum instead of discrete choices. MacNeil (1978), for example, distinguishes discrete and relational exchanges, while Håkansson (1982) speaks of exchange episodes opposed to relationships. Williamson (1985) contrasts markets with relational contracting, a term which best describes our model approach. Shapiro (1985) considers governance modes as a continuum from traditional adversarial approach, to new adversarial approach, to buyer-supplier partnerships, to "conduit for innovation". Like McIvor et al. (1998), the author emphasizes the adversarial character of transactional procurement.

After a first introduction to the literature on business relations management, our next goal is to approach the outstanding features of buyer-supplier relationships. For this purpose, we take a closer look at the framework proposed by Spekman et al. (1998). Step by step, the authors describe the transition from open market negotiations, to cooperation, to coordination, to collaboration (see Fig. 1.1). After Spekman et al. (1998), open market negotiations are geared above all towards single transactions and price optimization, resulting in adversarial attitudes of the parties involved. Interaction is primarily directed towards arm's-length negotiation of conditions. The business relation may be called cooperative, as soon as fewer suppliers and long-term contracts are present. To take the next step to coordination, the firms need to harmonize their IT systems to provide the basis for regular information exchange. In the state of collaboration, supplier and buyer typically engage in supply chain integration, joint planning, and technology sharing. What this thesis shows is

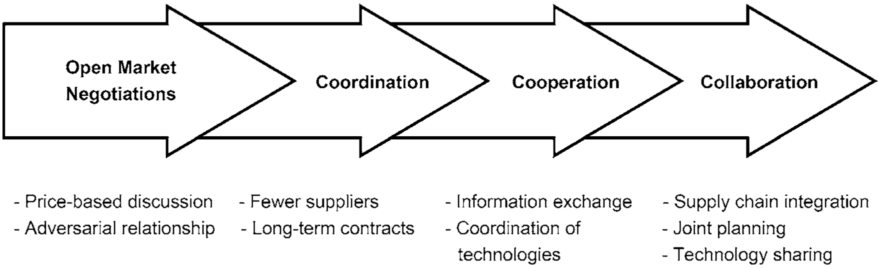

Fig. 1.1 The key transition from open-market negotiations to collaboration (adapted from Spekman et al. 1998)

that buyer-supplier relationships do not stop here. Spekman et al.'s framework does not consider continuous improvement or joint value creation through innovation. It stops at sharing order information and not customer information. We will pick up on these relationship features in the treatise at hand. Our model approach suggests that partnerial buyer-supplier relationships tend to be unsuccessful unless they are geared towards joint value creation with both firms continuously inducing effort. This is also supported by examples from the consumer goods and automotive industry (see Sect. 1.4, p. 7). Clearly, next phases of Spekman et al.'s framework would be co-development and co-ownership with vertical integration being the ultimate step.

Table 1.2 explores the characteristics of buyer-supplier relationships further. The table shows that as firms emphasize performance over price, they will apply new roles of engagement. In order to deepen their relationships with master suppliers, buyers move towards non-binding, long-term agreements, and cost standards. The scope of the relationship is "long-term-oriented, reciprocal, and extending beyond mere buying and selling" (Li and Dant 1997). Furthermore, both firms assume responsibility for improvement and equitably share risks, costs, and gains of improvement initiatives.

Goals pursued by entering into partnerships include cost reduction, process and quality optimization, enhancement of innovation capability and flexibility, and risk reduction. But both firms have to meet certain conditions to achieve these goals. Frequently cited obstacles to collaboration are, for instance, confidentiality concerns, limited interest by suppliers, legal barriers, and resistance to change (Monczka et al. 2005). As previously indicated, there are structural prerequisites for collaboration, i.e., infrastructure for communication and information sharing. Moreover, there are relational prerequisites like trust and alignment of goals and incentives. Trust[2] is key, when it comes to relationship commitment. Or, put differently, lack of trust has been identified as a major obstacle for collaboration (Ireland and Bruce 2000).

[2] Since trust is "a multi-faceted concept" (McEvily et al. 2003), definitions of trust abound (see Rotter 1967; Williamson 1993; Gambetta 1988; Lewicki and Bunker 1996). Throughout this treatise, we will refer to trust as a firm's "confidence in the goodwill" of an exchange partner (Ring and van de Ven 1994).

Table 1.2 Characteristics of buyer-supplier relationships (adapted from Monczka et al. 2005 and Schramm-Klein and Morschett 2006)

Attributes	Transactional procurement	Relational procurement
Contract type	Formal, multiple short-term contracts	Tendency towards informal agreements
Time horizon	Short term	Medium to long-term
Supplier base	Multiple sources played off against each other	One of a few preferred suppliers for each major item
Supplier costs	Low transaction costs and supplier switching costs	Rather high transaction costs and supplier switching costs
Focus of exchange	Price and transaction	Increasing importance given to technology, quality, (process) costs, and other services
Inter-organizational communication	Ad hoc information exchange	Regular information exchange
Problem solving behavior	Re-active conflict-solving behavior	Pro-active avoidance of conflict
Cost sharing	Buyer takes all cost savings, supplier hides cost savings	Win-win shared rewards
Joint improvement efforts	Little or none	Joint improvement driven by mutual interdependence

Besides trust, it is indisputable that successful collaboration requires creating the right incentives. Narayanan and Raman (2004) identify three main causes for incentive problems: Hidden actions by partner firms, hidden information like data or knowlege that only some of the firms in the supply chain possess, and badly designed incentives. They say that in the case where misalignment results from hidden actions, executives can bring those actions to the surface, by creating a contract that rewards or penalizes partners based on outcomes. In the same vein, Lee (2004) emphasizes that the best supply chains align the interest of all participating firms. Thus, as each player maximizes its own interest, it optimizes the chain's performance as well.

We close this section by giving an outlook on our buyer-supplier model. Our model studies buyer-supplier relationships with the help of relational contracting theory (see Chap. 3, p. 35). The underlying business relation may be characterized by the right column of Table 1.2. More precisely, we consider two firms, whose business relationship is long-term-oriented and provides room for informal agreements to induce effort. Like in Zhu et al. (2007), the focus of the buyer-supplier relationship is on quality improvement. Speaking in terms of Spekman et al. (1998), the firms have at least reached the stage of cooperation and are heading for collaboration: not only do they have sophisticated quality systems to exchange information on the current quality level, they also engage in joint effort for quality improvement and share resulting rewards. The challenge now lies in designing the buyer-supplier relationship in an optimal way. One might wonder, why the firms do not sign a contract that is contingent on the achieved quality level. In our framework, such contracts might not be enforceable by a third party, i.e., be infeasible, since quality does not depend deterministically on the induced efforts. Also note that our model makes assumptions on the underlying trust pattern. We assume that supplier and buyer employ trigger strategies. That is, a firm is willing to cooperate as long as the other firm cooperates. If cooperation breaks down, it will not be restored forever after. In terms of trust, our trigger strategy can be interpreted as follows: First of all, trust is a prerequisite for entering into informal agreements (if a firm presumes that the other firm tries to inflict damage on it, it will avoid informal agreements that are harmful for it in case of non-compliance). Second, trust will not be rebuilt, once it has been violated through refusal of cooperation.

1.4 Relational Procurement in Practice

It is no secret that procurement has gained in strategic importance. Globalization and the concentration on core competencies, accompanied by higher degrees of outsourcing, entails that buying firms highly rely on their suppliers as regards product innovation and quality. Adding the fact that these dimensions of procurement are often non-contractible, closer collaborations with strategic suppliers play a decisive role today, especially in competitively intense industries.

Monczka et al. (2005) note that the majority of buying firms within the United States are generally moving towards closer buyer-supplier relationships, yet find it difficult to take the ultimate step to collaboration. But there are also examples like IBM and American Airlines, which have moved aggressively towards collaboration, reducing their supply base and developing closer relations with remaining suppliers. A recent survey of over 350 global executives conducted by The Economist Intelligence Unit confirms that the trend towards supplier consolidation and deeper supplier relationships will continue in future (Jacoby 2005). A full 59% of survey respondents within procurement say that the company's total number of suppliers will have decreased further by 2015. Moreover, leaders responded that they are radically restructuring the nature of their relationships to support more collaborative supply chains, moving towards consensus on mutual objectives and co-investment in a long-term relationship. In this connection, growing emphasis is given to non-binding agreements, or, like the report puts it: "Contracts are out, while covenants are in".

A firm's industry also affects the degree of collaboration a firm establishes with its suppliers. Competitively intense industries, such as semiconductors, automotive, and electronics, tend to maintain deeper buyer-supplier relationships than less competitively intense industries (Monczka et al. 2005). Especially the high-tech industries depend heavily on collaboration with suppliers. Here, the capabilities of suppliers are leveraged to be able to drive product development, innovation, and fast cycle times in the supply chain.

For the aforementioned reasons, the systematic coordination, maintenance, and development of supplier relationships, collectively termed as Supplier Relationship

Management[3] (SRM), become the focal point of interest of procurement. Companies constantly refine their concepts for managing strategic supplier relationships (Handfield 2006), while IT providers like SAP and Oracle offer a range of SRM solutions for handling the procurement process.

1.4.1 Consumer Goods Industry

The following examples of relational procurement are taken from the consumer goods industry. The examples make clear that collaboration is on the agenda of both manufacturers and retailers and show how collaboration is actually realized in practice.

The increased efforts for collaboration in the consumer goods industry are reflected in two recent studies by the *Global Commerce Initiative* (GCI) and *ECR Europe*, both representing bodies of manufacturers and retailers on a global and European level respectively. For the GCI study, companies including Carrefour Group, Coca-Cola, Dairy Farm, DHL, Kraft Foods, Metro Group, Nestlé, Philips, Pick'n Pay, Procter & Gamble, Royal Ahold, Unilever, and Wal-Mart worked together to define a unique vision of the future value chain (Global Commerce Initiative 2006). The companies agreed that future challenges like issues of ecology, new technologies, the regulatory environment, and shifts in the global economy can only be addressed collaboratively by the members of the value chain. Joint efforts of trading partners include substantial changes in culture, collaborative business planning, information sharing, and new measures and rewards. "We can only do this by changing our cultures internally and rethinking the sustainability of the relationships that bind us", comments Ruud van der Pluijm, Vice President B2B eCommerce, Royal Ahold. "This will affect the development of our organizations and the rewards we use to identify new measures of performance. At the heart of the vision for 2016 is a fundamental principle of collaborative commercial trust". The study initiated by ECR Europe focuses on "Jointly Agreed Growth" (ECR Europe 2008). Starting from a consumer-centric approach, the goal is to reduce the time and effort spent negotiating on price, by bundling instead joint resources to bring innovations to market faster and more effectively. The study recommends three-year Jointly Agreed Growth plans (JAGs) with an annual review, which focus both manufacturer and retailer on mutually agreed growth targets. More precisely, joint business planning involves joint definition of the category strategy, the role of innovation, and growth criteria.

[3] The marketing counterpart "Customer Relationship Management" (CRM) may be more familiar to the reader. Indeed, the problems investigated in SRM and CRM literature are strongly related. As an example, take van Doorn and Verhoef (2008) who study satisfaction and loyalty issues in long-term customer relationships and show that critical incidents strongly affect the maintenance of customer relationships – an approach which is not too far away from our research considering amongst others the impact of quality on the maintenance of long-term supply relationships.

Metro Group has piloted a “Supplier Relationship & Collaboration Management Program” (SRCM) in 2007 (Rode 2008). After the advancement of EDI and RFID standards, Metro identifies SRCM as the next step towards collaboration with its major suppliers. Board member Zygmunt Mierdorf describes SRCM as “a strategic program to deepen supplier relationships and to achieve better solutions for the customers that way”. Metro has started the SRCM with Procter & Gamble, Coca-Cola, Kraft Foods, SCA, L’Oréal, Philips, Mattel, Red Bull, and Leifheit. The program has a global and long-term perspective. According to Mierdorf, the midterm target will be to manage the top 100 suppliers. According to the head of procurement, the definition of common goals with suppliers is at the core of the program. Here, the major topics are growth opportunities, expansion in new countries, improved services, new products, coordination of logistics, and quality of exchanged data. Another building block represent “Supplier Interest Groups”, where suppliers can propose focus topics for enhanced collaboration. Transparency is another important component, involving exchange of most important parameters and KPIs. The program puts particular emphasis on defined responsibilities: There are dedicated relationship managers per supplier, who meet regularly with the supplier to track and measure the performance on an international level. Not only Metro, but also its suppliers appreciate SRCM. Steve Newiss, Vice President Global Customers at Kraft International, particularly praises “the creation of a structured, long-term relationship”. He says that already shortly after the start of the SRCM collaboration, the common scorecard has established for both sides “one version of truth”. Similarly, Dr. Stefan Scholl, Vice President Customer Business Development at Procter & Gamble, positively points out that the relationship is geared towards continuity.

Johnson & Johnson has established a similar program, the so-called “Supplier Relationship Management Program”, which manages every aspect of the relationship between Johnson & Johnson and its suppliers (Thomson et al. 2008). This program is applied, for instance, in the relationship with its strategic supplier LEK Inc. Recognizing that Johnson & Johnson has as much influence as suppliers on the success or failure of the relationship, the company quarterly reviews the relationship on the basis of a two-way scorecard, enhanced by a quality audit and a review of improvement plans. Moreover, the program offers resource-sharing models for strategic suppliers and annual supplier recognition conferences to expose peer performance and best practices to the supplier base. A two-way scorecard is a bilateral performance measurement tool: It measures supplier and buyer results across a balanced set of categories, thereby enabling both sides to take joint responsibility for improvement. This type of scorecard can be regarded as a tool to achieve “dual accountability between a buyer and its strategic suppliers”, a relatively new approach used amongst others by The Home Depot Inc. to improve supply chain relationships (Slobodow et al. 2008).

Creating sustainable supply chains has forced many companies to rethink their relationships with suppliers. *Starbucks Corporation* is an example of how increasing focus on sustainable sourcing brings about fundamental changes in the complete supply chain, including a longer-term view of supply relationships (Lee et al. 2007). Together with Conservation International, an environmental non-profit organization, Starbucks developed so-called C.A.F.E. Practices (Coffee and Farmer Equity Practices) for environmentally sound, socially responsive, and sustainable supply of high quality coffee beans. Starbucks' main objectives were to reduce price and supply volatility of high quality coffee beans, improve its reputation among suppliers, and thereby gain a competitive advantage over other coffee roasters in the industry. The solution provided by C.A.F.E. Practices was to create longer-term supply relationships with a consistent set of strategic and high quality suppliers, offering preferential contract terms to those suppliers that best met the program's criteria.

1.4.2 Automotive Industry

The automotive industry is currently coined by high degrees of outsourcing and increasing supplier consolidation. OEMs (Original Equipment Manufacturers) depend heavily on suppliers as regards innovation and quality, and at the same time they are challenged to prevail within the limits of product differentiation and cost reduction. Seen against this background, it is particularly interesting to study buyer-supplier relationships in the automotive industry.

First, we will compare OEMs' purchasing philosophies on a global level. The example of the US auto industry makes clear that buyer-supplier relationships are not set in stone, but are constantly redefined. Ro et al. (2008) suggest that the "Big Three", Ford, General Motors und Chrysler, have moved to a hybrid of partnerial and adversarial supplier relationships after a phase of rapprochement in the 1990s. Contrary to this trend in US auto, recent studies predict that deeper relationships with strategic suppliers will be indispensable to stay competitive in future (The Boston Consulting Group 2004; Mercer Management Consulting and Fraunhofer 2004).

OEM–Supplier Relationship Styles

To understand to which degree the automotive industry shares a relationship-based view of procurement, we have collected assessments of leading OEMs. At first sight, all OEMs stress the importance of long-term supplier relationships nowadays. However, as we will see afterwards, OEMs still adopt different approaches to Supplier Relationship Management.

On the occasion of DaimlerChrysler's "Global Supplier Meeting" in July 2006, Dr. Dieter Zetsche, Chairman of the Board of Management, emphasized the company's commitment to foster strong, open and mutually successful relationships with its suppliers. He stressed that the company continued to optimize its organization for even more collaboration and would also be providing excellent business opportunities for top performing suppliers worldwide. Moreover, Thomas Sidlik, Member of the Board of Management of DaimlerChrysler, responsible for Global Procurement and Supply, underlined the importance of innovation and flexibility for mutual success: "Innovation and flexibility are key factors in dealing with global competition and for generating long-lasting, successful partnerships between DaimlerChrysler and its suppliers" (DaimlerChrysler 2006).

According to Dr. Herbert Diess, Member of the Board of Management of BMW AG, responsible for Purchasing and Supplier Network, BMW places particular emphasis on quality in its supplier relationships: "Our primary goal is quality. We perform very good by comparison, but we want to get even better. When we agree on objectives with our suppliers, we talk about quality improvements first and only afterwards about cost reductions" (Automobilwoche 2008).

In his address to the "Porsche Supplier Award 2008", Dr. Wiedeking, President of Porsche AG, pointed out that the Porsche success story could not be imagined without efficient and reliable suppliers. According to Wiedeking, Porsche suppliers take on a high level of responsibility due to the company's low vertical range of manufacture of about 15% (Beschaffung aktuell 2008). Porsche presented the award to Bose Automotive Systems Division, a supplier which has been developing and producing premium sound systems for each model produced by Porsche since 2001. Bose was able to fulfill this responsibility and meet Porsche's highest quality standards and strictest deadlines. Brandon Westley, President of the Bose Automotive Systems Division, underlined that "this achievement is the result of tremendous collaborative efforts, and both companies' mutual commitment to excellence" (Reuters 2008).

A look at Fig. 1.2 helps to better classify the statements just cited. In 2003, suppliers from Europe, Japan, and North America were asked to evaluate the negotiation styles of leading OEMs (The Boston Consulting Group 2004). Each OEM's purchasing philosophy was assessed along two major dimensions: bargaining through market power and bargaining through technology and process analysis.

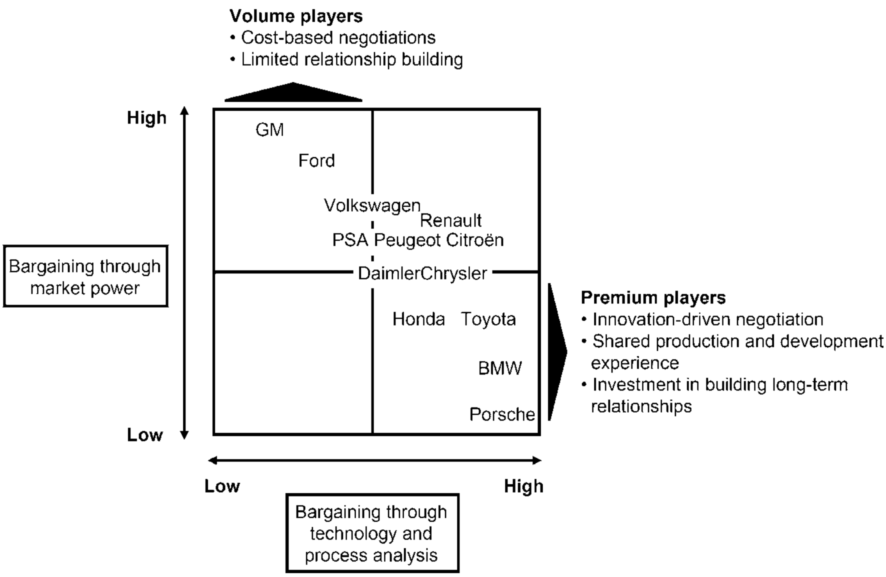

Fig. 1.2 OEMs' purchasing philosophies (adapted from The Boston Consulting Group 2004)

With the former approach, OEMs use their sourcing volume to exert unilateral price pressure. With the latter approach, OEMs analyze product contents and production processes together with their supplier in order to jointly identify opportunities to reduce costs – apparently an approach more suitable to foster collaboration. The interviews uncovered three types of purchasing philosophies. The first group of OEMS comprised the volume players Ford, GM, and Volkswagen, which ranked high on the dimension "bargaining through market power" and considerably lower on the dimension "bargaining through technology and process analysis". The report does not only capture the critical attitude of suppliers towards such negotiation practices, but also observes that the OEMs of this group were starting to implement product- and process-redesign initiatives with their key suppliers at that time: "Team Value Management" at Ford and "Partner Process Optimization" at Volkswagen. For the second group of OEMs, namely BMW, Honda, Porsche, and Toyota, the picture was totally reversed. In this group, OEMs focused strongly on technology-based negotiation and process analysis. For BMW and Porsche, this is presumably due to their limited volume leverage and strong engineering focus. For Honda and Toyota, this cooperative negotiation style can be attributed to the Japanese "keiretsu" system, characterized by the close ties exhibited between the buying firm and its direct suppliers. The third group contained OEMs such as DaimlerChrysler, PSA Peugeot Citroën, and Renault. In the suppliers' view, these OEMs were adopting a differentiated sourcing approach, practicing a cooperative negotiation style for critical, brand-differentiating systems and components on the one hand, and using more cost-driven negotiations for standard parts on the other hand.

The US Model

Along the same line, Ro et al. (2008) study the dynamics of buyer-supplier relationships in the US automotive industry. Between 1998 and 2001, the authors interviewed senior engineers, managers, and directors in the design, production, and product development departments of first-tier automotive suppliers as well as major US automakers, i.e., the Big Three. The study suggests that there is a new emerging US model of supplier management that can be characterized as "close but adversarial". Apparently, US automakers have moved back to a more adversarial interpretation of supplier management, after a trend towards collaboration in the 1990s. We will summarize the main findings of the study in the following.

Traditionally, Japanese OEM–supplier relationships are characterized as long-term partnerships that provide a solid basis for cooperation in key areas like quality management and product development. By contrast, US OEM–supplier relationships are often described as adversarial relationships resorting to coercive bureaucratic mechanisms. When looking at recent history, however, a far more differentiated picture emerges.

During the 1950s, both Japanese and US automakers were still vertically integrated (Nishiguchi 1994). The Japanese automakers, however, began to adopt a subcontracting strategy with their suppliers. This trend was primarily due to a more

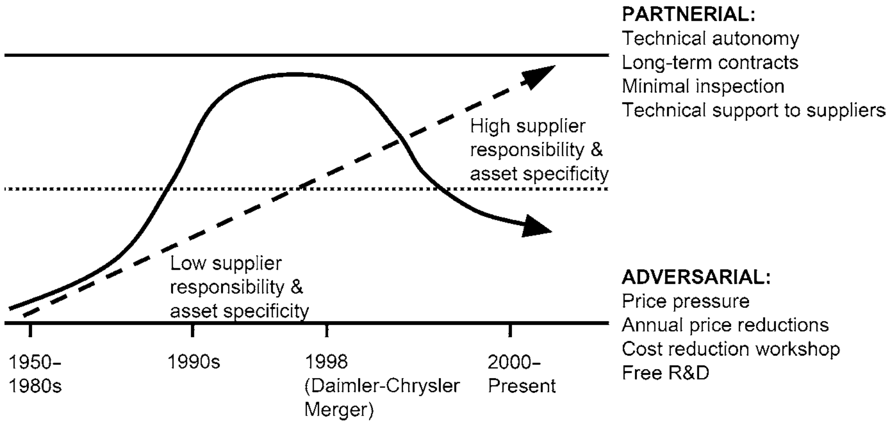

Fig. 1.3 Relationship evolution in the US auto industry (adapted from Ro et al. 2008)

organized labor force, lack of capital, and lack of financing for new capacity at that time. Successively, Japanese automakers established supplier relationships that were governed not primarily by market and hierarchy, but by trust (Bensaou and Anderson 1999). In addition to contract mechanisms, Japanese supplier relationships were strongly supported by equity holdings and the tremendous purchasing power of automakers. Finally, the Japanese automakers gave their suppliers extensive responsibility for the quality of parts and components.

When US automakers noticed the Japanese success in the late 1980s, they started to imitate the partnerial supplier management model. As illustrated by Fig. 1.3, US automakers moved towards closer supplier relationships in the 1990s. In 1998, at the beginning of the study, Chrysler was the closest to the partnerial model with its "Supplier Cost Reduction Effort" program (SCORE), GM stayed closest to the traditional adversarial model, with Ford taking a middle position. At the end of the study, however, the move towards more adversarial relationship styles became apparent, even in relationships with so-called "partner" suppliers. Also favored by the Daimler-Chrysler merger in 1998, that was accompanied by the replacement of most key executives, DaimlerChrysler began to move back towards the adversarial relationship style by the late 1990s and early 2000s. One contributing factor for this reversion was the discovery that some suppliers were opportunistically charging DaimlerChrysler more than their other Big Three customers.

According to Ro et al. (2008), one observes major differences, when comparing buyer-supplier relationships in the American and Japanese auto industries today. US companies concentrate on minimizing risk, use market mechanisms to motivate suppliers to reduce price, and closely monitor suppliers on key performance indicators. Complex tasks that might create a high degree of dependency on an outside supplier tend to be in-housed. Japanese automakers, however, source relatively whole tasks to outside suppliers, and then build effective mechanisms for integrating the suppliers into the product development process.

Ro et al. (2008) conclude that the US OEM purchasing strategy is still in a state of transition and describe the current supplier management model as a "hybrid of market control and relational contracting". The authors even worry that the Big Three approach to supplier relationship management will become the dominant organizational design in America. Namely for the following reasons: First, the current American automaker practices seem inappropriate to support the great responsibility assigned to suppliers. For instance, one may still observe adversarial relationships in US auto, when a partnership model is required for true integration of design efforts. Second, the US supplier model has not only resulted in general lack of trust, but has also broken technical and organizational systems that will not go away just by coercive market forces.

The Japanese Model

The Japanese supplier management model demonstrates how companies can create extraordinary value by fostering long-term, partnerial relationships with their suppliers. According to Dyer (1996), the best-practice models for supply chain management from a performance point of view still remain those based on the Japanese "keiretsu" relationships. And, although other OEMs have tried to imitate the Japanese model reinforcing their lean and six sigma programs, Japanese automaker plants continue to outperform the US plants from productivity and quality perspectives (Liker et al. 1999).

As already mentioned above, Japanese automakers are known to outsource large portions of the vehicle, even modules and systems. They put particular emphasis on partnerial relationships with their suppliers, but not surprisingly, the types of supplier relationships may differ. Japanese firms distinguish simple build-to-print component suppliers, module suppliers, and system suppliers. This implies that negotiations with simple build-to-print component suppliers are rather customer dominant. For modules, Japanese firms make an investment in the supplier and do not seem too concerned about risk, and for system suppliers, a partnership model is employed (Ro et al. 2008).

A recent study by The Boston Consulting Group (2007) explores Toyota's approach towards Supplier Relationship Management. The authors observe that Toyota puts considerable effort in helping its suppliers improve their performance. For this purpose, Toyota employs a few key tools:

1. Toyota monitors its suppliers' performance extensively and insists that senior managers and key executives of each supplier organization assume responsibility for all quality and performance issues.
2. Every six months, Toyota performs thorough quality audits.
3. Toyota employs proprietary processes that facilitate the rapid resolution of quality problems and prevent problems from recurring.
4. A robust knowledge-sharing network ensures that suppliers share their best practices and thereby enhance their capabilities.

A supplier entering an ongoing relationship with Toyota can be assured that Toyota will take responsibility for helping this supplier develop its capabilities and will ensure this supplier earns reasonable returns. However, suppliers should never take their relationships with the company for granted. Suppliers that perform well, will be rewarded with increased business, but those that disappoint Toyota will lose their business. Moreover, Toyota has repeatedly demonstrated that it cares for suppliers going through difficult times. During the financial crisis in Thailand in 1997, for example, Toyota helped its suppliers financially by making large up-front payments. Toyota also gave its suppliers in Thailand automatic price increases, when urgently needed. Immediately after the crisis, when suppliers were unable to meet their cash-flow requirements, Toyota increased its prices by 8% and then by another 5% half a year later, while other OEMs did not support their suppliers during the crisis.

But what makes the so-called "Toyota Way" of close supplier relationships so successful? And why do US automakers not simply replicate this apparently so successful model? One major reason certainly is that "the Toyota Way is first and foremost about culture [. . .]. At the core it is about respect for people and continuous improvement, and this has not changed since the company's founding" (Liker and Hoseus 2008). Hence, if American companies want to learn from the Toyota Way, they should take into account that the foundation of this approach is long-term thinking. In fact, Liker and Hoseus (2008) observe that the biggest barriers in American companies wishing to implement the Japanese model result from their short-term orientation and need for every action to pay for itself very quickly.

Future Trends Affecting the OEM–Supplier Interface

The automotive industry is becoming a leaner, fast-paced, and more competitive industry. Automakers are constantly reducing product development cycles and advances in information technology such as the Internet and e-business initiatives have brought greater speed of communication and data exchange (Liker and Hoseus 2008). Today, automakers buy more components and services from suppliers than they used to. According to Mercer Management Consulting and Fraunhofer IPA (2004), there will be another significant shift in the proportions of value-added to suppliers: Globally, automakers will reduce their proportion of value-added from ca. 35% in 2004 to ca. 25% in 2015. All these developments will affect the OEM–supplier interface in the future. On the whole, one may identify ten major trends impacting OEM–supplier relationships over the coming years, suggesting that OEMs will have to reframe their supply relations to become more partnerial (The Boston Consulting Group 2004):

1. *Supplier consolidation*
   The number of tier one and tier two suppliers worldwide is likely to shrink from between 1,500 and 2,000 in 2004 to between 500 and 700 by the end of the decade, of which only some 100 will be system integrators that deal directly with the OEMs.

2. *Suppliers as system integrators*
   As suppliers increasingly assume the role of system integrators, OEMs will need to include engineers in purchasing staffs. Moreover, systems are much harder to price and evaluate than simple parts.
3. *Suppliers as drivers of innovation*
   The suppliers' responsibility for innovation is constantly growing, which increases the dependence of OEMs on their suppliers.
4. *Partnership programs*
   Various partnership programs launched by OEMs to foster closer relationships with suppliers often fail to create the promised value.
5. *New business models*
   New business models (like pay-on-production models and supplier parks) gain in importance. OEMs will have to develop new competencies to handle these models effectively.
6. *Slower-than-anticipated adoption of e-procurement*
   The share of parts that can be sourced easily by online bidding is much lower than anticipated and processes must become more consistent.
7. *Shortening innovation cycles*
   This trend brings about that OEMs and suppliers get involved in each other's design and development processes much earlier than before.
8. *Increasing challenges on quality*
   To protect the OEMs' brand image as well as suppliers' finances, firms will have to work together to reduce the number of costly recalls.
9. *Increasing product differentiation*
   Proliferation of vehicle types entails that purchasing organizations must buy smaller lots of components.
10. *Global sourcing*
    Major challenges associated with global sourcing range from local supplier development to quality control and logistics.

Recent studies agree that in order to face the challenges ahead, OEMs and suppliers will have to rely increasingly on long-term partnerial relationships that distribute chances and risks in a fair manner (Mercer Management Consulting and Fraunhofer IPA 2004).

Another interesting question is the future interplay between relational procurement and e-marketplaces. First of all, Mitra and Singhal (2008) find that the announcement to form or join industry exchanges has a positive effect on shareholder value. The authors study consortium based industry exchanges and reveal that the abnormal market reaction to a founder's announcement to form an exchange is about 1% and significant. It remains to be answered how firms can reap the benefits of market-based coordination and at the same time preserve the value associated with long-term supply chain relationships - according to Grey, Olavson and Shi (2005) one of the greatest challenges associated with the introduction of e-marketplaces.

In fact, buyers benefit from participating in online exchanges through the inventory-pooling effects resulting in reduced costs, as Milner and Kouvelis (2007) show. But the authors also suggest that “a supplier acting strategically will counteract such benefits by restricting availability of goods to the spot market, sacrificing short-term spot-market revenue for long-term contract volume”.

Chapter 2
Literature Review on Supply Chain Contracts

This chapter gives an introduction to the supply chain contracting literature. Departing from a classic supply chain model, we classify the parameters over which supply chain contracts are usually observed. The focus of this chapter is on contracts coordinating a two-firm supply chain in a single-period newsvendor setting. Hence, a basic newsvendor model as well as contracts coordinating this setting are introduced. Moreover, extensions to the basic newsvendor model and current research on supply chain contracts are presented. Particular attention is given to research on QF and buy-back contracts. The goal is to contrast these two returns mechanisms in the light of formal supply chain contracting to provide a basis for later analysis and discussion in the context of relational contracting.

2.1 Introduction to Supply Chain Contracts

"Supply Chain Management deals with the management of material, information, and financial flows in a network consisting of vendors, manufacturers, distributors, and customers" (Anupindi and Bassok 1999a). Exchange of flows can be regarded as a routine transaction, occurring between any pair of suppliers and buyers in the network. Ideally, the quantity and pricing decisions in the supply chain, as shown in Fig. 2.1, would be made by a single decision maker who has all information at hand. Researchers in Supply Chain Management generally refer to this situation as the *centralized* or *integrated* supply chain and call the single decision maker the integrated firm. Respectively, a supply chain is called *decentralized* if the network consists of multiple decision makers having different information and incentives. Due to globalization and outsourcing, decentralized supply chains are prevalent today. Outsourcing of production, for example, automatically spreads decision rights among multiple decision makers. And often, even highly vertically integrated firms decentralize decision rights to set incentives and structure the flow of information.

To measure the performance of supply chains, coordination is an important assessment criterion. The terms *network, channel* or *supply chain coordination* all refer to the same situation: "A single decision maker optimizes the network with the union of information that the various decision makers

M.I. Höhn, *Relational Supply Contracts*, Lecture Notes in Economics and Mathematical Systems 629, DOI 10.1007/978-3-642-02791-8_2,

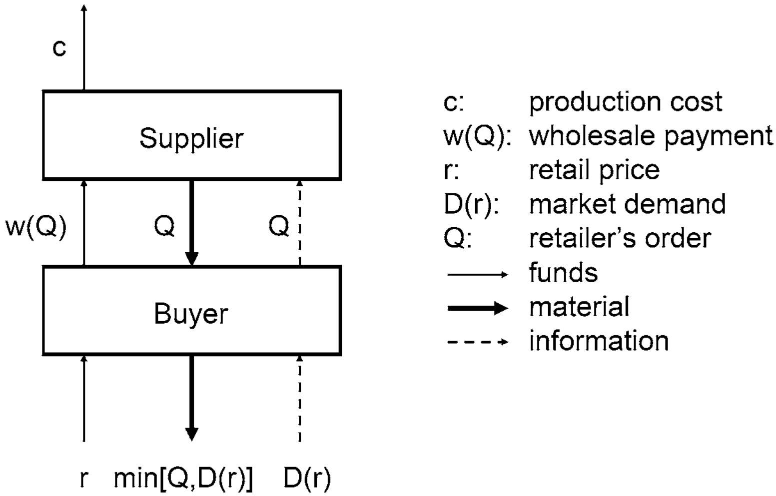

Fig. 2.1 The basic one-period supply chain model (adapted from Tsay et al. 1999b)

have" (Anupindi and Bassok 1999a). Coordination - and hence supply chain performance - is at risk as soon as there are multiple decision makers in the network who may have different private information and incentives. For instance, decision makers are often reluctant to share private information regarding cost and demand, which may lead to suboptimal supply chain performance (Corbett and Tang 1999; Corbett et al. 2004). Even if information asymmetry can be ruled out, lack of coordination and hence suboptimal supply chain performance may still occur. Since each decision maker optimizes a private objective function, the local optima need not be globally optimal for the whole supply chain. The problem of *double marginalization* is a prominent example of this phenomenon, first described by Spengler (1950) in the Economics literature. It can be shown that when operating independently, supplier and buyer will produce less than a vertically integrated monopolist, because they receive less than the total contribution margin at any given quantity (Tirole 1990). This clearly is a case where locally optimal decisions of supplier and buyer do not optimize the global supply chain problem. Or, in other words, the decentralized supply chain is inefficient, since the total expected profit Π^d of the decentralized supply chain is smaller than Π^c, the expected profit of the centralized supply chain.

To enable coordination, the supply chain resorts to contracts. In general, the goal is to write contracts that induce coordination through appropriate provisions for information and incentives such that supply chain performance will be optimized. This type of approach recurs in a broad range of settings. Cachon (2003) and Chen (2003) review the respective research on supply chain contracts. Early overviews on supply chain coordination with contracts were given by Whang (1995), Cachon (1999), Lariviere (1999), and Tsay et al. (1999b). Beyond, similar approaches can be found in related fields of research like the Economics literature on Vertical

Restraints (Mathewson and Winter 1984; Katz 1989) and the Marketing literature on Channel Coordination (Jeuland and Shugan 1983; Moorthy 1987). Tightly linked are the papers by Bergen et al. (1992), and van Ackere (1993), who study agency relationships.

We record that an important objective of supply chain contracts is system-wide performance improvement. Another motive that is pursued by entering into supply chain contracts is sharing the risk arising from the uncertainty in the supply chain (in the contracting model presented in this chapter, the notion of risk should be handled with care, given that the firms are assumed to be risk neutral; the model maximizes expected profits and does not study risk hedging). After Tsay et al. (1999b), another contracting motive is facilitating long-term relationships. By entering persistent business partnerships, supplier and buyer can reduce transaction costs since costly searches and renegotiations are reduced.

The premise of this chapter is that the management of buyer-supplier interactions within a supply chain is governed by formal contracts. Furthermore, most of the papers presented restrict to single-period models. Relaxing this constraint to incorporate more realistic settings with repeated buyer-supplier interactions will be very useful in understanding how to optimally structure the material, information, and financial flows in a supply chain. This perspective on supply chain contracts will be the focus of the following chapters.

2.1.1 Supply Chain Structure

In this section, we describe a one-period supply chain model that forms the basis for a wide range of supply chain analyses (Tsay et al. 1999b). Figure 2.1 depicts two consecutive nodes of the supply chain, here referred to as supplier and buyer, together with the material, information, and financial flows involved. As shown in Fig. 2.1, standard simplifying assumptions in the Supply Chain Management literature can be summarized as follows: In a one-period framework, the supplier produces or acquires a product at a constant unit cost of c and charges the buyer the wholesale or transfer payment $w(Q)$ per delivery, where $w(Q)$ may either be exogenous or a decision variable of one of the parties. On the other side, the buyer sells the product to the market at retail price r per unit.

In reality, market demand $D(r)$ is both price-sensitive and uncertain. Although some models include both features, it is common to fix either the order quantity or the retail price. In the Operations Research literature, the primary decision variable is the order quantity Q, the retail price is often assumed to be fixed, and market demand is stochastic. In the Economics and Marketing literature, however, the decision is primarily the retail price r. In the latter case, a common assumption is a deterministic, downward-sloping demand function. Moreover, most papers on supply chain contracts assume only a one-period problem, since the related models are often too complex to be tractable in a multi-period setting.

2.1.2 Classification of Contracts

Typically, a supply chain contract should capture the three types of flows encountered between the members of a supply chain, i.e., material, information, and financial flows. Yet, classifying supply chain contracts is not straightforward. To date, no commonly accepted taxonomy appears to exist. Anupindi and Bassok (1999a), for example, classify supply chain contracts according to eight contract parameters: Horizon length, pricing, periodicity of ordering, quantity commitment, flexibility, delivery commitment, quality, and information sharing. In contrast, Tsay et al. (1999b) classify the literature on supply chain contracts by eight contract clauses including specification of decision rights, pricing, minimum purchase commitments, quantity flexibility, buy-back or returns policies, allocation rules, lead time, and quality. Without preferring one classification over the other, we specify the latter one here. In addition, we explain the contract terms horizon length, periodicity of ordering, and information sharing, since these play an important role in relational supply contracts:

1. *Specification of decision rights*
   Here, reassigning control of the decision variables is at the center of the analysis. Exemplary agreements are Resale Price Maintenance or Quantity Fixing. Although the buyer typically chooses the order quantity Q and the retail price r given the transfer payment $w(Q)$ specified by the supplier, these mechanisms shift control to the supplier. Moreover, the issue of assigning control to local or global entities may also be interpreted as an issue of decision rights.
2. *Pricing*
   This contract category considers $w(Q)$, the financial component of the contract between the supplier and the buyer. Often, the wholesale price can be written as $w(Q) = F + w_t Q$ for constants F and w_t. Linear Pricing implies $F = 0$, perhaps the most commonly assumed pricing structure. In contrast, a positive F, also called a franchise fee, results in two-part tariff pricing. Also more complex pricing schemes, such as quantity discounting, may fall into this category.
3. *Minimum purchase commitments*
   Such an agreement requires the buyer to purchase a minimum quantity, either within each single transaction, or cumulatively over a specific time horizon. The supplier may reduce $w(Q)$ to provide an incentive to the buyer to agree to this arrangement.
4. *Quantity flexibility*
   In this type of supply chain contract, the buyer is granted a certain degree of quantity flexibility in the sense that the ultimate purchasing quantity may deviate from the initial order quantity.
5. *Buy-back or returns policies*
   A buy-back clause specifies that the buyer may return a percentage of unsold goods to the supplier. The buy-back price will typically be less or equal to the wholesale price. As in the case of quantity flexibility, mismatches between the

buyer's purchase and the market demand are only of interest when demand is assumed random.

6. *Allocation rules*
   This stream of contracting literature investigates the allocation of the supplier's available stock or production capacity among multiple buyers in a shortage scenario.
7. *Lead time*
   The lead time for delivery of the product from the supplier to the buyer may also be a contractual clause. In traditional inventory models, the lead time is either treated as a fixed constant or the realization of a random variable.
8. *Quality*
   Quality of the delivered product is a major prerequisite of any supply relationship. The specific dimensions of quality may be specified within the supply chain contract.
9. *Horizon length*
   The horizon length specifies the duration for which the contract is valid.
10. *Periodicity of ordering*
    This contract term specifies how often the buyer can place orders. One distinguishes fixed and random periodicity of ordering. The former term means that the buyer may place orders only on predetermined dates (it is not essential that orders are non-zero). The latter one means that orders can be placed any day of the week.
11. *Information sharing*
    This contract term specifies what type of information will be shared between buyer and supplier.

2.1.3 The Newsvendor Model

One of the standard building blocks for modeling order quantity decisions under stochastic demand is the newsvendor model. Since the newsvendor model represents the starting point for the analysis of supply chain contracts, a standard one-period one-product model will be introduced in this subsection (Cachon 2003). For more extensive treatments of the newsvendor model see Silver et al. (1998) or Nahmias (2008).

The sequence of events is:

1. The supplier offers a contract to the buyer.
2. The buyer accepts or rejects the contract. Under the assumption that the buyer accepts the contract, the buyer orders a quantity Q.
3. The supplier produces and fills the buyer's order before the start of the selling season.
4. Season demand materializes.
5. Finally, transfer payments are made between the firms according to the agreed contract.

In the newsvendor model, the action to coordinate is the buyer's order quantity. Facing stochastic demand, the buyer must determine an order quantity Q before the start of the selling season. As before, let r be the retail price and $D > 0$ be the demand realization during the selling season. Let F be the distribution function of demand with f being its density function. Standard assumptions are that F is differentiable, strictly increasing, and $F(0) = 0$. Furthermore, let $E[D]$ denote expected demand. The supplier's production cost per unit is c_s and the buyer's marginal cost per unit is c_b with $c_s + c_b < r$ (note that these definitions of F, c_s, and c_b only refer to this subsection). Goodwill penalty costs are incurred for every unit of unsatisfied demand: g_b for the buyer and analogously g_s for the supplier. For notational convenience, define $c = c_s + c_b$ and $g = g_s + g_b$. The buyer earns $s < c$ per salvaged unit at the end of the selling season. A standard assumption here is that the supplier's salvage value does not exceed s. Hence, one can assume that all left over inventory is salvaged at the buyer.

To continue with the description of the model, assume that each firm is risk neutral. That is, each firm maximizes expected profit. Let $S(Q)$ denote expected sales $\min(Q, D)$. Then $S(Q)$ can be written in the form:

$$\begin{aligned} S(Q) &= Q(1 - F(Q)) + \int_0^Q yf(y)dy \\ &= Q - \int_0^Q F(y)dy. \end{aligned}$$

Similarly, let $I(Q)$ be the expected left over inventory. Then:

$$I(Q) = (Q - D)^+ = Q - S(Q).$$

Let $L(Q)$ be the lost sales function:

$$L(Q) = (D - Q)^+ = E[D] - S(Q).$$

Furthermore, let P denote the expected transfer payment from the buyer to the supplier. This transfer payment will depend on the agreed supply chain contract.
The buyer's profit function is:

$$\begin{aligned} \pi_b(Q) &= rS(Q) + sI(Q) - g_b L(Q) - c_b Q - P \\ &= (r - s + g_b)S(Q) - (c_b - s)Q - g_b E[D] - Q. \end{aligned}$$

The supplier's profit function is

$$\pi_s(Q) = g_s S(Q) - c_s Q - g_s E[D] + P,$$

and the total profit of this two-firm supply chain is

$$\Pi(Q) = \pi_b(Q) + \pi_s(Q) = (r - s + g)S(Q) - (c - s)Q - gE[D].$$

Let Q^c be the supply chain optimal order quantity in the centralized supply chain with $Q^c = \arg\max_Q \Pi(Q)$. Analogously, let Q^d be the buyer's optimal order quantity in the decentralized supply chain, i.e., $Q^d = \arg\max_Q \pi_b(Q)$. Clearly, the buyer's order Q^d will depend on the transfer payment P determined by the agreed contract. It can be shown that the wholesale price contract generally does not optimize total expected supply chain profit. More complex contracts, e.g., buy-back and quantity flexibility (QF) contracts, may achieve coordination. These contracts typically combine a wholesale price with an adjustment depending on realized demand. Various contract types and their ability to coordinate the supply chain will be discussed in the following subsection.

2.1.4 Contract Types

We have already mentioned the drawbacks of wholesale price contracts regarding supply chain coordination. The goal of this subsection is to present supply chain contracts that find a remedy, i.e., that coordinate the newsvendor model presented in the previous subsection. To describe the different contract types, we will refer primarily to Cachon (2003) and adopt the notation introduced above. Moreover, equivalences and parallels between the different contract types will be discussed. The remainder of the subsection touches on related research and recent research papers studying extensions to the standard newsvendor model.

To evaluate a contract's strengths and weaknesses, the following criteria are of particular importance (Cachon 2003):

1. *Supply chain coordination*
   In the standard newsvendor model, the action to coordinate is the buyer's order quantity. That is, the buyer's quantity decision should optimize total supply chain profit and no firm should have a unilateral incentive to deviate from supply chain optimal actions.
2. *Arbitrary split of supply chain profit*
   A supply chain contract should offer sufficient flexibility to allow for any division of total supply chain profit. This is an important contract feature: If contract parameters can be adjusted to allocate rents arbitrarily, there always exists a contract that Pareto dominates a non-coordinating contract.
3. *Administrative costs*
   Not least, there will be a tradeoff between the *efficiency* of a supply chain contract (the ratio of supply chain profit under the respective contract to the optimal supply chain profit) and the administrative costs associated with it. Administrative costs are usually driven by the type and extent of material and informational flows specified by the contract.

Although the newsvendor model presented in Sect. 2.1.3 is not complex, it is sufficiently rich to study these three important criteria. In this framework, several different contract types can be shown to achieve coordination and to arbitrarily divide supply chain profit. As we will see in the following, these include revenue-sharing contracts, buy-back contracts, quantity flexibility contracts, sales-rebate contracts, and quantity discount contracts. As regards administrative costs, the wholesale price contract and the quantity discount contract will turn out to be equally costly to administer, since they only require a single transaction. Other supply chain contracts like revenue sharing, buy-back, or QF contract are more costly to administer, since they require additional material or informational flows between the firms. While the third criterion may explain the selection of a wholesale price contract or a quantity discount contract, it is more difficult to explain contract selection among revenue sharing, buy back or QF contract in practice.

Wholesale Price Contract

With a wholesale price contract, the buyer is charged a wholesale payment of w per unit purchased. Thus, the transfer payment P between buyer and supplier takes a simple form:

$$P_w(Q, w) = wQ.$$

Lariviere and Porteus (2001) provide a complete analysis of the wholesale price contract in the context of the newsvendor problem, while Bresnahan and Reiss (1985) study the contract under the assumption of deterministic demand. It can be shown that the wholesale price contract coordinates the newsvendor problem only if the wholesale price is smaller or equal to the supplier's production cost, i.e., only if the supplier's profit is non-positive, which is clearly not worthwhile for the supplier.

To quantify the performance of wholesale price contracts, researchers in Supply Chain Management have applied several performance measures to this contract type. These are typically worst-case analyses of supply chain performance. Lariviere and Porteus (2001), for example, quantify the efficiency of this contract type under various demand distributions. Perakis and Roels (2007) measure efficiency with the so-called *Price of Anarchy* (PoA). This performance measure computes the ratio of the performance of a centralized system over the worst performance of a decentralized system. The authors define the PoA as

$$\text{PoA} = \sup_{F \in \mathcal{F}} \frac{-cQ^c + rE[\min(Q^c, D)]}{-cQ^d + rE[\min(Q^d, D)]},$$

where Q^c is the optimal solution of the centralized supply chain, Q^d the quantity decision in the decentralized supply chain, and $\mathcal{F}$ the set of nonnegative demand distributions that have the IGFR property.[1] Perakis and Roels show that the PoA is at least 1.71 for a two-firm supply chain model, concluding that the inefficiency

[1] IGFR stands for increasing generalized failure rate. This assumption is commonly imposed on the demand distribution because it guarantees that the supplier's profit is well-behaved in the newsvendor problem (see Lariviere and Porteus 2001; Lariviere 2006).

of wholesale price contracts has not been overstated in the literature and justifies the whole stream of research on the design of more elaborate contracts improving coordination in supply chains.

Contrary to one-period models, Debo and Sun (2004) show that with repeated interaction, supply chain efficiency can be achieved with the wholesale price contract, given a sufficiently high discount factor. Cui et al. (2007) in turn consider fairness concerns. They demonstrate that when channel members are concerned about fairness, the supplier can use a wholesale price above her marginal cost to coordinate the channel both in terms of achieving the maximum channel profit and in terms of attaining the maximum channel utility.

Buy-Back Contract

With a buy-back contract, the buyer purchases Q units for the price w_b per unit at the start of the season and may return up to Q units at the end of the season for a refund of $b < w_b$ per unit. This yields:

$$P_b(Q, w_b, b) = w_b Q - bI(Q) = bS(Q) + (w_b - b)Q.$$

Pasternack (1985) was the first to identify that buy-back contracts coordinate the fixed-price newsvendor, while allowing for any split of total supply chain profit. For the set of buy-back parameters $\{w_b, b\}$ such that for $\lambda \geq 0$,

$$\begin{aligned} r - s + g_b - b &= \lambda(r - s + g) \\ w_b - b + c_b - s &= \lambda(c - s) \end{aligned}$$

it can be shown, that the buy-back contract coordinates the supply chain as long as $\lambda \leq 1$ (also see Cachon 2003). Moreover, Cachon and Lariviere (2005) show that buy backs are equivalent to revenue-sharing contracts, when the retail price is fixed. Interestingly, the two contracts are no longer equivalent under the assumption of a price-setting newsvendor. There is a substantial literature on buy-back contracts, which will be discussed in Sect. 2.3.

Revenue-Sharing Contract

With a revenue-sharing contract, the buyer pays w_r per unit purchased. In addition, the supplier is granted a fraction $(1 - \phi)$ of the buyer's revenue. The buyer, in turn, keeps a fraction ϕ of his revenue. Under the assumption that all revenue is shared, including the salvage revenue, the transfer payment with revenue sharing is

$$P_r(Q, w_r, \phi) = (w_r + (1 - \phi)s)Q + (1 - \phi)(r - s)S(Q).$$

Like buy backs, revenue-sharing contracts are able to coordinate the fixed-price newsvendor and arbitrarily divide the resulting profits. Consider the set of revenue-sharing parameters $\{w_r, \phi\}$ such that for $\lambda \geq 0$,

$$\phi(r - s) + g_b = \lambda(r - s + g),$$
$$w_b + c_b - \phi s = \lambda(c - s).$$

This set of revenue-sharing contracts induces the buyer to choose the optimal order quantity as long as $\lambda \leq 1$. Moreover, revenue-sharing contracts are equivalent to buy backs under the assumption of a fixed-price newsvendor. With revenue sharing, the buyer pays $w_r + (1 - \phi)s$ for each unit purchased and $(1 - \phi)(r - s)$ for each unit sold. This is equivalent to the coordinating buy-back contract $\{w_b, b\}$, which can be interpreted as paying $w_b - b$ for each unit purchased and an additional b per unit sold. (The usual description has the buyer paying w_b per unit purchased and receiving b per unit not sold to the market.) Consequently, revenue-sharing and buy-back contracts are equivalent when

$$w_b - b = w_b + (1 - \phi)s$$
$$b = (1 - \phi)(r - s).$$

Cachon and Lariviere (2005) provide a detailed analysis of revenue sharing and show that revenue sharing still coordinates the price-setting newsvendor, while buy backs, quantity flexibility contracts, and sales rebates cannot. Other analytical papers on revenue-sharing contracts include Dana and Spier (2001), Pasternack (2002), and Gerchak et al. (2006). The benefits of revenue-sharing are also observed in practice. Mortimer (2008), for example, estimates that the introduction of revenue sharing in the video rental industry increased total profit by 7%.

Quantity Flexibility Contract

With a QF contract, the supplier charges w_q per unit purchased but then compensates the buyer for his losses on unsold units up to ΔQ, where $\Delta \in [0, 1)$ is a contract parameter and Q is the number of units purchased. In the newsvendor model presented in Sect. 2.1.3, the buyer receives a credit from the supplier at the end of the season equal to $(w_q + c_b - s)\min(I, \Delta Q)$, where I is the amount of left over inventory.

With the QF contract, the transfer payment is

$$P_q(Q, w_q, \Delta) = w_q Q - (w_q + c_b - s)\int_{(1-\Delta)Q}^{Q} F(y)dy.$$

Compared to the buy-back contract, the QF contract fully protects the buyer on a portion of his order, whereas the buy back contract gives partial protection on the buyer's entire order. QF contracts also coordinate the fixed-price newsvendor and arbitrarily allocate profit (Tsay 1999). But unlike coordinating buy backs, coordinating QF contracts are not independent of the buyer's demand distribution.

Tsay and Lovejoy (1999a) study quantity flexibility contracts in a more complex setting than the one considered here: they have multiple locations, multiple demand periods, lead times, and demand forecast updates. Bassok and Anupindi (2008) provide an in-depth analysis of these contracts for a single stage system with more general assumptions than in Tsay and Lovejoy (1999a). Since our research focuses on QF contracts, this contract type will be discussed more in detail in Sect. 2.2.

Sales-Rebate Contract

Here, the supplier charges the buyer a per unit wholesale price w_s, but then gives the buyer a rebate m per unit sold above a fixed threshold n. So, the transfer payment with the sales rebate contract is

$$P_s(Q, w_s, m, n) = \begin{cases} w_s Q & Q < n, \\ (w_s - m)Q + m(n + \int_n^Q F(y)dy) & Q \geq n. \end{cases}$$

This contract type is studied by Krishnan et al. (2004) and Taylor (2002). Like buy backs or QF contracts, sales-rebate contracts coordinate the fixed-price newsvendor when properly designed, but struggle with the price-setting newsvendor. The authors give numerous examples for the prevalence of sales rebates in the hardware, software, and auto industries.

Quantity Discount Contract

There are many types of quantity discount schedules (Moorthy 1987). One example is an "all unit" quantity discount contract. In this case, the supplier charges the buyer $w_d(Q)$ per order, where $w_d(Q)$ is the per unit wholesale price that is decreasing in Q. Hence, the transfer payment is

$$P_d(Q) = w_d(Q)Q.$$

For the fixed-price newsvendor, the quantity discount achieves coordination and allows to allocate supply chain profit arbitrarily. Quantity discount contracts are similar to revenue-sharing contracts, because with both contracts, the buyer's expected profit is proportional to the supply chain's expected profit. Still, these contract types are not equivalent (Cachon and Lariviere 2005).

Comprehensive literature reviews on quantity discounts are given by Dolan and Frey (1987) and Boyaci and Gallego (1997). Wilson (1993) provides a broad discussion of non-linear pricing, while Tomlin (2003) examines both quantity discount and quantity premium contracts.

Model Extensions and Other Contract Types

There are several model extensions of the classic newsvendor problem that prove insightful, but have not been addressed so far. We have already mentioned the price-setting newsvendor, i.e., the case where the buyer chooses his retail price in addition to his stocking quantity. Moreover, one may consider costly retail effort, that increases demand. Coordination becomes more challenging in this case if the buyer's effort is non-contractible. On the one hand, the model can be extended to incorporate multiple competing buyers. On the other hand, one may want to let the buyer choose between different replenishment opportunities. Making the supplier hold inventory is another extension to the model, which is not trivial to coordinate. Finally, it remains the whole issue of asymmetric information and information sharing, which was already referred to in the introduction of this chapter. Cachon (2003) gives an extensive overview of research papers approaching these model extensions. One example for recent enhancements in this area is Chen (2007) who applies auction theory to a newsvendor problem with supply-side competition to study well-known supply chain contracts in procurement auctions.

Likewise, our review is not intended to present all possible types of supply chain contracts. We can only present an extract of contracting literature here, without claiming to be exhaustive. One type of supply chain contract that has been neglected so far is the two-part tariff. With a two-part tariff, the supplier charges a per unit wholesale price w_t and a fixed fee F. Here, the fixed fee serves to allocate profit between the supplier and the buyer. It can be shown that coordination is achieved with marginal cost pricing, because then the buyer's profit is total supply chain profit minus F. Since two-part tariffs achieve the same results as revenue-sharing contracts in the classic newsvendor model (Cachon and Lariviere 2005), we do not provide a detailed description of this contract type. Also price-discount contracts were not considered so far. Like a buy-back contract, a price-discount contract has a wholesale price and a buy-back rate. However, both contract terms are conditional on the chosen retail price which achieves coordination (Bernstein and Federgruen 2005). Furthermore, Cachon and Lariviere (2005) show that revenue sharing and price-discount contracts are equivalent in the price-setting newsvendor model. Another set of research papers that is not addressed is the literature on franchise contracts. A franchise contract combines revenue sharing with a two-part tariff. That is, the supplier charges a fixed fee, a per-unit wholesale price, and a revenue share per transaction. See Lafontaine (2001) for a review on this stream of literature.

Finally, we will address supply chain contracts with options. Similar to financial options (Black and Scholes 1973; Hull 2008), options used in supply chain contracts constitute the right, but not the obligation to receive or deliver a good, product, or service by a certain date and predetermined price. Options involving real assets are also called real options (Kester 1984). These are usually classified by the type of flexibility that they offer (Trigeorgis 1995) and include, for instance, the option to defer, the option to abandon, the option to switch, and the option to improve (Huchzermeier and Loch 2001). Under certain conditions, supply options achieve coordination and allow for an arbitrary split of total supply chain profit

(Cachon and Terwiesch 2005). Barnes-Schuster et al. (2002) investigate supply chain contracts with options in a two-period model, where the buyer has to make quantity commitments, but may purchase ahead of time some options. Options enable the buyer to procure in a future period additional quantity of goods at the exercise price. The authors show, inter alia, that QF contracts are special cases of their general model. Similarly, Cheng et al. (2008) consider a combination of quantity commitments and options. By considering call and put options, the authors point out the analogy to buy-back and return policies. In a real options approach, Kamrad and Siddique (2004) study supply chain contracts in a setting with exchange rate uncertainty, supplier-switching options, order-quantity flexibility, profit sharing, and supplier reaction options. Although most of the literature on supply chain contracts is concerned with the buyer's decisions, the authors concentrate on the supplier's decisions and show how flexibility can be mutually beneficial to both the buyer and the suppliers. Recent research on supply options include Lee and Whang (2002), Wu et al. (2002), Kleindorfer and Wu (2003), Spinler (2003), Jaillet et al. (2004), and Martínez-de-Albéniz and Simchi-Levi (2007).

2.2 Quantity Flexibility Contract

Tsay et al. (1999b) summarize the motives for entering a QF contract as follows: "[. . .] a QF clause has risk-sharing intent, and the hope is that the agreement can make both parties better off". Indeed, a QF contract achieves coordination of the fixed-price newsvendor problem by compensating the buyer for a certain amount of unsold goods. On the one hand, the benefit for the buyer clearly is a gain in quantity flexibility. On the other hand, the buyer should be prepared to pay a higher cost per unit compared to a pure wholesale price contract to compensate the supplier for the increased exposure to demand uncertainty. QF contracts can bring about further benefits for a supplier: Often, buyers benefit from over-production without bearing the immediate costs, hence provide higher planning forecasts. If supplier and buyer agree that the final order must be in a range predefined by a QF contract, this may be a means to improve forecasts.

QF contracts have emerged as a response to certain supply chain inefficiencies (Lee et al. 1997) and their widespread use has been documented in literature (see for example Bassok and Anupindi 1997; Li and Kouvelis 1999; Tsay and Lovejoy 1999a; Cachon and Lariviere 2001; and Barnes-Schuster et al. 2002). Farlow et al. (1996), for example, report the use of QF contracts at Sun Microsystems for the purchase of various workstation components. QF-type contracts have also been used by Toyota Motor Corporation (Lovejoy 1999), IBM (Connors et al. 1995), Hewlett Packard, and Compaq (Tsay 1999). They even appear internally at the interface between the manufacturing and marketing functions (Magee and Boodman 1967).

Models studying QF contracts typically require at least two decisions on the part of the buyer: An initial inventory decision, and then a revision conditional on whatever new information becomes available. Tsay (1999) study a model similar to the newsvendor model presented in this chapter. They determine optimal parameters of a coordinating contract and show that the profit share increases in the responsibility a firm takes for excess inventory. Bassok and Anupindi (2008) consider the buyer's side of the contract, where the planning for multiple future periods is performed in a rolling-horizon model. Tsay and Lovejoy (1999a) study a more complex variant of Tsay (1999) with multiple locations, periods, lead times, and forecast updates. In these multiperiod models, it is observed that QF contracts decrease order variability, thus retard the bullwhip effect. Cachon and Lariviere (2001) and Lariviere (2002) study the interplay of QF contracts and forecast sharing. In Cachon and Lariviere (2001), the buyer has to convince the supplier that the own forecast is better. In Lariviere (2002), the supplier wishes to incentivize forecasting effort. Cachon and Lariviere (2005) compare QF contracts to revenue-sharing and demonstrate several differences. Unlike the revenue-sharing contract, the QF contract cannot coordinate the price-setting newsvendor. Moreover, a coordinating QF contract depends on the demand distribution, which implies that one cannot simply write down a contract that works for all markets. Plambeck and Taylor (2007b) consider renegotiation in a system with one supplier and two buyers and identify conditions under which QF contracts achieve coordination.

2.3 Buy-Back Contract

As previously indicated, buy-back contracts are able to coordinate the fixed-price newsvendor (Pasternack 1985) and are equivalent to revenue-sharing contracts under the assumption of a fixed retail price (Cachon and Lariviere 2005). But unlike revenue sharing, the coordinating buy-back parameters depend on the retail price. Hence, it is not surprising that in contrast to revenue sharing, buy backs struggle with the price-setting newsvendor. In this case, as Kandel (1996) notes from an economist's perspective, buy backs cannot achieve coordination unless the supplier can impose resale price maintenance. In addition, Marvel and Peck (1995) and Bernstein and Federgruen (2005) demonstrate that buy backs coordinate the price-setting newsvendor only if the supplier earns zero profit. Emmons and Gilbert (1998) also study buy-back contracts with a price-setting newsvendor, while Taylor (2002) and Krishnan et al. (2004) combine a buy-back contract with a sales-rebate contract to coordinate the newsvendor with effort-dependent demand.

Padmanabhan and Png (1995) highlight applications of buy backs for products with a limited life expectancy, for example pharmaceuticals, computer hardware and software, magazines, newspapers, or books. Furthermore, the authors describe several motivations for entering buy-back contracts that are not included in the newsvendor model. One reason may be the supplier's wish to protect his brand image. By offering a buy-back contract, he makes the buyer return left over items instead of discounting them. Another motive may be the supplier's wish to rebalance inventory among several buyers (for literature on stock rebalancing see Lee 1987; Tagaras and Cohen 1992; Rudi et al. 2001; and Anupindi et al. 2001).

Padmanabhan and Png (1997) demonstrate how a supplier can use a buy-back contract to manipulate the competition between buyers. The buy-back model of Donohue (2000) incorporates multiple production opportunities and improving demand forecasts. Anupindi and Bassok (1999b) consider a two-buyer supply chain with consumers searching for inventory among those buyers and demonstrate that buy-back contracts can coordinate this setting. Strongly connected to the literature on buy-back contracts are the papers by Lee et al. (2000) and Taylor (2001) on price protection. Taylor (2001) shows that under the assumption of a declining retail price, coordination with arbitrary allocation of profit requires price protection in addition to a buy-back contract.

As Cachon (2003) notes, there are parallels between buy-back contracts and supply options studied in the context of capital intensive industries. In the two-firm supply chain of Wu et al. (2002), for example, the buyer reserves Q units for a fee h and pays a fee u for each unit of capacity utilized. If the spot price is less than u, the buyer does not exercise his option. But if the spot price is higher, the buyer has to determine an optimal mixture of capacity reservation and spot market. A buy-back analogous to the contract studied by Wu et al. (2002) would have a wholesale price $w_b = h + u$ and a buy-back price $b = u$, paying $h + u$ for each unit of capacity reserved and receiving u per unit of capacity not utilized. Cheng et al. (2008) consider quantity commitments with options and also highlight the analogy to buy-back contracts.

2.4 Quantity Flexibility vs. Buy-Back Contract

We have already noted that well-designed QF and buy-back contracts can greatly improve the performance of a supply chain for a short life-cycle product. Insofar, it is not surprising that these contracts have found numerous applications in industry such as the computer industry or publishing. But are there any reasons to say that one contract is better than the other? Although the literature on supply chain contracts has studied both contract types, answers to this question are scarce (see Lariviere 1999 and Lariviere 2002 for exceptions). The goal of this section is to identify circumstances under which one contract type might be preferable to the other.

First of all, both contract types are partial returns policies. While the buy-back contract is a price-based returns mechanism, the QF contract is a quantity-based one. In contrast to the wholesale price contract, both contracts are able to coordinate the fixed-price newsvendor and to split profits arbitrarily. Furthermore, both struggle with the price-setting newsvendor. However, there is a clear difference between coordinating buy-back and coordinating QF contracts: While optimal contract parameters of the buy back do not depend on the demand distribution, this is the case with QF contracts (which can be relaxed for some distributional families). One may argue, if dependence on the distribution is rather an advantage or a disadvantage. At first sight, independence to the demand distribution appears to be an advantage, since the supplier does not need to know a buyer's demand distribution to coordinate the supply chain with a buy back contract. Consider another example: A supplier is generally obliged to offer the same contractual terms to all buyers. If only one contract is offered, coordination can only be achieved if the contract does not depend on parameters that differ from buyer to buyer. Hence, in such a situation where buyers face different demand distributions, the buy back would clearly be the contract of choice (Cachon 2003). On the other hand, the dependence of a contract on the demand distribution may also be advantageous. Lariviere (2002) studies a model with one supplier selling to a buyer that may exert effort to improve his demand forecast. It is in the interest of the supplier to screen the buyers who are capable of forecasting from those who are not. Since coordinating buy backs do not depend on the demand distribution, they cannot be used to differentiate between forecasters and nonforecasters, while coordinating QF contracts can. Furthermore, the author shows that the supplier is better off using a buy back if forecasting is inexpensive.

Despite their obvious power, returns mechanisms have not completely displaced non-coordinating contracts like the wholesale price contract in practice. Researchers have found several explanations for this. First, although returns mechanisms are simple contractual forms, they are more costly to administer than wholesale price contracts. Second, Cachon and Lariviere (2001) raise the question of the enforcement mechanism. They distinguish forced compliance (where the supplier must fill every order that is feasible under the agreed contract) and voluntary compliance regimes (where the supplier cannot be forced to fill an order). In a setting with symmetric information, the authors show that returns policies can collapse under voluntary compliance. Marvel and Peck (1995) offer another explanation. They note that a buyer has to cope with two forms of market uncertainty: the number of customers in the store and the price they are willing to pay for the product. The newsvendor model presented in this chapter only takes the first form of uncertainty into account, completely neglecting the second one. Assume that the number of customers was deterministic and the price they are willing to pay uncertain. Then a buyer might set higher prices resulting in declining sales and higher returns which would clearly hurt the supplier.

Our research shows that optimal contract parameters of returns policies may change in the face of long-term buyer-supplier relationships with quality improvement efforts. The model predicts that buyers will incentivize their suppliers to induce effort for quality improvement, for example in the form of contract adaptations or transfer payments. Moreover, we investigate if QF or buy-back contracts have a relative advantage when applied in long-term business relationships. The numerical studies carried out in Chap. 6 do not advocate exclusively one type of contract in any circumstance, but disclose differences of QF and buy-back contract regarding the leeway for contract adaptation and the continuance of the supply relationship. We will see, for example, that a buy back tends to offer more leeway than the QF contract when actual demand is low and that a QF contract tends to be better for higher values of the demand realization.

Chapter 3
Relational Contracts

While relational contracts have been vastly studied in Sociology, Law, and Economics, these kinds of contracts represent a rather new stream of research in the Operations and Supply Chain Management literature. Hence, the main objectives of this chapter are to shed light on the concept of relational contracting and to give an overview of recent developments in this field. The first section aims at explaining the general concept of relational contracting. The second section addresses the roots of relational contracts in Economics. The third section presents recent publications in the field of Operations and Supply Chain Management. The last section provides an introduction to the game-theoretic background of relational contracts.

3.1 The Concept

Over the last 25 years, a great deal has been written about relational contracts, especially in Sociology, Law, and Economics. As there are various definitions of what a relational contract is depending on the respective discipline, it is important to clarify that the present treatise refers to the following characterization of relational contracts. Baker et al. (2002) describe relational contracts as "informal agreements and unwritten codes of conduct that powerfully affect the behavior of individuals", clearly emphasizing the informal nature of the contracts.

Relational contracts help firms to overcome difficulties in formal contracting. A formal contract must be specified ex ante in a way that terms can be verified ex post by a third party. By contrast, a relational contract can be based on aspects that are observed only by the contracting parties ex post or are too costly to be specified ex ante. For the same reasons, relational contracts cannot be enforced by a third party, hence must be self-enforcing.

Self-enforcement is essential to sustain a relational contract. When the relation between the contracting parties is finite, the only enforceable contracts are formal court-enforceable contracts (see, for example, Bolton and Dewatripont 2005). But when the contracting parties are engaged in a repeated, open-ended relationship, the situation changes. Now, any formal court-enforceable contract can be extended with informal self-enforced provisions and become a self-enforced relational contract.

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To create a basic understanding for relational contracts, we follow Gibbons (1997) and Gibbons (2008). The author gives a concise formalization of relational contracts, by developing a very simple repeated-game model. Gibbons first describes a one-time interaction between two parties, the so-called *Trust Game* (Kreps 1990), and then analyzes the associated relational contract, an ongoing relationship in which these interactions occur repeatedly.

The Trust Game

We start with the description of the trust game. As depicted in the game tree of Fig. 3.1, the trust game begins with a decision of player 1, who can choose either to trust or not to trust player 2. If player 1 chooses "Not Trust", he automatically terminates the relationship, i.e., the game ends. If player 1 chooses "Trust", then player 2 can choose to either honor or betray player 1's trust. The numbers at the end of each branch denote the players' payoffs for every outcome of the game. That is, in the case where player 1 ends the relationship, both players' payoffs are zero. If player 1 chooses to trust player 2 and player 2 honors player 1's trust, then both players receive payoffs of one. In the case where player 2 betrays player 1's trust, player 1 receives −1 and player 2 receives two.

Backwards induction yields that player 1 will end the relationship right in the beginning, by choosing not to trust player 2: If player 2 is given the move, he will prefer a payoff of two to a payoff of one, hence betray player 1's trust. Consequently, player 1 will receive zero if he chooses "Not Trust" and will receive −1 if he chooses "Trust". Since zero exceeds −1, player 1 should choose "Not Trust" and end the relationship.

The Relational Contract

Suppose the two players play the trust game repeatedly, with all previous outcomes observed by both players before the next period the game is played. Now, the sharp contrast between a one-period interaction and repeated long-term interactions

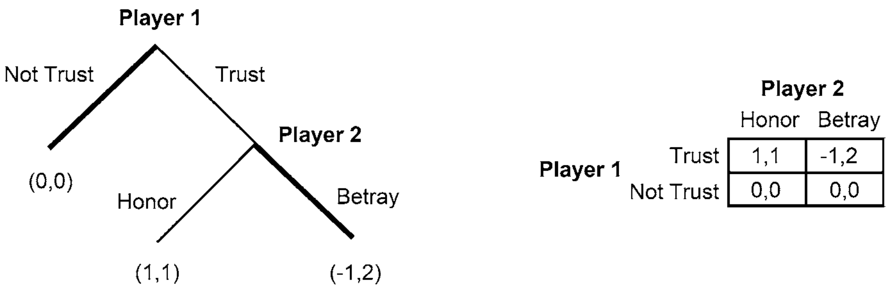

Fig. 3.1 The trust game (adapted from Gibbons 1997)

becomes evident. In repeated interactions, the players will make a tradeoff between their short-term and long-term interests. Hence, player 1 will not necessarily end the relationship right in the beginning, but will weigh up instead if choosing "Trust" may be beneficial to him in the long run. Likewise for player 2, to whom choosing "Betray" in the first period may no longer be attractive due to the prospect of future benefits.

Formally, Gibbon's relational contract is an infinitely repeated trust game, in which the players share an interest rate r and payoffs are discounted across periods (for an introduction to game-theoretic concepts, we refer the reader to Sect. 3.4.1 starting on p. 46). One interpretation for the "infinite" horizon of the game is that the game ends at a random date. Under this interpretation, the interest rate r reflects not only the time value of money but also the probability that the players will meet again after the current period.

Before we start analyzing this very simple relational contract, we make assumptions on the players' strategies. Mostly for analytical simplicity, we will consider the following trigger strategies:

- **Player 1** *In the first period, play "Trust". Thereafter, if all moves in all previous periods have been "Trust" and "Honor", play "Trust"; otherwise, play "Not Trust".*
- **Player 2** *If player 1 plays "Trust" this period, play "Honor" if all moves in all previous periods have been "Trust" and "Honor"; otherwise, play "Betray".*

As already mentioned before, these types of strategies are "not forgiving": If cooperation breaks down once, it will not be restored forever after. Or, in other words, the players react to a breakdown of cooperation with a reversion to short-term self-interest for the rest of the game.[1] Gibbons shows that these trigger strategies are a Nash equilibrium of the infinitely repeated game, provided that player 2 is sufficiently patient. That is, given that player 1 is playing his trigger strategy, it is in player 2's interest to play his trigger strategy as well, provided that the interest rate r is sufficiently small. To see that, suppose that player 1 follows his trigger strategy and chooses "Trust" in the first period. Player 2 then faces the following decision: He may choose "Betray", which maximizes his payoff in the first period of the game. In this case, player 1 will react by playing "Not Trust" forever after, resulting in a payoff of zero for player 2 in each subsequent period. Otherwise, player 2 may choose "Honor", in which case his payoff in the first period is smaller. But as player 1 is playing a trigger strategy, he will react by playing "Trust" in the next period, enabling player 2 to obtain non-zero payoffs in subsequent periods. Hence, player 2 has to trade off the short-term temptation of receiving 2 instead of 1 against the long-term cost of receiving 0 instead of 1 forever after. His decision will depend on the interest rate r. Given a sufficiently low interest rate r, the long-term self-interest will dominate, inducing player 2 to honor the relationship.

[1] The assumption of trigger strategies is not farfetched. Schweitzer, Hershey and Bradlow (2006) performed a laboratory study using a repeated trust game, which confirms that a subject actually refuses to cooperate in all subsequent periods once it has been deceived by the other party.

The repeated trust game illustrates how the prospect of future cooperation may induce two parties to sustain a relationship. The amount of payoffs per period as well as the interest rate r play a decisive role for the players' actions. To get a clearer picture of the players' willingness to cooperate, suppose that the players' payoffs per period are C from cooperation, D from defection, and P from punishment with $\mathsf{D} > \mathsf{C} > \mathsf{P}$. Here, punishment means that the other player reacts to defection with non-cooperation forever after. As before, a potential defector must weigh the present value of continued cooperation against the short-term gain from defection followed by the long-term loss from punishment. Now, the players' decisions amount to evaluating two time paths of payoffs. The players have to trade off the case of continued cooperation $(\mathsf{C}, \mathsf{C}, \mathsf{C}, \ldots)$ against the case of non-cooperation $(\mathsf{D}, \mathsf{P}, \mathsf{P}, \mathsf{P}, \ldots)$, as depicted in Fig. 3.2.

The time path of payoffs $(\mathsf{C}, \mathsf{C}, \mathsf{C}, \ldots)$ yields a higher present value than the time path $(\mathsf{D}, \mathsf{P}, \mathsf{P}, \mathsf{P}, \ldots)$ if

$$1 + \frac{1}{\mathsf{r}}\mathsf{C} > \mathsf{D} + \frac{1}{\mathsf{r}}\mathsf{P},$$

since the present value of \$1 received every period starting tomorrow is $\$\frac{1}{\mathsf{r}}$. Rearrangement of this inequality implies that it is optimal for a player to cooperate if $\mathsf{r} < (\mathsf{C} - \mathsf{P})/(\mathsf{D} - \mathsf{C})$. If this is the case, the players will forego the short-term temptation $\mathsf{D} - \mathsf{C}$ for the long-term gain $\mathsf{C} - \mathsf{P}$ in every subsequent period.

Even this simple relational contract offers one lesson: The higher the value of the relationship $\mathsf{C} - \mathsf{P}$, the more likely the relationship will continue. To allow for further conclusions about relational contracts, we can extend this model to incorporate fluctuating payoffs over time. Hence, suppose that at the beginning of period t the parties observe the payoffs C_t, D_t, and P_t. Also assume that each period's payoffs are independently drawn from a given joint probability distribution $H(\mathsf{C}, \mathsf{D}, \mathsf{P})$.

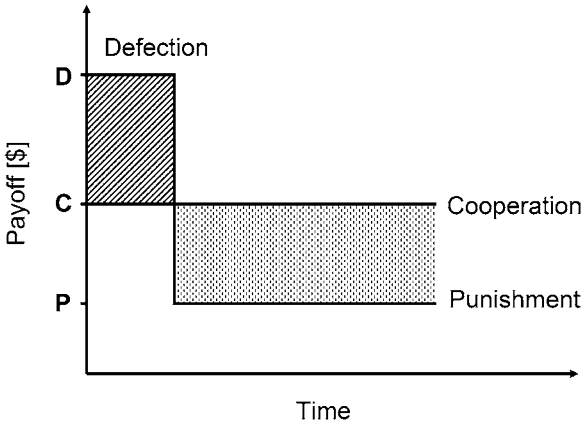

Fig. 3.2 Payoffs from defection (D), cooperation (C), and punishment (P) (adapted from Gibbons, 2008)

Then, one can show that cooperation is optimal for a player in period t if

$$\mathsf{C}_t + \frac{1}{\mathsf{r}} E[\mathsf{C}] > \mathsf{D}_t + \frac{1}{\mathsf{r}} E[\mathsf{P}].$$

Here, $E[\mathsf{C}]$ and $E[\mathsf{P}]$ denote the expected values of C and P respectively. This enriched model offers a second lesson: the relationship is particularly in danger, when the defection temptation (or the gain $\mathsf{D}_t - \mathsf{C}_t$ associated with defection) is largest relative to the cooperation payoff.

To summarize, Gibbon's model illustrates the core feature of a relational contract: "When people interact over time, threats and promises concerning future behavior may influence current behavior" (Gibbons 2008). Unlike one-time interactions, repeated interactions allow the parties involved to enforce cooperation without formal contracts. Thereby, the parties' actions will be guided by the tradeoff between short-term and long-term interests. In particular, parties will be more likely to honor a relationship, the higher the value of the relationship and the smaller the short-term gain from defection.

3.2 The Economists' Approach

The concept of relational contracts has its roots in Law, where by definition complex contracts are just regarded as frameworks to govern transactions. We will focus on the way economists try to integrate this notion of relational contracts in their models, differentiating between Transaction Cost Economics, Principal Agent, and Incomplete Contract Theory. We will see that in terms of classic Principal Agent and Incomplete Contract Theory, relational contracts are introduced as repeated games. As an illustration, two approaches by Levin (2003) and Baker et al. (2002) will be highlighted. The former is an extension of the classic Principal Agent model, while the latter is closer to the incomplete contract model according to Grossman and Hart (1986). Finally, recent findings by Brown et al. (2004) will be presented. Their experimental study gives rise to understanding, how the absence of third party enforcement actually fosters the adoption of long-term relationships.

Transaction Cost Economics

Transaction Cost Economics relate relational contracts to the notion of incompleteness. The incompleteness of a contract has the following meaning: There are decisions that have to be made ex post, e.g., by negotiation among the contracting parties. Therefore, contracts in Transaction Cost Economics are already designed to allow for adaptations and to simultaneously impede opportunism. As an application, Crocker and Masten (1991) present a price formula for long-term contracting in the gas industry (also see Masten and Crocker 1985). More precisely, the authors apply relational contracting theory to explain the price adjustment processes adopted in a sample of long-term natural gas contracts. Based on the contract data, the authors show that price adjustment processes tend to be more flexible the longer the duration of the contract (which is presumably due to the greater uncertainty associated with performance at more distant dates). Moreover, their work predicts that the contracting parties will value flexible arrangements most, when uncertainty about circumstances at the time of performance is greatest. In environments where opportunism is prevalent or where economic conditions are relatively simple and static, parties will tend to favor more precise and rigid agreements over flexible ones.

Principal Agent Theory

Principal Agent Theory views contracts and especially relational contracts differently. Provided that agents are rational, they can implement complete contracts, i.e., contracts that do not require revision. The main concern consists in the ex ante incentive alignment of the parties. This can be ensured if some relevant variables are assumed to be verifiable by a third party. In this case, the contract only comprises the verifiable variables and neglects relational aspects of the arrangement. Thus, relational contracts can only be introduced by considering repeated interactions among agents. Accordingly, Levin (2003) studies an infinitely repeated agency relation, where the incentive contract contains both court-enforced provisions and informal components. Performance measures are assumed to be non-verifiable. One main result of the paper is that the design of the optimal contract depends on the monitoring technology. Levin shows that optimal relational contracts often can take a simple stationary form, but that the absence of third party enforcement limits promised compensation and affects incentive provision. In his moral hazard model, optimal contracts terminate following poor performance.

Incomplete Contracts

The concept of Incomplete Contracts represents a similar approach. A contract is called incomplete when some relevant variables are non-verifiable, hence non-contractible. The corresponding model neglects ex post renegotiation because, even if renegotiations are necessary, those are assumed to be cost-free. The ex ante allocation of property rights plays an important role, as it affects the parties' incentives to invest in a specific asset. Like in the case of Principal Agent Theory, the relational aspect becomes irrelevant in the classic incomplete contract framework, and one can only establish relational contracts by considering repeated interactions. In the same line, Baker et al. (1994) introduce relational contracts in the incomplete contract framework of Grossman and Hart (1986). Baker et al. (2001, 2002) show that parties may shape their interactions through transfer payments and derive that the extent of the parties' incentives to renege and therewith the optimal relational contract depends on the ownership structure. Baker et al. (2002), for example, combine asset ownership with relational contracting. The authors develop repeated-game models that illustrate why optimal relational contracts differ, depending on whether firms are vertically integrated or not. Their research proposes that integration changes the parties' temptations to renege on a given relational contract and thus affects the optimal relational contract the parties can sustain. In fact, the relational contract produces the same actions and total surplus under either ownership structure, but the maximum total reneging temptation may differ: In some situations, the reneging temptation is lower between integrated parties. In others, the reneging temptation is lower between nonintegrated parties. As a result, Baker et al. derive that vertically integrated firms cannot replicate spot-market outcomes.

A vast literature suggests that repeated interactions may affect the structure of economic relationships by allowing implicit contracts to be sustained (see Klein and Leffler 1981; Williamson 1985; Shapiro and Stiglitz 1984; Bull 1987; MacLeod and Malcomson 1988; Baker et al. 1994; Klein 1996, 2002; and Gibbons 2005). Using an experimental approach, Brown et al. (2004) provide evidence that the absence of third party enforcement (entailing the use of incomplete contracts) results in fundamental changes in the nature of market interactions: "In the absence of third party enforcement, markets resemble a collection of bilateral trading islands rather than a competitive market". The researchers distinguish three different contract environments: Complete contracts enforceable by a third party (C), incomplete contracts under the absence of third party enforcement (ICF), and ICF contracts with the restriction that long-term relations between contracting parties are forbidden (ICR). In an experiment with over 200 participants, Brown et al. observe that in the ICF condition, the vast majority of trades are initiated with private offers (see Fig. 3.3, p. 41). By contrast, public offers dominate in the C condition. Hence, when third party enforcement is absent, participants use private offers to establish long-term relationships, which is not the case in a complete contract environment. As Fig. 3.4

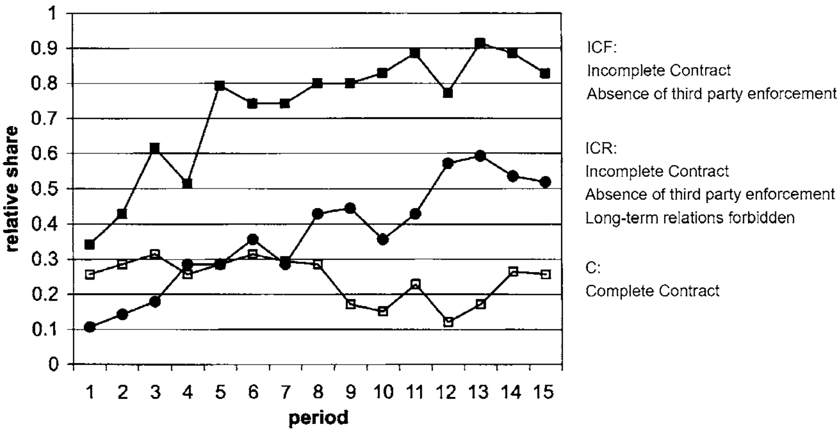

Fig. 3.3 Private vs. public offers: relative share of trades initiated by private offers (adapted from Brown et al. 2004)

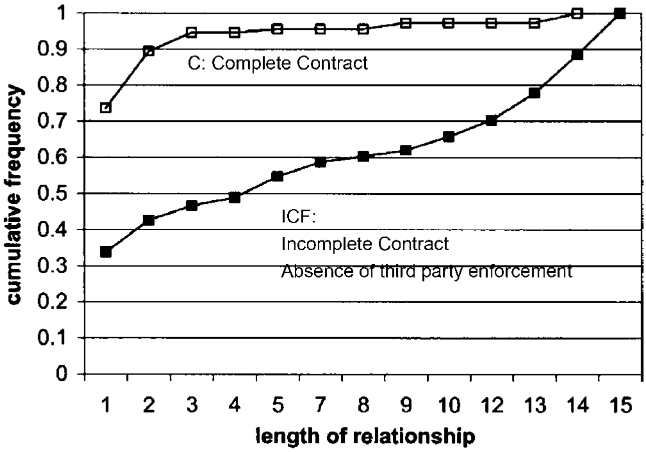

Fig. 3.4 The relevance of long-term relations: cumulative frequency of trades in relationships of different length in the C and the ICF condition (adapted from Brown et al. 2004)

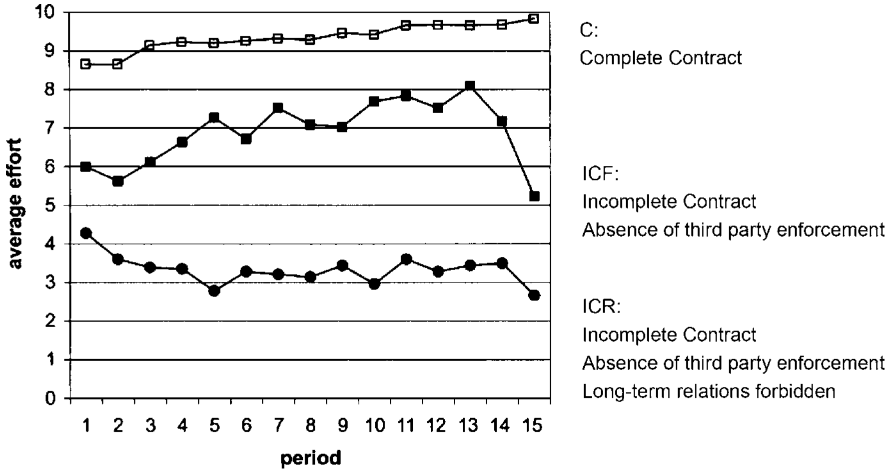

Fig. 3.5 The effort consequences of contingent contract renewals: Evolution of average effort over time (adapted from Brown et al. 2004)

shows, long-term relationships emerge endogenously under the absence of third party enforcement. Again, one observes that long-term relationships develop under the absence of third party enforcement, whereas long-term relationships rarely occur when complete contracts can be written. As regards relationship maintenance under self-enforcement, Brown et al. show that low effort (or bad quality) is penalized by the termination of the relationship. And finally, long-term relationships exhibit generous rent sharing and relatively high effort, as illustrated by the difference between the ICF and the ICR condition in Fig. 3.5 (clearly, the C condition represents a benchmark here, since effort is enforceable under this condition).

3.3 Review of Operations and Supply Chain Management Literature

Having addressed the roots of relational contracts in Economics, we proceed to a literature survey in Operations and Supply Chain Management. Although relational contracts persist in practice, research on these types of contracts is only emerging in this field. Therefore, much of the material in this section draws on fairly recent research. The concepts and methods presented may well evolve significantly in response to future research breakthroughs.

As illustrated in Chap. 2, research in Operations and Supply Chain Management has identified a number of contracts that coordinate the supply chain in a newsvendor setting. However, current models often turn out to be too dependent on one-time contracting and neglect supply chain interactions occurring over long periods of time. Therefore, recent publications like Cachon (2003) and Terwiesch et al. (2005) postulate the analysis of repeated interaction:

> First, we believe additional research is needed to analyze supply chain coordination in repeated game settings. While repeated games have been extensively studied in the economics literature, most of the contracting research in operations management has taken a rather static perspective, ignoring effects of trust building and reputation. (Terwiesch et al. 2005)

Researchers making first steps in this direction are for example Kranton and Minehart (2001), who analyze buyer-supplier networks and long-term relationships, and Plambeck and Zenios (2000), who investigate performance-based incentives in a dynamic principal agent model. Several papers explore renegotiation of formal contracts (see Laffont and Tirole 1990; Plambeck and Taylor 2007a,b). At the same time, a growing number of papers apply game theory to supply chain analysis, thereby demonstrating how repeated interactions may influence supply chain contracting. Cachon and Netessine (2004) summarize such approaches up to the year 2003.

The treatise at hand follows the framework constituted by recent papers on relational contracts in the field of Operations and Supply Chain Management. These papers will be reviewed in detail in the following. The reader will notice that especially Plambeck and Taylor have shaped this area of research significantly over the past years (Plambeck and Taylor 2006; Taylor and Plambeck 2007a,b).

We will start with a paper that examines the wholesale price contract in a setting with repeated interaction. More precisely, Debo and Sun (2004) investigate how repeated interaction affects coordination with wholesale price contracts. Although wholesale price contracts cannot achieve supply chain coordination in a single period interaction (Lariviere and Porteus 2001), Debo and Sun show that this is possible with repeated interaction if the supply chain members share a sufficiently high discount factor. In this case, the supplier decreases the wholesale price in return for a larger order quantity from the buyer. Moreover, Debo and Sun consider the impact of fluctuating demand on the parties' expected profits, the ability to coordinate, and the reneging temptation. They reveal that information about a parameter of the demand distribution available to both players in the beginning of each period may decrease the buyer's expected profit.

Ren et al. (2006) study relational contracts in the context of demand forecasting. They show that truthful information sharing can be achieved via a repeated forecasting game. Assuming that the buyer-supplier relationship is long-term, Ren et al. establish conditions under which a buyer operating with a linear price contract reveals demand information truthfully.

Tunca and Zenios (2006) model the competition between relational contracts and supply auctions emerging from the increased usage of electronic marketplaces. They investigate the interplay between price-based reverse auctions for low-quality parts and relational long-term contracts for high-quality parts, with multiple suppliers differing in quality. Market and product parameters that increase the economic value of both procurement modes are determined. Tunca and Zenios show that the competition from the auction market can either facilitate or undermine the relational contract, implying that the interplay of the different procurement modes plays an important role in the supply of high-quality products.

Plambeck and Taylor (2006) determine an optimal relational contract for joint production in a dynamic system with double moral hazard. Here, the output of joint production depends on the actions of both firms. These actions are unobservable and the output uncontractible with finitely many states characterizing system dynamics. Plambeck and Taylor provide an optimal relational contract that has a simple contract design and does not depend on the past history. Formal payments may be extended by discretionary ones to induce costly actions from the supplier's side. Moreover, the relational contract may require termination of the relationship with positive probability following poor performance. The authors show that process visibility can substantially improve system performance.

Taylor and Plambeck (2007a) consider repeated new product introduction and capacity investment and give guidance when to use relational price-only or relational quantity commitment contracts. In their framework, a supplier must invest in capacity when the product development effort is ongoing. Because the product is still ill-defined at this point of time, the buyer informally promises future terms of trade to provide incentives for capacity investment. Taylor and Plambeck show that the value of future relationship creates an incentive for the buyer to pay the supplier as promised. In a similar vein, Taylor and Plambeck (2007b) study relational contracts for capacity investment and procurement in a situation where the supplier has to invest in capacity during the design and production process of an innovative product. The authors show that the gain from relational contracting is substantial over a broad range of parameter values and is greatest when the capacity cost is moderate and the bargaining power is evenly distributed. They conclude that by properly structuring informal procurement agreements, the firms can avoid having the buyer monitor the supplier's capacity.

When it comes to relational contracting, an important question is how the parties deal with noncooperative behavior. Experimental and empirical research suggests that breaking a promise may have a significant negative impact on the business relationship: Helper (1991), for example, supports the prevalent assumption of trigger strategies by highlighting that US auto manufacturers that broke relational contracts for capacity investment in the 1970s had great difficulty building trust and cooperative relationships with suppliers later on. Yet, Atkins et al. (2006) have considered a richer behavioral model where the duration of punishment length is proportional to the magnitude of the deviation from the agreement.[2]

We conclude this section by sorting our model into the stream of literature presented. First of all, the aim of our research is to extend the analysis of well-established supply chain contracts like QF and buy-back contracts to a framework with repeated interactions, where joint effort for quality improvement and the prospect of future trade influences the parties' actions. In this sense, the model directly pursues current research on relational contracts and especially the work by Debo and Sun (2004), who consider wholesale price contracts in a setting with repeated interactions. Our model is closest to the double moral hazard problem of Plambeck and Taylor (2006). It also incorporates a Markov decision process, but introduces more complex supply chain contracts between the contracting parties as well as demand uncertainty. Another important aim is to gain more insights into the differences between QF and buy-back contracts in a relational contracting setting (see Lariviere 2002 for a comparison of QF and buy-back contracts for inducing forecast revelation).

Like most of the papers presented, we assume trigger strategies. That is, noncooperation is punished by excluding the unreliable party from trade in all subsequent periods. In conformity with Plambeck and Taylor (2006), a limit on the value the firms are willing to destroy in order to punish noncooperation can be captured in the model, whereas introducing a finite punishment length in terms of Atkins et al. (2006) will significantly complicate the relational contract.

To explain the role of renegotiation in our model, we draw on Plambeck and Taylor (2007b). In their setting, two buyers contract for capacity with a common supplier. The buyers invest in innovation and the supplier builds capacity. Finally, the firms may renegotiate to allow a buyer facing poor (favorable) market conditions to buy less (more) as contracted. Their objective is to design QF contracts that anticipate renegotiation, so-called *renegotiable* QF contracts. Our research, in contrast, examines the relational aspect of a long-term buyer-supplier relationship and its impact on the design of *relational* QF contracts. The objective is to show how long-term business relationships may allow for adaptation of QF contract parameters to induce effort for quality improvement from the supplier's side. Still, renegotiation is not disregarded in our model. According to Laffont and Tirole (1990) and Rey and Salanie (1996), allowing renegotiation is equivalent to considering long-term contracts that are immune to renegotiation (in the sense that in every period the parties cannot achieve greater profit by substituting the contract). This immunity to renegotiation can be imposed in our model, too. Also note that the potential benefit of renegotiation is greatest if new information becomes available or if the parties

[2] For an introduction to the Economics literature on renegotiation, we refer the reader to Abreu and Pearce (1991).

can overcome an existing information asymmetry. If the access to new information is limited, the potential benefit from renegotiation may be relatively small like in Cachon and Zhang (2006).

This section was meant to illustrate how relational contracts have found their way into Operations and Supply Chain Management in recent years and to relate our work to existing literature. The next section will address the game-theoretic background.

3.4 Analytical Tools

In general, designing an efficient relational contract means selecting and supporting a particular equilibrium of an infinitely repeated game (an infinite game without time dependence where the exact same game is played repeatedly). In our case, the analyzed game is actually time-dependent in the sense that it incorporates a Markov decision process. Since repeated games play an important role in the formulation and analysis of relational contracts, we present these games and discuss their strengths and limitations when applying them to SCM settings. The action strategies of the optimal relational contracts developed in this treatise boil down to a dynamic programming recursion. For this reason, the last subsection gives a brief introduction into infinite horizon dynamic problems.

3.4.1 Game Theory

In this subsection, game-theoretic notation is introduced exemplifying infinitely repeated games, followed by a brief discussion of strengths and limitations of applying repeated games and dynamic games with Markovian dynamics to Supply Chain Management issues.

3.4.1.1 Introduction to Game Theory

Game theory aims at mathematically capturing behavior in strategic situations, in which an individual's success in making choices is influenced by the choices of others. Modern game theory resides on the work by von Neumann and Morgenstern (1944) who summarize the basic concepts existing at that time. In the 1950s, the theory was advanced significantly by many scholars. Milestones in the development of game theory include the concept of equilibrium (Nash 1950), games with imperfect information (Kuhn 1953), cooperative games (Aumann 1959; Shubik 1962) and auctions (Vickrey 1961) inter alia. Nowadays, game theory is widely recognized as an important analytical tool in many fields, including Social Sciences (especially Economics), Biology, Engineering, Political Science, Computer Science, and Philosophy.

Exemplifying infinitely repeated games, we introduce basic game-theoretic notation and concepts in line with Gibbons (1992). For an elaborate treatment of game theory we refer the reader to texts like Friedman (1986) and Fudenberg and Tirole (1991).

We start by introducing static games to construct repeated games later on. Let $(s_1, \ldots, s_n)$ denote a combination of strategies, one for each player, and let u_i denote player i's payoff function, i.e., $u_i(s_1, \ldots, s_n)$ is the payoff to player i if the players choose the strategies $(s_1, \ldots, s_n)$. Then, we can represent a game G as follows:

Definition 3.1 (Normal-Form Representation). *The normal-form representation of an n-player game specifies the players' strategy spaces $S_1, \ldots, S_n$ and their payoff functions $u_1, \ldots, u_n$. We denote this game by $G = \{S_1, \ldots, S_n; u_1, \ldots, u_n\}$.*

Now, we can define the Nash equilibrium for the game G:

Definition 3.2 (Nash Equilibrium). *In the n-player normal-form game $G = \{S_1, \ldots, S_n; u_1, \ldots, u_n\}$, the strategies $(s_1^*, \ldots, s_n^*)$ are a Nash equilibrium if for each player i, s_i^* is player i's best response to the strategies specified for the $n-1$ other players, $(s_1^*, \ldots, s_{i-1}^*, s_{i+1}^*, \ldots, s_n^*)$:*

$$u_i(s_1^*, \ldots, s_{i-1}^*, s_i^*, s_{i+1}^*, \ldots, s_n^*) \geq u_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*)$$

for every feasible strategy s_i in S_i; that is, s_i^* solves

$$\max_{s_i \in S_i} u_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*).$$

In other words, saying that the strategies $(s_1', \ldots, s_n')$ are not a Nash equilibrium of the game G means that there exists a player i such that s_i' is not a best response to $(s_1', \ldots, s_{i-1}', s_{i+1}', \ldots, s_n')$. In fact, there exists some strategy s_i'' in S_i such that

$$u_i(s_1', \ldots, s_{i-1}', s_i', s_{i+1}', \ldots, s_n') < u_i(s_1', \ldots, s_{i-1}', s_i'', s_{i+1}', \ldots, s_n').$$

Consequently, strategies prescribed by a convention of how to play a given game must be a Nash equilibrium. Otherwise, at least one player will have an incentive to deviate from the convention.

To construct an infinitely repeated game, we consider a static game of complete information G that is repeated infinitely in every stage t, $t = 1, 2, 3, \ldots$, with the outcomes of all previous stages observed before the current stage begins. Note that complete information means that the players' payoff functions are common knowledge. G will also be called the *stage game* of the infinitely repeated game. Before giving the exhaustive definition of the infinitely repeated game, let's first specify the discount factor δ and the players' payoffs from the infinitely repeated game. The discount factor $\delta = \frac{1}{1+\mathsf{r}}$ is generally defined as the value today of a dollar to be received one stage later, where r is the interest rate per stage. Then the players' payoffs from the infinitely repeated game can be defined as the present value of the players' payoffs from the stage games:

Definition 3.3 (Present Value). Given the discount factor δ, the present value of the infinite sequence of payoffs $\pi_1, \pi_2, \pi_3, \ldots$ is

$$\pi_1 + \delta\pi_2 + \delta^2\pi_3 + \cdots = \sum_{t=1}^{\infty} \delta^{t-1}\pi_t.$$

Definition 3.4 (Infinitely Repeated Game). Given a stage game G, let $G(\infty, \delta)$ denote the infinitely repeated game in which G is repeated forever, and the players share the discount factor δ. For each t, the outcomes of the $t-1$ preceding plays of the stage game are observed before the t-th stage begins. Each player's payoff in $G(\infty, \delta)$ is the present value of the player's payoffs from the infinite sequence of stage games.

In any game, be it static or repeated, a player's strategy is a complete plan of action. More precisely, a player's strategy specifies a feasible action for the player in every event the player might be called upon to act. Before we define the term strategy in the context of repeated games, we will clarify what the term history of the game stands for:

Definition 3.5 (History of the Game). In the infinitely repeated game $G(\infty, \delta)$, the history of the game through stage t is the record of the players' choices in stages 1 through t.

Definition 3.6 (Strategy). In the infinitely repeated game $G(\infty, \delta)$, a player's strategy specifies the action the player will take in each stage, for each possible history of play through the previous stage.

A widely accepted assumption when dealing with repeated games (or relational contracts) is that players adhere to trigger strategies. Therefore, we pick up the term at this point. Since in repeated games players can choose their current action contingent on observed actions in previous periods, they may choose one strategy until the opponent changes his play and then react by reverting to a different strategy:

Definition 3.7 (Trigger Strategy). If a player cooperates until someone fails to cooperate, which triggers a switch to noncooperation forever after, this is called a trigger strategy.

The threat of reverting to a different strategy must be credible though, i.e., the players' strategies should represent an equilibrium for the whole horizon of the game. To distinguish between credible and non-credible threats, Selten (1965) introduced the subgame and the related notion of subgame-perfect equilibrium:

Definition 3.8 (Subgame). In the infinitely repeated game $G(\infty, \delta)$, each subgame beginning at stage $t+1$ is identical to the original game $G(\infty, \delta)$. As in the finite-horizon case, there are as many subgames beginning at stage $t+1$ of $G(\infty, \delta)$ as there are possible histories of play through stage t.

Definition 3.9 (Subgame-Perfect Nash Equilibrium, Selten 1965). A Nash equilibrium is subgame-perfect if the players' strategies constitute a Nash equilibrium in every subgame.

We can now proceed to the famous Folk Theorem (its source is unknown with Friedman being one of the first to treat the Folk Theorem in detail). Before stating the theorem, we define the term average payoff:

Definition 3.10 (Average Payoff). Given the discount factor δ, the average payoff of the infinite sequence of payoffs $\pi_1, \pi_2, \pi_3, \ldots$ is

$$(1-\delta)\sum_{t=1}^{\infty} \delta^{t-1}\pi_t .$$

Theorem 3.1 (Friedman 1971). *Let G be a finite, static game of complete information. Let $(e_1, \ldots, e_n)$ denote the payoffs from a Nash equilibrium of G, and let $(f_1, \ldots, f_n)$ denote any other feasible payoffs from G. If $f_i > e_i$ for every player i and if δ is sufficiently close to one, then there exists a subgame-perfect Nash equilibrium of the infinitely repeated game $G(\infty, \delta)$ that achieves $(f_1, \ldots, f_n)$ as the average payoff.*

The theorem proves that any convex combination of the feasible payoffs is attainable in the infinitely repeated game as an equilibrium. Hence, on the one hand, the study of subgame perfect equilibria reduces the set of equilibria when treating repeated games. On the other hand, the Folk Theorem suggests that infinitely repeated games feature multiplicity of equilibria.

3.4.1.2 Game Theory in Supply Chain Analysis

This subsection is named after a publication by Cachon and Netessine (2004), in which the authors survey the applications of game theory to supply chain analysis and sound out the potential of game-theoretic concepts for future application. Since the nature of game-theoretic concepts may influence the design and tractability of SCM models significantly, we will summarize some main observations here.

Game-theoretic models help to better understand the interactions of independent agents within and across firms. Accordingly, the application of game-theoretic concepts to Operations and Supply Chain Management has gained in importance significantly over the last years, suggesting that these fields are ideal candidates for game-theoretic applications (see literature surveys by Li and Whang 2001 and Cachon and Netessine 2004). While many SCM models like newsvendor-based models are static, a significant portion of literature examines dynamic models in which decisions are made over time. Often, the solution concept for these dynamic games are comparable to – but still differ from – the backwards induction used when treating dynamic programming problems.

The focus of this treatise is on dynamic games and especially those types of dynamic games that are used in the analysis of relational contracts. We have learned in the beginning of this chapter, that self-enforcement is essential to sustain a relational contract. Since informal agreements characteristic for relational contracts cannot be enforced when considering a finite number of periods, the games used to model relational contracts typically exhibit an infinite number of periods. Like many economists doing research on relational contracts, Debo and Sun (2004) apply infinitely repeated games to their study of relational wholesale price contracts. Other researchers studying relational contracts in SCM settings incorporate a Markov decision process into their game-theoretic models (see Plambeck and Taylor 2006).

These are exactly the two types of dynamic games of major interest in the analysis of relational contracts: infinitely repeated games and Markov games. Unlike the well-known Stackelberg game (the simplest possible dynamic game which has found many applications in SCM), these games look at situations where both players take actions not only once, but in multiple periods. The major difference between these types of games is that Markov games are time-dependent, while infinitely repeated games are not – a fundamental difference when it comes to modeling SCM problems.

We will take a closer look at the infinitely repeated game first. An interesting property of this game is that the set of equilibria is much larger than the set of equilibria in a static game. Furthermore, it may include equilibria that are not possible in the static game. Although concentrating on subgame-perfect equilibria may reduce the number of equilibria, their multiplicity still poses a major problem, as already mentioned in the previous section (see the Folk Theorem on p. 49). Another drawback of repeated games when applying them to SCM settings is that typical components of SCM models like the transfer of inventory cannot be modeled, since this would require time dependence of the game.

The other type of games allowing for time dependence is the so-called stochastic or Markov game, which essentially combines a static game with a Markov decision process. That is, the set of players with strategies and payoffs is supplemented by a set of states and a transition mechanism $p(x'|x, y)$, describing the probability for a transition from state x to state x' given action y. These games exhibit similar difficulties like non-stationary inventory models. Hence, a standard simplifying assumption is that demands are independent and identical across periods. Similar to repeated games, these games get rather complicated as soon as there is more than one decision maker involved. So, a standard approach is to focus on stationary equilibria, since non-stationary policies are hard to implement in practice.

In summary, this subsection revealed several implications for modelling relational contracts by means of game theory. Especially in SCM applications, special attention should be given to modelling time dependence, to making assumptions on the demand distribution, and to handling multiplicity of equilibria.

3.4.2 Dynamic Programming

Dynamic Programming looks at a very wide range of problems where decisions are made in stages. Characteristically, the outcome of each decision is not fully predictable but can be observed before the next decision is made. Another key aspect of such problems is that decisions cannot be viewed in isolation because low costs in earlier stages may imply high costs in later stages.

Dynamic Programming comprises deterministic and stochastic dynamic programming. In deterministic dynamic programming, given a state and a decision, both the immediate payoff and the next state are deterministic. In stochastic dynamic programming by contrast, either of these are stochastic. In the following, we will present a general stochastic dynamic programming definition and the respective solution algorithm.

3.4.2.1 Basic Dynamic Problem

For many optimization problems, the following principle of optimality applies: The partial solution of an optimal solution is itself optimal. Dynamic Programming takes advantage of this property by successively composing the optimal solution of partial solutions.

Formally speaking, the procedure can be described as follows (Bertsekas 1987). We consider a discrete-time dynamic system

$$x_{k+1} = f_k(x_k, u_k, w_k), \quad k = 0, 1, \ldots, N-1,$$

where in every period $k = \{0, 1, \ldots, N-1\}$ the system assumes a state x_k (e.g., the current inventory level). The decision variable u_k denotes the control to be selected at time k with knowledge of the state x_k (e.g., u_k may denote the current order quantity). The parameter w_k is stochastic and captures disturbance or noise (e.g., the current demand realization). N describes the time horizon of the system or the number of times control is applied.

Likewise, there is a cost functional that is to be minimized. This cost functional is additive over time. That is, a cost $g_k(x_k, u_k, w_k)$ is incurred in each period k and the total cost results from

$$g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, w_k),$$

where $g_N(x_N)$ is a terminal cost incurred at the end of the process. Since cost is generally a random variable, the model formulation aims at minimizing expected cost $E\left[g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, w_k)\right]$ by determining an optimal policy $\pi = \{\mu_0^*, \ldots, \mu_{N-1}^*\}$, which maps the states x_k into controls $u_k = \mu_k(x_k)$.

The principle of optimality can thus be applied by constructing optimal partial controls starting at time $N-1$ and putting these recursively together. The following proposition states the DP algorithm for the basic problem and shows its optimality.

Proposition 3.1 (Optimality of the DP Algorithm). *Let $J^*(x_0)$ be the optimal cost. Then*

$$J^*(x_0) = J_0(x_0),$$

where the function J_0 is given by the last step of the following algorithm, which proceeds backward in time from period $N-1$ to period 0*:*

$$J_N(x_N) = g_N(x_N) \tag{3.1}$$

$$J_k(x_k) = \min_{u_k \in U_k(x_k)} E_{w_k}\Big[g_k(x_k, u_k, w_k) + J_{k+1}[f_k(x_k, u_k, w_k)]\Big]$$
$$k = 0, 1, \ldots, N-1. \tag{3.2}$$

Furthermore, if $u_k^* = \mu_k^*(x_k)$ minimizes the right-hand side of (3.2) for each x_k and k, the control law $\pi^* = \{\mu_0^*, \ldots, \mu_{N-1}^*\}$ is optimal.

3.4.2.2 Infinite Horizon Dynamic Problems

The remainder of the chapter covers infinite horizon dynamic problems, i.e., dynamic problems with an infinite number of stages (Bertsekas 1987). The focus is on problems where the system is stationary in the sense that the system equation, the cost per stage, and the random disturbance statistics remain unchanged from one stage to the next. Clearly, the assumption of an infinite number of stages is a mathematical formalization. For problems involving a finite but very large number of stages it represents a reasonable approximation. The second assumption, stationarity, is often fulfilled in practice, and in other cases it constitutes a reasonable approximation for situations where the system parameters vary slowly with time.

The analysis of infinite horizon problems is much more complex than the analysis of the finite horizon counterparts. The analysis involves the study of limiting behavior which is often nontrivial. The convergence of the DP algorithm and the corresponding optimal policies for example need special attention, a crucial point in Chaps. 4 and 5 as well. Nevertheless, the analysis of infinite horizon problems has also several advantages. The analysis is often elegant and the implementation of optimal policies is often simple, because they are typically stationary, i.e., do not change from one stage to the next.

We will see in the following that especially the form of the cost functional may substantially affect tractability of an infinite horizon dynamic program. Thus, when dealing with this type of problems (like in our relational contracting model), special attention should be paid to the design of the cost functional.

Classes of Problems

Traditionally, three classes of infinite horizon problems have been of major interest:

(a) In the discounted case with bounded cost per stage, the cost functional takes the form

$$J_\pi(x_0) = \lim_{N\to\infty} E_{w_k, k=0,1,\ldots}\Big[\sum_{k=0}^{N-1} \delta^k g[x_k, \mu(x_k), w_k]\Big], \tag{3.3}$$

where $J_\pi(x_0)$ denotes the cost associated with an initial state x_0 and a policy $\pi = \{\mu_0, \mu_1, \ldots\}$, and δ is a scalar with

$$0 < \delta < 1, \tag{3.4}$$

called the discount factor. The cost per stage $g(x, u, w)$ is uniformly bounded from above and below. More precisely, the cost per stage g satisfies

$$0 \leq g(x, u, w) \leq M, \textit{for all } (x, u, w), \tag{3.5}$$

where M is some scalar.
Due to the presence of a contraction mapping underlying the Dynamic Programming recursion, case (a) is the simplest infinite horizon problem. There are no pathologies here, and effective computational methods are available for solution.

(b) In this class of problems, the costs per stage are allowed to be unbounded either from above or from below. The cost functional has the same form as in (a) except that δ may be a positive scalar greater or equal to 1. The resulting complications are substantial, and the analysis is much more complex compared to case (a).
(c) In the last class of problems, it cannot be guaranteed that $J_\pi(x_0)$ is finite, although minimization of $J_\pi(x_0)$ makes sense only if $J_\pi(x_0)$ is finite for at least some admissible policies π and some initial states x_0. The problems of interest have $J_\pi(x_0) = \infty$, but the limit

$$\lim_{N\to\infty} \frac{1}{N} E_{w_k, k=0,1,\ldots}\Big[\sum_{k=0}^{N-1} \delta^k g[x_k, \mu(x_k), w_k]\Big] \tag{3.6}$$

is finite for every policy $\pi = \{\mu_0, \mu_1, \ldots\}$ and initial state x_0. In this case, one tries to minimize the above expression, interpreted as an average cost per stage.

Computational Methods

The class of problems (a) being most relevant in the later analysis of our model, this subsection presents approaches for solving the infinite horizon problem (a) with cost functional (3.3) under assumptions (3.4) and (3.5). Computational methods for classes (b) and (c) are treated in Bertsekas (1987) for example.

The first approach is successive approximation. Essentially, it is the DP algorithm from Sect. 3.4.2.1, but the algorithm proceeds from lower to higher values of the index k (see Bertsekas 1987, p. 180). Starting from an arbitrary bounded function J, the DP algorithm yields in the limit the function J^*, and the rate of convergence is equal or faster than the rate of convergent geometric progression. Therefore, the DP algorithm is adequate for computing or at least approximating the function J^*. Policy iteration and linear programming offer possibilities to accelerate the computation in a finite number of iterations under the condition that the involved spaces are finite sets. If the number of states is large, this approach is not advisable due to a large overhead per iteration. The best features of both approaches successive approximation and policy iteration can be combined by adaptive aggregation (Bertsekas and Castanon 1989).

Chapter 4
Relational Contracts and Optimal Quantity Flexibility

So far in this treatise, we have seen that relational contracts persist in practice. A reason for this is that unobservable or uncontractible performance is hard to capture in formal contracts. In this case, relational contracts may provide an effective remedy. More precisely, in long-term business relationships, these contracts help to overcome the problem of incentivizing performance by introducing informal agreements. We have also learned that formal court-enforceable contracts are the only enforceable contracts when the relation between the contracting parties is finite. But in the case of a repeated, open-ended relationship, the situation changes. Now, any formal court-enforceable contract may be extended with informal self-enforced agreements, and hence become a relational contract. That is exactly what we will do in this chapter: A supplier and a buyer contract on the basis of a formal quantity flexibility (QF) contract. With the goal of preserving and improving quality, they engage in a long-term business relationship, thereby making room for informal agreements on the actual utilization of quantity flexibility. We will see that an optimal relational contract exists. Moreover, a simple stationary contract design can be determined giving guidance to the supply chain partners to organize their business relationship in an optimal way.

4.1 The Quantity Flexibility Model

The model which will be presented and analyzed in the following combines a formal QF contract with relational aspects of a buyer-supplier relationship. Figure 4.1 illustrates the succession of events. At the start of the relationship, the two parties negotiate a formal QF contract, which is enforceable by a third party. This QF contract regulates that the retailer purchases Q units for the price w_{Δ} per unit at the start of the period and may return up to ΔQ units at the end of the period for a full refund, $\Delta \in [0, 1)$. The scope of this contract is long-term. Moreover, buyer and supplier collaborate with the joint objective of improving quality. But their efforts for quality are unobservable and uncontractible (such that the firms cannot write a contract contingent on actions or quality level). In order to induce effort from the supplier's side, the buyer holds out the prospect of reducing his quantity flexibility,

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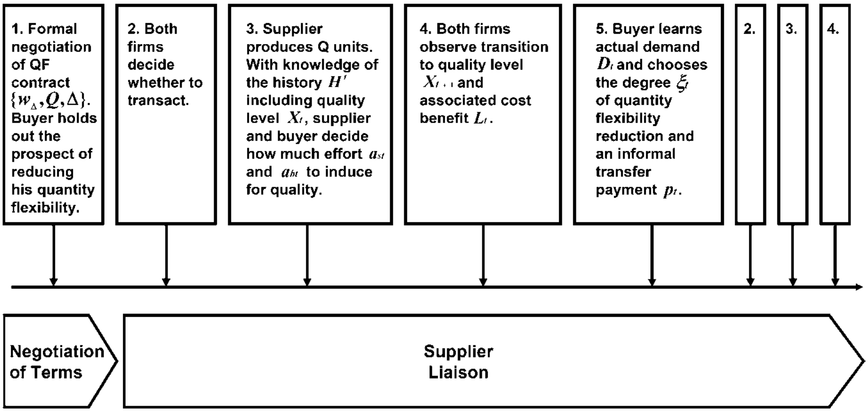

Fig. 4.1 Succession of events for the quantity flexibility model

thus of increasing his responsibility for excess inventory. In addition, the buyer can offer a supplementary transfer payment. These promises are informal and cannot be enforced by court.

Hence, the relational contract we are studying consists of four parts:

1. A *formal* (court-enforceable) QF contract
2. An *informal* agreement on the actual utilization of quantity flexibility, possibly supplemented by an informal transfer payment (not enforceable by a third party)
3. A *strategy for the supplier* regarding effort decisions and relationship management
4. A *strategy for the buyer* regarding effort decisions and relationship management

After a first model description, we will now take a closer look at the sequence of events. At the beginning of the buyer-supplier relationship, the parties negotiate a formal QF contract $\{w_\Delta, Q, \Delta\}$ whose scope is long-term. In the following business relationship, the stage game, i.e., steps 2–5 in Fig. 4.1, is repeated infinitely often. In period t of the game, the sequence of events looks as follows:

- In step 2, both firms decide whether to transact in the current period.
- In step 3, the supplier produces Q units. Knowing the history of the game H^t including the current quality level X_t, supplier and buyer decide how much effort a_{st} and a_{bt} to induce for quality.
- In step 4, both parties observe the impact of their effort decisions, i.e., the transition to the new quality level X_{t+1} and the associated cost benefit for the buyer L_t.
- In step 5, the buyer observes the actual demand D_t and chooses the degree of reduction $\xi_t \in [0, 1]$ of exploitable quantity flexibility and the amount of the supplementary transfer payment p_t.

In this dynamic game, assume that both parties employ trigger strategies, as is standard in the Economics literature on relational contracts (Baker et al. 2001, 2002; Levin 2003). A trigger strategy is to adhere to the relational contract in every period until the other firm does not stick to its promise for the first time and then to refuse to transact for the rest of the game. To a large extent, these strategies reflect behavioral realism: if cooperation breaks down, then it is finished and replaced by spot contracting forever after.

An interpretation for the "infinite" horizon of the game is that the relationship is open-ended, i.e., the firms do not know in advance the exact number of times the stage game will be played. If the game is repeated only a finite number of times, no cooperation will take place. Or, put differently, when the relation between the contracting parties is finite, a relational contract is not enforceable. To see that, consider playing the trust game from Chap. 3 exactly N times. If the number of times the stage game is repeated was known in advance, player 2 would opportunistically play "Betray" in the last period of the game. By backwards induction, one can show that player 1 will anticipate this behavior by playing "Not Trust" in the first period. Hence, no cooperation takes place.

We close the introduction to the model with a summary of the most important model assumptions. First, we consider a two-firm supply chain with one supplier, one buyer, and one product. In the dynamic game, the stage game is repeated infinitely with market demand materializing at the end of each period. In period t, every firm observes X_t and X_{t+1}, the old and the new quality level, and the cost benefit L_t, but cannot observe the other firm's action. The history of the game, the action sets, cost functions, and the transition matrix specifying the transition from quality level to quality level are common knowledge. Furthermore, it is assumed that both firms maximize expected profits, discount future profit streams, and employ trigger strategies. For tractability reasons, there is no carry-over of inventory between periods and demand is assumed to be independent and identically distributed across periods. And finally, quality is modeled dynamically (via Markovian dynamics) yielding an infinite horizon dynamic program.

4.2 Model Analysis

With the objective of determining an optimal self-enforcing relational contract, we now proceed to the analysis of the model. First, the nomenclature is introduced (see Sect. 4.2.1). Then the discounted expected profits of supplier and buyer are described in Sect. 4.2.2.

Having laid the foundations for the model analysis, we dedicate the remainder of the chapter to the characterization of an optimal relational contract. Section 4.2.3 deals with conditions for self-enforcement of the sought-after contract, while Sects. 4.2.4 and 4.2.5 take a closer look at the total expected discounted profit both in the case of perfect coordination and relational contracting. We will see that the total expected discounted profit under an optimal relational contract can be written in the form of a dynamic programming recursion. Convergence of this underlying dynamic programming recursion is taken care of in Sect. 4.2.6. Finally, we deduce a simple optimal relational contract and explain the managerial implications for the supply chain partners (see Sect. 4.2.7 and Theorem 4.1 in particular).

4.2.1 Nomenclature

The following list summarizes the symbols used to describe and analyze the model. Note that τ_{st}, e_{st}, and a_{st} are the supplier's decision variables and that τ_{bt}, e_{bt}, a_{bt}, ξ_t, and p_t are the buyer's decision variables.

t:	Period
c:	Production cost
Q:	Production quantity
r:	Retail price
w_Δ:	Wholesale price of the quantity flexibility contract
Δ:	Quantity flexibility
ξ_t:	Degree of reduction of exploitable quantity flexibility in period t
p_t:	Supplementary transfer payment in period t
δ:	Discount factor
D_t:	Demand realization in period t
G:	Continuous distribution function of D_t
s:	Salvage value
g:	Goodwill cost
$\mathcal{X}$:	Finite, discrete state space reflecting quality
$X_t \in \mathcal{X}$:	Initial quality level in period t
L_t:	Buyer's cost benefit from quality transition in period t
F_{xz}:	Continuous distribution function of L_t with support $[\underline{l}_{xz}, \overline{l}_{xz}]$ where $\underline{l}_{xz} < \overline{l}_{xz}$
$L(x,z)$:	Expected value of L_t given $(X_t, X_{t+1}) = (x,z)$, i.e., $L(x,z) \equiv E[L_t \mid (X_t, X_{t+1}) = (x,z)] = \int_{\underline{l}_{xz}}^{\overline{l}_{xz}} l dF_{xz}(l)$
γ_{st}:	Supplier's outside option in period t
γ_{bt}:	Buyer's outside option in period t
a_{st}:	Supplier's effort for quality in period t
a_{bt}:	Buyer's effort for quality in period t
$c_s(a_{st}, X_t, \gamma_{st})$:	Supplier's cost from effort in period t
$c_b(a_{bt}, X_t, \gamma_{bt})$:	Buyer's cost from effort in period t
$P_{xz}(a_s, a_b) = Pr\{X_{t+1} = z \mid X_t = x; a_s, a_b\}$:	Probability for a transition from state $X_t = x$ to state $X_{t+1} = z$, given efforts a_s and a_b
$\tau_{st}, \tau_{bt} \in \{0,1\}$:	Decision to transact in period t
$e_{st}, e_{bt} \in \{0,1\}$:	Decision to execute the informal transaction at the end of period t
$H^t = \{X_1, \ldots, X_t; L_1, \ldots, L_{t-1}; \tau_{s1}, \ldots, \tau_{s(t-1)}; e_{s1}, \ldots, e_{s(t-1)}; \tau_{b1}, \ldots, \tau_{b(t-1)}; e_{b1}, \ldots, e_{b(t-1)}\}$:	$H^t \in \mathcal{X}^t \times \mathbb{R}_+{}^{t-1} \times \{0,1\}^{4(t-1)}$ is the public history of the game at the beginning of period t

4.2.2 Discounted Expected Profits

This subsection specifies discounted expected profits for supplier and buyer starting from the beginning of period T with $0 < \delta < 1$. Discounted expected profits due to collaboration for quality improvement are given as follows:

$$\Pi_{sT} = \sum_{t=T}^{\infty} \delta^{t-T} \tau_{st} \tau_{bt} \Big[e_{st} e_{bt} [(w_\Delta - s)\xi_t \min(\Delta Q, \max(Q - D_t, 0)) + p_t] - c_s(a_{st}, X_t, \gamma_{st}) \Big], \tag{4.1}$$

$$\Pi_{bT} = \sum_{t=T}^{\infty} \delta^{t-T} \tau_{st} \tau_{bt} \Big[L_t + e_{st} e_{bt} [(s - w_\Delta)\xi_t \min(\Delta Q, \max(Q - D_t, 0)) - p_t] - c_b(a_{bt}, X_t, \gamma_{bt}) \Big]. \tag{4.2}$$

That is, in each period t, supplier and buyer decide whether they want to transact by setting τ_{st} and τ_{bt} to zero or one. Each transaction involves costs $c_s(a_{st}, X_t, \gamma_{st})$ for the supplier and costs $c_b(a_{bt}, X_t, \gamma_{bt})$ for the buyer, depending on the actions a_{st} and a_{bt} taken in period t, the current quality level X_t, and the current outside alternatives γ_{st} and γ_{bt}. Also note that the cost benefit L_t from the current quality level goes to the buyer. If both parties want to transact in period t, they can decide whether to execute the informal transaction $(w_\Delta - s)\xi_t \min(\Delta Q, \max(Q - D_t, 0)) + p_t$ by setting e_{st} and e_{bt} to zero or one. Here, shifting $(w_\Delta - s)\xi_t \min(\Delta Q, \max(Q - D_t, 0))$ to the supplier means that the buyer renounces ξ_t of exploitable quantity flexibility. Only if both are willing to do so, the informal transaction actually takes place.

Together with the discounted expected profits from the formal QF contract, this yields cumulative discounted expected profits (starting from the beginning of period T) for supplier and buyer of

$$\begin{aligned} \Pi_{sT}^{cum} = \sum_{t=T}^{\infty} \delta^{t-T} \tau_{st} \tau_{bt} \Big[& -cQ + w_\Delta [(1-\Delta)Q + \min(\Delta Q, \max(D_t - (1-\Delta)Q, 0))] \\ & + s(Q - [(1-\Delta)Q + \min(\Delta Q, \max(D_t - (1-\Delta)Q, 0))]) \\ & + e_{st} e_{bt} [(w_\Delta - s)\xi_t \min(\Delta Q, \max(Q - D_t, 0)) + p_t] - c_s(a_{st}, X_t, \gamma_{st}) \Big], \end{aligned} \tag{4.3}$$

$$\begin{aligned} \Pi_{bT}^{cum} = \sum_{t=T}^{\infty} \delta^{t-T} \tau_{st} \tau_{bt} \Big[& L_t - w_\Delta [(1-\Delta)Q + \min(\Delta Q, \max(D_t - (1-\Delta)Q, 0))] \\ & + r \min(Q, D_t) + s \max(0, (1-\Delta)Q - D_t) - g \max(0, D_t - Q) \\ & + e_{st} e_{bt} [(s - w_\Delta)\xi_t \min(\Delta Q, \max(Q - D_t, 0)) - p_t] - c_b(a_{bt}, X_t, \gamma_{bt}) \Big]. \end{aligned} \tag{4.4}$$

According to the formal QF contract, the supplier earns $w_\Delta[(1-\Delta)Q+\min(\Delta Q, \max(D_t-(1-\Delta)Q,0))]$ from the wholesale payment and $s(Q-[(1-\Delta)Q+\min(\Delta Q, \max(D_t-(1-\Delta)Q,0))])$ from salvaging leftover units. In addition, he incurs production costs of cQ. Thus, (4.3) combines the supplier's discounted expected profits from the formal QF contract with the supplier's discounted expected profits from collaboration captured in (4.1). The buyer, on the other hand, incurs costs of $w_\Delta[(1-\Delta)Q+\min(\Delta Q, \max(D_t-(1-\Delta)Q,0))]$ for the wholesale payment plus goodwill costs of $g\max(0, D_t-Q)$ in case he cannot fully satisfy market demand. Moreover, he earns $r\min(Q, D_t)$ from selling the product to the market and $s\max(0,(1-\Delta)Q-D_t)$ from salvaging unsold units. Together with the discounted expected profits from collaboration specified in (4.2), this yields the buyer's cumulative discounted expected profits in (4.4).

4.2.3 Self-Enforcing Contract

In the model at hand, the buyer may promise a reduction ξ_t of his quantity flexibility and a supplementary transfer payment p_t. Typically, such informal agreements cannot be enforced by a third party. Consequently, supplier and buyer must have incentives to sustain the relational contract voluntarily or in other words: the relational contract has to be self-enforcing. This is governed by the following equations. In the given framework, a self-enforcing contract must satisfy for every period $t=1,2,\ldots$, public history H^t, initial state $X_t\in\mathcal{X}$, and quality level L_t:

$$0 \le E[\Pi_{st}|H^t], \tag{4.5}$$

$$0 \le E[\Pi_{bt}|H^t], \tag{4.6}$$

$$a_{st} \in \arg\max_a \Big\{-c_s(a,X_t,\gamma_{st}) + \sum_{z\in\mathcal{X}} P_{X_t z}(a,a_{bt})E[(w_\Delta - s)\xi_t \min(\Delta Q, \max(Q-D_t,0)) + p_t + \delta\Pi_{s(t+1)}|H^t, X_{t+1}=z]\Big\}, \tag{4.7}$$

$$a_{bt} \in \arg\max_a \Big\{-c_b(a,X_t,\gamma_{bt}) + \sum_{z\in\mathcal{X}} P_{X_t z}(a_{st},a)\Big[L(X_t,z) + E[(s-w_\Delta)\xi_t \min(\Delta Q, \max(Q-D_t,0)) - p_t + \delta\Pi_{b(t+1)}|H^t, X_{t+1}=z]\Big]\Big\}, \tag{4.8}$$

$$0 \le \underbrace{(w_\Delta - s)\xi_t \min(\Delta Q, \max(Q-D_t,0)) + p_t}_{\text{benefit from informal transaction}} + \underbrace{\delta E[\Pi_{s(t+1)}|H^t, X_{t+1}, L_t]}_{\text{future profits}}, \tag{4.9}$$

$$0 \le \underbrace{(s - w_\Delta)\xi_t \min(\Delta Q, \max(Q-D_t,0)) - p_t}_{\text{loss from informal transaction}} + \underbrace{\delta E[\Pi_{b(t+1)}|H^t, X_{t+1}, L_t]}_{\text{future profits}}, \tag{4.10}$$

where $H^t=\{X_1,\ldots,X_t;L_1,\ldots,L_{t-1};\tau_{s1},\ldots,\tau_{s(t-1)};e_{s1},\ldots,e_{s(t-1)};\tau_{b1},\ldots,\tau_{b(t-1)};e_{b1},\ldots,e_{b(t-1)}\}$ represents the public history of the game at the beginning of period t. $\mathcal{X}$ is a finite, discrete state space reflecting quality, and $P_{xz}(a_s,a_b)=Pr\{X_{t+1}=z|X_t=x;a_s,a_b\}$ denotes the probability for a transition from quality

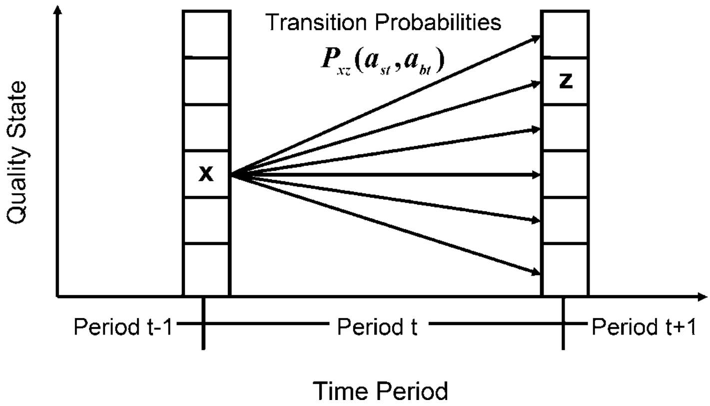

Fig. 4.2 State transition from quality level x to quality level z in the finite, discrete state space $\mathcal{X}$

state $X_t = x$ to quality state $X_{t+1} = z$, given efforts a_s and a_b. As illustrated in Fig. 4.2, these system dynamics go as follows:

- In the first period, an initial quality state is given.
- Subsequently, for every state x and every action pair (a_s, a_b) transitions to state z appear with transition probability $P_{xz}(a_s, a_b)$.

If either firm refuses to transact in period t, then both supplier and buyer incur zero cost, the cost benefit $L_t = 0$, and the distribution of X_{t+1} is governed by the transition matrix $P_{xz}(0, 0)$. Also note that the feasible actions a_s and a_b form real intervals $A_s(x) = [\underline{a}_s(x), \overline{a}_s(x)]$ and $A_b(x) = [\underline{a}_b(x), \overline{a}_b(x)]$ for every quality state $x \in \mathcal{X}$.

According to Abreu (1988), these conditions are sufficient for a relational contract with trigger strategies to be self-enforcing. In particular, (4.5) and (4.6) guarantee positive discounted expected profits for supplier and buyer. Under the assumption that the buyer chooses action a_{bt} in the current period and that both firms stick to the relational contract for the rest of the game, (4.7) specifies that the supplier's action a_{st} maximizes the discounted expected profit. Equation (4.8) states the analogous property for the buyer's action a_{bt}. The last two inequalities (4.9) and (4.10) ensure that executing the informal transaction is more advantageous than terminating the relationship.

4.2.4 *Total Expected Discounted Profit with Perfect Coordination*

Sections 4.2.4, 4.2.5, and 4.2.6 treat the total expected discounted profit generated by joint effort for quality improvement. The formulae and results presented in these sections are based on work by Plambeck and Taylor (2006). First, the integrated supply chain with perfect coordination will be covered. The remainder of this chapter is devoted to the identification of an optimal relational contract in the case of a decentralized supply chain.

In the case of perfect coordination where a single agent owns and operates the supply chain, the total expected discounted profit starting from state x can be written in the form of a dynamic programming recursion:

$$\begin{aligned}\bar{V}(x) = \max\Big[\delta \sum_{z\in\mathcal{X}} P_{xz}(0,0)\bar{V}(z);\\ \max_{a_s\in A_s(x), a_b\in A_b(x)} \{-c_s(a_s,x,\gamma_s) - c_b(a_b,x,\gamma_b)\\ + \sum_{z\in\mathcal{X}} P_{xz}(a_s,a_b)[L(x,z) + \delta\bar{V}(z)]\}\Big].\end{aligned}$$

We will shortly explore how this is connected to the decentralized system.

4.2.5 *Total Expected Discounted Profit under an Optimal Relational Contract*

We now take a closer look at the decentralized supply chain. Assume that supplier and buyer have put an optimal relational contract in place. Then, the total expected discounted profit under this relational contract is given by the following dynamic programming recursion:

$$\begin{aligned}&\forall x\in\mathcal{X}\ \forall v:\mathcal{X}\to\mathbb{R}^+\\ &T(v)(x) = \max\Big[\delta\sum_{z\in\mathcal{X}} P_{xz}(0,0)v(z);\\ &\max_{a_s\in A_s(x), a_b\in A_b(x)}\{-C(a_s,a_b,v,x)\\ &+\sum_{z\in\mathcal{X}} P_{xz}(a_s,a_b)[L(x,z)+\delta v(z)]\}\Big] \qquad (4.11)\end{aligned}$$

subject to

$$\begin{aligned}V_s(x,z) &\geq 0,\\ V_b(x,z) &\geq 0,\\ V_s(x,z)+V_b(x,z) &\leq v(z) \quad \text{for } z\in\mathcal{X},\\ a_s &\in \arg\max_{a\in A_s(x)}\Big\{-c_s(a,x,\gamma_s)+\sum_{z\in\mathcal{X}} P_{xz}(a,a_b)\delta V_s(x,z)\Big\},\\ a_b &\in \arg\max_{a\in A_b(x)}\Big\{-c_b(a,x,\gamma_b)+\sum_{z\in\mathcal{X}} P_{xz}(a_s,a)[L(x,z)+\delta V_b(x,z)]\Big\},\end{aligned}$$

where

$$C(a_s, a_b, v, x) \equiv c_s(a_s, x, \gamma_s) + c_b(a_b, x, \gamma_b) + \min_{V_s, V_b} \sum_{z \in \mathcal{X}} P_{xz}(a_s, a_b) Pr(x, z) \delta v(z),$$

$$Pr(x, z) \equiv \frac{[v(z) - V_s(x, z) - V_b(x, z)]}{v(z)} \,,$$

and where $V_s(x, z)$ and $V_b(x, z)$ are the portions of the total expected discounted profit starting from the next period allocated to the supplier and the buyer respectively.

Here, Tv maximizes the total discounted expected profit under a self-enforcing relational contract, i.e., on condition that each firm has an incentive to execute informal transactions. These incentives are modelled by constraints $V_s(x, z) \geq 0$ and $V_b(x, z) \geq 0$. In this connection, the cost function $C(a_s, a_b, v, x)$ captures two cost components: the direct cost of actions a_s and a_b and the expected cost of possible termination. More specifically, possible termination is characterized by the termination probability $Pr(x, z)$. This is the probability with which the supply chain partners terminate their relationship, conditional on a state transition from quality level x to quality level z. $Pr(x, z)$ is simply needed to allow for termination to occur with positive probability. The definition of $Pr(x, z)$ derives from Plambeck and Taylor (2006) who also provide a detailed analysis of probabilistic termination in the context of relational contracting.

Given this representation of the total expected discounted profit, it is natural to focus on the structural properties of the operator T. We will see that this operator has useful properties that guarantee effective computation of V^*, the total expected discounted profit under an optimal relational contract, and the optimal actions for supplier and buyer as well. These important structural properties of T will be examined in the following subsection.

4.2.6 Convergence

The preceding Sect. 4.2.5 expresses the total expected discounted profit under an optimal relational contract in the form of a dynamic programming recursion. Convergence of this dynamic programming recursion is essential for effective computation of the total expected discounted profit under an optimal relational contract V^* and hence for the computation of the optimal policies for supplier and buyer.

Before going into detail, let's sketch the most important steps and results of the following analysis: Lemmas 4.1 and 4.2 together with Propositions 4.1 and 4.2 ensure that value iteration yields in the limit V^*, primarily due to the presence of a contraction mapping (see Lemma 4.2, p. 64). As stated in Proposition 4.2, the rate of convergence is determined by the discount factor δ.

We will now get granular on the individual results. As already mentioned, the operator T has a special form that permits to obtain convergence. First of all, Lemma 4.1 indicates that the operator T is isotone. We will refer to this useful property, when proving the contraction property of T in Lemma 4.2 later on:

Lemma 4.1 (T is isotone). *The operator T is isotone, i.e.,*

$$\textit{if} \quad v_1 \geq v_2 \quad \textit{then} \quad Tv_1 \geq Tv_2.$$

Proof of Lemma 4.1. If $v_1 \geq v_2$, then for every $x \in \mathcal{X}$, $a_m \in A_m(x)$ and $a_b \in A_b(x)$,

$$\begin{aligned}
&-C(a_s, a_b, v_1, x) + \sum_{z \in \mathcal{X}} P_{xz}(a_s, a_b)[L(x,z) + \delta v_1(x)] \\
&\geq -C(a_s, a_b, v_2, x) + \sum_{z \in \mathcal{X}} P_{xz}(a_s, a_b)[L(x,z) + \delta v_2(x)].
\end{aligned}$$

Hence,

$$\begin{aligned}
T(v_1)(x) &= \max \Big[\delta \sum_{z \in \mathcal{X}} P_{xz}(0,0) v_1(z); \\
&\qquad \max_{a_s \in A_s(x), a_b \in A_b(x)} \{ -C(a_s, a_b, v_1, x) \\
&\qquad + \sum_{z \in \mathcal{X}} P_{xz}(a_s, a_b)[L(x,z) + \delta v_1(z)] \} \Big] \\
&\geq \max \Big[\delta \sum_{z \in \mathcal{X}} P_{xz}(0,0) v_2(z); \\
&\qquad \max_{a_s \in A_s(x), a_b \in A_b(x)} \{ -C(a_s, a_b, v_2, x) \\
&\qquad + \sum_{z \in \mathcal{X}} P_{xz}(a_s, a_b)[L(x,z) + \delta v_2(z)] \} \Big] \\
&= T(v_2)(x).
\end{aligned}$$

□

Next, Lemma 4.1 shows that T is a contraction mapping. This contraction property will be key to establishing convergence:

Lemma 4.2 (T is a contraction mapping). *The operator T is a contraction mapping, i.e.,*

$$\textit{if} \quad v_1 \geq v_2 \quad \textit{then} \quad \rho(Tv_1, Tv_2) \leq \delta \rho(v_1, v_2)$$

with $\rho(v_1, v_2) \equiv \sup_{x \in \mathcal{X}} |v_1 - v_2|$.

Proof of Lemma 4.2. Assume $v_1 \geq v_2$ and define

$$(\bar{a}_s, \bar{a}_b) \equiv \text{argmax}_{(a_s,a_b)\in A_s(x)\times A_b(x)}\{-C(a_s, a_b, v_1, x) + \sum_{z\in\mathcal{X}} P_{xz}(a_s, a_b)[L(x,z) + \delta v_1(z)]\}.$$

Case 1. Suppose that $\forall x \in \mathcal{X}$

$$\delta \sum_{z\in\mathcal{X}} P_{xz}(0,0)v_1(z) \leq -C(\bar{a}_s, \bar{a}_b, v_1, x) + \sum_{z\in\mathcal{X}} P_{xz}(\bar{a}_s, \bar{a}_b)[L(x,z) + \delta v_1(z)]\}.$$

Then $\forall x \in \mathcal{X}$

$$\begin{aligned}
0 &\leq Tv_1(x) - Tv_2(x) \quad \text{(since T is isotone; see Lemma 4.1)} \\
&\leq \underbrace{-C(\bar{a}_s, \bar{a}_b, v_1, x) + \sum_{z\in\mathcal{X}} P_{xz}(\bar{a}_s, \bar{a}_b)[L(x,z) + \delta v_1(z)]}_{=Tv_1(x)} \\
&\quad - \underbrace{\Big[-C(\bar{a}_s, \bar{a}_b, v_2, x) + \sum_{z\in\mathcal{X}} P_{xz}(\bar{a}_s, \bar{a}_b)[L(x,z) + \delta v_2(z)]\Big]}_{\leq Tv_2(x)} \\
&= C(\bar{a}_s, \bar{a}_b, v_2, x) - C(\bar{a}_s, \bar{a}_b, v_1, x) \\
&\quad + \delta \sum_{z\in\mathcal{X}} P_{xz}(\bar{a}_s, \bar{a}_b)[v_1(z) - v_2(z)] \\
&\leq \delta \sum_{z\in\mathcal{X}} P_{xz}(\bar{a}_s, \bar{a}_b)[v_1(z) - v_2(z)] \\
&\leq \delta \underbrace{\sum_{z\in\mathcal{X}} P_{xz}(\bar{a}_s, \bar{a}_b)}_{\leq 1} \sup_{z\in\mathcal{X}}[v_1(z) - v_2(z)] \\
&\leq \delta \sup_{z\in\mathcal{X}}[v_1(z) - v_2(z)].
\end{aligned}$$

Case 2. Suppose that $\forall x \in \mathcal{X}$

$$\delta \sum_{z\in\mathcal{X}} P_{xz}(0,0)v_1(z) \geq -C(\bar{a}_s, \bar{a}_b, v_1, x) + \sum_{z\in\mathcal{X}} P_{xz}(\bar{a}_s, \bar{a}_b)[L(x,z) + \delta v_1(z)]\}.$$

Then $\forall x \in \mathcal{X}$

$$\begin{aligned}
0 &\leq Tv_1(x) - Tv_2(x) \quad \text{(since T is isotone)} \\
&\leq \underbrace{\delta \sum_{z\in\mathcal{X}} P_{xz}(0,0)v_1(z)}_{=Tv_1(x)} - \underbrace{\delta \sum_{z\in\mathcal{X}} P_{xz}(0,0)v_2(z)}_{\leq Tv_2(x)} \\
&= +\delta \underbrace{\sum_{z\in\mathcal{X}} P_{xz}(0,0)}_{\leq 1}[v_1(z) - v_2(z)] \\
&\leq \delta \sup_{z\in\mathcal{X}}[v_1(z) - v_2(z)].
\end{aligned}$$

Thus, in both cases, the following inequality holds $\forall x \in \mathcal{X}$:

$$\begin{aligned}
|Tv_1(x) - Tv_2(x)| &= Tv_1(x) - Tv_2(x) \quad \text{(since T is isotone)} \\
&\leq \delta \sup_{z\in\mathcal{X}}[v_1(z) - v_2(z)] \\
&= \delta \sup_{z\in\mathcal{X}} |v_1(z) - v_2(z)| \quad (\text{since } v_1 \geq v_2).
\end{aligned}$$

Therefore, taking the supremum over x yields

$$\begin{aligned}
\rho(Tv_1 - Tv_2) &= \sup_{x\in\mathcal{X}} |Tv_1(x) - Tv_2(x)| \\
&\leq \delta \sup_{z\in\mathcal{X}} |v_1(z) - v_2(z)| \\
&= \delta\rho(v_1, v_2).
\end{aligned}$$

□

The contraction property of T allows for the application of Tarski's fixed point theorem implying that T has a largest fixed point:

Proposition 4.1 (T has a largest fixed point). *Existence of a largest fixed point follows from Tarski's fixed point theorem (Tarski 1955):*
The operator T has a largest fixed point V^*. That is,

$$V^* = TV^*,$$

and for any other fixed point $V = TV$, $V^*(x) \geq V(x)$ for all $x \in \mathcal{X}$. Furthermore, $V^* \in [0, \bar{V}(x)]$ for all $x \in \mathcal{X}$.

Proof of Proposition 4.1. $\bar{V}(x)$ is defined as the maximal discounted expected profit starting from state x (see Sect. 4.2.4, p. 61). Therefore, for any fixed point of T, $v(x) \in [0, \bar{V}(x)]$ holds for all $x \in \mathcal{X}$. Together with the isotonicity of T asserted by Lemma 4.1, this implies that Tarski's fixed-point theorem (Tarski 1955) is applicable, yielding that the operator T has a largest fixed point. □

The following proposition states that value iteration beginning with $\bar{V}$, the total expected discounted profit with perfect coordination, yields in the limit V^*, the total expected discounted profit under an optimal relational contract. In addition, it specifies the rate of convergence.

Proposition 4.2 (Convergence). *Value iteration beginning with* $\bar{V}$ *(the total expected discounted profit with perfect coordination) converges to the optimal value function* V^**:*

$$V^* = \lim_{n\to\infty} T^n \bar{V}.$$

Value iteration converges geometrically to the optimal value function at the rate of the discount factor:

$$\sup_{x\in\mathcal{X}}\{T^n\bar{V}(x) - V^*(x)\} \leq \delta^n \sup_{x\in\mathcal{X}}\{\bar{V}(x) - V^*(x)\}.$$

Proof of Proposition 4.2. In strict analogy to Plambeck and Taylor (2006), this is a proof by induction. Because T is isotone from Lemma 4.1, the following can be derived by induction:

$$T^n\bar{V} \leq T^{n-1}\bar{V} \quad \text{for } n = 1, 2, \ldots. \tag{4.12}$$

By induction on (4.12) and on the contraction property of T established in Lemma 4.2,

$$\begin{aligned} \rho(T^{n+1}\bar{V}, T^n\bar{V}) &\leq \delta\rho(T^n\bar{V}, T^{n-1}\bar{V}) \\ &\leq \delta^n \rho(T\bar{V}, \bar{V}) \\ &\leq \delta^n ||\bar{V}|| \\ &\to 0 \text{ as } n \to \infty, \end{aligned}$$

i.e., $T^n\bar{V}$ forms a Cauchy sequence, which has a limit

$$\hat{V} = \lim_{n\to\infty} T^n\bar{V} \tag{4.13}$$

satisfying

$$\hat{V} \leq T^n\bar{V}. \tag{4.14}$$

To see that $\hat{V}$ is a fixed point of T, choose $\epsilon > 0$ and select $N \geq 0$ such that $n \geq N$ implies $\rho(T^n\bar{V}, \hat{V}) < \epsilon/2$. Then for any $n \geq N$,

$$\begin{aligned} \rho(\hat{V}, T\hat{V}) &\leq \rho(\hat{V}, T^{n+1}\bar{V}) + \rho(T^{n+1}\bar{V}, T\hat{V}) \\ &\leq \frac{\epsilon}{2} + \rho(T^{n+1}\bar{V}, T\hat{V}) \\ &\leq \frac{\epsilon}{2} + \delta\rho(T^n\bar{V}, T\hat{V}) \\ &\leq \frac{\epsilon}{2} + \delta\frac{\epsilon}{2} \\ &\leq \epsilon\ , \end{aligned}$$

where the third inequality follows from Lemma 4.2 and inequality (4.14). Therefore we conclude that

$$\hat{V} = T\hat{V}.$$

Let V_2 denote another fixed point of T. From Proposition 4.1 $V_2 \leq \bar{V}$. Because T is isotone,

$$T^n\bar{V} \geq T^n V_2 = V_2 \tag{4.15}$$

for all n. Letting $n \to \infty$, we conclude from (4.13) and (4.15) that

$$\hat{V} \geq V_2,$$

so $\hat{V}$ must be the largest fixed point of T. That is, $\hat{V} = V^*$. □

In summary, the most important results of this subsection are as follows. First, we conclude that T has a largest fixed point V^* due to the contraction property of T and with the help of Tarski's fixed point theorem. Second, one can establish that value iteration beginning with $\bar{V}$ (the total expected discounted profit with perfect coordination) converges geometrically to V^* (the total expected discounted profit under an optimal relational contract) at the rate of the discount factor δ.

4.2.7 A Simple Optimal Relational Contract

To proceed, we introduce an optimal relational contract for the present setting. This self-enforcing contract is optimal in the sense that no other self-enforcing contract generates higher expected joint surplus. More precisely, Theorem 4.1 states two main points: First, a complete plan for the business relationship can be given. Second, a simple contract design can be determined which makes the relational contract more easily applicable in practice.

In the following theorem, for each $x \in \mathcal{X}$, $(a_s^*(x), a_b^*(x))$ denotes the optimal actions obtained by solving the dynamic programming recursion (4.11) on p. 62.

Theorem 4.1 (Simple Optimal Relational Contract). *The total expected discounted profit under an optimal relational contract is $V^*(X_1)$, and a simple optimal relational contract is characterized as follows. The firms' strategies for whether or not to transact are*

$$\tau_{st} = \begin{cases} \tau_s^*(X_t) & \textit{if } t \leq \Psi \textit{ and } e_{su} = e_{bu} = 1 \textit{ for all } u < t, \\ 0 & \textit{if } t > \Psi \textit{ or } e_{su}e_{bu} = 0 \textit{ for some } u < t; \end{cases}$$

$$\tau_{bt} = \begin{cases} \tau_b^*(X_t) & \textit{if } t \leq \Psi \textit{ and } e_{su} = e_{bu} = 1 \textit{ for all } u < t, \\ 0 & \textit{if } t > \Psi \textit{ or } e_{su}e_{bu} = 0 \textit{ for some } u < t. \end{cases}$$

That is, the firms terminate the relationship at the end of period $\Psi \equiv \inf\{t : L_t < F^{-1}_{X_t,X_{t+1}}(Pr^*(X_t, X_{t+1}))\}$*. In each period that the firms transact, the informal transaction depends only on the observed transition to the new quality level, the cost benefit thereof, and the current demand realization. If a fraction* $\alpha \in [0, 1]$ *of total expected profit was agreed to be allocated to the supplier, the degree of quantity flexibility reduction and supplementary payment should be chosen as follows:*

$$(w_\Delta - s)\xi_t \min(\Delta Q, \max(Q - D_t, 0)) + p_t$$
$$= \begin{cases} [1 - Pr^*(X_t, X_{t+1})]^{-1}\delta V_s^*(X_t, X_{t+1}) \\ -\sum_{z\in\mathcal{X}} P_{X_t z}(a_s^*(X_t), a_b^*(X_t))\delta V_s^*(z) \\ +\alpha[V^*(X_t) - \delta V^*(X_{t+1})] + c_{st} & \text{if} \quad L_t \geq F^{-1}_{X_t,X_{t+1}}(Pr^*(X_t, X_{t+1})), \\ \\ 0 \quad \text{implying } \xi_t = 0,\ p_t = 0 & \text{otherwise.} \end{cases} \tag{4.16}$$

The action strategies depend only on the current quality state:

$$a_{st} = a_s^*(X_t) \quad a_{bt} = a_b^*(X_t) \quad \text{for } t = 1, 2, \ldots$$

and each firm is willing to execute the informal transaction:

$$e_{st} = e_{bt} = 1 \quad \text{for } t = 1, 2, \ldots.$$

Proof of Theorem 4.1. The proof proceeds in three steps in the style of Plambeck and Taylor (2006). The first step compiles basic properties that have to be met by any relational contract. The second step shows that for any relational contract with these properties, there exists a simple self-enforcing variant that achieves the same total expected discounted profit. Finally, the third step establishes the simple optimal relational contract by solving the dynamic program in Sect. 4.2.5.

Step 1: Characteristics of any Optimal Relational Contract. Let o denote a relational contract with the following terms for the first period: degree of reduction of exploitable quantity flexibility ξ^o, supplementary transfer payment p^o, strategy for the supplier of $\{\tau_s^o, a_s^o\}$, and strategy for the buyer of $\{\tau_b^o, a_b^o\}$, conditional on $(X_1, X_2, L_1) = (x, z, l)$. Define V_1^o as the total expected discounted profit, conditional on $X_1 = x$:

$$\begin{aligned} V_1^o(x) &= E^o[\Pi_{s1}^o + \Pi_{b1}^o | X_1 = x] \\ &= E^o\Big[\sum_{t=1}^{\infty} \delta^{t-1}\tau_{st}^o\tau_{bt}^o[L_t - c_s(a_{st}^o, X_t, \gamma_{st}) \\ &\qquad -c_b(a_{bt}^o, X_t, \gamma_{bt})] | X_1 = x\Big], \end{aligned}$$

where the notation E^o indicates that the expectation is taken with respect to the distribution induced by the relational contract. Likewise, define $V_2^o(x, z)$ as the total expected discounted profit under the optimal relational contract starting from period 2, conditional on $(X_1, X_2) = (x, z)$.

$$V_2^o(x,z) = E^o\Big[\sum_{t=2}^{\infty} \delta^{t-1} \tau_{st}^o \tau_{bt}^o [L_t - c_s(a_{st}^o, X_t, \gamma_{st}) \\ - c_b(a_{bt}^o, X_t, \gamma_{bt})] | (X_1, X_2) = (x,z)\Big].$$

To be considered as a candidate for optimality, the relational contract o must satisfy the following two properties.

Property 1: A necessary condition for optimality of the relational contract o is

$$V_2^o(x,z) \le V_1^o(z) \quad \forall x, z \in \mathcal{X} \tag{4.17}$$

because if $V_2^o(x,z) > V_1^o(z)$, the firms could achieve a strictly greater total expected discounted profit by starting with the continuation contract from period 2, rather than the initial contract for state z. Equation (4.17) may be a strict inequality to create incentives for action in period 1. In the case $\tau_s^o(x)\tau_b^o(x) = 0$, we will obtain

$$V_2^o(x,z) = V_1^o(z) \quad \forall x, z \in \mathcal{X}.$$

Property 2: Another necessary condition for optimality of the relational contract o is self-enforcement in the first period, which implies that

$$E^o[\Pi_{s1}^o | X_1 = x] \ge 0 \quad \text{and} \quad E^o[\Pi_{b1}^o | X_1 = x] \ge 0 \quad \forall x \in \mathcal{X}. \tag{4.18}$$

And for every x such that $\tau_s^o(x) = \tau_b^o(x) = 1$ so that the firms transact in the first period, the contract must satisfy

$$a_s^o = \arg\max_{a \in A_s(x)} \Big\{ - c_s(a, x, \gamma_{s1}) \\ + \sum_{z \in \mathcal{X}} P_{xz}(a, a_b^o) E^o[(w_\Delta - s)\xi^o \min(\Delta Q, \max(Q - D_1, 0)) \\ + p^o + \delta \Pi_{s2}^o | X_1 = x, X_2 = z] \Big\}, \tag{4.19}$$

$$a_b^o = \arg\max_{a \in A_b(x)} \Big\{ - c_b(a, x, \gamma_{b1}) \\ + \sum_{z \in \mathcal{X}} P_{xz}(a_s^o, a) \Big[L(x,z) + E^o[(s - w_\Delta)\xi^o \min(\Delta Q, \max(Q - D_1, 0)) \\ - p^o + \delta \Pi_{b2}^o | X_1 = x, X_2 = z] \Big] \Big\}, \tag{4.20}$$

$$0 \le (w_\Delta - s)\xi^o(x,z,l) \min(\Delta Q, \max(Q - D_1, 0)) + p^o(x,z,l) \\ + \delta E^o[\Pi_{s2}^o | (X_1, X_2, L_1) = (x,z,l)], \tag{4.21}$$

$$0 \le (s - w_\Delta)\xi^o(x,z,l) \min(\Delta Q, \max(Q - D_1, 0)) - p^o(x,z,l) \\ + \delta E^o[\Pi_{b2}^o | (X_1, X_2, L_1) = (x,z,l)]. \tag{4.22}$$

Step 2: *The Equivalent Simple Relational Contract.* The second step constructs a simple, self-enforcing relational contract with the same expected total discounted profit as the relational contract o. For this purpose define

$$Pr(x,z) \equiv [V_1^o(z) - V_2^o(z)]/V_1^o(z)$$
$$\Psi \equiv \inf\{t : L_t < F_{X_t,X_{t+1}}^{-1}(Pr(X_t, X_{t+1}))\}.$$

Now, the firms' strategies for whether or not to transact can be written as follows. For $t = 1, 2, \ldots,$

$$\tau_{st} = \begin{cases} 1 & \text{if } t \le \Psi,\ e_{su} = e_{bu} = 1 \text{ for } u < t \text{ and} \\ & X_t \in \{x : \tau_s^o(x) = \tau_b^o(x) = 1\}, \\ 0 & \text{otherwise;} \end{cases}$$

$$\tau_{bt} = \begin{cases} 1 & \text{if } t \le \Psi,\ e_{su} = e_{bu} = 1 \text{ for } u < t \text{ and} \\ & X_t \in \{x : \tau_s^o(x) = \tau_b^o(x) = 1\}, \\ 0 & \text{otherwise.} \end{cases}$$

Furthermore, action strategies are, for $t = 1, 2, \ldots,$

$$a_{st} = a_s^o(X_t),$$
$$a_{bt} = a_b^o(X_t).$$

The informal adaptation of the contract is $(w_\Delta - s)\xi_t \min(\Delta Q, \max(Q - D_t, 0)) + p_t = 0$ if $L_t < F_{X_t,X_{t+1}}^{-1}(Pr(X_t, X_{t+1}))$, and otherwise is

$$\begin{aligned}&(w_\Delta - s)\xi_t \min(\Delta Q, \max(Q - D_t, 0)) + p_t \\ &= (w_\Delta - s)\xi(X_t, X_{t+1}) \min(\Delta Q, \max(Q - D_t, 0)) + p(X_t, X_{t+1}),\end{aligned}$$

where for each $(x,z) \in \mathcal{X} \times \mathcal{X}$,

$$\begin{aligned}&(w_\Delta - s)\xi(x,z) \min(\Delta Q, \max(Q - D_1, 0)) + p(x,z) \\ &= [1 - Pr(x,z)]^{-1} E^o[(w_\Delta - s)\xi^o \min(\Delta Q, \max(Q - D_1, 0)) + p^o \\ &\quad + \delta \Pi_{s2}^o | X_1 = x, X_2 = z] - \delta E^o[\Pi_{s1}^o | X_1 = z].\end{aligned}$$

From the definition of the simple relational contract, we get that

$$\begin{aligned}&[1 - Pr(x,z)]\Big[(w_\Delta - s)\xi(x,z) \min(\Delta Q, \max(Q - D_1, 0)) \\ &\quad + p(x,z) + \delta E^o[\Pi_{s1}^o | X_1 = z]\Big] \\ &= E^o[(w_\Delta - s)\xi^o \min(\Delta Q, \max(Q - D_1, 0)) \\ &\quad + p^o + \delta \Pi_{s2}^o | X_1 = x, X_2 = z] \end{aligned} \tag{4.23}$$

and

$$\begin{aligned}[1 - Pr(x,z)]\Big[&(s - w_\Delta)\xi(x,z)\min(\Delta Q, \max(Q - D_1, 0)) \\ &- p(x,z) + \delta E^o[\Pi^o_{b1}|X_1 = z]\Big] \\ = E^o[(s - w_\Delta)&\xi^o \min(\Delta Q, \max(Q - D_1, 0)) \\ &- p^o + \delta \Pi^o_{b2}|X_1 = x, X_2 = z]. \end{aligned} \tag{4.24}$$

As shown by (4.23), the simple relational contract achieves the same expected discounted profit for the supplier as the relational contract o, for each initial state $X_1 \in \mathcal{X}$. Equation (4.24) states the analogous result for the buyer.

In the remainder, we will make sure that the simple relational contract is self-enforcing. The two inequalities under (4.18) imply that the simple relational contract satisfies (4.5)–(4.6). Together, (4.19) and (4.23) imply that the simple relational contract satisfies (4.7). Similarly, (4.20) and (4.24) imply that the simple relational contract satisfies (4.8). Inequality (4.21) implies

$$\delta E^o[\Pi^o_{s1}|X_1 = z] \geq (s - w_\Delta)\xi(x,z)\min(\Delta Q, \max(Q - D_1, 0)) - p(x,z),$$

which implies that the simple relational contract satisfies (4.9). In the same way, inequality (4.22) implies

$$\delta E^o[\Pi^o_{b1}|X_1 = z] \geq (w_\Delta - s)\xi(x,z)\min(\Delta Q, \max(Q - D_1, 0)) + p(x,z),$$

which implies that the simple relational contract satisfies (4.10). Finally, since the contract satisfies (4.5)–(4.10), we demonstrate that the simple relational contract is self-enforcing.

Step 3: *The Simple Optimal Relational Contract.* In searching for an optimal relational contract, we can restrict attention to self-enforcing relational contracts with the simple structure described in step 2. We can also assume without loss of generality that the supplier is allocated a fraction $\alpha \in [0, 1]$ of the total expected discounted profit. Let $V(z)$ denote the maximal total expected discounted profit that can be achieved with such a relational contract, starting in state z. Suppose that the firms will adopt this relational contract in the second period, and would like to develop an informal adaptation of the formal QF-contract, a supplementary transfer payment, action strategies for the two parties, and a termination function for the first period, that are self-enforcing and maximize expected total discounted profit. Given that the system is initially in state x, this must result in total expected discounted profit of $V(x)$ with

$$\begin{aligned} V(x) = \max\Big[&\delta \sum_{z\in\mathcal{X}} P_{xz}(0,0)V(z); \\ &\max_{\xi,p,Pr,a_s,a_b} \{-c_s(a_s,x,\gamma_s) + c_b(a_b,x,\gamma_b) \\ &+ \sum_{x\in\mathcal{X}} P_{xz}(a_s,a_b)[L(x,z) + \delta[1 - Pr(x,z)]V(z)]\}\Big] \end{aligned}$$

subject to

$$a_s \in \max_{a \in A_s(x)} \{-c_s(a, x, \gamma_s) + \sum_{x \in \mathcal{X}} P_{xz}(a, a_b)[1 - Pr(x, z)]$$
$$\times [(w_\Delta - s)\xi(x, z) \min(\Delta Q, \max(Q - D, 0)) + p(x, z) + \delta\alpha V(z)]\},$$
$$a_b \in \max_{a \in A_b(x)} \{-c_b(a, x, \gamma_b) + \sum_{x \in \mathcal{X}} P_{xz}(a_s, a)\Big[L(x, z) + [1 - Pr(x, z)]$$
$$\times [(s - w_\Delta)\xi(x, z) \min(\Delta Q, \max(Q - D, 0)) - p(x, z) + \delta(1 - \alpha)V(z)]\Big]\},$$

$$\delta\alpha V(z) \geq (s - w_\Delta)\xi(x, z) \min(\Delta Q, \max(Q - D, 0)) - p(x, z),$$
$$\delta(1 - \alpha)V(z) \geq (w_\Delta - s)\xi(x, z) \min(\Delta Q, \max(Q - D, 0)) + p(x, z)$$
$$0 \leq Pr(x, z) \leq 1,$$

$$TV = V.$$

According to Proposition 4.1, T has a largest fixed point V^*, and therefore $V = V^*$. Hence, the optimal terms are as given in the statement of Theorem 4.1. □

The relational contract presented in Theorem 4.1 is a complete plan for the buyer-supplier relationship. The intuition of the theorem is as follows: As long as the termination period Ψ is not reached, the firms transact in every period and stick to the informal agreement. That is, the buyer shifts an amount of $(w_\Delta - s)\ \xi_t \min(\Delta Q, \max(Q - D_t, 0)) + p_t$ to the supplier, giving him a certain fraction of current surplus and compensating him for his cost of action. Their optimal policies, i.e., the optimal effort decisions a_{st} and a_{bt}, are the solutions of the dynamic programming recursion (4.11) on p. 62. Another key point here is that joint surplus can be divided in any way that respects the parties' participation constraints (via the parameter α).

Equation (4.16) shows that the buyer adjusts contractually guaranteed quantity flexibility. In contrast to spot contracting or business relationships with a finite scope, those informal adaptations become possible when there is the prospect of an ongoing business relationship. In our relational contracting framework, adaptability can be regarded as a key feature of a successful long-term buyer-supplier relationship: By adjusting the quantity flexibility to current conditions, the buyer creates incentives for action, thereby creating the prerequisites for a high performance business relationship. Hence – in contrast to spot contracting – actual contract parameters may deviate from contractually agreed ones. This may furnish an explanation why we sometimes observe contract parameters in practice that are different from those predicted by one-period contracting models.

We have seen that the relational contract terminates following poor performance. The essential role of termination is to create incentives for action. By jointly punishing the firms for low performance, the menace of termination provides the contracting parties with stronger incentives to induce effort. This way, the optimal relational contract balances the near-term benefits from opportunistic behavior against future losses resulting from termination.

One implication of Theorem 4.1 is that it may be essential for the maintenance of the relationship to allow for informal payments p_t additional to a reduction ξ_t of quantity flexibility. In fact, the flexibility introduced by quantity flexibility Δ is limited to the amount $(w_\Delta - s)\min(\Delta Q, \max(Q - D_t, 0))$ per period. Without the possibility to make informal payments p_t, the business relationship may break down earlier. Namely, if the right-hand side of (4.16) exceeds the maximally exploitable quantity flexibility $(w_\Delta - s)\min(\Delta Q, \max(Q - D_t, 0))$. Interpretations of the additional transfer payment p_t include buyer-specific quality training for the supplier's personnel, process capability studies, corrective action planning, exchange of personnel, direct capital investment (Carr and Pearson 1999), or information systems.

There is an interesting connection between the cost benefit from the current quality level L_t and the termination probability $Pr^*(X_t, X_{t+1})$. To understand the argument, consider Fig. 4.3 as an example. If the termination probability $Pr^*(X_t, X_{t+1})$ increases, the minimal cost benefit necessary to continue the relationship rises accordingly. This result reflects a frequently observed business reality: If a relationship does not generate high performance, expressed by a high quality level and the associated cost benefit, the relationship is more likely to break down. In this case, the continuation of the relationship necessitates higher efforts a_{st} and a_{bt} to improve performance. Minahan (1998) supports this connection. The author reports on a manufacturer and a supplier who will continue to cooperate as long as the supplier observes certain quality standards and the manufacturer pays a good price and provides sufficient volume:

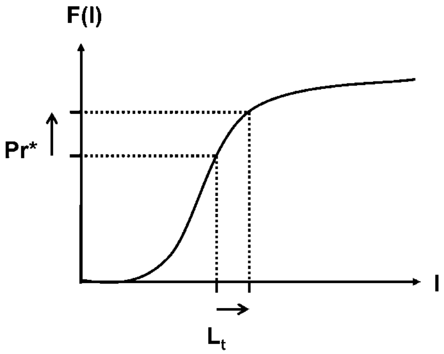

Fig. 4.3 Connection between termination probability and minimal cost benefit from current quality level necessary to maintain the relationship: if the termination probability $Pr^*(X_t, X_{t+1})$ is high, the cost benefit from the current quality level L_t has to be high as well in order to continue the business relationship. [F: distribution function of random variable L_t with $l \to F_{L_t}(l) = P(L_t \leq l)$]

> Suppliers who consistently rank poorly in quality measurement are often placed on some type of probation, which can include barring them from participating in any new business. [. . .] If for some reason quality improvement is not achievable with a particular supplier, PPG is free to let that supplier go.

The dynamic programming formulation in Sect. 4.2.5 opens up the possibility to extend the studied relational contract. By adding supplementary constraints, one can capture richer relational contract environments. One extension proposed by Plambeck and Taylor (2006) is to introduce renegotiation. As discussed in Sect. 3.3, p. 43, one can allow for renegotiation by making the relational contract "immune to renegotiation" (see Laffont and Tirole 1990; and Rey and Salanie 1996). Consequently, if we want to introduce renegotiation into the model, we simply have to modify the operator T such that $V_s(x,z) + V_b(x,z) = v(z)$ instead of $V_s(x,z) + V_b(x,z) \leq v(z)$. Hence, introducing renegotiation into the model means that the termination probability $Pr = 0$. A second model extension proposed by the authors is to limit the value the firms are willing to destroy in order to punish noncooperation. If firms are willing to destroy at most a fraction $\theta(z)$ of the value that can be created in state z, this can be captured in the dynamic program by adding the constraint $V_s(x,z) + V_b(x,z) \geq [1 - \theta(z)]v(z)$, for $z \in \mathcal{X}$. Introducing a finite punishment length in terms of Atkins et al. (2006), however, will significantly complicate the dynamic program (Plambeck and Taylor 2006).

In Chap. 6, we will analyze the optimal relational contract specified in Theorem 4.1 in more detail. Especially (4.16), which determines the informal transaction per period, will be investigated. For equivalent coordinating QF and buy-back contracts, we will show that QF and buy-back contracts differ significantly as regards the leeway they provide for informal contract adaptation and the necessity for additional transfer payments.

Chapter 5
Relational Contracts and Optimal Buy-Back Price

In Chap. 4, we considered a contracting problem where two firms interact repeatedly on the basis of a QF contract. In particular, the long-term buyer-supplier relationship was geared towards quality improvement and allowed for informal adaptations of contract parameters. The goal of the analysis was to determine an optimal relational contract specifying optimal policies for supplier and buyer. In this chapter, we shall keep the previous setup, but replace the QF contract by a buy-back contract. That is, we consider a price-based returns mechanism instead of a quantity-based one. The objectives are to identify an optimal relational contract for this setting, to examine the effect on the optimal buy-back price, and to lay the ground for the comparison of QF and buy-back contract in the context of relational contracting.

5.1 The Buy-Back Model

Consider a two-firm supply chain where supplier and buyer contract on the basis of a buy-back contract with relational aspects influencing buyer-supplier interactions. Figure 5.1 describes the sequence of events. Suppose that the two firms negotiate a formal buy-back contract at the beginning of the business relation. This long-term contract specifies that the buyer purchases Q units for the price w_b per unit at the start of the period and may return up to Q units at the end of the period for a refund of b per unit. Also note that this contract is court-enforceable. In the course of their business relationship, buyer and supplier both induce effort for quality improvement. But their efforts for quality are unobservable and uncontractible. To induce effort from the supplier's side, the buyer holds out the prospect of reducing the buy-back price. In addition, the buyer can offer a supplementary transfer payment. Unlike the parameters of the formal buy-back contract, these promises are informal and cannot be enforced by court.

During the business relationship, the stage game (i.e., steps 2–5 in Fig. 5.1) is repeated infinitely often:

- In step 2, both firms decide whether to transact in the current period.
- In step 3, the supplier produces Q units. Knowing the history of the game H^t including the current quality level X_t, supplier and buyer decide how much effort a_{st} and a_{bt} to induce for quality.

M.I. Höhn, *Relational Supply Contracts*, Lecture Notes in Economics and Mathematical Systems 629, DOI 10.1007/978-3-642-02791-8_5,

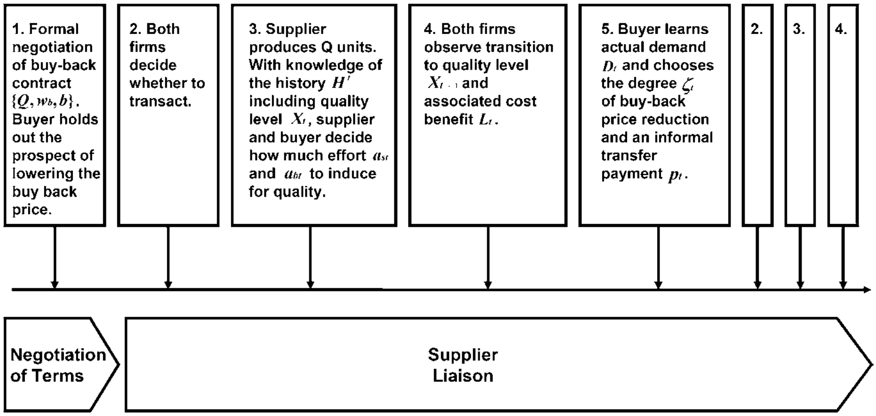

Fig. 5.1 Succession of events for the buy-back model

- In step 4, both parties observe the impact of their effort decisions, i.e., the transition to the new quality level X_{t+1}. Furthermore, they learn the associated cost benefit L_t for the buyer.
- In step 5, the buyer observes the actual demand and chooses the degree of buy-back price reduction $\zeta_t \in [0, 1]$ and the amount of the supplementary transfer payment p_t.

Hence, we essentially keep the framework presented in the previous chapter, but substitute the QF contract for a buy-back contract.

Again – like in the QF framework – assume that both parties employ trigger strategies, as is standard in the Economics literature on relational contracts (Baker et al. 2001, 2002; Levin 2003). Remember that a trigger strategy is to adhere to the relational contract in every period until the other firm does not stick to its promise for the first time, and then to refuse to transact for the rest of the game. All the other model assumptions made in the QF framework apply in the buy-back model as well (see Sect. 4.1, p. 55).

5.2 Model Analysis

This paragraph is meant to give an overview of the remainder of the chapter. To begin with, the nomenclature can be found in Sect. 5.2.1. The actual model analysis starts on p. 80 and comprises the following steps. First, the discounted expected profits of supplier and buyer are given in Sect. 5.2.2. Second, conditions for self-enforcement of the relational contract are developed in Sect. 5.2.3. Then, this chapter is structured in the same way as Chap. 4, studying the firms' total expected discounted profit both in the case of perfect coordination and the case of relational contracting in a decentralized system (see Sects. 5.2.4 and 5.2.5). Again, the total expected discounted profit under an optimal relational contract can be written in the form of a dynamic programming recursion. Convergence of this underlying dynamic programming recursion can be deduced in analogy to the QF framework (see Sect. 5.2.6). Finally, Sect. 5.2.7 presents a simple optimal relational contract for the current model. In particular, Theorem 5.1 gives the details of the relational contract. Its direct implications for the action strategies of supplier and buyer as well as immediately resulting differences to the QF framework are discussed.

5.2.1 Nomenclature

The following symbols are used to describe and analyze the buy-back model. Compared to the quantity flexibility model presented in Chap. 5, the symbols w_b and b are introduced to model the wholesale price and the buy-back price of the formal buy-back contract. In analogy to ξ_t, the reduction of quantity flexibility in period t, ζ_t now denotes the degree of buy-back price reduction in period t. Similar to the quantity flexibility model, τ_{st}, e_{st}, and a_{st} are the supplier's decision variables and τ_{bt}, e_{bt}, a_{bt}, ζ_t, and p_t are the buyer's decision variables.

t:	Period
c:	Production cost
Q:	Production quantity
r:	Retail price
w_b:	Wholesale price
b:	Buy-back price
ζ_t:	Degree of buy-back price reduction in period t
p_t:	Supplementary transfer payment in period t
δ:	Discount factor
D_t:	Demand realization in period t
G:	Continuous distribution of D_t
s:	Salvage value
g:	Goodwill cost
$\mathcal{X}$:	Finite, discrete state space reflecting quality
$X_t \in \mathcal{X}$:	Initial quality level in period t
L_t:	Buyer's cost benefit from quality transition in period t
F_{xz}:	Continuous distribution function of L_t with support $[\underline{l}_{xz}, \overline{l}_{xz}]$ where $\underline{l}_{xz} < \overline{l}_{xz}$
$L(x,z)$:	Expected value of L_t given $(X_t, X_{t+1}) = (x,z)$, i.e., $L(x,z) \equiv E[L_t \mid (X_t, X_{t+1}) = (x,z)] = \int_{\underline{l}_{xz}}^{\overline{l}_{xz}} l\,dF_{xz}(l)$
γ_{st}:	Supplier's outside option in period t
γ_{bt}:	Buyer's outside option in period t
a_{st}:	Supplier's effort for quality in period t
a_{bt}:	Buyer's effort for quality in period t
$c_s(a_{st}, X_t, \gamma_{st})$:	Supplier's cost from effort in period t
$c_b(a_{bt}, X_t, \gamma_{bt})$:	Buyer's cost from effort in period t

$P_{xz}(a_s, a_b) = Pr\{X_{t+1} = z | X_t = x; a_s, a_b\}$: Probability for a transition from state $X_t = x$ to state $X_{t+1} = z$, given efforts a_s and a_b

$\tau_{st}, \tau_{bt} \in \{0, 1\}$: Decision to transact in period t

$e_{st}, e_{bt} \in \{0, 1\}$: Decision to execute the informal transaction at the end of period t

$H^t = \{X_1, \ldots, X_t; L_1, \ldots, L_{t-1}; \tau_{s1}, \ldots, \tau_{s(t-1)}; e_{s1}, \ldots, e_{s(t-1)}; \tau_{b1}, \ldots, \tau_{b(t-1)}; e_{b1}, \ldots, e_{b(t-1)}\}$ $H^t \in \mathcal{X}^t \times \mathbb{R}_+{}^{t-1} \times \{0, 1\}^{4(t-1)}$ is the public history of the game at the beginning of period t

5.2.2 Discounted Expected Profits

This subsection specifies discounted expected profits for supplier and buyer starting from the beginning of period T with $0 < \delta < 1$. Discounted expected profits due to collaboration for quality improvement are given as follows:

$$\Pi_{sT} = \sum_{t=T}^{\infty} \delta^{t-T} \tau_{st} \tau_{bt} \Big[e_{st} e_{bt} [\zeta_t b \max(Q - D_t, 0) + p_t] - c_s(a_{st}, X_t, \gamma_{st}) \Big], \tag{5.1}$$

$$\Pi_{bT} = \sum_{t=T}^{\infty} \delta^{t-T} \tau_{st} \tau_{bt} \Big[L_t + e_{st} e_{bt} [-\zeta_t b \max(Q - D_t, 0) - p_t] - c_b(a_{bt}, X_t, \gamma_{bt}) \Big]. \tag{5.2}$$

Hence, in each period t, supplier and buyer decide whether they want to transact by setting τ_{st} and τ_{bt} to zero or one. Each transaction implies costs $c_s(a_{st}, X_t, \gamma_{st})$ for the supplier and costs $c_b(a_{bt}, X_t, \gamma_{bt})$ for the buyer, contingent on the actions a_{st} and a_{bt} taken in period t, the current quality level X_t, and the current outside options γ_{st} and γ_{bt}. In each period the firms transact, the cost benefit L_t from the current quality level goes to the buyer. If both parties want to transact in period t, they can decide whether to execute the informal transaction $\zeta_t b \max(Q - D_t, 0) + p_t$ by setting e_{st} and e_{bt} to zero or one. Here, shifting $\zeta_t b \max(Q - D_t, 0)$ to the supplier means that the buyer renounces a fraction ζ_t of the contractually agreed on buy-back price. Only if both are willing to do so, the informal transaction actually takes place.

Together with the discounted expected profits from the formal buy-back contract, this yields cumulative discounted expected profits (starting from the beginning of period T) for supplier and buyer of

$$\Pi_{sT}^{cum} = \sum_{t=T}^{\infty} \delta^{t-T} \tau_{st} \tau_{bt} \Big[-cQ + w_b Q - b \max(Q - D_t, 0) + e_{st} e_{bt} [\zeta_t b \max(Q - D_t, 0) + p_t] - c_s(a_{st}, X_t, \gamma_{st}) \Big], \tag{5.3}$$

$$\Pi_{bT}^{cum} = \sum_{t=T}^{\infty} \delta^{t-T} \tau_{st} \tau_{bt} \Big[L_t - w_b Q + r \min(Q, D_t) + (b+s) \max(Q - D_t, 0)$$
$$-g \max(D_t - Q, 0) + e_{st} e_{bt} [-\zeta_t b \max(Q - D_t, 0) - p_t] - c_b(a_{bt}, X_t, \gamma_{bt}) \Big]. \tag{5.4}$$

Like in the QF framework, (5.3) and (5.4) combine the discounted expected profits from the formal buy-back contract with the discounted expected profits from collaboration specified by (5.1) and (5.2). In (5.3), cQ are the supplier's production costs. According to the formal buy-back contract, $w_b Q$ is the wholesale payment and $b \max(Q - D_t, 0)$ the supplier's costs associated with units returned by the buyer. On the other hand, the buyer pays the wholesale payment $w_b Q$, earns $r \min(Q, D_t)$ from serving the market, incurs goodwill costs of g per unit of unsatisfied demand, and receives a payment $b \max(Q - D_t, 0)$ for unsold units from the supplier. As denoted by the term $(b+s) \max(Q - D_t, 0)$ in (5.4), it is assumed without loss of generality that the buyer still collects the salvage revenue per unit returned.

5.2.3 *Self-Enforcing Contract*

In the given framework, a self-enforcing contract must satisfy for every period $t = 1, 2, \ldots$, public history H^t, initial state $X_t \in \mathcal{X}$, and quality level L_t:

$$0 \le E[\Pi_{st} | H^t] \,, \tag{5.5}$$

$$0 \le E[\Pi_{bt} | H^t] \,, \tag{5.6}$$

$$a_{st} \in \arg\max_a \Big\{ -c_s(a, X_t, \gamma_{st}) + \sum_{z \in \mathcal{X}} P_{X_t z}(a, a_{bt}) E[\zeta_t b \max(Q - D_t, 0)$$
$$+ p_t + \delta \Pi_{s(t+1)} | H^t, X_{t+1} = z] \Big\} \,, \tag{5.7}$$

$$a_{bt} \in \arg\max_a \Big\{ -c_b(a, X_t, \gamma_{bt}) + \sum_{z \in \mathcal{X}} P_{X_t z}(a_{st}, a) \Big[L(X_t, z)$$
$$+ E[-\zeta_t b \max(Q - D_t, 0) - p_t + \delta \Pi_{b(t+1)} | H^t, X_{t+1} = z] \Big] \Big\} \,, \tag{5.8}$$

$$0 \le \underbrace{\zeta_t b \max(Q - D_t, 0) + p_t}_{\text{benefit from informal transaction}} + \underbrace{\delta E[\Pi_{s(t+1)} | H^t, X_{t+1}, L_t]}_{\text{future profits}} \,, \tag{5.9}$$

$$0 \le \underbrace{-\zeta_t b \max(Q - D_t, 0) - p_t}_{\text{loss from informal transaction}} + \underbrace{\delta E[\Pi_{b(t+1)} | H^t, X_{t+1}, L_t]}_{\text{future profits}} \,. \tag{5.10}$$

Here, (5.5) and (5.6) assure that the firms' discounted expected profits are positive. Under the assumption that the buyer chooses action a_{bt} in the current period and that both firms stick to the relational contract for the rest of the game, (5.7) specifies that the supplier's action a_{st} maximizes the discounted expected profit. The analogous property for the buyer is stated by (5.8). Executing the informal transaction should always be more advantageous than terminating the relationship. This condition is expressed by inequalities (5.9) and (5.10).

According to Abreu (1988), these conditions are sufficient for a relational contract with trigger strategies to be self-enforcing.

5.2.4 *Total Expected Discounted Profit with Perfect Coordination*

In the case of perfect coordination, the total expected discounted profit from joint effort for quality improvement can be written in the form of a dynamic programming recursion. If the initial quality level is x, the recursion looks as follows:

$$\begin{aligned}\bar{V}(x) = \max\Big[&\delta \sum_{z\in\mathcal{X}} P_{xz}(0,0)\bar{V}(z);\\ &\max_{a_s\in A_s(x), a_b\in A_b(x)} \{-c_s(a_s, x, \gamma_s) - c_b(a_b, x, \gamma_b)\\ &+ \sum_{z\in\mathcal{X}} P_{xz}(a_s, a_b)[L(x,z) + \delta\bar{V}(z)]\}\Big].\end{aligned}$$

5.2.5 *Total Expected Discounted Profit under an Optimal Relational Contract*

Now, let's consider putting an optimal relational contract in place. The total expected discounted profit under an optimal relational contract is given by the following dynamic programming recursion:

$$\begin{aligned}&\forall x \in \mathcal{X}\ \forall v : \mathcal{X} \to \mathbb{R}^+\\ &T(v)(x) = \max\Big[\delta \sum_{z\in\mathcal{X}} P_{xz}(0,0)v(z);\\ &\qquad \max_{a_s\in A_s(x), a_b\in A_b(x)} \{-C(a_s, a_b, v, x)\\ &\qquad + \sum_{z\in\mathcal{X}} P_{xz}(a_s, a_b)[L(x,z) + \delta v(z)]\}\Big] \qquad (5.11)\end{aligned}$$

subject to

$$V_s(x,z) \geq 0 \,,$$
$$V_b(x,z) \geq 0 \,,$$

$$V_s(x,z) + V_b(x,z) \leq v(z) \quad \text{for } z \in \mathcal{X} \,,$$
$$a_s \in \arg \max_{a \in A_s(x)} \Big\{ -c_s(a,x,\gamma_s) + \sum_{z \in \mathcal{X}} P_{xz}(a,a_b)\delta V_s(x,z) \Big\} \,,$$
$$a_b \in \arg \max_{a \in A_b(x)} \Big\{ -c_b(a,x,\gamma_b) + \sum_{z \in \mathcal{X}} P_{xz}(a_s,a)[L(x,z) + \delta V_b(x,z)] \Big\},$$

where

$$C(a_s,a_b,v,x) \equiv c_s(a_s,x,\gamma_s) + c_b(a_b,x,\gamma_b) + \min_{V_s,V_b} \sum_{z \in \mathcal{X}} P_{xz}(a_s,a_b) Pr(x,z)\delta v(z),$$
$$Pr(x,z) \equiv \frac{[v(z) - V_s(x,z) - V_b(x,z)]}{v(z)},$$

and where $V_s(x,z)$ and $V_b(x,z)$ are the portions of the total expected discounted profit starting from the next period allocated to the supplier and the buyer respectively.

5.2.6 Convergence

The lemmas and propositions developed in Chap. 4 also apply in the buy-back setting. For the sake of completeness, we cite the core propositions constituting convergence again below.

Due to the contraction property of the operator T, Lemmas 5.1 and 5.2 together with Propositions 5.1 and 5.2 ensure that the dynamic programming recursion presented in Sect. 5.2.5 converges to V^*, the total expected discounted profit under an optimal relational contract.

Lemma 5.1 (T is isotone). *The operator T is isotone, i.e.,*

$$\textit{if} \quad v_1 \geq v_2 \quad \textit{then} \quad Tv_1 \geq Tv_2.$$

Lemma 5.2 (T is a contraction mapping). *The operator T is a contraction mapping, i.e.,*

$$\textit{if} \quad v_1 \geq v_2 \quad \textit{then} \quad \rho(Tv_1, Tv_2) \leq \delta\rho(v_1, v_2)$$

with $\rho(v_1, v_2) \equiv \sup_{x \in \mathcal{X}} |v_1 - v_2|$.

Proposition 5.1 (T has a largest fixed point). *Existence of a largest fixed point follows from Tarski's fixed point theorem (Tarski 1955):*
The operator T has a largest fixed point V^*. That is,

$$V^* = TV^*,$$

and for any other fixed point $V = TV$, $V^*(x) \geq V(x)$ *for all* $x \in \mathcal{X}$. *Furthermore,* $V^* \in [0, \bar{V}(x)]$ *for all* $x \in \mathcal{X}$.

Proposition 5.2 (Convergence). *Value iteration beginning with* $\bar{V}$ *(the total expected discounted profit with perfect coordination) converges to the optimal value function* V^**:*

$$V^* = \lim_{n\to\infty} T^n \bar{V}.$$

Value iteration converges geometrically to the optimal value function at the rate of the discount factor:

$$\sup_{x\in\mathcal{X}}\{T^n\bar{V}(x) - V^*(x)\} \leq \delta^n \sup_{x\in\mathcal{X}}\{\bar{V}(x) - V^*(x)\}.$$

5.2.7 A Simple Optimal Relational Contract

We are now able to state the optimal relational contract in the case where the firms contract on the basis of a buy-back contract:

Theorem 5.1 (Simple Optimal Relational Contract). *The total expected discounted profit under an optimal relational contract is* $V^*(X_1)$*, and a simple optimal relational contract is characterized as follows. The firms' strategies for whether or not to transact are*

$$\tau_{st} = \begin{cases} \tau_s^*(X_t) & \text{if } t \leq \Psi \text{ and } e_{su} = e_{bu} = 1 \text{ for all } u < t, \\ 0 & \text{if } t > \Psi \text{ or } e_{su}e_{bu} = 0 \text{ for some } u < t; \end{cases}$$

$$\tau_{bt} = \begin{cases} \tau_b^*(X_t) & \text{if } t \leq \Psi \text{ and } e_{su} = e_{bu} = 1 \text{ for all } u < t, \\ 0 & \text{if } t > \Psi \text{ or } e_{su}e_{bu} = 0 \text{ for some } u < t. \end{cases}$$

That is, the firms terminate the relationship at the end of period $\Psi \equiv \inf\{t : L_t < F^{-1}_{X_t,X_{t+1}}(Pr^*(X_t, X_{t+1}))\}$*. In each period that the firms transact, the informal transaction depends only on the observed transition to the new quality level, the cost benefit thereof, and the current demand realization. If a fraction* $\alpha \in [0, 1]$ *of total expected profit was agreed to be allocated to the supplier, the degree of buy-back price reduction and supplementary payment should be chosen as follows:*

$$\zeta_t b \max(Q - D_t, 0) + p_t$$
$$= \begin{cases} [1 - Pr^*(X_t, X_{t+1})]^{-1}\delta V_s^*(X_t, X_{t+1}) \\ -\sum_{z\in\mathcal{X}} P_{X_t z}(a_s^*(X_t), a_b^*(X_t))\delta V_s^*(z) \\ +\alpha[V^*(X_t) - \delta V^*(X_{t+1})] + c_{st} & \text{if } L_t \geq F^{-1}_{X_t,X_{t+1}}(Pr^*(X_t, X_{t+1})), \\ 0 \quad \text{implying } \zeta_t = 0,\ p_t = 0 & \text{otherwise.} \end{cases} \tag{5.12}$$

The action strategies depend only on the current quality state:

$$a_{st} = a_s^*(X_t) \quad a_{bt} = a_b^*(X_t) \quad \textit{for } t = 1, 2, \ldots.$$

and each firm is willing to execute the informal transaction:

$$e_{st} = e_{bt} = 1 \quad \textit{for } t = 1, 2, \ldots$$

Proof of Theorem 5.1. We proceed as specified in the proof of Theorem 4.1 for the QF framework, emphasizing the parts of the proof that have to be changed when considering a buy-back instead of a QF contract.

Step 1: Characteristics of any Optimal Relational Contract. This part is organized in analogy to step 1 of the proof of Theorem 4.1; only properties (4.19)–(4.22) ensuring self-enforcement of the relational contract change to

$$\begin{aligned}
a_s^o = \arg\max_{a \in A_s(x)} \Big\{ & - c_s(a, x, \gamma_{s1}) \\
& + \sum_{z \in \mathcal{X}} P_{xz}(a, a_b^o) E^o[\zeta^o b \max(Q - D_1, 0) \\
& + p^o + \delta \Pi_{s2}^o | X_1 = x, X_2 = z] \Big\},
\end{aligned}$$

$$\begin{aligned}
a_b^o = \arg\max_{a \in A_b(x)} \Big\{ & - c_b(a, x, \gamma_{b1}) \\
& + \sum_{z \in \mathcal{X}} P_{xz}(a_s^o, a) \Big[L(x, z) + E^o[-\zeta^o b \max(Q - D_1, 0) \\
& - p^o + \delta \Pi_{b2}^o | X_1 = x, X_2 = z] \Big] \Big\},
\end{aligned}$$

$$\begin{aligned}
0 \le\ & \zeta^o(x, z, l) b \max(Q - D_1, 0) + p^o(x, z, l) \\
& + \delta E^o[\Pi_{s2}^o | (X_1, X_2, L_1) = (x, z, l)],
\end{aligned}$$

$$\begin{aligned}
0 \le\ & -\zeta^o(x, z, l) b \max(Q - D_1, 0) - p^o(x, z, l) \\
& + \delta E^o[\Pi_{b2}^o | (X_1, X_2, L_1) = (x, z, l)].
\end{aligned}$$

Step 2: The Equivalent Simple Relational Contract. The beginning of step 2 of the proof of Theorem 4.1 also holds in the buy-back framework. However, the informal adaptation of the contract now changes to $\zeta_t b \max(Q - D_t, 0) + p_t = 0$ if $L_t < F^{-1}_{X_t, X_{t+1}}(Pr(X_t, X_{t+1}))$, and otherwise is

$$\begin{aligned}
& \zeta_t b \max(Q - D_t, 0) + p_t \\
& = \zeta(X_t, X_{t+1}) b \max(Q - D_t, 0) + p(X_t, X_{t+1}),
\end{aligned}$$

where for each $(x, z) \in \mathcal{X} \times \mathcal{X}$,

$$\begin{aligned}&\zeta(x,z)b\max(Q-D_1,0)+p(x,z)\\&=[1-Pr(x,z)]^{-1}E^o[\zeta^o b\max(Q-D_1,0)+p^o\\&\quad+\delta\Pi^o_{s2}|X_1=x,X_2=z]-\delta E^o[\Pi^o_{s1}|X_1=z].\end{aligned}$$

Equations (4.23) and (4.24) now take the form:

$$\begin{aligned}&[1-Pr(x,z)]\Big[\zeta(x,z)b\max(Q-D_1,0)+p(x,z)+\delta E^o[\Pi^o_{s1}|X_1=z]\Big]\\&=E^o[\zeta^o b\max(Q-D_1,0)+p^o+\delta\Pi^o_{s2}|X_1=x,X_2=z]\end{aligned}$$

and

$$\begin{aligned}&[1-Pr(x,z)]\Big[-\zeta(x,z)b\max(Q-D_1,0)-p(x,z)+\delta E^o[\Pi^o_{b1}|X_1=z]\Big]\\&=E^o[-\zeta^o b\max(Q-D_1,0)-p^o+\delta\Pi^o_{b2}|X_1=x,X_2=z].\end{aligned}$$

Similar to the QF framework, we get

$$\delta E^o[\Pi^o_{s1}|X_1=z]\geq-\zeta(x,z)b\max(Q-D_1,0)-p(x,z)$$

and

$$\delta E^o[\Pi^o_{b1}|X_1=z]\geq\zeta(x,z)b\max(Q-D_1,0)+p(x,z),$$

which implies that the simple relational contract satisfies (5.9) and (5.10). We conclude that the contract satisfies conditions (5.5)–(5.10), i.e., the simple relational contract is self-enforcing.

Step 3: The Simple Optimal Relational Contract. In contrast to the proof of Theorem 4.1, the dynamic program specifying the simple optimal relational contract now looks as follows:

$$\begin{aligned}V(x)=\max\Big[&\delta\sum_{z\in\mathcal{X}}P_{xz}(0,0)V(z);\\&\max_{\xi,p,Pr,a_s,a_b}\{-c_s(a_s,x,\gamma_s)+c_b(a_b,x,\gamma_b)\\&+\sum_{x\in\mathcal{X}}P_{xz}(a_s,a_b)[L(x,z)+\delta[1-Pr(x,z)]V(z)]\}\Big]\end{aligned}$$

subject to

$$\begin{aligned}a_s\in\max_{a\in A_s(x)}&\{-c_s(a,x,\gamma_s)+\sum_{x\in\mathcal{X}}P_{xz}(a,a_b)[1-Pr(x,z)]\\&\times[\zeta(x,z)b\max(Q-D,0)+p(x,z)+\delta\alpha V(z)]\},\end{aligned}$$

$$a_b \in \max_{a \in A_b(x)} \{-c_b(a, x, \gamma_b) + \sum_{x \in \mathcal{X}} P_{xz}(a_s, a)\Big[L(x, z) + [1 - Pr(x, z)]$$
$$\times[-\zeta(x, z)b \max(Q - D, 0) - p(x, z) + \delta(1 - \alpha)V(z)]\Big]\},$$
$$\delta\alpha V(z) \geq -\zeta(x, z)b \max(Q - D, 0) - p(x, z),$$

$$\delta(1 - \alpha)V(z) \geq \zeta(x, z)b \max(Q - D, 0) + p(x, z)$$
$$0 \leq Pr(x, z) \leq 1.$$

This is equivalent to

$$TV = V.$$

According to Proposition 5.1, we know that T has a largest fixed point V^*, and therefore $V = V^*$. Hence, the optimal terms are as given in the statement of Theorem 5.1. □

As Theorem 5.1 highlights, there are a lot of similarities to the QF framework. First, as long as the termination period Ψ is not reached, the firms exert effort. That is, both firms induce effort for quality, and the buyer sticks to the informal promise of reducing the buy-back price in every period. In particular, the buyer reduces the buy-back price according to condition (5.12), voluntarily shifting an amount of $\zeta_t b \max(Q - D_t, 0) + p_t$ to the supplier in all periods $t \leq \Psi$. Again, actual contract parameters may deviate from contractually agreed ones. Like in the QF case, the optimal effort strategies a_{st} and a_{bt} for supplier and buyer are the solutions of the dynamic programming recursion (5.11) on p. 82.

Also the contract specifications regarding the cost benefit from quality improvement are alike: If the termination probability is low, the cost benefit from quality improvement does not need to be elevated to continue the relationship. In case the termination probability is high, both parties have to make greater quality-improvement efforts to maintain the business relation. For an illustration of this connection between cost benefit from the current quality level and termination probability see Fig. 4.3 on p. 74. As previously discussed in the QF framework, joint surplus from collaboration for quality improvement can be divided in any way that respects the parties' participation constraints.

As before, it may be essential for the maintenance of the relationship to allow for informal payments p_t additional to a reduction ζ_t of buy-back price. Since the flexibility introduced by the buy-back price is limited to the amount $b \max(Q - D_t, 0)$ per period, a supplementary transfer payment may be necessary to satisfy condition (5.12).

We close our discussion of the optimal relational contract with an outlook on the following chapter. The objective of the prior discussion was to highlight immediate similarities between buy-back and QF contract in the context of relational contracting. We have seen that one can derive optimal relational contracts for both frameworks that ultimately only differ in the left-hand sides of (4.16), p. 69, and

(5.12), p. 84, which specify the adaptation of actual quantity flexibility and buy-back price respectively. These differences derive from the different returns mechanisms inherent in buy-back and QF contracts. Accordingly, the next chapter will extend the comparison of the optimal relational contracts by investigating the left-hand sides of (4.16) and (5.12). The results of this analysis will be illustrated by numerical examples.

Chapter 6
QF vs. Buy-Back Contract in Buyer-Supplier Relationships

In Chaps. 4 and 5, we are able to determine optimal relational contracts in the case where repeated transactions between supplier and buyer are governed by a supply chain contract. We distinguish QF and buy-back contracts and find out that the respective relational contracts only differ in the returns mechanisms. This observation raises the question which type of contract is better with respect to relational contracting. First, we characterize how much leeway for adaptation QF and buy-back contracts grant to the supply chain parties and how the two contract types differ in this sense. Second, we analyze the implications of contract choice for the need for extra transfer payments in the course of business relationships. Finally, we illustrate by the help of numerical examples how the amount of supplementary transfer payments depends on the share of expected supply chain profit, realized demand, and market characteristics.

6.1 Construction and Analysis of Equivalent Contracts

To elaborate differences between QF and buy-back contracts in the context of relational contracting, we have to make sure that the contracts we are comparing are in a sense equivalent. This is done in Sect. 6.1.1. Later on, in Sect. 6.1.2, we use the results to contrast the leeway for contract adaptation offered by (equivalent) QF and buy-back contracts.

6.1.1 Construction of Equivalent Contracts

The objective of this subsection is to construct equivalent supply chain contracts that achieve channel coordination. Before we proceed to the actual analysis, some remarks on the notation: In this chapter, coordinating supply chain contracts are regarded as equivalent if they shift the same fraction of total expected supply chain profit to the buyer. The buyer's share will be referred to as η later on. Also recall that c stands for the production cost per unit and that r denotes the retail price per unit.

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For the construction of a coordinating buy-back contract, we fall back on the well-known theorem by Pasternack (1985) in which the author characterizes a coordinating buy-back contract. The proof can be found in Pasternack (1985) as well.

Theorem 6.1 (Buyer's Share under a Coordinating Buy-Back Contract). *Suppose the supplier offers $\{w_\epsilon, b_\epsilon\}$ for $0 \leq \epsilon < r - c$ where*

$$w_\epsilon = c + \epsilon \quad \text{and} \quad b_\epsilon = \epsilon \frac{r}{r-c}.$$

Then the buyer orders the integrated channel quantity, i.e., $Q^* = Q^I$. The buyer profit is $\Pi_b^*(w_\epsilon, b_\epsilon) = (1 - \frac{\epsilon}{r-c})\Pi^I$, where Π^I denotes integrated system profit. The supplier profit is $\Pi_s^*(w_\epsilon, b_\epsilon) = \frac{\epsilon}{r-c}\Pi^I$.

We see from the theorem that the buyer's share of total expected supply chain profit amounts to $1 - \frac{\epsilon}{r-c}$.

We now turn to coordinating QF contracts. In order to determine a coordinating QF contract, we have to make assumptions on the demand distribution. This particularity already stated in Chap. 2 represents a main difference between coordinating buy-back and QF contracts: Whereas the demand distribution does not appear in the description of a coordinating buy-back contract, it is required in the characterization of a coordinating QF contract. Hence, it is not surprising that in contrast to the buy back, one cannot always give a simple expression for the buyer's share under a coordinating QF contract. However, the case of power function demand is feasible as illustrated in the following theorem (Lariviere 2002).

Theorem 6.2 (Buyer's Share under a Coordinating QF Contract). *Let demand random variable D_k have a power function distribution with parameter $k > 0$. Then the integrated system profit is $\Pi_k^I = E[D_k](1 - c/r)^{1/k}$. The coordinating wholesale price is $w_\Delta(k) = r[1 + (1-\Delta)^{k+1}\frac{(r-c)}{c}]^{-1}$. If the supplier offers the QF contract $\{w_\Delta(k), \Delta\}$, the resulting buyer profit is*

$$\Pi_k^* = \frac{r}{c(1-\Delta)^{-k-1} + (r-c)} \Pi_k^I.$$

According to Theorem 6.2, the buyer's share of total expected supply chain profit can be stated as $\frac{r}{c(1-\Delta)^{-k-1}+(r-c)}$. One observes that the buyer's share is decreasing in quantity flexibility Δ and in the power function parameter k. We will make use of this result by assuming a power function distribution in our analysis.

To summarize, the buyer's share is $\eta_b = 1 - \frac{\epsilon}{r-c}$ under the coordinating buy-back contract presented in Theorem 6.1 and amounts to $\eta_\Delta = \frac{r}{c(1-\Delta)^{-k-1}+(r-c)}$ under the coordinating QF contract of Theorem 6.2. Hence, to construct equivalent contracts, it suffices to set cost parameter c and retail price r, to make assumptions on the demand distribution, to fix the buyer's share, and then to determine the respective coordinating contract parameters of QF and buy-back contract.

6.1.2 The Leeway for Contract Adaptation: A Comparison

Having laid the foundations for constructing equivalent coordinating QF and buy-back contracts, we can calculate how much leeway for adaptation is offered by the respective contracts contingent on realized demand D_t. Here, the term *leeway for adaptation* denotes the full flexibility that the buyer has to adapt the supply chain contract.

Before we take a closer look at the formulae, let's bring together direct implications of Theorems 4.1 and 5.1 describing the optimal relational contracts in the QF and buy-back framework. Figure 6.1 illustrates the leeway for contract adaptation provided by a QF contract $\{w_\Delta, \Delta\}$ and a buy-back contract $\{w_b, b\}$ with production quantity Q. As regards the buy-back contract, the leeway for contract adaptation decreases linearly in realized demand D_t on the interval $[0, Q]$. It is at its maximum bQ, when actual demand is zero and reaches its minimum zero as soon as demand is equal to the production quantity Q. As regards the QF contract, the leeway for adaptation also decreases in realized demand, but in a non-linear manner. The leeway for contract adaptation is constant at $(w_\Delta - s)\Delta Q$ for $D_t \leq (1 - \Delta)Q$ and then decreases linearly until $D_t = Q$. Hence, one difference between QF and buy-back contract is that the relation between realized demand and leeway for contract adaptation is linear on the interval $[0, Q]$ in the buy-back case and non-linear in the QF case.

Let's get back to the coordinating contracts of the previous subsection. If we compare the returns mechanisms of Theorems 4.1 and 5.1 [i.e., the left-hand sides of (4.16), p. 69, and (5.12), p. 84] for equivalent coordinating contracts, we get a clearer picture. Let's first consider the coordinating QF contract introduced in Sect. 6.1.1.

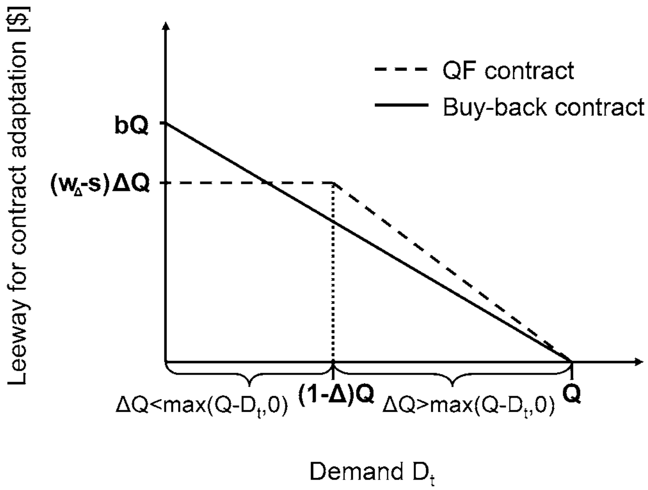

Fig. 6.1 Leeway for contract adaptation: a comparison of QF and buy-back contract

Leeway for Adaptation Under a Coordinating QF Contract

According to Theorem 4.1, the maximal leeway for adaptation of a general QF contract is $w_\Delta \min(\Delta Q, \max(Q - D_t, 0))$ in period t. Now, let's consider the coordinating QF contract introduced in Sect. 6.1.1 and determine the associated leeway for contract adaptation L_Δ. Without loss of generality, assume that $s = 0$. If we insert the coordinating wholesale price in the expression above, we get the leeway for adaptation for the mentioned QF contract:

$$\begin{aligned} L_\Delta &= w_\Delta \min(\Delta Q, \max(Q - D_t, 0)) \\ &= r\left[1 + (1-\Delta)^{k+1}\frac{(r-c)}{c}\right]^{-1} \min(\Delta Q, \max(Q - D_t, 0). \end{aligned} \tag{6.1}$$

It remains to calculate the leeway for contract adaptation for the equivalent coordinating buy-back contract.

Leeway for Adaptation Under the Equivalent Coordinating Buy-Back Contract

Assume that the parameters Δ and k of the coordinating QF contract are fixed and therewith the buyer's share of expected supply chain profit. Then the leeway for contract adaptation L_b offered by the equivalent coordinating buy-back contract can be expressed as follows:

$$\begin{aligned} L_b &= b \max(Q - D_t, 0) \\ &= \epsilon \frac{r}{r-c} \max(Q - D_t, 0) \\ &= \left[1 - \frac{r}{c(1-\Delta)^{-k-1} + r - c}\right] r \max(Q - D_t, 0). \end{aligned} \tag{6.2}$$

Hence, when considering coordinating supply chain contracts, the differences between QF and buy-back contract regarding the intrinsic flexibility to adapt are abounding more clearly.

For the closing analysis, we distinguish two cases. First, assume that min $(\Delta Q, \max(Q - D_t, 0)) = \Delta Q$, i.e., D_t is relatively low. This implies that the right part $\max(Q - D_t, 0)$ of (6.2) is greater or equal to the right part of (6.1) amounting to $\min(\Delta Q, \max(Q - D_t, 0)) = \Delta Q$. This explains why – with respect to the leeway for contract adaptation – the buy-back contract is often superior to the QF contract, when realized demand D_t is relatively low.

In the second case where $\min(\Delta Q, \max(Q - D_t, 0)) = \max(Q - D_t, 0)$, i.e., where D_t is relatively high, the situation changes. We can directly determine which contract offers a greater leeway for contract adaptation by comparing the left parts of (6.1) and (6.2). If the term $r\left[1 + (1 - \Delta)^{k+1}\frac{(r-c)}{c}\right]^{-1}$ is greater than $\left[1 - \frac{r}{c(1-\Delta)^{-k-1}+r-c}\right]r$, the QF contract offers more leeway than the buy-back contract and vice versa.

We conclude that the leeway for contract adaptation decreases in realized demand D_t with outstanding differences between QF and buy-back contracts. The immediate difference between the two contract types is that the relation between realized demand and leeway for contract adaptation is linear on the interval $[0, Q]$ in the buy-back case and non-linear in the QF case. A comparison of equivalent coordinating QF and buy-back contracts shows that the buy back tends to offer more leeway for contract adaptation than the equivalent QF contract, when realized demand D_t is relatively low. For higher values of D_t, the QF contract tends to perform better than the buy back.

6.2 Numerical Results

To illustrate how QF and buy-back contracts differ in the context of relational contracting, this section provides numerical results for a range of contract parameters and market conditions.

Equivalent coordinating contracts are constructed according to the results of Sect. 6.1.1 on p. 89. Like in Lariviere (2002), suppose that demand in market i for $i = 1, 2$ follows a power function distribution with parameter k_i and assume that $k_1 = 0.5$ and $k_2 = 2$, $G_i(x) = x^{k_i}$ for $0 \leq x \leq 1$. The salvage value s is assumed to be zero, the production cost per unit is $c = 3$, and the retail price is $r = 8$. The latter values are borrowed from Pasternack (1985).

Table 6.1 summarizes the numerical results. A look at the table reveals that Δ and k are fixed in the beginning, therewith the buyer's share η of system profit. For the resulting equivalent coordinating supply chain contracts (which both offer the same share $\eta_b = \eta_\Delta = \eta$ to the buyer), the leeway for contract adaptation is compared, depending on the actual demand realization D_t with $t \leq \Psi$.

One observes that the leeway for adaptation increases in contractually agreed on quantity flexibility Δ and decreases in the buyer's share η of total expected supply chain profit. This result is not surprising. On the one hand, if quantity flexibility is high, the flexibility to adapt is high as well. On the other hand, a QF contract induces that the firm's profit increases in the responsibility it takes for excess inventory. That is, if quantity flexibility is high, the buyer's share is correspondingly low.

Another implication is that the leeway for adaptation increases in the parameter k of the demand distribution. This connection can be directly verified on the basis of (6.1) and (6.2), describing the leeway for contract adaptation for equivalent coordinating QF and buy-back contracts.

Concerning the effect of realized demand D_t, the numerical results confirm the findings of Sect. 6.1.2. In the array where the demand realization D_t is low, the

Table 6.1 Leeway for contract adaptation: QF vs. buy-back contract

Δ	k	η	D_t	L_Δ	L_b
0.1	0.5	0.94	0	0.13	0.19
0.1	0.5	0.94	0.9Q	0.13	0.02
0.1	2.0	0.88	0	0.29	0.77
0.1	2.0	0.88	0.9Q	0.29	0.08
0.5	0.5	0.59	0	0.98	1.27
0.5	0.5	0.59	0.9Q	0.20	0.13
0.5	2.0	0.28	0	2.62	4.58
0.5	2.0	0.28	0.9Q	0.52	0.46
0.9	0.5	0.08	0	2.67	2.87
0.9	0.5	0.08	0.9Q	0.30	0.30
0.9	2.0	0.003	0	5.68	6.31
0.9	2.0	0.003	0.9Q	0.63	0.63

η: Buyer's share of total expected supply chain profit as guaranteed by the formal supply contract
L_Δ: Leeway for contract adaptation with coordinating QF contract
L_b: Leeway for contract adaptation with equivalent coordinating buy-back contract

buy-back contract offers more leeway than the QF contract. In the array where the demand realization D_t is high, the QF contract performs better than the buy back.

In summary, the numerical results show that the share of expected supply chain profit granted by the respective contracts, the current realization of demand, and market characteristics influence the extent of flexibility the buyer has to adapt the supply chain contract. Accordingly, the difference between QF and buy-back contract can be significant. With a parameter choice of $\Delta = 0.1$, $k = 0.5$, and $D_t = 0$ for instance, the leeway granted by the buy-back contract is twice as high as under the equivalent QF contract. These relationships and especially the impact on the extent of supplementary transfer payments will be described in more detail in the following subsections.

6.2.1 Impact of Buyer's Profit Share

This subsection refers to Figs. 6.2–6.7 on pp. 95 and 96. Figures 6.2–6.4 illustrate how the leeway for contract adaptation decreases in the buyer's share η of total supply chain profit and in the same way increases in quantity flexibility Δ. Figures 6.5–6.7 treat the extent of supplementary transfer payments.

Let's take a closer look at Figs. 6.2–6.4 first. While the leeway for contract adaptation is rather limited in the case where $\Delta = 0.1$ (and $\eta = 0.94$), it is much higher when $\Delta = 0.9$ (and $\eta = 0.08$). The explanation for this is rather simple, since η and Δ are directly interlinked: The buyer's share η of total expected supply chain profit increases in the responsibility he takes for excess inventory, thus decreases in quantity flexibility Δ, and: The higher the quantity flexibility, the higher the flexibility the buyer has to adapt the contract.

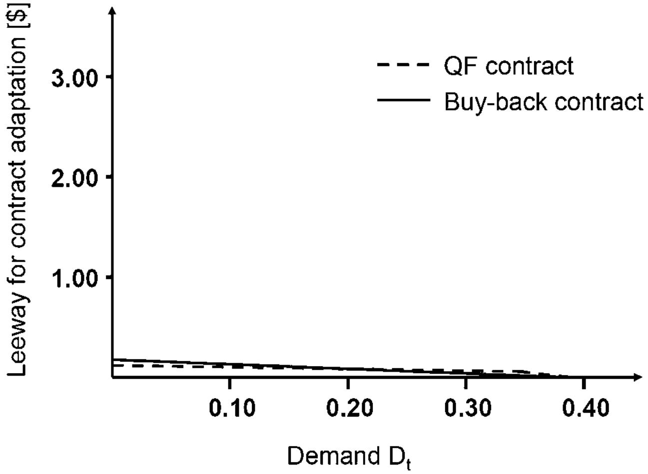

Fig. 6.2 Leeway for contract adaptation when Δ takes a low value ($r = 8$, $c = 3$, $k = 0.5$, $\Delta = \mathbf{0.1}$)

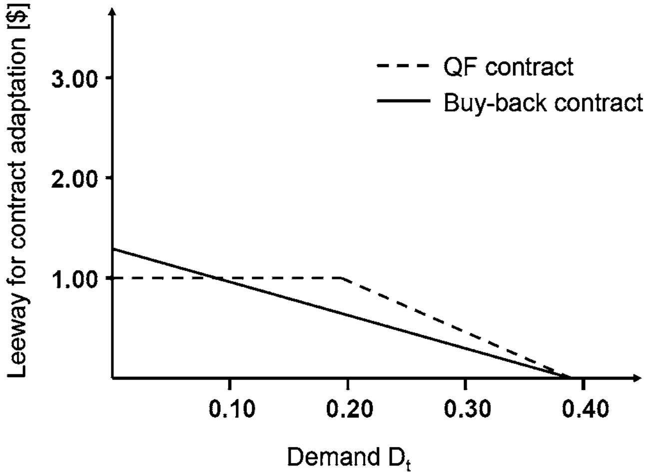

Fig. 6.3 Leeway for contract adaptation when Δ takes a medium value ($r = 8$, $c = 3$, $k = 0.5$, $\Delta = \mathbf{0.5}$)

As already mentioned in Sect. 6.1.2, the leeway for contract adaptation depends linearly on realized demand D_t under a buy-back contract, which is not the case under a QF contract. Another interesting observation is that the buy-back contract offers more leeway than the QF contract for small values of realized demand D_t and offers less leeway for higher values of D_t.

Let's turn to Fig. 6.5–6.7 which show the extent of the supplementary transfer payment for increasing values of quantity flexibility Δ (or similarly for decreasing

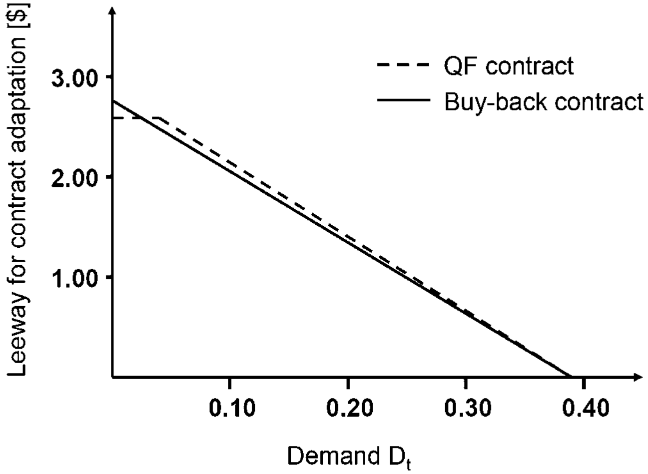

Fig. 6.4 Leeway for contract adaptation when Δ takes a high value ($r = 8$, $c = 3$, $k = 0.5$, $\Delta = \mathbf{0.9}$)

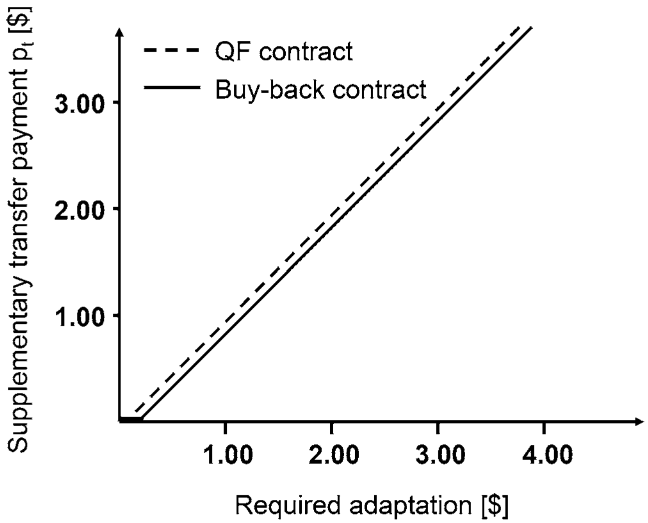

Fig. 6.5 Supplementary transfer payment when Δ takes a low value ($r = 8$, $c = 3$, $k = 0.5$, $\Delta = \mathbf{0.1}$, $D_t = 0$)

values of the buyer's share η). To be more precise, the abscissa depicts the adaptation claimed by the optimal relational contracts of Theorems 4.1 and 5.1. The ordinate displays the supplementary transfer payment needed under a QF contract on the one hand and the equivalent buy-back contract on the other hand.

Recall from Chaps. 4 and 5 that a supplementary transfer payment may be necessary to maintain the business relationship in the case where the flexibility provided by the QF or buy-back contract has already been fully utilized. In other words, the

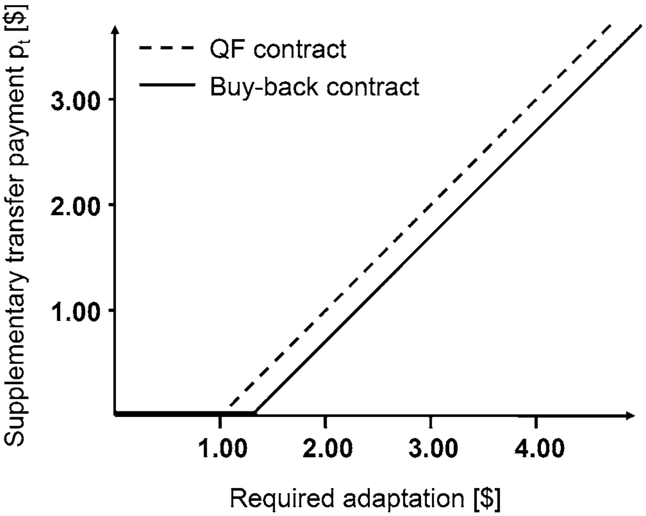

Fig. 6.6 Supplementary transfer payment when Δ takes a medium value ($r = 8$, $c = 3$, $k = 0.5$, $\Delta = \mathbf{0.5}$, $D_t = 0$)

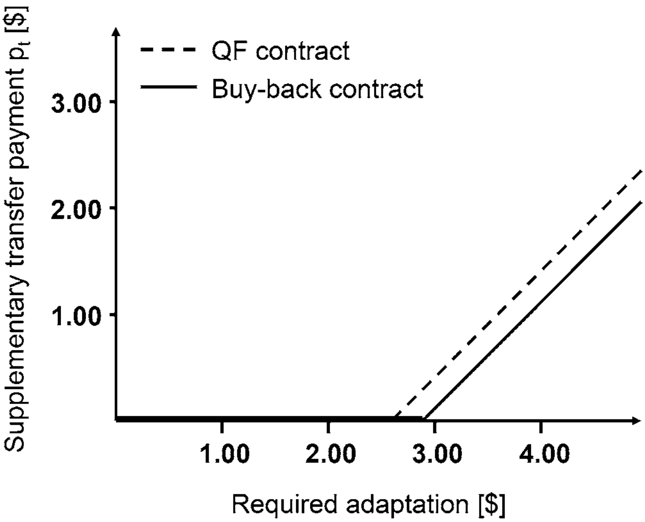

Fig. 6.7 Supplementary transfer payment when Δ takes a high value ($r = 8$, $c = 3$, $k = 0.5$, $\Delta = \mathbf{0.9}$, $D_t = 0$)

optimal relational contract calls for a nonzero supplementary transfer payment only when the required adaptation exceeds the current leeway for contract adaptation. Consistently, the figures show that the supplementary transfer payment depends on Δ and η in the following way: The extent of the supplementary transfer payment is higher in the case where quantity flexibility Δ is low, and it is lower when Δ is high. Similarly, the extent of the supplementary transfer payment is higher in the case where the buyer's share η is high, it is lower when η is low.

The discussion of the numerical examples leads to the conclusion that the buyer's share of total expected system profit has a major impact both on the leeway for contract adaptation and on the extent of the supplementary transfer payment. In the remainder of the chapter, we will address the impact of realized demand and market characteristics by means of numerical examples.

6.2.2 Impact of Realized Demand

This subsection refers to Figs. 6.8–6.10 on p. 98. Since the impact of the demand realization on the leeway for contract adaptation has already been discussed in Sect. 6.2, we will focus on the supplementary transfer payment in the case where the buyer-supplier relationship is still ongoing.

The three figures reveal that the need for a supplementary transfer payment increases according to realized demand D_t. A central purpose of this example is to illustrate that the graph moves from right to left for $D_t < Q$ with $D_t \rightarrow Q$ and finally reaches the origin for $D_t \geq Q$. In the former case, there is still leeway for contract adaptation implying that a supplementary transfer payment is not necessary at all events. In the latter case, there is no leeway for contract adaptation at all. Hence, a supplementary payment is definitely needed to continue the business relationship (as long as the adaptation required by the relational contract is non-zero, of course).

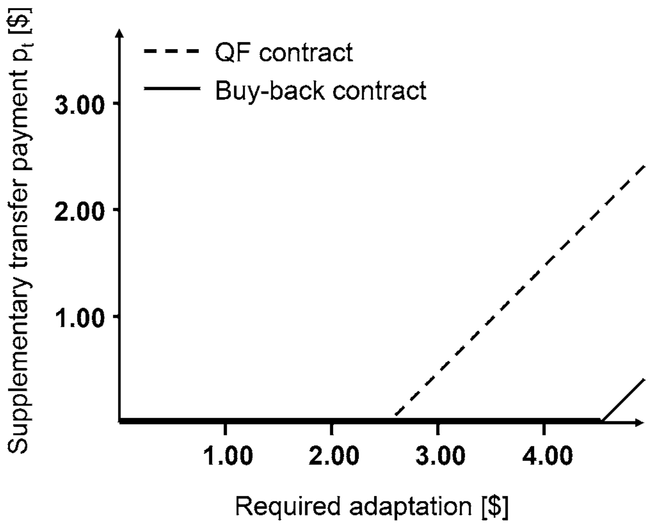

Fig. 6.8 Supplementary transfer payment when D_t takes a low value ($r = 8$, $c = 3$, $k = 2$, $\Delta = 0.5$, $\mathbf{D_t = 0}$)

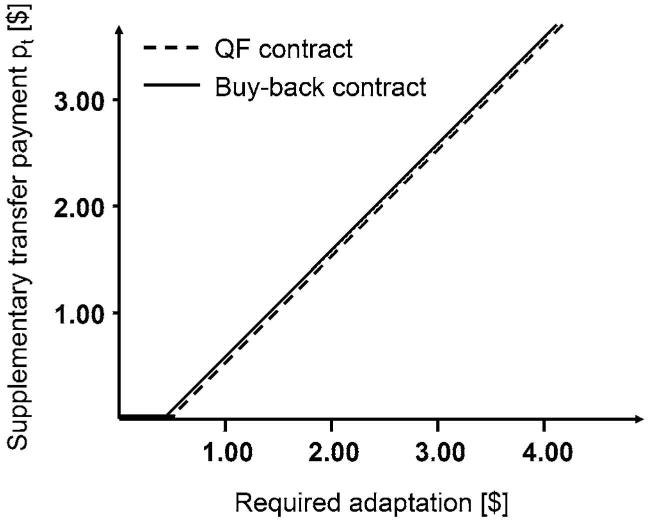

Fig. 6.9 Supplementary transfer payment when D_t takes a high value with $D_t < Q$ ($r = 8$, $c = 3$, $k = 2$, $\Delta = 0.5$, $\mathbf{D_t = 0.9Q}$)

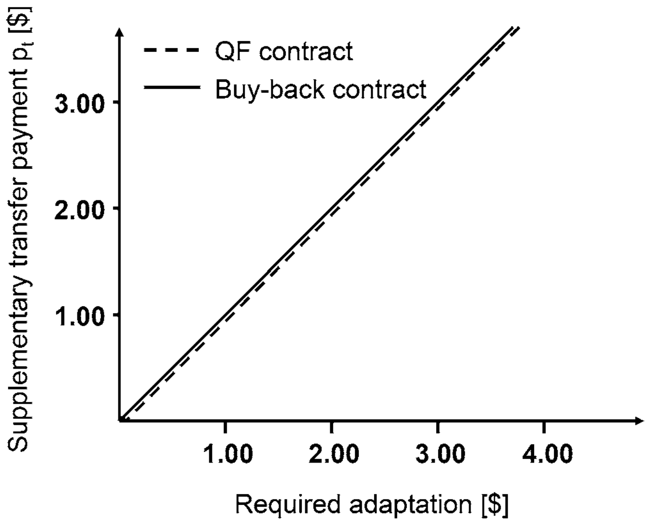

Fig. 6.10 Supplementary transfer payment when D_t takes a high value with $D_t \geq Q$ ($r = 8$, $c = 3$, $k = 2$, $\Delta = 0.5$, $\mathbf{D_t \geq Q}$)

6.2.3 Impact of Market Characteristics

The purpose of this subsection is to illustrate how the supplementary transfer payment depends on market characteristics. Again, suppose that demand in market i for $i = 1, 2, 3$ follows a power function distribution with parameter k_i, $G_i(x) = x^{k_i}$

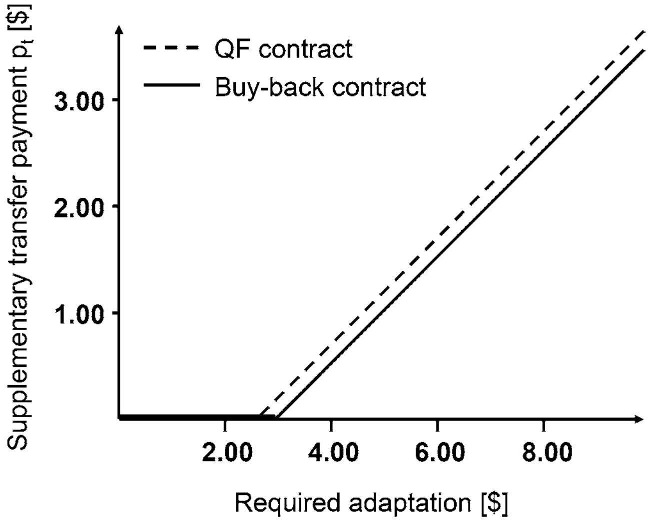

Fig. 6.11 Supplementary transfer payment when k takes a low value ($r = 8$, $c = 3$, $\mathbf{k=0.5}$, $\Delta = 0.9$, $D_t = 0$)

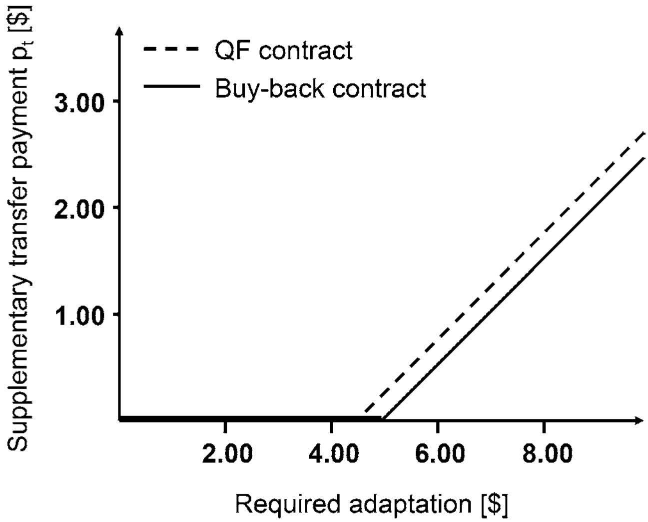

Fig. 6.12 Supplementary transfer payment when k takes a medium value ($r = 8$, $c = 3$, $\mathbf{k=1}$, $\Delta = 0.9$, $D_t = 0$)

for $0 \leq x \leq 1$ and $k_i > 0$. Assume that $k_1 = 0.5$, $k_2 = 1$, and $k_3 = 2$. Hence we move from a small market, where demand concentrates around zero (market 1), to a bigger market, where demand concentrates around 1 (market 3).

Figures 6.11–6.13 on p. 100 demonstrate that the amount of the supplementary transfer payment decreases in the parameter k, as already suggested by (6.1)

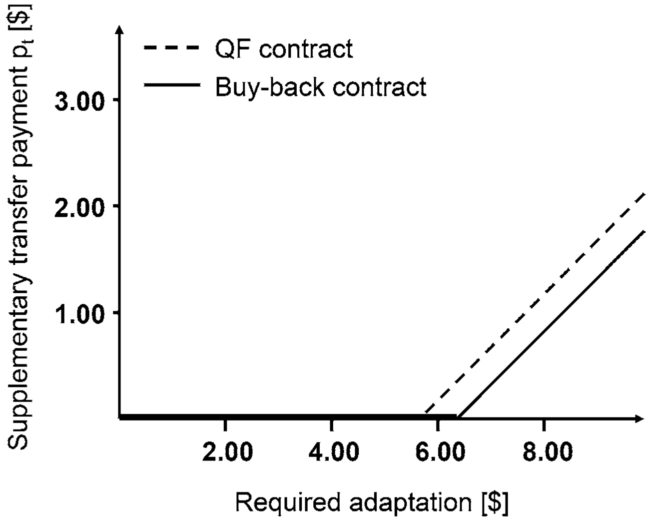

Fig. 6.13 Supplementary transfer payment when k takes a high value ($r = 8$, $c = 3$, $\mathbf{k=2}$, $\Delta = 0.9$, $D_t = 0$)

and (6.2), i.e., the buyer's supplementary transfer payments are greater in smaller markets and lower in bigger markets. Thus, the demand distribution the buyer is facing plays an important role with respect to the extra payments he has to grant to the supplier in order to maintain the business relationship.

6.3 Summary

We commenced this chapter with the objective to clarify which contract is better in the context of relational contracting, the QF or the buy-back contract. We observe that the algebraic analyses and the numerical studies carried out in this chapter do not advocate exclusively one type of contract in any circumstance. Instead, looking at the results of this chapter, a differentiated picture emerges. Be it the leeway for contract adaptation offered by the respective contracts or the need for supplementary transfer payments, the dimensions of these quantities do not only depend on the buyer's share of total expected supply chain profit ensured by QF and buy-back contract, but are also influenced by the actual demand realization. So, how can we support managerial decision making in this context?

First of all, the use of QF and buy-back contracts in the context of a buyer-supplier relationship differs due to the different returns mechanisms. A buyer should be aware that the leeway for contract adaptation decreases linearly in realized demand in the buy-back case, whereas it decreases in a non-linear way under a QF contract. This entails that the buy back tends to offer more leeway than the QF contract when actual demand is low and that the QF contract tends to be better for higher values of the demand realization. Directly connected to the leeway for contract adaptation, the supplementary transfer payment which is necessary to maintain the buyer-supplier relationship has to be higher when the leeway for adaptation is limited and reads lower when the leeway is elevated.

The parameters of the formal supply chain contract negotiated at the beginning of the business relation have further implications on the continuity of this relationship. Assume that supplier and buyer have agreed on a coordinating supply chain contract in the beginning. Then, the amount of the buyer's share of total expected supply chain profit determined by the respective contract is crucial for the flexibility the buyer has to adapt the supply chain contract later on. For the QF contract, where the buyer's share of system profit is decreasing in quantity flexibility Δ and the market parameter k, this has the following implication: For higher values of Δ and a greater market, the buyer has more leeway for adaptation and consequently has to resort to supplementary transfer payments more rarely.

Hence, when negotiating a QF or buy-back contract, the contracting parties should already keep an eye on the compensation mechanisms required in the course of an ongoing buyer-supplier relationship. If they enter a contract where the buyer's share of expected system profit is high, this may necessitate more frequent and higher transfer payments due to the restricted leeway for contract adaptation. This is in sharp contrast to the case where the buyer's share of system profit is low.

Chapter 7
Case Study: Supplier Relationship Management at Volkswagen Group

On the occasion of the VW Group Supplier Awards in 2005, Mr. Garcia Sanz, Member of the Board of Management of Volkswagen AG responsible for procurement, honored the award winning suppliers, saying: "Partnership is a key element of our long-term strategy. In these turbulent times, the Volkswagen Group Award recognizes successful cooperation" (BNET 2005). This declaration made us curious, how Volkswagen Group manages supplier relationships. The case study at hand stems from interviews with experts in procurement at Volkswagen Group, Wolfsburg, Germany, in August 2008. Given our model approach, we were particularly interested in the management of buyer-supplier relationships, the contracts employed and the handling of quality issues. In the following, we will document Volkswagen Group's current approach to Supplier Relationship Management including the different models for supplier management.

7.1 Volkswagen Group

Volkswagen AG is a growing global corporation with eight car brands, ca. €105 billion sales revenue, over 5.7 million vehicles produced per year, ca. 325,000 employees, and 42 production facilities located in Europe, Asia, Africa, and America (Garcia Sanz 2007). Based in Wolfsburg, Germany, Volkswagen Group is Europe's largest and one of the world's leading car manufacturers: As regards sales volume, Volkswagen Group is the fourth largest automaker worldwide (2006). The purchase quota is around 60–70%.

7.2 Key Elements of Supplier Relationship Management

Volkswagen Group has developed a vision for 2015, "Together – Best in Class in Customer Value and Cost", which defines the aims, the essential elements, and the guidelines for the group's strategy in purchasing. The primary goal of this strategy is long-term maximization of customer value regarding costs, quality, and innovation.

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As part of an integrated supply chain management, called "Value Net Management" at Volkswagen Group, the firm integrates suppliers more and more in the purchasing process and tries to combine its own innovation strength with suppliers to gain best results on the market. On the matrix depicted in Fig. 1.2, p. 11, the group is moving to the right (i.e. towards bargaining through technology and process analysis), which was also confirmed by our contacts at Volkswagen Group.

7.2.1 Supplier Collaboration Platform

The intensive integration of the suppliers in the core processes of the firm is effected in the form of a holistic approach concerning costs, quality, and innovation. For this purpose, dedicated platforms fostering more intensive collaboration with suppliers are established. Figure 7.1 depicts the key aspects of collaboration (Garcia Sanz 2007):

1. *Cost Optimization*
   Supplier workshops in the scope of the "Material Costs Forum".
2. *Quality Improvement*
   Start-up Management and a special "Supplier Quality Forum" as part of the "Quality Forum".
3. *Innovative Strength*
   "Innovation Forum" for the brand Volkswagen and "Audi Value Management" for the brand Audi.

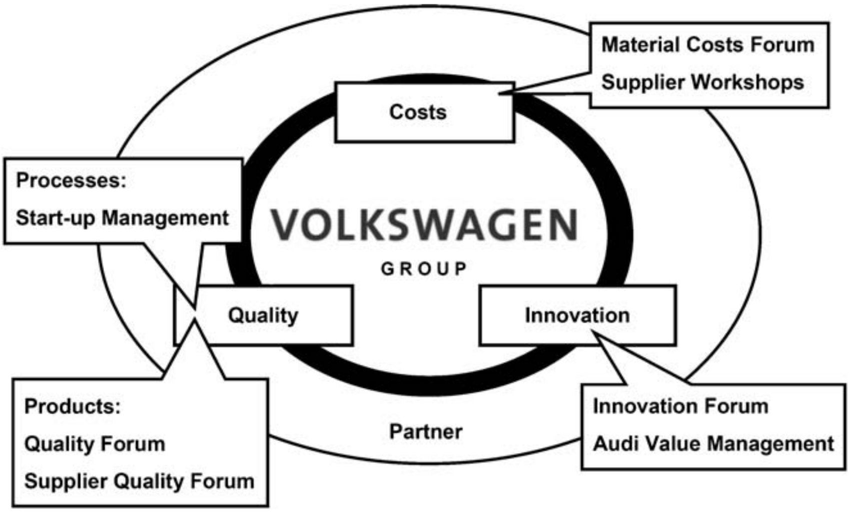

Fig. 7.1 Volkswagen group's supplier collaboration platform (adapted from Garcia Sanz 2007)

Through supplier workshops alone, saving potentials of over half a billion euros could be identified and have already been realized in large part. At the same time, field quality was fundamentally improved, while incidents in the production hall were halved. Currently, Volkswagen Group establishes a program named "Excellence in the Value Chain". The goal of this initiative is to achieve long-term efficiency gains for Volkswagen Group and its suppliers by simultaneously optimizing internal and external value-adding processes (Automobil-Produktion 2007).

7.2.2 Supplier Management Models

As regards supplier relationships, Volkswagen Group covers the whole range of governance modes as depicted in Fig. 1.1, p. 5. The depth of the OEM–supplier relationship depends on the complexity of the product and the structure of the supplier market. Bulk material, for instance, falls in the category of transactional procurement, while complete seats are an example for relationship-based procurement. Volkswagen Group essentially distinguishes three models of supplier management (Garcia Sanz 2007):

1. *Central Control*
   This supplier management model is geared towards transparency and effective process control. Selection and coordination of partners is effected by the OEM. Since central control limits the leeway for suppliers, Volkswagen Group uses additional forms of supplier collaboration.
2. *Modularization*
   This model shifts more responsibility to the supplier and is supposed to shorten time-to-market processes and synergies within modules.
3. *Strategic Partnership*
   The concept of strategic partnerships with suppliers is even more far-reaching. The supplier assumes extensive responsibility for development and/or value creation. Basic prerequisites for successful implementation of this model are a detailed target agreement, binding rules, and mutual trust. Since this model provokes strong dependencies between OEM and supplier, this form of collaboration is used very selectively.

7.2.3 Incentives and Breach of Trust

During the interview, we asked for the motivation to enter a strategic partnership with a supplier and how Volkswagen Group incentivizes effort and commitment from the supplier's side. The answers are summarized in Table 7.1. For both parties, the major reason for entering long-term relationships seems to be cost reduction. The supplier is incentivized through planning reliability, turnover increase, follow-up projects, and reputation building. We also asked the experts how they deal with

Table 7.1 Incentives for maintaining long-term buyer-supplier relationships

Volkswagen group	Supplier
– Long-term cost reduction, quality improvement and innovative strength – Sustainable supply relations in light of increasing supplier consolidation	– Long-term cost reduction, quality improvement and innovative strength – Planning reliability – Turnover increase – Follow-up projects (if competitive) – Reputation (e.g., “VW Group Supplier Award”)

breach of trust from the supplier’s side. Following breach of trust, a supplier may indeed be suspended from negotiations for a longer time. But it is also observed that trust can be rebuilt.

7.2.4 Contracts

At Volkswagen Group, the standard contract for production material with a contract volume of ca. €60 billion is essentially the quantity flexibility contract. The quantity flexibility contract employed allows for an adjustment of order quantity of typically 15% upwards and downwards.

Contracts are as standardized as possible, but may be supplemented with additional agreements. The leeway for contract adaptations is relatively low, because adaptations would be associated with high transaction costs. Nevertheless, price adaptations may be effected in case of increasing commodity prices. Additional to contractual agreements, the OEM may make concessions to a supplier in the context of so-called strategy meetings. The outcome of such a meeting may be the suspension of global sourcing over a certain period of time, for example.

7.2.5 Quality

As regards quality, Supplier Relationship Management follows a systematic approach. The concretion of the supplier strategy is based on key performance indicators from the departments Procurement, Technical Development, Quality and Logistics. Supplier quality rating systems, e.g. balanced scorecards, are employed. The performance of a supplier can be tracked over a certain period of time and comparisons between several suppliers are possible. Moreover, suppliers get valuable feedback via a supplier scorecard, meant to improve communication and relationship quality (Garcia Sanz 2007).

Although Volkswagen Group views quality rather as a contract parameter (whereas our model assumes that quality is uncontractible), our interviews confirmed that relationship maintenance strongly depends on the success the supplier has in delivering quality (which coincides with the results of our model approach). The supplier quality rating system supports this direct connection between quality and relationship maintenance: If a supplier gets an unacceptable quality rating for a certain site, this site will be blocked across the whole Volkswagen Group.

7.2.6 VWGroupSupply.com

Cooperation with the suppliers is enhanced by the private B2B supplier platform VWGroupSupply.com. In 2001, the portal was founded under the domain VWGroupSupply.com. It started with seven online applications. Today, more than 30 applications and many information services connect Volkswagen Group with its global suppliers. The most important components of the platform, which are used by all brands and regions of the Volkswagen Group, are "Online Catalogs", "Online Inquiries", "Online Negotiations", and "Capacity Management". That way, the Volkswagen Group already manages nearly its complete procurement volume of more than €72 billion via the Internet (VWGroupSupply.com 2008).

Volkswagen Group perceives the platform as "a dynamic tool that we will continually align to our needs and to those of our suppliers" (VWGroupSupply.com 2008). The overall goal is to optimize business processes to become more economic, efficient, and transparent. Main advantages of the platform are indeed bundling of all international sourcing activities, the reduction of administrative tasks, the acceleration of processes, improved planning accuracy, and improved transparency in the collaboration with the suppliers.

7.2.7 Sustainable Development

Naturally, Volkswagen Group's activities for sustainable development are part of the supplier relations. With the concept "Sustainability in Supplier Relations", the group aims at improving production and plant-related environmental and social standards together with its suppliers. Essentially, this concept resides on six pillars (Volkswagen AG 2006):

1. Supplier requirements for sustainability
2. Early detection to minimize risks
3. Contact point for sustainability
4. Monitoring and supplier development
5. Communication via VWGroupSupply.com
6. Supplier program "Priority A"

The concept involves the exchange of environmental data, certificates, and reports. Material recommendations, for instance, are incorporated in the product-related specifications for components and modules as quality standards for all parts. The supplier program "Priority A" fosters efforts for sustainability via supplier training courses, symposia, and an award for green innovations at product or plant level.

7.3 Outlook

We conclude the case study with an outlook on Volkswagen Group's future approach towards Supplier Relationship Management. According to Mr. Schmidt, Group Purchasing, Head of Department Supplier Management, SRM will continue to play a central role at Volkswagen Group. This is demonstrated by the fact that SRM is part of the group's "Strategy 2018". Mr. Schmidt especially highlights the importance of the "Innovation Forum", concept competitions, the annual supplier award, and the newly-established initiative "Excellence in the Value Chain" as key elements of SRM at Volkswagen Group.

Chapter 8
Conclusion and Outlook

A number of contracts have been identified that coordinate the supply chain in a newsvendor setting, but it is often observed that non-coordinating contracts or non-optimal contract parameters are chosen in practice. A standard argument is that complex coordinating contracts are more expensive to administer. Thus, one potentially fruitful research direction to understand contract choice better is the quantification of administration costs in practice (Cachon 2003). Another important research area is the refinement of existing models to incorporate important business realities such as long-term relationships. Those may include buyer-supplier networks, the need for renegotiation or the interaction with spot markets. Recent papers on relational contracts have investigated the interaction of formal contracts and relational aspects of an arrangement, suggesting that a relational perspective of buyer-supplier interaction may provide new insights. The model presented in this treatise puts a quantity flexibility contract between two parties in a relational contracting framework. We are able to characterize an optimal relational contract, i.e., to develop policies for supplier and buyer that structure investments in quality and flexibility in a way that no other self-enforcing contract generates higher expected joint surplus. A first observation is that the dynamics introduced by repeated interaction and joint responsibility for quality improvement may change contractually agreed parameters of the quantity flexibility contract. As long as the business relationship is still ongoing and the current quality level is sufficiently high, the buyer will incentivize the supplier to induce effort by informally offering an adaptation of contract parameters, possibly supplemented by an additional transfer payment. Hence, our game-theoretic model confirms the common observation that formal contract parameters are exposed to change, given the scope of the supply chain relationship is long-term.

This work designs buyer-supplier relationships and supply chain contracts in a way that is efficient for the whole value chain. In the light of increasing consolidation of the supplier base in many industries and the simultaneously growing emphasis on Supplier Relationship Management, our approach aims at providing insights to supply chain practice. Especially in quality-driven industries, there is a particular need to reconcile shaping a long-term supply relation with setting the right incentives. Our model provides a framework for determining optimal investments in quality and flexibility in this context, but also allows for general conclusions. In line

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with best practices in the automotive industry, our research shows that continuously improving performance necessitates joint commitment by supplier and buyer and underlines the importance of continuous supplier development. The model predicts that buyers will incentivize their suppliers if quality suffers, for example in the form of contract adaptations or transfer payments (Baker et al. 2001, 2002 have already emphasized that firms may shape their relational contracts through transfer payments). In the case where quality issues cannot be resolved, the relationship will be more likely to break down.

Moreover, our work shows that contract choice may have several implications on the design and continuance of buyer-supplier relationships: choosing a quantity flexibility contract or a buy-back contract may influence the leeway for contract adaptation as well as the need for supplementary transfer payments. When comparing equivalent QF and buy-back contracts, the different returns mechanisms entail that the buy back tends to offer more leeway for adaptation than the QF contract, when actual demand is low, and that the QF contract tends to be better for higher values of demand realization. Directly connected to the leeway for contract adaptation is the supplementary transfer payment necessary to maintain the buyer-supplier relationship. This additional payment will be higher when the leeway for adaptation is limited and will be lower when the leeway is elevated. In general, the contracting parties should already keep an eye on the compensation mechanisms required in the course of an ongoing buyer-supplier relationship, when negotiating a QF or buy-back contract. The buyer's share of total supply chain profit typically increases in the responsibility the firm takes for excess inventory. Hence, if the contracting parties enter a contract where the buyer's share of expected system profit is high, this may necessitate more frequent and higher transfer payments due to a restricted leeway for contract adaptation.

The approach of our model formulation is unique because it introduces supply chain contracts and demand uncertainty in the relational contracting model of Plambeck and Taylor (2006), which combines Levin's moral hazard model (Levin 2003) with Markovian dynamics. A standard simplifying assumption in these types of dynamic game-theoretic models with an infinite time horizon is that demands are independent and identical across periods. Still, it would be of interest to consider buyer-supplier relationships in an environment where stochastic properties adjust over time. Another natural extension to the model would be the introduction of inventory transitions from period to period. Our approach opens up the possibility to study numerous variations of the model by introducing additional constraints in the dynamic program, which preserve the overall structure of the optimal relational contract. In our view, the presented model is both general and sufficiently rich to demonstrate effects of relational contracting on optimal contract parameters of QF and buy-back contracts and to highlight differences of the two returns mechanisms in this context.

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